Imperfect Monitoring in Multi-agent Opportunistic Channel Access

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(ABSTRACT)

In recent years, extensive research has been devoted to opportunistically exploiting spectrum in a distributed cognitive radio network. In such a network, autonomous secondary users (SUs) compete with each other for better channels without instructions from a centralized authority or explicit coordination among SUs. Channel selection relies on channel occupancy information observed by SUs, including whether a channel is occupied by a PU or an SU. Therefore, the SUs' performance depends on the quality of the information. Current research in this area often assumes that the SUs can distinguish a channel occupied by a PU from one occupied by another SU. This can potentially be achieved using advanced signal detection techniques but not by simple energy detection. However, energy detection is currently the primary detection technique proposed for use in cognitive radio networks. This creates a need to design a channel selection strategy under the assumption that, when SUs observe channel availability, they cannot distinguish between a channel occupied by a PU and one occupied by another SU. Also, as energy detection is simpler and less costly than more advanced signal detection techniques, it is worth understanding the value associated with better channel occupancy information.

The first part of this thesis investigates the impact of different types of imperfect information on the performance of secondary users (SUs) attempting to opportunistically exploit spectrum resources in a distributed manner in a channel environment where all the channels have the same PU duty cycle. We refer to this scenario as the homogeneous channel environment. We design channel selection strategies that leverage different levels of information about channel occupancy. We consider two sources of imperfect information: partial observability and sensing errors. Partial observability models SUs that are unable to distinguish the activity of PUs from SUs. Therefore, under the partial observability models, SUs can only observe whether a channel was occupied or not without further distinguishing it was

occupied by a PU or by SUs. This type of imperfect information exists, as discussed above, when energy detection is adopted as the sensing technique. We propose two channel selection strategies under full and partial observability of channel activity and evaluate the performance of our proposed strategies through both theoretical and simulation results. We prove that both proposed strategies converge to a stable orthogonal channel allocation when the missed detection rate is zero. The simulation results validate the efficiency and robustness of our proposed strategies even with a non-zero probability of missed detection.

The second part of this thesis focuses on computing the probability distribution of the number of successful users in a multi-channel random access scheme. This probability distribution is commonly encountered in distributed multi-channel communication systems. An algorithm to calculate this distribution based on a recursive expression was previously proposed. We propose a non-recursive algorithm that has a lower execution time than the one previously proposed in the literature.

The third part of this thesis investigates secondary users (SUs) attempting to opportunistically exploit spectrum resources in a scenario where the channels have different duty cycles, which we refer to as the heterogeneous channel environment. In particular, we model the channel selection process as a one shot game. We prove the existence of a symmetric Nash equilibrium for the proposed static game and design a channel selection strategy that achieves this equilibrium. The simulation results compare the performance of the Nash equilibrium to two other strategies(the random and the proportional strategies) under different PU activity scenarios.

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(General Audience Abstract)

In recent years, the idea of cognitive radio network is brought up to better utilize the spectrum resources. In such a network, there are two types of users: primary users (PUs) and secondary users (SUs). Primary users are the radios that are preassigned to use the spectrum resources and secondary users are the radios that can opportunistically access the spectrum when the spectrum is not occupied by the primary users. In a centralized network, secondary users receive instructions from a central controller to take turns to access the available channels. In a distributed network, secondary users do not receive any instructions and need to compete with each other for better channels opportunities.

In a distributed cognitive radio network, the channel occupancy information observed by SUs is critical for decision making, i.e., the SUs' performance depends on the quality of the information. Current research in this area often assumes that the SUs can perfectly observe the channel occupancy status. For example, an SU can observe whether a channel is occupied by a PU, by another SU or by multiple other SUs. However, this assumption is not realistic as it requires advanced signal detection techniques while the primary detection technique proposed for use in cognitive radio networks is simple energy detection. Therefore, there is a need to design a channel selection strategy without assuming that SUs can distinguish different channel occupancies. Also, as energy detection is simpler and less costly than more advanced signal detection techniques, it is worth understanding the value associated with better channel occupancy information.

This thesis investigates the impact of different types of imperfect information on the performance of secondary users (SUs) attempting to opportunistically exploit spectrum resources in a distributed manner in a channel environment. We design channel selection strategies that leverage different levels of information about channel occupancy. We con-

sider two sources of imperfect information: partial observability and sensing errors. Partial observability models SUs that are unable to distinguish the activity of PUs from SUs. Therefore, under the partial observability models, SUs can only observe whether a channel was occupied or not without further distinguishing it was occupied by a PU or by SUs. This type of imperfect information exists, as discussed above, when energy detection is adopted as the sensing technique. We propose two channel selection strategies under full and partial observability of channel activity and evaluate the performance of our proposed strategies through both theoretical and simulation results. We prove that both proposed strategies converge to a stable orthogonal channel allocation when the missed detection rate is zero. The simulation results validate the efficiency and robustness of our proposed strategies even with a non-zero probability of missed detection. The result of this paper can be used as a guide for the wireless manufacturers to compare the tradeoff between the cost and performance. For example, when wireless manufacturers consider deploying the cognitive radios in a distributed network, they can user the results from this thesis the decide whether to adopt an advanced detection technology, which provides better performance, or adopt the basic energy detection technology, which provides more economic cost.

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Chapter 1

Introduction

In this chapter, we will introduce the background, motivation, and main contributions of our research. In particular, section 1.1 will introduce related background. The motivation will be explained in section 1.2. Finally, section 1.3 summarizes the contributions and structure of the rest of the thesis.

1.1 Background

This section introduces some background related to opportunistic spectrum access. We will first introduce some related terminology including cognitive radio (CR), cognitive radio networks (CRN), and dynamic spectrum access (DSA) . We then define opportunistic spectrum access (OSA) and provide a classification of OSA systems. Finally, we introduce the problem of designing a strategy for channel selection in OSA systems.

The explosive growth in demand for wireless service is becoming a real challenge to the wireless industry. Two primary approaches have been developed to solve this problem with respect to the radio spectrum:

- Make a bigger cake: The idea here is to increase the amount of the radio spectrum that can be used for communication. In particular, research under this approach is focused on how to develop wireless technologies that operate at higher frequencies (above 60 GHz). Recent topics such as small cells technology belong to this branch.

- Split the cake more wisely: The idea here is to increase the efficiency of spectrum usage. One way to increase efficiency is to let the unoccupied spectrum be used by other users. Research on spectrum auctions and dynamic spectrum access and spectrum belongs to this branch.

To improve the efficiency of spectrum usage, spectrum sharing is proposed. The idea of spectrum sharing is to let currently unoccupied spectrum be used by others. This reuse can be achieved either through a spectrum lease—assign the unoccupied spectrum through auctions or contracts—or through spectrum opportunity detection—let network users detect and use the unoccupied spectrum. Our research belongs to the opportunity detection branch of spectrum sharing.

To detect spectrum opportunities, the concepts of cognitive radio (CR) and cognitive radio networks (CRN) are proposed. CR is first proposed by Mitola in 1999 [1], in which a CR is described as an intelligent communication device that is aware of its environment and application needs, and can reconfigure itself to optimize quality of service. This description indicates the fundamental functions of a CR: awareness of the transmission environment and the ability to adapt and reconfigure.

One application of CR is to enable spectrum sharing. One particular spectrum sharing modality that uses spectrum opportunity decision is primary/secondary use of spectrum. Primary users (PUs) are licensed users that have existing rights to use a portion of the radio spectrum (e.g., a particular channel or channels). Secondary users (SUs), often employing CRs, sense the spectrum and, if it appears to be unused, access it while seeking to avoid harming the PUs.

Dynamic spectrum access (DSA) is used to refer to this type of spectrum sharing. According to [2], DSA adjusts spectrum resource usage in a near-real-time manner in response to the changing environment and objective of SUs. Refer to [3–5] for introductions to DSA.

DSA can be categorized from either an architecture or spectrum access technique perspective [3]:

1. From an architecture perspective, DSA can be categorized into centralized or distributed systems. In a centralized DSA system, a central controller is in charge of the spectrum allocation and access procedures. In this scenario, individual SUs are often required to forward their sensing information to a central controller and receive instructions from the central controller. In a distributed DSA system, a central controller is not available. Instead, autonomous SUs decide which channel(s) to access based on observed information or information exchanged with other SUs.

2. From a spectrum access technique perspective, DSA can be categorized into underlay and overlay systems. In an underlay DSA scheme, SUs and PUs can coexist on the same frequency channel as long as the total interference from SUs to the PU network is below a given constraint. In an overlay DSA system, SUs can only access spectrum that is not being occupied by PUs. An overlay DSA system is also called an opportunistic spectrum access (OSA) system. In particular, according to [6], OSA is the process of seeking and opportunistically utilizing "spectrum holes" that are not being utilized by the licensed owners.

Our research is about channel selection decision making in a distributed OSA system. Typically OSA channel selection involves three components: information collection, channel selection, and operation parameter update [7]. During information collection, the SUs collect information on spectrum occupancy, location, network, and traffic status. In channel selection, the SUs make channel selection decisions based on the observed information. Finally, the SUs update their operating parameters to implement their decisions. Our research focus is on the channel selection process—determining whether and which channels to access based on accumulated information to achieve a optimization goal.

OSA can be categorized into parallel sensing and sequential sensing schemes based on the sensing perspective. In a parallel sensing scheme each SU senses multiple channels simultaneously in each sensing period [8–18]; in a sequential sensing scheme SUs sense channels sequentially according to a scheduling policy in each sensing period [19–32]. Approaches based on multi-armed bandit theory [33–37] may be viewed as an extreme case of sequential sensing. In this case, each SU can only sense one channel in each time slot. Our research belongs to the category of parallel OSA sensing schemes.

1.2 Motivation

1.2.1 Imperfect Monitoring in Distributed OSA

In recent years, extensive research has been devoted to opportunistically exploiting spectrum in a distributed cognitive radio network. In such a network, autonomous secondary users (SUs) compete with each other for better channels without instructions from a centralized authority or explicit coordination among SUs. Channel selection relies on channel occupancy information observed by SUs. Therefore, the SUs' performance depends on the character and quality of the information.

Most existing work on channel access strategies design assume that SUs can distinguish signals transmitted by a PU from signals transmitted by an SU and from collision events [8–11, 17, 18, 33, 34, 37–43]. To distinguish between PU and SU signals, advanced detection technologies (e.g., cyclostationary feature detection [33] [44]) would need to be adopted. Some research also assumes no sensing errors (no false alarms nor missed detections) [12–14, 16, 19, 38–40, 44]. Other papers focus on designing channel selection strategies that are robust in the presence of false alarms and/or missed detections [8–11, 15, 17, 18, 34, 37, 43]. Sometimes, complex data fusion techniques, such as cooperative spectrum sensing [42] [41], are adopted to reduce sensing errors. However, improving the information and reducing sensing errors increases system design complexity and implementation cost.

Our previous paper [45] was the first to define imperfect information in terms of the inability of SUs to discriminate between SU and PU activity in an overlay cognitive radio network. The channel selection strategies presented in this thesis extend the work in [45] to be robust in the presence of other types of imperfect information, in particular sensing errors. Moreover, we compare the impact of different sources of imperfect information on SU performance. As the different sources of imperfect information represent constraints on different aspects of system design, the results of this paper can guide the system designer regarding trade-offs between implementation cost and system efficiency.

It is important to note that different definitions of imperfect information have been explored in the literature on cognitive radio systems. In [46, 47], imperfect information represents an SU's inability to observe other SUs' transmission power in an underlay cognitive radio system. In [48], the authors define imperfect monitoring as private monitoring where SUs do not have perfect observation of other SUs' past actions. Our research differs from these works in two aspects. First, the schemes in [46, 47] are based on an underlay cognitive radio network structure and [48] is based on a sequential sensing structure. Second, the imperfect information in [46–48] refers to the inability to observe other SUs' past actions while in our work imperfect information refers both to sensing errors and to the inability to distinguish SU and PU transmissions from each other and from collisions.

We consider two types of imperfect information: partially observable channel activity and channel occupancy sensing errors. When SUs can only observe whether a channel is idle or not and cannot distinguish signals transmitted by a PU from signals transmitted by other SUs, we call this "partially observable channel activity". To evaluate partially observable channel activity, we compare it to "fully observable channel activity" in which SUs are equipped with technology that allows them to distinguish signals transmitted by a PU from signals transmitted by another SU, as well as to distinguish successful transmissions from collision events. Under both partial and full observability, the channel occupancy information sensed by SUs is vulnerable to sensing errors including false alarms and missed detections.

1.2.2 Calculating the distribution of successful users in multi-channel random access

Random access is commonly adopted in protocol design in distributed multi-channel communication systems. A random access process requires the radios to randomly pick one channel to transmit data on. If a radio is the only one accessing a channel, this user can successfully transmit data on that channel. If multiple users transmit simultaneously on the same channel, a collision occurs and none of the users successfully delivers data. The probability distribution of the number of successful radios is of great interest in random access strategies analysis.

Recently, an algorithm was proposed in [49] to calculate the distribution of the number of successful users in random access. Specifically, the algorithm computes non-trivial combinatorics based on a recursion expression, which typically is not efficient with respect to execution time. In this thesis, we develop a non-recursive algorithm to calculate the same probability distribution in a more efficient way, which enables us to better apply such a computation in channel access strategies design.

1.2.3 Channel selection strategy design in a heterogeneous channel environment

In real scenarios, different PUs may use their channels with different intensity. We refer to the proportion of time that a PU is using its channel as the channel's duty cycle. When different channels have different duty cycles, we refer to this as a heterogeneous channel environment.

The channel selection strategy design in such a heterogeneous channel environment is more challenging, as SUs will prefer to access channels with lower PU duty cycles, to increase their probability of successfully accessing a channel. However, higher popularity of channels with low PU duty cycles among SUs may increase the likelihood of collisions between SUs on such channels. In this case, the optimal channel selection strategy requires SUs to randomize across multiple channels for channel exploration and collision avoidance.

The heterogeneous channel environment also changes the goal of the strategy design. In a homogeneous channel scenario, the goal of strategy design is to avoid collisions among SUs and converge to an orthogonal channel allocation. An orthogonal allocation is not necessarily optimal in the heterogeneous channel environment, as it is unfair to SUs assigned to high duty cycle channels. Strategies designed for a heterogeneous channel environment should involve both channel exploration and collision avoidance.

1.3 Contributions

The research presented in this thesis can be situated within the literature on opportunistic channel access strategy design in distributed cognitive radio networks. Our main contributions are:

- We design channel selection strategies for autonomous SUs in distributed cognitive radio networks under both fully and partially observable channel activity.
- We prove the convergence of the two proposed strategies in the presence of no sensing errors and calculate the expected convergence rate.
- We show the robustness and efficiency of our strategies in the presence of imperfect monitoring through both theoretical analysis and simulation results, and provide an analysis of the impact of different sources of imperfect information on the ability of SUs to dynamically exploit the band.

- We propose a non-recursive algorithm to calculate the distribution of the number of successful users in a multi-channel random access scheme, and compare its computational efficiency to that of a previously proposed recursive algorithm.
- We apply a game theoretic model to analyze channel selection for autonomous SUs in a heterogeneous channel environment. We propose a symmetric Nash equilibrium strategy and evaluate its performance.

Chapter 2 reviews the literature on opportunistic channel selection. Chapter 3 introduces and examines our channel selection strategies in a homogeneous network environment where all channels have the same duty cycle. Chapter 4 examines the computation of the probability distribution of the number of successful users in a multi-channel random access system. Chapter 5 revisits channel selection strategy in a heterogeneous network, where the channels have different duty cycles, in a static game framework. Chapter 6 concludes the thesis.

Chapter 2

Literature Review

This chapter reviews research literature relevant to this thesis. In section 2.1 we review the literature on channel selection in a distributed OSA environment. We focus on the scenario where observed information is imperfect and place our research in this context. In section 2.2 we review the literature on applying game theory in distributed channel selection.

2.1 Channel Selection in Distributed OSA systems with Imperfect Monitoring Information

Our research is on channel selection in a distributed overlay spectrum access system. In this section, we will review the concept of channel selection, provide a classification scheme for research in this area and review the existing literature.

In a distributed OSA system, in order to protect PUs' transmissions, SUs are required to adopt a check-before-access approach. It is common to assume that time is slotted, and each time slot is divided into two periods—one for sensing and one for channel access. Channel selection strategies are rules that SUs use to decide which channel to sense (and possibly access) in each time slot based on previously observed information.

Channel selection strategies can be classified from both the sensing and channel access perspectives. From the sensing perspective, channel selection strategies can be classified into parallel sensing and sequential sensing schemes. In a parallel sensing scheme each SU senses multiple channels simultaneously in each sensing period [8–18, 38–44]. The channel selection model in [18] is an example of parallel channel sensing. The authors consider a slotted time system. There are N potential primary channels. In each slot, a channel is free (i.e., without primary activities) with a fixed but unknown probability. For each channel, the channel states (busy or free) vary independently from one slot to another and across channels. Each slot consists of a sensing period with a fixed duration and a data transmission period with duration T. For each slot, during the sensing period the secondary user senses all the N channels. Among all the sensed-free channels, the secondary user can access (i.e., transmit its data over) up to K channels in the data transmission period. Since the secondary user senses all N channels, the only decision that the secondary user needs to make is which channel(s) to access. To protect primary users, only channels sensed free can be accessed.

In a sequential sensing scheme SUs sense channels according to a scheduling policy in each sensing period [19–32]. The channel selection strategy in [20] is an example sensing order strategy for a distributed cognitive radio (CR) network, where two or more autonomous CRs sense the channels sequentially (in some sensing order) for spectrum opportunities. An adaptive persistent sensing order strategy to reduce the likelihood of collisions is proposed and proved to converge in [20].

Approaches based on multi-armed bandit theory [33–37] may be viewed as an extreme case of sequential sensing. In this case, each SU can only sense one channel in each time slot. The authors in [36] apply decentralized multi-user on-line learning to model the channel selection process in OSA systems. A distributed algorithm is proposed to enable the secondary users to learn the optimal allocation with logarithmic regret, which is proved to achieve the fastest convergence rate to the optimal allocation.

From the channel access perspective, channel selection strategies can be classified into overlay and underlay schemes. In underlay cognitive radio systems, PUs and SUs can transmit simultaneously in the same band as long as the interference caused to incumbents is lower than a given threshold [46, 47, 50]. The authors in [46] design a framework for dynamic distributed spectrum sharing among secondary users (SUs) who adjust their power levels to compete for spectrum opportunities while satisfying the interference temperature (IT) constraints. In an overlay cognitive radio network [33, 38–42, 45], an SU is allowed to access a channel only when the PU is not using the channel.

Previous work on channel access strategies often assumes that SUs can distinguish signals transmitted by a PU from signals transmitted by an SU and collision events [8–11, 17, 18, 33, 34, 37–43]. To distinguish between PU and SU signals, advanced detection technologies (e.g., cyclostationary feature detection [33] [44]) would be needed. Some research also assumes no sensing errors (no false alarms nor missed detections) [12–14, 16, 19, 38–40, 44]. Other papers focus on designing channel selection strategies that are robust in the presence of false alarms and/or missed detections [8–11, 15, 17, 18, 34, 37, 43]. Sometimes, complex data fusion techniques, such as cooperative spectrum sensing [42] [41], are adopted to reduce sensing errors. However, improving the quality of information due to partially observable channel activity and sensing errors increases system design complexity and implementation cost.

Our previous paper [45] was the first to define imperfect information in terms of the inability of SUs to discriminate between SU and PU activity in a overlay cognitive radio network. The channel selection strategies presented in this thesis extend the work in [45] to be robust in the presence of other types of imperfect information, in particular sensing errors. Moreover, we compare the impact of different sources of imperfect information on SU performance. As the different sources of imperfect information represent constraints on different aspects of system design, the results of this thesis can guide the system designer regarding trade-offs between implementation cost and system efficiency.

Imperfect information refers to different concepts in the existing literature. In [46, 47], imperfect information refers to an SU's inability to observe other SUs' transmission power in an underlay cognitive radio system. The imperfect information in [48] refers to the private monitoring where SUs do not have perfect observation of other SUs' past channel selection actions. In [51–53], the authors propose two channel selection strategies for distributed opportunistic spectrum access with no information exchange. The SUs do not know the PU activity within the network. This represents an extreme case of imperfect information.

Our work differs from past work on imperfect information in channel selection in two aspects. First, our work is based on a distributed OSA system with parallel sensing while the schemes in [46, 47] are based on an underlay cognitive radio network structure and [48] is based on sequential sensing. Second, imperfect information in our research refers to the inability to distinguish signals transmitted by a PU from signals transmitted by an SU and collision events, while the imperfect information in [46–48] refers to the inability to observe other SUs' past actions (either transmission power or channel selection decision).

2.2 Game Theoretic Models of Channel Selection in DSA Networks

In this section, we summarize the literature in applying game theory to channel selection strategy design in DSA networks. We first briefly review the related literature in applying game theory in both centralized and distributed DSA systems. Then, we focus on the distributed DSA networks. In particular, we classify the literature based on the amount of communication allowed within the system. Finally, we are particularly interested in the scenario of distributed DSA networks where with no communication allowed. We summarize the literature in this area based on whether the scheme requires perfect observed information.

Game theory is a set of mathematical tools that have been widely applied to analyze wireless communication systems. According to the survey in [54], several game models including non-cooperative/cooperative, static/dynamic, and complete/incomplete information have been developed to study different multiple access schemes in wireless networks such as TDMA systems [55, 56], FDMA systems [57, 58], CDMA systems [59, 60], ALOHA systems [61, 62] and CSMA systems [63, 64]. Game theory has also been applied to model distributed networks such as wireless sensor networks and ad-hoc networks [65, 66]. In [67], the authors model the radio competition for opportunistic spectrum access as both centralized and distributed stochastic games. The same authors model the bidding process for secondary users competing for spectrum in [68]. According to [66], dynamic game models can be adopted to study how the radio's actions are affected by past experiences.

Recently, game theory has been applied to model channel access in both centralized and distributed DSA networks. In a centralized DSA network, a central controller is in charge of assigning channel selection decisions to the SUs. Examples of applying game theory in a centralized DSA system include [69–75]. The aim of applying game theory in these scenarios is often to achieve a Nash Equilibrium [70, 74]. After finding the Nash equilibrium, a central controller broadcasts the channel selection decisions to the SUs. All of the SUs should be willing to abide by the decision made by the central controller, as there is no benefit from unilaterally deviating for such a strategy. Game theory can also been applied to distributed DSA networks [76–82]. In a distributed DSA network, SUs autonomously decide which channel to choose based on observed information and history. The objective of applying game theory in such network scenarios is for SUs to independently select a set of channels while achieving close-to-optimal equilibrium performance [76, 81]. Without the instructions from the central controller, to achieve the Nash equilibrium, SUs need to adopt some learning algorithms [77, 80].

Among the literature in applying game theory in distributed DSA networks, most research works require information exchange within the network [72, 80, 83–90]. In such networks, SUs communicate with each other to get channel observation information. For example, in [83] and [85] the SUs share sensing information with each other. The authors in [89] studied the channel selection process in a multi-channel distributed DSA system. In their model, during the spectrum sensing process, multiple SUs coordinated with each other to sense the channels owned by the PUs. The sharing of the available channels by SUs after sensing is modelled by a channel access game, based on a weighted congestion game. An algorithm for SUs to select access channels to achieve the Nash equilibrium (NE) is proposed. Simulation results are presented to validate the performance of the proposed algorithms. Another example of applying game theory in channel selection in distributed DSA networks is [83]. In [83], the channel selection problem is modelled as a coordination game and a learning algorithm is proposed to achieve the optimal solution. The trade-off between sensing cost and achievable system throughput is investigated in the simulations.

In real distributed DSA scenarios, SUs often need to make channel selection decisions based on their own observations [75, 77, 82, 86, 91–95]. The work in [91] investigates the channel selection problem in a distributed DSA system with no communication allowed within the system. The channel selection process is formulated as a non-cooperative game, in which the utility of each SU is based on the expected weighted experienced interference. This game is proved to be a potential game, and a stochastic learning algorithm is proposed and shown to converge to pure strategy Nash equilibrium (NE). Another example of applying game theory in a distributed DSA network with no information exchange is [92]. In [92], joint channel selection and power allocation is formulated as a potential game. Under an interference constraint, a nonlinear optimization problem is formulated for improving the throughput and fairness. The Nash equilibria of this potential game are investigated. It is shown that the distributed sequential play converges to a Nash equilibrium point and quickly satisfies the interference constraint.

Furthermore, among the literature of applying game theory to distributed DSA networks in which no communication is allowed, most existing research requires SUs have perfect observation information. Perfect information may refer to knowledge of the PU activity, other SUs' past channel selections, or accurate sensing information [77, 86, 94, 96]. Paper [77] investigates the problem of distributed channel selection in a distributed DSA system with no information exchange among SUs. A MAC-layer interference minimization game is proposed, in which the utility of an SU is defined as a function of the number of neighbours competing for the same channel. The game is proved to be a potential game with the optimal Nash equilibrium (NE) point minimizing the aggregate MAC-layer interference. In their model, the SUs need to observe the exact number of SUs nearby. Another example of game theory application in this area is given in [77]. A learning algorithm, with which the SUs intelligently learn the desirable actions from their individual action-utility history is proposed to asymptotically minimize the aggregate MAC-layer interference. The utility function of game model in [77] involves the interference from other SUs. Therefore, the model requires the SUs to perfectly observe the interference information from other SUs. Similarly in [86] and [96], the utility functions involve the interference from all other SUs. Therefore, [86] and [96] assume that SUs can perfectly monitor each other's transmission power. The authors in [94] investigate the problem of distributed channel selection, where mutual interference occurs among nearby SUs. A local congestion game is proposed and proved to be a potential game. A stochastic learning scheme is proposed for SUs to learn the desirable channel selections from their action-payoff history. The proposed learning algorithm is proved to converge to pure strategy Nash equilibrium (NE) points without information exchange. The proposed algorithm is shown to minimize the aggregate collision level globally or locally, and hence achieves higher network throughput. However, the proposed scheme in [94] requires the SUs to know each others' past channel selections.

In real distributed DSA networks, limited sensing abilities and unknown channel occupancy information render perfect information unavailable. Imperfect information is considered in strategy design in several papers [75, 78, 82, 87, 95, 97, 98]. Imperfect monitoring in [87] represents localized observations, and the paper investigates the problem of achieving a global optimum for distributed channel selection in cognitive radio networks (CRNs). The problem of opportunistic spectrum access in a non-homogeneous channel environment is addressed in [97]. One significant contribution of this paper lies in considering private monitoring in which SUs are not able to observe each other's channel selections. A cognitive radio network (CRNs) with unknown PU activity is considered in [95]. The authors model such spectrum mobility by proposing Singleton Bayesian Spectrum Mobility Games. The paper considers both complete and incomplete information scenarios. Under complete information scenario, pure Nash equilibria are proved to exist. Under the incomplete information scenario, Bayesian equilibria are proved to exist. The authors further provide a polynomial-time algorithm for finding the socially optimal equilibrium among all possible equilibria. [82] is another example of applying game theory in distributed DSA systems with imperfect monitoring information. In their network, SUs adjust their power levels to compete for spectrum opportunities while satisfying the interference temperature (IT) constraints imposed by the PUs. The SUs only observe whether the IT constraints are violated, and their observation is imperfect due to the possibility of erroneous measurements. The authors model the interaction of the SUs as a repeated game with imperfect monitoring. The authors first characterize the set of Pareto optimal payoffs that can be achieved by deviation-proof spectrum sharing policies. And then they propose a deviation policy for any given payoff in this set. The proposed policy is shown to achieve Pareto optimality even when the SUs have limited and imperfect monitoring ability.

To sum up, game theory has recently been applied to channel selection in both centralized and distributed DSA networks. Most existing work in applying game theory in distributed DSA networks requires the information exchange within the system. When information exchange is not available, perfect information is usually assumed in the proposed scheme. Existing work considering imperfect monitoring often refers to an inability to observe neighbours' past actions or unknown PU activities. An inability to distinguish SU from PU transmissions needs to be considered in channel selection; this is the focus of our work.

Chapter 3

Homogeneous Primary User Activities

3.1 Introduction

In recent years, extensive research has been devoted to opportunistically exploiting spectrum in distributed cognitive radio networks. In such networks, autonomous secondary users (SUs) compete with each other for better channels without instructions from a centralized authority or explicit coordination among SUs. Channel selection relies on channel occupancy information observed by SUs. Therefore, the SUs' performance depends on the character and quality of the information.

We consider two types of imperfect information: partially observable channel activity and channel sensing errors. When SUs can only observe whether a channel is idle or not and cannot distinguish signals transmitted by a PU from signals transmitted by other SUs, we call this "partially observable channel activity". To evaluate partially observable channel activity, we compare it to "fully observable channel activity" in which SUs are equipped with technology that allows them to distinguish signals transmitted by a PU from signals transmitted by another SU, as well as to distinguish successful transmissions from collision events. Under both partial and full observability, the channel occupancy information sensed by SUs is vulnerable to sensing errors including false alarms and missed detections.

The goal of the SU's channel selection strategy is to select channels in order to maximize the long-term successful transmission rate. We propose a channel selection strategy for the full observability case and one for the partial observability case. In both strategies the main idea is to help SUs avoid collisions with each other. With this in mind, both strategies allow SUs that experience successful transmission to remain on the same channel until they experience a collision; other SUs avoid accessing channels that have been "claimed" by an SU based on observed occupancy information. We show that both strategies converge to an orthogonal allocation of the channels across SUs. Furthermore, our simulation results show that our strategies are robust under low missed detection and false alarm rates.

The focus of this work is on evaluating the impact of different types of channel observations on the SUs' channel selection performance. In real scenarios, the observed information about whether a channel is occupied is always imperfect. This imperfect information affects the efficiency of SUs' decisions. Imperfect information can be controlled to a certain degree by equipping the SUs with advanced signal detection technology, which might move an SU from partial to full observability or decrease the false alarm or missed detection rate. However, improving the precision of the observed information may be costly. We therefore seek to understand how different sources of imperfect information affect the channel selection performance.

The research presented in this chapter can be situated within the literature on opportunistic channel access strategy design in distributed cognitive radio networks. Our main contributions are:

- designs of channel selection strategies for autonomous SUs in distributed cognitive radio networks under both fully and partially observable channel activity,
- a proof of the convergence of the proposed strategies in the presence of no sensing errors and the derivation of the expected convergence rate of the full observability strategy,

and

- simulation results that show the robustness and efficiency of our strategies in the presence of imperfect monitoring and an assessment of the impact of different sources of imperfect information on the ability of SUs to dynamically exploit the band.

We begin with a brief discussion of the state of the literature considering imperfect monitoring in opportunistic channel selection in Section **??**. Section 3.2 introduces the system model and related notation. The strategies are described in Section 3.3 for both fully observable and partially observable channel activity monitoring with no sensing errors. The theoretical performance analysis of the strategies is presented in Section 3.4. Section 3.5 described the modified channel selection strategies with presence of sensing errors and Section 3.6 provides the simulation results. We summarize our conclusions and point towards future work in Section 4.5.

3.2 System Model and Notation

We consider a time-slotted cognitive radio system consisting of M homogeneous channels and N SUs. Each SU is equipped with two radios. Radio one is used to transmit data only. Radio two is used to continuously assess channel conditions (occupied/idle). In each time slot, an SU chooses exactly one channel to transmit data on. Just prior to transmission, the SU needs to verify that the selected channel is not being used by a PU in that time slot. If no PU is sensed as being active in the channel, the SU can then transmit data using that channel. If a PU is found to be active in the selected channel, the SU will wait during that time slot without transmitting. If multiple SUs sense the same channel and transmit data simultaneously, a collision will occur and none of the transmissions will be successful. Thus, a successful transmission occurs on a given channel if only one SU has chosen this channel and no PU is active on this channel. At the end of each time slot, an SU gets feedback on whether its transmission was successful (for instance, in the form of an ACK). Figure 3.1

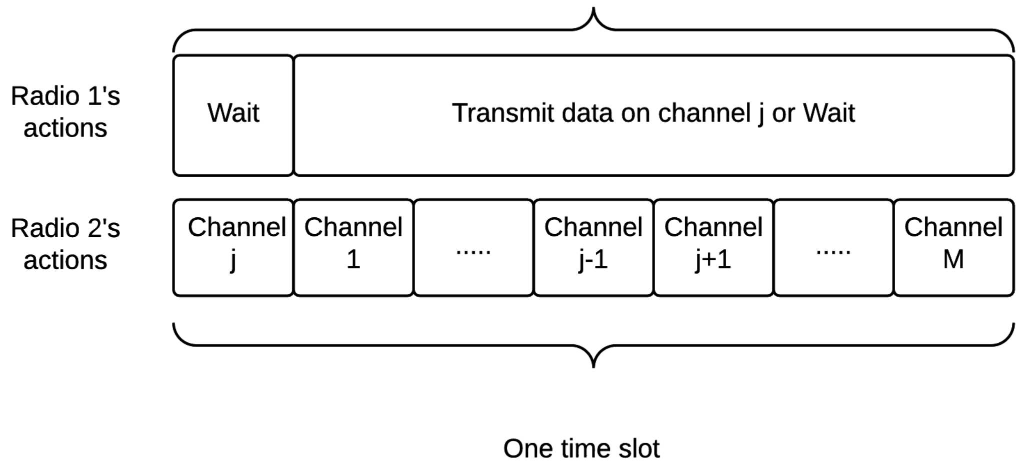


Figure 3.1: Actions of an SU's two radios in one time slot.

illustrates the actions of an SU's two radios in one time slot.

The key decision for each SU to make is which channel, j, to transmit on in each time slot to maximize its long term payoff, i.e., which channel is most likely to be vacant from PU activity and free of conflicts with the other SUs. This decision can be made based on previous channel occupancy history. In each transmission period, a channel can be: idle (neither occupied by a PU nor SUs), occupied by a PU, occupied by one SU, or occupied by more than one SU (resulting in a collision). We explore two scenarios: (i) SUs are capable of classifying the signals they encounter, distinguishing signals transmitted by a PU from signals transmitted by an SU and collision events (fully observable channel activity); (ii) SUs can only observe whether a channel is idle or not (partially observable channel activity). We also consider two types of sensing errors: false alarm and missed detection. Our objective is to compare the effect of these two types of imperfect information on the SUs' long-term channel selection payoffs. We further describe these scenarios in the following paragraphs.

In the case of fully observable channel activity $\sigma_j^i(t) \in \{0, 1, 2, 3\}$ denotes the state of channel j observed by SU i in time slot t, where $\sigma_j^i(t) = 0$ indicates that channel j was sensed idle, $\sigma_j^i(t) = 1$ indicates that channel j was sensed to be occupied by a PU, $\sigma_j^i(t) = 2$ indicates that channel j was sensed to be occupied by one SU, and $\sigma_j^i(t) = 3$ indicates that a collision was detected in channel j. In the case of partially observable channel activity, the occupancy signal for channel j observed by SU i at time slot t is denoted as $\eta_j^i(t) \in \{0, 1\}$, where $\eta_j^i(t) = 0$ indicates that j is observed to be idle and $\eta_j^i(t) = 1$ indicates that channel j is observed to be occupied (by a PU, an SU, or due to a collision between multiple SUs).

As previously mentioned, the second source of imperfect information is sensing errors. We denote the false-alarm probability, which is the probability that an SU erroneously detects an idle channel as occupied by a PU, by e_f. The missed-detection probability, e_m, is the probability that an SU erroneously detects an occupied channel as idle. We assume that e_f and e_m are the same for all SUs across all channels.

The payoff of SU i in slot t is denoted by $u^i(t) \in \{0, 1, -c\}$ with $c \in [0, 1)$. $u^i(t) = 0$ represents the inability to transmit in the chosen channel during time slot t due to the selected channel being occupied; $u^i(t) = 1$ represents a successful transmission; $u^i(t) = -c$ represents a collision with other SUs. The payoff of transmission failure is negative due to the waste of SUs' transmission power. At the beginning of time slot t, SU i decides which channel to access ($a^i(t)$) based on the observed channel occupancy history $\{\sigma^i(k)\}$ (or $\{\eta^i(k)\}$) and the SU's own past payoff ($u^i(k)$), for $k \in \{0, ..., t-1\}$. The goal of the channel selection strategy is to maximize the expected long term individual utility $E[u^i] = (1-\delta)\sum_{k=0}^{\infty}\delta^k E[u^i(k)]$, where $\delta \in (0, 1)$ is a discount factor. This factor can be understood as an indication of how much SUs care about future payoffs. The more patient an SU is, the closer its discount factor value is to one.

In this paper, we assume that the PU activity is independent and identically distributed (i.i.d.) across the channels as well as across the time slots. We assume all channels have the same PU duty cycle (d). We further assume the number of channels is greater than the number of SUs, i.e., $N < M$.

Table 3.1 summarizes the notation used in this Chapter.

Table 3.1: Notation Summary

Symbol	Definition
M	number of channels.
N	number of SUs.
$\mathcal{M}$	secondary users' action set. $\mathcal{M}$ =$\{1, 2, ...M\}$.
$a^i(t)$	SU $i's$ action at time t, with $a^i(t) \in \mathcal{M}$, $i \in \{1, 2, ..., N\}$.
$u^i(t)$	SU $i's$ payoff at time t: $u^i(t) \in \{-c, 0, 1\}$, with $i \in \{1, 2, ..., N\}$.
$E[u^i]$	SU $i's$ expected long-term individual utility.
$\sigma^i(t)$	channel occupancy information under fully observable channel activity monitoring: $\sigma^i(t) = (\sigma^i_1(t), \sigma^i_2(t), ..., \sigma^i_M(t))$, with $\sigma^i_j \in \{0, 1, 2, 3\}$, $j \in \mathcal{M}, i \in \{1, 2, ..., N\}$.
$\eta^i(t)$	channel occupancy information under partially observable channel activity monitoring: $\eta^{\mathbf{i}}(\mathbf{t}) = (\eta^i_1(t), \eta^i_2(t), ..., \eta^i_M(t))$, with $\eta^i_j \in \{0, 1\}$, $j \in \mathcal{M}, i \in \{1, 2, ..., N\}$.
e_f	false alarm probability. e_f is the same for all SUs and all channels.
e_m	missed detection probability. e_m is the same for all SUs and all channels.

3.3 Channel Selection Strategy under no Sensing Errors

In this section, we present our proposed channel selection strategies under fully and partially observable channel activity with no sensing errors. The main goal of channel selection is to decide which channel to access in each time slot to maximize the long-term individual utility as defined in the previous section. The probability that an SU successfully transmits on a chosen channel depends on the PU activity level (duty cycle) and whether or not any other SUs attempt to transmit on that channel. We consider a simple but widely adopted model of channel occupancy by PUs, namely that all channels are equally likely to be occupied by a PU, and that PU activity is independent between time slots. Therefore, the design of channel selection strategies focuses on avoiding collisions among SUs.

Under both fully and partially obsevable channel activity, the main idea behind the strategies is to reach, without coordination, an orthogonal channel allocation, under which each SU occupies its own channel that it continues attempting to access. After converging to the orthogonal allocation, there will be no collisions among the SUs. To achieve this orthogonal allocation, the channel selection strategies dictate that an SU that successfully transmitted data on a certain channel should keep accessing the same channel while other SUs try to avoid accessing this channel through observed channel occupancy information.

3.3.1 The channel selection strategy under fully observable channel activity

Under fully observable channel activity, SU i records its current state in a binary variable s^i where value 0 represents the "Stay" state and value 1 represents the "Randomize" state. Meanwhile, SU i updates a binary vector $\mathbf{c}^i$ of length M (which is initialized with all zeros), to indicate which channels can be explored in the "Randomized" state. Specifically, c^i_j is set to one when SU i observes channel j occupied by exactly one SU, and c_j^i is set to zero when SU i observes channel j idle. The update of $\mathbf{c}^i$ is independent of SU $i's$ current state.

An SU $i's$ channel selection decision is based on its current s^i and $\mathbf{c}^i$. SU i starts in the "Randomize" state. An SU i in the "Randomize" state selects a channel uniformly at random from those channels j with $c_j^i = 0$. An SU i in the "Stay" state continues to access the same channel as in the previous time slot. An SU in the "Randomize" state that experiences a successful transmission switches to the "Stay" state. Algorithm 3 describes the channel selection process under fully observable channel activity.

3.3.2 The channel selection strategy under partially observable channel activity

Under partially observable channel activity, an SU i will record its current state by updating a binary variable s^i as under the fully observable channel activity scenario.

The channel selection decision is based on the current state and the observed channel occupancy in the previous time slot. An SU i randomly selects a channel to access in the first time slot. From the second time slot, an SU i in the "Stay" state continues to access the same channel as it did in the previous time slot. An SU in the "Randomize" state selects a channel at random from those that were observed idle in the previous time slot; if all channels were occupied in the previous slot, then it continues accessing the same channel. An SU in the "Randomize" state that experiences a successful transmission switches to the "Stay" state. Algorithm 4 describes the channel selection process under partial observability.

3.4 Strategy Analysis

In this section, we conduct a theoretical analysis of the performance of the two strategies described in the previous section. In particular, we will show in subsection V-A that with no sensing errors, both strategies converge to an orthogonal channel allocation. We assume simultaneous entry of all N SUs. In subsection V-B, we will further investigate the convergence rate of the proposed strategy under fully observable channel activity with no sensing errors.

Under the i.i.d. PU activity channel model with duty cycle d, the maximum long-term expected payoff of an SU is $1-d$. This is the expected payoff of an SU that never selects the same channel as another SU. Therefore, the goal of any strategy under the i.i.d. PU activity model should be to converge to an orthogonal SU channel allocation as quickly as possible, as there will be no collisions among SUs once such an allocation is established.

3.4.1 Convergence of the Proposed Algorithms

In this section, we prove that when there are no sensing errors, the proposed strategies converge to an orthogonal channel allocation under both full and partial observability.

When there are no sensing errors, all SUs perfectly observe the channels occupied by other SUs in the "Stay" state (under full observability) or just occupied (under partial observability). Therefore, when all SUs follow the proposed strategy, an SU in the "Stay" state will remain in the "Stay" state permanently after a successful transmission. To prove convergence, we simply need to prove that all SUs will reach the "Stay" state. Fortunately, this is straightforward.

Theorem 1. *Under either proposed strategy (for partial or full observability) with no sensing errors, all users will converge to the "Stay" state almost surely.*

Proof. Let $S(t)$ denote the number of SUs in the "Stay" state at the end of time slot t. Then, under either the full observability strategy or the partial observability strategy, when there are no sensing errors, we have $S(1) \leq S(2) \leq \cdots \leq S(t) \leq N$. Hence, by the monotone convergence theorem, every realization of the sequence $\{S(t)\}$ is convergent.

However, if $S(t) < N$, then there is a nonzero probability that $S(t+1) > S(t)$ under both strategies. That is, under both strategies, if there are users in the "Randomize" state, then there is a non-zero probability that one or more users transition to the "Stay" state.[1] Thus, with probability 1, $\{S(t)\}$ converges to N. □

3.4.2 Convergence Time under Full Observability

If, as in the proof above, we let $S^f(t)$ be the number of SUs in the "Stay" state at the end of time slot t under full observability with no sensing errors, then $\{S^f(t)\}$ forms a Markov chain. Namely, it forms the absorbing Markov chain shown in Figure 3.2, where T_{ij} is the probability that there are j SUs in the "Stay" state at the end of time slot t given that there are i SUs in the "Stay" state at the beginning of time slot t. Observe that this is an absorbing Markov chain with absorbing state N and that, as we observed above, the chain is non-decreasing.

By definition, then, T_{ij} is the probability that exactly $j-i$ of the $N-i$ SUs that are in the "Randomize" state experience a successful transmission in time slot t under the full observability strategy. Let $Pr(m,n,k,d)$ be the probability that exactly k SUs successfully transmit out of n SUs randomly selecting among m channels with PU duty cycle d in one time slot. Then $T_{ij} = Pr(M-i, N-i, j-i, d)$. We will now seek an expression for $Pr(m,n,k,d)$.

Let $F(m,n,k)$ denote the probability that exactly k SUs do not experience a collision with other SUs when n SUs randomly select among m channels. (That is, $F(m,n,k)$ is the probability that exactly k SUs out of n choose unique channels when each of the n SUs

[1] We could see this, for instance, by computing the probability that no PUs transmit on one of the $M-S(t)$ channels that are not currently occupied by SUs and that all $N-S(t)$ SUs in the "Randomize" state choose different channels. This probability, the probability that $S(t+1) = N$, will be non-zero.

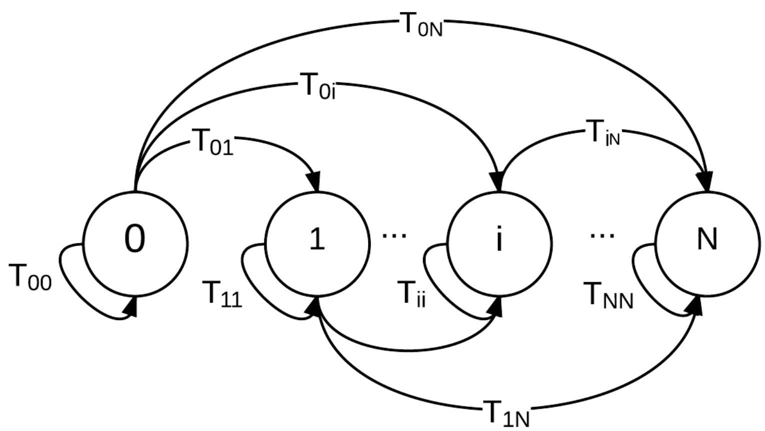


Figure 3.2: Markov Chain $\{S^f(t)\}$

randomly select among m channels.)

Lemma 2. *An expression for $Pr(m, n, k, d)$ is given by*

$$Pr(m, n, k, d) = \sum_{j=k}^{n} F(m, n, j) \binom{j}{k} (1 - d)^k d^{j-k}.$$

Proof. In the case where there are n SUs randomly selecting among m channels, let Y denote the number of these SUs that do not collide with other SUs and let X denote the number of SUs that successfully transmit data. Then $Pr(Y = k) = F(m, n, k)$. If the probability that a PU is active on each channel is d, then $Pr(X = k) = Pr(m, n, k, d)$.

Applying the law of total probability

$$\begin{aligned} Pr(m, n, k, d) &= Pr(X = k) \\ &= \sum_{j=0}^{n} Pr(X = k|Y = j) Pr(Y = j). \end{aligned}$$

Now, observe that $Pr(X = k|Y = j)$ is the probability that there are k successful transmissions when j SUs are not involved in collisions. This is the probability that exactly k of the j SUs that are not involved in collisions choose channels on which PUs are not active. Further, $Pr(X = k|Y = j) = 0$ if $k < j$. Hence, we can continue

$$Pr(m, n, k, d) = \sum_{j=k}^{n} \binom{j}{k} (1-d)^k d^{j-k} F(m, n, j).$$

□

Now, we need an expression for $F(m, n, j)$; we use an expression provided by [99]. Let $\Psi^c_{a,b}$ be the number of ways that we can partition $c \geq 1$ elements into $a \geq 1$ subsets of which $b \leq a$ contains more than one element. In other words, $\Psi^c_{a,b}$ is the number of ways to partition $c \geq 1$ distinguishable elements into $a - b$ subsets that each contain exactly one element and b subsets that each contain more than one element.

Lemma 3. *[99] An expression for $F(m, n, k)$ is given by*

$$F(m, n, k) = \sum_{j=k}^{\min(\lfloor \frac{n+k}{2} \rfloor, m)} \frac{\Psi^n_{j,j-k} \binom{m}{j} j!}{m^n}.$$

The proof is given in [99], which also provides a recursive technique to compute $\Psi^c_{a,b}$.

With these results, we can compute the transition probabilities of the Markov chain $\{S^f(t)\}$,

$$\begin{aligned} T_{ij} &= Pr(M-i, N-i, j-i, d) \\ &= \sum_{k=j-i}^{N-i} F(M-i, N-i, j-i) \binom{k}{j-i} (1-d)^{j-i} d^{k-j+i} \end{aligned}$$

For an absorbing Markov chain P with 1 absorbing state, the transition matrix can be written as $\begin{bmatrix} Q & R \\ 0 & 1 \end{bmatrix}$. The matrix $N = (I - Q)^{-1}$ is called the fundamental matrix for P. The entry n_{ij} of N gives the expected number of times the process will visit the transient state j before absorption if the chain is started in state i.

Theorem 4. *[100] Let t_i be the expected number of steps before the chain reaches an absorbing state, given that the chain starts in state s_i, and let t be a column vector whose ith entry is t_i. Then $t = Nc$, where c is a column vector whose entries are all equal to 1.*

The expected convergence time for our proposed strategy under full observability and perfect monitoring is then the summation of the first row of N, where

$$N = (I - T')^{-1}.$$

Here I is the $N \times N$ identity matrix and T' is the transition probability matrix with the last row and column (which correspond to the absorbing state N) removed.

3.5 Channel Selection Strategy under Sensing Errors

In this section, we modify our proposed channel selection strategies under sensing errors.

The modified strategies under fully and partially observable channel activity are similar to the algorithms proposed in Section 3.3.1 and 3.3.2. The only difference in the modified algorithm is that an SU in the "Stay" state that experiences a collision switches to the "Randomize" state with probability p.

The modified proposed channel selection strategies can be represented by a two-state machine, as shown in Figure 3.3. After a successful transmission, an SU in the "Randomize" state transitions to the "Stay" state. An SU in the "Stay" state, after experiencing collisions with other SUs will change to the "Randomize" state with a predefined probability p or remain in the "Stay" state with probability $1 - p$. An SU in the "Stay" state, will keep accessing the same channel as in the previous time slots.

A positive state transition probability p is necessary to make the strategies robust to sensing errors, avoiding a scenario in which multiple SUs in the "Stay" state wind up repeatedly colliding in the same channel.

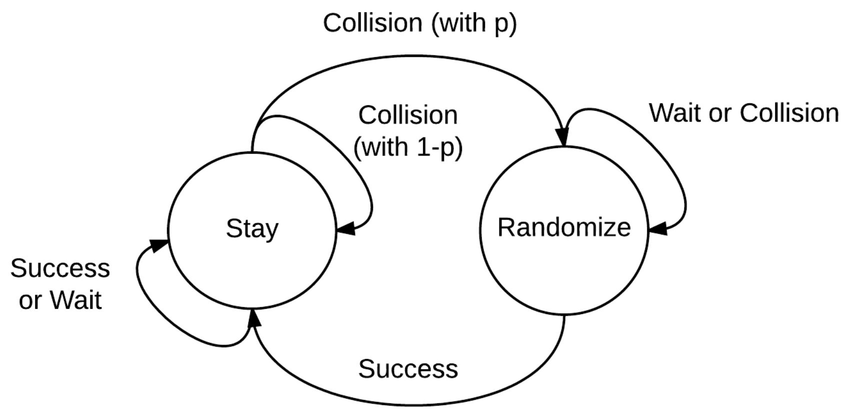


Figure 3.3: State Machine Description

In a real scenario, SUs may become active in different time slots. We will show with an example that without a positive state transition probability (i.e., with $p = 0$), SUs arriving in different time slots and a positive false alarm probability may lead to more than one SU being in the “Stay” state on the same channel.

Consider a cognitive radio system consisting of two SUs (1 and 2) and three channels (1, 2 and 3). SU 1 became active at time t_1 and successfully transmitted data on channel 1 during that time slot. According to both channel selection strategies, SU 1 will keep accessing channel 1 in the subsequent time slots. Suppose SU 2 joins the system later, at time $t_2 > t_1$. According to both strategies, SU 2 randomly selects a channel to access and therefore has a positive probability of selecting channel 1. Suppose SU 2 decides to access channel 1, and both SU 1 and 2 sense channel 1’s availability at the beginning of time t_2. Suppose channel 1 is idle from PU activity at t_2 and this is correctly detected by SU 2 but not by SU 1 due to a false alarm. In this case, SU 2 will access 1 and successfully transmit data on that channel. Both SU 1 and 2 will now be in the “Stay” state, and if $p = 0$ then they will both continue attempting to exploit channel 1 in all subsequent time slots (as a result, they will often collide). A positive state transition probability $p > 0$ will resolve this collision scenario.

In general, a positive state transition probability p is necessary to resolve the conflict when two SUs enter the "Stay" state on the same channel. For this to occur, there must exist a time slot t_2 such that an SU n_2 in the "Randomize" state successfully transmits data on a channel m_1, which is occupied by another SU n_1 in the "Stay" state. This scenario can only occur when the following three conditions are all met:

1. channel m_1 is idle from PU activity at time t_2.
2. SU n_1, which is in the "Stay" state on channel m_1, does not access channel m_1 at time t_2.
3. SU n_2, which is in the "Randomize" state, accesses and successfully transmits data on channel m_1.

Detection errors and stochastic SU arrival affect conditions 2) and 3). In particular, 2) occurs only with a false alarm: SU n_1 fails to access channel m_1 when it is idle from PU, due to a false alarm. Condition 3) can occur under one of the following three circumstances:

- stochastic arrival: SU n_2 becomes active after t_1 and accesses channel m_1 in its first time slot.
- false alarm: n_1 fails to access channel m_1 when it is idle from PU in slot $t_2 - 1$, due to a false alarm. SU n_2 observes that channel m_1 is idle and has a positive probability to access it in the time slot t_2.
- missed detection: n_2 fails to observe that n_1 has entered the "Stay" state on channel m_1, due to a missed detection. Therefore, SU n_2 accesses channel m_1 time slot t_2.

To conclude, the existence of false alarms is sufficient and necessary for two SUs to wind up on the same channel in the Stay state (in which case p $>$ 0 is needed to resolve the conflict). However, in conjunction with false alarms, missed detections and/or stochastic arrivals can make this conflict scenario more likely.

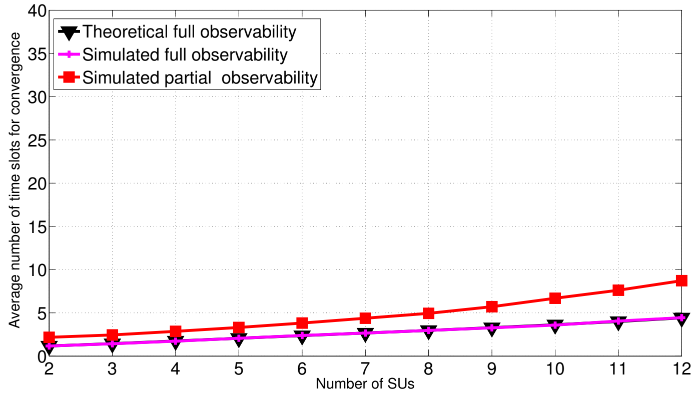
3.6 Simulation Results

In this section, we will evaluate the convergence of the proposed strategies with no sensing errors and analyze the impact of false alarms and missed detections on the performance of the proposed strategies. All simulations are based on a system operating on $M = 16$ channels and 1000 independent repeated experiments.

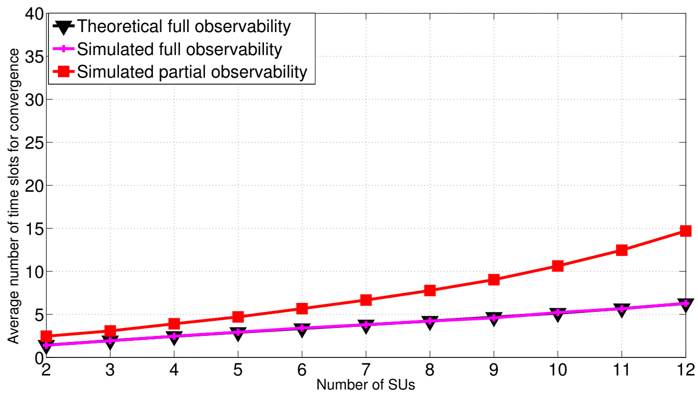
We first analyze the convergence time of the two proposed strategies with no sensing errors. In particular, we consider three duty cycle scenarios: the low PU activity level ($DC = 0.3$), shown in figure 3.4a, the medium PU activity level ($DC = 0.5$), shown in figure 3.4b, and the heavy PU activity level ($DC = 0.7$), shown in figure 4.1. Under each scenario, we compare the convergence time for strategies under both full and partial observability with respect to the number of SUs within the system. We notice that the simulation results for the strategy under full observability match the theoretical expected convergence time, thus validating the theoretical analysis in Section 3.4. Moreover, we observe that the convergence time increases with duty cycle, d, and the number of SUs, N, for both strategies.

We next investigate the convergence rates of the two proposed strategies separately with non-zero detection errors.

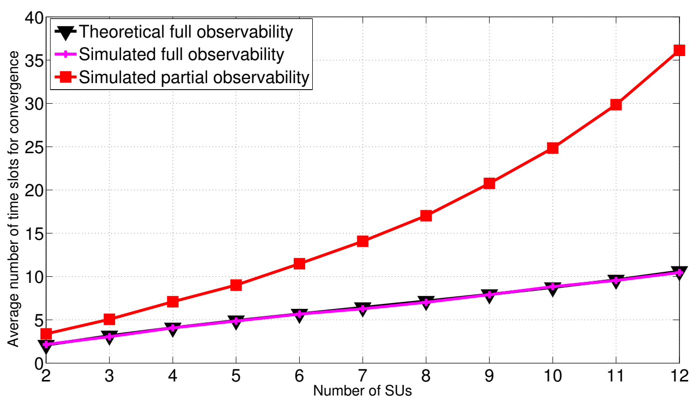
Figure 3.5a and 3.5b compare the convergence rates of fully and partially observable strategies with sensing errors. The PU duty cycle (d) is set to 0.3 and the state transition probability (p) is set to 0.1. It is clear from Figure 3.5a and 3.5b that the convergence time increases with the number of SUs under both channel selection algorithms. Figure 3.5a shows that false alarms slightly increase the convergence time under fully observable channel activity. The impact of missed detections (e_m) on convergence time is insignificant in this scenario and is therefore omitted from Figure 3.5a. Figure 3.5b shows that the



(a) $d = 0.3$



(b) $d = 0.5$



(c) $d = 0.7$

Figure 3.4: The convergence time of strategies under full and partial observability depends on the PU duty cycle (d) and number of SUs (N).

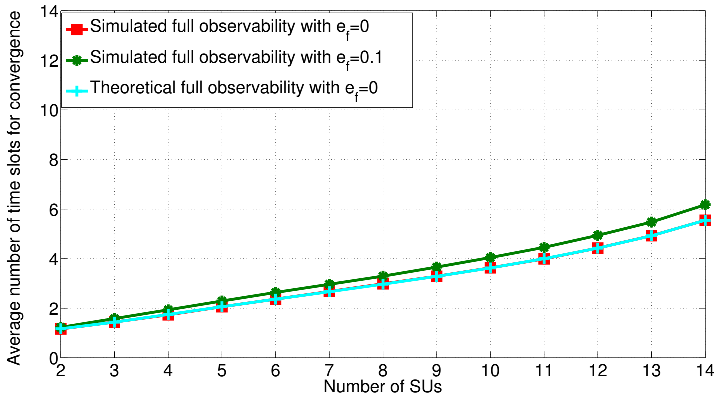
impact of both false alarm and missed detection on convergence rates is small when the SU competition level is low. Under high SU competition, the algorithm converges faster under both detection errors (the purple line) than under false alarm only (the black line). This is because the missed detections increase the number of channels for SUs in the "Randomize" state to select and therefore decrease the collisions. This phenomenon is more significant under high SU competition. Finally, the gap between expected convergence time under partial and full observability confirms the loss associated with the inability to distinguish channel occupancy information. Figure 3.5a and 3.5b together show that this loss further increases with the number of SUs.

3.7 Conclusion

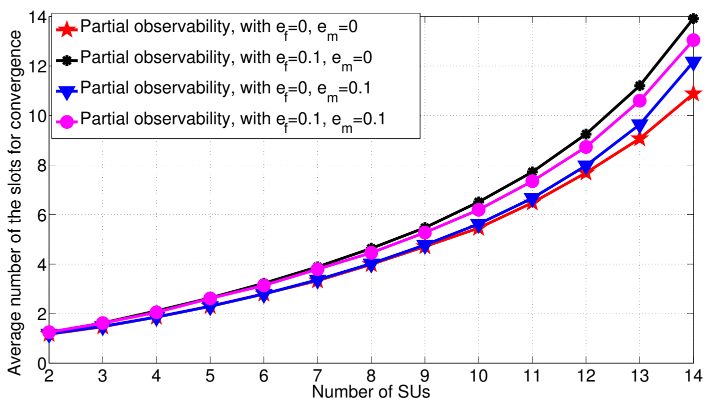
In this chapter, we have examined the impact of different sources of imperfect information on the performance of autonomous secondary users attempting to opportunistically exploit spectrum resources. We consider two sources of imperfect information: partially observable channel activity and sensing errors. Partially observable channel activity exists when SUs adopt a sensing methodology, like energy detection, that does not allow them to distinguish between PU and SU transmissions. Sensing errors are inevitable in real scenarios due to limitations on practical sensing. This chapter has shown how the quality of the monitoring information about channel activity affects the long-term throughput of each SU.

We have devised channel selection strategies for SUs under full and partial observability. Our theoretical findings showed that both strategies converge to a static orthogonal allocation of the channels with no sensing errors. We have also provided an analysis of convergence time under full observability with no sensing errors. Our simulation results generalize the convergence conclusions with non-zero sensing errors. We also use our simulations to evaluate the strategies' performance under different environmental parameters.

In this section, we focused on a homogeneous channel environment. Chapter 5 will



(a) Full observability with $e_m = 0$



(b) Paritial observability

Figure 3.5: The convergence time of strategies under full and partial observability scenario depends on the false alarm rate (e_f) and number of SUs (N). PU duty cycle $d = 0.3$, missed detection rate $e_m = 0$ and state transition probability $p = 0.1$

describe a heterogeneous channel environment.

Algorithm 1 Channel Selection Strategy for Node i under Full Observability

Initizlize: $t := 0$, $c_j^i := 0$ for all $j \in \mathcal{M}$, $s^i := 0$.
while $t < \infty$ **do**
    //Channel selection
    if $s^i = 1$ **then**
        $a^i(t) := a^i(t-1)$.
    else
        randomly select j from channels with $c_j^i = 0$, $a^i(t)$:=j.
    end if
    Attempt to access channel $a^i(t)$, observe outcome and $\sigma^i(t)$.
    //State update
    if $s^i = 0$ and outcome was success **then**
        $s^i := 1$.
    end if
    //Update c_j^i
    for all $j \in \mathcal{M}$ **do**
        if $\sigma_j^i(t) = 2$ **then**
            $c_j^i := 1$.
        else if $\sigma_j^i(t) = 0$ **then**
            $c_j^i := 0$.
        end if
    end for
    $t := t + 1$.
end while

Algorithm 2 Channel Selection Strategy for Node i under Partial Observability

Initizlize: $t := 0$, $s^i := 0$.
while $t < \infty$ **do**
    //Channel selection
    if $s^i = 1$ **then**
        $a^i(t) := a^i(t-1)$.
    else if $\eta^i_j = 1$ for all j **then**
        $a^i(t) := a^i(t-1)$.
    else
        randomly select j from channels with $\eta^i_j = 0$, $a^i(t)$:=j.
    end if
    Attempt to access channel $a^i(t)$, observe outcome and $\eta^{\mathbf{i}}(t-1)$.
    //State update
    if $s^i = 0$ and outcome was success **then**
        $s^i := 1$.
    end if
    $t := t + 1$.
end while

Chapter 4

Calculating the distribution of the number of successful transmission in multi-channel random access

4.1 Introduction

In this Chapter, we provide a non-recursive algorithm to calculate the probability distribution of the number of successful transmitters in multi-channel random access. This distribution can be used to model and analyze multi-channel communication systems such as orthogonal frequency-division multiple access (OFDMA) systems, multi-channel ALOHA, and overlay cognitive radio networks. Empirical results indicate that the proposed algorithm requires significantly less execution time and fewer operations than a recursive algorithm.

Multi-channel communication is an efficient way to alleviate interference on a wireless medium by enabling parallel transmissions over multiple channels. Examples of multi-channel wireless networks include OFDMA wireless systems [101], multi-channel ALOHA networks [102], multi-channel wireless sensor networks, and overlay cognitive radio networks [45]. Multi-channel systems can be categorized into centralized and distributed systems. In a centralized multi-channel system, a central controller makes channel access decisions for the radios. In a distributed multi-channel system, the individual radios autonomously decide which channel to access.

In multi-channel random access, each radio randomly selects a channel on which to transmit. If a radio is the only one accessing a channel, it can successfully transmit data on that channel. If multiple radios transmit simultaneously on the same channel, a collision occurs.

As described in Chapter 3 we have considered a multi-agent opportunistic channel access scenario in an homogeneous channel environment where all channels have the same PU duty cycle [103]. We proposed two channel selection strategies for cognitive radios to autonomously exploit the spectrum. A common feature of the two strategies is the behavior of cognitive radios after a successful transmission. In particular, under both strategies, a cognitive radio that has successfully transmitted data on a channel will continue to exploit the same channel in subsequent time slots. Unsuccessful cognitive radios randomly select channels from among the channels that were not successfully used by any radio in the previous time slots. The theoretical analysis in Chapter 3 aims to prove both strategies converge to a static orthogonal channel allocation, i.e., all cognitive radios finally occupy different channels. Analysis of the convergence time of such algorithms requires knowledge of appropriate conditional probabilities which relate to the probability distribution of the number of successful radios in multi-channel random access.

In this chapter, we propose a non-recursive algorithm to calculate this probability distribution. We compare our algorithm to a recursive algorithm proposed in [49], showing that the non-recursive algorithm exhibits significantly shorter execution time. The performance gap between the non-recursive and the recursive algorithms is more significant as the number of radios competing for channels increases.

The research presented in this chapter can be situated within the broader class of random channel access in multi-channel networks. The primary contribution of this chapter is a non-recursive algorithm to calculate this probability distribution and analyze the performance of the proposed algorithm as compared to a recursive algorithm already proposed in the literature.

We begin with a formulation of the problem in Section 4.2. Our non-recursive algorithm is described in Section 4.3. Section 4.4 validates the proposed algorithm numerically. Finally, we summarize our conclusions in Section 4.5.

4.2 Problem Formulation

We consider a time slotted multi-channel random access system consisting of m channels and n radios. The radios make their channel selection decisions independently. Multiple radios transmitting on the same channel results in a collision, causing no data to be delivered. Thus, a successful transmission occurs on a given channel only if exactly one radio transmits on channel.

In such a network, we define a random variable $K \in \{0, 1, ..., n\}$ as the number of successful radios in a time slot. We further define $p_K(k; m, n)$ as the probability mass function of K with respect to m and n. In other words, $p_K(k; m, n)$ is the probability that k out of n radios successfully transmit data when they randomly select one among m channels.

The calculation of $p_K(k; m, n)$ depends on the ways to partition n different radios into multiple subsets with exactly k subsets containing a single radio. We define $\Psi^c_{a,b}$ as the number of ways to partition $c \geq 1$ different elements into $1 \leq a \leq c$ subsets, among which $0 \leq b \leq a$ contain multiple elements. Thus $a - b$ subsets contain a single element.

Here is an example of $\Psi^4_{3,1}$, which represents the ways to divide 4 different elements into 3 subsets with 1 subset containing more than one element. Suppose the four elements are $\{x_1, ..., x_4\}$. We can easily list all the possible partitions:

- $x_1, x_2, \{x_3, x_4\}$,
- $x_1, x_3, \{x_2, x_4\}$,
- $x_1, x_4, \{x_2, x_3\}$,
- $x_2, x_3, \{x_1, x_4\}$,
- $x_2, x_4, \{x_1, x_3\}$,
- $x_3, x_4, \{x_1, x_2\}$.

Given this definition of $\Psi_{a,b}^c$, the expression for $p_K(k; m, n)$ can be generated through the total probability theorem. We denote the total number of channels occupied by n radios as j. Given j, the number of ways for k radios to successfully transmit is $\Psi_{j,j-k}^n$. Therefore, the probability of k successful radios in one time slot given j is: $\frac{\Psi_{j,j-k}^n \binom{m}{j} j!}{m^n}$. Note that $j!$ is necessary as the channels are distinguishable. The expression for $p_K(k; m, n)$ is [49]:

$$p_K(k; m, n) = \sum_{j=k}^{\min(\lfloor \frac{n+k}{2} \rfloor, m)} \frac{\Psi_{j,j-k}^n \binom{m}{j} j!}{m^n}. \tag{4.1}$$

We briefly explain the range of j in expression 4.1. The lower bound of j is k, which occurs when all radios successfully transmit. On the other hand, j is at most $\lfloor \frac{n-k}{2} \rfloor + k$, which occurs when the $n-k$ radios collide in pairs of two. As the number of channels is m, the upper bound of j is $min(\lfloor \frac{n-k}{2} \rfloor + k, m)$.

In summary, the calculation of $p_K(k; m, n)$ depends on $\Psi_{a,b}^c$. In the next section, we first generate an expression for $\Psi_{a,b}^c$ and propose a non-recursive algorithm to calculate $\Psi_{a,b}^c$.

4.3 Computation of $\Phi_{a,b}^c$.

4.3.1 An Expression for $\Psi_{a,b}^c$

We define a vector $\mathbf{r} = [r_1, ..., r_c]$ with r_j representing the number of subsets that contain j elements.

According to the definition of $\Psi_{a,b}^c$, $a - b$ subsets contain a single element, so

$$r_1 = a - b. \tag{4.2}$$

Also, as the total number of subsets in the partition is a, we have:

$$\sum_{j=1}^{c} r_j = a. \tag{4.3}$$

Finally, the total number of elements is c, which leads to:

$$\sum_{j=1}^{c} j r_j = c. \tag{4.4}$$

We further adopt $\mathbf{R}_{a,b}^c$ to represent the set that contains all $\mathbf{r}$ satisfying equations (4.2), (4.3) and (4.4).

We next derive an expression for $\Psi_{a,b}^c$ as a function of $\mathbf{r}$. The number of ways to partition c distinguishable elements for a given $\mathbf{r}$ is

$$\frac{c!}{\prod_{j=1}^{c} j^{r_j} r_j!}, \tag{4.5}$$

in which $r_j!$ in the denominator avoids counting the same configuration repeatedly. Therefore, an expression for $\Psi_{a,b}^c$ is:

$$\Psi_{a,b}^c = \sum_{\mathbf{r} \in \mathbf{R}_{a,b}^c} \frac{c!}{\prod_{j=1}^{c} j^{r_j} r_j!} \tag{4.6}$$

Equation (4.6) indicates that $\Psi^c_{a,b}$ depends on $\mathbf{R}^c_{a,b}$. In the next subsection, we propose an algorithm to enumerate $\mathbf{R}^c_{a,b}$.

4.3.2 The proposed algorithm to calculate $\Psi^c_{a,b}$

We define a vector $\mathbf{y} = [y_1, ..., y_b]$ to record the number of elements in each of the b subsets which contain more than one element. Therefore:

$$y_k \geq 2. \tag{4.7}$$

for $k \in \{1, .., b\}$. And

$$\sum_{k=1}^{b} y_k = c - (a - b). \tag{4.8}$$

Without loss of generality and to avoid counting the same configuration repeatedly, we further require that

$$y_k \leq y_{k+1} \tag{4.9}$$

for $k \in \{1, .., b-1\}$.

The proposed non-recursive algorithm to enumerate all possible $\mathbf{y}$ satisfying conditions (4.7), (4.8) and (4.9) is given in Algorithm 3. The idea behind the proposed algorithm is straightforward. It generates the first $b-1$ elements of $\mathbf{y}$ such that $2 \leq y_1 \leq ...y_{b-1}$. The algorithm then checks to see if there exists a bth element of $\mathbf{y}$ that would satisfy conditions (4.8) and (4.9). If there is, it computes this element y_b and prints the vector $\mathbf{y}$. If not, it generates the next possibility for the first $b-1$ elements. The algorithm 3 starts with recording the initial vector $\mathbf{y} = [2, ..., 2, c - a + b - 2(b-1)]$ (line 2 to 4).

The while loop, from line 11 to line 25, aims to find all other possible $\mathbf{y}$ based on the initial vector. The search chooses a position $k \in 1, ..., b-1$ and sets the values of the elements $y_k, ..., y_{b-1}$ such that $[y_1, ..., y_{b-1}]$ forms a non-decreasing sequence (line 10 to line 19). For each generated sequence $[y_1, ..., y_{b-1}]$, the algorithm further verifies the existence of y_b satisfying conditions (4.8) and (4.9) (from line 20 to 23).

Algorithm 4 transforms all qualified **y** generated by Algorithm 3 to the equivalent **r** vectors.

4.4 Evaluation and Comparison

In this section, we compare our proposed non-recursive algorithm to the algorithm proposed in [49], which is based on a recursion expression for $\Psi^{c}_{a,b}$:

$$\Psi^{c}_{a,b} = b\Psi^{c-1}_{a,b} + (a-b+1)\Psi^{c-1}_{a,b-1} + \Psi^{c-1}_{a-1,b} \tag{4.10}$$

with initial conditions: $\Psi^{a}_{a,0} = 1$, $\Psi^{1}_{1,0} = 1$, $\Psi^{1}_{1,1} = 0$ and $\Psi^{c\geq 1}_{1,1} = 1$.

4.4.1 Validation of our algorithm

We first validate our proposed non-recursive algorithm for calculating $\Psi^{c}_{a,b}$. Figure 4.1 shows the values of $\Psi^{c}_{a,b}$ generated by the recursive and the non-recursive algorithms with respect to c. In particular, we vary the c value from 5 to 14 and fix a and b to be 3 and 2. We have validated this over a wide variety parameters, not only with fixed a and b. The results of our proposed non-recursive algorithm match those of the recursive algorithm, thus validating the algorithm in Section 4.3.

4.4.2 Efficiency of our algorithm

In this subsection, we present numerical results to assess the efficiency of our proposed algorithm. We first analyze the computational complexity of the recursive algorithm with respect to c. We use $f(c)$ to represent the number of addition and multiplication operations required by the recursive algorithm in calculating $\Psi^{c}_{a,b}$. The expression in (4.10) indicates

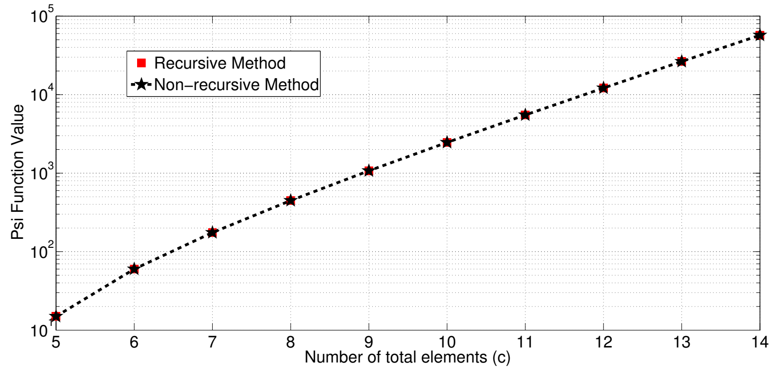


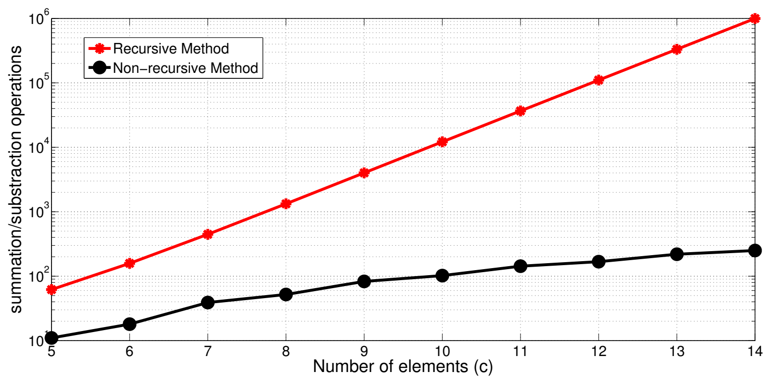
Figure 4.1: $\Psi_{a,b}^c$ under recursive and non-recursive algorithms

that $f(c)$ depends on three different $c-1$ scale Ψ functions ($f(c-1)$), 4 additions/subtractions and 2 multiplications. Therefore,

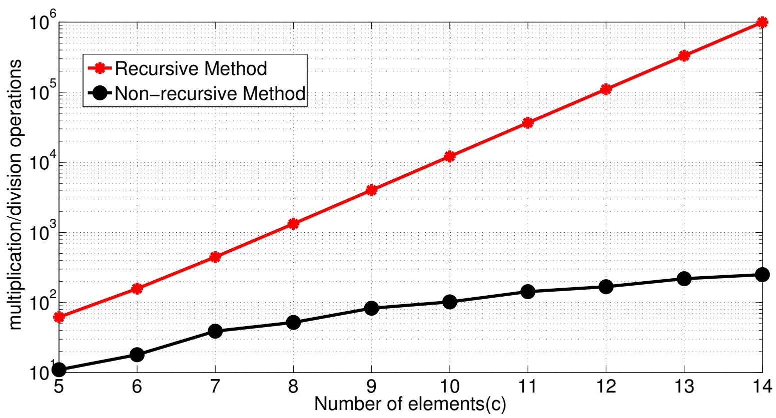
$$
\begin{aligned}
f(c) =& 3f(c-1) + 6 \\
=& 3^2 f(c-2) + 2 * 6 \\
& \dots \\
=& 3^{c-1} f(1) + 6(c-1) \\
=& 3^{c-1} + 6(c-1)
\end{aligned} \tag{4.11}
$$

which indicates the computing complexity of the recursive algorithm is $O(3^c)$.

Figure 4.2 shows the number of operations of the recursive and the non-recursive algorithms in calculating $\Psi_{a,b}^c$, as a function of c. The parameters a and b are fixed to be 3 and 2. The results are presented with c ranging from 5 to 14. Figure 4.2a shows the number of additions/subtractions under the two algorithms. The number of multiplications required by the two algorithms is compared in Figure 4.2b. It is observed that the non-recursive algorithm requires a dramatically lower number of both addition and multiplication operations. We further note in Figure 4.2 that as c increases, the number of operations required by the recursive algorithm increases exponentially, which can be expected from expression



(a) Number of addition/subtraction operations



(b) Number of multiplication/division operations

Figure 4.2: Number of operations to calculate $\Psi^c_{a,b}$.

(4.11). We also note that the gap between the number of operations for the two algorithms increases with respect to c, which indicates the efficiency of the non-recursive algorithm for large values of c.

Finally, we compare the execution time of the recursive and the non-recursive algorithms in generating $p_K(k; m, n)$. The algorithm is run on Matlab (version 2014a). The operating system is Mac OSX Yosemite run on a dual-core 1.4GHz Intel Core i5 processor. Figure 4.3 presents, in logarithmic scale, the running time of both algorithms. In particular, we demonstrate the dependency of the execution time on the number of channels (m) and the number of radios (n) in figures 4.3a and 4.3b separately. In Figure 4.3b, we set the number of radios to be 4 and vary the number of channels from 5 to 10. In Figure 4.3a, we set the number of channels to be 12 and vary the number of radios from 2 to 10. We note that the proposed non-recursive algorithm is faster than the recursion algorithm in both cases. Moreover, the performance improvement is more significant with respect to the number of radios than to the number of channels.

4.5 Conclusion

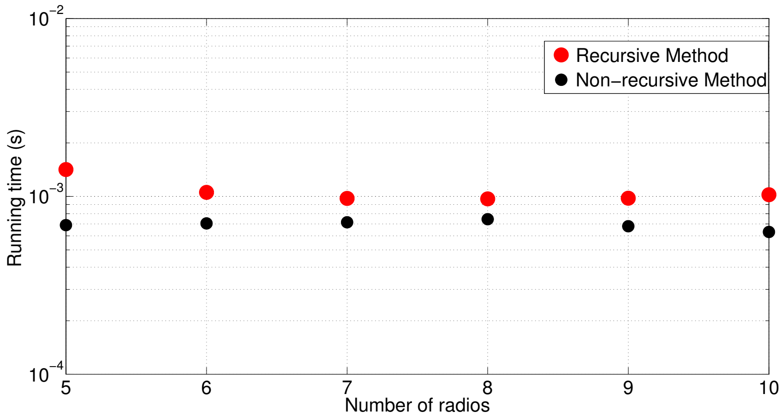
In this chapter, we proposed an non-recursive algorithm to calculate the probability distribution of the number of successful radios in multi-channel random access. The numerical results show the proposed algorithm requires significantly less running time and fewer operations than a recursion based algorithm.

Algorithm 3 Non-recursive Algorithm to find $\mathbf{y}$

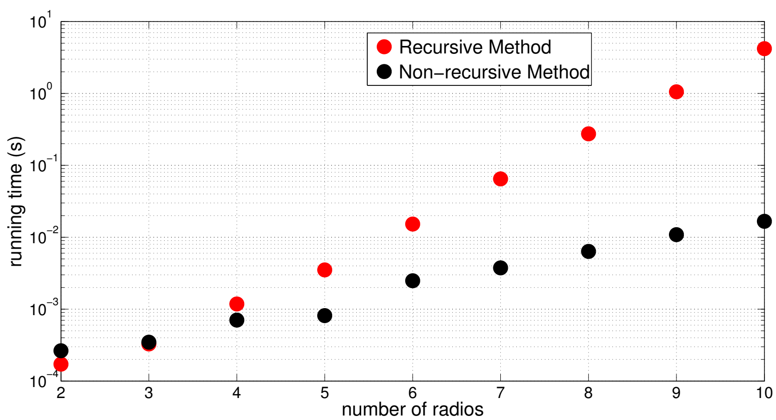
Input: a, b, c. Output: $\mathbf{y}$.
for all $k \in \{1, ..., b-1\}$ **do**
    $y_k = 2$
end for
$y_b = c - a + b - (b-1) * 2$.
record $\mathbf{y}$
while $k > 0$ **do**
    $k = b - 1$.
    while $k > 1$ and $y_k == c - a + b - (b-1) * 2$ **do**
        $k = k - 1$.
    end while
    if $k > 0$ **then**
        $y_k = y_k + 1$.
        $k = k + 1$.
        while $k < b$ **do**
            $y_k = y_{k-1}$.
            $k = k + 1$.
        end while
        if $c - a + b - \sum_{k=1}^{b-1} y_k >= y_{b-1}$ **then**
            $y_b = c - a + b - \sum_{k=1}^{b-1} y_k$
            record $\mathbf{y}$.
        end if
    end if
end while

Algorithm 4 Algorithm to find $\mathbf{R}_{a,b}^{c}$

```
Input: y Output: R^c_{a,b}.
//initialize r.
for all i ∈ {1, ..., c} do
    r_i = 0.
end for
//translate a valid y to r.
for all y_k ∈ y do
    r_{y_k} = r_{y_k} + 1.
end for
record r.
```



(a) $n = 4, m = 5 : 10$



(b) $n = 2 : 10, m = 12$

Figure 4.3: The running time of recursive and non-recursive algorithms in calculating $p_K(k; m, n)$.

Chapter 5

Heterogeneous Primary User Activity

5.1 Introduction

Chapter 3 considered the channel selection process in distributed OSA networks with homogeneous PU activity. In a more realistic scenario, the channels present the secondary user with different access opportunities. In particular, the channels have different PU duty cycles; we refer to this scenario as the heterogeneous channel environment. In this chapter, we consider channel access strategy design in a heterogeneous channel environment.

Channel selection in a heterogeneous channel environment is more challenging than in a homogeneous channel environment. As channels have different PU duty cycles, SUs have preferences over channels. When there is only one SU in the system, such a preference will lead the SU to access the channel with the lowest duty cycle to maximize the expected success rate. However, in a multi-user environment, SUs also compete with each other for channel access. If multiple SUs access the channel with the lowest duty cycle, then none of them transmits successfully due to a collision.

The goal of strategy design in a heterogeneous channel environment involves both channel exploration and collision avoidance. In a homogeneous channel environment, the goal of strategy design was simply to avoid collisions among SUs, finally converging to a orthogonal channel allocation. This orthogonal allocation scheme is not sufficient in the heterogeneous scenario due to fairness considerations.

In this chapter, we model channel selection as a one-shot game. We assume the duty cycle information is known to all the SUs. In the one-shot game model, we show the existence of the Nash equilibrium and propose an algorithm to achieve this equilibrium outcome.

We compare the proposed algorithm with two other strategies. We show numerically that all three strategies have the same performance in the homogeneous channel environment. We also show through simulations that when the duty cycles of the channels are linearly distributed from 0.1 to 0.9, the Nash equilibrium strategy outperforms the other two baseline strategies when there are few SUs in the system, but performs worse than the other two strategies as the number of SUs increases. When the duty cycles are randomly distributed from 0.1 to 0.9, the Nash equilibrium always performs worse than the other two baseline strategies.

The structure of this chapter is as follows. We begin with an introduction of the system model in section 5.2. A symmetric Nash equilibrium strategy is described in section 5.3. Section 5.4 presents the numerical results of the Nash equilibrium. We summarize our conclusions in Section 5.5.

5.2 System Model and Notation

We consider a time-slotted cognitive radio system consisting of M channels and N SUs. In each time slot, an SU chooses exactly one channel to transmit data on. Just prior to transmission, the SU needs to verify that the selected channel is not being used by a PU in that time slot. If no PU is sensed as being active in the channel, the SU can then transmit data using that channel. If a PU is found to be active in the selected channel, the SU will wait during that time slot without transmitting. If multiple SUs sense the same channel and transmit data simultaneously, a collision will occur and none of the transmissions will be successful. Thus, a successful transmission occurs on a given channel if only one SU has chosen this channel and no PU is active on this channel. At the end of each time slot, an SU gets feedback on whether its transmission was successful (for instance, in the form of an ACK).

PU activity is independent across the channels. In particular, the PU activity on each channel is modeled as a Bernoulli process. The duty cycle of channel j is denoted as d_j, which is the probability that channel j is occupied by a PU in each time slot. Without loss of generality, we assume $d_j \leq d_{j+1}$. We assume that $(d_1, ..., d_M)$ are known to the SUs. We further assume the number of channels is greater than the number of SUs, i.e., $N < M$.

A static game contains three elements: the set of players, the pure strategy space for each player, and payoff functions that assigns a utility for each strategy profile. In our scenario, we define the set of players as $\mathcal{N} = \{1, ..., N\}$. The pure strategy space for each SU is $\mathcal{M} = \{1, ..., M\}$, and the payoff function for SU i is denoted as u_i. As we focus on symmetric strategy design in this chapter, we omit the subscript i and simply write u to represent the utility of an SU.

The payoff of an SU is denoted by $u(\mathbf{s}^{'}, \mathbf{s}^{*}) \in (0, 1)$. $u(\mathbf{s}^{'}, \mathbf{s}^{*})$ represents the utility when one SU adopts strategy profile $\mathbf{s}^{'}$ while other SUs all adopt strategy profile $\mathbf{s}^{*}$. In particular, $u(k, \mathbf{s}^{*})$ represents the expected utility of an SU that always chooses channel $k \in \{1, ..., M\}$ with probability 1.

In a static game, a mixed strategy $\mathbf{s}$ is a probability distribution over pure strategies. The support of a mixed strategy is the set of pure strategies to which $\mathbf{s}$ assigns positive probability. We denote a mixed strategy profile by $\mathbf{s} = (s_1, ..., s_M)$. $s_j \in [0, 1]$ represents the probability for an SU to access channel j . Thus, $\sum_{j=1}^{M} s_j \leq 1$.

5.3 Symmetric Nash Equilibrium Strategy

We model the channel selection process as a one-shot game. We focus on a symmetric equilibrium to guarantee fairness among SUs. We first prove the existence of a symmetric Nash equilibrium and then propose an algorithm to achieve this equilibrium.

We denote a symmetric Nash equilibrium strategy profile as $\mathbf{s}^* = (s_1^*, ..., s_M^*)$. We will first prove the existence of the symmetric equilibrium $\mathbf{s}^*$ and then propose an algorithm to find $\mathbf{s}^*$.

We denote by $S(\mathbf{s}^*)$ the support of $\mathbf{s}^*$, which contains all the actions in $\mathcal{M}$ that have positive probability under $\mathbf{s}^*$, i.e., $S(\mathbf{s}^*) = \{j|\, j \in \mathcal{M},\ \mathbf{s}_j^* > 0\}$. Lemma 5 characterizes a property of $\mathbf{s}^*$.

Lemma 5. *For a symmetric Nash equilibrium profile* $\mathbf{s}^*$*, the sequence of probabilities* $s_1^*, ..., s_M^*$ is non-increasing.

Proof. We prove the Lemma by contradiction.

Suppose the sequence $s_1^*, ..., s_M^*$ is not non-increasing, then there exists $j \in \mathcal{M}$ such that $s_{j-1}^* < s_j^*$. We create a new mixed strategy profile $\mathbf{s}'$ by exchanging the order of s_{j-1}^* and s_j^* of $\mathbf{s}^*$, i.e., $\mathbf{s}' = (s_1^*, ..., s_{j-2}^*, s_j^*, s_{j-1}^*, s_{j+1}^* ..., s_M^*)$, and we show that an SU can get a better expected payoff by adopting $\mathbf{s}'$ while all other SUs adopt $\mathbf{s}^*$.

$$\begin{aligned}
u(\mathbf{s}', \mathbf{s}^*) - u(\mathbf{s}^*, \mathbf{s}^*) &= \sum_{k=1}^{M} s_k' u(k, \mathbf{s}^*) - \sum_{k=1}^{M} s_k^* u(k, \mathbf{s}^*) \\
&= \sum_{k=j-1}^{j} s_k' u(k, \mathbf{s}^*) - \sum_{k=j-1}^{j} s_k^* u(k, \mathbf{s}^*) \\
&= s_j^* u(j-1, \mathbf{s}^*) + s_{j-1}^* u(j, \mathbf{s}^*) - s_{j-1}^* u(j-1, \mathbf{s}^*) - s_j^* u(j, \mathbf{s}^*) \\
&= (s_j^* - s_{j-1}^*)(u(j-1, \mathbf{s}^*) - u(j, \mathbf{s}^*)) \qquad (5.1)
\end{aligned}$$

The expected payoff for an SU accessing channel j while all other SUs follow strategy profile $\mathbf{s}^*$ is:

$$u(j,\mathbf{s}^*) = (1-d_j)(1-s_j^*)^{N-1} \tag{5.2}$$

Therefore, expression (5.1) is:

$$u(\mathbf{s}^{'},\mathbf{s}^*) - u(\mathbf{s}^*,\mathbf{s}^*) = (s_j^* - s_{j-1}^*)((1-d_{j-1})(1-s_{j-1}^*)^{N-1} - (1-d_j)(1-s_j^*)^{N-1}) \tag{5.3}$$

As $d_{j-1} \leq d_j$, $s_{j-1}^* < s_j^*$, expression (5.3) is positive, which contradicts the fact that $\mathbf{s}^*$ is an equilibrium profile. Therefore, the sequence $s_1^*, ..., s_M^*$ is non-increasing. □

An immediate consequence of Lemma 5 is that if $\mathbf{s}^*$ is a symmetric Nash equilibrium strategy then, there exists an $m \in \mathcal{M}$ such that $S(\mathbf{s}^*) = \{1, ..., m\}$.

Theorem 6. *For the given static game, a symmetric Nash equilibrium $\mathbf{s}^*$ always exists.*

1. *If $M-1 \leq \sum_{k=1}^{M}(\frac{1-d_M}{1-d_k})^{\frac{1}{N-1}}$, $s_j^* = 1 - \frac{M-1}{\sum_{k=1}^{M}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}$ for $j \in \mathcal{M}$ is a Nash equilibrium.*

2. *If $M-1 > \sum_{k=1}^{M}(\frac{1-d_M}{1-d_k})^{\frac{1}{N-1}}$, there exists an m satisfying $\sum_{k=1}^{m}(\frac{1-d_{m+1}}{1-d_k})^{\frac{1}{N-1}} + 1 < m \leq \sum_{k=1}^{m}(\frac{1-d_m}{1-d_k})^{\frac{1}{N-1}} + 1$ and*

$$s_j^* = \begin{cases} 1 - \frac{m-1}{\sum_{k=1}^{m}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}} & \text{for } j \leq m \\ 0 & \text{for } j > m \end{cases} \tag{5.4}$$

is a symmetric Nash equilibrium.

Proof. We will prove the two parts of the theorem separately. In the first case ($M-1 \leq \sum_{k=1}^{M}(\frac{1-d_M}{1-d_k})^{\frac{1}{N-1}}$), we will first show that the presented $\mathbf{s}^*$ is a probability distribution and then prove it to be a Nash equilibrium profile. In the second case ($M-1 > \sum_{k=1}^{M}(\frac{1-d_M}{1-d_j})^{\frac{1}{N-1}}$), we will first show the existence of m followed by proving the presented $\mathbf{s}^*$ as a probability distribution and a Nash equilibrium profile.

- Case 1: $M-1 \leq \sum_{k=1}^{M}(\frac{1-d_M}{1-d_k})^{\frac{1}{N-1}}$.

  We will show that $\mathbf{s}^*$ forms a probability distribution. As $d_1 \leq ... \leq d_M$,

$$M-1 \leq \sum_{k=1}^{M}(\frac{1-d_M}{1-d_k})^{\frac{1}{N-1}} \leq ... \leq \sum_{k=1}^{M}(\frac{1-d_1}{1-d_k})^{\frac{1}{N-1}},. \tag{5.5}$$

  which guarantees $\frac{M-1}{\sum_{k=1}^{M}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}} \in [0,1]$. Therefore, $s_j^* = 1 - \frac{M-1}{\sum_{k=1}^{M}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}} \in [0,1]$.

  Also,

$$\begin{aligned}\sum_{j=1}^{M} s_j^* &= \sum_{j=1}^{M}(1 - \frac{M-1}{\sum_{k=1}^{M}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}) \\ &= M - \sum_{j=1}^{M}\frac{M-1}{\sum_{k=1}^{M}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}} \\ &= M - \frac{M-1}{\sum_{k=1}^{M}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\sum_{j=1}^{M}(\frac{1}{1-d_j})^{\frac{1}{N-1}} \\ &= 1. \end{aligned} \tag{5.6}$$

  Thus $\mathbf{s}^*$ is a probability profile.

  We now show that $\mathbf{s}^*$ is a symmetric Nash equilibrium by showing that $u(\mathbf{s}^*) \geq u(j, \mathbf{s}^*)$ for $\forall j \in \mathcal{M}$.

  The expected payoff for an SU accessing channel j while all other SUs follow strategy profile $\mathbf{s}^*$ is:

$$u(j, \mathbf{s}^*) = (1-d_j)(1-s_j^*)^{N-1} \tag{5.7}$$

  As $s_j^* = 1 - \frac{M-1}{\sum_{k=1}^{M}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}$, (5.7) becomes:

$$
\begin{aligned}
u(j, s^*) &= (1-d_j)\left(\frac{M-1}{\sum_{k=1}^{M}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= (1-d_j)\left(\frac{M-1}{(1-d_j)^{\frac{1}{N-1}}\sum_{k=1}^{M}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= (1-d_j)\left(\frac{1}{(1-d_j)^{\frac{1}{N-1}}}\right)^{N-1}\left(\frac{M-1}{\sum_{k=1}^{M}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= \left(\frac{M-1}{\sum_{k=1}^{M}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \qquad (5.8)
\end{aligned}
$$

which is independent of j. As $\mathbf{s}^*$ is a mixed strategy over all actions $j \in \mathcal{M}$, (5.8) shows that $\mathbf{s}^*$ is a best response when all other players are playing $\mathbf{s}^*$. Therefore $\mathbf{s}^*$ is a symmetric Nash equilibrium.

- Case 2: $M-1 > \sum_{k=1}^{M}(\frac{1-d_M}{1-d_k})^{\frac{1}{N-1}}$.

  We define a_j as a sequence with $a_j = \sum_{k=1}^{j}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}$ for $j \in \{1, ..., M-1\}$. Therefore,

$$a_{j+1} = \sum_{k=1}^{j+1}(\frac{1-d_{j+1}}{1-d_k})^{\frac{1}{N-1}} = \sum_{k=1}^{j}(\frac{1-d_{j+1}}{1-d_k})^{\frac{1}{N-1}} + 1. \qquad (5.9)$$

  Thus, to prove there exists an m satisfying $\sum_{k=1}^{m}(\frac{1-d_{m+1}}{1-d_k})^{\frac{1}{N-1}}+1 < m \leq \sum_{k=1}^{m}(\frac{1-d_m}{1-d_k})^{\frac{1}{N-1}}+1$ is equivalent to proving the existence of $m \in \{1, ..., M-1\}$ such that

$$a_{m+1} < m \leq a_m + 1 \qquad (5.10)$$

  If $a_2 < 1$, then,

$$a_2 < 1 \leq a_1 + 1 \qquad (5.11)$$

  where the second inequality is because $a_1 > 0$. In this case, clearly $m = 1$ satisfies 5.10.

Otherwise, $a_2 \geq 1$. In this case, if $a_3 < 2$, then

$$a_3 < 2 \leq a_2 + 1 \tag{5.12}$$

and clearly $m = 2$ satisfies 5.10.

Otherwise $a_3 \geq 2$. In this case, if $a_4 < 3$, then

$$a_4 < 3 \leq a_3 + 1. \tag{5.13}$$

and so $m = 3$.

We continue this process. At each step, we have either found $m = k$, in which case the process can stop, or we have shown that $a_{k+1} > k$. However, we know that $a_M < M - 1$, by assumption. Thus, the process above must terminate (finding $m = k$) when $k = M - 1$. Therefore, we are guaranteed to find $m \leq M - 1$.

We next show $s_j^* = 1 - \frac{m-1}{\sum_{k=1}^{m}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}$ for $j \leq m$ and $s_j^* = 0$ for $j > m$ forms a probability distribution.

As m satisfies $m - 1 \leq \sum_{j=1}^{m}(\frac{1-d_m}{1-d_j})^{\frac{1}{N-1}}$, $1 - s_m^* = \frac{m-1}{\sum_{j=1}^{m}(\frac{1-d_k}{1-d_j})^{\frac{1}{N-1}}} \in [0, 1]$. Also,

$$\begin{aligned}
\sum_{j=1}^{m} s_j^* &= \sum_{j=1}^{m}(1 - \frac{m-1}{\sum_{k=1}^{m}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}) \\
&= m - \sum_{j=1}^{m} \frac{m-1}{\sum_{k=1}^{m}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}} \\
&= m - \frac{m-1}{\sum_{k=1}^{m}(\frac{1}{1-d_k})^{\frac{1}{N-1}}} \sum_{j=1}^{m}(\frac{1}{1-d_j})^{\frac{1}{N-1}} \\
&= 1
\end{aligned} \tag{5.14}$$

Therefore, $\mathbf{s}^*$ is probability distribution.

We next prove $\mathbf{s}^*$ is a Nash equilibrium by showing that $u(\mathbf{s}^*) \geq u(j, \mathbf{s}^*)$ for $j \in \mathcal{M}$.

The expected payoff of adopting strategy $\mathbf{s}^*$ is:

$$
\begin{aligned}
u(\mathbf{s}^*) &= \sum_{j=1}^{M} s_j^*(1-d_j)(1-s_j^*)^{N-1} \\
&= \sum_{j=1}^{m} s_j^*(1-d_j)\left(\frac{m-1}{\sum_{k=1}^{m}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= \sum_{j=1}^{m} s_j^*\left(\frac{m-1}{\sum_{k=1}^{m}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= \left(\frac{m-1}{\sum_{k=1}^{m}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1}
\end{aligned}
\tag{5.15}
$$

As m satisfies $\sum_{j=1}^{m}(\frac{1-d_{m+1}}{1-d_j})^{\frac{1}{N-1}} \leq m-1$, we have $(1-d_{m+1})^{\frac{1}{N-1}} \sum_{j=1}^{m}(\frac{1}{1-d_j})^{\frac{1}{N-1}} \leq m-1$. Therefore,

$$1-d_{m+1} \leq \left(\frac{m-1}{\sum_{j=1}^{m}(\frac{1}{1-d_j})^{\frac{1}{N-1}}}\right)^{N-1}. \tag{5.16}$$

For $j > m$, $s_j^* = 0$. Therefore,

$$u(j,\mathbf{s}^*) = (1-d_j)(1-s_j^*)^{N-1} = 1-d_j \leq 1-d_{m+1}. \tag{5.17}$$

Combining (5.15), (5.16) and (5.17), we have

$$u(\mathbf{s}^*) = \left(\frac{m-1}{\sum_{j=1}^{m}(\frac{1}{1-d_j})^{\frac{1}{N-1}}}\right)^{N-1} \geq 1-d_{m+1} \geq u(j,\mathbf{s}^*) \ for\ j > m \tag{5.18}$$

On the other side, for $j \leq m$, $s_j^* = 1 - \frac{m-1}{\sum_{k=1}^{m}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}$

$$\begin{aligned}
u(j,\mathbf{s}^*) &= (1-d_j)(1-s_j^*)^{N-1} \\
&= (1-d_j)\left(\frac{m-1}{\sum_{k=1}^{m}(\frac{1-d_j}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= (1-d_j)\left(\frac{m-1}{(1-d_j)^{\frac{1}{N-1}}\sum_{k=1}^{m}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= (1-d_j)\left(\frac{1}{(1-d_j)^{\frac{1}{N-1}}}\right)^{N-1}\left(\frac{m-1}{\sum_{k=1}^{m}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= \left(\frac{m-1}{\sum_{k=1}^{m}(\frac{1}{1-d_k})^{\frac{1}{N-1}}}\right)^{N-1} \\
&= u(\mathbf{s}^*) \qquad (5.19)
\end{aligned}$$

5.18 and 5.19 indicate $u(\mathbf{s}^*) \geq u(j,\mathbf{s}^*)$ for $j \in \mathcal{M}$, which proves that $\mathbf{s}^*$ is a Nash equilibrium profile.

□

5.4 Numerical Results

In this section, we introduce two baseline strategies to compare the symmetric Nash equilibrium against, namely the random strategy and the proportional strategy. We then show that the symmetric Nash equilibrium and the two baseline strategies have the same performance in a homogeneous channel environment. Finally, we compare the performance of the three strategies in a heterogeneous channel environment.

We denote the random strategy profile as $\mathbf{s}^R$. Under the random strategy, an SU randomly selects a channel to explore, i.e., $s_j^R = \frac{1}{M}$ for $j \in \mathcal{M}$. The expected individual success rate under $\mathbf{s}^R$ is:

$$E[u(\mathbf{s}^R)] = \sum_{j=1}^{M} \frac{1}{M}(1-d_j)(1-\frac{1}{M})^{N-1} = (1-\frac{1}{M})^{N-1}(1-\frac{1}{M}\sum_{j=1}^{M} d_j). \tag{5.20}$$

The proportional strategy is denoted $\mathbf{s}^P$. Under this strategy, the probability that an SU accesses the channel j, s_j^P, is proportional to $1-d_j$, i.e., $s_j^P = \frac{1-d_j}{\sum_{k=1}^{M}(1-d_k)}$ for $j \in \mathcal{M}$. The expected individual success rate under $\mathbf{s}^P$ is:

$$\begin{aligned} E[u(\mathbf{s}^P)] &= \sum_{j=1}^{M} s_j^P(1-d_j)(1-s_j^P)^{N-1} \\ &= \sum_{j=1}^{M} \frac{1-d_j}{\sum_{k=1}^{M}(1-d_k)}(1-d_j)(1-\frac{1-d_j}{\sum_{k=1}^{M}(1-d_k)})^{N-1} \\ &= \frac{1}{\sum_{k=1}^{M}(1-d_k)} \sum_{j=1}^{M}(1-d_j)^2(1-\frac{1-d_j}{\sum_{k=1}^{M}(1-d_k)})^{N-1} \end{aligned} \tag{5.21}$$

We next show that when the duty cycle of all channels is the same, the symmetric Nash equilibrium strategy, the random strategy, and the proportional strategy all give the same expected payoff.

When all the duty cycles are the same for all channels, the expected payoff under the symmetric Nash equilibrium strategy reduces to:

$$E[u(\mathbf{s}^*)] = (\frac{m-1}{\sum_{k=1}^{m}(\frac{1}{1-d_k})^{\frac{1}{N-1}}})^{N-1} = (1-d)(1-\frac{1}{m})^{N-1}$$

In this scenario, the expected payoff under the random strategy is:

$$E[u(\mathbf{s}^R)] = (1-\frac{1}{M})^{N-1}(1-\frac{1}{M}\sum_{j=1}^{M} d_j) = (1-d)(1-\frac{1}{M})^{N-1}. \tag{5.22}$$

And the expected payoff under the proportional strategy is:

$$E[u(\mathbf{s}^P)] = \frac{1}{\sum_{k=1}^{M}(1-d_k)}\sum_{j=1}^{M}(1-d_j)^2(1-\frac{1-d_j}{\sum_{k=1}^{M}(1-d_k)})^{N-1}$$
$$= (1-d)(1-\frac{1}{M})^{N-1}. \tag{5.23}$$

We next compare the expected individual success rates of SUs under the three strategies, when different channels exhibit different duty cycles. First, we vary PU duty cycles from 0.1 to 0.9 with uniform gaps. It is clear from Figure 5.1 that the individual success rate decreases with respect to the number of SUs. This is because increasing SU competition increases the collision possibility and therefore decreases the individual SU's probability of success. It can also been observed from Figure 5.1 that the when the number of SUs is small, the Nash equilibrium strategy outperforms the other two strategies. As the number of SUs increases, the Nash equilibrium strategy performs worse than the Proportional strategy. Finally, when the SU competition level is high, the expected individual payoff of the Nash Equilibrium Strategy drops below those of the random and the proportional strategies.

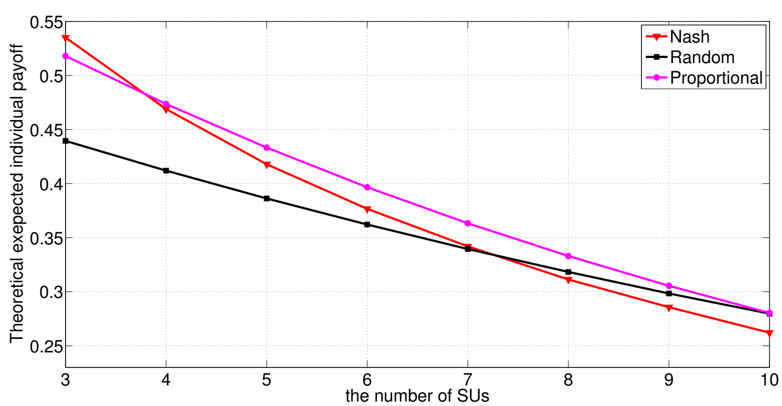


Figure 5.1: The average individual success rate of SUs for $M = 16$ with duty cycle linearly distributed between 0.1 and 0.9.

Figure 5.2 shows the average individual success rate of SUs when the duty cycles of channels are randomly distributed between 0.1 and 0.9. The results are averaged over 1000 independent repetitions. We observe that the proportional strategy gives the best performance among the three strategies under all SU competition levels. And the Nash equilibrium strategy performs worse than the other two strategies under all SU competition levels.

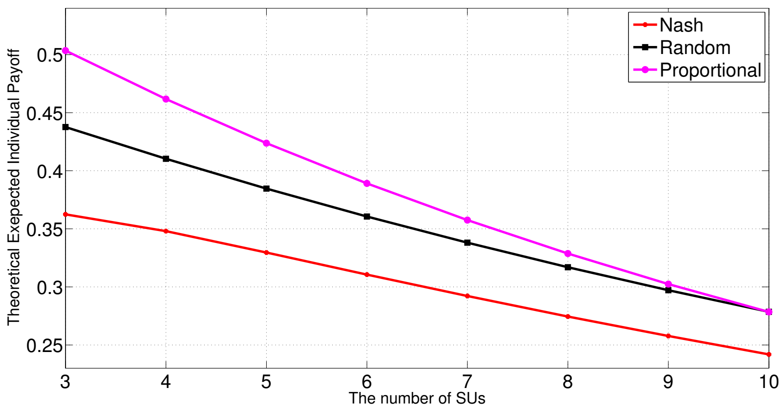


Figure 5.2: The average individual success rate of SUs for $M = 16$ with duty cycles randomly distributed between 0.1 and 0.9.

We further validate theoretical expressions for individual success rate for each of the three strategies through simulations. All simulations are based on a system operating on $M = 16$ channels and 10000 independent repeated experiments. The PU duty cycles in all simulations are linearly distributed from 0.1 to 0.9.

Figure 5.3 shows the individual success rates for the Nash strategy. Figure 5.4 shows the individual success rate for the random strategy. Figure 5.5 shows the individual success rates for the proportional strategy. It is clear from all three plots that the individual success rates given by the theoretical expressions match the simulated results, validating the theoretical expressions.

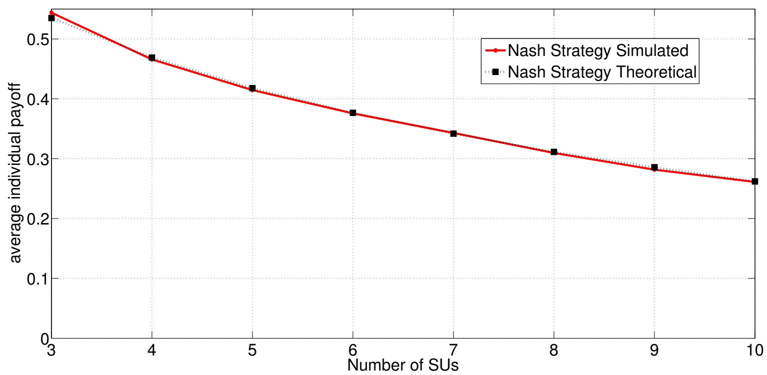


Figure 5.3: The average individual success rates of the Nash strategy with the duty cycle linearly distributed between 0.1 and 0.9.

5.5 Conclusion

In this chapter, we have modeled the channel selection in a heterogeneous channel environment as a one-shot game. In the one-shot game model, we show the existence of a symmetric Nash equilibrium and provide expressions for this equilibrium strategy profile.

We compared the symmetric Nash equilibrium profile with two baseline strategies. We found that all three strategies perform the same in a homogeneous channel environment. We have also provided a numerical analysis of performance of three strategies under a heterogeneous channel environment. Our simulation results show that when the duty cycles of the channels are linearly distributed from 0.1 to 0.9, the Nash equilibrium strategy outperforms the other two baseline strategies under small SU competition level, but performs worse than the other two strategies as the number of SUs increases. When the duty cycles are randomly distributed among 0.1 to 0.9, the Nash equilibrium always performs worse than the other two baseline strategies.

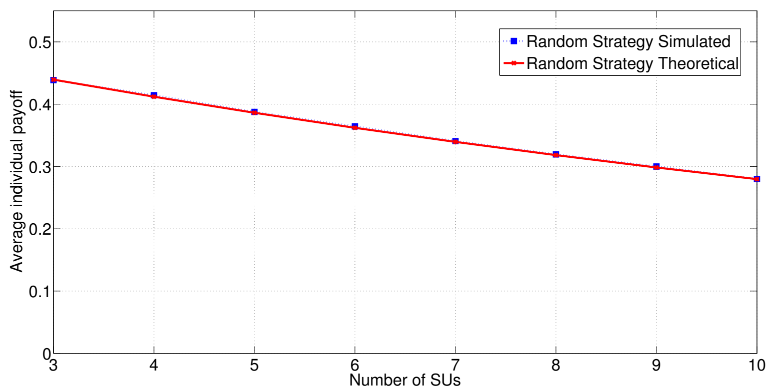


Figure 5.4: The average individual success rates of the Nash strategy with the duty cycle linearly distributed between 0.1 and 0.9.

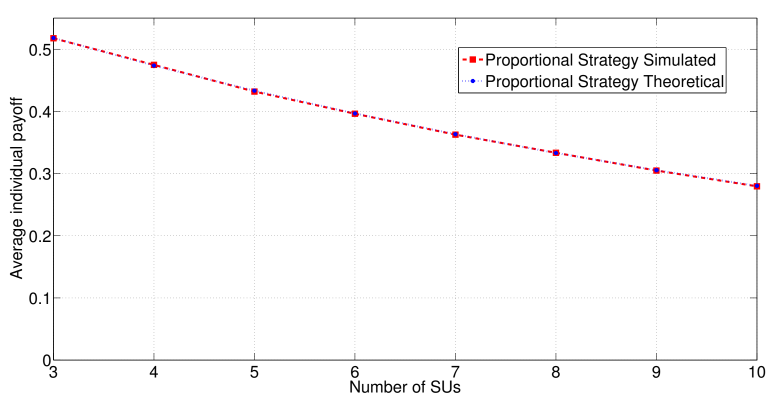


Figure 5.5: The average individual success rates of the Nash strategy with the duty cycle linearly distributed between 0.1 and 0.9.

Chapter 6

Conclusions

This chapter concludes the thesis. We will revisit the main contents and contributions of the previous chapters and list our related publications. Finally, we will give an overview of the further work.

The first part of this thesis investigated the impact of different types of imperfect information on the performance of SUs attempting to opportunistically exploit spectrum resources in a distributed manner in a homogeneous channel environment. We designed channel selection strategies that leverage different levels of information about channel occupancy. We considered partial observability and sensing errors as two sources of imperfect information. We proposed two channel selection strategies under both partial and full observability of channel activity and evaluated their performance through both theoretical and simulation results. We proved that both proposed strategies converge to a stable orthogonal channel allocation when the missed detection rate is zero. The simulation results also supported the robustness of our proposed strategies even with a non-zero probability of missed detection.

The second part of this thesis computed the probability distribution of the number of successful users in multi-channel communication. This probability distribution can be applied in a multi-agent opportunistic channel access scenario for strategy evaluation, e.g., the theoretical analysis of Chapter 3 relates to the probability distribution of the number of successful radios in multi-channel random access. We proposed a non-recursive algorithm to calculate this probability distribution. We compared our algorithm to a recursive algorithm proposed in the previous literature, showing that the non-recursive algorithm exhibits significantly shorter execution time. The simulation results showed that the performance gap between the non-recursive and the recursive algorithms is more significant as the number of radios competing for channels increases.

The third part of this thesis investigated secondary users (SUs) attempting to opportunistically exploit spectrum resources in a heterogeneous channel environment. We modelled the channel selection process as a one-shot game. We proved the existence of a symmetric Nash equilibrium and proposed a channel selection strategy that achieves this equilibrium. We compare the proposed algorithm with two other strategies (random strategy and proportional strategy) and showed numerically that all three strategies had the same performance under the homogeneous channel environment. We also showed through simulations the performance of different strategies under different PU duty cycles. We found that when the PU duty cycles are linearly distributed from 0.1 to 0.9, the Nash equilibrium strategy outperformed the other two baseline strategies under small SU competition level, but performed worse than the other two strategies as the number of SUs increases. When the duty cycles are randomly distributed among 0.1 to 0.9, the Nash equilibrium always performed worse than the other two baseline strategies.

The main contributions of the thesis are summarized as followed:

- We designed channel selection strategies for autonomous SUs in distributed cognitive radio networks in a homogeneous channel environment under both fully and partially observable channel activity.
- We proved the convergence of the two proposed strategies in the presence of no sensing errors and characterized the expected convergence rate of the full observability strategy.

- We showed the robustness and efficiency of our strategies in the presence of imperfect monitoring through both theoretical analysis and simulation results and provided an analysis of the impact of different sources of imperfect information on the ability of SUs to dynamically exploit the band.
- We proposed a non-recursive algorithm to calculate the probability distribution of the number of successful users in a multi-channel random access scheme and compared it to a previously proposed recursive algorithm to demonstrate its efficiency.
- We applied a game theoretic model to analyze channel selection for autonomous SUs in a heterogeneous channel environment. We propose a symmetric Nash equilibrium strategy and evaluate its performance compared with other strategies.

Here are the list of the related publications related to this thesis:

- J. Wang, I. Macaluso and L. A. DaSilva, "Perfect versus imperfect monitoring in multi-agent opportunistic channel access," *Proc. of the International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM)*, Oulu, 2014, pp. 480-485.
- J. Wang, I. Macaluso, L. A. DaSilva and Allen B. MacKenzie, "Partial Observability in Multi-agent Opportunistic Channel Access," submitted to *IEEE Transactions on Cognitive Communications and Networking*, 2016.
- J. Wang, L. A. DaSilva and Allen B. MacKenzie, "Calculating the distribution of the number of successful transmission in multi-channel random access," *submitted to IEEE Communications Letters.*

The work of this thesis can be extended to the scenario where SUs compete for the channel access in a heterogeneous channel access environment with unknown PU duty cycles and imperfect information. Channel selection in this scenario is more challenging compared to that in the heterogeneous channel environment with known PU duty cycles. As channels' duty cycles are unknown, SUs do not know which channels are preferable. Therefore, SUs need to adopt learning schemes in channel exploration to learn the PU activities. When there is only one SU in the system, such learning scheme is easy to design: The SU accesses each channel and estimates the PU duty cycle on each channel based on the success rate on that channel. However, in a multi-user environment with imperfect monitoring information, the design of a learning scheme becomes complicated. For example, due to the partial observability, learning PU activity is more difficult. Repeated games with imperfect monitoring can be adopted to model the channel access process in this scenario. In particular, channel selection needs to be based on past observations where the PU activities and other SUs' actions may not be directly observable.

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