

Damage Detection and Location in Large Space Trusses

by

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(ABSTRACT)

Researchers pursuing the goal to design and construct a large orbiting space structure are directing considerable effort toward many issues, including the ability to maneuver a flexible structure. In particular, basic research is underway into the technologies of control system design and structural modeling to support this effort. On-orbit verification of the structure is another issue under investigation. The thesis of this research is that structural damage can be detected and located with the control system of a large space structure.

A concept for damage location was developed and demonstrated in simulated tests. The control system "tests" the structure and measures the response. The measurements are then used in a system identification algorithm to produce a model of the damaged structure. The model is compared to one for the undamaged structure to find regions of reduced stiffness which indicate the location of damage.

Truss structures were considered for this research since they have been proposed for two NASA programs, including the Space Station. Damage was limited to catastrophic failure of a single member which produces regions of reduced or zero stiffness in the truss. In a simplified model, the truss member was deleted entirely.

Alvar M. Kabe developed a stiffness matrix adjustment algorithm that was selected, after evaluation tests, to be the central identification algorithm in the damage location method. The strength of Kabe's method is that it preserves the zero-nonzero pattern of the original stiffness matrix in the stiffness matrix for the damaged truss. Thus, the method uses information from the undamaged

structure and requires minimal measured data. This is important because measured data from orbiting structures will be limited.

Simulation studies were performed on two truss models. The members of both and the design of the second were borrowed from the concept design for the Space Station. Exact- and inexact-data simulated tests with the two structures indicated that damage can be located with this approach. With some exceptions due to the truss design, the ability to locate damage with exact data is excellent. Inexact data for the method degraded the performance, but allowed damage location in many cases.

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1.0 Introduction

The community of scientists and engineers that support and advance space exploration is pursuing a goal to design and construct a large orbiting space structure. Considerable effort is being directed to address the many issues that confront these researchers. Among others, attention is focussed on problems of maneuvering flexible structures, including orientation and vibration suppression. In particular, basic research is underway into the technologies of control system design and structural modeling.

The control system of a space structure consists of sensors and actuators and the capability to direct their use for maneuvering the structure. The system monitors the structure response to determine how to react if the response is not as expected. The control system also initiates inputs for reorienting and repositioning the structure.

An important aspect of the ability to control a flexible structure is the model used in the controller. This model must reflect changes to the structure, whether the changes are by design or are unintentional. In addition, models developed in the gravity environment on earth must be adjusted in the operational environment in space. The field of system identification is involved with the problems of structural modeling. System identification is the name given to the class of problems

where the response of a structure is used to determine the structure characteristics, or in other words, the structure model.

On-orbit verification of the structure is another issue for large space structures. Properties of composite materials degrade with time in a space environment to a point that is unacceptable. Elements of the structure also fail catastrophically.

The question of how best to provide on-orbit verification has not been answered. Sensors for each structure element requiring assessment could be included in the design, along with a system to monitor them. Fiber optic sensors have been suggested in this capacity. However, any system uses a part of the weight budget and may use computer or communications time. Including a verification system could exclude other capabilities.

The thesis of this research is that structural damage can be detected and located with the control system of a large space structure.

A reasonable program was obtained by considering a subset of all large space structures. Truss structures were an obvious choice for this subset since they have been proposed for two NASA programs concerned with issues for large space structures. The concept design for the International Space Station is a large truss structure. The Control of Flexible Structures (COFS) program was designed to provide knowledge that could be applied to the Space Station or to controlling other large space structures. Again, a large truss structure was proposed.

Other limitations of the research include standard first assumptions. Linear response was assumed, so the theory of superposition holds. Actual truss structures exhibit nonlinear response, especially for deployable structures. Complications that arise from considering nonlinear response are prohibitive at this time. The situation is considered to be stationary, so system parameters are constants, not time varying. Also, the system is considered to be deterministic, so stochastic analyses are not necessary.

Damage is a term that describes the many situations that occur in a large space truss structure. This work deals with the failure of a single member of a large truss. None of the work, however, excludes the possibility of future consideration of degraded properties or of multiply damaged members.

Space structures of the type considered exhibit certain characteristics that become features of the problem of damage location in large space trusses. Damping is small, so proportional damping is assumed in some instances and damping is neglected entirely in others. Also, these large structures typically have low, closely-spaced frequencies. In addition, orbiting structures have limitations for instrumentation, communication and calculation that must be considered.

The general approach to the problem of damage location in large space trusses is the determination of areas of reduced or zero stiffness in the truss structure. The control system is used to "test" the structure and produce response measurements. These measurements are then used in a system identification algorithm to produce a model of the damaged structure. The model is compared to one for the undamaged structure to locate any damage.

The following chapters of this dissertation present the research on damage location for large space trusses in a chronological order. First, Chapter 2 contains system identification background in the form of a literature review. Damage detection literature is presented as well. Chapter 3 is an evaluation of some promising methods of system identification with emphasis on damage detection. Identification methods are designed to perform well when considering specific objectives, but often these objectives are different than those needed for damage detection. Chapter 4 presents the damage location algorithm and simulated tests on truss structures whose designs were borrowed from the Space Station concept. Further demonstrations of the algorithm performance, including imperfect data to simulate errors which occur in measured data, are presented in Chapter 5. Finally, Chapter 6 contains the conclusions that resulted from this research and recommendations for future work.

The phrases, “damage detection” and “damage location” are used synonymously throughout this dissertation. In other contexts, damage detection might imply system verification, the determination that damage exists somewhere in the structure. Damage location would be the process of finding and specifying the damaged member. For this work, however, damage has occurred whenever the response of the truss is not as expected for the undamaged structure. “Damage detection” and “damage location” are used interchangeably to mean the process of determining which member is damaged.

2.0 Background

2.1 Introduction

This literature review had two main goals at the onset. The primary goal was to produce a representative collection of system identification methods. The secondary goal was a survey of the damage detection and system identification literature as a necessary first step of the research program.

The research program was organized to begin with comparison tests of available system identification methods. One or several would be selected to use in the main body of the research. The collection of representative system identification methods, that was the primary goal of this review, was to be used in these comparison tests. Deriving a new method of system identification for truss structures was not a part of the research plan. So, this literature review is more than a summary of the related work. It led to the selection of a method which formed the basis for the remainder of the program.

The secondary goal of this review was to acquaint the researcher with methods of damage detection and system identification for structures. This acquaintance included some knowledge of the subjects' history to lend understanding of their development. Works of importance and methods of importance were noted for future reference.

During the review process, an additional goal was defined. The system identification portion of the review had to be limited in some ways to methods of system identification for structures. If references for control systems were included in the literature review, a vast number of papers would be encompassed. In the preface to *Identification of Systems* (1st edition in 1972), Graupe mentions the "hundreds of papers on identification (that) have been published in recent years in professional journals." His purpose for the text was to present various methods of identification as a reference for controls or systems design. It would be wrong to completely exclude the references for control systems and other non-structural applications. Therefore, texts on these topics are included in the literature review as collections of important approaches. Survey papers, conference papers and journal articles were limited to structural system identification.

Even within these limitations, the review is not exhaustive or comprehensive. At a recent International Modal Analysis Conference (Los Angeles, 1986), more than 15% of 237 papers involved structural identification and parameter estimation methods (from the titles). Often the work presented is a derivative of a fundamental approach or the application of an established method to a new problem. Since the selection of a method of system identification is only the first part of the research program in damage detection for large space trusses and because the method selected after the comparison tests is used essentially as a tool for the remainder of the program, this review concentrated on representative methods of fundamental approaches for structural system identification.

2.2 System Identification

2.2.1 Texts

Several texts have been published on the subjects of identification and estimation. Graupe [1], mentioned previously, presents methods of system identification for control systems, predictors and filters. The methods are for problems of all degrees of difficulty from linear, stationary problems with much a priori information to problems of stochastic systems with little a priori knowledge. Kagiwada [2] assembled a diverse set of problems to demonstrate the application of system identification methods. These “inverse problems” include examples from celestial mechanics, optics, physiology and wave propagation, among others. A collection of applications as examples is also presented in Junkins [3]. Methods to “determine ‘best’ estimates of all poorly known parameters so that the mathematical model provides an ‘optimal estimate’ of the system’s actual behavior” are developed in separate chapters of this text. An extensive bibliography is included. Ljung and Soderstrom [4] presented recursive identification, also known as adaptive or on-line identification, in their text. Some recursive algorithms were developed from off-line methods, so this text includes both options to some extent. Finally, Graupe [5] has a recent text for stochastic problems or problems which tend to stochastic analysis “even if deterministic in origin.”

2.2.2 Surveys and Reviews

There are also a number of survey papers that have contributed collections of various identification methods and discussions of their relationships. Three early surveys of interest were presented at the 1972 Winter Annual Meeting of the ASME. Flannelly and Berman [6] reviewed primarily modal techniques including methods of resonance testing for the normal modes of a system. Collins,

Young, and Kiefling [7] concentrated on frequency domain methods, but included a summary of time domain methods as well. They developed a “technology tree” for both frequency and time domain methods. A weighted least squares, iterative, frequency domain method that “is particularly suited to finite element modeling” was highlighted. Finally, Schiff [8] presented an identification method survey for the large structures of the time - buildings. He included discussion of the identification problem with closely spaced frequencies.

In a later work, Hart and Yao [9] updated the technology trees from Collins, et. al. A detailed review of system identification methods in the civil engineering structural dynamics area was included. Methods were classified as with or without prior structural models and as with or without quantification of experimental or modeling errors.

Several survey papers have been published recently which add to the understanding of structural system identification. Ibrahim [10], noted for his work in time domain methods, has prepared a review of modal identification techniques. He presents resonance testing, multi-shaker methods, and frequency domain methods in a clear and historical review of structural identification. In the section on time domain methods, the author’s work is highlighted, although a few alternate and derivative methods are presented and discussed.

Another excellent review is *Identification of Large Space Structures on Orbit* [11]. A task committee, formed to “develop a state-of-the-art report on methods for identification of large structures in space”, assembled a reference that addresses the issues of on-orbit identification along with the methods. The bibliography is extensive. Chapters 5 and 6 are of particular interest because they present methods for modal parameter identification and structural model parameters identification, respectively. Structural model parameters are characterized as physical (densities, lengths, areas, etc.), or non-physical (i.e. matrix elements from finite element modeling). Detailed discussions of methods are presented for several forms of parameterization, including direct and iterative methods, least squares techniques, and perturbation methods, among others. Numerical examples are included for eight methods, although four different systems were used in the examples.

Most recently, Juang and Pappa [12] presented a review of modal testing and modal parameter identification that contributes a historical perspective for these two areas. They also present “principal references” to highlight key developments in the two fields.

2.2.3 Journal Articles and Conference Papers on Structural Identification

The damage detection problem was defined to limit its scope to a reasonable research program. Many of the research program limitations affect the options for system identification methods. For example, a nonlinear identification method is not needed for a system that is assumed to be linear.

In addition, some forethought would seem to indicate that knowledge of the space truss structure undamaged model would be available as a priori knowledge for the system identification method used to locate the damage. While this is most likely the situation that will give the most accurate results, methods of identification that use various degrees of prior information for the system were reviewed. Similarly, a discrete model of the truss structure would seem more likely to be used to detect damage to an individual truss member. Still, continuum models are included in this review, because ultimately a combination of continuous and discrete modeling may prove most effective for damage detection. A final selection of an appropriate identification method was made after comparison tests were complete. So, there was no attempt to limit this review with a presupposed direction of the damage detection program.

The papers in the following review are grouped by their parameterization method following the lead of *Identification of Large Space Structures on Orbit* [11]. Models are constructed with physical parameters, non-physical parameters, and, for repeating substructure trusses, equivalent continuum parameters. In addition, modal quantities - frequencies, mode shapes, and damping values - serve as parameters to be identified by many methods.

2.2.3.1 Physical Parameters

Physical parameters are often referred to as the measurable or observable parameters. These include, but are not limited to, discrete masses, lengths, cross-sectional areas, Young's moduli, spring stiffnesses, and mass densities. In a spring-mass system, the physical parameters are the spring constants and the masses. For a truss structure, the physical parameters could be mass densities, member lengths, cross-sectional areas and member stiffnesses (EI).

Hendricks, Hayes, and Junkins [13] use frequency measurements in an iterative least squares formulation to improve initial estimates of physical model parameters. Beliveau [14] uses Bayesian methods and modal data to identify damping in structures.

2.2.3.2 Equivalent Continuum Parameters

A large truss structure could obviously have an unmanageable number of physical parameters, even if each member has only length, area and stiffness parameters (density assumed constant) to identify. This is why equivalent continuum models for truss structures with repeated substructures are so attractive. The unwieldy number of physical parameters is greatly reduced.

Continuum modeling of large truss structures has been presented by many authors. Nayfeh and Hefzy [15] use sets of parallel members in the structure which have "unidirectional 'equivalent continuum' properties." The equivalent continuum model is produced with orthogonal transformations of the unidirectional properties. Repeating substructures are not used as elements in this method.

Noor, Anderson and Greene [16] and Noor and Andersen [17] develop continuum models by equating the kinetic and strain energies of the repeating substructure to that of the continuum element. Strain gradient variables are used to "account for local effects in the repeating element of

the actual structure” and produce more accurate results. Saw and Tamma [18] emphasize tetrahedral structures using a method based on Noor’s technique. Most recently, Dow et. al. [19] have developed a method which uses Noor’s strain gradient variables, but allows the analyst to provide the more easily produced repeating cell stiffness matrix, thereby reducing the “possibility of introducing errors by requiring the analyst to supply the strain gradient terms directly.” Equivalent parameters for complex structures are developed more easily with this method.

A least-squares formulation to identify the “physical” equivalent continuum parameters has been presented in *Identification of Large Space Structures on Orbit* [11]. Also, Baruh and Meirovitch [20,21] present a method to identify the physical parameters of a distributed-parameter system after identifying the eigensolution. Continuous parameters are functions of the spatial variable and are expanded in terms of orthogonal functions of this variable. A least-squares problem is developed for undetermined coefficients in the expansion. Prior information about the system can be included with modifications. Also, Meirovitch and Norris [22] use time domain measurements of distributed structures to identify physical parameters based on the finite element method.

Finally, Juang and Sun [23] present two identification techniques for the parameters of equivalent continuum models.

2.2.3.3 Non-physical Parameters

Non-physical parameters are of many types. The elements of the mass, damping and stiffness matrices that appear in the equations of motion of a spring-mass system or result from a finite element model of a truss structure are non-physical parameters. Scale factors or small perturbations of these matrix elements are method parameters as well.

Rajaram and Junkins [24] and Hendricks, et. al. [25,26] present a direct method for identifying the mass, stiffness and damping matrix elements from time domain responses. No prior

information beyond the model dimension is needed, but measurements of displacement, velocity, and acceleration are required at each degree-of-freedom at numerous times. A least squares problem is constructed for the elements' identification with either free or forced response data.

Leuridan, et. al. [27,28] also developed a direct least squares formulation, but from the Fourier transformed equations of motion. Prior knowledge of the matrix structure reduces the identification problem which is solved using frequency domain measurements. Frequency domain methods often experience difficulty with closely spaced frequencies, but multiple force inputs overcome this problem. Denman and Leyva-Ramos [29] also transform the matrix equations of motion, but use Laplace transform variables. Measurements of input and output are taken at a small number of prescribed times depending on the quadrature formula used for the transformation. The structure of the matrices is used to reduce the problem here, as well.

The problem of optimally adjusting the stiffness matrix has been addressed by several authors and provides an example of the difficulties that are encountered with limited data identification. Some prior work served as inspiration for the activity that followed, but the majority of the effort in this area began with Baruch and Bar Itzhack [30]. Their paper on an optimal method for adjusting measured modes to satisfy the requirement of orthogonality with respect to the mass matrix also included a section on optimally correcting the stiffness matrix.

A symmetric stiffness matrix is found that is "closest" to an original stiffness matrix and subject to constraints which express the dynamic equilibrium conditions of the structure. A weighted Euclidean norm is used to measure the closeness of the adjusted stiffness matrix to the original stiffness matrix. It is this norm which is minimized, subject to linear constraints. An extended cost function is obtained for the optimization problem using Lagrange multipliers to augment the weighted norm with the constraint equations.

Berman and Nagy [31] incorporated this method in a procedure to correct the analytical mass and stiffness matrices for a large model. However, the corrected matrices may be fully populated where

considerable sparsity existed in the original model. Therefore, coupling elements in the stiffness matrix are produced that are not physical couplings of the structure. This characteristic leads to doubts about using the adjusted matrices in dynamic analyses and expecting improved results for modes other than those used in the identification.

Wei [32] presented another derivation leading to the same method to obtain corrections for the stiffness matrix elements. He assumes that the mass matrix is known and an initial stiffness matrix approximation is obtained from measurements or finite element analysis.

Kabe [33,34] addressed the problem of unrealistic coupling in the result with an optimal adjustment procedure for the stiffness matrix that constrains the adjusted matrix to have the connectivity of the initial matrix, as well as to be symmetric and to satisfy the equilibrium equations. No zero elements of the initial stiffness matrix (modeling the physical load paths of the structure) become nonzero with this identification method. Again, constrained minimization theory is used, with the sum of the stiffness elements' percentage change as the error function. Closely spaced frequencies were included in the simulation example.

Kabe's stiffness matrix adjustment method and Baruch's and Bar Itzhack's stiffness update method use the same inputs, an initial model and few modes, but produce very different stiffness matrices. This confirms the assertion that non-unique solutions result, thus demonstrating difficulties encountered using limited modal data.

White and Maytum [35] developed a method that preserves the model coupling by using submatrices to adjust the initial model. These submatrices are multiplied by scaling factors to adjust the model. The identification procedure solves for these non-physical scaling factors.

Finally, Creamer [36] presented a method to determine the mass, damping and stiffness matrices from a small set of measurements. A unique identification of the structure is provided by using forced-response data in conjunction with free-response data.

2.2.3.4 Modal Quantities

Modal parameters are the frequencies, damping factors, and modal vectors of the structure and are identified by many methods. A well known structural identification method for modal parameters is the Ibrahim Time Domain (ITD) method presented by Ibrahim and Mikulcik [37,38]. Measured free response is used in an eigenvalue problem formulated from a hypothetical lumped parameter system. Displacement of the masses in the hypothetical system match the displacements measured on the actual system.

More recently, Ibrahim [39] presented a method to eliminate bias errors in damping values when a least-squares algorithm is used. The ITD method of the previous paper results in two alternate formulations for the eigenvalue problem. These two formulations develop two “oppositely biased solutions” which are averaged to reduce the overall errors.

Another well known structural identification method is the Eigensystem Realization Algorithm (ERA) of Juang and Pappa [40 – 43]. State variable equations are the starting point for this technique which uses the singular value decomposition. Repeated eigenvalues are recognized and two parameters are defined to indicate the identification accuracy. An optimum model order is also obtained from the identification results.

Sundararajan and Montgomery [44] presented a method of identification that also requires no prior specification of the model order. This method is a recursive, on-line identification with lattice filters used in signal processing. The model order and the mode shapes are found.

Voss [45] recently developed a related method for determining the modal model of a large space structure. Recursive lattice least squares was selected as the baseline algorithm. Then, several modifications were implemented to improve its performance for real time identification. The dual keel U.S. Space Station was used as a realistic simulation example.

Frequency domain methods have a well documented problem with closely spaced frequencies, a characteristic of large space structures. Craig and Blair [46] address the problems of mode shape identification of closely spaced modes and noise influence on identification of modal parameters. Multiple excitations are used and reduced system matrices are calculated with an algorithm similar to Leuridan. These reduced system matrices are used to produce the modal parameters.

2.3 Damage Detection

In direct contrast with the subject of system identification, the subject of damage location for large space structures has had very little attention. For other specific structures, literature is more available. However, our simplification of the problem to a situation with damage in one member of a large space truss removes the possibility of applying results from these related works.

Detecting and locating damage has been widely explored with work in nondestructive evaluation techniques for composite materials. Henneke [47] presented a survey of these techniques that included a discussion of the conflict between damage detection and structural verification. Inability to establish exact definitions for damage causes difficulty in developing or selecting a method for damage detection. For example, a flawed composite structure may still satisfy the system requirements and, thus, may be verified as acceptable. It is inappropriate, in this case, to use a method of damage detection that pinpoints each of the flaws. A method is needed which evaluates the structure, and its performance, as a whole. However, the field of nondestructive evaluation for composite materials has not matured to this point, yet.

Yao [48 – 50], Toussi, Yao and Chen [51] and Toussi and Yao [52] also discussed this conflict between damage detection and system verification in a series of papers on damage assessment of buildings. Limitations in the understanding of cumulative damage, combined with difficulty in

determining the damage state, create an extremely complex problem. Yao presented the idea of system identification for damage detection, but concluded that this approach may only be of benefit when used with other sources of data in an “expert system” for the assessment of damage.

For this research, the simplifying assumption to only consider damage to a single truss member, eliminates the conflict between damage detection and verification. Consequently, no method from these other areas of damage detection is directly applicable for large space trusses.

Three studies for the Space Station support consideration of damage to a single member of a large space truss as a simplified problem in damage detection. Dorsey [53] examined the structural performance of candidate trusses with missing members. Batla [54] discusses hypervelocity impacts of meteoroids and space debris on the walls of the Space Station. Conceivably, a single member of the truss could be damaged by an impact. Lepanto [55] considered instrumentation requirements for identification and mentions improper assembly of the Space Station as one situation requiring structural verification.

Finally, an approach for damage location could be similar to an approach adopted by Sidhu and Ewins [56] to establish areas of inaccuracy in a finite element model. Comparison of two reduced stiffness matrices for a structure, the first from a finite element model and the second from limited structure response measurements, leads to location of poorly modeled areas.

2.4 Summary

Graupe [1] presented six identification situations that call for different treatments. These six form a basis for summarizing the review and presenting a list of candidates for damage detection. The first three of Graupe’s situations were addressed by limitations of the current research which

included standard assumptions about linear response, constant parameters, and deterministic systems. Methods were included in the review considering these limitations. Even with the preceding restrictions, Graupe's remaining identification situations indicate that the problem retains several options. Structures are modeled as discrete or continuous systems. Response is excited by single input or multi-input methods. Finally, and most important in damage detection, many degrees of prior knowledge are available. All of these options are exhibited in the various identification methods included in the review. From here, then, a list of candidate identification methods was developed.

Candidate methods for damage detection include the following:

Hendricks' [13] Least Squares Method

Rajaram's [24 – 26] Least Squares Method

Leuridan's [27,28] Frequency Domain Method

Baruch's and Bar Itzhack's [30] Stiffness Matrix Update Method

Kabe's [33,34] Stiffness Matrix Adjustment Method

White's and Maytum's [35] Perturbation Method

Creamer's [36] Structure Model Identification (SMI) Method.

These methods were evaluated qualitatively with respect to the characteristics desired for damage detection. Then a reduced list was evaluated quantitatively to select an identification method for large space truss damage detection. Results of the evaluations are presented in the next chapter.

3.0 System Identification for Damage Detection

3.1 Introduction

Throughout *Identification of Large Space Structures on Orbit* [11], comparative evaluations of system identification methods are encouraged. Few comparison studies are available in the literature. In those that are available, simulation test results are usually used for comparison. Evaluations which demonstrate performance with experimental data are more scarce. Most of these present modal identification methods.

Consequently, a comparison study of structural model identification methods to use in selecting a method for damage detection was difficult to find. Although Chapter 6 of *Identification of Large Space Structures on Orbit* does provide several simple simulation examples to highlight differences between methods, different structures are used in different examples.

The performance required of a system identification method for damage detection is often different than the performance reported in comparison evaluations or original presentations aimed at other

uses for identification. Therefore, a comparison study of structural model system identification methods was conducted with performance for damage detection as the focus of the evaluation.

3.2 Background for Comparison of Identification Methods

3.2.1 Damage Location Approach

Deleting a member of a truss structure causes a region of reduced stiffness in the structure. The underlying idea of the method of damage detection is to find the region of reduced stiffness, thus locating the “damaged” member.

If each truss member is modeled as an element in a discrete model, it contributes uniquely to the model stiffness matrix. In the same fashion, element stiffness matrices in a finite element formulation are “assembled” into a global stiffness matrix. Each element stiffness matrix adds into the global stiffness matrix in a unique pattern, depending on the degrees of freedom associated with the element and the geometry of the structural model.

The identified stiffness matrix for the damaged structure is compared to an original stiffness matrix for the undamaged structure to produce a pattern of stiffness change. If this pattern matches the change that would occur from deleting a member of the truss, damage is located.

Damage detection is to be performed with the stiffness matrices from models of the undamaged truss and the damaged truss. The first requirement for a method of system identification, then, is to produce a stiffness matrix with measurements of the structure response. Other features desired for the method result from the approach, the simplified damage problem that has been adopted, and from characteristics of large space trusses.

3.2.2 Features Required for Damage Detection

From the approach selected for damage detection, system identification must produce a stiffness matrix for the damaged structure. It is not sufficient to just produce a stiffness matrix, though. The identified stiffness matrix must be consistent with the physical configuration of the structure to enable the location of the damaged member.

The stiffness matrix for a structure is not unique. It depends on the modeling method and decisions for discretization. Within the finite element method, the resulting stiffness matrix is affected by the type of element selected to model a part of the structure, as well as the number of elements chosen. Therefore, the identified stiffness matrix must be consistent with the modeling method, also.

The simplified damage detection problem assumed for this research requires that the method must be specific enough in its results to distinguish between damage to individual truss members. Continuum models do not satisfy this requirement and were excluded from the list of candidates for that reason.

Consideration of typical characteristics of large space trusses leads to the requirements that the method of system identification perform in the presence of low, closely spaced frequencies, little or no damping, and symmetry. Measurement capability is restricted on orbit, so the identification method should use minimal inputs. Related requirements include the desire for limited computational effort or limited communication of data if the computations must be performed on earth.

3.3 Qualitative Comparison of Candidate Methods

Candidate methods from Chapter 2 were evaluated considering the requirements for damage detection. Table 3-1 is a presentation of the methods and their performance with respect to these requirements. Some entries in the table are blank, indicating that the method's performance could not be assessed for that requirement from the available information. Simulation studies would have to be conducted to complete the table.

The first four candidate methods are eliminated from consideration by their shortcomings with respect to one of the requirements above. Rajaram's Least Squares Method uses measurements of displacement, velocity, acceleration and force at each degree of freedom. This is an impossible amount of data for an on-orbit situation. Leuridan's Frequency Domain Method requires less data because symmetry of the stiffness matrix is considered, but is "computationally cumbersome" [27]. Hendricks' Least Squares Method was discovered to have problems with symmetry in some applications [57]. Finally, Baruch's and Bar Itzhack's Stiffness Matrix Update Method produces non-physical couplings in the stiffness matrix which would prevent damage location.

Creamer's Structure Model Identification Method appears to be promising for damage detection. However, the method was under development during the evaluation portion of this research. At the time, there was not enough information to make an evaluation or to consider performing numerical studies. Therefore, this method was dropped from consideration.

Kabe's Stiffness Matrix Adjustment Method and White's and Maytum's Perturbation Method fit the requirements outlined for damage detection. Both methods are initially attractive because they avoid the problem of unrealistic coupling in the identified stiffness matrices. Kabe's method, which restricts the results to only coupling terms that are present in an original model, and White's and Maytum's method, which also produces only realistic couplings, were selected for further consideration.

Table 3-1. Performance of the Candidate Identification Methods with Respect to Damage Detection Requirements

Methods	Requirements						
	1	2	3	4	5	6	7
Hendricks' [13] Least Squares	no		yes		no	yes	yes
Rajaram's [24-26] Least Squares	yes		yes		yes	no	yes
Leuridan's [27,28] Frequency Domain	yes		yes	yes			no
Baruch's and Bar Itzhack's [30] Stiffness Matrix Update	yes	no	yes		yes	yes	yes
Kabe's [33,34] Stiffness Matrix Adjustment	yes	yes	yes	yes	yes	yes	
White's and Maytum's [35] Perturbation	yes	yes	yes	yes	yes	yes	
Cremer's [36] Structure Model Identification	yes		yes		yes		

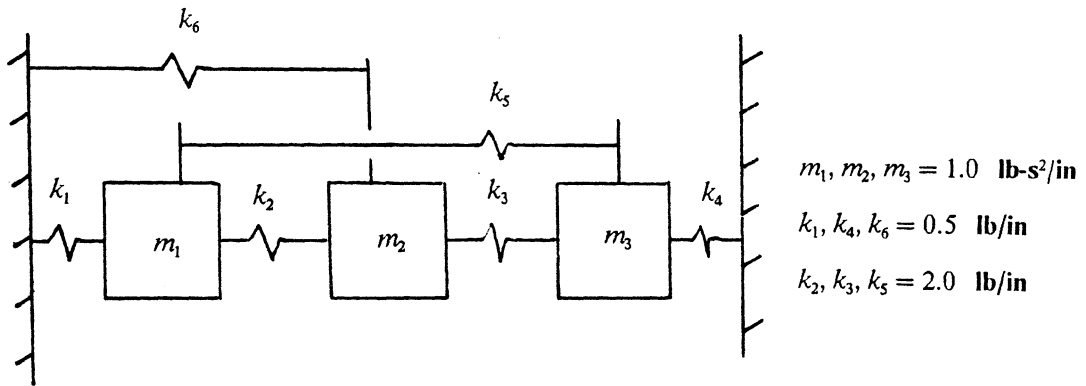
1. to produce stiffness matrix elements
2. to match physical configuration
3. can distinguish individual truss members
4. to perform in the presence of low, closely spaced frequencies
5. to perform in the presence of symmetry
6. minimal inputs
7. limited computational effort

“Further consideration” of these methods took the form of simulations with two models of different complexity. Kabe presented his method with an elaborate spring-mass model as an example, while White and Maytum presented a truss structure that was designed to produce response “typical to that encountered in the Aerospace industry”. Each example illustrated points of interest to the authors. Comparisons of the methods’ qualities and discovery of the method’s shortcomings were difficult without a common structure to analyze. Therefore, the idea of applying the different methods to standard problems was adopted.

3.4 Evaluation Models for Quantitative Comparison

Two models were used to study the relative merits and disadvantages of these system identification methods for the specific application of truss damage detection. Smith and Hendricks [58] presented the evaluation results in detail. Here, a summary of the evaluation tests is included. A linear spring-mass problem was designed as the simplest system. A more complex model, a planar truss with members borrowed from the Space Station concept design [59], was used for the primary evaluations.

The spring-mass model is a linear, undamped system of three masses and six springs as shown in Figure 3-1. The values of the masses and springs were assigned to produce a problem with repeated eigenvalues and a rigid body mode (although this mode has a non-zero frequency due to the three identical springs to ground). These requirements were specified to include some of the characteristics of the more complex truss model. But, the most important requirement was to have this simplest model provide the greatest possible understanding of the response of the system and of the application of the identification methods.



$$K = \begin{bmatrix} k_1 + k_2 + k_5 & -k_2 & -k_5 \\ -k_2 & k_2 + k_3 + k_6 & -k_3 \\ -k_5 & -k_3 & k_3 + k_4 + k_5 \end{bmatrix}$$

Figure 3-1. Spring-Mass Model

The planar truss model was designed as a model of intermediate complexity to use for the evaluation of system identification methods. The 51 elements of the ten bay structure are borrowed from the proposed Space Station design, so each bay is a 5 meter square with diagonal members. The model is shown in Figure 3-2. Member properties are presented in Table 3-2.

Repeated substructure, no damping and rigid body modes are characteristics desired for this model. This model and motion are restricted to one plane to allow better understanding of the problem and the system response.

Modal analyses were performed for the models in both undamaged and damaged conditions to provide understanding of the structure response and to generate response data to be used for identification method evaluations. For the truss structure, the finite element program ANSYS was used to generate the frequencies and normalized eigenvectors which were used in the identification process.

The undamaged modal response for these models was obtained to further the understanding of the test structures and their sensitivity to different damage states. The results of analysis of a damaged state of a system were used as the “measured data” for the identification methods. Two types of damage were considered for the spring-mass model: 1) removal of an intermass spring and 2) removal of a mass-to-ground spring. For the planar truss model, three damage states were created, each by deleting a single member of the fifth bay of the truss. A horizontal, vertical or diagonal member was removed for the three cases. Appendix A contains response data for the evaluation models, for undamaged and damaged states.

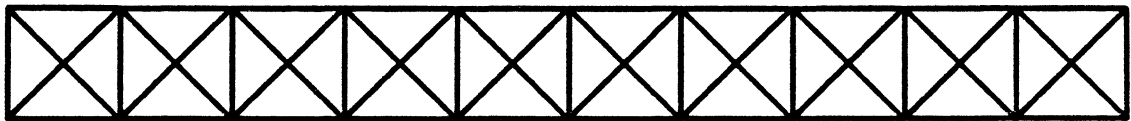


Figure 3-2. Planar Truss Model

Table 3-2. Model Properties for the Planar Truss

L_h	=	196.85 in (5 m)	Horizontal Member Length
L_v	=	196.85 in (5 m)	Vertical Member Length
E_x	=	40×10^6 psi	Axial Elastic Modulus
ρ	=	0.063 lb/in ³	Density
A	=	0.3657 in ²	Member Cross-Section Area

3.5 Evaluation of System Identification Methods

3.5.1 Kabe's Stiffness Matrix Adjustment Method

3.5.1.1 Description of Method

A non-iterative (or direct) method to identify the elements of the stiffness matrix has been developed to use the known physical connectivity of the structure to preclude unreasonable couplings in the result. Alvar M. Kabe of the Aerospace Corporation, El Segundo, California presented this stiffness matrix adjustment procedure in the September 1985 AIAA Journal [33,34].

Kabe's method uses an initial estimate of the stiffness matrix, the known mass matrix, a limited set of measured modal data, and the connectivity of the structure (assumed correct in the initial stiffness matrix) to produce an "adjusted stiffness matrix." Therefore, this method identifies nonphysical parameters, i.e. the elements of the stiffness matrix.

Kabe uses a "scalar matrix multiplication operator", \odot , for which two matrices are multiplied, element by element, to produce a third matrix. This unusual matrix multiplication provides that zero elements in the original stiffness matrix do not become non-zero elements in the final result. Each element of the adjusted stiffness matrix, $[K]$, is the product of the corresponding elements of the original stiffness matrix, $[k]$, and an adjustment matrix, $[\gamma]$ as follows:

$$[K] = [k] \odot [\gamma] \rightarrow K_{ij} = k_{ij} \gamma_{ij}. \quad (3-1)$$

A constrained optimization problem is developed where the percentage change of each stiffness element is minimized. The error function is derived to represent the percentage change of each

stiffness matrix element, while constraints are provided from the modal analysis equations and the problem symmetry. Lagrange multipliers are used to expand the error function to include the constraints. The resulting problem to solve for the Lagrange multipliers becomes a linear, symmetric, indefinite system, $Ax=b$, whose solution is available in various ways. Using the following equation, one easily obtains the adjusted stiffness matrix from the original stiffness matrix and the modal data once the Lagrange multipliers are known:

$$[K] = [k] - 1/4[k^2] \odot ([\lambda][\phi]^T + [\phi][\lambda]^T) \quad (3-2)$$

where

$[K]$ and $[k]$ are as before,

$[k^2]$ is the matrix scalar product $[k] \odot [k]$,

$[\lambda]$ is the matrix of Lagrange multipliers,

$[\phi]$ is the $n \times m$ matrix of m measured mode shapes, and

n is the number of degrees of freedom in the model.

Implementation of Kabe's procedure is straightforward. The size of the problem, original stiffness and mass matrices, number of measured modes and modal data are input for the structure. A sequence of matrix manipulations is used to assemble a symmetric indefinite system of equations which is solved for the Lagrange multipliers. The adjusted stiffness matrix is easily calculated from these values with Equation (3-2).

A difficulty with the implementation comes from the symmetric indefinite system which has a dimension equal to the product of the number of degrees of freedom and the number of modes used for the identification. This system of equations becomes very large. Fuh and Berman [60] consider this as a major drawback of Kabe's method. For a three mass system when three modes are used for the identification, this set of equations has a 9×9 symmetric indefinite coefficient matrix. For a small truss with 132 degrees of freedom and five modes used in the identification, this set has a coefficient matrix that is 660×660 .

Kabe presented a spring-mass system with eight degrees of freedom as an example of the method. Only three modes of the system are required to successfully identify all sixteen stiffness elements that are non-zero. Closely spaced frequencies were included in the example and handled without problem.

The stiffness matrix adjustment method developed by Kabe provides an identification method for stiffness matrix elements that does not have the problem of unrealistic couplings in the result. This is achieved by using the additional information of physical connectivity of the structure to limit the resulting stiffness matrix to changes of only nonzero values of the original stiffness matrix.

3.5.1.2 Results of Kabe's Identification Method

Kabe's identification method was first applied to the spring-mass model and achieved excellent results. Two cases of damage were considered. In the first, one of the mass-to-ground springs (spring number 4) was deleted to produce the damaged state. For the second, an intermass spring (spring number 2) was removed. Detailed results for both cases are presented in Reference 58.

Case 1 damage changed only one diagonal element of the undamaged stiffness matrix. This is seen in the stiffness matrix inset in Figure 3-1 where spring number 4 affects only the (3,3) element. The undamaged stiffness matrix was used as the initial guess (or original) stiffness matrix in the identification process. The results of Kabe's identification procedure, using first one mode, then two modes and finally three modes, show success in identifying all elements of the stiffness matrix using two modes.

Depending on the number of non-zero elements of the stiffness matrix to be identified and the size of the model, a limit is established for the number of modes required for complete identification with this method. Beyond this limit, any additional mode used in the identification process does not significantly improve the accuracy of the identification results. The spring-mass model

identification results illustrate this characteristic of Kabe's method. When three modes are used for the identification, no significant improvement over the results using two modes is noted. One mode is insufficient for this case; the process produces the best results possible, but not the exact values for the elements.

The second case of damage for the spring-mass model illustrates an interesting result with Kabe's method. Three of the damaged (exact) symmetric stiffness matrix elements differ from the undamaged (original) coefficients when spring number 2 is deleted. However, the first two frequencies and corresponding normal modes of the response are unaffected by this damage. This is understood by recognizing that masses 1 and 2, the two masses which are usually connected by the damaged spring, move in unison in the first two modes of the undamaged structure. The spring is not deformed during these motions, so its removal does not affect the system response for these two modes. As a consequence, all three modes were required for the complete identification of this second damage case. Appendix A presents the damaged and undamaged response data for the spring-mass evaluation tests.

The planar truss model was considered next for the evaluation of Kabe's stiffness matrix adjustment method. The results show potential for damage detection with this method.

Table 3-3 presents the original, adjusted, and exact values of the non-zero, upper triangular elements of the fifth bay portion of the stiffness matrix for the first damage case. In this case, the bottom horizontal member of the fifth bay was deleted, affecting three of the symmetric coefficients of the global stiffness matrix. Since the entire matrix is 44 x 44, only this section, where the damage affects the stiffness matrix elements, is included. Original (or undamaged) element values are presented in the left-hand column. The column labeled "exact elements" contains the damaged truss values. Results of Kabe's method are presented in the center column of adjusted coefficients. Finally, the percent difference between the adjusted and original values is presented in the right-hand column. Arrows have been added to direct attention to the three elements changed by the structure damage.

Table 3-3. Nonzero, Upper Triangular Stiffness Matrix Elements (Planar Truss - Damage Case 1)

Element Location	Original Element (lb/in)	Adjusted Elements (lb/in)	Exact Element (lb/in)	Percent Adjusted	
		3 modes*			
17,17	0.20116D+06	0.12878D+06	0.12685D+06	-35.98	←
17,21	-0.74310D+05	-0.34321D+04	0.00000D+00	-95.38	←
17,23	-0.26272D+05	-0.29181D+05	-0.26272D+05	11.07	
17,24	-0.26272D+05	-0.26175D+05	-0.26272D+05	-0.37	
18,18	0.12685D+06	0.12627D+06	0.12685D+06	-0.46	
18,20	-0.74310D+05	-0.74007D+05	-0.74310D+05	-0.41	
18,23	-0.26272D+05	-0.25100D+05	-0.26272D+05	-4.46	
18,24	-0.26272D+05	-0.26130D+05	-0.26272D+05	-0.54	
19,19	0.20116D+06	0.20721D+06	0.20116D+06	3.01	
19,21	-0.26272D+05	-0.26487D+05	-0.26272D+05	0.82	
19,22	0.26272D+05	0.26869D+05	0.26272D+05	2.27	
19,23	-0.74310D+05	-0.79733D+05	-0.74310D+05	7.30	
20,20	0.12685D+06	0.12608D+06	0.12685D+06	-0.61	
20,21	0.26272D+05	0.25353D+05	0.26272D+05	-3.50	
20,22	-0.26272D+05	-0.25882D+05	-0.26272D+05	-1.48	
21,21	0.20116D+06	0.12742D+06	0.12685D+06	-36.66	←
22,22	0.12685D+06	0.12665D+06	0.12685D+06	-0.16	
22,24	-0.74310D+05	-0.74572D+05	-0.74310D+05	0.35	
23,23	0.20116D+06	0.20135D+06	0.20116D+06	0.10	
24,24	0.12685D+06	0.12701D+06	0.12685D+06	0.12	

* first three elastic modes

Preliminary studies of the identification method for the planar truss showed that using one or all of the rigid body modes of the structure did not produce any adjustment of the original stiffness values. Therefore, the first three elastic modes of the structure were used for the identification process. The first two elastic modes of the damaged structure are bending modes; the third is an axial mode. Appendix A presents planar truss response data for the damaged and undamaged states used in the evaluation tests.

All elements of the stiffness matrix are adjusted by the identification process, not just the three elements that should change to give the exact result. Still, results show reduced stiffness for these three elements. Kabe's method was designed to minimize the percent change to all the non-zero elements of the stiffness matrix, so this result where all the stiffness elements are slightly adjusted should not be unexpected. Examination of the percent difference between the original values of the elements and the identified values reveals that the largest adjustments occur in the vicinity of the damage - for the three elements of interest in this case.

3.5.2 White's and Maytum's Matrix Perturbation Method

3.5.2.1 Description of Method

Linear perturbations of submatrices and an energy distribution analysis are the basis for this identification method to determine the elements of the global stiffness and mass matrices. Charles W. White and Bruce D. Maytum of the Martin Marietta Corporation in Denver, Colorado presented the method in the August 1976 *Shock and Vibration Bulletin* (Bulletin 46) [35]. The results of the perturbation and energy analyses are presented as a correlation fit (or identification) method, but mainly as a method to evaluate the effect of slight changes in the nominal model.

A nominal or original model of the structure and measured structure frequencies are all that is required for this identification method. The procedure has been simplified considerably with the assumption that the eigenvectors of the actual structure are essentially similar to the eigenvectors of the nominal model. Although White and Maytum have provided for identification of mass matrix elements, stiffness element identification is the main interest for the application of damage detection. Therefore, the mass element identification is not presented.

A reduction in problem size occurs with the grouping of model elements. Rather than identifying a perturbation parameter for each element, groups of similar elements are defined. The global stiffness matrix is assembled from the model element stiffness matrices expressed in terms of the global coordinate system. Therefore, the global stiffness matrix is separated into P group stiffness matrices, where groups of model elements are created to reduce the problem size.

Perturbations of the original model are expressed as linear variations of the group matrices,

$$[K] = [K_o] + \sum_{p=1}^P \delta_p [k]_p, \quad (3-3)$$

where δ_p is the unknown perturbation parameter. White and Maytum develop the more general problem with perturbations of the mass and stiffness matrices and measured eigenvectors. However, a significant simplification occurs when the eigenvectors of the perturbed problem are essentially the same as the eigenvectors of the original problem. This is a common occurrence for the evaluation models. If the mass matrix of the original model is assumed to be correct, their result is a system of equations to solve for the perturbation parameters,

$$\{\Omega_i^2 - \omega_o^2\} = [E_o]\{\delta\}, \quad (3-4)$$

where

$\{\Omega_i^2\}$ is a $p \times 1$ vector of the measured test eigenvalues,

$\{\omega_o^2\}$ is a $p \times 1$ vector of the corresponding original eigenvalues,
 $\{\delta\}$ is a $p \times 1$ vector of the unknown perturbation parameters, and
 $[E_o]$ is the energy distribution matrix.

Each element of the energy distribution matrix is calculated using an eigenvector and a group stiffness matrix of the original model.

The energy distribution analysis is performed on the original model of the structure and used to assess the structure sensitivity to changes. The “modal potential energy matrix”, $[E_{total}]$, is from the potential energy for the structure transformed into modal coordinates. Recognizing the stiffness matrix orthogonality condition, this energy matrix is diagonal. If the global stiffness matrix is separated into group stiffness matrices, as mentioned above, the total energy matrix is separated into group energy matrices as follows:

$$[E_{total}] = [\phi_o]^T \sum_{p=1}^P [k]_p [\phi_o] = [\phi_o]^T [k]_1 [\phi_o] + \dots + [\phi_o]^T [k]_p [\phi_o], \quad (3-5)$$

where

$[\phi_o]$ = eigenvector matrix and

$[k]_i$ = group global stiffness matrices.

In this equation, the index p is not for each element of the model of the structure, but for each group of similar elements. Note that the columns of the energy distribution matrix in Equation (3-4) are the appropriate diagonal elements of the group energy matrices.

Diagonal elements and off-diagonal elements of these group energy matrices are assigned meanings that are useful in assessing the sensitivity of the structure to changes in the model element groups. Diagonal elements of these energy matrices sum to give the total energy for each mode, i.e. the eigenvalue. So, divided by the respective eigenvalue, diagonal elements represent the fractional part

of the mode energy that is associated with the particular model group represented by the stiffness matrix, $[k]_i$, $i = 1, p$. Off-diagonal elements represent the coupling between modes for the group. Therefore, if a group energy matrix shows that the group has a large percentage of the energy of a mode, changes to that group of model elements affect the mode more than changes to other groups. In this way, structure sensitivity is evaluated.

Results of the energy distribution analysis are also used to assess groupings, since groups of similar elements produce group energy matrices with larger diagonal values and less coupling.

The number of modes used for the identification determines the character of the problem to solve for the perturbation parameters. If m , the number of measured eigenvalues, is greater than p , the number of perturbation parameters, a linear least squares solution is used to solve the overdetermined system. For $m = p$, the system of equations is linear and is solved with any one of the standard algorithms. Additional constraint equations must be added to solve for the perturbation parameters if m is less than p .

Implementation of the method requires considerable involvement of the analyst. First the elements must be grouped to reduce the problem size. Then, the energy distributions are examined to evaluate the groups and assess the structure sensitivity. The results of the identification depend on the set of modes selected, so there is a requirement for intuition or prior experience. The coupling results, along with the energy distribution, provide assistance.

An example structure that consists of two rigid masses and 30 truss members is presented to illustrate the correlation fit method in the article. The structure is designed to create uncoupled modes and modes with significant coupling. However, the measure of success of the identification method is reproduction of the structure response rather than the model matrix elements.

White and Maytum have developed an identification method that preserves the physical coupling of the structure through the use of submatrices of the original model. These are varied by linear

scale factors that serve as the unknowns in the derived system of equations. Once the solution is complete, elements of the global stiffness matrix are calculated with the linear perturbation definition, Equation (3-3).

3.5.2.2 Results of White's and Maytum's Identification Method

Grouping the model elements to reduce the problem size is a key ingredient of White's and Maytum's method. This grouping causes difficulties for the correct determination of the stiffness matrix, even though the method correctly reproduces the modes used for the identification. A perturbation parameter multiplies all the elements in the group stiffness matrix, thus indicating changes to all the model elements of the group. Isolation to a single element is impossible with the group definitions and a single analysis. To overcome this limitation, a second application of the procedure with different groups was tried to isolate the damaged element. This approach was not successful for the spring-mass problem tests, or for the trials with the planar truss model. Again, results of tests for both models are presented in detail in Reference 58.

Ideally, all perturbation parameters would be identified as zero except the one for the group with the damaged member. This value would be negative to reflect the reduced stiffness and would be bounded by zero and -1. If a group contained only undamaged members, the parameter would equal zero; if a group contained only the deleted member, the parameter would equal -1.

The identification approach is less than ideal with a linear perturbation assumption and groups of model elements. Still, it was hoped that the parameters would be identified in a recognizable pattern. Perhaps, a larger magnitude would result for the parameter of the group with the damaged member and smaller magnitudes for the rest. Also, the group containing the damaged element would have a negative parameter as the result of the procedure.

In the simulations, this hope was not realized. The identified perturbation parameters for the groups are both positive and negative. The groups containing the deleted members often have a negative parameter result, but not in a consistent pattern to use for damage detection. Also, the results are dependent on the modes used for the identification. The spring-mass and planar truss examples demonstrated these results.

The spring-mass model was used for the first evaluation of this identification method. An energy distribution analysis of the undamaged or original system was the first step. Elements were grouped by type into two groups - springs to ground and intermass springs. The group energy matrices were calculated and both were found to be diagonal. Consequently, no coupling exists between any modes for either group.

All of the energy of the first mode (a 'rigid-body' mode with a non-zero frequency) is contained in group A, the springs to ground. This is as expected, since the intermass springs do not deform for this mode. The majority of the energy for the second (and third) mode is contained in group B, although some deformation of the group A springs occurs. From these results and the lack of coupling, a change to all group A members would largely affect the first mode and slightly affect the other two. A change to group B members would not affect the first mode. Note that the change to the group would be to all elements, as in the perturbation equation.

Spring number 4 was deleted to produce damage case 1. Therefore, group A is the affected group in this trial. Results from this first test support the idea of a useful pattern for damage detection. The group A perturbation parameter has a negative result in the expected range of zero to -1. The parameter for group B, with no damage, is smaller in magnitude if two modes are used for the identification and considerably smaller if all three modes are used. Both positive and negative values of this parameter occur, though.

These results were used in Equation (3-3) to produce a stiffness matrix which was then used in the eigenvalue problem to validate the identification results. Frequencies used for the identification are

reproduced exactly with the new stiffness matrix. However, the third frequency is not correctly reproduced unless all three modes are used for the identification.

The second damage case for the spring-mass model (spring 2 deleted) lends support to the idea of damage detection as well. Perturbation parameter results with three modes produced a zero value for the parameter for group A, where no damage occurred. The group B parameter is negative and in the expected range. Note, modes 1 and 2 are the same as those of the undamaged structure, so the first identification produced zeros for both parameters. The original stiffness matrix produces the correct frequencies for the first two modes without any changes.

A second application of White's and Maytum's method was performed with a different grouping of the springs to isolate the damage to a single spring, rather than a group of similar springs. For this test, three groups of two springs each were created. The groups were designed so that no pair of springs are grouped as in the first application of the identification procedure. The resulting energy distribution matrix, $[E_o]$, is nearly singular causing an ill-conditioned problem. Since the energy distribution matrix is derived from the undamaged structure, neither of the damage cases for the spring-mass model produce satisfactory results. Consequently, the approach to isolate damage with a second application of the method failed for the spring-mass model.

The planar truss model tests were performed with the same multi-analysis approach to try to isolate damage. First, the elements were grouped into ten bays. The tenth bay included the final vertical member. In the second application of the identification analysis, the truss members were grouped into three element types: vertical, horizontal, and diagonal.

Energy distribution analysis for the undamaged structure with the elements grouped by bays showed that the energy percentages for the modes are spread across the groups, resembling deformed mode shapes. The first elastic mode for the structure is the first bending mode. Maximum displacement occurs for the center two bays and the maximum energies are for these bays as well. All the energies are symmetric about the center of the truss (as they should be). This

feature of the energy distribution suggests combining two bays with like energies. Therefore, Group A contains bays 1 and 10, Group B contains bays 2 and 9, and so on. Five groups are the result.

In addition to the energy distribution information from the group energy matrices, the off-diagonal terms provide coupling information. Examination of the group energy matrices revealed coupling between the bending modes for all groups. The axial modes are coupled for all groups as well, but uncoupled from the bending modes in all cases. Modal coupling or lack of coupling reduces the problem further and provides insight for the selection of modes to use for the identification.

The first test is for damage case 1, a deleted horizontal member of the fifth bay. Five, then six, then seven elastic mode frequencies were used to solve for the five perturbation parameters corresponding to the five groups. The results are presented in Table 3-4.

Parameter results from this application of the method illustrate that a pattern for damage detection is not evident. Group E contains the damaged member in the fifth bay, but the Group E parameters are not negative in all cases. For trials where this parameter is negative, the Group C parameter is also negative and sometimes larger in magnitude.

The second application of the method provided further evidence for the conclusion that a pattern for damage detection is unavailable. The second grouping for the members of the planar truss was by their orientation in the structure. Energy distribution analysis revealed that vertical members contain none of the energy for the bending modes (modes 4, 5, 7, 8, 10) and little for the axial modes (modes 6, 9). Also, no coupling exists between the modes for the Group B, the vertical members. Based on these results, the group could be eliminated from the identification problem. Of course, damage in this group would be difficult to detect.

Damage case 1, with a deleted horizontal member, was selected as the simulation for this application of the method to provide a comparison with the previous results. Again, no pattern is

Table 3-4. Perturbation Parameters (Planar Truss Damage Case 1)

Group	Group Description	Perturbation Parameters		
		5 modes	6 modes	7 modes
A	bays 1 and 10	-0.47138	0.00986	0.02164
B	bays 2 and 9	-1.4183	0.01131	0.04847
C	bays 3 and 8	-2.4692	-0.01108	-0.07521
D	bays 4 and 7	-2.2574	0.00782	0.06915
E	bays 5 and 6	0.87174	-0.02054	-0.04603

←

apparent for damage detection. Perturbation parameters found with five, six and seven modes in the identification are presented in Table 3-5.

The parameter for the damaged group, Group A, is negative for all cases. However, the parameters for the other groups are both positive and negative with no useful pattern.

3.6 Summary

Damage detection for large space truss structures prompted an evaluation of methods of system identification. Two methods were evaluated quantitatively, after several were eliminated by a qualitative comparison. Both methods identify the elements of the global stiffness matrix. Both methods use an original stiffness matrix and limited modal data from the damaged structure. Both methods preserve the physical coupling in the structure: Kabe's method by design and White's and Maytum's method as a result of assuming linear perturbations of original structure submatrices.

Kabe's method shows promise for use in damage detection. Remarkable results occurred in identifying the pattern of change to the stiffness matrix elements for a damage case where a member of the planar truss is deleted. Only three elements were changing in the upper triangle of a 44 x 44 matrix formulated with the finite element method. Kabe's method produced the largest percent adjustment in exactly the three elements.

White's and Maytum's method, on the other hand, is not suitable for damage detection in large space truss structures. The initial assumption of grouping like truss members is a key to reducing the size of the identification problem. However, this prevents the isolation of damage to a single member because the identification process did not produce recognizable patterns that could be used

Table 3-5. Perturbation Parameters (Planar Truss Damage Case 1)

Group	Group Description	Perturbation Parameters		
		5 modes	6 modes	7 modes
A	horizontal	-0.00297	-0.00443	-0.00118
B	vertical	-0.05540	0.04289	0.01512
C	diagonal	0.00049	-0.00215	-0.00212

←

in repeated analyses to locate the damaged member. The method does accurately reproduce the modal data used for the identification, though.

Based on the results of these evaluations, Kabe's method of stiffness matrix adjustment was selected as the single method of identification to use in developing a procedure for damage location. Further studies of the method, including some with a 10-bay simulation model of the Space Station truss, are presented in the next chapter. The damage location algorithm is presented. Also, some characteristics of Kabe's method that inhibit damage detection are discussed.

4.0 Damage Location

4.1 Introduction

Once the identification method was selected, the complete damage location algorithm was designed. Kabe's method requires modal data as the measured input from the structure. Therefore, a second identification process must precede the key identification. At first, a two-step identification algorithm seems inefficient when many methods exist to produce stiffness matrix models directly from measurements on the structure. However, modal data for the structure is required for other purposes, including structure control. *Identification of Large Space Structures on Orbit* [11] reports that "significantly greater than ten" modes are needed for control design studies and model verification. Kabe's method requires very few modes, in comparison, because much information is used from the original model. Therefore, involving a second identification process to produce modal data required for damage location is not inappropriate.

A flow chart which illustrates the total damage location method is presented in Figure 4-1. The first step is to establish an original model of the structure in the undamaged condition. This model consists of stiffness and mass matrices, such as those defined in a finite element model. Initial

adjustment of the model is assumed, so at the onset, it is a good representation of the structure. Modal data from the damaged truss is the next input requirement. Although few modes are needed, the mode shapes must be orthogonalized with respect to the original model mass matrix. These initial requirements are represented by boxes 1 to 3 in the flow diagram. Kabe's system identification method (box 4) produces a model of the damaged truss. The damaged structure model is compared to the undamaged structure model to determine areas of reduced stiffness. Finally, the pattern of reduced stiffness specifies the damaged truss member. These last two processes are shown in boxes 5 and 6, respectively.

In this chapter, a detailed presentation of the damage location method is followed by results of simulated tests with exact data. Identification performance is discussed, as well.

4.2 Damage Location Method

4.2.1 System Identification

4.2.1.1 Modal Data

Modal data required for stiffness matrix identification is available from several methods. The Eigensystem Realization Algorithm (ERA) of Juang and Pappa [40 – 43] is one. Voss [45] presents another and uses a Space Station model as a simulation test.

Obtaining the required modal data for the damage location process from measurements on the space structure is limited by instrumentation. The best modal data will be for the first modes. Juang

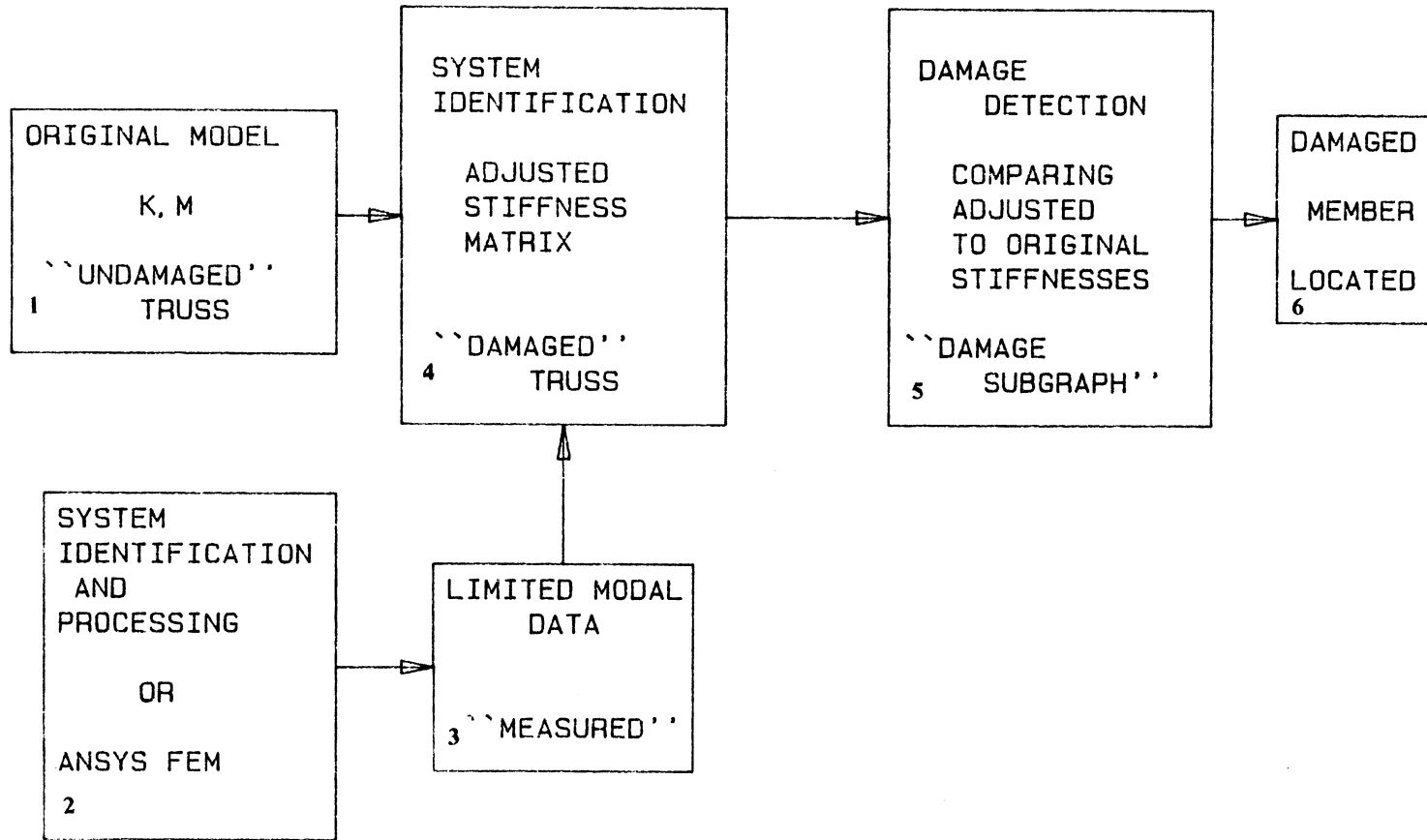


Figure 4-1. Illustration of the Damage Location Process

and Pappa [42] determined that the lowest modes of a structure are less influenced by errors in data for their Eigensystem Realization Algorithm.

Therefore, modal data is available for the damaged structure. Most likely, the best data is for the first modes, so these are used in the second identification process.

4.2.1.2 Stiffness Matrix Identification

Kabe's [33,34] method of stiffness matrix adjustment is the key to producing the damaged model of a large space truss. The method directly identifies the stiffness matrix elements by solving a constrained optimization problem. In addition, the zero-nonzero structure of the stiffness matrix, which represents the physical connectivity of the truss, is preserved in the adjusted stiffness matrix.

A constrained minimization problem is developed to determine the symmetric adjusted stiffness matrix that is "closest" to the original matrix while satisfying the dynamic response constraints and while preserving the zero-nonzero structure. The measure of the closeness between the original and adjusted stiffness matrices is the sum of the percentage change of each matrix element, squared to guarantee a positive function. Symbolically,

$$\sum_{i=1}^n \sum_{j=\text{adj}(i)}^n \left(\frac{K_{ij} - k_{ij}}{k_{ij}} \right)^2 \quad (4-1)$$

is minimized, subject to the constraints $K\phi = M\phi\omega^2$, $K = K^T$, $\text{sparsity}(K) = \text{sparsity}(k)$. $[K]$ is the adjusted stiffness matrix; $[k]$, the original stiffness matrix; $[M]$, the mass matrix; $[\phi]$, the matrix of measured modal vectors; and $[\omega^2]$, the diagonal matrix of measured eigenvalues. "j = adj(i)" represents all j indices where $k_{ij} \neq 0$. Lagrange multipliers are used to expand this error

function to include the dynamic equilibrium and symmetry constraints. After minimization, the adjusted stiffness matrix is easily obtained from Equation 3-2.

A linear, symmetric, indefinite system, $Ax = b$, must be solved for the Lagrange multipliers. Details for constructing this auxiliary problem are contained in References 33 and 34. The auxiliary problem order, as reported in the previous chapter, is equal to the product of the number of degrees of freedom and the number of modes used for the identification. Having to solve a large indefinite system of equations is a drawback of Kabe's method. This issue, among others, is addressed in Appendix B, which presents performance details of Kabe's method.

4.2.2 Damage Location

In essence, Kabe's method optimally adjusts each nonzero stiffness matrix value. To compare the adjusted stiffness values (the damaged model) to the original stiffness values (the undamaged model), the percentage change for each coefficient is calculated,

$$\text{percent adjusted} = \frac{K_{ij} - k_{ij}}{k_{ij}} \times 100. \quad (4-2)$$

A list of stiffness matrix elements that receive the most adjustment in the identification process is produced by searching the percent-adjusted values with a threshold value. Assuming the undamaged model stiffness matrix is a good representation of the undamaged structure, areas of significant adjustment reflect damage, i.e. the deleted truss member.

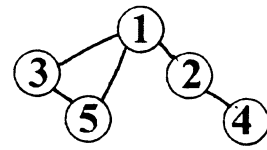
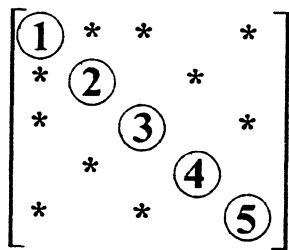
An improved approach follows from examining characteristics of the original stiffness matrix. Two thresholds improve the method's performance. Diagonal and off-diagonal elements of the stiffness matrix are affected differently by damage to the truss. Stiffness reductions occur for both, but within different adjustment percentage ranges. Therefore, one threshold is used to search the diagonal

element adjustments, while a second threshold is used to search the off-diagonal element adjustments. Two lists are generated, from which the damage patterns of reduced stiffness are recognized, enabling the location of the damage.

The damage location method described above has a significance when viewed with ideas from graph theory for matrices [61]. A graph for a matrix represents the zero-nonzero structure of the matrix. Figure 4-2 is an example of a 5x5 symmetric matrix and its graph. The graph has a node for each diagonal matrix element and an edge for each off-diagonal element pair. Two integer vectors are required to store the information for a graph. The first, *adj*, contains the coupled (or adjacent) nodes for each node of the graph. Nodes 2, 3 and 5 are adjacent to node 1. This is the identical adjacency set referenced in Equation 4-1. The second list, *xadj*, contains pointers for the first list. Adjacent nodes to node 2 start in position 4 of the first list. Therefore, *xadj* has a 4 in its second position. Figure 4-2 also shows the data required to store the example graph.

The graph of the original stiffness matrix of Kabe's method represents the connectivity of the undamaged structure. A graph of the adjusted stiffness matrix is identical to that for the original stiffness matrix because it was constrained to have the same zero-nonzero structure. And so, a graph of the matrix of adjustment percentages is the same as well.

If two real-valued vectors parallel to the graph integer vectors store the respective node and edge adjustment percentages, two lists result - just as described in the detection method above. One list contains the adjustments to the diagonal elements; the second contains the adjustments to the nonzero off-diagonal elements. To locate damage, these two vectors are filtered with two threshold values. Areas of the most adjustment form a subgraph of the original graph of the stiffness matrix. Thus, a "damage subgraph" is created. Since each truss member contributes uniquely to the structure stiffness matrix and its graph, the damage subgraph indicates the location of the deleted truss member.



```

adj [ 235 14 15 2 13 ]
xadj [ 1 4 6 8 9 ]

```

Figure 4-2. A Matrix, Its Graph and the Graph Data Structure

4.3 Simulated Tests

The damage location method was tested with two simulated truss structures. The first simulation model is the planar truss (44 degrees of freedom) presented in Chapter 3. The second is a ten-bay orthogonal tetrahedral truss (132 degrees of freedom) whose members and design are borrowed from the Space Station concept design [59]. Simulation models are shown in Figures 4-3 and 4-4, respectively. Table 4-1 is a listing of the model properties. The figure for the planar truss has been repeated in this chapter for reference.

The finite element program ANSYS was used to generate the original model of the truss structures for these simulations. Response data was calculated with the EISPAK subroutine, RSG, using the original mass matrix and a damaged stiffness matrix. A single truss member was deleted entirely to represent damage. The damaged stiffness matrix was obtained by altering the original stiffness matrix to reflect this damage. "Exact measured data" was generated with the solution of the eigenvalue problem with the mass and damaged stiffness matrices. Modal analyses were also performed for both models in an undamaged condition to provide understanding of the structure response. In keeping with the idea of limited modal data, the first elastic modes were used in order for each case of damage.

Typical results are presented in Table 4-2. The results may seem, at first glance, to be identical to those presented in Table 3-3. The damage case is the same, the lower horizontal member of the fifth bay. However, data for evaluations in Chapter 3 was not exact. Slight errors in the original model, approximating an actual damage detection situation, account for the slightly different results.

Adjusted percentages were separated into two lists, one for the diagonal elements (or nodes) and one for the off-diagonal elements (or edges). This reorganization reemphasizes the relative magnitude of the damage percentages to the other adjustments, as well as illustrates the two real-valued lists that are stored in parallel to the graph structure.

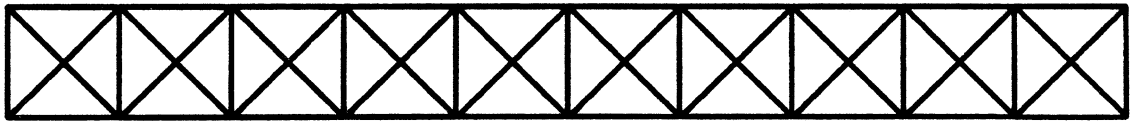


Figure 4-3. Planar Truss Model

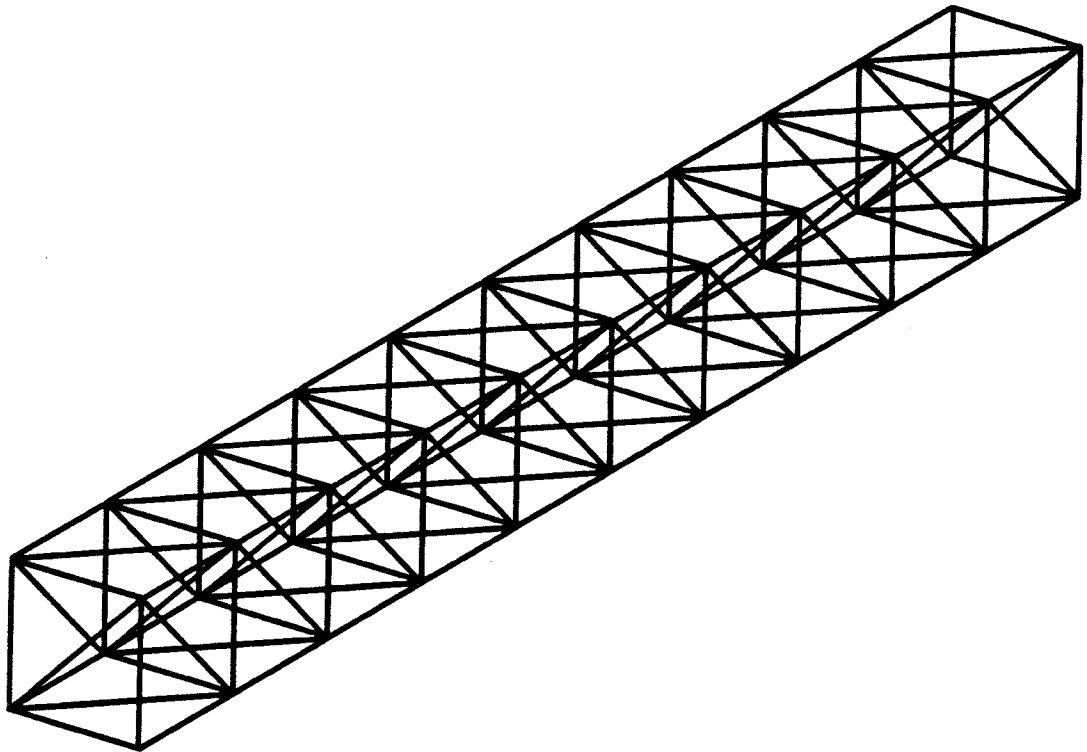


Figure 4-4. Space Station Truss Model

Table 4-1. Model Properties for the Planar and Space Station Truss Models

L_h	= 196.85 in (5 m)	Horizontal Member Length
L_v	= 196.85 in (5 m)	Vertical Member Length
L_t	= 196.85 in (5 m)	Transverse Member Length
	(Space Station Truss only)	
E_x	= 40×10^6 psi	Axial Elastic Modulus
ρ	= 0.063 lb/in ³	Density
A	= 0.3657 in ²	Member Cross-Section Area

Table 4-2. Typical Results of Kabe's Identification Method (Planar Truss - B5cas1)

Element Location	Original Element (lb/in)	Adjusted Elements (lb/in)	Exact Element (lb/in)	Percent Adjusted	
		3 modes*			
17,17	0.20116D+06	0.12859D+06	0.12685D+06	-36.08	←
17,21	-0.74310D+05	-0.36318D+04	0.00000D+00	-95.11	←
17,23	-0.26272D+05	-0.28827D+05	-0.26272D+05	9.73	
17,24	-0.26272D+05	-0.26023D+05	-0.26272D+05	-0.95	
18,18	0.12685D+06	0.12635D+06	0.12685D+06	-0.40	
18,20	-0.74310D+05	-0.74405D+05	-0.74310D+05	0.13	
18,23	-0.26272D+05	-0.25636D+05	-0.26272D+05	-2.42	
18,24	-0.26272D+05	-0.25878D+05	-0.26272D+05	-1.50	
19,19	0.20116D+06	0.20700D+06	0.20116D+06	2.90	
19,21	-0.26272D+05	-0.26706D+05	-0.26272D+05	1.65	
19,22	0.26272D+05	0.27020D+05	0.26272D+05	2.85	
19,23	-0.74310D+05	-0.79448D+05	-0.74310D+05	6.91	
20,20	0.12685D+06	0.12623D+06	0.12685D+06	-0.49	
20,21	0.26272D+05	0.25354D+05	0.26272D+05	-3.49	
20,22	-0.26272D+05	-0.25759D+05	-0.26272D+05	-1.95	
21,21	0.20116D+06	0.12707D+06	0.12685D+06	-36.83	←
22,22	0.12685D+06	0.12644D+06	0.12685D+06	-0.33	
22,24	-0.74310D+05	-0.74787D+05	-0.74310D+05	0.64	
23,23	0.20116D+06	0.20219D+06	0.20116D+06	0.51	
24,24	0.12685D+06	0.12671D+06	0.12685D+06	-0.11	

* first three elastic modes

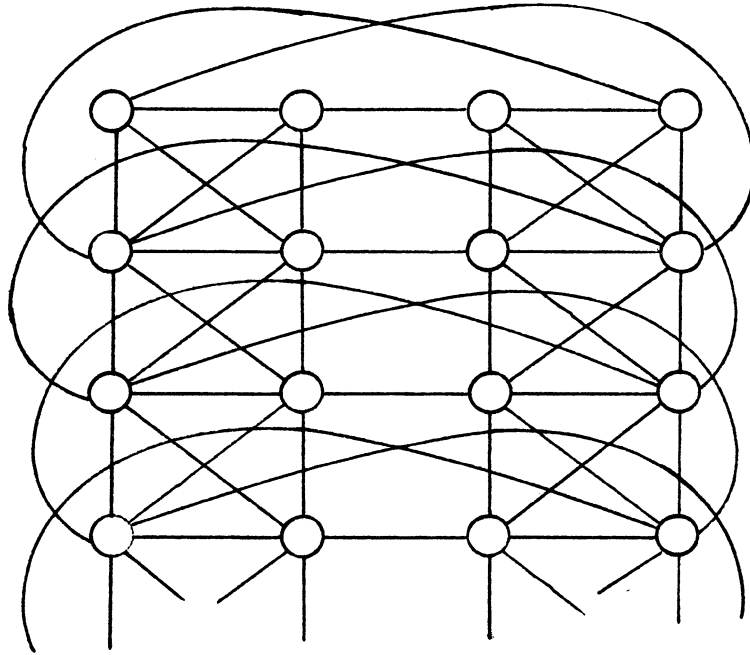
Each list is searched with an appropriate threshold to construct the damage subgraph and locate the damage. A portion of the planar truss stiffness matrix graph is shown in Figure 4-5, along with typical damage subgraphs. A horizontal or vertical member of the truss, because of its orientation in the global reference frame, has a damage subgraph with two nodes connected by one edge. Diagonal truss members have subgraphs consisting of four nodes and six edges, as shown in Figure 4-5 c).

Truss designs determine ranges for the thresholds. Within these ranges values must be selected based on limits of the method's performance.

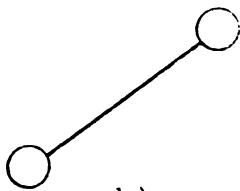
4.3.1 Planar Truss Studies

To illustrate limits of the method in application to the planar truss, tests were repeated with three different sets of threshold values. With low thresholds, the damage subgraphs may include extra nodes or edges; with high thresholds, the subgraphs may be incomplete. In other words, the filter passes through errors if the value is set too low. If set too high, the filter blocks the smallest adjustments. Small adjustments are a reflection of the structure insensitivity to certain damaged members, especially when using only the lowest modes in the identification.

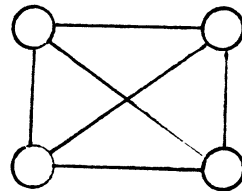
For either truss design in these simulated tests, the range for an appropriate threshold for the list of off-diagonal element percentages is zero to -100 percent. Each edge of the graph is associated with only one truss member which provides all coupling between any two degrees of freedom. Therefore, if the member is deleted, the off-diagonal stiffness element changes by -100 percent. An exception to this occurs for the planar truss diagonal members. However, damage detection for these diagonal members is prevented by a separate condition in the algorithm which is discussed later. Therefore, the planar truss diagonal members are excluded from consideration for edge threshold determination. Inexact results from the identification method occur at both ends of the



a)



b)



c)

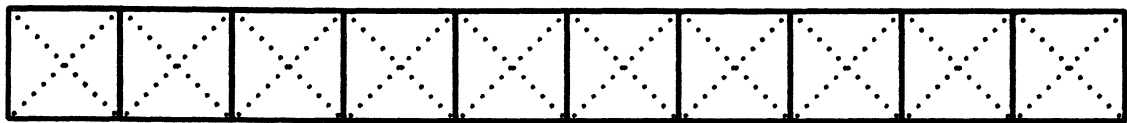
Figure 4-5. The Graph and Typical Subgraphs of the Planar Truss Stiffness Matrix
 a) A Portion of the Planar Truss Graph
 b) Damage Subgraph for a Horizontal or Vertical Member
 c) Damage Subgraph for a Diagonal Member

threshold range. Consequently, an intermediate threshold was used for the first tests. Minus fifty percent was used as the threshold for the off-diagonal adjustment percentages.

Similarly, the diagonal element (node) threshold acts as a filter for errors from the identification process. The minimum change that occurs in a diagonal element due to deleting a member establishes the maximum threshold value. For the planar truss, the threshold range is zero to -36.9 percent. The first test threshold was -20 percent.

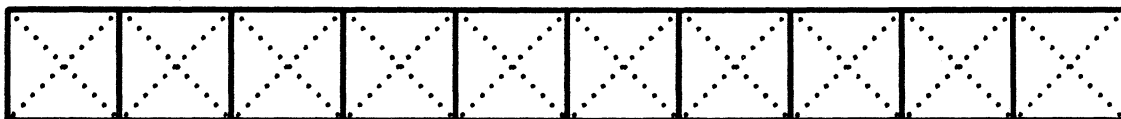
Using thresholds of -20 and -50 percent, respectively, for the node and edge adjustment percentages, damaged subgraphs of the planar truss were constructed for each available damage case. Twenty-three truss members were deleted in separate tests. Recognizing symmetry, the entire 51 member truss has been analyzed. Figure 4-6 a) presents the ability to locate damage in the planar truss. A member is drawn with a solid line only if damage to the member is "exactly located" with the process. "Exactly located" means that the damaged subgraph is exactly correct. All of the changed elements of the exact damaged stiffness matrix, as compared to the original matrix, and only those changed elements, are selected by the threshold searches. Damage detection and location performance that is less than exact is drawn with an interrupted line as indicated in the figure.

Damage detection and location for horizontal or vertical members of the planar truss is excellent. Using only the first three elastic structure modes, which include the first two bending modes and the first axial mode, even damage to the endmost vertical is easily located. However, this simulation truss exposes a drawback of Kabe's identification process for damage detection due to the preservation of the zero-nonzero structure of the original stiffness matrix. If this is not the correct zero-nonzero pattern for the damaged stiffness matrix, Kabe's method does not alter the pattern. Diagonal planar truss members produce zero elements in the stiffness matrix where coupling actually exists in the structure. These zero elements are not adjusted by the identification procedure, so adjustments are spread among adjacent nonzero elements. Threshold searches then exclude these spread-out adjustments and no damage is detected. Not all truss designs experience this problem. The second simulation truss does not have this difficulty.



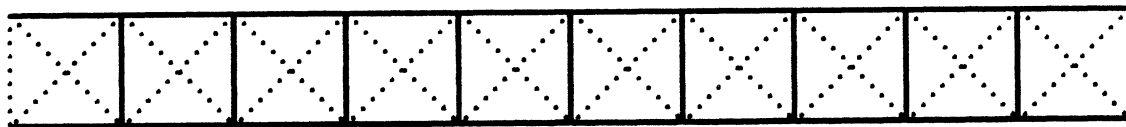
—— PERFECT DETECTION
 NO DETECTION

a)



—— PERFECT DETECTION
 NO DETECTION

b)



—— PERFECT DETECTION
 NO DETECTION

c)

Figure 4-6. Ability to Locate Damage in the Planar Truss
 a) Intermediate Thresholds
 b) Low Thresholds
 c) High Thresholds

Figure 4-6, Part b) presents the results for tests with -10 and -25 percent for the diagonal and off-diagonal thresholds, respectively. These are thresholds that are low, which could admit extra nodes or edges to the damage subgraph. Again, damage is detected exactly for all members, except diagonals.

With high thresholds, damage subgraphs could be incomplete. Values of -25 and -75 percent for the diagonal and off-diagonal thresholds, respectively, lead to results where damage is not detected in the endmost vertical members of the first and tenth bays. Damage is not detected for diagonal members, either. Figure 4-6, part c) presents these results.

Damage detection and location is possible for the planar truss as indicated, except for those members that require an altered zero-nonzero structure of the stiffness matrix. Damage subgraphs are constructed with filtering thresholds which exclude errors that may enter the identification process results. However, these thresholds could exclude damage information as well, if not selected appropriately.

4.3.2 Space Station Model Tests

Simulation tests for the Space Station truss were performed in an identical fashion to construct a figure indicating the ability to locate damage for each member. In separate tests, 78 of 135 members were damaged. Recognizing symmetry in the truss, damage to all members is considered. Thresholds of -10 and -50 percent were selected to search the diagonal and off-diagonal adjustment percentages, respectively. Figure 4-7 presents the ability to detect damage in the simulation Space Station truss.

In this figure, detected damage includes that which is perfectly detected and that which is detected after evaluation of damaged subgraphs which include (or exclude) minor deviations from a perfect subgraph. Damage subgraphs may show damage to a member, but also include extra nodes.

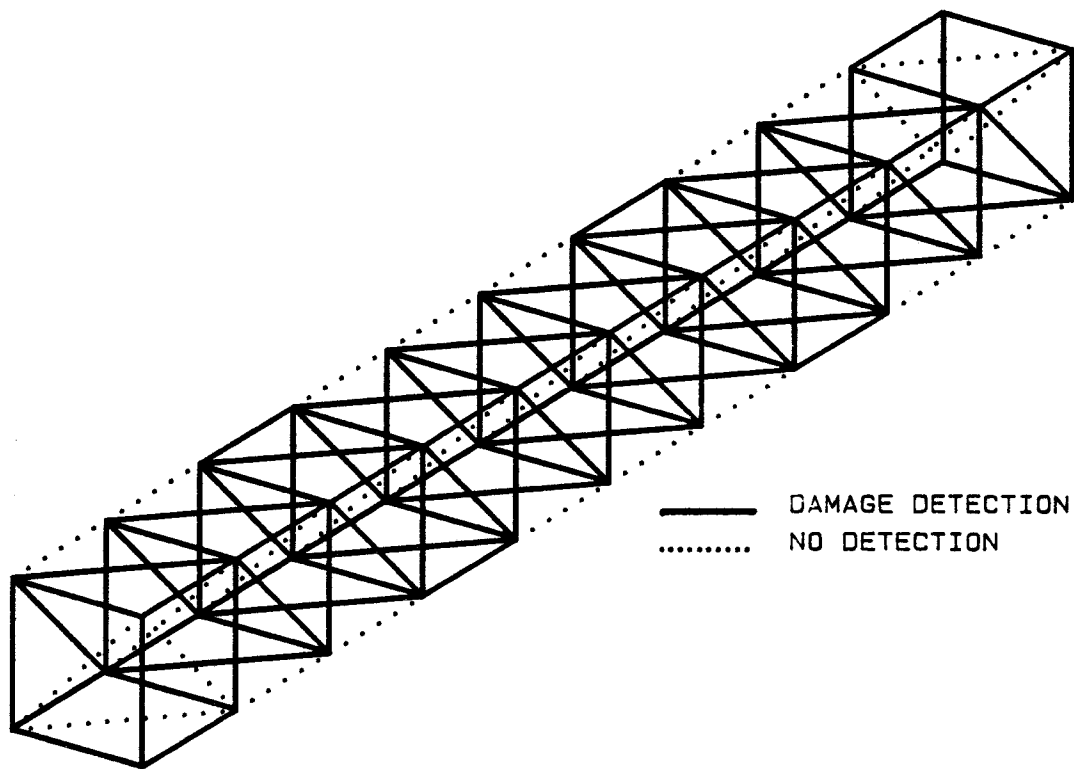


Figure 4-7. Ability to Locate Damage in the Space Station Truss

Without connecting edges, these nodes are not part of a damage pattern, and are discounted. Also, damage subgraphs for diagonal truss members may be slightly incomplete. With one or two nodes or edges missing, the subgraph is still recognizable, enabling damage detection.

Again, location of damage is excellent, using only the first three elastic modes of the 132 degree-of-freedom structure. However, damage to certain truss members is not detected. Specifically, diagonal members at the bay interfaces do not contribute significantly to the structure response. Also, damage to longitudinal elements along two edges of the truss is not identified with enough accuracy to be detected by the algorithm. This unexpected result is related to the structure symmetry. Damage to these members causes separation of closely spaced frequency pairs for the structure. The identification method did not produce results which successfully separate the frequency pairs in all cases, preventing location of the damage.

More details of the performance of Kabe's method are presented in Appendix B. Among these are characteristics of the auxiliary problem and further discussion of the number of modes required for identification.

4.4 Summary

In summary, an algorithm was developed for the detection and location of damaged members of a large truss structure. Structure response is used in a system identification process to generate an updated stiffness matrix. Comparing the updated matrix to the original model stiffness matrix produces adjustment percentages for each nonzero element. Searching these adjustment percentages with appropriate filtering thresholds creates a list of reduced stiffnesses that indicate the location of a damaged element. Using graph theory for matrices, this process constructs a "damaged" subgraph of the stiffness matrix graph, enabling location of the damaged member.

Simulations with exact data show that the algorithm locates damaged members. In keeping with the idea of limited modal data, the first three elastic modes were used in each case for the identification. For the majority of members in the two simulation structures, damage is detected easily. However, insensitivity in the structure response prevents the location of damage to certain truss members.

Tests for the Space Station truss were conducted using only the first three elastic structure modes. Tests have not been done with other combinations from the lowest modes because criteria for selecting the modes is not established. Kabe [34] explored the idea of mode selection, as well as the effect of errors in the modal data. Both of these affect the ability to locate damage in the Space Station truss. Improved detection is expected using modes which display the structure sensitivity to damaged members. Errors in the data are expected to reduce the ability to detect and to locate damage. Errors and their effect is explored in Chapter 5.

5.0 Damage Location with Imperfect Data

5.1 Introduction

Chapter 3 included quantitative evaluations of two methods of system identification. From the results, Kabe's method was selected as showing more promise for damage detection. The damage location method was subsequently developed with Kabe's identification algorithm as the key.

The one evaluation test that demonstrated the potential success of Kabe's identification method was with a planar truss structure. That simulation was designed to represent the circumstances of a damage detection problem and was not an exact-data situation like those in Chapter 4. Comparison of results presented in Tables 3-3 and 4-2 shows that the evaluation test for the same damaged member was slightly different.

In exact-data simulations, "measured" modal data was derived by adjusting the original model to reflect the damage. Together, damaged stiffness matrices and original mass matrices generated the modal data. As a result, eigenvectors satisfied conditions of orthonormality with respect to the original mass matrix.

Modal data for the evaluation tests of Chapter 3 was produced by the finite element program ANSYS. Considering "measured" ANSYS data as representing the true system, the evaluation simulation presented a case which included inexact data in the original model. Errors were from two sources. The damaged truss (true system) was modeled with one member deleted entirely, affecting both mass and stiffness matrices in comparison with an undamaged model. Therefore, the original mass matrix in the damage detection process was incorrect because the damaged member mass was present. A second source of errors in the evaluation test was separate development of the original model. Mass and stiffness matrices were constructed using fewer significant digits than the ANSYS model.

Figure 5-1 is a portion of the damage detection method flow chart originally presented in Figure 4-1. Boxes 1 and 3 represent the inputs to the method - the original model and the measured modal data. Exact data simulations resulted when the sources of these two inputs were combined. Recall that the original model was used to create the damaged model and generate the modal data. Evaluation simulations of Chapter 3 have these boxes separated, providing a better representation of true damage detection circumstances. However, inexact data was present only in the original model, not in both inputs to the identification method.

Effects of inexact values in both original model and measured data are examined in this chapter. First, further discussion of anticipated errors is presented, including published assessments and estimates. To demonstrate the effect of original model errors, ANSYS simulation results are presented in full for the planar truss and for the space station truss. Finally, the results of simulation tests with inexact modal data are discussed, as well.

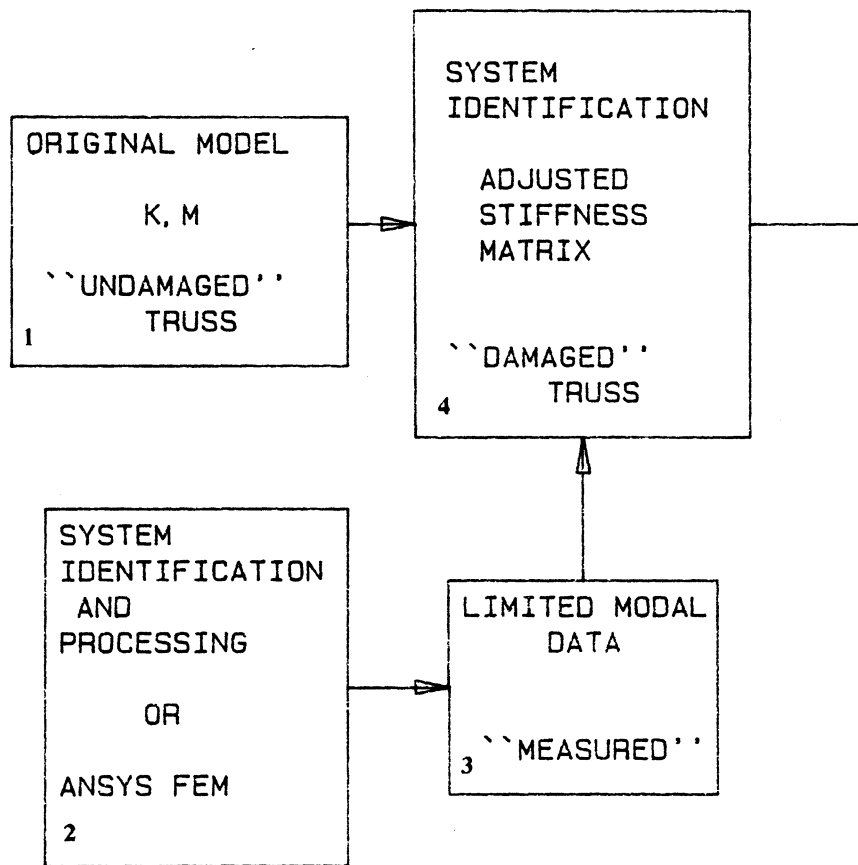


Figure 5-1. Inputs to the Identification Algorithm of the Damage Detection Method

5.2 Characterization of Expected Errors

Characterization of errors anticipated for the damage detection problem is difficult. With a real structure, the exact model is unknown for any situation, damaged or undamaged. Therefore, typical errors are not available in quantitative terms as if they had been produced from a simulation.

The original model, then, is evaluated by its matching of the undamaged system response. Conveniently, control systems require models that accurately predict the system response for numerous modes. Consequently, an original model developed with information from the control system model will be a "close" representation of the true system for the comparatively few modes needed for damage detection. For simulations in this chapter, an original model was developed separately and with less accuracy than the ANSYS models representing the true damaged and undamaged systems. Response predicted from the original model is compared using the eigenvalues and eigenvectors to response obtained from the undamaged ANSYS model. Discrepancies in the response predictions serve as the measure of the original model errors.

On the other hand, inexact modal data can be represented in more quantitative terms, even though true quantities are not available. Researchers in identification occasionally publish studies considering errors. More are needed, along with more studies with empirical data. However, Juang and Pappa [40 – 43] have presented both for their Eigensystem Realization Algorithm. Kabe [33,34] also presented results from his Stiffness Matrix Adjustment Method with simulated errors in the modal data. Their experience with testing actual structures supports acceptance of these authors' assumptions for numerical characterization of modal data errors.

Kabe uses the unit-normalized generalized mass matrix, $\phi'M\phi$, to assess the acceptability of empirical modes in conjunction with an analytical mass matrix. Off-diagonal terms in the result, which represent mutual contamination of the modes, must be less than 0.10 to consider the test a

success and to consider the mass matrix a good model. Measured modes are then orthogonalized with respect to the mass matrix before proceeding with the identification:

$$[\phi^c] = [\phi][\phi]^t[M][\phi]^{-1/2} \quad (5-1)$$

where $[\phi]$ is the $n \times m$ matrix of measured modes and $[\phi^c]$ the matrix of orthogonalized modes. Although Kabe references another source, this method is identical to that presented by Baruch and Bar Itzhack [30], which corrects only symmetric errors in the modes. The matrix quantity $(\phi^t M \phi)^{-1/2}$ must be computed in order to obtain the orthogonalized mode shapes. This is accomplished by finding the eigensolution for an error matrix, E , defined as

$$[E] = [\phi]^t[M][\phi] - I. \quad (5-2)$$

With the eigenvalues of E denoted by μ_i and the eigenvectors of E as the columns of P , an orthonormal matrix,

$$(\phi^t M \phi)^{-1/2} = P \left[\frac{1}{+\sqrt{1 + \mu_i}} \right] P^t. \quad (5-3)$$

Kabe [34] simulated errors in modal data by corrupting normal modes with noise. The i th value of a mode shape vector is adjusted by multiplying it by $1.0 + r_i$, where r is a vector of random errors, uniformly distributed between the values of ± 0.05 , ± 0.1 , or ± 0.2 . Orthogonalization of these "measured" modes followed. Five percent corruption of the normal modes led to acceptable identification results for the cases presented, but 10 and 20 percent corruption produced unacceptable results.

Juang and Pappa [42] studied the effects of errors in the data for the Eigensystem Realization Algorithm. Uniformly distributed errors were added to simulated free-response functions before identifying the frequencies. Results from 100 data sets in a Monte Carlo simulation showed that errors in the data affect identification of higher modes more than identification of lower ones. Identified frequencies from the recommended reduced models were slightly scattered about their

true values, with scattering on the order of the noise or less. No results were presented for identified mode shapes.

Measured data from structural dynamics testing contain errors in two basic forms. The first is contamination from higher frequency structure modes. Both studies above mention this form. Also, errors are introduced by noise in the sensors from cross talk, ground loops, bias and other sources. Noise levels of 10 percent or less on each channel could be expected in an acceptable test. Juang and Pappa concluded that their reduced model method “undoubtedly acted as an effective noise filter.” Considering this result, no errors modeled in this study exceed ± 5 percent. For eigenvalues, errors are limited to ± 1 percent.

5.3 Original Model Errors

Simulated tests with “measured” data from ANSYS models of the planar and space station trusses demonstrate the influence of original model errors on the damage detection results. The original model used in the damage detection method differed from the undamaged ANSYS model. Although identical properties were used to form the two models, fewer significant digits were retained in the original model leading to discrepancies in the eigensolutions. Table 5-1 presents eigenvalues for the two models of the planar truss undamaged state. Eigenvalues from the original model are listed in the first column, while the second column presents eigenvalues from the undamaged ANSYS model of the same truss.

No original model eigenvalues differ more than 0.2 of one percent from those of the ANSYS model, representing the true structure. Angles between respective eigenvectors also serve as a comparison of the models. Data for the first seven elastic modes reveals that original model eigenvectors are directed exactly along their respective ANSYS model vectors.

Table 5-1. Eigenvalues from Two Models of the Undamaged Planar Truss

Mode Number	Inaccurate Original Model	ANSYS Undamaged Model
1	-0.114952D+03	0.000000D+00
2	-0.445250D+02	0.734501D-08
3	-0.153366D+02	0.130808D-07
4	0.224988D+05	0.225442D+05
5	0.142995D+06	0.143065D+06
6	0.251053D+06	0.251270D+06
7	0.448869D+06	0.448930D+06
8	0.994553D+06	0.994461D+06
9	0.101611D+07	0.101658D+07
10	0.180163D+07	0.180107D+07
11	0.232072D+07	0.232138D+07
12	0.284839D+07	0.284687D+07
13	0.405640D+07	0.405339D+07
14	0.415778D+07	0.415814D+07
15	0.524849D+07	0.524395D+07
16	0.577739D+07	0.577399D+07
17	0.618437D+07	0.617967D+07
18	0.625723D+07	0.625599D+07
19	0.659851D+07	0.659250D+07
20	0.670547D+07	0.670210D+07
21	0.694475D+07	0.694121D+07
22	0.728149D+07	0.727773D+07
23	0.729177D+07	0.728779D+07
24	0.739873D+07	0.739521D+07
25	0.831756D+07	0.831298D+07
26	0.941859D+07	0.941337D+07
27	0.101037D+08	0.100989D+08
28	0.102810D+08	0.102758D+08
29	0.105179D+08	0.105119D+08
30	0.114883D+08	0.114815D+08
31	0.118144D+08	0.118085D+08
32	0.120674D+08	0.120619D+08
33	0.122401D+08	0.122329D+08
34	0.126761D+08	0.126698D+08
35	0.127435D+08	0.127364D+08
36	0.129991D+08	0.129918D+08
37	0.130446D+08	0.130378D+08
38	0.131971D+08	0.131905D+08
39	0.146092D+08	0.146001D+08
40	0.162583D+08	0.162472D+08
41	0.180610D+08	0.180489D+08
42	0.211336D+08	0.211158D+08
43	0.241547D+08	0.241287D+08
44	0.258673D+08	0.258355D+08

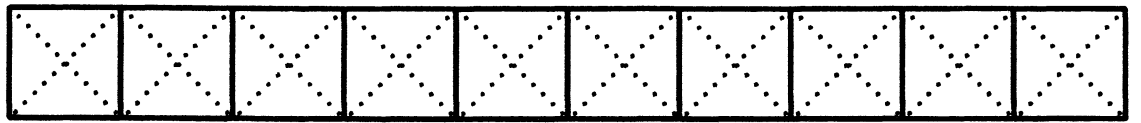
A similar comparison of the eigenvalue results for the undamaged space station truss from the ANSYS model and from the damage detection algorithm original model determines that none of the eigenvalues differ more than 0.5 of one percent. Comparison of the first 14 elastic modes shows that original model eigenvectors are nearly aligned with ANSYS model eigenvectors for this undamaged case, as well. These angles between respective eigenvectors of the two models are all less than 1.72 degrees.

Essential characteristics of the undamaged system are represented by the detection algorithm original model. However, this model differs from the true undamaged model. Smith and Hendricks [62] examined damage to each member of the planar and space station trusses by simulation test or by symmetry. Inaccuracies in the original model become reduced ability to detect damage in insensitive areas of the trusses. In particular, damage to remote members of the structures is difficult or impossible to locate. These results are presented in the next sections.

5.3.1 Planar Truss Studies

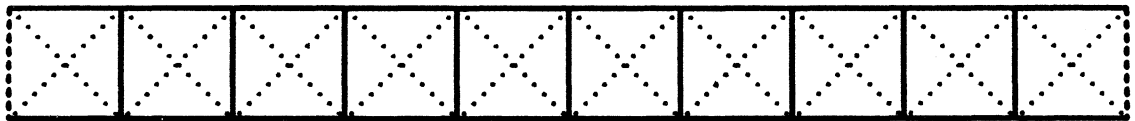
The series of exact-data tests conducted for the planar truss were repeated for these imperfect-data tests. Limits of the damage detection method were examined by altering the thresholds used to create the damage subgraphs. In each case, damage detection was performed with the first three elastic modes of the damaged structure.

Intermediate threshold values selected for the exact-data tests were used to filter the node and edge adjustment percentages in the first imperfect-data tests. Thresholds of -20 and -50 percent, respectively, were used to search the diagonal and off-diagonal adjustments. Figure 5-2 a) is a presentation of ability to detect damage in the planar truss. As before, a member is drawn with a solid line only if damage to the member is perfectly detected by the method. Damage subgraphs for these cases are exactly correct.



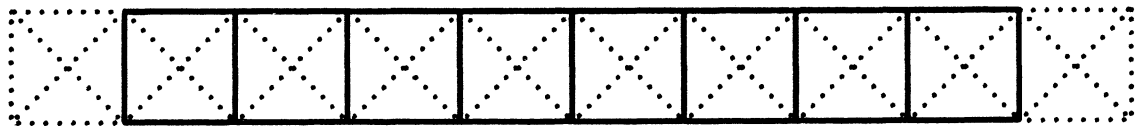
——— PERFECT DETECTION
 NO DETECTION

a)



——— PERFECT DETECTION
 - - - - - DETECTION +
 NO DETECTION

b)



——— PERFECT DETECTION
 NO DETECTION

c)

Figure 5-2. Ability to Locate Damage in the Planar Truss - ANSYS Data Cases
 a) Intermediate Thresholds
 b) Low Thresholds
 c) High Thresholds

For the first tests, damage detection algorithm performance does not appear to be affected by inaccuracies in the original model. Detection of damage to horizontal or vertical members of the truss is excellent, including damage to members in the first and tenth bays. Diagonal members of this planar truss, as before, create a situation requiring alterations to the stiffness matrix zero-nonzero pattern. Kabe's identification method is precluded from handling this situation. Damage to the diagonal members is not detected.

Figure 5-2, Part b) presents the results for tests with low thresholds. Values of -10 and -25 percent for the node and edge adjustments, respectively, were used. Excluding the diagonals, damage is detected exactly for all members except the end verticals. Their damage subgraphs show the damage to the end vertical member, but also include extra nodes. Without connecting edges, these extra nodes are not in a damage pattern, so they were discounted. Damage in these end vertical members was perfectly detected with exact data.

With high thresholds, damage subgraphs are incomplete because the filters exclude small adjustments that result from structure insensitivity. Figure 5-2, Part c) presents the results for tests with -25 and -75 percent for the diagonal and off-diagonal thresholds, respectively. Damage is not detected in the first or tenth bay truss members. Again, damage is not detected for diagonal members, either. This result is significantly different from the exact data study with the same thresholds. There, detection of damage in only the endmost verticals is lost.

Original model errors affect the limits of the damage detection method performance. With exact data, damage to all vertical or horizontal planar truss members was usually detected, regardless of threshold selection. Inaccuracies in the original model lead to a loss of ability to detect damage in the extremities of the truss for any but intermediate threshold values. High threshold detection results were significantly degraded.

5.3.2 Space Station Model Tests

Simulation tests for the Space Station truss were performed in an identical fashion to construct a figure indicating the ability to locate damage for each member. In separate tests, 54 of the 135 members were damaged. Recognizing symmetry in the truss, damage to all but 14 members is considered. Thresholds of -10 and -50 percent were selected to search the diagonal and off-diagonal adjustment percentages, respectively. These are identical to those used for the exact-data tests. Figure 5-3 presents the ability to detect damage in the Space Station truss, with errors included in the original model.

“Detected damage” in the figure includes that which is perfectly detected and that which is detected after evaluation of imperfect subgraphs. Imperfect subgraphs are damaged subgraphs which include (or exclude) minor deviations from a perfect subgraph, but clearly indicate the damaged truss member.

Again, tests were conducted using only the first three elastic modes of the structure. Damage detection for a majority of the members is possible, but considerably inhibited in the first and tenth bays. Where damage is not detected in the exact-data tests, it is also not detected here. Interface diagonals and members along two longitudinal edges do not affect the response of the first modes, so are not detected when deleted.

Results of damage detection with inaccurate original models of the planar or Space Station trusses show a decreased ability to locate damage in truss members which contribute less to the first modes of the response. In general, this affected the remote members of the trusses, in the first and tenth bays.

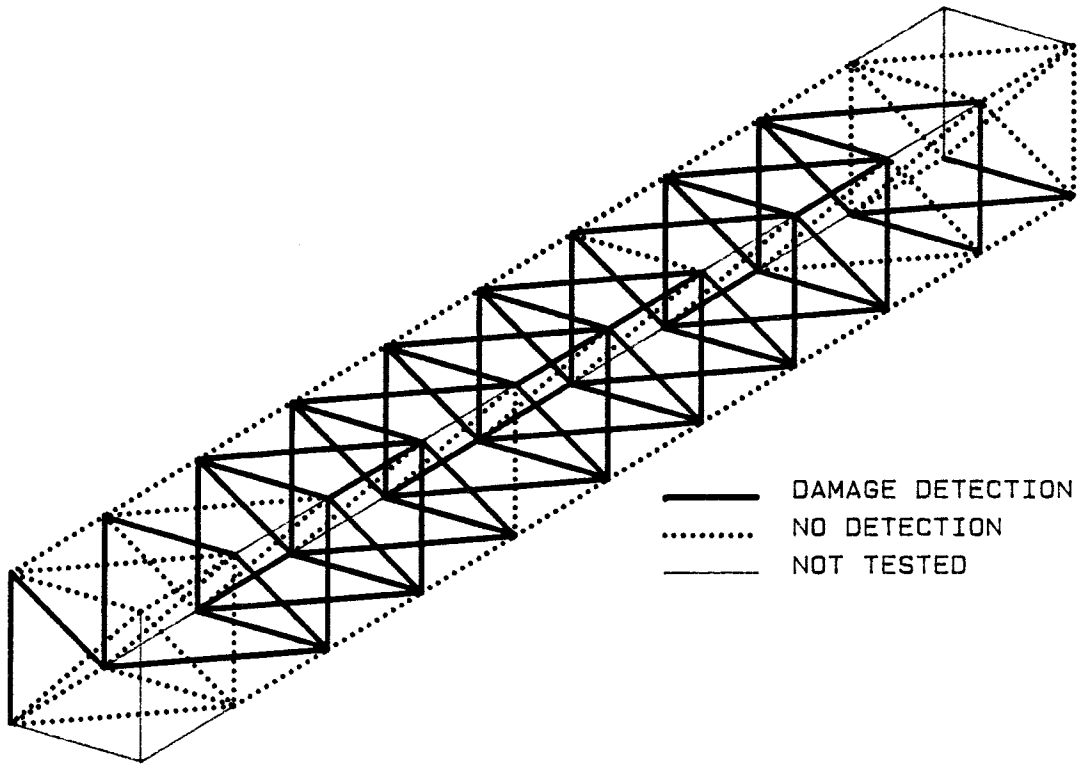


Figure 5-3. Ability to Locate Damage in the Space Station Truss - ANSYS Data Cases

5.4 Modal Data Errors

Errors in the modal data were simulated by corrupting the eigenvalues and normal modes from the exact solution with uniformly distributed noise. Zero mean, uniformly distributed vectors of random errors, $\{r\}$, were generated by the IMSL subroutine, GGUBS, to use in corrupting the exact eigenvalues and eigenvectors for a damage case. The values in the vector are uniformly distributed between $\pm x$ percent. x is indicated for each study which follows.

Error is created in the i th eigenvalue by multiplying by $(1.0 + r_i)$. Therefore, each eigenvalue is adjusted by a random percent about its true value. Eigenvectors are handled differently. The maximum value in each mode shape vector is used to scale the errors before they are imposed. Considering the maximum response level of any mode as ± 1.0 unit of amplitude, the errors are uniformly distributed over amplitude ± 0.01 , if x equalled 1, for example. Orthogonalization of the corrupted eigenvectors is necessary before proceeding with the damage location algorithm. Equation 5-1 is used to calculate the orthogonalized modes.

Three modes of the response are used for damage location in the following studies, except where indicated. Studies with the planar truss are more extensive, mainly because the size of the problem is more manageable. First, different levels of eigenvalue and eigenvector errors are examined. Then, a study of one damage case led to parameters used in assessing the ability to locate damage in the entire planar truss.

A study of the effects of eigenvalue and eigenvector errors revealed that damage detection performance is not affected by eigenvalue errors, but by errors in the eigenvectors. Table 5-2 is a presentation of damage detection performance for the undamaged planar truss model. Intermediate threshold values of -20 percent and -50 percent for the node and edge adjustments, respectively, were used. With no errors in the eigenvalues or eigenvectors, the original, adjusted and exact models of the truss are identical. The damage subgraph is blank; all adjustment percentages are zero.

As various levels of errors were added, the subgraph was reexamined. Regardless of the level of eigenvalue errors up to ± 50 percent, with perfect eigenvectors, the subgraph remains blank. However, having ± 2 percent errors in the eigenvectors, for any level of error in the eigenvalues, creates damage subgraphs with nodes and edges, some of which form apparent damage patterns. Damage detection performance is unacceptable with these results.

As the results are relatively unaffected by errors in the eigenvalues, studies for the remainder of this section were performed with only mode shape errors. The first study examined damage detection for the lower horizontal member of the fifth bay. With intermediate thresholds to search the adjustment percentages, damage location is perfect with ± 1 percent error in the modes, but impossible with ± 2 percent. Using higher thresholds to act as a filter for the errors, damage detection is possible, though not perfect, with this level of error in the data. Thresholds of -25 and -75 percent were used for the diagonal and off-diagonal adjustments, respectively. With an increased level of error, ± 3 percent, the thresholds can not be adjusted to exclude the erroneous results without excluding the damage, as well. A complete study of the effect of errors in the modes for the planar truss was then conducted, with ± 2 percent errors and threshold values of -25 and -75 for the nodes and edges, respectively.

Figure 5-4 presents the ability to locate damage in the planar truss with errors in the modal data, as described above. "Damage detection" includes perfect detection and detection after evaluation of imperfect subgraphs. Extraneous nodes appear in many of the graphs, but damage detection is still possible.

Ability to locate damage is considerably reduced in the planar truss. Damage to the diagonal members could not be detected even with exact data, but here, damage in half of the horizontal and vertical members is undetected, also. The inability to detect the damage is not in a symmetric pattern like that observed for the studies with original model errors, either. The errors added to the mode shape vectors have destroyed the essential character of the modes.

Table 5-2. Damage Detection Performance for the Undamaged Planar Truss with Varying Levels of Errors in the Modal Data

Damage Subgraph		Eigenvalue Error (Uniform Distribution Between These Values)					
		0.00	± 0.01	± 0.02	± 0.05	± 0.20	± 0.50
Mode	0.00	0 nodes	0 nodes	0 nodes	0 nodes	0 nodes	0 nodes
		0 edges	0 edges	0 edges	0 edges	0 edges	0 edges
Shape	± 0.01	0 nodes	0 nodes	0 nodes	0 nodes	0 nodes	0 nodes
		0 edges	0 edges	0 edges	0 edges	0 edges	0 edges
Error	± 0.02	5 nodes	5 nodes	5 nodes	5 nodes	5 nodes	4 nodes
		3 edges	3 edges	3 edges	3 edges	3 edges	1 edge

A blank subgraph is perfectly correct for the undamaged case.

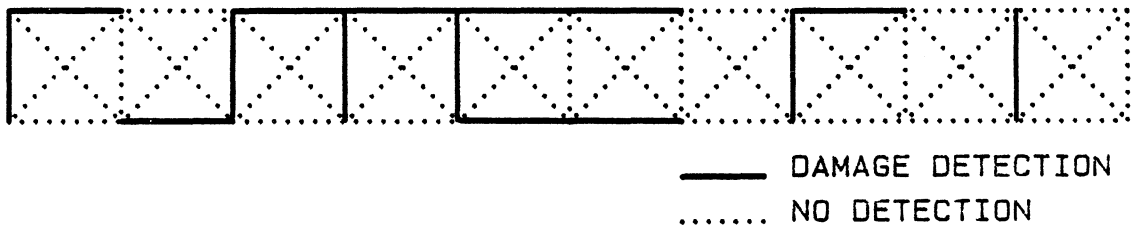


Figure 5-4. Ability to Locate Damage in the Planar Truss with Errors in the Modal Data

Varying the thresholds within the ranges allowed by a truss design is an option for improving the damage location performance in the face of errors. However, these errors exceeded the ranges established by the design preventing any further adjustment to improve performance.

With this discouraging result, testing of the planar truss with higher levels of noise in the mode shape data is superfluous. Results for the Space Station model are expected to be at least as discouraging, as well. When more information about the quality of the mode shape data is available, further testing would be warranted.

5.4 Summary

Performance of the damage location method was investigated with respect to errors in the data. Data for the central identification algorithm takes two forms, the original model of the undamaged truss and the measured modal data of the damaged structure. Errors in the original model were demonstrated by studies with the planar truss and the Space Station truss used in the previous chapter. Results revealed a reduced ability to detect damage in the most remote members of the structures.

Errors in the modal data were also investigated. The damage location method is insensitive to errors in the eigenvalues, but extremely sensitive to errors in the eigenvectors. Although the results are discouraging, more information about the accuracy of available mode shape data is required before deciding to abandon the method.

6.0 Conclusions

Structural verification for large space truss structures can be accomplished by measuring the structure response with components of the control system. This assertion was investigated by developing a concept for damage location and by demonstrating the resulting method in simulated tests. Success in locating deleted members of two truss structures verifies the concept and supports the original claim that structural verification is possible using the response. However, performance of the damage location method in simulated tests also leads to the conclusions that development of a system identification method tailored for damage detection is desirable and that more work is required to determine the availability and quality of measured modal data.

Truss structures were selected as a subset of all large space structures to reduce the problem of structural verification to one of reasonable size. Other assumptions, including linear response of the structure, constrained the research as well. Finally, damage was defined as the failure of a single member of a large truss. In a simplified model, the truss member was deleted entirely.

The general approach to the problem of damage location in large space trusses was determination of areas of reduced or zero stiffness in the truss structure. The control system “tests” the structure and produces response measurements. These measurements are then used in a series of two system

identification algorithms to construct a model of the damaged structure. The model is compared to one for the undamaged structure to locate any damage.

The first step is to establish an original model, consisting of mass and stiffness matrices, for the structure in the undamaged condition. Prior adjustment of the model is assumed, so at the onset, it accurately represents the lowest modes of the structure response. The original model and modal data identified from measurements of the damaged truss response are input to Kabe's system identification method which produces a model of the damaged truss. A comparison of the two models' stiffness matrices, damaged and undamaged, leads to location of areas of reduced stiffness in the damaged truss. Finally, the pattern of reduced stiffness specifies the damaged truss member.

Kabe's identification method to adjust the stiffness matrix of a model was selected as the key element of the damage location algorithm after consideration of many methods of system identification. First, a literature review on the subject generated a list of candidate methods. Then, qualitative and quantitative analyses of the candidates led to the selection of Kabe's method.

The strength of Kabe's method, which enables damage detection, comes from its use of information from the undamaged model. Measured data from orbiting structures is limited. Although modal data is required for design and operation of the control system, only a few of the modes are likely to be known with the accuracy needed for damage location. Kabe's method alters the undamaged truss stiffness matrix to form the closest stiffness matrix which satisfies constraints imposed by the modal data. In the process, the sparsity of the stiffness matrices is unchanged. Truss structures, like that proposed for the Space Station, have considerable sparsity in their stiffness matrices. This information from the undamaged model enables damage detection with only a few modes.

Simulation studies to locate damage were performed on two truss models. The members of both models and the design of the second model were borrowed from the concept design for the International Space Station. The first is a ten-bay planar truss (44 degrees of freedom), while the second simulation model is a ten-bay orthogonal tetrahedral truss (132 degrees of freedom).

Exact- and inexact-data simulated tests of the two structures demonstrated that damage can be located with this approach. With exact data, the ability to locate damage is excellent. Using only three modes for either of the trusses, damage to the farthest members in the first and tenth bays is detected. Damage to certain members throughout the truss is not detected, though, because these members do not contribute significantly to the response or because the design of the truss creates a damage situation that violates an assumption of the system identification method.

Inexact-data studies investigated the effect of errors in the original model and the effect of errors in the modal data. Errors in the original model are reflected in the detection results as loss of ability to detect damage in the remote members of the structures. Adding this loss of ability to the inability to detect certain members with exact data, damage location was still achieved for a majority of the members. Errors in the modal data degraded the results, as well. These effects were much more discouraging, though. Noise as small as ± 2 percent in the mode shapes considerably reduced the ability to detect damage in the planar truss. Large variations in the measured frequencies were tolerated, though. Unfortunately, mode shape measurements are far less accurate than frequency measurements.

In summary, a concept for structural verification of large space trusses was developed and demonstrated. Significant results include the following:

1. Damage location can be accomplished by determining areas of reduced stiffness in a model of the damaged truss compared to a model of the undamaged structure.
2. Kabe's system identification method produces the damaged model and is the key to the success of the algorithm. Minimal measured data is required because information from the original model is used in the identification process.
3. Ability to locate damage was excellent for most members in two simulated truss structures when exact data was used. Difficulties arose from aspects of the truss design.
4. Ability to locate damage is inhibited by errors in the original model, mainly for the most remote members.

5. Ability to locate damage is reduced by errors in the modal data, especially by those in the measured mode shapes.

Several problems with the implementation or performance of the damage location method were discovered during the development and testing of this approach. Most of these have been presented in the previous discussion of results, without mention of ways to overcome them. Briefly, major difficulties are as follows:

1. Ability to locate damage is sensitive to mode shape data errors.
2. Truss structure designs may be insensitive to damage in certain members, prohibiting their location.
3. Truss structure designs may also create damage situations which require a different zero-nonzero structure of the stiffness matrix, violating an assumption of Kabe's identification method.
4. For large structures, the auxiliary problem of Kabe's identification method becomes computationally unmanageable since the coefficient matrix is large and indefinite.

The problems with the method are not insurmountable, though. Levels of errors expected in the mode shape data are not well defined. Options from the field of signal processing may be useful in improving the quality of the measured modes, as well. For members that do not contribute significantly to the response of the first modes, a judicious selection of modes to use for the identification may enable damage location. Consideration of the quality of the modes selected will have to be a consideration, though.

Finally, difficulties that result from characteristics of Kabe's method can be addressed by developing a new method of system identification tailored for damage detection in large space trusses. Beattie and Smith [63] have investigated optimal-update system identification methods, which include Kabe's method, and have developed a new method to avoid problems 3 and 4 above. While similar in derivation to Kabe's method, a different cost function produces significant improvements. First,

connectivity information is obtained from the graph of the stiffness matrix, not from the zero-nonzero pattern. Therefore, the diagonal members of the planar truss do not create a situation violating an assumption of the identification process. More significantly, the auxiliary problem created is positive definite, enabling solution by conjugate gradient methods. No more storage than that needed for the original stiffness matrix is required.

Recommended areas for further research mostly follow from the problem areas identified above. An understanding of the availability and quality of measured data is needed to assess feasibility for actual verification. Empirical studies with scale model structures might be a first step in this understanding. In conjunction, further studies of the method, perhaps with a different central identification algorithm, could improve the performance. Refinement of the methods for threshold selection and for mode selection should be investigated, as well.

The preceding suggestions are within the framework of the concept adopted for this research. Outside that framework, different definitions of damage should be considered. Resolution of the conflict between structural verification and damage detection for space truss structures should be attempted. In addition, a thorough study of truss response under different damage conditions would be helpful in applying the inverse problem for damage detection. Finally, a continuing evaluation of system identification methods, with simulated and actual data, is necessary to discover the best use of these methods for damage detection or other large space structure applications.

An underlying goal of this research was to develop a concept for damage location that is compatible with the control system of a large space structure. The results of the numerical studies indicate that damage detection can be accomplished within this goal. However, considerable work remains to make both of these a reality.

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Appendix A. Evaluation Models

Spring-Mass Model

Three system models have been designed to study the relative merits and disadvantages of system identification methods for the specific application of truss damage detection. A linear spring-mass problem has been designed as the simplest system. More complex models, including a planar truss and finally the Space Station orthogonal tetrahedral truss, will be used for later evaluations.

The spring-mass model is used primarily for those methods which identify the elements of the matrices of the equations of motion (i.e. mass and stiffness matrices). Physical parameters, such as individual masses and spring constants, can be identified with this model as well.

Consideration of the space station truss, the final object of the study, and review of the literature led to several requirements for the spring-mass model. Closely spaced frequencies and little damping are characteristics of large space structures and important requirements for the model. Rigid body modes should be possible. But, the most important requirement is to have this simplest model

provide the greatest possible understanding of the response of the system and of the application of the identification methods.

The spring-mass model is shown in Figure 3-1. The matrix equations of motion for the undamped, linear system are

$$[M]\ddot{\underline{x}} + [K]\underline{x} = \underline{F} \quad (A-1)$$

where \underline{x} is the vector of generalized coordinates, x_i ; \underline{F} is the generalized force vector; M is the symmetric positive definite mass matrix and K is the symmetric stiffness matrix. Elements of the mass and stiffness matrices are as follows:

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \quad \text{and} \quad K = \begin{bmatrix} k_1 + k_2 + k_5 & -k_2 & -k_5 \\ -k_2 & k_2 + k_3 + k_6 & -k_3 \\ -k_5 & -k_3 & k_3 + k_4 + k_5 \end{bmatrix} \quad (A-2)$$

Values for the masses and spring constants are summarized in Table A-1 for three tests of the model. The first test is the full (undamaged) model. The second and third tests each have one spring removed. In the second test, removing the spring changes only a diagonal element of the stiffness matrix; in the third test, both diagonal and off-diagonal elements are affected. Different tests were considered in anticipation of the truss model evaluations where members may have different influence in the global stiffness matrix.

A modal analysis was performed to examine the response for each test of the spring-mass system. Eigenvalues and eigenvectors were determined using a power iteration method with deflation for the higher modes. The results were checked for accuracy with the orthonormality conditions for the modal matrix with the mass and stiffness matrices. Results of these analyses are presented in Table A-2.

Table A-1. Data for Three Tests of the Spring-Mass Model

	Test 1	Test 2	Test 3
m₁	1.0	1.0	1.0
m₂	1.0	1.0	1.0
m₃	1.0	1.0	1.0
k₁	0.5	0.5	0.5
k₂	2.0	2.0	0.0
k₃	2.0	2.0	2.0
k₄	0.5	0.0	0.5
k₅	2.0	2.0	2.0
k₆	0.5	0.5	0.5

Table A-2. Modal Analysis Results for Tests of the Spring-Mass Model

Mode	Eigenvalue	Mode Shape		
Test 1 Undamaged				
1	0.5	.5774	.5774	.5774
2	6.5	.4082	.4082	-.8165
3	6.5	.7071	-.7071	.0000
Test 2 Spring 4 Removed				
1	0.234	.5604	.5604	.6098
2	6.176	.4311	.4311	-.7926
3	6.5	.7071	-.7071	.0000
Test 3 Spring 2 Removed				
1	0.5	.5774	.5774	.5774
2	6.5	.4082	.4082	.4082
3	2.5	.7071	-.7071	.0000

The undamaged model has repeated eigenvalues due to symmetry. The fundamental frequency is a “rigid body mode” where the masses move in unison and the intermass springs are unstretched. This mode would have a zero frequency if the system did not include the springs to ground. Neither of the “damaged” tests have repeated eigenvalues, but corresponding modes between the damaged and undamaged tests are evident.

The spring-mass model is a linear, undamped system that will be used to evaluate system identification methods which determine the elements of the mass or stiffness matrices. It is the simplest of the evaluation models, but includes the important characteristics of the more complex systems - repeated eigenvalues, little damping, rigid body modes, and global stiffness matrix sensitive to different members.

Planar Truss Model

The planar truss model has been designed as a model of intermediate complexity to use for the evaluation of system identification methods. This model can be used for methods which identify the parameters of a discrete model as well as for methods which identify equivalent continuum parameters.

Repeated substructure, little damping and rigid body modes are characteristics desired for this model. Closely spaced frequencies were desired as well, but they will be considered with the most complex truss model. This model and motion are restricted to one plane to allow better understanding of the problem and the system response.

The planar truss model and its properties are shown in Figure 3-2 and Table 3-2, respectively.

Modal analysis was performed for the planar truss model using the finite element code ANSYS. The ten bay structure has 22 nodes and therefore has 44 degrees of freedom. The truss was modeled with each member represented with a truss element (no moments transmitted at the joints). None of the 44 degrees of freedom are restrained.

The first ten modes of the structure include 3 rigid body modes, 5 bending modes and 2 axial modes. The frequencies range from 0.0 to 213.6 cycles/sec. A summary of these modes and their corresponding frequencies is presented in Table A-3. Figure A-1 shows the first 3 bending modes and the first 2 axial modes for the planar truss.

Damaged Model Results

As with the spring-mass model, the planar truss structure was “damaged” and the modal analysis was repeated. Members of the truss were deleted to create the damaged states. Three cases or types of damage were analyzed - a horizontal, vertical, or diagonal member of the planar truss was deleted for the respective cases. Figure A-2 shows the three damaged truss models.

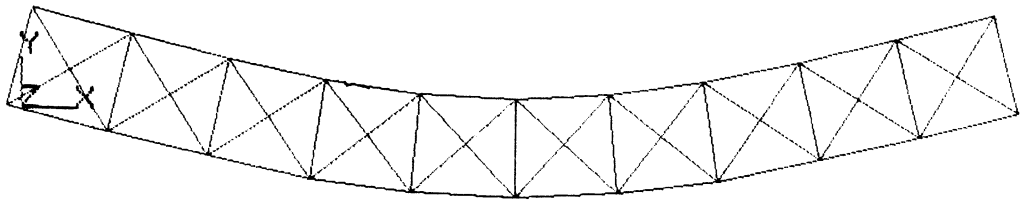
Again, the first 10 modes of the structure are considered for each damaged case to reveal the effect of the damage on the planar truss response. For the Case 1 damage, with a horizontal member removed from the fifth bay of the truss, three rigid body modes, 5 bending modes and 2 axial modes make up the first ten modes of the structure. A summary of these modes and their frequencies are presented in Table A-4, while two bending modes and the first two axial modes are shown in Figure A-3. Note that the axial modes of the truss are distorted, so coupling with the bending modes seems to be indicated for this damaged state.

A vertical member of the truss is deleted for Case 2 damage. Some axial and bending modes of the structure are exchanged by this type of damage. Notice that the second axial mode now has a slightly lower frequency than the fourth bending mode. Also, the tenth mode of the structure is an

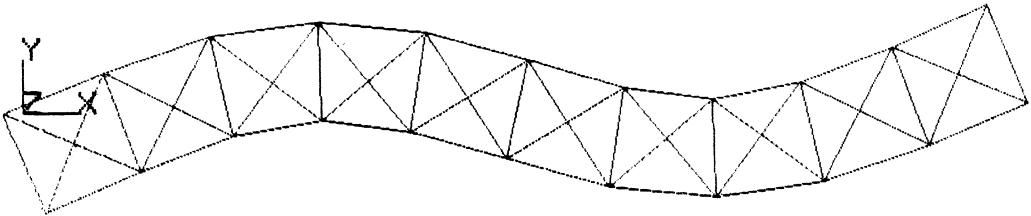
Table A-3. Modal Analysis Results for the Undamaged Planar Truss

Mode No.	Frequency cycles/sec	Comments
1-3	0.0	rigid body modes
4	23.897	B1 *
5	60.199	B2
6	79.779	A1
7	106.637	B3
8	158.714	B4
9	160.468	A2
10	213.592	B5

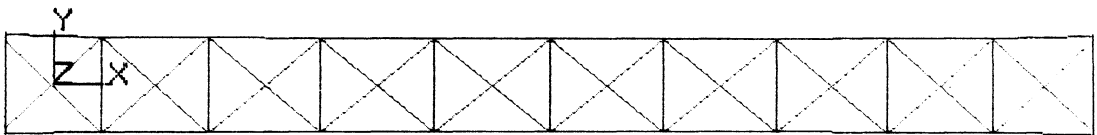
* Bx is the xth bending mode
 Ax is the xth axial mode



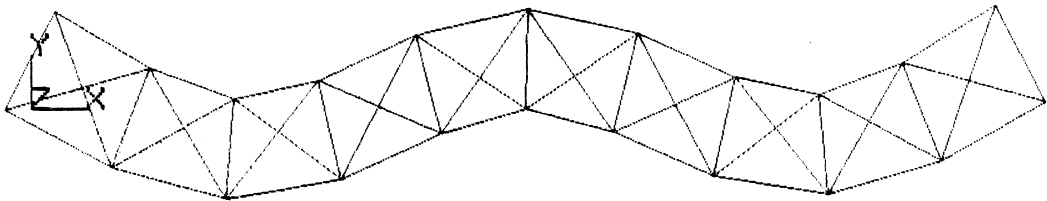
mode 4



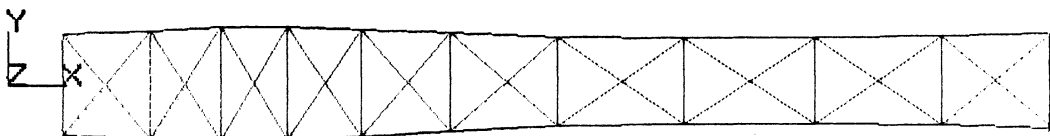
mode 5



mode 6



mode 7



mode 9

Figure A-1. Selected Modes of the Undamaged Planar Truss

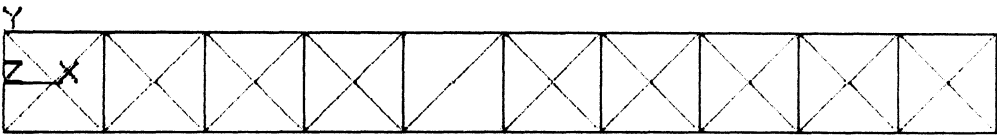
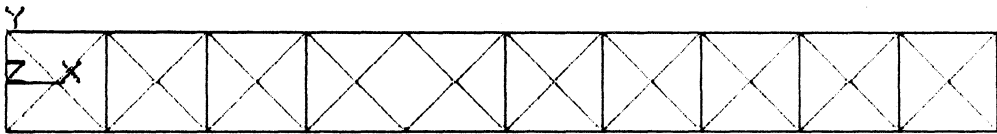
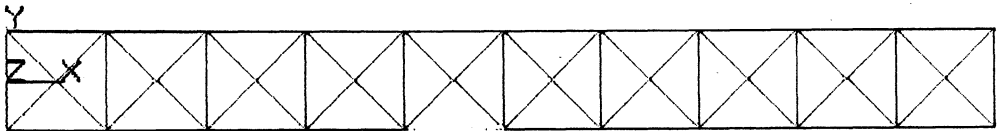


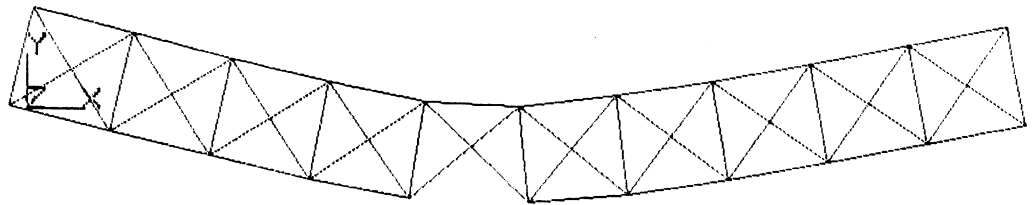
Figure A-2. Three Cases of Damage for the Planar Truss

Table A-4. Modal Analysis Summary - Planar Truss Damage Cases

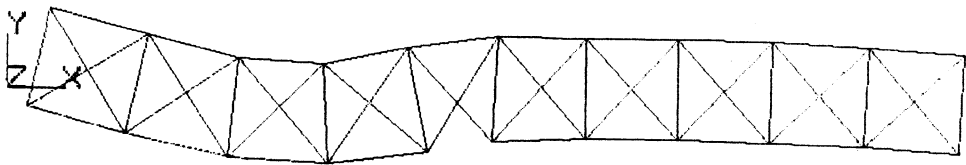
Mass No.	Frequency cycles/sec	% Difference *	Comments
Damage Case 1 - Lower Horizontal Member Deleted			
1-3	0.0	-	rb modes
4	16.739	29.95	B1
5	57.795	3.99	B2
6	68.809	13.75	A1 **
7	102.957	3.45	B3
8	154.586	2.60	B4
9	161.782	-0.82	A2 **
10	205.818	3.64	B5
Damage Case 2 - Vertical Member Deleted			
1-3	0.0	-	rb modes
4	24.122	-0.94	B1
5	60.697	-0.83	B2
6	76.938	3.56	A1
7	107.166	0.50	B3
8	158.301	1.35	A2
9	161.958	-2.04	B4
10	211.510	-	A3
Damage Case 3 - Lower Horizontal Member Deleted			
1-3	0.0	-	rb modes
4	24.255	-1.50	E1
5	57.610	4.30	B2
6	78.671	1.39	A1 **
7	106.313	0.30	B3
8	151.495	4.55	B4
9	163.939	-2.16	A2 **
10	203.052	4.93	B5

* Compared to corresponding mode of undamaged truss (A1 to A1, etc.)

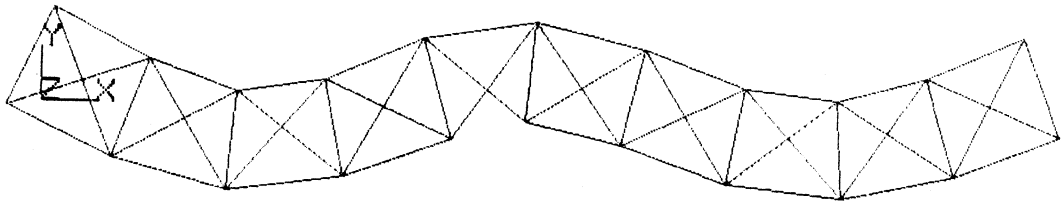
** Distorted



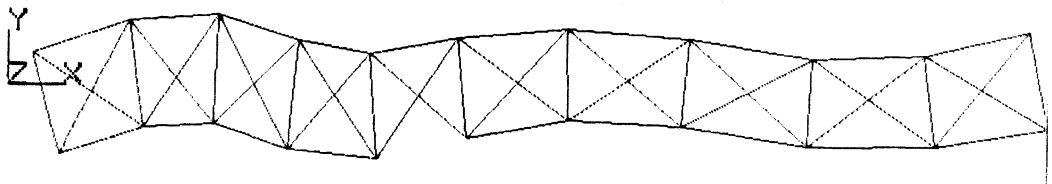
mode 4



mode 6



mode 7



mode 9

Figure A-3. Selected Modes of the Damaged Planar Truss - Case 1

axial mode now, where before the fifth bending mode had a lower frequency. Table A-4 and Figure A-4 present the frequencies and selected modes for this damage case.

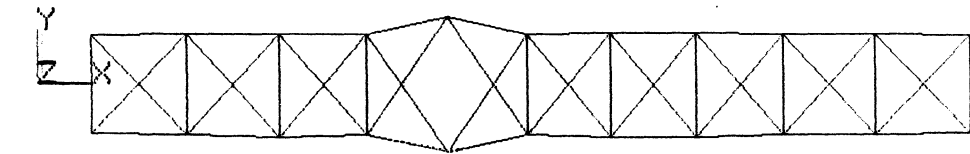
Finally, a diagonal member was deleted from the truss to produce the third damaged case. The modal analysis results showed that this damage produced more distortion of the mode shapes than the other damage types. The fifth bending mode is the tenth mode of the structure in this case. Also, coupling of bending and axial modes as can be seen in the summaries presented in Table A-4 and Figure A-5.

Stiffness Matrix Structure

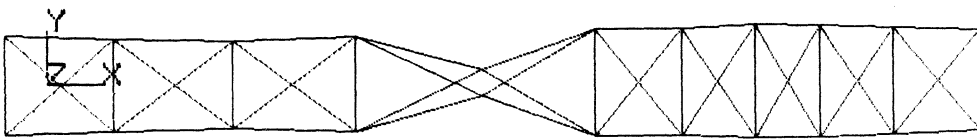
The global stiffness matrix that results from the finite element modeling of this planar truss may be used to locate the “damaged” member of the truss. The stiffness matrix structure is revealed by examining the local stiffness matrix for a truss element and the global matrix assembly procedure. Element stiffness matrices are transformed to the proper orientation in the global reference frame by a coordinate rotation. The assembly procedure then places each element stiffness contribution in its correct position in the global stiffness matrix. A schematic of the zero-nonzero structure of the global stiffness matrix for the planar truss is presented in Figure A-6. This figure also shows patterns of contributions to the stiffness matrix from horizontal, vertical and diagonal elements. Identification of these patterns in a damaged truss may lead to location of the damaged member.

Space Station Truss Model

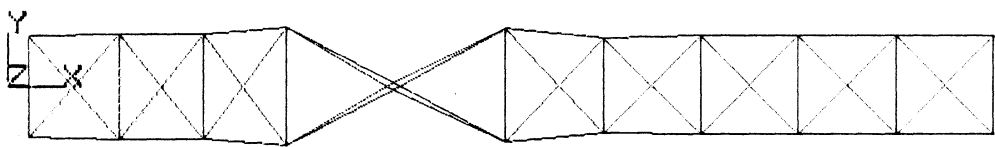
The space station truss model has been designed as the most complex of the three models used for evaluation of system identification methods. This model can be used for methods which identify



mode 6

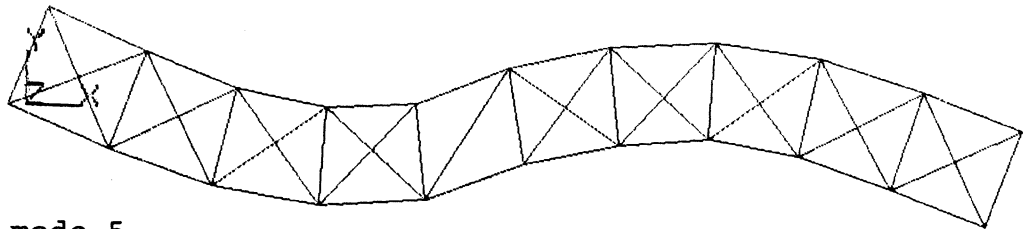


mode 8

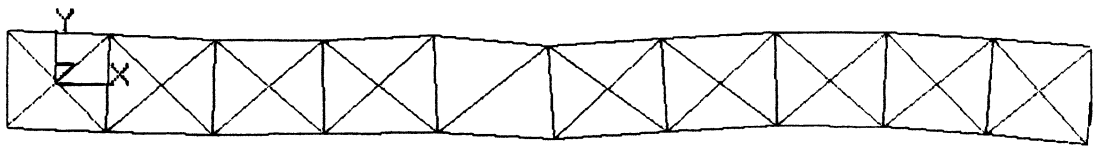


mode 10

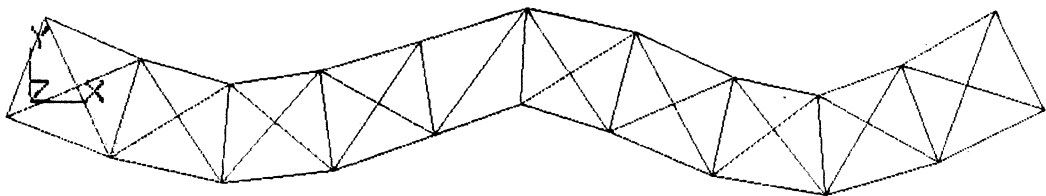
Figure A-4. Selected Modes of the Damaged Planar Truss - Case 2



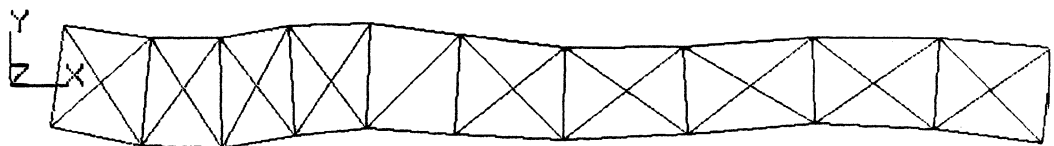
mode 5



mode 6



mode 7



mode 9

Figure A-5. Selected Modes of the Damaged Planar Truss - Case 3

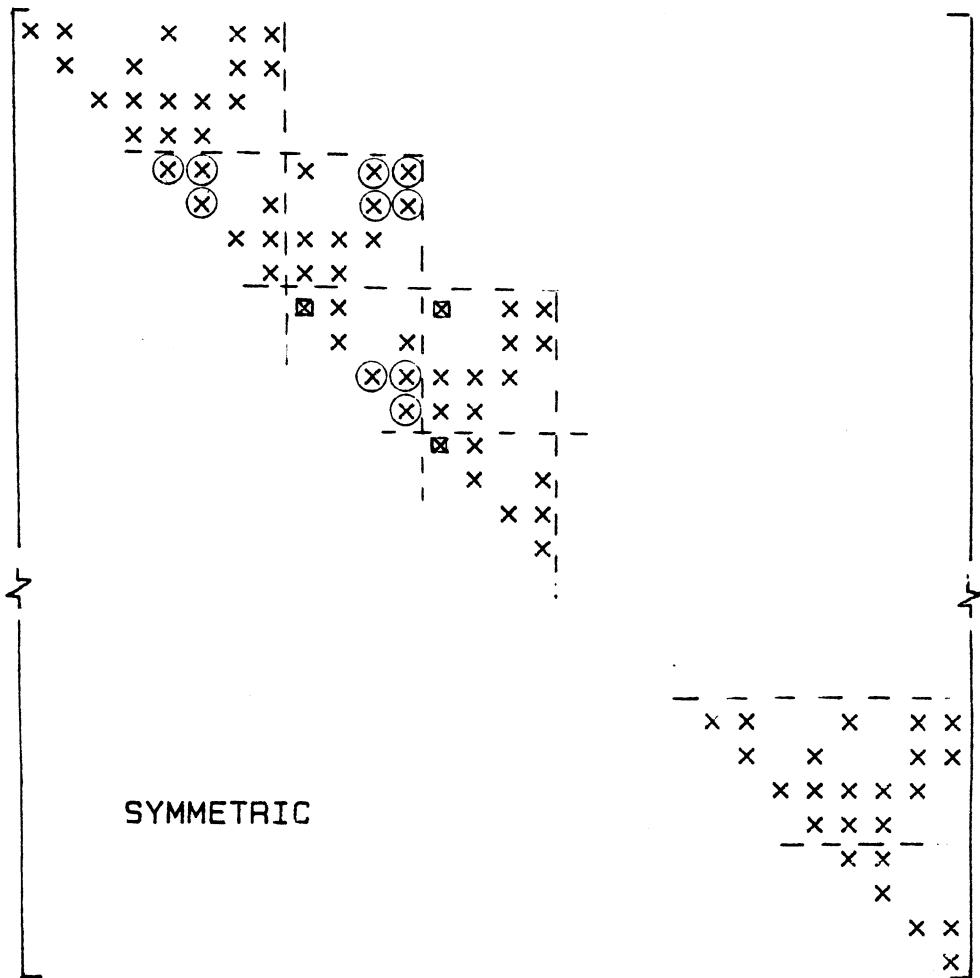


Figure A-6. Global Stiffness Matrix Schematic for the Planar Truss

the parameters of a discrete model as well as for methods which identify equivalent continuum parameters. It is a ten bay, three dimensional truss structure constructed of the members that will be used for the space station truss. The pattern of the truss, an orthogonal tetrahedral truss, is from the space station concept as well.

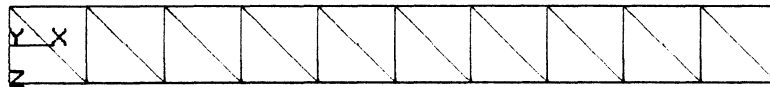
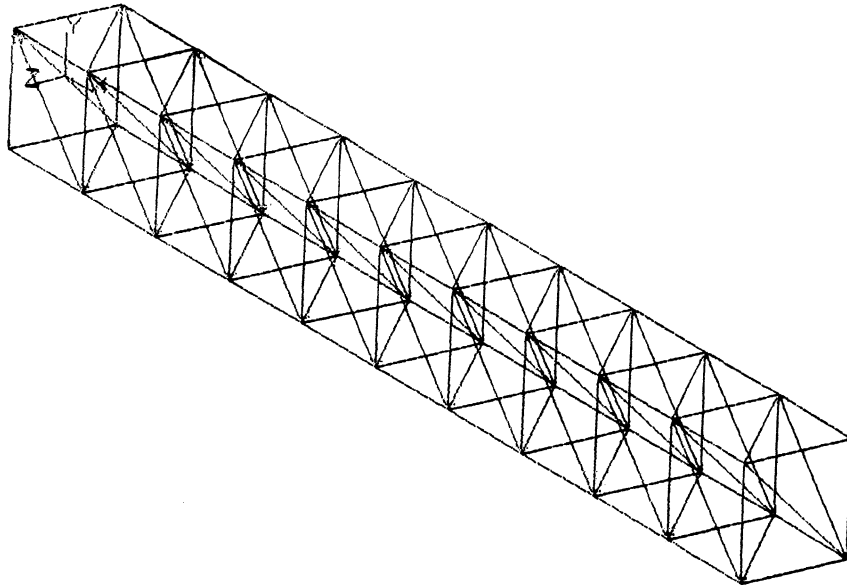
The space station will be constructed in sections on orbit. The first flight configuration consists of a ten bay truss with several appendage structures (antennas, etc.). The evaluation model designed for this damage detection program is comparable in size to the first flight structure.

As with the planar truss model, characteristics of repeated substructure, little damping and rigid body modes were desired. In addition, similarities in the vertical and transverse directions (y and z, respectively) ensure that closely spaced frequencies are a characteristic, as well.

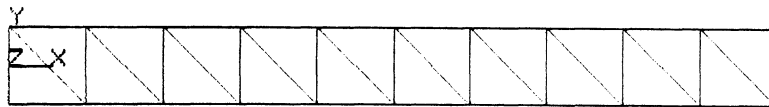
The space station truss model is shown in Figure A-7. Figure A-7 also presents the front and top views of the model for later comparison with these views of the deformed mode shapes. Table 4-1 presents the properties for the model.

Modal analysis was performed for the truss model using the finite element code ANSYS. The structure has 44 nodes with three degrees of freedom at each node. No constraints are used, so the system has 132 total degrees of freedom. Truss elements that do not include bending were used to construct the model. This is consistent with the previous model and truss joint designs that approximate pinned connections.

The first fifteen modes of the space station truss structure are presented from the modal analysis results. These modes include 6 rigid body modes, 5 bending mode pairs, 3 twisting modes and 1 axial mode. The frequencies for the deformed body modes ranged from 18.2 to 111.9 cycles/sec. A summary of these modes and their corresponding frequencies is presented in Table A-5. As would be expected from the similarity of the structure in the vertical and transverse directions, the bending modes occur in pairs with very closely spaced frequencies.



top view



front view

Figure A-7. Space Station Truss Model

Table A-5. Modal Analysis Results for the Undamaged Space Station Truss

Mode No.	Frequency cycles/sec	Comments
1-6	0.0	rigid body modes
7	18.177	B1 *
8	18.775	B1
9	27.712	T1
10	40.600	B2
11	41.446	B2
12	55.519	T2
13	62.096	A1
14	65.404	B3
15	66.100	B3

* Bx is the xth bending mode
Tx is the xth twisting mode
Ax is the xth axial mode

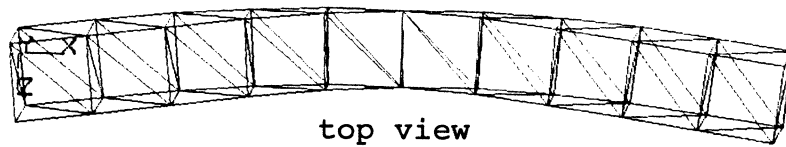
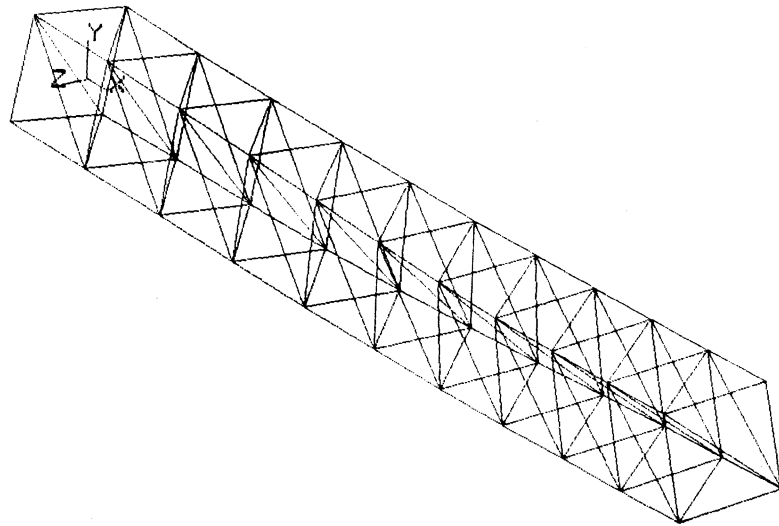
Four modes have been selected for presentation to illustrate the dynamic response of the truss structure: the first bending mode pair, the first twisting mode and the first axial mode. Figure A-8, Parts a through d, each show a mode shape from three views. To help visualization, the top near edge of the structure is highlighted in each view.

Damaged Model Results

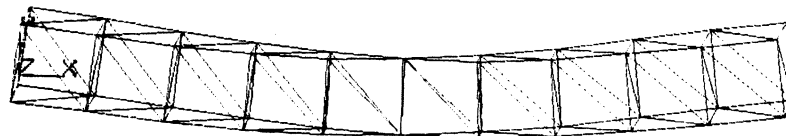
A study of the space station model response with respect to different types of damage was performed. As with the planar truss model, members were deleted to create the damaged states. However, for this three dimensional model, six cases or types of damage are studied. One member of the truss is deleted for each of the damage cases as follows: 1) longitudinal (x direction), 2) vertical (y direction), 3) transverse (z direction), 4) interface diagonal, 5) diagonal of front or back face (z faces), and 6) diagonal of top or bottom face (y faces). The first fifteen modes of the damaged systems were considered in each study.

The first type or case of damage is with a horizontal (x direction) member of the truss deleted. The lower front horizontal member of the fifth bay of the truss was removed for this study. Modal analysis of this model resulted with the frequencies presented in Table A-6. The “repeated frequencies” of several of the bending mode pairs are separated by this damage. Also, the first axial mode has a slightly reduced frequency. None of the mode shapes are substantially affected by this type of damage, however. Figure A-9 presents the first axial mode for comparison with the corresponding mode of the undamaged truss.

The front vertical member of the fifth bay of the truss was removed for the second case of damage. Again, the frequencies of the bending mode pairs were separated by the damage. However, the twisting modes, modes 9 and 12, were most affected in this case. Frequencies for each of these two modes are lower by more than 10 percent than the corresponding undamaged frequencies. The

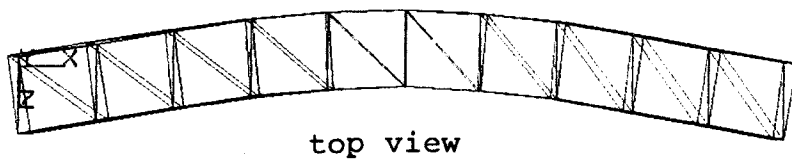
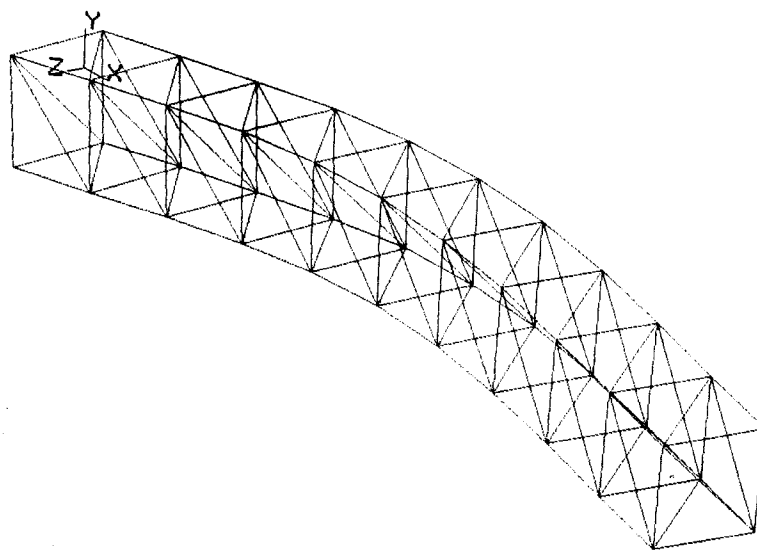


top view

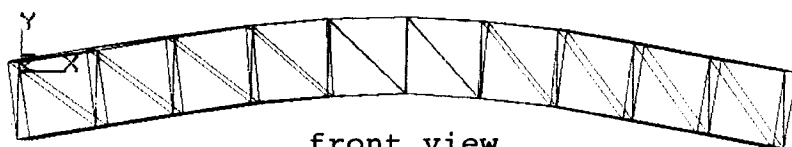


front view

Figure A-8 a). Selected Modes of the Undamaged Space Station Truss Mode 7

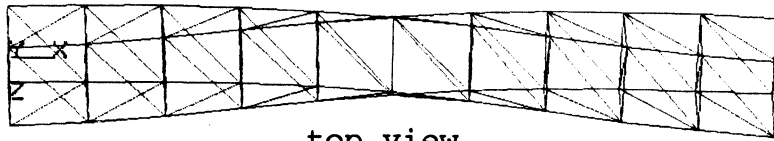
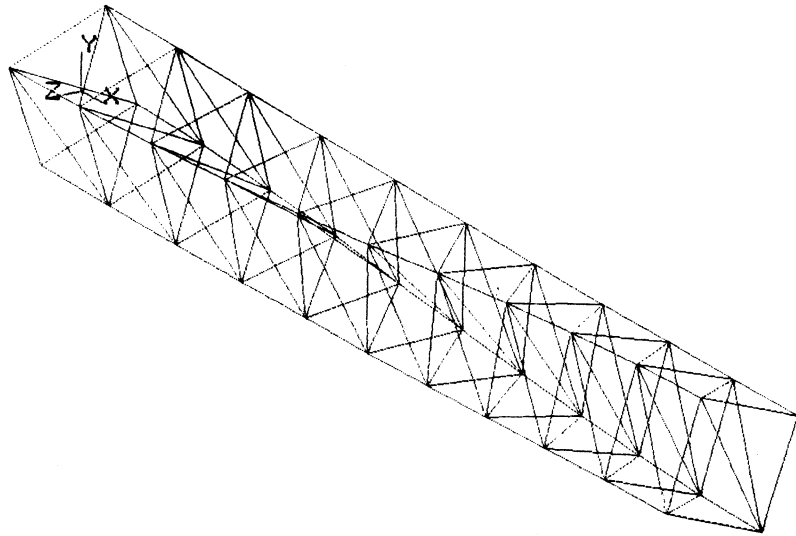


top view

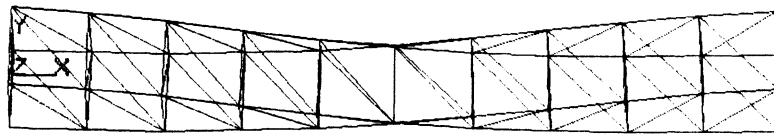


front view

Figure A-8 b). Selected Modes of the Undamaged Space Station Truss
Mode 8



top view



front view

Figure A-8 c). Selected Modes of the Undamaged Space Station Truss
Mode 9

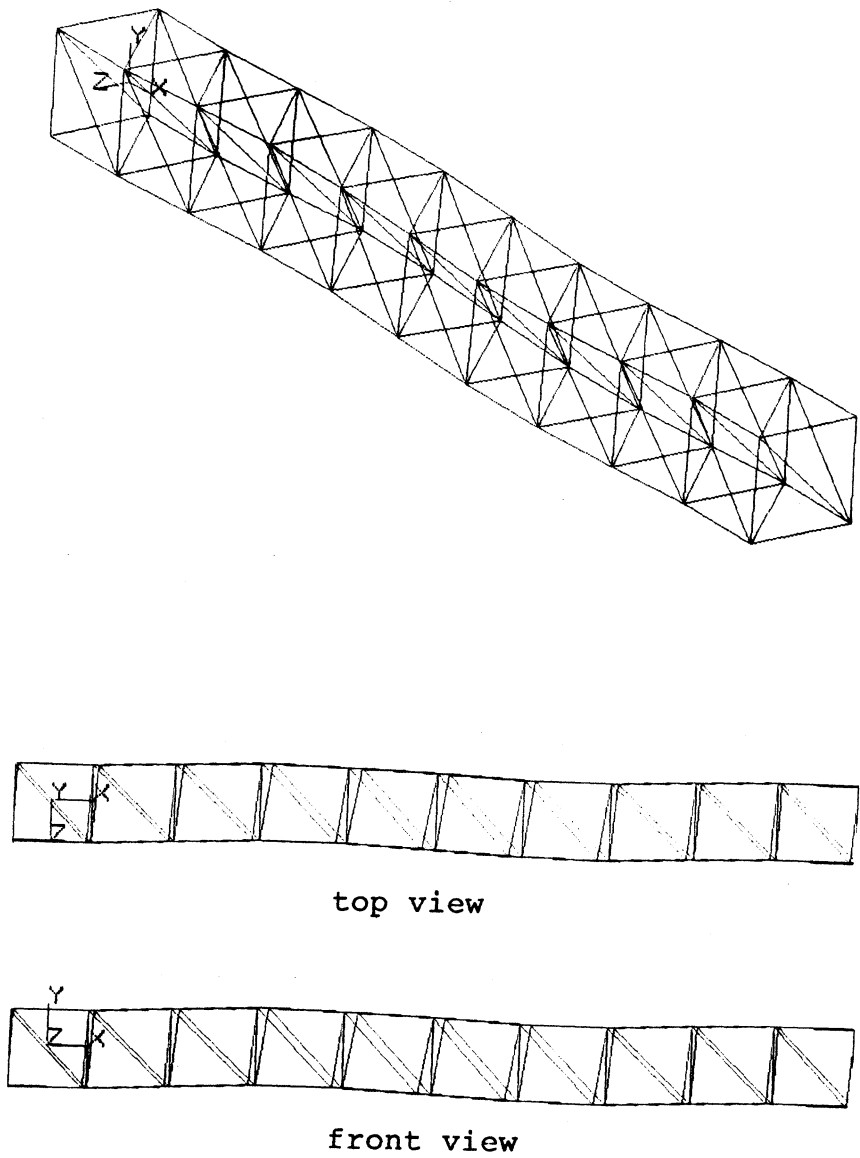


Figure A-8 d). Selected Modes of the Undamaged Space Station Truss
Mode 13

Table A-6. Modal Analysis Summary - Space Station Truss Damage Cases

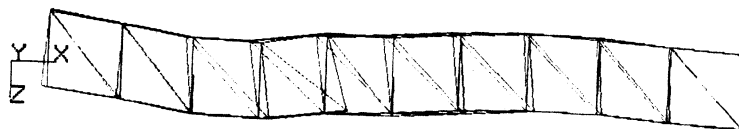
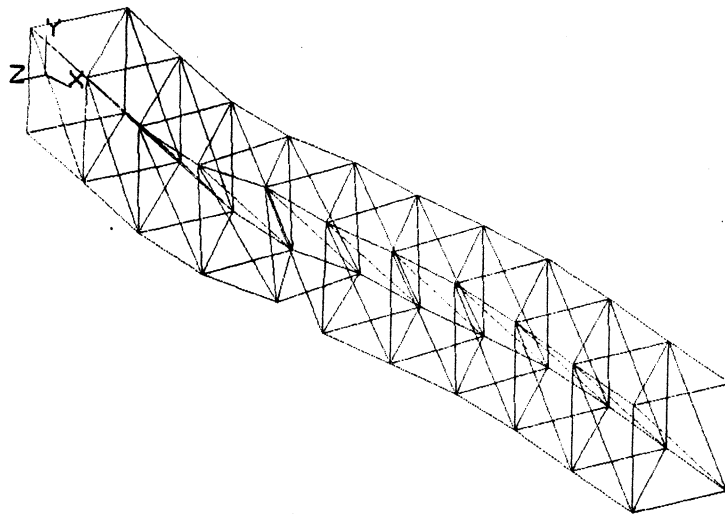
Mass No.	Frequency cycles/sec	% Difference *	Comments
a) Damage Case 1 - 5th Bay Longitudinal Member Deleted			
1-6	0.0	-	rb modes
7	13.937	23.33	B1
8	18.225	2.93	B1
9	27.769	-0.21	T1
10	37.182	8.42	B2
11	40.807	1.54	B2
12	55.673	-0.28	T2
13	58.690	5.49	A1
14	65.459	-0.08	B3
15	66.297	-0.30	B3
b) Damage Case 2 - 5th Bay Vertical Member Deleted			
1-6	0.0	-	rb modes
7	16.880	7.14	B1
8	18.735	0.21	B1
9	24.257	12.47	T1
10	37.958	6.51	B2
11	40.928	1.25	B2
12	46.321	16.57	T2
13	59.560	8.94 *	B3
14	62.182	-0.14 *	A1
15	65.961	0.21	B3
c) Damage Case 3 - 5th Bay Transverse Member Deleted			
1-6	0.0	-	rb modes
7	18.114	0.35	B1
8	18.648	0.68	B1
9	21.522	22.34	T1
10	36.302	10.59	B2
11	41.235	0.51	B2
12	55.804	-0.51	T2
13	59.124	9.60 *	B3
14	62.201	-0.17 *	A1
15	65.686	0.63	B3

* Compared to corresponding mode of undamaged truss (A1 to A1, etc.)

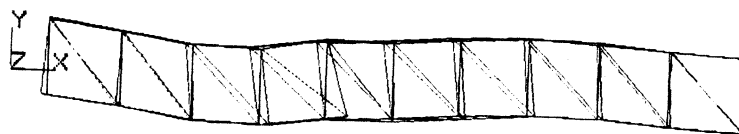
Table A-6. Modal Analysis Summary - Space Station Truss Damage Cases (Continued)

Mass No.	Frequency cycles/sec	% Difference *	Comments
d) Damage Case 4 - 4th/5th Bay Interface Diagonal Deleted			
1-6	0.0	-	rb modes
7	18.256	-0.43	B1
8	18.872	-0.52	B1
9	27.742	-0.11	T1
10	40.762	-0.40	B2
11	41.633	-0.45	B2
12	55.710	-0.34	T2
13	62.152	-0.09	A1
14	65.581	-0.27	B3
15	66.253	-0.23	B3
e) Damage Case 5 - 5th Bay Front Diagonal Deleted			
1-6	0.0	-	rb modes
7	17.316	4.74	B1
8	18.828	-0.28	B1
9	23.320	15.85	T1
10	36.581	9.90	B2
11	41.008	1.06	B2
12	50.664	8.74	T2
13	60.480	7.53 *	B3
14	62.217	-0.19 *	A1
15	66.375	-0.42	B3
f) Damage Case 6 - 5th Bay Top Diagonal Deleted			
1-6	0.0	-	rb modes
7	17.995	1.00	B1
8	18.865	-0.48	B1
9	21.766	21.46	T1
10	35.989	11.36	B2
11	41.108	0.82	B2
12	55.393	0.23	T2
13	61.787	0.50	A1
14	63.845	2.38	B3
15	66.207	-0.16	B3

* Compared to corresponding mode of undamaged truss (A1 to A1, etc.)



top view



front view

Figure A-9. Selected Mode of the Damaged Space Station Truss - Case 1
Mode 13

mode shape for the first twisting mode is presented in Figure A-10, while the results of the modal analysis are presented in Table A-6 b).

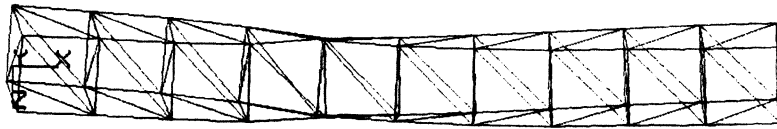
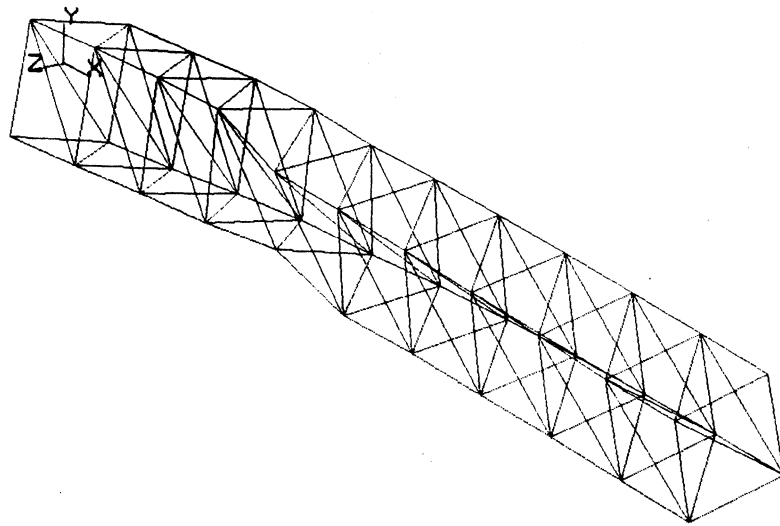
Damage case three was created by deleting the top transverse member of the fifth bay of the undamaged truss. Table A-6 c) is a summary of the frequencies, percent change of the frequencies (with respect to their corresponding mode in the undamaged truss), and character of the modes. The first bending mode pair is unaffected by the damage, but the next two bending mode pairs are split, with approximately 10 percent reduction of the frequency of one mode in each pair. The frequency of the first twisting mode has the largest percent change, though.

Damage case four is remarkable because this damage does not appreciably affect the response of the truss. The interface diagonal of the fifth bay is deleted for this case. Table A-6 d) is the summary of the modal analysis results and shows percent changes of less than 1% for the nine modes considered.

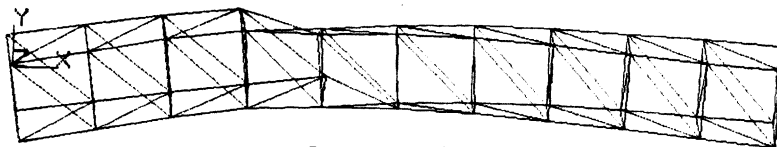
The diagonal member of the front face of the fifth bay of the truss was deleted to produce the fifth case of damage. Again, the bending mode pairs frequencies were split in the modal analysis results and the twisting modes frequencies are considerably changed. The summary of the analysis results is in Table A-6 e).

Finally, the diagonal member of the top face of the truss was deleted for damage case six. Modes 9 and 10, the first twisting mode and a second bending mode, are the most affected by the damage. Table A-6 f) presents the frequency results.

Each case of damage has a different effect on the response of the truss. Some of the modes are affected by one type of damage and not another. However, the bending and twisting modes appear, in general, to be more susceptible to damage changes.



top view



front view

Figure A-10. Selected Mode of the Damaged Space Station Truss - Case 2 Mode 9

Stiffness Matrix Structure

The global stiffness matrix that results from the finite element representation of this space station truss model led to the six damage classifications in the sensitivity study. Assembly of the local stiffnesses into the global matrix causes different patterns of change in the global stiffness matrix for each of the damage cases considered. A schematic of the zero-nonzero structure of the stiffness matrix single bay of the space station truss is presented in Figure A-11. As an example of the pattern of change that occurs for a damaged truss, the changed elements for Case 6, where a diagonal member is deleted from the top face of the truss, are highlighted. Note that the off-diagonal elements of the bay stiffness matrix associated with the deleted member of the truss will change from non-zero to zero. Even when the bay stiffness matrices are assembled into the global stiffness matrix, this same non-zero to zero change results. This pattern of change should be useful in detecting and locating damage to the truss structure.

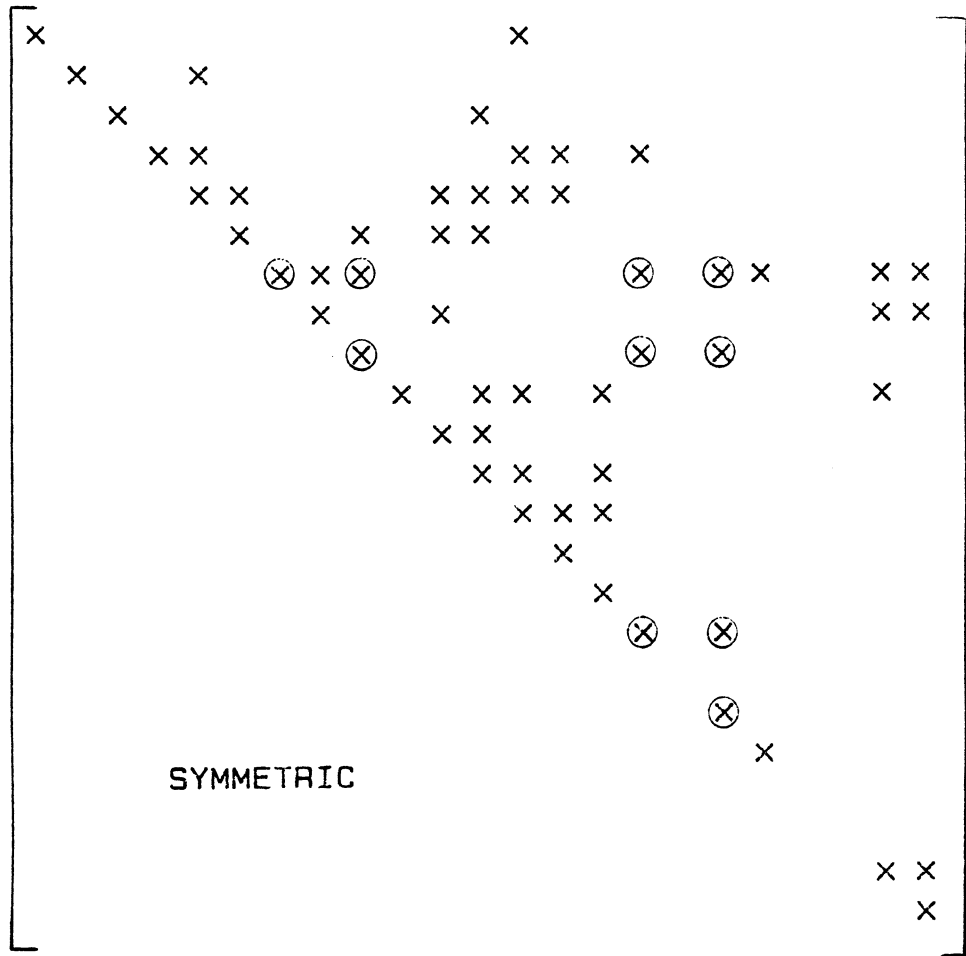


Figure A-11. Bay Stiffness Matrix Schematic for the Space Station Truss

Appendix B. Kabe's System Identification

Performance

Kabe's method of stiffness matrix adjustment was selected to be the central algorithm in the damage location process. Some aspects of the performance of this identification method are revealing, although not directly related to the process of damage detection. Details of the method's derivation and solution are found in References 33 and 34. Some are presented here as background for the performance discussion. A discussion of the number of modes needed for complete identification follows, including eigenvalue and eigenvector results for a typical damage case. When Kabe's method is applied to a classical identification situation, the stiffness matrix element results are not as expected, considering the excellent damage detection results. Modes used for the identification are matched, but stiffness elements are not adjusted correctly. Finally, traits of the auxiliary problem are presented as well.

Lagrange multipliers are used to expand the error function,

$$\sum_{i=1}^n \sum_{j=\text{adj}(i)}^n \left(\frac{K_{ij} - k_{ij}}{k_{ij}} \right)^2, \quad (B-1)$$

with the constraints,

$$K\phi = M\phi\omega^2, \quad K = K^t, \quad \text{and sparsity}(K) = \text{sparsity}(k), \quad (B-2)$$

where

$[K]$ is the nxn adjusted stiffness matrix;

$[k]$ is the nxn original stiffness matrix;

$[M]$ is the nxn original mass matrix;

$[\phi]$ is the nxp matrix of p measured modal vectors;

$[\omega^2]$ is the pxp diagonal matrix of measured eigenvalues;

“j = adj(i)” represents all j indices where $k_{ij} \neq 0$; and

n is the number of degrees of freedom in the model.

Kabe uses a “scalar matrix multiplication operator”, \odot , to incorporate the sparsity constraint into the dynamic equilibrium constraint. Each element of the adjusted stiffness matrix, $[K]$, is the product of the corresponding elements of the original stiffness matrix, $[k]$, and an adjustment matrix, $[\gamma]$ as follows:

$$[K] = [k] \odot [\gamma] \rightarrow K_{ij} = k_{ij}\gamma_{ij}. \quad (B-3)$$

Therefore, zero elements in the original stiffness matrix are constrained to remain zero elements in the final result,

$$(k \odot \gamma) \phi = M \phi \omega^2. \quad (B-4)$$

Minimization with respect to γ and the Lagrange multipliers produces a set of equations which are assembled into an auxiliary problem. The resulting problem to solve for the Lagrange multipliers is an $np \times np$, linear, symmetric, indefinite system, $A\lambda = b$. Symbolically,

$$([\alpha] + [\beta]) \{\lambda\} = \{b\}. \quad (B-5)$$

The coefficient matrix is formed as the sum of two symmetric matrices, $[\alpha]$ and $[\beta]$, which are defined as follows:

$$[\alpha] = \begin{bmatrix} [G^1] & 0 & \dots & 0 \\ 0 & [G^2] & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & [G^n] \end{bmatrix} \quad (B-6)$$

where $[G^i] = -[\phi]^t [k^2]_i [\phi]$ and $[k^2]_i$ is an nxn diagonal matrix of the i th row of $[k] \odot [k]$ and

$$[\beta] = \begin{bmatrix} [H]_{11} & [H]_{12} & \dots & [H]_{1n} \\ [H]_{21} & [H]_{22} & \dots & [H]_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ [H]_{n1} & [H]_{n2} & \dots & [H]_{nn} \end{bmatrix} \quad (B-7)$$

where

$$\begin{aligned} [H]_{ij} &= -\{\hat{\phi}\}_j \{D^j\}_i^t \\ \{\hat{\phi}\}_j &= \text{jth column of } [\phi]^t \\ \{D^j\}_i^t &= \text{ith column of } [\phi]^t [k^2]_j. \end{aligned}$$

Elements of the right-hand-side vector, b , are the elements, row by row, of the matrix, B , assembled from original model data and measured modal data,

$$[B] = 4([M][\omega^2] - [k][\phi]), \quad (B-8)$$

then

$$[B] = \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1p} \\ B_{21} & B_{22} & \dots & B_{2p} \\ \vdots & \vdots & \ddots & \vdots \\ B_{n1} & B_{n2} & \dots & B_{np} \end{bmatrix} \rightarrow \{b\} = \{B_{11} \ B_{12} \ \dots \ B_{1p} \ B_{21} \ \dots \ B_{np}\}^t \quad (B-9)$$

λ is a column vector of the unknown Lagrange multipliers, also loaded row by row from the $n \times p$ matrix used in the expanded cost function. Once the Lagrange multipliers are known, one easily obtains the adjusted stiffness matrix from the original stiffness matrix and the modal data:

$$[K] = [k] - 1/4[k^2] \odot ([\lambda][\phi]^T + [\phi][\lambda]^T). \quad (B-10)$$

In summary, implementation of Kabe's procedure is straightforward. The size of the problem, original stiffness and mass matrices, number of measured modes and modal data are input for the structure. A sequence of matrix manipulations assembles the symmetric indefinite system of equations which is solved for the Lagrange multipliers. The adjusted stiffness matrix is easily calculated from these values with Equation (B-10).

One feature of the method is that inclusion of additional modes, past a certain threshold, does not improve the identification performance. The number of independent stiffness matrix elements to be identified is approximated by the number of diagonal elements plus the number of nonzero elements on one side of the diagonal. For the planar truss, this number is 159. Considering each value in the mode shape data and the eigenvalue data as separate bits of information for the solution, between three and four modes should be needed to completely determine the stiffness matrix elements. In fact, with four modes the system is over constrained.

Original, adjusted and exact values of the stiffness matrix elements are presented in Table 4-2. Three elements show the most adjustment, matching the damage pattern created by deleting the lower

member of the fifth bay of the planar truss. Most important, for this 44 degree-of-freedom model, only three modes of the structure are necessary to achieve this result. With four exact-data modes used for the identification, only slightly better results occur.

Further illustration of this feature comes from examining the eigenvalue results from modal analyses with the stiffness and mass matrices. Table B-1 presents the eigenvalues for the first 25 modes of the planar truss from the original model, the exact model and adjusted models using 2, 3, 4 and 5 exact-data modes for the identification. The damage case described above was used for this example.

Several items should be noticed in the result. First, the three rigid body mode frequencies are corrupted by the identification method, so that they are no longer zero-valued. Using four and five modes in the identification procedure over constrains the problem and reduces this corruption somewhat. Secondly, exact-data modes imposed as constraints are reproduced in the eigenvalue results for the adjusted model. However, all frequencies are adjusted, generally toward the exact values, as demonstrated by the results in Table B-1. Finally, modes 15 and 16 illustrate the threshold for the number of modes needed for identification. Using two modes for the identification has improved these eigenvalues from the original values, but using three modes results with essentially the exact values. Adding a four and a fifth mode only slightly better the result over that with three modes.

The eigenvectors from these modal analyses illustrate this feature more clearly. A comparison of the direction of the eigenvectors from the original model to the direction of the corresponding eigenvectors from the exact model is accomplished by calculating the cosine of the angle between the vectors. If the cosine is 1.0, the two corresponding vectors are aligned. Table B-2 presents the alignment of the first seven elastic mode eigenvectors for each of the identification tests above. The left column presents the cosine for each original model vector with respect to the corresponding exact model vector. The same calculations were performed for the vectors resulting from identification with 2, 3, 4, and 5 modes. None of the original vectors are aligned with the exact

Table B-1. Comparison of Eigenvalue Results from Kabe's Identification Method (Planar Truss Damage Case B5cas1)

Mode Number	Original Model	Adjusted Models (no. of modes used for identification)				Exact Model
		2	3	4	5	
1	0.588460D-08	-0.136173D + 06	-0.687235D + 05	0.282061D + 01	-0.268198D + 01	0.657785D-08
2	0.149303D-07	-0.929099D + 02	0.162829D + 03	0.128518D + 02	0.255052D + 01	0.142022D-07
3	0.181086D-07	0.784084D + 04	0.919105D + 03	0.737272D + 03	0.343314D + 01	0.164019D-07
4	0.225407D + 05	0.107476D + 05*	0.107676D + 05*	0.107426D + 05*	0.107706D + 05*	0.107684D + 05
5	0.143051D + 06	0.131046D + 06*	0.131041D + 06*	0.131039D + 06*	0.131043D + 06*	0.131040D + 06
6	0.251212D + 06	0.186975D + 06	0.186088D + 06*	0.186083D + 06*	0.186085D + 06*	0.186087D + 06
7	0.448910D + 06	0.413146D + 06	0.410500D + 06	0.409314D + 06*	0.409322D + 06*	0.409319D + 06
8	0.994478D + 06	0.860447D + 06	0.889759D + 06	0.925340D + 06	0.925275D + 06*	0.925276D + 06
9	0.101637D + 07	0.962028D + 06	0.959070D + 06	0.101166D + 07	0.101053D + 07	0.101053D + 07
10	0.180124D + 07	0.169663D + 07	0.164563D + 07	0.164376D + 07	0.164361D + 07	0.164360D + 07
11	0.232104D + 07	0.196499D + 07	0.199657D + 07	0.199451D + 07	0.199439D + 07	0.199439D + 07
12	0.284737D + 07	0.276066D + 07	0.277948D + 07	0.277448D + 07	0.277450D + 07	0.277450D + 07
13	0.405448D + 07	0.383590D + 07	0.384122D + 07	0.383123D + 07	0.383124D + 07	0.383124D + 07
14	0.415785D + 07	0.405283D + 07	0.404436D + 07	0.406408D + 07	0.406373D + 07	0.406372D + 07
15	0.524586D + 07	0.512422D + 07	0.504167D + 07	0.504550D + 07	0.504458D + 07	0.504458D + 07
16	0.577623D + 07	0.548372D + 07	0.553087D + 07	0.553145D + 07	0.553071D + 07	0.553069D + 07
17	0.618207D + 07	0.570047D + 07	0.570305D + 07	0.578126D + 07	0.578077D + 07	0.578078D + 07
18	0.625601D + 07	0.613998D + 07	0.616904D + 07	0.619716D + 07	0.619652D + 07	0.619652D + 07
19	0.659527D + 07	0.647672D + 07	0.652436D + 07	0.653345D + 07	0.653282D + 07	0.653282D + 07
20	0.669967D + 07	0.668548D + 07	0.678159D + 07	0.670063D + 07	0.669963D + 07	0.669962D + 07
21	0.693902D + 07	0.693899D + 07	0.697148D + 07	0.689973D + 07	0.689891D + 07	0.689892D + 07
22	0.727606D + 07	0.703623D + 07	0.703166D + 07	0.702932D + 07	0.702764D + 07	0.702766D + 07
23	0.728989D + 07	0.723463D + 07	0.730533D + 07	0.728059D + 07	0.727947D + 07	0.727947D + 07
24	0.739457D + 07	0.733639D + 07	0.734696D + 07	0.732481D + 07	0.732435D + 07	0.732434D + 07
25	0.831481D + 07	0.795289D + 07	0.797357D + 07	0.793408D + 07	0.793291D + 07	0.793289D + 07
26	0.941458D + 07	0.903896D + 07	0.900551D + 07	0.898544D + 07	0.898531D + 07	0.898534D + 07
27	0.100979D + 08	0.989209D + 07	0.984135D + 07	0.978608D + 07	0.978635D + 07	0.978631D + 07
28	0.102753D + 08	0.101522D + 08	0.101456D + 08	0.101191D + 08	0.101206D + 08	0.101206D + 08
29	0.105121D + 08	0.103301D + 08	0.103153D + 08	0.102849D + 08	0.102885D + 08	0.102884D + 08
30	0.114806D + 08	0.110297D + 08	0.109273D + 08	0.108455D + 08	0.108467D + 08	0.108468D + 08

* modes used for identification

eigenvectors. With four modes imposed, all of the vectors from the adjusted model are essentially aligned with those of the exact model. Adding a fifth mode does not improve the result.

For the exact data cases, using four modes in the identification does produce slightly better results than using three modes. Alignment of the eigenvectors illustrates this. However, the constraints in the problem are consistent, so over constraining the problem does not create an impossible solution. Numerical studies with inexact data in the original model or in the modal data, produced better results with three modes in all cases. Therefore, three modes were used for all damage detection cases presented in the body of this dissertation.

Kabe's method constrains the adjusted stiffness matrix to match the imposed modes, but many stiffness matrices could satisfy these requirements. The optimization problem produces the "closest" solution to the original matrix that satisfies all of the constraints. In some identification situations, the closest matrix may not be the correct result. For example, the original model for the undamaged planar truss was constructed to approximate a classical identification problem. All of the stiffness elements were 5 percent low, as if the available data for the material was incorrect by that amount. Although all of the eigenvectors of the original model are aligned with those of the exact model, the eigenvalues of the original model are 5 percent low, as well. Kabe's method applied to this problem does not adjust the stiffness matrix as it should (5 percent for every element), but adjusts the matrix to reproduce the three imposed modes. The strength of Kabe's method for damage detection is its ability to use limited modal data. For classical identification problems, more data is apparently required to achieve comparable results.

The auxiliary problem to solve for the Lagrange multipliers is a drawback of Kabe's method. The size of the symmetric, indefinite coefficient matrix is the product of the number of modes used for the identification and the number of degrees of freedom in the model. Numerical studies have shown that the coefficient matrix is essentially singular for problems when more than one mode is used in the identification process. This is true with exact or with inexact mode shape data. Note that

Table B-2. Comparison of Eigenvector Cosine Results from Kabe's Method (Planar Truss Damage Case B5cas1)

Mode Number	Cosine of (Original to Exact)	Cosine of (Adjusted To Exact) (no. of modes used for identification)			
		2	3	4	5
4	0.983892D+00	0.999990D+00*	0.100000D+01*	0.999999D+00*	0.100000D+01*
5	0.931325D+00	0.100000D+01*	0.100000D+01*	0.100000D+01*	0.100000D+01*
6	0.851722D+00	0.982304D+00	0.100000D+01*	0.100000D+01*	0.100000D+01*
7	0.962681D+00	0.995904D+00	0.999975D+00	0.100000D+01*	0.100000D+01*
8	0.883281D+00	-0.800153D+00	-0.825951D+00	0.999993D+00	0.100000D+01*
9	0.850096D+00	0.810950D+00	0.828574D+00	0.999991D+00	0.100000D+01
10	0.849492D+00	0.992587D+00	0.999821D+00	0.100000D+01	0.100000D+01

* modes used for identification

only the original stiffness matrix and the mode shape data is used to construct the auxiliary problem coefficient matrix.

Solution of the auxiliary problem was accomplished with the DSISL and DSIFA subroutines from the LINPAK library. These routine are designed to handle the indefinite system. Kabe [33,34] presents a different, but equivalent, solution technique. Neither of these exploit the structure of the coefficient matrix of the auxiliary problem or the sparsity that develops in the problem with added modes in the identification. A conjugate gradient solution, designed for symmetric, indefinite problems could be used to solve the auxiliary system without explicit assembly of the large coefficient matrix. The predictable structure allows computation of a matrix-vector product, the necessary operation for a conjugate gradient method. However, in implementing a solution of this type, the singular nature of the coefficient matrix and its effect on the solution should be considered.

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