A General Optimal Substation Coverage Algorithm for Phasor Measurement Unit Placement in Practical Systems

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Abstract—The primary objective of the conventional optimal phasor measurement unit (PMU) placement problem is the minimization of the number of PMU devices that, when placed in a power system, measure all bus voltages. However, due to advancements in the field of relay technology, digital relays can now act as PMUs. This has significantly reduced device costs. Moreover, although the goal is to observe all the buses, the devices themselves can only be placed in substations, whose upgrade costs are much higher than those of the devices. Considering these factors, the approach proposed here *simultaneously* optimizes the number of substations where traditional PMUs and dual-use line relay PMUs can be placed. The general optimal substation coverage (GOSC) algorithm presented in this paper is also able to incorporate practical requirements such as redundancy in the measurement of critical elements of the system, and estimation of the tap ratios of the transformers present. Simulation results indicate that the GOSC algorithm provides significant techno-economic benefits.

Keywords—Criticality, integer linear programming (ILP), observability, optimal substation coverage, phasor measurement unit (PMU) placement, redundancy, tap setting estimation.

1. INTRODUCTION

Phasor measurement units (PMUs) are devices that provide real-time voltage and current phasor measurements at those locations of a power system network where they are placed. When present at a sufficiently large number of locations inside the grid, these devices are capable of creating a power system state estimator that is completely linear and non-iterative. Called a linear state estimator (LSE), it has numerous advantages over the classical state estimator in terms of speed, accuracy, and reliability [1]. However, PMUs cannot be placed at random inside the power system. This is primarily because of the associated communication infrastructure costs, as well as the costs incurred in upgrading substations. A US Department of Energy (DOE) report identified the communication infrastructure cost as the major portion of a PMU installation cost [2]. Labor and substation outage costs were identified as the next most significant cost drivers. The report concluded by noting that the PMU device cost was less than 5% of the total installed synchrophasor system cost. In light of this report, it is clear that the objective of the traditional optimal PMU placement (OPP) problem of "minimizing the number of devices that must be added to the system for its complete observability" is no longer valid. An alternate formulation called "general optimal substation coverage" (GOSC) is proposed in this paper to minimize the overall synchrophasor installation cost while simultaneously catering to practical constraints of realistic systems.

The rest of the paper is structured as follows. An overview of the state-of-the-art with respect to the OPP problem as well as the salient features of the proposed GOSC algorithm are presented in Section 2. Section 3 explains the GOSC formulation and associated constraints that it can address. Results obtained by applying the GOSC algorithm to standard IEEE systems as well as a 2383-bus Polish system are described in Section 4. Our conclusions are provided in Section 5.

2. THE OPTIMAL PMU PLACEMENT (OPP) PROBLEM

The goal of the traditional OPP problem is to ensure complete network observability. Observability is defined as the ability to measure the complex voltages (known as states) of a power network. A bus x is directly observable if a PMU is placed on x. A bus y is "indirectly observable" if y is connected to a directly observed bus by transmission lines whose parameters are assumed to be known. From a graph-theoretic perspective, where buses are nodes and transmission lines are edges, the traditional OPP problem is a variant of the minimum dominating set problem [3].

Mathematical techniques applied to solve the OPP problem include genetic algorithms [4], linear programming [5], semi-definite programming [6], particle swarm optimization [7], Tabu search [8], etc. However, integer linear programming (ILP) has emerged as the most popular choice for solving OPP problems [9]. The reason for this is that unlike meta-heuristic approaches, ILP always gives an optimal solution. Due to its inherently heavy computational burden, the application of ILP to large systems was a concern [10]. However, with the emergence of efficient optimizers such as GUROBI and CPLEX, the computational time required for finding an OPP set is no longer substantial even for large systems.

In terms of practical constraints, popular topics have been the presence of zero injection (ZI) buses [11], incorporation of conventional measurements [12], consideration of communication infrastructure costs [13], provisions for redundancy [14], and accounting for measurement channel capacity [15]. Most of the papers published on the topic of OPP have tried to address one or more of these constraints in their formulations. For a more detailed description of PMU-based applications and their placement methodologies, we refer the reader to [16].

All papers on PMU placement mentioned above considered the following formulation: an optimal placement set must satisfy all the specified constraints and use the least number of PMUs for doing so. This is because the cost of the device was assumed to be the primary reason for not placing PMUs at all buses. However, in a real system, there are multiple voltage levels (buses) at a particular substation and the tap settings between the different voltages are not usually known. Thus, different voltage levels are decoupled from the point of view of observability. Secondly, although the buses must be observed by PMUs, the PMUs themselves can only be placed inside the substations. Therefore, a distinction must be made between buses and substations when choosing optimal locations for PMU placement. It has already been shown in [2], [7] that minimizing the number of PMUs does not minimize the cost of PMU deployment. Likewise, during an actual implementation at Dominion Virginia Power (DVP), a US-based utility, it was observed that the majority cost associated with synchrophasor deployment was not due to the devices themselves but rather due to substation outage and infrastructure/labor costs. Motivated by this, in this paper, we study the problem of minimizing the total number of substation locations where installations are performed to observe all the buses of the network. In [17], the task of placing PMUs was done substation-wise. However, the goal of [17] was to minimize the total number of devices. As pointed in [2] and [7], the cost of the devices does not represent the major portion of the synchrophasor deployment cost. Therefore, from a cost minimization perspective the formulation developed in [17] is not optimal.

We refer to the formulation for the minimization of substation installations as the optimal *substation coverage* problem, and develop the general optimal substation coverage (GOSC) algorithm for this problem. Ref. [18] had also proposed an optimal substation coverage algorithm. The differences between this paper and [18] are as follows:

1. In [18], the optimization was performed using binary particle swarm optimization (BPSO), while the proposed approach uses ILP. The latter always gives an optimal solution, while BPSO is not guaranteed to always give an optimal solution.
2. This paper gives a detailed description of how the ZI bus condition should be appropriately considered. In [18], all ZI buses were treated the same way.
3. The algorithm developed in [18] was meant for optimizing locations of traditional (bus) PMUs only. The locations of the dual use line relay (branch) PMUs were assumed to be "known" in that paper. The primary focus of this paper is the simultaneous optimization of traditional PMU (TPMU) and dual-use line relay PMU (DULRP) locations. To the best of our knowledge, no other publication on PMU placement has performed such a simultaneous optimization.

To summarize, the key features of the GOSC algorithm are:

1. GOSC ensures complete observability of all buses by placing PMUs in a minimum number of substations under the assumption that all transformer tap ratios are unknown.

2. Once a substation is chosen for PMU placement using GOSC, all lines present inside the substation as well as the ones that connect it to other substations are monitored by PMUs.
3. Numbers and locations of TPMUs and DULRPs are *simultaneously* optimized.

3. THE GENERAL OPTIMAL SUBSTATION COVERAGE (GOSC) ALGORITHM

A mathematical formulation of GOSC is developed as follows.

3.1. Terminology Used

Let the power network be represented by an undirected graph $G(V, E)$, where V is the set of nodes (buses) and E is the set of edges (transmission lines or transformers). Let the node set V be partitioned into $k \geq 2$ blocks $B_1, B_2, \dots B_k$ where each block represents a substation. Let r_i denote $|B_i|$ for $1 \leq i \leq k$. Now, each node of V can be denoted by a pair of integers (x, y) where $x \in \{1, 2, \dots k\}$ is the block number and $y \in \{1, 2, \dots r_x\}$ is an index number within block B_x. Next, a lexicographic ordering amongst the nodes is introduced: Given two nodes $v_1 = (x_1, y_1)$ and $v_2 = (x_2, y_2)$ from two different blocks B_{x_1} and B_{x_2}, we define $v_1 \prec v_2$ if $x_1 < x_2$. For two nodes $v_1 = (x_1, y_1)$ and $v_2 = (x_1, y_2)$ lying within the same substation, $v_1 \prec v_2$ if $y_1 < y_2$. An edge $e = \{v_1, v_2\} \in E$ can either join two nodes inside the same block, or two nodes lying in two different blocks. In the former case, $v_1 = (x_1, y_1)$ and $v_2 = (x_1, y_2)$, while in the latter case, $v_1 = (x_1, y_1)$ and $v_2 = (x_2, y_2)$. When specifying an edge e as $\{v_1, v_2\}$, it will be assumed that $v_1 \prec v_2$; thus, v_1 and v_2 can be referred to as the low end and the high end of edge e, respectively. For any node $v \in V$, the neighborhood of v, denoted by N_v, contains the node v itself and all nodes that are adjacent to v. As an extension of this definition, the neighborhood of a set of nodes X, denoted by N_X, is defined by $N_X = \bigcup_{v \in X} N_v$.

Two types of PMUs are considered in this formulation: a traditional PMU (TPMU), and a dual-use-line-relay PMU (DULRP). When placed at a node $v_1 = (x_1, y_1)$, a TPMU observes the nodes in N_{v_1} subject to the number of measurement channels available, while disrupting block x_1. When a DULRP is placed on an edge $e = \{v_1, v_2\}$, it can observe nodes v_1 and v_2. If it is placed on the $v_1 = (x_1, y_1)$ end, then block x_1 must be disrupted. Likewise, if it is placed on the $v_2 = (x_2, y_2)$ end, then block x_2 must be disrupted. As one DULRP monitors only one edge, the issue of measurement channel limitation does not arise. Finally, considering measurement channel limitations of a TPMU, and by definition for a DULRP, from an observability perspective, these devices can be thought of as being *placed* on one (in case of a DULRP) or on many (in case of a TPMU) *edges* of a network. A mathematical formulation based on the terminology described here is developed below.

3.2. Mathematical Formulation

The basic version of the optimization assumes that the given system does not have any pre-installed TPMUs or DULRPs. Its goal is to place PMUs inside the network so that the following conditions are satisfied: *(a)* all the buses are observed; and *(b)* the number of affected substations is minimized. An ILP-based formulation of this problem is developed as follows.

For each substation B_i for $1 \leq i \leq k$, there is a binary valued variable y_i such that

$$y_i = \begin{cases} 1 & \text{if substation } B_i \text{ is disrupted} \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

For each edge e, there are two binary valued variables w_e^l and w_e^h such that

$$w_e^l = \begin{cases} 1 & \text{if PMU observes low end of edge e} \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

$$w_e^h = \begin{cases} 1 & \text{if PMU observes high end of edge e} \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

Using (1)-(3), the basic objective function can be defined as

$$\text{Minimize} \left(\sum_{i=1}^{k} y_i \right) \tag{4}$$

Three sets of constraints that must be imposed on this basic objective function are defined as follows. For any node v, let L_v denote the set of edges incident on v such that for each edge in L_v, v is the low end of that edge. Similarly, for any node v, let H_v denote the set of edges incident on v such that for each edge in H_v, v is the high end of that edge. Let $E_v = L_v \cup H_v$ denote the set of all edges incident on node v. Then, for each node $v \in V$

$$\sum_{e \in E_v} \{w_e^h + w_e^l\} \geq 1 \tag{5}$$

Eq. (5) ensures that each node v is observed by PMUs placed at any of the edges that are incident to v. For each node $v = (i, j)$, $1 \leq i \leq k$ and $1 \leq j \leq r_i$, and each edge $e \in L_v$

$$y_i \geq w_e^l \tag{6}$$

Eq. (6) ensures that if a PMU observes an edge for which $v = (i, j)$ is the low end, then B_i must be disrupted. Similarly, each node $v = (i, j)$, $1 \leq i \leq k$ and $1 \leq j \leq r_i$, and edge $e \in H_v$

$$y_i \geq w_e^h \tag{7}$$

Eq. (7) ensures that if a PMU observes an edge for which $v = (i, j)$ is the high end, then B_i must be disrupted. This completes the formulation of the basic version of the optimization. Practical constraints further imposed on the basic objective function are defined as follows:

3.2.1 *Redundancy to critical buses:* Measurement redundancy under $N - 1$ contingency has been proposed in many PMU placement papers [4]-[6], [11]. However, in order to provide redundancy to *all* phasor measurements, a large number of substations (> 50%) must be disrupted. A more practical scheme is to provide redundancy in the phasor measurements of *only* the most important buses of the system [14]. To do so, the basic GOSC formulation is modified as follows.

When a TPMU on node v fails, it loses observability of the edges it was monitoring. When a DULR on an edge e fails, it cannot observe any end point of e. Consider a given set $C_B \subseteq V$ of critical buses and an integer $t \geq 1$ that represents the maximum number of edges that lose observability (due to TPMU or DULRP failures). Then, the goal is to ensure that each node $v \in C_B$ is observed by at least one PMU even when any subset of t or fewer edges lose observability. Moreover, since the failures can be due to the device itself and/or outage of the line which that device observes, this constraint ensures observability under $N - t$ contingency. In the proposed formulation, this requirement is accommodated by ensuring that each critical bus is observed by at least $t + 1$ PMUs. This is done by replacing the constraint specified by (5) with (8).

$$\sum_{e \in E_v} \{w_e^h + w_e^l\} \geq t + 1 \quad \forall\, v \in C_B \tag{8}$$

3.2.2 Ensuring observability of important lines: Typically, these are the high-voltage (HV) lines present inside the system or the tie-lines that join one system to another. To the best of our knowledge, [18] was the first paper that placed PMUs while considering the fact that some lines needed to be observed by them. In this paper, the idea is extended further by proposing two levels of line criticality: (a) Critical Lines – those lines which must be observed by PMUs under normal conditions; and (b) Super Critical Lines – those lines which must be observed by PMUs under $N - 1$ contingency conditions. Thus, if $C_L \subseteq E$ is the set of critical lines present in a given system, then for all $e \in C_L$, (9a) must hold.

$$w_e^h + w_e^l \geq 1 \tag{9a}$$

Similarly, if $SC_L \subseteq E$ is the set of super critical lines present in a given system, then for all $e \in SC_L$, (9b) must hold.

$$w_e^h + w_e^l = 2 \tag{9b}$$

The concept of observing super-critical lines at both ends by PMUs is especially relevant for synchrophasor measurement-based fault detection and localization [19].

3.2.3 Handling prohibited substations: In the field, there exist some substations where synchrophasor installations cannot be made in the specified planning horizon. Suppose $S \subseteq \{B_1, B_2, \ldots B_k\}$ is the set of substations that cannot be disrupted. Then, for each substation $B_i \in S$, the constraint $y_i = 0$ must be added to the ILP formulation described in (4)-(7).

3.2.4 Handling pre-installed PMUs: Let $P \subseteq V$ be the set of nodes at which PMUs have already been placed. Then, PMUs placed at the nodes in P observe all the nodes in N_P. In other words, the placement of new PMUs needs to observe only the nodes in $V - N_P$. Therefore, to account for P, the constraint specified by (5) in the ILP formulation must be applied to each node $v \in V - N_P$.

3.2.5 *Considering measurement channel limitations:* Let the number of channels for measuring three-phase currents present on the j^{th} TPMU be c_j. Then, if the k^{th} substation which has l_k lines to be monitored is selected for PMU placement, the equations for computing the number of TPMUs and DULRPs required for that substation are as follows:

$$\begin{aligned} &\text{Number of TPMUs required} = \text{Quotient}(l_k, c_j) \\ &\text{Number of DULRPs required} = \text{Remainder}(l_k, c_j) \end{aligned} \tag{10}$$

In (10), $\text{Quotient}(l_k, c_j)$ denotes the (integer) quotient when l_k is divided by c_j while $\text{Remainder}(l_k, c_j)$ denotes the remainder when l_k is divided by c_j.

3.2.6 *Transformer tap ratio estimation:* In a real-system, transformers are often under local control, and so, the tap position is not communicated to the control center. An erroneous tap measurement or the presence of an unmeasured tap can lead to a high error in the state estimator [20]. Thus, utilities often wish to estimate tap settings using phasor measurements. Fig. 1 shows the equivalent circuit of an off-nominal two winding transformer with voltages and currents at sending and receiving ends denoted by $\overrightarrow{V_1}, \overrightarrow{I_1}$ and $\overrightarrow{V_2}, \overrightarrow{I_2}$, respectively. Then, considering the most general case, if the tap setting be a complex number $\vec{a}$, then using the measurements obtained from PMUs on either ends, the tap setting can be estimated using (11), where $\vec{y} = \frac{1}{R_{eq} + jX_{eq}}$ [21].

$$\begin{bmatrix} \overrightarrow{I_1} \\ \overrightarrow{I_2} \end{bmatrix} = \begin{bmatrix} \frac{\vec{y}}{|a|^2} & -\frac{\vec{y}}{\vec{a}^*} \\ -\frac{\vec{y}}{\vec{a}} & \vec{y} \end{bmatrix} \begin{bmatrix} \overrightarrow{V_1} \\ \overrightarrow{V_2} \end{bmatrix} \tag{11}$$

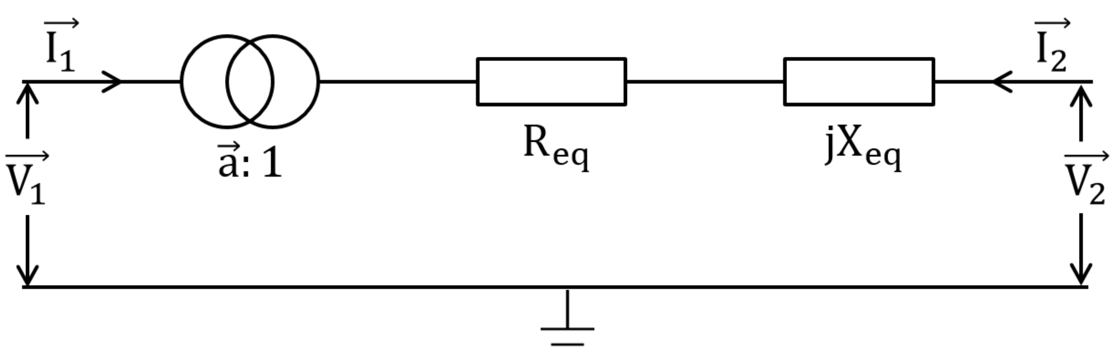

Fig. 1. Equivalent circuit of an off-nominal two winding transformer

In [22], the OPP problem was solved with the additional goal of estimating all transformer tap settings. However, it was done by minimizing number of devices/buses instead of substations. In the proposed GOSC algorithm, since all lines inside a substation selected for PMU placement are monitored, tap ratios of the transformers present in those substations can also be determined. We note that our algorithm for GOSC does not guarantee observability of tap settings of all the transformers present in the system.

3.2.7 *Considering presence of zero injection (ZI) buses:* In this paper, we consider a bus v to be a ZI bus if it satisfies the following two conditions: *(i)* bus v does not have any load or generation connected to it, and *(ii)* bus v and all the buses connected to v (i.e., the buses in $\mathrm{N_v}$) are at the same voltage level. The definition of ZI bus used by other researchers [6]-[8], [11] considered only condition *(i)* above. We added condition *(ii)* to ensure that for a ZI bus v, it is sufficient to place PMUs to observe $|\mathrm{N_v}| - 1$ of the buses in $\mathrm{N_v}$; the remaining bus in $\mathrm{N_v}$ can be observed using KCL [12]. Our new definition of ZI bus allows us to prove the following lemma.

Lemma 1: Let v be a ZI bus satisfying conditions (i) and (ii) mentioned above. Then, there cannot be any transformer with unknown tap ratio on any edge that connects v to another bus $w \in N_v$.

Proof: We prove this lemma by contradiction. Suppose there is a transformer with an unknown tap ratio on the line that connects v to w. Since the tap ratio of this transformer is unknown, even if we know the current leaving v, the current entering w is unknown. This contradicts the observability condition *(ii)* in our definition of ZI bus and the lemma follows.

In the rest of the paper, these two conditions will be collectively referred to as the $\mathrm{iso-voltage\ zero\ injection\ bus}$ (IvZIB) condition. The following methodology was implemented to reformulate (5) for the buses which satisfy the IvZIB condition [12], [23].

Let PZI be the set of buses that satisfy the IvZIB condition. Let $|\mathrm{PZI}| = \mathrm{k}$. Depending on the value of k, a set R of objects are constructed from which all the observability constraints are generated as shown below.

For $k = 1$: Let i be the single element of PZI and N_i be the neighborhood of i. For each pair of elements p and q in N_i, R contains the 2-element set $\{p, q\}$.

For $k \geq 2$: Let i and j be two elements of PZI. Let N_i and N_j denote the neighborhoods of i and j, respectively. Note that N_i and N_j may have elements in common. Let $N_{i,j} = N_i \cap N_j$, that is, $N_{i,j}$ is the set of elements that occur in both N_i and N_j. Let $N_i' = N_i - N_{i,j}$, that is, N_i' is the set of elements that are in N_i but not in N_j. Similarly, define $N_j' = N_j - N_{i,j}$. Construct $R_{i,j}$ as follows:

a. For each pair of elements p and q in N_i', add the 2-element set $\{p, q\}$ to $R_{i,j}$.
b. For each pair of elements p and q in N_j', add the 2-element set $\{p, q\}$ to $R_{i,j}$.
c. For each pair of elements p and q in $N_{i,j}$, add the 2-element set $\{p, q\}$ to $R_{i,j}$.
d. Construct the cross-product set $Q_{i,j} = N_i' \times N_j' \times N_{i,j}$. It is to be noted that $Q_{i,j}$ contains all triples (p, q, r) such that $p \in N_i'$, $q \in N_j'$ and $r \in N_{i,j}$. For each triple $(p, q, r) \in Q_{i,j}$, add the 3-element set $\{p, q, r\}$ to $R_{i,j}$.

For the elements i and j of PZI, every 2-element or 3-element set in $R_{i,j}$ leads to an observability constraint. Now, consider each pair of elements i and j in PZI and generate the set $R_{i,j}$ for that pair using steps a-d given above. The collection R from which observability constraints can be generated for all the $k \geq 2$ elements of PZI is given by (12).

$$R = \bigcup_{1 \leq i \leq j \leq k} R_{i,j} \tag{12}$$

In (12), it is assumed that the union operator eliminates duplicate entries. Finally, for every 2-element set $\{p, q\}$ and 3-element set $\{p, q, r\}$ in R, the modified observability constraints are given by (13) and (14), respectively.

$$\sum_{e \in E_p} \{w_e^h + w_e^l\} + \sum_{e \in E_q} \{w_e^h + w_e^l\} \geq 1 \tag{13}$$

$$\sum_{e \in E_p} \{w_e^h + w_e^l\} + \sum_{e \in E_q} \{w_e^h + w_e^l\} + \sum_{e \in E_r} \{w_e^h + w_e^l\} \geq 1 \tag{14}$$

This completes the ILP formulation of GOSC.

4. RESULTS

In the first set of simulations, the GOSC algorithm was applied to a modified IEEE 30-bus system having 26-substations as shown in Fig. 2. In the figure, numbers with the prefix ′S′ denote substations, while those without the prefix are buses. Substations S4, S6, and S24 are multi-bus substations with transformers in them (having unknown tap ratios), while the other substations are one-bus substations. The system has three voltage levels depicted by the colors red, blue, and green, respectively. The results obtained using the proposed method are also shown in Fig. 2, in which the lines which must be observed by TPMUs or DULRPs have blue dots on them. From the figure, it can be seen that the GOSC algorithm is able to identify the tap ratios of the transformers in substations S4 and S6. The basic observability results for this system are shown in Table 1. The costs given in Table 1 were computed based on the information from [2] and [26]. It was also realized from [26] that for each j, a suitable value of c_j (the number of measurement channels) for the TPMUs is 6. A stand-alone PDC was also assumed to be located at every substation where a PMU was placed. This was done to ensure that *(a)* the synchrophasor system becomes independent of failures in the SCADA system, and *(b)* if data loss happens in a downstream application, then the substation PDC can act as a back-up archive. The cost of a single PDC was set at 4 times the cost of a DULRP [27]. In Table 1, the results obtained using the proposed algorithm are compared with a TPMU-based approach with all tap ratio estimation capability [22], a TPMU-based approach with infinite number of measurement channels [24], and a DULRP-only based observability method [25]. From the table it becomes clear that the GOSC algorithm provides the most cost-optimal results.

TABLE 1: Cost comparison of GOSC algorithm with other OPP formulations for the IEEE 30-bus system

Method	#TPMU	#DULRP	#Substation Affected	Transformer Tap Ratios Observed	Total Cost*
Only TPMUs with all tap ratios estimated [22]	11	N/A	7	S4, S6, S24	223x
Only TPMUs with infinite number of measurement channels [24]	10	N/A	10	N/A	290x
Only DULRPs [25]	N/A	16	12	N/A	304x

GOSC Algorithm	3	20	7	S4, S6	203x

* x is the cost of a single DULRP; 1 TPMU = 5x [26]; Outage cost of 1 Substation = 20x [2]; 1 PDC = 4x [27]

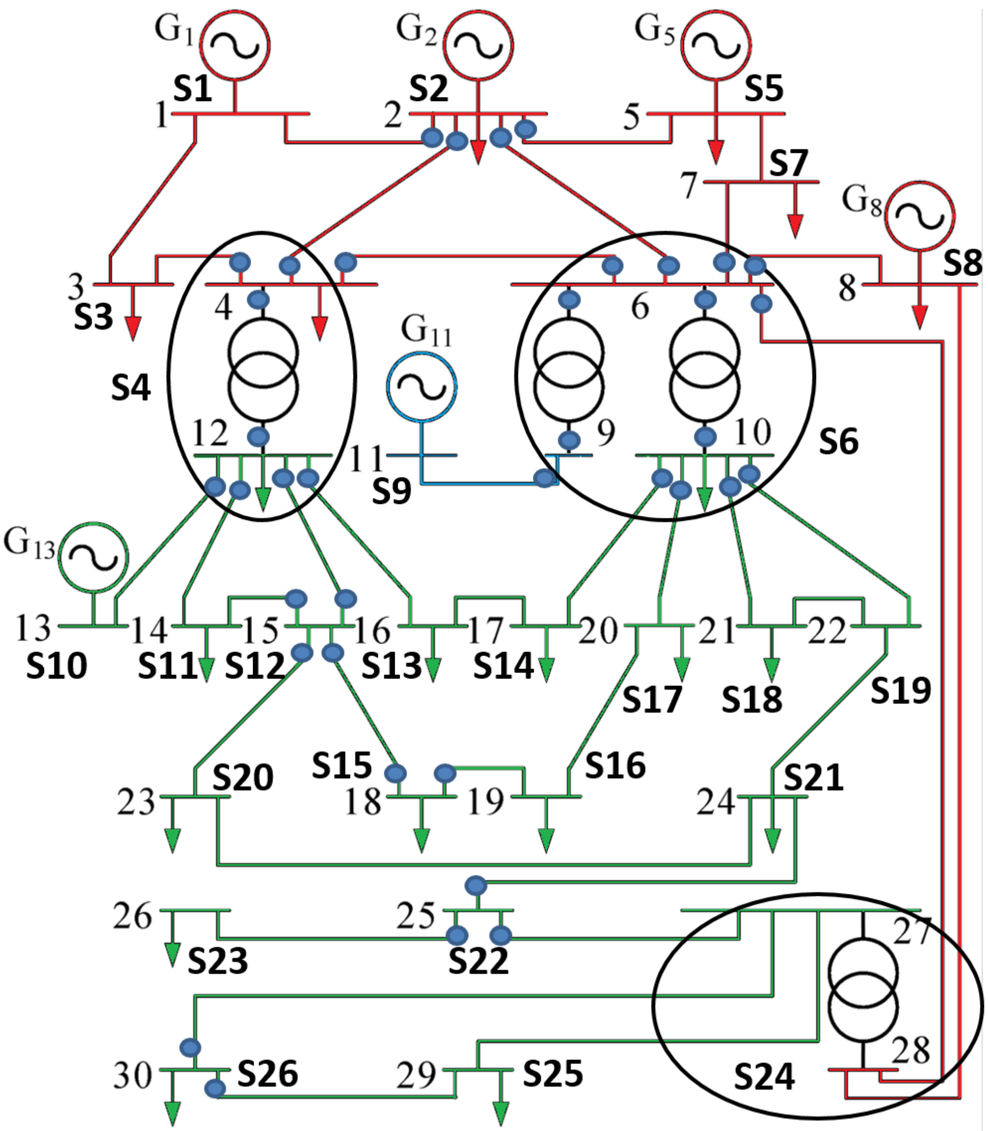

Fig. 2. Modified IEEE 30-bus system with different colors indicating different voltage levels; the blue dots indicate the lines which must be monitored by PMUs in accordance with the GOSC algorithm

The proposed methodology ensures that buses located at either end of a transformer are observed by different PMUs. Therefore, even if the transformer tap ratios are not known initially, voltages of all the buses of the network can be accurately estimated. An example of this can be found in Fig. 2 where although PMUs are not placed in substation S24, buses 27 and 28 are observed by the PMUs placed at substations S22 and S6, respectively. The fact that *many* of the transformer tap settings become *known* in the process of reaching the final solution (such as the tap settings of transformers in S4 and S6 of Fig. 2) is an added benefit of the proposed approach.

In the next set of simulations, the following constraints are applied to the modified IEEE 30-bus system: (a) Substation S2 is unsuitable for PMU placement; (b) Bus 28 inside Substation S24 is a critical bus and needs $\mathrm{N}-1$ redundancy; (c) The line between buses 23 and 24 is critical and must be observed under normal operating conditions; (d) The line between buses 12 and 13 is super critical and must be observed with $\mathrm{N}-1$ redundancy; and (e) Substation S26 has a pre-installed TPMU that monitors the voltage of bus 30 and currents flowing in lines 30-27 and 30-29. Section 3.2 explains how the proposed GOSC algorithm is able to handle these constraints. The results obtained are shown in Fig. 3 where the red dots indicate the lines observed by the pre-installed PMU while the blue dots indicate additional lines which must be observed in accordance with the proposed scheme. The total cost for this synchrophasor installation set-up is $252\mathrm{x}$, where x is the cost of one DULRP. In this set-up, all transformer tap ratios also become observable. These results indicate that the GOSC algorithm is able to incorporate a wide variety of practical constraints and provide techno-economic benefits.

The proposed GOSC algorithm is now applied to the IEEE 118-bus, the IEEE 300-bus, and a 2383-bus Polish system. By combining buses into substations (based on the locations of transformers), the three test systems became a 107-substation, a 184-substation, and a 2215-substation system, respectively [28]. Table 2 shows the results obtained when basic observability for the three systems was analyzed using different approaches. From the results, it becomes clear that the most cost-optimal results are obtained using the proposed GOSC approach. Moreover, although the methodology developed here was not meant to observe all tap ratios, it did observe most of them at a significantly lower total cost. This proves that the proposed method provides a good balance between costs incurred and benefits gained.

In the next set of simulations, the highest voltage buses of the three systems were assumed to be critical buses; while the high voltage transmission lines were assumed to be super critical lines that, from a PMU placement perspective, required observability under $\mathrm{N}-1$ contingency conditions. The results obtained for this scenario are summarized in Table 3. The table compares the results obtained using the proposed approach with the one developed in [14]. From the table it is clear that the GOSC algorithm is able to address practical constraints associated with OPP at a lower total cost.

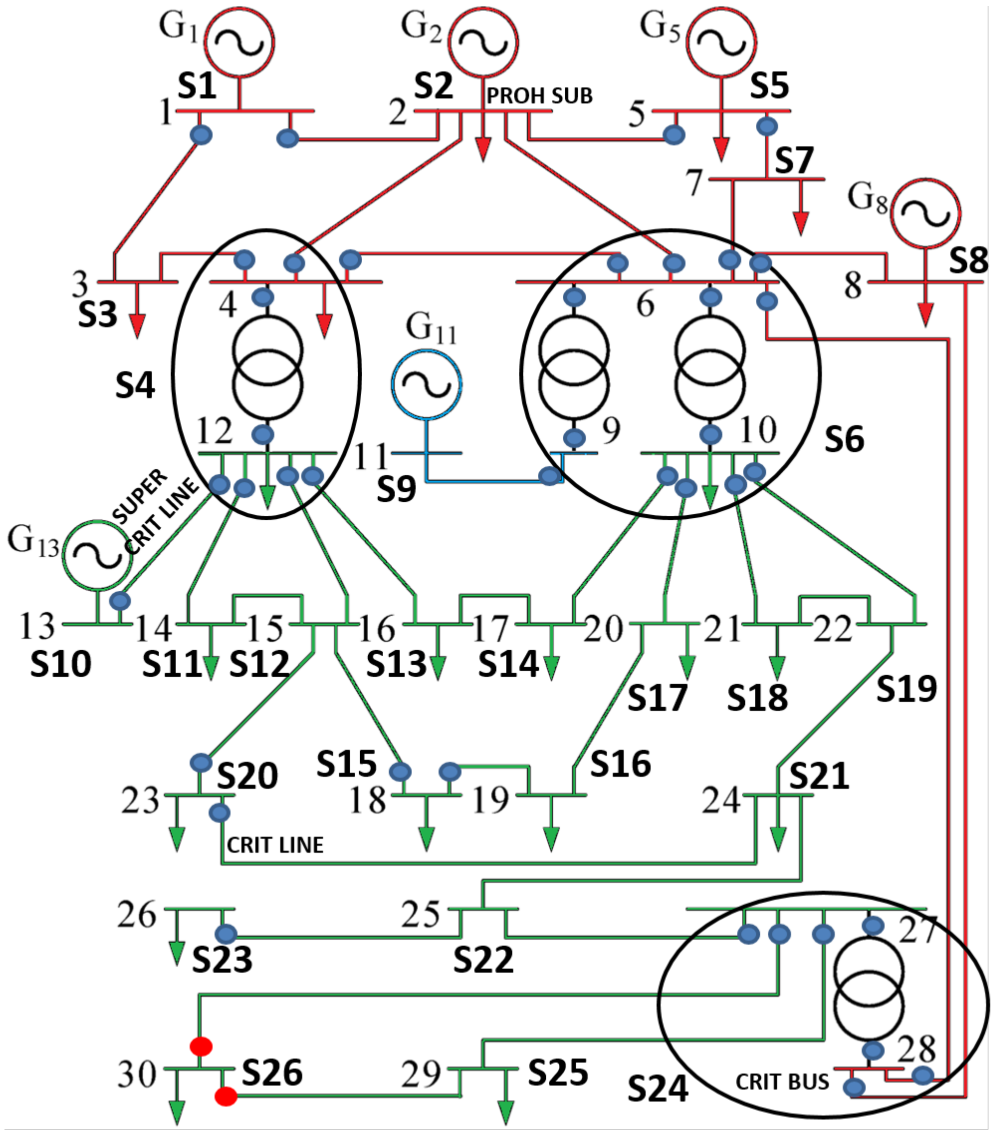

Fig. 3. Modified PMU placement-based edge observability for the IEEE 30-bus system in presence of practical constraints; red dots indicate lines that were observed by pre-installed PMU at Bus 30

For the last set of simulations, the buses of the three large test systems that satisfied the IvZIB condition described in Section 3.2.7 were treated as ZI buses, and the optimizations described in Tables 2 and 3 were repeated as shown in Table 4. From the table it is realized that when properly handled, ZI buses can significantly reduce the number of substations that are to be disrupted as well as the number of devices that are required for complete observability of the systems. Therefore, the IvZIB condition ensures that the total cost of synchrophasor deployment is reduced without compromising on the accuracy of state observability. The information about the substations that were disrupted and the lines on which the PMUs were placed can be found in [28]. The last column of Table 4 states the time required for performing the optimization. The computations were performed on an Intel (R) Core i7 processor having a speed of 2.70 GHz and an installed memory (RAM) of 16 GB.

TABLE 2: Cost comparison of GOSC algorithm with other OPP formulations for larger systems

Test System	Method	#TPMU	#DULRP	#Substation Affected	#Transformer Tap Ratios Observed	Total Cost*
IEEE 118-bus system	Only TPMUs with all tap ratios estimated [22]	44	N/A	33	11	1012x
	Only TPMUs with infinite number of measurement channels [24]	33	N/A	31	N/A	909x
	Only TPMUs with three measurement channel limits [29]	44	N/A	41	N/A	1204x
	Only TPMUs with four measurement channel limits [29]	38	N/A	35	N/A	1030x
	Only DULRPs [25]	N/A	63	60	N/A	1503x
	GOSC Algorithm	9	94	31	8	883x
IEEE 300-bus system	Only TPMUs with all tap ratios estimated [22]	198	N/A	82	120	2958x
	Only TPMUs with infinite number of measurement channels [24]	129	N/A	89	N/A	2781x
	Only TPMUs with infinite number of measurement channels [30]	95	N/A	80	N/A	2395x
	Only DULRPs [25]	N/A	191	127	N/A	3239x
	GOSC Algorithm	45	211	75	104	2236x
2383-bus Polish system	Only TPMUs with all tap ratios estimated [22]	887	N/A	719	174	21691x
	Only TPMUs with infinite number of measurement channels [24]	775	N/A	728	N/A	21347x
	Only TPMUs with infinite number of measurement channels [30]	799	N/A	753	N/A	22067x
	Only DULRPs [25]	N/A	1321	1155	N/A	29041x
	GOSC Algorithm	163	1860	704	149	19571x

* x is the cost of a single DULRP; 1 TPMU = 5x [26]; Outage cost of 1 Substation = 20x [2] ; 1 PDC = 4x [27]

TABLE 3: Cost comparison of GOSC algorithm with other OPP formulations for larger systems when the HV system is considered critical

Test System	Method	#TPMU	#DULRP	#Substation Affected	#Transformer Tap Ratios Observed	Total Cost*
IEEE 118-bus system	Ref. [14]	47	N/A	42	N/A	1243x
	GOSC Algorithm	10	101	34	10	967x

IEEE 300-bus system	Ref. [14]	138	N/A	92	N/A	2898x
	GOSC Algorithm	47	217	77	106	2300x
2383-bus Polish system	Ref. [14]	809	N/A	748	N/A	21997x
	GOSC Algorithm	168	1878	712	153	19806x

* x is the cost of a single DULRP; 1 TPMU = 5x [26]; Outage cost of 1 Substation = 20x [2] ; 1 PDC = 4x [27]

TABLE 4: Effect of ZI buses on results obtained using proposed approach

Test System	Scenario	#PZI	#TPMU	#DULRP	#Substation Affected	#Transformer Tap Ratios Observed	Total Cost*	Time (seconds)
IEEE 118 Bus	Basic Observability considering ZI Buses	2	9	92	30	9	857x	0.007774
	Basic Observability considering ZI Buses and HV buses as Critical	2	10	96	33	10	938x	0.008135
IEEE 300 Bus	Basic Observability considering ZI Buses	20	46	191	68	102	2053x	0.012909
	Basic Observability considering ZI Buses and HV buses as Critical	20	45	206	71	101	2135x	0.013249
Polish 2383 Bus	Basic Observability considering ZI Buses	376	103	1228	567	142	15351x	5.484777
	Basic Observability considering ZI Buses and HV buses as Critical	376	99	1279	577	140	15622x	8.095593

* x is the cost of a single DULRP; 1 TPMU = 5x [26]; Outage cost of 1 Substation = 20x [2] ; 1 PDC = 4x [27]

5. CONCLUSIONS

This paper describes a PMU placement procedure to optimize the number of substations where installations must be made to observe all the buses when traditional bus-type PMUs (TPMUs) as well as dual-use line relay branch-type PMUs (DULRPs) can be added into the network. The proposed approach also handles the additional constraint that all tap settings are unknown to start with. The formulation described here was developed to aid utilities such as DVP that wish to create a linear state estimator for their whole system. Therefore, conventional measurements obtained from the SCADA network were not incorporated into the proposed framework. The constraints that were considered in the proposed approach are providing redundancy in measurements of critical elements of the system, acknowledging presence of prohibited substations and substations with pre-installed PMUs, considering the presence of ZI buses, and accounting for measurement channel limitations. The results obtained indicate that the GOSC algorithm developed in this paper provides a cost-optimal solution while simultaneously addressing a variety of practical constraints. The methodology is also flexible because with minor modifications to the formulation, scenarios such as variable costs of substation upgrades, and TPMU-specific measurement channel numbers, can be successfully incorporated.

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