Using Continuous Simulation to Reduce Costs at a Recirculating Aquaculture Facility

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Abstract

The Recirculating Aquaculture System can be considered as a dynamic system in which the system variables change instantaneously during the growing period. The variables that can be altered by the user include the physical configuration of the tank, initial number of fish, amount of feed given, number of feeds per day, temperature (if indoor), water recirculation rate, fresh water intake rate and oxygen input rate. The manipulation of these variables will in turn have an effect on another group of variables(response) which include ammonia removal rates, ammonia generation rates, carbonate levels, pH of the system and growth rate of fish.

Changing the system configuration will change the operational cost of the system. For example, if the fresh water intake is reduced 5%, the cost of fresh water intake will decrease, but it will increase the ammonia level in the system. This will, in turn, affect the pH levels and so on. Under an ordinary linear model, it would be impossible to factor in all the interrelationships at all stages of the growing period.

In a continuous simulation model, the interrelationships between all the required variables are first defined, describing how a primary (regressor) set of variables(variables which are not affected by other variables, but which affect other variables; e.g., fresh water intake rate) change with time and then set the initial values for all these regressor variables. When the program is run, the variables are updated at every instant (in practice, at very small time intervals) until the end of the growing period. From this information, one can obtain the cost of running the facility during the growing period. The program also has an built in feature to try out different system configurations (i.e., different values of the regressor variables) and identify the one that minimizes the costs without violating any of the constraints(e.g., maximum ammonia levels, minimum oxygen level required).

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1. Introduction

Close to a billion people worldwide depend on the sea for their subsistence. Seafood provides an ideal source of dietary protein for much of the impoverished peoples of the world. Even in urbanized cities, they can supplement meat and poultry to meet the energy needs of humans. For example, according to the Food and Agricultural Organization (FAO,1998) wing of the United Nations, a city with about 10 million in population will need about 6000 tons of food each day. Thus, it is difficult to feed this population without alternate sources like seafood, given that most of the world population will live in such cities by 2010. However, 73% of the world's most productive natural fisheries have experienced steady declines since the 1970s. Given this background, the development of recirculating aquaculture assumes great importance in-order to maintain food security in the world.

The aquaculture industry today is a very fragile industry, operating on very thin profit margins. Hence, it is imperative to have an optimal configuration for a recirculating aquaculture facility that will minimize the overall cost. The overall goal of this work is to use continuous simulation to arrive at this low cost configuration.

Towards this end, the biological component of the system consisting of the fish tank and the biofiltration unit was simulated to identify an optimal water replenishment rate and oxygen supply rate. Continuous simulation was the method chosen because the variables involved change continuously over time and reach steady state only when the fish reached maturity at the end of the growing season.

2. The biological process overview

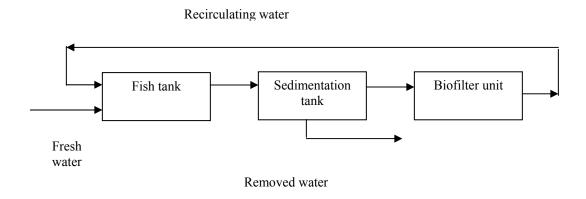


Fig.1. Block diagram of a single tank RAS facility

As mentioned earlier, this modeling effort assumes a three component system consisting of a fish tank, a sedimentation tank and a biofilter unit (Fig.1.).

At the very beginning, the biofilter (whichever type it may be) is acclimatized until the bacterial density on it reaches steady state. Then, the fish are introduced into the fish tank as fingerlings. Oxygen is supplied either through atmospheric diffusion, the direct addition of pressurized oxygen and/or mechanical aeration. The fish are fed on a particular diet depending on the species and the stage of growth. The quantity of feed is also changed as a percentage of body weight according to the stage of growth. Fish metabolism consumes oxygen, water and most of the feed and introduces carbon dioxide through respiration and ammonia through excretion. Carbon dioxide exists in an equilibrium with carbonic acid, bicarbonates and carbonates and ammonia exists in equilibrium with unionized ammonia. Both sets of equilibrium are a function of temperature, alkalinity and pH.

As the water flows continuously (or intermittently) through the system, it removes the carbon dioxide, TAN (total ammonia nitrogen) and other solids from the fish tank. A part of this water flows out of the system (about 3-15%) while the rest passes through the

sedimentation tank where the bulk of the suspended solids are removed. The water then flows through a biofilter where ammonia is converted into nitrites and subsequently into nitrates. This process of nitrification also consumes oxygen and produces carbon dioxide. The purified water is then fed back into the fish tank along with some new fresh water to compensate for the water wasted(Fig.1.).

This process continues until the fish are harvested at the end of approximately 8 months.

3. Description of the physical components

3.1 The fish tank

One of the main reasons why recirculating aquaculture systems(RAS) fail is, the inability of the tanks to self clean. So the tank design should facilitate the easy removal of waste matter. Fish tanks can have different sizes, shapes and depths. Generally, the tank shape is kept circular or oval to facilitate easy cleaning in the corners by the rotating motion of water and to better enable the fish to swim against the current. For Tilapia, the water velocities should be limited to 20-30 cm/s (Balarin and Haller, 1982)

The management practice can also vary. In the simplest approach there is only one fish tank in which the fish fingerlings are held until harvest. However, this method underutilizes the capacity of the facility for most of the growing period. Hence, it is a common practice to move the fish out into different tanks at various growth stages. But there will be some additional fish mortality and loss of growth each time the fish are transferred.

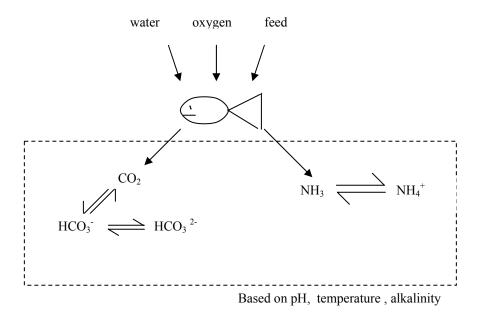


Fig. 2. Input to fish and output from fish

The configuration of the physical components of the RAS(size of tank, biofilter and sedimentation units, water pumping rate) depend on the feeding rates, which in turn depends on the number of fish and their mass. The number of fish depends on the fish species and the size of the fish. Timmons et al. (2002) suggest the following relation to estimate the number of fish per unit volume of the tank:

$$D_{density} = L/\ C_{density} \eqno(1)$$
 Where:
$$D_{density} = Stocking\ density\ (kg/m3);$$

$$L = length\ of\ the\ fish\ (in\ cm);$$

$$C_{density} = 0.24\ (for\ tilapia);$$

Some of the most modern farms have tanks that are 10 meters in diameter and about a meter in depth. There is huge reduction in cost as the size of the tank increases, but the risk of catastrophic system failure also increases.

3.2 The Biofilter unit

The main metabolic by-products from fish are ammonia and carbon-dioxide. The main sources of nitrogenous wastes in an RAS are:

- 1. Urea, uric acid and amino acids excreted by fish.
- 2. Organic debris from dead organisms.
- 3. Uneaten food.
- 4. Feces.
- 5. Nitrogen gas from the atmosphere.

As mentioned before, ammonia is extremely toxic to fish and needs to be removed. There are three ways to remove ammonia: air stripping, ion exchange and biological oxidation. Air stripping requires large stripping columns and a pH in excess of 10 in order to be effective. Ion exchange with clinoptilolite has been used to remove ammonia from fish culture systems, but the operation and maintenance of these ion exchange systems are cumbersome. Hence, biofilters are the most commonly used systems.

The biofilter removes ammonia produced from the recirculating water via nitrification. There are various kinds of biofilters including rotating biological contactors (RBC), fluidized sand reactors, trickling biofilters, bead/micro-bead filters and dynamic bead biofilters. While each biofilter has its own advantages and disadvantages there is generally a move towards the use of granular filters. The important characteristics of biofilters are:

- 1. void space- space in the biofilter that is not filled with the media
- 2. cross-sectional area- area perpendicular to the direction of water flow
- 3. hydraulic loading rate- volume of water flowing through per day per unit cross-sectional area
- 4. specific surface area- surface area of the media per unit volume of biofilter

While all these characteristics have an effect on the optimal configuration, this modeling effort considers only the effect of hydraulic loading rate. Further, in this effort we have assumed the biofilter to be an RBC.

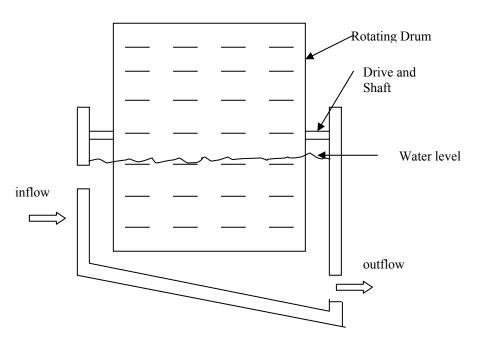


Fig.3. Rotating Biological Contactor(Biofilter)

An RBC consists of a rotating drum that contains a plastic medium on which the nitrifying bacteria grow(Fig.2.). The rotating medium is typically submerged to about 40% of its diameter in the water and rotates at the rate of about 1.5 to 2.0 rpm. RBCs have several advantages, some of which are:

- Due to the rotation, bacteria attached to the rotating media are exposed alternately to the wastewater and the atmosphere, which provides oxygen to the biofilm.
- The rotation also facilitates carbon dioxide stripping to some extent.

 The amount of carbon dioxide stripped becomes important as the rearing densities in the fish tank increases.
- The RBCs are low head devices, which minimizes pumping energy needs.
- 4) RBCs also tend to be better self-cleaning than other types of biofilters.

3.3 Sedimentation tank

The main sources of suspended solids in the RAS unit are feces, biofloc (dead and living bacteria), and uneaten food. While the total size of the particles can range between

0.01 micron to 1,000 microns, the majority of the particles by weight will be less than 100 micrometers in size(Timmons et al.,2002). The longer the particles spend in the tank, the smaller they become and the more difficult they are to remove. Removing the particles from the tank as soon as possible is therefore critical. However, such small particles cannot be removed by mechanical filtration. Hence the need for sedimentation tanks in RAS. In these tanks the sedimentation is done by gravity separation. Granular media filter can control the widest range of solids and so are a popular choice of sedimentation unit.

Total suspended solids (TSS) concentration is defined as the mass of particles above one micrometer in diameter that occur in known volume of water (APHA, 1989). TSS can be organic or inorganic. The organic part consumes oxygen and can create biofouling problems. Various authors have recommended an upper limit for TSS levels between 15(FIFAC, 1980) and 40 mg/L(Muir, 1982). A general rule of thumb (Timmons et al.,2002) is that the amount of TSS produced will be equal to:

0.25 * dry weight of fish feed

Another rule of thumb is that 10 mg of oxygen consumed by the fish will produce 10 - 20 mg of suspended solids.

3.4 Oxygenation/Aeration unit

Aquaculture Systems require a dissolved oxygen concentration of at least 5 mg/L in the tank. Assuming fish consume oxygen at the rate of 250 gms of oxygen per kilogram of feed, the oxygen demand in the tank increases as the stocking density increases. However, as the fish stocking density increases, it becomes difficult to sustain these levels of oxygen by just atmospheric diffusion. Hence, a mechanical aeration unit is used to sustain the oxygen levels. As the stocking density increases above 30-60 kg/m³, the dynamics between pH, carbon dioxide and other gases must also be considered in determining the oxygen levels. For example, if the carbon dioxide concentrations are higher in the water, it makes it difficult for the fish to exchange the carbon dioxide in its blood with oxygen in the water. In such cases, oxygen transfer systems need to be used. Air stones, packed towers, blowers, air pumps and compressors are examples of aeration devices used in RAS. There are three types of oxygen transfer systems: high- pressure

oxygen gas, liquid oxygen (LOX) and on-site oxygen generations. However, high pressure oxygen gas is expensive and is therefore used primarily as a back-up.

3.5 Carbon dioxide stripping unit

Carbon dioxide is introduced into the system through fish and bacterial respiration, decaying organic matter and atmospheric diffusion. The fish and nitrifying bacteria produce 14 mg of carbon dioxide for every 10 mg of oxygen consumed Timmons et al.,2002). As fish metabolism peaks between 1-4 hours after feeding, oxygen consumption and carbon dioxide production also increase during this period. Since carbon dioxide exists in an equilibrium with bicarbonates, carbonic acid and carbonates, the exact amount of carbon dioxide depends on the pH, temperature and the other compounds. As mentioned earlier, it is critical to remove carbon dioxide from the system because a high level of CO₂ can hamper intake of oxygen by the fish. Carbon dioxide levels can be controlled either by adding bases which shift the equilibrium towards carbonates and bicarbonates or by using CO₂ stripping units.

CO₂ stripping can easily be done through a gas exchange process using air stripping columns. But prediction of the exact removal rate is difficult because of the carbonate equilibrium system and because CO₂ concentration in the atmosphere can affect the stripping.

4. Literature Review

4.1 Fish growth and Metabolism

A typical feed consists of about 10 different amino acids, lipids, minerals(e.g., phosphorus) and carbohydrates. The growth rate of fish depends on the composition, digestibility and quantity of feed, temperature, oxygen supply, pH, ammonia toxicity, type of fish and mobility of fish in the tank. Tilapia typically take about 8-9 months to grow to market size. The feeding regimen changes according to the growth stage of the

fish. For example, the daily feed may be 5-8% of body weight for fingerlings but may be less than 2% of body weight for mature fish (Van Gorder and Dujakovic, 1996).

Fish consume oxygen and most of the feed, and produce carbon dioxide and TAN. The daily consumption and production of these materials varies according to their growth stage and the other factors described above.

4.2 Nitrification

There are some species of bacteria like *Nitrosomonas*, *Nitrosospira*, *Nitrosoglora* and *Nitrosococus* that are able to oxidize ammonia (O'Shaughnessy, 1982),but the most common ones are Nitrobacter and Nitrosomonas. These are grown on suitable media to be used for nitrification.

Nitrification takes place in two stages. In the first stage *Nitrosomonas* converts NH_4^+ into nitrite (NO_2^-) according to the equation:

$$NH_4^+ + 1.5 O_2 \rightarrow 2H^+ + H_2O + NO_2^-$$

In the second stage, *Nitrobacter* converts nitrite into nitrate according to the equation:

$$NO_2 + 0.5 O_2 -> NO_3$$

There are two zones, of varying thickness, between the biofilm surface and the bulk solution. The amount of oxygen, ammonia and nitrites available for nitrification depends on the concentration gradient across these two zones. Thickness of the waterfilm zone has been reported as 1-100 microns (Hocheimer, 1990) and as 56 microns (Williamson and McCarty, 1976) and the thickness of the biofilm zone was reported by Atkinson and Fowler (1994) as 0.07-4 mm and under flow rates, as about .2 mm. The rate of nitrification depends on the ammonia and oxygen concentration gradient across these zones.

If the amount of organic compounds in a system is high, heterotrophic bacteria will outcompete the nitrifying bacteria for space and oxygen, but in a recirculating aquacultural facility this organic loading is controlled.

The efficiency of a biofilter(RBC) depends on several factors such as the inflow substrate concentration, the flow rate, the rotational velocity of the RBC, temperature, pH, the population of the nitrogenous bacteria and the oxygen supply. Also, the following factors change dynamically during operation:

- a) The substrate concentration in the biofilm decreases as the film leaves the reactor liquid and approaches zero before reentry. Upon reentry, there is a sudden increase in the concentration.
- b) Variation in influent waste concentration, flow rate and constituents.
- c) Sloughing of the film.
- d) Changes in temperature and pH.

An ideal model will take all these factors into consideration. There are several models available in the literature. Although none of them are perfect, each offers several advantages and disadvantages. In the absence of real data from RBCs to fit equations and to validate, the best that could be managed is to apply principles from published biological models. Three different models are presented here:

1. Modified Monod equation- Assuming that the population of bacteria are at steady state, a simple Monod equation can be constructed for any given temperature to model the variation of TAN and nitrite removal efficiencies with inflow TAN and nitrite concentrations. Zhu and Chen et al.(1999) suggested the following modification to the Monod equation for the fixed film reactor:

Ammonia removal rate,
$$mg/L/m^2 = R_{max} * (S-S_{min})/(S+K_s-S_{min})$$

At 27°C (which is the approximate temperature at which tilapia are reared)this reduces to:

$$1859*(S-.07)/(S+1.93)$$
 (2)

Where,

S= Inflow ammonia concentration (mg/L);

Even though this model is simple in application, it makes the overall model less robust. For example, the constants have to be re-evaluated for each temperature change. Moreover, the actual RBC does not remain under water like the fixed film reactor, but alternates between rotating in the air and water.

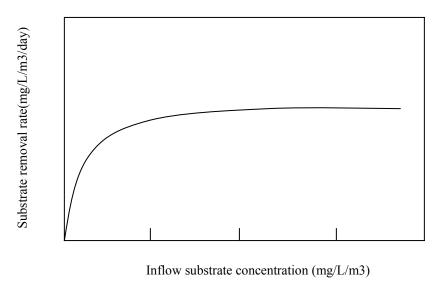


Fig.4. Monod equation for nitrification rate

2. Zhu and Chen et al. (2002) further developed a fixed-film model that incorporates the flow rate, oxygen limitation, TAN limitation and temperature. When TAN was limiting, the ammonium removal rate was given as:

```
= (Dws/L) * \sqrt{(Ds * K_{20} * Q^{T-20})} * S / (Dws/L + \sqrt{(Ds * K_{20} * Q^{T-20})})  (3) Where,  T = temperature (^{0}C);  Dws = 0.0000886*1.026^{T};  L = thickness of water film at temperature T;  = 1.23 * (u/\rho)^{1/3} * (d/v)^{1/2} * \epsilon^{3/2} * (Dwc/86400)^{1/3};  Where,  u = water \ viscosity(g/m/day);   p = water \ density \ (g/m^{3});   d = characteristic \ length \ of \ film = 0.01 \ m;   v = water \ velocity \ (m/sec);   \epsilon = void \ fraction = .98;
```

```
Ds = 0.8 * Dws;

K<sub>20</sub>= 390000 (/day);

Q = 1.1;

S = inflow TAN concentration;
```

Even though this equation(Eqn.3.) predicts a linear increase in the TAN removal rate with increase in TAN concentration, it is considered valid at lower concentrations of TAN.

Zhu and Chen et al.(2002)also gives another equation for the oxygen-limited case, but since the aquaculture system always maintains a high level of oxygen to sustain the fish (much higher than that required for the biofilter), this equation is ignored.

The main short coming of this fixed film model is that it is suited more for a continuously submerged fixed-film than for an RBC.

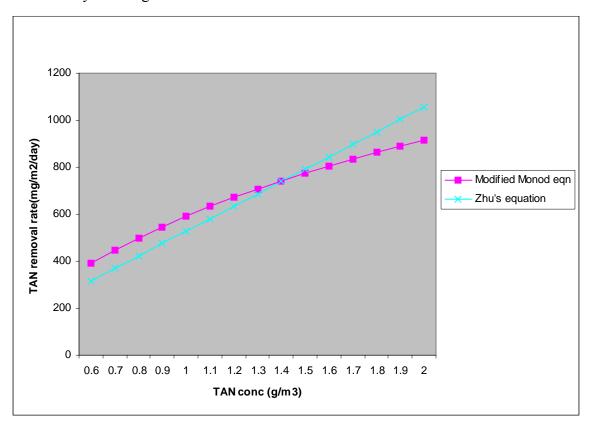


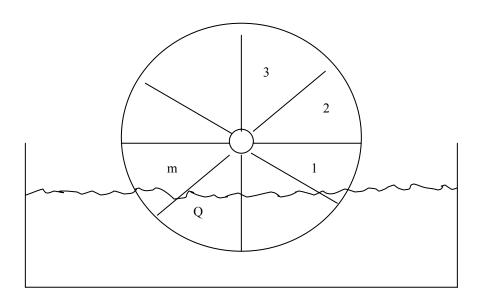
Fig.5. Comparison between Monod equation and the equation by Zhu and Chen (2002)

3. Greaves (1972) built the following models specifically for the RBC and reported excellent correlation with observed data, especially for short runs. The models are

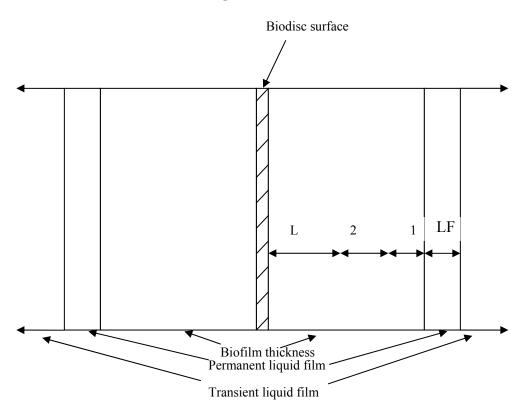
heterogeneous model, pseudo homogeneous model and steady-state model. The assumptions made are:

- a) There is complete mixing in the bulk liquid of the reactor.
- b) The effect of suspended solids in the bulk liquid on substrate removal is negligible.
 - c) There is no density gradient within the active part of the biofilm.
- d) When the biofilm enters air, it carries with it a liquid film. The relative position of the liquid film does not change with respect to the biofilm.
- e) The liquid film does not drip off from the RBC as it rotates through the air.
 - f) There is complete mixing in the liquid film.
 - g) The liquid film mixes instantly with the bulk liquid upon reentry.
- h) Substrate utilization by individual micro organisms in the biofilm is modeled by a Monod function. Its coefficients remain constant during periods of transient operation.
 - i) The mass of substrate consumed by organisms for maintenance metabolism is much less than that used for growth.
 - j) Substrate diffusion along the circumference of the RBC is negligible compared to diffusion into the biofilm.

Based on these assumptions, he divided the RBC into the following segments(Fig.6.):



Rotating disc- Side view



Rotating Disc – front view

Fig.6. Front and side view of a rotating biodisc as modeled by Greaves (1972). Numbers 1 to L, 1 to M and 1 to Q represents segments into which the biofilm is divided

Each disc is divided into 'm' sectors above the water and 'l' segments into the thickness of the biofilm. The thickness of each segment 'l' increases with the depth of the biofilm because the concentration also drops in proportion to the depth.

Now, according to the heterogeneous model, the rate of change of concentration in each of these 1 * m segments, the m liquid film segments, and the bulk liquid is given by the following equations:

For the bulk liquid,

$$dC_b/dt = F * (C_o - C_b)/V_b - F_f * (C_b - C_{LF,Q-1})/V_b - [\Sigma^n_O K_L * A * (C_b - C_{1,Q})]/V_b$$
(4)

Where,

 $F = flow (m^3/hr);$

C_o= inflow substrate concentration;

C_b= substrate concentration in bulk liquid;

V_b= Volume of bulk liquid;

 F_f = mass of liquid film attached to the biofilm entering the reactor per unit time;

n= number of sectors submerged in the bulk liquid;

 $C_{LF,Q-1}$ = substrate concentration in the liquid film in the sector just entering into the bulk liquid;

A = surface area of the biofilm submerged on each sector;

 K_L = liquid film coefficient = 0.01 to 0.03 cm/sec;

For the liquid film,

$$dC_{LF}/dt = -K_L * (C_{LF,m} - C_{L,m}) / \delta_L$$

$$\delta_L = \text{Thickness of the liquid film;}$$
(5)

For the biofilm segments,

$$DC_{L,m}/dt = D/\Delta Z_L * \{ (C_{L+1,m} - C_{L,m}) * 2/(\Delta Z_{L+1} + \Delta Z_L) - (C_{L,m} - C_{L-1,m}) * 2/(\Delta Z_L + \Delta Z_{L-1}) \} - \prod * x * C_{L,m} / Y(K_c + C_{L,m})$$
(6)

For the first layer this becomes,

$$DC_{l,m}/dt = K_L / \Delta Z_1 (C^* - C_{l,m}) - D / \Delta Z_1 * \{C_{l,m} - C_{l,m} * 2 / (\Delta Z_1 + \Delta Z_2)\} - C_{l,m}/dt = K_L / \Delta Z_1 (C^* - C_{l,m}) - D / \Delta Z_1 * \{C_{l,m} - C_{l,m} * 2 / (\Delta Z_1 + \Delta Z_2)\} - C_{l,m}/dt = K_L / \Delta Z_1 (C^* - C_{l,m}) - D / \Delta Z_1 * \{C_{l,m} - C_{l,m} * 2 / (\Delta Z_1 + \Delta Z_2)\} - C_{l,m}/dt = K_L / \Delta Z_1 (C^* - C_{l,m}) - D / \Delta Z_1 * \{C_{l,m} - C_{l,m} * 2 / (\Delta Z_1 + \Delta Z_2)\} - C_{l,m}/dt = K_L / \Delta Z_1 (C^* - C_{l,m}) - D / \Delta Z_1 * \{C_{l,m} - C_{l,m} * 2 / (\Delta Z_1 + \Delta Z_2)\} - C_{l,m}/dt = K_L / \Delta Z_1 (C^* - C_{l,m}) - D / \Delta Z_1 * \{C_{l,m} - C_{l,m} * 2 / (\Delta Z_1 + \Delta Z_2)\} - C_{l,m}/dt = K_L / \Delta Z_1 (C^* - C_{l,m}) + C_{l,m}/dt = K_L / \Delta Z_1 (C^*$$

$$\prod * x * C_{1,m} / (Y(K_c + C_{1,m}))$$

Where,

D = Substrate diffusivity of the biofilm= 0.64* 10⁻⁵ cm²/sec;

 ΔZ_L = Thickness of biofilm segment 'L';

 $C^* = C_{LF,m}$ when the element is in the reactor atmosphere;

= C_b when submerged in the bulk liquid;

$$\prod *x/Y = 7.12 \text{ mg/L sec};$$

 K_c = saturation constant for the Monod equation = 40 mg/L;

For the pseudo-homogeneous model, the different segments of the biofilm and the liquid film are assumed to be one homogeneous layer. Thus, the equations for the concentration change in the biofilm and liquid film reduces to:

$$DC_{l,m}/dt = K_L / \Delta Z (C^* - C_{l,m}) - \prod^* x^* C_{l,m} / (Y(K_c * n + C_{l,m}))$$
Where, $n = \text{effectiveness factor} \sim 5$

When the effectiveness factor was appropriately chosen, the pseudo-homogeneous model was reported to be as effective as the more complicated heterogeneous model.

Through a series of approximations,, the following steady-state model was also derived by Greaves(1972):

$$C_b/C_0 = 1/\left\{1 + N/F[P_1 * A_s + P_2 * rpm * (1 - e^{-(P_1 * Aa/(P_2 * rpm))})]\right\}$$
(8)

Where,

N = number of discs on the RBC;

F = flow rate of the substrate into the reactor;

$$P_1 = K_L * K_1 / (1 + K_1)$$

$$K_1 = \prod *_X *_{\Delta Z} /(Y *_{C} *_{n} *_{K_L})$$

 A_s = Area of each disc submerged;

 A_a = Area of each disc in the air;

$$P_2 = K_2 * \P * (R_1^2 - R_2^2) * rpm * \Delta L$$

 $K_2 - 1$, if there is no dripping;

 R_1 – Disc radius;

 R_2 – length of radius not submerged;

 ΔL – Thickness of liquid film if discs were; submerged to its mid-diameter.

The only two disadvantages of Greaves models are:

- a) The models do not predict long-term performances well, probably because this equation does not model the changes in the bacterial population very well.
- b) Even though Grieves' models can be effectively modeled using a professional version of the 'Arena' simulation software, it is beyond the scope of the version used in this effort.

4.3. Relationship between variables

4.3.1. Ammonia, nitrite and nitrate levels

The rates of nitrification depends on the temperature, pH, water flow-rate through the system, dissolved oxygen concentration, alkalinity, amount of bacteria and the ammonia and nitrite concentration. The amount of TAN available for nitrification depends on the proportion of ammonia (NH₃), which depends on the temperature, salinity and pH.

Some papers suggest that 4.2 grams of oxygen is consumed in oxidizing one gram of nitrate from ammonia (Gaudy and Gaudy, 1980), while others suggest 4.3 and even 4.57 (Chen, 2002).

Because the growth rate of Nitrosomonas is less than Nitrobacter, oxidation of ammonia is usually considered the rate limiting step. Because the bacterial population is already acclimatized and has already reached steady-state, and since the ammonia levels in the tank are low, the size of the bacterial population is generally not considered a limitation.

The rate of nitrification also depends on the concentration of ammonia and nitrite in the tank, as modeled by a Monod type equation. Alkalinity and temperature affects the pH which in turn affects the proportion of ammonia in TAN and so indirectly affects the nitrification rate.

Nitrification consumes bicarbonates (HCO₃⁻) and produces carbon dioxide. This elevated CO₂ level near the biofilm causes a decrease in the pH levels near the biofilm as a function of temperature. For example, at a temperature of 10-15°C, a pH of 7 in the bulk solution means a pH of <6 near the biofilm (Boller *etal.*, 1994)

Even though the nitrate levels are typically not monitored in aquaculture facilities, at elevated levels it can reduce the immunity of the fish and can even result in fish death Hrubec et al., 1996, Spotte, 1979)

4.3.2. pH

Even though the optimal pH range for nitrification is wide (6.5-9) it might be narrower for a specific filter. Nitrification (ammonia > nitrites > nitrates) efficiency reduces drastically between a pH of about 6.5 and 7.5 and becomes close to zero below 6.(Boller et al., (1994) as shown in Fig.7. An elevated carbon dioxide level from fish metabolism can reduce the pH, thus affecting nitrification rates (Grace and Piedrahita,1994). But, at high levels of pH, the equilibrium between ammonium(NH₄⁺) and unionized ammonia (NH₃)shifts towards unionized ammonia, increasing the proportion of unionized ammonia in the system. Unionized ammonia(NH₃) is toxic to the fish, so it is necessary to keep pH in the middle of the tolerable range.

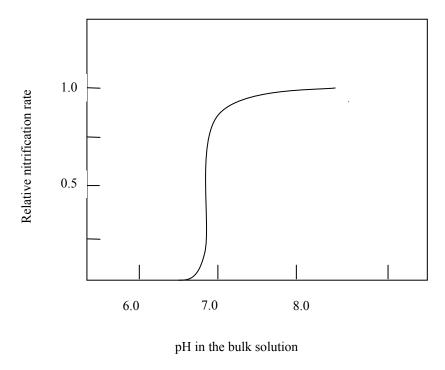


Fig.7. Change in the nitrification rate with change in the pH of the bulk solution

4.3.3. Alkalinity

Fish produces carbon dioxide and excretes TAN in proportion to the oxygen and feed consumed. However, the carbonates, bicarbonates and carbon dioxide (as carbonic acid) exist in an equilibrium with each other and so does ammonia and ammonium. The alkalinity of water affects this equilibrium.

Conversion of NH₄+ to nitrate consumes alkalinity(HCO₃⁻). The complete oxidation of 1 mg of ammonia to nitrate consumes 7.14 mg of alkalinity expressed as CaCO₃. Alkalinity levels (concentration of carbonates and bicarbonates) affect nitrifier growth (Malone and Burden, 1988). Gujer and Boller (1986) stated that 75 mg/L of CaCO₃ alkalinity was sufficient to maintain maximum nitrification rates.

4.3.4. Oxygen

Both fish and nitrifying bacteria consume oxygen. The limiting oxygen concentration is a function of temperature, concentration of organics in the feed water, bacterial biomass, fish density and dissolved gases especially carbon dioxide.

4.3.5 Temperature

After feed, temperature is probably the most important factor affecting fish growth. It affects almost every other variable in the system. Temperature directly affects the nitrification rate in the biofilter. It affects the equilibrium between ionized and unionized ammonia and the carbonate system. It affects the amount of oxygen and carbon dioxide dissolved in the water. As the temperature increases, the growth rate (measured in terms of mass gained or length gained) increases to a maximum and then reduces. For every 9°C increase in temperature, the metabolism of the fish doubles, thereby increasing the oxygen demand. However, the saturation capacity of the water decreases as the temperature increases. Thus, the metabolism of the fish decreases after a certain temperature due to a reduction in the oxygen supply.

4.3.6 *Salinity*

Salinity, as mentioned earlier, affects the proportion of ammonia in the system and helps prevent fish diseases. However, as far as nitrification is concerned, the bacteria are known to acclimate to salinity ranges from fresh water to 40 parts per thousand (Easter, 1992), given sufficient time. But, abrupt changes in salinity of greater than 5 g/L will shock the bacteria and decrease the reaction rate for both ammonia-nitrogen and nitrite-nitrogen removal (Wheaton, 1991).

4.3.7 Other minerals

Other minerals like phosphate, magnesium, iron, calcium, copper and sodium also affect nitrification, but are not considered in this model.

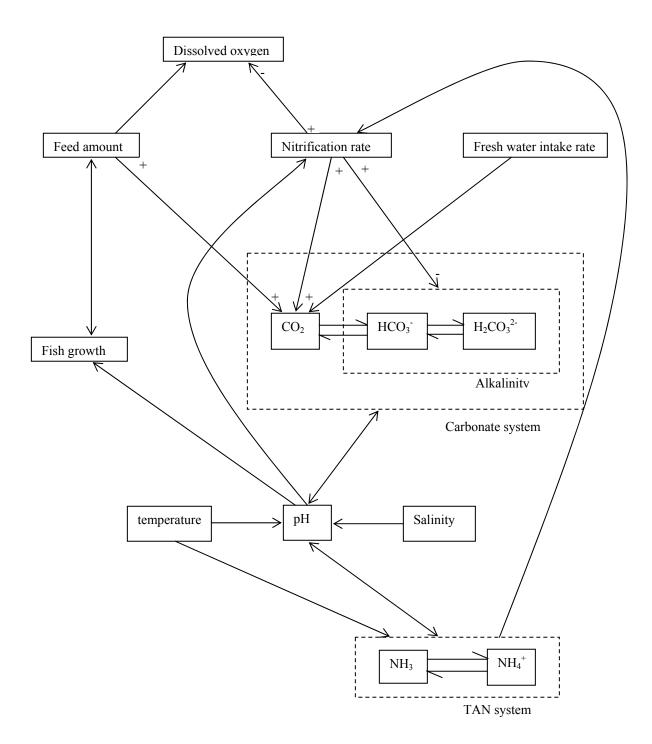


Fig.8. Interaction between the major system components

-___

4.4. Acceptable ranges for the variables

In an RAS system the primary variables of interest are temperature, pH, ammonia, nitrite, nitrate, dissolved oxygen concentration, dissolved carbon dioxide, size and concentration of suspended solids, alkalinity and hardness. Changing the system configuration(regressor variables) affects these variables. These variables are also affected by each other. The optimal values for some of these variables for tilapia farming are given in table.1.

Table.1. Factors that affect fish health and their tolerance limits as found in literature

Variable	Value	Cited by:
Ammonia	<.05	Colt and Armstrong,1981
concentration(mg/L)	<1	Malone 1999
	<0.6	Timmons et al., 2002
Unionized ammonia	~12.5	Colt and Orwicz,1991
$conc(\mu g/L)$		
nitrite concentration (mg/L)	<1	Losordo,1991
nitrate concentration (mg/L)	<100	Losordo,1991
Dissolved oxygen (mg/L)	>5	Kaiser and Wheaton,1983;
		Losordo, 1991
DO-minimum concentration	2	Hochheimer and Wheaton
for nitrification (mg/L)		,1991
CO ₂ (mg/L)	40-50	Timmons, et al., 2002
pH level (mg/L)	6.5-8	Meade,1989
	6.7-9	Klontz,1979
Total Suspended Solids	<15	FIFAC, 1980
(mg/L)	20-40	Muir,1982
T (C.1:)	22-23	Schmitz,1999
Temperature (Celsius)	20-30	Malone, 1999
Alkalinity (mg/L) (as	>100	Meade, 1989
CaCO ₃)		Losordo, 1991
Hardness (mg/L)(as CaCO ₃)	>100	Meade, 1989

5. Model development

In a continuous simulation model we first define the interrelationships between all the required variables, describe how a 'regressor' group of variables (variables which are not affected by other variables, but which affect other 'response' variables, eg: fresh water intake rate) change with time and then set the initial values for all the variables. When the program is run, the variables are updated at every instant(in practice at very small time intervals) until the end of the growing period. From this we can obtain the cost of running the facility during the growing period. The 'Optquest' feature is used to try out different system configurations(ie, different values of the primary variables) and find out the one that minimizes the costs without violating any of the constraints(for e.g.; maximum ammonia levels, minimum oxygen level required).

5.1 Performance metric

The model needs to be evaluated based on:

- 1. Statistical correlation between the model output and the observed data under similar environmental conditions.
- 2. Percentage reduction in operating costs.

5.2. Modeling the biofilter unit

The efficiency of the biofilter depends on the several factors including the inflow substrate concentration, the flow rate, the RPM(revolutions per minute) of the RBC, temperature, pH, the population of the nitrogenous bacteria and the oxygen supply.

Zhu and Chen (1999) suggested the following modification to the Monod equation for the fixed film reactor as described in a previous section:

Even though this model is simple in application, it reduces the utility of the overall model. So, this project also uses the Zhu and Chen's equation (2002) shown below:

$$(Dws/L)* \sqrt{(Ds * K_{20}* Q^{T-20})} * S / (Dws/L + \sqrt{(Ds * K_{20}* Q^{T-20})})$$

5.3 Modeling the fish growth

Several authors have developed equations to model the fish growth. For example, Ursin's growth model, which is used here, is of the form:

However, for an aquaculture facility, the term for catabolism (KWⁿ) can be ignored since the fish are regularly fed. Also, the term b is not a constant but decreases as the amount of feed available increases.

Now, dRa/dt =
$$\tau \delta \rho \gamma F$$
 (10)
Where,
 $F = \text{feed available (g/day)};$
 $\tau = \text{temperature factor, } 0 < \tau < 1;$
 $\delta = \text{dissolved oxygen factor, } 0 < \delta < 1;$
 $\gamma = \text{unionized ammonia factor, } 0 < \gamma < 1;$
 $\rho = \text{photo period factor, } 0 < \rho < 2 = \text{number of sunlight hours/12}$

It is assumed that the fish will not be fed more than they can eat.

Brett (1979), Colt and Armstrong (1981), Svirezhev et al. (1984) Cuenco et al. (1985a), and Bolte et al. (1995) suggested the following equations for τ , δ , ρ and γ :

$$\begin{split} \tau &= e^{\{-4.6[(Topt-T)/(Topt-Tmin)]4\}} & \text{if } T < T_{opt}; \\ &= e^{\{-4.6[(T-Topt)/(Tmax-Topt)]4\}} & \text{if } T >= T_{opt}; \\ \delta &= 1 & \text{if } DO > DO_{crit}; \end{split}$$

```
= (DO-DO_{min})/ (DO_{crit} - DO_{min}) \qquad if DO_{min} <= DO <= DO_{crit};
= 0 \qquad if DO < DO_{min};
\gamma = 1 \qquad if UIA < UIA_{Crit};
= (UIA_{max} - UIA)/(UIA_{max}-UIA_{crit}) \qquad if UIA_{crit} <= UIA <= UIA_{max};
= 0 \qquad if UIA > UIA_{max};
```

where:

 T_{max} = maximum temperature tolerable for the fish (${}^{\circ}$ C);

 T_{min} = minimum temperature tolerable for the fish (${}^{\circ}C$);

 T_{opt} = optimal temperature for the fish (24°C for tilapia);

DO_{min} = minimum dissolved oxygen levels required (mg/L);

DO = current oxygen levels (mg/L);

 $UIA_{max} = maximum unionized ammonia permitted (mg/L);$

 UIA_{crit} = unionized ammonia level below which feeding is not affected by UIA concentration (mg/L);

The range of the food assimilation efficiency (b) was reported to range between 0.53 and 0.7, decreasing with increase in the feed intake (Meyer-Burgdorff et al., 1989). However, for modeling purposes, the food assimilation efficiency was taken to be the mean, i.e., 0.62. The parameter for the effect of feeding catabolism, 'a', was taken to be random variable with mean 0.53 (Nath et al.. 1994) and variance 0.1 (normally distributed). This was done because the fish display varying activity levels each day. The minimum temperature tolerable for Nile tilapia was reported by Gannam and Phillips (1993) to be 15°C. The optimal temperature is the temperature at which the fish appetite is high and maintenance requirements (energy expenditure) are low. This was given as 33° C by Caulton (1982). The maximum temperature was 41° C (Denzer, 1967). Tilapia, due to their ability to use atmospheric oxygen can survive with dissolved oxygen concentrations as low as 0.1 - 0.3 mg/L (Ahmed and Magid, 1968, Magid and Babiker, 1975). However DO_{Crit} was calculated using Teichert-Coddington and Green's (1993) suggestion that the threshold DO for Nile tilapia was not greater than 10 % of saturation and thus was taken to be 1.0 mg/L at 15 °C. However, it should be noted that this amount decreases with increasing temperature and increasing altitude. Abdalla (1989) determined that UIA_{Max}

was 1.4 mg/L and UIA_{Crit} was 0.06 mg/L for Nile tilapia. Values for the aforementioned parameters as found in literature are tabulated below(Table.2.):

Table.2. Values for parameters in Ursin's growth model as found in literature

Parameters	Value	Source
a	0.53	Nath et al. 1994
b	0.53- 0.7	Meyer-Burgdorff et al. 1989
DO _{Crit}	1.0	Teichert-Coddington and
		Green, 1993
	~6 (preferred)	Timmons et al., 2002
$DO_{Min}(g/m^3)$	0.3	Ahmed and Magid, 1968
	2-3 (preferred)	Timmons et al., 2002
$T_{Min}(^{0}C)$	15	Gannam and Phillips, 1993
$T_{\text{Max}}(^{0}\text{C})$	41	Denzer, 1967
$T_{opt}(^{0}C)$	33	Caulton, 1982
UIA _{crit} (g/m ³)	0.06	Abdalla, 1989
UIA _{max} (g/m³)	1.4	Abdalla,1989
	0.025	Europea Inland Fishery
		Advisory Commission
		(EIFAC)

While the above set of equations can be used to model the growth rate of tilapia, and to make the changes in the feeding regimen, the daily ammonia excretion can be modeled by the following equation given by Hargreaves (1997):

Ammonia consumed, N _{input rate} = (dRa/dt)/10 * PROT * .16

Where,

DRa/dt - daily feed intake (g/day);

PROT – protein percentage in feed ($\sim 32 \%$);

0.16 – approximate nitrogen content in the protein(%);

Now, the ammonia excretion rate is,

$$N_{amm excr rate} = N_{input rate} (1-N_{retn}) * Diss_{frac}$$

Where:

 N_{retn} = proportion of ammonia retained in the fish (~25%);(different from efficiency of feed assimilation)

 $Diss_{frac}$ = fraction of ammonia excreted as dissolved ammonia(~80%);

Timmons et al. (2002) gave the following relation for the TAN production per day(kg/day) = daily feed intake * PROT * 0.16 * Diss_{frac} * Nitrogen assimilated * assimilated nitrogen that is excreted

Where:

Nitrogen assimilated = 0.8;

Assimilated nitrogen that is excreted = 0.8;

Thus Hargreaves predicts about 8 times less TAN than Timmons.

5.4. Modeling the nitrogen excretion and oxygen consumption

Though no data or models were found in the literature for nitrogen excretion and oxygen consumption specific to tilapia, there was some information for salmon. For example, Brett and Zala (1975) observed the following pattern(Fig.8.) of diurnal variation in ammonia excretion in fingerling sockeye salmon (*Oncorhynchus nerka*) that was fed a 3 % diet(as percentage of body weight) at 15 °C. The fish were fed only once daily at 8.00 am. Thus, it is seen that the ammonia excretion is roughly normally distributed as a function of time, peaking a couple of hours after feeding. However, it is expected that both the amplitude and the time lag to peak will vary with the quantity of feed given. For example, if the amount of feed per day remains the same, but feeding is done twice a day, the amplitude of the peak will be reduced. Moreover, the time lag to peak will also be reduced because the feed will be digested faster.

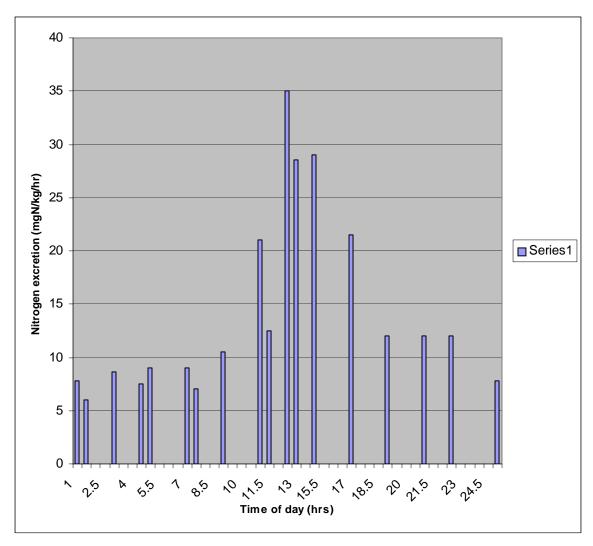


Fig. 9. Mean nitrogen excretion rates Vs time for sockeye salmon(adapted from Brett and Zala, 1975)

To incorporate these effects, the nitrogen (TAN) loading was modeled by the following equation:

TAN loading rate =
$$(0.0035 * Nitrogen consumed) / (fd * Sqr(2 * 3.1415 * sigma ^ 2))$$

* Exp(-(t - mean) ^ 2 / (2 * sigma ^ 2)))

Where,

Nitrogen consumed = $0.16 * 0.32 * 0.64 * 0.9 * Daily feed intake;$

fd = feeds per day;

t = time since the last feeding;

Timmons et al. (2001) mentions that a kilogram of feed requires about 250 grams of oxygen to metabolize. (The authors also say that 10 mg/L of oxygen consumed produces

about 1.4 mg/L of ammonia). Thus the constant of 0.0035 was obtained by trial and error and plugged into the above equation, so that 35 grams of ammonia was produced for every kilogram of feed consumed.

Limited data from Easter (1992) also validates this model. The two spikes are due to feedings at 8.30 am and mid-afternoon. The entire data set is provided in the appendix.

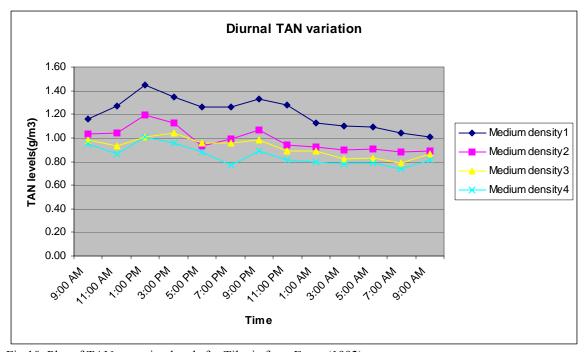


Fig. 10. Plot of TAN excretion levels for Tilapia from Easter(1992)

Even though the upper tolerance limit for TAN for tilapia is 1 mg/L, all TAN is not harmful to the fish. Only the unionized ammonia is harmful. At a given temperature and pH, the proportion of un-ionized ammonia is calculated as:

$$pk = .09018 + 2729.92/T \tag{12}$$
 where,
$$T = temperature (^{0} Kelvin)$$

$$K = 10^{-pk}$$

$$[H+] = 10^{-pH}$$
 So, the fraction of TAN in unionized form is given by Chapra as:
$$F = 1/[1+\{[H+]/K]\}$$

Since the conversion of ammonia to nitrite is the rate limiting step in the nitrification process, all the ammonia that is converted into nitrite will also be converted to nitrate. Hence it was considered unnecessary to model the nitrite-to-nitrate reaction.

The oxygen in the system was modeled as follows. Writing a mass balance equation for oxygen in the system:

Rate of oxygen loading into the system = Rate of oxygen coming in through fresh water + Rate of oxygen supply from the aeration tank – Rate of oxygen consumption by the fish

Rate of oxygen consumption by bacteria – Rate of oxygen loss through released water.

Rate of oxygen coming in through fresh water = FW * FWOxyConc

Where,

FW = Rate of fresh water flowing in (m³/day);

FWOxyConc- oxygen concentration in the freshwater coming in = 1000 * K * β * X *(P_{bp} - P_{wv})/760;

Where,

X = proportion of oxygen in the atmosphere = .20946;

K = 1.42903 for oxygen;

 $\beta = e^{[-58.388 + 85807.9/(T + 273.15) + 23.844*ln(T/100 + 273.15/100)]}$

Where,

 $T = temperature (^{0}C);$

 $P_{bp} = Barometric pressure = 10^{[2.8808 - h/19748.2]};$

Where.

h = altitude (m):

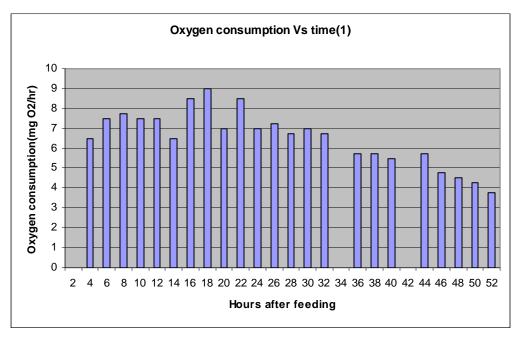
 $P_{wv} = \text{Water vapor pressure} = 4.76* e^{[0.0645*T]};$

Rate of oxygen supply from tank is a constant but depends on the type of system.

The rate of oxygen consumption by fish is modeled similar to the nitrification as follows.

The total oxygen consumption in the fish can be divided into three categories (Brett and Groves, 1979): Standard metabolism, active metabolism and feeding metabolism. Standard metabolism refers to the oxygen consumed to sustain the basic metabolic activities. Active metabolism is the oxygen consumed in swimming and feeding metabolism is oxygen consumed as food is digested. Altman and Dittmer (1974) reported an average standard metabolism for 57 species of 89 +- 34 (std.dev) mg O₂/kg of fish/hour. Data on active metabolism were not so forth coming. The active metabolism for Bass at 25 °C (350 mg O₂/kg of fish/hr) was used in this model. Moreover, in this model, the active metabolism is taken to be a normally distributed random

variable(variance 10%) because it changes daily. Feeding metabolism is seen to be roughly normally distributed as a function of time, rising to a peak value at the time of feeding and then falling.



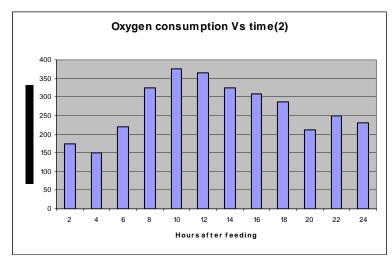


Fig.11. Oxygen consumption rate Vs time after feeding for 2 species of fish ((1)-Oxygen consumption plot following a single feeding for aholehole given a 4.7% ration at 23°C(Muir and Niimi,1972), (2)Oxygen consumption plot following a single feed for sockeye salmon given a 3% ration at 150C(Brett and Zala, 1975)adapted from Brett and Groves, 1979)

The oxygen consumed in feeding metabolism was modeled as:

$$= (0.025 * Nitrogen consumed) / (fd * Sqr(2 * 3.1415 * sigma ^ 2)) * Exp(-(t - mean) ^ 2 / (2 * sigma ^ 2)))$$
(13)

Again the equation is similar to the one for nitrogen excretion except that the constant 0.025 was plugged in to match the Timmons et al. (2001) rule of thumb; i.e., 250 gms of oxygen is consumed to oxidize 1 kg of feed.

Standard, active and feeding metabolism change with fish size and temperature. While the changes due to fish size are considered in this model, the temperature effect is ignored, mostly because the effect of temperature on fish metabolism is too complex to be included.

The rate of oxygen consumed by the bacteria is proportional to the rate of nitrification. From the nitrification equation given earlier, and using the molar weight of ammonia and oxygen, it was found that roughly 4.5 grams of oxygen is required in the complete oxidation of one gram of ammonia into nitrate. Thus, the rate of oxygen consumption by the bacteria is roughly 4.5 times the rate of nitrification.

The rate of oxygen loss through released water is simply the concentration of oxygen in the tank * the rate of removal of water from the system.

5.5. Modeling the Carbonate system

The carbonate system, the TAN equilibrium, temperature, pH and alkalinity are interrelated. So the proportion of the components of the carbonate system are calculated by assuming alkalinity to be a constant. In the actual RAS, this is easily achieved by adding additives to the water.

The total carbon level in the fish tank can be calculated by doing a mass balance on the rate for total carbonate carbon transfer. i.e.,

rate of change of total carbon in the fish tank = Rate of total carbon through fresh water + Rate of carbon influx from biofilter unit + Rate of carbon added by fish – Rate of carbon flowing out of fish tank.

Similarly, the rate of carbon change in the biofilter = Rate of carbon influx from fish tank + Rate of carbon dioxide produced by the nitrifiers – Rate of carbon dioxide stripping by the biofilter – Rate of total carbon flowing out of the biofilter unit.

In both the mass balances, atmospheric diffusion is ignored.

The amount of total carbonate carbon coming in through the fresh water is calculated as follows:

$$\begin{split} K_0 &= e^{[-58.0931 \,+\, 90.5069(100/(T+273.15)) + 22.294*ln((T+273.15)/100)]} \\ Beta &= 22.263 \, * \, K_0 \end{split}$$

So saturation concentration, Cs = $1000 * Beta * K * X * (P_{bp}-P_{wv})/760$

Where, K = 1.97681, X=.00032 and P_{bp} and P_{wv} are as derived in

So the rate of carbon dioxide influx through fresh water = Cs * freshwater intake rate.

The carbon influx from biofilter is explained in the next section.

The rate of carbon added by the fish is modeled as follows:

Sanni et al.(1996) gave the following relation for the amount of carbon dioxide produced by the fish for a unit amount of oxygen consumed:

$$MCO_2$$
 (mg CO_2 /kg fish/min)= 1.375 * MO_2 * RQ (14)

Where,

the previous section.

 MO_2 = Fish oxygen consumption rate (mg O_2 / kg fish/min);

RQ= Respiratory Quotient =

$$(.7 E_f + .9 E_p + E_c)/(E_f + E_p + E_c)$$

where, E_f , E_p & E_c are the metabolizable feed energy (MJ/kg) from fat, protein & carbohydrates respectively.

This evaluates to $1.126 \text{ mg CO}_2/\text{kg fish/min}$ produced for every mg O_2/kg fish/min consumed. Timmons et al. (2001) gave a figure of 14 mg of CO_2 produced for every 10 mg of O_2 consumed.

Since we assume that there is no time lag between oxygen consumption and carbon dioxide production, the rate of addition of carbon dioxide by fish can be modeled as eqn (13) * 1.4.

The carbon flowing into the biofilter unit is also explained in the next section.

The carbon dioxide production rate by the biofilter is also assumed to be 1.4 times the oxygen consumption rate.

Since it is impossible to accurately model the carbon dioxide stripping by the RBC, it is assumed to be 10 % of the carbon dioxide in the carbonate system.

From the above equations we can calculate the amount of total carbonate carbon in the fish tank and the biofilter unit. From this we can calculate the proportion of carbon dioxide using the temperature, alkalinity, salinity and TAN as follows:

```
Chlorinity = Salinity / 1.80655
TBorate = 0.000029 * Chlorinity
 If (Salinity < 5) Then
x = 0.016 * Salinity + 0.0532
 Else
x = 0.0007 * Salinity + 0.131
pK1 = 3404.71 / (273.15 + temperature) + 0.032786 * (temperature + 273.15) - 14.7122 -
0.19178 * Chlorinity ^ 0.3333
pK2 = 2902.39 / (273.15 + temperature) + 0.02379 * (temperature + 273.15) - 6.471 -
0.4693 * Chlorinity ^ 0.3333
pkB = 2291.9 / (273.15 + temperature) + 0.01756 * (temperature + 273.15) - 3.385 -
0.32051 * Chlorinity ^ 0.3333
pkN = 9.245 + 0.138 * (19.9273 * Salinity / (1000 - 1.005109 * Salinity)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034 * (298)) + (0.034
- temperature - 273.15)) + x
pkW = 4470.99 / (temperature + 273.15) - 6.0875 + 0.01706 * (temperature + 273.15)
k1 = 10 ^ (-1 * pK1)
k2 = 10 ^ (-1 * pK2)
kB = 10 ^ (-1 * pkB)
kN = 10 \land (-1 * pkN)
kW = 10 ^ (-1 * pkW)
a3 = (Alkalinity * (k1 + kN + kB) - Total carbonate carbon * k1 - Total Ammonia
Nitrogen * kN - TotalBorate * kB) / Alkalinity
```

```
a2 = (Alkalinity * (k1 * k2 + kN * kB + k1 * kN + k1 * kB) - Total carbonate carbon *
k1 * (2 * k2 + kN + kB) - Total Ammonia Nitrogen * kN * (k1 + kB) - TBorate * kB *
(k1 + kN) / Alkalinity
a1 = k1 * (Alkalinity * (k2 * kN + k2 * kB + kN * kB) - Total carbonate carbon * (kN *
kB + 2 * k2 * (kN + kB)) - Total Ammonia Nitrogen * kN * (k2 + kB) - TBorate * kB *
(k2 + kN) / Alkalinity
a0 = (k1 * k2 * kN * kB) * (Alkalinity - 2 * Total carbonate carbon - Total Ammonia
Nitrogen - TBorate) / Alkalinity
b = -a2
c = a1 * a3 - 4 * a0
d = -1 * (a1 ^2 + a0 * a3 ^2 - 4 * a0 * a2)
p = (3 * c - b ^ 2) / 9
qu = (2 * b ^ 3 - 9 * b * c + 27 * d) / 54
z = tan^{-1}(-((-qu/((-p)^1.5)))/Sqr(-((-qu/((-p)^1.5)))*((-qu/((-p)^1.5)))+1))+2*
tan^{-1}(1)
u1 = 2 * (-p) ^ 0.5 * Cos(z/3) - b/3
D0 = u1 / 2 - ((u1 / 2) ^ 2 - a0) ^ 0.5
D1 = a3 / 2 - (a3 ^ 2 / 4 + u1 - a2) ^ 0.5
H = (-D1 * (D1 ^ 2 - 4 * D0) ^ 0.5) / 2
pH = -Log(H)
        where,
        temperature in <sup>0</sup>C
        alkalinity in eq/L
        total borate = .000029 * Chlorinity
Once the exact pH is obtained the following equation from Summerfelt et al. (2003) can
be used to obtain amount of dissolved CO<sub>2</sub>:
CO_2 \text{ (mg/L)} = 44.000 \{\text{alkalinity/50000} - 10^{(\text{pH} - \text{pKw})} - 10^{(\text{-pH})}\} * \{1/(10^{(\text{pH} - \text{p(k}_0^K))} + 2*)\}
10^{(2pH-p(k_0K_1)-pK_2)})
        Where,
        Alkalinity in mg/L of CaCO<sub>3</sub>
```

Equations for K_0 , K_1 , K_2 were obtained from Gieskes (1974) and for Kw obtained from Stumm & Morgan(1981).

5.6. Modeling the mass transfer between system components

'Arena' uses two elements called 'levels' and 'rates' to model systems using differential equations. Every variable in the model whose value changes continuously with time is modeled as a 'rate' and 'level' element pair. The 'rate' element contains, in the form of a differential equation, the rate at which the variable changes its value. Using this 'rate', 'Arena' computes the value of the variable at any given point in the simulation and is stored within the 'level' element.

Since sedimentation is ignored in this effort, the RAS is modeled as a two component system consisting of the fish tank and a biofilter unit. As an example, consider the ammonia concentration in the fish tank. The differential equation used is:

 $V_t * dC_{t,a} / dt = WF * C_{b,a} - C_{t,a} * (WF + FW) + TAN \ loading \ rate$

Where,

 V_t = Volume of fish tank;

 $dC_{t,a}$ / dt = rate of change of ammonia concentration in the fish tank;

WF = water recirculation rate;

FW = Fresh water intake rate;

 $C_{b,a}$ = Concentration of ammonia in the biofilter;

 $C_{t,a}$ = concentration of ammonia in the fish tank;

TAN loading rate = as described in the previous section;

Weatherly et al. developed a very simple simulation model along these lines in 1993.

5.7. Plugging the cost factors

The water(2.82 \$/ kilogallon) and sewer cost(3.22 \$/kilogallon) was obtained from a project done by Ms. Stephanie Smith for the Agricultural Engineering Dept at Virginia Tech. The oxygen cost(.01 Cents/ m³ of oxygen)was calculated from a figure of 32 cents/ gallon of liquid oxygen and using the approximate relation that 1 gallon of liquid oxygen is equivalent to 3.26 m³ of gaseous oxygen (Timmons et al., 2001). The feed cost(44 cents/ kilogram) was also obtained from Ms. Smith's work. The power cost for running the pumps was taken to be 2 cents/ m³ of water pumped.

5.8. Optimizing the model

Using the optimizer feature of 'Arena' the low cost configuration was obtained. Since the algorithm is a heuristic and not an exact method, the output will not necessarily be a global optimum. However, the greater the time given for the program to run the greater the likelihood of attaining the global solution. The objective function and constraints used are as follows:

Minimize:

water cost + sewer cost + oxygen cost + pumping(power) cost

Subject to the constraints:

Maximum tank ammonia level <= 1 mg/L

Minimum tank oxygen level >= 5 mg/L

Maximum Carbon dioxide level <= 45 mg/L

6. Results and discussion

With the model as described previously, a single tank RAS facility is simulated for 8 months starting from the introduction of fish fingerlings on day one. Starting from an initial configuration of 1 feed/day, daily fresh water intake of 3.2 m³/day (15% of total water in the system), water recirculation rate of 100 m³/day (i.e., the water recirculates through the system 5 times a day) and oxygen input of 12000 grams/day, which costs about \$14,052, the optimizer (Optquest) arrived at a configuration of 12 feed/day, daily fresh water intake of 1 m³/day, water recirculation rate of 63 m³/day and an oxygen input of 11,000 grams/day. i.e.,the output of the model shows that the freshwater intake rate can be brought down from 15% to 5%, the daily water recirculation rate can be reduced from 5 times/day to 3 times/day and the oxygen supply reduced by 10 %, even while maintaining the ammonia, carbon dioxide and oxygen tolerance levels. This translates to a water, sewage, oxygen and power cost savings of about \$1,045.00 over the 8 month period.

Since the random variables in the model change daily, and the simulation is run for 8 months, there is no significant variation in costs across runs. Hence rather than

building a regression model with the amount of fish as the regressor, it is sufficient to validate using a paired t-test on the difference between the simulated cost of the given facility configuration and the actual cost. The rationale being that under null (H_0) hypothesis most of the differences will be clustered around zero. However, in the absence of substantial data from real facilities to validate the model, the following guidelines are used to validate:

- Eyeballing shows nitrogen & CO₂ generation & O₂ consumption rates can be approximated by a normal distribution. A Kolmogorov- Smirnov goodness-of-fit test on a limited data set gives a p-value of .04 suggesting that it is a reasonable though not an ideal approximation.
- By choosing an appropriate value for the 'feed assimilation efficiency' it is seen that the fish become full grown in 8 months. This roughly corresponds to what is observed in real facilities, confirming that fish growth is modeled accurately.
- From the model it is clear that the water replenishment rate can be reduced to about 5 % without compromising fish health. This percentage is equal to the best reported practices in the industry.

Since the total cost depends on the value of the individual unit costs, it necessary to see how sensitive the optimal cost is to changes in the individual costs. The robustness of the model is tested for using a sensitivity analysis. As is clear from the table below, a 10 % change in the control (unit cost of water, power, sewer and oxygen) produces a change of less than 1.6% in the total cost. Thus the model is quite robust.

Table.3. Percentage change in optimal cost of a 10% change in control value

10% change in value of con	trol new total cost	% change in total cost
.9*UnitWaterCost	1932	0.92%
1.1*UnitWaterCost	1968	-0.92%
.9*UnitPowerCost	1919	1.59%
1.1*UnitPowerCost	1980	-1.54%
.9*UnitSewerCost	1929	1.08%
1.1*UnitSewerCost	1970	-1.03%
.9*UnitOxyCost	1923	1.38%
1.1*UnitOxyCost	1976	-1.33%

7. Summary and Conclusion

In the absence of real data to validate the model, the benefit of this work is limited to demonstrating the potential of computer simulation to cut costs in an RAS facility. Theoretically, this model predicts a potential \$ 1,000.00 savings in water, sewage and oxygen costs by identifying a near optimal water and oxygen replenishment rate. This translates to a savings of 12,000.00 dollars for a medium sized commercial facility.

The model can be further enhanced to optimize feed costs as well by better modeling fish growth. Future efforts would also attempt to incorporate the effect of temperature on fish growth and the overall system efficiency. This can also improve the accuracy of the water and oxygen requirement predictions. It is also possible to model the transfer of fish between tanks at various stages of growth. Since most existing aquaculture facilities do this to maximize capacity utilization, incorporating this into the model will make it more realistic. It will also open up more variables for optimization based on mortality rates and capacity utilization.

Paper

Using continuous simulation to reduce costs at a Recirculating Aquaculture facility

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Abstract

The Recirculating Aquaculture System can be considered as a dynamic system in which the system variables change instantaneously during the growing period. This effort uses a combination of discrete and continuous simulation to model this system. Changing the values of a certain set of variables (regressors) result in changes to another set of variables (responses). So the inter relationship between the regressors and responses are first defined. Then the changes to the regressors are defined as a function of time. The computer program then simulates time from the beginning to the end of the growing period. By attaching cost factors, the cost of running the system during the growing period can be obtained.

In a real facility, some of the regressors can be changed by the operator, affecting the overall cost. So, in this model, different values for this subset of regressors are tried out using a proprietary heuristic called 'Optquest' to arrive at a low cost configuration. Some of the responses are also constrained while trying to minimize the cost, e.g., ammonia level.

The output of the model gives a cost savings of about \$10,000 over a period of 8 months for a facility with 12 tanks by suggesting optimal values for the water replenishment rate, water recirculation rate, oxygen supply rate and feeds per day.

Nomenclatu	re
S	inflow ammonia concentration
T	temperature
L	thickness of water film at temperature T
u	water viscosity
ρ	water density
d	characteristic length of film
V	water velocity
3	void fraction
dW/dt	rate of change of fish weight
dRa/dt	daily feed intake
a	effect of food catabolism
b	efficiency of feed assimilation
K	coefficient of fasting catabolism
W	body weight

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n exponent of weight for fasting catabolism

F feed available

τ temperature factor

δ dissolved oxygen factor

γ unionized ammonia factor

photo period factor

 T_{max} maximum temperature tolerable for the fish minimum temperature tolerable for the fish

 T_{opt} optimal temperature for the fish

DO_{min} minimum dissolved oxygen levels regrd

DO current oxygen levels

UIA_{max} maximum unionized ammonia permitted

UIA_{crit} unionized ammonia level below which feeding is not

affected by UIA conc.

PROT protein percentage in feed N amm exer rate ammonia excretion rate

N_{retn} proportion of ammonia retained in the fish

Diss_{frac} fraction of ammonia excreted as dissolved ammonia

N input rate ammonia consumed through feed

fd feeds per day

t time since the last feeding FW rate of fresh water flowing in

FWOxyConc oxygen concentration in the freshwater coming in

X proportion of oxygen in the atmosphere

P_{bp} barometric pressure

h altitude

 P_{wv} water vapor pressure Saturation concentration MO_2 fish oxygen consumption rate

RQ respiratory quotient

E_f, E_p & E_c metabolizable feed energy from fat, protein &

carbohydrates

 V_t volume of fish tank

dC_{ta} / dt rate of change of ammonia concentration in the fish tank

WF water recirculation rate FW fresh water intake rate

 $C_{b,a}$ concentration of ammonia in the biofilter concentration of ammonia in the fish tank

1. Introduction

Close to a billion people world wide depend on the sea for their subsistence. Seafood provides an ideal source of dietary protein for much of the impoverished peoples of the world. According to the Food and Agricultural Organization (FAO,1998) wing of the UN, a city with about 10 million in population will need about 6000 tons of food each day. Thus, it is impossible to feed this population without alternate sources like seafood,

given that most of the world population will live in such cities by 2010. Moreover, 73% of the world's most productive natural fisheries have experienced steady declines since the 1970s. Given this background the development of recirculating aquaculture assumes great importance in order to maintain enough food in the world.

The aquaculture industry today is a very fragile industry operating on very thin profit margins. Hence, it is imperative to have an optimal configuration for the recirculating aquaculture facility that will minimize the overall cost. The goal of this work is to use continuous simulation to arrive at this low cost configuration.

Towards this end the biological component of the system, consisting of the fish tank and the biofiltration unit was simulated. Continuous simulation was the method chosen because the variables involved change continuously over time and reached steady state only when the fish reached maturity at the end of the growing season.

2. The biological process overview

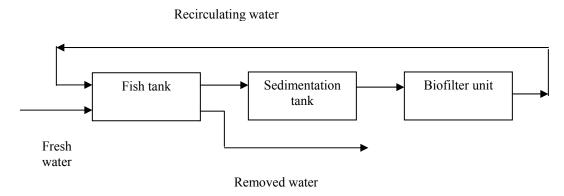


Fig. 1. Block diagram of a single tank RAS facility

This modeling effort assumes a three unit system configuration as shown in the figure.

3. Literature Review

3.1 Nitrification

Only ammonium part (and not unionized ammonia, NH₃) can be nitrified.

The efficiency of the biofilter depends on several factors like the inflow substrate concentration, the flow rate, the RPM of the RBC, temperature, pH the population of the nitrogenous bacteria, the oxygen supply, etc. Of these, the following factors change dynamically during each trial:

- a) The substrate concentration in the biofilm decreases as the film leaves the reactor liquid and might reach zero before reentry. Upon reentry there is a sudden increase in the concentration.
- b) Variation in influent waste concentration, flow rate and constituents.
- c) Sloughing of the film.
- d) Changes in temperature and pH.

An ideal model will take all these factors into consideration. There are several models found in the literature. Although none of them are perfect, each offers several advantages and disadvantages. Three different models are presented here:

1. Monod equation- Assuming that the population of the bacteria are at steady state, a simple Monod equation can be constructed for any given temperature to model the variation of TAN and nitrite removal efficiencies with inflow TAN and nitrite concentrations. Zhu and Chen (1999) suggested the following modification to the Monod equation for the fixed film reactor:

Ammonia removal rate, $mg/L/m^2 = R_{max} * (S - S_{min})/(S + Ks - S_{min})$ (1) At 27°C (which is the approximate temperature at which tilapia are reared) this reduces to:

$$1859*(S-.07)/(S+1.93)$$
Where S is inflavy TAN concentration (mg/L)

Where, S is inflow TAN concentration (mg/L)

Even though this model is simple in application, it reduces the utility of the overall model. For example, the constants have to be evaluated for each temperature change. Moreover, the actual RBC does not always remain under water like a fixed-film reactor, but alternates between rotating in the air and water.

2. Zhu and Chen (2002) further developed a fixed-film model that incorporated flow rate, oxygen limitation, TAN limitation and temperature as well into an equation. When TAN was limiting, the ammonia removal rate was given as:

(Dws/L)*
$$\sqrt{(Ds * K_{20} * Q^{T-20})} * S / (Dws/L + \sqrt{(Ds * K_{20} * Q^{T-20})})$$
 (3) Where, T is Temperature (0 C), Dws = .0000886*1.026 T , L is thickness of water film at temperature T. L is calculated as:

$$1.23 * (u/\rho)^{1/3} * (d/v)^{1/2} * \varepsilon^{3/2} * (Dwc/86400)^{1/3}$$
(4)

Where, u is water viscosity(g/m/day), ρ is water density (g/m³), d is characteristic length of film (0.01 m), v is water velocity (m/sec), ϵ is void fraction (0.98, dimensionless), Ds is 0.8 * Dws, K_{20} is 390000 (/day), Q = 1.1

Even though this equation predicts a linear increase in the TAN removal rate with increases in TAN concentration, it is considered valid at lower concentrations of TAN.

The main short coming of this model is that it is suited more for a fixed-film than for an RBC.

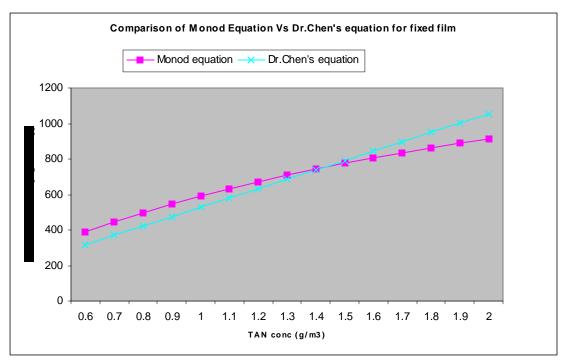


Fig.2. Comparison between Monod equation and the equation by Zhu and Chen (2002)

- 3. Greaves (1972) built a heterogeneous model, a pseudo homogeneous model and a steady state model specifically for the RBC and reported excellent correlation with observed data especially for short runs. The only two disadvantages of Grieves' models are:
 - a) The model does not predict long term performances well, probably because his equation does not model the changes in the bacterial population very well
 - b) The number of differential equations are too high for the simulation software used in this effort (trial version) to handle. This however can be handled by a professional version of the same software.

3.2. Relationship between variables

3.2.1. Ammonia, Nitrite and Nitrate levels

The rate of nitrification depends on the temperature, pH, water flow rate through the system, dissolved oxygen concentration, alkalinity, amount of bacteria and the ammonia and nitrite concentration. The amount of TAN available for nitrification depends of the proportion of ammonia (NH₃), which depends on the temperature, salinity and pH.

Some papers suggest that 4.2 grams of oxygen is consumed in oxidizing one gram of ammonia to nitrate(Gaudy & Gaudy, 1980) while others suggest 4.3 and even 4.57 (Zhu and Chen, 2002)

Because the growth rate of Nitrosomonas is less than Nitrobacter, oxidation of ammonia is usually considered the rate limiting step.

Nitrification consumes bicarbonates (HCO₃⁻) and produces carbon dioxide. This elevated CO₂ level near the biofilm causes a drop in the pH level near the biofilm as a function of temperature. For example, at 10-15 ⁰ Celsius a pH of 7 in the bulk solution means a pH of <6 near the biofilm (Boller et al., 1994)

Even though the nitrate levels are typically not monitored in aquaculture facilities, at elevated levels it can reduce the immunity of the fish and can even result in fish death (Hrubec et al. 1996; Spotte 1979)

3.2.2. pH

Even though the optimal pH range for nitrification is wide(6.5-9), it might be narrower for a specific filter. Nitrification efficiency reduces drastically between 6.5 and about 7.5, and becomes close to zero below 6(Boller et al. (1994). Elevated carbon dioxide from fish metabolism can reduce the pH, thus affecting nitrification rates (Grace and Piedrahita,1994). But, at high levels of pH, the equilibrium between ammonium(NH₄⁺) and unionized ammonia (NH₃)shifts towards unionized ammonia, increasing the proportion of unionized ammonia in the system. Since UIA is harmful to the fish, the pH must be maintained in the middle of the acceptable range.

3.2.3. Alkalinity

Fish produce carbon dioxide and excrete TAN in proportion to the oxygen and feed consumed. However, carbonates, bicarbonates, carbon dioxide and carbonic acid exist in an equilibrium with each other, as does ammonia and ammonium. The alkalinity of water affects this equilibrium.

Conversion of NH₄⁺ to nitrate consumes alkalinity. The complete oxidation of 1 mg of ammonia to nitrate consumes 7.14 mg of alkalinity expressed as CaCO₃. Alkalinity levels (concentration of carbonates and bicarbonates) affect nitrifier growth (Malone and Burden, 1988). Gujer and Boller (1986) stated that 75 mg/L of CaCO₃ alkalinity was sufficient to maintain maximum nitrification rates.

3.2.4. Oxygen

Both fish and nitrifying bacteria consume oxygen. The limiting oxygen concentration is a function of temperature, concentration of organics in the feed water and bacterial biomass.

3.2.5 Temperature

Temperature is probably the most important factor affecting fish growth after feed. It affects almost every other variable in the system. Temperature directly affects the nitrification rate in the biofilter, the equilibrium between ionized and unionized ammonia and the amount of oxygen and carbon dioxide dissolved in the water. As the temperature increases the growth rate increases to a maximum and then reduces.

3.2.6 *Salinity*

Salinity affects the proportion of UIA in the system. Abrupt changes in salinity of greater than 5 g/L will also shock the bacteria and decrease the reaction rate for both ammonianitrogen and nitrite-nitrogen removal. However over time bacteria are known to acclimatize.

3.3. Acceptable ranges for the variables

The optimal values for the variables of concern for tilapia in an RAS as found in the literature are as follows:

Table.1. Factors that affect fish health and their tolerance limits as found in literature

Variable	Value	Cited by:
Ammonia (TAN)	< 0.05	Colt and Armstrong,1981
concentration(mg/L)	<1	Malone 1999
	< 0.6	Timmons et al., 2002
Unionized ammonia	~12.5	Colt and Orwicz,1991
conc(µg/L)		
nitrite concentration (mg/L)	<1	Losordo,1991
nitrate concentration (mg/L)	<100	Losordo,1991
Dissolved oxygen (mg/L)	>5	Kaiser and Wheaton, 1983;
		Losordo, 1991
DO-minimum concentration	2	Hochheimer and Wheaton
for nitrification (mg/L)		,1991
CO_2 (mg/L)	40-50	Timmons, et al., 2002
pH level (mg/L)	6.5-8	Meade,1989
	6.7-9	Klontz,1979
Total Suspended Solids	<15	FIFAC, 1980
(mg/L)	20-40	Muir,1982
	22-23	Schmitz,1999
Temperature (⁰ C)	20-30	Malone, 1999
Alkalinity (mg/L as CaCO ₃)	>100	Meade, 1989
		Losordo, 1991
Hardness (mg/L as CaCO ₃)	>100	Meade, 1989

4. Model development

4.1. Modeling the biofilter unit

This model uses Zhu and Chen's modification to the Monod equation and Zhu and Chen's fixed-film equation, as described in the previous section to model the RBC.

4.2 Modeling the fish growth

Several authors have developed several equations to model fish growth. For example, Ursin's growth model (Yi, 1998), which is used here, is of the form:

$$dW/dt = [b(1-a)dRa/dt] - KW^{n}$$
(5)

Where, dW/dt is the rate of change of fish weight (g/day), dRa/dt is the daily feed intake (g/day), a is the effect of food catabolism, b is efficiency of feed assimilation(dimensionless), K is coefficient of fasting catabolism, W is body weight (g), n is exponent of weight for fasting catabolism.

However, for an aquaculture facility, the term for catabolism (KWⁿ) can be ignored since the fish are regularly fed. Also, the term b is not a constant, but decreases as the amount of feed available increases.

Now,
$$dRa/dt = \tau \delta \rho \gamma F$$
 (6)

Where, F is feed available (g/day), τ is temperature factor(0< τ < 1, dimensionless), δ is dissolved oxygen factor (0 < δ < 1, dimensionless), γ is unionized ammonia factor (0 < γ < 1, dimensionless), ρ is photo period factor [0 < ρ < 2, number of sunlight hours/12 (1 for RAS facility), dimensionless].

Here the assumption is that fish will not be fed an amount (feed available) more than what they can eat.

Based on work by Brett(1979), Colt and Armstrong (1981), Svirezhev et al. (1984) and Cuenco et al. (1985a), Bolte et al. (1995) suggested the following equations for τ , δ , ρ and γ :

$$\begin{array}{ll} \tau = e^{\{-4.6[(Topt-T)/(Topt-Tmin)]4\}} & \text{if } T < T_{opt} & (7) \\ = e^{\{-4.6[(T-Topt)/(Tmax-Topt)]4\}} & \text{if } T >= T_{opt} \\ \delta = 1 & \text{if } DO > DO_{crit} & (8) \\ = (DO-DO_{min})/(DO_{crit}-DO_{min}) & \text{if } DO < DO_{min} \\ = 0 & \text{if } DO < DO_{min} \\ \gamma = 1 & \text{if } UIA < UIA_{Crit} & (9) \\ = (UIA_{max}-UIA)/(UIA_{max}-UIA_{crit}) & \text{if } UIA_{crit} <= UIA <= UIA_{max} \\ = 0 & \text{if } UIA > UIA_{max} \end{array}$$

Where, T_{max} is maximum temperature tolerable for the fish (°C), T_{opt} is optimal temperature for the fish (24°C for tilapia), DO_{min} is minimum dissolved oxygen levels required (mg/L),

DO is current oxygen levels (mg/L), UIA_{max} is maximum unionized ammonia permitted (mg/L) and UIA_{crit} is the unionized ammonia level below which feeding is not affected by UIA concentration (mg/L)

The range in food assimilation efficiency (b) was reported to be between 0.53 and 0.7, decreasing with increase in feed intake (Meyer-Burgdorff et al., 1989). However, for modeling purposes, the food assimilation efficiency was taken to be the mean; i.e., 0.62. The parameter for the effect of feeding catabolism, 'a', was taken to be random variable with a mean of 0.53 (Nath et al. 1994) and variance of 0.1 (normally distributed). This was done because the fish display varying activity levels each day. The minimum temperature tolerable for Nile tilapia was reported by Gannam and Phillips (1993) to be 15°C. The optimal temperature is the temperature at which the fish appetite is high and maintenance requirements (energy expenditure) are low. This was given as 33° C by Caulton (1982). The maximum temperature was 41° C (Denzer, 1967). Tilapia, due to their ability to use atmospheric oxygen can survive with dissolved oxygen concentrations as low as 0.1 - 0.3 mg/L (Ahmed and Magid, 1968, Magid and Babiker, 1975). However DO_{Crit} was calculated using Teichert-Coddington and Green's (1993) suggestion that the threshold DO for Nile tilapia was not greater than 10 % of saturation, and thus was taken to be 1.0 mg/L at 15 °C. However, it should be noted that this amount decreases with increasing temperature and increasing altitude. Abdalla (1989) determined that UIA_{Max} was 1.4 mg/L and UIACrit was 0.06 mg/L for Nile tilapia. Values for the aforementioned parameters, as found in literature are tabulated below(Table 2.):

Table 2. Values for parameters in Ursin's growth model as found in literature

Parameters	Value	Source
a	0.53	Nath et al. 1994
b	0.53- 0.7	Meyer-Burgdorff et al. 1989
$\mathrm{DO}_{\mathrm{Crit}}$	2.0	Teichert-Coddington and
		Green, 1993
	~6 (preferred)	Timmons et al., 2002
$DO_{Min}(g/m^3)$	0.3	Ahmed and Magid, 1968
	2-3 (preferred)	Timmons et al., 2002
$T_{Min}(^{0}C)$	15	Gannam and Phillips, 1993
$T_{\text{Max}}(^{0}\text{C})$	41	Denzer, 1967
$T_{opt}(^{0}C)$	33	Caulton, 1982
UIA _{crit} (g/m ³)	0.06	Abdalla, 1989
$UIA_{max} (g/m^3)$	1.4	Abdalla,1989
, ,	0.025	Europea Inland Fishery
		Advisory Commission
		(EIFAC)

While the above set of equations can be used to model the growth rate of tilapia, and to decide the change to the feeding regimen, the daily ammonia excretion can be modeled by the following equation given by Hargreaves (1997):

Ammonia consumed, N
$$_{input rate} = (dRa/dt)/10 * PROT * 0.16$$
 (10)

Where, dRa/dt is daily feed intake (g/day), PROT is protein percentage in feed (~ 32 %), 0.16 is the approximate nitrogen content in the protein (%)

Now, the ammonia excretion rate is,

$$N_{\text{amm excr rate}} = N_{\text{input rate}} (1-N_{\text{retn}}) * \text{Diss}_{\text{frac}}$$
 (11)

Where, N_{retn} = proportion of ammonia retained in the fish (~25%); (different from efficiency of feed assimilation)

 $Diss_{frac}$ = fraction of ammonia excreted as dissolved ammonia (\sim 80%)

Timmons et al. (2002) gave the following relation for the TAN production per day (kg/day) = daily feed intake * PROT * 0.16 * Diss_{frac} * Nitrogen assimilated * assimilated nitrogen that is excreted (12)

Where, nitrogen assimilated is taken to be 0.8 and assimilated nitrogen that is excreted is taken to be 0.8.

Thus, Hargreaves predicts about 9 times less TAN than Timmons.

4.3. Modeling the nitrogen excretion and oxygen consumption

Though no data or models were found in the literature for nitrogen excretion and oxygen consumption specific to Tilapia, there was some information for salmon. For example, Brett and Zala (1975) observed the following pattern of diurnal variation in ammonia excretion in fingerling sockeye salmon (*Oncorhynchus nerka*) that was fed a 3 % diet at 15 0 C. The fish were fed only once daily at 8.00 am. Thus, it is seen that ammonia excretion is roughly normally distributed as a function of time, peaking a couple of hours after feeding. However, it is expected that both the amplitude and the time lag to peak will vary with the quantity of feed given. For example, if the amount of feed per day remains the same, but feeding is done twice a day then, the amplitude of the peak will come down. Moreover, the time lag to peak will also reduce because the feed will be digested faster.

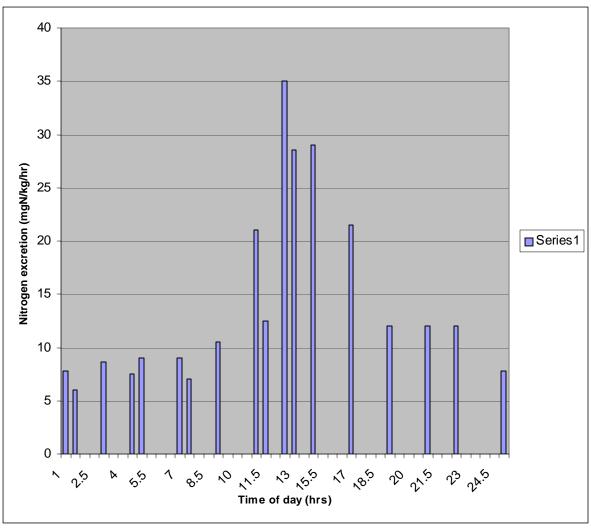


Fig.3.Mean nitrogen excretion rates vs time for sockeye salmon (adapted from Brett and Zala, 1975)

To incorporate these effects, the nitrogen(TAN) loading was modeled by the following equation:

TAN loading rate = $(0.0035 * Nitrogen consumed) / (fd * Sqr(2 * 3.1415 * sigma ^ 2)) * Exp(-(t - mean) ^ 2 / (2 * sigma ^ 2)))$ (13)

Where, nitrogen consumed is 0.16 * 0.32 * 0.64 * 0.9 * Daily feed intake, fd is feeds per day, t is time since the last feeding.

Timmons et al. (2001) mentions that a kilogram of feed requires about 250 grams of oxygen to metabolize. The authors also say that 10 mg/L of oxygen consumed produces about 1.4 mg/L of ammonia. Thus, the constant of 0.0035 was obtained by trial and error and plugged into the above equation such that 35 grams of ammonia was produced for every kilogram of feed consumed.

Limited data from Easter (1992) also validates this model. The two spikes are due to the feeding 8 30 am mid afternoon. at

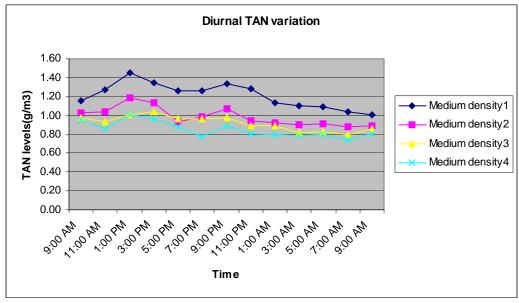


Fig.4. Plot of TAN excretion levels for Tilapia from Easter(1992)

Even though the upper tolerance limit for TAN for Tilapia is 1 mg/L, all TAN is not harmful to the fish. Only the un-ionized ammonia is harmful. At a given temperature and pH, the proportion of un-ionized ammonia is calculated as:

$$pK = 0.09018 + 2729.92/T$$

$$K = 10^{-pk}$$

$$[H+] = 10^{-pH}$$
So, the fraction of TAN in unionized form is given by (Chapra):
$$F = 1/[1+\{[H+]/K\}]$$
Where, T is the temperature (${}^{0}K$)

Since the conversion of ammonia to nitrite is the rate limiting step in the nitrification process, all the ammonia that is converted into nitrite will also be converted to nitrate. Hence it was considered unnecessary to model the nitrite-to-nitrate reaction.

The oxygen in the system was modeled as follows.

Writing a mass balance equation for oxygen in the system:

Rate of oxygen loading into the system =

Rate of oxygen coming in through fresh water + Rate of oxygen supply from oxygen tank - Rate of oxygen consumption by the fish - Rate of oxygen consumption by bacteria – Rate of oxygen loss through released water (16)

Rate of oxygen coming in through fresh water = FW * FWOxyConc (17)

Where, FW is rate of fresh water flowing in (m³/day), FWOxyConc is oxygen concentration in the influent freshwater. This is obtained using the equation:

$$1000 * K * \beta * X * (P_{bp}-P_{wv})/760$$
 (18)

Where, X is proportion of oxygen in the atmosphere (0.20946), k is 1.42903 for oxygen and β is as given below: $\beta = e^{[-58.388 + 85807.9/(1+273.15) + 23.844*ln(T/100+273.15)]}$

$$\beta = e^{[-58.388 + 85807.9/(T + 273.15) + 23.844*\ln(T/100 + 273.15)]}$$
(19)

Where, T is temperature (⁰C), P_{bp} is Barometric pressure. Again, barometric pressure is obtained by the following equation:

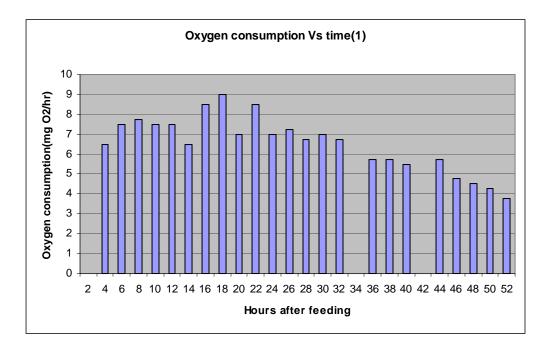
$$P_{bp} = 10^{[2.8808 - h/19748.2]}$$
 (20)

Where, h is altitude (m), P_{wv} is water vapor pressure given by: $P_{wv} = 4.76*~e^{[0.0645*T]}$

$$P_{\rm wv} = 4.76 * e^{[0.0645 * 1]} \tag{21}$$

Rate of oxygen supply from tank is a constant, but depends on the type of system. Rate of oxygen consumption by fish is modeled similar to the nitrification as follows:

The total oxygen consumption in the fish can be divided into 3 categories (Brett and Groves ,1979): standard metabolism, active metabolism and feeding metabolism. The standard metabolism refers to the oxygen consumed to sustain the basic metabolic activities. Active metabolism is the oxygen consumed during swimming and feeding metabolism is oxygen consumed in digesting the feed. Altman and Dittmer(1974) reported a 57 species average standard metabolism of 89 +- 34 (std.dev) mg O₂/kg of fish/hour. Data on active metabolism were not so forth coming. So, the active metabolism for Bass at 25 °C (350 mg O₂/kg of fish/hr) was used in this model. Moreover, in this model, the active metabolism is taken to be a random variable because it changes daily. The feeding metabolism is seen to be roughly normally distributed as a function of time, rising to a peak value after the time of feeding and then falling.



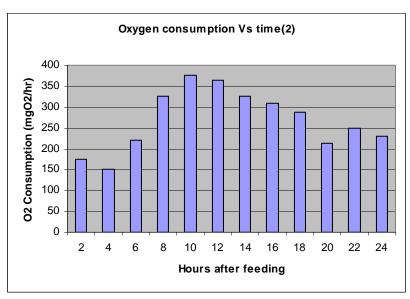


Fig.5. Oxygen consumption rate Vs time after feeding for 2 species of fish ((1)-Oxygen consumption plot following a single feeding for aholehole given a 4.7% ration at 23°C(adapted from Muir and Niimi,1972), (2)Oxygen consumption plot following a single feed for sockeye salmon given a 3% ration at 15°C(adapted from Brett and Zala, 1975))

So, the Oxygen consumed in feeding metabolism was modeled as:

$$= (0.025 * Nitrogen consumed) / (fd * Sqr(2 * 3.1415 * sigma ^ 2)) * Exp(-(t - mean) ^ 2 / (2 * sigma ^ 2)))$$
 (22)

The equation is similar to the one for nitrogen excretion except that the constant 0.025 was plugged in to match the Timmons (2001) rule of thumb; i.e, 250 gms of oxygen is consumed to oxidize 1 kg of feed.

The standard, active and feeding metabolisms change with fish size and temperature. While the changes due to fish size are considered in this model, the temperature effect is ignored.

The rate of oxygen consumed by the bacteria is proportional to the rate of nitrification. From the nitrification equation given below, and using the molar weight of ammonia and oxygen, it was found that roughly 4.5 grams of oxygen is required in the complete oxidation of one gram of ammonia into nitrate. Thus, the rate of oxygen consumption by the bacteria is roughly 4.5 times the rate of nitrification.

The rate of oxygen loss through released water is simply the concentration of oxygen in the tank * the rate of removal of water from the system.

4.4. Modeling the Carbonate system

The carbonate system, the TAN equilibrium, temperature, pH and alkalinity are interrelated. So, the proportion of the components of the carbonate system are calculated

by assuming alkalinity to be a constant. In the actual RAS, this is easily achieved by adding additives to the water.

The total carbon level in the fish tank can be calculated by doing a mass balance on the rate for total carbonate carbon transfer; i.e.,

Rate of change of total carbon in the fish tank = Rate of total carbon through fresh water + Rate of carbon influx from biofilter unit + Rate of carbon added by fish - Rate of carbon flowing out of fish tank (23)

Similarly, the rate of carbon change in the biofilter = Rate of carbon influx from fish tank + Rate of carbon dioxide produced by the nitrifiers – Rate of carbon dioxide stripping by the biofilter – Rate of total carbon flowing out of the biofilter unit (24) In both the mass balances, atmospheric diffusion is ignored.

The amount of total carbonate carbon coming in through the fresh water is calculated as follows:

$$K_0 = e^{[-58.0931 + 90.5069(100/(T+273.15)) + 22.294*\ln((T+273.15)/100)]}$$
(25)

Beta =
$$22.263 * K_0$$
 (26)

So saturation concentration, Cs =
$$1000 * Beta * K * X * (P_{bp}-P_{wv})/760$$
 (27)

Where, K is 1.97681, X is 0.00032 and P_{bp} and P_{wv} are as derived in the previous section.

The rate of carbon dioxide influx through fresh water = Cs * freshwater intake rate (28)

The carbon influx from biofilter is explained in the next section.

The rate of carbon added by the fish is modeled as follows:

Sanni et al.(1996) gave the following relation for the amount of carbon dioxide produced by the fish for a unit amount of oxygen consumed:

$$MCO_2$$
 (mg CO_2 /kg fish/min)= 1.375 * MO_2 * RQ (29)

Where, MO_2 is fish oxygen consumption rate (mg O_2 / kg fish/min), RQ is respiratory quotient = ($.7 E_f + .9 E_p + E_c$)/ ($E_f + E_p + E_c$) (30)

Where, E_f , E_p & E_c are the metabolizable feed energy (MJ/kg) from fat, protein & carbohydrates respectively.

This yields 1.126 mg CO_2/kg fish/min produced for every mg O_2/kg fish/min consumed. Timmons et al. (2001) gave a figure of 14 mg of CO_2 produced for every 10 mg of O_2 consumed.

Since we assume that there is no time lag between oxygen consumption and carbon dioxide production, the rate of addition of carbon dioxide by fish can be modeled by eqn (22) * 1.4 (31)

The carbon flowing into the biofilter unit is also explained in the next section.

The carbon dioxide production rate by the biofilter is also assumed to be 1.4 times the oxygen consumption rate.

Since it is impossible to accurately model the carbon dioxide stripping by the RBC, it is assumed to be 10 % of the carbon dioxide in the carbonate system.

From the above equations we can calculate the amount of total carbonate carbon in the fish tank and the biofilter unit. From this we can calculate the proportion of carbon dioxide using the temperature, alkalinity, salinity and TAN as follows:

(32)

Chlorinity = Salinity / 1.80655

```
TBorate = 0.000029 * Chlorinity
                                                                                 (33)
If (Salinity < 5) Then
x = 0.016 * Salinity + 0.0532
Else
x = 0.0007 * Salinity + 0.131
pK1 = 3404.71 / (273.15 + temperature) + 0.032786 * (temperature + 273.15) - 14.7122 -
0.19178 * Chlorinity ^ 0.3333
                                                                                 (34)
pK2 = 2902.39 / (273.15 + temperature) + 0.02379 * (temperature + 273.15) - 6.471 -
0.4693 * Chlorinity ^ 0.3333
                                                                                 (35)
pkB = 2291.9 / (273.15 + temperature) + 0.01756 * (temperature + 273.15) - 3.385 -
0.32051 * Chlorinity ^ 0.3333
pkN = 9.245 + 0.138 * (19.9273 * Salinity / (1000 - 1.005109 * Salinity)) + (0.034 * (298))
- temperature - 273.15)) + x
pkW = 4470.99 / (temperature + 273.15) - 6.0875 + 0.01706 * (temperature + 273.15)
                                                                                 (38)
k1 = 10 ^ (-1 * pK1)
                                                                                 (39)
k2 = 10 \land (-1 * pK2)
                                                                                 (40)
kB = 10 ^ (-1 * pkB)
                                                                                 (41)
kN = 10 ^ (-1 * pkN)
                                                                                 (42)
kW = 10 ^ (-1 * pkW)
                                                                                 (43)
a3 = (Alkalinity * (k1 + kN + kB) - Total carbonate carbon * k1 - Total Ammonia
Nitrogen * kN - TotalBorate * kB) / Alkalinity
a2 = (Alkalinity * (k1 * k2 + kN * kB + k1 * kN + k1 * kB) - Total carbonate carbon *
k1 * (2 * k2 + kN + kB) - Total Ammonia Nitrogen * kN * (k1 + kB) - TBorate * kB *
(k1 + kN) / Alkalinity
a1 = k1 * (Alkalinity * (k2 * kN + k2 * kB + kN * kB) - Total carbonate carbon * (kN *
kB + 2 * k2 * (kN + kB)) - Total Ammonia Nitrogen * kN * (k2 + kB) - TBorate * kB *
(k2 + kN) / Alkalinity
a0 = (k1 * k2 * kN * kB) * (Alkalinity - 2 * Total carbonate carbon - Total Ammonia
Nitrogen - TBorate) / Alkalinity
                                                                                 (47)
b = -a2
                                                                                 (48)
c = a1 * a3 - 4 * a0
                                                                                 (49)
d = -1 * (a1 ^2 + a0 * a3 ^2 - 4 * a0 * a2)
                                                                                 (50)
p = (3 * c - b ^ 2) / 9
                                                                                 (51)
qu = (2 * b ^ 3 - 9 * b * c + 27 * d) / 54
                                                                                 (52)
z = tan^{-1}(-((-qu/((-p)^{1.5}))/Sqr(-((-qu/((-p)^{1.5})))*((-qu/((-p)^{1.5}))) + 1)) + 2*
tan<sup>-1</sup>(1)
                                                                                 (53)
u1 = 2 * (-p) ^ 0.5 * Cos(z/3) - b/3
                                                                                 (54)
D0 = u1 / 2 - ((u1 / 2)^2 - a0)^0 0.5
                                                                                 (55)
D1 = a3 / 2 - (a3 ^ 2 / 4 + u1 - a2) ^ 0.5
                                                                                 (56)
H = (-D1 * (D1 ^ 2 - 4 * D0) ^ 0.5) / 2
                                                                                 (57)
pH = -Log(H)
                                                                                 (58)
       Where, temperature in <sup>0</sup>C, alkalinity in eq/L and total borate given by:
       Total Borate = 0.000029 * Chlorinity
                                                                                 (59)
```

Once the exact pH is obtained, the following equation from Summerfelt et al. (2003) can be used to obtain amount of dissolved CO₂:

CO₂ (mg/L) = 44,000 {alkalinity/50000 -
$$10^{(pH - pKw)}$$
 - $10^{(-pH)}$ } * { $1/(10^{(pH - p(k_0 K_1))} + 2* 10^{(2pH - p(k_0 K_1) - pK_2)})$ } (60)

Where, alkalinity is in mg/L of CaCO₃

 K_0 , K_1 and K_2 equations were obtained from Gieskes (1974) and the Kw equation was obtained from Stumm & Morgan (1981).

4.5. Modeling the mass transfer between system components

'Arena' uses two elements called 'levels' and 'rates' to model systems using differential equations. Every variable in the model whose value changes continuously with time is modeled as a 'rate' and 'level' element pair. The 'rate' element contains, in the form of a differential equation, the rate at which the variable changes its value. Using this 'rate', 'Arena' computes the value of the variable at any given point in the simulation and is stored within the 'level' element.

Since sedimentation is ignored in this effort, the RAS is modeled as a two component system consisting of the fish tank and a biofilter unit. As an example, consider the ammonia concentration in the fish tank. The differential equation used is:

$$V_t * dC_{t,a} / dt = WF * C_{b,a} - C_{t,a} * (WF + FW) + TAN loading rate$$
 (61)

Where, V_t is volume of fish $tank(m^3)$, $dC_{t,a}$ / dt is rate of change of ammonia concentration in the fish tank, WF is water recirculation rate, FW is fresh water intake rate, $C_{b,a}$ is concentration of ammonia in the biofilter (g/m^3) , $C_{t,a}$ is concentration of ammonia in the fish tank (g/m^3) and TAN loading rate is as described in the previous section.

Weatherly et al. developed a very simple simulation model along these lines in 1993.

4.6. Plugging the cost factors

The water (2.82 \$/kilogallon) and sewer costs (3.22 \$/kilogallon) were obtained from a project done by Ms. Stephanie Smith and Dr. Charlie Coale of the Agricultural Economics Dept at Virginia Tech(personal communication, 2004). The oxygen cost(0.01 cents/ m³ of oxygen) was calculated from a figure of 32 cents/ gallon of liquid oxygen and using the approximate relation that 1 gallon of liquid oxygen is equivalent to 3.26 m³ of gaseous oxygen (Timmons et al., 2001). The feed cost(44 cents/ kilogram) was also obtained from their work. The power cost for running the pumps was taken to be 2 cents/ m³ of water pumped.

4.7. Optimizing the model

Using the optimizer feature of 'Arena', the low cost configuration was obtained. Since the algorithm is a heuristic and not an exact method, the output will not necessarily be a global optimum. However, the greater the time given for the program to run, the greater the likelihood of attaining the global solution. The objective function and constraints used are as follows:

Minimize:

Total cost

Subject to the constraints:

Maximum tank ammonia level <= 1 mg/L

Minimum tank oxygen level >= 5 mg/L

Where,

Total cost =

water cost + sewer cost + feed cost + oxygen cost + pumping cost

(62)

5. Results and discussion

With the model as described previously, a single tank RAS facility is simulated for 8 months starting from the introduction of fish fingerlings on day one. Starting from an initial configuration of 1 feed/day, daily fresh water intake of 3.2 m³/day (15% of total water in the system), water recirculation rate of 100 m³/day (i.e., the water recirculates through the system 5 times a day) and oxygen input of 12,000 grams/day, which costs about \$ 14,052, the optimizer (Optquest) arrived at a configuration of 12 feed/day, daily fresh water intake of 1 m³/day, water recirculation rate of 63 m³/day and an oxygen input of 11,000 grams/day. The output of the model shows that the freshwater intake rate can be decreased from 15% to 5%, the daily water recirculation rate can be reduced from 5 times/day to 3 times/day and the oxygen supply reduced by 10 %, while maintaining the ammonia, carbon dioxide and oxygen tolerance levels. This translates to a water, sewage, oxygen and power cost savings of about \$1,045.00 over the 8 month period. Increasing the number of feeds also makes intuitive sense because it reduces the peak in the ammonia level thus reducing the load on the biofilter. Multiple feeds is better also for the fish. In trying to identify the least cost configuration, the effect of the carbonate system on the oxygen intake by the fish was not considered because it was not possible to accurately model it. It was also not possible to quantify the carbon dioxide stripping efficiency of the RBC because of varying concentration of carbon dioxide in the atmosphere. The model itself assumes a fixed carbon dioxide stripping rate, but it is ignored while identifying the optimal configuration

Since the random variables in the model change daily, and the simulation is run for 8 months, there is no significant variation in costs across runs. Hence, rather than building a regression model with the amount of fish as the regressor, it is sufficient to validate using a paired t-test on the difference between the simulated cost of the given facility configuration and the actual cost. The rationale being that under null (H₀) hypothesis most of the differences will be clustered around zero. However, in the absence of substantial data from real facilities to validate the model, the following guidelines were used to validate the model:

Eyeballing a limited dataset (Brett and Groves, 1979) shows nitrogen & CO2 generation & O2 consumption rates can be approximated by a normal distribution. A Kolmogorov- Smirnov goodness-of-fit test on this data set gives a p-value of 0.04 suggesting that it is a reasonable, though not an ideal approximation(i.e., acceptable at α = .99 but not at .95).

By choosing an appropriate value for the 'feed assimilation efficiency', it is seen that the fish become full grown in 8-9 months. This roughly corresponds to what is observed in real facilities, confirming that fish growth is modeled accurately.

From the model it is clear that the water replenishment rate can be reduced to about 5 % without compromising fish health. This percentage is equal to the best reported practices in the industry.

Since the total cost depends on the value of the individual unit costs, it was necessary to see how sensitive the optimal cost is to changes in the individual costs. The robustness of the model is tested for using a sensitivity analysis. As is clear from the table below, a 10 % change in the control (unit cost of water, power, sewer and oxygen) produces a change of less than 1.6% in the total cost. Thus, the model is quite robust.

Table.3. Percentage change in optimal cost of a 10% change in control value

10% change in value of control	new total cost	% change in total cost
.9*UnitWaterCost	1932	0.92%
1.1*UnitWaterCost	1968	-0.92%
.9*UnitPowerCost	1919	1.59%
1.1*UnitPowerCost	1980	-1.54%
.9*UnitSewerCost	1929	1.08%
1.1*UnitSewerCost	1970	-1.03%
.9*UnitOxyCost	1923	1.38%
1.1*UnitOxyCost	1976	-1.33%

6. Conclusion

In the absence of real data to validate the model, the benefit of this work is limited to demonstrating the potential of computer simulation to cut costs in a RAS facility. Theoretically, this model predicts a potential \$1,000 savings in water, sewage and oxygen costs by identifying a near optimal water and oxygen replenishment rate. This translates to a savings of \$12,000 for a medium-sized commercial facility with 12 tanks.

The model can be further enhanced to optimize feed costs as well by better modeling fish growth. Future efforts should also attempt to incorporate the effect of temperature on fish growth and the overall system efficiency. This can also improve the accuracy of the water and oxygen requirement predictions. It is also possible to model the transfer of fish between tanks at various stages of growth. Since most existing aquaculture facilities do this to maximize capacity utilization, incorporating this into the model will make it more realistic. It will also open up more variables for optimization such as mortality rate and capacity utilization.

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10. APPENDIX

Appendix .1. TAN, Nitrite, Dissolved oxygen and alkalinity in High, Medium and Low density systems

High Density system							
Time		TA	٨N		NO2	DO	Alk
	Tank	Stage 1	Stage 2	Stage 3			
9:00 AM	1.99	1.7	1.62	1.63	0.350	7.9	75
11:00 AM	2.28	1.84	1.72	1.68		5.1	
1:00 PM	2.63	2.13	1.94	1.86	0.540	4.90	76.00
3:00 PM	2.51	0.46	1.28	1.44	0.550	5.20	
5:00 PM	2.21	1.67	1.54	1.52	0.500	5.40	118.00
7:00 PM	2.12	1.60	1.45	1.41	0.434	4.60	110.00
9:00 PM	2.26	1.72	1.61	1.54	0.514	5.10	103.00
11:00 PM	2.24	1.72	1.53	1.54	0.512	6.00	96.00
1:00 AM	2.05	1.68	1.60	1.61	0.496	6.20	89.00
3:00 AM	2.08	1.62	1.54	1.51	0.436	7.30	81.00
5:00 AM	2.06	1.68	1.57	1.57	0.414	8.00	76.00
7:00 AM	1.90	1.63	1.46	1.56	0.356	8.50	71.00
9:00 AM	1.91	1.63	1.59	1.53	0.414	6.90	70.00

Medium Density System								
Time		•	TAN		NO2	DO	Alk	
	Tank	Stage 1	Stage 2	Stage 3				
9:00 AM	1.16	1.03	0.98	0.95	0.330	9.40	107.00	
11:00 AM	1.27	1.04	0.93	0.86		7.40		
1:00 PM	1.45	1.19	1.01	1.01	0.500	7.80	99.00	
3:00 PM	1.35	1.13	1.04	0.96	0.430	8.10		
5:00 PM	1.26	0.93	0.96	0.88	0.440	7.60	143.00	
7:00 PM	1.26	0.99	0.96	0.77	0.380	7.10	134.00	
9:00 PM	1.33	1.07	0.98	0.89	0.450	8.20	131.00	
11:00 PM	1.28	0.94	0.89	0.81	0.444	9.00	125.00	
1:00 AM	1.13	0.92	0.89	0.80	0.430	9.40	120.00	
3:00 AM	1.10	0.90	0.82	0.78	0.350	9.70	113.00	
5:00 AM	1.09	0.91	0.83	0.79	0.318	10.00	110.00	
7:00 AM	1.04	0.88	0.79	0.74	0.284	10.10	107.00	

		9:00 AM	1.01	0.89	0.86	0.81	0.356	8.80	110.0
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Low Density System							
Time		TA	λN		NO2	DO	Alk
	Tank	Stage 1	Stage 2	Stage 3			
9:00 AM	0.76	0.73	0.61	0.64	0.140	10.70	61.00
11:00 AM	0.80	0.72	0.65	0.62		9.40	
1:00 PM	1 1.08	0.91	0.76	0.73	0.210	9.70	58.00
3:00 PM	0.99	0.90	0.71	0.64	0.200	9.30	
5:00 PM	0.86	0.76	0.67	0.62	0.200	9.00	113.00
7:00 PM	0.80	0.66	0.48	0.52	0.136	9.30	110.00
9:00 PM	0.76	0.69	0.62	0.60	0.134	9.50	104.00
11:00 PM	0.75	0.66	0.61	0.56	0.120	10.00	101.00
1:00 AM	0.73	0.65	0.59	0.55	0.111	9.90	100.00
3:00 AM	0.73	0.65	0.61	0.55	0.100	10.00	97.00
5:00 AM	0.67	0.61	0.50	0.53	0.095	10.30	95.00
7:00 AM	0.71	0.65	0.56	0.57	0.091	10.60	93.00
9:00 AM	0.55	0.52	0.47	0.44	0.158	10.30	91.00

Appendix.2. Nitrogen excretion Vs time after feeding(adapted fromm Brett & Zala, 1975)

Time		Nitrogen Excretion
	1	7.75
	1.5	6
	3	8.6
	4.5	7.5
	5	9
	7	9
	7.5	7
	9	10.5
	11	21
	11.5	12.5
	12.5	35
	13	28.5
	15	29
	17	21.5
	19	12
	21	12
	22.5	12
	23	9.5

Appendix.3. VBA code for continuous simulation

Level Name	Rate Name	Number
BF ammonia level	BF ammonia rate	1
Tank ammonia level	Tank ammonia rate	2
FishGrowth Level	FishGrowth rate	3
Tank Oxy Level	Tank Oxy Rate	4
BF Oxy Level	BF Oxy Rate	5
Tank carbon level	Tank carbon rate	6
Biofilter carbon level	Biofilter carbon rate	7

Private Sub ModelLogic_UserContinuousEquations()

Dim oSIMAN As Arena.SIMAN

Dim temperature As Double

Dim wviscosity As Double

Dim charalength As Double

Dim wvelocity As Double

Dim wdensity As Double

Dim ephsilon As Double

Dim Dwc As Double

Dim Dc As Double

Dim Dws As Double

Dim Ds As Double

Dim Mu As Double

Dim K20 As Double

Dim q As Double

Dim wthickness As Double

Dim Js As Double

Dim BFArea As Double

Dim WFRate As Double

Dim OFAmm As Double

Dim IFAmm As Double

Dim BasO2Mtblsm As Double

Dim ActiveMtblsm As Double

Dim OxyTrfrCpty As Double

Dim FishOxyConsRate As Double

Dim BFOxyConsRate As Double

Dim Altitude As Double

Dim Pbp As Double

Dim Pwv As Double

Dim Beta As Double

Dim OxyDiss As Double

Dim mean As Double

Dim sigma As Double

Dim Oxymean As Double

Dim Oxysigma As Double

Dim n As Double

Dim Alkalinity As Double

Dim Chlorinity As Double

Dim Salinity As Double

Dim TBorate As Double

Dim CarbDiss As Double

Dim x As Double

Dim pH As Double

Dim pHF As Double

Dim pHB As Double

Dim pK0 As Double

Dim pK1 As Double

Dim pK2 As Double

Dim pkB As Double

Dim pkN As Double

Dim pkW As Double

Dim pK0K1 As Double

Dim k0 As Double

Dim k1 As Double

Dim k2 As Double

Dim kB As Double

Dim kN As Double

Dim kW As Double

Dim a0F As Double

Dim a1F As Double

Dim a2F As Double

Dim a3F As Double

Dim bF As Double

Dim cF As Double

Dim dF As Double

Dim pF As Double

Dim quF As Double

Dim zF As Double

Dim u1F As Double

Dim D1F As Double

Dim D0F As Double

Dim HF As Double

Dim CO2F As Double

Dim a0B As Double

Dim a1B As Double

Dim a2B As Double

Dim a3B As Double

Dim bB As Double

Dim cB As Double

Dim dB As Double

Dim pB As Double

Dim quB As Double

Dim zB As Double

Dim u1B As Double

Dim D1B As Double

Dim D0B As Double

Dim HB As Double

Dim CO2B As Double

Set oSIMAN = ThisDocument.Model.SIMAN

q = 1.1

Ktwenty = 390000 'day

ephsilon = 0.98

wvelocity = 0.03 'm/s

charalength = 0.01 'm

BFArea = 100 'm2

Altitude = oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("Altitude"))

temperature = oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("temperature"))

'celsius

WFRate = oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("WaterFlowRate"))

'm3/day

FwFRate =

oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("FrshwtrFlowRate"))

'm3/day

```
oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("BiofilterVol"))
BFVol
'm3
TankVol
                 oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("TankVol"))
'm3
FeedAmt = oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("FeedAmount"))
FeedFactor = oSIMAN. VariableArrayValue(oSIMAN.SymbolNumber("Feed Factor"))
n = oSIMAN. Variable Array Value (oSIMAN. Symbol Number ("Feeds Per Day"))
BasO2Mtblsm
                                                                              =
oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("BasicO2FishKg"))
                                                                              'g
O2/Kg fish
OxyTrsfrCpty
                                                                              =
oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("OxyTrsfrCpty"))
               oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("Alkalinity"))
' eq/liter
                  oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("Salinity"))
Salinity
           =
' %
                             1
'wviscosity = 1.7373 * 0.97449 ^ temperature * 86400
                                                  'g m /day
'wdensity = 1001627 * 0.9997585 ^ temperature
                                                 'g/m3
'Dws = 0.0000886 * 1.026 ^ temperature
                                              'm2/day
'D_S = 0.8 * Dw_S
'Dwc = 0.0000878 * 1.035 \temperature
                                              'm2/day
Dc = 0.8 * Dwc
'wthickness = 1.23 * (wviscosity / wdensity) ^{(1/3)} * (charalength / wvelocity) ^{0.5} *
(ephsilon) ^ 1.5 * (Dwc / 86400) ^ (1 / 3)
                               1
```

```
Pbp = 10 ^ (2.880814 - Altitude / 19748.2)
Pwv = 4.7603 * Exp(0.0645 * temperature)
Beta = Exp(-58.3877 + 85.8079 * 100 / (273.15 + temperature) + 23.8439 * Log((273.15 + temperature)))
+ temperature) / 100))
OxyDiss = 1000 * 1.42903 * Beta * 0.20946 * (Pbp - Pwv) / 760
If (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (1 / n) Then
Oxymean = 0.043
Oxysigma = 0.1
mean = (1 / n) / 2
sigma = 0.0625
ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (2 / n) Then
Oxymean = 0.043 + 1 / n * 1
Oxysigma = 0.1
mean = (1/n)/2 + 1/n * 1
sigma = 0.0625
ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (3 / n) Then
Oxymean = 0.043 + 1 / n * 2
Oxysigma = 0.1
mean = (1/n)/2 + 1/n * 2
sigma = 0.0625
ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (4 / n) Then
Oxymean = 0.043 + 1 / n * 3
Oxysigma = 0.1
mean = (1/n)/2 + 1/n * 3
sigma = 0.0625
ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (5 / n) Then
Oxymean = 0.043 + 1 / n * 4
Oxysigma = 0.1
mean = (1/n)/2 + 1/n * 4
```

```
sigma = 0.0625
```

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (6 / n) Then

Oxymean =
$$0.043 + 1 / n * 5$$

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 5$$

$$sigma = 0.0625$$

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (7 / n) Then

Oxymean =
$$0.043 + 1 / n * 6$$

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 6$$

$$sigma = 0.0625$$

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (8 / n) Then

Oxymean =
$$0.043 + 1 / n * 7$$

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 7$$

$$sigma = 0.0625$$

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) \leq (9 / n) Then

Oxymean =
$$0.043 + 1 / n * 8$$

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 8$$

$$sigma = 0.0625$$

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (10 / n) Then

Oxymean =
$$0.043 + 1 / n * 9$$

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 9$$

$$sigma = 0.0625$$

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (11 / n) Then

Oxymean =
$$0.043 + 1 / n * 10$$

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 10$$

$$sigma = 0.0625$$

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (12 / n) Then

Oxymean = 0.043 + 1 / n * 11

Oxysigma = 0.1

mean = (1/n)/2 + 1/n * 11

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (13 / n) Then

Oxymean = 0.043 + 1 / n * 12

Oxysigma = 0.1

mean = (1/n)/2 + 1/n * 12

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (14 / n) Then

Oxymean = 0.043 + 1 / n * 13

Oxysigma = 0.1

mean = (1/n)/2 + 1/n * 13

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (15 / n) Then

Oxymean = 0.043 + 1 / n * 14

Oxysigma = 0.1

mean = (1/n)/2 + 1/n * 14

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (16 / n) Then

Oxymean = 0.043 + 1 / n * 15

Oxysigma = 0.1

mean = (1/n)/2 + 1/n * 15

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (17 / n) Then

Oxymean = 0.043 + 1 / n * 16

Oxysigma = 0.1

mean = (1/n)/2 + 1/n * 16

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (18 / n) Then

Oxymean = 0.043 + 1 / n * 17

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 17$$

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (19 / n) Then

Oxymean = 0.043 + 1 / n * 18

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 18$$

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (20 / n) Then

Oxymean = 0.043 + 1 / n * 19

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 19$$

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (21 / n) Then

Oxymean = 0.043 + 1 / n * 20

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 20$$

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (22 / n) Then

Oxymean = 0.043 + 1 / n * 21

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 21$$

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (23 / n) Then

Oxymean = 0.043 + 1 / n * 22

Oxysigma = 0.1

mean =
$$(1/n)/2 + 1/n * 22$$

sigma = 0.0625

ElseIf (oSIMAN.RunCurrentTime - Int(oSIMAN.RunCurrentTime)) < (24 / n) Then

Oxymean = 0.043 + 1 / n * 23

```
Oxysigma = 0.1

mean = (1 / n) / 2 + 1 / n * 23

sigma = 0.0625

End If
```

```
OFAmm = WFRate * oSIMAN.LevelValue(1) / BFVol
                                                                                                                                    'g /day
IFAmm = WFRate * oSIMAN.LevelValue(2) / TankVol
                                                                                                                                    'g /day
'Js = (Dws / wthickness) * Sqr(Ds * Ktwenty * Q \land (temperature - 20)) / (Dws /
wthickness + Sqr(Ds * Ktwenty * Q ^ (temperature - 20)))
                                                                       1
oSIMAN.RateValue(2) = (OFAmm - oSIMAN.LevelValue(2) * (WFRate + FwFRate) /
TankVol
                                                                                                                                                                    0.9
                                       (0.16)
                                                                       0.32
                                                                                                     0.64
                                                                                                                                    0.005
oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("Feed Factor")) * FeedAmt) /
(n * Sqr(2 * 3.1415 * sigma ^ 2)) * Exp(-((oSIMAN.RunCurrentTime - 
Int(oSIMAN.RunCurrentTime)) - mean) ^ 2 / (2 * sigma ^ 2))) 'g/day
FishOxvConsRate = ((0.16 * 0.32 * 0.64 * 0.9 * 0.035 * FeedFactor * FeedAmt) / (n * 0.035 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 0.045 * 
TankVol * Sqr(2 * 3.1415 * Oxysigma ^ 2)) * Exp(-((oSIMAN.RunCurrentTime -
Int(oSIMAN.RunCurrentTime)) - Oxymean) ^ 2 / (2 * Oxysigma ^ 2))) + BasO2Mtblsm
* oSIMAN.LevelValue(3) + ActiveMtblsm * oSIMAN.LevelValue(3)
                                                                                                                      OxyTrsfrCpty
oSIMAN.RateValue(4)
oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("OxyTrsfrEff")) + WFRate *
oSIMAN.LevelValue(5) / BFVol + FwFRate * OxyDiss - (WFRate + FwFRate) /
TankVol * oSIMAN.LevelValue(4) - FishOxyConsRate
BFOxyConsRate = (4.57 * 0.16 * 0.32 * 0.64 * 0.005 * 0.9 * FeedFactor * FeedAmt) / (n)
* TankVol * Sqr(2 * 3.1415 * sigma ^ 2)) * Exp(-((oSIMAN.RunCurrentTime -
Int(oSIMAN.RunCurrentTime)) - mean) ^ 2 / (2 * sigma ^ 2))
oSIMAN.RateValue(5) = WFRate * oSIMAN.LevelValue(4) / TankVol - WFRate *
oSIMAN.LevelValue(5) / BFVol - BFOxyConsRate
```

```
oSIMAN. VariableArrayValue(oSIMAN.SymbolNumber("FshOxyConsRate")) = (0.16 *
                                          0.9
0.32
                     0.64
                                                               0.035
oSIMAN.VariableArrayValue(oSIMAN.SymbolNumber("Feed Factor")) * FeedAmt) /
(n * TankVol * Sqr(2 * 3.1415 * Oxysigma ^ 2)) * Exp(-((oSIMAN.RunCurrentTime -
Int(oSIMAN.RunCurrentTime)) - Oxymean) ^ 2 / (2 * Oxysigma ^ 2)) + BasO2Mtblsm *
oSIMAN.LevelValue(3)
If oSIMAN.LevelValue(4) / tankvol > 43 Then
oSIMAN.LevelValue(4) = 43 * tankvol
End If
If oSIMAN.LevelValue(5) / BFvol > 43 Then
oSIMAN.LevelValue(5) = 43 * BFvol
End If
'oSIMAN.RateValue(1) = (IFAmm - oSIMAN.LevelValue(1) * Js * BFArea - OFAmm) /
BFVol
oSIMAN.RateValue(1) = (IFAmm - OFAmm - (oSIMAN.LevelValue(1) / BFVol - 0.07)
/ (oSIMAN.LevelValue(1) / BFVol + 1.93) * 10 * BFArea)
CarbDiss = Exp(-58.0931 + 90.5069 * (100 / (temperature + 273.15)) + 22.294 *
Log((temperature + 273.15) / 100)) * 22.263 * 1000 * 0.00032 * 1.97681 * (Pbp - Pwv) /
760
oSIMAN.RateValue(6) = FwFRate * CarbDiss + WFRate * oSIMAN.LevelValue(7) /
BFVol + 1.4 * FishOxyConsRate - (WFRate + FwFRate) * oSIMAN.LevelValue(6) /
TankVol
oSIMAN.RateValue(7) = oSIMAN.LevelValue(6) / TankVol * WFRate + 1.4 *
BFOxyConsRate - 0.02 * oSIMAN.LevelValue(7) - oSIMAN.LevelValue(7) / BFVol *
WFRate
```

```
Chlorinity = Salinity / 1.80655
TBorate = 0.000029 * Chlorinity
If (Salinity < 5) Then
x = 0.016 * Salinity + 0.0532
Else
x = 0.0007 * Salinity + 0.131
End If
pK0 = -2622.38 / (temperature + 273.15) + 15.5873 - 0.0178471 * (temperature + 273.15)
(273.15) + Chlorinity * (0.011795 + 2.77676 * 10 ^ (-5) * (temperature + 273.15))
pK1 = 3404.71 / (273.15 + temperature) + 0.032786 * (temperature + 273.15) - 14.7122 -
0.19178 * Chlorinity ^ 0.3333
pK2 = 2902.39 / (273.15 + temperature) + 0.02379 * (temperature + 273.15) - 6.471 -
0.4693 * Chlorinity ^ 0.3333
pkB = 2291.9 / (273.15 + temperature) + 0.01756 * (temperature + 273.15) - 3.385 -
0.32051 * Chlorinity ^ 0.3333
pkN = 9.245 + 0.138 * (19.9273 * Salinity / (1000 - 1.005109 * Salinity)) + (0.034 * (298))
- temperature - 273.15)) + x
pkW = 4470.99 / (temperature + 273.15) - 6.0875 + 0.01706 * (temperature + 273.15)
k0 = 10 ^ (-1 * pK0)
k1 = 10 ^ (-1 * pK1)
k2 = 10 ^ (-1 * pK2)
kB = 10 ^ (-1 * pkB)
kN = 10 ^ (-1 * pkN)
kW = 10 ^ (-1 * pkW)
pK0K1 = -(Log(k0 * k1) / Log(10))
a3F = (Alkalinity * (k1 + kN + kB) - oSIMAN.LevelValue(6) / TankVol * k1 -
oSIMAN.LevelValue(2) * 0.000059 / TankVol * kN - TBorate * kB) / Alkalinity
```

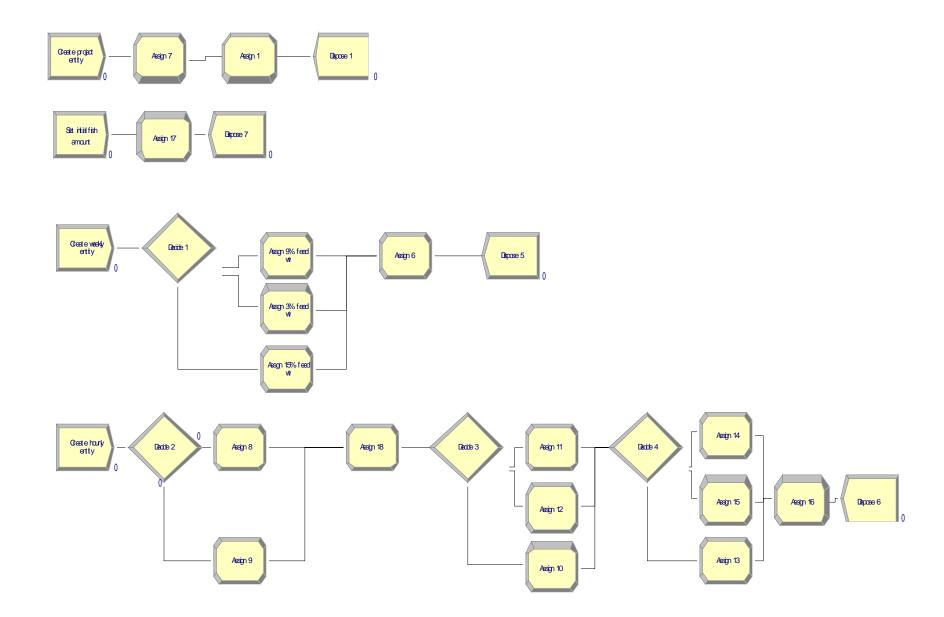
```
a2F = (Alkalinity * (k1 * k2 + kN * kB + k1 * kN + k1 * kB) - oSIMAN.LevelValue(6) /
TankVol * k1 * (2 * k2 + kN + kB) - oSIMAN.LevelValue(2) * 0.000059 / TankVol *
kN * (k1 + kB) - TBorate * kB * (k1 + kN)) / Alkalinity
a1F = k1 * (Alkalinity * (k2 * kN + k2 * kB + kN * kB) - oSIMAN.LevelValue(6) /
TankVol * (kN * kB + 2 * k2 * (kN + kB)) - oSIMAN.LevelValue(2) * 0.000059 /
TankVol * kN * (k2 + kB) - TBorate * kB * (k2 + kN)) / Alkalinity
a0F = (k1 * k2 * kN * kB) * (Alkalinity - 2 * oSIMAN.LevelValue(6) / TankVol -
oSIMAN.LevelValue(2) * 0.000059 / TankVol - TBorate) / Alkalinity
bF = -a2F
cF = a1F * a3F - 4 * a0F
dF = -1 * (a1F ^ 2 + a0F * a3F ^ 2 - 4 * a0F * a2F)
pF = (3 * cF - bF ^ 2) / 9
quF = (2 * bF ^ 3 - 9 * bF * cF + 27 * dF) / 54
zF = Atn(-((-quF / ((-pF) ^ 1.5))) / Sqr(-((-quF / ((-pF) ^ 1.5))) * ((-quF / ((-pF) ^ 1.5))) + ((-q
1)) + 2 * Atn(1)
u1F = 2 * (-pF) ^ 0.5 * Cos(zF / 3) - bF / 3
D0F = u1F / 2 - ((u1F / 2)^2 - a0F)^0 0.5
D1F = a3F / 2 - (a3F^2 / 4 + u1F - a2F)^0 0.5
HF = (-D1F * (D1F ^ 2 - 4 * D0F) ^ 0.5) / 2
pHF = -(Log(HF) / Log(10))
CO2F = 44000 * (Alkalinity - 10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW) - 10 ^ (-pHF)) * (1 / (10 ^ (pHF - pkW
pK0K1) + 2 * 10 ^ (2 * pHF - pK0K1 - pK2)))
a3B = (Alkalinity * (k1 + kN + kB) - oSIMAN.LevelValue(7) / BFVol * k1 -
oSIMAN.LevelValue(1) * 0.000059 / BFVol * kN - TBorate * kB) / Alkalinity
a2B = (Alkalinity * (k1 * k2 + kN * kB + k1 * kN + k1 * kB) - oSIMAN.LevelValue(7) /
BFVol * k1 * (2 * k2 + kN + kB) - oSIMAN.LevelValue(1) * 0.000059 / BFVol * kN *
(k1 + kB) - TBorate * kB * (k1 + kN) / Alkalinity
a1B = k1 * (Alkalinity * (k2 * kN + k2 * kB + kN * kB) - oSIMAN.LevelValue(7) /
BFVol * (kN * kB + 2 * k2 * (kN + kB)) - oSIMAN.LevelValue(1) * 0.000059 / BFVol
* kN * (k2 + kB) - TBorate * kB * (k2 + kN)) / Alkalinity
```

```
a0B = (k1 * k2 * kN * kB) * (Alkalinity - 2 * oSIMAN.LevelValue(7) / BFVol -
 oSIMAN.LevelValue(1) * 0.000059 / BFVol - TBorate) / Alkalinity
bB = -a2B
cB = a1B * a3B - 4 * a0B
dB = -1 * (a1B ^ 2 + a0B * a3B ^ 2 - 4 * a0B * a2B)
pB = (3 * cB - bB ^ 2) / 9
quB = (2 * bB ^ 3 - 9 * bB * cB + 27 * dB) / 54
zB = Atn(-((-quB / ((-pB) ^ 1.5))) / Sqr(-((-quB / ((-pB) ^ 1.5))) * ((-quB / ((-pB) ^ 1.5)))
+1)) + 2 * Atn(1)
u1B = 2 * (-pB) ^ 0.5 * Cos(zB / 3) - bB / 3
D0B = u1B / 2 - ((u1B / 2)^2 - a0B)^0
D1B = a3B / 2 - (a3B^2 + u1B - a2B)^0 0.5
HB = (-D1B * (D1B ^ 2 - 4 * D0B) ^ 0.5) / 2
pHB = -(Log(HB) / Log(10))
CO2B = 44000 * (Alkalinity - 10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB)) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) - 10 ^ (-pHB))) * (1 / (10 ^ (pHB - pkW) 
pK0K1) + 2 * 10 ^ (2 * pHB - pK0K1 - pK2)))
```

oSIMAN. Variable Array Value (oSIMAN. Symbol Number ("pH")) = pHF

End Sub

Appendix.4. Model Logic for the discrete simulation part



Vita

John Simon was born in the city of Kochi in the south Indian state of Kerala on Nov 3rd, 1979. He obtained his undergraduate degree in Mechanical Engineering from Kerala university, one of the better known universities in the state. After a brief stint as a software programmer in the same state, he decided to pursue higher education in the United States. Having gotten admission into Virginia Tech for a Master's in Industrial Engineering, he arrived in Virginia on Aug 7th, 2002.

Growing up in the coastal state of Kerala, fishermen and fish farms were a continuous part of the landscape for him. However, it was only after coming to the US that he became aware of the possibility of using simulation tools from his Industrial Engineering courses to improve efficiencies in the fish industry.

After finishing this work he plans to graduate in May 2005 and join the work force thereafter.