Behavior-based Incentives for Node Cooperation in Wireless Ad Hoc Networks

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(ABSTRACT)

A Mobile Ad Hoc Network (MANET) adopts a decentralized communication architecture which relies on cooperation among nodes at each layer of the protocol stack. Its reliance on cooperation for success and survival makes the ad hoc network particularly sensitive to variations in node behavior. Specifically, for functions such as routing, nodes which are limited in their resources may be unwilling to cooperate in forwarding for other nodes. Such selfish behavior leads to degradation in the performance of the network and possibly, in the extreme case, a complete cessation of operations. Consequently it is important to devise solutions to encourage resource-constrained nodes to cooperate.

Incentive schemes have been proposed to induce selfish nodes to cooperate. Though many of the proposed schemes in the literature are payment-based, nodes can be incentivized to cooperate by adopting policies which are non-monetary in nature, but rather are based on the threat of retaliation for non-cooperating nodes. These policies, for which there is little formal analysis in the existing literature on node cooperation, are based on observed node behavior. We refer to them as behavior-based incentives. In this work, we analyze the effectiveness of behavior-based incentives in inducing nodes to cooperate.

To determine whether an incentive scheme is effective in fostering cooperation we develop a game-theoretic model. Adopting a repeated game model, we show that nodes may agree to cooperate in sharing their resources and forward packets, even if they perceive a cost in doing so. This happens as the nodes recognize that refusing to cooperate will result in similar behavior by others, which ultimately would compromise the viability of the network as a whole.

A major shortcoming in the analysis done in past works is the lack of consideration of practical constraints imposed by an ad hoc environment. One such example is the assumption that a node, when making decisions about whether to cooperate, has perfect knowledge of every other node's actions. In a distributed setting this is impractical. In our work, we analyze behavior-based incentives by incorporating such practical considerations as imperfect monitoring into our game-theoretic models. In modeling the problem as a game of imperfect public monitoring (nodes observe a common public signal that reflects the actions of other nodes in the network) we show that, under the assumption of first order stochastic dominance of the public signal, the grim trigger strategy leads to an equilibrium for nodes to cooperate.

Even though a trigger-based strategy like grim-trigger is effective in deterring selfish behavior it is too harsh in its implementation. In addition, the availability of a common public signal in a distributed setting is rather limited. We, therefore, consider nodes that individually monitor the behavior of other nodes in the network and keep this information private. Note that this independent monitoring of behavior is error prone as a result of slow switching between transmit and promiscuous modes of operation, collisions and congestion due to the wireless medium, or incorrect feedback from peers. We propose a probability-based strategy that induces nodes to cooperate under such a setting. We analyze the strategy using repeated games with imperfect private monitoring and show it to be robust to errors in monitoring others' actions. Nodes achieve a near-optimal payoff at equilibrium when adopting this strategy.

This work also characterizes the effects of a behavior-based incentive, applied to induce cooperation, on topology control in ad hoc networks. Our work is among the first to consider selfish behavior in the context of topology control. We create topologies based on a holistic view of energy consumption – energy consumed in forwarding packets as well as in maintaining links. Our main results from this work are to show that: (a) a simple forwarding policy induces nodes to cooperate and leads to reliable paths in the generated topology, (b) the resulting topologies are well-connected, energy-efficient and exhibit characteristics similar to those in small-world networks.

Dedicated to my parents and my brother, Pankaj
For their love, sacrifice and blessings.

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

Chapter 1. INTRODUCTION

A wireless ad hoc network is characterized by a distributed, dynamic, self-organizing architecture. Each device (henceforth referred to as a “node”) in the network is capable of independently adapting its operation based on the current environment according to predetermined algorithms and protocols. In addition, the network must rely on cooperation among all nodes to achieve successful and error-free communication between peers. This cooperation is essential across all layers of the protocol stack.

At the physical layer, a transmitting node may increase its transmit power to improve its signal-to-noise ratio (SNR) at the intended receiver. However this will also lead to increased interference at nearby nodes in the network. Therefore, it is essential for nodes to cooperate and adopt a distributed power control scheme so that nodes achieve satisfactory SNR values with minimum interference. Another facet of cooperative communication at the physical layer involves two or more intermediate nodes acting as repeaters by boosting and re-transmitting signals received from their neighbors [1]. This requires each intermediate node to cooperate in expending its energy to provide spatial diversity at the next hop receiver.

Cooperation among nodes is also required in medium access since nodes are required to back off for random periods after each unsuccessful transmission. Failure to do so results in every node’s repeatedly attempting to retransmit at the same time, thereby increasing the number of collisions. At the network layer, nodes that do not lie within communication radio range of one another rely on peer nodes to ensure that their packets reach the intended destination. This requires a node to share its resources such as bandwidth, energy and other processing capabilities when establishing routes or forwarding data packets for its peers. Therefore, for successful delivery of packets along multi-hop routes willingness of nodes to share resources and cooperate is essential. This work focuses on cooperation at the network layer, particularly on the forwarding functionality. We investigate the problem of sustaining cooperation at the network layer in the presence of energy-constrained nodes.

We address the problem by applying behavior-based incentives and analyzing them to determine their efficacy in inducing selfish nodes to cooperate. Specifically, we consider practical scenarios where nodes are unable to perfectly assess the behavior of other nodes in the network. We propose a probability-based cooperation strategy that can be applied to induce cooperation in the network. We also consider the presence of selfish nodes in topology control and analyze the trade-offs a node faces in staying connected and cooperating vis-à-vis limiting its energy expenditure. We apply a behavior-based incentive to induce cooperation and characterize the topologies that emerge as a result of such trade-offs.

1.1 Motivation

Cooperation in ad hoc networks is impacted by selfish behavior. But *what really constitutes selfish behavior and how does it affect cooperation?* Since nodes in an ad hoc network are typically energy-constrained, one or more nodes may be unwilling to cooperate in forwarding packets for others to limit use of their resources. In the extreme case where all nodes stop cooperating, the network may cease to operate. Such behavior where a node seeks to maximize its individual utility by reducing the use of its own resources and in the process inadvertently affects the network is defined as *selfish*.

The presence of selfish nodes is likely to have a pronounced effect in commercial deployments of ad hoc networks due to the lack of a single centralized monitoring authority. An example of a commercial deployment of an ad hoc network is a mesh network. Figure 1.1 shows the network architecture of a proprietary mesh networking solution developed by Motorola [2]. It adopts ad hoc networking concepts to communicate among the client nodes. In the figure, Bob can program his laptop to refuse to forward packets for other users (or to forward only a portion of such packets), thereby affecting the throughput experienced by Sarah in communicating with

George. In the absence of any centralized control, Bob's behavior goes unchecked, affecting the overall throughput. Therefore, a distributed solution is required which will dissuade users like Bob from such actions and induce them to cooperate.

This is in contrast to behavior we characterize as malicious. Let us again consider Figure 1.1. For illustration purposes, let us consider a simple denial of service attack where Bob programs his laptop to drop every packet it is requested to forward (also known as the black hole attack). Bob can further increase the damage by ensuring that all routes originating from Sarah and Stephen would use Bob as an intermediate hop in communicating with George. To do so he would falsely advertise shortest routes to every destination in the network and prevent packets from Sarah and Stephen from reaching their intended destinations.

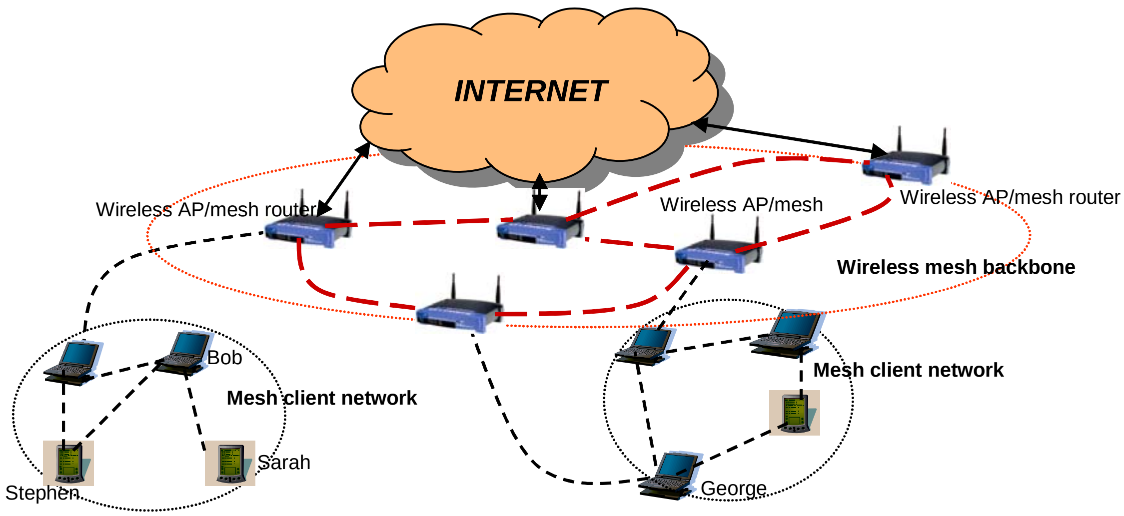

Figure 1.1 Mesh network architecture

Note that a malicious node is characterized by an active intent in disrupting communication among nodes, as compared to selfish nodes where disruption of network operations is only a side effect of the node's limiting use of its resources. Also, in achieving its objective of causing maximum harm, a malicious node is typically not concerned with conserving its own resources.

In this research we focus on selfish behavior. Given this basic characterization of selfish behavior and its impact on cooperation, the next question we consider is *how do we reduce this* *impact and induce selfish nodes to cooperate?* We provide incentives to increase the willingness of nodes to cooperate.

We broadly classify different incentive techniques into:

- Credit-based incentives: These incentives (such as those adopted in the works of [3-6]) are based on a concept of charge-and-reward, in which a node charges for providing any service and, in turn, rewards the others for their service. This concept is implemented using economic instruments introduced as add-ons to the network. One main drawback of a credit-based incentive is its susceptibility to cheating when nodes may falsely report the amount of credit involved during cooperation. Therefore, one must devise payment schemes that are cheat-proof. This leads to additional overhead making credit-based schemes complicated and tedious to implement.
- Behavior-based incentives: Here, incentives are provided by nodes' adopting a policy that makes cooperation the best option for every node to exercise. Such policies are normally a function of the observed behavior of nodes in the network. Also, behavior-based incentives do not involve the use of external monetary or economic methods and are, therefore, simpler to implement than credit-based incentives.

Research in applying behavior-based incentives for ad hoc networks has been active for the past few years, but most formal analysis concentrates on credit-based mechanisms. In this work, we consider the effectiveness of behavior-based incentives for network layer cooperation and propose a probability-based strategy that nodes can adopt in inducing cooperation even under imperfect monitoring conditions. We adopt a game-theoretic approach to assess the effectiveness of the behavior-based incentives considered.

1.2 Objectives

Our main objective in this work is to conduct formal analysis to develop an understanding of the effect of selfish node behavior and to evaluate the efficacy of incentives to induce cooperation among such nodes in ad hoc networks. To achieve this goal the following objectives must be accomplished.

- ***Characterization:*** We formally characterize selfish behavior and study its effect on node cooperation. The characterization highlights the trade-offs experienced by a selfish node, modeled by its utility function, in rational cooperation decisions.
- ***Efficiency analysis under perfect observations:*** We assess the effectiveness of a behavior-based incentive, the grim-trigger strategy and some variations of it, in incentivizing nodes to cooperate. For this purpose, we develop game theoretic models incorporating this incentive as a repeated game node strategy. The analysis provides us with answers to performance-related issues such as: (a) the effectiveness of the incentive in inducing nodes to cooperate; and (b) the robustness of the resulting equilibrium stable to deviations by one or more nodes.
- ***Efficiency analysis under imperfect observations:*** Furthermore, we consider practical challenges imposed by an ad hoc environment (such as the lack of a way to perfectly monitor other nodes' behavior) in our game-theoretic models. In lieu of directly monitoring others' actions, a node may need to rely on other signals that provide information about other nodes' behavior in the network. Based on whether such signals are the same for all nodes (public monitoring) or vary with each node (private monitoring), we develop different game-theoretic models that account for these imperfections. The analysis of these models provide further insight on the performance of behavior-based incentives, particularly, under what conditions cooperation can be sustained and by how much imperfect monitoring impacts the effectiveness of the incentive.
- ***Stability analysis under imperfect observations***: In a behavior-based incentive such as the grim-trigger, the trigger mechanism is extremely sensitive to node deviations. This, coupled with the inherent characteristic of the grim-trigger to 'punish' all nodes even if a single node stops cooperating, motivates us to consider non-trigger incentives. We consider a probability-

based strategy in the presence of monitoring imperfections and evaluate its efficacy in inducing cooperation. We further compare its robustness to the grim-trigger under one or more node deviations.

- ***Application:*** Finally, we apply a simple behavior-based incentive to a traditional ad hoc networking problem – topology control – in the presence of selfish nodes. We characterize the topologies that emerge in terms of energy and topological properties such as connectivity, path length and clustering coefficient.

1.3 Contributions

The contributions of this research include:

- In this work, we model selfish behavior in a repeated game structure and derive conditions to sustain cooperation at equilibrium when nodes adopt the grim trigger strategy. In addition, we present a stability analysis of the network under one or more node deviations and show that the stability depends on a node's perception of game repeatability.
- We are among the first to analyze behavior-based incentives for cooperation in ad hoc networks that takes into account imperfection in observing other nodes' actions. In this research, we develop a repeated game model of imperfect public monitoring and derive conditions for cooperation for the grim trigger strategy. We show that the grim-trigger is undesirably sensitive to one or more node deviations.
- Since nodes tend to be independent and distributed, there is a limited amount of shared public information which truly reflects the behavior of all nodes. Hence, it is practical to consider each node to possess private information about the behavior of other nodes. Furthermore due to the wireless nature of the medium such information is noise-ridden, thereby the monitoring of other nodes' behavior is imperfect. We propose a probability-based strategy that provides adequate incentives for cooperation and is robust to errors in observation. We leverage recent developments in game theory—repeated games with imperfect private monitoring—in proving

that this strategy induces nodes to cooperate at equilibrium and in assessing the range of observation error for which such cooperation can be sustained. Also, in the absence of observation error we show that nodes adopting this strategy achieve an optimal payoff when fully cooperating.

- This work also presents a simulation-based analysis to understand the effect of selfish nodes on topology control. We show that a simple distributed forwarding policy adopted by self-interested nodes is successful in inducing cooperation, resulting in highly connected topologies with reliable paths. In our model, nodes take a holistic view of energy consumption, the topologies that emerge achieve significant gains in energy efficiency while still maintaining power efficiency comparable to traditional power-aware topology control algorithms.

1.4 Organization

The dissertation document is divided into six chapters. Chapter two presents related work, along with a basic primer on how ad hoc networks can be modeled using game theory. Chapter three defines behavior-based incentives and presents our game-theoretic model for the analysis of the grim trigger strategy under perfect observation of nodes' actions. We relax the assumption of perfect monitoring and propose a probability-based strategy to induce cooperation under imperfections in Chapter four. Chapter five describes the application of a behavior-based incentive to topology control and Chapter six concludes the work with suggestions for potential future work.

Chapter 2. MODELING AD HOC NETWORKS

In this chapter we present the basics of game theory and explain key concepts that have been adopted in this research. Also, we highlight the main features of game theory when applied in the analysis of ad hoc networking problems. We also briefly discuss existing research on the analysis and application of incentive schemes for ad hoc networks, and position our work in relation to the existing literature.

2.1 Basics of game theory

Game theory is a field of applied mathematics that describes and analyzes interactive decision situations. It provides analytical tools to predict the outcome of complex interactions among rational entities, where rationality demands strict adherence to a strategy based on perceived or measured results. The main areas of application of game theory include economics, political science, biology and sociology. From the early 1990s, engineering and computer science have been added to this list. In this section, we highlight some basic concepts in game theory.

2.1.1 Normal form game and Nash equilibrium

We limit our discussion to non-cooperative models that address the interaction among individual rational decision makers. Such models are called "games" and the rational decision makers are referred to as "players." In the most straightforward approach, players select a single action from a set of feasible actions. Interaction between the players is represented by the influence that each player has on the resulting outcome after all players have selected their actions. Each player evaluates the resulting outcome through a payoff or "utility" function representing her objectives. There are two ways of representing the different components (players, actions and payoffs) of a game: normal or strategic form, and extensive form. In our work we adopt the normal form representation. Formally, a normal form of a game G is given by $G = \langle N, A, \{u_i\} \rangle$ where $N = \{1, 2, \cdots, n\}$ is the set of players (decision makers), A_i is the action set for player i, $A = A_1 \times A_2 \times \cdots \times A_n$ is the Cartesian product of the sets of actions available to each player, and $\{u_i\} =$

$\{u_1, \cdots, u_n\}$ is the set of utility functions that each player *i* wishes to maximize, where $u_i: A \rightarrow \mathbb{R}$. Note that $\mathbb{R}$ denotes the set of real numbers. For every player i, the utility function is a function of the action chosen by player i, a_i, and the actions chosen by all the players in the game other than player i, denoted as $\boldsymbol{a}_{-i}$. Together, a_i and $\boldsymbol{a}_{-i}$ make up the action tuple $\boldsymbol{a}$. An action tuple is a unique choice of actions by each player. From this model, steady-state conditions known as *Nash equilibria* can be identified. Before describing the Nash equilibrium we define the best response of a player as an action that maximizes her utility function for a given action tuple of the other players. Mathematically, $\bar{a}$ is a best response by player *i* to $\boldsymbol{a}_{-i}$ if

$$\bar{a} \equiv \{\arg\max u_i(a_i, \boldsymbol{a}_{-i})\}$$

A Nash equilibrium (NE) is an action tuple that corresponds to the mutual best response: for each player *i*, the action selected is a best response to the actions of all others. Equivalently, a NE is an action tuple where no individual player can benefit from unilateral deviation. Formally, the action tuple $\boldsymbol{a}^* = (a_1^*, a_2^*, a_3^*, \cdots, a_n^*)$ is a NE if $u_i(a_i^*, \boldsymbol{a}_{-i}^*) \geq u_i(a_i, \boldsymbol{a}_{-i}^*)$ $\forall a_i \in A_i$ and $\forall i \in N$. The action tuples corresponding to the Nash equilibria are a consistent prediction of the outcome of the game, in the sense that if all players predict that Nash equilibrium will occur then no player has any incentive to choose a different strategy. There are difficulties with using the Nash equilibrium as a prediction of likely outcomes (for instance, what happens when multiple such equilibria exist?). There are also refinements to the concept of Nash equilibrium tailored to certain classes of games. We shall discuss some refinements as applied to repeated games in Chapter 3.

There is no guarantee that a Nash equilibrium, when one exists, will correspond to an efficient or desirable outcome for a game (indeed, sometimes the opposite is true). Pareto optimality is often used as a measure of the efficiency of an outcome. An outcome is Pareto optimal if there is no other outcome that makes every player at least as well off while making at least one player better off. Mathematically, we can say that an action tuple $\boldsymbol{a} = (a_1, a_2, a_3, \cdots, a_n)$ is Pareto optimal if and only if there exists no other action tuple $\boldsymbol{b} = (b_1, b_2, b_3, \cdots, b_n)$ such that $u_i(\boldsymbol{b}) \geq u_i(\boldsymbol{a})$ $\forall i \in N$, and for some $k \in N, u_k(\boldsymbol{b}) > u_k(\boldsymbol{a})$.

To illustrate these basic concepts, consider a peer-to-peer file sharing network modeled as a normal form game [7]. The players of the game are individual users who experience a trade-off in sharing their files with others. For simplicity consider a network of three users. Each user has the option of either sharing her files or not sharing. Thus the action set of each player is $\{Share, Not\ share\}$. The payoff to each user is given by the sum of the benefits she experiences when other users share their files, and the cost she incurs by sharing her own files. We assume the users to be limited in resources. We assign the payoffs such that each user benefits by 1 unit for each other user that shares files and incurs a cost of 1.5 units in sharing her own files (these values are chosen somewhat arbitrarily and do not affect the basic results described here). The payoff matrix can be represented as in Table 2-1. In the payoff matrix, the payoff for user 1 is listed first, the payoff for user 2 is listed second, and the payoff for user 3 is listed third. Rather than attempting to represent the three dimensional action space as a single object, we have presented the action space in two two-dimensional slices.

Table 2-1. A payoff matrix for a three-player peer-to-peer file sharing game

User 1 \ User 2	Share	Not share
Share	0.5,0.5,0.5	-0.5,2,-0.5
Not share	2,-0.5,-0.5	1,1,-1.5

User 3 = Share

User 1 \ User 2	Share	Not share
Share	-0.5,-0.5,2	-1.5,1,1
Not share	1,-1.5,1	0,0,0

User 3 = Not share

From the payoffs we observe that the best response of each user irrespective of other users' actions is to not share. The unique NE is the action tuple (Not share, Not share, Not share). Also, it is evident that no user accrues any benefit by unilaterally deviating and sharing her files. One should note that the Nash equilibrium is *not* Pareto optimal in this case. The outcome (Share,

Share, Share) would make all three players better off than the NE action tuple. Those familiar with game theory will recognize this formulation as a three-player version of the Prisoners' Dilemma game [8].

2.1.2 Repeated games

As seen in the example above, it is possible that the Nash equilibrium of a single-stage normal form game is non-optimal. Also, players in a file sharing game or nodes in an ad hoc network repeatedly interact with each other either for a finite amount of time or indefinitely. Therefore it is essential to capture such interactions by repeating the single stage game. A repeated game models multiple plays between the players. A repeated game is structured as a sequence of the single stage game with the following properties [9]:

- The strategy of any player in the repeated game comprises the actions played at every period. The action at any period is, typically, a function of the past actions of all players.
- The payoff to any player is a sum (discounted by a factor δ) of the individual stage payoffs.

In our work we consider a *stationary* repeated game structure. In other words, for each play the action set of any player is the same regardless of the stage in which the game is in; and the payoffs to any player at any stage of the game depend on the action profile which is played in that stage.

We can denote a generic model of a repeated game as (note that we refer to notation for a single stage normal form game as defined in section 2.1.1):

G^T - repeated game of duration T stages;

$\boldsymbol{a}^t$ - the joint action profile at stage *t;*

$H^t = (\boldsymbol{a}^1, \boldsymbol{a}^2, \cdots, \boldsymbol{a}^{t-1})$ - history of actions at stage *t*;

$\sigma_i(t): H^t \rightarrow A_i$ - strategy of player *i* at stage t;

$\boldsymbol{\sigma}(t) := \left(\sigma_1(t), \sigma_2(t), \sigma_3(t), \cdots, \sigma_n(t)\right)$ - strategy profile at stage t;

$\boldsymbol{\sigma} = (\boldsymbol{\sigma}(1), \boldsymbol{\sigma}(2), \cdots \boldsymbol{\sigma}(T))$ - joint strategy profile of the repeated game;

δ - discount factor, $0 < \delta < 1$; and

$v_i(\boldsymbol{\sigma}) = \sum_{t=0}^{T} \delta^t u_i\left(\boldsymbol{\sigma}(t)\right)$ - repeated game payoff for player i;.

Based on the observed action at the end of each stage, we can broadly define two types of repeated games:

- Repeated games with perfect monitoring: In this repeated game structure each player observes the actions exercised by the other players at the end of each stage of the game. This enables a player to adapt accurately to the observed behavior of other players in the game.
- Repeated games with imperfect monitoring: In this repeated game structure each player observes a signal at the end of each stage instead of the actions played by the others. The signal observed is randomly correlated to the actions of other players. Depending on whether all players observe the same signal value or individually evaluate the signal, we consider imperfect public monitoring or private monitoring games, respectively [10].

2.2 Applicability of game theory to ad hoc networks

From the example in Table 2-1, we observe that players are independent decision makers whose payoffs depend on other players' actions. Nodes in an ad hoc network are characterized by the same feature. This similarity leads to a strong mapping between traditional game theory components and elements of an ad hoc network. Table 2-2 shows typical components of an ad hoc networking game.

Game theory can be applied to the modeling of an ad hoc network at the physical layer (distributed power control and waveform adaptation), link layer (medium access control) and network layer (packet forwarding). Applications at the transport layer and above exist also, although less pervasive in the literature. Some of that work includes analysis of the impact of selfish behavior in congestion control adapted by TCP, and studies of selfishness and incentives for data sharing in peer-to-peer systems. A question of interest in all those cases is that of how to provide the appropriate incentives to discourage selfish behavior. Selfishness is generally detrimental to overall network performance; examples include a node's increasing its power without regard for interference it may cause on its neighbors (layer 1), a node's immediately retransmitting a frame in case of collisions without going through a back-off phase (layer 2), or a node's refusing to forward packets for its neighbors (layer 3). In this research we focus primarily on the development of game-theoretic models characterizing selfish node behavior for voluntarily participating in network layer functions. Although game theory is a promising tool, there are certain trade-offs with regards to its application. We present some of the benefits and potential challenges in its application.

Table 2-2. Typical mapping of ad hoc network components to a game

Components of a game	**Elements of an ad hoc network**
Players	Nodes in the network
Strategy	Action related to the functionality being studied (e.g. the decision to forward packets or not, the setting of power levels, the selection of waveform/modulation scheme)
Utility function	Performance metrics (e.g. throughput, delay, target signal-to-noise ratio)

2.2.1 Benefits in the application of game theory to ad hoc networks

Game theory offers certain benefits to analyze distributed algorithms and protocols for ad hoc networks. Those benefits include:

a. Analysis of distributed systems: Game theory allows us to investigate the existence, uniqueness and convergence to a steady state operating point when network nodes perform independent adaptations. Hence it serves as a tool for a rigorous analysis of distributed protocols.

b. Cross layer optimization: Often in ad hoc networking games, node decisions at a particular layer are made with the objective of optimizing performance at some of the other layers. With an appropriate formulation of the action space, game theoretic analysis can provide insight into approaches for cross layer optimization.

c. Design of incentive schemes: Mechanism design is an area of game theory that concerns itself with how to engineer incentive mechanisms that will lead independent, self-interested participants towards outcomes that are desirable from a system-wide point of view. One of the key applications of mechanism design is in the development of cheat-proof incentive schemes. We will briefly discuss some of the recent literature applying mechanism design to ad hoc networks later in the chapter.

2.2.2 Challenges in the application of game theory to ad hoc networks

The use of game theory to analyze the performance of ad hoc networks is not without its challenges. We point out three particularly challenging areas:

a. Assumption of rationality: Game theory is founded on the hypothesis that players act rationally, in the sense that each player has an objective function that she tries to optimize given imposed constraints on her choices of actions by conditions in the game. Although nodes in an ad hoc network can be programmed to act in a rational manner, the steady state outcome of rational behavior need not be socially desirable. Indeed, a major contribution of game theory is that it formally shows that individually rational, objective-maximizing behavior does not necessarily lead to socially optimal states.

   The assumption of perfect rationality, on some practical occasions, does not accurately reflect empirically observed behavior (e.g., wide-spread existence of peer-to-peer file sharing networks in the absence of any punishment/reward schemes). The work in [11] considers an extension of the NE concept in order to accurately model nodes that deviate slightly from their expected optimal behavior. This form of weakened rationality is known as near-rationality.

b. Realistic scenarios require complex models: The dynamic nature of ad hoc networks leads to imperfection or noise in actions observed by a node. Such imperfections need to be modeled with reasonably complex games of imperfect information and/or games of imperfect monitoring. In addition, modeling of wireless channel models and interactions between protocols at the different layers involves complex and, at times, non-linear mathematical analysis.

c. Choice of utility functions: It is difficult to assess how a node will value different levels of performance and what trade-offs it is willing to make. Since different nodes may have different preferences over their actions it is difficult to represent this heterogeneity with a single utility function. As a result the formulation of a utility function must be dictated by a bare minimum set of assumptions about the nodes' preferences. In addition, the formulation problem is exacerbated by a lack of analytical models that map each node's available actions to higher layer metrics such as throughput.

2.2.3 Why use game theory?

The problem of node cooperation in ad hoc networks comprises two sub problems:

a. Study of selfish node behavior and node adaptations: To understand node behavior it is important to identify the trade-offs a node experiences in sharing its resources with other nodes while providing services in an autonomous setting. These trade-offs influence the different adaptations which are exercised by a node while interacting with other nodes in the network. A game theoretic model formally captures these trade-offs in the form of preference relations (or utility functions), with the adaptations modeled as action sets. In addition, repeated interactions between nodes can be modeled as repeated games with different node forwarding policies as complex strategies.

b. Application of an incentive scheme and its analysis: It is necessary to analyze the effectiveness of the incentive scheme in encouraging nodes to cooperate. Its application should typically lead to a socially optimal state which is stable. The Nash equilibrium and its refinements as adapted to repeated games can be used to determine the optimality and stability of the applied incentive scheme to the network. This renders game theory an appropriate tool to analyze the problem of node cooperation.

2.3 Related work

Broadly, game theory helps us address four basic issues in the modeling and analysis of networking algorithms: (a) Does a steady-state or equilibrium exist for the algorithm?; (b) By solving the model, can the steady-state be identified and if so, what constitutes it?; (c) Is the steady-state optimal and stable?; and (d) Do individual nodes making distributed and independent decisions converge to the equilibrium point? In the recent literature pertaining to ad hoc networks, game theory has been successfully applied to answer some of these questions for distributed power control algorithms [12, 13], interference avoidance schemes [14] and for medium access protocols [15, 16]. At the networking layer, game theory has been applied to ad hoc routing [17], in which the authors focus on the analysis of the effectiveness of three ad hoc routing techniques, namely link state routing, distance vector routing and multicast routing (reverse path forwarding), in the event of frequent route changes.

Specifically, game theory has been used to model selfish node behavior while forwarding packets for other nodes in an ad hoc network. Here, one of the main applications of game theory is to analyze the effectiveness of different incentive schemes in encouraging selfish nodes to cooperate. Mechanism design principles have also been applied to design cheat-proof payment-based incentive schemes. In the next section we provide a review of the literature pertaining to incentive schemes, with our main focus on behavior-based or non-monetary based schemes. For a detailed review of the aforementioned game theoretic applications we point to our recent paper on using game theory to analyze wireless ad hoc networks [7].

2.3.1 Credit-based incentive schemes

Credit-based incentive schemes, described in Chapter 1, adopt a mechanism of charge and reward for services offered by a node. Different ways of implementing this mechanism have been proposed in [3-6, 18]. In such a scheme, a node is credited for cooperating with the other nodes towards a common network goal, and is debited when requesting service from others. One way of implementing the charge and reward scheme is by the introduction of 'virtual currency' as in [4]. In this method each node is rewarded with 'tokens' for providing service, which are then used by the node for seeking services from others. One criticism of this method is that it requires a tamper-proof hardware module to prevent nodes from cheating during 'token' exchange. In addition, such techniques may be cumbersome to implement, as charges and rewards are calculated on a per packet basis [19].

To prevent nodes from falsely reporting the costs incurred and cheating, the concept of algorithmic mechanism design can be leveraged to design pricing policies that lead to truthful reporting. The works in [3, 5, 6, 18, 20, 21] develop incentive compatible, cheat-proof mechanisms that apply the principles of mechanism design to enforce node collaboration for routing in ad hoc networks. As compared to a credit-based mechanism, behavior-based schemes tend to be easier to implement and do not require management of any external logical entity such as credits or physical tokens.

2.3.2 Behavior-based incentive schemes

As described in Chapter 1, behavior-based incentives act on observed node behavior. One commonly adopted technique to evaluate a node's behavior involves maintaining a reputation indicator for nodes; this indicator reflects the nodes' past behavior in cooperating with others. Here each node builds a positive reputation for itself by cooperating with others and is tagged as "misbehaving" otherwise. The nodes that gain a bad reputation are then isolated from the network over time. The fear of being isolated provides the necessary incentive. In our work reputation can be considered as one of the metrics in the assessment of a node's behavior. We explore some of the other possible ways of assessing node behavior based on nodes' actions in Chapter 3.

Several reputation mechanisms can be found in the recent literature (such as those surveyed in [22]). Specifically, game theory has been used in [23] for the analysis of a reputation exchange mechanism. According to this mechanism, a node assigns reputation values to its neighbors based on its direct interactions with them and on indirect reputation information obtained from other nodes. Further, this reputation mechanism is modeled as a complex node strategy in a repeated game model. The analysis of the game helps to assess the robustness of the reputation scheme against different node strategies and derive conditions for cooperation. Even though reputation schemes are effective in providing incentives, the design of a good reputation system to judge a node is often complex and involves trust management between the individual nodes. Our reputation scheme in [24] tries to simplify some of the complexities by eliminating the need for trust between nodes.

In our work, we propose incentives that take into account the 1-stage history of nodes' actions based on individual observations without the requirement of implementing a system-wide reputation and trust management overlay. Based on periodic observations a node adapts its strategies while interacting with other nodes in the network. The research undertaken in [25] is one of the early works to adopt such a technique. The authors use game theory to analyze the adopted behavior mechanism. Nodes are classified into different energy classes based on their individual energy constraints. This classification is useful in defining policies that each node within a class might undertake. The authors identify a Pareto optimal point in terms of throughput experienced by a node and show how the generous tit-for-tat (GTFT) [26] strategy can be employed to achieve Pareto optimality. In GTFT, a node in the present stage of a repeated game mimics the actions of other nodes in the previous stages. However, any node employing the GTFT strategy is required to be generous on occasion and is required to cooperate even if other nodes had not previously done so. Our research differs from that work by relaxing the assumption of perfect knowledge of observed behavior. The work in [27] differs only slightly from [25], as they develop a generic model of selfish node behavior and identify possible equilibrium strategies for nodes with two different levels of selfishness. Among the recent works, [28] considers a slight variation of a tit-for-tat strategy in which a node, if it reduces its own cooperation level, is punished by other nodes' reducing their own cooperation level. The authors derive conditions for the strategy to lead to an equilibrium in which nodes cooperate at some steady level. The research reported in [29] considers different repeated game strategies and identifies equilibrium conditions. Specifically, the authors focus on tit-for-tat and modified tit-for-tat strategies in their analysis. Again, our work differs since one of the key assumptions in [29] is perfect knowledge of other nodes' behavior. As an extension to this work the authors identify equilibrium conditions under limited mobility situations with nodes arranged in a ring-like topology [30].

The majority of the research effort in analyzing the efficacy of behavior-based incentives assumes nodes have a perfect knowledge of each other's actions after every interaction. However, in a distributed network a perfect assessment is difficult to obtain. Researchers in [31] are among the first to tackle this issue for the case of selfish nodes in forwarding packets. The imperfection in monitoring opponents' actions is modeled as noise in the observed action for each node. The authors adopt repeated games with imperfect private monitoring in their analysis and prove the efficacy of a belief-based incentive scheme in inducing cooperation. This incentive scheme dictates that a node will cooperate or not based on its belief of its opponent's private history. A node's private history is a sequence of the opponent's actions as observed by the node. Hence, according to this incentive scheme, each node is required to track its opponent's private history, in other words, to evaluate the likelihood that its opponent might have observed a particular action at the end of each stage of the game, and react to that. This differs from our adopted scheme, described in Chapter 4, which is probability-based and absolves the node of keeping track of beliefs of its opponent's history.

In the next chapter we continue our discussion of behavior-based incentives by analyzing them in single-stage and multi-stage game theoretic models to prove their efficacy in inducing cooperation.

2.4 Chapter summary

In this chapter we explore the basics of game theory including the definition of a game, the concept of Nash Equilibrium, and its application to analyzing ad hoc networks.

We also highlight the benefits and limitations in the applicability of game theory to ad hoc networks. The key benefit of using game theory is the strong suite of analytical tools that help in the evaluation of ad hoc networking algorithms. The main limitation is in the design of the utility function along with a need to incorporate practical realities in the game-theoretic models which, in turn, make the models fairly complex.

Finally, we perform a literature review of existing work that employs incentives – credit-based and behavior-based – in inducing nodes to cooperate. We point out the key differences in our work with respect to the recent literature, further underscoring our contributions.

Chapter 3. BEHAVIOR-BASED INCENTIVES

In this chapter we explain the concept of a behavior-based incentive scheme to sustain cooperation and analyze the effectiveness of such a scheme in inducing nodes to cooperate. In particular, we consider a simple behavior-based incentive: the grim trigger strategy. In the scenario studied in this chapter nodes can perfectly assess the behavior of other nodes in the network by observing their actions. In the next chapter, we relax that assumption by studying behavior-based incentives under imperfect monitoring.[1]

3.1 Behavior-based incentive schemes

In Chapter 1, we briefly introduced behavior-based incentive schemes and positioned them with respect to credit-based schemes. In this section we define and further characterize behavior-based incentives.

3.1.1 Basic concept

Behavior-based incentives for cooperation rely on observed behavior without the use of external economic or monetary instruments. The incentive here is typically provided through the threat of retaliation for non-cooperative behavior.

Some examples of behavior-based schemes found in the literature include the tit-for-tat [26], generous tit-for-tat [32] and other solutions such as the ones proposed in [23, 33]. In our work, we consider both, strict (grim-trigger) and forgiving (probability-based) behavior-based incentives to induce node cooperation.

We have also studied the effects of behavior-based incentives on topology control in the presence of selfish nodes, investigating topology properties such as connectivity, node degree, and path reliability. We present our results for the study of topology control in Chapter 5.

3.1.2 Game-theoretic approach

In this research we apply game theory to analyze the effectiveness of a behavior-based scheme in sustaining cooperation in packet forwarding in an ad hoc network. We summarize the steps we

[1] The game theoretic model and results for perfect monitoring presented in this chapter appear in [32].

followed to analyze the behavior-based incentive schemes for node cooperation. These steps are depicted in Figure 3.1.

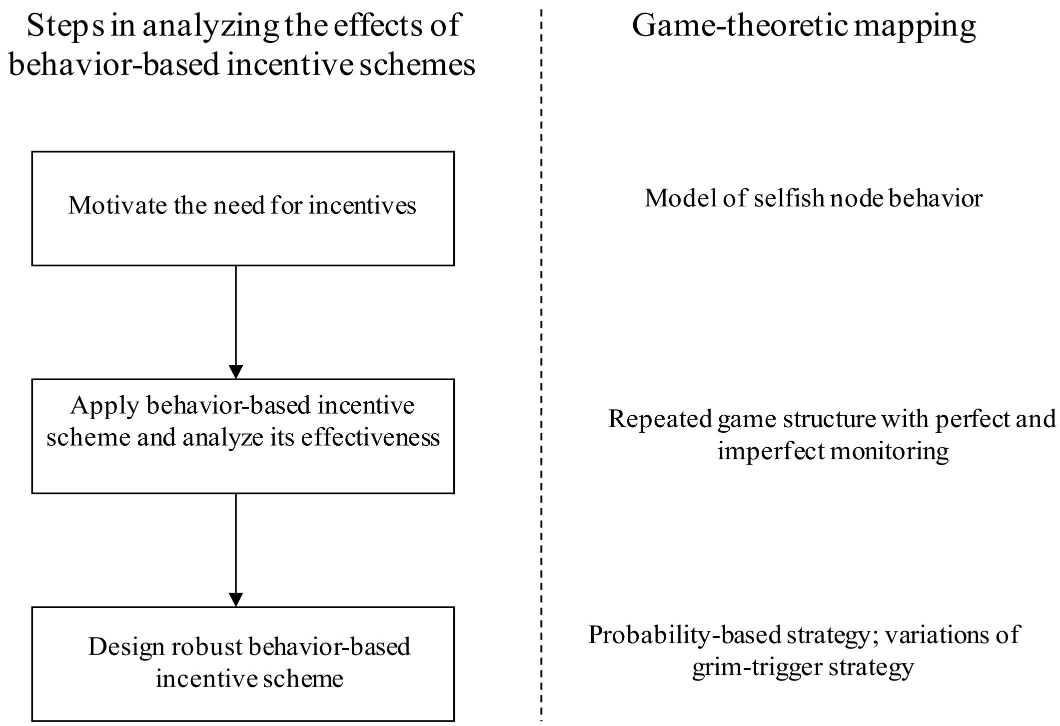

Figure 3.1. Steps in analyzing the effects of behavior-based incentive schemes for node cooperation.

- *Model of selfish behavior* – We begin by evaluating the effect of selfish node behavior on the operation of an ad hoc network, with emphasis on the forwarding functionality. We model selfish behavior and analyze the model in the absence of an incentive scheme, determining that the resulting equilibrium is for none of the nodes to cooperate, clearly an undesirable result from the point of view of the network as well as of individual nodes.
- *Performance analysis of a behavior-based incentive scheme* – We then consider the effects of behavior-based incentive schemes on node cooperation. The analysis provides answers to key questions related to: (a) the effectiveness of the scheme in discouraging selfish behavior; and (b) the stability of the network when nodes implement the scheme.

  We model the grim trigger strategy in a repeated game structure assuming perfect knowledge of other nodes' actions at every stage and derive conditions for nodes to cooperate.

We then relax the assumption of perfect knowledge and analyze the same strategy under imperfect monitoring of other nodes' actions. We perform a stability analysis of the network when one or more nodes deviate from the equilibrium strategy.

- *Design of a robust behavior-based incentive scheme* – One of the observations from our stability analysis (described in Chapter 4) is the intuitive result that the grim-trigger strategy is overly sensitive to node deviations. Therefore, we propose a forgiving strategy that is robust to the lack of perfect knowledge and evaluate its efficacy in inducing cooperation.

3.2 Game-theoretic analysis

3.2.1 Single-stage game for node cooperation

We model node cooperation in an ad-hoc network as a strategic-form game G, as described in Chapter 2. We consider homogeneous actions to be available to all nodes: to cooperate $\left(a_j = 1\right)$ or to refrain from cooperating $\left(a_j = 0\right)$. We denote the cardinality of set of nodes as $n = |N|$.

In such a game-theoretic formulation, the utility function is often the "weakest link," due to the difficulty in assessing tradeoffs as perceived by individual nodes. In this work, we adopt general, intuitive assumptions about the utility function, without attempting to completely characterize such functions. In particular, we consider a node's utility function to be the difference of two components, benefit and cost:

$$u_j(\boldsymbol{a}) = \alpha_j(\boldsymbol{a}) - \beta_j(\boldsymbol{a}) \tag{3.1}$$

- $\alpha_j(\boldsymbol{a}) = \sum_{i \in N, i \neq j} a_i$ is the benefit accrued by a node when other nodes cooperate. We assume $\alpha_j(\mathbf{0}) = 0$ and $\alpha_j(\boldsymbol{a}) > 0$ if $\exists\, k \neq j$ such that $a_k \neq 0$, as it is intuitive that a node will accrue positive benefit from others' willingness to perform services for it.
- $\beta_j(\boldsymbol{a}) = \beta_j\left(a_j\right)$ is the cost incurred by node j in cooperating. We assume this part of the utility function to be dependent only on the node's own chosen strategy. (Note that in an ad hoc network the cost of cooperation may depend on how many other nodes also cooperate, as

the number of routing requests may increase if few nodes in the network are willing to cooperate. These effects are considered later in this chapter.) Also, $\beta_j(\mathbf{0}) = 0$.

A joint strategy $\boldsymbol{a}$ is a Nash equilibrium (NE) if no node can benefit from unilaterally deviating. For this formulation it is fairly simple to show that the only Nash equilibrium is for no nodes to cooperate. We point to a similar conclusion reached in [27, 32] where, in the absence of incentives, cooperation among nodes is not possible. The analysis of the single stage game results in a non-optimum equilibrium from an overall network's perspective. However, other equilibria are achievable when the game is repeated, provided that nodes do not know a priori how many repetitions of the game there will be. This provides the inspiration for the development below.

3.2.2 Repeated game model for node cooperation

Consider a repeated game, played K times, where K is a geometrically distributed discrete random variable with parameter $0 < p < 1$. Therefore, $P[K = k] = p(1-p)^k$, $k = 0,1,2,\cdots$ and $E[K] = \frac{1-p}{p}$. Note that, as $p \to 1$, the probability that the game will be repeated approaches 0.The geometric distribution is chosen for its memoryless property.

3.2.2.1 Grim trigger strategy

Consider a grim-trigger strategy [8] adopted by all nodes: cooperate in forwarding packets as long as all other nodes are willing to cooperate; do not cooperate if any of the others have deviated in the previous round. The trigger is activated when any one node decides to switch from the desired behavior (cooperating in sharing resources) and is grim as the nodes do not switch their action back once the punishment (not cooperating) is initiated. If no node deviates, at any time a node's expected payoff from that point onwards is:

$$\left[\alpha_j(N-1) - \beta_j(1)\right] \cdot [1 + \sum_{k=0}^{\infty} p \cdot k \cdot (1-p)^k] \tag{3.2}$$

If, on the other hand, a node unilaterally deviates, its expected payoff from that point onwards is simply $\alpha_j(N-1)$. So, it is a Nash equilibrium for all nodes to cooperate as long as, $\forall\, j$,

$$\alpha_j(N-1) > \frac{\beta_j(1)}{1-p} \tag{3.3}$$

We offer an interpretation of this result. If $\beta_i(1) < 0$, i.e., if the node derives some benefit or satisfaction from cooperating with others, then, not surprisingly, it is always an equilibrium to cooperate. More interestingly, when $\beta_i(1) > 0$ (i.e., there is a cost in cooperation), then a socially optimal equilibrium is still sustainable. The precise cost/benefit tradeoff is given by the inequality above. We also note that, in an ad hoc network, the time horizon for the repetitions of a game (characterized by parameter p) can be interpreted as having a direct relationship with a node's mobility. In this sense, the more mobile nodes are (the closer p is to 1), fewer are the interactions between nodes and lower is the incentive to cooperate.

The results above assume an all-or-nothing policy: if any node deviates (refuses to cooperate) in one round, all others will deviate in the next round. In the next section, we explore the robustness of a "softer" policy that does not require *all* nodes to cooperate.

3.2.2.2 An alternative strategy

Let us denote by $a_j^{(k)}$ the strategy adopted by node j in the k^{th} round of the game. Suppose a node i adopts the following strategy: in every round k of the repeated game, node i decides to cooperate as long as the following condition is satisfied:

$$\alpha_i\left(\sum_{j\in N, i\neq j} a_j^{(k-1)}\right) > \frac{\beta_i(1)}{1-p} \tag{3.4}$$

We study the stability of this desirable equilibrium in relation to the probability of the game being repeated. We perform a simulation to determine whether a deviation in a single node's strategy (due to variation in nodes' perceptions of whether the game is likely to be repeated) will result in a cascading effect on the other nodes and lead to a shift to an equilibrium where all nodes decide not to cooperate. It is clear that if nodes do not believe the game will be repeated ($p = 1$) the game reduces to a single stage game and all nodes adopt a non-cooperation strategy. However, we wish to determine, as a function of the number of nodes in the network, the value of p at which the cascading behavior is observed and the equilibrium shifts for every node. In the simulation, for a fixed value of cost incurred by a node in cooperating (i.e., $\beta_i(1) = \beta$), we vary the number of nodes in the system and establish what values of p will still support a socially desirable equilibrium. In our simulation, each node accrues a different benefit that it derives from other node's willingness to cooperate in sharing its resources. The function $\alpha_j(\boldsymbol{a}) = \varphi_j \sum_{i \in N, i \neq j} a_i$, with φ_j being taken from a uniform distribution between [0,1]. However, the cost incurred by each node in cooperating is the same. The simulation is repeated for different values of β. Each point plotted is the average of 200 repetitions of the simulation.

The plot in Figure 3.2 can be interpreted as follows. For each value of β, we plot the maximum value of p that will still lead to a desirable equilibrium. When more nodes are present, the desirable equilibrium is more robust to players' exogenous beliefs about the repeatability of the game (and, in a practical interpretation for ad hoc networks, that equilibrium is more robust to node mobility). Also, as the cost of cooperation (β) increases, the desirable equilibrium requires players to believe that the game has a high probability of being repeated (corresponding to a low value of p). As discussed before, $p = 1$ corresponds to a single stage game; the node knows that the game will not be repeated (at least, not with the same neighbors), always leading to non-cooperation. It is interesting to note, however, that if the number of nodes is high enough (i.e., if the network is dense enough), a socially desirable equilibrium is achievable even for values of p arbitrarily close to 1.

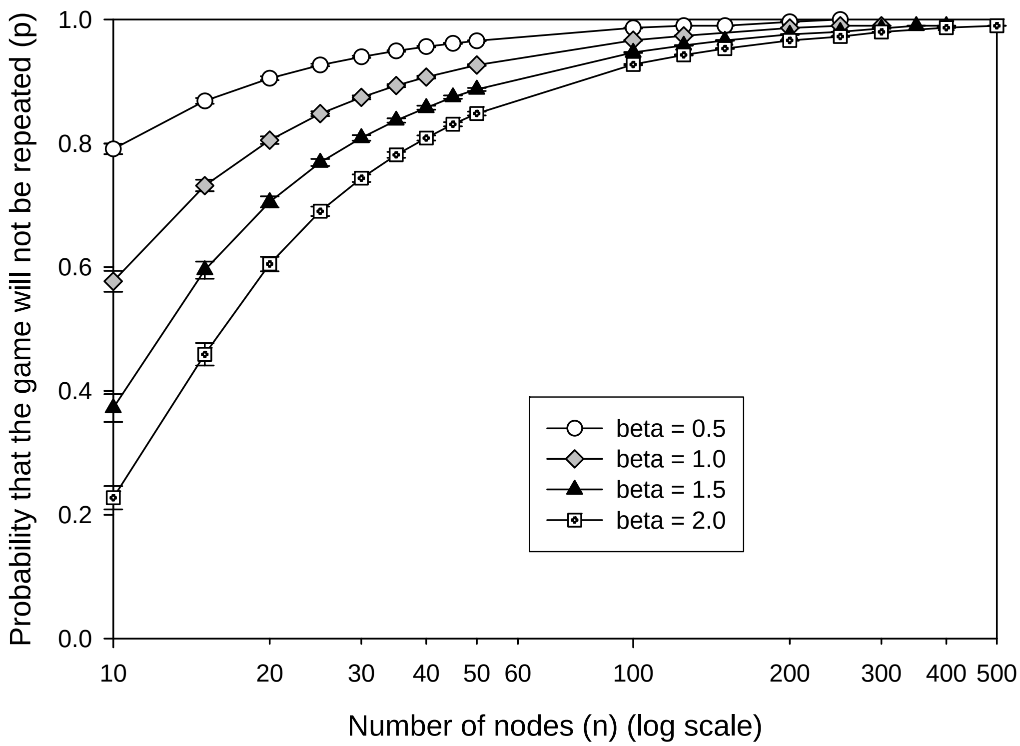

Figure 3.2. Depending on node density, desirable equilibria are achieved even if nodes think there is a low likelihood of the game being repeated (corresponding to $p \to 1$)

As discussed earlier, the number of forwarding requests to a node in an ad hoc network may increase if few nodes in the network are willing to cooperate in forwarding packets. Therefore we refine the model to account for this fact. We modify the cost incurred by node j, $\beta_j(\boldsymbol{a})$, such that it is dependent on the number of nodes willing to cooperate, instead of on node j's actions only. We set, $\forall\, j$, $\beta_j(\boldsymbol{a}) = \frac{1}{(1+\sum_{i \neq j, i \in N} a_i)}$. We compare the stability of the equilibrium for the modified $\beta_j(\boldsymbol{a})$ function against the fixed value function ($\beta_j(\boldsymbol{a}) = \beta_j(a_j = 1) = \beta = 1$) by calculating the maximum value of p at which equilibrium is still observed. For the modified utility we observe (see Figure 3.3) that the equilibrium is stable even at higher values of p than for the fixed cost case. This is to be expected, since the cost incurred by a node decreases if a significant percentage of nodes within the network cooperate. As a result, the equilibrium condition fails only at relatively higher values of p than when the cost is constant.

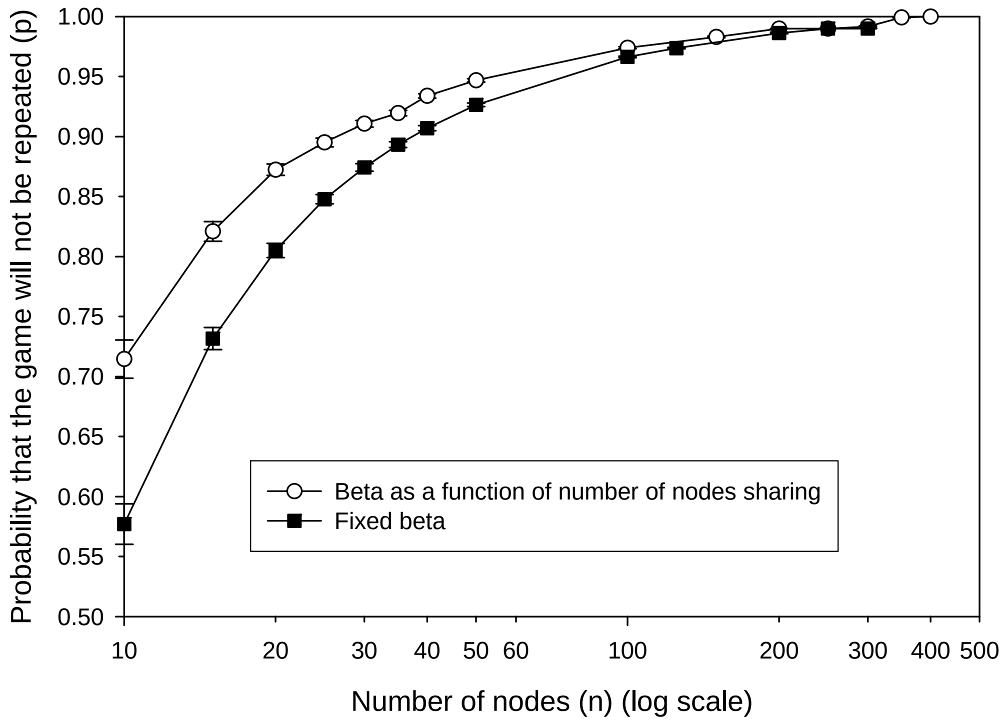

Figure 3.3. Comparison of the effect of a modified cost function on the maximum value of p for sustenance of optimal equilibrium.

A non-trivial conclusion from the results presented here is that nodes may agree to cooperate in sharing their resources and forward packets, even if they perceive a cost in doing so. This happens as the nodes recognize that refusing to cooperate will result in similar behavior by others, which ultimately would compromise the viability of the network as a whole. In addition, when the cost is dependent on other nodes' actions it is possible for a node to sustain a socially optimal equilibrium even when the chances of game repeatability are low.

One of the key assumptions in this model is that each node has perfect knowledge of every other node's forwarding decisions. To obtain this information, a node would need to constantly monitor its neighbors' transmissions, periodically switching the network interface to promiscuous mode of operation. In practice, this approach is costly from the energy consumption stand point and is liable to errors. We therefore relax the assumption of perfect knowledge and analyze the efficacy of behavior-based incentives in which nodes do not have exact information regarding other nodes' actions. Instead, a node observes a random signal which is correlated to the actions of all other nodes in the game.

In the next chapter, we explore games of imperfect monitoring depending on whether the random signal observed by a node is public or private information.

3.3 Chapter summary

In this chapter we characterize behavior-based incentives and outline the steps to be employed in analyzing their efficacy in inducing cooperation. We present a simple behavior-based incentive - the grim-trigger strategy, and analyze its efficacy in inducing cooperation.

A non-trivial conclusion from the results presented here is that nodes may agree to cooperate even if they perceive a cost in doing so. This happens as the nodes recognize that refusing to participate will result in similar behavior by others, which ultimately would compromise the viability of the network as a whole.

Chapter 4. IMPACT OF IMPERFECT MONITORING ON BEHAVIOR-BASED INCENTIVES

In practice, nodes cannot accurately monitor the actions of other nodes in the network. In this chapter we evaluate the efficacy of behavior-based incentives under such imperfections in monitoring. We consider two distinct scenarios: (a) each node monitors the actions of other nodes in the network through a public signal, commonly observed by all other nodes in the network as well, and (b) each node individually monitors, though imperfectly, the actions of others and such information is private. We propose a probability-based incentive and prove its efficacy in inducing cooperation. We further show it to be robust to imperfections in monitoring when compared with the grim-trigger strategy.[2]

4.1 Repeated games with imperfect public monitoring

In Chapter 3, we analyzed the grim-trigger strategy for node cooperation, modeling it as a node strategy for repeated games with perfect monitoring. However, nodes in a wireless network are susceptible to errors in monitoring. In this section, we relax the assumption of perfect monitoring by nodes and develop a game theoretic model in which nodes monitor other nodes' actions as a signal that is publicly observable. This random public signal correlates to the actions of all other nodes in the game. The random public signal can be considered a proxy for the actions of other nodes, providing imperfect and indirect information. An example of a feasible public signal is the number of users connected in a peer-to-peer network at a given instance. This reflects the likelihood of observing cooperation and of resource availability in the network.

4.1.1 Single stage model of imperfect monitoring

We extend the single stage game, as defined in Chapter 2 for perfect monitoring, to include the random public signal and associated changes to the utility function. Let P be the random public signal which takes values, p, in $\Omega \subseteq \mathbb{R}$. Here, $\mathbb{R}$ denotes the set of real numbers. For games with imperfect monitoring, the single stage utility, $u_i(a_i, p)$, is stochastic in nature and depends on the realization of the random public signal. As defined earlier, we express the utility as the difference between the two components, benefit and cost:

[2]We publish results for imperfect public monitoring presented in this chapter in [35].

$$u_i(a_i, p) = \alpha_i(p) - \beta_i(a_i) \tag{4.1}$$

- $\alpha_i(p)$ - the benefit accrued by node i depends on the random public signal, which reflects the actions of all other nodes.
- $\beta_i(a_i)$ - the cost accrued from forwarding packets for other nodes. Here, we assume the cost to be a function of the node's own strategy only.

One of the key challenges in the design of an imperfect monitoring game is the selection of a public signal, as it should possess the following properties [34] :

- The signal reflects the actions selected by each node in the network;
- The realization of the signal after each stage is known publicly by all nodes of the game; and
- The probability distribution function of the signal is correlated to the joint action profile.

For an ad hoc network, one can consider the public signal to be a performance metric that reflects the overall state of the network. From the perspective of forwarding levels, the aggregate network goodput can be one such candidate. It reflects the actions selected by the individual nodes and, in the absence of a centralized mechanism, nodes may exchange an indication of goodput experienced by each node and thereby estimate the aggregate goodput. The correlation of the public signal to nodes' actions is captured in the parameterization of the distribution function of p by the joint action profile of all nodes, $\boldsymbol{a}$, and denoted by $F(p; \boldsymbol{a})$. Since the utility is non-deterministic, we define the expected utility, π_i, for node i as a function of the joint action profile. It is expressed as:

$$\pi_i(\boldsymbol{a}) = \int_{p \in \Omega} u_i(a_i, p) \cdot dF(p; \boldsymbol{a}) \tag{4.2}$$

The expected utility is a probabilistic valuation of what each node "expects" to receive as a payoff for its selected action a_i. As in Chapter 3, each node selects its cooperation level, i.e. the proportion of packets it is willing to forward for others, from the action space $A_i = [0,1]$. We consider homogenous action sets for all nodes. The NE for G is expressed in terms of the expected utility; $\boldsymbol{a}$ is a NE if

$$\pi_i(\boldsymbol{a}) \geq \pi_i(a_i', \boldsymbol{a_{-i}}) \quad \forall a_i' \in A_i \quad \forall i \tag{4.3}$$

For our formulation, it is straight-forward to show that in the absence of incentive schemes it is best for nodes to not cooperate in forwarding packets. Hence, the Nash equilibrium for the single stage strategic form game is non-optimal.

4.1.2 Repeated game model

Let G^{∞}denote the infinitely repeated game with imperfect public monitoring. We consider the strategy of node i at each stage, t, as a function of the past values of the public signal, alone. We denote such a *public* strategy as $\sigma_i(t): \Omega^{t-1} \to A_i$, with $\boldsymbol{\sigma}(t)$ denoting the joint strategy vector at stage t. Let Σ_i be the set of repeated game strategies for node i such that for any stage t, $\sigma_i(t) \in \Sigma_i$. We discount the payoffs, since a node associates a lower value to the utility derived at later stages of the game. Let $v_i(\boldsymbol{\sigma})$ denote the repeated game payoff for node i, where $\boldsymbol{\sigma}$ is the joint public strategy profile. The (average) discounted expected game payoff for a node *i* is given by:

$$v_i(\boldsymbol{\sigma}) = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1}\pi_i\big(\boldsymbol{\sigma}(t)\big) \tag{4.4}$$

A public strategy is a perfect public equilibrium (PPE) if after any stage t, $\boldsymbol{\sigma}$ forms a Nash equilibrium from that stage on. In other words, if we denote $\boldsymbol{\varphi} \equiv \boldsymbol{\sigma}^{(-2)}$ (where $\boldsymbol{\sigma}^{(-t)}$ is the joint strategy profile after stage *t*) to be the remainder of $\boldsymbol{\sigma}$ after stage 2, then $\boldsymbol{\varphi}$ must form a Nash equilibrium for all subsequent stages. Generalizing, $\boldsymbol{\sigma}$ is a PPE if, after every stage *t*,

$$v_i(\boldsymbol{\sigma}) \geq v_i(\sigma_i', \boldsymbol{\sigma_{-i}}) \quad \forall i \quad \forall \sigma_i' \in \Sigma_i \tag{4.5}$$

4.1.2.1 Reduction to a single stage game

In order to simplify the evaluation of a PPE, we exploit the recursive nature of the repeated game and reduce it to an equivalent single stage game. The resulting single stage game can be denoted as $G_w = \langle N, A, \pi_w^i \rangle$ where: N is the set of nodes in the network; $\boldsymbol{A} = A_1 \times A_2 \times \cdots \times A_n$ is the joint action space; and π_w^i is the equivalent expected utility. We express the equivalent utility as:

$$\pi_w^i(\boldsymbol{a}) = (1-\delta)\pi_i(\boldsymbol{a}) + \delta \int_{p\in\Omega} w_i(p) \cdot dF(p; \boldsymbol{a}) \tag{4.6}$$

where $w_i(p): \Omega \to W$ is a continuation payoff function that maps every possible value of the public signal to a continuation payoff selected from $W \subset \mathbb{R}^n$ for a node i. For the repeated game, the equivalent utility represents a sum of the expected payoff for the first stage of the game (due to action $\boldsymbol{a}$ being played) and an expected continuation payoff for the remaining stages that results from the correlation of $\boldsymbol{a}$ with the public signal. Applying this representation to the repeated game payoff we can then write it as:

$$v_i(\boldsymbol{\sigma}) = (1-\delta)\pi_i(\boldsymbol{a}) + \delta \int_{p\in\Omega} w_i(p) \cdot dF(p; \boldsymbol{a}) \tag{4.7}$$

If $\boldsymbol{\sigma}$ is to be a PPE of the repeated game, then $\boldsymbol{a}$ must be a Nash equilibrium in the first stage of the game. Therefore, we can say that $\boldsymbol{\sigma}$ is a PPE if:

$$v_i(\boldsymbol{\sigma}) \geq (1-\delta)\pi_i(a_i, \boldsymbol{a_{-i}}) + \delta \int_{p\in\Omega} w_i(p) \cdot dF(p; a_i, \boldsymbol{a_{-i}}) \quad \forall i \quad \forall a_i \in A_i \tag{4.8}$$

4.1.2.2 Grim trigger strategy

Consider the grim trigger strategy ($\hat{\sigma}_i$) again, where each node forwards all the packets it receives ($\hat{a}_i = 1$) as long as the value of the public signal is above a certain threshold (x), and, when below, it ceases to forward. In other words, each node switches to the NE of the single stage game when the signal is below the selected threshold. Again, for simplicity we assume that each node adopts the same threshold. The trigger, which is activated as soon as the value of the signal falls below the threshold, is considered grim as the nodes do not switch their action back to forwarding all packets once the punishment (drop every packet) phase has been initiated. Let $\hat{\boldsymbol{\sigma}} = \langle \hat{\boldsymbol{\sigma}}(1), \hat{\boldsymbol{\sigma}}(2), \cdots, \hat{\boldsymbol{\sigma}}(n) \rangle$ denote such a strategy with $\hat{\boldsymbol{a}} = \langle \hat{\boldsymbol{a}}(1), \hat{\boldsymbol{a}}(2), \cdots, \hat{\boldsymbol{a}}(n) \rangle$ corresponding to the desirable action where every node forwards all packets. The associated payoff received by node i is given by:

$$v_i(\hat{\boldsymbol{\sigma}}) = (1-\delta)\pi_i(\hat{\boldsymbol{a}}) + \delta \int_{p\in\Omega} w_i(p) \cdot dF(p; \hat{\boldsymbol{a}}) \tag{4.9}$$

Therefore, a node following this strategy continues to receive a payoff v_i until the trigger is activated, after which it receives the NE payoff. Since every node follows the same strategy, we can express the continuation payoff function as:

$$w(p) = \begin{cases} v_i & if\ p > x \\ \pi^{NE} & if\ p \leq x \end{cases} \tag{4.10}$$

where π^{NE} is the Nash equilibrium payoff for the single stage game. The repeated game payoff now reduces to:

$$v_i(\widehat{\boldsymbol{\sigma}}) = (1-\delta)\pi_i(\widehat{\boldsymbol{a}}) + \delta\left[v_i\left(1 - F(x;\widehat{\boldsymbol{a}})\right) + \pi^{NE}F(x;\widehat{\boldsymbol{a}})\right] \tag{4.11}$$

For $\widehat{\boldsymbol{\sigma}}$ to be a PPE, it must be a Nash equilibrium for each node to forward all packets i.e. $\widehat{\boldsymbol{a}}$ should be a Nash equilibrium in the single stage game $\forall i$. Therefore,

$$v_i(\widehat{\boldsymbol{\sigma}}) \geq (1-\delta)\pi_i(a_i, \widehat{\boldsymbol{a}_{-i}}) + \delta\left[v_i\left(1 - F(x; a_i, \widehat{\boldsymbol{a}_{-i}})\right) + \pi^{NE}F(x; a_i, \widehat{\boldsymbol{a}_{-i}})\right] \tag{4.12}$$

Since at Nash equilibrium none of the nodes forward any packets ($\pi^{NE} = 0$), combining (4.11) and (4.12) we get:

$$\frac{1-\delta+\delta F(x;a_i,\widehat{\boldsymbol{a}_{-i}})}{1-\delta+\delta F(x;\widehat{\boldsymbol{a}})} \geq \frac{\pi_i(a_i,\widehat{\boldsymbol{a}_{-i}})}{\pi_i(\widehat{\boldsymbol{a}})} \tag{4.13}$$

Hence the grim trigger strategy is a PPE if the above inequality is satisfied for any deviation a_i and $\forall i$. We base the above derivation on a similar analysis done to determine collusion in repeated games in [35].We assume that $F(\cdot\,;\widehat{\boldsymbol{a}})$ stochastically dominates $F(\cdot\,; a_i, \widehat{\boldsymbol{a}_{-i}})$ for any profitable deviation a_i , i.e. $F(\cdot\,;\widehat{\boldsymbol{a}}) \leq F(\cdot\,; a_i, \widehat{\boldsymbol{a}_{-i}})$ for $\pi_i(a_i, \widehat{\boldsymbol{a}_{-i}}) \geq \pi_i(\widehat{\boldsymbol{a}})$. Therefore, for a given threshold value, if a node expects to benefit by deviating from $\widehat{a_i}$, it increases the chances of triggering the grim period earlier by risking the public signal to fall below the threshold.

We offer an intuitive interpretation of the above result. If a node i expects a better utility by reducing its cooperation level and deviating from $\widehat{a_i}$, the chances of the aggregate throughput reducing below a set threshold increase. This acts as an intrinsic incentive for node i to maintain its cooperation level for 'fear' of potentially triggering the grim phase. The inequality above expresses the relationship between the probability distribution of the public signal (i.e., the likelihood of triggering a grim trigger phase by deviating), the discount factor attached to future repetitions of the game, and the initial benefit and expected future cost from deviating.

It is clear that the stochastic dominance property of the distribution function satisfies a necessary condition for the grim trigger strategy to be an equilibrium under a profitable deviation. In addition, under the assumption of stochastic dominance, the equilibrium also holds for non-profitable deviations from $\widehat{a_i}$.

4.1.2.3 Folk theorem for imperfect public monitoring

Folk theorems, broadly, identify sets of payoff vectors which can be supported by some strategy profile at equilibrium. For repeated games, folk theorems that support subgame perfect equilibrium are of interest.

For games with imperfect public monitoring the folk theorem [36] states that any strict, individually rational payoff vector, for patient players, can lead to a PPE provided certain identifiability conditions, described below, are satisfied. This is a slight departure from the folk theorem for repeated game with perfect monitoring [10], which requires no such conditions to be satisfied.

The identifiability conditions require that the action profile being evaluated for equilibrium be *pairwise full rank*. In other words, the action profile for all player pairs induces different probability distributions on the public signal outcomes for any deviation by the players. As a result, the applicability of folk theorem is strongly dependent on the probability distribution function of the public signal.

4.1.2.4 Determination of the threshold range to sustain equilibrium

The condition (inequality (4.13)) for the grim trigger strategy to be a PPE only holds for a certain range of threshold values. In order determine this range, we vary the threshold values and check whether the inequality condition is satisfied under a single node deviation from $\hat{a_i} = 1$ to $a_i = 0$.

Prior to the determination of the range, we need to specifically characterize the benefit and the cost functions. Since the benefit accrued by a node is expected to increase with an increase in the forwarding levels of the other nodes, we assume it to be linearly correlated to the public signal i.e. $\alpha_i(p) = k_i p$. Here, $k_i \in (0,1)$ parameterizes the perceived benefit for each node. The cost incurred by a node is a function of its own actions and is equal to the node's forwarding level, i.e. $\beta_i(a_i) = a_i$. We realize that there may be more than one probability distribution which possess the first-order stochastic dominance property. One such distribution is the gamma distribution. For illustration purposes we assume the public signal (which, as discussed previously, could be some indicator of aggregate goodput for the network) to be gamma distributed with the shape parameter of the distribution equal to the sum of the joint actions of the nodes. Hence, the cumulative distribution function is given by $F(p; \boldsymbol{a}) = \frac{\Gamma_p(a')}{\Gamma(a')}$, where $a' = \sum_{i \in N} a_i$ is the shape parameter of the gamma distributed random variable. Here, $\Gamma_p(a')$ is the incomplete gamma function, $\Gamma_p(a') = \int_0^p t^{a'-1} \cdot e^{-t} dt$. We select the signal to be gamma distributed as it satisfies the necessary requirement of stochastic dominance such that $F(\cdot\,;\gamma) \le F(\cdot\,;\mu)$ for $\gamma > \mu$ with $\gamma, \mu > 0$.

Substituting the expression for the gamma distribution into $\pi_i(\boldsymbol{a}) = \int_{p \in \Omega} u_i(a_i, p) \cdot dF(p; \boldsymbol{a})$ (4.2), the expected utility to any node i when all nodes cooperate, i.e. $\hat{a_i} = 1 \quad \forall i$ is

$$\pi_i(\hat{\boldsymbol{a}}) = \int_{p \in \Omega} (k_i p - 1) dF(p; \hat{\boldsymbol{a}}) - 1 = k_i E[p] - 1 \qquad (4.14)$$

Therefore, we have $\pi_i(\hat{\boldsymbol{a}}) = k_i a' - 1 = k_i n - 1,\ \forall i$. Now, if any node deviates from $\hat{a_i} = 1$ to $a_i = 0$ the expected utility reduces to $\pi_i(a_i, \widehat{\boldsymbol{a}_{-i}}) = k_i a' = k_i(n-1),\ \forall i$. Replacing the expected utilities, (4.13) now reduces to:

$$n\delta\Gamma_x(n-1)(nk_i - 1) - \delta\Gamma_x(n)(nk_i - k_i) \geq n!\,(1-\delta)(1-k_i) \qquad (4.15)$$

Revisiting the folk theorem, in our case, the shape of the gamma distribution is parameterized by the sum of the actions of the nodes. For the grim trigger strategy any deviation by one node cannot be distinguished from a similar deviation by another. As a result our game construct does not necessarily satisfy the folk theorem conditions.

We determine the range of threshold values for different network sizes for which (4.15) holds (see Figure 4.1). The range is obtained by averaging over 50 different random values of k_i for a fixed value of the discount factor ($\delta = 0.7$). For the evaluation the incomplete gamma function is numerically computed according to the numerical algorithm presented in [37]. From Figure 4.1 we conclude that for larger network sizes, the range of threshold values for which the condition is satisfied is greater. It is easy to deduce this from the condition above since for higher values of n, the inequality $n\Gamma_x(n-1) > \Gamma_x(n)$ holds true over a larger range of x. In addition, this also means that as the size of the network increases a node experiences greater benefit in cooperating and switches to a grim action only for much lower thresholds. This leads to a much wider choice in the selection of the threshold without violating the condition for equilibrium. We also varied the discount factor and observed that for higher values, the range of the threshold tends to increase for a given network size. This is expected, as for higher values of δ inequality (4.15) holds true for a broader range of values of x. This means that as a node's value for its future payoff improves, it perceives greater benefit in cooperating.

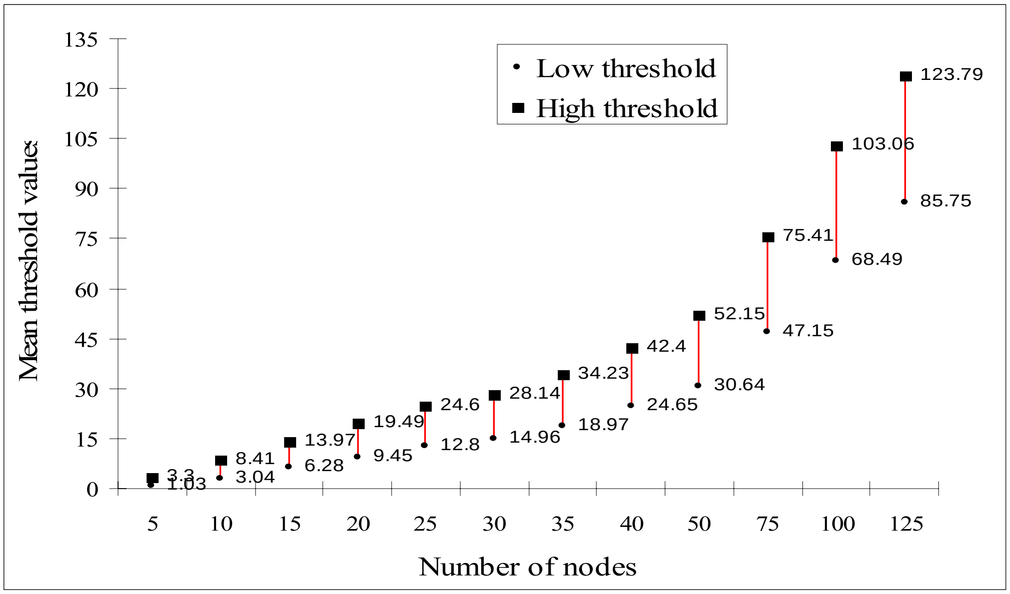

Figure 4.1. Mean threshold values for which the equilibrium condition is satisfied for different network sizes

4.1.2.5 Network stability

Apart from evaluating the effectiveness of an incentive scheme in discouraging selfish behavior it is also important to study its effect on network stability, namely how sensitive individual nodes are to changes triggered by actions of their peers. For the grim trigger strategy under consideration a node is triggered to change its action based on a comparison of the realized value of the random signal to a set threshold. We consider the network to be stable if a deviation by a node (or nodes) from $\hat{a_i} = 1$ to $a_i = 0$ at any stage does not lead to a cascading effect of other node deviations in the subsequent stages. The stability is evaluated in terms of the average number of stages, after nodes (or a node) deviate(s), for which a cascading effect is observed. Here, we provide a comparison of the stability, under the grim trigger strategy, between an imperfect and a perfect monitoring situation. In addition, we also rate the stability by calculating the percentage of total runs for which a cascading event is observed.

For imperfect monitoring, we perform the evaluation by randomly selecting a threshold value for every node from the pre-determined range (shown in Figure 4.1) for which the strategy is an equilibrium. For every stage we generate a gamma distributed random value, using algorithms proposed in [38] and [39], whose distribution function is parameterized by the joint action of the nodes played in that stage. A node deviates if the generated value is lower than its selected threshold. As a result of this deviation the probability of generating a lower random value in the subsequent stage increases, thereby increasing the chances of further node deviations. We adopt a similar approach in evaluating the stability for a perfect monitoring situation by studying the effect of node deviations on the equilibrium condition derived in Chapter 2. Figure 4.2 shows the average number of stages, for different network sizes, after which all nodes decide to not cooperate for imperfect and perfect monitoring. From the figure we can conclude that for imperfect monitoring the trigger point for the grim trigger strategy is too sensitive to other nodes' deviations. This is because slight node deviations coupled with the random nature of the public signal causes premature triggering, thereby leading to a cascading effect. In addition, as compared to perfect monitoring, the random nature of the public signal leads to a faster cascade. We also observe that the average number of stages until all nodes deviate for a perfect monitoring situation depends on the number of nodes for smaller sized networks before stabilizing to a fixed value. On the other hand, for imperfect monitoring the cascading effect is largely independent of network size, with a slightly faster cascade for larger networks. For large networks a significant number of nodes deviate within the first stage causing a drastic change in the random signal which eventually leads to a faster cascade.

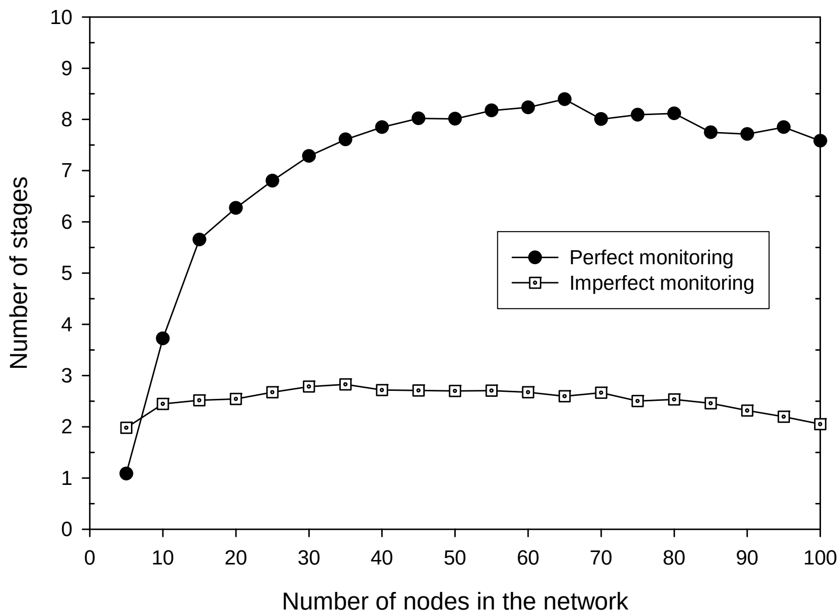

Figure 4.2. Plot of the average number of stages after which cascading is observed for the grim trigger strategy

We observe, from Figure 4.3 , that cascading also occurs for a high percentage of runs for a given network size. The number of nodes has little effect on whether the cascading effect occurs.

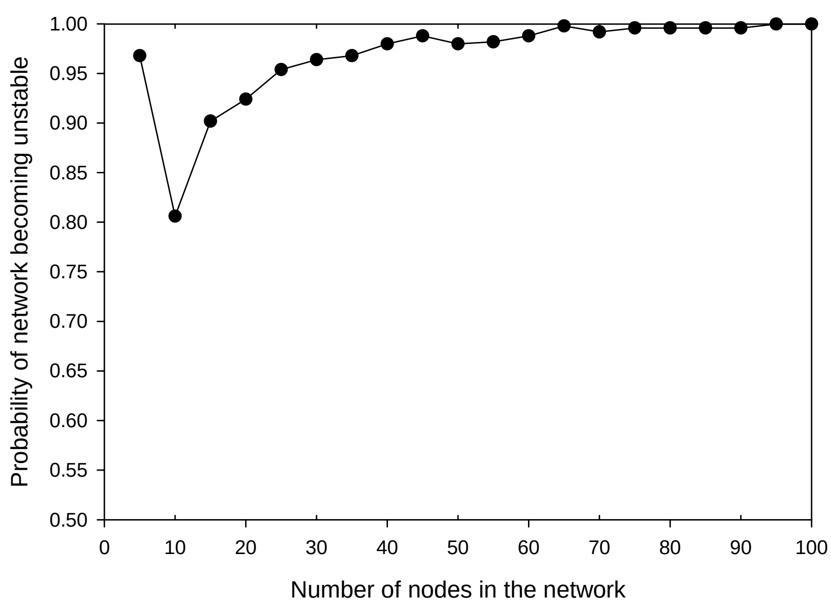

Figure 4.3. Probability that the network becomes unstable, with all nodes not cooperating, for traditional grim trigger strategy

We therefore "soften" the sensitivity of the trigger by considering two modifications to the grim trigger strategy:

a. The trigger for a node is activated only when the n-stage running average of the realized public signal is below the set threshold; and

b. The trigger for a node is activated when the random signal is below the set threshold for each of the n consecutive stages.

For the numerical evaluation of the strategies above, we consider n = 3. Figure 4.4 includes a comparison of the probability of the network becoming unstable, when all nodes switch to not cooperating, for two different variations of the grim trigger strategy. We see that nodes following the modified strategies are less sensitive to the randomness of the public signal, making them less sensitive to changes in other nodes' behaviors, as expected. Therefore, even though the grim trigger strategy is effective in providing a socially optimal equilibrium, it has a drawback with regards to the stability of the network for nodes following that strategy. A small modification in the trigger leads to significantly more stable behavior.

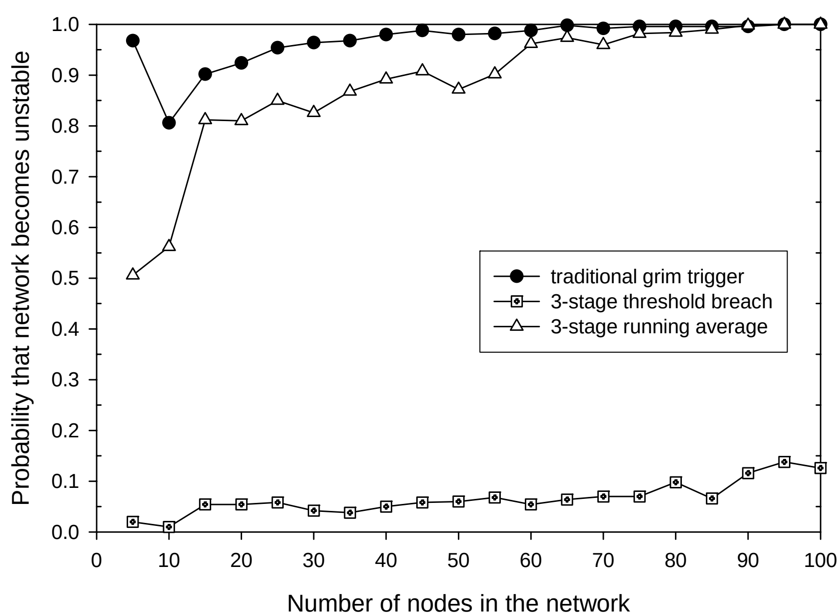

Figure 4.4. Probability that the network becomes unstable for the grim trigger and modified grim trigger strategies

Therefore we can conclude that under the assumption of first order stochastic dominance of the public signal, the grim trigger strategy leads to an equilibrium for nodes to cooperate. For an imperfect public monitoring environment we observe that the trigger point for the grim trigger strategy is overly sensitive to slight changes in other nodes' actions, potentially making the network unstable. By considering a smoother condition for activation of the trigger we can reduce this sensitivity caused by the randomness of the public signal.

There are two main limitations of adopting this approach of imperfect public monitoring to induce cooperation in ad hoc networks:

- The grim-trigger strategy is strict and sensitive to slight variations in the observation of the public signal. This leads the entire network to cease cooperation. We need nodes to adopt a forgiving strategy that accounts for imperfections in monitoring. This lack of robustness is also harmful in the presence of a malicious node: by not cooperating a single "bad apple" can bring the entire network down.
- In an ad hoc environment the choices of a public signal are limited due to the distributed, self-organized nature of the network. It is a practical challenge to find a public signal which ensures that all nodes possess common information that reflects the actions of every node in the network at the end of each stage of the repeated game. Therefore we consider a distributed information structure where each node obtains individual information about the behavior of other nodes in the network. We refer to the distributed information as the *private signal* of the node.

4.2 Repeated games with imperfect private monitoring

The distributed nature of ad hoc networks limits the possibility of observing a common public signal. Each node being an independent entity individually observes the actions of other nodes in the network. This information is private and not shared. However, such observations are error-prone. We need to account for these errors in observation as a result of slow switching between transmit and promiscuous modes of operation, collisions and congestion due to the wireless medium, or incorrect feedback from peers.

We propose a probability-based strategy that provides adequate incentives for cooperation and is robust to errors in observation. We leverage recent developments in game theory—repeated games with imperfect private monitoring—in proving that this strategy induces nodes to cooperate at equilibrium and in assessing the range of observation error for which such cooperation can be sustained. Also, in the absence of observation error we show that nodes adopting this strategy achieve an optimal payoff when fully cooperating.

A cooperation strategy guides the node on how to decide whether or not to forward packets for its neighbors. We propose a probability-based cooperation strategy such that during any interaction a node probabilistically decides to cooperate or not in forwarding packets based on its own past action and the observed action of its neighbor from their previous interaction. We define *observation error* as the inconsistency between the actual and observed actions of a neighbor. The proposed strategy accounts for such observation errors and helps strike a balance between providing incentives for cooperation and issuing threats to a non-cooperating node. This results in nodes adopting a more forgiving approach than trigger-based and tit-for-tat type strategies.

Each node *i*, when asked to forward a packet for its neighbor j, decides whether or not to cooperate according to a set of probabilities described below. These probabilities depend on node i's previous actions and on i's observation of j's previous actions in their past interaction. Each node keeps a memory of the previous interaction with each neighbor (one bit per neighbor is sufficient).

Describing this in a more formal manner, for any current interaction between i and j let a_{ij} denote the action taken by node *i* to cooperate $(a_{ij} = C \equiv 1)$ or not $(a_{ij} = NC \equiv 0)$ when asked by j to forward a packet. Let ω_{ij} denote node *i*'s observation of neighbor *j*'s action (to cooperate $(\omega_{ij} = c \equiv 1)$ or not $(\omega_{ij} = nc \equiv 0)$) when j is asked to forward a packet. We denote the probability with which node *i* cooperates as $p_i^{\bar{a}_{ij},\bar{\omega}_{ij}}$. Here, $\bar{a}_{ij}$ denotes the action that was chosen by node *i* the previous time it was asked to forward by node *j* and $\bar{\omega}_{ij}$ denotes the observation by *i* of neighbor *j*'s actions the previous time node *j* was asked to forward a packet by *i*.

Each node accrues some utility which, like the strategy itself, is a function of the node's own actions and the observed actions. Unlike games with imperfect public monitoring where the utility is a function of the common public signal, the utility in this case is a function of the observed actions. As is the case so far in all our games in this work, we express the utility as the difference between two components: the benefit accrued by node *i* when *j* cooperates in forwarding and the cost *i* incurs by cooperating. We define a *stage* to be any interaction in which node *i* asks *j* to forward and/or is asked to forward by *j*. Then, the per-stage utility received by node *i* is defined as $u_i(a_{ij}, \omega_{ij}) = k_i\omega_{ij} - a_{ij}$ where k_i is the benefit perceived by node *i* due to *j*'s cooperation. In any given stage if node *i* is not asked to forward, then a_{ij} is zero. Similarly, node *j* accrues the same individual *stage* utility. Since nodes interact repeatedly the overall utility is a discounted sum of the individual stage utilities. The overall utility is discounted by a discount factor, δ, $0 < \delta < 1$, and is expressed as:

$$V_i = \sum_{t=0}^{\infty} \delta^t u_i(a_{ij}, \omega_{ij})$$

With the actions and utility now defined, we propose the probability-based strategy adopted by nodes in the presence of observation error, ε, as:

$$\left.\begin{aligned} p_i^{C,c} &= 1 \\ p_i^{C,nc} &= \frac{(2\varepsilon-1)^2-\frac{1}{\delta k_i}}{(2\varepsilon-1)^2-\frac{1}{k_i}} \\ p_i^{NC,c} &= \frac{(1-\delta)}{\delta k_i\left[(2\varepsilon-1)^2-\frac{1}{k_i}\right]} \\ p_i^{NC,nc} &= 0. \end{aligned}\right\} \quad (4.16)$$

Nodes cooperate with a positive probability on observing non-cooperative behavior ($p_i^{C,nc} > 0$), making the strategy forgiving and as we will show later, robust to observation errors. Nodes adopting this strategy achieve a near-optimal payoff when fully cooperating at equilibrium in the presence of an appreciable range of observation error. The probability-based strategy defined here is a result of analyzing node cooperation as a repeated game of imperfect private monitoring. In the next sections, we present the game-theoretic analysis that led to the design of this probabilistic cooperation strategy.

4.2.1 Efficacy of the probability-based strategy

We model the probability-based strategy in the presence of observation errors using repeated games of imperfect private monitoring. In these games each player observes a private signal that reflects the opponent's action. We model the repeated interactions between a node and its neighbor as a 2-node, $\{i, j\}$, game. From a network-wide perspective, each node is involved in one such game with each of its neighbors.

The effectiveness of any behavior-based strategy depends on there being sufficient traffic in the network, so that nodes are able to observe their neighbors' actions when asked to forward packets. In the analytical model of our proposed probabilistic strategy, we assume there is sufficient traffic that enables each node to observe its neighbor's action at every stage. This is consistent with assumptions in [27, 30-33].

Let $\pi(\boldsymbol{\omega}|\boldsymbol{a})$ be the probability that the private signal profile, $\boldsymbol{\omega} = (\omega_{ij}, \omega_{ji})$, is realized given that the joint action profile, $\boldsymbol{a} = (a_{ij}, a_{ji})$, is chosen. For every such action profile we determine the probability distribution of $\boldsymbol{\omega}$. So for example, for $\boldsymbol{a} = (C, C)$ and observation error denoted as ε, we have: $\pi\big((c,c)|(C,C)\big) = (1-\varepsilon)^2$; $\pi\big((c,nc)|(C,C)\big) = (1-\varepsilon)\varepsilon$; $\pi\big((nc,c)|(C,C)\big) = \varepsilon(1-\varepsilon)$; $\pi\big((nc,nc)|(C,C)\big) = \varepsilon^2$. We evaluate node i's single-stage expected payoff as a function of the joint action profile. The single-stage expected payoff is expressed as $f_i(\boldsymbol{a}) =$

$\sum_{\boldsymbol{\omega}} u_i(a_{ij}, \omega_{ij}). \pi(\boldsymbol{\omega}|\boldsymbol{a})$. For instance, for $\boldsymbol{a} = (C, C)$ we have: $f_i((C, C)) = k_i - k_i\varepsilon(1-\varepsilon)^2 - k_i\varepsilon^2 - 1$. We determine the expected payoff for all such $\boldsymbol{a}$ and normalize it to the payoff experienced by nodes when they mutually cooperate. The expected payoffs to both nodes reduce to the values shown in Table 4-1.

Table 4-1. Normalized expected single-stage payoff for nodes *i* and *j*

Node *j* / Node *i*	C	NC
C	$1,1$	$-g, 1+g$
NC	$1+g, -g$	$0,0$

Here, $g > 0$ and $g = \frac{1}{k_i(1-2\varepsilon)-1}$.

Since node *i*'s observation of *j*'s action is independent of *j*'s observation of *i*'s actions, the private signals for the two nodes are independent. We express the marginal distribution of ω_{ij} for every possible action profile as, $\pi_i\left(\omega_{ij}|(a_{ij}, a_{ji})\right)$: $\pi_i(c|(\cdot, C)) = 1-\varepsilon$; $\pi_i(nc|(\cdot, NC)) = 1-\varepsilon$; $\pi_i(nc|(\cdot, C)) = \varepsilon$; $\pi_i(nc|(., NC)) = \varepsilon$. For the sake of simplicity in notation, we will denote the marginal probability that node *i* observes a cooperate signal given that (a_{ij}, a_{ji}) was played as $\pi_i^{a_{ij}a_{ji}}$ and similarly, $\pi_j^{a_{ji}a_{ij}}$ for node *j*.

Note the similarity in the models of repeated games with perfect monitoring and those with imperfect private monitoring. In the absence of observation error, $\varepsilon = 0$, the observed action is the actual action itself with $\omega_{ij} = a_{ij}$.

We briefly introduce the solution approach adopted in repeated games of imperfect monitoring. The Nash equilibrium concept is refined for repeated games with perfect monitoring to define sub-game perfect equilibrium, since it eliminates those equilibria in which the players' threats are not credible. For games with imperfect monitoring a different construct of Nash equilibrium – the sequential equilibrium – is considered. The sequential equilibrium comprises not only of the players' strategy but, to account for the imperfection in monitoring, defines a belief system for every player. The belief system is consistent with a player's continuation strategy and characterizes the beliefs a player has about its opponents' future actions after every history of the repeated game. These beliefs, which are calculated based on knowledge of an opponent's past history of actions, get difficult to compute in the case of imperfect private monitoring, as each opponent's history is private information [10].

In our model for imperfect private monitoring, we derive a mixed-strategy equilibrium for our probability-based strategy. For any mixed-strategy equilibrium to exist a node must receive the same payoff irrespective of what pure-strategy action it chooses [8]. This leads each node to randomize between its pure-strategy actions during any stage of the game. Since a node receives the same payoff from choosing any one of its pure strategies, a node's beliefs about its opponent's future actions do not come into play and hence at any stage, irrespective of the beliefs, whatever continuation strategy a node chooses will be a best reply [10]. This eases the computation of the equilibrium as compared to evaluating beliefs at every stage – a technique adopted in [31].

At any stage of the game, we express the repeated game payoff to a node as a sum of the expected single-stage payoff obtained in the first stage and the continuation payoff for the remaining stages, which in turn depends on the joint action adopted by nodes in the first stage. Let V_i^C be the average discounted repeated game payoff to node *i* when *j* chooses to cooperate and V_i^{NC} be the average discounted repeated game payoff when *j* chooses to not cooperate. The average discounted repeated game payoff to node *i* when both nodes cooperate in the current stage is given by: –

$$V_i^C = (1-\delta) + \delta V_i^C\left[\pi_j^{CC} p_j^{C,c} + \left(1-\pi_j^{CC}\right) p_j^{C,nc}\right] + \delta V_i^{NC}\left[\left(1-\pi_j^{CC}\right)\left(1-p_j^{C,nc}\right) + \pi_j^{CC}\left(1-p_j^{C,c}\right)\right] \quad (4.17)$$

An equilibrium in mixed strategies is sustainable when a node receives the same payoff at the current stage irrespective of choosing to cooperate or not. The average discounted repeated game payoff to node $\underline{i}$ when it did not cooperate in the current stage but node *j* did is equated to V_i^C and can be expressed as:

$$V_i^C = (1-\delta)(1+g) + \delta V_i \ \left[\pi_j^{CNC} p_j^{C,c} + \left(1-\pi_j^{CNC}\right) p_j^{C,nc}\right] + \ \delta V_i^{NC}\left[\left(1-\pi_j^{CNC}\right)\left(1-p_j^{C,nc}\right) + \pi_j^{CNC}\left(1-p_j^{C,c}\right)\right] \quad (4.18)$$

On a similar note, the average discounted repeated game payoff to node *i* when it cooperates in the current stage but *j* did not, is given by: –

$$V_i^{NC} = -g(1-\delta) + \delta V_i^C\left[\pi_j^{NCC} p_j^{NC,c} + \left(1-\pi_j^{NCC}\right) p_j^{NC,nc}\right] + \ \delta V_i \ \left[\left(1-\pi_j^{NCC}\right)\left(1-p_j^{NC,nc}\right) + \pi_j^{NCC}\left(1-p_j^{NC,c}\right)\right] \quad (4.19)$$

The average discounted repeated game payoff to node *i* when both did not cooperate in the current stage is expressed as: –

$$V_i^{NC} = \delta V_i^C\left[\pi_j^{NCNC} p_j^{NC,c} + \left(1-\pi_j^{NCNC}\right) p_j^{NC,nc}\right] + \ \delta V_i^{NC}\left[\left(1-\pi_j^{NCNC}\right)\left(1-p_j^{NC,nc}\right) + \pi_j^{NCNC}\left(1-p_j^{NC,c}\right)\right] \quad (4.20)$$

These four expressions for repeated game payoff are now solved to evaluate $p_i^{\bar{a}_{ij},\bar{\omega}_{ij}}$.

4.2.1.1 Analysis of probability-based strategy for perfect monitoring

We begin by considering the special case of perfect monitoring. In the absence of observation errors, $\varepsilon = 0$ and $\omega_{ij} = a_{ij}$. Therefore, equations (4.17) – (4.20) reduce to:

$$V_i^C = (1-\delta) + \delta[V_i^C p_j^{C,C} + \ V_i^{NC}\left(1-p_j^{C,C}\right)] \quad (4.21)$$

$$V_i^C = (1-\delta)(1+g) + \delta[V_i^C p_j^{C,NC} + V_i^{NC}(1-p_j^{C,NC})] \quad (4.22)$$

$$V_i^{NC} = -g(1-\delta) + \delta\left[V_i^C p_j^{NC,C} + \ V_i^{NC}\left(1-p_j^{NC,C}\right)\right] \quad (4.23)$$

$$V_i^{NC} = \delta[V_i^C p_j^{NC,NC} + V_i^{NC}(1-p_j^{NC,NC})] \quad (4.24)$$

Note that, $g > 0$ and $g = \frac{1}{k_i - 1}$. We claim that:

Proposition 1: If $k_i > \left(1/_{\delta}\right)$ *the symmetric cooperation strategy profile given by:*

$$p_i^{C,C} = 1; p_i^{C,NC} = \frac{\left(\mathrm{k_i} - \frac{1}{\delta}\right)}{(\mathrm{k_i}-1)}; p_i^{NC,C} = \frac{(1-\delta)}{\delta(\mathrm{k_i}-1)}; p_i^{NC,NC} = 0$$

is an equilibrium and results in nodes i and j achieving the optimal repeated game payoff when fully cooperating.

We find a mixed strategy for node *j* that satisfies equations (4.21) – (4.24). By the symmetric argument we find a strategy for node *i* that satisfies analogous equations for node *j*. Since these equations imply that each node should receive the same payoff when playing *C* or *NC*, each node's mixed strategy is a mutual best response at any stage of the game and it constitutes an equilibrium. Nodes adopting these strategies achieve an optimal payoff of $V_i^C = 1$ and the probabilities are defined as long as $k_i > \left(1/_{\delta}\right)$.

We can pictorially represent the probability strategy for perfect monitoring in a state diagram, with each state representing the actions taken by the two nodes at a given stage. The cooperation strategy leads to the transition probabilities shown in Figure 4.5.

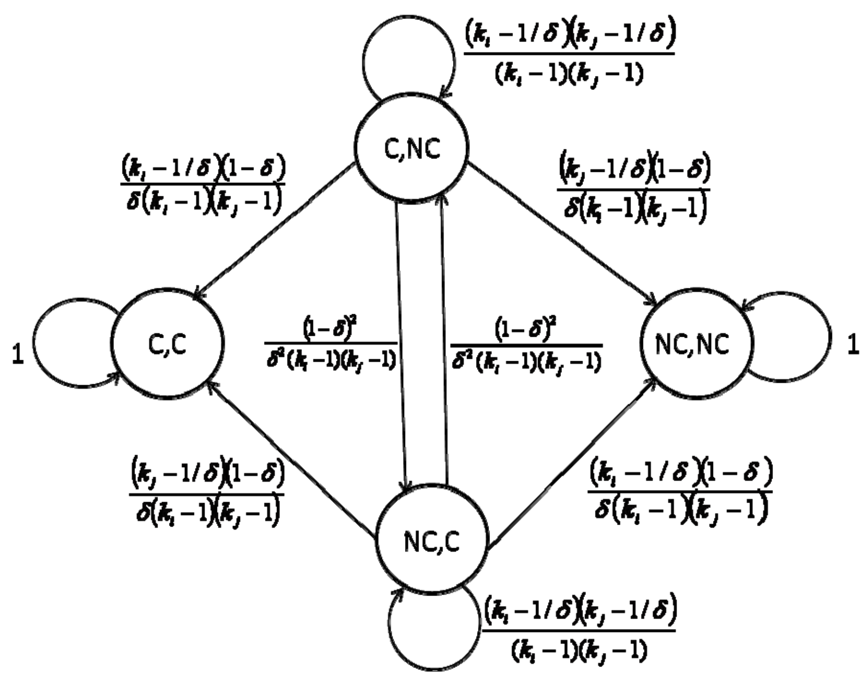

Figure 4.5. State diagram to illustrate the cooperation strategy.

States (C, C) and (NC, NC) are absorbing, so when selfish nodes begin their interactions by initially forwarding for each other, they are incentivized to continue cooperation. Nodes may find themselves in states (C, NC) or (NC, C) under two possible scenarios: (a) if one of the two nodes decides to not cooperate initially, or (b) if a node deviates in its action from the equilibrium path. In both cases, unlike trigger-based or tit-for-tat strategies, the cooperating node continues to cooperate with some positive probability, $p_i^{C,NC} = \frac{\left(\mathrm{k_i} - \frac{1}{\delta}\right)}{(\mathrm{k_i} - 1)}$. This positive probability ensures that there exists a potential transition back to state (C, C) or (NC, NC). The transition to (C, C) is needed to reward the cooperating node while the other node is not cooperating and the potential transition to (NC, NC) is needed as a threat to the non-cooperating node such that is incentivized to switch to cooperation. This inherent forgiving characteristic makes the strategy well-suited to imperfections in monitoring errors as discussed below.

4.2.1.2 Analysis of the probability-based strategy for imperfect monitoring

Let us now turn our attention to the more general case of imperfect monitoring where $\varepsilon \neq 0$. In the presence of observation errors, we claim that:

Proposition 2: For $0 \leq \varepsilon < \frac{1}{2} - \frac{1}{2\sqrt{k_i}}$*, the symmetric cooperation strategy presented in (4.16) is an equilibrium with nodes i and j achieving a near-optimal payoff.*

We adopt the same procedure followed in the previous section to determine the probability for which equations (4.17) – (4.20) are satisfied. The probabilities exist as long as the error is limited to $(\frac{1}{2} - \frac{1}{2\sqrt{k_i}})$. For a node adopting this strategy, the expected repeated game payoff for cooperating equals $V_i^C = 1 - \frac{g\varepsilon}{(1-2\varepsilon)}$ with $g = \frac{1}{k_i(1-2\varepsilon)-1}$.

The variation of the cooperation probability for a node with observation error is shown in Figure 4.6. We observe that as the error at any given stage increases, the probability that the node cooperates given that it cooperated (or *did not cooperate*) and observed its opponent not cooperate (or *cooperate*) in the prior stage, decreases (or *increases*). This seemingly non-intuitive behavior can be attributed to a node employing a “correction mechanism” on noticing an increase in the observation error at any given stage. This prompts the node to reverse its action employed in the prior stage. In addition, $p_i^{C,c} = 1$ implies that a node continues to cooperate as long as it observes (with the error in observation falling within the allowable error limits defined above) that its opponent cooperated in the previous stage. A similar rationale holds true for non-cooperation as well. This is consistent with behavior as seen in the absence of observation error and also with behavior seen in nodes when a network may not comprise of any selfish nodes.

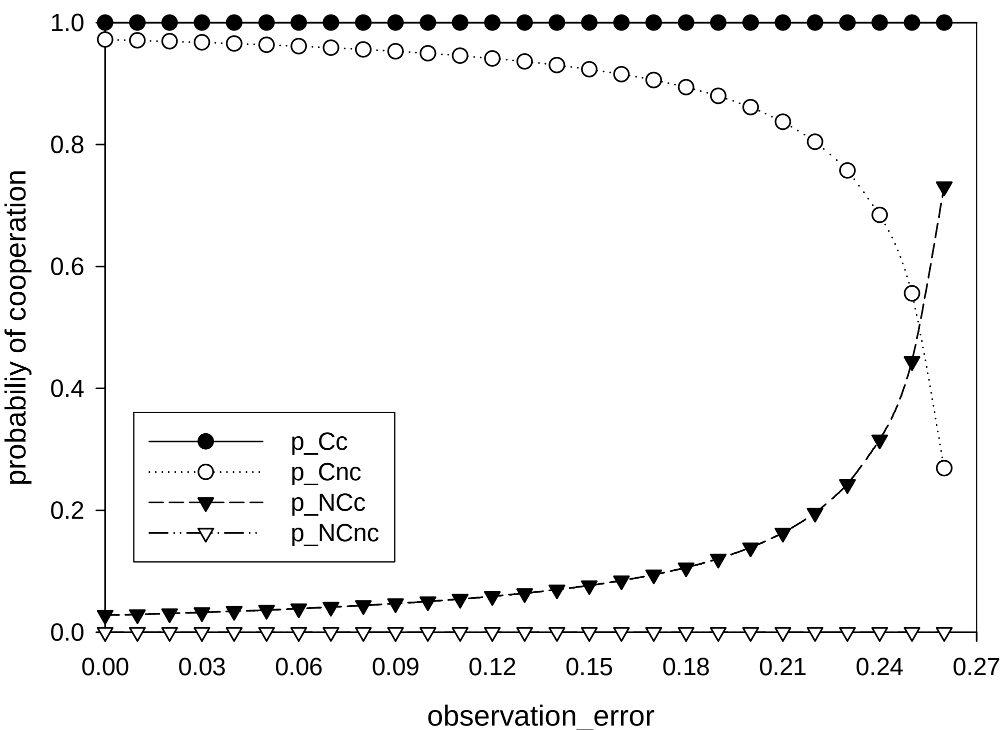

Figure 4.6. Variation of the probability-based strategy and the cooperation payoff with different values of observation error. The plots are for $k_i = 5.0$ and $\delta = 0.9$.

4.2.2 Robustness of probability-based strategy to observation errors

We evaluate the robustness of the probability-based strategy by tracking the variation in a node's probability of cooperation in the presence of observation error as a game progresses, and compare it with that for a node adopting the grim-trigger strategy. Let A_i^k be the event that node *i* cooperates in stage k. Let B_i^k be the event that node *i* perceives that node *j* cooperated in stage *k*. The probability that node *i* cooperates in stage *k* can be expressed as:

$P[A_i^k] = p_i^{C,c} \cdot P[A_i^{k-1} \cdot B_i^{k-1}] + p_i^{NC,c} \cdot P[\bar{A}_i^{k-1} \cdot B_i^{k-1}] + p_i^{C,nc} \cdot P[A_i^{k-1} \cdot \bar{B}_i^{k-1}] + p_i^{NC,nc} \cdot P[\bar{A}_i^{k-1} \cdot \bar{B}_i^{k-1}]$.

Note that for the first stage ($k = 0$) we have $P[B_i^0] = (1 - \varepsilon)$ and $P[A_i^0] = 1$. Also, for the probability-based strategy, at equilibrium, $p_i^{C,c} = 1$, $p_i^{NC,nc} = 0$. On the other hand, recall that for the grim-trigger strategy, a node cooperates as long as it observes its opponent cooperate. Therefore, $p_i^{C,nc} = 0$. Also, it is evident that if a node is not cooperating then it will continue to do so. Hence, $p_i^{NC,c} = p_i^{NC,nc} = 0$. Therefore, the probability of cooperation in stage k for a node adopting the grim-trigger strategy reduces to: $P[A_i^k] = P[A_i^{k-1} \cdot B_i^{k-1}]$.

Figure 4.7 plots the variation in the probability of cooperation in stage k as the game progresses for different error values for nodes adopting these two different strategies.

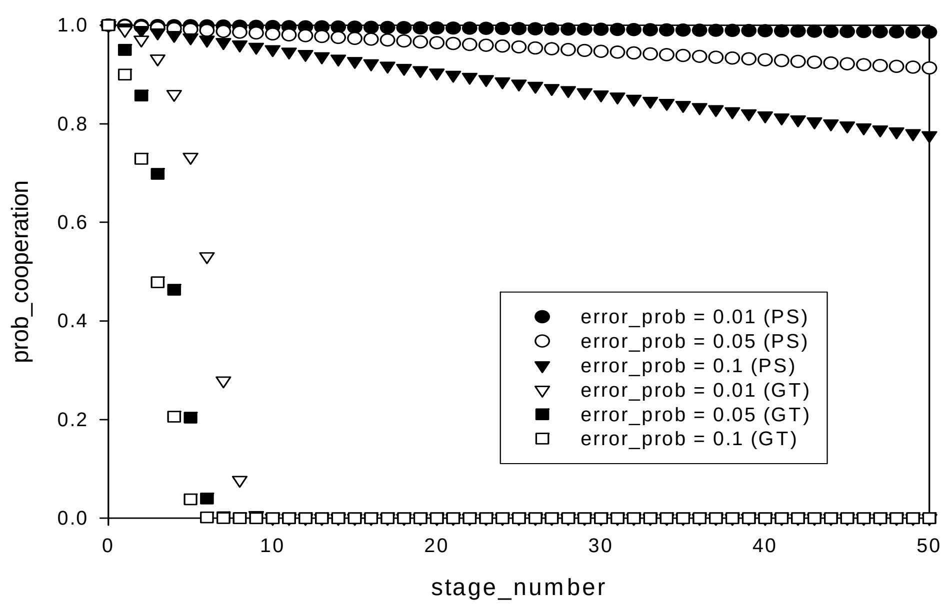

Figure 4.7. Probability of cooperation at any given stage for a node adopting probability-based (PS) or grim-trigger strategy (GT)

We notice that a node adopting the probability-based strategy changes its cooperation level gradually and maintains a significant probability of cooperation longer into the game. This high level of sustained cooperation as the game progresses lends the system a significant level of robustness to observation errors. The slow decay of the cooperation probability helps to reduce the chances of a misbehaving node to trigger a denial-of-service attack in a reputation management scheme. Furthermore, the continued cooperation of a node in spite of perceiving its opponent to not cooperate fully can be interpreted as a node providing its selfish opponent an incentive to improve upon its cooperation level.

For the tit-for-tat strategy a node cooperates as long as it observes its opponent cooperate. So we have $p_i^{C,c} = 1$ and $p_i^{NC,c} = 1$, with $p_i^{C,nc} = p_i^{NC,nc} = 0$.

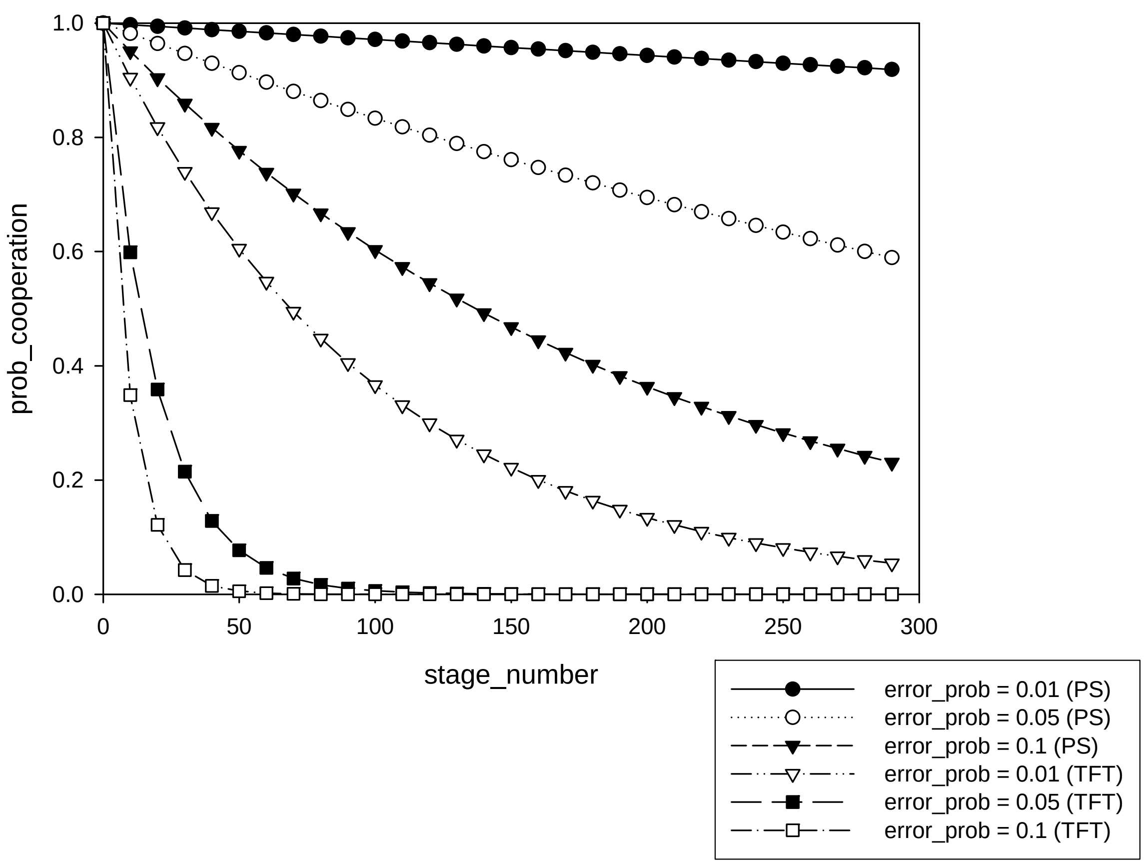

Figure 4.8. Probability of cooperation at any given stage for a node adopting probability-based (PS) or tit-for-tat strategy (TFT)

Figure 4.8 compares the variation in probability of cooperation values for the probability-based strategy with the tit-for-tat strategy. We observe that while the tit-for-tat strategy shows better robustness than grim-trigger, the probability-based strategy still outperforms it.

4.3 Chapter summary

In this chapter we consider the practical difficulties in observing other nodes' actions and relax the assumption of perfect monitoring from our repeated games model. We show that the grim trigger strategy is effective in inducing cooperation under imperfect observation with nodes observing a common signal that reflects the actions of its opponents. We also show that the grim trigger strategy has limitations due to its harsh nature. It is overly sensitive to perceived node deviations and can lead the entire network to instability. We therefore propose a probability-based strategy that is individualistic in implementation (private) and forgiving in nature. We prove the efficacy of the strategy in inducing cooperation where each node's observation of its opponents action is private and imperfect. We also show the probability-based strategy to be robust to sudden behavioral changes when compared to grim trigger and tit-for-tat strategies.

Chapter 5. APPLICATION OF BEHAVIOR-BASED INCENTIVES: TOPOLOGY CONTROL

In this chapter we consider the application of behavior-based incentives to topology control in ad hoc networks. We study the problem of selfish node behavior and energy efficiency in topology control. Specifically, we characterize the trade-off between global objectives of the network (e.g. connectivity, cooperation among nodes to forward packets for one another) and self-motivated objectives of nodes (e.g. minimum energy consumption, reliable packet delivery). We show that a simple behavior-based incentive is effective in inducing network-wide cooperation. Furthermore, our energy-aware topologies achieve significant gains in energy efficiency, as compared to other published topology control solutions, with high degree of connectivity[3].

5.1 Introduction

Topology control in an ad hoc network is the study of how nodes adjust their transmit power to achieve an optimal balance between connectivity and energy consumption goals. As nodes decrease their transmit power, the network gets more sparsely connected, and the energy savings in the transmission of a single frame may be offset by energy expenditures in forwarding due to longer end-to-end paths. There is also the possibility that some nodes will refuse to cooperate in packet forwarding to conserve their limited resources (such as battery lifetime) and only compete for the medium when transmitting their own packets.

These two considerations – *total energy consumption*, including energy consumed in both forwarding and sourcing packets and *effects of selfish behavior* – are missing from most of the literature on topology control and are explicitly addressed in our study. In such cases the conventional topology control objective of connectivity is no longer sufficient. The objective now is to induce nodes to cooperate and form an energy-efficient network that exhibits good connectivity.

In our model, nodes make independent decisions regarding their transmit power and willingness to cooperate by taking into account energy spent in sourcing as well as in forwarding

[3] The work in this chapter was performed in collaboration with Dr. Ramakant S. Komali and is partially reported in [40]. The idea of redefining topology control with selfish nodes was proposed by me. We worked jointly in defining the system model and characterizing the utility function. We split the simulation work with my contribution being in the analysis of the cooperation efficacy of our proposed scheme, along with charactering the topologies that emerge.

packets for other nodes, in addition to the benefits perceived in achieving a reliable and well-connected topology.

We show that, when selfish nodes take this holistic view of energy consumption, the topologies that emerge achieve significant gains in energy efficiency while still maintaining power efficiency comparable to that observed in power-aware topology control work reported in the literature [40], [41]. We also show that a simple distributed forwarding policy adopted by self-interested nodes is successful in inducing cooperation, resulting in highly connected topologies with reliable paths. Nodes adopting this policy are induced to cooperate at a level equal to that of the most cooperative node in the network. Such conditions mean that a small percentage of nodes that always cooperate will induce complete cooperation among all others. In this work we also observe similarities in topological attributes such as path length and clustering coefficient, to those found in small world networks. These form our main contributions.

5.2 Related work

Our work belongs to the body of research that analyzes the impact of selfish node behavior on network performance. As seen in Chapter 3, even when nodes are self-interested, some amount of cooperation is required to sustain a completely autonomous ad hoc network. One important problem is how to stimulate the nodes to cooperate, when they are driven by self-interest. The need for appropriate incentives for cooperation can be observed at multiple layers in the protocol stack [7]. We consider cooperation in topology control at the network layer. Other facets of cooperation, such as cooperative communications at the physical layer [42], are outside the scope of this chapter.

Research efforts to address the problem of topology control in the presence of selfish nodes are fairly recent and a widely adopted approach is the use of game theory and mechanism design. The work in [43] is among the first to pose the topology control problem for ad hoc networks as a non-cooperative game. The authors derive bounds on the cost of anarchy: the performance that results from the Nash equilibrium compared to the social optimum. Much of the work is devoted to algorithmic aspects and complexity analysis of finding the Nash equilibrium, when it exists. In [44], the authors formulate topology control games as potential games. The potential game formulation guarantees the existence of at least one Nash equilibrium. Moreover, if the nodes employ a best response algorithm, convergence to one of these equilibria is guaranteed.

In both [43] and [44] nodes are modeled as selfish in conserving energy solely by varying their transmit power levels. Once power levels are chosen, nodes are assumed to fully cooperate in forwarding packets. This is significantly different from our characterization of selfish behavior, as we allow nodes to aggressively conserve energy not only by varying their power level but also by selecting an appropriate level of cooperation when forwarding for others.

Mechanism design seeks to achieve global efficiency by aligning the selfish objectives of individual users with the socially desirable outcome. In the context of topology control, mechanism design is employed to provide the appropriate incentives to individual users so that they maximize their objective functions when the network minimizes total energy consumption, subject to connectivity constraints. This approach has been adopted in [45] and [46] by engineering a payment system that leads selfish nodes to forward packets for others. The utility function proposed in [45] requires that each node declare the per-edge price that it intends to charge in exchange for forwarding packets. Mechanism design also incentivizes nodes to truthfully report their costs incurred in establishing links with their neighbors or in forwarding packets. This approach has been adopted in [47] in the design of a truthful topology control algorithm.

The approach of assigning prices on a per-link basis, as suggested in [45] and [47], does not account for the wireless broadcast advantage. The cost to a node in maintaining links with all its one hop neighbors is a function, among other factors, of the link length; typically the neighbor that is the farthest dictates the cost that the node incurs in maintaining that neighborhood. The assignment of per-link prices may reasonably reflect costs in a wired network, or in the case of per-packet power control, but it does not capture the costs involved in the establishment of individual wireless links, due to their broadcast nature. In our model we evaluate costs as a function of the transmit power, and do not assume any link-based charges.

In addition, charge-based incentive mechanisms such as the ones studied by [45] and [46] impose considerable overhead (and may require additional hardware modules) to keep track of payments to individual nodes. The incentive policy we consider does not require the intervention of a third party arbitrator to keep track of payment exchanges and to validate transactions. Nevertheless, like any other non-pricing based reputation schemes (e.g. see [7] and the references therein), the assessment of cooperation levels introduces some overhead as nodes need to maintain and periodically update information about other nodes' behavior.

5.3 System model

5.3.1 Selfishness and path reliability

As in most research on topology control for ad hoc networks, we assume that each node $i \in V \equiv \{1,2,\cdots,n\}$ can autonomously set its transmit power $p_i \in [0, p_{i,max}]$. These individual settings can be collected into a power vector $p = (p_1, p_2, \cdots p_n)$, which induces a certain network topology $g(p)$. If every node i transmits at $p_{i,max}$, we call the induced topology g_{max} the maximum power graph. The topology is a graph consisting of vertices in V and edges that represent realizable links between pairs of vertices. A bi-directional link between nodes $i, j \in V$ can be realized if the power setting at node i is sufficient to meet the signal to noise ratio requirements at node j in the absence of any interferer, and vice-versa. When a bi-directional link exists between two nodes, the nodes are said to be neighbors. The neighborhood of any node i is formally defined as: $N_i = \{j | p_i \geq p_{ij} \wedge p_j \geq p_{ji}\}$ where p_{ij} is the minimum power required to establish a link from i to j. A topology $g(p)$ is connected if there exists a path - a collection of realizable links - between every node pair $i, j \in V$. Note that we ignore unidirectional links between nodes, as wireless medium access protocols typically require a bi-directional link for handshakes between communicating parties (such as link layer acknowledgements).

For successful multi-hop operation the underlying topology must not only be connected but, ideally, there should be a path between any two nodes $i, j \in V$ with every intermediate node k in the path willing to forward packets for others. Here, our study differs significantly from most other work on topology control, as in addition to controlling its transmit power each node can autonomously decide its level of cooperation with others in the forwarding of packets. A node can decide to cooperate intermittently, forwarding packets for others with some probability $q_i \in [0,1]$. The cooperation level vector is denoted by $q = (q_1, q_2, \cdots, q_n)$.

In the absence of appropriate incentives, selfish nodes are expected to refrain from cooperation, setting $q_k = 0$ (see [48] for a more formal analysis). Therefore, incentives are needed to encourage selfish nodes to increase their cooperation level, improving the reliability of available paths between any source/destination pair.

We consider nodes to adopt a forwarding policy whose goals include mutual cooperation and isolation of free-riders. *Each node agrees to forward traffic only for those nodes whose cooperation level is greater than or equal to its own*. This policy is dependent on evaluating the behavior of other nodes in the network by assessing their cooperation levels. The forwarding policy provides an implicit incentive that stimulates nodes to increase their cooperation level, since packets transmitted by any node with a zero cooperation level will not be forwarded. In later sections, we show that this policy leads to the creation of a topology with reliable paths and nodes cooperating at a level equal to the most cooperative node in the network.

We assume, for the purposes of this work, that a node can perfectly assess the current cooperation level of others. However, it is feasible to apply repeated games with imperfect private monitoring under conditions when nodes cannot perfectly assess the cooperation and power level of other nodes in the network. In other words, the models developed in Chapter 4 can be extended to look at the effect of observation error on cooperation-aware topology control; we leave this for future work.

Let us now describe our characterization of path reliability. Consider a path P_{ij} between nodes i and j. If i and j are neighbors, the path reduces to a single link, to which we assign reliability of 1. If the two nodes are not neighbors, the reliability $\Phi_{P_{ij}}$ of a path P_{ij} is defined as:

$$\Phi_{P_{ij}} = \prod_{k \epsilon P_{ij}, k \neq i,j} q_k \qquad (5.1)$$

A node cooperating at a given power level faces a trade-off between the benefit it accrues from a connected topology with reliable paths and the cost it incurs in the energy consumed to form and support such a topology. For every node i, we propose a *utility function* that captures these trade-offs. The utility function maps the joint power and cooperation levels to a payoff value for each node i. In general, we can express the utility as:

$$u_i(p,q) = \alpha_i \lambda_i(p,q) - (1 - \alpha_i)\mu_i(p,q) \qquad (5.2)$$

The term $\lambda_i(p,q)$ represents the benefit derived by node i and is a function of its connectivity to other network nodes and the reliability of paths connecting it to each possible destination. The term $\mu_i(p,q)$ is the cost incurred by node i and is a function of the energy it consumes in sourcing its own packets as well as in forwarding packets for other nodes at a given power level. We weigh the two terms in the utility function by a scalar $0 < \alpha_i < 1$, which indicates the relative importance that nodes attach to the benefit and cost components. The specific utility function that we adopt is discussed in the following subsection.

5.3.2 Utility function

We consider a utility function, illustrated by an example, that characterizes the costs of cooperation and the benefits of a reliable, connected topology. We begin by describing the benefit component of the function.

5.3.2.1 Benefit function

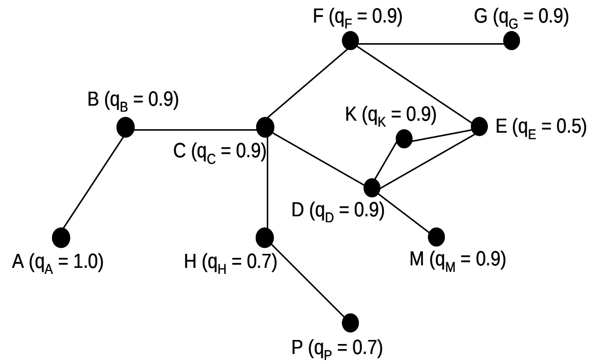

Figure 5.1. An 11-node topology with cooperation level depicted as q_{node_id} next to each node. All links are bi-directional.

The benefit function formulation can perhaps be best explained by an example. Consider node B as a source node in Figure 5.1. For destinations A and C that are in B's neighborhood, the reliability of the links B-A and B-C equals 1. In other words, at its chosen power level, B need not rely on any other node to deliver its packets to A and C. For destinations that are outside B's radio range, such as node E, B must rely on intermediate nodes to forward its packets. In such a scenario node B will choose a path to E with maximum reliability among all available paths.

Based on the forwarding policy described in the previous subsection, a path from B to E is *feasible* only if it passes through nodes whose cooperation level is less than or equal to q_B. In the figure, there exist three such feasible paths: B-C-D-E, B-C-F-E and B-C-D-K-E; the path reliabilities are 0.81, 0.81, and 0.729 respectively. Therefore, to maximize the proportion of packets reaching E, B should select the path with reliability 0.81. We have two such paths and we break ties by selecting the path with the lowest hop count. Since both paths B-C-D-E and B-C-F-E have equal hop count, we choose one arbitrarily. We define such a shortest path with maximum path reliability to be an *optimal* path. In a similar manner, we calculate the optimal path reliabilities for paths from B to remaining destination nodes. Intuitively, the sum of optimal path reliabilities from B to each of its potential destinations is an indication of how much benefit B derives from the network. (As traffic patterns are typically not known at topology formation time, each node implicitly assumes all destinations to be equally likely.)

Extending our example to a general setting, let F_{ij} denote the set of feasible paths from node i to node j and P_{ij}^{opt} denote *an optimal* path from i to j. The reliability of the optimal path P_{ij}^{opt} is given by:

$$r_{ij}(p,q) = \begin{cases} 1 & if\ j \in N_i \\ \max_{P_{ij} \in F_{ij}} \Phi_{P_{ij}} & if\ j \notin N_i \end{cases} \tag{5.3}$$

The total benefit derived by node i is the sum of the reliabilities of the optimal paths to each possible destination. Since the sum scales as $O(n)$, the benefit is normalized to lie in the range [0,1), making it independent of network size n, and is given by:

$$\lambda_i(p,q) = \frac{\sum_{j \in V, i \neq j} r_{ij}(p,q)}{n} \tag{5.4}$$

5.3.2.2 Cost function

The benefit to a node is characterized in terms of reliability of paths to all potential destinations in the network. To realize this benefit, nodes experience a cost in the form of energy spent forwarding packets for other nodes in order to take advantage of their relay services. Thus, a node's cost function takes into account energy spent to: (a) source the node's own packets, and (b) forward packets for others.

Again, we do not assume a priori knowledge of traffic pattern, and, since nodes are unable to assess exact traffic demand, they act on the premise that all destinations are equally likely. Hence, the expressions for (a) and (b) can be obtained in terms of node connectivity. This is explained in more detail in the context of the example in Figure 5.1.

Recall that, in the example, node B transmits at a power level p_B and cooperates with a level $q_B = 0.9$. The total energy spent by node B in sourcing its packets to each of its 10 potential destinations is proportional to $10p_B$. Node B also expends energy in forwarding traffic: based on the forwarding policy described earlier, B forwards packets sourced by A and those destined to A from all nodes except E, H and P. There are 15 such potential flows – 9 originating from A and 6 destined to A – that go through B. Of these, all packets originating from A and C reach B, whereas only a fraction of those originating from other sources reach B. (The remaining packets are dropped en route by other nodes.) For instance, D forwards only 90% of packets sourced by M destined for A, out of which C forwards 90% to B. Therefore, only 81% of packets sourced by M reach B. Again, since B further forwards 90% of the fraction it receives, the energy consumed by B in forwarding M's packets to A is proportional to the fraction of packets it receives and its own cooperation level, i.e. it is proportional to $(0.9 \times 0.9) \times 0.9p_B = 0.729p_B$. Extending this to all flows along feasible paths to and from A, the total energy consumed by B in forwarding packets for others is found to be proportional to $12.807p_B$. This, combined with the energy consumed by B when acting as a source, gives us the total cost incurred by B as proportional to $22.807p_B$.

We now generalize this example. When acting as a source, node i transmits packets for c_i destinations – the number of nodes for which there exists a feasible path with positive reliability from i; the energy consumed by node i in transmitting its own packets is then proportional to $c_i p_i$. When acting as an intermediate node on any optimal path P_{jk}^{opt} between distinct nodes j and k, unless i is the next hop from j or the path reliability is 1, some portion of the traffic from j is dropped before it reaches i. Moreover, different optimal paths drop packets with different probabilities, according to the cooperation level of other intermediate nodes on those paths. Taking such varied path reliabilities into account, let m_i denote the total fraction of traffic received by node i along all such optimal paths. As node i further forwards only a q_i fraction of the traffic it receives, the energy consumed in forwarding is proportional to $m_i p_i q_i$. The overall cost is a function of the total energy spent, hence it is proportional to $c_i p_i + m_i p_i q_i$. Since this cost scales as $O(n^2)$, we normalize the cost to lie within the same range [0,1) as the benefit. We express the cost function as:

$$\mu_i(p,q) = \frac{c_i p_i + m_i p_i q_i}{n^2 \cdot p_{i.max}} \qquad (5.5)$$

5.4 Results

The utility function reflects the benefits of a connected topology with reliable paths and characterizes the costs to form and support it. In this section we characterize the steady-state topology resulting from nodes' maximizing their utility. We evaluate the topology in terms of its energy efficiency, path reliability and connectivity properties.

5.4.1 Simulation set-up

We consider a 50-node network with nodes randomly positioned according to a uniform distribution in a fixed area ([-1,1] x [-1,1]). We initialize each node i with a maximum power level, $p_{i,max}$, such that the initial topology (maximum power graph) is connected. Each node is initialized with the same maximum power value necessary to achieve 1-connectivity with 95% probability [49]. In addition, each node is also initialized with a cooperation level, q_i, selected at random according to a uniform distribution in the interval [0,1]. We carry out 400 runs to ensure the steady state results are statistically significant and, when applicable, we plot the 95% confidence interval for the results reported below. For each run a different random initial topology is generated.

In each run, a node initially calculates its utility, $u_i(p_{max}, q)$ where p_{max} denotes the maximum power vector. We maintain the same benefit factor α for all nodes. The simulation adopts an iterative process for nodes to update their power and cooperation levels. Within each iteration, nodes, in a round-robin manner, select an appropriate power level p_i' and cooperation level q_i' that will maximize their utility function. In other words, each node follows a *best response dynamic*, given the settings of other nodes in the network. Mathematically, p_i' and q_i' are the best response transmit power and cooperation levels if:

$$(p_i', q_i') = arg\max_{(p_i,q_i)} u_i(p_i, p_{-i}, q_i, q_{-i}) \quad (5.6)$$

Here, p_{-i} and q_{-i} refer to the joint power and cooperation level profiles, respectively, of all nodes except i. Only one node adapts its settings at a time and other nodes are made aware of this adaptation. (We assume the presence of a higher layer mechanism that allows nodes to inform one another of their new settings through the exchange of control messages.)

In this manner, at each iteration, nodes update their settings and re-evaluate their utilities. The best response dynamic continues until none of the nodes improves on its utility by modifying its settings. At this point, the process is said to have converged to a steady state.

It is important to note that the best response dynamic is not guaranteed to lead to convergence. In some cases, we observe that a subset of nodes cycle in their power and cooperation settings. On detecting a cycle, nodes re-initialize their cooperation levels and continue the best response dynamic until they converge.

The best response dynamic that we adopt is simple but can lead to update cycles. While there is no common approach proven in the literature that guarantees convergence to a Nash equilibrium for distributed networks, there are specific robust approaches such as those suggested in [50, 51] that take into account the knowledge of players' past actions and histories. These are an improvement over the best response dynamic and, provided certain requirements are satisfied, have been shown to lead to a Nash equilibrium.

We begin the characterization of the topology by presenting the energy and power efficiency performance.

5.4.2 Energy and power efficiency

We evaluate the energy and power efficiency of the topology at steady-state by comparing it with power-aware topology algorithms minimum spanning tree (MST) [40] and k-closest neighbors (kCN) [41], and when no topology control is applied. Note that for MST and kCN algorithms the energy consumption is calculated by assuming nodes to be fully cooperative. In the scenario when no topology control is applied, nodes transmit at a sufficiently high power level necessary to ensure connectivity. (This corresponds to the initial transmit power setting, $p_{i,max}$, in our simulation.) Also, since the network may not always converge to a connected state, for comparison purposes we average across those runs for which the network is connected.

Table 5-1 presents the ratio of the average power (and average energy) consumed by each node in various power-aware topology control algorithms including the minimum spanning tree (MST) and k-closest neighbors (kCN) algorithms, as well as with no topology control, to the average power (and average energy) consumed by each node that follows the cooperation-aware topology control mechanism considered in this work.

We notice that nodes adopting the MST algorithm exhibit better power-efficiency than those adopting the cooperation-aware topology control. This is expected, since the MST algorithm constructs a minimum power topology by minimizing the aggregate power consumed. However, minimum power topologies are generally sparse and result in long paths. This creates heavy loads on individual links, and therefore, increases the forwarding cost and total energy consumed by nodes. This is reflected in Table 5-1. MST is at least 19% less energy-efficient than the cooperation- and energy-aware mechanism.

Table 5-1. Normalized average power/energy consumed by each node for MST and kCN algorithms, as well as with no topology control

Normalized Energy Consumption				
Benefit factor	*no TC*	*kCN-5*	*kCN-9*	*MST*
0.9	1.493	1.208	1.302	1.187
0.7	1.606	1.274	1.378	1.268
Normalized Power Consumption				
0.9	5.006	1.920	3.731	0.777
0.7	2.939	1.122	2.178	0.455

On the other hand, the kCN algorithm has an objective of constructing a connected topology such that every node has k neighbors. Both kCN-5 (i.e. $k = 5$) and kCN-9 (i.e. $k = 9$) yield poor energy and power efficiency. One reason is that kCN requires a constant degree k at all nodes. Nodes at the periphery are generally more sparsely connected than nodes in the center of a network, so in order to satisfy the node degree constraint, the peripheral nodes have to increase their power levels, leading to inefficient power assignments.

Finally, for the case when no topology control is applied, the poor energy and power efficiency performance underscores the motivation for adopting topology control algorithms.

We conclude that, under a holistic view of energy consumption, the topologies that emerge from cooperation-aware topology control achieve significant performance gains in energy efficiency as compared to the kCN and MST algorithms while achieving comparable power efficiency.

Nodes conserve energy but at the same time adopt a forwarding policy which serves as an intrinsic-incentive policy for cooperation. We next evaluate how effective the policy is in inducing nodes to cooperate.

5.4.3 *Efficacy of the forwarding policy*

Under the effect of the forwarding policy described in Section 5.3.1, we observe that within 2-3 iterations nodes starting from random initial cooperation levels converge to a steady state cooperation value of 0 or q_{max}, where q_{max} is the maximum initial cooperation level among all nodes. Further, we observe that when nodes form a connected topology, every node is incentivized to cooperate at q_{max} and none of the nodes are at 0. This guarantees high path reliability in the network. On the flip side, for cases when the network is partially connected, an isolated node plays no role in forwarding packets and hence reduces its cooperation level to 0. (For a detailed discussion on the steady-state connectivity property, see subsection 5.4.4 below.)

Given this result, it stands to reason that the presence of fully-cooperative ("altruistic") nodes in the network can steer selfish nodes towards cooperation. We characterize this further by considering a worst case scenario wherein we insert one or more altruistic nodes and the remaining nodes are initially set to not cooperate at all ($q_i = 0$). The simulation is carried out by varying the percentage of altruistic nodes. The outcome of this experiment is promising: only a small percentage of altruistic nodes are sufficient to steer the entire network towards full cooperation.

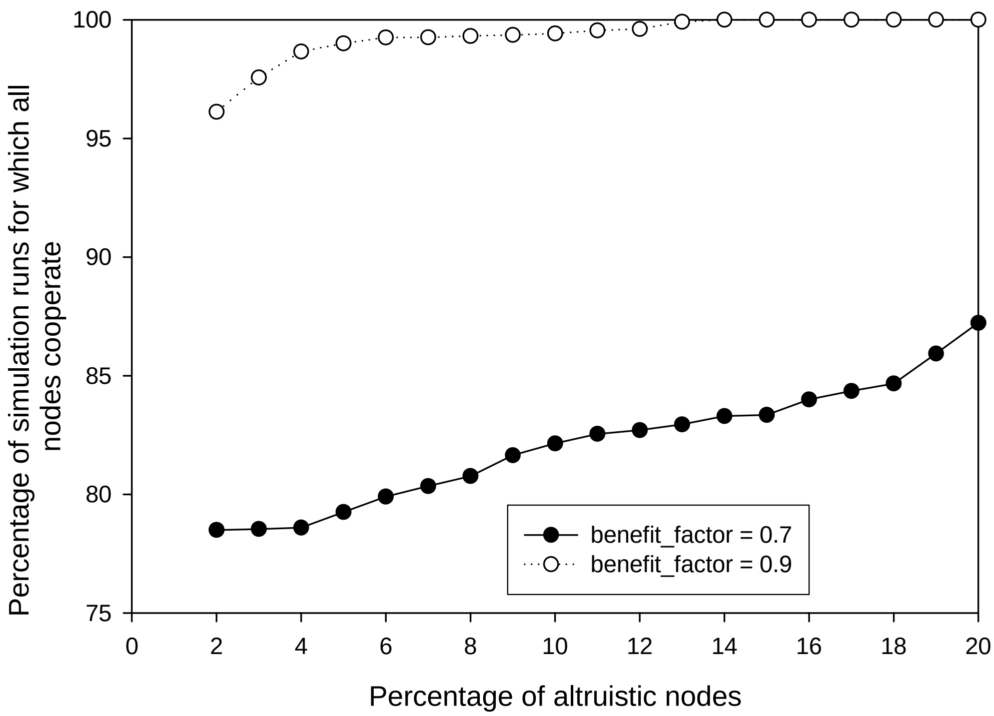

Figure 5.2. Effect of the introduction of altruistic nodes on inducing overall cooperation in the network. The y-axis represents the percentage of simulation runs in which all nodes fully cooperate ($q_i = 1$ for all i) in steady state.

We observe that an initial network with at least 14% of altruistic nodes is sufficient to guarantee an all-cooperating network for $\alpha = 0.9$ (as seen in Figure 5.2). This critical number is significantly higher for $\alpha = 0.7$, as one would expect, because nodes perceive higher cost and hence require greater incentives to cooperate. Thus, with decreasing benefit factor, the likelihood of altruistic nodes' driving the entire network to cooperate decreases.

Even though we observe some runs where we do not achieve complete cooperation from all nodes, the final fraction of nodes that do fully cooperate is high (see Figure 5.3). We notice that at least 98% of nodes cooperate in steady state, when the initial topology contains as few as 4% altruistic nodes.

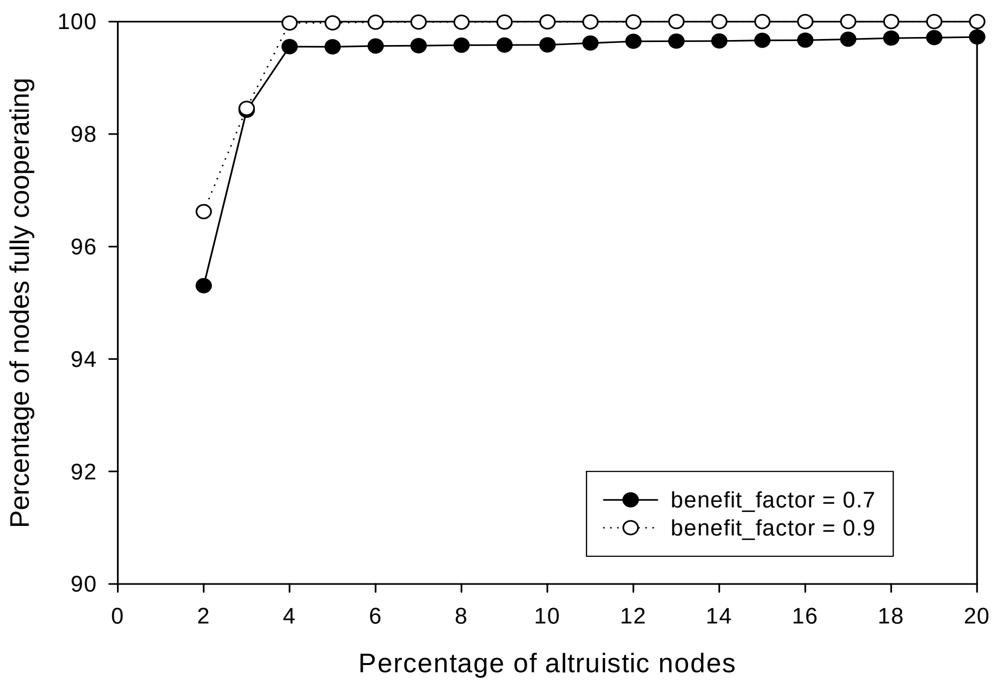

Figure 5.3. Characterization of the impact of altruistic nodes. The y-axis represents the percentage of nodes in the network that fully cooperate, averaged across all runs.

We next examine the robustness of the behavior-based incentive by introducing selfish, non-adaptive nodes in the network and determining the percentage of such nodes that would lead all nodes to not cooperate. The results further confirm the efficacy of our behavior-based incentive: it takes a large percentage of selfish nodes to pull the entire network towards non-cooperation. The simulations are carried out by varying the percentage of selfish, non-adaptive nodes in the network for different values of the benefit factor.

Irrespective of the benefit factor, in fewer than 10% of the total runs we observe the entire network to be completely non-cooperative when 90% of the nodes are selfish and non-adaptive (see Figure 5.4). In fact, a large percentage of selfish, non-adaptive nodes (around 98%) is required to ensure that all nodes refuse to cooperate for all runs.

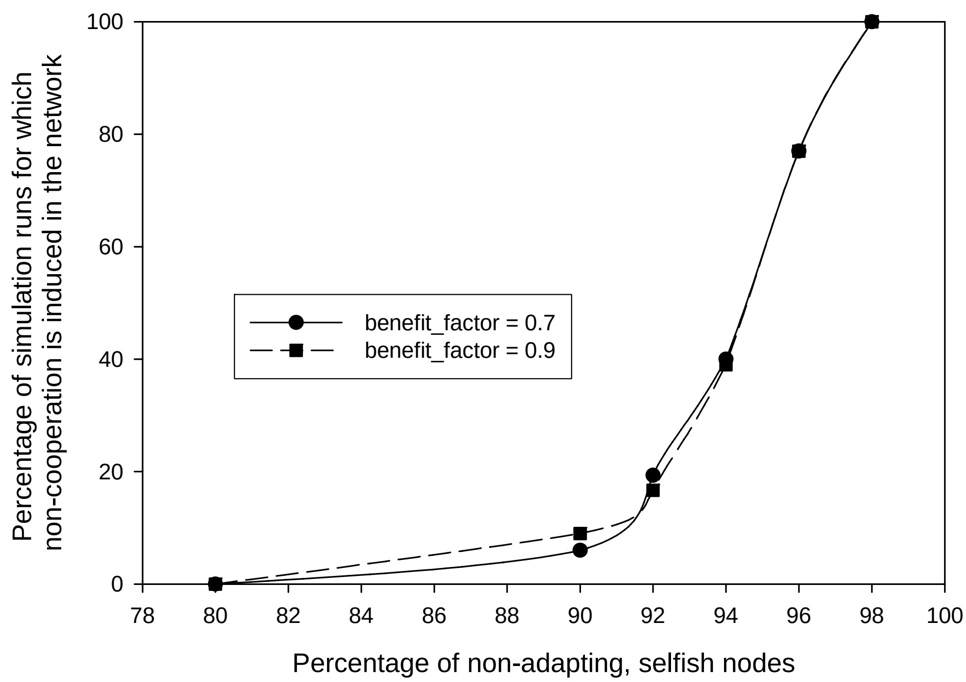

Figure 5.4. Percentage of simulation runs for which the presence of selfish, non-adaptive nodes induces the entire network towards non-cooperation.

We further characterize individual runs by identifying the percentage of adaptive nodes that continue to cooperate at equilibrium. We note in Figure 5.5 that irrespective of the benefit factor, close to 60% of the adaptive nodes continue to cooperate even when 90% of the network consists of selfish, non-adaptive nodes; the number drops to 0 only when the network is composed of 98% selfish, non-adaptive nodes.

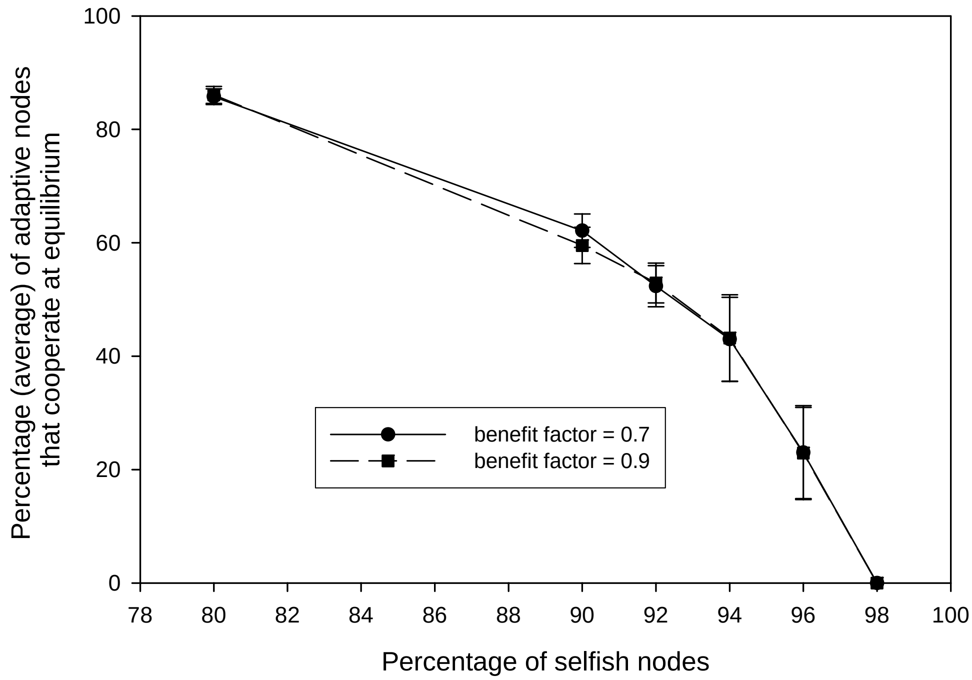

Figure 5.5. Percentage of adaptive nodes that cooperate at equilibrium. The y-axis represents the average value across all runs, plotted with a 95% confidence interval

This further attests to the robustness of the incentive policy and corroborates our aforementioned result that the incentive policy coupled with our utility function provides a strong incentive for cooperation and connectivity.

We conjecture that the effect of the forwarding policy can be further enhanced by strategically placing the altruistic nodes in the network; this is a subject for future investigation. Also, while the forwarding policy nodes adopt is effective in inducing cooperation we have not investigated the incentive compatibility of this strategy.

5.4.4 Topology characteristics

In this section, we present various topological properties such as connectivity, path length and clustering coefficient (cliquishness of nodes) observed at steady-state.

Connectivity: Typically, the problem of topology control is constrained by requiring a connected network. However, at times the cost of connecting to an isolated node is not justifiable. In this work, we balance the objectives of complete network connectivity and reduced energy consumption costs. Hence, the network is not guaranteed to be connected at steady-state.

We present the connectivity properties of the steady-state topology in Figure 5.6 and Figure 5.7 as nodes' perception of benefit and cost varies. As nodes experience a greater benefit in connecting to other nodes (for higher values of the benefit factor), the contribution of a connected topology to the overall utility increases and the probability for which the network is connected in steady-state improves (shown in Figure 5.6).

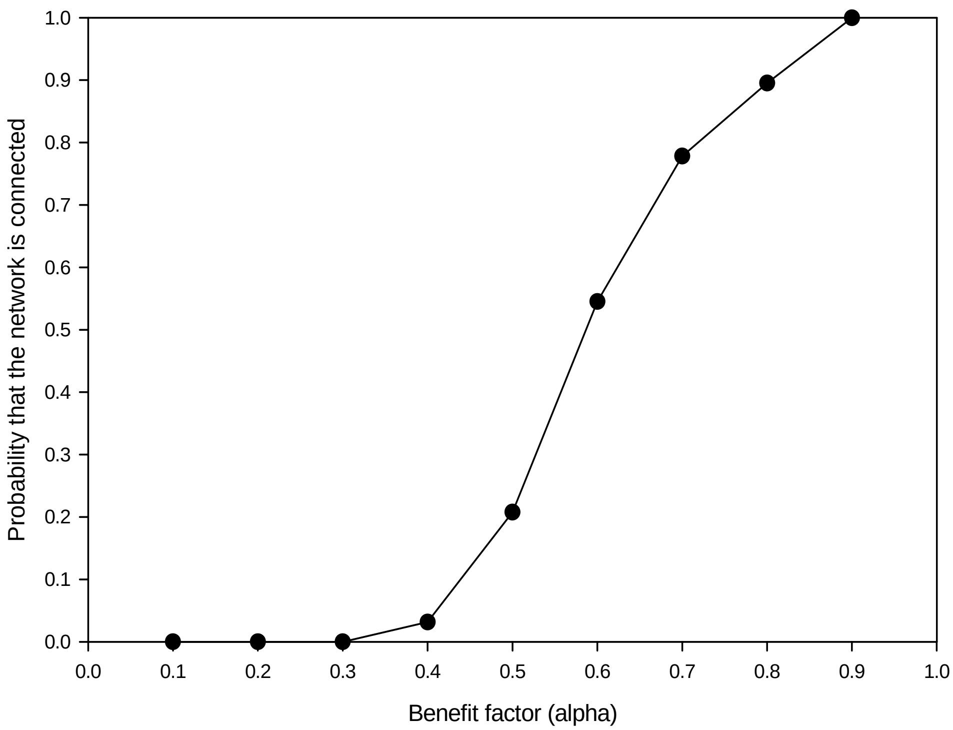

Figure 5.6. Probability that the network is connected

For the case when the network is not connected, we characterize node connectivity by evaluating the largest connected node component. We refer to the ratio of the number of nodes forming the largest connected component to the total network size as the *connectivity fraction* of the network. We observe that even when nodes do not perceive a high benefit to form a connected network, a substantial percentage of nodes remain connected (on an average 95% of nodes are connected for a benefit factor of 0.5 as shown in Figure 5.7).

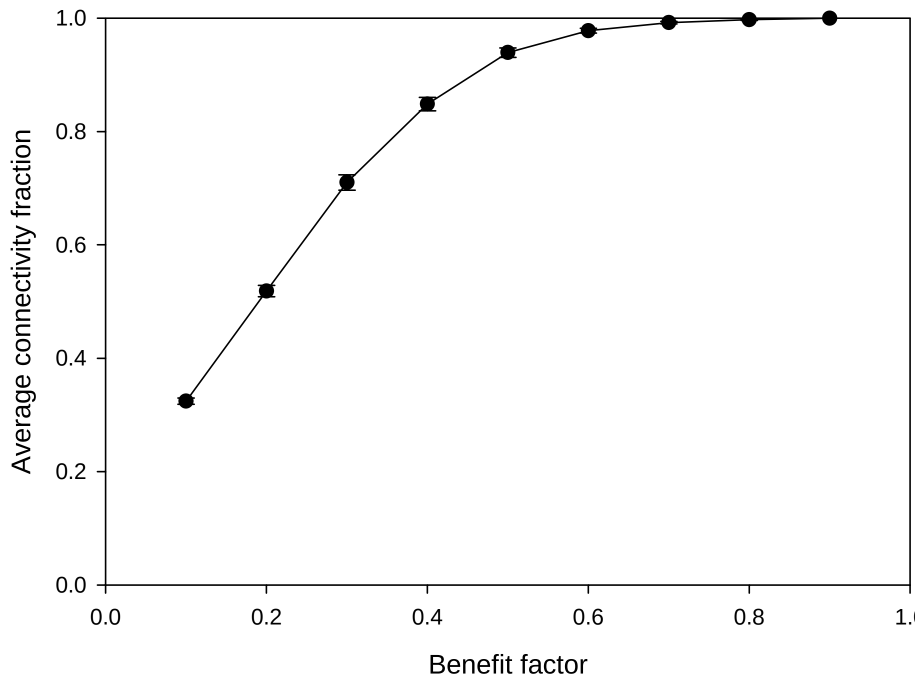

Figure 5.7. Average connectivity fraction for different values of the benefit factor. The y-axis represents connectivity fraction value averaged over all runs, plotted with a 95% confidence interval

Small world properties: As Watts and Strogatz point out in [52], small world properties include a high clustering coefficient (C) – as high as is seen in regular lattice structures of the same size – and small characteristic path length (l) – comparable to those of random graphs. The clustering coefficient, as defined in [52], represents the degree with which neighbors of a node are connected to one another. Formally, if any node, k, has n neighbors then the maximum feasible number of edges among those n nodes is $n(n-1)/2$. The clustering coefficient is the fraction of the possible number of edges that actually exist. If there are m such actual edges, the clustering coefficient for node k is expressed as $C_k = \frac{m}{n(n-1)/2}$. A clustering coefficient near 1 indicates that the node has a densely connected neighborhood. Specifically, comparing these two attributes to those of random graphs the relationship can be characterized as $l \geq l_{rand}$ with $C \gg C_{rand}$. Here l_{rand} and C_{rand} denote the characteristic path length and clustering coefficient for a random graph. Therefore, a network to be classified as a small world network should exhibit these comparative properties.

For our constructed topologies, the average characteristic path length and the clustering coefficient at steady-state are tabulated for two different values of the benefit factor in Table 5-2. We compare these two metrics with those of a random graph of same size and with the same average node degree. Note that the characteristic path length and the clustering coefficient for the random graph was calculated based on equations proposed in [52] (and references within).

Table 5-2. Average path length and clustering coefficient for the network along with 95% confidence interval values

α	l	l_{rand}	l_{rand}/l	C	C_{rand}	C_{rand}/C
0.7	6.97 (±0.22)	3.776	0.54	0.337 (±0.028)	0.056	0.167
0.9	5.16 (±0.25)	2.357	0.46	0.516 (±0.037)	0.105	0.203

From the table we observe that irrespective of the benefit factor the average path length obtained at steady-state is substantially greater than that of a random graph of the same size and the clustering coefficient is close to 5 times higher. The results of these comparisons indicate that the generated topologies exhibit characteristics typically observed in small-world networks.

5.5 Chapter summary

Our work is one of the first to address the problem of topology control in presence of selfish nodes in ad-hoc networks. The objective of this work is to observe topologies (and their properties) that emerge when self-interested nodes consider trade-offs between cooperation and energy consumption.

In topology control, transmit power is typically used as a proxy for energy and optimal network design is achieved through transmit power control. Our model admits a more holistic view of energy conservation – by regulating the energy spent in transmitting at a given transmit power level as well as in forwarding packets at a certain cooperation level. This shows benefits in energy efficiency when compared to power-aware topology control algorithms. Also, we show that an incentive policy based on the observation of other nodes' behavior leads to strong cooperation among selfish nodes. In addition, the steady-state topologies exhibit clustering and path lengths characteristically observed in small-world networks.

We conclude the work carried out in this doctoral research and provide suggestions for potential future research in Chapter 6.

Chapter 6. CONCLUSIONS AND FUTURE WORK

In this chapter we conclude our work by presenting a summary of our main results. We also reiterate our contributions and present some ideas towards extending this work in the future.

6.1 Summary of results

In this section we summarize the key results achieved in the earlier chapters.

- The grim-trigger strategy, through its intrinsic characteristic of threatening non-cooperating nodes, is effective in inducing nodes to cooperate.

  a. For perfect monitoring this requires the nodes to interact with each other repeatedly with high probability, and

  b. In addition to (a), for imperfect public monitoring a necessary requirement is that the probability distribution of the public signal is first order stochastically dominant for profitable deviations by a node. That is, $F(\cdot\,;\widehat{\boldsymbol{a}}) \leq F(\cdot\,; a_i, \widehat{\boldsymbol{a}_{-i}})$ for $\pi_i(a_i, \widehat{\boldsymbol{a}_{-i}}) \geq \pi_i(\widehat{\boldsymbol{a}})$. Here, $F(\cdot\,;\widehat{\boldsymbol{a}})$ is the probability distribution function of the public signal and $\pi_i(\widehat{\boldsymbol{a}})$ is the expected payoff to node i, for the joint action profile, $\widehat{\boldsymbol{a}}$.

     Therefore, for a given threshold value, if a node expects to benefit by deviating from $\widehat{a_i}$, it increases the chances of triggering the grim period earlier by risking the public signal to fall below the threshold. In other words, if a node expects a better utility by reducing its cooperation level and deviating from the equilibrium, the chances of the aggregate throughput reducing below a set threshold increase. This acts as an intrinsic incentive for node i to maintain its cooperation level for 'fear' of potentially triggering the grim phase.

- We show that the grim-trigger strategy is not robust to deviations in node behavior in the presence of monitoring noise and a single instance of misbehavior can lead all nodes to stop cooperating.

- The probability-based strategy proposed in this work is effective in inducing nodes to cooperate under imperfect monitoring conditions and is more robust than the grim trigger strategy. Nodes achieve a near-optimal payoff at equilibrium when adopting this strategy.
- In topology control the application of a behavior-based incentive produces topologies at equilibrium that are energy-efficient and well-connected. The topologies also exhibit properties consistent with those seen in small world networks. Namely, the path length is slightly greater than and the clustering coefficient is much larger than those exhibited by random graphs of a similar size.

From this set of concrete results, we can draw some general conclusions that potentially extend the contribution of this work to apply to broader issues in modeling selfish behavior and cooperation in self-organizing networks, as well as the independent adaptation of autonomous network nodes under incomplete or noisy information.

6.2 Conclusions

Repeated games are appropriate for modeling selfish behavior in ad hoc networks. We derive conditions under which a trigger-based strategy such as grim-trigger can sustain equilibrium when actions of every node are perfectly monitored. We also show that when nodes adopt the grim-trigger strategy the network remains stable as long as the nodes perceive the game will be repeated for a long duration with a high probability.

We are among the first to analyze incentives for selfish behavior in ad hoc network under imperfect monitoring conditions. The distributed nature of ad hoc networks renders the assumption of perfect monitoring questionable at best. We do away with this assumption and show the efficacy of trigger-based and non-trigger-based strategies under imperfect monitoring conditions. We can envision the application of games of imperfect monitoring to model other autonomous adaptations that nodes in a self-organizing network are expected to perform.

We show that in situations where it is feasible for nodes to assess the general health of the network and observe a common public signal that reflects the forwarding actions of others, a simple strategy such as grim-trigger is incentivizing enough to lead nodes to cooperate at equilibrium. However, the grim-trigger strategy is sensitive and not robust to node deviations resulting due to monitoring errors. In addition, the availability of a public signal upon which all nodes can rely on, in a distributed network, is rather limited.

We propose a probability-based strategy that is effective in inducing cooperation and robust to monitoring errors when compared to grim-trigger strategy. We show that at equilibrium cooperation leads to a near-optimal payoff to nodes. By adopting a probability-based strategy ours is among the early research efforts to employ repeated games of imperfect private monitoring in a way that nodes do not need to maintain beliefs regarding their opponents' private information.

We are also the first ones to analyze the problem of topology control by (a) considering the presence of selfish nodes in the network, and (b) optimizing power and cooperation levels based on a holistic view of energy consumption. We show that by adopting this approach the topologies that emerge are energy-efficient when compared to those that result from power-aware algorithms. We also show that by adopting a behavior-based incentive nodes not only achieve well-connected topologies, hence satisfying the traditional topology control objective, but also establish reliable, energy-efficient paths.

The application of imperfect private monitoring games to analyze behavior-based incentives has wider scope in the analysis of ad hoc networking algorithms, particularly those pertaining to networks of cognitive radios. Cognitive radios are devices that are characterized by the capability to observe, learn and adapt. The modeling and analysis approach adopted in this work can be used to model similar imperfections in observations made by cognitive radios. The subsequent analysis will provide us with much needed insight into the impact of noise and other imperfections on existing algorithms designed for such smart radios.

Incentives for resource-sharing are also widely used in P2P networks such as BitTorrent to induce users to participate in sharing files, and in inter-domain routing. This work therefore has a broader scope in its applicability beyond wireless ad hoc networks. The nature of peer-to-peer networks is inherently distributed thereby making it to difficult to track a user's participation level in sharing its resources. In such a scenario, one or more strategies and their analyses carried out in this work could be applied to determine the equilibrium point.

6.3 Future work

Distributed networks in the near-future are expected to comprise independent, decision-making nodes capable of observing, learning and adapting to dynamic system changes. Ignorance in such networks has been defined [53] as information that is missing, incomplete or imperfect. One potential application of our work is in the applicability of imperfect private and public monitoring games, beyond node cooperation, to capture ignorance in information in the analysis and design of cognitive network algorithms and protocols.

Nodes may tend to adopt more than one cooperation strategy in dealing with selfish behavior. The models and solution techniques employed in this research can be used to analyze, mathematically or through simulations, the interaction between different cooperation strategies. The MANIAC Challenge [54] is one such implementation-based research effort; it is an ad hoc networking competition organized to create a real-life ad hoc network. The participating teams were given the objective of designing cooperation strategies to balance the trade-off of energy consumption versus cooperation in forwarding packets. The further analysis of how different cooperation strategies interact with one another is a logical area of extension of research on incentives for cooperation.

Our research effort is complementary to reputation management systems, both of which are based on the assessment of node behavior. There is potential in the application of behavior-based incentives in reputation management systems to induce node cooperation from selfish nodes.

How behavior-based incentives could be applied to reputation systems is a question worth pursuing in the future.

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