Three Essays on Economic Agents' Incentives and Decision Making

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(ABSTRACT)

This dissertation consists of three essays on theoretical analysis of economic agents' decision making and incentives. Chapter 1 gives an outline of the subjects to be examined in the subsequent chapters and shows their conclusions in brief.

Chapter 2 explores the decision problem of a superordinate (a principal) regarding whether to delegate its authority or right to make a decision to a subordinate (an agent) in an organization. We first study the optimal contracting problem of the superordinate that specifies the allocation of the authority and wage in a principal-agent setting with asymmetric information, focusing on two motives for delegation, "informative" and "effort-incentive-giving" delegation. Further, we suggest delegating to multiple agents as a way of addressing the asymmetric information problem within an organization, focusing on another motive for delegation, "strategic" delegation.

Chapter 3 analyzes the behavior of players in a particular type of contest, called "the weakest-link contest". Unlike a usual contest in which the winning probability of a group in a contest depends on the sum of the efforts of all the players in the group, the weakest-link contest follows a different rule: the winning probability of a group is determined by the lowest effort of the players in the group. We first investigate the effort incentives of the players in the weakest-link contest, and then check whether the hungriest player in each group, who has the largest willingness to exert effort, has an incentive to incentivize the other players in his group in order to make them exert more effort.

Chapter 4 examines the decision making of software programmers in the software industry between an open source software project and a commercial software project. Incorporating both intrinsic and extrinsic motivation on open source project participation into a stylized economic model based on utility theory, we study the decision problem of the programmers in the software industry and provide the rationale for open source project participation more clearly. Specifically, we examine the question of how the programmers' intrinsic motivation, extrinsic motivation, and abilities affect their project choices between an open source project and a commercial project, and effort incentives.

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Chapter 1

Introduction

An incentive is any factor that motivates a particular course of action. All the people respond to economic incentives. In other words, people make a decision in accordance with their incentives. Therefore, to understand the behavior of people or the phenomena shown in the real world, we first need to consider the incentives behind those. Once the incentives are found and understood well, we are able to obtain much insight to understand and predict the behavior of people and further design those toward the direction we want to head to. In this dissertation, while focusing on the incentives of economic agents in an organization or group, we try to understand how they make their decisions or choices in a certain environment. Specifically, we try to answer the following three questions of economic agents by examining the incentives they are facing in each situation:

1. In an organization, should an uninformed superordinate delegate its authority or decision right to an well-informed subordinate (or subordinates)?

2. In the weakest-link contest where the performance of a group is crucially dependent on the weakest player in the group, how much effort do players exert to win a prize in the contest?

3. In computer software industry, where does an individual programmer participate in either an open source project or a commercial project?

First, we consider the decision problem of an superordinate (a principal) within an organization whether to delegate its authority to a subordinate (an agent) or not. Delegating an authority to someone is a ubiquitous phenomenon in our reality. Especially, delegation of authority to subordinates is an essential feature of a hierarchically decentralized organization. There are several reasons why a person having initially the right to make a decision over something delegates her authority to others. At first, an economic agent can benefit by delegating its authority to others who have more information (or ability) to make a decision than herself. Delegation done from this motive is called "informative" delegation. The strand of literature on "informative" delegation treats delegation as a means of addressing informational asymmetries within an organization. Second, the motive for delegation comes from the fact that an economic agent can provide others with a strong effort incentive by delegating. This is because people tend to work harder when they work with their own decision or idea than when they are forced to work with other person's decision or idea. Delegation done from this motive is called "effort-incentive-giving" delegation. Finally, another motive for delegation is that an economic agent can benefit by committing its behavior and hence achieving strategic advantages by delegating its authority to a person whose preference differs from hers. This sort of delegation is called "strategic" delegation. The literature on "strategic" delegation views delegation as a means for a principal to commit to a course of action. It is normally modeled as a two-stage game of complete information: in the first stage, the principal appoints an agent from a set of potential agents with different types; in the second stage, the agent plays a game with other players (possibly other agents). Applications have ranged from oligopoly (Vickers (1985), Fershtman and Judd (1987)) and central bank independence (Rogoff (1985)) to representative democracy (Persson and Tabellini (1994), Besley and Coate (2001)).

In chapter 2, we seek to answer the question of whether a superordinate within an organization should delegate its authority to a subordinate who has more information but different preferences from the perspective of "informative" and "effort-incentive-giving" delegation. In order to answer the question, we study an optimal contracting problem of the superordinate in a principal-agent setting with asymmetric information. Much literature (see chapter 2) studied a principal's decision of delegating its authority to an agent or keeping it from the perspective of either "informative" or "effort-incentive-giving" delegation. In this dissertation, we link these two views of delegation in the setting of a hierarchically decentralized organization. In the organization, the uninformed principal faces the choice problem between retaining its authority and delegating it to an informed agent (possibly multiple agents). That is, we incorporate the effort-incentive-giving motive for delegation with the informative motive in a principal-agent setting with asymmetric information. In such a setting, we try to understand why an uninformed principal (e.g., company owners and senior management) may grant formal decision rights to an agent or possibly multiple agents (e.g., senior or middle management) who are better informed but have different objectives. We also suggest a new type of delegation, delegation to multiple agents, as a way of addressing the asymmetric information problem the principal faces. That is, we show that the principal can indirectly extract information from the agents by strategically delegating its authority to multiple agents who have different preferences. In a sense, hence, we study the principal's decision problem in view of "strategic" delegation as well as "informative" and "effort-incentive-giving" delegation.

In chapter 3, we examine the behavior of the economic agents participating in a certain type of contest. A contest is a situation where each player or group competes against each other to win a prize or award, and that situation is an easily observed phenomenon in the real world. In this dissertation, we consider a particular type of group contest where each group competes with each other to win a prize and the likelihood of each group's winning in the contest is dependent on the minimum performance of the players in that group. In other worlds, the performance of a group participating in a contest depends on the overall performance of all the players in that group and especially the performance of the weakest player in that group. We call this type of group contest the "weakest-link contest", using the term of Hirshleifer (1985). Much literature (see chapter 3) analyzed a group contest. They assume that the winning probability of each group participating in the contest depends on the total performance (sum) of all the players in the group. That is, they assume that the efforts of the players in a group are perfectly substitutive. Due to the perfect substitutability among efforts of the players in a group, we can intuitively expect that each player may have much incentive to free ride on the others in his group. Therefore, in equilibrium, only the hungriest player in each group exerts some efforts and the others in the group free ride on the hungriest one in their group (Baik (1993)).

However, in the weakest-link contest, we assume that the players' efforts in a group are not perfectly substitutive but perfectly complementary. Then what happens to the players' incentives in the contest? Given the fact that the players' efforts are perfectly complementary in a group, each player in the group will realize that his extra effort, which exceeds any other player's effort in his group, is wasteful. Consequently, in equilibrium, all the players in a group match their effort to the effort level exerted by the least hungry player in their group. That is, there is no incentives for each player to free ride on others and the least hungry player in each group has an important role in determining the success of its group in the weakest-link contest. Considering these points, we can also cast the following question: Is there any incentive for the hungriest player, who has more willingness to expend effort, in each group to motivate the least hungry one, who has less willingness to expend effort, in his group in order to increase his group's winning probability in the contest? Interestingly, we find that the answer is "Yes.". We show that the hungriest player in each group has an incentive to subsidize the other players' effort in his group.

Finally, in chapter 4, we analyze an interesting phenomenon observed in the computer software industry. There are primarily two types of software: commercial software and open source software. Commercial software is software that is distributed under commercial license agreements with the purpose of making profit. On the contrary, Open Source Software (OSS) is software whose source code is publicly known to everyone, enabling anyone to copy, modify and redistribute its source code. The success of the open source software project cannot be achieved without the tremendous efforts made by self-motivated individual programmers who are willing to spend their time without getting paid for their effort and time. Then, the question is "What motivates these programmers to join in open source software projects that never give them any monetary benefit?". In this dissertation, we study the incentives of the economic agents who participate in open source software projects and examine their decision problems whether to join either open source software projects or commercial software projects.

From a behavioral perspective grounded on survey data, the various motivations to participate in open source project have been grouped under two broad categories: intrinsic and extrinsic motivation (Lakhani and Wolf (2003), Rossi (2004), Robers et al. (2006)). Intrinsic motivation is defined as the performing of an activity for its inherent satisfactions rather than for some separable consequence. In the context of open source software, intrinsic motivations can be the enjoyment of programming, satisfaction, accomplishment as a member of the community, altruism, generalized reciprocity, and a gift-giving attitude (Rossi (2004)). On the other hand, extrinsic motivation refers to motivation that stems from factors outside an individual. Rewards like reputation and monetary compensation are examples of extrinsic motivation. In the context of the open source software, peer recognition or potential job offers may motivate the open source software developers extrinsically.

Learner and Tirole (2002), from an economic perspective, argue that a programmer participates in a project only if she derives a net benefit. They also argue that existing economic theory can explain the motivation for open source project participation as long as a programmer's benefits and costs are articulated in her utility function. However, very little research in the economic literature attempts to explain the motivation of the open source software developers with an economic model based on utility theory. In this dissertation, we aim to bridge the gap between economic literature and behavioral science on the motivation for open source project participation by examining the programmers' decision making between open source and proprietary software projects in a stylized economic model.

Chapter 2

Information, Incentives, and Delegation of Authority

2.1 Introduction

Delegation of authority to subordinates is an essential feature of a hierarchically decentralized organization. Why delegation within an organization? There are two main motives for delegation. First, a superordinate (she) can benefit by delegating authority to a subordinate (he) who has more information than herself and having him make a decision on behalf of her. That is, delegation can be used as a means of addressing informational asymmetries in an organization. Second, delegation can be used as a way to solve the moral hazard problem because, under delegation, the subordinate makes his own decision and hence he may work harder for the success of his decision than if he was forced to work on the superordinate's decision. Along with these two benefits from delegation, delegation entails a cost on the superordinate. Once authority is delegated to the subordinate, the superordinate cannot control the subordinate's behavior, and hence the subordinate will make his most favorite decision that is different from the superordinate's one. That is, delegation results in a loss of control. Comparing these two rationales for delegation with the cost of delegation, we study the optimal allocation of authority within an organization, i.e., the question of whether a superordinate in an organization should delegate or not.

For instance, consider project choices within a firm that consists of a firm owner and several branch managers. The firm owner wants to select and implement a project that maximizes the firm's economic profit or market value but she does not have any information about an economic environment that determines the return of each project. That is, the firm owner does not know which project generates the maximum profit of the firm. Each of the branch managers is well-experienced and then well-informed about the economic environment but he wants to undertake a project that maximizes the benefit of his branch rather than a profit-maximizing project because he is concerned with his career. Hence, if the firm owner delegates her authority to select a project to one of the branch managers, the manager tends to undertake an inefficient project that is not the best for the firm owner. However, the manager is willing to exert high effort in implementing the project, because it gives him high private benefit. In other words, the manager may be highly motivated to expend his effort on the project. Should the firm owner delegate her authority to one of the managers or not? If she delegate, to whom and how?

To assess how the allocation of authority within an organization depends on the information structure and the effort incentives, we study a principal-agent model in which a principal contracts with an agent (possibly multiple agents). In our main model, a principal and an agent chosen by the principal should select a project and implement it. The principal is ignorant about an economic environment (or a state of the world) which determines the payoff of each project when it succeeds, while agents have the exact information about that. Initially, the authority of selecting a project is given to the principal, but the principal can delegate the authority to an agent. The probability that a selected project succeeds or fails depends on the agent's non-observable effort. Therefore, the principal should design the optimal allocation of authority and the optimal effort incentive to solve a problem involving both adverse selection (asymmetric information) and moral hazard (non-observable effort).

Following the incomplete contracts approach, we assume that a principal and an agent cannot make a contract on the agent's private information, the selection of a project, and their payoffs.[1] However, the allocation of the authority over project selection is contractible. Thus, the contract between the principal and the agent specifies the allocation of the authority to select a project and a wage schedule. A wage schedule cannot be conditioned on the selection of a project or the principal's payoff. It is only contingent on the outcome of the project, that is, whether the project succeeds or fails. Since the authority is initially given to the principal, the principal has an option to keep the authority of selecting a project or delegate the authority to an agent. In this setting, we first consider the optimal contract between a principal and a single agent. Then, we study the

[1] See Aghion, Dewatripont, and Rey (2002) for the incomplete contract literature.

optimal contract in the case where a principal delegates the authority to multiple agents.

Our main finding is that consideration of effort incentives and asymmetric information within an organization makes the principal more likely to delegate the authority over project selection to an agent. However, if information is symmetric, the principal never delegates. We also find that a well-informed agent has an incentive to reveal the information about an economic environment to the principal truthfully and voluntarily but the information transmission from the agent to the principal does not happen because of the time inconsistency problem (commitment problem) in the agent's behavior. Furthermore, we suggest delegation to multiple agents as a way of addressing the asymmetric information problem, and find that the principal can extract the information about an economic environment indirectly by delegating the authority to multiple agents biased in opposite directions.

Many papers study the optimal allocation of authority within an organization, i.e., the choice between delegation and centralization, in a principal-agent setting. Focusing on the informational benefits of delegation, some papers consider a trade-off between the loss of control resulting from delegation and the loss of information in case of centralization. Other papers consider the trade-off between delegation and centralization, focusing on increasing the agent's effort incentive as the benefit of delegation. In this chapter we study the principal's choice between delegation and centralization from both the information and the effort-incentive perspective of delegation. For doing this, we combine the models of two papers, Dessein (2002) based on the information view of delegation and Bester and Krähmer (2008) based on the effort-incentive view of delegation. Specifically, we extend the model of Bester and Krähmer (2008), where there is no asymmetric information between a principal and an agent, to the case of asymmetric information as examined in Dessein (2002). This is one of the main contributions of the current chapter. The other is to suggest several ways of inducing information revelation by the well-informed agents.

The chapter proceeds as follows. In Section 2.2, we review some literature related to our work. In Section 2.3, we present the basic model and set up the game. In Section 2.4, we examine the optimal contracts between a principal and a single agent. Section 2.5 suggests three different forms of delegation with multiple agents and analyzes the optimal contract between the principal and multiple agents for each case. We discuss an open question that is important and interesting to study for our future work in Section 2.6, and finally conclude in Section 2.7.

2.2 Related Literature

Many papers compare delegation and centralization from the information perspective of delegation. Dessein (2002) considers a principal-agent relationship where they have diverging preferences on project selection and have asymmetric information about a state of the world that determines the optimal project for both the principal and the agent. When the principal delegates her authority to the agent, she faces a cost, a loss of control, because the biased agent selects his favorite project that is different from the principal's favorite. If the principal keeps her authority, there occurs communication between the principal and the agent, i.e., the principal asks the agent to advise her on project selection. In this case, communication is a cheap talk game (Crawford and Sobel (1982)) because the agent does not truthfully report the state of the world. Hence, the principal faces a cost, a loss of information, in case of keeping her control. In such a setting, Dessein (2002) shows that delegation is preferred by the principal when the loss of control is relatively small, that is, the agent's bias is not too large relative to the principal's uncertainty about the state of the world. With a setting similar to Dessein (2002), Harris and Raviv (2005) examine the question of what determines the allocation of investment decision within a firm consisting of a CEO (a principal) and a division manager (an agent). Their model differs from Dessein (2002) in that both the principal and the agent have private information regarding the profit maximizing investment level. In this setting, Harris and Raviv (2005) show that the likelihood of delegation decreases with the importance of the principal's information. Acemoglu et al. (2007) analyze the relationship between the diffusion of new technologies and the allocation of authority of firms empirically, and show that firms closer to the technological frontier, firms in more heterogeneous environments, and younger firms are more likely to delegate.

Some papers compare delegation and centralization from the effort-incentive perspective of delegation. Aghion and Tirole (1997) study a principal-agent model where a principal hires an agent to collect information about projects. The principal and the agent acquire information about projects' payoffs with some probability, which depends on their effort. They show that delegating the formal authority to the agent increases his incentives to expend effort for acquiring information about projects' payoffs because the principal cannot overrule the agent's choice under delegation. Stein (2002) takes into account a distinction between soft and hard information relevant to a principal's and an agent's decision, and shows that when information is soft, delegation increases the agent's incentive to search for information. On the contrary, when information is hard (verifiable), centralization always dominates delegation. Zábojník (2002) finds that delegating the authority to the agent may be optimal even if the principal has better ability to choose a profitable project, because if the agent chooses a project in his favor, he will be more optimistic about the success of the project and be more encouraged in exerting effort. Bester and Krähmer (2008) study the relation between authority and incentives in a standard principal-agent model where a principal and an agent have different preferences over project selection and the agent's effort level depends not only on the selected project but also on monetary incentives. They find that the consideration of effort incentives makes the principal less likely to delegate the authority to the agent.

In this chapter we study the principal's choice between delegation and centralization from both the information and the effort-incentive perspective of delegation. We combine the two literatures, the information view of delegation taken by Dessein (2002) and the effort-incentive view of delegation assumed by Bester and Krähmer (2008). We then suggest several ways of inducing information revelation by the well-informed agents. Specifically, we suggest delegation to multiple agents as a tool to induce information revelation by the well-informed agents. Hence our work is related to Legros (1993) and Gautier and Paolini (2007) who consider delegation as a tool to transmit information. Legros (1993) considers a two-period model of repeated delegation with asymmetric information. In his model, the principal delegates the decision of the first project to a better informed agent chosen from a given set of agents. In the second period, the principal can either reselect the same agent and let him choose the second project or she can hire a new agent to implement the second decision. The agent chosen in the first period faces a trade-off between the immediate benefit of implementing his preferred policy (revealing his private information) and the probability of being reselected that increases when he can convince the principal that his preference is close to her. So, the first period decision is used by the agent to signal the agent's private information (his type) to the principal. Gautier and Paolini (2007) compare two different organizational structures: centralization and partial delegation. Under centralization, the agent transmit a message about the state of the world to the principal, and the principal then chooses projects. Under partial delegation, the principal delegates the decision of the first period to the agent. After observing the agent's choice in the first period, the principal chooses a project in the second period. By observing the agent's choice in the first period, the principal may acquire the agent's private information. They show that partial delegation is a mechanism to induce full revelation by the well-informed agent.

Our work is also related to the literature studying the optimal form of delegation. Krähmer (2006) considers a partially incomplete contracting environment where the principal's commitment is limited by non-contractibility of actions, but message from the agent to the principal and decision right are contractible. Since decision rights and message are contractible, the principal can transfer control on a contingent basis, depending on a report by the agent (contingent delegation). He shows that contingent delegation provides the principal with an additional instrument to structure the agent's incentives to reveal information. Mylovanov (2008) establishes the veto-power principle in a principal-agent model with hidden information and no monetary transfers. He argues that veto-based delegation is an attractive decision mechanism because with a proper choice of a default decision it achieves the optimal separation of decision initiation and decision control. Under veto-based delegation, the principal encourages the agent to use his knowledge through delegation of formal rights to initiate and implement decisions, while she prevents him from being excessively opportunistic by holding the right to block his decision. Alonso and Matouschek (2008) study the characterization of optimal decision rules by a principal who faces an informed but biased agent and who is unable to commit to contingent monetary transfers. They show that the principal benefits from delegation if and only if the principal and the agent are minimally aligned. They also consider how much discretion the agent should have, i.e., which decisions the agent should be allowed to make and which should be ruled out, and show that interval delegation (Holmström, 1984), specifying an interval of decisions from which the agent is allowed to choose his preferred one, is optimal when the agent is sufficiently aligned with the principal.

2.3 The Basic Model

Consider an organization in which a risk-neutral principal and risk-neutral agents should jointly undertake a project $d \in D$, where D is a set of projects. We adopt an incomplete contracting approach by assuming that project selection is not verifiable to the third party and hence cannot be contracted upon but the right over project selection can be assigned contractually either to the principal or the agent. The right over project selection is initially given to the principal. We call this decision right *authority.* If the principal keeps authority, she retains the right to select a project. On the other hand, if she delegates her authority, she grants the right over project selection to an agent. There exist various types of agents. Agent i, denoted by A_i, is represented by his type $b_i \in B$. $B = [-\bar{b}, -\underline{b}] \cup [\underline{b}, \bar{b}]$ is a set of possible agents' types where $0 < \underline{b} < \bar{b}$. The type of each agent is observable to the principal.

The success or failure of a project depends on the agent's effort e. The agent exerts his effort after a project is chosen. Thus, at the stage where the agent exerts his effort, he knows which project is chosen, i.e., a selected project d. If the agent chooses his effort level e, he incurs the effort cost $c(e) = e^2/2$ and the project succeeds with probability $p(e) = e \in [0, 1]$. Following a standard principal-agent model with moral hazard, we assume that the agent's effort is not verifiable.

If the project fails both the principal and the agent get zero. If a project succeeds the principal and the agent receive the private benefits u_P and u_A, respectively. These benefits depend on a state of the world described by a parameter $\theta \in \Theta$. A state of the world θ is a random variable which has twice differentiable distribution $F(\theta)$ with positive density $f(\theta)$ supported on $[-L, L]$, where $L > 0$. We assume that only the agents observe the realization of θ and the principal only knows $\Theta = [-L, L]$ and the distribution $F(\theta)$. The private benefits of the principal and the agent, when a project d succeeds, are defined as

$$u_P(d) = r_P - k_P(\theta - d)^2$$

and

$$u_{A_i}(d) = r_{A_i} - k_{A_i}(\theta + b_i - d)^2,$$

where the project-space D is defined as $[-(L + \bar{b}), L + \bar{b}]$.[2] These benefits are not verifiable to the third party and hence are not contractible. In order to guarantee the nonnegativity of private benefits, we assume that $r_P > k_P(2L + \bar{b})^2$ and $r_{A_i} > 4k_A(L + \bar{b})^2$. The parameters $k_P > 0$ and $k_{A_i} > 0$ describe how much the principal and the agent care about project selection. Thus the principal's benefit reaches a unique maximum when the project $d = \theta$ is chosen and the agent's benefit is maximized when the project $d = \theta + b_i$ is chosen. The principal and the agent have conflicting interests over the selection of a project by assuming $b_i \neq 0$. We refer to b_i as the bias of agent i because, for a given state of the world θ, the most favorite project of the principal (θ) is different from the one of the agent ($\theta + b_i$). In other words, the principal and the agent have different ideal projects, which are dependent on θ.

The agents are identical except for their own biases in the sense that, regardless of agents' types (biases), each agent gets the same maximum private benefit when his ideal project is selected and it succeeds, and each agent has the same weight of how much he cares about the selection of

[2]This utility function is used in Crawford and Sobel (1982), Dessein (2002), and Bester and Krähmer (2008)

a project. This implies that $r_{A_i} = r_A$ and $k_{A_i} = k_A$ for all i.

Let $w = (w_s, w_f)$ be an incentive scheme contingent on success and failure of a selected project. If the project succeeds, the principal pays the agent the wage w_s. If the project fails, the principal pays the agent the wage w_f. Then the expected payoffs of the principal and agent, for given a state of the world θ, are

$$U_P(d, e, w) = e\big(r_P - k_P(\theta - d)^2 - w_s\big) - (1 - e)w_f$$

and

$$U_{A_i}(d, e, w) = e\big(r_A - k_A(\theta + b_i - d)^2 + w_s\big) + (1 - e)w_f - \frac{1}{2}e^2.$$

Each agent's outside option payoff, or reservation utility, is $\overline{U}_{A_i} = 0$. We assume that the agent has limited liability, i.e., the agent cannot be paid a negative wage in any case. This means that $w_s \geq 0$ and $w_f \geq 0$.

Since the selection of a project is not contractible, the principal offers an agent a contract which specifies the allocation of the authority to select a project and the wage schedule w. We describe the allocation of authority by $h \in \{P, A\}$.[3] If $h = P$, the principal keeps the right to select a project $d \in D$ and we call this case *centralization*. If $h = A$, she delegates the right to an agent and we call this case *delegation*.

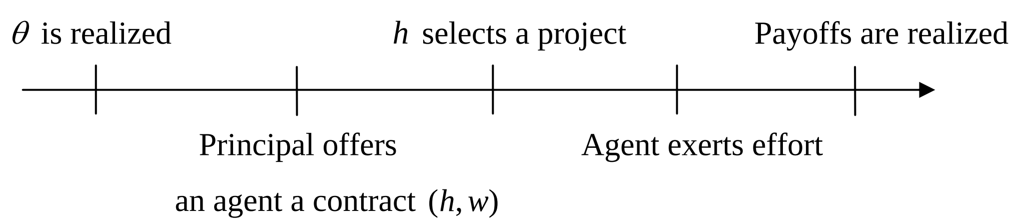

Figure 2.1: The Sequence of Events

The time structure of the model is summarized in Figure 2.1: First, a state of the world (θ) is realized. Note that only the agents observe the realization of θ. Second, the principal offers an agent (chosen from a set of agents) a contract (h, w) that specifies the allocation of authority and the wage schedule. If the agent accepts the principal's offer, according to the contract, the party h who has the authority selects a project at the subsequent stage. If the agent does not accept the offer the game ends and both the principal and agent get nothing. Next, the agent chooses his

[3]This notation is used in Bester and Krähmer (2008).

effort e after observing the selected project by the party h. The effort exerted by the agent affects the project's probability of success and failure and the payoffs of the principal and the agent are realized in the final stage.

2.4 Optimal contracts

2.4.1 Optimal contracts when information is symmetric

As a benchmark, we consider a symmetric information case where both the principal and agents can observe the realization of θ, a state of the world. To find the optimal allocation of authority and wage schedules in the view of the principal, we consider two scenarios: one case where the principal keeps her authority to select a project and the other case where the principal delegates her authority to an agent.

The optimal contract under centralizaton

We examine the case where the principal keeps her authority, that is, the principal offers a contract $\big(h = P, w = (w_s, w_f)\big)$ to agent i. For doing this, we work backwards. At first, we study the optimal wage scheme (w_s, w_f) which the principal offers to agent i and a project she chooses. Then, we consider the principal's problem of choosing the type of an agent.

After observing a state of the world θ, the principal solves the optimal contracting problem under moral hazard:

$$\max_{\{w_s, w_f, d\}} U_P = e\left(r_P - k_P(\theta - d)^2 - w_s\right) + (1-e)(-w_f) \tag{2.1}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of the agent:

$$U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2 \geq \overline{U}_{A_i};$$

(c) The incentive-compatibility constraint, which stipulates that the effort level maximizes the agent's expected payoff given w_s, w_f, and d:

$$e = \arg\max_{e \in [0,1]} U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2.$$

The incentive-compatibility constraint (c) can be simplified to:

$$e = r_A - k_A(\theta + b_i - d)^2 + w_s - w_f. \tag{2.2}$$

Substituting (2.2) into (2.1) and solving the principal's optimal contracting problem, we obtain the following wage schedule and a project the principal chooses:

$$w_s = \frac{1}{2}\left(r_P - r_A + \frac{k_A k_P (k_P - k_A){b_i}^2}{(k_A + k_P)^2}\right), w_f = 0 \text{ and } d = \theta + \frac{k_A b_i}{k_A + k_P}. \tag{2.3}$$

Now we consider the principal's problem of choosing an agent from a set of agents. Substituting (2.2) and (2.3) into (2.1) and solving the principal's maximization problem with respect to $b_i \in B$, we obtain the optimal type of agent, b_i^*, the principal chooses. The following lemma summarizes the optimal contract under centralization. All proofs are presented in the Appendix.

Lemma 2.1

(a) Under centralization, the principal offers the agent whose bias is $b_i^* = \underline{b}$ (-$\underline{b}$) the contract $\big(h = P, w = (w_s^*, w_f^*)\big)$ such that $w_s^* = \frac{1}{2}\big(r_P - r_A + \frac{k_A k_P (k_P - k_A)\underline{b}^2}{(k_A+k_P)^2}\big)$ and $w_f^* = 0$. The principal selects a project $d^* = \theta + \frac{k_A \underline{b}}{k_A+k_P}$ ($d^* = \theta - \frac{k_A \underline{b}}{k_A+k_P}$) and the agent then exerts his effort $e^* = \frac{1}{2}\big(r_P + r_A - \frac{k_A k_P \underline{b}^2}{k_A+k_P}\big)$.

(b) The expected payoffs of the principal and the agent are $U_P^* = \frac{1}{4}\big(r_P + r_A - \frac{k_A k_P \underline{b}^2}{k_A+k_P}\big)^2$ and $U_{A_i}^* = \frac{1}{8}\big(r_P + r_A - \frac{k_A k_P \underline{b}^2}{k_A+k_P}\big)^2$, respectively.

To complete the analysis of the optimal contract under centralization when information is symmetric, we also consider the case where neither the principal nor the agents can observe the realization of θ. In this case, the principal solves the optimal contracting problem under moral hazard:

$$\max_{\{w_s, w_f, d\}} U_P = \int_{-L}^{L} \left\{e\left(r_P - k_P(\theta - d)^2 - w_s\right) + (1 - e)(-w_f)\right\} dF(\theta) \tag{2.4}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of the agent:

$$U_{A_i} = \int_{-L}^{L} \left\{ e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2 \right\} dF(\theta) \geq \overline{U}_{A_i};$$

(c) The incentive-compatibility constraint, which stipulates that the effort level maximizes the agent's expected payoff given w_s, w_f, and d:

$$e = \arg\max_{e\in[0,1]} U_{A_i} = \int_{-L}^{L} \left\{ e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2 \right\} dF(\theta).$$

The incentive-compatibility constraint (c) can be simplified to:

$$e = r_A - k_A \int_{-L}^{L} (\theta + b_i - d)^2 dF(\theta) + w_s - w_f. \tag{2.5}$$

Substituting (2.5) into (2.4) and solving the principal's optimal contracting problem, we obtain the following first-order conditions for the optimal incentive scheme and project the principal chooses:

$$\left.\begin{array}{l} w_s = \frac{1}{2}\left(r_P - r_A + k_A \int_{-L}^{L}(\theta + b_i - d)^2 dF(\theta) - k_P \int_{-L}^{L}(\theta - d)^2 dF(\theta)\right), \\ w_f = 0 \quad \text{and} \quad -\frac{k_A \int_{-L}^{L}(\theta + b_i - d)dF(\theta)}{k_P \int_{-L}^{L}(\theta - d)dF(\theta)} = \frac{r_A - k_A \int_{-L}^{L}(\theta+b_i-d)^2 dF(\theta) + w_s}{r_P - k_P \int_{-L}^{L}(\theta-d)^2 dF(\theta) - w_s}. \end{array}\right\} \tag{2.6}$$

As one of many examples, we now assume that a state of the world θ is uniformly distributed on $[-L, L]$. By using this assumption and solving the first-order conditions in (2.6) simultaneously, we obtain the following incentive scheme and a project the principal chooses:

$$w_s = \frac{1}{2}\left(r_P - r_A + \frac{k_A k_P (k_P - k_A) {b_i}^2}{(k_A + k_P)^2} - \frac{L^2}{3}(k_P - k_A)\right), w_f = 0 \quad \text{and} \quad d = \frac{k_A b_i}{k_A + k_P}. \tag{2.7}$$

Now we consider the principal's problem of choosing an agent from a set of agents. Substituting (2.5) and (2.7) into (2.4) and solving the principal's maximization problem with respect to $b_i \in B$, we obtain the optimal type of agent, b_i°, the principal chooses. The following lemma summarizes the optimal contract under centralization. All proofs are presented in the Appendix.

Lemma 2.2

(a) If both the principal and agent are ignorant about the realization of θ, the principal, under centralization, offers the agent whose bias is $b_i^\circ = \underline{b}$ ($-\underline{b}$) the contract $\left(h = P, w = (w_s^\circ, w_f^\circ)\right)$

such that $w_s^\circ = \frac{1}{2}\big(r_P - r_A + \frac{k_A k_P (k_P - k_A) \underline{b}^2}{(k_A+k_P)^2} - \frac{L^2}{3}(k_P - k_A)\big)$ *and* $w_f^\circ = 0$. *The principal selects a project* $d^\circ = \frac{k_A \underline{b}}{k_A+k_P}$ *(*$d^\circ = -\frac{k_A \underline{b}}{k_A+k_P}$*) and the agent then exerts his effort* $e^\circ = \frac{1}{2}\big(r_P + r_A - \frac{k_A k_P \underline{b}^2}{k_A+k_P} - \frac{L^2}{3}(k_P + k_A)\big)$.

(b) The expected payoffs of the principal and the agent are $U_P^\circ = \frac{1}{4}\big(r_P + r_A - \frac{k_A k_P \underline{b}^2}{k_A+k_P} - \frac{L^2}{3}(k_P + k_A)\big)^2$ *and* $U_{A_i}^\circ = \frac{1}{8}\big(r_P + r_A - \frac{k_A k_P \underline{b}^2}{k_A+k_P} - \frac{L^2}{3}(k_P + k_A)\big)^2$, *respectively.*

The optimal contract under delegation

Now we examine the case where the principal delegates her authority to an agent, that is, the principal offers a contract $\big(h = A, w = (w_s, w_f)\big)$ to agent i. For doing this, we again work backwards. At first, we study the optimal wage scheme (w_s, w_f) which the principal offers to agent i. Then, we consider the principal's problem of what type of agent she chooses.

After observing a state of the world θ, the principal solves the optimal contracting problem under moral hazard:

$$\max_{\{w_s, w_f\}} U_P = e\left(r_P - k_P(\theta - d)^2 - w_s\right) + (1-e)(-w_f) \tag{2.8}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of the agent:

$$U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2 \geq \overline{U}_{A_i};$$

(c) The incentive-compatibility constraint, which stipulates that the project and the effort level maximize the agent's expected payoff given w_s and w_f:

$$d = \arg\max_{d \in D} U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2$$

and

$$e = \arg\max_{e \in [0,1]} U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2.$$

The incentive-compatibility constraints (c) can be simplified to:

$$d = \theta + b_i \quad \text{and} \quad e = r_A + w_s - w_f. \tag{2.9}$$

Substituting (2.9) into (2.8) and solving the principal's optimal contracting problem with respect to w_s and w_f, we obtain the following incentive scheme the principal makes:

$$w_s = \frac{1}{2}\left(r_P - r_A - k_P {b_i}^2\right) \quad \text{and} \quad w_f = 0. \tag{2.10}$$

Now we consider the principal's problem of choosing an agent from a set of agents. Substituting (2.9) and (2.10) into (2.8) and solving the principal's maximization problem with respect to $b_i \in B$, we obtain the optimal type of agent, b_i^{**}, the principal chooses. The following lemma summarizes the optimal contract under delegation. All proofs are presented in the Appendix.

Lemma 2.3

(a) Under delegation, the principal offers the agent whose bias is $b_i^{**} = \underline{b}$ (-$\underline{b}$) the contract $\big(h = A, w = (w_s^{**}, w_f^{**})\big)$ such that $w_s^{**} = \frac{1}{2}\big(r_P - r_A - k_P \underline{b}^2\big)$ and $w_f^{**} = 0$. The agent selects a project $d^{**} = \theta + \underline{b}$ ($d^{**} = \theta - \underline{b}$) and then exerts his effort $e^{**} = \frac{1}{2}\big(r_P + r_A - k_P \underline{b}^2\big)$.

(b) The expected payoffs of the principal and the agent are $U_P^{**} = \frac{1}{4}\big(r_P + r_A - k_P \underline{b}^2\big)^2$ and $U_{A_i}^{**} = \frac{1}{8}\big(r_P + r_A - k_P \underline{b}^2\big)^2$, respectively.

Lemma 2.3 says that the optimal incentive w_s^{**}, effort level of the agent e^{**}, and the expected payoffs of the principal and agent increase as $\underline{b}$ decreases. This is intuitively true, because, as the agent's bias becomes small, the private benefit of the principal in the case of project success increases, and hence the principal pays the agent more to induce him to exert more effort in implementing the project. This also implies that the principal differentiates the wages of agents according to their types. Finally, note that the expected payoffs of the principal and agent, U_P^{**} and $U_{A_i}^{**}$, have the same maximum values in the hypothetical case when $\underline{b} = 0$, i.e., the preference of the agent is perfectly aligned with the principal's.

From Lemma 2.1 and Lemma 2.3, we obtain the following proposition that characterizes the optimal allocation of authority in the view of the principal when information is completely symmetric.

Proposition 2.4 *When information is symmetric, i.e. both the principal and agent can observe the realization of θ, centralization is optimal and efficient. That is, delegation never happens under symmetric information.*

Proof. Let us compare the expected payoffs of the principal and agent under centralization with those under delegation. Since $k_P > 0$ and $k_A > 0$, we can easily show that $U_P^* > U_P^{**}$. This means that centralization is optimal for the principal. Trivially, we can show that $U_{A_i}^* > U_{A_i}^{**}$. Hence, centralization is efficient for the principal and agent. *Q.E.D.*

2.4.2 Optimal contracts when information is asymmetric

Now we consider an asymmetric information case where agents can observe the realization of θ, a state of the world, but the principal cannot. To find the optimal allocation of authority and incentives in the view of the principal, we consider two scenarios: one case where the principal keeps her authority to select a project and the other case where the principal delegates her authority to an agent.

The optimal contract under centralizaton

We examine the case where the principal keeps her authority, that is, the principal offers a contract $\big(h = P, w = (w_s, w_f)\big)$ to agent i. For doing this, we work backwards. At first, we study the optimal wage scheme (w_s, w_f) which the principal offers to agent i and a project she chooses. Then, we consider the principal's problem of what type of agent she chooses. Since the principal cannot observe the realized state of the world θ, the principal solves the following optimal contracting problem under moral hazard:

$$\max_{\{w_s, w_f, d\}} U_P = \int_{-L}^{L} \left\{ e\left(r_P - k_P(\theta - d)^2 - w_s\right) + (1-e)(-w_f) \right\} dF(\theta) \tag{2.11}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of the agent:

$$U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2 \geq \overline{U}_{A_i};$$

(c) The incentive-compatibility constraint, which stipulates that the effort level maximizes the agent's expected payoff given w_s, w_f, and d:

$$e = \arg\max_{e \in [0,1]} U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2.$$

The incentive-compatibility constraint (c) can be simplified to:

$$e = r_A - k_A(\theta + b_i - d)^2 + w_s - w_f. \tag{2.12}$$

Substituting (2.12) into (2.11) and solving the principal's optimal contracting problem, we obtain the following first-order conditions for the optimal incentive scheme and project the principal chooses:

$$\left.\begin{array}{c} w_s = \frac{1}{2}\left(r_P - r_A + k_A \int_{-L}^{L}(\theta + b_i - d)^2 dF(\theta) - k_P \int_{-L}^{L}(\theta - d)^2 dF(\theta)\right), \\ w_f = 0 \;\text{ and }\; \frac{k_P \int_{-L}^{L}(\theta-d)\left(r_A - k_A(\theta+b_i-d)^2 + w_s\right)dF(\theta)}{k_A \int_{-L}^{L}(\theta+b_i-d)(r_P - k_P(\theta-d)^2 - w_s)dF(\theta)} = -1. \end{array}\right\} \tag{2.13}$$

Assuming that a state of the world θ is uniformly distributed on $[-L, L]$ and solving the first-order conditions in (2.13) simultaneously, we obtain the following incentive scheme and project the principal chooses:

$$\left.\begin{array}{c} w_s = \frac{1}{2}\left(r_P - r_A + \frac{k_A k_P (k_P - k_A) b_i{}^2}{(k_A + k_P)^2} - \frac{4 k_A k_P b_i \Delta(b_i)}{k_A + k_P} - (k_P - k_A)\left(\Delta(b_i)^2 + \frac{L^2}{3}\right)\right), w_f = 0 \\ \text{and } \; d = \frac{k_A b_i}{k_A + k_P} + \Delta(b_i), \end{array}\right\} \tag{2.14}$$

where $\Delta(b_i) \equiv \frac{A - (B + \sqrt{A^3 + B^2})^{2/3}}{(B + \sqrt{A^3 + B^2})^{1/3}}$, $A = (k_A + k_P)^2\Big(L^2\big(L(k_A - k_P)^2 + 12 k_A k_P\big) - 3\big((r_A + r_P)(k_A + k_P) - b_i{}^2 k_A k_P\big)\Big)$ and $B = 9 b_i L^2 (3 - L) k_A k_P (k_A - k_P)(k_A + k_P)^3$.

Now we consider the principal's problem of choosing an agent from a set of agents. Substituting (2.12) and (2.14) into (2.11) and solving the principal's maximization problem with respect to $b_i \in B$, we obtain the optimal type of agent, $b_i^\star$, the principal chooses. The following lemma summarizes the optimal contract under centralization. All proofs are presented in the Appendix.

Lemma 2.5

(a) Under centralization, the principal offers the agent whose bias is $b_i^\star = \underline{b}$ (-$\underline{b}$) the contract $\big(h = P, w = (w_s^\star, w_f^\star)\big)$ such that

$$w_s^\star = \frac{1}{2}\Big(r_P - r_A + \frac{k_A k_P (k_P - k_A)\underline{b}^2}{(k_A + k_P)^2} - \frac{4 k_A k_P \underline{b}\Delta(\underline{b})}{k_A + k_P} - (k_P - k_A)\big(\Delta(\underline{b})^2 + \frac{L^2}{3}\big)\Big) \text{ and } w_f^\star = 0.$$

The principal selects a project $d^\star = \frac{k_A \underline{b}}{k_A + k_P} + \Delta(\underline{b})$ ($d^\star = -\frac{k_A \underline{b}}{k_A + k_P} - \Delta(\underline{b})$) and the agent then exerts his effort $e^\star = r_A - k_A(\theta + b_i^\star - d^\star)^2 + w_s^\star$.

(b) The expected payoffs of the principal and the agent are

$$U_P^\star = \frac{1}{2L}\int_{-L}^{L}\left(r_A - k_A(\theta + b_i^\star - d^\star)^2 + w_s^\star\right)\left(r_P - k_P(\theta - d^\star)^2 - w_s^\star\right)d\theta$$

and

$$U_{A_i}^\star = \frac{1}{4L}\int_{-L}^{L}\left(r_A - k_A(\theta + b_i^\star - d^\star)^2 + w_s^\star\right)^2 d\theta, \text{ respectively.}$$

The optimal contract under delegation

We examine the case where the principal delegates her authority to an agent, that is, the principal offers a contract $\left(h = A, w = (w_s, w_f)\right)$ to agent i. For doing this, we work backwards. At first, we study the optimal wage scheme (w_s, w_f) which the principal offers to agent i. Then, we consider the principal's problem of what type of agent she chooses. Since the principal cannot observe the realized state of the world θ, the principal solves the following optimal contracting problem under moral hazard:

$$\max_{\{w_s, w_f\}} U_P = \int_{-L}^{L}\left\{e\left(r_P - k_P(\theta - d)^2 - w_s\right) + (1-e)(-w_f)\right\}dF(\theta) \tag{2.15}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of the agent:

$$U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2 \geq \overline{U}_{A_i};$$

(c) The incentive-compatibility constraints, which stipulates that the project and the effort level maximize the agent's expected payoff given w_s and w_f:

$$d = \arg\max_{d \in D} U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2$$

and

$$e = \arg\max_{e \in [0,1]} U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2.$$

The analysis of this principal's optimal contracting problem can be done by the same way as in case of delegation under symmetric information. Consequently, we obtain Lemma 2.6 which shows the same results as those in Lemma 2.3.

Lemma 2.6

(a) Under delegation, the principal offers the agent whose bias is $b_i^{\star\star} = \underline{b}$ *(*$-\underline{b}$*) the contract* $\big(h = A, w = (w_s^{\star\star}, w_f^{\star\star})\big)$ *such that* $w_s^{\star\star} = \frac{1}{2}\big(r_P - r_A - k_P\underline{b}^2\big)$ *and* $w_f^{\star\star} = 0$*. The agent selects a project* $d^{\star\star} = \theta + \underline{b}$ *(*$d^{\star\star} = \theta - \underline{b}$*) and then exerts his effort* $e^{\star\star} = \frac{1}{2}\big(r_P + r_A - k_P\underline{b}^2\big)$*.*

(b) The expected payoffs of the principal and the agent are $U_P^{\star\star} = \frac{1}{4}\big(r_P + r_A - k_P\underline{b}^2\big)^2$ *and* $U_{A_i}^{\star\star} = \frac{1}{8}\big(r_P + r_A - k_P\underline{b}^2\big)^2$*, respectively.*

From Lemma 2.5 and Lemma 2.6, we obtain the following proposition that characterizes the optimal allocation of authority in the view of the principal when information is asymmetric.

Proposition 2.7 *When information is asymmetric, there exists* $\widehat{L}$*, a critical value of* L*, so that the principal delegates her authority to the agent as long as* $L \geq \widehat{L}$*.*

Proof. From Lemma 2.2 and 2.5, we can see that $U_P^\star$ is continuous and decreasing in L, and that it converges to its maximum value U_P^* as L goes to 0, because $w_s^\star \to w_s^*$, $d^\star \to d^*$, and $e^\star \to e^*$ as $L \to 0$. In addition, it holds that $U_P^* \geq U_P^\star \geq U_P^\circ$ where the equalities hold when $L = 0$. By the way, since it always holds that $U_P^* > U_P^{\star\star}$, there exists $\widehat{L}$, a critical value of L, such that $U_P^\star \leq U_P^{\star\star}$ for $L \geq \widehat{L}$. *Q.E.D.*

By Proposition 2.4 and 2.7, we now conclude that the optimal allocation of authority depends on the information structure. If information is symmetric between the principal and agent, the principal keeps her authority to select a project: centralization is the optimal choice in the view of the principal. If information is asymmetric, however, she delegates her authority to an agent, i.e. delegation is the optimal choice. We also have the following proposition from these results.

Proposition 2.8 *If information is asymmetric and the uncertainty the principal faces is large enough (i.e.* $L \geq \widehat{L}$*), there exists the agent's incentive to transmit his private information truthfully to the principal before principal's offering a contract.*

Proof. By Proposition 2.7 the principal delegates her authority to the agent if the uncertainty about a state of the world she faces is large enough. When information is asymmetric, consequently, the agent gets his expected payoff $U_{A_i}^{\star\star}$, which is less than $U_{A_i}^*$, the expected payoff he gets in the case of symmetric information. This implies that the agent is better off under centralization and symmetric information than under delegation and asymmetric information. Therefore, if information is asymmetric, the agent has an incentive to transmit his information truthfully to the principal and hence make her centralize a project selection. *Q.E.D.*

However, the information transmission from the agent to the principal does not work because the agent's commitment to tell the information truthfully is unenforceable: the principal would interpret the true information incorrectly because she is also aware of the incentive for the agent to lie, and the agent cannot remove this incentive within the confines of our model.

2.5 Information revelation by delegating to multiple agents

In the previous section we have studied the optimal contract between a principal and a single agent. We have shown that under asymmetric information, delegation is the optimal choice of the principal, while centralization is optimal under symmetric information. We also have found that under asymmetric information, the agent has an incentive to transmit his private information to the principal but this information transmission is impossible because of the agent's commitment problem or time inconsistency problem. Consequently, the impossibility of the information flow from the agent to the principal results in the regime of delegation under asymmetric information.

Our next question is how the principal can extract the information from the agent. As one answer for this question, in this section, we suggest the regime of *delegation to multiple agents*. Namely, we consider the optimal contract in a setting where the principal delegates her authority of selection a project to several agents.

We first suggest a simple mechanism design under delegation to multiple agents as a way of extracting agents' information. The principal offers agents a certain rule under which the agents decide a project to be implemented. Specifically, we consider an average rule as a mechanism through which a project is selected by the agents. That is, with the average rule, the project that will be implemented is the arithmetic mean of projects which are proposed by all the agents. In such a setting, we show that the principal can extract more information by delegating to multiple agents with this simple mechanism (the average rule) than by delegating to a single agent.

Second, we consider a situation where a principal delegates her authority to multiple agents and she does not offer any mechanism or any rule. We hypothetically adopt a bargaining procedure as a way of agents' determining a project to be implemented. We find, interestingly and surprisingly, that under this hypothetical setting, the principal can extract all the information by delegating to multiple agents and not providing any kind of mechanism.

Finally, we suggest the division of labor between agents as a way of extracting information. Under the regime of *delegation with division of labor*, a principal assigns a different task to each agent. That is, the principal gives her authority to an agent and then have the other agent implement a project chosen by the agent having the authority. We find that the principal can extract all the information under this regime. In order to keep our analysis simple without changing our focus, we consider the optimal contract when the principal offers a contract to **two agents**, called *agent 1* and *agent 2*.

2.5.1 Delegation to multiple agents with a mechanism design (Joint decision making of agents)

In the previous section we see that the agent chooses his favorite project if a principal delegates her authority of choosing a project to a single agent. Then what project will be chosen if the principal delegates her authority to two agents? How can the principal extract the private information from two agents? To answer these questions, we suggest an average rule as a mechanism design or decision rule under which a project to be implemented is determined by agents. The average rule prescribes that the principal should commit to implement the arithmetic mean of projects which are proposed by the agents.

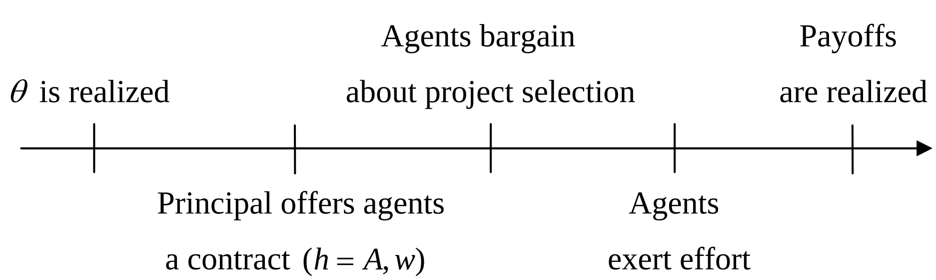

Figure 2.2: The Sequence of Events

The time structure of the model is summarized in Figure 2.2. First, a state of the world θ is realized and only the agents observe the realization of θ. Second, the principal offers two agents from a set of agents a contract ($h = A, w, r$) that specifies the allocation of authority (delegation), the wage schedule, and a mechanism that specifies a rule under which a project to be implemented is selected by the agents, where r denotes an average rule. If the agents accept the principal's offer, each agent proposes a project to the principal and a project to be implemented is chosen according to the mechanism r at the subsequent stage. Note that, under the mechanism r, the final project is the arithmetic mean of the projects the agents propose to the principal. If the agents do not accept the offer, the game ends and both the principal and the agents get nothing. Next, the agents choose their effort levels. Finally, the efforts exerted by the agents affect the probability of the project success and the payoffs of the principal and agents are realized.

Optimal contracts

We follow the basic settings of Section 2.4 about a state of nature, a set of agents, the project space, and so on other than the probability of succeeding a project. We now assume that $p(e_1, e_2) = \frac{1}{2}(e_1 + e_2)$ where e_i is the effort level of agent i and $e_i \in [0, 1]$.

Since the principal cannot observe the realized state of the world θ, the principal solves the following optimal contracting problem under moral hazard:

$$\max_{\{w_s, w_f\}} U_P = \int_{-L}^{L} \left\{p(e_1, e_2)\left(r_P - k_P(\theta - d)^2 - 2w_s\right) + (1 - p(e_1, e_2))\left(-2w_f\right)\right\} dF(\theta) \tag{2.16}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of each agent:

$$U_{A_i} = p(e_1, e_2)\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1 - p(e_1, e_2))w_f - \frac{1}{2}e_i^2 \geq \overline{U}_{A_i};$$

(c1) The incentive-compatibility constraint, which stipulates that the effort level of each agent maximizes the agent's expected payoff given w_s, w_f, d and the other agent's effort:

$$e_i = \arg\max_{e_i \in [0,1]} U_{A_i};$$

(c2) The incentive-compatibility constraint, which stipulates that a project each agent proposes to the principal maximizes the agent's expected payoff given w_s, w_f and the project the other agent proposes:

$$d_i = \arg\max_{d_i \in D} U_{A_i}.$$

The incentive-compatibility constraint of each agent (c1) can be simplified to:

$$e_i = \frac{1}{2}\left(r_A - k_A(\theta + b_i - d)^2 + w_s - w_f\right). \tag{2.17}$$

Substituting (2.17) into U_{A_i} and solving agent i's maximization problem with respect to d_i that is a project proposed by agent i, the incentive-compatibility constraint (c2) can be simplified to:

$$d_1 = 2B_1 - d_2 \ \text{ and } \ d_2 = 2B_2 - d_1, \tag{2.18}$$

where $B_1 = \theta + \frac{1}{3}(2b_1 + b_2) + \epsilon_+$ and $B_2 = \theta + \frac{1}{3}(b_1 + 2b_2) + \epsilon_-$. The derivation of the result and $\epsilon_\pm$ are presented in the Appendix.

Now we consider the stage where each agent proposes a project to the principal. Employing the concept of Nash equilibrium and solving the equations in (2.18), we find what projects the agents propose in equilibrium. Lemma 2.9 shows the results. Without loss of generality, we assume $b_1 \geq b_2$. All proofs are presented in the Appendix.

Lemma 2.9

(a) If $b_1 = b_2 (= b)$, then the project combination the agents propose at (Nash) equilibrium is any (d_1, d_2) such that $\frac{d_1+d_2}{2} = \theta + b$ and d_1, $d_2 \in D$. Consequently, a project to be implemented is $\theta + b$.

(b) If $b_1 \neq b_2$ or equivalently $b_1 > b_2$, then the projects the agents propose at equilibrium is either $(d_1 = L + \bar{b}, d_2 = 2B_2 - L - \bar{b})$, $(d_1 = L + \bar{b}, d_2 = -L - \bar{b})$, or $(d_1 = 2B_1 + L + \bar{b}, d_2 = -L - \bar{b})$. Consequently, a project to be implemented is either B_2, 0 or B_1.

From the outcomes in Lemma 2.9 we obtain the following proposition that characterizes the optimal type of agents to whom the principal offers a contract for extracting information under the average rule. All proofs are presented in the Appendix.

Proposition 2.10 *The principal can extract more information from agents when she delegates her authority to the agents biased in opposite directions and uses an average rule as a mechanism than when she delegates to a single agent. Specifically, when the principal delegates her authority to the agents with bias $\underline{b}$ and $-\underline{b}$, i.e. $b_1 = \underline{b}$ and $b_2 = -\underline{b}$, she obtains the maximum level of information through the average rule.*

Finally, by using the result in Proposition 2.10 and solving the principal's optimal contracting problem in (2.16), we can get the optimal incentive scheme the principal offers to the agents. We omit to find the optimal incentive scheme here because of its irrelevancy to our purpose and the complexity of computation.

2.5.2 Delegation to multiple agents without any mechanism

In the previous section we have shown that the principal can extract some information by delegating her authority to multiple agents and using a simple mechanism, called an average rule, as a decision rule under which a project to be implemented is selected by agents. Then what happens if the principal delegates the authority to multiple agents without giving them any mechanism or any decision rule? Is delegating to multiple agents still beneficial to the principal in that case? To answer to this question, we hypothetically adopt a bargaining procedure as a way of agents' selecting a project to be implemented. That is, we assume to design a bargaining procedure by which the multiple agents make a decision on project selection.

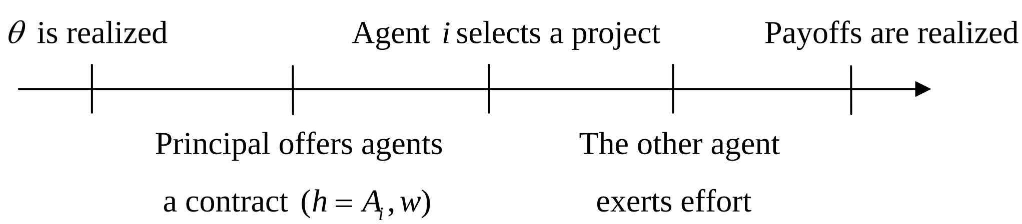

Figure 2.3: The Sequence of Events

The time structure of the model is summarized in Figure 2.3. First, a state of the world θ is realized and only the agents observe the realization of θ. Second, the principal offers two agents from a set of agents a contract $(h = A, w)$ that specifies the allocation of authority (delegation) and the wage schedule. If the agents accept the principal's offer, according to the contract, the agents select a project at the subsequent stage. Note that the decision on project choice is made in a bargaining procedure between the agents. If the agents do not accept the offer, the game ends and both the principal and the agents get nothing. Next, the agents choose their effort levels. Finally, the effort exerted by the agents affect the probability of the project success and the payoffs of the principal and agents are realized.

Optimal contracts

To solve the principal's optimal contracting problem, we work backwards. At first, we consider the optimal wage scheme (w_s, w_f) the principal offers to two agents given arbitrarily. And then, we solve the principal's problem of choosing the types of agents.

Since the principal cannot observe the realized state of the world θ, the principal solves the following optimal contracting problem under moral hazard:

$$\max_{\{w_s,w_f\}} U_P = \int_{-L}^{L} \left\{p(e_1,e_2)\left(r_P - k_P(\theta-d)^2 - 2w_s\right) + (1-p(e_1,e_2))\left(-2w_f\right)\right\} dF(\theta) \tag{2.19}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of each agent:

$$U_{A_i} = p(e_1,e_2)\left(r_A - k_A(\theta+b_i-d)^2 + w_s\right) + (1-p(e_1,e_2))w_f - \frac{1}{2}e_i^2 \geq \overline{U}_{A_i};$$

(c) The incentive-compatibility constraint, which stipulates that the effort level of each agent maximizes the agent's expected payoff given w_s, w_f, d and the other agent's effort:

$$e_i = \arg\max_{e_i\in[0,1]} U_{A_i} = p(e_1,e_2)\left(r_A - k_A(\theta+b_i-d)^2 + w_s\right) + (1-p(e_1,e_2))w_f - \frac{1}{2}e_i^2;$$

(d) The Nash bargaining solution requirement:

$$d = \arg\max_{d\in D} \left(U_{A_1} - \overline{U}_{A_1}\right)^{\alpha}\left(U_{A_2} - \overline{U}_{A_2}\right)^{1-\alpha},$$

where $\alpha = 0.5$, i.e., we assume that all the agents have equal bargaining power.

The incentive-compatibility constraint of each agent (c) can be simplified to:

$$e_i = \frac{1}{2}\left(r_A - k_A(\theta+b_i-d)^2 + w_s - w_f\right). \tag{2.20}$$

Substituting (2.20) into U_{A_i} and solving the bargaining problem between agents, the Nash bargaining solution requirement (d) can be simplified to:

$$d = \theta + \frac{1}{2}(b_1+b_2). \tag{2.21}$$

Note that the Nash bargaining solution in (2.21), $d = \theta + \frac{1}{2}(b_1 + b_2)$, is (first-best) efficient because the project maximizes the overall expected surplus $U_{A_1} + U_{A_2}$. The detail for the result is presented in the Appendix.

Substituting (2.20) and (2.21) into (2.19) and solving the principal's optimal contracting problem, we obtain the following incentive scheme:

$$w_s = \frac{1}{4}\left(r_P - 2r_A - \frac{1}{4}k_P(b_1 + b_2)^2 + \frac{1}{2}k_A(b_1 - b_2)^2\right) \text{ and } w_f = 0. \tag{2.22}$$

Now we consider the principal's problem of choosing the types of agents. Substituting (2.20), (2.21), and (2.22) into (2.19) and solving the principal's maximization problem with respect to $b_1, b_2 \in B$, we obtain the optimal type of agents the principal chooses. The following lemma summarizes the optimal contract under delegation to multiple agents. All proofs are presented in the Appendix.

Lemma 2.11

(a) *Under delegation to the agents biased in the same direction, the principal offers the agents whose biases are* $(\underline{b}, \underline{b})$ *or* $(-\underline{b}, -\underline{b})$ *the contract* $\left(h = A, w = (w_s^\diamond, w_f^\diamond)\right)$ *such that* $w_s^\diamond = \frac{1}{4}\left(r_P - 2r_A - k_P\underline{b}^2\right)$ *and* $w_f^\diamond = 0$. *Then agents choose the project* $d^\diamond = \theta + \underline{b}$ $(\theta - \underline{b})$ *and they exert their effort* $e_i^\diamond = \frac{1}{8}\left(r_P + 2r_A - k_P\underline{b}^2\right)$. *Consequently, the expected payoffs of the principal and the agents are* $U_P^\diamond = \frac{1}{16}\left(r_P + 2r_A - k_P\underline{b}^2\right)^2$ *and* $U_{A_i}^\diamond = \frac{3}{128}\left(r_P + 2r_A - k_P\underline{b}^2\right)^2$*, respectively.*

(b) *Under delegation to the agents biased in the opposite direction, the principal offers the agents whose biases are* $(\underline{b}, -\underline{b})$ *or* $(-\underline{b}, \underline{b})$ *the contract* $\left(h = A, w = (w_s^\bullet, w_f^\bullet)\right)$ *such that* $w_s^\bullet = \frac{1}{4}\left(r_P - 2r_A + 2k_A\underline{b}^2\right)$ *and* $w_f^\bullet = 0$. *Then agents choose the project* $d^\bullet = \theta$ *and they exert their effort* $e_i^\bullet = \frac{1}{8}\left(r_P + 2r_A - 2k_A\underline{b}^2\right)$. *Consequently, the expected payoffs of the principal and the agents are* $U_P^\bullet = \frac{1}{16}\left(r_P + 2r_A - 2k_A\underline{b}^2\right)^2$ *and* $U_{A_i}^\bullet = \frac{3}{128}\left(r_P + 2r_A - 2k_A\underline{b}^2\right)^2$*, respectively.*

Lemma 2.11 says that under delegation to the agents biased in the same direction, the equilibrium wage $w_s^\diamond$, effort levels of agents $e_i^\diamond$, and the expected payoffs of the principal and agents increase as $\underline{b}$ decreases. This is intuitively true, because, as the agents' bias becomes small, the private benefit of the principal in the case of project success increases, and hence the principal pays the agents more to motivate them to exert more effort in implementing the project. This also implies that the principal differentiates the wages of agents according to their biases. On the other hand, under delegation to the agents biased in the opposite direction, the equilibrium wage $w_s^\bullet$ increases as the size of agents' biases, $\underline{b}$, increases. This is also intuitively true. Note that under delegation to the agents biased in the opposite direction, the selected project in the equilibrium is $d^\bullet = \theta$, that is, the principal indirectly extracts the information about a state of the world by making the agents with different biases compete against each other. However, the principal has to pay for the price of this informational benefit. In other words, delegating her authority to the agents with different types weakens the agent's incentives to expend their effort levels in implementing a project. Therefore, the principal has to pay the agents more as the size of the agents' biases increases in order to induce them to exert more effort in implementing the project.

By comparing the principal's expected payoff in (a) with (b) of Lemma 2.11, we obtain the following proposition.

Proposition 2.12 *If $k_P \geq 2k_A$, i.e. the principal cares about the project selection relatively much compared to the agents, delegating her authority to the biased agents with opposite directions is the optimal choice of the principal. Otherwise, i.e. $k_P < 2k_A$, delegating to the biased agents with same directions is optimal for the principal.*

Proof. Trivially true. *Q.E.D.*

2.5.3 Information revelation through division of labor

Finally, we consider another way of a principal's addressing the informational asymmetry between her and agents. We suggest the regime of *division of labor* as the solution. Under this regime, the principal assigns a different task to each agent. The contract the principal offers to agents specifies each agent's task. In our model, an agent's task is either selecting a project to be implemented or implementing a selected project.

The time structure of the model is summarized in Figure 2.4. First, a state of the world θ is realized and only the agents observe the realization of θ. Second, the principal offers two agents from a set of agents a contract $(h = A_i, w)$ that specifies the task of each agent and the wage schedule. Without loss of generality, we assume that the principal offers a contract $(h = A_1, w)$, that is, agent 1 has a task to select a project and agent 2 has a task to implement the project chosen by agent 1. If the agents accept the principal's offer, according to the contract, agent i selects a project at the subsequent stage. If the agents do not accept the offer, the game ends and

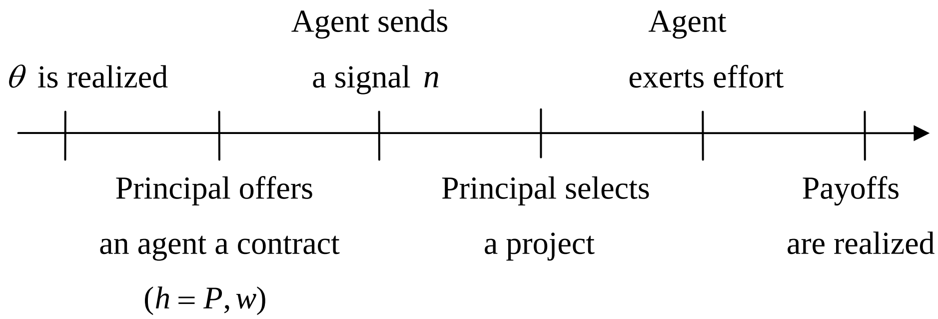

Figure 2.4: The Sequence of Events

both the principal and the agents get nothing. Next, the other agent, or agent $-i$, chooses effort level. Finally, the effort exerted by the agent affect the probability of the project success and the payoffs of the principal and agents are realized.

Optimal contracts

To solve the principal's optimal contracting problem, we work backwards. At first, we consider the optimal wage scheme (w_s, w_f) the principal offers to two agents. And then, we solve the principal's problem of choosing the types of agents.

Since the principal cannot observe the realized state of world θ, the principal solves the following optimal contracting problem under moral hazard:

$$\max_{\{w_s,w_f\}} U_P = \int_{-L}^{L} \left\{e\left(r_P - k_P(\theta - d)^2 - 2w_s\right) + (1-e)\left(-2w_f\right)\right\} dF(\theta) \tag{2.23}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of each agent:

$$U_{A_1} = e\left(r_A - k_A(\theta + b_1 - d)^2 + w_s\right) + (1-e)w_f \geq \overline{U}_{A_1}$$

and

$$U_{A_2} = e\left(r_A - k_A(\theta + b_2 - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2 \geq \overline{U}_{A_1};$$

(c1) Agent 1's incentive-compatibility constraint, which stipulates that the project agent 1 selects maximizes his expected payoff given w_s, w_f, and the effort level of agent 2:

$$d = \arg\max_{d \in D} U_{A_1};$$

(c2) Agent 2's incentive-compatibility constraint, which stipulates that the effort level of agent 2 maximizes his expected payoff given w_s, w_f, and d chosen by agent 1:

$$e = \arg\max_{e \in [0,1]} U_{A_2}.$$

The incentive-compatibility constraint of agent 2 (c2) can be simplified to:

$$e = r_A - k_A(\theta + b_2 - d)^2 + w_s - w_f. \tag{2.24}$$

Substituting (2.24) into U_{A_1} and solving the maximization problem of agent 2 with respect to d, the incentive-compatibility constraint of agent 1 (c1) can be simplified to:

$$d = \theta + \frac{1}{2}(b_1 + b_2). \tag{2.25}$$

Substituting (2.24) and (2.25) into (2.23) and solving the principal's optimal contracting problem, we obtain the following incentive scheme:

$$w_s = \frac{1}{4}\left(r_P - 2r_A - \frac{1}{4}k_P(b_1 + b_2)^2 + \frac{1}{2}k_A(b_1 - b_2)^2\right) \text{ and } w_f = 0. \tag{2.26}$$

Now we consider the principal's problem of choosing the types of agents. Substituting (2.24), (2.25), and (2.26) into (2.23) and solving the principal's maximization problem with respect to $b_1, b_2 \in B$, we obtain the optimal type of agents the principal chooses. The following lemma summarizes the optimal contract under the regime of division of labor. All proofs are presented in the Appendix.

Lemma 2.13

(a) If the principal offers a contract to the agents biased in the same direction, the principal offers the agents whose biases are $(\underline{b}, \underline{b})$ or $(-\underline{b}, -\underline{b})$ the contract $\left(h = A_1, w = (w_s^\diamond, w_f^\diamond)\right)$ such that $w_s^\diamond = \frac{1}{4}\left(r_P - 2r_A - k_P\underline{b}^2\right)$ and $w_f^\diamond = 0$. Then agent 1 chooses the project $d^\diamond = \theta + \underline{b}$ $(\theta - \underline{b})$ and agent 2 exerts effort $e^\diamond = \frac{1}{4}\left(r_P + 2r_A - k_P\underline{b}^2\right)$. Consequently, the expected payoffs of the principal and the agents are $U_P^\diamond = \frac{1}{8}\left(r_P + 2r_A - k_P\underline{b}^2\right)^2$, $U_{A_1}^\diamond = \frac{1}{16}\left(r_P + 2r_A - k_P\underline{b}^2\right)^2$, and $U_{A_2}^\diamond = \frac{1}{32}\left(r_P + 2r_A - k_P\underline{b}^2\right)^2$, respectively.

(b) If the principal offers a contract to the agents biased in the opposite direction, the principal offers the agents whose biases are $(\underline{b}, -\underline{b})$ or $(-\underline{b}, \underline{b})$ the contract $\big(h = A_1, w = (w_s^\bullet, w_f^\bullet)\big)$ such that $w_s^\bullet = \frac{1}{4}\big(r_P - 2r_A + 2k_A\underline{b}^2\big)$ and $w_f^\bullet = 0$. Then agent 1 chooses the project $d^\bullet = \theta$ and agent 2 exerts effort $e^\bullet = \frac{1}{4}\big(r_P + 2r_A - 2k_A\underline{b}^2\big)$. Consequently, the expected payoffs of the principal and the agents are $U_P^\bullet = \frac{1}{8}\big(r_P + 2r_A - 2k_A\underline{b}^2\big)^2$, $U_{A_1}^\bullet = \frac{1}{16}\big(r_P + 2r_A - 2k_A\underline{b}^2\big)^2$, and $U_{A_2}^\bullet = \frac{1}{32}\big(r_P + 2r_A - 2k_A\underline{b}^2\big)^2$, respectively.

By comparing the principal's expected payoff in (a) with (b) of Lemma 2.13, we obtain the following proposition.

Proposition 2.14 *If $k_P \geq 2k_A$, i.e. the principal cares about the project selection relatively much compared to the agents, making a contract with the agents biased in the opposite directions is the optimal choice of the principal. Otherwise, i.e. $k_P < 2k_A$, making a contract with the agents biased in the same directions is optimal for the principal.*

Proof. Trivially true. *Q.E.D.*

2.6 Further Research

In Section 2.4 we have implicitly assumed that communication between the principal and agents is impossible under centralization. That is, if a principal decides to retain her authority, she selects a project which maximizes her expected payoff conditional on her prior information (belief) about a state of the world without interacting with an agent. However, we can also think of the case where communication between the principal and agent is feasible under centralization. If communication between the principal and agents is feasible, the principal then may change her belief upon a state of the world by communicating with the agents. This kind of communication is often referred to as cheap talk, which was first analyzed by Crawford and Sobel (1982). In Crawford and Sobel (1982), a well-informed sender (the agent) may reveal some of his information by sending a signal to a receiver (the principal), who then takes an action (chooses a project) which affects the payoffs of both. Following the communication structure in Crawford and Sobel (1982), we can consider the optimal contract under centralization with communication.

The time structure of the model is summarized in Figure 2.5: First, a state of the world (θ) is realized. Note that only the agent observes the realization of θ. Second, the principal offers an agent

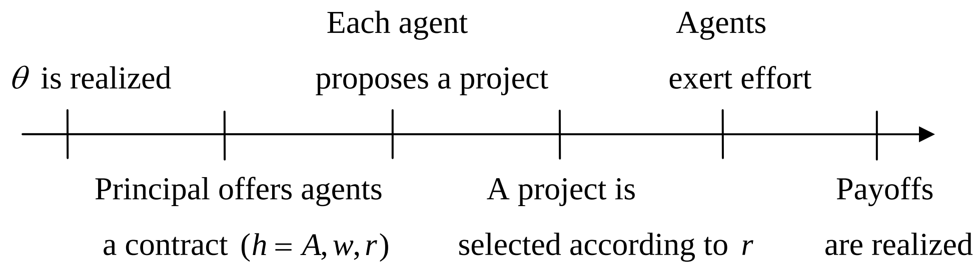

Figure 2.5: The Sequence of Events

from a set of agents a contract $(h = P, w)$ that specifies the allocation of authority (centralization) and the wage schedule. If the agent accepts the principal's offer, he sends a signal n to the principal. After receiving the agent's signal, the principal selects a project at the subsequent stage. If the agent does not accept the offer the game ends and both the principal and agent get nothing. Next, the agent chooses his effort e after observing the selected project by the principal. The effort exerted by the agent affects the project's probability of success and failure and the payoffs of the principal and the agent are realized in the final stage.

The solution concept for the stage where the agent sends a message and the principal chooses a project is the Perfect Bayesian Nash equilibrium. Formally, an equilibrium consists of a family of signaling rules for the agent, denoted $q(n|\theta)$, and action rule for the principal, denoted $d(n)$. Then the principal solves the following optimal contracting problem under moral hazard:

$$\max_{\{w_s, w_f\}} U_P = \int_{-L}^{L} \left\{ e \left(r_P - k_P(\theta - d(n))^2 - w_s \right) + (1-e)(-w_f) \right\} dF(\theta) \tag{2.27}$$

subject to:

(a) The limited-liability constraint:

$$w_s \geq 0, w_f \geq 0;$$

(b) The participation constraint of the agent:

$$U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2 \geq \overline{U}_{A_i};$$

(c) The incentive-compatibility constraint, which stipulates that the effort level maximizes the agent's expected payoff given w_s, w_f, and d:

$$e = \arg\max_{e \in [0,1]} U_{A_i} = e\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2;$$

(d) The Perfect Bayesian Nash equilibrium requirements:

- for each $\theta \in [-L, L]$, $\int_N q(n|\theta)dn = 1$, where the Borel set N is the set of feasible signals, and if n^* is in the support of $q(\cdot|\theta)$, then n^* solves

$$\max_{n \in N} U_{A_i} = e\left(r_A - k_A(\theta + b_i - d(n))^2 + w_s\right) + (1-e)w_f - \frac{1}{2}e^2; \text{and}$$

- for each n, $d(n)$ solves

$$\max_{d} U_P = \int_{-L}^{L} \left\{e\left(r_P - k_P(\theta - d)^2 - w_s\right) + (1-e)(-w_f)\right\} p(\theta|n)d\theta,$$

where $p(\theta|n) = q(n|\theta)f(\theta)/\int_{-L}^{L} q(n|t)f(t)dt$.

One of the key issues we can consider in this model is about the quality of communication, i.e., the informativeness of the signals the agent sends in equilibrium, because the optimal decision of the principal about whether to delegate or not depends on the quality of the information the agent sends under centralization with communication. Intuitively, the more informative signal of the agent will make the principal more likely keep her authority.

In the communication game of Crawford and Sobel (1982), all equilibria are characterized by a partition of $[-L, L]$, where the agent introduces noise into his signal by only specifying to which partition element the realized state of world belongs. By using the result of Crawford and Sobel (1982), Dessein (2002) showed that a principal prefers to delegate the authority to an informed agent rather than to communicate with the agent as long as the bias of the agent is not too large relative to the principal's uncertainty about a state of the world. However, the communication game in our model is slightly different from those in Crawford and Sobel (1982) and Dessein (2002). They consider the strategic communication between the uninformed principal and the informed agent from the perspective of information, while, in our model, we have to consider the communication not only from the perspective of information and but also from the perspective of effort incentives the agent faces. Therefore, the equilibria in our model should be different from those in Crawford and Sobel (1982) and Dessein (2002) - especially in the informativeness of the agent's signal. According to the informativeness of the signal, centralization with communication could be the optimal choice of the principal rather than delegation. We leave this for the future work.

2.7 Conclusion

To find the optimal choice of an organization between delegation and centralization in the views of both information and effort incentives, we have studied the optimal allocation of authority in a principal-agent setting where the uninformed principal provides the informed agent(s) with the incentives to exert a non-observable effort on a project. We first have studied the optimal contract between a principal and a single agent that specifies which party has the right to select a project and a wage scheme that is contingent on the outcome of the project. Thereafter, we have extended our basic model to the cases where the principal delegates her authority over project selection to multiple agents to answer for the question of how the principal can make the agents reveal their information.

Our main findings are as follows: 1) The optimal allocation of authority depends on the information structure. That is, if the information is asymmetric, the consideration of effort incentives and the information asymmetries between the principal and the agent makes the principal more likely to delegate her authority over project to the agent. However, if the information is symmetric, centralization is an optimal choice of the principal. 2) If the information is asymmetric, there exists the agent's incentive to reveal his information to the principal truthfully before making a contract but the information transmission between them is impossible due to the agent's commitment problem (time-inconsistency problem). 3) The principal can indirectly address the informational asymmetry problem by delegating her authority to the agents biased in opposite directions. 4) Delegating the authority to the agents entirely may be better for the principal than trying to design a mechanism for the purpose of the information revelation by the agents. 5) Under delegation, the regime of division of labor can be used as a way of addressing the information asymmetry.

Appendix

Proof of equations in (2.3) and Lemma 2.1. From (2.2) we know that $\frac{\partial e}{\partial w_f} < 0$ and $\frac{\partial U_P}{\partial w_f} < 0$. Then, by the limited-liability constraint $w_f \geq 0$, we obtain $w_f = 0$. Substituting (2.2) and $w_f = 0$ into (2.1), the principal's optimal contracting problem is written as follows:

$$\max_{\{w_s,d\}} U_P = \left(r_A - k_A(\theta + b_i - d)^2 + w_s\right)\left(r_P - k_P(\theta - d)^2 - w_s\right).$$

Solving the first-order conditions of maximizing U_P, i.e. $\frac{\partial U_P}{\partial w_s} = 0$ and $\frac{\partial U_P}{\partial d} = 0$, we have $w_s = \frac{1}{2}\left(r_P - r_A + \frac{k_A k_P (k_P - k_A) b_i{}^2}{(k_A + k_P)^2}\right)$ and $d = \theta + \frac{k_A b_i}{k_A + k_P}$. The second-order condition is satisfied, that is, the Hessian of U_P at this solution is negative definite.

Substituting (2.3) into (2.1), we get the following principal's problem in choosing the type of agent:

$$\max_{b \in B} U_P = \frac{1}{4}\left(r_P + r_A - \frac{k_A k_P b_i^2}{k_A + k_P}\right)^2.$$

Solving this maximization problem, we obtain $b_i^* = \underline{b}$ or $-\underline{b}$. By substituting b_i^* into (2.2), (2.3), U_P, and U_{A_i}, we get the results in Lemma 2.1.

Proof of equations in (2.6) and (2.7) and Lemma 2.2. From (2.5) we know that $\frac{\partial e}{\partial w_f} < 0$ and $\frac{\partial U_P}{\partial w_f} < 0$. Then, by the limited-liability constraint $w_f \geq 0$, we obtain $w_f = 0$. Substituting (2.5) and $w_f = 0$ into (2.4), the principal's optimal contracting problem is written as follows:

$$\max_{\{w_s,d\}} U_P = \left(r_A - k_A \int_{-L}^{L} (\theta + b_i - d)^2 dF(\theta) + w_s\right)\left(r_P - k_P \int_{-L}^{L} (\theta - d)^2 dF(\theta) - w_s\right).$$

The first-order conditions of maximizing U_P, i.e. $\frac{\partial U_P}{\partial w_s} = 0$ and $\frac{\partial U_P}{\partial d} = 0$, are as follows:

$$w_s = \frac{1}{2}\left(r_P - r_A + k_A \int_{-L}^{L} (\theta + b_i - d)^2 dF(\theta) - k_P \int_{-L}^{L} (\theta - d)^2 dF(\theta)\right)$$

and

$$\frac{-k_A \int_{-L}^{L} (\theta + b_i - d) dF(\theta)}{k_P \int_{-L}^{L} (\theta - d) dF(\theta)} = \frac{r_A - k_A \int_{-L}^{L} (\theta + b_i - d)^2 dF(\theta) + w_s}{r_P - k_P \int_{-L}^{L} (\theta - d)^2 dF(\theta) - w_s}.$$

The second-order condition is satisfied, that is, the Hessian of U_P at the solution satisfying the first-order conditions is negative definite.

By substituting the former first-order condition (w_s) into the latter, we get the following optimal condition for the project selection:

$$k_A \int_{-L}^{L} (\theta + b_i - d) dF(\theta) + k_P \int_{-L}^{L} (\theta - d) dF(\theta) = 0.$$

Now we assume that θ is uniformly distributed on $[-L, L]$. With this assumption, solving the above equation, we have $d = \frac{k_A b_i}{k_A + k_P}$. Substituting d into w_s of the first-order condition, we obtain $w_s = \frac{1}{2}\left(r_P - r_A + \frac{k_A k_P (k_P - k_A) b_i^2}{(k_A + k_P)^2} - \frac{L^2}{3}(k_P - k_A)\right)$.

Substituting (2.7) into (2.4), we get the following principal's problem in choosing the type of agent:

$$\max_{b \in B} U_P = \frac{1}{4}\left(r_P + r_A - \frac{k_A k_P b_i^2}{k_A + k_P} - \frac{L^2}{3}(k_A + k_P)\right)^2.$$

Solving this maximization problem, we obtain $b_i^\circ = \underline{b}$ or $-\underline{b}$. By substituting b_i° into (2.5), (2.7), U_P, and U_{A_i}, we get the results in Lemma 2.2.

Proof of equations in (2.10) and Lemma 2.3. From (2.9) we know that $\frac{\partial e}{\partial w_f} < 0$ and $\frac{\partial U_P}{\partial w_f} < 0$. Then, by the limited-liability constraint $w_f \geq 0$, we obtain $w_f = 0$. Substituting (2.9) and $w_f = 0$ into (2.8), the principal's optimal contracting problem is written as follows:

$$\max_{w_s} U_P = (r_A + w_s)(r_P - k_P b_i^2 - w_s).$$

Solving the first-order conditions of maximizing U_P, i.e. $\frac{\partial U_P}{\partial w_s} = 0$, we have $w_s = \frac{1}{2}(r_P - r_A k_P b_i^2)$. The second-order condition at this solution is satisfied.

Substituting (2.10) into (2.8), we get the following principal's problem in choosing the type of agent:

$$\max_{b \in B} U_P = \frac{1}{4}(r_P + r_A - k_P b_i^2)^2.$$

Solving this maximization problem, we obtain $b_i^{**} = \underline{b}$ or $-\underline{b}$. By substituting b_i^{**} into (2.9), (2.10), U_P, and U_{A_i}, we get the results in Lemma 2.3.

Proof of equations in (2.13) and (2.14) and Lemma 2.5. From (2.12) we know that $\frac{\partial e}{\partial w_f} < 0$ and $\frac{\partial U_P}{\partial w_f} < 0$. Then, by the limited-liability constraint $w_f \geq 0$, we obtain $w_f = 0$. Substituting (2.12) and $w_f = 0$ into (2.11), the principal's optimal contracting problem is written as follows:

$$\max_{\{w_s, d\}} U_P = \int_{-L}^{L} \left(r_A - k_A(\theta + b_i - d)^2 + w_s\right)\left(r_P - k_P(\theta - d)^2 - w_s\right) dF(\theta).$$

The first-order conditions of maximizing U_P, i.e. $\frac{\partial U_P}{\partial w_s} = 0$ and $\frac{\partial U_P}{\partial d} = 0$, are as follows:

$$w_s = \frac{1}{2}\left(r_P - r_A + k_A \int_{-L}^{L} (\theta + b_i - d)^2 dF(\theta) - k_P \int_{-L}^{L} (\theta - d)^2 dF(\theta)\right)$$

and

$$-\frac{k_A}{k_P} = \frac{\int_{-L}^{L}(\theta - d)\left(r_A - k_A(\theta + b_i - d)^2 + w_s\right)dF(\theta)}{\int_{-L}^{L}(\theta + b_i - d)\left(r_P - k_P(\theta - d)^2 - w_s\right)dF(\theta)}.$$

The second-order condition is satisfied, that is, the Hessian of U_P at the solution satisfying the first-order conditions is negative definite.

By substituting the former first-order condition (w_s) into the latter, we get the following optimal condition for the project selection:

$$-\frac{k_A}{k_P} = \frac{\int_{-L}^{L}(\theta - d)\left\{r_P + r_A - 2k_A(\theta + b_i - d)^2 + k_A\int_{-L}^{L}(\theta + b_i - d)^2 dF(\theta) - k_P\int_{-L}^{L}(\theta - d)^2 dF(\theta)\right\}dF(\theta)}{\int_{-L}^{L}(\theta + b_i - d)\left\{r_P + r_A - 2k_P(\theta - d)^2 - k_A\int_{-L}^{L}(\theta + b_i - d)^2 dF(\theta) + k_P\int_{-L}^{L}(\theta - d)^2 dF(\theta)\right\}dF(\theta)}.$$

Now we assume that θ is uniformly distributed on $[-L, L]$. With this assumption, solving the above equation, we have $d = \frac{k_A b_i}{k_A + k_P} + \Delta(b_i)$. Substituting d into w_s of the first-order condition, we obtain w_s in (2.14).

Substituting (2.14) into (2.11) and solving the principal's problem in choosing the type of agent, we obtain $b_i^\star = \underline{b}$ or $-\underline{b}$. By substituting $b_i^\star$ into (2.12), (2.14), U_P, and U_{A_i}, we get the results in Lemma 2.5.

Proof of equations in (2.18), Lemma 2.9, and Proposition 2.10. Substituting the incentive-compatibility constraints of agents in (2.17) into U_{A_i} and differentiating U_{A_i} with respect to d_i, we obtain the following first-order condition for maximizing the expected payoff of agent i:

$$\frac{1}{4}k_A(P_i Q_i + P_{-i}Q_i + P_i Q_{-i}) = 0$$

where $P_i = \theta + b_i - \frac{d_1+d_2}{2}$ and $Q_i = r_A - k_A\left(\theta + b_i - \frac{d_1+d_2}{2}\right)^2 + w_s - w_f$. The second-order condition is $-\frac{1}{8}k_A\left\{3(r_A + w_s - w_f) - k_A\left(3\theta + 2b_i + b_{-i} - \frac{3}{2}(d_1 + d_2)\right)^2\right\}$ and this is negative because we assume that r_A is sufficiently large. From the first-order condition for each agent, we have the incentive-compatibility constraints of the agents in (2.18), where ϵ_+ and ϵ_- are defined as follows:

$$\epsilon_\pm = \frac{1}{6k_A}\left\{\frac{6\ 2^{1/3}k_A(r_A + w_s - w_f)}{(\sqrt{k_A^3(k_A^3(b_1 - b_2)^6 - 108(r_A + w_s - w_f)^3)} \pm k_A^3(b_1 - b_2)^3)^{1/3}} + 2^{2/3}(\sqrt{k_A^3(k_A^3(b_1 - b_2)^6 - 108(r_A + w_s - w_f)^3)} \pm k_A^3(b_1 - b_2)^3)^{1/3}\right\}.$$

By examining the first-order conditions of the agents, it is shown that $B_1 = B_2 = \theta + b$ if $b_1 = b_2 = b$ and $\theta + b_2 < B_2 < \theta + \frac{b_1+b_2}{2} < B_1 < \theta + b_1$ if $b_1 > b_2$. Note that B_1 and B_2 depend on θ and that the set of strategy of each agent, d_i, is assumed to be between $-(L + \bar{b})$ and $L + \bar{b}$. Using these facts and solving simultaneously the two equations in (2.18), that are virtually the best responses of the agents, we obtain the results in Lemma 2.9. That is, if $b_1 = b_2 = b$, the best response of an agent corresponds with the one of the other and hence any (d_1, d_2) satisfying $\frac{d_1+d_2}{2} = \theta + b$ is an equilibrium. If $b_1 \neq b_2$, then we obtain the following projects the agents propose in equilibrium:

$$\begin{cases} (d_1, d_2) = (L + \bar{b},\ 2B_2 - L - \bar{b}) \Rightarrow d = B_2 & \text{if } B_1 > B_2 > 0, \\ (d_1, d_2) = (L + \bar{b}, -L - \bar{b}) \Rightarrow d = 0 & \text{if } B_1 \geq 0 \text{ and } B_2 \leq 0, \\ (d_1, d_2) = (2B_1 + L + \bar{b}, -L - \bar{b}) \Rightarrow d = B_1 & \text{if } 0 > B_1 > B_2. \end{cases}$$

Considering the results in Lemma 2.9, we now solve the principal's problem of choosing the types of agents. If the principal chooses the same type of agents, i.e. $b_1 = b_2 = b$, the project selected through the average rule is $\theta + b$, which is no more than a project selected under delegation to a single agent with bias b. Therefore, choosing the same type of agents does not help the principal to extract some information from agents. Eliminating this case of choosing the same type of agents, we have the following three possible cases according to the types of the agents chosen by the principal:

1. $b_1 > b_2 > 0$ (The agents biased in same direction)

2. $0 > b_1 > b_2$ (The agents biased in same direction)

3. $b_1 > 0$ and $b_2 < 0$ (The agents biased in opposite directions)

For each case, by using the results in Lemma 2.9 and the fact that $\theta + b_2 < B_2 < \theta + \frac{b_1+b_2}{2} < B_1 < \theta + b_1$ if $b_1 > b_2$, we find the following projects (d_1, d_2) the agents propose and a project (d) selected through the average rule in equilibrium:

1. $b_1 > b_2 > 0$

$$\begin{cases} (d_1, d_2) = (L + \bar{b},\ 2B_2 - L - \bar{b}) \Rightarrow d = B_2 \prec \theta + b_2 & \text{if } B_1 > B_2 > 0, \\ (d_1, d_2) = (L + \bar{b}, -L - \bar{b}) \Rightarrow d = 0 \prec \theta + b_2 & \text{if } B_1 \geq 0 \text{ and } B_2 \leq 0, \\ (d_1, d_2) = (2B_1 + L + \bar{b}, -L - \bar{b}) \Rightarrow d = B_1 \prec \theta + b_2 & \text{if } 0 > B_1 > B_2. \end{cases}$$

2. $0 > b_1 > b_2$

$$\begin{cases} (d_1, d_2) = (L + \bar{b},\ 2B_2 - L - \bar{b}) \Rightarrow d = B_2 \prec \theta + b_1 & \text{if } B_1 > B_2 > 0, \\ (d_1, d_2) = (L + \bar{b}, -L - \bar{b}) \Rightarrow d = 0 \prec \theta + b_1 & \text{if } B_1 \geq 0 \text{ and } B_2 \leq 0, \\ (d_1, d_2) = (2B_1 + L + \bar{b}, -L - \bar{b}) \Rightarrow d = B_1 \prec \theta + b_1 & \text{if } 0 > B_1 > B_2. \end{cases}$$

3. $b_1 > 0$ and $b_2 < 0$

$$\begin{cases} (d_1,d_2) = (L+\bar{b},\ 2B_2 - L - \bar{b}) \Rightarrow d = B_2 \succ \theta + b_1, \theta + b_2 & \text{if } B_1 > B_2 > 0, \\ (d_1,d_2) = (L+\bar{b}, -L-\bar{b}) \Rightarrow d = 0 \succ \theta + b_1, \theta + b_2 & \text{if } B_1 \geq 0 \text{ and } B_2 \leq 0, \\ (d_1,d_2) = (2B_1 + L + \bar{b}, -L-\bar{b}) \Rightarrow d = B_1 \succ \theta + b_1, \theta + b_2 & \text{if } 0 > B_1 > B_2, \end{cases}$$

where $\prec$ and $\succ$ represents the principal's preference over projects.

As we can see the results above, the project to be selected through the average rule for the case 1 and 2 is always dominated to the project $\theta + b_1$ or $\theta + b_2$. This implies that delegation to a single agent is more beneficial to the principal rather than delegation to multiple agents biased in the same direction. That is, for the principal to exploit the informational benefit under delegation to multiple agents with the average rule, she has to delegate her authority to the agents biased in the opposite directions. Note that the project selected in case 3 is preferred to the projects $\theta + b_1$ and $\theta + b_2$. To extract the maximum level of information, furthermore, the principal delegates her authority to the agents who are biased in the opposite directions and each of whom has the smallest bias, i.e. $b_1 = \underline{b}$ and $b_2 = -\underline{b}$, because the distance between B_1 and B_2 increases with the distance between b_1 and b_2. Then, according to the realization of θ, a project to be selected under delegation to multiple agents with the average rule is as follows:

$$\begin{cases} (d_1,d_2) = (L+\bar{b},\ 2B_2 - L - \bar{b}) \Rightarrow d = B_2 & \text{if } \theta \in [\frac{1}{3}\underline{b} + \epsilon_{\oplus}, L], \\ (d_1,d_2) = (L+\bar{b}, -L-\bar{b}) \Rightarrow d = 0 & \text{if } \theta \in (-\frac{1}{3}\underline{b} + \epsilon_{\ominus}, \frac{1}{3}\underline{b} + \epsilon_{\oplus}), \\ (d_1,d_2) = (2B_1 + L + \bar{b}, -L-\bar{b}) \Rightarrow d = B_1 & \text{if } \theta \in [-L, -\frac{1}{3}\underline{b} + \epsilon_{\ominus}], \end{cases}$$

where $\epsilon_{\oplus(\ominus)}$ is the value of $\epsilon_{+}(\epsilon_{-})$ when $b_1 = \underline{b}$ and $b_2 = -\underline{b}$.

Proof of equations in (2.21) and (2.22) and Lemma 2.11. Substituting (2.20) into U_{A_i}, we have $U_{A_i} = \frac{1}{8}\big(r_A - k_A(\theta+b_i-d)^2 + w_s - w_f\big)\big(3(r_A+w_s-w_f) - k_A(\theta+b_i-d)^2 - 2k_A(\theta+b_{-i}-d)^2\big) + w_f$. Then, the Nash bargaining solution requirement is written as follows:

$$d = \arg\max_{d\in D} \left(\frac{1}{8}A_1(A_1+2A_2)+w_f\right)^{1/2} \left(\frac{1}{8}A_2(A_2+2A_1)+w_f\right)^{1/2}$$

where $A_i = r_A - k_A(\theta + b_i - d)^2 + w_s - w_f$. From the first-order condition for maximizing the product of agents' surplus, we obtain $d = \theta + \frac{1}{2}(b_1 + b_2)$. The second-order condition at this solution is satisfied. We can also show that $d = \theta + \frac{1}{2}(b_1 + b_2)$ maximizes the overall surplus of the agents, $U_{A_1} + U_{A_2}$, because their expected payoffs are symmetric.

From (2.20) we know that $\frac{\partial e}{\partial w_f} < 0$ and $\frac{\partial U_P}{\partial w_f} < 0$. Then, by the limited-liability constraint $w_f \geq 0$, we obtain $w_f = 0$. Substituting (2.20), (2.21), and $w_f = 0$ into (2.19), the principal's optimal contracting problem is written as follows:

$$\max_{w_s} U_P = \frac{1}{2}\left(r_A - \frac{1}{4}k_A(b_1 - b_2)^2 + w_s\right)\left(r_P - \frac{1}{4}k_P(b_1 + b_2)^2 - 2w_s\right).$$

Solving the first-order conditions of maximizing U_P, i.e. $\frac{\partial U_P}{\partial w_s} = 0$, we have $w_s = \frac{1}{4}\left(r_P - 2r_A - \frac{1}{4}k_P(b_1 + b_2)^2 + \frac{1}{2}k_A(b_1 - b_2)^2\right)$. The second-order condition is satisfied.

Substituting (2.20), (2.21), and (2.22) into (2.19), we get the following principal's problem in choosing the type of agent:

$$\max_{b_1, b_2 \in B} U_P = \frac{1}{16}\left(2r_A + r_P - \frac{1}{2}k_A(b_1 - b_2)^2 - \frac{1}{4}k_P(b_1 + b_2)^2\right)^2.$$

Solving this maximization problem, we obtain the following optimal type of agents:

1. the same types of agents: $(b_1, b_2) = (\underline{b}, \underline{b})$ or $(-\underline{b}, -\underline{b})$
2. the different types of agents: $(b_1, b_2) = (\underline{b}, -\underline{b})$ or $(-\underline{b}, \underline{b})$.

By substituting (b_1, b_2) in each case into (2.20), (2.21), (2.22), U_P, and U_{A_i}, we get the results in Lemma 2.11.

Proof of equations in (2.25) and (2.26) and Lemma 2.13. Substituting (2.24) into U_{A_1}, we have $U_{A_1} = \left(r_A - k_A(\theta + b_2 - d)^2 + w_s - w_f\right)\left(r_A - k_A(\theta + b_1 - d)^2 + w_s - w_f\right) + w_f$. From the first-order condition for maximizing U_{A_1} in terms of d, we obtain $d = \theta + \frac{1}{2}(b_1 + b_2)$. The second-order condition at $d = \theta + \frac{1}{2}(b_1 + b_2)$ is strictly negative and so satisfied.

Substituting $d = \theta + \frac{1}{2}(b_1 + b_2)$ into (2.24), we have $e = r_A - \frac{1}{4}k_A(b_2 - b_1)^2 + w_s - w_f$. We know that $\frac{\partial e}{\partial w_f} < 0$ and $\frac{\partial U_P}{\partial w_f} < 0$. Then, by the limited-liability constraint $w_f \geq 0$, we obtain $w_f = 0$. Substituting (2.24), (2.25), and $w_f = 0$ into (2.23), the principal's optimal contracting problem is written as follows:

$$\max_{w_s} U_P = \left(r_A - \frac{1}{4}k_A(b_1 - b_2)^2 + w_s\right)\left(r_P - \frac{1}{4}k_P(b_1 + b_2)^2 - 2w_s\right).$$

Solving the first-order conditions of maximizing U_P with respect to w_s, i.e. $\frac{\partial U_P}{\partial w_s} = 0$, we have $w_s = \frac{1}{4}\left(r_P - 2r_A - \frac{1}{4}k_P(b_1 + b_2)^2 + \frac{1}{2}k_A(b_1 - b_2)^2\right)$. The second-order condition is satisfied.

Substituting (2.24), (2.25), and (2.26) into (2.23), we get the following principal's problem in choosing the type of agent:

$$\max_{b_1, b_2 \in B} U_P = \frac{1}{8}\left(2r_A + r_P - \frac{1}{2}k_A(b_1 - b_2)^2 - \frac{1}{4}k_P(b_1 + b_2)^2\right)^2.$$

Solving this maximization problem, we obtain the following optimal type of agents:

1. the same types of agents: $(b_1, b_2) = (\underline{b}, \underline{b})$ or $(-\underline{b}, -\underline{b})$

2. the different types of agents: $(b_1, b_2) = (\underline{b}, -\underline{b})$ or $(-\underline{b}, \underline{b})$.

By substituting (b_1, b_2) in each case into (2.24), (2.25), (2.26), U_P, and U_{A_i}, we get the results in Lemma 2.13.

Chapter 3

Weakest-link Contests with Group-specific Public Good Prizes

> "PART OF THE BEAUTY and mystery of basketball rests in the variety of its team requirements. Championships are not won unless a team has forged a high degree of unity, attainable only through the selflessness of each of its players. Statistics don't always measure teamwork; holding the person you're guarding scoreless doesn't show up in your stats. But when you're "taking care of business," you're working to produce a championship team, and "We won" comes to mean more and lasts longer than the ephemeral "I scored." Solidarity becomes an essential part of your professionalism.
>
> The society we live in glorifies individualism, what Ross Perot used to champion with the expression "eagles don't flock." Basketball teaches a different lesson: that untrammeled individualism destroys the chance for achieving victory. Players must have sufficient self-knowledge to take the long view-to see that what any one player can do alone will never equal what a team can do together." (Bradley, Bill (1998), p. 43)

3.1 Introduction

In most of the public goods literature, it has been assumed that the socially available amount of a public good is the simple sum of the separate amounts produced by the individuals in the community. Departing from this traditional assumption, Hirshleifer (1983) suggested different possible ways, which are called *social composition functions*, of combining individual contributions into a socially available amount of a public good. One of the social composition functions he concentrated is the weakest-link function. The weakest-link function describes various 'linear' situations where each member in a society successively has a kind of veto power over the total achievement of the society. As an example of a weakest-link public good, Hirshleifer (1983) imagined a low-lying island, Anarchia, where individuals with an extreme aversion to collective action live along the coast line. Each individual builds his own dike, and then the overall protection of the island depends on the lowest dike in the island because once any dike was breached the whole island would be flooded. A similar example is the protection of a military.

In most of the contest literature, it is also assumed that a group's probability of winning the prize depends on the aggregate effort level of its members. Following this assumption, Katz et al. (1990), Ursprung (1990), Baik (1993), Riaz et al. (1995), Dijkstra (1998), Baik et al. (2001), and Baik (2008) study contests with group-specific public good prizes. Among them, Baik (1993) and (2008) are closely related to this chapter. Baik (1993) and (2008) consider the contests where the individual players in each group choose their effort levels noncooperatively to win their public-good prize, and show that in the equilibrium, only the highest-valuation player in each group expend positive effort and the rest in the group free ride on the player.

In this chapter we incorporate the weakest-link rule of Hirshleifer (1983) into contests with group-specific public good prizes. Specifically, we consider a contest where the individual players in each group choose their effort levels noncooperatively to win their public-good prize and each group's probability of winning the prize depends on the minimum effort levels of the other groups as well as its own, not the aggregate effort levels of the groups. In other words, we use the weakest-link technology in computing the socially available effort level of each group in the contest; we assume that each player in a group is responsible for one link of a chain.

Many examples of teamwork involve the contests with the weakest-link technology. For example, consider a contest between several research teams, where each research team consists of some experts in different fields and the expert's input from each field is indispensable to the research. Then the success of one research team may depends on the weakest performer in the team. Also, the contests in team sports which require an organizational team play, such as basketball, baseball, and a team race, are other examples of the contests with the weakest-link technology.

In our model of the contests with the weakest-link rule, we show the following. First, contrary to the results in Baik (1993, 2008), the lowest-valuation players in each group play decisive roles in determining the Nash equilibria of the game and no free riding problem exists in equilibrium. Second, similar to the results analyzed in Hirshleifer (1983), there exist incentives for the high-valuation players in each group to subsidize the low-valuation players in their group. Finally, we find the equilibrium subsidy rates of the groups in the contest.

The chapter proceeds as follows. In Section 3.2, we develop the general model, and find the Nash equilibria of the game in Section 3.3. Section 3.4 presents and analyzes a simple model where the players in the contest are budget-constrained. In Section 3.5, we develop and analyze a simple model in which the players in each group subsidizes others in their group, and we discuss about further research in 3.6. Finally, Section 3.6 presents conclusions.

3.2 The model

We follow the basic settings of Baik (2008) to develop the general model in this chapter. Consider a contest in which n groups compete to win a prize, where $n > 1$. Group i consists of m_i risk-neutral players who expend effort to win the prize, where $m_i \geq 1$. The prize is a public good within each group−thus, it is called a group-specific public good prize. The individual player's valuations for the prize may differ. Let v_{ik} represent the valuation for the prize of player k in group i. Each player's valuation for the prize is positive and publicly known.

Assumption 3.1 *Without loss of generality, we assume that* $v_{i1} \geq v_{i2} \geq \cdots \geq v_{im_i} > 0$.

Let x_{ik} represent the effort level expended by player k in group i, and let X_i represent the minimum effort level expended by the players in group i, so that $X_i = min\{x_{i1}, x_{i2}, \ldots, x_{im_i}\}$. Each player's effort is irreversible−each player cannot recover his effort expended whether or not his group wins the prize. Effort levels are nonnegative, and are measured in units commensurate with the prize. Let p_i denote the probability that group i wins the prize. We assume that each group's probability of winning depends on the other groups' minimum effort levels as well as its own. (the weakest-link contest success function) Specifically, we assume that the contest success function for group i is

$$p_i = p_i(X_1, \ldots, X_n),$$

where $0 \leq p_i \leq 1$, the function p_i has the properties specified in Assumption 3.2 below, and $\sum_{j=1}^{n} p_j = 1$.

Assumption 3.2 *We assume that* $\partial p_i/\partial X_i \geq 0$, $\partial^2 p_i/\partial X_i^2 \leq 0$, $\partial p_i/\partial X_j \leq 0$, *and* $\partial^2 p_i/\partial X_j^2 \geq 0$. *We further assume that* $\partial p_i/\partial X_i > 0$ *and* $\partial^2 p_i/\partial X_i^2 < 0$ *when* $X_j > 0$ *for some* j, *and* $\partial p_i/\partial X_j < 0$ *and* $\partial^2 p_i/\partial X_j^2 > 0$ *when* $X_i > 0$.

Assumption 3.2 says that, given the rival groups' minimum effort levels, each group's probability of winning is increasing in its minimum effort level at a decreasing rate. It also says that each group's probability of winning is decreasing in a rival group's minimum effort level at a decreasing rate, given that the minimum effort levels of the rest remain constant. Under Assumption 3.2, the group expending the largest minimum effort level does not win the prize with certainty−that is, it may lose the prize−when there are at least two groups which expend positive minimum effort levels.

Let π_{ik} represent the expected payoff for player k in group i. Then the payoff function for player k in group i is

$$\pi_{ik} = v_{ik} p_i(X_1, \ldots, X_n) - x_{ik}.$$

We assume that all the players in the contest choose their effort levels independently and simultaneously. Finally, we assume that all of the above is common knowledge among the players, and employ Nash equilibrium as the solution concept.

3.3 The Nash equilibria of the game

Let x_{ik}^b denote the best response of player k in group i when he can make the other players in his group expend any effort level he wants, given effort levels of the players in the other groups. In other words, x_{ik}^b is the effort level which maximizes his expected payoff

$$\pi_{ik}^b = v_{ik} p_i(X_1, \ldots, X_{i-1}, x_{ik}, X_{i+1}, \ldots, X_n) - x_{ik}$$

subject to the nonnegativity constraint, $x_{ik} \geq 0$. Thus, x_{ik}^b satisfies the first-order condition for maximizing π_{ik}^b:

$$v_{ik} \frac{\partial p_i}{\partial x_{ik}} = 1.$$

Note that under Assumption 3.1 and 3.2, it holds that $x_{i1}^b(X_{-i}) \geq x_{i2}^b(X_{-i}) \geq \cdots \geq x_{im_i}^b(X_{-i})$ for all $X_{-i} \geq (0, \ldots, 0)$, where $X_{-i} = (X_1, \ldots, X_{i-1}, X_{i+1}, \ldots, X_n)$. Now let x_{ik}^B denote the best response of player k in group i, given effort levels of all the other players in the contest. By definition, it is the effort level which maximizes his expected payoff, π_{ik}, subject to $x_{ik} \geq 0$. By considering the characteristics of the minimum function, i.e. $min\{x_{i1}, \ldots, x_{im_i}\}$, and using the definition of x_{ik}^b, we obtain the best responses of players in group i:

$$x_{i1}^B(x_{-i1}, X_{-i}) = min\left\{x_{-i1}, x_{i1}^b(X_{-i})\right\},$$

$$x_{i2}^B(x_{-i2}, X_{-i}) = min\left\{x_{-i2}, x_{i2}^b(X_{-i})\right\},$$

$$\vdots$$

$$x_{im_i}^B(x_{-im_i}, X_{-i}) = min\left\{x_{-im_i}, x_{im_i}^b(X_{-i})\right\},$$

where $x_{-ik} = (x_{i1}, x_{i2}, \ldots, x_{ik-1}, x_{ik+1}, \ldots, x_{im_i})$. From the best responses of the players, we can see that the players in each group match their effort levels in equilibrium. Lemma 3.3 summarize this result.

Lemma 3.3 *Given effort levels of the players in the other groups, X_{-i}, the best responses of the players in group i are the following vectors of effort levels:*

$$(x_{i1}, \ldots, x_{im_i}) \text{ such that } x_{i1} = \cdots = x_{im_i} \text{ and } 0 \leq X_i \leq x_{im_i}^b(X_{-i}).$$

Lemma 3.3 implies that, given effort levels of the other groups, the players in a group expend the same effort level, that is, they match their effort levels. It also says that the matched effort level of a group can be any value between 0 and the best response of the player whose valuation is the lowest in that group.

Now we are ready to obtain the pure-strategy Nash equilibria of the game. Using the result in Lemma 3.3, we obtain Proposition 3.4.

Proposition 3.4

(a) There exist multiple pure-strategy Nash equilibria in the game.

(b) To obtain the Nash equilibria of the game, one only needs to consider a reduced n-player game where the lowest-valuation players in each group compete against each other to win the prize.

(c) There exists an unique coalition-proof Nash equilibrium of the game at which the players in any group do not have an incentive to coordinate and deviate from the equilibrium. The following

strategy profile constitutes the coalition-proof Nash equilibrium: $(x_{11}^N, \ldots, x_{1m_1}^N, \ldots, x_{n1}^N, \ldots, x_{nm_n}^N) = (x_1^N, \ldots, x_1^N, \ldots, x_n^N, \ldots, x_n^N)$, *where* x_i^N *is the equilibrium effort level of the lowest-valuation player in group* i *when the original game is reduced to the* n*-player game consisting of the lowest-valuation players in each group.*

Proof. By considering Lemma 3.3 and the definition of Nash equilibrium, (1) and (2) are obvious. We know that, given effort levels of the players in the other groups (X_{-i}), the largest matched effort level of group i, or $x_{im_i}^b(X_{-i})$, dominates the others matched effort levels belonging to $[0, x_{im_i}^b(X_{-i}))$ in a sense that it results in the highest expected payoffs of the players in group i. In other words, if the players in each group can cooperate with each other in choosing their effort levels, the best response of the players in group i shrinks from $[0, x_{im_i}^b(X_{-i})]$ to $x_{im_i}^b(X_{-i})$. Consequently, the original $(\sum_{j=1}^n m_j)$-player game is reduced to the n-player game where n players, each of whom is the lowest-valuation player in a group, compete to win the prize, and the reduced n-player game has an unique Nash equilibrium in our model. *Q.E.D.*

Proposition 3.4 implies that the Nash equilibrium of the game is determined by the interaction of players with the lowest valuation in each group. That is, the lowest-valuation players in each group play decisive roles in determining the Nash equilibria of the contest. Also, it says that, unlike the result of Baik (2008), i.e. only the highest-valuation players expend positive effort and the rest free ride on them, there is no free riding problem in our model.

3.3.1 An example

As an example we consider a simple contest where there are two groups, group 1 and group 2, and each group consists of two players. We also assume that all the players in the contest do not have any budget constraint, or equivalently, each player has sufficiently large wealth. Keeping all the notations and assumptions in Section 3.2, x_{ik} represents the effort level expended by player k in group i and X_i represents the minimum effort level expended by players in group i, i.e. $X_i = min\,\{x_{i1}, x_{i2}\}$. As a specific form of the contest success function for group i, we use the simplest logit-form contest success function that is extensively used in the literature on the theory of contests. Specifically, the contest success function for group i is defined as follows:

$$p_i(X_1, X_2) = \begin{cases} \frac{X_i}{\sum_{j=1}^2 X_j} = \frac{min\{x_{i1}, x_{i2}\}}{\sum_{j=1}^2 min\{x_{j1}, x_{j2}\}} & \text{if } X_1 + X_2 > 0 \\ \quad 1/2 & \text{if } X_1 + X_2 = 0. \end{cases}$$

This specifies that group i's probability of winning the prize is equal to its minimum effort level divided by the sum of the minimum effort levels of the groups in the contest if the sum of the minimum effort levels is positive, and it is equal to 1/2 if the sum of the minimum effort levels is zero. Then the payoff function for player k in group i is

$$\pi_{ik} = v_{ik} \frac{X_i}{X_1 + X_2} - x_{ik}.$$

To obtain the pure-strategy Nash equilibria of the game, we first consider the best responses of player k in group i, given effort levels of all the other players in the contest. First, let x_{ik}^b denote the best response of player k in group i when the other player's effort level in group i is equal to that of player k in group i, given effort levels of the players in the other group. That is, x_{ik}^b is the effort level which maximizes his expected payoff

$$\pi_{ik}^b = v_{ik} \frac{x_{ik}}{x_{ik} + X_{-i}} - x_{ik}$$

subject to the nonnegativity constraint, $x_{ik} \geq 0$, where $X_{-i} = min\,\{x_{j1}, x_{j2}\}$ for $j \neq i$. Thus, x_{ik}^b satisfies the first-order condition for maximizing π_{ik}^b:

$$(x_{ik} + X_{-i})^2 = X_{-i} v_{ik}.$$

Using the first-order condition for maximizing π_{ik}^b, we obtain the following x_{ik}^b:

$$x_{ik}^b(X_{-i}) = \sqrt{X_{-i} v_{ik}} - X_{-i}.$$

Note that under Assumption 3.1, it holds that $x_{i1}^b(X_{-i}) \geq x_{i2}^b(X_{-i})$ for all $X_{-i} \geq 0$. Now let x_{ik}^B denote the best response of player k in group i, given effort levels of all the other players in the contest. By definition, it is the effort level which maximizes his expected payoff, π_{ik}, subject to $x_{ik} \geq 0$. By considering the characteristics of the minimum function, i.e. $min\,\{x_{i1}, x_{i2}\}$, and using the definition of x_{ik}^b, we obtain the best responses of players in group i, x_{ik}^B:

$$x_{i1}^B(x_{i2}, X_{-i}) = min\left\{x_{i2}, x_{i1}^b(X_{-i})\right\} = \begin{cases} x_{i2} & \text{for } x_{i2} \leq x_{i1}^b(X_{-i}) \\ x_{i1}^b(X_{-i}) & \text{for } x_{i2} > x_{i1}^b(X_{-i}) \end{cases}$$

and

$$x_{i2}^B(x_{i1}, X_{-i}) = min\left\{x_{i1}, x_{i2}^b(X_{-i})\right\} = \begin{cases} x_{i1} & \text{for } x_{i1} \leq x_{i2}^b(X_{-i}) \\ x_{i2}^b(X_{-i}) & \text{for } x_{i1} > x_{i2}^b(X_{-i}). \end{cases}$$

We are ready to obtain the pure-strategy Nash equilibria of the game. From the best responses of the players, we can see that the players in each group match their effort levels in equilibrium.

More specifically, given effort levels of the players in the other group, i.e. X_{-i}, the best responses of the players in group i are the following vectors of effort levels:

$$(x_{i1}, x_{i2}) \text{ such that } x_{i1} = x_{i2} \text{ and } 0 \leq X_i \leq x_{i2}^b(X_{-i}).$$

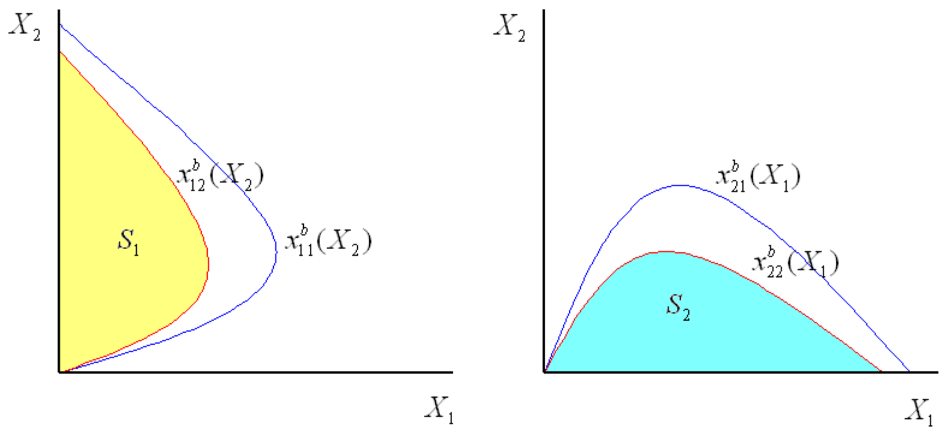

Figure 3.1: The best response of players in group i, given X_{-i}

The shadowed area S_i in Figure 3.1 represents the best responses of players in group i, given effort levels of the players in the other group, X_{-i}. Then, we know that the pure-strategy Nash equilibria of the game are consist of the vectors of effort levels which belong to the overlapped area from S_1 and S_2, i.e. $S_1 \cap S_2$. Figure 3.2 shows the pure-strategy Nash equilibria of the game. Note that, although we consider the Nash equilibria of the game in 2-dimensional space, the 2-tuple vectors in Figure 3.1 and 3.2 intrinsically denote 4-tuple vectors of effort levels of all the players in the contest.

At a Nash equilibrium, each player does not have any incentive to change his effort level, given all the other players' effort levels. We can see that every vector of effort levels, $(x_1^*, x_1^*, x_2^*, x_2^*)$, which is belonging to the shadowed area in Figure 3.2, constitutes a Nash equilibrium of the game. However, we focus on the Nash equilibrium at N, $(v_{12}^2 v_{22}/(v_{12}+v_{22})^2, v_{12}^2 v_{22}/(v_{12}+v_{22})^2, v_{12}v_{22}^2/(v_{12}+v_{22})^2, v_{12}v_{22}^2/(v_{12}+v_{22})^2)$, which is defined at the intersection of two curves, x_{12}^b and x_{22}^b. The reason why we focus on the Nash equilibrium at N is that, at any equilibrium within the shadowed area other than the equilibrium at N, the players in at least a group have incentives to cooperate with each other and hence increase their effort levels, given the effort levels of the players in the other group. Namely, given the effort levels of the players in the other group, $x_{i2}^b(X_{-i})$ results

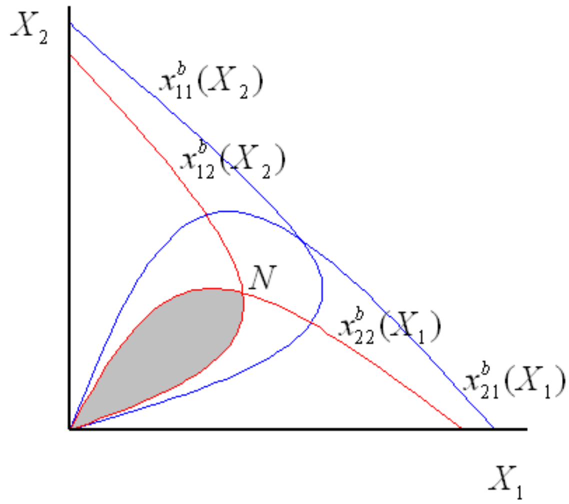

Figure 3.2: The pure-strategy Nash equilibria of the game

in the maximum expected payoffs of the players in group i. In conclusion, there exist multiple pure-strategy Nash equilibria of the game in a sense that each individual player in the contest has no incentive to change his effort level in the equilibria. But there is only one coalition-proof Nash equilibrium at which neither each player nor group has any incentive to deviate from the equilibrium, and the coalition-proof Nash equilibrium is determined at the intersection of the two curves, x^b_{12} and x^b_{22}.

3.4 Budget-constrained players

We have shown in Section 3.3 that, in each group, the lowest-valuation players play decisive roles in determining the equilibrium effort levels of the players in the contest. In this section, we consider a model in which the players in the contest are budget-constrained.

We consider a simple model which is the same as in Section 3.3, the contest between two groups with two players, with the exception that the players are budget constrained. Let b_{ik} represent the budget of player k in group i. Each player's budget is positive and publicly known. In this game, then, player k in group i maximizes his payoff given his budget constraint:

$$\max_{0 \leq x_{ik} \leq b_{ik}} \pi_{ik} = v_{ik} p_i(X_1, X_2) - x_{ik}.$$

For the simplicity of our analysis without losing our focus on budget-constrained players, we assume that the players in each group are identical. More specifically, we assume that $v_{11} = v_{12} = v_1$, $v_{21} = v_{22} = v_2$, $b_{11} = b_{12} = b_1$, and $b_{21} = b_{22} = b_2$.

To obtain the best responses of players with budget constraint, we first consider the best responses of the players without budget constraint. Using the results in Section 3.3, we get the following best responses of the players without budget constraint, x_{ik}^B:

$$x_{11}^B(X_2) = x_{12}^B(X_2) = \sqrt{X_2 v_1} - X_2$$

and

$$x_{21}^B(X_1) = x_{22}^B(X_1) = \sqrt{X_1 v_2} - X_1.$$

As you can see, the best response of players in group i, $x_{ik}^B(X_{-i})$, has its maximum value $v_i/4$ at $X_{-i} = v_i/4$. If $b_1 \geq v_1/4$ and $b_2 \geq v_2/4$ hold, the Nash equilibrium of the game with budget-constrained players is exactly the same as the equilibrium of the game with players without budget constraint. Hence, we assume $b_1 < v_1/4$ and $b_2 < v_2/4$ from now on to focus on the game with budget-constrained players. We then obtain the following best responses of the budget-constrained players in each group:

$$\overline{x}_{1k}^B(X_2) = \begin{cases} \sqrt{X_2 v_1} - X_2 & \text{for } 0 \leq X_2 \leq \frac{v_1 - 2b_1 - \sqrt{v_1(v_1 - 4b_1)}}{2} \\ b_1 & \text{for } \frac{v_1 - 2b_1 - \sqrt{v_1(v_1 - 4b_1)}}{2} < X_2 \leq \frac{v_1 - 2b_1 + \sqrt{v_1(v_1 - 4b_1)}}{2} \\ \sqrt{X_2 v_1} - X_2 & \text{for } \frac{v_1 - 2b_1 + \sqrt{v_1(v_1 - 4b_1)}}{2} < X_2 \end{cases}$$

and

$$\overline{x}_{2k}^B(X_1) = \begin{cases} \sqrt{X_1 v_2} - X_1 & \text{for } 0 \leq X_1 \leq \frac{v_2 - 2b_2 - \sqrt{v_2(v_2 - 4b_2)}}{2} \\ b_2 & \text{for } \frac{v_2 - 2b_2 - \sqrt{v_2(v_2 - 4b_2)}}{2} < X_2 \leq \frac{v_2 - 2b_2 + \sqrt{v_2(v_2 - 4b_2)}}{2} \\ \sqrt{X_1 v_2} - X_1 & \text{for } \frac{v_2 - 2b_2 + \sqrt{v_2(v_2 - 4b_2)}}{2} < X_1. \end{cases}$$

Now we are ready to find the Nash equilibrium of the contest with budget-constrained players. Note that, though there exists multiple Nash equilibria of the game, we are focusing on the coalition-proof Nash equilibrium. We obtain the following coalition-proof Nash equilibrium of the game by using the best responses of the budget-constrained players obtained above:

- **Case 1.** If $\frac{v_1^2 v_2}{(v_1+v_2)^2} \leq b_1 < \frac{v_1}{4}$ and $\frac{v_1 v_2^2}{(v_1+v_2)^2} \leq b_2 < \frac{v_2}{4}$, we get the following Nash equilibrium:

$$(x_{11}^N, x_{12}^N, x_{21}^N, x_{22}^N) = (x_1^*, x_1^*, x_2^*, x_2^*), \text{where } x_1^* = \frac{v_1^2 v_2}{(v_1 + v_2)^2} \text{ and } x_2^* = \frac{v_1 v_2^2}{(v_1 + v_2)^2}.$$

- **Case 2.** If $\frac{v_1^2 v_2}{(v_1+v_2)^2} \leq b_1 < \frac{v_1}{4}$ and $b_2 < \frac{v_1 v_2^2}{(v_1+v_2)^2}$, we get the following Nash equilibrium:

$$(x_{11}^N, x_{12}^N, x_{21}^N, x_{22}^N) = (x_1^*, x_1^*, x_2^*, x_2^*), \text{where } x_1^* = \sqrt{b_2 v_1} - b_2 \text{ and } x_2^* = b_2.$$

- **Case 3.** If $b_1 < \frac{v_1^2 v_2}{(v_1+v_2)^2}$ and $\frac{v_1 v_2^2}{(v_1+v_2)^2} \leq b_2 < \frac{v_2}{4}$, we get the following Nash equilibrium:

$$(x_{11}^N, x_{12}^N, x_{21}^N, x_{22}^N) = (x_1^*, x_1^*, x_2^*, x_2^*), \text{where } x_1^* = b_1 \text{ and } x_2^* = \sqrt{b_1 v_2} - b_1.$$

- **Case 4.** If $b_1 < \frac{v_1^2 v_2}{(v_1+v_2)^2}$ and $b_2 < \frac{v_1 v_2^2}{(v_1+v_2)^2}$, we get the following Nash equilibrium:

$$(x_{11}^N, x_{12}^N, x_{21}^N, x_{22}^N) = (x_1^*, x_1^*, x_2^*, x_2^*)$$

$$\text{where } \begin{cases} x_1^* = b_1 \text{ and } x_2^* = \sqrt{b_1 v_2} - b_1 & \text{for } 0 < b_1 \leq \frac{v_2 - 2b_2 - \sqrt{v_2(v_2 - 4b_2)}}{2} \\ x_1^* = b_1 \text{ and } x_2^* = b_2 & \text{for } \frac{v_2 - 2b_2 - \sqrt{v_2(v_2 - 4b_2)}}{2} < b_1 \leq \sqrt{b_2 v_1} - b_2 \\ x_1^* = \sqrt{b_2 v_1} - b_2 \text{ and } x_2^* = b_2 & \text{for } \sqrt{b_2 v_1} - b_2 < b_1 < \frac{v_1^2 v_2}{(v_1+v_2)^2}. \end{cases}$$

3.5 Subsidizing in the contest

We have shown that, in Section 3.3, the lowest-valuation players in each group plays decisive roles in determining the Nash equilibrium of the weakest-link contest, that is, the (coalition-proof) Nash equilibrium of the game is determined at the intersection of the best responses of the lowest-valuation players in each group.

Then the next natural question we can have is about whether the high-valuation players in each group have incentives to behave strategically in favor of themselves, e.g. subsidizing or transferring resources to others, in order to influence on the decision of the lowest-valuation player in the group. In this section, we examine the strategic behavior of high-valuation players in each group.

3.5.1 The high-valuation player's incentive to subsidize the lowest-valuation player

For the simplicity of our analysis, we again consider the example in Section 3.3, where each group consists of two players without budget constraints having different valuation on the prize. To find whether there exists the incentive of the high-valuation player in one group to subsidize the low-valuation player in his group, we consider the following game. In the first stage, the high-valuation player in group i offers to subsidize the effort level of the low-valuation player in the group at the rate s. In the second stage, knowing the subsidy rate s, all the players in the contest expend their effort levels simultaneously and independently. Finally, the winning group is chosen, and the payoffs of the players are realized. Before analyzing this, we summarize the equilibrium outcomes of the contest without subsidizing the other player analyzed in Section 3.3. Lemma 3.5 shows the results.

Lemma 3.5

(a) In the equilibrium of the game, the players in group 1 expend $x_1^* = v_{12}^2 v_{22}/(v_{12}+v_{22})^2$, and the players in group 2 expend $x_2^* = v_{12}v_{22}^2/(v_{12}+v_{22})^2$. Accordingly, group 1's probability of winning the prize is $p_1^* = v_{12}/(v_{12}+v_{22})$, and group 2's winning probability is $p_2^* = v_{22}/(v_{12}+v_{22})$.

(b) The expected payoff for player 1 in group 1 is $\pi_{11}^* = v_{12}(v_{11}v_{12}+v_{11}v_{22}-v_{12}v_{22})/(v_{12}+v_{22}^2)$, that for player 2 in group 1 is $\pi_{12}^* = (v_{12})^3/(v_{12}+v_{22})^2$, that for player 1 in group 2 is $\pi_{21}^* = v_{22}(v_{21}v_{12}+v_{21}v_{22}-v_{12}v_{22})/(v_{12}+v_{22})^2$, and that for player 2 in group 2 is $\pi_{22}^* = (v_{22})^3/(v_{12}+v_{22})^2$.

Now we consider the game where the high-valuation player in one of groups subsidize the low-valuation in that group. Without loss of generality, we assume that the high-valuation player in group 1, player 1, subsidizes the low-valuation player in the group, player 2. We solve this game backwards. Observing s chosen by player 1 in group 1 in the first stage, in the second state, each player in the contest chooses his effort level which maximizes his expected payoff. The payoff of each player in the contest is as follows, given the subsidy rate s:

$$\pi_{11} = v_{11}p_1(X_1, X_2) - x_{11} - sx_{12},$$

$$\pi_{12} = v_{12}p_1(X_1, X_2) - (1-s)x_{12},$$

and

$$\pi_{21} = v_{21}p_2(X_1, X_2) - x_{21},$$

$$\pi_{22} = v_{22}p_2(X_1, X_2) - x_{22}.$$

Let x_{ik}^b denote the best response of player k in group i when he can make the other players in his group expend any effort level he wants, given effort levels of the players in the other group.

Using the first-order condition for maximizing the expected payoffs, we obtain the following best responses of the players in a group, given the other players' effort levels in the other group:

$$x_{11}^b(X_2) = \sqrt{(1+s)^{-1}v_{11}X_2} - X_2,$$

$$x_{12}^b(X_2) = \sqrt{(1-s)^{-1}v_{12}X_2} - X_2,$$

and

$$x_{21}^b(X_1) = \sqrt{v_{21}X_1} - X_1,$$

$$x_{22}^b(X_1) = \sqrt{v_{22}X_1} - X_1.$$

Assumption 3.6 *We assume that there exists the upper bound of s, i.e. $0 < s \le (v_{11}-v_{12})/(v_{11}+v_{12})$.*

Note that under Assumption 3.6, it holds that $x_{11}^b(X_2) \ge x_{12}^b(X_2)$ for all $X_2 \ge 0$. From Assumption 3.6 and Lemma 3.3, we obtain the best responses of the players in a group, given effort levels of the players in the other group:

$$(x_{11}, x_{12}) \text{ such that } x_{11} = x_{12} \text{ and } 0 \le X_1 \le x_{12}^b(X_2)$$

and

$$(x_{21}, x_{22}) \text{ such that } x_{21} = x_{22} \text{ and } 0 \le X_2 \le x_{22}^b(X_1).$$

By using these results and (c) in Proposition 3.4, we obtain the effort levels of players, each group's winning probability, and expected payoffs of players in the second stage, given the subsidy rate, s. Lemma 3.7 summarizes these results.

Lemma 3.7

(a) In the second stage of the game, given the subsidy rate s, each player in group 1 expends $x_1(s) = v_{12}^2 v_{22}/(v_{12}+(1-s)v_{22})^2$, and each player in group 2 expends $x_2(s) = (1-s)v_{12}v_{22}^2/(v_{12}+(1-s)v_{22})^2$. Accordingly, group 1's probability of winning the prize is $p_1(s) = v_{12}/(v_{12}+(1-s)v_{22})$, and group 2's winning probability is $p_2(s) = (1-s)v_{22}/(v_{12}+(1-s)v_{22})$.

(b) The expected payoff for player 1 in group 1 is $\pi_{11}(s) = v_{12}(v_{11}v_{12}+(1-s)v_{11}v_{22}-(1+s)v_{12}v_{22})/(v_{12}+(1-s)v_{22}^2)$, that for player 2 in group 1 is $\pi_{12}(s) = (v_{12})^3/(v_{12}+(1-s)v_{22})^2$, that for player 1 in group 2 is $\pi_{21}(s) = (1-s)v_{22}(v_{21}v_{12}+(1-s)v_{21}v_{22}-v_{12}v_{22})/(v_{12}+(1-s)v_{22})^2$, and that for player 2 in group 2 is $\pi_{22}(s) = (1-s)^2(v_{22})^3/(v_{12}+(1-s)v_{22})^2$.

Before we analyze the problem of player 1 in group 1, in the first stage, in choosing the optimal subsidy rate s which maximizes his payoff, we consider two extreme cases, $s = 0$ and $s = 1$. If $s = 0$ holds, of course, we get the same results in Lemma 3.5. If $s = 1$ holds, we obtain $x_1^* = v_{22}$, $x_2^* = 0$, $p_1^* = 1$, $p_2^* = 0$, $\pi_{11}^* = v_{11} - 2v_{22}$, $\pi_{21}^* = v_{12}$, $\pi_{21}^* = 0$, and $\pi_{22}^* = 0$.

Now we consider the first stage in which player 1 in group 1 chooses the optimal subsidy rate, s^*. Having perfect foresight about $x_1(s)$ and $x_2(s)$, in the first stage, player 1 in group 1 chooses s which maximizes

$$\pi_{11}(s) = v_{11}(x_1(s)/(x_1(s) + x_2(s))) - (1 + s)x_1(s).$$

From the first-order condition for maximizing $\pi_{11}(s)$, we obtain the following equilibrium subsidy rate, s^*:

$$s^* = (v_{12}v_{11} - v_{12}^2 + v_{11}v_{22} - 3v_{12}v_{22})/v_{22}(v_{11} + v_{12}).$$

Substituting s^* into the results in Lemma 3.7, we obtain Lemma 3.8 summarizing the equilibrium outcomes of the game.

Lemma 3.8

(a) *In the equilibrium of the game, in the first stage, player 1 in group 1 chooses* $s^* = (v_{12}v_{11} - v_{12}^2 + v_{11}v_{22} - 3v_{12}v_{22})/v_{22}(v_{11} + v_{12})$. *Then, in the second stage, each player in group 1 expends* $x_1^s = v_{22}(v_{11}+v_{12})^2/4(v_{12}+2v_{22})^2$, *and each player in group 2 expends* $x_2^s = v_{22}(v_{11}+v_{12})(4v_{22} + v_{12} - v_{11})/4(v_{12} + 2v_{22})^2$. *Accordingly, group 1's probability of winning the prize is* $p_1^s = (v_{11} + v_{12})/2(v_{12} + 2v_{22})$, *and group 2's winning probability is* $p_2^s = (4v_{22} + v_{12} - v_{11})/2(v_{12} + 2v_{22})$.

(b) *The expected payoff for player 1 in group 1 is* $\pi_{11}^s = (v_{11} + v_{12})^2/4(v_{12} + 2v_{22})$, *that for player 2 in group 1 is* $\pi_{12}^s = v_{12}(v_{11} + v_{12}^2)/4(v_{12} + 2v_{22})^2$, *that for player 1 in group 2 is* $\pi_{21}^s = (4v_{22}+v_{12}-v_{11})(2v_{21}v_{12}+4v_{21}v_{22}-v_{22}v_{11}-v_{22}v_{12})/4(v_{12}+2v_{22})^2$, *and that for player 2 in group 2 is* $\pi_{22}^s = v_{22}(4v_{22} + v_{12} - v_{11})^2/4(v_{12} + 2v_{22})^2$.

By comparing the expected payoff of player 1 in group 1 in Lemma 3.8, π_{11}^s, with the one in Lemma 3.5, π_{11}^*, we obtain the following Proposition 3.9.

Proposition 3.9 *The high-valuation player in each group has an incentive to subsidize the low-valuation player's effort level in his group.*

3.5.2 The equilibrium subsidies of high-valuation players in the contest

In the previous subsection, we have shown that there exists the high-valuation player's incentive to subsidize the low-valuation player in each group. Then, in this section, we examine the case where the high-valuation player in each group subsidizes the low-valuation player in that group. Formally, we consider the following game. In the first stage, the high-valuation player in each group offers to subsidize the low-valuation player's effort level in his group at the rate, s_i. In the second stage, observing the subsidy rates s_1 and s_2, all the players in the contest expend their effort levels simultaneously and independently. Finally, the winning group is chosen, and the payoffs of the players are realized.

To solve this two-stage game, we work backwards. Given s_1 and s_2 chosen in the first stage, in the second stage, each player exerts his effort level which maximizes his expected payoff. Let x_{ik}^b denote the best response of player k in group i when he can make the other players in his group expend any effort level he wants, given effort levels of the players in the other group. Using the first-order condition for maximizing the expected payoffs, we obtain the following best responses of the players in a group, given the other players' effort levels in the other group:

$$x_{11}^b(X_2) = \sqrt{(1+s_1)^{-1}v_{11}X_2} - X_2,$$

$$x_{12}^b(X_2) = \sqrt{(1-s_1)^{-1}v_{12}X_2} - X_2,$$

and

$$x_{21}^b(X_1) = \sqrt{(1+s_2)^{-1}v_{21}X_1} - X_1,$$

$$x_{22}^b(X_1) = \sqrt{(1-s_2)^{-1}v_{22}X_1} - X_1.$$

Assumption 3.10 *We assume that there exists the upper bound of s_i, i.e. $0 < s_i \leq (v_{i1} - v_{i2})/(v_{i1} + v_{i2})$ for $i = 1, 2$.*

Note that under Assumption 3.10, it holds that $x_{i1}^b(X_{-i}) \geq x_{i2}^b(X_{-i})$ for all $X_{-i} \geq 0$. From Assumption 3.10 and Lemma 3.3, we obtain the best responses of the players in a group, given effort levels of the players in the other group:

$$(x_{11}, x_{12}) \text{ such that } x_{11} = x_{12} \text{ and } 0 \leq X_1 \leq x_{12}^b(X_2)$$

and

$$(x_{21}, x_{22}) \text{ such that } x_{21} = x_{22} \text{ and } 0 \leq X_2 \leq x_{22}^b(X_1).$$

By using these results and (c) in Proposition 3.4, we obtain the effort levels of players, each group's winning probability, and expected payoffs of players in the second stage, given the subsidy rate, s_1 and s_2. Lemma 3.11 summarizes these results.

Lemma 3.11

(a) In the second stage of the game, given the subsidy rates s_1 and s_2, each player in group 1 expends $x_1(s_1, s_2) = (1 - s_2)v_{12}^2 v_{22}/((1 - s_2)v_{12} + (1 - s_1)v_{22})^2$, and each player in group 2 expends $x_2(s_1, s_2) = (1 - s_1)v_{12}v_{22}^2/((1 - s_2)v_{12} + (1 - s_1)v_{22})^2$. Accordingly, group 1's probability of winning the prize is $p_1(s_1, s_2) = (1 - s_2)v_{12}/((1 - s_2)v_{12} + (1 - s_1)v_{22})$, and group 2's winning probability is $p_2(s_1, s_2) = (1 - s_1)v_{22}/((1 - s_2)v_{12} + (1 - s_1)v_{22})$.

(b) The expected payoff for player 1 in group 1 is $\pi_{11}(s_1, s_2) = (1 - s_2)v_{12}((1 - s_2)v_{11}v_{12} + (1 - s_1)v_{11}v_{22} - (1 + s_1)v_{12}v_{22})/((1 - s_2)v_{12} + (1 - s_1)v_{22})^2$, that for player 2 in group 1 is $\pi_{12}(s_1, s_2) = (1 - s_2)^2(v_{12})^3/((1 - s_2)v_{12} + (1 - s_1)v_{22})^2$, that for player 1 in group 2 is $\pi_{21}(s_1, s_2) = (1-s_1)v_{22}((1-s_2)v_{21}v_{12}+(1-s_1)v_{21}v_{22}-(1+s_2)v_{12}v_{22})/((1-s_2)v_{12}+(1-s_1)v_{22})^2$, and that for player 2 in group 2 is $\pi_{22}(s_1, s_2) = (1 - s_1)^2(v_{22})^3/((1 - s_2)v_{12} + (1 - s_1)v_{22})^2$.

Now we consider the first stage in which player 1 in group 1 and player 1 in group 2 choose the optimal subsidy rates, s_1^* and s_2^*. Having perfect foresight about $x_1(s_1, s_2)$ and $x_2(s_1, s_2)$, in the first stage, player 1 in group 1 chooses s_1, given the subsidy rate s_2, which maximizes

$$\pi_{11}(s_1, s_2) = v_{11}(x_1(s_1, s_2)/(x_1(s_1, s_2) + x_2(s_1, s_2))) - (1 + s_1)x_1(s_1, s_2).$$

From the first-order condition for maximizing $\pi_{11}(s_1, s_2)$, we obtain the following subsidy rate of player 1 in group 1, $s_1(s_2)$:

$$s_1(s_2) = \frac{v_{11}(v_{12} + v_{22}) - v_{12}(v_{12} + 3v_{22})}{v_{22}(v_{11} + v_{12})} - \frac{v_{12}(v_{11} - v_{12})s_2}{v_{22}(v_{11} + v_{12})}.$$

Similarly, we obtain the following subsidy rate of player 1 in group 2, $s_2(s_1)$:

$$s_2(s_1) = \frac{v_{21}(v_{12} + v_{22}) - v_{22}(v_{22} + 3v_{12})}{v_{12}(v_{21} + v_{22})} - \frac{v_{22}(v_{21} - v_{22})s_1}{v_{12}(v_{21} + v_{22})}.$$

Now we are ready to find the equilibrium subsidy rates of the high-valuation players in each group. By solving the two equations, $s_1(s_2)$ and $s_2(s_1)$, simultaneously, we find the equilibrium subsidy rates of the players in the contest.

Proposition 3.12 *In the equilibrium of the game, in the first stage, the high-valuation player in each group chooses the following equilibrium subsidy rate:*

$$s_1^* = \frac{v_{11}v_{22} + (2v_{11} - 2v_{22} - v_{21})v_{12} - 2v_{12}^2}{v_{11}v_{22} + v_{12}v_{21}}$$

and

$$s_2^* = \frac{v_{12}v_{21} + (2v_{21} - 2v_{12} - v_{11})v_{22} - 2v_{22}^2}{v_{11}v_{22} + v_{12}v_{21}}.$$

3.6 Further Research

In the previous section, we have shown that the high-valuation player in each group has an incentive to subsidize the low-valuation player in his group and have found the subsidy rates the high-valuation player offers to the low-valuation player in his group at Nash equilibrium. However, subsidizing the other player's effort may be impossible in our reality because, for instance, each agent in a group may have a different kind of role that requires different kind of ability or position. Even though subsidizing the effort of the other player is possible, it is very hard to be implemented when the effort of the players is non-observable. For these reasons, we can think an alternative way for the high-valuation player in a group to motivate the other player in his group; a contingent payment scheme. That is, to motivate the low-valuation player, the high-valuation player offers an outcome-contingent bonus to the low-valuation player in his group.

Specifically, we can construct the following game. In the first stage, the high-valuation player in group 1 offers an outcome-contingent bonus, b, which will be given to the low-valuation player in his group in case of his group's winning the contest. If that group loses the contest, the high-valuation player does not give anything to the low-valuation player. In the second stage, observing the bonus b, all the players in the contest expend their effort simultaneously and independently. Finally, the winning group is chosen and the payoffs of the players are realized. Analyzing this game and comparing the results under being incentivized with those under being subsidized should be interesting works.

In the current version, we considered a simple contest model in which two groups consisting of two players compete to win the prize. It should be interesting to extend the model to a more general case where several group consisting of several players with different valuations on the prize compete with one another to win the prize. Furthermore, it will be also interesting to study endogenous formation of groups in the weakest-link contests by allowing the players in the contest to move from one group to others before expending their effort levels. Finally, we can consider another type of contest, “Best-Shot contests with group-specific public good prizes”, where each group’s winning probability depends on the maximum effort level expended by the players in the group. We leave all these works for our future research.

3.7 Conclusion

We have examined the equilibrium effort levels of individual players and groups in a contest in which n groups compete to win a group-specific public good prize, the individual players choose their effort levels simultaneously and independently, and each group’s probability of winning the prize follows a weakest-link rule or weakest-link contest success function. In the general model, we first showed that the lowest-valuation players in each group play decisive roles in determining the Nash equilibrium of the contest. There are multiple pure-strategy Nash equilibria of the game but there exists an unique coalition-proof Nash equilibrium where neither an individual player nor group has any incentive to deviate from the equilibrium. No free riding problem exists in the equilibrium. We then studied the equilibrium effort levels of individual players and groups in the model where the players in the contest are budget-constrained. We also found that high-valuation players in each group have an incentive to subsidize low-valuation players in their group because subsidizing the lowest-valuation player in their group increases their payoffs in equilibrium. Finally, we examined the equilibrium subsidy rates of the groups in a model where first high-valuation players in each group decide how much to subsidize low-valuation players in their group and then the individual players in the contest choose their effort levels simultaneously and independently.

Chapter 4

Endogenous Decisions of the Agents in the Software Industry Between Open Source and Proprietary Projects

4.1 Introduction

Open source software projects have been successful despite the skeptical prediction of their future in the early days of the open source phenomenon. Linux, Apache, and MySQL are the examples of the successful open source software. The remarkable success of the open source software projects cannot be achieved without the tremendous efforts made by self-motivated individual programmers who are willing to spend hours for this non-paying effort. It is startling to observe the voluntary contributions of the highly motivated developers in the open source software projects. In his "Open Letter to Hobbyists" written in 1976, Bill Gates expressed his skepticism about the success of the hobbyist community. He mentioned the unfairness of gaining the benefits of software developers' resources without paying anything. He wrote: "*Who can afford to do professional work for nothing? What hobbyist can put 3-man years into programming, finding all bugs, documenting his product and distributing for free?*" (Gates 1976).

The open source software phenomenon in general, has been drawing broad attention of academic researchers as well as industry practitioners. In particular, academic researchers from various fields such as management, psychology, and economics, have attempted to reveal the motivations of the individual developers to participate in open source software projects, which do not customarily offer monetary compensation. Examining the motivations of the individual programmers for open source project participation has been a popular theme among researchers (Fitzgerald and Feller (2001)). In the early days of open source software research, Ghosh (1998) identifies enjoyment and creativity as main motives for the open source project participation while Raymond (1999) argues that reputation gives them an incentive to participate in open source projects. Other motivations for open source project participation include the developers' own desire to solve the problem they cope with (Franke and von Hippel (2003)), and career concerns such as potential good job offers in the future (Hann et al. (2006)).

Under a widely accepted theme in the literature, the various motivations for open source project participation have been grouped under two broad categories: intrinsic and extrinsic (Lakhani and Wolf (2003), Rossi (2004), Roberts et al. (2006), Shah (2006)). Ryan and Deci (2000) define intrinsic motivation as the doing of an activity for its inherent satisfactions rather than for some separable consequence. Workers with intrinsic motivation pursue goals because they enjoy more benefits from doing the task per se, than the reward for any service they offer. Economists call such workers motivated agents (Besley and Chatak (2005)). Examples of this include police officers being motivated to promoting justice, fire fighters being motivated to saving lives, and soldiers being motivated to defending their country. In the context of open source software, intrinsic motivations can be the enjoyment of programming, satisfaction and accomplishment as a member of the community, altruism, generalized reciprocity, and a gift-giving attitude (Rossi (2004)). On the other hand, extrinsic motivation refers to motivation that stems from factors outside an individual. Rewards like reputation and monetary compensation are the examples of the extrinsic motivation. In the context of the open source software, peer recognition or potential job offers may motivate the open source software developers extrinsically.

Prior studies in the stream of research on the open source project participants' motivation examine the problem from a behavioral perspective grounded on survey data. Learner and Tirole (2002) argue that a programmer participates in a project only if she derives a net benefit. They also argue that existing economic theory can explain the motivation for open source project participation as long as a programmer's benefits and costs are articulated in her utility function. However, very little research in the economic literature attempts to explain the motivation of the open source software developers with an economic model based on utility theory. Inspired by economists' view championing versatility of economic theory, we aim to bridge the gap between economic literature and behavioral science on the motivation for open source project participation, by examining the programmers' choice between open source and proprietary software projects using a stylized economic model.

Existing literature focuses on identifying the motivations for open source project participation. An important yet unanswered question is how intrinsic and extrinsic motivations affect a software programmer's decision on which project to participate in: open source or proprietary. As suggested by Lerner and Tirole (2002), we attempt to model the factors of the benefit and the cost in a programmer's utility function, and logically explain the programmer's choice of the software project. Our model captures two dimensions of a programmer's type: intrinsic motivation and ability. We assume no correlation between these two dimensions, implying that being a motivated agent does not mean that she is a highly capable programmer. We also consider two extrinsic motivations for software project participation: a future job offer to open source developers, and monetary compensation to proprietary project participants. With this setting, we aim to answer the three major questions: First, how does the interplay among these three factors: intrinsic motivation, extrinsic motivation, and ability affect software programmers' choice between an open source project and a proprietary software project. Second, how does the ability to observe programmers' type affect a proprietary software vendor's strategy? Third, under what conditions, can each programmer's choice between an open source project and a proprietary project be an equilibrium outcome?

We find that motivations, both intrinsic and extrinsic, and the ability level positively affect the programmers' effort level in the open source project. In the commercial project, we find evidence that the publicity of information changes neither the contract offered by the commercial company, nor the effort level of the programmers. Our result indicates that the open source project is an effective compensation mechanism. Our equilibrium analysis shows that any combination of the programmers' decisions can occur as an equilibrium outcome. We show that there exists a pooling equilibrium where all the programmers choose either the open source or the commercial project. We also find the equilibria at which the programmers are sorted by their ability levels and the levels of intrinsic motivations. Our finding implies that an open source project is more likely to have skillful and motivated programmers than a commercial project.

We aim to contribute to the literature of information systems and economics in the following ways. First, we provide rationale for open source project participation grounded on an economic theory, and show that it is not a startling phenomenon as in a way it might look in its early days. Second, we model the ability dimension of software programmers along with the motivation factor, both intrinsic and extrinsic, and examine the impact of the interplay between these two dimensions of a programmer type on her choice of the software project. Third, we provide software programmers with guidelines which can help them make the right decision given their costs and benefits. Finally, our results can give managerial implications to proprietary software companies who want to recruit capable programmers in order to increase the likelihood of the success of their projects.

The rest of the article is organized as follows. Section 4.2 reviews the literature. We present the model in Section 4.3. In Section 4.4, we analyze the subgames that appears after the first stage of the game, i.e., the optimal efforts of programmers in each project and the optimal contract offered by a commercial company, given the decisions of the programmers made at the first stage. In Section 4.5, we study the programmers' project choices at the first stage of the game. Finally, Section 4.6 concludes by discussing the contributions and the limitations of our model.

4.2 Related Literature

our work is grounded on two research streams of economic literature: intrinsic motivation of economic agents and economics of open source software. We share the topic with the aforementioned behavioral studies that identify different motivations for contributing to open source projects, but we analyze it from an economic perspective. In this section, we survey the economic literature.

The literature on intrinsic motivation of economic agents is growing in economics. Besley and Ghatak (2005) study the behavior of motivated agents who have different intrinsic benefits according to the mission of the organization for which they work. They emphasize the importance of matching the mission of the organization and the agents for the efficiency of the organization. Benabou and Tirole (2003) examine the relationship between the two motives of an agent, extrinsic and intrinsic motivation, in a setting where an informed principal selects a policy (extrinsic incentives) which reveals information about the agent's ability or his task (intrinsic incentives) the agent does not know. They show that the extrinsic incentives have negative effects in the long run. Benabou and Tirole (2006) further consider the three components of an agent's motivation: altruistic motivation, material self-interest, and self-image concerns.

Examining open source software markets with economic tools is becoming a popular venue among researchers in the domain of information systems. Raghunathan et al. (2005) uses a stylized economic model and finds evidence that open source software quality is not necessarily lower than commercial software quality. Casadesus-Masanell and Ghemawat (2006) analyze a dynamic mixed duopoly in which a proprietary software vendor interacts with an open source software vendor which offers free software when the demand-side learning effects are present. Kim et al. (2006) examine the ways to make money from open source software, and analyze the optimal pricing strategies under a dual-licensing scheme or a service support model. Economides and Katsamakas (2006) analyze the optimal two-sided pricing strategy of a platform firm and compare industry structures based on a proprietary platform inspired by the real-world operating systems competition. August et al. (2007) examine the economic viability of the open source software business model grounded on an analytical model. They identify the market conditions under which the open source model is successful.

4.3 The Model

There are two types of projects in the software industry: open source software projects and commercial software projects. Both types of projects are run by a group of economic agents, or a group of software programmers.

Programmers differ with respect to two key dimensions: abilities and intrinsic motivation. Each programmer either has a high ability or a low ability in terms of a skill-set required for a certain task, and is either highly motivated or poorly motivated in terms of intrinsic motivation. Let us denote $A = \{a_H, a_L\}$ as a set of ability parameters where a_H and a_L represent high ability and low ability, respectively. Let us denote $\Theta = \{\theta_H, \theta_L\}$ as a set of motivation parameter where θ_H and θ_L represent high motivation and low motivation, respectively. Then there are four types of programmers in the industry: programmers with high ability and high intrinsic motivation ($HH \equiv (a_H, \theta_H)$), with high ability and low intrinsic motivation ($HL \equiv (a_H, \theta_L)$), with low ability and high intrinsic motivation ($LH \equiv (a_L, \theta_H)$), and with low ability and low intrinsic motivation ($LL \equiv (a_L, \theta_L)$). The number of each type of programmer is normalized to unity. The commercial software company cannot observe these types of the programmers while the information about the programmer types is common knowledge among programmers in the industry.

Whether a project succeeds or fails depends on the total effort of the programmers who are participating in the project. Let us denote e_i as the effort of a programmer whose type is i where

$i \in \{HH, HL, LH, LL\}$. If programmer i exerts his effort, she incurs cost $c_i(e_i) = \frac{a_i e_i^2}{2}$, where $a_i > 0$ is her ability parameter and measures the degree of disutility in exerting his effort . Hence the ability parameter of programmers with high ability ($i = HH$ or HL) is less than the one of programmers with low ability ($i = LH$ or LL). That is, we have $a_{HH} = a_{HL} = a_H < a_L = a_{LH} = a_{LL}$. Let us denote OS and CS as a set of programmers who are participating in an open source software project and in a commercial software project, respectively. Then the probability that the open source software project will succeed is $P(\sum_{j \in OS} e_j)$. Similarly, the commercial software project succeeds with probability $P(\sum_{j \in CS} e_j)$.

If the open source project succeeds, the programmers participating in the project receive two benefits: intrinsic benefit and extrinsic benefit. Programmer i in the open source project gets her motivation parameter θ_i as the intrinsic benefit. The motivation parameter of the highly motivated programmers ($i = HH$ or LH) is greater than the one of the poorly motivated programmers ($i = HL$ or LL). That is, we have $\theta_{HH} = \theta_{LH} = \theta_H > \theta_{HL} = \theta_{LL} = \theta_L$. Programmer i in the open source project also obtains the extrinsic benefit in case of the success of the project. Outsiders can see the contribution of each programmer working on the open source project simply because the project is open to the public. Thus, a programmer whose contribution toward the success of the project is relatively large will have a high likelihood of getting a good reputation, and consequently getting a good job offer in the future. Hence, programmer i in the open source project receives the monetary values of a future job offer,V, with some probability which increases with her effort level relative to the total effort level of the programmers in the project. This future job offer gives the participants of an open source project an extrinsic motivation. Note that both benefits are attainable only when the open source project succeeds. If the project fails, the programmers get nothing. Then the utility of programmer i from working in the open source project is given by

$$U_i = P(\sum_{j \in OS} e_j) \left(\theta_i + \frac{e_i}{\sum_{j \in OS} e_j} V \right) - \frac{a_i e_i^2}{2}, \tag{4.1}$$

where $P(\cdot)$ is a function with properties $P'(\cdot) > 0$ and $P''(\cdot) \leq 0$.

The programmers participating in the commercial project receive monetary gain according to a contract made with the commercial software company. Since the effort levels of programmers are non-contractible (or non-observable), the commercial company offers an incentive scheme that is contingent on success or failure of the project in order to induce the programmers' effort. Let $w = (w_s, w_f)$ be a contract made by the commercial company and the programmers. If the project succeeds, the commercial company pays each programmer the wage w_s. In the case of failure, the commercial company pays each programmer the wage w_f. Then the utility of programmer i from working in the commercial project is given by

$$U_i = P(\sum_{j \in CS} e_j) w_s + \left(1 - P(\sum_{j \in CS} e_j)\right) w_f - \frac{a_i e_i^2}{2}. \tag{4.2}$$

Let π be the revenue the commercial project makes when the project succeeds. If the project fails the revenue is 0. Then the utility of the commercial company is given by

$$U_{CS} = P(\sum_{j \in CS} e_j)(\pi - \# w_s) + (1 - P(\sum_{j \in CS} e_j))(-\# w_f), \tag{4.3}$$

where # is the number of programmers working on the commercial project.

We formally construct the following game: At first, all the programmers in the industry make choices of either participating in an open source project or participating in a commercial project simultaneously and independently. Then, in the open source project, the programmers exert their effort to make the project successful. In the commercial project, the commercial company offers a contract $w = (w_s, w_f)$ to the programmers and then each programmer in the project accepts or rejects the offer. If she rejects the offer, she leaves the industry and gets nothing. Conversely, the programmers accepting the offer exert their effort to make the project successful. Finally, both projects succeed or fail and the payoffs of all the programmers and the commercial company are realized.

4.4 Programmers' Optimal Levels of Effort

4.4.1 Open Source Project

In this section, we examine the optimal levels of effort that the participants of the open source project exert, and the corresponding utility levels. We investigate how different types of programmers make their decisions on the effort level, i.e., how intrinsic motivation (θ_i), extrinsic motivation (V), and ability (a_i) affect how much effort a programmer is willing to make in the open source project (e_i^*). Recall from equation (4.1) that the utility of a programmer of type i from working in the open source project is given by $U_i = P(\sum_{j \in OS} e_j)\left(\theta_i + \frac{e_i}{\sum_{j \in OS} e_j} V\right) - \frac{a_i e_i^2}{2}$ where $P(\cdot)$ is a function with properties $P'(\cdot) > 0$ and $P''(\cdot) \leq 0$. For simplicity we assume that the probability of success is the sum of the participants' efforts, i.e., $P(\sum_{j \in OS} e_j) = \sum_{j \in OS} e_j$ where $e_i \in [0, 0.25]$. Then the utility becomes

$$U_i = \sum_{j \in OS} e_j \left(\theta_i + \frac{e_i}{\sum_{j \in OS} e_j} V \right) - \frac{a_i e_i^2}{2}. \tag{4.4}$$

An analysis to solve for the optimal level of effort that programmer i exerts leads to Proposition 4.1.

Proposition 4.1 *An open source project participant's optimal level of effort is $e_i^* = \frac{\theta_i + V}{a_i}$. As the programmer has higher intrinsic ($\theta \uparrow$) and extrinsic motivations ($V \uparrow$), and higher ability ($a_i \downarrow$), she makes more efforts ($e_i^* \uparrow$) and enjoys higher utility ($U_i^* \uparrow$).*

Proof. See the Appendix.

Proposition 4.1 implies that in an open source project, a programmer with higher motivations, and higher ability makes more efforts and thus enjoys higher utility. This result is intuitive in the sense that both the motivation factors and the ability factors positively affect a programmer's effort level. Our findings share the view with the existing literature in that the open source project is an efficient way to incentivize the programmer with high intrinsic motivation which is not appreciated in the commercial project.

4.4.2 Commercial Project

Under Complete Information

In this section, we examine the commercial project where programmers exert their efforts for extrinsic benefit, i.e. monetary incentives, offered by the commercial software company. We first study the optimal contract, i.e. incentive scheme, of the commercial company with complete information, meaning that programmers' type in terms of ability is publicly known. The commercial company designs an incentive scheme $w = (w_s, w_f)$ that is contingent on success and failure of the commercial software project in order to induce the programmers' efforts. Recall from equation (4.2) that programmer i enjoys utility $U_i = P(\sum_{j \in CS} e_j) w_s + (1 - P(\sum_{j \in CS} e_j)) w_f - \frac{a_i e_i^2}{2}$. Consistent with the open source case, we assume that the probability of success is the sum of the participants' efforts, i.e., $P(\sum_{j \in CS} e_j) = \sum_{j \in CS} e_j$ where $e_i \in [0, 0.25]$. Given the contract $w = (w_s, w_f)$, the optimal level of effort programmer i exerts is $e_i = \frac{w_s - w_f}{a_i}$ which can be easily obtained by solving the first-order condition.

We now consider the commercial company's optimal contract. The commercial company gets profit π if the project succeeds while it gets nothing otherwise. Then the utility of the commercial company is given by $U_{CS} = \sum_{j\in CS} e_j(\pi - \#w_s) + (1 - \sum_{j\in CS} e_j)(-\#w_f)$, where # is the number of programmers working on the commercial project. The maximization problem that the commercial company faces is formulated as follows:

$$\max_{\{w_s,w_f\}} U_{CS} = \sum_{j\in CS} e_j(\pi - \#w_s) + (1 - \sum_{j\in CS} e_j)(-\#w_f) \tag{4.5}$$

$$\text{subject to} \quad w_s, w_f \geq 0 \quad \text{(LLC)}$$

$$U_{i\in CS} \geq 0 \quad \text{(PC)}$$

$$e_i = \arg\max_{e_i} U_i \text{ for } i \in CS \quad \text{(ICC)}$$

Solving for the above maximization problem leads to the commercial company's optimal contract and programmer i's optimal effort level as follows[1]:

$$w^* = (w_s^*, w_f^*) = (\frac{\pi}{2\#}, 0) \quad \text{and} \quad e_i^* = \frac{\pi}{2\#a_i}.$$

Hence, programmer i in the commercial project gets the following utility:

$$U_{i\in CS}^* = \sum_{j\in CS} e_j^* w_s^* - \frac{a_i(e_i^*)^2}{2} = \frac{\pi^2}{4\#^2}\left(\sum_{j\in CS}\frac{1}{a_j} - \frac{1}{2a_i}\right) = \frac{\pi^2}{4\#^2}\left(\sum_{j\neq i}\frac{1}{a_j} + \frac{1}{2a_i}\right). \tag{4.6}$$

Under Incomplete Information

In the real-world software industry, a programmer's ability is often not clearly observable to the prospective employers. This incomplete information can explain a variety of efforts that the employers make to have better prior knowledge about the job seekers' skill levels such as internships. We model the information asymmetry between the commercial company and the programmers about the programmers' ability levels. We investigate how incomplete information affects the commercial company's decision on the optimal contract, and the programmers' effort levels. Assume that the commercial company knows the probability distribution of the programmers' ability. Denote a cumulative distribution of the programmer's ability level by $F(a)$ with $a \in \{a_H, a_L\}$. Then the commercial company's optimal contracting problem under moral hazard becomes the following:

$$\max_{\{w_s,w_f\}} E[U_{CS}] = \int \left\{ \sum_{j\in CS} e_j(\pi - \#w_s) + (1 - \sum_{j\in CS} e_j)(-\#w_f) \right\} dF(a) \tag{4.7}$$

[1] The proof is available in the Appendix.

$$\text{subject to} \quad w_s, w_f \geq 0 \quad \text{(LLC)}$$

$$U_{i \in CS} \geq 0 \quad \text{(PC)}$$

$$e_i = \arg\max_{e_i} U_i \text{ for } i \in CS \quad \text{(ICC)}$$

A further analysis seeking the optimal solutions to the above maximization problem yields Proposition 4.2.

Proposition 4.2 *A commercial software company offers the same contract to the participating programmers under incomplete information about the programmers' ability levels as the contract under complete information. The optimal level of effort a programmer makes does not change with the publicity of information.*

Proof. See the Appendix.

In an open source software project, there is no hiring process. Rather, programmers self-select projects. Since programmers are the ones who know the exact level of their skill sets, the open source software market is under complete information. On the other hand, a commercial software company goes through a hiring process, and the key factor is the skill levels of the potential employees. In the real world, employers are often misinformed about the ability levels of the candidates despite their attempts to correctly evaluate the qualifications such as interviews. Interestingly, our findings show that, even in the commercial project, information about the ability level affects neither the optimal contract provided by the company, nor the effort levels made by the programmers. The optimal contract under complete information maximizes the profit for the commercial company even in the presence of information asymmetry.

4.4.3 Open Source vs. Commercial as a Compensation Mechanism

Whether an agent who puts forth more efforts always gets a higher payoff under a certain mechanism is an interesting question to both the employer and the prospective employees. This question is particularly interesting to the software industry where the probability of success of a project heavily depends on the efforts that the participating programmers make. In this section, we examine the effectiveness of both types of software projects as compensations mechanisms for the programmers' efforts. We have found that a programmer with higher ability always makes more of an effort in a software project regardless of whether it's open source or commercial. By examining the payoffs for the programmers, we aim to answer the following questions: Will a programmer with higher ability get higher compensation? We identify the conditions under which a certain type of software project is effective as a compensation mechanism for the programmers' efforts.

Recall from the proof of Proposition 4.1 that programmer i in the open source project enjoys her utility $U^*_{i\in OS} = \theta_i \sum_{j\in OS}\left(\frac{\theta_j+V}{a_j}\right) - \frac{(\theta_i+V)(\theta_i-V)}{2a_i}$. The utility of a programmer of each type is as follows:

$$U^*_{HH\in OS} = \theta_H \sum_{j\in OS}\left(\frac{\theta_j+V}{a_j}\right) - \frac{(\theta_H+V)(\theta_H-V)}{2a_H},$$

$$U^*_{LH\in OS} = \theta_H \sum_{j\in OS}\left(\frac{\theta_j+V}{a_j}\right) - \frac{(\theta_H+V)(\theta_H-V)}{2a_L},$$

$$U^*_{HL\in OS} = \theta_L \sum_{j\in OS}\left(\frac{\theta_j+V}{a_j}\right) - \frac{(\theta_L+V)(\theta_L-V)}{2a_H},$$

$$U^*_{LL\in OS} = \theta_L \sum_{j\in OS}\left(\frac{\theta_j+V}{a_j}\right) - \frac{(\theta_L+V)(\theta_L-V)}{2a_L}.$$

It is straightforward that $U^*_{HH\in OS} \geq U^*_{LH\in OS}$ when $V \geq \theta_H$, and that $U^*_{HL\in OS} \geq U^*_{LL\in OS}$ when $V \geq \theta_L$. Note from (4.6) that programmer i in the commercial project gets her utility $U^*_{i\in CS} = \frac{\pi^2}{4\#^2}\left(\sum_{j\in CS}\frac{1}{a_j} - \frac{1}{2a_i}\right)$. Thus, a programmer of each type enjoys the following utility:

$$U^*_{HH\in CS} = U^*_{HL\in CS} = \frac{\pi^2}{4\#^2}\left(\sum_{j\in CS}\frac{1}{a_j} - \frac{1}{2a_H}\right),$$

$$U^*_{LH\in CS} = U^*_{LL\in CS} = \frac{\pi^2}{4\#^2}\left(\sum_{j\in CS}\frac{1}{a_j} - \frac{1}{2a_L}\right).$$

Since $a_H < a_L$, we have $U^*_{HH\in CS} = U^*_{HL\in CS} < U^*_{LH\in CS} = U^*_{LL\in CS}$. Proposition 4.3 summarizes the outcomes of our analysis.

Proposition 4.3 *When the extrinsic benefit is larger than the intrinsic benefit ($V > \theta_i$), a programmer with higher ability, who thus makes more of an effort, gets a higher payoff in the open source project. In the commercial project, a programmer with higher ability is not compensated for his efforts.*

Proposition 4.3 implies that the open source project is an effective compensation mechanism which pays more for programmers with higher ability who make more contributions to the project as long as the benefit of getting recognized in the open source project is sufficiently large. On the other hand, the commercial project does not compensate for the efforts made by skillful programmers in a fair way. Programmers with higher ability enjoy less utility than the others in spite of the more valuable input they make. Our findings may explain one of the programmers' incentives to join the open source project instead of the commercial project. Particularly, skillful programmers may choose an open source project over a commercial one when they realize sufficient benefit of being a prominent participant in the open source project, for example, a future job offer while being an outstanding programmer in a commercial project does not bring much benefit to themselves.

4.5 Programmers' Choice of Project Types

In the previous section, we have examined the optimal effort levels of the programmers participating in each project and the optimal contract of the commercial company, given the programmers' choices between participating in an open source project and participating in a commercial project. Now, we consider each programmer's decision problem of which project she will participate in.

According to each programmer's choice between the open source and the commercial project made at the first stage of the game, there appear 16 subgames. Table 1 shows what type of programmer participates into which project for each subgame:

Insert Table 1 Here

From the results obtained in Section 4.4, we can compute all the programmers' utilities which will be derived in each subgame. Remind that $U^*_{i \in OS} = \theta_i \sum_{j \in OS} \left(\frac{\theta_j + V}{a_j} - \frac{(\theta_i + V)(\theta_i - V)}{2a_i} \right)$ and $U^*_{i \in CS} = \frac{\pi^2}{4\#^2} \left(\sum_{j \in CS} \frac{1}{a_j} - \frac{1}{2a_i} \right)$. To make our analysis simple without losing our focus, we assume that θ_L is normalized to 0, and denote θ_H as θ. We define a_H as $\frac{a_L}{k}$ where $k > 1$, and denote a_L as a. With these notations we compute all the programmers' utilities in each subgame. Table 2 summarizes the utilities of the programmers in each subgame.

Insert Table 2 Here

Now we are ready to see the endogenous decision of the programmers about their choice of the software project. We examine how each type of programmer makes her choice between the open source project and the commercial project in the first stage of the game. That is, we analyze the conditions under which each subgame becomes an equilibrium outcome. In order to specify the equilibrium conditions under which a certain subgame is an equilibrium outcome, we have to find some conditions under which each programmer in that subgame does not have any incentive to deviate from a project she is currently participating to the other project, given the other programmers' choices between the open source project and the commercial project. Solving simultaneously these non-deviation conditions of the programmers in that subgame, we obtain the equilibrium condition under which that subgame will appear as an equilibrium. Executing this procedure for all the subgames, we find the equilibrium conditions for each subgame. The following propositions summarize our findings.

Proposition 4.4 *All the subgames possibly appear as one of the equilibrium outcomes. That is, for each subgame, there exist equilibrium conditions under which that subgame becomes an equilibrium outcome.*

Proof. See the Appendix.

Interestingly, we have found that for each subgame, there exist equilibrium conditions for the subgame to be an equilibrium outcome, and that there exist multiple equilibria under some equilibrium conditions. Which subgame (possibly subgames) will appear in equilibrium depends on the parameters in our model, V, π, k, and θ. It is generally believed that the open source software project is tempting to only the programmers with high intrinsic motivations. In other words, the perception about such programmers with high intrinsic motivation is that they may never participate in the commercial software project. Our result indicates that any case is possible, which may encourage the commercial software company which is interested in recruiting programmers with high ability as well as high intrinsic motivation. We further examine the equilibrium conditions.

Proposition 4.5 *(Pooling Equilibria)*

(a) If $V \geq \frac{\pi}{2}$, all the programmers in the industry participate in the open source project at equilibrium.

(b) If $V + \theta \leq \frac{\pi}{8}\sqrt{\frac{3k+4}{k}}$, all the programmers in the industry participate in the commercial project at equilibrium.

Proof. See Table 3 at the appendix.

Proposition 4.5 shows the pooling equilibria at which all the programmers make the same decision regardless of their own types. The findings in Proposition 4.5 are intuitively true. If the future benefit coming from getting famous in the open source project (V) is large enough, all the programmers in the industry participate in the open source project. On the contrary, if the sum of the extrinsic benefit and the intrinsic benefit is sufficiently small, there is no incentive for anyone to go to the open source project and hence all the programmers participate in the commercial project. This result implies that an open source software project can be successful only if it provides the prospective participants with sufficient benefits, which can be either intrinsic or extrinsic or both.

Proposition 4.6 *(Equilibria at which the programmers are sorted out by abilities)*

(a) There possibly exist two equilibria at which the programmers are sorted out according to their abilities. At one of the equilibria, subgame 6 appears, i.e., the high-ability programmers (HH, HL) participate in the open source project and the low-ability programmers (LH, LL) participate in the commercial project. At the other equilibrium, there appears subgame 11, which is inversely symmetric to subgame 6.

(b) If the equilibrium conditions for subgame 11 to be an equilibrium outcome are satisfied, then those for subgame 6 to be an equilibrium outcome are always satisfied. That is, if subgame 11 is an equilibrium outcome, subgame 6 is also. However, the converse is not true.

Proof. By comparing the equilibrium conditions for subgame 6 with those for subgame 11 shown in Table 3, we can see that the equilibrium conditions for subgame 11 is a sufficient condition of the equilibrium conditions for subgame 6.

Proposition 4.6 shows the equilibria at which the programmers in the industry are separated according to their abilities and the relation between the equilibrium conditions for those equilibria. Interestingly, the equilibrium conditions under which the highly-abled programmers go to the commercial project are a subset of those under which the lowly-abled programmers go to the commercial project. This implies that the case where the highly-abled programmers participate in the commercial project and the lowly-abled programmers participate in the open source project cannot be a *unique* equilibrium. In other words, if the case where all the highly-abled programmers participate in the commercial project is an equilibrium, the opposite case where they work in the open source project is an equilibrium as well. Our finding supports the argument that an open source project is more likely to have knowledgeable and skillful programmers than a commercial project, which makes the commercial company be concerned about recruiting highly-abled programmers.

Proposition 4.7 *(Equilibria at which the programmers are sorted out by intrinsic motivations)*

(a) (i) There possibly exist two equilibria at which the programmers are sorted out according to their intrinsic motivations. At one of the equilibria, subgame 7 appears, i.e., the highly motivated programmers (HH, LH) *participate in the open source project and the poorly motivated programmers* (HL, LL) *participate in the commercial project. At the other equilibrium, there appears subgame 10 that is inversely symmetric to subgame 7.*

(b) (ii) If the equilibrium conditions for subgam 10 to be an equilibrium outcome are satisfied, then those for subgame 7 to be an equilibrium outcome are always satisfied. That is, if subgame 10 is an equilibrium outcome, subgame 7 is also. However, the converse is not true.

Proof. By comparing the equilibrium conditions for subgame 7 with those for subgame 10 shown in Table 3, we can see that the equilibrium conditions for subgame 10 is a sufficient condition of the equilibrium conditions for subgame 7.

Proposition 4.7 show the equilibria at which the programmers are separated according to their intrinsic motivations, and the relation between the equilibrium conditions for those equilibria. The proposition says, interestingly, that the strange-looking case where the poorly-motivated programmers participate in the open source project and the highly-motivated programmers participate in the commercial project can be an equilibrium outcome. However, this case can be an equilibrium only when there is no big difference in the ability parameters between the programmers (k) and the future benefit derived from participating in the open source project (V) is sufficiently small. That is, this case requires very restricted equilibrium conditions. Besides, similar to the results in Proposition 4.6, the equilibrium conditions for this case is a subset of those for the case where the highly-motivated programmers go to the open source project and the poorly-motivated ones in the commercial project. This implies that the case where the poorly-motivated programmers participate in the open source project and the highly-motivated programmers in the commercial project cannot be a *unique* equilibrium.

We have also found the relationships between the equilibrium conditions for subgame 8 and 7, and between the equilibrium conditions for subgame 9 and 6 as follows. The equilibrium conditions for subgame 8 (subgame 9) to be an equilibrium outcome is a subset of those for subgame 7 (subgame 6). This implies that the case where the programmers are not sorted out by either abilities or intrinsic motivations cannot be a *unique* equilibrium in our model. From this finding, Proposition 4.6, and Proposition 4.7, we conclude that the equilibrium conditions for subgame 6 are a necessary condition of the equilibrium conditions for subgame 9 and 11, and that the equilibrium conditions for subgame 7 are a necessary condition of the equilibrium conditions for subgame 8 and 10. This implies that subgame 6 and 7 have the most comprehensive equilibrium conditions, i.e., the most unrestricted equilibrium conditions, among subgames 6, 7, 8, 9, 10, and 11.

Proposition 4.8 *(Equilibria at which the programmers are biased toward the open source project)*

(a) There possibly exist four equilibria at which only one programmer participates in the commercial project and the others participate in the open source. At these equilibria, subgame 2, 3, 4, and 5 appear.

(b) If the equilibrium conditions for either subgam 3 or 4 or 5 to be an equilibrium outcome are satisfied, then those for subgame 2 to be an equilibrium outcome are always satisfied. That is, if at least one of subgames 3, 4, and 5 is an equilibrium, subgame 2 is also.

Proof. By scrutinizing the equilibrium conditions for subgames 2, 3, 4, and 5, we can see that the equilibrium conditions for subgame 2 is a necessary condition of the equilibrium conditions for subgames 3, 4, and 5. Additionally, we can also see that the equilibrium condition for subgame 4 is a necessary condition of the equilibrium conditions for subgame 5.

Proposition 4.8 shows the equilibria at which only one type of the programmers in the industry participates in the commercial project and the others in the open source project, and the relation between the equilibrium conditions for those equilibria. Interestingly, the equilibrium conditions for subgame 2 where only the lowly-abled-and-poorly-motivated programmer goes to the commercial project is a necessary condition of the equilibrium conditions for subgames 3, 4, and 5. This means that subgame 2 has the most comprehensive equilibrium conditions, i.e., the most unrestricted equilibrium conditions, among subgames 2, 3, 4, and 5. Specifically, if $k > \frac{3}{2}$ and $V < \left(6(k+1) + \sqrt{3(12k^2 + 26k + 13)}\right)\theta$, subgames 3, 4, and 5 never appear in equilibrium.

Proposition 4.9 *(Equilibria at which the programmers are biased toward the commercial project)*

(a) There possibly exist four equilibria at which only one programmer participates in the open source project and the others participate in the commercial project. At these equilibria, subgames 12, 13, 14, and 15 appear.

(b) If the equilibrium conditions for subgam 15 to be an equilibrium outcome are satisfied, then those for subgame 13 to be an equilibrium outcome are always satisfied. That is, if subgame 15 is an equilibrium outcome, subgame 13 is also.

Proof. By comparing the equilibrium conditions for subgame 13 with those for subgame 15 shown in Table 3, we can see that the equilibrium conditions for subgame 15 is a sufficient condition of the equilibrium conditions for subgame 13.

Finally, Proposition 4.9 shows the equilibria at which only one of the programmers in the industry participates in the open source project and the others in the commercial project, and the relation between the equilibrium conditions for those equilibria. We have found that the equilibrium conditions for subgame 15 are a subset of those for subgame 13. This implies that the case where only the lowly-abled-and-poorly-motivated programmer participates in the open source project cannot be a *unique* equilibrium. We have also found that if $k > \frac{7+\sqrt{561}}{24}$ or $V < \frac{36k^2+51k+22}{-36k^2+21k+32}\theta$, subgame 14 and 15 never appear in equilibrium.

4.6 Conclusion

Examining the motivations of the programmers who participate in the open source software projects is an interesting and important question. The motivations of the open source developers have not been fully explained by the utility theory of economics. We study the endogeneous decision of the programmers between the open source and the commercial software projects from an economic perspective grounded on the utility theory. We consider the types of the programmers in two dimensions: intrinsic motivation and ability. Our model captures how extrinsic motivation as well as the aforementioned two factors, intrinsic motivation and ability affect the programmers' project choices. We aim to provide the programmers and the commercial software company with the guidelines for project participation and recruiting, respectively.

We find that the optimal level of effort made by the programmers in the open source project increases with the levels of motivations and ability. Interestingly, our result shows that the optimal contract by the commercial company and the programmers' effort levels are the same under complete and incomplete information. Our finding supports the argument that the open source project is an effective compensation mechanism. We show that any subgame can be an equilibrium outcome. We find the equilbria at which the programmers are sorted by their ability levels and the levels of intrinsic motivations. Our finding implies that an open source project is more likely to have programmers with high ability and high intrinsic motivation than a commercial project.

Our main contribution is to bridge the gap between economic literature and behavioral science on the motivation for open source project participation. We studied the decision problems of the programmers in the software industry, whose types are different in terms of ability and intrinsic motivation, with an economic model based on utility theory. By analyzing the behaviors of the programmers grounded on an economic theory, we provided the rationale for open source project participation more clearly. We also provided software programmers with guidelines which help them make their decisions, and proprietary software companies with managerial implications of recruiting and compensating programmers.

In our model we assumed that all the programmers in the industry simultaneously make decisions on which project to participate in. However, in our real world, the decision of programmers could depend on who are participating in each project. Hence the case where the programmers choose one of projects sequentially is an interesting topic to study. We also assumed that there is no correlation between each programmer's ability and her intrinsic motivation. Relaxing this assumption is also one of interesting topics. In the current model, the commercial company offers a wage contract after all the programmers decide on which project to participate in. What happens if the commercial company moves first, that is, it announces a wage contract before the programmers' decisions? We leave all these questions for the future works.

Appendix A. Mathematical Proofs

Proof of Proposition 4.1.

Note that the first-order condition for maximizing U_i with respect to e_i is as follows:

$$\theta_i + \frac{e_i}{\sum_{j\in OS} e_j}V + \frac{\sum_{j\neq i} e_j}{\sum_{j\in OS} e_j}V - a_i e_i = 0.$$

The second order condition is then

$$2\frac{\sum_{j\neq i} e_j}{(\sum_{j\in OS} e_j)^2}V - 2\frac{\sum_{j\neq i} e_j}{(\sum_{j\in OS} e_j)^2}V - a_i = -a_i < 0.$$

From the first-order condition, $e_i^* = \frac{\theta_i+V}{a_i}$. It is trivial that e_i^* increases with $\frac{1}{a_i}$, θ_i, and V. At equilibrium, programmer i enjoys the following utility:

$$U^*_{i\in OS} = \sum_{j\in OS} e_j^* \left(\theta_i + \frac{e_i^*}{\sum_{j\in OS} e_j^*}V\right) - \frac{a_i(e_i^*)^2}{2}$$

$$= \sum_{j\in OS} \frac{(\theta_j+V)}{a_j}\left(\theta_i + \frac{a_i^{-1}(\theta_i+V)}{\sum_{j\in OS} a_j^{-1}(\theta_j+V)}V\right) - \frac{(\theta_i+V)^2}{2a_i}$$

$$= \theta_i \sum_{j\neq i}\left(\frac{\theta_j+V}{a_j}\right) + \frac{(\theta_i+V)^2}{2a_i}$$

$$= \theta_i \sum_{j\in OS}\left(\frac{\theta_j+V}{a_j}\right) - \frac{(\theta_i+V)(\theta_i-V)}{2a_i}.$$

Note that $\frac{\partial U^*_{i\in OS}}{\partial \theta_i} = \sum_{j\neq i}\left(\frac{\theta_j+V}{a_j}\right) + \frac{\theta_i+V}{a_i} > 0$, $\frac{\partial U^*_{i\in OS}}{\partial V} = \theta_i\sum_{j\neq i}\frac{1}{a_j} + \frac{\theta_i+V}{a_i} > 0$, and $\frac{\partial U^*_{i\in OS}}{\partial a_i} = -\frac{(\theta_i+V)^2}{2a_i^2} < 0$, which completes the proofs.

Proof of Optimal Contract and Effort Level under Complete Information.

Since $e_i = \frac{w_s-w_f}{a_i}$ and $P(\sum_{j\in CS} e_j) = \sum_{j\in CS} e_j$ by assumption, the maximization problem that the commercial company faces can be rearranged as

$$\max_{w_s,w_f\geq 0} U_{CS} = (w_s-w_f)\sum_{j\in CS}\frac{1}{a_j}(\pi - \#w_s) + \left(1-(w_s-w_f)\sum_{j\in CS}\frac{1}{a_j}\right)(-\#w_f).$$

It is straightforward that $w_f^* = 0$. Thus, the optimal contracting problem can be simplified as

$$\max_{w_s,w_f\geq 0} U_{CS} = w_s(\pi - \#w_s)\sum_{j\in CS}\frac{1}{a_j}.$$

Note that the first- and second-order conditions are as follows:

$$\text{F.O.C.}: \ (\pi - 2\# w_s) \sum_{j \in CS} \frac{1}{a_j} = 0,$$

$$\text{S.O.C.}: \ -2\# \sum_{j \in CS} \frac{1}{a_j} < 0.$$

From the first-order condition, we obtain the commercial company's optimal contract as

$$w^* = (w_s^*, w_f^*) = \left(\frac{\pi}{2\#}, 0 \right).$$

Then the optimal level of effort made by programmer i is $e_i^* = \frac{\pi}{2\# a_i}$. Recall from equation (4.2) that programmer i's utility in a commercial project under complete information is $U_i = P(\sum_{j \in CS} e_j) w_s + \left(1 - P(\sum_{j \in CS} e_j)\right) w_f - \frac{a_i e_i^2}{2}$. Substituting w_s^*, w_f^*, and e_i^* into U_i, the utility of programmer i is as follows:

$$U_{i \in CS}^* = \sum_{j \in CS} e_j^* w_s^* - \frac{a_i (e_i^*)^2}{2} = \frac{\pi^2}{4\#^2} \left(\sum_{j \in CS} \frac{1}{a_j} - \frac{1}{2a_i} \right).$$

Proof of Proposition 4.2.

Recall that we find the optimal contract and the optimal level of effort under complete information as $w^* = (w_s^*, w_f^*) = \left(\frac{\pi}{2\#}, 0\right)$ and $e_i^* = \frac{\pi}{2\# a_i}$. Now, we examine the case under incomplete information. From (4.6), the commercial company's optimal contracting problem is

$$\max_{\{w_s, w_f\}} E[U_{CS}] = \int \left\{ \sum_{j \in CS} e_j (\pi - \# w_s) + (1 - \sum_{j \in CS} e_j)(-\# w_f) \right\} dF(a)$$

$$\text{subject to } \ w_s, w_f \geq 0 \ \ \text{(LLC)}$$

$$U_{i \in CS} \geq 0 \ \ \text{(PC)}$$

$$e_i = \arg\max_{e_i} U_i \text{ for } i \in CS \ \ \text{(ICC)}$$

From ICC, we obtain the optimal level of effort as $e_i = \frac{w_s - w_f}{a_i}$. Thus, the maximization problem can be simplified to

$$\max_{\{w_s, w_f\}} E[U_{CS}] = \int \left\{ (w_s - w_f)(\pi - \# w_s) \sum_{j \in CS} \frac{1}{a_j} + \left(1 - (w_s - w_f) \sum_{j \in CS} \frac{1}{a_j} \right) (-\# w_f) \right\} dF(a).$$

Straightforwardly, $w_f^* = 0$. Thus, the optimal contracting problem becomes

$$\max_{\{w_s, w_f\}} E[U_{CS}] = w_s(\pi - \# w_s) \int \sum_{j \in CS} \frac{1}{a_j} dF(a).$$

The first- and second-order conditions are as follows:

$$\text{F.O.C.}: \quad (\pi - 2\# w_s) \inf \sum_{j \in CS} \frac{1}{a_j} dF(a) = 0,$$

$$\text{S.O.C.}: \quad -2\# \inf \sum_{j \in CS} \frac{1}{a_j} dF(a) < 0.$$

We obtain the commercial company's optimal contract and programmer i's optimal effort level as follows:

$$w^* = (w_s^*, w_f^*) = \left(\frac{\pi}{2\#}, 0\right) \quad \text{and} \quad e_i^* = \frac{\pi}{2\# a_i}.$$

Proof of Proposition 4.4.

We examine non-deviation conditions of all the programmers under each subgame. At first, let us consider the conditions for subgame 1 to be an equilibrium outcome. For subgame 1 to be an equilibrium outcome, each programmer should not have any incentive to deviate from the open source project to the commercial project, given the fact that the other programmers participate in the open source project. Specifically, the following non-deviation conditions should be satisfied for subgame 1 to become an equilibrium outcome:

- Non-deviation condition of a programmer of type HH;
  U_{HH}^* at subgame 1 $\geq U_{HH}^*$ at subgame 5 $\Leftrightarrow \frac{2\theta\{(k+2)V+\theta\}+k(\theta+V)^2}{2ka} \geq \frac{\pi^2}{8a}$,
- Non-deviation condition of a programmer of type HL;
  U_{HL}^* at subgame 1 $\geq U_{HL}^*$ at subgame 4 $\Leftrightarrow \frac{V^2}{2a} \geq \frac{\pi^2}{8a}$,
- Non-deviation condition of a programmer of type LH;
  U_{LH}^* at subgame 1 $\geq U_{LH}^*$ at subgame 3 $\Leftrightarrow \frac{2\theta\{(2k+1)V+k\theta\}+(\theta+V)^2}{2ka} \geq \frac{\pi^2}{8ka}$,
- Non-deviation condition of a programmer of type LL;
  U_{LL}^* at subgame 1 $\geq U_{LL}^*$ at subgame 2 $\Leftrightarrow \frac{V^2}{2ka} \geq \frac{\pi^2}{8ka}$.

Solving these four inequalities simultaneously, we obtain the equilibrium condition for subgame 1, $\pi \leq 2V$. That is, if $\pi \leq 2V$ then all the programmers in subgame 1 do not have any incentive to change their decisions and hence subgame 1 is an equilibrium outcome.

For another example, let us consider the equilibrium conditions for subgame 6 to be an equilibrium outcome. The following non-deviation conditions should be satisfied for subgame 6 to become an equilibrium outcome:

- Non-deviation condition of a programmer of type HH;
  U^*_{HH} at subgame 6 $\geq U^*_{HH}$ at subgame 13 $\Leftrightarrow \frac{2\theta V+(\theta+V)^2}{2a} \geq \frac{(k+4)\pi^2}{72ka}$,
- Non-deviation condition of a programmer of type HL;
  U^*_{HL} at subgame 6 $\geq U^*_{HL}$ at subgame 12 $\Leftrightarrow \frac{V^2}{2a} \geq \frac{(k+4)\pi^2}{72ka}$,
- Non-deviation condition of a programmer of type LH;
  U^*_{LH} at subgame 6 $\geq U^*_{LH}$ at subgame 2 $\Leftrightarrow \frac{3\pi^2}{32ka} \geq \frac{2k\theta(2V+\theta)+(\theta+V)^2}{2ka}$,
- Non-deviation condition of a programmer of type LL;
  U^*_{LL} at subgame 6 $\geq U^*_{LL}$ at subgame 3 $\Leftrightarrow \frac{3\pi^2}{32ka} \geq \frac{V^2}{2ka}$.

Solving these four inequalities simultaneously, we obtain the following equilibrium conditions for subgame 6:

$$\pi \in \left[4\sqrt{\frac{2k\theta(2V+\theta)+(\theta+V)^2}{3}}, 6\sqrt{\frac{kV^2}{k+4}}\right]$$

and

$$V \geq \frac{4(k+4)(2k+1)+2\sqrt{k(k+4)(2k+1)(8k+59)}}{23k-16}\theta.$$

By the exactly same way as in subgame 1 and 6, we find the equilibrium conditions for the other subgames. The equilibrium conditions for each subgame are summarized at Table 3.

Appendix B. Tables

Table 1. Types of the Programmers Participating into Each Project in Each subgame

	Open source project	Commercial project
Subgame 1	HH, HL, LH, LL	$\varnothing$
Subgame 2	HH, HL, LH	LL
Subgame 3	HH, HL, LL	LH
Subgame 4	HH, LH, LL	HL
Subgame 5	HL, LH, LL	HH
Subgame 6	HH, HL	LH, LL
Subgame 7	HH, LH	HL, LL
Subgame 8	HH, LL	HL, LH
Subgame 9	HL, LH	HH, LL
Subgame 10	HL, LL	HH, LH
Subgame 11	LH, LL	HH, HL
Subgame 12	HH	HL, LH, LL
Subgame 13	HL	HH, LH, LL
Subgame 14	LH	HH, HL, LL
Subgame 15	LL	HH, HL, LH
Subgame 16	$\varnothing$	HH, HL, LH, LL

Table 2. The Utilities of the Programmers in Each Subgame

	HH	*HL*	*LH*	*LL*
Subgame 1	$\frac{2\theta\{(k+2)V+\theta\}+k(\theta+V)^2}{2ka}$	$\frac{V^2}{2a}$	$\frac{2\theta\{(2k+1)V+k\theta\}+(\theta+V)^2}{2ka}$	$\frac{V^2}{2ka}$
Subgame 2	$\frac{2\theta\{(k+1)V+\theta\}+k(\theta+V)^2}{2ka}$	$\frac{V^2}{2a}$	$\frac{2k\theta(2V+\theta)+(\theta+V)^2}{2ka}$	$\frac{\pi^2}{8ka}$
Subgame 3	$\frac{2\theta(k+1)V+k(\theta+V)^2}{2ka}$	$\frac{V^2}{2a}$	$\frac{\pi^2}{8ka}$	$\frac{V^2}{2ka}$
Subgame 4	$\frac{2\theta(2V+\theta)+k(\theta+V)^2}{2ka}$	$\frac{\pi^2}{8a}$	$\frac{2\theta\{(k+1)V+k\theta\}+(\theta+V)^2}{2ka}$	$\frac{V^2}{2ka}$
Subgame 5	$\frac{\pi^2}{8a}$	$\frac{2\theta(2V+\theta)+kV^2}{2ka}$	$\frac{2\theta(k+1)V+(\theta+V)^2}{2ka}$	$\frac{V^2}{2ka}$
Subgame 6	$\frac{2\theta V+(\theta+V)^2}{2a}$	$\frac{V^2}{2a}$	$\frac{3\pi^2}{32ka}$	$\frac{3\pi^2}{32ka}$
Subgame 7	$\frac{(\theta+V)\{(k+2)\theta+kV\}}{2ka}$	$\frac{(k+2)\pi^2}{32ka}$	$\frac{(\theta+V)\{(2k+1)\theta+V\}}{2ka}$	$\frac{(2k+1)\pi^2}{32ka}$
Subgame 8	$\frac{2\theta V+k(\theta+V)^2}{2ka}$	$\frac{(k+2)\pi^2}{32ka}$	$\frac{(2k+1)\pi^2}{32ka}$	$\frac{V^2}{2ka}$
Subgame 9	$\frac{(k+2)\pi^2}{32ka}$	$\frac{V^2}{2a}$	$\frac{2\theta kV+(\theta+V)^2}{2ka}$	$\frac{(2k+1)\pi^2}{32ka}$
Subgame 10	$\frac{(k+2)\pi^2}{32ka}$	$\frac{V^2}{2a}$	$\frac{(2k+1)\pi^2}{32ka}$	$\frac{V^2}{2ka}$

Subgame 11	$\frac{3\pi^2}{32a}$	$\frac{3\pi^2}{32a}$	$\frac{2\theta V+(\theta+V)^2}{2ka}$	$\frac{V^2}{2ka}$
Subgame 12	$\frac{(\theta+V)^2}{2a}$	$\frac{(k+4)\pi^2}{72ka}$	$\frac{(2k+3)\pi^2}{72ka}$	$\frac{(2k+3)\pi^2}{72ka}$
Subgame 13	$\frac{(k+4)\pi^2}{72ka}$	$\frac{V^2}{2a}$	$\frac{(2k+3)\pi^2}{72ka}$	$\frac{(2k+3)\pi^2}{72ka}$
Subgame 14	$\frac{(3k+2)\pi^2}{72ka}$	$\frac{(3k+2)\pi^2}{72ka}$	$\frac{(\theta+V)^2}{2ka}$	$\frac{(4k+1)\pi^2}{72ka}$
Subgame 15	$\frac{(3k+2)\pi^2}{72ka}$	$\frac{(3k+2)\pi^2}{72ka}$	$\frac{(4k+1)\pi^2}{72ka}$	$\frac{V^2}{2ka}$
Subgame 16	$\frac{(3k+4)\pi^2}{128ka}$	$\frac{(3k+4)\pi^2}{128ka}$	$\frac{(4k+3)\pi^2}{128ka}$	$\frac{(4k+3)\pi^2}{128ka}$

Table 3. The Equilibrium Conditions for Each subgame

	Equilibrium Conditions
Subgame 1	$\pi \le 2V$
Subgame 2	$\pi \in \left[2V, \min\left\{4\sqrt{\frac{kV^2}{k+2}}, 4\sqrt{\frac{2k\theta(2V+\theta)+(\theta+V)^2}{3}}\right\}\right]$
Subgame 3	$\pi \le \left[2\sqrt{2\theta\{(2k+1)V+k\theta\}+(\theta+V)^2}, 4\sqrt{\frac{V^2}{3}}\right]$, $V \ge \left(6(k+1)+\sqrt{3(13+26k+12k^2)}\right)\theta$
Subgame 4	$\pi \le \left[2V, 4\sqrt{\frac{V^2}{2k+1}}\right]$, $k \le \frac{3}{2}$
Subgame 5	$\pi \in \left[2\sqrt{\frac{2\theta\{(k+2)V+\theta\}+k(\theta+V)^2}{k}}, 4\sqrt{\frac{V^2}{2k+1}}\right]$, $V \ge \frac{2(k+1)(2k+1)+\sqrt{(2k+1)\{4+k(22+19k+6k^2)\}}}{k(3-2k)}\theta$, $k < \frac{3}{2}$
Subgame 6	$\pi \in \left[4\sqrt{\frac{2k\theta(2V+\theta)+(\theta+V)^2}{3}}, 6\sqrt{\frac{kV^2}{k+4}}\right]$, $V \ge \frac{4(k+4)(2k+1)+2\sqrt{k(k+4)(2k+1)(8k+59)}}{23k-16}\theta$
Subgame 7	$\pi \in \left[4\sqrt{\frac{kV^2}{k+2}}, 6\sqrt{\frac{(\theta+V)\{(k+2)\theta+kV\}}{3k+2}}\right]$, $V < 2\theta$ **or** $\pi \in \left[4\sqrt{\frac{kV^2}{k+2}}, 6\sqrt{\frac{(\theta+V)\{(2k+1)\theta+V\}}{2k+3}}\right]$, $V \ge 2\theta$, $k \le \frac{3}{16}(\sqrt{65}-1)$ **or** $\pi \in \left[4\sqrt{\frac{kV^2}{k+2}}, 6\sqrt{\frac{(\theta+V)\{(2k+1)\theta+V\}}{2k+3}}\right]$, $V \in \left[2\theta, \frac{9(k+1)(k+2)+3\sqrt{k(k+2)(12+50k+25k^2)}}{8k^2+3k-18}\right]$, $k > \frac{3}{16}(\sqrt{65}-1)$

Subgame 8	$\pi \in \left[4\sqrt{\frac{2\theta\{(k+1)V+k\theta\}+(\theta+V)^2}{2k+1}}, 6\sqrt{\frac{V^2}{2k+3}}\right]$, $V \in \left[\frac{8(k+1)(2k+3)+2\sqrt{(2k+3)\{45+4k(33+33k+8k^2)\}}}{10k-3}\theta, \frac{k(k+2)+\sqrt{(k+2)(5k^2-4k+2)}}{2(k^2-1)}\theta\right]$ **or** $\pi \in \left[4\sqrt{\frac{kV^2}{k+2}}, 6\sqrt{\frac{V^2}{2k+3}}\right]$, $V \geq \frac{k(k+2)+\sqrt{(k+2)(5k^2-4k+2)}}{2(k^2-1)}\theta$, $k \leq \frac{3}{16}(\sqrt{65}-1)$
Subgame 9	$\pi \in \left[4\sqrt{\frac{2\theta\{(k+1)V+\theta\}+k(\theta+V)^2}{k+2}}, 6\sqrt{\frac{kV^2}{3k+2}}\right]$, $V \in \left[\frac{4(2+7k+6k^2)+2\sqrt{(3k+2)\{8+k(45k^2+84k+64)\}}}{k(10-3k)}\theta, \frac{(k+1)(3k+2)+\sqrt{k(k+1)(3k+2)(3k+7)}}{2(k^2-1)}\theta\right]$, $k < 1.27423$ **or** $\pi \in \left[4\sqrt{\frac{2\theta\{(k+1)V+\theta\}+k(\theta+V)^2}{k+2}}, 6\sqrt{\frac{2\theta kV+(\theta+V)^2}{2k+3}}\right]$, $V \geq \max\left\{\frac{(k+1)(3k+2)+\sqrt{k(k+1)(3k+2)(3k+7)}}{2(k^2-1)}\theta, \frac{7k^2+5k-6+\sqrt{144-k(k+2)(15k^2+76k-132)}}{-8k^2-3k+18}\theta\right\}$, $k \leq \frac{3}{16}(\sqrt{65}-1)$
Subgame 10	$\pi \in \left[4\sqrt{\frac{2\theta(k+1)V+k(\theta+V)^2}{k+2}}, 6\sqrt{\frac{V^2}{2k+3}}\right]$, $V \geq \frac{4(3+8k+4k^2)+2\sqrt{(2k+3)\{12+k(74+77k+24k^2)\}}}{-8k^2-3k+18}\theta$, $k \leq \frac{3}{16}(\sqrt{65}-1)$

Subgame 11	$\pi \in \left[4\sqrt{\frac{2\theta(2V+\theta)+k(\theta+V)^2}{3k}}, 6\sqrt{\frac{V^2}{4k+1}}\right]$, $V \geq \frac{4(k+2)(4k+1)+2\sqrt{(k+2)(4k+1)(59k+8)}}{k(23-16k)}\theta,\ k<\frac{23}{16}$
Subgame 12	$\pi \in \left[6\sqrt{\frac{(\theta+V)\{(2k+1)\theta+V\}}{2k+3}}, 8\sqrt{\frac{k(\theta+V)^2}{3k+4}}\right]$, $V \in \left[\frac{22k^2+51k+36}{32k^2+21k-36}\theta, \frac{(k+1)(k+4)+\sqrt{k(k+4)(3+12k+5k^2)}}{2(k-1)(k+2)}\theta\right]$ **or** $\pi \in \left[6\sqrt{\frac{kV^2}{k+4}}, 8\sqrt{\frac{k(\theta+V)^2}{3k+4}}\right],\ V \geq \frac{(k+1)(k+4)+\sqrt{k(k+4)(3+12k+5k^2)}}{2(k-1)(k+2)}\theta,\ k<\frac{28}{11}$ **or** $\pi \in \left[6\sqrt{\frac{kV^2}{k+4}}, 8\sqrt{\frac{k(\theta+V)^2}{3k+4}}\right]$, $V \in \left[\frac{(k+1)(k+4)+\sqrt{k(k+4)(3+12k+5k^2)}}{2(k-1)(k+2)}\theta, \frac{16(k+4)+12\sqrt{(k+4)(3k+4)}}{11k-28}\theta\right],\ k>\frac{28}{11}$
Subgame 13	$\pi \in \left[6\sqrt{\frac{2\theta V+(\theta+V)^2}{k+4}}, 8\sqrt{\frac{kV^2}{3k+4}}\right]$, $V \geq \max\left\{\left(k+2+\sqrt{(k+1)(k+3)}\right)\theta, \frac{18(3k+4)+3\sqrt{(3k+4)(108+145k+16k^2)}}{16k^2+37k-36}\theta\right\}$ **or** $\pi \in \left[6\sqrt{\frac{2\theta kV+(\theta+V)^2}{2k+3}}, 8\sqrt{\frac{kV^2}{3k+4}}\right]$, $V \in \left[\frac{9(k+1)(3k+4)+3\sqrt{k(3k+4)(120+122k+27k^2)}}{32k^2+21k-36}\theta, \left(k+2+\sqrt{(k+1)(k+3)}\right)\theta\right],\ k \geq 1.3870$

Subgame 14	$\pi \in \left[6\sqrt{\frac{(\theta+V)\{(k+2)\theta+kV\}}{3k+2}}, 8\sqrt{\frac{(\theta+V)^2}{4k+3}}\right]$, $V \geq \frac{36k^2+51k+22}{-36k^2+21k+32}\theta$, $k \leq \frac{7+\sqrt{561}}{24}$
Subgame 15	$\pi \in \left[6\sqrt{\frac{2\theta V+k(\theta+V)^2}{3k+2}}, 8\sqrt{\frac{V^2}{4k+3}}\right]$, $V \geq \frac{9(k+1)(4k+3)+3\sqrt{(4k+3)(27+122k+120k^2)}}{-36k^2+21k+32}\theta$, $k < \frac{7+\sqrt{561}}{24}$
Subgame 16	$\pi \geq 8\sqrt{\frac{k}{3k+4}}(\theta+V)$

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