

Large Scale Homogeneous Turbulence and Interactions with a Flat-Plate Cascade

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(Abstract)

The turbulent flow through a marine propulsor was experimentally modeled using a large cascade configuration with six 33 cm chord flat plates spanning the entire height of the test section in the Virginia Tech Stability Wind Tunnel. Three-component hot-wire velocity measurements were obtained ahead, throughout and behind both an unstaggered and a 35° staggered cascade configuration with blade spacing and onset turbulence integral scales on the order of the chord. This provided a much needed data-set of much larger Taylor Reynolds number than previous related studies and allowed a thorough investigation of the blade-blocking effects of the cascade on the incident turbulent field.

In order to generate the large scale turbulence needed for this study, a mechanically rotating “active” grid design was adopted and placed in the contraction of the wind tunnel at a streamwise location sufficient to cancel out the relatively large inherent low frequency anisotropy associated with this type of grid. The resulting turbulent flow is one of the largest Reynolds number ($Re_\lambda \approx 1000$) homogeneous near-isotropic turbulent flows ever created in a wind tunnel, and provided the opportunity to investigate Reynolds number effects on turbulence parameters, especially relating to inertial range dynamics. Key findings include 1) that the extent of local isotropy is solely determined by the turbulence generator and the size of the wind-tunnel that houses it; and 2) that the turbulence generator operating conditions affect the shape of the equilibrium range at fixed Taylor Reynolds number. The latter finding suggests that grid turbulence is not necessarily self-similar at a given Reynolds number independent of how it was generated.

The experimental blade-blocking data was compared to linear cascade theory and showed good qualitative agreement, especially for wavenumbers above the region of influence of the wind tunnel and turbulence generator effects. As predicted, the turbulence is permanently modified by the presence of the cascade after which it remains invariant for a significant downstream distance outside the thin viscous regions. The obtained results support the claim that Rapid Distortion Theory (RDT) is capable of providing reasonable estimates of the flow behind the cascade even though the experimental conditions lie far outside the predicted region of validity.

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Nomenclature

Upper-case Roman

A	Constants; Local wind tunnel area (m^2)
B	Constants
C	Kolmogorov constant
C_i	One-dimensional Kolmogorov constants
C_{i^*}	Modified one-dimensional Kolmogorov constants
C_D	Drag coefficient
C_P	Pressure coefficient
D_{iii}	Third-order structure function
E	Three-dimensional energy spectrum (m^2/s^1); Hot-wire anemometer output voltage (V)
E_1	Corrected hot-wire voltage (V)
E_2	Uncorrected hot-wire voltage (V)
E_{ii}	One-dimensional energy spectrum (m^2/s^1)
F	Fourier transform of f
G_{ff}	Periodogram of function f
H	Wind tunnel height and width (m)
I	Anisotropy ratio, u/v
K	Kurtosis
L	Macroscale (m)
L_{11}	Longitudinal integral scale (m)
M	Grid mesh length (m)
N	Number of data points
R_{ii}	Two-point correlation
Re_L	Turbulence Reynolds number, $k^{1/2}L/\nu$
Re_M	Reynolds number, $U_\infty M/\nu$
Re_T	Turbulence Reynolds number, uL_{11}/ν
Re_x	Streamwise Reynolds number, $U_\infty x/\nu$
Re_λ	Taylor Reynolds number, $u\lambda/\nu$
S	Skewness
T	Average grid cruise time: $\langle \tilde{T} \rangle$ (s); Period of a function (s)
$\tilde{T}$	Instantaneous grid rod cruise time (s)
T_c	Hot-wire calibration temperature ($^\circ\text{C}$)
T_∞	Free-stream flow temperature ($^\circ\text{C}$)
U	Local mean convective velocity (m/s)
U_{eff}	Effective velocity as sensed by a hot-wire sensor (m/s)
U_∞	Test section flow velocity (m/s)
V	Local vertical mean velocity (m/s)
W	Local lateral mean velocity (m/s)
X	Streamwise grid-based coordinate (m)
X_0	Virtual Origin (m)

Nomenclature

Y	Vertical grid-based coordinate (m)
Z	Lateral grid-based coordinate (m)

Lower-case Roman

a	Lagrangian position vector (m)
a_i	Fourier series constants
b	Agitator wing width (m)
b_i	Fourier series constants
c	Cascade blade chord length (m); Agitator wing chord length (m); Contraction ratio, U_2/U_1 or A_1/A_2
d	Grid Rod Diameter (m)
f	Frequency (s^{-1}); Function of time
f_s	Sampling frequency (s^{-1})
g	Function of frequency
g_m	Components of the DFT
h	Incremental step size
i	Index subscript indicating direction: 1: streamwise, 2: vertical, 3: lateral; Imaginary number: $(-1)^{1/2}$
k	Turbulence kinetic energy (m^2/s^2)
l	Characteristic body dimension (m)
n	Decay exponent; Integer value
p	Probability of an event
r	Separation distance (m)
r_c	Relative humidity during hot-wire calibration (%)
r_∞	Ambient Relative humidity (%)
s	Cascade blade spacing (m)
t	Maximum cruise time deviation (s); time (s)
u	Streamwise RMS velocity (m/s)
$\tilde{u}$	Instantaneous streamwise velocity (m/s)
u'	Streamwise fluctuating velocity (m/s)
$\tilde{u}_i$	Generalized instantaneous velocity: $\tilde{u}_1 = \tilde{u}$, $\tilde{u}_2 = \tilde{v}$, $\tilde{u}_3 = \tilde{w}$ (m/s)
v	Vertical RMS velocity (m/s)
$\tilde{v}$	Instantaneous vertical velocity (m/s)
v'	Vertical fluctuating velocity (m/s)
w	Local rms velocity in the lateral direction (m/s)
$\tilde{w}$	Instantaneous lateral velocity (m/s)
w'	Lateral fluctuating velocity (m/s)
x	Streamwise cascade-based coordinate (m)
y	Vertical cascade-based coordinate (m)
z	Lateral cascade-based coordinate (m)

Nomenclature

Upper-case Greek

Δ	Incremental change
Δh	Distance between grid planes (m)
Φ_{ij}	Velocity spectrum tensor
Ω	Average absolute grid rotational speed: $\langle \tilde{\Omega} \rangle$ (Hz)
$\tilde{\Omega}$	Instantaneous grid rod rotational speed (Hz)

Lower-case Greek

α	Grid acceleration rate (Hz/s); Contraction parameter: $[1 - c^{-3}]^{1/2}$
δ	Boundary layer thickness (m)
δ^*	Displacement thickness (m)
δ_{ij}	Kronecker delta
ε	Viscous (pseudo-) dissipation rate (m^2/s^3)
η	Kolmogorov scale (m)
θ	Hot-wire overheat ratio; Momentum thickness (m)
κ	Wavenumber magnitude: $(\kappa_1^2 + \kappa_2^2 + \kappa_3^2)^{1/2}$ (m^{-1})
$\vec{\kappa}$	Vector wavenumber: $\kappa_1 \mathbf{i} + \kappa_2 \mathbf{j} + \kappa_3 \mathbf{k}$ (m^{-1})
κ_d	Characteristic wavenumber of dissipative eddies: $1/\eta$ (m^{-1})
κ_e	Characteristic wavenumber of energetic eddies: $3/(4L_{11})$ (m^{-1})
κ_i	Directional wavenumber (m^{-1})
$\kappa_{H,i}$	Wind tunnel wavenumber of the i^{th} harmonic (m^{-1})
λ_f	Longitudinal Taylor microscale (m)
λ_g	Transverse Taylor microscale (m)
λ_H	Wind tunnel wavelength (m)
ν	Kinematic viscosity (m^2/s)
ξ	Temperature dependent coefficient
σ	Standard deviation; Grid Solidity
τ	Two-dimensional wavenumber magnitude: $(\kappa_1^2 + \kappa_2^2)^{1/2}$ (m^{-1})
φ	Cascade stagger angle ($^\circ$)
ω	Maximum rotational speed deviation (s^{-1}); Vorticity (m/s^2); Angular frequency (rad/s)

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Chapter 1

Introduction

1.1 Motivation and Application

As a submarine glides through a body of water, the boundary layer shed off the hull will get ingested into the propulsion system. In the case of a shrouded propeller, the inlet guide vanes will alter the incoming turbulent field in two steps. The main effect of the cascade is the modification of the incoming turbulent flow through blade blocking effects, acceleration and flow turning while the inevitable wakes from the vanes will add new vorticity to the flow which interacts with the original turbulence as it is convected towards the propeller. When this highly inhomogeneous turbulence impinges on the rotating propeller, noise and vibration is generated and propagated into the surroundings. This does not only affect the overall efficiency and life expectancy of the propulsor, but can also prove detrimental in naval applications where silence is often the key to survival.

The ultimate goal with respect to noise prediction is to establish how the turbulent flow field appears from the propeller's viewpoint. Although the instantaneous flow can not be predicted as it is highly unsteady, the average flow-field appears stationary in time. Such a flow description, which can account for the alterations of the free-stream flow, would become a useful aid towards the development of new propeller designs.

This scenario can be extended to commercial applications as well, such as the situation in which turbulent fluid is ingested by a turbofan aircraft engine, modified by the rotating fan blades and ultimately interacts with downstream structural objects, contributing to engine noise which can be remedied thorough the proper understanding of the flow characteristics.

It is not realistic to require experimental testing of each and every design modification, and it is therefore highly desirable to develop computer codes which can be used to predict the various flow fields. In lieu of an extremely expensive Direct Numerical Simulation (DNS) calculation which will only give one possible instantaneous flow rendition, one would want to use simpler methods that capture the physical essence of the problem and that give reasonable predictions in a relatively short time. In order to develop such computer codes it is extremely useful to be able to compare the results to experimental results for certain benchmark conditions in order to calibrate the code to the point where a useful engineering solution can be obtained.

Complete datasets documenting the blade blocking effects due to the non-penetration condition imposed by the solid surfaces of single blades or cascade configurations already exist; however, the vast majority of previous experimental studies have been restricted to relatively small Reynolds number models with weak ambient turbulence of much smaller scales than is generally found in practical applications. The motivation behind this research is therefore to design and conduct an experiment to realistically model the dominant physics of the flow

through the first stage of a marine propulsor unit in order to provide data for the fine-tuning of potential computational methods.

This work presents the experimental results of such a study along with a comparison with linear cascade theory which has previously shown some merit in predicting the initial blade blocking behavior in shearless boundary layers (Hunt & Graham, 1978) as well as through cascades of blades with small spacing (Graham, 1998). Additionally, the relatively unique method of generating the large-scale free stream turbulence for this work provided a substantial amount of data which called for an in-depth analysis of some of the properties of high Reynolds number homogeneous near-isotropic turbulence. A significant portion of this material has also been included.

1.2 Brief Review of Basic Turbulence Quantities and Parameters

This section provides a short review of some of the most basic quantities and parameters often utilized in the study of turbulence. It is included here in order to serve as a reference as many of these relations will be referred to numerous times throughout the text. It can be skipped, by readers familiar with these concepts, with no loss of continuity.

The instantaneous streamwise velocity ($\tilde{u}$) in a turbulent flow can be decomposed into a mean part (U) and a fluctuating part (u') according to $\tilde{u} = U + u'$. In the case of free-stream turbulence sufficiently away from a wall or other obstacle, the mean flow takes on a value denoted by U_∞ . The rms (root-mean-square) velocity fluctuation is defined via Equation 1.1 where the over-bar signifies averaging. From this the turbulence intensity relative to the free-stream can be obtained as $TI = u/U_\infty$ which is usually expressed as a percentage. The lateral velocity component $\tilde{v}$ and $\tilde{w}$ can be defined similarly, but in contrast to the streamwise velocity, $V_\infty = W_\infty = 0$ for decaying free-stream turbulence in a wind tunnel.

$$u = \sqrt{\overline{u'^2}} \quad (1.1)$$

One of the most important concepts in fluids dynamics is the Reynolds number, and turbulent flows are no exception. The Reynolds number is effectively a measure of the relative importance between inertial and viscous forces in a flow. Besides the standard flow Reynolds number, $Re_l = U_\infty l / \nu$, where l is any characteristic lengthscale and ν is the free stream velocity, a frequently encountered parameter in the study of turbulence is the Taylor Reynolds number (Equation 1.2). In a given fluid this non-dimensional number can be interpreted as a measure of the size and intensity of the turbulent fluctuations present in the flow. The Taylor Reynolds number is based on the (transverse) Taylor microscale, λ_g , which will be discussed in more detail in Section 4.9.2. It is, however, a relatively small lengthscale whose associated velocity scale is the streamwise rms velocity fluctuation u . In isotropic turbulence the Taylor microscale can be experimentally obtained from Equation 1.3.

$$Re_\lambda = \frac{u \lambda_g}{\nu} \quad (1.2)$$

$$\lambda_g = \sqrt{\frac{u^2}{\left(\partial u' / \partial x\right)^2}} \quad (1.3)$$

The integral scale, L_{11} , is a measure of the largest eddies in a turbulent flow. Traditionally, the integral lengthscale is obtained from the low-frequency limit of the energy spectrum according to Equation 1.4 where E_{11} is the streamwise one-dimensional energy spectrum.* As will be shown in Chapter 4 this proves to be a rather uncertain method for large-scale laboratory turbulence. Mydlarski & Warhaft (1996) solved this problem by utilizing an alternative method which determines the wavenumber associated with the peak (κ_{peak}) of $\kappa_1 E_{11}(\kappa_1)$ and takes $L_{11} = 1/\kappa_{\text{peak}}$. This method was also utilized by Lumley & Panofsky (1964) in their book on atmospheric turbulence. The approach will be discussed in more detail in Section 4.7.3, but in summary it proves to be a much more consistent method than the estimates provided through Equation 1.4.

$$L_{11} = \frac{\pi E_{11}(\kappa_1 \rightarrow 0)}{2u^2} \quad (1.4)$$

The macroscale, L , in a turbulent flow is defined by Pope (2000) in terms of the turbulence kinetic energy (k)[†] and the dissipation rate (ε) according to Equation 1.5. This characteristic lengthscale is another measure of the largest eddies present in a flow, but is generally about a factor of two larger than the integral lengthscale. The macroscale is defined using the concept that there is a scaling law for the turbulence cascade:

$$L = \frac{k^{3/2}}{\varepsilon} \quad (1.5)$$

Mydlarski & Warhaft (1996) utilized an alternative form of this as given below in Equation 1.6 where A is a constant with a value close to unity. In isotropic turbulence A is by definition given in Equation 1.7. There is, however, a problem with this equation at low Taylor Reynolds since L_{11}/L is actually a function of Re_λ . This will be discussed in more detail in Section 4.7.3.

$$L_{11} = A(u^2)^{3/2} / \varepsilon \quad (1.6)$$

$$A = \left(\frac{3}{2}\right)^{3/2} \frac{L_{11}}{L} \quad (1.7)$$

It is also necessary to define a couple of different turbulence Reynolds numbers. The first one, Re_L , is based on the macroscale, which via Equation 1.5 can be rewritten in terms of the dissipation rate, while Re_T uses the more conventional integral lengthscale.

* The energy spectrum will be discussed in greater detail in Section 4.6.

[†] The turbulence kinetic energy is defined as $k = (u^2 + v^2 + w^2)/2$. In true isotropic turbulence where $u^2 = v^2 = w^2$, this simply reduces to $k = 3u^2/2$.

$$\text{Re}_L = \frac{k^{1/2} L}{\nu} = \frac{k^2}{\varepsilon \nu} \quad (1.8)$$

$$\text{Re}_T = \frac{u L_{11}}{\nu} \quad (1.9)$$

The two turbulence Reynolds numbers and the Taylor Reynolds number are related as follows:

$$\text{Re}_T = \sqrt{\frac{2}{3}} \frac{L_{11}}{L} \text{Re}_L \quad (1.10)$$

$$\text{Re}_L = \frac{3}{20} \text{Re}_\lambda^2 \quad (1.11)$$

The Kolmogorov scale (η), which is a measure of the size of the turbulent eddies that are responsible for energy dissipation, can be obtained through the macroscale by combining Equation 1.8 and the definition $\eta = (\nu^3/\varepsilon)^{1/4}$ which results in Equation 1.12.*

$$\frac{L}{\eta} = \text{Re}_L^{3/4} \quad (1.12)$$

Finally, the anisotropy ratio needs to be defined. This is an indication of how isotropic the flow is based on second order statistics. It is a relatively crude measure, but good for initial evaluation of a turbulent free-stream flow. The anisotropy ratio assumes axisymmetric flow in which $v = w$ and is simply defined as $I = u/v$. In general, the closer this ratio is to unity, the more isotropic the flow.

1.3 Background

Returning to the problem at hand this section summarizes some of the available computational methods and reviews theoretical and experimental work of previous blade-blocking studies.

1.3.1 Computational Methods

In order to select an efficient computational tool for predicting the blade blocking effects of the problem in question, one needs to evaluate the methods available. One of the more

* The Kolmogorov scale depends solely on the viscosity of the fluid, ν , and the viscous dissipation rate, ε , of the turbulent flow field. The methods for obtaining values for the viscous dissipation are discussed in detail in section 4.8.

straightforward approaches would be to perform a Reynolds-Averaged Navier-Stokes (RANS) calculation of the flow field around the cascade. The RANS method consists of averaging the Navier-Stokes equations and modeling the resulting Reynolds stress tensor to obtain a solution for the mean flow field. This would take care of all the averaged flow parameters. The correlation between points in the flow on the other hand, would be completely lost during the averaging making it impossible to track the evolution of the space-time correlation through the cascade passage which is critical in order to predict the noise emitted from the interaction with the blade row.

A Direct Numerical Simulation (DNS) would provide all the necessary information as every term of the Navier-Stokes equations is calculated with all scales resolved. Although a great tool at low Reynolds numbers ($Re_\lambda < 100$) it provides only a single realization per calculation and several rounds of calculations may be necessary to obtain a good average. Computational cost, however, rises quickly with Reynolds number. In the range that is necessary for this application a DNS simulation would take on the order of 90 years at $Re_\lambda \approx 400$ and 5000 years at $Re_\lambda \approx 800$ on a computer performing at one gigaflop (as estimated in Pope, 2000, pp. 349). This effectively removes DNS from the list of available CFD methods.

Large-Eddy Simulation (LES) has often been called “the poor man’s DNS” because it only calculates the low wavenumber range directly while models the rest of the turbulence spectrum. This significantly cuts down on computer cost. While excellent for homogeneous turbulence, in the presence of a solid boundary it becomes necessary to provide the calculations with a very finely resolved grid close to the wall. This is a highly Reynolds number dependent situation which again could become an insurmountable task for the present application. Another approach would be to model the near-wall behavior and apply it to the calculations close to the wall. This keeps the computational costs under control even for large Reynolds numbers, but the task of modeling the flow behavior close to the wall is usually a very difficult process.

Another method, known as Rapid-Distortion Theory (RDT), offers reasonable computational expense and the ability of calculating space-time correlations by linearizing and solving the governing equations for the convection of the turbulence. One of the key assumptions of RDT is that the distortion (in this case the introduction of the cascade blades) must occur over a relatively short time for the solution to be valid. For large scale applications however, the validity of RDT for this purpose is still an open question. The following two subsections will cover some of the main aspects of the theory and a brief historical overview of the evolution of the theory.

1.3.2 Rapid Distortion Theory and its Assumptions

Although originally devised for a flow with an associated mean strain, the assumptions and underlying concepts can be used in a blade blocking scenario without such a condition (i.e. a flow entering a cascade). The cascade problem encountered in this project, involving modified velocity potentials and frozen vorticity transport, can be made linear by imposing the Rapid Distortion Theory assumptions in which the solution should be valid for a sufficiently short time behind the leading edges of the cascade. This is the form of RDT used by Graham (1998) which will be briefly mentioned below in Section 1.3.4 and eventually considered in Chapter 7. As an introduction to the subject, however, the original RDT theory is summarized below.

Chapter 1: Introduction

Classic Rapid Distortion Theory is mainly concerned with the distortion (or strain) imposed on a turbulent field for example by the introduction of one or more rigid boundaries. If the timescale over which this strain is applied is sufficiently small, certain simplifying assumptions can be made which leads to a linearized version of Cauchy's equation (Eq. 1.13) which is the Lagrangian form of the vorticity transport equation for incompressible inviscid fluids. This equation is expressed in index notation where repeated indices signify summation. The vorticity of the fluid at the beginning of the distortion is denoted $\omega'(a)$ at position a ; $\omega(a)$ is the vorticity associated with the same element at any subsequent time during the course of the distortion while $\partial x_i / \partial a_j$ is the strain or distortion tensor.

$$\omega_i(a) = \frac{\partial x_i}{\partial a_j} \omega'_j(a) \quad (1.13)$$

In order to use Cauchy's equation, the assumption that viscous forces are negligible has already been made. This is generally not the case in a turbulent flow as viscous forces are an important part of the problem. However, when a large strain is applied to a turbulent flow, the large scale implications will for a short time be of much greater importance than the effects of viscosity. The definition of a sufficiently "short" time is important here as it is the fundamental idea behind Rapid Distortion Theory. Mathematically this assumption can be expressed in terms of the dissipation rate, ε , as:

$$\int_{t_1}^{t_2} \varepsilon(t) dt \ll \frac{1}{2} u^2 \quad (1.14)$$

This requirement can be simplified by using the initial value of the dissipation rate and Equation 1.6 such that Equation 1.14 turns into:

$$t_2 - t_1 \ll \frac{L_{11}}{2A\sqrt{u^2}} \quad (1.15)$$

Invoking Taylor's hypothesis, $t_2 - t_1$ can be related to a distortion lengthscale, $l = (t_2 - t_1)U_\infty$, such that the final requirement for RDT (with some rearrangement) becomes:

$$2A \frac{u}{U_\infty} \ll \frac{L_{11}}{l} \quad (1.16)$$

The constant A is on the order of unity, so this requirement really states that twice the turbulence intensity must be much smaller than the ratio between the integral lengthscale and the length over which the distortion takes place. In their paper Batchelor & Proudman (1954) acknowledged that very few situations will satisfy the assumptions of RDT. However it was shown by for example Uberoi (1956) that the results of RDT calculations describe experimentally obtained fluctuating velocity data quite well in the case of flows through the initial parts of contractions, up to an order of magnitude beyond the supposed limits of the theory.

The second assumption, necessary to make the problem linear, is that the fluctuating particles' effect on their neighboring particles is very small compared to the effect of the external distortion. In other words the effect of the turbulence on itself is very small. This is achieved by having a large strain rate compared to the ratio u/L_{11} . However, Kevlahan & Hunt (1997) showed that even in the presence of weak straining the non-linear terms are significantly inhibited, allowing RDT to work even if the non-linear terms are initially on the same order as the linear terms.

1.3.3 Brief History of Rapid Distortion Theory

Batchelor (1953) introduced Rapid Distortion Theory to solve the case of a turbulent stream through screens and the effect of a contraction on initially isotropic turbulence.* The concepts were extended in Batchelor & Proudman (1954) before Goldstein & Durbin (1980) expanded the predictions of the effect of a contraction to include flows with large scale turbulence. Hunt (1973) utilized the rapid distortion assumptions to calculate the turbulent velocities upstream of and around two-dimensional bluff bodies outside the region of separated flow. A variation of this was used to develop the theory used in Hunt & Graham (1978) which strived to describe the blade blocking effects of a shearless boundary layer on a flat plate in a uniform turbulent stream.

Goldstein (1978) modified the generalized Hunt (1973) version of RDT to include the interaction of entropy fluctuations on potential flow in expansion fans in external flows through incorporating a linear wave equation. This was also applied to non-uniform upstream conditions for internal flows in Goldstein (1979) and obstacles with stagnation points by Atassi & Grzedzinski (1989).

In cascade scenarios, RDT has to date mostly been used in aeroacoustic studies due to the theory's ability to describe the unsteady surface pressure field in turbulent flows. Examples of such applications include predictions of the evolution of disturbances through loaded flat blade cascades such as in Peake & Kerschen (1997); the acoustic response of a cascade of thin airfoils at zero angle of attack subjected to a homogeneous turbulent inflow as described in Kullar & Graham (1986); or the Majumdar & Peake (1998) study of the noise signature of ingested contracting atmospheric free stream turbulence into both open and ducted fan blades. On the other hand, the application of RDT for the purpose of predicting turbulent statistics throughout and behind a cascade is largely absent from the literature. There are also several studies that are significantly related in nature and hence form a foundation for the current work; these will be reviewed in the next subsection.

1.3.4 Previous Relevant Experimental and Theoretical Work

Although not in a cascade scenario, an early study on the effects of blade blocking was performed by Uzkan and Reynolds (1967) who experimentally investigated the shear-free boundary layer of a moving wall in a nearly isotropic turbulent flow. This study eliminated the effects associated with the production of new turbulence along the wall and could therefore focus

* The theory behind the effect of a contraction will be applied in section 3.1.2 in order to place the active grid in the wind tunnel.

on the attenuating effect from the presence of the wall on streamwise fluctuations. The motivation for this study was based on the observation that the outer (wake) portion of turbulent boundary layers are very similar to free turbulent shear layers, which suggests that the presence of a wall is confined to only a small region close to the wall. The results of the study show that the region of the inhibiting effects of the wall is smaller than the size of a regular boundary layer at the same condition. Although at the lower streamwise Reynolds numbers ($Re_x \approx 10^4$) tested, the thickness of the attenuating layer was on the order of the turbulent boundary layer,* the authors predicted that at a streamwise Reynolds number on the order of 10^6 , the inhibiting effects would be contained within the first 10% of the boundary layer.

In a similar experiment in a wind tunnel Thomas & Hancock (1977) revealed substantially different results than Uzkan & Reynolds (1967) as a magnification of the streamwise normal stresses were found instead of a monotonic decrease which was the case of the earlier study. This experiment also included lateral components in order to obtain experimental evidence of the suppression of the component normal to the wall.

Simultaneously, Hunt & Graham (1978) attempted to mathematically describe the effect seen by the experiments of Uzkan & Reynolds (1967) and Thomas & Hancock (1977) through the use of Rapid Distortion Theory. The region of wall influence on the turbulence was divided into two parts. The first is a thin viscous layer adjacent to the wall where the turbulent velocities decrease towards a value of zero at the wall. The second region is a much thicker kinematic, or “source,” layer. In this region the streamwise and spanwise fluctuations are amplified equally whereas the normal fluctuations are dampened. The theory was in overall good agreement with the Thomas & Hancock (1977) data set. It was hypothesized that due to the very low Taylor Reynolds number ($Re_\lambda \approx 25$) associated with the Uzkan & Reynolds (1967) study, the viscous layer completely overshadowed the kinematic layer. Although the theory completely ignores the effect of the shear in the presence of a velocity mismatch between the plate and the free-stream, the theory is still useful as the same blade blocking effects observed in the shear-free case will remain in effect outside the boundary layer for the more practical case of a stationary plate in a wind tunnel stream.

The Hunt & Graham (1978) estimates were used in the above-mentioned paper by Thomas & Hancock (1977) who observed that strong turbulence[†] adhered to the predictions of RDT much longer than in the case of weaker turbulence. Evidence was also provided which showed that the upwash component remained consistent with the predictions over a considerably longer time/distance than the streamwise component.

In a Direct Numerical Simulation (DNS) of the problem, Perot & Moin (1995) identified the regions responsible for transferring energy from the normal components to the spanwise and streamwise components in a shear-free boundary layer as “splats”. The splats are simply local instantaneous stagnation regions where fluid particles are forced to change direction due to the presence of the wall. The authors also introduced the concept of an “antisplat” which is the region where two splats meet and where the fluid is ejected perpendicularly away from the wall. In equal amounts, the presence of both these structures would cause very little pressure-strain close to the wall, however it was found that due to viscous effects the splats dominate the antisplats, causing significant pressure-strain and resulting in significant inter-component energy transfer. The Reynolds numbers of the DNS study were relatively small and comparable to the

* Although in a shear flow the turbulence production would overshadow and hence conceal most of the attenuating layer.

[†] Thomas & Hancock (1978) suggested $Re_\lambda \approx 100$ to be the differentiation between strong and weak turbulence.

Uzkan & Reynolds (1967) experimental study. It showed qualitative agreement with the Hunt & Graham (1978) theory, but only for a short time corresponding to immediately after the insertion of the wall, after which viscous effects close to the wall would take over as in the measurements seen by Uzkan & Reynolds (1967). The region of validity of RDT was not estimated, but documented to be at least smaller than one eddy turnover distance ($L_{11}U_\infty/u$) which is not inconsistent with the Hunt & Graham (1978) theory's region of validity.

More recently Aronson *et al.* (1997) performed another experiment with a shearless boundary layer, with turbulent Reynolds numbers (Re_T) on the order of 200, which together with the Uzkan & Reynolds (1967) results with $Re_T \approx 90$ cover the DNS range offered by Perot & Moin (1995). In comparison the experiment of Thomas & Hancock (1977) exhibited turbulent Reynolds numbers around 2000.* This new study found good agreement with the predictions of Hunt & Graham (1978), but only for about one integral scale downstream of the leading edge. Beyond this, viscous effects were found to dampen out the peaks seen in both the streamwise and the spanwise components. This then essentially agreed with Uzkan & Reynolds (1967) and Perot & Moin (1995), but not with Thomas & Hancock (1977) where the streamwise peak remained downstream. An explanation was offered by Aronson *et al.* (1997) that this discrepancy could be attributed to frictional heating of the wall and its effect on hot-wire measurements in the case of Thomas & Hancock (1978). The fact that the heating of the wall would primarily affect the streamwise component measurement gives this hypothesis more credibility since Thomas & Hancock's (1978) spanwise measurement did not exhibit the same behavior as the streamwise component, and if anything was in more agreement with the findings of both Aronson *et al.* (1997), Uzkan & Reynolds (1967) as well as the calculations of Perot & Moin (1995).

When it comes to work on actual cascade configurations, Basuki (1983) measured a turbulent stream generated by a conventional grid before and after it passed through a cascade of airfoils. Although limited to measurements directly behind the blades as well as in the center of the passages, and with some conflicting results, it was concluded that the normal (upwash) fluctuating component undergoes significant attenuation as it passes through the cascade. Basuki (1983) also compared the results to the linear problem of a turbulent stream flowing through wire gauze presented in Batchelor (1953). Although this approach, together with the problem of a sudden area change on a turbulent stream which will be discussed in Section 3.1.2, constitutes some of the earliest RDT applications, it is only remotely relevant to the cascade problem and this was shown by the results given in Basuki (1983) which were highly dependent on integral lengthscale. The experimental ratio of the integral lengthscale to the cascade spacing (L_{11}/s) only varied from 0.7 to 2.1 whereas the ratio for real wire gauzes would generally be several orders of magnitude larger.

In a parallel effort, Haidos (1983), essentially repeated parts of Basuki's (1983) experiment, but focused on the mid passage values at different streamwise locations behind two cascades. Haidos (1983) reported an approximate reduction in the streamwise turbulence intensity of about 15% whereas the upwash and spanwise intensities decreased 45% and 25% respectively. Most notably, however, was the presence of a significant "return to isotropy" downstream of the cascade.

Following the theory of Hunt & Graham (1978), Graham (1998) developed a theoretical RDT estimate of the modification of initially isotropic turbulence behind an infinite unstaggered cascade of flat blades. For simplicity, single-point results were evaluated for a very small blade spacing compared to the integral scale (although the chord dimension was much larger than the

* This can be compared to the present results which are more than one magnitude larger with $Re_T \approx 6.4 \times 10^4$.

integral scale). The results were compared with the experimentally obtained data from Haidos (1983) and Basuki (1983). Downstream of the initial region of validity, Graham (1998) found that the suppression of the normal components was fairly well predicted by theory whereas the streamwise and spanwise components were underpredicted by the theory. Graham (1998) also showed that the theory predicted a significant effect of the blade spacing on the mid-passage suppression of the blade-normal velocity fluctuations.

A generalized version of Graham's (1998) estimates can be reached by applying the cascade response function described in Glegg (1999) which considers a finite chord cascade with arbitrary stagger with or without compressibility effects. By adding extra terms to the formulation in order to deal with leading edge effects while simultaneously preserve the Kutta-condition at the trailing edge, the function can be used for any blade spacing to chord ratio (s/c) as opposed to the explicit form Graham (1998) used in which the restriction $s/c \ll 1$ was applied in order for leading edge discontinuities to decay by the end of the chord. The Glegg (1999) approach can be used to evaluate the cascade response to a given inflow for any location in and behind the cascade which gives it much more flexibility than its counterpart. The computations are quite expensive compared to the simplified Graham (1998) theory, and for everything but the lowest wavenumbers, Graham's (1998) theory sufficiently describes the downstream response, even when $s/c \approx 1$ (Glegg, private communication).

As a predecessor to the current work, de la Riva (2001) measured the evolution of turbulent stresses through a highly staggered cascade configuration. The measurements were compared to a relatively simple RDT-calculation which ignored the non-penetration condition. With the addition of viscous consideration, agreement within 10% for all three normal stresses was found far from the blades. This was encouraging given the fact that the physical restrictions on the experiment did not conform to the RDT assumptions and was therefore far outside the region of validity. An extension of this work was provided in de la Riva *et al.* (2004) in which the stress profiles and space-time correlations were compared to Hunt & Graham's (1978) RDT predictions. The results close to the blades were found to be at least quantitatively well described by the theory.

1.4 Requirements for a New Experiment

Consider the flow through a marine propulsor such as the idealized situation shown in Figure 1.1. Large scale turbulence originating from the thick boundary layers at the rear of a marine vehicle gets ingested into the propulsor before being severely distorted by the inlet guide vanes as it convects further downstream towards the propeller. How can this scenario be realistically modeled in a laboratory setting? First of all, instead of a water-facility, a conventional wind tunnel can be used to provide the mean flow through the idealized propulsor. This approach generally yields larger Reynolds numbers as wind tunnels are generally available in greater dimensions and, perhaps more important, the range of obtainable speeds is significantly larger. In addition data acquisition is often simplified when working in a gas rather than a liquid. Good wind tunnel facilities are commonly designed to provide very low background turbulence intensity levels, while in this case a great deal of turbulence is obviously desired.

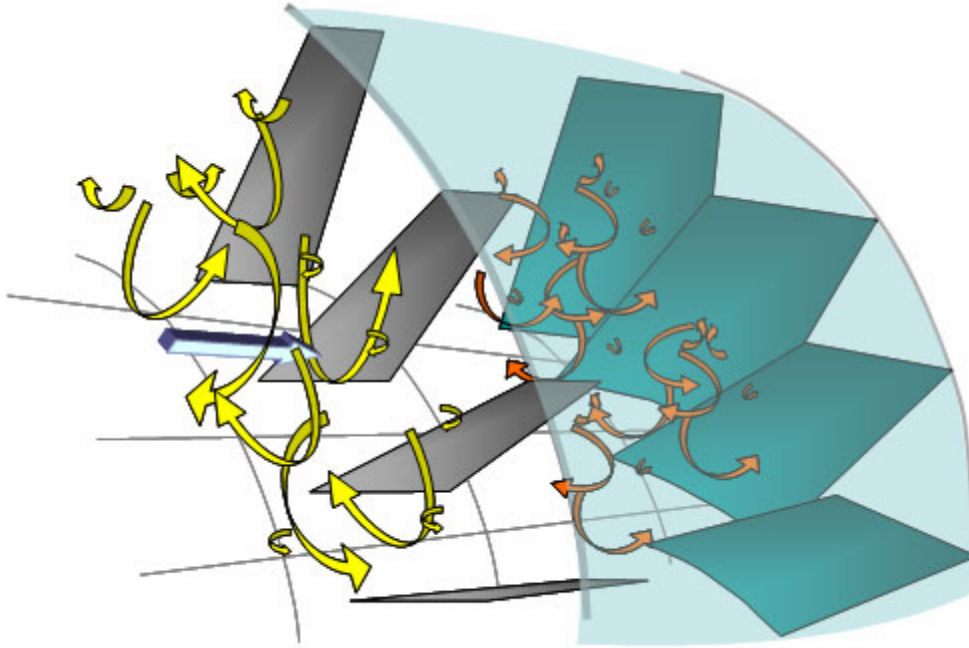


Figure 1.1. Schematic of flow through an idealized marine propulsor.

Due to the impracticality of placing an actual submarine hull upstream in a wind tunnel in order to provide a turbulent boundary layer to work with, a more common approach would be to install a turbulence generator inside the facility instead. To idealize this turbulence to ease the mathematical comparisons it is extremely desirable to obtain a certain level of isotropy in which the turbulence properties are invariant through coordinate rotation. Additionally, obtaining a closely laterally homogeneous* flow-field some distance downstream of the generator is also a requirement for a precise definition of the two-point statistics. For realism, the Reynolds number of the turbulence should also be as large as possible. This would imply large lengthscales and/or large turbulence intensities. As will be clear below, there is a restriction as to how strong the turbulence intensity can be. Initially, an integral lengthscale on the order of 30 cm and turbulence intensities around 10% were envisioned. Hence a turbulence generator capable of providing large scale, moderately intense homogeneous isotropic turbulence was sought to provide the turbulent flow for this problem.

The first stage in a shrouded marine propulsor could be a set of inlet guide vanes as in Figure 1.1 or alternatively a stator or rotor row. This stage needs to be modeled in the wind tunnel. Any other subsequent stage, be it another stator/rotor or a propeller, will be considered the noise producing device. Hence the turbulent field downstream of the first stage requires the most attention, but in order to capture the entire blade blocking effect it is also desirable to measure the turbulence in front, throughout as well as behind the cascade. It is well known (see for example Peake & Kerschen, 1997) that in the absence of strong radial flows the unwrapping of an annular cascade can be approximated well by a two-dimensional linear cascade of blades. However a set of straight inlet guide vanes would be the equivalent of an unstaggered linear cascade whereas a stator/rotor would transform into a staggered configuration. Both

* Attaining a homogeneous flow is easier than an isotropic flow since isotropic flows form a subset of homogeneous flows.

configurations are of fundamental interest and it is therefore imperative that the linear cascade can be mounted into the wind-tunnel either staggered or unstaggered.

1.5 Turbulence Generation

In order to provide the cascade configuration with the large scale, relatively high intensity turbulence needed to obtain realistic Taylor Reynolds numbers, the choice of turbulence generator is of paramount importance. This section reviews some of the existing designs from well-known conventional grids to some of the more advanced (yet not necessarily more effective) jet and mechanically agitated turbulence generator concepts available.

1.5.1 Conventional Grid Generated Turbulence

The idea of creating a mean turbulent flow field through installing arrays of circular or rectangular bars to form a grid in a wind tunnel has been around since at least the experiments of Simmons & Salter (1934). The interaction between the jets and wakes created through the use of such a grid turned out to create a uniform transversely homogeneous decaying turbulent flow after the initial “wind-shadow” right behind the grid. After the revolutionary theory of Taylor (1935a) emerged, it turned out that the turbulence generated by such a grid was also nearly isotropic. The anisotropy ratio (I) behind such grids is generally about 1.10 (or 10%).

Due to facility restrictions, conventional grid generated turbulence is generally limited to Taylor Reynolds numbers up to a few hundred. The classic grid turbulence experiments of Comte-Bellot & Corrsin (1970) for example, reports a range of Taylor Reynolds numbers from 36 to 72. A few examples of larger scale Taylor Reynolds numbers include the measurements by G.R. Stegen at Colorado State University described in Helland *et al.* (1977) in which a wind tunnel size on the order of the Virginia Tech Stability Tunnel was used in combination with a large mesh width ($M = 22.9$ cm) and a free stream velocity of $U_\infty = 29$ m/s resulted in a Taylor Reynolds number of 237 at $X/M = 38$, while Schedwin *et al.* (1974) reported the same flow to have reached $Re_\lambda = 280$ at $X/M = 41$.

Even higher Taylor Reynolds number values were obtained in the experiments by Kistler & Vrebalovich (1966) where a 2.59 m by 3.51 m wind tunnel section was used. Although the grid meshlength was only 17.2 cm; it produced Taylor Reynolds numbers of about 670* due to a large free stream velocity of 61 m/s and the ability to vary the barometric pressure inside the facility. The actual lengthscales and turbulence intensities were far smaller than the measurements of G. R. Stegen described above.

One of the restrictions on conventional grids is that the solidity† should be kept well below a factor of 0.5 since a solidity above this have been shown through experiments to result in a transversely inhomogeneous flowfield downstream of the grid (see Villiermaux *et al.*, 1991 for a brief review). Comte-Bellot & Corrsin (1966), for example, used grids with solidities of 0.34 for their square bar grids and 0.44 for grids made from round rods. The reason for the difference was

* As evaluated by Makita (1991)

† Solidity = ratio of open to total area.

motivated by the desire to create approximately the same pressure drop due to the different wake dynamics between cylindrical and square cross-sections.

1.5.2 Alternative Approaches; Dynamic Vs Passive Generators

Gad-el-Hak & Corrsin (1973) defined a “dynamic”^{*} turbulence generator to be one that 1) has moving boundaries or 2) adds mean momentum to the turbulent flow, beyond that of the conventional passive or static grid. This definition will be adopted in this work, and any turbulence generator that doesn’t fit this definition will be considered passive (or static). Even though the turbulence is arguably generated dynamically behind a conventional grid due to the coalescence of the wakes stemming from the grid, the device itself is simply acting as an obstacle to the flow and is therefore a prime example of a passive grid.

The next three sections, which largely follow the review given by Poorte (1998), will briefly discuss some of the alternative turbulence generators previously used. It is by no means an exhaustive list, but attempts to highlight some of the conceptual designs used to remedy the inherent limitations encountered in the use of a conventional static grid.

1.5.3 Unconventional Passive Turbulence Generator Configurations

Uberoi & Wallis (1967) tested four different grid geometries and assessed the uniformity and homogeneity of each. Two of these were simply standard biplane grids with or without roughened surfaces. The third grid exhibited inclined rods to make equal angles with the three coordinate axes in order to produce vorticity in equal amounts in all directions. The attempt resulted in lower anisotropy downstream of the grid than compared with a conventional grid. The fourth geometry tested was a honeycomb shaped grid with small flap like surfaces angled at 34° at the trailing edge of the metal sheets making up the square honeycomb to induce the shedding of vortices in the streamwise direction. This resulted in a vast increase in turbulence intensity (25% increase compared to the biplane conventional grids) and lengthscales. The anisotropy ratio was found to be unity behind the grid, but with a large increase with downstream distance in the initial period of decay. The greatest problem however, was the lack of transverse homogeneity behind the grid, a condition which only worsened with streamwise distance, but Uberoi & Wallis (1967) recognized the potential lifting surfaces have in the production of turbulence.

A different approach was taken by Roach (1986) who created a grid made out of a streamwise oriented bundle of tubes in order to achieve turbulence intensities on the order of 10%. Such an apparatus is generally used in order to reduce turbulence intensities and to straighten the flow going into the inlet of wind tunnels in the form of honeycombs. When applied to low background turbulence flows, the effect is reversed and larger turbulent fluctuations are created. This is analogous to the difference between screens and grids. Roach (1986) was mostly concerned with the decay of the turbulence and did not measure the anisotropy level or the degree of uniformity of the downstream flow.

^{*} The definition actually referred to “active” generators, but this term has been reserved for the Makita design described in Section 1.16.

To bridge into the next category of jet generators, Villiermaux *et al.* (1991) experimented with perforated plates with either square or triangular arranged holes to create high solidity grids ($\sigma = 0.87$ and 0.92 respectively). Here the wakes generated by the dead zones in between the holes play a much less significant role than with a conventional grid structure. Even though the homogeneity was found to be good in the region immediately behind the merging of the individual jets, this breaks down to create the expected inhomogeneous region further downstream.

1.5.4 Turbulence Generators Involving Injection of Fluid

Although most grids involve some combination of jets and wakes, others have taken this to a different level. Thole *et al.* (1994) developed a grid where high velocity jets were inserted into the flow vertically across the test section in a wind tunnel perpendicular to the mean flow. A water tunnel version was also tested. The concept was initiated by the need to realistically model the flow found inside gas turbines which typically call for turbulence intensities between 20% and 30%. It was found that turbulence intensities up to about 20% could be generated by varying the Reynolds number and the ratio between the jet and the free stream velocity. Anisotropy levels were approximately 30% and stayed constant downstream of the grid. Compared to a static grid this approach clearly is capable of generating much higher turbulence intensities, but at the cost of lower lateral mean flow uniformity and second order homogeneity. The Taylor Reynolds numbers considered in the study ranged between 159 and 271.

A similar concept was independently developed by Shavit & Chigier (1995). Here jet injection was utilized inside a “transverse-jets-atomizer” nozzle in order to actively control the turbulence levels at the nozzle exit in order to study the effects of turbulence on water atomization.

A completely different concept is described in Gad-el-Hak & Corrsin (1973) in which a conventional grid was outfitted with arrays of controllable nozzles capable of injecting jets aligned either with or against the free stream flow. Homogeneity studies showed the flows to be generally quite acceptable, especially with co-flow injection and anisotropy was around 15% which is slightly higher than the 10% generally found in conventional grids. Turbulence intensities were slightly higher resulting in an increase in Taylor Reynolds number compared to the case of a conventional grid (i.e. no jet injection). An essentially identical setup but with a different configuration to supply the secondary air-flow was used by Tassa & Kamotani (1974) to complement the data set of Gad-el-Hak & Corrsin (1973) for lower injection ratios.

1.5.5 Mechanically Agitated Turbulence Generators

Other types of dynamic generators include standard grids that are mechanically agitated such as that of Ling & Wan (1972) which featured rectangular bars mounted on vertical rods. The attached bars were positioned in a staggered formation such that when in the neutral position the apparatus would form a plane grid with a square mesh. Each rod would then be rotated sinusoidally through an angle of $\pm 19^\circ$ in the opposite direction of the adjacent rod. The resulting turbulence was found to reach its near-isotropic value faster than conventional grids, with only about 5% anisotropy observed and turbulence intensities from 3% to 7% controlled by the

rotation rate of the rods. It should be noted however that this apparatus was used in a water tunnel with a very low free stream velocity and Reynolds number. Scaling this design up for a wind tunnel would mean prohibitively high rotation rates in order to obtain the desired conditions

Thompson & Turner (1975) took a slightly different approach where a conventional grid was mechanically oscillated in a water tank to create turbulence in a water tunnel without a mean flow. It was found that the fluctuating velocity* away from the grid was related to the stroke and the frequency of oscillation. Srdic *et al.* (1995) extended this idea to include two oscillating grids in the hope that the region between them would exhibit even closer isotropy. With only one grid operating the anisotropy was assessed to be between 20% and 50% whereas for both grids in operation it was reduced to about 4%. Homogeneity was also found to be fairly good even though the measurement domain was very close behind the two grids.

A third type of mechanical generator was introduced by Makita (1991) in which agitator wings attached to rotating rods created a time-varying solidity which on average is much higher than that found with most conventional grids. This enables the design to create large scale high intensity turbulence previously unparalleled. Poorte (1998) noted that this design in essence combined the lifting surfaces of Uberoi & Wallis (1967) with the concept of moving boundaries of the oscillating grids. Figure 1.2 shows the Taylor Reynolds numbers obtained by previous grids as a function of their grid Reynolds numbers. Active and static grids exhibit vastly different slopes which indicate that a small low-speed facility equipped with an active grid is capable of much larger Taylor Reynolds numbers than with a conventional turbulence generator. The scatter shown in Figure 1.2 is undoubtedly partly due to the fact that no consideration of the streamwise distance from the grid has been taken into account.

Throughout the rest of this work the term “active turbulence grid” or “active grid” will solely refer to the Makita-style design, which will be introduced in more detail in the following section.

* It would not make sense to speak of a turbulence intensity in this case.

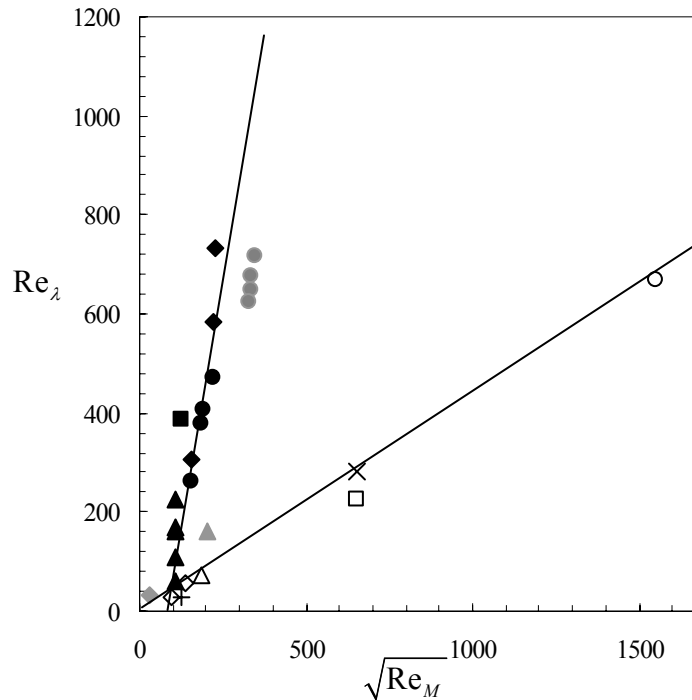


Figure 1.2. Taylor Reynolds number as a function of grid Reynolds number. Static Grids: ○, Kistler & Vrebalovich (1966); □, Helland *et al.* (1977); ◇, Frenkiel as reported by Poorte (1998); △, Comte-Bellot & Corrsin (1966 and 1971); ×, Schedvin *et al.* (1974); +, Makita (1991). Makita style grids: ■, Makita (1991); ●, Mydlarski & Warhaft (1996); ◆, Mydlarski & Warhaft (1998); ▲, Poorte (1998); ●, Kang *et al.* (2003). Other types of turbulence generators: ◆, Ling & Wan (1972); ▲, Gad-el-Hak & Corrsin (1974); Solid lines indicate trends for active and static grids. After Poorte (1998).

1.6 The Active Turbulence Grid Concept

The active grid concept was developed at the Department of Energy Engineering at the Toyohashi University of Technology in Japan in the early eighties by Makita and Miyamoto (1983). It wasn't until eight years later, however, with Makita (1991) and Makita & Sassa (1991) that the worldwide scientific community was introduced to this new type of turbulence generator capable of generating very large, high intensity turbulence in relatively small facilities.

1.6.1 Previous Experiments

The Makita active turbulence grid consists of 15 vertical and horizontal rods in a biplanar configuration. Each rod is equipped with agitator vanes and is independently controlled by a separate stepper motor. The grid was mounted in a relatively small wind tunnel with a square cross section with 0.7 m side dimensions. Some initial tests were documented in Makita & Miyamoto (1983) while a closer investigation of the turbulent flows the grid produced was

presented in Makita & Sassa (1991) and Makita (1991). The latter documents an integral lengthscales of 19.7 cm, turbulence intensities of 16% resulting in a Taylor Reynolds numbers, Re_λ , of 387 for a flow speed of 5 m/s.

A smaller grid (0.45 m square cross section) based on the same design was used by Mydlarski & Warhaft (1996) at Cornell University in order to systematically test the design's capability of generating high Reynolds number turbulence. In the process Taylor Reynolds numbers of up to 473 were reached, a number which was extended to 731 for a scaled up version of the grid in Mydlarski & Warhaft (1998). Poorte (1998) and Poorte & Biesheuvel (2002) investigated a slight modification of the grid design in a water tunnel, and also documented how the turbulent flow was affected by changing the operational parameters (also known as the "forcing protocol") of the grid. Kang *et al.* (2003) used the Makita grid design in a larger wind tunnel (1.22 m by 0.91 m cross-section) in order to compare the resulting turbulence ($Re_\lambda \approx 720$) to results obtained through large-eddy simulation (LES). A summary of the previously published active grid experiments is provided in Table 1.1 together with some of the corresponding turbulence parameters.

Study and Test Case	x/M	U_∞ (m/s)	Ω (Hz)	u/U_∞ (%)	L_{11} (m)	Re_λ	u/v
Makita (1991) 1	50	5.0	2	16.4	0.20	387	1.22
M&W (1996) 1	68	7.1	2	7.4	0.12	262	1.21
M&W (1996) 2	68	10.4	2	8.6	0.14	377	
M&W (1996) 3	68	14.3	2	9.5	0.15	473	
M&W (1998) 1	68	11.4	1 or 2	8.9	0.16	407	
M&W (1998) 2	62	3.3	1 or 2	9.1	0.30	306	
M&W (1998) 3	62	7.0	1 or 2	10.9	0.43	582	
M&W (1998) 4	31	6.9	1 or 2	17.4	0.40	731	
Kang <i>et al.</i> (2003) 1	20	11.9	3.5 to 7	16.0	0.27	755	1.19
Kang <i>et al.</i> (2003) 2	30	11.2	3.5 to 7	12.9	0.30	696	1.17
Kang <i>et al.</i> (2003) 3	40	11.0	3.5 to 7	10.8	0.32	654	1.17
Kang <i>et al.</i> (2003) 4	48	11.1	3.5 to 7	9.5	0.33	624	1.16

Table 1.1. Overview of previous active grid experimental conditions and resulting turbulence parameters.

1.6.2 Forcing Protocols

The biggest differences between the previous active grid efforts lies not mainly in the way they have been designed, but in the forcing protocols driving their dynamic behavior. Poorte (1998) was the first person to investigate the effect of the active grids operating parameters, or "forcing protocol", on the resulting turbulent flow. Adopting this author's terminology makes it easier to describe the different grid forcing protocols that have been used to date. The three major different schemes utilized are "synchronous", "single random", and "double random". All of the previous efforts have used one or more of these protocols with some minor modifications. Figure 1.3 illustrates these three different grid forcing modes as well as one variation, which are all described below.

The synchronous mode (Figure 1.3a) refers to each grid bar rotating with the same continuous grid rotational speed Ω . Makita & Miyamoto (1983), Mydlarski & Warhaft (1996,

1998) and Poorte (1998) all investigated this forcing scheme in which adjacent bars were rotating in opposite directions in order to avoid introducing net vorticity into the flow (Mydlarski & Warhaft, 1996). With this forcing protocol, however, the initial positioning of the grid rods will affect the actual flow through the grid. Poorte (1998) used a randomly distributed probability density function to define the starting position of each rod, while Mydlarski & Warhaft (1996, 1998) placed all rods in the same orientation. It appears Makita and Miyamoto (1983) initialized their horizontal wings perpendicular to the vertical wings which creates a third configuration.

The synchronous motion was found to produce turbulence with spectral properties similar to the random modes described below, but with smaller turbulence levels and integral scales (both of which result in a smaller Re_λ) for a given flow condition. The biggest problem with this forcing protocol was found in the low wavenumber region, however. Large non-turbulent vortices were periodically shed off the vanes which contaminated the weak turbulence field that was actually present. These coherent structures resulted in a shorter inertial range for the energy cascade and a much slower downstream decay of turbulence energy, as shown by Makita and Miyamoto (1983). In addition these authors showed evidence of relatively poor lateral homogeneity and uniformity for this protocol.

The three different investigations all revealed large periodicities in the generated turbulence, creating distinctive spikes in the power spectrum at the frequencies associated with the periodic grid rotation. One would expect the spike to show up at frequencies corresponding to the first harmonic of the rotation rate ($f = 2\Omega$) since the effective period of each rod is only half a revolution. Mydlarski & Warhaft (1996) found this to be indeed the case. In a test where all bars were rotating in the same direction, Makita and Miyamoto (1983) also documented the presence of significant energy associated with the next three harmonics. When alternating the direction on adjacent rods the second and third harmonics were still discernable in the spectrum, but contained much less relative energy compared to the unidirectional case.

Poorte (1998) offers a good treatment of the synchronous forcing protocol based on experimental results, and concludes that the autocorrelation, turbulence intensities and anisotropy ratios (u/v) are all highly dependent on Ω and U_∞ . Makita & Miyamoto (1983) tried to remedy the presence of the spectral peaks by running a synchronous scheme where the vertical rods would have a slightly different speed from the horizontal (2 Hz and 2.2 Hz). This resulted in a much smoother spectrum by spreading the vortical structures over a wider range. It fixed the decay problem also, but the protocol discriminated against frequencies below $1/2\Omega$. This so called “mixed” mode naturally led to the development of the more advanced “single random” mode.

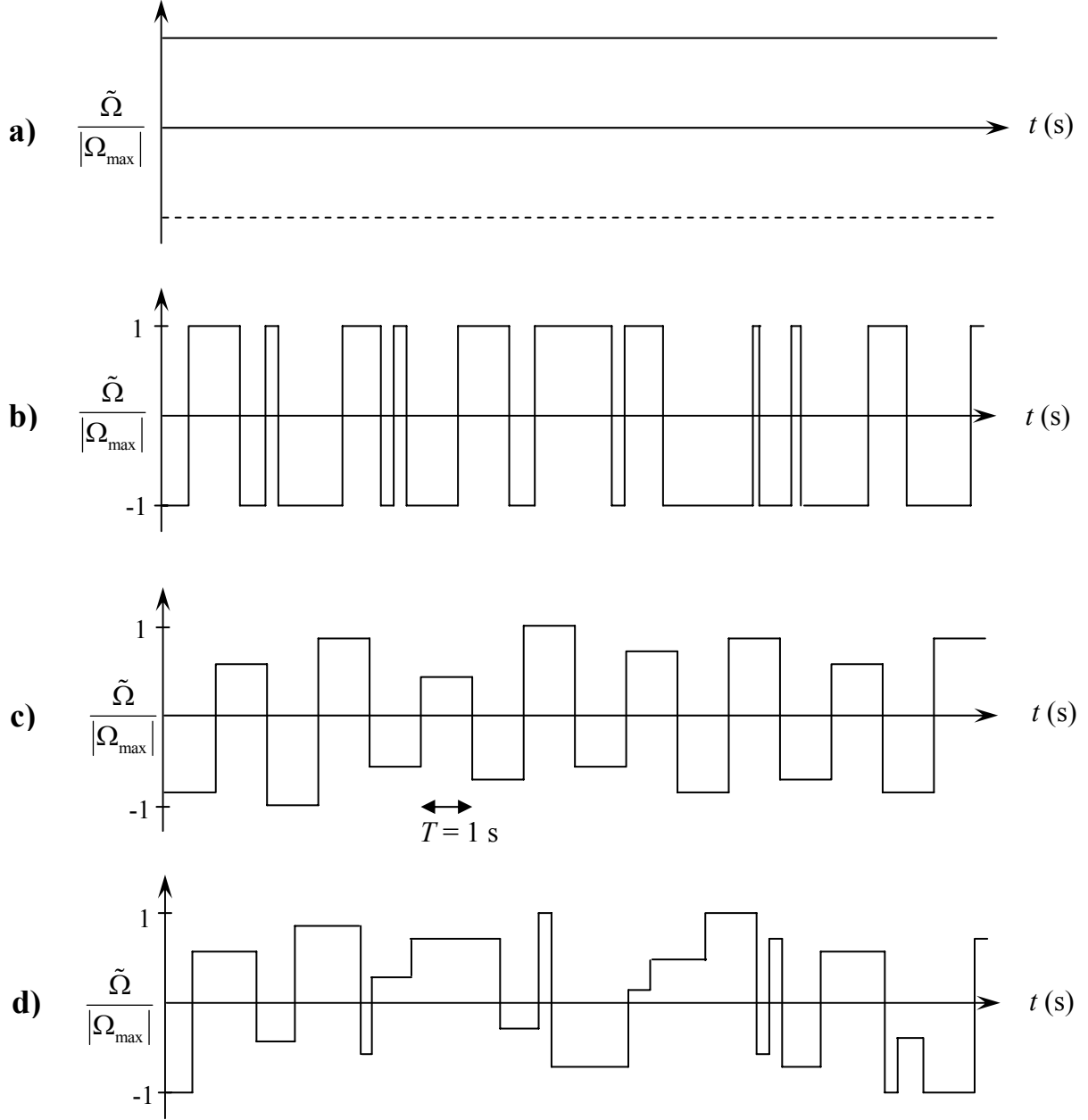


Figure 1.3. Grid forcing schemes from previous efforts: a) synchronous; b) single random; c) modified single random; d) double random.

The single random protocol attempts to excite the lower frequencies in the spectrum uniformly by keeping the rotation rate, Ω , constant while randomizing the *cruise time* $\tilde{T}$ which is the instantaneous time between direction changes (Figure 1.3b). Makita & Miyamoto (1983), Makita (1991), Makita & Sassa (1991), Mydlarski & Warhaft (1996, 1998), and Poorte (1998) all utilized this protocol while Kang *et al.* (2003) used the opposite scheme where the rotation rate was randomized with a fixed cruise time of 1 second (Figure 1.3c). In the first case, there are still peaks present at frequencies corresponding to the first and second harmonic of the rotation rate as shown by Mydlarski & Warhaft (1996), but their relative magnitude are much smaller

compared to that of the synchronous forcing protocol. The spectral distribution resulting from the modified single random mode of Kang *et al.* (2003) showed even less evidence of the peaks as the energy is spread over a wide range of lower wavenumbers

With the addition of the randomized parameter the spectrum shows a monotonic decrease throughout the wavenumber range. The turbulence intensities and integral lengthscales also increased significantly, and the lateral homogeneity and uniformity appeared much better than that of the synchronous motion (Makita and Miyamoto, 1983). Mydlarski & Warhaft (1998) updated their single random mode to accommodate a slightly different rotation rate on each rod, analogous to the synchronous mixed mode of Makita & Miyamoto (1983) described above.

Poorte (1998, 2002) introduced the double random protocol which is shown in Figure 1.3d. In this case the rotational speed, the cruise time, and even the directional change parameters are all randomized within certain parameters. Poorte (1998) found that the macro structure of the turbulence could be manipulated through the choice of rotation rate as well as the cruise time parameter through the resulting integral time scale. Some experiments were performed to document trends at a fixed Reynolds number, but a systematic approach beyond this was not presented. The prospect of generating a myriad of different turbulent flows by manipulating flow speed, rotation rates and cruise times makes the double random forcing motion ideal for active grids.

1.6.3 Isotropy

One of the biggest drawbacks with the active grid design is the relatively large anisotropy associated with it. As opposed to a conventional turbulence grid which consistently yield values of anisotropy around 10%, Makita (1991) recorded an anisotropy ratio (u/v) ranging between 1.8 just downstream of the grid and 1.22 at $X/M = 50$ for his active grid, where X is the streamwise distance from the grid and M is the mesh length of the grid. Although both of these were measured relatively close to the grid in absolute terms, the downstream value was confirmed by Mydlarski & Warhaft (1996) who documented an average anisotropy of 21% at $X/M = 68$. Kang *et al.* (2003) measured the isotropy ratio to be 1.13 and 1.16 for their grid with measurements at X/M stations of 20 and 48 respectively. Note here the apparent lack of a “return to isotropy” which is observed in most conventional grid experiments. In this case the opposite trend is actually seen. Poorte (1998) recorded the same departure from isotropy trend for his single random forcing protocols. When double random forcing was utilized the anisotropy ratio remained constant throughout the length of the test section. A distinctive relationship between rotational speed and isotropy was also found. Low rotation rates (~ 1 Hz) as well as very high rotation rates (~ 10 Hz) generally produced larger anisotropy than moderate rotation rates (~ 6 Hz) for the single random protocols. Ranging from about 0.94 to 1.26, the anisotropy ratios measured by Poorte (1998) were found to be much smaller than other active grids with the same forcing. This was attributed to the agitator wings being mounted in a staggered configuration as opposed to parallel like in the other grids. A staggered configuration has adjacent wings placed perpendicularly along every rod, effectively reducing the time average solidity of the grid and causing a more uniform obstruction in both the streamwise and lateral directions, resulting in anisotropy levels on the order of a static grid for double random forcing. The downside of this is of course a strong decrease in both the integral lengthscales as well as the turbulence intensity compared to an equivalent design with the agitator wings in a parallel configuration.

The large anisotropy generally observed with the previous active grid efforts were found to be restricted to the lower wavenumbers, and that above a Taylor Reynolds number of about 200 the resulting active grid turbulence showed distinct evidence of locally isotropic behavior (Mydlarski & Warhaft, 1996). In comparison, appreciable local isotropy is questionable even at $Re_\lambda \approx 600$ in a boundary layer shear flow as the shear stresses affect the wavenumbers throughout the inertial subrange. Saddoughi and Veeravalli (1994) show that in such shear flows it takes a Re_λ of around 1450 to obtain one decade of a locally isotropic inertial subrange. This makes active grid turbulence very attractive since local isotropy can be obtained relatively easily even in very small facilities.

In conclusion, based on Poorte's (1998) work, the anisotropy ratio seems to be a strong function of geometry, but also varies with the forcing protocol used; Generally the more random the grid operates, the closer the anisotropy ratio will be to unity which serves as an explanation as to why Poorte (1998), and even Kang *et al.* (2003), observed closer isotropy than for example Makita (1991) and Mydlarski & Warhaft (1996).

1.7 Objectives of the Present Work

The primary objective of the present study is to document the experimental results of the interaction between a large-scale high Reynolds number turbulent flow with a cascade of flat plates. This requires the design and construction of:

- 1) A turbulence generator capable of producing high Reynolds number homogeneous turbulence with integral scales on the order of 30 cm in a facility capable of accommodate such large coherent structures.
- 2) A modular flat plate cascade which can be mounted in both staggered and unstaggered configurations without causing unsteady stall due to the intensity of the onset turbulent flow.

Upon completion of these logistical tasks the following objectives can be defined

- 1) Expansion of already existing datasets in order to investigate the properties of high-Reynolds number grid-generated homogeneous turbulence.
- 2) Providing a calibration dataset for computational methods of large scale blade blocking effects throughout both an unstaggered and a staggered cascade configuration, both close to the blade surfaces as well as in the mid passage.
- 3) Comparison between the experimental results and linear, inviscid RDT predictions based on Graham's (1998) cascade theory of the flow field behind an unstaggered cascade.

The third objective deserves some additional comments. In the current study, the lengthscale will be on the order of the chord which means that with moderate turbulence intensity the left hand side of Equation 1.16 will be around 0.2 whereas the right hand side will be at least unity within the cascade passage. Downstream of the cascade, the assumption will no longer hold as the two sides of Equation 1.16 will equalize. But as previous experiments have shown, there are plenty of examples where RDT, as an engineering model, has successfully been applied over

much longer distances than the theory calls for. The question is that given the large turbulence scales envisioned in the present experiment, will RDT hold all the way downstream where a propulsion device would be located?

1.8 Experimental Approach

A cascade wind tunnel such as the one used by de la Riva (2001) is a natural choice when studying a cascade configuration experimentally. It does not suit the needs for the current study however, due to the fact that the aspect ratio of the blades tends to be too small for the integral lengthscales desired in this experiment. This would restrict the level of two-dimensionality of the cascade flow and create additional blade-blocking in the spanwise direction. Additionally a long development length is wanted to ensure that the large scale turbulence has reached a laterally homogeneous state. This also permits the lengthscales to grow and allows control of the magnitude of the turbulence intensities by choosing the streamwise location of the cascade in the test section as opposed to the predetermined location of a permanently attached cascade configuration.

Instead of designing and constructing a scaled up version of such a cascade tunnel, a custom made cascade was tailored for the Virginia Tech Stability Wind Tunnel. This wind tunnel facility, described in more detail in Chapter 2, has a 7.3 m long test-section with a 1.8 m x 1.8 m square cross-section. This allows for the convection of large turbulent eddies with significantly less wall influence as compared to more standard sized working sections.

In order to keep the model simple, a cascade of thin flat plates at zero angle of attack can be used to mimic the first propulsor stage. This removes any unwanted effects stemming from a loaded cascade and will ease the subsequent analysis of the data. This decision can be justified by the results of de la Riva (2001) and de la Riva *et al.* (2004) which showed that RDT is capable of predicting the distorted flow field of a loaded cascade sufficiently far away from the blades.

For realism, the chord length should be kept on the order of the blade spacing and the undisturbed integral scale of the turbulence, which was initially envisioned at around 30 cm, depending on the performance of the turbulence generator. As mentioned above, large aspect ratio blades are desired in order to keep the flow strictly two-dimensional. All of these requirements call for a very large wind tunnel. Additionally, unsteady stall effects should be avoided. This can be controlled by properly rounding the leading edges of the blades as well as limiting the turbulent fluctuations while at the same time attempting to maximize the Taylor Reynolds number. Sharp trailing edges are also desirable to keep the viscous wakes as narrow as possible behind the cascade.

The most daunting task, however, is to actually generate the required large Reynolds number turbulent flow. Based on the designs and findings of Makita (1991) and Mydlarski & Warhaft (1996, 1998) the active turbulence grid design was adopted for the current study due to its proven ability to generate large scale, high intensity turbulent flows in relatively small tunnels. By placing the grid inside the contraction prior to the test section following the approach used by Comte-Bellot and Corrsin (1966), an improvement over the documented large anisotropy ratios associated with this type of grid was anticipated.

1.9 Outline

This dissertation which documents the results of an experimental study on high Reynolds number large-scale turbulence and the blade blocking effects as a consequence of the turbulence being convected through a cascade of flat plates, is organized as follows: Chapter 2 introduces the Stability Wind Tunnel and the data acquisition hardware used for this study. Chapter 3 focuses on the design and development of the Active Turbulence Grid. A discussion on the characteristics of the free stream turbulence generated by the Active Turbulence Grid can be found in Chapter 4. Chapter 5 covers the physical aspects of the modular flat plate cascade, while Chapter 6 exhibits the experimental results of the blade blocking study. Chapter 7 gives a comparison between the experimental results with those obtained from Rapid Distortion Theory calculations before the main conclusions are reached in Chapter 8.

Although this dissertation does attempt to follow a natural progression from the experimental hardware, through the turbulent inflow into the cascade experiments and predictions, most of the chapters can be read as standalone units. The one exception is possibly Chapter 7 which relies heavily on material in Sections 1.3.2 through 1.3.4.

Chapter 2

Wind Tunnel and Instrumentation

This chapter describes the experimental facility and the instrumentation used. First, the Virginia Tech Stability Tunnel will be presented in Section 2.1, followed by a description of the hot-wire anemometry system utilized for velocity measurements in Section 2.2. Finally Section 2.3 describes the methods used for flow visualization of the flow over the cascade blades. The Active Turbulence Grid and the modular flat plate cascade, both of which were constructed as part of this investigation, are presented in Chapter 3 and Chapter 5 respectively. All experiments presented in this dissertation were conducted in the Virginia Tech Stability Wind Tunnel between November 2001 and May 2003.

2.1 Virginia Tech Stability Wind Tunnel

2.1.1 Overview

The Stability Wind Tunnel was constructed in 1941 at NACA Langley Aeronautical Laboratory. The wind tunnel was originally built to measure stability derivatives and is therefore commonly known as the “Stability Tunnel.” In 1958 the tunnel was moved to Virginia Tech. Figure 2.1 shows the wind tunnel as it appears on the Virginia Tech Campus today.

The Virginia Tech Stability Tunnel is a closed-loop wind tunnel with an air-exchange tower which allows for temperature stabilization. Figure 2.2 displays a schematic of the wind tunnel. The fan, measuring 14' in diameter, consists of 8 custom made constant pitch blades, and is powered by a 600 hp Westinghouse Model No. 28767 motor generator which rotates at a maximum speed of 900 rpm (Figure 2.3). The resulting top speed of the facility is some 80 m/s.

Each corner is equipped with large turning vanes, spaced 11.8" apart along the diagonal, to ensure a uniform flow. The corner immediately upstream of the test section, however, has turning vanes spaced every 3". Upstream of the contraction, in the 18' by 18' settling chamber, seven successive stainless steel anti-turbulence grids have been installed. The settling chamber extends far enough downstream to allow the small scale turbulence inevitably created from the screens to dissipate before the flow accelerates through the 9:1 contraction and into the test section.

The 24' long, 6' by 6' wide test section is configured with removable steel panels on three sides while the side facing the control room features Plexiglas panels. The control room is contained within a pressure sealed room and its pressure is equal to the static pressure level of the test section.



Figure 2.1. Stability Wind Tunnel on the campus of Virginia Tech. Courtesy of the AOE Department, Virginia Tech.

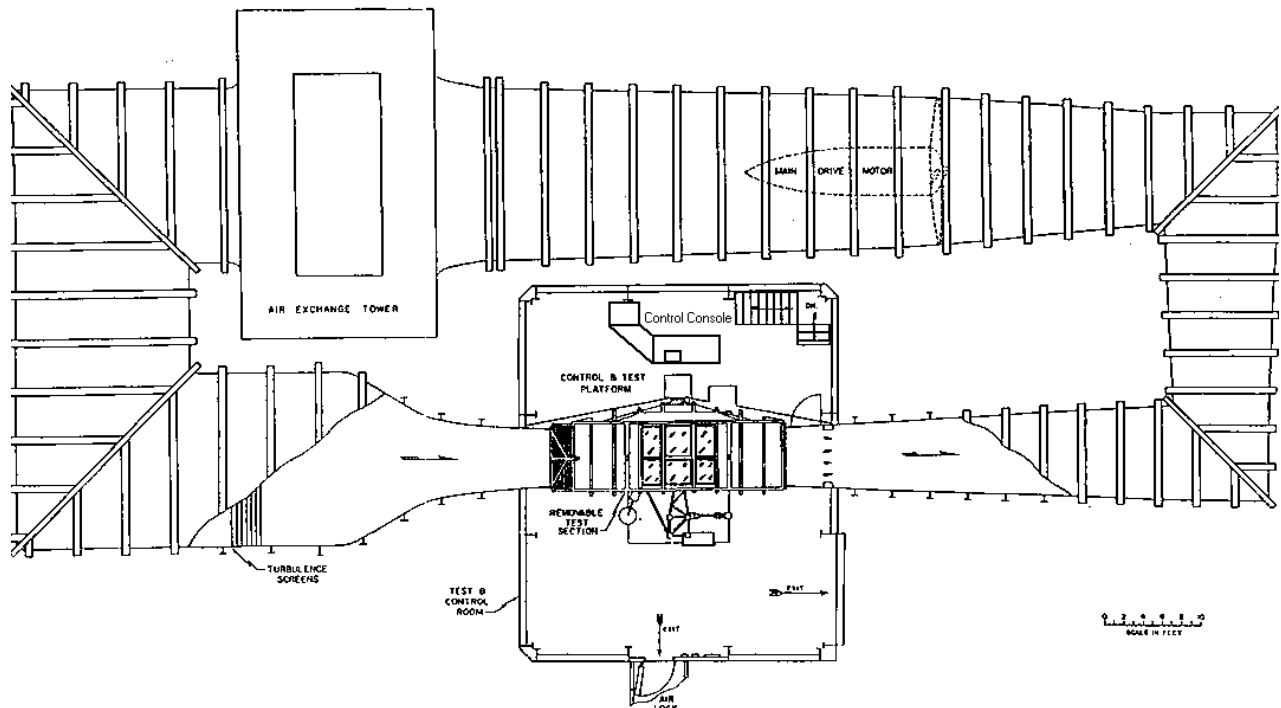


Figure 2.2. Schematic of the Virginia Tech Stability Tunnel. Courtesy of the AOE Department, Virginia Tech.



Figure 2.3. Stability Wind Tunnel fan. Courtesy of the AOE Department, Virginia Tech.

2.1.2 Flow Quality

Flow quality measurements were performed by Choi & Simpson (1987) who found that the turbulence intensity levels for the empty tunnel are extremely low: less than 0.05% for flow speeds up to 15 m/s, and less than 0.1% for flows up to 38 m/s. Most of the turbulence intensity is contained within frequencies below 40 Hz at a flowspeed of 15 m/s, and below 200 Hz at 38 m/s. It should be noted, however, that since this study took place prior to the exchange of the original fan blades, which should result in further improvement of the flow quality. Flow angularities (yaw and pitch) were measured to be less than $\pm 2^\circ$ and the position of minimum flow angle increasingly deviates from the center of the test section with velocity.

Choi and Simpson (1987) also documented some unsteadiness at the blade passing frequencies of the eight-bladed propeller. The vibration of the wind tunnel and the motor generator creates fluctuations at 3-4 Hz and 15 Hz respectively.

In addition, even though this facility was not intended to be a quiet facility, a recent study by Larssen & Devenport, (1999) showed that the A-weighted sound levels in the Stability wind tunnel are very much comparable to other facilities specifically designed for acoustic measurements.

2.1.3 Tunnel Instrumentation and Hardware

Existing wind tunnel instrumentation was used to monitor the reference flow parameters. This included a Validyne DB-99 digital barometer (resolution: 0.01"Hg), an Omega thermistor type 44004 (accuracy: $\pm 0.2^{\circ}\text{C}$), and a 7.5" long Dwyer Pitot-Static tube connected to a Setra 239 pressure transducer (accuracy: $\pm 0.14\%$). An additional Dwyer Pitot-static probe was used to measure mean flow uniformity. A small flattened Pitot-probe, custom-made in house, was used for initial evaluation of the flat plate cascade boundary layers.

The wind tunnel traverse system (Figure 2.4a) was used for both hot-wire and Pitot measurement. The traverse consists of two vertical frames connected by four airfoil shaped sections to reduce the drag experienced by the construction. The total blockage due to the system amounts to about 10% when installed in the tunnel; however any measurement probes attached to the traverse were mounted at least 36" upstream of the main structure by means of a streamlined sting support. The traverse is powered by three Compumotor model 08309-1-8-040-010 stepper motors and controlled by the data acquisition software through an RS-232 interface. The system is accurate to less than 0.01," and is capable of traversing most of the cross-sectional area of the tunnel.

When only measurements on the tunnel centerline were needed (or when 2-point measurements were performed), the hot-wire probes were mounted on a steel strut (Figure 2.4b) made out of an airfoil section bar with a blockage of less than 0.4%. Two guy wires (not shown in Figure 2.4b) were used to tie the strut to the walls of the tunnel in order to counteract the effects of unsteady lift and vibrations from gusts. The wires were weighted to avoid a standing resonant wave forming in the wire between the wall and the strut.

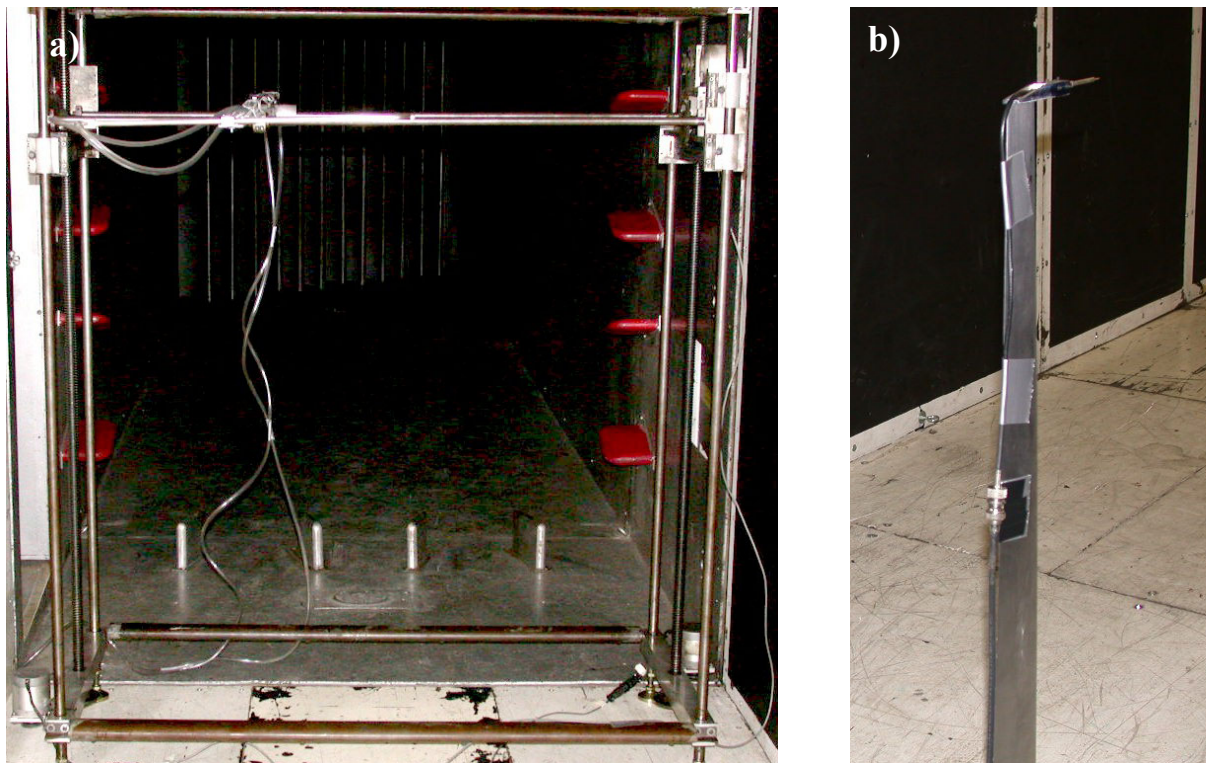


Figure 2.4. Traverse system (a) and airfoil strut (b) in the Stability Wind Tunnel.

2.2 Hot-Wire Anemometry

The vast majority of velocity measurements were acquired by means of hot-wire anemometry. Two different systems were utilized: a single component system was used for calibration of the Active Grid, while a three-component system was utilized for detailed description of the baseline flow condition and the cascade flows. This section describes briefly the hot-wire systems and probes used.

2.2.1 Overview

Hot wire anemometry is, together with laser Doppler anemometry, the most used measurement technique utilized for experiments in turbulent single phase flows. An idealized hot wire probe consists of a thin sensor made out of a material with a high temperature coefficient of resistance. As the sensor is exposed to a flow, the sensor resistance changes as the temperature fluctuates. When the sensor is connected to a hot-wire bridge circuit (essentially a Wheatstone bridge) a voltage proportional to the sensed velocity is produced. There are two types of anemometers widely available: Constant Current Anemometers (CCA) and Constant Temperature Anemometers (CTA). In the former, the bridge provides a constant current which heats the sensor. The temperature and corresponding resistance changes in the wire caused by the cooling of the flow will cause the voltage across the wire to vary with the instantaneous flow velocity. CTA on the other hand attempts to keep the temperature of the sensor (and hence its resistance) constant through a feedback loop. The current and hence the voltage required to keep the resistance constant is then the effective measure of flow velocity. CTA is to prefer over CCA as it provides much higher frequency response.

2.2.2 Hot-Wire Probes

Three different type probes were used for the turbulence measurements. For the calibration of the ATG and initial cascade periodicity tests a TSI 1210T1.5 single wire probe, with a 1.7-mm long 5- μm diameter etched tungsten sensor wire was utilized for streamwise velocity measurements. This wire can be seen mounted on the strut in Figure 2.4b. At certain points the results were checked using two custom built Auspex single-wire probes. One probe consisted of a 0.5-mm long, 2.5- μm diameter sensor, while the other was 1 mm long with a 5- μm diameter. Both hot-wires were operated using a Dantec 55M01 anemometer unit using an overheat factor of 1.7. Since the Kolmogorov scales associated with laboratory turbulent flows are generally at least one order of magnitude smaller than the length of a general hot-wire these different wirelengths were used in order to calculate dissipation rates from the extrapolation to zero wire-length as detailed in Azad & Kassab (1989). The single hot-wire system was optimized to give a flat frequency response to 12.5kHz (TSI probe) and 33kHz (Auspex probes).

Chapter 2: Wind Tunnel and Instrumentation

Three-component velocity measurements were made downstream of the ATG and throughout the cascade using the computerized hot-wire system described in detail by Wittmer *et al.* (1997). Miniature four-sensor Kovasznay type hot-wire probes (Figure 2.5) manufactured by Auspex Corporation were used for both single- and 2-pt measurements. These probes are custom made versions based on the Auspex model AVOP-4-100 design and consist of two orthogonal X-wire arrays with each sensor inclined at a nominal 45° angle to the probe axis. The four $5\mu\text{m}$ etched tungsten wires are close to 1.2 mm in length, yielding a length to diameter ratio of 240 and a total measurement volume (Figure 2.6) of approximately 0.75 mm^3 . Each of the four hot wire sensors was operated separately using a Dantec 56C01/56C17 constant temperature anemometer (CTA) unit with the overheat factor set at 1.7 and the system was optimized to yield a flat frequency response beyond 22 kHz. Figure 2.7 depicts two of these three-component probes during a 2-pt. measurement behind the flat plate cascade, while the Dantec bridges are shown in Figure 2.8 together with the buck-and-gain amplifiers described below.

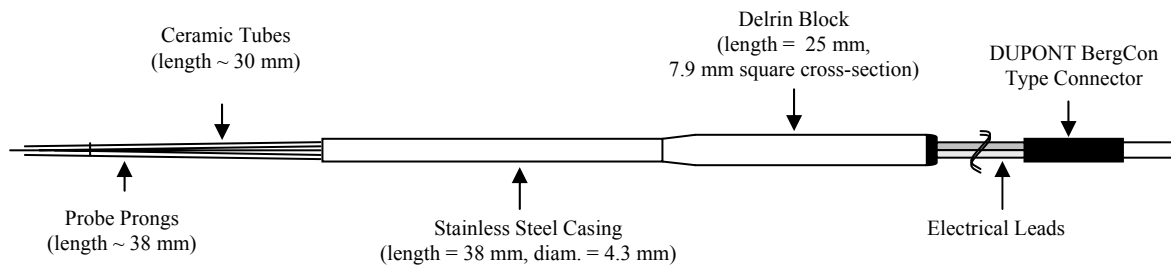


Figure 2.5. Auspex custom AVOP 4-100 Kovasznay type four sensor hot-wire.

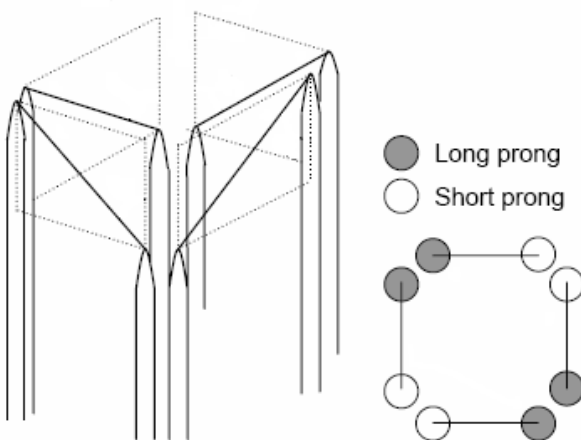


Figure 2.6. Detail of four sensor wire measurement volume.

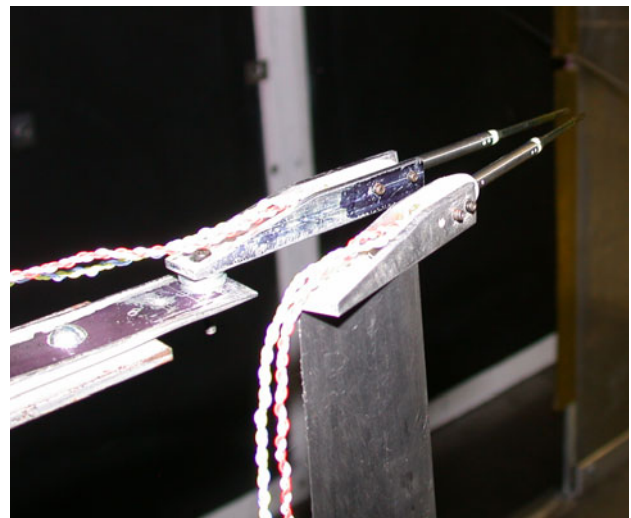


Figure 2.7. Two four-sensor probes set up for 2-pt. measurements.

2.2.3 Data Acquisition System

The hot-wire output voltages from the anemometers were buffered by four $\times 10$ buck-and-gain amplifiers (Figure 2.8) containing calibrated RC-filters to limit their frequency response to 50 kHz. The signals were subsequently digitized by a Hewlett Packard E1432A 16-bit A/D converter. This device is capable of sampling at up to 51.2 kHz on each of its 16 channels, and offers signal conditioning and anti-aliasing capabilities. The VXI mainframe (Figure 2.9) can be populated with up to 12 of these modules for a total of 192 channels. The A/D converter also sampled voltage outputs from the wind tunnel digital thermometer and reference Pitot-static pressure transducer, as well as the amplifier offset voltage. Data was finally transmitted by a HP E8491A module via an IEEE 1394 interface to a Dell Latitude C810 laptop computer running Agilent VEE data acquisition software programmed in house. A schematic of the signal flow from the hot-wire probe to the recorded signal is shown in Figure 2.10.

Several different sampling schemes were used. For the calibration of the Active Grid generally 100 records, each 5.12 seconds in length were obtained at a sampling rate of 51.2 kHz. This resulted in a good resolution of frequencies in the low end of the spectrum, while simultaneously allowing for further averaging of the energy contained in the higher wavenumbers during post-processing.

Most of the cascade measurements and baseline free stream streamwise measurements of turbulence stresses and spectra were generally sampled 100 times for 2 seconds at a rate of 25.6 kHz which proved sufficient for the measurements in question. During post-processing the spectra underwent further averaging in the high-frequency domain.

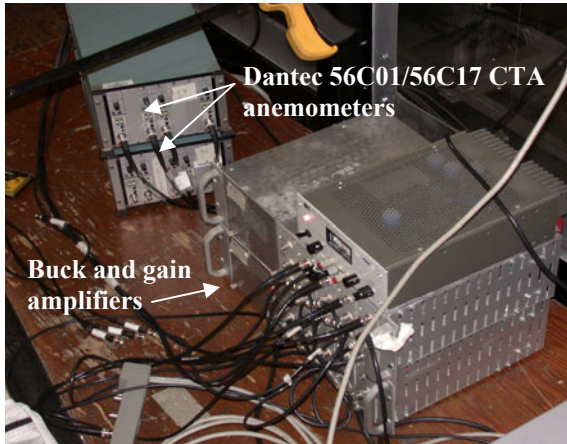


Figure 2.8. Amplifiers and Dantec anemometer bridges.

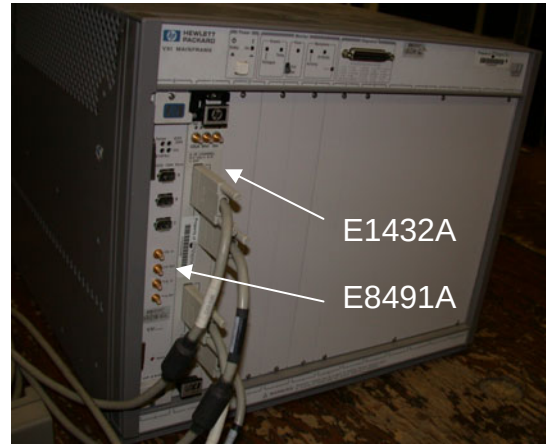


Figure 2.9. HP VXI 16-bit A/D converter.

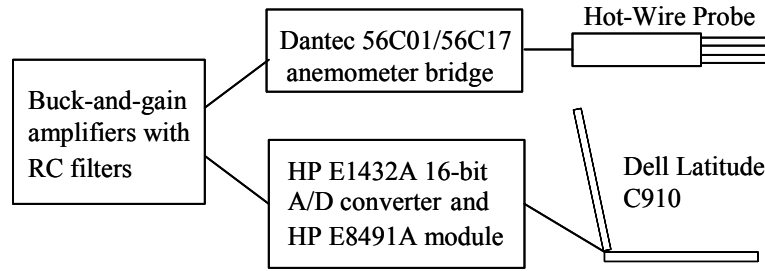


Figure 2.10. Schematic of data acquisition system.

2.2.4 Velocity and Angle Calibration

All hot-wire probes were velocity calibrated before and after each sequence of measurements by placing them in the 6.4 mm uniform jet of a TSI 1125 calibrator (Figure 2.11). King's law (Equation 2.1), where A and B are constants, was applied to correlate the bridge output voltages with the effective flow velocities. During long measurement sessions an interpolation between several velocity calibrations was calculated for higher accuracy.

$$E^2 = A + BU_{eff}^{0.45} \quad (2.1)$$

For the single wires the process of converting U_{eff} to streamwise velocity is trivial as the wires are mounted perpendicular to the flow which yields $U = U_{eff}$. The four-sensor probe conversion from U_{eff} must take the measurement volume geometry into account as well as a direct angle calibration scheme described in detail by Wittmer *et al.* (1997). This calibration is specific to each probe and assumes that the probe undergoes no rotation around its own axis during use. The angle calibration is generated by simultaneously pitching and yawing the probe in the TSI calibrator jet through a grid of angles ranging from -45° to 45° . By comparing these known pitch and yaw angles with the probe outputs, a relationship between flow angle and the effective velocities can be formed and the result can be interpolated or surface fitted to yield a solution for all points within the boundaries of the calibration.

Another calibration problem encountered in this experiment is due to the fact that the air temperature in the Stability Wind Tunnel is highly dependent on outside weather conditions which are generally very different from the control room environment where the hot-wires were calibrated for velocity. The temperature differences between the calibration and the measurement could result in large bias errors. This problem was remedied by 1) running the experiments at night when the tunnel temperature was relatively constant with time and 2) compensating for the difference between ambient and calibration temperature, as well as any ambient drift shown in Equation 2.2 which is a slightly modified version of the method described by Bearman (1971). In this equation θ is the hot-wire overheat ratio (set at 1.7) and E_2 and E_1 are the corrected and uncorrected voltages respectively. T_c is the temperature in which the hot-wire is calibrated and T_∞ is the ambient temperature of the measured flow. This correction scheme performs quite well, especially for temperature differences of less than 4°C , but can be used with satisfactory results for differences up to about 10°C .

$$E_2 = \sqrt{\left(\frac{\theta - 1}{\theta - T_c / T_\infty}\right)} E_1 \quad (2.2)$$

The final calibration consideration is the need for ambient humidity corrections. Larsen & Busch (1974) presented a method for quantifying the systematic error introduced by the failure to account for the relative humidity in the air. A modified version of the relation given by Larsen & Busch (1974) is given in Equation 2.3. Here ξ is a temperature dependent coefficient and r is the relative humidity in percent where the subscripts indicate calibration (c) and ambient (∞) temperatures.

$$\left. \frac{\Delta U_\infty}{U_\infty} \right|_{\text{hum}} = \frac{1.2\xi}{100} \left(1 - \frac{1.7}{\sqrt{U_\infty}} \right) (r_c - r_\infty) \quad (2.3)$$

To assess the maximum effect of the error due to humidity, one can assume that the measurements are performed at the worst case scenario of $r_c = 0$ and $r_\infty = 100$ which assumes that the probe was calibrated in a humidity free environment whereas the measurements were obtained in a situation where there was 100% relative humidity. Additionally, for ambient temperatures between 0°C and 45°C the values Larsen & Busch (1974) present for ξ follow the relation $\xi \approx 0.06 \times \exp(0.0625 T_\infty)$ quite closely. Substituting this into Equation 2.3 yields the following:

$$\left. \frac{\Delta U_\infty}{U_\infty} \right|_{\text{hum}} = 0.0072 e^{0.0625 T_\infty} \left(1 - \frac{1.7}{\sqrt{U_\infty}} \right) \quad (2.4)$$

From this equation it can be seen that low speed experiments performed at low ambient temperatures are the least susceptible to humidity errors. The most detrimental factor is of course the exponential trend of the ambient temperature and if one is dealing with for example a wind tunnel exposed to exterior sunlight the velocity error due to humidity will start compiling quickly. For example at a flow speed of $U_\infty = 50$ m/s in an ambient environment at $T_\infty = 40^\circ\text{C}$ the worst case scenario in Equation 2.4 predicts an error of about 6.6% on U_∞ . In the present data however the ambient temperature was generally less than room temperature ($\approx 15^\circ\text{C}$) and most of the data was obtained at a mean flow velocity of $U_\infty = 12$ m/s. The worst case scenario for this situation introduces an error of only 0.88%. When considering that most of the data was acquired at or below 50% relative humidity this further reduces the error by a factor of two even when retaining the assumption of a humidity free calibration environment which is hardly realistic.* The true error due to humidity is therefore insignificant compared to the other sources of uncertainties associated with the hot-wire probes themselves and the correction has therefore been omitted in this work. Instead the overall uncertainty on the mean flow has been increased by an estimated 0.2 percentage point.

* The calibration conditions were not measured, but given the location of the calibrator jet inside the control room of the wind tunnel it is unlikely that the humidity differences were large enough to significantly impact the overall uncertainty calculations.

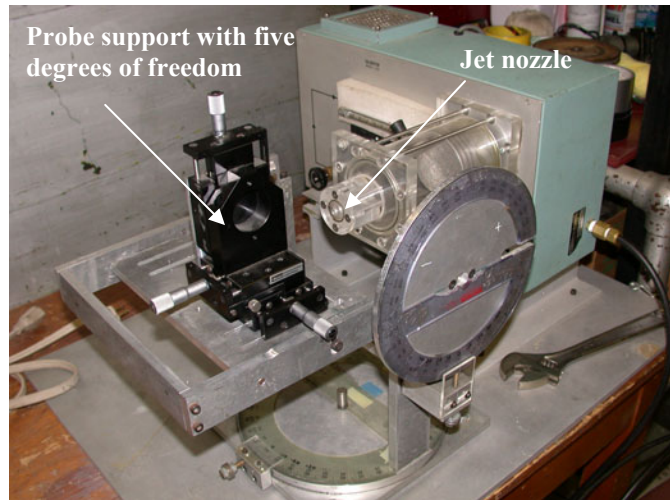


Figure 2.11. TSI 1125 gas probe calibrator.

2.2.5 Hot-wire Uncertainties

The overall uncertainties on mean velocities, turbulence intensities, and turbulent stress associated with the four-sensor hot-wire system, given 20:1 odds, are presented in Table 2.1 and were calculated based on the method presented in Ma (2003). These uncertainties stem from bias and random errors introduced by the data acquisition system as well as from probe calibration and data averaging. Single wire uncertainties for the streamwise velocity and its mean square fluctuation take on similar values.

Measurement Quantity	Uncertainty (20:1 odds)	Relative to
U, V, W	$\pm 1.8\%, \pm 1.5\%, \pm 1.5\%$	U_∞
u^2, v^2, w^2	$\pm 5\%, \pm 8\%, \pm 8\%$	u^2, v^2, w^2
uv, vw, uw^*	$\pm 3.5\%, \pm 3.5\%, \pm 3.5\%$	$(u^2 v^2)^{0.5}$

Table 2.1. Four-sensor hot-wire uncertainties.

2.3 Flow Visualization

During the testing of the flat plate cascade described in Chapter 3, flow visualization was used to get a rough idea of the surface flow on one of the center passage blades. First a black plastic coated self adhesive paper was attached around midspan of one of the cascade blades such that it extended over the chord on both sides of the blade. An oil flow mixture based on nominally 15 parts kerosene to 5 parts titanium dioxide (TiO_2), and 1 part oleic acid was subsequently applied using a sponge brush. The turbulent wind tunnel flow was passed over the

* Note that the Reynolds stresses, uv , vw , and uw are *not* equivalent to the rms velocities multiplied together, but rather $uv = vu = \overline{u'v'}$, $vw = wv = \overline{v'w'}$, and $uw = wu = \overline{u'w'}$

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blade for about 10 minutes until most of the kerosene had evaporated and the changes in oil flow patterns were minimal.

In addition, tuft visualization was performed by attaching 1.5” pieces of yellow cotton yarn with transparent tape to a 40cm section at midspan on the same blade used for the oil flow. The tufts were attached at approximately 2” intervals in both the chordwise and spanwise direction on the side facing the wind tunnel control room. Flow visualization results as well as general footage and pictures of all experiments were documented using a SONY DCR-TRV 740 digital video camera recorder and a Nikon Coolpix 995 digital camera.

Chapter 3

The Active Turbulence Grid

The turbulent flows presented in this work were generated using an active turbulence grid, which was specially designed for this study. An overview of the concept design stage of the grid development is given in Section 3.1 followed by the final grid hardware and layout in Section 3.2. The mechanical forcing system is described in Section 3.3, while Section 3.4 documents the operational envelope of the grid to conclude the chapter.

3.1 Concept Design Stage

As discussed in Chapter 1, the active grid concept was developed in Japan in the early eighties, and the work was first published by Makita & Miyamoto (1983). However, the work was not widely published until Makita (1991) and Makita & Sassa (1991). At the time of the design and fabrication of the Virginia Tech Active Turbulence Grid (ATG) in early 2001 only the grids of Makita (1991) and Mydlarski & Warhaft (1996, 1998) were identified. Later on the author became aware of the similar efforts by Poorte (1998) as well as Kang *et al.* (2003).

3.1.1 Grid Sizing

The process of grid sizing was based on the results of Makita (1991) and Mydlarski & Warhaft (1996, 1998). Makita used a 30 rod configuration (15 vertical and 15 horizontal rods) in a tunnel with test section dimensions of 0.7 m x 0.7 m. The grids built by Mydlarski & Warhaft on the other hand used 14 rods (7 + 7) for their vertical wind tunnel with a cross section of 0.41 m x 0.41 m, and a larger 15 rod configuration (8 + 7) in their horizontal 0.91 m x 0.91 m test section. All these tunnels are significantly smaller than the test section of the Virginia Tech Stability Tunnel with its 1.8 x 1.8 m² cross section. This would perhaps suggest twice as many axes compared to previous designs, however this option was discarded early on due to the added expense associated with the extra motors and relevant hardware needed to drive a 60 axis grid.

The mesh width and consequently the number of independent axes were decided upon through the following considerations. Lengthscales inherently scale on streamwise distance from the grid, as well as the mesh width, M . The smaller the integral lengthscales, the greater the chance of having good transverse homogeneity, however one should not lose sight of the fact that the underlying purpose of this work is to generate large scale turbulence.

Large Taylor Reynolds numbers are obtained, not only by achieving large Taylor microscales, but also significant turbulence intensities. This, however, could prove detrimental for the cascade application where large scale unsteady stall could destroy the study of the blade blocking phenomena which is the main goal of this work. It would therefore be desirable to have

turbulence intensities nominally less than 10% in magnitude. Active grids have previously shown to produce larger turbulence intensities than this. However, the magnitude of the relative velocity fluctuations can be controlled by either moving the cascade to a point in the test section where the turbulence kinetic energy has decayed adequately. Another approach could be to manipulate the grid operation, or forcing software protocol driving the grid. In fact Mydlarski & Warhaft (1996) mentioned that changing the rotational speed affect Re_λ to a certain degree, which indicates that turbulence intensity could be controlled by adjusting the proper grid operational parameters.

Intuitively, turbulence intensity, and especially integral scales are inverse functions of rotation rate due to the coherent large structures passing through the grid at low rotation rates. The time a grid cell is considered “open” relative to the flow decreases as the rotational speed is increased which results in a smaller lengthscale. Hence, the contribution to the low-frequency (energy-containing) part of the velocity spectrum will also decrease. This causes the total integral to yield a lower RMS velocity and results in lower turbulence intensities as well. Based on the above consideration, and the sectional size of Stability Wind Tunnel, the final design for the Virginia Tech version of the active turbulence generator yielded a 20-axis configuration with a mesh size, M , of 8.25".* This was thought to be enough to preserve transverse homogeneity given a properly calibrated grid operation scheme, while simultaneously achieving the desired large turbulent lengthscales.

3.1.2 Improving the Isotropy

One of the main drawbacks of the previous active grids is the large anisotropy associated with them. Mydlarski & Warhaft (1996) observed that the 20% anisotropy ratio (u/v) was largely contained within the low wavenumber region. Poorte (1998) improved the isotropy ratio by mounting the agitator wings in a staggered formation, but suffered in return great losses in the turbulence kinetic energy values and lengthscales produced by the grid. The approach used in the present work is different and is based on Rapid Distortion Theory (RDT)[†] where an estimate of the required contraction ratio needed to cancel out an initial anisotropy can be calculated.

Prandtl (1933) was the first to predict the ratio of mean square velocity fluctuation before and after passing through an axisymmetric contraction. Following Helmholtz’s vortex theorems, the analysis is based on the assumption that most of the lateral energy stems from discrete vortex tubes with axes running in the streamwise direction, while streamwise fluctuations are governed by vortex tubes in the lateral direction. By allowing the vortex tubes to stretch and deform through a contraction c , where $c = U_2/U_1$ or alternatively $c = A_1/A_2$ through continuity, Prandtl (1933) obtained the relations listed in Equations 3.1 and 3.2. Letting u' and v' be the peripheral velocity from the lateral and streamwise oriented vortex tubes respectively, the derivation of these relations is presented below.

When passing a vortex filament through a contraction c , the volume of the tube will remain constant. For an axisymmetric contraction such as the one depicted in Figure 1.1 the vortex filament centered on the streamwise axis will experience a radius decrease by a factor $\sqrt{c}$ which results in an elongation by a factor c and a reduction of area by this same factor. The total

* As will be clear in the next section, the grid was not placed in the test section which explains the discrepancy between the mesh size, number of axis and the width of the Stability wind tunnel test section.

[†] Refer to Section 1.3.2 for a brief overview of RDT.

circulation or strength must be constant and is equal to the product of the angular velocity and the cross-sectional area of the vortex tube. This results in the peripheral velocity (product of radius and angular velocity) increasing by a factor of $\sqrt{c}$. Squaring the ratio of peripheral velocities yields Equation 3.1. The same analysis can be made for the vortex filament that lies perpendicular to the tunnel axis. The effective radius in the resulting elliptical cross-section will decrease by a factor of $\sqrt{c}$ along with a decrease of $\sqrt{c}$ in the angular velocity. Squaring the peripheral velocity ratio yields the factor of $1/c^2$ for the contraction effect on the streamwise fluctuations in Equation 3.2.

$$\frac{\overline{v_2'^2}}{\overline{v_1'^2}} = \frac{\overline{w_2'^2}}{\overline{w_1'^2}} = c \quad (3.1)$$

$$\frac{\overline{u_2'^2}}{\overline{u_1'^2}} = \frac{1}{c^2} \quad (3.2)$$

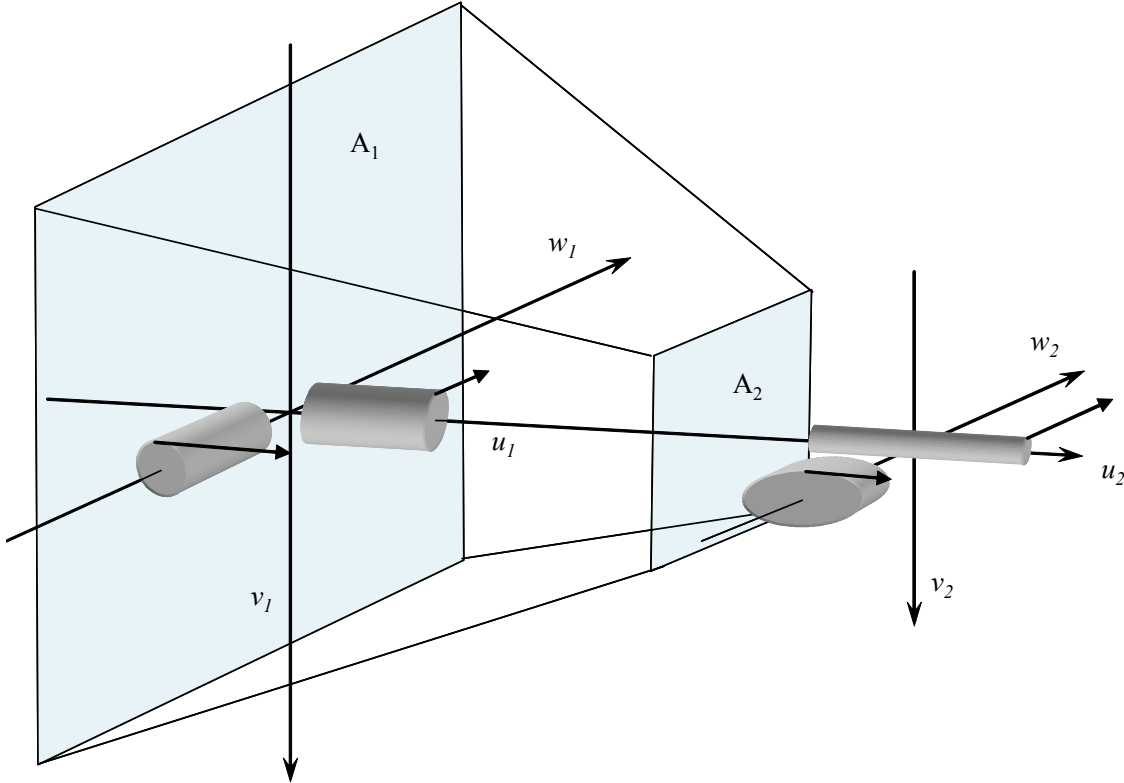


Figure 3.1. Sketch of discrete vortex tubes passing through a contraction. Peripheral velocities indicated by arrows. After Uberoi (1956).

The above discrete vortex tube theory by Prandtl (1933) must be regarded as a qualitative method, only useful to predict the general effect a contraction has on the turbulence energy content. By invoking the RDT assumptions, where the straining is assumed to occur in such a short time that viscous forces and turbulence interaction can be neglected, Taylor (1935b) derived a linear relationship between a sinusoidal disturbance and an axisymmetric contraction.

The results exhibited the same form as the predictions made by Prandtl, but with different numerical factors.

Based on this initial work by Taylor, Batchelor (1953)* provided a full RDT estimate of the turbulence energy spectrum in both the lateral and streamwise directions for an initially isotropic turbulent flow, by integrating over all possible wavenumbers. The same results (in a slightly different form) were also independently derived by Ribner & Tucker (1953). By using an axisymmetric contraction the relations simplify significantly, and by forming the ratios of the post contraction turbulence energies to those of the initial free stream values Batchelor (1953) derived Equations 3.3 and 3.4.

$$\frac{\overline{u_2'^2}}{\overline{u_1'^2}} = \frac{3}{4c^2} \left[\frac{1+\alpha^2}{2\alpha^3} \ln \left(\frac{1+\alpha}{1-\alpha} \right) - \frac{1}{\alpha^2} \right], \text{ where } \alpha^2 = 1 - c^{-3} \quad (3.3)$$

$$\frac{\overline{v_2'^2}}{\overline{v_1'^2}} = \frac{\overline{w_2'^2}}{\overline{w_1'^2}} = \frac{3c}{4} + \frac{3}{4c^2} \left[\frac{1}{2\alpha^2} - \frac{1-\alpha^2}{4\alpha^3} \ln \left(\frac{1+\alpha}{1-\alpha} \right) \right] \quad (3.4)$$

Figure 3.2 shows the results of both Prandtl's (1933) and Batchelor's (1953) predictions. One can easily verify that the linear theory follows the general trend predicted by the vortex tube theory, but that the change in energy is slower both for streamwise and lateral components. From this the ideal location for a turbulence generator in the contraction in order to counteract the initial anisotropy, can easily be calculated. Figure 3.3 shows the required contraction ratio, c , needed to cancel out initial anisotropy up to 30% ($1 < u/v < 1.3$) for both Prandtl's (1933) and Batchelor's (1953) predictions. For design purposes it should be clear that the linear theory should be used over the rather simplistic qualitative method.

Taking the 20% pre-contraction anisotropy which is expected from a Makita-type active grid one obtains a required contraction ratio of approximately 1.3 (using the black RDT line) in Figure 3.3. This is a good initial guess, but it must be kept in mind that the central postulate in RDT does not hold true for most contractions nor is Equation 1.15 valid for an active grid with inherently large turbulence intensities.

Uberoi (1956) tested the validity of the linear theory by performing several experiments with conventional grid turbulence accelerating through three different contractions. Although somewhat inconclusive for large contractions, the results clearly showed that the larger the contraction ratio, the more significant the deviation from theory. Good agreement was observed for contraction ratios less than two (especially within the larger nozzles). By increasing the grid Reynolds number, the results suggested even better agreement with theory for moderate contraction ratios. Nevertheless, the linear RDT theory was shown to overestimate the effect of the contraction.

Although a relatively small contraction ratio is being considered for the current application, the convergence of the tunnel section is quite slow at this point and one should envision a larger contraction ratio than the 1.3 given from Batchelor's (1953) theory. Furthermore the theory initially assumes isotropic turbulence evolving into anisotropy. For this case the opposite is being attempted, and the theory can hence only provide an approximation unless it was to be re-derived

* The RDT work presented in Batchelor (1953) is a summary of the more complete coverage found in Batchelor & Proudman (1954)

for anisotropic turbulence. In accordance with Uberoi (1956) it will be assumed that the resulting changes from a modified theory will be too small to warrant the new calculations.

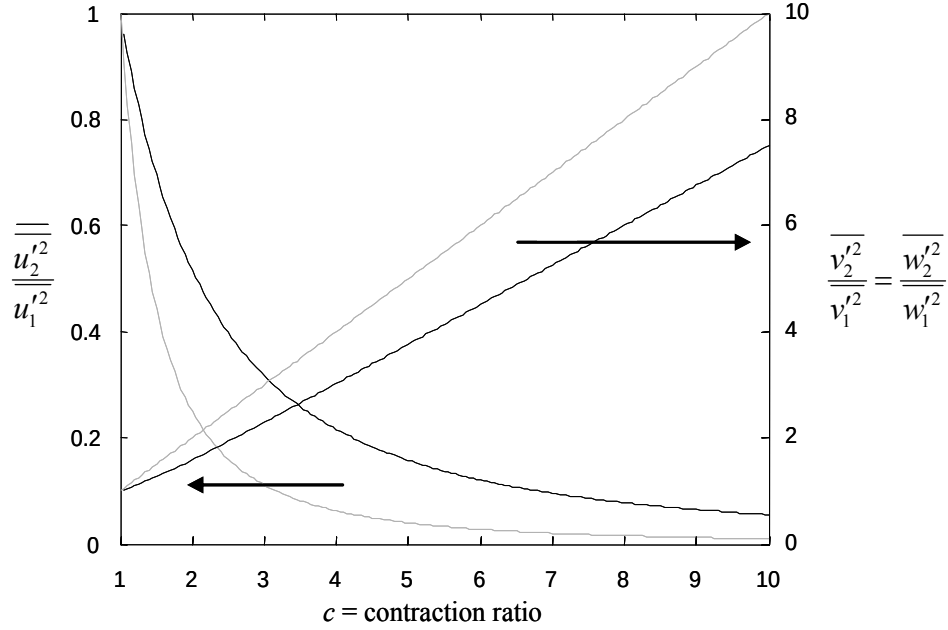


Figure 3.2. Ratio of turbulence energies before and after passing through a contraction. The discrete vortex theory of Prandtl (1933) is indicated by grey lines while the black lines are Batchelor's (1956) RDT predictions.

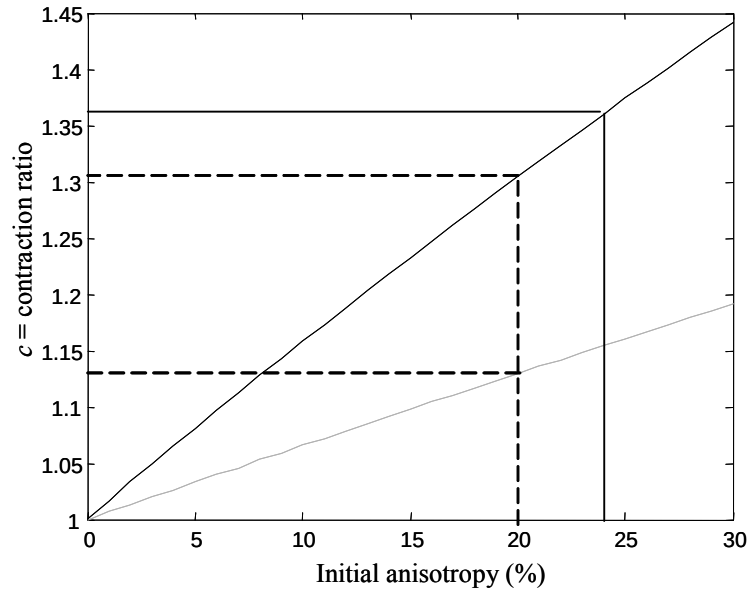


Figure 3.3. Required contraction ratio, c , for a given pre-contraction anisotropy level ($u > v$) in order to achieve an isotropy ratio of unity. Prandtl's discrete vortex theory (1933) and Batchelor's (1953) RDT predictions are indicated in grey and black lines while locations of theoretical and actual grid placement are indicated by dashed and solid lines respectively.

Measurements in a small contraction ($c = 1.27$) were made by Comte-Bellot & Corrsin (1966) with conventional grid turbulence. The contraction was utilized to obtain isotropy ratios close to unity in the test section. Based on the RDT solution in Figure 3.3 it can be estimated that an initial anisotropy level of 17.5% would be cancelled out by a contraction ratio of 1.27. The experimental data showed a smaller reduction than this, where anisotropy ratios between 12% and 15% were cancelled out. This data-set gives an estimate of the magnitude of the error associated with the linear theory discussed above and was used to extrapolate towards the design point of the current grid. Instead of using a contraction ratio of 1.3, as dictated by the linear theory, a contraction ratio of 1.36 was finally decided upon. This puts the grid in a location where the cross section is 2.14 m x 2.14 m, 2.74 m upstream of the test section entrance, which increases the effective development distance to 10.05m.

3.2 Detailed Grid Design and Construction

The basic concept design for the Virginia Tech Active Turbulence Grid (ATG) is shown in Figure 3.4. The grid is shown in its fully closed state (a) and in an instantaneous snapshot during randomized operation (b). A bi-planar design was adopted in which all the vertical rods were placed in the same streamwise plane with the horizontal rods placed 2.5" upstream of the vertical rods. This configuration reduces the drag on the grid, decreasing the turbulence intensity slightly compared to previous grid designs. It also greatly diminishes the danger of the agitator vanes ever hitting each other in the event of a malfunctioning coupling or bearing.

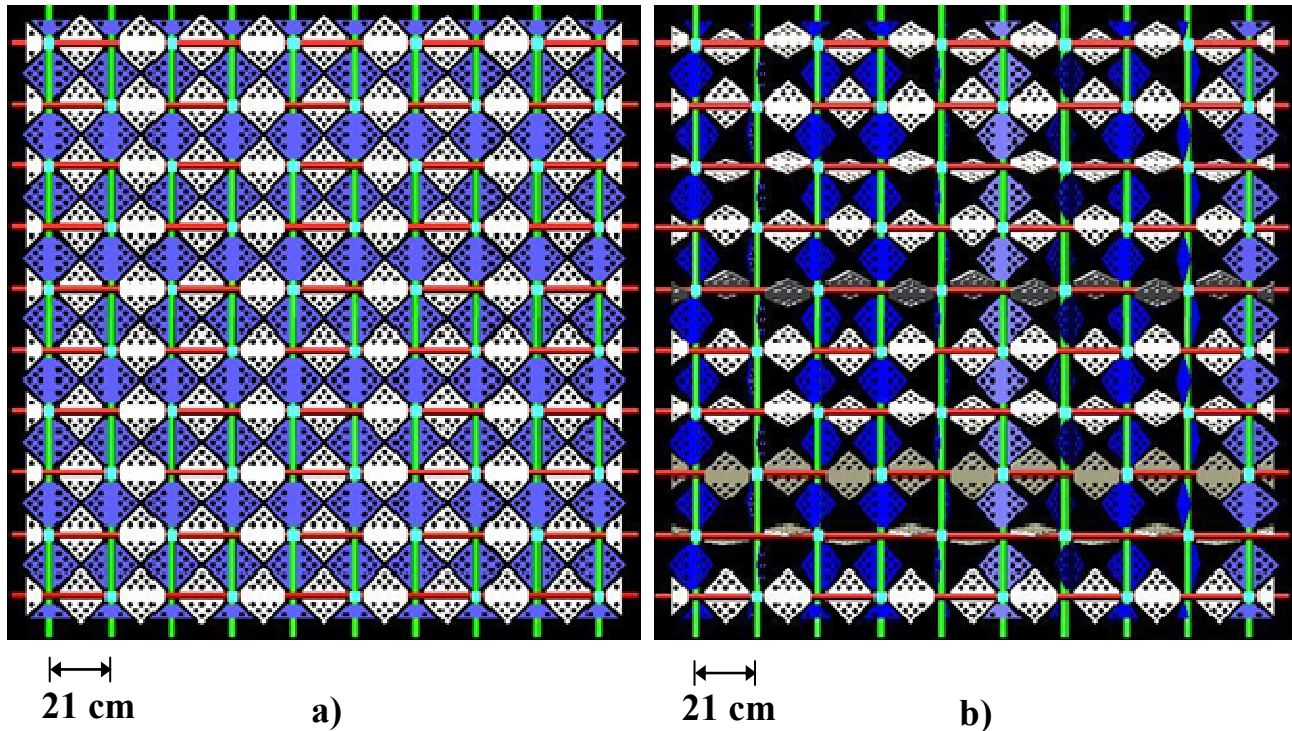


Figure 3.4. Concept design of the Active Turbulence Grid in a) completely closed configuration and b) instantaneous snapshot in random operation.

3.2.1 Shafts and Vanes

The grid utilizes rotating half-vanes adjacent to the walls. Makita (1991) used this same layout, while Mydlarski & Warhaft (1996) attached stationary half vanes to the tunnel walls instead to complete the grid. Uniformity and homogeneity appeared to be better in the experiments by Makita (1991) and this could possibly be due to these rotating half vanes since, when accounting for images in the test section wall, an infinite rotating grid will be simulated which would not be the case with static end vanes. The final 10 by 10 rod configuration therefore yielded 180 (20 x 9) full agitator vanes and 40 (20 x 2) rotating half vanes.

The grid in this study is more than four times the size of the grid used by Kang *et al.* (2003) which was the largest of previous active grid efforts. Kang *et al.* used square aluminum tubing for their grid, but other previous active grids have used solid rods. The inertia associated with solid rods was reduced significantly by settling on the use of thin-walled tubes instead. In addition this pushes the resonant frequency further away from the operational envelope. Based on weight, yield strength, bending properties, and moment of inertia, 1" nominal diameter aluminum tubes with a wall thickness of 1/16" were used for the grid axes in this work. In order to keep the moment of inertia symmetric for each rod, all agitator vanes were alternately positioned on opposite sides of the rod.*

To further reduce the inertial load on each rod assembly, holes were drilled in each of the 180 full vanes. The four perforated designs under initial consideration can be seen in Figure 3.5. The design in Figure 3.5c reduced the inertia of a vane by 27% followed by designs d, b, and a (at 20%, 14%, and 13% respectively). Furthermore, the two corners overlapping the rod of each vane were shaved off to allow for mounting space of the stabilizing couplings described below.

* The mounting is still considered parallel and should not be confused with the staggered vane mounting utilized by Poorte (1998) which was described in Chapter 1.

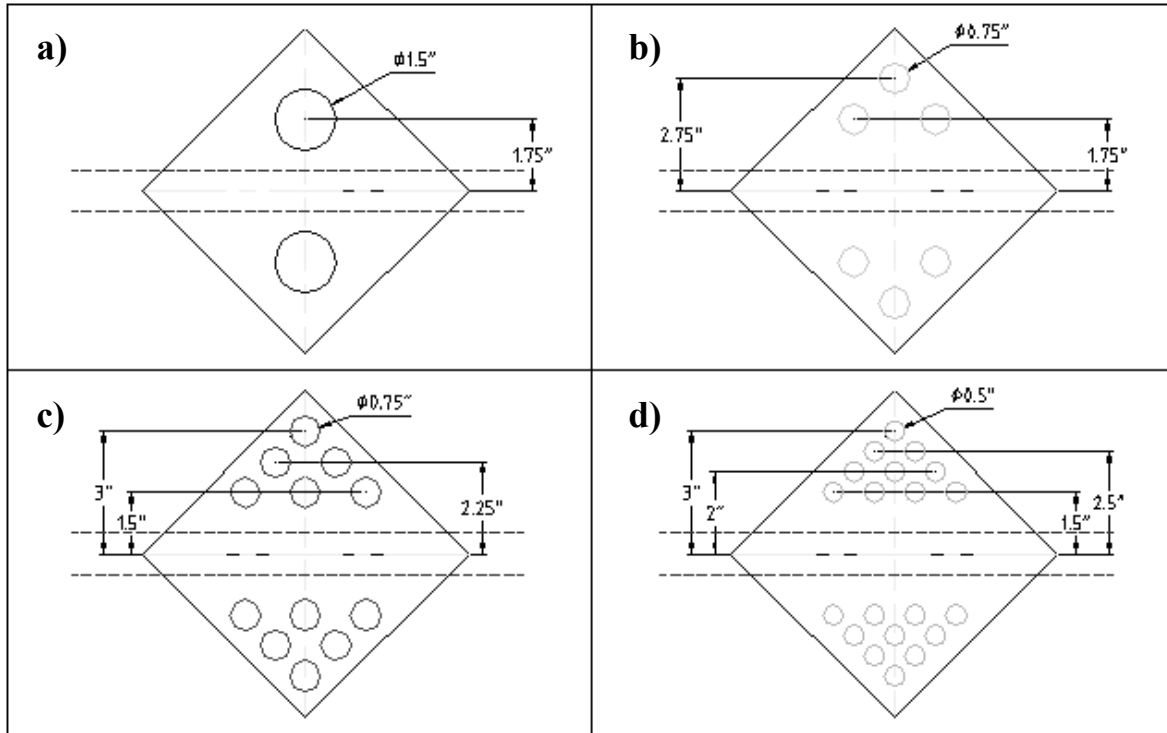


Figure 3.5. The four perforated vane layouts under consideration. The dashed lines indicate the path of the aluminum rod.

Two different materials were considered for the agitator vanes: $3/32''$ thick aluminum and $3/16''$ thick plywood. After a trade-off analysis, the wooden version proved to only require about $1/3$ of the torque compared to that of the aluminum. Added benefits of using wood also arise in the case of a grid malfunction, as the easily replaceable wooden vanes will fracture without destroying the rest of the grid. Based on these considerations it was decided to use Luan plywood as the material for the vanes.

Figure 3.6 shows the final vane design with six $3/4''$ holes drilled on each side of the axis of rotation (a) and the final manufactured version in Luan plywood (b). All the holes were subsequently covered with self-adhesive plastic sheets and reinforced with a layer of clear paint in order to seal the vane as well as providing the option of tuning the grid for improved performance as done by Mydlarski & Warhaft (1996).

Prototype tests showed that the original end vanes left too big of a gap between the wall and the edge of the grid (due to the tunnel wall irregularities) which yielded a smaller region of uniform flow. This was remedied by custom fitting each rod with extended half vanes to conform better to the somewhat curved wind tunnel walls.

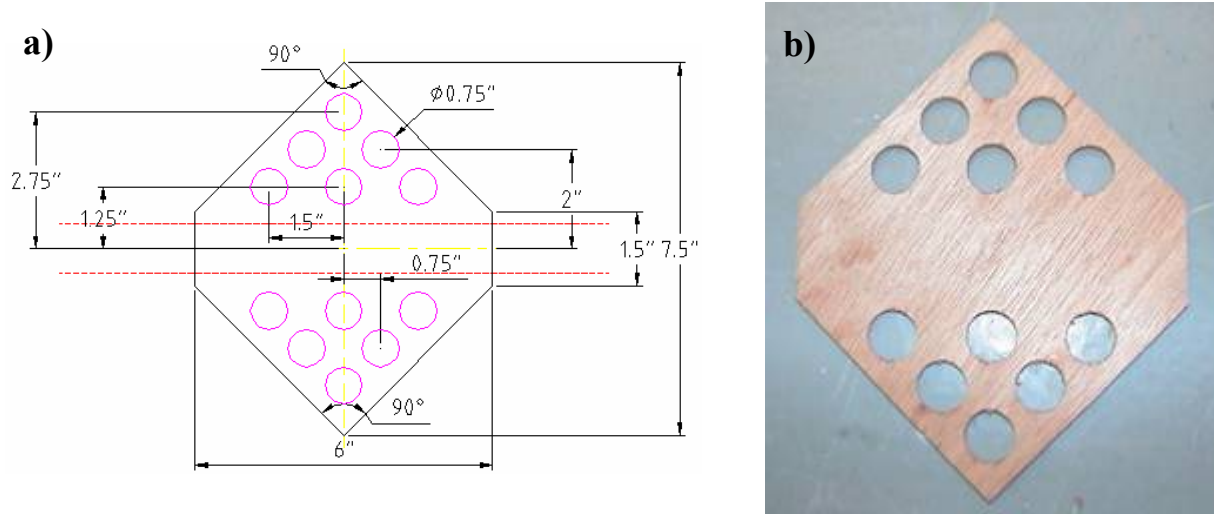


Figure 3.6. The final agitator vane design.

3.2.2 Couplings, Bearings, and Tunnel Fitting

The heart of the ATG is the stepper motors which are mounted outside the Stability tunnel underneath an aluminum weather cover duct. Ten 1.25" holes were drilled through the tunnel walls spaced $M = 8.25"$ apart through the ceiling and the starboard outside wall of the tunnel. Two custom made 80" long brackets, which can hold ten (size 34) motors each, were fitted over these holes on the exterior surfaces. The brackets are made from C-beam aluminum sections with the flanges cut to compensate for the approximately 7° local contraction angle. This allows for flush mounting of the motors, independent of the irregularities of the local contraction wall. The brackets have been raised just enough to prevent the motor shafts from protruding into the tunnel section, and by mounting them with countersunk bolts from the inside, they can be left permanently on the outside of the tunnel without affecting the flow in the absence of the grid. In addition, mounting the motors on such channels prevents accidental immersion of the motors in the case of a leak, and will also act as a heat sink for the motors during prolonged operation. A picture displaying the mounting of the motors on the stability tunnel can be seen in Figure 3.7a as well as a close-up of the side mounted channel in Figure 3.7b.



Figure 3.7. Motors and outside aluminum brackets: a) outside view of the contraction of the Stability Wind Tunnel; b) side mounted motors and bracket.

Figure 3.7 also shows the catwalk located some 12' above ground. The catwalk was mounted to the wind tunnel in order to provide safe access to the side wall of the contraction. In order to provide the motors with protection from inclement weather, an aluminum weather cover duct was designed to attach to the tunnel walls around the motor brackets on the outside of the wind tunnel. Enclosing both arrays of motors, the duct extends from the catwalk floor, wraps around the two upper corners of the wind tunnel roof before exhausting through a downward facing exit on the opposite tunnel wall.

Inside, on the non-motor side of the test section (floor and port side), there are two 80" long, 3" wide and $\frac{1}{2}$ " thick brackets with 10 ball bearings mounted into the wall to accommodate the end bearings which mount into the grid rods. These brackets are fitted directly onto the contraction wall, but the ball bearings have been accommodated at the 7° required angle to compensate for the local contraction angle. In order to properly place the hardware inside the tunnel, a RoboSquare 3-coordinate laser level was used to ensure full alignment with the externally mounted motor supports.

The open, (single row, deep groove type) ball bearings used (displayed in Figure 3.8a) are manufactured by SKF (model #6204) and have inner and outer diameters of 2.2 cm and 4.6 cm respectively. The end bearings, shown in Figure 3.8b, slide into the ball bearings with one end and match the inner diameter of the grid rods at the other end. Slotted holes at the ends of each rod allow for local adjustment before the rods are secured into place. Figure 3.8b also shows the aluminum couplings which slide onto the motor shafts and allow for mounting of the grid rods in the same manner as the end bearings.

As mentioned previously the bi-planar design leaves a 2.5" gap between the centers of the two sets of rods. To reduce the vibration and resulting friction during operation 25 vinyl couplings (Figure 3.8c) with lubricated Nyliner flanged sleeve bearings were inserted in strategic positions* throughout the grid joining the horizontal and vertical rods. This addition increased the wind-off torque requirement by a relatively large factor, but was critically necessary for dynamic operation of the grid.

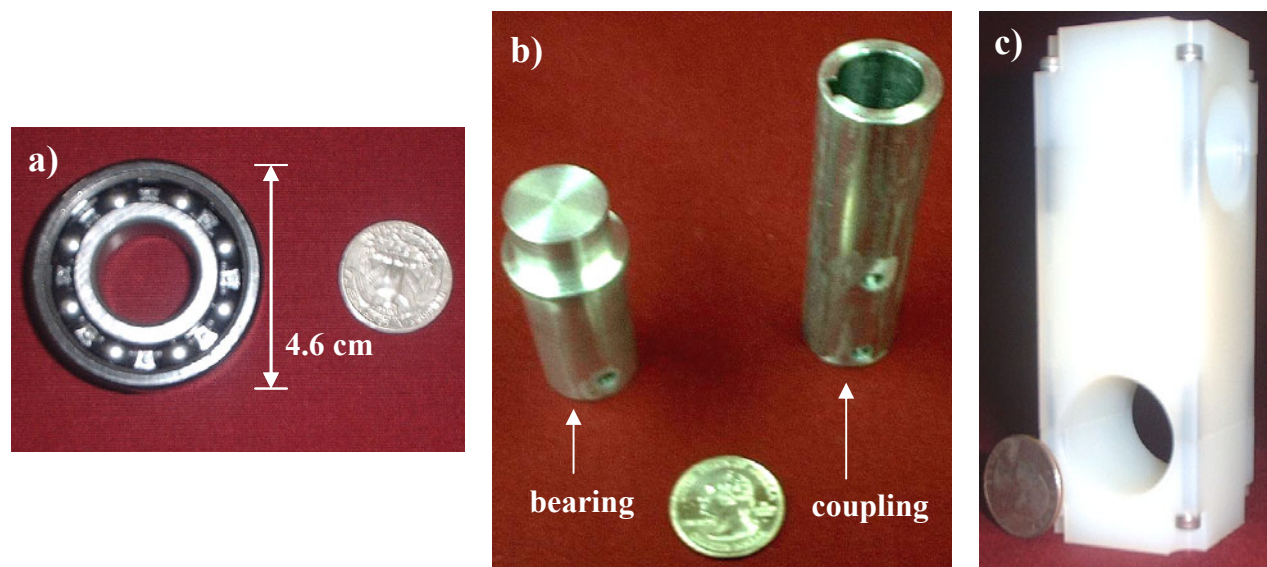


Figure 3.8. Grid Supporting Hardware: a) ball bearings; b) motor to rod coupling and end bearing; c) vinyl support coupling.

Figure 3.9 shows a conceptual drawing of the proposed grid with nylon couplings installed between every agitator vane and the resulting pathway of the vanes during rotation. Although this concept design displays couplings between every linkage in the grid, it became clear during grid assembly that only half the number of couplings were needed to stiffen the grid sufficiently to withstand the required dynamic forces. Limiting the number of couplings substantially reduces the extra frictional load on each individual rod. Detailed views of actual grid cells are shown in Figure 3.10 while the fully assembled grid prototype, with the final configuration of vinyl couplings is pictured as seen from downstream in Figure 3.11a. Figure 3.11b on the other hand shows the prototype without the nylon couplings in an isometric view.

* The exact location of each coupling in the grid structure can be seen in figure 3.11a.

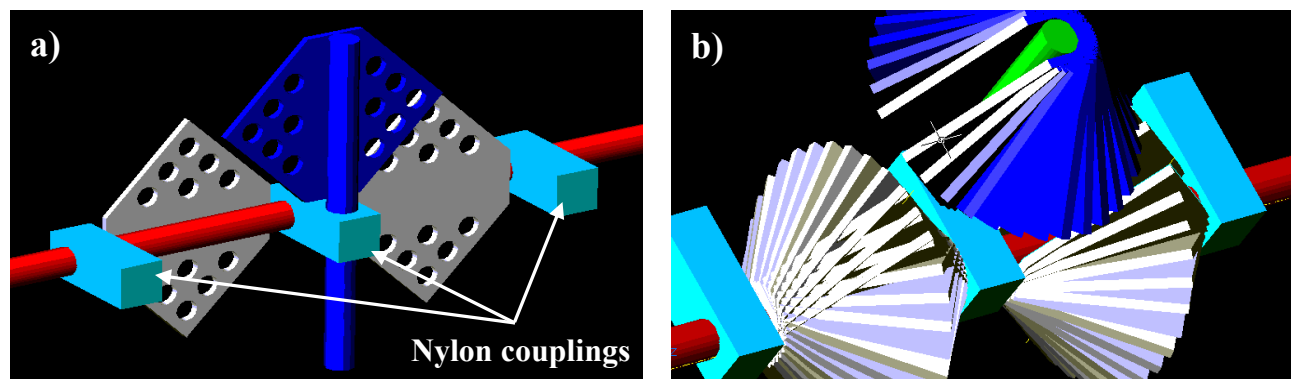


Figure 3.9. Conceptual drawings of a) grid linkage showing the mounting of the agitator vanes and couplings; b) agitator vane paths during rotation.

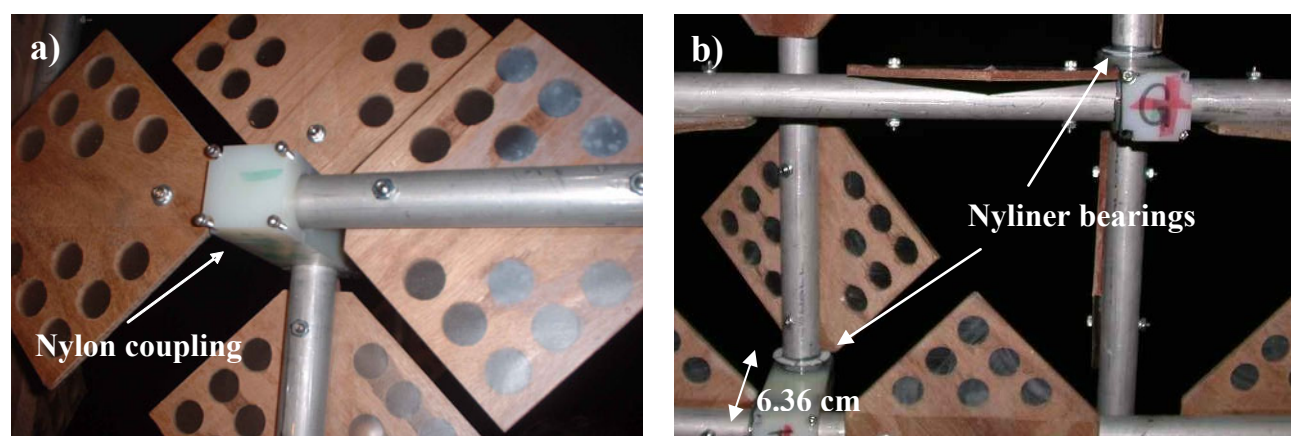


Figure 3.10. Detailed views of the grid cells including nylon couplings with Nyliner sleeve bearings.

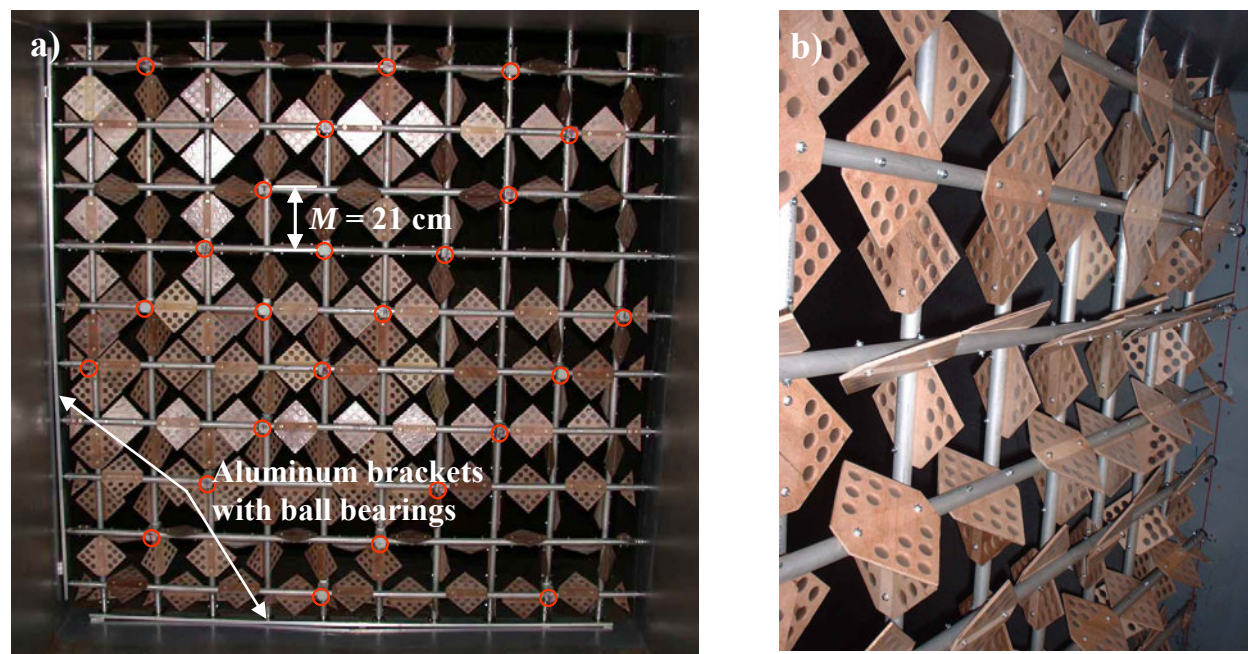


Figure 3.11. Grid prototype: a) downstream view; b) isometric view.

3.3 Forcing Hardware and Software

The most important part of the Active Turbulence Grid is the power plant. This section describes the sizing of the motors and the associated hardware and software necessary to realize the force required to drive the grid.

3.3.1 Motors, Drivers and Power Supplies

Initially the grid was sized to operate with rotation rates up to 5 Hz and accelerations of up to 1000 Hz/s. The torque requirement for this scheme under no dynamic load is about 500 oz-in for the selected rod and vane configuration. It was found, however, that the acceleration requirement was vastly overestimated and that much smaller parameters could be used with satisfactory grid performance. By relaxing the acceleration requirement, much larger rotational speeds can obviously be obtained if needed.

The dynamic load on the grid during regular operation is of obvious importance when selecting a motor. Free stream velocities up to 20 m/s were desired, which can put a total drag load of up to 5500 N across the grid in a worst case scenario*. The extra torque required due to the frictional forces that are generated during dynamic operation will also increase the requirements significantly. Additionally, for smoother operation at lower speeds it was found necessary to half-step the motors, which lowers the available torque at any speed by a factor of about 1.4.

The decision of utilizing step motors instead of servo motors was made early on in order to exercise better control of the grid operation by the ability of tracking the motor position at any given time. The three different step motors in Figure 3.12 were tested as candidates for the power plant of the active grid. All three features a resolution of 200 steps per revolution. The smallest motor is a Pacific Scientific (PacSci) N31HRHK-series rated at 665 oz-in of torque. This motor was briefly used during initial laboratory testing, but a larger motor was clearly needed. Subsequently a PacSci K33HRHJ-series motor was obtained along with an Applied Motion Products (AMP) HT34-438. These two motors are rated very similarly with a holding torque of 2145 and 2140 oz-in respectively. The PacSci has slightly smaller rotor inertia, but a larger inductance (7 mH to 2.6 mH of the AMP motor) with a current rating of up to 5 A per phase as opposed to the rival at 6 A (unipolar rating). Both motors were found to perform the job satisfactorily, and although slightly more expensive, the choice fell on the HT34-438. Nevertheless, the single red PacSci motor, having the exact same mounting hardware and shaft dimensions, was used in the final grid configuration as a replacement for a malfunctioning AMP motor.

Each of the 20 motors is wired in bipolar parallel configuration and separately controlled in half-stepping mode by an Intelligent Motion System (IMS) IB106 step motor driver (Figure 3.13) which can provide up to 6 A of electric current per phase. Initially the drivers were mounted on the outside of the wind tunnel, but significantly better performance was obtained by placing the drivers on a panel inside the control room (Figure 3.14). This allowed for shorter logic and power input wiring at the expense of the motor to driver cabling. By using high quality

* This would only occur in the rare event of all the agitator vanes being positioned normal to the flow.

twisted shielded 16-gauge cabling for the typically 20' long cables, the integrity of the signal was kept intact.

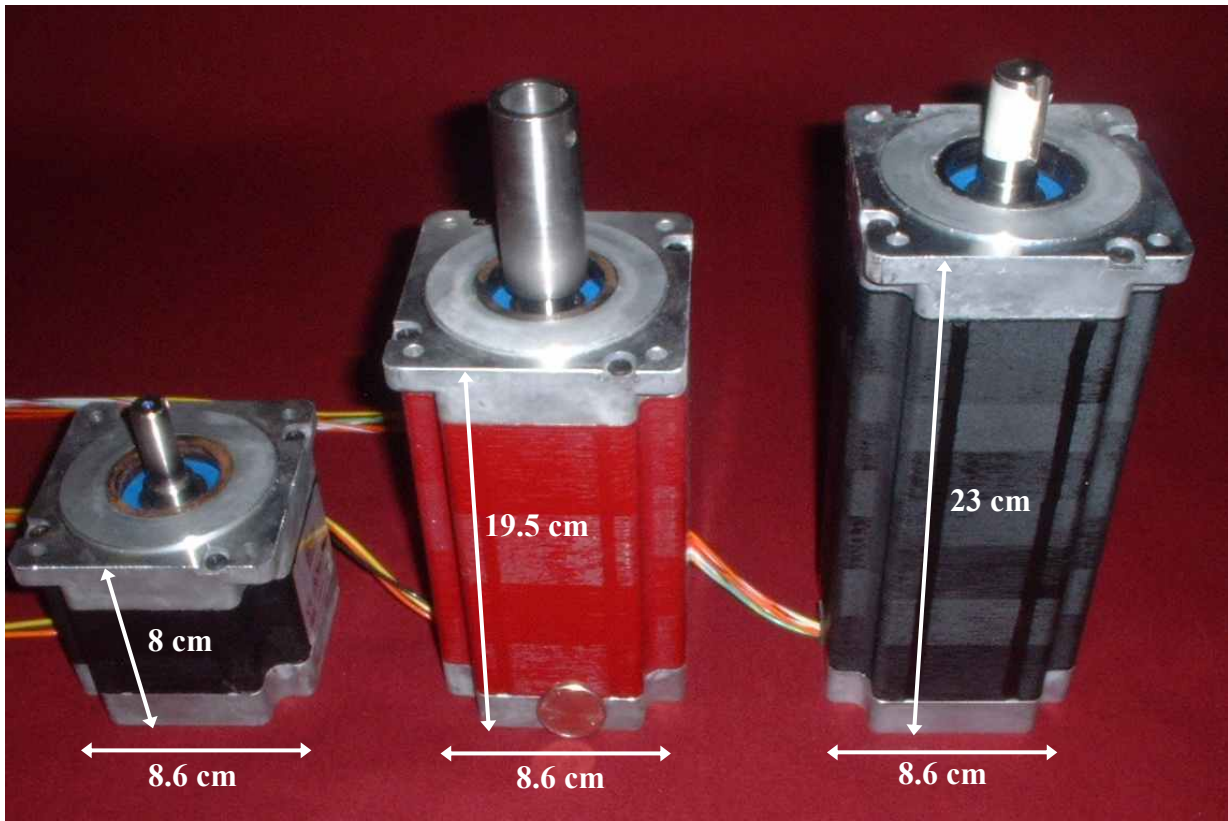


Figure 3.12. Step motors (from left to right): PacSci N31HRHK, PacSci K33HRHJ, and AMP HT34-438.



Figure 3.13. IMS IB106 step motor driver



Figure 3.14. Complete array of 20 IMS IB 106 step motor drivers

The circuit diagram for the IB 106 connections is shown in Figure 3.15. By connecting a hybrid step motor in bipolar parallel configuration, more torque is obtained at higher rotational

speeds. Note the external resistors (R_1) necessary in order to reduce the current into each of the logic inputs. A 500 Ω resistor is sufficient to get the current provided by the standard 5 V computer signal down to the required 5-15 mV range.

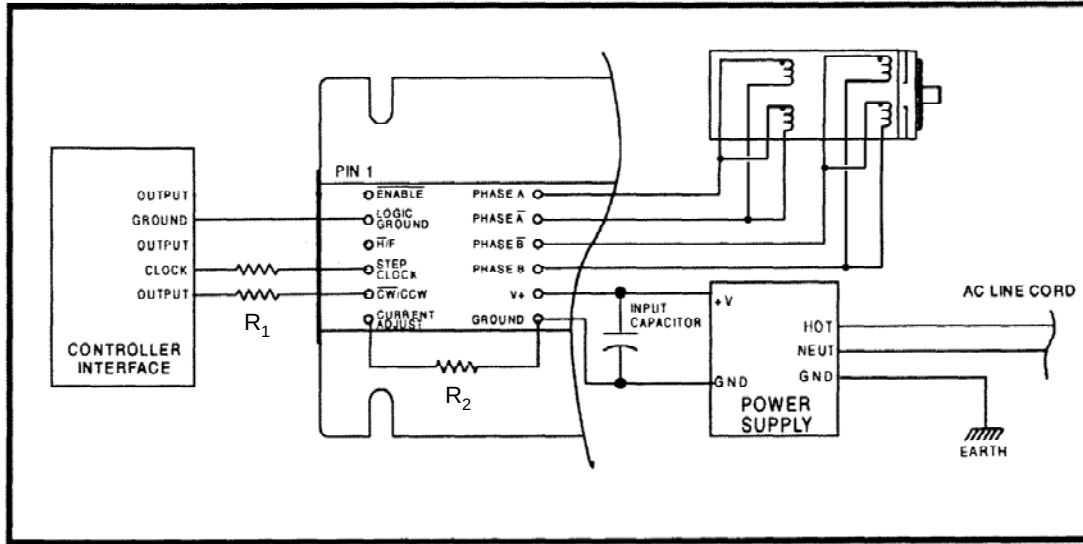


Figure 3.15. Circuit Diagram for the IB106 with the motor wired in bipolar parallel. Figure adapted from IMS documentation. Optional connections can be made to enable/disable the drive as well as to toggle full and half stepping. The default settings are *enabled* and *half* stepping respectively.

The prolonged use of the grid during testing cause the motors to produce significant heat and although the IB106 is rated up to 6A per phase, reducing the current alleviates some of the heat problems. This is done by connecting a current reducing resistor, $R_2 = 750 \Omega$, into the circuit as shown in Figure 3.15 in order to limit the current to 4.5 A. For shorter duration tests, this resistor can be omitted, resulting in increased grid performance. Four outside mounted thermistor type indoor/outdoor thermometers monitor the temperature of four strategically chosen motor casings and can be read from inside the control room. To provide additional cooling of the motors, a Pelonis type PM1751-7 seven-bladed fan (Figure 3.16) rated at 227 ft³/min and 2650 RPMs was installed in one end of the weather cover duct. The fan has mounting screws which lets it fit into the webbing of the outside tunnel catwalk right at the entrance of the weather cover, and can be manually switched on from the control room in order to keep the motor case temperature well below the critical temperature of 80°C.

One of the greatest advantages of the IB106 drive is the high input voltage range (24 to 80 VDC). Generally a higher input voltage yields better motor performance. As a result the power supplies were purchased from the same company to take advantage of this. The IMS IP806 unregulated linear power supplies shown in Figure 3.17 provide a nominal 80 V output voltage with no load. Ten of these power supplies were rack mounted, each providing power to a pair of IB 106 drives. When loading the power supplies, the output voltage dropped to approximately 70 V. As indicated in the circuit diagram (Figure 3.15) a 1200 μ F capacitor rated for 100 V is included between the power terminals to reduce the effect of back EMF. The 1200 μ F rating ensures that the entire current range up to 6 A can be used with this capacitor. The relatively high input voltage and output currents associated with the IB 106 modules create excessive heat

which the drivers are not able to dissipate fast enough despite their built in cooling fins. A standard table fan was found to be sufficient for providing convective cooling to keep the driver array below its maximum case temperature of 70°C.



Figure 3.16. Pelonis PM1751 7-bladed cooling fan.

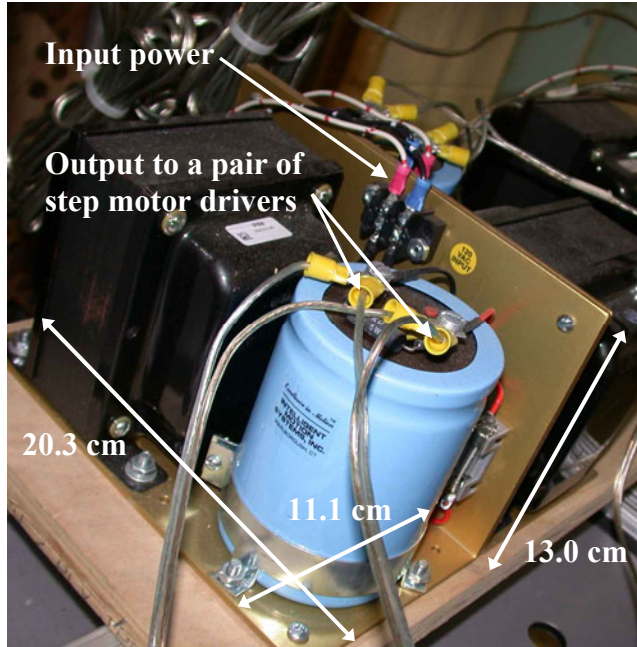


Figure 3.17. IMS IP806 80 V power supply.

3.3.2 Computer Interface

The step motor drivers are controlled through two coupled National Instruments (NI) PCI-6534 digital 32 channel I/O cards with 32Mb RAM and Direct Memory Access transfer from the motherboard memory of the Pentium II based system running Windows NT. At a rate of 140 kHz the cards can output the required pulses much faster than the inertia of the motors will allow for, and hence the speed of the grid is limited by the friction/load attached to the motors as well by the internal windings in the motors.

The two NI PCI cards supply 5V per channel and are connected to ten step motor drivers each through HD68 pin SCSI cables connected to a separate SCSI terminal strip (Figure 3.18) manufactured by Dynamic Engineering. Generic computer ribbon cables are used to interface the logic signals to the driver array.

3.3.3 Grid Control Software

With a C-based computer control program developed in house, it is possible to operate the ATG in synchronous mode as well as randomly with full control of the range of speeds, cruise times as well as the linear acceleration/deceleration. While synchronous operation, with its phase delay capability, has a lot of potential for generating deterministic periodic disturbances

into the flow (see for example Grissom & Devenport, 2004), it is a poor generator of turbulence.* A few tests were performed using a synchronous forcing scheme, but with any significant dynamic force applied, the grid would create large periodic gusts close to the resonance of the facility itself.

The random motion used almost exclusively throughout the measurements in this work is similar to the “double random” scheme adopted by Poorte (1998) in Figure 1.3d, but with a forced directional change introduced at every rotational speed change. In short, a grid-rod is accelerated from rest at a constant rate α to an angular velocity $\tilde{\Omega}$. The vanes then remain at this speed for a “cruise” time $\tilde{T}$ after which they decelerate at the same rate to zero. The rod then immediately reaccelerates in the reverse direction and performs a qualitatively identical maneuver with a new $\tilde{\Omega}$ and $\tilde{T}$. This is shown schematically in Figure 3.19. The cruise time and rotation rate are varied randomly between successive maneuvers according to a uniform probability density functions (pdf) defined by $T \pm t$ and $\Omega \pm \omega$ respectively. In other words, the capitalized letters describe the mean value of the pdfs while the lower case letters refer to the maximum deviation from the average in each direction with any value in the domain having the same associated probability. The motion of each rod is completely independent of the others.

The ATG was also used as a static grid by aligning all the agitator vanes with the flow. At flow-speeds larger than 15 m/s it was found to be necessary to lock the rods in place by supplying current through the motors, as even the slightest angle of attack would cause the entire grid to close shut resulting in a rapid and dramatic loss of free stream velocity.

3.4 Operational Envelope

An attempt was made to quantify the operational envelope in terms of where the grid would operate reliably for the duration of a measurement session (up to 10 minutes) without any grid axis failing[†] more than once. A myriad of different operating conditions were tested in the initial calibration of the active turbulence grid. A few parameters really do not play much into the grid operation such as the cruise time parameters T and, t . Additionally taking $\omega \leq \Omega/2$ the limitations due to the rotation rate can be measured solely in terms of the mean grid rotational rate Ω . The three major parameters that control the operational envelope are therefore: U_∞ , Ω , and α . Initial tests made it fairly clear that the acceleration of the system posed the greatest limitation, and that by keeping $\alpha \leq 20$ Hz/s the integrity of the system would not be compromised. It is possible to increase the acceleration beyond this, but as a result the other two parameters will be severely limited. With the flow off, the grid can reliably be pushed to around 15 Hz at $\alpha = 20$, and up to 20 to 30 Hz if α is sufficiently low. This is not very useful for the intended measurement, but creates, if nothing else, an unforgettable visual experience.

The lower end operation is also limited to 2 Hz as anything below this mean speed will create a lot of noise from the motors. If lower end operation is needed, the wiring can be switched from parallel to serial to alleviate some of these symptoms. As for the free stream velocity, this is the parameter that can potentially break the grid, rather than just make it fail and bind up due to motor limitations. It was found that at low grid speeds (2 Hz) the grid can withstand free stream velocities up to 20 m/s, although prolonged operation at such conditions will require some grid

* Refer to Section 1.6.2 for a survey of different forcing protocols.

[†] Failing is defined as one or more grid bars binding up during any maneuver regulated by the forcing protocol.

maintenance following the test due to the immense force which continuously bends the entire grid dramatically out of plane as the drag coefficient of the grid fluctuates. As the grid speed is increased the fluctuations of the drag coefficient become so rapid that it is necessary to reduce the total force on the grid. Although the grid was never tested until actual fracture, it is not recommended that the top speed exceeds the following simple relationship which at least applies for the domain $2 \text{ Hz} < \Omega < 10 \text{ Hz}$ with Ω in Hz and U_∞ in m/s:

$$U_\infty \leq -\frac{3}{2}\Omega + 23 \quad (3.5)$$

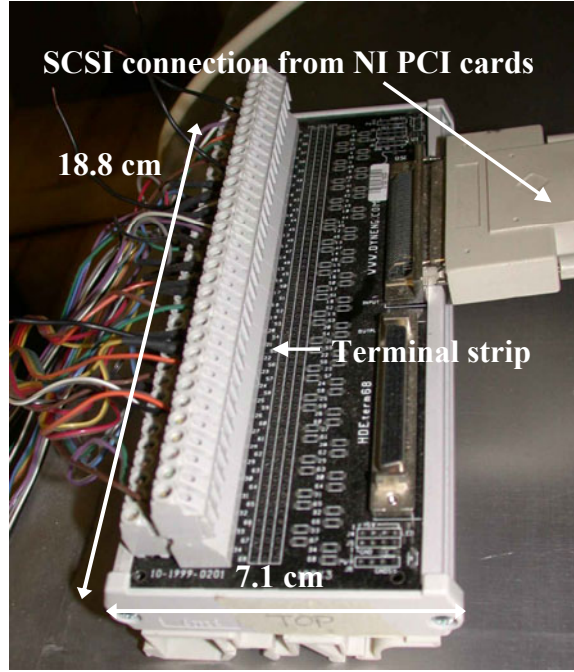


Figure 3.18. Dynamic Engineering HD68 SCSI terminal strip interface.

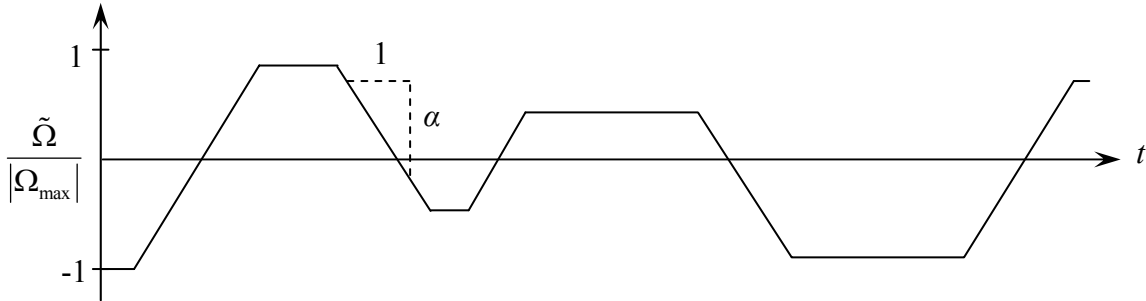


Figure 3.19. Modified double random forcing protocol used for the active grid. Acceleration rate has been greatly reduced for clarity.

Chapter 4

Active Grid-Generated Free-Stream Turbulence

A large portion of the current research has been dedicated to investigating the free stream turbulence produced by the active turbulence grid. After a brief introduction of the grid coordinate system in Section 4.1 and the non-dimensionalization of the different grid parameters in Sections 4.2 the mean flow quality of the turbulence in the Stability Wind Tunnel will be addressed in Section 4.3. The time averaged higher order statistics are discussed in Section 4.4. Section 4.5 deals with the initial period of decay of turbulence kinetic energy while Section 4.6 discusses some general aspects of the experimentally obtained velocity spectra and compares them with a model spectrum. Section 4.7 focuses in detail on the spectral inertial range and compares the current dataset to the detailed analysis presented in Mydlarski & Warhaft (1996). Some of the different methods of obtaining the viscous dissipation rate are discussed in Section 4.8 while Section 4.9 deals with the autocorrelation. Section 4.10 introduces the two-point correlation while Section 4.11 wraps up the chapter by attempting to map out the dependence of different turbulence properties on the active grid forcing parameters.

4.1 Grid Coordinate System

The global grid coordinate system (shown schematically in Figure 4.1 and Figure 4.2) is expressed in capitalized coordinates, where the origin lies in the center of the grid, midway between the streamwise planes defined by the two sets of rods. X describes the downstream distance from the grid, Y is positive down, and Z completes the right-handed system in the lateral direction. For the cascade measurements a local streamwise and laterally translated coordinate system is used in conjunction with the global system. These cascade coordinates are configuration specific and explained in more detail in Section 5.3.

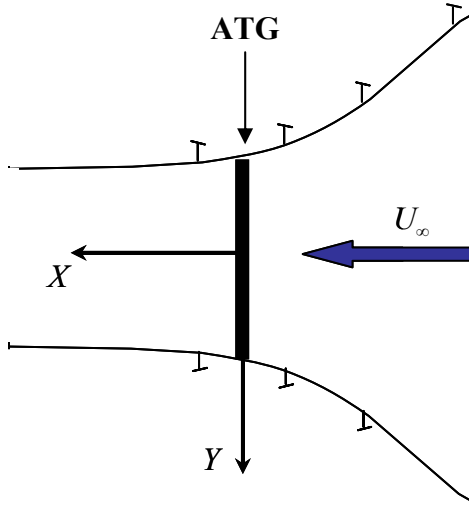


Figure 4.1. Global grid coordinate system, side view.

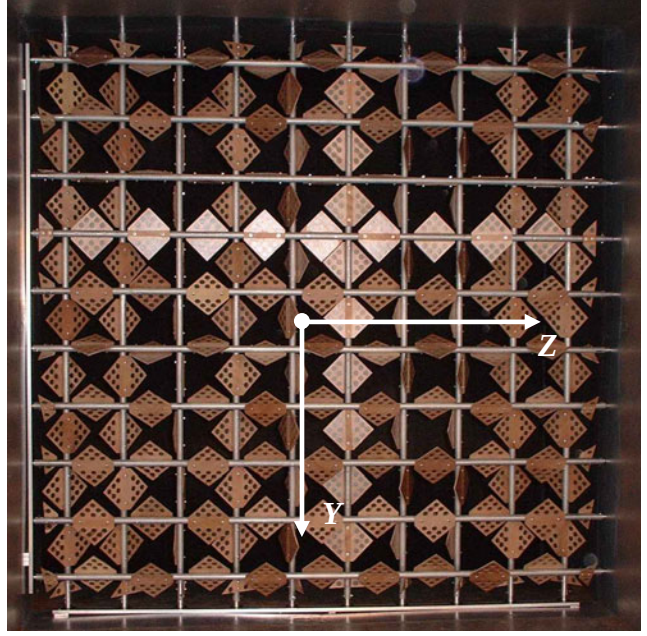


Figure 4.2. Global grid coordinate system; downstream view.

4.2 Non-Dimensional Grid Parameters

The fundamental parameters defining the grid operation were introduced in Section 3.3.3. There are five major non-dimensional parameters one can define that could control the properties of the grid turbulence: 1) the normalized distance from the grid 2) the grid Reynolds number, based on free stream velocity and grid mesh length, 3) the vane Rossby number,^{*} 4) the average number of vane revolutions in each maneuver, 5) the normalized maximum deviations in cruise time and 6) the normalized maximum deviations in rotation rate. Symbolically these are

$$\frac{X}{M}, \quad \frac{MU_\infty}{\nu}, \quad \frac{U_\infty}{M\Omega}, \quad T\Omega, \quad \frac{t}{T}, \quad \frac{\omega}{\Omega} \quad (4.1)$$

The effect each of these parameters has on the turbulent flow the active grid produces will be discussed in detail in Section 4.11. Not included in the above non-dimensional parameters is the acceleration rate α . Some limited study of the effects of acceleration rate was made, but for the majority of the test cases this rate was kept constant at 20 Hz/s due to mechanical considerations.

^{*} The Rossby number is the ratio of inertial forces to rotational forces, usually applied to the Coriolis force in the atmosphere. It has been extended here to obtain a non-dimensional grid rotation rate.

4.2.1 Baseline Flow Condition

One particular grid generated flow was selected as the baseline case for all measurements. This flow was chosen based on initial results and the physical limitations of the grid. The major parameters of the grid operation, namely flow velocity and grid rotation rate, were set to nominal values of 12.5 m/s and 4 Hz respectively. The rest of the parameters were chosen somewhat arbitrarily which gave the following non dimensional grid parameters for the baseline flow:

$$\frac{X}{M} = 37.3, \quad \frac{MU_{\infty}}{\nu} = 1.71 \times 10^5, \quad \frac{U_{\infty}}{M\Omega} = 14.79, \quad T\Omega = 8.00, \quad \frac{t}{T} = \frac{\omega}{\Omega} = 0.5 \quad (4.2)$$

The baseline flow was also conveniently chosen as the onset undisturbed turbulent flow for the cascade experiments described in Chapter 6.

4.2.2 Alternate Flow Conditions

As mentioned above a myriad of different flow conditions were tested in the initial calibration phase of the active turbulence grid. 39 different test conditions, including their non-dimensional grid parameters and most relevant turbulence properties which will be discussed throughout this chapter, are listed in Table 4.1 for quick reference. The baseline condition described above corresponds to test case number 21.

Chapter 4: Active Grid-Generated Free-Stream Turbulence

Test Case	x/M	U_∞ (m/s)	Ω (Hz)	$\frac{U_\infty}{M\Omega}$	$\frac{\omega}{\Omega}$	$T\Omega$	$\frac{t}{T}$	α (Hz/s)	u/U_∞ (%)	L_{11} (m)	Re_λ	u/v
1	37.3	7.6	0.0	∞	-	-	-	20	2.1	0.07	101	
2	37.3	12.4	0.0	∞	-	-	-	20	2.0	0.06	124	
3	37.3	15.0	0.0	∞	-	-	-	20	2.0	0.05	137	
4	37.3	21.9	0.0	∞	-	-	-	20	1.9	0.07	171	
5	37.3	8.6	2.0	20.5	0.5	4.0	0.5	20	7.2	0.51	599	
6	37.3	12.3	2.0	29.3	0.5	4.0	0.5	20	8.1	0.40	771	
7	37.3	15.1	2.0	35.9	0.5	4.0	0.5	20	8.1	0.52	919	1.07
8	37.3	19.2	2.0	45.8	0.5	4.0	0.5	20	8.4	0.56	1053	
9	37.3	8.2	4.0	9.7	0.5	4.0	0.5	20	6.3	0.26	440	
10	37.3	10.7	4.0	12.7	0.5	4.0	0.5	20	6.8	0.31	565	
11	37.3	12.7	4.0	15.1	0.5	4.0	0.5	20	7.0	0.35	664	1.00
12	37.3	14.8	4.0	17.6	0.5	4.0	0.5	20	7.5	0.34	722	1.01
13	37.3	12.3	4.0	14.7	0.5	12.0	0.2	20	6.9	0.30	560	
14	37.3	12.3	4.0	14.6	0.5	4.0	1.0	20	7.1	0.39	624	
15	37.3	12.4	4.0	14.8	0.0	4.0	0.5	20	7.1	0.25	585	
16	37.3	12.6	4.0	15.0	0.5	4.0	0.5	10	7.3	0.31	612	
17	37.3	12.6	4.0	15.0	0.5	0.0	-	20	8.1	0.43	754	
18	37.3	12.4	4.0	14.7	0.3	4.0	0.5	20	7.3	0.31	611	
19	37.3	12.5	4.0	14.9	0.5	4.0	0.5	5	7.7	0.33	669	
20	37.3	12.6	4.0	15.0	0.5	12.0	0.5	20	7.1	0.36	574	
21	37.3	12.5	4.0	14.9	0.5	8.0	0.5	20	7.2	0.34	599	1.01
22	37.3	12.5	4.0	14.9	0.5	8.0	1.0	20	7.0	0.38	611	
23	37.3	12.5	4.0	14.9	0.8	8.0	0.3	20	7.1	0.41	637	
24	37.3	8.1	6.0	6.5	0.5	4.0	0.5	20	5.8	0.27	433	
25	37.3	12.2	6.0	9.7	0.5	4.0	0.5	20	6.7	0.28	565	1.02
26	37.3	8.3	8.0	4.9	0.5	4.0	0.5	20	5.9	0.26	421	
27	37.3	8.0	8.0	4.8	0.5	5.3	0.4	20	5.2	0.25	377	
28	37.3	8.5	8.0	5.0	0.5	0.0	-	20	6.5	0.30	452	
29	37.3	10.7	8.0	6.4	0.5	4.0	0.5	20	6.2	0.31	533	
30	37.3	8.2	10.0	3.9	0.2	5.0	0.5	20	5.7	0.25	429	
31	47.5	12.4	4.0	14.8	0.5	4.0	0.5	20	6.3	0.39	589	
32	47.5	15.3	4.0	18.2	0.5	4.0	0.5	20	6.7	0.34	715	
33	41	12.2	4.0	14.5	0.5	4.0	0.5	20	6.9	0.39	621	
34	41	15.5	4.0	18.4	0.5	4.0	0.5	20	7.1	0.42	725	
35	29.3	12.4	4.0	14.8	0.5	4.0	0.5	20	8.2	0.36	629	
36	29.3	15.2	4.0	18.1	0.5	4.0	0.5	20	8.8	0.43	798	
37	21.3	12.5	4.0	14.9	0.5	4.0	0.5	20	9.9	0.37	715	
38	21.3	15.6	4.0	18.5	0.5	4.0	0.5	20	10.5	0.37	900	
39	21.3	20.2	2.0	48.0	0.5	4.0	0.5	20	12.0	0.67	1362	

Table 4.1. The 39 different test cases with corresponding grid operating parameters and relevant turbulence parameters. Experiments performed in air with a kinematic viscosity of nominally $1.53 \times 10^{-5} \text{ m}^2/\text{s}$. Grid mesh spacing $M = 0.21 \text{ m}$. The baseline condition (test case 21) has been highlighted.

4.3 Flow Quality

4.3.1 Wind Tunnel Boundary Layer Predictions

In order to estimate the size of the boundary layers forming on the wind tunnel walls a simple numerical scheme was utilized. By assuming a linear contraction from the grid to the beginning of the wind tunnel section, the velocity profile throughout the entire test section could be predicted.* The turbulent mean flow integral method by Moses was used for the entire sectional length, with the exception of the initial section which was computed using the Thwaites-Walz momentum integral method starting at the grid in order to provide initial conditions for the turbulent integral method. Refer to Schetz (1993) for an in-depth description of these two methods. The assumed streamwise velocity variation together with the calculated streamwise variation of the boundary layer (δ), displacement (δ^*) and momentum (θ) thicknesses for all distances from the grid to the end of the test section are shown in Figure 4.3 for a nominal free stream velocity of 12.5 m/s which corresponds to the baseline condition. Although it can be seen from this figure that the boundary layers are quite thick at the end of the test section (approximately 13 cm), the displacement thickness is much smaller and will only cause a 2% total increase in free stream velocity between the entrance and the end of the constant test section.

This method does not take into account the free stream turbulence levels and should only be regarded as a rough estimate of the boundary layers present in the wind tunnel as they are really only a function of the approximate wind tunnel sectional shape, the viscosity of the air and the free stream velocity. This is obviously an oversimplification of the problem, but is more than sufficient as a guideline for what to expect. For the baseline condition at $X/M = 37.3$ ($U_\infty = 12.5$ m/s), the boundary layer thickness of the tunnel sections were calculated to be 10.1 cm. How this corresponds to the measurements will be discussed below.

* Since only an initial estimate was required, the velocity profile was not iterated upon using displacement thickness results from earlier iterations.

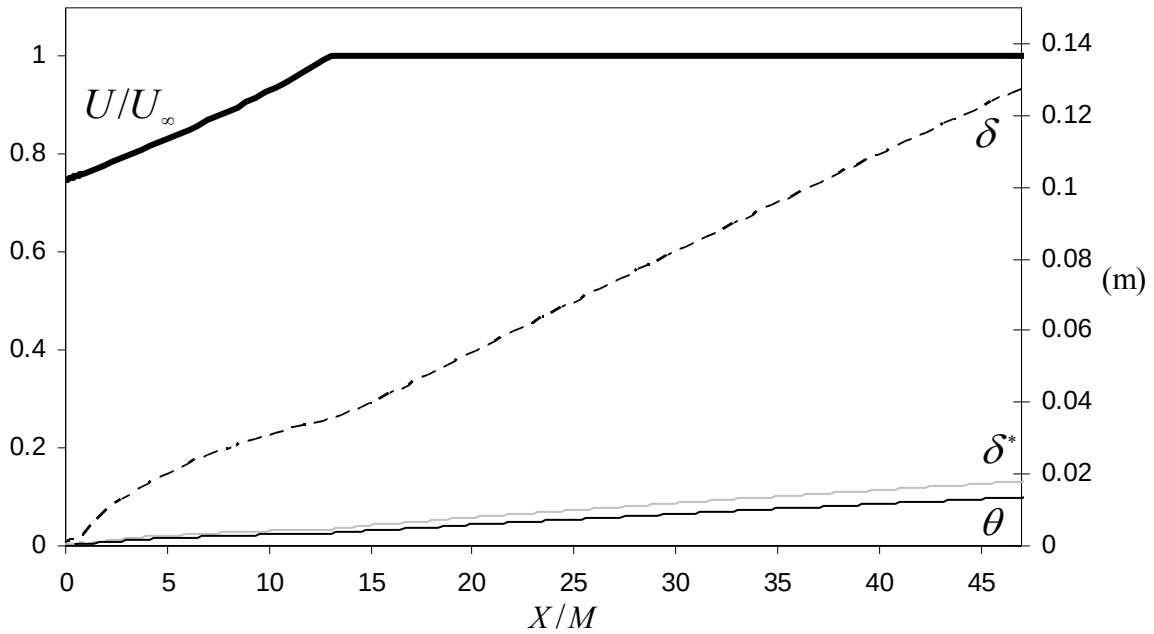


Figure 4.3. Estimated velocity profile through test section and calculated boundary layer thicknesses.

4.3.2 Uniformity and Boundary Layer Measurements

The uniformity of the mean flow was mapped out by traversing a Pitot-static probe in each direction from the center of the cross section at $X/M = 37.3$ for four different grid conditions.* The result can be seen in Figure 4.4 where U/U_∞ has been plotted as a function of Z/H ($Y/H = 0$) where H is the wind tunnel sectional dimension (1.85 m). Similarly, Figure 4.5 shows the vertical uniformity at $Z/H = 0$ for two of these conditions. In both figures the measurements extend out to 7.6 cm from each wall. Overall, the flow field looks uniform with the exception of the boundary-layers close to the wall. There is also a very slight jet-effect centered on $Z/H = \pm 0.34$. This is believed to be caused by the air escaping through the gaps between the grid and the wall. The problem is a little more pronounced on the positive Z side and negative Y side where the grid does not have a bearing strip and thus there is less flow blockage. The uniformity test was also repeated for the central horizontal tunnel core section at mid-height for the baseline condition (also at $X/M = 37.3$) with the four-sensor hot wire, which confirmed the experimental Pitot-static probe results.

* The conditions, at which the uniformity study was conducted, are listed in Table 4.1 where the applicable test cases are numbered 5, 7, 10 and 12.

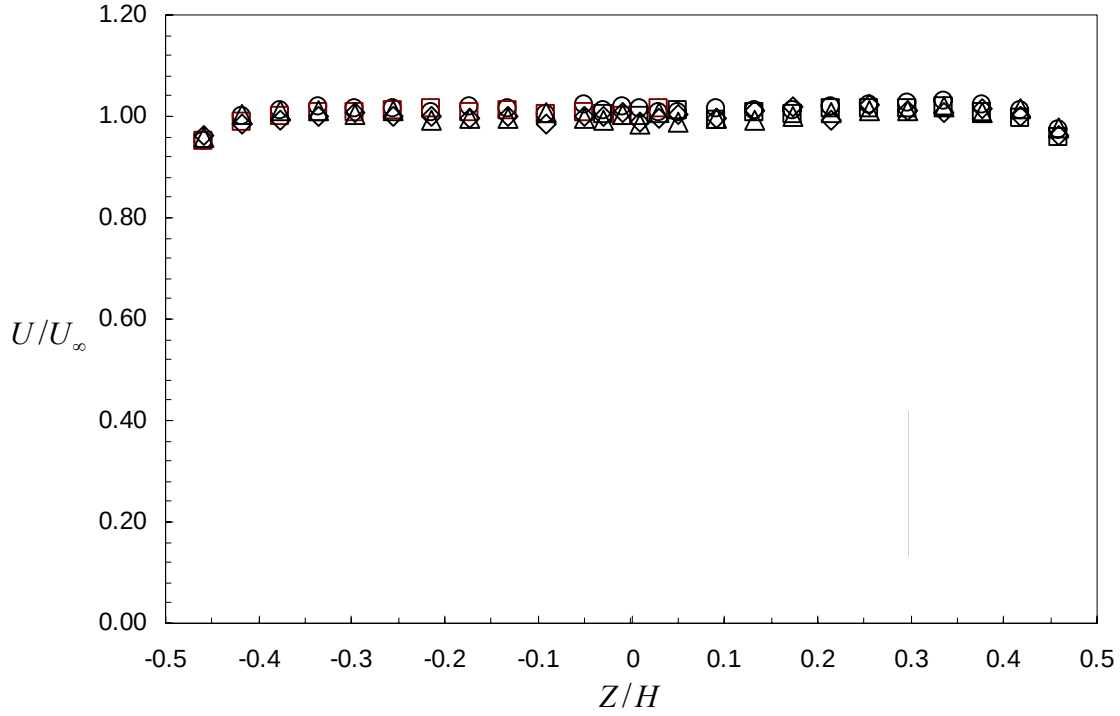


Figure 4.4. Horizontal mean velocity uniformity at $X/M = 37.3$ and $Y/H = 0$. Grid conditions: $\circ$, $\Omega = 2$ Hz, $U_\infty \approx 10$ m/s; $\square$, $\Omega = 2$ Hz, $U_\infty \approx 15$ m/s; $\triangle$, $\Omega = 4$ Hz, $U_\infty \approx 10$ m/s; $\diamond$, $\Omega = 4$ Hz, $U_\infty \approx 15$ m/s.

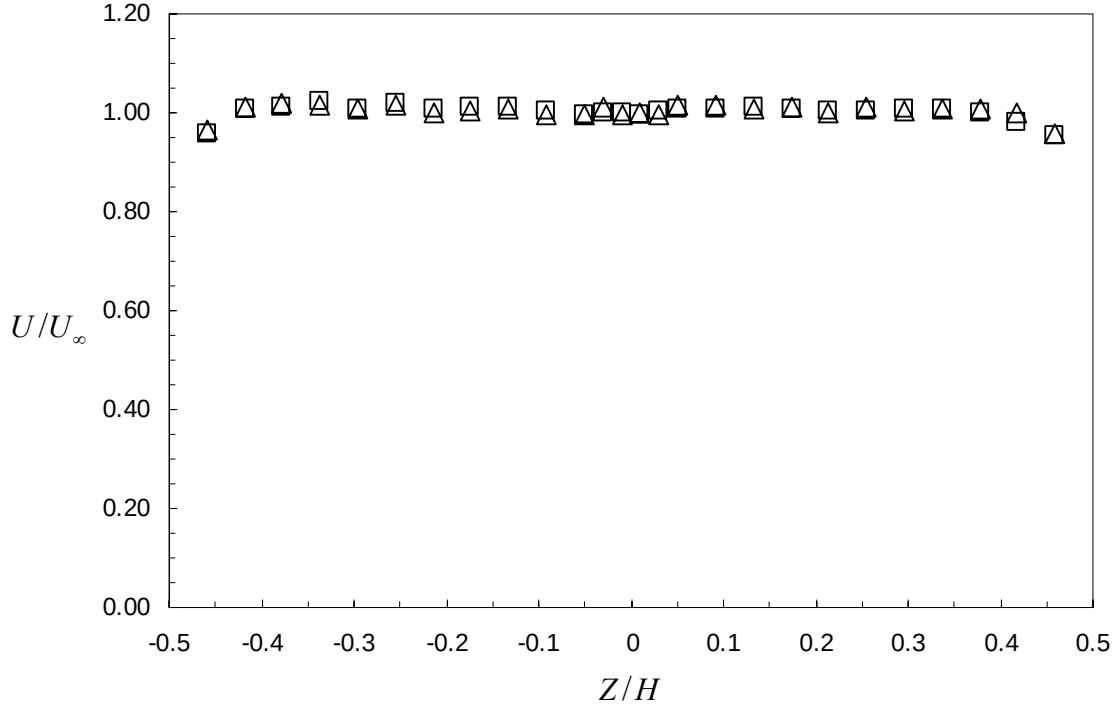


Figure 4.5. Vertical mean velocity uniformity at $X/M = 37.3$ and $Z/H = 0$. Grid conditions: $\square$, $\Omega = 2$ Hz, $U_\infty \approx 15$ m/s; $\triangle$, $\Omega = 4$ Hz, $U_\infty \approx 10$ m/s.

Going back to the boundary layer calculations of the previous section, it would be desirable to see how closely the measurements and the predicted boundary layer calculations correspond. Assuming the common velocity profile shape (see Schetz, 1993, p. 240) for turbulent flow over a flat plate as given by Equation 4.3, it is possible to calculate the thickness of the boundary layer, δ , based on the measured velocity ratio and the position Y from the wall.

$$\frac{U}{U_{\infty}} = \left(\frac{Y}{\delta} \right)^{1/7} \quad (4.3)$$

From Figure 4.4 and Figure 4.5 the average velocity ratio, at a distance 7.6 cm from the wall (the outermost points in each profile), is $U/U_{\infty} = 0.960$. Through Equation 4.3 this yields a boundary layer thickness of 10.11 cm which is remarkably close to the numerical estimate of 10.1 cm.

4.3.3 Homogeneity and Isotropy

The homogeneity and anisotropy ratios of the test section were mapped out at $X/M = 37.3$ for test case numbers 11 and 12 (see Table 4.1) by traversing a four sensor hot wire in the test section. The three-component velocity fluctuations as a function of the Z -coordinate ($Y/H = 0$) for case 11* ($\Omega = 4$ Hz and $U_{\infty} = 12.42$ m/s) can be seen in Figure 4.6. A few conclusions can immediately be drawn: The turbulent flow field is closely homogeneous and isotropic within 38 cm of the centerline with turbulence intensity values close to 7% which is quite a bit lower than that of previous efforts, but expected due to the RDT predictions in Figure 3.2.

Close to the wind tunnel walls the w -component fluctuations reduce in value while similar profiles in the Y -direction show a similar fall-off in v -component fluctuations. This behavior is an inevitable result of the large integral scale in the flow ($L_{11} = 0.39$ m) and the imposition of the non-penetration condition on the wind tunnel walls. Based on the RDT cascade theory of Graham (1998), which will be discussed in Chapter 7, this fall-off should occur over a distance equivalent to roughly two integral scales from the wall ($Z/H < -0.19$ and $Z/H > 0.19$).[†] This is consistent with what is seen here.

The fact that the active turbulence grid was placed in the contraction seems to have alleviated most of the problems with anisotropy experienced in earlier studies, confirming that Comte-Bellot & Corrsin's (1966) method of placing the grid in the contraction is effective also with this type of grid. Maximum anisotropy ratios u/v and u/w for the inner regions in each direction ($-0.19 > Z/H > 0.19$ in Figure 4.6) lie between 1.0 and 1.02 for the measured cases.

* This case is very similar to the baseline condition but with some variation in the normalized time variation.

[†] This estimate is obtained by treating the wind tunnel walls as part of an infinite cascade and a blade spacing of $4.8L_{11}$.

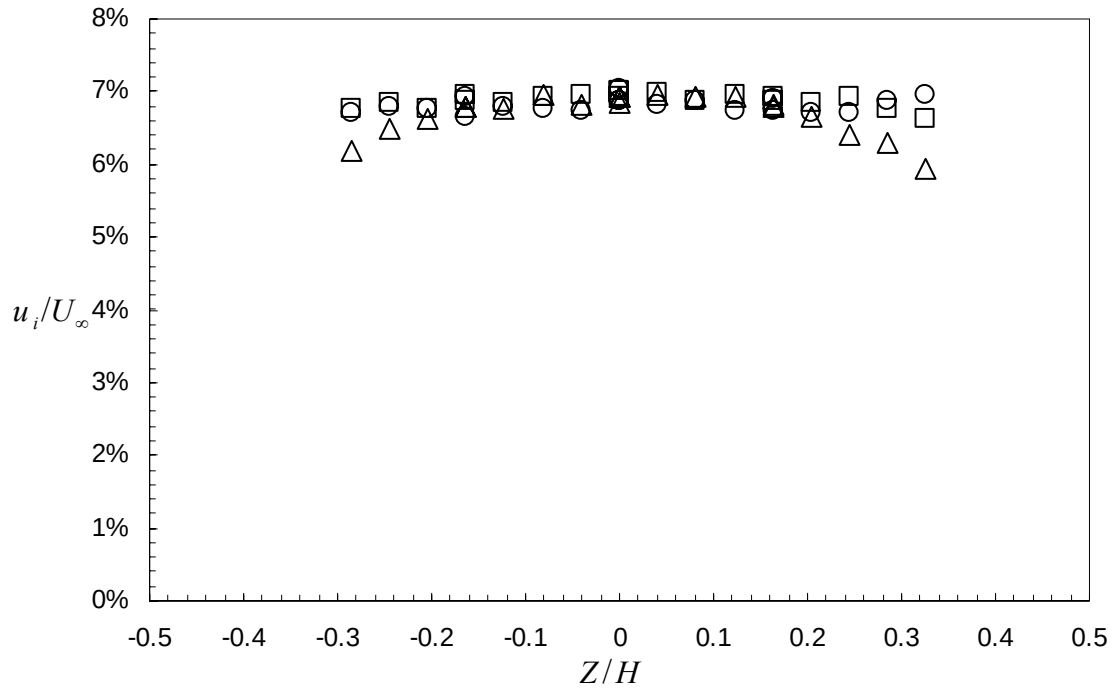


Figure 4.6. Turbulence intensities at $X/M = 37.3$ and $Y/H = 0$ for the baseline condition. $\circ$, u ; $\square$, v ; $\triangle$, w .

4.3.4 The Anisotropy Ratio

Section 1.6.3 pointed out the lack of evidence of a “return to isotropy” downstream of the previous active grid efforts (Kang *et al.*, 2003 and Poorte, 1998) which is contrary to most conventional grid experiments. Although Makita (1991) recorded a drop in the anisotropy ratio from 1.8 to 1.22 between $X/M = 10$ and $X/M = 50$, one must assume that the first streamwise station was not yet part of the homogeneous initial period of decay described in more detail in Section 4.5, in which case the anisotropy ratio does not convey much useful information.. As discussed in Chapter 1, much of the variation between different active grids lies in the forcing protocols and the vane configuration of the grid. However, given the large lengthscales associated with active grid flows, the ratio H/L_{11} will also play a significant role. It turns out that $H/L_{11} \approx 3.6$ for Makita (1991), and Kang *et al.* (2003) which makes it easier to compare these grids in terms of forcing protocols (since they are all geometrically configured the same). Mydlarski & Warhaft (1996) with their small tunnel has an associated H/L_{11} ratio of only 2.7 at $X/M = 68$.

This also implies however, that there exists somewhat of a limit as to how large the emitted integral lengthscales can be for an active grid, and that this limit is controlled by the size of the tunnel, rather than the mesh length as is common with static grids. In addition this enforces a restriction on how large the scales can grow downstream without negatively affecting the anisotropy ratio. Although Kang *et al.*’s (2003) furthest downstream position has $H/L_{11} \approx 2.7^*$ it

* This is using the smaller dimension of the test facility. By using the average of the two, the ratio would be $L_{11}/H = 3.2$

is clear that the anisotropy ratio at this point is quickly departing from the initial value of 1.13 for $H/L_{11} \approx 3.6$. Poorte's (1998) does not state the size of the integral scales obtained, but with the different vane configuration it would be hard to assess the independent contribution of the two effects.

The present data has a ratio of about $H/L_{11} = 4.8$ for the baseline condition at $X/M = 37.3$ (note that ratios of about $H/L_{11} \approx 3.0$ were obtained for certain conditions (see Table 4.1 test case 7 for example) which is significantly higher than the previous efforts. According to the theory of Graham (1998), mentioned in the previous section, $H/L_{11} = 4.8$ will induce an anisotropy ratio of $u/v = u/w = 1.015$ in the middle of the wind tunnel section. The induced anisotropy ratio for $H/L_{11} \approx 3.6$ is 1.024. Moreover, regular static grids have typically anisotropy ratios around 1.1 whereas most static grids have very small integral scales compared to the wind tunnel sectional size. For example, for $H/L_{11} = 10$ the anisotropy ratio has reached unity (within five significant digits). This suggests that tunnel confinement contributes to almost 25% of the difference between the static and active grids, and much more so when the forcing protocol is randomized properly as done by Poorte (1998) and Kang *et al.* (2003). The relative contribution will also grow much larger as the H/L_{11} is further reduced, which inherently happens as the integral lengthscales grow downstream of the grid.

For the current data the anisotropy ratio grows slightly from 1.00 to 1.02 between $X/M = 25$ to $X/M = 47$. Since the lengthscales haven't reached the same relative size compared to the tunnel, in contrast to for example the Kang *et al.* (2003) dataset, the downstream change in isotropy is not quite as severe in comparison. It can then be concluded, that the combined effects of the grid geometry, forcing protocols, and resulting H/L_{11} on the anisotropy ratio for the baseline condition were indeed cancelled out by placing the active grid in the contraction where $c = A_1/A_2 = 1.36$.

4.4 Time Series Statistics

The following subsection deals with the probability density functions of the streamwise and lateral velocity fluctuations as well as the corresponding velocity derivatives. Also, with the inherent large lengthscales associated with the resulting turbulent flow, the issue of sample independence, especially for the higher order statistics involved, is also investigated. A discussion of filtered higher order statistics parameters concludes this section.

4.4.1 Probability Density Functions

Velocity fluctuations

It has been known for a long time (see Batchelor, 1953, pp 169-170) that the three components of velocity in grid turbulence are distributed very closely to a normal (Gaussian) probability density function. While the second order statistic, or variance, describes the absolute width of such a distribution, the higher order statistics describe the shape of the normalized probability density function. The third-order statistic, as given in Equation 4.4 is called the

skewness and reveals whether there tends to be a bias to either side of the mean. A positive skewness, perhaps contrary to intuition, indicates a bias towards the left of the mean. The fourth-order statistic, commonly known as the kurtosis or flatness factor, is given in Equation 4.5. This parameter indicates whether the distribution is flat and wide, or tall and narrow when normalized on the variance. A lower value for the kurtosis indicates a flatter, or less peaked, shape. A standard normal probability distribution with a given variance will have a skewness value of zero and a kurtosis of 3.0. Already in the mid-thirties Simmons & Salter (1934) established that the fluctuating velocity originating from a decaying turbulent flow behind a grid exhibits a nearly Gaussian distributed probability density function.

$$S(u'_i) = \frac{\overline{u'^3_i}}{\overline{u'^2_i}^{3/2}} \quad (4.4)$$

$$K(u'_i) = \frac{\overline{u'^4_i}}{\overline{u'^2_i}^2} \quad (4.5)$$

In true isotropic turbulence the skewness, $S(u_i)$ should be zero. Even if grid turbulence were isotropic in planes perpendicular to the streamwise direction, there would still be the issue of decay which makes it impossible for grid turbulence to be completely isotropic. It is well known that decaying grid turbulence has a slight positive skewness* associated with it in the streamwise direction which generally decreases with downstream distance from the grid (e.g. Mohamed & LaRue, 1990). This, coupled with the evidence of a “return to isotropy” with streamwise distance and the inherent statistical properties associated with the skewness yielding a value of zero for isotropic turbulence, leads to the somewhat ambiguous notion that the skewness is a measure of the anisotropy of a given turbulent flow (see for example Poorte, 1997). Although technically correct, this explanation can be slightly misleading since isotropy (or lack thereof) can be quantified in many ways. When using the term “isotropic” in connection with grid generated turbulent flow it is not assumed that there is no decay involved. The inherent streamwise inhomogeneity dictates that isotropy can only occur in planes perpendicular to the mean flow or at least limited to small streamwise distances where the decay of turbulence kinetic energy can be considered negligible.

Maxey (1987) reviewed several publications which suggest that skewness can be induced due to non-linear response of hot-wires as well as filtering of the signal, but even after applying necessary correction, significant values of skewness remain. This author showed that the positive velocity skewness is attributed to a turbulent flux of turbulence kinetic energy in the downstream direction and that the value of the skewness should be on the order of the turbulence intensity (generally a few percent) and hence will diminish with downstream distance, just as seen through experiments. This relationship between the skewness and the turbulent flux of energy will become important when obtaining dissipation estimates for the turbulent flow. This will be discussed in more detail in Section 4.8.1.

Poorte (1998) arrived at a similar conclusion, but emphasized that the skewness is generated at the grid in the initial inhomogeneous section where the amount of mean velocity and Reynolds

* The skewness is positive for time series analysis, but will become negative if Taylor’s hypothesis is invoked.

stress shearing is large. This shearing can be shown to be a function of solidity and mesh length and is obviously much larger for an active grid than for a passive grid, which in turn induces a larger skewness. Makita (1991) measured a skewness value of 0.19 which is the same as documented by Mydlarski & Warhaft (1996) for active grids while the skewness associated with static grids are generally an order of magnitude smaller.

Although the skewness and subsequent odd-order moments are affected in grid generated turbulence, Maxey (1987) also demonstrates how the even-ordered moments such as the kurtosis is relatively unaffected by the presence of turbulent energy flux, and that the kurtosis should therefore be very close to that expected of a normal distribution ($K(u'_i) = 3.0$).

The probability density functions for the streamwise and lateral velocity fluctuations for the baseline case are shown in Figure 4.7 where the abscissa has been normalized by the standard deviation (σ) which in this case is equivalent to the corresponding rms velocity. The current results differ significantly from previous experimental evidence. First of all the magnitude of $S(u') \approx 0.12$ at $X/M = 37.3$ in Figure 4.7a is larger than conventional static grids, but smaller than most previous active grids. Given that the anisotropy ratio is so close to unity for this flow, and with the skewness being associated to isotropy, one might expect that $S(u')$ for this flow to be significantly closer to zero. However, following the argument of Poorte (1998) that the skewness is generated upstream at the grid, we can conclude that even though the contraction reduces the anisotropy ratio, the effect on the skewness is much smaller and hence there is still significant skewness observed in the test section.

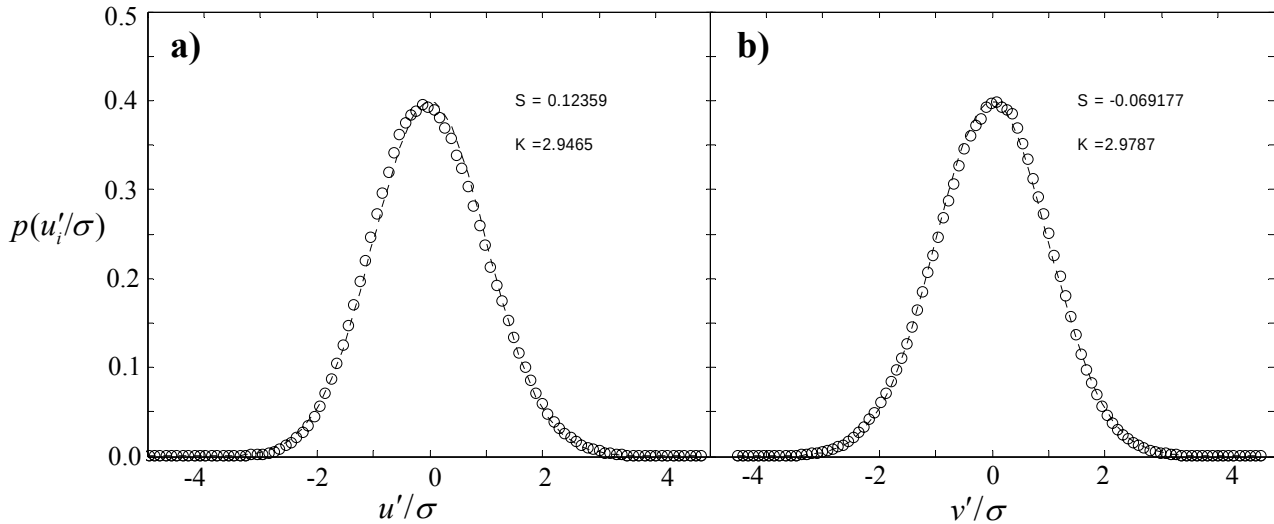


Figure 4.7. Probability density functions for a) the streamwise velocity component u' and b) the lateral velocity component v' . The standard Gaussian distribution is indicated by the dashed lines.

The kurtosis $K(u')$ is consistent with previous experiments, with the value falling between 2.9 and 3.0 as reported by Batchelor (1953). This value will remain constant throughout the streamwise measurement domain as expected. Experimental support for this is given in Figure 4.9.

The lateral (v') velocity distribution is shown in Figure 4.7b for $X/M = 37.3$. With a measured skewness of -0.07 and kurtosis of 2.98, this distribution shows a closer fit to a Gaussian curve as expected compared to the streamwise component. The w' -distribution displays very similar

attributes, but with a positive skewness of comparable value. Another, perhaps more illustrative way of displaying the probability density functions are to plot them on semi-logarithmic axes as shown in Figure 4.8. This gives a much better picture of the tails of the distribution. Whereas the standard linear axes show the inherent skewness as a tilt around the mean, this figure makes the skewness manifest itself by expanding the low-probability regions far from the mean.

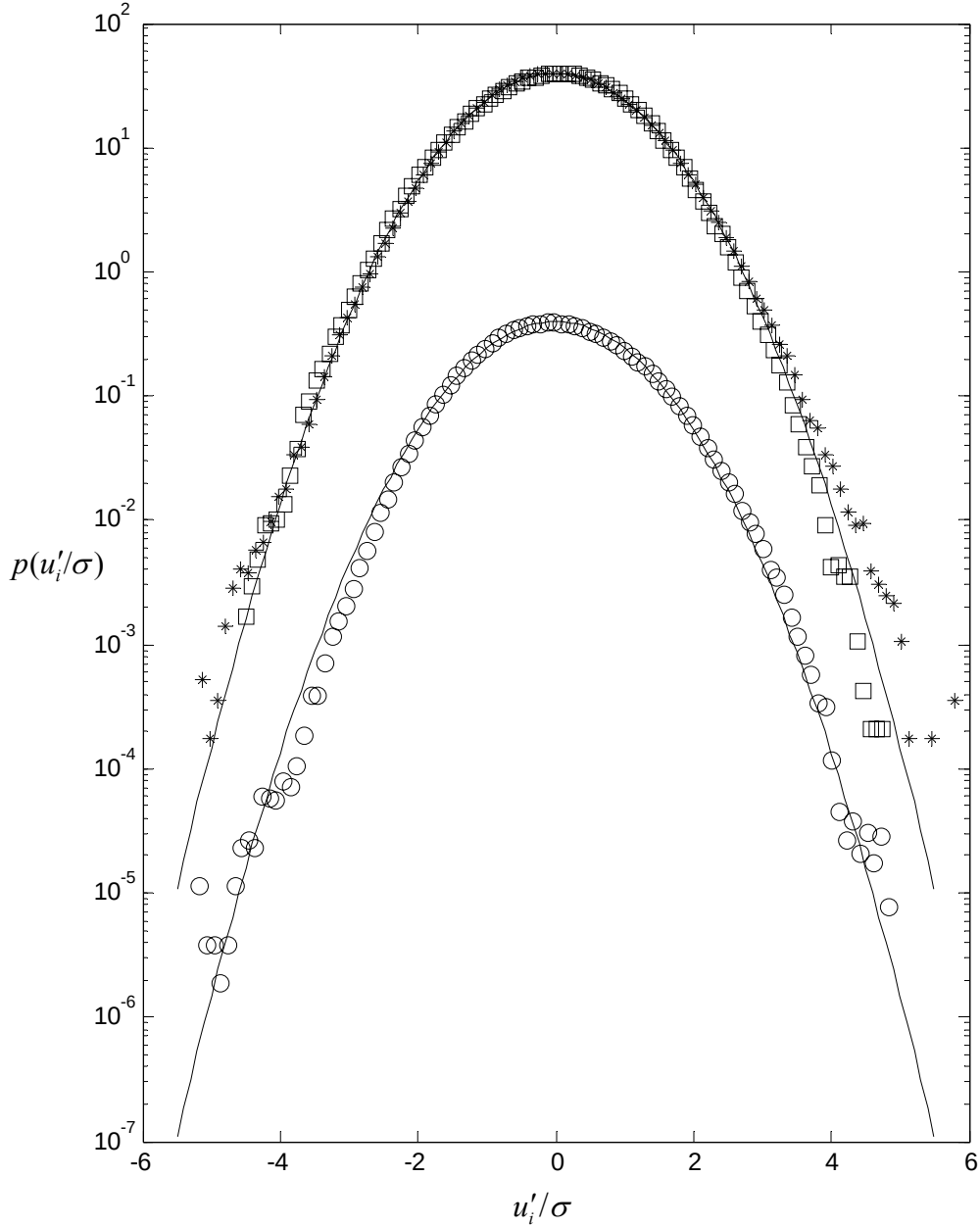


Figure 4.8. Probability density functions for the velocity fluctuations: $\circ$, u' ; $\square$, v' ; $*$, w' . The standard Gaussian distribution is indicated by the solid lines. The streamwise distribution has been shifted up two decades.

As opposed to conventional static grids, Figure 4.9 shows that the skewness remains constant over the entire streamwise measurement range for the baseline condition. Previous experiments

have consistently reported a diminishing skewness value with downstream distance, which is completely absent in the current measurements. This goes hand in hand with the lack of a “return to isotropy” since the anisotropy ratio is already very close to unity at the entrance of the test section. Poorte (1998) did not measure a “return to isotropy” either, but did see the values of skewness diminish with downstream distance, although it must be kept in mind that the initial anisotropy ratios associated with that flow were significantly larger than one.

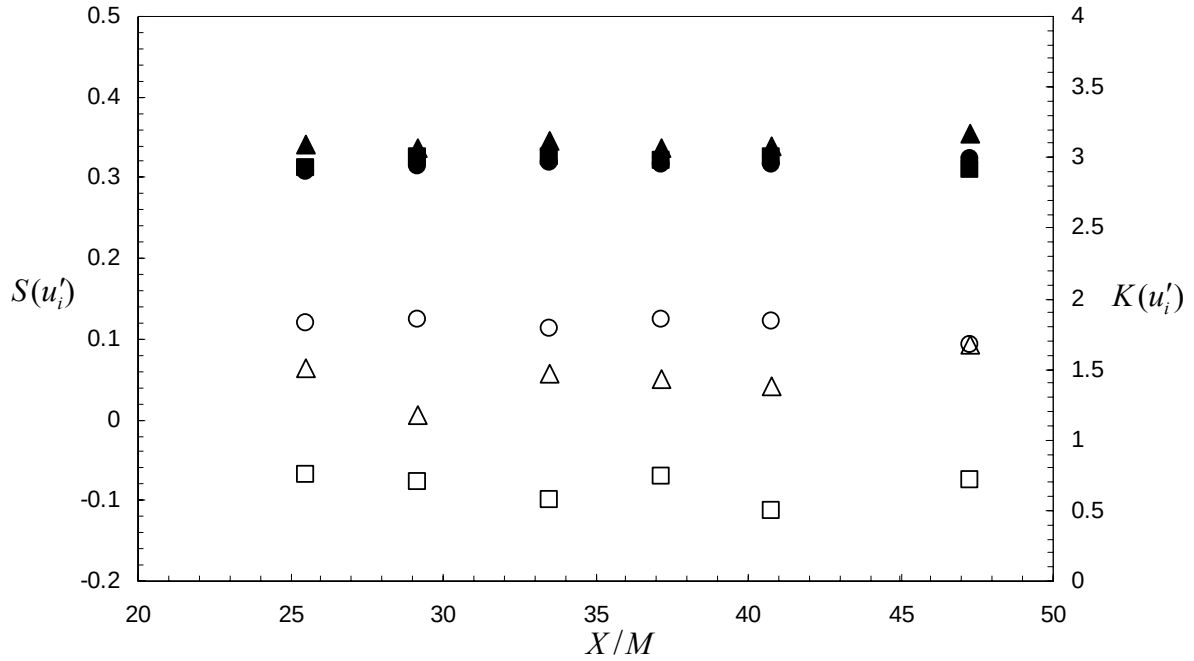


Figure 4.9. Streamwise variation of velocity fluctuation skewness and kurtosis. $\circ$, $S(u')$; $\square$, $S(v')$; $\triangle$, $S(w')$; $\bullet$, $K(u')$; $\blacksquare$, $K(v')$; $\blacktriangle$, $K(w')$.

Velocity derivative fluctuations

From the time-series, the derivative of the turbulent fluctuations with respect to time can be obtained by simple numerical differentiation, according to Equation 4.6, making sure each record is handled independently due to the fact that two consecutive records are not continuous:

$$\frac{du'_i}{dt} = \frac{d\tilde{u}_i}{dt} \approx \frac{\tilde{u}_i(t) - \tilde{u}_i(t + \Delta t)}{\Delta t} \quad (4.6)$$

In most cases the spatial derivative is a lot more useful than the time derivative. Taylor’s hypothesis according to Equation 4.7 can be used when converting the derivative in the following manner, although in the following treatment, the temporal derivative in Equation 4.6 will be used.

$$\frac{du'_i}{dx} = \frac{d\tilde{u}_i}{dx} \approx -\frac{1}{U_\infty} \frac{d\tilde{u}_i}{dt} \quad (4.7)$$

Figure 4.10 shows the streamwise and transverse probability density function for the velocity derivatives, again at $X/M = 37.3$. The general shape of the curve is quite different than that of the velocity fluctuations as the intermittent nature of turbulence comes across with a high peak at the mean and longer tails, which simply states that the gradients on each of the velocity fluctuations mostly will be close to the mean value, although they will also have a much higher probability on taking on values far from the mean compared to the more Gaussian distribution of the velocity fluctuations. Another name for this type of distribution is “super-Gaussian”. This is especially the case for the lateral fluctuations, as the kurtosis value of 10.0 is larger than that of the streamwise counterpart at 7.08. The skewness of the lateral distribution is zero within experimental uncertainty, but the streamwise skewness displays a considerable value of 0.52. These values are similar to those reported by other active grid investigators, although Makita (1991) recorded a kurtosis of only 8.44 for the lateral distribution. Mydlarski & Warhaft (1996), on the other hand, reported a kurtosis of 9.7.

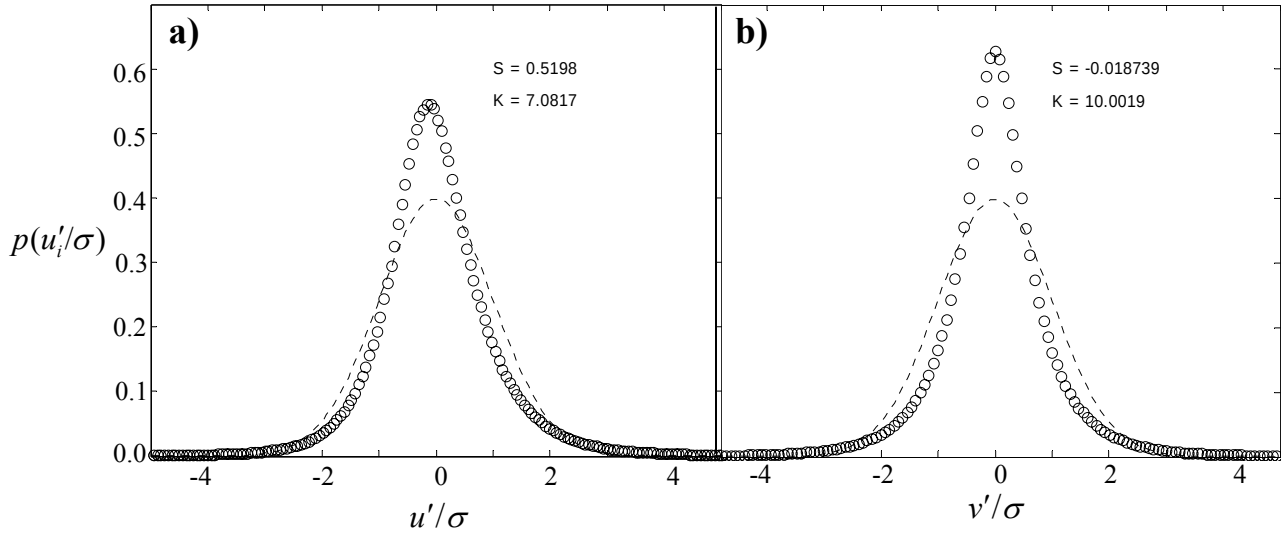


Figure 4.10. Probability density functions for a) the streamwise velocity derivative component $d\tilde{u}/dt$ and b) the lateral velocity derivative component $d\tilde{v}/dt$. The standard Gaussian distribution is indicated by the dashed lines.

Again, the probability functions are plotted on semi-logarithmic axes in Figure 4.11. The exponential tails which appear as straight lines can be seen forming about four standard deviations away from the mean which is consistent with the high Taylor Reynolds number atmospheric boundary layer data presented in Pope (2000, p. 258) and the data of Mydlarski & Warhaft (1996). As discussed by Pope (2000), these exponential tails and their non-Gaussian nature are responsible for the very large values associated with the higher moments of the distribution function.

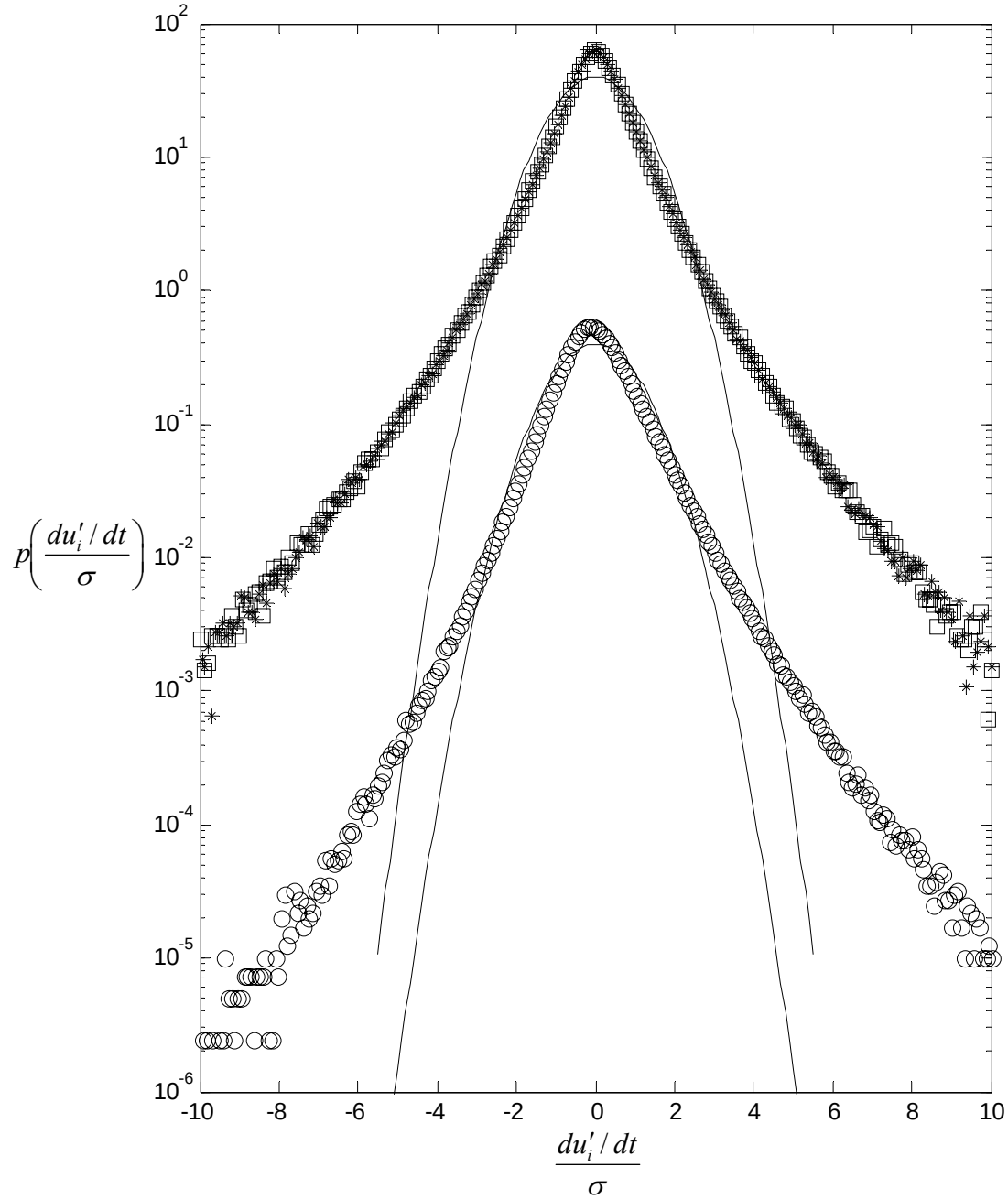


Figure 4.11. Probability density functions for the velocity derivative fluctuations: $\circ$, u' ; $\square$, v' ; $*$, w' . The standard Gaussian distribution is indicated by the solid lines. The streamwise distribution has been shifted up two decades.

Returning to the third moment, Batchelor (1953, pp. 119) states that for sufficiently high Reynolds numbers, the skewness of the velocity derivative should be a constant for a locally isotropic flow. Mohamed and LaRue (1990) suggests using this as a means to determine the streamwise location where the flow becomes locally isotropic, and shows that for their conventional grid this value becomes a streamwise constant somewhere between $X/M = 25$ and 55 (increasing with Re_M). The value of this constant decreases with Re_λ , which is consistent with

Batchelor (1953), but was reported as 20% larger for the same Taylor Reynolds number range*. This decreasing trend, and hence Batchelor's claim regarding the approach towards a universal constant value for $S(\partial u'/\partial t)$ of approximately 0.3 seems to be only valid for low Re_λ flows as measurements by Wyngaard & Tennekes (1970) in mixing layers and atmospheric boundary layers indicate that for higher Taylor Reynolds number flows (>100) there is a clear increase in the value of the skewness which violates the universal equilibrium theory. This claim is also supported by Van Atta & Antonia (1980), which came to the same conclusion by means of a different analysis and obtained a relationship as follows for $Re_\lambda > 100$:

$$S(\partial u'/\partial t) \propto Re_\lambda^{0.15} \quad (4.8)$$

The value of the constant of proportionality can be estimated from the data presented in Van Atta & Antonia (1980) as approximately 0.20.

The kurtosis is also an increasing function of Re_λ with Van Atta & Antonia (1980) presenting data suggesting an initial kurtosis of 3.8 for very small Taylor Reynolds numbers with a subsequent increase according to:

$$K(\partial u'/\partial t) \propto Re_\lambda^{0.41} \quad (4.9)$$

The constant of proportionality in this case is approximately 0.61, with measurements at $Re_\lambda = 30,000$ exhibiting Kurtoses around 40. Pope (2000) suggests an exponent of 3/8 instead of the 0.41 used in Equation 4.9. Combining Equation 4.8 and 4.9 the skewness can be defined in terms of the kurtosis thus eliminating the Taylor Reynolds number from the equation. The result is shown below and is only valid for $Re_\lambda > 100$:

$$S(\partial u'/\partial t) = 0.24K(\partial u'/\partial t)^{0.366} \quad (4.10)$$

For the baseline case it is clear from Figure 4.12 that the skewness of the streamwise velocity derivative remains at a constant value of about 0.52 throughout the measurement range with a constant kurtosis of 7.08. According to Equation 4.10, the skewness associated with this kurtosis is 0.49 which is fairly consistent with the measurements. A few things should be noted however: The data given by Wyngaard & Tennekes (1970) has a rather large spread and the trends obtained are simply empirical best fits. Although the overall skewness and kurtosis levels obtained in the current study are overall consistent with the trends given by Equations 4.8 through 4.10, they are also very similar to the values obtained by Mydlarski & Warhaft (1996) at a Taylor Reynolds number of 262 which is almost a factor of two lower than that considered here. Besides, the skewness of the velocity derivative does not decrease with downstream distance (although the Taylor Reynolds number naturally decreases), which leads to the hypothesis that the Re_λ trend does not apply within the decay of a locally isotropic flow, but can

* There is a lot of confusion in the literature regarding sign conventions. Generally when speaking of the time derivative there will be a positive skewness, which becomes a negative skewness when dealing with the spatial derivative according to equation 4.6. Several authors do not follow this convention and will use the opposite sign convention without explicitly stating so.

only be used as a tool to estimate what the constant actually is for any given flow based on a representative Taylor Reynolds number for that flow.

$S(\partial v'/\partial t)$ and $S(\partial w'/\partial t)$ take on experimental values of -0.02 and 0.01 respectively while the corresponding kurtoses are 10.00 and 9.96 for $X/M = 37.3$ with comparable values for all streamwise stations. This is consistent with Mydlarski & Warhaft's (1996) reported values of 0.05 and 9.72 for the skewness and kurtosis of their lateral components. Makita (1991) on the other hand reported $S(\partial v'/\partial t) = 0.03$ and $K(\partial w'/\partial t) = 8.44$.

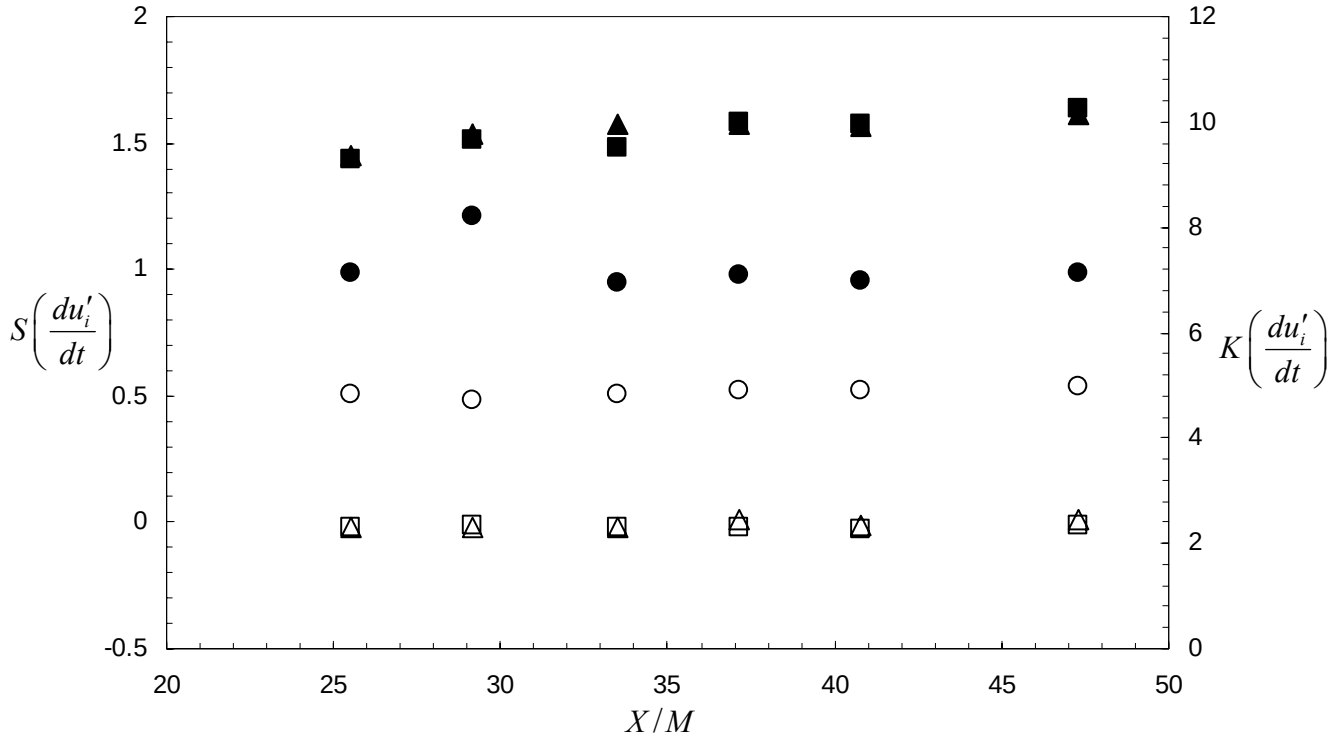


Figure 4.12. Streamwise variation of velocity derivative fluctuation skewness and kurtosis. ○, $S(u')$; □, $S(v')$; △, $S(w')$; ●, $K(u')$; ■, $K(v')$; ▲, $K(w')$.

4.4.2 Sample Independence

An important issue in data acquisition is whether enough samples are obtained to reach true mean values. The large scales associated with the generated flow limits the number of integral lengthscales that pass by the measurement probe during the acquisition time. The sample schemes described in Chapter 2 used for this study yield more than 5 million samples per measurement location. The progressively calculated rms values, skewness, and kurtosis for all three velocity components are shown in Figure 4.13 through Figure 4.15. As for the rms velocity, good approximations of the actual mean values are reached after only 3.5 million samples while for the higher order statistics there is still some oscillation present around the mean even after all the obtained samples, although the distance to a discernable mean value is much smaller than the associated experimental uncertainty associated with the data acquisition. The mean square velocity derivative, displayed in Figure 4.16 as a function of sample points, reaches an acceptable mean value after approximately 4 million samples.

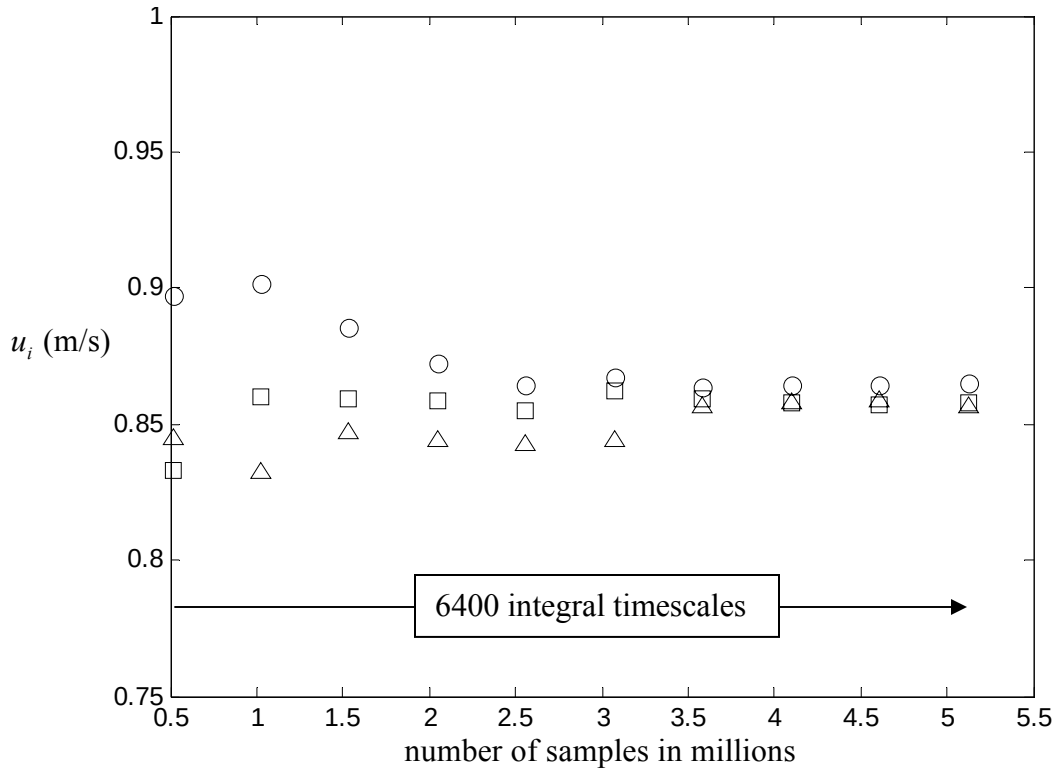


Figure 4.13. Rms velocities as a function of number of samples for the baseline condition. $\circ$, u ; $\square$, v ; $\triangle$, w .

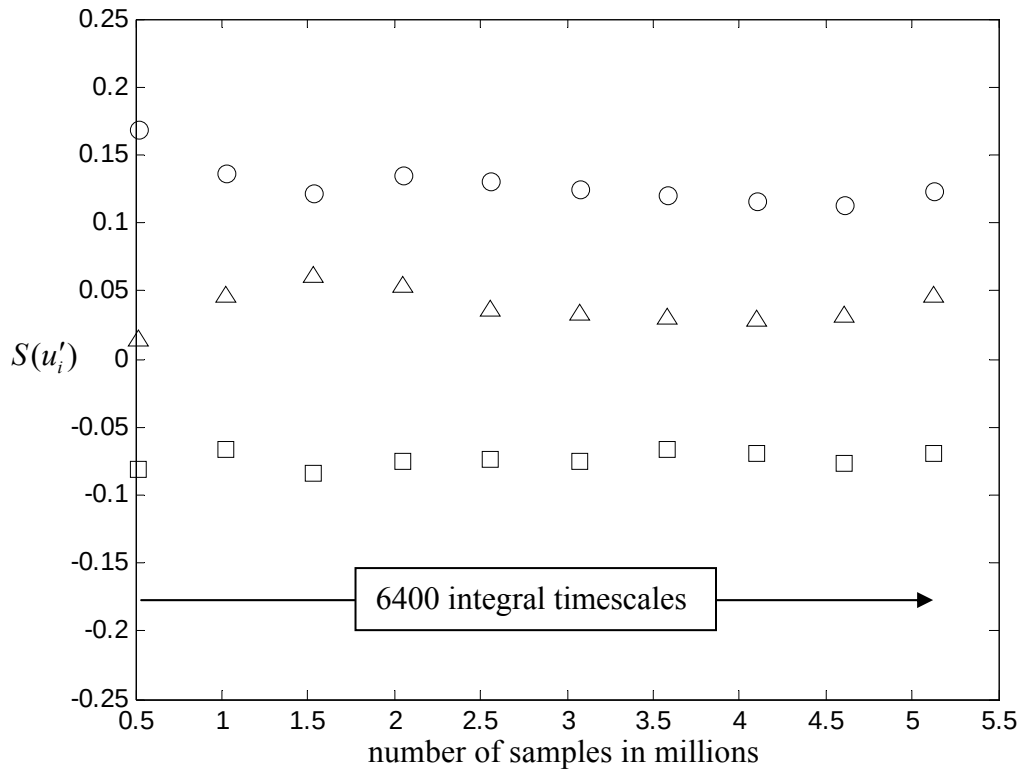


Figure 4.14. Velocity skewness as a function of number of samples for the baseline condition. $\circ$, $S(u')$; $\square$, $S(v')$; $\triangle$, $S(w')$.

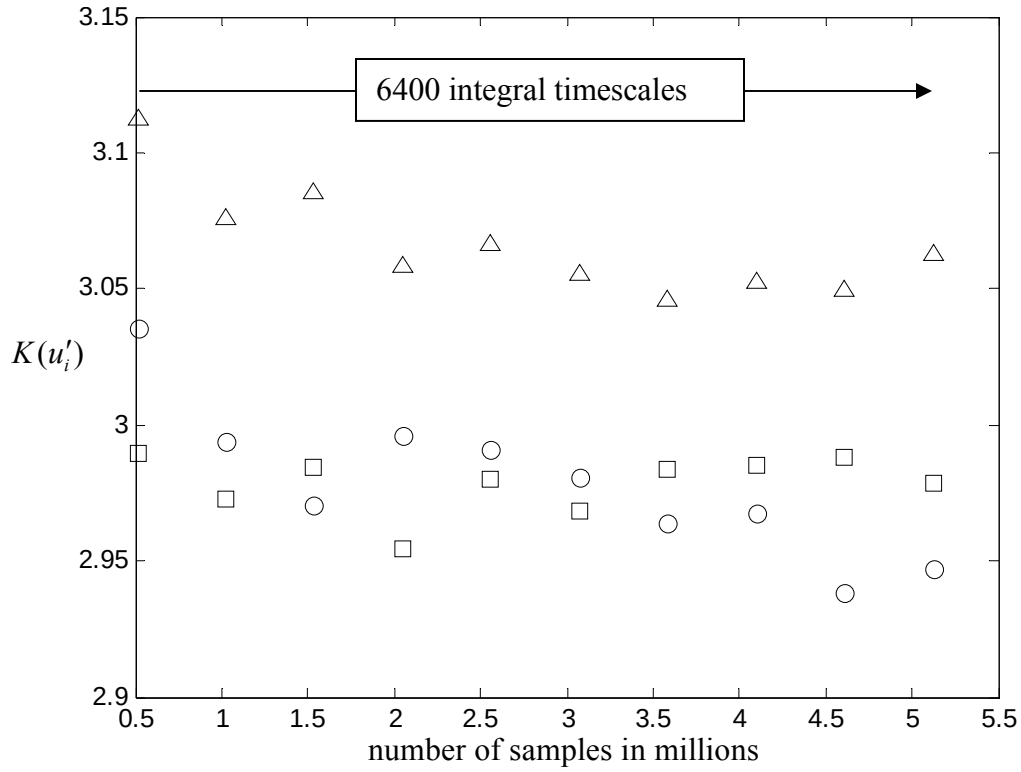


Figure 4.15. Velocity kurtosis as a function of number of samples for the baseline condition. $\circ$, $K(u')$; $\square$, $K(v')$; $\triangle$, $K(w')$.

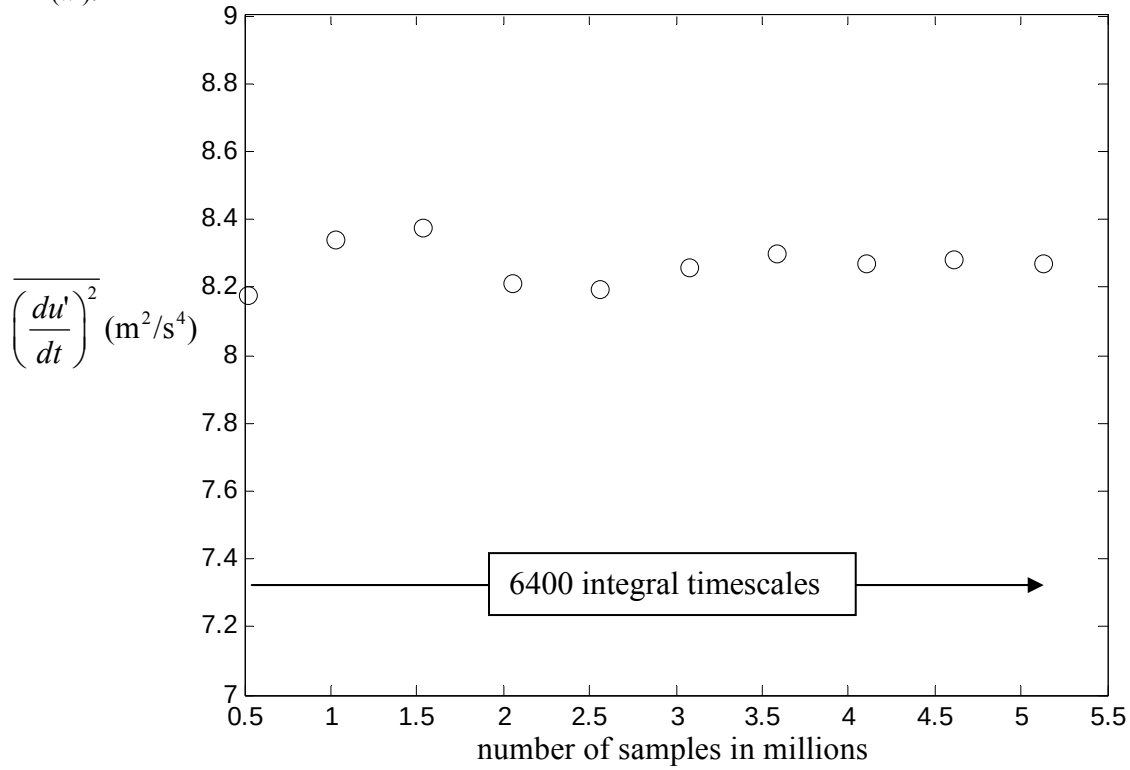


Figure 4.16. Mean square streamwise velocity derivative as a function of number of samples for the baseline condition.

4.4.3 Filtered Statistics

One of the issues plaguing the concept of large scale turbulence in a laboratory setting is the fact that the confinement due to the wind tunnel walls inherently will affect the largest eddies present in the flow. This is much less of a problem in static grid turbulence where the integral scale is generally on the order of 10 cm or less. With active grid turbulence, where the largest eddies can be up to an order of magnitude larger, this can pose quite a problem. This is an issue which will be addressed throughout the chapter, but will be looked at in this section with respect to the higher order statistic parameters discussed above.

The approach for the calculation of the filtered statistical parameters is to convert the data to the frequency domain via the Fourier Transform,^{*} high-pass filtering the result with a prescribed cut-off frequency and converting the data back into the time domain for calculations of turbulence intensity, skewness and kurtosis. This gives much more flexibility than the filtering during the time of data acquisition, performed by analog filters in, for example, Simmons, Salter & Taylor (1938) which was one of the first experiments dealing with spectral analysis of a turbulent flow.

Filtering the baseline condition data, Figure 4.17 displays turbulence intensities against cut-off frequency. Starting with the unfiltered data ($f_c = 0$), the three components appear isotropic as they all have a value of about 6.9%. By high-pass filtering one would not expect the streamwise components to be the same as the lateral components due to the way the isotropic turbulence spectra are defined (see Section 4.6 for more detail on this). One would however expect a relatively smooth behavior when progressively increasing the cut off frequency f_c . Figure 4.17 does show a relatively smooth behavior of the turbulence intensity values as a function of f_c , but at around $f_c \approx 5$ Hz there appears to be somewhat of an inflection point where the data start converging towards a different value than what seems evident from the later trend. Attempting to fit a curve to the data for $f_c > 5$ Hz suggest that barring any low frequency distortions, the turbulence intensity would have been in the vicinity of 10 % at this location for both the lateral as well as longitudinal velocity components. This of course would be in the absence of wind tunnel wall effects as well as grid-related contamination which both occur in this low wavenumber region.

As for the filtered skewness effects, this can be seen in Figure 4.18. The most evident artifact is that the lateral skewness contribution is contained within the very few frequency bins and they reach a minimum at just about $f_c = 7.5$ Hz. The actual level is much closer to zero (which it should be) for the w fluctuations than for the v fluctuations, but it is believed that this difference is simply due to experimental error. It is also seen that the vast majority of the streamwise skewness is generated at frequencies below 15 Hz with a particularly large contribution at $f_c < 5$ Hz.

The same dependence is shown in Figure 4.19 for the kurtosis of the three components. Again at $f_c \approx 7$ Hz there is a large inflection point, especially for the streamwise kurtosis which completely veers of its projected path based on the higher frequency trend. There is also some

^{*} The Fourier Transform is an essential tool used in translating data from the time domain to the frequency domain and vice versa. It is frequently encountered in the study of turbulent flows and therefore needs to be understood thoroughly. Unfortunately, in this author's experience, most references describing this material can be very confusing for students unfamiliar with the fundamental concepts behind the Fourier Transform. Appendix A should provide newcomers to the subject a much needed introduction to the material and attempts to shed some light over what the Fourier Transform is and how it works.

deviation of the $K(w')$ at this point where it stops following the same trend as the v -component (albeit as a slightly higher level). The difference between the two lateral components is again contributed to experimental uncertainty. Since a discrepancy showed up in the skewness it is expected to be amplified in the kurtosis. Again, the confinement still forces all three unfiltered components to converge to a somewhat consistent value as seen in the analysis above. Following the high frequency trend and extrapolating down to lower frequencies it can be deduced, however, that the kurtoses from all components would end up at a similar value in the absence of low frequency distortion effects.

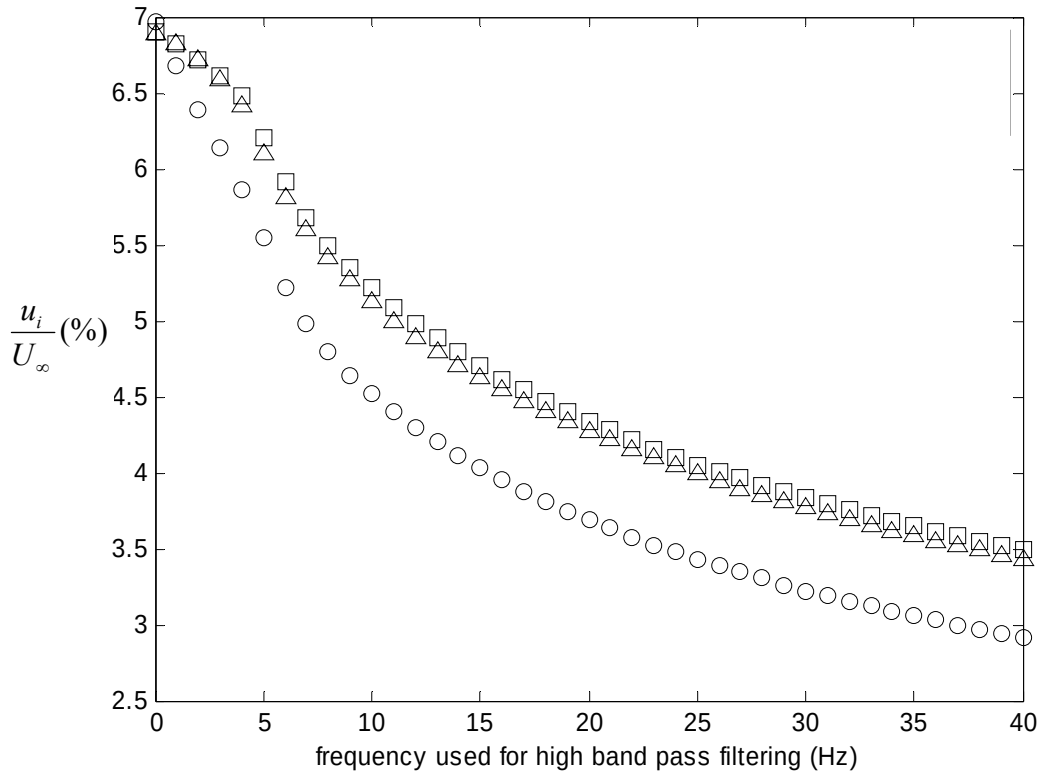


Figure 4.17. High-passed filtered rms velocity fluctuations as a function of cut-off frequency for the baseline condition. $\circ$, u ; $\square$, v ; $\triangle$, w .

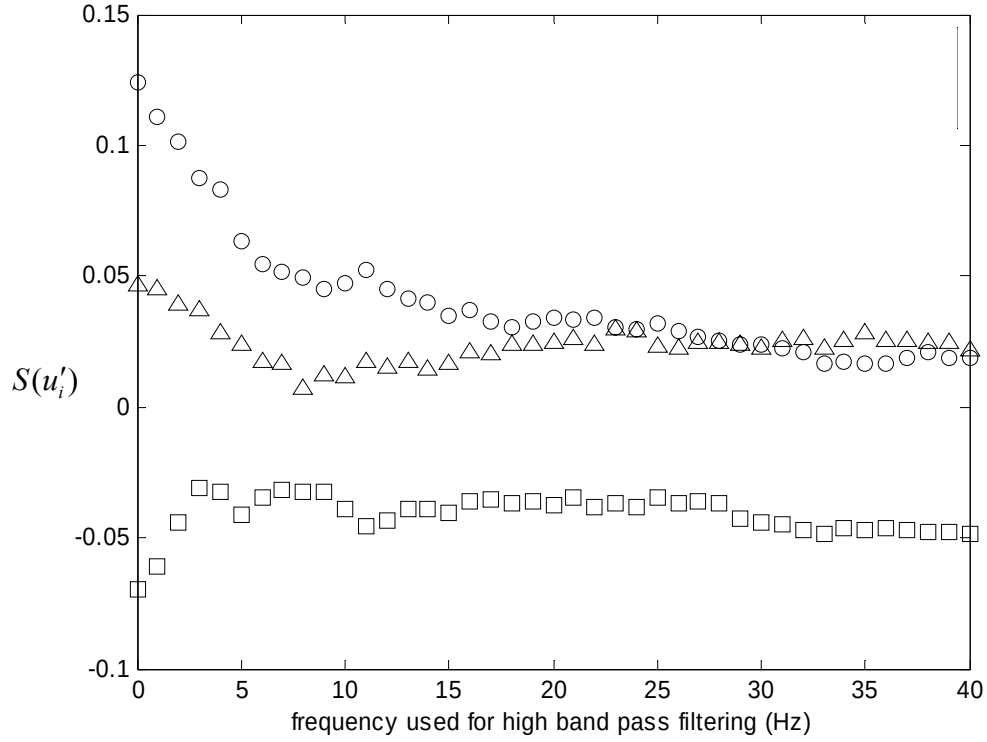


Figure 4.18. High-passed filtered skewness as a function of cut-off frequency for the baseline condition. $\circ$, u' ; $\square$, v' ; $\triangle$, w' .

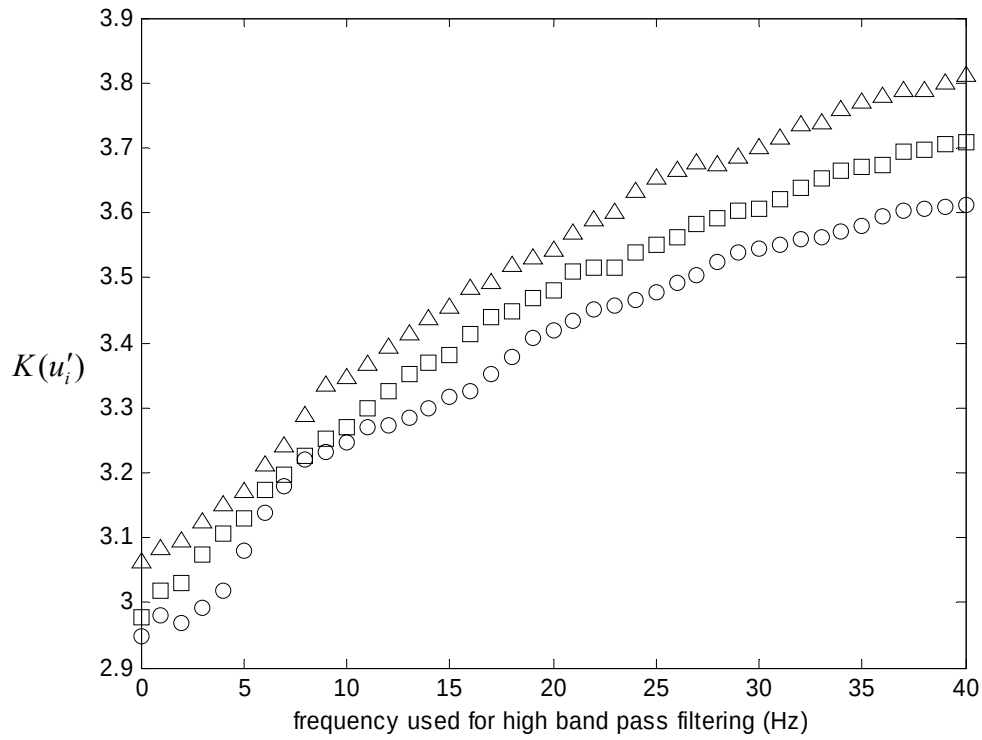


Figure 4.19. High-passed filtered kurtosis as a function of cut-off frequency for the baseline condition. $\circ$, u' ; $\square$, v' ; $\triangle$, w' .

4.4.4 The Wind Tunnel Wavenumber

In order to assess the interference of the wind tunnel walls on the large scale turbulence, it is instructive to define a wind tunnel wavenumber. This parameter can be formed by taking a standing wave propagating between two closed ends in which the tunnel wavelength (λ_H) is given by Equation 4.11 where U_∞ is the propagation velocity, H is the sectional height (or width) of the wind tunnel and N denotes the n^{th} harmonic. By solving this equation for frequency (f_H), applying it to the first fundamental ($N = 1$) and converting back to wavenumber space as shown in Equation 4.12 the wind tunnel wavenumber, $\kappa_{H,1}$ can be defined. This signifies a waveform spanning the entire tunnel section height from node to node (half a complete wavelength). Similarly $\kappa_{H,2}$ is obtained by setting $N = 2$ and is the wavenumber corresponding to an entire wavelength fitting inside the tunnel test section. N can be any integer, but the first few harmonics are the most important for assessing how the tunnel interferes with the spectrum.

$$\lambda_H = \frac{2H}{N} \quad (4.11)$$

$$\kappa_{H,N} = \frac{2\pi}{U_\infty} f_H = \frac{2\pi}{U_\infty} \left(\frac{U_\infty}{\lambda_H} \right) = \frac{\pi N}{H} \quad (4.12)$$

For the stability wind tunnel with its sectional size of 1.87 m, the two first modes give wind tunnel wavenumbers of $\kappa_{H,1} = 1.68 \text{ m}^{-1}$ and $\kappa_{H,2} = 3.36 \text{ m}^{-1}$. For the baseline condition this corresponds to frequencies of 3.32 Hz and 6.64 Hz, the latter which was found to have a large effect on the filtered statistics in the preceding section. As will be clear in Section 4.9.3, $\kappa_{H,2}$ is also the cutoff point where the flow field can no longer support the isotropic relationship between the longitudinal and transverse correlations.

Applying this concept to a grid condition in which $\Omega = 2 \text{ Hz}$ and $U_\infty = 16 \text{ m/s}$, it was found that the skewness and kurtosis divergence occurred at approximately 8 Hz. The corresponding frequency to $\kappa_{H,2}$ would in this case be 8.55 Hz which supports that this parameter directly influences the behavior of the skewness and kurtosis at low frequencies.

4.5 Initial Period of Decay

The initial period of decay is traditionally described by Equation 4.13 where the exponent, n , must be a negative number on the order of unity. The virtual origin X_0 is the location at which the energy would be infinite if the relationship was valid everywhere. Due to the large Reynolds numbers involved with the active grid experiments, the behavior associated with the final period of decay (where the non-linear inertial and pressure terms in the Navier-Stokes equations are negligible and only the viscous terms remain) can obviously not be observed within the length of the stability tunnel test section.

$$\frac{u^2}{U_\infty^2} = A \left(\frac{X - X_0}{M} \right)^n \quad (4.13)$$

Throughout the literature there seems to be little evidence supporting that the parameters in Equation 4.13 are universal constants. On the contrary, there long seemed to be evidence supporting the opposite (e.g. George, 1988). Based on a myriad of experiments documenting the decay of grid turbulence (a survey is available in Mohamed & LaRue, 1990) it seemed like the parameters of Equation 4.13 are all dependent upon the design of each grid where the exponent, n , took on values between -1.00 and -1.47. Mohamed & LaRue (1990) on the other hand suggested that prior use of datapoints in the inhomogeneous (anisotropic) region close to the grid where such a power law isn't warranted, together with a lack of consistency in the approach to determine the virtual origin could be the reason for such a lack of consistency. In their paper they suggest a simple method of determining the virtual origin by only considering data from the homogeneous and locally isotropic region of decay. This is done by evaluating the skewness of both the streamwise velocity fluctuations and the streamwise velocity derivative fluctuations (see Section 4.4.1) followed by an attempt to minimize the resulting error by curve fitting the streamwise normal stresses through varying the virtual origin (X_0/M) and calculating the corresponding decay exponent (n), and constant A . The authors reevaluated the parameters from several apparently inconsistent static grid datasets and indeed found a consistent trend. From this they concluded that the virtual origin and the decay exponent were both independent of initial conditions with values of zero and -1.3 respectively. In contrast, the constant A was found to be highly dependent on the grid itself and would vary with such parameters as Reynolds number, mesh size, solidity, rod shapes etc. Applied to the current work the question whether this trend is indeed universal in which case it should also work for active grids immediately arises.

Makita (1991) used the same technique for his baseline active grid and obtained a virtual origin at $X_0/M = -9.2$ and $n = -1.43$. The virtual origin was here found to be negative which means it is located upstream of the grid. This suggests the presence of a shorter development time, and that it is possible to obtain homogeneous turbulence measurements much closer to an active grid than it is to a conventional passive grid.

For the current dataset, which was obtained using the baseline grid condition, the value of the exponent, n , was found to be -1.285 which is within 1% of the estimate provided for static grids. The virtual origin was found to be located upstream of the grid, at $X_0/M = -6.67$ with an associated constant $A = 0.600$ (in Equation 4.13). The exact form of the expression is given in Equation 4.14 and shown together with the experimental data in Figure 4.21. If the virtual origin is artificially placed further upstream of the grid, the exponent naturally becomes larger as well. For the current data with a forced virtual origin of $X_0/M = -9.0$, the corresponding value of n becomes -1.36 which corresponds better to the Makita (1991) value.

$$\frac{\overline{u^2}}{U_\infty^2} = 0.6 \left(\frac{X}{M} - 6.67 \right)^{-1.285} \quad (4.14)$$

Applying the same method to the published data of Kang *et al.* (2003), although of only limited use with only four data points, a very similar result is obtained with a decay exponent of -1.29, virtual origin at $X_0/M = -8.6$ with a corresponding $A = 1.78$. Mydlarski & Warhaft (1996) assumed a virtual origin of zero and fitted the data with a decay exponent of $n = -1.21$. This

triggered the question whether or not this data set could be fitted to a similar function which would appear indistinguishable in the measurement domain between $X/M = 27$ and $X/M = 68$. Given the relationship provided in Mydlarski & Warhaft (1996) it became clear that this was possible for this interval with a decay coefficient of $n = -1.3$ with an associated values of $X_0/M = -6.0$ and $A = 2.1$. Although the fitting was here done with a certain bias, this is mentioned to show that the Mydlarski & Warhaft (1996) dataset is not inconsistent with the current observations nor with the data of Kang *et al.* (2003).

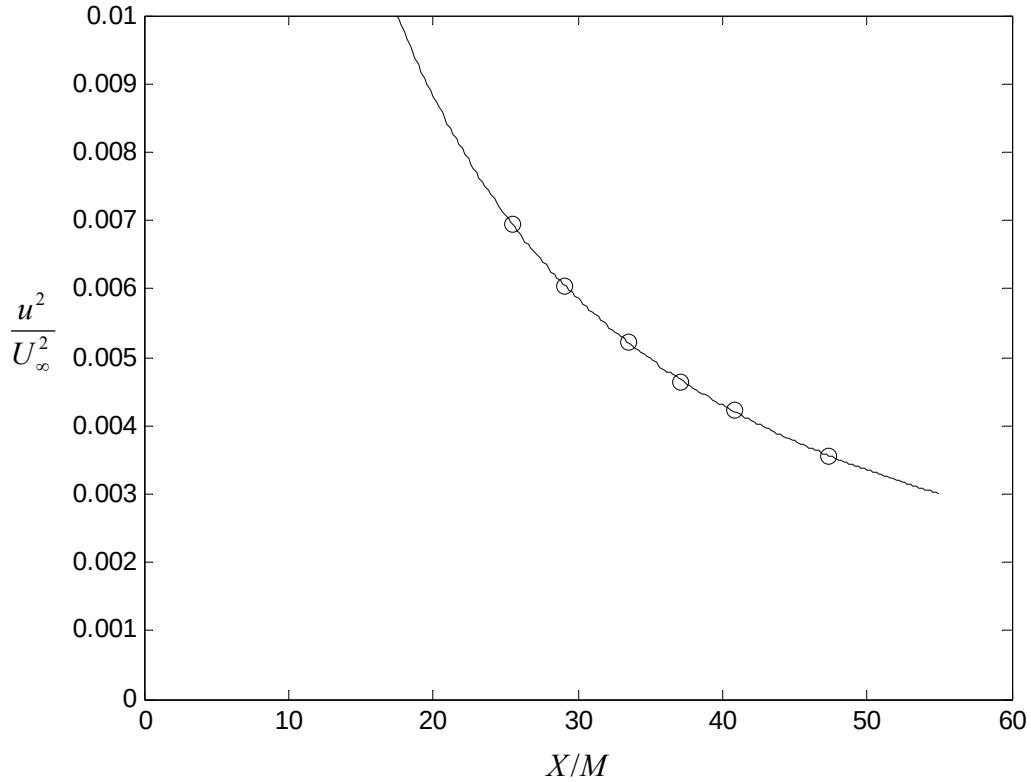


Figure 4.20. Decay of streamwise Reynolds stress normalized on free stream velocity for the baseline condition. $\circ$, experimental quad-wire data; —, Equation 4.14.

From the above discussion it becomes clear however, that Mohamed & LaRue's (1990) suggestion that the power law is indeed universal for all homogeneous grid turbulence may have some bearing in that the exponent seems to have a universally constant value of -1.3 (within experimental error) for any grid. The virtual origin on the other hand is clearly a function of grid type, and possibly geometry, although the current available data can not confirm this assertion. All the documented active grid designs unmistakably have their virtual origin located upstream of the grid in contrast to conventional static grids which have zero or possibly a small positive values associated with this parameter. The constant, A , is a function of both grid design and Reynolds number and could therefore very well be a function of the drag coefficient (C_D) acting on the grid as suggested by Batchelor (1953).

Inviscidly, the drag coefficient based on only pressure drag for a conventional static grid will equal the static pressure drop coefficient across the grid which can be calculated according to

Equation 4.15* as given by Gad-el-Hak & Corrsin (1974) where σ is the solidity of the grid. Considering the typical gridcell in a static grid (shown in Figure 4.21a) the area blocked by the grid can be expressed as $2(d \times M) - d^2$, where d is the diameter of the rods. The solidity σ is subsequently defined by normalizing this area by that of the gridcell according to Equation 4.16.

$$C_D = C_p \approx \left(\frac{\sigma}{1 - \sigma} \right)^2 \quad (4.15)$$

$$\sigma_{static} = \frac{2dM - d^2}{M^2} = \frac{d}{M} \left(2 - \frac{d}{M} \right) \quad (4.16)$$

For a static grid we then have that the decay coefficient A is simply a function of the rod diameter and the meshwidth. The exact relationship has up to now been experimentally obtained from the grid in question, although it is ultimately desirable to find a universal law which can predict the decay coefficient. Batchelor (1953) assumes that A is proportional to C_D while Gad-el-Hak & Corrsin (1973) proposed an alternative proportionality with $C_D + 2\sqrt{C_D}$.

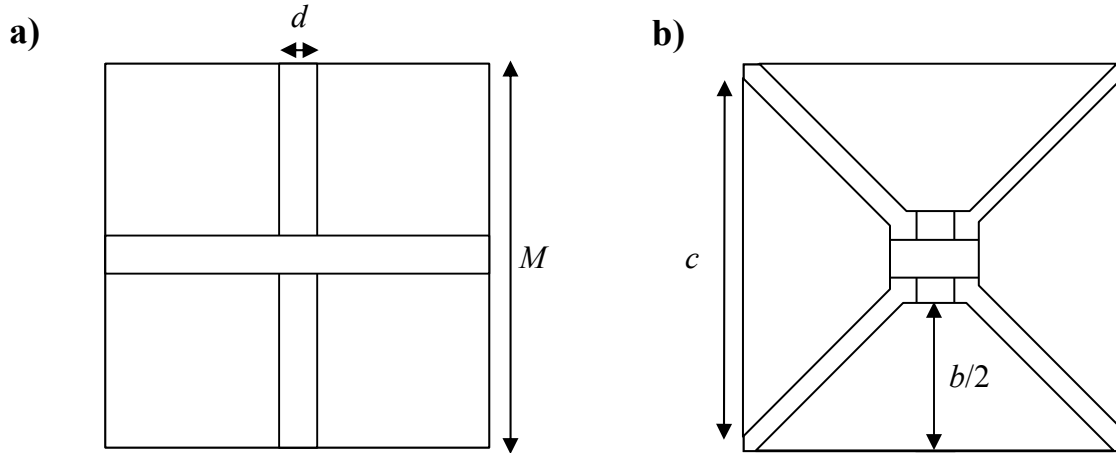


Figure 4.21. Minimum (a) and maximum (b) grid cell solidities for an active grid cell in parallel configuration. The minimum solidity case corresponds to a static grid cell.

No free stream velocity variation is expected for static grids as long as the separation location is kept constant on the grid bars. This can be ensured by using rods with rectangular cross-sections whereas for circular rods, even though the flow is still sub-critical at very low rod Reynolds numbers the drag coefficient does not stay perfectly constant as there may still be some variation of separation point and hence the pressure drag coefficient associated with the grid will show a slight decrease when the Reynolds number (i.e. flow speed) is increased. The latter case

* This equation is derived with several assumptions such as the separation location on the rods and a lack of viscous forces. It will therefore correspond better with the low Reynolds number flow over circular cylinders rather than rectangular bars. The difference in actual drag coefficient between the two cases should be a constant and can be compensated for by including the effect in A .

can be seen in the present static grid where the turbulence intensity decreases about ten percent when tripling the free stream velocity. The static grid used by Mish (2001) with $\sigma_{\text{static}} = \sigma_{\text{max}}$ of 0.31 and a mesh length of 0.304 m utilized square sectional bars and the associated turbulence levels remained, as expected, essentially constant when tripling the Reynolds number. Staid

For active grids the analysis becomes much more difficult as the grid solidity is a function of time. The time average solidity for a randomly operating active grid can be assumed to be the average of a completely closed and a completely open configuration. The open configuration corresponds to that of a static grid with the only difference that the width of the agitator wings will have to be taken into account. In the fully closed configuration the solidity of the grid can be obtained from the wing width b and the chordlength c in addition to the rod-diameter d and mesh-size M . After adding up all the contributing areas, dividing by the grid cell area and simplifying we are left with Equation 4.17.

$$\sigma_{\text{max}} = \frac{2(bc - bd + dM) - b^2 - d^2}{M^2} \quad (4.17)$$

In all earlier efforts the wing width is the same as the chordlength (a square shaped wing) in which case Equation 4.17 reduces to:

$$\sigma_{\text{max}} = \frac{c^2 + 2d(M - c) - d^2}{M^2} \quad (4.18)$$

Note that Equation 4.18 reduces to Equation 4.16 if $b = c = 0$ which is the case for a conventional static grid.

Equations 4.17 and 4.18 may define the maximum solidity for an active grid, but the pressure drop is no longer accurately described by Equation 4.15 due to the fact that the two grid planes are positioned somewhat apart, decreasing the apparent blockage, and hence the effective maximum solidity is slightly smaller. Additionally the equation above does not take into account mounting hardware and links inserted into the grid for structural stability, nor does it include the effect of irregular walls.

As will become clear in Section 4.11.1, the magnitude of the velocity fluctuations are found to be affected by grid parameters such as the grid Rossby number, controlled by the free stream velocity and the average rotation rate. This results in a complex relationship in which the decay law constant A is now a function of the active grid parameters as well as the drag coefficient. We nevertheless proceed, assuming naively that Equation 4.15 still holds (with σ now replaced by $\sigma(t)$) which arguably is the case for slower rotation rates (Poorte, 1998). In order to compute a time average drag coefficient one would need to specify the probability density function of the solidity of the grid, $\text{pdf}(\sigma)$, in order to calculate the mean drag from Equation 4.19 as suggested by Poorte (1998) where $\text{pdf}(\sigma)$ would be defined in terms of σ_{static} , σ_{max} , as well as the distance between the vertical and horizontal grid plane, Δh , non-dimensionalized by the local tunnel sectional dimension, H .

$$\overline{C_D} \approx \int_0^1 \left(\frac{\sigma}{1-\sigma} \right)^2 \text{pdf}(\sigma) d\sigma, \quad \sigma = \sigma(\sigma_{\text{static}}, \sigma_{\text{max}}, \Delta h / H) \quad (4.19)$$

Although a definition of the probability density function of σ has not been attempted here, it is clear that in order to maximize the time average drag coefficient one needs large, obtainable $\sigma_{\max}$ occurring. Table 4.2 presents values for both σ_{static} , $\sigma_{\max}$ of active grids calculated from Equation 4.17, as well as values for $\Delta h/H$. Apart from the grids of Makita (1991)* and Poorte (1998) who reported a gap of 1.0 mm between the rod planes, no documentation of this parameter was found for the other grids and the values in Table 4.2 (with the exception of the current grid) have been estimated by adding 1.0 mm to the outer diameter (or diagonal of the square cross section in the case of Kang *et al.*, 2003) of the rods.

Comparing the grids we see that with the exception of Poorte's (1998) staggered grid all the active grids have comparable values for σ_{static} . The differences are however significant in the plane separation and $\sigma_{\max}$ where Makita's (1991) grid clearly represents the largest time average solidity and hence the largest drag coefficient. Although the minimum solidity is much larger in the staggered case of Poorte (1998) a quick look at Equation 4.19 shows that the possibility of the very large solidities in the parallel configurations result in a much larger time-averaged drag coefficient and hence the resulting turbulence intensity generated by the grid. As mentioned above other factors play a role in determining the resulting downstream grid turbulence intensity created by a grid, but Table 4.2 goes a long way in explaining the large turbulence levels associated with the active grids and also the difference between the grids.

Grid	σ_{static}	$\sigma_{\max}$	$\Delta h/H$	$\sigma_{\max}^2(1-\Delta h/H)$
Makita (1991)	0.24	0.93	0.0100	0.86
Mydlarski & Warhaft (1996)	0.24	0.91	0.0182	0.81
Mydlarski & Warhaft (1998a)	0.21	0.91 [†]	0.0150	0.82
Poorte (1998) Staggered/Parallel	0.63 / 0.25	0.75 / 0.89	0.0133	0.56 / 0.78
Kang <i>et al.</i> (2003)	0.23	0.90	0.0262	0.79
Present Work	0.23	0.84	0.0297 [‡]	0.68

Table 4.2. Grid specific solidities based on grid solidity (Equation 4.17) as well as grid plane separations for active grids. Equation 4.17 does not apply to the staggered configuration of Poorte (1998) in which case the values reported by the author are repeated here.

Attempting to create an empirical fit for the constant A for active grids is deemed next to impossible at this point since the available data from different grids was not obtained using consistent forcing protocols, but it is fairly clear that it will be dependent on grid geometry through the time averaged drag coefficient as well as the forcing protocol (especially through the Rossby number[§] since this affects how the grid appears to the fluid particles traveling through it) together with the grid Reynolds number. One would need a modular grid where the effective solidity can easily be changed in order to assess this, and in addition make sure that the tunnel confinement effect discussed in Section 4.4.4 is accounted for.

* Rod separation value for the Makita (1991) active grid was documented in Makita and Miyamoto (1983).

[†] Chordlength and wing width dimensions were not reported in Mydlarski & Warhaft (1998) and hence it is assumed that the grid would have the same c/M and b/M ratios as their previous effort, resulting in the same maximum solidity as in Mydlarski & Warhaft (1996).

[‡] This value is obtained by using the local wind tunnel section: $H = 2.13$ m.

[§] See section 4.2.

4.6 Velocity Spectra

4.6.1 Overview

So far the discussion has revolved around statistical values that are obtained by assessing the time series directly. Although extremely informative, the time series data does not tell the entire story about how the fluctuating energy is distributed in frequency or wavenumber space. In Section 4.4.3 this problem was partly addressed by filtering the statistics to determine the effect of the wind tunnel walls on the statistical values. This was possible through the use of the Fourier Transform in order to convert the data from time to frequency and back to the time domain again via the inverse Fourier Transform. In this section the frequency domain will be considered, although the frequencies will be converted to spatial frequency, or wavenumber space, through the simple transformation $\kappa = 2\pi f/U_\infty$.

The process of transforming a time series into a velocity spectrum or power spectral density where energy per frequency is plotted against frequency is explained in detail in Appendix A. For the streamwise velocity component, $\tilde{u}$, the procedure will result in the streamwise one-dimensional velocity spectrum, $E_{11}(f)$. The two numbers indicate which two velocity components are being used in the Fourier Transform. In this case it simply means that the streamwise component $\tilde{u}$ ($i = 1$) is being used. $E_{11}(f)$ can now be converted from frequency space to wavenumber space, again through invoking Taylor's hypothesis:

$$E(\kappa_1) = \frac{U_\infty E_{11}(f)}{2\pi} \quad (4.20)$$

In this equation κ_1 is the streamwise wavenumber. The lateral spectra $E_{22}(\kappa_1)$ and $E_{33}(\kappa_1)$ can similarly be obtained from the time-series of $\tilde{v}$ and $\tilde{w}$ respectively. The one dimensional spectra are related to the three-dimensional velocity spectra and each other through Equation 4.21 and 4.22 where κ is the vector sum of κ_1 , κ_2 and κ_3 . See Pope (2000) for details.

$$E(\kappa) = \frac{1}{2} \kappa^3 \frac{d}{d\kappa} \left(\frac{1}{\kappa_1} \frac{dE_{11}(\kappa_1)}{d\kappa_1} \right) \quad (4.21)$$

$$E_{22}(\kappa_1) = \frac{1}{2} \left(E_{11}(\kappa_1) - \kappa_1 \frac{dE_{11}(\kappa_1)}{d\kappa_1} \right) \quad (4.22)$$

4.6.2 The Pope Model Spectrum

The three-dimensional velocity spectrum is the Fourier Transform of the two-point velocity correlation which will be discussed in greater detail in Section 4.10. Several attempts have been made to describe the three-dimensional velocity spectra mathematically based on the Kolmogorov hypotheses and available experimental data. One of the most famous such model

spectra was proposed by von Kármán whose spectrum exhibited the $-5/3$ law in the inertial subrange and a power law according to κ^4 . Pope (2000) proposes the same basic shape in the inertial range, but with low wavenumbers taking on a κ^2 power law towards zero as well as an exponentially decaying dissipation range as shown in Equation 4.23 to 4.25. This model, which will from now on be referred to as the Pope model spectrum is Taylor Reynolds number independent above $Re_\lambda = 300$. Below this threshold one needs to use slightly modified constants in order for the spectrum $E(\kappa)$ to integrate to k , while at the same time the integration of $2\nu\kappa^2 E(\kappa)$ with wavenumber must equal to the dissipation rate ε . Refer to Pope (2000) for more details on these spectrum functions and their associated numerical constants. The constant C in Equation 4.23 is the Kolmogorov constant which will be discussed in more detail in Section 4.7.4, which takes on a value of approximately 1.5.

$$E(\kappa) = C\varepsilon^{2/3}\kappa^{-5/3}f_L(\kappa L)f_\eta(\kappa\eta) \quad (4.23)$$

$$f_L(\kappa L) = \left(\frac{\kappa L}{((\kappa L)^2 + 6.78)^{1/2}} \right)^{5/3+2} \quad (4.24)$$

$$f_\eta(\kappa\eta) = \exp \left[-2.1 \left[\left((\kappa\eta)^4 + 0.4^4 \right)^{1/4} - 0.4 \right] \right] \quad (4.25)$$

Based on the Pope-spectrum Figure 4.22 shows the theoretical three dimensional spectrum for $Re_\lambda = 500$ with an associated macroscale $L = 0.89$ m together with the associated streamwise and lateral one-dimensional spectra.

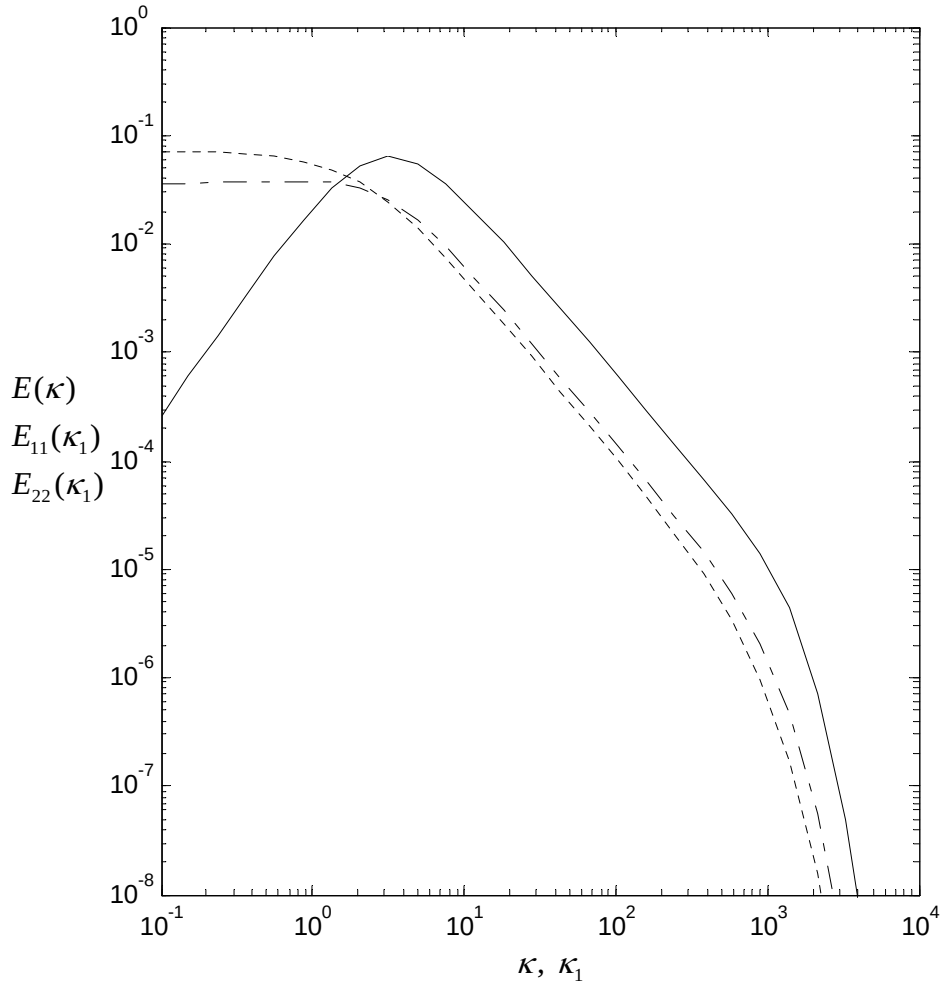


Figure 4.22. Pope model spectra with arbitrary units applied to the abscissa: —, $E(\kappa)$; ---, $E_{11}(\kappa_1)$; - · - ·, $E_{22}(\kappa_1)$.

Concentrating on the streamwise one-dimensional spectrum, the Pope model now can be compared to the spectra obtained experimentally. Figure 4.23 shows a wide array of turbulent flows obtained using the active grid with experimentally obtained Taylor Reynolds numbers ranging from 100 to over 1350. By taking advantage of Kolmogorov scaling, such as in this figure, the universal equilibrium range (higher wavenumbers) will scale equally for each spectrum, while the low wavenumber level will grow larger with Taylor Reynolds number. The streamwise one-dimensional Pope spectrum has also been added to Figure 4.23 for several Taylor Reynolds for comparative purposes.

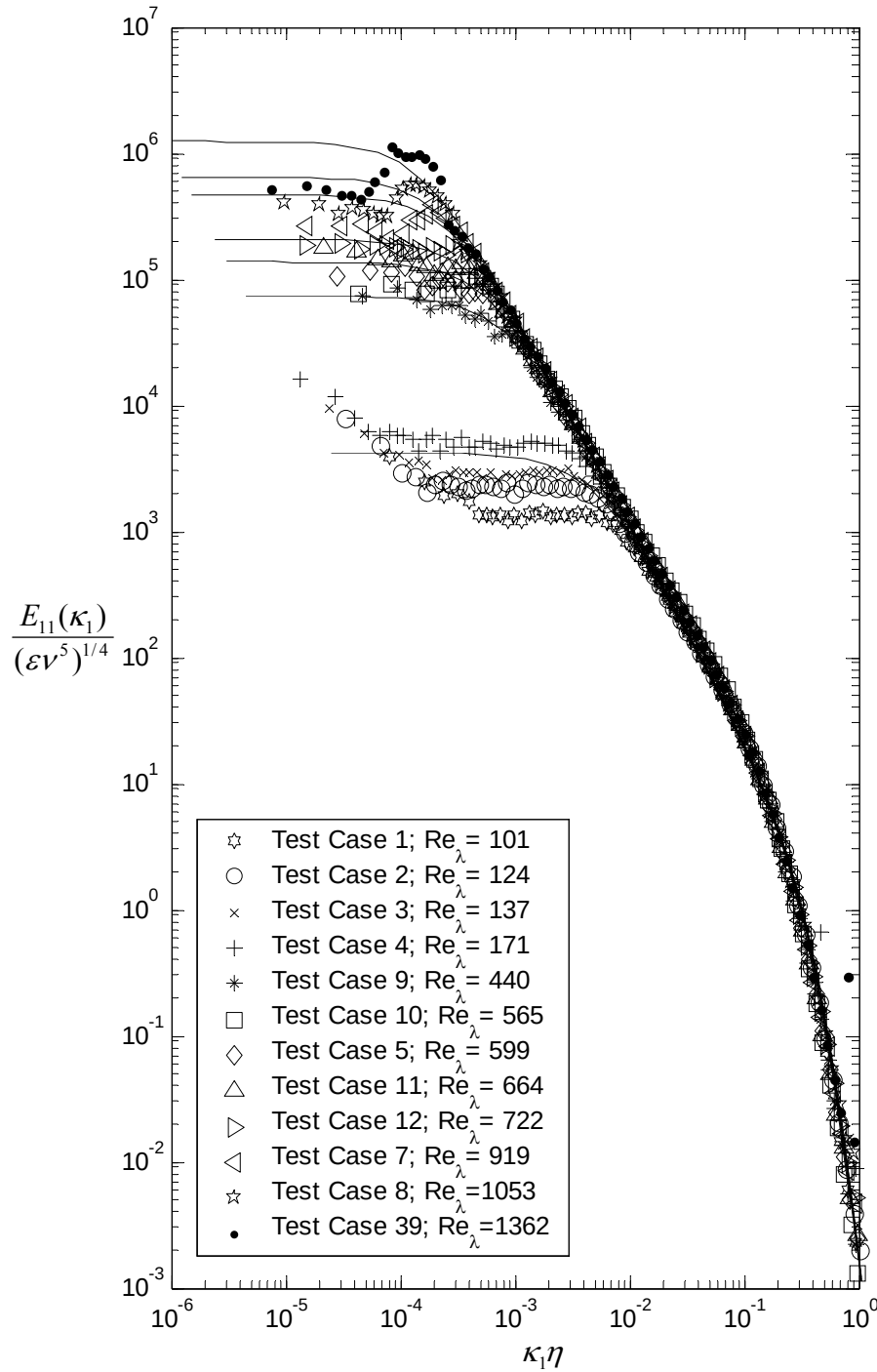


Figure 4.23. Experimental streamwise velocity spectra with Kolmogorov scaling. The four lowest Taylor Reynolds numbers are associated with the grid in the static configuration. Test case 11 (Δ) corresponds closely to the baseline condition. The Pope-spectrum functions have been added as solid lines for the following Taylor Reynolds numbers: 140, 440, 565, 665, 920, 1050, and 1360. Refer to Table 4.1 for more information on each test case.

A few general remarks should be made about the low wavenumber region. For the static cases it is evident that some low frequency wind tunnel unsteadiness is affecting the turbulence

intensity. These are fluctuations that correspond to less than 1 cycle per second and are visible in all the four static grid cases shown in Figure 4.23. The presence of these impurities will drag the integrated velocity fluctuation variance up, and create an artificially high value of the Taylor Reynolds number. The model spectrum is plotted for $Re_\lambda = 130$ (Equations 4.23 through 4.25 have been modified slightly to account for a Taylor Reynolds number less than 300) and although there is decent agreement, it seems that the corresponding experimentally obtained Taylor Reynolds number is about 20% too large. Nevertheless the obtained values are in quite close agreement with the data of Helland, Van Atta & Stegen (1977) where a grid and wind tunnel comparable to the current static grid configuration was used, but with a larger free stream velocity of 29 m/s. For that particular experiment the authors reported a Taylor Reynolds number of 237 which follows the trend shown in Figure 4.23.

Figure 4.24 shows a magnified version focusing on the low-wavenumber region of some of the active grid data from Figure 4.23. With the exception of the few lowest Taylor Reynolds numbers it becomes clear that the experimentally obtained values are underestimated by the fact that the low wavenumbers are affected in two ways. The wind-tunnel tends to attenuate the lowest wavenumbers, which artificially decreases the Taylor Reynolds number. Additionally the energy from the grid is shown to add energy into the spectrum close to the inertial range which is consistent with the prediction for even higher Taylor Reynolds number flows. This is especially evident when the grid rotation rate is set at $\Omega = 2$ Hz. This infusion of energy creates a longer equilibrium range which would normally be associated with much higher Taylor Reynolds numbers, but the spectra suffer an extra strong attenuation effect as the levels are forced back down by the tunnel at low wavenumbers. This suggests that even though the lower grid rotation rates produce inherently larger Taylor Reynolds numbers, they do so by severally departing from the standard isotropic spectral form, and one should exercise care in interpreting this data at low wavenumbers.

The data obtained using a grid rotation rate of 4 Hz will have inherently lower Taylor Reynolds numbers than the spectral shape at the beginning of the universal equilibrium range call for, due to the attenuation of low wavenumbers. The advantage however is that the general shape of the low wavenumber range stays truer to theoretical isotropic turbulence, which means that the grid itself has much smaller interfering effect on the resulting turbulence.

The baseline condition described in Section 4.2.1 was used as the turbulent inflow for the blade blocking experiments discussed in Chapter 6. When comparing the experimental results to theoretical predictions which will be done in Chapter 7 the integral lengthscale of the turbulence becomes of paramount importance. As mentioned earlier, the uncertainty of the low wavenumber region makes the definition of the exact integral scale difficult. From the four-sensor hot-wire measurement at the baseline condition at $X/M = 37.3$ in Figure 4.24 it can be seen that the Pope spectrum based on the calculated Taylor Reynolds number and integral scales does not conform very well to the data in the low wavenumber problem area. As a matter of fact there are really three different sets of parameters that can be defined for this particular flow. In the first method a low Taylor Reynolds number, corresponding to the “apparent” integral lengthscale from the classic definition in Equation 1.4 can be used. The second method, used in the majority of calculations in this chapter, utilizes the entire wavenumber range according to Equation 1.6 which results in the measured Reynolds number of about 650 and an integral lengthscale of 38.2 cm which is plotted in Figure 4.24; The third method involves fitting the high wavenumber region in and below the inertial range and extrapolating the spectrum towards zero wavenumber according to the Pope spectrum function. Although an exact Taylor Reynolds number and

integral scale is difficult to define accurately this way, it is clear that this would yield much larger lengthscales as well.

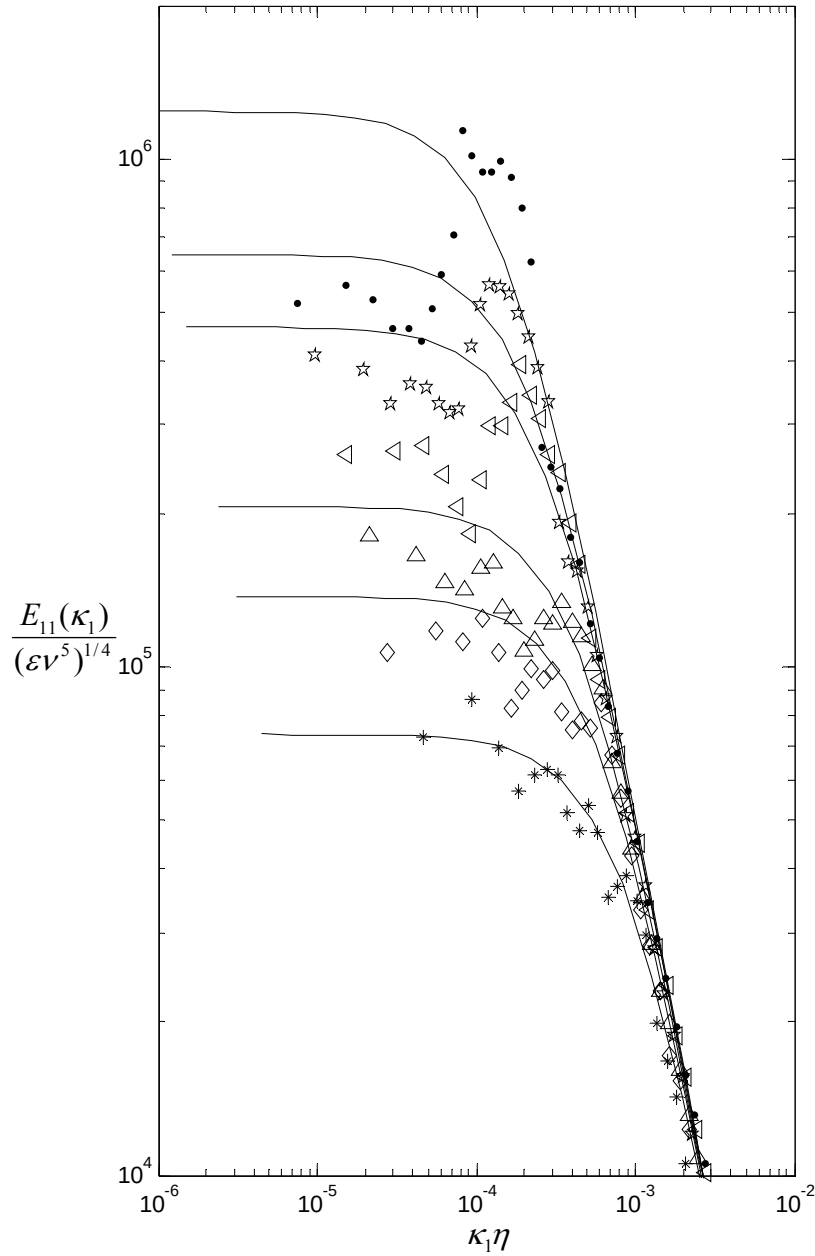


Figure 4.24. Detailed view of the low-wavenumber region of Figure 4.23 with the same symbol convention. Δ corresponds to the baseline condition. The Pope-spectrum functions have been added as solid lines for the following Taylor Reynolds numbers: 440, 565, 665, 920, 1050, and 1360.

Figure 4.25 shows an example of this where the baseline condition (here as obtained with the four sensor hot wire) has been plotted together with Pope-spectra for three different Taylor Reynolds numbers. Also shown in this figure is the $\pm 5\%$ measurement uncertainty (see Section

2.2.5) displayed as a grey band. Clearly the measurement uncertainty itself, although larger in the low-end of the spectrum compared to the inertial subrange, does not account for the discrepancy observed with the model spectrum.

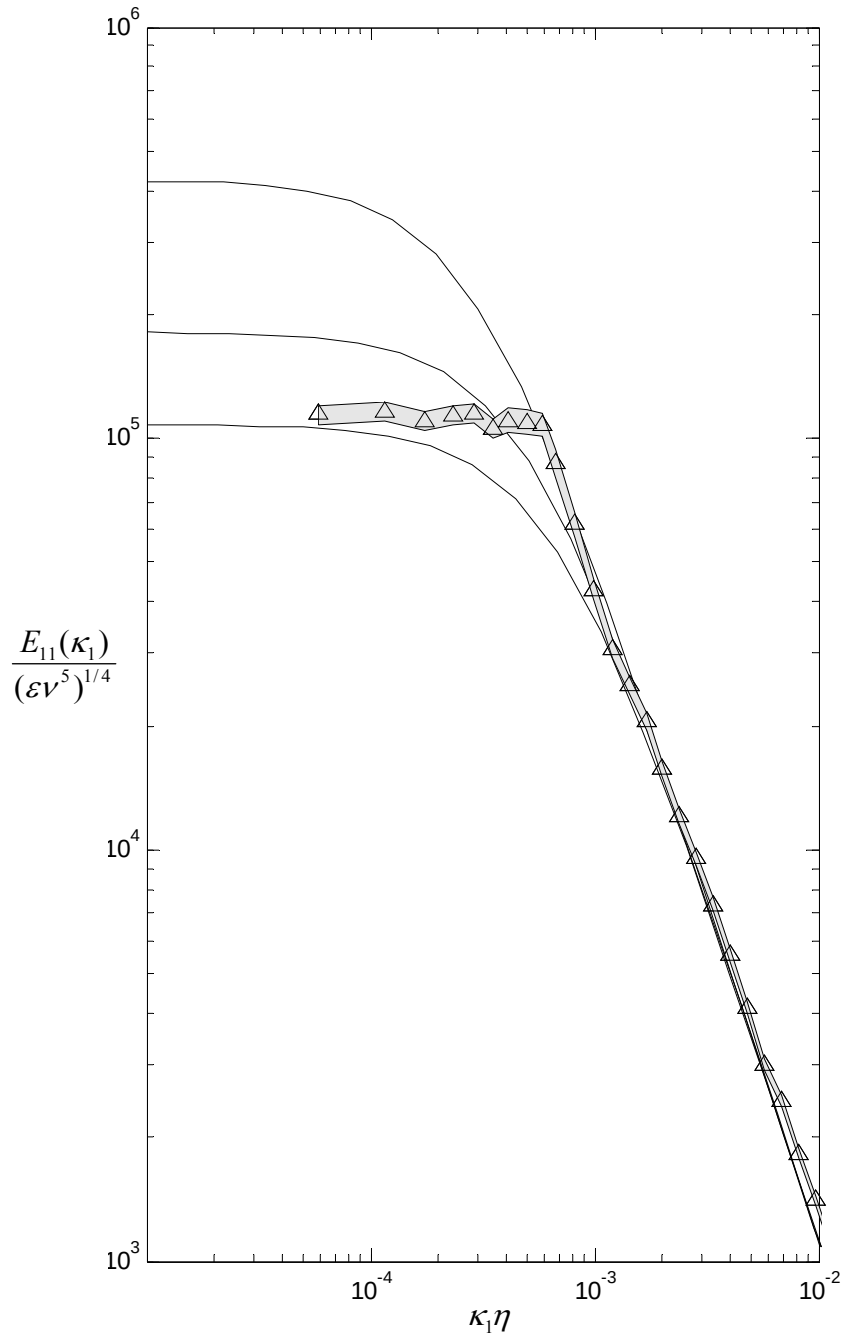


Figure 4.25. Streamwise velocity spectrum (Δ) obtained with a quad-wire superimposed on the Pope-spectra (solid lines) based on Taylor Reynolds numbers of 512, 630 and 886. The grey band shows the measurement uncertainty.

At a theoretical Taylor Reynolds number of 512 ($L_{11} = 28.0$ cm) the low-end asymptote fits the data very well, but a large part of the inertial range is not accounted for and hence the turbulence kinetic energy is severely underpredicted. The second Pope spectrum shows the

regularly obtained Taylor Reynolds number of 630 ($L_{11} = 38.2$ cm). This method overpredicts the very lowest wavenumbers while cutting off part of the apparent inertial range. It must be kept in mind that this occurs close to the region of grid influence. In other words this method could be viewed as an estimate of what the low-end spectrum would look like in the absence of both the confinement of the walls and the active grid. The third spectrum, obtained by extrapolating the spectrum yield a Taylor Reynolds number of 886 ($L_{11} = 63.6$ cm) which assumes that the entire inertial range belongs to the isotropic turbulence and that anything below a certain wavenumber is attenuated by the tunnel. This does not take the grid into consideration at all, and as such is of the least significance. The bottom line however, is that the true Taylor Reynolds number and hence integral lengthscale of the turbulence generated by an active grid in a wind tunnel is not simply defined by any one single model spectrum. This will become very important for the theoretical cascade calculation presented in Chapter 7.

4.7 The Inertial Subrange

4.7.1 Overview

One of the most characteristic properties of the velocity spectrum associated with large scale turbulence is the presence of a substantial inertial subrange. The inertial subrange is the intermediate part of the energy spectrum above the energy containing eddies and below the dissipation range where energy is being transferred from (generally) lower to higher wavenumbers. In this region there is no production or dissipation occurring and hence the spectrum at this point is independent of the viscosity of the fluid. According to the two similarity hypotheses of Kolmogorov this inertial subrange* is only dependent on the dissipation rate, ε , which for all practical purposes is equal to the energy transfer rate. The second Kolmogorov hypothesis also states that the wavenumber (or frequency) spectrum will have a slope of $n = -5/3$ in the inertial subrange when viewed on a logarithmic plot according to Equation 4.26. The same is the case for the one-dimensional spectra, but with different constants (Equations 4.27 and 4.28).

$$E(\kappa) = C\varepsilon^{2/3}\kappa^{-n} \quad (4.26)$$

$$E_{11}(\kappa_1) = C_1\varepsilon^{2/3}\kappa_1^{-n_1} \quad (4.27)$$

$$E_{22}(\kappa_1) = E_{33}(\kappa_1) = C_2\varepsilon^{2/3}\kappa_1^{-n_2} \quad (4.28)$$

Mydlarski & Warhaft (1996) investigated in detail the behavior of the inertial subrange as a function of Taylor Reynolds number. This section compares some of their results with the present data in addition to some supplemental observations.

* The term “subrange” is used since this portion of the spectrum is together with the higher wavenumber dissipation range forms the universal equilibrium range. See Pope (2000) for a brief, yet concise, treatment on the Kolmogorov hypotheses.

4.7.2 The Inertial Range Slope

It is a well known fact in grid generated turbulence that the slope of the inertial range is generally much smaller than the high Reynolds number value of $-5/3$ as stated in the Kolmogorov hypotheses. Mydlarski & Warhaft (1996) had the opportunity to investigate the asymptotic nature of the inertial slope with Re_λ and proposed Equation 4.29 with $B = 5.25$ which described their data with little scatter. Pope (2000) slightly modified this equation, introducing a $Re_\lambda^{-3/4}$ dependence instead which makes the curve asymptote towards $-5/3$ faster at higher Taylor Reynolds numbers but leaves Equation 4.29 essentially intact for $Re_\lambda < 300$.

$$n_1 = -5/3 + B Re_\lambda^{-2/3} \quad (4.29)$$

For the current measurements, n_1 was determined by selecting two points that belong to the inertial subrange and computing the least-squares linear curve fit based on the logarithm of the data. A more conventional method, used by for example Mydlarski & Warhaft (1996) is to plot the compensated spectrum (as described in Section 4.7.4) and by trial and error choose the value of n_1 which yields a constant value for the inertial range in log-log coordinates.

Figure 4.26 shows the current dataset at several different grid rotation rates together with the function given in Equation 4.29 with $B = 5.25$ as suggested. It is immediately evident that the only datapoints that conform to Equation 4.29 are the cases that were obtained using a grid rotation rate of 2 Hz. This is the same parameter as was used by Mydlarski & Warhaft (1996) during their data acquisition. Other rotation rates appear to follow a similar trend, but with a different constant. This gave rise to the suspicion that the value of the inertial slope is flow dependent. In order to show this, the rest of the data was fitted according to Equation 4.29 but with varying values of the constant B . The result can be seen in Figure 4.27. The constant B appears to increase exponentially according to the following equation, whose trend is also shown as an insert in Figure 4.27. For a conventional static turbulence grid, B is in other words estimated to be 4.59.

$$B = 4.588 e^{0.0715\Omega} \quad (4.30)$$

One must be somewhat careful to adopt these relationships as there is very little data available for large ranges in Taylor Reynolds numbers at different grid rotation rates to support them, but the data at hand seems too consistent to be an artifact of random uncertainty. The estimated absolute uncertainties related to extracting the slope values is about 0.01 which more than covers the variation of the shown datapoints from their respective trend lines. Also, the form of Equation 4.29 was adopted for describing each rotation rate was picked solely due to its ability to describe active grid data at a rotation rate of 2 Hz (both from the current data and the dataset of Mydlarski & Warhaft (1996)). There is no supplemental evidence present that dictates that other rotation rates must adhere to the same form.

This discovery that that the grid-generated turbulence may actually not be completely self-similar in the inertial range for different flow condition is quite curious and should be validated with further testing. It implies that a flow at a Taylor Reynolds number of 4 Hz is a completely different flow than one generated by a grid operating at 10 Hz at identically the same Taylor

Reynolds number. The lower rotation rate implies a more “ideal” or high-Reynolds number behavior than that produced by the latter operating condition.

One might think that this should have been caught when scaling the spectra with Kolmogorov scaling such as the one in Figure 4.23, but it is important to remember that with the forgiving nature of a log-log plot, relatively small differences such as the ones revealed here would go largely unnoticed.

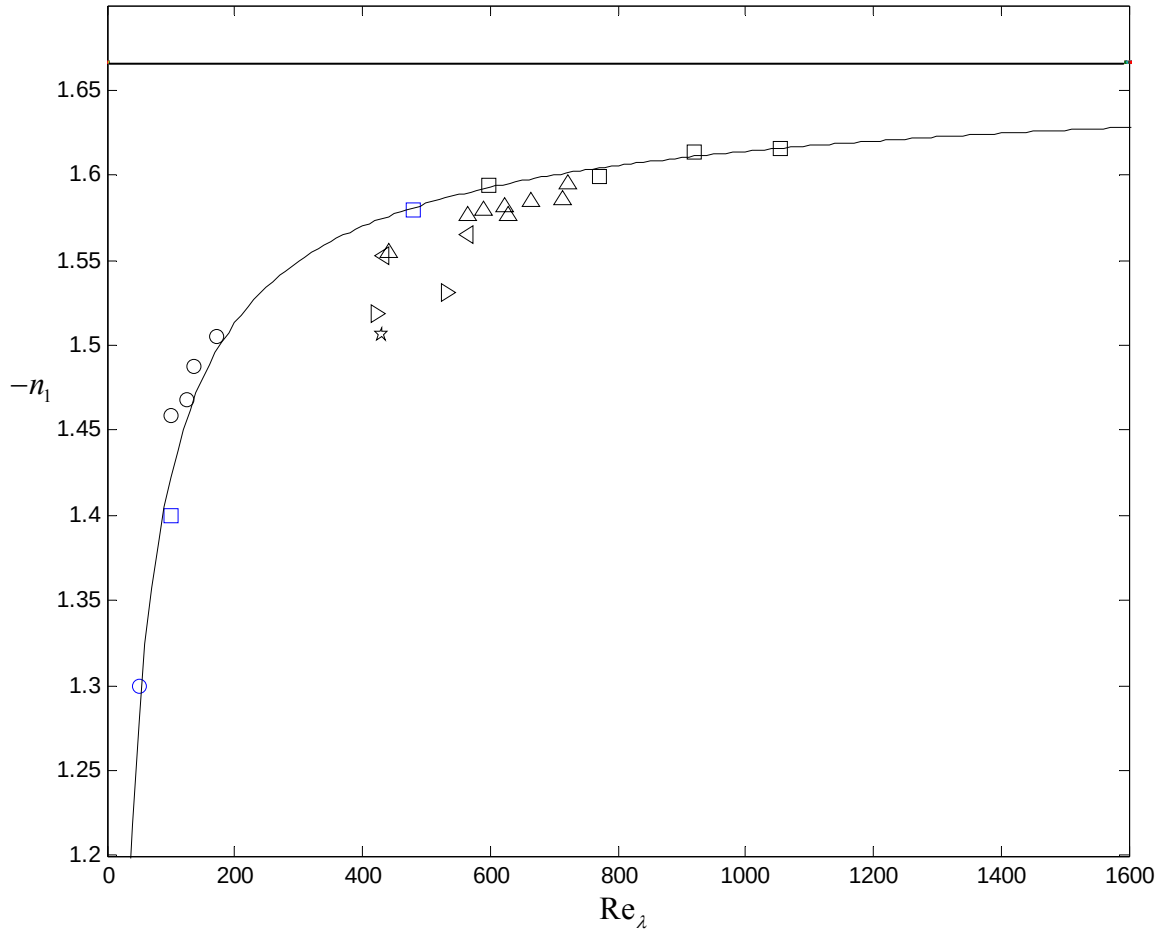


Figure 4.26. Inertial range slope n_1 as a function of Re_λ for several different rotation rates. Black datapoints: current study, blue datapoints: Mydlarski & Warhaft (1996). $\circ$, $\Omega = 0$ Hz; $\square$, $\Omega = 2$ Hz; $\triangle$, $\Omega = 4$ Hz; $\triangleleft$, $\Omega = 6$ Hz; $\triangleright$, $\Omega = 8$ Hz; $\star$, $\Omega = 10$ Hz.

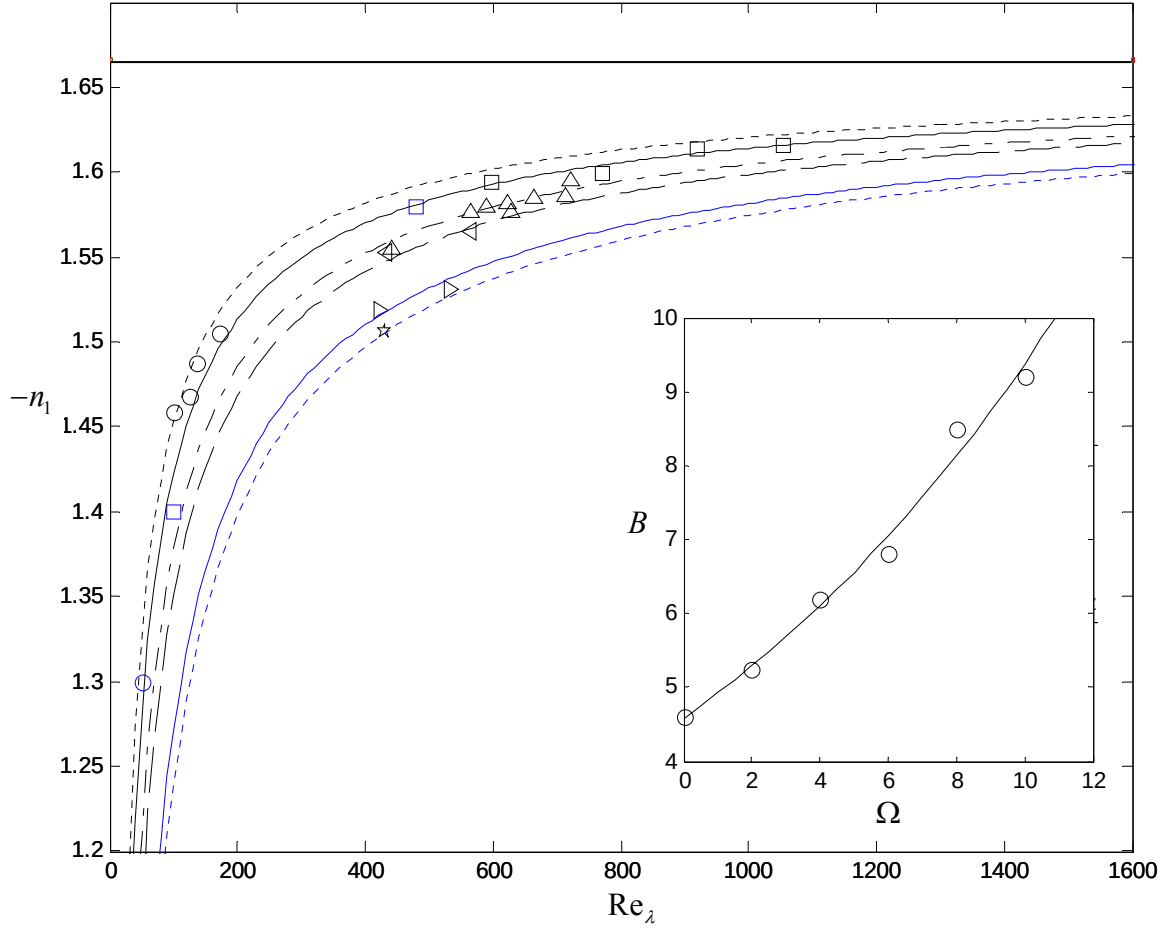


Figure 4.27. Inertial range slope n_1 as a function of Re_λ for several different rotation rates. Black datapoints: current study, blue datapoints: Mydlarski & Warhaft (1996). $\circ$, $\Omega = 0$ Hz; $\square$, $\Omega = 2$ Hz; $\triangle$, $\Omega = 4$ Hz; $\triangleleft$, $\Omega = 6$ Hz; $\triangleright$, $\Omega = 8$ Hz; $\star$, $\Omega = 10$ Hz. Lines according to Equation 4.29 are shown with empirically fitted constants B shown in the insert. The trend in the insert is given in Equation 4.30.

4.7.3 Separation of Scales and Inertial Range Dilation

As the Taylor Reynolds number increases the energy containing scales progressively separate from the dissipation scales, and even though the Kolmogorov scale grows, the macroscale grows even faster. As Mydlarski & Warhaft (1996) points out, Equation 1.6 can be written in terms of the integral scale L_{11} and the Kolmogorov scale η .

$$L_{11} / \eta = A^{1/4} Re_T^{3/4} \quad (4.31)$$

Mydlarski & Warhaft (1996) presented evidence to support a value of 0.9 for the constant A . Pope (2000) on the other hand predicts the high Reynolds number value of A to be closer to 0.79. The problem is that A is not a true constant for $Re_\lambda < 500$. This is due to the fact that at low Taylor Reynolds numbers the scale separation rate for the integral scale is not the same as that

for the macroscale. For the macroscale, Equation 1.12 is valid throughout the Taylor Reynolds number range whereas Equation 4.31 is not.

For Taylor Reynolds numbers less than 500, the constant A will be larger due to the non-constant relationship between the macroscale L and the integral lengthscale L_{11} . This ratio L_{11}/L is presented in Figure 4.28 as a function of Re_λ for the current dataset where Pope's (2000) prediction has been superimposed on the data. Note the difference between the two methods of obtaining the integral scale. The uncertainty of the zero-bin method (Equation 1.4) manifests itself as an immense scatter while the spectral peak method introduced in Chapter 1 gives a relatively consistent result with Pope's (2000) model spectrum predictions.

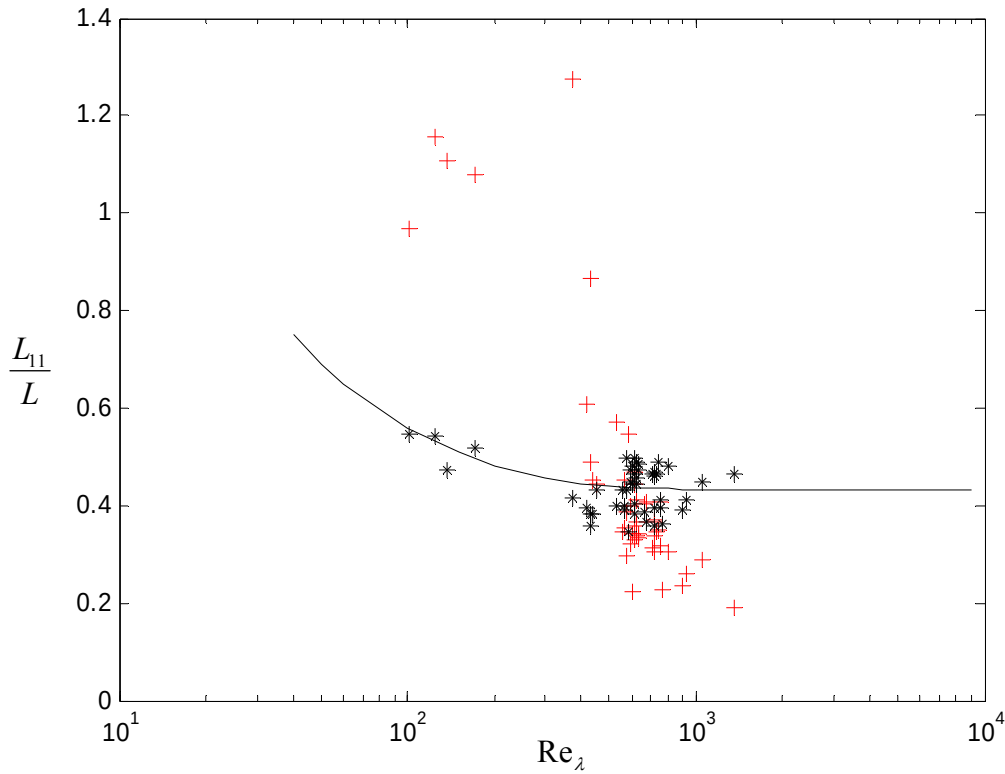


Figure 4.28. The ratio of the integral scale to the macroscale as a function of Re_λ . +, L_{11} obtained via Equation 1.4; *, L_{11} obtained via the spectrum peak method. The solid line gives the ratio as determined from the Pope model spectrum.

To demonstrate the spectral peak method the longitudinal one-dimensional spectrum is multiplied by the wavenumber and plotted as shown in Figure 4.29. The peak is taken as the wavenumber equivalent of the integral lengthscale through $L_{11} = 1/\kappa_{\text{peak}}$. In most cases there is an additional peak located at the first harmonic of the mean rotation rate and for less random forcing protocols also at higher harmonics (Makita and Miyamoto, 1983). Hence one must exercise caution in selecting the correct peak corresponding to the integral lengthscale. Given the nature of the method, the lower the grid speed, the less reliable the method is. The second parameter which contributes to the accuracy of the method is the frequency spacing of the spectrum (in other words the number of samples obtained). The method can only estimate the wavenumber to an accuracy of about half a frequency bin which for really large lengthscales can introduce

quite a significant bias for low resolution spectra. This undoubtedly accounts for some of the scatter seen in the trend of L_{11}/L in Figure 4.28.

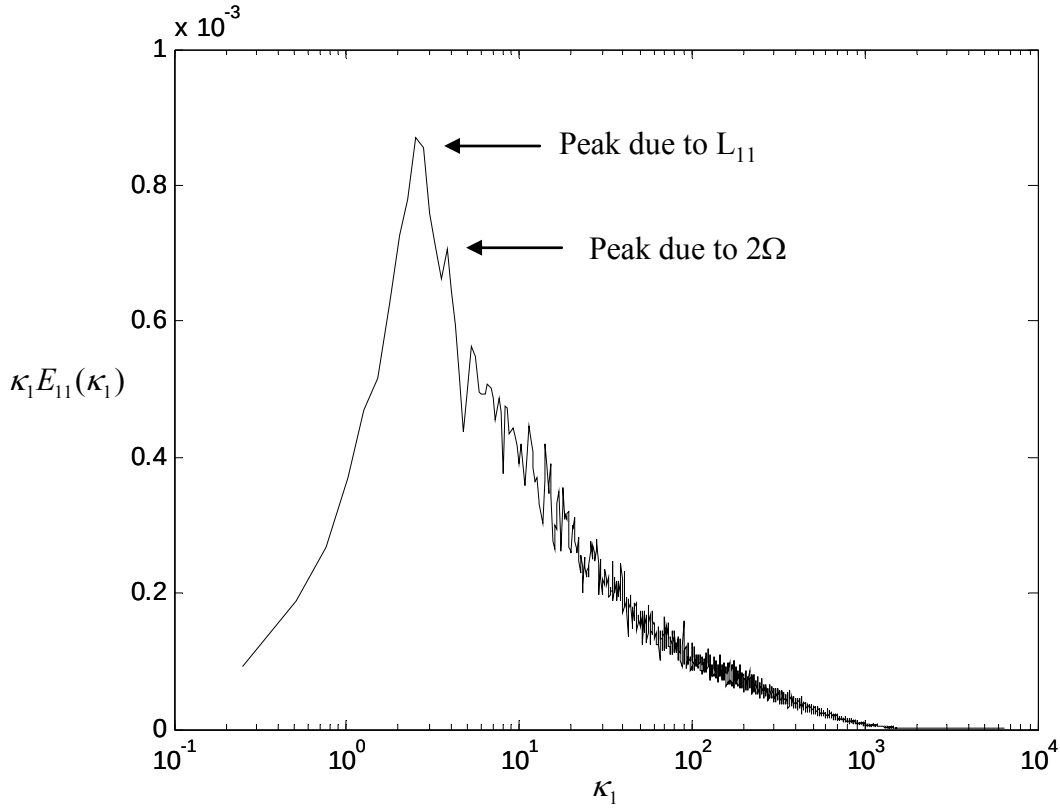


Figure 4.29. Demonstration of the spectrum peak method for obtaining the integral lengthscale.

When using Equation 4.31 one must be careful to include the Taylor Reynolds number effect on the constant A . Alternatively one could simply utilize Equation 1.12 which has no such dependence. Figure 4.30 shows L_{11}/η vs. Re_T with the Pope prediction superimposed on top. A close agreement is easily verified. From this figure it appears that all the datapoints fall on the same line with minimal scatter. This is not quite the case as the scaling of this figure does not reveal inconsistencies very well.

Another way of showing the same thing is by plotting the turbulence Reynolds number against the square of the Taylor Reynolds number as shown in Figure 4.31. The relationship will be related according to Equation 4.32 which is obtained by simply inserting $\eta = (\nu^3/\varepsilon)^{1/4}$ back into Equation 4.31. It is obvious from this figure ($A = 0.79$) that the static grid measurements do not adhere onto this line due to the non-constant behavior of A at low Taylor Reynolds numbers. One could also plot Re_L (as opposed to Re_T) in the same fashion and all the points would fall on the same line, by definition.

$$Re_T = (A/15) Re_\lambda^2 \quad (4.32)$$

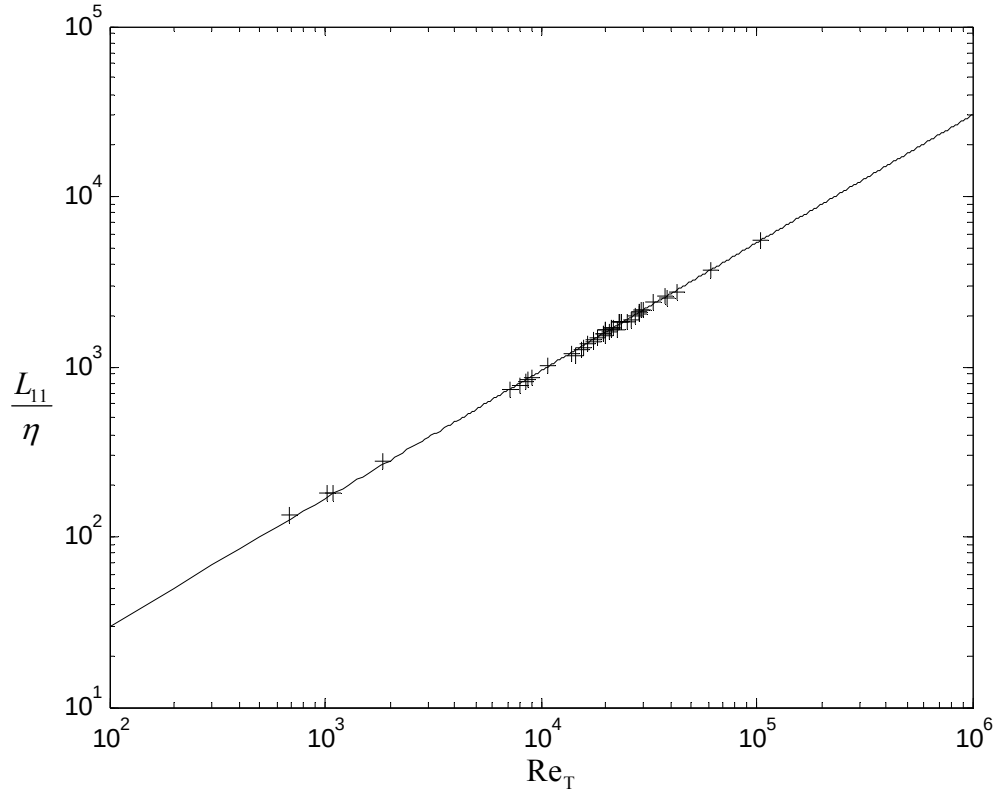


Figure 4.30. Separation of scales as a function of turbulence Reynolds number.

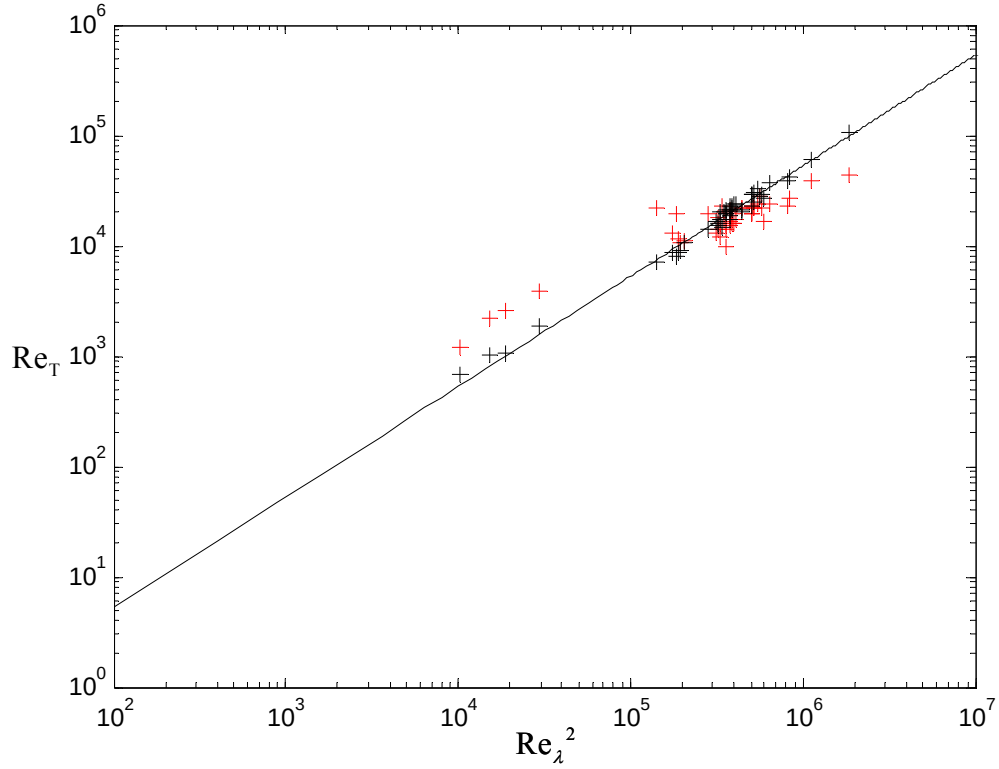


Figure 4.31. Separation of scales as a function of Taylor Reynolds number squared. +, L_{11} obtained via Equation 1.4; +, L_{11} obtained via the spectrum peak method. The solid line gives the high Reynolds number Pope prediction.

It can be seen in Figure 4.31 that for the static grid effort with Taylor Reynolds numbers in the range of 100 to 200 the values are higher than the high Re_λ prediction and indeed consistent with a constant A of 0.9 which would seemingly suggests that there is a bias towards the low Taylor Reynolds numbers in the data of Mydlarski & Warhaft (1996). However it is important to remember that the definition of A in Equation 1.7 assumes isotropic turbulence. Mydlarski & Warhaft (1996) reported an anisotropy ratio of 1.21 which means that the turbulence kinetic energy is severely overpredicted by using the isotropic relation $k = (u^2 + v^2 + w^2) = 3u^2/2$. This will tend to increase the observed value of A . This, coupled with the possible use of data stemming from the unknown behavior of the synchronous forcing protocol as well the inclusion of a questionable datapoint at very low Reynolds numbers in Mydlarski & Warhaft's (1996) Figure 7, go a long way explaining the differences. In conclusion it can be stated that the current data supports the Pope (2000) model spectrum behavior with regards to the relationship between different length scales, even at low Taylor Reynolds numbers although the available data is limited for $Re_\lambda < 300$.

Related to the separation of the integral lengthscale and the Kolmogorov scale is the concept of inertial range dilation. Separating the energy-containing eddies and the dissipative eddies, the inertial range should logically grow as fast as the scales are separating. As derived above in Equation 4.31 the scales separate at a rate of $Re_\lambda^{3/4}$ which when combined with Equation 4.32 gives a growth of $Re_\lambda^{3/2}$. Hence when plotting the ratio $\kappa_{max}/\kappa_{min}$, which signifies the end of the inertial subrange over the beginning, against Re_λ on a logarithmic scale, it should appear as a straight line. Mydlarski & Warhaft (1996) used Equation 4.33 as a best fit to their data. This fitted line has been reproduced in Figure 4.32 together with the current data. The data agrees closely with the Mydlarski & Warhaft (1996) dataset which clearly shows the predicted trend of $Re_\lambda^{3/2}$. The Taylor Reynolds number dependence of the integral scale does not seem to be an issue at the wavenumbers involved in the inertial range since all datapoints seem to fit the line with some inherent scatter due to the intricacy of determining the cutoff points of the inertial subrange.

$$\frac{\kappa_{max}}{\kappa_{min}} = 0.071 Re_\lambda^{2/3} \quad (4.33)$$

Another common way of measuring the scale separation is to take the ratio of a wavenumber associated with the dissipative eddies (κ_d) to the energetic eddies (κ_e). The larger the resulting ratio, the larger is the separation of scales. These wavenumbers are most commonly defined as: $\kappa_d = 1/\eta$ and $\kappa_e = 3/(4L_{11})$. For the baseline condition the ratio κ_d/κ_e is close to 2250 whereas smaller facilities with equipped with static grids generally produce ratios one order of magnitude smaller.

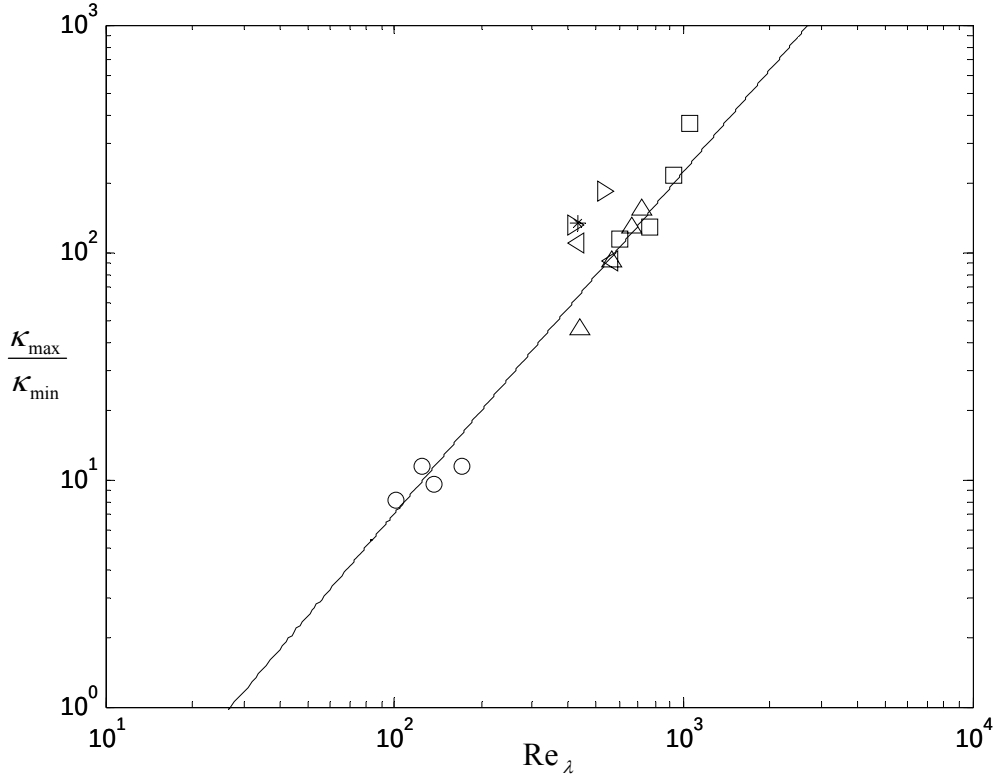


Figure 4.32. Inertial subrange dilation as a function of Taylor Reynolds number. $\circ$, $\Omega = 0$ Hz; $\square$, $\Omega = 2$ Hz; $\triangle$, $\Omega = 4$ Hz; $\triangleleft$, $\Omega = 6$ Hz; $\triangleright$, $\Omega = 8$ Hz; $*$, $\Omega = 10$ Hz. The relation given in Equation 4.33 is indicated with a solid line.

4.7.4 The Kolmogorov Constant

The Kolmogorov constant, C , has for sufficiently high Reynolds numbers a value of 1.5 according to significant experimental evidence as surveyed by for example Sreenivasan (1995), and consistent with the boundary layer data of Saddoughi and Veeravalli (1994). The one dimensional Kolmogorov constants C_1 and C_2 can be obtained through Equations 4.21 and 4.22 coupled with the inertial range descriptions (Equations 4.26 through 4.28) to obtain the following relationships, as shown by Pope (2000), where n is the slope of the spectra in the inertial subrange.

$$C = \frac{1}{2}n(n-2)C_1 \quad (4.34)$$

$$C_2 = -\frac{1}{2}(n-1)C_1 \quad (4.35)$$

However for high Taylor Reynolds number turbulence, $n = -5/3$ which yields:

$$C_1 = \frac{18}{55}C \approx 0.49 \quad (4.36)$$

$$C_2 = \frac{4}{3}C_1 = \frac{24}{55}C \approx 0.65 \quad (4.37)$$

These values of C , C_1 and C_2 can be confirmed in the model spectrum shown in Figure 4.22 where the level of the inertial ranges between the three spectra can be seen to be in this proportion.

Obtaining the Kolmogorov constant C_1 is traditionally done by either curve-fitting the inertial range using Equation 4.26, or alternatively by plotting the compensated spectrum $\varepsilon^{-2/3} \kappa_1^{5/3} E_{11}(\kappa_1)$ against $\kappa_1 \eta$. For high Taylor Reynolds number spectra the inertial range should appear flat on such a plot with its value equal to the Kolmogorov constant. Figure 4.33 shows such a compensated streamwise velocity spectrum for $Re_\lambda = 660$ which demonstrates some of the difficulties associated with obtaining the Kolmogorov constant this way. The inertial range is far from flat due to the fact that the slope is not quite equal to the Kolmogorov value of $-5/3$ for this Reynolds number. Even the atmospheric boundary layer measurements presented in Praskovsky & Oncley (1994) show that for Taylor Reynolds numbers between 2000 and 12700 the inertial range slope takes on values less than $-5/3$ since the compensated spectra show a small, yet clear increase with $\kappa_1 \eta$.

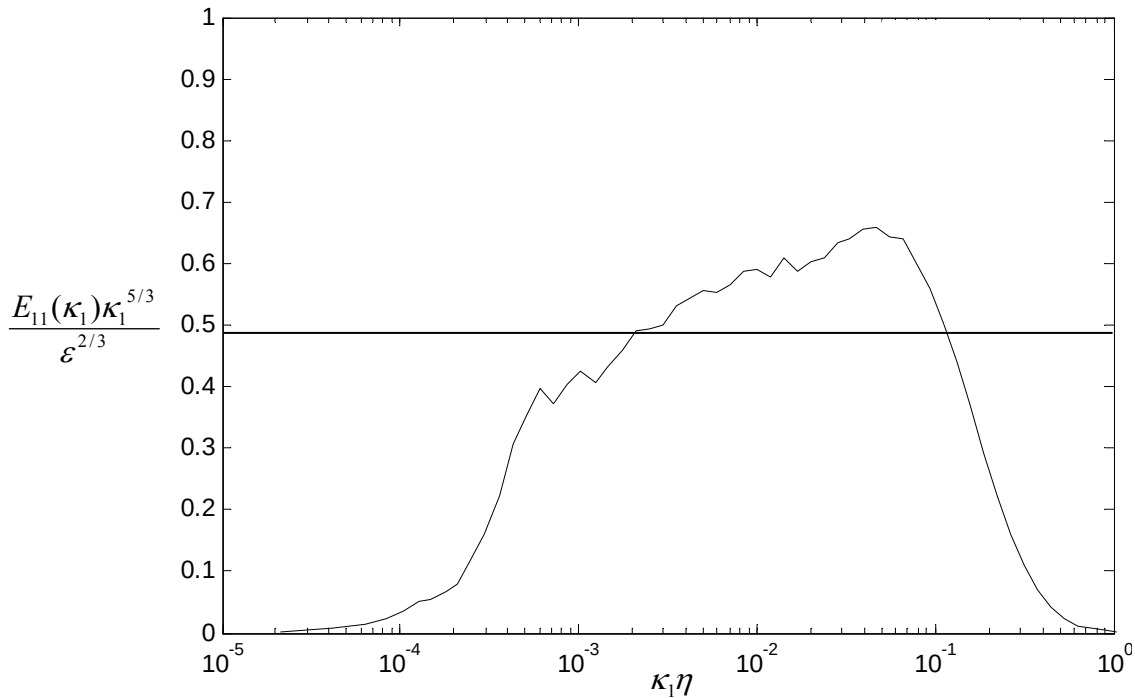


Figure 4.33. Standard compensated spectrum at $Re_\lambda = 660$

The strong relationship between the inertial range slope and the value of the Kolmogorov constant can however be utilized to deduce the trend of the constants as a function of Reynolds number. Mydlarski & Warhaft (1996) plotted a modified similarity form according to Equation 4.38.

$$E_{11}(\kappa_1) = C_{1*} \varepsilon^{-2/3} \kappa_1^{-5/3} (\kappa_1 \eta)^{5/3-n_1} \quad (4.38)$$

This form of the compensated spectrum ensures that as $Re_\lambda \rightarrow \infty$, n_1 approaches $-5/3$ which will eliminate the last factor in Equation 4.38 and ensures that $C_{1*} \rightarrow C_1$. In addition, since the exponent now reflects the actual slope of the inertial range the compensated spectrum will be flat as long as the value of n_1 is picked correctly. The method for determining the value of the inertial range slope was described above. The result applied to the spectrum for $Re_\lambda = 660$ with a corresponding n_1 of -1.584 is shown in Figure 4.34. The level of the flat portion of these modified compensated spectra is the value of the modified Kolmogorov constant C_{1*} and will be significantly higher than the accepted value for C_1 for low Taylor Reynolds numbers. This method gives a much better view of the trend towards a truly universal Kolmogorov constant than previous efforts that attempt to obtain a direct value of C_1 . Sreenivasan (1995) present some of these efforts which, with significant scatter, show a general increase of C_1 for $Re_\lambda < 50$ followed by a decrease for subsequent Taylor Reynolds numbers with an asymptotic trend towards a value consistent with the accepted value of 0.49.

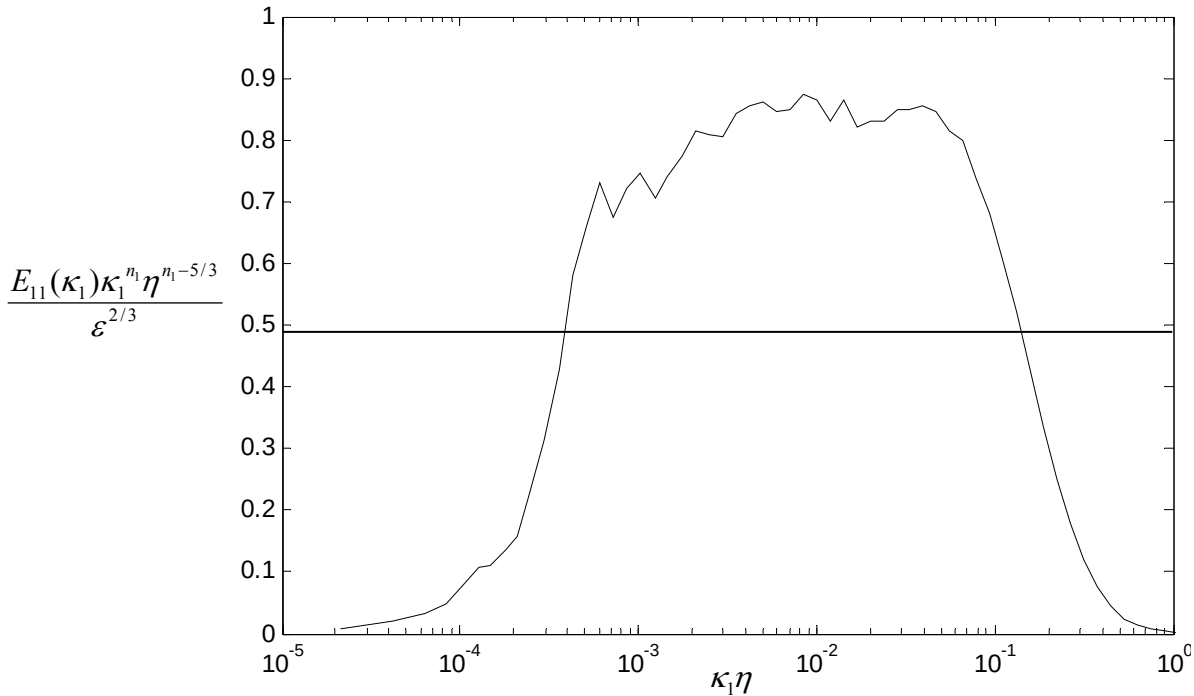


Figure 4.34. Modified compensated spectrum at $Re_\lambda = 660$ with $n_1 = -1.584$.

Current single-wire results are presented in Figure 4.35 for a range of Taylor Reynolds numbers and mean rotation rates. By plotting the difference between the infinite Reynolds number inertial slope and the actual slope against the difference between the modified Kolmogorov constant and the accepted value it is possible to get an idea of the asymptotic behavior of the experimental results. At first glance it looks like the data in Figure 4.35 follow a power-law given by the solid black line according to Equation 4.39.

$$C_{1*} - 0.49 = 1.6426(5/3 + n_1)^{2/3} \quad (4.39)$$

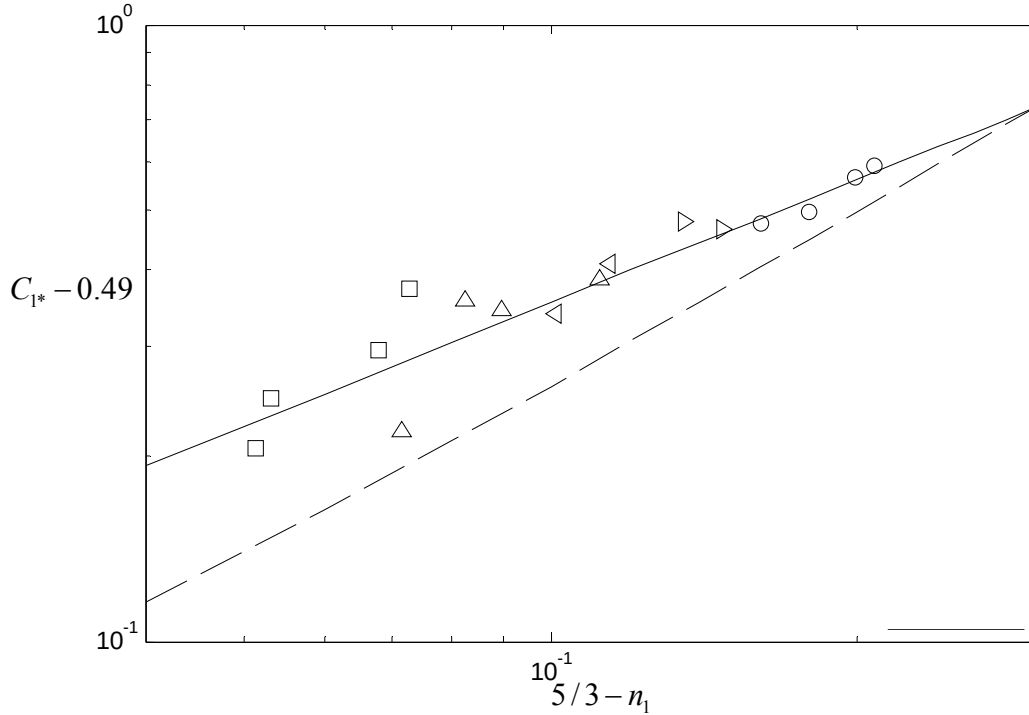


Figure 4.35. Relationship between the modified Kolmogorov constant and the inertial range slope. $\circ$, $\Omega = 0$ Hz; $\square$, $\Omega = 2$ Hz; $\triangle$, $\Omega = 4$ Hz; $\triangleleft$, $\Omega = 6$ Hz; $\triangleright$, $\Omega = 8$ Hz; The solid line is the best fit to all the displayed data given in Equation 4.39 while the dashed line is the line fitted by Mydlarski & Warhaft (1996).

As noted in Section 4.7.2 there is a strong dependence on the grid rotation rate on the slope of the inertial range and one would expect the same to be the case for the modified Kolmogorov constant. By examining the datapoints that were obtained with the same forcing scheme it becomes clear that Equation 4.39 fits the static grid data relatively well, but when isolating the data for which $\Omega = 2$ Hz it becomes evident that a completely different line (Equation 4.40) fits better.

$$C_{1*} - 0.49 = 13.291(5/3 + n_1)^{1.3845} \quad (4.40)$$

By examining each of the rotation rates separately it is possible to estimate separate trends although with the lack of data points available there is obviously a large extent of uncertainty associated with this type of curve-fitting. An attempt of this is given in Figure 4.36, which shows

that there is a very high probability that the slope of the trend lines increase with grid rotation rate.

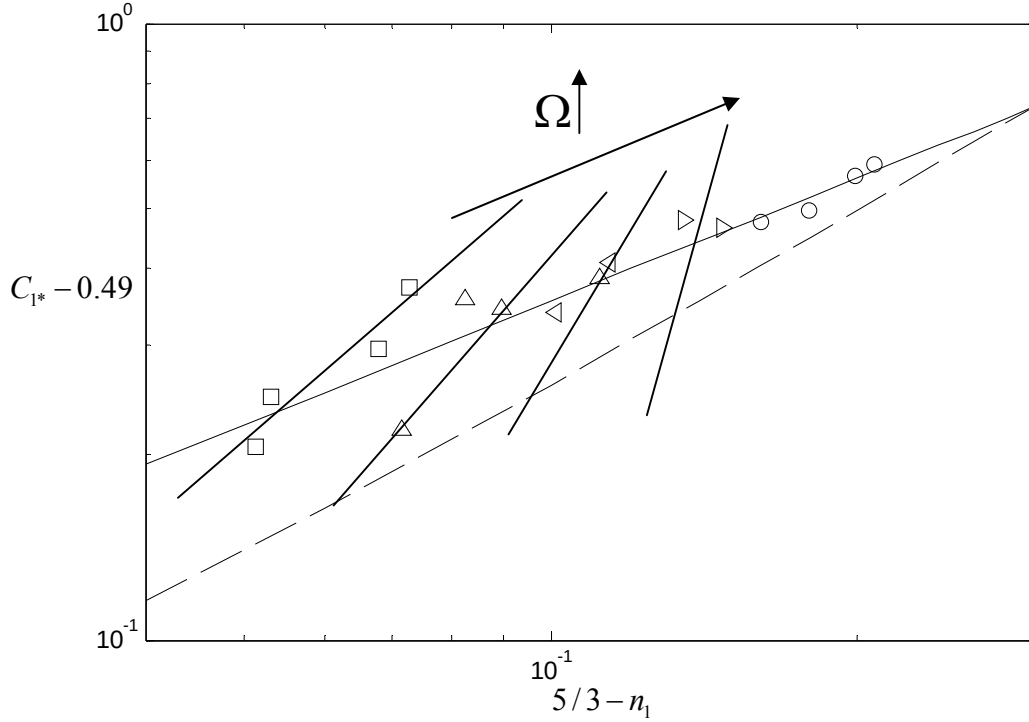


Figure 4.36. Relationship between the modified Kolmogorov constant and the inertial range slope. $\circ$, $\Omega = 0$ Hz; $\square$, $\Omega = 2$ Hz; $\triangle$, $\Omega = 4$ Hz; $\triangleleft$, $\Omega = 6$ Hz; $\triangleright$, $\Omega = 8$ Hz; The solid line is the best fit to all the displayed data given in Equation 4.39 while the dashed line is the line fitted by Mydlarski & Warhaft (1996). Best fits for increasing rotation rates are superimposed to show dependence on Ω .

Based on the preceding analysis it is now possible to find the Taylor Reynolds number dependence of the modified Kolmogorov constant by taking Equation 4.39 or 4.40 and combining them with Equation 4.29. It is, however, very important to combine the equations with the proper constant B to compensate for the grid rotation rate dependence. The result can be shown in Figure 4.37 and the relationship is given in Equation 4.41 for static grids ($B = 4.59$) and in Equation 4.42 for a grid rotation rate of 2 Hz ($B = 5.29$). Mydlarski & Warhaft's (1996) prediction $C_{1*} = 0.51 + 12.6\text{Re}_\lambda^{-2/3}$ is also given in Figure 4.37 for reference. The reason for the lower values for the modified Kolmogorov constants obtained by these authors is unknown, but since the trends are flow dependent there are many factors that could cause this discrepancy, for example inclusion of data obtained using synchronous grid forcing,

$$C_{1*} = 0.49 + 4.987\text{Re}_\lambda^{-4/9} \quad (4.41)$$

$$C_{1*} = 0.49 + 133.41\text{Re}_\lambda^{-0.923} \quad (4.42)$$

Although Equations 4.41 and 4.42 give one estimate on how fast the Kolmogorov constant converges to 0.49 in grid turbulence, it does depend a lot on especially Equation 4.29 which is hard to verify with the limited data available. Another more *ad hoc* approach would be to fit a straight line to the active grid data in Figure 4.37 which would reach a value of 0.49 at approximately $Re_\lambda = 2500$. In contrast, Saddoughi & Veeravalli (1994) obtained convergence in their boundary layer flows at $Re_\lambda \approx 1450$, while the atmospheric boundary layer data of Praskovsky & Oncley (1994) shows the Kolmogorov constant dropping from 0.61 to 0.52 between $2000 > Re_\lambda > 12700$, supporting the notion that different type of flows will exhibit wildly different Taylor Reynolds number trends with respect to the convergence of the Kolmogorov constant as well as the inertial range slope value.

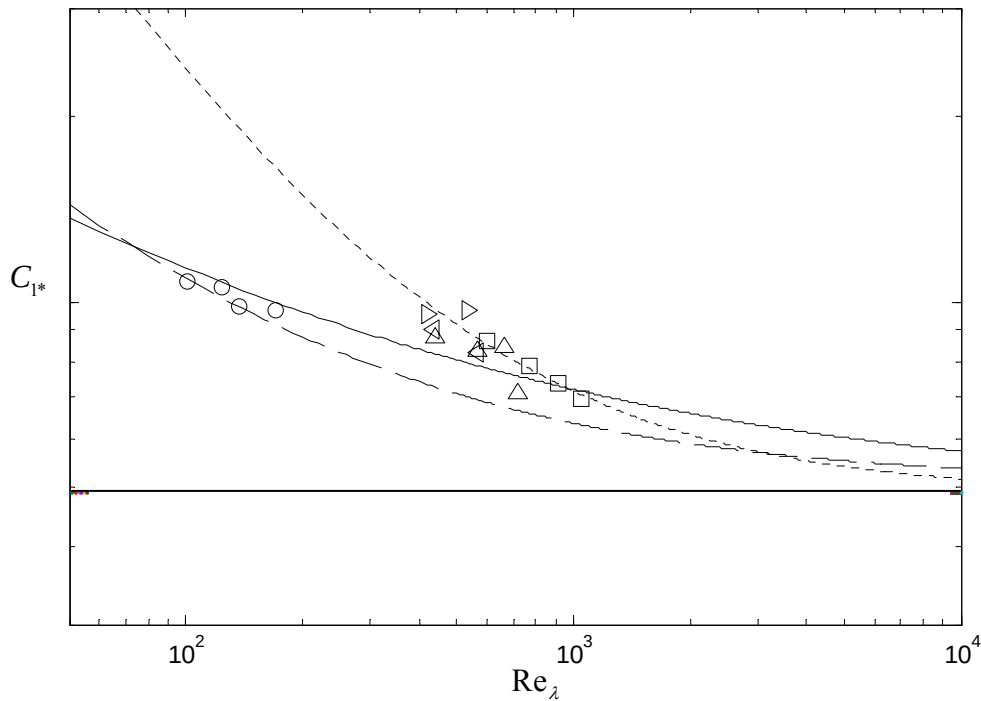


Figure 4.37. The Kolmogorov constant as a function of Taylor Reynolds number. $\circ$, $\Omega = 0$ Hz; $\square$, $\Omega = 2$ Hz; $\triangle$, $\Omega = 4$ Hz; $\triangleleft$, $\Omega = 6$ Hz; $\triangleright$, $\Omega = 8$ Hz; The solid line is the best fit to all the displayed data given in Equation 4.41. The dotted line uses the inertial range slopes for $\Omega = 2$ Hz (Equation 4.42) while the dashed line is the line fitted by Mydlarski & Warhaft (1996).

4.7.5 The “Bottleneck” Effect

A spectral “bump” is often seen, especially in wall bounded flow, at the transition between the inertial range and the dissipation range. It manifests it self as a region with a slope less than that of the inertial range and have been attributed to a “bottleneck effect” as described by Sreenivasan (1995) where the constant transferred energy piles up at the end of the inertial range prior to reaching the dissipating effects at larger wavenumbers. A good example of the presence of such spectral bumps is the boundary layer data given in Saddoughi & Veeravalli (1994) where the compensated spectrum clearly show an increase in wavenumber at the end of the inertial

range for $Re_\lambda = 600$ and 1450. Mydlarski & Warhaft (1996) suggested that the effect of this bump could be diminished by compensating the spectrum with a value smaller than $-5/3$ following the method used in Section 4.7.4. Their paper shows the outcome from applying this technique to the Saddoughi & Veeravalli (1994) dataset which resulted in Kolmogorov constants consistent, but slightly lower than the Mydlarski & Warhaft (1996) trend shown in Figure 4.37. However from the dimensional spectra given in Saddoughi & Veeravalli (1994) it becomes clear that the existence of the spectral bump in those measurements can not be completely hidden by scaling as there is a clearly bulbous trend of the inertial range prior to the dissipation range.

The spectral bump is usually particularly evident in compensated spectra and can possibly be discerned in the current measurements as seen in Figure 4.33, but is not substantially larger than other irregularities in the inertial range. When using the modified compensated method (Equation 4.38) any evidence of the bump somewhat drowns as seen in Figure 4.34. This seems to be consistent throughout the entire data set obtained from the lowest Taylor Reynolds number static grid data to the highest Taylor Reynolds number upstream active grid measurements. From the data presented in Mydlarski & Warhaft (1996) it seems like only the synchronous forcing protocols resulted in spectral bumps, even at only $Re_\lambda = 99$, whereas the single random forcing protocol for the most part display little or no such effect. This also agrees with the measurements of Makita (1991) which shows no discernable spectral bump either.

The larger scale active grid measurements of Kang *et al.* (2003), on the other hand, presents significant evidence of the bottleneck behavior which seems to increase with Taylor Reynolds numbers. The generation of the turbulence in Kang *et al.* (2003) study is very similar to the one used in for the current results and it is very curious that such a difference would exist. The spectral bump is not really discernable in the velocity spectrum, compared to the data of Saddoughi & Veeravalli (1994). The compensated spectrum presented shows a significant bump close to the dissipation range for the most upstream location ($X/M = 20$). However it is believed that if the spectrum was compensated properly with the actual inertial range slope the presence of the bump would be much less noticeable. In addition Kang *et al.* (2003) seems to have a larger difference between the streamwise and the lateral inertial range slope exponents which, when plotting the compensated spectra together amplifies the bottleneck effect appearance.

In conclusion, it is suggested that large scale active grid turbulence, when generated using a sufficiently randomized forcing protocol, should display much less evidence of a bottleneck effect, and that increasing the Taylor Reynolds number overall does very little to enhance this effect. This is in stark contrast to boundary layer measurements where the presence of the spectral bump is quite pronounced and can even be seen by the naked eye in the velocity spectrum (refer to Saddoughi & Veeravalli, 1994 for a good example of this)

4.7.6 Lateral Spectral Components

So far in the discussion only the streamwise component has been considered. Mydlarski & Warhaft (1996) also offered some insight into the behavior of the lateral components. One of their key findings were that the slope of the inertial range for the lateral component was consistently less than that of the streamwise component, but that the difference between them decreased with Taylor Reynolds number to a value of about 0.02 at $Re_\lambda \approx 500$. In the following section the relationship between the lateral and the streamwise components of the current data will be investigated.

Figure 4.38 shows the result of dividing the lateral spectrum by the streamwise spectrum with Kolmogorov scaling at $X/M = 37.3$ for $\Omega = 4$ Hz and $U_\infty = 12.4$ m/s. This effectively plots C_3/C_1 which according to Equation 4.37 is $4/3$ in the inertial range. According to the Pope (2000) model spectrum the ratio of these two spectra should be 0.5 at the lowest wavenumbers. Figure 4.38 supports these claims to a high degree even though it has already been shown that the high Reynolds number predictions are still not quite valid at this Taylor Reynolds number. The fact that n_2 is always smaller than n_1 for Taylor Reynolds numbers in this range can be seen from the fact that the spectral ratio increases throughout the inertial range consistent with the findings of Mydlarski & Warhaft (1996). This can be remedied by utilizing the same approach that was used to achieve a flat inertial range in the compensated spectra which will be discussed below.

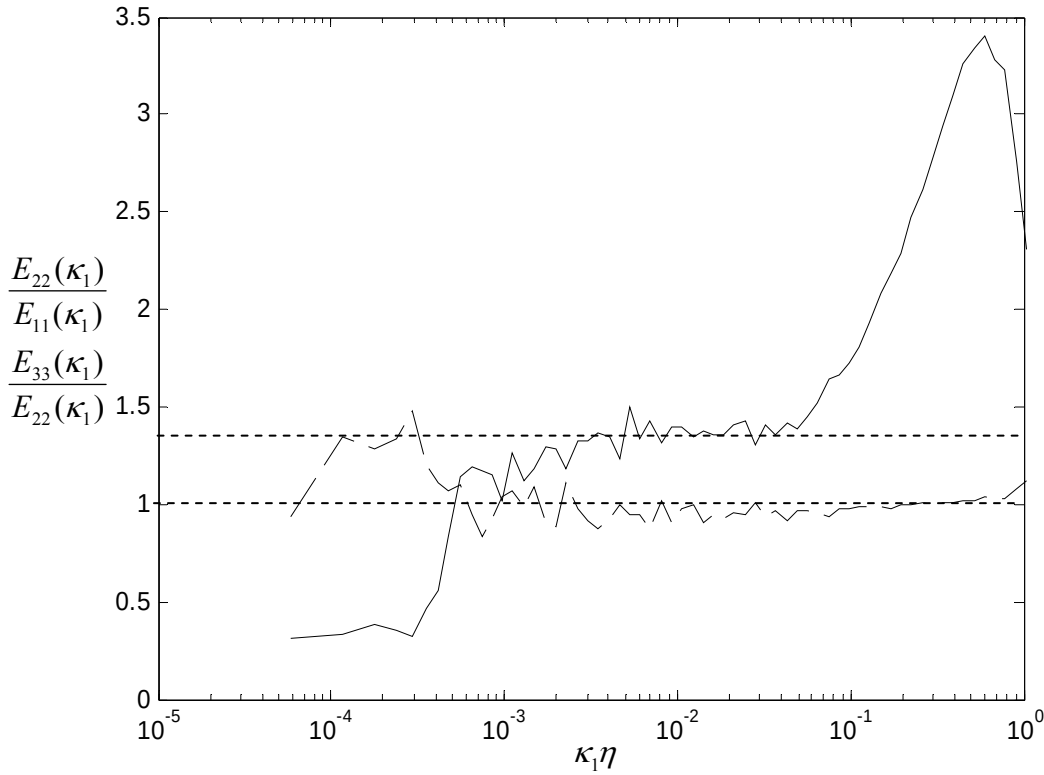


Figure 4.38. Ratio of one-dimensional velocity spectra: —, $E_{22}(\kappa_1)/E_{11}(\kappa_1)$; --, $E_{33}(\kappa_1)/E_{22}(\kappa_1)$ for the baseline case. Dashed lines are superimposed for expected high Reynolds number values at $4/3$ and 1 .

The ratio of the two lateral spectra is also plotted in Figure 4.38. Ideally this curve should remain at unity all throughout the wavenumber range as there should be no difference between these components. It can be clearly seen that this is roughly the case for all wavenumbers above the wind tunnel wavenumber ($\kappa_1\eta \approx 7.8 \times 10^{-4}$). The difference between the two lateral spectra below this fundamental wind tunnel wavenumber can be attributed to 1) random uncertainty in the probe 2) random effects of the tunnel on the large eddies in both directions and 3) lack of a sufficient sampling time. It is expected that by increasing the number of records, this uncertainty would go sharply down as there seems to be no pattern between different measurement locations in whether w is larger than v or vice versa. In the particular case $v > w$ by approximately 1.4%.

Even though the large scale fluctuations of w appear significantly larger, the slightly smaller value all throughout the inertial subrange makes up for this difference.

Going back to estimating the difference in inertial range slopes between the streamwise and lateral components, Equation 4.38 and equivalent equations* for the two lateral components were taken to form the ratio $E_{22}(\kappa_1) / E_{11}(\kappa_1)$ which yields Equation 4.43.

$$\frac{E_{22}(\kappa_1)}{E_{11}(\kappa_1)} = \frac{C_{2*}}{C_{1*}} (\kappa_1 \eta)^{n_1 - n_2} \quad (4.43)$$

Again as the Taylor Reynolds number goes to infinity, $n_2 \rightarrow n_1 \rightarrow -5/3$ which leaves $C_{2*}/C_{1*} \rightarrow C_2/C_1 \rightarrow 4/3$. A properly compensated spectral ratio is plotted in Figure 4.39. A flat plateau was obtained by applying a difference of 0.02 between the two slopes which is perfectly consistent with the findings of Mydlarski & Warhaft (1996) This suggests that the relationship between the streamwise and lateral spectra is not a function of grid rotation rate, at least not in the inertial range. A best guess estimate of C_2/C_1 from Figure 4.38 is 1.37 which is quite similar to the high Reynolds number value. The more easily found ratio C_{2*}/C_{1*} obtained from Figure 4.39[†] is 1.46 which is somewhat smaller than, but still comparable to the Mydlarski & Warhaft (1996) estimate of 1.5 for a Taylor Reynolds numbers of 660.

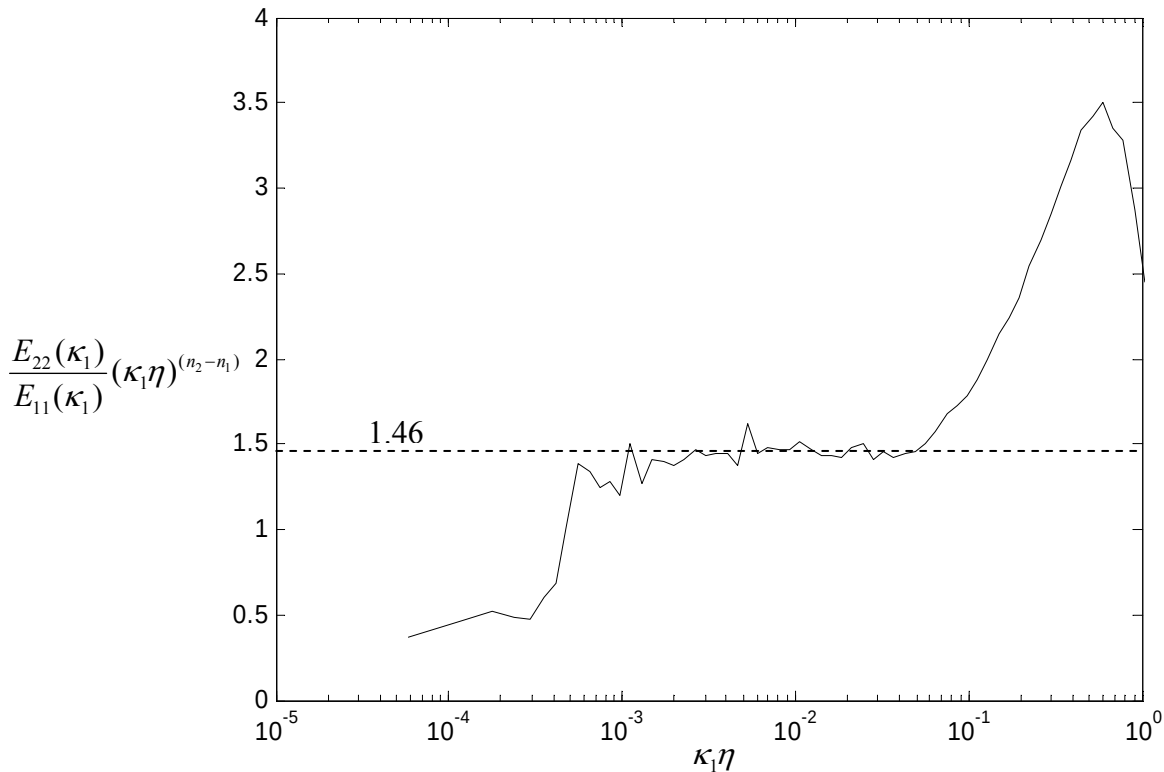


Figure 4.39. Ratio of the transverse and streamwise one-dimensional velocity spectra for the baseline condition.

* Where C_{2*} and C_{3*} as well as n_2 and n_3 replace C_{1*} and n_1 respectively. Theoretically there should be no difference between the two lateral components, but some experimental difference is nevertheless found as explained above.

[†] $E_{22}(\kappa_1)$ and $E_{33}(\kappa_1)$, have been averaged to yield one “representative” lateral spectrum which is used in place of the actual vertical component spectrum.

4.8 Dissipation

The dissipation of turbulence kinetic energy is performed by the turbulent eddies on the order of the Kolmogorov scales, η , and is therefore next to impossible to measure with a hot-wire probe whose measurement volume is generally many orders of magnitude larger than the scales in question. Even with large scale turbulence, which inherently has much larger Kolmogorov scales associated with it than conventional grid turbulence, these small scales are still about one order of magnitude smaller than the distance between the hot wire prongs,^{*} and the dissipation (and hence the Kolmogorov scales themselves) must be estimated in some fashion

To determine a reliable method for estimating the dissipation rate associated with the different flows generated by the active grid, four different approaches were considered. These will all be discussed in this section. The first method simply makes use of the streamwise decay described above. The second approach utilizes the theoretical high Reynolds number inertial range prediction to obtain a value for the dissipation rate. The third method is based on a spectral integration while the fourth method was adopted from Kang *et al.* (2003) and uses the third order structure function.

4.8.1 Decay of Turbulence Kinetic Energy

The most intuitive way of estimating the dissipation rate is simply found through using the streamwise decay of turbulence kinetic energy of the flow. The transport equation for the turbulence kinetic energy downstream of the grid can be written (by ignoring the negligible production, and pressure diffusion terms) as[†]:

$$\frac{Dk}{Dt} = -\frac{1}{2} \frac{\partial(u^3 + uv^2 + uw^2)}{\partial x} - \varepsilon \quad (4.44)$$

Ideally, for truly homogeneous isotropic turbulence, this equation would simply be $Dk/Dt = -\varepsilon$, but due to the nature of grid turbulence, and especially active grid turbulence, this is not the case. Through some algebraic manipulation Equation 4.44 can be written in terms of the skewness $S(u)$ which results in the following equation:[‡]

$$\frac{Dk}{Dt} = -\frac{1}{2} S(u) \left(u^{1/2} \frac{2}{3} \frac{\partial k}{\partial x} + u^2 \sqrt{U_\infty} \frac{\partial(u/U)^{1/2}}{\partial x} \right) - \varepsilon \quad (4.45)$$

^{*} The Kolmogorov scales for the baseline condition are estimated to be approximately 0.23 mm. This can be compared to the measurement volume of the quad-wire probes, which in Section 2.2.2 was listed as 0.75 mm³, and the distance between each prong which is approximately 0.85 mm.

[†] The dissipation, ε , as referred to in this work, really is considered to be the “pseudo-dissipation” (see for example Pope, 2000, p. 132) as the second derivative of the streamwise decay of the has been ignored in equation 4.44 for simplicity. The difference between the “real” dissipation and the pseudo-dissipation was calculated to be on the order of 10^{-7} for the baseline condition at $X/M = 37.3$ which certainly warrants the use of the pseudo-dissipation in lieu of the actual dissipation.

[‡] For the details of this derivation, see Appendix B.

All of these derivatives can be calculated from the equation of streamwise decay (Equation 4.13). The result as seen in appendix C yields for the baseline condition ($X/M = 37.3$) $\varepsilon = 1.864 \text{ m}^2/\text{s}^3$ of which 99.6% (1.860) originates from Dk/Dt . This shows that the skewness has very little effect on the total value of the dissipation rate. As an independent check of Equation 4.44 the streamwise variation of the triple product u^3 was estimated and the result yielded a consistent result of $\varepsilon = 1.865 \text{ m}^2/\text{s}^3$. The effect of the turbulence kinetic energy transport in the current dataset is even smaller than that of Mydlarski & Warhaft (1996) who observed an effect on the order of 5% of the measured dissipation rate. Much of this difference is simply due to the lower value of $S(u')$ present in this flow as well as the overall much slower rate of decay than observed in previous active grid efforts.

The advantages of this method for obtaining dissipation estimates is that it is fairly simple and does not require equipment with incredible dynamic range, and by taking data at a few streamwise stations it gives a good first estimate of the dissipation rate. The disadvantages however are quite staggering. First of all the estimates are incredibly sensitive to the way the data is fitted. Fitting power laws using different datapoints and different constants in Equation 4.13 gave, in the case of the baseline condition, dissipation rate values from 1.28 to $2.25 \text{ m}^2/\text{s}^3$. Secondly, since most of the energy is contained in the large scales, this is the data that is used to estimate the dissipation rate. As discussed throughout this chapter there is considerable uncertainty associated with the energy contained within the lower wavenumbers and this would tend to increase the uncertainty on the streamwise decay significantly. A portion of this energy is no doubt large non-turbulent grid-generated low frequency motions that never make it down the cascade of energy through the inertial range and therefore are not dissipated by the Kolmogorov scales. Therefore one must look towards slightly more advanced methods of obtaining an estimate of the viscous dissipation rate.

4.8.2 Inertial Range Fitting

The streamwise energy spectrum can also be used in several ways to estimate the dissipation rate. This particular method is fairly simple as it takes the theoretical high Taylor Reynolds number prediction of the inertial range (Equation 4.27), compares it with the measured spectrum and uses the dissipation rate which fits the data.

The inertial range fitting approach requires relatively high Taylor Reynolds number turbulence since a clear definition of the inertial range is required. Another requirement is that the measurement equipment needs to have a dynamic range that covers at least the frequencies associated with the inertial range. Again the method only produces an estimate of the dissipation rate as it has been shown above that neither the Kolmogorov constant nor the slope of the inertial range are equal to their infinite Taylor Reynolds number asymptotic values for the flows encountered in active (and especially not for static) grid generated turbulence, but it does utilize information drawn from the actual spectral transfer and should therefore be regarded as a more trustworthy estimate than the previous method. For the baseline case at $X/M = 37.15$, by averaging the values obtained from 11 different positions within the inertial range, $\varepsilon = 1.27 \text{ m}^2/\text{s}^3$ with a standard deviation of 0.02.

How good of an estimate is this? To determine this, one has to look at the two variables in play in Equation 4.27. The first is the Kolmogorov constant C_1 which is difficult to determine accurately for $Re_\lambda < 2000$. As mentioned in Section 4.7.4 there is some evidence that in the range

of Taylor Reynolds numbers of the current data, C_1 tends to be overestimated, which would cause an underestimation of the dissipation rate. Much more important however is the variation of the slope since a less negative value for the slope will cause the whole term to become larger and hence be associated with a smaller dissipation rate, which results in an overestimate of ε through the use of Equation 4.27. Hence it is believed that even though the value of the dissipation rate using this method is much smaller (by 32%) than that obtained from the streamwise decay of energy, that the true value of the dissipation rate is slightly less than $1.27 \text{ m}^2/\text{s}^3$.

4.8.3 Spectral Integration

This method also uses the spectral data, but instead of fitting an equation, the second moment of the spectrum is computed. The basis for this approach is Equation 4.46 which only assumes local isotropy and a homogeneous flow field.*

$$\varepsilon = 15\nu \overline{\frac{\partial u'^2}{\partial x}} \quad (4.46)$$

This equation can be used to calculate the dissipation rate from a time series of velocity components through numerical differentiation and by invoking Taylor's hypothesis as described in Section 4.4.1. Ensuring sample independence is again very important for the derivative to reach an acceptable mean value and the compliance with this for the baseline condition was shown in Figure 4.16. It should be noted that in case of significant high frequency noise, some low-pass filtering may be needed prior to using Equation 4.46. Alternatively, as Thoroddsen (1995) suggests, the lateral fluctuation derivatives can be used instead of the streamwise velocity derivative according to Equation 4.47.

$$\varepsilon = 7.5\nu \overline{\frac{\partial v'^2}{\partial x}} = 7.5\nu \overline{\frac{\partial w'^2}{\partial x}} \quad (4.47)$$

By using the baseline case time-series and determining both the streamwise and the lateral estimate of the dissipation rate resulted in values of $1.23 \text{ m}^2/\text{s}^3$ and $1.31 \text{ m}^2/\text{s}^3$ for Equations 4.46 and 4.47 respectively, but no adjustment for high-frequency noise was taken into account.

Another way of obtaining the square of the streamwise derivative which avoids this problem is to integrate the second moment of the streamwise velocity spectrum according to Equation 4.48 (alternatively the lateral spectra can be used in a similar way).

$$\overline{\frac{\partial u'^2}{\partial x}} = \int_0^\infty \kappa_1^2 E_{11}(\kappa_1) d\kappa_1 \quad (4.48)$$

Equations 4.48 and 4.46 can then be combined which yields:

* See for example Pope (2000) pp 132-134 for a derivation.

$$\varepsilon = 15 \int_0^{\infty} \nu \kappa_1^2 E_{11}(\kappa_1) d\kappa_1 \quad (4.49)$$

As with the time series, this integration must be done with care as minor contamination of the spectrum by high-frequency noise can result in a large overestimate. From the cumulative sum of the integrand, it was found in all cases that the integral reached an asymptotic value at high frequency before noise became a factor. This asymptotic value was taken as the dissipation rate.

For the baseline condition this yielded an estimate of $\varepsilon = 1.21 \text{ m}^2/\text{s}^3$ which is similar, but a little lower (as expected) than the estimate obtained from the inertial range equation. This method, although requiring a large dynamic range on the measurement probes used, puts very little importance on the contribution of the low wavenumbers and is therefore barely affected by the uncertainty occurring in this region.

4.8.4 Third Order Structure Function

Kang *et al.* (2003) utilized a complete different scheme for obtaining an estimate of the dissipation rate. The value of ε is found based on Equation 4.50* where $D_{uuu}(r, t)$ is the third order structure function given by Equation 4.51.

$$\frac{D_{uuu}(r, t)}{\varepsilon r} = -\frac{4}{5} + 2.0 \left[4 \left(\frac{r}{\eta} \right)^{-4/3} + \frac{4\sqrt{15}(1.92)}{17} \frac{\nu}{u\lambda} \left(\frac{r}{\eta} \right)^{2/3} \right] \quad (4.50)$$

$$D_{uuu}(r, t) = \overline{(u(x+r, t) - u(x, t))^3} \quad (4.51)$$

This method was applied to the baseline case time series in which $D_{uuu}(r, t)$ was calculated for a variety of values for the separation r from 0.48 mm to 0.48 m (representing different sampling schemes through Taylor's hypothesis). One would expect the smaller the separation r , the better the dissipation estimate should get. This did not turn out to be the case as seen in Figure 4.40. At the smallest separation, no value ε would solve Equation 4.50. Additionally above $r = 97 \text{ mm}$, no solution could be found either. In between these separation distances however, the dissipation rate values did indeed provide a ballpark estimate consistent with the other methods discussed, with a local minimum at about $\varepsilon = 1.3 \text{ m}^2/\text{s}^3$. This said however, the other viable values at other separation distances ranged anywhere from the minimum to about $\varepsilon = 2.2 \text{ m}^2/\text{s}^3$. Kang *et al.* (2003) seems to arrive at more consistent results using this approach, which may be due to a few corrections mentioned by the authors which have not been applied to the current dataset. On the other hand it was reported that the streamwise decay of their flow only differed from the third order structure function by at the most 10%. Given the large scales of their experiment one would expect the discrepancy to be close to the consistent 30% seen with the current data.

The conclusion seems to be that the third order structure function method certainly works to give a ball park estimate for the dissipation rate, similarly to the method of streamwise decay.

* See Kang *et al.* for a more in-depth description of the method and the constants used.

Nevertheless, the complexity of the calculations and the unusual variation with separation distance does not justify the use of this method in the current work.

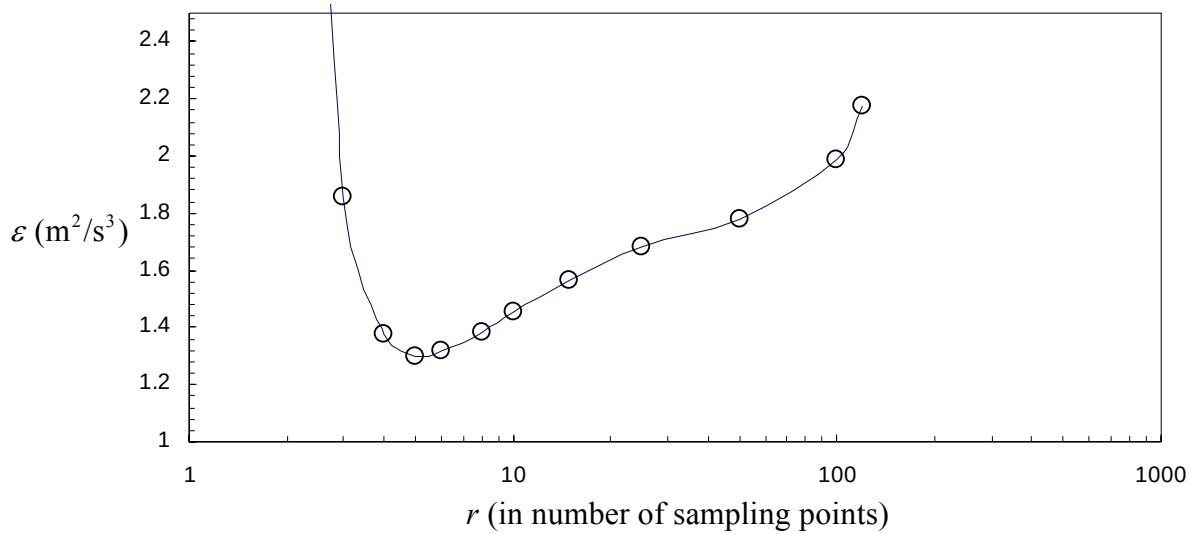


Figure 4.40. Dissipation rate estimates according to Equation 4.50 as a function of separation distance (given here in terms of number of sampling points) for the baseline condition.

4.8.5 Zero Wire Length Extrapolation

Since the Kolmogorov scales are much smaller than the measurement volume of the probe (the reason one has to estimate the dissipation rate instead of measuring it directly) there is some concern regarding the interference of the probe itself. Since the probe has hot-wires of finite length one could raise the question whether the dissipation estimates are a function of wire-length. Azad & Kassab (1989) investigated this issue and found that by using the spectral integration method described above, the estimate of the dissipation rate would decrease linearly with the wire length of the measurement probe. However at a certain point the wire lengths were so short that the probe itself started interfering with the measurements and the dissipation estimates fell below the linear trend. Azad & Kassab (1989) therefore proposed that the true dissipation rate would hence be the value at which the linear curvefit intersected the theoretical value for zero wire length. The authors reported good agreement with theoretical values. In order to test this assertion several different wire lengths were used to estimate the dissipation rate according to Equation 4.49. All cases were operated at the baseline grid condition, but at different free stream velocities and streamwise locations. In order to cancel out any effects of the free stream velocity all values were non-dimensionalized by U_∞^3 and an arbitrary lengthscale. The results are shown in Figure 4.41. Ignoring the 1-mm hot-wire data at 21.3 for which there seems to be some suspicious frequency response issues, the data looks remarkably consistent. Scaled on free stream velocity, both datasets (at nominally 12 m/s and 16 m/s) fall right on top of each other. There is not any evidence that the values of the estimated dissipation rate decreases with wire length. On the contrary: regardless of the wire-length the non-dimensionalized

dissipation rate appears to remain a constant value. This is apparent both at $X/M = 21.3$ as well as at 37.3 and indicates that the estimate given by integrating the second moment of the velocity spectrum is indeed a viable way of obtaining the dissipation rate.

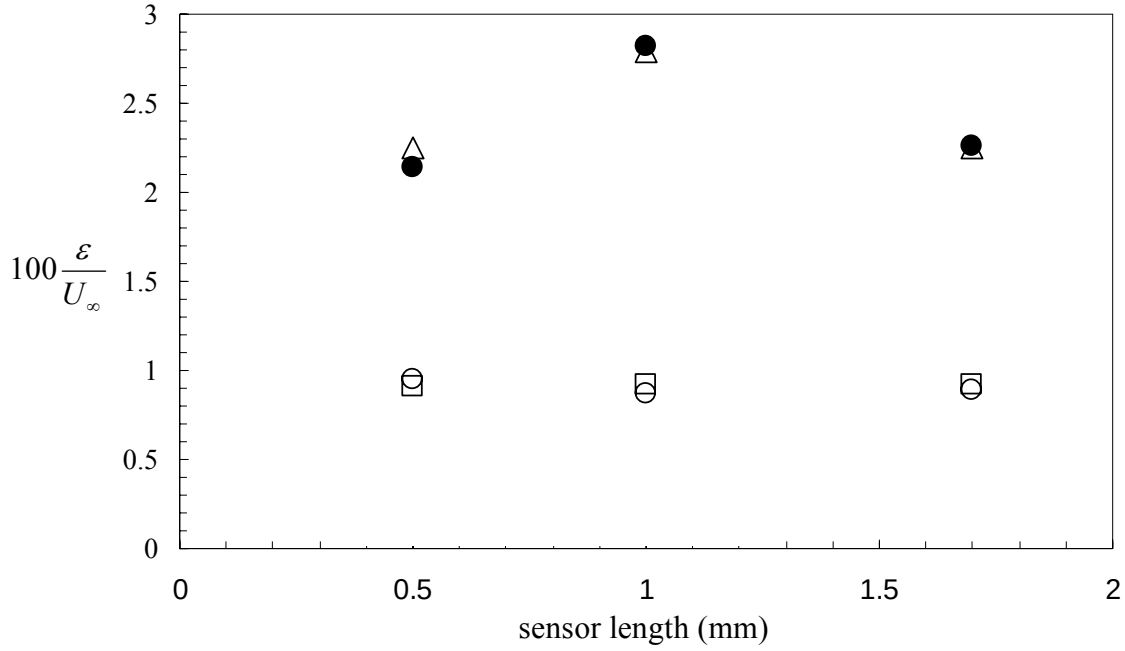


Figure 4.41. Dissipation rate estimates as a function of hot-wire sensor length. $\circ$, $X/M = 37.3$, $U_\infty = 12$; $\square$, $X/M = 37.3$, $U_\infty = 16$; $\triangle$, $X/M = 21.3$, $U_\infty = 12$; $\bullet$, $X/M = 21.3$, $U_\infty = 16$.

4.8.6 Summary

Table 2.1 shows the results of applying the four different approaches to the estimation of the viscous dissipation rate for the baseline case at $X/M = 37.3$. Based on the above analysis the spectral integration method was selected for use with the entire dataset. This is consistent with the analysis of Mydlarski & Warhaft (1996, 1998) who adopted the same method for their dissipation estimates. It is interesting to note that Makita (1991) used the streamwise decay equation (4.13) in order to obtain the values used in that work. This would tend to lead to smaller Taylor microscales and hence smaller Taylor Reynolds numbers, which may suggest that Makita (1991) obtained larger Taylor Reynolds numbers than reported. The same could then be true for the data of Kang *et al.* (2003).

The difference between the values obtained through the streamwise decay of energy and the spectral integration deserves some comment. The current results show a 35% smaller value while the corresponding value of Mydlarski & Warhaft (1996) was about 30% for a Taylor Reynolds number of 262 when a value for the dissipation rate is obtained using the streamwise decay parameters put forth in Section 4.5. Schedvin, Stegen & Gibson (1974) using a variation of

Equation 4.46 also found that the result was about 35% smaller than that of the streamwise decay of energy for their static grid measurements of $Re_\lambda = 280$ after correcting their hot-wire results for finite wire length (40% prior). These authors blamed the discrepancy on additional wire-length problems and attempted to correct the data based on universal reference spectra. Due to the wire length study completed with the current data, and the fact that the spectra do not seem to be universal at these low Taylor Reynolds numbers there seems to be little reason to apply hot-wire correction to the current data. Additionally since the skewness has been shown to have very little effect on the streamwise decay, there is clearly another low-frequency phenomenon which affects the streamwise decay of energy approach for obtaining a dissipation rate estimate.

Method	$\varepsilon (\text{m}^2/\text{s}^3)$
Streamwise Decay of Energy	1.87
Inertial Range Fitting	1.27
Spectral Integration	1.21
Unfiltered time series ($\partial u'/\partial t$)	1.23
Unfiltered time series ($\partial v'/\partial t$)	1.31
Third Order Structure Function	1.30-2.20

Table 4.3. The six methods used to estimate the viscous dissipation rate and their corresponding values for the baseline condition.

4.9 The Autocorrelation

Experimentally the temporal autocorrelation, $R_{ij}(x, \Delta t)$, is obtained using one probe by correlating the velocity components at the same location but at different time according to Equation 4.52. The two-point correlation can similarly be obtained by varying the spacing between the probes (using two probes), but sampling simultaneously. This will be further discussed in Section 4.10. The two point spatial correlation can alternatively be estimated through invoking Taylor's hypothesis. This method is used below to obtain a value for the integral lengthscale.

$$R_{ij}(X, \Delta t) = \frac{\overline{u_i'(X, t) u_j'(X, t + \Delta t)}}{\overline{u_i u_j}} \quad (4.52)$$

In order to simplify the notation the longitudinal autocorrelation $R_{11}(X, \Delta t)$ will be addressed as $f(X, \Delta t)$ and likewise the lateral autocorrelation $R_{22}(X, \Delta t) = R_{33}(X, \Delta t)$ will be denoted $g(X, \Delta t)$. This is done in accordance with the notation of Pope (2000) and will simplify the notation convention for the Taylor Microscale in Section 4.9.2.

The result of the autocorrelation indicates how well correlated the data is at each time delay Δt which can be used to assess some of the lengthscales associated with the turbulent flow as well as a measure of how far down into the large scales the local isotropy assumption holds. These issues will all be addressed in this section.

A representative autocorrelation is shown in Figure 4.42 for the baseline flow at $X/M = 37.3$. The domain of Δt can not be larger than each individually sampled window. As can be seen from Figure 4.42 the autocorrelation goes from unity, passes zero and then oscillates back and forth

around zero with progressively smaller amplitudes. The autocorrelation is actually defined for both positive and negative time-steps, which results in two symmetrical parts which are centered on the abscissa. The negative portion of the autocorrelation has been removed from Figure 4.42 for clarity.

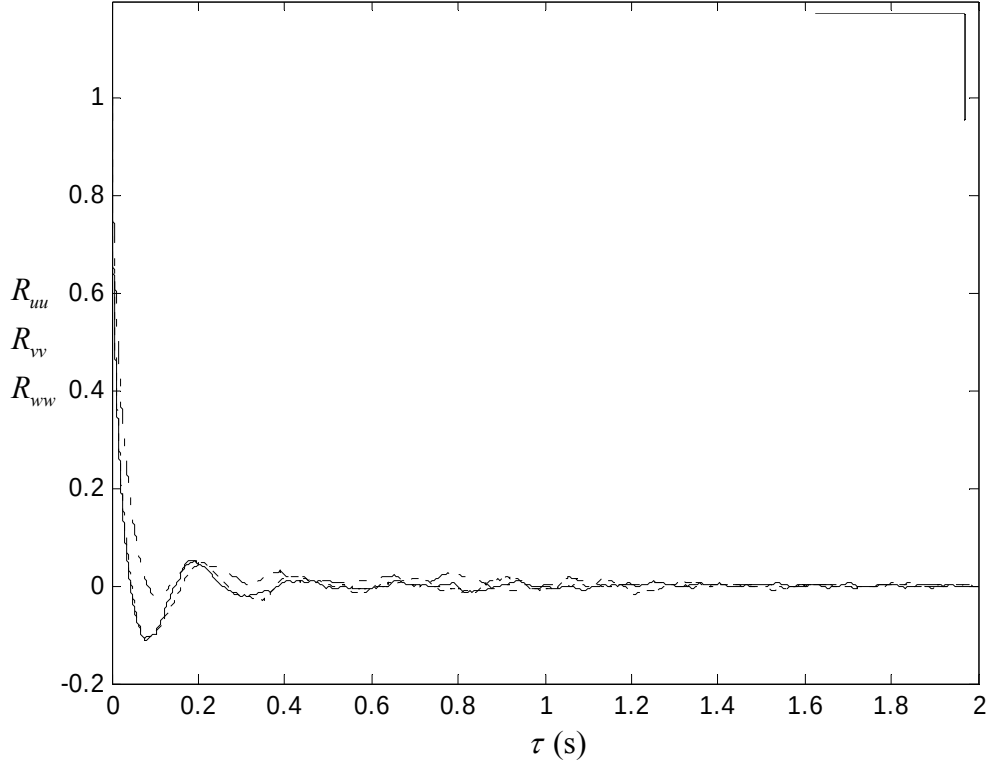


Figure 4.42. Autocorrelations for the baseline condition. ---, R_{uu} ; —, R_{vv} ; - · -, R_{ww} .

4.9.1 The Integral Lengthscale

The integral lengthscale can, by definition, be calculated from the longitudinal autocorrelation $f(X, \Delta t)$ by integrating the area under the curve according to Equation 1.4. Similarly, the lateral lengthscales L_{22} and L_{33} can be obtained by integrating under $g(X, \Delta t)$. In isotropic turbulence $L_{22} = L_{33} = L_{11}/2$ (see Pope, 2000, p. 197-198).

$$L_{11} = U_{\infty} \int_0^{\infty} f(X, t) d\Delta t \quad (4.53)$$

The integral lengthscale was previously defined in Equation 1.4 as a function of the lowest bin in the power spectrum. This is due to the fact that the longitudinal autocorrelation is related to the velocity spectrum through the Fourier Transform as shown by Equation 4.54 where Δt and the distance r_1 are simply related by Taylor's hypothesis such that $U_{\infty} f(X, \Delta t) = f(r_1, t)$.*

* Both the negative and the positive time-steps must be present in the autocorrelation before the Fourier transform is applied.

$$E_{11}(\kappa_1) = \frac{2}{\pi} u^2 \int_0^\infty f(r_1, t) \cos(\kappa_1 r_1) dr_1 \quad (4.54)$$

By setting the wavenumber κ_1 to zero in this expression, substituting in Equation 1.4 and rearranging, Equation 1.4 can be recovered.

Another common way of defining the integral lengthscale is to only integrate Equation 1.4 up to the first zero crossing of $f(X, \Delta t)$ and $g(X, \Delta t)$. This reduces the dependence on the lowest wavenumbers which are highly uncertain both due to limited averaging as well as wind tunnel distortion effects. For the baseline case at $X/M = 37.3$ the lengthscale estimates from the different methods are listed in Table 4.4. The lengthscale estimates obtained via the dissipation rate in Equation 1.6, are as expected very consistent with the spectral peak method while the two autocorrelation methods and the low-frequency limit spectral method are quite different. Another supporting argument in that the low wavenumbers are not adhering to the laws of local isotropy can be seen in the fact that for the integral lengthscales determined from the first zero crossing of the autocorrelation L_{22} does appear roughly as $L_{11}/2$ while this is far from the truth when the entire autocorrelation is considered. Generally the spectral limiting method would yield the same exact result as that from the total integration of the autocorrelation, but due to the way the autocorrelation was calculated, this is not the case since the autocorrelation ends up consisting of twice as many points as the spectral function^{*} which, when Fourier Transformed, gives fictitious information[†] further down into the wavenumber range which yields a higher value of $E_{11}(0)$. The difference between the two bins is a quite significant 58% which is the exact relationship between the two values of 0.43 and 0.25 seen in Table 4.4 as well. This shows that although defined according to Equation 1.4, this relation must be used with extreme care, and that the integration only to the first zero crossing is a much better estimate of the real integral scale. In this case however, since the large eddies are sufficiently suppressed, the actual integration in the longitudinal direction actually gives a fairly good estimate of the true integral scale, although the estimate of the transverse integral scale is rather abysmal.

Method	L_{11} (m)	$L_{22} = L_{33}$ (m)
Integration of autocorrelation	0.43	0.10
Integration to first zero crossing	0.27	0.15
Low frequency limit spectral (Equation 1.4)	0.25	
Spectral peak determination	0.40	
From Dissipation (Equation 1.6)	0.39	

Table 4.4. The five methods used to estimate the integral lengthscale and their corresponding values for the baseline condition.

^{*} This is a consequence of using the Matlab function “xcorr” which significantly reduces the time required to computer the autocorrelation.

[†] The second half of the autocorrelation simply mirrors the positive time around the ordinate and hence creates the appearance of twice the sampling time.

4.9.2 The Taylor Microscale

Taylor (1935a) somewhat incorrectly associated the Taylor microscale with the size of the dissipative eddies, but as discussed above, the Kolmogorov scales are the ones responsible for this. Hence the physical interpretation of the Taylor microscale is rather obscure, but it has nevertheless remained one of the most important parameters in quantifying turbulence.

The autocorrelation can also be used to obtain an estimate of the Taylor microscale. Equation 1.3 already provided a definition of the lateral Taylor microscale* in terms of the velocity fluctuations and their streamwise derivative. This definition actually comes from the reasoning described in detail by Pope (2000) and summarized below.

When examining the very beginning of the autocorrelation it can be seen that it has a parabolic shape. The autocorrelation starts out at $f(X,0) = 1$ with a derivative of $f'(X,0) = 0$ and a negative second derivative $f''(X,0) < 0$. Taylor then defined λ_f (the longitudinal Taylor timescale) as:[†]

$$\lambda_f(X) = U_\infty \left(-\frac{1}{2} f''(X,0) \right)^{-1/2} \quad (4.55)$$

This is related to the autocorrelation by fitting a parabola of the shape $p(\Delta t)$ in Equation 4.56 for the first few points close to zero time delay. The number of applicable points vary greatly with the actual size of the microscale as well as the sampling scheme utilized. The parabola can be merged with Equation 4.55 which yields Equation 4.57. This concept has been applied to the autocorrelation for the baseline case at $X/M = 37.3$ and can be seen in Figure 4.43.

$$p(\Delta t) = 1 + \frac{1}{2} f''(X,0) (\Delta t)^2 \quad (4.56)$$

$$p(\Delta t) = 1 - \frac{(\Delta t)^2}{(\lambda_f / U_\infty)^2} \quad (4.57)$$

When fitting this equation to the applicable points of the autocorrelation, the parabola $p(\Delta t)$ will intersect the time delay axis at the Taylor timescale λ_f / U_∞ from which the longitudinal Taylor Microscale can easily be recovered. The lateral Taylor microscale, λ_g , can be obtained in the same way from the lateral autocorrelation $g(X,0)$. As Pope (2000, p. 198-199) shows, for isotropic turbulence, the two are related by

$$\lambda_g = \frac{\lambda_f}{\sqrt{2}} \quad (4.58)$$

* The “Taylor microscale” (without mention of “longitudinal” or “transverse/lateral”) is considered to be referring to λ_g .

[†] Note that Taylor’s hypothesis has been invoked here since the original definition was in terms of the two point spatial correlation.

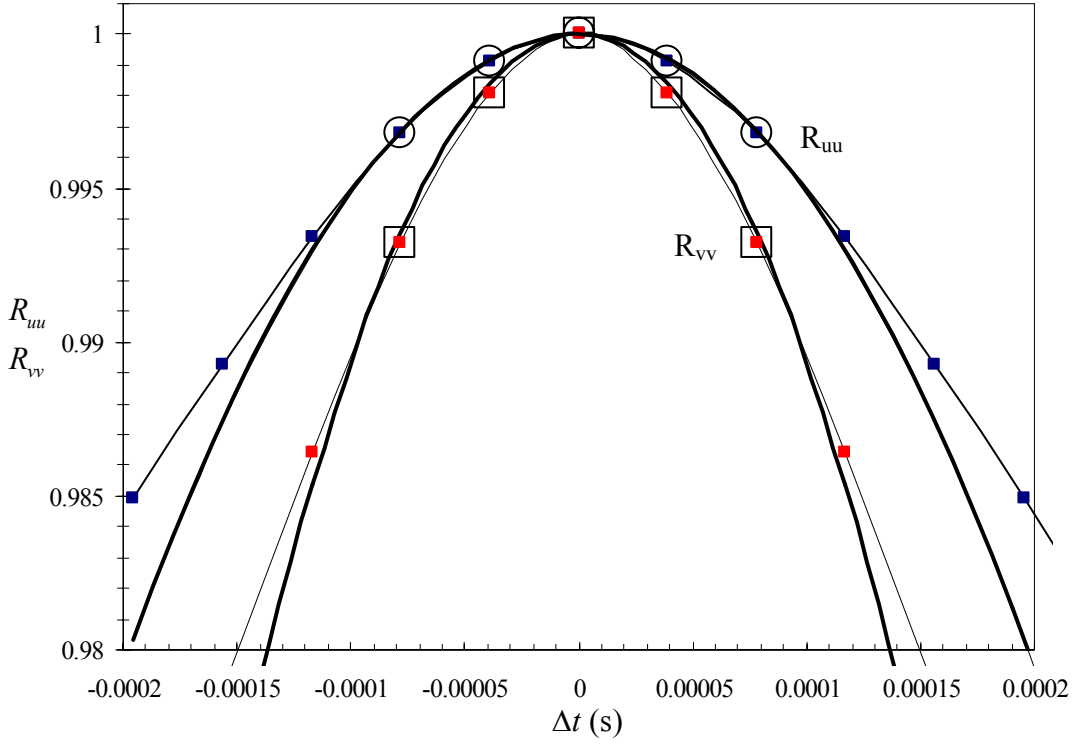


Figure 4.43. Streamwise and lateral autocorrelations for the baseline condition. Parabolas are fitted at 5 points and are described by $p(\Delta t) = 1 - \Delta t^2 / 0.001393$ and $p(\Delta t) = 1 - \Delta t^2 / 0.000970$ for $f(X, \Delta t)$ and $g(X, \Delta t)$ respectively

The longitudinal Taylor microscale can also be related to the velocity fluctuation derivative through Equation 4.59, which when rearranged together with Equation 4.58 is identical to the definition of λ_g in Equation 1.3.

$$\overline{\left(\frac{\partial u'}{\partial x}\right)^2} = \frac{2u^2}{\lambda_g} \quad (4.59)$$

The Taylor microscale can now be related directly to the dissipation rate through Equation 4.46 as:

$$\lambda_g^2 = 15\nu \frac{u^2}{\varepsilon} \quad (4.60)$$

The three estimates stemming from the longitudinal and transverse autocorrelations as well as the calculated value based on Equation 4.60 are shown in Table 4.5 for the baseline condition at $X/M = 37.3$. A much higher consistency is seen here compared to that of the integral lengthscales which again is due to the extra averaging and the lack of influence by the low wavenumber uncertainty. The three estimates are all within 5 % of each other which is within the experimental uncertainty.

Method	λ_q (m)
Fitting $f(X, \Delta t)^*$	0.0123
Fitting $g(X, \Delta t)$	0.0121
From dissipation (Equation 4.60)	0.0118

Table 4.5. The three methods used to estimate the Taylor microscale and their corresponding values for the baseline condition.

4.9.3 Local Isotropy

As mentioned earlier the autocorrelation can be used to assess the level of local isotropy associated with the turbulent flow. For isotropic turbulence it can be shown that Equation 4.61 holds.[†] In short this equation reveals that if one can define the longitudinal autocorrelation function, $f(X, \Delta t)$, the expected transverse correlation, $g(X, \Delta t)$, can be obtained.

$$g(X, \Delta t) = f(X, \Delta t) + \frac{1}{2} \Delta t \frac{\partial}{\partial \Delta t} f(X, \Delta t) \quad (4.61)$$

Since an independently measured transverse correlation was measured, the prediction can be plotted together with the obtained data, where Equation 4.61 was evaluated directly from the data by numerical differentiation similarly to that used in Equation 4.6. The two traces should be very closely related up to time delays associated with the wavenumbers on the order of the wind tunnel where the high energy eddies are being distorted by the wind tunnel walls. The result applied to the baseline condition at $X/M = 37.3$ can be seen in Figure 4.44, using the transformation $U_\infty f(X, \Delta t) = f(r, t)$ according to Taylor's hypothesis, where the longitudinal and the two transverse auto-correlations are shown together with the calculated version of $g(r, t)$ prior to the first zero crossing of $f(r, t)$. Since $f(r, t)$ is not a perfectly smooth function, it is therefore obvious that the calculated $g(r, t)$ will be even less smooth, which is evident in this figure. Note also that the ordinate has been non-dimensionalized using the grid mesh length, M , which is common practice. Although slightly overpredicting the lateral correlation, Equation 4.61 is following the data closely up to a separation of r/M between 4 and 5, which is in the vicinity of the first zero crossing of the longitudinal correlation. This corresponds to a dimensional separation distance of between 0.84 m and 1.05 m which indicates close local isotropy up to these large scales which are on the order of half the wind tunnel height. Translating this to wavenumbers through $\kappa_r = \pi/r^\dagger$ one can get a feel for where the flow becomes locally anisotropic. This appears to occur somewhere between $\kappa_1 = 2.99 \text{ m}^{-1}$ and 3.74 m^{-1} or in the frequency domain between $f = 6 \text{ Hz}$ and 7.39 Hz . These values are in strong agreement with the observations made on the cumulative filtering effects on the skewness and the kurtosis in

* The value of λ_q is obtained through equation 4.60.

[†] See for example Libby (1996) for a summary of the derivation of the spatial equivalent of this equation originally given by von Kármán & Howarth (1938).

[‡] See section 4.4.4 on the wind tunnel wavenumber for information on how this relation was obtained. Only the fundamental mode is considered here such that $N = 1$ and $H = r$ in equation 4.11.

the kurtosis in Section 4.4.3 where the large scale effects of the tunnel were believed to occur for frequencies below 7 Hz. It is also perfectly consistent with the second harmonic of the wind tunnel wavenumber which was shown in Section 4.4.4 to be $\kappa_{H,2} = 3.36 \text{ m}^{-1}$ or $f = 6.64 \text{ Hz}$ for the baseline flow. This also suggests that the mean grid rotation rate, Ω , should be kept below what corresponds to $\kappa_{H,2}$ in order not to restrict the range of locally isotropic wavenumbers further than what is dictated by the size of the wind tunnel. A thorough investigation of the isotropy as a function of flow speed and rotation rate should be undertaken to verify these hypotheses.

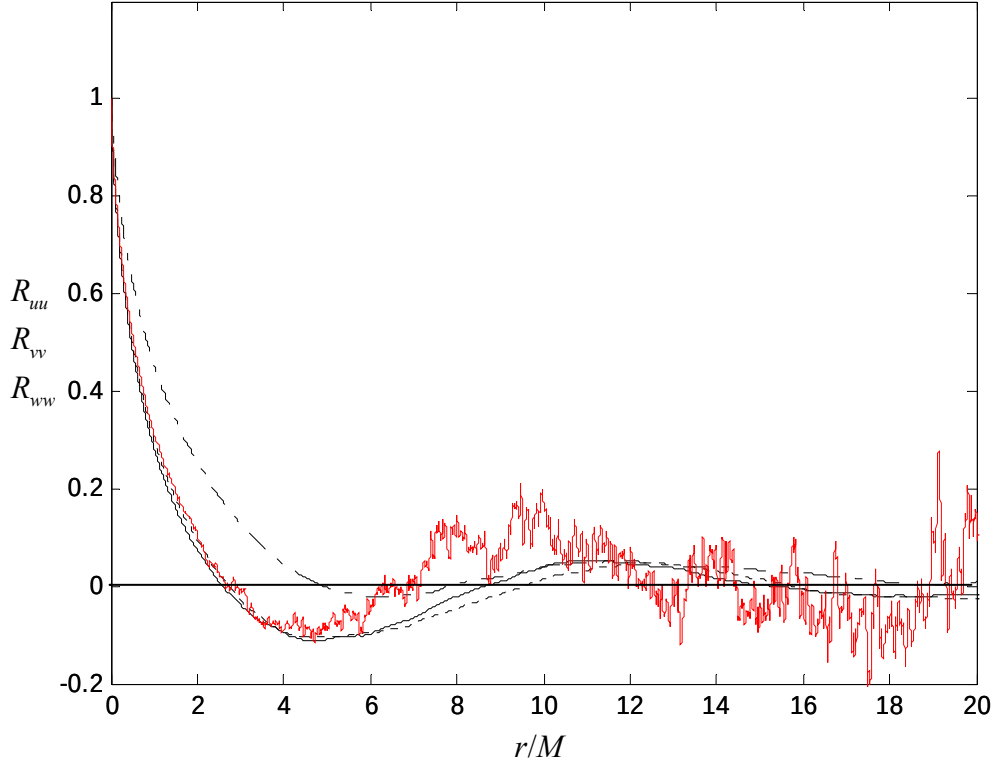


Figure 4.44. The autocorrelations from Figure 4.42 with Equation 4.61 superimposed as a function of streamwise distance. ---, R_{uu} ; —, R_{vv} ; ---, R_{ww} ; —, R_{uu} from Equation 4.61.

Even though the above observations give a very good idea of how much of the energy spectrum adheres to the laws of local isotropy, the total anisotropy ratio of the entire wavenumber range is still unity as was shown in Section 4.3.3. This is a completely different issue since the distortion of the wind tunnel contraction forces the largest eddies to conform to this condition, even though the low end wavenumber spectral components are in no way related by the laws of isotropic turbulence below $\kappa_{H,2}$.

4.10 Two-Point Correlation

As discussed in Section 4.9 the two-point correlation is related to the autocorrelation, but allows the measurement of spatial correlations without the need of Taylor's hypothesis by utilizing two independent hot-wire probes (although due to probe interference the autocorrelation is still the preferred method for obtaining streamwise correlations). The two point method is also the only way of physically measuring the spatial correlations in the lateral directions. Based on the Von Kármán spectrum which was mentioned in Section 4.6, Hinze (1976) presents interpolation functions which will predict the 2-point correlation for a given integral lengthscale L_{11} . These functions are given in Equation 4.62 and 4.63 where $f_1(x_i)$ is the correlation function corresponding to the direction traversed, $f_2(x_i)$ corresponds to the two directions perpendicular to the traversed direction, $\kappa_e = 3L_{11}/4$, and Γ and K signify the Gamma function and the modified second order Bessel function respectively.

$$f_1(x_i) = \frac{2^{2/3} (\kappa_e x_i)^{1/3}}{\Gamma_{1/3}} K_{1/3}(\kappa_e x_i) \quad (4.62)$$

$$f_2(x_i) = \frac{2^{2/3} (\kappa_e x_i)^{1/3}}{\Gamma_{1/3}} \left(K_{1/3}(\kappa_e x_i) - \frac{\kappa_e x_i}{2} K_{-2/3}(\kappa_e x_i) \right) \quad (4.63)$$

These interpolation functions assume the Von Kármán model spectrum. As was seen in Figure 4.23* the measured spectra do not resemble the model in the low wavenumber region due to tunnel wall interference. One can therefore not expect these interpolation functions to predict the correlation between the two probes in this flow. Figure 4.45 shows the correlation for all three components in the Z-direction for the baseline case with an integral lengthscale $L_{11} = 0.39$ m as determined through Equation 1.6. The predicted correlation function clearly overestimates the real correlation which indicates that the integral lengthscale is too large. A much better fit however is found when using an integral lengthscale of 0.28 m in Equations 4.62 and 4.63 as seen in Figure 4.46. This is of course very similar to the lengthscales obtained by integrating the autocorrelation to the first bin as well as obtaining the lengthscale from the low end of the velocity spectrum as described in Section 4.9.1. This would in turn correspond to a turbulent flow with a Taylor Reynolds number of barely over 500 as opposed to the value of around 650 determined using the high wavenumber region (see Section 4.6.2 for an in-depth discussion on this topic). This again shows how the uncertainty of the low-wavenumber region can influence the turbulence parameters in large scale laboratory turbulence.

* The theoretical spectra in Figure 4.23 correspond to the Pope spectral model, which is slightly different at low wavenumbers, however, the general trend between the two are very similar, yet vastly different from the experimental data.

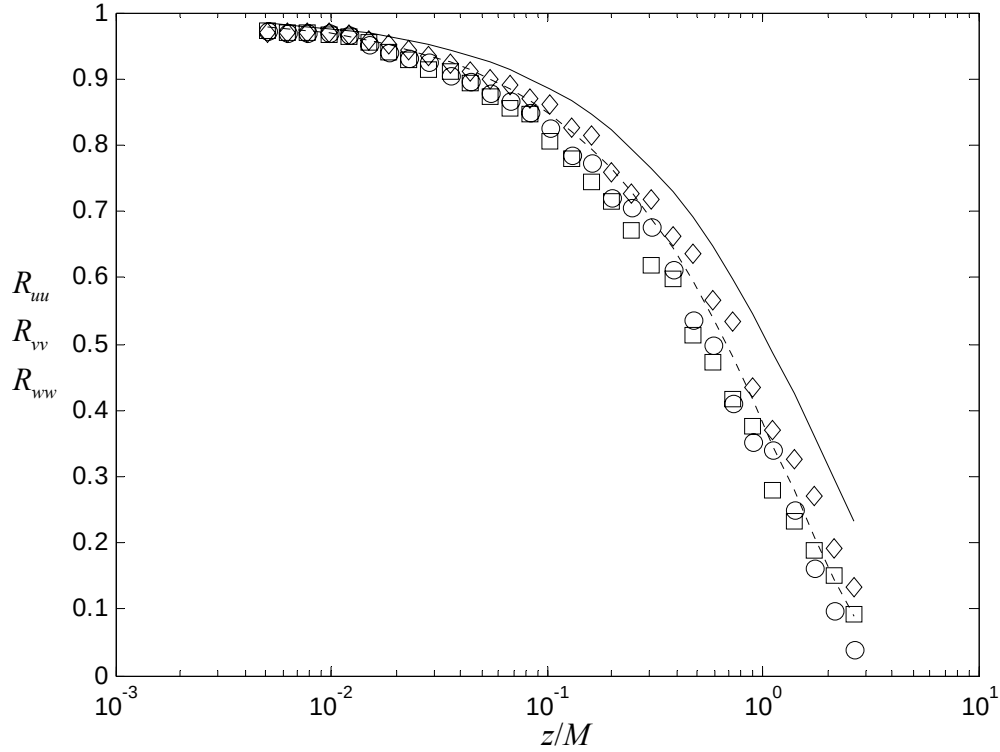


Figure 4.45. Two point autocorrelation with the Von Kármán model spectrum estimates given by Equations 4.62 and 4.63 for an integral lengthscale $L_{11} = 0.39\text{m}$. $\circ$, R_{uu} ; $\square$, R_{vv} ; $\diamond$, R_{ww} ; —, f_1 ; ---, f_2 .

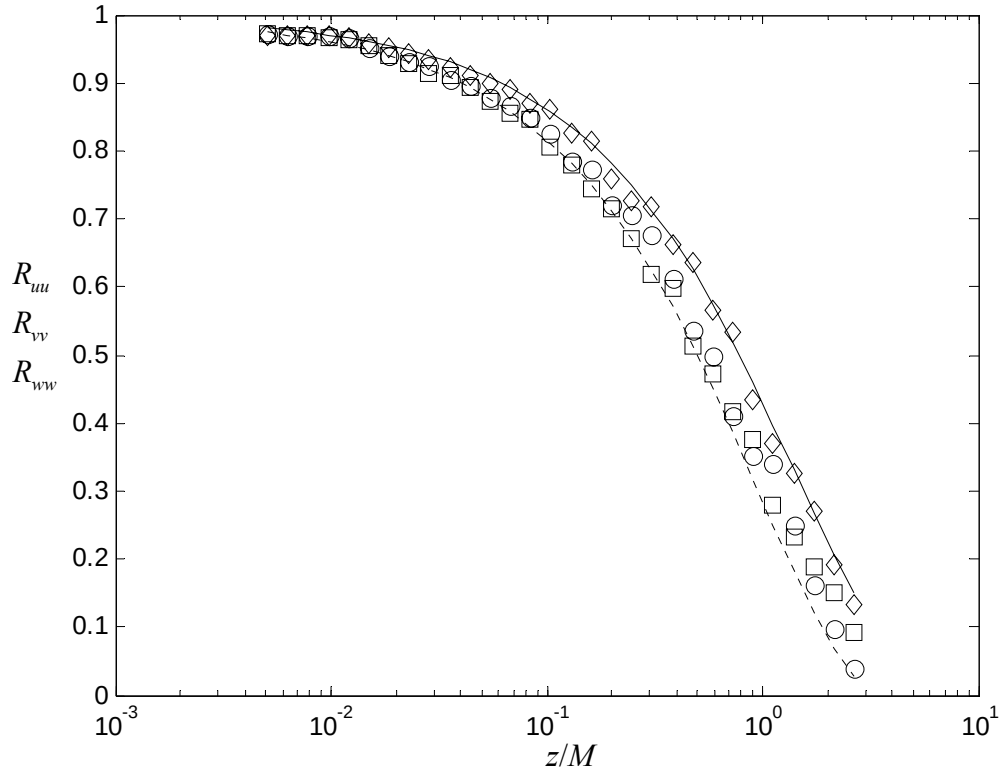


Figure 4.46. Two point autocorrelation with the Von Kármán model spectrum estimates given by Equations 4.62 and 4.63 for an integral lengthscale $L_{11} = 0.28\text{m}$. $\circ$, R_{uu} ; $\square$, R_{vv} ; $\diamond$, R_{ww} ; —, f_1 ; ---, f_2 .

4.11 Active Control of the Turbulent Flow Field

One of the greatest advantages of an active grid is that depending on the forcing scheme driving the grid, a large range of different turbulent flows can be produced. Without motorized control it can simulate a standard passive grid by aligning all the vanes with the free stream flow,^{*} or a number of different defined static obstructions defined by the angular position of each of the rods in the grid. Depending on the degrees of operational freedom, all the possible parameters can be fine tuned to ring up a flow suitable for the problem at hand. Section 4.2 introduced the different non-dimensional groups that will be used to relate grid operation with the resulting turbulent flow.

Very little work has previously been presented in this area, although Poorte (1998) attempted to quantify certain parameters such as the isotropy, integral lengthscale and turbulence intensity as a function of non-dimensional time and rotational ratios. The problem with the data put forth in Poorte (1998) however, is that, the integral time scale of the rotation rate,[†] was used to define the non-dimensional time scale which did not seem to result in very consistent trends. A completely different grid timescale, with a much simpler definition, was independently chosen and presented in Section 4.2. This timescale, $T\Omega$, is purely based on the average time of repeated motions per axis and is a parameter which can be fed to the grid controller directly, while the Poorte (1998) definition is an indirect parameter which must be calculated for random grid operation. Most of the variations presented will be in terms of the grid Reynolds number and the grid Rossby number as these are by far the two most dominating of the non-dimensional parameters together with the non-dimensional streamwise location which has already been covered in detail throughout this chapter. For that reason, unless noted otherwise, the variations showed in this section will keep the streamwise location fixed at $X/M = 37.3$ while other parameters are allowed to take on varying values. The nominal flowspeeds tested were 8, 10, 12, 16, and 20 m/s. Mean grid rotation rates were set to 2, 4, 6, 8, and 10 Hz (rev/s). It should also be noted that for the minor non-dimensional parameters all other (possible) non-dimensional parameters are fixed. This causes very few test cases to remain, and the trends described from these data points must be used with a large degree of skepticism.

The vast majority of the following data was obtained using the TSI single-wire probe, with the obvious exception of the isotropy measurements which were obtained using the four-sensor hot-wire probes and a few of the basic results are available numerically in Table 4.1 which also lists the conditions and locations of all the considered measurements.

The range of the turbulence properties produced with the present active grid in Table 4.1 deserves some comment. The turbulence intensity varies from 5.2 to 12% depending on location and grid parameters. (The highest levels were measured closest to the grid for the highest flow speeds and lowest grid rotation rates.). Perhaps more remarkable are the integral scales and Taylor Reynolds numbers, which reach 0.67m and 1362 respectively in test case 39, and 0.56m and 1053 in test case 8 further downstream.

^{*} This requires a parallel vane configuration as opposed to the staggered design used by Poorte (1998)

[†] This integral time scale was defined by Poorte (1996) as the integral of the normalized autocorrelation for the rotation rate. A synchronous forcing protocol for example gives a rotational integral time scale of infinity as the autocorrelation is defined as 1 due to constant correlation. For more information refer to the original work.

4.11.1 Turbulence Intensity

The variation of turbulence intensity, u/U_∞ , with streamwise distance from the grid was covered in great detail in Section 4.5 and will not be repeated here. Instead the variation with the grid Reynolds number and Rossby number at $X/M = 37.3$ have been plotted in Figure 4.47 and Figure 4.48. First of all the turbulence intensity does not appear to be a consistent function of MU_∞/ν , although, as can be seen from Figure 4.47, the variation with this parameter does appear linearly bounded from below within the experimental domain. In other words, there is a clear minimum value which can be obtained at a given grid Reynolds number, but the exact value will depend on other factors. Also shown on this plot is the trend of the static grid cases which are shown to slowly decrease with an increase in the grid Reynolds number. This is in stark contrast with the active grid data where the lower boundary shows a clear increase in turbulence intensity with MU_∞/ν .

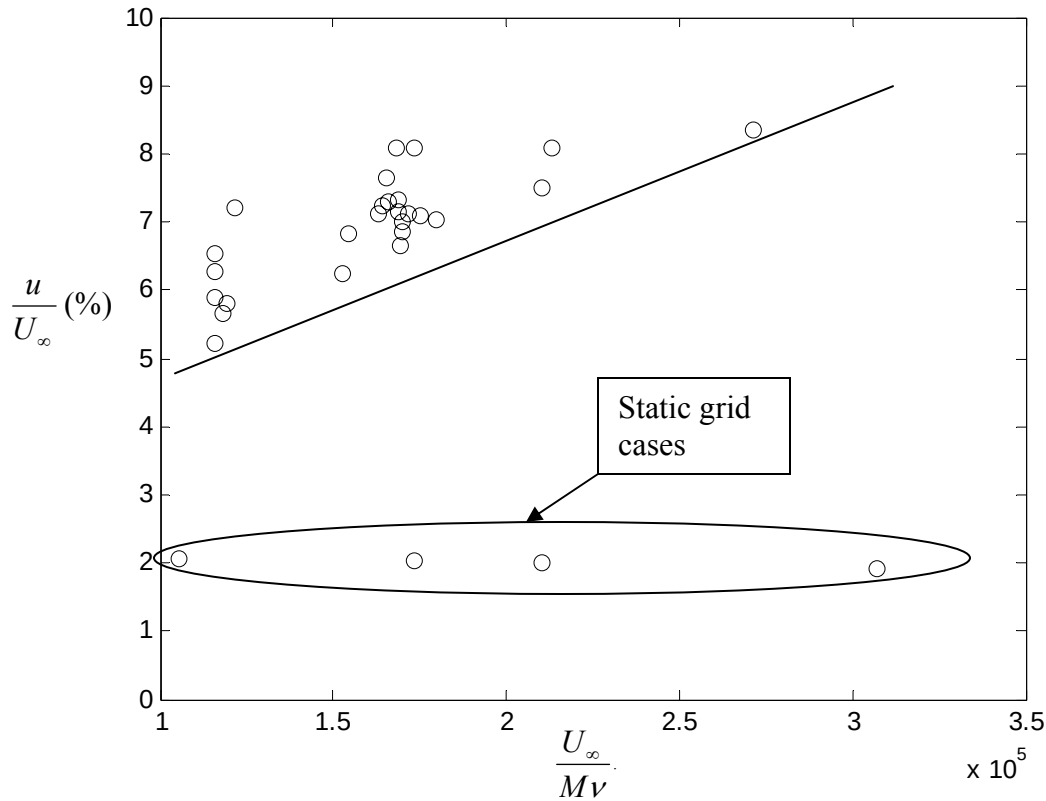


Figure 4.47. Turbulence intensity as a function of grid Reynolds number

Turning to the variation with the grid Rossby number in Figure 4.48 a more defined relationship can be found where turbulence intensity can be seen to be an exponential function of $U_\infty/M\Omega$.^{*} This effectively means that the turbulence intensity will decrease with an increasing rotation rate. There is still quite some scatter in the data (although less than seen in Figure 4.47)

^{*} Note that the static grid conditions are not included in Figure 4.48 since the grid Rossby number is not defined for such a flow

which can be attributed to the combined effect of some of the minor non-dimensional parameters which will be discussed next.

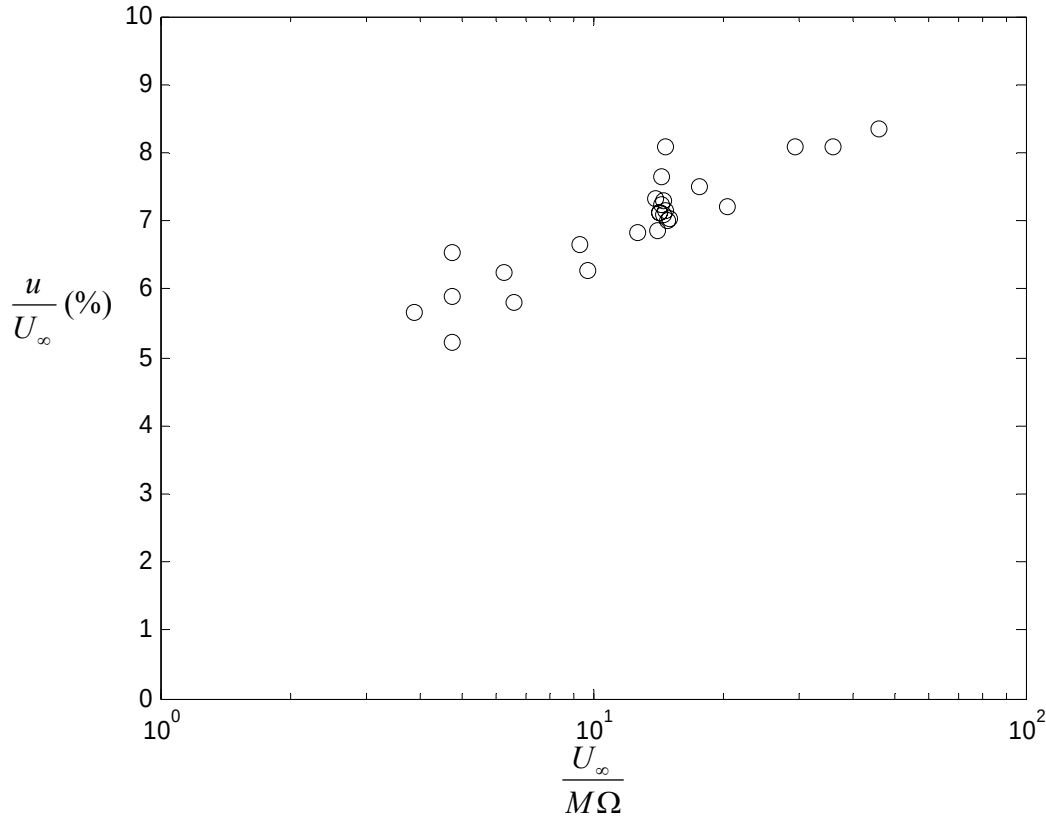


Figure 4.48. Turbulence intensity as a function of grid Rossby number

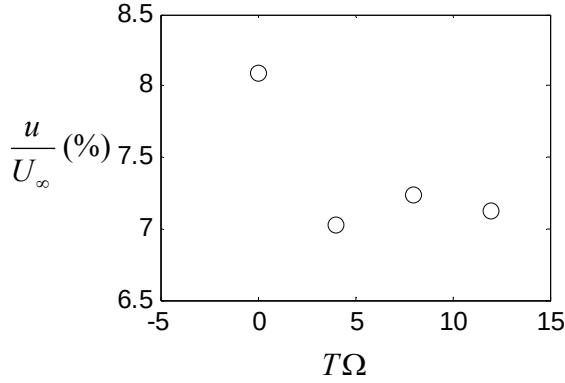
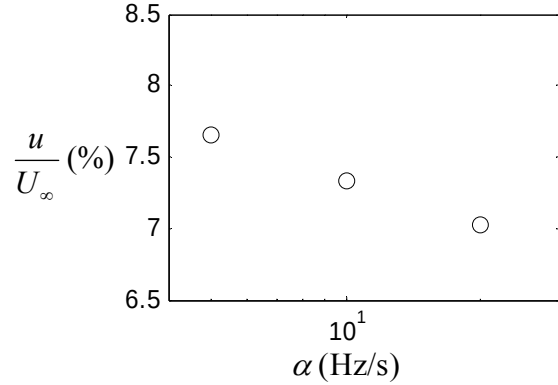
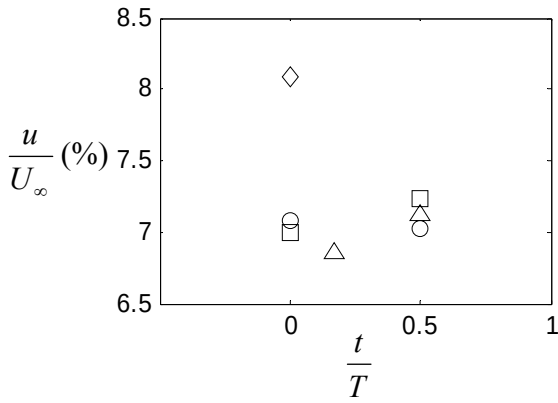
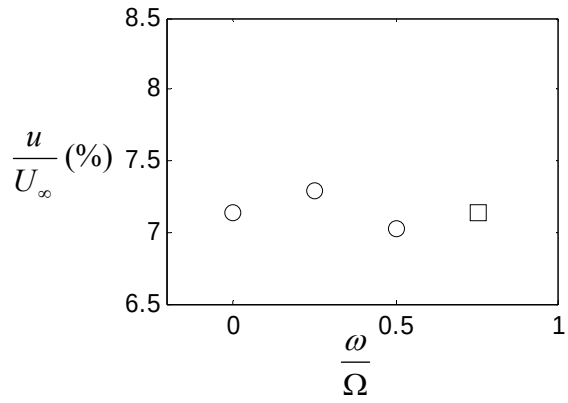
Intuitively the average number of rotations in a maneuver, $T\Omega$, should account for some of the scatter seen in Figure 4.48, but as can clearly be seen in Figure 4.49*, this parameter hardly has any effect whatsoever on the turbulence intensity with one exception: when $T\Omega = 0$ ($T = 0$ with $t = 0$) there is a large jump in turbulence intensity. Physically this implies no cruise time for any of the grid bars and hence a more random grid state as the grid is constantly accelerating and decelerating to different speeds, but with no correlated constant maneuvers in between. It may appear that the data-point at $T\Omega = 4$ is an outlier of a decreasing exponential trend, but this point was measured several times and was extremely repeatable.

The dependence on the grid acceleration is shown in Figure 4.50. Here there is an apparent logarithmic drop in turbulence intensity with increasing acceleration. A lower acceleration is comparable to slower grid speeds which will mimic the same effect as if the rotation rate was lowered. This is therefore perfectly consistent with the grid Rossby number observations.

Figure 4.51 shows the effect of the normalized maximum standard deviation of cruise time t/T . For the most part this parameter does not affect the turbulence intensity much, again unless $T = 0$ where the parameter t/T in reality is not defined (yet it is plotted as $t/T = 0$ for reference).

* Ω has been held constant to keep the Rossby number from differing between the datapoints.

Lastly, the variation of the normalized maximum standard deviation of the rotation rate ω/Ω is shown in Figure 4.52. The turbulence intensity, although slightly scattered, shows no specific dependency on this parameter. It can therefore be concluded that the turbulence intensity is mainly dependent on the grid acceleration, and the Grid Rossby number, with alternatively some of the dependence also captured by the grid Reynolds number. The three most important dimensional parameters for determining the turbulence intensity become: U_∞ , Ω , α . The change between $T = 0$ and $T > 0$ also makes a significant difference.


 Figure 4.49. Turbulence intensity as a function of $T\Omega$

 Figure 4.50. Turbulence intensity as a function of α

 Figure 4.51. Turbulence intensity as a function of t/T . $\diamond$, $T = 0$; $\circ$, $T = 1$ s; $\square$, $T = 2$ s; $\triangle$, $T = 3$ s.

 Figure 4.52. Turbulence intensity as a function of ω/Ω at $\Omega = 4$ Hz. $\circ$, $t/T = 0.5$; $\square$, $t/T = 0.25$.

4.11.2 The Macroscale

Similarly to the discussion on turbulence intensity, the macroscale, L , from which the integral lengthscale can be defined appears to be a weak function of MU_∞/ν , but a relatively strong function of $U_\infty/M\Omega$ which can be seen from Figure 4.53 and Figure 4.51. This is not surprising as it is evident that the lower the rotation rate, the larger the scales. To a smaller extent the macroscales also increase with free stream velocity.

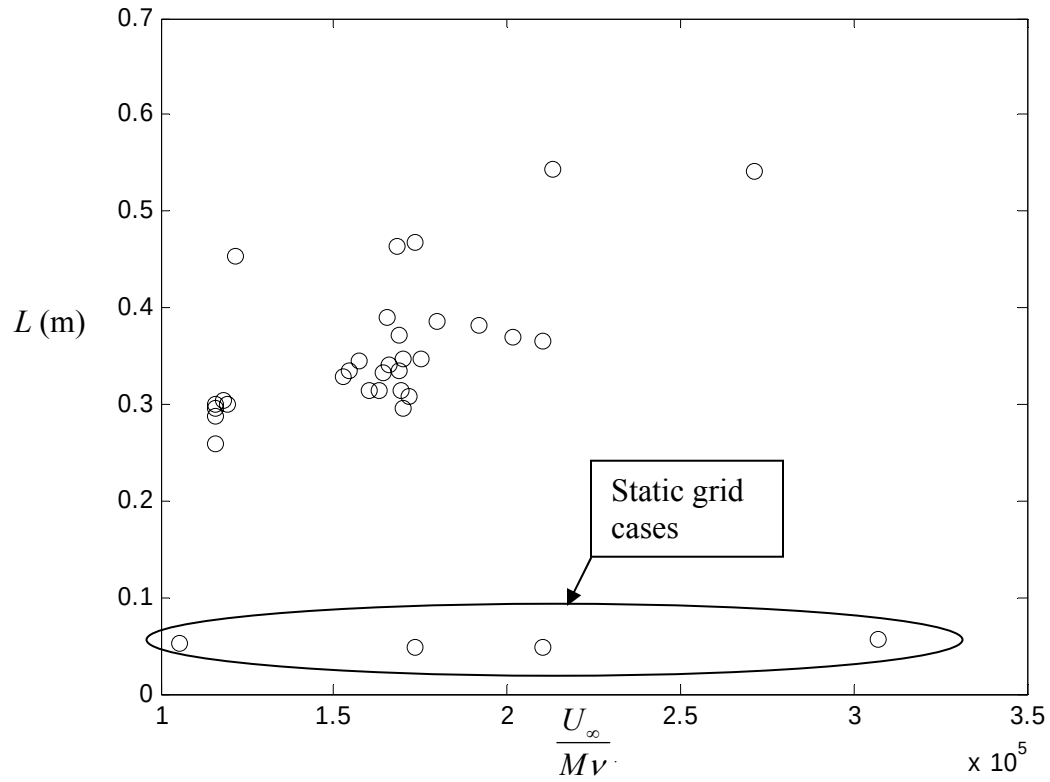


Figure 4.53. Macroscale as a function of grid Reynolds number

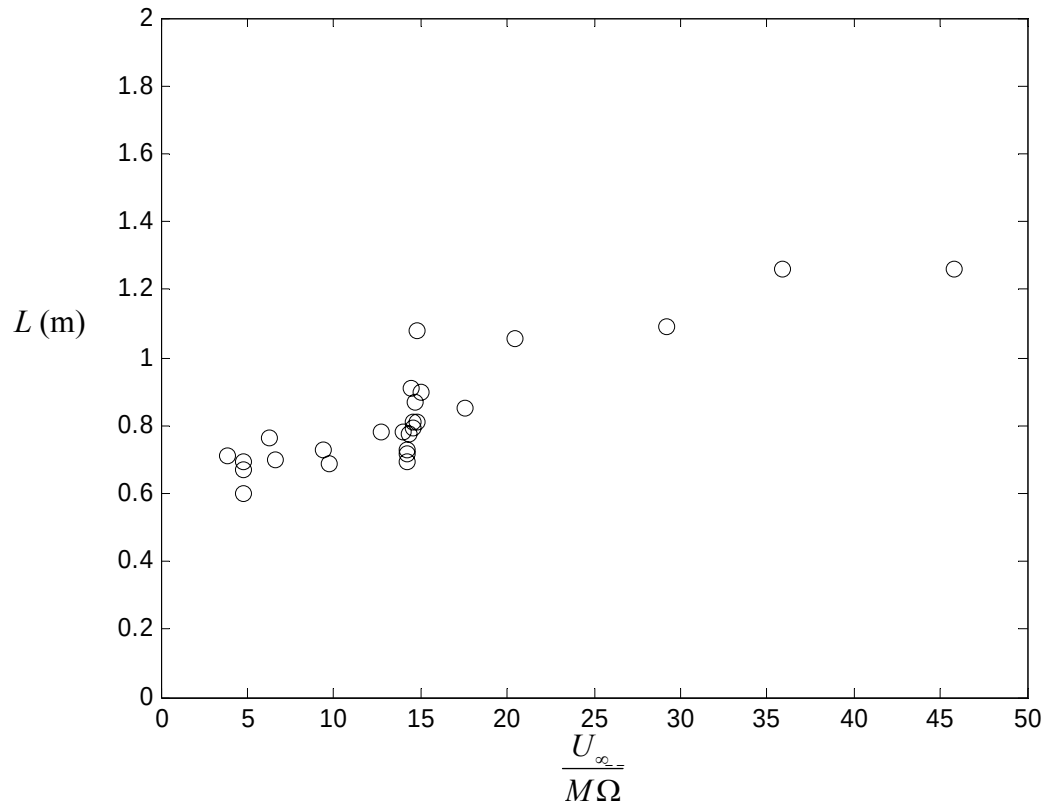
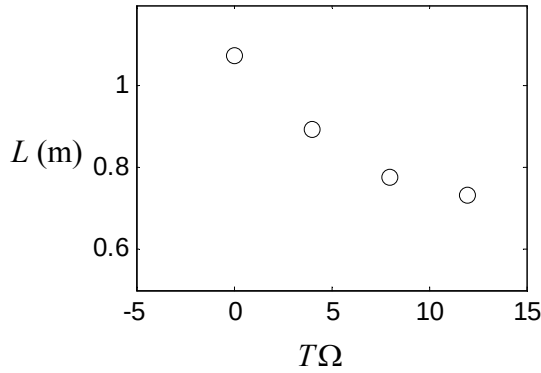
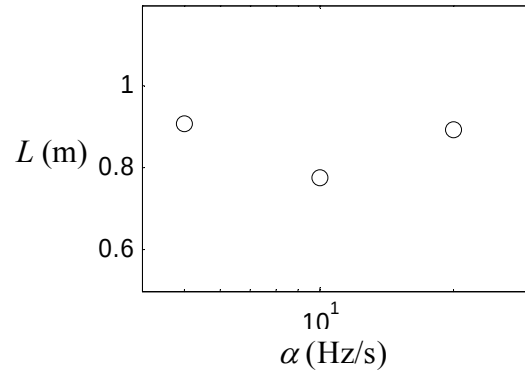
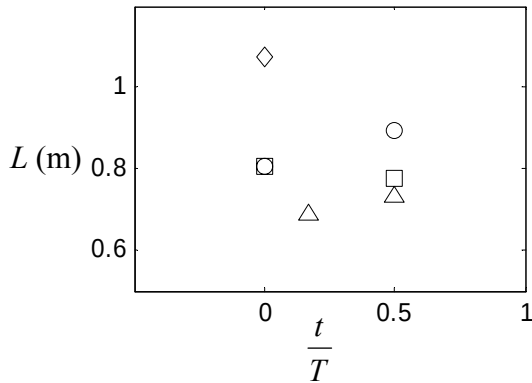
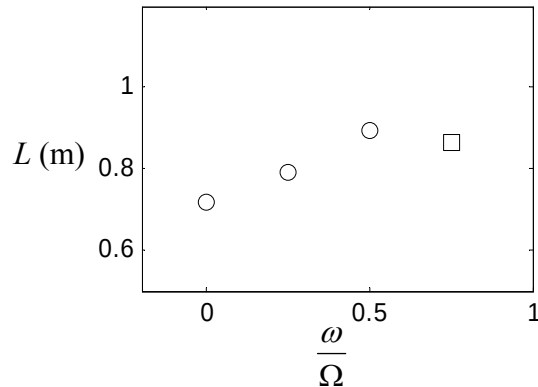


Figure 4.54. Macroscale as a function of grid Rossby number

Figure 4.55 show a clear dependence on T as the macroscale decreases almost exponentially as $T\Omega$ increases. A consistent trend can not be found in Figure 4.56 or Figure 4.57 for neither the acceleration parameter nor the t/T parameter although again it is evident that keeping $T = 0$ increases the lengthscale significantly. Figure 4.58 on the other hand shows that for the macroscale there is a marked increase with ω . Since the turbulence intensity did not show such a trend, this implies that the dissipation rate is directly affected by this parameter.


 Figure 4.55. Macroscale as a function of $T\Omega$

 Figure 4.56. Macroscale as a function of α

 Figure 4.57. Macroscale as a function of t/T . $\diamond$, $T = 0$; $\circ$, $T = 1$ s; $\square$, $T = 2$ s; $\triangle$, $T = 3$ s.

 Figure 4.58. Macroscale as a function of ω/Ω at $\Omega = 4$ Hz. $\circ$, $t/T = 0.5$; $\square$, $t/T = 0.25$.

In conclusion, it is evident that the important parameters for determining the macroscale, and hence the integral lengthscale, are Ω , U_∞ , T , and ω . The very large integral scales that can be created using the grid (and thus the extreme Taylor Reynolds numbers which will be discussed below), are roughly twice the values reported by Mydlarski & Warhaft (1998) and Kang *et al.* (2003), who had test section widths about half the present size, and four times those reported by Mydlarski & Warhaft (1996) who had a test section approximately one quarter the size. Note that the grid cell sizes M for these studies are not quite in this proportion (Kang *et al.* (2003) used a grid cell size 1.6 times that of Mydlarski & Warhaft (1998)). It thus appears that the maximum turbulence scale produced by this type of active grid scales approximately on the test section size and not the grid cell size.

A simplistic interpretation of this result is that the largest integral scale may simply reflect the largest scale of turbulence that can be contained within the test section. However, this doesn't explain how such scales are generated. It is suspected that scales on the order of the wind tunnel size are generated because, although they are in random motion, each vane row rotates together, across the entire width or height of the section. Hence the largest lengthscale will always be the tunnel size

4.11.3 Dissipation Rate

Figure 4.59 shows there is a definite grid Reynolds number dependence on the value of the viscous dissipation rate, which appears to effectively follow a power-law with free stream velocity. Again, note the different the levels of the active and the static grid curves. When plotted as a function of grid Rossby number on the other a relationship can be observed, although much weaker than with Reynolds number. This can be seen in Figure 4.60.

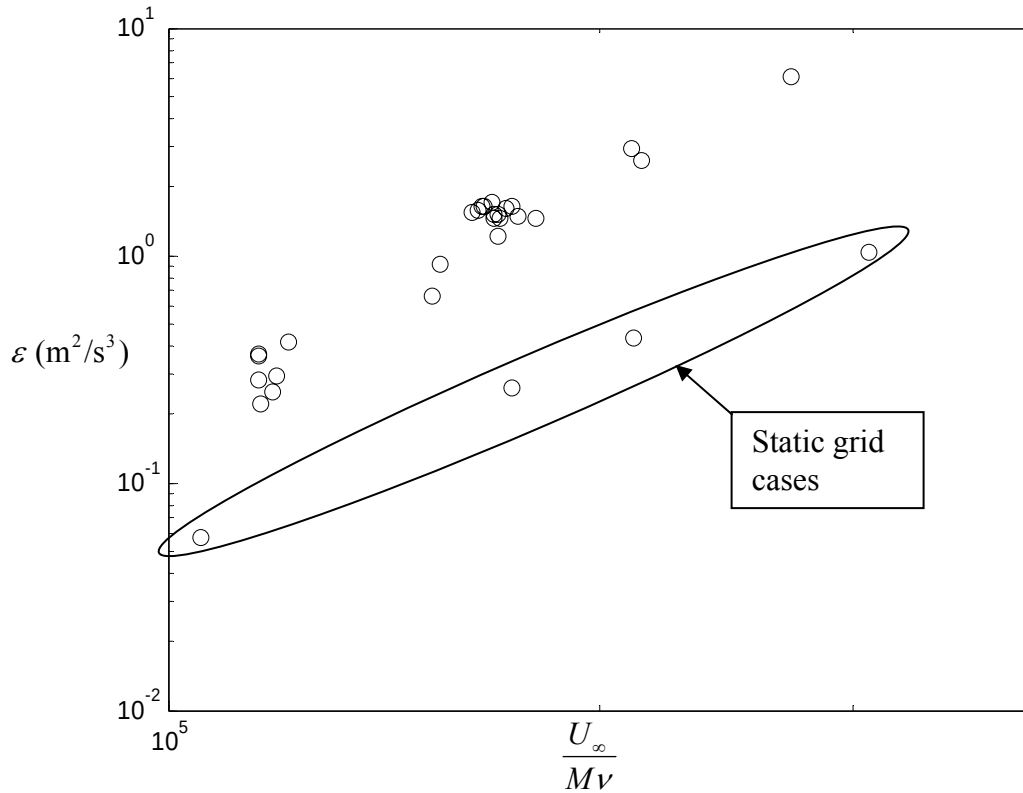


Figure 4.59. Dissipation rate as a function of grid Reynolds number

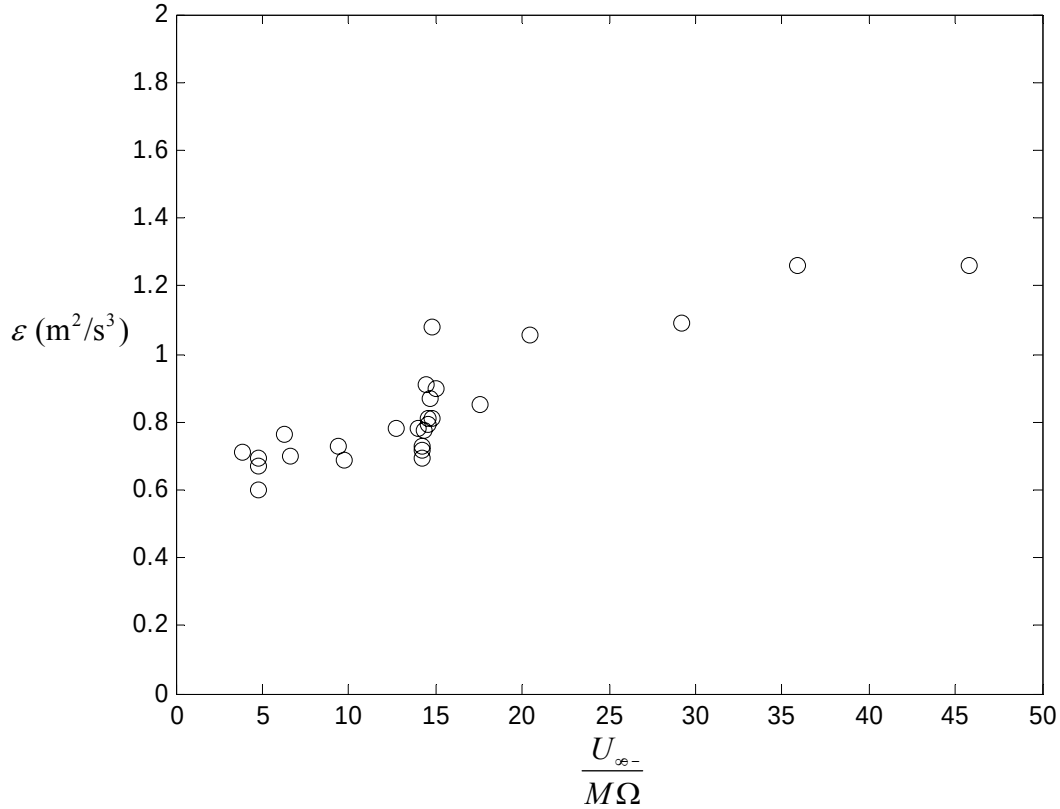
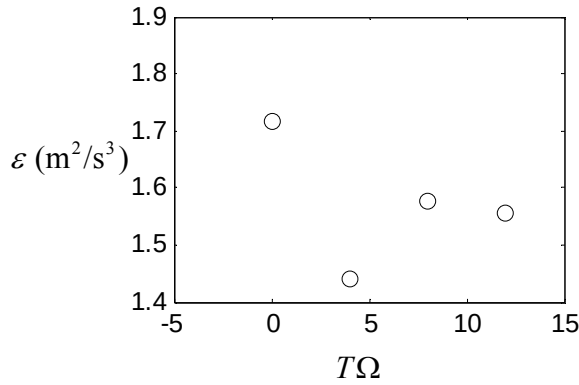
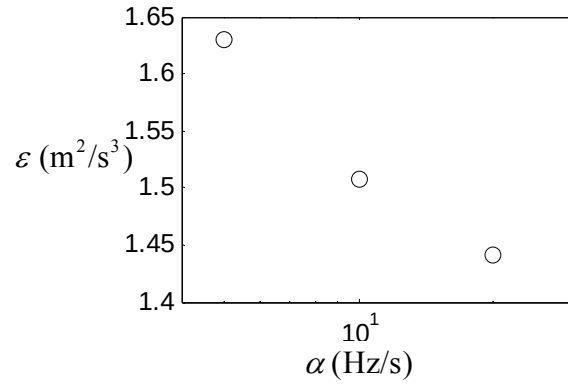
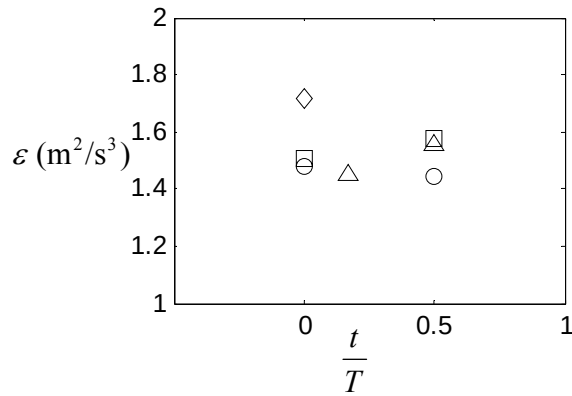
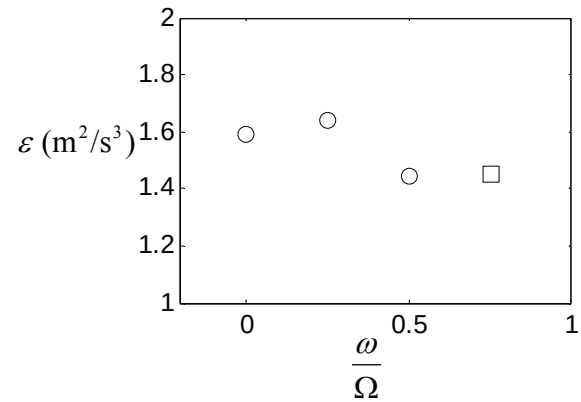


Figure 4.60. Dissipation rate as a function of grid Rossby number

The dependence of acceleration, $T\Omega$, t/T and ω/Ω can be found in Figure 4.61 though Figure 4.64. It appears that dissipation rate decreases with an increase in acceleration, which goes hand in hand with the decrease in turbulence intensity as well. Dissipation rate is relatively insensitive to variation of t (for $T > 0$), but decreases, as expected from the macroscale observation, with an increase in ω . Most notable is however the trend with T . It looks very similar to the trend observed with the turbulence intensity, but this time the value $T\Omega = 4$ takes on an even smaller value relative to the other datapoints suggesting quite a complex relationship between T and dissipation rate, which is necessary in order to set up the smooth curve of $T\Omega$ vs. ε shown in Figure 4.61.

Looking at all four minor parameters ($T\Omega$, α , t/T and ω/Ω) it is evident how remarkably similar the trends are to the ones stemming from the turbulence intensity in Section 4.11.1. This is also the case for the two major parameters ($U_{\infty}/M\Omega$ and MU_{∞}/ν) if one only compare the points involved in Figure 4.61 Figure 4.64 separately. The reason for these similar trends is that although independently obtained, the turbulence intensity and the dissipation rate are inherently related (an increase in turbulence intensity is followed by an increase in dissipation, if all other parameters remain the same).

From this section it can then be concluded that the value of the dissipation rate is a function of U_{∞} , T , α and ω . Since the dissipation rate and the turbulence intensity are directly related to the macroscale (Equation 1.5) it is therefore no surprise that the combined relevant parameter set between the dissipation rate and the turbulence intensity equals that of the macroscale.


 Figure 4.61. Macroscale as a function of $T\Omega$

 Figure 4.62. Macroscale as a function of α

 Figure 4.63. Macroscale as a function of t/T . $\diamond$, $T = 0$; $\circ$, $T = 1$ s; $\square$, $T = 2$ s; $\triangle$, $T = 3$ s.

 Figure 4.64. Macroscale as a function of ω/Ω at $\Omega = 4$ Hz. $\circ$, $t/T = 0.5$; $\square$, $t/T = 0.25$.

4.11.4 Taylor Reynolds Number

The variations for the Taylor Reynolds number are expected to display traits from all the previous sections as Re_λ is a function of L , u , and ε . The dependence of the Taylor Reynolds number with the grid Reynolds and Rossby number is shown in Figure 4.65 and Figure 4.66 respectively. The static cases show a very linear trend with grid Reynolds number as discussed in Section 1.6.1. The active cases are also displaying somewhat of a linear trend but with large scatter which is explained by the fact that the Taylor Reynolds number is also a relatively strong function of the grid Rossby number.

Figure 4.67 adds some of the present grid data for the static grid, $\Omega = 2$ Hz, $\Omega = 4$ Hz to Figure 1.2. The static grid data conforms quite well to the empirical fit, but the active grid data falls short. This is expected due to the limiting effects of 1) the contraction, 2) the tunnel walls effect on the large eddies in the test section, and 3) the large dimensional streamwise distance from the grid.

Turning to the four minor parameters ($T\Omega$, α , t/T and ω) in Figure 4.68 through Figure 4.71 the trends are very similar to the ones stemming from the macroscale in Section 4.11.2. This

stems from the fact that the Taylor Reynolds numbers is obtained from the macroscale, the turbulence intensity and the dissipation rate, and with the dissipation rate canceling out the turbulence intensity effects, it adopts the trend seen by the macroscale when all but one parameter are kept constant.

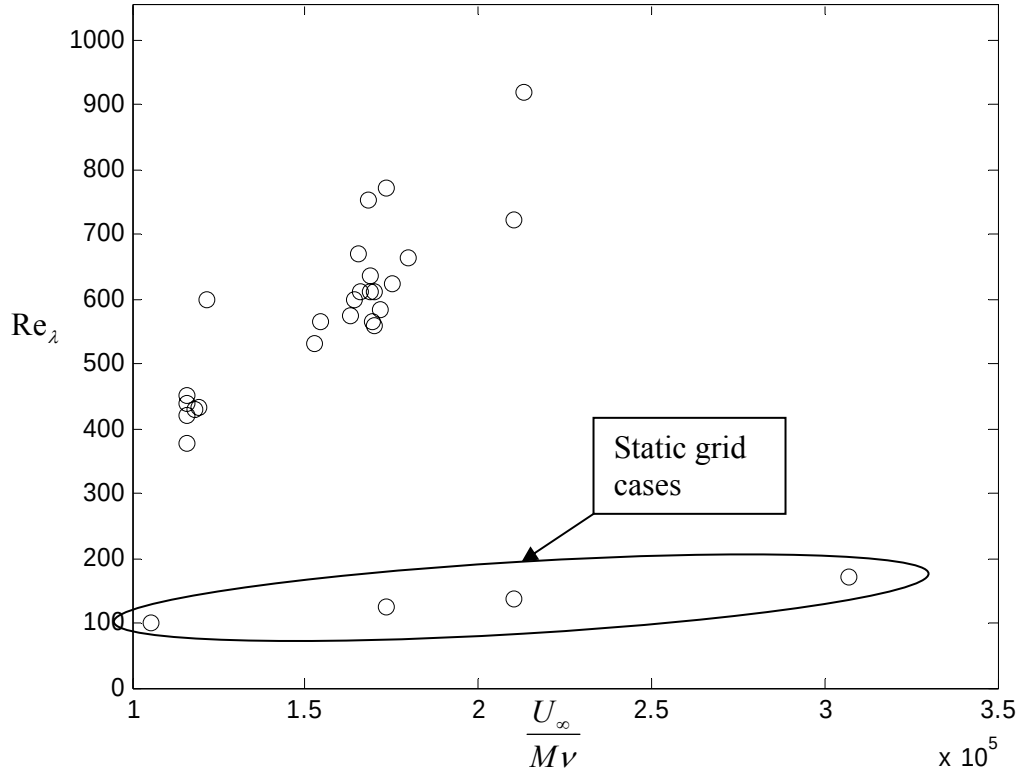


Figure 4.65. Taylor Reynolds number as a function of grid Reynolds number

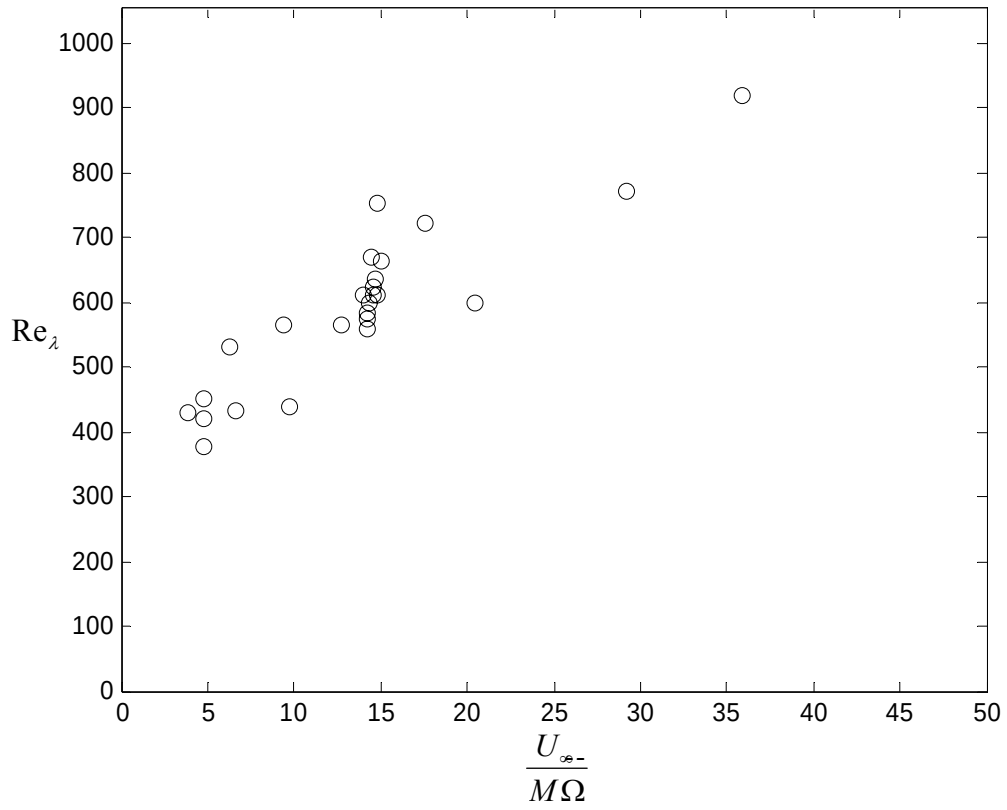


Figure 4.66. Taylor Reynolds number as a function of grid Rossby number

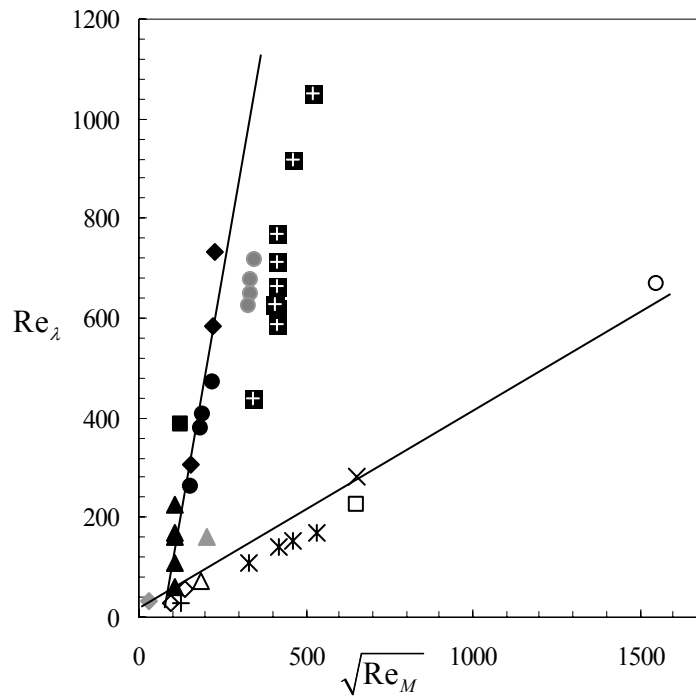
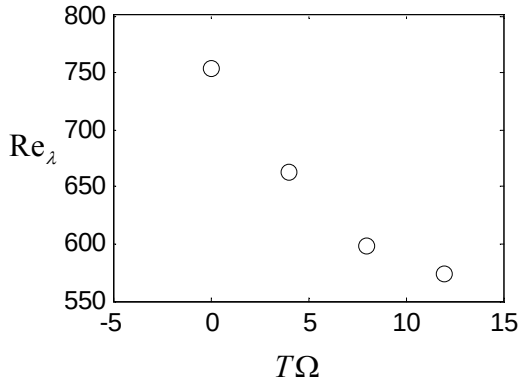
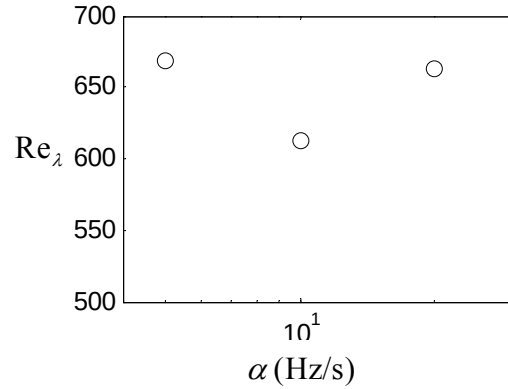
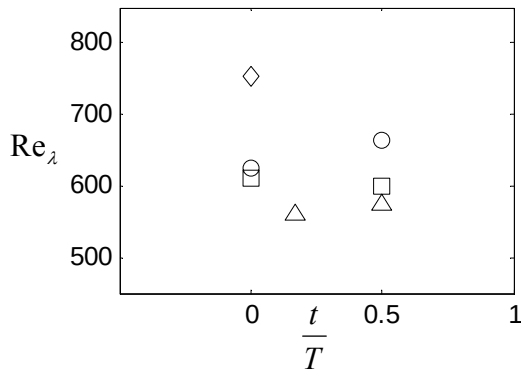
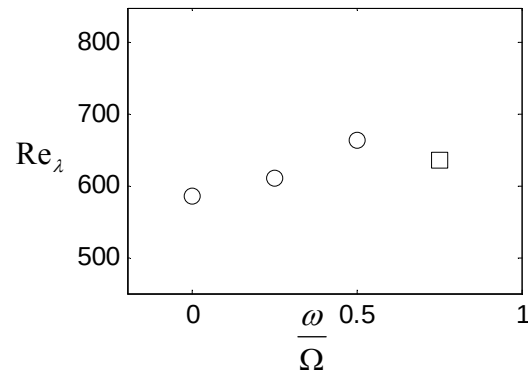


Figure 4.67. Taylor Reynolds number as a function of grid Reynolds number. *, present static grid results; $\boxplus$, present active grid results. See Figure 1.2 for complete symbol description.


 Figure 4.68. Macroscale as a function of $T\Omega$

 Figure 4.69. Macroscale as a function of α

 Figure 4.70. Macroscale as a function of t/T . $\diamond$, $T = 0$; $\circ$, $T = 1$ s; $\square$, $T = 2$ s; $\triangle$, $T = 3$ s.

 Figure 4.71. Macroscale as a function of ω/Ω at $\Omega = 4$ Hz. $\circ$, $t/T = 0.5$; $\square$, $t/T = 0.25$.

4.11.5 The Anisotropy Ratio

Poorte (1998) presented some results that linked rotation rates to the anisotropy ratio for a synchronous forcing protocol. Originally this data was presented using vane tip velocity over free stream velocity as the non dimensional parameter, but in Figure 4.72 Poorte's (1998) data is shown as a function of Rossby number together with data points obtained from Makita (1991), Mydlarski & Warhaft (1996), Kang *et al.* (2003) and the current dataset. Poorte's (1998) data was all obtained using a constant free stream velocity, and the data shows a quite complex relationship between the grid Rossby number and the anisotropy ratio. At very high rotation rates there seem to be slight local maximum before a local minimum is reached close to an anisotropy ratio of unity after which the anisotropy ratio again increases to values comparable to other active grid efforts. Poorte (1998) contributed the favorable range of isotropy ratios mostly to the presence of the random forcing protocol as well as the staggered configuration of the agitator vanes. It can be seen however, that the grid Rossby number is also a large contributor. The presence of a local minimum is confirmed by the current data, but it occurs at a different Rossby number which is likely to be a function of tunnel size and possibly Reynolds number. Data from

other active grids have also been inserted into Figure 4.72 which backs up the trend since they are produced using similar Reynolds number and grid configurations as the current dataset. This dependence on grid Rossby number strongly suggests that the mean rotation rate is extremely important in obtaining an anisotropy ratio close to unity.

Changing the free stream value in the present results from 11.4 m/s to 15.0 m/s has insignificant effect on the anisotropy ratio but halving the rotation rate however from 4 to 2 Hz on the other hand changed the anisotropy ratio from unity to approximately 1.07. Increasing the rotation rate to 6 Hz however has a slight adverse effect, albeit small. In other words, each active grid facility (with a given forcing protocol) seems to have an optimal mean rotation rate in terms of anisotropy. By choosing a rotation rate vastly different than this optimum, the anisotropy will suffer greatly. Kang *et al.* (2003) used a rotation rate of $\Omega = 5.25$ which is not far enough from the optimum to have much adverse effect. This coupled with a largely randomized forcing protocol* which also further reduces additional active grid related anisotropy shows that the vast majority of the additional anisotropy as compared to that generated by a static grid is stemming from the tunnel confinement whereas the rest can be attributed to vane configuration ala Poorte (1998)

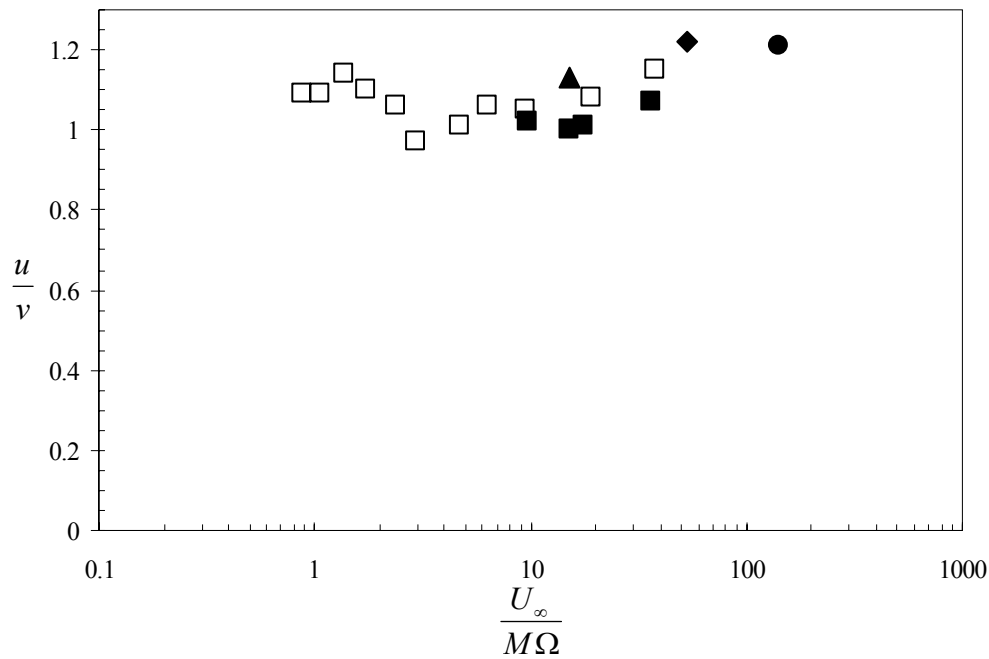


Figure 4.72. Anisotropy ratio as a function of Rossby number. $\square$, Poorte (1998); $\blacksquare$, Present data; $\bullet$, Mydlarski & Warhaft (1996); $\blacktriangle$, Kang *et al.* (2003); $\blacklozenge$, Makita (1991).

* Even though the modified single random forcing protocol was used by Kang *et al.* (2003) it was shown in section 4.11.1 through 4.11.4 that the cruise time parameters have very little effect on the overall turbulence which indicates that the modified single random scheme is almost equivalent to a double random forcing protocol in terms of the turbulence it produces as long as T is not set to zero.

Chapter 5

The Flat Plate Cascade

Providing realistic turbulence such as that generated by the active turbulence grid was only the first stage of the research project. The ultimate goal is the prediction of the flow inside a simple marine propulsion system. In order to mimic the flow throughout a propulsor pump or a shrouded propeller, a model of the first stage of blades (rotor or stator) or alternatively a set of inlet guide vanes had to be created. This was done by installing a custom made flat plate linear cascade in the Virginia Tech Stability Wind Tunnel. This chapter discusses the design and construction as well as some initial tests of the cascade used for the blade blocking study found in Chapter 6.

5.1 Design Considerations

The requirements for the cascade were relatively simple, yet crucial in order to create a useful study of the blade blocking effects on large scale turbulence. As mentioned in Chapter 1, the spacing and the chord of the cascade should be on the order of the integral scale, which physically limits the number of blades that can be inserted into the 1.8 meter wide test section of the wind tunnel due to the large scale turbulence created by the active turbulence grid. Also, in order to avoid complications during analysis, flat plates with rounded leading edges and sharp trailing edges were chosen for the design.

The decision to use flat plates prevents the presence of pressure gradients over the majority of the blade surface when properly installed at zero angle of attack. The rounded leading edges also minimize the effect of leading edge separation caused by unsteady stall due to the relatively high intensity large scale active grid turbulence. The sharp trailing edges are included to limit the vortex shedding off the cascade blades which should result in narrower wakes. To simplify installation and eliminate the possibility of any spanwise camber due to gravity it was decided to install the blades vertically in the wind tunnel test section. Additionally the cascade blades would have to run the entire height of the test section in order to ensure a close two-dimensionality of the flow.

The last requirement was that the cascade hardware had to be of a modular design such that the cascade itself could be installed in both a staggered as well as an un-staggered configuration, without radically changing the streamwise placement in the wind tunnel. With these requirements in mind a few design iterations yielded the cascade design described in the next section.

5.2 The Cascade

The completed cascade, which can be seen in Figure 5.1 in the unstaggered configuration, consists of six flat blades. Six blades were chosen instead of five in order to obtain a central passage as opposed to a central blade, although this reduced the overall chord/spacing ratio. The unswept cascade is designed to be mounted at zero degrees angle of attack relative to the geometric axis of the wind tunnel test section. The modular hardware design allows the cascade to be installed either at 0° or 35° stagger. In both cases the mounting hardware permits the cascade to be positioned at the correct blade spacing before each blade is set in tension from outside the test section. This was required to reduce the inherent chord- and spanwise camber present in the material, as well as reducing vibration during tunnel and grid operation. Additionally, two sets of dual aluminum wires covered in airfoil-section tubing of chord 0.25" and thickness 0.125" run the width of the tunnel through each blade. These wires are secured to the blades via set-screws in the leading and trailing edges, which stiffens the cascade, significantly reducing vibration. The two sets of wires were located at approximately one third and two thirds of the span, the exact intervals being uneven in order to avoid the possibility of any low-order spanwise vibrational modes in each blade (see Figure 5.1 and Figure 5.2).

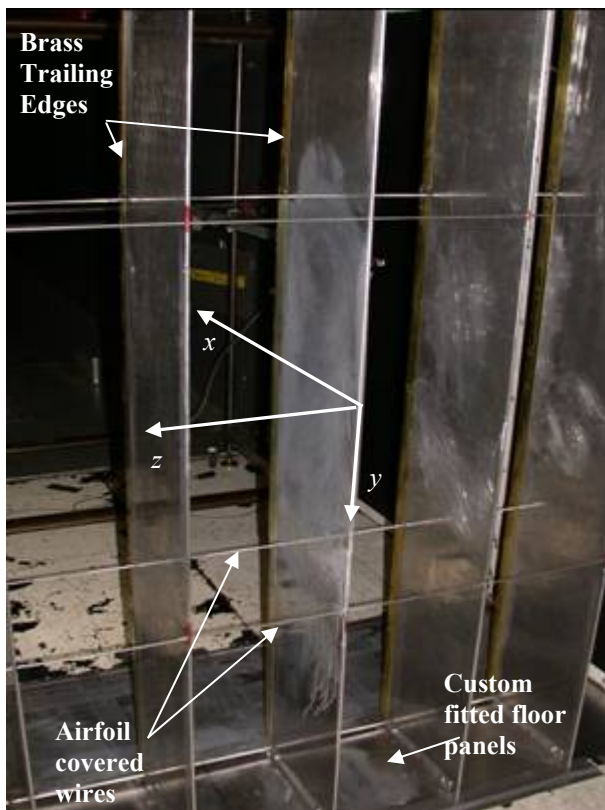


Figure 5.1. Non-staggered cascade configuration in the Virginia Tech Stability Tunnel

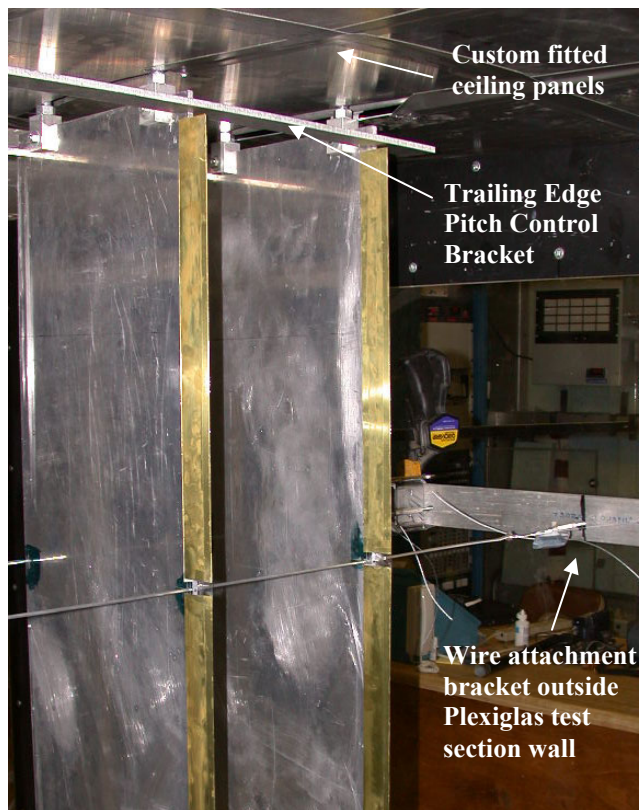


Figure 5.2. Detail of cascade trailing edges and select cascade mounting hardware

Each plate, constructed from 0.25" thick aluminum, spans the entire height of the test section in order to ensure two-dimensionality as far as possible. Two large threaded pins are attached to

the top of each blade through a linking bracket which is secured by aluminum C-beam sections resting on the top of the frame. This allows for each blade to be tensioned independently as can be seen in Figure 5.3a and Figure 5.3b. From below, the blades are secured by two aluminum L-brackets mounted on aluminum I-beams that lie flush next to the tunnel frame (Figure 5.3c and Figure 5.3d).

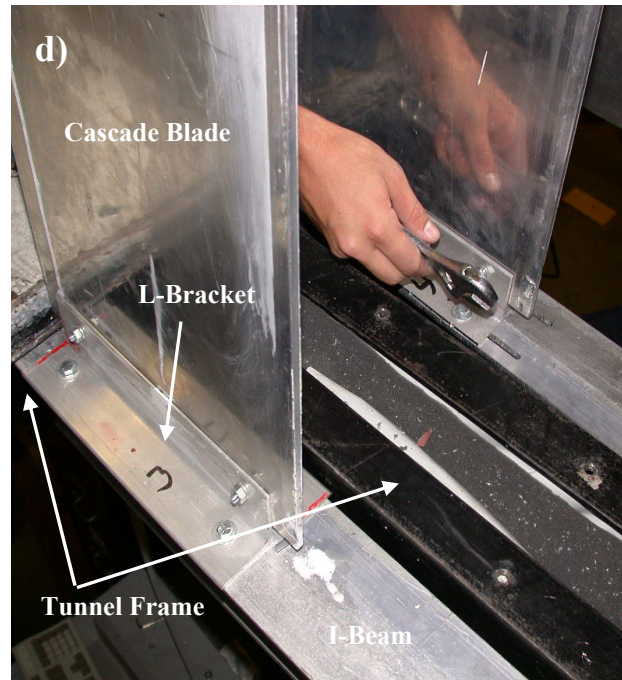
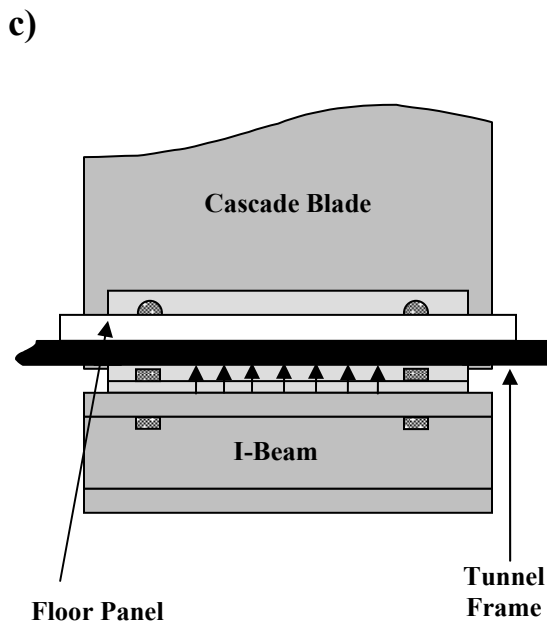
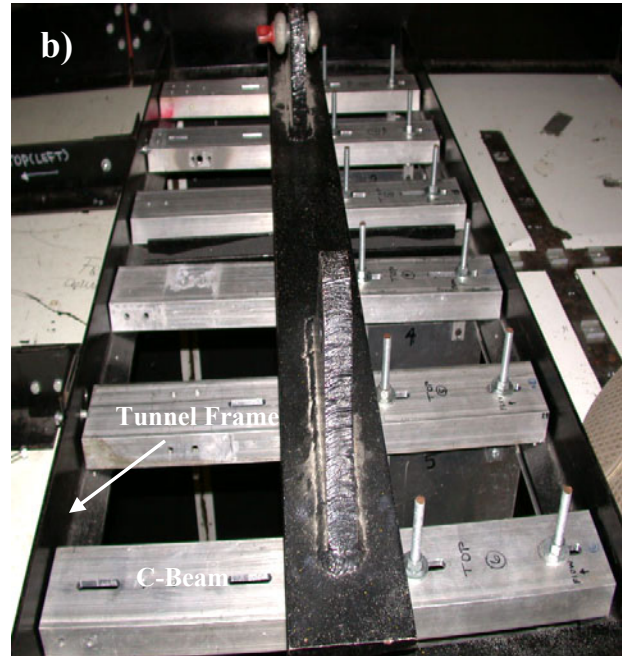
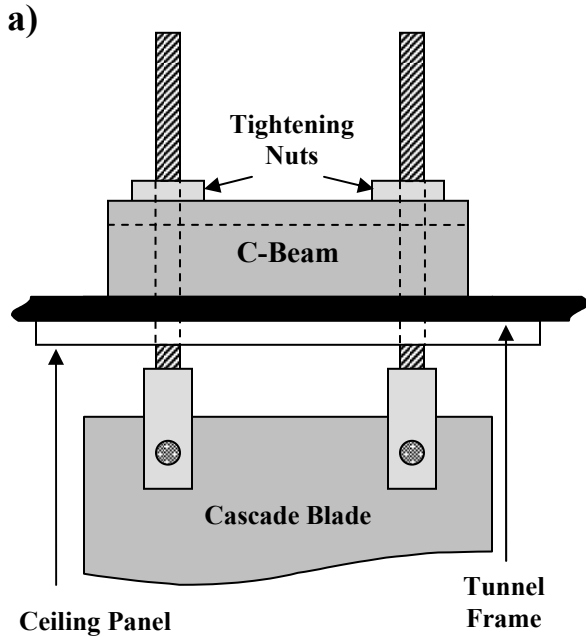


Figure 5.3. Cascade mounting hardware; a) ceiling and tensioning assembly; b) top view of cascade mounted in the tunnel with ceiling panels removed; c) floor assembly; d) view of floor assembly with floor panels removed. Schematics in a) and c) are not drawn to scale.

The gaps in the tunnel ceiling and floor are covered with rectangular 1/8" thick steel panels which are clamped to the exterior tunnel frame by passing two oppositely positioned screws through countersunk holes from the inside of the panels. Seals between the panels are provided by strips of duct-tape.

The chord of 12.9 ± 0.05 " was chosen in order to obtain a ratio of turbulence integral scale to chord length on the order of unity, while retaining reasonable Reynolds numbers. The six blades resulted in a blade spacing* of 10.4" which gives the cascade a solidity of $c/s = 1.24$.

The semi-circular leading edge requirement was fulfilled by placing 0.25" thick wooden dowel rods next to the rectangular blade edge and wrapping a layer of self adhesive aluminum shim tightly over the assembly which resulted in a smooth surface. The trailing edges are made from brass shim and account for 0.9" of the total chord. The edges were formed by attaching a rectangular strip of shim on both sides of the blade and forming a sharp trailing edge at their intersection. The resulting trailing edge thickness was about 0.025". The leading edge of the shim produced steps in the blade profile at the 88% chord location of less than 0.008" in height which was faired using a layer of 0.002" thick self-adhesive tape.

Four thin aluminum brackets (one of which is visible in Figure 5.2) run the width of the cascade placed at the spanwise limits of the blades (adjacent to the tunnel walls) in order to accurately fix the streamwise location of the blades, the blade spacing and zero angle of attack as well as improve the rigidity of the entire structure. Spanwise and blade-to-blade variations in blade spacing were estimated to be less than 0.1". Variations in the streamwise position of the blades were less than 0.05". Deviations in angle of attack from zero were less than 0.2 degrees.

5.3 Cascade Coordinate System

In addition to the global grid coordinate system described in Section 4.1, the cascade calls for the inclusion of another local system, expressed here in lower case coordinates. The coordinates x , y , and z , have the same directional senses as the corresponding capitalized global coordinates as can be seen in Figure 5.1. In both cascade configurations (staggered and un-staggered) the origin lies in the leading edge plane at midspan of the third blade from the left when viewed from downstream. For the non-staggered configuration the leading edges of the cascade ($x = 0$) were situated at $X/M = 38.0$. This location was chosen since detailed free stream turbulence statistics were obtained for the desired operating conditions at a streamwise location of $X/M = 37.3$. The corresponding streamwise location for the staggered cascade configuration is defined with the local cascade coordinate located at $X/M = 37.0$. This is shown schematically in Figure 5.4 which reveals the relative positioning of the two coordinate systems. The stagger angle is given by ϕ which is zero for the unstaggered and 35.8° for the staggered configuration.

* The blade spacing (s) is defined as the distance between the centerline of two adjacent blades.

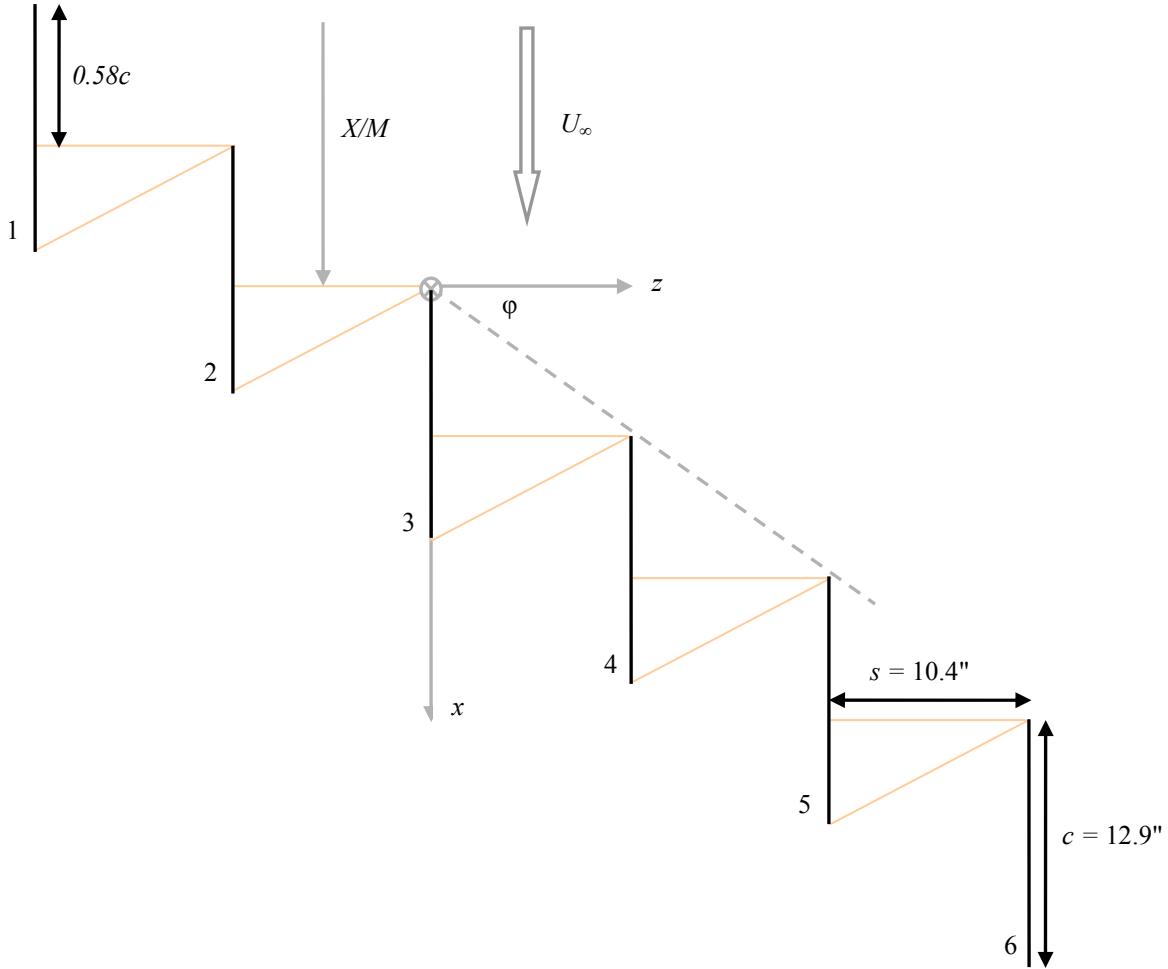


Figure 5.4. Generic Cascade Coordinate System. The unstaggered cascade is fully defined by $X/M = 38.0$ and $\phi = 0$. The equivalent numbers for the staggered cascade are $X/M = 37.0$ and $\phi = 35.8^\circ$. The free stream velocity (U_∞) is nominally 12.5 m/s for both configurations with an associated Reynolds number of $Re_c = 2.7 \times 10^5$. The schematic shows the staggered cascade configuration to scale, without the presence of the wind tunnel walls.

5.4 Flow Visualization

As mentioned earlier there were some initial concerns that the large turbulence would produce unsteady, large-scale stall on the blades, and thus produce very thick viscous regions making the experiment unrealistic, and the following comparison with theory meaningless. Some initial tests were in other words required to confirm the nature of the flow over the central part of the cascade. This was done by checking the boundary layers developing on a single cascade prototype blade placed in the middle of the wind tunnel section using the flattened Pitot-probe. By measuring the velocity profile at $x/c = 0.89$, the boundary layer thickness was found to be around 1.60 cm for free stream velocities between 9 and 15 m/s at grid speeds of both 2 and 4 Hz as can be seen in Figure 5.5.

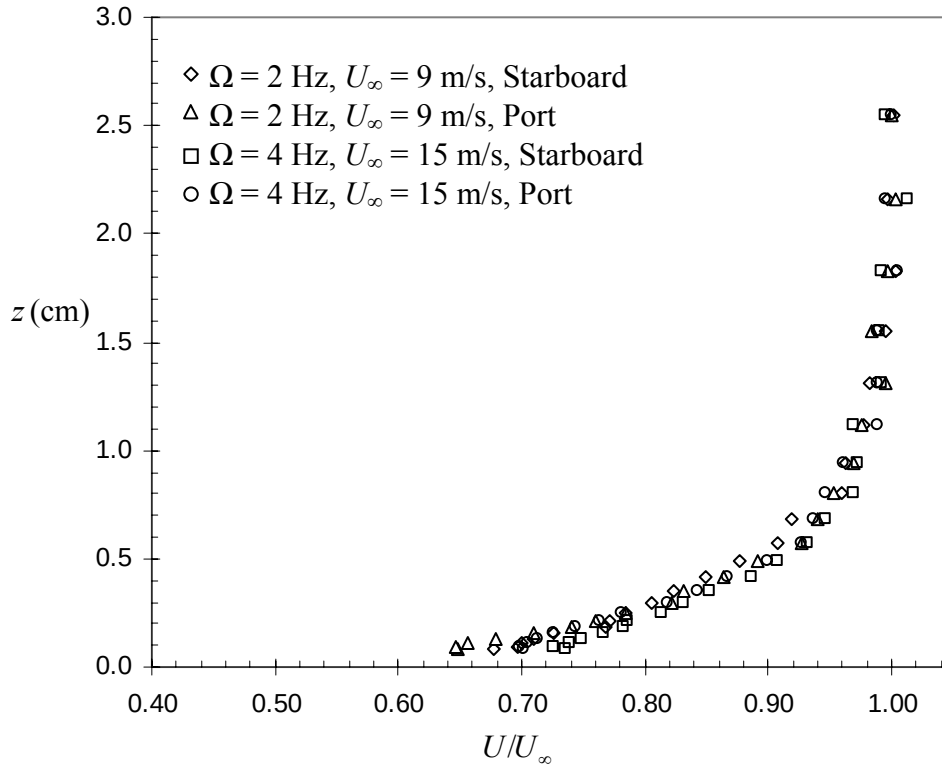


Figure 5.5. Boundary layer measurements on both sides of cascade blade obtained at $x/c = 0.90$ at two different free stream and grid speeds.

Although promising, a more visual picture of the flow behavior on the cascade blades was obtained through flow visualization after the initial assembly of the unstaggered cascade. By performing an oil flow experiment on the surface of blade 3 some more insight was given of the overall flow behavior. During the visualization the cascade was subjected to the baseline free stream condition described in Section 4.2.1. The result of the study can be seen in Figure 5.6 where the central vertical axis represents the leading edge and the left and right sides correspond to the starboard and port side of the blade respectively as seen from downstream. The flow does indeed separate and forms a separation bubble at the leading edge as the oil can be seen to pile up here. This only accounts for about 2% of the chord however as skin friction lines start to develop over the rest of the surface. The experiment was somewhat flawed by a liberal application of oil over parts of the surface causing gravity to play in and make the skin friction lines take a downward turn. This can especially be seen on the lower surface whereas the left top part of the oil flow shows a better oil application which resulted in more useful results

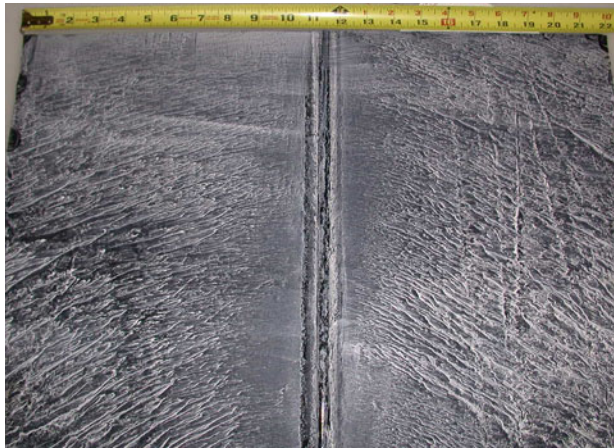


Figure 5.6. Oil Flow Visualization of blade 3 for the baseline condition.

Whereas oil-flow visualization gives a good quantitative idea of the mean flow, the instantaneous flow-field can be assessed visually by attaching tufts over the cascade surface. This was done according to the description in Section 2.3 on the starboard side of blade 3 in the staggered cascade configuration. Video footage shows no sign of adverse flow behavior at various speeds and grid conditions including the baseline condition. What can clearly be shown, however, is the presence of very large eddies as tufts placed more than 30 cm apart would display very concurrent behavior, confirming the presence of considerable correlation even at such large separation distances.

5.5 Other Flow Characteristics

With the confirmation of a well behaved flow field over the blade surface itself, a few more measurements were obtained in order to document some general characteristics of the cascade. This included the mean flow field in and around the cascade passages, as well as the characteristics of the wakes to assess the pitchwise periodicity behind both cascade configurations. Additionally a simple method to verify the level of spanwise 2-dimensionality was utilized on the unstaggered cascade

5.5.1 Mean Flow Field

The mean flow data in this section was obtained using three-component four-sensor hot-wires, and measurement profiles were generally obtained starting at blade 3 (or behind the centerline in the case of the wake locations) extending to the middle of the passage for the unstaggered cascade and covering as much of the passage as possible for the unstaggered case. Complete spectral data was obtained at every measurement point. The turbulent stress profiles and associated velocity spectra are presented in Chapter 6.

The mean velocity measurements for the flow through the unstaggered cascade show, as expected, essentially uniform flow. Deviations from uniformity greater than the measurement uncertainty included only: a slight 2.3% acceleration of the flow coming into the blade passage, consistent with the blockage of the blades and their boundary layers, a slight deflection of the flow around the blade leading edges, measured near $z/s = 0$ at $x = 0$, and the reduction in velocity in the viscous blade boundary layers and wakes.

Figure 5.7 shows mean flow profiles through one half of the central passage as well as immediately behind cascade blade 3 for the unstaggered cascade. Note that a different scale has been applied to the lateral component (V) where the reference free stream velocity appears ten times larger than for the streamwise component (U). The velocity scales are indicated in both figures in terms of the free stream velocity U_∞ . Likewise, Figure 5.8 displays the measured mean velocity field in the central blade passage of the staggered cascade configuration.

By symmetry the unstaggered cascade mean velocity field can be mirrored around $z/s = 0$ for a more intuitive view of the overall mean flow. This has been done in Figure 5.9 which exhibits a vector version of Figure 5.7 which includes both measured and mirrored data. It should be noted that the vertical scale in this figure has been greatly exaggerated for clarity. Close to the leading edge the velocity vector appears to be angled almost 45 degrees, although the actual deflection angle is closer to 11 degrees. Similarly, by taking advantage of the repeatable nature of an ideal infinite staggered cascade, the data from Figure 5.8 has been streamwise shifted and applied to the adjacent passage to achieve a more complete picture of the velocity field around the blades. The result is shown in Figure 5.10. Again, the vertical scale has been exaggerated to enhance the spanwise detail.

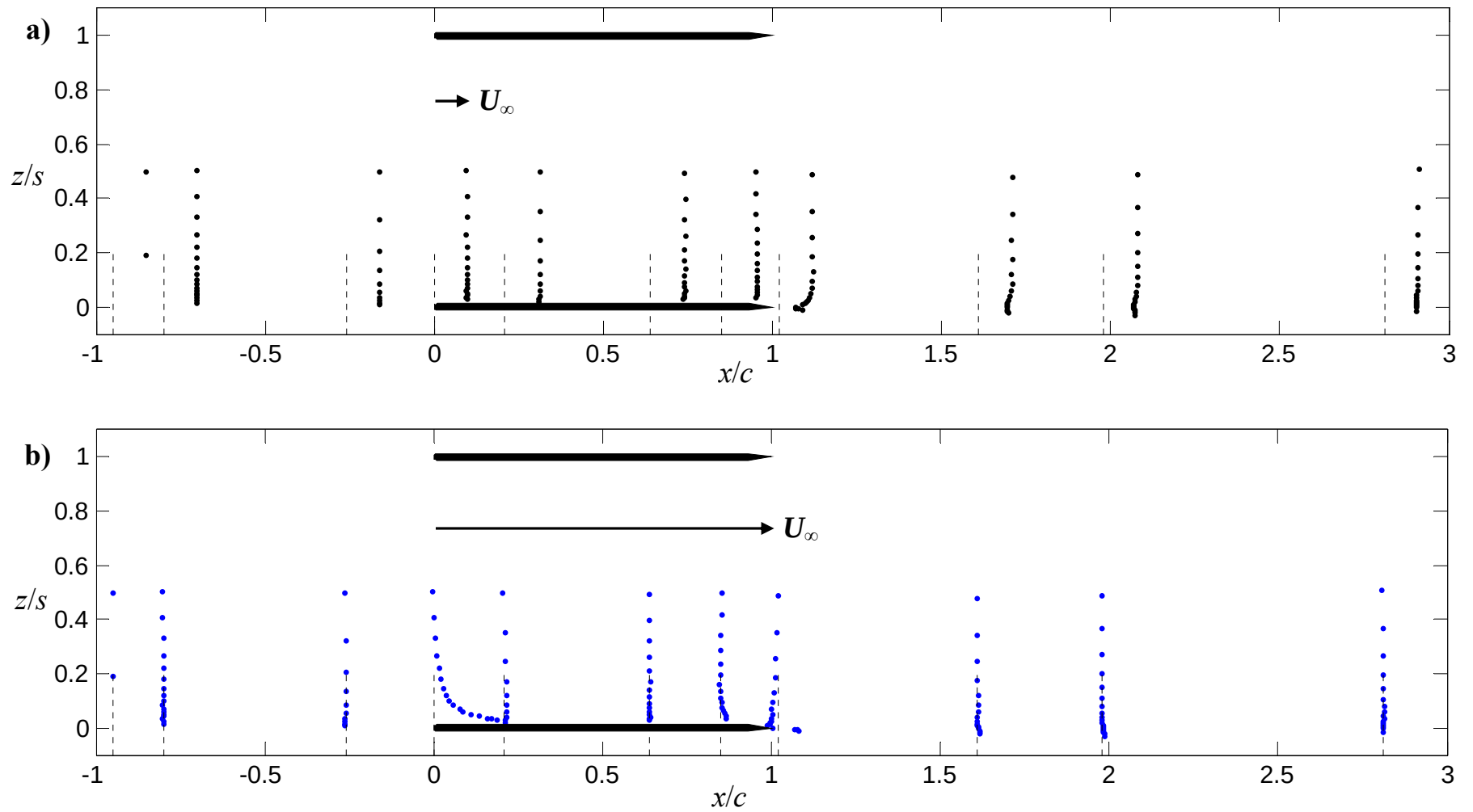


Figure 5.7. Measured mean velocity field over the starboard side of blade 3 in the unstaggered cascade. Measurement stations are indicated with a dashed vertical line. The streamwise component (U , ●) of mean velocity is shown in a) where the applied scale is $U_\infty = 0.1x/c$ while the lateral component (W , ●) is displayed in b) with $U_\infty = x/c$ as a scale reference.

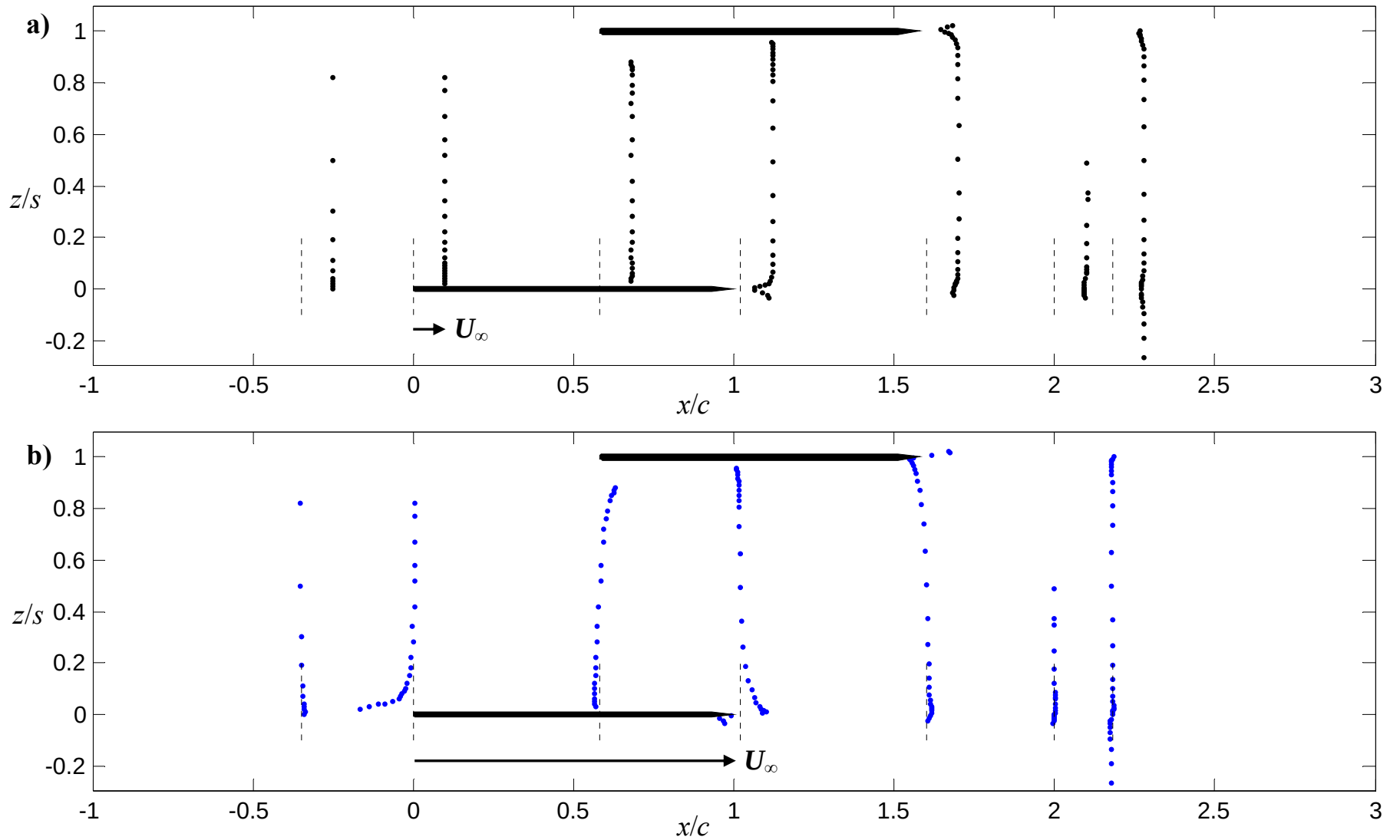


Figure 5.8. Measured mean velocity field through the two central passages in the staggered cascade. Measurement stations are indicated with a dashed vertical line. The streamwise component (U , ●) of mean velocity is shown in a) where the applied scale is $U_\infty = 0.1x/c$ while the lateral component (W , ●) is displayed in b) with $U_\infty = x/c$ as a scale reference.

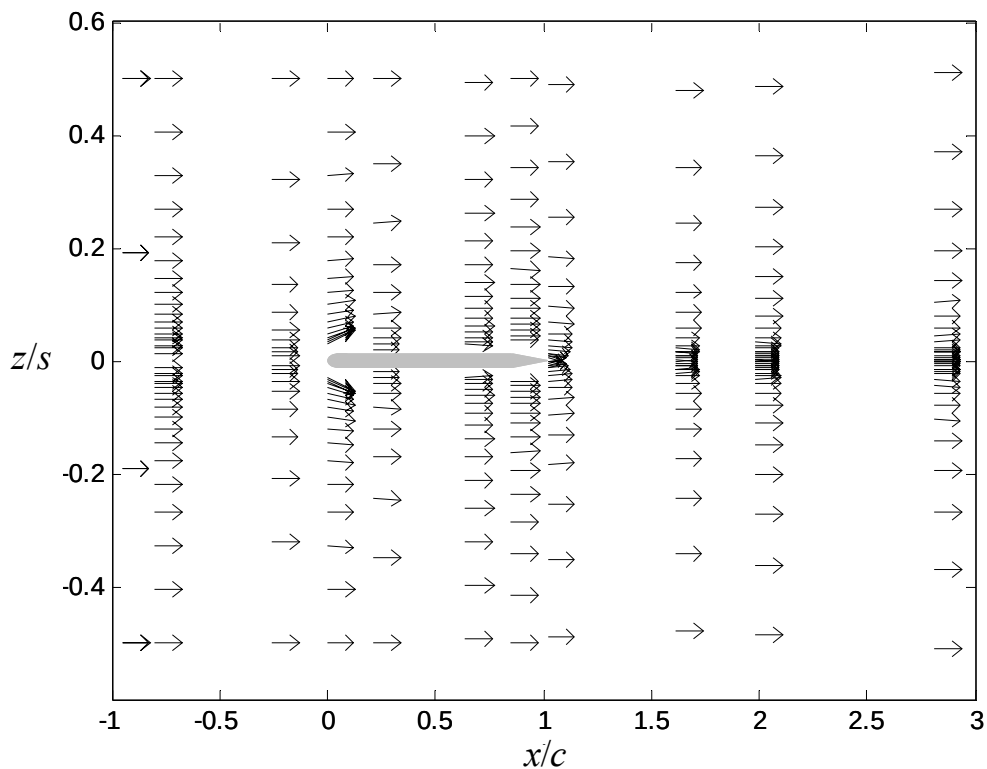


Figure 5.9. Complete mean velocity field around blade 3 in the unstaggered cascade obtained by mirroring Figure 5.7 around $z/s = 0$. Vertical scale has been greatly exaggerated.

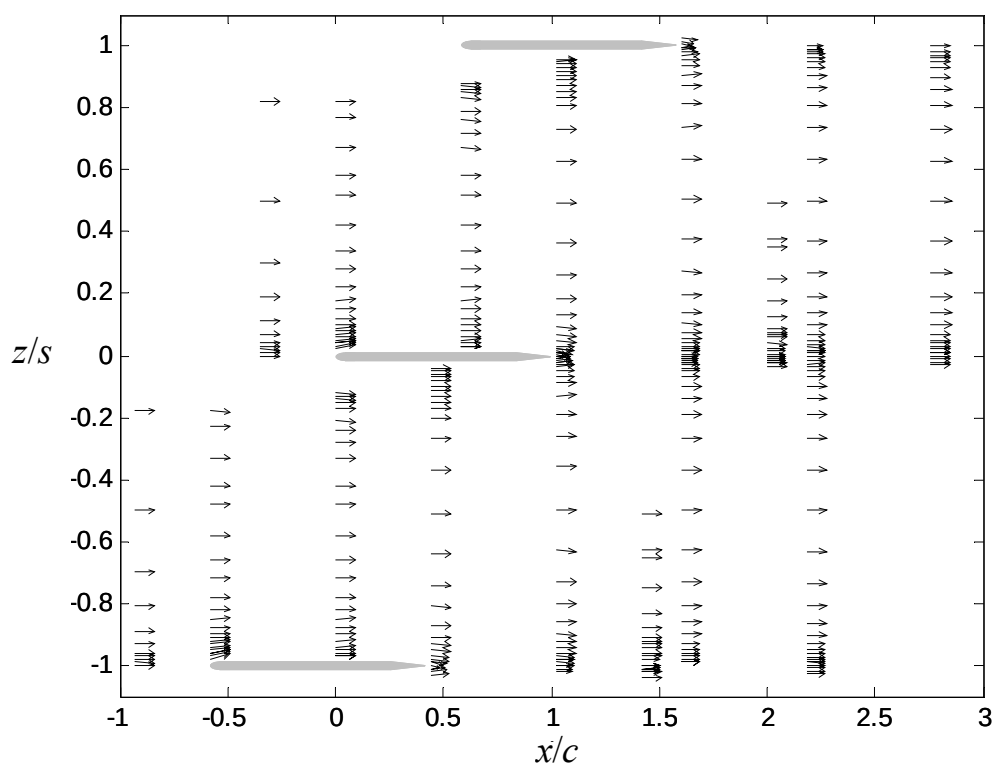


Figure 5.10. Complete mean velocity field through the two central passages in the staggered cascade obtained by applying the data in Figure 5.8 to the adjacent passage. Again the vertical scale has been exaggerated for clarity.

5.5.2 Periodicity and Two-Dimensionality

One way of assessing the cascade flow is to look at the pitchwise periodicity in the mean and fluctuating flow immediately behind the cascade blades. If the flat plate unstaggered cascade is placed at zero angle of attack the wakes should be perfectly symmetric and appear at even intervals equal to the blade spacing. Additionally, by comparing at least two different spanwise wake profiles at the same streamwise location the level of two-dimensionality can be assessed. As discussed above three-dimensional effects were reduced by extending each cascade blade to the tunnel walls. Due to the large integral scales present in the active turbulent flow, however, a large part of the tunnel section will experience wall effects as discussed in Section 4.3.3.

In order to document the periodicity and the level of two-dimensionality of the unstaggered cascade, the TSI single hot-wire was traversed in the wake at $x/c = 1.21$ over the three central passages (4 blades) at mid-span ($y = 0$) and over the central passage (2 blades) at $y = -22.9$ cm for the baseline condition. The two profiles have been plotted in Figure 5.11 which shows both the mean and fluctuating velocities normalized on the free stream velocity and the z -coordinate has been normalized on blade spacing, s .

By examining the profile at $y = 0$, it can be concluded that the flow is fully periodic over the three central passages with the wakes centered at integer multiples of the blade spacing from the origin within the accuracy allowed by the resolution of the measurements. Comparing this to the second profile at $y = -22.9$ cm, it is clear that the mean flow measurements differ by less than 2% between the two spanwise locations while the turbulence intensity displays a maximum difference of approximately 1%. This confirms that the flow is closely two-dimensional between these positions, at least for the streamwise component. It should be expected, based on the results from Section 4.3.3 which showed homogeneity up to approximately one and a half integral lengthscales from the wall, that two-dimensionality will hold for the same measurement domain. The mean flow as well as streamwise and upwash fluctuations (x - and z -direction) are expected to display two-dimensional behavior even closer to the wall, the extent of which can be determined based on the wall blocking prediction methods which will be discussed in Section 7.2.

The central passage for the staggered cascade is similarly shown in Figure 5.12 immediately behind ($2c/100$) the trailing edge of the blades at $y = 0$. Again the pitchwise periodicity is evident. The two datasets shown in this figure were obviously acquired at two different streamwise locations in order to achieve the same relative position behind the blades. This creates the discontinuity shown at $z/s = 0.5$. The difference between the level of turbulence intensity found in the two wakes can be attributed to local trailing edge imperfections as well as the fact that the size of the hot-wires used in this experiment and the resolution and positioning of the traverse mechanism can cause some data averaging where the turbulence intensity gradients are very large.

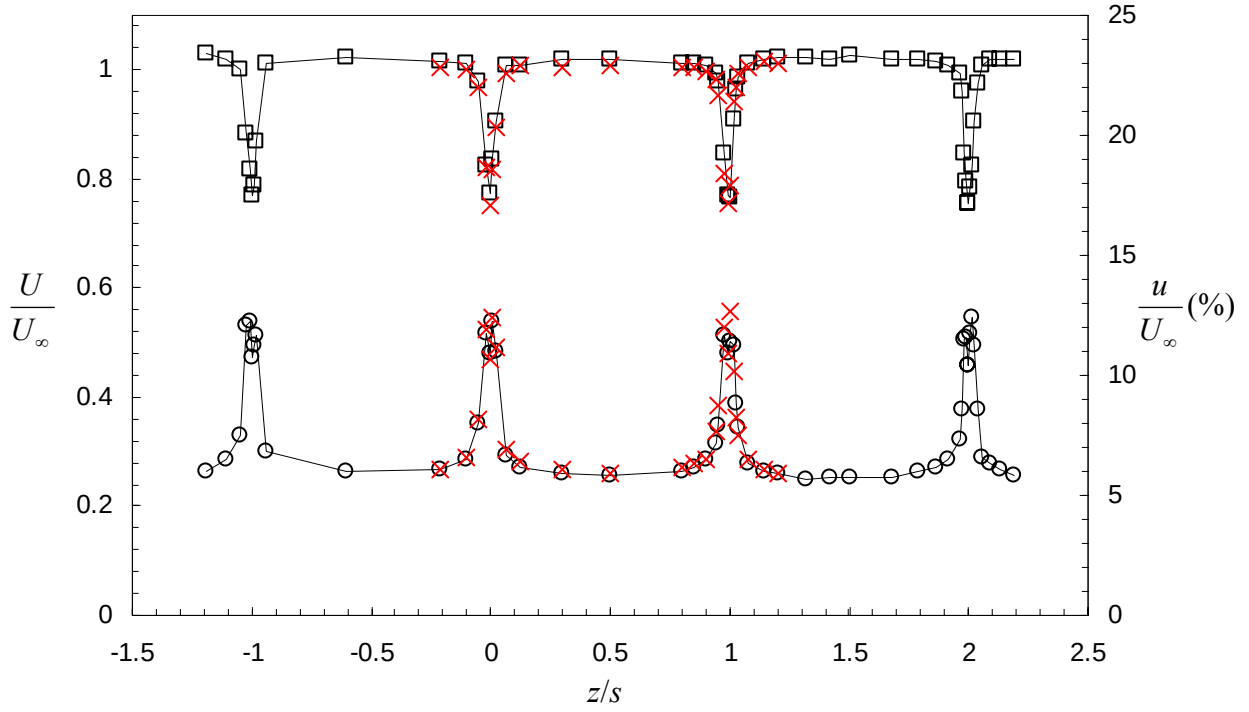


Figure 5.11. Normalized mean velocity levels (squares) and turbulence intensity (circles) across the three central passages behind ($x/c = 1.21$) the unstaggered cascade at $y = 0$ (black) and at $y = -22.9$ cm (red).

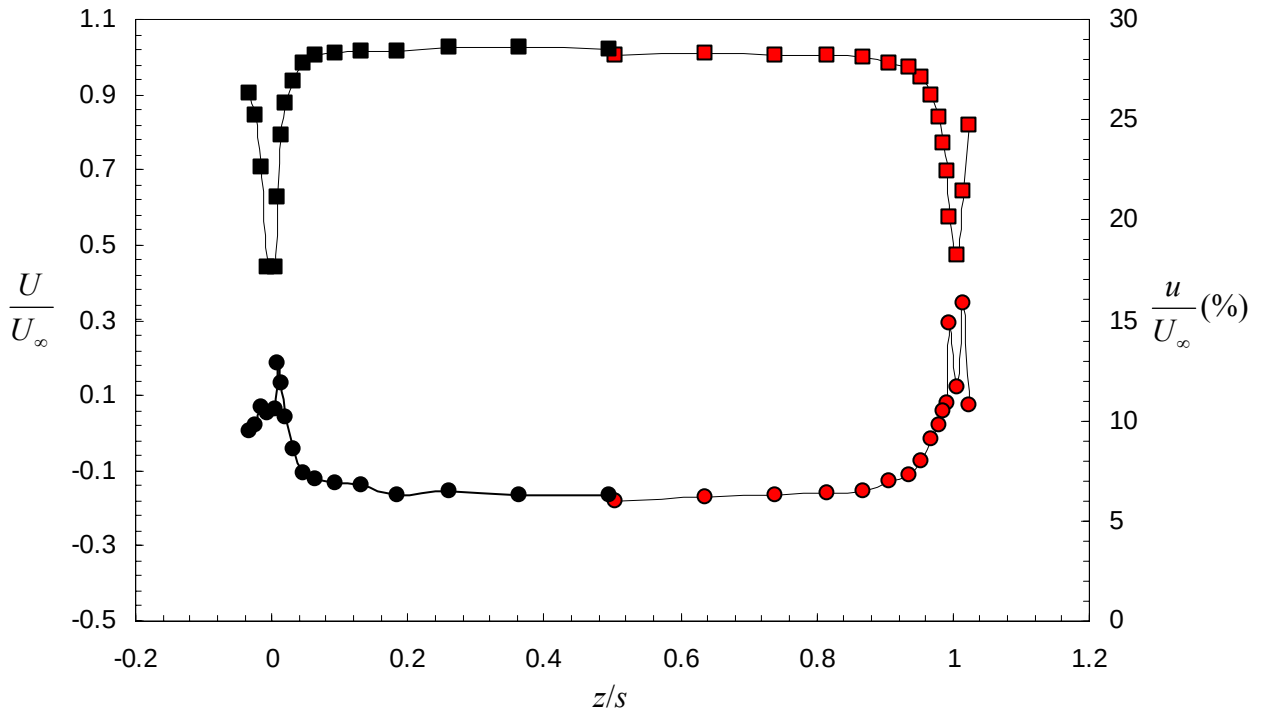


Figure 5.12. Normalized mean velocity levels (squares) and turbulence intensity (circles) across the central passage behind the staggered cascade. Data obtained at two different streamwise positions due to the stagger, hence creating the discontinuity seen at $z/s = 0.5$; $x/c = 1.02$ (black) and $x/c = 1.60$ (red); $y = 0$.

Chapter 6

Cascade Effects: Experiments

This chapter presents the results of the blade blocking study where large scale turbulence was passed through a cascade of flat plates. After a brief introduction in Section 6.1, both the unstaggered and staggered cascade configuration will be discussed in terms of averaged Reynolds stresses in Section 6.2, spectral considerations in Section 6.3 and a look at the modified two point correlation will be shown in Section 6.4.

6.1 Introduction

The baseline condition described in Section 4.2.1 was used as the turbulent inflow for the blade blocking experiments discussed in this chapter. As mentioned in Section 4.6.2 the low frequency region is subject to severe uncertainty due to the wind tunnel wall effects on the largest eddies present, and hence makes the definition of an exact integral scale difficult. To make matters worse, based on the literature presented in Section 1.3.4, it is expected that the vast majority of cascade effects will be observed in the low frequency region of the velocity spectrum. As will become clear in Chapter 7 the problem of a well-defined in-flow integral scale must be addressed when comparing with theoretical predictions. In the presentation of the experimental data by itself, however, this is generally of less importance, and could largely be ignored for now. Nevertheless, for reference, the turbulent inflow is defined as having a Taylor Reynolds number of 512 and an integral scale L_{11} of 28 cm immediately upstream of the cascade entrance ($X/M = 37.3$).^{*} With the chord length, $c = 32.8$ cm and blade spacing, $s = 26.5$ cm, this results in the following cascade parameters: the chord to spacing ratio $c/s = 1.27$, the spacing to integral lengthscale ratio $s/L_{11} = 0.95$ and the chord to integral lengthscale ratio $c/L_{11} = 1.17$. As initially required (see Section 1.8) these ratios are all close to unity. These parameters will be revisited in Chapter 7 as they are of paramount importance in the RDT predictions of the cascade blade blocking effect.

Three-component hot-wire measurements were made prior to, inside and behind the central cascade passage in both the staggered and unstaggered configurations and both Reynolds stresses and spectral results will be presented in this chapter. For the unstaggered case, the ten different streamwise stations were obtained during two separate wind tunnel entries eight months apart, and the earlier session has therefore been normalized such that the free stream turbulence kinetic energy of the two sessions match in an attempt to eliminate quantitative differences between the

^{*} As shown in Section 4.6.2 these parameters better describe the low frequency end of the actual measured turbulence, and is more representative for the blade blocking study than the parameters given in table 1.1 for the baseline condition which takes into account the entire frequency spectrum.

two datasets. The cascade coordinate system was introduced in Section 5.3 and all the normalized coordinates and references used herein refer to the definitions found schematically in Figure 5.4.

For the staggered cascade a total of seven different streamwise stations were obtained. The data was acquired during the same wind tunnel entry as the second measurement session described above and the turbulent onset flow hence roughly matches that of the unstaggered cascade. Apart from the obvious geometric differences between the two cascade configurations, the reference blade, at which the cascade coordinate system described in Figure 5.4 originates, is located slightly further upstream ($X/M = 37.0$ vs. $X/M = 38.0$) in the tunnel. The relative isotropic reference free stream values presented in the following subsections take this into account, although the difference is vanishingly small given the relatively low dissipation rate of the free stream.

6.2 Throughflow: Reynolds Stresses

This section describes Reynolds stress profiles behind both cascade configurations. Emphasis has been put on the behavior of the normal components, but inside and behind the cascade turbulence shear stress profiles have also been included for completeness.

6.2.1 Unstaggered Cascade

Each part of this subsection features a detailed look at each measurement station. A summary is provided at the end to tie everything together.

Ahead of the unstaggered cascade

In the flow prior to the cascade, one would at first expect very few differences from the free stream flow in the absence of the cascade. It turns out, however, that there were distinct differences attributed to the presence of the cascade. Figure 6.1 shows the three normal stresses at $x/c = -0.80$ with the isotropic value obtained from the empirical fit given by Equation 4.13 included for reference. Although almost one chordlength ahead of the cascade, the isotropy has already been impaired in comparison with the free stream value. The pitchwise stresses have been slightly reduced while the spanwise stresses are comparatively amplified. The streamwise stresses seem relatively unaffected by the cascade yielding no net decrease in overall turbulence kinetic energy (tke) at this location. Although intuitively one would might think that the cascade is of no effect this far upstream, it is conceivable that due to the large lengthscales present, the effects of the cascade is felt already at this point.

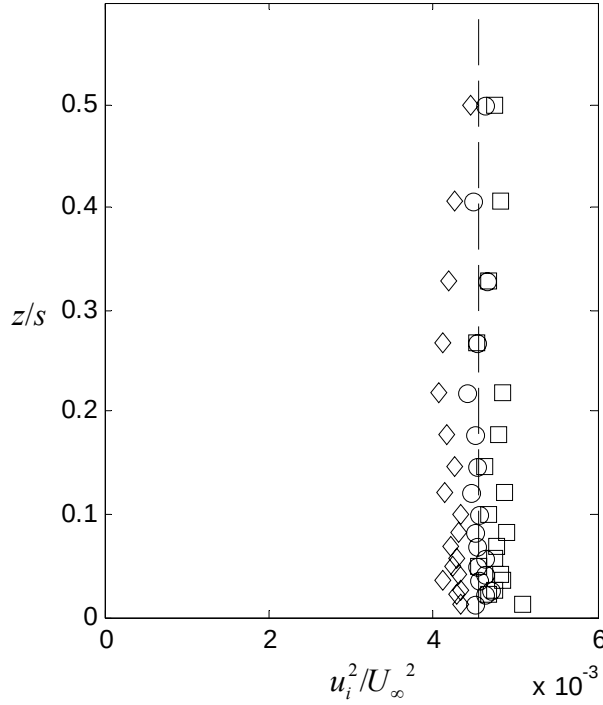


Figure 6.1. Reynolds normal stresses at $x/c = -0.80$ ($X/M = 36.75$) for the unstaggered cascade. $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 . The reference free stream level for the same location is given by the dashed line.

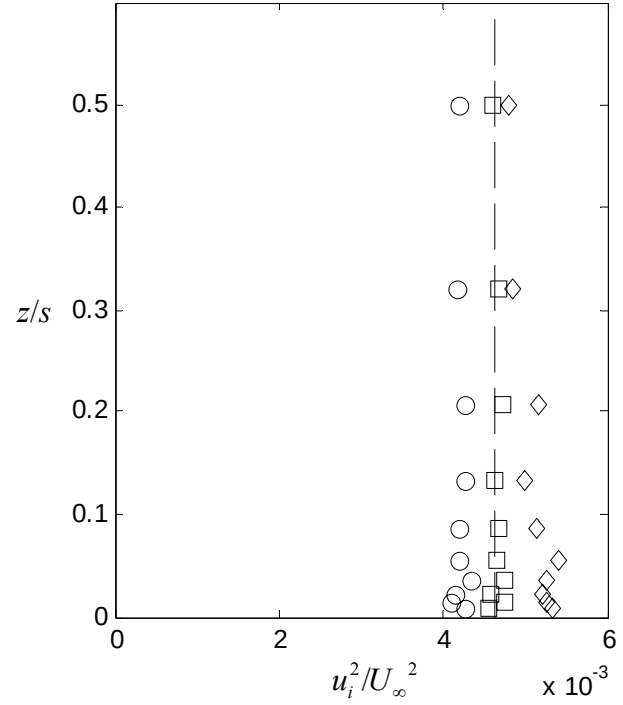


Figure 6.2. Reynolds normal stresses at $x/c = -0.26$ ($X/M = 37.59$) for the unstaggered cascade. $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 . The reference free stream level for the same location is given by the dashed line.

As the flow nears the cascade, the streamwise stresses decrease by 9% relative to the spanwise and reference free stream values as can be seen in Figure 6.2. Most curious however is the sudden burst in the pitchwise stresses which in the center of the passage yield levels consistent with the isotropic free-stream values, but with a marked increase towards the cascade blade, keeping the total tke constant relative to the isotropic value. Comparing Figure 6.1 and Figure 6.2 it is clear that the cascade effects are propagated at least one integral scale upstream with complex dynamic results, especially in the pitchwise stresses. This effect is generally not observed in similar cascade studies, as most measurement sets generally do not include upstream data in the presence of the cascade, and even if it did, the region of interest would be very close to the leading edge due to much smaller free stream integral scales.

At the leading edge plane

Figure 6.3 shows all six unique Reynolds stresses at the leading edge plane of the cascade. As expected the shear stresses are all zero in the mid-passage. While the perpendicular shear stresses remain zero all the way to the blade, the parallel uw shear stress can be found to monotonically increase as the leading edge is approached. This is due to the deflection of the streamlines in the vicinity of the leading edge and would be expected to be equal, but opposite in sign on the other side of the blade.

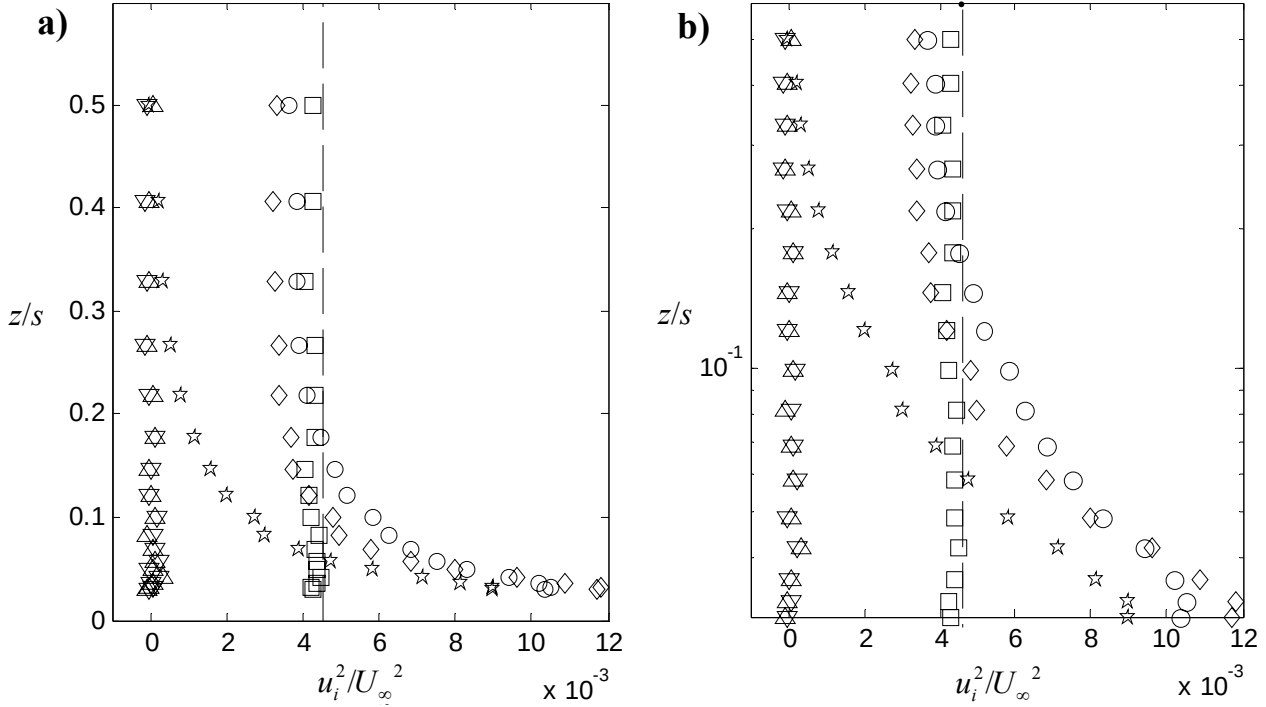


Figure 6.3. Reynolds stresses at $x/c = 0.00$ ($X/M = 38.0$) for the unstaggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line.

As for the normal stresses, the spanwise stress still remains fairly similar to the reference freestream values, although a slight 6% drop can be observed. The level is however consistently preserved from the mid-passage location all the way to the blade as expected due to the lack of apparent blocking in the spanwise direction prior to the leading edge plane. The mid-passage value of the streamwise component continues to decrease, here with a 19% suppression compared to the free stream value. As the blade is approached, however, the streamwise stress monotonically increases up to three times the mid-passage value. The upwash component behaves similarly to the streamwise normal stress, but the mid-passage level has severely dropped, especially considering the apparent boost just upstream. At this location it has experienced a total drop of 27% compared to the isotropic free stream value. The increase in upwash stress towards the blade is also distinctively more rapid than that of the streamwise component. This vast increase exhibited by the streamwise and upwash stresses must be mostly attributed to the mean flow acceleration and deflection caused in the x - z plane by the presence of the leading edge, as the real blade blocking effects with transfer from the normal component to the parallel components obviously has not yet set in (given the behavior of the spanwise stress profile).

Inside the unstaggered cascade passage

Inside the passage, at $x/c = 0.21$, Figure 6.4 shows that the shear stresses remain at zero throughout the passage with the exception of uw which as expected takes on increasingly negative values as the boundary layer is traversed towards the blade surface. The viscous boundary layer and wake regions indicated in this and subsequent figures were estimated from

the mean flow results from Section 5.5.1 in addition to the cues given by the stress profiles in this chapter. The blade thickness is also shown in Figure 6.4a (and subsequent figures containing linear ordinates) for reference.

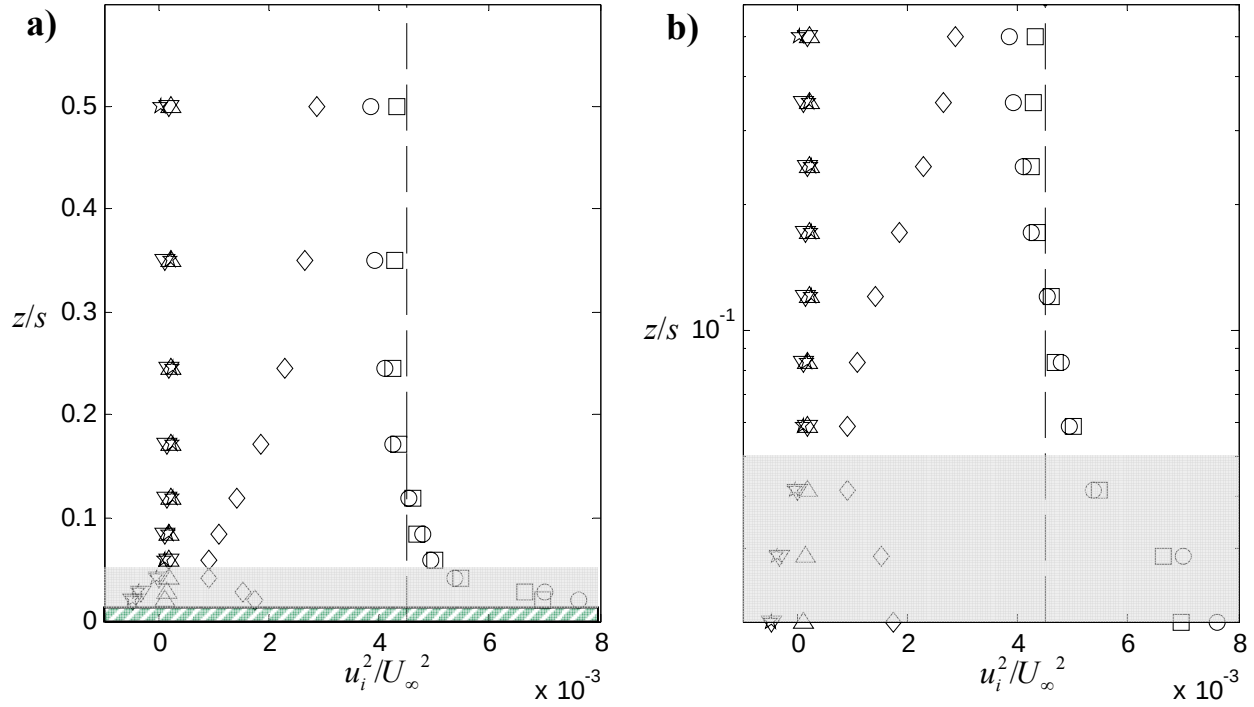


Figure 6.4. Reynolds stresses at $x/c = 0.21$ ($X/M = 38.33$) for the unstaggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

At this location the blade blocking of the upwash component manifests itself very clearly as the mid-passage value of w^2 is only 64% of the isotropic freestream value (36% attenuation). As the blade is approached the attenuation reaches a level corresponding to 80% attenuation before the viscous production in the boundary layer overshadow the blockage effects, causing an increase in overall stress levels. In the absence of a viscous region, the profile would be expected to monotonically approach zero.* The mid-passage attenuation of the streamwise and spanwise components amounts to 14% and 4% respectively, with both components increasing towards the blade, the streamwise components somewhat more rapidly. The transition into the viscous region appears smooth, but with an increased growth rate.

The boundary layer acts to naturally restrict the fluctuations within it due to the wall as well as simultaneously increase the stresses due to the new production caused by the large velocity gradient present in stationary wall boundary layers. For more detail on the restrictive effect on the streamwise fluctuations, see e.g. Uzkan & Reynolds (1967) who performed an experiment on shear-free wall bounded turbulence. Uzkan & Reynolds estimated the wall effect region to grow roughly as $\delta_v = 1.8(\nu x/U_\infty)^{1/2}$. Hunt & Graham's (1978) calculations resulted in an adjustment of the constant to 4.0. In any case this would confine the so called "inhomogeneity" layer to approximately 2.5 mm away from the blade, which is far too close to the blade surface to be

* See Section 7.2.

resolved in the current measurements. On the other hand, the dominating effect of the turbulence production manifests itself throughout the measured boundary layer and shows up as vastly increased stress levels throughout the passage.

The same trend seen at $x/c = 0.21$ is also observed at $x/c = 0.64$ (Figure 6.5) where the streamwise, spanwise and upwash components are attenuated to 84%, 92% and 50% respectively relative to the original free stream levels. This shows that the two blade-parallel components steadily lose energy although at a lower rate compared to the upwash component. In addition it can be noted that due to a thicker boundary layer at this location, and hence a weaker velocity gradient, the shear stress uw is noticeably smaller in magnitude compared to Figure 6.4.

The profiles at $x/c = 0.85$, which can be seen in Figure 6.6, appear very similar. The measured attenuation levels relative to the free stream value in the mid-passage are 84%, 93% and 53% which suggests that the turbulent flow was fully modified by the previous station, indicating that the cascade effects were fully enforced somewhere between $x/c = 0.21$ and $x/c = 0.64$.

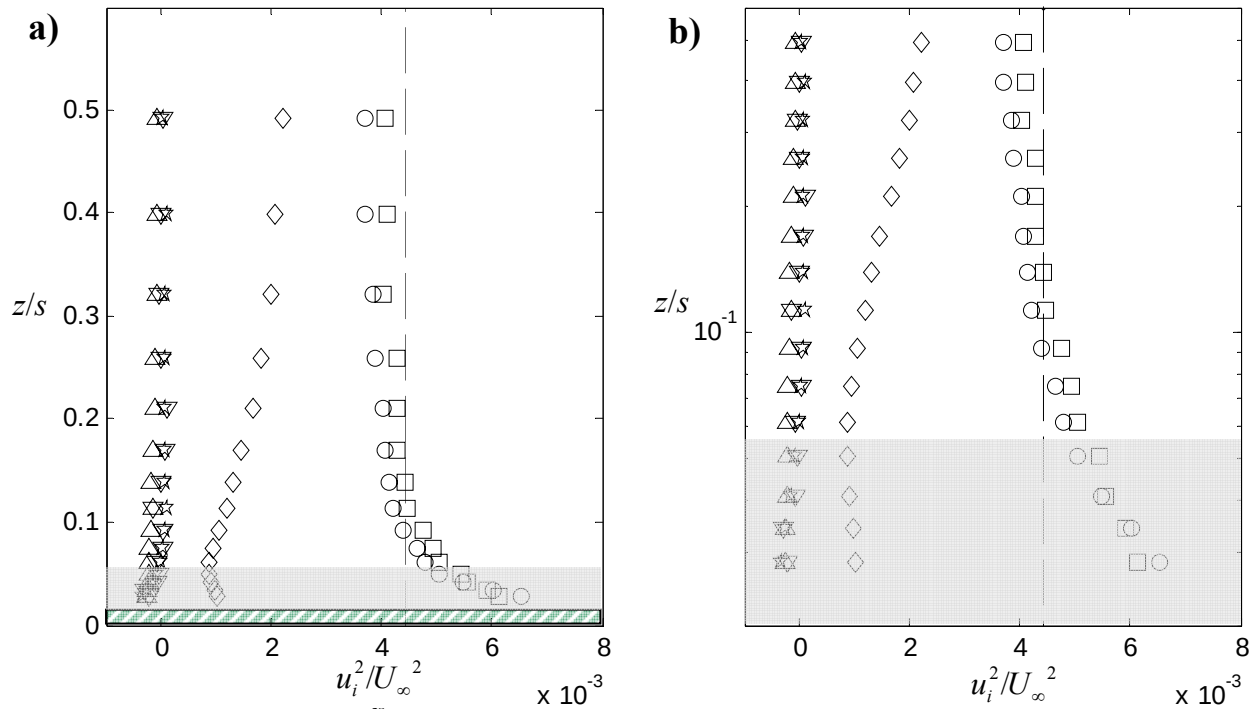


Figure 6.5. Reynolds stresses at $x/c = 0.64$ ($X/M = 39.00$) for the unstaggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

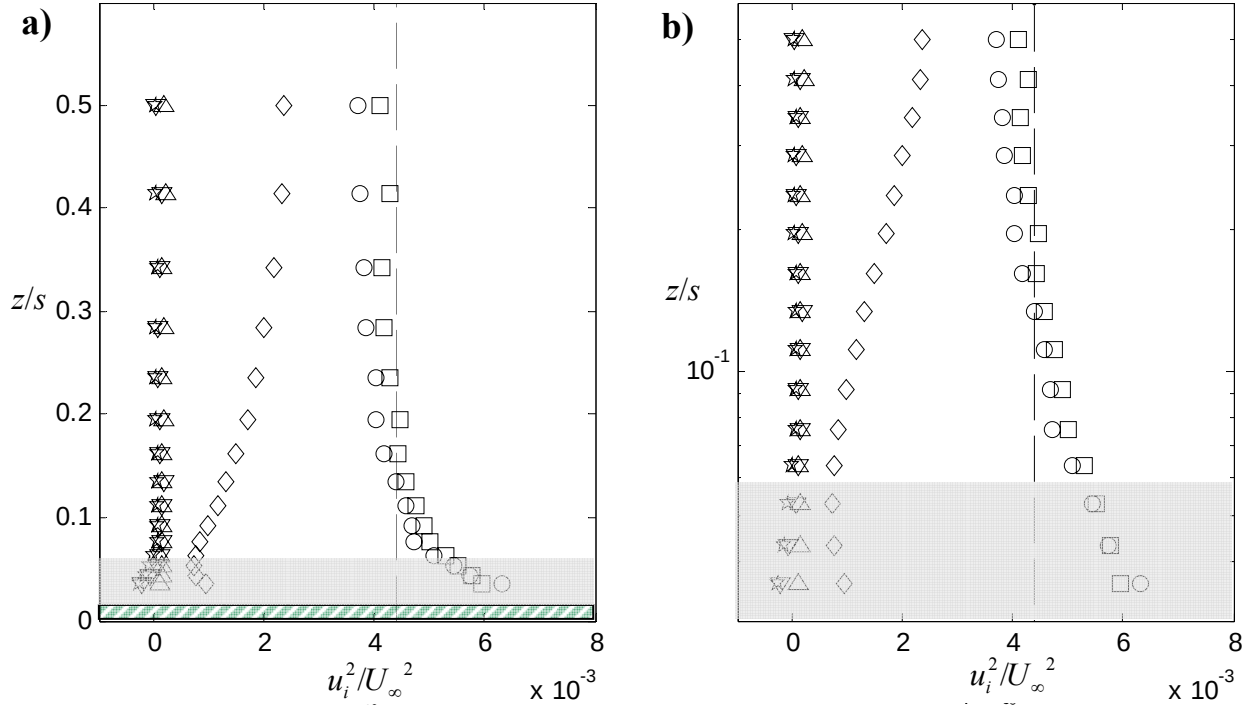


Figure 6.6. Reynolds stresses at $x/c = 0.85$ ($X/M = 39.33$) for the unstaggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

At the trailing edge plane

Figure 6.7 shows the Reynolds stresses immediately after the flow leaves the trailing edge of the cascade. The wake becomes slightly thicker here due to the trailing edge effects which further attenuates the normal stresses. The relative ratio between the normal stresses and the ideal free stream reference is at this point in the mid-passage 81%, 87%, and 46% for the streamwise, spanwise and upwash components. Just before the boundary layer the total suppression of the upwash component has reached 82%. All three normal stresses increase rapidly in value in the highly turbulent region inside the wake. Note the great increase in shear stresses in the wake of the cascade. The uw -stress is appropriately negative on this side of the plate, but returns to zero right behind the trailing edge before switching sign on the other side of the blade.

In the wake of the unstaggered cascade

The rest of the wake region behaves very similarly to what was seen in the trailing edge plane. At $x/c = 1.61$ (Figure 6.8) the attenuation levels are nearly unchanged as the three normal stresses take on values of 79%, 89%, and 45% relative to the isotropic free stream. Due to thicker viscous wakes the max attenuation is reduced to 78% at the onset of the boundary layer. This value is further reduced to 73% at $x/c = 1.98$ (Figure 6.9), and 70% at $x/c = 2.81$ (Figure 6.10). The stresses within the viscous wakes also steadily reduce in magnitude as the flow convects

downstream, however, the mid-passage values of the normal stresses are somewhat enhanced with downstream distance. Relative to the isotropic free stream value the streamwise stress reach a level of 85% by $x/c = 1.98$ and remains at this level for the $x/c = 2.81$ station. The spanwise component follows the same trend at 94%. This increase is also reflected in the upwash component where the level is increased to 50% of the isotropic free stream value. The reason for this is most likely due to a different decay law (modified dissipation rate) on the centerline, resulting in the cascade flow slowly approaching the free stream level estimates for a given location. Over a short distance like the chord itself, this effect is completely overshadowed by the attenuation of all three normal stresses, but as soon as the cascade has permanently altered the flow, this effect slowly becomes important, especially over considerable downstream distances such as the two last streamwise stations.

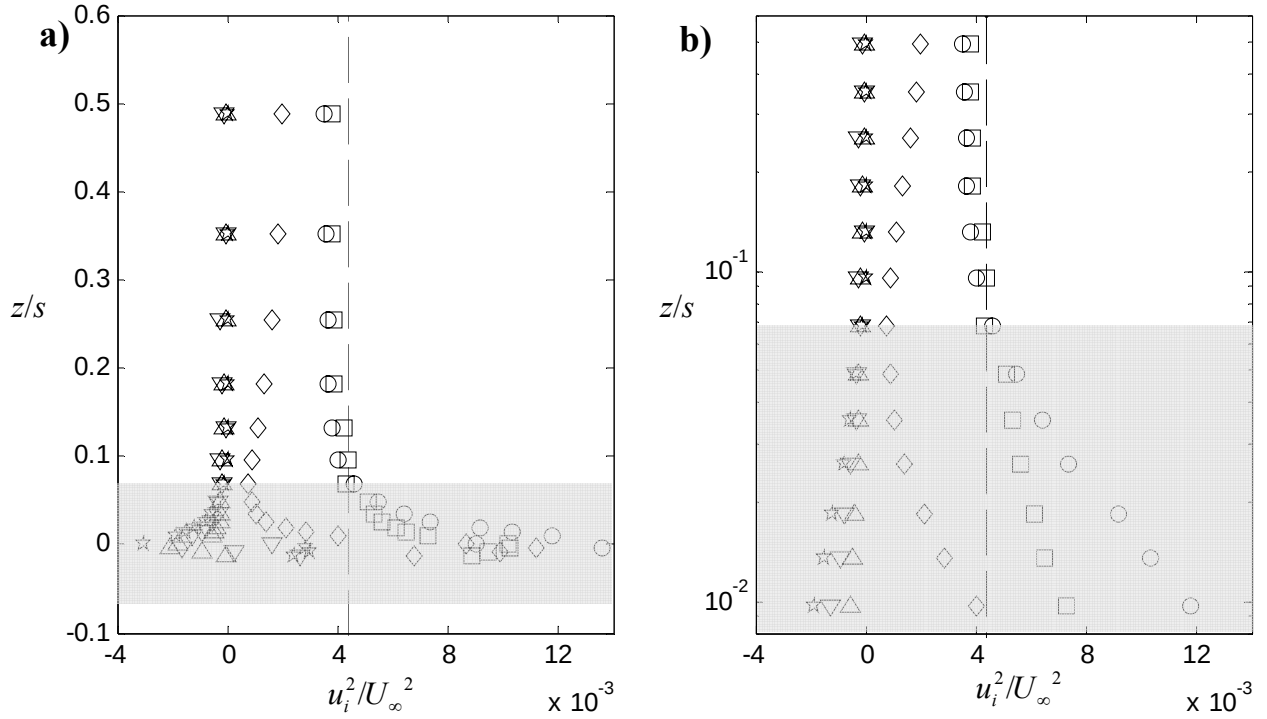


Figure 6.7. Reynolds stresses at $x/c = 1.02$ ($X/M = 39.59$) for the unstaggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

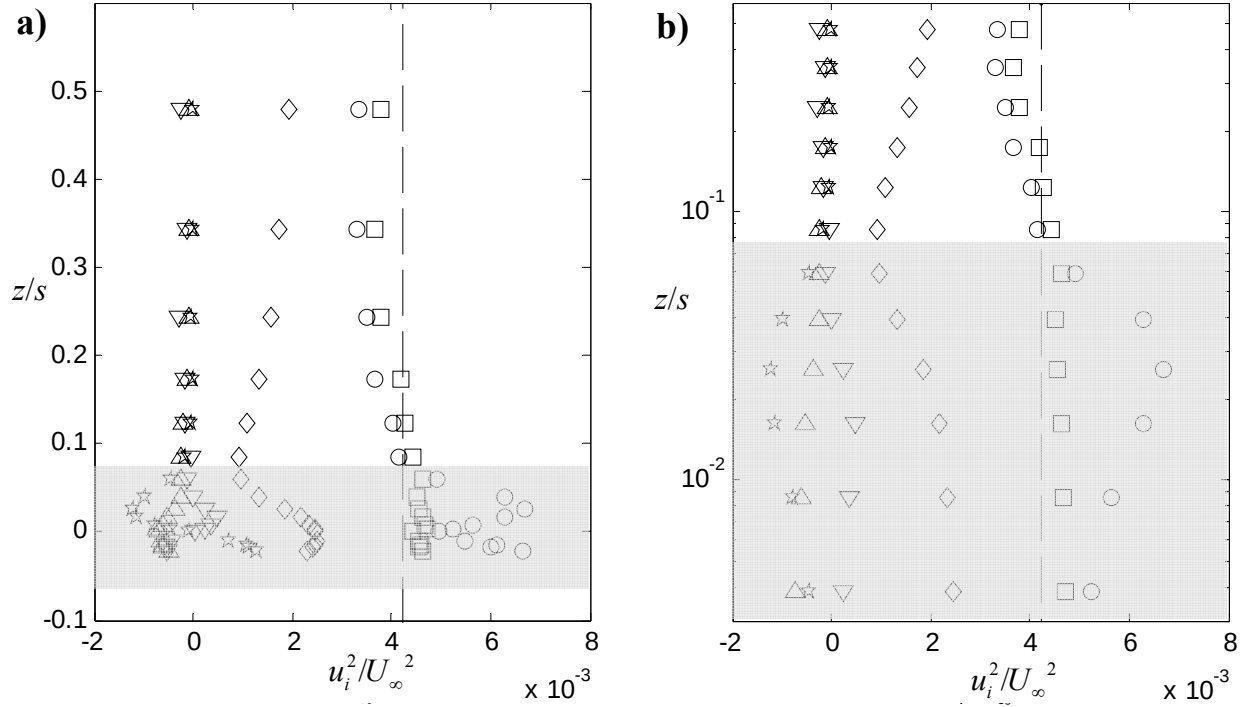


Figure 6.8. Reynolds stresses at $x/c = 1.61$ ($X/M = 40.52$) for the unstaggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

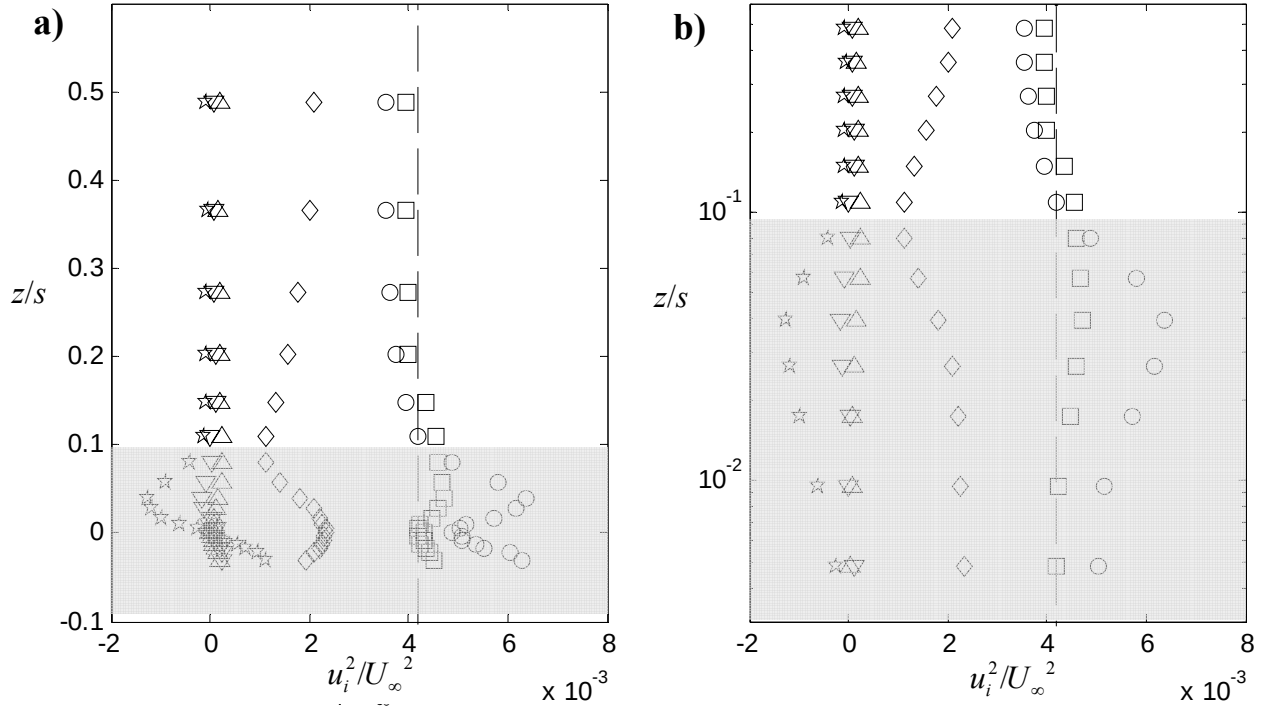


Figure 6.9. Reynolds stresses at $x/c = 1.98$ ($X/M = 41.10$) for the unstaggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

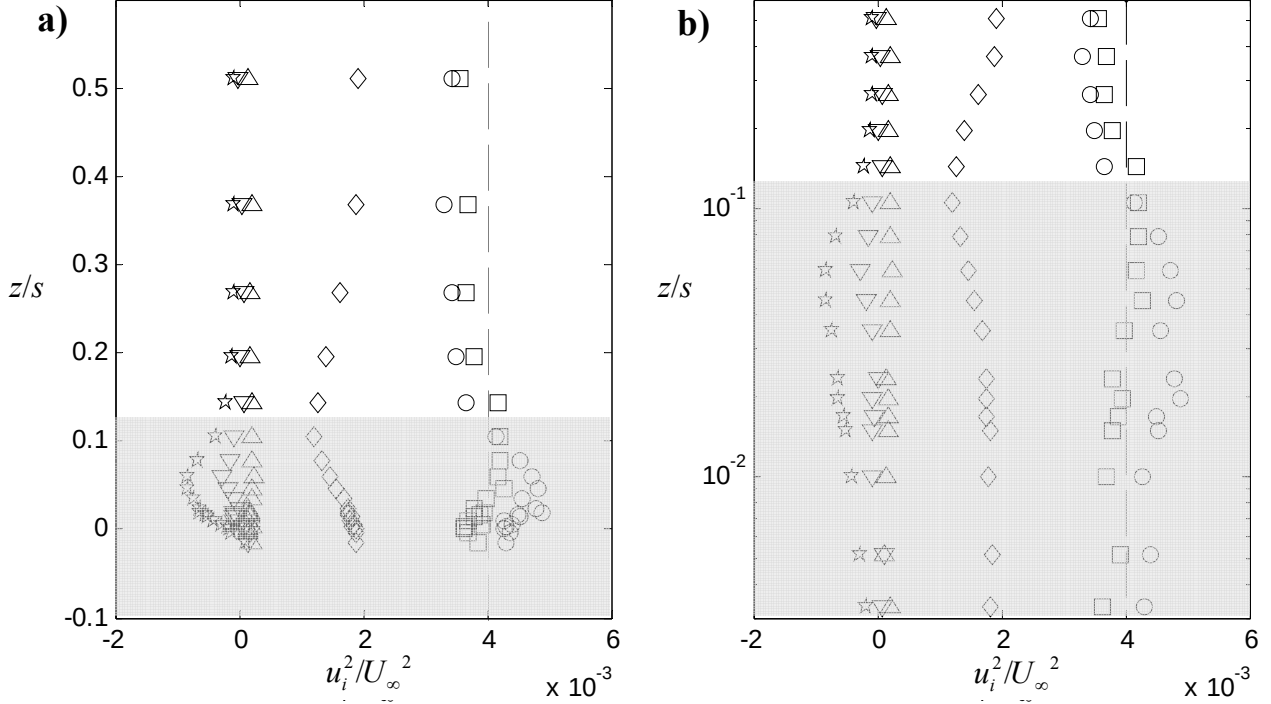


Figure 6.10. Reynolds stresses at $x/c = 2.81$ ($X/M = 42.39$) for the unstaggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

Summary

To sum up this section, the overall behavior of the normal stresses contained in Figure 6.1 through Figure 6.10 have been consolidated into Figure 6.11 which gives a better view of the overall modification of the turbulent stresses. The most important conclusions to draw from this is that the vast majority of cascade effects occur within about half a chordlength (and hence on the order of half an integral scale) of the cascade leading edge plane. Although minor changes do occur as the flow convects further along the chord and exits behind the cascade, the shape and magnitude of the stress profiles remain largely unchanged outside of the relatively thin viscous regions. This suggests that the viscous wakes act like an extension of the cascade blades which do not allow the stress components attempt to return to isotropy as is often seen in conventional homogeneous anisotropic grid flows.

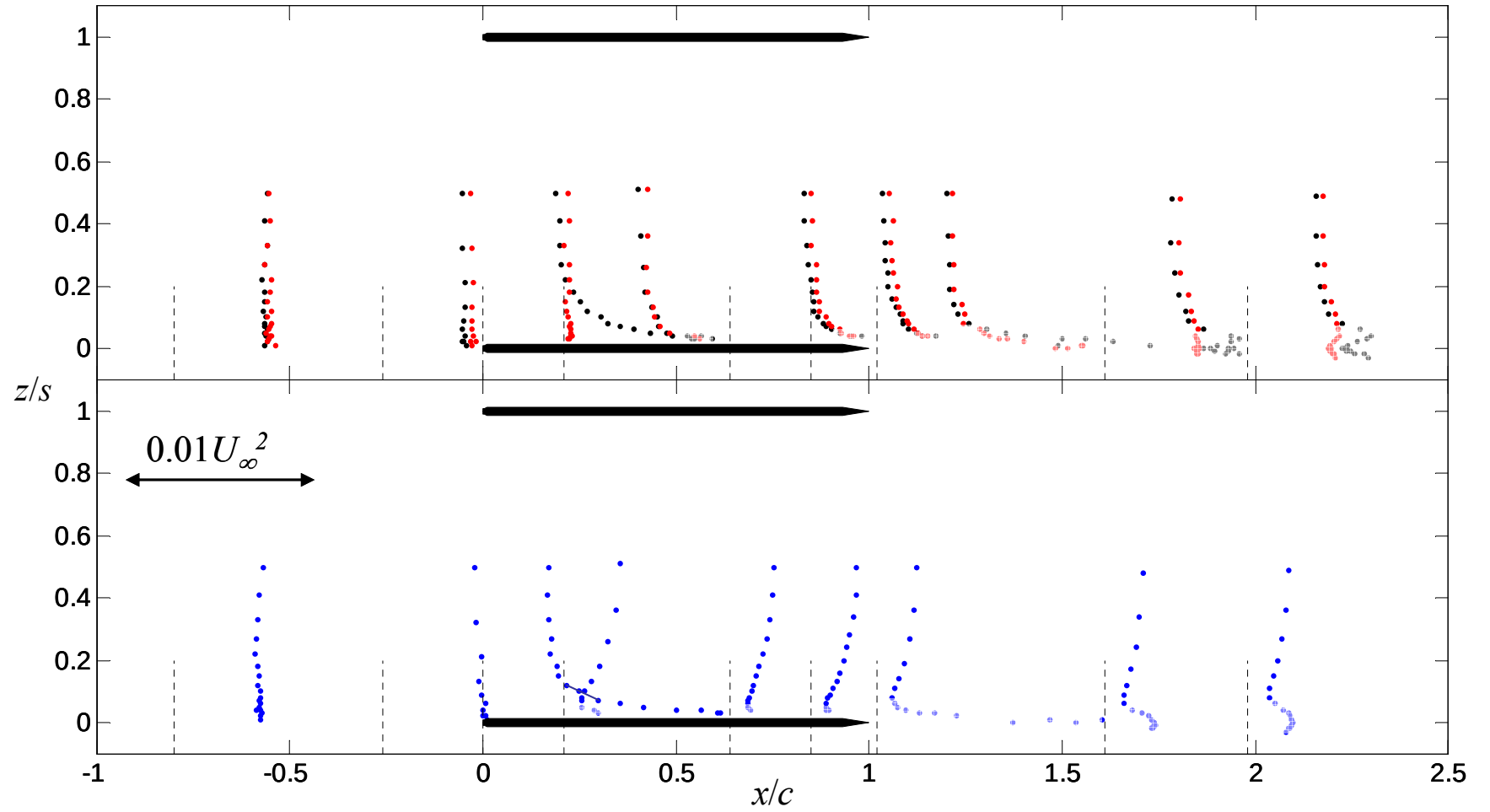


Figure 6.11. Reynolds normal stress profiles throughout the unstaggered cascade. Each station is shown as a dashed line with the stress levels indicated by the colored profiles. Viscous regions are marked in faded colors. ●, u^2/U_∞^2 ; ●, u^2/U_∞^2 ; ●, u^2/U_∞^2 .

6.2.2 Staggered Cascade

With the results of the unstaggered cascade in mind it can be assumed that given the effect of the blades and the cascade wakes, most of the difference (if any) between the unstaggered and staggered cascade will lie in the initial modification where the blade blocking effect transitionally sets in for the staggered cascade. In that respect one would also expect the leading edge effects to be less severe and spread out over a larger distance. By the time the flow passes the trailing edge of a blade the flow field within that particular passage should be fully modified and only very minor changes should be expected to occur further downstream.

Ahead of the staggered cascade

Figure 6.12 shows the Reynolds normal stresses at $x/c = -0.35$. As opposed to the unstaggered case, the total kinetic energy is markedly lower than the reference free stream value. At this location the flow has in reality already entered the cascade and is experiencing some blade blocking due to blades 1 and 2 (refer to Figure 5.4) which makes it difficult to explicitly separate these effects from the leading edge anticipation, but a small decrease in the streamwise and spanwise component seems clear compared to the unstaggered case in Figure 6.1. The actual attenuation of u^2 , v^2 , and w^2 based on the isotropic free stream level is 94%, 99% and 96% respectively. An amplification of the upwash component ahead of the cascade entrance as seen in Figure 6.2 is completely absent at this location.

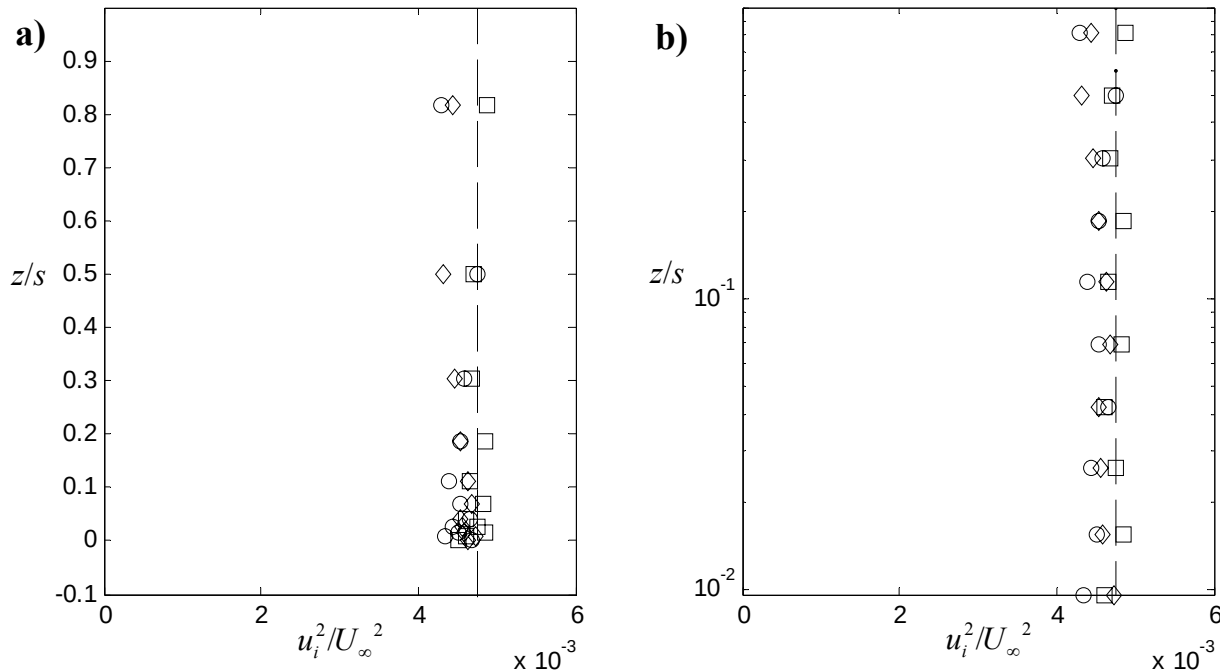


Figure 6.12. Reynolds stresses at $x/c = -0.35$ ($X/M = 36.45$) for the staggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line.

Inside the staggered cascade center passage

As the flow enters the leading edge plane of blade 3, the flow is obstructed close to the blade, but at this location ($x/c = 0.00$) there are only upstream effects applied from blade 4. Figure 6.13 shows the Reynolds normal and shear stresses as a function of most of the central passage. Close to blade 3 the flow appears to behave very similarly to the unstaggered case with the shear stress deviating from zero starting at about $z/s = 0.4$. The mid-passage values of the streamwise and spanwise components are on the other hand back to the isotropic levels and the upwash attenuation is only 10% compared to 27% for the unstaggered counterpart.

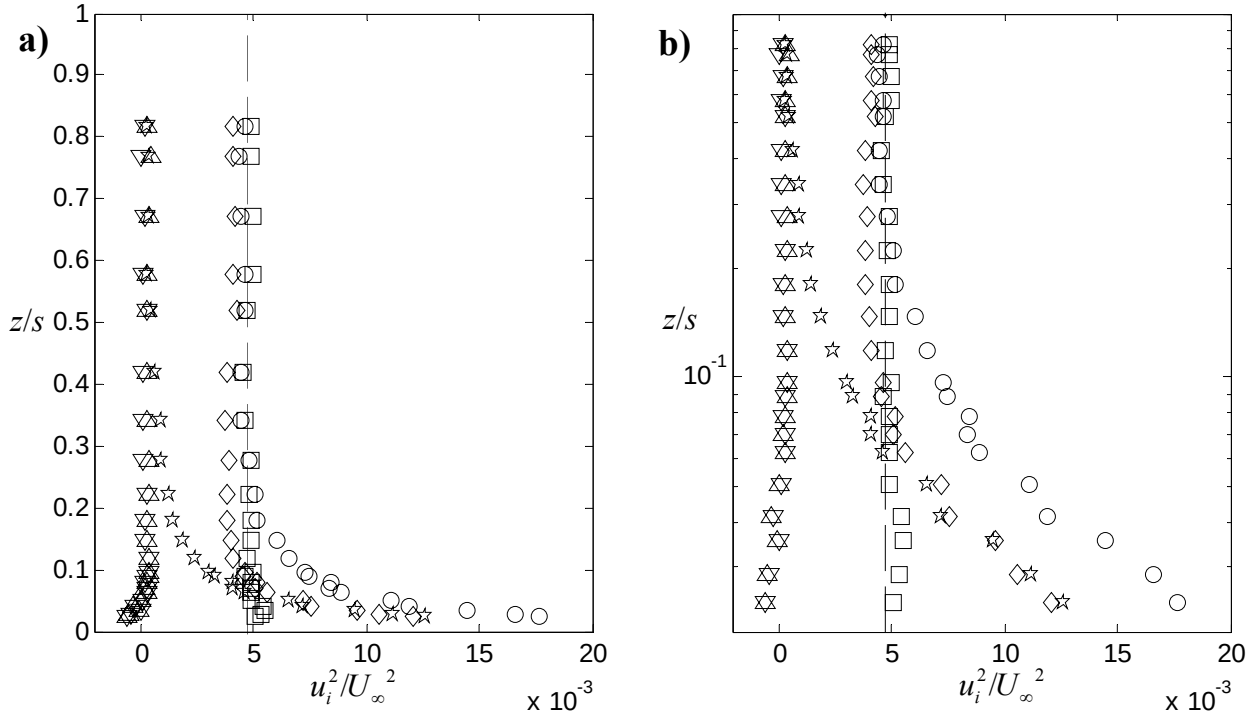


Figure 6.13. Reynolds stresses at $x/c = 0.00$ ($X/M = 37.0$) for the staggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line.

Figure 6.14 shows the Reynolds stresses further into the passage as the flow passes the leading edge plane of blade 4. Beyond this location the flow is now fully enveloped by blades on both sides which from now on is expected to be reflected in large attenuation of the normal stresses. At this location however, the cascade's major contributor is still only blade 3 which is reflected in the figure as an enhancement of the spanwise component in the mid-passage. The streamwise component is attenuated 11% compared to the reference free stream while the upwash component is now attenuated 31%. The attenuation is of course decreased as blade 4 is approached as the stress profile here changes into a scenario similar to the one seen in Figure 6.13 for blade 3. Note the negative sign on the shear stress close to blade 4.

Close to the boundary layer of blade 3 the maximum attenuation of w^2 has reached 85% which is very comparable to 84% of the unstaggered cascade at $x/c = 0.64$ in Figure 6.5.

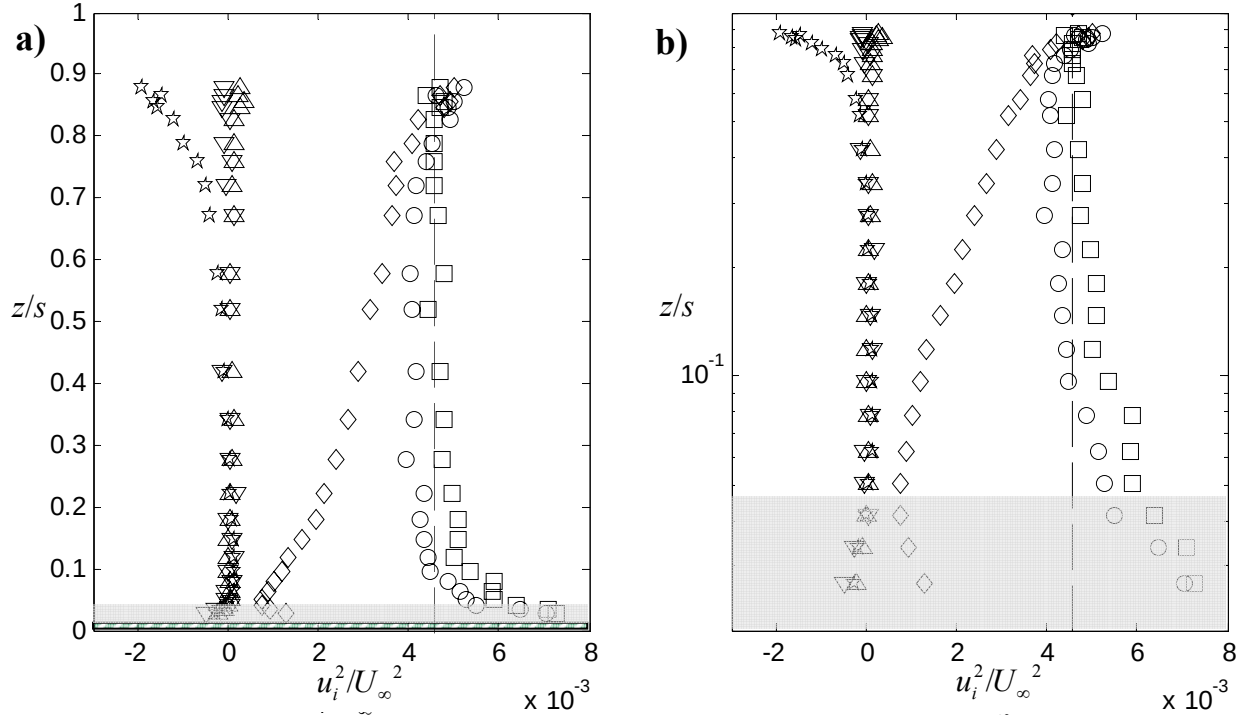


Figure 6.14. Reynolds stresses at $x/c = 0.58$ ($X/M = 37.91$) for the staggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

At the trailing edge plane of blade 3 it can be seen from Figure 6.15 that the more familiar attenuation trends manifested in the unstaggered cascade are developing also in the staggered cascade passage. Representative mid-passage attenuation has reached 13% for the streamwise and 46% for the upwash components respectively while the spanwise components take on values which surpass the free stream. Maximum attenuation of the upwash component is 83% close to blade 3, but only 75% near blade 4 which is slightly lower than expected from the unstaggered cascade, but consistent with the larger mid-passage values present in the staggered configuration.

In the wake of the staggered cascade center passage

As the trailing edge of the fourth blade is reached ($x/c = 1.6$) the mid-passage attenuation remains at 46% for w^2 , but has increased to 17% for u^2 as shown in Figure 6.16. The spanwise component follows the same trend as earlier, but the mid-passage level is now back to the free-stream level.

Figure 6.17 displays the Reynolds stresses at $x/c = 2.18$. At this station, the profiles between blades 2 and 3 have also been included. The behavior in the neighborhood of blade 2 is equivalent to a streamwise position of $x/c = 2.76$. Taking this into account the total mid-passage attenuation for the streamwise normal stresses are 20% and 17% for the two stations. The spanwise components are also reduced to 97% and 94% of the reference free stream. The upwash attenuation remains fairly constant at 45% and 46%. The maximum attenuation steadily decreases as the wake gets thicker. By $x/c = 2.76$ the upwash component has increased to 33% of the reference free-stream value just outside the wake.

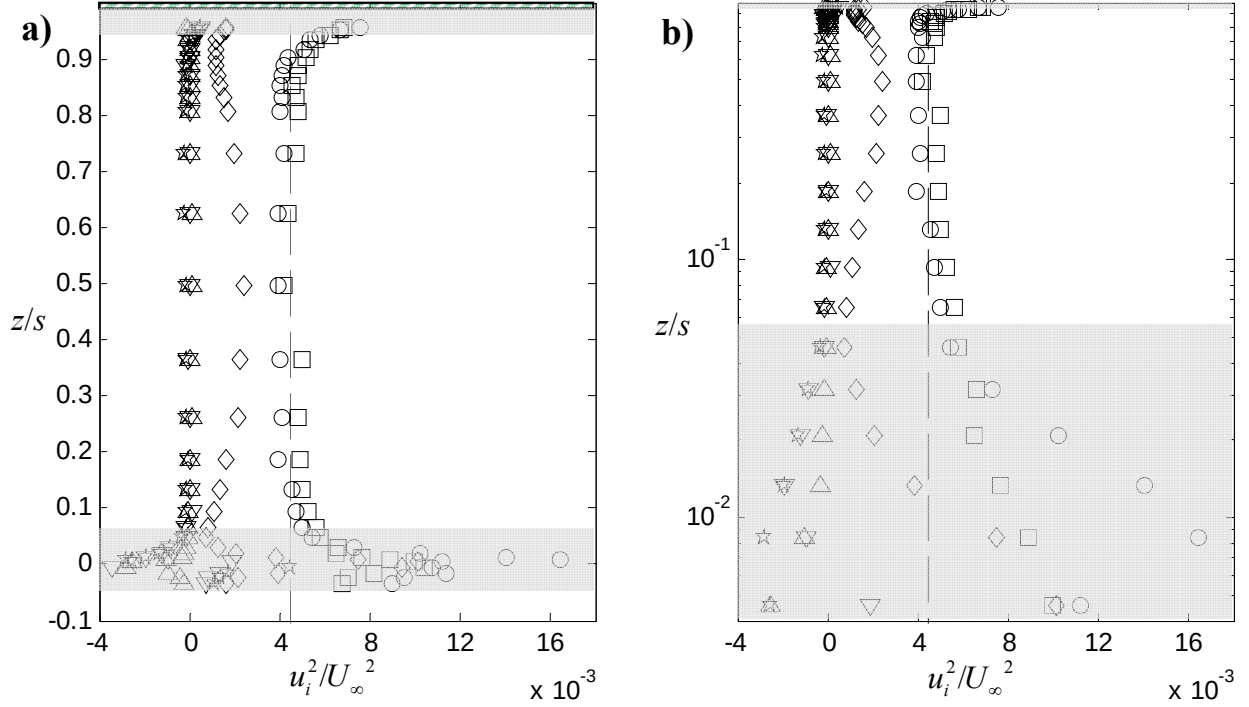


Figure 6.15. Reynolds stresses at $x/c = 1.02$ ($X/M = 38.59$) for the staggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

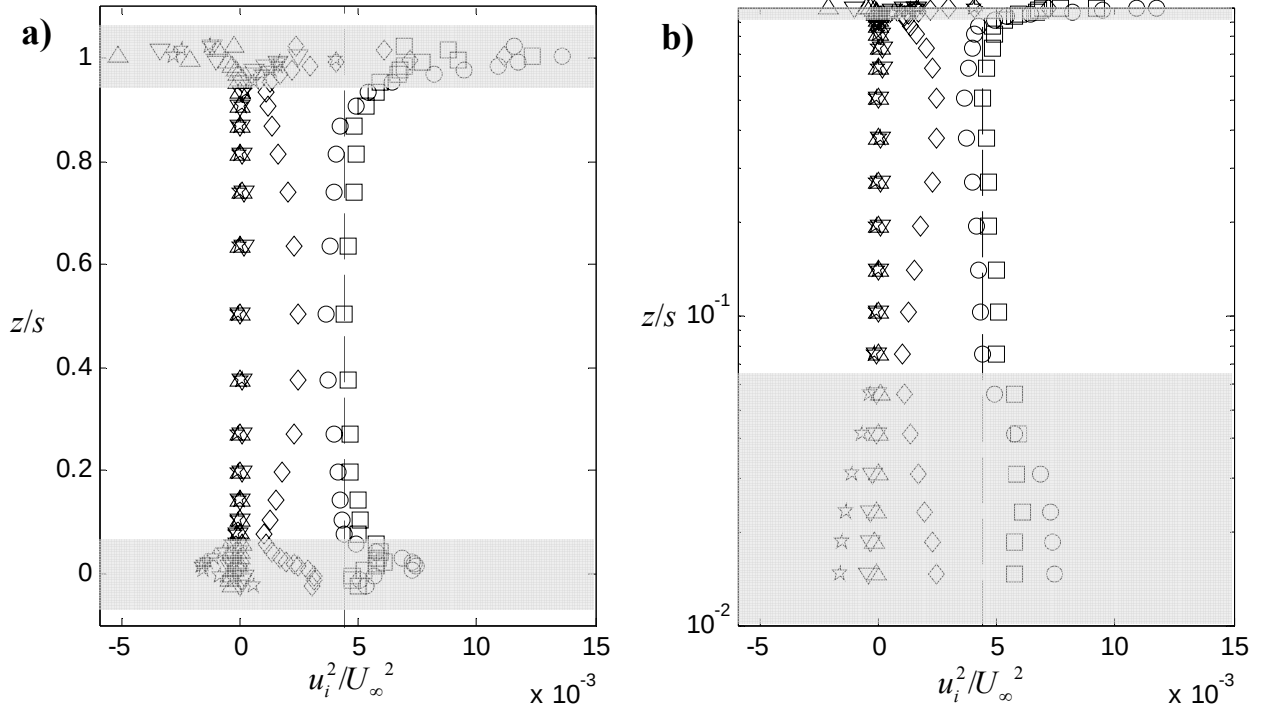


Figure 6.16. Reynolds stresses at $x/c = 1.60$ ($X/M = 39.50$) for the staggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

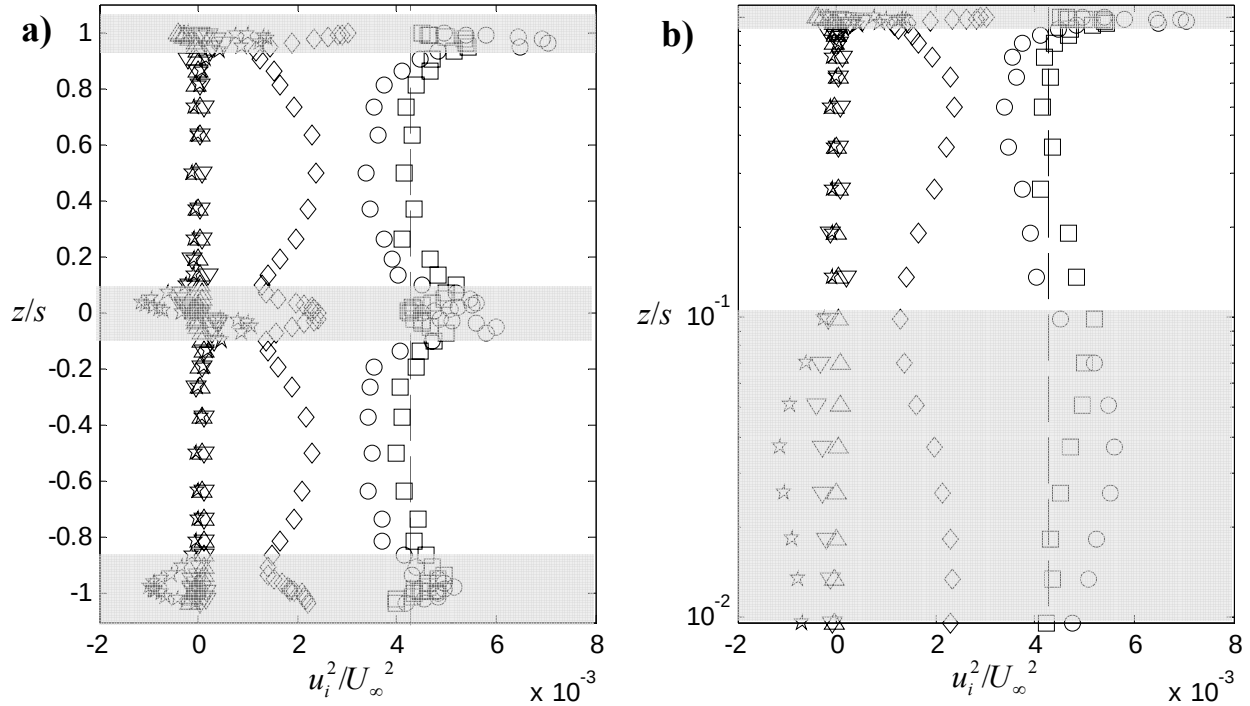


Figure 6.17. Reynolds stresses at $x/c = 2.18$ ($X/M = 40.41$) for the staggered cascade. Linear scale (a) and logarithmic scale (b). $\circ$, u^2/U_∞^2 ; $\square$, v^2/U_∞^2 ; $\diamond$, w^2/U_∞^2 ; ∇ , uv/U_∞^2 ; $\triangle$, vw/U_∞^2 ; $\star$, uw/U_∞^2 . The reference free stream level for the same location is given by the dashed line. Viscous regions indicated by the shaded area.

Summary

In order to gauge the similarity between the flow behind blades three and four, Figure 6.18 shows the data behind the upper end of the passage with an average of the results from the lower end superimposed in the domain $x/c = 1.0$ to 2.2 . The upwash datapoints consistently fall on top of each other. The streamwise and spanwise components change somewhat during this domain and are therefore more scattered than in the case of the upwash component. Due to the streamwise difference between the trailing edges of the blades there is some additional decay behind blade four which is reflected by a slightly reduced profile compared to blade 3. This can be seen in Figure 6.18 which does not compensate for the natural dissipation rate occurring between the two measurement sets. In light of this, the data must be considered to be very consistent between the two locations. This can possibly be explained by considering that the two wakes are essentially uncorrelated due to the decrease in lateral integral scales acting across the passage. With most of the turbulence in the wakes stemming from viscous production occurring behind the blades, this new energy further acts to drown any residual correlated energy actually present in the wake.

Figure 6.19 provides a better view of the complete evolution of the Reynolds normal stress evolution throughout the staggered cascade. Note the larger anisotropy between the streamwise and spanwise components (in comparison to Figure 6.11) throughout most of the staggered cascade passage. This and other differences between the two cascade configurations will be further discussed next.

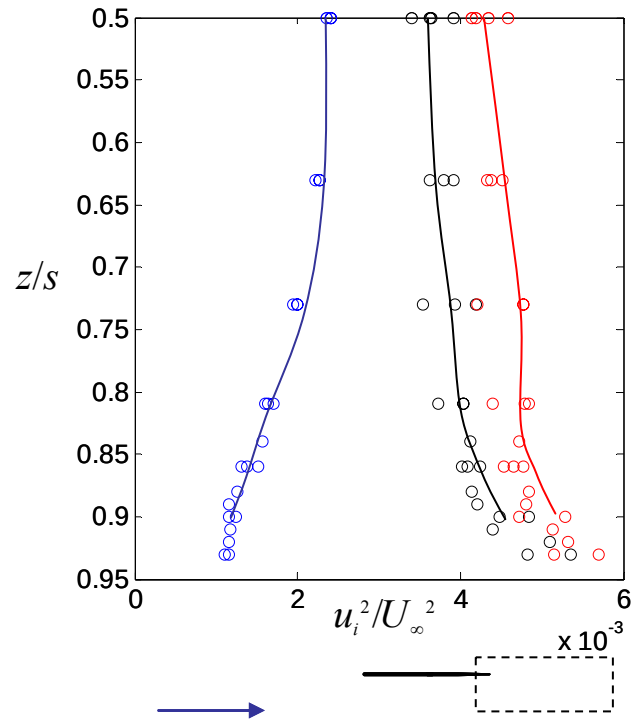


Figure 6.18. Reynolds stresses at $x/c = 1.0$ to 2.6 for the staggered cascade in the vicinity of blade 4. Averages of the results from blade 3 (also shown in Figure 6.21) in the same domain are shown in solid lines. $\circ$, streamwise stresses; $\circ$, spanwise stresses; $\circ$, upwash stresses.

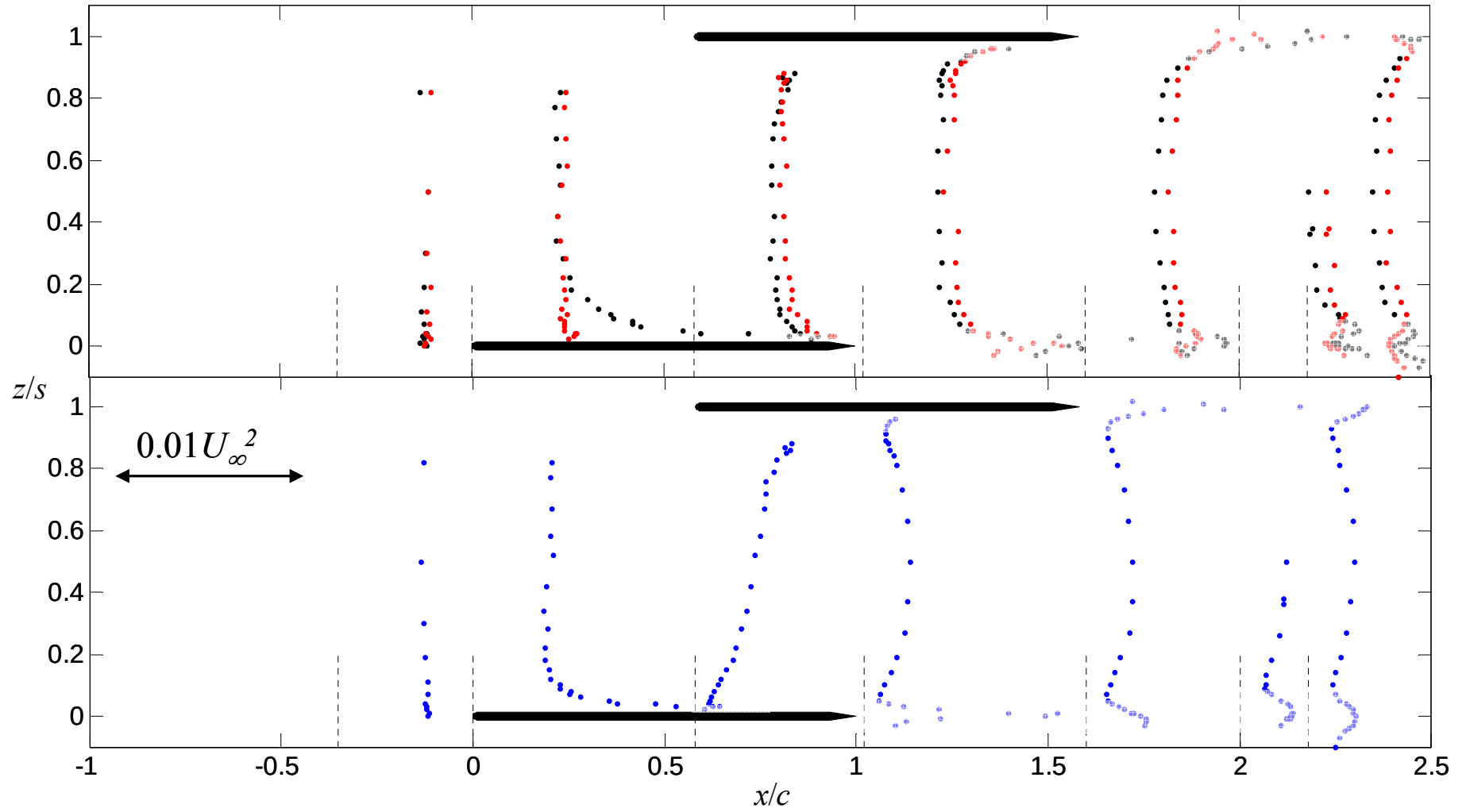


Figure 6.19. Reynolds normal stress profiles throughout the staggered cascade. Each station is shown as a dashed line with the stress levels indicated by the colored profiles. Viscous regions are marked in faded colors. ●, u^2/U_∞^2 ; ●, u^2/U_∞^2 ; ●, u^2/U_∞^2 .

6.2.3 Discussion

Although the Reynolds stress distributions in the unstaggered and staggered cascade share a lot of similarities at first glance, there are also some marked differences. Both cascade configurations act to distort the normal turbulence stresses compared to the reference isotropic free stream levels. This is particularly evident in the normal component to the blade, but the blade-parallel components also undergo significant attenuation. The majority of the blockage occurs within about half an integral scale of the leading edge plane of the cascade for the unstaggered cascade and over a significant longer distance for the staggered cascade. However by the cascade exit the stress profiles have taken on a new permanent shape (outside of the thin viscous wake) and convect downstream with no return to isotropy observed.

With a distinctive loss in dissipation rate associated with the modifications of the unstaggered cascade, the mid-passage values decay slower than the reference turbulent inflow. This means that far downstream of the cascade, both the blade-parallel (spanwise and streamwise) components may indeed be higher than in the absence of the cascade. The upwash component also increases in value compared to the reference free stream, but due to a severe attenuation inside the cascade passages, it would take a very long development time for the total turbulence kinetic energy to surpass the reference stream.

This effect is not seen in the staggered cascade. Behind the cascade the levels remain fairly consistent in the mid-passage, or if anything, a small decline compared to the reference freestream values, indicating a comparable decay rate. Additionally, due to the more gradual introduction of the blade blocking effects, the flow suffers less of a distortive shock as it enters the staggered cascade. This leads to a smaller attenuation across all the normal stress components, and the relative levels behind the cascade are therefore higher than in the case of the unstaggered cascade. This applies to all the normal components, but particularly the spanwise component escapes the attenuation, and the initial level coming out of the cascade is actually higher than the reference free stream. This causes a larger gap between the streamwise and the spanwise components than is seen in the unstaggered case. The cascade acts to equalize this effect further down behind the blade row as the two blade parallel components slowly start approaching each other in value in the middle of the passage. The upwash component however remains attenuated at a constant level

In order to visualize the differences between the cascade cases, Figure 6.20 shows the three normal components behind the unstaggered cascade for a variety of different streamwise positions between $x/c = 1.0$ to 2.0 . The average trends from this figure have been superimposed in forms of dashed lines in Figure 6.21. The greater anisotropy of the staggered cascade is obvious together with a lower attenuation level of all three components which is consistent from the mid-passage all the way to the viscous wake regions. As mentioned above the difference between the streamwise and upwash components are in proportion to each other while the spanwise stress component shows a much greater difference between the staggered and unstaggered configurations.

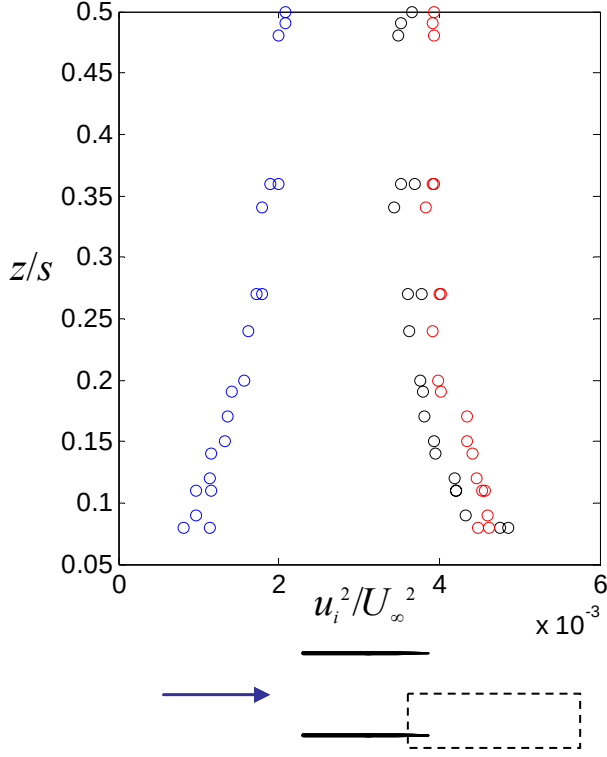


Figure 6.20. Reynolds stresses at $x/c = 1.0$ to 2.0 for the unstaggered cascade in the vicinity of blade 3. $\circ$, streamwise stresses; $\circ$, spanwise stresses; $\circ$, upwash stresses.

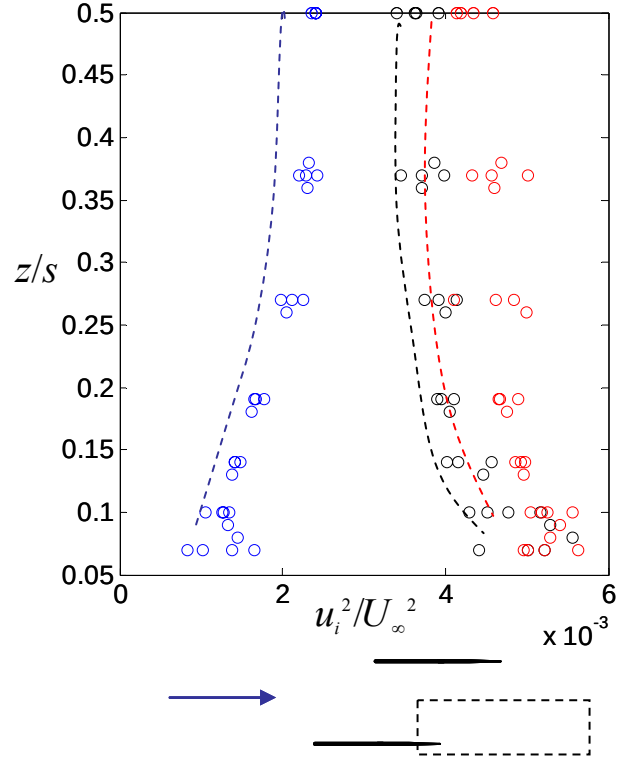


Figure 6.21. Reynolds stresses at $x/c = 1.0$ to 2.18 for the staggered cascade in the vicinity of blade 3. An average of the unstaggered results in the same domain (from Figure 6.20) given by the dashed lines. $\circ$, streamwise stresses; $\circ$, spanwise stresses; $\circ$, upwash stresses.

6.3 Throughflow: Velocity Spectra

The previous section maps out the attenuation levels on each of the three normal Reynolds stresses. The question is of course how the attenuation acts across different wavenumbers? It is expected that most of the attenuation will occur in the low-end of spectrum due to the non-penetration condition on the blades, but how far up into the spectrum can the blade blocking effects be felt? This topic will be explored in this section. Due to the documented similarity between the two cascade configurations, only the unstaggered cascade data will be used as it presents a more straightforward analysis due to the inherent symmetry of the flow. Spectra are plotted as $E_{ii}(\kappa_1)/U_\infty^2$ vs. κ_1 where $E_{ii}(\kappa_1)$ is the one-dimensional energy spectrum (in m^3/s^2), and κ_1 is the streamwise wavenumber.*

Figure 6.22 shows the three one-dimensional velocity spectra at the centerline of the passage ($z/s = 0.5$) for four different streamwise positions. E_{11} (Figure 6.22a) and E_{22} (Figure 6.22b) remain practically unchanged throughout the cascade centerline, following the initial attenuation

* Refer to Section 4.6 and Appendix A for more information about the velocity spectrum and how it is obtained.

experienced prior to the leading edge, but are provided as a reference.* The upwash spectrum (E_{33}), on the other hand, undergoes a severe attenuation as seen in Figure 6.22c from the free stream value, to the leading edge and finally through the passage. In the lowest frequency range the upwash spectrum exhibits a drop of about a factor of four from the leading edge to the $x/c = 0.64$ location. The attenuation effect can be seen to extend up to around $\kappa_1 \approx 20 \text{ m}^{-1}$. Since most of the attenuation was shown above to occur prior to $x/c = 0.64$ there is no surprise that the spectra downstream of this location look relatively unchanged.

Moving further towards the blade, Figure 6.23 displays the same scenario at $z/s = 0.12$ which is just outside the viscous wakes for the stations located furthest downstream. At this location the enhancement of the streamwise and spanwise components give spectra roughly comparable to the reference free stream levels which can be verified in the spectral trends in Figure 6.23a and Figure 6.23b (i.e. no or little change as a function of downstream distance). However, at this location in the passage the upwash attenuation is much larger than in the mid-passage as the low-wavenumber range is reduced by a factor of 16 (more than one decade) as seen in Figure 6.23c. The actual attenuation effect can be seen to extend further up into the wavenumber range where the spectrum does not reach the free stream shape until a wavenumber of approximately 100 m^{-1} .

As the probe penetrates the boundary layer, this trend will no longer be valid as all three normal components are strongly enhanced due to viscous production effects. Figure 6.24 displays how this enhancement is distributed at $z/s = 0.04$ which, with the exception of the leading edge location ($x/c = 0.00$), is entirely within the viscous region (boundary layer or wake) of the passage. Disregarding the leading edge effects, it appears from Figure 6.24a that the increase in the streamwise stress levels are generated over a fairly wide range of wavenumbers.

Below $\kappa_1 \approx 20 \text{ m}^{-1}$ the viscous streamwise spectra appear to have increased slightly and returned to a level similar to the reference free stream spectrum due to energy transfer from the upwash component. The increase in spectral levels above this limit appears more significant, and is caused by viscous production. Since the Reynolds number of the newly generated turbulence is significantly smaller than the grid turbulence, the analysis in Section 4.7.2 suggests that the inertial range should be notably flatter than the external turbulence. When superimposed on the modified grid turbulence the low-frequency components of the boundary layer and wake production nearly drowns in the much larger scales energy already present in the flow. Higher up in the wavenumber range on the other hand, the large amount of small scale turbulence created becomes more significant and boosts the overall level in this region. This should be expected to occur in all three components, which is confirmed in both Figure 6.24b and Figure 6.24c. The difference in the lateral components is that the visible effect of the viscous turbulence occurs at a larger wavenumber ($\kappa_1 \approx 70 \text{ m}^{-1}$). Also note that the boundary layer turbulence consists of much smaller scale turbulence than the vortical structures shed from the trailing edge. This manifests itself in Figure 6.24c as the low-frequency range of the $x/c = 0.64$ location continues the attenuation process inside the boundary layer (as compared to Figure 6.23c), whereas this is not the case in the wake where the spectrum levels have returned to values associated with locations outside the viscous regions.

To better show the attenuation of the lower frequencies and the enhancement throughout the rest of the spectrum within the boundary layer as a function of z/s , the upwash spectra for a range of z -locations have been plotted for $x/c = 0.64$ in Figure 6.25.

* Small changes that can easily be observed in a stress profile such as the ones given in Section 6.2 may go largely unnoticed in velocity spectra due to the logarithmic scales used. This explains why the streamwise and spanwise look like they match perfectly up behind the leading edge even though definite change was documented earlier.

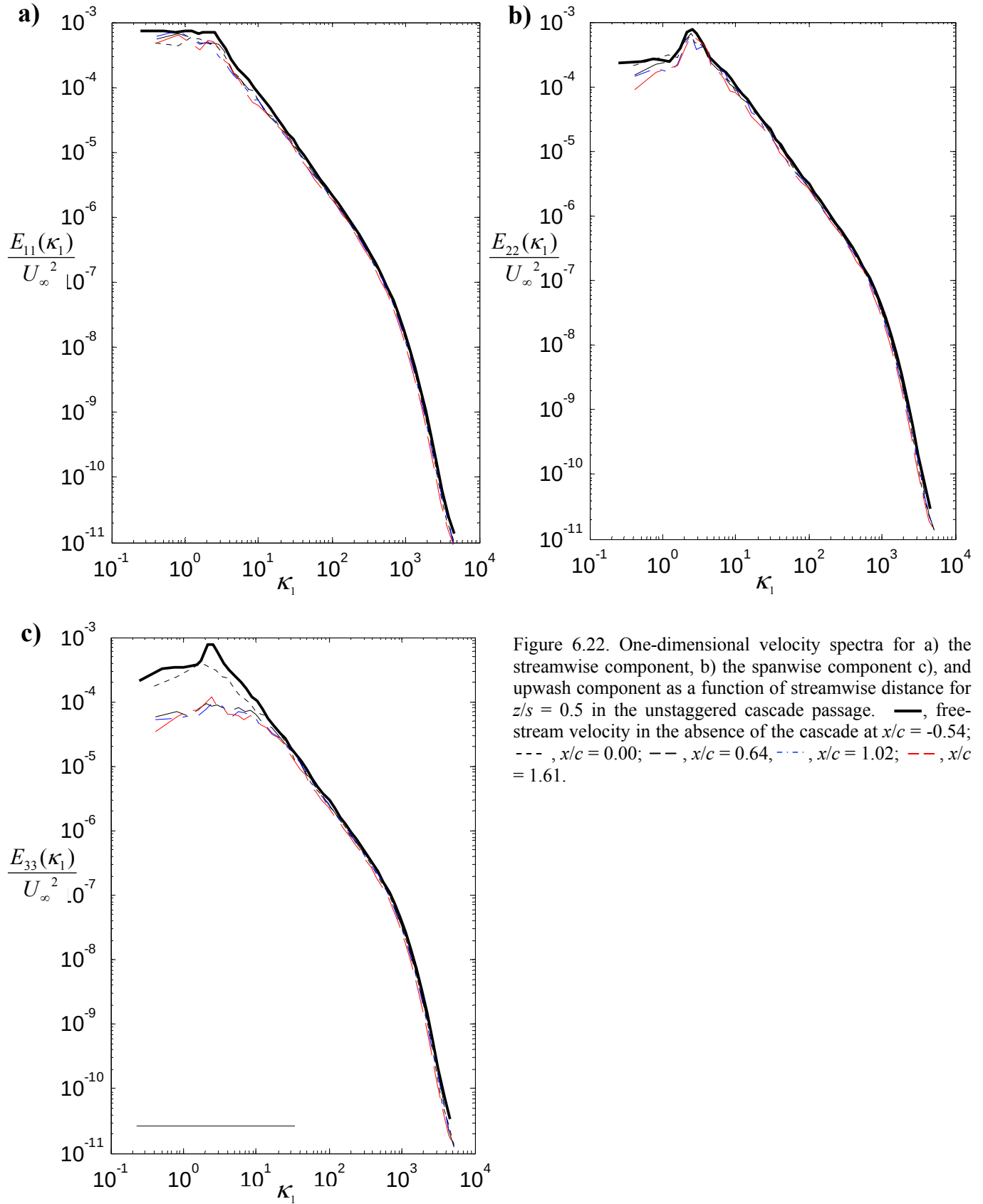


Figure 6.22. One-dimensional velocity spectra for a) the streamwise component, b) the spanwise component c), and upwash component as a function of streamwise distance for $z/s = 0.5$ in the unstaggered cascade passage. —, free-stream velocity in the absence of the cascade at $x/c = -0.54$; ---, $x/c = 0.00$; — —, $x/c = 0.64$, - · - ·, $x/c = 1.02$; - - -, $x/c = 1.61$.

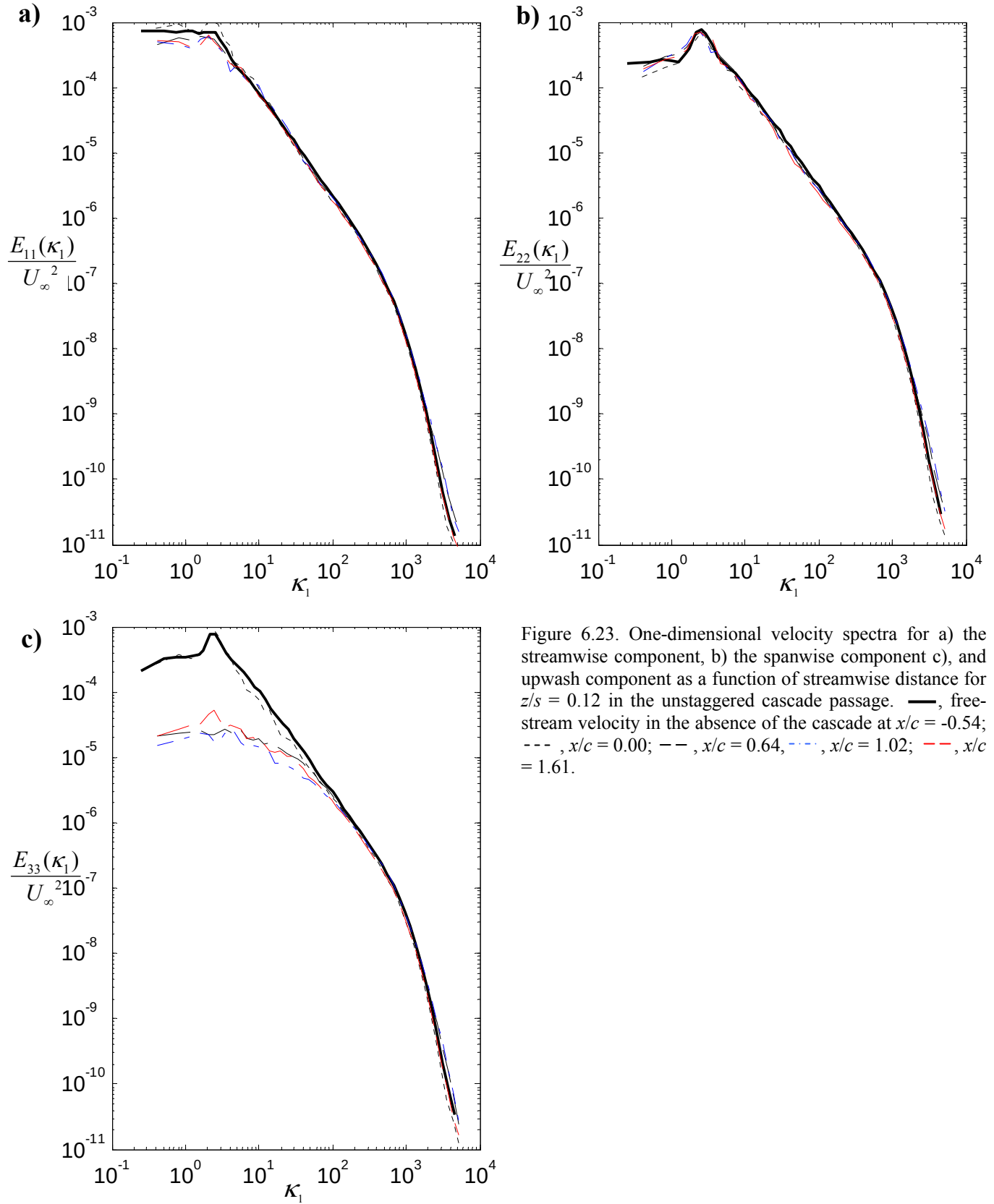


Figure 6.23. One-dimensional velocity spectra for a) the streamwise component, b) the spanwise component c), and upwash component as a function of streamwise distance for $z/s = 0.12$ in the unstaggered cascade passage. —, free-stream velocity in the absence of the cascade at $x/c = -0.54$; ---, $x/c = 0.00$; — —, $x/c = 0.64$, - · - ·, $x/c = 1.02$; - · - ·, $x/c = 1.61$.

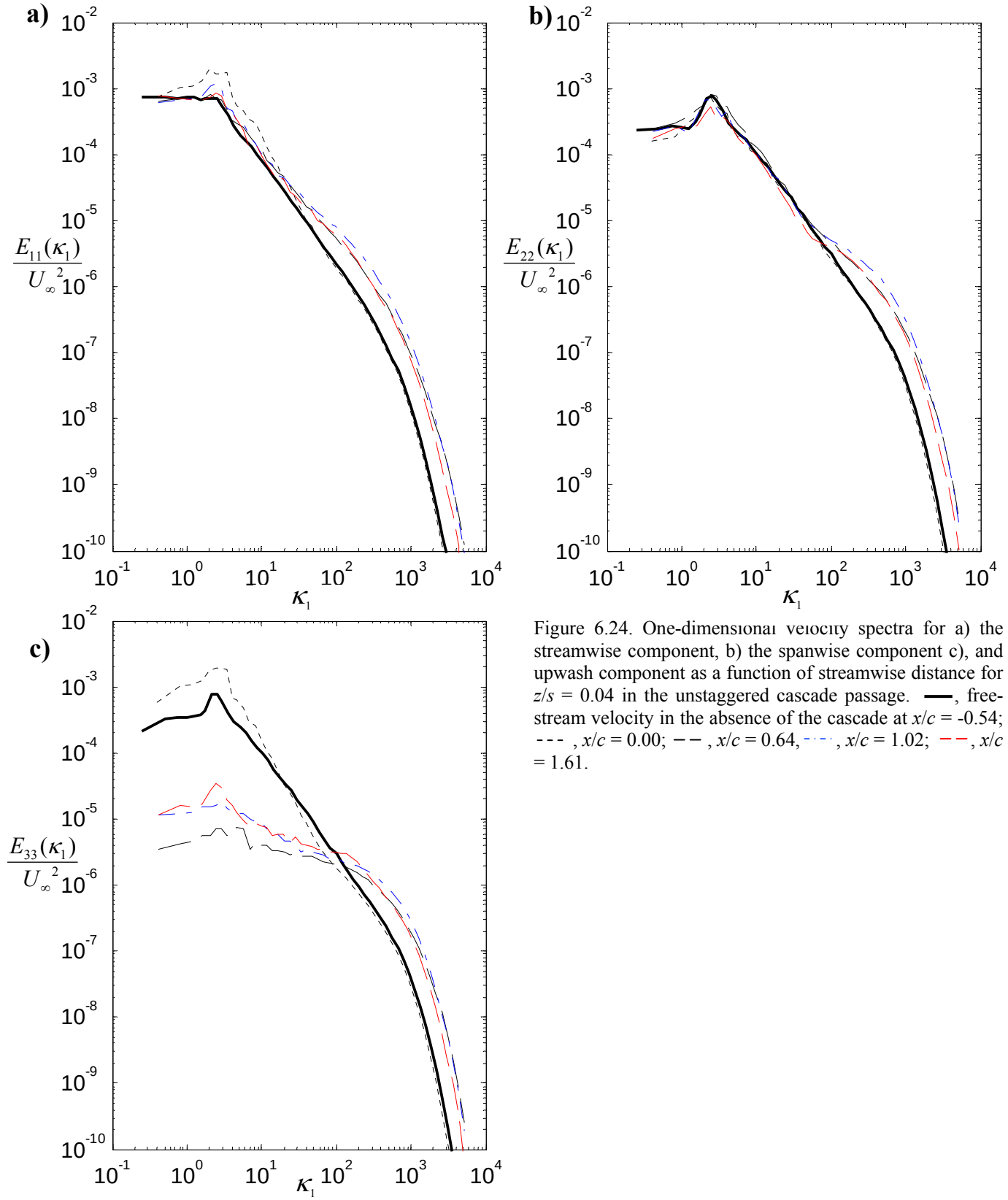


Figure 6.24. One-dimensional velocity spectra for a) the streamwise component, b) the spanwise component c), and upwash component as a function of streamwise distance for $z/s = 0.04$ in the unstaggered cascade passage. —, free-stream velocity in the absence of the cascade at $x/c = -0.54$; ---, $x/c = 0.00$; — —, $x/c = 0.64$; - · - ·, $x/c = 1.02$; — —, $x/c = 1.61$.

The large low-frequency attenuation from the free stream level, to the mid-passage level to the blade adjacent station can be clearly seen in this figure and amounts to a factor of about 270 which manifests itself in Figure 6.25 as a drop in spectral level of more than two decades. The high-frequency range remains consistent with the reference free stream all the way until the boundary layer is reached which in Figure 6.25 occurs at $z/s = 0.07$. As the blade is progressively approached the enhancement of the equilibrium range gets stronger which is consistent with larger production.

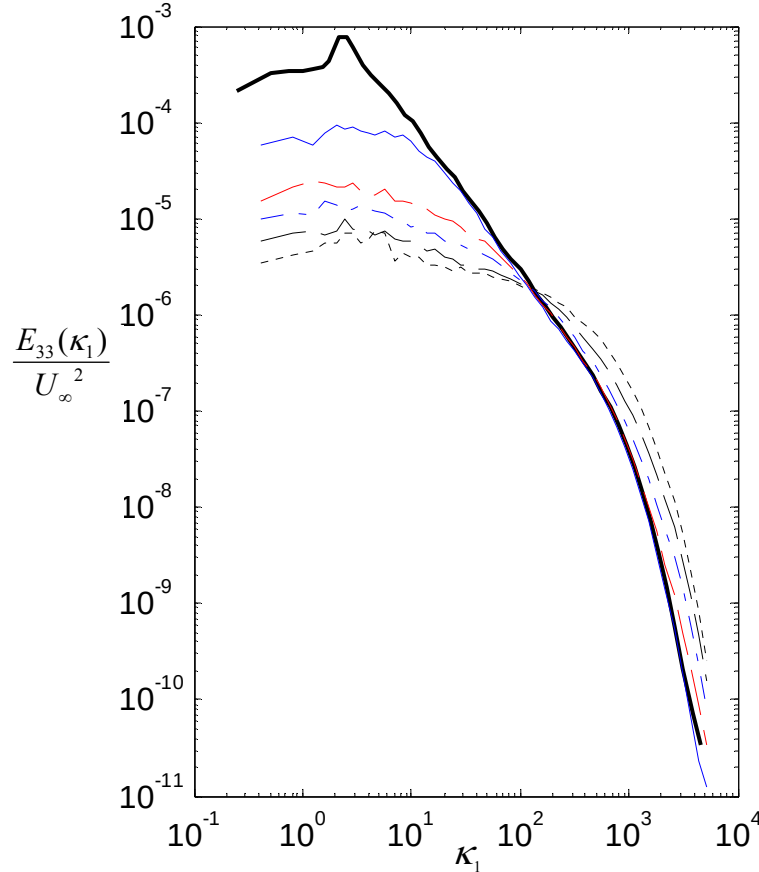


Figure 6.25. One-dimensional velocity spectra for the upwash component as a function of transverse distance (z/s) inside the unstaggered cascade passage at $x/c = 0.64$. —, free-stream velocity in the absence of the cascade at $x/c = -0.54$; ---, $z/s = 0.04$; --, $z/s = 0.05$; - · - ·, $z/s = 0.07$; - - -, $z/s = 0.10$; —, $z/s = 0.50$.

So far the spectral discussion has revolved around the unstaggered cascade for simplicity. The staggered case behaves very similarly, but with less of a drastic change in levels. For comparative purposes, the lateral trend inside the blade passage is shown below in Figure 6.26. When comparing this to Figure 6.25 there are obvious similarities, but the most striking difference is the distinctly higher level of the mid-passage value, indicating lower attenuation which is consistent with the turbulent normal stress observations. As expected, as the blade is approached, the attenuation levels start converging towards the ones observed in the unstaggered cascade.

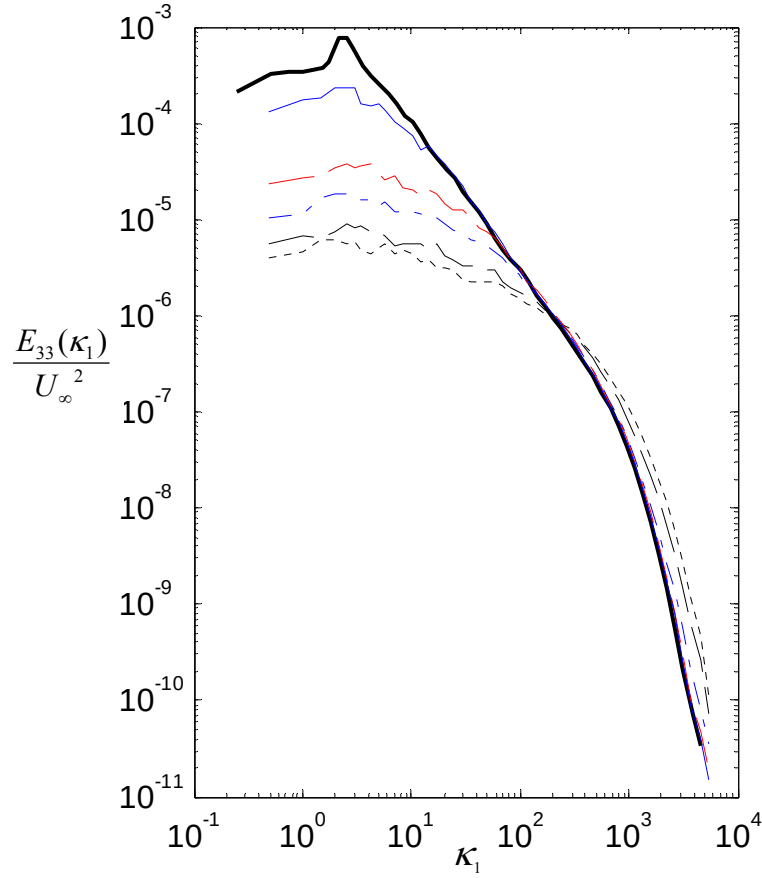
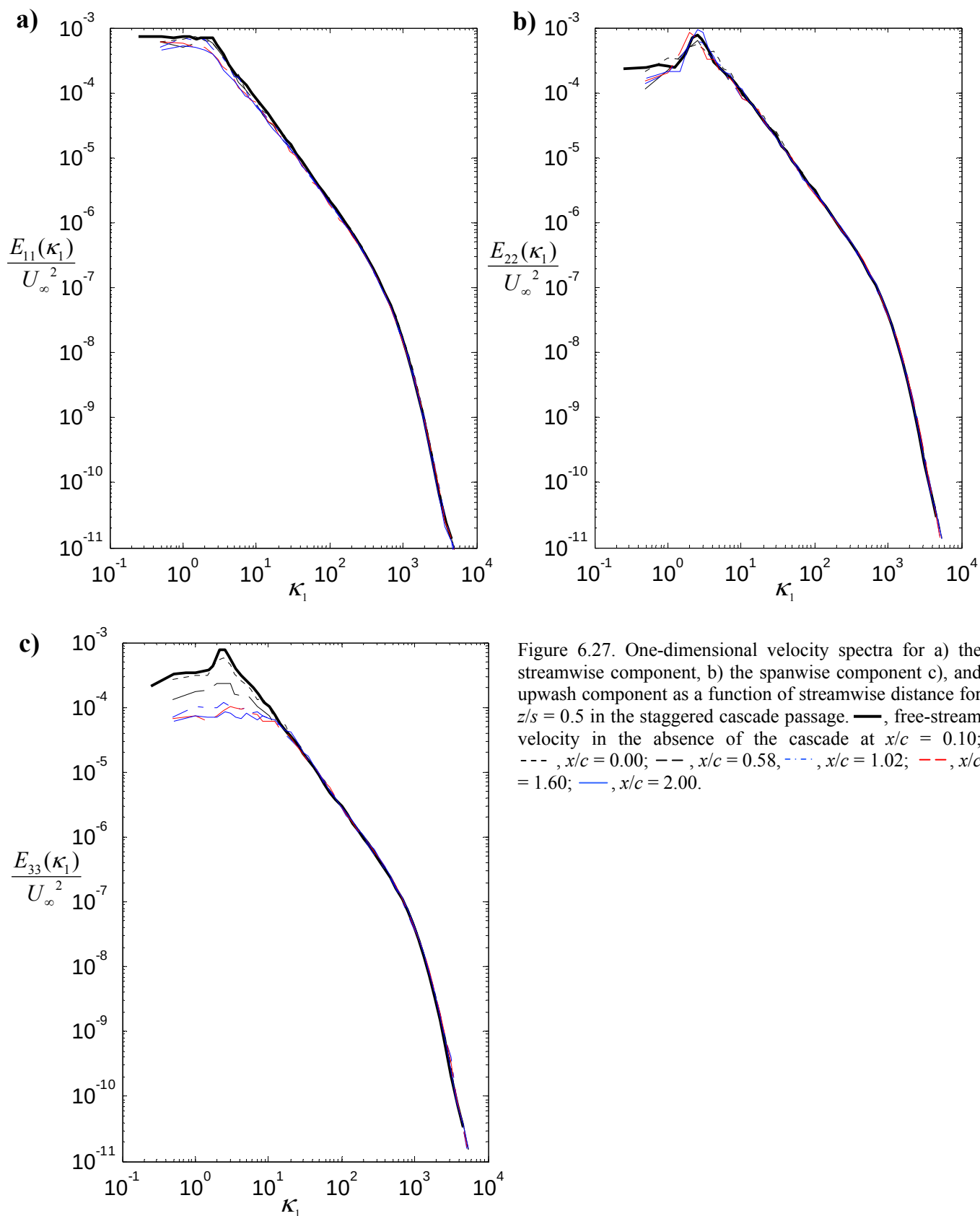


Figure 6.26. One-dimensional velocity spectra for the upwash component as a function of transverse distance (z/s) inside the staggered cascade passage at $x/c = 0.58$. —, free-stream velocity in the absence of the cascade at $x/c = 0.10$; ---, $z/s = 0.04$; — —, $z/s = 0.05$; - · - ·, $z/s = 0.08$; - - -, $z/s = 0.12$; — — —, $z/s = 0.52$.

The mid-passage variation throughout the staggered cascade is also shown below in Figure 6.27. Again, the trends are similar to that of the unstaggered cascade. The most prominent difference compared to the unstaggered case in Figure 6.22 is that the upwash attenuation occurs over a much longer streamwise distance in the staggered cascade. Whereas the unstaggered cascade shows very little change after the initial distortion prior to $x/c = 0.64$, the staggered case is far from completely modified at this location.



6.4 Two-Point Correlations

Section 4.10 introduced the two-point correlation in terms of the free stream experimental data, and how it corresponded to the Von Kármán interpolation function. Using the proper integral lengthscale (0.28 m) in Equations 4.52 and 4.63 a good correspondence could be found to the obtained free stream data in Figure 4.46. A similar profile was obtained inside the cascade passage for both the unstaggered and staggered cascade configurations. The profiles were obtained in the region of large upwash suppression, but outside the viscous wakes.

Figure 6.28 shows the two-point correlation obtained by traversing in the vertical ($-y$) direction of the unstaggered cascade. The location of the fixed probe was $x/c = 1.98$, $y/H = 0.086$, and $z/s = 0.108$. The moving probe was initially located at the same streamwise location, but translated 0.64 cm in the vertical direction ($y/H = 0.082$) and due to the nature of the experimental setup also 0.32 cm further towards the blade ($z/s = 0.096$). Data was obtained for spatial separation distances up to 36.9 cm which in the figure amounts to $y/s = 1.39$ or consistent with previous notation, up to $y/H = -0.111$.

The free stream Von Kármán predictions for the three components are included in Figure 6.28 for reference. Due to the different direction of data acquisition compared to that of Figure 4.46 the streamwise and upwash spectra should in this case be identical in the absence of the cascade while the spanwise components should exhibit larger correlation coefficients for a given separation. Figure 6.28 does indeed show that the streamwise and spanwise components retain their separation, but that both are attenuated slightly compared to their free stream levels. As expected, the upwash component has been severely attenuated by the cascade. Where the streamwise component experiences almost a 90% correlation at the lowest spacing, the equivalent upwash coefficient is here only 0.6. The correlation drops off at about the same rate as the other components until $y/s = 0.2$ away from the origin when the correlation asymptotically starts tending to zero.

Similarly Figure 6.29 shows the two-point correlation obtained by traversing in the vertical ($-y$) direction of the staggered cascade. The location of the fixed probe was $x/c = 1.98$, $y/h = 0.089$, and $z/s = 0.135$. A position further out in the passage was used to ensure that the viscous wake was avoided as it was slightly thicker at this location for the staggered cascade configuration. The moving probe was initially located at the streamwise location, at the same vertical position ($y/h = 0.089$) but 0.64 cm further into the passage ($z/s = 0.144$). Data was obtained for spatial separation distances up to 44.0 cm which in the figure amounts to $y/s = 1.66$ or consistent with previous notation, $y/h = -0.148$.

Overall, Figure 6.29 is very reminiscent of the unstaggered two-point correlation in Figure 6.28, but a few differences should be pointed out. First of all the first few points in the profile of all three correlation functions are artificially low due to the forced offset in the z -direction. This effect soon drowns as the spanwise gap is increased. Second, the profiles are smoother than the unstaggered case, which is most likely due to the fact that it was obtained further into the passage with less danger of the unsteady wake entering into the measurement volume. The level of attenuation of the upwash correlation coefficient is also somewhat lower than seen in the unstaggered cascade which is consistent with the results seen in Section 6.2.

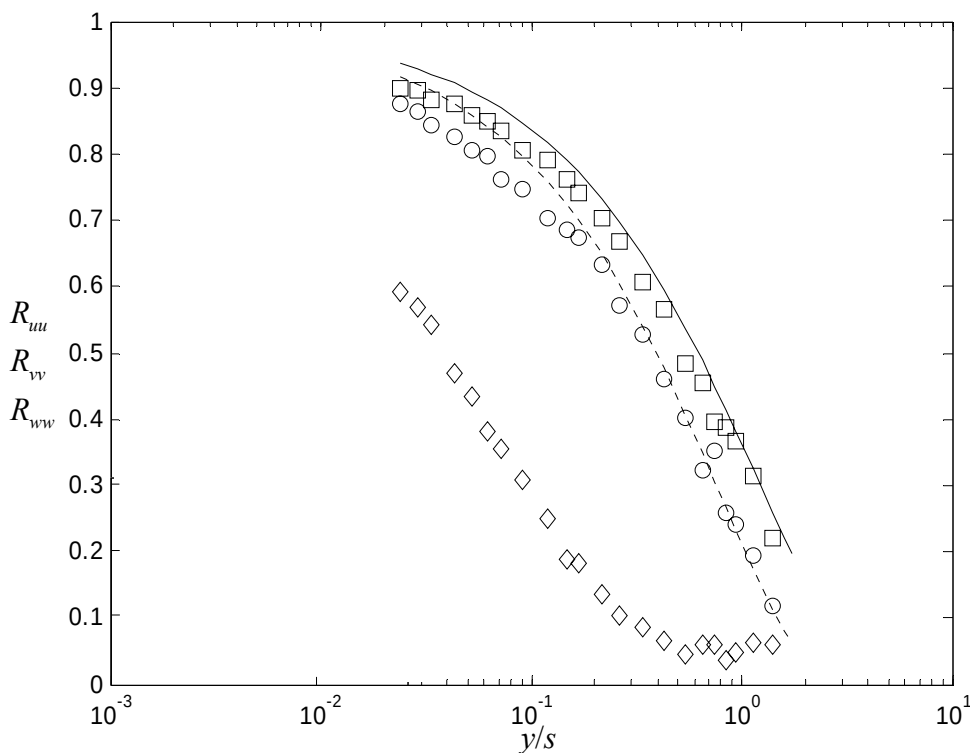


Figure 6.28. Two-point spatial correlation inside the unstaggered cascade center passage. $\circ$, R_{uu} ; $\square$, R_{vv} ; $\diamond$, R_{ww} ; —, f_1 (Eq. 4.62); ---, f_2 (Eq. 4.63). Note that normalization and traverse direction are different from Figure 4.46.

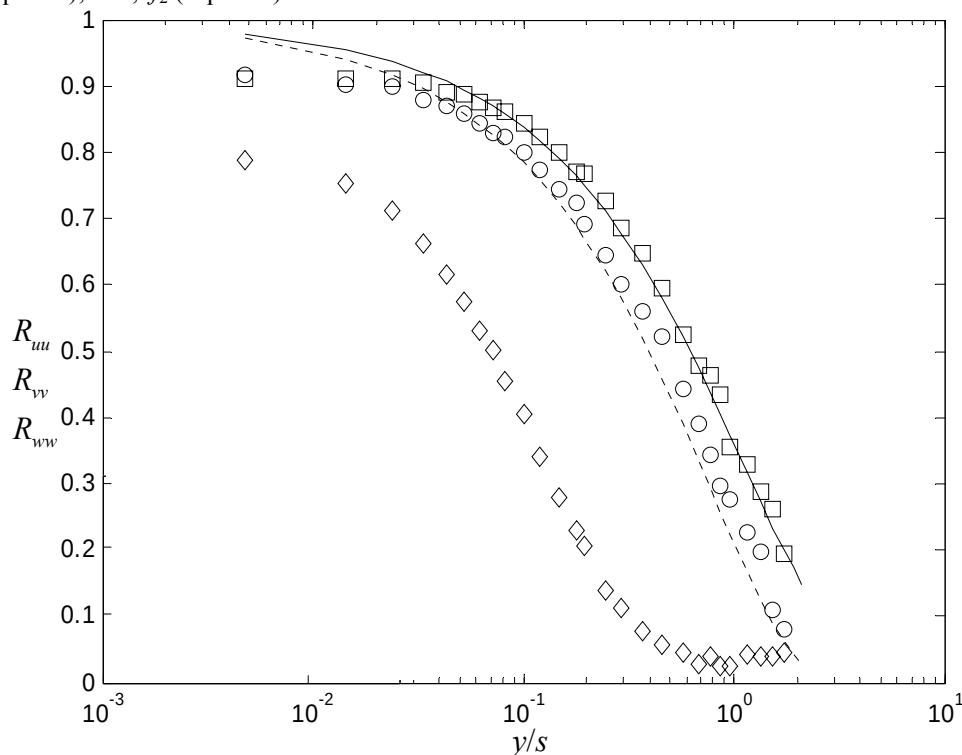


Figure 6.29. Two-point spatial correlation inside the staggered cascade center passage. $\circ$, R_{uu} ; $\square$, R_{vv} ; $\diamond$, R_{ww} ; —, f_1 (Eq. 4.62); ---, f_2 (Eq. 4.63). Note that normalization and traverse direction are different from Figure 4.46.

Chapter 7

Cascade Effects: RDT Predictions and Comparison with Experimental Data

This chapter will compare some of the results found in the previous chapter with theoretical estimates obtained through Rapid Distortion Theory (RDT). Besides some introductory words, Section 7.1 features the definition of the velocity-spectrum tensor before the utilized RDT cascade theory is presented in Section 7.2. The Reynolds stress comparison between measurements and theory is the topic of Section 7.3, while the theoretical effects of the blade blocking on the velocity spectra are investigated in Section 7.4. Sections 7.5 through 7.7 return to the comparison between predictions and the experimental data from Chapter 6, before Section 7.8 wraps up the chapter with a short summary.

7.1 Introduction

The tools to predict the blade blocking effect on the three-dimensional velocity spectrum for an unstaggered flat-plate cascade can be obtained from the theory of Graham (1998). Based heavily on the earlier work of Hunt & Graham (1978), this theory incorporates more than one blade and, instead of a semi-infinite chord, assumes that the chordlength is simply larger than the leading edge region of influence. This region of influence was shown to be on the order of an integral scale on a single blade (Hunt & Graham, 1978). In this respect Graham (1998) did point out that since the region of validity of the theory is generally on the order of an integral scale from the leading edge for a cascade configuration (as shown by Perot & Moin, 1995 and Aronson *et al.*, 1997). In other words, the theoretical region of validity of RDT in a cascade configuration will be vanishingly small. However, the stress attenuation normal to the blade has exhibited a tendency to agree with predictions over much longer distances than warranted by the theory (as shown by for example the shearless boundary layer data of Thomas & Hancock, 1977).

The RDT predictions in this chapter are defined in terms of their three-dimensional wavenumber spectra described briefly in Chapter 4, but in order to proceed, it is necessary to expand upon the material found in Section 4.6.

7.1.1 The Velocity-Spectrum Tensor

The velocity-spectrum tensor $\Phi_{ij}(\kappa)$ is the Fourier transform of the two-point velocity correlation. It is also related to the three-dimensional energy spectrum, but all the directional

information is generally lost since the energy spectrum is a scalar function of a scalar, instead of a second order tensor or a vector. In isotropic turbulence, on the other hand, the velocity spectrum tensor and the three-dimensional energy spectrum are related according to Equation 7.1.* The arrow overscript above the wavenumber on the left hand side in this equation distinguishes between the vector wavenumber $\vec{\kappa} = \kappa_1\mathbf{i} + \kappa_2\mathbf{j} + \kappa_3\mathbf{k}$, its scalar components, as well as the magnitude $\kappa = (\kappa_1^2 + \kappa_2^2 + \kappa_3^2)^{1/2}$.† The Kronecker delta δ_{ij} signifies the 3×3 identity matrix and the indices take on values between 1 and 3 signifying the three directions (streamwise, spanwise, and upwash).

$$\Phi_{ij}(\vec{\kappa}) = \frac{E(\kappa)}{4\pi\kappa^2} \left(\delta_{ij} - \frac{\kappa_i\kappa_j}{\kappa^2} \right) \quad (7.1)$$

Instead of determining the three one-dimensional spectra through Equations 4.21 and 4.22, they can be obtained directly from the velocity spectrum tensor through integration over κ_2 and κ_3 as shown in Equation 7.2. The repeated indices in this equation do not imply summation.

$$E_{ii}(\kappa_1) = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Phi_{ii}(\vec{\kappa}) d\kappa_2 d\kappa_3 \quad (7.2)$$

Another important property of the velocity-spectrum tensor is that it describes the expected value of the product of two Fourier transformed velocity components as provided in Graham (1998) and shown below. The hat overscript indicates that the Fourier transform has been applied, the asterisk signifies a complex conjugate, while the overbar implies the expected value.

$$\Phi_{ij}(\vec{\kappa}) = \overline{\hat{u}_i \hat{u}_j^*} \quad (7.3)$$

7.2 Graham's Cascade Theory

Graham (1998) himself provided the necessary equations to calculate the results for any given blade spacing, but restricted his own calculations to the limiting case where $s/L_{11} \rightarrow 0$. The following treatment will not make this restriction such that a custom value of the blade spacing can be utilized in the calculations even though Graham (1998) showed that the limiting case does indeed provide good ballpark estimates for the experimental data of Basuki (1983) and Haidos (1983) for relatively dense cascade configurations with s/L_{11} values between 0.8 and 0.45 and chords about $2L_{11}$.

Graham's theory makes a couple of assumptions necessary to form a solution. The first assumption states, as mentioned above, that the chord is greater or equal to the free-stream integral lengthscale. This is necessary in order to escape the leading edge effect region. The next

* See Pope (2000), p.222 for more detail.

† It is easy to forget, however, that when making numerical calculations, these wavenumber scalars will be assembled into vectors in order to compute results over the entire spectrum for a representative subset of wavenumbers.

assumption is that the Reynolds number of the free stream flow is sufficiently large in order to ensure thin viscous regions. The theory also assumes that the cascade does not affect the mean flow throughout. When applying the RDT assumptions (which neglect the non-linear and viscous terms of the vorticity transport) a solution based on the modified velocity potential due to the blade blocking can be obtained when taking into account the divergence free properties of the free stream.*

Taking the Fourier transform of the RDT solution given by Graham (1998), Equations 7.4 through 7.6 can be obtained. These equations describe the Fourier transform of the three velocity components (denoted by hat overscripts) valid behind the leading edge region of influence. In these equations $i = (-1)^{1/2}$, $\tau = (\kappa_1^2 + \kappa_2^2)^{1/2}$ and $x_3' = x_3 - ns$ where n is an integer such that $0 \leq x_3' \leq s$ and $f(k_3, \tau, x_3')$ is given by Equation 7.7.

The trailing edge is dealt with by assuming a coplanar vortex sheet being shed from behind each blade. Graham (1998) applied the Kutta-Joukowski condition[†] and continuity of pressure in the vortex sheet and arrived at the conclusion that to the order of the solution, the wake contributes in exactly the same way as the cascade itself. In other words, the wakes are simply extensions of the chord, effectively setting up a cascade with infinite chords. The solution then suggests that outside the region of leading edge influence, the flow-field is identical in the passage and far downstream. This is consistent with the experimental data of Chapter 6. Of course, the actual downstream distance over which the solution is valid is limited due to non-linear terms, and viscous dissipation.

$$\hat{u}_1 = \hat{u}_{1\infty} + \frac{ik_1}{\tau} \hat{u}_{3\infty} f(k_3, \tau, x_3') e^{ik_3 x_3'} \quad (7.4)$$

$$\hat{u}_2 = \hat{u}_{2\infty} + \frac{ik_2}{\tau} \hat{u}_{3\infty} f(k_3, \tau, x_3') e^{ik_3 x_3'} \quad (7.5)$$

$$\hat{u}_3 = \hat{u}_{3\infty} - \frac{\sinh(\tau x_3')}{\sinh(\tau s)} e^{-ik_3(s-x_3')} - \frac{\sinh(\tau(s-x_3'))}{\sinh(\tau s)} e^{ik_3 x_3'} \quad (7.6)$$

$$f(k_3, \tau, x_3') = \frac{\cosh(\tau x_3') e^{-ik_3 s} - \cosh(\tau(s-x_3'))}{\sinh(\tau s)} \quad (7.7)$$

The above equations can further be manipulated in order to obtain the predicted blade blocking effects at any transverse location in the passage. This can be done in terms of the spectral distributions, or Reynolds stresses through integration for any flat plate cascade, given the free-stream turbulence and blade spacing, s . Instead of working with separate Fourier transformed velocity fields, Equation 7.8 shows the same equations generalized in terms of the velocity-spectrum tensor:

* See Graham (1998) for details.

[†] The Kutta-Joukowski condition specifies zero pressure difference on either side of the trailing edge.

$$\begin{aligned} \Phi_{ij}(\vec{\kappa}) = & \Phi_{ij\infty}(\vec{\kappa}) + \Phi_{33\infty}(\vec{\kappa}) \left[A_i A_j^* + B_i B_j^* + A_i B_j^* e^{-ik_3 s} + B_i A_j^* e^{ik_3 s} \right] + \\ & \Phi_{33\infty}(\vec{\kappa}) \left[A_j^* e^{ik_3(s-x_3')} + B_j^* e^{-ik_3 x_3'} \right] + \Phi_{33\infty}(\vec{\kappa}) \left[A_i e^{-ik_3(s-x_3')} + B_i^* e^{ik_3 x_3'} \right] \end{aligned} \quad (7.8)$$

The free-stream velocity-spectrum tensor, $\Phi_{ij\infty}$, can be obtained for example via a pre-defined isotropic free-stream three-dimensional spectrum model such as the Pope-spectrum* given in Section 4.6.2 coupled with equation 7.1. The coefficients in the above equation can with the proper use of hyperbolic identities be shown to reduce to the following:

$$A_1 = \frac{i\kappa_1}{\tau} \frac{\cosh(\tau x_3')}{\sinh(\tau s)} \quad A_2 = \frac{i\kappa_2}{\tau} \frac{\cosh(\tau x_3')}{\sinh(\tau s)} \quad A_3 = -\frac{\sinh(\tau x_3')}{\sinh(\tau s)} \quad (7.9)$$

$$B_1 = \frac{-i\kappa_1}{\tau} \frac{\cosh(\tau(s-x_3'))}{\sinh(\tau s)} \quad B_2 = \frac{-i\kappa_2}{\tau} \frac{\cosh(\tau(s-x_3'))}{\sinh(\tau s)} \quad B_3 = -\frac{\sinh(\tau(s-x_3'))}{\sinh(\tau s)} \quad (7.10)$$

It should be noted however, that when implementing a numerical code some of the hyperbolic arguments in the above coefficients can be too large for the computer to handle. This can be remedied by recognizing that $\cosh(x \rightarrow \infty) \approx \sinh(x \rightarrow \infty) \approx e^x/2$.

The above theory is in reality a generalization of Hunt & Graham's (1978) theory of an instantaneously introduced semi-infinite plate in a free stream flow for more than one plate. As the blade spacing goes to infinity, the results given from the above theory should match perfectly with its predecessor. To check this, the two theoretical predictions were plotted as shown in Figure 5.5 for a blade spacing of $10L_{11}$ in which the two approaches are found to be, for all practical purposes, identical.

* Another, more widely used, spectrum function would be the one by Von Kármán which models the low-wavenumbers slightly differently. See Section 4.6.2 for the differences between the two models.

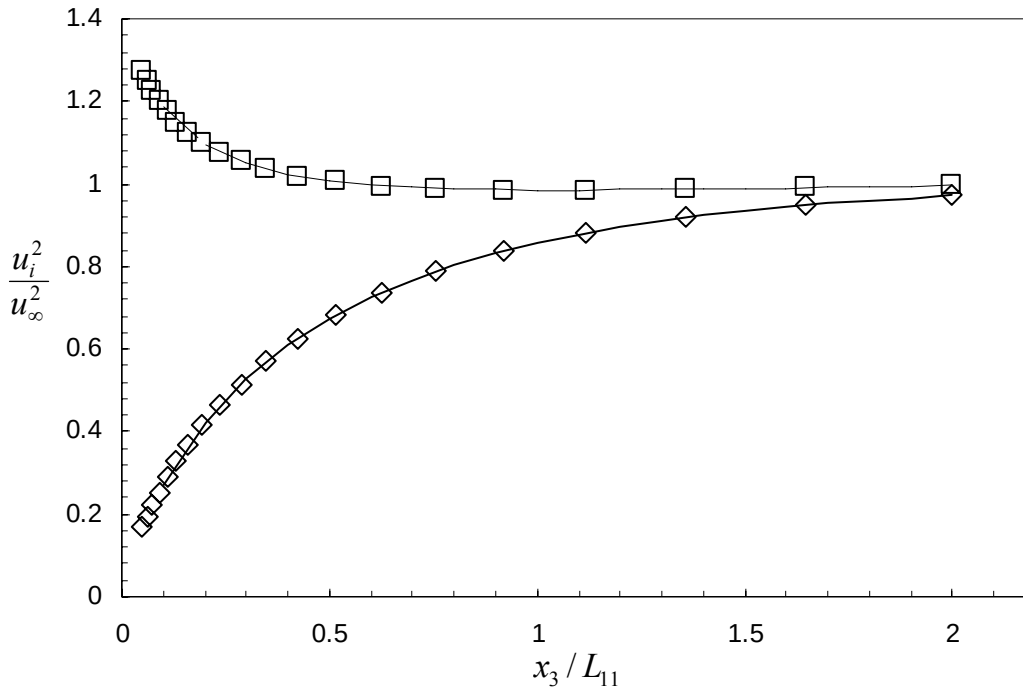


Figure 7.1. Comparison of Hunt & Grahams (1978) single plate predictions ($\square$, tangential stresses; $\diamond$, normal stress) and Grahams (1998) cascade theory with $s/L_{11} = 10$ (—, tangential stresses; —, normal stress).

7.3 Reynolds Stress Comparison

7.3.1 Mid-Passage Stresses

Clearly, as shown in numerous previous studies and the calculations shown in Figure 7.1, the largest effect of blade blocking is on the normal stress perpendicular to the blade surface. Based on the theory in Section 7.2, the mid-passage stresses can be calculated for any given cascade. The mid-passage values of the upwash stress are given below in Figure 7.2 as a function of blade spacing. In this figure both the Von Kármán free stream spectrum as well as the Pope spectrum for high Reynolds number ($\kappa_d/\kappa_e = 1700^*$) appear. This is representative of the active grid turbulence with a Kolmogorov lengthscale $\eta = 0.23$ mm and an integral lengthscale $L_{11} = 28$ cm. The difference is quite substantial between the two spectral models. This is entirely due to the difference in the very low end of the spectrum as explained in Section 4.6.2. When the κ_d/κ_e ratio is substantially lower (in the low hundreds) the Pope-spectrum mid-passage levels drop below the Von Kármán estimates for small blade spacings, but remain slightly larger around $s/L_{11} \approx 2$.

Included in Figure 7.2 is also experimental data provided by Graham's (1998) Figure 9 as well as results from the current experiment at various downstream locations behind the unstaggered cascade. According to this figure, the current data is more consistent with the Von

* See Section 4.7.3 for the definition of κ_d and κ_e .

Kármán estimates, however one must keep in mind that the real integral lengthscale of the flow (39 cm) is significantly larger than the 28 cm which was used because of its compliance with the low end of the spectrum. This would push the recorded data from $s/L_{11} = 0.95$ to about 0.70 in which case the datapoints in question would fall right in between the two theoretical predictions.

The last issue which influences the results is of course the low frequency uncertainty effects. This problem will be looked at in subsequent sections, but in conventional facilities Figure 7.2 should be expected to provide a good ballpark estimate of the upwash stress attenuation immediately downstream of the cascade.

As for the blade-parallel stresses, the streamwise and spanwise components behave identically as the blade spacing is decreased as seen in Figure 7.3, which intuitively seems correct as the cascade would act to two-dimensionalize the turbulence in the limit of zero blade spacing. It was however seen in Chapter 6 that there is indeed some difference in the way the two blade-parallel components behave, but it is believed that this discrepancy is attributed more to the fact that the streamwise component have a larger percentage of its energy in the low-frequency region of the spectra compared to both the lateral stresses. In other words, the streamwise component is in its spectral nature more prone to tunnel attenuation than the spanwise and upwash stresses.

Graham's theory using the high Reynolds number Pope spectrum function predicts the two parallel stresses to take on a value of 86% of the free stream value behind the unstaggered cascade. This is again quite consistent with the experimental data in which, at $x/c = 1.98$, the stresses take on values corresponding to 85% of the reference free stream. The experimentally measured attenuation immediately behind the cascade is again slightly larger. The Von Kármán estimate and some of Graham's (1998) own measurements have also been included in Figure 7.3 together with current unstaggered cascade data. Note that the high Reynolds number Pope and Von Kármán spectral models yield very comparable results as the streamwise and spanwise components are only slightly affected such that the low-frequency spectral difference makes less of a contribution. As expected, however, the discrepancy increases with blade spacing.

An interesting point was made by Dr. Glegg (private communication), who applied his cascade theory found in Glegg (1999), to the current problem and found a distinct difference in the way the streamwise and the parallel stresses behave as a function of blade spacing. By assuming $c = L_{11}$ he found that for a blade spacing of $s/L_{11} = 1$, the two parallel stresses behave the same. By decreasing the blade spacing, however, the two stresses were found to diverge. The spanwise stress would increase after an initial suppression before asymptoting to the free-stream levels, whereas the streamwise stress would decrease monotonically with blade spacing.

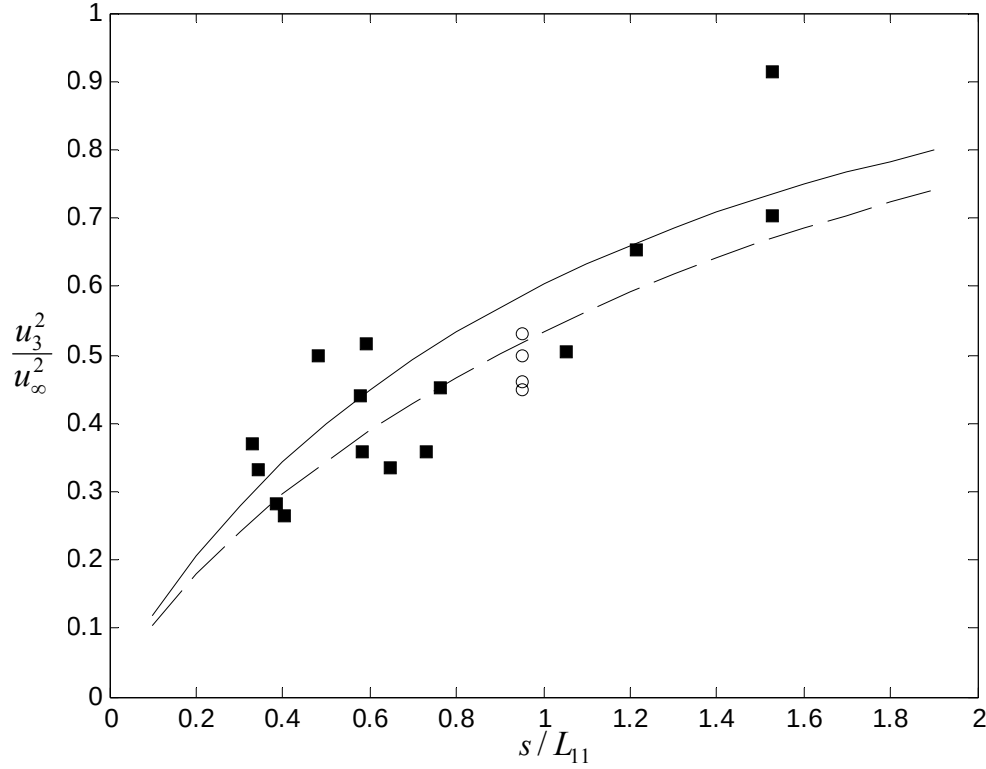


Figure 7.2. Mid-passage normal stress ratios using Grahams (1998) cascade theory with: —, the Pope model spectrum ($\kappa_d/\kappa_e = 1700$) and --, the Von Kármán model spectrum; ■, previous experimental data provided in Figure 9 of Graham (1998); ○, current unstaggered cascade data at various downstream locations.

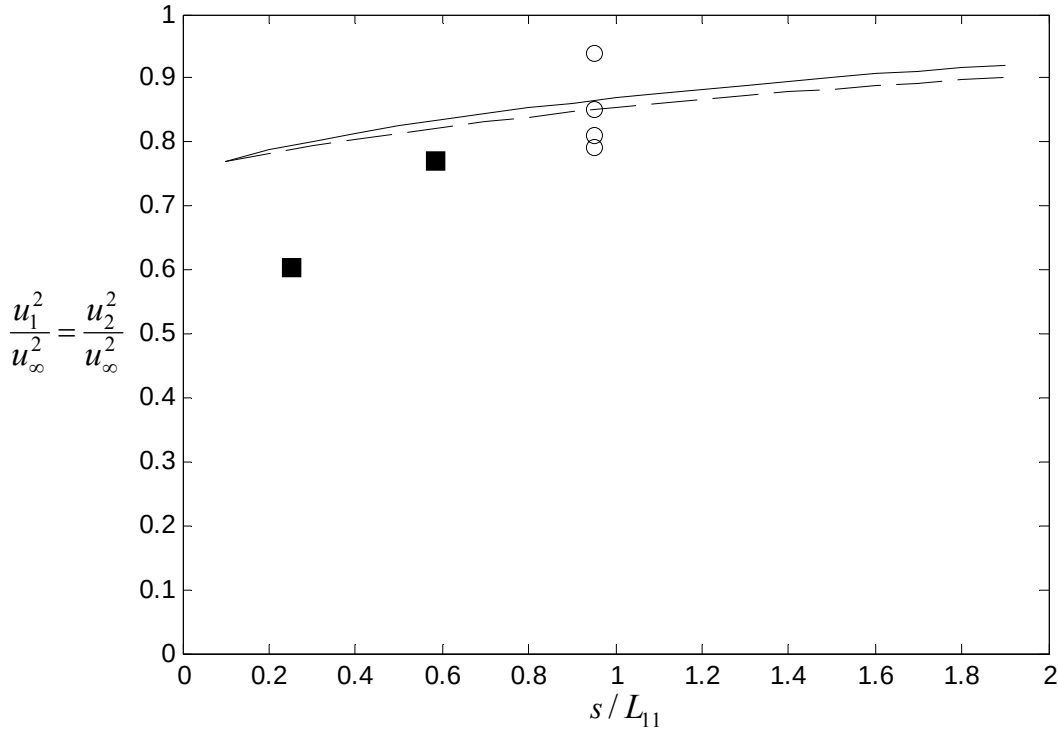


Figure 7.3. Mid-passage parallel stress ratios using Grahams (1998) cascade theory with: —, the Pope model spectrum ($\kappa_d/\kappa_e = 1700$) and --, the Von Kármán model spectrum; ■, previous experimental data provided in Figure 9 of Graham (1998); ○, current unstaggered cascade data at various downstream locations.

7.3.2 Complete Stress Profiles

Figure 7.4 displays the theoretical predictions for all three Reynolds stresses in the passage together with the measurements made behind the unstaggered cascade at $x/c = 1.61$. As expected from the previous section, all three stress components are over-predicted by the theory by a significant margin. Although not very encouraging quantitatively, the general trends calculated across the passage appear very consistent with the behavior of the experimental data. This suggests that the theoretical model is at least modeling the physics of the situation correctly. However, prior to forming a hypothesis on the quantitative discrepancy, the actual spectral distribution of the theoretical attenuation effects will be investigated.

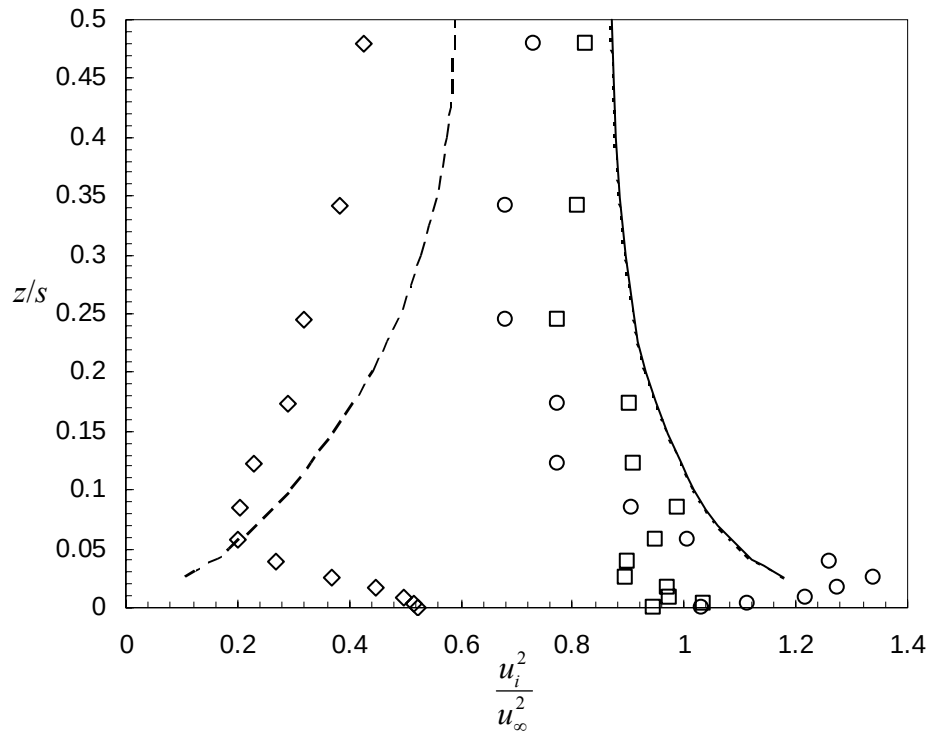


Figure 7.4. Reynolds stress profiles behind the unstaggered cascade. Theoretical predictions are based on the Graham cascade theory for a blade spacing of $s/L_{11} = 0.95$ using the Pope model spectrum ($\kappa_d/\kappa_e = 1700$); —, RDT streamwise and spanwise component; ---, RDT upwash component; Measurements behind the cascade ($x/c = 1.61$): $\circ$, streamwise component; $\square$, spanwise component; $\diamond$, upwash component.

7.4 Spectral Effects

This section will take a look at the spectral distribution of energy before and after the turbulence passes through the cascade, according to Graham's theory. Figure 7.5 shows the low-wavenumber part of the streamwise velocity spectrum for a variety of normalized distances away from the blade together with the free stream reference. According to the theory (at least for practical distances from the blade surface) the attenuation is found to only affect the energy-

containing eddies while modestly extending into the inertial range. A noticeable drop in spectrum levels is observed between the free stream and the mid-passage location. However, as the blade is being approached, note how the energy grows back in a different manner from how it was lost. The streamwise component never regains the energy attenuated from the lowest-wavenumbers, but compensates by regaining energy at slightly higher wavenumbers which significantly extends the inertial range behavior sufficiently close to the blade.

As for the spanwise component, the situation is depicted in Figure 7.6. Most of the energy-loss occurs at extremely low wavenumbers, but as opposed to the streamwise component, most of this is regained far from the mid-passage. The extension of the inertial range is much less obvious in this case, but a slight enhancement is nevertheless evident at $z/s = 0.1$.

By now it should be no surprise that the predicted upwash spectra show the largest energy loss. Figure 7.7 shows a substantial energy loss between the free-stream and the mid-passage plane, and the trend continues for lower values of z/s . Qualitatively this is very much the same trend as seen in the experimental data in Figure 6.25 as over a decade is lost at the lowest wavenumbers between the free stream and $z/s = 0.1$ which is still outside the viscous wake for most of the streamwise measurement stations. Also note how the inertial range is affected at progressively higher wavenumbers as the blade surface plane is approached.

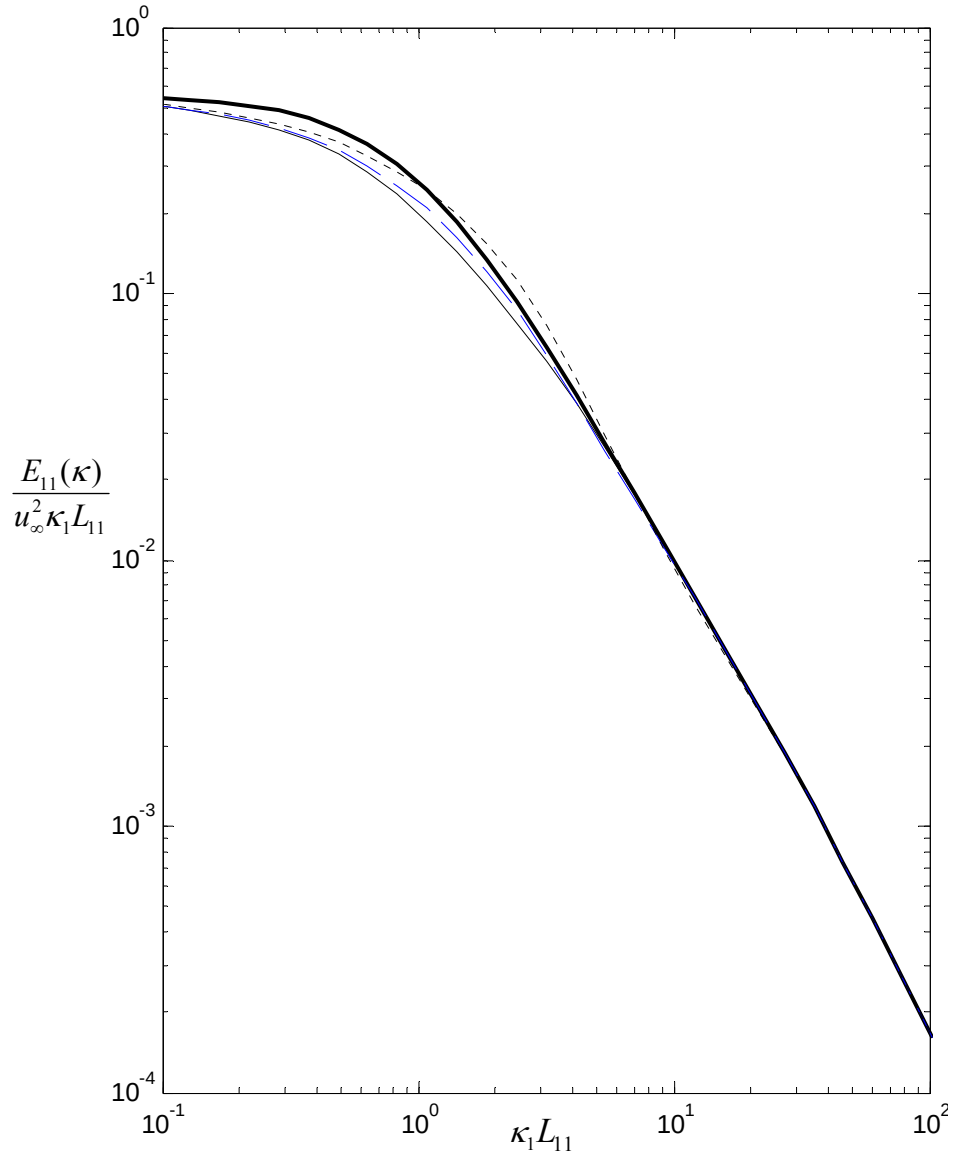


Figure 7.5. One-dimensional streamwise velocity spectra behind the cascade using Grahams (1998) cascade theory with the Pope model spectrum ($\kappa_d/\kappa_e = 1700$) for a blade spacing of $s/L_{11}=0.95$. —, Free stream; —, $x/s = 0.5$; - - , $x/s = 0.25$; - - - , $x/s = 0.1$.

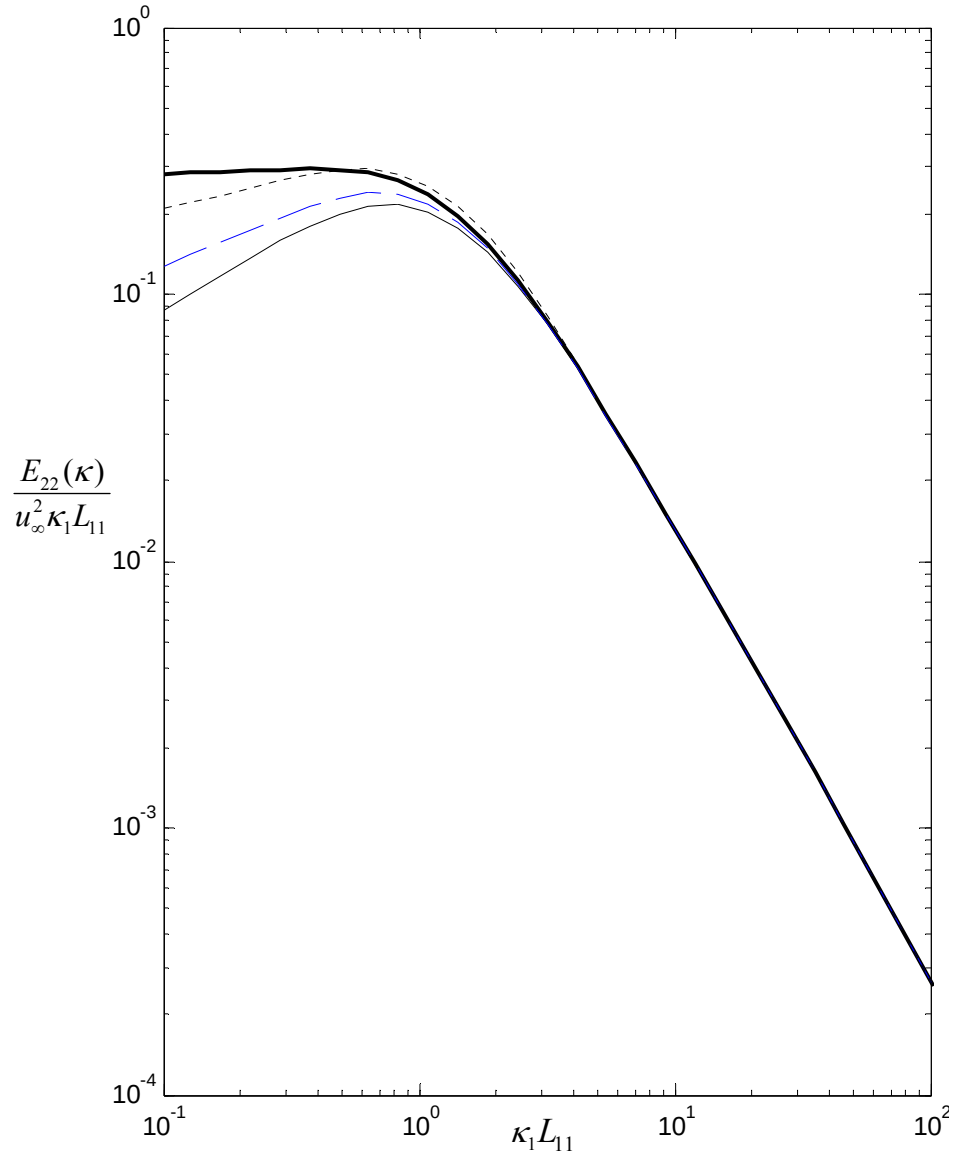


Figure 7.6. One-dimensional spanwise velocity spectra behind the cascade using Grahams (1998) cascade theory with the Pope model spectrum ($\kappa_d/\kappa_e = 1700$) for a blade spacing of $s/L_{11}=0.95$. —, Free stream; —, $x/s = 0.5$; - - , $x/s = 0.25$; - - - , $x/s = 0.1$.

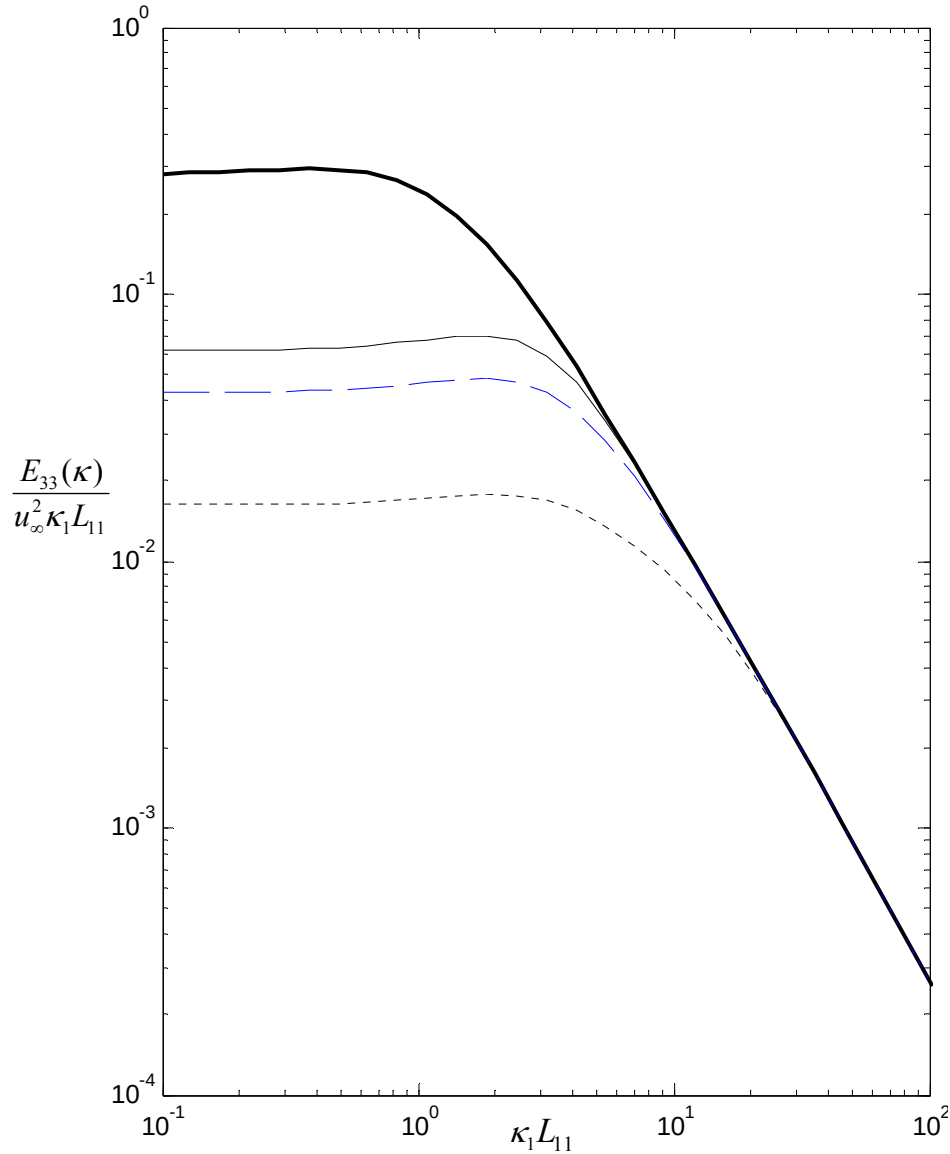


Figure 7.7. One-dimensional upwash velocity spectra behind the cascade using Grahams (1998) cascade theory with the Pope model spectrum ($\kappa_d/\kappa_e = 1700$) for a blade spacing of $s/L_{11}=0.95$. —, Free stream; —, $x/s = 0.5$; - - , $x/s = 0.25$; - - - , $x/s = 0.1$.

7.5 Spectral Comparison

In order to compare the theoretically attenuated spectra with those experimentally obtained (presented in Chapter 6), it is helpful to focus on the normal component as this one offers the greatest attenuation which eases the task of a qualitative comparison between the two. Figure 7.8 shows the measured free-stream and mid-passage spectral values at $x/c = 1.61$ together with the Pope model spectrum as well as the mid-passage Graham prediction. Some of the previously evident discrepancy between the theory and the measurements is now becoming clearer. Since the theory is based on the free stream measurements, the difference in the inertial range between

the two measurements sets (due to the presence of the cascade and the inherent repeatability issues between wind tunnel entries) is quite substantial. In addition the streamwise decay of grid turbulence has to be taken into account between the measurement positions. Although it may not make much of a difference in the equilibrium range itself due to the energy distribution, the discrepancies do propagate to the lower frequency end of the spectrum, contributing to a large difference between the theoretical and measured stresses when the spectrum is integrated.

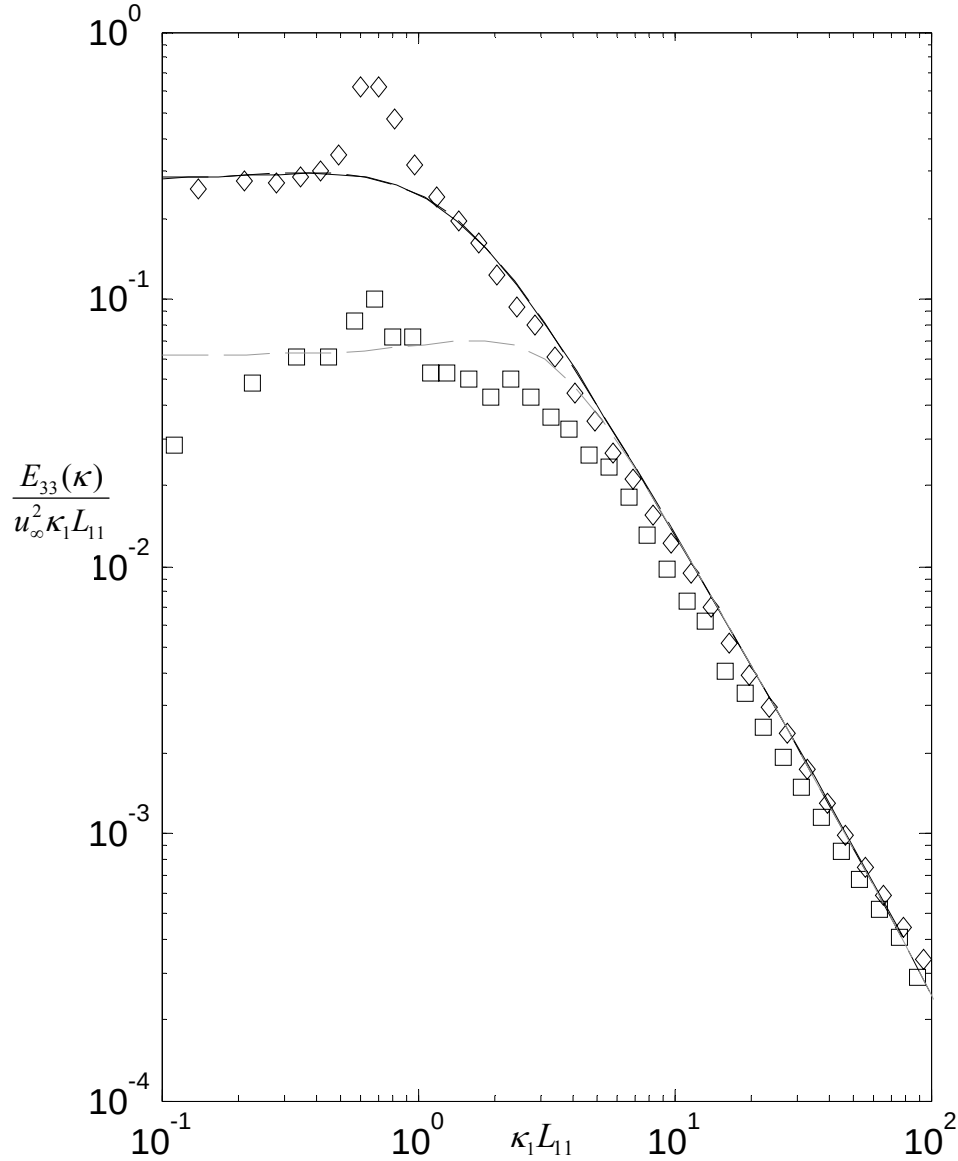


Figure 7.8. One-dimensional upwash velocity spectra. —, free-stream according to the Pope model spectrum ($\kappa_d/\kappa_e = 1700$); --, Grahams (1998) cascade theory for a blade spacing of $s/L_{11} = 0.95$ at $x/s = 0.5$; $\diamond$, free stream measurements at $X/M = 37.15$ without the cascade present; $\square$, measurements behind the unstaggered cascade ($x/c = 1.61$) at $x/s = 0.5$.

The above-mentioned discrepancy can be remedied by boosting the spectral levels such that the inertial range conforms to the free stream data. Dividing through by a factor of about 0.75 is

sufficient, and the result of this compensated measurement can be seen in Figure 7.9. At this point the attenuation is almost impeccably describing the attenuation of the upwash spectrum for $\kappa_1 L_{11} > 1.0$. With an integral scale of 0.28 m this corresponds to a wavenumber of 3.57 which is exactly the cutoff determined in Section 4.9.3 where local isotropy fails and tunnel and grid effects start influencing the spectrum. This effect can clearly be seen in Figure 7.9 as the free stream measurement and model spectrum start diverging below $\kappa_1 L_{11} < 1.0$.

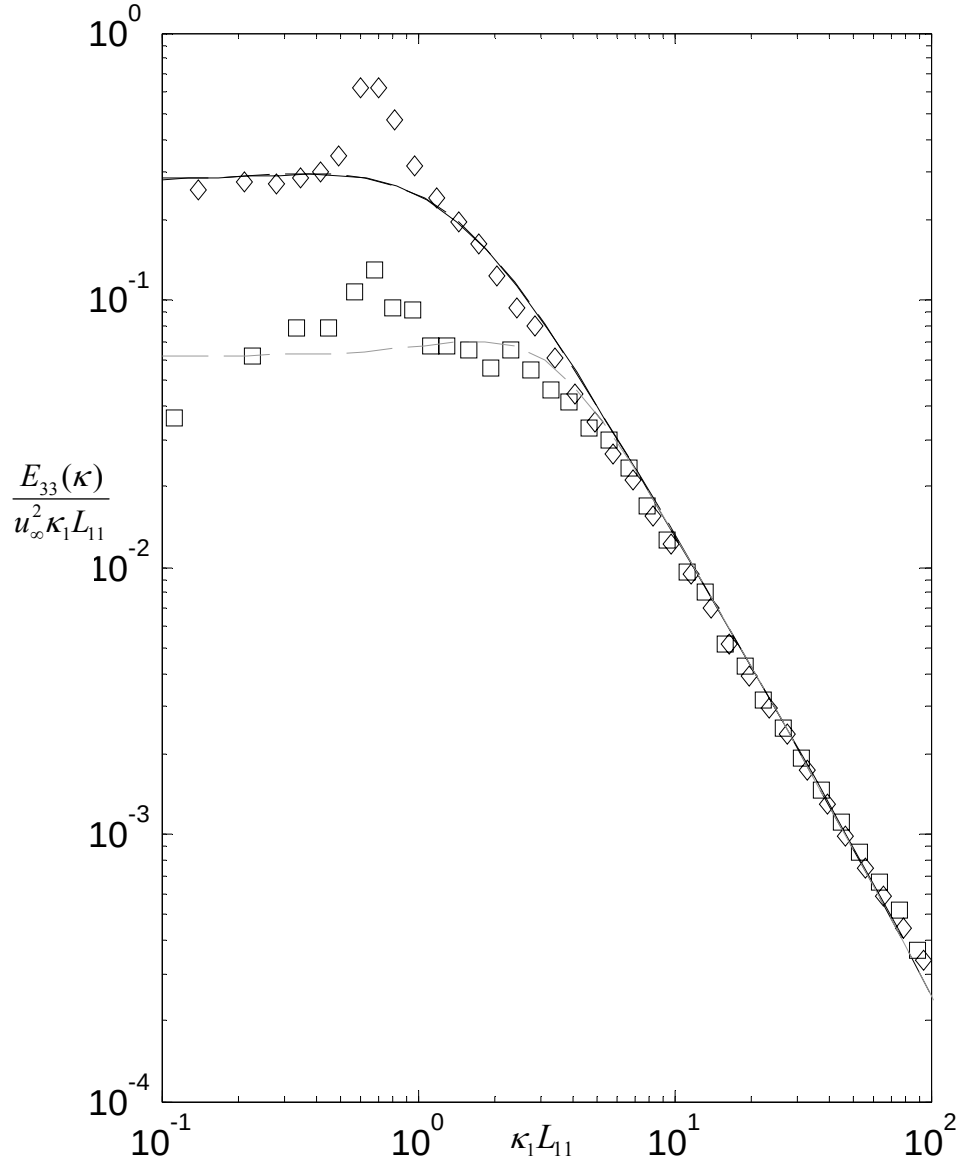


Figure 7.9. One-dimensional upwash velocity spectra. —, free-stream according to the Pope model spectrum ($\kappa_d/\kappa_e = 1700$); --, Grahams (1998) cascade theory for a blade spacing of $s/L_{11} = 0.95$ at $x/s = 0.5$; $\diamond$, free stream measurements at $X/M = 37.15$ without the cascade present; $\square$, compensated measurements behind the unstaggered cascade ($x/c = 1.61$) at $x/s = 0.5$.

Although not explicitly shown, the mid-passage experimental results for the spanwise and streamwise spectral components reveal very little attenuation compared to the free-stream

spectra. The reason for this can be seen in Figure 7.5 and Figure 7.6 as the vast majority of attenuation is seen below the cutoff wavenumber of $\kappa_1 L_{11} = 1.0$. However, the necessary compensation factor still has to be applied to both of the blade parallel spectral components in order to reach the free stream levels in the inertial range.

To summarize, there are two main effects influencing the comparison between theory and measurements. First of all, the inability of reproducing the exact same free-stream flow field into the cascade. This is important as the theory is based on the completely undisturbed turbulent free-stream flow. Secondly, the low frequency interaction between the flow, tunnel and turbulence generator severely disturbs the region with highest energy content, in other words creating an incorrect stress level when integrated. This goes a long way in explaining why the experimentally obtained mid-passage stresses in Figure 7.2 and Figure 7.3 were significantly lower than the predicted theoretical estimates. The first problem can be remedied as shown above by compensating the cascade measurements back to free stream high-frequency levels. The second predicament can not directly be solved, but it is possible to bypass it by comparing measurements filtered above the problematic wavenumbers and comparing it to a high-pass filtered version of the theory.

7.6 Reynolds Stress Comparison: Filtered Components

Figure 7.10 shows the results of applying the spectral filtering to all three principal stress components in Graham's theory. The cutoff wavenumber has been chosen as $\kappa_1 = 3.5$ which for the baseline condition corresponds to a frequency of 7 Hz, and the compensation for the difference between the cascade and free-stream measurements has been applied. The agreement between the theory and the measurements is now much improved over that seen in Figure 7.4. The spanwise component is seen to have the best agreement,* although this is dependent on the actual compensation level applied. The required level is somewhat difficult to determine accurately from the spectral data. Although the streamwise and the upwash components are about 6% larger than the theoretical spectra in the mid-passage, the qualitative trend appears to be almost identical to the prediction outside the viscous region. As the wakes are approached the trends between the measurement and the theory start to differ which is consistent with the presence of an unsteady wake, in which packets of turbulence from the wake occasionally pass on the outside of the mean viscous boundary. This results in larger values of all three stresses which the inviscid theory obviously doesn't account for.

Now the question arises. What happens if only the free-stream level correction is applied while leaving the data unfiltered? As can be seen in Figure 7.11, the behavior of the upwash component is actually predicted even better than for the filtered results, but the penalty is paid in the two other components as the streamwise stresses, and especially the spanwise stresses, are grossly underpredicted. This trend is in very good agreement with the Thomas & Hancock (1978) data which shows that the normal components conform to RDT over longer distances than in the case of streamwise or spanwise components.

* With the exception of one obvious outlier. When plotted unfiltered, this point appears to take on a more reasonable value, which suggests that an unusually large portion of the energy at this measurement location was present below the cutoff wavenumber.

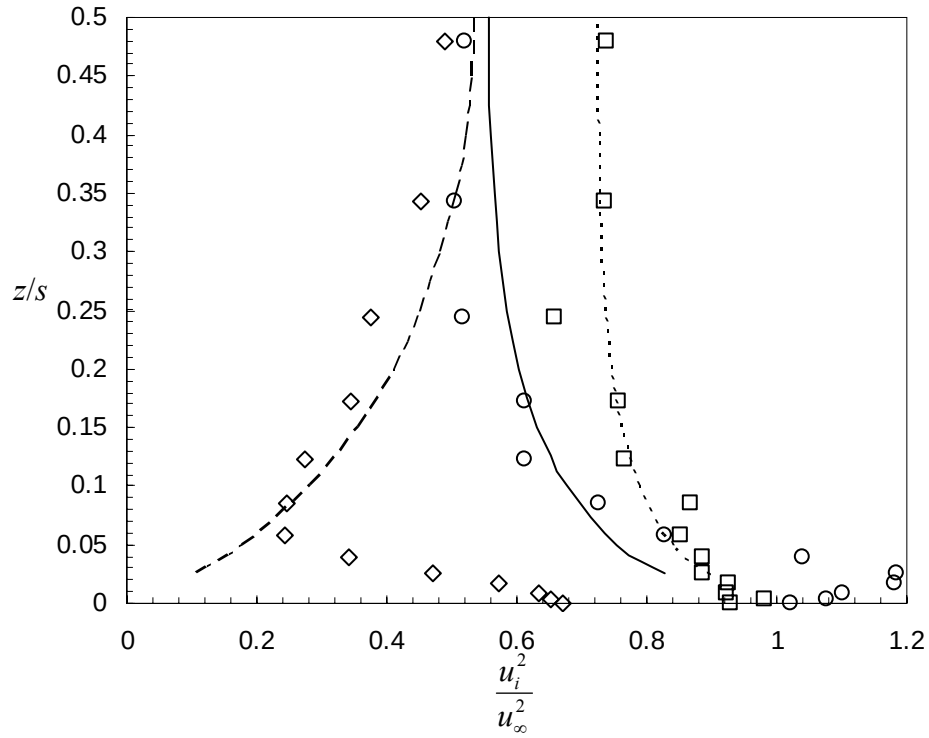


Figure 7.10. Reynolds stress profiles behind the unstaggered cascade based on high-pass filtered spectra. Theoretical predictions are based on the Graham cascade theory for a blade spacing of $s/L_{11} = 0.95$ using the Pope model spectrum ($\kappa_d/\kappa_e = 1700$); —, RDT streamwise component; ---, RDT spanwise component; - - -, RDT upwash component; Compensated and filtered measurements behind the cascade ($x/c = 1.61$): ○, streamwise component; □, spanwise component; ◇, upwash component.

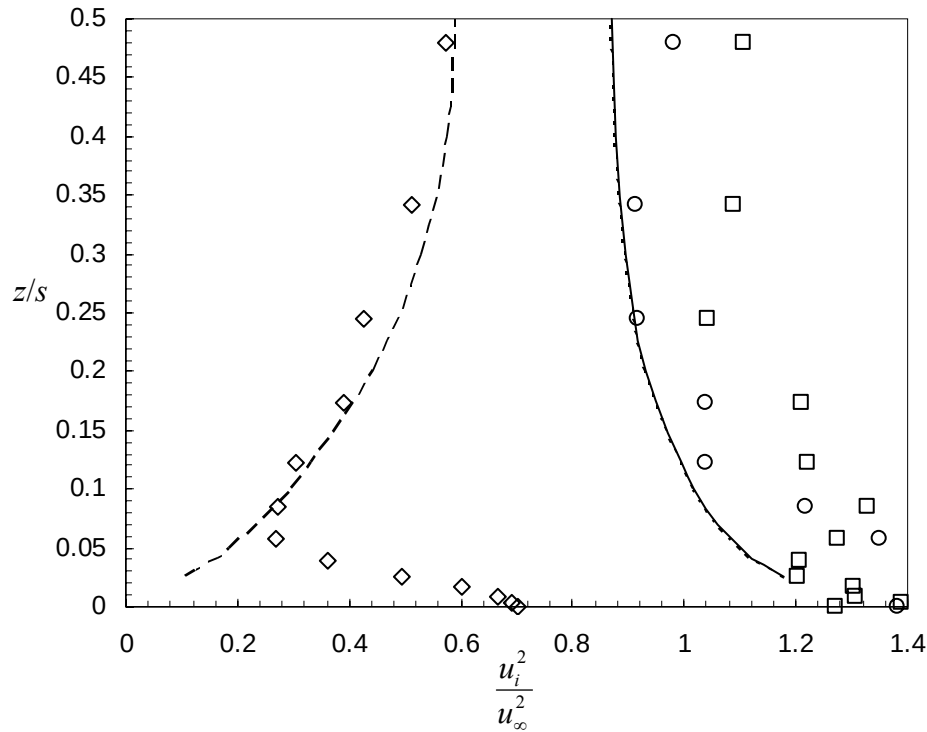


Figure 7.11. Reynolds stress profiles behind the unstaggered cascade based on unfiltered spectra. Theoretical predictions are based on the Graham cascade theory for a blade spacing of $s/L_{11} = 0.95$ using the Pope model spectrum ($\kappa_d/\kappa_e = 1700$); —, RDT streamwise and spanwise component; --, RDT upwash component; Compensated unfiltered measurements behind the cascade ($x/c = 1.61$): ○, streamwise component; □, spanwise component; ◇, upwash component.

7.7 Staggered Cascade Results

The theory of Graham (1998) is only explicitly valid for an unstaggered cascade configuration; however, the theory of Glegg (1999) is a more generalized version of the Graham theory which, in addition to incorporating leading edge effects, also allows an arbitrary stagger angle. It turns out that for the cascade parameters used in the present experiment, the difference in predicted high-pass filtered levels behind the cascade are for all practical purposes invariant from the unstaggered cascade (Glegg, private communication). As mentioned earlier, adjacent blades are essentially uncorrelated and with the wakes acting infinite chord-extensions, the symmetry across the passage as seen in Figure 6.19 results. Taking this into consideration, the predictions used for the unstaggered cascade will be utilized for the staggered cascade as well.

In order to assess the discrepancy between the data in the wake of the staggered cascade and the free stream, the comparison between the theoretical and experimentally obtained upwash spectra for the non-cascade case as well as the mid-passage value at $x/c = 1.60$ has been plotted in Figure 7.12. The inertial range of the two experiments line up very well, which means no compensation is required. Also note that even though there is some scatter, the mid-passage attenuation of the experimental data seems to follow the predictions well below the cutoff wavenumber.

Figure 7.13 and Figure 7.14 show unfiltered and filtered versions of both predicted and experimental Reynolds stresses in half the passage of the staggered cascade ($x/c = 1.60$). The unfiltered data (Figure 7.13) again shows good conformity for the upwash components although a slight over-prediction can be noted. Qualitatively however, the experimental upwash data fits the theory very well. The streamwise and spanwise components again behave in the same way as expected, but compared to theory the profiles are far off the predicted values. Apparently, the experimental spanwise component are about the same level as for the unstaggered cascade, but the streamwise component has dropped significantly below the predictions due to the increase in anisotropy between these components.

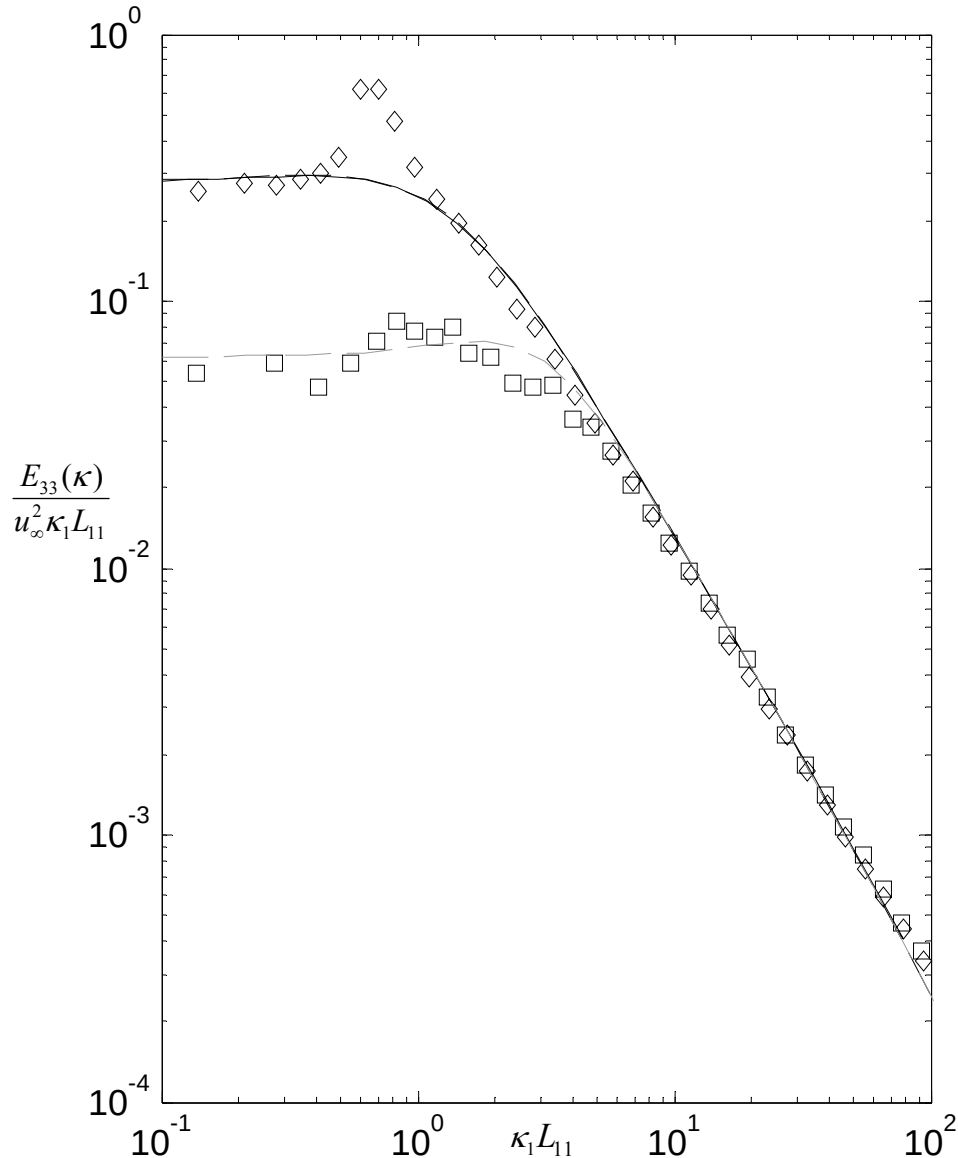


Figure 7.12. One-dimensional upwash velocity spectra. —, free-stream according to the Pope model spectrum ($\kappa_d/\kappa_e = 1700$); --, Grahams (1998) cascade theory for a blade spacing of $s/L_{11} = 0.95$; $\diamond$, free stream measurements at $X/M = 37.15$ without the cascade present; $\square$, measurements behind the staggered cascade ($x/c = 1.61$) at $x/s = 0.5$.

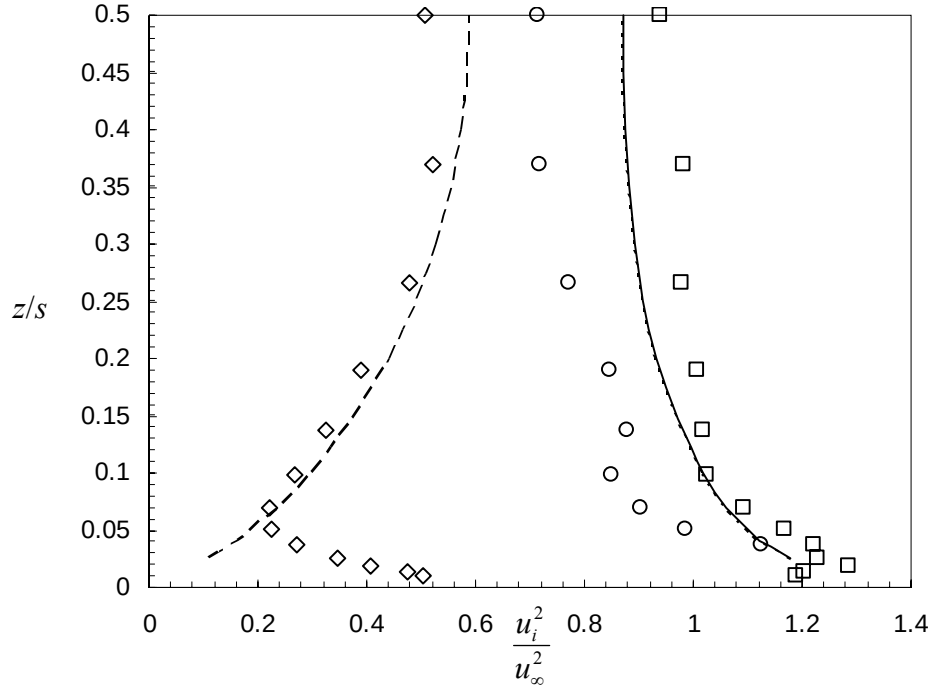


Figure 7.13. Reynolds stress profiles behind the staggered cascade. Theoretical predictions are based on the Graham cascade theory for a blade spacing of $s/L_{11} = 0.95$ using the Pope model spectrum ($\kappa_d/\kappa_e = 1700$); —, RDT streamwise and spanwise component; ---, RDT upwash component; Measurements behind the cascade ($x/c = 1.60$): $\circ$, streamwise component; $\square$, spanwise component; $\diamond$, upwash component.

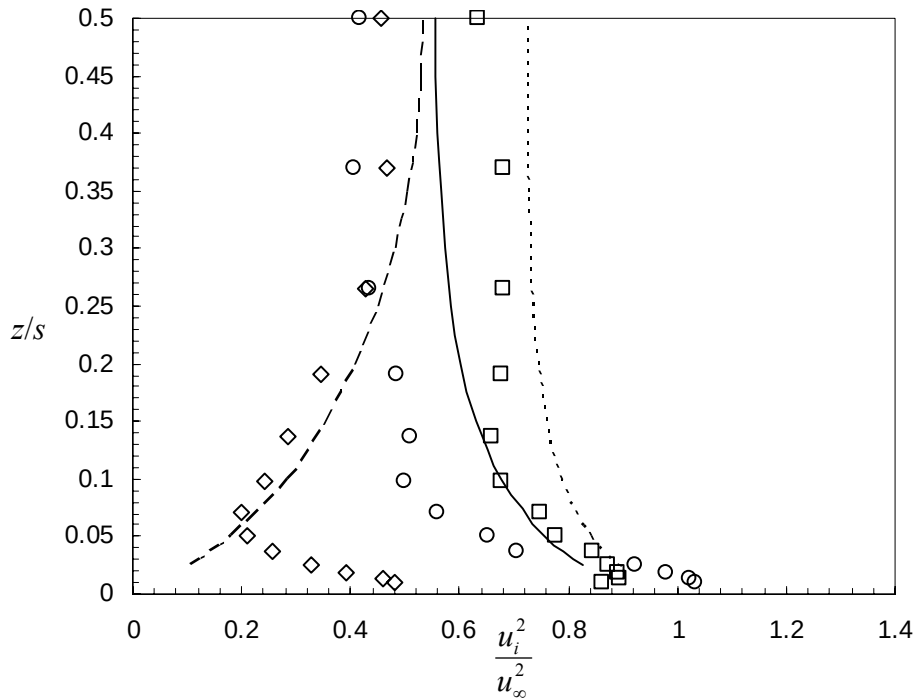


Figure 7.14. Reynolds stress profiles behind the staggered cascade based on high-pass filtered spectra. Theoretical predictions are based on the Graham cascade theory for a blade spacing of $s/L_{11} = 0.95$ using the Pope model spectrum ($\kappa_d/\kappa_e = 1700$); —, RDT streamwise component; ---, RDT spanwise component; - - -, RDT upwash component; Compensated measurements behind the cascade ($x/c = 1.60$): $\circ$, streamwise component; $\square$, spanwise component; $\diamond$, upwash component.

The filtered version in Figure 7.14 tells a somewhat different story. The upwash stress description remains about the same as when no filter is used. The streamwise component is slightly better described using the filter, and the spanwise component is now again over-predicted rather than the other way around. The differences between the two blade-parallel components and their corresponding predictions are of about the same magnitude, but overall, the staggered case is not nearly as well described by the theory as the unstaggered case

7.8 Summary

In conclusion, the relatively simple, inviscid linear cascade theory offered by Graham's (1998) Rapid Distortion Theory describes the stress field behind especially the unstaggered cascade surprisingly well, even far outside the region of applicability of the solution. However, in order to get a faithful estimate, it is extremely important to take the limitations of the experiment into account when comparing with theory. For the current experiment, the low-frequency region of the velocity spectrum is contaminated by wind tunnel attenuation and active grid energy not present in ideal isotropic free-stream turbulence, and this region must be disregarded, especially in the streamwise and spanwise direction.

When the above limitation is considered, the current data supports that all three components behave roughly according to the theoretical estimates. If the entire wavenumber region is considered on the other hand it is found that the upwash component adheres to the RDT predictions much further than is the case for the two other Reynolds normal stress components. This was also previously shown to be the case on a single flat plate by Thomas & Hancock (1977), but in this case only shows that the effects of the low-wavenumber region tends to cancel itself out which results in a consistent result with theory. As can be clearly seen in Figure 7.9 this is more of a coincidence than anything else, as the model spectrum does in no way represent the actual experimentally obtained data in this region.

It is believed that the underestimates of the quantitative values are due to the fact that the application no longer adheres to the fundamental assumptions of the theory. Nevertheless the theory provides reasonable estimates, but much in the same way as for the contraction estimates, falls a bit short of the actual values.

The Reynolds stresses behind the staggered cascade are qualitatively well described by the Graham theory, but in terms of actual values it tends to over-predict stress values by as much as 20%. One way to improve the validity of the downstream predictions, especially in a large scale experiment as this one, would be to incorporate the streamwise decay of turbulence kinetic energy inside the passage into the theoretical predictions. This would also alleviate some of the need to "boost" the downstream spectral levels in order to correspond with the inflow.

Chapter 8

Conclusions

Measurements of the interaction between a large scale homogeneous turbulent flow and a cascade of flat plates have been made in an attempt to realistically model the flow conditions within a marine propulsor unit. The cascade configuration with chord-lengths of 32.8 cm, blade-spacing-to-chord ratio of 0.81 and a selectable stagger angle of 0 or 35.8 degrees was mounted in the Virginia Tech Stability Wind Tunnel test section. The cascade was subjected to an incoming near-isotropic turbulent flow generated by an active turbulence grid placed in the contraction of the wind tunnel. Three-component single and two-point spectral hot-wire data was obtained ahead, throughout, and behind the cascade for both available stagger angles, as well as a detailed measurement set in the absence of the cascade.

Based on its previously documented* ability to create large scale, high-intensity homogeneous turbulent flows in a laboratory setting, an active turbulence generator, which to the author's knowledge is the largest existing generator of its type, was developed for the Virginia Tech Stability Tunnel. An active grid flow has many advantages over conventional grids, but is not without its problems especially since the turbulent scales it produces can be on the order of the tunnel. Nevertheless, this type of turbulence generator was used for the blade-blocking experiment which provided a realistic Reynolds number and a fairly well described incident flow-field which allowed the measurements to be compared with the linear cascade predictions by Graham (1998).

The results of the study have shed more light on the subject of large scale homogeneous laboratory turbulence, and have shown that linear cascade theory can indeed be used to predict the evolution of such flows through a cascade over a much longer distance than the assumptions of the theory initially allow for. The main conclusions reached in Chapters 4 through 7 are summarized in the following two sections. Section 8.1 relates to the active grid itself and the free stream turbulence it produces. The second set of conclusions is presented in Section 8.2 and deals with the experimental blade-blocking study in terms of both the experiment itself and how it compares to predictions based upon linear cascade theory.

8.1 Active Grid Generated Free-stream Turbulence

By varying the grid operating parameters, flow speed, and streamwise location, numerous different flow conditions were created, some of which are among the largest Taylor Reynolds number homogeneous and near-isotropic turbulent flows ever created in a regular wind tunnel. The following are conclusions that can be drawn about the active grid itself:

* Makita (1991) and Mydlarski & Warhaft (1996, 1998)

- 1) An active turbulence grid produced a much higher Taylor Reynolds number compared to a static grid with the same flow condition and mesh length. During the present study, Taylor Reynolds numbers up to 1360 and integral lengthscales up to 67 cm were generated, although flows with Taylor Reynolds numbers above 500 are severally attenuated at low frequencies due to tunnel confinement.
- 2) The contamination of the low wavenumber range of the velocity spectrum associated with previous active grids can be reduced by providing a sufficiently randomized grid forcing protocol. It is more important to randomize the rotation rate than the cruise time between successive maneuvers.
- 3) The most important grid parameters are grid rotation rates and free stream velocity. Cruise time plays a small role as long as $T > 0$. Larger scales and intensities result if T is close to 0, but due to a loss in randomization it is questionable how well the low-wavenumber region behaves in this scenario. The maximum deviation in cruise time is almost insignificant for the resulting turbulence as long as $T \neq 0$, while a wide range of possible rotation rates naturally increases the obtainable lengthscales for a given mean rotation rate due.
- 4) The lower the mean rotation rate, the larger the turbulence intensity and integral scale.
- 5) Comparing with earlier studies it appears that the largest lengthscale that can be produced by this type of grid scales with the size of the wind tunnel, not the grid cell size.
- 6) Three different Taylor Reynolds numbers can be defined for each grid flow realization due to the low frequency ambiguity. First of all the low-wavenumber part of the spectrum can be used to yield one estimate. A second method takes the high-wavenumber dynamics and fits an ideal low-wavenumber behavior based on an ideal model spectrum to obtain an unaffected Taylor Reynolds number. This estimate will generally be much larger than the first. A compromise between these two methods would be to rely on the scaling of the integral scale with dissipation rate and turbulence kinetic energy. This method adopted by Mydlarski & Warhaft (1996) has also been used in this study in order to properly compare the experimental data with previous experiments.
- 7) Placing the active grid in the contraction improves the anisotropy ratio, but does not appear to have much effect on the extent of actual local isotropy.
- 8) Local isotropy appears to be limited by the size of the test section due to the attenuation of large scales. The second wind-tunnel wavenumber $\kappa_{H,2} = 2\pi/H$, where H is the characteristic dimension of the test section, is suggested as the cut-off point for local isotropy.
- 9) Dissipation rate estimates should not be based on streamwise decay in the case of active grid turbulence since the tunnel confinement plays a large role on the part of the velocity spectrum that contains most of the energy. Using such a method has been shown to yield estimates that can be 50% too large compared to other methods that utilize the unaffected dissipation range to estimate the dissipation rate.

As for the dynamics of the turbulent free stream velocity the following can be said:

- 1) Second, third and fourth order statistics produced by the grid are in general agreement with previous studies, but the tunnel confinement clearly plays a significant role at low wavenumbers.

- 2) The streamwise decay law suggested by Mohamed & Larue (1990) is supported by the current data as well as previous active grids, with the exception of the virtual origin which invariably takes on a negative value.
- 3) The Pope estimate of $A = 0.79$ in Equation 1.7 is supported by the current data as opposed to the value of 0.9 used by Mydlarski & Warhaft (1996, 1998) and Kang *et al.* (2003).
- 4) After growing rapidly for Taylor Reynolds numbers below 400, the inertial range slope, n_1 , increases relatively slowly and has reached a value of only 1.62 for $Re_\lambda \approx 1000$.
- 5) There appears to be evidence that the inertial range slope is not only a function of Taylor Reynolds number, but also grid-dependent parameters such as grid rotation rates. The lower the rotation rate, the larger the value of n_1 for a given Taylor Reynolds number. The same is the case for the Kolmogorov constant, C_1 .

8.2 Blade Blocking of Large-Scale Homogeneous turbulence

The cascade study was executed by producing a turbulent inflow with a turbulent intensity of 6.8% and a low-wavenumber behavior of a flow with an associated integral lengthscale of 0.28 m which is on the order of the chord. Chapter 5 showed good mean flow characteristics with the flow behind the cascade exhibiting a closely two-dimensional behavior with good pitchwise periodicity. Viscous boundary layers and wakes remained thin over the measurement domain which indicates no appreciable unsteady stall around the leading edges of the cascade blades. As for the blade-blocking effects described in Chapter 6, the following conclusions are made (referring to the unstaggered cascade unless otherwise noted):

- 1) In anticipation of the cascade, the normal stresses are affected slightly already about one chord-length ahead of the cascade. This effect manifests itself as a uniform loss of isotropy as the upwash stresses get slightly suppressed, then get magnified again as the cascade is approached. Streamwise stresses are also suppressed, while spanwise stresses appear to be enhanced in order to keep the total kinetic energy relatively constant.
- 2) The blade-blocking effects of the cascade permanently modify the incoming turbulent field well before the trailing edge and only minor changes occur downstream as the turbulence continues to dissipate, albeit at a slower rate than the free-stream as it convects downstream. No appreciable return to isotropy is seen as the wakes act like extensions of the cascade blades themselves.
- 3) In the middle of the passage behind the cascade the total attenuation of the three stresses is on the order of 16%, 6% and 50% for streamwise, spanwise and upwash stress components respectively. This cascade-generated anisotropy between the two blade-parallel components is thought to be caused by the low-frequency ambiguity, as the streamwise component has significantly more energy stored in this part of the spectrum compared to the lateral components and hence is more affected by this problem.
- 4) Upwash spectra show almost two and a half orders of magnitude attenuation between the free stream value and the blade-adjacent station inside the passage.

- 5) Due to the lower effective blockage associated with the staggered cascade, the total mid-passage attenuation is less severe for all three normal stress components. The reduction in attenuation is about the same magnitude for the upwash and streamwise components, while the spanwise component almost sees no effective attenuation at all. This causes an even greater anisotropy in the exit plane of the cascade than in the unstaggered case. Apart from this, the flow behavior is very similar in the two cascade configurations.

When comparing to Graham's (1998) theoretical cascade calculations, it becomes evident that:

- 1) Previous blade blocking studies have shown some ambiguity about the behavior of the spanwise component, but the current data leaves little doubt that the RDT prediction, which calls for a strong increase of both blade-parallel components close to the blades, is correct.
- 2) Graham's theory qualitatively describes the cascade attenuation of the three normal stresses very well but the numerical factors do not match. This is largely attributed to the fact that the incoming turbulence does not adhere to the model spectrum for the lowest wavenumbers due to the lack of local isotropy below $\kappa_{H,2}$.
- 3) High-pass filtered results that only take into account isotropic data show a much closer agreement with the theory. The spanwise component is very well predicted whereas the upwash and streamwise attenuation is still slightly under-predicted, but the difference is at most 8%.
- 4) As for the filtered staggered cascade, the upwash component is well predicted (within 11%) while the two blade-parallel components fall on either side of the predictions with a maximum difference of 20%.
- 5) Even far outside the theoretical limits of RDT, the current study shows that the predictions still provide a fairly reliable estimate of the stress distribution downstream of the cascade. The blade-blocking trend itself is in fact very accurately captured for all three components, although numerically the mid-passage values may be a little off compared to the experimentally obtained values. The underestimation of the stresses is to be expected since the same issue occurred when using RDT to predict the contraction ratio needed to cancel out initial anisotropy.

Appendix A

The Fourier Transform

The Fourier Transform is one of the cornerstones in signal processing and the author feels that most references describing this material can be very confusing for students unfamiliar with the underlying math, and the entire transform seems like a black box which is used with very little understanding as to why and how it works. This appendix will attempt to shed some light over the Fourier transform applied to discrete data.

A.1 Fourier Series and Fourier Transforms

In 1807 Jean Baptiste Joseph Fourier introduced the idea that any arbitrary periodic function (subjected to some mild continuity restrictions to ensure convergence) can be represented as an infinite series of elementary periodic functions (sines and cosines). Such a sum of superimposed periodic functions is what is known as a Fourier series. The most common form of the Fourier series is that of Equation A.1 with the coefficients given in Equations A.2 and A.3.

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{2\pi n}{T}t\right) + b_n \sin\left(\frac{2\pi n}{T}t\right) \right) \quad (\text{A.1})$$

$$a_n = \frac{2}{T} \int_0^T f(t) \cos\left(\frac{2\pi n}{T}t\right) dt \quad (\text{A.2})$$

$$b_n = \frac{2}{T} \int_0^T f(t) \sin\left(\frac{2\pi n}{T}t\right) dt \quad (\text{A.3})$$

The time it takes for the series to repeat itself is defined as the period, T . By taking advantage of Euler's formula given in Equation A.4, the above Fourier series can also be written in the form of Equation A.5 with the coefficients F_n given by Equation A.6

$$e^{i\theta} = \cos \theta + i \sin \theta \quad (\text{A.4})$$

$$f(t) = \sum_{n=0}^{\infty} F_n e^{i \frac{2\pi n}{T}t} \quad (\text{A.5})$$

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$$F_n = \frac{1}{T} \int_0^T f(t) e^{-i \frac{2\pi n}{T} t} dt \quad (\text{A.6})$$

Equation A.6, which describes the coefficients of the Fourier series, is the Fourier transform of $f(t)$. Equation A.5, which represents the Fourier series expression, is in reality a version of the inverse Fourier transform described by an infinite sum rather than with an integral. The inverse Fourier transform serves to undo the transformation, which makes the process reversible for a continuous signal. A second and much more useful property of the Fourier transform in signal processing is that the transform gives information about the periodic nature of an arbitrary waveform. It returns the frequency content from the time-series $f(t)$. A more common representation of the Fourier transform and the inverse Fourier transform (together called the Fourier Transform Pair) is given in Equations A.7 and A.8. Essentially the Fourier transform pair is simply a generalized version of the Fourier series, expressed in complex exponential notation.

$$g(\omega) = F\{f(t)\} = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \quad (\text{A.7})$$

$$f(t) = F^{-1}\{g(\omega)\} = \int_{-\infty}^{\infty} g(\omega) e^{i\omega t} d\omega \quad (\text{A.8})$$

There is a lot of inconsistency as to where the factor of $1/2\pi$ should appear. In the turbulence literature both Pope (2000) and Batchelor (1953) uses the above convention although most references in other fields generally would apply this factor to the inverse Fourier transform. As can be verified from the transform pair (Equations A.7 and A.8), it doesn't really matter as long as it appears in one or the other. Some authors even take the square root of the factor and apply it to both transforms in order to avoid confusion all together.

The Fourier transform above integrates over all possible times and works wonders for a given continuous function. However, in the world of experimentally obtained data one cannot deal with an infinite signal and we therefore need to somewhat limit the wave. This is done by restricting the signal to a period of 2π (or any multiple thereof) which is the natural choice since this will ensure a periodic nature which is necessary in the following derivation. Then we can rewrite Equation A.7 as:

$$g(\omega) = F\{f(t)\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-i\omega t} dt \quad (\text{A.9})$$

A.2 The Discrete Fourier Transform (DFT)

Equation A.9 is still a continuous function, and in order to apply it to experimental data it is necessary to discretize it. This forms the basis of the Discrete Fourier Transform (DFT) which is given in its final form in Equation A.19. The derivation of the DFT from the general Fourier

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transform is given below. First consider the entire integrand of Equation A.9 and define it as $f(t)$. Applying the trapezoidal rule to the full equation with N incremental time segments we get:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt \approx \frac{1}{2\pi} \left(\frac{h}{2} (f(-\pi) + 2f(-\pi + h) + \dots + 2f(-\pi + (N-1)h) + f(-\pi + Nh)) \right) \quad (\text{A.10})$$

where the step size is defined as $h = 2\pi/N$ and originates at $t = -\pi$. Only N discrete datapoints are present, but we can make use of the fact that for a wave with a 2π period, $f(Nh) = f(0)$ by definition. This results in:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt \approx \frac{1}{2\pi} \left(\frac{2\pi/N}{2} (2f(-\pi) + 2f(-\pi + h) + \dots + 2f(-\pi + (N-1)h)) \right) \quad (\text{A.11})$$

which simplifies into:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) dt \approx \frac{1}{N} \sum_{n=0}^{N-1} f(-\pi + nh) = \frac{1}{N} \sum_{n=-N/2}^{N/2-1} f(nh) \quad (\text{A.12})$$

If instead $f(0) = f(Nh)$ was used (which is strictly more proper since we don't have a measurement at time zero, the series would run from $-N/2+1$ to N instead, but when implementing the computer algorithm later on, the current form will prove more beneficial.

When applying the form of Equation A.12 to Equation A.9, we have:

$$g(\omega) = \frac{1}{N} \sum_{n=-N/2}^{N/2-1} f(nh) e^{-i\omega nh} \quad (\text{A.13})$$

However, this implies a continuous range of frequencies which is not ideal for discretized functions. In order to deal with this we discretize the result as well:

$$g(m\Delta\omega) = \frac{1}{N} \sum_{n=-N/2}^{N/2-1} f(nh) e^{-im\Delta\omega nh} \quad (\text{A.14})$$

Now we should recognize that $\Delta\omega = 2\pi/T$, but the period has been forced to be 2π , which simplifies the expression to:

$$g(m) = \frac{1}{N} \sum_{n=-N/2}^{N/2-1} f(nh) e^{-imnh} \quad (\text{A.15})$$

The above function would suffice well as the discrete version of the Fourier transform, or DFT, however it usually appears in a different way. If we change the counting in Equation A.15 to run from 1 to N we will get a more common form. This is done by decomposing Equation A.15 into:

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$$\frac{1}{N} \sum_{n=-N/2}^{N/2-1} f(nh)e^{-imnh} = \frac{1}{N} \sum_{n=-N/2}^{-1} f(nh)e^{-imnh} + \frac{1}{N} \sum_{n=0}^{N/2-1} f(nh)e^{-imnh} \quad (\text{A.16})$$

and then add $N+1$ to the n count in the first term on the RHS and just 1 to n in the second term. In order not to change the value of the expression this means that the same value must be subtracted from the function itself wherever n appears. The stepsize h is also converted to its actual value of $2\pi/N$. We also need to subtract 1 from the count of m as well. This yields:

$$g(m-1) = \frac{1}{N} \sum_{n=-N/2+N+1}^{-1+N+1} f((n-N-1)\frac{2\pi}{N})e^{-i2\pi(m-1)(n-N-1)/N} + \frac{1}{N} \sum_{n=0+1}^{N/2-1+1} f(n-1)\frac{2\pi}{N}e^{-i2\pi(m-1)(n-1)/N} \quad (\text{A.17})$$

Rearranging the limits and recognizing that the N/N term in the exponentials disappears due to the multiplication of a constant (m) and 2π makes the entire term go away, we have:

$$g(m-1) = \frac{1}{N} \sum_{n=N/2+1}^N f(-2\pi + (n-1)\frac{2\pi}{N})e^{-i2\pi(m-1)(n-1)/N} + \frac{1}{N} \sum_{n=+1}^{N/2} f(n-1)\frac{2\pi}{N}e^{-i2\pi(m-1)(n-1)/N} \quad (\text{A.18})$$

The periodic nature of the signal also eliminates the -2π offset in the first sum. If we also define $g_m = g(m-1)$ and $f_n = f((n-1)h)$, then we have:

$$g_m = \frac{1}{N} \sum_{n=1}^N f_n e^{-i2\pi(m-1)(n-1)/N} \quad (\text{A.19})$$

Equation A.19 is the definition of the DFT. Another common form is a sum running from $n = 0$ to $n = N-1$. The conversion from one to the other should be clear at this point.

A.3 The Fast Fourier Transform (FFT)

Calculating the DFT from Equation A.19 is a rather computationally expensive process with complexity N^2 . In the mid-1960's the "Fast Fourier Transform" or FFT was developed which cuts out some of the redundancies associated with calculating the DFT, and the resulting complexity of $N\log_2 N$ vastly reduces computational times, especially for very large values of N . Matlab utilizes the FFT algorithm and defines its DFT similarly to Equation A.19, but without the factor of $1/N$. As long as we are aware of the definition we can utilize Equation A.20 to compute the DFT. It is important however to use the FFT function with the appropriate number of points (N) in order to get the frequency scale correct.

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$$g_m = \frac{1}{N} FFT(f_n, N) \quad (A.20)$$

Equation A.19 or alternatively Equation A.20 returns N complex valued points of what is known as the Fourier Spectrum. It is a two sided spectrum, in which only the first $N/2+1$ points are unique. The power spectrum estimate, G_{ff} (also known as a Periodogram), which is simply g_m multiplied by its complex conjugate is more common to work with. At this point only the first half of the power spectrum, is retained (the identical part is deleted). In order to conserve the total power the remaining points are multiplied by 2 according to Equation A.21.

$$G_{ff} = 2g_m g_m^* \quad (A.21)$$

We also need a meaningful frequency scale m which is given in Equation A.22 in which T is the total sampling period. This can alternatively be expressed in terms of the total number of points, N , over the sampling frequency, f_s . If radial frequency is desired instead, then Equation A.22 is simply multiplied by a factor of 2π .

$$m = \frac{1}{T}(1, 2, \dots, N/2 + 1) = \frac{N}{f_s}(1, 2, \dots, N/2 + 1) \quad (A.22)$$

The first value of m corresponds to a frequency of 0 Hz (due to the $m-1$ factor in g_m) which gives an indication of the mean (or DC) value of the signal according to Equation A.23:

$$DC(f(t)) = \sqrt{G_{ff}(1)/2} \quad (A.23)$$

The variance of the original time signal can also be extracted from the power spectrum by summing up all the power starting from the second bin according to Equation A.24:

$$\text{var}(f(t)) = \sum_{i=2}^{N/2+1} G_{ff}(i) \quad (A.24)$$

In addition to the power spectrum another common representation is the Power Spectral Density (PSD) which is simply the power spectrum multiplied by the sampling frequency over the number of points. If $f(t)$ is a velocity component, the PSD would be known as a velocity spectrum:

$$PSD = G_{ff} \frac{N}{f_s} = 2g_m g_m^* \frac{N}{f_s} = 2g_m g_m^* T \quad (A.25)$$

The difference between Equations A.25 and A.21 is just that the area under the PSD curve will integrate to the variance (i.e. PSD is defined per Hz).

A.4 Example Application

A simple example should serve to illustrate the previous ideas. Equation A.26 describes a waveform as observed on an oscilloscope. The function has frequency content at 5, 15, and 100 Hz and a DC voltage level of 1. By sampling this wave at $f_s = 400\text{Hz}$ and $T = 3\text{ s}$ we get $N = f_s T = 1200$. The time series is shown in Figure A.1 while the associated power spectral density obtained using Equation A.25 is shown in Figure A.2 (the zero-frequency bin is not shown). The energy content is here shown in the frequency domain, which makes it obvious that the majority of energy is contained in a wave with frequency 100 Hz. This would be next to impossible to determine from the time series in Figure A.1.

$$f(t) = \sin(2\pi 5t) + 2\sin(2\pi 15t) + 3\sin(2\pi 100t) + 1 \quad (\text{V}) \quad (\text{A.26})$$

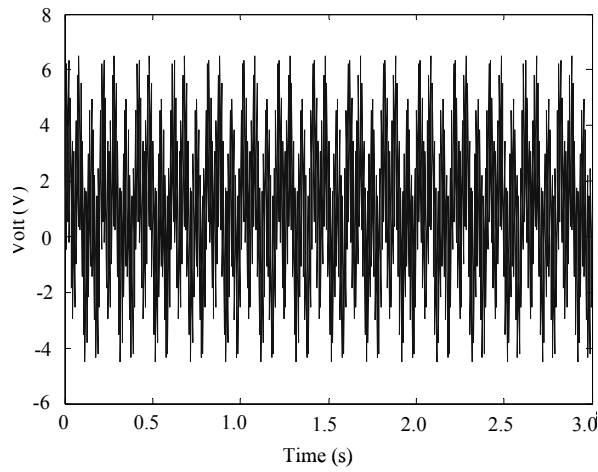


Figure A.1. Timeseries of $f(t)$ in Equation A.26

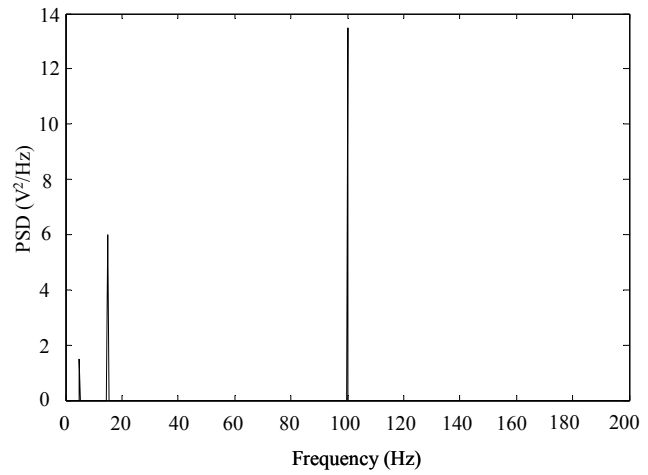


Figure A.2. PSD of timeseries in Equation A.26.

Appendix B

Derivation of the Dissipation Rate through the Decay of TKE Equation

This appendix outlines the steps needed to derive Equation 4.45 starting with Equation 4.44. Equation 4.44 is obtained from the full transport equation for turbulence kinetic energy equation via the Reynolds Averaged Navier Stokes (RANS) equations and the Reynolds stress transport equation. For a detailed description of the steps leading up to Equation 4.44, refer to for example Durbin & Pettersson Reif (2001) p. 47-51. As mentioned earlier the pressure-diffusion is assumed to be small compared to the turbulent transport and the production is non existent due to the lack of a streamwise variation of the mean convective velocity, U_∞ . Equation 4.44, where these simplifications have already been invoked, is reproduced below.

$$\frac{Dk}{Dt} = -\frac{1}{2} \frac{\partial(u'^3 + u'v'^2 + u'w'^2)}{\partial x} - \varepsilon \quad (\text{B.1})$$

The two terms $u'v'^2$ and $u'w'^2$ are negligible compared to the u'^3 and will therefore be ignored.

$$\frac{Dk}{Dt} = -\frac{1}{2} \frac{\partial(u'^3 + u'w'^2)}{\partial x} - \varepsilon \quad (\text{B.2})$$

In order to assess the extra contribution made by the triple-products one can either try to estimate a streamwise decay for u'^3 or take advantage of the skewness of the velocity fluctuations discussed in Section 4.4.1. By recognizing that the contribution of $u'v'^2$ and $u'w'^2$ is negligible compared to that of u'^3 and that $S(u') = \overline{u'^3}/(\overline{u'^2})^{3/2}$, then:

$$\frac{Dk}{Dt} = -\frac{1}{2} \frac{\partial((\overline{u'^2})^{3/2} S(u'))}{\partial x} - \varepsilon \quad (\text{B.3})$$

$$\frac{Dk}{Dt} = -\frac{1}{2} \left((\overline{u'^2})^{3/2} \frac{\partial S(u')}{\partial x} + S(u') \frac{\partial((\overline{u'^2})^{3/2})}{\partial x} \right) - \varepsilon \quad (\text{B.4})$$

The first term of Equation B.4 vanishes as Section 4.4.1 showed that the skewness is independent of downstream distance. The next steps are:

Appendix B: Derivation of the Dissipation Rate through the Decay of TKE Equation

$$\frac{Dk}{Dt} = -\frac{1}{2}S(u')\frac{\partial(u^2u^{1/2})}{\partial x} - \varepsilon \quad (\text{B.5})$$

$$\frac{Dk}{Dt} = -\frac{1}{2}S(u')\left(u^{1/2}\frac{\partial(u^2)}{\partial x} + u^2\frac{\partial(u^{1/2})}{\partial x}\right) - \varepsilon \quad (\text{B.6})$$

The first term can now be rewritten in terms of turbulence kinetic energy by assuming isotropy which has been shown to be a valid assumption with the flows generated by the active grid. In addition the second term can be expressed in terms of turbulence intensity. This results in Equation 4.45 which is repeated below.

$$\frac{Dk}{Dt} = -\frac{1}{2}S(u')\left(u^{1/2}\frac{2}{3}\frac{\partial k}{\partial x} + u^2\sqrt{U_\infty}\frac{\partial(u/U_\infty)^{1/2}}{\partial x}\right) - \varepsilon \quad (\text{B.7})$$

The left hand side of this equation can be modified through the following relationship by recognizing that k is a function of space, but not time.

$$\frac{dk}{dx} = \frac{Dk}{Dt}/U_\infty \quad (\text{B.8})$$

Now the left hand side of Equation B.7 can be obtained experimentally (see Section 4.5) and yields the following for the baseline flow condition:

$$\frac{Dk}{Dt} = U_\infty \frac{dk}{dx} = \frac{3}{2}U_\infty^3 \frac{d(u^2/U_\infty^2)}{dx} = \frac{3}{2}U_\infty^3 \frac{d}{dx}\left(0.60026\left(\frac{X}{M} + 6.6679\right)^{-1.285}\right) \quad (\text{B.9})$$

The right hand side of Equation B.7 is similarly obtained by observing that the partial derivatives can be replaced using simple derivatives, and that:

$$\frac{d(u/U_\infty)^{1/2}}{dx} = \frac{d}{dx}\left(0.60026^{1/4}\left(\frac{X}{M} + 6.6679\right)^{-1.285/4}\right) \quad (\text{B.10})$$

Applying the baseline flow variables as follows for $X/M = 37.3$ and $U_\infty = 12.42$ m/s, then $Dk/Dt = -1.857 \text{ m}^2/\text{s}^3$ which makes $dk/dx = -0.15$ and $d(u/U_\infty)^{1/2}/dx = -0.0091$. The value of u at this point is 0.85 m/s while $S(u')$ is 0.12. Substituting all this into Equation B.7 gives:

$$\varepsilon = 1.857 - \frac{0.12}{2}\left(\sqrt{0.85}\frac{2}{3}(-0.15) + 0.85^2\sqrt{12.42}(-0.0091)\right) = 1.864 \text{ m}^2/\text{s}^3 \quad (\text{B.11})$$

References

- ARONSON, D., JOHANSSON, A.V. & LÖFDAHL, L. 1997 Shear-free turbulence near a wall. *J. Fluid Mech.* **338**, 363-385.
- ATASSI, H.M. & GRZEDZINSKI, J. 1989 Unsteady disturbance of streaming motions around bodies. *J. Fluid Mech.* **209**, 385-403.
- AZAD, R.S. & KASSAB, S.Z. 1989 New method of obtaining dissipation. *Exp. in Fluids*, **7**, 81-87.
- BASUKI, J. 1983 *Unsteady flow over aerofoils and cascades*. Ph.D. thesis, London University.
- BATCHELOR, G.K. 1953 *The Theory of Homogeneous Turbulence*. Cambridge University Press, Cambridge.
- BATCHELOR, G.K. & PROUDMAN, I. 1954 The effect of rapid distortion on a fluid in turbulent motion. *Quart. J. Mech. Appl. Math.* **7** 1, 83-103.
- BEARMAN, P.W. 1971 Corrections for the effect of ambient temperature drift on hot-wire measurements in incompressible flow. *DISA Information* **11**, 25-30.
- CHOI, K. & SIMPSON, R.L. 1987 *Some mean velocity, turbulence, and unsteadiness characteristics of the VPI & SU Stability Wind Tunnel*. AOE Dept., Virginia Tech.
- COMTE-BELLOT, G. & CORRSIN, S. 1966 The use of a contraction to improve the isotropy of grid generated turbulence. *J. Fluid Mech.* **25**, 657-682.
- COMTE-BELLOT, G. & CORRSIN, S. 1971 Simple Eulerian time correlation of full- and narrow-band velocity signals in grid-generated "isotropic" turbulence. *J. Fluid Mech.* **48**, 273-337.
- DE LA RIVA, D.H. 2001 *Turbulence interaction in a highly staggered cascade-propulsor configuration*. M.S. thesis, Virginia Tech, Virginia.
- DE LA RIVA, D.H., DEVENPORT, W.J., MUTHANNA, C. & GLEGG, S.A.L. 2004 Behavior of turbulence flowing through a compressor cascade. *AIAA J.* **42** 7, 1302-1313.
- GAD-EL-HAK, M. & CORRSIN, S. 1974 Measurements of the nearly isotropic turbulence behind a uniform jet grid. *J. Fluid Mech.* **62**, 115-143.
- GEORGE, W.K. 1988 The decay of homogeneous isotropic turbulence. In *Transport Phenomena in Turbulent Flows* (eds. M. Hirata & N. Kasagi), pp. 3-16. Hemisphere Pub. Corp., NY.
- GLEGG, S.A.L. 1999 The response of a swept blade row to a three-dimensional gust. *J. Sound Vib.* **227** 1, 29-64.
- GOLDSTEIN, M.E. 1978 Unsteady vortical and entropic distortions of potential flows round arbitrary obstacles. *J. Fluid Mech.* **89**, 433-468.
- GOLDSTEIN, M.E. 1979 Turbulence generated by the interaction of entropy fluctuations with non-uniform mean flows. *J. Fluid Mech.* **93**, 209-224.
- GOLDSTEIN, M.E. & DURBIN, P.A. 1980 The effect of finite turbulence spatial scale on the amplification of turbulence by a contracting stream. *J. Fluid Mech.* **98**, 473-508.
- GRAHAM, J.M.R. 1998 The effect of a two-dimensional cascade of thin streamwise plates on homogeneous turbulence. *J. Fluid Mech.* **356**, 125-147.

References

- GRISSOM, D. L. & DEVENPORT, W.J. 2004 Development and testing of a deterministic disturbance generator. 10th AIAA-CEAS Aeroacoustics conference, Manchester, UK.
- HAIDOS, C. 1983 *Study of turbulence interacting with a cascade*. M.Sc. thesis, Imperial College, University of London.
- HELLAND, K.N., VAN ATTA, C.W. & STEGEN, G.R. 1977 Spectral energy transfer in high Reynolds number turbulence. *J. Fluid Mech.* **79**, 337-359.
- HINZE, J.O. 1975 *Turbulence (2nd edition)*. McGraw-Hill, NY.
- HUNT, J. C. R. 1973 A theory of turbulent flow round two-dimensional bluff bodies. *J. Fluid Mech.* **61**, 625-706.
- HUNT, J. C. R. & GRAHAM, J. M. R. 1978 Free-stream turbulence near plane boundaries. *J. Fluid Mech.* **84**, 209-235.
- KANG, H.S., CHESTER, S. & MENEVEAU, C. 2003 Decaying turbulence in an active-grid-generated flow and comparisons with large-eddy simulation. *J. Fluid Mech.* **480**, 129-160.
- KEVLAHAN, N.K.-R. & HUNT, J.C.R. 1997 Nonlinear interactions in strained turbulence. *J. Fluid Mech.* **337**, 333-364.
- KISTLER, A.L. & VREBALOVICH, T. 1966 Grid turbulence at large Reynolds numbers. *J. Fluid Mech.* **26**, 37-47.
- KULLAR, I. & GRAHAM, J.M.R. 1986 Acoustic effects due to turbulence passing through cascades of thin airfoils. *J. Sound Vib.* **110** 1, 143-160.
- LARSEN, S.E. & BUSCH, N.E. 1974 Hot-wire measurements in the atmosphere. *DISA Information* **16**, 15-34.
- LIBBY, P.A. 1996 *Introduction to Turbulence*. Taylor & Francis, Washington DC.
- LING, S.C. & WAN, C.A. 1972, Decay of isotropic turbulence generated by a mechanically agitated grid. *Phys. Fluids* **15** 8, 1363-1369.
- LUMLEY, J.L. & PANOFKY, H.A. 1964 *The Structure of Atmospheric Turbulence*. Interscience, NY.
- MA, R. 2003 *Unsteady turbulence interaction in a tip leakage flow downstream of a simulated axial compressor rotor*. Ph.D. dissertaion, Virginia Tech, Virginia.
- MAJUMDAR, S.J. & PEAKE, N. 1998 Noise generation by the interaction between ingested turbulence and a rotating fan. *J. Fluid Mech.* **359**, 181-216.
- MAKITA, H. 1991 Realization of a large-scale turbulence field in a small wind tunnel. *Fluid Dynamics Research* **8**, 53-64.
- MAKITA, H. & MIYAMOTO, S. 1983 Generation of high intensity turbulence and control of its structure in a low speed wind tunnel. In *Proc. 2nd Asian Congress on Fluid Mechanics*, pp. 101-106.
- MAKITA, H. & SASSA, K. 1991 Active turbulence generation in a laboratory wind tunnel. In *Advances in Turbulence 3* (eds. A.V. Johansson & P.H. Alfredsson), pp. 497-505. Springer-Verlag, Berlin.
- MAXEY, M. R. 1987 The velocity skewness measured in grid turbulence. *Phys. Fluids* **30** 4, 935-938.
- MISH, P. F. 2001 Mean Loading and Turbulence Scale Effects on the Surface Pressure Fluctuations Occurring on a NACA 0015 Airfoil Immersed in Grid Generated Turbulence. M.S. Thesis, Virginia Tech, Virginia.
- MOHAMED, M. S. & LARUE, C. 1990 The decay power law in grid-generated turbulence. *J. Fluid Mech.* **219**, 195-214.

References

- MYDLARSKI, L. & WARHAFT, Z. 1996 On the onset of high-Reynolds-number grid-generated wind tunnel turbulence. *J. Fluid Mech.* **320**, 331-368.
- MYDLARSKI, L. & WARHAFT, Z. 1998 Passive scalar statistics in high-Péclet-number grid turbulence. *J. Fluid Mech.* **358**, 135-175.
- PEAKE, N. & KERSCHEN, E.J. 1997 Influence of mean loading on noise generated by the interaction of gusts with a flat-plate cascade: upstream radiation. *J. Fluid Mech.* **347**, 315-346.
- PEROT, B. & MOIN, P. 1995 Shear-free turbulent boundary layers. Part 1. Physical insights into near-wall turbulence. *J. Fluid Mech.* **295**, 199-227.
- POORTE, R.E.G. 1998 *On the motion of bubbles in active grid generated turbulent flows*. Ph.D. thesis, University of Twente, Holland.
- POORTE, R.E.G. & BIESHEUVEL, A. 2002 Experiments on the motion of gas bubbles in turbulence generated by an active grid. *J. Fluid Mech.* **461**, 127-154.
- POPE, S.B. 2000 *Turbulent Flows*. Cambridge University Press, Cambridge.
- PRANDTL, L. 1933 Attaining a steady air stream in wind tunnels. *NACA TM-726*.
- PRASKOVSKY, A. & ONCLEY, S. 1994 Measurements of the Kolmogorov constant and intermittency exponent at very high Reynolds numbers. *Phys. Fluids* **6** 9, 2886-2888.
- RIBNER, H.S. & TUCKER, M. 1953 Spectrum of turbulence in a contracting stream. *NACA-1113*.
- ROACH, P.E. 1986 The generation of nearly isotropic turbulence downstream of streamwise tube bundles. *Int. J. Heat Fluid Flow* **7** 2, 117-125.
- ROACH, P.E. 1987 The Generation of nearly isotropic turbulence by means of grids. *Int. J. Heat Fluid Flow* **8** 2, 82-92.
- SADDOUGH, S.G. & VEERAVALLI, S.V. 1994 Local isotropy in turbulent boundary layers at high Reynolds number. *J. Fluid Mech.* **268**, 333-372.
- SCHETZ, J.S. 1993 *Boundary Layer Analysis*. Prentice Hall, NJ.
- SHAVIT, U. & CHIGIER, N. 1995 Development and evaluation of a new turbulence generator for atomization research. *Exp. in Fluids* **20**, 291-301.
- SIMMONS, L.F.G. & SALTER, C. 1934 Experimental investigation and analysis of the velocity variations in turbulent flow. *Proc. R. Soc. London A* **145**, 212-234.
- SIMMONS, L.F.G., SALTER, C. & TAYLOR, G.I. 1938 An experimental determination of the spectrum of turbulence. *Proc. R. Soc. London A* **165**, 73-89.
- SRDIC, A., FERNANDO, H.J.S. & MONTENEGRO, L. 1995 Generation of nearly isotropic turbulence using two oscillating grids. *Exp. in Fluids* **20**, 395-397.
- SREENIVASAN, K.R. 1995 On the universality of the Kolmogorov constant. *Phys. Fluids* **7** 11, 2778-2784.
- TASSA, Y. & KAMOTANI, Y. 1974 Experiments on turbulence behind a grid with jet injection in downstream and upstream direction. *Phys. Fluids* **18** 4, 411-414.
- TAYLOR, G.I. 1935a Statistical theory of turbulence, parts I-IV. *Proc. R. Soc. London A* **151**, 421-478.
- TAYLOR, G.I. 1935b Turbulence in a contracting stream. *Z. Angew. Math. Mech.* **15**, 91-96.
- THOLE, K.A., BOGARD, D.G. & WHAN-TONG, J.L. 1994 Generating high freestream turbulence levels. *Exp. in Fluids* **17**, 375-380.
- THOMAS, N.H. & HANCOCK, P.E. 1977 Grid turbulence near a moving wall. *J. Fluid Mech.* **82**, 481-496.

References

- THOMPSON, S.M. & TURNER, J.S. 1975 Mixing across an interface due to turbulence generated by an oscillating grid. *J. Fluid Mech.* **67**, 349-368.
- THORODDSEN, S.T. 1995 Reevaluation of the experimental support for the Kolmogorov refined similarity hypothesis, *Phys. Fluids*, **7** 4, 691-693.
- UBEROI, M.S. 1956 Effect of wind-tunnel contraction on free-stream turbulence. *J. Aero. Sci.* **23**, 754-764.
- UBEROI, M.S. & WALLIS, S., 1967 Effects of grid geometry on turbulence decay. *Phys. Fluids*, **10** 6, 1216-1224.
- UZKAN, T. & REYNOLDS, W.C. 1967 A shear-free turbulent boundary layer. *J. Fluid Mech.* **28**, 803-821.
- VILLERMAUX, E., SOMMERIA, J., GAGNE, Y. & HOPFINGER, E.J. 1991 Oscillatory instability and genesis of turbulence behind a high solidity grid. *Eur. J. Mech. B/Fluids* **10** 4, 427-439.
- VAN ATTA, C.W. & ANTONIA, R.A. 1980 Reynolds number dependence of skewness and flatness factors of turbulent velocity derivatives. *Phys. Fluids* **23** 2, 252-257.
- VON KÁRMÁN, T. & HOWARTH, L. 1938 On the statistical theory of isotropic turbulence. *Proc. R. Soc. London A*, **164**, pp. 192-215.
- WITTMER, K.S., DEVENPORT, W.J. & ZSOLDOS, J.S. 1998 A four-sensor hot-wire probe system for three-component velocity measurement. *Exp. in Fluids* **24**, 416-423. See also: 1999, *Exp. in Fluids*, **27**, U1.
- WYNGAARD, J.C. & TENNEKES, H. 1970 Measurements of the small-scale structure of turbulence at moderate Reynolds numbers. *Phys. Fluids* **13** 8, 1962-1969.