Phenomenological Consequences of Heavy Right Handed Neutrinos

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(ABSTRACT)

The discovery of neutrino mixing provides the possibility of a non vanishing CP violating phase in the neutrino mixing matrix. CP violation in the leptonic sector can be large enough to explain the matter-antimatter asymmetry in the universe. An indirect probe of CP violation is the experimental measurement of Electric Dipole Moment (EDM). CP violation has been discovered in the quark sector,but it contributes to lepton EDM at the 3-loop level.

Neutrino masses can be generated in the standard model via the see-saw mechanism where heavy right-handed neutrinos mix with the weak-basis states. The Majorana nature of the seesaw type neutrinos generates new 2-loop diagrams that lead to a non-vanishing lepton EDM. Only estimates of the resulting EDM have been done in the literature. A full calculation of the 2-loop diagrams and the exact result is presented.

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Dedication

To my father Ramadan, and my mother Serriyyeh

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Introduction

CP violation was discovered in 1964 in kaon decays by Cronin and Fitch [1]. It was a big surprise; although weak interactions violate both C and P, CP was long believed to be conserved. But then it has been noted by Kobayashi and Maskawa that CP violation can be understood in the quark sector if the quark mixing matrix (CKM matrix) contains a complex phase [4]. Since then several experiments discovered more evidence of CP violation in K and B meson decays [23].

The importance of CP violation arises in understanding the Matter-Antimatter asymmetry in the universe. Sakharov indicated that such asymmetry requires baryon number violating interactions, C and CP violation, and a departure from thermal equilibrium [2].However, the CP violating phase in the CKM matrix is not large enough to account for the asymmetry. For a good review on the subject, see Ref [3].

The discovery of neutrino oscillations indicates that a similar mixing matrix exists for neutrinos (the MNS matrix). A complex phase in the MNS matrix violates CP, just as in the CKM matrix. Fukugita and Yanagida suggested that baryon asymmetry in the universe can be explained by first generating lepton asymmetry . The lepton asymmetry then can be converted into baryon asymmetry at the weak scale [6].

Another signature of CP violation would be a non-zero electric dipole moment (EDM). Several experiments are looking for Neutron and Lepton EDM, but non have been observed yet. The contribution to lepton EDM from quarks appears at the 3-loop level so it is heavily suppressed. An estimate of the electron EDM from quark loops [12] predicts the EDM to be of order $10^{-38} e \cdot cm$. The latest experimental upper limit on the electron EDM is $(0.069 \pm 0.074) \times 10^{-26} e \cdot cm$ [23], which puts the theoretical prediction way beyond the reach of experiment. Even with a CP violating phase in the MNS matrix, using the analysis of Shabalin [7], it can be shown that the contribution from Dirac neutrinos to lepton EDM vanishes at the two loop level. At the three loop level the contribution (if not zero) would be comparable to the that from the quark loops.

It was noted by Ng and Ng that if Neutrinos are Majorana, new two loops diagrams contribute to the lepton EDM [15]. Only estimates and approximate calculations of the new diagrams have been performed in the literature [15],[18],[19]. In the popular seesaw mechanism [13] ; heavy right handed Majorana neutrinos mix with the weak basis states, this results in the tiny masses of the observable neutrinos and all the neutrinos are Majorana in nature. A naive seesaw type mass matrix results in GUT scale heavy right handed neutrinos and very small mixing with the left handed neutrinos. The small mixing leads to the suppression of the contribution of the 2-loop diagram, and estimates puts the resulting EDM below that from the quark loops [18].

However, it was suggested by Chang , Ng, and Ng [16] that the mixing can be enhanced when there is more than one generation of neutrinos. Various interesting phenomenological consequences can result from this enhancement. A seesaw type texture was suggested by Takeuchi et al. [11] where three TeV right handed Majorana neutrinos are added. In this texture, the masses and the mixings between the right- and the left-handed neutrinos are made independent. A similar model was suggested by Glashow [22]. The large mixing and the TeV masses of the heavy Majorana's could lead to a huge amplification to the Majorana 2-loop diagrams leading to lepton EDM.

In order to calculate the EDM resulting from the new diagrams, a complete calculation is needed. The goal of this thesis is to provide the full calculation of the diagrams in terms of general Yukawa couplings and mixing's so an estimate can be provided for a variety of TeV Majorana neutrino models.

In Chapter 1, I will discuss the difference between Dirac and Majorana mass terms and introduce the Charge and Parity symmetries. Next, I will discuss how different mass terms can be incorporated in the standard model via the See-Saw Mechanism. In Chapter 2, I will show how CP violation leads to a non-zero EDM. Chapter 3 introduces the diagrams that contribute to the charged lepton EDM. A detailed calculation is carried out to determine the contribution of the diagrams in terms of general choice of the mass textures. Chapter 4 presents the exact result.

Chapter 1

Massive Neutrinos in the Standard Model

1.1 Parity and Charge conjugation transformations

Parity transformation is a Unitary transformation that reverses the spatial dimensions. The action of the Parity operator on a particle results in reversing the direction of momentum of the particle but not its spin.

$$P\psi(t,\vec{x})P^{-1} = \eta_P \gamma_0 \psi(t,-\vec{x}) \tag{1.1}$$

where $\eta_p = \pm 1$.

Charge conjugation is a Unitary operator that changes a fermion into an antifermion [8].

$$C\psi C^{-1} = \eta_C C\bar{\psi}^T \tag{1.2}$$

where η_C is a phase . In the chiral representation C can be written as:

$$C = \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix} \tag{1.3}$$

1.2 Dirac and Majorana Mass terms

Let's take a Dirac spinor and write is as two Weyl spinors:

$$\psi = \begin{pmatrix} \xi \\ \zeta \end{pmatrix} \tag{1.4}$$

ξ transforms under the $(\frac{1}{2}, 0)$ representation of the Lorentz group while ζ transforms under the $(0, \frac{1}{2})$ representation. It can be shown that $i\sigma_2\xi^*$ transforms under the $(0, \frac{1}{2})$ representation. so we can rewrite ψ:

$$\psi = \begin{pmatrix} \xi_1 \\ i\sigma_2\xi_2^* \end{pmatrix} \tag{1.5}$$

Where both ξ_1 and ξ_2 transform under the $(\frac{1}{2}, 0)$ representation.

Now we can write mass terms in the Lagrangian that are invariant under Lorentz transformations. First we write what is called the Dirac mass term.

$$\begin{aligned}\mathcal{L} = -m\overline{\psi}\psi &= -m\begin{pmatrix} \xi_1^\dagger & -\xi_2^T i\sigma_2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \xi_1 \\ i\sigma_2\xi_2^* \end{pmatrix} \\ &= -m\xi_1^\dagger i\sigma_2\xi_2^* + m\xi_2^T i\sigma_2\xi_1 \end{aligned} \tag{1.6}$$

In terms of "left" and "right" handed fields, we can define:

$$\psi = \psi_L + \psi_R \tag{1.7}$$

Where

$$\begin{aligned}\psi_L &= P_L\psi = \frac{1}{2}(1-\gamma_5)\psi = \begin{pmatrix} \xi_1 \\ 0 \end{pmatrix}, \\ \psi_R &= P_R\psi = \frac{1}{2}(1+\gamma_5)\psi = \begin{pmatrix} 0 \\ i\sigma_2\xi_2^* \end{pmatrix}\end{aligned} \tag{1.8}$$

Notice that (ignoring the phase):

$$\begin{aligned}\psi^C &= C\overline{\psi}^T \\ &= C(\psi^\dagger\gamma_0)^T \\ &= C\gamma_0\psi^* \\ &= \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \xi_1^* \\ i\sigma_2\xi_2 \end{pmatrix} \\ &= \begin{pmatrix} \xi_2 \\ i\sigma_2\xi_1^* \end{pmatrix},\end{aligned} \tag{1.9}$$

and:

$$\begin{aligned}(\psi_L)^C &= \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} \xi_1^* \\ 0 \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ i\sigma_2\xi_1^* \end{pmatrix} = (\psi^C)_R.\end{aligned} \tag{1.10}$$

Similarly,

$$(\psi_R)^C = \begin{pmatrix} \xi_2 \\ 0 \end{pmatrix} = (\psi^C)_L. \tag{1.11}$$

In the same way we can show that:

$$\begin{aligned}\overline{\psi} &= \begin{pmatrix} \xi_2^T i\sigma_2 & \xi_1^\dagger \end{pmatrix} \\ \overline{\psi}^C &= \begin{pmatrix} -\xi_1^T i\sigma_2 \\ -\xi_2^\dagger \end{pmatrix} \\ (\overline{\psi_L})^C &= \begin{pmatrix} 0 \\ -\xi_2^\dagger \end{pmatrix} = (\overline{\psi^C})_R \\ (\overline{\psi_R})^C &= \begin{pmatrix} -\xi_1^T i\sigma_2 \\ 0 \end{pmatrix} = (\overline{\psi^C})_L\end{aligned} \tag{1.12}$$

A Majorana spinor can be written in terms of a Dirac spinor as:

$$\psi_M = \begin{pmatrix} \xi \\ i\sigma_2\xi^* \end{pmatrix}, \tag{1.13}$$

where the two Weyl components are constructed from the same spinor. Notice that:

$$\begin{aligned}
\psi_M^C &= C\overline{\psi}_M^T \\
&= C(\psi_M^\dagger \gamma_0)^T \\
&= C\gamma_0 \psi_M^* \\
&= \begin{pmatrix} -i\sigma_2 & 0 \\ 0 & i\sigma_2 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \xi^* \\ i\sigma_2 \xi \end{pmatrix} \\
&= \begin{pmatrix} \xi \\ i\sigma_2 \xi^* \end{pmatrix} = \psi_M \qquad (1.14)
\end{aligned}$$

So a Majorana fermion is its own antiparticle. Now we can write Majorana mass terms:

$$\begin{aligned}
\mathcal{L} &= -m\overline{\psi}_M \psi_M \\
&= m(\xi^T i\sigma_2 \xi - \xi^\dagger i\sigma_2 \xi^*) \qquad (1.15)
\end{aligned}$$

Again the Lagrangian is invariant under Lorentz transformations. However, it is not invariant under $U(1) : \xi \to e^{i\phi}\xi$ [9]. As a consequence; Majorana mass terms cannot exist for charged leptons since they violate charge conservation. Since neutrinos are neutral; the Lagrangian can include Majorana mass terms for the neutrinos.

1.3 The Electroweak Lagrangian

The covariant derivative for the electroweak sector is:

$$\mathcal{D} = \partial + ieAQ - i\frac{g}{\sqrt{2}}(W^+T_+ + W^-T_-) - i\frac{g}{\cos\theta_w} Z(T_3 - Q\sin^2\theta_w) \qquad (1.16)$$

We consider the Standard Model with the regular Higgs doublet ($Y = \frac{1}{2}$):

$$H = \begin{bmatrix} \phi^+ \\ \phi^0 \end{bmatrix} \qquad (1.17)$$

with the conjugate($Y = \frac{-1}{2}$):

$$\tilde{H} = \begin{bmatrix} \phi^{0\dagger} \\ -\phi^- \end{bmatrix} \qquad (1.18)$$

The kinetic part of the Lagrangian of the Higgses result in the masses of the W and the Z after the Higgs develops a vacuum expectation value (VEV).

The charged and neutral currents can be derived from the Kinetic Lagrangian of the Lepton doublet (Left Handed):

$$L = \begin{bmatrix} \nu \\ l \end{bmatrix} \qquad (1.19)$$

$$\overline{L}\gamma^\mu D_\mu L \to \mathcal{L}_W = \frac{g}{\sqrt{2}}(W_\mu^+ \bar{\nu}\gamma^\mu l + W_\mu^- \bar{l}\gamma^\mu \nu) \qquad (1.20)$$

The mass terms for leptons come from the Yukawa interactions with the Higgs, the invariant terms under $SU(2) \otimes U(1)$:

$$\mathcal{L}_Y = -\lambda\overline{L}Hr - \lambda^*\overline{r}H^\dagger L, \qquad (1.21)$$

where r is the right-handed charged lepton singlet, λ is the Yukawa coupling matrix, which in general can be complex.

$$\begin{aligned}
\mathcal{L}_Y &= -\lambda \left[\begin{array}{cc} \overline{\nu} & \bar{l} \end{array} \right] \left[\begin{array}{c} \phi^+ \\ \phi^0 \end{array} \right] r - \lambda^* \overline{r} \left[\begin{array}{cc} \phi^- & \phi^{0*} \end{array} \right] \left[\begin{array}{c} \nu \\ l \end{array} \right] \\
\mathcal{L}_Y &= -\lambda \bar{l} \phi^0 r - \lambda^* \overline{r} \phi^{0*} l \\
&\quad -\lambda \overline{\nu} \phi^+ r - \lambda^\dagger \overline{r} \phi^- \nu
\end{aligned} \tag{1.22}$$

The first line will generate the lepton masses after the Higgs develops a VEV , the second line describes the Yukawa interactions. Neutrinos are massless in this description.

Generalization to 3 families is straightforward, assuming that λ is a general complex matrix.

$$\begin{aligned}
\mathcal{L}_Y &= -\lambda_{ij} \frac{v}{\sqrt{2}} \bar{l}_i r_j - \lambda^\dagger{}_{ij} \frac{v}{\sqrt{2}} \overline{r}_i l_j \\
&\quad -\lambda_{ij} \overline{\nu}_i \phi^+ r_j - \lambda^\dagger{}_{ij} \overline{r}_i \phi^- \nu_j \\
&\quad +\text{Higgs interactions terms}
\end{aligned} \tag{1.23}$$

1.4 Neutrino Mass Terms and the See-Saw Mechanism

There are several mechanisms where neutrino masses are introduced. In the see-saw mechanism, Heavy masses for Majorana right-handed neutrino (RHN) singlets are generated at a higher mass scale. The mixing between the Heavy states and the left-handed neutrinos results in the small mass of the light neutrinos.

The Lagrangian now can include more terms that are invariant under $SU(2) \otimes U(1)$:

$$\begin{aligned}
&-\Lambda_{ij} \overline{L}_i \tilde{H} \chi_j - \Lambda^\dagger_{ij} \overline{\chi}_i \tilde{H}^\dagger L \\
&-\frac{1}{2} \overline{\chi}_i M_{ij} \chi^c_j - \frac{1}{2} \chi^c_i M^T_{ij} \overline{\chi}_j
\end{aligned} \tag{1.24}$$

Λ_{ij} is the Yukawa coupling matrix with the new singlets. In several GUT models (e.g. Pati-Salam [17]) the Yukawas of the right handed neutrinos are the same as the up type quarks (i.e. electroweak scale). M_{ij} is the Majorana mass matrix of the RHN that are generated at a higher scale (in Left-Right symmetric Models that is the scale at which W_R acquires its mass). χ_i are the RHN singlets.

The first line reads:

$$\begin{aligned}
&= -\Lambda_{ij} \left[\begin{array}{cc} \overline{\nu}_i & \bar{l}_i \end{array} \right] \left[\begin{array}{c} \phi^{0*} \\ -\phi^- \end{array} \right] \chi_j - \Lambda^\dagger_{ij} \overline{\chi}_i \left[\begin{array}{cc} -\phi^+ & \phi^0 \end{array} \right] \left[\begin{array}{c} \nu_j \\ l_j \end{array} \right] \\
&= -\Lambda_{ij} \overline{\nu}_i \phi^{0*} \chi_j - \Lambda^\dagger_{ij} \overline{\chi}_i \phi^0 \nu_j \\
&\quad +\Lambda_{ij} \bar{l}_i \phi^- \chi_j + \Lambda^\dagger_{ij} \overline{\chi}_i \phi^+ l_j
\end{aligned} \tag{1.25}$$

The vacuum expectation value of the Higgs gives the Dirac mass terms of the neutrinos.

$$\begin{aligned}
&= -\Lambda_{ij} \frac{v}{\sqrt{2}} \overline{\nu}_i \chi_j - \Lambda^\dagger_{ij} \frac{v}{\sqrt{2}} \overline{\chi}_i \nu_j \\
&\quad +\Lambda_{ij} \bar{l}_i \phi^- \chi_j + \Lambda^\dagger_{ij} \overline{\chi}_i \phi^+ l_j \\
&\quad + \text{Higgs interaction terms..}
\end{aligned} \tag{1.26}$$

Call:

$$D_{ij} = \frac{v}{\sqrt{2}} \Lambda^\dagger_{ij}, \tag{1.27}$$

now the neutrino mass terms look like:

$$\begin{aligned}
&-\overline{\chi_i}D_{ij}\nu_j + \frac{1}{2}\overline{\chi_i}M_{ij}\chi_j^c + h.c. \\
&= -\frac{1}{2}\left(\overline{\chi_i}D_{ij}\nu_j + \overline{\nu_i^c}D_{ij}^T\chi_j^c + \overline{\chi_i}M_{ij}\chi_j^c\right) + h.c. \\
&= -\frac{1}{2}\left[\begin{array}{cc} \overline{\nu_i^c} & \overline{\chi_j} \end{array}\right]\left[\begin{array}{cc} 0_{ik} & D_{il}^T \\ D_{jk} & M_{jl} \end{array}\right]\left[\begin{array}{c} \nu_k \\ \chi_l^c \end{array}\right] + h.c. \qquad (1.28)
\end{aligned}$$

Upon diagonalizing the mass matrix the mass states will split into three light and three heavy (Majorana) states, hence the name seesaw . The mixing depends on the texture of the mass matrix as explained in the next section.

1.5 See-Saw Mass Textures

1.5.1 One Genereation Case

For one generation, the see-saw mass matrix can only be

$$\left[\begin{array}{cc} \overline{\nu^c} & \overline{\chi} \end{array}\right]\left[\begin{array}{cc} 0 & m \\ m & M \end{array}\right]\left[\begin{array}{c} \nu \\ \chi^c \end{array}\right] ., \qquad (1.29)$$

with only two free parameters: m and M. we can diagonalize the mass matrix using the unitary matrix U:

$$U = \left[\begin{array}{cc} i\cos(\theta) & \sin(\theta) \\ -i\sin(\theta) & \cos(\theta) \end{array}\right] \qquad (1.30)$$

where

$$\tan(2\theta) = \frac{2m}{M} \qquad (1.31)$$

This which yields

$$U^{\mathrm{T}}\left[\begin{array}{cc} 0 & m \\ m & M \end{array}\right]U = \left[\begin{array}{cc} m\,\tan(\theta) & 0 \\ 0 & m/\tan(\theta) \end{array}\right] . \qquad (1.32)$$

when $m \ll M$ the two mass eigenvalues are

$$\begin{aligned}
m_{\text{light}} &\equiv m\,\tan(\theta) \approx \frac{m^2}{M} , \\
m_{\text{heavy}} &\equiv \frac{m}{\tan(\theta)} \approx M . \qquad (1.33)
\end{aligned}$$

The mass eigenstates and the original generation eigenstates are related through

$$\left[\begin{array}{c} \nu \\ \chi \end{array}\right] = U\left[\begin{array}{c} n \\ N \end{array}\right] = \left[\begin{array}{c} i\cos(\theta)\,n + \sin(\theta)\,N \\ -i\sin(\theta)\,n + \cos(\theta)\,N \end{array}\right] , \qquad (1.34)$$

where the light and heavy states are denoted n and N, respectively. Observe that

$$\theta^2 \approx \frac{m^2}{M^2} \approx \frac{m_{\text{light}}}{m_{\text{heavy}}} , \qquad (1.35)$$

Since we started with only two free parameters m, M in the original mass matrix , the two mass eigenvalues $m_{\text{light}} \sim m^2/M$, $m_{\text{heavy}} \sim M$, and the mixing angle $\theta \sim m/M$ are necessarily related. If we wanted the mass of the light state to be $\sim \mathcal{O}(eV)$ and if we assume that $m \approx 100\,GeV$ this will lead to

$$M \approx 10^{13}\,GeV$$

$$\theta^2 \approx 10^{-13} \tag{1.36}$$

This mass scale is way beyond the current limits of experiment. And the phenomenological consequenses will be suppressed by a factor proportional to θ^2.

1.5.2 Two Generations Case

In a two generation sea-saw mass texture there are more free parameters in the mass matrix. In some specific choices of the mass matrix the mass values and the mixing can be made independent. The most general seesaw mass matrix for two generations is

$$\begin{bmatrix} \mathbf{0}_{2\times2} & \mathbf{D}^T_{2\times2} \\ \mathbf{D}_{2\times2} & \mathbf{M}_{2\times2} \end{bmatrix}, \tag{1.37}$$

Consider a specific mass texture:

$$[\, \nu_1\ \nu_2\ \chi_1\ \chi_2 \,] \begin{bmatrix} 0 & 0 & m & -m \\ 0 & 0 & m & -m \\ m & m & M & 0 \\ -m & -m & 0 & -M \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \chi_1 \\ \chi_2 \end{bmatrix}. \tag{1.38}$$

This mass matrix is manifestly rank two, so two of the mass eigenvalues will be zero regardless the values of m and M. This matrix can be diagonalized as

$$O^{\mathrm{T}} \begin{bmatrix} 0 & 0 & m & -m \\ 0 & 0 & m & -m \\ m & m & M & 0 \\ -m & -m & 0 & -M \end{bmatrix} O = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & m/\sin(\theta)\cos(\theta) & 0 \\ 0 & 0 & 0 & -m/\sin(\theta)\cos(\theta) \end{bmatrix}, \tag{1.39}$$

$$\tan 2\theta = \frac{2m}{M}. \tag{1.40}$$

In this prescription the masses and the mixing angle are

$$\begin{aligned} m_{\text{light}} &= 0 \\ m_{\text{heavy}} &= \pm\sqrt{M^2+4m^2} \approx \pm M. \\ \theta^2 &= \frac{m^2}{M^2} \end{aligned} \tag{1.41}$$

Adding small perturbations to the mass matrix generates the small masses of the light neutrinos. The masses of the heavy neutrinos and the mixing angle can be independently chosen. if we choose $m \approx 100\ GeV$ and $M \approx 1\ TeV$ then $\theta^2 \approx \mathcal{O}(10^{-2})$. The large mixing angle and the relatively lower scale of the heavy masses lead to interesting phenomenological consequences [10] , [11] , [16].

1.5.3 Three Generations: The Okamura Texture

A generalization to the two generations case discussed in the last section; it was suggested by [11]. The mass matrix loos like:

$$[\, \nu_1\ \nu_2\ \nu_3\ \chi_1\ \chi_2\ \chi_3 \,] \begin{bmatrix} 0 & 0 & 0 & \alpha m & \beta m & \gamma m \\ 0 & 0 & 0 & \alpha m & \beta m & \gamma m \\ 0 & 0 & 0 & \alpha m & \beta m & \gamma m \\ \alpha m & \alpha m & \alpha m & \alpha M & 0 & 0 \\ \beta m & \beta m & \beta m & 0 & \beta M & 0 \\ \gamma m & \gamma m & \gamma m & 0 & 0 & \gamma M \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \\ \chi_1 \\ \chi_2 \\ \chi_3 \end{bmatrix}, \tag{1.42}$$

where $\alpha+\beta+\gamma=0$. This mass matrix provides the same advantages of the specific two generations case discussed earlier, namely: mixings that are independent of the masses of the light neutrino states. This texture was proposed initially with values $m = 100\ GeV$, $M = 3\ TeV$ which results into $\theta \approx 0.055$. The large mixing was used to explain the NuTeV anomaly [10] . Later I will show how it can also lead to a relatively large electron EDM.

1.6 Back to the Lagrangian

To calculate the contribution from the new class of diagrams, let's go back to the Lagrangian and write down the interactions. I will denote the left- and right-handed charged lepton fields with l and r, and the left- and right-handed neutrino fields with ν and χ. Then, the interactions we need to consider are

$$\begin{aligned} -\mathcal{L} \;=\;& \frac{g}{\sqrt{2}}\left(\overline{l_i}\gamma^\mu\nu_i\right)W_\mu^- + h.c. \\ &+\lambda_{ij}\,\overline{r_i}\left(\phi^-\nu_j+\phi^{0*}l_j\right)+h.c. \\ &+\Lambda^\dagger_{ij}\,\overline{\chi_i}\left(\phi^0\nu_j-\phi^+l_j\right)+h.c. \\ &+\frac{1}{2}\,\overline{\chi_i}\,M_{ij}\,\chi^c_j+h.c. \end{aligned} \tag{1.43}$$

Without loss of generality, M_{ij} can be taken to be (complex) symmetric. After the neutral Higgs develops a VEV,

$$\left\langle\phi^0\right\rangle=\left\langle\phi^{0*}\right\rangle=\frac{v}{\sqrt{2}}\ , \tag{1.44}$$

the Yukawa couplings λ_{ij} and Λ_{ij} lead to the mass matrix of the charged leptons and the Dirac mass matrix of the neutrinos:

$$m_{ij}=\frac{v}{\sqrt{2}}\lambda_{ij}\ , \qquad D_{ij}=\frac{v}{\sqrt{2}}\Lambda^\dagger_{ij}\ . \tag{1.45}$$

Shifting the ϕ^0 field to:

$$\phi^0\rightarrow\frac{v+h+i\varphi^0}{\sqrt{2}}\ , \tag{1.46}$$

the Lagrangian is now:

$$\begin{aligned} -\mathcal{L} \;=\;& \frac{g}{\sqrt{2}}\left(\overline{l_i}\gamma^\mu\nu_i\right)W_\mu^- + h.c. \\ &+\overline{r_i}\,m_{ij}\,l_j+\lambda_{ij}\,\overline{r_i}\left(\phi^-\,\nu_j+\frac{h+i\varphi^0}{\sqrt{2}}\,l_j\right)+h.c. \\ &+\overline{\chi_i}\,D_{ij}\,\nu_j+\Lambda^\dagger_{ij}\,\overline{\chi_i}\left(\frac{h+i\varphi^0}{\sqrt{2}}\,\nu_j-\phi^+\,l_j\right)+h.c. \\ &+\frac{1}{2}\,\overline{\chi_i}\,M_{ij}\,\chi^c_j+h.c. \end{aligned} \tag{1.47}$$

The neutrino mass terms can be written as

$$\begin{aligned} &\overline{\chi_i}\,D_{ij}\,\nu_j+\frac{1}{2}\overline{\chi_i}\,M_{ij}\,\chi^c_j+h.c. \\ =\;& \frac{1}{2}\left(\overline{\chi_i}\,D_{ij}\,\nu_j+\overline{\nu^c_i}\,D^T_{ij}\chi^c_j+\overline{\chi_i}\,M_{ij}\,\chi^c_j\right)+h.c. \\ =\;& \frac{1}{2}\begin{bmatrix}\overline{\nu^c_i} & \overline{\chi_j}\end{bmatrix}\begin{bmatrix}0_{ik} & D^T_{i\ell}\\ D_{jk} & M_{j\ell}\end{bmatrix}\begin{bmatrix}\nu_k\\ \chi^c_\ell\end{bmatrix}+h.c. \end{aligned} \tag{1.48}$$

The mass matrix for the charged leptons is diagonalized with unitary transformations of the l and r fields:

$$r_i=A_{i\alpha}r_\alpha\ , \qquad l_j=B_{j\beta}l_\beta\ , \qquad \overline{r_i}\,m_{ij}\,l_j=\overline{r_\alpha}\underbrace{\left(A^\dagger_{\alpha i}\,m_{ij}\,B_{j\beta}\right)}_{diagonal}l_\beta\ . \tag{1.49}$$

The fields with a greek index are the mass eigenfields. Similarly, the mass matrix for the neutrinos is diagnalized with a unitary transformation involving the ν and χ^c fields:

$$\begin{bmatrix} \nu_i \\ \chi_j^c \end{bmatrix} = U \begin{bmatrix} \nu_\alpha \\ \nu_\beta \end{bmatrix} , \tag{1.50}$$

so that

$$\begin{bmatrix} \overline{\nu_i^c} & \overline{\chi_j} \end{bmatrix} \begin{bmatrix} 0_{ik} & D^T_{i\ell} \\ D_{jk} & M_{j\ell} \end{bmatrix} \begin{bmatrix} \nu_k \\ \chi_\ell^c \end{bmatrix} = \begin{bmatrix} \overline{\nu_\alpha^c} & \overline{\nu_\beta^c} \end{bmatrix} \underbrace{\left(U^T \begin{bmatrix} 0 & D^T \\ D & M \end{bmatrix} U \right)}_{diagonal} \begin{bmatrix} \nu_\gamma \\ \nu_\delta \end{bmatrix} . \tag{1.51}$$

The ν_α fields with greek indices are the left-handed mass eigenfields, (Note that this notation is different from the usual one where greek indices are used for the flavor eigenstates.) with $\alpha = 1, 2, 3$ being the light states, and $\alpha = 4, 5, 6$ being the heavy states. Decomposing the 6×6 matrix U into four 3×3 matrices as

$$U = \begin{bmatrix} P & Q \\ R & S \end{bmatrix} , \tag{1.52}$$

we can write

$$\begin{aligned} \nu_i &= P_{i\rho}\nu_\rho + Q_{i,\sigma}\nu_{\sigma+3} , \\ \chi_i^c &= R_{i\rho}\nu_\rho + S_{i,\sigma}\nu_{\sigma+3} , \end{aligned} \tag{1.53}$$

The second line can also be written

$$\chi_i = R^*_{i\rho}\nu^c_\rho + S^*_{i\sigma}\nu^c_{\sigma+3} . \tag{1.54}$$

The relevant interaction terms in the Lagrangian are then:

$$\begin{aligned} \frac{g}{\sqrt{2}} \left(\overline{l_i}\gamma^\mu\nu_i\right) W^+_\mu &\rightarrow \frac{g}{\sqrt{2}} \left\{ \overline{l_\beta} \left(B^\dagger_{\beta i} P_{i\rho} \right) \gamma^\mu \nu_\rho + \overline{l_\beta} \left(B^\dagger_{\beta i} Q_{i\sigma} \right) \gamma^\mu \nu_{\sigma+3} \right\} W^+_\mu , \\ \left(\overline{r_i}\,\lambda_{ij}\,\nu_j\right)\phi^- &\rightarrow \left\{ \overline{r_\alpha} \left(A^\dagger_{\alpha i}\lambda_{ij} P_{j\rho} \right) \nu_\rho + \overline{r_\alpha} \left(A^\dagger_{\alpha i}\lambda_{ij} Q_{j\sigma} \right) \nu_{\sigma+3} \right\} \phi^- , \\ \left(\overline{\chi_i}\,\Lambda^\dagger_{ij}\,l_j\right)\phi^+ &\rightarrow \left\{ \overline{\nu^c_\rho} \left(R^T_{\rho i}\Lambda^\dagger_{ij} B_{j\beta} \right) l_\beta + \overline{\nu^c_{\sigma+3}} \left(S^T_{\sigma i}\Lambda^\dagger_{ij} B_{j\beta} \right) l_\beta \right\} \phi^+ . \end{aligned} \tag{1.55}$$

Interactions involving the neutral Goldstone or the physical Higgs do not contribute. The combination

$$V_{\beta\rho} = \left(B^\dagger_{\beta i} P_{i\rho} \right) , \tag{1.56}$$

in the first line corresponds to the MNS matrix, which is not necessarily unitary. Define the Dirac lepton field as

$$\ell_\alpha \equiv l_\alpha + r_\alpha , \qquad (\alpha = e, \mu, \tau) , \tag{1.57}$$

so that

$$l_\alpha = P_L\,\ell_\alpha , \quad r_\alpha = P_R\,\ell_\alpha , \qquad P_{L/R} = \frac{1 \mp \gamma_5}{2} . \tag{1.58}$$

We also define the Majorana fields for the neutrino mass eigenstates:

$$\begin{aligned} n_\alpha &\equiv \nu_\alpha + \nu^c_\alpha , \\ N_\alpha &\equiv \nu_{\alpha+3} + \nu^c_{\alpha+3} , \end{aligned} \tag{1.59}$$

for $\alpha = 1, 2, 3$. Note that

$$n^c_\alpha = n_\alpha , \qquad N^c_\alpha = N_\alpha , \qquad \nu_\alpha = P_L\,n_\alpha , \qquad \nu_{\alpha+3} = P_L\,N_\alpha , \qquad \nu^c_\alpha = P_R\,n_\alpha , \qquad \nu^c_{\alpha+3} = P_R\,N_\alpha . \tag{1.60}$$

In terms of these mass eigenfields, the interaction Lagrangian becomes:

$$
\begin{aligned}
-\mathcal{L}_{\rm int} &= \frac{g}{\sqrt{2}}\left\{\overline{\ell_\beta}\left(B^\dagger P\right)_{\beta\rho}\gamma^\mu P_L\, n_\rho + \overline{\ell_\beta}\left(B^\dagger Q\right)_{\beta\sigma}\gamma^\mu P_L\, N_\sigma\right\} W_\mu^+ + h.c. \\
&+ \left\{\overline{\ell_\alpha}\left(A^\dagger \lambda P\right)_{\alpha\rho} P_L\, n_\rho + \overline{\ell_\alpha}\left(A^\dagger \lambda Q\right)_{\alpha\sigma} P_L\, N_\sigma\right\}\phi^- + h.c. \\
&+ \left\{\overline{n_\rho}\left(R^T \Lambda^\dagger B\right)_{\rho\beta} P_L\,\ell_\beta + \overline{N_\sigma}\left(S^T\Lambda^\dagger B\right)_{\sigma\beta} P_L\,\ell_\beta\right\}\phi^+ + h.c.
\end{aligned}
\tag{1.61}
$$

In particular, the terms involving the N fields are:

$$
\begin{aligned}
-\mathcal{L}_N &= \frac{g}{\sqrt{2}}\left\{\overline{\ell_\beta}\left(B^\dagger Q\right)_{\beta\sigma}\gamma^\mu P_L\, N_\sigma\right\} W_\mu^+ + h.c. \\
&+ \left\{\overline{\ell_\alpha}\left(A^\dagger \lambda Q\right)_{\alpha\sigma} P_L\, N_\sigma\right\}\phi^- + h.c. \\
&+ \left\{\overline{N_\sigma}\left(S^T\Lambda^\dagger B\right)_{\sigma\beta} P_L\,\ell_\beta\right\}\phi^+ + h.c.
\end{aligned}
\tag{1.62}
$$

To simplify the notation, define

$$
\begin{aligned}
\tilde{V}_{\alpha\beta} &\equiv (B^\dagger Q)_{\alpha\beta}\ , \\
\tilde{\lambda}_{\alpha\beta} &\equiv (A^\dagger \lambda Q)_{\alpha\beta}\ , \\
\tilde{\Lambda}^\dagger_{\alpha\beta} &\equiv (S^T \Lambda^\dagger B)_{\alpha\beta}\ .
\end{aligned}
\tag{1.63}
$$

Note that $\tilde{V} = (B^\dagger Q)$ is different from the MNS matrix $V = (B^\dagger P)$. This allows us to write:

$$
\begin{aligned}
-\mathcal{L}_N &= \frac{g}{\sqrt{2}}\left\{\overline{\ell_\beta}\tilde{V}_{\beta\sigma}\gamma^\mu P_L\, N_\sigma\right\} W_\mu^+ + \frac{g}{\sqrt{2}}\left\{\overline{N_\sigma}\tilde{V}^\dagger_{\sigma\beta}\gamma^\mu P_L\,\ell_\beta\right\} W_\mu^- \\
&+ \left\{\overline{\ell_\alpha}\tilde{\lambda}_{\alpha\sigma} P_L\, N_\sigma\right\}\phi^- + \left\{\overline{N_\sigma}\tilde{\lambda}^\dagger_{\sigma\alpha} P_R \ell_\alpha\right\}\phi^+ \\
&+ \left\{\overline{N_\sigma}\tilde{\Lambda}^\dagger_{\sigma\beta} P_L\,\ell_\beta\right\}\phi^+ + \left\{\overline{\ell_\beta}\tilde{\Lambda}_{\beta\sigma} P_R\, N_\sigma\right\}\phi^-\ .
\end{aligned}
\tag{1.64}
$$

Note that

$$
M_W = \frac{gv}{2}\ , \qquad m \sim \frac{\lambda v}{\sqrt{2}}\ , \qquad D \sim \frac{\Lambda v}{\sqrt{2}}\ . \tag{1.65}
$$

Also, the mixings between the light and heavy states given by the matrix Q enters into $\tilde{V}$ and $\tilde{\lambda}$, but not $\tilde{\Lambda}$. Assuming $Q \sim O(\varepsilon) \ll 1$, we can expect:

$$
\tilde{V} \sim \varepsilon\ , \qquad \frac{\tilde{\lambda}}{g} \sim \frac{m}{M_W}\varepsilon\ , \qquad \frac{\tilde{\Lambda}}{g} \sim \frac{D}{M_W}\ , \tag{1.66}
$$

So we can neglect the $\tilde{\lambda}$ terms. The $\tilde{\Lambda}$ terms will be largest if we assume $D \sim m_t$.

Chapter 2

CP Violation in the leptonic sector

2.1 CP Violation and Charged Lepton EDM

Consider the effective Lagrangian:

$$\mathcal{L} = \overline{\psi}\Big[\gamma_\mu(i\partial^\mu - qA^\mu) - m + \frac{q}{2m}\sigma^{\mu\nu}(F + iG\gamma_5)(\partial_\nu A_\mu)\psi\Big] \tag{2.1}$$

The Lagrangian is invariant under C transformation:

$$C\Big[\overline{\psi}\sigma^{\mu\nu}(F + iG\gamma_5)(\partial_\nu A_\mu)\psi\Big]C^\dagger = \bar{\psi}\sigma^{\mu\nu}(F + iG\gamma_5)(\partial_\nu A_\mu)\psi \tag{2.2}$$

since

$$\begin{aligned} C\overline{\psi}\sigma^{\mu\nu}\psi C^\dagger &= -\overline{\psi}\sigma^{\mu\nu}\psi \\ C\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi C^\dagger &= -\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi \\ CA_\mu C^\dagger &= -A_\mu \end{aligned} \tag{2.3}$$

Under P transformation:

$$P\Big[\overline{\psi}\sigma^{\mu\nu}(F + iG\gamma_5)(\partial_{nu} A_\mu)\psi\Big]P^\dagger = \overline{\psi}\sigma^{\mu\nu}(F - iG\gamma_5)(\partial_\nu A_\mu)\psi \tag{2.4}$$

since

$$\begin{aligned} P\overline{\psi}\sigma^{\mu\nu}\psi P^\dagger &= -\overline{\psi}\sigma^{\mu\nu}\psi \\ P\overline{\psi}\sigma^{\mu\nu}\gamma_5\psi P^\dagger &= \overline{\psi}\sigma^{\mu\nu}\gamma_5\psi \\ PA_\mu P^\dagger &= -A_\mu \end{aligned} \tag{2.5}$$

so $\mathcal{L}$ is not invariant under P if $G \neq 0$. The term with non-zero G is a CP violating term[14].

2.2 Calculating the EDM for Charged Leptons

The most general form factors for the interaction of a vector boson with a fermion is given by

$\overline{u}(p_2)\,\Gamma^\mu(p_1, p_2)\,u(p_1)$

$$= \overline{u}(p_2)\left[\left\{F_1(q^2)+G_1(q^2)\gamma_5\right\}\gamma^\mu+\left\{F_2(q^2)+G_2(q^2)\gamma_5\right\}\frac{i\sigma^{\mu\nu}q_\nu}{2m}+\left\{F_3(q^2)+G_3(q^2)\gamma_5\right\}q^\mu\right]u(p_1)\ , \tag{2.6}$$

where $q^\mu = (p_1 - p_2)^\mu$. To see that this is the most general form, first note that Γ^μ has one Lorentz index, and it can only depend on $q^\mu = (p_1 - p_2)^\mu$ and $(p_1 + p_2)^\mu$. There are 16 independent 4×4 matrices, and they can be expressed as:

$$\mathbf{1}\ , \qquad \gamma^\mu\ , \qquad \sigma^{\mu\nu} = \frac{i}{2}\left[\,\gamma^\mu . \gamma^\nu\,\right]\ , \qquad \gamma_5\gamma^\mu\ , \qquad \gamma_5\ . \tag{2.7}$$

Of these, γ^μ and $\gamma_5\gamma^\mu$ already have one Lorentz index so they can be used as they are. The matrices $\mathbf{1}$ and γ_5 can be given a Lorentz index by multiplying them with q^μ:

$$q^\mu\ , \qquad \gamma_5 q^\mu\ . \tag{2.8}$$

The extra index on $\sigma^{\mu\nu}$ can be gotten rid of by contracting with q_ν:

$$\sigma^{\mu\nu}q_\nu\ , \qquad \varepsilon_{\mu\nu\kappa\lambda}\sigma^{\kappa\lambda}q^\nu\ . \tag{2.9}$$

As will be shown below, $\varepsilon_{\mu\nu\kappa\lambda}\sigma^{\kappa\lambda} = 2i\gamma_5\sigma_{\mu\nu}$, so the second term can be replaced by $\gamma_5\sigma^{\mu\nu}q_\nu$.

Instead of q^μ, we can also use $(p_1 + p_2)^\mu$ to obtain

$$(p_1+p_2)^\mu\ , \qquad \gamma_5(p_1+p_2)^\mu\ , \qquad \sigma^{\mu\nu}(p_1+p_2)_\nu\ , \qquad \gamma_5\sigma^{\mu\nu}(p_1+p_2)_\nu\ . \tag{2.10}$$

However, using the Gordon identities, these can be rewritten using the terms we already have. To see this, note that

$$\begin{aligned} g^{\mu\nu} &= \frac{1}{2}\left\{\gamma^\mu,\gamma^\nu\right\} = \frac{1}{2}\left(\gamma^\mu\gamma^\nu+\gamma^\nu\gamma^\mu\right)\ , \\ -i\sigma^{\mu\nu} &= \frac{1}{2}\left[\gamma^\mu,\gamma^\nu\right] = \frac{1}{2}\left(\gamma^\mu\gamma^\nu-\gamma^\nu\gamma^\mu\right)\ , \end{aligned} \tag{2.11}$$

so that

$$\begin{aligned} \gamma^\mu\gamma^\nu &= g^{\mu\nu} - i\sigma^{\mu\nu}\ , \\ \gamma^\nu\gamma^\mu &= g^{\mu\nu} + i\sigma^{\mu\nu}\ . \end{aligned} \tag{2.12}$$

Also:

$$\overline{u}(p_2)\not{p}_2 = \bar{u}(p_2)m_2\ , \qquad \not{p}_1 u(p_1) = m_1 u(p_1)\ . \tag{2.13}$$

Therefore,

$$\begin{aligned} \bar{u}(p_2)i\sigma^{\mu\nu}p_{1\nu}u(p_1) &= \bar{u}(p_2)\left(g^{\mu\nu}-\gamma^\mu\gamma^\nu\right)p_{1\nu}u(p_1) \\ &= \bar{u}(p_2)\left(p_1^\mu-\gamma^\mu\not{p}_1\right)u(p_1) \\ &= \bar{u}(p_2)\left(p_1^\mu-m_1\gamma^\mu\right)u(p_1)\ , \\ \\ \bar{u}(p_2)i\sigma^{\mu\nu}p_{2\nu}u(p_1) &= \bar{u}(p_2)\left(-g^{\mu\nu}+\gamma^\nu\gamma^\mu\right)p_{2\nu}u(p_1) \\ &= \bar{u}(p_2)\left(-p_2^\mu+\not{p}_2\gamma^\mu\right)u(p_1) \\ &= \bar{u}(p_2)\left(-p_2^\mu+m_2\gamma^\mu\right)u(p_1)\ , \\ \\ \bar{u}(p_2)\gamma_5 i\sigma^{\mu\nu}p_{1\nu}u(p_1) &= \bar{u}(p_2)\gamma_5\left(g^{\mu\nu}-\gamma^\mu\gamma^\nu\right)p_{1\nu}u(p_1) \\ &= \bar{u}(p_2)\gamma_5\left(p_1^\mu-\gamma^\mu\not{p}_1\right)u(p_1) \\ &= \bar{u}(p_2)\left(\gamma_5 p_1^\mu-m_1\gamma_5\gamma^\mu\right)u(p_1)\ , \end{aligned}$$

$$\begin{aligned}
\bar{u}(p_2)\gamma_5 i\sigma^{\mu\nu}p_{2\nu}u(p_1) &= \bar{u}(p_2)\gamma_5\left(-g^{\mu\nu}+\gamma^\nu\gamma^\mu\right)p_{2\nu}u(p_1) \\
&= \bar{u}(p_2)\left(-\gamma_5 g^{\mu\nu}-\gamma^\nu\gamma_5\gamma^\mu\right)p_{2\nu}u(p_1) \\
&= \bar{u}(p_2)\left(-\gamma_5 p_2^\mu-\not{p}_2\gamma_5\gamma^\mu\right)u(p_1) \\
&= \bar{u}(p_2)\left(-\gamma_5 p_2^\mu-m_2\gamma_5\gamma^\mu\right)u(p_1)\ ,
\end{aligned} \tag{2.14}$$

from which we find

$$\begin{aligned}
\bar{u}(p_2)i\sigma^{\mu\nu}(p_1+p_2)_\nu u(p_1) &= \bar{u}(p_2)\left[(p_1-p_2)^\mu-(m_1-m_2)\gamma^\mu\right)u(p_1)\ , \\
\bar{u}(p_2)\gamma_5 i\sigma^{\mu\nu}(p_1+p_2)_\nu u(p_1) &= \bar{u}(p_2)\left[\gamma_5(p_1-p_2)^\mu-(m_1+m_2)\gamma_5\gamma^\mu\right)u(p_1)\ , \\
\bar{u}(p_2)i\sigma^{\mu\nu}q_\nu u(p_1) = \bar{u}(p_2)i\sigma^{\mu\nu}(p_1-p_2)_\nu u(p_1) &= \bar{u}(p_2)\left[(p_1+p_2)^\mu-(m_1+m_2)\gamma^\mu\right)u(p_1)\ , \\
\bar{u}(p_2)\gamma_5 i\sigma^{\mu\nu}q_\nu u(p_1) = \bar{u}(p_2)\gamma_5 i\sigma^{\mu\nu}(p_1-p_2)_\nu u(p_1) &= \bar{u}(p_2)\left[\gamma_5(p_1+p_2)^\mu-(m_1-m_2)\gamma_5\gamma^\mu\right)u(p_1)\ .
\end{aligned} \tag{2.15}$$

Therefore, the form factors shown in Eq. (2.6) is the most general form. They can only depend on q^2 since the other Lorentz scalars are $(p_1+p_2)^2 = 2(m_1^2+m_2^2)-q^2$, and $(p_1+p_2)q = m_1^2-m_2^2$. Of course, when the vector boson is the photon, gauge invariance requires $\bar{u}(p_2)\,(\,q_\mu\Gamma^\mu\,)\,u(p_1)=0$, and this will impose various conditions on the form factors.

The EDM is $d = G_2(0)/2mi$. To see this, we first note that

$$\begin{aligned}
\Big\{\,\bar{u}(p_2)\gamma_5\, i\sigma^{\mu\nu}q_\nu u(p_1)\,\Big\}A_\mu(-q) &= \Big\{\,\bar{u}(p_2)\gamma_5\,\sigma^{\mu\nu}u(p_1)\,\Big\}\Big\{iq_\nu A_\mu(-q)\Big\} \\
&= \Big\{\,\bar{u}(p_2)\gamma_5\,\sigma^{\mu\nu}u(p_1)\,\Big\}\Big\{\partial_\nu A_\mu(-q)\Big\} \\
&= -\frac{1}{2}\Big\{\,\bar{u}(p_2)\gamma_5\,\sigma^{\mu\nu}u(p_1)\,\Big\}\Big\{\partial_\mu A_\nu-\partial_\nu A_\mu\Big\} \\
&= -\frac{1}{2}\Big\{\,\bar{u}(p_2)\gamma_5\,\sigma^{\mu\nu}u(p_1)\,\Big\}F_{\mu\nu} \\
&= -\frac{1}{2}\Big\{\,\bar{u}(p_2)\gamma_5\,\sigma_{\mu\nu}u(p_1)\,\Big\}F^{\mu\nu}\ .
\end{aligned} \tag{2.16}$$

Since $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu,\gamma_\nu]$, we find:

$$\begin{aligned}
\gamma_5\,\sigma_{\mu\nu} &= i\gamma^0\gamma^1\gamma^2\gamma^3\sigma_{\mu\nu} \\
&= -\frac{1}{2}\gamma^0\gamma^1\gamma^2\gamma^3\,[\gamma_\mu,\gamma_\nu] \\
&= +\frac{1}{4}\varepsilon_{\mu\nu\kappa\lambda}\left[\gamma^\kappa,\gamma^\lambda\right] \\
&= -\frac{i}{2}\varepsilon_{\mu\nu\kappa\lambda}\sigma^{\kappa\lambda}\ .
\end{aligned} \tag{2.17}$$

To make sure this is correct, let's write out the expressions for the matrices in the Dirac representation:

$$\gamma^0 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \qquad \gamma^i = \begin{bmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{bmatrix}, \qquad \gamma_5 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{2.18}$$

$$\begin{aligned}
\sigma^{0i} = -\sigma^{i0} = -\sigma_{0i} = \sigma_{i0} &= \frac{i}{2}[\gamma^0,\gamma^i] \;=\; i\begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix} \\
\sigma^{ij} = -\sigma^{ji} = \sigma_{ij} = -\sigma_{ji} &= \frac{i}{2}[\gamma^i,\gamma^j] \;=\; \varepsilon_{ijk}\begin{bmatrix} \sigma_k & \\ 0 & \sigma_k \end{bmatrix}
\end{aligned} \tag{2.19}$$

$$\begin{aligned}
\gamma_5\sigma_{0i} &= -i\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{bmatrix} = -i\begin{bmatrix} \sigma_i & 0 \\ 0 & \sigma_i \end{bmatrix} = -\frac{i}{2}\varepsilon_{ijk}\sigma^{jk} \;=\; -\frac{i}{2}\varepsilon_{0ijk}\sigma^{jk}\ , \\
\gamma_5\sigma_{ij} &= \varepsilon_{ijk}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \sigma_k & 0 \\ 0 & \sigma_k \end{bmatrix} = \varepsilon_{ijk}\begin{bmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{bmatrix} = -i\varepsilon_{ijk}\sigma^{0k} \;=\; -i\varepsilon_{ij0k}\sigma^{0k} \;=\; -\frac{i}{2}\varepsilon_{ij\mu\nu}\sigma^{\mu\nu}\ .
\end{aligned}$$

(2.20)

Therefore,

$$\begin{aligned} -\frac{1}{2}\Big\{\,\bar{u}(p_2)\gamma_5\,\sigma_{\mu\nu}u(p_1)\,\Big\}F^{\mu\nu} &= -\frac{1}{2}\left\{-\frac{i}{2}\varepsilon_{\mu\nu\kappa\lambda}\,\bar{u}(p_2)\sigma^{\kappa\lambda}u(p_1)\,\right\}F^{\mu\nu} \\ &= \frac{i}{2}\Big\{\,\bar{u}(p_2)\sigma^{\kappa\lambda}u(p_1)\,\Big\}\left\{\frac{1}{2}\varepsilon_{\mu\nu\kappa\lambda}F^{\mu\nu}\right\} \\ &= \frac{i}{2}\Big\{\,\bar{u}(p_2)\sigma^{\kappa\lambda}u(p_1)\,\Big\}\tilde{F}_{\kappa\lambda}\ . \end{aligned} \tag{2.21}$$

The electromagnetic fields are:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -B_z & B_y \\ E_y & B_z & 0 & -B_x \\ E_z & -B_y & B_x & 0 \end{bmatrix}, \qquad F_{\mu\nu} = \begin{bmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{bmatrix},$$
$$\tilde{F}_{\mu\nu} = \begin{bmatrix} 0 & -B_x & -B_y & -B_z \\ B_x & 0 & -E_z & E_y \\ B_y & E_z & 0 & -E_x \\ B_z & -E_y & E_x & 0 \end{bmatrix}, \qquad \tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & B_x & B_y & B_z \\ -B_x & 0 & -E_z & E_y \\ -B_y & E_z & 0 & -E_x \\ -B_z & -E_y & E_x & 0 \end{bmatrix}. \tag{2.22}$$

In the non-relativistic limit, only the σ^{ij} terms survive:

$$\begin{aligned} \frac{i}{2}\Big\{\,\bar{u}(p_2)\sigma^{\kappa\lambda}u(p_1)\,\Big\}\tilde{F}_{\kappa\lambda} &\to \frac{i}{2}\Big\{\,\bar{u}(p_2)\sigma^{ij}u(p_1)\,\Big\}\tilde{F}_{ij} \\ &= \frac{i}{2}\Big\{\,\bar{u}(p_2)\varepsilon_{ijk}\sigma_k u(p_1)\,\Big\}\left(-\varepsilon_{ij\ell}E_\ell\right) \\ &= -i\,\bar{u}(p_2)\left(\vec{\sigma}\cdot\vec{E}\right)u(p_1)\ . \end{aligned} \tag{2.23}$$

So what we have found is:

$$\frac{G_2(q^2)}{2m}\,\gamma_5\,i\sigma^{\mu\nu}q_\nu A_\mu \;\longrightarrow\; \frac{G_2(q^2)}{2mi}\left(\vec{\sigma}\cdot\vec{E}\right)\ , \tag{2.24}$$

which let's us identify

$$d = \frac{G_2(0)}{2mi}\ . \tag{2.25}$$

(The i in the denominator cancels the i coming from the imaginary parts of the diagrams.)

Using the Gordon identities

$$\begin{aligned} \bar{u}_2(p_2)\,(p_1+p_2)^\mu u_1(p_1) &= (m_1+m_2)\,\bar{u}_2(p_2)\gamma^\mu u_1(p_1) + i\bar{u}_2(p_2)\sigma^{\mu\nu}(p_1-p_2)_\nu u_1(p_1)\ , \\ \bar{u}_2(p_2)\,(p_1+p_2)^\mu\gamma_5 u_1(p_1) &= -(m_1-m_2)\,\bar{u}_2(p_2)\gamma^\mu\gamma_5 u_1(p_1) + i\bar{u}_2(p_2)\sigma^{\mu\nu}(p_1-p_2)_\nu\gamma_5 u_1(p_1)\ , \end{aligned} \tag{2.26}$$

with $m_1 = m_2 = m$, we can also write:

$$\begin{aligned} &\bar{u}_2(p_2)\,\Gamma^\mu(p_1,p_2)\,u_1(p_1) \\ &= \bar{u}_2(p_2)\left[\{(F_1-F_2)+G_1\gamma_5\}\,\gamma^\mu + \{F_2+G_2\gamma_5\}\,\frac{(p_1+p_2)^\mu}{2m} + \{F_3+G_3\gamma_5\}\,q^\mu\right]u_1(p_1)\ , \end{aligned} \tag{2.27}$$

so to find $G_2(0)$, we need to calculate the coefficient of $\gamma_5(p_1+p_2)^\mu$.

Chapter 3

Lepton EDM from See-Saw Textures

3.1 CP violating diagrams at the two loop order

In order to have a non zero G in Eq.(2.1) the imaginary part of the diagrams contributing to the EDM must not vanish. The complex phases come from diagonalizing the mass matrix and appear in the Yukawa couplings. Let's consider the different classes of diagrams that might contribute to the EDM.

First consider the diagrams that involves only Dirac neutrinos. At 1-loop we can only havethe diagram in figure 3.1.

The contribution from this diagram is proportional to $|U_{\alpha i}|^2$ which is real; and the theory is CP invariant at the 1-loop level.

At 2-loops we have the diagram in figure 3.2 . The contribution of the diagram is $\propto (U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j})$. Since the diagram is symmetric under interchanging the labels i and j, the contribution will be proportional to:

$$
\begin{aligned}
& (U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j}) + (U^*_{\alpha j}U_{\beta j})(U_{\alpha i}U^*_{\beta i}) \\
= & (U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j}) + ((U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j}))^* \\
= & 2Re(U^*_{\alpha i}U_{\beta i})(U_{\alpha j}U^*_{\beta j})
\end{aligned}
\tag{3.1}
$$

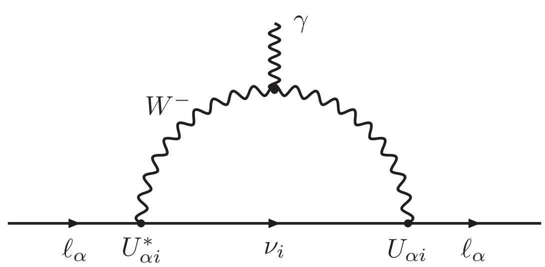
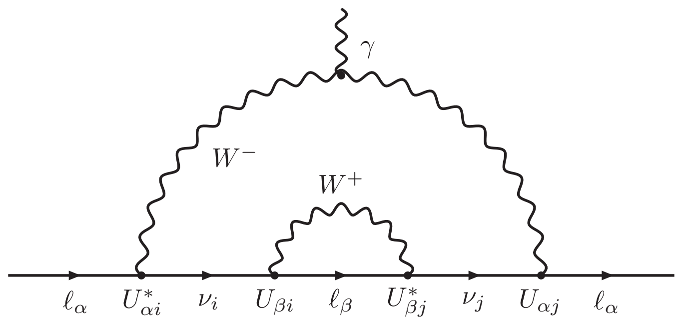
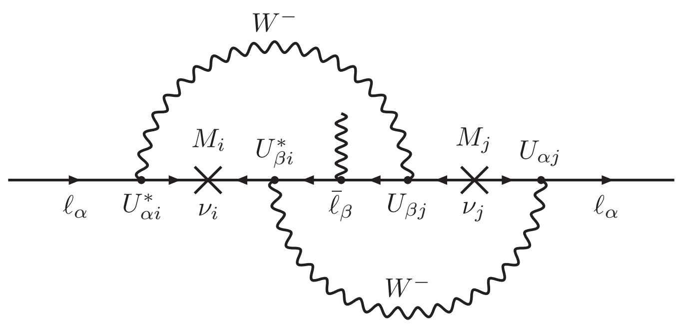


Figure 3.1: **One loop diagram.** At one loop the contribution to the EDM is zero



$$\propto (U^*_{\alpha i} U_{\beta i})(U_{\alpha j} U^*_{\beta j})$$

Figure 3.2: **Two loop diagrams, Dirac neutrinos.** At the two loop level, two neutrinos are involved in each diagram. However,summing over all the neutrinos the contribution to EDM vanishes



$$\propto (U^*_{\alpha i} U^*_{\beta i})(U_{\alpha j} U_{\beta j})$$

Figure 3.3: **Two loop diagrams, Majorana neutrinos.** new two loop diagrams exist for Majorana neutrinos with possibly non zero contribution to EDM

and the imaginary pieces cancel out.

At the 3-loop level and beyond; the contribution will be highly suppressed even if it does not vanish.

However if neutrinos are Majorana particles, a new class of diagrams exists at the 2-loop level as in figure 3.3. The diagrams are not symmetric under the exchange of the neutrino labels, which might lead to a non-vanishing complex phase depending on the choice of the mass matrix parameter.

Notice that the contribution of the diagrams picks the Majorana masses of the neutrinos, in the case of the heavy states, that is counteracted by a suppression due to the mixing between the heavy and the light neutrinos. In models where the mixing and the masses are independent the contribution of the diagrams can be significant.

3.2 The Contributing Diagrams

In the Feynman-t'Hooft gauge there are 20 diagrams we have to consider, they are displayed in figure 3.4.

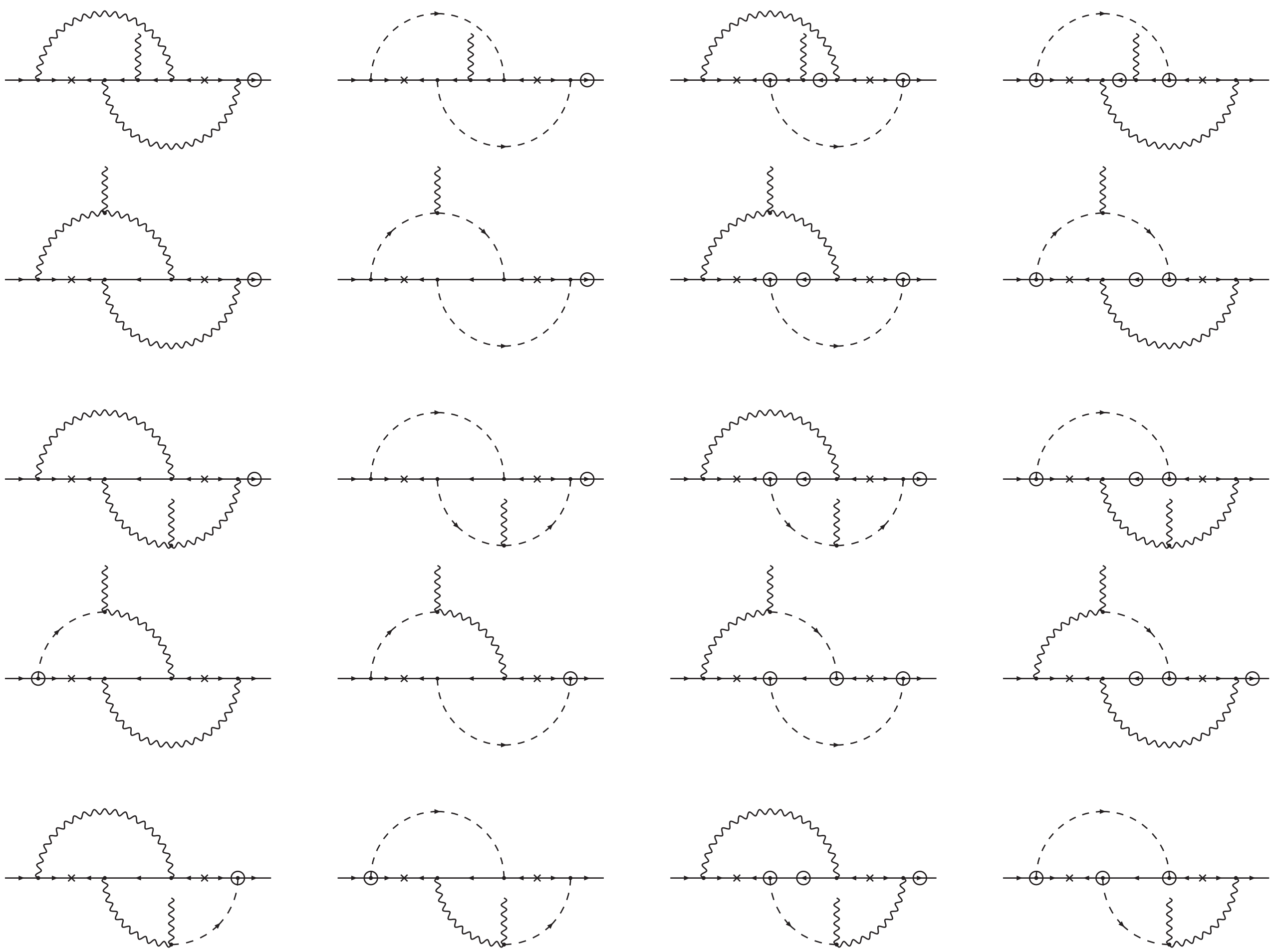


Figure 3.4: **Contributing two loop diagrams.** In the Feynman-t'Hooft gauge there exists 20 diagrams, they are labeled by a number matching the row (1-5) and a letter corresponding to the column (A-D). The contribution from diagrams in the C and D columns is suppressed by a factor of the lepton mass squared over the W mass squared; so they can be neglected.

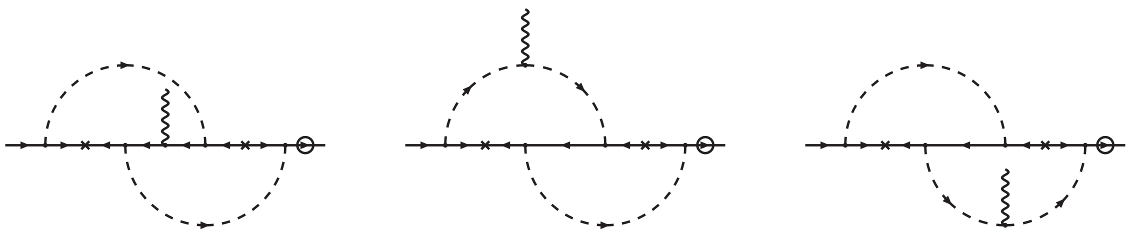


Diagram 1B Diagram 2B Diagram 3B

Figure 3.5: **Diagrams with largest contribution to the EDM.** Diagrams 1B, 2B, 3B contribute the greatest to EDM in see-saw models with large mixing that is independent from the masses (i.e. Okamura texture).

We refer to these diagrams by a number from 1 to 5; which designates the row, and a letter from A to D; which designates the column of this array of diagrams. We are only interested in contributions that pick up the Majorana masses of both heavy neutrals on the internal lines. This imposes certain chirality flips to occur at the vertices and propagators of these diagrams. The $\tilde{\lambda}$ interactions and chirality flips on the charged lepton line that occur due to this requirement are indicated by small circles. the diagrams on the 3rd and 4th (C and D) columns all have three circles whereas those on the 1st and 2nd (A and B) columns have only one. So the diagrams in the C and D columns are suppressed by a factor of $(m/M_W)^2$ compared to those in the A and B columns, and we can ignore them.

Notice that in models that unify the quarks and leptons, the Λ interactions are the greatest, so for that class of models the largest contribution comes from the diagrams involving the Goldstone bosons; figure 3.5

3.3 The Calculation

I will present the details of the calculation of the diagrams that contribute most to EDM due to seesaw textures where the mixing between the light and the heavy states is amplified. I will only present the final results for the rest of the diagrams in the next chapter. The details will be presented in Appendix A.

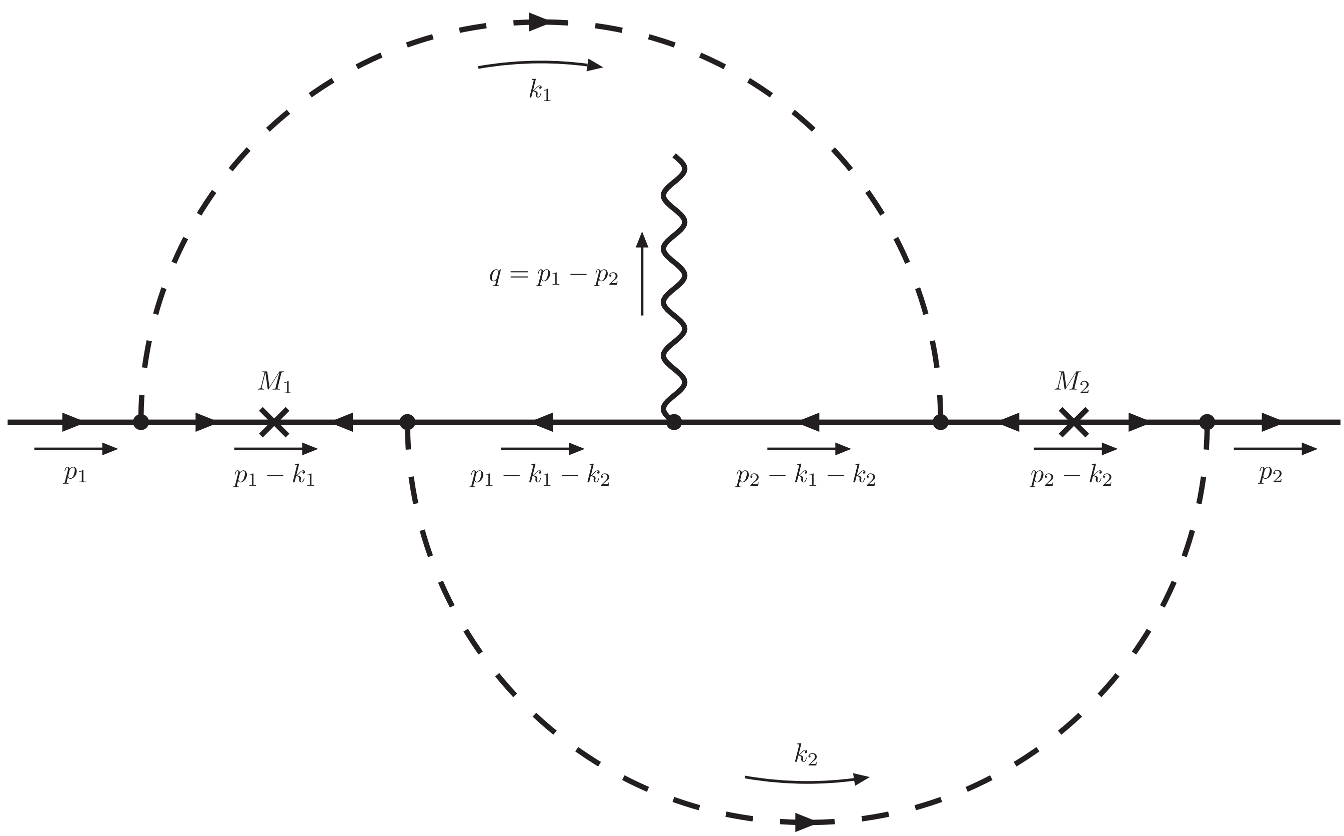


Figure 3.6: **Diagram 1B**

3.3.1 Diagram 1B

The coupling constants at the five vertices give the factor

$$\left(-i\tilde{\Lambda}_{\alpha 2}\right)\left(-i\tilde{\Lambda}_{\beta 2}\right)(ie)\left(-i\tilde{\Lambda}^{\dagger}_{1\beta}\right)\left(-i\tilde{\Lambda}^{\dagger}_{1\alpha}\right) = ie\left(\tilde{\Lambda}_{\alpha 2}\tilde{\Lambda}_{\beta 2}\tilde{\Lambda}^{*}_{\beta 1}\tilde{\Lambda}^{*}_{\alpha 1}\right) \ . \tag{3.2}$$

The fermion line:

$$\begin{aligned}
&\langle \ell_\alpha(p_2)| \left(\overline{\ell_\alpha} P_R N_2 \phi^-\right)\left(\overline{\ell_\beta} P_R N_2 \phi^-\right)\left(\overline{\ell_\beta}\gamma^\lambda \ell_\beta\right)\left(\overline{N_1} P_L \ell_\beta \phi^+\right)\left(\overline{N_1} P_L \ell_\alpha \phi^+\right)|\ell_\alpha(p_1)\rangle \\
&\quad = \quad -\langle \mu(p_2)| \left(\overline{\mu} P_R N \phi^-\right)\left(\overline{N} P_R \mu^c \phi^-\right)\left(\overline{\mu^c}\gamma^\lambda \mu^c\right)\left(\overline{\mu^c} P_L N \phi^+\right)\left(\overline{N} P_L \mu \phi^+\right)|\mu(p_1)\rangle \\
&\quad = \quad -\langle \mu(p_2)| \left(\overline{\mu} P_R N\right)\left(\overline{N} P_R \mu^c\right)\left(\overline{\mu^c}\gamma^\lambda \mu^c\right)\left(\overline{\mu^c} P_L N\right)\left(\overline{N} P_L \mu\right)|\mu(p_1)\rangle \left\langle \phi^-\phi^+\right\rangle\left\langle \phi^-\phi^+\right\rangle \\
&\quad = \quad -\bar{u}(p_2) P_R \left\langle N\overline{N}\right\rangle P_R \left\langle \mu^c\overline{\mu^c}\right\rangle \gamma^\lambda \left\langle \mu^c\overline{\mu^c}\right\rangle P_L \left\langle N\overline{N}\right\rangle P_L u(p_1) \times \left\langle \phi^-\phi^+\right\rangle\left\langle \phi^-\phi^+\right\rangle \\
\\
&\quad \to \quad -\bar{u}(p_2) P_R \left[\,(\not{p}_2 - \not{k}_2) + M_2\,\right] P_R \,(\not{p}_2 - \not{k}_1 - \not{k}_2)\,\gamma^\lambda\,(\not{p}_1 - \not{k}_1 - \not{k}_2)\, P_L \left[\,(\not{p}_1 - \not{k}_1) + M_1\,\right] P_L u(p_1) \\
&\quad = \quad -M_1 M_2\, \bar{u}(p_2)\,(\not{p}_2 - \not{k}_1 - \not{k}_2)\,\gamma^\lambda P_R\,(\not{p}_1 - \not{k}_1 - \not{k}_2)\, u(p_1)
\end{aligned}$$

$$\begin{aligned} &= -M_1 M_2\, \bar{u}(p_2)\,(m - \not{k}_1 - \not{k}_2)\,\gamma^\lambda P_R\,(m - \not{k}_1 - \not{k}_2)\,u(p_1) \\ &= -M_1 M_2\, \bar{u}(p_2)\,(m - \not{K})\,\gamma^\lambda P_R\,(m - \not{K})\,u(p_1) \end{aligned} \tag{3.3}$$

where

$$K \equiv k_1 + k_2 \tag{3.4}$$

$$= -M_1 M_2\, \bar{u}(p_2)\left(\not{K}\gamma^\lambda P_R \not{K} - m\not{K}\gamma^\lambda P_R - m\gamma^\lambda P_R \not{K}\right) u(p_1) \tag{3.5}$$

The $m^2\gamma^\lambda$ does not produce any p^μ term so it does not contribute.

$$= -M_1 M_2\, \bar{u}(p_2)\left(K^\kappa K^\rho \gamma_\kappa \gamma^\lambda P_R \gamma_\rho - mK^\kappa \gamma_\kappa \gamma^\lambda P_R - mK^\kappa \gamma^\lambda P_R \gamma_\kappa\right) u(p_1) \tag{3.6}$$

After integration, the K's will be replaced by p_1 and p_2. We define the functions:

$$\begin{aligned} &p_1^\kappa A_1 + p_2^\kappa A_2 \\ &\quad= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa}{[(p_2-k_2)^2 - M_2^2](p_2-K)^2(p_1-K)^2[(p_1-k_1)^2 - M_1^2](k_1^2 - M_W^2)(k_2^2 - M_W^2)}, \\ &g^{\kappa\rho}B_0 + p_1^\kappa p_1^\rho B_1 + p_2^\kappa p_2^\rho B_2 + (p_1^\kappa p_2^\rho + p_2^\kappa p_1^\rho)B_3 \\ &\quad= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\rho}{[(p_2-k_2)^2 - M_2^2](p_2-K)^2(p_1-K)^2[(p_1-k_1)^2 - M_1^2](k_1^2 - M_W^2)(k_2^2 - M_W^2)}. \end{aligned} \tag{3.7}$$

So the expression becomes:

$$\begin{aligned} &= -M_1 M_2\, \bar{u}(p_2)\big((g^{\kappa\rho}B_0 + p_1^\kappa p_1^\rho B_1 + p_2^\kappa p_2^\rho B_2 + (p_1^\kappa p_2^\rho + p_2^\kappa p_1^\rho)B_3)\gamma_\kappa \gamma^\lambda P_R \gamma_\rho \\ &\quad -m(p_1^\kappa A_1 + p_2^\kappa A_2)\gamma_\kappa \gamma^\lambda P_R - m(p_1^\kappa A_1 + p_2^\kappa A_2)\gamma^\lambda P_R \gamma_\kappa\big)\, u(p_1) \\ &= -M_1 M_2\, \bar{u}(p_2)\big((4B_0 + \not{p}_1 \gamma^\lambda \not{p}_1 B_1 + \not{p}_2 \gamma^\lambda \not{p}_2 B_2 + (\not{p}_1 \gamma^\lambda \not{p}_2)B_3 + (\not{p}_2 \gamma^\lambda \not{p}_1)B_3)P_L \\ &\quad -m(\not{p}_1 A_1 + \not{p}_2 A_2)\gamma^\lambda P_R - m\gamma^\lambda P_R(\not{p}_1 A_1 + \not{p}_2 A_2)\big)\, u(p_1) \\ &= -M_1 M_2\, \bar{u}(p_2)\big(m2p_1^\lambda P_R B_1 + m2p_2^\lambda P_L B_2 + (-m2p_1^\lambda P_R - m2p_2^\lambda P_L)B_3 \\ &\quad -m(2p_1^\lambda A_1)P_R - m(2p_2^\lambda A_2)P_L\big)\, u(p_1) \end{aligned} \tag{3.8}$$

where in the last line I dropped all the terms that do not contribute to the EDM.

$$\begin{aligned} &= -M_1 M_2\, \bar{u}(p_2)\Bigg(m2p_1^\lambda \frac{1}{2}(1+\gamma_5)B_1 + m2p_2^\lambda \frac{1}{2}(1-\gamma_5)B_2 + (-m2p_1^\lambda \frac{1}{2}(1+\gamma_5) \\ &\quad -m2p_2^\lambda \frac{1}{2}(1-\gamma_5))B_3)\ -m(2p_1^\lambda A_1)\frac{1}{2}(1+\gamma_5) - m(2p_2^\lambda A_2)\frac{1}{2}(1-\gamma_5)\Bigg)\, u(p_1) \\ &= -M_1 M_2\, \bar{u}(p_2)\big(mp_1^\lambda (1+\gamma_5)B_1 + mp_2^\lambda (1-\gamma_5)B_2 + (-mp_1^\lambda (1+\gamma_5) - mp_2^\lambda (1-\gamma_5))B_3 \\ &\quad -m(p_1^\lambda A_1)(1+\gamma_5) - m(p_2^\lambda A_2)(1-\gamma_5)\big)\, u(p_1) \end{aligned} \tag{3.9}$$

Collecting the γ_5 terms

$$\begin{aligned} &= -M_1 M_2\, \bar{u}(p_2)\big(mp_1^\lambda (+\gamma_5)B_1 + mp_2^\lambda (-\gamma_5)B_2 + (-mp_1^\lambda (+\gamma_5) - mp_2^\lambda (-\gamma_5))B_3 \\ &\quad -m(p_1^\lambda A_1)(+\gamma_5) - m(p_2^\lambda A_2)(-\gamma_5)\big)\, u(p_1) \\ &= -M_1 M_2\, \bar{u}(p_2)\big(mp_1^\lambda (+\gamma_5)B_1 + mp_2^\lambda (-\gamma_5)B_2 + (-m(p_1^\lambda - p_2^\lambda)(+\gamma_5))B_3 \\ &\quad -m(p_1^\lambda A_1)(+\gamma_5) - m(p_2^\lambda A_2)(-\gamma_5)\big)\, u(p_1) \end{aligned} \tag{3.10}$$

On-shell $(p_1^\lambda - p_2^\lambda)\epsilon_\lambda = 0$

$$= -mM_1 M_2\, \bar{u}(p_2)(\gamma_5)u(p_1)\big((+p_1^\lambda B_1 - p_2^\lambda B_2 - (p_1^\lambda A_1) + (p_2^\lambda A_2)\big)$$

$$
\begin{aligned}
&= -mM_1M_2\,\bar{u}(p_2)(\gamma_5)u(p_1)\left(+p_1^\lambda B_1 - \frac{1}{2}(p_1^\lambda - p_2^\lambda)B_2 - p_2^\lambda B_2 - \frac{1}{2}(p_1^\lambda - p_2^\lambda)B_1\right.\\
&\quad\left. -(p_1^\lambda A_1) + \frac{1}{2}(p_1^\lambda - p_2^\lambda)A_1 + (p_2^\lambda A_2) + \frac{1}{2}(p_1^\lambda - p_2^\lambda)A_2\right)\\
&= -mM_1M_2\,\bar{u}(p_2)(\gamma_5)u(p_1)\left(-\frac{1}{2}(p_1^\lambda + p_2^\lambda)B_2 + \frac{1}{2}(p_1^\lambda + p_2^\lambda)B_1)\right.\\
&\quad\left. -\frac{1}{2}(p_1^\lambda + p_2^\lambda)A_1 + \frac{1}{2}(p_1^\lambda + p_2^\lambda)A_2\right)\\
&= -\frac{1}{2}mM_1M_2\,\bar{u}(p_2)(\gamma_5)u(p_1)(p_1^\lambda + p_2^\lambda)\left(+B_1 - B_2 - A_1 + A_2\right)
\end{aligned}
\tag{3.11}
$$

So

$$
d = \frac{1}{2}e\,mM_1M_2\left(i\tilde{\Lambda}_{\alpha 2}\tilde{\Lambda}_{\beta 2}\tilde{\Lambda}^*_{\beta 1}\tilde{\Lambda}^*_{\alpha 1}\right)\Big[(A_1 - A_2) - (B_1 - B_2)\Big]\,. \tag{3.12}
$$

Now we evaluate the A functions:

$$
\begin{aligned}
&A_1\,p_1^\kappa + A_2\,p_2^\kappa\\
&\quad= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(p_1-K)^2(p_2-K)^2(k_1^2-M_W^2)(k_2^2-M_W^2)[(p_1-k_1)^2-M_1^2][(p_2-k_2)^2-M_2^2]}\\
&\quad= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(K^2-2p_1\cdot K)(K^2-2p_2\cdot K)(k_1^2-M_W^2)(k_2^2-M_W^2)[(k_1^2-M_1^2)-2p_1\cdot k_1]}\\
&\qquad\times\frac{1}{[(k_2^2-M_2^2)-2p_2\cdot k_2]}\\
&\quad= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
&\qquad\times\left[1+\frac{2p_1\cdot K}{K^2}+\cdots\right]\left[1+\frac{2p_2\cdot K}{K^2}+\cdots\right]\left[1+\frac{2p_1\cdot k_1}{k_1^2-M_1^2}+\cdots\right]\left[1+\frac{2p_2\cdot k_2}{k_2^2-M_2^2}+\cdots\right]\\
&\quad= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
&\qquad\times\left[1+\frac{2p_1\cdot K}{K^2}+\frac{2p_2\cdot K}{K^2}+\frac{2p_1\cdot k_1}{k_1^2-M_1^2}+\frac{2p_2\cdot k_2}{k_2^2-M_2^2}+\cdots\right]\\
&\quad= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
&\qquad\times\left[1+2\left(\frac{K^\lambda}{K^2}+\frac{k_1^\lambda}{k_1^2-M_1^2}\right)p_{1\lambda}+2\left(\frac{K^\lambda}{K^2}+\frac{k_2^\lambda}{k_2^2-M_2^2}\right)p_{2\lambda}+\cdots\right]\,.
\end{aligned}
\tag{3.13}
$$

Therefore,

$$
\begin{aligned}
A_1\,p_1^\kappa &= 2p_{1\lambda}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\left(\frac{K^\lambda}{K^2}+\frac{k_1^\lambda}{k_1^2-M_1^2}\right)\,,\\
A_2\,p_2^\kappa &= 2p_{2\lambda}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\left(\frac{K^\lambda}{K^2}+\frac{k_2^\lambda}{k_2^2-M_2^2}\right)\,.
\end{aligned}
\tag{3.14}
$$

The integrals on the right-hand side can be nothing but $g^{\kappa\lambda}$ times a scalar function:

$$
\begin{aligned}
\tilde{A}_1\,g^{\kappa\lambda} &\equiv 2\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\left(\frac{K^\lambda}{K^2}+\frac{k_1^\lambda}{k_1^2-M_1^2}\right)\,,\\
\tilde{A}_2\,g^{\kappa\lambda} &\equiv 2\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{K^\kappa}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\left(\frac{K^\lambda}{K^2}+\frac{k_2^\lambda}{k_2^2-M_2^2}\right)\,,
\end{aligned}
$$

(3.15)

and it is easy to see that $A_1 = \tilde{A}_1$, and $A_2 = \tilde{A}_2$. Acting on both sides of the equation with $g_{\kappa\lambda}$, we find

$$\begin{aligned}
4\tilde{A}_1 &= 2\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\left(1+\frac{K\cdot k_1}{k_1^2-M_1^2}\right)\,, \\
4\tilde{A}_2 &= 2\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\left(1+\frac{K\cdot k_2}{k_2^2-M_2^2}\right)\,.
\end{aligned} \tag{3.16}$$

Therefore,

$$\begin{aligned}
&(A_1-A_2) \\
&= (\tilde{A}_1-\tilde{A}_2) \\
&= \frac{1}{2}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\left(\frac{K\cdot k_1}{k_1^2-M_1^2}-\frac{K\cdot k_2}{k_2^2-M_2^2}\right)\,.
\end{aligned} \tag{3.17}$$

Using

$$\begin{aligned}
2K\cdot k_1 &= K^2+k_1^2-k_2^2\,, \\
2K\cdot k_2 &= K^2-k_1^2+k_2^2\,,
\end{aligned} \tag{3.18}$$

we find

$$\begin{aligned}
(A_1-A_2) &= \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\quad\times\left[K^2\left\{\frac{1}{(k_1^2-M_1^2)}-\frac{1}{(k_2^2-M_2^2)}\right\}+(k_1^2-k_2^2)\left\{\frac{1}{(k_1^2-M_1^2)}+\frac{1}{(k_2^2-M_2^2)}\right\}\right] \\
&= \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{K^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad\times\left[\frac{1}{(k_1^2-M_1^2)}-\frac{1}{(k_2^2-M_2^2)}\right] \\
&\quad+\frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{(k_1^2-k_2^2)}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad\times\left[\frac{1}{(k_1^2-M_1^2)}+\frac{1}{(k_2^2-M_2^2)}\right]
\end{aligned} \tag{3.19}$$

Define:

$$\begin{aligned}
X &\equiv \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{K^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad\times\left[\frac{1}{(k_1^2-M_1^2)}-\frac{1}{(k_2^2-M_2^2)}\right]\,, \\
Y &\equiv \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{(k_1^2-k_2^2)}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad\times\left[\frac{1}{(k_1^2-M_1^2)}+\frac{1}{(k_2^2-M_2^2)}\right]\,, \\
&.
\end{aligned} \tag{3.20}$$

The same integrals will show up in the B function so let's look at the B's:

$$B_0 g^{\kappa\lambda}+B_1\,p_1^\kappa p_1^\lambda+B_2\,p_2^\kappa p_2^\lambda+B_3\,(p_1^\kappa p_2^\lambda+p_2^\kappa p_1^\lambda)$$

$$
\begin{aligned}
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(p_1-K)^2(p_2-K)^2(k_1^2-M_W^2)(k_2^2-M_W^2)[(p_1-k_1)^2-M_1^2][(p_2-k_2)^2-M_2^2]} \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(K^2-2p_1\cdot K)(K^2-2p_2\cdot K)(k_1^2-M_W^2)(k_2^2-M_W^2)[(k_1^2-M_1^2)-2p_1\cdot k_1]} \\
&\qquad \times \frac{1}{[(k_2^2-M_2^2)-2p_2\cdot k_2]} \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\quad \times \left[1+\frac{2p_1\cdot K}{K^2}+\frac{(2p_1\cdot K)^2}{(K^2)^2}+\cdots\right]\left[1+\frac{2p_2\cdot K}{K^2}+\frac{(2p_2\cdot K)^2}{(K^2)^2}+\cdots\right] \\
&\quad \times \left[1+\frac{2p_1\cdot k_1}{k_1^2-M_1^2}+\frac{(2p_1\cdot k_1)^2}{(k_1^2-M_1^2)^2}+\cdots\right]\left[1+\frac{2p_2\cdot k_2}{k_2^2-M_2^2}+\frac{(2p_2\cdot k_2)^2}{(k_2^2-M_2^2)^2}+\cdots\right] \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\quad \times \Bigg[1+(\text{terms linear in } p_1 \text{ and } p_2) \\
&\qquad +4p_{1\mu}p_{2\nu}\left\{\frac{K^\mu K^\nu}{(K^2)^2}+\frac{k_1^\mu k_2^\nu}{(k_1^2-M_1^2)(k_2^2-M_2^2)}+\frac{k_1^\mu K^\nu}{(k_1^2-M_1^2)K^2}+\frac{K^\mu k_2^\nu}{K^2(k_2^2-M_2^2)}\right\} \\
&\qquad +4p_{1\mu}p_{1\nu}\left\{\frac{K^\mu K^\nu}{(K^2)^2}+\frac{K^\mu k_1^\nu}{K^2(k_1^2-M_1^2)}+\frac{k_1^\mu k_1^\nu}{(k_1^2-M_1^2)^2}\right\} \\
&\qquad +4p_{2\mu}p_{2\nu}\left\{\frac{K^\mu K^\nu}{(K^2)^2}+\frac{K^\mu k_2^\nu}{K^2(k_2^2-M_2^2)}+\frac{k_2^\mu k_2^\nu}{(k_2^2-M_2^2)^2}\right\}+\cdots\Bigg] .
\end{aligned}
\tag{3.21}
$$

Comparing both sides, we can conclude that

$$
\begin{aligned}
&B_1\, p_1^\kappa p_1^\lambda \\
&\quad = 4p_{1\mu}p_{1\nu}\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad\qquad \times\left[\frac{K^\mu K^\nu}{(K^2)^2}+\frac{K^\mu k_1^\nu}{K^2(k_1^2-M_1^2)}+\frac{k_1^\mu k_1^\nu}{(k_1^2-M_1^2)^2}\right] \\
&B_2\, p_2^\kappa p_2^\lambda \\
&\quad = 4p_{2\mu}p_{2\nu}\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad\qquad \times\left[\frac{K^\mu K^\nu}{(K^2)^2}+\frac{K^\mu k_2^\nu}{K^2(k_2^2-M_2^2)}+\frac{k_2^\mu k_2^\nu}{(k_2^2-M_2^2)^2}\right]
\end{aligned}
\tag{3.22}
$$

The first term in the brackets is symmetric in M_1 and M_2, so it will not contribute to the EDM.

Take the difference

$$
\begin{aligned}
&B_1\, p_1^\kappa p_1^\lambda - B_2\, p_2^\kappa p_2^\lambda \\
&\quad = 4\Bigg[p_{1\mu}p_{1\nu}\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad\qquad \times\left[\frac{K^\mu k_1^\nu}{K^2(k_1^2-M_1^2)}+\frac{k_1^\mu k_1^\nu}{(k_1^2-M_1^2)^2}\right] \\
&\qquad -p_{2\mu}p_{2\nu}\int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad\qquad \times\left[\frac{K^\mu k_2^\nu}{K^2(k_2^2-M_2^2)}+\frac{k_2^\mu k_2^\nu}{(k_2^2-M_2^2)^2}\right]\Bigg]
\end{aligned}
\tag{3.23}
$$

Notice that the B_2 terms are the same as B_1 after interchanging $M_1 \to M_2$ and $k_1 \to k_2$.

Consider the integrals for B_1.The result can only be some scalar functions multiplied by some combination of g's

$$\begin{aligned} & B^{'} g^{\kappa\lambda} g^{\mu\nu} + B^{''} g^{\kappa\mu} g^{\lambda\nu} + B^{'''} g^{\kappa\nu} g^{\lambda\mu} \\ = & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^\kappa K^\lambda}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\ & \times \left[\frac{K^\mu k_1^\nu}{K^2(k_1^2 - M_1^2)} + \frac{k_1^\mu k_1^\nu}{(k_1^2 - M_1^2)^2} \right] \end{aligned} \tag{3.24}$$

If we plug that back in:

$$\begin{aligned} B_1\, p_1^\kappa p_1^\lambda &= 4 p_{1\mu} p_{1\nu} \left[B^{'} g^{\kappa\lambda} g^{\mu\nu} + B^{''} g^{\kappa\mu} g^{\lambda\nu} + B^{'''} g^{\kappa\nu} g^{\lambda\mu} \right] \\ &= 4 \left[B^{'} g^{\kappa\lambda} m^2 + B^{''} p_1^\kappa p_1^\lambda + B^{'''} p_1^\kappa p_1^\lambda \right]. \end{aligned} \tag{3.25}$$

Notice that the $B^{'}$ piece belongs to the expression for B_0 so when we evaluate our integrals we need to subtract it out. the rest will give B_1

$$B_1 = 4(B^{''} + B^{'''}). \tag{3.26}$$

To figure out the B's we contract both sides with $g_{\kappa\lambda} g_{\mu\nu}$, $g_{\kappa\mu} g_{\lambda\nu}$ and $g_{\kappa\nu} g_{\lambda\mu}$.
Contract with $g_{\kappa\lambda} g_{\mu\nu}$

$$\begin{aligned} & 16B^{'} + 4B^{''} + 4B^{'''} \\ = & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^2}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\ & \times \left[\frac{K \cdot k_1}{K^2(k_1^2 - M_1^2)} + \frac{k_1^2}{(k_1^2 - M_1^2)^2} \right] \end{aligned} \tag{3.27}$$

Contract with $g_{\kappa\mu} g_{\lambda\nu}$

$$\begin{aligned} & 4B^{'} + 16B^{''} + 4B^{'''} \\ = & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\ & \times \left[\frac{K^2 K \cdot k_1}{K^2(k_1^2 - M_1^2)} + \frac{(K \cdot k_1)^2}{(k_1^2 - M_1^2)^2} \right] \end{aligned} \tag{3.28}$$

Contract with $g_{\kappa\nu} g_{\lambda\mu}$

$$\begin{aligned} & 4B^{'} + 4B^{''} + 16B^{'''} \\ = & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\ & \times \left[\frac{K^2 K \cdot k_1}{K^2(k_1^2 - M_1^2)} + \frac{(K \cdot k_1)^2}{(k_1^2 - M_1^2)^2} \right] \end{aligned} \tag{3.29}$$

Subtract (3.29) from (3.28) gives

$$B'' = B''' \tag{3.30}$$

plug back in (3.27) and (3.28)

$$\begin{aligned}
16B^{'} + 8B^{''} & \\
= & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^2}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\
& \times \left[\frac{K \cdot k_1}{K^2(k_1^2 - M_1^2)} + \frac{k_1^2}{(k_1^2 - M_1^2)^2}\right] \\
4B^{'} + 20B^{''} & \\
= & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\
& \times \left[\frac{K^2 K \cdot k_1}{K^2(k_1^2 - M_1^2)} + \frac{(K \cdot k_1)^2}{(k_1^2 - M_1^2)^2}\right]
\end{aligned} \tag{3.31}$$

Multiply the second equation by 4

$$\begin{aligned}
16B^{'} + 8B^{''} & \\
= & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{K^2}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\
& \times \left[\frac{K \cdot k_1}{K^2(k_1^2 - M_1^2)} + \frac{k_1^2}{(k_1^2 - M_1^2)^2}\right] \\
16B^{'} + 80B^{''} & \\
= & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\
& \times \left[\frac{4K^2 K \cdot k_1}{K^2(k_1^2 - M_1^2)} + \frac{4(K \cdot k_1)^2}{(k_1^2 - M_1^2)^2}\right]
\end{aligned} \tag{3.32}$$

Subtract the first line from the second line

$$\begin{aligned}
72B^{''} & \\
= & \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\
& \times \left[\frac{3K \cdot k_1}{(k_1^2 - M_1^2)} + \frac{4(K \cdot k_1)^2 - K^2 k_1^2}{(k_1^2 - M_1^2)^2}\right]
\end{aligned} \tag{3.33}$$

So B_1 becomes

$$\begin{aligned}
B_1 = 8B^{''} & \\
= & \frac{1}{9} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\
& \times \left[\frac{3K \cdot k_1}{(k_1^2 - M_1^2)} + \frac{4(K \cdot k_1)^2 - K^2 k_1^2}{(k_1^2 - M_1^2)^2}\right]
\end{aligned} \tag{3.34}$$

B_2 is the same as B_1 after interchanging $M_1 \to M_2$ and $k_1 \to k_2$.

$$\begin{aligned}
B_1 - B_2 & \\
= & \frac{1}{9} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)} \\
& \times \left[\frac{3K \cdot k_1}{(k_1^2 - M_1^2)} + \frac{4(K \cdot k_1)^2 - K^2 k_1^2}{(k_1^2 - M_1^2)^2}\right] \\
- & \frac{1}{9} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(K^2)^2(k_1^2 - M_W^2)(k_2^2 - M_W^2)(k_1^2 - M_1^2)(k_2^2 - M_2^2)}
\end{aligned}$$

$$\times\left[\frac{3K\cdot k_2}{(k_2^2-M_2^2)}+\frac{4(K\cdot k_2)^2-K^2k_2^2}{(k_2^2-M_2^2)^2}\right]. \tag{3.35}$$

Expanding the expression

$$\begin{aligned}
B_1-B_2 & \\
= & \frac{1}{6}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{K^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{1}{(k_1^2-M_1^2)}-\frac{1}{(k_2^2-M_2^2)}\right]\\
+ & \frac{1}{6}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{(k_1^2-k_2^2)}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{1}{(k_1^2-M_1^2)}+\frac{1}{(k_2^2-M_2^2)}\right]\\
+ & \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{1}{(k_1^2-M_1^2)^2}-\frac{1}{(k_2^2-M_2^2)^2}\right]\\
+ & \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{K^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{k_1^2}{(k_1^2-M_1^2)^2}-\frac{k_2^2}{(k_2^2-M_2^2)^2}\right]\\
- & \frac{2}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{K^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{k_2^2}{(k_1^2-M_1^2)^2}-\frac{k_1^2}{(k_2^2-M_2^2)^2}\right]\\
+ & \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{(k_1^2-k_2^2)}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{k_1^2}{(k_1^2-M_1^2)^2}+\frac{k_2^2}{(k_2^2-M_2^2)^2}\right]\\
- & \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{(k_1^2-k_2^2)}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{k_2^2}{(k_1^2-M_1^2)^2}+\frac{k_1^2}{(k_2^2-M_2^2)^2}\right].
\end{aligned} \tag{3.36}$$

Now define:

$$\begin{aligned}
X \equiv & \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{K^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{1}{(k_1^2-M_1^2)}-\frac{1}{(k_2^2-M_2^2)}\right],\\
Y \equiv & \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{(k_1^2-k_2^2)}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{1}{(k_1^2-M_1^2)}+\frac{1}{(k_2^2-M_2^2)}\right],\\
W \equiv & \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)}\\
& \times\left[\frac{1}{(k_1^2-M_1^2)^2}-\frac{1}{(k_2^2-M_2^2)^2}\right],
\end{aligned}$$

$$\begin{aligned}
Z &\equiv \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{K^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2-M_1^2)^2}-\frac{k_2^2}{(k_2^2-M_2^2)^2}\right] , \\
Z^{'} &\equiv \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{K^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad \times\left[\frac{k_2^2}{(k_1^2-M_1^2)^2}-\frac{k_1^2}{(k_2^2-M_2^2)^2}\right] , \\
V &\equiv \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{(k_1^2-k_2^2)}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2-M_1^2)^2}+\frac{k_2^2}{(k_2^2-M_2^2)^2}\right] , \\
V^{'} &\equiv \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{(k_1^2-k_2^2)}{(K^2)^2(k_1^2-M_W^2)(k_2^2-M_W^2)(k_1^2-M_1^2)(k_2^2-M_2^2)} \\
&\qquad \times\left[\frac{k_2^2}{(k_1^2-M_1^2)^2}+\frac{k_1^2}{(k_2^2-M_2^2)^2}\right] \\
&.
\end{aligned} \tag{3.37}$$

$$B_1-B_2=\frac{1}{6}(X+Y)+\frac{1}{9}W+\frac{1}{9}Z-\frac{2}{9}Z^{'}+\frac{1}{9}V-\frac{1}{9}V^{'} . \tag{3.38}$$

Now Wick rotating and expanding in Gegenbauer polynomials (4-D analogous of the Legendre expansion) ,we replace:

$$\begin{aligned}
\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{K^2} \;\longrightarrow\; & \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\,k_1^2\int_0^\infty dk_2^2\,k_2^2\,\frac{1}{k_>^2}=\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\,k_<^2 \\
= & \frac{1}{(4\pi)^4}\left[\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2+\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\,k_1^2\right] .
\end{aligned} \tag{3.39}$$

and

$$\begin{aligned}
& \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{(k_1^2-k_2^2)}{(K^2)^2} \;\longrightarrow \\
& \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\,k_1^2\int_0^\infty dk_2^2\,k_2^2\,\frac{(k_1^2-k_2^2)}{k_>^2(k_>^2-k_<^2)} \\
= & \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\,\frac{k_<^2(k_1^2-k_2^2)}{(k_>^2-k_<^2)} \\
= & \frac{1}{(4\pi)^4}\left[\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,\frac{k_2^2(k_1^2-k_2^2)}{(k_1^2-k_2^2)}+\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\,\frac{k_1^2(k_1^2-k_2^2)}{(k_2^2-k_1^2)}\right] \\
= & \frac{1}{(4\pi)^4}\left[\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2-\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\,k_1^2\right] .
\end{aligned} \tag{3.40}$$

So the integrations become:

$$\begin{aligned}
X &= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
&\qquad \times\left[\frac{1}{(k_1^2+M_1^2)}-\frac{1}{(k_2^2+M_2^2)}\right]
\end{aligned}$$

$$
\begin{aligned}
& + \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{1}{(k_1^2+M_1^2)} - \frac{1}{(k_2^2+M_2^2)}\right] , \\
Y = & \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{1}{(k_1^2+M_1^2)} + \frac{1}{(k_2^2+M_2^2)}\right] \\
& - \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{1}{(k_1^2+M_1^2)} + \frac{1}{(k_2^2+M_2^2)}\right] , \\
W = & \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{\infty} dk_2^2\; k_2^2 k_1^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{1}{(k_1^2+M_1^2)^2} - \frac{1}{(k_2^2+M_2^2)^2}\right] , \\
Z = & \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{k_1^2}{(k_1^2+M_1^2)^2} - \frac{k_2^2}{(k_2^2+M_2^2)^2}\right] \\
& + \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{k_1^2}{(k_1^2+M_1^2)^2} - \frac{k_2^2}{(k_2^2+M_2^2)^2}\right] , \\
Z' = & \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{k_2^2}{(k_1^2+M_1^2)^2} - \frac{k_1^2}{(k_2^2+M_2^2)^2}\right] \\
& + \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{k_2^2}{(k_1^2+M_1^2)^2} - \frac{k_1^2}{(k_2^2+M_2^2)^2}\right] , \\
V = & \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{k_1^2}{(k_1^2+M_1^2)^2} + \frac{k_2^2}{(k_2^2+M_2^2)^2}\right] \\
& - \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{k_1^2}{(k_1^2+M_1^2)^2} + \frac{k_2^2}{(k_2^2+M_2^2)^2}\right] , \\
V' = & \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
& \qquad \times \left[\frac{k_2^2}{(k_1^2+M_1^2)^2} + \frac{k_1^2}{(k_2^2+M_2^2)^2}\right] \\
& - \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}
\end{aligned}
$$

$$\times\left[\frac{k_2^2}{(k_1^2+M_1^2)^2}+\frac{k_1^2}{(k_2^2+M_2^2)^2}\right] . \tag{3.41}$$

to simplify the expression, first we look at $X+Y$:

$$\begin{aligned}
X+Y &= \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\; k_2^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\left[\frac{1}{(k_1^2+M_1^2)}\right]\\
&- \frac{-2}{(4\pi)^4}\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\; k_1^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\left[\frac{1}{(k_2^2+M_2^2)}\right]\\
&= \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_W^2)(k_1^2+M_1^2)^2}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&- \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_W^2)(k_1^2+M_2^2)^2}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)} ,
\end{aligned} \tag{3.42}$$

Now $Z+V$

$$\begin{aligned}
Z+V &= \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\; k_2^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\left[\frac{k_1^2}{(k_1^2+M_1^2)^2}\right]\\
&- \frac{-2}{(4\pi)^4}\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\; k_1^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\left[\frac{k_2^2}{(k_2^2+M_2^2)^2}\right] ,\\
&= \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&- \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)} ,
\end{aligned} \tag{3.43}$$

What's left is $2Z^{'}+V^{'}$:

$$\begin{aligned}
2Z^{'}+V^{'} &= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\; k_2^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\\
&\qquad\times\left[\frac{2k_2^2}{(k_1^2+M_1^2)^2}-\frac{2k_1^2}{(k_2^2+M_2^2)^2}\right]\\
&+ \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\; k_1^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\\
&\qquad\times\left[\frac{2k_2^2}{(k_1^2+M_1^2)^2}-\frac{2k_1^2}{(k_2^2+M_2^2)^2}\right] ,\\
&+ \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\; k_2^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\\
&\qquad\times\left[\frac{k_2^2}{(k_1^2+M_1^2)^2}+\frac{k_1^2}{(k_2^2+M_2^2)^2}\right]\\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\; k_1^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\\
&\qquad\times\left[\frac{k_2^2}{(k_1^2+M_1^2)^2}+\frac{k_1^2}{(k_2^2+M_2^2)^2}\right]
\end{aligned}$$

$$\begin{aligned}
&= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
&\qquad\qquad \times\left[\frac{3k_2^2}{(k_1^2+M_1^2)^2} - \frac{k_1^2}{(k_2^2+M_2^2)^2}\right] \\
&+ \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)} \\
&\qquad\qquad \times\left[\frac{k_2^2}{(k_1^2+M_1^2)^2} - \frac{3k_1^2}{(k_2^2+M_2^2)^2}\right] \\
&= \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\; \frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_2^2)} \\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)}\int_0^{k_1^2} dk_2^2\; \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3} \\
&+ \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2\, \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\int_0^{k_2^2} dk_1^2\; \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3} \\
&- \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3}\int_0^{k_2^2} dk_1^2\; \frac{k_1^4}{(k_1^2+M_W^2)(k_1^2+M_1^2)}\,,
\end{aligned} \tag{3.44}$$

$$\begin{aligned}
2Z^{'}+V^{'} &= \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\; \frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_2^2)} \\
&- \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\; \frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_1^2)} \\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)}\int_0^{k_1^2} dk_2^2\; \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3} \\
&+ \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)}\int_0^{k_1^2} dk_2^2\; \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)^3}
\end{aligned} \tag{3.45}$$

Combine:

$$\begin{aligned}
Z+V-2Z^{'}-V^{'} &= \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\; \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)} \\
&- \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\; \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)} \\
&- \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\; \frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_2^2)} \\
&+ \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\; \frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_1^2)} \\
&+ \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)}\int_0^{k_1^2} dk_2^2\; \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3} \\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)}\int_0^{k_1^2} dk_2^2\; \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)^3}
\end{aligned} \tag{3.46}$$

Now W:

$$\begin{aligned}
W &= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\; k_2^2k_1^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)}\\
&\qquad\qquad\times\left[\frac{1}{(k_1^2+M_1^2)^2}-\frac{1}{(k_2^2+M_2^2)^2}\right]\\
&= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\; k_2^2k_1^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)^3(k_2^2+M_2^2)}\\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\; k_2^2k_1^2\,\frac{1}{(k_1^2+M_W^2)(k_2^2+M_W^2)(k_1^2+M_1^2)(k_2^2+M_2^2)^3}\\
&= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^\infty dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)}\int_0^\infty dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3}\\
&= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^\infty dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^\infty dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)}\\
&= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&+ \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2\,\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\int_0^{k_2^2} dk_1^2\;\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)}\\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2\,\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)}\int_0^{k_2^2} dk_1^2\;\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\;. \qquad (3.47)
\end{aligned}$$

So

$$\begin{aligned}
W &= \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&+ \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)^3}\\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)}\\
&- \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3},
\end{aligned} \qquad (3.48)$$

and

$$\begin{aligned}
W+Z+V-2Z^{'}-V^{'} &= \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&- \frac{-2}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)}\\
&- \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&+ \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_1^2)}
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3}\\
&- \quad \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)^3}\\
&+ \quad \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&+ \quad \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)^3}\\
&- \quad \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)}\\
&- \quad \frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3},
\end{aligned}
\tag{3.49}
$$

$$
\begin{aligned}
W+Z+V-2Z^{'}-V^{'} \quad &= \quad \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&- \quad \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)}\\
&- \quad \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_2^2)}\\
&+ \quad \frac{-3}{(4\pi)^4}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_W^2)(k_1^2+M_2^2)^3}\int_0^{k_1^2} dk_2^2\;\frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_1^2)}
\end{aligned}
\tag{3.50}
$$

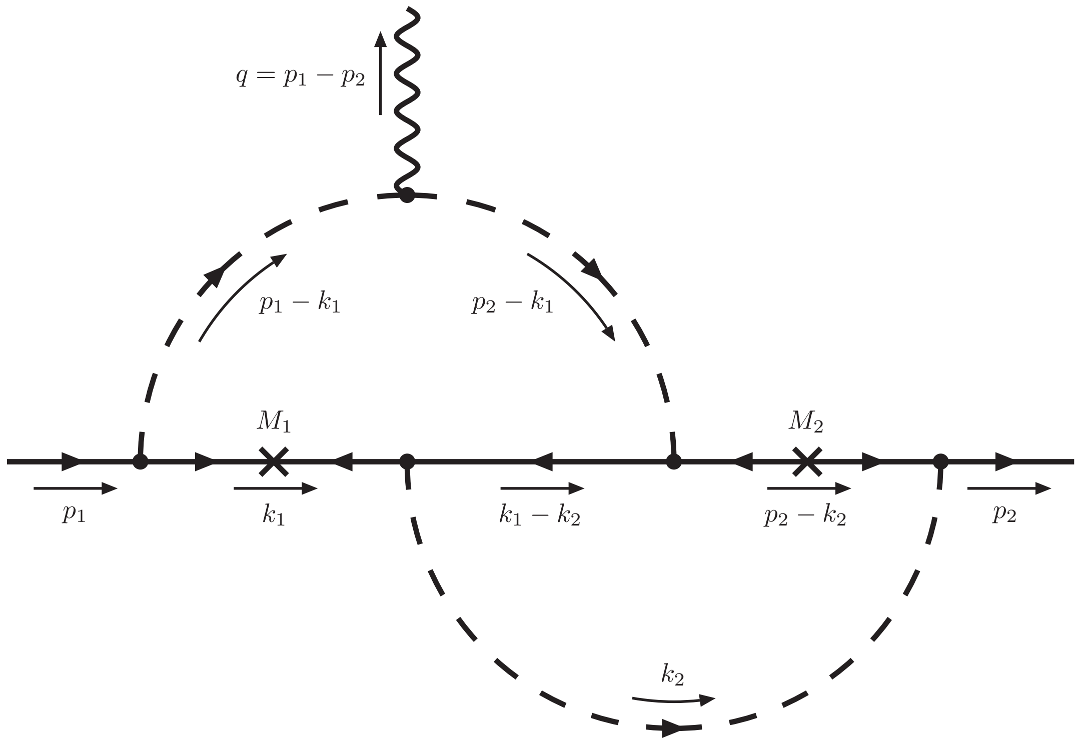


Figure 3.7: **Diagram 2B**

3.3.2 Diagram 2B

The $\gamma\phi\phi$ vertex is

$$(-ie)\left\{(p_1 - k_1)_\lambda - (-p_2 + k_1)_\lambda\right\} = (-ie)\left(p_1 + p_2 - 2k_1\right)_\lambda \ . \tag{3.51}$$

The coupling constant, $(-ie)$, and those along the fermion line combine to the factor

$$(-ie)\left(-i\tilde{\Lambda}_{\alpha 2}\right)\left(-i\tilde{\Lambda}_{\beta 2}\right)\left(-i\tilde{\Lambda}^\dagger_{1\beta}\right)\left(-i\tilde{\Lambda}^\dagger_{1\alpha}\right) = -ie\left(\tilde{\Lambda}_{\alpha 2}\tilde{\Lambda}_{\beta 2}\tilde{\Lambda}^*_{\beta 1}\tilde{\Lambda}^*_{\alpha 1}\right) \ . \tag{3.52}$$

The six propagators will contribute $i^6 = -1$ and get rid of the minus sign.

The fermion line gives

$$M_1 M_2\, \bar{u}(p_2)(\not{k}_1 - \not{k}_2) P_L u(p_1). \tag{3.53}$$

Define:

$$\begin{aligned}
&C_1 p_1^\kappa + C_2 p_2^\kappa \\
&\quad= \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\, \frac{(k_1-k_2)^\kappa}{[(p_2-k_2)^2 - M_2^2](k_1-k_2)^2(k_1^2 - M_1^2)[(p_1-k_1)^2 - M_W^2]} \\
&\qquad\qquad\times\frac{1}{[(p_2-k_1)^2 - M_W^2](k_2^2 - M_W^2)} \\
&D_0 g^{\kappa\mu} + D_{11}p_1^\kappa p_1^\mu + D_{22}p_2^\kappa p_2^\mu + D_{12}p_1^\kappa p_2^\mu + D_{21}p_2^\kappa p_1^\mu \\
&\quad= \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\, \frac{k_1^\kappa (k_1-k_2)^\mu}{[(p_2-k_2)^2 - M_2^2](k_1-k_2)^2(k_1^2 - M_1^2)[(p_1-k_1)^2 - M_W^2]}
\end{aligned}$$

$$\times \frac{1}{[(p_2-k_1)^2-M_W^2](k_2^2-M_W^2)} \tag{3.54}$$

so what we have becomes:

$$\begin{aligned} & M_1M_2\,\bar{u}(p_2)(\not{k}_1-\not{k}_2)(p_1+p_2-2k_1)_\lambda P_L u(p_1) \\ = \; & M_1M_2\,\bar{u}(p_2)(k_1-k_2)^\kappa\gamma_\kappa(p_1+p_2)_\lambda P_L u(p_1) \\ - \; & 2M_1M_2\,\bar{u}(p_2)(k_1-k_2)^\mu\gamma_\mu(k_1)^\kappa g_{\kappa\lambda} P_L u(p_1)\,. \end{aligned} \tag{3.55}$$

After integrations we can replace the k's with the p's:

$$\begin{aligned} = \; & M_1M_2\,\bar{u}(p_2)(C_1p_1^\kappa+C_2p_2^\kappa)\gamma_\kappa(p_1+p_2)_\lambda P_L u(p_1) \\ - \; & 2M_1M_2\,\bar{u}(p_2)(D_0g^{\kappa\mu}+D_{11}p_1^\kappa p_1^\mu+D_{22}p_2^\kappa p_2^\mu+D_{12}p_1^\kappa p_2^\mu+D_{21}p_2^\kappa p_1^\mu)\gamma_\mu g_{\kappa\lambda} P_L u(p_1)\,. \end{aligned} \tag{3.56}$$

Collect only the terms that depend on p^λ and neglecting the rest:

$$\begin{aligned} = \; & M_1M_2(p_1+p_2)_\lambda\,\bar{u}(p_2)(C_1\not{p}_1+C_2\not{p}_2)P_L u(p_1) \\ - \; & 2M_1M_2\,\bar{u}(p_2)(D_{11}p_{1\lambda}\not{p}_1+D_{22}p_{2\lambda}\not{p}_2+D_{12}p_{1\lambda}\not{p}_2+D_{21}p_{2\lambda}\not{p}_1)P_L u(p_1)\,. \end{aligned} \tag{3.57}$$

$$\begin{aligned} = \; & M_1M_2(p_1+p_2)_\lambda\,\bar{u}(p_2)(C_1\not{p}_1)P_L u(p_1) \\ + \; & M_1M_2(p_1+p_2)_\lambda\,\bar{u}(p_2)(C_2\not{p}_2)P_L u(p_1) \\ - \; & 2M_1M_2\,\bar{u}(p_2)(D_{11}p_{1\lambda}\not{p}_1+D_{21}p_{2\lambda}\not{p}_1)P_L u(p_1) \\ - \; & 2M_1M_2\,\bar{u}(p_2)(D_{22}p_{2\lambda}\not{p}_2+D_{12}p_{1\lambda}\not{p}_2)P_L u(p_1)\,. \end{aligned} \tag{3.58}$$

$$\begin{aligned} = M_1M_2m\,[\,p_{1\lambda} \quad & (\,(C_2-2D_{12})\,\bar{u}(p_2)P_L u(p_1) \\ + \quad & (C_1-2D_{11})\,\bar{u}(p_2)P_R u(p_1)\;\;) \\ +p_{2\lambda} \quad & (\,(C_2-2D_{22})\,\bar{u}(p_2)P_L u(p_1) \\ + \quad & (C_1-2D_{21})\,\bar{u}(p_2)P_R u(p_1)\;\;)\,]\,. \end{aligned} \tag{3.59}$$

Collect the γ^5 terms:

$$\begin{aligned} = M_1M_2m\bar{u}(p_2)\,[\,p_{1\lambda} \quad & (\,(C_2-2D_{12})\,(-\frac{1}{2}\gamma^5) \\ + \quad & (C_1-2D_{11})\,(\frac{1}{2}\gamma^5)\;\;) \\ +p_{2\lambda} \quad & (\,(C_2-2D_{22})\,(-\frac{1}{2}\gamma^5) \\ + \quad & (C_1-2D_{21})\,(\frac{1}{2}\gamma^5)\;\;)\,]\,u(p_1) \\ = M_1M_2m\bar{u}(p_2)(-\frac{1}{2}\gamma^5)u(p_1)\,[\,p_{1\lambda} \quad & (\,(C_2-2D_{12})-(C_1-2D_{11})\;\;) \\ +p_{2\lambda} \quad & (\,(C_2-2D_{22})-(C_1-2D_{21})\,)\,] \\ = M_1M_2m\bar{u}(p_2)(-\frac{1}{2}\gamma^5)u(p_1)\,[\,p_{1\lambda} \quad & (\,(C_2-C_1-2D_{12}+2D_{11})\;\;) \\ +p_{2\lambda} \quad & (\,(C_2-C_1-2D_{22}+2D_{21})\,)\,] \\ = M_1M_2m\bar{u}(p_2)(-\frac{1}{2}\gamma^5)u(p_1)\,[\,\frac{1}{2}(p_{1\lambda}+p_{2\lambda}) \quad & (\;2C_2-2C_1-2D_{12}+2D_{21}+2D_{11}-2D_{22}\;) \\ +\frac{1}{2}(p_{1\lambda}-p_{2\lambda}) \quad & (\;-2D_{12}-2D_{21}+2D_{11}+2D_{22}\;)\,]\,. \end{aligned} \tag{3.60}$$

Since $(p_{1\lambda}-p_{2\lambda})$ is zero on-shell; the last equation becomes:

$$M_1M_2m\bar{u}(p_2)(-\frac{1}{2}\gamma^5)u(p_1)\,[\,\frac{1}{2}(p_{1\lambda}+p_{2\lambda})\,(\;2C_2-2C_1-2D_{12}+2D_{21}+2D_{11}-2D_{22}\;)\,]$$

$$\begin{aligned}
&= \frac{1}{2} M_1 M_2 m \bar{u}(p_2)(\gamma^5) u(p_1)(p_{1\lambda}+p_{2\lambda})\left[\ (\ C_1 - C_2 + D_{12} - D_{21} - D_{11} + D_{22}\ )\ \right] \\
&= \frac{1}{2} M_1 M_2 m \bar{u}(p_2)(\gamma^5) u(p_1)(p_{1\lambda}+p_{2\lambda})\left[(C_1 - C_2) + (D_{12} - D_{21}) - (D_{11} - D_{22})\right].
\end{aligned} \tag{3.61}$$

Now we evaluate the C and D functions.

The C functions

$$\begin{aligned}
&C_1 p_1^\kappa + C_2 p_2^\kappa \\
&\quad = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{(k_1-k_2)^\kappa}{[(p_2-k_2)^2 - M_2^2](k_1-k_2)^2(k_1^2-M_1^2)[(p_1-k_1)^2-M_W^2]} \\
&\qquad\qquad \times \frac{1}{[(p_2-k_1)^2 - M_W^2](k_2^2-M_W^2)} \\
&\quad = \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{(k_1-k_2)^\kappa}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^2(k_2^2-M_W^2)} \\
&\qquad \times\left[1+\frac{2(p_2\cdot k_2)}{k_2^2-M_2^2}+\cdots\right]\left[1+\frac{2(p_1\cdot k_1)}{k_1^2-M_W^2}+\cdots\right]\left[1+\frac{2(p_2\cdot k_1)}{k_1^2-M_W^2}+\cdots\right]
\end{aligned} \tag{3.62}$$

Comparing the terms we can write:

$$\begin{aligned}
&C_1 p_1^\kappa \\
&\quad = 2p_{1\mu} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\mu (k_1-k_2)^\kappa}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)} \\
&C_2 p_2^\kappa \\
&\quad = 2p_{2\mu} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_2^\mu (k_1-k_2)^\kappa}{(k_2^2-M_2^2)^2(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^2(k_2^2-M_W^2)} \\
&\quad + 2p_{2\mu} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\mu (k_1-k_2)^\kappa}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)}
\end{aligned} \tag{3.63}$$

Since the integrals can only be a scalar multiplied by $g^{\mu\kappa}$

$$\begin{aligned}
g^{\mu\kappa} C_1^{'} &= 2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\mu (k_1-k_2)^\kappa}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)} \\
g^{\mu\kappa} C_2^{'} &= 2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_2^\mu (k_1-k_2)^\kappa}{(k_2^2-M_2^2)^2(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^2(k_2^2-M_W^2)} \\
&+ 2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\mu (k_1-k_2)^\kappa}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)}
\end{aligned} \tag{3.64}$$

Contracting both sides with $g^{\mu\kappa}$

$$\begin{aligned}
4C_1^{'} &= 2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)} \\
4C_2^{'} &= 2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_2\cdot(k_1-k_2)}{(k_2^2-M_2^2)^2(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^2(k_2^2-M_W^2)} \\
&+ 2 \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)}
\end{aligned} \tag{3.65}$$

Plug back

$$C_1 = \frac{1}{2} \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)}$$

$$\begin{aligned}
C_2 &= \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_2\cdot(k_1-k_2)}{(k_2^2-M_2^2)^2(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^2(k_2^2-M_W^2)} \\
&+ \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)}
\end{aligned} \tag{3.66}$$

Let's define $K = k_1 - k_2$ and rearrange the terms:

$$\begin{aligned}
C_1 &= \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot K}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)} \\
C_2 &= \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_2\cdot K}{(k_2^2-M_2^2)^2(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^2(k_2^2-M_W^2)} \\
&+ \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot K}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)^3(k_2^2-M_W^2)}
\end{aligned} \tag{3.67}$$

$$\begin{aligned}
C_1 &= \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1\cdot K}{(k_1^2-M_W^2)^2}\right] \\
C_2 &= \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_2\cdot K}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right] \\
&+ \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1\cdot K}{(k_1^2-M_W^2)^2}\right]
\end{aligned} \tag{3.68}$$

Since:

$$\begin{aligned}
K &= k_1 - k_2 \\
K - k_1 &= -k_2 \\
K^2 + k_1^2 - 2k_1\cdot K &= k_2^2 \\
k_1\cdot K &= \frac{1}{2}(K^2 + k_1^2 - k_2^2)\ ,
\end{aligned} \tag{3.69}$$

and

$$\begin{aligned}
K &= k_1 - k_2 \\
K + k_2 &= k_1 \\
K^2 + k_2^2 + 2k_2\cdot K &= k_1^2 \\
k_2\cdot K &= \frac{1}{2}(k_1^2 - k_2^2 - K^2)\ ,
\end{aligned} \tag{3.70}$$

$$\begin{aligned}
C_1 &= \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{K^2+k_1^2-k_2^2}{(k_1^2-M_W^2)^2}\right] \\
C_2 &= \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2-k_2^2-K^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right] \\
&+ \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{K^2+k_1^2-k_2^2}{(k_1^2-M_W^2)^2}\right]
\end{aligned} \tag{3.71}$$

$$C_1 = \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{1}{(k_1^2-M_W^2)^2}\right]$$

$$
\begin{aligned}
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right]\\
C_2 &= \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-1}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{1}{(k_1^2-M_W^2)^2}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right]
\end{aligned}
\tag{3.72}
$$

$$
\begin{aligned}
&C_1-C_2\\
&= \quad -\frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-1}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&- \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&- \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]
\end{aligned}
\tag{3.73}
$$

After the usual wick rotation and expanding in Gegenbauer polynomials we can replace:

$$
\begin{aligned}
\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{K^2} \quad\longrightarrow\quad & \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\,k_1^2\int_0^\infty dk_2^2\,k_2^2\,\frac{1}{k_>^2}=\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\,k_<^2\\
= \quad & \frac{1}{(4\pi)^4}\left[\int_0^\infty dk_1^2\int_0^{k_1^2}dk_2^2\,k_2^2+\int_0^\infty dk_2^2\int_0^{k_2^2}dk_1^2\,k_1^2\right].
\end{aligned}
\tag{3.74}
$$

$$
\begin{aligned}
&C_1-C_2\\
&= \quad \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2}dk_2^2\,k_2^2k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{-1}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]\\
&+ \quad \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2}dk_2^2\,k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]\\
&+ \quad \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2}dk_2^2\,k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]\\
&+ \quad \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2\int_0^{k_2^2}dk_1^2\,k_1^2k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{-1}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]\\
&+ \quad \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2\int_0^{k_2^2}dk_1^2\,k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]\\
&+ \quad \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2\int_0^{k_2^2}dk_1^2\,k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]
\end{aligned}
$$

$$\tag{3.75}$$

$$\begin{aligned} &C_1 - C_2 \\ &= \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\, k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\ &+ \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\, k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\ &+ \frac{1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\, k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \end{aligned} \tag{3.76}$$

Interchanging k_1 and k_2 in the last two lines

$$\begin{aligned} &C_1 - C_2 \\ &= \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\, k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\ &+ \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\, k_2^2\, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)(k_1^2+M_W^2)}\left[\frac{k_2^2}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right] \\ &+ \frac{1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\, k_2^2\, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)(k_1^2+M_W^2)}\left[\frac{-k_1^2}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right] \end{aligned} \tag{3.77}$$

$$\begin{aligned} C_1 - C_2 &= \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_W^2)^2(k_1^2+M_1^2)}\int_0^{k_1^2} dk_2^2 \frac{-k_2^4}{(k_2^2+M_W^2)(k_2^2+M_2^2)^2} \\ &+ \frac{1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_W^2)(k_1^2+M_2^2)^2}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_W^2)^2(k_2^2+M_1^2)} \\ &+ \frac{1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_W^2)(k_1^2+M_2^2)^2}\int_0^{k_1^2} dk_2^2 \frac{-k_2^2}{(k_2^2+M_W^2)^2(k_2^2+M_1^2)} \end{aligned} \tag{3.78}$$

Scale all masses and momenta to M_W^2

$$\begin{aligned} C_1 - C_2 &= \frac{1}{4}\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^2(k_1^2+M_1^2)}\int_0^{k_1^2} dk_2^2 \frac{-k_2^4}{(k_2^2+1)(k_2^2+M_2^2)^2} \\ &+ \frac{1}{4}\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_2^2)^2}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+1)^2(k_2^2+M_1^2)} \\ &+ \frac{1}{2}\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+1)(k_1^2+M_2^2)^2}\int_0^{k_1^2} dk_2^2 \frac{-k_2^2}{(k_2^2+1)^2(k_2^2+M_1^2)} \end{aligned} \tag{3.79}$$

These are the same integrals as C_a , C_d and C_e

$$C_1 - C_2 = \frac{1}{(4\pi)^4 M_W^4}\left[\frac{-1}{4}C_a + \frac{1}{4}C_d + \frac{1}{2}C_e\right] \tag{3.80}$$

The contribution from the C functions is completely symmetric in the neutrino masses, so it does not contribute to the EDM.

The D functions

$$
\begin{aligned}
&D_0 g^{\kappa\mu} + D_{11} p_1^\kappa p_1^\mu + D_{22} p_2^\kappa p_2^\mu + D_{12} p_1^\kappa p_2^\mu + D_{21} p_2^\kappa p_1^\mu \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\kappa (k_1-k_2)^\mu}{[(p_2-k_2)^2-M_2^2](k_1-k_2)^2(k_1^2-M_1^2)[(p_1-k_1)^2-M_W^2]} \\
&\qquad \times \frac{1}{[(p_2-k_1)^2-M_W^2](k_2^2-M_W^2)} \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\kappa (k_1-k_2)^\mu}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\left[1+\frac{2(p_2\cdot k_2)}{(k_2^2-M_2^2)}+\frac{4(p_2\cdot k_2)^2}{(k_2^2-M_2^2)^2}+\cdots\right]\left[1+\frac{2(p_1\cdot k_1)}{(k_1^2-M_W^2)}+\frac{4(p_1\cdot k_1)^2}{(k_1^2-M_W^2)^2}+\cdots\right] \\
&\qquad \times\left[1+\frac{2(p_2\cdot k_1)}{(k_1^2-M_W^2)}+\frac{4(p_2\cdot k_1)^2}{(k_1^2-M_W^2)^2}+\cdots\right] \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\kappa (k_1-k_2)^\mu}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\left[1+\frac{2(p_2\cdot k_2)}{(k_2^2-M_2^2)}+\frac{2(p_1\cdot k_1)}{(k_1^2-M_W^2)}+\frac{2(p_2\cdot k_1)}{(k_1^2-M_W^2)}\right. \\
&+\frac{4(p_1\cdot k_1)^2}{(k_1^2-M_W^2)^2} \\
&+\frac{4(p_2\cdot k_2)^2}{(k_2^2-M_2^2)^2}+\frac{2(p_2\cdot k_1)}{(k_1^2-M_W^2)}\frac{2(p_2\cdot k_2)}{(k_2^2-M_2^2)}+\frac{4(p_2\cdot k_1)^2}{(k_1^2-M_W^2)^2} \\
&\left.+\frac{2(p_2\cdot k_2)}{(k_2^2-M_2^2)}\frac{2(p_1\cdot k_1)}{(k_1^2-M_W^2)}+\frac{2(p_2\cdot k_1)}{(k_1^2-M_W^2)}\frac{2(p_1\cdot k_1)}{(k_1^2-M_W^2)}\right].
\end{aligned}
\tag{3.81}
$$

Omitting the first line in the brackets

$$
\begin{aligned}
&D_0 g^{\kappa\mu} + D_{11} p_1^\kappa p_1^\mu + D_{22} p_2^\kappa p_2^\mu + D_{12} p_1^\kappa p_2^\mu + D_{21} p_2^\kappa p_1^\mu \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\kappa (k_1-k_2)^\mu}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\left[4p_{1\lambda}p_{1\nu}\left[\frac{k_1^\lambda k_1^\nu}{(k_1^2-M_W^2)^2}\right]\right. \\
&+4p_{2\lambda}p_{2\nu}\left[\frac{k_2^\lambda k_2^\nu}{(k_2^2-M_2^2)^2}+\frac{k_1^\lambda k_2^\nu}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^\lambda k_1^\nu}{(k_1^2-M_W^2)^2}\right] \\
&\left.+4p_{1\lambda}p_{2\nu}\left[\frac{k_1^\lambda k_2^\nu}{(k_2^2-M_2^2)(k_1^2-M_W^2)}+\frac{k_1^\lambda k_1^\nu}{(k_1^2-M_W^2)^2}\right]\right].
\end{aligned}
\tag{3.82}
$$

$$
\begin{aligned}
&D_{11} p_1^\kappa p_1^\mu \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\kappa (k_1-k_2)^\mu}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\times\left[4p_{1\lambda}p_{1\nu}\left[\frac{k_1^\lambda k_1^\nu}{(k_1^2-M_W^2)^2}\right]\right] \\
&D_{22} p_2^\kappa p_2^\mu \\
&= \int \frac{d^4k_1}{(2\pi)^4} \int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\kappa (k_1-k_2)^\mu}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\times\left[4p_{2\lambda}p_{2\nu}\left[\frac{k_2^\lambda k_2^\nu}{(k_2^2-M_2^2)^2}+\frac{k_1^\lambda k_2^\nu}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^\lambda k_1^\nu}{(k_1^2-M_W^2)^2}\right]\right]
\end{aligned}
$$

$$\begin{aligned}
&D_{12}p_1^\kappa p_2^\mu + D_{21}p_2^\kappa p_1^\mu \\
&= \int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\kappa(k_1-k_2)^\mu}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\quad\times\left[4p_{1\lambda}p_{2\nu}\left[\frac{k_1^\lambda k_2^\nu}{(k_2^2-M_2^2)(k_1^2-M_W^2)} + \frac{k_1^\lambda k_1^\nu}{(k_1^2-M_W^2)^2}\right]\right].
\end{aligned} \tag{3.83}$$

Again the integrals can only be some scalar functions multiplied by g's:

$$\begin{aligned}
&D_{11}^{'}g^{\kappa\mu}g^{\lambda\nu} + D_{11}^{''}g^{\kappa\lambda}g^{\mu\nu} + D_{11}^{'''}g^{\kappa\nu}g^{\lambda\mu} \\
&= 4\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{k_1^\kappa(k_1-k_2)^\mu}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^\lambda k_1^\nu}{(k_1^2-M_W^2)^2}\right].
\end{aligned} \tag{3.84}$$

To figure out the D's we contract both sides with $g_{\kappa\mu}g_{\lambda\nu}$, $g_{\kappa\lambda}g_{\mu\nu}$ and $g_{\kappa\nu}g_{\lambda\mu}$
Contracting with $g_{\kappa\mu}g_{\lambda\nu}$

$$\begin{aligned}
&16D_{11}^{'} + 4D_{11}^{''} + 4D_{11}^{'''} \\
&= 4\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].
\end{aligned} \tag{3.85}$$

Contracting with $g_{\kappa\lambda}g_{\mu\nu}$

$$\begin{aligned}
&4D_{11}^{'} + 16D_{11}^{''} + 4D_{11}^{'''} \\
&= 4\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].
\end{aligned} \tag{3.86}$$

Contracting with $g_{\kappa\nu}g_{\lambda\mu}$

$$\begin{aligned}
&4D_{11}^{'} + 4D_{11}^{''} + 16D_{11}^{'''} \\
&= 4\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].
\end{aligned} \tag{3.87}$$

$D_{11}^{'}$ does not contribute to D_{11}. Subtracting (3.86) from (3.87) gives:

$$D_{11}^{''} = D_{11}^{'''} \tag{3.88}$$

Now (3.85) and (3.86) look like

$$\begin{aligned}
&16D_{11}^{'} + 8D_{11}^{''} \\
&= 4\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].
\end{aligned} \tag{3.89}$$

$$\begin{aligned}
&4D_{11}^{'} + 20D_{11}^{''} \\
&= 4\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].
\end{aligned} \tag{3.90}$$

Multiplying the last equation by 4

$$\begin{aligned} &16D_{11}^{'}+8D_{11}^{''} \\ &= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\end{aligned} \tag{3.91}$$

$$\begin{aligned} &16D_{11}^{'}+80D_{11}^{''} \\ &= 16\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\end{aligned} \tag{3.92}$$

Now we can extract the expressions for$D_{11}^{''}$ which is the only term that contribute to the EDM.

$$\begin{aligned} &72D_{11}^{''} \\ &= 12\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\end{aligned} \tag{3.93}$$

$$\begin{aligned} &D_{11}^{''} \\ &= \frac{1}{6}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\end{aligned} \tag{3.94}$$

This means that

$$\begin{aligned} &D_{11} \\ &= \frac{1}{3}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\end{aligned} \tag{3.95}$$

Define $K=k_1-k_2$

$$\begin{aligned} &D_{11} \\ &= \frac{1}{3}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot K}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\end{aligned} \tag{3.96}$$

Since $k_1\cdot K=\frac{1}{2}(K^2+k_1^2-k_2^2)$

$$\begin{aligned} &D_{11} \\ &= \frac{1}{6}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\end{aligned} \tag{3.97}$$

Now let's do D_{22}

$$\begin{aligned} &D_{22}^{'}g^{\kappa\mu}g^{\lambda\nu}+D_{22}^{''}g^{\kappa\lambda}g^{\mu\nu}+D_{22}^{'''}g^{\kappa\nu}g^{\lambda\mu} \\ &= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1^\kappa(k_1-k_2)^\mu}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\ &\quad\times\left[\frac{k_2^\lambda k_2^\nu}{(k_2^2-M_2^2)^2}+\frac{k_1^\lambda k_2^\nu}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^\lambda k_1^\nu}{(k_1^2-M_W^2)^2}\right].\end{aligned}$$

(3.98)

To figure out the D's we contract both sides with $g_{\kappa\mu}g_{\lambda\nu}$, $g_{\kappa\lambda}g_{\mu\nu}$ and $g_{\kappa\nu}g_{\lambda\mu}$
Contracting with $g_{\kappa\mu}g_{\lambda\nu}$

$$\begin{aligned}
&16D_{22}^{'}+4D_{22}^{''}+4D_{22}^{'''}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{k_2^2}{(k_2^2-M_2^2)^2}+\frac{k_1\cdot k_2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{k_2^2(k_1\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}+\frac{(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\right].
\end{aligned}\tag{3.99}$$

Contracting with $g_{\kappa\lambda}g_{\mu\nu}$

$$\begin{aligned}
&4D_{22}^{'}+16D_{22}^{''}+4D_{22}^{'''}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{(k_1\cdot k_2)(k_2\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}+\frac{k_1^2(k_2\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\right].
\end{aligned}\tag{3.100}$$

Contracting with $g_{\kappa\nu}g_{\lambda\mu}$

$$\begin{aligned}
&4D_{22}^{'}+4D_{22}^{''}+16D_{22}^{'''}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{(k_1\cdot k_2)(k_2\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}+\frac{(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\right].
\end{aligned}\tag{3.101}$$

$D_{22}^{'}$ does not contribute to D_{22}. Adding the last two equations gives:

$$\begin{aligned}
&8D_{22}^{'}+20D_{22}^{''}+20D_{22}^{'''}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{2(k_1\cdot k_2)(k_2\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}+\frac{k_1^2(k_2\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{2k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\right].
\end{aligned}\tag{3.102}$$

So what we have now is

$$\begin{aligned}
&16D_{22}^{'}+40D_{22}^{''}+40D_{22}^{'''}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{4(k_1\cdot k_2)(k_2\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}+\frac{2k_1^2(k_2\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{2(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{4k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\right].\\
&16D_{22}^{'}+4D_{22}^{''}+4D_{22}^{'''}
\end{aligned}$$

$$
\begin{aligned}
= \quad & 4\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\, \frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \times\Bigg[\frac{k_2^2(k_1\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}+\frac{(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\Bigg]\,.
\end{aligned} \tag{3.103}
$$

Subtracting the second equation from the first one:

$$
\begin{aligned}
& 36D_{22}^{''}+36D_{22}^{'''} \\
= \quad & 4\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\, \frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \times\Bigg[\frac{4(k_1\cdot k_2)(k_2\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}+\frac{2k_1^2(k_2\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
& \qquad +\frac{3k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}-\frac{k_2^2(k_1\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}\Bigg]\,.
\end{aligned} \tag{3.104}
$$

$$
\begin{aligned}
& D_{22} \\
= \quad & \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\, \frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \times\Bigg[\frac{4(k_1\cdot k_2)(k_2\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}+\frac{2k_1^2(k_2\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
& \qquad +\frac{3k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}-\frac{k_2^2(k_1\cdot(k_1-k_2))}{(k_2^2-M_2^2)^2}\Bigg]\,.
\end{aligned} \tag{3.105}
$$

Define $K=k_1-k_2$

$$
\begin{aligned}
& D_{22} \\
= \quad & \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\, \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \times\Bigg[\frac{4(k_1\cdot k_2)(k_2\cdot K)}{(k_2^2-M_2^2)^2}+\frac{2k_1^2(k_2\cdot K)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{(k_1\cdot k_2)(k_1\cdot K)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{3k_1^2(k_1\cdot K))}{(k_1^2-M_W^2)^2}-\frac{k_2^2(k_1\cdot K)}{(k_2^2-M_2^2)^2}\Bigg]\,.
\end{aligned} \tag{3.106}
$$

Since $k_1\cdot K=\dfrac{1}{2}(K^2+k_1^2-k_2^2)$, $k_2\cdot K=\dfrac{1}{2}(k_1^2-k_2^2-K^2)$ and $k_2\cdot k_1=\dfrac{1}{2}(k_1^2+k_2^2-K^2)$

$$
\begin{aligned}
& D_{22} \\
= \quad & \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\, \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \times\Bigg[\frac{4(k_1\cdot k_2)(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_2^2-M_2^2)^2}+\frac{2k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{(k_1\cdot k_2)(\frac{1}{2}(K^2+k_1^2-k_2^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
& +\frac{3k_1^2(\frac{1}{2}(K^2+k_1^2-k_2^2)))}{(k_1^2-M_W^2)^2}-\frac{k_2^2(\frac{1}{2}(K^2+k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}\Bigg]\,.
\end{aligned} \tag{3.107}
$$

In the first and third term , integrating $(k_1\cdot k_2)K^2$ will give zero.

$$D_{22}$$

$$
\begin{aligned}
= \quad & \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \Big[\frac{4(k_1\cdot k_2)(\frac{1}{2}(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}+\frac{2k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{(k_1\cdot k_2)(\frac{1}{2}(k_1^2-k_2^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
& +\frac{3k_1^2(\frac{1}{2}(K^2+k_1^2-k_2^2)))}{(k_1^2-M_W^2)^2}-\frac{k_2^2(\frac{1}{2}(K^2+k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}\Big]\,.
\end{aligned}
\tag{3.108}
$$

So

$$
\begin{aligned}
& D_{22} \\
= \quad & \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \Big[\frac{4(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}+\frac{2k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
& +\frac{3k_1^2(\frac{1}{2}(K^2+k_1^2-k_2^2)))}{(k_1^2-M_W^2)^2}-\frac{k_2^2(\frac{1}{2}(K^2+k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}\Big]\,.
\end{aligned}
\tag{3.109}
$$

Remember that:

$$
\begin{aligned}
& D_{11} \\
= \quad & \frac{1}{6}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\Big[\frac{k_1^2}{(k_1^2-M_W^2)^2}\Big]\,. \\
= \quad & \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{3(K^2+k_1^2-k_2^2)}{2(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\Big[\frac{k_1^2}{(k_1^2-M_W^2)^2}\Big]\,.
\end{aligned}
\tag{3.110}
$$

The combination that we need is $D_{11}-D_{22}$

$$
\begin{aligned}
& D_{11}-D_{22} \\
= \quad & \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \times\Big[-\frac{4(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}-\frac{2k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}-\frac{(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
& +\frac{k_2^2(\frac{1}{2}(K^2+k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}\Big]\,.
\end{aligned}
\tag{3.111}
$$

$$
\begin{aligned}
& D_{11}-D_{22} \\
= \quad & \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \times\Big[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}-\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}-\frac{1}{4}\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
& +\frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Big]\,.
\end{aligned}
$$

(3.112)

Now consider D_{12} , D_{21}

$$
\begin{aligned}
&D_{12}p_1^{\kappa}p_2^{\mu}+D_{21}p_2^{\kappa}p_1^{\mu}\\
&= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1^{\kappa}(k_1-k_2)^{\mu}}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[4p_{1\lambda}p_{2\nu}\left[\frac{k_1^{\lambda}k_2^{\nu}}{(k_2^2-M_2^2)(k_1^2-M_W^2)}+\frac{k_1^{\lambda}k_1^{\nu}}{(k_1^2-M_W^2)^2}\right]\right]
\end{aligned}
\tag{3.113}
$$

Again the integrals can only be some scalar functions multiplied by g's:

$$
\begin{aligned}
&D_{12}^{'}g^{\kappa\mu}g^{\lambda\nu}+D_{12}^{''}g^{\kappa\lambda}g^{\mu\nu}+D_{12}^{'''}g^{\kappa\nu}g^{\lambda\mu}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1^{\kappa}(k_1-k_2)^{\mu}}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{k_1^{\lambda}k_2^{\nu}}{(k_2^2-M_2^2)(k_1^2-M_W^2)}+\frac{k_1^{\lambda}k_1^{\nu}}{(k_1^2-M_W^2)^2}\right]
\end{aligned}
\tag{3.114}
$$

Notice that $D_{12}^{''}=D_{12}$ and $D_{21}=D_{12}^{'''}$

$$
\begin{aligned}
&D_{12}^{'}g^{\kappa\mu}g^{\lambda\nu}+D_{12}g^{\kappa\lambda}g^{\mu\nu}+D_{21}g^{\kappa\nu}g^{\lambda\mu}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1^{\kappa}(k_1-k_2)^{\mu}}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{k_1^{\lambda}k_2^{\nu}}{(k_2^2-M_2^2)(k_1^2-M_W^2)}+\frac{k_1^{\lambda}k_1^{\nu}}{(k_1^2-M_W^2)^2}\right]
\end{aligned}
\tag{3.115}
$$

To figure out the D's we contract both sides with $g_{\kappa\mu}g_{\lambda\nu}$, $g_{\kappa\lambda}g_{\mu\nu}$ and $g_{\kappa\nu}g_{\lambda\mu}$
Contracting with $g_{\kappa\mu}g_{\lambda\nu}$

$$
\begin{aligned}
&16D_{12}^{'}+4D_{12}+4D_{21}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1\cdot(k_1-k_2)}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{k_1\cdot k_2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2}{(k_1^2-M_W^2)^2}\right].\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\right].
\end{aligned}
\tag{3.116}
$$

Contracting with $g_{\kappa\lambda}g_{\mu\nu}$

$$
\begin{aligned}
&4D_{12}^{'}+16D_{12}+4D_{21}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{k_1^2(k_2\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\right].
\end{aligned}
\tag{3.117}
$$

Contracting with $g_{\kappa\nu}g_{\lambda\mu}$

$$\begin{aligned}
&4D'_{12}+4D_{12}+16D_{21}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{(k_1\cdot k_2)(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2(k_1\cdot(k_1-k_2))}{(k_1^2-M_W^2)^2}\right].
\end{aligned} \tag{3.118}$$

Subtracting the last equation from the one before gives:

$$\begin{aligned}
&12D_{12}-12D_{21}\\
&= 4\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{-(k_1\cdot k_2)(k_1\cdot(k_1-k_2))+k_1^2(k_2\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned} \tag{3.119}$$

So what we have now is

$$\begin{aligned}
&D_{12}-D_{21}\\
&= \frac{1}{3}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1-k_2)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{-(k_1\cdot k_2)(k_1\cdot(k_1-k_2))+k_1^2(k_2\cdot(k_1-k_2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned} \tag{3.120}$$

Define $K=k_1-k_2$

$$\begin{aligned}
&D_{12}-D_{21}\\
&= \frac{1}{3}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{-(k_1\cdot k_2)(k_1\cdot K)+k_1^2(k_2\cdot K)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned} \tag{3.121}$$

Since $k_1\cdot K=\frac{1}{2}(K^2+k_1^2-k_2^2)$, $k_2\cdot K=\frac{1}{2}(k_1^2-k_2^2-K^2)$ and $k_2\cdot k_1=\frac{1}{2}(k_1^2+k_2^2-K^2)$

$$\begin{aligned}
&D_{12}-D_{21}\\
&= \frac{1}{3}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{-(k_1\cdot k_2)(\frac{1}{2}(K^2+k_1^2-k_2^2))+k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned} \tag{3.122}$$

In the first term , integrating $(k_1\cdot k_2)K^2$ will give zero.

$$\begin{aligned}
&(D_{12}-D_{21})\\
&= \frac{1}{3}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\left[\frac{-(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))+k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned}$$

(3.123)

Remember

$$
\begin{aligned}
&D_{11} - D_{22} \\
&= \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\quad\times\Bigg[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2} - \frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} - \frac{1}{4}\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
&\qquad + \frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Bigg]\,.
\end{aligned} \tag{3.124}
$$

So

$$
\begin{aligned}
&(D_{11} - D_{22}) - (D_{12} - D_{21}) \\
&= \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\quad\times\Bigg[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2} - \frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} - \frac{1}{4}\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
&\qquad + \frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Bigg]\,. \\
&- \frac{1}{3}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\quad\times\Bigg[\frac{-(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + \frac{k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\Bigg]\,.
\end{aligned} \tag{3.125}
$$

$$
\begin{aligned}
&(D_{11} - D_{22}) - (D_{12} - D_{21}) \\
&= \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\quad\times\Bigg[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2} - \frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} - \frac{1}{4}\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
&\qquad + \frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Bigg]\,. \\
&- \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\quad\times\Bigg[\frac{-3}{4}\frac{((k_1^2+k_2^2-K^2))(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + \frac{3}{2}\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\Bigg]\,.
\end{aligned} \tag{3.126}
$$

$$
\begin{aligned}
&(D_{11} - D_{22}) - (D_{12} - D_{21}) \\
&= \frac{1}{9}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\quad\times\Bigg[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2} - \left[1+\frac{3}{2}\right]\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} \\
&- \left[\frac{1}{4}-\frac{3}{4}\right]\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + \frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Bigg]\,.
\end{aligned}
$$

(3.127)

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21})\\
&= \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\Big[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}-\frac{5}{2}\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\\
&\quad+\frac{1}{2}\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Big]\,.
\end{aligned}
\tag{3.128}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21})\\
&= \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\Big[\frac{K^2(k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}-\frac{k_1^4-k_2^4}{(k_2^2-M_2^2)^2}+\frac{5}{2}\frac{k_1^2K^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}-\frac{5}{2}\frac{k_1^2(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\\
&\quad-\frac{1}{2}\frac{K^2(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{1}{2}\frac{k_1^4-k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{1}{2}\frac{k_2^2K^2}{(k_2^2-M_2^2)^2}+\frac{1}{2}\frac{k_2^2(k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Big]\,.
\end{aligned}
\tag{3.129}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21})\\
&= \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\Big[2\frac{K^2(k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}-2\frac{k_1^4-k_2^4}{(k_2^2-M_2^2)^2}+5\frac{k_1^2K^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}-5\frac{k_1^2(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\\
&\quad-\frac{K^2(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^4-k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_2^2K^2}{(k_2^2-M_2^2)^2}+\frac{k_2^2(k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Big]\,.
\end{aligned}
\tag{3.130}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21})\\
&= \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\quad\times\Big[\frac{K^2(2k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2-M_2^2)^2}+\frac{K^2(4k_1^2+k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{5k_1^2k_2^2-4k_1^4-k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\Big]\,.
\end{aligned}
\tag{3.131}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21})\\
&= \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\times\\
&\qquad\times\Big[\frac{K^2(2k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Big]\,.\\
&+ \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\qquad\times\Big[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2-M_2^2)^2}\Big]\,.\\
&+ \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}
\end{aligned}
$$

$$
\begin{aligned}
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\times\left[\frac{K^2(4k_1^2+k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].\\
+&\quad \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\times\left[\frac{5k_1^2k_2^2-4k_1^4-k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned}
\tag{3.132}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21})\\
&= \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\times\left[\frac{(2k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\right].\\
&+ \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2-M_2^2)^2}\right].\\
&+ \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\times\left[\frac{(4k_1^2+k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].\\
&+ \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\times\left[\frac{5k_1^2k_2^2-4k_1^4-k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned}
\tag{3.133}
$$

Now doing the usual wick rotation and expanding in Gegenbauer polynomials we can replace:

$$
\begin{aligned}
\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{K^2}\ \longrightarrow\ &\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\,k_1^2\int_0^\infty dk_2^2\,k_2^2\,\frac{1}{k_>^2}=\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\ k_<^2\\
=\ &\frac{1}{(4\pi)^4}\left[\int_0^\infty dk_1^2\int_0^{k_1^2}dk_2^2\ k_2^2+\int_0^\infty dk_2^2\int_0^{k_2^2}dk_1^2\ k_1^2\right].
\end{aligned}
\tag{3.134}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21})\\
&= \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2}dk_2^2\ k_2^2k_1^2\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\times\left[\frac{(2k_1^2-k_2^2)}{(k_2^2+M_2^2)^2}\right].\\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2}dk_2^2\ k_2^2\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2+M_2^2)^2}\right].\\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2}dk_2^2\ k_2^2k_1^2\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\\
&\qquad\qquad\qquad\qquad\qquad\qquad\times\left[\frac{(4k_1^2+k_2^2)}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right].
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[\frac{5k_1^2k_2^2-4k_1^4-k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_2^2k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[\frac{(2k_1^2-k_2^2)}{(k_2^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_2^2k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[\frac{(4k_1^2+k_2^2)}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[\frac{5k_1^2k_2^2-4k_1^4-k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right].
\end{aligned}
\tag{3.135}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21}) \\
&= \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[\frac{(2k_1^4-k_2^2k_1^2)}{(k_2^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[\frac{(4k_1^4+k_2^2k_1^2)}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[\frac{5k_1^2k_2^2-4k_1^4-k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad\qquad \times\left[\frac{(2k_1^2k_2^2-k_2^4)}{(k_2^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}
\end{aligned}
$$

$$
\begin{aligned}
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[-\frac{2k_1^4 - k_1^2k_2^2 - k_2^4}{(k_2^2+M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(4k_1^2k_2^2 + k_2^4)}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{5k_1^2k_2^2 - 4k_1^4 - k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right].
\end{aligned}
\tag{3.136}
$$

$$
\begin{aligned}
& (D_{11}-D_{22})-(D_{12}-D_{21}) \\
= \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{k_2^4}{(k_2^2+M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{6k_1^2k_2^2 - k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(3k_1^2k_2^2 - 2k_1^4)}{(k_2^2+M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{9k_1^2k_2^2 - 4k_1^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right].
\end{aligned}
\tag{3.137}
$$

$$
\begin{aligned}
& (D_{11}-D_{22})-(D_{12}-D_{21}) \\
= \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{k_2^4}{(k_2^2+M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{6k_1^2k_2^2 - k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(3k_1^2k_2^2 - 2k_1^4)}{(k_2^2+M_2^2)^2}\right].
\end{aligned}
$$

$$+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \times\left[\frac{9k_1^2k_2^2-4k_1^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \tag{3.138}$$

$$\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21}) \\
&= \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{k_2^6}{(k_2^2+M_2^2)^2}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)^2(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{6k_1^2k_2^4-k_2^6}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{(3k_1^4k_2^2-2k_1^6)}{(k_2^2+M_2^2)^2}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{9k_1^4k_2^2-4k_1^6}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right].
\end{aligned} \tag{3.139}$$

$$\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21}) \\
&= \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{k_2^6}{(k_2^2+M_2^2)^2}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{6k_1^2k_2^4-k_2^6}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{(3k_2^4k_1^2-2k_2^6)}{(k_1^2+M_2^2)^2}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{9k_2^4k_1^2-4k_2^6}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right].
\end{aligned} \tag{3.140}$$

$$\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21}) \\
&= \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{k_2^6}{(k_2^2+M_2^2)^2}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{6k_1^2k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{-k_2^6}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{(3k_2^4k_1^2)}{(k_1^2+M_2^2)^2}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{(-2k_2^6)}{(k_1^2+M_2^2)^2}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{9k_2^4k_1^2}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right]. \\
&+ \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{-4k_2^6}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right].
\end{aligned}$$

(3.141)

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21}) \\
&= \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,; \frac{1}{(k_1^2+M_1^2)(k_1^2+M_W^2)^2}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3}\,. \\
&+ \frac{6}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+M_W^2)^3}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_2^2)^2(k_2^2+M_W^2)}\,. \\
&+ \frac{-1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_1^2)(k_1^2+M_W^2)^3}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_W^2)(k_2^2+M_2^2)^2}\,. \\
&+ \frac{3}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+M_W^2)^2}\,. \\
&+ \frac{-2}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_2^2)^3(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+M_W^2)^2}\,. \\
&+ \frac{9}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+M_W^2)^3}\,. \\
&+ \frac{-4}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_2^2)^2(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+M_W^2)^3}\,.
\end{aligned}
\tag{3.142}
$$

Scaling all masses and momenta to M_W^2

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21}) \\
&= \frac{1}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\,; \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+1)(k_2^2+M_2^2)^3}\,. \\
&+ \frac{6}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_2^2)^2(k_2^2+1)}\,. \\
&+ \frac{-1}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+1)(k_2^2+M_2^2)^2}\,. \\
&+ \frac{3}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^2}\,. \\
&+ \frac{-2}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+1)^2}\,. \\
&+ \frac{9}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\, \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^3}\,. \\
&+ \frac{-4}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+1)^3}\,.
\end{aligned}
\tag{3.143}
$$

In terms of the integrals in Appendix B:

$$
\begin{aligned}
&(D_{11}-D_{22})-(D_{12}-D_{21}) \\
&= \frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\left[\frac{1}{18}D_a+\frac{6}{18}D_b-\frac{1}{18}D_c+\frac{3}{18}D_d-\frac{2}{18}D_e+\frac{9}{18}D_f-\frac{4}{18}D_g\right]
\end{aligned}
\tag{3.144}
$$

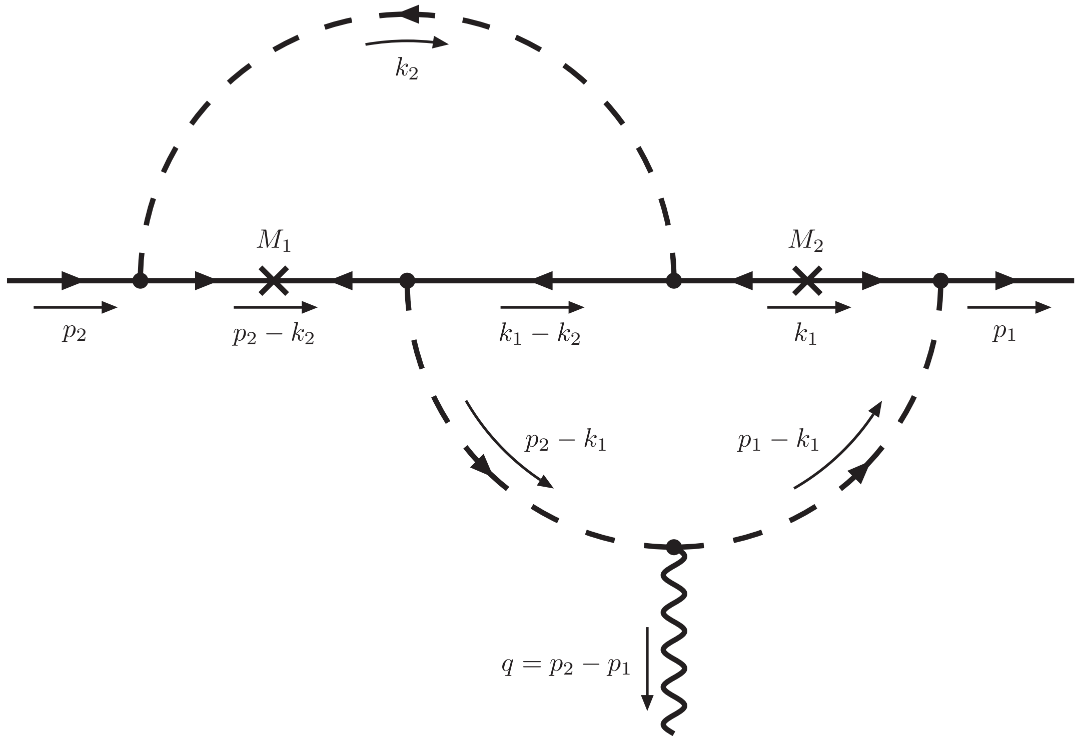


Figure 3.8: **Diagram 3B**

3.3.3 Diagram 3B

The $\gamma\phi\phi$ vertex is

$$(-ie)\left\{(p_1 - k_1)_\lambda - (-p_2 + k_1)_\lambda\right\} = (-ie)\,(p_1 + p_2 - 2k_1)_\lambda \ , \tag{3.145}$$

which is the same as diagram 2B. The coupling constant, $(-ie)$, and those along the fermion line combine to the factor

$$(-ie)\left(-i\tilde{\Lambda}_{\alpha 2}\right)\left(-i\tilde{\Lambda}_{\beta 2}\right)\left(-i\tilde{\Lambda}^{\dagger}_{1\beta}\right)\left(-i\tilde{\Lambda}^{\dagger}_{1\alpha}\right) = -ie\left(\tilde{\Lambda}_{\alpha 2}\tilde{\Lambda}_{\beta 2}\tilde{\Lambda}^{*}_{\beta 1}\tilde{\Lambda}^{*}_{\alpha 1}\right) \ , \tag{3.146}$$

which is again the same as diagram 2B. The six propagators will contribute $i^6 = -1$ and get rid of the minus sign. The fermion line gives

$$\begin{aligned}
&\langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}P_R N_2\phi^-)(\overline{\ell_\beta}P_R N_2\phi^-)(\overline{N}_1 P_L \ell_\beta\phi^+)(\overline{N}_1 P_L \ell_\alpha\phi^+)\,|\ell_\alpha(p_1)\rangle \\
&\quad = \quad \langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}P_R N_2\phi^-)(\overline{N}_2 P_R \ell^c_\beta\phi^-)(\overline{\ell^c_\beta}P_L N_1\phi^+)(\overline{N}_1 P_L \ell_\alpha\phi^+)\,|\ell_\alpha(p_1)\rangle \\
&\quad = \quad \langle \ell_\alpha(p_2)|\,(\overline{\mu_\alpha}P_R N_2)(\overline{N}_2 P_R \ell^c_\beta)(\overline{\ell^c_\beta}P_L N_1)(\overline{N}_1 P_L \ell_\alpha)\,|\ell_\alpha(p_1)\rangle \left\langle \phi^-\phi^+\right\rangle \left\langle \phi^-\phi^+\right\rangle \\
&\quad = \quad \bar{u}(p_2)P_R\left\langle N_2\overline{N}_2\right\rangle P_R\left\langle \ell_\beta\overline{\ell^c_\beta}\right\rangle P_L\left\langle N_1\overline{N}_1\right\rangle P_L u(p_1) \times \left\langle \phi^-\phi^+\right\rangle \left\langle \phi^-\phi^+\right\rangle \\
&\quad \to \quad \bar{u}(p_2)P_R\left[\not{k}_1 + M_2\right]P_R(\not{k}_1 - \not{k}_2)P_L\left[(\not{p}_1 - \not{k}_2) + M_1\right]P_L u(p_1)
\end{aligned}$$

$$= \quad M_1 M_2\, \bar{u}(p_2)(\not{k}_1 - \not{k}_2) P_L u(p_1)\ . \tag{3.147}$$

The numerator is exactly the same as in diagram 2B. Put together, we are looking at

$$(p_1 + p_2 - 2k_1)_\lambda (\not{k}_1 - \not{k}_2) P_L\ . \tag{3.148}$$

These terms can be written as:

$$(k_1 - k_2)^\kappa (p_1 + p_2)_\lambda \gamma_\kappa P_L - 2k_1^\kappa (k_1 - k_2)^\mu g_{\kappa\lambda} \gamma_\mu P_L\ . \tag{3.149}$$

Using the $\overline{C}$- and $\overline{D}$-functions we introduced for diagram 3A, the result after loop-integration can be written as:

$$\begin{aligned}
&(\overline{C}_1 p_2^\kappa + \overline{C}_2 p_1^\kappa)(p_1+p_2)_\lambda \gamma_\kappa P_L - 2(\overline{D}_0 g^{\kappa\mu} + \overline{D}_{11} p_2^\kappa p_2^\mu + \overline{D}_{22} p_1^\kappa p_1^\mu + \overline{D}_{12} p_2^\kappa p_1^\mu + \overline{D}_{21} p_1^\kappa p_2^\mu) g_{\kappa\lambda}\gamma_\mu P_L \\
&= \quad (p_1+p_2)_\lambda (\overline{C}_1 \not{p}_2 + \overline{C}_2 \not{p}_1) P_L - 2(\overline{D}_0 \gamma_\lambda + \overline{D}_{11} p_{2\lambda} \not{p}_2 + \overline{D}_{22} p_{1\lambda} \not{p}_1 + \overline{D}_{12} p_{2\lambda} \not{p}_1 + \overline{D}_{21} p_{1\lambda} \not{p}_2) P_L \\
&= \quad (p_1+p_2)_\lambda (\overline{C}_1 \not{p}_2 P_L + \overline{C}_2 P_R \not{p}_1) - 2(\overline{D}_0 \gamma_\lambda P_L + \overline{D}_{11} p_{2\lambda} \not{p}_2 P_L + \overline{D}_{22} p_{1\lambda} P_R \not{p}_1 + \overline{D}_{12} p_{2\lambda} P_R \not{p}_1 \\
&\qquad\qquad + \overline{D}_{21} p_{1\lambda} \not{p}_2 P_L) \\
&\to \quad m\Big[(p_1+p_2)_\lambda (\overline{C}_1 P_L + \overline{C}_2 P_R) - 2\left(\overline{D}_{11} p_{2\lambda} P_L + \overline{D}_{22} p_{1\lambda} P_R + \overline{D}_{12} p_{2\lambda} P_R + \overline{D}_{21} p_{1\lambda} P_L\right)\Big] - 2\overline{D}_0 \gamma_\lambda P_L \\
&= \quad -2\overline{D}_0 \gamma_\lambda P_L + m\Big[(p_1+p_2)_\lambda (\overline{C}_1 P_L + \overline{C}_2 P_R) \\
&\qquad -(p_1+p_2)_\lambda \Big\{\overline{D}_{11} P_L + \overline{D}_{22} P_R + \overline{D}_{12} P_R + \overline{D}_{21} P_L\Big\} \\
&\qquad -(p_1-p_2)_\lambda \Big\{-\overline{D}_{11} P_L + \overline{D}_{22} P_R - \overline{D}_{12} P_R + \overline{D}_{21} P_R\Big\}\Big]\ .
\end{aligned} \tag{3.150}$$

Collecting the coefficients of $m(p_1+p_2)_\lambda \gamma_5$, we find:

$$-\frac{\overline{C}_1 - \overline{C}_2}{2} + \frac{(\overline{D}_{11} - \overline{D}_{22}) - (\overline{D}_{12} - \overline{D}_{21})}{2}\ , \tag{3.151}$$

which is the same as diagram 2B with M_1 and M_2 interchanges, and the change of the overall sign. So the sum of diagrams 2B and 3B will be anti-symmetric under the interchange $M_1 \leftrightarrow M_2$. Multiply by $M_1 M_2$ to get the contribution to the EDM:

$$-mM_1M_2(p_1+p_2)_\lambda \gamma_5\, \frac{(\overline{C}_1 - \overline{C}_2) - (\overline{D}_{11} - \overline{D}_{22}) + (\overline{D}_{12} - \overline{D}_{21})}{2}\ . \tag{3.152}$$

The complete expression including the coupling constants is

$$-iemM_1M_2 \left(\tilde{\Lambda}_{\alpha2} \tilde{\Lambda}_{\beta2} \tilde{\Lambda}^*_{\beta1} \tilde{\Lambda}^*_{\alpha1}\right) \left[\frac{(\overline{C}_1 - \overline{C}_2) - (\overline{D}_{11} - \overline{D}_{22}) + (\overline{D}_{12} - \overline{D}_{21})}{2}\right] (p_1+p_2)_\lambda \gamma_5\ , \tag{3.153}$$

from which we can deduce

$$d = +e\, mM_1M_2 \left(i\tilde{\Lambda}_{\alpha2} \tilde{\Lambda}_{\beta2} \tilde{\Lambda}^*_{\beta1} \tilde{\Lambda}^*_{\alpha1}\right) \left[\frac{(\overline{C}_1 - \overline{C}_2) - (\overline{D}_{11} - \overline{D}_{22}) + (\overline{D}_{12} - \overline{D}_{21})}{2}\right]\ . \tag{3.154}$$

Chapter 4

Results

4.1 The Total Contribution

Collect the contributions of all ten diagrams, summing over internal states:

$$\begin{aligned}
d_{1\mathrm{A}} &= +\frac{eg^4}{2}\,m\sum_{i,j,\beta} M_iM_j\left(i\tilde{V}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right)\Big[(A_1-A_2)-(B_1-B_2)\Big]_{(M_i,M_j)}\\
d_{2\mathrm{A}} &= -\frac{eg^4}{4}\,m\sum_{i,j,\beta} M_iM_j\left(i\tilde{V}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right)\Big[(5C_1+C_2)-(D_{11}-D_{22})+3(D_{12}-D_{21})\Big]_{(M_i,M_j)}\\
d_{3\mathrm{A}} &= +\frac{eg^4}{4}\,m\sum_{i,j,\beta} M_iM_j\left(i\tilde{V}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right)\Big[(5\overline{C}_1+\overline{C}_2)-(\overline{D}_{11}-\overline{D}_{22})+3(\overline{D}_{12}-\overline{D}_{21})\Big]_{(M_i,M_j)}\\
d_{1\mathrm{B}} &= +2e\,m\sum_{i,j,\beta} M_iM_j\left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right)\Big[(A_1-A_2)-(B_1-B_2)\Big]_{(M_i,M_j)}\\
d_{2\mathrm{B}} &= -e\,m\sum_{i,j,\beta} M_iM_j\left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right)\left[\frac{(C_1-C_2)-(D_{11}-D_{22})+(D_{12}-D_{21})}{2}\right]_{(M_i,M_j)}\\
d_{3\mathrm{B}} &= +e\,m\sum_{i,j,\beta} M_iM_j\left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right)\left[\frac{(\overline{C}_1-\overline{C}_2)-(\overline{D}_{11}-\overline{D}_{22})+(\overline{D}_{12}-\overline{D}_{21})}{2}\right]_{(M_i,M_j)}\\
d_{4\mathrm{A}} &= +\frac{eg^3}{2\sqrt{2}}M_W\sum_{i,j,\beta} M_iM_j\left(i\tilde{\lambda}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right)\Big[\,C_1+C_2\,\Big]_{(M_i,M_j)}\\
d_{5\mathrm{A}} &= -\frac{eg^3}{2\sqrt{2}}M_W\sum_{i,j,\beta} M_iM_j\left(i\tilde{\lambda}_{\alpha j}\tilde{V}_{\beta j}\tilde{V}^*_{\beta i}\tilde{V}^*_{\alpha i}\right)\Big[\,\overline{C}_1+\overline{C}_2\,\Big]_{(M_i,M_j)}\\
d_{4\mathrm{B}} &= +\frac{eg}{2\sqrt{2}}M_W\sum_{i,j,\beta} M_iM_j\left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{V}_{\beta j}\tilde{\lambda}_{\alpha j}\right)C_2(M_i,M_j)\\
d_{5\mathrm{B}} &= -\frac{eg}{2\sqrt{2}}M_W\sum_{i,j,\beta} M_iM_j\left(i\tilde{\Lambda}_{\alpha j}\tilde{\Lambda}_{\beta j}\tilde{V}^*_{\beta i}\tilde{\lambda}^*_{\alpha i}\right)\overline{C}_2(M_i,M_j)
\end{aligned} \tag{4.1}$$

the A and B functions have the following properties :

$$\begin{aligned}
A_2(M_i,M_j) &= A_1(M_j,M_i) \equiv \overline{A}_1(M_i,M_j)\,,\\
B_2(M_i,M_j) &= B_1(M_j,M_i) \equiv \overline{B}_1(M_i,M_j)\,.
\end{aligned} \tag{4.2}$$

So the combinations of functions appearing in $d_{1\mathrm{A}}$ and $d_{1\mathrm{B}}$ are anti-symmetric under the interchange $M_i \leftrightarrow M_j$ and can be written

$$(A_1 - A_2) - (B_1 - B_2) = (A_1 - \overline{A}_1) - (B_1 - \overline{B}_1) \,. \tag{4.3}$$

The sum of $d_{1\mathrm{A}}$, $d_{2\mathrm{A}}$, and $d_{3\mathrm{A}}$ is then

$$d_{1\mathrm{A}} + d_{2\mathrm{A}} + d_{3\mathrm{A}} = +\frac{eg^4}{4}\, m \sum_{i,j,\beta} M_i M_j \left(i\tilde{V}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right) R_1(M_i, M_j)\,, \tag{4.4}$$

where

$$\begin{aligned} R_1(M_i, M_j) \;\; \equiv \;\; & \Big[2(A_1 - \overline{A}_1) - 2(B_1 - \overline{B}_2) - 5(C_1 - \overline{C}_1) - (C_2 - \overline{C}_2) \\ & +(D_{11} - \overline{D}_{11}) - (D_{22} - \overline{D}_{22}) - 3(D_{12} - \overline{D}_{12}) + 3(D_{21} - \overline{D}_{21}) \Big]_{(M_i, M_j)} \,. \end{aligned} \tag{4.5}$$

Note that $R_1(M_i, M_j) = -R_1(M_j, M_i)$. Therefore,

$$\begin{aligned} & d_{1\mathrm{A}} + d_{2\mathrm{A}} + d_{3\mathrm{A}} \\ = \;\; & +\frac{eg^4}{4}\, m \sum_{\beta}\sum_{i,j} M_i M_j \left(i\tilde{V}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right) R_1(M_i, M_j) \\ = \;\; & +\frac{eg^4}{8}\, m \sum_{\beta}\sum_{i,j} M_i M_j \left[\left(i\tilde{V}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right) R_1(M_i, M_j) + \left(i\tilde{V}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) R_1(M_j, M_i) \right] \\ = \;\; & +\frac{eg^4}{8}\, m \sum_{\beta}\sum_{i,j} M_i M_j \left[\left(i\tilde{V}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right) - \left(i\tilde{V}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) \right] R_1(M_i, M_j) \\ = \;\; & +\frac{eg^4}{4}\, m \sum_{\beta}\sum_{i,j} M_i M_j\, \Im \left(\tilde{V}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) R_1(M_i, M_j) \\ = \;\; & +\frac{eg^4}{2}\, m \sum_{\beta}\sum_{i>j} M_i M_j\, \Im \left(\tilde{V}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) R_1(M_i, M_j) \,. \end{aligned} \tag{4.6}$$

Similarly, the sum of $d_{1\mathrm{B}}$, $d_{2\mathrm{B}}$, and $d_{3\mathrm{B}}$ is

$$d_{1\mathrm{B}} + d_{2\mathrm{B}} + d_{3\mathrm{B}} = +e\, m \sum_{i,j,\beta} M_i M_j \left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right) R_2(M_i, M_j)\,, \tag{4.7}$$

where

$$\begin{aligned} R_2(M_i, M_j) \;\; \equiv \;\; & \Bigg[2(A_1 - \overline{A}_1) - 2(B_1 - \overline{B}_1) - \frac{1}{2}\left\{ (C_1 - \overline{C}_1) - (C_2 - \overline{C}_2) \right\} \\ & + \frac{1}{2}\left\{ (D_{11} - \overline{D}_{11}) - (D_{22} - \overline{D}_{22}) - (D_{12} - \overline{D}_{12}) + (D_{21} - \overline{D}_{21}) \right\} \Bigg]_{(M_i, M_j)} \,. \end{aligned} \tag{4.8}$$

Again, we have $R_2(M_i, M_j) = -R_2(M_j, M_i)$. Therefore,

$$\begin{aligned} & d_{1\mathrm{A}} + d_{2\mathrm{A}} + d_{3\mathrm{A}} \\ = \;\; & +e\, m \sum_{\beta}\sum_{i,j} M_i M_j \left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right) R_2(M_i, M_j) \\ = \;\; & +\frac{e}{2}\, m \sum_{\beta}\sum_{i,j} M_i M_j \left[\left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right) R_2(M_i, M_j) + \left(i\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{\Lambda}^*_{\beta j}\tilde{\Lambda}^*_{\alpha j}\right) R_2(M_j, M_i) \right] \end{aligned}$$

$$
\begin{aligned}
&= +\frac{e}{2}\, m \sum_{\beta}\sum_{i,j} M_i M_j \left[\left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{\Lambda}_{\beta j}\tilde{\Lambda}_{\alpha j}\right) - \left(i\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{\Lambda}^*_{\beta j}\tilde{\Lambda}^*_{\alpha j}\right)\right] R_2(M_i, M_j) \\
&= +e\, m \sum_{\beta}\sum_{i,j} M_i M_j\, \Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{\Lambda}^*_{\beta j}\tilde{\Lambda}^*_{\alpha j}\right) R_2(M_i, M_j) \\
&= +2e\, m \sum_{\beta}\sum_{i>j} M_i M_j\, \Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{\Lambda}^*_{\beta j}\tilde{\Lambda}^*_{\alpha j}\right) R_2(M_i, M_j)\ . \qquad (4.9)
\end{aligned}
$$

Before we sum $d_{4\mathrm{A}}$ and $d_{5\mathrm{A}}$, I rewrite $d_{5\mathrm{A}}$ as follows:

$$
\begin{aligned}
d_{5\mathrm{A}} &= -\frac{eg^3}{2\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j \left(i\tilde{\lambda}_{\alpha j}\tilde{V}_{\beta j}\tilde{V}^*_{\beta i}\tilde{V}^*_{\alpha i}\right)\left[\,\overline{C}_1 + \overline{C}_2\,\right]_{(M_i, M_j)} \\
&= -\frac{eg^3}{2\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j \left(i\tilde{\lambda}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right)\left[\,\overline{C}_1 + \overline{C}_2\,\right]_{(M_j, M_i)} \\
&= -\frac{eg^3}{2\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j \left(i\tilde{\lambda}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right)\left[\,C_1 + C_2\,\right]_{(M_i, M_j)}\ . \qquad (4.10)
\end{aligned}
$$

Then,

$$
\begin{aligned}
&d_{4\mathrm{A}} + d_{5\mathrm{A}} \\
&\quad = +\frac{eg^3}{2\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j \left[\left(i\tilde{\lambda}^*_{\alpha i}\tilde{V}^*_{\beta i}\tilde{V}_{\beta j}\tilde{V}_{\alpha j}\right) - \left(i\tilde{\lambda}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right)\right]\left[\,C_1 + C_2\,\right]_{(M_i, M_j)} \\
&\quad = +\frac{eg^3}{\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j\, \Im\left(\tilde{\lambda}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right)\left[\,C_1 + C_2\,\right]_{(M_i, M_j)}\ . \qquad (4.11)
\end{aligned}
$$

let's define

$$
R_3(M_1, M_2) \equiv 2\Big[\,C_1 + C_2\,\Big]_{(M_i, M_j)}\ , \qquad (4.12)
$$

so we can write

$$
\begin{aligned}
&d_{4\mathrm{A}} + d_{5\mathrm{A}} \\
&\quad = +\frac{eg^3}{2\sqrt{2}} M_W \sum_{\beta}\sum_{i,j} M_i M_j\, \Im\left(\tilde{\lambda}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) R_3(M_i, M_j)
\end{aligned}
\qquad (4.13)
$$

To add $d_{4\mathrm{B}}$ and $d_{5\mathrm{B}}$, I first rewrite $d_{5\mathrm{B}}$:

$$
\begin{aligned}
d_{5\mathrm{B}} &= -\frac{eg}{2\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j \left(i\tilde{\Lambda}_{\alpha j}\tilde{\Lambda}_{\beta j}\tilde{V}^*_{\beta i}\tilde{\lambda}^*_{\alpha i}\right)\overline{C}_2(M_i, M_j) \\
&= -\frac{eg}{2\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j \left(i\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{V}^*_{\beta j}\tilde{\lambda}^*_{\alpha j}\right) C_2(M_i, M_j)\ . \qquad (4.14)
\end{aligned}
$$

Then

$$
\begin{aligned}
&d_{4\mathrm{B}} + d_{5\mathrm{B}} \\
&\quad = +\frac{eg}{2\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j \left[\left(i\tilde{\Lambda}^*_{\alpha i}\tilde{\Lambda}^*_{\beta i}\tilde{V}_{\beta j}\tilde{\lambda}_{\alpha j}\right) - \left(i\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{V}^*_{\beta j}\tilde{\lambda}^*_{\alpha j}\right)\right] C_2(M_i, M_j) \\
&\quad = +\frac{eg}{\sqrt{2}} M_W \sum_{i,j,\beta} M_i M_j\, \Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{V}^*_{\beta j}\tilde{\lambda}^*_{\alpha j}\right) C_2(M_i, M_j)\ . \qquad (4.15)
\end{aligned}
$$

Let's define

$$
R_4(M_1, M_2) \equiv C_2(M_i, M_j)\ , \qquad (4.16)
$$

so we can write

$$\begin{aligned} & d_{4\mathrm{B}} + d_{5\mathrm{B}} \\ = & \; +\frac{eg}{\sqrt{2}} M_W \sum_\beta \sum_{i,j} M_i M_j \, \Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{V}^*_{\beta j}\tilde{\lambda}^*_{\alpha j}\right) R_4(M_i, M_j) \end{aligned} \tag{4.17}$$

Putting everything together, we get

$$\begin{aligned} d \;=\; & +\frac{eg^4}{2}\, m \sum_\beta \sum_{i>j} M_i M_j \, \Im\left(\tilde{V}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) R_1(M_i, M_j) \\ & +2e\, m \sum_\beta \sum_{i>j} M_i M_j \, \Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{\Lambda}^*_{\beta j}\tilde{\Lambda}^*_{\alpha j}\right) R_2(M_i, M_j) \\ & +\frac{eg^3}{2\sqrt{2}} M_W \sum_\beta \sum_{i,j} M_i M_j \, \Im\left(\tilde{\lambda}_{\alpha i}\tilde{V}_{\beta i}\tilde{V}^*_{\beta j}\tilde{V}^*_{\alpha j}\right) R_3(M_i, M_j) \\ & +\frac{eg}{\sqrt{2}}\, M_W \sum_\beta \sum_{i,j} M_i M_j \, \Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{V}^*_{\beta j}\tilde{\lambda}^*_{\alpha j}\right) R_4(M_i, M_j)\,. \end{aligned} \tag{4.18}$$

The explicit form of the R functions is:

$$\begin{aligned} R_1 \;=\; & -\frac{1}{(4\pi)^4 M_W^4}\frac{1}{18}\frac{1}{\left({M_1}^2-1\right)^4}\frac{1}{\left({M_2}^2-1\right)^4} \\ & \left(-141\,{M_1}^4 - 55\,{M_2}^2 + 55\,{M_1}^2 + 141\,{M_2}^4 + 36\,Li_2\left(1-{M_2}^2\right) - 119\,{M_2}^6\right. \\ & -36\,Li_2\left(1-{M_1}^2\right) + 119\,{M_1}^6 - 117\,{M_2}^6{M_1}^4 - 33\,{M_1}^8 - 27\,{M_1}^6 Li_2\left(1-{M_2}^2\right){M_2}^6 \\ & +18\,{M_1}^2{M_2}^2 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right) - 180\,{M_2}^2{M_1}^2\ln\left({M_1}^2\right) + 18\,{M_1}^2{M_2}^2 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right) \\ & +180\,{M_1}^2{M_2}^2\ln\left({M_2}^2\right) - 18\,{M_2}^2{M_1}^2 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right) + 117\,{M_2}^4{M_1}^6 - 236\,{M_2}^2{M_1}^6 \\ & -27\,{M_1}^2{M_2}^4 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right) - 120\,{M_1}^2 Li_2\left(1-{M_2}^2\right) + 74\,{M_1}^2\ln\left({M_1}^2\right) \\ & -225\,{M_2}^4{M_1}^2 + 27\,{M_1}^6 Li_2\left(1-{M_1}^2\right){M_2}^6 - 27\,{M_1}^6 Li_2\left(1-{M_1}^2\right){M_2}^4 \\ & -18\,{M_1}^4 Li_2\left(1-{M_1}^2\right){M_2}^6 + 177\,{M_1}^4 Li_2\left(1-{M_1}^2\right){M_2}^2 - 33\,{M_1}^{10} Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right){M_2}^2 \\ & +51\,{M_1}^8 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right){M_2}^4 + 27\,{M_1}^6 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right){M_2}^6 \\ & -228\,{M_1}^4 Li_2\left(1-{M_2}^2\right){M_2}^2 - 162\,{M_1}^4{M_2}^2 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right) + 21\,{M_2}^6{M_1}^4\ln\left({M_1}^2\right) \\ & +57\,{M_2}^6\ln\left({M_2}^2\right){M_1}^2 - 57\,{M_1}^6\ln\left({M_1}^2\right){M_2}^2 + 228\,{M_2}^4 Li_2\left(1-{M_1}^2\right){M_1}^2 \\ & +18\,{M_2}^4 Li_2\left(1-{M_2}^2\right){M_1}^6 - 177\,{M_2}^4 Li_2\left(1-{M_2}^2\right){M_1}^2 - 27\,{M_2}^2{M_1}^4 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right) \\ & +27\,{M_2}^2{M_1}^4 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right) + 27\,{M_1}^2{M_2}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right) - 297\,Li_2\left(1-{M_1}^2\right){M_2}^2{M_1}^2 \\ & -108\,Li_2\left(1-{M_1}^2\right){M_2}^4{M_1}^4 + 297\,Li_2\left(1-{M_2}^2\right){M_2}^2{M_1}^2 + 132\,{M_1}^6 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right){M_2}^2 \\ & -132\,{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2 + 33\,{M_1}^{10} Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2 - 51\,{M_1}^8 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^4 \\ & -27\,{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^6 + 33\,{M_1}^4{M_2}^8\ln\left({M_2}^2\right) + 84\,{M_1}^4{M_2}^6\ln\left({M_2}^2\right) \\ & +45\,{M_1}^8{M_2}^2 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right) - 27\,{M_1}^4{M_2}^6 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right) \end{aligned}$$

$$
\begin{aligned}
&-222\,{M_1}^6{M_2}^4Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)-27\,{M_2}^4{M_1}^6Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)+45\,{M_2}^8{M_1}^2Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\\
&+27\,{M_2}^4{M_1}^6Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)-45\,{M_2}^8{M_1}^2Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+165\,{M_1}^4{M_2}^2\ln\left({M_1}^2\right)\\
&+198\,{M_1}^4{M_2}^4Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)+27\,Li_2\left(1-{M_2}^2\right){M_2}^6{M_1}^4+108\,Li_2\left(1-{M_2}^2\right){M_2}^4{M_1}^4\\
&-165\,{M_1}^2{M_2}^4\ln\left({M_2}^2\right)+171\,{M_2}^4{M_1}^2\ln\left({M_1}^2\right)-162\,{M_2}^4{M_1}^2Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\\
&+162\,{M_2}^4{M_1}^2Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+198\,{M_1}^4{M_2}^4Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\\
&+222\,{M_2}^6{M_1}^4Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)-198\,{M_2}^4{M_1}^4Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)\\
&+132\,{M_2}^6Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_1}^2-33\,{M_2}^{10}Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_1}^2\\
&+33\,{M_2}^{10}Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right){M_1}^2-45\,{M_1}^8{M_2}^2Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)\\
&+27\,{M_1}^4{M_2}^6Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+222\,{M_1}^6{M_2}^4Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)-132\,{M_2}^6\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right){M_1}^2\\
&-75\,{M_1}^6\ln\left({M_2}^2\right){M_2}^6-33\,{M_1}^8\ln\left({M_2}^2\right){M_2}^2-21\,{M_1}^6\ln\left({M_2}^2\right){M_2}^4-84\,{M_2}^4{M_1}^6\ln\left({M_1}^2\right)\\
&-98\,{M_2}^6{M_1}^2\ln\left({M_1}^2\right)-48\,{M_2}^4{M_1}^4\ln\left({M_1}^2\right)+33\,{M_2}^8{M_1}^2\ln\left({M_1}^2\right)-33\,{M_2}^4{M_1}^8\ln\left({M_1}^2\right)\\
&-33\,{M_2}^8{M_1}^4\ln\left({M_1}^2\right)+75\,{M_2}^6{M_1}^6\ln\left({M_1}^2\right)+48\,{M_1}^4{M_2}^4\ln\left({M_2}^2\right)-66\,{M_1}^2{M_2}^8\ln\left({M_2}^2\right)\\
&+33\,{M_1}^8{M_2}^4\ln\left({M_2}^2\right)+27\,{M_1}^6{M_2}^6Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)+51\,{M_1}^4{M_2}^8Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\\
&-27\,{M_1}^6{M_2}^6Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)-51\,{M_1}^4{M_2}^8Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+98\,{M_1}^6\ln\left({M_2}^2\right){M_2}^2\\
&-18\,{M_2}^6\left(1-{M_1}^2\right){M_1}^2+18\,{M_1}^6Li_2\left(1-{M_2}^2\right){M_2}^2+33\,{M_2}^8{M_1}^4+225\,{M_2}^2{M_1}^4\\
&-66\,{M_2}^8{M_1}^2-33\,{M_2}^4{M_1}^8+66\,{M_2}^2{M_1}^8-66\,{M_2}^6\ln\left({M_2}^2\right)+33\,{M_2}^8+66\,{M_1}^8\ln\left({M_1}^2\right){M_2}^2\\
&+162\,{M_1}^4{M_2}^2Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)-33\,{M_1}^8\ln\left({M_1}^2\right)-105\,{M_1}^4\ln\left({M_1}^2\right)+87\,{M_1}^2Li_2\left(1-{M_1}^2\right)\\
&+120\,{M_2}^2Li_2\left(1-{M_1}^2\right)+9\,{M_1}^4Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)-9\,{M_1}^4Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)\\
&-87\,{M_2}^2Li_2\left(1-{M_2}^2\right)-9\,{M_1}^2Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+9\,{M_1}^2Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)\\
&+9\,{M_2}^2Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-9\,{M_2}^2Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)-9\,{M_2}^4Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\\
&+9\,{M_2}^4Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)-74\,{M_2}^2\ln\left({M_2}^2\right)-198\,{M_1}^4{M_2}^4Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)\\
&-222\,{M_2}^6{M_1}^4Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-171\,{M_1}^4{M_2}^2\ln\left({M_2}^2\right)-93\,{M_2}^4Li_2\left(1-{M_1}^2\right)\\
&-18\,{M_1}^2{M_2}^2Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+105\,{M_2}^4\ln\left({M_2}^2\right)+236\,{M_2}^6{M_1}^2-9\,Li_2\left(1-{M_2}^2\right){M_1}^6\\
&+51\,Li_2\left(1-{M_2}^2\right){M_2}^4-33\,{M_1}^{10}Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+96\,{M_1}^8Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)\\
&+33\,{M_2}^{10}Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-96\,{M_2}^8Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-33\,{M_2}^{10}Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)\\
&+96\,{M_2}^8Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+9\,{M_2}^6Li_2\left(1-{M_1}^2\right)-51\,Li_2\left(1-{M_1}^2\right){M_1}^4
\end{aligned}
$$

$$
\begin{aligned}
&-63\,{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+63\,{M_1}^6 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)+93\,{M_1}^4 Li_2\left(1-{M_2}^2\right)\\
&+63\,{M_2}^6 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-63\,{M_2}^6 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+33\,{M_2}^8\ln\left({M_2}^2\right)\\
&+66\,{M_1}^6\ln\left({M_1}^2\right)+33\,{M_1}^{10} Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)-96\,{M_1}^8 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)\Bigg)
\end{aligned} \tag{4.19}
$$

Expanding in terms of the difference of the masses and their sum:

$$
R_1 \approx -\frac{1}{(4\pi)^4 M_W^4}\left(\frac{352}{3}-\frac{160}{9}\,\pi^2-192\ln\left(\frac{M}{2}\right)\right)\frac{d}{M^5}+O(d^3) \tag{4.20}
$$

Where

$$
\begin{aligned}
M &= M_1+M_2\\
d &= M_1-M_2
\end{aligned} \tag{4.21}
$$

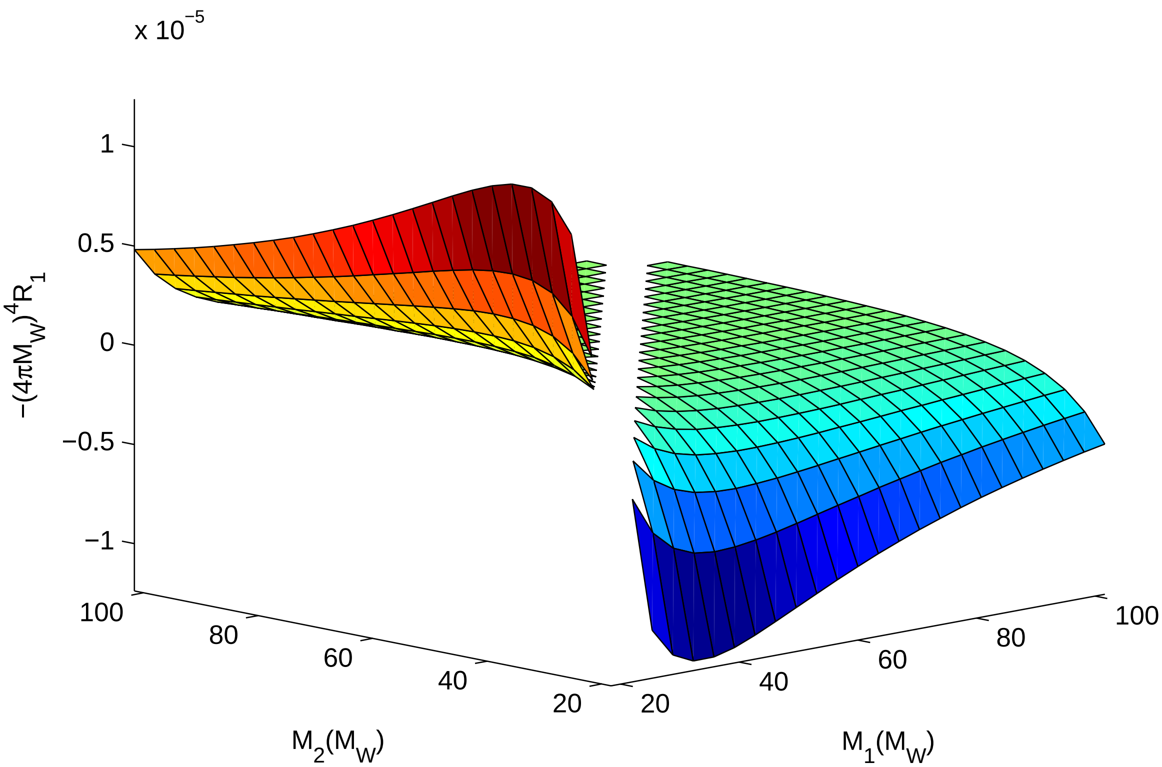


Figure 4.1: $R1$ **Function**

$$
\begin{aligned}
R_2 \quad = \quad & -\frac{1}{(4\pi)^4 M_W^4}\frac{1}{36}\frac{1}{\left({M_2}^2-1\right)^4}\frac{1}{\left({M_1}^2-1\right)^4} \\
& \left(-18\,{M_2}^2{M_1}^2 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+18\,{M_2}^2{M_1}^2 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\right. \\
& -18\,{M_1}^2{M_2}^2 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+123\,{M_1}^2{M_2}^2\ln\left({M_2}^2\right)-123\,{M_1}^2{M_2}^2\ln\left({M_1}^2\right) \\
& +18\,{M_1}^2{M_2}^2 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)+96\,{M_2}^4\ln\left({M_2}^2\right)+9\,{M_2}^2 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right) \\
& +9\,Li_2\left(1-{M_2}^2\right)+9\,{M_1}^2 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)-9\,{M_1}^2 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right) \\
& -43\,{M_2}^2\ln\left({M_2}^2\right)-96\,{M_1}^4\ln\left({M_1}^2\right)-9\,{M_2}^2 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right) \\
& -63\,{M_1}^4 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)+63\,{M_1}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+43\,{M_1}^2\ln\left({M_1}^2\right) \\
& -21\,{M_1}^2 Li_2\left(1-{M_2}^2\right)+24\,{M_1}^2 Li_2\left(1-{M_1}^2\right)-63\,{M_2}^4 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)
\end{aligned}
$$

$$
\begin{aligned}
&+21\,{M_2}^2 Li_2\left(1-{M_1}^2\right)+63\,{M_2}^4 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)-24\,{M_2}^2 Li_2\left(1-{M_2}^2\right)\\
&+102\,{M_2}^4-38\,{M_2}^2+24\,{M_1}^{10} Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2-15\,{M_1}^8 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^4\\
&-192\,{M_1}^4{M_2}^4\ln\left({M_1}^2\right)+24\,{M_1}^4{M_2}^8-141\,{M_1}^4{M_2}^2\ln\left({M_2}^2\right)+66\,{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2\\
&-85\,{M_2}^6\ln\left({M_1}^2\right){M_1}^2-27\,{M_2}^2{M_1}^4 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)+228\,{M_1}^4{M_2}^2\ln\left({M_1}^2\right)\\
&-63\,{M_1}^4{M_2}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+84\,{M_1}^4{M_2}^6\ln\left({M_1}^2\right)+69\,{M_1}^6{M_2}^4\ln\left({M_1}^2\right)\\
&+147\,{M_2}^6\ln\left({M_2}^2\right){M_1}^2-228\,{M_1}^2{M_2}^4\ln\left({M_2}^2\right)+72\,{M_1}^8 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right){M_2}^2-24\,{M_1}^8\\
&+178\,{M_1}^2{M_2}^6-24\,{M_1}^8{M_2}^4-51\,{M_1}^6{M_2}^4 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)+21\,{M_1}^4 Li_2\left(1-{M_2}^2\right)\\
&-178\,{M_1}^6{M_2}^2-48\,{M_1}^2{M_2}^8+38\,{M_1}^2-72\,{M_2}^8{M_1}^2 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)\\
&+51\,{M_2}^6{M_1}^4 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)-54\,Li_2\left(1-{M_1}^2\right){M_1}^2{M_2}^2+63\,{M_2}^4{M_1}^4 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\\
&-66\,{M_1}^6 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right){M_2}^2+66\,{M_2}^6 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right){M_1}^2\\
&-24\,{M_1}^{10} Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right){M_2}^2+15\,{M_1}^8\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right){M_2}^4+24\,{M_2}^8{M_1}^2\ln\left({M_1}^2\right)\\
&-18\,{M_2}^6 Li_2\left(1-{M_1}^2\right){M_1}^2-63\,{M_2}^4{M_1}^4 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)-33\,{M_2}^4 Li_2\left(1-{M_2}^2\right){M_1}^2\\
&+9\,{M_2}^6 Li_2\left(1-{M_1}^2\right){M_1}^4+72\,{M_2}^8{M_1}^2 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-51\,{M_2}^6{M_1}^4 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\\
&-24\,{M_2}^{10}{M_1}^2 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)+15\,{M_2}^8{M_1}^4 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)+117\,{M_1}^6 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)\\
&-117\,{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+9\,Li_2\left(1-{M_1}^2\right){M_2}^6-15\,Li_2\left(1-{M_1}^2\right){M_1}^4\\
&+24\,{M_2}^{10} Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-87\,{M_2}^8 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-9\,{M_1}^6 Li_2\left(1-{M_2}^2\right)\\
&-24\,{M_1}^{10} Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+87\,{M_1}^8 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)-75\,{M_2}^6\ln\left({M_2}^2\right)\\
&+117\,{M_2}^6 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-117\,{M_2}^6 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+15\,Li_2\left(1-{M_2}^2\right){M_2}^4\\
&+24\,{M_2}^8\ln\left({M_2}^2\right)+24\,{M_1}^{10} Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)-87\,{M_1}^8 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)\\
&-24\,{M_2}^{10} Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+87\,{M_2}^8 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right)+75\,{M_1}^6\ln\left({M_1}^2\right)+88\,{M_1}^6\\
&+24\,{M_2}^8-27\,Li_2\left(1-{M_1}^2\right){M_1}^4{M_2}^4-21\,{M_2}^4 Li_2\left(1-{M_1}^2\right)-15\,{M_2}^8 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right){M_1}^4\\
&+24\,{M_2}^{10} Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right){M_1}^2-3\,{M_2}^6\ln\left({M_2}^2\right){M_1}^6+24\,{M_2}^8\ln\left({M_2}^2\right){M_1}^4\\
&-69\,{M_2}^6\ln\left({M_2}^2\right){M_1}^4-48\,{M_2}^8\ln\left({M_2}^2\right){M_1}^2+27\,Li_2\left(1-{M_2}^2\right){M_1}^4{M_2}^4\\
&+27\,{M_1}^2{M_2}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)-27\,{M_1}^2{M_2}^4 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right)+192\,{M_1}^4{M_2}^4\ln\left({M_2}^2\right)
\end{aligned}
$$

$$
\begin{aligned}
&+85\,{M_1}^6 \ln\left({M_2}^2\right){M_2}^2 + 24\,{M_1}^8 \ln\left({M_2}^2\right){M_2}^4 - 24\,{M_1}^8{M_2}^2 \ln\left({M_2}^2\right) - 84\,{M_1}^6{M_2}^4 \ln\left({M_2}^2\right)\\
&+141\,{M_2}^4{M_1}^2 \ln\left({M_1}^2\right) + 63\,{M_1}^4{M_2}^4 Li_2\left(\frac{{M_1}^2-{M_2}^2}{{M_1}^2}\right) + 54\,Li_2\left(1-{M_2}^2\right){M_1}^2{M_2}^2\\
&+18\,Li_2\left(1-{M_2}^2\right){M_1}^6{M_2}^2 - 147\,{M_1}^6 \ln\left({M_1}^2\right){M_2}^2 - 48\,{M_1}^4 Li_2\left(1-{M_2}^2\right){M_2}^2\\
&-9\,{M_1}^6 Li_2\left(1-{M_2}^2\right){M_2}^4 + 27\,{M_2}^2{M_1}^4 Li_2\left(-\frac{{M_1}^2-{M_2}^2}{{M_2}^2}\right) - 88\,{M_2}^6 - 9\,Li_2\left(1-{M_1}^2\right)\\
&-102\,{M_1}^4 - 168\,{M_1}^2{M_2}^4 - 24\,{M_1}^8 \ln\left({M_1}^2\right){M_2}^4 + 3\,{M_1}^6 \ln\left({M_1}^2\right){M_2}^6 - 24\,{M_1}^4{M_2}^8 \ln\left({M_1}^2\right)\\
&+48\,{M_1}^8{M_2}^2 \ln\left({M_1}^2\right) + 48\,{M_2}^4 Li_2\left(1-{M_1}^2\right){M_1}^2 - 66\,{M_2}^6 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_1}^2\\
&+33\,{M_1}^4 Li_2\left(1-{M_1}^2\right){M_2}^2 - 72\,{M_1}^8 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2 + 51\,{M_1}^6{M_2}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)\\
&-24\,{M_1}^8 \ln\left({M_1}^2\right) - 90\,{M_1}^4{M_2}^6 + 90\,{M_1}^6{M_2}^4 + 168\,{M_1}^4{M_2}^2 + 48\,{M_1}^8{M_2}^2\ )
\end{aligned}
\tag{4.22}
$$

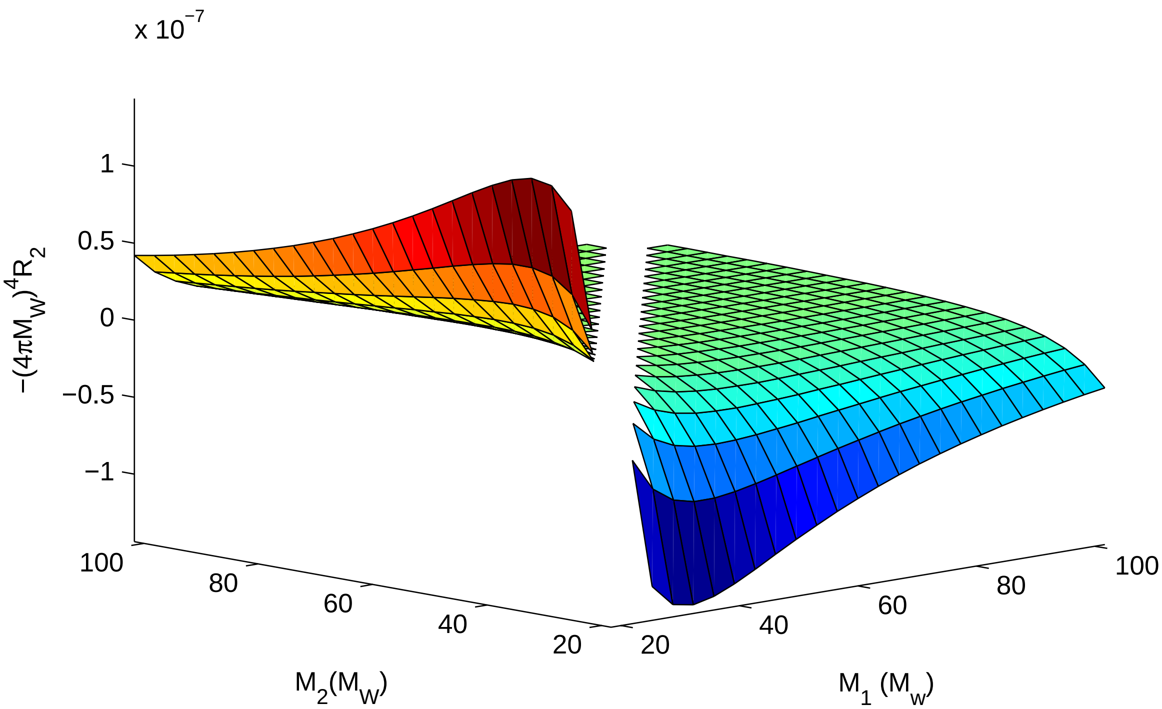


Figure 4.2: $R2$ **Function**

$$R_2 \approx \left(-\frac{592}{3} + \frac{176}{9}\,\pi^2\right)\frac{d}{M^5} + O(d^3) \tag{4.23}$$

Where

$$\begin{aligned} M &= M_1 + M_2 \\ d &= M_1 - M_2 \end{aligned} \tag{4.24}$$

$$\begin{aligned}
R_3 \;=\; & -\frac{1}{(4\pi)^4 M_W^4}\frac{1}{2}\,\frac{1}{\left({M_1}^2-1\right)^3}\,\frac{1}{\left({M_2}^2-1\right)^3} \\
& \left(-2+2\,{M_1}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^4 - 7\,{M_1}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2\right. \\
& -2\,{M_1}^4 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right){M_2}^4 + 7\,{M_1}^4 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right){M_2}^2 \\
& +{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2 + 2\,{M_1}^2 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2 - 3\,{M_2}^6 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_1}^2 \\
& +4\,{M_2}^2{M_1}^2\ln\left({M_1}^2\right) - {M_1}^2\ln\left({M_1}^2\right) + 6\,{M_1}^2 + 2\,{M_1}^2\ln\left({M_1}^2\right){M_2}^6 - {M_1}^4\ln\left({M_1}^2\right){M_2}^4 \\
& -5\,{M_1}^2\ln\left({M_1}^2\right){M_2}^4 + 2\,{M_1}^4\ln\left({M_1}^2\right){M_2}^2 - 5\,Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right){M_2}^4{M_1}^2 \\
& -4\,Li_2\left(1-{M_1}^2\right){M_2}^4{M_1}^2 - 2\,\ln\left({M_2}^2\right){M_2}^4{M_1}^2 + 3\,{M_2}^6 Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right){M_1}^2 \\
& -3\,{M_2}^4\ln\left({M_2}^2\right) + 5\,{M_2}^4 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right) - 2\,{M_2}^8 Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right)
\end{aligned}$$

$$
\begin{aligned}
&+5\,{M_2}^6 Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right)-3\,{M_1}^2 Li_2\left(1-{M_1}^2\right)-{M_1}^4\ln\left({M_1}^2\right)-5\,{M_2}^4 Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right)\\
&+5\,{M_1}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)-5\,{M_2}^6 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)-8\,{M_2}^4-{M_1}^6 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right){M_2}^2\\
&-2\,{M_1}^2 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right){M_2}^2+2\,{M_1}^4 Li_2\left(1-{M_2}^2\right){M_2}^4-2\,{M_1}^2{M_2}^2 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)\\
&+5\,Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_2}^4{M_1}^2+2\,{M_1}^2{M_2}^2 Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right)-5\,{M_1}^4 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right)\\
&+{M_1}^6 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right)+2\,{M_1}^4 Li_2\left(1-{M_2}^2\right)+2\,Li_2\left(1-{M_1}^2\right){M_2}^4-3\,Li_2\left(1-{M_1}^2\right){M_2}^2\\
&-{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+2\,{M_2}^8 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)+10\,{M_2}^2{M_1}^4-3\,\ln\left({M_2}^2\right){M_2}^2{M_1}^4\\
&+7\,Li_2\left(1-{M_1}^2\right){M_2}^2{M_1}^2+2\,{M_2}^6+Li_2\left(1-{M_1}^2\right)+5\,{M_2}^4\ln\left({M_2}^2\right){M_1}^4\\
&-2\,\ln\left({M_2}^2\right){M_2}^6{M_1}^2-4\,Li_2\left(1-{M_2}^2\right){M_2}^2{M_1}^4-Li_2\left(1-{M_2}^2\right){M_2}^2{M_1}^2\\
&-2\,{M_2}^6{M_1}^2+2\,\ln\left({M_2}^2\right){M_2}^6+Li_2\left(1-{M_2}^2\right){M_2}^2+Li_2\left(1-{M_2}^2\right){M_1}^2+14\,{M_2}^4{M_1}^2\\
&+8\,{M_2}^2-6\,{M_2}^4{M_1}^4+3\,{M_2}^2\ln\left({M_2}^2\right)-2\,Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_2}^2+2\,Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right){M_2}^2\\
&-2\,{M_1}^2 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+2\,{M_1}^2 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right)-18\,{M_2}^2{M_1}^2-4\,{M_1}^4-Li_2\left(1-{M_2}^2\right)\Bigg)
\end{aligned}
\tag{4.25}
$$

$$
\begin{aligned}
R_4 \;=\;& -\frac{1}{(4\pi)^4 M_W^4}\frac{1}{16}\frac{1}{\left({M_1}^2-1\right)^3}\frac{1}{\left({M_2}^2-1\right)^3}\\
&\Big(-9+4\,{M_1}^4 Li_2\left(1-{M_2}^2\right)+12\,{M_2}^2{M_1}^4-4\,{M_1}^2\ln\left({M_1}^2\right)-40\,{M_2}^2{M_1}^2\\
&-8\,Li_2\left(1-{M_1}^2\right){M_2}^4{M_1}^2-4\,{M_1}^2 Li_2\left(1-{M_1}^2\right)+8\,\ln\left({M_2}^2\right){M_2}^4{M_1}^2\\
&-4\,\ln\left({M_2}^2\right){M_2}^2{M_1}^4+8\,{M_2}^2\ln\left({M_2}^2\right)-4\,Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_2}^6{M_1}^2\\
&-4\,{M_1}^4 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right){M_2}^4-4\,{M_1}^6 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right){M_2}^2\\
&-8\,{M_1}^2 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right){M_2}^2+4\,{M_1}^4 Li_2\left(1-{M_2}^2\right){M_2}^4-8\,{M_1}^4 Li_2\left(1-{M_2}^2\right){M_2}^2\\
&-20\,{M_1}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2+4\,{M_1}^2 Li_2\left(1-{M_2}^2\right)-5\,{M_1}^4\\
&+20\,{M_1}^4 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right){M_2}^2+4\,Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_2}^4{M_1}^2+6\,{M_2}^4{M_1}^4\ln\left({M_2}^2\right)\\
&+8\,\ln\left({M_2}^2\right){M_2}^6-27\,{M_2}^4-4\,Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right){M_2}^4{M_1}^2+8\,{M_2}^2{M_1}^4\ln\left({M_1}^2\right)\\
&+4\,{M_1}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^4+4\,{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2+8\,{M_1}^2 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right){M_2}^2\\
&+12\,Li_2\left(1-{M_1}^2\right){M_2}^2{M_1}^2+4\,{M_1}^6 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right)-4\,Li_2\left(1-{M_2}^2\right)\\
&-4\,{M_2}^2{M_1}^2\ln\left({M_2}^2\right)+16\,{M_2}^2{M_1}^2\ln\left({M_1}^2\right)-20\,Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_2}^6\\
&+4\,Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right){M_2}^6{M_1}^2+8\,Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_2}^8+8\,{M_2}^6{M_1}^2\ln\left({M_1}^2\right)\\
&-4\,{M_2}^4{M_1}^4\ln\left({M_1}^2\right)-20\,{M_2}^4{M_1}^2\ln\left({M_1}^2\right)+14\,{M_1}^2+28\,{M_2}^2-14\,{M_2}^4\ln\left({M_2}^2\right)\\
&-7\,{M_2}^4{M_1}^4+34\,{M_2}^4{M_1}^2-8\,{M_2}^6{M_1}^2-12\,Li_2\left(1-{M_1}^2\right){M_2}^2+8\,Li_2\left(1-{M_1}^2\right){M_2}^4
\end{aligned}
$$

$$+20\,{M_2}^4 Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right)+4\,Li_2\left(1-{M_2}^2\right){M_2}^2-4\,{M_1}^4\ln\left({M_1}^2\right)-8\,Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right){M_2}^8$$
$$+20\,Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right){M_2}^6-4\,{M_1}^6 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+16\,{M_1}^4 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)$$
$$-16\,{M_1}^4 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right)-20\,{M_2}^4 Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right)+8\,{M_2}^6-8\,Li_2\left(\frac{{M_2}^2-1}{{M_2}^2}\right){M_2}^2$$
$$+8\,Li_2\left(\frac{{M_2}^2-{M_1}^2}{{M_2}^2}\right){M_2}^2-8\,{M_1}^2 Li_2\left(\frac{{M_1}^2-1}{{M_1}^2}\right)+8\,{M_1}^2 Li_2\left(-\frac{{M_2}^2-{M_1}^2}{{M_1}^2}\right)$$
$$-4\,Li_2\left(1-{M_2}^2\right){M_2}^2{M_1}^2-8\,\ln\left({M_2}^2\right){M_2}^6{M_1}^2+4\,Li_2\left(1-{M_1}^2\right)\Bigg) \tag{4.26}$$

The R_3 and the R_4 are not anti-symmetric in the masses. However, after summing over the different internal states it picks the anti-symmetric part of the functions in M_i and M_j

4.2 Electron EDM in Large Mixing Textures

Now we look at the Okamura texture as an example of models with relatively large mixing.

In this class of models the Λ's are the largest of all the couplings. So we can keep the term with all the Λ's and neglect the rest.

$$d = +2e\,m\sum_\beta\sum_{i>j} M_i M_j\,\Im\left(\tilde{\Lambda}_{\alpha i}\tilde{\Lambda}_{\beta i}\tilde{\Lambda}^*_{\beta j}\tilde{\Lambda}^*_{\alpha j}\right)R_2(M_i,M_j)\,. \tag{4.27}$$

Looking at the approximate expression, and after some manipulations:

$$\begin{aligned} d &\propto e m_e G_F^2 dM\frac{m^4}{M^4} \\ &\propto e m_e G_F^2 dM(mixing)^4. \end{aligned} \tag{4.28}$$

In models with large and independent mixings, the contribution will be enhanced by several orders of magnitude.

Performing the sum of the exact expression numerically for certain choices of the parameters in the Okamura texture (ignoring the charged leptons diagonalization matrix) we get the following results:

- For $(\alpha,\beta,\gamma)=(0.3,1.2i,-0.3-1.2i)\Rrightarrow d=1.5*10^{-32}e.cm.$
- For $(\alpha,\beta,\gamma)=(0.3,1.2i,-0.3-1.2i)\Rrightarrow d=-3.7*10^{-32}e.cm.$
- For $(\alpha,\beta,\gamma)=(0.5,.1-.4*i,-0.6+0.4i)\Rrightarrow d=-6.6*10^{-33}e.cm.$

Scanning over several values of α and β, we find that alot of points fall between $10^{-31}-10^{-32}e\cdot cm$. The contribution is highly enhanced compared with the estimated contribution from the quark sector (of order $10^{-38}e\cdot cm$). It is still a few orders of magnitude below the current upper bound $(0.069\pm0.074)\times10^{-26}e\cdot cm$ [23] , [26] . However, next generation experiments are expected to improve the current upper bound by a few orders of magnitude [24] , [25] .

Conclusion

In this thesis, I presented a full 2-loop calculation of the diagrams that contribute to the lepton Electric Dipole Moment and involve Majorana neutrinos.

In the first Chapter, I introduced Charge and Parity transformations. Dirac and Majorana neutrinos can both be included in the Lagrangian of the Standard Model via the seesaw mechanism to generate the small masses for the light neutrinos.

An interesting case is when the see-saw mass matrix involves more than one generation of neutrinos. In certain textures the masses and the mixings can be made independent. The masses of the heavy right handed neutrinos can then be anywhere above the current experimental limits (TeV scale). A complex phase in the mixing matrix of the neutrinos leads to CP violation and a non-vanishing Electric Dipole Moment for leptons. If the mixing between the heavy right handed and the light neutrinos is made large enough, the contribution to the EDM can be large.

In the second Chapter, I showed how a CP violating term in the Lagrangian leads to a non-vanishing EDM for leptons. Then I showed how to extract the EDM from the calculation of the form factors of the interaction between the photon and leptons.

The third Chapter discusses a new class of diagrams that only exists if neutrinos are Majorana. I first show that the contribution does not necessarily vanish. After that I move to calculate the exact expression of the EDM.

The Feynman-t'Hooft gauge was used, where twenty sub-diagrams were to be calculated. However, chirality flips required on the internal lepton lines and Goldstone-lepton vertices reduce the number of diagrams needed to be calculated to ten. The rest of the diagrams are suppressed by order of the mass of the lepton squared over the mass of the W squared.

In the specific case where the masses and the mixings of the neutrinos in a seesaw texture can be made independent, three of these diagrams will be amplified by a factor related to the mixings. A detailed calculation showing all the intermediate steps and tricks is the rest of the content of Chapter 3.

The rest of the diagrams are calculated in Appendix A. In Chapter 4, the total contribution to the EDM is summarized.

Since the result is general in terms of the Yukawas and the Masses, they can be applied to various models with heavy right handed neutrinos, one of them would be the Takeuchi et. al. model [11]. Applying the result to the texture in [11] ; the electron EDM can be expected to be as large as $\mathcal{O}(10^{-31} - 10^{-30})$. Next generation experiments expect to reach that range [24] , [25] .

Finally, it would be exciting if future experiments discover a non zero EDM, or see TeV neutrinos. Even if no big discoveries are made; the improved limits will provide sever constraints on several theoretical models at the TeV scale, including models with TeV Majorana neutrino.

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Appendix A

The remaining diagrams

A.1 Diagram 1A

The couplings at the five vertices combine to give:

$$\left(-i\frac{g}{\sqrt{2}}\tilde{V}_{\alpha 2}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}_{\beta 2}\right)(+ie)\left(-i\frac{g}{\sqrt{2}}\tilde{V}^{\dagger}_{1\beta}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}^{\dagger}_{1\alpha}\right) = i\frac{eg^4}{4}\left(\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^{*}_{\beta 1}\tilde{V}^{*}_{\alpha 1}\right). \tag{A.1}$$

The fermion line is:

$$\begin{aligned}
&\langle\ell_\alpha(p_2)|\left(\overline{\ell_\alpha}\gamma^\mu P_L N_2\right)\left(\overline{\ell_\beta}\gamma^\nu P_L N_2\right)\left(\overline{\ell_\beta}\gamma^\lambda \ell_\beta\right)\left(\overline{N_1}\gamma_\mu P_L \ell_\beta\right)\left(\overline{N_1}\gamma_\nu P_L \ell_\alpha\right)|\ell_\alpha(p_1)\rangle \\
&= \quad -\langle\ell_\alpha(p_2)|\left(\overline{\ell_\alpha}\gamma^\mu P_L N_2\right)\left(\overline{N_2}P_L\gamma^\nu {\ell_\beta}^c\right)\left(\overline{{\ell_\beta}^c}\gamma^\lambda {\ell_\beta}^c\right)\left(\overline{{\ell_\beta}^c}P_L\gamma_\mu N_1\right)\left(\bar{N}_1\gamma_\nu P_L \ell_\alpha\right)|\ell_\alpha(p_1)\rangle \\
&= \quad -\bar{u}_\alpha(p_2)\gamma^\mu P_L\left\langle N_2\overline{N_2}\right\rangle P_L\gamma^\nu\left\langle \ell^c_\beta\overline{\ell^c_\beta}\right\rangle\gamma^\lambda\left\langle \ell^c_\beta\overline{\ell^c_\beta}\right\rangle P_L\gamma_\mu\left\langle N_1\overline{N_1}\right\rangle\gamma_\nu P_L u_\alpha(p_1) \\
&\to \quad -\bar{u}_\alpha(p_2)\gamma^\mu P_L\left[(\not{p}_2-\not{k}_2)+M_2\right]P_L\gamma^\nu\left(\not{p}_2-\not{k}_1-\not{k}_2\right)\gamma^\lambda\left(\not{p}_1-\not{k}_1-\not{k}_2\right)P_L\gamma_\mu \\
&\qquad\qquad \times\left[(\not{p}_1-\not{k}_1)+M_1\right]\gamma_\nu P_L u_\alpha(p_1) \\
&= \quad -M_1M_2\,\bar{u}_\alpha(p_2)\gamma^\mu\gamma^\nu\left(\not{p}_2-\not{k}_1-\not{k}_2\right)\gamma^\lambda\left(\not{p}_1-\not{k}_1-\not{k}_2\right)\gamma_\mu\gamma_\nu P_L u_\alpha(p_1)\,.
\end{aligned} \tag{A.2}$$

The last two lines show only the numerators of the fermion propagators. I neglect the charged lepton mass, m_β, in the internal propagators since they only contribute at order m^2, the same order at which the neglected diagrams contribute. Let $K \equiv k_1 + k_2$, $P_1 \equiv p_1 - K$, $P_2 \equiv p_2 - K$ to simplify our notation. Then,

$$\begin{aligned}
\gamma^\mu\gamma^\nu\not{P}_2\gamma^\lambda\not{P}_1\gamma^\mu\gamma^\nu P_L &= \left[\gamma^\mu\gamma^\nu\not{P}_2\gamma^\lambda\not{P}_1\gamma^\mu\right]\gamma^\nu P_L \\
&= 2\left[\not{P}_2\gamma^\lambda\not{P}_1\gamma^\nu+\gamma^\nu\not{P}_1\gamma^\lambda\not{P}+2\right]\gamma^\nu P_L \\
&= 2\left[4\not{P}_2\gamma^\lambda\not{P}_1-2\not{P}_2\gamma^\lambda\not{P}_1\right]P_L \\
&= 4\not{P}_2\gamma^\lambda\not{P}_1 P_L \\
&= 4\not{P}_2\gamma^\lambda P_R\not{P}_1 \\
&= 4(\not{p}_2-\not{K})\gamma^\lambda P_R(\not{p}_1-\not{K}) \\
&\to 4(m_\alpha-\not{K})\gamma^\lambda P_R(m_\alpha-\not{K}) \\
&= 4\left[m_\alpha^2\gamma^\lambda P_R-m_\alpha\left(\not{K}\gamma^\lambda P_R+\gamma^\lambda\not{K}P_L\right)+\not{K}\gamma^\lambda\not{K}P_L\right]
\end{aligned} \tag{A.3}$$

That is the same combination that we had in diagram $1B$ except for the 4 factor. so we expect the result to be:

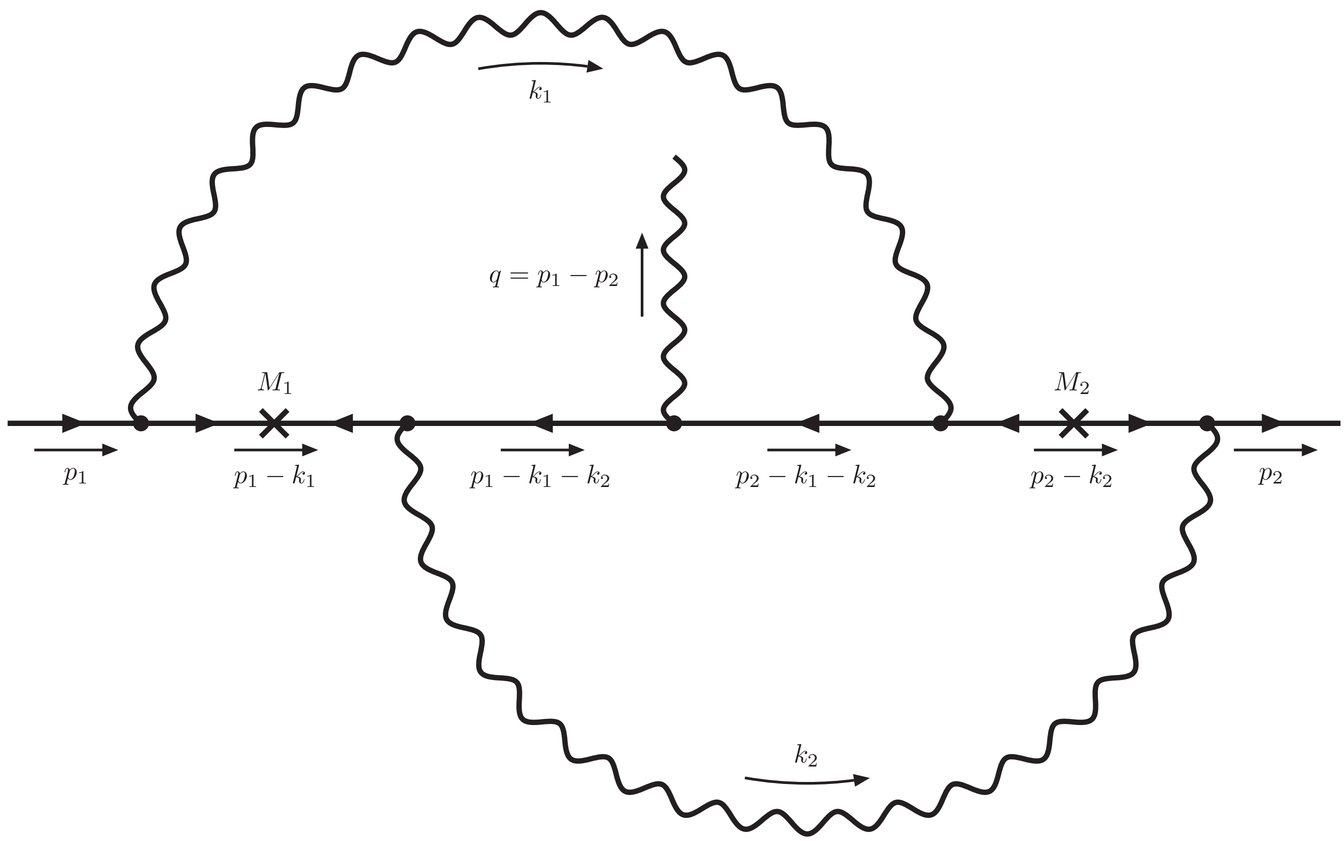

Figure A.1: **Diagram 1A**

$$2mM_1M_2\Big[-(A_1-A_2)+(B_1-B_2)\Big](p_1+p_2)^\lambda\gamma_5\ . \tag{A.4}$$

Multiplied by the factor

$$i\,\frac{eg^4}{4}\left(\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^*_{\beta 1}\tilde{V}^*_{\alpha 1}\right) \tag{A.5}$$

and we obtain

$$i\,\frac{eg^4}{2}mM_1M_2\left(\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^*_{\beta 1}\tilde{V}^*_{\alpha 1}\right)\Big[-(A_1-A_2)+(B_1-B_2)\Big](p_1+p_2)^\lambda\gamma_5\ . \tag{A.6}$$

This needs to be identified with

$$iG_2(0)\,\frac{(p_1+p_2)^\lambda\gamma_5}{2m}=-d\,(p_1+p_2)^\lambda\gamma_5\ , \tag{A.7}$$

so we find

$$d=\frac{e}{2}\,g^4mM_1M_2\left(i\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^*_{\beta 1}\tilde{V}^*_{\alpha 1}\right)\Big[(A_1-A_2)-(B_1-B_2)\Big]\ . \tag{A.8}$$

A.2 Diagram 2A

The γWW-vertex is

$$[(-p_1+2p_2-k_1)_\kappa g_{\lambda\nu}+(-p_1-p_2+2k_1)_\lambda g_{\kappa\nu}+(2p_1-p_2-k_1)_\nu g_{\kappa\lambda}]\ . \tag{A.9}$$

The fermion line gives

$$4M_1M_2\,\bar{u}(p_2)(k_1-k_2)^\nu\gamma^\kappa P_L u(p_1)\ . \tag{A.10}$$

Combining with the γWW-vertex factor, we find

$$\begin{aligned}
& 4M_1M_2\,\bar{u}(p_2)(k_1-k_2)_\lambda(-\not{p}_1+2\not{p}_2-\not{k}_1)P_L u(p_1)\ .\\
+\ & 4M_1M_2\,\bar{u}(p_2)(\not{k}_1-\not{k}_2)(-p_1-p_2+2k_1)_\lambda P_L u(p_1)\ .\\
+\ & 4M_1M_2\,\bar{u}(p_2)(k_1-k_2)\cdot(2p_1-p_2-k_1)\gamma^\lambda P_L u(p_1)\ .\\
=\ & 4M_1M_2\,\bar{u}(p_2)(k_1-k_2)_\lambda(2m-\not{k}_1)P_L u(p_1)-4M_1M_2\,m\bar{u}(p_2)(k_1-k_2)_\lambda P_R u(p_1)\ .\\
+\ & 4M_1M_2\,\bar{u}(p_2)(\not{k}_1-\not{k}_2)(-p_1-p_2+2k_1)_\lambda P_L u(p_1)\ .\\
+\ & 4M_1M_2\,\bar{u}(p_2)(k_1-k_2)\cdot(2p_1-p_2-k_1)\gamma^\lambda P_L u(p_1)\ .
\end{aligned} \tag{A.11}$$

The last term does not contribute, now define:

$$\begin{aligned}
& C_1p_1^\kappa+C_2p_2^\kappa\\
&= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{(k_1-k_2)^\kappa}{[(p_2-k_2)^2-M_2^2](k_1-k_2)^2(k_1^2-M_1^2)[(p_1-k_1)^2-M_W^2][(p_2-k_1)^2-M_W^2]}\\
&\qquad\times\frac{1}{(k_2^2-M_W^2)}
\end{aligned}$$

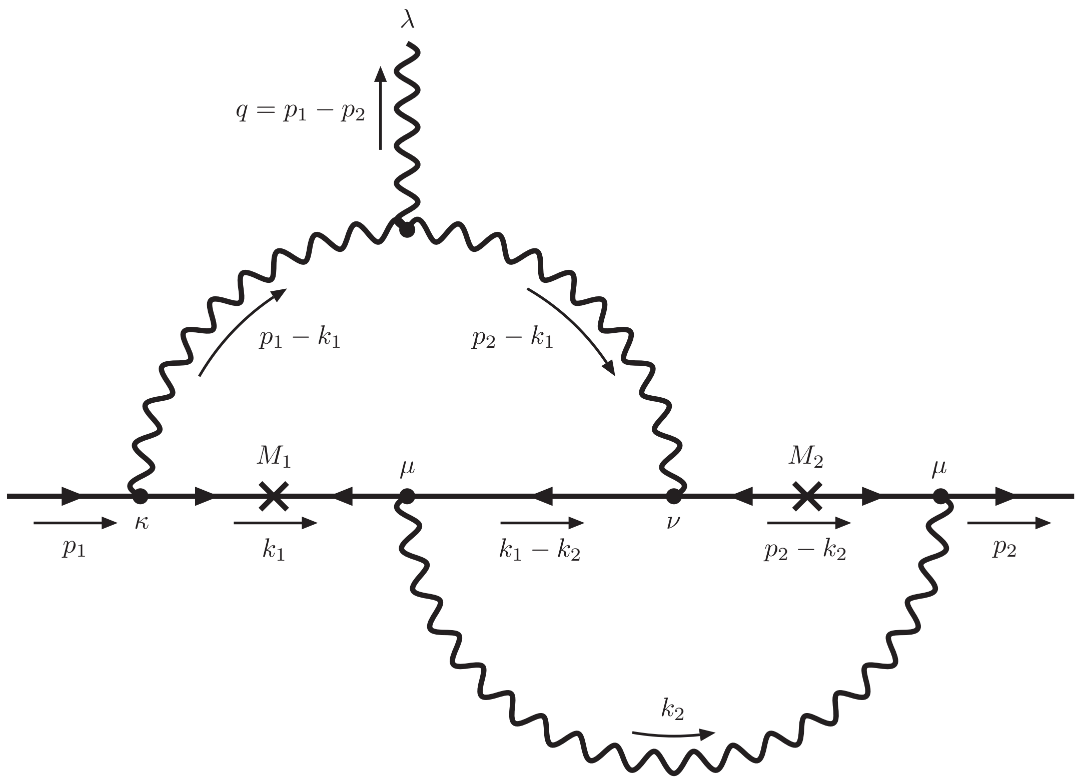

Figure A.2: **Diagram 2A**

$$
\begin{aligned}
&D_0 g^{\kappa\mu} + D_{11} p_1^\kappa p_1^\mu + D_{22} p_2^\kappa p_2^\mu + D_{12} p_1^\kappa p_2^\mu + D_{21} p_2^\kappa p_1^\mu \\
&\quad = \int \frac{d^4 k_1}{(2\pi)^4} \int \frac{d^4 k_2}{(2\pi)^4} \frac{k_1^\kappa (k_1 - k_2)^\mu}{[(p_2 - k_2)^2 - M_2^2](k_1 - k_2)^2 (k_1^2 - M_1^2)[(p_1 - k_1)^2 - M_W^2][(p_2 - k_1)^2 - M_W^2]} \\
&\qquad \times \frac{1}{(k_2^2 - M_W^2)}
\end{aligned} \tag{A.12}
$$

so what we have becomes:

$$
\begin{aligned}
& 4M_1 M_2 \, \bar{u}(p_2)(k_1 - k_2)_\lambda (2m - \not{k}_1) P_L u(p_1) - 4M_1 M_2 \, m\bar{u}(p_2)(k_1 - k_2)_\lambda P_R u(p_1) \, . \\
+ \;& 4M_1 M_2 \, \bar{u}(p_2)(\not{k}_1 - \not{k}_2)(-p_1 - p_2 + 2k_1)_\lambda P_L u(p_1) \, . \\
= \;& 4M_1 M_2 \, \bar{u}(p_2)(k_1 - k_2)_\lambda (2m) P_L u(p_1) - 4M_1 M_2 \, m\bar{u}(p_2)(k_1 - k_2)_\lambda P_R u(p_1) \, . \\
- \;& 4M_1 M_2 \, \bar{u}(p_2)(k_1 - k_2)_\lambda (\not{k}_1) P_L u(p_1) \, . \\
+ \;& 4M_1 M_2 \, \bar{u}(p_2)(\not{k}_1 - \not{k}_2)(-p_1 - p_2)_\lambda P_L u(p_1) \, . \\
+ \;& 4M_1 M_2 \, \bar{u}(p_2)(\not{k}_1 - \not{k}_2)(2k_1)_\lambda P_L u(p_1) \, . \\
= \;& 4M_1 M_2 \, \bar{u}(p_2) g_{\kappa\lambda} (k_1 - k_2)^\kappa (2m) P_L u(p_1) \\
- \;& 4M_1 M_2 \, m\bar{u}(p_2) g_{\kappa\lambda} (k_1 - k_2)^\kappa P_R u(p_1) \, . \\
- \;& 4M_1 M_2 \, \bar{u}(p_2) g_{\mu\lambda} (k_1 - k_2)^\mu (k_1)^\kappa \gamma_\kappa P_L u(p_1) \, . \\
+ \;& 4M_1 M_2 \, \bar{u}(p_2)(k_1 - k_2)^\kappa \gamma_\kappa (-p_1 - p_2)_\lambda P_L u(p_1) \, . \\
+ \;& 8M_1 M_2 \, \bar{u}(p_2)(k_1 - k_2)^\mu \gamma_\mu (k_1)^\kappa g_{\kappa\lambda} P_L u(p_1) \, .
\end{aligned} \tag{A.13}
$$

After integrations we can replace the k's with the p's:

$$
\begin{aligned}
= \;& 4M_1 M_2 \, \bar{u}(p_2) g_{\kappa\lambda} (C_1 p_1^\kappa + C_2 p_2^\kappa)(2m) P_L u(p_1) \\
- \;& 4M_1 M_2 \, m\bar{u}(p_2) g_{\kappa\lambda} (C_1 p_1^\kappa + C_2 p_2^\kappa) P_R u(p_1) \, . \\
- \;& 4M_1 M_2 \, \bar{u}(p_2) g_{\mu\lambda} (D_0 g^{\kappa\mu} + D_{11} p_1^\kappa p_1^\mu + D_{22} p_2^\kappa p_2^\mu + D_{12} p_1^\kappa p_2^\mu + D_{21} p_2^\kappa p_1^\mu) \gamma_\kappa P_L u(p_1) \, . \\
+ \;& 4M_1 M_2 \, \bar{u}(p_2)(C_1 p_1^\kappa + C_2 p_2^\kappa) \gamma_\kappa (-p_1 - p_2)_\lambda P_L u(p_1) \, . \\
+ \;& 8M_1 M_2 \, \bar{u}(p_2)(D_0 g^{\kappa\mu} + D_{11} p_1^\kappa p_1^\mu + D_{22} p_2^\kappa p_2^\mu + D_{12} p_1^\kappa p_2^\mu + D_{21} p_2^\kappa p_1^\mu) \gamma_\mu g_{\kappa\lambda} P_L u(p_1) \, .
\end{aligned} \tag{A.14}
$$

collecting only the terms that have p^λ and neglecting the rest:

$$
\begin{aligned}
= \;& 8M_1 M_2 m C_1 p_{1\lambda} \, \bar{u}(p_2) P_L u(p_1) \; + 8M_1 M_2 m C_2 p_{2\lambda} \, \bar{u}(p_2) P_L u(p_1) \\
- \;& 4M_1 M_2 m C_1 p_{1\lambda} \, \bar{u}(p_2) P_R u(p_1) \; - 4M_1 M_2 m C_2 p_{2\lambda} \, \bar{u}(p_2) P_R u(p_1) \\
- \;& 4M_1 M_2 \, \bar{u}(p_2)(D_{11} \not{p}_1 p_{1\lambda} + D_{22} \not{p}_2 p_{2\lambda} + D_{12} \not{p}_1 p_{2\lambda} + D_{21} \not{p}_2 p_{1\lambda}) P_L u(p_1) \, . \\
- \;& 4M_1 M_2 (p_1 + p_2)_\lambda \, \bar{u}(p_2)(C_1 \not{p}_1 + C_2 \not{p}_2) P_L u(p_1) \, . \\
+ \;& 8M_1 M_2 \, \bar{u}(p_2)(D_{11} p_{1\lambda} \not{p}_1 + D_{22} p_{2\lambda} \not{p}_2 + D_{12} p_{1\lambda} \not{p}_2 + D_{21} p_{2\lambda} \not{p}_1) P_L u(p_1) \, .
\end{aligned} \tag{A.15}
$$

$$
\begin{aligned}
= \;& 8M_1 M_2 m C_1 p_{1\lambda} \, \bar{u}(p_2) P_L u(p_1) \; + 8M_1 M_2 m C_2 p_{2\lambda} \, \bar{u}(p_2) P_L u(p_1) \\
- \;& 4M_1 M_2 m C_1 p_{1\lambda} \, \bar{u}(p_2) P_R u(p_1) \; - 4M_1 M_2 m C_2 p_{2\lambda} \, \bar{u}(p_2) P_R u(p_1) \\
- \;& 4M_1 M_2 \, \bar{u}(p_2)(D_{11} \not{p}_1 p_{1\lambda} + D_{12} \not{p}_1 p_{2\lambda}) P_L u(p_1) \, . \\
- \;& 4M_1 M_2 \, \bar{u}(p_2)(D_{22} \not{p}_2 p_{2\lambda} + D_{21} \not{p}_2 p_{1\lambda}) P_L u(p_1) \, . \\
- \;& 4M_1 M_2 (p_1 + p_2)_\lambda \, \bar{u}(p_2)(C_1 \not{p}_1) P_L u(p_1) \, . \\
- \;& 4M_1 M_2 (p_1 + p_2)_\lambda \, \bar{u}(p_2)(C_2 \not{p}_2) P_L u(p_1) \, . \\
+ \;& 8M_1 M_2 \, \bar{u}(p_2)(D_{11} p_{1\lambda} \not{p}_1 D_{21} p_{2\lambda} \not{p}_1) P_L u(p_1) \, . \\
+ \;& 8M_1 M_2 \, \bar{u}(p_2)(D_{22} p_{2\lambda} \not{p}_2 + D_{12} p_{1\lambda} \not{p}_2) P_L u(p_1) \, .
\end{aligned} \tag{A.16}
$$

$$
= \; 8M_1 M_2 m C_1 p_{1\lambda} \, \bar{u}(p_2) P_L u(p_1) \; + 8M_1 M_2 m C_2 p_{2\lambda} \, \bar{u}(p_2) P_L u(p_1)
$$

$$
\begin{aligned}
- &\quad 4M_1M_2mC_1p_{1\lambda}\,\bar{u}(p_2)P_Ru(p_1)\ -4M_1M_2mC_2p_{2\lambda}\,\bar{u}(p_2)P_Ru(p_1)\\
- &\quad 4M_1M_2m\,\bar{u}(p_2)(D_{11}p_{1\lambda}+D_{12}p_{2\lambda})P_Ru(p_1)\ .\\
- &\quad 4M_1M_2m\,\bar{u}(p_2)(D_{22}p_{2\lambda}+D_{21}p_{1\lambda})P_Lu(p_1)\ .\\
- &\quad 4M_1M_2m(p_1+p_2)_\lambda\,\bar{u}(p_2)(C_1)P_Ru(p_1)\ .\\
- &\quad 4M_1M_2m(p_1+p_2)_\lambda\,\bar{u}(p_2)(C_2)P_Lu(p_1)\ .\\
+ &\quad 8M_1M_2m\,\bar{u}(p_2)(D_{11}p_{1\lambda}+D_{21}p_{2\lambda})P_Ru(p_1)\ .\\
+ &\quad 8M_1M_2m\,\bar{u}(p_2)(D_{22}p_{2\lambda}+D_{12}p_{1\lambda})P_Lu(p_1)\ .
\end{aligned}
\tag{A.17}
$$

$$
\begin{aligned}
= 4M_1M_2m\,[&\quad 2C_1p_{1\lambda}\,\bar{u}(p_2)P_Lu(p_1)\ +2C_2p_{2\lambda}\,\bar{u}(p_2)P_Lu(p_1)\\
- &\quad C_1p_{1\lambda}\,\bar{u}(p_2)P_Ru(p_1)\ -C_2p_{2\lambda}\,\bar{u}(p_2)P_Ru(p_1)\\
- &\quad \bar{u}(p_2)(D_{11}p_{1\lambda}+D_{12}p_{2\lambda})P_Ru(p_1)\ .\\
- &\quad \bar{u}(p_2)(D_{22}p_{2\lambda}+D_{21}p_{1\lambda})P_Lu(p_1)\ .\\
- &\quad (p_1+p_2)_\lambda\,\bar{u}(p_2)(C_1)P_Ru(p_1)\ .\\
- &\quad (p_1+p_2)_\lambda\,\bar{u}(p_2)(C_2)P_Lu(p_1)\ .\\
+ &\quad 2\,\bar{u}(p_2)(D_{11}p_{1\lambda}+D_{21}p_{2\lambda})P_Ru(p_1)\ .\\
+ &\quad 2\,\bar{u}(p_2)(D_{22}p_{2\lambda}+D_{12}p_{1\lambda})P_Lu(p_1)\ .\ ]
\end{aligned}
\tag{A.18}
$$

$$
\begin{aligned}
= 4M_1M_2m\,[\ p_{1\lambda} &\quad (\ 2C_1\,\bar{u}(p_2)P_Lu(p_1)\\
- &\quad C_1\,\bar{u}(p_2)P_Ru(p_1)\\
- &\quad \bar{u}(p_2)(D_{11})P_Ru(p_1)\ .\\
- &\quad \bar{u}(p_2)(D_{21})P_Lu(p_1)\ .\\
- &\quad \bar{u}(p_2)(C_1)P_Ru(p_1)\ .\\
- &\quad \bar{u}(p_2)(C_2)P_Lu(p_1)\ .\\
+ &\quad 2\,\bar{u}(p_2)(D_{11})P_Ru(p_1)\ .\\
+ &\quad 2\,\bar{u}(p_2)(D_{12})P_Lu(p_1)\ .\ )\\
+p_{2\lambda} &\quad (\ 2C_2\,\bar{u}(p_2)P_Lu(p_1)\\
- &\quad C_2\,\bar{u}(p_2)P_Ru(p_1)\\
- &\quad \bar{u}(p_2)(D_{12})P_Ru(p_1)\ .\\
- &\quad \bar{u}(p_2)(D_{22})P_Lu(p_1)\ .\\
- &\quad \bar{u}(p_2)(C_1)P_Ru(p_1)\ .\\
- &\quad \bar{u}(p_2)(C_2)P_Lu(p_1)\ .\\
+ &\quad 2\,\bar{u}(p_2)(D_{21})P_Ru(p_1)\ .\\
+ &\quad 2\,\bar{u}(p_2)(D_{22})P_Lu(p_1)\ .\ )\ ]
\end{aligned}
\tag{A.19}
$$

$$
\begin{aligned}
= 4M_1M_2m\,[\ p_{1\lambda} &\quad (\ (2C_1-C_2-D_{21}+2D_{12})\,\bar{u}(p_2)P_Lu(p_1)\\
+ &\quad (-2C_1+D_{11})\,\bar{u}(p_2)P_Ru(p_1)\ \ )\\
+p_{2\lambda} &\quad (\ (C_2+D_{22})\,\bar{u}(p_2)P_Lu(p_1)\\
+ &\quad (-C_2-C_1-D_{12}+2D_{21})\,\bar{u}(p_2)P_Ru(p_1)\ \ )\ ]
\end{aligned}
\tag{A.20}
$$

collecting the γ^5 terms:

$$
\begin{aligned}
= 4M_1M_2m\bar{u}(p_2)u(p_1)\,[\ p_{1\lambda} &\quad (\ (2C_1-C_2-D_{21}+2D_{12})\,(-\frac{1}{2}\gamma^5)\\
+ &\quad (-2C_1+D_{11})\,(\frac{1}{2}\gamma^5)\ \ )\\
+p_{2\lambda} &\quad (\ (C_2+D_{22})\,(-\frac{1}{2}\gamma^5)\\
+ &\quad (-C_2-C_1-D_{12}+2D_{21})\,(\frac{1}{2}\gamma^5)\ \ )\ ]\\
= 4M_1M_2m\bar{u}(p_2)u(p_1)(-\frac{1}{2}\gamma^5)\,[\ p_{1\lambda} &\quad (\ (2C_1-C_2-D_{21}+2D_{12})-(-2C_1+D_{11})\ \ )
\end{aligned}
$$

$$
\begin{aligned}
&+p_{2\lambda} && (\,(C_2+D_{22})-(-C_2-C_1-D_{12}+2D_{21})\,)\,] \\
&= 4M_1M_2m\bar{u}(p_2)u(p_1)(-\frac{1}{2}\gamma^5)\,[\,p_{1\lambda} && (\,(2C_1-C_2-D_{21}+2D_{12})+(2C_1-D_{11})\ ) \\
&+p_{2\lambda} && (\,(C_2+D_{22})+(+C_2+C_1+D_{12}-2D_{21})\,)\,] \\
&= 4M_1M_2m\bar{u}(p_2)u(p_1)(-\frac{1}{2}\gamma^5)\,[\,p_{1\lambda} && (\,(4C_1-C_2-D_{21}+2D_{12}-D_{11})\ ) \\
&+p_{2\lambda} && (\,(2C_2+C_1+D_{12}-2D_{21}+D_{22})\,)\,] \\
&= 4M_1M_2m\bar{u}(p_2)u(p_1)(-\frac{1}{2}\gamma^5)\,[\,\frac{1}{2}(p_{1\lambda}+p_{2\lambda}) && (\,5C_1+C_2-3D_{21}+3D_{12}-D_{11}+D_{22}\,) \\
&+\frac{1}{2}(p_{1\lambda}-p_{2\lambda}) && (\,3C_1-3C_2+D_{21}+D_{12}-D_{11}-D_{22}\,)\,]
\end{aligned} \tag{A.21}
$$

since $(p_{1\lambda}-p_{2\lambda})$ is zero onshell;

$$
\begin{aligned}
&= 4M_1M_2m\bar{u}(p_2)u(p_1)(-\frac{1}{2}\gamma^5)\,[\,\frac{1}{2}(p_{1\lambda}+p_{2\lambda})\,(\,5C_1+C_2-3D_{21}+3D_{12}-D_{11}+D_{22}\,)\,] \\
&= M_1M_2m\bar{u}(p_2)u(p_1)(\gamma^5)(p_{1\lambda}+p_{2\lambda})\,[-\,(\,5C_1+C_2-3D_{21}+3D_{12}-D_{11}+D_{22}\,)\,] \\
&= M_1M_2m\bar{u}(p_2)u(p_1)(\gamma^5)(p_{1\lambda}+p_{2\lambda})\,[-(5C_1+C_2)-3(D_{12}-D_{21})+(D_{11}-D_{22})]
\end{aligned} \tag{A.22}
$$

<u>The C functions</u>

The calculation is done in a similar way to diagram2B, remember eqn (A.78)

$$
\begin{aligned}
C_1 &= \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{1}{(k_1^2-M_W^2)^2}\right] \\
&+ \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right] \\
&+ \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right] \\
C_2 &= \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-1}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right] \\
&+ \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right] \\
&+ \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right] \\
&+ \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{1}{(k_1^2-M_W^2)^2}\right] \\
&+ \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right] \\
&+ \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right]
\end{aligned} \tag{A.23}
$$

$$
\begin{aligned}
5C_1 &= \frac{5}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{1}{(k_1^2-M_W^2)^2}\right] \\
&+ \frac{5}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
& + && \frac{5}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right]\\
C_2 & = && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-1}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
& + && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
& + && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
& + && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{1}{(k_1^2-M_W^2)^2}\right]\\
& + && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right]\\
& + && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right]
\end{aligned}
\tag{A.24}
$$

$$
\begin{aligned}
5C_1+C_2 & = && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&&& \times\left[\frac{-1}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
& + && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&&& \times\left[\frac{k_1^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
& + && \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&&& \times\left[\frac{-k_2^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
& + && \frac{3}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&&& \times\left[\frac{1}{(k_1^2-M_W^2)^2}\right]\\
& + && \frac{3}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&&& \times\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right]\\
& + && \frac{3}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\
&&& \times\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right]
\end{aligned}
\tag{A.25}
$$

Now doing the usual wick rotation and expanding in Gegenbauer polynomials we can replace:

$$
\begin{aligned}
\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{K^2} \quad\longrightarrow\quad & \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\,k_1^2\int_0^\infty dk_2^2\,k_2^2\,\frac{1}{k_>^2} = \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\,k_<^2\\
= \; & \frac{1}{(4\pi)^4}\left[\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2+\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\,k_1^2\right].
\end{aligned}
\tag{A.26}
$$

$$\begin{aligned}
5C_1 + C_2 \quad = \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-1}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{1}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-1}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{1}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}$$

$$
\begin{aligned}
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}
\tag{A.27}
$$

$$
\begin{aligned}
5C_1+C_2 \quad = \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & (-3)\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}
\tag{A.28}
$$

$$
\begin{aligned}
5C_1+C_2 \quad = \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & (-3)\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
&+ \quad \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
&+ \quad \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}
\tag{A.29}
$$

$$
\begin{aligned}
5C_1+C_2 \quad = \quad &\frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad &(-3)\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad &\frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad &\frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)(k_1^2+M_W^2)} \\
&\qquad \times\left[\frac{k_2^2}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right] \\
+ \quad &\frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_1^2\; k_2^2\, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)(k_1^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_1^2}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right] \\
+ \quad &\frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_1^2} dk_2^2\; k_2^2\, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)(k_1^2+M_W^2)} \\
&\qquad \times\left[\frac{k_2^2}{(k_2^2+M_W^2)^2}\right]
\end{aligned}
\tag{A.30}
$$

$$
\begin{aligned}
5C_1+C_2 \quad = \quad &\frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_1^2)(k_1^2+M_W^2)^2}\int_0^{k_1^2} dk_2^2\;\; \frac{-k_2^4}{(k_2^2+M_2^2)^2(k_2^2+M_W^2)} \\
+ \quad &(-3)\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+M_W^2)^3}\;\int_0^{k_1^2} dk_2^2\;\; \frac{k_2^2}{(k_2^2+M_2^2)(k_2^2+M_W^2)}
\end{aligned}
$$

$$
\begin{aligned}
&+ \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+M_W^2)^3}\int_0^{k_1^2} dk_2^2 \ \frac{-k_2^4}{(k_2^2+M_2^2)(k_2^2+M_W^2)} \\
&+ \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2 \ \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+M_W^2)^2} \\
&+ \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2 \ \frac{-k_2^2}{(k_2^2+M_1^2)(k_2^2+M_W^2)^2} \\
&+ \frac{-3}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2 \ \frac{-k_2^4}{(k_2^2+M_1^2)(k_2^2+M_W^2)^3}
\end{aligned}
\tag{A.31}
$$

scale all masses and momenta to M_W^2

$$
\begin{aligned}
5C_1+C_2 &= \frac{-1}{4}\frac{1}{(4\pi)^4M_W^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2 \ \frac{-k_2^4}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
&+ (-3)\frac{1}{(4\pi)^4M_W^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3}\int_0^{k_1^2} dk_2^2 \ \frac{k_2^2}{(k_2^2+M_2^2)(k_2^2+1)} \\
&+ \frac{-3}{2}\frac{1}{(4\pi)^4M_W^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3}\int_0^{k_1^2} dk_2^2 \ \frac{-k_2^4}{(k_2^2+M_2^2)(k_2^2+1)} \\
&+ \frac{-1}{4}\frac{1}{(4\pi)^4M_W^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)}\int_0^{k_1^2} dk_2^2 \ \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^2} \\
&+ \frac{-1}{2}\frac{1}{(4\pi)^4M_W^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)}\int_0^{k_1^2} dk_2^2 \ \frac{-k_2^2}{(k_2^2+M_1^2)(k_2^2+1)^2} \\
&+ \frac{-3}{2}\frac{1}{(4\pi)^4M_W^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)}\int_0^{k_1^2} dk_2^2 \ \frac{-k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^3}
\end{aligned}
\tag{A.32}
$$

In terms of the integrals in Appendix B:

$$
5C_1+C_2 = \frac{-1}{(4\pi)^4M_W^4}\left[\frac{-1}{4}C_a+3C_b+\frac{3}{2}C_c+\frac{1}{4}C_d+\frac{1}{2}C_e+\frac{3}{2}C_f\right]
\tag{A.33}
$$

<u>**The D functions**</u> Remember Eqn's (3.112),(3.123):

$$
\begin{aligned}
&D_{11}-D_{22} \\
&= \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\ \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
&\quad \left[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}-\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}-\frac{1}{4}\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right. \\
&\qquad \left.+\frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\right].
\end{aligned}
\tag{A.34}
$$

$$
\begin{aligned}
&(D_{12}-D_{21}) \\
&= \frac{1}{3}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\ \frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}
\end{aligned}
$$

$$\Big[\frac{-(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))+k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\Big]\,. \tag{A.35}$$

$$\begin{aligned} &3(D_{12}-D_{21})\\ &= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\ &\quad\Big[\frac{-(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))+k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\Big]\,. \end{aligned} \tag{A.36}$$

so

$$\begin{aligned} &(D_{11}-D_{22})-3(D_{12}-D_{21})\\ &= \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\ &\quad\Big[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}-\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}-\frac{1}{4}\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\\ &\qquad+\frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Big]\,.\\ &- \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\ &\quad\Big[\frac{-(\frac{1}{2}(k_1^2+k_2^2-K^2))(\frac{1}{2}(k_1^2-k_2^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{k_1^2(\frac{1}{2}(k_1^2-k_2^2-K^2))}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\Big]\,. \end{aligned} \tag{A.37}$$

$$\begin{aligned} &(D_{11}-D_{22})-3(D_{12}-D_{21})\\ &= \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\ &\quad\Big[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2}-\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}-\frac{1}{4}\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\\ &\qquad+\frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\Big]\,.\\ &- \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\\ &\quad\Big[\frac{-9}{4}\frac{((k_1^2+k_2^2-K^2))(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}+\frac{9}{2}\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\Big]\,. \end{aligned} \tag{A.38}$$

$$\begin{aligned} &(D_{11}-D_{22})-3(D_{12}-D_{21})\\ &= \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \end{aligned}$$

$$\left[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2} - \left[1+\frac{9}{2}\right]\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right.$$
$$- \quad \left.\left[\frac{1}{4}-\frac{9}{4}\right]4\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + \frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\right]. \tag{A.39}$$

$$(D_{11}-D_{22})-3(D_{12}-D_{21})$$
$$= \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}$$
$$\left[-\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2))}{(k_2^2-M_2^2)^2} - \frac{11}{2}\frac{k_1^2(k_1^2-k_2^2-K^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right.$$
$$\left.+2\frac{(k_1^2+k_2^2-K^2)(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + \frac{1}{2}\frac{k_2^2(K^2+k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\right]. \tag{A.40}$$

$$(D_{11}-D_{22})-3(D_{12}-D_{21})$$
$$= \frac{1}{9}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}$$
$$\left[\frac{K^2(k_1^2-k_2^2)}{(k_2^2-M_2^2)^2} - \frac{k_1^4-k_2^4}{(k_2^2-M_2^2)^2} + \frac{11}{2}\frac{k_1^2K^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)} - \frac{11}{2}\frac{k_1^2(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right.$$
$$\left.-2\frac{K^2(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + 2\frac{k_1^4-k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + \frac{1}{2}\frac{k_2^2K^2}{(k_2^2-M_2^2)^2} + \frac{1}{2}\frac{k_2^2(k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\right]. \tag{A.41}$$

$$(D_{11}-D_{22})-3(D_{12}-D_{21})$$
$$= \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}$$
$$\left[2\frac{K^2(k_1^2-k_2^2)}{(k_2^2-M_2^2)^2} - 2\frac{k_1^4-k_2^4}{(k_2^2-M_2^2)^2} + 11\frac{k_1^2K^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)} - 11\frac{k_1^2(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right.$$
$$\left.-4\frac{K^2(k_1^2-k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + 4\frac{k_1^4-k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + \frac{k_2^2K^2}{(k_2^2-M_2^2)^2} + \frac{k_2^2(k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\right]. \tag{A.42}$$

$$(D_{11}-D_{22})-3(D_{12}-D_{21})$$
$$= \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}$$
$$\left[\frac{K^2(2k_1^2-k_2^2)}{(k_2^2-M_2^2)^2} - \frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2-M_2^2)^2} + \frac{K^2(7k_1^2+4k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)} + \frac{11k_1^2k_2^2-7k_1^4-4k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]. \tag{A.43}$$

$$(D_{11}-D_{22})-3(D_{12}-D_{21})$$

$$\begin{aligned}
= \quad & \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{K^2(2k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2-M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{K^2(7k_1^2+4k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]. \\
+ \quad & \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{11k_1^2k_2^2-7k_1^4-4k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned} \tag{A.44}$$

$$\begin{aligned}
& (D_{11}-D_{22})-3(D_{12}-D_{21}) \\
= \quad & \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(2k_1^2-k_2^2)}{(k_2^2-M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2-M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(7k_1^2+4k_2^2)}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]. \\
+ \quad & \frac{1}{18}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{(k_2^2-M_2^2)K^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_1^2-M_W^2)(k_2^2-M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{11k_1^2k_2^2-7k_1^4-4k_2^4}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right].
\end{aligned} \tag{A.45}$$

Now doing the usual wick rotation and expanding in Gegenbauer polynomials we can replace:

$$\begin{aligned}
\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{1}{K^2} \quad\longrightarrow\quad & \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\,k_1^2\int_0^\infty dk_2^2\,k_2^2\,\frac{1}{k_>^2} = \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\,k_<^2 \\
= \quad & \frac{1}{(4\pi)^4}\left[\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2+\int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\,k_1^2\right].
\end{aligned} \tag{A.46}$$

$$\begin{aligned}
& (D_{11}-D_{22})-3(D_{12}-D_{21}) \\
= \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2k_1^2\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}
\end{aligned}$$

$$
\begin{aligned}
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(2k_1^2-k_2^2)}{(k_2^2+M_2^2)^2}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2+M_2^2)^2}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(7k_1^2+4k_2^2)}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{11k_1^2k_2^2-7k_1^4-4k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_2^2k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(2k_1^2-k_2^2)}{(k_2^2+M_2^2)^2}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2+M_2^2)^2}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_2^2k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(7k_1^2+4k_2^2)}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{11k_1^2k_2^2-7k_1^4-4k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] .
\end{aligned}
\tag{A.47}
$$

$$
\begin{aligned}
& (D_{11}-D_{22})-3(D_{12}-D_{21}) \\
= \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(2k_1^4-k_2^2k_1^2)}{(k_2^2+M_2^2)^2}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2+M_2^2)^2}\right] . \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad \times\left[\frac{(7k_1^4+4k_2^2k_1^2)}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] .
\end{aligned}
$$

$$
\begin{aligned}
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{11k_1^2k_2^2-7k_1^4-4k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{(2k_1^2k_2^2-k_2^4)}{(k_2^2+M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[-\frac{2k_1^4-k_1^2k_2^2-k_2^4}{(k_2^2+M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{(7k_1^2k_2^2+4k_2^4)}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{11k_1^2k_2^2-7k_1^4-4k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right].
\end{aligned} \tag{A.48}
$$

$$
\begin{aligned}
& (D_{11}-D_{22})-3(D_{12}-D_{21}) \\
= \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{k_2^4}{(k_2^2+M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; k_2^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{15k_1^2k_2^2-4k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{(3k_1^2k_2^2-2k_1^4)}{(k_2^2+M_2^2)^2}\right]. \\
+ \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \; k_1^2 \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{18k_1^2k_2^2-7k_1^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right].
\end{aligned} \tag{A.49}
$$

$$
\begin{aligned}
& (D_{11}-D_{22})-3(D_{12}-D_{21}) \\
= \quad & \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \; \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{k_2^6}{(k_2^2+M_2^2)^2}\right].
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)^2(k_1^2+M_W^2)(k_2^2+M_W^2)}\left[\frac{15k_1^2k_2^4-4k_2^6}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{(3k_1^4k_2^2-2k_1^6)}{(k_2^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2 \, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{18k_1^4k_2^2-7k_1^6}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right].
\end{aligned} \tag{A.50}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-3(D_{12}-D_{21}) \\
&= \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{k_2^6}{(k_2^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{15k_1^2k_2^4-4k_2^6}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{(3k_2^4k_1^2-2k_2^6)}{(k_1^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{18k_2^4k_1^2-7k_2^6}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right].
\end{aligned} \tag{A.51}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-3(D_{12}-D_{21}) \\
&= \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{k_2^6}{(k_2^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{15k_1^2k_2^4}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)^2(k_2^2+M_W^2)}\left[\frac{-4k_2^6}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{(3k_2^4k_1^2)}{(k_1^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{(-2k_2^6)}{(k_1^2+M_2^2)^2}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{18k_2^4k_1^2}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right]. \\
&+ \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2 \, \frac{1}{(k_1^2+M_2^2)(k_2^2+M_1^2)(k_2^2+M_W^2)^2(k_1^2+M_W^2)}\left[\frac{-7k_2^6}{(k_2^2+M_W^2)(k_1^2+M_2^2)}\right].
\end{aligned} \tag{A.52}
$$

$$
\begin{aligned}
&(D_{11}-D_{22})-3(D_{12}-D_{21}) \\
&= \quad \frac{1}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\,;\frac{1}{(k_1^2+M_1^2)(k_1^2+M_W^2)^2}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_W^2)(k_2^2+M_2^2)^3}. \\
&+ \quad \frac{15}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \, \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+M_W^2)^3}\int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_2^2)^2(k_2^2+M_W^2)}. \\
&+ \quad \frac{-4}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+M_W^2)^3}\int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_W^2)(k_2^2+M_2^2)^2}.
\end{aligned}
$$

$$
\begin{aligned}
&+\quad \frac{3}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\ \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+M_W^2)}\int_0^{k_1^2}dk_2^2\frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+M_W^2)^2}\ .\\
&+\quad \frac{-2}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\ \frac{1}{(k_1^2+M_2^2)^3(k_1^2+M_W^2)}\int_0^{k_1^2}dk_2^2\frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+M_W^2)^2}\ .\\
&+\quad \frac{18}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\ \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+M_W^2)}\int_0^{k_1^2}dk_2^2\frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+M_W^2)^3}\ .\\
&+\quad \frac{-7}{18}\frac{-1}{(4\pi)^4}\int_0^\infty dk_1^2\ \frac{1}{(k_1^2+M_2^2)^2(k_1^2+M_W^2)}\int_0^{k_1^2}dk_2^2\frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+M_W^2)^3}\ .
\end{aligned}
\tag{A.53}
$$

Scaling all masses and momenta to M_W^2

$$
\begin{aligned}
&(D_{11}-D_{22})-3(D_{12}-D_{21})\\
&=\quad \frac{1}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\,;\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2}dk_2^2\frac{k_2^6}{(k_2^2+1)(k_2^2+M_2^2)^3}\ .\\
&+\quad \frac{15}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\ \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3}\int_0^{k_1^2}dk_2^2\frac{k_2^4}{(k_2^2+M_2^2)^2(k_2^2+1)}\ .\\
&+\quad \frac{-4}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\ \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3}\int_0^{k_1^2}dk_2^2\frac{k_2^6}{(k_2^2+1)(k_2^2+M_2^2)^2}\ .\\
&+\quad \frac{3}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\ \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)}\int_0^{k_1^2}dk_2^2\frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^2}\ .\\
&+\quad \frac{-2}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\ \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)}\int_0^{k_1^2}dk_2^2\frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+1)^2}\ .\\
&+\quad \frac{18}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\ \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)}\int_0^{k_1^2}dk_2^2\frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^3}\ .\\
&+\quad \frac{-7}{18}\frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\int_0^\infty dk_1^2\ \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)}\int_0^{k_1^2}dk_2^2\frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+1)^3}\ .
\end{aligned}
\tag{A.54}
$$

In terms of the integrals in Appendix B:

$$
\begin{aligned}
&(D_{11}-D_{22})-3(D_{12}-D_{21})\\
&\quad=\quad \frac{-1}{(4\pi)^4}\frac{1}{M_W^4}\left[\frac{1}{18}D_a+\frac{15}{18}D_b-\frac{4}{18}D_c+\frac{3}{18}D_d-\frac{2}{18}D_e+D_f-\frac{7}{18}D_g\right]
\end{aligned}
\tag{A.55}
$$

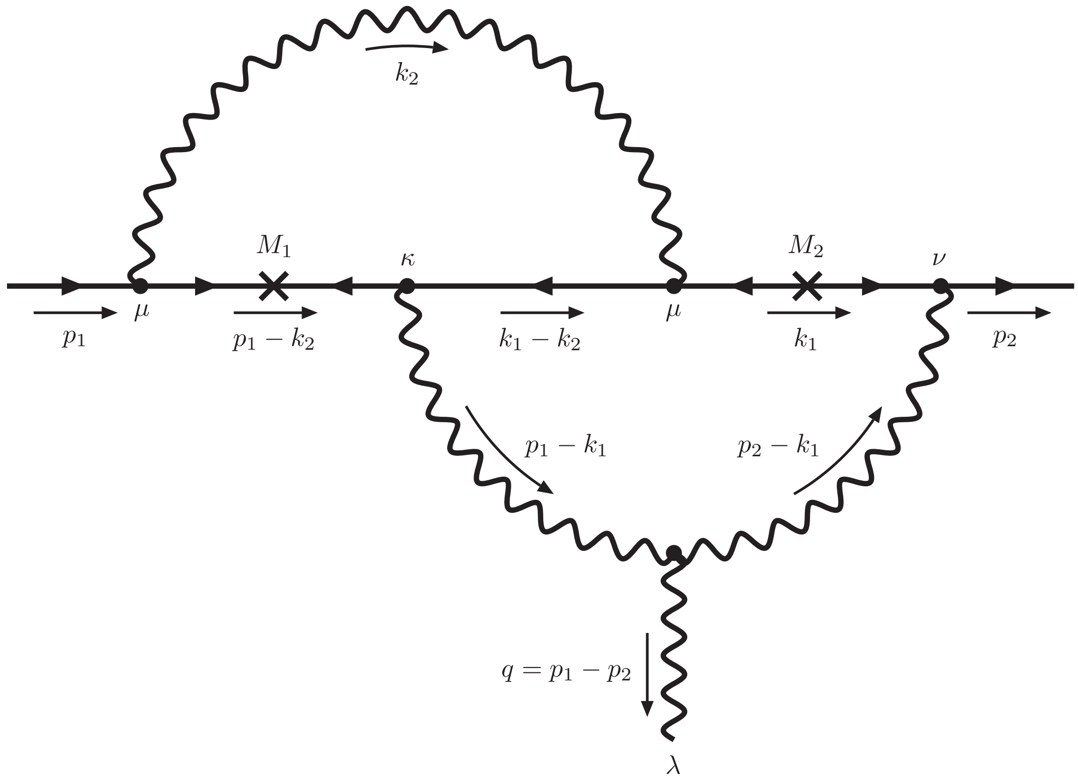

Figure A.3: **Diagram 3A**

A.3 diagram 3A

We labeled the loop-momentum carried by the W which connects to the photon k_1, and the loop-momentum carried by the other W is labeled k_2. With this labeling, the γWW-vertex is exactly the same as the one in diagram 2A:

$$(-ie)\Big[(-p_1 + 2p_2 - k_1)_\kappa g_{\lambda\nu} + (-p_1 - p_2 + 2k_1)_\lambda g_{\kappa\nu} + (2p_1 - p_2 - k_1)_\nu g_{\kappa\lambda} \Big] \,. \tag{A.56}$$

The coupling constant, $(-ie)$, combines with the coupling constants along the fermion line to give:

$$(-ie)\left(-i\frac{g}{\sqrt{2}}\tilde{V}_{\alpha 2}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}_{\beta 2}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}^\dagger_{1\beta}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}^\dagger_{1\alpha}\right) = -i\frac{eg^4}{4}\left(\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^*_{\beta 1}\tilde{V}^*_{\alpha 1}\right) \,, \tag{A.57}$$

which is also exactly the same as diagram 2A. The fermion line gives

$$\begin{aligned}
&\langle \ell_\alpha(p_2)| \, (\overline{\ell_\alpha}\gamma^\nu P_L N_2)(\overline{\ell_\beta}\gamma^\mu P_L N_2)(\overline{N_1}\gamma^\kappa P_L \ell_\beta)(\overline{N_1}\gamma_\mu P_L \ell_\alpha) \, |\ell_\alpha(p_1)\rangle \\
&= \langle \ell_\alpha(p_2)| \, (\overline{\ell_\alpha}\gamma^\nu P_L N_2)(\overline{N}_2 P_L \gamma^\mu \ell^c_\beta)(\overline{\ell^c_\beta} P_L \gamma^\kappa N_1)(\overline{N}_1 \gamma_\mu P_L \ell_\alpha) \, |\ell_\alpha(p_1)\rangle \\
&= \bar{u}(p_2)\gamma^\nu P_L \left\langle N_2 \overline{N}_2 \right\rangle P_L \gamma^\mu \left\langle \ell_\beta \overline{\ell^c_\beta} \right\rangle P_L \gamma^\kappa \left\langle N_1 \overline{N}_1 \right\rangle \gamma_\mu P_L u(p_1)
\end{aligned}$$

$$\begin{aligned}
&\rightarrow \bar{u}(p_2)\gamma^\nu P_L\,[\not{k}_1 + M_2]\,P_L\gamma^\mu(\not{k}_1 - \not{k}_2)P_L\gamma^\kappa\,[(\not{p}_1 - \not{k}_2) + M_1]\,\gamma_\mu P_L u(p_1) \\
&= M_1 M_2\,\bar{u}(p_2)\gamma^\nu\gamma^\mu(\not{k}_1 - \not{k}_2)\gamma^\kappa\gamma_\mu P_L u(p_1) \\
&= 4M_1 M_2\,\bar{u}(p_2)\gamma^\nu(k_1 - k_2)^\kappa P_L u(p_1) \\
&= 4M_1 M_2\,\bar{u}(p_2)(k_1 - k_2)^\kappa\gamma^\nu P_L u(p_1)\;. \qquad \text{(A.58)}
\end{aligned}$$

This is almost the same as diagram 2A: only the Lorentz indices κ and ν are interchanged. The three fermion propagators contribute a factor of $i^3 = -i$, which I haven't explicitly written in this expression, but they will cancel against the factor of $(-i)^3 = i$ coming from the three W propagators. Combining with the γWW-vertex factor, we find

$$\begin{aligned}
&\Big[(-p_1 + 2p_2 - k_1)_\kappa g_{\lambda\nu} + (-p_1 - p_2 + 2k_1)_\lambda g_{\kappa\nu} + (2p_1 - p_2 - k_1)_\mu g_{\kappa\lambda}\Big](k_1 - k_2)^\kappa\gamma^\nu P_L \\
&= \Big[(-p_1 + 2p_2 - k_1)\cdot(k_1 - k_2)\gamma_\lambda + (-p_1 - p_2 + 2k_1)_\lambda(\not{k}_1 - \not{k}_2) + (k_1 - k_2)_\lambda(2\not{p}_1 - \not{p}_2 - \not{k}_1)\Big]P_L \\
&= (-p_1 + 2p_2 - k_1)\cdot(k_1 - k_2)\gamma_\lambda P_L + (-p_1 - p_2 + 2k_1)_\lambda(\not{k}_1 - \not{k}_2)P_L + (k_1 - k_2)_\lambda(2\not{p}_1 - \not{p}_2 - \not{k}_1)P_L \\
&\rightarrow (-p_1 + 2p_2 - k_1)\cdot(k_1 - k_2)\gamma_\lambda P_L + (-p_1 - p_2 + 2k_1)_\lambda(\not{k}_1 - \not{k}_2)P_L \\
&\qquad + (k_1 - k_2)_\lambda(2mP_R - mP_L - \not{k}_1 P_L) \\
&= (-p_1 + 2p_2 - k_1)\cdot(k_1 - k_2)\gamma_\lambda P_L - (p_1 + p_2)_\lambda(\not{k}_1 - \not{k}_2)P_L + 2k_{1\lambda}(\not{k}_1 - \not{k}_2)P_L \\
&\quad + m(k_1 - k_2)_\lambda(2P_R - 2P_L) - (k_1 - k_2)_\lambda\not{k}_1 P_L\;. \qquad \text{(A.59)}
\end{aligned}$$

The first term does not contribute to the EDM so it can be dropped. The other terms can be combined into

$$(k_1 - k_2)^\kappa\Big[mg_{\kappa\lambda}(2P_R - P_L) - (p_1 + p_2)_\lambda\gamma_\kappa P_L\Big] + k_1^\kappa(k_1 - k_2)^\mu\Big[-g_{\lambda\mu}\gamma_\kappa + 2g_{\kappa\lambda}\gamma_\mu\Big]P_L\;. \qquad \text{(A.60)}$$

The integrals we have to do are:

$$\begin{aligned}
&\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{(k_1 - k_2)^\kappa}{[(p_1 - k_2)^2 - M_1^2]\,(k_1 - k_2)^2(k_1^2 - M_2^2)\,[(p_2 - k_1)^2 - M_W^2]} \\
&\qquad\times\frac{1}{[(p_1 - k_1)^2 - M_W^2]\,(k_2^2 - M_W^2)}\;, \\
&\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1^\kappa(k_1 - k_2)^\mu}{[(p_1 - k_2)^2 - M_1^2]\,(k_1 - k_2)^2(k_1^2 - M_2^2)\,[(p_2 - k_1)^2 - M_W^2]} \\
&\qquad\times\frac{1}{[(p_1 - k_1)^2 - M_W^2]\,(k_2^2 - M_W^2)}\;, \qquad \text{(A.61)}
\end{aligned}$$

which are the same as the ones we have to do for diagram 2A, except for the interchanges $p_1 \leftrightarrow p_2$, and $M_1 \leftrightarrow M_2$. Recalling that

$$\begin{aligned}
&C_1(M_1, M_2)\,p_1^\kappa + C_2(M_1, M_2)\,p_2^\kappa \\
&= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{(k_1 - k_2)^\kappa}{[(p_2 - k_2)^2 - M_2^2]\,(k_1 - k_2)^2(k_1^2 - M_1^2)\,[(p_1 - k_1)^2 - M_W^2]} \\
&\qquad\times\frac{1}{[(p_2 - k_1)^2 - M_W^2]\,(k_2^2 - M_W^2)}\;, \\
&D_0(M_1, M_2)\,g^{\kappa\mu} + D_{11}(M_1, M_2)\,p_1^\kappa p_1^\mu + D_{22}(M_1, M_2)\,p_2^\kappa p_2^\mu + D_{12}(M_1, M_2)\,p_1^\kappa p_2^\mu + D_{21}(M_1, M_2)\,p_2^\kappa p_1^\mu \\
&= \int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1^\kappa(k_1 - k_2)^\mu}{[(p_2 - k_2)^2 - M_2^2]\,(k_1 - k_2)^2(k_1^2 - M_1^2)\,[(p_1 - k_1)^2 - M_W^2]} \\
&\qquad\times\frac{1}{[(p_2 - k_1)^2 - M_W^2]\,(k_2^2 - M_W^2)}\;, \qquad \text{(A.62)}
\end{aligned}$$

we can write our integrals as

$$\begin{aligned}
&\int \frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{(k_1-k_2)^\kappa}{[(p_1-k_2)^2-M_1^2]\,(k_1-k_2)^2(k_1^2-M_2^2)\,[(p_2-k_1)^2-M_W^2]\,[(p_1-k_1)^2-M_W^2]\,(k_2^2-M_W^2)}\\
&= \quad C_1(M_2,M_1)\,p_2^\kappa + C_2(M_2,M_1)\,p_1^\kappa\\
&\equiv \quad \overline{C}_1 p_2^\kappa + \overline{C}_2 p_1^\kappa \ ,
\end{aligned}$$

$$\begin{aligned}
&\int \frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{k_1^\kappa(k_1-k_2)^\mu}{[(p_1-k_2)^2-M_1^2]\,(k_1-k_2)^2(k_1^2-M_2^2)\,[(p_2-k_1)^2-M_W^2]\,[(p_1-k_1)^2-M_W^2]\,(k_2^2-M_W^2)}\\
&= \quad D_0(M_2,M_1)\,g^{\kappa\mu} + D_{11}(M_2,M_1)\,p_2^\kappa p_2^\mu + D_{22}(M_2,M_1)\,p_1^\kappa p_1^\mu + D_{12}(M_2,M_1)\,p_2^\kappa p_1^\mu + D_{21}(M_2,M_1)\,p_1^\kappa p_2^\mu\\
&\equiv \quad \overline{D}_0\, g^{\kappa\mu} + \overline{D}_{11}\, p_2^\kappa p_2^\mu + \overline{D}_{22}\, p_1^\kappa p_1^\mu + \overline{D}_{12}\, p_2^\kappa p_1^\mu + \overline{D}_{21}\, p_1^\kappa p_2^\mu \ .
\end{aligned} \tag{A.63}$$

Then, the terms in Eq. (A.60) lead to

$$\begin{aligned}
&\left(\overline{C}_1 p_2^\kappa + \overline{C}_2 p_1^\kappa\right)\Big[m g_{\kappa\lambda}(2P_R-P_L) - (p_1+p_2)_\lambda\gamma_\kappa P_L\Big]\\
&\qquad + \left(\overline{D}_0 g^{\kappa\mu} + \overline{D}_{11}p_2^\kappa p_2^\mu + \overline{D}_{22}p_1^\kappa p_1^\mu + \overline{D}_{12}p_2^\kappa p_1^\mu + \overline{D}_{21}p_1^\kappa p_2^\mu\right)\Big[-g_{\lambda\mu}\gamma_\kappa + 2g_{\kappa\lambda}\gamma_\mu\Big]P_L\\
&= \quad m(\overline{C}_1 p_{2\lambda} + \overline{C}_2 p_{1\lambda})(2P_R-P_L) - (p_1+p_2)_\lambda(\overline{C}_1 \not{p}_2 + \overline{C}_2\not{p}_1)P_L\\
&\qquad +\{\overline{D}_0\gamma_\lambda + \overline{D}_{11}\,p_{2\lambda}\not{p}_2 + \overline{D}_{22}\,p_{1\lambda}\not{p}_1 + \overline{D}_{12}(-p_{1\lambda}\not{p}_2 + 2p_{2\lambda}\not{p}_1) + \overline{D}_{21}(-p_{2\lambda}\not{p}_1 + 2p_{1\lambda}\not{p}_2)\}P_L\\
&\to \quad \frac{m}{2}\left[(\overline{C}_1+\overline{C}_2)(p_1+p_2)_\lambda - (\overline{C}_1-\overline{C}_2)(p_1-p_2)_\lambda\right](2P_R-P_L) - m\overline{C}_1(p_1+p_2)_\lambda P_L\\
&\qquad\qquad\qquad\qquad - m\overline{C}_2(p_1+p_2)_\lambda P_R\\
&\qquad +\overline{D}_0\gamma_\lambda P_L + m\overline{D}_{11}p_{2\lambda}P_L + m\overline{D}_{22}p_{1\lambda}P_R + m\overline{D}_{12}(-p_{1\lambda}P_L + 2p_{2\lambda}P_R) + m\overline{D}_{21}(-p_{2\lambda}P_R + 2p_{1\lambda}P_L)\\
&= \quad m(p_1+p_2)_\lambda\left[\frac{1}{2}(\overline{C}_1+\overline{C}_2)(2P_R-P_L) - \overline{C}_1 P_L - \overline{C}_2 P_R\right] - m(p_1-p_2)_\lambda\left[\frac{1}{2}(\overline{C}_1-\overline{C}_2)(2P_R-P_L)\right]\\
&\qquad +\overline{D}_0\gamma_\lambda P_L + m\,p_{1\lambda}\left(\overline{D}_{22}P_R - \overline{D}_{12}P_L + 2\overline{D}_{21}P_L\right) + m\,p_{2\lambda}\left(\overline{D}_{11}P_L + 2\overline{D}_{12}P_R - \overline{D}_{21}P_R\right)\\
&= \quad m(p_1+p_2)_\lambda\left[\overline{C}_1 P_R - \frac{1}{2}(3\overline{C}_1+\overline{C}_2)P_L\right] - m(p_1-p_2)_\lambda\left[\frac{1}{2}(\overline{C}_1-\overline{C}_2)(2P_R-P_L)\right]\\
&\qquad +\overline{D}_0\gamma_\lambda P_L + \frac{m}{2}(p_1+p_2)_\lambda\left[\overline{D}_{22}P_R + \overline{D}_{11}P_L + \overline{D}_{12}\,(2P_R-P_L) + \overline{D}_{21}(-P_R+2P_L)\right]\\
&\qquad\quad +\frac{m}{2}(p_1-p_2)_\lambda\left[\overline{D}_{22}P_R - \overline{D}_{11}P_L + \overline{D}_{12}\,(-2P_R-P_L) + \overline{D}_{21}(P_R+2P_L)\right] \ .
\end{aligned} \tag{A.64}$$

Collect the coefficients of $m(p_1+p_2)_\lambda\gamma_5$:

$$+\left(\frac{5\overline{C}_1+\overline{C}_2}{4}\right) - \left\{\frac{(\overline{D}_{11}-\overline{D}_{22}) - 3(\overline{D}_{12}-\overline{D}_{21})}{4}\right\} \ . \tag{A.65}$$

This result is the same as that for diagram 2A except for the interchange $M_1 \leftrightarrow M_2$ and the overall sign. The sum will be anti-symmetric in $M_1 \leftrightarrow M_2$, which is what we need.

Multiply this by $4M_1M_2$ to obtain contribution to EDM:

$$mM_1M_2\Big[(5\overline{C}_1+\overline{C}_2) - (\overline{D}_{11}-\overline{D}_{22}) + 3(\overline{D}_{12}-\overline{D}_{21})\Big](p_1+p_2)_\lambda\gamma_5 \ . \tag{A.66}$$

Multiply by the coupling constants

$$-i\frac{eg^4}{4}\left(\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^*_{\beta 1}\tilde{V}^*_{\alpha 1}\right) \ , \tag{A.67}$$

and we get

$$-i\frac{eg^4}{4}mM_1M_2\left(\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^*_{\beta 1}\tilde{V}^*_{\alpha 1}\right)\Big[(5\overline{C}_1+\overline{C}_2) - (\overline{D}_{11}-\overline{D}_{22}) + 3(\overline{D}_{12}-\overline{D}_{21})\Big](p_1+p_2)_\lambda\gamma_5 \ . \tag{A.68}$$

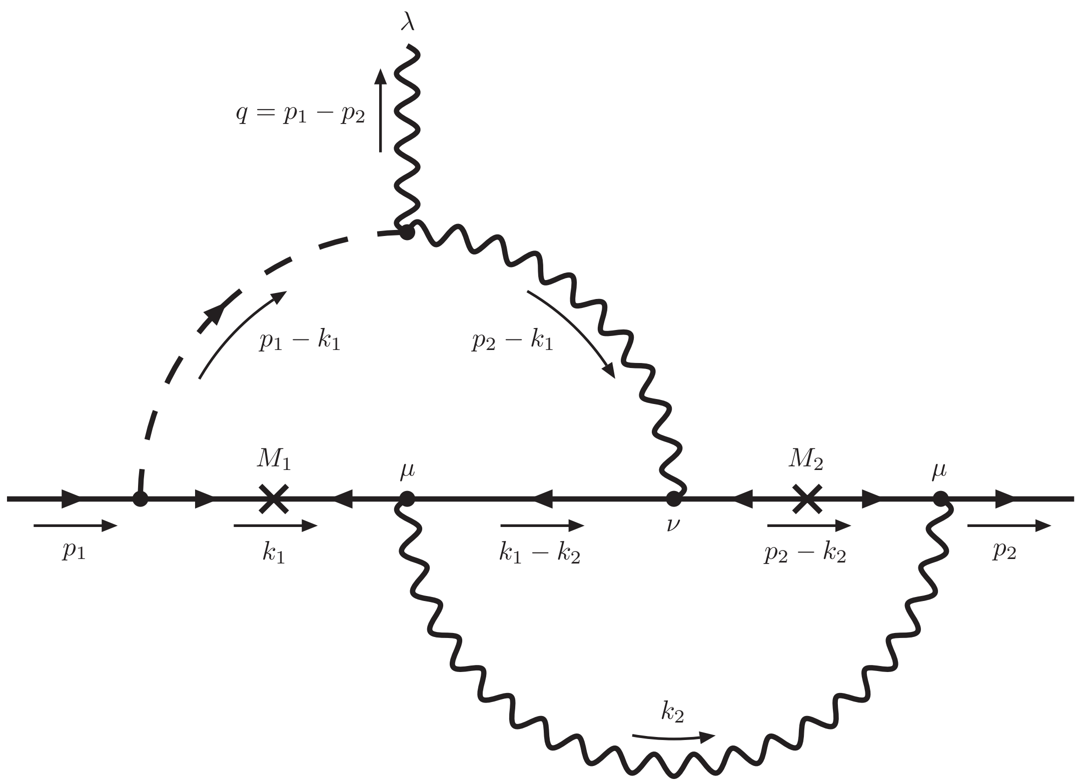

Figure A.4: **Diagram 4A**

Identify this with

$$i\frac{G_2(0)}{2m}(p_1+p_2)_\lambda\gamma_5 = -d\,(p_1+p_2)_\lambda\gamma_5\ . \tag{A.69}$$

We find:

$$d = \frac{eg^4}{4}mM_1M_2\left(i\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^*_{\beta 1}\tilde{V}^*_{\alpha 1}\right)\left[(5\overline{C}_1+\overline{C}_2)-(\overline{D}_{11}-\overline{D}_{22})+3(\overline{D}_{12}-\overline{D}_{21})\right]\ . \tag{A.70}$$

Summing this with the contribution of diagram 2A, and summing over internal states will give us the imaginary part of $(\tilde{V}_{\alpha 2}\tilde{V}_{\beta 2}\tilde{V}^*_{\beta 1}\tilde{V}^*_{\alpha 1})$.

A.4 Diagram 4A

The $\gamma\phi W$ vertex is just $+ieM_W g_{\lambda\nu}$. The coupling constants in this diagram are

$$(ieM_W)\left(-i\tilde{\lambda}^\dagger_{1\alpha}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}^\dagger_{1\beta}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}_{\beta 2}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}_{\alpha 2}\right) = i\frac{eg^3}{2\sqrt{2}}M_W\left(\tilde{\lambda}^*_{\alpha 1}\tilde{V}^*_{\beta 1}\tilde{V}_{\beta 2}\tilde{V}_{\alpha 2}\right)\ . \tag{A.71}$$

$M_W = gv/2$ combined with the Yukawa coupling $\lambda = \sqrt{2}m/v$ from the left-most vertex yields a term of order $m/\sqrt{2}$. The fermion line gives:

$$\begin{aligned}
&\langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}\gamma^\mu P_L N_2)(\overline{\ell_\beta}\gamma^\nu P_L N_2)(\overline{N}_1\gamma_\mu P_L \ell_\beta)(\overline{N}_1 P_R \ell_\alpha)\,|\ell_\alpha(p_1)\rangle \\
&= \langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}\gamma^\mu P_L N_2)(\overline{N}_2 P_L \gamma^\nu \ell^c_\beta)(\overline{\ell^c_\beta} P_L \gamma_\mu N_1)(\overline{N}_1 P_R \ell_\alpha)\,|\ell_\alpha(p_1)\rangle \\
&= \bar{u}(p_2)\gamma^\mu P_L \left\langle N_2 \overline{N}_2 \right\rangle P_L \gamma^\nu \left\langle \ell_\beta \overline{\ell^c_\beta} \right\rangle P_L \gamma_\mu \left\langle N_1 \overline{N}_1 \right\rangle P_R u(p_1) \\
&\to \bar{u}(p_2)\gamma^\mu P_L \left[(\not{p}_2 - \not{k}_2) + M_2\right] P_L \gamma^\nu (\not{k}_1 - \not{k}_2) P_L \gamma_\mu \left[\not{k}_1 + M_1\right] P_R u(p_1) \\
&= M_1 M_2\, \bar{u}(p_2)\gamma^\mu \gamma^\nu (\not{k}_1 - \not{k}_2)\gamma_\mu P_R u(p_1) \\
&= 4 M_1 M_2\, \bar{u}(p_2)(k_1 - k_2)^\nu P_R u(p_1)\ .
\end{aligned} \tag{A.72}$$

Contraction with the $\gamma\phi W$ vertex only changes the Lorentz index from ν to λ. Propagators will give $i^4(-i)^2 = -1$ as the overall sign. Integration will yield the C functions we introduced to calculate diagram 2A:

$$\begin{aligned}
(C_1 p_1^\lambda + C_2 p_2^\lambda)P_R &= \left[C_1\left\{\frac{(p_1+p_2)^\lambda + (p_1-p_2)^\lambda}{2}\right\} + C_2\left\{\frac{(p_1+p_2)^\lambda - (p_1-p_2)^\lambda}{2}\right\}\right] P_R \\
&= \left(\frac{C_1 + C_2}{2}\right)(p_1+p_2)^\lambda P_R + \left(\frac{C_1 - C_2}{2}\right)(p_1-p_2)^\lambda P_R\ .
\end{aligned} \tag{A.73}$$

So the coefficient of $(p_1+p_2)^\lambda \gamma_5$ is

$$-M_1 M_2 (C_1 + C_2)\ , \tag{A.74}$$

where the we have flipped the sign as mentioned above, and together with the coefficients, we have

$$\frac{iG_2(0)}{2m} = -i\frac{eg^3}{2\sqrt{2}} M_W M_1 M_2 \left(\tilde{\lambda}^*_{\alpha 1}\tilde{V}^*_{\beta 1}\tilde{V}_{\beta 2}\tilde{V}_{\alpha 2}\right)(C_1 + C_2)\ , \tag{A.75}$$

or

$$d = \frac{eg^3}{2\sqrt{2}} M_W M_1 M_2 \left(i\tilde{\lambda}^*_{\alpha 1}\tilde{V}^*_{\beta 1}\tilde{V}_{\beta 2}\tilde{V}_{\alpha 2}\right)(C_1 + C_2)\ . \tag{A.76}$$

To evaluate $C_1 + C_2$ use the result from Diagram$2B$

$$\begin{aligned}
C_1 &= \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2 - M_2^2)(K)^2(k_1^2 - M_1^2)(k_1^2 - M_W^2)(k_2^2 - M_W^2)}\left[\frac{K^2 + k_1^2 - k_2^2}{(k_1^2 - M_W^2)^2}\right] \\
C_2 &= \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2 - M_2^2)(K)^2(k_1^2 - M_1^2)(k_1^2 - M_W^2)(k_2^2 - M_W^2)}\left[\frac{k_1^2 - k_2^2 - K^2}{(k_1^2 - M_W^2)(k_2^2 - M_2^2)}\right] \\
&+ \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2 - M_2^2)(K)^2(k_1^2 - M_1^2)(k_1^2 - M_W^2)(k_2^2 - M_W^2)}\left[\frac{K^2 + k_1^2 - k_2^2}{(k_1^2 - M_W^2)^2}\right]
\end{aligned} \tag{A.77}$$

$$\begin{aligned}
C_1 &= \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2 - M_2^2)(k_1^2 - M_1^2)(k_1^2 - M_W^2)(k_2^2 - M_W^2)}\left[\frac{1}{(k_1^2 - M_W^2)^2}\right] \\
&+ \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2 - M_2^2)(K)^2(k_1^2 - M_1^2)(k_1^2 - M_W^2)(k_2^2 - M_W^2)}\left[\frac{k_1^2}{(k_1^2 - M_W^2)^2}\right] \\
&+ \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2 - M_2^2)(K)^2(k_1^2 - M_1^2)(k_1^2 - M_W^2)(k_2^2 - M_W^2)}\left[\frac{-k_2^2}{(k_1^2 - M_W^2)^2}\right] \\
C_2 &= \frac{1}{4}\int \frac{d^4k_1}{(2\pi)^4}\int \frac{d^4k_2}{(2\pi)^4} \frac{1}{(k_2^2 - M_2^2)(k_1^2 - M_1^2)(k_1^2 - M_W^2)(k_2^2 - M_W^2)}\left[\frac{-1}{(k_1^2 - M_W^2)(k_2^2 - M_2^2)}\right]
\end{aligned}$$

$$
\begin{aligned}
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{1}{(k_1^2-M_W^2)^2}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right]
\end{aligned}
\tag{A.78}
$$

$$
\begin{aligned}
&C_1+C_2\\
&= \quad \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{1}{(k_1^2-M_W^2)^2}\right]\\
&+ \quad \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)^2}\right]\\
&+ \quad \frac{1}{2}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)^2}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-1}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{k_1^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]\\
&+ \quad \frac{1}{4}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{(k_2^2-M_2^2)(K)^2(k_1^2-M_1^2)(k_1^2-M_W^2)(k_2^2-M_W^2)}\left[\frac{-k_2^2}{(k_1^2-M_W^2)(k_2^2-M_2^2)}\right]
\end{aligned}
\tag{A.79}
$$

after the usual wick rotation and expanding in Gegenbauer polynomials we can replace:

$$
\begin{aligned}
\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\,\frac{1}{K^2} \quad\longrightarrow\quad & \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\,k_1^2\int_0^\infty dk_2^2\,k_2^2\,\frac{1}{k_>^2} = \frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^\infty dk_2^2\,k_<^2\\
= \quad & \frac{1}{(4\pi)^4}\left[\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2 + \int_0^\infty dk_2^2\int_0^{k_2^2} dk_1^2\,k_1^2\right].
\end{aligned}
\tag{A.80}
$$

$$
\begin{aligned}
C_1+C_2 \quad = \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\\
&\times\left[\frac{-1}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]\\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\\
&\times\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]\\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\int_0^{k_1^2} dk_2^2\,k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)}\\
&\times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right]
\end{aligned}
$$

$$
\begin{aligned}
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\ k_2^2k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{1}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\ k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\ k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\ k_1^2k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{-1}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\ k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\ k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\ k_1^2k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{1}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\ k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right] \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\ k_1^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}
\tag{A.81}
$$

$$
\begin{aligned}
C_1+C_2 \quad = \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\ k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
+ \quad & (-1)\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\ k_2^2\,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
& \qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}
$$

$$
\begin{aligned}
&+ \quad \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right] \\
&+ \quad \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
&+ \quad \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
&+ \quad \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}
\tag{A.82}
$$

$$
\begin{aligned}
C_1+C_2 \quad = &\quad \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
&+ \quad (-1)\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right] \\
&+ \quad \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \int_0^{k_1^2} dk_2^2\; k_2^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)^2}\right] \\
&+ \quad \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
&+ \quad \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{-k_2^2}{(k_1^2+M_W^2)(k_2^2+M_2^2)}\right] \\
&+ \quad \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_2^2 \int_0^{k_2^2} dk_1^2\; k_1^2 \,\frac{1}{(k_2^2+M_2^2)(k_1^2+M_1^2)(k_1^2+M_W^2)(k_2^2+M_W^2)} \\
&\qquad \times\left[\frac{k_1^2}{(k_1^2+M_W^2)^2}\right]
\end{aligned}
\tag{A.83}
$$

This reduces to:

$$
\begin{aligned}
C_1 + C_2 \quad = \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_1^2)(k_1^2+M_W^2)^2}\int_0^{k_1^2} dk_2^2\, \frac{-k_2^4}{(k_2^2+M_2^2)^2(k_2^2+M_W^2)} \\
+ \quad & (-1)\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+M_W^2)^3}\int_0^{k_1^2} dk_2^2\, \frac{k_2^2}{(k_2^2+M_2^2)(k_2^2+M_W^2)} \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_1^2)(k_1^2+M_W^2)^3}\int_0^{k_1^2} dk_2^2\, \frac{-k_2^4}{(k_2^2+M_2^2)(k_2^2+M_W^2)} \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2\, \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+M_W^2)^2} \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2\, \frac{-k_2^2}{(k_2^2+M_1^2)(k_2^2+M_W^2)^2} \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+M_W^2)}\int_0^{k_1^2} dk_2^2\, \frac{-k_2^4}{(k_2^2+M_1^2)(k_2^2+M_W^2)^3}
\end{aligned}
\tag{A.84}
$$

scale all masses and momenta to M_W^2

$$
\begin{aligned}
C_1 + C_2 \quad = \quad & \frac{-1}{4}\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\, \frac{-k_2^4}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
+ \quad & (-1)\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3}\int_0^{k_1^2} dk_2^2\, \frac{k_2^2}{(k_2^2+M_2^2)(k_2^2+1)} \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2\, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3}\int_0^{k_1^2} dk_2^2\, \frac{-k_2^4}{(k_2^2+M_2^2)(k_2^2+1)} \\
+ \quad & \frac{-1}{4}\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)}\int_0^{k_1^2} dk_2^2\, \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^2} \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)}\int_0^{k_1^2} dk_2^2\, \frac{-k_2^2}{(k_2^2+M_1^2)(k_2^2+1)^2} \\
+ \quad & \frac{-1}{2}\frac{1}{(4\pi)^4 M_W^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)}\int_0^{k_1^2} dk_2^2\, \frac{-k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^3}
\end{aligned}
\tag{A.85}
$$

In terms of the integrals in Appendix B:

$$
C_1 + C_2 \quad = \quad \frac{-1}{(4\pi)^4 M_W^4}\left[\frac{-1}{4}C_a + C_b + \frac{1}{2}C_c + \frac{1}{4}C_d + \frac{1}{2}C_e + \frac{1}{2}C_f\right]
\tag{A.86}
$$

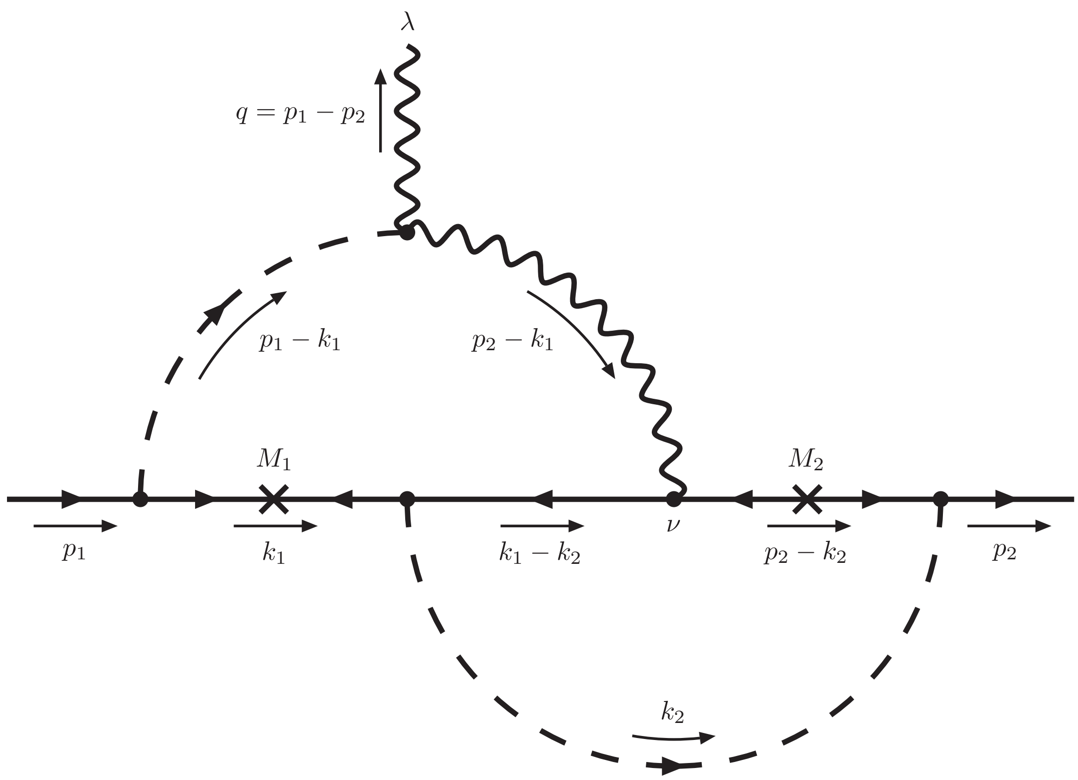

Figure A.5: **Diagram 4B**

A.5 Diagram 4B

The $\gamma\phi W$ vertex is just $+ieM_W g_{\lambda\nu}$. The coupling constants in this diagram are

$$(ieM_W)\left(-i\tilde{\Lambda}^{\dagger}_{1\alpha}\right)\left(-i\tilde{\Lambda}^{\dagger}_{1\beta}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}_{\beta 2}\right)\left(-i\tilde{\lambda}_{\alpha 2}\right) = i\frac{eg}{\sqrt{2}}M_W\left(\tilde{\Lambda}^{*}_{\alpha 1}\tilde{\Lambda}^{*}_{\beta 1}\tilde{V}_{\beta 2}\tilde{\lambda}_{\alpha 2}\right) . \tag{A.87}$$

The fermion line gives:

$$\begin{aligned}
&\langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}P_L N_2)(\overline{\ell_\beta}\gamma^\nu P_L N_2)(\overline{N}_1 P_L \ell_\beta)(\overline{N}_1 P_L \ell_\alpha)\,|\ell_\alpha(p_1)\rangle \\
&\quad = \quad \langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}P_L N_2)(\overline{N}_2 P_L \gamma^\nu \ell^c_\beta)(\overline{\ell^c_\beta}P_L N_1)(\overline{N}_1 P_L \ell_\alpha)\,|\ell_\alpha(p_1)\rangle \\
&\quad = \quad \bar{u}(p_2)P_L \left\langle N_2\overline{N}_2\right\rangle P_L\gamma^\nu \left\langle \ell^c_\beta\overline{\ell^c_\beta}\right\rangle P_L \left\langle N_1\overline{N}_1\right\rangle P_L u(p_1) \\[1ex]
&\quad \to \quad \bar{u}(p_2)P_L\left[(\not{p}_2 - \not{k}_2) + M_2\right] P_L\gamma^\nu(\not{k}_1 - \not{k}_2)P_L\left[\not{k}_1 + M_1\right] P_L u(p_1) \\
&\quad = \quad M_1 M_2\,\bar{u}(p_2)\gamma^\nu(\not{k}_1 - \not{k}_2)P_L u(p_1) .
\end{aligned} \tag{A.88}$$

The propagators give an overall sign of $i^5(-i) = 1$. Integration yields the C functions:

$$\gamma_\lambda(C_1\not{p}_1 + C_2\not{p}_2)P_L \quad = \quad C_1\gamma_\lambda P_R\not{p}_1 + C_2\gamma_\lambda\not{p}_2 P_L$$

$$\begin{aligned} &= C_1\gamma_\lambda P_R \not{p}_1 + C_2(2p_{2\lambda} - \not{p}_2\gamma_\lambda)P_L \\ &\rightarrow m(C_1\gamma_\lambda P_R - C_2\gamma_\lambda P_L) + C_2[(p_1+p_2)_\lambda - (p_1-p_2)_\lambda]P_L\ . \end{aligned} \tag{A.89}$$

The coefficient of $m(p_1+p_2)_\lambda\gamma_5$ will be

$$-\frac{1}{2}M_1M_2C_2\ . \tag{A.90}$$

Together with the coupling constants, we find

$$i\frac{G_2(0)}{2m} = -i\frac{eg}{2\sqrt{2}}M_W M_1 M_2\left(\tilde{\Lambda}^*_{\alpha 1}\tilde{\Lambda}^*_{\beta 1}\tilde{V}_{\beta 2}\tilde{\lambda}_{\alpha 2}\right)C_2\ , \tag{A.91}$$

or

$$d = \frac{eg}{2\sqrt{2}}M_W M_1 M_2\left(i\tilde{\Lambda}^*_{\alpha 1}\tilde{\Lambda}^*_{\beta 1}\tilde{V}_{\beta 2}\tilde{\lambda}_{\alpha 2}\right)C_2\ . \tag{A.92}$$

Repeating the same procedure as in Diagram $4A$ we reach to the result:

$$C_2 = \frac{-1}{(4\pi)^4M_W^4}\left[\frac{-1}{4}C_a + \frac{1}{2}C_b + \frac{1}{4}C_c + \frac{1}{4}C_d + \frac{1}{2}C_e + \frac{1}{4}C_f\right] \tag{A.93}$$

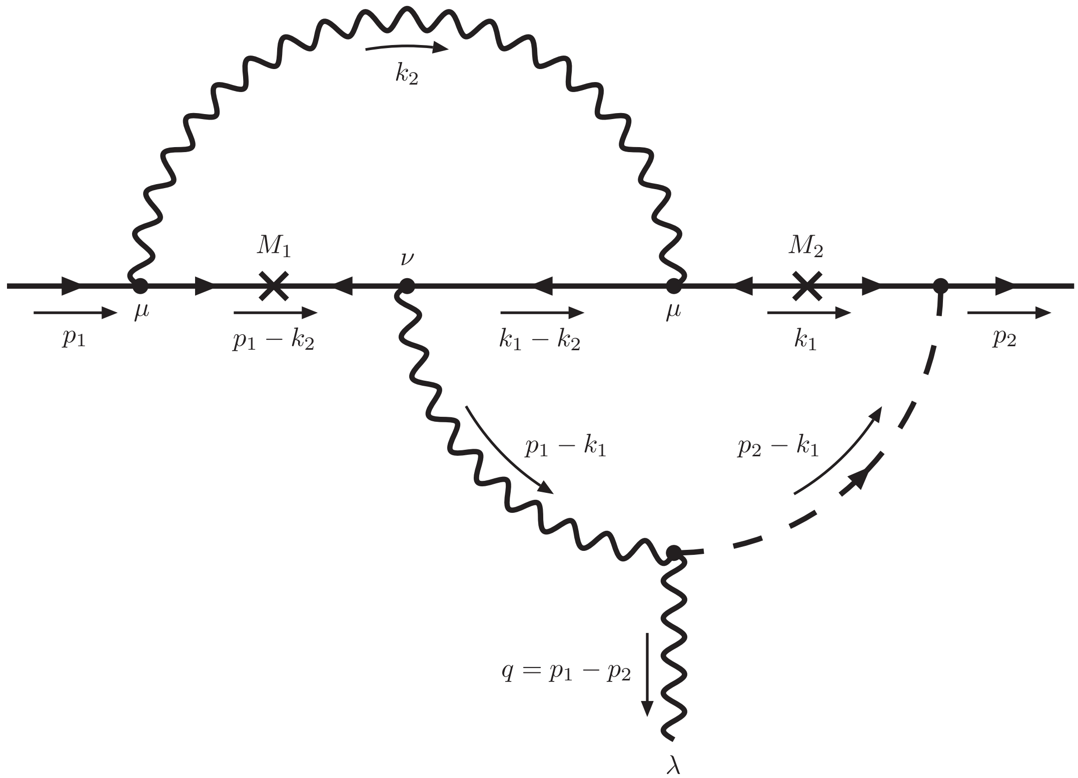

Figure A.6: **Diagram 5A**

A.6 Diagram 5A

Diagrams 5A can be obtained from 4A and 4B by interchanging M_1 and M_2, and flipping the signs of p_1 and p_2, I think. Let's check. The $\gamma\phi W$ vertex is just $+ieM_W g_{\lambda\nu}$. The coupling constants in this diagram are

$$(ieM_W)\left(-i\frac{g}{\sqrt{2}}\tilde{V}^\dagger_{1\alpha}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}^\dagger_{1\beta}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}_{\beta 2}\right)\left(-i\tilde{\lambda}_{\alpha 2}\right) = i\frac{eg^3}{2\sqrt{2}}M_W\left(\tilde{V}^*_{\alpha 1}\tilde{V}^*_{\beta 1}\tilde{V}_{\beta 2}\tilde{\lambda}_{\alpha 2}\right) . \tag{A.94}$$

This is different from diagram 4A: the labels 1 and 2 are interchanged, and the entire combination is complex conjugated. This is what you need since you want the diagrams with M_1 and M_2 interchanged to be complex conjugates of each other. The fermion line gives:

$$\begin{aligned}
&\langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}P_L N_2)(\overline{\ell_\beta}\gamma^\mu P_L N_2)(\overline{N}_1\gamma^\nu P_L\ell_\beta)(\overline{N}_1\gamma_\mu P_L\ell_\alpha)\,|\ell_\alpha(p_1)\rangle \\
&= \quad \langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}P_L N_2)(\overline{N}_2 P_L\gamma^\mu\ell^c_\beta)(\overline{\ell^c_\beta}P_L\gamma^\nu N_1)(\overline{N}_1\gamma_\mu P_L\ell_\alpha)\,|\ell_\alpha(p_1)\rangle \\
&= \quad \bar{u}(p_2)P_L\left\langle N_2\overline{N}_2\right\rangle P_L\gamma^\mu\left\langle \ell_\beta\overline{\ell^c_\beta}\right\rangle P_L\gamma^\nu\left\langle N_1\overline{N}_1\right\rangle\gamma_\mu P_L u(p_1) \\
&\to \quad \bar{u}(p_2)P_L\left[\not{k}_1 + M_2\right]P_L\gamma^\mu(\not{k}_1 - \not{k}_2)P_L\gamma^\nu\left[(\not{p}_1 - \not{k}_2) + M_1\right]\gamma_\mu P_L u(p_1)
\end{aligned}$$

$$
\begin{aligned}
&= M_1 M_2\, \bar{u}(p_2)\gamma^\mu(\not{k}_1 - \not{k}_2)\gamma^\nu \gamma_\mu P_L u(p_1) \\
&= 4M_1 M_2\, \bar{u}(p_2)(k_1 - k_2)^\nu P_L u(p_1)\ . \qquad \text{(A.95)}
\end{aligned}
$$

This is the same as diagram 4A, except P_R has been replaced by P_L. Contraction with the $\gamma\phi W$ vertex only changes the Lorentz index from ν to λ. Propagators will give $i^4(-i)^2 = -1$ as the overall sign. Integration will yield the $\overline{C}$ functions we introduced to calculate diagram 3A:

$$
\begin{aligned}
(\overline{C}_1 p_2^\lambda + \overline{C}_2 p_1^\lambda)P_L &= \left[\overline{C}_1 \left\{\frac{(p_1+p_2)^\lambda - (p_1-p_2)^\lambda}{2}\right\} + \overline{C}_2 \left\{\frac{(p_1+p_2)^\lambda + (p_1-p_2)^\lambda}{2}\right\}\right] P_L \\
&= \left(\frac{\overline{C}_1 + \overline{C}_2}{2}\right)(p_1+p_2)^\lambda P_L - \left(\frac{\overline{C}_1 - \overline{C}_2}{2}\right)(p_1-p_2)^\lambda P_L\ . \qquad \text{(A.96)}
\end{aligned}
$$

So the coefficient of $(p_1+p_2)^\lambda \gamma_5$ is

$$
+M_1 M_2 (\overline{C}_1 + \overline{C}_2)\ , \qquad \text{(A.97)}
$$

where the we have flipped the sign as mentioned above, and together with the coefficients, we have

$$
\frac{iG_2(0)}{2m} = +i\frac{eg^3}{2\sqrt{2}} M_W M_1 M_2 \left(\tilde{V}^*_{\alpha 1}\tilde{V}^*_{\beta 1}\tilde{V}_{\beta 2}\tilde{\lambda}_{\alpha 2}\right)(\overline{C}_1 + \overline{C}_2)\ , \qquad \text{(A.98)}
$$

or

$$
d = -\frac{eg^3}{2\sqrt{2}} M_W M_1 M_2 \left(i\tilde{V}^*_{\alpha 1}\tilde{V}^*_{\beta 1}\tilde{V}_{\beta 2}\tilde{\lambda}_{\alpha 2}\right)(\overline{C}_1 + \overline{C}_2)\ . \qquad \text{(A.99)}
$$

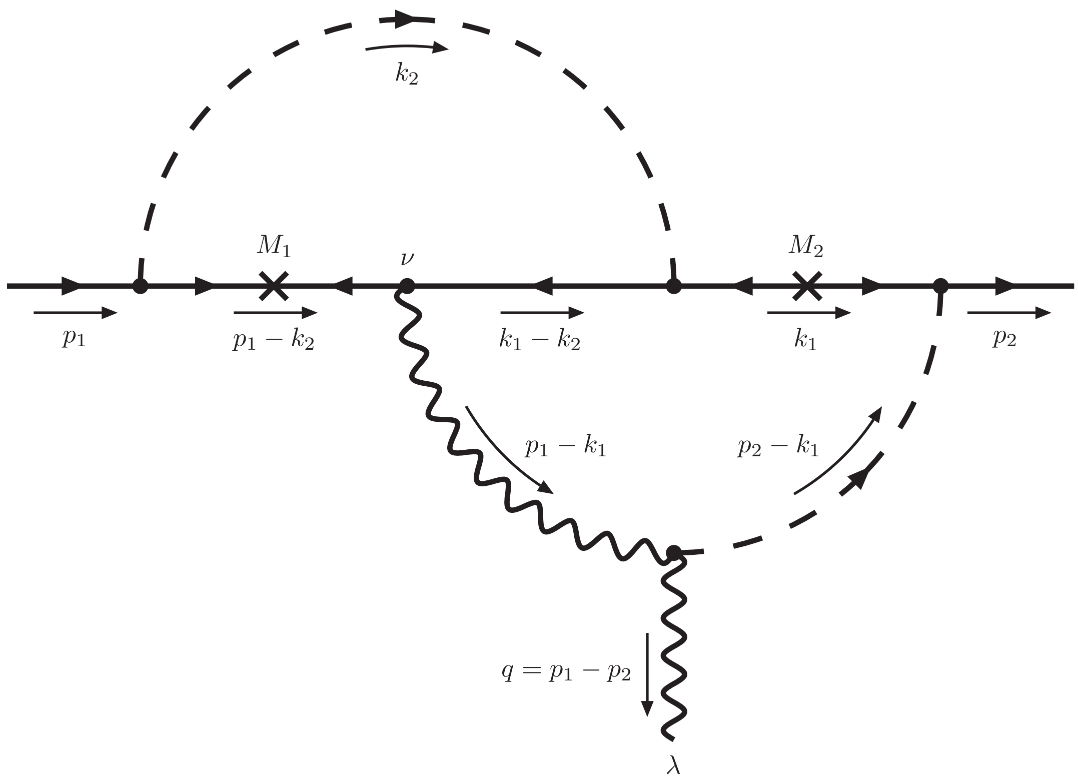

Figure A.7: **Diagram 5B**

A.7 Diagram 5B

The $\gamma\phi W$ vertex is just $+ieM_W g_{\lambda\nu}$. The coupling constants in this diagram are

$$(ieM_W)\left(-i\tilde{\lambda}^{\dagger}_{1\alpha}\right)\left(-i\frac{g}{\sqrt{2}}\tilde{V}^{\dagger}_{1\beta}\right)\left(-i\tilde{\Lambda}_{\beta 2}\right)\left(-i\tilde{\Lambda}_{\alpha 2}\right) = i\frac{eg}{\sqrt{2}}M_W\left(\tilde{\lambda}^{*}_{\alpha 1}\tilde{V}^{*}_{\beta 1}\tilde{\Lambda}_{\beta 2}\tilde{\Lambda}_{\alpha 2}\right) . \tag{A.100}$$

The fermion line gives:

$$\begin{aligned}
&\langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}P_R N_2)(\overline{\ell_\beta}P_R N_2)(\overline{N}_1\gamma^\nu P_L \ell_\beta)(\overline{N}_1 P_R \ell_\alpha)\,|\ell_\alpha(p_1)\rangle \\
&= \langle \ell_\alpha(p_2)|\,(\overline{\ell_\alpha}P_R N_2)(\overline{N}_2 P_R \ell^c_\beta)(\overline{\ell^c_\beta}P_L\gamma^\nu N_1)(\overline{N}_1 P_R \ell_\alpha)\,|\ell_\alpha(p_1)\rangle \\
&= \bar{u}(p_2)P_R\left\langle N_2\overline{N}_2\right\rangle P_R\left\langle \ell^c_\beta\overline{\ell^c_\beta}\right\rangle P_L\gamma^\nu\left\langle N_1\overline{N}_1\right\rangle P_R u(p_1) \\
&\to \bar{u}(p_2)P_R\left[\not{k}_1 + M_2\right]P_R(\not{k}_1 - \not{k}_2)P_L\gamma^\nu\left[(\not{p}_1 - \not{k}_2) + M_1\right]P_R u(p_1) \\
&= M_1 M_2\,\bar{u}(p_2)(\not{k}_1 - \not{k}_2)\gamma^\nu P_R u(p_1) .
\end{aligned} \tag{A.101}$$

The propagators give an overall sign of $i^5(-i) = 1$. Integration yields the $\overline{C}$ functions:

$$(\overline{C}_1\not{p}_2 + \overline{C}_2\not{p}_1)\gamma_\lambda P_R = \overline{C}_1\not{p}_2\gamma_\lambda P_R + \overline{C}_2(2p_{1\lambda} - \gamma_\lambda\not{p}_1)P_R$$

$$
\begin{aligned}
&= \overline{C}_1 \not{p}_2 \gamma_\lambda P_R + \overline{C}_2 (2p_{1\lambda} P_R - \gamma_\lambda P_L \not{p}_1) \\
&\to m(\overline{C}_1 \gamma_\lambda P_R - \overline{C}_2 \gamma_\lambda P_L) + \overline{C}_2 [(p_1 + p_2)_\lambda + (p_1 - p_2)_\lambda] P_R \ . \qquad \text{(A.102)}
\end{aligned}
$$

The coefficient of $m(p_1 + p_2)_\lambda \gamma_5$ will be

$$
+\frac{1}{2} M_1 M_2 \overline{C}_2 \ . \qquad \text{(A.103)}
$$

Together with the coupling constants, we find

$$
i \frac{G_2(0)}{2m} = +i \frac{eg}{2\sqrt{2}} M_W M_1 M_2 \left(\tilde{\lambda}^*_{\alpha 1} \tilde{V}^*_{\beta 1} \tilde{\Lambda}_{\beta 2} \tilde{\Lambda}_{\alpha 2} \right) \overline{C}_2 \ , \qquad \text{(A.104)}
$$

or

$$
d = -\frac{eg}{2\sqrt{2}} M_W M_1 M_2 \left(i \tilde{\lambda}^*_{\alpha 1} \tilde{V}^*_{\beta 1} \tilde{\Lambda}_{\beta 2} \tilde{\Lambda}_{\alpha 2} \right) \overline{C}_2 \ . \qquad \text{(A.105)}
$$

Appendix B

Integrals

B.1 The di-logarithm integration

We need to evaluate integrals that look like

$$\begin{aligned} I &= M_k^2 \int_0^\infty dk \ln(1+\frac{k}{M_k^2})\frac{1}{(k+1)(k+M_i^2)} \\ &= \int_0^\infty dk \ln(1+k)\frac{1}{(k+\frac{1}{M_k^2})(k+\frac{M_i^2}{M_k^2})} \end{aligned} \tag{B.1}$$

Define $\quad k^{'} = 1+k$

$$I = \int_1^\infty dk^{'} \frac{\ln(k^{'})}{(k^{'}+\frac{1}{M_k^2}-1)(k^{'}+\frac{M_i^2}{M_k^2}-1)} \tag{B.2}$$

Redefine $\quad k = \frac{1}{k^{'}}$

$$I = \int_0^1 dk \frac{\ln(k)}{(1+k(\frac{1}{M_k^2}-1))(1+k(\frac{M_i^2}{M_k^2}-1))} \tag{B.3}$$

Rename

$$a = 1-\frac{1}{M_k^2} \qquad b \quad = 1-\frac{M_i^2}{M_k^2} \tag{B.4}$$

$$\begin{aligned} I &= \int_0^1 dk \frac{\ln(k)}{(1-ak)(1-bk)} \\ I &= \int_0^1 dk \frac{a}{b-a}\frac{\ln(k)}{(1-ak)} + \frac{b}{b-a}\frac{\ln(k)}{(1-bk)} \end{aligned} \tag{B.5}$$

Integrating by parts

$$
\begin{aligned}
I^{'} &= \int_0^1 dk \frac{\ln(k)}{(1-ak)} \\
&= -\frac{1}{a}\ln(k)\ln(1-ak)\Big|_0^1 + \int_0^1 dk \frac{1}{a}\frac{\ln(1-ak)}{(k)} \\
&= -\frac{1}{a}\ln(k)\ln(1-ak)\Big|_0^1 + \int_0^a dk \frac{1}{a}\frac{\ln(1-k)}{(k)} \\
&= -\frac{1}{a}\ln(k)\ln(1-ak)\Big|_0^1 - \frac{1}{a}Li_2(a)
\end{aligned}
\tag{B.6}
$$

Where $Li_2(x) = -\int_0^x dk \frac{\ln(1-k)}{(k)} = \sum_{n=1}^{\infty}\left(\frac{x^n}{n^2}\right)$ if 0<x<1.

$$
\begin{aligned}
I &= \frac{a}{a-b}\left[-\frac{1}{a}\ln(k)\ln(1-ak)\Big|_0^1 - \frac{1}{a}Li_2(a)\right] + \frac{b}{b-a}\left[-\frac{1}{b}\ln(k)\ln(1-ak)\Big|_0^1 - \frac{1}{b}Li_2(b)\right] \\
I &= \frac{1}{b-a}\left[Li_2(a) - Li_2(b)\right]
\end{aligned}
\tag{B.7}
$$

B.2 list of different integrations

B.2.1 The A and B integrals

$$\begin{aligned} X + Y \quad = \quad & -\frac{2}{(4\pi)^4}\left[\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_W^2)(k_1^2+M_1^2)^2}\int_0^{k_1^2} dk_2^2\, \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_2^2)} \right. \\ & \left. - \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_W^2)(k_1^2+M_2^2)^2}\int_0^{k_1^2} dk_2^2\, \frac{k_2^2}{(k_2^2+M_W^2)(k_2^2+M_1^2)}\right] . \end{aligned} \tag{B.8}$$

It is enough to evaluate the first integral, since the second is the same after interchanging M_1 and M_2

$$\int_0^\infty dk_1^2 \frac{1}{\left(k_1^2+M_w^2\right)\left(k_1^2+M_1^2\right)^2}\int_0^{k_1^2} dk_2^2 \frac{k_2^2}{\left(k_2^2+M_w^2\right)\left(k_2^2+M_2^2\right)}$$

Scale all masses and momenta to M_w^2

$$\frac{1}{M_w^4}\int_0^\infty dk_1^2 \frac{1}{\left(k_1^2+1\right)\left(k_1^2+M_1^2\right)^2}\int_0^{k_1^2} dk_2^2 \frac{k_2^2}{\left(k_2^2+1\right)\left(k_2^2+M_2^2\right)}$$

Name I

$$I = \int_0^\infty dk_1^2 \frac{1}{\left(k_1^2+1\right)\left(k_1^2+M_1^2\right)^2}\int_0^{k_1^2} dk_2^2 \frac{k_2^2}{\left(k_2^2+1\right)\left(k_2^2+M_2^2\right)}$$

Do the k_2 integration

$$\begin{aligned} & \int_0^{k_1^2} dk_2^2 \frac{k_2^2}{\left(k_2^2+1\right)\left(k_2^2+M_2^2\right)} \\ & = \int_0^{k_1^2} dk_2^2 \left(\frac{A}{\left(k_2^2+M_2^2\right)} + \frac{B}{\left(k_2^2+1\right)}\right) \\ & = A\ln(1+\frac{k_1^2}{M_2^2}) + B\ln(1+k_1^2) \end{aligned}$$

Where A and B are

$$A = \frac{M_2^2}{M_2^2-1} \qquad B \quad = \frac{-1}{M_2^2-1}$$

Plug back

$$I = \int_0^\infty dk_1^2 \frac{1}{\left(k_1^2+1\right)\left(k_1^2+M_1^2\right)^2}\left(A\ln(1+\frac{k_1^2}{M_2^2}) + B\ln(1+k_1^2)\right)$$

$$I = \int_0^\infty dk_1^2 \left(\frac{C}{\left(k_1^2+M_1^2\right)^2} + \frac{D}{\left(k_1^2+1\right)\left(k_1^2+M_1^2\right)}\right)\left(A\ln(1+\frac{k_1^2}{M_2^2}) + B\ln(1+k_1^2)\right)$$

$$C \quad = \frac{-1}{M_1^2-1} \qquad D \quad = \frac{1}{M_1^2-1}$$

doing the integrals yields

$$1 = \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+M_1^2)^2} = \frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)}$$

$$2 = \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_1^2)^2} = \frac{\ln(M_1^2)}{(M_1^2-1)}$$

$$3 = \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_1^2)} = \frac{-Li_2(1-M_1^2)}{(M_1^2-1)}$$

$$4 = \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2})\frac{1}{(k_1^2+1)(k_1^2+M_1^2)} = \frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}$$

Where $Li_2(x) = -\int_0^x dk \frac{\ln(1-k)}{(k)} = \sum_{n=1}^{\infty}\left(\frac{x^n}{n^2}\right)$ if 0<x<1.

$$\begin{aligned} I = \quad & \left(\left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{-1}{M_1^2-1}\right)\left(\frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)}\right)\right. \\ + \quad & \left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{1}{M_1^2-1}\right)\left(\frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right) \\ + \quad & \left(\frac{-1}{M_2^2-1}\right)\left(\frac{-1}{M_1^2-1}\right)\left(\frac{\ln(M_1^2)}{(M_1^2-1)}\right) \\ + \quad & \left.\left(\frac{-1}{M_2^2-1}\right)\left(\frac{1}{M_1^2-1}\right)\left(\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}\right)\right) \end{aligned}$$

the second piece of the integral is obtained by interchanging M_1 and M_2

$$\begin{aligned} I^* = \quad & \left(\left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{-1}{M_2^2-1}\right)\left(\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right)\right. \\ + \quad & \left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{1}{M_2^2-1}\right)\left(\frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right) \\ + \quad & \left(\frac{-1}{M_1^2-1}\right)\left(\frac{-1}{M_2^2-1}\right)\left(\frac{\ln(M_2^2)}{(M_2^2-1)}\right) \\ + \quad & \left.\left(\frac{-1}{M_1^2-1}\right)\left(\frac{1}{M_2^2-1}\right)\left(\frac{-Li_2(1-M_2^2)}{(M_2^2-1)}\right)\right) \end{aligned}$$

so

$$
\begin{aligned}
X+Y=-\frac{2}{(4\pi)^4}\Bigg[\quad & \Bigg[\left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{-1}{M_1^2-1}\right)\left(\frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)}\right) \\
+ \quad & \left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{1}{M_1^2-1}\right)\left(\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right) \\
+ \quad & \left(\frac{-1}{M_2^2-1}\right)\left(\frac{-1}{M_1^2-1}\right)\left(\frac{\ln(M_1^2)}{(M_1^2-1)}\right) \\
+ \quad & \left(\frac{-1}{M_2^2-1}\right)\left(\frac{1}{M_1^2-1}\right)\left(\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}\right)\Bigg] \\
- \quad & \Bigg[\left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{-1}{M_2^2-1}\right)\left(\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right) \\
+ \quad & \left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{1}{M_2^2-1}\right)\left(\frac{Li_2\left(1-\frac{1}{M_1^2}\right)-Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right) \\
+ \quad & \left(\frac{-1}{M_1^2-1}\right)\left(\frac{-1}{M_2^2-1}\right)\left(\frac{\ln(M_2^2)}{(M_2^2-1)}\right) \\
+ \quad & \left(\frac{-1}{M_1^2-1}\right)\left(\frac{1}{M_2^2-1}\right)\left(\frac{-Li_2(1-M_2^2)}{(M_2^2-1)}\right)\Bigg]\Bigg] .
\end{aligned}
$$

Now the rest of the integrals

the first part of the integral looks like

$$I_a = \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2 + M_W^2)(k_1^2 + M_1^2)^3} \int_0^{k_1^2} dk_2^2 \frac{k_2^2}{(k_2^2 + M_W^2)(k_2^2 + M_2^2)}$$

Scale all masses and momenta to M_w^2

$$I_a = \frac{1}{M_w^4} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2 + 1)(k_1^2 + M_1^2)^3} \int_0^{k_1^2} dk_2^2 \frac{k_2^2}{(k_2^2 + 1)(k_2^2 + M_2^2)}$$

Do the k_2 integration

$$II = \int_0^{k_1^2} dk_2^2 \frac{k_2^2}{(k_2^2 + 1)\,(k_2^2 + M_2^2)}$$
$$II = \int_0^{k_1^2} dk_2^2 \left(\frac{A}{(k_2^2 + M_2^2)} + \frac{B}{(k_2^2 + 1)} \right)$$
$$II = A \ln(1 + \frac{k_1^2}{M_2^2}) + B \ln(1 + k_1^2)$$

Where A and B are

$$A = \frac{M_2^2}{M_2^2 - 1} \qquad B = \frac{-1}{M_2^2 - 1}$$

Plug back

$$I_a = \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2 + 1)\,(k_1^2 + M_1^2)^3} \left(A \ln(1 + \frac{k_1^2}{M_2^2}) + B \ln(1 + k_1^2) \right)$$
$$I_a = \int_0^\infty dk_1^2 \left(\frac{C}{(k_1^2 + M_1^2)^3} + \frac{D}{(k_1^2 + M_1^2)^2} + \frac{E}{(k_1^2 + 1)\,(k_1^2 + M_1^2)} \right) \left(A \ln(1 + \frac{k_1^2}{M_2^2}) + B \ln(1 + k_1^2) \right)$$

$$C = \frac{M_1^2}{M_1^2 - 1} \qquad D = \frac{1}{(M_1^2 - 1)^2} \qquad E = \frac{-1}{(M_1^2 - 1)^2}$$

doing the integrals yields

$$1 = \int_0^\infty dk_1^2 \frac{\ln(1 + \frac{k_1^2}{M_2^2})}{(k_1^2 + M_1^2)^3} = \frac{1}{2} \left(\frac{\ln(M_1^2) - \ln(M_2^2)}{(M_1^2 - M_2^2)^2} - \frac{1}{M_1^2\,(M_1^2 - M_2^2)} \right)$$
$$2 = \int_0^\infty dk_1^2 \frac{\ln(1 + k_1^2)}{(k_1^2 + M_1^2)^3} = \frac{1}{2} \left(\frac{\ln(M_1^2)}{(M_1^2 - 1)^2} - \frac{1}{M_1^2\,(M_1^2 - 1)} \right)$$
$$3 = \int_0^\infty dk_1^2 \frac{\ln(1 + \frac{k_1^2}{M_2^2})}{(k_1^2 + M_1^2)^2} = \frac{\ln(M_1^2) - \ln(M_2^2)}{(M_1^2 - M_2^2)}$$
$$4 = \int_0^\infty dk_1^2 \frac{\ln(1 + k_1^2)}{(k_1^2 + M_1^2)^2} = \frac{\ln(M_1^2)}{(M_1^2 - 1)}$$
$$5 = \int_0^\infty dk_1^2 \frac{\ln\left(1 + k_1^2\right)}{(k_1^2 + 1)\,(k_1^2 + M_1^2)} = \frac{-Li_2(1 - M_1^2)}{(M_1^2 - 1)}$$

$$6 = \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2})\frac{1}{(k_1^2+1)(k_1^2+M_1^2)} = \frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)} \tag{B.9}$$

$$\begin{aligned}
I_a = & \left(\left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{1}{2}\left(\frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)^2}-\frac{1}{M_1^2\,(M_1^2-M_2^2)}\right)\right)\right. \\
& + \left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{1}{(M_1^2-1)^2}\right)\left(\frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)}\right) \\
& + \left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{-1}{(M_1^2-1)^2}\right)\left(\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right) \\
& + \left(\frac{-1}{M_2^2-1}\right)\left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{1}{2}\left(\frac{\ln(M_1^2)}{(M_1^2-1)^2}-\frac{1}{M_1^2\,(M_1^2-1)}\right)\right) \\
& + \left(\frac{-1}{M_2^2-1}\right)\left(\frac{1}{(M_1^2-1)^2}\right)\left(\frac{\ln(M_1^2)}{(M_1^2-1)}\right) \\
& + \left.\left(\frac{-1}{M_2^2-1}\right)\left(\frac{-1}{(M_1^2-1)^2}\right)\left(\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}\right)\right)
\end{aligned} \tag{B.10}$$

Interchanging M_1 and M_2

$$\begin{aligned}
I_a^* = & \left(\left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{1}{2}\left(\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)^2}-\frac{1}{M_2^2\,(M_2^2-M_1^2)}\right)\right)\right. \\
& + \left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{1}{(M_2^2-1)^2}\right)\left(\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right) \\
& + \left(\frac{M_1^2}{M_1^2-1}\right)\left(\frac{-1}{(M_2^2-1)^2}\right)\left(\frac{Li_2\left(1-\frac{1}{M_1^2}\right)-Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right) \\
& + \left(\frac{-1}{M_1^2-1}\right)\left(\frac{M_2^2}{M_2^2-1}\right)\left(\frac{1}{2}\left(\frac{\ln(M_2^2)}{(M_2^2-1)^2}-\frac{1}{M_2^2\,(M_2^2-1)}\right)\right) \\
& + \left(\frac{-1}{M_1^2-1}\right)\left(\frac{1}{(M_2^2-1)^2}\right)\left(\frac{\ln(M_2^2)}{(M_2^2-1)}\right) \\
& + \left.\left(\frac{-1}{M_1^2-1}\right)\left(\frac{-1}{(M_2^2-1)^2}\right)\left(\frac{-Li_2(1-M_2^2)}{(M_2^2-1)}\right)\right)
\end{aligned} \tag{B.11}$$

The second piece is

$$I_b = \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_W^2)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\,\frac{k_2^4}{(k_2^2+M_W^2)(k_2^2+M_2^2)} \tag{B.12}$$

Scale all masses and momenta to M_w^2

$$I_b = \frac{1}{M_w^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_1^2)^3}\int_0^1 dk_2^2\,\frac{k_2^4}{(k_2^2+1)(k_2^2+M_2^2)} \tag{B.13}$$

$$
\begin{aligned}
I_b &= \frac{1}{M_w^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\, 1 \\
&+ \frac{-M_2^4}{(M_2^2-1)}\frac{1}{M_w^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_1^2)^3}\int_0^{k_1^2} dk_2^2\, \frac{1}{(k_2^2+M_2^2)} \\
&+ \frac{1}{(M_2^2-1)}\frac{1}{M_w^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_1^2)^3}\int_0^{1} dk_2^2\, \frac{1}{(k_2^2+1)}
\end{aligned}
\tag{B.14}
$$

$$
\begin{aligned}
I_b &= \frac{1}{M_w^4}\int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+1)(k_1^2+M_1^2)^3} \\
&+ \frac{-M_2^4}{(M_2^2-1)}\frac{1}{M_w^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_1^2)^3}\ln(1+\frac{k_1^2}{M_2^2}) \\
&+ \frac{1}{(M_2^2-1)}\frac{1}{M_w^4}\int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_1^2)^3}\ln(1+k_1^2)
\end{aligned}
\tag{B.15}
$$

$$
\begin{aligned}
I_b &= \frac{1}{M_w^4}\int_0^\infty dk_1^2 \Big[\frac{-1}{(M_1^2-1)^3}\frac{1}{(k_1^2+1)} + \frac{1}{(M_1^2-1)^3}\frac{1}{(k_1^2+M_1^2)} + \frac{1}{(M_1^2-1)^2}\frac{1}{(k_1^2+M_1^2)^2} \\
&\qquad + \frac{M_1^2}{(M_1^2-1)}\frac{1}{(k_1^2+M_1^2)^3}\Big] \\
&+ \frac{-M_2^4}{(M_2^2-1)}\frac{1}{M_w^4}\int_0^\infty dk_1^2 \Big[\frac{1}{(M_1^2-1)^2}\frac{1}{(k_1^2+1)(k_1^2+M_1^2)} + \frac{-1}{(M_1^2-1)^2}\frac{1}{(k_1^2+M_1^2)^2} \\
&\qquad + \frac{-1}{(M_1^2-1)}\frac{1}{(k_1^2+M_1^2)^3}\Big]\ln(1+\frac{k_1^2}{M_2^2}) \\
&+ \frac{1}{(M_2^2-1)}\frac{1}{M_w^4}\int_0^\infty dk_1^2 \Big[\frac{1}{(M_1^2-1)^2}\frac{1}{(k_1^2+1)(k_1^2+M_1^2)} + \frac{-1}{(M_1^2-1)^2}\frac{1}{(k_1^2+M_1^2)^2} \\
&\qquad + \frac{-1}{(M_1^2-1)}\frac{1}{(k_1^2+M_1^2)^3}\Big]\ln(1+k_1^2)
\end{aligned}
\tag{B.16}
$$

doing the integrals yields

$$
\begin{aligned}
1 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{\left(k_1^2+M_1^2\right)^3} = \frac{1}{2}\left(\frac{\ln(M_1^2)-\ln(M_2^2)}{\left(M_1^2-M_2^2\right)^2} - \frac{1}{M_1^2\left(M_1^2-M_2^2\right)}\right) \\
2 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{\left(k_1^2+M_1^2\right)^3} = \frac{1}{2}\left(\frac{\ln(M_1^2)}{\left(M_1^2-1\right)^2} - \frac{1}{M_1^2\left(M_1^2-1\right)}\right) \\
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{\left(k_1^2+M_1^2\right)^2} = \frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)} \\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{\left(k_1^2+M_1^2\right)^2} = \frac{\ln(M_1^2)}{(M_1^2-1)}
\end{aligned}
$$

$$
\begin{aligned}
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{\left(k_1^2+1\right)\left(k_1^2+M_1^2\right)} = \frac{-Li_2(1-M_1^2)}{(M_1^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2})\frac{1}{(k_1^2+1)(k_1^2+M_1^2)} = \frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}
\end{aligned}
\tag{B.17}
$$

$$
\begin{aligned}
I_b &= \frac{1}{M_w^4}\left[\frac{-\ln(M_1^2)}{(M_1^2-1)^3}+\frac{1}{M_1^2(M_1^2-1)^2}+\frac{1}{2M_1^2(M_1^2-1)}\right] \\
&+ \frac{-M_2^4}{(M_2^2-1)}\frac{1}{M_w^4}\left[\frac{1}{(M_1^2-1)^2}\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}+\frac{-1}{(M_1^2-1)^2}\frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)}\right. \\
&\qquad\left.+\frac{-1}{(M_1^2-1)}\frac{1}{2}\left(\frac{\ln(M_1^2)-\ln(M_2^2)}{\left(M_1^2-M_2^2\right)^2}-\frac{1}{M_1^2\left(M_1^2-M_2^2\right)}\right)\right] \\
&+ \frac{1}{(M_2^2-1)}\frac{1}{M_w^4}\left[\frac{1}{(M_1^2-1)^2}\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}+\frac{-1}{(M_1^2-1)^2}\frac{\ln(M_1^2)}{(M_1^2-1)}\right. \\
&\qquad\left.+\frac{-1}{(M_1^2-1)}\frac{1}{2}\left(\frac{\ln(M_1^2)}{\left(M_1^2-1\right)^2}-\frac{1}{M_1^2\left(M_1^2-1\right)}\right)\right]
\end{aligned}
\tag{B.18}
$$

B.2.2 The C integrals

Lets start with

$$
\begin{aligned}
C_a &= \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{k_2^4}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
&= \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{k_2^2(k_2^2+1-1)}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
&= \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{k_2^2}{(k_2^2+M_2^2)^2} \\
&- \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{k_2^2}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
&= \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{k_2^2+M_2^2-M_2^2}{(k_2^2+M_2^2)^2} \\
&- \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{k_2^2+1-1}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
&= \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{1}{(k_2^2+M_2^2)} \\
&- \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{M_2^2}{(k_2^2+M_2^2)^2} \\
&- \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\,\frac{1}{(k_2^2+M_2^2)^2}
\end{aligned}
$$

$$
\begin{aligned}
&+ \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \; \frac{1}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
&= \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \ln(\frac{k_2^2}{M_2^2}+1) \\
&- \frac{M_2^2+1}{M_2^2} \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \; \frac{1}{(k_2^2+M_2^2)^2(k_2^2+1)}
\end{aligned} \tag{B.19}
$$

$$
\begin{aligned}
&\int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \ln(\frac{k_2^2}{M_2^2}+1) \\
&= \frac{1}{M_1^2-1} \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+1)^2} \ln(\frac{k_2^2}{M_2^2}+1) \\
&+ \frac{-1}{M_1^2-1} \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)} \ln(\frac{k_2^2}{M_2^2}+1)
\end{aligned} \tag{B.20}
$$

The second piece is

$$
= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2}) \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} = \frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)} \tag{B.21}
$$

The first piece gives:

$$
\int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2}) \frac{1}{(k_1^2+1)^2} = \frac{\ln(M_2^2)}{(M_2^2-1)} \tag{B.22}
$$

so

$$
\begin{aligned}
&\int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \ln(\frac{k_2^2}{M_2^2}+1) \\
&= \frac{1}{M_1^2-1} \frac{\ln(M_2^2)}{(M_2^2-1)} \\
&+ \frac{-1}{M_1^2-1} \frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}
\end{aligned} \tag{B.23}
$$

Now

$$
\begin{aligned}
&\int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \; \frac{1}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
&= \frac{-1}{M_2^2-1} \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \; \frac{1}{(k_2^2+M_2^2)^2} \\
&+ \frac{1}{M_2^2-1} \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \; \frac{1}{(k_2^2+M_2^2)(k_2^2+1)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{-1}{M_2^2(M_2^2-1)}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\,\frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{1}{(M_2^2-1)^2}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\left[\ln(k_1^2+1)-\ln(\frac{k_1^2}{M_2^2}+1)\right]
\end{aligned}
\tag{B.24}
$$

Also

$$
= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_1^2)} = \frac{-Li_2(1-M_1^2)}{(M_1^2-1)} \tag{B.25}
$$

So

$$
\begin{aligned}
&\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\int_0^{k_1^2} dk_2^2\;\frac{1}{(k_2^2+M_2^2)^2(k_2^2+1)} \\
&= \frac{-1}{M_2^2(M_2^2-1)}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\,\frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{1}{(M_2^2-1)^2}\int_0^\infty dk_1^2\left[\frac{1}{M_1^2-1}\frac{1}{(k_1^2+1)^2}-\frac{1}{M_1^2-1}\frac{1}{(k_1^2+M_1^2)(k_1^2+1)}\right]\left[\ln(k_1^2+1)-\ln(\frac{k_1^2}{M_2^2}+1)\right] \\
&= \frac{-1}{M_2^2(M_2^2-1)}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\,\frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{1}{(M_2^2-1)^2}\left[\frac{1}{M_1^2-1}\left[1-\frac{\ln(M_2^2)}{(M_2^2-1)}\right]\right. \\
&- \left.\frac{1}{M_1^2-1}\left[\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}-\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right]\right]
\end{aligned}
\tag{B.26}
$$

SO

$$
\begin{aligned}
C_a &= \frac{1}{M_1^2-1}\frac{\ln(M_2^2)}{(M_2^2-1)} \\
&+ \frac{-1}{M_1^2-1}\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)} \\
&- \frac{M_2^2+1}{M_2^2}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\,\frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{-1}{M_2^2(M_2^2-1)}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\,\frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{1}{(M_2^2-1)^2}\left[\frac{1}{M_1^2-1}\left[1-\frac{\ln(M_2^2)}{(M_2^2-1)}\right]\right. \\
&\qquad\qquad -\left.\frac{1}{M_1^2-1}\left[\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}-\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right]\right]
\end{aligned}
\tag{B.27}
$$

The last piece to evaluate is:

$$
= \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\,\frac{k_1^2}{(k_1^2+M_2^2)}
$$

$$
\begin{aligned}
&= \frac{-1}{(M_1^2-1)(M_2^2-1)}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+1)^2} \\
&\quad+\frac{M_1^2M_2^2-1}{(M_1^2-1)^2(M_2^2-1)^2}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+1)} \\
&\quad+\frac{M_1^2}{(M_1^2-M_2^2)(M_1^2-1)^2}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)} \\
&\quad+\frac{M_2^2}{(M_2^2-M_1^2)(M_2^2-1)^2}\int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_2^2)} \\
&= \frac{-1}{(M_1^2-1)(M_2^2-1)}-\frac{M_1^2\ln(M_1^2)}{(M_1^2-M_2^2)(M_1^2-1)^2}-\frac{M_2^2\ln(M_2^2)}{(M_2^2-M_1^2)(M_2^2-1)^2}
\end{aligned}
\tag{B.28}
$$

Finally

$$
\begin{aligned}
C_a &= \frac{1}{M_1^2-1}\frac{\ln(M_2^2)}{(M_2^2-1)} \\
&+ \frac{-1}{M_1^2-1}\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)} \\
&- \frac{M_2^2}{M_2^2-1}\left[\frac{-1}{(M_1^2-1)(M_2^2-1)}-\frac{M_1^2\ln(M_1^2)}{(M_1^2-M_2^2)(M_1^2-1)^2}-\frac{M_2^2\ln(M_2^2)}{(M_2^2-M_1^2)(M_2^2-1)^2}\right] \\
&+ \frac{1}{(M_2^2-1)^2}\left[\frac{1}{M_1^2-1}\left[1-\frac{\ln(M_2^2)}{(M_2^2-1)}\right]\right. \\
&\qquad\qquad\left.-\frac{1}{M_1^2-1}\left[\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}-\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right]\right]
\end{aligned}
\tag{B.29}
$$

Now the second integral

$$
\begin{aligned}
C_b &= \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \; \frac{k_2^2}{(k_2^2+M_2^2)(k_2^2+1)} \\
&= \frac{-1}{M_2^2-1} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \left[\ln(k_1^2+1) - M_2^2 \ln(\frac{k_1^2}{M_2^2}+1)\right] \\
&= \frac{-1}{M_2^2-1} \frac{-1}{M_1^2-1} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^3} \left[\ln(k_1^2+1) - M_2^2 \ln(\frac{k_1^2}{M_2^2}+1)\right] \\
&+ \frac{-1}{M_2^2-1} \frac{M_1^2}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^2} \left[\ln(k_1^2+1) - M_2^2 \ln(\frac{k_1^2}{M_2^2}+1)\right] \\
&+ \frac{-1}{M_2^2-1} \frac{-M_1^2}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)} \left[\ln(k_1^2+1) - M_2^2 \ln(\frac{k_1^2}{M_2^2}+1)\right]
\end{aligned}
\tag{B.30}
$$

doing the integrals yields

$$
\begin{aligned}
1 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+1)^3} = \frac{1}{2}\left(\frac{-\ln(M_2^2)}{(1-M_2^2)^2} - \frac{1}{(1-M_2^2)}\right) \\
2 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+1)^3} = \frac{1}{4} \\
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+1)^2} = \frac{-\ln(M_2^2)}{(1-M_2^2)} \\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+1)^2} = 1 \\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_1^2)} = \frac{-Li_2(1-M_1^2)}{(M_1^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2}) \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} = \frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}
\end{aligned}
\tag{B.31}
$$

So what we have becomes:

$$
\begin{aligned}
C_b &= \frac{-1}{M_2^2-1} \frac{-1}{M_1^2-1} \left[\frac{1}{4} - M_2^2 \frac{1}{2}\left(\frac{-\ln(M_2^2)}{(1-M_2^2)^2} - \frac{1}{M_1^2\,(1-M_2^2)}\right)\right] \\
&+ \frac{-1}{M_2^2-1} \frac{M_1^2}{(M_1^2-1)^2} \left[1 - M_2^2 \frac{-\ln(M_2^2)}{(1-M_2^2)}\right] \\
&+ \frac{-1}{M_2^2-1} \frac{-M_1^2}{(M_1^2-1)^2} \left[\frac{-Li_2(1-M_1^2)}{(M_1^2-1)} - M_2^2 \frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right]
\end{aligned}
\tag{B.32}
$$

The third integral

$$
\begin{aligned}
C_c &= \int_0^\infty dk_1^2 \, \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \, \frac{-k_2^4}{(k_2^2+M_2^2)(k_2^2+1)} \\
&= \frac{-1}{M_2^2-1} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \left[\ln(k_1^2+1) - M_2^4 \ln(\frac{k_1^2}{M_2^2}+1) \right] \\
&- \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \\
&= \frac{-1}{M_2^2-1} \frac{1}{M_1^2-1} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^3} \left[\ln(k_1^2+1) - M_2^4 \ln(\frac{k_1^2}{M_2^2}+1) \right] \\
&+ \frac{-1}{M_2^2-1} \frac{-1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^2} \left[\ln(k_1^2+1) - M_2^4 \ln(\frac{k_1^2}{M_2^2}+1) \right] \\
&+ \frac{-1}{M_2^2-1} \frac{1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)} \left[\ln(k_1^2+1) - M_2^4 \ln(\frac{k_1^2}{M_2^2}+1) \right] \\
&- \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3}
\end{aligned}
\tag{B.33}
$$

Which looks similar to what we had before

$$
\begin{aligned}
C_c &= \frac{-1}{M_2^2-1} \frac{1}{M_1^2-1} \left[\frac{1}{4} - M_2^4 \frac{1}{2} \left(\frac{-\ln(M_2^2)}{\left(1-M_2^2\right)^2} - \frac{1}{(1-M_2^2)} \right) \right] \\
&+ \frac{-1}{M_2^2-1} \frac{-1}{(M_1^2-1)^2} \left[1 - M_2^4 \frac{-\ln(M_2^2)}{(1-M_2^2)} \right] \\
&+ \frac{-1}{M_2^2-1} \frac{1}{(M_1^2-1)^2} \left[\frac{-Li_2(1-M_1^2)}{(M_1^2-1)} - M_2^4 \frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)} \right] \\
&- \left[\frac{M_1^4-1}{2(M_1^2-1)^3} - \frac{M_1^2\ln(M_1^2)}{(M_1^2-1)^3} \right]
\end{aligned}
\tag{B.34}
$$

The fourth integral

$$
\begin{aligned}
C_d &= \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \ \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^2} \\
&= \frac{1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \ \left[\frac{M_1^4}{k_2^2+M_1^2} + \frac{1-2M_1^2}{k_2^2+1} + \frac{M_1^2-1}{(k_2^2+1)^2}\right] \\
&= \frac{1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \left[(1-2M_1^2)\ln(k_1^2+1) + M_1^4 \ln(\frac{k_1^2}{M_1^2}+1) + \frac{k_1^2(M_1^2-1)}{(k_1^2+1)}\right] \\
&= \frac{1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \left[(1-2M_1^2)\ln(k_1^2+1) + M_1^4 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \left[\frac{k_1^2(M_1^2-1)}{(k_1^2+1)}\right] \\
&= \frac{1}{(M_1^2-1)^2} \frac{1}{(M_2^2-1)} \int_0^\infty dk_1^2 \left[-\frac{1}{(k_1^2+M_2^2)^2} + \frac{1}{(k_1^2+M_2^2)(k_1^2+1)}\right] \\
&\qquad \times \left[(1-2M_1^2)\ln(k_1^2+1) + M_1^4 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)^2}
\end{aligned}
\tag{B.35}
$$

Using (in the B's calculation)

$$
\begin{aligned}
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_1^2})}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)} \\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)}{(M_2^2-1)} \\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\left(k_1^2+M_2^2\right)} = \frac{-Li_2(1-M_2^2)}{(M_2^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_1^2}) \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} = \frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}
\end{aligned}
\tag{B.36}
$$

$$
\begin{aligned}
C_d &= \frac{1}{(M_1^2-1)^2} \frac{1}{(M_2^2-1)} \int_0^\infty dk_1^2 \left[-\frac{1}{(k_1^2+M_2^2)^2}\right] \left[(1-2M_1^2)\ln(k_1^2+1) + M_1^4 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)^2} \frac{1}{(M_2^2-1)} \int_0^\infty dk_1^2 \left[\frac{1}{(k_1^2+M_2^2)(k_1^2+1)}\right] \left[(1-2M_1^2)\ln(k_1^2+1) + M_1^4 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)^2}
\end{aligned}
\tag{B.37}
$$

$$
C_d = \frac{-1}{(M_1^2-1)^2} \frac{1}{(M_2^2-1)} \left[(1-2M_1^2)\frac{\ln(M_2^2)}{(M_2^2-1)} + M_1^4 \frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right]
$$

$$
\begin{aligned}
&+ \quad \frac{1}{(M_1^2-1)^2}\frac{1}{(M_2^2-1)}\left[(1-2M_1^2)\frac{-Li_2(1-M_2^2)}{(M_2^2-1)} + M_1^4\frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right] \\
&+ \quad \frac{1}{(M_1^2-1)}\left[\frac{(M_2^2+1)\ln(M_2^2)}{(M_2^2-1)^3} + \frac{2(-M_2^2+1)}{(M_2^2-1)^3}\right]
\end{aligned}
\tag{B.38}
$$

The fifth integral

$$
\begin{aligned}
C_e &= \int_0^{\infty} dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \; \frac{-k_2^2}{(k_2^2+M_1^2)(k_2^2+1)^2} \\
&= \frac{1}{(M_1^2-1)^2} \int_0^{\infty} dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \; \left[\frac{M_1^2}{k_2^2+M_1^2} + \frac{-M_1^2}{k_2^2+1} + \frac{M_1^2-1}{(k_2^2+1)^2}\right] \\
&= \frac{1}{(M_1^2-1)^2} \int_0^{\infty} dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \left[(-M_1^2)\ln(k_1^2+1) + M_1^2 \ln(\frac{k_1^2}{M_1^2}+1) + \frac{k_1^2(M_1^2-1)}{(k_1^2+1)}\right] \\
&= \frac{1}{(M_1^2-1)^2} \int_0^{\infty} dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \left[(-M_1^2)\ln(k_1^2+1) + M_1^2 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)^2} \int_0^{\infty} dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \left[\frac{k_1^2(M_1^2-1)}{(k_1^2+1)}\right] \\
&= \frac{1}{(M_1^2-1)^2} \frac{1}{(M_2^2-1)} \int_0^{\infty} dk_1^2 \left[\frac{M_2^2}{(k_1^2+M_2^2)^2} - \frac{1}{(k_1^2+M_2^2)(k_1^2+1)}\right] \\
&\qquad \times \left[(-M_1^2)\ln(k_1^2+1) + M_1^2 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)} \int_0^{\infty} dk_1^2 \frac{k_1^4}{(k_1^2+M_2^2)^2(k_1^2+1)^2}
\end{aligned}
\tag{B.39}
$$

Using

$$
\begin{aligned}
3 &= \int_0^{\infty} dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_1^2})}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)} \\
4 &= \int_0^{\infty} dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)}{(M_2^2-1)} \\
5 &= \int_0^{\infty} dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_2^2)} = \frac{-Li_2(1-M_2^2)}{(M_2^2-1)} \\
6 &= \int_0^{\infty} dk_1^2 \ln(1+\frac{k_1^2}{M_1^2}) \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} = \frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}
\end{aligned}
\tag{B.40}
$$

$$
\begin{aligned}
C_e &= \frac{1}{(M_1^2-1)^2} \frac{1}{(M_2^2-1)} \int_0^{\infty} dk_1^2 \left[\frac{M_2^2}{(k_1^2+M_2^2)^2}\right] \left[(-M_1^2)\ln(k_1^2+1) + M_1^2 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)^2} \frac{1}{(M_2^2-1)} \int_0^{\infty} dk_1^2 \left[\frac{-1}{(k_1^2+M_2^2)(k_1^2+1)}\right] \left[(-M_1^2)\ln(k_1^2+1) + M_1^2 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)} \int_0^{\infty} dk_1^2 \frac{k_1^4}{(k_1^2+M_2^2)^2(k_1^2+1)^2}
\end{aligned}
\tag{B.41}
$$

$$
C_e = \frac{1}{(M_1^2-1)^2} \frac{M_2^2}{(M_2^2-1)} \left[(-M_1^2)\frac{\ln(M_2^2)}{(M_2^2-1)} + M_1^2 \frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right]
$$

$$
\begin{aligned}
+ &\quad \frac{-1}{(M_1^2-1)^2}\frac{1}{(M_2^2-1)}\left[(-M_1^2)\frac{-Li_2(1-M_2^2)}{(M_2^2-1)} + M_1^2\frac{Li_2\left(1-\dfrac{1}{M_1^2}\right) - Li_2\left(1-\dfrac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right] \\
+ &\quad \frac{1}{(M_1^2-1)}\left[\frac{(-2M_2^2)\ln(M_2^2)}{(M_2^2-1)^3} + \frac{(M_2^4-1)}{(M_2^2-1)^3}\right]
\end{aligned} \tag{B.42}
$$

The last one

$$\begin{aligned}
C_f &= \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \; \frac{-k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^3} \\
&= \frac{1}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \; \Bigg[\frac{M_1^4}{k_2^2+M_1^2} + \frac{-M_1^4}{k_2^2+1} + \frac{(2M_1^2-1)(M_1^2-1)}{(k_2^2+1)^2} \\
&\qquad + \frac{-(M_1^2-1)^2}{(k_2^2+1)^3}\Bigg] \\
&= \frac{1}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)} \Bigg[(-M_1^4)\ln(k_1^2+1) + M_1^4 \ln(\frac{k_1^2}{M_1^2}+1) \\
&\qquad + \frac{k_1^2(2M_1^2-1)(M_1^2-1)}{(k_1^2+1)} + \frac{-k_1^2(k_1^2+2)(M_1^2-1)^2}{2(k_2^2+1)^2}\Bigg] \\
&= \frac{1}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)} \left[(-M_1^4)\ln(k_1^2+1) + M_1^4 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{1}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)} \left[\frac{k_1^2(2M_1^2-1)(M_1^2-1)}{(k_1^2+1)}\right] \\
&+ \frac{1}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)} \left[\frac{-k_1^2(k_1^2+2)(M_1^2-1)^2}{2(k_2^2+1)^2}\right] \\
&= \frac{1}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)(k_1^2+1)} \left[(-M_1^4)\ln(k_1^2+1) + M_1^4 \ln(\frac{k_1^2}{M_1^2}+1)\right] \\
&+ \frac{(2M_1^2-1)}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)(k_1^2+1)^2} \\
&+ \frac{-1}{2(M_1^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2(k_1^2+2)}{(k_1^2+M_2^2)(k_1^2+1)^3}
\end{aligned} \tag{B.43}$$

Using (in the B's calculation)

$$\begin{aligned}
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_2^2)} = \frac{-Li_2(1-M_2^2)}{(M_2^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_1^2}) \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} = \frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}
\end{aligned} \tag{B.44}$$

$$\begin{aligned}
C_f &= \frac{1}{(M_1^2-1)^3}\left[(-M_1^4)\frac{-Li_2(1-M_2^2)}{(M_2^2-1)} + M_1^4 \frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right] \\
&+ \frac{(2M_1^2-1)}{(M_1^2-1)^2}\left[\frac{(M_2^2)\ln(M_2^2)}{(M_2^2-1)^2} - \frac{(M_2^2-1)}{(M_2^2-1)^2}\right] \\
&+ \frac{-1}{2(M_1^2-1)}\left[\frac{(M_2^4-2M_2^2)\ln(M_2^2)}{(M_2^2-1)^3} - \frac{(M_2^4-4M_2^2+3)}{2(M_2^2-1)^3}\right]
\end{aligned} \tag{B.45}$$

B.2.3 The D Integrals

The first integral

$$
\begin{aligned}
D_a &= \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+1)(k_2^2+M_2^2)^3} \\
&= \frac{M_2^6}{(M_2^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)^3} \\
&+ \frac{-2M_2^6+3M_2^4}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)^2} \\
&+ \frac{M_2^6-3M_2^4+3M_2^2}{(M_2^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)} \\
&+ \frac{-1}{(M_2^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)} \\
&= \frac{M_2^2}{2(M_2^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \frac{k_1^2(k_1^2+2M_2^2)}{(k_1^2+M_2^2)^2} \\
&+ \frac{-2M_2^4+3M_2^2}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{M_2^6-3M_2^4+3M_2^2}{(M_2^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \ln(\frac{k_1^2}{M_2^2}+1) \\
&+ \frac{-1}{(M_2^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \ln(k_1^2+1)
\end{aligned}
\tag{B.46}
$$

The second and third piece is done in the C's file

$$
\begin{aligned}
&= \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \frac{k_1^2}{(k_1^2+M_2^2)} \\
&= \frac{-1}{(M_1^2-1)(M_2^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^2} \\
&\quad + \frac{M_1^2M_2^2-1}{(M_1^2-1)^2(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)} \\
&\quad + \frac{M_1^2}{(M_1^2-M_2^2)(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)} \\
&\quad + \frac{M_2^2}{(M_2^2-M_1^2)(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)} \\
&= \frac{-1}{(M_1^2-1)(M_2^2-1)} - \frac{M_1^2\ln(M_1^2)}{(M_1^2-M_2^2)(M_1^2-1)^2} - \frac{M_2^2\ln(M_2^2)}{(M_2^2-M_1^2)(M_2^2-1)^2}
\end{aligned}
\tag{B.47}
$$

$$
\begin{aligned}
&\int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2} \ln(\frac{k_2^2}{M_2^2}+1) \\
&= \frac{1}{M_1^2-1} \frac{\ln(M_2^2)}{(M_2^2-1)}
\end{aligned}
$$

$$
+ \quad \frac{-1}{M_1^2-1}\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)} \tag{B.48}
$$

The fourth piece

$$
\begin{aligned}
& \int_0^\infty dk_1^2\,\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\ln(k_2^2+1) \\
= \quad & \frac{1}{M_1^2-1} \\
+ \quad & \frac{-1}{M_1^2-1}\frac{-Li_2\left(1-M_1^2\right)}{(M_1^2-1)}
\end{aligned} \tag{B.49}
$$

So all we need to evaluate is the first piece:

$$
\begin{aligned}
& \int_0^\infty dk_1^2\frac{1}{(k_1^2+M_1^2)(k_1^2+1)^2}\frac{k_1^2(k_1^2+2M_2^2)}{(k_1^2+M_2^2)^2} \\
= \quad & \int_0^\infty dk_1^2\frac{1}{(k_1^2+1)^2}\frac{1-2M_2^2}{(M_2^2-1)^2(M_1^2-1)} \\
& +\int_0^\infty dk_1^2\frac{1}{(k_1^2+1)}\frac{1-3M_2^2+2M_2^4M_1^2}{(M_2^2-1)^3(M_1^2-1)^2} \\
& +\int_0^\infty dk_1^2\frac{1}{(k_1^2+M_2^2)^2}\frac{M_2^4}{(M_2^2-1)^2(M_2^2-M_1^2)} \\
& +\int_0^\infty dk_1^2\frac{1}{(k_1^2+M_2^2)}\frac{M_2^4(3M_2^2-2M_1^2-1)}{(M_2^2-1)^3(M_2^2-M_1^2)^2} \\
& +\int_0^\infty dk_1^2\frac{1}{(k_1^2+M_1^2)}\frac{-M_1^2(2M_2^2-M_1^2)}{(M_1^2-1)^2(M_2^2-M_1^2)^2} \\
= \quad & \frac{1-2M_2^2}{(M_2^2-1)^2(M_1^2-1)}+\frac{M_2^2}{(M_2^2-1)^2(M_2^2-M_1^2)} \\
+ \quad & \frac{M_2^4(3M_2^2-2M_1^2-1)\ln(M_2^2)}{(M_2^2-1)^3(M_2^2-M_1^2)^2}+\frac{-M_1^2(2M_2^2-M_1^2)\ln(M_1^2)}{(M_1^2-1)^2(M_2^2-M_1^2)^2}
\end{aligned} \tag{B.50}
$$

$$
\begin{aligned}
& D_a \\
= \quad & \frac{M_2^2}{2(M_2^2-1)}\Bigg[\frac{1-2M_2^2}{(M_2^2-1)^2(M_1^2-1)}+\frac{M_2^2}{(M_2^2-1)^2(M_2^2-M_1^2)}-\frac{M_2^4(3M_2^2-2M_1^2-1)\ln(M_2^2)}{(M_2^2-1)^3(M_2^2-M_1^2)^2} \\
& \qquad -\frac{-M_1^2(2M_2^2-M_1^2)\ln(M_1^2)}{(M_1^2-1)^2(M_2^2-M_1^2)^2}\Bigg] \\
+ \quad & \frac{-2M_2^4+3M_2^2}{(M_2^2-1)^2}\left[\frac{-1}{(M_1^2-1)(M_2^2-1)}-\frac{M_1^2\ln(M_1^2)}{(M_1^2-M_2^2)(M_1^2-1)^2}-\frac{M_2^2\ln(M_2^2)}{(M_2^2-M_1^2)(M_2^2-1)^2}\right] \\
+ \quad & \frac{M_2^6-3M_2^4+3M_2^2}{(M_2^2-1)^3}\left[\frac{1}{M_1^2-1}\frac{\ln(M_2^2)}{(M_2^2-1)}+\frac{-1}{M_1^2-1}\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right]
\end{aligned}
$$

$$+ \quad \frac{-1}{(M_2^2-1)^3}\left[\frac{1}{M_1^2-1}+\frac{-1}{M_1^2-1}\frac{-Li_2\left(1-M_1^2\right)}{(M_1^2-1)}\right] \tag{B.51}$$

The second integral

$$
\begin{aligned}
D_b &= \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+1)(k_2^2+M_2^2)^2} \\
&= \frac{-M_2^4}{(M_2^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)^2} \\
&+ \frac{-2M_2^2+M_2^4}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)} \\
&+ \frac{1}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)} \\
&= \frac{-M_2^4}{(M_2^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{-2M_2^2+M_2^4}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \ln(\frac{k_1^2}{M_2^2}+1) \\
&+ \frac{1}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \ln(k_1^2+1) \\
&= \frac{-M_2^4}{(M_2^2-1)} \int_0^\infty dk_1^2 \Big[\frac{-M_1^2}{(M_1^2-1)} \frac{1}{(k_1^2+1)^3} + \frac{M_1^2}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)^2} \\
&\qquad + \frac{-M_1^2}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} \Big] \frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{-2M_2^2+M_2^4}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{-M_1^2}{(M_1^2-1)} \frac{1}{(k_1^2+1)^3} + \frac{M_1^2}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)^2} \\
&\qquad + \frac{-M_1^2}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} \Big] \ln(\frac{k_1^2}{M_2^2}+1) \\
&+ \frac{1}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{-M_1^2}{(M_1^2-1)} \frac{1}{(k_1^2+1)^3} + \frac{M_1^2}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)^2} \\
&\qquad + \frac{-M_1^2}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} \Big] \ln(k_1^2+1)
\end{aligned} \tag{B.52}
$$

The last two lines are done in the C integrals.

$$
\begin{aligned}
1 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+1)^3} = \frac{1}{2}\left(\frac{-\ln(M_2^2)}{(1-M_2^2)^2} - \frac{1}{M_1^2\,(1-M_2^2)} \right) \\
2 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+1)^3} = \frac{1}{4} \\
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+1)^2} = \frac{-\ln(M_2^2)}{(1-M_2^2)} \\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+1)^2} = 1 \\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_1^2)} = \frac{-Li_2(1-M_1^2)}{(M_1^2-1)}
\end{aligned}
$$

$$6 \;=\; \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2})\frac{1}{(k_1^2+1)(k_1^2+M_1^2)} = \frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)} \tag{B.53}$$

The first line integrals:

$$\begin{aligned}
7 \;=\;& \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^3}\frac{k_1^2}{(k_1^2+M_2^2)} \\
=& \int_0^\infty dk_1^2 \Big[\frac{-1}{(M_2^2-1)}\frac{1}{(k_1^2+1)^3}+\frac{M_2^2}{(M_2^2-1)^2}\frac{1}{(k_1^2+1)^2}+\frac{-M_2^2}{(M_2^2-1)^3}\frac{1}{(k_1^2+1)} \\
&+\frac{M_2^2}{(M_2^2-1)^3}\frac{1}{(k_1^2+M_2^2)}\Big] \\
=& \Big[\frac{-1}{(M_2^2-1)}\frac{1}{2}+\frac{M_2^2}{(M_2^2-1)^2}+\frac{M_2^2}{(M_2^2-1)^3}(-\ln(M_2^2))\Big] \\
8 \;=\;& \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^2}\frac{k_1^2}{(k_1^2+M_2^2)} \\
=& \int_0^\infty dk_1^2 \Big[\frac{-1}{(M_2^2-1)}\frac{1}{(k_1^2+1)^2}+\frac{M_2^2}{(M_2^2-1)^2}\frac{1}{(k_1^2+1)}+\frac{-M_2^2}{(M_2^2-1)^2}\frac{1}{(k_1^2+M_2^2)}\Big] \\
=& \Big[\frac{-1}{(M_2^2-1)}+\frac{-M_2^2}{(M_2^2-1)^2}(-\ln(M_2^2))\Big] \\
9 \;=\;& \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_1^2)}\frac{k_1^2}{(k_1^2+M_2^2)} \\
=& \int_0^\infty dk_1^2 \Big[\frac{-1}{(M_2^2-1)(M_1^2-1)}\frac{1}{(k_1^2+1)}+\frac{M_1^2}{(M_2^2-M_1^2)(M_1^2-1)}\frac{1}{(k_1^2+M_1^2)} \\
&+\frac{-M_2^2}{(M_2^2-M_1^2)(M_2^2-1)}\frac{1}{(k_1^2+M_2^2)}\Big] \\
=& \Big[\frac{M_1^2}{(M_2^2-M_1^2)(M_1^2-1)}(-\ln(M_1^2))+\frac{-M_2^2}{(M_2^2-M_1^2)(M_2^2-1)}(-\ln(M_2^2))\Big]
\end{aligned} \tag{B.54}$$

Now carefully adding everything together:

$$\begin{aligned}
D_b \;=\;& \frac{-M_2^2}{(M_2^2-1)}\Big[\frac{-M_1^2}{(M_1^2-1)}(7)+\frac{M_1^2}{(M_1^2-1)^2}(8)+\frac{-M_1^2}{(M_1^2-1)^2}(9)\Big] \\
+\;& \frac{-2M_2^2+M_2^4}{(M_2^2-1)^2}\Big[\frac{-M_1^2}{(M_1^2-1)}(1)+\frac{M_1^2}{(M_1^2-1)^2}(3)+\frac{-M_1^2}{(M_1^2-1)^2}(6)\Big] \\
+\;& \frac{1}{(M_2^2-1)^2}\Big[\frac{-M_1^2}{(M_1^2-1)}(2)+\frac{M_1^2}{(M_1^2-1)^2}(4)+\frac{-M_1^2}{(M_1^2-1)^2}(5)\Big]
\end{aligned} \tag{B.55}$$

$$\begin{aligned}
D_b \;=\;& \frac{-M_2^2}{(M_2^2-1)}\Big[\frac{-1}{(M_1^2-1)}\Big[\frac{-1}{(M_2^2-1)}\frac{1}{2}+\frac{M_2^2}{(M_2^2-1)^2}+\frac{M_2^2}{(M_2^2-1)^3}(-\ln(M_2^2))\Big] \\
&+\frac{M_1^2}{(M_1^2-1)^2}\Big[\frac{-1}{(M_2^2-1)}+\frac{-M_2^2}{(M_2^2-1)^2}(-\ln(M_2^2))\Big]
\end{aligned}$$

$$
\begin{aligned}
& +\frac{-M_1^2}{(M_1^2-1)^2}\left[\frac{M_1^2}{(M_2^2-M_1^2)(M_1^2-1)}(-\ln(M_1^2))+\frac{-M_2^2}{(M_2^2-M_1^2)(M_2^2-1)}(-\ln(M_2^2))\right]\Bigg] \\
+ \quad & \frac{-2M_2^2+M_2^4}{(M_2^2-1)^2}\left[\frac{-1}{(M_1^2-1)}\left[\frac{1}{2}\left(\frac{-\ln(M_2^2)}{(1-M_2^2)^2}-\frac{1}{M_1^2\,(1-M_2^2)}\right)\right]+\frac{M_1^2}{(M_1^2-1)^2}\left[\frac{-\ln(M_2^2)}{(1-M_2^2)}\right]\right. \\
& \left. +\frac{-M_1^2}{(M_1^2-1)^2}\left[\frac{Li_2\left(1-\frac{1}{M_2^2}\right)-Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right]\right] \\
+ \quad & \frac{1}{(M_2^2-1)^2}\left[\frac{-1}{(M_1^2-1)}\left[\frac{1}{4}\right]+\frac{M_1^2}{(M_1^2-1)^2}\left[1\right]+\frac{-M_1^2}{(M_1^2-1)^2}\left[\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}\right]\right]
\end{aligned}
\tag{B.56}
$$

The Third integral

$$
\begin{aligned}
D_c &= \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+1)(k_2^2+M_2^2)^2} \\
&= \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 (1) \\
&+ \frac{M_2^6}{(M_2^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)^2} \\
&+ \frac{-2M_2^6+3M_2^4}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)} \\
&+ \frac{-1}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)} \\
&= \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \\
&+ \frac{M_2^6}{(M_2^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{-2M_2^6+3M_2^4}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \ln(\frac{k_1^2}{M_2^2}+1) \\
&+ \frac{-1}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_1^2)(k_1^2+1)^3} \ln(k_1^2+1) \\
&= \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_1^2)(k_1^2+1)^3} \\
&+ \frac{M_2^6}{(M_2^2-1)} \int_0^\infty dk_1^2 \Bigg[\frac{1}{(M_1^2-1)} \frac{1}{(k_1^2+1)^3} + \frac{-1}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)^2} \\
&\qquad + \frac{1}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} \Bigg] \frac{k_1^2}{(k_1^2+M_2^2)} \\
&+ \frac{-2M_2^6+3M_2^4}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \Bigg[\frac{1}{(M_1^2-1)} \frac{1}{(k_1^2+1)^3} + \frac{-1}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)^2} \\
&\qquad + \frac{1}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} \Bigg] \ln(\frac{k_1^2}{M_2^2}+1) \\
&+ \frac{-1}{(M_2^2-1)^2} \int_0^\infty dk_1^2 \Bigg[\frac{1}{(M_1^2-1)} \frac{1}{(k_1^2+1)^3} + \frac{-1}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)^2} \\
&\qquad + \frac{1}{(M_1^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} \Bigg] \ln(k_1^2+1)
\end{aligned}
\tag{B.57}
$$

Except for the first line the remaining integrals are the same as the second integral which are:

$$
\begin{aligned}
1 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+1)^3} = \frac{1}{2}\left(\frac{-\ln(M_2^2)}{(1-M_2^2)^2} - \frac{1}{(1-M_2^2)} \right) \\
2 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+1)^3} = \frac{1}{4} \\
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+1)^2} = \frac{-\ln(M_2^2)}{(1-M_2^2)}
\end{aligned}
$$

$$\begin{aligned}
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{{(k_1^2+1)}^2} = 1 \\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_1^2)} = \frac{-Li_2(1-M_1^2)}{(M_1^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2})\frac{1}{(k_1^2+1)(k_1^2+M_1^2)} = \frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}
\end{aligned} \tag{B.58}$$

The Second line integrals:

$$\begin{aligned}
7 &= \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)^3}\frac{k_1^2}{(k_1^2+M_2^2)} \\
&= \int_0^\infty dk_1^2 \Bigg[\frac{-1}{(M_2^2-1)}\frac{1}{{(k_1^2+1)}^3} + \frac{M_2^2}{(M_2^2-1)^2}\frac{1}{{(k_1^2+1)}^2} + \frac{-M_2^2}{(M_2^2-1)^3}\frac{1}{(k_1^2+1)} \\
&\qquad + \frac{M_2^2}{(M_2^2-1)^3}\frac{1}{(k_1^2+M_2^2)}\Bigg] \\
&= \left[\frac{-1}{(M_2^2-1)}\frac{1}{2} + \frac{M_2^2}{(M_2^2-1)^2} + \frac{M_2^2}{(M_2^2-1)^3}(-\ln(M_2^2))\right] \\
8 &= \int_0^\infty dk_1^2 \frac{1}{{(k_1^2+1)}^2}\frac{k_1^2}{(k_1^2+M_2^2)} \\
&= \int_0^\infty dk_1^2 \left[\frac{-1}{(M_2^2-1)}\frac{1}{{(k_1^2+1)}^2} + \frac{M_2^2}{(M_2^2-1)^2}\frac{1}{(k_1^2+1)} + \frac{-M_2^2}{(M_2^2-1)^2}\frac{1}{(k_1^2+M_2^2)}\right] \\
&= \left[\frac{-1}{(M_2^2-1)} + \frac{-M_2^2}{(M_2^2-1)^2}(-\ln(M_2^2))\right] \\
9 &= \int_0^\infty dk_1^2 \frac{1}{(k_1^2+1)(k_1^2+M_1^2)}\frac{k_1^2}{(k_1^2+M_2^2)} \\
&= \int_0^\infty dk_1^2 \Bigg[\frac{-1}{(M_2^2-1)(M_1^2-1)}\frac{1}{(k_1^2+1)} + \frac{M_1^2}{(M_2^2-M_1^2)(M_1^2-1)}\frac{1}{(k_1^2+M_1^2)} \\
&\qquad + \frac{-M_2^2}{(M_2^2-M_1^2)(M_2^2-1)}\frac{1}{(k_1^2+M_2^2)}\Bigg] \\
&= \left[\frac{M_1^2}{(M_2^2-M_1^2)(M_1^2-1)}(-\ln(M_1^2)) + \frac{-M_2^2}{(M_2^2-M_1^2)(M_2^2-1)}(-\ln(M_2^2))\right]
\end{aligned} \tag{B.59}$$

The first line is just integral (7) after interchanging M_1 and M_2.

$$\begin{aligned}
10 &= \int_0^\infty dk_1^2 \frac{1}{{(k_1^2+1)}^3}\frac{k_1^2}{(k_1^2+M_1^2)} \\
&= \int_0^\infty dk_1^2 \Bigg[\frac{-1}{(M_1^2-1)}\frac{1}{{(k_1^2+1)}^3} + \frac{M_1^2}{(M_1^2-1)^2}\frac{1}{{(k_1^2+1)}^2} + \frac{-M_1^2}{(M_1^2-1)^3}\frac{1}{(k_1^2+1)} \\
&\qquad + \frac{M_1^2}{(M_1^2-1)^3}\frac{1}{(k_1^2+M_1^2)}\Bigg] \\
&= \left[\frac{-1}{(M_1^2-1)}\frac{1}{2} + \frac{M_1^2}{(M_1^2-1)^2} + \frac{M_1^2}{(M_1^2-1)^3}(-\ln(M_1^2))\right]
\end{aligned}$$

(B.60)

Now carefully adding everything together:

$$
\begin{aligned}
D_c \quad = \quad & (10) \\
+ \quad & \frac{M_2^6}{(M_2^2-1)}\left[\frac{1}{(M_1^2-1)}(7) + \frac{-1}{(M_1^2-1)^2}(8) + \frac{1}{(M_1^2-1)^2}(9)\right] \\
+ \quad & \frac{-2M_2^6+3M_2^4}{(M_2^2-1)^2}\left[\frac{1}{(M_1^2-1)}(1) + \frac{-1}{(M_1^2-1)^2}(3) + \frac{1}{(M_1^2-1)^2}(6)\right] \\
+ \quad & \frac{-1}{(M_2^2-1)^2}\left[\frac{1}{(M_1^2-1)}(2) + \frac{-1}{(M_1^2-1)^2}(4) + \frac{1}{(M_1^2-1)^2}(5)\right]
\end{aligned}
\tag{B.61}
$$

$$
\begin{aligned}
D_c \quad = \quad & \left[\frac{-1}{(M_1^2-1)}\frac{1}{2} + \frac{M_1^2}{(M_1^2-1)^2} + \frac{M_1^2}{(M_1^2-1)^3}(-\ln(M_1^2))\right] \\
+ \quad & \frac{M_2^4}{(M_2^2-1)}\left[\frac{1}{(M_1^2-1)}\left[\frac{-1}{(M_2^2-1)}\frac{1}{2} + \frac{M_2^2}{(M_2^2-1)^2} + \frac{M_2^2}{(M_2^2-1)^3}(-\ln(M_2^2))\right]\right. \\
& \qquad + \frac{-1}{(M_1^2-1)^2}\left[\frac{-1}{(M_2^2-1)} + \frac{-M_2^2}{(M_2^2-1)^2}(-\ln(M_2^2))\right] \\
& \qquad \left. + \frac{1}{(M_1^2-1)^2}\left[\frac{M_1^2}{(M_2^2-M_1^2)(M_1^2-1)}(-\ln(M_1^2)) + \frac{-M_2^2}{(M_2^2-M_1^2)(M_2^2-1)}(-\ln(M_2^2))\right]\right] \\
+ \quad & \frac{-2M_2^6+3M_2^4}{(M_2^2-1)^2}\left[\frac{1}{(M_1^2-1)}\left[\frac{1}{2}\left(\frac{-\ln(M_2^2)}{(1-M_2^2)^2} - \frac{1}{(1-M_2^2)}\right)\right] + \frac{-1}{(M_1^2-1)^2}\left[\frac{-\ln(M_2^2)}{(1-M_2^2)}\right]\right. \\
& \qquad \left. + \frac{1}{(M_1^2-1)^2}\left[\frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}\right]\right] \\
+ \quad & \frac{-1}{(M_2^2-1)^2}\left[\frac{1}{(M_1^2-1)}\left[\frac{1}{4}\right] + \frac{-1}{(M_1^2-1)^2}\left[1\right] + \frac{1}{(M_1^2-1)^2}\left[\frac{-Li_2(1-M_1^2)}{(M_1^2-1)}\right]\right]
\end{aligned}
\tag{B.62}
$$

The fourth integral

$$
\begin{aligned}
D_d &= \int_0^\infty dk_1^2 \, \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^2} \\
&= \frac{1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)^2} \\
&+ \frac{-2M_1^2+1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)} \\
&+ \frac{M_1^4}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_1^2)} \\
&= \frac{1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)} \frac{k_1^2}{(k_1^2+1)} \\
&+ \frac{-2M_1^2+1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)} \ln(k_1^2+1) \\
&+ \frac{M_1^4}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)} \ln(\frac{k_1^2}{M_1^2}+1) \\
&= \frac{1}{(M_1^2-1)} \int_0^\infty dk_1^2 \Big[\frac{M_2^4}{(M_2^2-1)^2} \frac{1}{(k_1^2+M_2^2)^3} + \frac{2M_2^2}{(M_2^2-1)^3} \frac{1}{(k_1^2+M_2^2)^2} + \frac{2M_2^2+1}{(M_2^2-1)^4} \frac{1}{(k_1^2+M_2^2)} \\
&\qquad + \frac{1}{(M_2^2-1)^3} \frac{1}{(k_1^2+1)^2} + \frac{-2M_2^2-1}{(M_2^2-1)^4} \frac{1}{(k_1^2+1)} \Big] \\
&+ \frac{-2M_1^2+1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{M_2^2}{(M_2^2-1)} \frac{1}{(k_1^2+M_2^2)^3} + \frac{1}{(M_2^2-1)^2} \frac{1}{(k_1^2+M_2^2)^2} \\
&\qquad + \frac{-1}{(M_2^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} \Big] \ln(k_1^2+1) \\
&+ \frac{M_1^4}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{M_2^2}{(M_2^2-1)} \frac{1}{(k_1^2+M_2^2)^3} + \frac{1}{(M_2^2-1)^2} \frac{1}{(k_1^2+M_2^2)^2} \\
&\qquad + \frac{-1}{(M_2^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} \Big] \ln(\frac{k_1^2}{M_1^2}+1)
\end{aligned}
\tag{B.63}
$$

As done before

$$
\begin{aligned}
1 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+M_1^2)^3} = \frac{1}{2}\left(\frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)^2} - \frac{1}{M_1^2\,(M_1^2-M_2^2)} \right) \\
2 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_1^2)^3} = \frac{1}{2}\left(\frac{\ln(M_1^2)}{(M_1^2-1)^2} - \frac{1}{M_1^2\,(M_1^2-1)} \right) \\
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_2^2})}{(k_1^2+M_1^2)^2} = \frac{\ln(M_1^2)-\ln(M_2^2)}{(M_1^2-M_2^2)} \\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_1^2)^2} = \frac{\ln(M_1^2)}{(M_1^2-1)} \\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_1^2)} = \frac{-Li_2(1-M_1^2)}{(M_1^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_2^2}) \frac{1}{(k_1^2+1)(k_1^2+M_1^2)} = \frac{Li_2\left(1-\frac{1}{M_2^2}\right) - Li_2\left(1-\frac{M_1^2}{M_2^2}\right)}{(M_1^2-1)}
\end{aligned}
$$

(B.64)

if we interchange M_1 and M_2 it becomes the integrals in the last two lines

$$\begin{aligned}
1 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_1^2})}{(k_1^2+M_2^2)^3} = \frac{1}{2}\left(\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)^2} - \frac{1}{M_2^2\,(M_2^2-M_1^2)}\right)\\
2 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_2^2)^3} = \frac{1}{2}\left(\frac{\ln(M_2^2)}{(M_2^2-1)^2} - \frac{1}{M_2^2\,(M_2^2-1)}\right)\\
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_1^2})}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)}{(M_2^2-1)}\\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_2^2)} = \frac{-Li_2(1-M_2^2)}{(M_2^2-1)}\\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_1^2})\frac{1}{(k_1^2+1)(k_1^2+M_2^2)} = \frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}
\end{aligned} \tag{B.65}$$

So our integral becomes

$$\begin{aligned}
D_d &= \frac{1}{(M_1^2-1)}\int_0^\infty dk_1^2\Bigg[\frac{M_2^4}{(M_2^2-1)^2}\frac{1}{(k_1^2+M_2^2)^3} + \frac{2M_2^2}{(M_2^2-1)^3}\frac{1}{(k_1^2+M_2^2)^2} + \frac{2M_2^2+1}{(M_2^2-1)^4}\frac{1}{(k_1^2+M_2^2)}\\
&\qquad + \frac{1}{(M_2^2-1)^3}\frac{1}{(k_1^2+1)^2} + \frac{-2M_2^2-1}{(M_2^2-1)^4}\frac{1}{(k_1^2+1)}\Bigg]\\
&+ \frac{-2M_1^2+1}{(M_1^2-1)^2}\left[\frac{M_2^2}{(M_2^2-1)}\left[2\right] + \frac{1}{(M_2^2-1)^2}\left[4\right] + \frac{-1}{(M_2^2-1)^2}\left[5\right]\right]\\
&+ \frac{M_1^4}{(M_1^2-1)^2}\left[\frac{M_2^2}{(M_2^2-1)}\left[1\right] + \frac{1}{(M_2^2-1)^2}\left[3\right] + \frac{-1}{(M_2^2-1)^2}\left[6\right]\right]
\end{aligned} \tag{B.66}$$

$$\begin{aligned}
D_d &= \frac{1}{(M_1^2-1)}\left[\frac{1}{2(M_2^2-1)^2} + \frac{2}{(M_2^2-1)^3} + \frac{2M_2^2+1}{(M_2^2-1)^4}(-\ln(M_2^2)) + \frac{1}{(M_2^2-1)^3}\right]\\
&+ \frac{-2M_1^2+1}{(M_1^2-1)^2}\left[\frac{M_2^2}{(M_2^2-1)}\left[\frac{1}{2}\left(\frac{\ln(M_2^2)}{(M_2^2-1)^2} - \frac{1}{M_2^2\,(M_2^2-1)}\right)\right] + \frac{1}{(M_2^2-1)^2}\left[\frac{\ln(M_2^2)}{(M_2^2-1)}\right]\right.\\
&\qquad \left. + \frac{-1}{(M_2^2-1)^2}\left[\frac{-Li_2(1-M_2^2)}{(M_2^2-1)}\right]\right]\\
&+ \frac{M_1^4}{(M_1^2-1)^2}\left[\frac{M_2^2}{(M_2^2-1)}\left[\frac{1}{2}\left(\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)^2} - \frac{1}{M_2^2\,(M_2^2-M_1^2)}\right)\right]\right.\\
&\qquad + \frac{1}{(M_2^2-1)^2}\left[\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right]\\
&\qquad \left. + \frac{-1}{(M_2^2-1)^2}\left[\frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right]\right]
\end{aligned} \tag{B.67}$$

The fifth integral

$$
\begin{aligned}
D_e &= \int_0^\infty dk_1^2 \; \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+1)^2} \\
&= \int_0^\infty dk_1^2 \; \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 (1) \\
&+ \frac{-1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)^2} \\
&+ \frac{3M_1^2-2}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)} \\
&+ \frac{-M_1^6}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_1^2)} \\
&= \int_0^\infty dk_1^2 \; \frac{k_1^2}{(k_1^2+M_2^2)^3(k_1^2+1)} \\
&+ \frac{-1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)} \frac{k_1^2}{(k_1^2+1)} \\
&+ \frac{3M_1^2-2}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)} \ln(k_1^2+1) \\
&+ \frac{-M_1^6}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^3(k_1^2+1)} \ln(\frac{k_1^2}{M_1^2}+1) \\
&= \int_0^\infty dk_1^2 \; \Big[\frac{M_2^2}{(M_2^2-1)} \frac{1}{(k_1^2+M_2^2)^3} + \frac{1}{(M_2^2-1)^2} \frac{1}{(k_1^2+M_2^2)^2} + \frac{1}{(M_2^2-1)^3} \frac{1}{(k_1^2+M_2^2)} \\
&\qquad + \frac{-1}{(M_2^2-1)^3} \frac{1}{(k_1^2+1)} \Big] \\
&+ \frac{-1}{(M_1^2-1)} \int_0^\infty dk_1^2 \Big[\frac{-M_2^2}{(M_2^2-1)^2} \frac{1}{(k_1^2+M_2^2)^3} + \frac{-2M_2^2-1}{(M_2^2-1)^3} \frac{1}{(k_1^2+M_2^2)^2} + \frac{-2M_2^2-2}{(M_2^2-1)^4} \frac{1}{(k_1^2+M_2^2)} \\
&\qquad + \frac{-1}{(M_2^2-1)^3} \frac{1}{(k_1^2+1)^2} + \frac{2M_2^2+2}{(M_2^2-1)^4} \frac{1}{(k_1^2+1)} \Big] \\
&+ \frac{3M_1^2-2}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{-1}{(M_2^2-1)} \frac{1}{(k_1^2+M_2^2)^3} + \frac{-1}{(M_2^2-1)^2} \frac{1}{(k_1^2+M_2^2)^2} \\
&\qquad + \frac{1}{(M_2^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} \Big] \ln(k_1^2+1) \\
&+ \frac{-M_1^6}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{-1}{(M_2^2-1)} \frac{1}{(k_1^2+M_2^2)^3} + \frac{-1}{(M_2^2-1)^2} \frac{1}{(k_1^2+M_2^2)^2} \\
&\qquad + \frac{1}{(M_2^2-1)^2} \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} \Big] \ln(\frac{k_1^2}{M_1^2}+1)
\end{aligned}
\tag{B.68}
$$

The integrals are the same as in D_d

$$
\begin{aligned}
1 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_1^2})}{(k_1^2+M_2^2)^3} = \frac{1}{2}\left(\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)^2} - \frac{1}{M_2^2\,(M_2^2-M_1^2)} \right) \\
2 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_2^2)^3} = \frac{1}{2}\left(\frac{\ln(M_2^2)}{(M_2^2-1)^2} - \frac{1}{M_2^2\,(M_2^2-1)} \right)
\end{aligned}
$$

$$
\begin{aligned}
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_1^2})}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)} \\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)}{(M_2^2-1)} \\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_2^2)} = \frac{-Li_2(1-M_2^2)}{(M_2^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_1^2})\frac{1}{(k_1^2+1)(k_1^2+M_2^2)} = \frac{Li_2\left(1-\frac{1}{M_1^2}\right)-Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}
\end{aligned}
\tag{B.69}
$$

So our integral becomes

$$
\begin{aligned}
D_e &= \int_0^\infty dk_1^2 \left[\frac{M_2^2}{(M_2^2-1)}\frac{1}{(k_1^2+M_2^2)^3}+\frac{1}{(M_2^2-1)^2}\frac{1}{(k_1^2+M_2^2)^2}+\frac{1}{(M_2^2-1)^3}\frac{1}{(k_1^2+M_2^2)}\right. \\
&\qquad \left.+\frac{-1}{(M_2^2-1)^3}\frac{1}{(k_1^2+1)}\right] \\
&+ \frac{-1}{(M_1^2-1)}\int_0^\infty dk_1^2\left[\frac{-M_2^2}{(M_2^2-1)^2}\frac{1}{(k_1^2+M_2^2)^3}+\frac{-2M_2^2-1}{(M_2^2-1)^3}\frac{1}{(k_1^2+M_2^2)^2}+\frac{-2M_2^2-2}{(M_2^2-1)^4}\frac{1}{(k_1^2+M_2^2)}\right. \\
&\qquad \left.+\frac{-1}{(M_2^2-1)^3}\frac{1}{(k_1^2+1)^2}+\frac{2M_2^2+2}{(M_2^2-1)^4}\frac{1}{(k_1^2+1)}\right] \\
&+ \frac{3M_1^2-2}{(M_1^2-1)^2}\left[\frac{-1}{(M_2^2-1)}\left[2\right]+\frac{-1}{(M_2^2-1)^2}\left[4\right]+\frac{1}{(M_2^2-1)^2}\left[5\right]\right] \\
&+ \frac{-M_1^6}{(M_1^2-1)^2}\left[\frac{-1}{(M_2^2-1)}\left[1\right]+\frac{-1}{(M_2^2-1)^2}\left[3\right]+\frac{1}{(M_2^2-1)^2}\left[6\right]\right]
\end{aligned}
\tag{B.70}
$$

$$
\begin{aligned}
D_e &= \left[\frac{1}{2M_2^2(M_2^2-1)}+\frac{1}{M_2^2(M_2^2-1)^2}+\frac{1}{(M_2^2-1)^3}(-\ln(M_2^2))\right] \\
&+ \frac{-1}{(M_1^2-1)}\left[\frac{-1}{2M_2^2(M_2^2-1)^2}+\frac{-2M_2^2-1}{M_2^2(M_2^2-1)^3}\frac{1}{(k_1^2+M_2^2)^2}+\frac{-2M_2^2-2}{(M_2^2-1)^4}(-\ln(M_2^2))+\frac{-1}{(M_2^2-1)^3}\right] \\
&+ \frac{3M_1^2-2}{(M_1^2-1)^2}\left[\frac{-1}{(M_2^2-1)}\left[\frac{1}{2}\left(\frac{\ln(M_2^2)}{(M_2^2-1)^2}-\frac{1}{M_2^2\,(M_2^2-1)}\right)\right]+\frac{-1}{(M_2^2-1)^2}\left[\frac{\ln(M_2^2)}{(M_2^2-1)}\right]\right. \\
&\qquad \left.+\frac{1}{(M_2^2-1)^2}\left[\frac{-Li_2(1-M_2^2)}{(M_2^2-1)}\right]\right] \\
&+ \frac{-M_1^6}{(M_1^2-1)^2}\left[\frac{-1}{(M_2^2-1)}\left[\frac{1}{2}\left(\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)^2}-\frac{1}{M_2^2\,(M_2^2-M_1^2)}\right)\right]\right. \\
&\qquad \left.+\frac{-1}{(M_2^2-1)^2}\left[\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right]+\frac{1}{(M_2^2-1)^2}\left[\frac{Li_2\left(1-\frac{1}{M_1^2}\right)-Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right]\right]
\end{aligned}
\tag{B.71}
$$

The sixth Integral

$$
\begin{aligned}
D_f &= \int_0^\infty dk_1^2 \, \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{k_2^4}{(k_2^2+M_1^2)(k_2^2+1)^3} \\
&= \frac{1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)^3} \\
&+ \frac{-2M_1^2+1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)^2} \\
&+ \frac{M_1^4}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)} \\
&+ \frac{-M_1^4}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)} \\
&= \frac{1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \frac{k_1^2(k_1^2+2)}{2(k_1^2+1)^2} \\
&+ \frac{-2M_1^2+1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \frac{k_1^2}{(k_1^2+1)} \\
&+ \frac{M_1^4}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \ln(k_1^2+1) \\
&+ \frac{-M_1^4}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{k_1^2}{(k_1^2+M_2^2)^2(k_1^2+1)} \ln(\frac{k_1^2}{M_1^2}+1) \\
&= \frac{1}{(M_1^2-1)} \int_0^\infty dk_1^2 \Bigg[\frac{1}{2(M_2^2-1)^2} \frac{1}{(k_1^2+1)^3} + \frac{-M_2^2-1}{2(M_2^2-1)^3} \frac{1}{(k_1^2+1)^2} + \frac{-M_2^4+4M_2^2}{2(M_2^2-1)^4} \frac{1}{(k_1^2+1)} \\
&\qquad + \frac{M_2^2(M_2^2-2)}{2(M_2^2-1)^3} \frac{1}{(k_1^2+M_2^2)^2} + \frac{M_2^4-4M_2^2}{2(M_2^2-1)^4} \frac{1}{(k_1^2+M_2^2)} \Bigg] \\
&+ \frac{-2M_1^2+1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \Bigg[\frac{M_2^4}{(M_2^2-1)^2} \frac{1}{(k_1^2+M_2^2)^2} + \frac{2M_2^2}{(M_2^2-1)^3} \frac{1}{(k_1^2+M_2^2)} + \frac{1}{(M_2^2-1)^2} \frac{1}{(k_1^2+1)^2} \\
&\qquad + \frac{-2M_2^2}{(M_2^2-1)^3} \frac{1}{(k_1^2+1)} \Bigg] \\
&+ \frac{M_1^4}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \Bigg[\frac{M_2^2}{(M_2^2-1)} \frac{1}{(k_1^2+M_2^2)^2} + \frac{-1}{(M_2^2-1)} \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} \Bigg] \ln(k_1^2+1) \\
&+ \frac{-M_1^4}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \Bigg[\frac{M_2^2}{(M_2^2-1)} \frac{1}{(k_1^2+M_2^2)^2} + \frac{-1}{(M_2^2-1)} \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} \Bigg] \ln(\frac{k_1^2}{M_1^2}+1)
\end{aligned}
\tag{B.72}
$$

The last two lines are the same integrals from before

$$
\begin{aligned}
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_1^2})}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)} \\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)}{(M_2^2-1)} \\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\,(k_1^2+M_2^2)} = \frac{-Li_2(1-M_2^2)}{(M_2^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_1^2}) \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} = \frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}
\end{aligned}
$$

(B.73)

So our integral becomes

$$
\begin{aligned}
D_f \quad &= \quad \frac{1}{(M_1^2-1)}\left[\frac{1}{4(M_2^2-1)^2}+\frac{-M_2^2-1}{2(M_2^2-1)^3}+\frac{(M_2^2-2)}{2(M_2^2-1)^3}+\frac{M_2^4-4M_2^2}{2(M_2^2-1)^4}(-\ln(M_2^2))\right] \\
&+ \quad \frac{-2M_1^2+1}{(M_1^2-1)^2}\left[\frac{M_2^2}{(M_2^2-1)^2}+\frac{2M_2^2}{(M_2^2-1)^3}(-\ln(M_2^2))+\frac{1}{(M_2^2-1)^2}\right] \\
&+ \quad \frac{M_1^4}{(M_1^2-1)^3}\left[\frac{M_2^2}{(M_2^2-1)}\left[\frac{\ln(M_2^2)}{(M_2^2-1)}\right]+\frac{-1}{(M_2^2-1)}\left[\frac{-Li_2(1-M_2^2)}{(M_2^2-1)}\right]\right] \\
&+ \quad \frac{-M_1^4}{(M_1^2-1)^3}\left[\frac{M_2^2}{(M_2^2-1)}\left[\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right]+\frac{-1}{(M_2^2-1)}\left[\frac{Li_2\left(1-\frac{1}{M_1^2}\right)-Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right]\right]
\end{aligned}
\tag{B.74}
$$

The last integral:

$$\begin{aligned}
D_g &= \int_0^\infty dk_1^2 \; \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{k_2^6}{(k_2^2+M_1^2)(k_2^2+1)^3} \\
&= \frac{-1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)^3} \\
&+ \frac{3M_1^2-2}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)^2} \\
&+ \frac{-3M_1^4+3M_1^2-1}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+1)} \\
&+ \frac{M_1^6}{(M_1^2-1)^3} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \int_0^{k_1^2} dk_2^2 \frac{1}{(k_2^2+M_2^2)} \\
&= \frac{-1}{(M_1^2-1)} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \frac{k_1^2(k_1^2+2)}{2(k_1^2+1)^2} \\
&+ \frac{3M_1^2-2}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \frac{k_1^2}{(k_1^2+1)} \\
&+ \frac{-3M_1^4+3M_1^2-1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \ln(k_1^2+1) \\
&+ \frac{M_1^6}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \frac{1}{(k_1^2+M_2^2)^2(k_1^2+1)} \ln(\frac{k_1^2}{M_1^2}+1) \\
&= \frac{-1}{(M_1^2-1)} \int_0^\infty dk_1^2 \Big[\frac{-1}{2(M_2^2-1)^2}\frac{1}{(k_1^2+1)^3} + \frac{1}{(M_2^2-1)^3}\frac{1}{(k_1^2+1)^2} + \frac{M_2^4-2M_2^2-2}{2(M_2^2-1)^4}\frac{1}{(k_1^2+1)} \\
&\qquad + \frac{-M_2^2(M_2^2-2)}{2(M_2^2-1)^3}\frac{1}{(k_1^2+M_2^2)^2} + \frac{-M_2^4+2M_2^2+2}{2(M_2^2-1)^4}\frac{1}{(k_1^2+M_2^2)} \Big] \\
&+ \frac{3M_1^2-2}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{-M_2^2}{(M_2^2-1)^2}\frac{1}{(k_1^2+M_2^2)^2} + \frac{-M_2^2-1}{(M_2^2-1)^3}\frac{1}{(k_1^2+M_2^2)} + \frac{-1}{(M_2^2-1)^2}\frac{1}{(k_1^2+1)^2} \\
&\qquad + \frac{M_2^2+1}{(M_2^2-1)^3}\frac{1}{(k_1^2+1)} \Big] \\
&+ \frac{-3M_1^4+3M_1^2-1}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{-1}{(M_2^2-1)}\frac{1}{(k_1^2+M_2^2)^2} + \frac{1}{(M_2^2-1)}\frac{1}{(k_1^2+1)(k_1^2+M_2^2)} \Big] \ln(k_1^2+1) \\
&+ \frac{M_1^6}{(M_1^2-1)^2} \int_0^\infty dk_1^2 \Big[\frac{-1}{(M_2^2-1)}\frac{1}{(k_1^2+M_2^2)^2} + \frac{1}{(M_2^2-1)}\frac{1}{(k_1^2+1)(k_1^2+M_2^2)} \Big] \ln(\frac{k_1^2}{M_1^2}+1)
\end{aligned} \tag{B.75}$$

The last two lines are the same integrals from before

$$\begin{aligned}
3 &= \int_0^\infty dk_1^2 \frac{\ln(1+\frac{k_1^2}{M_1^2})}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)} \\
4 &= \int_0^\infty dk_1^2 \frac{\ln(1+k_1^2)}{(k_1^2+M_2^2)^2} = \frac{\ln(M_2^2)}{(M_2^2-1)} \\
5 &= \int_0^\infty dk_1^2 \frac{\ln\left(1+k_1^2\right)}{(k_1^2+1)\left(k_1^2+M_2^2\right)} = \frac{-Li_2(1-M_2^2)}{(M_2^2-1)} \\
6 &= \int_0^\infty dk_1^2 \ln(1+\frac{k_1^2}{M_1^2}) \frac{1}{(k_1^2+1)(k_1^2+M_2^2)} = \frac{Li_2\left(1-\frac{1}{M_1^2}\right) - Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}
\end{aligned} \tag{B.76}$$

So our integral becomes

$$
\begin{aligned}
D_g \quad = \quad & \frac{-1}{(M_1^2-1)}\left[\frac{-1}{4(M_2^2-1)^2}+\frac{1}{(M_2^2-1)^3}+\frac{-(M_2^2-2)}{2(M_2^2-1)^3}+\frac{-M_2^4+2M_2^2+2}{2(M_2^2-1)^4}(-\ln(M_2^2))\right] \\
+ \quad & \frac{3M_1^2-2}{(M_1^2-1)^2}\left[\frac{-1}{(M_2^2-1)^2}+\frac{-M_2^2-1}{(M_2^2-1)^3}(-\ln(M_2^2))+\frac{-1}{(M_2^2-1)^2}\right] \\
+ \quad & \frac{-3M_1^4+3M_2^2-1}{(M_1^2-1)^3}\left[\frac{-1}{(M_2^2-1)}\left[\frac{\ln(M_2^2)}{(M_2^2-1)}\right]+\frac{1}{(M_2^2-1)}\left[\frac{-Li_2(1-M_2^2)}{(M_2^2-1)}\right]\right] \\
+ \quad & \frac{M_1^6}{(M_1^2-1)^3}\left[\frac{-1}{(M_2^2-1)}\left[\frac{\ln(M_2^2)-\ln(M_1^2)}{(M_2^2-M_1^2)}\right]+\frac{1}{(M_2^2-1)}\left[\frac{Li_2\left(1-\frac{1}{M_1^2}\right)-Li_2\left(1-\frac{M_2^2}{M_1^2}\right)}{(M_2^2-1)}\right]\right]
\end{aligned}
\tag{B.77}
$$