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LIQUIDITY AS A LATENT VARIABLE – AN APPLICATION OF THE MIMIC MODEL

*Aris Spanos**

I. INTRODUCTION

Since the late 1960's mainly because of the revival of interest in monetary policy, the question of 'how to measure money' has become of considerable importance. An important implication of this has been a renewed interest in the role of financial assets, broadly categorized as near-monies, at the theoretical and policy levels. The issue was first raised by Gurley and Shaw (1953) who suggested a broadening of the 'measure of money' to include interest-bearing financial assets because such assets possess 'moneyness' in varying degree. The Radcliffe Report (1959) defined this abstract property possessed by other financial assets apart from Currency and Demand Deposits (CDD) as 'liquidity'. At the policy level the Radcliffe Report suggested that the effectiveness of a monetary policy based on controlling CDD might be impaired because of the existence of other financial assets contributing to the 'liquidity' of an economy. For monetary policy to be effective it should influence the general 'liquidity' of an economy. The suggestion to broaden the measure of money was taken up by Friedman and Meiselman (1963) in their discussion of the effectiveness of monetary policy by including time deposits (TD) in commercial banks together with CDD.

At the theoretical level the Gurley and Shaw suggestion for a more comprehensive monetary theory which considers the role of near monies in relation to the general liquidity of an economy was taken up by mainstream monetary theory (see Tobin (1958), Tobin and Brainard (1968), Hicks (1967), *inter alia*). As far as the measurement of money was concerned, Chetty (1969) used a utility maximization model for financial assets to derive an even broader definition of money which includes not only CDD and TD but deposits in mutual savings banks and savings and loan association shares as well. The concept of money was defined to be a weighted average of these 'liquid' assets with the

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weights being related to the substitution parameters. For a critique and extension of Chetty's results see Boughton (1981). This approach to measuring money can be described as a *multiple-indicators* (MI) approach where the various financial assets are used as indicators of 'liquidity' leading to an empirical definition of 'liquidity' as an estimated weighted average. This multiple-indicators approach has been extended by Barnett (1980) in a way which avoids estimating the weights by using statistical index numbers. As a measure of money he suggested the Tornquist–Theil Divisia index, which defines the rate of change of money as a weighted average of the changes of the 'liquid' assets with the weights related to the user cost price of these assets. For an application of the Tornquist–Theil Divisia index to the UK monetary aggregate see Mills (1983).

An alternative approach to deriving an empirical definition of money, which can be described as a *multiple-cause* (MC) approach, was proposed by Laidler (1969). This approach amounts to defining money by relating it to some 'key' causal variables such as nominal income; see also Kaufman (1969), Laumas (1968), *inter alia* and for an excellent survey of these results see Feige and Pearce (1977). The approach has been extended by Mills (1983), who emphasized the dynamics of such a relationship between money and income in an ARIMA framework.

One thing upon which all these studies related to the definition of money agree is that this definition should be extended to include all 'liquid' assets in ways which reflect their relative liquidity. A natural way to proceed in view of the above two broad approaches is to combine them in a multiple-indicator multiple-cause (MIMIC) framework, in which both indicators and causes are used. Given that what the financial assets have in common is liquidity in varying degrees, the modelling will be centred on liquidity as a latent variable. A latent variable is defined to be any theoretical variable which does not correspond one-to-one to a particular observed data series. A famous example of a latent variable is 'trade union pushfulness' (see Hines (1964)). In a MIMIC framework we could quantify such a latent variable using observable variables such as 'the number of strikes' and 'relative wage increases' as indicators and 'the number of trade union members' and unemployment as 'causes'.

The MIMIC framework enables us to quantify such variables by relating them jointly to their indicators and 'causes'. The MIMIC model also can be interpreted as providing a measure for the latent variable using a (canonical) correlation type of analysis between the two sets of observable random variables corresponding to indicators and causes. It constitutes a systematization of some of the attempts, discussed above, to quantify latent variables which consider only indicators or 'causes' (these last being interpreted as proximate determinants rather than 'fundamental causes').

II. THE MIMIC MODEL-SPECIFICATION

Let the latent variable be denoted by ξ_t, the indicators by m variables $\boldsymbol{y}_t$, and the 'causes' by $\boldsymbol{z}_t$, $k \times 1$. The MIMIC model consists of two sets of relationships. Since ξ_t is a scalar, the first set becomes a single behavioural equation assumed to take the form:

$$\xi_t = \boldsymbol{\beta}'\boldsymbol{z}_t + \epsilon_t \qquad \epsilon_t \sim \mathrm{IN}(0, \sigma_\epsilon^2) \tag{1}$$

where $E(\boldsymbol{z}_t\epsilon_t) = 0$, $t = 1, 2, \ldots, T$, and 'IN' stands for 'independent normal'. The second system of equations, referred to as measurement equations, take the form:

$$\boldsymbol{y}_t = \boldsymbol{\lambda}\xi_t + \boldsymbol{v}_t \qquad \boldsymbol{v}_t \sim N(0, \boldsymbol{\Sigma}), t = 1, 2, \ldots, T \tag{2}$$

where $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_m^2)$* and $E(\boldsymbol{v}_t\epsilon_t) = \mathbf{0}$. The parameters of interest in this model come in the form of the unknown parameters $(\boldsymbol{\beta}, \boldsymbol{\lambda}, \boldsymbol{\Sigma}, \sigma_\epsilon^2)$, which are $k + 2m + 1$ in number. The reduced form in terms of the observables is:

$$\boldsymbol{y}_t = \boldsymbol{\Pi}'\boldsymbol{z}_t + \boldsymbol{u}_t \tag{3}$$

where

$$\boldsymbol{\Pi} = \boldsymbol{\beta}\boldsymbol{\lambda}' \quad \text{and} \quad E(\boldsymbol{u}_t\boldsymbol{u}_t') \equiv \boldsymbol{\Omega} = \boldsymbol{\lambda}\boldsymbol{\lambda}'\sigma_\epsilon^2 + \boldsymbol{\Sigma} \tag{4}$$

Looking at (3) and (4) we can see that the model determines $\boldsymbol{\lambda}$ up to a scalar multiple because when $\boldsymbol{\lambda}$ is multiplied by a scalar c and $\boldsymbol{\beta}$ and σ_ϵ divided by the same scalar, $\boldsymbol{\Pi}$ and $\boldsymbol{\Omega}$ remain unchanged (see Goldberger (1974)). To remove this indeterminancy we need to adopt some normalization rule. Various normalization rules have been suggested in the literature, such as $\sigma_\epsilon^2 = 1$, $\lambda_i = 1$ for some $i = 1, 2, \ldots, m$,

$$\sum_{i=1}^{m} \lambda_i = 1, \quad \sum_{i=1}^{m} \frac{\lambda_i^2}{\sigma_i^2} = 1$$

The normalization $\sigma_\epsilon^2 = 1$ will be adopted in Section 4 for expositional purposes in order to bring out the role of:

$$\sum_{i=1}^{m} \frac{\lambda_i^2}{\sigma_i^2} = 1$$

in the 'estimation' of ξ_t.

* For a discussion of this assumption see Kalman (1982).

Before we consider the identification and estimation of the MIMIC model it is important to understand the structure of the model. Equation (1) is proposed as a behavioural equation for the latent variable ξ_t but (2) refers to a particular way ξ_t might be measured. Equation (2) as a measurement system for ξ_t constitutes a generalization of the errors-in-variables measurement equation:

$$x_t = \xi_t + e_t \qquad E(\xi_t e_t) = 0,\ t = 1, 2, \ldots, T \tag{5}$$

which is used in the proxy variables literature (see Maddala (1977), *inter alia*). Thus, instead of postulating one observable proxy (indicator) for ξ_t we postulate m of them in the form of (2), i.e.:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \\ \cdot \\ \cdot \\ \cdot \\ y_{mt} \end{pmatrix} = \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \cdot \\ \cdot \\ \cdot \\ \lambda_m \end{pmatrix} \xi_t + \begin{pmatrix} v_{1t} \\ v_{2t} \\ \cdot \\ \cdot \\ \cdot \\ v_{mt} \end{pmatrix} \tag{2}$$

Logically (1) and (2) are distinct propositions which are put together in (3) to generate the *a priori* non-linear restrictions in (4). Looking at these restrictions more closely we can see that the first set of restrictions, called *econometric-type* restrictions by Joreskog and Goldberger (1975), are:

$$\pi_{ij} = \beta_i \lambda_j \qquad i = 1, 2, \ldots, k, j = 1, 2, \ldots, m \tag{6}$$

Thus, all the coefficients of a given variable in the reduced form equations must not only have the same or opposite signs depending on the sign of the indicator coefficient (λ_i) but must also differ only by a factor of proportionality as determined by the value of λ_i. This should not be interpreted as suggesting that the individual indicators behave as if their behaviour differs only by a proportionality factor because (2) is nothing but a measurement equation. We should interpret (6) as suggesting that what the indicators have in common is ξ_t, and what we are trying to do in (1) is to model the behaviour of this common attribute. In the case of 'liquidity' the indicators ($\mathbf{y}_t$) come in the form of the various financial assets considered as near-monies, and their common attribute is liquidity (ξ_t). This implies that the extent to which we manage to capture the behaviour of ξ_t depends crucially on the form of (1) in view of the indicators $\mathbf{y}_t$. As argued below, specifying (1) appropriately in an attempt to capture the common behaviour of the indicators $\mathbf{y}_t$, is not easy and it is not usually a matter of *a priori* information only.

The second set of restrictions, usually called *factor-analytic* type restrictions are:

$$\omega_{ij} = \begin{cases} \lambda_i + \sigma_i^2 & i = 1, 2, \ldots, m \\ \lambda_i \lambda_j & i \neq j; i, j = 1, 2, \ldots, m \end{cases} \tag{7}$$

These restrictions are commonly encountered in factor analysis (see Lawley and Maxwell (1971)). They enhance the informational content of the MIMIC model by relating the coefficient parameters to the co-variance of the reduced form error. In econometrics, *a priori* information is usually introduced into the statistical model in the form of restrictions on the coefficients defining (1).

III. THE STATE-SPACE FORMULATION

The MIMIC model can be viewed as a very special case of a general latent variables modelling scheme, widely used in engineering, the so-called state-space model. The state-space model takes the general form:

$$\boldsymbol{\xi}_t = \boldsymbol{A}_t\boldsymbol{\xi}_{t-1} + \boldsymbol{B}_t\boldsymbol{z}_t + \boldsymbol{G}_t\boldsymbol{\epsilon}_t \tag{8}$$

$$\boldsymbol{y}_t = \boldsymbol{\Lambda}_t\boldsymbol{\xi}_t + \boldsymbol{v}_t \tag{9}$$

$$\begin{pmatrix}\boldsymbol{\epsilon}_t\\ \boldsymbol{v}_t\end{pmatrix} \sim N\left(\begin{pmatrix}0\\0\end{pmatrix}\begin{pmatrix}\boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12}\\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22}\end{pmatrix}\right) \tag{10}$$

where $\boldsymbol{\xi}_t$ is an $n \times 1$ vector of latent variables, $\boldsymbol{z}_t$ is a $k \times 1$ vector of observable extraneous variables, $\boldsymbol{y}_t$ is an $m \times 1$ vector of observable indicators of $\boldsymbol{\xi}_t$ and $\boldsymbol{A}_t$, $\boldsymbol{B}_t$, $\boldsymbol{G}_t$ and $\boldsymbol{\Lambda}_t$ are coefficient matrices of order $n \times n$, $n \times k$, $n \times l$ and $m \times n$, respectively. The system of equations in (8) is referred to as the set of *transition equations* and can be interpreted as behavioural equations for the latent vector $\boldsymbol{\xi}_t$. The equations in (9) are referred to as *measurement equations*; they purport to provide a way of 'measuring' $\boldsymbol{\xi}_t$ in terms of the observable indicators in $\boldsymbol{y}_t$.

The MIMIC model (1) and (2) is a special case of (8)–(10) with $n = 1, \boldsymbol{A}_t = 0, \boldsymbol{G}_t = \boldsymbol{I}, \boldsymbol{\Sigma}_{12} = 0, \boldsymbol{\Sigma}_{22} = \text{diag}(\sigma_1^2, \ldots, \sigma_m^2), \boldsymbol{\Lambda}_t = \boldsymbol{\lambda}, t = 1, 2, \ldots, T$.

In engineering applications the parameters in $(\boldsymbol{A}_t, \boldsymbol{B}_t, \boldsymbol{G}, \boldsymbol{\Lambda}_t, \boldsymbol{\Sigma}_{11}, \boldsymbol{\Sigma}_{12}, \boldsymbol{\Sigma}_{22})$ are usually known and the problem is seen as one of 'estimating' (predicting) the latent vector $\boldsymbol{\xi}_t$ given the information on $\boldsymbol{y}_t$ up to $t-1$ (technically, given the σ-field generated by $(y_{t-i}, i = 1, 2, \ldots, t-1)$, denoted by Y_{t-1}^0). The problem posed in the above form was 'solved' by Kalman (1960) who also suggested the state-space formulation (8)–(10). The solution was in the form of a recursive system of updating equations known as the *Kalman filter*. The basic idea underlying the Kalman filter is that 'the conditional expectation represents the best (in the Mean Square Error sense) linear predictor'. Let $\hat{X}_{t/t-h}$ represent the conditional expectation of X_t given information up to $t-h$. The information set at time $t-1$ for the state-space model (8)–(10) comprises the σ-field generated by the sequence of random variables y_{t-i}, $i = 1$, 2, ..., t denoted by $Y_{t-1}^0 = \sigma(y_{t-i},\ i = 1, \ldots, t)$ (see Spanos (1984)) as well as z_t, assumed to be a known sequence. The reason we use the σ-field Y_{t-1}^0 and not the past observed values of y_t as the appropriate information set is that the stochastic structure of the sequence $\{y_{t-i}, i = 1, \ldots, t\}$ is of primary importance. From (8), the conditional expectation of ξ_t at time $t-1$ takes the form:

$$\hat{\xi}_{t/t-1} = A_t \hat{\xi}_{t-1/t-1} + B_t z_t$$

and the prediction error is $\eta_t = \xi_t - \hat{\xi}_{t/t-1}$.

The conditional expectation of y_t from (9) is:

$$\hat{y}_{t/t-1} = \Lambda_t \hat{\xi}_{t/t-1}$$

and the prediction error is $e_t = y_t - \hat{y}_{t/t-1}$.

A revised 'predictor' (conditional expectation) of ξ_t at time t, denoted by $\hat{\xi}_{t/t}$, can be constructed by combining the information at $t-1$ and the 'new' information at t in the updating equation:

$$\hat{\xi}_{t/t} = \hat{\xi}_{t/t-1} + R_t e_t$$

where $R_t = [E(\eta_t e_t' | Y_{t-1}^0)][E(e_t e_t' | Y_{t-1}^0)]^{-1}$ is known as the *gain* matrix; for the formula of R_t see the Appendix. The gain matrix provides the weight for the 'new' information in the indicators y_t. Note that R_t can be viewed as a regression coefficient matrix. For an exposition of the Kalman filter see Anderson and Moore (1979) and Harvey (1981). The Kalman filter viewed as a Mean Square Estimator (MSE) is both unbiased,

$$E(\xi_t - \hat{\xi}_{t/t}) = 0, \forall t$$

and has minimum variance among the class of linear and unbiased MSE's, i.e.

$$\mathrm{Cov}(\hat{\xi}_t^e) \geqslant \mathrm{Cov}(\hat{\xi}_{t/t-1})$$

for any linear and unbiased estimator $\hat{\xi}_t^e$.

The Kalman filter viewed as a way of 'estimating' the latent vector ξ_t using information up to $t-1$ provides us with a 'solution' to the latent variables problem when the parameters of the state space model are known *a priori*. When the parameters are not known *a priori* but ξ_t is observable, the maximum likelihood method can be used to estimate these parameters, assuming that they are identifiable. Intuition suggests that we might be able to solve the latent variables and the unknown parameters problem by combining the Kalman filter and the likelihood function. The former provides an estimator of ξ_t given that the parameters are known and the latter yields estimators for the parameters given that ξ_t is observable.

The state-space formulation is of considerable interest in econometrics, being ideally suited for handling models with latent variables such as equilibrium and expectational variables (see Hendry and Spanos (1980)). Moreover, the state-space formulation can be used for missing and/or irregular observations as well as data revisions (see Harvey (1981)). For an application of the state-space model see Engle and Watson (1981).

It is interesting to note that the Kalman filter for the MIMIC model takes the form:

$$\hat{\xi}_t = (1 - \lambda'\Omega^{-1}\lambda)\,\beta' z_t + \lambda'\Omega^{-1} y_t \tag{12}$$

which reduces to:

$$\hat{\xi}_t = \lambda'\Omega^{-1} y_t \tag{13}$$

when the normalization $\lambda'\Omega^{-1}\lambda = 1$ is adopted. It will be shown below that (13) coincides with the 'natural' estimator of ξ_t first suggested by Bartlett (1937) in the context of factor analysis. For further details on the relationship between latent variables models and the Kalman filter see Spanos (1982).

IV. ESTIMATING THE MIMIC MODEL

In order to be able to estimate the MIMIC parameters $\theta \equiv (\beta, \lambda, \sigma_1^2, \ldots, \sigma_m^2, \sigma_\epsilon^2)$ subject to some normalization condition (say $\sigma_\epsilon^2 = 1$), we need to be able to solve the two sets of equations $\Pi = \beta\lambda'$ and $\Omega = \lambda\lambda' + \Sigma$ uniquely for θ. A necessary condition for the identification is:

$$(mk + \tfrac{1}{2}m(m+1) - 2m - k) \geqslant 0 \tag{14}$$

The log-likelihood function of the MIMIC model takes the form:

$$\log L = \text{const.} - \frac{T}{2}[\log(\det\Omega) + \operatorname{tr}(\Omega^{-1}S)] \tag{15}$$

where $S = (1/T)(y - Z\Pi)'(y - Z\Pi)$, $\Omega = \lambda\lambda' + \Sigma$.

The first order conditions for a maximum yield the equations for estimating θ:

$$\frac{\partial \log L}{\partial \beta} = 0 \;\Rightarrow\; \hat{\beta} = (\hat{\lambda}'\Omega^{-1}\hat{\lambda})^{-1}\,\hat{\Pi}\hat{\Omega}^{-1}\hat{\lambda} \tag{16}$$

where $\hat{\Pi} = (Z'Z)^{-1} Z'Y$:

$$\frac{\partial \log L}{\partial \lambda} = 0 \;\Rightarrow\; (S + (\hat{\lambda}'\hat{\Omega}^{-1}\hat{\lambda})^{-1}\, Y'Z\hat{\Pi})\,\hat{\Omega}^{-1}\hat{\lambda} = (1 + \hat{\beta}'Z'Z\beta)\,\hat{\lambda} \tag{17}$$

$$\frac{\partial \log L}{\partial \sigma_i^2} = 0 \;\Rightarrow\; \hat{\sigma}_i^2 = s_{ii} - (1 + \hat{\beta}'Z'Z\hat{\beta})\,\hat{\lambda}_i^2 \qquad i = 1, 2, \ldots, m \tag{18}$$

$S = [s_{ij}]$ (see Goldberger (1974)).

When the model is overidentified, one can test for the null hypothesis:

$$H_0 : \boldsymbol{\Pi} = \beta\boldsymbol{\lambda}' \quad \text{and} \quad \boldsymbol{\Omega} = \boldsymbol{\lambda\lambda}' + \boldsymbol{\Sigma}$$

against

$$H_1 : \boldsymbol{\Pi} \neq \beta\boldsymbol{\lambda}' \quad \text{and} \quad \boldsymbol{\Omega} \neq \boldsymbol{\lambda\lambda}' + \boldsymbol{\Sigma}$$

using the likelihood-ratio test statistic:

$$\eta_1(\gamma) = T \sum_{i=1}^{m} \log \hat{\sigma}_i^2 - \log(1 - \hat{\boldsymbol{\lambda}}'\hat{\boldsymbol{\Omega}}^{-1}\boldsymbol{\lambda}) - \log(\det S)] \tag{19}$$

Under H_0, for large T $\eta_1(\gamma)$ is distributed as $\chi^2(\gamma)$ (denoted by $\underset{\alpha}{\sim}$) where $\gamma = k(m-1) + \frac{1}{2}m(m-3)$ (see Joreskog and Goldberger (1975)).

The main problem with the above Full Information Maximum Likelihood (FIML) estimation procedure is that the specification of the behavioural equation for the latent variable ξ_t is assumed known. In practice postulating such a behavioural equation *a priori* which satisfies the restrictions (6) and (7) can prove to be very difficult. In order to be able to tackle this model selection facet of the problem we need to 'estimate' ξ_t at a preliminary stage. For this reason the author adopted an indirect way of estimating the MIMIC model which allows greater flexibility in regard to model selection.

This approach to estimating the MIMIC model proceeds by ignoring the restrictions (6) and estimating $\boldsymbol{\lambda}$ and $\boldsymbol{\Sigma}$ in a Multiple-Indicators (MI) model. The log-likelihood takes the form:

$$\log L_1 = \text{const.} - \frac{T}{2}[\log(\det \boldsymbol{\Omega}) + \operatorname{tr}(\boldsymbol{\Omega}^{-1}\boldsymbol{S}_1)] \tag{20}$$

where $\boldsymbol{S}_1 = (1/T)\, \boldsymbol{Y}'\boldsymbol{Y}$, $\boldsymbol{\Omega} = \boldsymbol{\lambda\lambda}' + \boldsymbol{\Sigma}$.

$$\frac{\partial \log L_1}{\partial \boldsymbol{\lambda}} = 0 \Rightarrow \boldsymbol{S}_1\boldsymbol{\Omega}^{-1}\tilde{\boldsymbol{\lambda}} = \tilde{\boldsymbol{\lambda}} \tag{21}$$

$$\frac{\partial \log L_1}{\partial \sigma_i^2} = 0 \Rightarrow \tilde{\sigma}_i^2 = s_{ii} - \tilde{\lambda}_i^2 \tag{22}$$

where s_{ii} is the ith diagonal element of $\boldsymbol{S}_1$.

The overidentifying restrictions whose number is:

$$d = \tfrac{1}{2}m(m+1) - 2m \tag{23}$$

can be tested using the likelihood-ratio test statistic:

$$\eta_2(d) = T[\log(\det \tilde{\boldsymbol{\Omega}}) + \operatorname{tr}(\tilde{\boldsymbol{\Omega}}^{-1}\boldsymbol{S}_1) - \log(\det \boldsymbol{S}_1) - m] \tag{24}$$

When the d overidentifying restrictions are valid,

$$\eta_2(d) \underset{\alpha}{\sim} \chi^2(d)$$

This test can be used in order to assess the appropriateness of the indicators chosen.

Using the estimators $(\tilde{\lambda}, \tilde{\Sigma})$ and the observations on y_t, we can 'estimate' $\xi \equiv (\xi_1, \ldots, \xi_T)'$ by minimizing the sum of mean-square errors:

$$l(\xi) = \sum_t E[(y_t - \tilde{\lambda}\xi_t)' \tilde{\Omega}^{-1}(y_t - \tilde{\lambda}\xi_t)] \tag{25}$$

$$\frac{\partial l(\xi)}{\partial \xi_t} = 0 \Rightarrow \tilde{\xi}_t = (\lambda'\Omega^{-1}\lambda)^{-1}\lambda'\Omega^{-1}y_t \tag{26}$$

for $t = 1, 2, \ldots, T$.

This is a Mean Square Estimator (MSE) of ξ_t which is unbiased (see Harvey (1981)) under the normalization:

$$\lambda'\Omega^{-1}\lambda = 1 \quad \text{since for} \quad \tilde{\xi}_t = \lambda'\Omega^{-1}y_t \tag{27}$$

$$E(\tilde{\xi}_t - \xi_t) = 0, \quad \forall t \tag{28}$$

We can see that under the same normalization, (27) coincides with the Kalman filter (13). The 'estimator' (26) was first proposed by Bartlett (1937) as an estimator of the factor scores in the context of factor analysis as a least-squares estimator when λ and Σ are known (see Lawley and Maxwell (1971)).

Using the operational form of (26):

$$\tilde{\xi}^* = (\tilde{\lambda}'\Omega^{-1}\tilde{\lambda})^{-1} y\tilde{\Omega}^{-1}\tilde{\lambda} \tag{29}$$

we can proceed to select the form of the behavioural equation (1) so as to satisfy the restrictions in (6). This will enable us to tackle the model spcification facet of the MIMIC model. This amounts to regressing $\tilde{\xi}^*$ on Z to get:

$$\tilde{\beta} = (Z'Z)^{-1}Z'\tilde{\xi}^* = (\tilde{\lambda}'\tilde{\Omega}^{-1}\tilde{\lambda})^{-1}\tilde{\Pi}\tilde{\Omega}^{-1}\tilde{\lambda} \tag{30}$$

which has the same form as the MLE of β in (16). Joreskog and Goldberger (1975) show that $\tilde{\beta}$ and $\hat{\beta}$ have the same properties apart from the fact that $\hat{\beta}$ is more efficient than $\tilde{\beta}$. In particular,

$$\frac{\det(\mathrm{Var}(\hat{\beta}))}{\det(\mathrm{Var}(\tilde{\beta}))} = (1 + \lambda'\Omega^{-1}\lambda)(1 + \beta'Z'Z\beta)^{m-1} < 1 \tag{31}$$

if $(\sigma_1^2, \ldots, \sigma_m^2)$ are known *a priori*.

Although one could stop at the estimation of β *via* (30) subject to the restrictions in (6), it might be preferable to use (30) as a preliminary stage in order to select the form of (1). Once the model selection is resolved, the MIMIC model can be estimated by FIML as discussed above, in order to get fully efficient estimators.

Joreskog and Goldberger (1975) suggested another indirect estimation procedure which they called the econometric-based approach. The name stems from the fact that the estimation proceeds from the reduced form (3) subject only to the restrictions (6), and then the estimators of $\boldsymbol{\beta}$ and $\boldsymbol{\lambda}$ so derived are used to estimate $\boldsymbol{\Sigma}$ in a second stage. For a generalization of this approach see Pudney (1980).

In Section 5 the MIMIC model is applied to the latent variable liquidity. The estimation proceeds with a Kalman-filter type estimator of $\boldsymbol{\xi}_t$ in an attempt to illustrate its use for model selection. The normalization adopted is $\boldsymbol{\lambda}'\boldsymbol{\Omega}^{-1}\boldsymbol{\lambda} = 1$ for the reasons discussed above.

V. AN EMPIRICAL EXAMPLE

After the above discussion of the MIMIC model let us consider applying it to the case of liquidity as a latent variable. As indicators of money the following observed Private Sector Liquidity Components (PSL2) for the UK were chosen:

- C_1 Notes and coins in circulation plus UK private sector sterling sight deposits with banks
- C_2 UK sterling time deposits and certificates of deposit
- C_3 Other money market instruments net; treasury bills, bank bills, deposits with local authorities and finance houses
- C_4 Saving deposits and securities net; shares and deposits with building societies, Trustees Savings Bank, the National Savings Bank and National Savings securities

(see *Financial Statistics*, 1981).

The publication of these statistics since 1980 signified an increasing realization on behalf of the Bank of England that broader aggregates which embrace institutions such as building societies and savings banks, are needed for policy decisions. The monetary aggregates M1 and £M3 are related directly to the above components in that $\text{M1} = C_1$ and $£\text{M3} = C_1 + C_2$. The two broader aggregates on which data are published are:

$$\text{PSL1} = £\text{M3} + C_3 + \text{Certificates of Tax Depoxits}$$

$$\text{PSL2} = \text{PSL1} + C_4$$

The choice of the components C_1–C_4 above for the present study was partly based on their relative magnitudes as well as the availability of data for the whole of the period 1963i–1981ii. Certificates of Tax Deposits were excluded because there are no data for the first part of the period.

As explanatory variables the following quarterly series for the same period were chosen:

X_t real consumers' expenditure (1975 prices)
P_t implicit price deflator for X_t
i_t short-run interest rate (local authority deposit yield, 3 months)
r_t long-run interest rate (British Government securities average yield on $2\frac{1}{2}$ per cent long-term consols)

(see *Economic Trends*, *Annual Supplement 1982*).

The choice of these explanatory variables was largely based on the fact that in the MIMIC framework we are trying to model the common behaviour of the indicators in an attempt to capture the property they share, 'liquidity'. Although interest rates directly related to the individual components C_1–C_4 are available, the choice of i_t was made as the closest to a 'common' interest rate. In several studies of the demand for money (see Artis and Lewis (1976)) it is preferred to most other short-run interest rates. As argued below the MIMIC model for liquidity can be used to consider the question of 'which monetary aggregate to use in demand for money studies'.

Given that the degree of liquidity possessed by the above indicators has changed with time it was decided to model the short run behaviour of the indicators in differenced form, i.e.:

$$y_{it} \equiv \Delta \ln(C_{it}), \quad i = 1, 2, 3, 4 \tag{32}$$

The relative liquidity of C_4 has changed considerably since the early 1950's. For the reasons explained in Section 4 the indirect estimation procedure was adopted for the model selection facet. Estimation of the MI model with (32) as indicators (and $\lambda'\Omega^{-1}\lambda = 1$ as the normalization condition) yielded:

$$\tilde{\lambda}' = \underset{(0.11)}{(0.321,}\ \underset{(0.09)}{-0.182,}\ \underset{(0.12)}{-0.04,}\ \underset{(0.09)}{0.235)} \tag{33}$$

$$\tilde{\Sigma}^{1/2} = \text{diag}\underset{(0.24)}{(0.34,}\ \underset{(0.06)}{0.848,}\ \underset{(0.06)}{0.99,}\ \underset{(0.08)}{0.719),}\ \sigma_\epsilon = 0.052 \tag{34}$$

$$\eta_2(2) = 13.83 \tag{35}$$

Estimated asymptotic standard errors are shown in parentheses beneath the estimates. Although the test statistic $\eta_2(\cdot)$ implies a rejection of the overidentifying restrictions it is interesting to discuss the estimates of λ in (33). The estimated coefficients of the first and fourth indicators have the same sign but opposite to those of the second and third components. This brings out the complementarity and substitutability of the liquidity components, and suggests that the simple sum aggregates such as M3 and £M3 can be very misleading measures of the liquidity in the economy. Components such as C_4 seem to possess a higher degree of liquidity than C_2 and/or C_3 but are excluded from the conventional aggregates. The order such monetary assets are given in PSL2 does not reflect their relative liquidity in any way. An important implication of such findings is that the control of monetary aggregates such as £M3 is called into question as an effective way of controlling liquidity in the economy. Any substitution effects between sight and time deposits induced by interest rate changes will leave £M3 unchanged. This suggests that the recent rediscovery of interest rates as a non-discriminatory monetary policy instrument should be accompanied by measures of liquidity where the effects of changes in interest rates are clearly discernible.

In view of the rejection of the overidentifying restrictions 'real' changes were chosen as indicators, i.e.:

$$y_{it} = \Delta \ln \left(\frac{C_i}{P}\right)_t, \quad i = 1, 2, 3, 4 \tag{36}$$

Estimation of the MI model with real changes as indicators yielded the following estimates for the indicator coefficients:

$$\tilde{\lambda}' = \underset{(0.18)}{(0.621}, \underset{(0.10)}{0.206}, \underset{(0.15)}{-0.05}, \underset{(0.11)}{0.256)} \tag{37}$$

The error standard deviations were:

$$\tilde{\Sigma}^{1/2} = \mathrm{diag}(\underset{(0.11)}{0.865}, \underset{(0.08)}{0.912}, \underset{(0.06)}{0.997}, \underset{(0.09)}{0.908}), \sigma_\epsilon = 0.042 \tag{38}$$

The asymptotic likelihood-ratio test statistics yielded:

$$\eta_2(2) = 2.94 \tag{39}$$

The value of $\eta_2(\cdot)$ is well within the acceptance region for a 5 per cent test. The estimated indicator coefficients confirm the high degree of liquidity possessed by the first and fourth components although the coefficient of the second component is now positive. The relative magnitudes of these coefficients are what one would expect *a priori*. The coefficient of the third component is clearly insignificant and that component can be dropped as an indicator of liquidity. Re-estimation after dropping $\Delta \ln(C_{3t}/P_t)$ as an indicator of liquidity yielded estimates of retained coefficients which differed from those in (37) and (38) only in the third decimal point. Treasury bills, bank bills and deposits with local authorities and finance houses are relatively illiquid financial assets in view of the fact that lack of transactions liquidity within the various financial institutions (including the Treasury), is commonly the the reason for their issue.

The use of £M3 between 1976–81 as the main monetary target was largely due to its relationship, *via* various balance sheet identities, to

Domestic Credit Expansion components and external flows. The growth of £M3 can be related, *via* certain accounting identities, to changes in bank lending to the private sector, the Public Sector Borrowing Requirements (PSBR), changes in Public Sector Debt and external flows (see Coghlan (1980)). This was interpreted as enabling the authorities to combine credit, fiscal and debt management policies which can be monitored *via* changes in £M3. Such an interpretation of accounting identities can be very misleading, however, because they are treated as behavioural equations (see Spanos (1981)). One important implication of such an interpretation has been that credit policy restrictions were concentrated almost exclusively on banks. Over the last 10–15 years, however, other financial institutions such as building societies and trustee savings banks have grown in importance and they contribute significantly to the liquidity of the economy. The publication of the broad monetary aggregates PSL1 and PSL2 can be seen as an attempt by the authorities to monitor the overall liquidity of the economy. Indeed in March 1982 target ranges for £M3 as well as M1 and PSL2 were announced for the period 1982–85. The results in (33) and (37) suggest that even the broader simple-sum aggregates such as PSL1 and PSL2 can be rather misleading measures of liquidity. The monetary component C_4 contributes significantly to the liquidity of the economy and any monetary aggregate purporting to reflect transactions liquidity such as M1 and M2 should be broadened to include some parts of C_4 as well.

The measurement of liquidity plays a very important role in the context of applied research on the demand for money where the results depend crucially on the particular simple sum aggregate chosen as a measure of "money" (see Coghlan (1980), Feige and Pearce (1977) *inter alia*). In the MIMIC model such as a measure of liquidity can be derived through the Kalman-filter type 'estimator' (29).

The estimated coefficients in (37) and (38) can be used in the Kalman filter type estimator (29) of ξ_t which yields:

$$\tilde{\xi}_t = 0.829y_{1t} + 0.248y_{2t} + 0.310y_{4t} \tag{40}$$

The 'derived' weights correspond closely to the weights expected on *a priori* grounds, whereas the simple sum aggregates use only weights of zero or one. Moreover, in the case of the nominal components with the derived components based on (33) and (34) the time deposits components carries a negative weight. If interest rates are used in conjunction with monetary targets as part of a monetary policy package such considerations can be of paramount importance. The form of (40) resembles closely the Torquist–Theil Divisia index suggested by Barnett (1980).

The estimated ξ_t from (40) can now be used to select a behavioural equation for ξ_t in view of the restrictions in (6). As argued above the aim of model selection is to specify a common behavioural equation for the various indicators. The behavioural equation selected using 'the general to the specific methodology' (see Hendry (1980)) was:

$$\hat{\xi}_t = -\underset{(0.24)}{1.9} + \underset{(0.01)}{0.107}\,\Delta x_t^* - \underset{(0.12)}{1.301}\,\Delta \ln P_t - \underset{(0.007)}{0.079}\ln(i/r)_t - \underset{(0.01)}{0.056}\ln r_{t-1}$$

$$+ \underset{(0.03)}{0.119}\,\Delta \ln r_{t-3} - \underset{(0.004)}{0.062}\,Q_{1t} - \underset{(0.004)}{0.027}\,Q_{2t} - \underset{(0.004)}{0.028}\,Q_{3t} \quad (41)$$

$R^2 = 0.886$, $\bar{R}^2 = 0.871$, $s = 0.012$, D.W. $= 1.99$, $\eta_3(8) = 9.82$, $\eta_4(2) = 1.4$.

In (41),

$$\Delta x_t^* = \sum_{i=0}^{4} \Delta \ln x_{t-i}$$

Q_{it} refers to the ith quarterly dummy variable, $i = 1, 2, 3$, and quoted standard errors are conditional on the 'estimated' ξ_t. Also,

$\eta_3(k)$ is a Lagrange multiplier test for kth order autocorrelation; under no autocorrelation, $\eta_3(k) \underset{\alpha}{\sim} \chi^2(k)$.

$\eta_4(\cdot)$ is the Jarque–Bera (1980) normality test statistic; under normality, $\eta_4(\cdot) \underset{\alpha}{\sim} \chi^2(2)$.

The differenced form of the estimation equation (41) was chosen in an attempt to model the short-run behaviour of liquidity. The error-correction term (see Hendry (1980)) was omitted in view of the fact that the interpretation of liquidity in the long-run can be very problematical. This is because of the change of institutional factors inducing a change of relative liquidity over time with building societies being an obvious example. The estimated equation (41) is interpreted as an adjustment equation for the latent variable 'liquidity' with the demand-side dominating the adjustment process. The temptation to interpret it as a demand equation is obvious but such an interpretation is not warranted in view of the fact that the observed data do not refer to intentions on behalf of the economic agents but to actual realizations. Moreover, the fact that the whole of the money stock in the economy is held is no indication of an equilibrium in the money market (see Hendry and Spanos (1980) for further discussion).

In order to see how (41) features as a common behavioural equation for the indicators, the unrestricted reduced form (3) was estimated using the regressors in (41):

	c	Δx_t^*	$\Delta \ln P_t$	$\ln(i/r)_t$	$\ln r_{t-1}$	$\Delta \ln r_{t-3}$
$\Delta \ln(C_1/P)_t$	-1.22	0.07	-0.88	-0.085	-0.03	0.13
	(0.31)	(0.017)	(0.15)	(0.01)	(0.014)	

R^2	D.W.	s	
0.81	2.2	0.016	(42)

	c	Δx_t^*	$\Delta \ln P_t$	$\ln(i/r)_t$	$\ln r_{t-1}$	$\Delta \ln r_{t-3}$	
$\Delta \ln(C_2/P)_t$	−2.02	0.17	−1.04	0.006	−0.12	0.09	
	(0.58)	(0.03)	(0.27)	(0.018)	(0.03)	(0.067)	
	R^2	D.W.	s				
	0.62	1.8	0.029				(43)

	c	Δx_t^*	$\Delta \ln P_t$	$\ln(i/r)_t$	$\ln r_{t-1}$	$\Delta \ln r_{t-3}$	
$\Delta \ln(C_4/P)_t$	−0.38	0.02	−1.0	−0.032	−0.008	−0.03	
	(0.23)	(0.013)	(0.11)	(0.007)	(0.01)	(0.03)	
	R^2	D.W.	s				
	0.8	2.0	0.012				(44)

These estimated reduced form equations suggest that (41) constitutes a common behavioural equation for the indicators of liquidity. The estimated coefficients in (41) represent a compromise of those of (42)–(44). If we were to consider the modelling of the various indicators separately we could easily improve these specifications by including regressors which are directly related to the individual indicators such as different interest rates (7 days' bank deposits and building societies' interest rates). The purpose of the exercise, however, is to model the common behaviour of the indicators in an attempt to capture the quality they all possess in different degrees, namely *liquidity*.

As argued above, one of the most important problems facing a research worker applying MIMIC type models is model selection; in particular the modelling of the time dimension of the observed data. If (1) is postulated *a priori* ignoring any possible dynamic misspecification, then any such misspecification will be erroneously passed over to the measurement facet of the problem. This is similar to the problem encountered in simultaneous equations models where dynamic misspecification can be erroneously interpreted as simultaneity. In the example above the D.W. and $\eta_3(\cdot)$ statistics indicate no dynamic misspecification of first and kth order.

It is interesting to note that estimation of the restricted form imposing only the econometric-type restrictions (6) yielded:

$$\tilde{\boldsymbol{\lambda}} = \underset{(0.06)\ (0.09)\ (0.06)}{(0.605, 0.701, 0.221)'} \tag{45}$$

$$\tilde{\boldsymbol{\beta}} = \underset{(0.23)(0.013)\ (0.14)(0.0007)(0.01)\ (0.02)}{(-1.53,\ 0.085, -1.04,\ 0.004, -0.013, 0.081)} \tag{46}$$

(see Joreskog and Goldberger (1975)). These show some differences in the estimates of $\boldsymbol{\lambda}$ and $\boldsymbol{\beta}$ but most of the estimates are similar.

Partialling out the seasonal variation and using:

$$\xi_t = \beta_0 + \beta_1 \left(\sum_{i=0}^{4} \Delta \ln X_{t-i} \right) + \beta_2 \Delta \ln P_t + \beta_3 \left(\frac{i_t}{r_t} \right)$$
$$+ \beta_4 \ln r_{t-1} + \beta_5 \Delta \ln r_{t-3} + \epsilon_t \tag{47}$$

as the form of (1) the MIMIC model was estimated by FIML. This estimation yielded:

$$\hat{\lambda}' = \underset{(0.15)}{(0.651}, \underset{(0.11)}{0.412}, \underset{(0.12)}{0.406)}, \sigma_\epsilon = 0.046 \tag{48}$$

$$\hat{\Sigma}^{1/2} = \text{diag}(\underset{(0.11)}{0.838}, \underset{(0.07)}{0.921}, \underset{(0.07)}{0.919}) \tag{49}$$

$$\hat{\beta} = (\underset{(0.18)}{-1.63}, \underset{(0.01)}{0.092}, \underset{(0.11)}{-1.07}, \underset{(0.00)}{0.008}, \underset{(0.012)}{-0.024}, \underset{(0.02)}{-0.086}) \tag{50}$$

$\eta_1(12) = 19.7$, and $\chi^2(12) = 21$ at the 5 per cent level.

The FIML estimates (47)-(49) are very similar to the estimates derived using the Kalman filter-type estimator to select (1) in (37) and (38). The statistic $\eta_1(\cdot)$ is close to the critical value but within the acceptance region.

These results suggest that in cases where the behavioural and measurement equations of a state-space formulation can be separated, at least at an initial stage, the Kalman filter-type estimator of the latent vector ξ_t can be used to tackle the model selection problem in econometric models with latent variables.

VI. CONCLUSION

The empirical results related to liquidity confirmed previous findings that simple sum aggregates can be rather misleading as monetary targets. The use of £M3 as a monetary target ignores not only the substitution effects between sight and time deposits induced by interest rate changes but also the highly liquid component C_4. The Kalman filter-type estimator provided an example of a weighted aggregate with the weights closely reflecting the relative liquidity of the components. The use of both indicators and causes in a MIMIC latent variables model provides a natural extension of previous attempts to measure 'moneyness'.

The MIMIC model was applied to the modelling of liquidity and special emphasis was placed on model selection, since selection is particularly difficult when ξ_t is a latent variable. The solution suggested in the paper was to separate the behavioural and measurement equations, at least in the initial stages, and use a Kalman filter-type estimator to 'generate' ξ_t which could be used to choose the form of the behavioural equation. Looking at the MIMIC model, one of the main reasons we were able to separate the behavioural and measurement equations, without losing consistency of the estimators, was the essentially static nature of the former. In the general state-space formulation this will be very difficult to achieve. Intuition, however, suggests that even crude transition equations might be adequate to 'generate' ξ_t for model selection. Moreover, economic theory could be used to give preliminary values to coefficients in order to generate ξ_t for model selection purposes at an initial stage.

Birkbeck College, London

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APPENDIX

The Gain Matrix R_t

Let

$$V_{t/t-1} \equiv E((\xi_t - \hat{\xi}_{t/t-1})(\xi_t - \hat{\xi}_{t/t-1})' \mid Y^0_{t-1})$$

Using the relationship

$$y_t - \hat{y}_{t/t-1} = \Lambda_t(\xi_t - \hat{\xi}_{t/t-1}) + v_t$$

we can deduce that:

$$E(\eta_t e_t \mid Y^0_{y-1}) = (V_{t/t-1}\Lambda_t' + G_t\Sigma_{12})$$

and

$$E(e_t e_t' \mid Y^0_{t-1}) = \Lambda_t V_{t/t-1}\Lambda_t' + \Sigma_{22}$$

Thus,

$$R_t = (V_{t/t-1}\Lambda_t' + G_t\Sigma_{12})(\Lambda_t V_{t/t-1}\Lambda_t' + \Sigma_{22})^{-1}$$
$$V_{t/t-1} = A_t V_{t-1/t-1}\Lambda_t' + G_t\Sigma_{11}G_t'$$

and

$$V_{t/t} = V_{t/t-1} - (V_{t/t-1}\Lambda_t' + G_t\Sigma_{12})(\Lambda_t V_{t/t-1}\Lambda_t' + \Sigma_{22})^{-1} \times (V_{t/t-1}\Lambda_t' + G_t\Sigma_{12})$$

(see Kalman (1960), Kalman and Bucy (1961), Kailath (1974)).

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