

Design of Active Structural Acoustic Control Systems Using Eigenassignment Approaches

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(ABSTRACT)

Active structural acoustic control (ASAC) in conjunction with the adaptive feedforward control has been proved to be an efficient practical approach to reduce structure-borne sound. ASAC works on the principles of reducing the vibration amplitude of the structure (modal reduction), as well as changing the vibration distributions of the structure so that the vibration distributions of each structural modes destructively interfere with one another in their associated radiating acoustic field (modal restructuring). Based on these observations, two different but related design strategies, namely the non-volumetric design and the minimum supersonic wavenumber design, were developed for designing efficient ASAC system. The eigenassignment method for feedforward control system serves as the fundamental design tool for both formulations.

In this study, the dynamic characteristics of a multiple-input, multiple-output (MIMO) feedforward controlled system was investigated both analytically and experimentally on a simply supported plate under harmonic excitation. It was demonstrated that, when the control system has equal number of control inputs and error sensor outputs, the feedforward controller can effectively modify the system dynamics (i.e., resonance frequencies and mode shapes). This provides the theoretical basis for the eigenassignment method.

For the non-volumetric design, the single-input, single-output (SISO) eigenassignment technique is used to modify the eigenproperties of a planar structure using structure actuators

and sensors so that all the controlled modes are non-volumetric (inefficient sound radiators at low frequencies, i.e., $k_0 a \ll 1$), leading large global sound attenuation in the far field. The effectiveness of this formulation was demonstrated through numerical simulations for the control of radiation from simply supported and clamped-free beams. The experimental validation of the non-volumetric design was also carried out on a simply supported beam using PZT actuators and shaped PVDF film as error sensor. The filtered- x LMS algorithm was used in the experiment. Excellent global sound attenuation was achieved in the low frequencies.

The minimum supersonic wavenumber design stems from the fact that only supersonic wavenumber components of the structural velocity spectra radiate to the far field. A SISO eigenassignment technique is used to modify the eigenproperties of a planar structure so that the eigenfunctions of the controlled system have minimum supersonic wavenumber in the frequency range of study. The sound pressure or sound power radiated by the structure is therefore reduced. The design was demonstrated on a simply supported beam to minimize the supersonic wavenumber components contributed by the odd-order modes only. Significant global sound attenuation was achieved in the frequency range of study.

The main advantage of the proposed design methods is that they do not depend on the characteristics of the external disturbance, such as the form, location and frequency contents. Also, the error sensor and control input are optimized simultaneously, resulting in better acoustic control performance. The practical implementations of the proposed designs require accurate system modeling, this is the major limitation of the proposed designs.

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Chapter 1

Introduction

Noise pollution is a general environmental issue in many industrialized countries. In recent years, this environmental issue has raised enough public attention that various government regulations have been issued to address this problem. While many industrialized countries are examining the impact of modern industries on our living environment, many underdeveloped countries are struggling to industrialized their countries. They believe that modernization will strengthen their country and improve the quality of their people's lives. With industrialization as the top priority, people's living environment often becomes the victim. Noise pollution, together with other environmental pollution, has become a global issue. Human beings, in some sense, have discovered that they are the victims of their own efforts in trying to improve the quality of their lives.

In the past few years, various noise control techniques have been developed in diverse fields to alleviate the noise pollution. Among those techniques, the traditional passive noise reduction techniques are widely used in industries and commercial products. Passive control methods typically use absorptive materials or vibration absorbers, etc. to achieve noise reduction. They are proved to be very effective in the middle to high frequency range. However, the passive control approaches are often impractical to implement in the low frequency range, where the acoustical wavelength is relative long and the passive control methods

require bulky/heavy installations. Efficient noise reduction approach in the low frequency range thus poses a challenging topic to noise control engineers.

In recent years, with rapid development in high speed digital signal processors as well as in fast, multi-channel analog-to-digital and digital-to-analog converters, active control of sound radiation from vibrating structures has emerged as a very promising alternative method to reduce noise, especially at low frequencies. Active Noise Control (ANC) is featured by introducing secondary sound sources (i.e., acoustic control sources) into the system [1]. It works on the principle of destructive interference between the sound fields generated by the primary sound sources and the secondary sound sources, whose acoustic outputs can be determined by the controller.

An alternative control approach is the widely adopted Active Structural Acoustical Control (ASAC) technique proposed by Fuller [2]. In this approach, control inputs are applied directly to the vibrating structures (responsible for the sound radiation) to reduce or change the vibration distributions, so that the sound pressure or a pressure related variable is minimized. ASAC works on the principles of reducing the vibration amplitude of the structure, as well as changing the vibration distributions of the structure so that the vibration distributions of the structural modes destructively interfere one another in their associated radiating acoustic field. These two mechanisms are termed as “modal reduction” and “modal restructuring”, respectively [3]. It is shown that, in general, this strategy requires a smaller number of control inputs for a global sound reduction in the acoustic field as compared to ANC, where acoustical transducers (i.e., loud speakers) are used to excite the acoustic medium. The efficiency of this approach in conjunction with adaptive feedforward control is now widely recognized.

In this introduction chapter, a review of the previous work on ASAC is presented. The scope and objectives of the current research, followed by a detailed outline of this dissertation, are then described.

1.1 Previous and current work

The attenuation of sound radiation from vibrating structures is a particularly important issue in practical engineering. With the emergence of active noise control technology, many different control algorithms have been developed as a consequence. In particular, the adaptive feedforward control algorithms (especially the filtered- x LMS algorithm by Widrow [4]) are widely used in active noise control when a reference signal characterizing the primary disturbance is available [5]. The traditional design approach in adaptive feedforward control is to select the number/location of the error sensors and actuators based on some physical understanding of the dynamic characteristics of the uncontrolled system or to use standard optimization techniques [6, 7]. Many systematic design approaches such as the eigenvalue assignment method that have been used by feedback control communities for years are not available for their feedforward counterpart. Recently, Burdisso and Fuller [8, 9] extensively studied the dynamics of the single-input, single-output (SISO) adaptive feedforward controlled system and revealed that the adaptive feedforward control, similar to the feedback control, will alter the eigenproperties of the controlled system whether vibration or acoustic radiation is being minimized. This conclusion was also confirmed by the independent study of Albert and the possibility to extend it to broadband excitation was further investigated by the same author [10]. Burdisso and Fuller [11, 12] further developed a design methodology, referred to as the eigenassignment design technique, based on their findings for the single-input, single-output feedforward controlled system. In this approach, the design process is separated into two distinct phases. The control system is first optimized in terms of general modal parameters such that the controlled system responds with desired eigenproperties, which are commonly derived from either vibrational or acoustical considerations. The performance of the controlled system can then be evaluated in the modal domain without knowledge of the physical sensors and actuators. The second stage of this design procedure is to design the physical system (realistic structural error sensors and actuators) that yields the desired modal parameters of the optimized control system. The significant advantage of this modal-based design approach is that the design is valid for both narrowband and broadband excitations and is independent of the disturbance characteristics (i.e., type, position and frequency content). Also, the design of both error sensors and actuators is carried out

simultaneously and in comparison requires a minimal computational effort.

Another important aspect of the active control system is the design of sensors and actuators. In the early years of ASAC, microphones in the acoustic far-field were frequently used as the error sensors, which is often impractical in real implementation. Over the past few years, a significant amount of research has been conducted in the development of piezoelectric actuators and sensors to control vibrations of planar structures [13, 14, 15, 16]. Structure bonded distributed error sensors/actuators, together with adaptive controller, form the compact “smart structure” which can compensate for changes in the input disturbance by itself. Piezoceramic patches embedded into or bonded to the structure are now widely used as actuators in ASAC systems [17, 18, 19]. These integrated, distributed actuators overcome many disadvantages of the traditional shakers. On the other hand, the polyvinylidene fluoride (PVDF) film, when bonded to the surface of the structure, can be served as an extremely effective sensor. It is light weight and flexible, thus causes little change to the system response. It has a high piezoelectric charge constant and can be shaped into complicated forms either by cutting the film or etching the electrodes. Lee and Moon [20] showed that the PVDF film could be configured to provide an output signal proportional to the integral of the in-plane strain of the structure covered by the piezoelectric film. They further demonstrated that it was possible to design distributed sensors that observe only one or several selected modes for one-dimensional plate and beam structures [21]. In other words, the distributed error sensors such as the PVDF film can serve as modal filters. Ever since then, PVDF materials have been widely used as error sensors in ASAC. Appropriately configured error sensors in active structural acoustical control should be able to provide signals that are only related to the radiating part of the out-of-plane vibration and filter out the others. For instance, Clark [22] used two narrow rectangular stripes of PVDF film positioned along the x and y direction of a simply-supported rectangular plate as error sensors. His results show that the odd-odd modes of the plate, which are efficient sound radiators, are the dominant contributors to the error sensor signal. Attenuation of the error signal therefore leads to sound radiation reduction in the far field. Guigou [23] designed a PVDF sensor for one-dimensional beams that only measured the volumetric modes of the beam by using modal decomposition. Good sound attenuation has been achieved in the low frequency range.

To successfully design an efficient ASAC system, one must understand the physics of the interaction between the structural vibration and acoustic medium. The sound power radiation from a structure is a function of the out-of-plane velocity distribution over the surface of the structure. It is well known that, below the critical frequency of the structure, certain velocity distributions radiate sound more efficiently than others. For example, it is well recognized that the odd-odd modes of a baffled plate radiate more efficiently than others [24, 25]. An efficient control scheme should therefore not only try to attenuate the amplitude of the out-of-plane vibration of the structure, but also try to alter the vibration distributions of the structural modes so that they destructively interfere with one another in the acoustic medium. In other words, it is more desirable to change the velocity distributions of the structure so that the structure responds with a set of inefficient sound radiators. The effects of modal coupling on acoustic power radiation from one-dimensional baffled panels have been studied by Kelti and Peng [26]. They concluded that although the modal coupling effect was not significant under resonant excitation, it could not be ignored at low frequency and under off-resonant excitation.

Recently, the concept of “radiation modes” has drawn significant attention in developing efficient design strategies for ASAC. Borgiotti, Cunefare and Koopman, and Elliott and Johnson [27, 28, 29] have shown that the sound power radiated by vibrating structures can be expressed as a linear contributions of the radiation modes, where the radiation modes are defined as surface velocity distributions that radiate sound power independently in the acoustic far-field. This conclusion is also implicit in Baumann’s work [30]. The radiation modes correspond to the eigenvectors of the sound power or radiation resistance matrix while the eigenvalues measure the associated radiation efficiencies of the radiation modes. However, there is an intrinsic limitation in implementing these design considerations since the radiation modes are functions of the excitation frequencies and the radiation resistance matrix is ill conditioned in general. Therefore, it is rather difficult to design a single or a set of error sensors to accurately observe several radiation modes in the complete design frequency range. However, it can be shown that, in the low frequency range, i.e., for dimensionless frequency $k_0 a \ll 1$ (where k_0 is the acoustic wavenumber in the air and a is the characteristic dimension of the structure), the shape of the radiation modes is reasonably independent of frequency

and only the first radiation mode is significant in terms of sound radiation [28, 29]. Both Snyder *et al.* [31] and Johnson and Elliott [32] have successfully developed PVDF sensors designed to only observe these radiation modes. Burdisso and Fuller [12] developed a different but related approach in which the system eigenfunctions have been modified by the controller so that they inefficiently couple vibrational energy to acoustic energy. It has been shown that the first radiation mode is the piston type, which represents the monopole behavior of the structure and whose amplitude corresponds to the net volume velocity of the structure. It can be further deduced that the first radiation mode is a combination of structural modes associated with a volumetric component (i.e., structural modes corresponding to a non-zero net volume velocity). These “volumetric modes” radiate sound efficiently at low frequencies as they behave as monopole radiators [22, 23, 32, 25]. On the other hand, “non-volumetric modes” (i.e., modes corresponding to a zero net volume velocity) radiate poorly in the low frequency region where their directivity patterns resemble dipoles, quadrupoles or higher-order patterns. These observations provide the physical background for part of the work in this thesis and the details will be discussed later in Chapter 3.

The wavenumber transform technique, i.e., spatial Fourier Transform of the out-of-plane velocity distribution of the structure, is widely used in ASAC due to its physical significance in comparison with the traditional Rayleigh integral approach. The relationship between the acceleration distribution over the surface of the planar structure and the far-field sound radiation is fully revealed by the wavenumber transform technique presented in Junger and Feit’s exceptional book [33]. Junger and Feit have shown that, for planar radiators, the far-field sound pressure is directly related to the wavenumber transformed out-of-plane acceleration distribution of the structure. To be more specific, the sound pressure from a planar structure at a particular angle is directly proportional to the corresponding discrete supersonic wavenumber component of the structure acceleration spectrum. Based on this observation, Fuller and Burdisso [34] developed a wavenumber control technique to minimize sound radiation toward a particular direction. This is done by expressing the cost function of the optimal controller in the wavenumber domain and suppressing the discrete wavenumber components corresponding to that specific direction. Another important conclusion revealed by the wavenumber transform method, as pointed out by Fahy, Clark and Fuller, and Fuller

and Burdisso [25, 35, 34], is that the far-field sound pressure is proportional to the supersonic wavenumber components (i.e., wavenumber smaller than the acoustic wavenumber in the acoustic medium), while the subsonic wavenumber components (i.e., wavenumber greater than the acoustic wavenumber in the acoustic medium) do not contribute to sound radiation in the far-field. Thus minimizing the supersonic wavenumber components of the structural acceleration spectrum will lead to sound pressure or sound power attenuation in the far-field. Moreover, as illustrated by Maillard [36], it is possible to reconstruct the far-field sound pressure at predetermined angles from measured surface velocity at discrete points in real time by using digital signal processing technique. This not only provides an efficient, robust sensing technique, but also provides a convenient facility for obtaining far-field sound pressure without using an anechoic chamber.

The primary goal of this work is, based on some physical considerations, to develop more efficient design strategies for planar ASAC systems in order to overcome some of the limitations of previous works. Two new design formulations, namely the non-volumetric design and the minimum supersonic wavenumber design, will be developed and tested in this project. The eigenassignment technique will be adopted as the primary design tool throughout the proposed work. As will be illustrated later in Chapter 3 and Chapter 5, the main advantage of the proposed non-volumetric design method is that the design does not depend on the characteristics of the external disturbances, such as the type, positions and frequency contents. Another advantage of the non-volumetric design is that the proposed method optimizes both the error sensor and control input simultaneously, a better performance in terms of acoustics suppression will be demonstrated. Although the design procedure will only be carried out for a simple one-dimensional structure located in rigid baffle, it is readily extensible to more general planar structures since it is based upon a normal mode analysis.

1.2 Scope and objectives of current research

The objective of this work is to develop and implement, both analytically and experimentally, efficient design strategies based on physical observations underlying the mechanisms of

structure-borne sound radiation. The eigenassignment technique is used as the design tool throughout the dissertation.

First, in order to complete the eigenproperty theory for the feedforward controlled system as proposed by Burdisso and Fuller [8], the dynamic characteristics of a multiple-input, multiple-output (MIMO) feedforward controlled system will be studied both analytically and experimentally. A two-input, two-output feedforward controlled simply-supported plate subject to a harmonic excitation will be used as an example to illustrate the idea, although the analysis is valid for any modal responding finite system. It will be shown that, when the number of control inputs is equal to the number of error sensors, the multi-input, multi-output feedforward controlled system, just as the single-input, single-output feedforward controlled system, effectively behaves as having new eigenproperties (i.e., natural frequencies and mode shapes). A new mathematical approach will be developed to compute the MIMO controlled system resonance frequencies and their associated mode shapes. It will be shown that the resonance frequencies of the MIMO feedforward controlled system can be obtained by solving the roots of the characteristic equation of the controlled system, which is given by the characteristic equation of the control compensator. The associated eigenfunctions of the controlled system are just linear combinations of those uncontrolled system eigenfunctions. A two-input, two-output feedforward controller using the filtered- x LMS algorithm will be used in the experiment. The control algorithm is coded in a PC based TMS320C30 DSP board. This study completes the feedforward control eigenproperty theory which serves as the bases of the eigenassignment method.

The eigenproperty theory suggests that, by properly configuring the control actuators and error sensors, the mode shapes of the feedforward controlled system can be modified in such a way that they radiate weakly in the far-field. In this work, two new formulations for the design of ASAC systems will be proposed. The first design strategy will be based on the fact that structural modes with zero integrals of normal velocity distributions over their surface (i.e., non-volumetric) are inefficient sound radiators at low frequencies, where their directivity patterns resemble dipoles, quadrupoles or other higher even-order patterns. To achieve significant sound attenuation in the low frequency range, the single-input, single-output eigenassignment technique will be used to modify the eigenproperties of a planar structure

so that all modes of the controlled system are non-volumetric, leading to an overall sound attenuation in the far-field. To demonstrate the design procedure and the effectiveness of the proposed design, both simulation and experimental implementation to a simply-supported beam located in a rigid baffle will be carried out. Two design schemes will be developed for designing the PZT actuators. A single-input, single-output (SISO) feedforward controller using the filtered- x LMS algorithm will be used in the experiment. This design formulation is readily extensible to more general planar structures.

Another formulation is to design the proper actuators and error sensors so that the mode shapes of the controlled system will lead minimal supersonic wavenumber components, thus reduce the sound pressure or power in the far-field. Only computer simulation will be presented for this design. It will be shown later that the proposed non-volumetric design is a special case of the minimum supersonic wavenumber design, as it corresponds to minimizing the supersonic components of the structural acceleration wavenumber spectrum at $k_x = 0, k_y = 0$, where $k_x = k_0 \sin \theta \cos \phi, k_y = k_0 \sin \theta \sin \phi$ are the structural wavenumber components in x and y directions, respectively. For these two design formulations proposed in this study, surface-bonded piezoceramic patches (PZT) will be used as the actuators and either accelerometers or PVDF films can be used as the error sensors. However, in favor of developing a compact smart system, shaped PVDF films will be used as error sensors in this dissertation.

1.3 Organization of the dissertation

This dissertation is organized as follows: Chapter 1 is the introduction. A review of the past work on active structural acoustical control is given. The scope and objective of this dissertation are also discussed. In Chapter 2, theoretical and experimental studies on the eigenproperties of the MIMO feedforward controlled system are presented. A brief review of the eigenassignment design technique is first presented in Chapter 3, and the non-volumetric concept and its acoustic implications are then presented in detail. The modal domain design of the non-volumetric ASAC system for one-dimensional planar structure is then discussed

in detail. Numerical simulations for simply-supported beam and clamped-free beam are presented. The second phase of the non-volumetric design, namely the design of the PZT actuators and PVDF sensors, is presented in Chapter 4 for simply-supported beam only. Sensitivity analysis associated with the optimal PZT actuators design is also addressed in this chapter. In Chapter 5, the design procedure and computer simulation of the minimum supersonic wavenumber design are carried out for a simply-supported beam. Experimental validation of the non-volumetric design on a simply-supported beam is discussed in Chapters 6. The experimental setup, experimental procedure and various equipment used in the experiment are first discussed in Chapter 6. The experiment results are then presented in details. The SISO filtered- x LMS algorithm is used in this experiment to achieve the optimal controller. The experimental results are then compared with those obtained from simulation. Finally, the main conclusion of this dissertation and some suggestions for the future work are discussed in Chapter 7.

Chapter 2

Eigenproperties of feedforward control system

As mentioned in Chapter 1, rigorous mathematical formulation and experimental study show that a single-input, single-output (SISO) feedforward controlled system effectively behaves as having new eigenproperties. A new mathematical approach to predict the dynamic characteristics of the controlled structure has been developed by Burdisso and Fuller [11, 12]. In this chapter, in an attempt to complete the eigenproperty theory for feedforward control system, the dynamic characteristics of multiple-input, multiple-output (MIMO) feedforward controlled system will be discussed. For completeness, a brief introduction to feedforward control theory is presented first. A review of the eigenproperty theory for the SISO feedforward controlled system follows. The mathematical formulation for the eigenproperties of a MIMO feedforward controlled system is presented in detail, and a two-input, two-output feedforward controlled simply supported plate under harmonic excitation is used as an example to illustrate the mathematical formulation. The associated experimental results are presented next. Finally, some important conclusions and remarks are presented in the last part of this chapter.

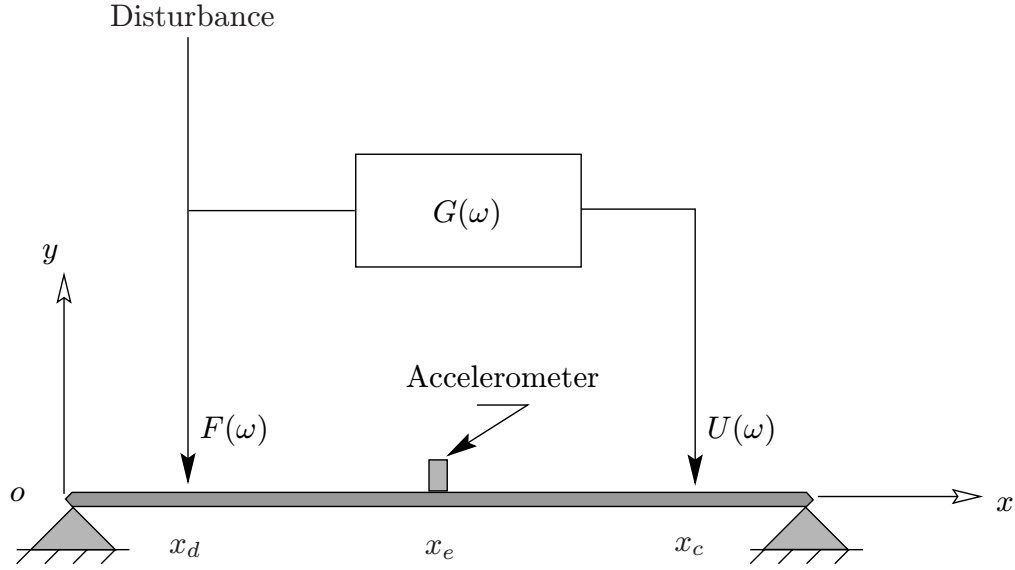


Figure 2.1: Schematic of a simply supported beam

2.1 Feedforward control system

Feedforward control, as mentioned in Chapter 1, is widely used in active noise control when a reference signal, characterizing the primary or disturbance source, is available. A typical feedforward control system, without loss of generality, is shown in Figure 2.1, where the plant is a simply supported beam, and $G(\omega)$ is the frequency response function of the feedforward control compensator. The feedforward control compensator as depicted in Figure 2.1 is characterized as being driven by a fully coherent signal from the disturbance source. The signal proportional to the response of the mechanical system (e.g., the signal from the accelerometer in Figure 2.1) plays no direct role in the control path, but could be used to adapt the response of the controller.

For the flexible beam shown in Figure 2.1, the primary source $F(\omega)$ is applied at location x_d , and the control source at location x_c . Let the flexible structure frequency response

function between an external input at location a and displacement at location b be given as

$$G_{ab}(j\omega) = \frac{n_{ab}(j\omega)}{d(j\omega)} \quad (2.1)$$

where $d(j\omega)$ is common to all frequency response functions since it contains the poles of the structure which are global properties of the system, and $n_{ab}(j\omega)$ gives the zeros of the system whose nature depend on the particular input and output locations. For simplicity, in the rest of the dissertation, the notation $G_{ab}(\omega)$, instead of $G_{ab}(j\omega)$, is used to denote frequency response function. For any location x on the beam, the normal displacement response of the controlled system can be written as

$$w(x, \omega) = \frac{n_{dx}(\omega)}{d(\omega)}F(\omega) + \frac{n_{cx}(\omega)}{d(\omega)}G(\omega)F(\omega) \quad (2.2)$$

where the subscript d and c are used to denote the disturbance and control inputs, respectively. The control compensator $G(\omega)$ is usually defined to render a desired controlled response. In ASAC, the control compensator $G(\omega)$ is often defined by minimizing the sum of the quadratic square of the signals from error sensors on the structure (e.g., accelerometers) or in the acoustic field (e.g., microphones).

The traditional view of the mechanism of feedforward control under steady state disturbances is “active cancellation” as suggested by equation (2.2), i.e., the response of the structure due to the primary sources is canceled out by that due to the control sources as a result of superposition. However, the experimental work of Snyder and Hansen [37] on active control of sound in ducts suggests that the control source modifies the input impedance of the duct upon which the primary source acts, and thus leads reduced energy input into the system by the primary source. This suggests the necessity to re-investigate the mechanism of feedforward control. It is generally believed that feedforward control can only change the system zeros and only feedback control can be used to change the system poles. Recently, Burdisso and Fuller [8, 9] pointed out that, just like their feedback counterpart, the feedforward control can alter the system eigenproperties (i.e., natural frequencies and mode shapes) by properly selecting the control configurations. In his independent study, Alberts [10] also

confirmed this conclusion and a more general mathematical proof based on transfer function analysis was presented. He also introduced the relationship that the control compensator's numerator and denominator must satisfy in order to alter the system eigenproperties. He further investigated the possibilities to extend this analysis to broadband excitation and other flexible structures. In the next section, a review of the eigenproperty theory for the feedforward control system is presented based upon the framework of references [8, 38, 10]

2.2 Eigenproperty of SISO feedforward controlled system

Before proceed any further, it is helpful to review some of the important assumptions underlining the current study:

- The system under control is linear, so the principle of superposition holds.
- The reference signal is fully coherent with the primary source signal (there is no feedback from the control sources to the reference detection sensors).
- There is no measurement noise in the error sensor path.

The frequency response function of the control compensator $G(\omega)$, as mentioned in Section 2.1, can be defined by driving the response at the error sensor location to zero:

$$w(x_e, \omega) = 0 \tag{2.3}$$

From equations (2.2) and (2.3), the control compensator is given by

$$G(\omega) = -\frac{n_{de}(\omega)}{n_{ce}(\omega)} \tag{2.4}$$

In order to analyze the dynamics of the controlled system, it is necessary to include the dynamics of the control compensator with that of the plant. Substituting equation (2.4)

into equation (2.2), the response of the controlled system at any location x on the structure can be written as

$$w(x, \omega) = \frac{n_{dx}(\omega)}{d(\omega)} F(\omega) - \frac{n_{cx}(\omega)}{d(\omega)} \frac{n_{de}(\omega)}{n_{ce}(\omega)} F(\omega) \quad (2.5)$$

In references [8, 38], and more generally in reference [10], it is proved that for the second term in equation (2.5) there exists the following identity:

$$\frac{n_{cx}(\omega)}{d(\omega)} \frac{n_{de}(\omega)}{n_{ce}(\omega)} = \frac{n_{dx}(\omega)}{d(\omega)} + \frac{\tilde{n}(\omega)}{n_{ce}(\omega)} \quad (2.6)$$

where $\tilde{n}(\omega)$ is a new function. With this simplification, equation (2.5) becomes

$$w(x, \omega) = -\frac{\tilde{n}(\omega)}{n_{ce}(\omega)} F(\omega) \quad (2.7)$$

Equations (2.4) and (2.7) state that the system characteristic function (hence the dynamics) is changed to $n_{ce}(\omega)$ for any arbitrary location on the controlled structure. In other words, the controlled system effectively has new eigenvalues (hence resonance frequencies) given by the poles of the control compensator $G(\omega)$.

For conciseness, a general proof modified from reference [10] is included in Appendix A. Interested reader should refer to that appendix for details. What we want to emphasize here is that the conclusions presented here are valid for any single-input, single-output dynamic, distributed structure. Later in Section 2.3 we will see that the proof given in the Appendix A is also valid for multiple-input, multiple-output feedforward control. For the simply supported beam depicted in Figure 2.1, the explicit expression of the control compensator $G(\omega)$ in equation (2.4) can be written as

$$G(\omega) = -\frac{\sum_{n=1}^N \xi_n \nu_n \prod_{\substack{m=1 \\ m \neq n}}^N (\omega_m^2 - \omega^2)}{\sum_{n=1}^N \xi_n \mu_n \prod_{\substack{m=1 \\ m \neq n}}^N (\omega_m^2 - \omega^2)} \quad (2.8)$$

where ω_m^2 is the eigenvalue of the uncontrolled system, ξ_n is the modal error sensor component, and ν_n and μ_n are the unit modal disturbance input and unit modal control input, respectively. Furthermore, equation (2.7) can be written as [8]:

$$w(x, \omega) = \sum_{l=1}^{N-1} \frac{f_{cl}}{m_{cl}} \phi_{cl}(x) H_{cl}(\omega) F(\omega) \quad (2.9)$$

where the subscript c is used to denote the controlled system, and m_{cl} and f_{cl} are the modal mass and modal components of the input distribution for the l^{th} controlled mode, respectively. Equation (2.9) states that the response of the controlled system can be expressed as a linear combination of a new set of $(N - 1)$ eigenfunctions $\phi_{cl}(x)$. The controlled system will show resonance if driven at the excitation frequency ω_{cl} , which is associated with the l^{th} controlled modal frequency response function $H_{cl}(\omega) = (\omega_{cl}^2 - \omega^2)^{-1}$. Equation (2.9) also suggests that the SISO feedforward controlled system has one less resonance frequency due to the constraint imposed by driving the response at the error sensor location to zero. In other words, the control input reduces by one the dynamic degrees of freedom of the original structure.

The eigenfunctions of the controlled system are shown to be just linear combinations of those of the uncontrolled modes and can be uniquely defined as [8]:

$$\phi_{cl}(x) = \sum_{n=1}^N \Gamma_{nl} \phi_n(x), \quad \text{for } l = 1, 2, \dots, N - 1 \quad (2.10)$$

where the expansion coefficients Γ_{nl} is given as

$$\Gamma_{nl} = \frac{\mu_n}{(\omega_{cl}^2 - \omega_n^2)} \left\{ \sum_{m=1}^N \left[\mu_m / (\omega_{cl}^2 - \omega_m^2) \right]^2 \right\}^{-1/2} \quad (2.11)$$

where p_n is the modal component of the spatial distribution of the control input corresponding to the n^{th} uncontrolled mode.

In summary, with a properly selected control compensator, the feedforward controlled system effectively behaves as having a new set of desired eigenproperties. The eigenvalues

(hence the resonance frequencies) of the controlled system are given by the poles of the control compensator $G(\omega)$. The eigenfunctions (mode shapes) of the controlled system are linear combinations of those uncontrolled modes. It is important to point out that the eigenproperties of the controlled system are only functions of the control input distribution, location of the error sensor and the structure eigenproperties. They are independent of the primary or disturbance source as suggested by equations (2.7) and (2.11) [8]. It is also important to point out that this system dynamics modification is for feedforward control where coherent information from the primary source is applied to the structure as control inputs. To any other incoherent disturbance, the system dynamics remains unchanged [8].

2.3 Eigenproperties of MIMO feedforward controlled system

In this section we will study the dynamic behavior of a multiple-input, multiple-output feedforward controlled finite system excited by a harmonic disturbance input. Without loss of generality, to show the characteristics of the MIMO feedforward control configuration, two-input, two-output feedforward control applied to a planar structure is depicted in Figure 2.2, where $F(\omega)$ is the disturbance input and $U_1(\omega)$ and $U_2(\omega)$ are the control inputs, respectively. The control inputs are obtained by feeding forward fully coherent information from the disturbance input $F(\omega)$ through the control channels $G_1(\omega)$ and $G_2(\omega)$. The control compensator $G_i(\omega)$ is usually obtained by minimizing a quadratic cost function of measurable variables of interest. A rigorous mathematical study of the dynamics of the control system is presented for a general, modal responding finite system. The approach adopted in this work is an extension to the work of Burdisso and Fuller [8].

2.3.1 Equations of motion

For a linear distributed system under harmonic excitation, following Meirovitch [39], the equation of motion of the system can be written in the form of the partial differential

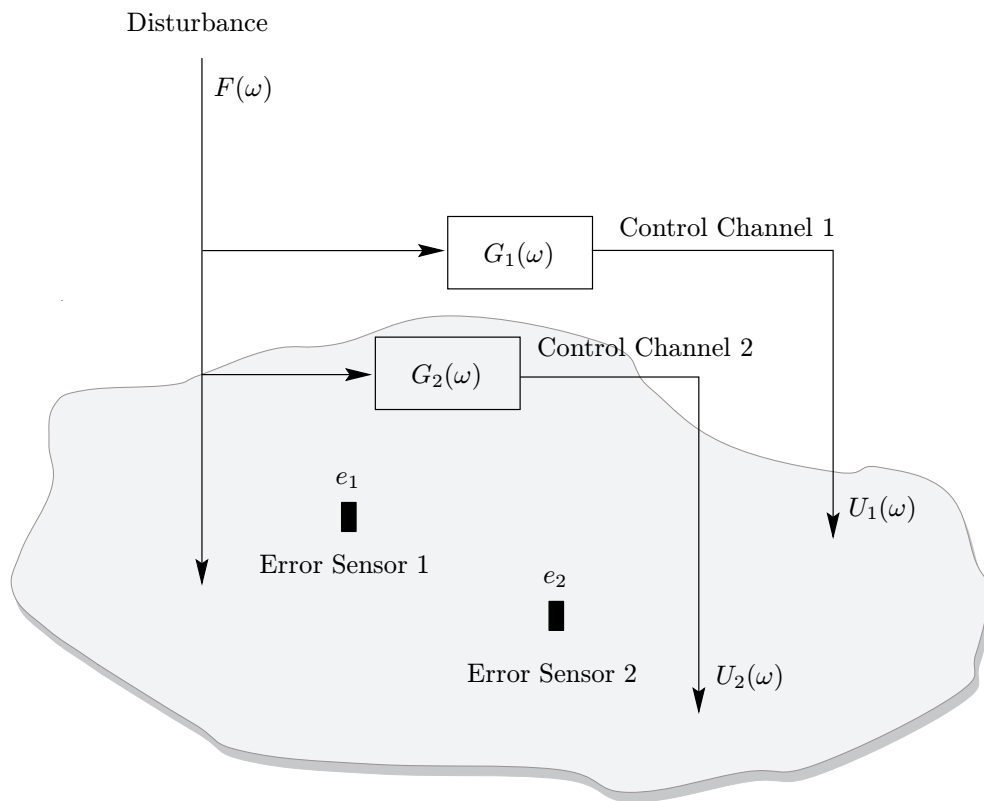


Figure 2.2: Typical MIMO feedforward control configuration

equation

$$Lw(D, t) + m(D)\ddot{w}(D, t) = (f_d(D)F + \sum_{p=1}^{M_c} f_{cp}(D)U_p)e^{j\omega t} \quad D \in \Omega \quad (2.12)$$

where

- $w(D, t)$ = Out-of-plane displacement,
- D = Typical point inside the domain Ω ,
- L = Homogeneous self-adjoint differential operator of order $2r$ and r is an integer,
- $m(D)$ = Mass density of the structure,
- $f_d(D)$ = Spatial distribution function of the disturbance input
- F = Amplitude of the disturbance input,
- $f_{cp}(D)$ = Spatial distribution function of the p^{th} control input,
- U_p = Amplitude of the p^{th} control input,
- M_c = Number of control inputs,
- ω = Frequency of the disturbance input in *rad/sec*

The solution $w(D, t)$ of equation (2.12) is subject to the boundary conditions

$$B_i w(D, t) = 0 \quad D \in \partial\Omega \quad \text{for } i = 1, 2, \dots, r \quad (2.13)$$

where B_i are boundary differential operators of maximum order $2r-1$ and $\partial\Omega$ is the boundary of Ω . The associated eigenvalues problem is given by the following differential equation

$$L\phi(D) = \lambda m(D)\phi(D) \quad (2.14)$$

where $\phi(D)$ is subject to the boundary conditions

$$B_i \phi(D) = 0 \quad D \in \partial\Omega \quad i = 1, 2, \dots, r \quad (2.15)$$

The solution of the eigenvalue problem consists of an infinite set of real eigenvalues λ_n and eigenfunctions $\phi_n(D)$ ($n = 1, 2, \dots$). Assuming that L is positive definite, all λ_n are positive. We denote them by $\lambda_n = \omega_n^2$ ($n = 1, 2, \dots$), where ω_n are the natural frequencies of the structure. The eigenfunctions are orthogonal and can be normalized so as to satisfy the orthonormality relations [39]

$$\int_{\Omega} \phi_m(D) m(D) \phi_n(D) d\Omega = \delta_{mn} \quad (2.16)$$

$$\int_{\Omega} \phi_m(D) L \phi_n(D) d\Omega = \omega_n^2 \delta_{mn} \quad m, n = 1, 2, \dots \quad (2.17)$$

where δ_{mn} is the Kronecker Delta function defined as

$$\delta_{mn} = \begin{cases} 1 & \text{for } m = n \\ 0 & \text{for } m \neq n \end{cases} \quad (2.18)$$

By using the expansion theorem, the solution of equation (2.12) can be represented by the series expansion of the form

$$w(D, t) = \sum_{n=1}^N \phi_n(D) q_n(t) \quad (2.19)$$

where $q_n(t)$ is the n^{th} generalized coordinate and N is the number of modes included in the analysis. The generalized coordinate $q_n(t)$ is the solution to the following equation [39]

$$\ddot{q}_n(t) + j2\zeta_n\omega_n\dot{q}_n(t) + \omega_n^2 = (\nu_n F + \sum_{p=1}^{M_c} \mu_{np} U_p) e^{j\omega t} \quad (2.20)$$

where

$$\begin{aligned}
\nu_n &= \text{Unit modal disturbance input} \\
\mu_{np} &= \text{Unit modal control input of the } p^{th} \text{ control input} \\
\zeta_n &= \text{Modal damping ratio}
\end{aligned}$$

and ν_n and μ_{np} are defined as

$$\nu_n = \int_{\Omega} \phi_n(D) f_d(D) d\Omega \quad (2.21)$$

$$\mu_{np} = \int_{\Omega} \phi_n(D) f_{cp}(D) d\Omega \quad (2.22)$$

Since the external loading is assumed stationary, the solution to the above equation can be conveniently carried out in the frequency domain by time Fourier transform method. The solution to equation (2.20) can be written as [39]:

$$q_n(\omega) = \left[\nu_n F(\omega) + \sum_{p=1}^{M_c} \mu_{np} U_p(\omega) \right] H_n(\omega) \quad (2.23)$$

where $H_n(\omega) = (\omega_n^2 + j2\zeta_n\omega_n - \omega^2)^{-1}$ is the n^{th} modal frequency response function.

The response of the structure as described by equation (2.19) can be rewritten as

$$w(D, \omega) = \sum_{n=1}^N \phi_n(D) q_n(\omega) \quad (2.24)$$

Once the complex amplitudes of the control inputs U_p , $p = 1, 2, \dots, M_c$ are determined, the response of the controlled system is completely described by equations (2.24) and (2.23). The complex amplitudes of the control inputs U_p can be obtained by utilizing the linear quadratic optimal control theory (LQOCT)[5].

2.3.2 Optimal control inputs

Traditionally, in active control, the optimal control inputs can be obtained by minimizing a quadratic cost function, which is usually defined as the sum of the mean-square value of the error outputs,

$$J = \boldsymbol{\epsilon}^H \boldsymbol{\epsilon} \quad (2.25)$$

where $\boldsymbol{\epsilon}$ denotes the error sensor output vector of dimension M_e by 1, M_e is the number of error sensor outputs and $(\cdot)^H$ is the Hermitian operator. The q^{th} component of the error sensor output vector $\boldsymbol{\epsilon}$ is related to the system response by the following relation

$$\epsilon_q(\omega) = w(D, \omega) b_q(D) = \sum_{n=1}^N q_n(\omega) \xi_{nq} \quad q = 1, 2, \dots, M_e \quad (2.26)$$

where $b_q(D)$ ($q = 1, 2, \dots, M_e$) are the weighting functions describing the characteristics of the q^{th} error sensor. For instance, if the error sensors consist of an array of accelerometers, each $b_q(D)$ is simply a spatial Dirac delta function denoting the locations of each accelerometer. ξ_{nq} is the modal error sensor component corresponding to the q^{th} error sensor, i.e.,

$$\xi_{nq} = \int_{\Omega} \phi_n(D) b_q(D) d\Omega \quad (2.27)$$

Substituting equation (2.23) into equation (2.26), the error sensor output vector $\boldsymbol{\epsilon}$ can be written as

$$\boldsymbol{\epsilon} = \mathbf{T}_{ce} \mathbf{U} + \mathbf{T}_{de} F(\omega) \quad (2.28)$$

where $\mathbf{T}_{de}(\omega)$ is the transfer function vector between the disturbance input and error sensor outputs, and $\mathbf{T}_{ce}(\omega)$ is the transfer function matrix between each control input and error sensor outputs. $\mathbf{T}_{de}(\omega)$ and $\mathbf{T}_{ce}(\omega)$ have dimensions of M_e by 1 and M_e by M_c , respectively.

They are defined as

$$T_{deq}(\omega) = \sum_{n=1}^N \xi_{nq} \nu_n H_n(\omega) \quad q = 1, 2, \dots, M_e \quad (2.29)$$

$$T_{cepq}(\omega) = \sum_{n=1}^N \xi_{nq} \mu_{np} H_n(\omega) \quad \begin{matrix} p = 1, 2, \dots, M_c \\ q = 1, 2, \dots, M_e \end{matrix} \quad (2.30)$$

Substituting equations (2.29) and (2.30) into equation (2.25), the cost function becomes

$$J = F^H (\mathbf{T}_{de}^H \mathbf{T}_{de}) F + \mathbf{U}^H (\mathbf{T}_{ce}^H \mathbf{T}_{de}) F + F^H (\mathbf{T}_{de}^H \mathbf{T}_{ce}) \mathbf{U} + \mathbf{U}^H (\mathbf{T}_{ce}^H \mathbf{T}_{ce}) \mathbf{U} \quad (2.31)$$

Provided the matrix $(\mathbf{T}_{ce})^H \mathbf{T}_{ce}$ is positive definite, the above cost function J represents a hyperbolic surface in terms of the real and imaginary parts of the control inputs $\mathbf{U}$ with a global minimum. The optimal control complex amplitudes $\mathbf{U}$ that render the minimum cost function can be obtained by differentiating equation (2.31) with respect to the real and imaginary part of each control input and setting them to zero, respectively. As discussed by Nelson and Elliott [5], the optimal control inputs that minimize the cost function defined in equation (2.31) are the solution of the linear system equation

$$(\mathbf{T}_{ce}^H \mathbf{T}_{ce}) \mathbf{U}(\omega) = -(\mathbf{T}_{ce}^H \mathbf{T}_{de}) F(\omega) \quad (2.32)$$

The solution to this equation, if exists, can be written as

$$\mathbf{U}(\omega) = -(\mathbf{T}_{ce}^H \mathbf{T}_{ce})^{-1} (\mathbf{T}_{ce}^H \mathbf{T}_{de}) F(\omega) \quad (2.33)$$

The linear system equations in equation (2.32) indicate that there must be equal or more error sensor outputs than control inputs in order to render a unique solution. If the number of error outputs is larger than control inputs, a solution of the control inputs is possible only in the least mean square sense. In other words, the solution given in equation (2.33) is the pseudo-inverse solution. As a consequence, the minimized cost function may not be zero after control. In the case when the number of error outputs is equal to the control inputs,

i.e., $M_e = M_c = M$, equation (2.33) reduces to

$$\mathbf{U}(\omega) = -\mathbf{T}_{ce}^{-1}\mathbf{T}_{de}F(\omega) \quad (2.34)$$

and the minimum cost function J after control is zero.

The response of the system after control can now be evaluated by substituting equation (2.34) into (2.24). The dynamic characteristics of the controlled system are investigated in the following section.

2.3.3 Dynamic modification of feedforward control

In this section, the dynamic characteristics of a MIMO feedforward controlled system are investigated. The controlled system response can be evaluated for any stationary disturbance once the control inputs are obtained. However, the global characteristics of the controlled system can be derived without the need to solve for the response of the system. These characteristics are the eigenproperties of the controlled system represented by its eigenvalues and the associated eigenfunctions or mode shapes. The main purpose of this section is to derive these properties for a general system. As we mentioned in Section 2.2, in order to analyze the dynamics of the controlled system, it is necessary to include the dynamics of the control compensator with that of the plant.

First, consider the case when the control system has equal number of error sensor outputs and control inputs, i.e., $M_c = M_e = M$. In this case the optimal control inputs are defined by equation (2.34), and it is important to note that the cost function after control is zero. The p^{th} control input, which is the solution of the linear system equation defined in equation (2.33), can be rewritten as the ratio of two determinants by using Cramer's rule [40]:

$$U_p(\omega) = -\frac{|\mathbf{T}_{ce}^p|}{|\mathbf{T}_{ce}|}F(\omega) = -G_p(\omega)F(\omega) \quad p = 1, 2, \dots, M_c \quad (2.35)$$

where $\mathbf{T}_{ce}^p$ is the determinant obtained on replacing the respective elements in the p^{th} column of $\mathbf{T}_{ce}$ by the vector $\mathbf{T}_{de}$ defined in equation (2.29).

It is convenient to rewrite equation (2.35) into a different form. Multiplying both the numerator and denominator of equation (2.35) by the product of the modal frequency response functions $\prod_{n=1}^N H_n^{-1}(\omega)$, after a series of manipulation, it is shown in Appendix B by the principle of finite induction that the p^{th} control input of the control compensator $G_p(\omega)$ can be written as

$$G_p(\omega) = \frac{\sum_{m=1}^N \sum_{n=1}^N \cdots \sum_{r=1}^N \xi_{m1} \xi_{n2} \cdots \xi_{rM} \cdot |\boldsymbol{\mu}_p| \cdot \prod_{\substack{\alpha=1 \\ \alpha \neq m, n, \dots, r}}^N (\omega_\alpha^2 - \omega^2)}{\sum_{m=1}^N \sum_{n=1}^N \cdots \sum_{r=1}^N \xi_{m1} \xi_{n2} \cdots \xi_{rM} \cdot |\boldsymbol{\mu}| \cdot \prod_{\substack{\beta=1 \\ \beta \neq m, n, \dots, r}}^N (\omega_\beta^2 - \omega^2)} = \frac{N_p(\omega)}{D(\omega)} \quad (2.36)$$

where $|\boldsymbol{\mu}|$ and $|\boldsymbol{\mu}_p|$ are the determinants of dimension M by M and they are defined in Appendix B by equations (B.7) and (B.8), respectively. The number of summation terms in equation (2.36) equals the number of control inputs M . The control compensator $G_p(\omega)$ is the ratio of two polynomials of the variable ω^2 of order $(N - m)$. The denominator $D(\omega)$ of the control compensator remains the same for all the M control channels $G_p(\omega)$. A close examination of equation (2.36) shows that $G_p(\omega)$ is a function of the control inputs, locations or placements of the error sensors and eigenproperties of the uncontrolled system, but it is independent of the disturbance input. Solving for the $(N - M)$ roots of the polynomials in equation (2.36), equation (2.36) can be rewritten as

$$G_p(\omega) = \kappa_p \frac{\prod_{l=1}^{N-M} (\rho_{lp}^2 - \omega^2)}{\prod_{l=1}^{N-M} (\omega_{cl}^2 - \omega^2)} \quad p = 1, 2, \dots, M \quad (2.37)$$

where κ_p is defined as

$$\kappa_p = \frac{\sum_{m=1}^N \sum_{n=1}^N \cdots \sum_{r=1}^N \xi_{m1} \xi_{n2} \cdots \xi_{rM} \cdot |\boldsymbol{\mu}_p|}{\sum_{m=1}^N \sum_{n=1}^N \cdots \sum_{r=1}^N \xi_{m1} \xi_{n2} \cdots \xi_{rM} \cdot |\boldsymbol{\mu}|} \quad p = 1, 2, \dots, M \quad (2.38)$$

The roots of the numerator and denominator in equation (2.37) give the zeros (ρ_{lp}^2) and poles (ω_{cl}^2) of the control compensators. By using partial fraction expansion, the control compensator $G_p(\omega)$ can be written in a parallel form representation as follows

$$G_p(\omega) = \frac{\kappa_p}{N-M} \sum_{l=1}^{N-M} (A_{lp} - \omega^2) H_{cl}(\omega) \quad p = 1, 2, \dots, M \quad (2.39)$$

where the coefficients A_{lp} of the partial fraction can be obtained by standard partial expansion procedure. The controller defined in the form of equation (2.39) is known as the parallel form realization. Each $H_{cl}(\omega) = (\omega_{cl}^2 - \omega^2)^{-1}$ is equivalent to the frequency response function of a single-undamped oscillator, where ω_{cl} is the corresponding natural frequency. These controlled natural frequencies are in general different from the natural frequencies of the original system, unless the error sensors are at nodal points and/or the control force distributions f_{cp} are orthogonal to the mode shape. This implies that the mode is unobservable and/or uncontrollable. Hereinafter we assume that all modes are observable and controllable.

Substituting equation (2.39) into equation (2.23), the response of the controlled system given by equation (2.24) can be written as:

$$w(D, \omega) = \sum_{n=1}^N \left[\nu_n H_n(\omega) - \sum_{p=1}^M \mu_{np} \frac{\kappa_p}{N-M} \sum_{l=1}^{N-M} (A_{lp} - \omega^2) H_{cl}(\omega) H_n(\omega) \right] \phi_n F(\omega) \quad (2.40)$$

The product of the control compensator and system modal frequency response functions in the above equation can be written in partial fractions as follows:

$$(A_{lp} - \omega^2) H_{cl}(\omega) H_n(\omega) = C_{pln} H_{cl}(\omega) + D_{pln} H_n(\omega) \quad (2.41)$$

where the partial fraction coefficients C_{pln} and D_{pln} are given by

$$C_{pln} = \frac{\omega_{cl}^2 - A_{lp}}{\omega_{cl}^2 - \omega_n^2} \quad D_{pln} = \frac{A_{lp} - \omega_n^2}{\omega_{cl}^2 - \omega_n^2} \quad (2.42)$$

Substituting equation (2.41) into equation (2.40) and reordering terms gives

$$\begin{aligned}
w(D, \omega) = & \sum_{n=1}^N \left[\nu_n - \sum_{p=1}^M \mu_{np} \frac{\kappa_p}{N-M} \sum_{l=1}^{N-M} D_{pln} \right] H_n(\omega) \phi_n F(\omega) \\
& - \sum_{n=1}^N \left[\sum_{p=1}^M \mu_{np} \frac{\kappa_p}{N-M} \left(\sum_{l=1}^{N-M} C_{pln} H_{cl}(\omega) \right) \right] \phi_n F(\omega)
\end{aligned} \tag{2.43}$$

It can be shown that the first term in equation (2.43) vanishes at all frequencies. For the sake of conciseness, the proof of this statement is presented in Appendix B.2.

The response of the controlled system can then be written as

$$w(D, \omega) = \sum_{l=1}^{N-M} \left[- \sum_{n=1}^N \sum_{p=1}^M \mu_{np} \frac{\kappa_p}{N-M} C_{pln} \phi_n \right] H_{cl}(\omega) F(\omega) \tag{2.44}$$

or in the typical form found in modal analysis [39] as follows:

$$w(D, \omega) = \sum_{l=1}^{N-M} \frac{f_{cl}}{m_{cl}} \phi_{cl} H_{cl}(\omega) F(\omega) \tag{2.45}$$

where

$$\frac{f_{cl}}{m_{cl}} \phi_{cl} = - \sum_{n=1}^N \sum_{p=1}^M \mu_{np} \frac{\kappa_p}{N-M} C_{pln} \phi_n \tag{2.46}$$

Equation (2.45) shows that the response of the system after control can be expressed in terms of a new set of $(N - M)$ mode shapes ϕ_{cl} with corresponding natural frequencies ω_{cl} . The modal mass is m_{cl} and f_{cl} is the l^{th} modal component of the input distribution. Equation (2.45) has exactly the same form as equation (2.9) for the single-input, single-output feedforward control system. Equation (2.46) suggests that the l^{th} mode shape ϕ_{cl} of

the controlled system is given as a linear expansion of the uncontrolled mode shapes:

$$\phi_{cl} = \sum_{n=1}^N \Gamma_{nl} \phi_n \quad l = 1, 2, \dots, N - M \quad (2.47)$$

where the expansion coefficient Γ_{nl} is defined as

$$\Gamma_{nl} = k_l \sum_{p=1}^M \mu_{np} \kappa_p C_{pln} \quad (2.48)$$

and the normalization constant k_l can be determined so that the controlled system mode shapes satisfy the orthonormal conditions represented by equation (2.16), i.e.,

$$\int_{\Omega} \phi_{cl}(D) m(D) \phi_{cl'}(D) d\Omega = \delta_{ll'} \quad l, l' = 1, 2, \dots, N - M \quad (2.49)$$

Substitution of equation (2.47) into equation (2.49) renders the normalization constant

$$k_l = \left[\sum_{n=1}^N \left(\sum_{p=1}^M \mu_{np} \kappa_p C_{pln} \right)^2 \right]^{-1/2} \quad (2.50)$$

The determination of the expansion coefficients Γ_{nl} as defined in equations (2.48) and (2.50) involves the calculation of the partial fraction coefficients defined in equations (2.39) and (2.42), which is a computationally complex process in general. For convenience, a recursive formulation is developed to calculate these expansion coefficients. The main idea of the recursive formulation is that we apply only one control input to the structure at a time, and then calculate the eigenproperties (i.e., natural frequencies and mode shapes) of the system consisting of only one control input. The “so obtained” set of eigenproperties are treated as the eigenproperties of the “uncontrolled system” for the next control input, and so forth. By this way we can repeatedly apply equation (2.11) to calculate the expansion coefficient Γ_{nl} for the current MIMO controlled system. This formula takes advantage of the fact that eigenproperties of a system are global properties of the system and independent of external forces. The main advantage of this formula is that it simplifies the computational

complexity involved in the calculation of the expansion coefficients in equations (2.48) and (2.50), and most of all, it clearly reveals the fact that the size of the digital filter necessary to implement the control compensator will be reduced with increasing number of control channels, as we will discuss later.

Equation (2.45) shows that the response of the system after control has resonances at the frequencies ω_{cl} , which are the poles of the control compensator $G_i(\omega)$ defined in equation (2.36). In other words, herein above we have proved that the poles of the control compensators give the eigenvalues (hence resonance frequencies) of the controlled system. The proof provided here is completely parallel to the proof presented in Appendix A, which is based on frequency response function approach. The polynomial $D(\omega)$ as defined in equation (2.36) is the characteristic equation of the controlled system. Moreover, $D(\omega)$ is a function of the control force distribution, location of the error sensor and the structure eigenproperties only. The controlled eigenproperties depend only on these factors and are independent of the external loading. The later statement may not be clear from equations (2.48) and (2.50), since the expansion coefficients Γ_{nl} as defined in these two equations are functions of the disturbance input. However, if we recall the recursive scheme developed above and examine equation (2.11) carefully, we can see that the expansion coefficients Γ_{nl} are actually independent of the disturbance input. Therefore, once the control forces and error sensors placements are specified, the frequency characteristics of the controlled system are fixed and remain the same for any coherent input disturbance. It is also important to mention that the applied control forces reduce by M the dynamic degrees of freedom of the original uncontrolled structure due to the constraint imposed by driving the responses at M error sensor locations to zero. This implies that the complexity of the control filters will be reduced with increasing number of control channels if one also notices that the denominator $D(\omega)$ is the same for all control compensators $G_i(\omega)$.

It should be noted that this system modal modification is for feedforward control where coherent information from the disturbance input is applied to the plate as a control action. To any other incoherent noise disturbance the system will exhibit its uncontrolled modal response. The implications of this are profound: a system can be made to appear totally different to a required noise disturbance (e.g., soft) while remaining unchanged to other

inputs (e.g., stiff). This behavior should be contrasted to a system with feedback control, in which case the system characteristics are generally changed for all disturbance inputs.

For the case in which the number of error outputs are larger than the number of control inputs, the control compensator can only be uniquely determined in a least mean square sense. As a direct consequence, the cost function after control (hence the responses at error sensor locations) is not equal to zero in general, and the above theory does not apply.

2.4 Simulation on a simply supported plate

The applicability of the formulation developed here is demonstrated on a simply supported uniform rectangular steel plate subject to two-input, two-output feedforward control, as shown in Figure 2.3.

2.4.1 Equations of motion

The plate is assumed made of homogeneous, linear elastic, isotropic material. The thickness of the plate is much smaller compared to other dimensions of the plate and flexural wavelength so that the rotary inertia and shear deformation can be ignored. The plate is excited by a concentrated harmonic force $F(\omega)$ located at $D(x_d, y_d)$. The two concentrated control forces $U_1(\omega)$ and $U_2(\omega)$, located at positions $C_1(x_{c1}, y_{c1})$ and $C_2(x_{c2}, y_{c2})$, are determined to minimize the mean square of the summation of the responses of the plate at error sensor locations. For simplicity, the error sensors and control forces are assumed to be collocated as shown in Figure 2.3. Following Meirovitch [39] the equation of motion can be written as

$$\rho_s h \ddot{w} + C \dot{w} + D \nabla^4 w = -p(x, y, \omega) \quad (2.51)$$

where

w = Out-of-plane displacement

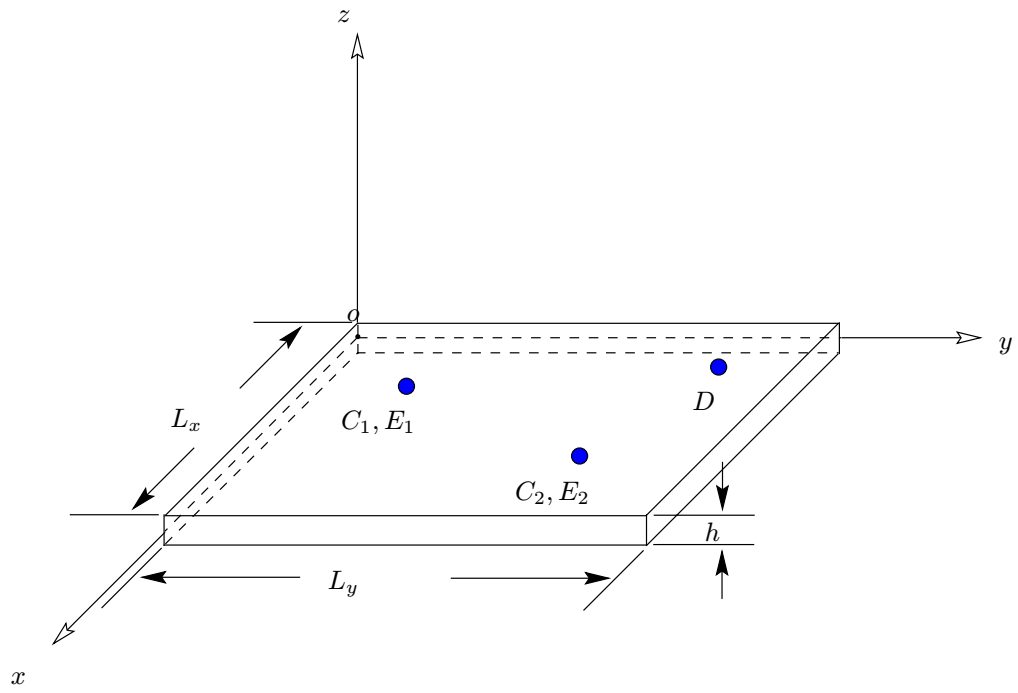


Figure 2.3: Control configuration of the simply supported plate

ρ_s	=	Mass density of the plate
h	=	Thickness of the plate
C	=	Damping coefficient
D	=	Flexural rigidity of the plate
∇^4	=	Biharmonic operator
$p(x, y, \omega)$	=	Applied external load

and D , ∇^4 and $p(x, y, \omega)$ are defined as

$$D = \frac{Eh^3}{12(1 - \nu^2)} \quad (2.52)$$

$$\nabla^4 = \nabla^2 \nabla^2 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \quad (2.53)$$

$$p(x, y, \omega) = F(\omega) \delta(x - x_d) \delta(y - y_d) + \sum_{p=1}^2 U_p(\omega) \delta(x - x_{cp}) \delta(y - y_{cp}) \quad (2.54)$$

where $\delta(\cdot)$ is the Dirac delta function defined as

$$\delta(x - x_0) = \begin{cases} 0 & \text{if } x \neq x_0 \\ 1 & \text{if } x = x_0 \end{cases} \quad (2.55)$$

and ν is the poisson's ratio and E is the Young's modulus of elasticity. Here, one must recognize that the equation of motion of the plate represented by equation (2.51) conforms to the general formulation presented in Section 2.3.1 with $L = D\nabla^4 + C$ and $m(D) = \rho_s h$.

For a simply supported plate, both the traverse displacements and moments must vanish at the plate boundaries. The boundary conditions are thus specified by

$$w(x, y) \Big|_{x=0, L_x} = \frac{\partial^2 w(x, y)}{\partial x^2} \Big|_{x=0, L_x} = 0 \quad (2.56)$$

$$w(x, y) \Big|_{x=0, L_y} = \frac{\partial^2 w(x, y)}{\partial x^2} \Big|_{x=0, L_y} = 0 \quad (2.57)$$

Solving the eigenvalue problem of the homogeneous equation associated with equation (2.51) gives the natural frequencies and the normalized mode shapes of the simply supported plate. They are

$$\omega_{mn} = \sqrt{\frac{D}{\rho_s h}} \left[\left(\frac{m\pi}{L_x} \right)^2 + \left(\frac{n\pi}{L_y} \right)^2 \right] \quad (2.58)$$

$$\phi_{mn}(x, y) = \frac{2}{\sqrt{L_x L_y \rho_s h}} \sin\left(\frac{m\pi}{L_x} x\right) \sin\left(\frac{n\pi}{L_y} y\right) \quad (2.59)$$

where m, n are modal indices.

The forced response of the plate can be obtained by modal decomposition [39]. Assuming the solution to equation (2.51) is in the form:

$$w(x, y, \omega) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \phi_{mn} q_{mn}(\omega) \quad (2.60)$$

Substitution of the above assumed solution into equation (2.51) and using the orthogonality property of the plate mode shapes in the x and y directions, the generalized coordinates $q_{mn}(\omega)$ can be obtained as

$$q_{mn}(\omega) = [\nu_{mn} F(\omega) + \sum_{p=1}^2 \mu_{mnp} U_p(\omega)] H_{mn}(\omega) \quad (2.61)$$

where

ν_{mn} = Unit modal disturbance input

μ_{mnp} = Unit modal control input of the p^{th} control channel

$H_{mn}(\omega)$ = Modal frequency response function

These are defined as

$$\nu_{mn} = \phi_{mn}(x_d, y_d) \quad (2.62)$$

$$\mu_{mnp} = \phi_{mn}(x_{cp}, y_{cp}) \quad p = 1, 2 \quad (2.63)$$

$$H_{mn}(\omega) = (\omega_{mn}^2 - j2\zeta_{mn}\omega_{mn}\omega + \omega^2)^{-1} \quad (2.64)$$

where $\zeta_{mn} = C/(2\rho_s h \omega_{mn})$ is the modal damping factor.

The optimal control inputs $U_p(\omega)$, $p = 1, 2$ are determined to minimize the sum of the mean square signals from the error sensors located at E_1 and E_2 , and are given by equation (2.34). Computer simulation of the responses of the control system will be presented in the next section.

2.4.2 Numerical simulations

The dimensions and material properties of the plate used in this study are listed in Table 2.1. The locations of the disturbance and control inputs are presented in Table 2.2. The simulations are performed over the frequency range of 0 Hz to 560 Hz. Only the first nine modes of the plate are included in the simulation since they provide adequate accuracy in predicting the system response within this frequency range. These first nine natural frequencies of the plate, obtained from equation (2.59), are presented in Table 2.3.

To illustrate the dynamic behavior of the plate before and after control, the plate normal acceleration response at the disturbance input location is computed. Assuming a unit disturbance input force, i.e., $F(\omega) = 1$, the acceleration response of the uncontrolled system is computed from equations (2.60) and (2.61) by setting $U_p(\omega) = 0$. The response corresponding to the controlled system can be obtained from the same equations with $U_p(\omega)$ given by equation (2.34). The amplitude and phase of the acceleration at the disturbance input location are shown in Figure 2.4 as functions of the excitation frequency ω . By convention, a solid line is used to represent the response of the plate without control and a dashed line is used to describe the response of the plate after control. From Figure 2.4, one can see that

Table 2.1: Dimensions and material properties of the plate

Parameter	Magnitude
Length, L_x (m)	0.38
Width, L_y (m)	0.3
Thickness, h (m)	1.96e-3
Young's Modulus, E (N/m)	2.0e+11
Poisson's Ratio, ν	0.29
Mass Density, ρ_s (kg/m ³)	7833
Damping, ζ_n	0.15%

Table 2.2: External inputs/error sensors locations on the simply supported plate

Type	Coordinate (m)
Disturbance Point Force, $D(x_d, y_d)$	(0.06, 0.25)
Control Point Force #1, $C_1(x_{c1}, y_{c1})$	(0.114, 0.09)
Control Point Force #2, $C_2(x_{c2}, y_{c2})$	(0.266, 0.21)
Error Sensor #1, $E_1(x_{e1}, y_{e1})$	(0.114, 0.09)
Error Sensor #2, $E_2(x_{e2}, y_{e2})$	(0.266, 0.21)

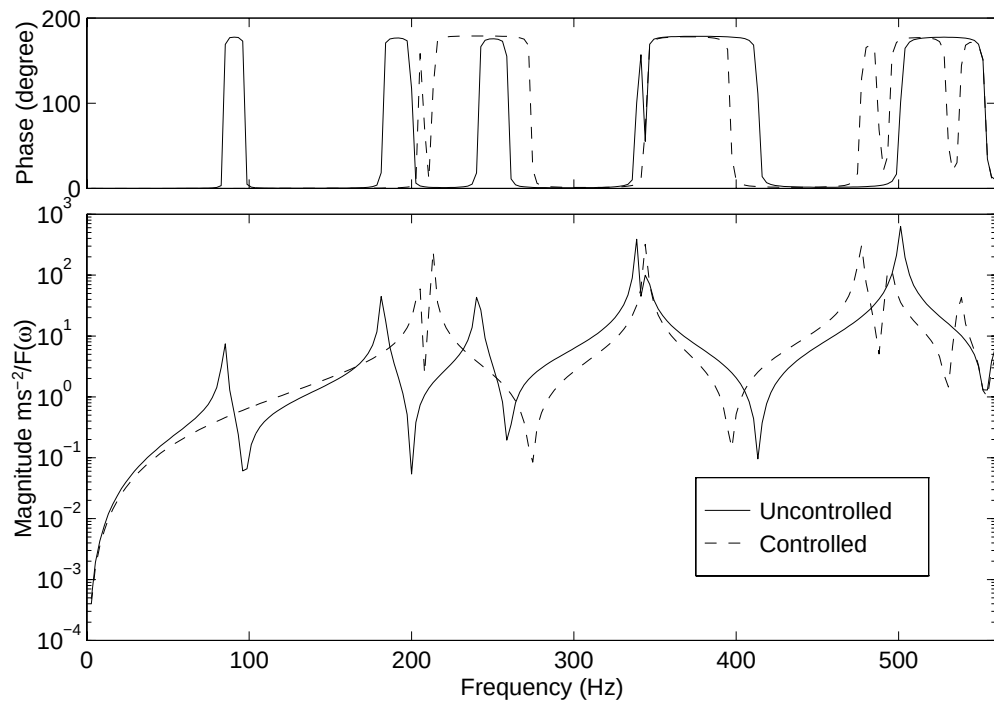


Figure 2.4: Theoretical acceleration response at disturbance input location

Table 2.3: Analytical natural frequencies of the plate

Index	Mode	$f_{mn} = \omega_{mn}/2\pi$ (Hz)
1	(1,1)	84.5
2	(2,1)	182.1
3	(1,2)	241.1
4	(2,2)	338.5
5	(3,1)	344.6
6	(3,2)	501.0
7	(1,3)	501.8
8	(4,1)	572.1
9	(2,3)	599.2

the uncontrolled plate shows resonant peaks at frequencies given in Table 2.3, while on the other hand, when the control forces $U_1(\omega)$ and $U_2(\omega)$ are applied, the response of the plate shows resonant behavior at excitation frequencies near 205, 213, 344, 477, 495, 538, and 594 Hz. The computed phase angle, in both uncontrolled and controlled system, also shows the typical 180° phase shift at the resonance and anti-resonance frequencies. Thus, it is clear from Figure 2.4 that the controlled system effectively behaves as having new resonance frequencies (to the disturbance) that are different from those of the uncontrolled plate.

As we have discussed in the previous sections, the eigenvalues (hence the resonance frequencies) of the feedforward controlled system are given by the poles of the control compensator. The roots of the polynomial $D(\omega)$ in equation (2.36), i.e., poles of the control compensator, are solved by using the IMSL [41] subroutine *ZPORC* (Jenkin -Traub three stage method), and they are presented in Table 2.4. Also, in order to show the validity of the recursive scheme proposed in Section 2.3.3, the above computations are performed again following the recursive scheme. The same IMSL subroutine is used. Exactly the same results are obtained with the direct and recursive approach. As we have shown in the formulation presented in Section 2.3.3, the controlled system has two less dynamic degrees of

Table 2.4: Analytical eigenvalues and natural frequencies of the controlled system

Mode	Eigenvalue ω_{cl}^2	Natural Frequencies f_{cl} (Hz)
1	0.1652e+7	204.6
2	0.1794e+7	213.2
3	0.4677e+7	344.2
4	0.8972e+7	476.7
5	0.9652e+7	494.5
6	0.1141e+8	537.6
7	0.1393e+8	593.9

freedom than the original plate due to the constraint imposed by driving the error sensors outputs to zero. Since we include nine uncontrolled modes in the analysis, there are seven controlled eigenvalues or roots. The last column in Table 2.4 gives the natural frequencies of the controlled system.

The amplitudes and phases of the control compensator $G_p(\omega)$, ($p = 1, 2$), using equation (2.34), are computed and shown in Figure 2.5. Comparing Figures 2.4 with 2.5, we conclude that large control effort is required to minimize the response at the resonance frequencies of the controlled system. Thus, in practice, the locations of the controlled plate resonance frequencies are important because they indicate possible points of control instability or control implementation difficulty for a particular arrangement.

The mode shapes corresponding to the controlled natural frequencies are obtained from equation (2.47). Due to the difficulties associated with the computation of the expansion coefficients as defined in equations (2.48) and (2.50), the recursive scheme discussed in Section 2.3.3 is adopted.

Figures 2.6 and 2.7 give the first seven mode shapes of the uncontrolled system versus those of the controlled system. We can see that all the controlled eigenfunctions have nodal points at the error sensor locations. This is in contrast to the conventional understanding

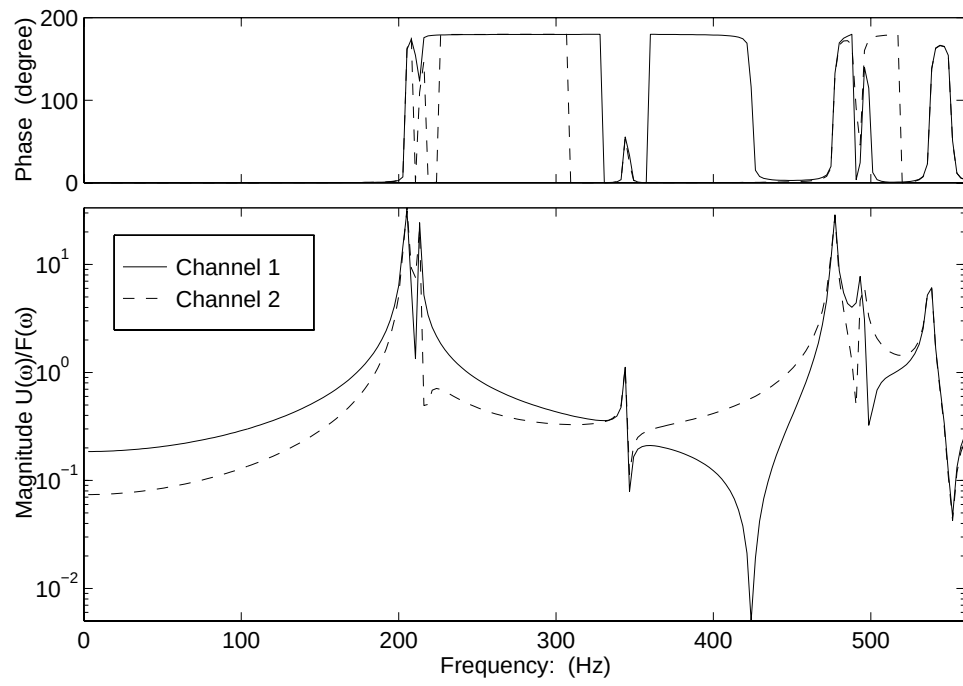


Figure 2.5: Analytical transfer function of the control compensators

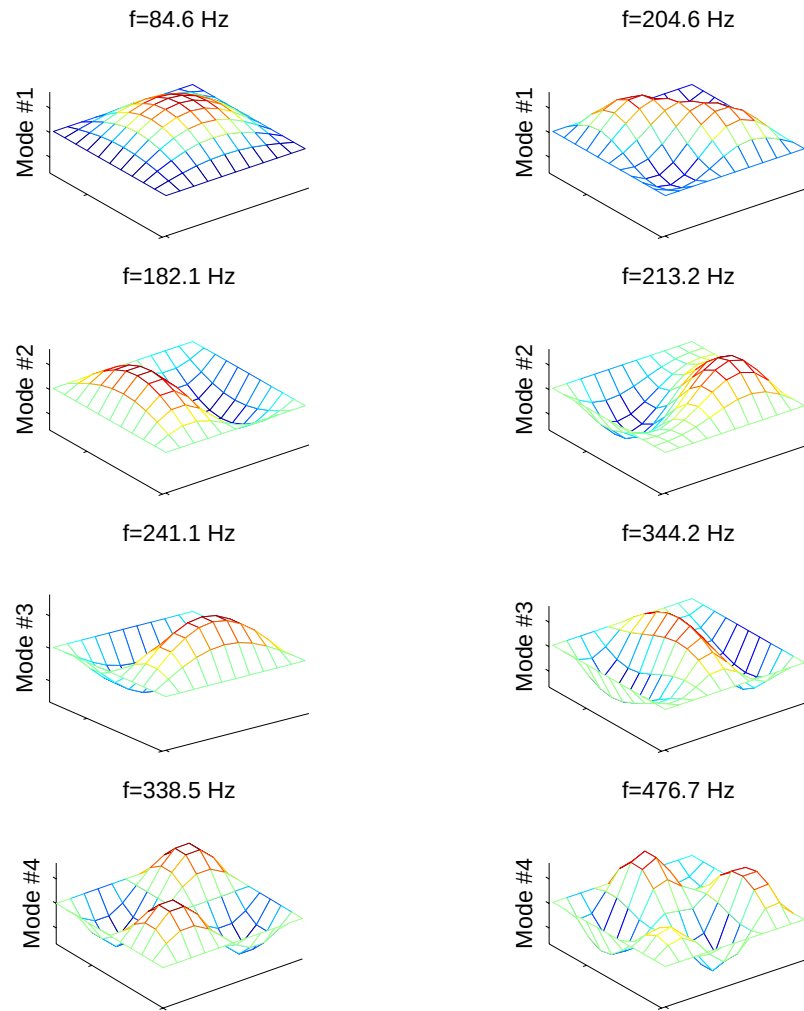


Figure 2.6: Analytical mode shapes of the first four modes: Uncontrolled versus Controlled system

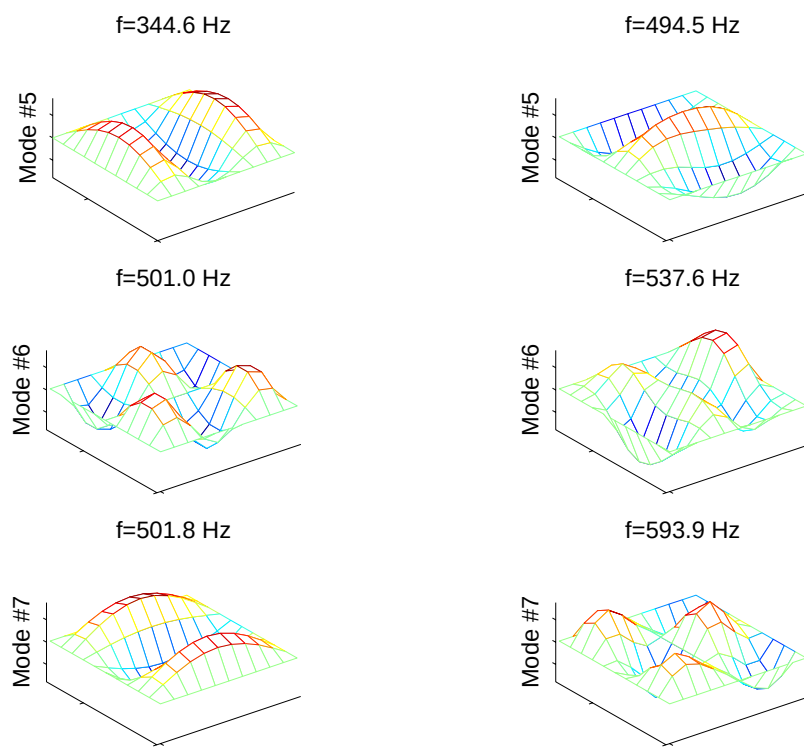


Figure 2.7: Analytical mode shapes of the fifth to seventh modes: Uncontrolled versus Controlled system

which explains this phenomenon in terms of the uncontrolled modes "phase canceling" the response to zero at those points. It can also be demonstrated that if the error sensors are located at the nodes of an uncontrolled mode, then that mode is undetectable and its mode shape will remain unchanged in the frequency response function of the controlled system.

2.5 Experimental verification on a simply supported plate

To validate the theoretical formulations proposed in this Chapter, an experimental investigation on a simply supported plate is performed.

2.5.1 Experiment test rig

The experimental rig used in this test consists of a thin carbon steel plate mounted in a heavy steel support stand. The plate is attached to the stand by shims at the edges of the plate. The shims are extremely stiff in extension but are soft in rotation. Thus, this arrangement reasonably approximates the simply supported boundary conditions. The plate's parameters, given in Table 2.1, are identical to the plate in the numerical example presented in Section 2.4.2.

The disturbance input and control forces are applied by three mini-shakers (Ling Dynamics Systems-Model 201) attached point-wise to the plate. The shakers are suspended by long elastic cords to minimize the introduction of additional dynamics due to the shaker mounting. Three B&K accelerometers (Model 4374) are mounted on the plate. One is used to measure the acceleration response at the disturbance input location and the other two are used as control error sensors. One Kistler (Model 9712A500) and two PCB (Model 208A04) force transducers are used to monitor the shaker forces. To experimentally verify the previous analytical results, a two-input, two-output adaptive feedforward control system is implemented. A schematic of the control system is given in Figure 2.8. The control approach

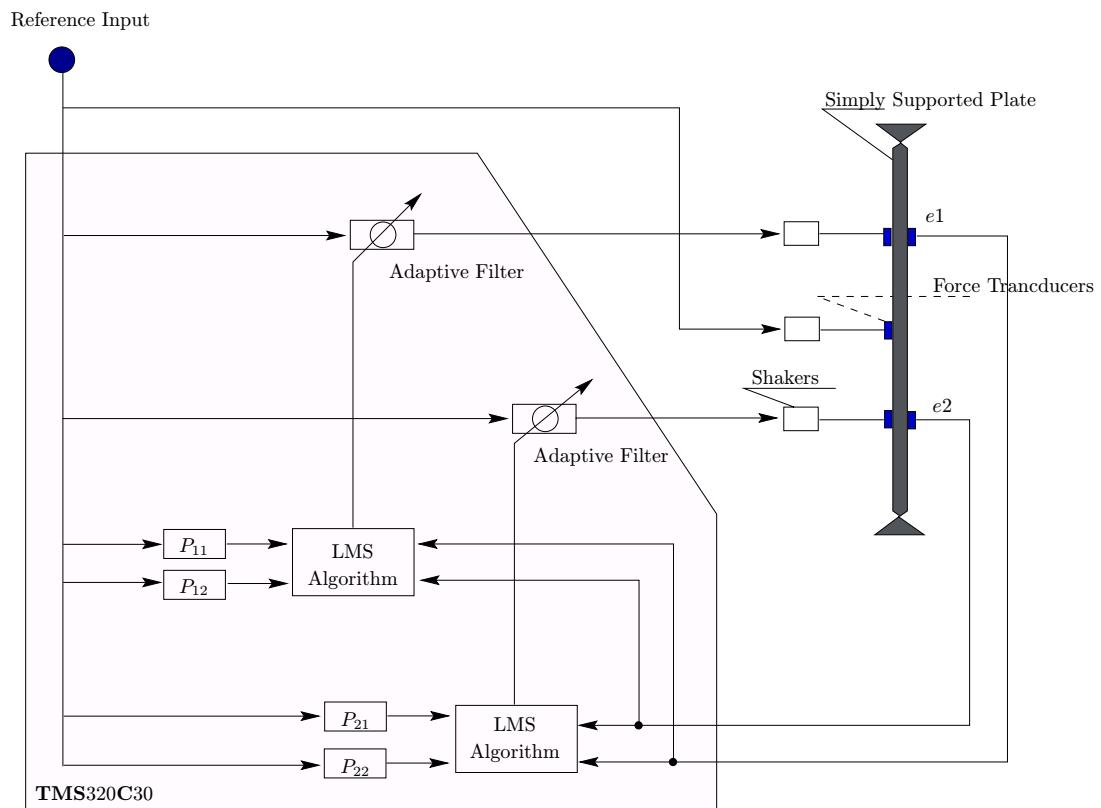


Figure 2.8: Schematic of 2I2O feedforward control of a simply supported plate

is based upon the filtered- x LMS algorithm as described in reference [4]. The algorithm is coded in assembly language and downloaded to a PC-based Texas Instrument TMS320C30 digital signal processing board. As can be seen from Figure 2.8, the control system adapts the coefficients of the FIR control filter based upon reference and error signals. As in the numerical example, the disturbance is a pure tone sinusoid and only two coefficients are needed in the FIR filter. Similar to the theoretical formulation presented in Section 2.3.3, the reference signal is taken from the same signal generator used for disturbance, providing a coherent control signal with the disturbance. As described in reference [4], the adaptive LMS algorithm uses a cost function which is based upon the sum of the mean square amplitude of the error signals. The standard control approach is to minimize the cost function. Thus, when the controller has converged, its optimal transfer function should be identical to the transfer functions of the control compensator derived in the above analysis. This implies that the eigenvalues and eigenfunctions of the controlled simply supported plate should be identical to those prescribed in the previous section.

In this experiment, structural vibrations are measured with a Politec laser vibrometer (Model OFV-2600). The laser head is mounted on a two dimensional traverse to allow measurement over the complete surface of the plate. The traverse allows the laser head to move in a plane parallel to the plate surface. The laser beam is kept perpendicular to the plate surface thus the output of the laser vibrometer is proportional to the out-of-plane velocity of the plate. The traverse is controlled through the parallel port of another PC. All the measurements are analyzed in the frequency domain with a two channel Brüel & Kjær dynamic analyzer (Model 2032). An in-house software program written in QBASIC is used to interface with the Brüel & Kjær analyzer through GPIB interface.

2.5.2 Experimental procedure

The measurement of the mode shapes of the plate after control poses a special challenge in this work. As we have discussed in Section 2.3.3, the system modal modification is for feedforward control where coherent information from the disturbance input is applied to the plate as a control action. For any other incoherent disturbance the system will exhibit

its uncontrolled modal response. For this reason, the standard experimental modal analysis procedure, which typically uses random or pseudo-random signals to excite the structure, will not give the desired mode shapes of the controlled system. In this study, at each frequency point, the response of the plate at the disturbance location (or any other point) before and after control are measured with a Brüel & Kjær accelerometer. Since the plate used in this experiment is lightly damped (damping factor $\zeta \leq 0.1\%$, i.e., see Maillard [36]), the peak frequencies of the measured response of the plate are considered to be a good approximation to the natural frequencies of the plate. The plate is then driven at the obtained resonance frequencies and the responses of the plate are measured with laser vibrometer over a grid containing 10 points along the x -axis and 9 points along the y -axis. The real and imaginary parts of the transfer functions between the laser vibrometer and disturbance force transducer are downloaded automatically into the PC for post processing. The auto-spectrum of the signals from the disturbance force transducer are always measured to monitor the disturbance force. The measured responses of the plate at resonance frequencies, after being normalized with respect to the mass distribution of the plate, are considered to be a good approximations to the associated mode shapes.

2.5.3 Experimental results

In this experiment, in order to examine the boundary conditions, two tap tests are carried out to experimentally determine the natural frequencies of the plate without control. The first test is conducted for the plate-frame system only. To evaluate the influence of the attached shakers and force transducers to the natural frequencies of the plate-frame system, a second test is conducted after they are attached to the plate. For both tests, a movable Brüel & Kjær accelerometer is used to measure the response of the plate at peak location of the interested mode. The peak frequency of the measured transfer function between the disturbance force transducer and the accelerometer is considered as the natural frequency of that mode. The experimentally measured natural frequencies are listed in Table 2.3, excellent agreement between the analytical and the first test is found. However, after the shakers and force transducers are attached, the results show difference of up to 10%. This

Table 2.5: Natural frequencies of the simply supported plate

Index	Mode	Analytical (Hz)	Test 1(Hz)	Test 2 (Hz)
1	(1,1)	84.5	87	85
2	(2,1)	182.1	184	172
3	(1,2)	241.1	245	232
4	(2,2)	338.5	334	307
5	(3,1)	344.6	347	346
6	(3,2)	501.0	483	469
7	(1,3)	501.8	501	495
8	(4,1)	572.1	578	564
9	(2,3)	599.2	595	646

arrangement will affect the results of the experiment somewhat, but we will see later that the main points of this chapter are still clearly demonstrated. The mode shapes of the plate-frame with shakers and force transducers attached to them are measured. The first 7 mode shapes are shown in Figures 2.9 and 2.10. As can be seen, for the first six modes the experiment results agree very well with the simulation results, confirming that the shims do simulate the simply supported boundary condition fairly well. For the higher order modes, however, the experimental results have some distortions as can be seen from Figure 2.10 due to the influence of the shakers and force transducers.

In order to determine the influence of the feedforward control compensators on the dynamics of the original simply supported plate, the acceleration response of the plate at disturbance input locations, before and after control, were measured at each of the selected frequencies. Figure 2.11 shows the acceleration response of the plate at the disturbance location before control.

It is apparent that there is good agreement between the experimental and simulation values over the complete frequency range (on- and off- resonance). The shift to lower frequencies

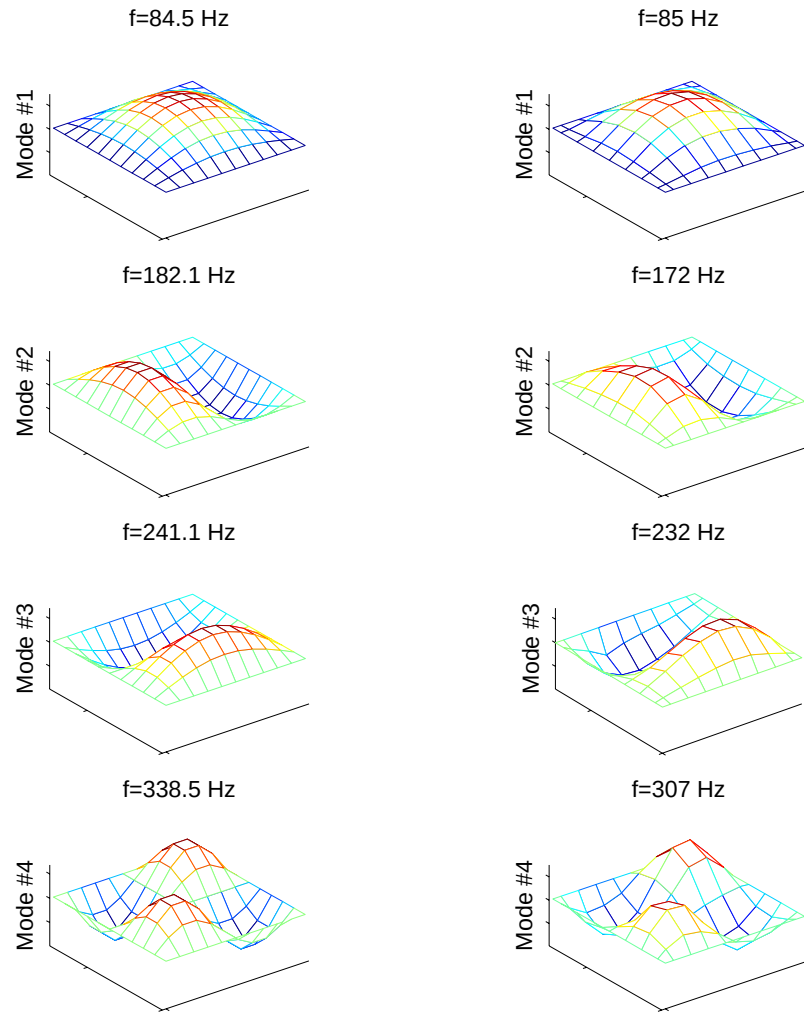


Figure 2.9: Experimental mode shapes of the four modes of the uncontrolled plate:
Analytical versus Experimental

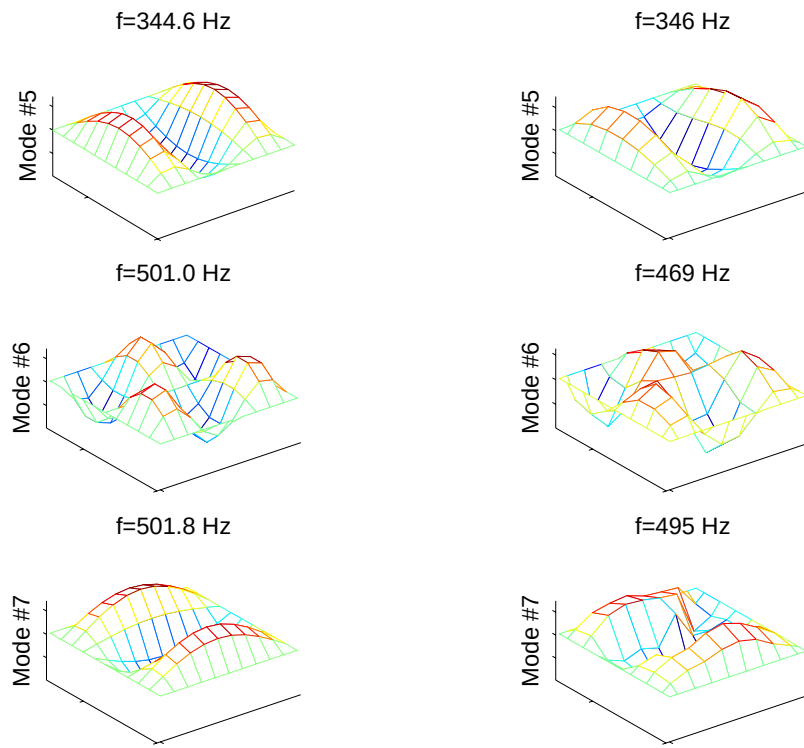


Figure 2.10: Experimental mode shapes of the fifth to seventh modes of the uncontrolled plate: Analytical versus Experimental

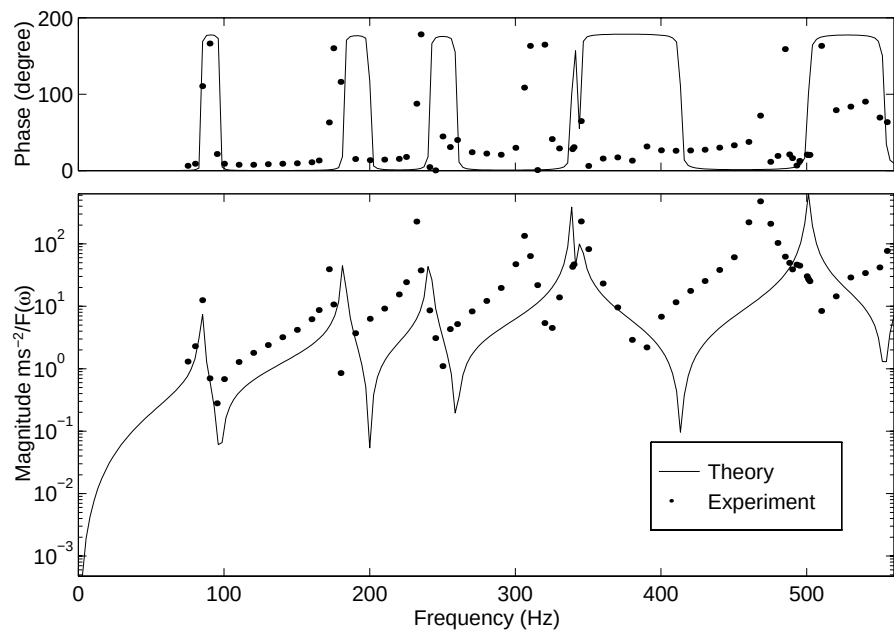


Figure 2.11: Acceleration response at the disturbance location of the uncontrolled system: Analytical versus Experimental results

in the experimental results is partly due to the influence of the shakers and force transducers attached to the plate, as expected from the above modal analysis tests. One also should note that the boundary conditions of the plate are not perfectly simply supported. From this, it can be expected that the acceleration response of the controlled system at disturbance input locations will also shift to the left a little. This deviation from the simulation results will become more significant at higher resonance frequencies. From Figure 2.12 we can see that this is indeed the case. However, the experimentally measured response shows the same number of resonant peaks as the numerical simulations, and the phase of the response has typical jump of 180° at these peak frequencies. Obviously, the trend of the experimental results agrees quite well with analytical predictions. The experimentally measured transfer

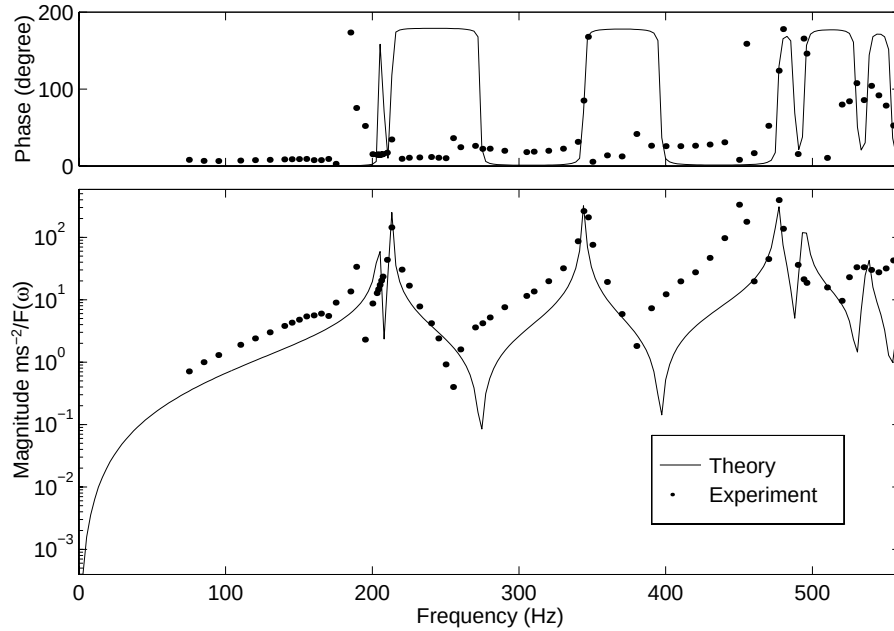


Figure 2.12: Acceleration response at disturbance location of the controlled system:
Analytical versus Experimental results

functions of the control compensator are presented in Figures 2.13 and 2.14. They were determined by measuring the transfer functions between the disturbance force transducer and control force transducers at each tested frequencies. Figures 2.13 and 2.14 show that

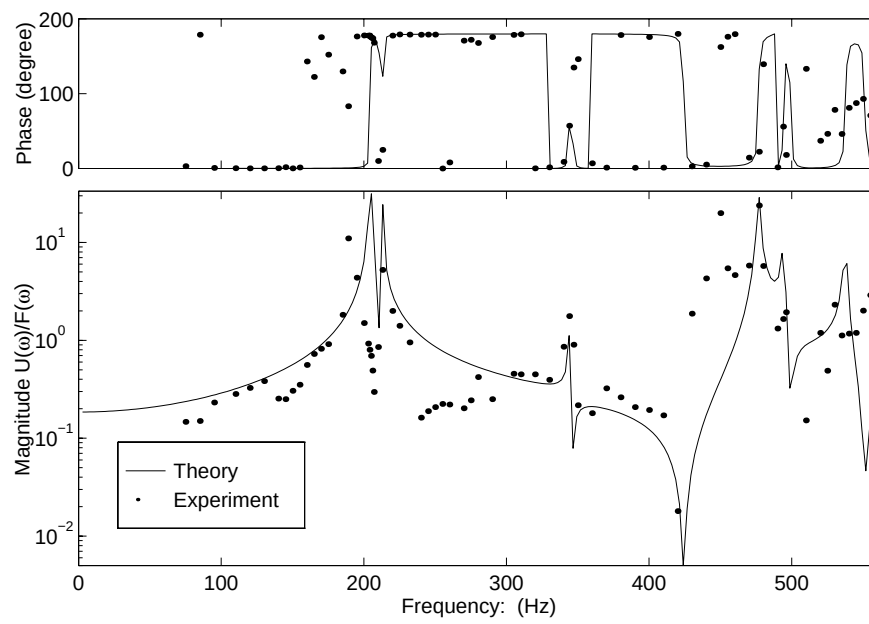


Figure 2.13: Transfer function of control compensator channel 1: Analytical versus Experimental results

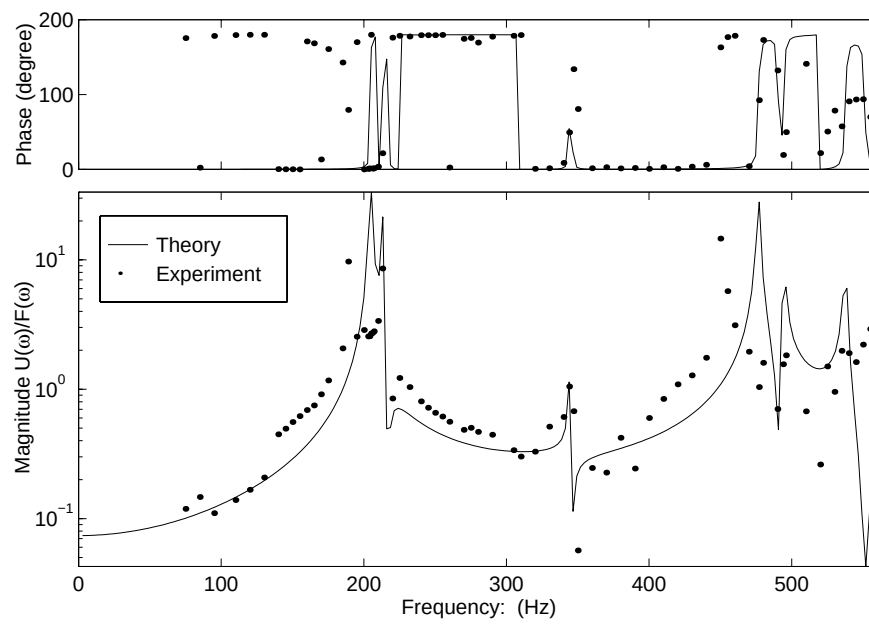


Figure 2.14: Transfer function of control compensator channel 2: Analytical versus Experimental results

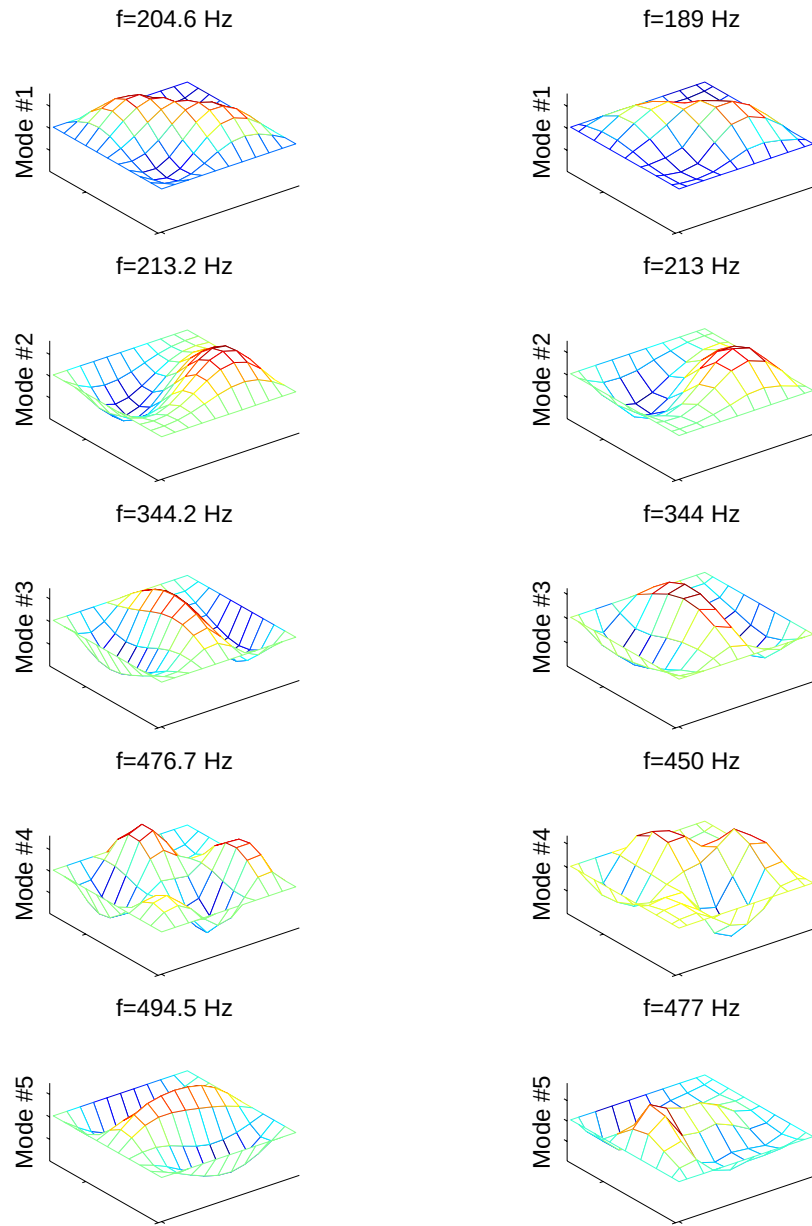


Figure 2.15: Experimental mode shapes of the first five modes of the controlled system: Analytical versus Experimental results

the trend of the experimental results is the same as the simulation results. However, we can see that the peaks of the transfer functions of the control compensator also shift to the left at higher resonance frequencies for the same reasons. One can see that large control effort is necessary to suppress the response of the plate at the resonance frequencies. Therefore, in designing the feedforward control system, these frequencies should be kept out of the frequency range of concern.

The mode shapes of the controlled system associated with the first five resonance frequencies as shown in Figure 2.12 were experimentally determined by driving the controlled system at these resonance frequencies and then measuring the plate responses at 10x9 equally spaced positions using the laser vibrometer. The measured first five mode shapes, after being normalized with respect to the mass distribution of the plate, are plotted in Figure 2.15, and excellent agreement with the computed mode shapes is found. These computed mode shapes are obtained from the individual characteristic functions defined by equation (2.47) instead of evaluating the total system response at the frequency of interest. We can see that the results of the experiment strongly support the analytical development of this work.

2.6 Summary

In this chapter, the dynamic behavior of a MIMO feedforward control system was investigated. The theoretical and experimental studies presented in this chapter show that, when the control system has equal number of error sensors and control actuators, the MIMO feedforward controlled system effectively behaves as having new eigenproperties. A new mathematical approach was developed to compute the MIMO controlled system resonance frequencies and mode shapes. The controlled system eigenvalues are obtained by solving for the poles of the control compensator. The characteristic equation of the control compensator is also the characteristic equation of the controlled system. The associated mode shapes of the controlled system are found to be a linear combinations of those of the uncontrolled system. These eigenproperties are functions of the applied control forces and error sensor locations but are independent of the characteristics of the disturbance input.

The theory was numerically illustrated by two-input, two-output control of a simply supported plate excited by a harmonic disturbance. In addition, the control algorithm was implemented on a digital signal processing board, and experimental data were obtained and compared to the analytical results. Good overall agreement between the analytical studies and the experimental measurements is found. The investigation presented here not only provides a new method of predicting the dynamic characteristics of the MIMO feedforward control systems, but also provides theoretical guidelines for designing such systems.

The eigenproperty theory of the feedforward controlled system suggests that, by properly configuring control actuators and error sensors, the dynamic properties of the feedforward controlled system can be modified in desired ways, for example, all controlled modes radiate weakly in the far-field. The eigenassignment method for designing SISO feedforward control system as proposed by Burdisso and Fuller [12, 11] will be adopted in the rest of this dissertation to design acoustically efficient systems.

Chapter 3

Design of non-volumetric ASAC systems

In this chapter, the design of ASAC systems using a non-volumetric eigenproperty assignment approach is discussed in detail. Since the eigenassignment design technique serves as the principal design tool throughout this dissertation, a review of the eigenassignment design procedure used in this dissertation is in order. The concept of the non-volumetric modes is then introduced and its implications in structural acoustic behavior of the beam system is discussed in detail. The rest of this chapter is dedicated to the design of the non-volumetric ASAC system in the modal domain. A simply supported beam and clamped-free beam are used as examples to illustrate the design procedure. The acoustic performance of the proposed non-volumetric system is thoroughly evaluated. Furthermore, for the simply supported beam, the performance of the non-volumetric controlled system is compared to two other control strategies. Finally, a summary for the non-volumetric design is presented to conclude this chapter.

3.1 The eigenassignment design method

The eigenassignment design approach was first introduced by Burdisso and Fuller [11, 12] for a SISO feedforward control system. Recalling Chapter 2, we have demonstrated that the feedforward control can change the dynamic properties of the structure. More specifically, the feedforward controlled system can effectively have new resonance frequencies and eigenfunctions (mode shapes), which are only functions of the control actuators and error sensors, and are independent of the disturbance characteristics. This implies that, by properly configuring the control actuators and error sensors, the controlled system can be actively modified to respond in a desired manner.

The eigenassignment design procedure generally consists of two steps: first, the design of the global control system parameters in the modal domain by eigenassignment; second, the design of the physical system (realistic structural error sensors and actuators) to implement the control strategy developed in the first step. This particular separation of the design process between the modal and physical domain offers the advantage that different transducers and configurations can be investigated in terms of system performance with minimum computational effort.

The eigenproperties of the SISO feedforward-controlled structure are governed by the characteristics of the control compensator $G(\omega)$ defined in equation (2.8). The controlled eigenvalues ω_{cl}^2 are the roots of the following characteristic equation of the controlled system

$$\sum_{n=1}^N \xi_n \mu_n \prod_{\substack{m=1 \\ m \neq n}}^N (\omega_m^2 - \omega_{cl}^2) = 0, \quad \text{for } l = 1, 2, \dots, N-1 \quad (3.1)$$

where ω_m^2 is the m^{th} eigenvalue of the uncontrolled system. The controlled eigenvalues ω_{cl}^2 are therefore only a function of the unit modal control input, modal error sensor components and eigenproperties of the uncontrolled structure and are independent of the disturbance loading, i.e., type, location and frequency content. It is also important to note that the applied control input reduces the dynamic degrees of freedom of the structure by one due to the constraint imposed by driving the response of error sensor to zero. The corresponding

eigenfunctions of the controlled system are linear combinations of those of the uncontrolled system represented by equation (2.10):

$$\phi_{cl} = \sum_{n=1}^N \Gamma_{nl} \phi_n(x), \quad \text{for } l = 1, 2, \dots, N-1$$

where the expansion coefficients Γ_{nl} ($n = 1, 2, \dots, N$, $l = 1, 2, \dots, N-1$) are defined in equation (2.11). Further analysis shows that, if the l^{th} controlled eigenvalue ω_{cl}^2 equals the n^{th} uncontrolled eigenvalue ω_n^2 , then the corresponding l^{th} controlled eigenfunction is identical to the n^{th} uncontrolled eigenfunction, regardless of the unit modal control input μ_n . Equation (2.11) can therefore be rewritten as

$$\Gamma_{nl} = \begin{cases} \frac{\mu_n}{\omega_{cl}^2 - \omega_n^2} \left\{ \sum_{n=1}^N \left[\mu_n / (\omega_{cl}^2 - \omega_n^2) \right]^2 \right\}^{-1/2} & \text{for } \omega_{cl}^2 \neq \omega_n^2, \\ \delta_{nl} & \text{for } \omega_{cl}^2 = \omega_n^2. \end{cases} \quad (3.2)$$

From equations (3.1) and (3.2), we can see that, once the control actuators and error sensors are selected, the eigenstructure of the controlled system is then completely determined, independent of the characteristics of the disturbance input. On the other hand, if the controlled system eigenstructure is specified, then the unit modal control inputs and modal error sensor components can be determined such that the controlled system responds with these characteristics.

In eigenassignment design technique, the determination of the desired expansion coefficients $\bar{\Gamma}_{nl}$ depends on the applications. For instance, Burdisso and Fuller [12] show that the desired expansion coefficients can be obtained to render a controlled system that inefficiently couples vibrational energy to acoustic energy. Once the desired expansion coefficients $\bar{\Gamma}_{nl}$ are found, the N unit modal control inputs μ_n and the $N-1$ controlled system eigenvalues ω_{cl}^2 can be found by matching the desired expansion coefficients with those defined by equation (3.2). Since there are $(N-1) \times N$ expansion coefficients to match and only $N + (N-1)$ unknowns, a solution can only be accomplished in some least-square sense. In other word, the controlled eigenvalues ω_{cl}^2 and unit modal control inputs μ_n can be obtained by solving

the following constrained non-linear least-square minimization problem:

$$\text{Minimize} \quad F(\mu_n, \omega_{cl}^2) = \sum_{l=1}^{N-1} \sum_{n=1}^N [\bar{\Gamma}_{nl} - \Gamma_{nl}]^2 \quad (3.3)$$

$$\text{Subject to} \quad \begin{cases} \sum_{n=1}^N \mu_n^2 = 1 \\ (\omega_{cl}^2)_{\text{lower}} \leq \omega_{cl}^2 \leq (\omega_{cl}^2)_{\text{upper}} \\ 0 \leq \omega_{c1}^2 \leq \omega_{c2}^2 \leq \dots \leq \omega_{c,N-1}^2 \end{cases} \quad (3.4)$$

where the equality constraint represents the normalization of modal control input (since the relative controllability of the modes is the only relevant information), and $(\omega_{cl}^2)_{\text{lower}}$ and $(\omega_{cl}^2)_{\text{upper}}$ are the lower and upper bounds for the l^{th} controlled eigenvalues, i.e., $(\omega_{cl}^2)_{\text{lower}}$ and $(\omega_{cl}^2)_{\text{upper}}$ are selected to ensure that the optimal expansion coefficients Γ_{nl} be as close as possible to the desired expansion coefficients $\bar{\Gamma}_{nl}$, and such that the following equation is satisfied:

$$\omega_1^2 \leq \omega_{c1}^2 \leq \omega_2^2 \leq \omega_{c2}^2 \leq \omega_3^2 \leq \dots \omega_{N-1}^2 \leq \omega_{c,N-1}^2 \leq \omega_{cN}^2 \quad (3.5)$$

If, for example, the excitation consists of multiple harmonics, the controlled eigenvalues can also be selected (by properly choosing the upper and lower bounds) such that they do not coincide with any of these harmonics. The above non-linear constrained optimization problem can be carried out by using any of the commercially available standard routines. In this dissertation, the program *constr* in MATLAB optimization toolbox [42], which adopts the sequential quadratic programming (SQP) algorithm, is used.

Once the optimal unit modal control inputs μ_n and the controlled system eigenvalues ω_{cl}^2 are determined, the modal error sensor components ξ_n can be obtained by solving a linear system of equations derived from equation (3.1). To show this, assuming that the N^{th} mode is fully observable and setting $\xi_n = 1$, the modal error sensor components are obtained by

solving the following linear system [11, 12]:

$$\begin{bmatrix} \eta_{11} & \eta_{12} & \cdots & \eta_{1,N-1} \\ \eta_{21} & \eta_{22} & \cdots & \eta_{2,N-1} \\ \vdots & \vdots & \cdots & \vdots \\ \eta_{N-1,1} & \eta_{N-1,2} & \cdots & \eta_{N-1,N-1} \end{bmatrix} \begin{Bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_{N-1} \end{Bmatrix} = \begin{Bmatrix} \eta_{1,N} \\ \eta_{2,N} \\ \vdots \\ \eta_{N-1,N} \end{Bmatrix} \quad (3.6)$$

where

$$\eta_{ln} = \mu_n \prod_{\substack{m=1 \\ m \neq n}}^N (\omega_m^2 - \omega_{cl}^2) \quad (3.7)$$

The vector $[\xi_1, \xi_2, \dots, \xi_{N-1}, 1]^T$ defines the error sensor in the modal domain.

The determination of the optimal modal error sensor components ξ_n and the unit modal control inputs μ_n completely defines the control system configuration in the modal domain. The performance of the controlled system can now be investigated without the need to actually determine the physical control actuators and error sensors.

The next stage of the eigenassignment design technique involves the design of the physical actuators and error sensors to implement the design strategy proposed in the modal domain. This process depends on characteristics of the actuators and error sensors and will be discussed in full detail in Chapter 4.

3.2 The non-volumetric concept

As we discussed in Section 1.1, sound radiation of planar radiators such as beams and plates is known to be directly related to the velocity distribution over the structure surface at low frequencies. For example, the non-volumetric modes correspond to poor sound radiators for $k_0 a \ll 1$ (where k_0 is the wave number in the air and a is the characteristic dimension of the structure). The “non-volumetric modes” are defined as structural modes with zero

integral of the normal velocity distributions over the surface of the planar structure (i.e., zero volume velocity). These modes radiate poorly at low frequencies where their directivity patterns resemble dipoles, quadrupoles, or higher-order patterns. If we can design a control system (using structural actuators and sensors) to modify the eigenproperties of a planar structure so that all of the vibration modes of the controlled system are non-volumetric, we can then markedly reduce the sound radiation in the low frequency range.

Furthermore, it is not difficult to show that the non-volumetric design will always lead to large sound pressure reduction in a direction normal to a finite planar structure. This is because the far-field sound pressure in a direction normal to the finite vibrating structure is proportional to the net volume velocity of the structure, as pointed out by Guigou, Li and Fuller [43]. To see this, from the wavenumber transform solution for planar radiators summarized in Appendix C, the sound pressure in the far-field can be written as (see equation (C.14)):

$$p(R, \theta, \phi) = \frac{j\rho_0\omega e^{jkR}}{2\pi R} \dot{W}(\bar{k}_x, \bar{k}_y) \quad (3.8)$$

where $\dot{W}(\bar{k}_x, \bar{k}_y)$ is the wavenumber transform of the out-of-plane velocity, and $\bar{k}_x, \bar{k}_y$ are defined in equation (C.15), that is

$$\begin{aligned} \bar{k}_x &= k_0 \sin \theta \cos \phi \\ \bar{k}_y &= k_0 \sin \theta \sin \phi \end{aligned} \quad (3.9)$$

The total volume velocity of the structure is defined as

$$V = j\omega \iint_{\Omega} w(x, y) dx dy \quad (3.10)$$

where Ω denote the surface of the planar structure. The out-of-plane velocity distribution

of the baffled planar structure as depicted in Figure C.1 is

$$\dot{w}(x, y) = \begin{cases} j\omega w(x, y) & \text{for } (x, y) \in \Omega \\ 0 & \text{otherwise} \end{cases} \quad (3.11)$$

If we apply the wavenumber transform defined by equation (C.1) to the vibration distribution of equation (3.11) and consider the special case of $k_x = 0, k_y = 0$, we have

$$\dot{W}(k_x = 0, k_y = 0) = j\omega \iint_D w(x, y) dx dy \quad (3.12)$$

From equation (3.10) it is apparent that the right hand side of equation (3.12) is the volume velocity of the structure. Thus the far field radiated sound pressure corresponding to the special case of $\bar{k}_x = 0, \bar{k}_y = 0$ is proportional to the volume velocity of the structure. Examining equation (3.9) reveals that this special case corresponds to radiation in a direction normal to the planar structure where $\theta = 0$ (ϕ can take any value). Therefore, if we can design a controlled system whose modes are all non-volumetric, theoretically, the far-field sound pressure in the direction normal to the structure will be zero. The above observation can also be viewed in a different aspect: designing a structural sensor to observe either a structural volume velocity or normal far-field radiation is equivalent, therefore the non-volumetric design can effectively replace a far-field microphone normal to the surface of the structure [43] (from sensing point of view). These observations inspired the non-volumetric design approach presented in this chapter.

3.3 Design of non-volumetric system in the modal domain

The non-volumetric design for the one-dimensional beam as depicted in Figure 3.1 is presented in this section. The goal of the non-volumetric design is to achieve a controlled structure that responds only with non-volumetric modes so as to attenuate the radiated

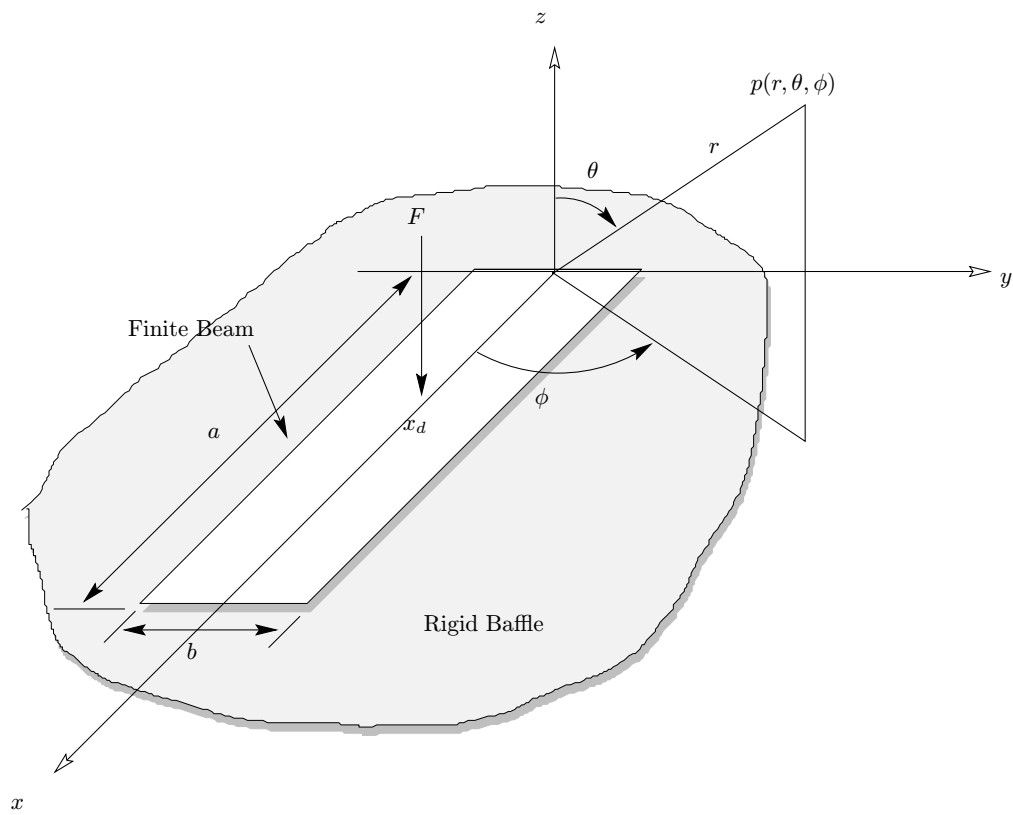


Figure 3.1: Schematic of beam and coordinate system

sound power. The structural eigenproperties have to be altered in such a way that the controlled eigenfunctions are all non-volumetric and preferably orthogonal. To allow flexibility in the design process, we choose to construct the l^{th} controlled eigenfunction as a linear combination of the first $(l + 1)$ uncontrolled eigenfunctions, i.e.,

$$\phi_{cl}(x) = \sum_{n=1}^{l+1} \bar{\Gamma}_{nl} \phi_n(x) \quad \text{for } l = 1, 2, \dots, N - 1 \quad (3.13)$$

where the coefficients $\bar{\Gamma}_{nl}$ are derived by imposing the non-volumetric and the orthonormal conditions (see equation (2.16)):

$$\begin{cases} \int_0^a \phi_{cl}(x) dx = 0 & \text{non-volumetric condition} \\ \int_0^a \phi_{cl}(x) m(x) \phi_{cl'}(x) dx = \delta_{ll'} & \text{for } l' \leq l, \text{ orthonormal condition} \end{cases} \quad (3.14)$$

where $\delta_{ll'}$ represents the Kronecker Delta function defined by equation (2.18). Thus, it can be noticed from equations (3.13) and (3.14) that the first controlled eigenfunction is constructed from a linear combination of the first and second uncontrolled eigenfunctions. In general, the l^{th} controlled eigenfunction is a linear combination of the first $(l + 1)$ uncontrolled eigenfunctions. If the number of uncontrolled modes (to be taken into account in the frequency range of concern) has to be increased, new controlled eigenfunctions are added without changing the lower order ones. Substituting equation (3.13) into equation (3.14), the desired expansion coefficients $\bar{\Gamma}_{nl}$ that render non-volumetric, orthonormal controlled eigenfunctions are given as

$$\bar{\Gamma}_{nl} = \begin{cases} \gamma_n \gamma_{l+1} \left[\sum_{j=1}^l \gamma_j^2 \cdot \sum_{i=1}^{l+1} \gamma_i^2 \right]^{-1/2} & \text{for } 1 \leq n \leq l \\ \left(\sum_{j=1}^l \gamma_j^2 \right) \left[\sum_{j=1}^l \gamma_j^2 \cdot \sum_{i=1}^{l+1} \gamma_i^2 \right]^{-1/2} & \text{for } n = l + 1 \\ 0 & \text{for } l + 1 < n \leq N \end{cases} \quad (3.15)$$

where γ_n is the volume velocity of the beam defined as

$$\gamma_n = \int_0^a \phi_n(x) dx \quad (3.16)$$

If γ_n is zero, the n^{th} mode then obeys the non-volumetric condition. Detailed study of equation (3.15) shows that the $(l+1)^{th}$ uncontrolled mode is the most significant contributor to the l^{th} controlled mode. Furthermore, the contribution of the n^{th} uncontrolled mode ($1 \leq n \leq l$) to the controlled mode decreases as the order of the controlled mode increases. This is related to the fact that the controlled eigenfunctions become more complex (i.e., with increasing number of nodes) as the mode order l increases. This may not be clear at this point, but readers can refer to the simulation results shown in Table 3.2 and Table 3.3 in Section 3.6 of this chapter for better understanding.

It is important to recognize that the expansion coefficients as defined by equation (3.15) are independent of frequency. Also recalling that the controlled eigenproperties are independent of the primary or disturbance source, we conclude that the proposed non-volumetric design presented here is independent of frequency. This poses a major advantage over some other design considerations where the design is in general depend on frequency, for example, the volume velocity sensing design as demonstrated by Johnson and Elliott [32] and the weakest modal radiators design as proposed by Burdisso and Fuller [12] to name a few.

Once the desired expansion coefficients rendering the non-volumetric, orthonormal controlled modes are found, the configuration of the control system can be determined in the modal domain by using the SISO eigenassignment technique discussed in Section 3.1.

3.4 System modeling

In this work, the non-volumetric design is applied to a finite beam structure shown in Figure 3.1. The baffled finite beam has length a and width b . Boundary conditions are applied at $x = 0$ and $x = a$. The beam is excited by a harmonic point force $F(\omega)e^{j\omega t}$ located at $x = x_d$. The analysis is carried out in frequency domain.

3.4.1 Structural response

The beam is assumed made of homogeneous, linear elastic, isotropic material. It is also assumed that the cross-sectional dimensions of the beam are small enough compared with the length of the beam and flexural wavelength so that the rotary inertia and shear deformation can be ignored. The equation of motion of the beam can then be written as

$$\frac{\partial^2}{\partial x^2} \left[EI(x) \frac{\partial^2 w(x)}{\partial x^2} \right] + C \frac{\partial w(x)}{\partial t} + m(x) \frac{\partial^2 w(x)}{\partial t^2} = F \delta(x - x_d) + f_c(x) U \quad (3.17)$$

where

$w(x)$	=	Out-of-plane displacement of the beam
$EI(x)$	=	Bending stiffness of the beam
C	=	Damping coefficient
$m(x)$	=	Mass density per unit length of the beam
$f_c(x)$	=	Spatial distribution vector of the control input
F	=	Amplitude of the disturbance input
U	=	Amplitude of the control input

and $\delta(\cdot)$ is the Dirac delta function (see equation (2.55)) and the time dependence $e^{j\omega t}$ is omitted for brevity. One can recognize that the equation of motion of the beam represented by equation (3.17) conforms to the general formulation presented in Section 2.3.1 with $L = \frac{\partial^2}{\partial x^2} [EI(x) \frac{\partial^2 w(x)}{\partial x^2}] + C$ and $m(D) = m(x)$.

From equation (2.19), the response of the beam in the frequency domain can generally be represented by a series expansion of the form

$$w(x, \omega) = \sum_{n=1}^N q_n(\omega) \phi_n(x) = \mathbf{q}^T \boldsymbol{\phi}(x) \quad (3.18)$$

where $w(x, \omega)$ is the out-of-plane displacement, and $\mathbf{q} = [q_1(\omega), q_2(\omega), \dots, q_N(\omega)]^T$, $\boldsymbol{\phi}(x) = [\phi_1(x), \phi_2(x), \dots, \phi_N(x)]^T$ are vectors of the generalized coordinates and eigenfunctions, respectively. The eigenfunctions of the beam depend on the boundary conditions and are normalized with respect to mass distribution of the beam, i.e.,

$$\int_0^a \phi_m(x) m(x) \phi_n(x) dx = \delta_{mn} \quad (3.19)$$

From equation (2.23), the generalized coordinates in the frequency domain, $q_n(\omega)$, can be expressed as

$$q_n(\omega) = [\nu_n F(\omega) + \mu_n U(\omega)] H_n(\omega) \quad (3.20)$$

where

$$\begin{aligned} \nu_n &= \text{Unit modal disturbance input} \\ \mu_n &= \text{Unit modal control input} \\ H_n(\omega) &= \text{Modal frequency response function} \end{aligned}$$

They are defined as

$$\nu_n = \phi_n(x_d) \quad (3.21)$$

$$\mu_n = \int_0^a \phi_n(x) f_c(x) dx \quad (3.22)$$

$$H_n(\omega) = (\omega_n^2 - \omega^2 + j2\zeta_n\omega_n\omega)^{-1} \quad (3.23)$$

where $\zeta_n = \frac{C}{2m(x)\omega_n}$ and ω_n are the n^{th} modal damping factor and natural frequency, respectively. The optimal control input $U(\omega)$ can be obtained by minimizing the square of an error sensor output $\epsilon(\omega)$, which is represented as the sum of a linear contribution of each

mode, i.e.,

$$\epsilon(\omega) = \sum_{n=1}^N q_n(\omega) \xi_n \quad (3.24)$$

where ξ_n is the n^{th} modal error sensor component and is in general a function of the characteristics of the error sensor (see Section 2.3.2). Minimizing equation (3.24) leads to the control input magnitude expressed as (see Section 2.3.2)

$$U(\omega) = -G(\omega)F(\omega) = -\frac{\sum_{n=1}^N \xi_n \nu_n H_n(\omega)}{\sum_{n=1}^N \xi_n \mu_n H_n(\omega)} F(\omega) \quad (3.25)$$

The response of the beam are now fully described by equations (3.18) and (3.20).

3.4.2 Acoustic radiation

The far-field acoustic radiation of a baffled planar radiator can be obtained by using the spatial Fourier transform (i.e., the wavenumber transform) approach as shown by Junger and Feit [33]. For brevity, the derivation for general planar radiator is summarized in Appendix C.

The sound pressure in the far-field for a baffled beam can be written as (see equation (C.14))

$$p(R, \theta, \phi) = \frac{j\rho_0\omega e^{jkR}}{2\pi R} \dot{W}(\bar{k}_x, \bar{k}_y) \quad (3.26)$$

and $\bar{k}_x, \bar{k}_y$ are defined by equation (C.15), i.e.,

$$\begin{aligned} \bar{k}_x &= k_0 \sin \theta \cos \phi \\ \bar{k}_y &= k_0 \sin \theta \sin \phi \end{aligned} \quad (3.27)$$

where $k_0 = \omega/c_0$ represents the acoustic wavenumber in the acoustic medium, ρ_0 and c_0 are the density of the acoustic medium (air in this case) and sound velocity, respectively; $\dot{W}(k_x, k_y)$ is the structural velocity wavenumber spectrum defined as the double spatial Fourier transform of the velocity distribution over the surface of the structure, i.e.,

$$\dot{W}(k_x, k_y) = \iint_{-\infty}^{+\infty} \dot{w}(x, y) e^{j(k_x x + k_y y)} dx dy \quad (3.28)$$

For the baffled beam, the system out-of-plane velocity depends only on the x -coordinate and is constant along the beam width. From equation (3.18), the spatial velocity distribution of the beam can be written as

$$\dot{w}(x, y) = \begin{cases} j\omega \mathbf{q}^T \boldsymbol{\phi}(x) & \text{for } 0 \leq x \leq a, -b/2 \leq y \leq b/2 \\ 0 & \text{otherwise} \end{cases} \quad (3.29)$$

Substituting equation (3.29) into equation (3.28) renders wavenumber transformed velocity of the beam:

$$\dot{W}(k_x, k_y) = j\omega \mathbf{q}^T \boldsymbol{\Phi}(k_x, k_y) \quad (3.30)$$

where the wavenumber transformed eigenfunction vector $\boldsymbol{\Phi}(k_x, k_y)$ is defined as

$$\boldsymbol{\Phi}(k_x, k_y) = \int_{-b/2}^{+b/2} \int_0^a \boldsymbol{\phi}(x, y) e^{j(k_x x + k_y y)} dx dy = b \left[\text{sinc}\left(\frac{k_y b}{2\pi}\right) \right] \boldsymbol{\Phi}(k_x) \quad (3.31)$$

where $\text{sinc}(x)$ is defined as [40]

$$\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x} = \begin{cases} \frac{\sin(\pi x)}{\pi x} & \text{for } x \neq 0 \\ 1 & \text{for } x = 0 \end{cases} \quad (3.32)$$

Substitute equation (3.30) into equation (3.26), the far-field sound pressure generated by the

beam can be written as

$$p(R, \theta, \phi) = -\frac{\rho_0 \omega^2 e^{jkR}}{2\pi R} \mathbf{q}^T \mathbf{\Phi}(\bar{k}_x, \bar{k}_y) \quad (3.33)$$

The total acoustic power radiated by the beam can be obtained by using the wavenumber transform technique as described in Appendix C.2 and is given by equations (C.29) and (C.30). Substitute equation (3.31) into equation (C.30), the sound power radiated by the beam becomes

$$\Pi(\omega) = \mathbf{q}^T \mathbf{B} \mathbf{q}^* \quad (3.34)$$

where the power matrix [26] $\mathbf{B}$ represents a $N \times N$ matrix of integrals of the form

$$\mathbf{B} = \frac{\rho_0 \omega^3}{8\pi^2} \int_0^{\pi/2} \int_0^{2\pi} \left[b \operatorname{sinc}\left(\frac{\bar{k}_y b}{2\pi}\right) \right]^2 \mathbf{\Phi}(\bar{k}_x) \mathbf{\Phi}^H(\bar{k}_x) k \sin \theta d\theta d\phi \quad (3.35)$$

where $\bar{k}_x, \bar{k}_y$ are given by equation (3.27). Examining equation (3.35) shows that $\mathbf{B} = \mathbf{B}^H$, i.e., the matrix $\mathbf{B}$ is a Hermitian matrix. Its diagonal terms represent the contributions to the radiated acoustic power made by each individual mode, and its off-diagonal terms (commonly referred to as modal coupling terms) represent the acoustic power due to the interference or coupling of two different modes.

The radiation efficiency is first defined by Wallace [24] as the ratio of the acoustic power that a structure radiates to the power radiated by a piston of equivalent area vibrating with an amplitude equal to the spatial average mean-square velocity of the structure. For the beam, it can be written as

$$\sigma = \frac{\Pi(\omega)}{\rho_0 c_0 a b \overline{\dot{w}^2(x)}} \quad (3.36)$$

where $\overline{\dot{w}^2(x)}$ is the spatial average mean-square velocity defined as

$$\overline{\dot{w}^2(x)} = \frac{\omega^2}{2a} \int_0^a |w(x)|^2 dx \quad (3.37)$$

3.5 Implementation for beams with typical boundary conditions

This section presents some simplifications of the previously developed general formulation in the case of lightly damped simply supported and clamped-free beams.

3.5.1 Simply supported beam

For simply supported beam, the out-of-plane displacements and bending moments at both ends of the beam vanish:

$$w(x)|_{x=0,a} = \frac{\partial^2 w(x)}{\partial x^2} \Big|_{x=0,a} = 0 \quad (3.38)$$

Solving the eigenvalue problem associated with equation (3.17), the n^{th} normalized eigenfunction (normalized according to equation (3.19)) is given as [39]

$$\phi_n(x) = \sqrt{\frac{2}{ma}} \sin(k_n x) \quad (3.39)$$

where $k_n = n\pi/a$ is the n^{th} structural wavenumber. The associated eigenvalue is given as

$$\omega_n^2 = k_n^4 \frac{EI}{m} \quad (3.40)$$

Substituting equation (3.39) into equation (3.16) renders the volume velocity of the simply supported beam:

$$\gamma_n = \sqrt{\frac{2}{ma}} \left[\frac{1 - (-1)^n}{k_n} \right] \quad n = 1, 2, \dots, N \quad (3.41)$$

Examining equation (3.41) reveals that the even-order modes of the beam are non-volumetric, i.e., $\gamma_n = 0$ when the index number n is even. Therefore, the dynamics of the even-order

uncontrolled modes do not need to be changed by the controller. From equations (3.15) and (3.41), it can be found that the controlled eigenfunctions can be expressed as

$$\begin{aligned}\phi_{c,2n-1}(x) &= \phi_{2n}(x) \\ \phi_{c,2n}(x) &= \sum_{k=1}^{n+1} \bar{\Gamma}_{2n,2k-1} \phi_{2k-1}(x)\end{aligned}\tag{3.42}$$

where the expansion coefficients $\bar{\Gamma}_{2n,2k-1}$ are defined by equation (3.15). Equation (3.42) states that the controlled eigenfunctions with odd order are identical to the uncontrolled ones with even order: for instance, the first controlled mode is $\phi_{c1}(x) = \phi_2(x)$. On the other hand, the controlled eigenfunctions with even order are linear combinations of the uncontrolled eigenfunctions with odd order only: for example, the second controlled modes is $\phi_{c2}(x) = \bar{\Gamma}_{11}\phi_1(x) + \bar{\Gamma}_{13}\phi_3(x)$. For a baffled, simply supported beam, the wavenumber transformed eigenfunctions can be obtained from equation (3.39) as:

$$\Phi_n(k_x) = \begin{cases} \sqrt{\frac{2}{ma}} \frac{k_n [(-1)^n e^{jk_x a} - 1]}{k_x^2 - k_n^2} & k_x \neq k_n \\ j \frac{a}{2} \sqrt{\frac{2}{ma}} & k_x = k_n \end{cases}\tag{3.43}$$

Thus from equations (3.35) and (3.43), the expression for the entry in the matrix $\mathbf{B}$ due to the interference of the m^{th} and n^{th} modes can be written as

$$B_{mn} = \begin{cases} \frac{\rho_0 \omega^3}{8\pi^2} \frac{2}{k_m k_n} \frac{2}{ma} \int_0^{\pi/2} \int_0^{2\pi} \left[b \operatorname{sinc}\left(\frac{\bar{k}_y b}{2\pi}\right) \right]^2 \frac{1 - (-1)^m \cos(\bar{k}_x a)}{[1 - (\bar{k}_x/k_m)^2][1 - (\bar{k}_x/k_n)^2]} k \sin \theta d\theta d\phi & \text{for } m, n \text{ in parity} \\ 0 & \text{for } m, n \text{ in opposite parity} \end{cases}\tag{3.44}$$

This equation states that, for simply supported beam, the interference between odd-even or even-odd (opposite parity) modes is identically zero, and only even-even or odd-odd (in

parity) modes will interact with each other with respect to the total radiated sound power [26, 28].

The sound power radiated by the simply supported beam is given by equations (3.34) and (3.35).

3.5.2 Clamped-free beam

The non-volumetric design can be applied to beam system with more complexed boundary conditions. To illustrate this, a clamped-free beam is studied in this section.

For the clamped-free beam, at the clamped end, $x = 0$, we have the imposed boundary conditions

$$w(x)|_{x=0} = \frac{\partial w(x)}{\partial x} \Big|_{x=0} = 0 \quad (3.45)$$

At the free end, $x = a$, the bending moment and shear force are zero, i.e.,

$$\frac{\partial^2 w(x)}{\partial x^2} \Big|_{x=a} = \frac{\partial^3 w(x)}{\partial x^3} \Big|_{x=a} = 0 \quad (3.46)$$

Solving the eigenvalue problem associated with equation (3.17), the n^{th} eigenfunction is given as

$$\phi_n(x) = A_n \left[\cos(k_n x) - \cosh(k_n x) + \alpha_n (\sin(k_n x) - \sinh(k_n x)) \right] \quad (3.47)$$

where

$$\alpha_n = \frac{\sin(k_n a) - \sinh(k_n a)}{\cos(k_n a) + \cosh(k_n a)} \quad (3.48)$$

the structural wavenumber k_n is the solution of the frequency equation

$$\cos(k_n a) \cosh(k_n a) = -1 \quad n = 1, 2, \dots \quad (3.49)$$

and A_n is the normalization constant which can be obtained from equation (3.19). It can be easily shown that the eigenfunctions $\phi_n(x)$ are orthogonal. They constitute a complete set of orthogonal eigenfunctions. The associated eigenvalues of the beam are given by equation (3.40) as before.

Substitute equation (3.47) into equation (3.16), the volume velocity of the clamped-free beam can be written as

$$\gamma_n = \frac{2A_n}{k_n} \frac{\sin(k_n a) - \sinh(k_n a)}{\cos(k_n a) + \cosh(k_n a)} \quad (3.50)$$

For the clamped-free beam, unlike the simply supported beam, none of the uncontrolled modes is non-volumetric. As a consequence, all the controlled eigenfunctions will be different from the uncontrolled ones. The desired expansion coefficients for the clamped-free beam can be obtained by substituting equation (3.50) into equation (3.15). The controlled eigenfunctions are defined by equation (3.13).

For the a baffled clamped-free beam, the wavenumber transformed eigenfunction can be obtained from equation (3.47) as

$$\begin{aligned} \Phi_n(k_x)/A_n = & \frac{[k_n \sin(k_n a) + jk_x \cos(k_n a)]e^{jk_x a} - jk_x}{k_n^2 - k_x^2} \\ & - \frac{[k_n \sinh(k_n a) - jk_x \cosh(k_n a)]e^{jk_x a} + jk_x}{k_n^2 + k_x^2} \\ & + \alpha_n \left[\frac{k_n - e^{jk_x a} [k_n \cos(k_n a) - jk_x \sin(k_n a)]}{k_n^2 - k_x^2} \right. \\ & \quad \left. - \frac{[k_n \cosh(k_n a) - jk_x \sinh(k_n a)]e^{jk_x a} - k_n}{k_n^2 + k_x^2} \right] \end{aligned} \quad (3.51)$$

Unlike the simply supported beam, elements of the power matrix $\mathbf{B}$ are in general not zero. In other words, all structure modes are coupled in terms of sound radiation.

After obtaining the desired expansion coefficients $\bar{\Gamma}_{nl}$ for the two finite beams considered here, the constrained non-linear least-square minimization problem as expressed in equations (3.3) and (3.4) can be solved for the controlled eigenvalues ω_{cl}^2 and unit modal control

inputs μ_n . The modal error sensor components ξ_n are then found using equations (3.6) and (3.7). Once these components are found correctly, the controlled system will respond with the desired non-volumetric eigenfunctions or mode shapes.

3.6 Numerical simulations

In this section, in order to demonstrate the design procedure and evaluate the performance of the proposed control strategy, numerical simulations for a simply supported and clamped-free beam are presented. The beam is constructed from carbon steel and its dimensions and material property are given in Table 3.1.

Table 3.1: Dimensions and material properties of the beam

Parameter	Magnitude
Length, a (m)	0.38
Width, b (m)	0.038
Thickness, h (m)	0.0048
Young's Modulus, E (N/m)	2.0e+11
Mass Density, ρ_s (kg/m ³)	7860
Damping, ζ_n	1.0%

The beam is assumed to be excited by a harmonic point force of 1 N in amplitude and located at $x_d = 0.1$ m. The frequency range of study is limited to 10 Hz to 2500 Hz. For the simply supported beam, the first 6 modes are included in the analysis. For the clamped-free beam, the first 7 modes are included. Note that the simulation performed in this section is based upon the design in the modal domain, without any knowledge of the physical actuators and sensors.

3.6.1 Optimal controller

For both simply supported and clamped-free beam, the desired expansion coefficients that render the non-volumetric controlled modes can be obtained from equation (3.15), with γ_n given by equation (3.41) for simply supported beam and equation (3.50) for clamped-free beam. They are listed in Table 3.2 and Table 3.3, respectively. The associated non-volumetric

Table 3.2: Desired expansion coefficients $\bar{\Gamma}_{nl}$ for simply supported beam

Uncontrolled Modes	Controlled Modes				
	1	2	3	4	5
1	0	0.3162	0	0.1768	0
2	1.0	0	0	0	0
3	0	-0.9487	0	0.0589	0
4	0	0	1.0	0	0
5	0	0	0	-0.9825	0
6	0	0	0	0	1.0

mode shapes for the simply supported beam are shown in Figure 3.2. Similarly, the non-volumetric mode shapes for the clamped-free beam are presented in Figure 3.3.

For simply supported beam, examining Table 3.2 and Figure 3.2 and recalling Section 3.5.1, we can see that the odd-order controlled modes correspond to the next even-order uncontrolled ones (i.e., the first controlled mode is the second mode of the uncontrolled system). On the other hand, the controlled modes with even order only depend on the uncontrolled ones with odd order (i.e., the second controlled mode is a combination of mode 1 and mode 3 of the uncontrolled system). It can also be noticed from Tables 3.2 and 3.3 that, for both the simply supported and clamped-free beam, the largest expansion coefficient for the controlled modes corresponds to the highest order uncontrolled mode included in the decomposition. This can be associated with the fact that the mode shape becomes more

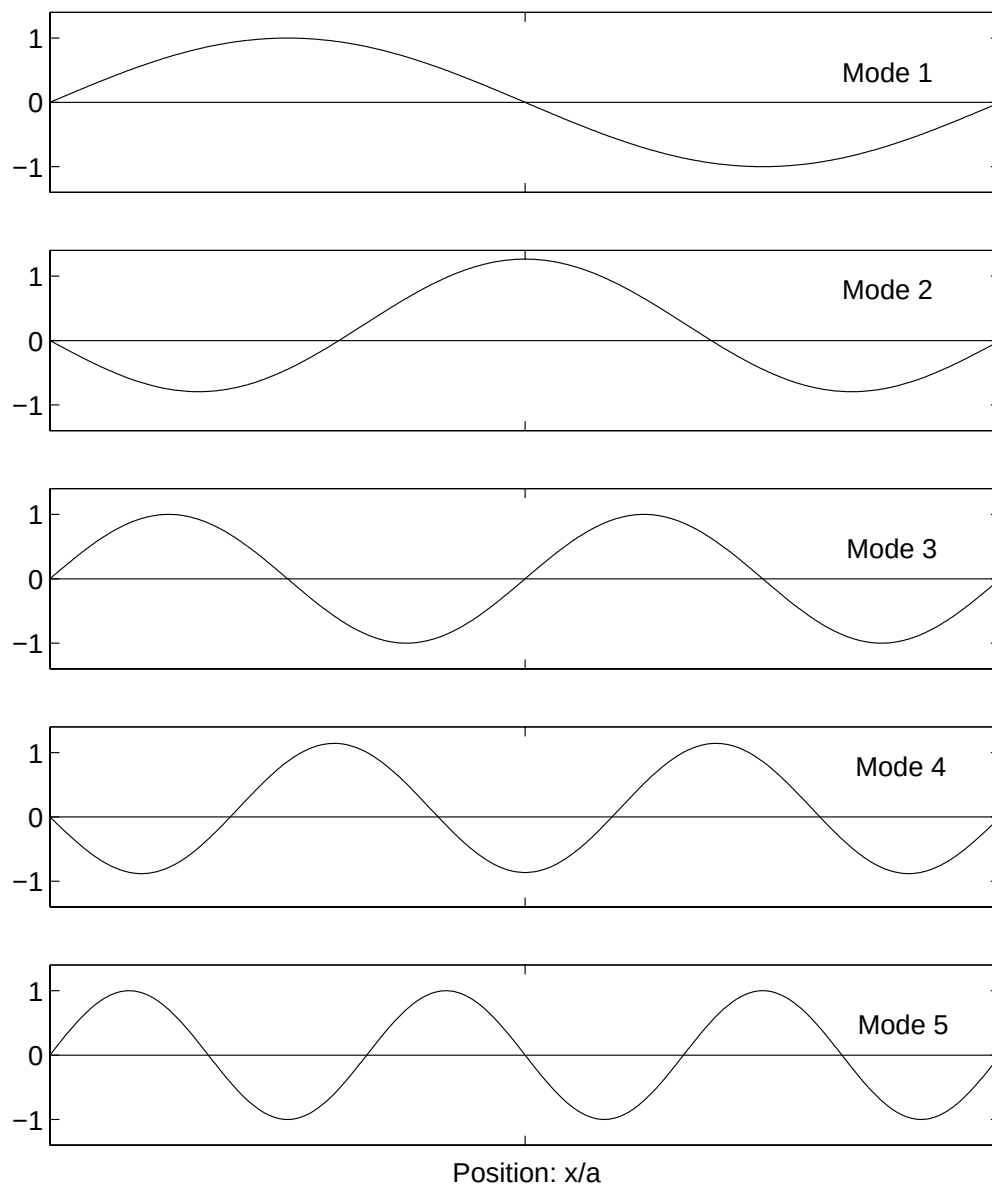


Figure 3.2: Controlled mode shapes of the simply supported beam

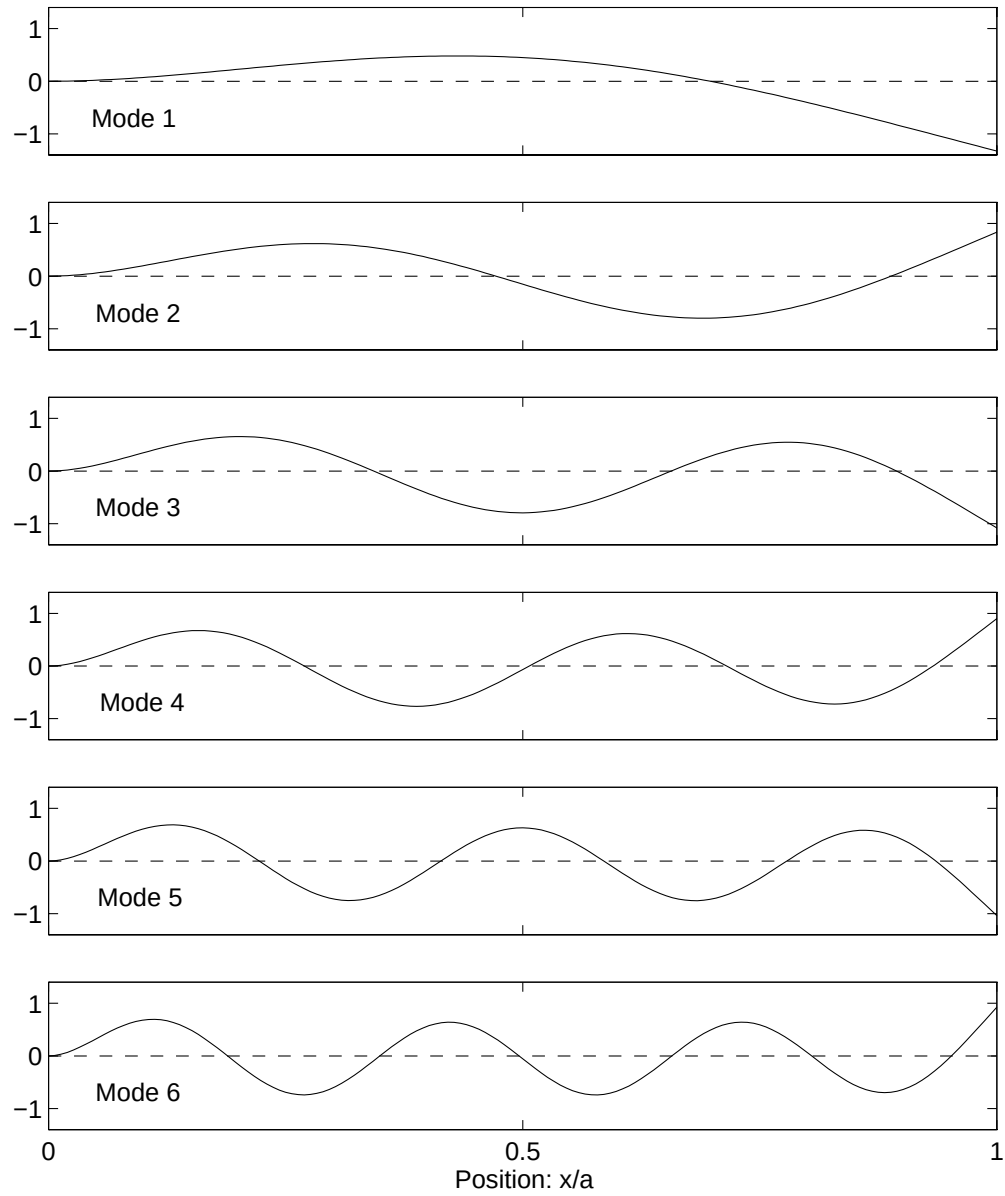


Figure 3.3: Controlled mode shapes of the clamped-free beam

Table 3.3: Desired expansion coefficients for clamped-free beam

Uncontrolled	Controlled Modes					
Modes	1	2	3	4	5	6
1	0.4847	0.2391	0.1614	0.1218	0.0979	0.0818
2	-0.8747	0.1325	0.0894	0.0675	0.0542	0.0453
3	0	-0.9619	0.0524	0.0396	0.0318	0.0266
4	0	0	-0.9814	0.0283	0.0227	0.0190
5	0	0	0	-0.9891	0.0177	0.0148
6	0	0	0	0	-0.9928	0.0121
7	0	0	0	0	0	-0.9949

complex (increasing number of nodes) with increasing mode order as seen from Figures 3.2 and 3.3.

The optimal controlled system eigenvalues ω_{cl}^2 ($l = 1, 2, \dots, N - 1$) and the unit modal control inputs μ_n ($n = 1, 2, \dots, N$) are obtained by solving the optimization problem described by equations (3.3) and (3.4). To ensure good optimal solution, the lower and upper bounds for the controlled eigenvalues are set to satisfy equation (3.5). The modal error sensor components ξ_n ($n = 1, 2, \dots, N$) are then obtained from equations (3.6) and (3.7). The results for simply supported beam are given in Table 3.4. For comparison, the natural frequencies of the uncontrolled system (obtained from equation (3.40)) are also listed in the same table. Similarly, the results for the clamped-free beam are listed in Table 3.5. Examining Table 3.4 for the simply supported beam, as we expected, the resonance frequencies of the odd-order controlled modes are identical to those of the even-order uncontrolled modes (as their associated eigenfunctions are the same). We can also conclude that, from the unit modal control inputs μ_n and error sensor components ξ_n in Table 3.4, we only need to observe and control (or actively modify) the odd-order modes. This makes sense since the even-order modes of the simply supported beam are already non-volumetric. These observations do not hold for clamped-free beam since all the uncontrolled modes are volumetric therefore need to

be controlled. The modal parameters for the control input and the error sensor in this case are presented in Tables 3.4 and 3.5 and they completely define the controlled system in the modal domain. As discussed previously, the optimization process is independent of the type, location and frequency content of the disturbance input. The design process is different from that presented in [12] where the eigenfunctions are found to minimize the radiation efficiency at one particular frequency. The eigenstructure of the system has been changed in such a way that all the controlled modes are non-volumetric therefore radiates poorly in the low frequency region.

Table 3.4: Optimal control configuration for the simply supported beam

Mode #	Natural Frequencies (Hz)		Modal Control	Modal Error Sensor
	Uncontrolled	Controlled	Components μ_n	Components ξ_n
1	76.03	304.13	0.954	0.932
2	304.13	653.60	-0.012	0.
3	684.30	1216.53	0.279	0.310
4	1216.53	1883.04	0.006	-0.001
5	1900.82	2737.18	0.100	0.186
6	2737.18		0.034	0.

To examine the accuracy of the optimal solution, the optimal expansion coefficients corresponding to the optimal control configurations presented in Tables 3.4 and 3.5 are shown in Table 3.6 for simply supported beam and in Table 3.7 for clamped-free beam, respectively. Compare Table 3.2 with Table 3.6, we can see excellent match between the desired and optimal expansion coefficients for simply supported beam. Similar conclusion holds true for the clamped-free beam. We conclude that, for both the simply supported and clamped-free beam, the optimization problem represented by equations (3.3) and (3.4) has been successfully solved. The controlled system should respond with the desired non-volumetric modes.

Table 3.5: Optimal control configuration for the clamped-free beam

Mode #	Natural Frequencies (Hz)		Modal Control	Modal Error Sensor
	Uncontrolled	Controlled	Components μ_n	Components ξ_n
1	27.09	150.43	0.858	0.810
2	169.75	461.20	0.434	0.444
3	475.30	920.25	0.219	0.259
4	931.40	1531.09	0.127	0.186
5	1539.67	2294.03	0.077	0.145
6	2300.00	3212.00	0.044	0.119
7	3213.39		0.002	0.100

Table 3.6: Optimal expansion coefficients for the simply supported beam

Uncontrolled Modes	Controlled Modes				
	1	2	3	4	5
1	0.	0.3160	0.	0.1765	0.
2	1.0	-0.0051	0.	-0.0023	0.
3	0.	-0.9487	0.	0.0594	0.
4	0.	0.	1.0	0.0018	0.
5	0.	-0.0044	0.	-0.9825	0.
6	0.	0.	0.	-0.0057	1.0

Table 3.7: Optimal expansion coefficients for the clamped-free beam

Uncontrolled	Controlled Modes					
Modes	1	2	3	4	5	6
1	0.4845	0.2352	0.1618	0.1244	0.1014	0.0855
2	-0.8747	0.1383	0.0854	0.0642	0.0520	0.0437
3	-0.0133	-0.9620	0.0562	0.0351	0.0270	0.0223
4	-0.0019	-0.0113	-0.9815	0.0292	0.0180	0.0138
5	0.	-0.0021	-0.0080	-0.9891	0.0165	0.0099
6	0.	0.	-0.0016	-0.0051	-0.9928	0.0090
7	0.	0.	0.	0.	0.	-0.9949

After carrying out the above steps, the eigenassignment has been completed and the controlled eigenstructure is fixed. To examine the performance of the proposed design strategy, the acoustic and vibrational responses of the uncontrolled and controlled systems are exclusively studied when the excitation frequency is varied from 10 to 2500 Hz (i.e., $0.069 \leq k_0 a \leq 17.28$).

3.6.2 Control acoustic performance

In this section, in order to evaluate the overall acoustic performance of the proposed non-volumetric design for both the simply supported and the clamped-free beam, the acoustic and vibrational responses of the sytem before and after control are extensively studied. The disturbance signal is a pure tone sinusoid and its frequency is varied from 10 Hz to 2500 Hz (i.e., $0.069 \leq k_0 a \leq 17.28$).

As expected, the controlled modes radiate less efficiently than the uncontrolled modes. For the simply supported beam, the radiation efficiencies of the first 4 modes (before and after control) are shown in Figure 3.4. Similar results are presented in Figure 3.5 for the

clamped-free beam. It can be observed that for both types of boundary conditions, at low frequencies, the radiation efficiencies of the controlled modes are much smaller than those of the uncontrolled modes. This decrease implies that the controlled modes are weaker radiators compared to the uncontrolled ones. Another interesting phenomenon that can be observed from Figure 3.4 is that, for example, the radiation efficiency of the second controlled mode, which is a linear combination of the first and third uncontrolled modes, is even lower than the radiation efficiency of the second uncontrolled modes, which itself is a dipole radiator. This is probably due to the fact that the second controlled mode has one more degree of freedom than the second uncontrolled mode. Similar observations hold true for other even-order controlled modes.

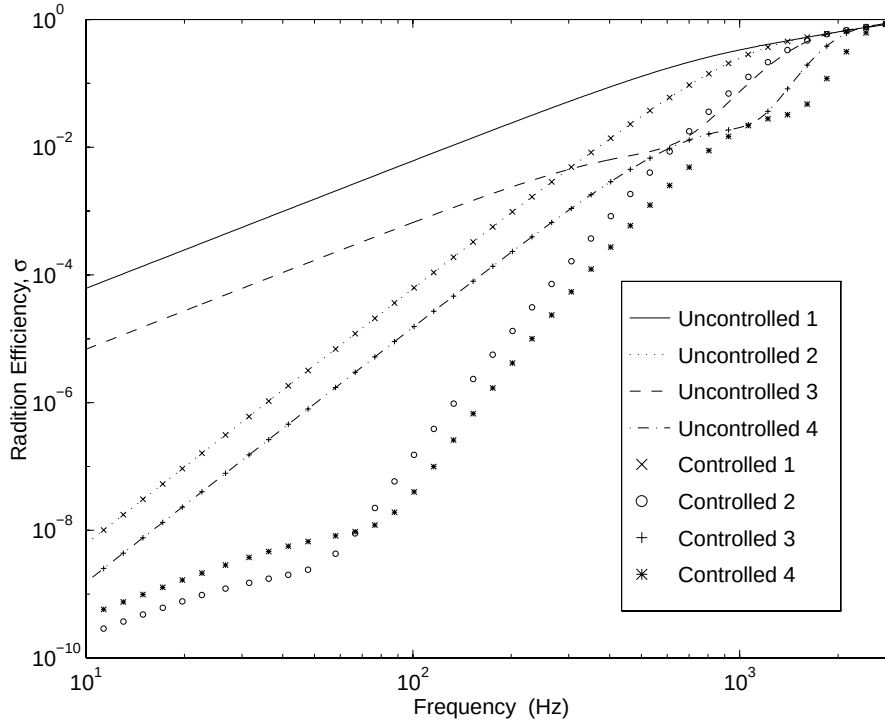


Figure 3.4: Modal radiation efficiencies of modes 1–4 of a simply supported beam

To illustrate the effectiveness of the control strategy, the far-field sound pressure levels

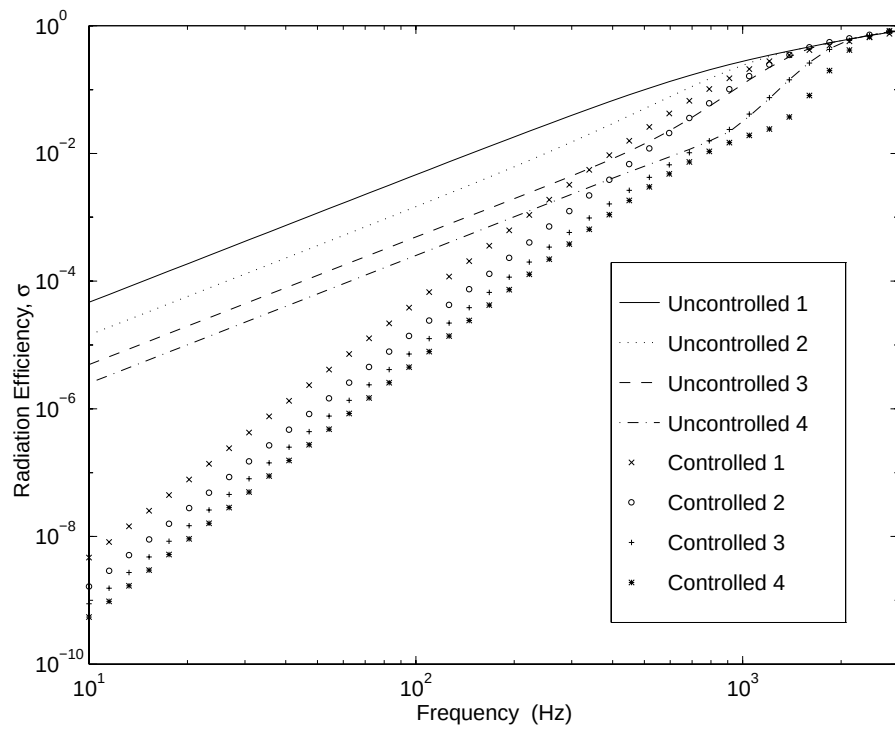


Figure 3.5: Modal radiation efficiencies of modes 1-4 of a clamped-free beam

(in dB ref. $20\mu\text{ Pa}$) before and after control at $\mathbf{r}(5 \times a, \theta = 0, \phi = 0)$, i.e., in the direction perpendicular to the surface of the finite beam, are shown in Figures 3.6 and 3.7 for simply supported beam and clamped-free beam, respectively. It can be observed that the sound

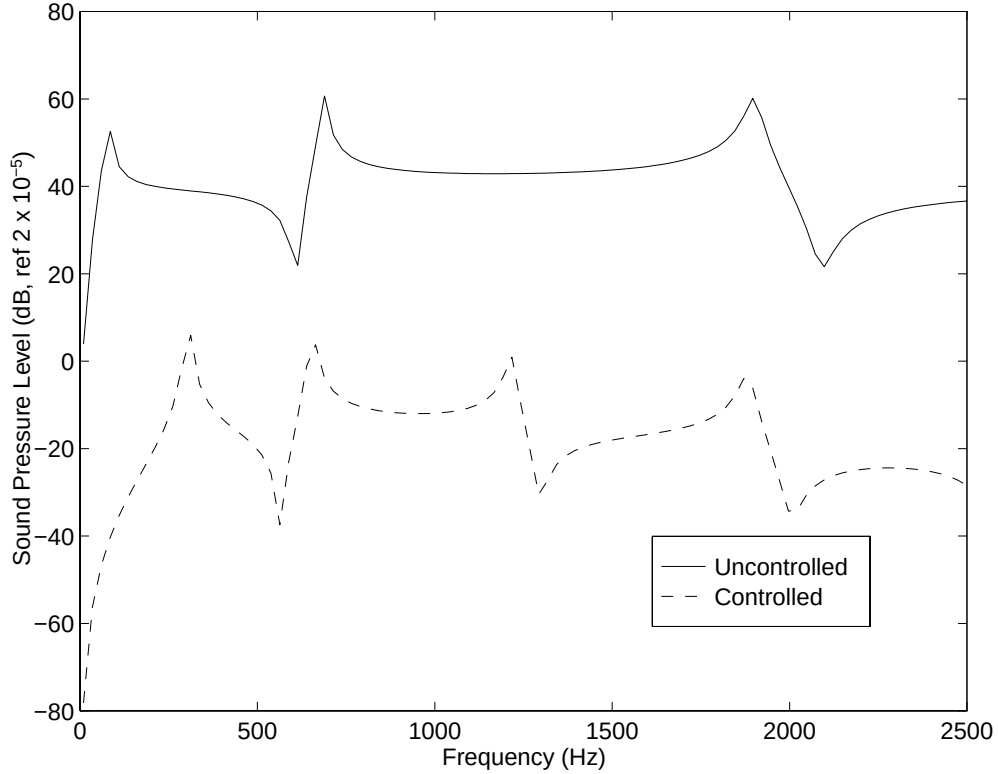


Figure 3.6: Far-field radiated pressure in a direction perpendicular to the simply supported beam

pressure level after control is significantly reduced within a wide frequency range ($0 \sim 2500$ Hz). This reduction is mainly due to the fact that the non-volumetric controlled modes are associated with a dipole radiation pattern in the far-field, and therefore have a minimum radiation in the direction perpendicular to the vibrating structure.

For simply supported beam, the far-field pressure directivity in the $x - z$ plane is shown in Figures 3.8 and 3.9 for frequencies of 76 Hz and 200 Hz respectively, where negative value

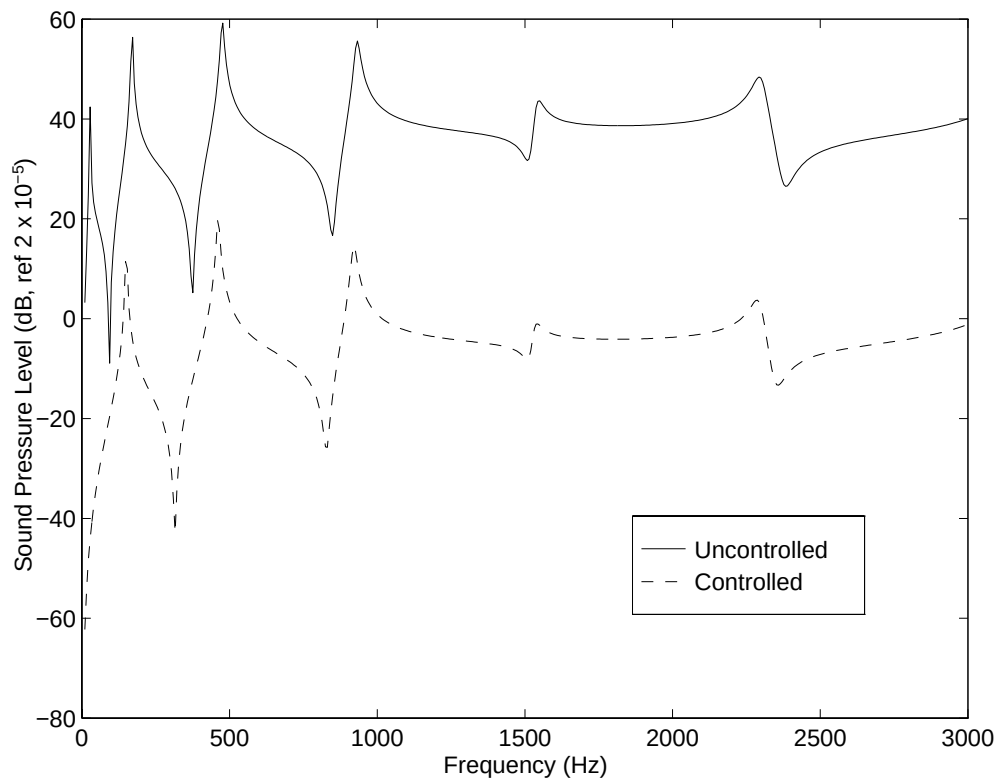


Figure 3.7: Far-field radiated pressure in a direction perpendicular to the clamped-free beam

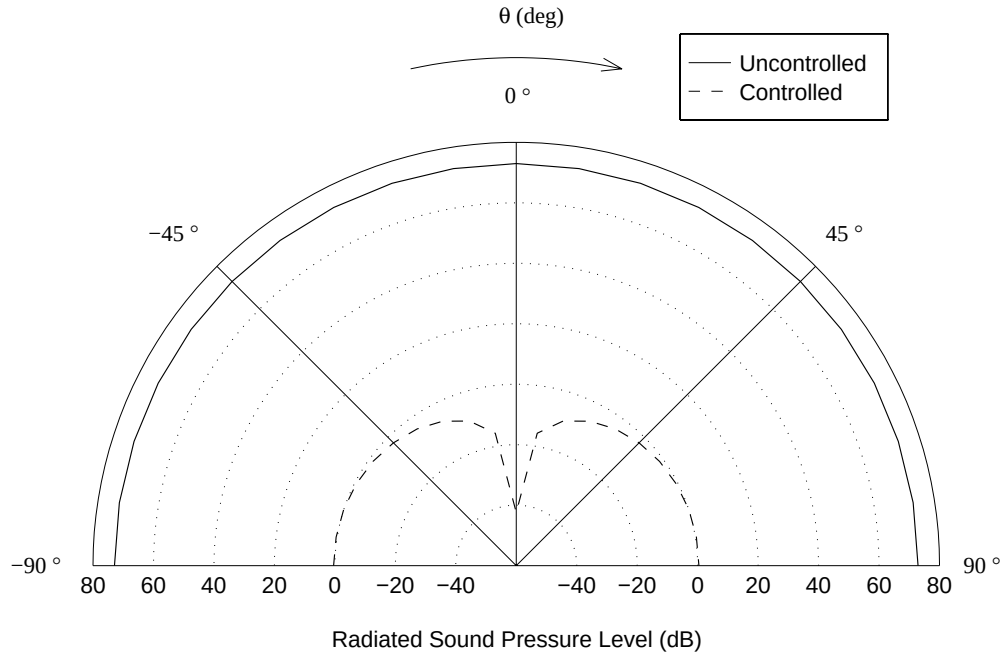


Figure 3.8: Radiation directivity of the simply supported beam at 76 Hz (first uncontrolled resonance)

of θ corresponds to $\phi = 0$ in the positive x plane. The sound pressure levels are calculated at a radius $R = 5 \times a$. For an excitation frequency of 76 Hz, which is the first resonance frequency of the uncontrolled system, significant global sound attenuation has been achieved. Similar results can also be observed when the system is driven at other uncontrolled resonance frequencies. At 200 Hz, i.e., an off-resonance frequency for both the uncontrolled and controlled system, excellent global sound attenuation is also obtained as seen in Figure 3.9. However, it should be noted that the global sound attenuation decreases as the frequency increases. Similar results can be observed for clamped-free beam in Figures 3.10 and 3.11 for frequencies of 169.8 Hz and 200 Hz. The results show that the controlled system behaves very much like a dipole type radiator. Sound radiation directivity at the controlled resonance frequencies is also studied for both beams. Figure 3.12 shows the far-field pressure directivity at 653.1 Hz, which corresponds to the second controlled resonance frequency of

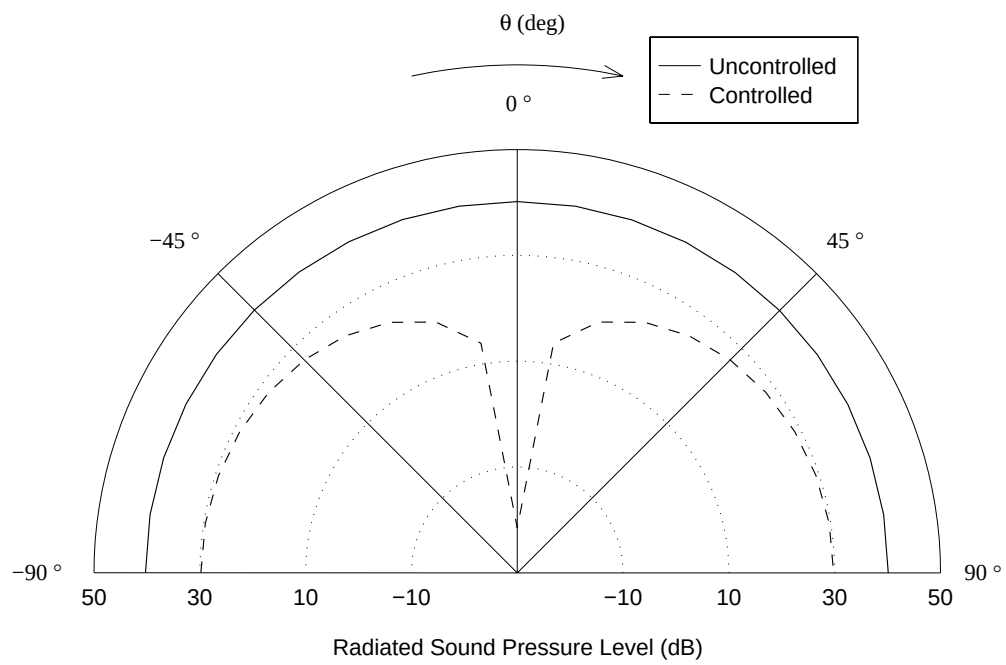


Figure 3.9: Radiation directivity of the simply supported beam at 200 Hz (off-resonance)

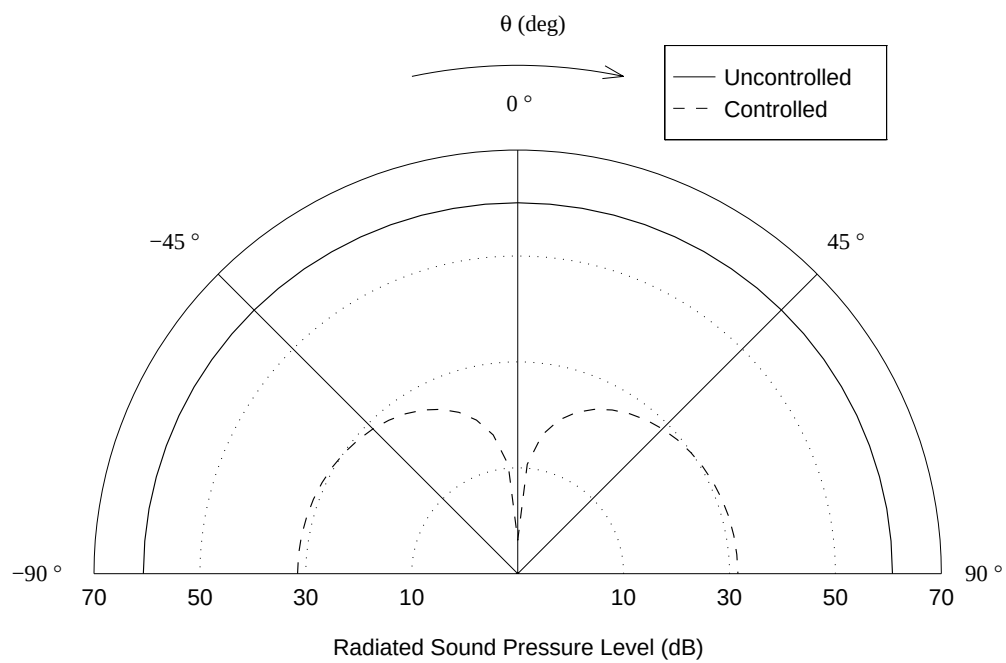


Figure 3.10: Radiation directivity of the clamped-free beam at 169.8 Hz (second uncontrolled resonance)

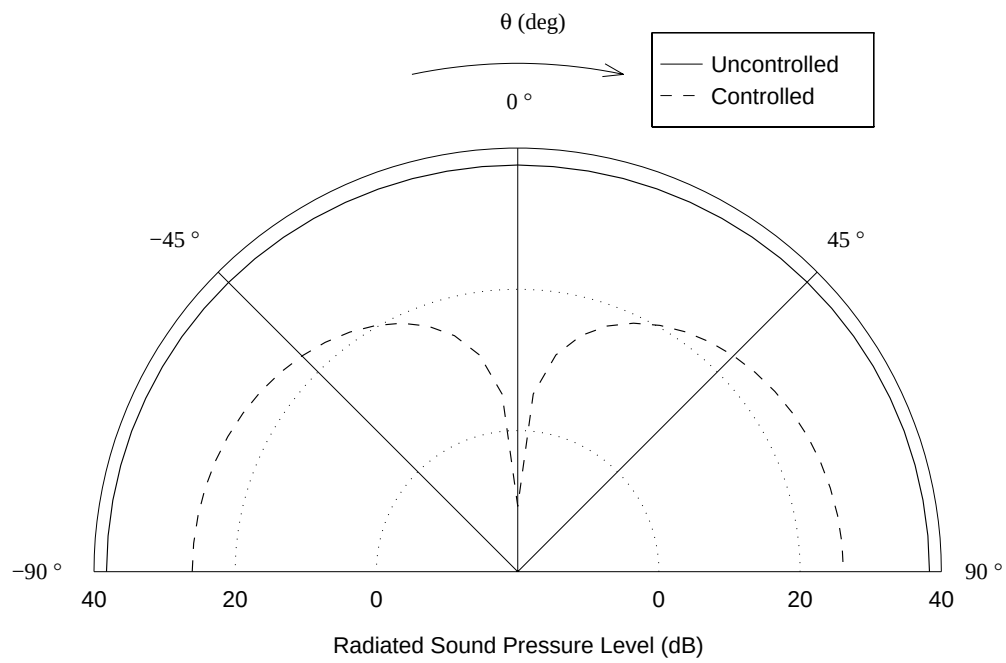


Figure 3.11: Radiation directivity of the clamped-free beam at 200 Hz (off-resonance)

the simply supported beam. Radiation directivity associated with the first controlled resonance frequency of the clamped-free beam (i.e., 150.4 Hz) is given in Figure 3.13. Close observation of these two plots suggests that the sound radiation decreases near $\theta = 0$ but increases elsewhere. However, the global radiation level does not increase significantly. Similar conclusion holds true for other controlled resonance frequencies.

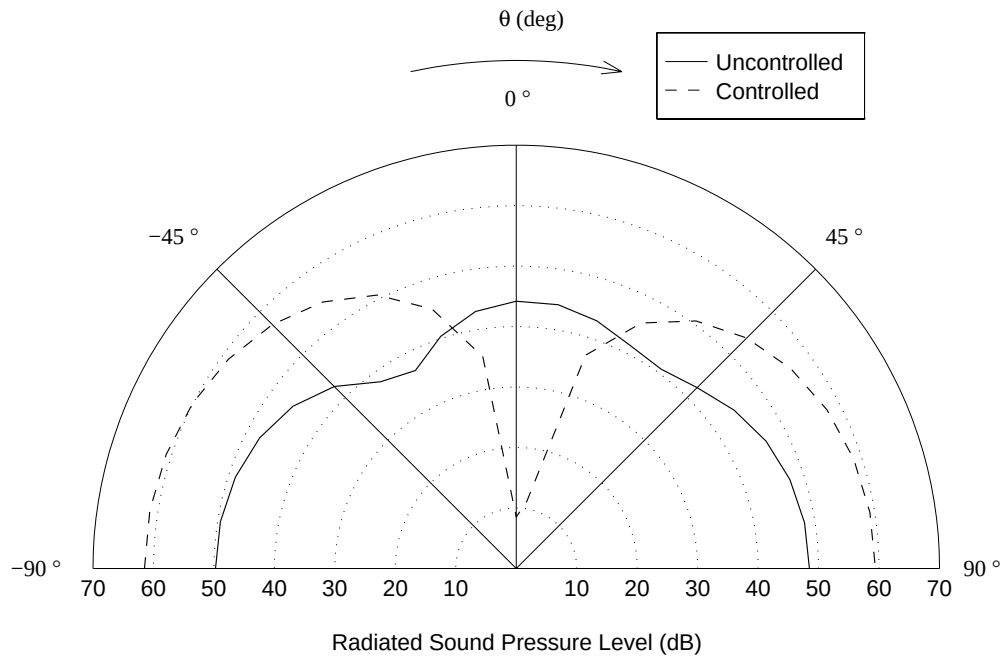


Figure 3.12: Radiation directivity of the simply supported beam at 653.6 Hz (Second controlled resonance)

To evaluate the overall performance of the controlled systems, the total radiated sound power (in dB ref. 10^{-12} W) and radiation efficiency for both the uncontrolled and controlled systems are calculated. The results for simply supported beam are shown in Figures 3.14 and 3.15, respectively. Figure 3.14 shows excellent sound power attenuation below the first controlled (or second uncontrolled) natural frequency. The radiation efficiency plot (see Figure 3.15) shows that the controlled system efficiency is greatly reduced when the frequency

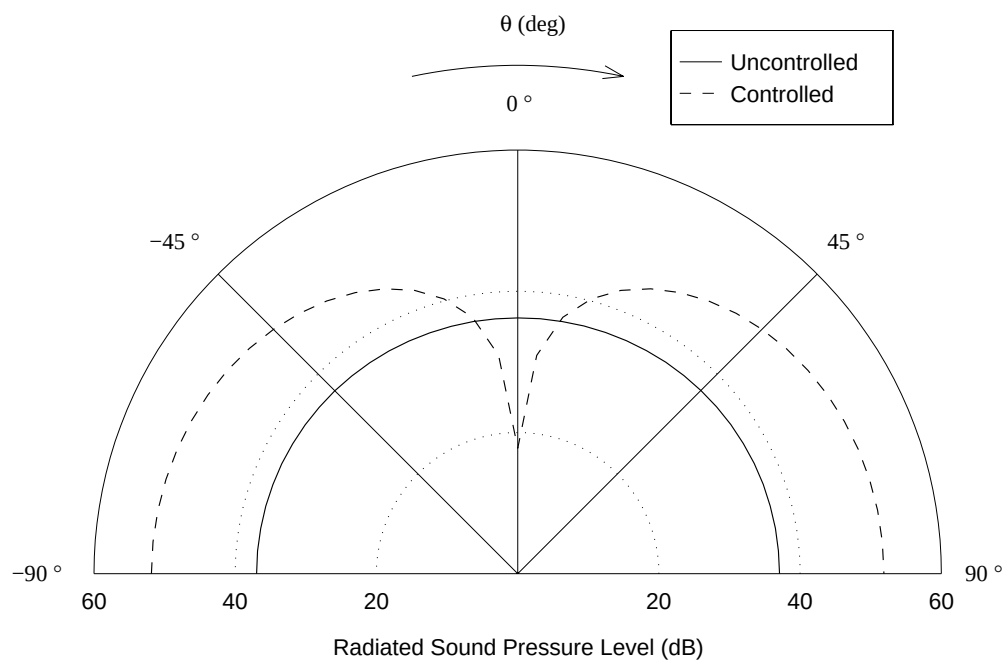


Figure 3.13: Radiation directivity of the clamped-free beam at 150.4 Hz (first controlled resonance)

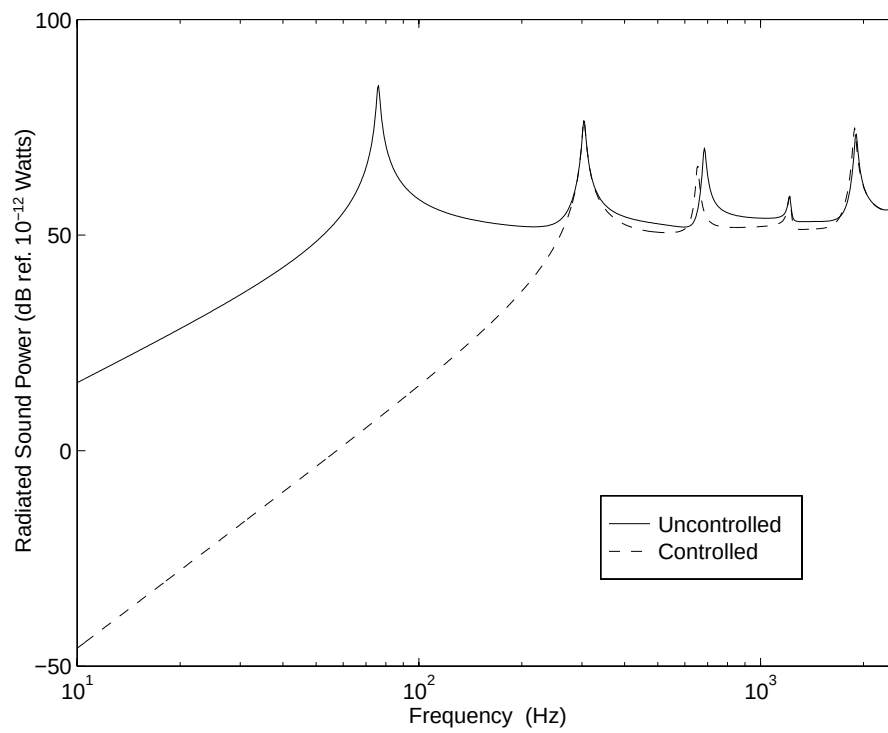


Figure 3.14: Radiated sound power of the simply supported beam

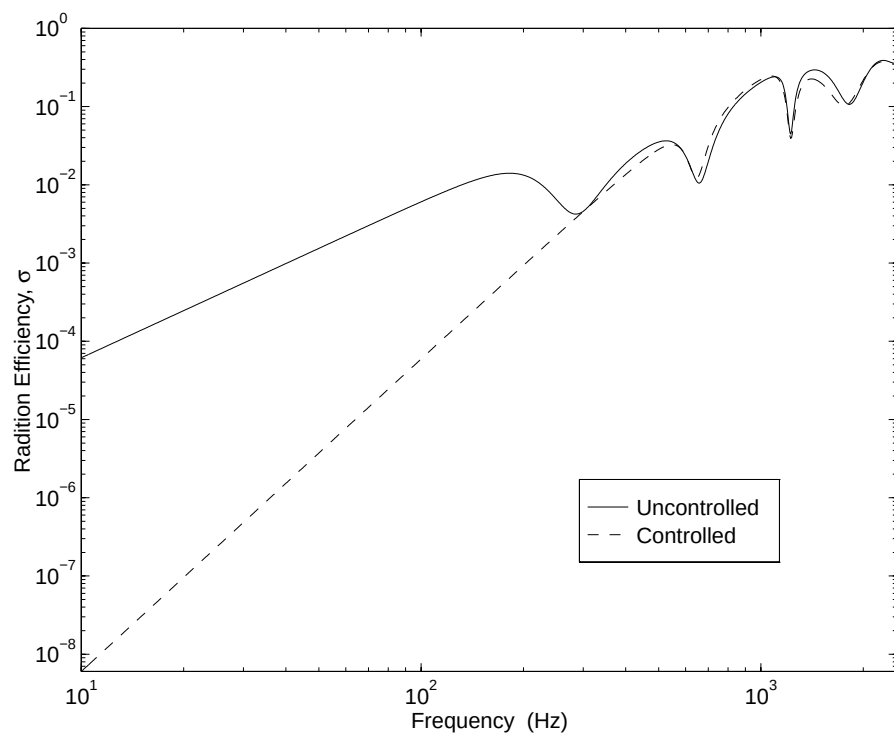


Figure 3.15: Radiation efficiency of the simply supported beam

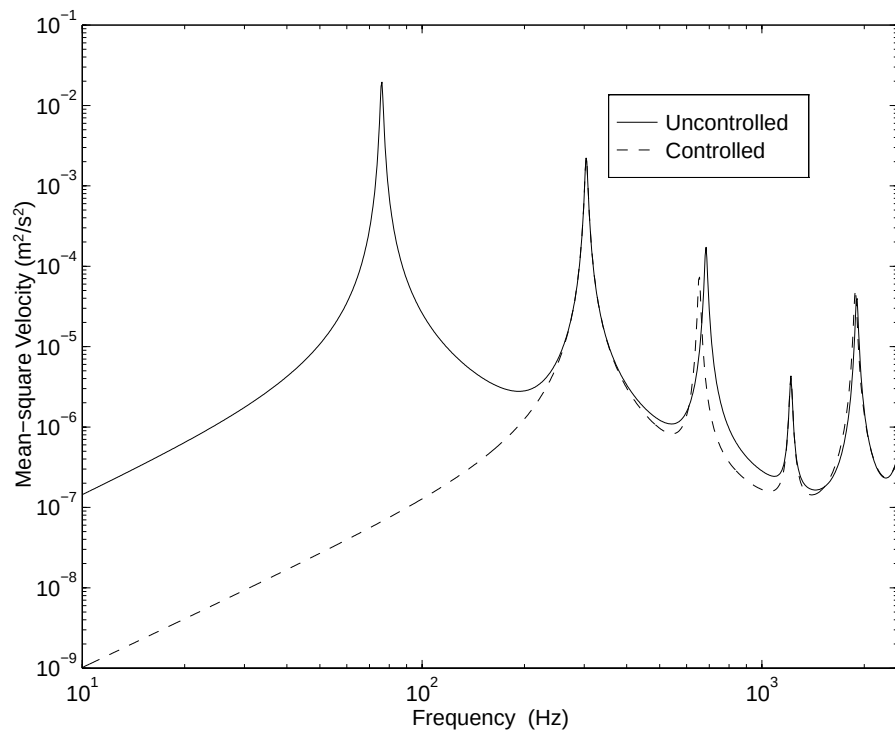


Figure 3.16: Mean-square velocity of the simply supported beam

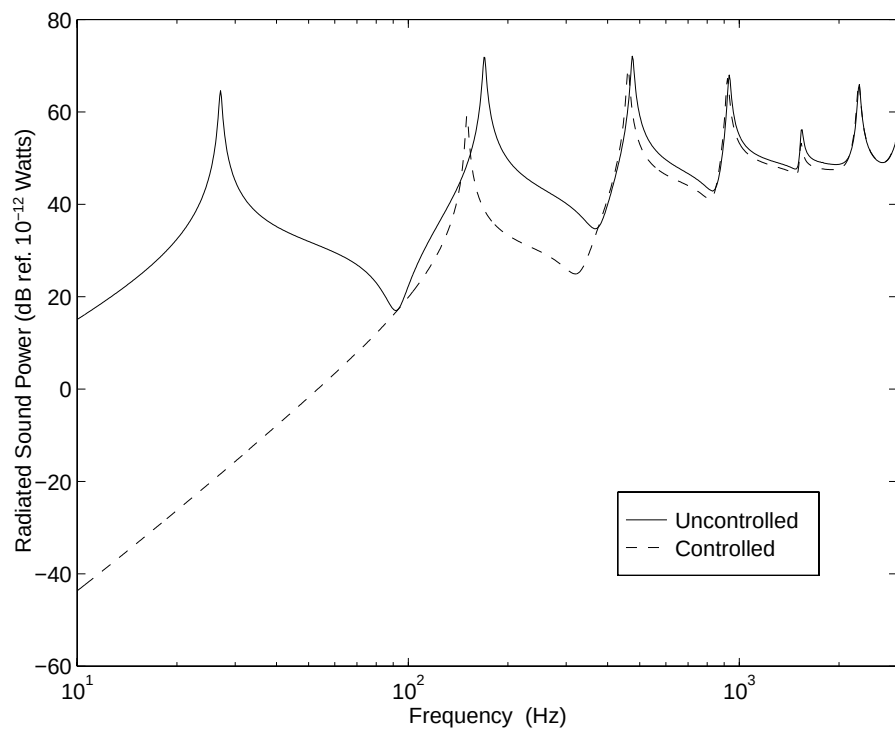


Figure 3.17: Radiated sound power of the clamped-free beam

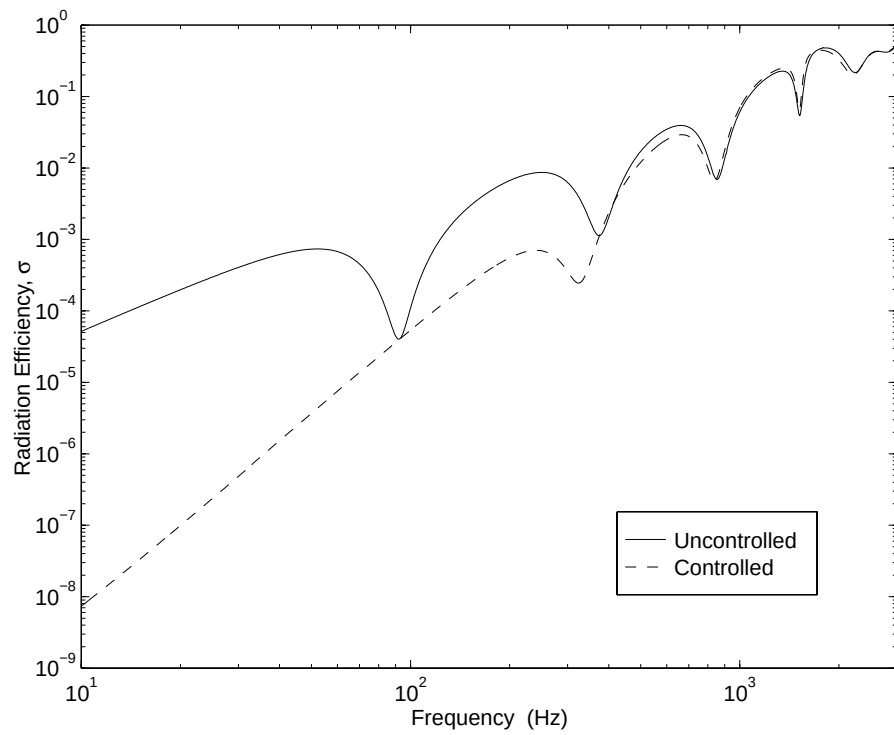


Figure 3.18: Radiation efficiency of the clamped-free beam

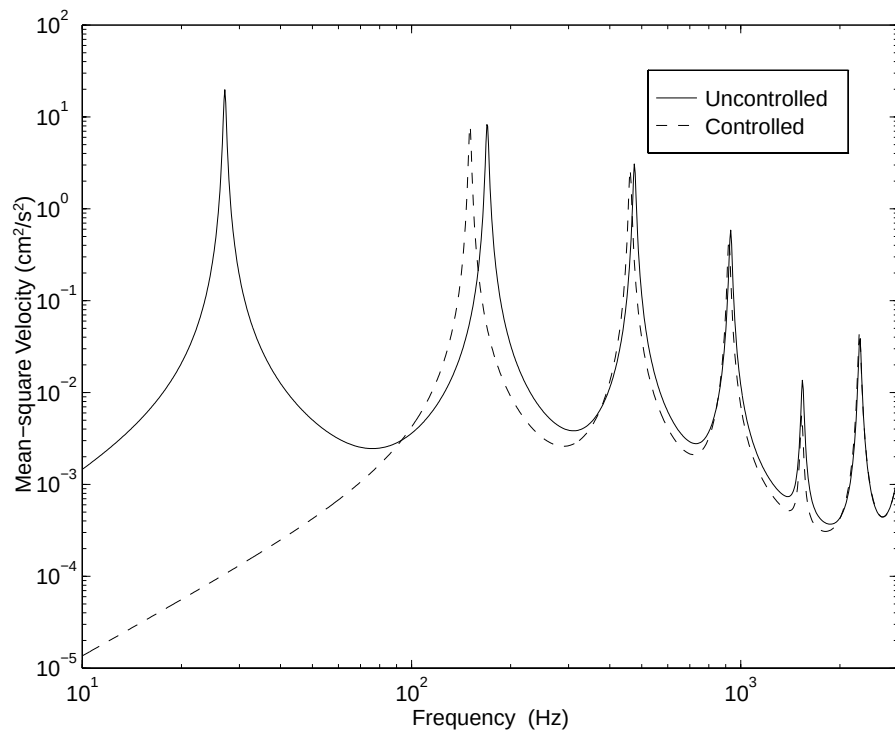


Figure 3.19: Mean-square velocity of the clamped-free beam

is below the second controlled (or third uncontrolled) natural frequency. In general, the controlled system corresponds to an inefficient radiator for frequencies below the first controlled resonance frequency. Figure 3.14 also shows good sound power attenuation for off-resonance frequencies, in contrast to the work described in reference [23]. To further understand the underlying mechanism for the controlled system, the spatial average mean-square velocity before and after control (see equation (3.37)) is presented in Figure 3.16. It can be seen that the controlled vibrational levels are generally reduced in the studied frequency range. This fact suggests that the “modal reduction” [3] is an important mechanism in the proposed control process. On the other hand, the controlled modes are a linear combination of the uncontrolled modes and the controller is designed to make all the controlled modes non-volumetric. This fact implies that “modal restructuring” [3] is also an important mechanism in the control behavior. In general, the overall performance of the controlled system is limited by the first natural frequency of the controlled system. The same conclusions hold true for clamped-free beam, as one can see from Figure 3.16 to Figure 3.19. To overcome this limit, one may try to increase this first controlled resonance frequency. However, as expressed in equation (3.5), the upper bound for the first controlled natural frequency is given by the second uncontrolled resonance frequency. Thus, for the present design, the control efficiency is restricted in frequency by the constraint on the controlled resonance frequencies. This limitation could be overcome by adopting a similar control design using a MIMO feed-forward controller and the recursive formulation as proposed in Chapter 2. However, this will not be included in this dissertation and should be explored in the future work.

3.6.3 Control performance comparison

In this section, different control methods are compared for the previous system of the simply supported beam. The “non-volumetric design” corresponds to the approach proposed in the previous section. The “volume velocity sensing design” considers the use of a volume velocity sensor on the simply supported beam as described in [23]. The “accelerometer sensing design” uses an accelerometer located at the center of the beam as error sensor. All other system parameters are identical. For the latter two approaches, the error sensor

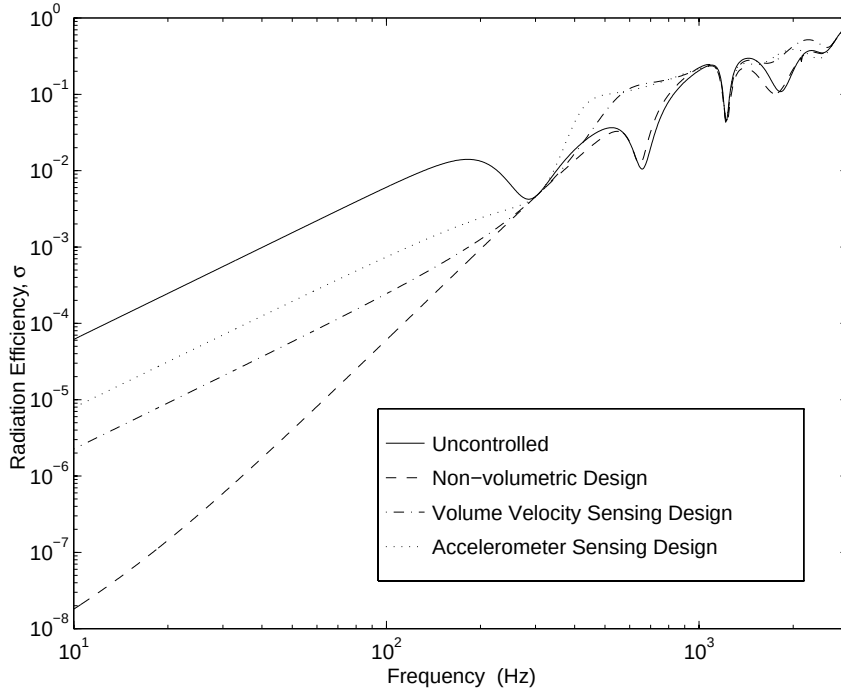


Figure 3.20: Comparison of radiation efficiency for different control methods

detects only odd-order modes as they are the only ones that are non-volumetric. The control input in these two cases is assumed to be a concentrated force located at the middle of the beam. This location was selected to prevent the control force from coupling into the even-order modes which are already non-volumetric and therefore radiate weakly. The last two methods do not include design of the actuator load; indeed, they are purely based upon a sensing strategy and the control load is chosen only based upon physical insight.

The radiation efficiencies of the three different controlled systems are shown in Figure 3.20. All methods provide a large decrease of the radiation efficiency in the low frequency domain. However, the use of the accelerometer as an error sensor leads the poorest results (an increase of the radiation efficiency can be noticed at high frequencies) as its signal is not directly related to sound radiation. The non-volumetric design proposed in this dissertation gives better results than the volume velocity sensing design throughout the frequency range

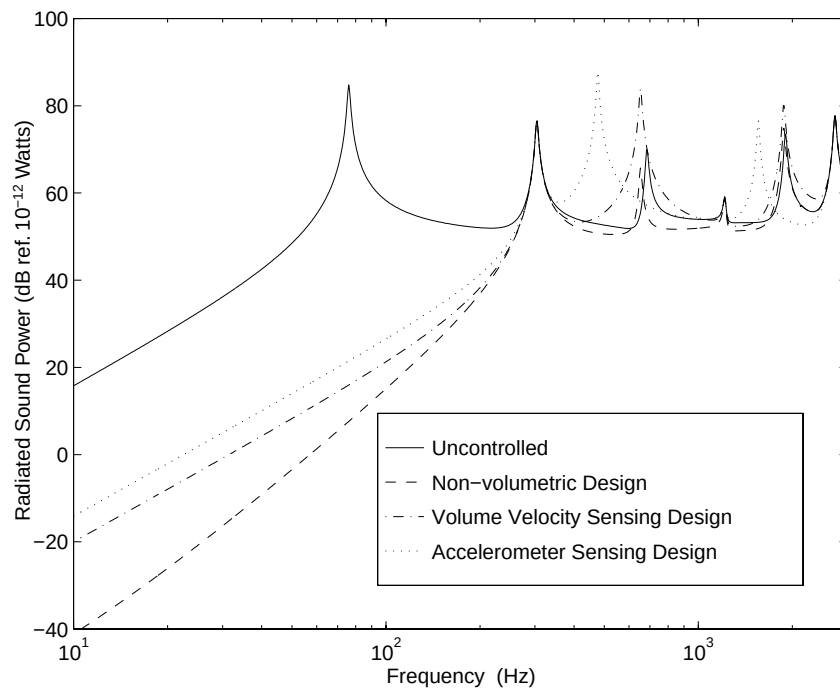


Figure 3.21: Comparison of radiated sound power for different control methods

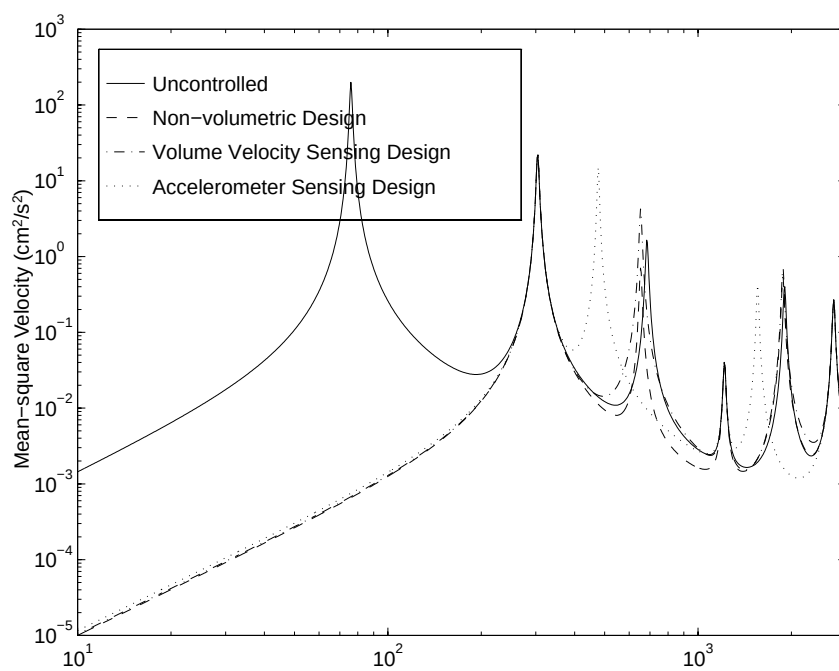


Figure 3.22: Comparison of mean-square velocity for different control methods

of study. This is due to the fact that the non-volumetric design provides the modal characteristics of both the sensor and the actuator simultaneously. The associated controlled radiated sound power and mean-square velocity are shown in Figures 3.21 and 3.22, respectively. The non-volumetric design is the most efficient in the low frequency region (up to 10 dB improvement) in terms of controlling sound power radiation. It is also apparent from Figure 3.16 that the non-volumetric design (the procedure of this dissertation) does not lead to a large increase of radiated sound and vibrational levels at off-resonance excitation of the uncontrolled system while the other two methods do. For the volume velocity sensing design, the sound radiation is increased significantly around 1 kHz because of a large increase of the displacement in the same frequency range (see Figure 3.22). It can be noticed in Figure 3.22 that the volume velocity sensing design and the accelerometer sensing design are associated with very large increase of vibrational levels in the higher frequency region. It appears that the method developed in this dissertation provides a better controlled system, because the vibrational and acoustic levels are generally not increased at off-resonance frequencies of both the controlled and uncontrolled system, while the low frequency performance is significantly improved. This improvement is due to the fact that the sensor and the control input are optimized at the same time regardless of frequency and excitation characteristics, thus limiting control spillover effects.

3.7 Summary

In this chapter a new formulation for the design of ASAC systems is proposed. The design methodology proposed here is based on the fact that elemental radiators with zero integrals of normal velocity over their surface (i.e., non-volumetric) are inefficient radiators of sound at low frequencies. A SISO eigenassignment method is thus developed in order to modify the system eigenstructure so that all controlled modes are non-volumetric, leading to an overall sound attenuation in the far field. Although the design procedure was carried out in the modal domain for simple one-dimensional structure located in rigid baffle, it is readily extensible to more general planar structures since it is based upon a normal modal analysis. One main advantage of this method is that the design does not depend on the characteristics

of the external disturbance, such as the form, location and frequency contents. The design is only based on the modal behavior of the structure. The effectiveness of this formulation was demonstrated through numerical simulations for the control of radiation from simply supported and clamped-free beams. Significant sound power attenuation is achieved in the low frequencies, up to the second uncontrolled resonance frequency. The present approach was also compared to other approaches employing a volume velocity sensor or an accelerometer as a sensor at the center of the beam. The control method discussed in this dissertation provides larger attenuation of both sound radiation and vibrational levels than the two latter cases. This increased control performance is associated with the fact that the proposed method optimizes both the error sensor and control input simultaneously.

Chapter 4

Implementation of the non-volumetric design

The design of the physical error sensors and actuators yielding the non-volumetric controlled system as proposed in the previous chapter is discussed in this chapter. First, a brief discussion of the selection of suitable types of actuators and error sensors is presented. The design of the shaped PVDF sensor rendering the non-volumetric modes for one-dimensional planar structure is described next. Then two different design approaches for the PZT actuators are developed. The design of the PZT actuators and PVDF error sensor rendering the non-volumetric controlled system is carried out for the simply supported beam studied in the previous chapter. The eigenassignment design process is then complete for the simply supported beam. Finally, a summary of the design procedure discussed in this chapter is presented.

4.1 Types of actuators and error sensors

The design of the physical actuators and sensors is an important issue in any practical control implementation. The determination of the appropriate types and positions of the

control transducers usually depends on the application. In general, control transducers can be divided into two categories: discrete and continuously distributed transducers. Discrete transducers consist of point transducers and/or array of point transducers. Distributed transducers commonly used in ASAC are the piezoelectric transducers, such as PZT ceramic actuators and PVDF film sensors. The major advantage of the distributed transducers over conventional transducers is due to their light weight and their ability to excite/observe only certain motions by choosing appropriate positions and shape. Detailed discussion over this topic with particular emphasis on ASAC applications can be found in Fuller *et al.* [44].

In this work, PZT (lead, zirconate, titanate) ceramic actuators are used due to their light weight and their ability to provide sufficient control authority. Also, because of their distributed nature, the shape and locations of the actuators can be chosen to excite only certain modes (e.g., odd-order modes in the case of the non-volumetric design for simply supported beam) without exciting the other modes, thus provides the potential to effectively reduce the control spillover.

For the non-volumetric design, the error sensor that yields the desired modal error sensor components as prescribed in the previous chapter can be obtained by using the traditional point sensors (accelerometers, for example) employed in array with different weights. The main disadvantage of this approach is due to the associated signal conditioning requirements and aliasing. On the other hand, distributed piezoelectric sensors show potential to overcome this disadvantage. They can be shaped so as to act as spatial filters which only observe the required one or several modes (e.g., odd-order modes in the case of the non-volumetric design for simply supported beam). Due to their continuous, distributed nature, this spatial filtering is achieved by effectively integrating the measured variable over the sensor surface covered by electrode. In other words, the distributed piezoelectric sensor such as the polymer polyvinylidene fluoride (PVDF) film can be tailored to yield the desired modal error sensor components. Thus, PVDF film becomes the natural choice for the non-volumetric design.

4.2 Design of the PVDF sensor

The design of the shaped PVDF sensor is discussed in this section. Although targeted on PVDF film, the development presented in this section is applicable to any other distributed strain material. Assume the finite beam shown in Figure 4.1 is covered with a thin layer of PVDF film and that the PVDF film is mechanically isotropic and its thickness is small compared to its length. It is also assumed that the PVDF film has uniform electro-mechanical property cross the area covered by the film. The assumption for the beam is the same as that in Section 3.4.1. Following Lee and Moon [21], the total charge $Q(\omega)$ generated by the PVDF film when it is deformed can be written as

$$Q(\omega) = -\tilde{z} \int_0^a e_{31} \mathcal{F}(x) \frac{\partial^2 w(x, \omega)}{\partial x^2} dx \quad (4.1)$$

where

$\tilde{z}$	=	z -coordinate of the mid-plane of the PVDF film
e_{31}	=	Piezoelectric stress constant
$\mathcal{F}(x)$	=	Shape function of the PVDF film
w	=	Out-of-plane displacement of the beam

and $\tilde{z}$, $\mathcal{F}(x)$ are defined as

$$\tilde{z} = \frac{h_b + h_s}{2} \quad (4.2)$$

$$\mathcal{F}(x) = \int_{-b/2}^{+b/2} F(x, y) P(x, y) dy \quad (4.3)$$

where b is the width of the beam, and h_b and h_s are the thickness of the beam and PVDF film, respectively. $\mathcal{F}(x)$ is introduced to represent the spatial dependent part of the interactions between PVDF and electric field summed along the y -direction. $F(x, y)$ models the effective surface electrode: $F(x, y)$ equals one if (x, y) is covered by the electrode, and zero otherwise.

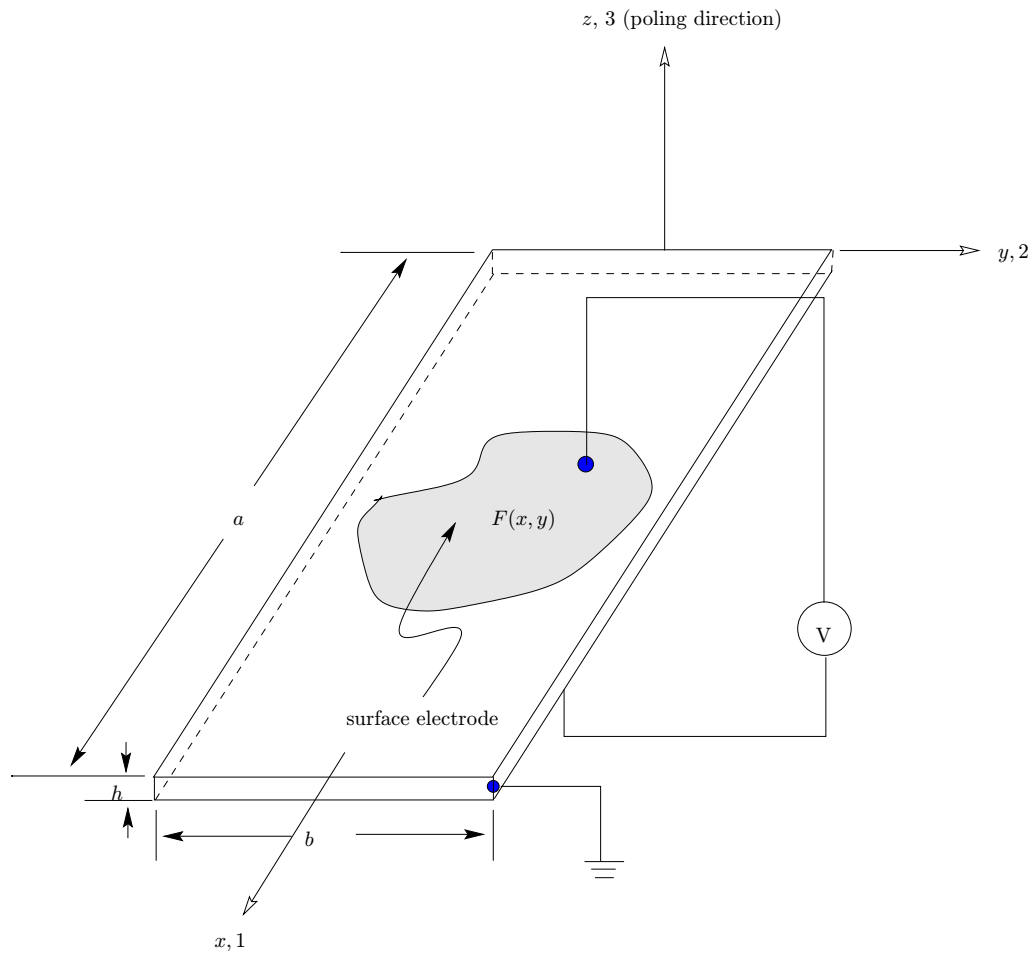


Figure 4.1: Schematic of PVDF and beam configuration

The polarization profile $P(x, y)$ is used to model the effect that PVDF is ferroelectric, which means that the piezoelectric strength can be changed or reversed by poling. In general, $P(x, y)$ will be ± 1 for piezoelectric material poled through its thickness direction.

For one-dimensional beam, the PVDF can be shaped to observe only one particular mode or several modes (i.e., the PVDF film can effectively act as modal sensors [21]). Recalling Chapter 3, the beam out-of-plane displacement can be represented by a series expansion of the form (see equation (3.18))

$$w(x, \omega) = \sum_{n=1}^N q_n(\omega) \phi_n(x) \quad (4.4)$$

Substituting equation (4.4) into equation (4.1) yields

$$Q(\omega) = \sum_{n=1}^N q_n(\omega) \xi_n \quad (4.5)$$

where the modal error sensor components ξ_n are defined as

$$\xi_n = -\tilde{z} \int_0^a e_{31} \mathcal{F}(x) \left[\frac{\partial^2 \phi_n(x)}{\partial x^2} \right] dx \quad (4.6)$$

Since the finite beam is a self-adjoint system, the modes are orthogonal to each other [39]. This important property facilitates the modal sensor design. Assuming that $\mathcal{F}(x)$ is proportional to the summation of the second derivatives of the modes:

$$\mathcal{F}(x) = \sum_{n=1}^N b_n EI(x) \frac{\partial^2 \phi_n(x)}{\partial x^2} \quad (4.7)$$

where b_n is the weighting coefficient for the n^{th} mode, substituting the above equation into equation (4.6) and recalling the orthogonormality conditions (see equations (2.16) and (2.17)), it can be found that

$$\xi_n = -\tilde{z} e_{31} b_n \omega_n^2 \quad (4.8)$$

The weighting coefficients b_n are therefore defined as

$$b_n = -\frac{\xi_n}{\tilde{z}e_{31}\omega_n^2} \quad (4.9)$$

For the non-volumetric design as prescribed in Chapter 3, the desired modal error sensor components ξ_n are given by equation (3.6). The shape of the piezoelectric sensor rendering the desired modal error sensor components is then defined by the unique, closed form solution represented by equations (4.7) and (4.9).

4.3 Design of the PZT actuators

The design of the control actuators is covered in this section. The reciprocal relationship of the distributed piezoelectric sensors and actuators [20, 21] ensures that the development for the modal sensors as described in the previous section can be used to design the modal actuators. However, PVDF film is not suitable for an actuator since it has low piezoelectric strain constant and internal stiffness and can not produce sufficient control authority to achieve control [14]. The PZT ceramic material is an excellent candidate for an actuator, but commercial etchant for PZT patch was not readily available at the time of the experiment. For these reasons, other design considerations for the PZT actuators were explored.

A brief discussion of the available models for piezoelectric actuators is discussed first. The static approach adopted in this work is then presented. Two design strategies based on the classical model are then described in detail.

4.3.1 Modeling of the PZT actuators

In general, the modeling of the piezoelectric actuators can be categorized into two different groups: the static (Newtonian) approach and the variational (Hamiltonian) approach.

The static approach is traditionally used to model the structural responses of the various systems excited by piezoelectric actuators and the mass and stiffness loading introduced by the transducers are generally ignored [13, 14, 45]. Dimitriadis *et al.* [45] developed a simplified model for a thin plate excited by a pair of PZT actuators bonded symmetrically to both sides of the plate and driven 180° out of phase (anti-symmetric actuator). Their analytical results show that the actuators can be modeled by equivalent line moments acting along the edges of the actuator. Experiments carried out by Clark *et al.* [46] on a simply supported beam show that this static model provides a reasonable approximation to the dynamic excitation of the beam by the anti-symmetric PZT pairs. Gibbs and Fuller [47] developed a static model for a beam driven by a single-sided PZT patch (asymmetric actuator). In this case both the flexural and extensional displacements are induced simultaneously whose relative amplitudes are functions of beam/actuator geometries and properties. For the flexural response, the asymmetric actuator can be effectively modeled as an external load consisting of a pair of line moments of opposite sign acting along the edges of the actuator. As for the induced extensional vibration, the asymmetric actuator is effectively equivalent to two equal and opposite line forces acting in the longitudinal direction along actuator edges. The extensional vibration is generally not significant in the low frequency range if the length of the actuator is small compared to the wavelength.

The variational approach provides more accurate model for the piezoelectric transducers. The dynamic effects of the piezoelectric transducers (mass and stiffness loading) can be included in the analysis [36]. Charette *et al.* [48] developed a detailed model for the asymmetric actuation of a beam with arbitrary boundary conditions. Their results show that the dynamic effects of the piezoelectric actuators on the beam response correspond mainly to a mass loading. They also demonstrated that the extensional actuation is considerably less than the flexural actuation.

In this work, the formulation based on the static approach is adopted for its simplicity. The effect of the mass loading due to the actuator can be compensated by means of experimental modal analysis, as will be discussed later in Chapter 6. Since one side of the beam surface is reserved for the PVDF sensor, an array of asymmetric PZT actuators is used on the opposite side.

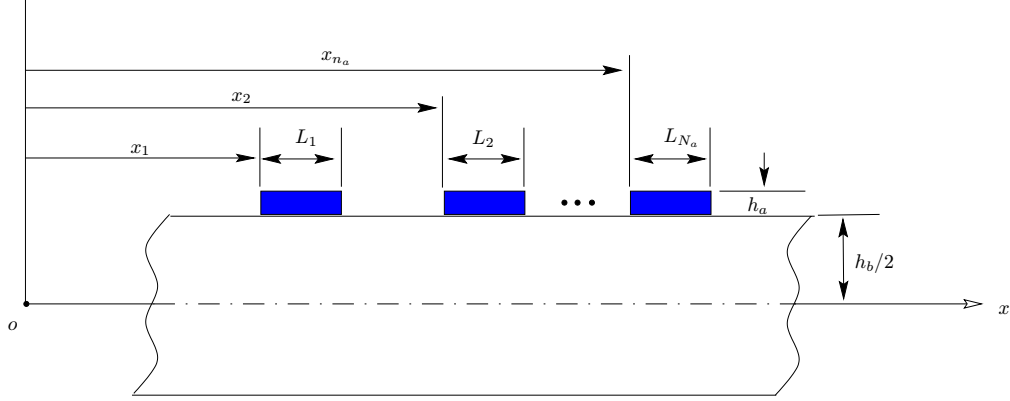


Figure 4.2: Schematic of an array of PZT actuators

Assume an array of N_a asymmetric actuators (voltage driven) is bonded to one side of a finite beam, as shown in Figure 4.2. The equation of motion of the flexural displacement of the beam driven by the PZT actuators can be written as [44]:

$$\begin{aligned} \frac{\partial^2}{\partial x^2} \left[EI(x) \frac{\partial^2 w(x)}{\partial x^2} \right] + C \frac{\partial w(x)}{\partial t} + m(x) \frac{\partial^2 w(x)}{\partial t^2} \\ = C_0 \epsilon_{pe}^p \left(\frac{b_a^p}{b} \right) \sum_{p=1}^{N_a} [\delta'(x - x_p) - \delta'(x - x_p - L_p)] \end{aligned} \quad (4.10)$$

where

- $EI(x)$ = Bending stiffness of the beam
- C = Damping coefficient of the beam
- $m(x)$ = Mass density per unit length of the beam
- L_p = Length of the p^{th} PZT actuator
- b_a^p = Width of the p^{th} PZT actuator
- b = Width of the beam
- x_p = Locations of the edge of the p^{th} actuator

$$\begin{aligned}
C_0 &= \text{Piezoelectric strain-beam moment coupling term} \\
\epsilon_{pe}^p &= \text{Induced strain of the } p^{th} \text{ unconstrained PZT actuator} \\
\delta'(\cdot) &= \text{Derivative of the Dirac Delta function}
\end{aligned}$$

and C_0 and ϵ_{pe}^p are defined as

$$C_0 = EI(x)K_f \quad (4.11)$$

$$\epsilon_{pe}^p = \frac{d_{31}V^p}{h_a} \quad (4.12)$$

where d_{31} is the piezoelectric strain constant, h_a is the thickness of the PZT actuator and V^p is the applied driving voltage to the p^{th} PZT actuator. The material-geometric constant K_f for the asymmetric actuator is given as [44]

$$K_f = \frac{6E_{pe}Eh_bh_a(h_b + h_a)}{E^2h_b^4 + E_{pe}E(4h_b^3h_a + 6h_b^2h_a^2 + 4h_bh_a^3) + E_{pe}^2h_a^4} \quad (4.13)$$

where E_{pe} and h_b are the Young's modulus of the PZT actuator and the thickness of the beam, respectively.

The out-of-plane displacement of the beam is represented by equation (4.4). Substituting equation (4.4) into equation (4.10) and using the orthogonormality conditions (see equations (2.16) and (2.17)), the generalized coordinates $q_n(\omega)$ can be written as

$$q_n(\omega) = \sum_{p=1}^{N_a} \frac{2C_0\epsilon_{pe}^p}{m(x)a} \left(\frac{b_a^p}{b} \right) [\phi'_n(x_p) - \phi'_n(x_p + L_p)] H_n(\omega) \quad (4.14)$$

where $H_n(\omega)$ is the n^{th} modal frequency response function given by equation (3.23).

It is assumed that all PZT actuators have identical material properties and are driven either in phase or 180° out of phase. Since the relative value of the modal control input is the only relevant information needed in the actuator design [12], the modal control input

μ_n , from equation (4.14), can be written as

$$\mu_n = \sum_{p=1}^{N_a} P_p \left[\phi'_n(x_p) - \phi'_n(x_p + L_p) \right] \quad (4.15)$$

where $P_p = \pm 1$ is used to denote the relative phase of the driving voltage to the p^{th} actuator.

Having obtained the expression for the modal control input for the PZT actuators, the design of the PZT actuators yielding the desired control configuration as discussed in previous chapter is presented next.

4.3.2 Optimal locations of the PZT actuators

The design of the PZT actuators rendering the desired modal control components $\bar{\mu}_n$ (for a simply supported beam, the desired modal control inputs $\bar{\mu}_n$ are presented in Table 3.2) can be carried out using a constrained optimization technique.

To simplify the optimization problem, it is assumed that all PZT actuators have same length, i.e., $L = L_1 = L_2 = \dots = L_{N_a}$. The design variables are the number of PZT actuators N_a , the positions of the PZT actuators x_p and the phase of the driving voltage P_p ($p = 1, 2, \dots, N_a$) for each actuator. The optimization problem can be constructed as

$$\text{Minimize} \quad F(x_p, P_p, N_a) = \sum_{n=1}^N [\bar{\mu}_n - \mu_n]^2 \quad (4.16)$$

$$\text{Subject to} \quad \begin{cases} \sum_{n=1}^N \mu_n^2 = 1.0 \\ 0 \leq x_p \leq a - L & \text{for } p = 1, 2, \dots, N_a \\ x_{p+1} - x_p \geq L & \text{for } p = 1, 2, \dots, N_a \end{cases} \quad (4.17)$$

where μ_n is the modal control input defined by equation (4.15). The equality constraint represents the normalization of modal control inputs (since the relative controllability of the modes is the only relevant information), and other constraints are introduced to ensure that

all PZT actuators will be located on the beam and there is no overlap between any two consecutive actuators.

The optimization represented by equations (4.16) and (4.17) is a *hybrid* type optimization problem, i.e., the design variables consist of both integer and continuous variables: N_a and P_p are integer variables while the positions of the PZT actuators x_p ($p = 1, 2, \dots, N_a$) are continuous variables. With the presence of the integer variables, the optimization problem becomes significantly complicated and in general requires very large computational efforts. Commercial software that can deal with the hybrid type optimization problem is not readily available. In this dissertation, a trial and error procedure based on the adaptive numerical optimization procedure as suggested by Arora [49] is adopted: first, assume the number of PZT actuators N_a is known and treat the phase of the driving voltage P_p as continuous variable, the optimization problem defined by equations (4.16) and (4.17) is solved by the program *constr* in MATLAB optimization toolbox [42], which adopts the sequential quadratic programming (SQP) algorithm; now the phases of the driving voltage P_p are treated as integer variables of value ± 1 , with their sign taken from the “optimal solution” obtained above, and the optimization problem defined by equations (4.16) and (4.17) is solved again. Repeat this process for various choice of N_a , until the desired modal control inputs have been obtained. The numerical example presented in Section 4.4.2 shows that excellent solution to this optimization problem can be obtained by using this procedure.

4.3.3 Sensitivity analysis for multiple PZT actuators

The sensitivity analysis associated with the optimal PZT actuator design as presented in the previous section is discussed in this section. The sensitivity of the cost function, and more importantly, the sensitivity of the normalized modal control inputs with respect to the variations of the PZT actuator positions are our primary concern.

The equality constraint presented in equation (4.17) can be incorporated into the cost

function defined by equation (4.16). The cost function can be rewritten as

$$F(x_p, P_p, N_a) = \sum_{n=1}^N [\bar{\mu}_n - \tau_n]^2 \quad (4.18)$$

where the normalized modal control input τ_n is defined as

$$\tau_n = \frac{\mu_n}{\sqrt{\sum_{s=1}^N \mu_s^2}} \quad (4.19)$$

and μ_n is defined by equation (4.15). Differentiating equations (4.18) and (4.19), it is found that

$$\frac{\partial F}{\partial x_p} = -2 \sum_{n=1}^N [\bar{\mu}_n - \tau_n] \frac{\partial \tau_n}{\partial x_p} \quad (4.20)$$

$$\frac{\partial \tau_n}{\partial x_p} = \frac{1}{\sqrt{\sum_{s=1}^N \mu_s^2}} \left[\frac{\partial \mu_n}{\partial x_p} - \tau_n \sum_{s=1}^N \tau_s \frac{\partial \mu_s}{\partial x_p} \right] \quad (4.21)$$

and $\frac{\partial \mu_n}{\partial x_p}$ can be obtained from equation (4.15) as

$$\frac{\partial \mu_n}{\partial x_p} = P_p [\phi_n''(x_p) - \phi_n''(x_p + L_p)] \quad (4.22)$$

The analytical expression of the gradient of the cost function represented by equation (4.20) can be adopted in the optimization program to facilitate the optimization process. One major concern of using the PZT as the control actuator is that the modal control inputs μ_n are sensitive to the variation of the positions of the PZT actuators, as the modal control inputs of the PZT are functions of the slopes of the structure mode shapes. For this reason, the design of the PZT actuators with different weights is proposed in the next section.

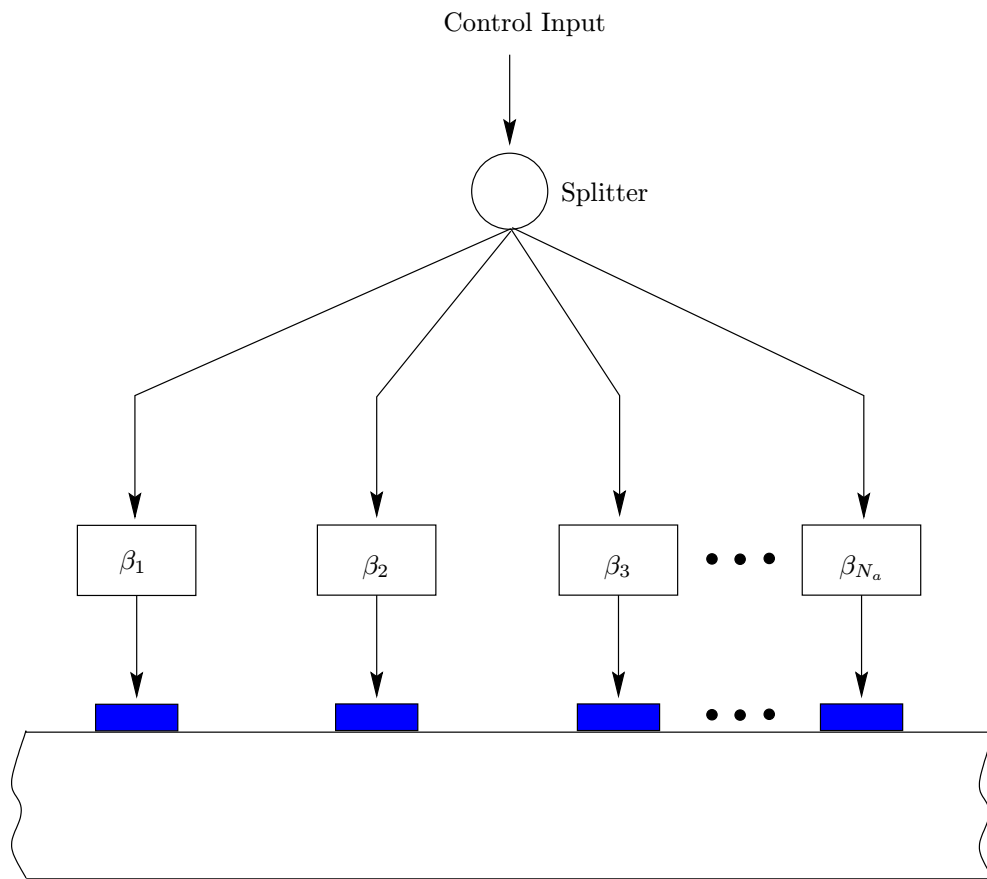


Figure 4.3: Schematic of an array of PZT actuators with different weights

4.3.4 PZT actuators with different weights

An alternative design strategy for the PZT actuators is presented in this section. Assume an array of PZT actuators of the same length are located at prescribed positions on the beam as shown in Figure 4.3. Each actuator is driven by the same control input but with different weights so that the desired modal control inputs $\bar{\mu}_n$ can be generated. Let β_p be the weight for the p^{th} actuator and assume that $N_a \geq N$, i.e., the number of the PZT actuators is at least equal to the number of modes included in the analysis. Equation (4.15) can be rewritten into the following linear system form

$$\begin{Bmatrix} \mu_1 \\ \vdots \\ \mu_N \\ 0 \\ \vdots \\ 0 \end{Bmatrix} = \begin{bmatrix} \psi_1(x_1) & \psi_1(x_2) & \dots & \psi_1(x_{N_a}) \\ \vdots & \vdots & \dots & \vdots \\ \psi_N(x_1) & \psi_N(x_2) & \dots & \psi_N(x_{N_a}) \\ \psi_{N+1}(x_1) & \psi_{N+1}(x_2) & \dots & \psi_{N+1}(x_{N_a}) \\ \vdots & \vdots & \dots & \vdots \\ \psi_{N_a}(x_1) & \psi_{N_a}(x_2) & \dots & \psi_{N_a}(x_{N_a}) \end{bmatrix} \begin{Bmatrix} \beta_1 \\ \vdots \\ \beta_N \\ \beta_{N+1} \\ \vdots \\ \beta_{N_a} \end{Bmatrix} \quad (4.23)$$

where

$$\begin{aligned} \psi_n(x_p) &= \phi'_n(x_p) - \phi'_n(x_p + L_p) & n &= 1, 2, \dots, N \\ & & p &= 1, 2, \dots, N_a \end{aligned} \quad (4.24)$$

The weight β_p for each PZT actuator rendering the desired modal control inputs $\bar{\mu}_n$ (for simply supported beam, the desired modal control inputs $\bar{\mu}_n$ are presented in Table 3.4) can be obtained by solving equation (4.23). By taking more actuators than the number of modes included in the analysis, the influence of the higher order modes that are not included in the analysis can be reduced. This can be done by setting the modal control inputs of the higher order modes to zero, i.e., zero padding the desired modal control inputs vector, as shown in equation (4.23).

Recalling the sensitivity analysis described in Section 4.3.3, the design of the PZT actuators with different weights are preferred, since the weight for each PZT actuator can be

changed easily to accommodate to the actual system properties (such as the experimentally measured mode shapes, actual positions of the PZT actuators, etc.).

4.4 Implementation for a simply supported beam

The design of the PVDF sensor and PZT actuators that yield the desired modal error sensor components and modal control inputs is carried out for the simply supported beam studied in Section 3.6. The dimensions and material properties of the simply supported beam are given in Table 3.1. The first 6 modes are included in the analysis.

4.4.1 PVDF error sensor

The PVDF film available at Vibration and Acoustics Laboratories (VAL) of Virginia Tech is the *Kynar* [50] piezoelectric film manufactured by AMP Incorporated. The properties of the *Kynar* piezoelectric film are listed in Table 4.1.

The shape of the PVDF error sensor can be tailored to yield the desired modal error sensor components as discussed in Section 4.2. For the simply supported beam studied in the previous chapter, the desired modal error sensor components ξ_n of the non-volumetric design are given in Table 3.4. The shape of the PVDF sensor is then described by equations (4.7) and (4.9), with the piezoelectric stress constant given by $e_{31} = d_{31}E_s$. The shaped PVDF sensor for the non-volumetric design is shown in Figure 4.4.

Table 4.1: Properties of the Kynar PVDF film

Thickness, h_a (m)	28×10^{-6}
Young's Modulus, E_s (N/m ²)	2×10^9
Piezoelectric Strain Constants, d_{31} (C/N)	23×10^{-12}



Figure 4.4: Shaped PVDF sensor for the non-volumetric design for the simply supported beam

4.4.2 PZT actuators

As discussed in Section 4.3, the design of the PZT actuators rendering the non-volumetric design is independent of the characteristics of the PZT actuators. Different PZT actuators are used in the experimental process. In order to keep the material concise, the properties of the PZT actuators used in this dissertation will be presented later in Chapter 6. Both design strategies for the PZT actuators discussed in this chapter are carried out for the simply supported beam.

Optimally positioned PZT actuators

Assume the length of each PZT actuator is 0.024 m. The optimization problem prescribed in Section 4.3.2 is then carried out for the simply supported beam. The optimal solution indicates that 6 PZT actuators driven in phase are necessary to generate the desired modal control inputs given in Table 3.4. The optimal locations of the PZT actuators and the associated modal control inputs are presented in Table 4.2. For comparison, the desired modal control inputs prescribed in Table 3.4 are also listed in the same table.

Table 4.2 shows that the optimization procedure described in Section 4.3.2 leads to an excellent solution. The controlled system should respond with the desired non-volumetric modes.

Table 4.2: Optimal positions of the PZT actuators for the simply supported beam

PZT #	Position x_p (m)	Phase of Driving Voltage P_p	Optimal Modal Control Components μ_n	Desired Modal Control Components $\bar{\mu}_n$
1	0.058	+1	0.954	0.954
2	0.130	+1	-0.012	-0.012
3	0.155	+1	0.279	0.279
4	0.223	+1	0.006	0.006
5	0.251	+1	0.101	0.101
6	0.343	+1	0.034	0.034

Sensitivity analysis

To visualize the sensitivity of the normalized modal control inputs with respect to the variations of the PZT actuator positions, a simplified sensitivity analysis is carried out for the simply supported beam for illustration purpose.

Assuming that all PZT actuators are exactly located at their prescribed positions as listed in Table 4.2 except the first one, whose position is perturbed around its optimal position. The variations of the first 6 modal control inputs with respect to this perturbation are presented in Figure 4.5, with the x-axis representing the normalized position of the first PZT actuator. The partial derivatives of the normalized modal control inputs with respect to this perturbation (see equation (4.21)) are also calculated and shown in Figure 4.6.

From Figures 4.5, it can be observed that the normalized control inputs of the PZT actuators are very sensitive to the positions of the PZT actuators. For instance, the modal control component of the first mode τ_1 (the dominant modal control component for the non-volumetric design, see Table 4.2) changes substantially for a small perturbation around its optimal position. This observation is also confirmed by Figure 4.6, which clearly shows that the associated gradient of the first modal control component $\partial\tau_1/\partial x_1$ is very steep. For this

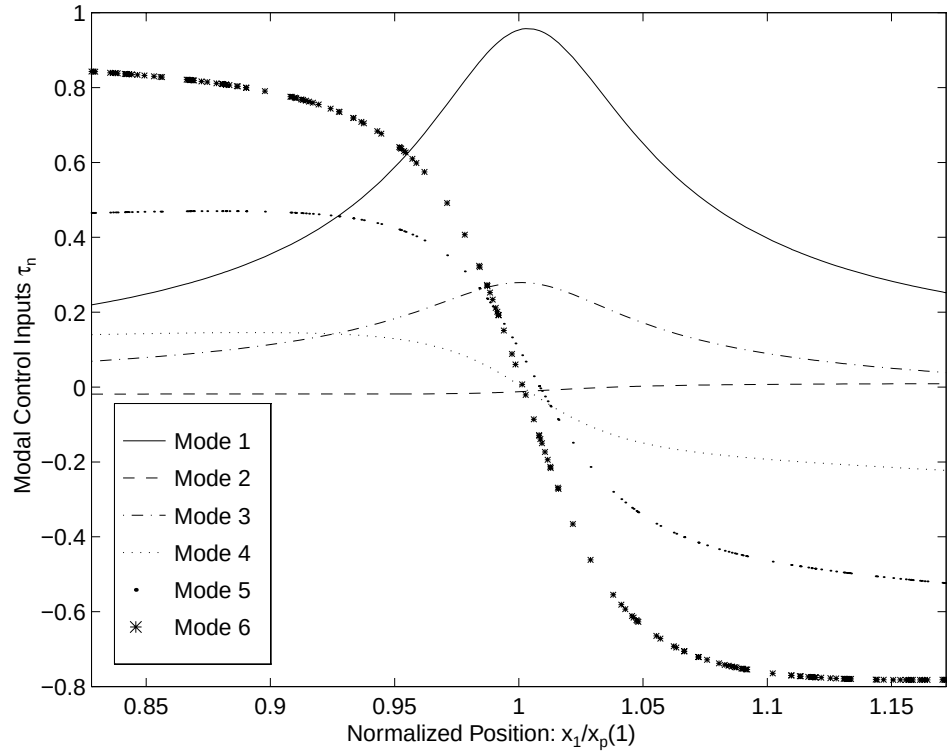


Figure 4.5: Variations of τ_n w.r.t the perturbation of the first PZT actuator position

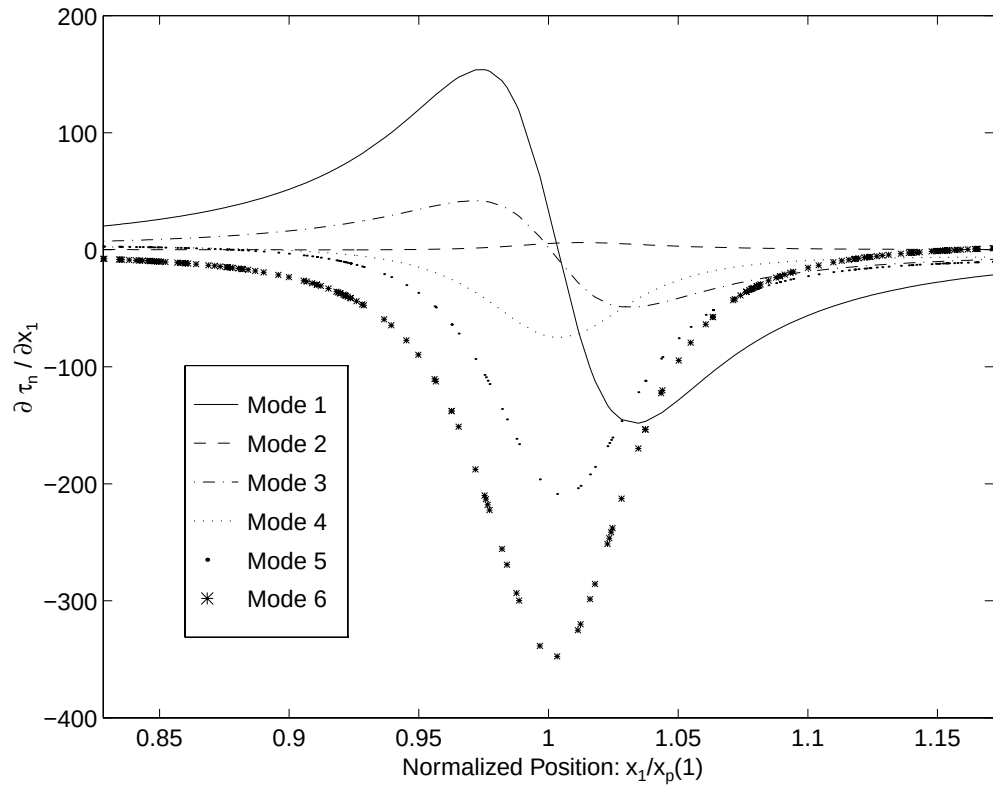


Figure 4.6: Gradients of modal control inputs w.r.t the perturbation of the first PZT actuator position

reason, the design of the PZT actuators with different weights is preferred, as we concluded in Section 4.3.4.

PZT actuators with different weights

The design of the PZT actuators with different weights is also carried out for the simply supported beam. The beam has 7 single-sided PZT patches of dimension $0.038\text{m} \times 0.024\text{m} \times 1.27e^{-4}\text{m}$ mounted on one side. Six of them are used as the control actuators. The other one is used as the disturbance actuator. The weights for each PZT actuator can be obtained by solving equation (4.23). Since the relative magnitudes of the weights are the only relevant information, the weights for each PZT actuator are normalized by setting the maximum weight to unity. The normalized weights as well as the positions of each control PZT actuator are listed in Table 4.3.

Table 4.3: Weights for each control PZT actuators for the simply supported beam

PZT #	Position x_p (m)	Weights β_p
1	0.04	0.5624
2	0.012	0.8958
3	0.156	0.5768
4	0.224	1.0000
5	0.260	0.4147
6	0.340	0.6266

The sign of each weight indicates the relative phase of the driving voltage applied to the corresponding PZT actuator. The PZT actuators should be driven either in phase or 180° out of phase. From Table 4.3, all PZT actuators should be driven in phase.

4.5 Summary

The design of the physical actuators and error sensors to implement the non-volumetric control strategy has been presented in this chapter. Distributed structural transducers such as those made from PZT ceramics and PVDF film were adopted due to their light weight and their ability to only observe/excite certain modes without the need of signal processing.

The design of the shaped PVDF sensor rendering the desired modal error sensor components is straight forward. The design of the PZT actuators, however, posed the major challenge in this chapter. Two design strategies were developed for the PZT actuators, namely the optimally positioned PZT actuators and PZT actuators with different weights. The optimization problem associated with the optimally positioned PZT actuators design was successfully solved. The solution to the optimization problem is not unique, and the true minimum for the objective function can not be guaranteed in general. However, the modal control inputs are the only concern in the thesis. As long as the optimal modal control inputs match closely with the desired ones, the associated solution can be regarded as a true optimum to the optimization problem. The associated sensitivity analysis of the cost function, and more importantly, the variations of the normalized modal control inputs due to the variations of the positions of the PZT actuators were also carried out. The analysis shows that the modal control inputs of the PZT actuator are sensitive to the variations of the PZT position. This fact, together with the consideration that once the PZT actuators were attached to the structure, their position could not be adjusted, stimulated the design of PZT actuators with different weights. The major advantage of this approach is that the weight for each PZT actuator can be adjusted as desired through simple signal conditioning.

The eigenassignment design process is now complete for the non-volumetric design for the simply supported beam. Experimental studies of this approach applied to the simply supported beam are presented later in Chapter 6.

Chapter 5

Design of ASAC system with minimum supersonic wavenumber

The design of ASAC systems by using the eigenassignment method to minimize the supersonic components of the structural wavenumber spectrum is developed in this chapter. The relationship between the supersonic wavenumber components of the structural velocity spectrum and the acoustic radiation from the structure is discussed first. A SISO controller is then designed to modify the eigenstructures of the system so that the eigenfunctions (i.e., mode shapes) of the controlled system have a minimum supersonic wavenumber distribution. As a direct consequence, the global sound radiation from the structure will be significantly attenuated in the low frequency range ($k_0 a \leq 1$, where a is the characteristic dimension of the structure). To this end, an optimization strategy is developed to find the desired expansion coefficients for the controlled eigenfunctions. The design process is then demonstrated to control the odd-order modes of a simply supported beam under harmonic disturbance. The acoustic performance of the proposed design is thoroughly evaluated and compared to that of the non-volumetric design discussed in Chapter 3. Finally, a summary for the minimum supersonic wavenumber design is presented to conclude this chapter.

5.1 Wavenumber components and acoustic radiation

Sound power radiated from vibrating structure can be obtained by applying the wavenumber transform technique as summarized in Appendix C. As discussed by Junger and Feit [33], the radiation pressure from a plate at a particular angle is directly proportional to the corresponding wavenumber component of the structural velocity spectrum (see equation (C.14)). Based on this observation, Fuller and Burdisso [34] developed a wavenumber control technique to minimize sound radiation in a particular direction. This is done by expressing the cost function of the optimal controller in the wavenumber domain and suppressing the discrete wavenumber component associated with that particular direction.

Furthermore, it is well known that only supersonic wavenumber components (i.e., structural wavenumbers smaller than the acoustic wavenumber) of the structural wavenumber spectrum contribute to the far-field radiation. The subsonic wavenumber components (i.e., structural wavenumbers greater than the acoustic wavenumber), on the other hand, are associated with decaying near field waves close to the surface of the vibrating structure, and do not radiate to the far field [51, 25, 44]. Clark and Fuller [35] experimentally and theoretically demonstrated a wavenumber domain controller that is designed to minimize several spectral lines of the structural wavenumber spectrum in the supersonic region.

It has been shown that global sound attenuation is always associated with reduction of the supersonic wavenumber components [44]. For instance, for the non-volumetric design for the simply supported beam discussed in Chapter 3, the wavenumber modulus spectra of each modes before and after control are shown in Figure 5.1 (for simplicity, only the wavenumber spectrum along the k_x -axis is shown). From Figure 5.1, it can be observed that the supersonic wavenumber components of each structural modes are significantly reduced in the low frequency range after the control is applied. In fact, the non-volumetric design is equivalent to minimizing the supersonic wavenumber spectral line corresponding to $k_x = 0, k_y = 0$, as discussed in Section 3.2.

In general, it can be concluded that reducing the supersonic wavenumber components of the structural wavenumber spectrum will equivalently lead global sound attenuation. The minimum supersonic wavenumber design discussed next stems from this observation.

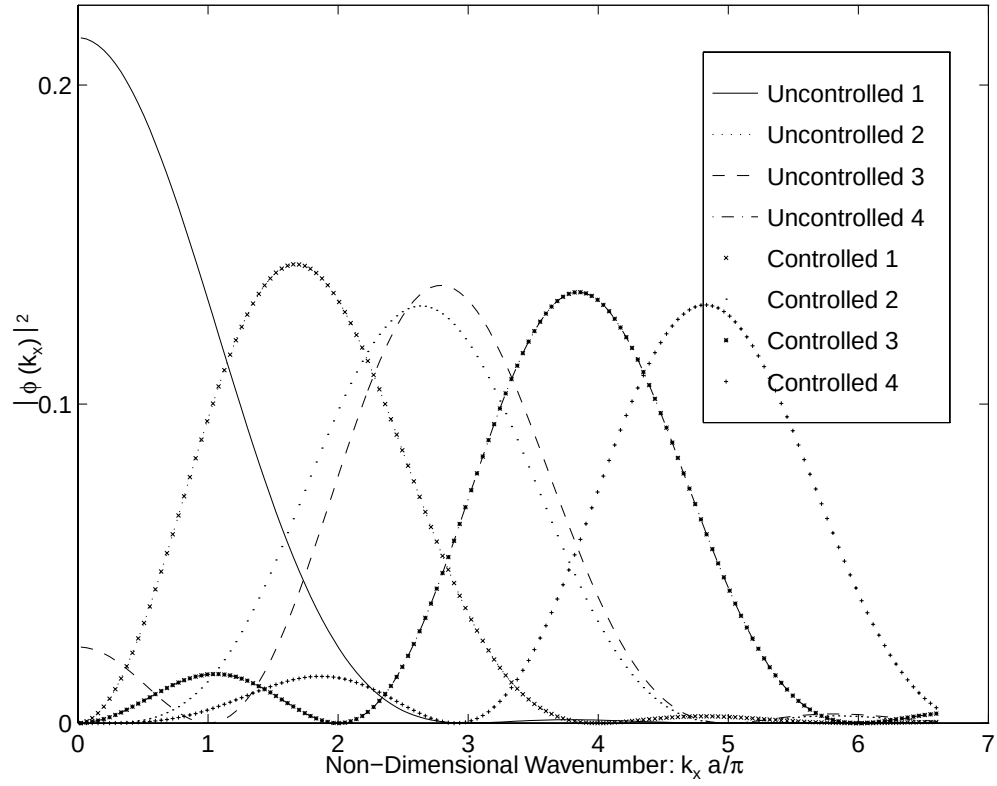


Figure 5.1: Non-volumetric design: wavenumber modulus spectra of a simply supported beam

5.2 Minimizing supersonic wavenumber components

From equation (C.21), it is easy to observe that the sound power radiated by the planar structure is proportional to the wavenumber components within the supersonic region, i.e.,

$$\Pi(\omega) \propto \iint_{k_x^2 + k_y^2 \leq k_0^2} |\dot{W}(k_x, k_y)|^2 dk_x dk_y \quad (5.1)$$

where the supersonic region of the structural velocity wavenumber spectrum corresponds to the circular region defined by equation (C.20), i.e.,

$$\sqrt{k_x^2 + k_y^2} \leq k_0 \quad (5.2)$$

If we can design a control system that minimizes the supersonic components of the structural velocity wavenumber spectrum, the sound power radiated by the structure will be reduced. Recalling Section C.2, the wavenumber transformed velocity of the planar structure is given by equation (C.27). The supersonic region of the structural velocity wavenumber spectrum can thus be written as

$$\iint_{k_x^2 + k_y^2 \leq k_0^2} |\dot{W}(k_x, k_y)|^2 dk_x dk_y = \omega^2 \bar{\mathbf{\Gamma}}^T \left[\iint_{k_x^2 + k_y^2 \leq k_0^2} \mathbf{\Phi}(k_x, k_y) \mathbf{\Phi}^H(k_x, k_y) dk_x dk_y \right] \bar{\mathbf{\Gamma}}^* \quad (5.3)$$

where $\bar{\mathbf{\Gamma}}$ is the $N \times 1$ vector of the generalized coordinates and N is the number of modes included in the analysis, $\mathbf{\Phi}(k_x, k_y)$ is the $N \times 1$ wavenumber transformed eigenfunction vector of the structure, and $()^T$ and $()^*$ denote the transpose and complex conjugate operators, respectively.

Similar to the non-volumetric design discussed in Chapter 3, the first step in the eigenasignment design formulation is to find the set of desired eigenfunctions with which the closed loop controlled system should respond. The desired eigenfunctions of the controlled system should have minimum supersonic wavenumber components. Since the eigenfunctions of the

controlled system are linear combinations of those of the uncontrolled system, this task is equivalent to find the desired set of expansion coefficients. This can be done by minimizing equation (5.3) (which is the integral of the structural velocity wavenumber components within the supersonic region) with respect to the expansion coefficient vector $\bar{\Gamma}$ as follows:

$$\text{Minimize } F(\bar{\Gamma}) = \bar{\Gamma}^T \mathbf{M}(\omega) \bar{\Gamma}^* \quad (5.4)$$

$$\text{Subject to } \bar{\Gamma}^T \bar{\Gamma}^* = 1.0 \quad (5.5)$$

where the supersonic wavenumber spectra matrix $\mathbf{M}(\omega)$ is defined as

$$\mathbf{M}(\omega) = \omega^2 \iint_{k_x^2 + k_y^2 \leq k_0^2} \Phi(k_x, k_y) \Phi^H(k_x, k_y) dk_x dk_y \quad (5.6)$$

and the equality constraint represents the normalization scheme for the desired expansion coefficient vectors $\bar{\Gamma}$.

The proposed optimization problem represents an eigenvalue problem for the matrix $\mathbf{M}(\omega)$. To show this, the optimization problem represented by equations (5.4) and (5.5) can be solved by Lagrange multiplier theorem [49]. The Lagrangian for this optimization problem is

$$L(\bar{\Gamma}, \lambda) = \bar{\Gamma}^T \mathbf{M}(\omega) \bar{\Gamma}^* - \lambda(\bar{\Gamma}^T \bar{\Gamma}^* - 1.0) \quad (5.7)$$

The stationary points of the Lagrangian at the optimal points can be found by setting its partial derivatives with respect to $\bar{\Gamma}$ and λ to zero:

$$\frac{\partial L(\bar{\Gamma}, \lambda)}{\partial \bar{\Gamma}} = \mathbf{M}(\omega) \bar{\Gamma}^* - \lambda \bar{\Gamma}^* = 0 \quad (5.8)$$

$$\frac{\partial L(\bar{\Gamma}, \lambda)}{\partial \lambda} = -(\bar{\Gamma}^T \bar{\Gamma}^* - 1.0) = 0 \quad (5.9)$$

The solution to the optimization problem therefore represents an eigenvalue problem subject

to a normalization condition:

$$\mathbf{M}(\omega)\bar{\mathbf{\Gamma}}^* = \lambda\bar{\mathbf{\Gamma}}^* \quad (5.10)$$

$$\bar{\mathbf{\Gamma}}^T \bar{\mathbf{\Gamma}}^* = 1.0 \quad (5.11)$$

It is easy to recognize that the supersonic wavenumber spectra matrix $\mathbf{M}(\omega)$ is a positive definite Hermitian matrix as the modulus squared of the structural wavenumber spectrum is always positive for any nontrivial vector $\bar{\mathbf{\Gamma}}$. This property ensures that all eigenvalues of the matrix $\mathbf{M}(\omega)$ are always real and positive and their associated eigenvectors are real and orthogonal to each other. The eigenvalues are ordered such that $\lambda_1 < \lambda_2 < \dots < \lambda_N$. The eigenvalue λ_l represents the minimum supersonic wavenumber achievable for the l^{th} controlled mode, which is a linear combination of the uncontrolled modes with the expansion coefficients given by the associated eigenvector $\bar{\mathbf{\Gamma}}_l$. The optimal set of controlled eigenfunctions is therefore chosen as that with the $N - 1$ lowest supersonic wavenumber, since the controlled system effectively has $N - 1$ new eigenvalues and their associated eigenfunctions due to the constraint imposed by driving the error signal to zero at all frequencies (see Chapter 2). The desired expansion coefficients for the $N - 1$ controlled eigenfunctions are then given by the first $N - 1$ eigenvectors $\bar{\mathbf{\Gamma}}_l$ of equation (5.10). These eigenfunctions form a set of independent base functions for the controlled system.

5.3 Optimal control configuration

Once the desired expansion coefficients $\bar{\mathbf{\Gamma}}_l$ rendering the minimum supersonic wavenumber are found, the configuration of the control system can be determined in the modal domain by using the SISO eigenassignment technique discussed in Chapter 3. More specifically, the controlled eigenvalues ω_{cl}^2 ($l = 1, 2, \dots, N - 1$) and the unit modal control inputs μ_n ($n = 1, 2, \dots, N$) can be obtained by solving the constrained non-linear least-square minimization problem defined by equations (3.3) and (3.4).

The determination of the lower and upper bounds for the l^{th} controlled eigenvalues ω_{cl}^2 , i.e., $(\omega_{cl}^2)_{\text{lower}}$ and $(\omega_{cl}^2)_{\text{upper}}$, however, requires discerning study and usually depends on the desired expansion coefficients Γ_{nl} . Some guidelines can be summarized as follow:

- The relative signs of each element of the desired expansion coefficient vectors (e.g., the eigenvectors $\bar{\Gamma}$ of equation (5.10) for the current design) determine the feasible bounds for the controlled eigenvalues ω_{cl}^2 . From equation (3.2), the sign of each expansion coefficients is determined by the relative magnitudes of the controlled eigenvalues ω_{cl}^2 and the uncontrolled eigenvalues ω_n^2 , as well as the sign of the modal control inputs μ_n . By studying the possible combinations of these three to match the relative sign of the desired expansion coefficients, the bounds for the controlled eigenvalues can often be determined.
- The bounds for the controlled eigenvalues should ensure that the controlled eigenvalues increase as the order of the controlled mode increases.

These two guidelines usually dominate the selection process. Furthermore, in order to improve the performance of the controlled system, the resonance frequencies (eigenvalues) of the controlled system should be detuned from the frequency contents of disturbance input as much as possible. However, the applicability of this latter guideline is usually severely limited by the first two considerations listed above. The first guideline may not be clear at this point. It will be illustrated through the design example presented in Section 5.5.2.

Once the optimal unit modal control inputs μ_n and the controlled system eigenvalues ω_{cl}^2 are determined, the modal error sensor components ξ_n can be obtained by solving equation (3.6). The determination of the optimal modal error sensor ξ_n and modal control inputs μ_n completely defines the control system in the modal domain. The structural and acoustic performance of the controlled system can then be evaluated by following the procedure discussed in Chapter 3.

5.4 Implementation for a simply supported beam

This section presents some simplifications of the previously developed general formulation in the case of a lightly damped simply supported beam.

For a simply supported beam, the wavenumber transformed eigenfunction vector $\Phi(k_x, k_y)$ of the uncontrolled system is defined by equations (3.31) and (3.43). Substituting these two equations into equation (5.6), a typical element of the supersonic wavenumber component matrix $\mathbf{M}(\omega)$ can be found as

$$M_{mn} = \begin{cases} \frac{2\omega^2}{k_m k_n} \frac{2}{ma} \iint_{k_x^2 + k_y^2 \leq k_0^2} \left[b \operatorname{sinc}\left(\frac{k_y b}{2\pi}\right) \right]^2 \frac{1 - (-1)^m \cos(k_x a)}{[1 - (k_x/k_m)^2][1 - (k_x/k_n)^2]} dk_x dk_y & \text{for } m, n \text{ in parity} \\ 0 & \text{for } m, n \text{ in opposite parity} \end{cases} \quad (5.12)$$

From this expression, the supersonic wavenumber component matrix $\mathbf{M}(\omega)$ can be rewritten as

$$\mathbf{M}(\omega) = \begin{Bmatrix} M_{11} & M_{31} & \dots \\ M_{31} & M_{33} & \dots \\ \vdots & \dots & \ddots \end{Bmatrix} + \begin{Bmatrix} M_{22} & M_{24} & \dots \\ M_{24} & M_{44} & \dots \\ \vdots & \dots & \ddots \end{Bmatrix} \quad (5.13)$$

Equations (5.12) and (5.13) clearly state that, for the supersonic wavenumber spectra matrix $\mathbf{M}(\omega)$ of a simply supported beam, the coupling between odd-even or even-odd (opposite parity) modes is identically zero, and only even-even or odd-odd (in parity) modes will interact with each other. This implies that, in terms of attenuating the supersonic wavenumber components (thus the attenuating of the sound power radiated from the structure), the mechanism of “modal restructuring” [3] works for the structural modes of the same parity. In other words, the attenuation of the supersonic wavenumber components can be achieved by the destructive interaction of the modes of the same parity. Equation (5.13) clearly indicates

that, for the simply supported beam, the control of the even-even, odd-odd, even-odd and odd-even modes should be treated independently.

In the next section, the minimum supersonic wavenumber design is applied to a simply supported beam to demonstrate the effectiveness of the proposed design strategy.

5.5 Numerical simulation

In this section, the minimum supersonic wavenumber design is applied to a simply supported beam. For simplicity, a SISO controller is designed to control the odd-order modes only, as they are the most efficient sound radiators. To this end, it is assumed that the odd-order modes are the only modes excited by the control inputs and observed by the error sensor. Mathematically this is equivalent to set the modal parameters μ_n and ξ_n to zero for all even-order modes.

The beam is constructed from carbon steel and its dimension and material property are given in Table 3.1. Similar to the non-volumetric design discussed in Chapter 3, the beam is assumed to be excited by a harmonic point force of 1 N in amplitude and located at $x_d = 0.1\text{m}$. The frequency range of study is limited to 10 Hz to 1600 Hz. The first 6 modes are included in the analysis. Note that the simulation performed in this section is carried out in the modal domain, without any knowledge of the physical actuators and sensors.

5.5.1 Optimal expansion coefficients

The desired expansion coefficients yielding the minimum supersonic wavenumber spectra can be obtained by solving the eigenvalue problem defined by equations (5.10) and (5.11), with the supersonic wavenumber spectra matrix $\mathbf{M}(\omega)$ given by the first matrix of equation (5.13),

i.e.,

$$\mathbf{M}(\omega) = \begin{Bmatrix} M_{11} & M_{31} & \dots \\ M_{31} & M_{33} & \dots \\ \vdots & \dots & \ddots \end{Bmatrix} \quad (5.14)$$

The solution to this eigenvalue problem is solved at the frequency of 400 Hz (i.e., $ka/\pi = 0.8799$) for illustration. As discussed in Section 5.2, since the SISO controller reduces the system's dynamic degree of freedom by 1, the desired expansion coefficient vectors for the two new controlled modes (to be constructed from the first three odd-order modes) are taken as the first two eigenvectors $\bar{\Gamma}_l$ ($l = 1, 2$) of matrix $\mathbf{M}(\omega)$. They are presented in Table 5.1.

Table 5.1: Desired expansion coefficients for the two constructed modes

Uncontrolled Mode Index	Eigenvectors	
	$\bar{\Gamma}_1$	$\bar{\Gamma}_2$
1	-0.0064	-0.3095
3	-0.5277	-0.8066
5	0.8494	-0.5035

In the above development, the even-order modes (i.e., modes 2, 4, 6) of the uncontrolled system are not affected by the controller, since the modal control inputs μ_n corresponding to these modes are set to zero. The odd-order modes (i.e., modes 1, 3, 5), which are efficient sound radiators [24, 25], however, are modified by the controller to minimize the associated supersonic wavenumber components. The eigenfunctions of the controlled system therefore consist of those original even-order modes and those two new modes constructed from the odd-order modes with the expansion coefficient vectors given in Table 5.1. The feasible set of controlled mode shapes is shown in Figure 5.2. The associated desired expansion coefficients $\bar{\Gamma}_{nl}$ are listed in Table 5.2.

Figure 5.2 shows that the odd-order modes of the controlled system correspond to the next

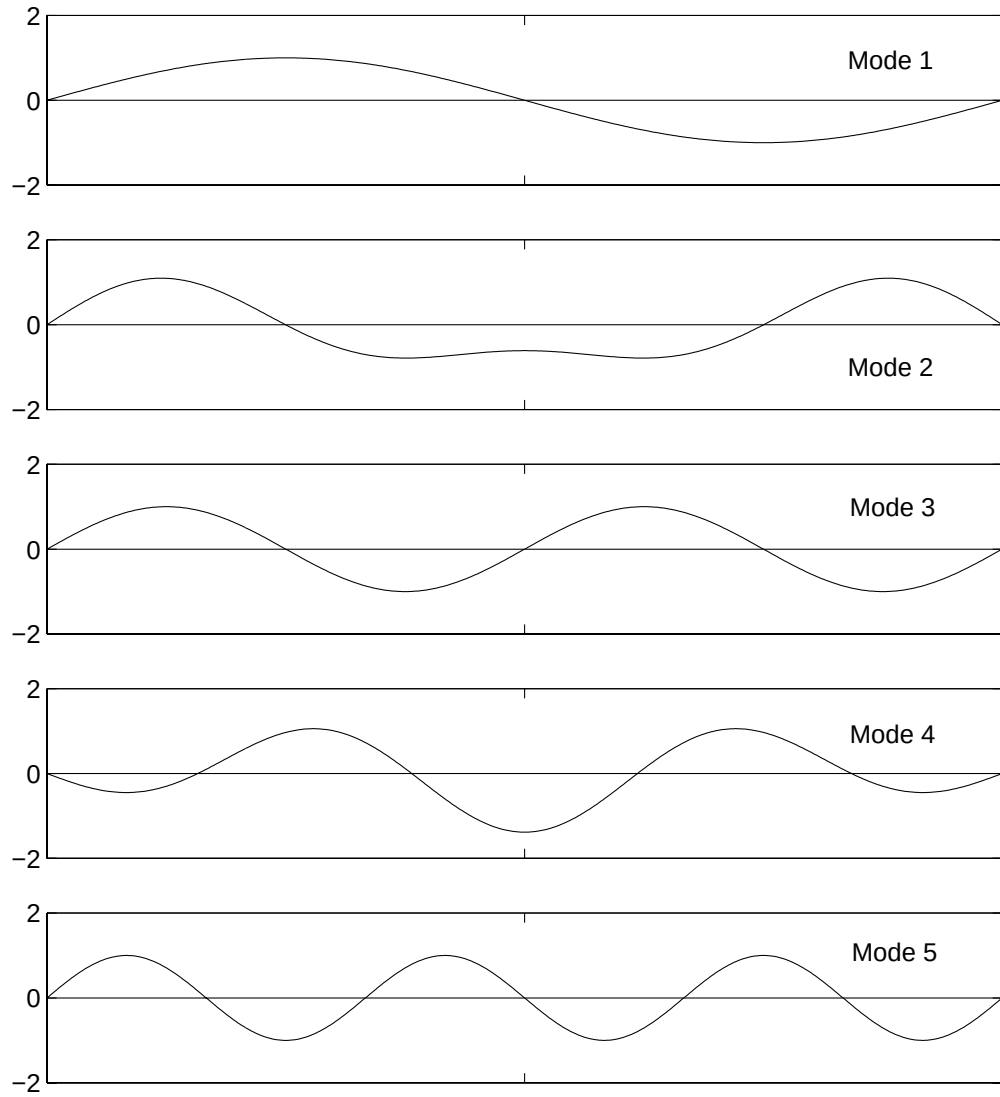


Figure 5.2: Controlled mode shapes of the simply supported beam

Table 5.2: Desired expansion coefficients for the simply supported beam

Uncontrolled	Controlled Modes				
Modes	1	2	3	4	5
1	0	-0.3095	0	-0.0064	0
2	1.0	0	0	0	0
3	0	0.8066	0	0.5277	0
4	0	0	1.0	0	0
5	0	0.5035	0	-0.8494	0
6	0	0	0	0	1.0

even-order modes of the uncontrolled system (e.g., the first controlled mode is the second mode of the uncontrolled system), while the controlled modes with even order are linear combinations of the odd-order uncontrolled modes (e.g., the second controlled mode is a combination of modes 1, mode 3 and mode 5 of the uncontrolled system), with the associated expansion coefficients given by Table 5.2. Close examination of Table 5.2 shows that the desired expansion coefficient vectors are orthonormal to each other. Since the controlled mode shapes are just linear combinations of those uncontrolled ones, the controlled mode shapes as presented in Figure 5.2 are orthonormal to each other.

It is interesting to compare the mode shapes of the minimum supersonic wavenumber design to those of the non-volumetric design discussed in Section 3.6.1. Comparing Figure 5.2 with Figure 3.2, it is clear that the controlled mode shapes of these two designs look similar but are actually different. Their mode shapes look similar, because the non-volumetric design is equivalent to minimizing only the supersonic wavenumber spectral line corresponding to $k_x = 0$, $k_y = 0$. However, unlike the non-volumetric design, numerical simulations show that the controlled mode shapes of the minimum supersonic wavenumber design are not non-volumetric, but their associated volume velocities (see equation (3.10)) are significantly reduced.

5.5.2 Optimal controller

As discussed in the previous section, the odd-order controlled modes correspond to the next even-order uncontrolled modes (e.g., $\phi_{c1}(x) = \phi_2(x)$). This suggests that the eigenvalues of the odd-order controlled modes are identical to the eigenvalues of the corresponding even-order uncontrolled modes (e.g., $\omega_{c1}^2 = \omega_2^2$). The corresponding expansion coefficients (calculated from equation (3.2), see the discussion associated with this equation for more details) for these odd-order modes are then identical to those presented in Table 5.2. The unit modal control inputs μ_n and the modal error sensor components ξ_n corresponding to the even-order uncontrolled modes are set to zero as a premise.

For the even-order controlled modes, the controlled system eigenvalues ω_{cl}^2 and the unit modal control inputs μ_n that yield the desired expansion coefficients $\bar{\Gamma}_{nl}$ in Table 5.1 can be obtained by solving the optimization problem described by equations (3.3) and (3.4).

The selection of the lower and upper bounds for the controlled eigenvalues, as discussed in Section 5.3, is critical to the success of the optimization. The relative sign of the expansion coefficients listed in Table 5.1 determines the feasible bounds for the controlled eigenvalues ω_{cl}^2 . From equation (3.2), the sign of the expansion coefficients Γ_{nl} is a function of the relative magnitudes of the controlled eigenvalues ω_{cl}^2 and the uncontrolled eigenvalues ω_n^2 , as well as the sign of the unit control modal components μ_n . By studying the possible combinations of these three to match this sign requirement, it is found that the controlled eigenvalues ω_{cl}^2 must satisfy the following condition:

$$\omega_3^2 \leq \omega_{c2}^2 \leq \omega_5^2 \leq \omega_{c4}^2 \quad (5.15)$$

and the corresponding unit modal control inputs μ_n should satisfy

$$\begin{aligned} \mu_1 &\leq 0 \\ \mu_3 &\geq 0 \\ \mu_5 &\leq 0 \end{aligned} \quad (5.16)$$

On the other hand, from Figure 5.2, to ensure the correct sequence of the controlled modes, the fourth controlled eigenvalue should not exceed the eigenvalues of the fifth controlled modes, i.e., $\omega_{c4}^2 \leq \omega_6^2$. The lower and upper bounds for the controlled eigenvalues are therefore given as

$$\omega_3^2 \leq \omega_{c2}^2 \leq \omega_4^2 \leq \omega_5^2 \leq \omega_{c4}^2 \leq \omega_6^2 \quad (5.17)$$

The optimal controlled system eigenvalues ω_{cl}^2 ($l = 2, 4$) and the unit modal control inputs μ_n ($n = 1, 3, 5$) can now be solved. The modal error sensor components ξ_n ($n = 1, 3, 5$) can be obtained from equations (3.6) and (3.7). They are presented in Table 5.3, where one must recall that the modal control inputs μ_n and modal error sensor components ξ_n of the even-order modes are set to zero. For comparison, the natural frequencies of the uncontrolled system (obtained from equation (3.40)) are also listed in the same table.

Table 5.3: Optimal control configuration for the simply supported beam

Mode #	Natural Frequencies (Hz)		Modal Control Components μ_n	Modal Error Sensor Components ξ_n
	Uncontrolled	Controlled		
1	76.03	304.13	-0.218	0.973
2	304.13	1200.00	0.	0.
3	684.30	1216.53	0.576	0.226
4	1216.53	1491.47	0.	0.
5	1900.82	2737.18	-0.788	0.0375
6	2737.18		0.	0.

The optimal expansion coefficients corresponding to the optimal control parameters as presented in Table 5.3 are listed in Table 5.4. These expansion coefficients are very close to the desired expansion coefficients presented in Table 5.2.

The determination of the optimal modal error sensor components ξ_n and the unit modal

Table 5.4: Desired expansion coefficients for the simply supported beam

Uncontrolled	Controlled Modes				
Modes	1	2	3	4	5
1	0	-0.2134	0	-0.1481	0
2	1.0	0	0	0	0
3	0	0.8334	0	0.4949	0
4	0	0	1.0	0	0
5	0	0.5098	0	-0.8562	0
6	0	0	0	0	1.0

control inputs μ_n completely defines the control system configuration in the modal domain. The performance of the controlled system can now be investigated.

5.5.3 Control acoustic performance

The acoustic performance of the minimum supersonic wavenumber components design for the simply supported beam is studied in this section. The disturbance signal is a pure tone sinusoid and the frequency range of study is limited to 0 to 1600 Hz.

The wavenumber modulus spectra of the first four modes of the simply supported beam before and after control are shown in Figure 5.3. For simplicity, the wavenumber spectra is plotted along the positive k_x -axis only (wavenumber spectra of the beam along the negative k_x -axis and the k_y -axis are not shown). Figure 5.3 clearly shows that the supersonic wavenumber components of each controlled mode are significantly reduced, where the supersonic region is defined by equation (5.2) with k_0 satisfying $k_0 a / \pi \leq 0.8799$ (i.e., 400 Hz). Since only supersonic wavenumber components radiate to the far-field, the controlled modes radiate less efficiently than the uncontrolled modes do. Figure 5.4 shows that the radiation efficiencies of the first 4 modes for both the uncontrolled and controlled system. It

can be observed that, at low frequencies, the radiation efficiencies of each controlled modes are much more smaller than those of the uncontrolled modes.

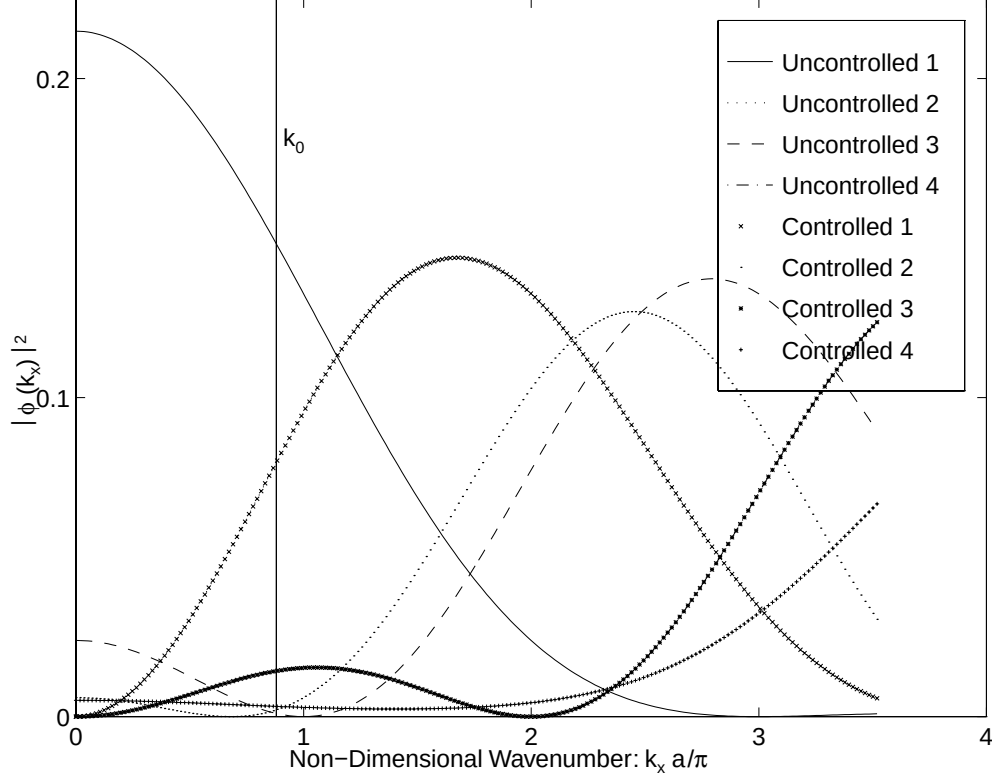


Figure 5.3: Wavenumber modulus spectra of the first four modes of the simply supported beam

The far-field sound pressure levels (in dB ref. 20μ Pa) before and after control versus frequency at $\mathbf{r}(5 \times a, \theta = 0, \phi = 0)$, i.e., in a direction perpendicular to the surface of the finite beam, are shown in Figure 5.5. Figure 5.5 shows that an average of 25.5 dB sound pressure level attenuation has been achieved in the frequency range of 10 Hz to 700 Hz. This can be explained from Figure 5.3, as the supersonic wavenumber component at $k_x = 0, k_y = 0$ (correspond to the direction of $\theta = 0$) are significantly reduced after control. At frequencies above 700 Hz, however, large control spillover occurs. This is associated with the fact that

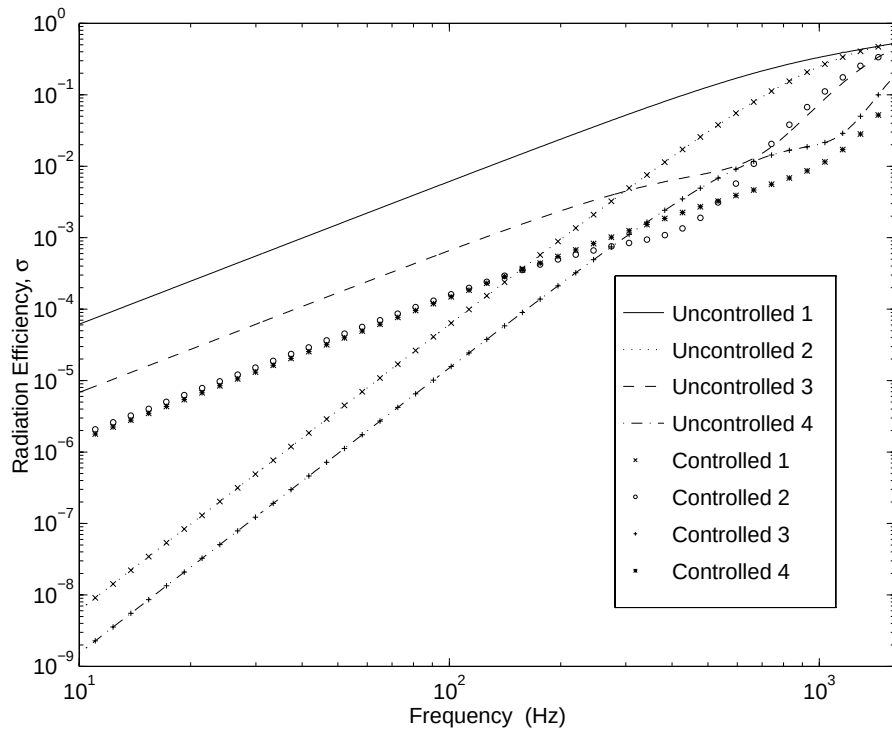


Figure 5.4: Radiation efficiency of the first four modes of the simply supported beam

the supersonic wavenumber design is carried out at the frequency of 400 Hz. In other words, only the supersonic wavenumber components within the frequency range below 400 Hz are minimized. This will be further discussed later.

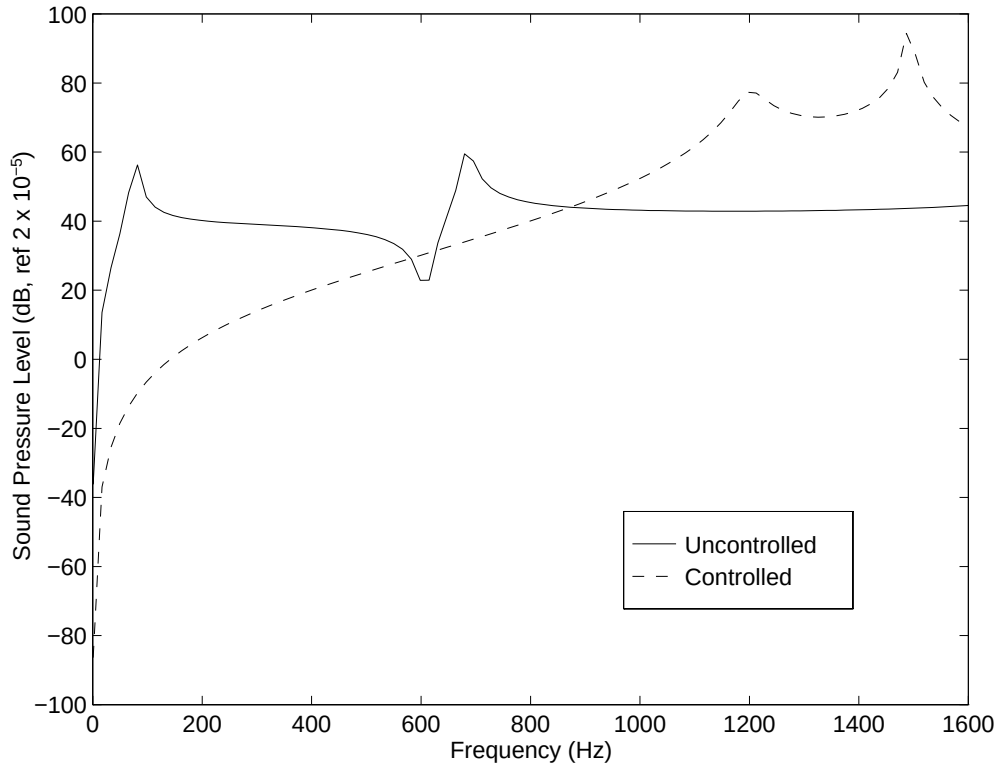


Figure 5.5: Far-field sound pressure in the direction $\theta = 0$

To show the radiation characteristics of the controlled system, the radiation directivities in the $x - z$ plane are evaluated at selected frequencies and they are shown in Figures 5.6 and 5.7. In all figures, the negative values of θ correspond to $\phi = 0$ in the positive x half-plane and the sound pressure levels are calculated at a distance of $R = 5 \times a$. Figure 5.6 shows the radiation directivity at 76 Hz, which corresponds to the first resonance frequency of the uncontrolled system. Substantial global sound attenuation has been achieved. Similar results can be observed when the system is driven at other uncontrolled resonance frequencies. At

200 Hz, an off-resonance frequency for both controlled and uncontrolled system, excellent global sound attenuation is also obtained as shown in Figure 5.7. However, it should be noted that the global sound attenuation decreases as the frequency increases. In fact, at frequencies above 700 Hz, the sound pressure levels actually increase after control (see Figure 5.5).

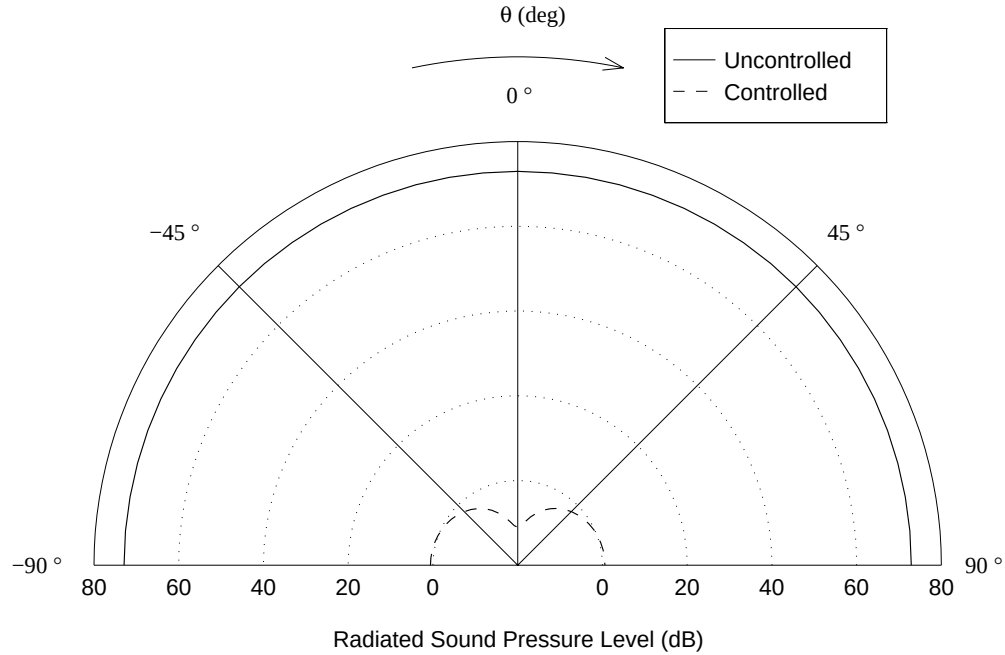


Figure 5.6: Radiation directivity of the simply supported beam at 76 Hz (first uncontrolled resonance)

Sound radiation directivities at the controlled resonance frequencies are also studied. Figure 5.8 shows the radiation directivity at 1200 Hz, corresponding to the second controlled resonance frequency of the simply supported beam. It can be observed that the sound pressure level increases significantly. This is associated with the fact that the current design is carried out by minimizing the supersonic region of the structure velocity spectra at frequency 400 Hz, and the second resonance frequency of the controlled system is well above this frequency range. This is clearly shown in Figure 5.3, where the supersonic wavenumber

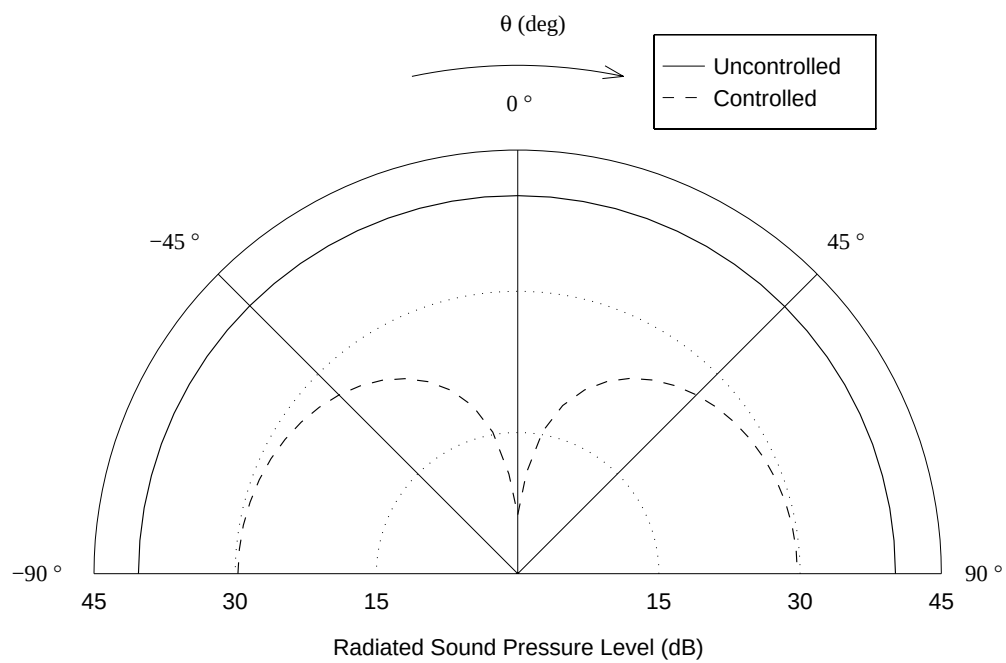


Figure 5.7: Radiation directivity of the simply supported beam at 200 Hz (off-resonance)

components of the controlled modes are not small at the frequency of 1200 Hz (i.e., $ka/\pi = 2.64$). Similar conclusion holds true for other controlled resonance frequencies.

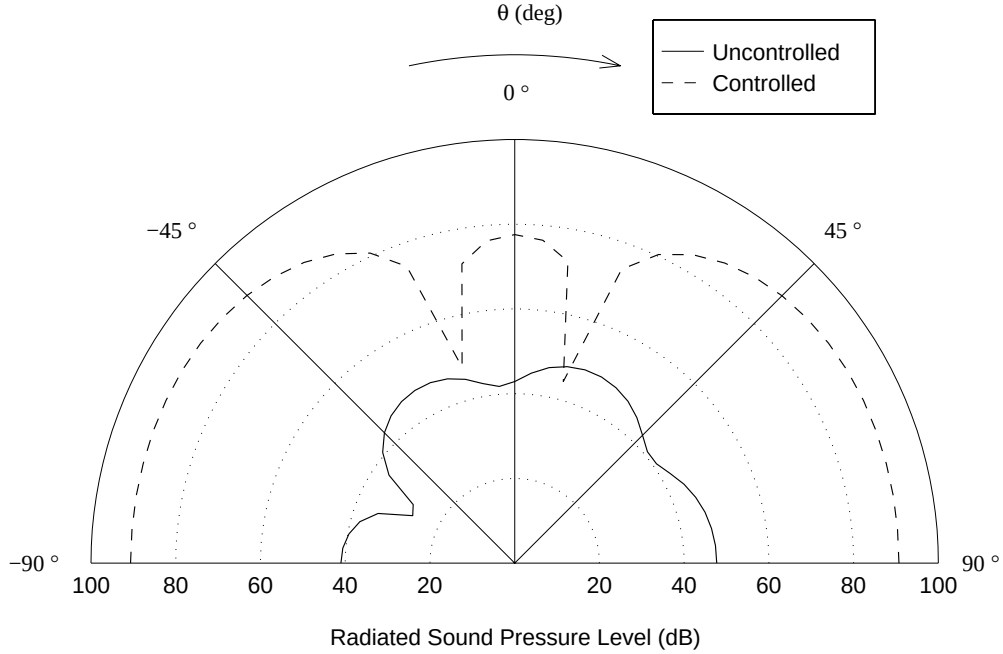


Figure 5.8: Radiation directivity of the simply supported beam at 1200 Hz (second controlled resonance)

It is very interesting to compare the radiation directivities of the current control strategy with those of the non-volumetric design discussed in Chapter 3. Compare Figures 5.6 and 5.7 with Figures 3.8 and 3.9, which correspond to the non-volumetric design for the same simply supported beam, it is easy to see that both designs lead large global sound pressure attenuation at low frequencies and the magnitudes of sound pressure attenuation are comparable. Furthermore, the radiation directivity patterns of both designs are very similar and they resemble dipole type radiators, while the minimum supersonic wavenumber design has slightly higher sound pressure in the direction of $\theta = 0$ after control. This is due to the fact that, for the minimum supersonic wavenumber design, the supersonic wavenumber components of each controlled modes are not zero at $k_x = 0, k_y = 0$ (which corresponds

to the direction of $\theta = 0$), in contrast to the non-volumetric design (see Figure 5.1 and Figure 5.3 for comparison). However, as the frequency increases, while the radiation directivity patterns of the non-volumetric design still resemble those of the dipole type radiators, the radiation directivity patterns of the minimum supersonic wavenumber design begin to show patterns similar to those of the monopole type radiators. As a consequence, for the supersonic wavenumber design, the sound pressure level reduction in the direction of $\theta = 0$ is smaller than that achieved by the non-volumetric design. This is clearly shown in Figure 5.9.

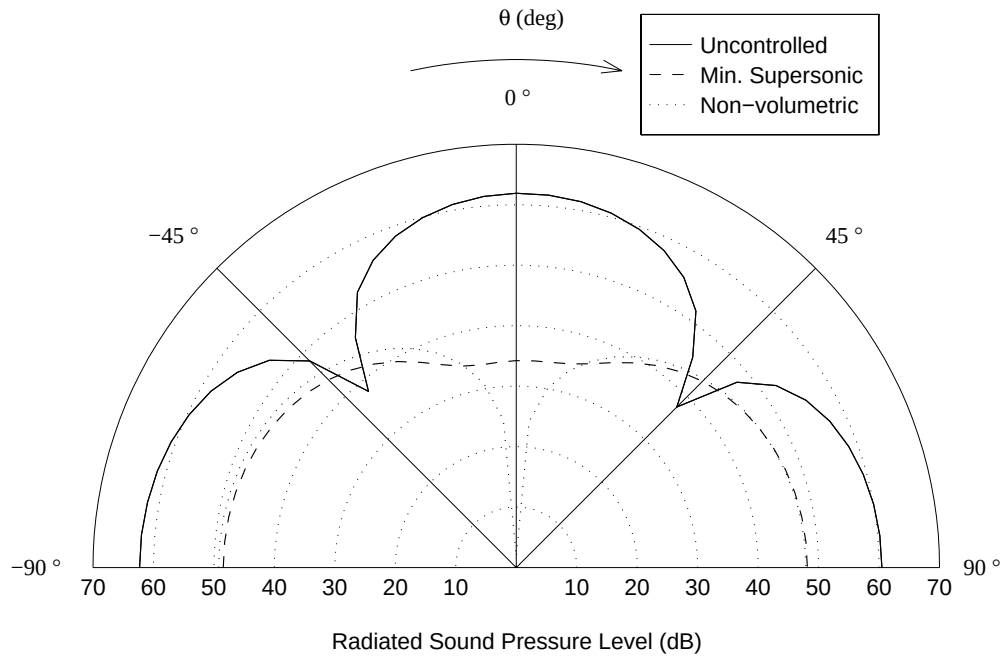


Figure 5.9: Radiation directivity of the simply supported beam at 684 Hz (third uncontrolled resonance): Minimum supersonic wavenumber design versus Non-volumetric design

To evaluate the overall performance of the controlled system, a calculation of the radiated sound power (in dB ref. 10^{-12} Watt) and radiation efficiencies for both the uncontrolled and controlled system are carried out. Since the sound power radiated by the vibrating

structure is proportional to the supersonic wavenumber components of the structural velocity spectrum, the attenuation of the supersonic wavenumber components of each controlled modes in the low frequencies as shown in Figure 5.3 implies that the sound power radiated by the structure will be reduced after control. This is indeed the case, as shown in Figure 5.10. An average of 33.4 dB sound power attenuation has been achieved after control in the frequency range up to 700 Hz. The radiation efficiency plot (see Figure 5.11) shows that the controlled system efficiency is greatly reduced in the same frequency range.

To further understand the underlying mechanism for the controlled system, the spatial average mean-square velocity as defined by equation (3.37) is calculated before and after control, and they are presented in Figure 5.12. It can be seen that the controlled vibration level is in general reduced in the frequency range of 10 Hz to 700 Hz. This fact suggests that, besides the “modal restructuring” [3], the “modal reduction” [3] is also an important mechanism in the proposed control process. Unlike the non-volumetric design (see Figures 3.13, 3.14 and 3.15 for comparison), the minimum supersonic wavenumber design shows large “control spillover” at high frequencies. Again this is associated with the fact that the current design is carried out at the frequency of 400 Hz, i.e., only the supersonic wavenumber components within the frequency range below 400 Hz are minimized. This demonstrates the frequency dependent nature of the minimum supersonic wavenumber design.

The design of the structural actuators and error sensors rendering the proposed minimum supersonic wavenumber design is entirely analogous to the procedure for the non-volumetric design as discussed in Chapter 4. They will not be thus carried out for the control approach of this chapter.

5.6 Summary

A new design strategy, namely the minimum supersonic wavenumber design, was developed in this chapter. A SISO feedforward controller was designed to modify the eigenproperties of a planar structure so that the mode shapes of the controlled system have minimum

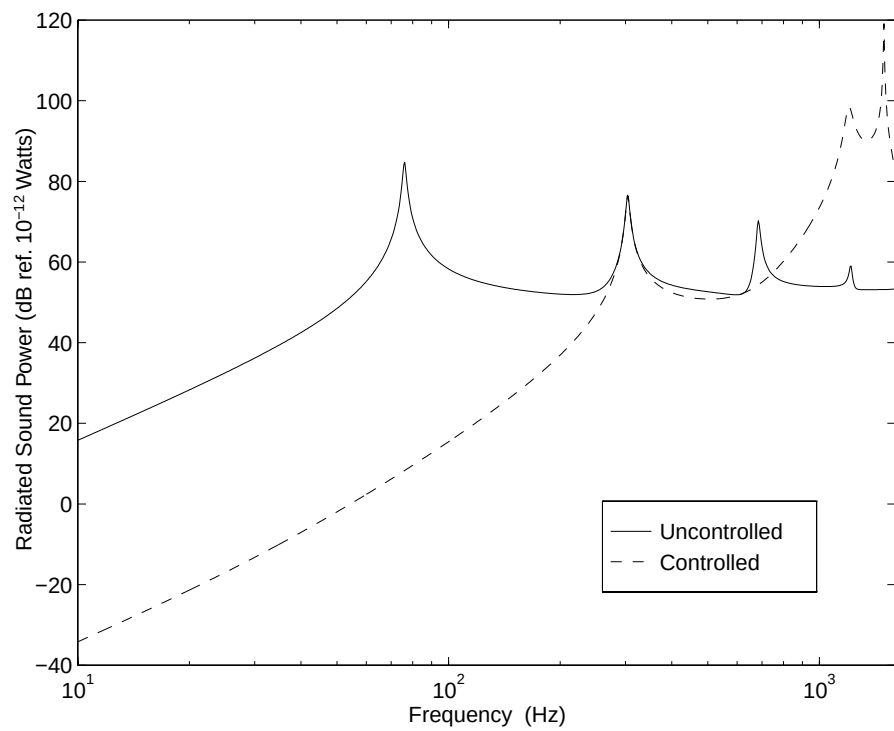


Figure 5.10: Radiated sound power of the simply supported beam

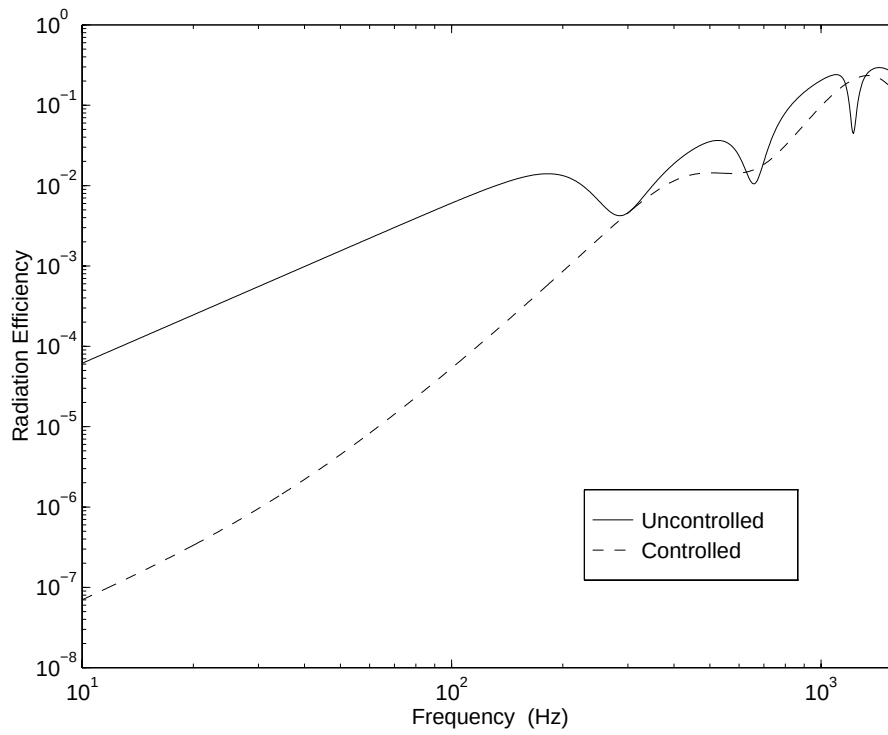


Figure 5.11: Radiation efficiency of the simply supported beam

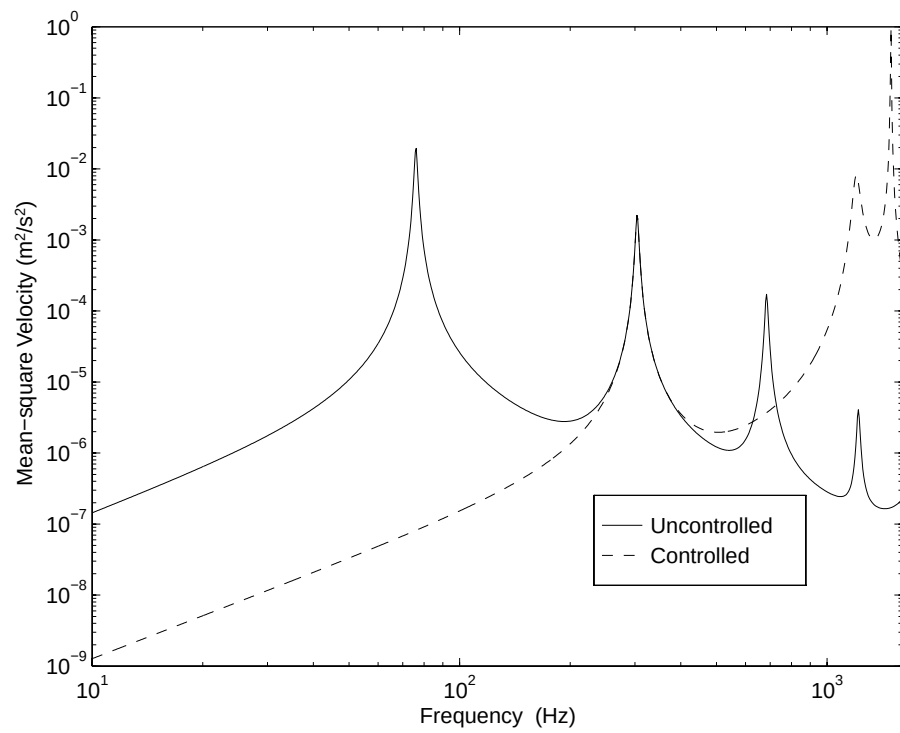


Figure 5.12: Mean-square velocity of the simply supported beam

supersonic wavenumber components in the frequency range of study. As a consequence, the sound pressure or sound power radiated by the structure is reduced. An optimization strategy has been developed to find the desired expansion coefficients for each controlled modes. It has been shown that, with a properly constructed optimization problem (for minimizing the supersonic wavenumber components of the structural velocity spectrum), the minimization of the supersonic wavenumber components reduces to an eigenvalue problem, where the eigenvalues give the minimum supersonic wavenumber components achievable for each controlled modes, and the associated eigenvectors provide the desired expansion coefficients for the corresponding controlled modes. The design process was demonstrated on a simply supported beam to minimize the supersonic wavenumber components contributed by the odd-order modes only. Significant global sound attenuation is achieved in the frequency range of study.

The present approach was also compared to the non-volumetric design discussed in Chapter 3. The comparison shows that, at low frequencies, the performance and radiation characteristics of the minimum supersonic wavenumber component design closely resemble those of the non-volumetric design. At high frequency (well above the design frequency), however, the minimum supersonic wavenumber component design shows large control spillover. This is because the supersonic wavenumber design minimizes only the wavenumber components within the supersonic region defined by the design frequency. Another limitation of the supersonic wavenumber design is associated with the fact that the supersonic wavenumber component matrix is ill-conditioned at all frequencies. Considering these limitations, the non-volumetric design is more favorable in terms of structural and acoustic performance.

Chapter 6

Experimental investigation of the non-volumetric design approach

In this chapter, the non-volumetric design discussed in Chapter 3 is experimentally validated on a simply supported beam under harmonic disturbance. A SISO feedforward controller using the filtered- x LMS algorithm is used in the experiment. Various aspects of the experimental setup and experimental procedures are first discussed. The experimental results, as well as the associated simulation results, are presented next. Plots of the attenuation of the radiated sound power before and after control provide a representation of the global sound reduction obtained. The sound radiation directivities before and after control at selected test frequencies are also presented to provide more information about the radiation characteristics of the control system. Finally, a summary of the main experimental results is presented to conclude the chapter.

6.1 Experimental setup and procedures

In this section, various aspects of the experimental setup and experimental procedure are discussed. The material properties, geometry and boundary conditions of the test structure

and the control actuators are presented first, followed by a description of the experimental setup, including the control system setup and the measurement setup. The experimental procedure is then summarized. A discussion of the measured data post-processing follows, with the emphasis on the far-field sound pressure reconstruction from the experimentally measured structural velocities by using the discrete wavenumber transform technique.

6.1.1 The experimental test structure

The structure used in the experiments is a simply supported steel beam of dimensions $380 \times 38 \times 4.8$ mm (see Table 3.1 for details). Thin, flexible metal shims connect the edges of the beam to a heavy support stand to approximate the simply supported boundary conditions. To examine the dynamic characteristics of the beam, an experimental modal analysis was carried out. The results will be presented in Section 6.2.1.

Both the primary (disturbance) input and the secondary (control) inputs are applied by single-sided (i.e., asymmetric) G1195 piezoceramic patches (PZT). All PZT actuators used in the experiments were obtained from Piezo-Systems, Inc. [52]. The piezoelectric actuators are made of lead zirconate titanate (Material designation PSI-5A-S4) with nickel electrodes on both sides. The dimensions and the nominal material properties of these PZT actuators as provided by the manufacturer are listed in Table 6.1.

Table 6.1: Dimensions and material properties of the PZT actuators

Parameter	Magnitude
Length, L_p (mm)	24
Width, b_a^p (mm)	36
Thickness, h_a (mm)	0.1270
Young's Modulus, E_{pe} (N/m ²)	6.6e10
Piezoelectric strain coefficients, d_{31} m/V	190e-12

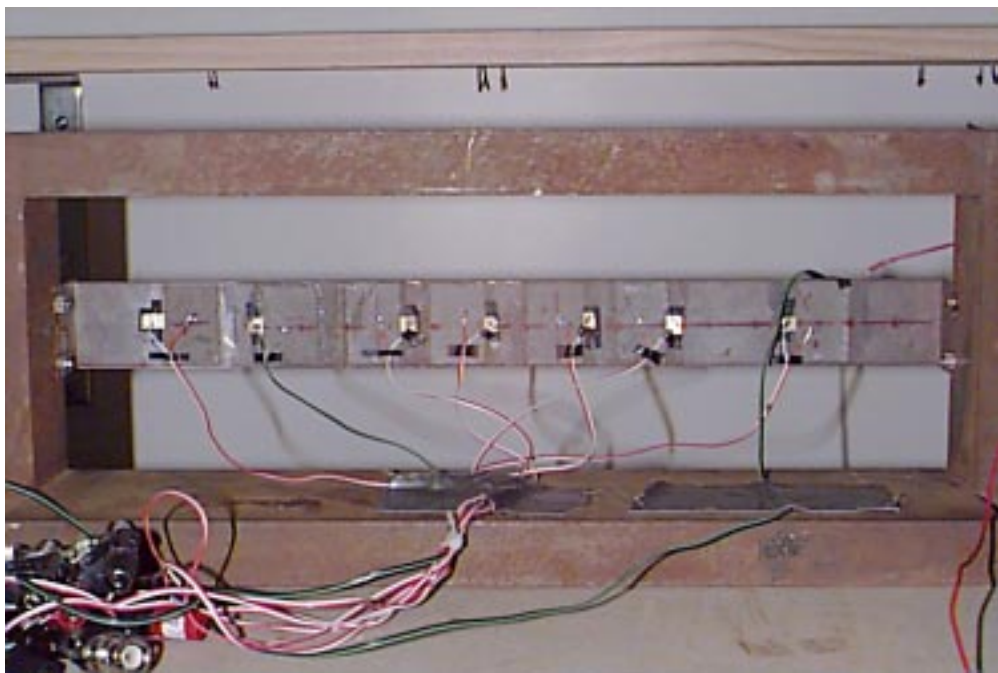


Figure 6.1: PZT actuators mounted on the beam

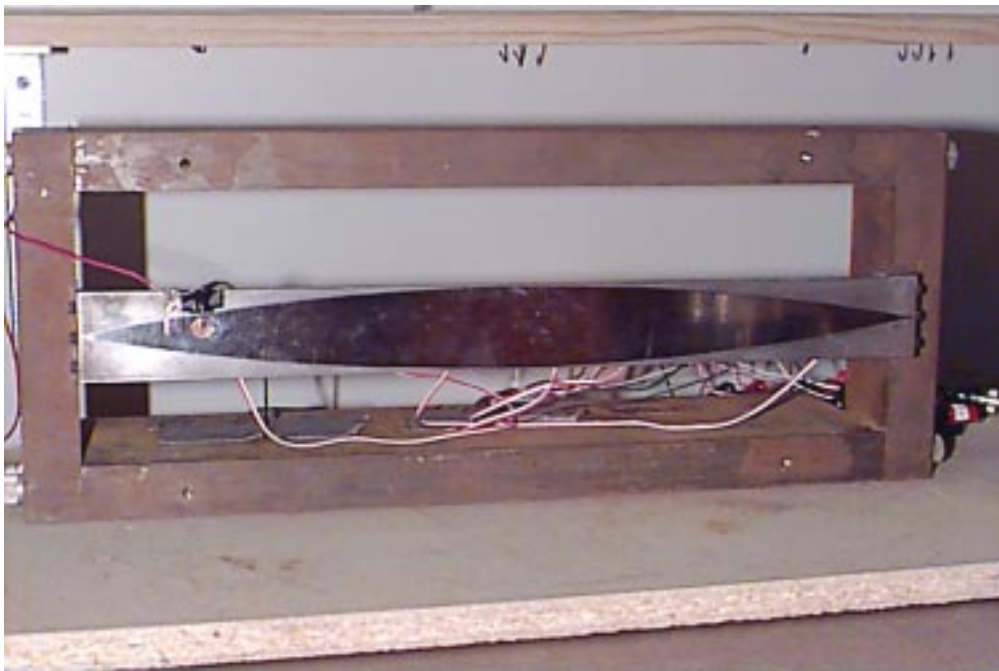


Figure 6.2: Configuration of the PVDF error sensor

The PZT actuators are bonded to the surface of the beam at prescribed positions by using a thin layer of dyanoacrylate based adhesive (M-Bond 200). A thin layer of spun-bounded polyester (Model 2250) from Freemay is used to provide the insulation between the actuators' electrodes and the surface of the beam. A total of 7 asymmetric PZT actuators (each measuring $0.024\text{m} \times 0.036\text{m}$) are used in the experiments, as can be seen from Figure 6.1, where the second PZT actuator from the left is used as the disturbance actuator. The control PZT actuators are bonded at prescribed positions and different weights are applied to each PZT actuator to generate the desired modal control inputs required by the non-volumetric design (see Section 4.4.2 for details). The positions of the PZT actuators and their associated weights are listed in Table 4.3 and they will be further discussed in Section 6.2.2.

PVDF film (Kynar[50] piezoelectric film from AMP Incorporated) is used as the error sensor in the experiments. The material properties of the Kynar PVDF film are presented in Table 4.1. The shape of the PVDF error sensor, shown in Figure 4.4, is designed to yield the desired non-volumetric design discussed in Chapter 4. The PVDF film is cut into the shape described by Figure 4.4 using a pair of scissors. The PVDF error sensor is then bonded to the other side of the beam by the 3M double-sided tape. The configuration of the PVDF error sensor is clearly shown in Figure 6.2.

6.1.2 Experimental setup

In this section, the control system configurations used in the experiment are described first. The experimental measurement setup and various equipment used in the experiment are then discussed.

Control system setup

Figure 6.3 shows a schematic of the control system setup. A SISO feedforward controller is used in the experiment. The well known filtered- x LMS algorithm [4] is used in the experiment. As can be seen from Figure 6.3, the controller adapts the coefficients of the

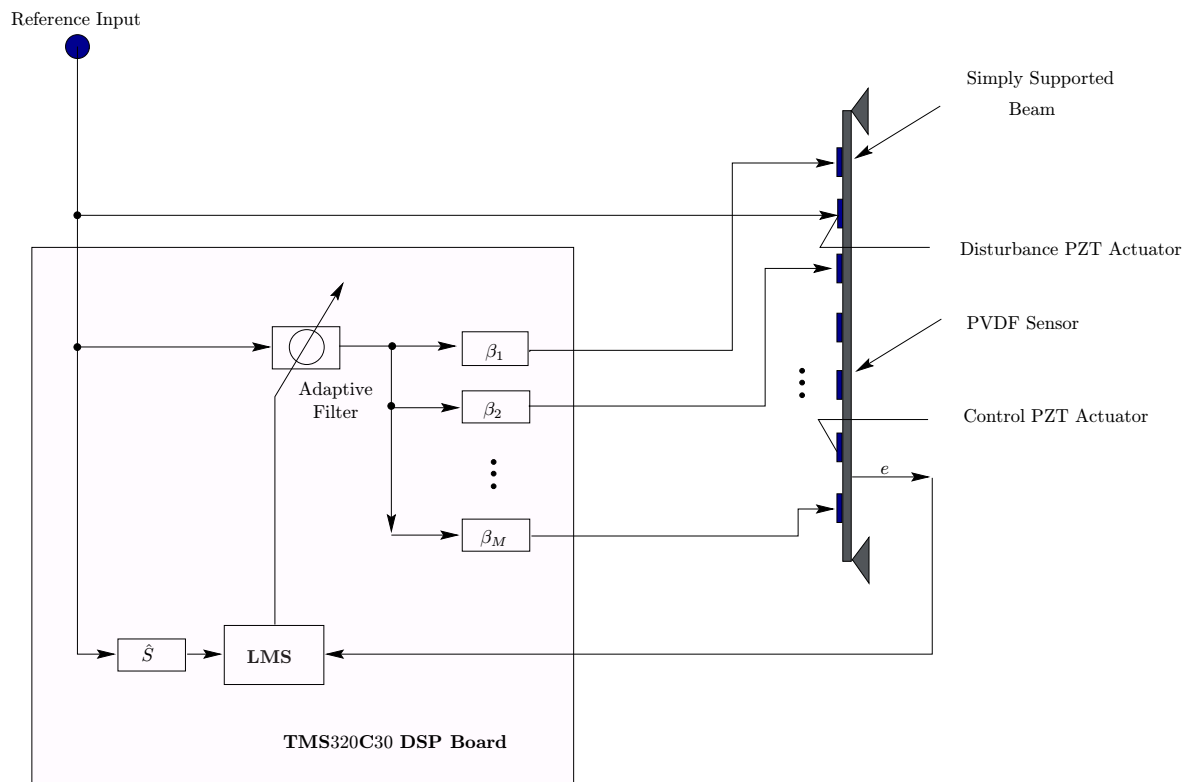


Figure 6.3: Schematic of the experiment setup

FIR control filter based upon reference and error signals. The reference signal is taken from the disturbance signal generator and thus is fully coherent with the primary source. Since the disturbance input is a pure tone sinusoid, only two coefficients are needed in the FIR control filter. As described in reference [4], the standard control approach is to minimize a cost function which is based upon the mean square amplitude of the error signals. Thus, when the controller has converged, its optimal transfer function should be identical to the transfer function of the control compensator derived in Section 3.4.1. This implies that the eigenvalues and eigenfunctions of the controlled system should be identical to those prescribed in Section 3.6.1. The control algorithm is written in C and assembly language and downloaded to a PC-based Texas Instrument TMS320C30 digital signal processing board [53]. The sample rate is taken as 4 to 5 times of the frequency to be controlled. For the filtered- x LMS algorithm, a user friendly graphic user interface was developed to set various aspects of the control parameters, such as the sampling frequency, convergence parameters etc. [53].

The harmonic disturbance signal is generated by the built-in generator of a dual channel Brüel & Kjær dynamic analyzer (Model 2032), and low-pass filtered through an Ithaco 4032 24 dB/octave filter with cutoff frequencies setting to 1.2 times of the disturbance frequencies. The disturbance signal is then fed into a single channel PCB power amplifier (Model 790A01). The reference signal is taken from the same signal generator used for the disturbance, and thus is fully coherent with the primary source. The control input generated by the controller is then multiplied by the appropriate weight, low-pass filtered through an array of low-pass filters (consisting of Frequency Devices 9002 and Ithaco 4032 filters), and then amplified through a six channel PCB power amplifier (Model 790A04). The PCB power amplifiers (Model 790A01, 790A04) are specially designed for driving the piezo-electric actuators, thus eliminating the need of the voltage transformers. The power amplifiers used for driving the control PZT actuators are carefully calibrated to ensure the gain for each channel is the same and remains unchanged throughout the experiments.

The shaped PVDF error sensor bonded to the surface of the beam is used to provide the error signal for the controller. The signal from the PVDF film is fed through an Ithaco power amplifier (Model 4302) specially designed for PVDF error sensor, and band-limited filtered

by an Ithaco Dual filter (Model 4032) to reduce the noise and high frequency contents of the signal which are out of the range of interest. The band-limited filter is implemented by using the two channels of an Ithaco Dual filter (Model 4032) in series: one as the high-pass filter and the other as the low-pass filter. Since the PVDF film is sensitive to the frequency of the main electric signal (i.e., 60 Hz), the beam is grounded.

Measurement setup

In the experiments, all measurements are analyzed in the frequency domain by a two channel Brüel & Kjær dynamic analyzer (Model 2032, 2034). Both channels of the Brüel & Kjær analyzer are calibrated so that all measurements are performed in engineering units (e.g., m/s for velocity and volt for voltage).

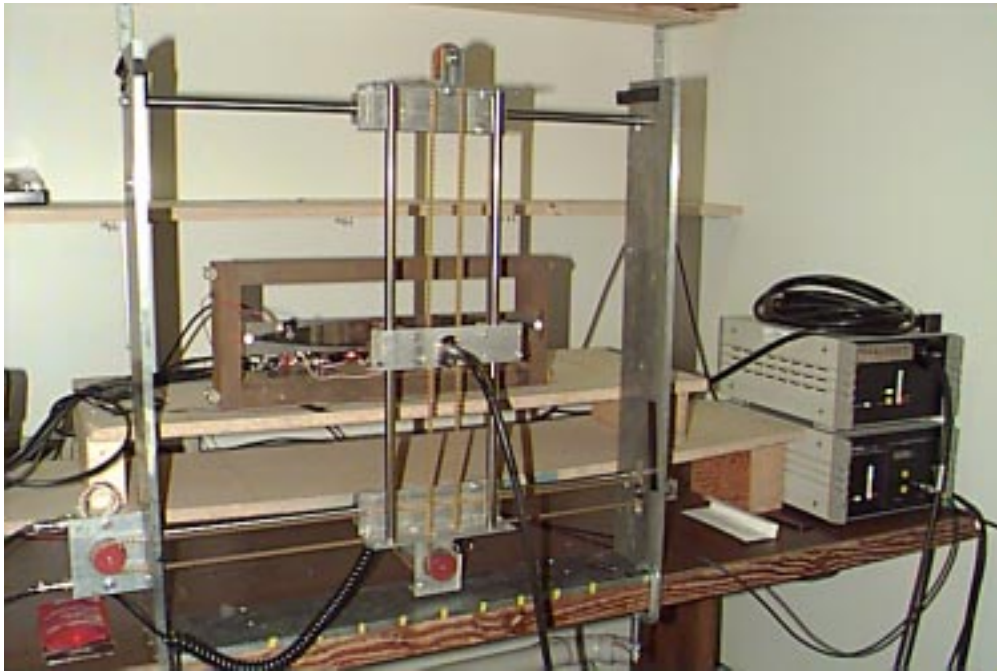


Figure 6.4: Experimental measurement setup

The out-of-plane structural vibrations are measured with a Politec Fiber Interferometer laser (Model OFV 501, OFV 2600). The laser head is mounted on a two dimensional traverse to allow measurement over the complete surface of the beam. The traverse allows the laser head to move in a plane parallel to the beam. The laser beam is kept perpendicular to the beam surface thus the output of the laser vibrometer is proportional to the out-of-plane velocity of beam. A photograph of the measurement setup is shown in Figure 6.4. The traverse is controlled by a PC through its parallel port. The Brüel & Kjær analyzer is interfaced with a PC through a GPIB card from National Instruments. An in-house software program written in MATLAB and C is used to control the Brüel & Kjær analyzer and the traverse simultaneously. This software enables the PC to set the measurement parameters for the Brüel & Kjær analyzer, start the measurement, automatically download the measured data from the Brüel & Kjær analyzer to the PC's hard disk and move the traverse to the next measurement point. Upon finishing all measurement points, the PC moves the traverse back to the starting position, and the whole measurement process can be repeated for the next testing. The measurement process is thus fully automatic. In this work, the output of plane velocities of the beam at selected test frequencies are measured with the laser at 38 equally spaced points along the x -axis before and after control. At each measurement point, the auto-spectra (i.e., G_{aa} and G_{bb}) and the cross-spectra (i.e., G_{ab}) measured by the two channel Brüel & Kjær analyzer in power unit are downloaded to the hard disk of the PC for post-processing.

6.1.3 Experimental procedure

Experimental modal analysis

Experimental modal analysis is performed on the test structure to determine its natural frequencies and mode shapes. The frequency response functions of the beam are obtained by exciting the structure with a Ling Dynamics shaker (Model 201). The shakers are suspended by long elastic cords to minimize the introduction of additional dynamics due to the shaker mounting. The shaker is driven by a pseudo-random signal with a band-limit of 0 Hz to

3200 Hz. A PCB force transducer (Model 208A04) is used to measure the applied force. A Brüel & Kjær accelerometer (Model 4374), mounted at the same position but on the other side of beam, is used to measure the responses at the driving point as reference. The out-of-plane velocities of the beam are measured with the laser at 38 equally spaced points along the x -axis, with the laser mounted on the two dimensional traverse. A uniform rectangular windowing function with zero overlap is used in the measurement. The auto- and cross-spectra of the signals from the laser and force transducer are then measured by the Brüel & Kjær analyzer. The experimental frequency response functions can then be derived from the measured auto- and cross-spectra. A frequency domain curve-fitting algorithm is used to identify the natural frequencies and mode shapes of the beam from these measured frequency response functions. The curve fitting algorithm uses the Forsythe polynomials [54, 55] and is implemented in MATLAB with graphic user interface by Maillard [36].

To study the influences of the PZT actuators, the experimental modal analysis is performed before and after the mounting of the PZT actuators.

System diagnostics

To better understand the dynamics of the experiment system, the following transfer functions are measured:

- The transfer function between the disturbance PZT actuator and the PVDF error sensor.
- The transfer function between the control PZT actuators and the response of the beam.
- The transfer function between the control PZT actuators and the PVDF error sensor.

All these measured transfer functions include the dynamics of various electronic equipment used in the corresponding transfer function paths. Throughout the experiments, the electronic equipment used in each electronic path is not changed, only its settings is modified for different tests.

To measure these transfer functions, a band-limited pseudo-random noise between 0 and 1600 Hz is used. The auto- and cross-spectra of the input and output signals are estimated by the Brüel & Kjær analyzer and downloaded to the hard disk of the PC for post-processing. By comparing the measured transfer functions with those obtained based on the theoretical model, the behavior of the control system can be better understood.

Control tests

The experimental procedure for the control tests proceeds as follows: First, the disturbance of an appropriate level is turned on. To fully utilize the maximum dynamic range of the DSP input A/D converters to obtain the best signal resolution, the disturbance signal was always tuned to an appropriate level such that the error signal from the PVDF error sensor was near to its maximum allowable voltage (3 Volts peak to peak) [19]. The out-of-plane velocity of the beam is measured using the laser vibrometer. Next, the control program is initialized: the convergence parameter and the sampling frequency are set. A system identification is then performed to determine the transfer function between the control input and the error sensor for use in the filtered- x LMS algorithm. The disturbance is turned on again with the same level, and the control is invoked. When the error signal is minimized, the out-of-plane velocity of the beam is again measured.

6.1.4 Experimental data post-processing

As mentioned before, for all tests, the Brüel & Kjær analyzer is calibrated so that all measured data are in engineering units. For each measurement, the auto- and cross-spectra of the driving signal and the response signal are estimated by the Brüel & Kjær analyzer and downloaded in power units for post-processing.

System diagnostics

The various transfer functions of interest can be obtained from the corresponding measured auto and cross-spectra of the input and response signal. The H_1 estimate of the transfer function between the input (input to channel A of the analyzer) and the measured response (input to channel B of the analyzer) can be expressed as [56]

$$H_1 = \frac{G_{ab}}{G_{aa}} \quad (6.1)$$

where G_{aa} is the auto-spectra of input signal and G_{ab} is the cross-spectra of the input signal and the measured response.

Structural vibrations

Provided that the auto- and cross-spectra estimated by the Brüel & Kjær analyzer are in power unit, the magnitudes of the out-of-plane velocities (RMS value) of the beam at measurement points are given as

$$\dot{w}(x_i, \omega) = \sqrt{G_{bb}} \quad (6.2)$$

If phase information is needed, the complex amplitudes of the out-of-plane velocities of the beam at measurement points x_i can be obtained from the measured spectra as follows:

$$\dot{w}(x_i, \omega) = \frac{G_{ab}(\omega)}{\sqrt{G_{aa}(\omega)}} \quad (6.3)$$

An estimate to the out-of-plane mean-square velocity $\overline{\dot{w}^2(x)}$ as defined by equation (3.37) can be obtained from a finite number of velocity measurements:

$$\overline{\dot{w}^2(x)} \approx \frac{1}{2N_x} \sum_{i=1}^{N_x} |\dot{w}(x_i)|^2 \quad (6.4)$$

where N_x is the number of measurement points along the beam (e.g., $N_x = 38$ in this experiment) and the measurement points x_i are defined as

$$x_i = \frac{\Delta x}{2} + i\Delta x \quad i = 0, 1, 2, \dots, N_x - 1, \quad \Delta x = \frac{a}{N_x} \quad (6.5)$$

Let $G_{bb}(x_i, \omega)$ be the auto-spectrum (in power unit) of the laser output measured at position x_i , the measured out-of-plane mean-square velocity defined by equation (6.4) is obtained as

$$\overline{\langle \dot{w}^2(x) \rangle}(\omega) \approx \frac{1}{N_x} \sum_{i=1}^{N_x} G_{bb}(x_i, \omega) \quad (6.6)$$

Sound pressure reconstruction

In this experiment, the far-field sound pressure is not directly measured in the anechoic chamber. This is because the anechoic chamber (of dimension $4 \times 2 \times 2.4\text{m}$) at VAL, Virginia Tech has a cutoff frequency of 250 Hz and the current experimental study is focused in the low frequency range below this limit. Also, the traverse microphone used in the anechoic chamber does not provide true far-field pressure measurements due to the limited size of the anechoic chamber relative to the dimensions of the beam and the acoustic wave-length within the frequency range of study.

Alternatively, in this experiment, the far-field sound pressure at prescribed direction is reconstructed from the wavenumber transformed out-of-plane structural velocity response measured by the laser vibrometer [36]. In this approach, the out-of-plane structural velocity distribution is measured at selected locations (sampling points), and by applying discrete wavenumber transform to the measured structural velocity response, the far-field sound pressure at predetermined direction can be found. To reduce the aliasing error introduced by the discrete wavenumber transform, for the simply supported beam, the number of measurement points along the length of the beam must be greater than the highest order mode with large response within the frequency range of study. Provided the structural velocity field

is measured accurately with sufficient spatial resolution, the reconstructed sound pressure from the measured surface velocity distribution accurately estimates the far-field radiated pressure assuming a perfect infinite baffle. The reader should refer to [36] for details.

The far-field acoustic radiation of a baffled planar structure can be obtained by using the wavenumber transform technique as summarized in Appendix C. The sound pressure in the far-field for a baffled planar structure can be obtained from equation (C.14), where $\bar{k}_x$, $\bar{k}_y$ are defined by equation (C.15). The wavenumber transformed velocity of the structure defined by equation (3.28) can be approximated by the discrete wavenumber transform [36]:

$$\dot{W}_d(k_x, k_y) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \dot{w}(x_i, y_j) e^{j(k_x x_i + k_y y_j)} \Delta x_i \Delta y_j \quad (6.7)$$

For a baffled simply supported beam with velocities measured at positions defined by equation (6.5), the above discrete wavenumber transform reduces to

$$\dot{W}_d(k_x, k_y) = b e^{jk_y b/2} \sum_{i=1}^{N_x} \dot{w}(x_i) e^{jk_x x_i} \Delta x \quad (6.8)$$

To reduce the aliasing error, the number of measurement points N_x must be greater than the highest order mode with resonance within the frequency range of study [36].

Substituting equation (6.8) into equation (C.14), the estimated far-field sound pressure radiated from the beam at frequency ω can be approximated as

$$p_d(R, \theta, \phi, \omega) = \frac{j\omega\rho_0 b \Delta x}{2\pi R} \exp \left[j \left(\frac{\omega}{c} \right) (R + b/2 \sin \theta \sin \phi) \right] \sum_{i=1}^{N_x} \dot{w}(x_i, \omega) \exp \left[j \left(\frac{\omega}{c} \right) x_i \sin \theta \cos \phi \right] \quad (6.9)$$

where the velocity complex amplitude $\dot{w}(x_i, \omega)$ is given by equation (6.3).

Radiated sound power

The radiated sound power from the baffled beam can be obtained from equation (C.25). Alternatively, it can be obtained by integrating the far-field (i.e., $k_0 R \gg 1$) intensity over a hemisphere ($0 \leq \phi \leq 2\pi$, $0 \leq \theta \leq \pi/2$) surrounding the structure:

$$\Pi(\omega) = \frac{R^2}{2\rho_0 c} \int_0^{\pi/2} \int_0^{2\pi} |p(R, \theta, \phi)|^2 \sin \theta d\theta d\phi \quad (6.10)$$

where $c = 345.5 \text{ m/s}$ is the phase speed of the acoustic medium (air in this work).

An estimate of the sound power radiated from the baffled beam can be obtained from the estimated sound pressure as follows

$$\Pi(\omega) = \frac{R^2 \Delta\theta \Delta\phi}{\rho_0 c} \sum_{i=1}^{N_\theta} \sum_{j=1}^{N_\phi} |p_d(R, \theta_i, \phi_j, \omega)|^2 \sin \theta_i \quad (6.11)$$

where

$$\begin{aligned} \theta_i &= \frac{\Delta\theta}{2} + i\Delta\theta & i &= 0, 1, 2, \dots, N_\theta - 1 & \Delta\theta &= \frac{\pi}{2N_\theta} \\ \phi_j &= \frac{\Delta\phi}{2} + j\Delta\phi & j &= 0, 1, 2, \dots, N_\phi - 1 & \Delta\phi &= \frac{2\pi}{N_\phi} \end{aligned} \quad (6.12)$$

In this study, the sound power is estimated over a grid consisting of $N_\theta = 10$ points in the azimuthal direction, θ , and $N_\phi = 18$ points along the circumferential direction, ϕ .

6.2 Experimental Results

The experimental results of the non-volumetric design for the simply supported beam are discussed in this section. The experiments are restricted to a harmonic disturbance. The results of the experimental modal analysis are presented first, the non-volumetric design is then carried out again based on the actual system dynamic characteristics identified by

the experimental modal analysis. The major experimental results are then discussed. To facilitate the experimental study, the associated simulation results for the simply supported beam studied in Chapter 3 with the physical actuators/error sensors are presented.

6.2.1 Experimental modal analysis

As discussed before, the simply supported beam (see Table 3.1 for details) used in this experiment is mounted to a heavy steel support stand by thin, flexible metal shims at the edges of the beam to provide the simply supported boundary conditions. To examine the dynamic characteristics of the beam, an experimental modal analysis was conducted for the bare beam (i.e., prior to the mounting of the PZT actuators) first. In order to study the influence of the PZT actuators, experimental modal analysis was performed again after the PZT actuators were mounted. The obtained first 6 natural frequencies and damping factors for both tests are presented in Table 6.2. For comparison, the theoretically predicted natural frequencies are also listed in the same table. The first 6 measured mode shapes of the simply supported beam for both tests are presented in Figure 6.5. For comparison, the theoretical mode shapes are plotted in the same figure.

Table 6.2: Natural frequencies and damping factors of the simply supported beam

Mode #	Natural Frequencies (Hz)			Damping factor ζ_n	
	Theory	w/o PZT	w. PZT	w/o PZT	w. PZT
1	76.0	78.7	77.4	1.88e-3	1.93e-3
2	304.1	301.5	298.4	1.01e-2	1.73e-2
3	684.3	675.8	669.8	3.16e-3	9.82e-4
4	1216.5	1194.7	1196.7	4.37e-3	1.00e-3
5	1900.8	1878.0	1876.3	1.19e-3	1.25e-3
6	2737.2	2709.7	2703.0	8.58e-4	1.06e-3

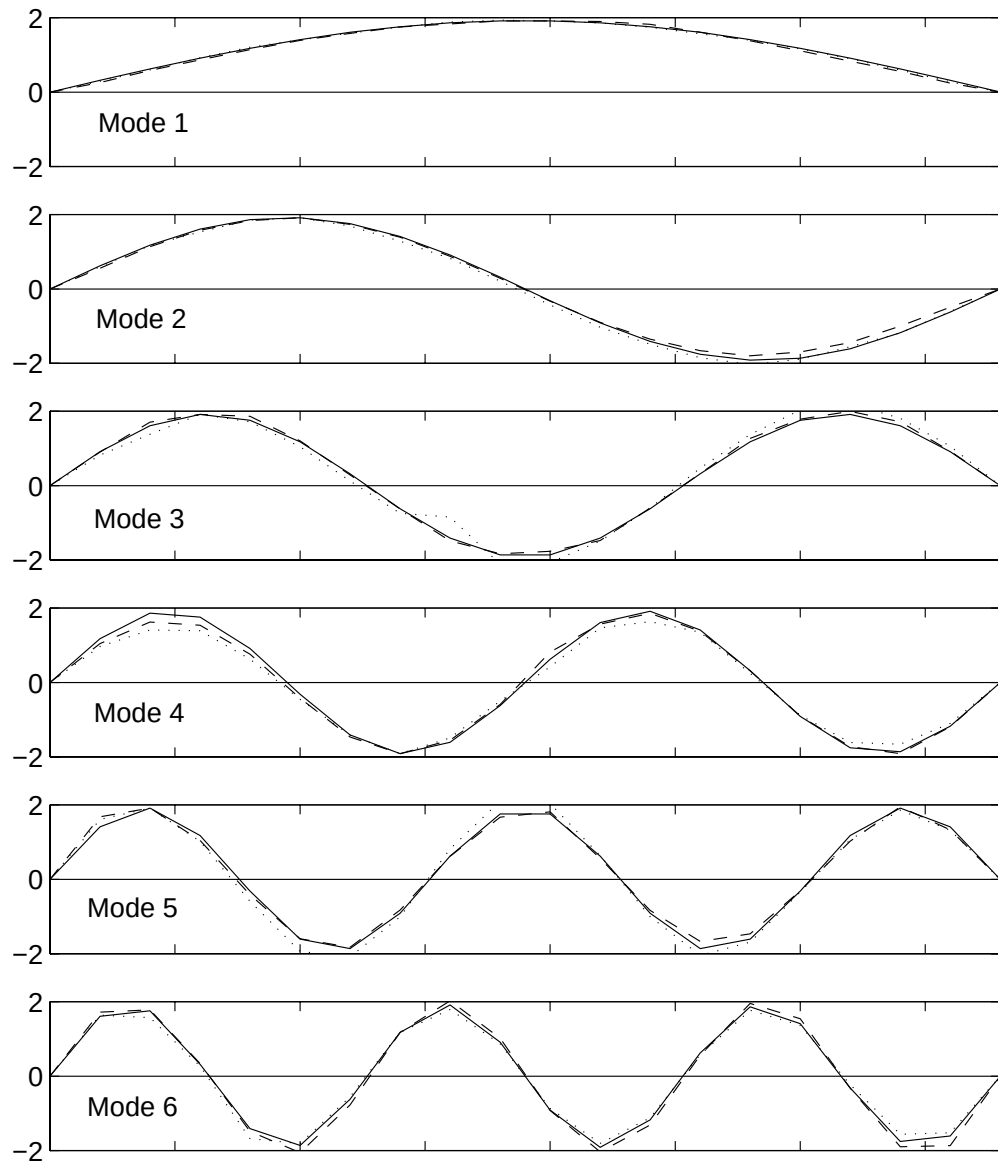


Figure 6.5: Mode shapes of the simply supported beam: Theoretical (—), Without PZT (---), With PZT (·····)

From Table 6.2, it can be observed that the experimentally determined natural frequencies of the bare beam are within 2% of the theoretically predicated natural frequencies. After the PZT actuators are attached to the beam, the natural frequencies of the beam become slightly smaller. This suggests that the dynamic effects of the PZT actuators on the response of the beam correspond mainly to a mass loading. This is in agreement with the conclusions of other studies [48, 36]. From Figure 6.5, it is obvious that the experimentally determined mode shapes of the bare beam approach closely to those of the simply supported boundary conditions. Furthermore, Figure 6.5 shows that the mode shapes of the beam change slightly after PZT actuators are mounted. This will influence the control performance as we will see later.

6.2.2 Modified non-volumetric design

The results of the experimental modal analysis discussed in the previous section clearly show that the boundary conditions of the beam reasonably approximate the simply supported boundary conditions. However, the experimentally determined natural frequencies, especially those of the higher order modes, deviate from the theoretically predicted ones. In this experimental study, in order to improve the control performance, the experimentally obtained natural frequencies of the bare beam are adopted in the design. The first 6 modes are included in the analysis. The design procedure is exactly the same as discussed in Chapters 3 and 4. For conciseness, they will not be discussed again, only the final results are presented in Table 6.3.

Both the disturbance input and the control inputs are applied by asymmetric PZT actuators. The dimensions and material properties of the PZT actuators are shown in Table 6.1. A total of 7 asymmetric PZT actuators are used in the experiments. A schematic of the configuration of the PZT actuators is shown in Figure 6.6, where the second PZT actuator (located at $x_d = 0.083\text{m}$) is used as the disturbance actuator. The positions of the control PZT actuators are presented in Table 4.3. The weights for each PZT actuators are obtained from equation (4.23), with the desired modal control inputs μ_n given by Table 6.3. The

Table 6.3: Optimal control configuration for the simply supported beam

Mode #	Natural Frequencies (Hz)		Modal Control	Modal Error Sensor
	Uncontrolled	Controlled	Components μ_n	Components ξ_n
1	78.7	301.5	0.934	0.933
2	301.5	645.9	0.001	0.
3	675.8	1194.7	0.269	0.309
4	1194.7	1836.9	-0.04	0.
5	1878.3	2709.7	0.235	0.187
6	2709.7		0.021	0.

obtained weights for each control PZT actuators as well as their associated positions are listed in Table 6.4.

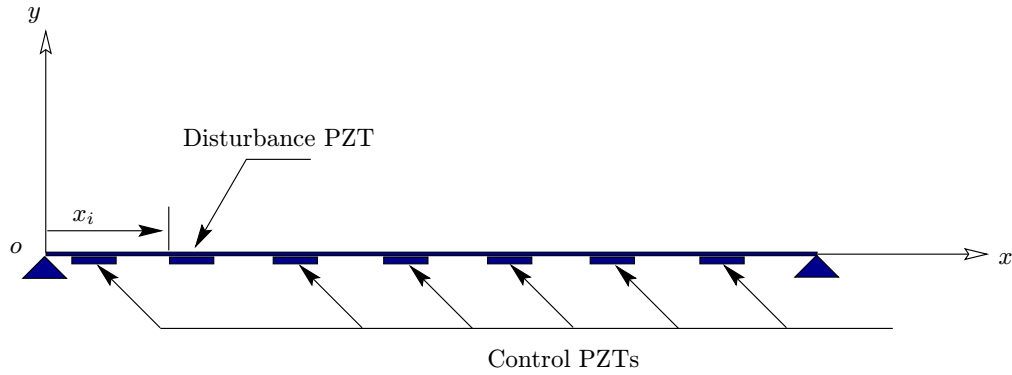


Figure 6.6: Schematic of the configuration of the PZT actuators

6.2.3 System diagnostics

As discussed in Section 6.1.3, to better understand the dynamics of the experiment system, various transfer functions were measured. They are presented in this section. To facilitate

Table 6.4: Weights for each PZT actuator

PZT #	Position x_p (m)	Weights β_p
1	0.04	0.5674
2	0.083	Disturbance
3	0.12	0.8940
4	0.156	0.5822
5	0.224	1.000
6	0.260	0.4071
7	0.340	0.6288

the study, the associated theoretical transfer functions are also presented in the same plots. All theoretical transfer functions are scaled with the corresponding experimental ones at the first uncontrolled resonance frequency to facilitate the comparison.

To evaluate the performance of the PVDF error sensor, the transfer function between the disturbance PZT actuator and the PVDF error sensor is presented in Figure 6.7. It can be observed that the experimental transfer function agrees well with the theoretical one, except at frequencies close to the second uncontrolled resonance frequency. At the second resonance frequency, around 304 Hz, the experimental transfer function shows a larger peak comparing to the theoretical results. In other words, the experimental error sensor responds to the even-order modes (in theory, it does not). This could be due to the errors in tailoring the PVDF sensor as well as in positioning the sensor on the beam. Another reason for this behavior is that the beam is not strictly symmetric/anti-symmetric about the center of the beam, especially after the PZTs are mounted (see Figure 6.5). In other words, the even-order modes of the beam are not strictly non-volumetric. However, at frequencies close to the second uncontrolled resonance frequency, the PVDF error signal is much smaller than at frequencies close to the odd-order resonance frequencies. This is associated with the fact that, for the non-volumetric design, the error sensor is mainly designed to observe the odd-order modes only (as shown in Table 6.3, the modal error sensor components for the

even-order modes are always zero or close to zero). This is clearly shown by the theoretical transfer function presented in Figure 6.7 (peaks appear close to the resonance frequencies of the odd-order modes). Due to this response of the experimental error sensor, some differences in control performance can be expected.

The transfer function between the control PZT actuators and the velocity response of the beam can be used to evaluate the performance of the actuator design. For instance, this transfer function at the location $x = 0.18$ m (one centimeter away from the center of the beam, and the coordinate system for the simply supported beam is shown in Figure 6.6) is presented in Figure 6.8. It can be seen that the measured transfer function show a resonance at the second uncontrolled resonance frequency (i.e., around 304 Hz). Figure 6.8 also shows that the third uncontrolled mode is highly excited by the control PZT actuators. This is believed to be due to the errors in positioning the PZT actuators along the beam. Also, the mode shapes of the simply supported beam is not strictly symmetric/anti-symmetric about the center of the beam. It has been shown in Sections 4.3.3 and 4.4.2 that the modal control inputs of the PZT actuators are very sensitive to the variations of the positions of the PZT actuators, or equivalently, to the variations of the mode shapes of the beam. Due to these discrepancies, the performance of the controlled system will be somewhat different from the theoretical results, as we will see later in Section 6.2.4.

The transfer function between the control PZT actuators and the PVDF error sensor was also measured and is presented in Figure 6.9. The associated theoretical transfer function is also shown for comparison. The experimental transfer function agrees well with the theoretical one. Differences can be seen at frequencies close to the second uncontrolled resonance frequency. This again is due to the errors in error sensor as well as the PZT actuators locations as discussed previously. Furthermore, the measured transfer function does not show significant anti-resonance around 646 Hz (right before the third mode), in contrast to the theoretical model. This anti-resonance frequency actually corresponds to the second controlled resonance (see Table 6.4 in the theoretical simulation results). As a consequence, the controlled system will not show a large resonance at this frequency, as we will see later in Section 6.2.4.

From the results discussed above, we can expect that the experimental results will deviate

from simulation results somewhat. The experimental control results are discussed in the next section.

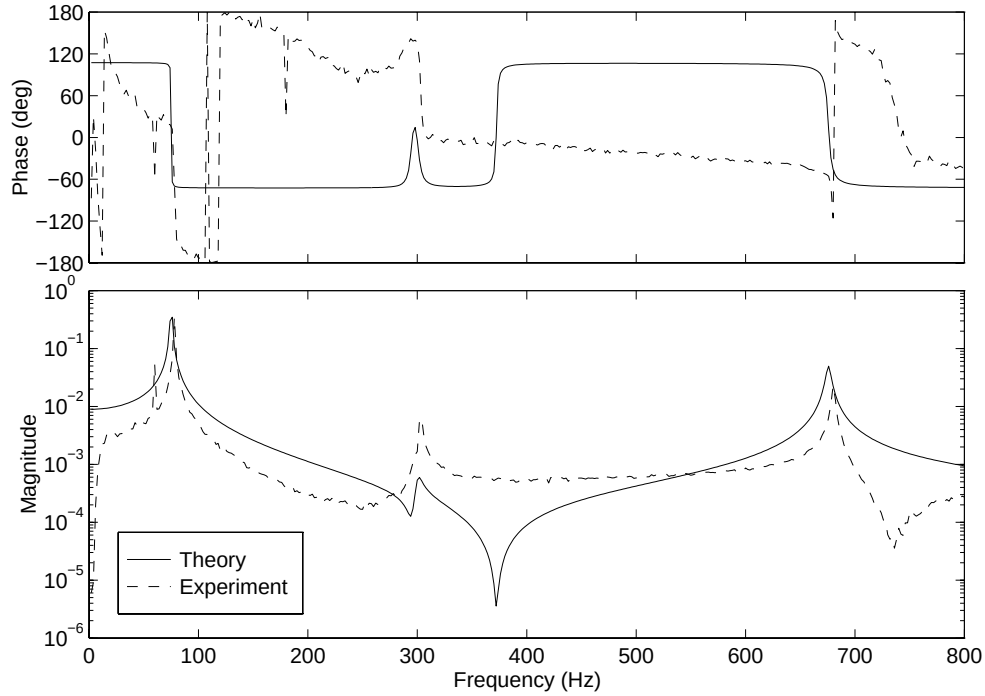


Figure 6.7: Transfer function between the disturbance PZT actuator and the PVDF error sensor

6.2.4 Experimental control results

The experimental control results of the non-volumetric design for the simply supported beam are presented in this section. The beam is excited by a harmonic disturbance. The performance of the control system is examined at selected frequencies. At each test frequencies, the out-of-plane velocities of the beam before and after control were measured by a laser vibrometer. The far-field sound pressure is then “reconstructed” from the measured structural velocity response as described in Section 6.1.4. The radiated sound power can then

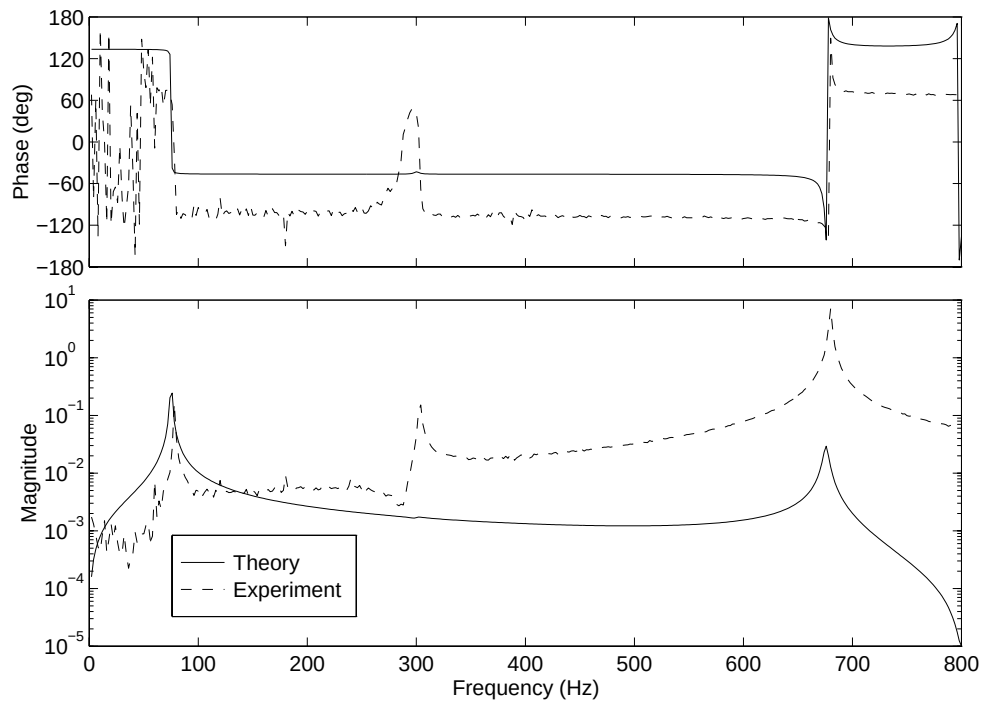


Figure 6.8: Transfer function between the control PZT actuators and the velocity response of the beam at location $x = 0.18$ m

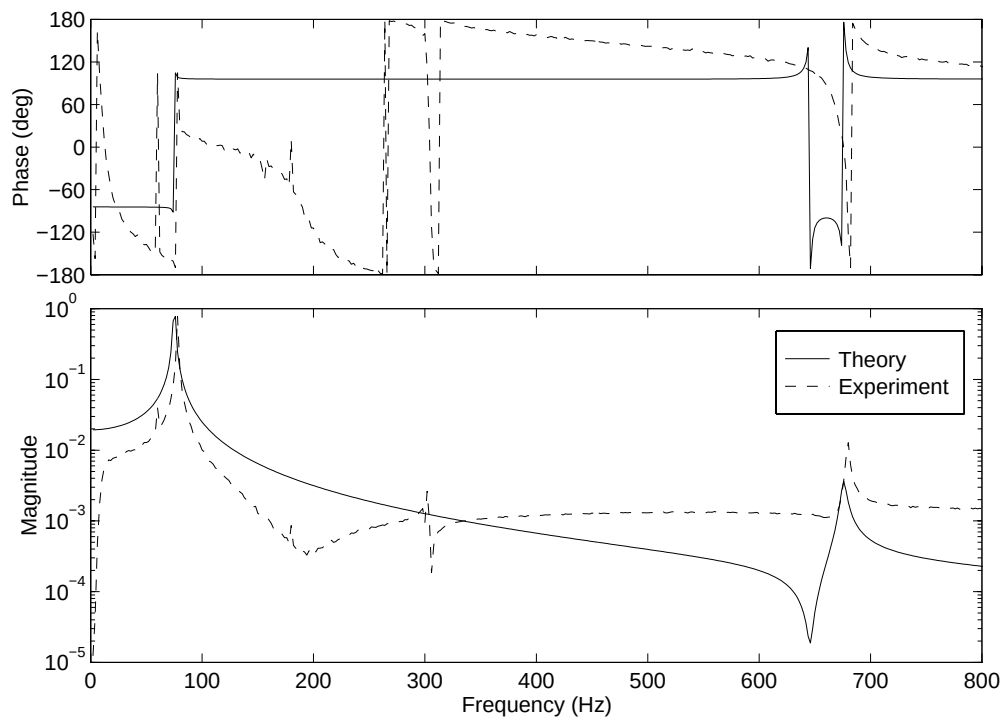


Figure 6.9: Transfer function between the control PZT actuators and the PVDF error sensor

be estimated from the reconstructed sound pressure evaluated over a grid of 10×18 points covering the entire hemisphere surrounding the beam (see Section 6.1.4 for details).

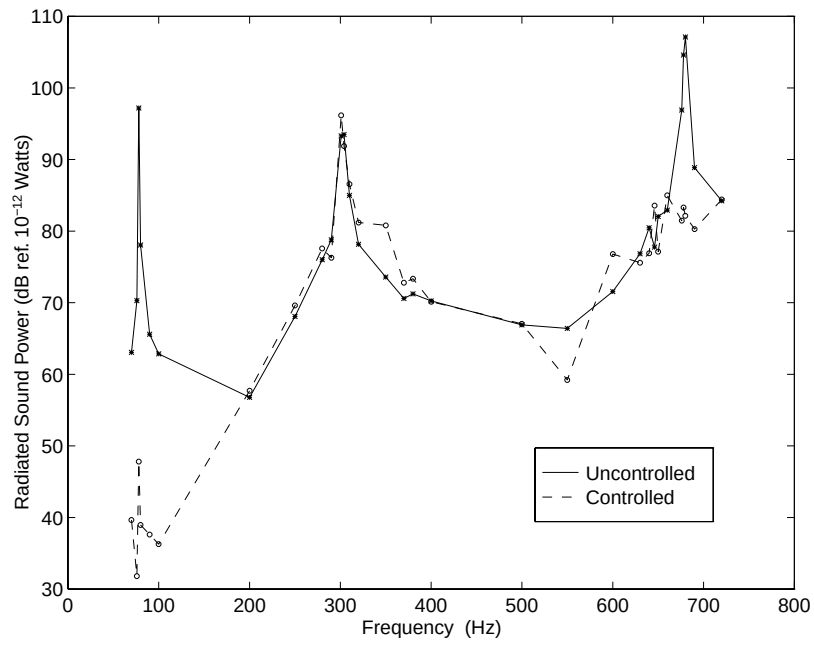
To facilitate the experimental studies, the associated simulation results for the simply supported beam with PZT actuators and PVDF error sensor are also presented for comparison. The dimensions and material properties of the beam are listed in Table 3.1. Note that the simulation results presented in this chapter are not a simple duplication of those presented in Chapter 3. The simulation performed in this section is based upon the design with physical actuators and error sensor, rather than the design in the modal domain without knowledge of the physical transducers. The beam is now excited by a disturbance PZT actuator located at $x_d = 0.083$ m, instead of a concentrated force of 1 N located at $x_d = 0.083$ m. The simulation is carried out by following the procedure described in Chapters 3 and 4. The frequency range of study is limited to 70 Hz to 800 Hz.

To evaluate the overall performance of the controlled system, the reconstructed radiated sound power (in dB ref. 10^{-12} Watt) of the beam before and after control are shown in Figures 6.10(a). For comparison, the associated theoretical radiated sound power of the simply supported beam is presented in Figure 6.10(b). From Figures 6.10(a) and 6.10(b), it can be seen that both figures show large sound power attenuation after control in the frequency range below 200 Hz. Also, both figures show that large sound power attenuation after control has been achieved at frequencies close to the third uncontrolled resonance frequency. Figure 6.10(a) shows that a global sound power attenuation of about 34 dB is achieved at the selected test frequencies below 200 Hz. For the test frequencies that are close to the second uncontrolled resonance frequency, however, small control spillover occurs. In fact, during the course of the experiment, at frequencies that are close to the second resonance frequency, the signal from the PVDF sensor was very small, causing difficulties in achieving control. In order to achieve control, the gain of the power amplifier for the PVDF error sensor had to be significantly increased, as a direct consequence, the uncorrelated signals were also increased. This phenomenon is associated with the fact that for the non-volumetric design for the simply supported beam, the error sensor is designed to “observe” the odd-order modes only (e.g., see Table 6.3, where the modal error sensor components for the even-order modes are always zero or close to zero). This is also clearly shown in Figure 6.7 and more detailed

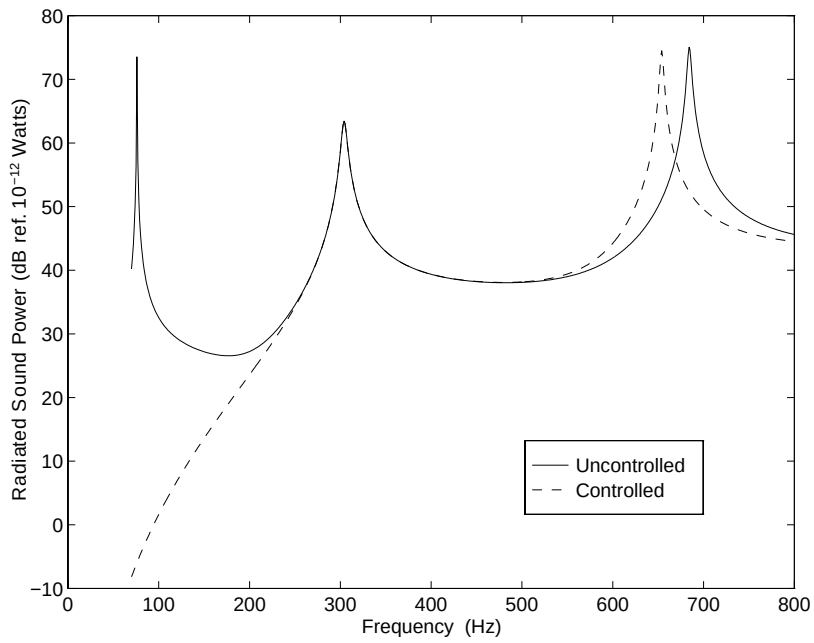
discussions can be found in Section 6.2.3. The result also explains the fact that sound power attenuation is again achieved at frequencies near the third uncontrolled resonance frequency (i.e., near 680 Hz). From Figure 6.10(a), it can be seen that the experimental results show resonance at 78 Hz, the first resonance frequency of the uncontrolled beam. However, a sound power attenuation of about 50 dB is still achieved at this frequency. The PVDF error signal is very small after the controller is converged, it is probably close to the resolution of the 12-bit A/D converter used in the DSP I/O board. It is believed that the control performance at this frequency can be further improved by adopting an A/D converter with higher resolution (e.g., 16-bit).

To further understand the underlying mechanism for the control system, the experimentally measured spatial average mean-square velocities (see equation (6.6)) before and after control are presented in Figure 6.11(a). For comparison, the simulated spatial average mean-square velocities are also shown in Figure 6.11(b). Comparing Figures 6.11(a) and 6.11(b), it can be observed that, similar to the radiated sound power shown in Figures 6.10, both figures show that the mean-square velocities of the beam are significantly reduced after control at frequencies below 200 Hz and at frequencies that are close to the third uncontrolled resonance frequency. At frequencies that are close to the second uncontrolled resonance frequency, the experimental results presented in Figure 6.11(a) show that there is small control spillover at these frequencies. This is because the PVDF error sensor cannot provide sufficiently large error signals at these frequencies. Also, as explained in Section 6.2.3, the modal control inputs are different from the theoretical ones as the mode shapes of the beam are different from the theoretical model. Also, unlike the simulation results, the experimentally measured mean-square velocity does not show resonance at frequencies around 646 Hz (the second controlled resonance frequency from simulation results). This is discussed in Section 6.2.3 and they will not be repeated here. Figure 6.11(a) clearly shows that “modal reduction” [3] is an important mechanism in the control.

The radiation efficiency of the simply supported beam calculated from the experimental measurements is shown in Figure 6.12(a). To facilitate the comparison, the corresponding simulation result is included in Figure 6.12(b). From Figures 6.12(a) and 6.12(b), the experimental radiation efficiency of the simply supported beam before control agrees well with



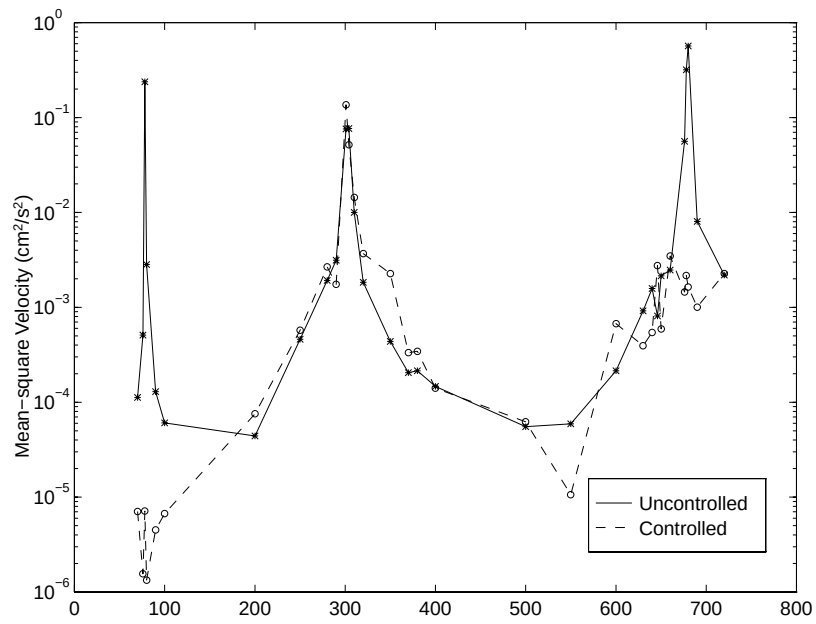
(a) Reconstructed



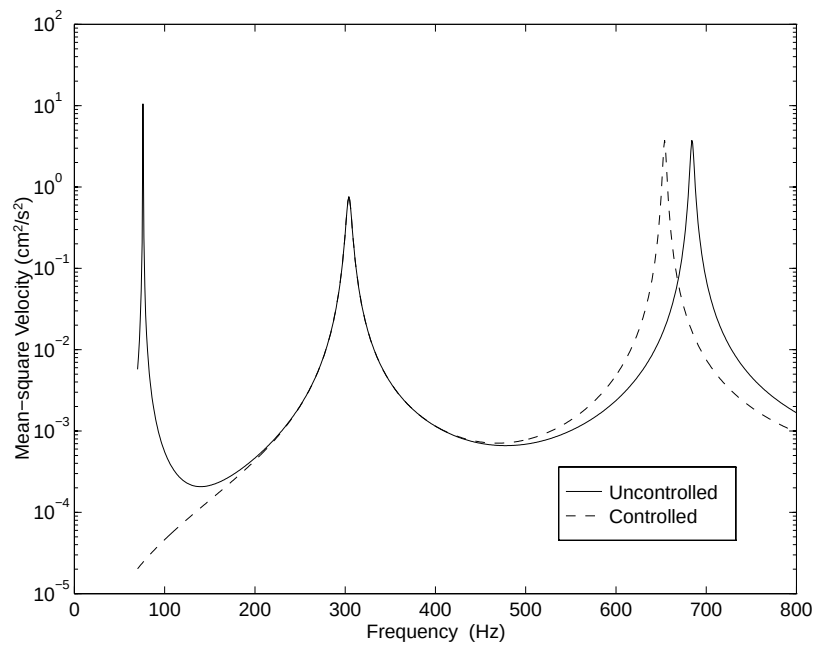
(b) Theoretical

the associated simulation result. However, the experimental radiation efficiency of the beam after control shows a large peak at 78 Hz (the first uncontrolled resonance frequency). This is due to the fact that the experimental controlled radiated power as well as the mean-square velocity both have a peak at this frequency. As mentioned previously, it may be possible to improve the results by using a higher bit A/D converter for the DSP I/O board (see the discussion for the radiated sound power). However, above 100 Hz, the after control radiation efficiency of the beam follows that of a dipole, i.e., a non-volumetric source. This implies that the non-volumetric design is achieved.

The reconstructed far-field sound pressure levels (in dB ref. 20μ Pa) before and after control at $\mathbf{r}(5 \times a, \theta = 0, \phi = 0)$, i.e., in a direction perpendicular to the surface of the finite beam, are shown in Figure 6.13(a). The associated simulation result is presented in Figure 6.13(b). Figure 6.13(a) shows that a large sound pressure attenuation, about 34 dB, is achieved at all selected test frequencies below 200 Hz. Large sound pressure attenuation is also found at test frequencies that are near the third uncontrolled resonance frequency. However, comparing Figures 6.13(a) and 6.13(b), the experimental results show large control spillover at test frequencies near the second uncontrolled natural frequency. There are several reasons for this phenomenon. First, as shown in Figure 6.7, the experimental PVDF sensor is more sensitive to the second uncontrolled resonance frequency, which is not desired. Second, the error signal provided by the PVDF error sensor at these frequencies was very small. In order to achieve control, the gain for the PVDF power amplifier had to be increased significantly. Therefore, the error signal to the controller contained some “noise” and the convergence of the controller was poor. Furthermore, recalling Figure 6.8, the modal control inputs of the second uncontrolled mode is not zero and the second mode of the beam is being excited by the controller (see Section 6.2.3 for discussion). The even-order mode shapes of the simply supported beam are not strictly non-volumetric, since the mode shapes of the beam are symmetric/anti-symmetric about the center of the beam, especially after the PZT actuators are mounted (see Figure 6.5). It is interesting to notice that the control spillover of the sound pressure level in the direction $\theta = 0$ is not associated with an important increase in radiated sound power in the frequency range close to the second uncontrolled resonance frequency (around 304 Hz, see Figure 6.10(a)). This can be explained by the sound pressure

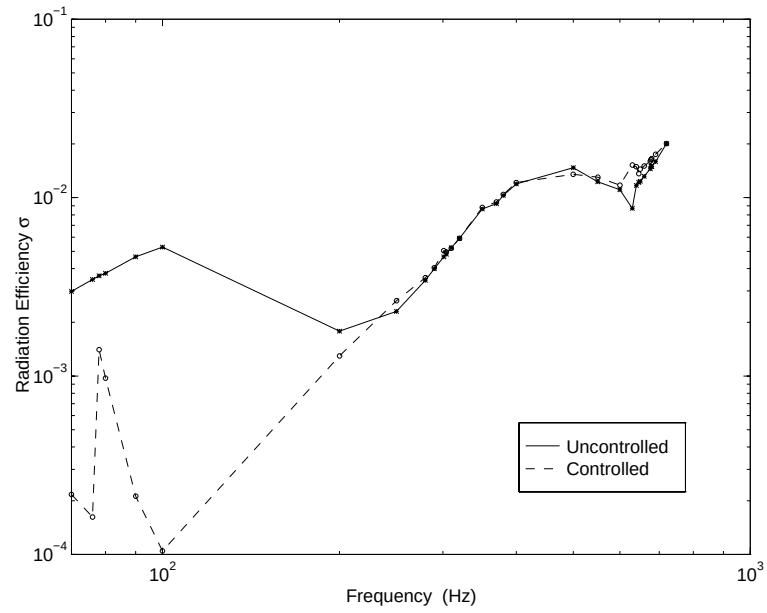


(a) Experimental

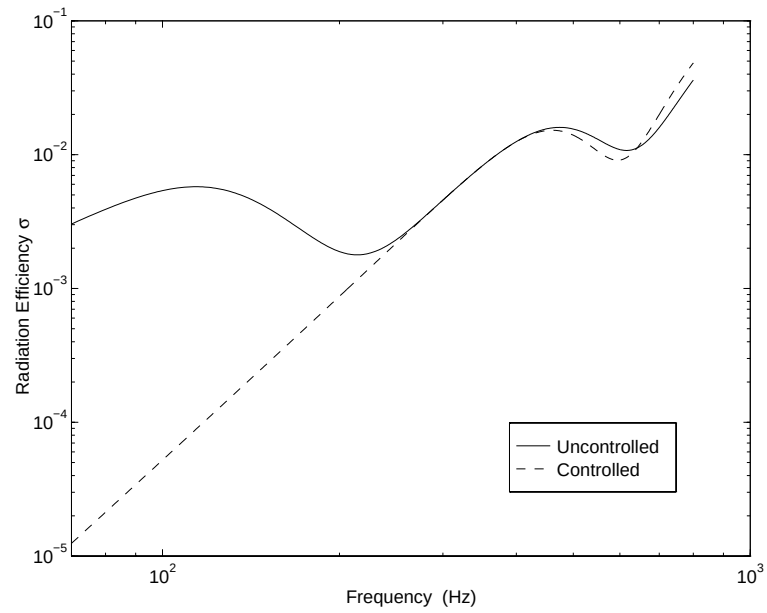


(b) Theoretical

185
Figure 6.11: Mean-square velocity of the simply supported beam



(a) Experimental



(b) Theoretical

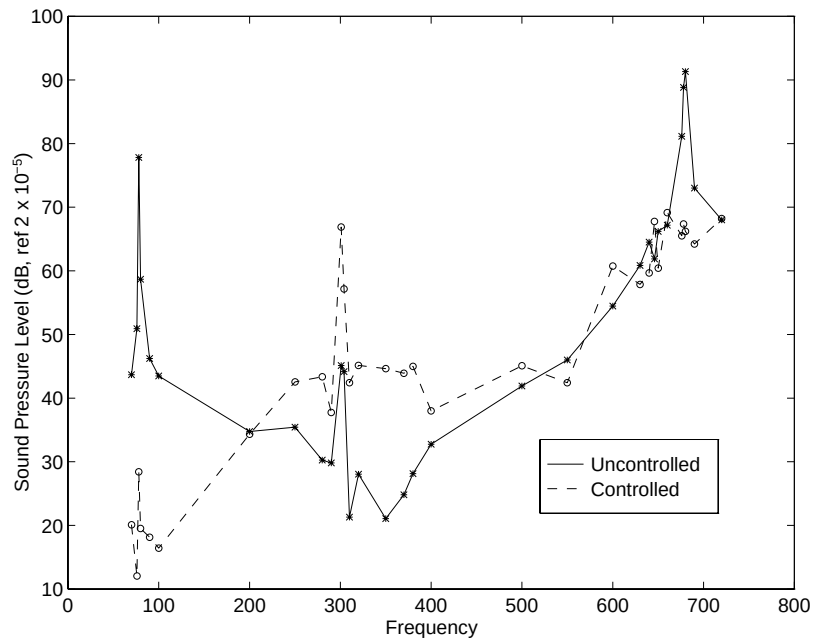
Figure 6.12: Radiation efficiency of the simply supported beam

directivity at 304 Hz presented in Figure 6.14, where the negative value of θ corresponds to $\phi = 0$ in the positive x half-plane. The sound pressure levels are calculated at a radius $R = 5 \times a$. It can be observed that although sound pressure in the $\theta = 0$ direction is increased at this frequency, the sound pressure level is reduced in all other directions. This explains the fact that there is no significant sound power control spillover at this frequency.

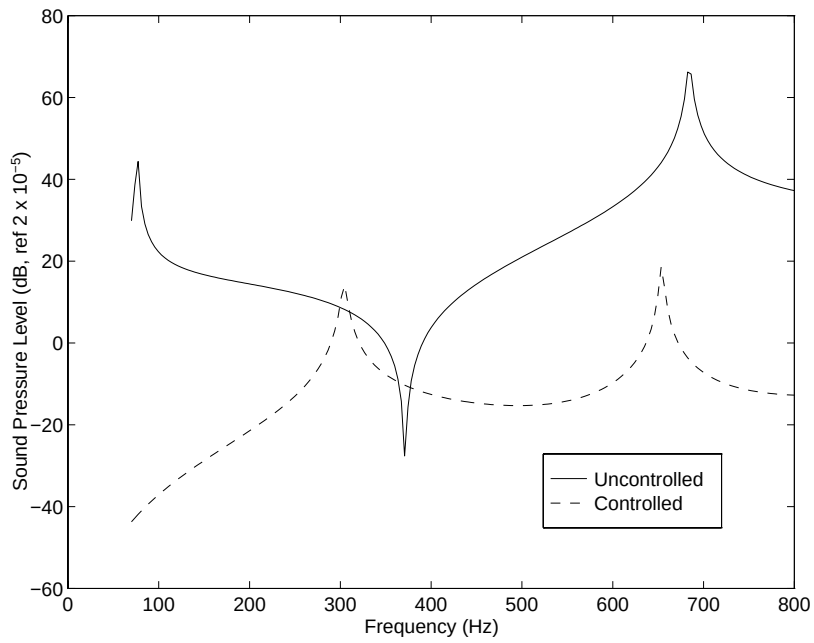
To further study the radiation characteristics of the control system, the reconstructed radiation directivity of the simply supported beam at 78 Hz (the first uncontrolled resonance frequency) is shown in Figure 6.15(a). The associated simulation result is given in Figure 6.15(b) for comparison. From Figure 6.15(a), it can be observed that significant global sound pressure attenuation after control has been achieved in the experiment. However, comparing Figures 6.15(a) and 6.15(b), the experimental radiation directivity of the controlled system still shows monopole behavior. As discussed previously, this is believed to be associated to the fact that although the error signal was minimized by about 30 dB (attenuation limited by the dynamic range and resolution of the A/D converter of the DSP I/O board), it could have been further reduced to change the beam from a monopole type radiator to a dipole type radiator. This is also because the experimental modal control inputs and modal error sensor components deviate from their associated theoretical one, as we have discussed in Section 6.2.3. Similar results can also be observed at the third uncontrolled resonance frequency (i.e., 680 Hz), as shown in Figures 6.16(a) and 6.16(b). Again, similar reasons as for the first mode are believed to apply. The reconstructed and theoretical sound radiation directivity of the simply supported beam at 100 Hz, i.e., an off-resonance frequency for both the uncontrolled and controlled system, are shown in Figures 6.17(a) and 6.17(b), respectively. Figure 6.17(a) shows that large global sound pressure attenuation is obtained in the experiment after control is applied. However, for the same reasons as discussed before, the radiation pattern of the beam after control is different from the theoretical prediction.

6.3 Summary

The experimental validation of the non-volumetric design was carried out on a simply supported beam using PZT actuators and shaped PVDF film as error sensor. In this chapter, the



(a) Reconstructed



(b) Theoretical

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Figure 6.13: Far-field sound pressure in the direction $\theta = 0$

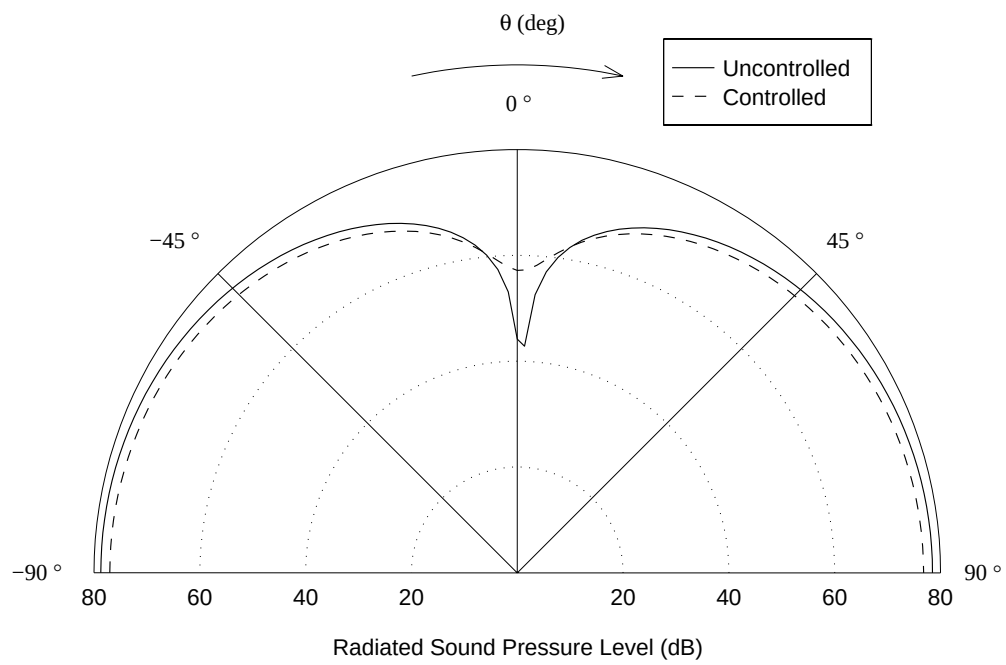
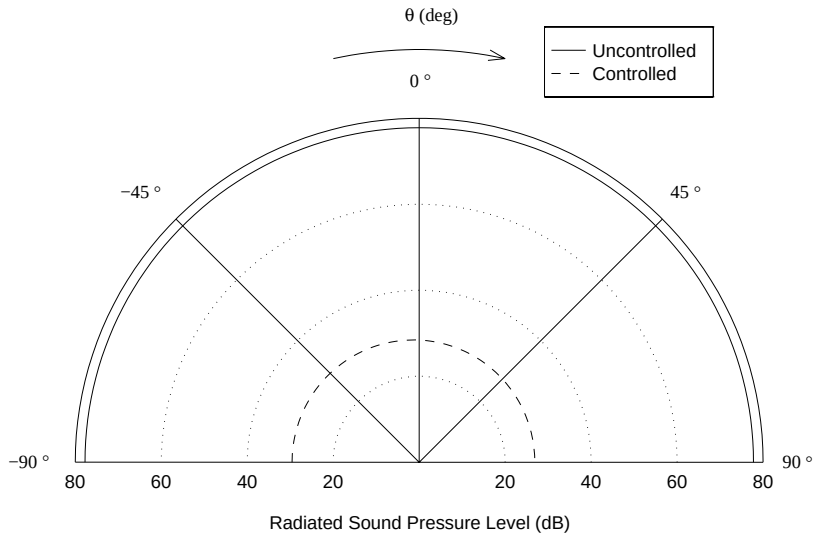
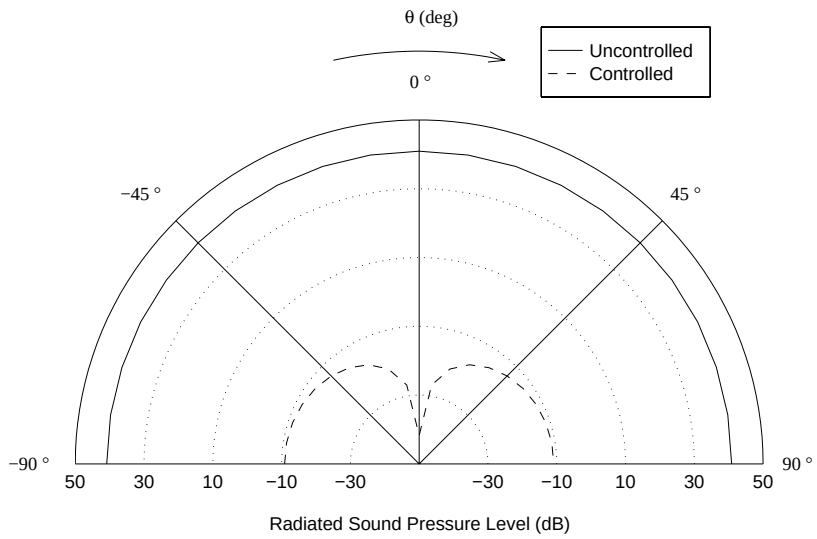


Figure 6.14: Reconstructed radiation directivity of the simply supported beam at 304 Hz (second uncontrolled resonance)

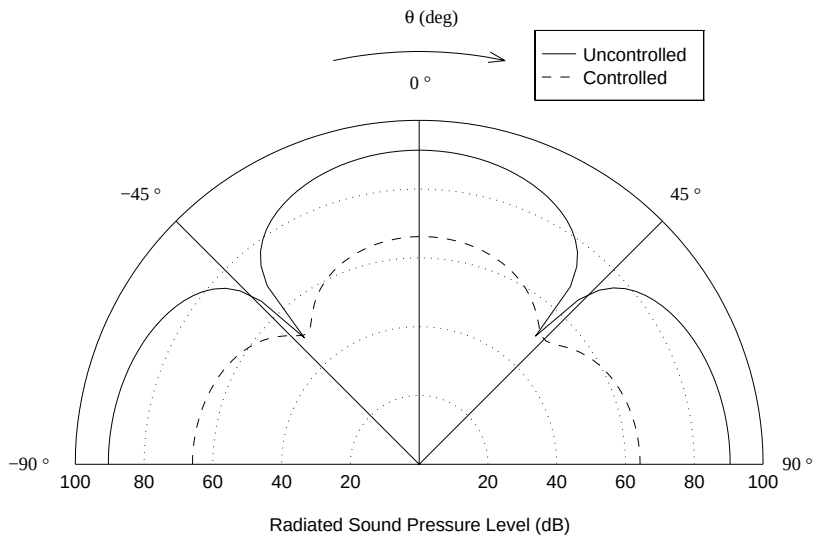


(a) Reconstructed

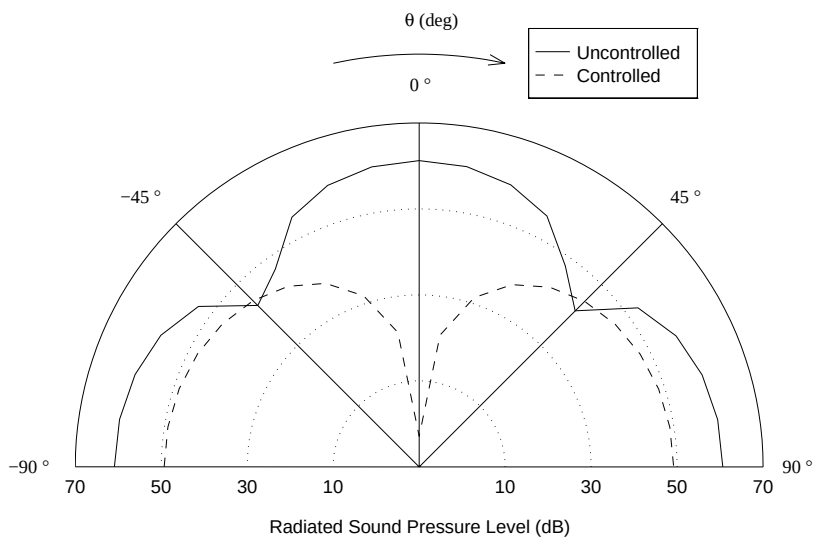


(b) Theoretical

Figure 6.15: Radiation directivity of the simply supported beam at 78 Hz

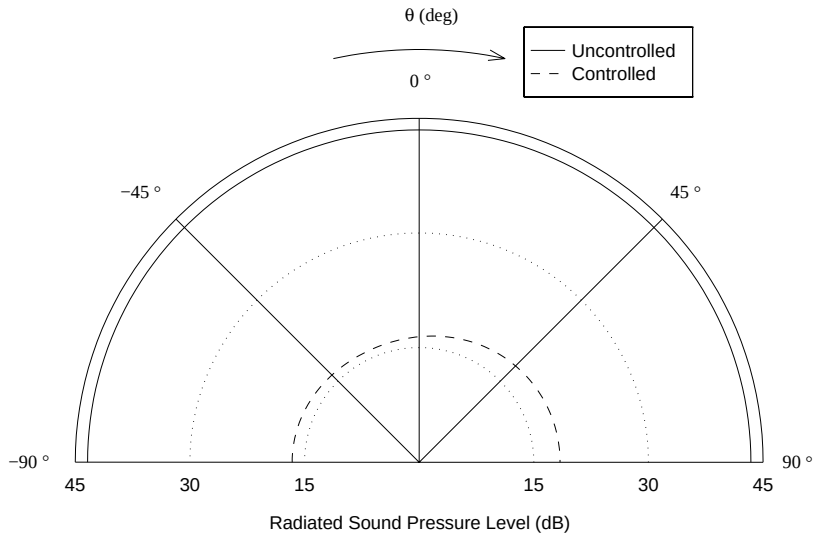


(a) Reconstructed

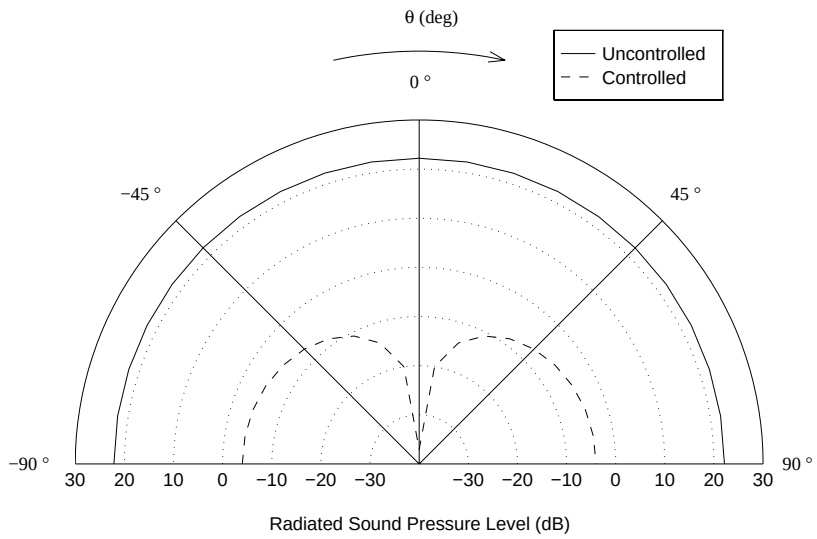


(b) Theoretical

Figure 6.16: Radiation directivity of the simply supported beam at 680 Hz



(a) Reconstructed



(b) Theoretical

Figure 6.17: Radiation directivity of the simply supported beam at 100 Hz

experimental setup and procedure were discussed. The experimental data post-processing, especially the procedure to reconstruct the far-field sound pressure from the experimentally measured structural velocities, was also discussed. The experimental results were then discussed in detail. The experimental results shows that significant global sound power attenuation has been achieved in the low frequency range. The experimental results also indicate that excellent sound power attenuation can be achieved at frequencies close to the resonance frequencies of the odd-order modes. However, the performance of the control system is poor at frequencies close to the second uncontrolled resonance frequency. This is associated with the fact that the non-volumetric design for the simply supported beam is designed to “observe/control” odd-order modes only.

Chapter 7

Conclusions and recommendations

This chapter presents the main conclusions of this study and the recommendations for the possible future work.

7.1 Conclusions

The goal of this work, aimed at efficient design of Active Structural Acoustic Control applied to planar sound radiators, can be divided into four main areas:

- Characterize the dynamic behavior of a multiple-input, multiple-output (MIMO) feedforward control controlled system.
- Use the eigenassignment technique for feedforward controlled system to design inefficient radiating structures base on some physical observations underlying the mechanisms of structure-borne sound radiation.
- Develop the tools to experimentally implement the proposed design,
- Validate experimentally such design on a simple beam structure.

General conclusions drawn from this work are as follows:

- A MIMO feedforward controlled system was demonstrated to have new dynamic characteristics (i.e., resonance frequencies and mode shapes) with respect to the disturbance when the control system has equal number of control inputs and error sensor outputs.
- A new mathematical approach was developed to compute the MIMO controlled system resonance frequencies and mode shapes. The eigenvalues of the controlled system are obtained by solving for the poles of the control compensator. The characteristic equation of the control compensator is also the characteristic equation of the controlled system. The mode shapes associated with the controlled system are found to be a linear combination of the uncontrolled system mode shapes.
- The new dynamic characteristics of the feedforward controlled system are found to depend only on the applied control inputs and the error sensor locations. They are independent of the characteristics of the disturbance input (type, location and frequency content).
- The theory was numerically illustrated by two-input, two-output (2I2O) control system on a simply supported plate excited by a harmonic disturbance. In addition, the control algorithm was implemented on a digital signal processing board and an experimental study was carried out on the simply supported plate. Two shakers were used as the control inputs and two accelerometers were used as the error sensors. The experimental results were then compared to the analytical results. Good overall agreement between the analytical studies and the experimental measurements was found.
- A single-input, single-output (SISO) eigenassignment technique was used to modify the eigenproperties of a planar structure using structure actuators and sensors such that all the controlled modes are non-volumetric (inefficient sound radiators at low frequencies, i.e., $k_0 a \ll 1$), leading large global sound attenuation in the far field. This design approach is referred to as the non-volumetric design.

- The effectiveness of the non-volumetric design was demonstrated through numerical simulations for the control of sound radiation from simply supported and clamped-free beams. Significant sound power attenuation was achieved in the low frequencies, up to the second uncontrolled resonance frequency. The advantage of this design was that the vibration level were not increased after control.
- Although the design procedure was carried out in the modal domain for simple one-dimensional structure located in rigid baffle, it is readily extensible to more general planar structures since it is based upon a normal modal analysis.
- The tools to experimentally implement the non-volumetric design, i.e., to obtain the desired modal control input and error sensor components, was developed. It was demonstrated that a closed form solution exists for the desired PVDF error sensor. Two strategies were investigated to achieve the desired modal control inputs for the PZT actuators: optimally positioning the PZT actuators or driving the PZT actuators with different weights. A sensitivity analysis showed that the modal control inputs of the PZT actuator are sensitive to the variations of the positions of the PZT actuators. To reduce their influence, an array of PZT actuators with different weights through simple signal processing were used in the experiment, as the weights for each PZT actuators can be changed at run time to accommodate the actual system dynamics. The filtered- x LMS algorithm was used in the experiment to achieve the optimal control compensator. Excellent global sound power attenuation was achieved in the low frequency range. Also, the control system shows good performance at frequencies close to the uncontrolled resonance frequencies of the odd-order modes. The control performance deteriorated at frequencies near the second uncontrolled resonance frequency. This was explained by the errors in the sensor and control input implementation.
- A SISO eigenassignment technique was used to modify the eigenproperties of a planar structure so that the eigenfunctions of the controlled system have minimum supersonic wavenumber in the frequency range of study. This design approach was referred to as the minimum supersonic wavenumber design. This design approach stems from the fact that only supersonic wavenumber components of the structural velocity spectra

radiate to the far field.

- The efficiency of the minimum supersonic wavenumber design was demonstrated on a simply supported beam to minimize the supersonic wavenumber components contributed by the odd-order modes only. Significant global sound attenuation was achieved in the frequency range of study.
- Unlike the non-volumetric design, large control spillover in vibrational field occurred at frequencies well above the design frequency. This demonstrated the frequency dependent nature of the minimum supersonic wavenumber design.
- The non-volumetric design can be considered as a special case of the minimum supersonic wavenumber design, as it corresponds to minimizing the supersonic components of the structural velocity wavenumber spectrum at $k_x = 0, k_y = 0$, where $k_x = k_0 \sin \theta \cos \phi, k_y = k_0 \sin \theta \sin \phi$ are the structural wavenumber components in x and y directions, respectively.
- The main advantage of the proposed design methods is that they do not depend on the characteristics of the external disturbance, such as the form, location and frequency contents. Also, the error sensor and control input are optimized simultaneously, resulting in better acoustic control performance. However, the practical implementations of the proposed designs require accurate system modeling, this is the major limitation of the proposed designs.

7.2 Recommendations

Recommendations for future investigation are as follows:

- To improve the experimental results, the design of the non-volumetric control system can be carried out based on the experimentally measured mode shapes of the structure.

- Different error sensors and actuators, such as array of accelerometers should be investigated to improve the experimental results. In this case simple signal conditioning may be necessary.
- The experimental control results have been shown to be limited by the resolution or dynamic range of the A/D converter of the DSP I/O board. Therefore, an A/D converter of 16- or 32-bit should help achieve better control performance
- Tools should be developed to use the experimentally measured transfer functions to optimize for the modal control inputs.
- The proposed non-volumetric design and the minimum supersonic wavenumber design are readily extensible to more general planar structures such as plates. In this case, distributed error sensors such as the PVDF film could be difficult to implement. However, arrays of accelerometers located at fixed positions could be used as error sensors. Similar to the design of the PZT actuators with different weights, different weights can be assigned to each accelerometers through simple signal conditioning to acquire the desired modal error sensor components.
- The SISO non-volumetric design conducted in this work provided an approach to eliminate the “piston mode”, the most efficient radiation mode in the low frequency range. Therefore the performance of this design is limited to the low frequency range. The next most efficient radiation mode is the “rocking mode”[29], which is volumetric. To improve the control performance over an extended frequency range, a MIMO eigenassignment technique may be investigated to eliminate both the piston and rocking radiation modes.

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Appendix A

Dynamics modification of feedforward control

In this appendix we will prove that in general, with a properly configured control compensator, feedforward control can change the dynamics of an elastic system. This theory was first demonstrated, both theoretically and experimentally, by Burdisso and Fuller [8, 38] on active vibration control of a simply-supported beam with a single-input, single-output feedforward controller, where a discrete control source (shaker) and error sensor (accelerometer) are used. This conclusion was also confirmed by Alberts [10]. The main purpose here, rather than a detailed study of the dynamics of the system before and after control, is to show that the dynamics of the system can be changed by a properly configured feedforward control compensator. For this reason the derivation presented in this appendix is based on Alberts's approach, but is extended to general planar structures.

For any flexible structure such as the simply-supported flexible beam as shown in Figure 2.1, assume the primary or disturbance source $F(\omega)$ is applied at location x_d , and the control source at location x_c . Let us denote the frequency response function between the

external input at location a and displacement at location b as

$$G_{ab}(\omega) = \frac{n_{ab}(\omega)}{d(\omega)} \quad (\text{A.1})$$

where $d(\omega)$ is common to all transfer functions since it gives the poles of the structure which are global properties of the system, and $n_{ab}(\omega)$ gives the zeros of the system whose nature depends on the particular input and output locations.

For any typical point x on the structure, the displacement response of the controlled system can be written as

$$w(x, \omega) = \frac{n_{dx}(\omega)}{d(\omega)} F(\omega) + \frac{n_{cx}(\omega)}{d(\omega)} G(\omega) F(\omega) \quad (\text{A.2})$$

where $d(\omega)$ represents the characteristic function of the original structure without control, and the control compensator $G(\omega)$ is usually defined by minimizing the sum of the quadratic square of the signals of the error sensors on the structure (e.g., accelerometers) or in the acoustic field (e.g., microphones).

For some system it is possible to force the response at the error sensor location to zero

$$w(x_e, \omega) = 0 \quad (\text{A.3})$$

by choosing the control compensator as

$$G(\omega) = -\frac{n_{de}(\omega)}{n_{ce}(\omega)} \quad (\text{A.4})$$

In order to analyze the dynamics of the controlled system, it is necessary to include the dynamics of the control compensator with that of the plant. Substitutes equation (A.4) into equation (A.2), the displacement response at any point x on the structure can be written as

$$w(x, \omega) = \frac{n_{dx}(\omega)}{d(\omega)} F(\omega) - \frac{n_{cx}(\omega)}{d(\omega)} \frac{n_{de}(\omega)}{n_{ce}(\omega)} F(\omega) \quad (\text{A.5})$$

We now prove that equation (A.5) can be further simplified as

$$w(x, \omega) = -\frac{\tilde{n}_x(\omega)}{n_{ce}(\omega)} F(\omega) \quad (\text{A.6})$$

where $\tilde{n}_x(\omega)$ is a new function.

The proof of this equation requires applying modal analysis technique. First we will show that equation (2.6) is true, i.e., the following identity holds:

$$\frac{n_{cx}(\omega)}{d(\omega)} \frac{n_{de}(\omega)}{n_{ce}(\omega)} = \frac{n_{dx}(\omega)}{d(\omega)} + \frac{\tilde{n}_x(\omega)}{n_{ce}(\omega)} \quad (\text{A.7})$$

The r.h.s of equation (A.7) can be expanded in partial fractions and a general expression can be written as

$$\frac{n_{cx}(\omega)}{d(\omega)} \frac{n_{de}(\omega)}{n_{ce}(\omega)} = \frac{\tilde{m}(\omega)}{d(\omega)} + \frac{\tilde{n}_x(\omega)}{n_{ce}(\omega)} \quad (\text{A.8})$$

We will show that $\tilde{m}(\omega)$ is identically $n_{dx}(\omega)$. From standard modal analysis, assume the modal frequency response function is $H_n(\omega) = (\omega_n^2 - \omega^2)^{-1}$, the first term of the r.h.s of equation (A.8) can be written as

$$\frac{\tilde{m}(\omega)}{d(\omega)} = \sum_{n=1}^{\infty} \frac{\tilde{m}_n(\omega)}{\omega_n^2 - \omega^2} \quad (\text{A.9})$$

where ω_n is the poles of $d(\omega)$. From standard partial fraction expansion, the expansion coefficient $\tilde{m}_n(\omega)$ is given by

$$\tilde{m}_n(\omega) = \left[(\omega_n^2 - \omega^2) \frac{n_{cx}(\omega)}{d(\omega)} \frac{n_{de}(\omega)}{n_{ce}(\omega)} \right] \Big|_{\omega=\omega_n} \quad (\text{A.10})$$

From modal analysis, the response of the system at any location x on the structure due to

the external input f at location x_a can be expressed as

$$w(x, \omega) = \sum_{n=1}^N f_n(x_a) H_n(\omega) \phi_n(x) \quad (\text{A.11})$$

where $f_n(x_a)$ is the n^{th} modal components of the spatial distribution of the external input,

$$f_n(x_a) = \int_D \phi_n(D) f(x_a) dD \quad (\text{A.12})$$

For two arbitrary points, say x_b and x_d , on the structure, the ratio of the responses at these two points can be obtained from equation (A.11) as

$$\frac{w(x_b, \omega)}{w(x_d, \omega)} = \frac{\sum_{n=1}^N f_n(x_a) H_n(\omega) \phi(x_b)}{\sum_{m=1}^N f_m(x_a) H_m(\omega) \phi(x_d)} \quad (\text{A.13})$$

Multiplying and dividing the product of the modal frequency response function to the r.h.s of equation (A.13), we have

$$\frac{w(x_b, \omega)}{w(x_d, \omega)} = \frac{\sum_{n=1}^N f_n(x_a) \phi(x_b) \prod_{\substack{r=1 \\ r \neq n}}^N (\omega_r^2 - \omega^2)}{\sum_{m=1}^N f_m(x_a) \phi(x_d) \prod_{\substack{s=1 \\ s \neq n}}^N (\omega_s^2 - \omega^2)} \quad (\text{A.14})$$

Evaluating equation (A.14) at $\omega = \omega_n$ gives

$$\left. \frac{w(x_b, \omega)}{w(x_d, \omega)} \right|_{\omega=\omega_n} = \frac{\phi_n(x_b)}{\phi_n(x_d)} \quad (\text{A.15})$$

Equation (A.15) states that for any two arbitrary points on the structure, the ratio of the responses at these two points evaluated at the resonant frequency is only a function of the locations of these two points, and independent of the input location. From equation (A.15) and

noting the denominator $d(\omega)$ is common for all frequency response functions, the following identity holds:

$$\left. \frac{n_{de}(\omega)}{n_{dx}(\omega)} \right|_{\omega=\omega_n} = \left. \frac{n_{ce}(\omega)}{n_{cx}(\omega)} \right|_{\omega=\omega_n} \quad (\text{A.16})$$

Applying this identity to equation (A.10), we have

$$\tilde{m}_n(\omega) = \left[(\omega_n^2 - \omega^2) \frac{n_{dx}(\omega)}{d(\omega)} \right] \Big|_{\omega=\omega_n} \quad (\text{A.17})$$

Equations (A.9) and (A.17) shows that the partial fractions of $\tilde{m}(\omega)/d(\omega)$ and $n_{dx}(\omega)/d(\omega)$ are the same, hence we have proved that

$$\frac{\tilde{m}(\omega)}{d(\omega)} = \frac{n_{dx}(\omega)}{d(\omega)} \quad (\text{A.18})$$

Substituting equation (A.18) into equation (A.8), the result is the identity defined in equation (A.7). The substitution of equation (A.7) into equation (A.5) proves the equation (A.6).

The above derivation can also be viewed from a different aspect. Note that the equation (A.5) can be rewritten as

$$w(x, \omega) = \frac{1}{d(\omega)} \left[n_{dx}(\omega) - n_{cx}(\omega) \frac{n_{de}(\omega)}{n_{ce}(\omega)} \right] F(\omega) \quad (\text{A.19})$$

From the above derivation, we can say that the poles of the original system has been canceled out by the zeros of the terms in the bracket [38, 10].

Equation (A.6) states that, with a properly configured feedforward control compensator, the system characteristic function (hence the dynamics of the system) can be effectively changed from $d(\omega)$ to $n_{ce}(\omega)$, the characteristic function of the feedforward control compensator. In other word, the controlled system effectively has new eigenvalues (hence the resonant frequencies) given by the poles of the feedforward control compensator $G(\omega)$. It is important to point out that this system dynamics modification is for feedforward control

where coherent information from the primary source is applied to the structure as control input. To any other incoherent disturbance, the system dynamics remain unchanged [8].

Appendix B

MIMO feedforward control compensator

In this appendix, the proof to equation (2.36) introduced in Section 2.3.3 is presented. We will also prove that the first term in equation (2.43) vanishes for all frequencies. All mathematical notations are the same as in Chapter 2.

B.1 Proof of equation (2.36)

For multiple-input, multiple-output feedforward controlled system, when the number of error sensors is equal to the number of the control inputs, the control compensator can be expressed as equation (2.36). Proof of this equation is presented in this section. When the number of error sensor is equal to the control inputs, i.e., $M_e = M_c = M$, equation (2.32) in Section 2.3.2 can be written as

$$\sum_{j=1}^M a_{ij}(\omega)U_j(\omega) = -b_i(\omega) \quad i = 1, 2, \dots, M \quad (\text{B.1})$$

where $a_{ij}(\omega)$ and $b_i(\omega)$ $i, j = 1, 2, \dots, M$ are defined as

$$a_{ij}(\omega) = \sum_{n=1}^N \xi_{ni} H_n(\omega) \mu_{nj} \quad (\text{B.2})$$

$$b_i(\omega) = \sum_{n=1}^N \xi_{ni} H_n(\omega) \nu_n F(\omega) \quad (\text{B.3})$$

The control compensator is defined as

$$G_i(\omega) = -\frac{U_i(\omega)}{F(\omega)}, \quad i = 1, 2, \dots, M \quad (\text{B.4})$$

From equations (B.1) and (B.4), the control compensator can be obtained by solving the following equation

$$\sum_{j=1}^M a_{ij}(\omega) G_j(\omega) = -b_i(\omega) \quad i = 1, 2, \dots, M \quad (\text{B.5})$$

We shall prove the solution to the above equation can be written as

$$G_i(\omega) = \frac{\sum_{m=1}^N \sum_{n=1}^N \dots \sum_{p=1}^N \xi_{m1} \xi_{n2} \dots \xi_{pM} \cdot |\boldsymbol{\mu}_i| \cdot \prod_{\substack{\alpha=1 \\ \alpha \neq m, n, \dots, p}}^N (\omega_\alpha^2 - \omega^2)}{\sum_{m=1}^N \sum_{n=1}^N \dots \sum_{p=1}^N \xi_{m1} \xi_{n2} \dots \xi_{pM} \cdot |\boldsymbol{\mu}| \cdot \prod_{\substack{\beta=1 \\ \beta \neq m, n, \dots, p}}^N (\omega_\beta^2 - \omega^2)} \quad (\text{B.6})$$

where the $M \times M$ determinant

$$|\boldsymbol{\mu}| = \begin{vmatrix} \mu_{m1} & \mu_{m2} & \dots & \mu_{mM} \\ \mu_{n1} & \mu_{n2} & \dots & \mu_{nM} \\ \vdots & \vdots & \vdots & \vdots \\ \mu_{p1} & \mu_{p2} & \dots & \mu_{pM} \end{vmatrix} \quad (\text{B.7})$$

and $|\boldsymbol{\mu}_i|$ is the $M \times M$ determinant obtained on replacing the respective elements of $\mu_{mi}, \mu_{ni}, \dots, \mu_{pi}$ in the i^{th} column of $|\boldsymbol{\mu}|$ by $\nu_m, \nu_n, \dots, \nu_p$:

$$|\boldsymbol{\mu}_i| = \begin{vmatrix} \mu_{m1} & \mu_{m2} & \cdots & \mu_{m,i-1} & \nu_m & \mu_{m,i+1} & \cdots & \mu_{mM} \\ \mu_{n1} & \mu_{n2} & \cdots & \mu_{n,i-1} & \nu_n & \mu_{n,i+1} & \cdots & \mu_{nM} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mu_{p1} & \mu_{p2} & \cdots & \mu_{p,i-1} & \nu_p & \mu_{p,i+1} & \cdots & \mu_{pM} \end{vmatrix} \quad (\text{B.8})$$

We attempt to prove this by the principle of finite induction [40]. This method is a strict mathematical description of heuristics. The basic idea of this method is that we shall prove, at first, the conclusion is right for the case of $M = 1$ and $M = 2$. We then assume that the conclusion is true for the case of $M = k$, where k is an arbitrary integer. If we can prove that the conclusion is true for the case of $M = k + 1$, we can conclude that the conclusion is correct for any M .

When $M = 1$ (one control input, one error sensor), it's trivial to see that equation (B.6) is the solution to equation (B.5).

For the case of $M = 2$, equation (B.5) can be written in matrix form as

$$\begin{Bmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n2} \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n1} & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n2} \end{Bmatrix} \begin{Bmatrix} G_1(\omega) \\ G_2(\omega) \end{Bmatrix} = \begin{Bmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \nu_n \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \nu_n \end{Bmatrix} \quad (\text{B.9})$$

Using Cramer's rule, the solution for this equation is

$$\begin{aligned} G_1(\omega) &= \frac{\mathbf{D}_{2 \times 2}^1(\omega)}{\mathbf{D}_{2 \times 2}(\omega)} \\ &= \frac{\begin{vmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \nu_n & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n2} \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \nu_n & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n2} \end{vmatrix}}{\begin{vmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n2} \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n1} & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n2} \end{vmatrix}} \\ &= \frac{\sum_{n=1}^N \sum_{m=1}^N \xi_{n1} \xi_{m2} H_n(\omega) H_m(\omega) \cdot \begin{vmatrix} \nu_n & \mu_{n2} \\ \nu_m & \mu_{m2} \end{vmatrix}}{\sum_{n=1}^N \sum_{m=1}^N \xi_{n1} \xi_{m2} H_n(\omega) H_m(\omega) \cdot \begin{vmatrix} \mu_{n1} & \mu_{n2} \\ \mu_{m1} & \mu_{m2} \end{vmatrix}} \end{aligned} \quad (\text{B.10})$$

Multiplying the product of the modal frequency response functions $\prod_{n=1}^N H_n^{-1}(\omega)$ to both the numerator and denominator of equation (B.10), it is found

$$G_1(\omega) = \frac{\sum_{n=1}^N \sum_{m=1}^N \xi_{n1} \xi_{m2} \cdot \begin{vmatrix} \nu_n & \mu_{n2} \\ \nu_m & \mu_{m2} \end{vmatrix} \cdot \prod_{\substack{s=1 \\ s \neq n,m}}^N (\omega_s^2 - \omega^2)}{\sum_{n=1}^N \sum_{m=1}^N \xi_{n1} \xi_{m2} \cdot \begin{vmatrix} \mu_{n1} & \mu_{n2} \\ \mu_{m1} & \mu_{m2} \end{vmatrix} \cdot \prod_{\substack{r=1 \\ r \neq n,m}}^N (\omega_r^2 - \omega^2)} \quad (\text{B.11})$$

Similarly, we can solve for $G_2(\omega)$ from equation (B.5)

$$\begin{aligned} G_2(\omega) &= \frac{\mathbf{D}_{2 \times 2}^2(\omega)}{\mathbf{D}_{2 \times 2}(\omega)} \\ &= \frac{\begin{vmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \sum_{n=1}^N \xi_{n1} H_n(\omega) \nu_n \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n1} & \sum_{n=1}^N \xi_{n2} H_n(\omega) \nu_n \end{vmatrix}}{\begin{vmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n2} \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n1} & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n2} \end{vmatrix}} \\ &= \frac{\sum_{n=1}^N \sum_{m=1}^N \xi_{n1} \xi_{m2} H_n(\omega) H_m(\omega) \cdot \begin{vmatrix} \mu_{n1} & \nu_n \\ \mu_{m1} & \nu_m \end{vmatrix}}{\sum_{n=1}^N \sum_{m=1}^N \xi_{n1} \xi_{m2} H_n(\omega) H_m(\omega) \cdot \begin{vmatrix} \mu_{n1} & \mu_{n2} \\ \mu_{m1} & \mu_{m2} \end{vmatrix}} \end{aligned} \quad (\text{B.12})$$

Multiplying the numerator and denominator of the above equation by the product of the modal frequency response functions $\prod_{n=1}^N H_n^{-1}(\omega)$, it is found that

$$G_2(\omega) = \frac{\sum_{n=1}^N \sum_{m=1}^N \xi_{n1} \xi_{m2} \cdot \begin{vmatrix} \mu_{n1} & \nu_n \\ \mu_{m1} & \nu_m \end{vmatrix} \cdot \prod_{\substack{s=1 \\ s \neq n,m}}^N (\omega_s^2 - \omega^2)}{\sum_{n=1}^N \sum_{m=1}^N \xi_{n1} \xi_{m2} \cdot \begin{vmatrix} \mu_{n1} & \mu_{n2} \\ \mu_{m1} & \mu_{m2} \end{vmatrix} \cdot \prod_{\substack{r=1 \\ r \neq n,m}}^N (\omega_r^2 - \omega^2)} \quad (\text{B.13})$$

Comparing equations (B.11) and (B.13) with equation (B.6), we conclude that equation (B.6) is the solution for equation (B.5).

Now assuming equation (B.6) is the solution for equation (B.5) when $M = k$, where k is any positive integer:

$$\begin{aligned}
G_i(\omega) &= \frac{\mathbf{D}_{k \times k}^i(\omega)}{\mathbf{D}_{k \times k}(\omega)} \\
&= \frac{\overbrace{\sum_{n=1}^N \sum_{m=1}^N \cdots \sum_{r=1}^N \sum_{p=1}^N \xi_{n1} \xi_{m2} \cdots \xi_{r,k-1} \xi_{pk} |\boldsymbol{\mu}_i|}^{k \text{ elements}} \prod_{\substack{s=1 \\ s \neq n, m, \dots, p}}^N (\omega_s^2 - \omega^2)}{\underbrace{\sum_{n=1}^N \sum_{m=1}^N \cdots \sum_{r=1}^N \sum_{p=1}^N \xi_{n1} \xi_{m2} \cdots \xi_{r,k-1} \xi_{pk} |\boldsymbol{\mu}|}^{k \text{ elements}} \prod_{\substack{r=1 \\ r \neq n, m, \dots, p}}^N (\omega_r^2 - \omega^2)} \quad (\text{B.14})
\end{aligned}$$

where $\mathbf{D}_{k \times k}(\omega)$ denotes the system determinant:

$$\begin{vmatrix}
\sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \cdots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{ni} & \cdots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{nk} \\
\sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n1} & \cdots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{ni} & \cdots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{nk} \\
\vdots & \cdots & \vdots & \cdots & \vdots \\
\sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{n1} & \cdots & \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{ni} & \cdots & \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{nk}
\end{vmatrix} \quad (\text{B.15})$$

and the subscript $k \times k$ denotes the order of the determinant. The determinant $\mathbf{D}_{k \times k}^i(\omega)$ is obtained on replacing the respective elements in the i^{th} column of $\mathbf{D}_{k \times k}(\omega)$ by $b_1, b_2, \dots, b_n$ defined in equation (B.3) and can be written as:

$$\begin{vmatrix}
\sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \cdots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \nu_n & \cdots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{nk} \\
\sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n1} & \cdots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \nu_n & \cdots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{nk} \\
\vdots & \cdots & \vdots & \cdots & \vdots \\
\sum_{n=1}^N \xi_{nk} H_n(\omega) \mu_{n1} & \cdots & \sum_{n=1}^N \xi_{nk} H_n(\omega) \nu_n & \cdots & \sum_{n=1}^N \xi_{nk} H_n(\omega) \mu_{nk}
\end{vmatrix} \quad (\text{B.16})$$

and $|\boldsymbol{\mu}|$, $|\boldsymbol{\mu}_i|$ are $k \times k$ determinants as defined in equations (B.7) and (B.8).

When $M = k + 1$, by Cramer's rule, the solutions to equation (B.5) are

$$G_i(\omega) = \frac{\mathbf{D}_{(k+1) \times (k+1)}^i(\omega)}{\mathbf{D}_{(k+1) \times (k+1)}(\omega)} \quad (\text{B.17})$$

where $\mathbf{D}_{(k+1) \times (k+1)}(\omega)$ and $\mathbf{D}_{(k+1) \times (k+1)}^i$, $i = 1, 2, \dots, k, k + 1$ are the same as before:

$$\mathbf{D}_{(k+1) \times (k+1)}(\omega) = \begin{vmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \dots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{ni} & \dots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n,k+1} \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n1} & \dots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{ni} & \dots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n,k+1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{n1} & \dots & \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{ni} & \dots & \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{n,k+1} \end{vmatrix} \quad (\text{B.18})$$

$$\mathbf{D}_{(k+1) \times (k+1)}^i(\omega) = \begin{vmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \dots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \nu_n & \dots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n,k+1} \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n2} & \dots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \nu_n & \dots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n,k+1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{n1} & \dots & \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \nu_n & \dots & \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{n,k+1} \end{vmatrix} \quad (\text{B.19})$$

For convinience, let us consider the expression of the system determinant as defined in equation (B.18) first. By simple Laplace development, the system determinant $\mathbf{D}_{(k+1) \times (k+1)}(\omega)$ can be represented in terms of the elements and cofactors of the last column:

$$\mathbf{D}_{(k+1) \times (k+1)}(\omega) = \sum_{i=1}^{k+1} (-1)^{i+(k+1)} \mathbf{A}_{i,k+1}(\omega) \left(\sum_{n=1}^N \xi_{ni} H_n(\omega) \mu_{n,k+1} \right) \quad (\text{B.20})$$

where the minor $\mathbf{A}_{i,k+1}(\omega)$ of the element $a_{i,k+1}$ in the $(k + 1)^{th}$ order determinant $\mathbf{D}_{(k+1) \times (k+1)}(\omega)$ is the k^{th} order determinant obtained from equation (B.18) on erasing the

i^{th} row and the $(k+1)^{th}$ column:

$$\mathbf{A}_{i,k+1}(\omega) = \begin{vmatrix} \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n1} & \dots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_n & \dots & \sum_{n=1}^N \xi_{n1} H_n(\omega) \mu_{n,k+1} \\ \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n2} & \dots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_n & \dots & \sum_{n=1}^N \xi_{n2} H_n(\omega) \mu_{n,k+1} \\ \vdots & \dots & \vdots & \dots & \vdots \\ \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{n1} & \dots & \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_n & \dots & \sum_{n=1}^N \xi_{n,k+1} H_n(\omega) \mu_{n,k+1} \end{vmatrix} \quad (\text{B.21})$$

Applying equation (B.14) to equation (B.21), equation (B.20) can be written in the following form

$$\begin{aligned} \mathbf{D}_{(k+1) \times (k+1)}(\omega) = & (-1)^{2k+2} \sum_{q=1}^N \left(\xi_{q,k+1} H_q(\omega) \mu_{q,k+1} \right) \sum_{n=1}^N \sum_{m=1}^N \dots \sum_{p=1}^N \xi_{n1} \xi_{m2} \dots \xi_{pk} H_n(\omega) H_m(\omega) \dots H_p(\omega) \boldsymbol{\mu}_{(k+1) \times (k+1)}^q \\ & (-1)^{2k+1} \sum_{p=1}^N \left(\xi_{pk} H_p(\omega) \mu_{p,k+1} \right) \sum_{n=1}^N \dots \sum_{r=1}^N \sum_{q=1}^N \xi_{n1} \dots \xi_{r,k-1} \xi_{q,k+1} H_n(\omega) \dots H_r(\omega) H_q(\omega) \boldsymbol{\mu}_{(k+1) \times (k+1)}^p \\ & \dots \\ & (-1)^{k+3} \sum_{m=1}^N \left(\xi_{m2} H_m(\omega) \mu_{m,k+1} \right) \sum_{n=1}^N \dots \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \dots \xi_{pk} \xi_{q,k+1} H_n(\omega) \dots H_p(\omega) H_q(\omega) \boldsymbol{\mu}_{(k+1) \times (k+1)}^m \\ & (-1)^{k+2} \sum_{n=1}^N \left(\xi_{n1} H_n(\omega) \mu_{n,k+1} \right) \sum_{m=1}^N \dots \sum_{p=1}^N \sum_{q=1}^N \xi_{m1} \dots \xi_{pk} \xi_{q,k+1} \cdot H_m(\omega) \dots H_p(\omega) H_q(\omega) \boldsymbol{\mu}_{(k+1) \times (k+1)}^n \end{aligned} \quad (\text{B.22})$$

where

$$\boldsymbol{\mu}_{(k+1) \times (k+1)}^q = \begin{vmatrix} \mu_{n1} & \mu_{n2} & \dots & \mu_{nk} \\ \mu_{m1} & \mu_{m2} & \dots & \mu_{mk} \\ \vdots & \vdots & \dots & \vdots \\ \mu_{p1} & \mu_{p2} & \dots & \mu_{pk} \end{vmatrix} \quad \boldsymbol{\mu}_{(k+1) \times (k+1)}^p = \begin{vmatrix} \mu_{n1} & \mu_{n2} & \dots & \mu_{nk} \\ \vdots & \vdots & \dots & \vdots \\ \mu_{r1} & \mu_{r2} & \dots & \mu_{rk} \\ \mu_{q1} & \mu_{q2} & \dots & \mu_{qk} \end{vmatrix}$$

$$\boldsymbol{\mu}_{(k+1) \times (k+1)}^m = \begin{vmatrix} \mu_{n1} & \mu_{n2} & \dots & \mu_{nk} \\ \vdots & \vdots & \dots & \vdots \\ \mu_{p1} & \mu_{p2} & \dots & \mu_{pk} \\ \mu_{q1} & \mu_{q2} & \dots & \mu_{qk} \end{vmatrix} \quad \boldsymbol{\mu}_{(k+1) \times (k+1)}^n = \begin{vmatrix} \mu_{m1} & \mu_{m2} & \dots & \mu_{mk} \\ \vdots & \vdots & \dots & \vdots \\ \mu_{p1} & \mu_{p2} & \dots & \mu_{pk} \\ \mu_{q1} & \mu_{q2} & \dots & \mu_{qk} \end{vmatrix}$$

If we note that the subscript m, n, p, q are just dummy index number, equation (B.22) can be further rewritten as

$$\begin{aligned} \mathbf{D}_{(k+1) \times (k+1)}(\omega) = & \sum_{n=1}^N \dots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \dots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} H_n(\omega) \dots H_r(\omega) H_p(\omega) H_q(\omega) (-1)^{2k+2} \boldsymbol{\mu}_{q,k+1} \boldsymbol{\mu}_{(k+1) \times (k+1)}^q \\ & + \sum_{n=1}^N \dots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \dots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} H_n(\omega) \dots H_r(\omega) H_p(\omega) H_q(\omega) (-1)^{2k+1} \boldsymbol{\mu}_{p,k+1} \boldsymbol{\mu}_{(k+1) \times (k+1)}^p \\ & + \dots \\ & + \sum_{n=1}^N \dots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \dots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} H_n(\omega) \dots H_r(\omega) H_p(\omega) H_q(\omega) (-1)^{k+3} \boldsymbol{\mu}_{m,k+1} \boldsymbol{\mu}_{(k+1) \times (k+1)}^m \\ & + \sum_{n=1}^N \dots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \dots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} H_m(\omega) \dots H_r(\omega) H_p(\omega) H_q(\omega) (-1)^{k+2} \boldsymbol{\mu}_{n,k+1} \boldsymbol{\mu}_{(k+1) \times (k+1)}^n \end{aligned} \quad (\text{B.23})$$

From the simple Laplace development, the above equation can be rewritten as

$$\begin{aligned} \mathbf{D}_{(k+1) \times (k+1)}(\omega) = & \sum_{n=1}^N \dots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \dots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} H_n(\omega) \dots H_r(\omega) \dots H_p(\omega) H_q(\omega) |\boldsymbol{\mu}| \end{aligned} \quad (\text{B.24})$$

where $|\boldsymbol{\mu}|$ is the $(k+1) \times (k+1)$ matrix as defined in (B.7). Similarly, we can prove that the numerator can also be written as

$$\mathbf{D}_{(k+1) \times (k+1)}^i(\omega) = \sum_{n=1}^N \dots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \dots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} H_n(\omega) \dots H_r(\omega) H_p(\omega) H_q(\omega) |\boldsymbol{\mu}_i| \quad (\text{B.25})$$

The control compensator can thus be expressed as

$$G_i(\omega) = \frac{\sum_{n=1}^N \cdots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \cdots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} H_n(\omega) \cdots H_r(\omega) H_p(\omega) H_q(\omega) |\boldsymbol{\mu}_i|}{\sum_{n=1}^N \cdots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \cdots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} H_n(\omega) \cdots H_r(\omega) H_p(\omega) H_q(\omega) |\boldsymbol{\mu}|} \quad (\text{B.26})$$

Multiplying both the numerator and denominator of equation (B.26) by the product of modal frequency response functions, $\prod_{n=1}^N H_n^{-1}(\omega)$, the above equation becomes

$$G_i(\omega) = \frac{\overbrace{\sum_{n=1}^N \cdots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \cdots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} |\boldsymbol{\mu}_i|}^{k+1 \text{ elements}} \prod_{\substack{s=1 \\ s \neq n, \dots, r, p, q}}^N (\omega_s^2 - \omega^2)}{\underbrace{\sum_{n=1}^N \cdots \sum_{r=1}^N \sum_{p=1}^N \sum_{q=1}^N \xi_{n1} \cdots \xi_{r,k-1} \xi_{pk} \xi_{q,k+1} |\boldsymbol{\mu}|}_{k+1 \text{ elements}} \prod_{\substack{r=1 \\ r \neq n, \dots, r, p, q}}^N (\omega_r^2 - \omega^2)} \quad (\text{B.27})$$

This is exactly the equation (B.6). Therefore it has been proved that equation (B.6) is true for $M = k + 1$. From the principle of finite induction, we can conclude that equation (B.6) is true for any given integer k . Equation (B.6) defines the control compensator for any number of control inputs and error sensor channels, as long as the number of control inputs and error sensor outputs is the same.

B.2 Proof of equation (2.44)

In this section it is proved that the first term of equation (2.43) in Section 2.3.3 vanishes for all frequencies, therefore equation (2.43) reduces to equation (2.44). Evaluating the control compensator $G_p(\omega)$ as defined in equations (2.36) and (2.39) at the resonant frequency ω_n and using equation (2.42), it is found that

$$G_p(\omega_n) = \frac{|\boldsymbol{\mu}_p|}{|\boldsymbol{\mu}|} = \frac{\kappa_p}{N - M} \sum_{l=1}^{N-M} D_{pln} \quad (\text{B.28})$$

We shall now prove, by using Cramer's rule, that the following identity holds true:

$$\nu_n = \sum_{p=1}^M \mu_{np} \frac{|\boldsymbol{\mu}_p|}{|\boldsymbol{\mu}|} \quad (\text{B.29})$$

To prove this, recalling the definition of $|\boldsymbol{\mu}_p|$ and $|\boldsymbol{\mu}|$ as described by equations (B.8) and (B.7), from Cramer's rule, we recognize that

$$x_p = \frac{|\boldsymbol{\mu}_p|}{|\boldsymbol{\mu}|} \quad p = 1, 2, \dots, M \quad (\text{B.30})$$

is the p^{th} solution to the following set of linear algebraic equations

$$\begin{aligned} \mu_{n1}x_1 + \mu_{n2}x_2 + \dots + \mu_{nM}x_M &= \nu_n \\ \mu_{m1}x_1 + \mu_{m2}x_2 + \dots + \mu_{mM}x_M &= \nu_m \\ \dots & \\ \mu_{p1}x_1 + \mu_{p2}x_2 + \dots + \mu_{pM}x_M &= \nu_p \end{aligned} \quad (\text{B.31})$$

Therefore equation (B.29) is just the first equation of equation (B.31).

Substituting equations (B.32) and (B.29) into equation (2.43), the first term of equation (2.43) becomes:

$$\sum_{n=1}^N \left(\nu_n - \sum_{p=1}^M \mu_{np} \frac{|\boldsymbol{\mu}_p|}{|\boldsymbol{\mu}|} \right) H_n(\omega) \phi_n F(\omega) = 0 \quad (\text{B.32})$$

Therefore we have proved that the first term of equation (2.43) vanishes for all frequencies.

Appendix C

Wavenumber transform solution for planar radiators

For two-dimensional planar structures, the far-field sound radiation can be solved by using the spatial Fourier transform (i.e., wavenumber transform) technique. Detailed derivation is beyond the scope of the current work and can be found in Junger and Feit's book [33].

C.1 Sound pressure field

For a two-dimensional surface described by rectangular coordinate system as shown in Figure C.1, the spatial Fourier transform and its inverse are defined as

$$F(k_x, k_y) = \iint_{-\infty}^{+\infty} f(x, y) e^{j(k_x x + k_y y)} dx dy \quad (\text{C.1})$$

$$f(x, y) = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} F(k_x, k_y) e^{-j(k_x x + k_y y)} dk_x dk_y \quad (\text{C.2})$$

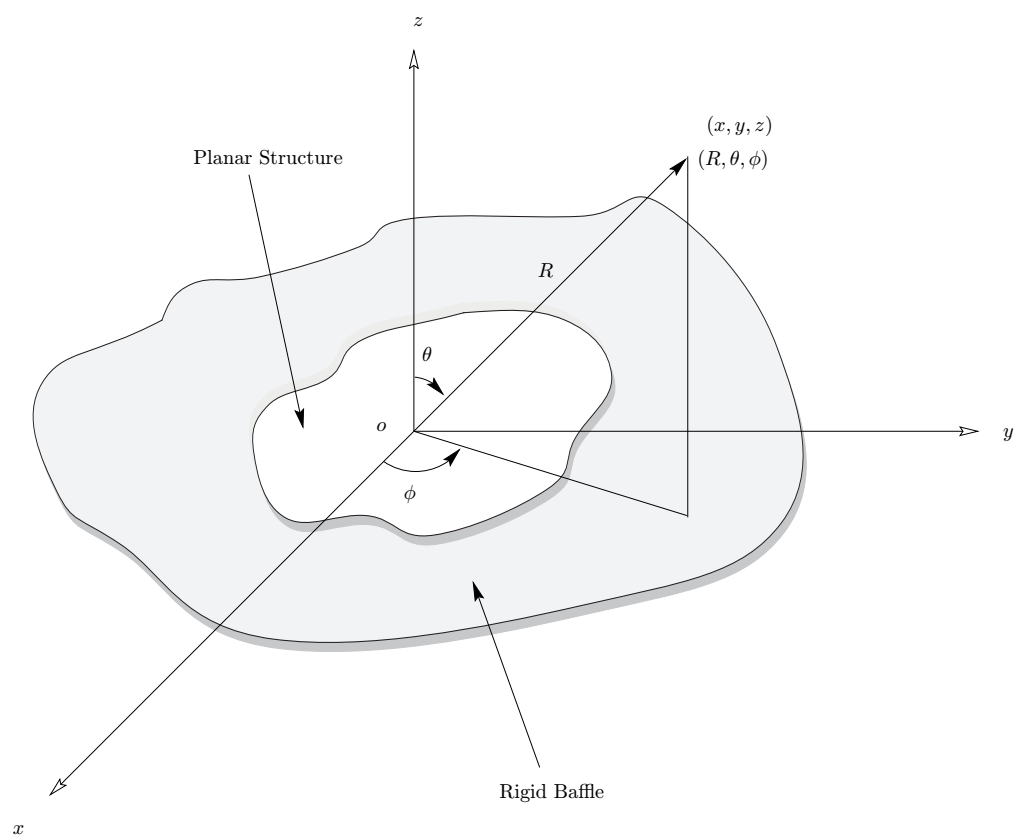


Figure C.1: Generic planar structure.

This is analogous to the usual Fourier transform from time to frequency domain. Here, the transform is from spatial to wavenumber domain. For planar radiators described by the Cartesian coordinate system as shown in Figure C.1, the Helmholtz equation describing a three dimensional sound pressure field is given as [33]:

$$(\nabla^2 + k_0^2)p(x, y, z) = 0 \quad (\text{C.3})$$

where $k_0 = \omega/c_0$ represents the acoustic wavenumber and ∇^2 is the Laplacian operator defined as

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \quad (\text{C.4})$$

At the surface of the planar radiator, the boundary condition must satisfy the linearized equation of conservation of momentum

$$j\omega\rho_0\dot{w}(x, y) + \left. \frac{\partial p(x, y, z)}{\partial z} \right|_{z=0} = 0 \quad (\text{C.5})$$

where $\dot{w}(x, y)$ is the velocity of the vibrating surface in the positive z -direction and ρ_0 is the density of the acoustic medium. Applying the wavenumber transform to the above Helmholtz equation, equation (C.3), we have

$$\iint_{-\infty}^{+\infty} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} + k_0^2 \right) p(x, y, z) e^{j(k_x x + k_y y)} dx dy = 0 \quad (\text{C.6})$$

Note that the z -derivative and the k_0^2 term in the above integrand are independent of x and y and can be taken outside the integral. The derivatives with respect to x and y integrate to $-k_x^2 P(k_x, k_y; z)$ and $-k_y^2 P(k_x, k_y; z)$, respectively. The above transformed Helmholtz equation reduces to

$$\left(k_0^2 - k_x^2 - k_y^2 + \frac{\partial^2}{\partial z^2} \right) P(k_x, k_y; z) = 0 \quad (\text{C.7})$$

The general solution to equation (C.7) can be written in the following form

$$P(k_x, k_y; z) = A \exp(j(k_0^2 - k_x^2 - k_y^2)^{1/2} z) \quad (\text{C.8})$$

Similarly, the transformed boundary conditions can be found as

$$j\omega\rho_0\dot{W}(k_x, k_y) + \left. \frac{\partial P(k_x, k_y; z)}{\partial z} \right|_{z=0} = 0 \quad (\text{C.9})$$

The unknown amplitude of the solution A can be found by substituting equation (C.8) into the transformed boundary condition, equation (C.9):

$$A = \frac{\rho_0\omega\dot{W}(k_x, k_y)}{(k_0^2 - k_x^2 - k_y^2)^{1/2}} \quad (\text{C.10})$$

Substitute equation (C.10) into equation (C.8), an explicit expression of the transformed sound pressure in terms of the transformed surface velocity can be expressed as

$$P(k_x, k_y; z) = \frac{\rho_0\omega\dot{W}(k_x, k_y)}{(k_0^2 - k_x^2 - k_y^2)^{1/2}} \exp(j(k_0^2 - k_x^2 - k_y^2)^{1/2} z) \quad (\text{C.11})$$

Sound pressure field produced by the vibrating planar structure can be found by applying the inverse wavenumber transform (i.e., equation (C.2)) to equation (C.11):

$$p(x, y, z) = \frac{\rho_0\omega}{(2\pi)^2} \iint_{-\infty}^{+\infty} \frac{\dot{W}(k_x, k_y) \exp(-j(k_x x + k_y y + k_z z))}{k_z} dk_x dk_y \quad (\text{C.12})$$

where the wavenumber k_z is given by

$$k_z = \sqrt{k_0^2 - k_x^2 - k_y^2} \quad (\text{C.13})$$

The solution to this integral is quite complicated and in general not tractable in the near-field. However, the integral can always be evaluated in the far-field (i.e., $k_0 R \gg 1$) by applying the stationary phase method. Details of this method will not be presented here,

interested reader can consult Junger and Feit's book [33]. Indeed, using the stationary phase method, the solution for the sound pressure in the far-field is given by

$$p(R, \theta, \phi) = \frac{j\rho_0\omega e^{jkR}}{2\pi R} \dot{W}(\bar{k}_x, \bar{k}_y) \quad (\text{C.14})$$

where $\bar{k}_x$ and $\bar{k}_y$ correspond to the wavenumber at the stationary point defined as

$$\begin{aligned} \bar{k}_x &= k_0 \sin \theta \cos \phi \\ \bar{k}_y &= k_0 \sin \theta \sin \phi \end{aligned} \quad (\text{C.15})$$

C.2 Radiated sound power

Sound power radiated by vibrating surface can be found by applying the Parseval's formula [40] in the wavenumber domain. The sound power radiated by a vibrating surface with normal velocity $\dot{w}(x, y)$ and surface acoustic pressure $p(x, y)$ is given by

$$\Pi(\omega) = \frac{1}{2} \mathcal{R}e \left[\iint_{-\infty}^{+\infty} p(x, y) \dot{w}^*(x, y) dx dy \right] \quad (\text{C.16})$$

where $()^*$ denotes the complex conjugate and $\mathcal{R}e$ stands for the real part of a complex variable. Substitution of the following Parseval's formula

$$\iint_{-\infty}^{+\infty} p(x, y) \dot{w}^*(x, y) dx dy = \frac{1}{(2\pi)^2} \iint_{-\infty}^{+\infty} P(k_x, k_y) \dot{W}^*(k_x, k_y) dk_x dk_y \quad (\text{C.17})$$

into equation (C.16) renders

$$\Pi(\omega) = \frac{1}{8\pi^2} \mathcal{R}e \left[\iint_{-\infty}^{+\infty} P(k_x, k_y) \dot{W}^*(k_x, k_y) dk_x dk_y \right] \quad (\text{C.18})$$

From equations (C.11) and (C.16), the radiated sound power by the vibrations of the surface can be further written as

$$\Pi(\omega) = \frac{\rho_0 \omega}{8\pi^2} \mathcal{R}e \left[\iint_{-\infty}^{+\infty} \frac{|\dot{W}(k_x, k_y)|^2}{\sqrt{k_0^2 - k_x^2 - k_y^2}} dk_x dk_y \right] \quad (\text{C.19})$$

From the above equation, it is obvious that only wavenumber components satisfying the condition

$$\sqrt{k_x^2 + k_y^2} \leq k_0 \quad (\text{C.20})$$

contribute to the real part of the integral and to sound power radiation. Equation (C.19) reduces to

$$\Pi(\omega) = \frac{\rho_0 \omega}{8\pi^2} \iint_{k_x^2 + k_y^2 \leq k_0^2} \frac{|\dot{W}(k_x, k_y)|^2}{\sqrt{k_0^2 - k_x^2 - k_y^2}} dk_x dk_y \quad (\text{C.21})$$

The significance of equation (C.20) is first discussed in some detail by Cremer and Heckel [51]. It has been fully discussed by Fuller, Elliott and Nelson [44]. Essentially equation (C.20) states that only supersonic wavenumber components (values of wavenumber satisfy equation (C.20)) radiate to the far-field, and the subsonic wavenumber components (values of wavenumber satisfy $k_0 > \sqrt{k_x^2 + k_y^2}$) are associated with decaying near-field waves only and do not contribute to the sound radiation in the far-field.

Before we can actually use equation (C.21) to calculate the sound power radiated by a structure, we need to deal with the singularity presented in this equation. We can use the coordinate transform technique to eliminate this singularity. From Figure C.2, Let

$$\begin{aligned} k_x &= k_0 \sin \theta \cos \phi \\ k_y &= k_0 \sin \theta \sin \phi \end{aligned} \quad (\text{C.22})$$

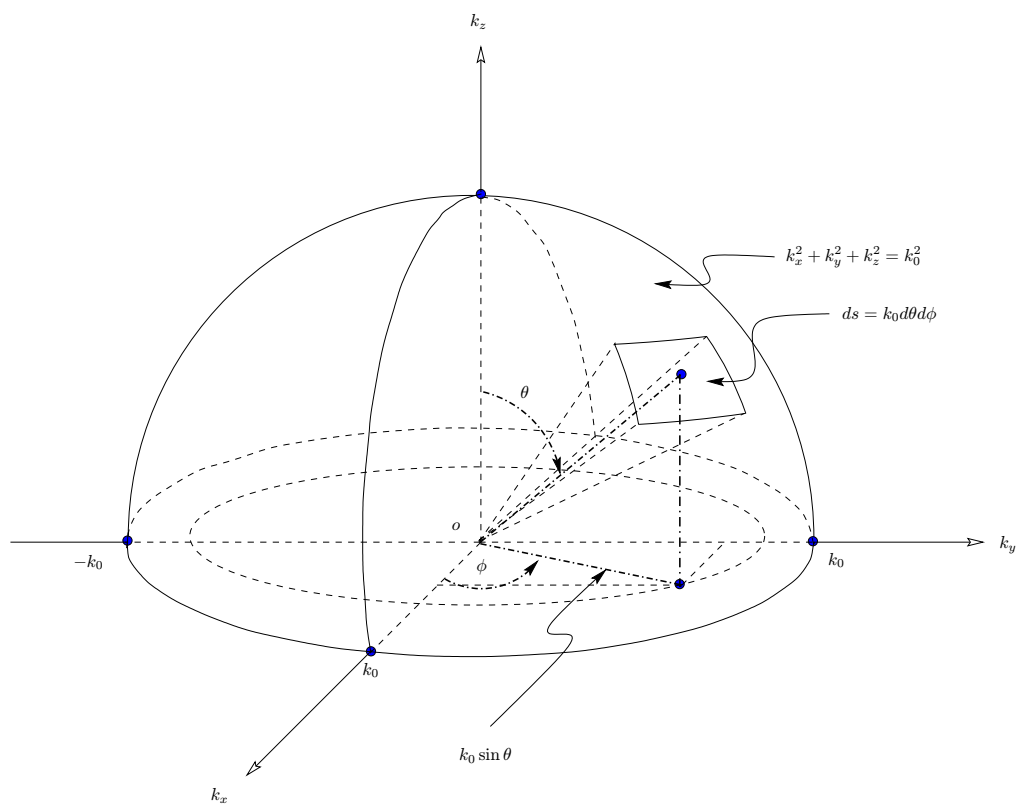


Figure C.2: Wavenumber domain

The differential areas $dk_x dk_y$ can be rewritten as

$$dk_x dk_y = \det(J) d\theta d\phi \quad (\text{C.23})$$

where $\det(J)$ is the Jacobian determinant defined as

$$\det(J) = \begin{vmatrix} \frac{\partial k_x}{\partial \theta} & \frac{\partial k_y}{\partial \theta} \\ \frac{\partial k_x}{\partial \phi} & \frac{\partial k_y}{\partial \phi} \end{vmatrix} = k_0^2 \sin \theta \cos \theta \quad (\text{C.24})$$

Applying this coordinate transform to equation (C.21) yields the sound power radiated by the vibrating surface expressed as

$$\Pi(\omega) = \frac{\rho_0 \omega}{8\pi^2} \int_0^{\pi/2} \int_0^{2\pi} |\dot{W}(k_0 \sin \theta \cos \phi, k_0 \sin \theta \sin \phi)|^2 k_0 \sin \theta d\theta d\phi \quad (\text{C.25})$$

For baffled planar structure, the out-of-plane velocity of the structure can be written as

$$\dot{w}(x, y) = j\omega \mathbf{q}^T \boldsymbol{\phi}(x, y) \quad (\text{C.26})$$

where $\mathbf{q} = [q_1, q_2, \dots, q_N]^T$ represents the generalized displacement coordinates vector and $\boldsymbol{\phi}(x, y) = [\phi_1(x, y), \phi_2(x, y), \dots, \phi_N(x, y)]^T$ represents the vector of mode shapes of the first N modes included in the analysis. Applying the wavenumber transform as defined in equation (C.1) to the above equation leads

$$\dot{W}(k_x, k_y) = j\omega \mathbf{q}^T \boldsymbol{\Phi}(k_x, k_y) \quad (\text{C.27})$$

where $\boldsymbol{\Phi}(k_x, k_y)$ represents vector of the wavenumber transformed mode shapes. Multiplying both sides of equation (C.27) by their Hermitians, we have

$$|\dot{W}|^2 = \omega^2 \mathbf{q}^T \boldsymbol{\Phi}(k_x, k_y) \boldsymbol{\Phi}^H(k_x, k_y) \mathbf{q}^* \quad (\text{C.28})$$

where $()^H$ and $()^*$ denote the Hermitian and complex conjugate operator, respectively. Sub-

stitute equation (C.28) into equation (C.25), the power radiated by the structure becomes

$$\Pi(\omega) = \mathbf{q}^T \mathbf{B} \mathbf{q}^* \quad (\text{C.29})$$

where the matrix $\mathbf{B}$ represents an $N \times N$ matrix of integrals of the form

$$\mathbf{B} = \frac{\rho_0 \omega^3}{8\pi^2} \int_0^{\pi/2} \int_0^{2\pi} \Phi(\bar{k}_x, \bar{k}_y) \Phi^H(\bar{k}_x, \bar{k}_y) k_0 \sin \theta d\theta d\phi \quad (\text{C.30})$$

where $\bar{k}_x, \bar{k}_y$ is given by equation (C.15). Examining equation (C.30) shows that $\mathbf{B} = \mathbf{B}^H$, i.e., the matrix $\mathbf{B}$ is a Hermitian matrix. Its diagonal terms represent the contributions to the radiated acoustic power made by each individual structural mode, and its off-diagonal terms (commonly referred to as modal coupling terms) represent the acoustic power due to the interference or coupling of two different structural modes.

Vita

Zhonglin Li was born on March 7, 1966 in Wulumuqi, People's Republic of China. He received his B.S from Tsinghua University in August, 1989, specialized in civil engineering. He moved to the United States and began his graduate studies at State University of New York at Stony Brook in August, 1991. He earned his M.S from the Mechanical Engineering Department of State University of New York at Stony Brook in August, 1993. He then began his Ph. D studies in the Mechanical Engineering Department of Virginia Polytechnic Institute and State University in September, 1993. With Professor Chris Fuller as his advisor, he completed his Doctoral degree in October, 1997.