A Multilevel Analysis of Student Engagement, Teacher Quality, and Math Achievement

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ABSTRACT

This study examined the relationships between math engagement, teacher quality, school factors, and math achievement in middle school students. This study used the Trends in International Mathematics and Science Study (TIMSS) data from the 2007 wave. The data were analyzed using exploratory factor analysis (EFA), confirmatory factor analysis (CFA), and hierarchical linear modeling (HLM). The results EFA and CFA showed that students' engagement in math classrooms consists of three dimensions: behavior, cognition, and emotion. The results provided evidence in supporting the multidimensional theory of student engagement, and provided a well-developed instrument that could measure students' math engagement. The findings of HLM analysis indicated that students' emotional engagement had a positive association with math achievement. In addition, teacher content knowledge displayed a positive effect on achievement, and teacher subject knowledge preparation and students' emotional engagement showed an interactional effect on achievement. What's more, school SES was a significant factor that influences math achievement. The findings suggested that students' math achievement was not only related to students' engagement, but also varied across class and school level factors. The study had both theoretical and practical significance, providing valuable insights for math education and math learning.

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GENERAL AUDIENCE ABSTRACT

Math achievement has been considered as a critical issue by policy makers, educators, and researchers. The math achievement of U.S. students lags behind their international peers. Moreover, there are significant and persistent math achievement gaps within U.S. Therefore, in order to improve U.S. students' math learning, it is important to figure out what factors are significantly related to math achievement and how these factors influence math achievement of different groups. Based on a nationally representative data set, this study examined the association of student math engagement, teacher quality, and school-related factors with student math achievement using the methods of Exploratory factor analysis (EFA), confirmatory factor analysis (CFA), and hierarchical linear modeling (HLM). It was found that students' math engagement consisted of three dimensions– behavioral engagement, cognitive engagement, and emotional engagement. In addition, students' emotional engagement, teacher's content knowledge, and school SES presented positive associations with students' math achievement. What's more, teacher subject knowledge preparation and students' emotional engagement showed an interactional effect on achievement. This study has implications for practice at individual, class, and school level. Enhancing engagement in math learning and increasing teacher quality for all students will have positive effect on math achievement for all students.

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Chapter 1 : Introduction

Background of the Study

The math achievement of America's middle and high school students is an issue of great concern to policy makers, educators, and researchers. Many believe that secondary math achievement is a significant predictor of a nation's long-term economic potential (e.g., Friedman, 2006). It was noted that math and science education are increasingly critical to the development of a nation (Technewsworld, 2006). "More investment in the science and math disciplines is necessary if the United States wants to remain competitive" (p. 1). Politicians and experts from industry agree that math and science education are foundation of a strong workforce. Further, math skills are assumed to enhance problem solving skills which are linked to innovation and technical advances in industry. Currently, individuals with abilities of identifying and analyzing patterns and using logic, along with critical thinking and problem solving skills are highly needed in industry. According to Porter (1998), if math and science levels are not maintained, the skill level in producing goods and services will diminish over time. Therefore, the U.S. may not be able to compete globally.

The math performance of U.S. students lags behind their international peers. Research results show that currently U.S. students' competence in basic subjects like math is still alarmingly low than that of other counties. According to the Program for International Student Assessment (PISA) study (Thomson, Cresswell, & De Bortoli, 2003), American 15-year-olds ranked 28th out of 40, behind western nations such as Australia, Canada, France, and Germany and far behind Asian nations such as Hong Kong, Korea, and Japan. In addition, according to the results of 2003 and 2007 Trends in

International Math and Science Study (TIMSS) math assessments, the scores of U.S. fourth graders and eighth graders were above international average, but they consistently lagged behind their counterparts in Asian countries such as Chinese Hong Kong, Japan, Korea, and Singapore (Mullis et al., 2004, 2008). In 2009, a little more than a third of fourth-graders in U.S. public schools were proficient in math. Students' performance gets worse as they enter secondary school: only 26 percent of U.S. high school students are proficient in math (NAEP Mathematics, 2009).

Beside the international comparisons, the problem of math performance within the United States is even more severe. There are significant differences between the achievement of White and middle-class students and that of minority and disadvantaged students, and the achievement gap is not diminishing. According to the National Assessment of Educational Progress (2007), 42% of White students scored proficient or better, compared to 11% of African American, 15% of Hispanic, and 16% of American Indian students. What's more, 42% of non-poor eighth graders scored proficient or higher compared to 15% of students who qualify for free lunch.

One problem resulting from the low math achievement is school dropout. In an examination of the standard school year, U.S. students attend school 40 weeks per year which is in line with most other industrialized nations (U.S. Department of Education, 2010). However, although 91% of all 15 year olds are enrolled in school, only 78% of all 17 year olds were still enrolled (Nationmaster, 2011). The high dropout rate not only influences the student leaving high school without a degree, but adds many problems to the society. According to Morial (2009), "The dropout rate is driving the nation's increasing prison population, and it's a drag on America's economic competitiveness".

Further, today mathematical knowledge is not only essential for school and workplace success, but also important for students to think critically and communicate using mathematical logic and ideas in their personal lives (Schoenfeld, 2002). Poor performance in math may lead to decreased opportunities in the fields of engineering, science, and technology. The lack of mathematics knowledge may further influence the tendency of a stratified society where those who perform well in middle and high school would generally succeed in college level and be easily employed after graduation, while students with negligible skills may only have access to the lowest paying jobs (Schoenfeld, 2002). In conclusion, improving students' math achievement has become necessary to deal with school dropout and to provide competitive workforce for the economic development of the nation.

On the basis of math education conditions, in the last two decades, several math curriculum reforms have been launched in the U.S. to enhance math teaching and learning. Early in 1989, the National Council of Teachers of Mathematics (NCTM) established the Commission on Standards for School Mathematics, stating that increased attention should be given to topics such as meanings of operations, use of calculators for complex computations, cooperative work, and use of manipulatives. The NCTM standards were adopted as national standards then. By 1997, most states adopted the NCTM standards as guidelines for their math state standard (National Council of Teachers of Mathematics, 2000). However, the math books published along with the NCTM standards failed to develop basic arithmetic and algebra skills (Klein, 2003). The results from the Third International Mathematics and Science Study (TIMSS) in 1995 showed a dismal performance of U.S. middle and high school students (Beaton, Mullis,

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Gonzales, Kelly, & Smith, 1996). One reason of this result is that the math curriculum was not sufficiently demanding as compared to other nations. Another reason is that math teaching relied on recalling information and was not directed toward an intellectual challenge such as reasoning and problem solving. In 2000, the NCTM published the Principles and Standards for School Mathematics (NCTM, 2000) as an update of the 1989 Standards. This document described in detail the standards and expectations for Pre-K12 grade levels. It listed the content standards for all students in terms of knowledge of problem solving, reasoning, reasoning and proof, communication, connections and representations.

Research studies on math education in high-performing countries have concluded that the math curriculum in the United States must be substantially focused and coherent in order to improve students' math achievement. In 2010, the National Governors Association and the Council of Chief State School Officers released Common Core State Standards (CCSS) for mathematics (Common Core State Standards Initiative, 2011). The CCSS provided all states with the standards of knowledge and skills students should have within K-12 education, so that students who graduate from high school would be fully prepared for college and careers. The middle school standards are regarded as a sound and complete preparation for high school mathematics. The high school standards need to pay more attention on application of mathematical ways of thinking and reasoning skills.

In addition to educational policies and curriculum reforms, considerable research has been conducted to investigate factors that influence students' math achievement. Earlier research mainly investigated the association of student characteristics to math achievement, such as student prior math achievement (Kiplinger, 2004; Scott, Rock,

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Pollack, & Ingels, 1995), coursetaking (Lee, Burkham, Smerdon, Chow-Hoy, & Geverdt, 1997; Owings, 2003), and students' demographic background including socio-economic status (SES), race, and gender (Berends, Lucas, Sullivan, & Briggs, 2005; Bussey & Bandura, 1999; Darling-Hammond, 1999; Finn, Gerber, & Wang, 2001; Jacobson, Olsen, King Rice, & Sweetland, 2001; Scott et al., 1995; Thomas & Stockton, 2003). Most of these studies found that student individual characteristics were significantly associated to math achievement. What's more, students' motivational and affective factors were also important factors that were associated to math achievement. Additionally, studies showed that students' family background and parental involvement had substantial association with students' math achievement (Hanushek, 1986; Mayer, 1997).

More recently, research on math achievement has increasingly showed the importance of teacher and school factors which are being considered as key elements to improve student achievement and educational quality (Darling-Hammond & Bransford, 2005; Ferguson, 1998; Goldhaber & Anthony, 2003; Hanushek, Kain, & Rivkin, 2002; 1998; Wright, Horn, & Sanders, 1997). Comparing to student demographic variables and family background variables, teacher and school level factors can be manipulated by interventions.

Among all the examined factors that influenced student achievement, student engagement has been recognized as a promising construct that explains students' learning process and school outcomes. Studies on examining the reasons of dropout pointed out that many youths reported a gradual psychological disengagement process from school-related activities (Entwisle, Alexander, & Olson, 2005). This process begins in early schooling and evolves over the years (Sameroff & Fiese, 1990). When students become

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alienated by school, they would withdraw and eventually dropout (Alexander, Entwisle, & Horsey, 1997). Adolescence is a period when engagement and academic motivation significantly decline for the majority of students (Eccles, Midgley, & Wigfield et al., 1993). Research has found that student motivation, self-concept, and positive attitudes toward school decrease dramatically during grades six and seven, especially during the transition into middle school (Berndt & Hawkins, 1988). Some researchers attributed that phenomenon to the uneven cognitive maturation during puberty. For instance, researchers have noted that student enjoyment of math declined during the transition into grade seven; while students' self-esteem was lowest in the first semester of grade seven (Eccles, Wigfield, Midgley, Reuman, MacIver, & Feldlaufer, 1993). Unlike factors such as gender or family background, student engagement is a dimension that can be modifiable and intervened by government and educators. Middle school years, especially the 8th grade, are a critical period in students' mathematics education. Students' mathematics achievement at this stage determines students' high school curricular choices and is an important indicator of students' high school enrollment and future success in college and workforce (Riley, 1997). Therefore, understanding how student engagement affects math achievement in middle school to promote school completion is highly important and is needed in the literature of math education.

In addition to the concept of student engagement, recently, teacher qualifications and teacher preparation issues have increasingly become important concerns of researchers and policy makers. In 2002, the No Child Left Behind (NCLB) Act was launched (U.S. Department of Education, 2002). The particular goal of NCLB was to improve student achievement and narrow the achievement gap. The NCLB led to higher

accountability and a high-stake testing system. Also, a great deal of responsibility for improving educational outcomes was placed on teachers (Goldhaber & Anthony, 2003). Many states started developing new educational policies, refining teacher education and teacher preparation programs, teacher evaluation systems, and teacher compensation systems. Schools have tried to implement programs and policies to enhance teacher quality in order to increase student achievement.

Teacher quality is regarded as a key element that influences student math achievement. Teachers who are well prepared with subject matter knowledge and pedagogical knowledge would lead to better math achievement of students (Strauss & Sawyer, 1986). What's more, since classroom is one of the most proximal contextual settings that influence students' behaviors and psychological factors, how teachers provide affordances and construct learning opportunities in the classroom affects student engagement. In this sense, teacher quality is an important aspect that needs to be examined to explain both student engagement and math achievement.

Statement of the Problem

Often low achievement in math during middle school years has been attributed to low engagement of students. The primary goal of this study is to examine 8th grade math achievement and how it is influenced by students' engagement in math learning, teaching effectiveness, and school level factors.

Among the ample studies on student's academic achievement, student engagement has been viewed as an increasingly important "meta" construct by researchers, educators, and policy makers (Finn, 1989; Fredricks, Blumenfeld, & Paris, 2004; Mosher & McGowan, 1985). Student engagement was primarily used as a

theoretical model to address the problem of school dropout and to promote school completion (Christenson, Reschly, Appleton, Berman, Spanjers, & Varro 2008), and it also provides a unique construct to study students' school outcomes, particularly how engagement relates to academic achievement. However, despite the current popularity of the concept, the conceptualization of student engagement among researchers is quite varied (Appleton, Christenson, Kim, & Reschly 2006; Fredricks et al., 2004; Finn, 1989; Skinner, Furrer, Marchand, & Kindermann, 2008). Although currently researchers have reached some agreement that engagement is a multidimensional construct, there is still debate on the types and numbers of dimensions and the measures of each dimension. Although literature is growing on academic engagement in general, studies on subject specific engagement are few. This study attempted to fill the gap in subject specific engagement in current knowledge.

Another limitation of current literature lies in that there's lack of measures of student engagement on specific subjects or contexts. For instance, little research has investigated math achievement with a measurement model of student engagement, which includes measures of students' activities and performance in math classroom. Therefore, developing a math engagement model would be important to deepen our understanding of engagement and its mechanism in improving math teaching, and to examine the consistency of student engagement concept across different contexts and subjects.

In addition, although most recent studies support a three-dimension model of student engagement in which behavior, cognition, and emotion are three subtypes that indicate this concept (Fredricks et al., 2004), research findings on the associations of each dimension to academic achievement are not consistent. For example, there has been

substantial evidence of the positive relationship between behavioral manifestation of student engagement and academic outcomes (Johnson, McGue, & Iacono, 2006; Marks, 2000), but studies that allow for investigation of the unique effect of emotional and cognitive engagement on academic outcomes are needed.

Beside student level variables, considerations of classroom and school level factors is critical in identifying different effects on achievement. In recent years, the teacher effectiveness has been considered as a key element in math achievement. For example, teacher qualification has been recognized as the most powerful school-related factor that influences students' academic achievement (National Academies, 2007). Some comparative studies showed that one of the main differences between U.S. and East Asian school systems is in teacher qualifications (Akiba, LeTendre, & Scribner, 2007; Mullis, Martin, Gonzalez, Gregory, Garden, O'Connor, Chrostowski, et al., 2000). Therefore, in addition to understanding the influence of student engagement on academic achievement, including teacher quality in models of math achievement is vitally important. However, teacher qualification is a very broad and complex concept. Literature on identifying significant indicators of teacher quality is not consistent. Additionally, research findings on the impact of teacher qualifications and practices on student achievement are mixed. Whether standard settings such as teacher certification, teacher experience and teacher education would improve teacher quality has been treated as a complicated issue that requires more research. Due to the lack of consensus on what factors contribute to a qualified teacher, developing measures of teacher quality has become a challenging task. Overall, consideration of student, teacher and school factors simultaneously is critical to understand the complete nature of math achievement. This

study is important because it examined both student variables and teacher and school variables simultaneously.

Purpose of the Study

The first goal of this study is to develop and examine a multi-dimensional measurement model of math engagement. It is hypothesized that student math engagement is indicated by three factors: behavior, cognition, and emotion. Further, the second purpose of this study is to examine the association of student engagement, teacher quality, and school-level factors with students' mathematic achievement. The research questions are:

1. What's the factor structure of math engagement?

2. Is there any relationship between math achievement and student engagement? Does this vary significantly across classes and schools? If there is significant variation, what class/teacher characteristics are associated with it?

3. At the school level, is there any relationship between teacher quality and math achievement? If there is any significant difference in the math achievement between teachers, what are the characteristics of teacher that can explain the variation?

4. Is there any significant difference in the math achievement between schools? what are the characteristics of school that can explain the variation among schools?

It is hypothesized that students' behavioral, cognitive, and emotional engagement in math class will have positive effects on math achievement; students' math engagement will differ significantly across classrooms and schools and it will be associated with teacher quality and school characteristics.

Overview of Methodology

Characteristics of the dataset

  The data set used for this study was the Treads in International Mathematics and Science Study (TIMSS: 2007). It provides data on the U.S. students' math and science achievement as well as data from other countries. The first round of TIMSS was conducted in 1995 based on a 4-year cycle, collecting data from students at 4th and 8th grade. TIMSS 2007 is the fourth study of math and science achievement conducted by the International Association for the Evaluation of Educational Achievement (IEA). Thirty-six countries participated at grade four and forty-eight countries participated at grade eight in the 2007 round. The TIMSS data of the United States was collected by the National Center for Education Statistics (NCES) and the National Science Foundation (NSF). It collected data on the math and science achievement of 4th and 8th grade students and detailed information at student, teacher, and school levels, including math and science curriculum, teacher preparation and instruction, school climate, and the use of technology.

Exploratory Factor Analysis (EFA)

  In this study, the exploratory factor analysis (EFA) was first employed to examine the factor structure of math engagement. Factor analysis is a statistical technique used to explore simple patterns in the relationships of relevant observed variables. EFA is used when the relationships between observed and latent variables are unobvious (Byrne, 1998), and when researchers are uncertain about the nature of the underlying factor structure of the measures. In this study, EFA is used to select items measuring behavioral, cognitive, and emotional engagement in math study, aiming to identify the underlying

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structure of the items and reduce the number of observed indicators of student engagement.

Confirmatory Factor Analysis (CFA)

In addition to EFA, the confirmatory factor analysis (CFA) was applied to specify alternative hypothesized models of behavioral engagement based on theory, and then to test how well the observed data fit the specified structure of behavioral engagement.

This study used LISREL 8.8 computer program (Jöreskog & Sörbom, 2006) to assess the hypothesized factor model of behavioral engagement. After selecting items that theoretically related to behavioral engagement, a measurement model was developed and examined to determine if the selected items had significant factor loadings on latent constructs. In addition to parameter estimates, measurement errors, and model modifications, the program also provides fit indices to assess how well the theoretical model fits the empirical data. Such fit indices make it possible to evaluate the adequacy of the model in explaining the data (Schumacker & Lomax, 2004).

Hierarchical Linear Modeling (HLM)

A second analytical approach used in this study is Hierarchical Linear Modeling (HLM). It is often a common phenomenon that data structures and organizations are grouped into hierarchical levels (Osborne, 2000). That means individual units are organized into larger groups; and groups of individuals are organized into higher order organizations; these organizations may be grouped into higher levels (Raudenbush & Bryk, 2002). For example, in education field, data are often organized at student, classroom, school, and school districts levels.

According to the nested structure in this study, a three-level hierarchical linear

model (Raudenbush & Bryk, 2002) was formulated to explain the class and school level variability, where the level-1 variables are dimensions of student math engagement; the level-2 variables are factors of teacher quality; and the level-3 units are school characteristics such as school climate and school SES. A hierarchical linear model allows simultaneous analysis of the multilevel components. It separately estimates the predictive coefficients for each level of the nested data. This method reduces measurement errors from ordinary least squares (OLS) regression models (Bryk & Raudenbush, 1988).

Significance of This Study

This study contributes to the literature in the following ways: First, measures and factor structures of student engagement in math are explored and examined, which provides empirical evidence to support the multidimensional theory of student engagement. And this confirmed model provides a subject-specific instrument of engagement in math learning. This instrument could be further specified and used by other researchers and teachers as an assessment of student engagement in math classrooms. For instance, with better understanding of students' math engagement, teachers are able to know which classroom activity is more efficient in improving math achievement, how students process math tasks using cognitive strategies, and what are students' emotional responses to math learning. So the increased understanding and knowledge could serve as references for designing appropriate classroom interventions.

Second, as it is acknowledged that teacher quality is the most important school variable affecting student achievement, policy makers and researchers have been grappling with how to measure teacher quality and how it can be used as an impetus for school reform, for bettering practice, and for enhancing student learning. Therefore, by

including indicators of teacher quality, this study provides evidence that measures of teachers' content knowledge are efficiently linked to teacher quality and it positively influences student math achievement. And teacher subject-matter knowledge displays a positive association with students' emotional engagement in math classrooms.

Third, research has noted that one reason for the achievement gap in different social groups and among students with different SES levels is the unequal learning opportunity. Teacher quality and school conditions have been recognized as main measures of learning opportunity. By including teacher quality and school factors such as school SES and school climate, this study provides a way to examine how students' variations in math achievement and engagement levels in math class are influenced by their unequal access to qualified teachers and better schools. This provides useful information for policy makers to attribute to educational resources and to narrow students' achievement gaps.

Fourth, in addition to the conceptual contribution described above, the main methodological strength is the use of multiple approaches to analyze the data such as exploratory factor analysis (EFA), confirmatory factor analysis (CFA), and hierarchical linear modeling (HLM). The use of EFA is efficient to explore the underlying factor structure of the measures. The confirmatory nature of CFA enables to postulate a priori, the interrelations of observed variables, relations between observed variables and latent constructs, and to test the plausibility of the specified hypotheses. Moreover, CFA provides the fit indices to assess how well the model fits the data. Such fit indices make it possible to evaluate the adequacy of the proposed model. Further, HLM allows variance in outcome variables to be analyzed at multiple hierarchical levels. The three-level

hierarchical linear model used in this study explains the schools and teachers' variability. Both methodologies in conjunction provide a deeper insight into the complexity of math achievement and provide a better understanding of why there is a large variability in math learning outcomes.

In conclusion, the results of this study are meaningful for providing sound educational policies that are likely to improve students' math achievement and school quality. The pedagogical and theoretical implications are useful in creating educational policies that can provide educational benefits to all students through more student engagement, teacher factors, and school practices.

Summary

This chapter provides context for the study and describes the importance of math achievement, current state of math performance among U.S. students, objectives and research questions of the study. It includes the significance and likely contributions of this study.

Chapter 2 : Literature Review

This chapter explores the theoretical literature and empirical research on student engagement, teacher quality, school climate and math achievement. The first section describes the importance of math achievement; the second section presents the definitions and measurement of student engagement and the effects of student engagement on math achievement. The perspectives of three-dimensional model (Fredricks et al., 2004) including behavioral, emotional, and cognitive engagement as well as measurement issues in student engagement are emphasized in this section. The third section introduces the definition and measurement of teacher quality and its effects on math achievement. In the last section, school SES and school climate are explained.

Explanation of the Search Process

In last decades, a substantial body of literature has emerged that examines the role of student engagement and teacher and school factors in learning. Broad searches of this literature on math achievement and student engagement, and teacher and school factors were carried out using several approaches. First, an initial search on the following databases was conducted: the Education Research Complete (EBSCOhost) database, the Education Resources Information Center (ERIC) database, Educational Abstract, PsycINFO, and WorldCat. The search process was aided by EndNote X.0. Google Scholar was also used for any published studies related to math achievement in secondary school. The utilized key words included "math achievement (performance)", "engagement", "teacher quality (effectiveness)", and "school climate".

After the initial search stage, the abstracts were downloaded from various databases. The abstracts were merged after deleting the duplicate entries. Based on the set

of abstracts, an EndNote library of those abstracts was created that described an empirical study about some aspect of influence in math achievement conducted between 1990 and current. Both qualitative and quantitative studies were included. The next step involved a comprehensive reference search based on the studies selected previously. In this step, specific studies and theories cited in these articles were searched. In addition to the review of the literature on math achievement, methodology articles and books on the subject of structural equation modeling and hierarchical linear model were also searched.

The following review of the current literature provides a conceptual framework for the study. It is organized under several sections on student engagement and teacher and school factors. It starts with the importance of math achievement and is followed by research and theory on student engagement, teacher experience and teacher quality, and school factors. It focuses on both theory and operationalization of the measures.

Importance of Mathematics Achievement

Middle school years, especially the 8th grade, are a critical period of mathematics education. Students' mathematics achievement at this stage determines students' high school curricular choices and is an important indicator of students' high school enrollment and future success in college and workforce (Riley, 1997). Today, mathematical knowledge is not only essential for school and workplace success, but also important for students to think critically and communicate using mathematical logic and ideas in their personal lives (Schoenfeld, 2002). Poor performance in math may lead to decreased opportunities in the fields of engineering, science, and technology. The lack of mathematics knowledge may further influence the tendency of a stratified society where those who perform well in middle and high school would generally succeed in college

level and be easily employed after graduation, while students with negligible skills may only have access to the lowest paying jobs (Schoenfeld, 2002).

One reason for low achievement in math could be a low engagement of students in math learning. When students are not engaged cognitively and affectively in teaching tasks, and are distracted and disengaged, they are unlikely to master the content of the mathematics. Students who are disengaged in schools may eventually drop out of the school and add many problems to the society.

Student Engagement

Definition of student engagement

Engagement is viewed as a multidimensional construct and it has several conceptualizations. Early conceptualizations of student engagement were rooted in literature on high school dropout prevention and high school completion. Engagement was defined as "students' psychological investment in and effort directed toward learning, understanding, or mastering the knowledge, skills, or crafts that academic work is intended to promote" (Newmann, 1992, p.12).

According to the Participation-Identification Model pointed out by Finn (1989), school dropout and completion result from long-term process of school disengagement or school engagement, which indicates that student engagement is comprised of both behavior and affect. The behavioral component is viewed as participation in classroom and extracurricular activities such as attendance, paying attention, homework completion, following school rules, and making effort to learn. The affective component refers to emotional responses to school such as interest, boredom, a sense of belonging, and valuing school.

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With the development of engagement theory and intervention within the dropout prevention literature, student engagement was gradually viewed as an important construct to explaining student achievement and behavior (Fredricks et al., 2004). Fredricks et al., (2004) extended previous literature and postulated a three-subtype typology that student engagement is made up of behavior, emotion, and cognition. Similar to Finn (1993, 1989), behavioral engagement in the three-dimensional model refers to participation in academic or social activities; emotional engagement indicates either positive or negative affect in student's interactions with peers, teachers, and the school. Cognitive engagement refers to students' self-regulation and their use of learning strategies (Fredricks et al., 2004). This highlights student's personal investment in learning that they make efforts for mastery and challenge (Connell & Wellborn, 1991; Newmann et al., 1992).

Fredricks et al. (2004) also argued that the three dimensions of student engagement are not isolated, but rather, they are dynamic and interrelated with each other. And the interactions across these three dimensions of engagement have long-term effects on students' academic achievement. Students with low levels of emotional identification with schools may be lacking of behavioral and cognitive involvement in learning, leading to unsuccessful school outcomes; while students who lack behavioral and cognitive engagement may show little emotional attachment to schools, which also results in low school achievement. The three aspects of engagement reciprocally influence each other as well as academic achievement in the long run.

Additionally, engagement was viewed as a mediator between context, individual, and outcomes (Appleton et al., 2006; Connell & Wellborn, 1991; Skinner, Furrer,

Marchand, & Kinderman, 2008). Besides its mediating role, given school completion and dropout are long-term outcomes of engagement and disengagement, student engagement can be treated as both process and outcome. For example, classroom attendance as an outcome at one-time point, may also be considered as an important indicator (process variable) for another outcome, such as grades or graduation.

Some researchers proposed a four-factor model in that they divided behavioral engagement into academic engagement and behavioral engagement (Appleton, Christenson, Kim, & Reschly, 2006). In their model, academic engagement refers to behaviors directly related to learning such as paying attention to, assignment completion, and classroom discussion activities; while behavioral engagement refers to behaviors related to classroom rules such as attendance, disruptive behaviors, and activity participation.

Although there's no consensus on the definition of student engagement, in order to enhance the clarity of this concept and to facilitate the ongoing study of engagement, Christenson, Reschly, and Wylie (2012) offered a specific definition of this construct:

> "Student engagement refers to the student's active participation in academic and co-curricular or school-related activities, and commitment to educational goals and learning. Engaged students find learning meaningful, and are invested in their learning and future. It is a multidimensional construct that consists of behavioral (including academic), cognitive, and affective subtypes. Student engagement drives learning; requires energy and effort; is affected by multiple contextual influences; and can be achieved for all learners." (p.816)

The present study adopted the three-subtype theory of student engagement from Fredericks et al. (2004) to explore the relationship of engagement to math learning. To understand and apply the construct of engagement to math learning, the following literature on the measurement of engagement is reviewed.

Measurement of student engagement

One important aspect of understanding the role of engagement in learning is to examine the measurement of student engagement and how it can be applied to measure math engagement. Engagement has been measured variously by different scholars and researchers. In recent studies, most researchers suggested that engagement be measured by three factors: behavior, cognition, and emotion. The following sections are detailed explanation of the three factors of engagement.

Behavioral engagement

Finn (1989) described four aspects of behavioral engagement (participation). The first level is basic learning behaviors, reflected by students' following classroom and school rules such as paying attention to the teacher (Finn & Rock, 1997; Johnson, Crosnoe, & Elder, 2001), answering teacher's questions (Newmann et al., 1992), and completing homework. The second level is initiative-taking behaviors such as doing beyond required work, trying alternative methods to study with classroom materials, and asking questions or communicating with teachers. The third level is reflected by students' participation in extracurricular activities in school. This type of participation may indicate no direct effects on school outcomes, but rather, increase students' psychological well-being and decrease risk behaviors (Gottfredson & Hirschi, 1990). The final level includes the social tasks in school, such as interacting appropriate by peers and teachers, not

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disrupting the class, and classroom attendance. The four aspects of behavior have been viewed as distinct in recent literature. Because this study aims to examine students' engagement in math learning and the direct effect of engagement on achievement, it is assumed that students' behavioral engagement is mostly reflected in their performance in math classroom. Therefore, in this study, the measurement of behavioral engagement focuses on students' participation in math class activities such as practicing math tasks, solving problems, explaining answers, and listening to the teachers' presentation.

Emotional engagement

The measurement of emotional engagement is highly related to its conceptualization. One indicator of this dimension is school belonging (Anderman & Anderman, 1999; Finn, 1989; Goodenow, 1993; Morrison, Cosden, O'Farrell, & Campos, 2003), or school attachment (Hoppe, Wells, Haggerty, Simpson, Gainey, & Catalano, 1998; Johnson et al., 2001). School belonging or attachment refers to student's perception of being accepted and respected in their school, and a network of relationships with peers and teachers in the school. Another similar indicator is identification with school (Finn, 1989; Voelkl, 1997), meaning student's belief that he or she belongs in school and how students value school and school-related outcomes. Most studies of emotional engagement adopt self-report scales to measure this concept. Students are asked to respond to questions regarding their affective reactions to school work, and/or the staff in school. Because this study mainly focuses on student engagement in math class, so in this study, emotional engagement is measured by students' affective reactions to math learning.

Cognitive engagement

Although the dimension of cognitive engagement is often mentioned, there is large variation in how it is defined and measured. Newmann et al. (1992) highlight that cognitive engagement is individual psychological investment in learning. So measures aligning with this aspect of engagement may ask about students' intrinsic motivation or about the degree to which they desire to go beyond the teachers' requirements.

Other researchers define cognitive engagement as a student's perception of school and educational outcomes. For example, in their study, Kenny, Blustein, and Haase, (2006) found that students who entered ninth grade with higher levels of career planfulness and more positive expectations regarding their career success might be more engaged in school over the course of the year. Corresponding measures may ask questions about students' thoughts on the meaningfulness of knowledge and skills learned in school and the significance of education for their future. The Student Engagement Instrument (SEI) developed by Appleton and Christenson (2004), is one of the instruments that includes questions related to cognitive engagement such as control and relevance of school work, future aspirations and goals, and commitment to learning.

A third conceptualization emphasizes cognition and self-regulation. Studies show that students who are cognitively engaged or self-regulated would use meta-cognitive strategies to plan or evaluate their performance in learning (Pintrich & De Groot, 1990; Zimmerman, 1990). In their study, Pintrich and De Groot (1990) found that seventh graders who used cognitive strategies such as rehearsal, elaboration, and organization were more cognitively engaged in trying to learn by organizing, memorizing, and transforming class material, and would have better academic achievement than students who tended not to use these cognitive strategies. Instruments related to this aspect assess

how students set goals, make plans, and self-regulate in the learning process. Additionally, students are asked to report their strategy use (Miller, Greene, Montalvo, Ravindran, & Nichols, 1996; Pintrich & De Groot, 1990). Overall, the measures of cognitive engagement include aspects of self-regulated learning, motivation, and strategy use in learning. In this study, the measures of cognitive engagement will include students' value of math for future education and occupational aspirations in math.

Measurement issues of engagement

In studies that focus on the measurement of engagement, there is lack of consensus in the definitions as well as the number of dimensions of this construct. In recent publications, although a majority of scholars endorsed the three-dimensional typology (Fredricks et al., 2004) that they view student engagement as multidimensional, including behavior, cognition, and emotion, there is little agreement on the definition of each dimension of engagement. For instance, Finn (2006) classified perceived relevance of school as emotional engagement, while some would categorize it as cognitive engagement (Appleton et al., 2006; Appleton, Christenson, & Furlong, 2008). Or some researchers viewed student-perceived relationships with peers and teachers and the sense of belonging as emotional engagement (Appleton et al., 2006), while some treated them as behavioral engagement (Yazzie-Mintz & McCormick, 2012). In addition, it is also difficult to make clear definitions when researchers attempt to label terms into one category. For example, there is no substantive evidence that suggest that terms such as liking for school and boredom belong to engagement construct (Fredricks et al., 2004). Therefore, it is necessary to clearly detail how engagement is operationalized in measurement across scholars.

Effects of engagement on math achievement

One purpose of this study is to investigate the effects of engagement on middle school students' math achievement. Although in recent years there have been many studies on examining the relationship between engagement and students' academic achievement, few of them specifically focused on math engagement and math achievement using multidimensional theory of engagement. Individual dimensions of student engagement or indicators of each dimension are identified as significant predictors for math achievement in some related research.

First, some studies mainly focused on the relationship between behavioral engagement and math achievement. For instance, in their study, Ripski and Gregory (2009) created a measure of behavioral engagement for eighth grade students using the Educational Longitudinal Study of 2002 (ELS: 2002). The measure was five combined scales from teacher ratings of students' behaviors including classroom attention, disruptive behaviors, and homework completion. The finding showed that there was significant positive correlation between engagement and mathematics test scores (r=0.39).

Second, studies of cognitive engagement and math achievement have produced inconsistent results. One reason for the inconsistency is due to the different methods of assessing students' internal processes of engagement. Generally, there are two types of assessments for cognitive engagement: direct assessment and indirect assessment. Direct assessments are to ask students to elaborate the processes they use in learning, while indirect assessments are to use students' self-responded or teacher-responded questionnaires. For Example, in their study, Peterson and colleagues (1984) used three

methods to collect data on fifth-grade students' cognitive engagement: stimulated recall as direct assessment, and video recording and student surveys as indirect assessment. They found that in terms of on-task behaviors, stimulated recall measures and the scales in cognitive questionnaire were more highly correlated with students' math achievement, which indicates that students' responses are more reliable and valid than observers' judgement of students' attention. Additionally, student reports of total number of specific cognitive strategies such as attention, understanding, and checking are positively related to math achievement. What's more, Miller and colleagues (1996) found that high school students' self-report of strategy use (e.g. self-regulation, effort, deep processing use) and goals for future consequences were significant positive predictors for math achievement: Effort ($\beta = .23$) and goals for future consequences ($\beta = .18$) explained the most variance in math achievement.

Like cognitive engagement, emotional engagement is often measured by indirect assessment. In their study, Sciarra and Seirup (2008) examined the effects of three dimensions of student engagement on math achievement among five racial groups. They found that instead of significantly predicting math achievement for all five racial groups like behavioral and cognitive engagement, emotional engagement only presented significant effects for White and Latino students. The authors suggested that this is probably because students' actual behaviors are more predictive of academic achievement than their feelings, which is consistent with other research (Booker, 2007, 2004; Trusty & Niles, 2003). Additionally, emotional engagement is associated with many motivational and behavioral outcomes (Osterman, 2000). Students who perceive high levels of belonging with school would also show high levels of motivation and effort

than those who perceive low levels of belonging (Goodenow & Grady, 1993). On the other hand, students who show low levels of belonging would display negative behaviors such as cheating (Voelkl & Frone, 2004), drug and alcohol use (Hawkins, Catalano, & Miller, 1992; Voelkl & Frone, 2000), and dropping out of school (Jessor, Turbin, & Costa, 1998; Rumberger & Lim, 2008). Therefore, some research suggests that this form of engagement exerts indirect effect on academic achievement (Woelkl, 2012). It serves as mediator between other forms of engagement (behavior, cognition) and learning (Osterman, 2000). Overall, few researchers to date have adopted a multidimensional model of student engagement to examine student academic achievement in a particular subject. Neither have they explored how the three dimensions differ in influencing student achievement. In the present study, we adopt a multidimensional theory of engagement and apply it to math learning. We also examine the effects of each dimension of engagement on math achievement.

Teacher Quality

Another critical construct that affects student learning is teacher quality. Teacher quality is widely recognized by policymakers, practitioners, and researchers as the most important school-related factor predicting student outcomes (Ferguson, 1998; Goldhaber, 2002; Hanushek, Kain, & Rivkin, 1999). Because the concept of teacher quality is very broad, the conceptualization and measurement of this construct varied among researchers. In the following sections, the recent changes in perceptions of teacher quality, how this construct is measured in different dimensions, and how it relates to student learning are explained.

Definition

The perceptions of teacher quality have been changing over time because of the changing interests and values of society. Current research on defining teacher quality is quite diverse and includes many perspectives and there has been little consensus on the conceptualization and implementation of this construct (Lewis, Parsad, Carey, Bartfai, & Farris, 1999). To some scholars, teacher quality can be identified and measured by specific criteria (Bennett, 2002; Hickok, 2002; Izumi & Evers, 2002). For instance, the No Child Left Behind (2002) proposed a specific definition of this construct that "a highly qualified teacher" should meet the following requirements: 1) a bachelor's degree 2) full state certification and 3) subject matter competency. However, some others argue that teacher quality may not derive from a set of criteria, but a multi-dimensional construct that consists of different forms of analysis and introspection (Clandinin, 1989; Lambert, Walker, Zimmerman, Cooper, Lambert, Gardner, & Szabo, 2002). According to literature, the various aspects that reflect teacher quality include teacher behaviors and personality traits, teacher preparation and teacher qualifications, teacher experience, and teacher professional development.

In fact, literature is quite mixed on identifying teacher quality and teacher qualifications. Teacher qualifications are mainly related to teacher's tangible and relatively easy to record competence such as teacher certificate and teacher education degree, while teacher quality includes teachers' subject matter expertise and pedagogical knowledge, which are less tangible and harder to measure. In this sense, teacher quality is more than teacher qualifications. In this study, indicators of teacher quality not only included teacher qualifications such as degree and experiences, but also teachers' pedagogical knowledge such as subject matter knowledge and content coverage.

The following section will provide a detailed description of research on how the different dimensions of teacher quality are measured.

Measurement of teacher quality

Teacher behaviors and personality traits

Although most of the recent research on teacher quality has focused on teacher qualifications or instructional practice, an interest on personal characteristics of teacher quality has developed in the last decade. Research has consistently highlighted a few personal characteristics related to teacher (Stronge, 2002). The measurement of teacher personality traits is mainly reflected in the aspects of caring, connecting with students, enthusiasm, and commitment to student learning. In the following section, each of these traits is elaborated.

Caring

Caring is defined as a sense of love and concern. The importance of caring have been suggested by several researchers when they describe effective teachers. Noddings (1992) noted caring as "a state of being in relation, characterized by receptivity, relatedness and engrossment". Stronge (2002) also pointed that caring teachers are those who know their students and tend to create a supportive and warm classroom environment, which may lead to more effective teaching. For example, in his study, Johnson (1997) interviewed several education stakeholders and asked them to share their perceptions of quality teaching. The interviewees reached on the consensus that quality teaching was a communicative process between teacher and students and the teacher should be well equipped with the knowledge of subject matter, caring, and the ability to maintain control.

Connecting with students

Connecting with students refers to establishing and maintaining relationships with students and knowing them personally. In the review of research on teacher characteristics, Stronge (2002) lists that teachers connect with students in various ways such as knowing students' personality and situations, being available for communication with students, listening to students' concerns and helping to solve their problems, and sharing teacher's personal experiences with students. In related studies on examining teacher characteristics affecting school achievement in urban environments, a positive teacher-student relationship was found to play a very important role (Baker, 1999; Peart & Cambell, 1999).

Respect for diversity and commitment to student learning

As the student population in America has become more culturally and ethically diverse, students' needs in learning process are also becoming more diverse (Howard, 2007). The requirement of teachers being respectful of diversity and having a commitment and belief for all students has become more important. Howard (2007) stated that teachers who have a respect for diversity and commitment to students are those who are able to build authentic and effective relationships across different groups. For instance, in their study, Kannapel and Clements (2005) found that "respectful relationships between students and adults" and a belief that all students can learn could narrow the gap between low- and middle- income students, between black and white students, and could increase students' achievement as a whole.

Enthusiasm in teaching

A teacher who is enthusiastic toward teaching could also influence their students'
learning. The study of this characteristic was reflected in research regarding student
motivation and achievement. In a mixed method study, researchers found that teachers
who had a greater intrinsic motivation regarding the class content would lead to a higher
degree of success of the students (Patrick, Hisley, & Kempler, 2000). Similarly, in a
qualitative study for high school students, it is found that enthusiastic teachers could
provide an exciting classroom environment for learning, thus motivating students to learn
more (Arnold, 2005).

Teacher professional development

Another important factor indicating teacher quality is teacher professional
development. There is a vast growing literature on the relationship of professional
development to teacher quality. According to the American Federation of Teachers
(AFT), teacher professional development is defined as

> "...continuous process of individual and collective examination and improvement
> of practice. It should empower individual educators and communities of educators
> to make complex decisions; to identify and solve problems; and to connect theory,
> practice, and student outcomes." (2001, p. 1)

The scope of professional development is very broad and can be represented in
many different aspects. In the review of teacher professional development, Resnick
(2005) noted that professional development can significantly affect teachers' classroom
practices and lead to improved student achievement when it focuses on the following
aspects: 1) the way that students learn particular subject matter; 2) instructional practices
that are related to the subject matter and how students understand it; 3) enhancing

teachers' subject-matter knowledge. Professional development provides a way to improve teachers' knowledge on both subject-matter content and pedagogical strategies, and it influences students' achievement indirectly. In their study, Carpenter and his colleagues randomly placed one group of first-grade teachers in a workshop that talked about research on how students understand simple math word problems such as addition and subtraction, and placed another group of teachers in professional development that focused on math problem-solving strategies but not on how students learn. The result showed that teachers who participated in the workshop would pose complex problems to students, listen to the processes students used to solve the problems, and encourage them to seek different methods to find answers. While teachers who participated in the professional development would emphasize basic fact recall, getting answers quickly, and allow students working alone rather than in groups. In addition, based on research, the Center for Teaching Quality (CTQ) reported that professional development was very important for high quality teaching (2007). However, in the survey collected by CTQ, only half of teachers responded that their professional development was meaningful and improved their teaching quality (2007). This result showed that there was a lack of teacher's own input and contributions when they were engaging in professional development. Further, when teacher's professional development focused on how students learn effectively, student achievement would be higher and students' basic and advanced reasoning and problem-solving skills would increase dramatically. This indicates that professional development can have a significant influence on student achievement when it emphasizes subject matter and student learning. Therefore, high quality professional development is needed, and "improving teachers' skills and knowledge is one of the most

important investments of time and money that local, state, and national leaders make in education" (Resnick, 2005, p. 1).

Teacher qualifications and preparation

Teacher qualifications and preparation are mainly about what teachers bring in before they enter the classroom. These two factors play a vital role in defining quality teacher. The measurement of teacher qualifications and preparation mainly includes: certification status, subject matter knowledge, instructional knowledge, teacher experience, and teacher education and preparation. This is also the focus of the present study in measuring teacher quality.

Certification status

Certification status is a major concern in discussing teacher qualifications. Due to the requirement in NCLB, certification status is treated as an indicator of teacher quality. However, certification issues are very complex, because each state may have different requirement for certification (Zumwalt & Craig, 2005). Teachers who are certified are viewed as to have met the minimum requirement set by a particular state, but it cannot reflect the prospective teacher's preparation (Zumwalt & Craig, 2005). In a research review, Wilson and Youngs (2005) examined eight studies on exploring the relationship between teacher certification and teacher quality. They found that seven of the studies were "in favor of certified teachers" (p.611). In addition, it was reported in several studies that unlicensed teacher were "less academically able than their licensed counterparts, achieving GPAs well below those of other college students, who in turn perform less well than teacher education graduates" (Darling-Hammond, 1999, p. 189). However, research findings on examining the influence of teacher certification are

conflicting. In one study of the National Center for Analysis of Longitudinal Data in

Education Research (CALDER), the researchers adopted similar quantitative methods

based on data from Florida and North Carolina. The results from North Carolina data

showed students in third, fourth, and fifth grades taught by nationally certified teachers

could learn significantly more in a school year than students who taught by uncertified

teachers (Viadero, 2007). While in the Florida study, it was found that nationally board

certified teachers were more effective than those who were not certified only in certain

grades or certain subjects (Viadero, 2007). This inconsistency may result from the

different demographics of teachers and different standards for being certified between the

two states (Viadero 2007).

Subject matter knowledge

In recent research, there has been lots of discussion on subject matter knowledge

when describing teacher quality. Because of the calling of No Child Left Behind

(NCLB), the U.S. department of Education suggests that to add the requirement of

subject matter knowledge to define highly qualified teachers is "the most dramatic policy

shift in No Child Left Behind" (U.S. Department of Education, 2002, p.4). Even though

few researchers have suggested that it is not important for a teacher to be equipped with

the content knowledge where they are teaching, results of research on studying the

association of teacher quality to subject matter knowledge were mixed (Darling-

Hammond, 1999). In their study, Floden and Meniketti (2005) examined the subject

matter coursework and teacher performance based on extensive review of survey data.

They found that there is a positive association between teachers' college study of math

and high school students' math learning, with the implication that teachers are obtaining

subject-related knowledge for teaching from those math courses. However, they didn't

find a clear link between subject matter knowledge and teacher quality. The current

greater effectiveness of teachers with more coursework may be due to other

characteristics of the teachers such as enjoyment of math, early experiences with math

teachers who taught them approaches to teaching. In these cases, formal teacher

preparation of subject-matter knowledge is not the source of teacher quality.

Additionally, it has been posited that few research has directly examined prospective

teachers' subject matter knowledge and the relationships between subject matter

preparation and students' learning (Wilson, Floden, & Ferrini-Mundy, 2001).

Pedagogical knowledge

Many researchers argue that there has been a lack of empirical research on

instructional knowledge as an important aspect of teacher quality (Cochran-Smith &

Fries, 2001; McDiarmid & Wilson, 1991; Wise, 2002). What's more, in their report,

Wilson, Floden, and Mundy (2002) mentioned that they found no research directly

examined what teachers learn in their instructional preparation and the association

between instructional knowledge and student learning or teacher behavior. However, a lot

of policy documents provide evidence that policymakers are aware that instructional

knowledge of math teaching is necessary for effective math teaching. For instance, the

U.S. National Council for Accreditation of Teacher Education (NCATE) Standards 8 for

Middle School Mathematics Teachers (2003) suggests that beginning math teachers

should have a deep understanding of how students learn math and specific pedagogical

knowledge of math. In addition, considerable research has noted that the knowledge U.S.

beginning math teachers bring to the classroom is insufficient for them to teach math

effectively. Researchers have conceptualized the pedagogical content knowledge (PCK) as a necessary aspect of teacher effectiveness (Marks, 1990; Shulman, 1986). For example, Marks (1990) pointed out that PCK for math teachers can be measured by four components: knowledge of student understanding, knowledge of subject matter for instructional purposes, knowledge of media for instruction, and knowledge of instructional processes. Research on pedagogical knowledge mainly focused on "value added by education coursework" (Wilson, Floden, & Mundy, 2002, p. 193). In a multiple regression study, researchers found that courses on mathematics education methods led to higher student gains than courses focusing on mathematics content only (Adams & Krockover, 1997). In another study, researchers found that education coursework could better predict teaching success than subject matter or GPA (Ferguson & Womack, 1993). It was found that 91% of graduates who participated teacher education preparation passed exams on content knowledge, while the pass rate of graduates who didn't receive education on pedagogy was 73% (ETS, 2004).

Teacher education and preparation

Teacher preparation and college requirements have direct links to pedagogical knowledge because how a teacher obtains knowledge about pedagogy is often associated with teacher preparation programs. The American Federation of Teachers (AFT) suggests that "the best way to bring an adequate supply of well-trained teachers into the classroom is not by avoiding collegiate teacher education, but rather by strengthening it" (AFT, 2001, p.1). The National Parent Teacher Association (NPTA) also addresses the importance of teacher preparation programs to initiate and strengthen teachers' professional performance including instruction, academic knowledge, teaching

techniques, critical thinking, and parent and community involvement and so on (NPTA, 2003). In her report, Darling-Hammond (1999) stated that teacher preparation had stronger correlation with student achievement than teacher salaries, or class size, and accounted for 40% to 60% of variance in achievement when taking students' demographics into account. Also, in their study, Monk and King (1994) conducted a multi-level analysis to examine the effect of teacher quality on secondary students' math achievement. They found that teachers' subject matter preparation in math and science presents a positive impact on student achievement in those subjects. Similarly, Goldhaber and Brewer (1997) assessed the impact of educational resources on student achievement using measures of teacher skill based on a national data set. The findings suggested that teachers' subject-specific training has a significant influence on students' math and science achievement.

Teaching experience

Teaching experience refers to the total number of years of teaching, and "is often used as a proxy for quality" (Zumwalt & Craig, 2005, p. 176). Teaching experience provides teachers with on-the-job learning opportunities and leads to teachers' better teaching practices (Rice, 2003). In addition, teacher experience is also associated with student test scores (Grissmer, Flanagan, Kavata & Williamson, 2000; Rice, 2003; Rivkin, Hanushek, & Kaine, 2002). For instance, two studies report that teachers with two years' teaching experiences don't have significant effects on student test scores (Hanushek, 1997; Rivkin et al., 2002). In another study, researchers collected data from 60 related studies and concluded that teaching experience was a significant indicator of educational outcomes such as student test scores (Greenwald, Hedges & Laine, 1996). Further

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research also investigated specific number of years of teaching experience that affect student achievement. In their study, Ferguson and Ladd (1996) examined the relationship of teacher experience and students' test scores using a state data set. The findings showed that teachers with five or more years' teaching experience had no significant effect on students' test scores comparing to those with less than five years' experience. While in another study, Grissmer and his colleagues (2000) examined state-level achievement scores on the National Assessment of Educational Progress (NAEP). One finding of their study was that states having a high proportion of teachers with more than two years of experience has a positive effect on student achievement. In her book, Rice (2003) examined the research evidence on indicators of teacher quality. She noted that the distribution of experienced teachers usually vary among schools. Schools with high student poverty or in urban locations would have more inexperienced teachers, which linked to lower student performance.

In conclusion, in the present study, three main measures of teacher quality will be used: teacher education, teacher experience, and teacher subject-matter preparation. These three measures belong to the aspects of teacher qualifications, preparation and teacher experience explained in the previous sections. First, the reason for using teacher experience as a measure is that although this factor has been shown as an important indicator of teacher quality in some studies, but research findings on the effect of teacher experience are mixed. There's a need to provide more evidence on the effect of teacher experience on student achievement. Second, a growing body of studies has treated teacher education, or teacher preparation as an important factor when examining the

effect of teacher quality on student achievement. So teacher education is a good measure to be included in this study.

Finally, teacher's preparation for subject-matter knowledge is another important indicator of teacher quality that predicts student achievement. In addition, according to the review of teachers' professional development and teachers' pedagogical knowledge, teachers' subject-matter knowledge is a main focus and reflection of professional development, and it also directly links to teacher's pedagogical knowledge. One main purpose of this study is to examine students' math achievement and its relationship to teacher preparation. Therefore, involving teacher preparation of subject-matter knowledge is necessary to explore how teacher preparation of math content knowledge contributes to math achievement and how it affects student engagement in math classroom.

Effects of teacher quality on math achievement

Many studies have been conducted to explore the effects of characteristics of teachers and teacher quality on student achievement. A meta-analysis found that there was a positive relationship between teacher quality and practices and student achievement (Hedges, Laine, & Greenwald, 1994). Previous studies have also suggested that the effect of teacher quality on student achievement is positive and real (Greenwald et al., 1996). According to Goe (2007), teacher quality is effective in identifying teachers who improve their students' scores, especially in mathematics. Several of these studies have confirmed that teacher certification, subject matter knowledge, instructional knowledge, and teaching experience have significant association with higher student

math achievement (Darling-Hammond & Youngs, 2002; Rice, 2003; Wayne & Youngs, 2003; Wilson, Floden, & Ferrini-Mundy, 2001, 2002).

Specifically, in terms of certification status, studies have found that students who were taught by teachers holding subject-specific certification would have higher academic achievement. In their study, Hawk, Coble, & Swanson (1985) collected data from 36 secondary teachers and 826 students based on a paired-comparison design. They found that students who were taught by teachers holding a mathematics certification would achieve higher scores in general mathematics and algebra than did students who were taught by teachers with other subjects' certifications. In another study, Darling-Hammond (2000) used the National Assessment of Educational Progress data and found that the percentage of teachers holding full certification and the percentage of teachers holding major in the subject area were significant predictors of student achievement in both mathematics and reading. However, Rowan, Correnti, & Miller (2002) reported that subject-specific certification had no significant effect on students' achievement growth in mathematics and reading in elementary school. This result may suggest that teacher certification status have impact in secondary schools but not in elementary schools (Rice, 2003). Second, in terms of subject matter knowledge and pedagogical knowledge, some researchers used the National Teachers Examination (NTE) to measure subject matter knowledge and explored its association with students' mathematics achievement. For instance, Strauss and Sawyer (1986) collected and analyzed district-based data and found that with every 1% increase in district average NTE scores, the rate of student failure on mathematics and reading in high school competency examinations would decline by 5%. On the other hand, some researchers found no or negative significant relationship

between NTE scores and student achievement in elementary school subjects (Sheehan &

Marcus, 1978) and in secondary school subjects (Summers & Wolfe, 1977). Furthermore,

in regards to teacher experience, many empirical studies have provided evidence that

there is a significant and positive relationship between total number of years of teaching

and student achievement (Greenwald et al., 1996; Rice, 2003). But the relationship is not

linear. The effectiveness of teaching experience in improving students' achievement

seems to increase in the first three years of teaching, while no major improvement in the

effectiveness occurs after three years of teaching (Boyd, Grossman, Lankford, Loeb, &

Wyckoff, 2006; Rivkin et al., 2005).

Therefore, based on literature, there's a lack of consistency in research findings

on the association of teacher quality and students' math achievement. More research is

needed to provide evidence that different dimensions of teacher quality have impact on

students' math achievement.

School SES, School Environment, and Student Achievement

Research shows that schools' socio-economic status (SES) and school

environment also effect student achievement and teacher quality. It was noted that

schools with high SES could provide more academic and more advanced courses, while

schools with low SES would provide more vocational courses (Lee & Smith, 2001).

Schools with high SES may attract high quality teachers that encourage all students to

learn regardless of their backgrounds (Wenglinsky, 2001).

In one study, Hanushek, Kain, and Rivkin (2002) investigated the contribution of

school racial composition to the racial and ethnic achievement gap using a state data set.

They found that the proportion of minority composition had no significant effect on white

and Hispanic students, while black students were negatively affected by the proportion of black students. However, this effect didn't appear to be driven by school quality differences and achievement differences of classmates, indicating that black students impose peer pressure on other black students not to achieve and that a higher proportion of black students may lead teachers to reduce their expectations for all blacks. These findings indicated that black students were more affected by the racial-ethnic backgrounds of the schools comparing with their white and Hispanic peers. In another study, researchers conducted a multilevel structural equation model (SEM) and they found that school-level factors such as school size, the proportion of students in poverty, and the racial/ethnic composition in high schools were important factors that influence student achievement (Everson & Millsap, 2004). Additionally, based on a multiple regression analysis, after controlling for students' race/ethnicity, gender, and English ability, Moscoso (2000) found that students whose schools had a high proportion of non-English speaking students or a high proportion of Hispanic students would achieve lower tests than peers in other schools.

Summary

This chapter presents the literature review on the main variables in the study. It provides a theoretical model for the study. It also describes the gaps in the literature and the inconsistent findings of the effects of student and teacher factors on math achievement, thus establishing a need for more empirical research in this area.

Chapter 3 : Methodology

This chapter provides an overview of the methodology of the study. The chapter addresses the following topics: Research questions, data source, sampling, weighting, research design, measures, and the data analysis procedures. Data analysis will include exploratory and confirmatory factor analysis and hierarchical linear modeling. The chapter is organized under the following sections.

Research Questions

The main foci of this study are to examine the three-dimensional model of student math engagement and to investigate what characteristics of students, teachers, and schools lead to a higher level math achievement. The purposes will be investigated through various research questions, as stated in Chapter One:

1. What is the factor structure of math engagement?

2. Is there any relationship between math achievement and student engagement? Does this significantly across classes and schools? If there is significant variation, what class/teacher characteristics are associated with it?

3. At the class level, is there any relationship between teacher quality and math achievement? If there is any significant difference in the math achievement between teachers, what are the characteristics of teacher that can explain the variation?

4. Is there any significant difference in the math achievement between schools? what are the characteristics of school that can explain the variation among schools?

Data

  The data set used in this study is the Treads in International Mathematics and Science Study (TIMSS) 2007. It provides data on the U.S. students' math and science achievement compared to that of other countries. The data was collected on a regular four-year cycle. The first round of TIMSS was conducted in 1995, collecting data from students at 4th and 8th grade, and the last round of data was collected in 2011. TIMSS 2007 is the fourth study of math and science achievement conducted by the International Association for the Evaluation of Educational Achievement (IEA). Fifty-nine countries and eight benchmarking participants were involved in the 2007 round. This database includes data from 433,785 students, 46,770 teachers, 14,753 schools, and the National Research Coordinators of each country. In the United States, TIMSS is sponsored by the Education's National Center for Education Statistics (NCES) and the National Science Foundation (NSF). It investigated the math and science achievement of 4th and 8th grade students as well as detailed information at student, teacher, and school levels, including math and science curriculum, teacher preparation and instruction, home contexts, school climate, and the use of technology. The United States TIMSS data is nationally representative and has a three-level nested data structure that links students to their teachers within a school. In other words, with this dataset, researchers are able to estimate student characteristics while exploring teacher effect and school effect at the same time.

Sampling

  Because the target population in TIMSS was students as well as their classes and schools, the hierarchical nature of the nested data required a sampling frame by stages (Joncas, 2008). Therefore, the TIMSS used a two-stage stratified cluster sample design.

The first stage involved school sampling that schools were sampled using the probability proportional-to-size (PPS) technique, which allows larger schools to have a greater chance of being selected for the sample. The second stage involved a sample of one or more than one classrooms from the target grade in sampled schools. Classrooms were selected based on equal probabilities. The full database included six aspects: 1) Students' responses to each of the mathematics and science assessment items administered in the study; 2) Student achievement scores in mathematics and science; 3) Students' responses to the student questionnaires; 4) Teachers' responses to the teacher questionnaires; 5) Principals' responses to the school questionnaires; 6) National Research Coordinators' responses to the curriculum questionnaires.

Because this study mainly focuses on U.S. middle school math education, the United States 8th grade students' questionnaire, math teacher questionnaire, and school principal questionnaire are used for the study. In addition, this study mainly examines the nested structure of the data that individual students are linked to their math teachers within a school. Students who had math test scores and who were linked to their math teachers within schools were initially selected as the samples for the current study. Initially selected samples for the study included 7381 8th grade students. The data file contained missing values. Therefore, the actual sample size dropped when students were linked to their math teachers and schools after removing missing cases for all the variables used in the analysis. Finally, a total of 4202 8th grade students, 359 math teachers, and 185 schools were actually analyzed for HLM analysis.

Weighting

In this study, the relative (or normalized) weights were used at each level in the analysis. First, the relative student weight was computed by dividing the total student weight (TOTWGT) by its mean. Likewise, the relative class weight was calculated by dividing the total class weight by its mean and the relative school weight was calculated by dividing the school weight (SCHWGT) by its mean. The relative student weight was used when the student-level data were aggregated by group mean to the class level; while the relative class weight was used when the class-level data were aggregated by group mean to the school level. The total student weight (TOTWGT) and the school weight (SCHWGT) were provided by the TIMSS 2007 data. The total class weight was obtained by multiplying the class weight (CLSWGT) and the school weight (SCHWGT) provided by the original data set.

Research Design

This study is a non-experimental quantitative research design. One type of non-experimental research refers to research that focuses on statistical relationships among variables but doesn't include the manipulation of an independent variable and random assignment of participants to conditions (Cook & Campbell, 1979). Correlational research is one of the basic forms of this kind of research. In correlational research, the researcher measures the two variables of interest with little or no attempt to control extraneous variables and then assesses the relationship between them. In addition, researchers rely on survey or observational data to gather information from a sample in order to generalize the results to a larger population drawn on the sample (Ferber, Sheatsley, Turner, & Waksberg, 1980). But the correlational research is seriously flawed

if researchers are interested in concluding that an observed relationship is a causal relationship.

However, over the past three decades, several methods of analysis for making causal inferences with observational data such as large-scale national datasets have been developed (e.g., Rubin, 1974, 1978; Rosenbaum, 1986). First, there are several advantages of using large-scale, nationally representative datasets. One advantage is that in contrast to randomized controlled experiments, large-scale national educational studies can be generalized to specific populations of students. Large-scale datasets thus provide rich descriptive information on students, teachers, and schools. In addition, such datasets make it possible to analyze achievement gains at both individual and group levels, and they can also be used to develop plausible hypotheses on the causes of differences in student achievement gains. What's more, when randomized controlled experiments are not feasible, large-scale nationally representative studies could provide the best source of data for educational policy decisions. Second, fixed effects models, instrumental variables, and regression discontinuity designs are several methods that can be used to approximate randomized controlled experiments that make causal inferences (Winship & Morgan, 1999).

This study is based on the analysis of a large-scale, nationally representative dataset. Fixed effects variable like school SES is also included in the HLM analysis to explore the effects of student, teacher, and school level factors on students' math achievement. With the use of such methods, tentative causal inferences can be made based on the observational data.

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Measures

     One of the main purposes of this study is to examine student math achievement using a hierarchical linear model. In the analysis, a three-level model is estimated. Reflective indicators of each variable in the three-level HLM were selected from the original TIMSS 2007 survey based on theory and operational definitions used in previous empirical studies. All the independent variables adopted in the three-level model are presented in Table 3.1. The descriptions of variable creation processes at each level are listed in the following sections.

     Because there were many items related to the constructs of student math engagement and school climate, the Exploratory Factor Analysis (EFA) was conducted to select good items that identify the underlying factors. The final selected items were presented in Appendix A.

Table 3.1 Summary of Independent Variables

Summary of Independent Variables

Level	Variable name	Short name	Description
Student	Behavioral Engagement	BEH	Mean of student behavioral engagement in math classrooms
	Emotional Engagement	EMO	Mean of student emotional engagement in math classrooms
	Cognitive Engagement	COG	Mean of student cognitive engagement in math classrooms
	Gender (female)	FEM	Female indicator of student's gender (0=Male, 1=Female)
	Parent Education	PARED	Mean of mother's and father's education level
	Race (Black)	BLACK	Black group of student's race (0=Non-Black, 1=Black)
	Race (Hispanic)	HISPA	Hispanic group of student's race (0=Non-Hispanic, 1=Hispanic)
	Race (Asian)	ASIAN	Asian group of student's race (0=Non-Asian, 1=Asian)

Teacher	Race (Other)	OTHER	Student who is native American, Pacific islander, or has two or more than two races (0=Non-Other, 1=Other)
	Teacher's Educational Level	EDU	Whether or not degree is Master's or beyond (0=No, 1=Yes)
	Teacher's Experiences	EXP	Years the teacher has been taught
	Teacher's Experiences Squared	EXP2SQ	Grand mean years of the teacher has been taught (Squared)
	Teacher's Subject Knowledge Preparation	PREP	Whether or not the teacher is well prepared for teaching math-related topics (0=No, 1=Yes)
	Teacher's Content Coverage	TOPIC	Total scores of topics addressed by the TIMSS math test taught in class
School	School Social Economic Status	SES	Percentage of students were eligible to receive free or reduced-price lunches at this school
	School Math Resource	RSOR	Mean of school math-related resources

Dependent variable

In this study, the dependent variable used is eighth-grade students' math achievement. This variable was measured by the National Math Rasch Score. It was computed by standardizing the math logits to create logit scores with a weighted mean of 150 and a weighted standard deviation of 10 within each country.

Independent variables

Student-level variables

Math behavioral engagement was indicated by 17 items in the questionnaire. As a result of the exploratory factor analysis, 16 items (see Appendix A) were selected and created as a composite to indicate the construct of behavioral engagement. For these

items, students were asked to answer questions of how often they participated in various activities in math class. All the items were 4-point likert scales where high score represented high level of behavioral engagement in math class and low score represented low level of behavioral engagement. All the items were reverse coded.

Math emotional engagement was measured by four items in the students' questionnaire. For each item, students were asked to report their level of agreement on a 4-point Likert scale (see Appendix A) that exhibited their attitudes toward math study. All the items were reverse coded so that high points reflect high levels of math engagement. In the HLM analysis, the average of the four items were calculated to represent the variable of math emotional engagement.

Math cognitive engagement consists of four items of 4-point likert scales (see Appendix A) that reflected students' expectations for their future math study and how they valued math study. Like emotional engagement, the four items were created as a composite by calculating their average to represent cognitive engagement variable in the HLM analysis.

In addition to the three main measures of student engagement, students' gender, race, and parents' education levels were selected as covariates to be included in the model. The research presented previously has shown that gender, race, and socioeconomic status (SES) were important factors related to students' math achievement. Including these covariates in the HLM model will help to provide more information about how gender, race, and SES affect student math achievement at individual, class, and school levels. Further, it can also provide more valid results of the main effects of student math engagement, teacher quality, and school-related factors on

student math achievement. In this study, the gender as female indicator (1=female, 0=male) was included as a covariate at the student level.

In terms of the race, four dummy groups were created that White student was the reference group (White = 0) and the Black, Hispanic, Asian, and Other (native American, Pacific islander, or student who has two or more than two races) groups were four indicators of students' race included in the model.

The parents' education level was selected to represent students' SES because there were no other appropriate variables indicating SES in the dataset. In fact, a SES variable is frequently created as a composite of parent education, parent occupational status, and family income. However, in the dataset, only parent education was available. The variable of parents' education level is an average of educational attainment of the mother and father. If one of them didn't report it, then information from one parent was used. The items in the questionnaire were questions asking how far in school did the student's father/mother go and they consisted of seven categories: 1) no elementary school, 2) finished elementary school, 3) finished senior high school, 4) finished high school, 5) some technical education after high school, 6) some college/university, and 7) bachelor degree. The composite variable used in the HLM model was treated as a continuous variable with high score representing high SES and low score representing low SES.

Teacher-level variables (Teacher quality)

Teacher education level was measured by the item that asked teachers to resport their highest level of formal education completed. This item originally consisted of six categories:1) Did not complete high school, 2) Completed high school, 3) Completed a

vocational/technical certificate after high school, 4) Completed an Associate's degree (AA) in a vocational/technical program, 5) Completed an academic Associate's or Bachelor's degree, and 6) Completed an academic Master's degree, post-graduate certificate program (e.g., teaching) or first professional degree (e.g., law, medicine, dentistry). After conducting a frequency analysis, it was found that almost all the teachers chose category five or category six. In order to make the result more interpretable, a dummy group (1= Completed an academic Master's degree, post-graduate certificate program or first professional degree) was created as a comparison of the reference group (0 = Completed an academic Associate's or Bachelor's degree).

Teacher experience was measured by the item that asked "by the end of this school year, how many years will you have been teaching altogether?" Teachers were asked to report their total teaching years. The item ranged from 1 to 45 with a normal distribution. As is shown in the literature review section, there was no consensus on the association of teacher experience to students' math achievement. Based on a regression examination with the data, it was found that there was a curvilinear relationship between teacher experience and student math achievement. Therefore, a squared teacher experience variable was created and was included in the HLM model to represent the curvilinear relationship.

Teacher subject knowledge preparation was indicated by 18 items (see Appendix A) in the questionnaire. The items were questions that asked teachers how well prepared they are to teach various math-related topics including number, algebra, geometry, and data and chance. All the questions were 3-point Likert Scales with 1=not well prepared, 2=somewhat prepared, and 3= very well prepared. By examining the frequencies of each

item, it was showed that most teachers chose "very well prepared" for all 18 items.

Therefore, in order to make the results more interpretable, two groups were created to

represent teacher subject-matter preparation. The dummy group (1 = very well prepared)

consisted of teachers who selected "very well prepared" for all the 18 items and the

reference group (0 = not very well prepared) consisted of teachers who selected other

choices for the items.

Teacher Content Coverage was a composite derived from the teachers'

questionnaire. This composite variable was measured by 39 items (see Appendix A) that

asked teachers to indicate whether and when students in the TIMSS class had been taught

each math-related topic. The original categories were: 1) Mostly taught before this year;

2) Mostly taught this year; and 3) Not yet taught or just introduced. In order to reflect the

amount of topics teacher covered in the current class, the items were reverse coded. The

composite variable of teacher knowledge coverage was computed as the total score of the

39 items and the scale of the composite ranged from 1 to 117.

School-level variables

School SES: This item was measured by school principal's response of

percentage of students that were eligible to receive free or reduced-price lunches through

the National School Lunch Program in their school. The item ranged from 0 to 100 that

high score reflected low school SES while low score reflected high school SES.

School math-related resources was a composite of the average of five items (see

Appendix A) in the school principals' questionnaire. Each of the five item was 4-point

likert scale asking school principals to choose how much is the school's capacity to

provide instruction in math affected by a shortage or inadequacy of any of the math-

related resources such as computer, calculators, and library materials relevant to mathematics instruction. The items were reverse coded so that low score would represent low math resources which high score would represent high math resources.

Statistical Method and Model Estimation

Descriptive statistics

Prior to implementing exploratory factor analysis (EFA), preliminary descriptive statistics such as mean, standard deviation, skewness and kurtosis were checked in order to ensure that the observed indicators were normally distributed and without excessive skewness and kurtosis. Additionally, the reliability coefficients of each latent construct: engagement, teacher quality, and school climate – as a whole and within each dimension of the constructs were estimated. Bivariate correlations among observed variables were examined in order to conduct a preliminary assessment of the correlation pattern. For example, correlation coefficients among indicator variables indicating common latent constructs were expected to be higher than the correlations among variables loading on different factors of math engagement.

Exploratory factor analysis (EFA)

Because there were many items related to student math engagement, a preliminary selection of items measuring behavioral, cognitive, and emotional engagement based on construct validity was carried out. Therefore, exploratory factor analysis (EFA) was used to select preliminary items and to identify the underlying structure of the items and reduce the number of observed indicators of student engagement. Based on EFA, items with high loadings and clustering in the same factors were selected for further analysis.

Factor analysis is a statistical technique used to discover simple patterns in the

relationships of observed variables. It aims to explain the relationships among many observed variables to determine the underlying structure of latent variables. There are two types of factor analysis approaches: exploratory factor analysis (EFA) and confirmatory factor analysis (CFA). EFA is used when the relationships between observed and latent variables are unobvious (Byrne, 1998), and when researchers are uncertain about the nature of the underlying factor structure of the measures. EFA is considered as one of the data reduction techniques and it answers the basic question that whether the covariance among observed variables could be indicated by a smaller number of factors (Fabrigar, Wegener, MacCallum, & Strahan, 1999). In EFA, it is assumed that the observed variables are correlated because they indicate one or more common latent constructs.

Confirmatory Factor Analysis (CFA)

In addition to EFA, confirmatory factor analysis is a statistical technique used to examine the factor structure of a set of observed variables. In EFA, researchers specify different alternative models, seeking to find a model that fits the data. In CFA, researchers seek to statistically test the significance of the hypothesized factor model, in other words, to test whether the sample data confirm the model (Schumacker & Lomax, 2010). Based on the purpose of this study, once the EFA is conducted to select the items of math behavioral engagement, the CFA was used to further examine the hypothesized factor model of behavioral engagement or specify alternative models of behavioral engagement.

To build confirmatory factor models, one should follow a logical sequence of given steps (Bollen & Long, 1993) using one or all three types of relationships

(association, direct, and indirect effects). The steps are: 1) model specification, 2) model identification, 3) model estimation, 4) model testing, and 5) model modification.

Model specification refers to developing a theoretical model using all the available related theory and information. In this step, the researcher determines which variables to be included in the specified model and specifies every relationship and parameter in the model. The measurement model specifies both observed variables and latent constructs (Mustafa, 1999) and identifies which observed variables define a latent construct.

Model identification is to ask whether a unique set of parameter estimates can be found based on sample covariance matrix S and population covariance matrix Σ in the theoretical model (Randall & Richard, 2010). Structural models may be just-identified, overidentified, or underidentified on the basis of the degree of freedom. The degree of freedom is represented by the difference between the total number of elements in variance covariance matrix and the number of parameters to be estimated. The model is just-identified when the degree of freedom equals zero; a negative degree of freedom indicates that the model is underidentified; a positive degree of freedom indicates that the model is over-identified. Only overidentified models can be further examined for model fits.

Once the model is specified and identified, the next step is to estimate model parameters. In this step, we aim to obtain estimates for each of the specified parameters such that the parameter values generate a matrix as close as possible to the sample covariance matrix of the observed variables. In the estimation procedure, there are three particular fitting functions that minimize the discrepancy between sample covariance

matrix and population covariance matrix: 1) unweighted or ordinary least squares (ULS or OLS), 2) generalized least squares (GLS), and 3) maximum likelihood (ML, Randall & Richard, 2010). There are other methods of estimation too. In this study, maximum likelihood will be used.

Once the parameter estimates are obtained for a specified model, one should examine how well the data fits the model. That is to say, to what extent is the theoretical model supported by the sample data? This step is achieved by comparing the hypothesized model covariance with the sample covariance matrix. The model is deemed fit when the predicted population covariance matrix is close to the sample covariance matrix. There are two ways to examine model fit. The first way is to consider the model fit criteria including a number of model fit indices. The specific model fit indices are explained in the following section (hypothesis testing). The second way is to check the individual parameters of the model. The model presents good fit when the parameter estimates and standard errors are significantly different from zero and the parameter estimates are consistent with relevant theory.

The last step is model modification. If the fit of the hypothesized model is not strong enough, then one could modify the model and subsequently assess the new modified model. In confirmatory analysis, the researcher establishes a hypothesized model based on theory, then collects data, tests the fit of the hypothesized model to the sample data, and make decision of whether to reject or keep the model. There's no need to make further model modifications. In alternative models, the researcher points out several alternative models, selects one model that better fits the sample data after analyzing the data. Finally, if the researcher rejects the model as a result of poor fit,

he/she proceeds to modify and re-estimate the model in an exploratory way (Byrne, 1998).

Hierarchical Linear Modeling (HLM)

In addition to EFA and CFA, another analytic methodology of this study is HLM. HLM is "a complex form of ordinary least squares (OLS) regression that is used to analyze variance in the outcome variables when the predictor variables are at varying hierarchical levels" (Woltman, Feldstain, MacKay, & Rocchi, 2012). Compared to most other statistical methodologies used in psychological research, HLM is relatively new and it is still in the process of development (Raudenbush & Bryk, 2002).

Prior to the development of HLM, researchers would use fixed parameter simple linear regression techniques to assess hierarchical data with the assumption that classroom effects and school effects are equal. However, this type of statistical analysis can be misleading when the effects vary among individuals and the contexts (Raudenbush & Bryk, 2002). Therefore, the advancement of HLM lies in that it allows for the estimation of covariance components for unbalanced data (Dempster, Rubin, & Tsutakawa, 1981). In other words, HLM accounts for the shared variance in hierarchical data and it can accurately estimate lower level slopes and their implementation in estimating higher-level variables. In addition, HLM provides an integrated approach to deal with problems such as aggregation bias in standard error estimates and erroneous probability values in hypothesis testing of school effects. The OLS regression in HLM breaks the total variance into the between- and within-school effects, and the between-school effect can be influenced by school-level variables. This study is in an effort to explain variations in student math achievement by decomposing observed relationships

into between- and within-school components.

In this study, a three-level HLM analysis is used, accounting for the variations in 8th graders math achievement that are associated with three levels: students, teachers, and schools. The variations in math achievement are from student math engagement, both between students within classes and within schools; teacher quality between classrooms within schools; and school characteristics across schools. When using the HLM analysis, part of the variability at each level can be explained by measured variables at each level. Indicators of student behavioral, cognitive, and emotional math engagement are treated as level-1 predictors that represent student effects on math achievement; teacher education, teacher experience, and teacher's subject-matter preparation are level-2 predictors that represent the teacher effects on student math achievement; school SES and school climate serve as level-3 indicators that represent the school effects on the outcome variable.

In the following HLM analysis, a three-level unconditional hierarchical linear model was first examined to estimate how much variance is attributed to the student level, teacher level, and school level. The unconditional model is the simplest model in HLM and it contains no predictor variables at any level (Raudenbush & Bryk, 2002). According to Lee (2000), if the account of variance that exists at a higher level of aggregation is more than 10% of the total variance in the outcome, then a HLM analysis is necessary to be processed. The following analyses use these guidelines.

The unconditional model

Student-level model

The level-1 model refers to math Achievement for each student as a function of the class mean plus fandom error (Raudenbush & Bryk, 2002):

$$Y_{ijk} = \pi_{0jk} + e_{ijk} \tag{1}$$

where

Y_{ijk} represents the math achievement of each student i in class j and school k.

π_{0jk} represents the mean math achievement of class j in school k.

e_{ijk} represents "student effect", that is, the random error of student i in class j and

school k. These effects are assumed normally distributed with a mean of 0 and variance

σ^2 .

i = 1, 2, 3,…, n_{jk} students in class j and school k.

j = 1, 2,…, J_k classes within school k.

k = 1,…, K schools.

Teacher-level model

In this level-2 model, π_{0jk} becomes mean math achievement of each classroom,

serving as an outcome and varying randomly around some school means:

$$\pi_{0jk} = \beta_{00k} + r_{0jk} \tag{2}$$

where

β_{00k} represents the mean math achievement in school k.

r_{0jk} represents the random classroom effect, that is the deviation of classroom

jk's mean math score from the school mean. These effects are assumed normally

distributed with a mean of 0 and variance τ_π. Within each school k, the variability among

classrooms is assumed the same.

School-level model

In the level-3 model, math school mean achievement varies randomly around a

grand mean for all schools.

$$\beta_{00k} = \gamma_{000} + u_{00k} \tag{3}$$

where

γ_{000} represents the grand mean math achievement for all schools.

u_{00k} represents the random school effect, the deviation of school k's mean from

the grand mean. These effects are assumed normally distributed with a mean of 0

and variance τ_β.

Placing Equations (2) and (3) into Equation (1) yields the following combined

model:

$$Y_{ijk} = \gamma_{000} + u_{00k} + r_{0jk} + e_{ijk} \tag{4}$$

where γ_{000} is the grand mean; and μ_{00k}, r_{0jk}, and e_{ijk} are random variations in school

mean math achievement. Equation (4), the combined unconditional model, allows for

estimating variations in math achievement at three levels. That is to say, the three-level

unconditional model constitutes three parts of variations: the variation between students

within class, the variation between class within a school, and the variation between

schools. If these random effects are significant, they suggest that a hierarchical linear

model of teacher- and school-level is necessary.

The conditional model

In the conditional model, predictors are sequentially added to the unconditional

model. Student, teacher, and school variables are used as predictors to examine variations

in math achievement.

In order to define the best-fitted models for the data, the conditional models in

this study were built by the following steps according to the instructions of model

specification of Raudenbush and Bryk (2002) and Singer and Willett's (2003):

1. Start small at level 1, adding one variable at a time. Add each predictor after all others are removed, to avoid collinearity.

2. Add level 2 predictors together and check p-values. Remove insignificant effects first from the slopes, and then the intercepts. The deviance difference test is used to evaluate the model changes.

3. Add level 3 predictors based on research hypotheses and check p-values.

These methods (EFA, CFA, and HLM) explained above will be used to answer the research questions in the study.

Summary

This chapter was organized in five sections, giving an overview of the methods of the study. The chapter covers three main aspects of methodology – research design, measurement, and data analysis.

Chapter 4 : Results

This chapter presents the results of the data analysis. This chapter begins with presenting the results of the Exploratory Factor Analysis (EFA) on a subset of the data. The Confirmatory Factor Analysis (CFA) was then provided to explain the dimensions of students' math engagement construct. Next, the results of descriptive statistics and correlation coefficients between the predictors and the dependent variable at each level are presented. Finally, Hierarchical Linear Models (HLM) are fitted to the data to explain the relationships between student-, class-, and school-level predictors and students' math achievement.

The results of the factor analysis are described first because it is necessary to create engagement variables, key independent variables in the present study. After engagement variables are created via factor analysis, descriptive analysis is conducted to understand distributions of variables used in the present study.

Exploratory Factor Analysis (EFA) and Confirmatory Factor Analysis (CFA)

Exploratory Factor Analysis (EFA) is a method that examines the number of underlying factors based on empirical analysis. In the EFA step in this study, principal components method with Varimax rotation was used to extract factors from a set of items. Scree plots and total explained variance (see Table 4.3) were adopted to determine the number of factors. As mentioned in Chapter 3, three sub-factors were considered for engagement and named as behavioral, cognitive, and emotional engagement respectively. As a test blue print, 17, 4, and 4 items were designated as indicators for each engagement sub-factor. At the first step, all the items of the three dimensions of engagement were included in the EFA to examine the factor loading patterns. As a result, the 25 items

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loaded on six factors. Seventeen items of behavioral engagement loaded on four factors, and four math cognitive engagement items and four emotional engagement items loaded on one factor respectively, indicating a good match among factors and items for cognitive and emotional engagements. Though four factors were extracted by this preliminary EFA, at least these seventeen items were not in either cognitive or emotional engagement factor groups. Then, in order to examine whether or not the unidimensionarity of each factor, separate EFA was conducted for cognitive engagement, emotional engagement, and behavioral engagement items. The Cronbach's Alpha of cognitive engagement and emotional engagement were .711 and .859 respectively, suggesting acceptable internal consistency reliability of the two factors. Table 4.4 and Table 4.5 present the factor loadings and the Cronbach's Alpha for cognitive engagement and emotional engagement.

Among 17 behavioral engagement items, one item ("I use calculator") was excluded based on the low values of loading compared to others' in the initial EFA of 17-item behavioral engagement. The rest 16 items loaded on four factors (see Table 4.6 for the loading patterns). However, according to the theoretical framework of engagement, behavioral engagement should be captured as one construct. With the EFA procedure, it was difficult to trim the items so that the rest of the items to indicate one factor of behavioral engagement. Therefore, the confirmatory factor analysis (CFA) was adopted to statistically test a hypothesized factor model. In the CFA step, a first-order one-factor model where math engagement was indicated by 16 items was specified. The results showed that the standardized factor loadings of 7 items were lower than .4, indicating poor factor loadings. Although the fit indices suggested that this model was acceptable and had relatively high values of global fit indices such as RMSEA, SRMR, GFI, CFI,

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and NFI (see Table 4.8 for the fit indices), the values of GFI, CFI, and NFI were lower

than the recommended level of 0.95. Therefore, in order to improve the model fits, those

7 items with relatively low factor loadings were deleted from the model, and then, a one-

factor nine-item model was refitted. The results showed that all the standardized factor

loadings were statistically significant and higher than .4 and the model fitted the data well

(see Table 4.7 for the factor loadings and Table 4.8 for the fit indices). The Cronbach's

Alpha of the total (or average) score of these 9 items was .783, suggesting acceptable

internal consistency reliability of the construct of behavioral engagement. Based on the

CFA results, a composite of math behavioral engagement was created by calculating the

mean of the 9 items.

Descriptive Statistics

Descriptive statistics for each variable used in the present study, including

minimum, maximum, mean, standard deviation, and correlations between independent

and the dependent variables are shown below. All the results of the descriptive statistics

were based on the relative sample weight so that they reflect the unbiased estimates of the

corresponding population values.

Table 4.1

Descriptive Statistics for Student-, Classroom-, and School-Level Variables

Level	Variables	Minimum	Maximum	Mean	SD
Student	Math Score (MATH)	103.38	194.14	150.73	9.85
	Behavioral Engagement (BEH)	1.00	4.00	2.95	0.49
	Emotional Engagement (EMO)	1.00	4.00	3.41	0.59
	Cognitive Engagement (COG)	1.00	4.00	2.72	0.75
	Gender (female) (FEM)	0.00	1.00	0.50	0.50
	Parent Education (PARED)	1.00	7.00	4.39	1.76
	Race (Black) (BLACK)	0.00	1.00	0.12	0.33
	Race (Hispanic) (HISPA)	0.00	1.00	1.22	0.41
	Race (Asian) (ASIAN)	0.00	1.00	0.03	0.16
	Race (Other) (OTHER)	0.00	1.00	0.06	0.24

Classroom	Teacher's Educational Level (EDU)	4.00	6.00	5.60	0.50
	Teacher's Experiences (EXP)	1.00	42.00	14.47	10.56
	Teacher's Experiences Squared (EXP2SQ)	1.00	1764.00	320.61	394.64
	Teacher's Subject Knowledge Preparation (PREP)	0.00	1.00	0.64	0.48
	Teacher's Content Coverage in Teaching (TOPIC)	36.00	117.00	89.43	12.96
	Class Mean Behavioral Engagement(GPMBEH)	1.00	3.54	2.14	0.25
	Class Mean Emotional Engagement(GPMEMO)	1.18	3.75	2.71	0.33
	Class Mean Cognitive Engagement(GPMCOG)	1.50	3.92	3.40	0.22
	Class Mean Gender (female)(GPMFEM)	0.00	1.00	0.50	0.17
	Class Mean Parent Education (GPMPERED)	1.63	6.67	4.43	0.99
	Class Mean Race (Black)(GPMBLACK)	0.00	1.00	0.12	0.22
	Class Mean Race (Hispanic)(GPMHISPA)	0.00	1.00	0.27	0.28
	Class Mean Race (Asian)(GPMASIAN)	0.00	0.61	0.02	0.07
	Class Mean Race (Other)(GPMOTHER)	0.00	0.75	0.62	0.09
School	School Social Economic Status (SES)	0.00	100.00	44.37	28.32
	School Math Resource (RSOR)	1.00	4.00	3.15	0.80
	School Mean Teacher's Education Level(GPMEDU)	4.00	6.00	5.55	0.48
	School Mean Teacher's Experiences (GPMEXP)	1.00	42.00	14.64	9.12
	School Mean Teacher's Experiences Squared (GPMEXP2SQ)	1.00	1764.00	327.66	350.23
	School Mean Teacher's Subject Knowledge Preparation (GPMPREP)	0.00	1.00	0.63	0.41
	School Mean Teacher's Content Coverage in Teaching (GPMTOPIC)	36.00	111.00	89.66	9.80

Note. Student-level variables, N=4,202. Classroom-level variables, J=359. School-level variables, K=185. All the variables are weighed by relative weights. EXP2= EXP-$\overline{EXP}$. EXP2SQ = $(EXP - \overline{EXP})^2$

As described in Chapter 3, relevant items were extracted and composites were

created by using the student, teacher, and school administrator questionnaires from the

TIMSS 2007 data. The average score of the relevant items for each composite variable

such as teacher's subject knowledge preparation, teacher's content coverage in teaching,

and school math resource were identified by the exploratory factor analysis (EFA) based on the principal component method followed by the Varimax rotation similar to constructing engagement variables. Table 4.1 provides descriptive statistics for the variables at each level. 4,202 eighth grade students, 359 math teachers, and 185 schools were analyzed for this study. About 49.8% of 4202 students (N=2091) were male and 50.2% students (N=2111) were female. Approximately 58.3% students (N=2449) were White (the reference group), 12.3% students (N=515) were Black (BLACK as the variable in Table 4.1), 20.8% students (N=873) were Hispanic (HISPA), 2.5% students (N=105) were Asian (ASIAN), and 6.2% were Native American or Pacific Islander (OTHER). Each type of students' engagement scores ranged from 1 to 4. The mean behavioral (BEH), emotional (EMO), and cognitive engagement (COG) scores were 2.95, 3.41, and 2.72 respectively, suggesting that students had relatively higher emotional engagement level and lower cognitive and behavioral engagement level. Students' math scores ranged from 103.38 to 194.14. The average math score was 150.73 with a standard deviation of 9.85.

Among the 359 8th grade math teachers, 59.3% held at least a Master's degree (EDU). Teachers' years of experience in teaching (EXP) ranged from 1 year to 42. The teachers' average years of teaching (EXP) was 14.47. An indicator of whether a teacher was well prepared in teaching math related topics such as "computing with fractions and decimals", "representing, comparing, ordering, and computing with integers", and "translation, reflection, and rotation" was constructed. The number of teachers who were well-prepared in teaching math-related topics (PREP) was 230 (64%), comparing with 129 teachers (36%) who were not well-prepared. By weighting the percentage changed

slightly. Teacher's content coverage variable was created by averaging the scores of 39 items that asked teachers to indicate whether and when students had been taught each math-related topics. The scores of teachers' content coverage in teaching math topics (TOPIC) ranged from 36 to 117 with a mean of 89.43.

School Socioeconomic Status (SES) captured by percent of free/reduced price lunch students within a school ranged from 0 to 100 with a mean of 44. 37 and a standard deviation of 28.32, meaning school SES differ widely among the 185 schools. Note that the higher value of the SES variable indicated lower school SES. The variable of school math resources was created by averaging the scores of five items that asked how much the school's capacity to provide instruction in math affected by a shortage or inadequacy of any of the math-related resources such as computer, calculators, and library materials. Scores of school math resources (RSOR) ranged from 1 to 4 and the average score was 3.15. Additionally, class and school level means were presented for the level 2 and level 3 variables derived from aggregating the corresponding lower level variables.

In order to obtain some insights on the relationship between the main predictors of all the three levels and students' math scores, the Pearson correlation matrix among the predictors at each level with the dependent variable was presented (see Table 4.2). It showed that all the main predictors at each level except behavioral engagement had a statistically significant relationship with the dependent variable, math score (MATH). Note that this correlation matrix was presented for the purpose of exploring potential significant associations among the variables. Since it ignored the nested structure of the data, the significance level of any pair of variables that involved higher level variables was exaggerated. Among the significant relationships, school SES showed a negative

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association to math score (-0.406**). This is because school SES was measured by the

percent of receiving free/reduced price lunch students in a school, which indicated that a

higher value suggested a lower school SES, thus the negative correlation indicated that

the higher the proportion of students who received the free/reduced price lunch, the lower

the math achievement was.

Table 4.2

Correlation *Matrix* between the Predictors and Math Achievement (weighted)

	MATH	BEH	COG	EMO	EDU	EXP	EXP2SQ	PREP	TOPIC	SES	RSOR
MATH	1										
BEH	.016	1									
COG	.055**	.271**	1								
EMO	.177**	.207**	.340**	1							
EDU	.046**	-.037**	-.056**	-.016	1						
EXP	.054**	-.060**	.002	.013	.247**	1					
EXP2SQ	.036**	-.057**	-.008	.006	.217**	.957**	1				
PREP	.066**	.013	.017	.020	.110**	.148**	.119**	1			
TOPIC	.331**	-.005	-.018	.076**	.115**	-.037*	-.058**	.056**	1		
SES	-.406**	-.002	.022	-.015	-.032*	-.026	-.028	.057**	-.276**	1	
RSOR	.034*	.021	.036**	.068**	-.100**	.096**	.090**	.019	-.035*	-.037*	1

**. Correlation is significant at the 0.01 level (2-tailed). *. Correlation is significant at the 0.05 level (2-tailed).

Table 4.3

Results of Exploratory Factor Analyses for Scales

Scale	Number of Items	Factors Extracted	Explained Variance	Total Explained Variance
Behavioral Engagement	16	4	25.10%	47.68%
			8.73%	
			7.44%	
			6.42%	
Cognitive Engagement	4	1		57.87%

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Emotional Engagement	4	1	54.93%

Table 4.4

Factor Loadings and Cronbach's Alpha for the Cognitive Engagement Items

Factor/Items	EFA factor Loadings	α
Cognitive Engagement		.711
I need to do well in mathematics to get the job I want.	.762	
I need to do well in mathematics to get into the university or college of my choice.	.739	
I need mathematics to learn other school subjects.	.681	
I think learning mathematics will help me in my daily life.	.666	

Table 4.5

Factor Loadings and Cronbach's Alpha for the Emotional Engagement Items

Factor/Items	EFA factor Loadings	α
Emotional Engagement		.859
I like mathematics.	.878	
I enjoy learning mathematics.	.870	
Mathematics is boring.	.798	
I would like to take more mathematics in school.	.725	

Table 4.6

Factor Loadings and Cronbach's Alpha for the Behavioral Engagement Items

Factors/Items	Factor Loadings	α
Behavioral Engagement		.703
1. Work on fractions and decimals.	.764	
2. Interpret data in tables, charts, or graphs.	.651	
3. Solve problems about geometric shapes, lines and angles	.638	

4. Practice adding, subtracting, multiplying, and dividing without using a calculator	.624		
5. Write equations and functions to represent relationships	.568		
6. Explain our answers		.632	
7. Review our homework		.593	
8. Listen to the teacher give a lecture-style presentation		.560	
9. Memorize formulas and procedures		.466	
10. Use computers			.686
11. Work together in small groups			.674
12. Decide on our own procedures for solving complex problems			.519
13. Relate what we are learning in mathematics to our daily lives			.472
14. Begin our homework in class			.739
15. Have a quiz or test			.563
16. Work problems on our own			.530

Table 4.7

Standardized Factor Loadings for the One-factor Nine-item Model of Math Behavioral

Engagement

Items	Factor Loadings
1. Work on fractions and decimals	.49
2. Interpret data in tables, charts, or graphs	.64
3. Solve problems about geometric shapes, lines and angles	.56
4. Write equations and functions to represent relationships	.62
5. Practice adding, subtracting, multiplying, and dividing without using a calculator	.44
6. Memorize formulas and procedures	.58
7. Decide on our own procedures for solving complex problems	.60
8. Relate what we are learning in mathematics to our daily lives	.61
9. Explain our answers	.46

Table 4.8

Global Fit Indices for the two Models for Behavioral Engagement by CFA

Index	One-factor Sixteen-item Model	One-factor Nine-item Model
Chi-square	2470.92	617.63
Degree of Freedom	103	26
Significance	.00	.00
Root Mean Square Error of Approximation (RMSEA)	.074	.074
Standardized Root Mean Square Residual (SRMR)	.052	.044
Goodness of Fit Index (GFI)	.93	.97
Comparative Fit Index (CFI)	.91	.96
Normed Fit Index (NFI)	.90	.96

Hierarchical Linear Modeling

Model 1: Unconditional model

The data analyses of this study began with a fully unconditional model (Model 1) in which there were no predictor variables at each level. The outcome variable, math achievement, was free to vary across student-, class-, and school-level. The unconditional model is the simplest three-level hierarchical linear model. It provided the extent of variation within classrooms, between classrooms, and between schools in students' math achievement. The equations of the unconditional model are described below:

Equation 1. Equations for Unconditional Model (Model 1)

Level-1 Model:
$$\gamma_{ijk} = \pi_{0jk} + e_{ijk}, \, e_{ijk} \sim N(0, \sigma^2),$$

Level-2 Model:
$$\pi_{0jk} = \beta_{00k} + r_{0jk}, \, r_{0jk} \sim N(0, \tau_\pi),$$

Level-3 Model:
$$\beta_{00k} = \gamma_{000} + u_{00k}, \, u_{00k} \sim N(0, \tau_\beta).$$

In the unconditional model, γ_{ijk} represents the math achievement of 8th grade student i in class j and school k in 2007. The intercept π_{0jk} represents the class mean math achievement of class j in school k. The level-1 random error of student i in j class and school k is represented by e_{ijk}. It is assumed that e_{ijk} is independent and identically distributed (i.i.d) and has a normally distribution with a mean of 0 and a variance σ^2, which was denoted $e_{ijk} \sim N(0, \sigma^2)$. At the second level, the intercept β_{00k} represents the mean math achievement in school k. The term r_{0jk}, called the level-2 random error, represents the deviation of class j within school k from the school mean β_{00k}. It is assumed that $r_{0jk} \sim N(0, \tau_\pi)$, where τ_π represents the variance of the class mean math achievement, i.e., $\text{Var}(\pi_{0jk}) = \tau_\pi$. At the third level, γ_{000} represents the grand mean math achievement for all schools. The term u_{00k} represents the random school effect, the deviation of school k's mean from the grand mean. It is assumed that $u_{00k} \sim N(0, \tau_\beta)$, where τ_β is the variance of the school mean math achievement, i.e., $\text{Var}(\beta_{00k}) = \tau_\beta$. It should be noted that the sample relative weights were applied to all the HLM analysis presented in this study.

The first column in Table 4.8 shows the results of the unconditional model. The results of the unconditional model indicated that the estimated grand mean of math achievement was $\gamma_{000}=150.44$ ($t=371.29$, $p <0.001$). In terms of variance partitioning, the estimates of the variance components at student level was $\sigma^2=57.90$. There were statistically significant variations in mean math achievement across classrooms ($\hat{\tau}_\pi=27.16$, $\chi^2=1112.50$, $p<0.001$) and across schools ($\hat{\tau}_\beta=12.72$, $\chi^2=323.35$, $p<0.001$).

The amount of variance explained by each level is presented by the variance component parameter estimates from the unconditional model (Model 1). The equation to calculate the proportion of variance between student is:

$$\sigma^2/(\sigma^2 + \tau_\pi + \tau_\beta),$$

The equation to estimate the proportion of variance between classrooms is:

$$\tau_\pi/(\sigma^2 + \tau_\pi + \tau_\beta), \text{ and}$$

The proportion of variance among schools can be estimated by the following equation:

$$\tau_\beta/(\sigma^2 + \tau_\pi + \tau_\beta).$$

According to the equations above, the estimates of the percentage of variance in math achievement at each level showed that the proportion of variance between students within classrooms in students' math achievement was 59.2% ($\hat{\sigma}^2/(\hat{\sigma}^2 + \hat{\tau}_\pi + \hat{\tau}_\beta)$ =57.90/(57.90+27.16+12.72)=57.90/97.78≈ 0.59215); the proportion of variance between classrooms within schools in students' math achievement was 27.78% ($\hat{\tau}_\pi/(\hat{\sigma}^2 + \hat{\tau}_\pi + \hat{\tau}_\beta)$=27.16/97.78≈0.2778); and the proportion of variance between schools in students' math achievement was 13.01% ($\hat{\tau}_\beta/(\hat{\sigma}^2 + \hat{\tau}_\pi + \hat{\tau}_\beta)$=12.72/97.78≈ 0.1301). The total variance of students' math achievement is 97.78, the sum of the three variance components. Since this is a three-level model, several minds of intraclass correlation coefficients can be considered. Intraclass correlation is a correlation coefficient of the units in the same contexts, representing a measure of similarity (or likeness), i.e., the size of clustering effect. For example, the likeness of students in the same classes and the same schools is 0.41 ($\hat{corr}(\gamma_{ijk}, \gamma_{ij'k})$= ($\hat{\tau}_\pi + \hat{\tau}_\beta)/(\hat{\sigma}^2 + \hat{\tau}_\pi +$

$\hat{\tau}_\beta) = 39.88/97.78 \approx 0.41, i' \neq j$), while the likeness of students in the same schools is

0.13 $(co\hat{r}r(\gamma_{ijk}, \gamma_{i'j'k}) = \hat{\tau}_\beta/(\hat{\sigma}^2 + \hat{\tau}_\pi + \hat{\tau}_\beta) = 12.72/97.78 \approx 0.13, i' \neq i, j' \neq j)$. In

addition, the likeness of classes in the same schools in terms of true mean math

achievement is 0.32 $(co\hat{r}r(\pi_{0jk}, \pi_{0j'k}) = \hat{\tau}_\beta/(\hat{\tau}_\pi + \hat{\tau}_\beta) = 12.72/39.88 \approx 0.32, j' \neq$

$j)$. The proportions of variance at both the class and school levels and all of the intraclass

correlations were considerably high, suggesting that the effects of classroom-level and

school-level factors were important in terms of improving student math achievement and

narrowing the achievement gap. Especially, it is interesting to know that between-

classrooms variance was more than twice larger than the between-school variance.

The following models built on the unconditional model by adding predictors at each of

the three levels. The predictor addition started at the student level and ended up with a

model that includes level-3 predictors. At each level, all the main variables from the level

were added at one time. Variables were dropped if they were found not to be statistically

significant by deviance test. It should be noticed that the four dummy variables of race

were considered as one set. That is, when conducting deviance test, the four variables

were examined together as one variable. All the level-1 and level-2 variables were group-

mean centered. One advantage of using group-mean centering is to ensure that the

regression coefficients of the predictors at the level of interest explain the unbiased

relationship between the predictors and the dependent variable. In addition, when cross-

level interactions can affect the instability of model estimates, group-mean centering is

advantageous because it removes correlations between variables across levels (Kreft & de

Leeuw, 1998), resulting in an intercept for each unit that is unadjusted for level 1 or level

2 predictors. Another advantage is that when accounting for variance, predictors with

group-mean centering entered will not affect the level and variability in the intercept across higher groupings, so that the variance accounted for at each subsequent level is only affected by predictors added at that level. In addition, all the level-3 predictors were grand-mean centered, to facilitate the interpretation of the results. Model 2: Level-1 main predictors without covariates model

After having fitted the unconditional model, the three level-1 variables, behavioral engagement, cognitive engagement, and emotional engagement, which were the key level-1 independent variable of interest to the current study, were added to the model. As mentioned before, whether or not each coefficient at level 1 should be kept as random or fixed was tested and determined by the deviance test. The predictors found significant at the 0.01 level were discussed considering large sample compared to those at level 2 and level 3.

In this study, the half-standardized coefficients (HSC) were calculated to indicate the effect sizes of the variable coefficients. The half-standardized coefficients were obtained by multiplying the standard deviation (SD) of the variable and its regression coefficient (Greenwald et al., 1996). For example, the effect size of teachers' content coverage in math class was 4.67 (SD=12.96, coefficient γ_{050}=0.36, 12.96*0.36=4.67), meaning with one standard deviation unit increase in teacher content coverage, the math score would increase by 4.67. The effect size of 4.67 point is easy to evaluate since math score had the standard deviation of 15. Thus, the half-standardized coefficient can easily be converted to the full standardized coefficient by 4.67/15 = 0.31, about one third standard deviation change.

Model 2: Level-1 main predictors without covariates model

All results for Model 2 can be found in column 3 of Table 4.9. The following equations were obtained as the final model 2.

Equation 2. Equations for level-1 Predictors without Covariates Model (Model-2)

Level-1 Model
$$\text{MATHSC}_{ijk} = \pi_{0jk} + \pi_{1jk}*(GPCBEH_{ijk}) + \pi_{2jk}*(GPCCOG_{ijk}) + \pi_{3jk}*(GPCEMO_{ijk}) + e_{ijk},$$

Level-2 Model
$$\pi_{0jk} = \beta_{00k} + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k}$$
$$\pi_{2jk} = \beta_{20k}$$
$$\pi_{3jk} = \beta_{30k} + r_{3jk},$$

Level-3 Model
$$\beta_{00k} = \gamma_{000} + u_{00k}$$
$$\beta_{10k} = \gamma_{100}$$
$$\beta_{20k} = \gamma_{200}$$
$$\beta_{30k} = \gamma_{300}.$$

The model found that student emotional engagement (EMO) presented a statistically significant positive effect on student math scores ($\hat{\gamma}_{300}$=1.72, $p \leq 0.001$) controlling for behavioral and cognitive engagement, meaning that for every unit increase that students showed positive emotions about math, there was a positive increase in math scores of 1.72 points (HSC=1.01). What's more, the variance of student's emotional engagement slope was found to be statistically significant ($\hat{var}(r_{3jk})$=0.60), suggesting that the effects of students' emotional engagement on math scores vary significantly across different classrooms. Additionally, the model also found that student behavioral engagement (BEH) tended to have a positive association with math achievement (MATHSC) ($\hat{\gamma}_{100}$=0.17. p>0.5, HSC=0.08), controlling for students' cognitive and emotional engagement levels. But the effect was not significant. The Cognitive engagement (COG) was found to have slight negative association with math scores, the effect was not significant ($\hat{\gamma}_{400}$=-0.05, $p \geq 0.5$, HSC=0.04) neither. Even though they were

not statistically significant predictors of students' math achievement, considering that

both behavioral engagement and cognitive engagement were main level-1 predictors of

interest in the present study, the two variables were kept in the following models.

In Table 4.10, the results for the amount of variance accounted for at each level is

presented. According to the table, the level-1 pseudo R^2 (R^2_{L-1}) for Model 2 was 5.16%.

This value meant that 5.16% of the student level variance of math scores, a relatively

small portion of total variance, was explained by student behavioral, cognitive, and

emotional engagement levels. The level-2 and level-3 intercept variances went down too

mainly because a random slope of group-mean centered emotional engagement

(GPCEMO) was introduced.

Model 3: Level-1 predictors with covariates model

Based on model 2, in this model, in addition to the three level-1 main predictors,

covariates including student gender, race, and student parents' education level were also

added to the level 1. All the results of this model are presented in column 4 of Table 4.9.

The equations of the model are represented as follows:

Equation 3. Equations for level-1 predictors with Covariates Model (Model 3)

Level-1 Model
$$MATH_{ijk} = \pi_{0jk} + \pi_{1jk}*(GPCBEH_{ijk}) + \pi_{2jk}*(GPCCOG_{ijk}) + \pi_{3jk}*(GPCEMO_{ijk}) + \pi_{4jk}*(GPCDFEMA_{ijk}) + \pi_{5jk}*(GPCPARED_{ijk}) + \pi_{6jk}*(GPCBLACK_{ijk}) + \pi_{7jk}*(GPCHISPA_{ijk}) + \pi_{8jk}*(GPCASIAN_{ijk}) + \pi_{9jk}*(GPCOTHER_{ijk}) + e_{ijk},$$

Level-2 Model
$$\pi_{0jk} = \beta_{00k} + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k}$$
$$\pi_{2jk} = \beta_{20k}$$
$$\pi_{3jk} = \beta_{30k} + r_{3jk}$$
$$\pi_{4jk} = \beta_{40k}$$
$$\pi_{5jk} = \beta_{50k}$$
$$\pi_{6jk} = \beta_{60k} + r_{6jk}$$
$$\pi_{7jk} = \beta_{70k} + r_{7jk}$$

$$\pi_{8jk} = \beta_{80k} + r_{8jk}$$
$$\pi_{9jk} = \beta_{90k} + r_{9jk},$$

Level-3 Model
$$\beta_{00k} = \gamma_{000} + u_{00k}$$
$$\beta_{10k} = \gamma_{100}$$
$$\beta_{20k} = \gamma_{200}$$
$$\beta_{30k} = \gamma_{300}$$
$$\beta_{40k} = \gamma_{400}$$
$$\beta_{50k} = \gamma_{500}$$
$$\beta_{60k} = \gamma_{600}$$
$$\beta_{70k} = \gamma_{700}$$
$$\beta_{80k} = \gamma_{800}$$
$$\beta_{90k} = \gamma_{900}.$$

Again, in order to facilitate the interpretation of the results, all the variables including the dummy coded predictors were group (i.e., classroom) mean centered, indicated by the prefix "GDC" before the original variable name, e.g., GPCDFEMA is the group mean centered DFEMALE, a dummy variable indicating female. The model found several level-1 covariates statistically significant. In the model, student parents' education level (PARED) had a significant positive association with math scores ($\hat{\gamma}_{500}$=0.37, $p{\leq}0.001$, HSC=0.65). Female students (FEM) scored significantly lower than male students ($\hat{\gamma}_{400}$=-1.29, $p{\leq}0.001$). Black students (BLACK) scored significantly lower than White students ($\hat{\gamma}_{600}$=-4.26, $p{\leq}0.001$). Likewise, Hispanic students (HISPA) showed significantly lower math achievement than White students ($\hat{\gamma}_{700}$=-3.24, $p{\leq}0.01$). These results were expected based on previous literature.

In terms of variance component parameters in Model 3, it was found that most of the level-1 variables had non-significant random slopes. Even though the slope variance of emotional engagement became non-significant at this time (V$\hat{a}$r(r_{3jk})=0.69, $p{\geq}0.5$) by the univariate x^2 test, the multivariance deviance test suggested that this term should be

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kept in the model. Likewise, since the groups of race were considered as one set, even though most of the groups had non-significant varying slopes (i.e., r_{6jk}, r_{7jk}, r_{8jk}, and r_{9jk}), they were determined to be kept in the model together according to the deviance test. Student with other race (OTHER) showed a significant random slope at the $p \leq 0.1$ level (V$\hat{a}$r(r_{9jk})=6.53), but it didn't meet the threshold of our level-1 criterion of $p \leq 0.01$, via the univariate χ^2 test.

In Table 4.10, the results for the amount of variance accounted for at each level are presented. According to the table, the level-1 pseudo R^2 (R^2_{L-1}) for Model 3 was 13.68%, an 8.52% increase from Model 2. This value meant that 13.68% of the student level variance of math scores was explained by student behavioral, cognitive, and emotional engagement levels and student gender, race, and their parents' education levels. Even though the four random slopes for race/ethnicity were added in model 3, the intercept variances for level-2 and level-3 were virtually unchanged from those in Model 2.

Model 4: Level-2 predictors without covariates model

Once the Level-1 model was built, as shown in Model 3, the level-2 main predictors of interest including teacher educational level (EDU), teacher experience (EXP), teacher experience squared (EXPSQ), teacher preparation (PREP), and teacher content coverage (TOPIC) were added to the level 2 to develop Model 4. After the level-2 main predictors were added to the model, the variability of the slopes for these predictors at level-3 were examined by the deviance test. It was found that all of the five slopes needed to be fixed. The results for Model 4 can be found in column 5 of Table 4.9. For this model, variables at level-2 were considered meaningfully significant at the

$p \leq 0.05$ level considering the sample size of J=359. The equations for this model are as follows:

Equation 4. Equation for Level-2 Predictors without Covariates Model (Model 4)

Level-1 Model
$$MATH_{ijk} = \pi_{0jk} + \pi_{1jk}*(GPCBEH_{ijk}) + \pi_{2jk}*(GPCCOG_{ijk}) + \pi_{3jk}*(GPCEMO_{ijk}) + \pi_{4jk}*(GPCDFEMA_{ijk}) + \pi_{5jk}*(GPCPARED_{ijk}) + \pi_{6jk}*(GPCBLACK_{ijk}) + \pi_{7jk}*(GPCHISPA_{ijk}) + \pi_{8jk}*(GPCASIAN_{ijk}) + \pi_{9jk}*(GPCOTHER_{ijk}) + e_{ijk},$$

Level-2 Model
$$\pi_{0jk} = \beta_{00k} + \beta_{01k}*(GPCEDU_{jk}) + \beta_{02k}*(GPCEXP_{jk}) + \beta_{03k}*(GPCEXPSQ_{jk}) + \beta_{04k}*(GPCPREP_{jk})$$
$$+ \beta_{05k}*(GPCTOPIC_{jk}) + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k}$$
$$\pi_{2jk} = \beta_{20k}$$
$$\pi_{3jk} = \beta_{30k} + r_{3jk}$$
$$\pi_{4jk} = \beta_{40k}$$
$$\pi_{5jk} = \beta_{50k}$$
$$\pi_{6jk} = \beta_{60k} + r_{6jk}$$
$$\pi_{7jk} = \beta_{70k} + r_{7jk}$$
$$\pi_{8jk} = \beta_{80k} + r_{8jk}$$
$$\pi_{9jk} = \beta_{90k} + r_{9jk},$$

Level-3 Model
$$\beta_{00k} = \gamma_{000} + u_{00k}$$
$$\beta_{01k} = \gamma_{010}$$
$$\beta_{02k} = \gamma_{020}$$
$$\beta_{03k} = \gamma_{030}$$
$$\beta_{10k} = \gamma_{100}$$
$$\beta_{20k} = \gamma_{200}$$
$$\beta_{30k} = \gamma_{300}$$
$$\beta_{40k} = \gamma_{400}$$
$$\beta_{50k} = \gamma_{500}$$
$$\beta_{60k} = \gamma_{600}$$
$$\beta_{70k} = \gamma_{700}$$
$$\beta_{80k} = \gamma_{800}$$
$$\beta_{90k} = \gamma_{900}.$$

Again, all the level-2 independent variables were group mean centered, denoted by the prefix "GPC" before the variable name, e.g., GPCEDU for EDU in order to capture the unbiased relationships at this level. At level 2, it was found that teacher

content coverage (TOPIC) presented a significant positive association with students'

math scores ($\hat{\gamma}_{050}$=0.36, $p\leq$0.001, HSC=4.67), controlling for teachers' educational level,

teacher experience, and teacher preparation. This can be interpreted as for every unit

increase in the teacher's content coverage in math class, students' math scores increased

0.36 point. According to Table 4.10, the model explains 13.78% of the variation at the

student level which referenced to Model 1 and 65.50% at the classroom level which

referenced to Model 3.

Model 5: Level-2 predictors with covariates model

In Model 5, the level-2 covariates including class mean student behavioral,

cognitive, and emotional engagement, and class mean student gender, race, and student

parents' education level were added to the level 2 intercept (π_{3jk}) on top of Model 4,

which only had the level-2 key independent variables of interest, i.e., teacher variables. In

addition, both level-2 predictors and covariates were added to random slope of emotional

engagement at level 2. The results of this model are presented in column 6 of Table 4.9.

The equations of the final level-2 model are as follows:

Equation 5. Equation for level-2 Predictors with Covariates Model (Model 5)

Level-1 Model
$$\text{MATH}_{ijk} = \pi_{0jk} + \pi_{1jk}*(GPCBEH_{ijk}) + \pi_{2jk}*(GPCCOG_{ijk}) + \pi_{3jk}*(GPCEMO_{ijk}) + \pi_{4jk}*(GPCDFEMA_{ijk}) + \pi_{5jk}*(GPCPARED_{ijk}) + \pi_{6jk}*(GPCBLACK_{ijk}) + \pi_{7jk}*(GPCHISPA_{ijk}) + \pi_{8jk}*(GPCASIAN_{ijk}) + \pi_{9jk}*(GPCOTHER_{ijk}) + e_{ijk}$$

Level-2 Model
$$\pi_{0jk} = \beta_{00k} + \beta_{01k}*(GPCEDU_{jk}) + \beta_{02k}*(GPCEXP_{jk}) + \beta_{03k}*(GPCEXPSQ_{jk}) + \beta_{04k}*(GPCPREP_{jk})$$
$$+ \beta_{05k}*(GPCTOPIC_{jk}) + \beta_{06k}*(GPCBEH_{jk}) + \beta_{07k}*(GPCCOG_{jk}) + \beta_{08k}*(GPCEMO_{jk})$$
$$+ \beta_{09k}*(GPCDFEMA_{jk}) + \beta_{010k}*(GPCPARED_{jk}) + \beta_{011k}*(GPCBLACK_{jk})$$
$$+ \beta_{012k}*(GPCHISPA_{jk}) + \beta_{013k}*(GPCASIAN_{jk}) + \beta_{014k}*(GPCOTHER_{jk}) + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k}$$
$$\pi_{2jk} = \beta_{20k}$$
$$\pi_{3jk} = \beta_{30k} + \beta_{31k}*(GPCEDU_{jk}) + \beta_{32k}*(GPCEXP_{jk}) + \beta_{33k}*(GPCEXPSQ_{jk}) +$$

$\beta_{34k}*(GPCPREP_{jk})$

$\quad + \beta_{35k}*(GPCTOPIC_{jk}) + \beta_{36k}*(GPCDFEMA_{jk}) + \beta_{37k}*(GPCBLACK_{jk}) +$
$\beta_{38k}*(GPCHISPA_{jk})$

$\quad + \beta_{39k}*(GPCASIAN_{jk}) + \beta_{310k}*(GPCOTHER_{jk}) + r_{3jk}$

$\pi_{4jk} = \beta_{40k}$

$\pi_{5jk} = \beta_{50k}$

$\pi_{6jk} = \beta_{60k} + r_{6jk}$

$\pi_{7jk} = \beta_{70k} + r_{7jk}$

$\pi_{8jk} = \beta_{80k} + r_{8jk}$

$\pi_{9jk} = \beta_{90k} + r_{9jk}$

Level-3 Model

$\beta_{00k} = \gamma_{000} + u_{00k}$

$\beta_{01k} = \gamma_{010}$

$\beta_{02k} = \gamma_{020}$

$\beta_{03k} = \gamma_{030}$

$\beta_{04k} = \gamma_{040}$

$\beta_{05k} = \gamma_{050}$

$\beta_{06k} = \gamma_{060}$

$\beta_{07k} = \gamma_{070}$

$\beta_{08k} = \gamma_{080}$

$\beta_{09k} = \gamma_{090}$

$\beta_{010k} = \gamma_{0100}$

$\beta_{011k} = \gamma_{0110}$

$\beta_{012k} = \gamma_{0120}$

$\beta_{013k} = \gamma_{0130}$

$\beta_{014k} = \gamma_{0140}$

$\beta_{10k} = \gamma_{100}$

$\beta_{20k} = \gamma_{200}$

$\beta_{30k} = \gamma_{300}$

$\beta_{31k} = \gamma_{310}$

$\beta_{32k} = \gamma_{320}$

$\beta_{33k} = \gamma_{330}$

$\beta_{34k} = \gamma_{340}$

$\beta_{35k} = \gamma_{350}$

$\beta_{36k} = \gamma_{360}$

$\beta_{37k} = \gamma_{370}$

$\beta_{38k} = \gamma_{380}$

$\beta_{39k} = \gamma_{390}$

$\beta_{310k} = \gamma_{3100}$

$\beta_{40k} = \gamma_{400}$

$\beta_{50k} = \gamma_{500}$

$\beta_{60k} = \gamma_{600}$

$\beta_{70k} = \gamma_{700}$

$\beta_{80k} = \gamma_{800}$

$\beta_{90k} = \gamma_{900}$

The results of Model 5 showed that class mean student emotional engagement had a statistically significant positive association with students' math scores ($\hat{\gamma}_{080}$=4.11, $p\leq0.001$, HSC=1.36). This coefficient can be interpreted as for students who had the same individual emotional engagement level, with every unit increase in the class-level emotional engagement, their math score increased 4.11 points. The class mean variable is a compositional variable and the difference between the regression coefficient on the compositional variable and its original variable at a lower level represents compositional effects. Compositional effects refer to the extent to which the size of the organizational-level effect differs from the size of the individual-level effect (Raudenbush & Bryk, 2002). The magnitude of the association between math achievement and emotional engagement at the classroom level represented by this value of 4.11 was more than twice larger than the individual level association (γ_{300}=1.72), which indicated a large average compositional effect of 2.40 (=4.11-1.72).

In addition, several covariates at level 2 showed statistically significant effects on students' math scores. Class mean parents' education level had a significant positive effect on math scores ($\hat{\gamma}_{0100}$=1.31, $p\leq0.01$, HSC=1.30). Classroom mean Asian students (i.e., proportion of Asian students in the classroom) had a coefficient of ($\hat{\gamma}_{0130}$=13.75, $p\leq0.01$, HSC=0.96), meaning that all Asian students classes had 13.75 points higher math scores compared to no Asian students on average, holding other independent variables in the model constant; while the classes with students in other race groups in the classroom scored 8.91 lower than those with no other race groups in the classrooms on average ($\hat{\gamma}_{0140}$ =-8.91, $p\leq0.01$, HSC=0.80), again holding other independent variables in the model constant.

In terms of the slope of emotional engagement, it was found that teacher's education level showed a significant negative effect on students' emotional engagement slope ($\hat{\gamma}_{310}$=-1.57, $p \leq 0.05$, HSC=0.79) controlling for other teacher-level and student-level predictors and covariates. This means that for every unit increase in teacher's educational level, there was a 1.57 units decrease in students' emotional engagement level when other factors in the model being equal. In addition, teacher's subject-matter preparation showed a significant positive effect on students' emotional engagement slope ($\hat{\gamma}_{340}$=2.51, $p \leq 0.05$, HSC=1.20), suggesting that teachers who were very well prepared to teach math-related topics would magnify the positive effect of students' emotional engagement significantly (1.66+2.51=4.17), holding other independent variables in the model constant.

As can be seen from Table 4.10, the model explained 14.11% of the variation at the student level and 86.01% at the classroom level, an increase of 20.51% from Model 4. This indicated that compositional variables such as class mean engagement and classroom racial composition variables had significant additional explanatory power for explaining between-class variation of math achievement.

Model 6: Level-3 predictors with covariates

In model 6, based on the final level-2 model (i.e., Model 5), the two level-3 predictors including school SES (SES) and school math resources (RSOR) and the aggregated means of all other level-1 and level-2 covariates were added to the model. As mentioned before, all the variables entered into the third level were grand-mean centered. At this level, variables that were significant at the $p \leq 0.1$ level, which is much liberal significance level compared to the level 1 and the level 2, were kept and discussed in the final model because of the relatively small sample size (K=185). The results of this

model can be found in the seventh column of Table 4.9. The following are the equations

of this model:

Equation 6. Equations for level-3 Predictors and Covariates Model (Model 6)

Level-1 Model
$$\text{MATH}_{ijk} = \pi_{0jk} + \pi_{1jk}*(GPCBEH_{ijk}) + \pi_{2jk}*(GPCCOG_{ijk}) + \pi_{3jk}*(GPCEMO_{ijk}) + \pi_{4jk}*(GPCDFEMA_{ijk}) + \pi_{5jk}*(GPCPARED_{ijk}) + \pi_{6jk}*(GPCBLACK_{ijk}) + \pi_{7jk}*(GPCHISPA_{ijk}) + \pi_{8jk}*(GPCASIAN_{ijk}) + \pi_{9jk}*(GPCOTHER_{ijk}) + e_{ijk},$$

Level-2 Model
$$\pi_{0jk} = \beta_{00k} + \beta_{01k}*(GPCEDU_{jk}) + \beta_{02k}*(GPCEXP_{jk}) + \beta_{03k}*(GPCEXPSQ_{jk}) + \beta_{04k}*(GPCPREP_{jk})$$
$$+ \beta_{05k}*(GPCTOPIC_{jk}) + \beta_{06k}*(GPCBEH_{jk}) + \beta_{07k}*(GPCCOG_{jk}) + \beta_{08k}*(GPCEMO_{jk})$$
$$+ \beta_{09k}*(GPCDFEMA_{jk}) + \beta_{010k}*(GPCPARED_{jk}) + \beta_{011k}*(GPCBLACK_{jk}) + \beta_{012k}*(GPCHISPA_{jk}) + \beta_{013k}*(GPCASIAN_{jk}) + \beta_{014k}*(GPCOTHER_{jk}) + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k}$$
$$\pi_{2jk} = \beta_{20k}$$
$$\pi_{3jk} = \beta_{30k} + \beta_{31k}*(GPCEDU_{jk}) + \beta_{32k}*(GPCEXP_{jk}) + \beta_{33k}*(GPCEXPSQ_{jk}) + \beta_{34k}*(GPCPREP_{jk})$$
$$+ \beta_{35k}*(GPCTOPIC_{jk}) + \beta_{36k}*(GPCDFEMA_{jk}) + \beta_{37k}*(GPCBLACK_{jk}) + \beta_{38k}*(GPCHISPA_{jk})$$
$$+ \beta_{39k}*(GPCASIAN_{jk}) + \beta_{310k}*(GPCOTHER_{jk}) + r_{3jk}$$
$$\pi_{4jk} = \beta_{40k}$$
$$\pi_{5jk} = \beta_{50k}$$
$$\pi_{6jk} = \beta_{60k} + r_{6jk}$$
$$\pi_{7jk} = \beta_{70k} + r_{7jk}$$
$$\pi_{8jk} = \beta_{80k} + r_{8jk}$$
$$\pi_{9jk} = \beta_{90k} + r_{9jk},$$

Level-3 Model
$$\beta_{00k} = \gamma_{000} + \gamma_{001}(GDCBEH_k) + \gamma_{002}(GDCCOG_k) + \gamma_{003}(GDCEMO_k) + \gamma_{004}(GDCDFEMA_k)$$
$$+ \gamma_{005}(GDCPARED_k) + \gamma_{006}(GDCBLACK_k) + \gamma_{007}(GDCHISPA_k) + \gamma_{008}(GDCASIAN_k)$$
$$+ \gamma_{009}(GDCOTHER_k) + \gamma_{0010}(GDCEDU_k) + \gamma_{0011}(GDCEXP_k) + \gamma_{0012}(GDCEXPSQ_k)$$
$$+ \gamma_{0013}(GDCPREP_k) + \gamma_{0014}(GDCTOPIC_k) + \gamma_{0015}(GDCSES_k) + \gamma_{0016}(GDCRSOR_k) + u_{00k}$$
$$\beta_{01k} = \gamma_{010}$$
$$\beta_{02k} = \gamma_{020}$$
$$\beta_{03k} = \gamma_{030}$$
$$\beta_{04k} = \gamma_{040}$$
$$\beta_{05k} = \gamma_{050}$$
$$\beta_{06k} = \gamma_{060}$$
$$\beta_{07k} = \gamma_{070}$$
$$\beta_{08k} = \gamma_{080}$$

$$\beta_{09k} = \gamma_{090}$$
$$\beta_{010k} = \gamma_{0100}$$
$$\beta_{011k} = \gamma_{0110}$$
$$\beta_{012k} = \gamma_{0120}$$
$$\beta_{013k} = \gamma_{0130}$$
$$\beta_{014k} = \gamma_{0140}$$
$$\beta_{10k} = \gamma_{100}$$
$$\beta_{20k} = \gamma_{200}$$
$$\beta_{30k} = \gamma_{300}$$
$$\beta_{31k} = \gamma_{310}$$
$$\beta_{32k} = \gamma_{320}$$
$$\beta_{33k} = \gamma_{330}$$
$$\beta_{34k} = \gamma_{340}$$
$$\beta_{35k} = \gamma_{350}$$
$$\beta_{36k} = \gamma_{360}$$
$$\beta_{37k} = \gamma_{370}$$
$$\beta_{38k} = \gamma_{380}$$
$$\beta_{39k} = \gamma_{390}$$
$$\beta_{310k} = \gamma_{3100}$$
$$\beta_{40k} = \gamma_{400}$$
$$\beta_{50k} = \gamma_{500}$$
$$\beta_{60k} = \gamma_{600}$$
$$\beta_{70k} = \gamma_{700}$$
$$\beta_{80k} = \gamma_{800}$$
$$\beta_{90k} = \gamma_{900}.$$

At the school level, school SES was found to have a statistically significant association with math scores ($\hat{\gamma}_{0015}$=-0.03, $p \leq 0.01$, HSC=0.85). This indicated that with every unit decrease in percent of students' receiving free/reduced lunch price at one school (i.e., 1%), students' math score increased 0.03 point. The school mean teacher experience squared showed a small but significant positive coefficient on math scores ($\hat{\gamma}_{0012}$=-0.004, $p \leq 0.1$, HSC=1.40), indicating that there was a quadratic relationship between the school mean teacher experience and students' math scores. The school mean teacher content coverage also showed a significant positive association with students' math scores ($\hat{\gamma}_{0014}$=0.05, $p \leq 0.05$, HSC=0.49). This suggested that with every unit increase

in the average teacher content coverage at the school level, students' math scores increased by 0.05 point.

In addition, other school-level covariates also presented statistically significant associations with students' math scores. At the school level, the proportion of female students in the school (in %) had positive coefficient ($\hat{\gamma}_{004}$=3.13, $p\leq0.1$, HSC=0.41). School mean parents' educational level showed a significant positive effect on math achievement ($\hat{\gamma}_{005}$=1.49, $p\leq0.001$, HSC=1.34). One unit increase in the proportion of Black students in the school (i.e., comparing all Black and all White schools) was associated 11.80 points decrease in math on average ($\hat{\gamma}_{006}$=-11.80, $p\leq0.001$, HSC=2.83); the proportion of Hispanic students in the school had a negative statistically significant coefficient ($\hat{\gamma}_{007}$=-5.17, $p\leq0.001$, HSC=1.24); the proportion of Asian students in the school, surprisingly, was associated 8.95 points decrease in math on average ($\hat{\gamma}_{008}$=-8.95, $p\leq0.01$, HSC=0.41); and the proportion of students with other races in the school was associated 7.53 points decrease in math on average ($\hat{\gamma}_{009}$=-7.53, $p\leq0.01$, HSC=7.30). At the school level, all aggregated school mean engagement variables became non-significant, though emotional engagement still held the positive sign.

As can be seen in Table 4.10, the level-3 pseudo R^2 (R^2_{L-3}) for this model was 92.79%, meaning that 92.79% of the original variance for the school mean math achievement in Model 5 was explained by the school mean SES, school math resources, and other school-level covariates.

Table 4.9

Results of HLM Analysis

	Model 1 Unconditional Model	Model 2 L-1 Predictors without Covariates Model	Model3 L-1 Predictors with Covariates Model	Model 4 L-2 Predictors without Covariates Model	Model 5 L-2 Predictors with Covariates Model	Model 6 L-3 Predictors with Covariates Model
Fixed Effects						
Model for School Mean						
Intercept, γ_{000}	150.44***	150.82***	150.83***	150.80***	150.84***	150.25***
School Mean Behavioral Engagement, γ_{001}						0.52
School Mean Cognitive Engagement, γ_{002}						2.01
School Mean Emotional Engagement, γ_{003}						1.52
School Mean Female, γ_{004}						3.13~
School Mean Parent Education, γ_{005}						1.49***
School Mean Black, γ_{006}						-11.80***
School Mean Hispanic, γ_{007}						-5.17***
School Mean Asian, γ_{008}						-8.95**
School Mean Other, γ_{009}						-7.54**
School Mean Teacher Education, γ_{0010}						0.46
School Mean Teacher Experience, γ_{0011}						-0.03
School Mean Teacher Preparation, γ_{0013}						0.36
School Mean Teacher Content Coverage, γ_{0014}						0.05*
School SES, γ_{0015}						-0.03**
School Math Resource, γ_{0016}						0.12
Model for Class Mean						
Teacher Education, γ_{010}				0.63	0.08	-0.10
Teacher Experience, γ_{020}				-0.03	-0.03	-0.01

	(1)	(2)	(3)	(4)	(5)
Teacher Experience Squared, γ_{030}			-0.01	-0.01	0.003
Teacher Preparation, γ_{040}			0.60	1.38	1.24
Teacher Content Coverage, γ_{050}			0.36***	0.27***	0.28***
Class Mean Student Behavioral Engagement, γ_{060}				0.45	0.30
Class Mean Student Cognitive Engagement, γ_{070}				2.05	1.71
Class Mean Student Emotional Engagement, γ_{080}				4.11***	4.19***
Class Mean Female, γ_{090}				-2.85*	-2.70~
Class Mean Parent Education, γ_{0100}				1.31**	1.27**
Class Mean Black, γ_{0110}				-0.77	-1.22
Class Mean Hispanic, γ_{0120}				-2.04	-1.50
Class Mean Asian, γ_{0130}				13.75**	13.07**
Class Mean Other, γ_{0140}				-8.91**	-9.40**
Behavioral Engagement, γ_{100}	0.17	0.09	0.09	0.12	0.14
Cognitive Engagement, γ_{200}	-0.05	-0.24	-0.25	-0.21	-0.18
Model for Emotional Engagement Slope					
Intercept, γ_{300}	1.72***	1.54***	1.57***	1.66***	1.65***
Teacher Education, γ_{310}				-1.57*	-1.58~
Teacher Experience, γ_{320}				-0.02	-0.02
Teacher Experience Squared, γ_{330}				-0.004	-0.003
Teacher Preparation, γ_{340}				2.51*	2.52**
Teacher Content Coverage, γ_{350}				-0.02	-0.02
Female, γ_{360}				-3.47~	-3.52
Black, γ_{370}				2.16	2.19
Hispanic, γ_{380}				0.48	0.70
Asian, γ_{390}				11.14~	11.19~
Other, γ_{3100}				-7.66	-7.58
Female, γ_{400}		-1.29***	-1.32***	-1.36***	-1.36***
Parent Education, γ_{500}		0.37***	0.37***	0.35***	0.35***
Black, γ_{600}		-4.26***	-4.24***	-4.31***	-4.42***
Hispanic, γ_{700}		-3.25**	-3.23**	-3.18**	-3.19**

Asian, γ_{800}			-0.97	-1.27	-1.45	-1.50
Other, γ_{900}			-0.38	-0.36	-0.41	-0.50
Random Effects						
School Variation, u_{00k}	12.72***	9.16***	9.41***	17.60***	20.81***	1.50***
Classroom Variation, r_{0jk}	27.16***	24.10***	24.52***	8.46***	3.43***	2.87***
Emotional Engagement Slope Variation, r_{3jk}		0.60**	0.69	0.83	0.50	0.56
Black Slope Variation, r_{6jk}			6.19	5.87	5.69	5.24
Hispanic Slope Variation, r_{7jk}			22.92	22.16	21.65	21.10
Asian Slope Variation, r_{8jk}			16.15	13.73	11.87	11.93
Other Slope Variation, r_{9jk}			6.53~	5.90~	6.50~	6.39~
Student Variation, e_{ijk}	57.90***	54.91***	49.98***	49.92***	49.73***	49.89***

*** Significant at the 0.001 level; ** Significant at the 0.01 level; * Significant at the 0.05 level; ~ Significant at the 0.1 level

Table 4.10

Proportion of Variance Explained

Model	Student Level variance Intercept Variance	R^2	Class Level Variance Intercept Variance	R^2	School Level Variance Intercept Variance	
Model 1	57.90***	(Base)	27.16***		12.72***	
Model 2	54.91***	5.16%	24.10***		9.16***	
Model 3	49.98***	13.68%	24.52***	(Base)	9.41***	
Model 4	49.92***	13.78%	8.46***	65.50%	17.60***	
Model 5	49.73***	14.11%	3.43***	86.01%	20.81***	(Base)
Model 6	49.89***	15.56%	2.87***	88.30%	1.50***	92.79%

Note. ~p≤.10. *p≤.05. **p≤.01. ***p≤.001. R^2 indicates the pseudo-R^2, or the proportion of explained variance by including independent variables compared to Model A, the unconditional model for the level-1 variance, Model 3 for the level-2 variance, and Model 5 for the level-3 variance. The values used for the comparison base is represented as (Base).

Sensitivity Analysis

After examining the results of the models above, it seemed that Model 6 was the best model to explain the research purposes and questions. In order to further examine the stability of Model 6, the sensitivity analysis was conducted by fitting several alternative models and the comparisons with the final model were made. Since the model was quite complex, decisions were made in the model building process when the statistical significance level was at the border line (e.g., the slope variation of emotional engagement). The results of sensitivity analysis and the equations of the sensitivity models are presented below:

Table 4.11

Results of Sensitivity Analysis

	Model 6 L-3 Predictors with Covariates Model	Model 7 Sensitivity Analysis Model 1	Model 8 Sensitivity Analysis Model 2	Model 9 Sensitivity Analysis Model 3
Fixed Effects				
Model for School Mean				
Intercept, γ_{000}	150.25***	150.25***	150.83***	150.26***
School Mean Behavioral Engagement, γ_{001}	0.52	0.47		0.38
School Mean Cognitive Engagement, γ_{002}	2.01	1.86		2.12
School Mean Emotional Engagement, γ_{003}	1.52	1.55		1.29
School Mean Female, γ_{004}	3.13~	3.10~		3.13~
School Mean Parent Education, γ_{005}	1.49***	1.51***		1.54***
School Mean Black, γ_{006}	-11.80***	-11.95***		-12.27***
School Mean Hispanic, γ_{007}	-5.17***	-5.25***		-5.45***
School Mean Asian, γ_{008}	-8.95**	-8.57*		-8.13*
School Mean Other, γ_{009}	-7.54**	-7.57**		-6.87**
School Mean Teacher Education, γ_{0010}	0.46	0.43		0.40
School Mean Teacher Experience, γ_{0011}	-0.03	-0.03		-0.03
School Mean Teacher Experience Squared, γ_{0012}	0.004~	0.004~		0.004~
School Mean Teacher Preparation, γ_{0013}	0.36	0.31		0.25
School Mean Teacher Content Coverage, γ_{0014}	0.05*	0.05*		0.05~
School SES, γ_{0015}	-0.03**	-0.03**		-0.03**
School Math Resource, γ_{0016}	0.12	0.12		0.16
Model for Class Mean				
Teacher Education, γ_{010}	-0.10	-0.17	-0.04	-0.17
Teacher Experience, γ_{020}	-0.01	-0.02	-0.03	-0.02
Teacher Experience Squared, γ_{030}	0.003	0.004	-0.001	0.003
Teacher Preparation, γ_{040}	1.24	1.25	1.36	1.20
Teacher Content Coverage, γ_{050}	0.28***	0.28***	0.27***	0.28***
Class Mean Student Behavioral Engagement, γ_{060}	0.30	0.29	0.25	0.04
Class Mean Student Cognitive Engagement, γ_{070}	1.71	1.78	2.19	1.72
Class Mean Student Emotional Engagement, γ_{080}	4.19***	4.13***	4.18***	4.22***
Class Mean Female, γ_{090}	-2.70~	-2.61~	-2.84*	-2.61~
Class Mean Parent Education, γ_{0100}	1.27**	1.28*	1.31**	1.25*
Class Mean Black, γ_{0110}	-1.22	-1.17	-1.07	-1.25
Class Mean Hispanic, γ_{0120}	-1.50	-1.48	-1.76	-1.58

Class Mean Asian, γ_{0130}	13.07**	13.09**	13.67**	12.84**
Class Mean Other, γ_{0140}	-9.40**	-9.54**	-8.79**	-9.39**
Behavioral Engagement, γ_{100}	0.14	0.16	0.12	0.12
Cognitive Engagement, γ_{200}	-0.18	-0.05	-0.09	-0.09
Emotional Engagement, γ_{300}	1.65***	1.65***	1.52***	1.52***
Teacher Education, γ_{310}	-1.58~	-1.71*		
Teacher Experience, γ_{320}	-0.02	-0.02		
Teacher Experience Squared, γ_{330}	-0.003	-0.004		
Teacher Preparation, γ_{340}	2.52**	2.42*		
Teacher Content Coverage, γ_{350}	-0.02	-0.01		
Female, γ_{360}	-3.52	-3.99~		
Black, γ_{370}	2.19	1.97		
Hispanic, γ_{380}	0.70	0.64		
Asian, γ_{390}	11.19~	9.44		
Other, γ_{3100}	-7.58	-7.22		
Female, γ_{400}	-1.36***	-1.35***	-1.29***	-1.29***
Parent Education, γ_{500}	0.35***	0.36**	-0.37***	0.37***
Black, γ_{600}	-4.42***	-4.80***	-4.85***	-4.85***
Hispanic, γ_{700}	-3.19**	-3.02***	-3.05**	-3.05**
Asian, γ_{800}	-1.50	-1.62	-1.37	-1.37
Other, γ_{900}	-0.50	-0.46	-0.50	-0.50
Random Effects				
School Variation, u_{00k}	1.50***	1.53***	21.48***	1.55***
Classroom Variation, r_{0jk}	2.87***	2.62***	2.82***	2.50***
Emotional Engagement Slope Variation, r_{3jk}	0.56	0.49*		
Black Slope Variation, r_{6jk}	5.24			
Hispanic Slope Variation, r_{7jk}	21.10			
Asian Slope Variation, r_{8jk}	11.93			
Other Slope Variation, r_{9jk}	6.39~			
Student Variation, e_{ijk}	49.89***	52.25***	52.76***	52.90***

*** Significant at the 0.001 level; ** Significant at the 0.01 level; * Significant at the 0.05 level; ~ Significant at the 0.1 level

Model 7: Sensitivity analysis model 1

Level-1 Model
$$MATH_{ijk} = \pi_{0jk} + \pi_{1jk}*(GPCBEH_{ijk}) + \pi_{2jk}*(GPCCOG_{ijk}) + \pi_{3jk}*(GPCEMO_{ijk})$$
$$+ \pi_{4jk}*(GPCDFEMA_{ijk}) + \pi_{5jk}*(GPCPARED_{ijk}) + \pi_{6jk}*(GPCBLACK_{ijk}) +$$
$$\pi_{7jk}*(GPCHISPA_{ijk}) + \pi_{8jk}*(GPCASIAN_{ijk}) + \pi_{9jk}*(GPCOTHER_{ijk}) + e_{ijk},$$

Level-2 Model
$$\pi_{0jk} = \beta_{00k} + \beta_{01k}*(GPCEDU_{jk}) + \beta_{02k}*(GPCEXP_{jk}) + \beta_{03k}*(GPCEXPSQ_{jk}) +$$
$$\beta_{04k}*(GPCPREP_{jk})$$
$$+ \beta_{05k}*(GPCTOPIC_{jk}) + \beta_{06k}*(GPCBEH_{jk}) + \beta_{07k}*(GPCCOG_{jk}) + \beta_{08k}*(GPCEMO_{jk})$$
$$+ \beta_{09k}*(GPCDFEMA_{jk}) + \beta_{010k}*(GPCPARED_{jk}) + \beta_{011k}*(GPCBLACK_{jk})$$

$\qquad + \beta_{012k}*(GPCHISPA_{jk}) + \beta_{013k}*(GPCASIAN_{jk}) + \beta_{014k}*(GPCOTHER_{jk}) + r_{0jk}$

$\pi_{1jk} = \beta_{10k}$

$\pi_{2jk} = \beta_{20k}$

$\pi_{3jk} = \beta_{30k} + \beta_{31k}*(GPCEDU_{jk}) + \beta_{32k}*(GPCEXP_{jk}) + \beta_{33k}*(GPCEXPSQ_{jk}) +$
$\beta_{34k}*(GPCPREP_{jk})$

$\qquad + \beta_{35k}*(GPCTOPIC_{jk}) + \beta_{36k}*(GPCDFEMA_{jk}) + \beta_{37k}*(GPCBLACK_{jk}) +$
$\beta_{38k}*(GPCHISPA_{jk})$

$\qquad + \beta_{39k}*(GPCASIAN_{jk}) + \beta_{310k}*(GPCOTHER_{jk}) + r_{3jk}$

$\pi_{4jk} = \beta_{40k}$

$\pi_{5jk} = \beta_{50k}$

$\pi_{6jk} = \beta_{60k}$

$\pi_{7jk} = \beta_{70k}$

$\pi_{8jk} = \beta_{80k}$

$\pi_{9jk} = \beta_{90k},$

Level-3 Model

$\beta_{00k} = \gamma_{000} + \gamma_{001}(GDCBEH_k) + \gamma_{002}(GDCCOG_k) + \gamma_{003}(GDCEMO_k) +$
$\gamma_{004}(GDCDFEMA_k)$

$\qquad + \gamma_{005}(GDCPARED_k) + \gamma_{006}(GDCBLACK_k) + \gamma_{007}(GDCHISPA_k) +$
$\gamma_{008}(GDCASIAN_k)$

$\qquad + \gamma_{009}(GDCOTHER_k) + \gamma_{0010}(GDCEDU_k) + \gamma_{0011}(GDCEXP_k) +$
$\gamma_{0012}(GDCEXPSQ_k)$

$\qquad + \gamma_{0013}(GDCPREP_k) + \gamma_{0014}(GDCTOPIC_k) + \gamma_{0015}(GDCSES_k) + \gamma_{0016}(GDCRSOR_k)$
$+ u_{00k}$

$\beta_{01k} = \gamma_{010}$

$\beta_{02k} = \gamma_{020}$

$\beta_{03k} = \gamma_{030}$

$\beta_{04k} = \gamma_{040}$

$\beta_{05k} = \gamma_{050}$

$\beta_{06k} = \gamma_{060}$

$\beta_{07k} = \gamma_{070}$

$\beta_{08k} = \gamma_{080}$

$\beta_{09k} = \gamma_{090}$

$\beta_{010k} = \gamma_{0100}$

$\beta_{011k} = \gamma_{0110}$

$\beta_{012k} = \gamma_{0120}$

$\beta_{013k} = \gamma_{0130}$

$\beta_{014k} = \gamma_{0140}$

$\beta_{10k} = \gamma_{100}$

$\beta_{20k} = \gamma_{200}$

$\beta_{30k} = \gamma_{300}$

$\beta_{31k} = \gamma_{310}$

$\beta_{32k} = \gamma_{320}$

$\beta_{33k} = \gamma_{330}$

$\beta_{34k} = \gamma_{340}$

$\beta_{35k} = \gamma_{350}$

$$\beta_{36k} = \gamma_{360}$$
$$\beta_{37k} = \gamma_{370}$$
$$\beta_{38k} = \gamma_{380}$$
$$\beta_{39k} = \gamma_{390}$$
$$\beta_{310k} = \gamma_{3100}$$
$$\beta_{40k} = \gamma_{400}$$
$$\beta_{50k} = \gamma_{500}$$
$$\beta_{60k} = \gamma_{600}$$
$$\beta_{70k} = \gamma_{700}$$
$$\beta_{80k} = \gamma_{800}$$
$$\beta_{90k} = \gamma_{900}.$$

Model 8: Sensitivity analysis model 2

Level-1 Model
$$MATH_{ijk} = \pi_{0jk} + \pi_{1jk}*(GPCBEH_{ijk}) + \pi_{2jk}*(GPCCOG_{ijk}) + \pi_{3jk}*(GPCEMO_{ijk})$$
$$+ \pi_{4jk}*(GPCDFEMA_{ijk}) + \pi_{5jk}*(GPCPARED_{ijk}) + \pi_{6jk}*(GPCBLACK_{ijk}) +$$
$$\pi_{7jk}*(GPCHISPA_{ijk}) \quad + \pi_{8jk}*(GPCASIAN_{ijk}) + \pi_{9jk}*(GPCOTHER_{ijk}) + e_{ijk},$$

Level-2 Model
$$\pi_{0jk} = \beta_{00k} + \beta_{01k}*(GPCEDU_{jk}) + \beta_{02k}*(GPCEXP_{jk}) + \beta_{03k}*(GPCEXPSQ_{jk}) +$$
$$\beta_{04k}*(GPCPREP_{jk})$$
$$+ \beta_{05k}*(GPCTOPIC_{jk}) + \beta_{06k}*(GPCBEH_{jk}) + \beta_{07k}*(GPCCOG_{jk}) + \beta_{08k}*(GPCEMO_{jk})$$
$$+ \beta_{09k}*(GPCDFEMA_{jk}) + \beta_{010k}*(GPCPARED_{jk}) + \beta_{011k}*(GPCBLACK_{jk})$$
$$+ \beta_{012k}*(GPCHISPA_{jk}) + \beta_{013k}*(GPCASIAN_{jk}) + \beta_{014k}*(GPCOTHER_{jk}) + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k}$$
$$\pi_{2jk} = \beta_{20k}$$
$$\pi_{3jk} = \beta_{30k}$$
$$\pi_{4jk} = \beta_{40k}$$
$$\pi_{5jk} = \beta_{50k}$$
$$\pi_{6jk} = \beta_{60k}$$
$$\pi_{7jk} = \beta_{70k}$$
$$\pi_{8jk} = \beta_{80k}$$
$$\pi_{9jk} = \beta_{90k},$$

Level-3 Model
$$\beta_{00k} = \gamma_{000} + u_{00k}$$
$$\beta_{01k} = \gamma_{010}$$
$$\beta_{02k} = \gamma_{020}$$
$$\beta_{03k} = \gamma_{030}$$
$$\beta_{04k} = \gamma_{040}$$
$$\beta_{05k} = \gamma_{050}$$
$$\beta_{06k} = \gamma_{060}$$
$$\beta_{07k} = \gamma_{070}$$
$$\beta_{08k} = \gamma_{080}$$
$$\beta_{09k} = \gamma_{090}$$

$$\beta_{010k} = \gamma_{0100}$$
$$\beta_{011k} = \gamma_{0110}$$
$$\beta_{012k} = \gamma_{0120}$$
$$\beta_{013k} = \gamma_{0130}$$
$$\beta_{014k} = \gamma_{0140}$$
$$\beta_{10k} = \gamma_{100}$$
$$\beta_{20k} = \gamma_{200}$$
$$\beta_{30k} = \gamma_{300}$$
$$\beta_{40k} = \gamma_{400}$$
$$\beta_{50k} = \gamma_{500}$$
$$\beta_{60k} = \gamma_{600}$$
$$\beta_{70k} = \gamma_{700}$$
$$\beta_{80k} = \gamma_{800}$$
$$\beta_{90k} = \gamma_{900}.$$

Model 9: Sensitivity analysis model 3

Level-1 Model
$$\text{MATH}_{ijk} = \pi_{0jk} + \pi_{1jk}*(GPCBEH_{ijk}) + \pi_{2jk}*(GPCCOG_{ijk}) + \pi_{3jk}*(GPCEMO_{ijk}) + \pi_{4jk}*(GPCDFEMA_{ijk}) + \pi_{5jk}*(GPCPARED_{ijk}) + \pi_{6jk}*(GPCBLACK_{ijk}) + \pi_{7jk}*(GPCHISPA_{ijk}) + \pi_{8jk}*(GPCASIAN_{ijk}) + \pi_{9jk}*(GPCOTHER_{ijk}) + e_{ijk},$$

Level-2 Model
$$\pi_{0jk} = \beta_{00k} + \beta_{01k}*(GPCEDU_{jk}) + \beta_{02k}*(GPCEXP_{jk}) + \beta_{03k}*(GPCEXPSQ_{jk}) + \beta_{04k}*(GPCPREP_{jk})$$
$$+ \beta_{05k}*(GPCTOPIC_{jk}) + \beta_{06k}*(GPCBEH_{jk}) + \beta_{07k}*(GPCCOG_{jk}) + \beta_{08k}*(GPCEMO_{jk})$$
$$+ \beta_{09k}*(GPCDFEMA_{jk}) + \beta_{010k}*(GPCPARED_{jk}) + \beta_{011k}*(GPCBLACK_{jk})$$
$$+ \beta_{012k}*(GPCHISPA_{jk}) + \beta_{013k}*(GPCASIAN_{jk}) + \beta_{014k}*(GPCOTHER_{jk}) + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k}$$
$$\pi_{2jk} = \beta_{20k}$$
$$\pi_{3jk} = \beta_{30k}$$
$$\pi_{4jk} = \beta_{40k}$$
$$\pi_{5jk} = \beta_{50k}$$
$$\pi_{6jk} = \beta_{60k}$$
$$\pi_{7jk} = \beta_{70k}$$
$$\pi_{8jk} = \beta_{80k}$$
$$\pi_{9jk} = \beta_{90k},$$

Level-3 Model
$$\beta_{00k} = \gamma_{000} + \gamma_{001}(GDCBEH_k) + \gamma_{002}(GDCCOG_k) + \gamma_{003}(GDCEMO_k) + \gamma_{004}(GDCDFEMA_k)$$
$$+ \gamma_{005}(GDCPARED_k) + \gamma_{006}(GDCBLACK_k) + \gamma_{007}(GDCHISPA_k) + \gamma_{008}(GDCASIAN_k)$$
$$+ \gamma_{009}(GDCOTHER_k) + \gamma_{0010}(GDCEDU_k) + \gamma_{0011}(GDCEXP_k) + \gamma_{0012}(GDCEXPSQ_k)$$
$$+ \gamma_{0013}(GDCPREP_k) + \gamma_{0014}(GDCTOPIC_k) + \gamma_{0015}(GDCSES_k) + \gamma_{0016}(GDCRSOR_k)$$

$$+ u_{00k}$$
$$\beta_{01k} = \gamma_{010}$$
$$\beta_{02k} = \gamma_{020}$$
$$\beta_{03k} = \gamma_{030}$$
$$\beta_{04k} = \gamma_{040}$$
$$\beta_{05k} = \gamma_{050}$$
$$\beta_{06k} = \gamma_{060}$$
$$\beta_{07k} = \gamma_{070}$$
$$\beta_{08k} = \gamma_{080}$$
$$\beta_{09k} = \gamma_{090}$$
$$\beta_{010k} = \gamma_{0100}$$
$$\beta_{011k} = \gamma_{0110}$$
$$\beta_{012k} = \gamma_{0120}$$
$$\beta_{013k} = \gamma_{0130}$$
$$\beta_{014k} = \gamma_{0140}$$
$$\beta_{10k} = \gamma_{100}$$
$$\beta_{20k} = \gamma_{200}$$
$$\beta_{30k} = \gamma_{300}$$
$$\beta_{40k} = \gamma_{400}$$
$$\beta_{50k} = \gamma_{500}$$
$$\beta_{60k} = \gamma_{600}$$
$$\beta_{70k} = \gamma_{700}$$
$$\beta_{80k} = \gamma_{800}$$
$$\beta_{90k} = \gamma_{900}.$$

In the first sensitivity analysis model (Model 7), the random effects of the race/ethnicity group were deleted, since most of these terms were not statistically significant in Model 6. The results of Model 7 were presented in column three of Table 4.11. It showed that without including the terms, the model didn't change significantly in the effect values comparing to Model 6. Based on Model 7, Model 8 was built by deleting all the school-level variables as well as the slope effects of emotional engagement. The results in column four Table 4.11 indicated that there were no significant changes of this model comparing to Model 6. Finally, based on Model 8, Model 9 was developed by adding all the school-level variables back. As can be seen from column five Table 4.11, there was still no significant changes.

In addition, Model 6 includes variables at all the three levels, and it also includes slope effects of teacher quality on student emotional engagement, which would provide adequate information to answer the research questions. In conclusion, Model 6 was examined to be the final model for the discussion and conclusion.

Chapter 5 : Discussions and Conclusions

Math achievement has been considered as a critical issue by policy makers, educators, and researchers. Students' math achievement during the middle school years determines their high school enrollment in STEM courses and future success in college and workforce. The math achievement of U.S. students lags behind their international peers. The U.S. eighth graders ranked 28th out of 40 western nations. Moreover, the problem of math achievement within the U.S. is also severe. There are significant and persistent math achievement gaps between White, middle-class students and minority, disadvantaged students. Despite consistent efforts of math reforms, these gaps in achievement have persisted. Therefore, in order to improve U.S. students' math learning, it is important to figure out what factors are significantly related to math achievement and how these factors influence math achievement of different groups.

The objective of this study was to examine the association of student math engagement, teacher quality, and school-related factors with student math achievement. Exploratory factor analysis (EFA) and confirmatory factor analysis (CFA) were used to analyze the factor structure of the three dimensions (behavioral, cognitive, and emotional) of math engagement; hierarchical linear modeling (HLM) was conducted to build models that explain the significance of student math engagement, teacher quality, and school-level factors on math achievement. This chapter presents a summary of the answers to the research questions and a discussion of the findings. In addition, significance, limitations, and implication for future research are included.

Summary of the Results

This study found that students' math engagement consisted of three dimensions–behavioral engagement, cognitive engagement, and emotional engagement, which is supported by the three-dimensional theory of student engagement (Appleton et al., 2008; Fredricks et al., 2004). Another finding was that students' emotional engagement presented a positive association with students' math achievement. At the teacher level, teacher's content knowledge showed a positive association with math achievement (Goldhaber & Brewer, 1997; Monk & King, 1994). In addition, teacher subject-matter preparation showed a positive relationship to math achievement through its interaction with students' emotional engagement, meaning that teachers who are well prepared for teaching math topics would be more likely to stimulate students' learning interests and to improve students' learning and achievement. Finally, school SES was found to be a significant factor that influences math achievement (Wenglinsky, 2001). Furthermore, the results of the study confirmed that there are significant group differences in math achievement.

Discussion and Implications

Structure of student engagement in math

Based on the results of exploratory factor analysis on a subset of the data and the follow-up confirmatory factor analysis, student engagement was indicated by three factors—behavioral, cognitive, and emotional engagement. In the CFA analysis of behavioral engagement, the first-order sixteen-item model and the first-order nine-item model were specified and compared. Based on fit indices, the first-order nine-item model was chosen as the final model to indicate behavioral engagement. Behavioral engagement

was indicated by nine items that reflected students' participation of math class activities such as "work on fractions and decimals", "memorize formulas and procedures", and "relate what we are learning in math to our daily lives" (see Table 4.7). Cognitive engagement was measured by four items that explained students' aspiration for future use and study in math such as "I need to do well in math to get the job I want", "I need math to learn other school subjects", and "I think learning math will help me in my daily life" (see Table 4.4). Emotional engagement was measured by four items that explained students' attitudes toward math such as "I like math", "I enjoy learning math", and "I would like to take more math in school" (see Table 4.5). The three factors showed relatively high internal reliabilities ranging from .70 to .86.

The three-factor model of students' math engagement in this study supports the three-dimensional conceptualization of student engagement, which is in accord with the predominant multidimensional theory of school engagement (Appleton et al., 2008; Fredricks et al., 2004). Based on the results, this study provides a well-developed instrument that could measure students' engagement in math class, which extends our understanding about students' engagement in math learning. It also provides strong evidence that students' engagement, as a global concept, can be applied to particular subjects and population.

The relationship of dimensions of engagement to math achievement

When looking at dimensions of students' engagement in math for promoting students' math achievement, it was found that students' emotional engagement in math displayed a statistically significant association with math achievement (γ_{300}=1.65, p<0.001)(from Model 6 column in Table 4.9) controlling for demographic factors

(gender, race, and parents' education). This suggests that when students show more interest and enjoyment in math learning, they would achieve higher scores in math. In addition, the variance of students' emotional engagement slope was statistically significant between classrooms, meaning that the association of students' emotional engagement with math achievement significantly varied across different classes.

This finding provides evidence in supporting previous research that students' emotional experiences are important in the learning process (Linnenbrink-Garcia & Pekrun, 2011). Emotions experienced in academic environment and classrooms have an important contribution to students' motivation and can influence academic performance (Pekrun, 2006). Promoting students' intrinsic motivation which derived from interest and pure pleasure and desire in academic settings could produce positive academic outcomes (Lepper, Corpus, & Ivengar, 2005). Conversely, negative emotions would lead to negative effects on students' academic achievement (Goetz, Pekrun, Hall, & Haag, 2006). According to Pekrun (2000), in the math and statistics courses, students' negative emotions emerge from their low self-concept, self-efficacy, and expectancy. For example, anxiety during math learning can occur when a student does not feel very competent towards the course materials. Therefore, based on the result of positive relationship between emotional engagement and math achievement, there's a need for teachers to stimulate students' positive emotions in the process of teaching. Teachers could attempt to increase the intensity of constructive emotions about a topic or subject and could integrate relevance of math learning into math curriculum. Constructive emotions are emotions that serve to focus student attention closely on the salient aspects of the subject matter (Rosiek, 2003). Teachers could choose to address students' emotional experiences

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explicitly or implicitly based on different teaching topics and activities. In addition, teaching materials prepared by teachers should be in line with students' abilities, which would help to build students' self-efficacy towards learning.

The results also showed that there was a positive relationship between students' behavioral engagement and math achievement, but the relationship was not significant. Theoretically, behavioral engagement should display significant association with math achievement (Fredricks et al., 2004; Ripski & Gregory, 2009), but the finding of this study doesn't support it. It is likely that there were high correlations among variables of behavioral, cognitive, and emotional engagement. After controlling cognitive engagement, emotional engagement, and student-level covariates, the effect of behavioral engagement on math achievement became smaller. Another reason for the non-significant result may be that behavioral engagement includes different dimensions with specific measures, but the variables derived from the data set are not valid measures of math behavioral engagement. Further research could draw on particular instruments that designed to measure math behavioral engagement to reexamine the relationship between behavioral engagement and achievement.

In addition, students' cognitive engagement showed a non-significant negative relationship to math achievement. This result fails to support previous research that cognitive engagement showed a positive relationship to achievement (Miller et al., 1996; Peterson et al., 1984). Like behavioral engagement, the non-significant result may due to the high correlations of cognitive engagement and the other two dimensions of engagement. In addition, in this study, the construct of cognitive engagement was indicated by students' values and aspirations of math. However, students' cognitive

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engagement can also be indicated by measures of learning strategies and self-regulation. The different measures used may also lead to the non-significant result.

The relationship of teacher quality and math achievement

According to the results, teachers' content coverage in math presented a positive and statistically significant association with students' math achievement when controlling for other teacher level variables and students' class mean characteristics. With one-unit increase in teacher content coverage, students' math achievement was expected to increase by a score of 0.28. This finding provides further evidence in supporting previous research that there was a strong relationship between content coverage and student achievement (Barr & Dreeben, 1983).

Teacher content coverage has been treated as an important factor that associated with students' learning. In their study, Rowan, Correnti, & Miller (2002) noted that when considering the factors that affect students' achievement performance, how teachers use the instructional time plays a more important role than the time allocation of learning particular subject or the time students are actively engaged in instruction in class. Teachers who are capable to maintain the continuity of the lesson could provide students with more opportunities to be exposed to the subject content and hence the opportunity to learn (OTL, Gillies & Quijada, 2008). In fact, previous study has showed that the math learning opportunities that students encounter were influenced by different districts and teachers (Schmidt, Cogan, Houang, & McKnight, 2011). Despite the influences of district/state curricular policies, teachers could make judgements about the teaching content, teaching time, and teaching order, which would lead to the variations in OTL that influence students' achievement. Furthermore, it is difficult for large-scale policies

such as No Child Left Behind (NCLB) to easily change the allocation of instructional time (Morton & Dalton, 2007). However, teacher content coverage is more mendable than other variables related to teacher qualification and teacher effectiveness such as pedagogical knowledge, allocation of instructional time, and teacher characteristics. It is quite manipulable to have teachers change their content coverage in the teaching process. Therefore, one implication for policy makers and educators is that teacher content coverage that is aligned with students' skill levels would facilitate student learning. Teacher preparation and teacher professional development programs could pay more attention on improving teachers' subject-matter knowledge, encouraging them to cover more topics in math teaching, thus providing more learning opportunities for students.

An interesting finding of this study was that there was an interaction effect between emotional engagement and teacher subject-matter preparation, which indicated that controlling for students' demographic factors and other teacher qualities, students who were under the instruction of well-prepared math teachers would have higher emotional engagement levels and achieve higher scores in math. This finding can be explained by the concept of emotional scaffolding (Rosiek, 2003) that teachers could use pedagogical strategies such as analogies, metaphors, and narratives to influence students' emotional response to specific aspects of the subject matter to improve student learning. In his article, Rosiek (2003) addressed a series of projects administrated by the Stanford Teacher Education Program to explore what should be included in teacher subject matter knowledge. One result of the projects was that most teachers who participated in the study reported that one important dimension of teacher content knowledge was to influence students' emotional response to an idea by scaffolding. Teachers who have

adequate knowledge of their subject matter and knowledge about the various effects on students' emotional reaction to the subject matter would be capable in transforming these knowledge into forms that are "pedagogical powerful and yet adaptive to the variations in ability and background presented by the students" (Shulman, 1987, p.15). In addition, Rosiek (2003) noted that teachers' consideration of students' emotions in class would help minority and disadvantaged students to overcome the feeling of cultural exclusion so that they are willing to participant into the learning activities in class.

In fact, there have been a lack of attention in research to the practical work of helping students build emotional relations to what they are learning (Rosiek, 2003). Except some qualitative studies, most studies only theoretically discussed about the significance of the association between students' emotions and teacher content knowledge. There is a dearth of empirical studies that show a relationship between learning and emotion states of students in the classroom. Few studies have examined the interaction effect of students' emotions and teacher content knowledge on learning achievement based on quantitative data. Based on a large data set, this study contributes to the literature by providing empirical evidence of the interactive effect of emotion and teacher content knowledge on learning achievement. In terms of implications, teacher preparation and teacher professional development programs should regard how to enhance students' emotional engagement in learning as an important part of promoting teacher subject matter knowledge.

Another finding at the teacher/class level was that there was a large compositional effect of students' mean emotional engagement on students' math achievement at the class level. When students have the same individual emotional engagement level,

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students' math score would increase by 4.19 points with every unit increase in their emotional engagement at the class level. This finding indicates that students situated in emotionally engaged classrooms could learn more than students who are situated in low emotionally engaged classrooms. In other words, students who feel a supportive and caring classroom climate would achieve more than those who are in classroom where the classroom environment is less supportive and thus less conducive to learning. Unlike individual student's emotional engagement, emotionally engaged classrooms mean that the classroom climate is harmonious, supportive, and positive so that most or all students in the classroom would be highly engaged in classroom learning activities.

Students' emotional engagement, as a classroom contextual factor, is more influential than students' individual emotional engagement level on promoting students' math achievement. Classroom has been considered as a social context for learning (Lan, Ponitz, Miller, Su, Cortina, Perry, & Fang, 2009). Previous study has noted that classrooms are communicative settings where students' behaviors are influenced by external events such as the behaviors of peers and teachers (Weinstein, 1991). Effective peer relationships and student-teacher relationships could create a social context in the classroom that prompts students to participate in learning activities and show interests in learning tasks. To create a positive classroom emotional climate, teachers should maintain a supporting and caring classroom environment. Additionally, teachers should encourage students to establish positive peer relationship that students tend to help each other and be respectful and kind to each other.

Combining the findings above together, students' emotional engagement is significantly associated with students' math achievement. It would be more beneficial

when enhancing students' emotional response to the subject matter at both the individual level and the class level. At the individual level, students' interests and enjoyment of math learning could be motivated by using instructional strategies discussed as emotional scaffolding (Rosiek, 2003); at the class level, a caring and supportive classroom environment should be built to incorporate students into the learning activities. The ways to promote emotional engagement was found to be an important part of teacher subject matter knowledge preparation.

In addition to the significant results, it was found that teacher education measured by teachers held a bachelor's degree or higher showed a negative but non-significant association with achievement. What's more, teacher experience was found not to be a significant factor for achievement, and its curvilinear relationship to achievement was non-significant either. These findings can be supported by relevant studies. The study of Croninger and his colleagues (2007) showed that there were no significant effects of teacher education and teacher experience on first-grade student achievement. In their study with NELS:88 data, Goldhaber and Brewer (1997) found that level of education was not significantly related to achievement, but a bachelor's degree or master's degree in math was significantly related to math achievement when they used the percentage of teachers with at least a master's degree as the education level. In another study, Rivkin and his colleagues (2005) found that there was no statistically significant relationship between teachers' experience and students' math achievement in elementary schools in Texas. Besides, the results of analysis of variance (ANOVA) showed that education level and teacher experience were only significantly related to math and science achievement for 12th graders (Darling-Hammond, Berry, & Thoreson, 2001). The mixed results of the

studies on teacher education and teacher experience imply that interpretation of the effects of teacher education and teacher experience on achievement should be done carefully because the results are different depending on statistical methods used, variables controlled, and population. Overall, teachers' education and experience were not found to be significant. Both of these results have been found in other studies.

Also, teacher subject knowledge preparation presented a positive but non-significant association with math achievement. In fact, at the class level, it was found that teacher subject knowledge preparation and students' emotional engagement had an interactional effect on achievement, indicating that teacher subject knowledge preparation might show an indirect effect on achievement through other factors such as students' engagement, teachers' instructional strategies, and classroom environment. Future research could examine the relationship with other methodologies such as structural equation modeling (SEM) and multilevel structural equation modeling (MSEM).

Relationship of school characteristics and math achievement

At the school level, as expected, school SES measured by the free/reduced lunch rates presented a significant negative association to students' mean math achievement at school, which means that schools with a large percentage of students of low SES would have a relatively low mean math achievement. This finding is supported by previous research. It has been shown that students' demographic factors such as race/ethnicity and family SES had significant associations with their math achievement. Social factors such as schools and classrooms also played an important role in influencing students' achievement (Schmidt et al., 2011). Students' mean math achievement varied by different schools. Such variation may be due to the inequalities in the learning opportunities

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related to content coverage (Schmidt et al., 2011). For instance, schools with low SES would have a hard time to recruit and retain highly qualified teachers. Less-qualified teachers who are not well-prepared would not be able to offer lessons that cover most math related topics, which would have influence on students' learning in math and will exacerbate the poor math achievement of students who come from lower SES communities (Mo, Singh, & Chang, 2013). It is noteworthy that many teachers who teach math in low SES urban schools may not be certified to teach math. On the other hand, schools located in the high SES districts have better work environment and school resource for teachers, which would be more likely to attract highly qualified teachers who can provide more learning opportunities to students. Therefore, in order to balance the unequal educational resource in different school districts, it is important for federal and state agencies to assist school districts in teacher preparation and teacher training programs to enhance teacher qualities.

In addition, school math resource had a positive relationship to students' math achievement, but the relationship was non-significant. In this study, the construct of school's math related resources was measured by math learning materials to assist students' math learning such as computers, calculators, and learning software. The reason for the non-significant result may be that school math resource is a less relevant determinant of academic achievement than student-level factors (Gamoran & Long. 2006). School resource may exhibit an indirect effect on achievement through influencing other factors such as students' behavioral, cognitive, and emotional engagement. Therefore, structural equation modeling (SEM) could be used in future research to future examine the relationship between school resource and achievement.

Significance of the Study

This study provides several main contributions to the literature. First, based on theory, it proposed a three-factor measurement model of math engagement, which extends the previous research and confirms the measurement model on large nationally representative data. The results provide strong empirical evidence that engagement is a multidimensional construct including behavior, cognition, and emotion (Fredricks et al., 2004). What's more, the confirmed model provides a subject-specific instrument of engagement in math learning. It could be further specified and used by other researchers and teachers as an assessment of student engagement in math classrooms. For example, with better understanding of students' math engagement, teachers are able to know which classroom activity is more efficient in improving math achievement, how students process math tasks using cognitive strategies, and what the students' emotional responses to math learning are. So the increased understanding and knowledge of dimensions of engagement could serve as references for designing appropriate classroom interventions and helping low-achieving students.

In addition, this study also expands our understanding of the differences in math achievement among classrooms and schools. Students in classrooms who were taught by teachers equipped with solid content knowledge and who could feel a supportive and caring environment would obtain higher scores in math; and schools with higher SES would achieve better mean math scores. The study included various variables and relationships in its model of math achievement. Class and school level variables were included to model the complex nested structures of the learning setting. The results of the study present a complex picture of how teacher qualities are related to students' math

engagement and how the different types of engagement in math learning lead to achievement.

This study is significant in providing practical implications. First, there's a need for teacher preparation and teacher professional development programs to pay more attention on improving teacher subject matter knowledge, which would enable teachers to cover more topics in math teaching. Also, being equipped with solid subject matter knowledge is an efficient way for teachers to have students emotionally engaged in learning activities. In addition, teachers should create a caring and supportive classroom environment in a way to promote students' engagement and learning. Finally, federal and state agencies are responsible to assist school districts in teacher preparation and teacher training programs to improve teacher qualities, thus balancing the unequal educational resources in different school districts.

Finally, the combined use of exploratory factor analysis (EFA), confirmatory factor analysis (CFA), and hierarchical linear modeling (HLM) also has methodological advantage. The findings were based on rigorous methodology. The use of EFA and CFA enabled the specification of relations between observed variables and latent constructs. These methods were efficient to identify the underlying structure of the items and the observed indicators of student engagement. CFA provides the fit indices to assess how well the estimated model fits the data. In this study, the fit indices made it possible to evaluate the adequacy of the theoretical model and to compare different models of behavioral engagement. HLM allowed simultaneous analysis of the multilevel data. The three-level hierarchical linear model used in this study explained the teachers' and schools' variability in math achievement. These methodologies in conjunction provided a

better insight into the complexity of math achievement and provided a better understanding in explaining the large variability in math learning outcomes.

Limitations of the Study

Despite some important contributions of the study, there are several limitations that should be considered. First, the data set used in this study is a secondary dataset. Secondary data set has several advantages. For instance, secondary data is available for public use, making it easier to be accessed by researchers. Researchers save time and resources when thy use extant data, instead of collecting data themselves. Furthermore, they generate new insights from previous analyses of the data (Vartanian, 2011). However, secondary data doesn't always provide measures that fit the research questions. Furthermore, when using the combination of items to create appropriate indicators drawing on secondary data, researchers must interpret the results with caution (Li, Ruiz-Primo, & Shavelson, 2006). The TIMSS data used in this study is a large nationally representative dataset that includes extensive background information of student learning, teacher instruction in math and science, and school administration. Therefore, due to the TIMSS data is a secondary data set, there were some limitations in creating composite variables and in interpreting the results. For example, in the data set, the dependent variable was students' Rasch score. However, other indicators of math achievement such as standardized math test score, grade point average, and advanced placement courses could also be used as the dependent variable in this study or could be used to create a more comprehensive measure of achievement. Likewise, there were no variables measuring students' socioeconomic status (SES) in the dataset. Therefore, the composite variable of parent's educational level was created as a covariate instead of students' SES.

Previous studies cited in the literature review showed that there could be some associations of parent's education level with students' achievement, which provides some justification for the use of this variable. In addition, the measures of students' behavioral, cognitive, and emotional engagement were derived from students' class activities in math, students' interest in math, and students' value and aspirations of math achievement. However, there can be many other measures of these dimensions of engagement. As discussed above, theoretically, behavioral engagement should display significant association with math achievement, but the findings of this study didn't support it. It is likely that the variables in the dataset are not comprehensive measures of math behavioral engagement. Behavioral engagement is a complex construct indicated by several dimensions. In this study, items of behavioral engagement derived from the dataset were related to what students do in math class. However, other measures such as participation in extracurricular activities, paying attention to the instructions, and doing homework are also considered as indicators of behavioral engagement in the literature, but there were no such items in the TIMSS survey. Likewise, the non-significant result of the effect of cognitive engagement on math achievement may also due to the measures of the original data. In this study, cognitive engagement was indicated by students' values and aspirations in math. However, students' cognitive engagement can also be indicated by measures of learning strategies and self-regulation. In addition, most measures were derived from self-reported items, which included measurement errors. Independent instruments of math engagement with high convergent and discriminant validity could be developed and examined by researchers in future studies. Despite these limitations, this

study contributes to a better understanding of math learning and has implications for practice.

The second issue that impacted the study was sample size. In order to conduct a three-level HLM analysis, there's a need to have a large sample size to examine variations that exist at each level. In this study, it was found that 88% of the variance in students' math achievement was explained by class/teacher level variables. The large amount of variance across classes suggested that math teacher quality had a strong effect on students' math achievement. However, in the dataset, the number of classes per school was too small to estimate a model with multiple random slopes at class/teacher level. There were only 1 to 3 classes that were selected in each sampled school. Therefore, there's a need for the survey designers to select more classes per school in order to investigate the variability of the classes.

Finally, the third issue is related to missing values from the data. The data for teacher and school level variables contained many missing values. Therefore, the sample size of the data for a HLM analysis is dramatically dropped when all missing cases were deleted. The reduced sample size due to missing values at class and school level data could limit the generalizability of the results. Considering these limitations of the data, future large-scale surveys should pay attention to the consistency of data structures.

Directions for Future Research

This study raises several questions for further research. First, the effect of students' behavioral engagement in math learning was found to be non-significant on math achievement. However, behavioral engagement has been found to be a crucial factor that affects achievement. As discussed in the limitations section, the non-

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significant results may be due to the measures of behavioral engagement derived from the national-level dataset. Future study could be conducted using other instruments related to math engagement to reexamine the relationship between behavioral engagement and achievement. More research is needed to parse the relationship of different dimensions of engagement to learning. There are many inconsistencies in the results of the current studies.

In addition, teacher content coverage was found to be an important factor associated with students' math achievement. However, in this study, the measure of number of topics introduced by the teacher was just one dimension of content coverage. Besides coverage of math topics, how teachers manage the difficulty of topics, teaching order, and teaching time remain to be explored. Future studies can be designed to take an in-depth look of the course content, breadth and depth of coverage of topics.

Finally, this study found that there was an interactional effect of students' emotional engagement and teacher's subject-matter preparation. Compared to those who were not well prepared, teachers who were very well prepared in teaching math topics showed a positive influence on students' emotional engagement, leading to higher students' math scores. Some covariates related to demographic factors also significantly affected math achievement. There might be mediating effects among the variables at different levels. For instance, students' demographic background such as gender, race/ethnicity, and family SES may affect math achievement through their engagement; engagement may also serve as a mediator between teacher quality and achievement. In order to further explain the complex relationships among math achievement and student engagement, teacher quality, and school factors, other statistical methods such as

structural equation modeling (SEM) or multilevel structural equation modeling (MSEM) could be applied in future study.

Conclusions

The present study expands our understanding of eighth-grade students' math achievement, students' engagement in math class, and the relationship between students' engagement, teacher qualities, and school factors and students' math learning achievement. It provided evidence that students' math achievement and the effects of various types of math engagement are varied across classes and schools due to different teacher qualities and school conditions. The results of the study revealed that emotional engagement in math learning significantly affected math achievement. This is a very significant finding and has important implications for the design of classroom instruction and pedagogy. Teacher subject-matter preparation, teacher content knowledge, and school SES also played an important role in influencing achievement. This study has implications for practice at individual, class, and school level. Enhancing engagement in math learning and increasing teacher quality for all students will have positive effect on math achievement for all students.

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Appendix A – Items used to create composite scores or categorical variables for behavioral engagement, cognitive engagement, emotional engagement, teacher subject knowledge preparation, teacher subject knowledge coverage, and school math related resources

Construct	TIMSS 2007 Item Name	Item Questions/Items	Options
1. Emotional Engagement		How much do you agree with these statements about learning mathematics?	1= Disagree a lot to 4=Agree a lot
	BS4MALIK	I like mathematics*	
	BS4MAENJ	I enjoy learning mathematics*	
	BS4MABOR	Mathematics is boring	
	BS4MAMOR	I would like to take more mathematics in school*	
2. Cognitive Engagement		How much do you agree with these statements about learning mathematics?	1= Disagree a lot to 4=Agree a lot
	BS4MAGET	I need to do well in mathematics to get the job I want*	
	BS4MAUNI	I need to do well in mathematics to get into the university or college of my choice*	
	BS4MAOSS	I need mathematics to learn other school subjects*	
	BS4MAHDL	I think learning mathematics will help me in my daily life*	
3. Behavioral Engagement		How often do you do these things in your mathematics lessons?	1= Never to 4=Every or almost every lesson
	BS4MHWFD	We work on fractions and decimals*	
	BS4MHGCT	We interpret data in tables, charts, or graphs*	
	BS4MHGSA	We solve problems about geometric shapes, lines and angles*	
	BS4MHASM	We practice adding, subtracting, multiplying, and dividing without using a calculator*	
	BS4MHEFR	We write equations and functions to represent relationships*	
	BS4MHEXP	We explain our answers*	
	BS4MHROH	We review our homework*	
	BS4MHLSP	We listen to the teacher give a lecture-style presentation*	
	BS4MHFRR	We memorize formulas and procedures*	

	BS4MHCOM	We use computers*	
	BS4MHWSG	We work together in small groups*	
	BS4MHSCP	We decide on our own procedures for solving complex problems*	
	BS4MHMDL	We relate what we are learning in mathematics to our daily lives*	
	BS4MHBHC	We begin our homework in class*	
	BS4MHHQT	We have a quiz or test*	
	BS4MHWPO	We work problems on our own*	
4.Teacher Subject-matter Preparation		How well prepared do you feel you are to teach the following topics?	1=Not applicable to 4=Very well prepared
	BT4MTT01	Computing, estimating or approximating with whole numbers*	
	BT4MTT02	Representing decimals and fractions using words, numbers, or models (including number lines)*	
	BT4MTT03	Computing with fractions and decimals*	
	BT4MTT04	Representing, comparing, ordering, and computing with integers*	
	BT4MTT05	Problem solving involving percents and proportions*	
	BT4MTT06	Numeric, algebraic, and geometric patterns or sequences (extension, missing terms, generalization of patterns)*	
	BT4MTT07	Simplifying and evaluating the algebraic expressions*	
	BT4MTT08	Simple linear equations and inequalities, and simultaneous (two variables) equations*	
	BT4MTT09	Equivalent representations of functions as ordered pairs, tables, graphs, words, or equations*	
	BT4MTT10	Geometric properties of angles and geometric shapes (triangles, quadrilaterals, and other common polygons)*	
	BT4MTT11	Congruent figures and similar triangles*	
	BT4MTT12	Relationship between three–dimensional shapes and their two-dimensional representation*	
	BT4MTT13	Using appropriate measurement formulas for perimeters, circumferences, areas of circles, surface areas and volumes*	
	BT4MTT14	Cartesian plane - ordered pairs, equations, intercepts, intersections, and gradient*	
	BT4MTT15	Translation, reflection, and rotation*	
	BT4MTT16	Reading and displaying data using tables, pictographs, bar graphs, pie charts and line graphs*	
	BT4MTT17	Interpreting data sets (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points)*	
	BT4MTT18	Judging, predicting, and determining the chances of possible outcomes*	

5.Teacher Subject Knowledge Coverage		The following list includes the main topics addressed by the TIMSS mathematics test. Choose the response that best describes when students in the TIMSS class have been taught each topic.	1=Mostly taught before this year to 3=Not yet taught or just introduced
	BT4MTP01	Whole numbers including place value, factorization, and the four operations*	
	BT4MTP02	Computations, estimations, or approximations involving whole numbers*	
	BT4MTP03	Common fractions including equivalent fractions and ordering of fractions*	
	BT4MTP04	Decimal including place value, ordering, and converting to common fractions (and vice versa)*	
	BT4MTP05	Representing decimals and fractions using words, numbers, or models (including number lines)*	
	BT4MTP06	Computations with fractions*	
	BT4MTP07	Computations with decimals*	
	BT4MTP08	Representing, comparing, ordering, and computing with integers*	
	BT4MTP09	Ratios (equivalence, division of a quantity by a given ratio)*	
	BT4MTP10	Conversion of percents to fractions or decimals and vice versa*	
	BT4MTP11	Numeric, algebraic, and geometric patterns or sequences (extension, missing terms, generalization of patterns)*	
	BT4MTP12	Sums, products, and powers of expressions containing variables*	
	BT4MTP13	Evaluating expressions for given numeric value*	
	BT4MTP14	Simplifying or comparing algebraic expressions*	
	BT4MTP15	Modeling situations using expressions*	
	BT4MTP16	Evaluating functions/formulas for given values of the variables*	
	BT4MTP17	Simple linear equations and inequalities, and simultaneous (two variables) equations*	
	BT4MTP18	Equivalent representations of functions as ordered pairs, tables, graphs, words, or equations*	
	BT4MTP19	Angles - acute, right, straight, obtuse, reflex*	
	BT4MTP20	Relationships for angles at a point, angles on a line, vertically opposite angles, angles associated with a transversal cutting parallel lines, and perpendicularity*	
	BT4MTP21	Properties of geometric shapes: triangles, quadrilaterals, and other common polygons*	
	BT4MTP22	Construct or draw triangles and rectangles of given dimensions*	
	BT4MTP23	Congruent figures (triangles, quadrilaterals) and their corresponding measures*	
	BT4MTP24	Similar triangles and recall their properties*	

	BT4MTP25	Relationships between two-dimensional and three-dimensional shapes*	
	BT4MTP26	Pythagorean theorem (not proof) to find length of a side*	
	BT4MTP27	Measurement, drawing, and estimation of the size of angles, the lengths of lines, areas, and volumes*	
	BT4MTP28	Measurement formulas for perimeters, circumferences, areas of circles, surface areas, and volumes*	
	BT4MTP29	Measures of irregular or compound areas (e.g., by covering with grids or dissecting and rearranging pieces)*	
	BT4MTP30	Cartesian plane - ordered pairs, equations, intercepts, intersections, and gradient*	
	BT4MTP31	Line and rotational symmetry for two-dimensional shapes*	
	BT4MTP32	Translation, reflection, and rotation*	
	BT4MTP33	Reading data from tables, pictographs, bar graphs, pie charts, and line graphs*	
	BT4MTP34	Organizing and displaying data using tables, pictographs, bar graphs, pie charts, and line graphs*	
	BT4MTP35	Characteristics of data sets including mean, median, range, and shape of distribution (in general terms)*	
	BT4MTP36	Interpreting data sets (e.g., draw conclusions, make predictions, and estimate values between and beyond given data points)*	
	BT4MTP37	Data displays that could lead to misinterpretation (e.g., inappropriate grouping and misleading or distorted scales)*	
	BT4MTP38	Using data from experiments to predict chances of future outcomes*	
	BT4MTP39	Using the chances of a particular outcome to solve problems*	
6.School Math Related Resources		How much is your school's capacity to provide instruction affected by a shortage or inadequacy of any of the following?	1=Not at all to 4=A lot
	BC4MST07	Computers for mathematics instruction*	
	BC4MST08	Computer software for mathematics instruction*	
	BC4MST09	Calculators for mathematics instruction*	
	BC4MST10	Library materials relevant to mathematics instruction*	
	BC4MST11	Audio-visual resources for mathematics instruction*	

* Reverse coded