

Research Article

Quasi-Brittle Fracture Modeling of Preflawned Bitumen Using a Diffuse Interface Model

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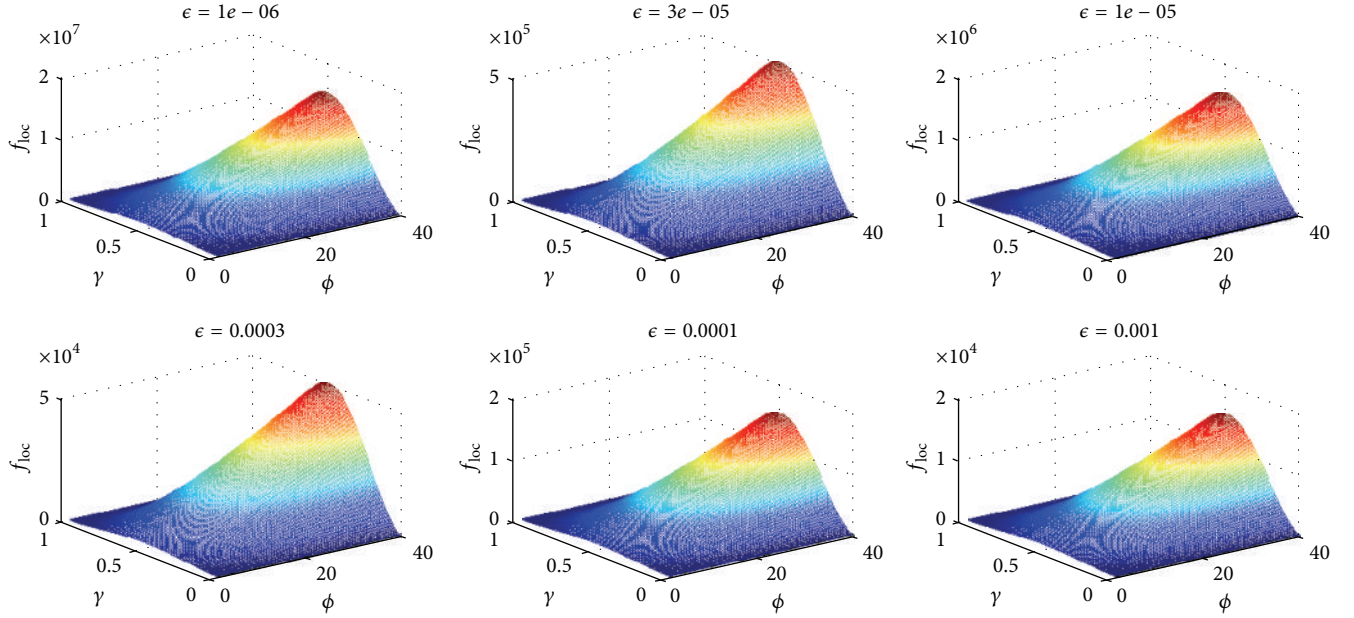
Fundamental understandings on the bitumen fracture mechanism are vital to improve the mixture design of asphalt concrete. In this paper, a diffuse interface model, namely, phase-field method is used for modeling the quasi-brittle fracture in bitumen. This method describes the microstructure using a phase-field variable which assumes one in the intact solid and negative one in the crack region. Only the elastic energy will directly contribute to cracking. To account for the growth of cracks, a nonconserved Allen-Cahn equation is adopted to evolve the phase-field variable. Numerical simulations of fracture are performed in bituminous materials with the consideration of quasi-brittle properties. It is found that the simulation results agree well with classic fracture mechanics.

1. Introduction

One of the most serious distresses in pavement structure is brittle cracking in bitumen [1, 2]. The Strategic Highway Research Program (SHRP) realized the importance of the cracking properties of bitumen for the first time [3]. Aya-tollahi and Pirmohammad (2013) studied the temperature effects on brittle fracture in cracked asphalt concrete [4]. Rowe et al. (2014) conducted researches on the influence of binder rheology on the cracking of asphalt mixes in airport and highway projects [5]. Dave et al. (2007) studied the thermal reflective cracking of asphalt concrete overlays [6]. Birgisson et al. (2002) predicted quasi-brittle response and crack growth in asphalt mixture with the Boundary Element Method [7]. Ameri et al. (2011) calculated the stress intensity factor in cracked asphalt pavement under traffic loading using 3D finite element analysis [8]. Tan and Guo (2014) studied the interfacial interaction between asphalt and mineral fillers and the influence on asphalt cracking [9]. Hou et al. (2014) conducted researches on the mixed mode fracture failure of asphalt binder at low temperature using the phase-field approach [10]. Feng et al. (2007) studied the anticracking

performance of the ATB30 asphalt treated base [11]. Mun and Lee (2011) modeled quasi-brittle crack growth in hot-mix asphaltic concrete mixtures using a disk-shaped compact tension test [12]. Zhao and Zhang (2010) conducted research on low temperature cracking of asphalt pavement based on Cohesive Zone Model [13]. Almost all these analyses are based on classic fracture mechanics first proposed by Griffith [14], which needs to clearly depict the crack front conditions, and thus may be very complicated. In order to consider the quasi-brittle properties of bitumen concisely, phase-field method is used in this paper. Besides, although the authors have previously analyzed the asphalt cracking using phase-field method [15], there still have been two limits: (1) the previous research is focused on the asphalt linear behavior and does not consider the quasi-brittle behavior; (2) there lacks analytical solutions on the stress during cracking.

The phase-field method (PFM) was originally proposed by Cahn and Hilliard [16]. In this model, a phase-field variable is introduced to identify different phases. In this paper, the development of an Allen-Cahn phase-field model is implemented in the finite element software COMSOL. The model was validated using the two-dimensional simulations

FIGURE 1: Local free energy density with respect to different ϵ .

of model I quasi-brittle fracture at one fixed temperature and comparison with Griffith's theory.

2. Theoretical Analysis

2.1. Free Energy in Quasi-Brittle Materials. In the simulation of preflawed bitumen cracking, the phase-field variable is set as $\phi = -1$ for the broken part and $\phi = +1$ for the unbroken part. The total free energy in the cracking system is

$$F = \int_{\Omega} (f_{gr} + f_{loc} + f_{el}) dV, \quad (1)$$

where f_{el} is the elastic energy density, $f_{gr} = 3\gamma\epsilon(\nabla\phi)^2/2$ is the gradient energy density, and $f_{loc} = 6\gamma\phi^2(1-\phi)^2/\epsilon$ is the local free energy density (double well potential). The rationale of free energy composition in asphalt material is well explained in Hou et al. (2014) [10]. γ is the surface free energy; and ϵ the interface coefficient.

Figure 1 shows the double well potential function with respect to different ϵ . It is observed that the smaller the interface coefficient ϵ is, the larger the peak value of f_{loc} is. Also, note that curvature is significantly larger for a large surface free energy γ , which indicates that bitumen has more cracking resistance for a large γ .

Figure 2 shows the double well potential function with respect to different γ . It is observed that the larger the surface free energy γ is, the larger the peak value of f_{loc} is. Also note for a fixed γ value, there exist two minima at $\phi = 0$ and $\phi = 1$, which indicates only these two states are metastable.

Figure 3 shows the double well potential function with respect to different ϕ . Note that there are three important states: $\phi = 0$ represents the cracking state, $\phi = 0.5$ represents the interface state, and $\phi = 1$ represents the unbroken state.

For these three states, the system is metastable; that is, $f_{loc} = 0$. Other than these states, the system is unstable.

Actually, for bitumen and bituminous materials, the materials are quasi-brittle since the visco- and plastic properties cannot be neglected no matter how small their magnitudes are. The cracking process is always initiated by the local plastic deformation at the crack tip. The plastic work needs to be done first before crack can advance to create new surfaces. The total stress is thus considered as the sum of the elastic part, the viscous part and, the plastic part as

$$\sigma_{ik}^{total} = \sigma_{ik}^{el}(\epsilon_{ik}) + \sigma_{ik}^{vis}(\dot{\epsilon}_{ik}) + \sigma_{ik}^{pl}. \quad (2)$$

For low temperature cracking on bitumen and bituminous materials, the latter two terms in (2) can be neglected for approximation.

The elastic part can be obtained according to Hooke's law as

$$\sigma_{ik}^{el} = \frac{E}{1+\nu} \left(\epsilon_{ik} + \frac{1}{1-2\nu} \delta_{ik} \epsilon_{ii} \right), \quad (3)$$

where δ_{ik} is Kronecker delta.

The viscous part could similarly be obtained as [17]

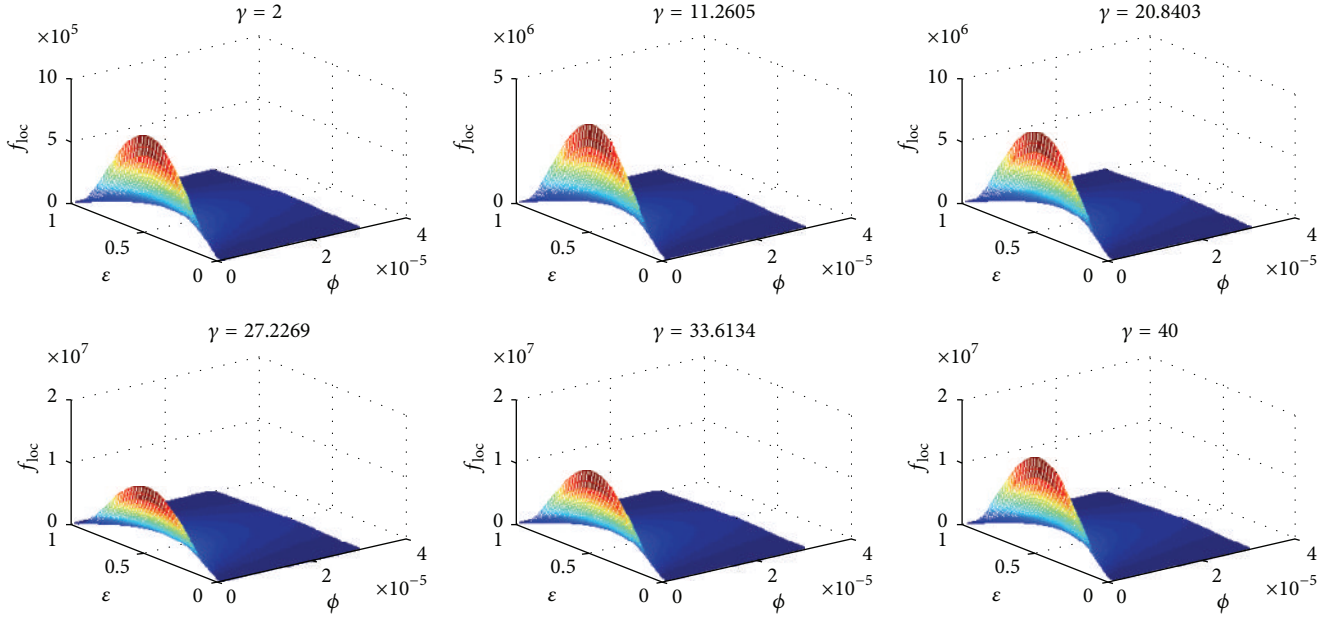
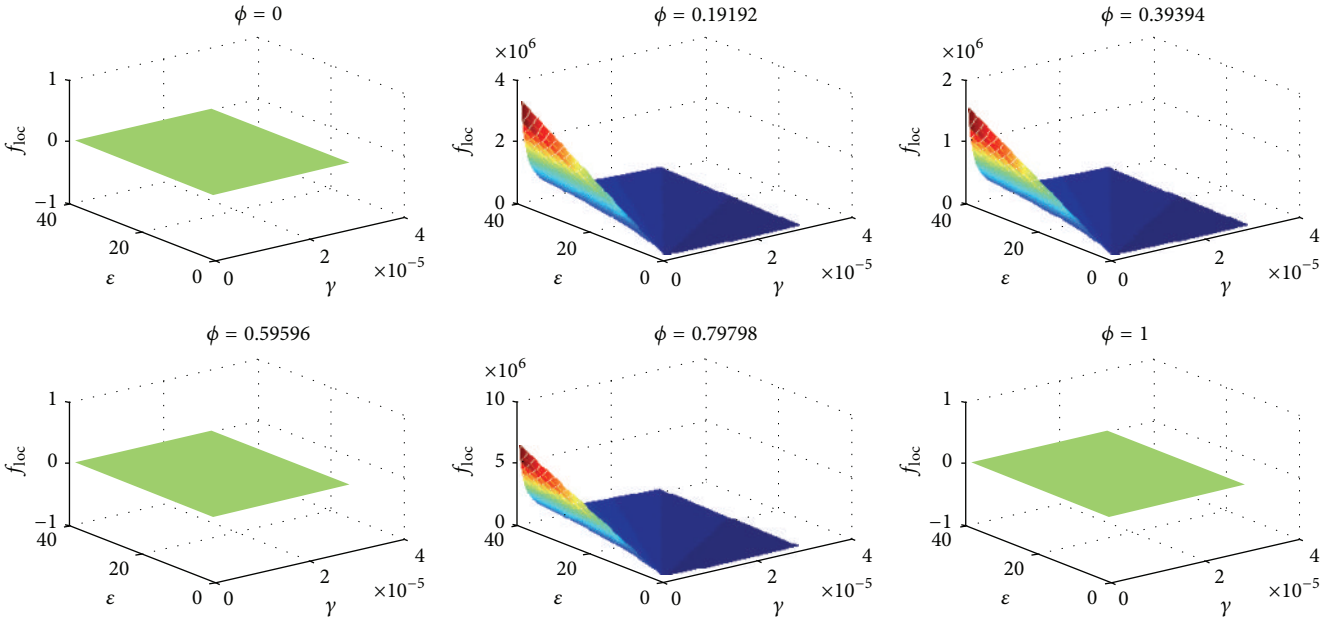
$$\sigma_{ik}^{vis} = \frac{\eta}{1+\zeta} \left(\dot{\epsilon}_{ik} + \frac{1}{1-2\zeta} \delta_{ik} \dot{\epsilon}_{ii} \right), \quad (4)$$

where η and ζ are two viscosity constants defined similar to the elastic constants in (3).

The plastic stress is obtained as

$$\sigma_{ik}^{pl} = \sqrt{\frac{1}{2} \left[(\sigma_x - \sigma_y)^2 + (\sigma_x - \sigma_z)^2 + (\sigma_y - \sigma_z)^2 \right]}, \quad (5)$$

where σ_x , σ_y , and σ_z are the localized stress.

FIGURE 2: Local free energy density with respect to different γ .FIGURE 3: Local free energy density with respect to different ϕ .

2.2. Momentum Equations. The evolution of the stress fields is determined by the principle of momentum conservation:

$$\rho \ddot{u}_i = \frac{\partial}{\partial x_i} (\sigma_{ik}^{\text{el}} + \sigma_{ik}^{\text{vis}} + \sigma_{ik}^{\text{pl}}). \quad (6)$$

In order to implement phase-field in the stress field, the elastic modulus should be modified as

$$\sigma_{ik} = \frac{E(\phi)}{1 + \nu} \left(\varepsilon_{ik} + \frac{1}{1 - 2\nu} \delta_{ik} \varepsilon_{ii} \right), \quad (7)$$

where $E(\phi)$ is the elastic modulus and ν is Poisson's ratio. And $E(\phi) = E + (E - E_0)(-(1/4)\phi^3 + (3/4)\phi + (1/2))$, where E and E_0 are the elastic moduli of the bitumen and the broken part, respectively.

Note that the viscous part and plastic part almost have no contributions to quasi-brittle cracking. Allen-Cahn equation is employed as the governing equation:

$$\frac{\partial \phi}{\partial t} = -\frac{D}{3\gamma\epsilon} \psi, \quad (8)$$

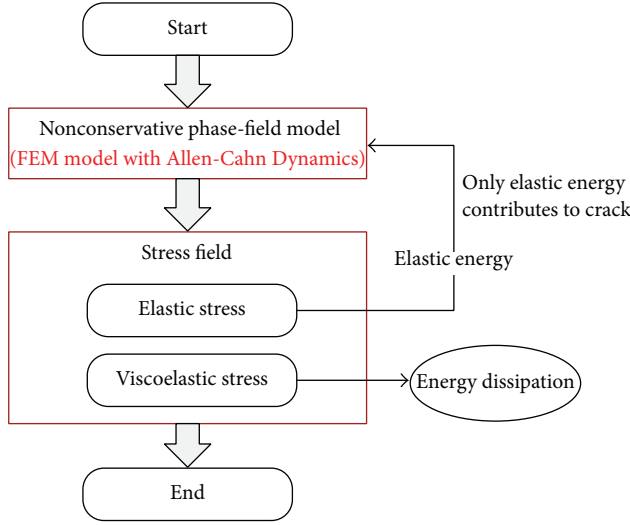


FIGURE 4: Quasi-brittle cracking calculation flowchart.



FIGURE 5: Asphalt mixture with coarse aggregates.

where D corresponds to the kinetic coefficient and ψ is the chemical potential:

$$\psi = \frac{\delta F}{\delta \phi} = -\nabla \cdot \lambda \nabla \phi + \frac{\lambda}{\epsilon^2} (\phi^2 - 1) \phi + \frac{\partial f_{el}}{\partial \phi}. \quad (9)$$

Figure 4 shows the whole computation scheme.

3. Results

3.1. Applicability. The derivations of our previous theoretical analysis based on PFM lies in that bituminous material can be considered isotropic and homogenous. However, in reality, there will exist large coarse aggregates in asphalt mixture, as shown in Figure 5. In this case, the accuracy of PFM will be reduced. Nevertheless, if the asphalt concrete could be approximately considered as isotropic and the crack goes through the bitumen part other than through the aggregate, the PFM theory can still be used for approximation.

The cracking simulation using the diffuse interface approach used in this paper could be not only good for preflawed bitumen quasi-brittle cracking but also good for bituminous mixture using fine aggregates (Figure 6(a)). This method will not have high accuracy for bituminous mixture using coarse aggregates (Figure 6(b)).

3.2. Theoretical Stress Analysis. The authors have previously analyzed the asphalt binder cracking using phase-field method at low temperature [15]. However, there still have been two limits: (1) the previous research is focused on the asphalt linear behavior and does not consider the quasi-brittle behavior; (2) there lacks analytical solutions on the stress during cracking. To solve the second problem, the following equations are derived.

Consider the preflaw in bitumen specimen as a rectangular crack with depth w and width d . The analytical expressions for stress are first obtained. For brittle cracking, Akono et al. (2011) suggested the specific J -integral expression for a rectangular crack propagation as [18]

$$J = \frac{1}{p} \int_A \left(W n_x - t_i \frac{\partial u_i}{\partial x} \right) dA, \quad (10)$$

where $p = w + 2d$ is the perimeter edge of the crack. w is the crack width and d is crack depth. Figure 7 shows the cracking contour. Note that in our research $\sigma_{xx} = 0$, and all of the elastic strain energy is contributed by σ_{yy} which is different from Akono et al.'s previous work [18].

Based on Hooke's law, we further have elastic strain energy density as $W = (\kappa/2E)\sigma_{yy}^2$; $dA = (w + 2d)ds$, and $ds = wdz$. Since the J -integral only has physical meaning when crack begins to propagate, we substitute the critical value $F = F_0$ into (11) and we have $\sigma_{yy} = F_0/(S - l_0 d)$.

Since there is no traction on the fracture surface, we thus have

$$J = \frac{1}{w + 2d} \int_0^d W w dz = \frac{\kappa w d \sigma_{yy}^2}{2E(w + 2d)}, \quad (11)$$

where $\kappa = 1 - \nu^2$ for plane stress and $\kappa = 1$ for plane strain.

Also, note that, during the linear elastic cracking process, the elastic strain energy will be totally transformed to the fracture energy G ; that is,

$$J = G = \frac{\kappa K_C^2}{E}. \quad (12)$$

Compare (11) and (12), we now have

$$\sigma_{yy} = \sqrt{2} K_C \sqrt{\frac{1}{d} + \frac{2}{w}}, \quad (13)$$

$$\sigma_{yy} = k\sigma = \frac{kF}{S}, \quad (14)$$

where k is the stress concentration factor due to the existence of visco- and plastic properties in bitumen.

Equation (14) indicates the relationship between the pre-made crack dimensions and stress based on the assumption of linear fracture mechanics, which needs to be further modified if the bitumen cracking is not quasi-brittle. It is observed that, for a given K_C , the smaller the initial crack is, the larger the stress is.

3.3. Numerical Simulations. A two-dimensional finite element model is established in COMSOL to simulate the quasi-brittle cracking process of bitumen under tension loading.



FIGURE 6: (a) Fine aggregates and (b) coarse aggregates.

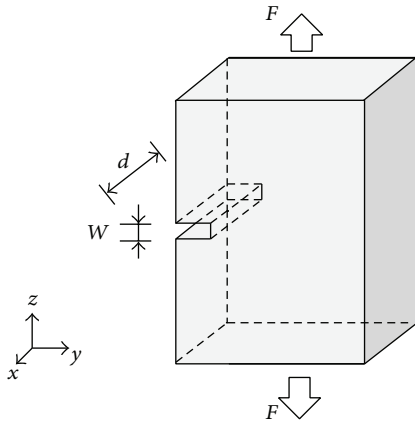


FIGURE 7: Cracking contour.

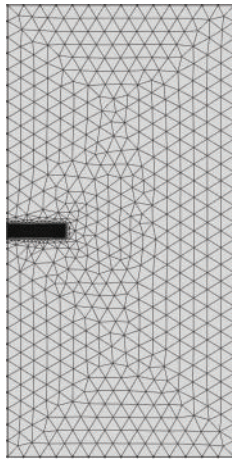


FIGURE 8: Fixed Eulerian mesh in preflawed bitumen model.

A fixed Eulerian mesh is used to describe the internal interfaces between the intact solid and crack void, as shown in Figure 8.

Actually the overall cracking process in preflawed bitumen should be simulated in three dimensions. However, it will be very costly to do such simulation. Previous researches

[10, 19] show that two-dimensional simplification, that is, plain strain or plain stress, will have sufficient accuracy for small size bitumen specimen. In our case, consider that the strain on the z direction almost remains the same during cracking, and the plain strain assumption is used for simplification and the computational domain is a rectangular with dimensions 0.04×0.02 m. The thickness of the preflawed bitumen model is set as 0.01 m. The interfacial thickness is $\epsilon = 0.001$ m and the initial crack length is $a = 0.005$ m. The finite crack width is $W = 0.001$ m. The six-node triangular elements (P2 element) are used in the finite element calculations. 78,061 elements are used in the initial mesh. To solve the phase-field variable ϕ across the interface conveniently, the self-adaptive meshing is employed. The whole computation is conducted on a PC with Intel Core i7-4702HQ CPU, which takes about 2 hours computational time. It is expected that the smaller the interface coefficient is and the more the meshing is used, the more accurate the results will be.

Pure tension loading is applied on top and bottom boundaries, which is set as $\sigma(t) = (\sigma_0/t_0) \cdot t$ where $\sigma_0/t_0 = 2 \times 10^6$ N/m²s. The time step Δt is set as 0.1 s, considering our computational ability.

To study the quasi-brittle cracking behavior and viscoelastic cracking behavior on bitumen, two simulations are conducted simultaneously. The first is to consider the bitumen as linear elastic, while the second is to consider bitumen as viscoelastic plastic.

Case 1. Quasi-brittle cracking: the following material properties are used: $\gamma = 183.77$ J/m², $E = 5.8$ MPa, and Poisson's ratio $\nu = 0.3$.

Case 2. Viscoelastic cracking is used to reflect the viscoelastic property of bitumen.

In order to study the differences between quasi-brittle cracking and viscoelastic cracking in our phase-field model on bitumen, the fracture results are first calculated and compared to classic fracture mechanics, where the Griffith criterion is $K_I \geq K_{IC}$, K_I is the mode I stress intensity factor and K_{IC} is the mode I fracture toughness which is a material parameter.

For bitumen at low temperature, $K_{IC} = 48 \text{ kNm}^{-3/2}$ by Ponniah et al. [20] is selected as fracture toughness. The elastic modulus of bitumen specimen is 5.8 MPa, which corresponds to a fracture energy $G_C = 368 \text{ J/m}^2$.

The critical load calculation based on the classic fracture mechanics without considering viscous property can be calculated as 313310.96 Pa. In phase-field method, the load value at the time instant when ϕ reaches -1 is considered to be the critical load.

For Case 1, where bitumen brittle-cracking occurs, our simulation results show that at time step = 16 Δt critical load happens, which corresponds to a load as 320000 Pa.

For Case 2, where bitumen brittle-cracking occurs, our simulation results show that at time step = 31 Δt critical load happens, which corresponds to a load as 620000 Pa.

The reason why the critical load in quasi-brittle phase-field model is much smaller than that in the viscoelastic model is that only the elastic stress part will contribute to fracture rather than the total stress (including elastic stress, viscous stress, and plastic stress) during brittle-cracking. It means bitumen has more resistance to fracture when considering viscoelastic properties than pure brittle linear elastic.

For infinitesimal deformation during bitumen brittle cracking, Cauchy stress is the same as Piola-Kirchhoff stress, which indicates the pure brittle cracking. However, considering the “quasi-brittle” property, finite deformations will occur at crack tip, and the second Piola-Kirchhoff stress is used to express the stress with Gauss-point evaluation. For the x component of second Piola-Kirchhoff stress at $t = 1.6 \text{ s}$ at the crack tip, the calculation result is $1.7 \times 10^6 \text{ pa}$, which is very large compared with other regions.

4. Summary

In this paper, we present a phase-field model to analyze the quasi-brittle fracture in bitumen subject to tension loading. In phase-field model, only the elastic stress will contribute to the fracture. Overall, compared with the classic fracture mechanics, the phase-field model does not need to explicitly treat the crack surface and can easily solve the quasi-brittle cracking of preflawed bitumen specimen.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

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