# Characterizing Long-Time Variations in Fully Developed Wind-Turbine Array Boundary-Layers using Proper Orthogonal Decomposition

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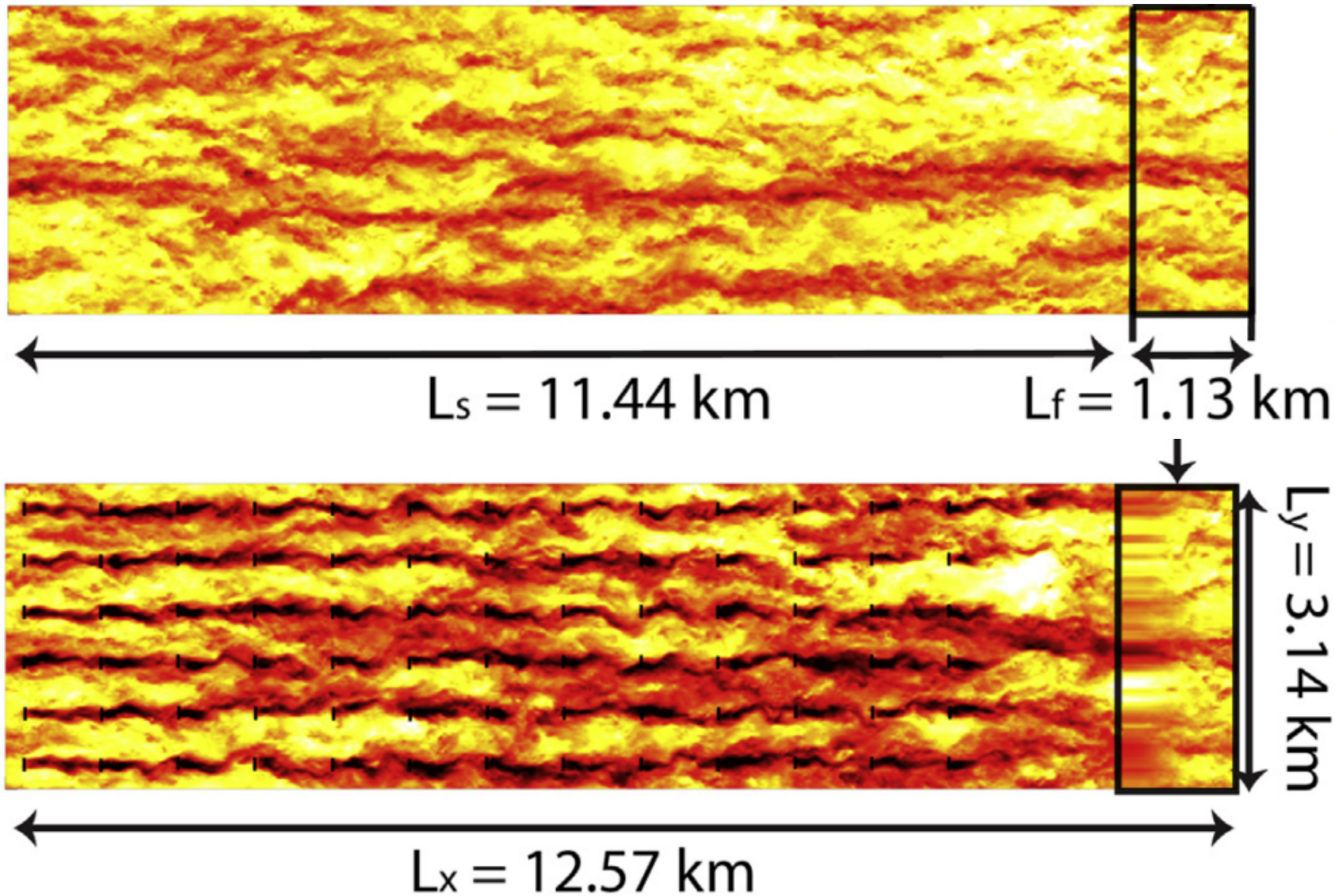
NSF IIA 1243482

# Fully developed wind-turbine array boundary-layers

Photograph by Christian Steiness of UniFly A/S on February 12, 2008

# Large-scale variations in the ABL



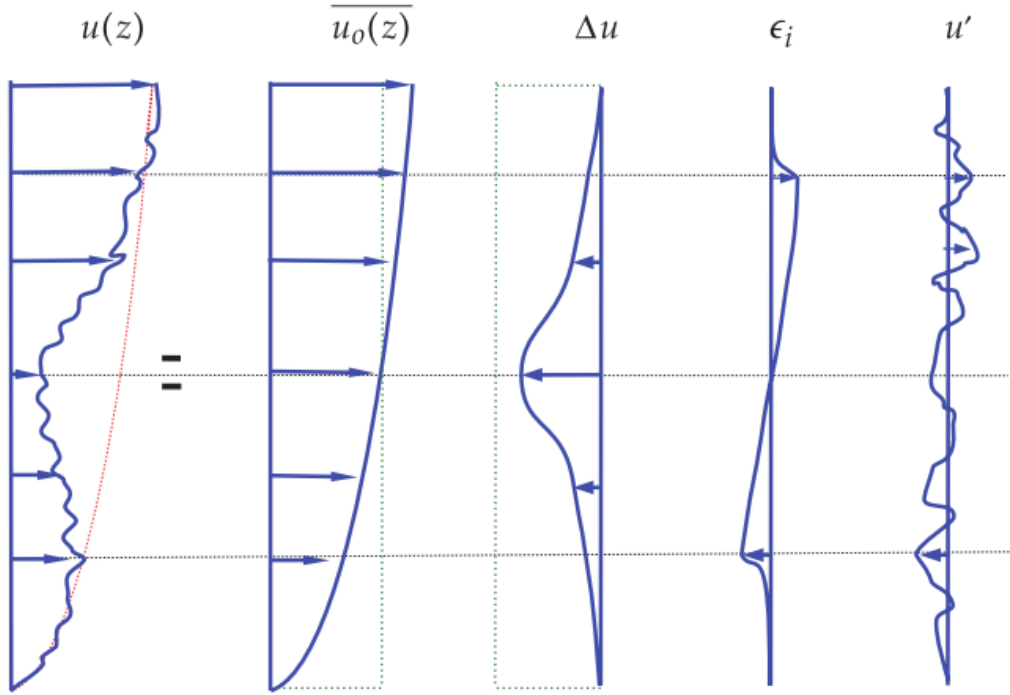

*R.J.A.M. Stevens et al., Renewable Energy 68 (2014) 46–50*

# Sample basis

**Degrees of freedom:**

- LES: 1+ million gridpoints for each variable at each time
- The real physical system: ?? (not so many - repeated info)

**Example of a simplified, intuitive basis:**




*Trujillo et al., "Light detection and ranging measurements of wake dynamics. Part II: Two-dimensional scanning," Wind Energy, 2011.*

# A new basis from POD

**Use such a basis to measure effect of the following:**

- Modeling scheme
- Wind speed and direction
- Ambient turbulence
- Atmospheric stability
- ...

**Compared to the sample basis (prev. slide), the POD basis is unambiguous and optimal (TKE capture), though less intuitive.**

**Possible uses**:

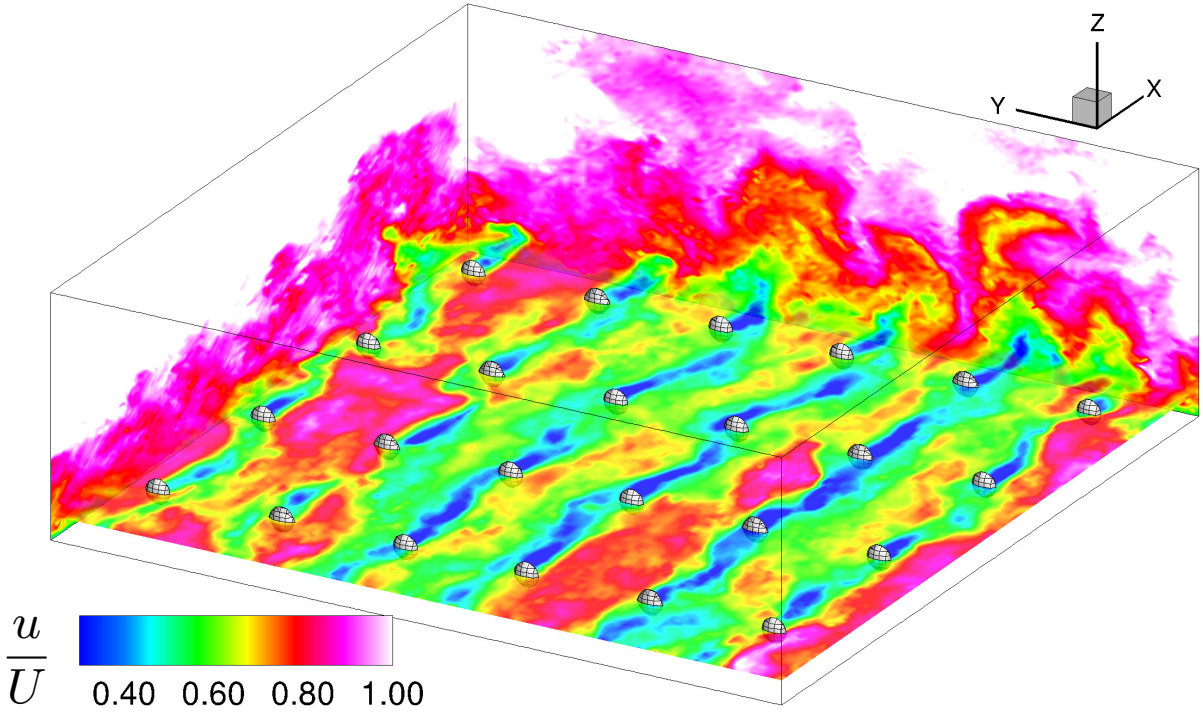
- Validation of code/experiments (captures dynamics)
- Evaluation of the influence of a parameter or operating condition
- Identify flow structures

# Outline

- ☐ **Review POD of 3D LES fields**
- ☐ Calculation of the POD mode time-coefficients
- ☐ How well do the POD modes reconstruct the velocity fields?
- ☐ How does each category of POD mode represent the flow?
- ☐ Summary and future directions

# LES dataset (input to the POD)

The following image is an instantaneous snapshot of streamwise velocity in a fully developed wind farm LES:




We see e.g. large-scale structures in the atmospheric flow and high- and low-speed streaks between turbine columns.

Input to the POD is a set of thousands of these 3D velocity fields.

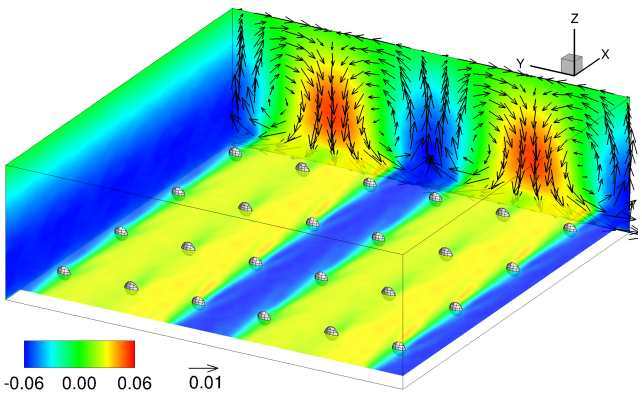
# 3D Shapshot Proper Orthogonal Decomposition (POD)

The POD method is based on the following decomposition:

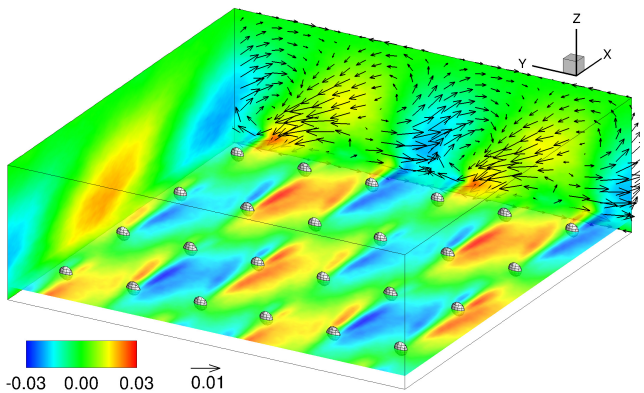
$$u_i(\mathbf{x},t) = \bar{u}_i(\mathbf{x}) + \sum_{k=1}^{N} a^k(t)\psi_i^k(\mathbf{x})$$

- *POD Input*: $u_i(\mathbf{x},t)$ is a single "snapshot" (3D velocity field)
- *POD Input*: $\bar{u}_i(\mathbf{x})$ is the time-averaged velocity
- *POD Input*: $N$ is the total number of snapshots
- *POD Output*: $a^k(t)$ is the time coefficient for the $k$th POD mode
- *POD Output*: $\psi_i^k(\mathbf{x})$ is the $k$th POD mode

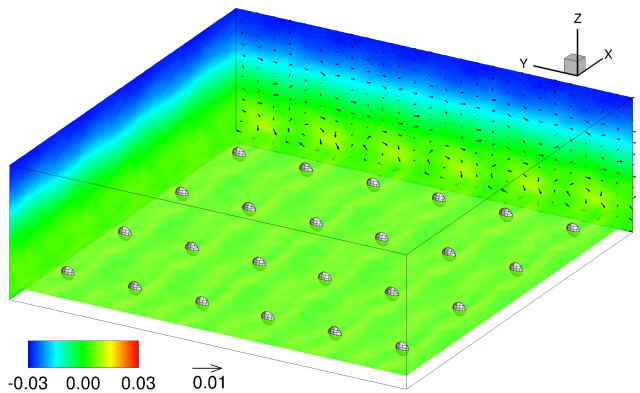
The POD modes are an optimal basis for the fluctuating velocity field.




roller mode type




streamwise-varying type



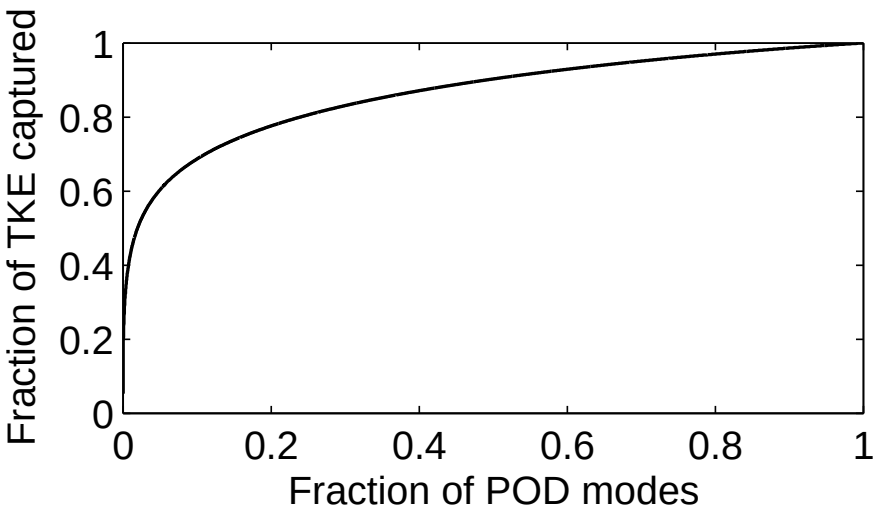

shear mode type

*VerHulst and Meneveau, Physics of Fluids, 025113 (2014)*

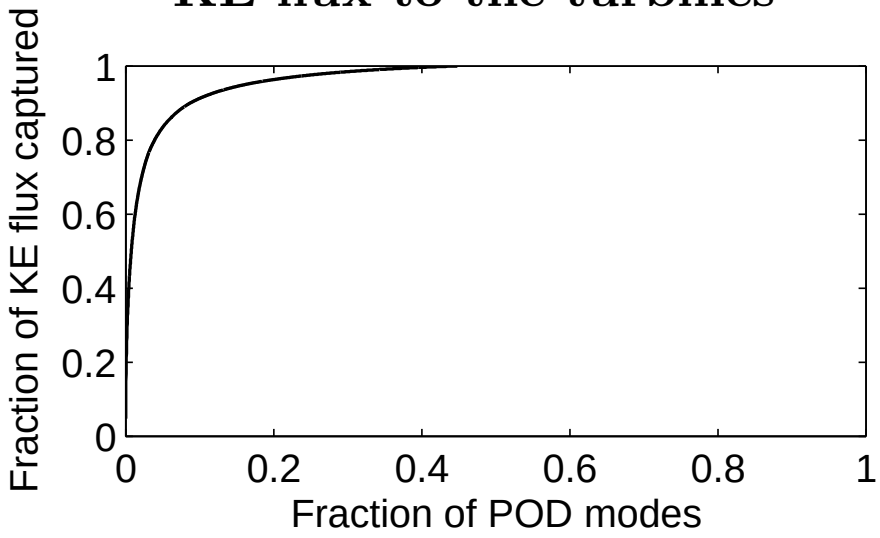
# POD mode strength

The mode's strength, $\overline{a^k(t)a^k(t)} = \lambda^k$, represents average contribution to TKE in the domain. The modes are ordered so $\lambda^k > \lambda^{k+1}$.

**20% of modes capture 80% of the TKE in the domain**




**6% of modes capture 80% KE flux to the turbines**




But the mode strength says nothing about *when* a POD mode is strongly present in the flow, or the *time scales* over which it varies in magnitude.

*VerHulst and Meneveau, Physics of Fluids, 025113 (2014)*

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# Calculation of the POD mode time-coefficients

Recall the decomposition:

$$u_i'(\mathbf{x},t) = u_i(\mathbf{x},t) - \bar{u}_i(\mathbf{x}) = \sum_{k=1}^{N} a^k(t)\psi_i^k(\mathbf{x}).$$

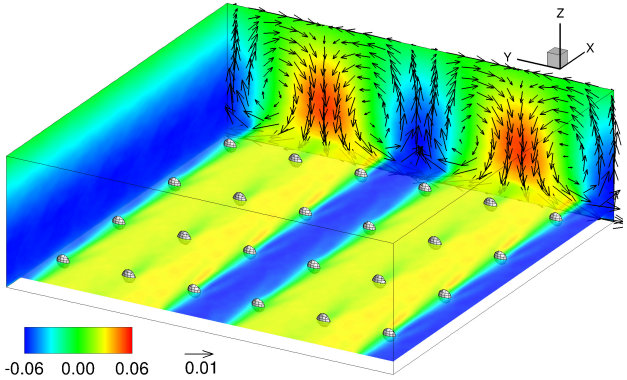
Note also that the POD modes are ortho-normal: $\langle \psi_i^k(\mathbf{x})\psi_i^l(\mathbf{x})\rangle_{xyz} = \delta_{kl}$.

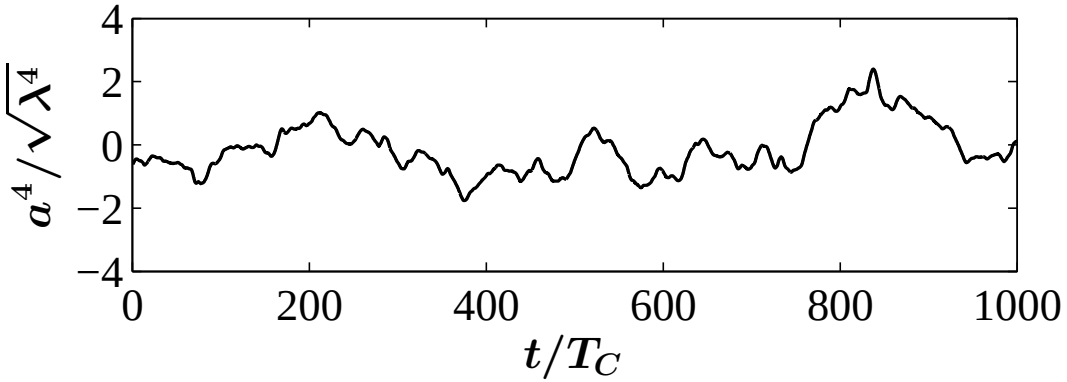
The coefficients $a^k(t)$ are calculated by projecting the fluctuating velocity field onto each POD mode:

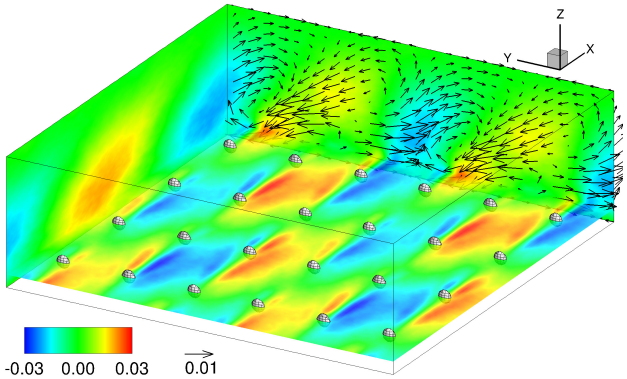
$$a^k(t) = \langle \psi_i^k(\mathbf{x})u_i'(\mathbf{x},t)\rangle_{xyz}.$$

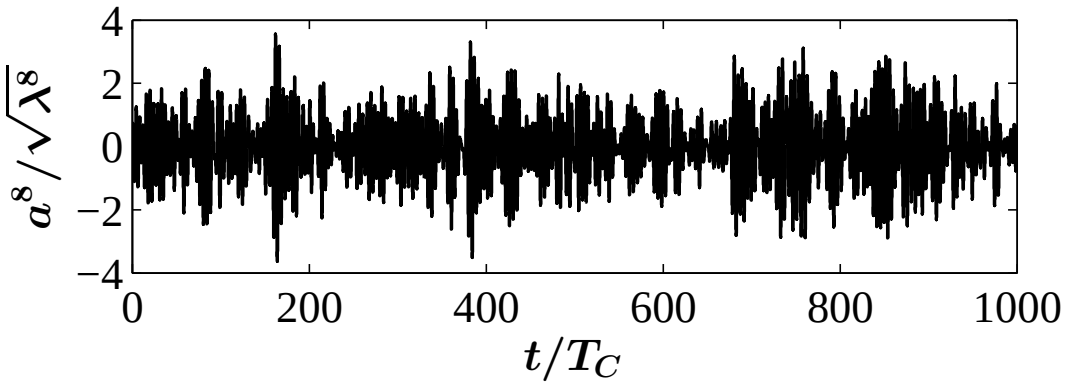
# POD time coefficients, three examples

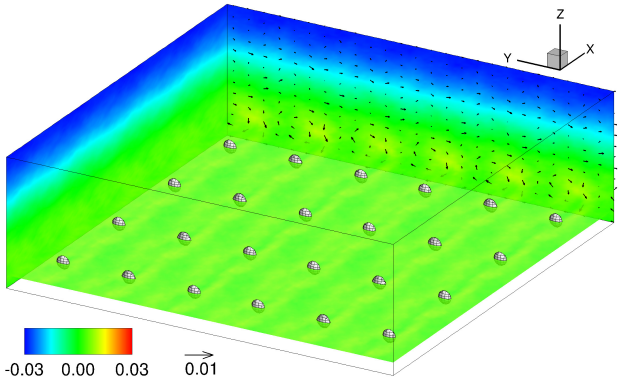
Normalized $a^k(t)$ for three modes (with $T_C$ avg. inter-row convective time):

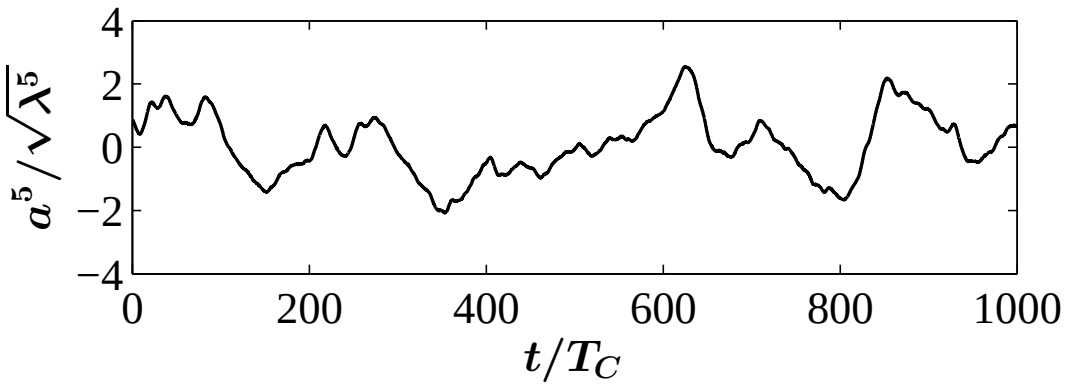


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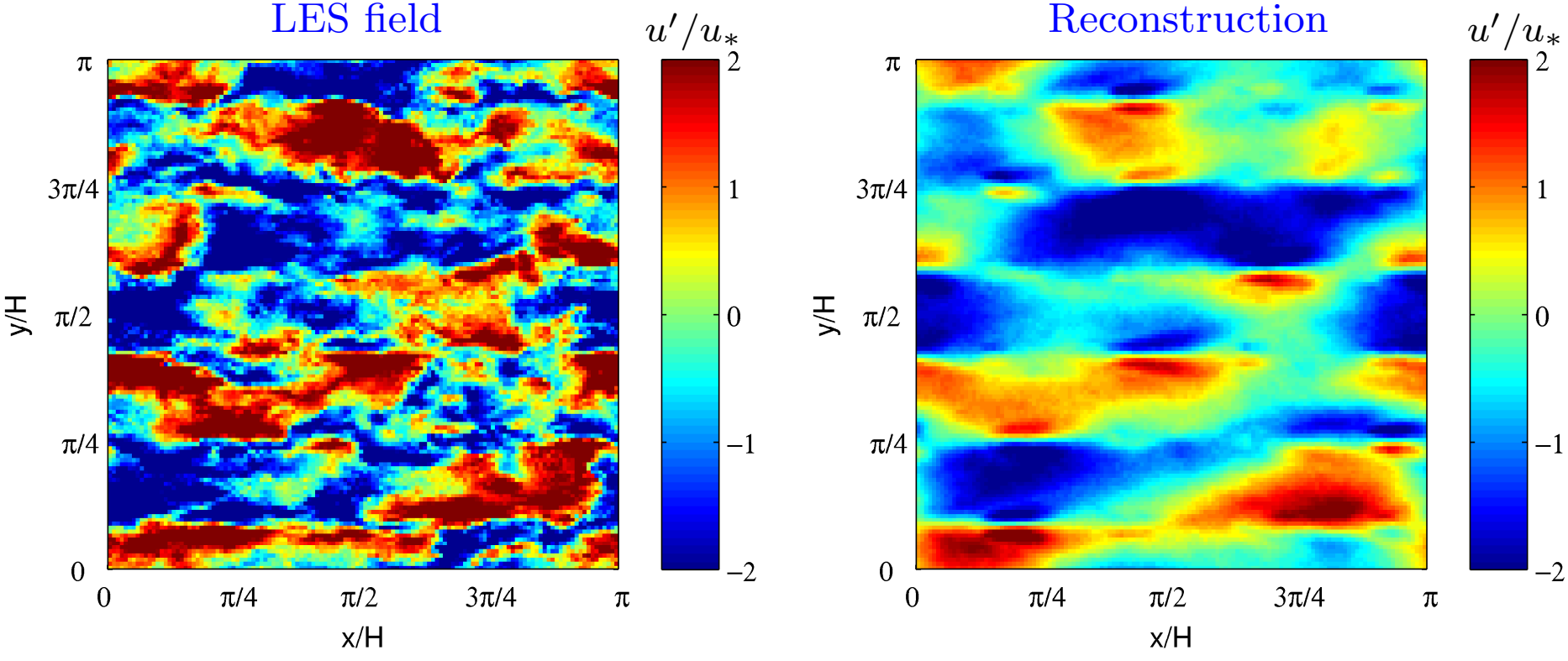
# Partial reconstructions at hub height

Instantaneous reconstruction of streamwise velocity field at hub height

$$u'(\mathbf{x},t) = u(\mathbf{x},t) - \overline{u}(\mathbf{x}) \approx \sum_{k=1}^{50} a^k(t)\psi_u^k(\mathbf{x})$$



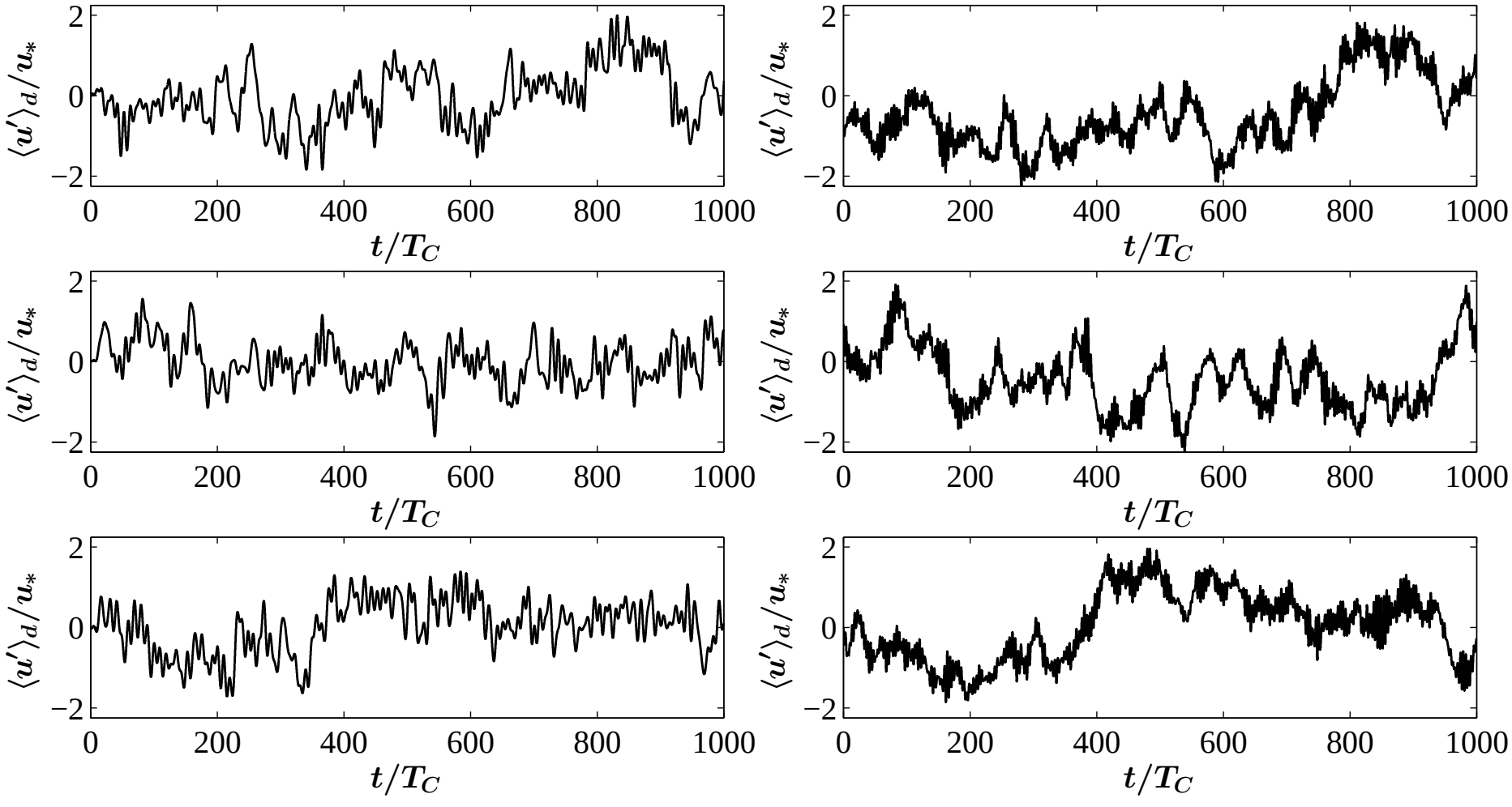

POD modes 1-50 capture some large-scale features in the atmospheric flow but miss small-scale fluctuations.

# Reconstructions of disk velocity using 20 POD modes (0.3% of total)

$$\langle u'(\mathbf{x},t)\rangle_d \approx \sum_{k=1}^{20} a^k(t)\langle \psi_i^k(\mathbf{x})\rangle_d$$

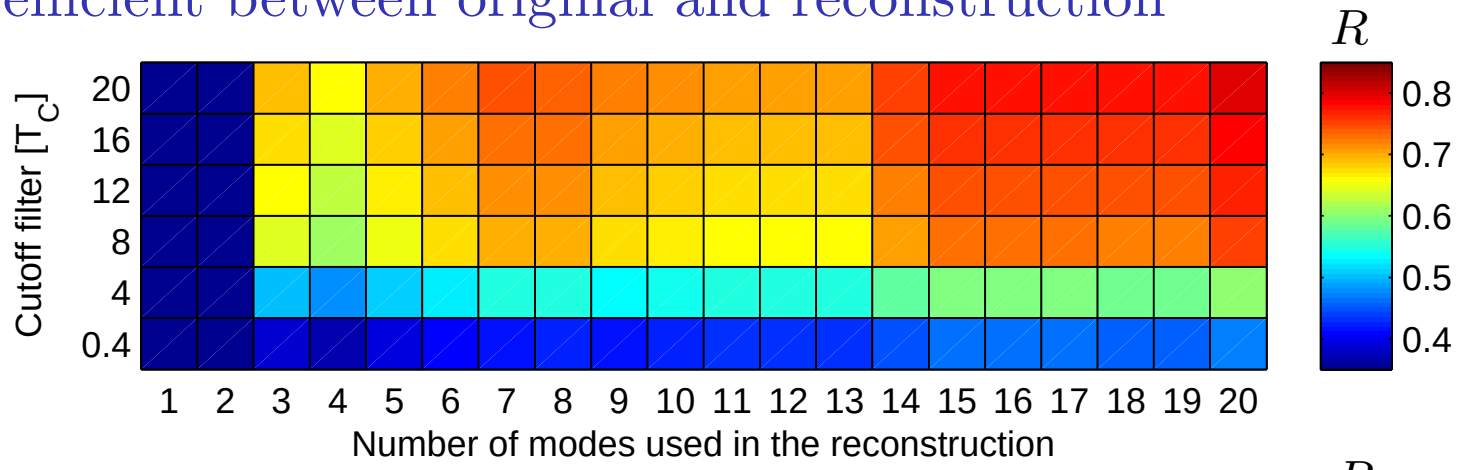
Measured from LES; filtered 8 $T_C$

POD reconstruction, 20 modes

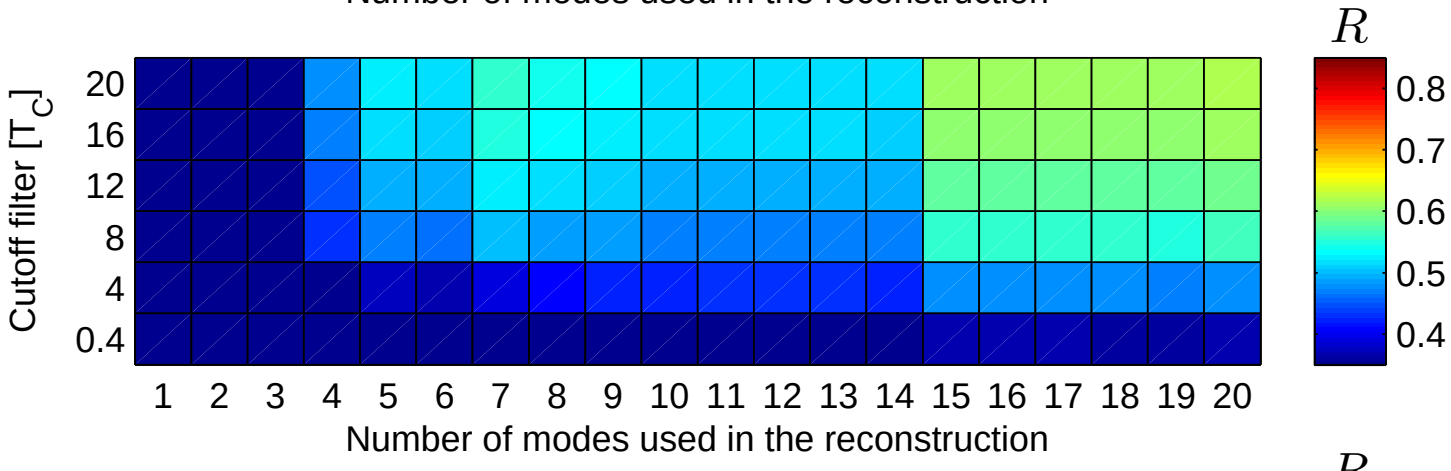


# Correlation coefficient between original and reconstruction

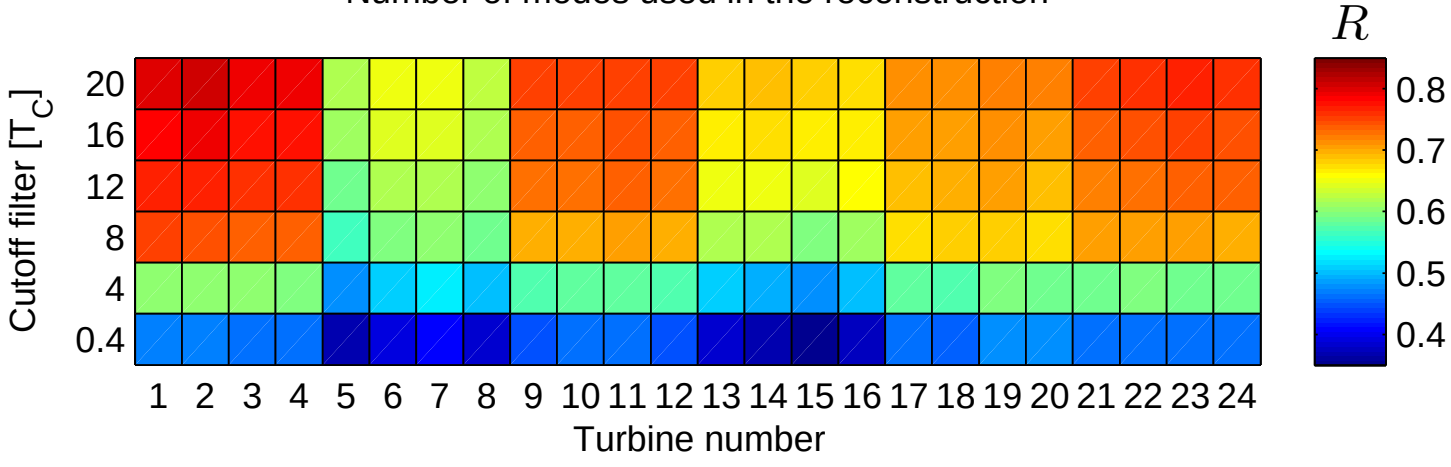
Turbine 1




Turbine 5




Using 20 modes



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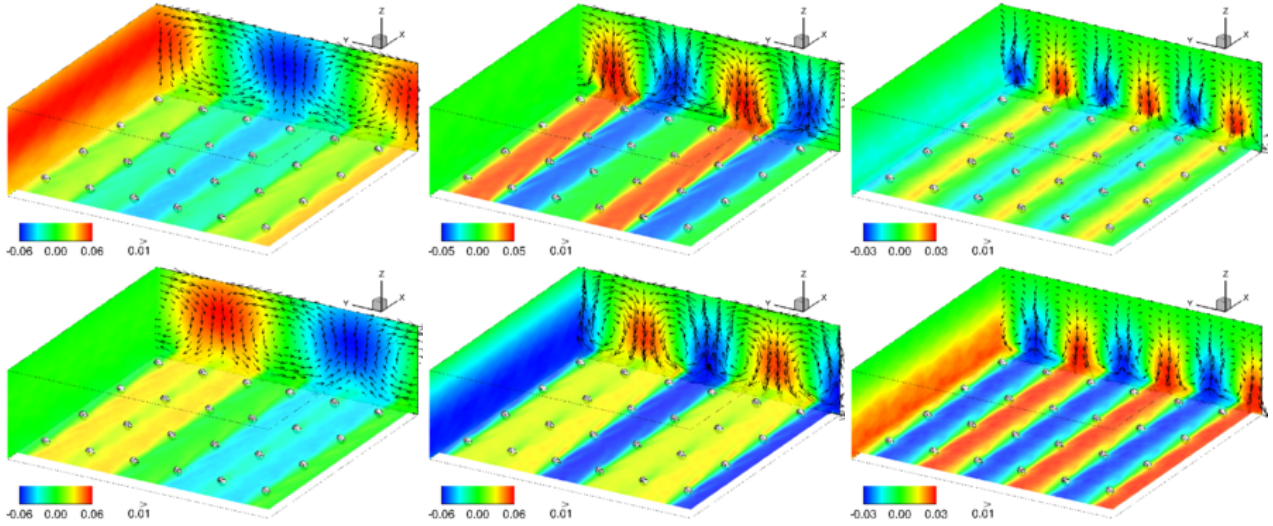
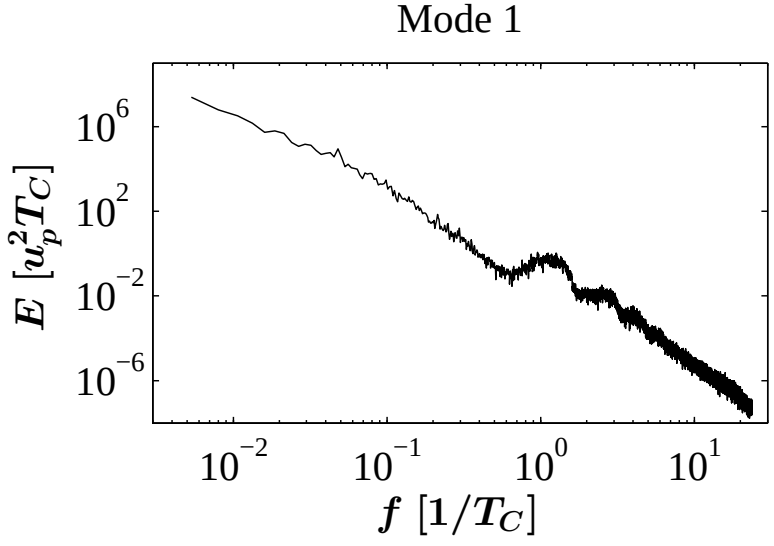
# (1) Roller modes

Examples of roller modes, ordered by pair:



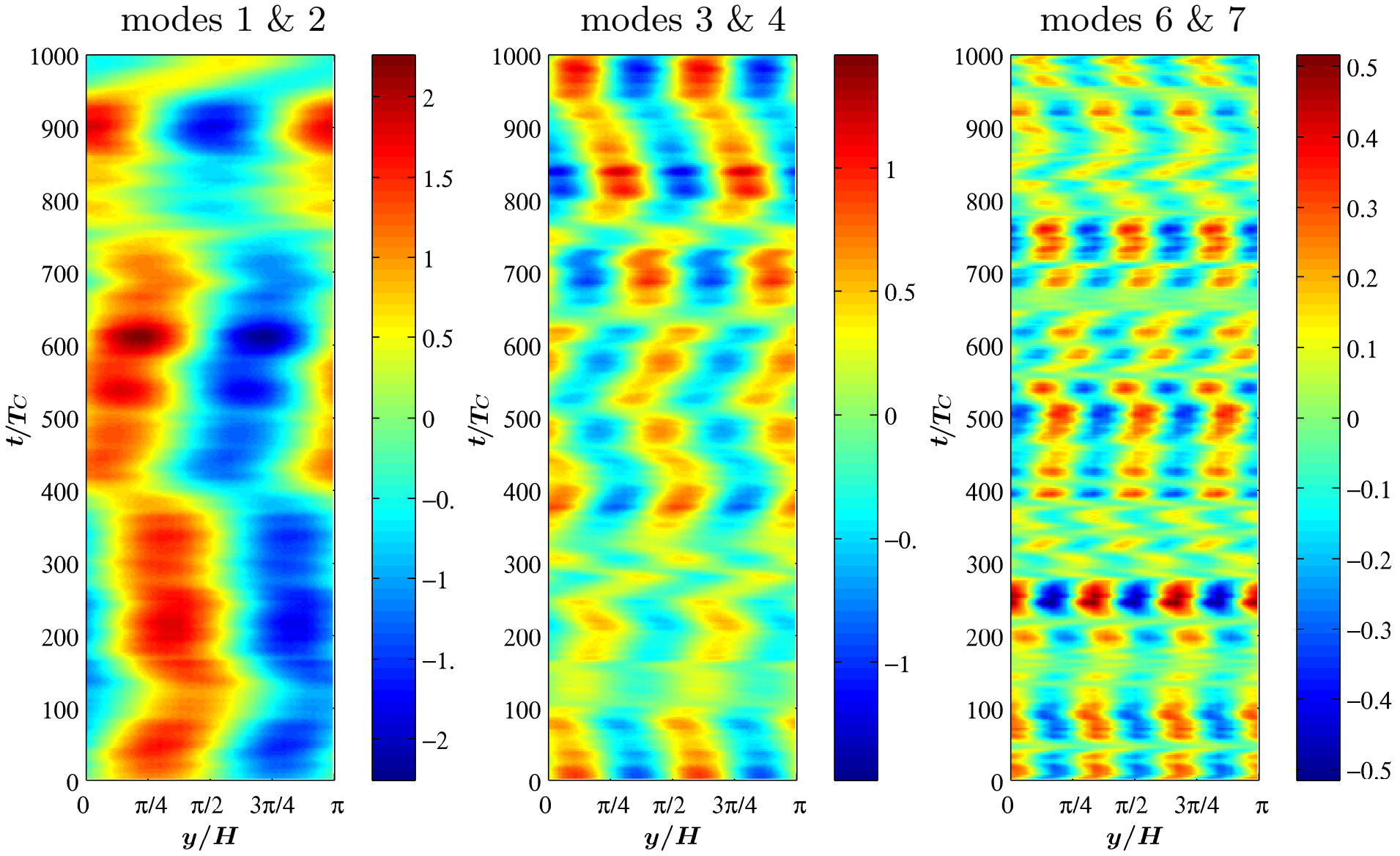



Effects of the roller modes:

- These roller modes capture the majority of the TKE and KE flux in the domain
- Roller mode pairs create meandering streaks in the flow

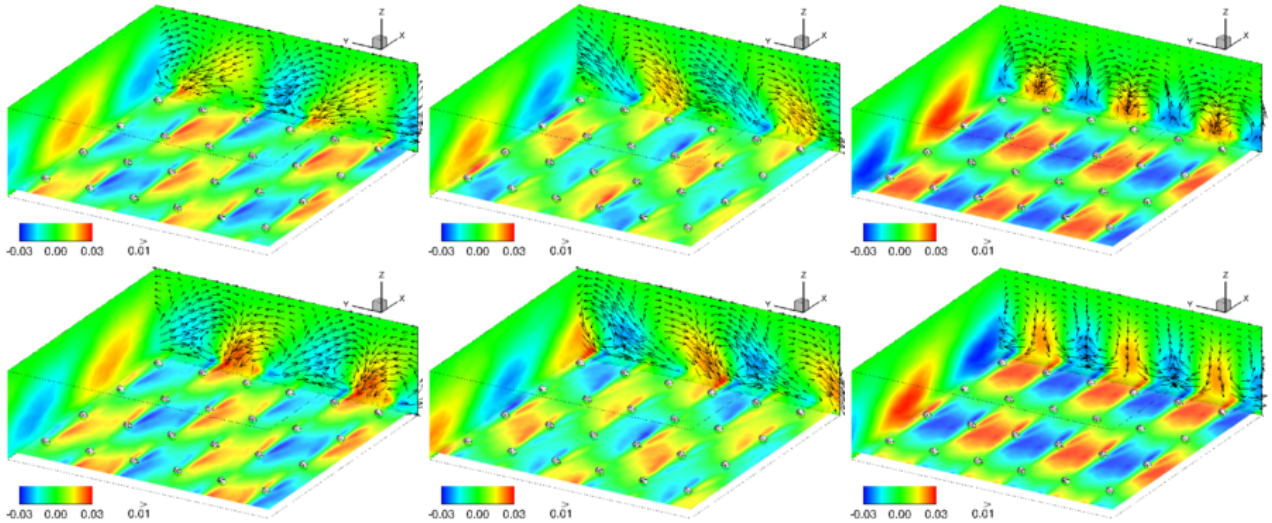
# (1) Roller modes

Spanwise cut as function of time; reconstruction from mode pairs

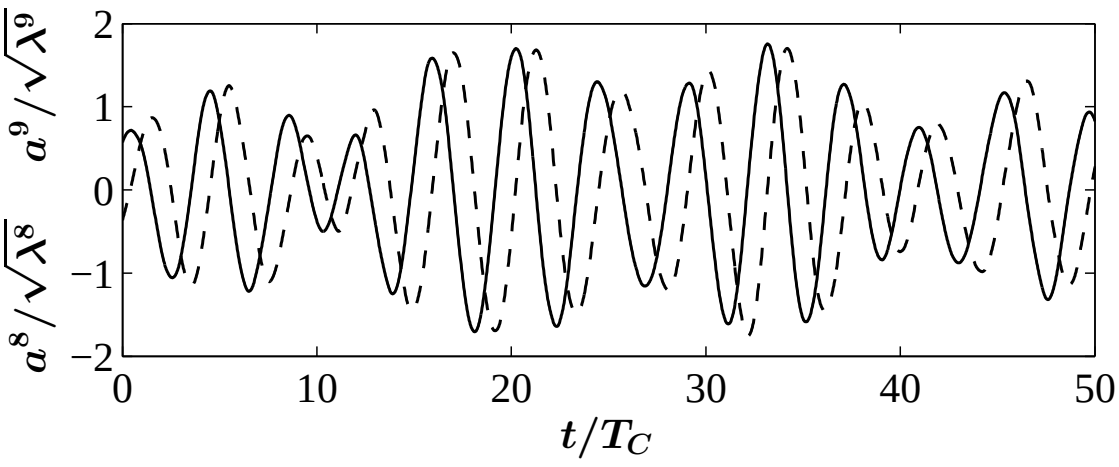


# (2) Streamwise-varying mode pairs

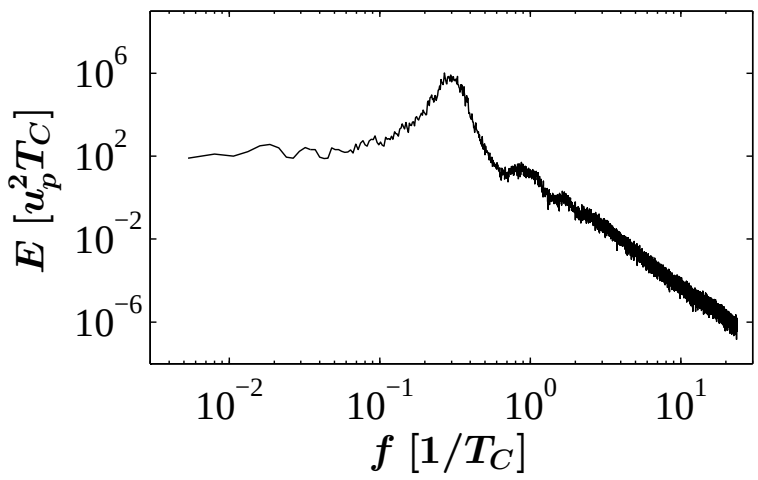
Examples of streamwise-varying modes, ordered by pair:



Example time-series for one pair:




Mode 8

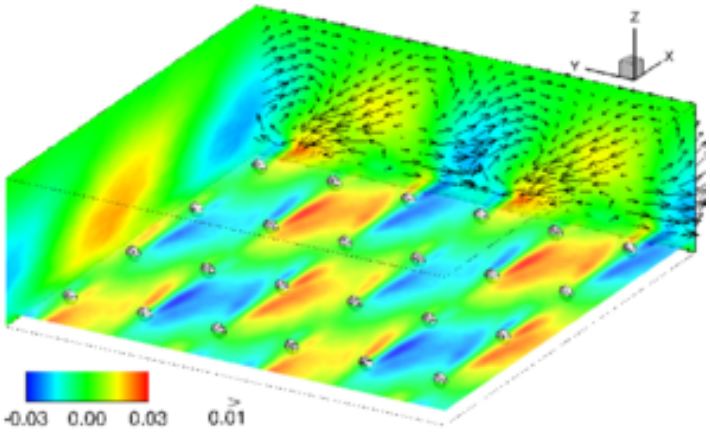


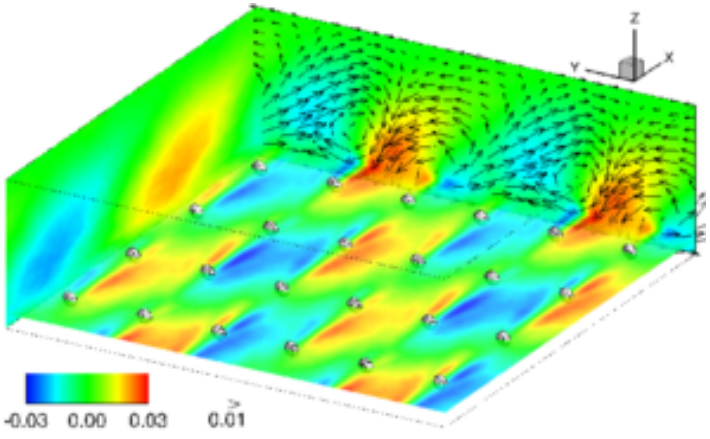

Effects of the streamwise-varying modes:

- **Represent advection of velocity perturbations with mean flow**

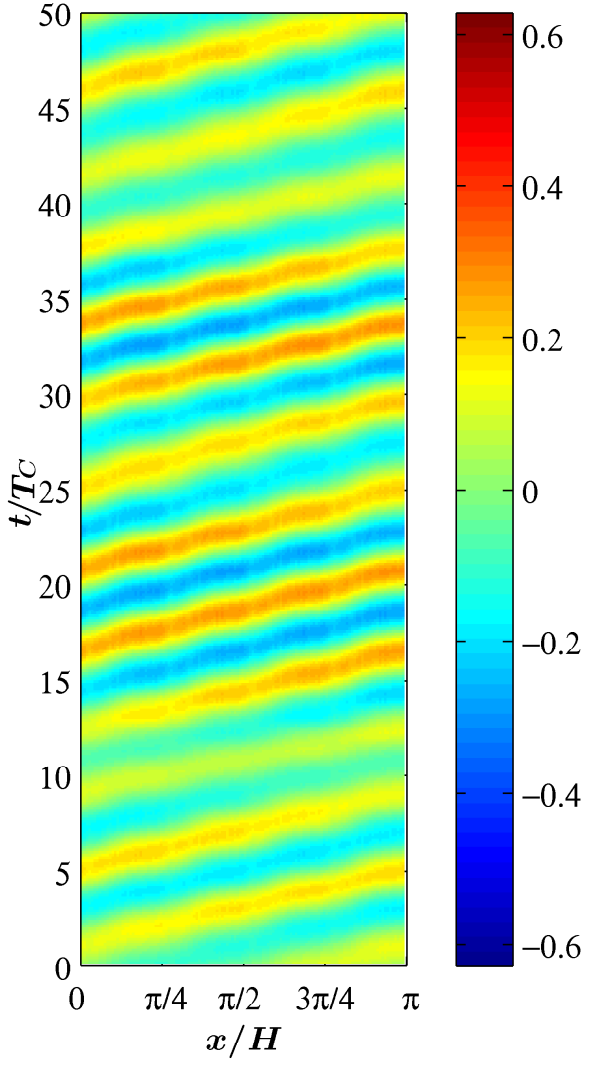
# (2) Streamwise-varying mode pairs

Streamwise cut as function of time; reconstruction from mode pairs

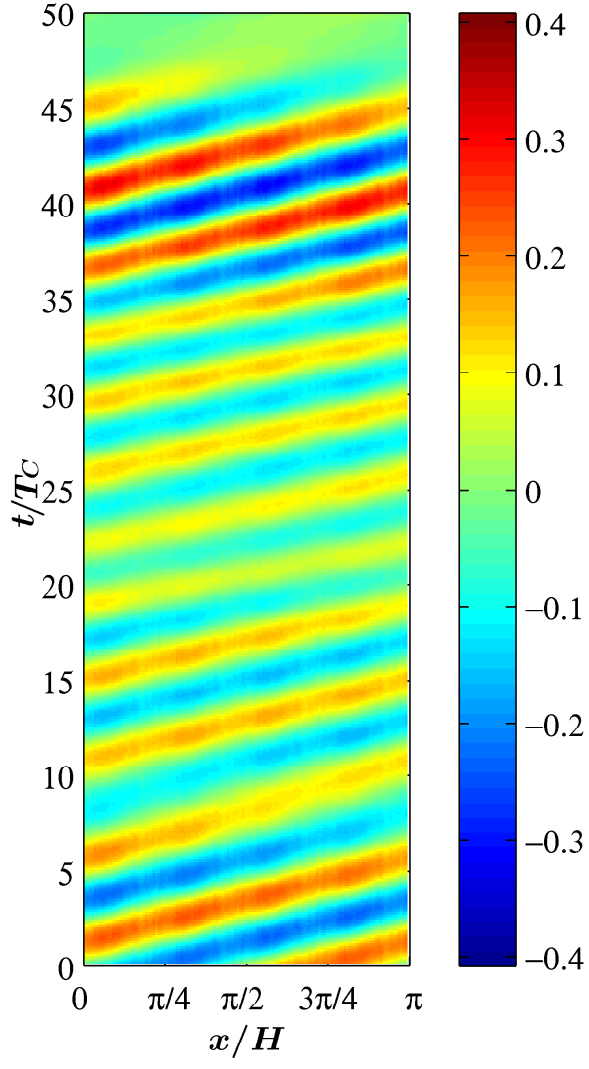



modes 8 & 9

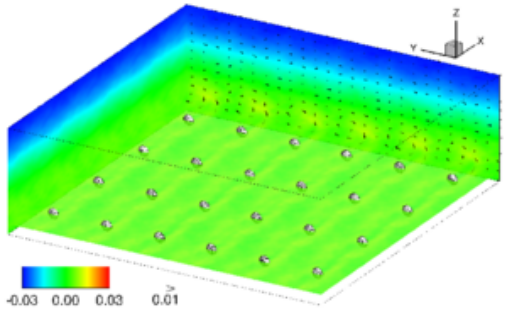



modes 10 & 11

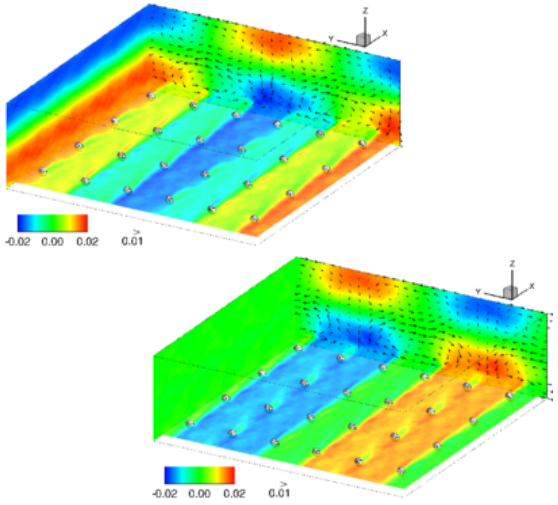


# (3) Shear modes

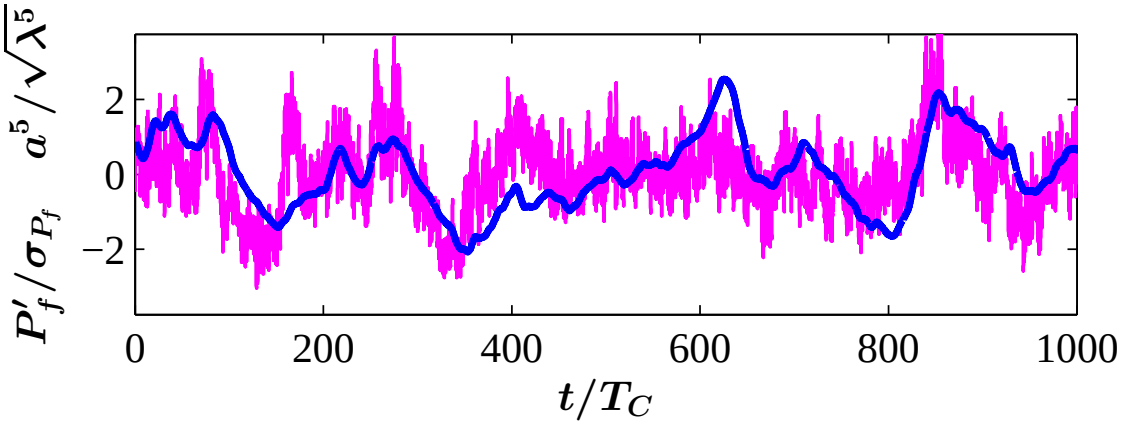
*global shear:*



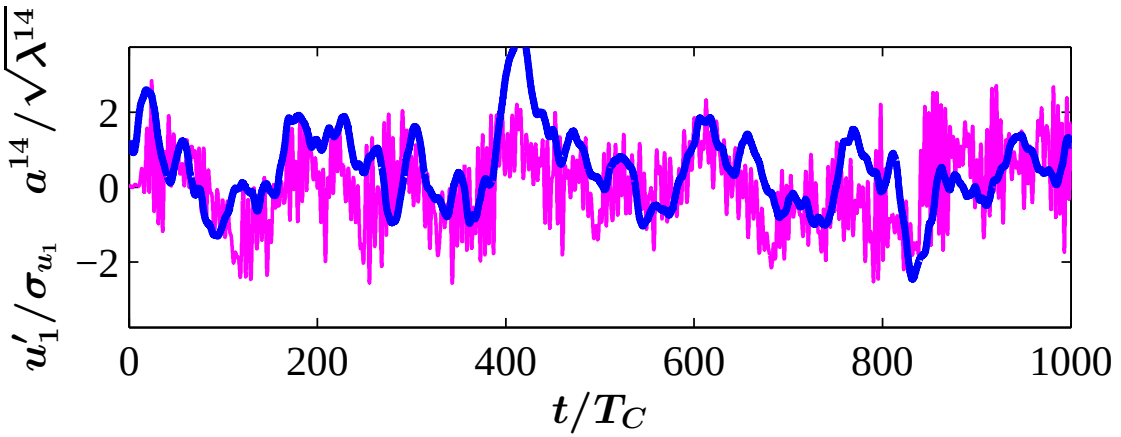

*local shear:*




Mean disk vel. (pink) and global shear mode (blue):




Turbine 1 filtered (8 $T_C$) disk vel. (pink) and local shear mode (blue):




Effects of the shear modes:

- **Captures much of the long-time variations in velocity (power)**

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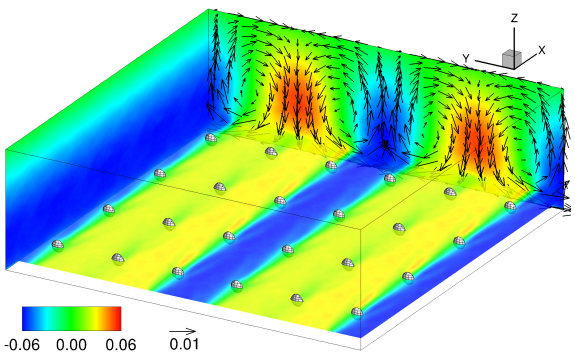
# Summary and future directions

**Summary**:

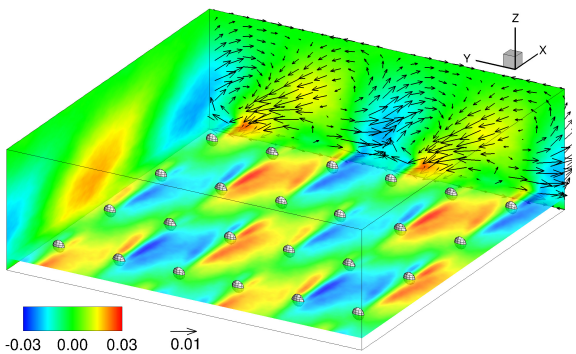
- Considered POD temporal variation and partial reconstructions
- Most energetic POD modes capture long-time variation
- Roller modes (majority TKE and KE flux): streak meandering
- Streamwise-varying modes: advective transport
- Shear mode: global streamwise velocity variation

**Possible future directions**:

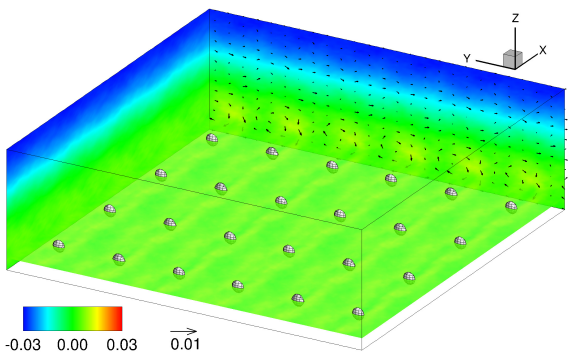
- Correlation with "important events" in the flow?
- Stochastic model for POD amplitudes (reduced order model for large-scale velocity variations)




roller mode type




streamwise-varying type




shear mode type

# Summary and future directions

Thank you for your attention.

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*of* ENGINEERING

WINDINSPIRE

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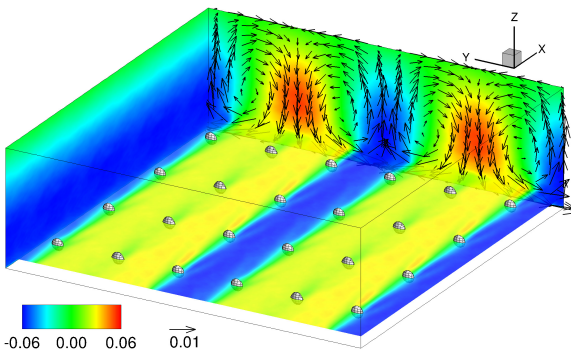
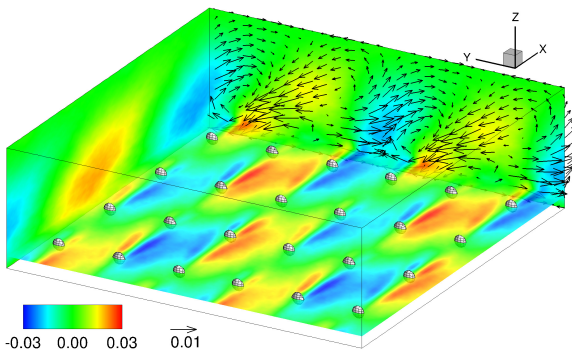
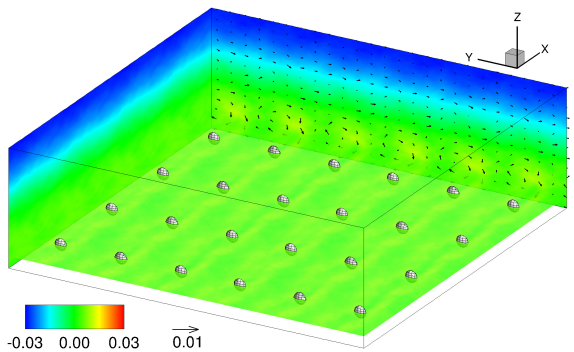



roller mode type




streamwise-varying type




shear mode type

# Top 20 POD-A (aligned) modes

The ordering is from left to right, then down.