CHAPTER 4 :

POST-FAULT TRAJECTORY PREDICTION

The main purpose of developing the Curve Fitting method was to be able to locate with accuracy the c.u.e.p. of the post-fault system. As shown in Chapter 3, it has also been successfully extended to the second swing transient stability analysis. Assuming that generator angles may be obtained from terminal voltages and current by synchronized phasor measurement devices placed at the terminal bus of each generator, the post-fault trajectory is predicted from the data given by the phasor measurements in the angle subspace. Further explanations on phasor measurements are given in appendix A. In this part, a second-order polynomial function and a second-order auto-regressive model are used and tested. A least-squares estimator is applied to estimate the parameters of both models. The model proposed by Y. Ohura, upgraded into an iterative algorithm, is also computed. As explained previously, they are compared with a direct integration of the dynamic equations using a fourth order Runga-Kutta method. The direct integration of the dynamic equations has already been implemented in a method called Time Domain Simulation method. Numerous papers [10] have already shown the advantages and drawbacks of such method. Therefore in this chapter, the direct integration is used as reference for comparison. Using these models, the post-fault trajectory is predicted until it reaches the stability boundary or, as explained in chapter 3, until it reaches a potential energy peak. The peak is detected thanks to a change in sign from negative to positive in the directional derivative of the potential energy gradient (3.1). The path, which yields the equation (3.1), is also developed in this chapter. In order to detect with efficiency the exit point located on the p.e.b.s., the first point at which the potential energy attains a local maximum on the ray starting at the s.e.p. and in the direction to the last point of the prediction is taken. The Ball-Drop method [22] here is used as back up when the local maximum method fails. This method, based on an analogy with a ball in a bowl, is also explained in this chapter.

4.1 Motivation of the approach

To obtain security limits for a variety of operating conditions, which exists on a specific network, a large number of transient stability studies are required. These studies simulate a large number of contingencies at different transmission interface flow levels and under different outage conditions. For example, studies made by Ontario Hydro [18] have indicated that derivation of operating security limits cannot be effectively made by off-line studies alone. The need for an on-line scheme to augment the off-line data was recognized.

One of the first transient stability method is the time domain simulation. One of its advantages is its unlimited modeling capability. Moreover, this method produces time responses for all quantities, some of which may be needed for detailed examination of the stability phenomenon or for simulating special protection schemes. Despite its advantages, the time domain simulation method has two shortcomings. Firstly, it is inherently slow due to the step-by-step integration process involved in solving the differential equations. Secondly, it only yields a yes-or-no type answer on the stability problem, with no indication on the degree of stability. The lack of sensitivity information on the system's degree of stability has to be compensated for by many stability runs in order to derive stability limits. This approach requires then much computational time and is hence not suitable for on-line applications.

Those drawbacks are the starting points for the elaboration of a new computationally fast method. As revealed in chapter 3, the proposed approach presented in this paper can be divided in two parts. The first part has brought its contribution in the detection of the exit point and the c.u.e.p. The second part is based on the energy approach that assesses the energy margin as index of stability. Instead of a step-by-step integration of the dynamic equations, which is inherently slow, simple models for the post-fault trajectory are used to simulate it and to detect in advance the appropriate c.u.e.p. Models are set up to give us the direction where the trajectory may go out of the stability region. An ideal model would give us the direction and insure us that the predicted trajectory crosses the stability boundary in this direction. Several different models will be presented in the next section. Regarding the second step of the method, starting at the exit point, the c.u.e.p. is computed solving a multidimensional optimization problem by a Newton's method. The different problems such as the convergence problems are treated in the next chapter.

4.2 Prediction

In this section, simulations on the Athay's 3-machine system are considered. The same kind of simulation could easily be reproduced on the 10-machine New-England Power System. A self-cleared fault is considered on bus 3. The impedance of the fault is equal to $Zf=(1+j)*10^{(-5)}$. The critical clearing time is computed and is equal to 0.42 sec. Applying the same fault, but with a clearing time of 0.433 sec, a simulation of the post-fault trajectory is shown in Figure 4.1. This simulation is obtained thanks to a direct integration of the dynamic equations using a fourth order Runga-Kutta method.



Figure 4.1: Prediction Case using tcl=0.433 sec

As explained before, the step-by-step simulation procedure used to determine system performance is based here on classical machine models and a reduced network matrix. The energy function is evaluated at each time step and the fourth order Runga-Kutta method is applied to numerically integrate the system differential equations. Critical clearing times are determined by repeated simulation and by varying the time at which the fault is cleared. As this determines the stability limit in the time domain, it forms the basis of comparison between the Curve-Fitting method, its different models for the prediction step, and the time domain simulation method.

4.2.1 The Ray Approximation

Thanks to its simplicity, the ray approximation is the first model utilized to predict the post-fault trajectory. The ray is calculated from the post-fault s.e.p. and in the direction of the clearing time point or the last post-fault machine angle measurements. The ray is stopped when a potential energy peak is encountered. This point is then considered to be the exit point located on the p.e.b.s. An example is shown in Figure 4.2.



Figure 4.2: Ray Approximation

The problem of the ray approximation has already been underlined in the previous chapters. As emphasized in the computation of the p.e.b.s., the ray meets a potential energy peak only if it intersects orthogonally with the stability boundary of the reduced system. For the other cases, we can resort to compute the Ball-Drop Method at each step of the prediction. This method is only considered as a back up in the case where the potential energy peak method may fail. As explained at the end of this chapter, the Ball-Drop method involves a step-by step integration of the dynamic equations. As shown in [18], a step size bigger than 0.05 second would cause appreciable error in the energy calculation. Then, the fact that the Ball-Drop method has to be used at each step of the prediction slows down significantly the computation of the exit point. Also, the ray approximation technique is not acceptable in the domain of the prediction. As shown in Figure 4.2, it did not fit with the real trajectory, which is smooth in the absence of inter-area oscillations. An acceptable mathematical model would at least include the speed or the acceleration. Integrating such value in the model will not insure us that the predicted trajectory crosses the stability boundary. But, as explained in chapter 3, some backup tools have already been set up.

4.2.2 A second-order polynomial function

The first model that has been implemented after the ray approximation is a second-order polynomial function. Looking at the plots of the machine angles versus time (Figure 4.3), a

second-order polynomial function seems an easy and simple way to approximate the post-fault trajectory.



Figure 4.3: Machine Angles versus time (tcl=0.2 sec & Zf2=0.032 pu)

The machine angles are estimated as:

$$\therefore d_i(t) = a_{0i} + a_{1i}t + a_{2i}t^2 + e_i \qquad \text{for i=1,...,n-1} \quad (4.1)$$

Then, for each angle, the least-squares estimator is used to estimate the parameters a_{0i} , a_{1i} , a_{2i} . The estimation consists of the following steps:

- <u>Step 1</u>: Record a data sample of machine angles and their time tags after the clearing time.
- <u>Step 2</u>: Build the model specification matrix \underline{T} for a quadratic model and the angle vector \underline{y} . They are given by

$$\mathbf{T} = \begin{bmatrix} 1 & t_1 & t_1^2 \\ 1 & t_2 & t_2^2 \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ 1 & t_p & t_p^2 \end{bmatrix} \text{ and } \mathbf{y} = \begin{bmatrix} \mathsf{d}_{1i} \\ \mathsf{d}_{2i} \\ \cdot \\ \cdot \\ \mathsf{d}_{pi} \end{bmatrix} \text{ for } \mathbf{i} = 1, .., \mathbf{n} - 1 \quad (4.2)$$