Essays on Contest Theory Experiments and Revealed Time Preference Models

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(ABSTRACT)

In this series of essays, we study the influence of weight and group size in the sequential multi-battle contest with laboratory experiences (Chapter 2 and Chapter 3). We then develop an empirical method to model perceptual present and time inconsistency (Chapter 4).

Chapter 2 examines how the weight and the ordered weights in battles affect the behavior in sequential multi-battle contests with an experiment. We find robustly that the weight of the current battle consistently influences contestants' efforts. Additionally, we discover the math-point-oriented behavior despite differences in history. In other words, the weight effect is expressed in two ways: influencing the effort of the current battle and transferring a contest to the next battle with a designated intensity.

Chapter 3 explores the group size effect and how the contest success functions influence the group size effect in sequential multi-battle contests with an experiment. We capture the negative group size effect on the leaders' efforts, participation and dropout rates; contrarily, the positive effect on the non-leaders' efforts. Compared to the Tullock lottery, the all-pay auction intensifies the group size effect of the high effort in the initial battle. It also enlarges the observed group size effects of the effort gaps between the leaders and the non-leaders.

Chapter 4 develops the quasi-hyperbolic discounting model into the general beta-delta model to parametrically detect and measure the inconsistency in revealed time preference. This method empirically classifies time preference into four categories, i.e., time consistent, present bias, future bias, and mixed inconsistent. Then we applied this method to the convex time budget data of seven experiments, including 3670 subjects. We discover empirical evidence supporting perceptual differences in the present-future threshold. Traditional present bias models may interpret the time preference imprecisely.

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(GENERAL AUDIENCE ABSTRACT)

Competition and Time are two essential aspects of life. Many decisions are made in a competitive environment. Some other decisions are made when time serves as a critical factor. We divide this dissertation into two parts. In the first part, we study strategic behavior in competitions. Specifically, we examine how (1) the importance of each round (weight), (2) the number of competitors, and (3) the ambiguity of the rule affect the result of a multi-round competition. In the second part, we study people's subjective understanding of time, generally the personal beliefs and preferences of the present and future.

In part one (Chapter 2 and Chapter 3), first, we find people are very responsive about the importance of a round in a multi-round competition. When a round is more important, people make more effort in such round. People are also sensible about the competition's current status (leading, behind, or tied) rather than the history. Second, at the beginning of a competition, we find an increase in the participation rate when fewer competitors exist. Suppose there are more competitors; the leading position players compete more brutally; on the contrary, the non-leading players are discouraged more. Third, people spend more energy when the rule is less ambiguous in a multi-round competition.

In part two (Chapter 4), We find a very diversified subjective belief in the word "present." The concept of "now" lasts longer than we conventionally thought. When the subjective "present" is captured at the individual level, we find the immediate now is not necessarily the best way to represent the "average present" for the population.

Dedication

To my dear grandfather,

a tough man who carried me on his shoulders on those rainy days,

in loving memory.

Acknowledgments

I'm sincerely thankful to those who helped me through this fantastic journey.

First of all, I would like to thank my advisors, Dr. Sheryl Ball and Dr. Alec Smith, for their guidance, advice, help, support, understanding, and kindness throughout those years. I have learned so much from them in the aspect of both academia and life. Those inspirations are life-changing, and I am grateful to be their student. I would also take a moment to thank the rest of the committee members, Dr. Klaus Moeltner, Dr. Matthew Kovach, and Dr. Walid Saad, for the advice and feedback that helped me through my Ph.D. studies.

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I remember watching shooting stars on the prairie of Kansas; I remember chasing waves at the bay of California; I remember catching fireflies in the mountains of Virginia. Many years later, I still love this journey, but "not in Kansas anymore."

A Summer's day in Maryland

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Chapter 1

Introduction

We focus our studies on two following economic aspects. First is the sequential multi-battle contest, where contestants compete for a prize with costly efforts in a dynamic setting. Second is the revealed time preference, where individuals make consumption decisions on a time horizon; usually, this process involves a trade-off between sooner and later.

Contest theory (Konrad, 2009) is a rich topic in game theory research. The theory work has been developed in three different models, the all-pay auction (Hillman and Samet, 1978), the Tullock lottery (Tullock, 1980), and the rank-order tournament (Lazear and Rosen, 1981). These models are applied to study real-world scenarios such as patent race, R&D, wars, sports events, rent-seeking, political debates and campaigns, college admissions, etc. We study a specific type of contest called race (Konrad and Kovenock 2009) or sequential multi-battle contest (SMBC), where the players have to compete with costly effort in a sequence of independent battles, and the first to reach a threshold wins the prize. The examples we can give for the SMBC can be the US primary election, Formula One, NASCAR, Tour de France, triathlon, biathlon, decathlon, heptathlon, college admission in the US, graduate school admission in China, the Apple-Samsung patent race.

The theoretical work of SMBC starts with Klumpp and Holborn (2006; using the Tullock lottery) and Konrad and Kovenock (2009; using the all-pay auction). Later development includes Mego et al. (2013; with intermediate prize), Gelder (2014; adding penalty), Doğan et al. (2018, increasing the group size). Our work compares theory prediction and empirical behavior in laboratory experiments.

Experimental studies in SMBC have been emerging in recent years. Zizzo (2002) first tested the theory in a protracted contest. Mego et al. (2013) discussed the intermediate prize and the decisiveness parameter (Hirshleifer, 1995). Mago and Sheremeta (2017 and 2019) separately tested all-pay and Tullock lottery in best-of-three games. Mago et al. (2019) extended the current literature into heterogeneities in the opponent's gender. In Descamps et al. (2022), they studied the possible source of observed momentum.

Our experimental research in SMBC mainly contributes to the research field in three directions, i.e., weighted battles, group size, and CSF. Currently, the asymmetrically weighted battles are only modeled in the Colonel Blotto game (Theory: Friedman, 1958; Duffy and Matros, 2015. Experiments: Duffy and Matros, 2017; Chowdhury et al., 202). The Blotto games model a resources allocating process in a simultaneous mechanism. In Chapter 2, we brought the idea of unequally weighted battles into SMBC and examined the weight and order of weight effect in contest experiments. A reach body of literature studies the group size effect in the contest (Theory: Baye et al., 1996; Doğan et al., 2018. Experiment: Harbring and Irlenbusch, 2003; Orrison et al., 2004; Gneezy and Smorodinsky 2006; Amaldoss and Rapoport; 2009; Sheremeta, 2011; Morgan et al., 2012; Ernst and Thöni, 2013; Lim et al., 2014; Booseya et al., 2017; List et al., 2020; Nelson, 2020; Hudja, 2021; Fallucchi et al., 2021; Peeters et al., 2021). All existing experiments study the group size of the contest in a one-shot sense. In Chapter 3, we study the group size effect in SMBC. There is no experimental study yet to compare all-pay and Tullock in SMBC (Mego et al., 2013 compared different decisiveness parameters but not for all-pay auction). In chapter 3, we provide a side-by-side comparison of the all-pay auction and the Tullock lottery in a dynamic setting.

Time is a foundational aspect of the decision-making process to consider. The standard way of modeling time preference can be traced back to streams of exponential discounting models (Samuelson, 1937; Koopmans, 1960). The two major stream models that capture the inconsistency from exponential discounting of time in psychology and economics studies are hyperbolic discounting (Mazur, 1987; Loewenstein and Prelec, 1992) and quasi-hyperbolic discounting (Phelps and Pollak, 1968; Laibson, 1997). Other attempts at making behavioral adjustments are proportional discounting (Ainslie and Herrnstein, 1981), power discounting (Harvey, 1986), and constant sensitivity discounting (Ebert and Prelec, 2007). A more detailed summary of the time discounting model can be seen in Frederick et al. (2002).

There are several famous approaches to eliciting time preference from experiments, such as intertemporal choice (Loewenstein, 1988), convex time budget (Andreoni and Sprenger, 2012a), and dynamic choice (Jackson and Yariv, 2014). Additionally, the studies on axiomatically characterizing time preferences are also rich (Koopmans, 1960; Fishburn and Rubinstein, 1982; Fishburn and Edwards, 1997; Prelec, 2004; Ok and Masatlioglu, 2007; Halevy, 2008; Bleichrodt et al., 2008, 2009; Attema, 2010; Noor, 2011; Montiel Olea et al., 2014; Dziewulski, 2018; Echenique et al., 2020). A recent meta-analysis (Imai et al., 2021) of convex time budget studies points out that the time inconsistency of quasi-hyperbolic discounting varies according to different categories. Jackson and Yariv (2014) suggest four basic types of time preference (time consistent, present bias, future bias, mixed inconsistent). We follow the recent trend in the time preference topic concerning "How soon is now" (Glimcher et al., 2007; DellaVigna, 2018; Ericson and Laibson, 2019; Balakrishnan et al., 2020). Then in Chapter 4, we developed Laibson's quasi-hyperbolic discounting model into a general beta-delta discounting model to find the perceptual present and its corresponding time inconsistency categories with the convex time budget experimental data.

Chapter 2

New Hampshire or Super Tuesday, Does the Weight Matter?

Kevin Zou, Yichuan Cai, Dongwoo Lee

Abstract

This paper studies the impact of the weight and the sequence order of the weights in the sequential multi-battle contest (SMBC) with the all-pay auction as the contest success function. Theoretically, we can reduce the New Hampshire effect by assigning more relative weight to the second battle in an SMBC. In the experiment, we compared three treatments in which the first two battles differed by weight. We find that the assigned weight has a positive effect on the effort. However, the effect is not long-lasting. Through the process of the sequential battles, we capture a path-independent strategic momentum. The players' behavior depends on the match-point-oriented intensity of the current state. Contestants bid more when the battle gets more intense, despite differences in the contest's path, ordered weight, and symmetry.

JEL Classification: C72; C73; C91; C92; D44; D72

Keywords: Weight; Sequential Contest; All-pay Auction; Experiments

2.1 Introduction

Weight [1] is an essential aspect of a multi-battle competition. Different battles have different importance in contributing to the final victory. The difference in weight ubiquitously exists in many occupations, such as sports events, interview process, college addition, patent race, and political campaigns. In this paper, we contest a laboratory experiment to examine the effects of the weight and the order of the weighted battles in a sequential multi-battle contest.

The sequential multi-battle contest (SMBC) model (Klumpp and Polborn, 2006; Konrad and Kovenock, 2009) is broadly studied to explain the phenomenon of the New Hampshire effect in the United States presidential primary election, in which the winner of the New Hampshire primary has a higher probability of winning the nomination within the party. The first theory of the sequential multi-battle contest model traces back to Klumpp and Holborn 2006, whereas they use Tullock (1980, 2001) as the contest success function (CSF). They showed that the winner of the first battle has a deterministic strategic advantage in winning the entire contest. Konrad and Kovenock (2009) reached a similar conclusion with an all-pay auction (Hillman and Riley, 1989; Baye et al., 1996) setting. Mego et al. (2013) further developed the model from Klumpp and Ploborn (2006) with intermediate prizes. Gelder (2014) added a penalty mechanism to the all-pay auction SMBC. Doğan et al. (2018) and Zou and Cai (2022) generalized the SMBC theory from a two-player game to an n-player game with Tolluck and all-pay auction as contest success function respectfully. In the latest theory work, Cai (2022) proved that players would only compete in the most deterministic battle regardless of the weight assignment in an all-pay auction SMBC. Namely, the most

[1]Battles usually are differed by their importance in a multi-battle contest (sequential or simultaneous). We call this importance in determining the result of a contest the "weight". For example, in the 2020 Democratic Party presidential primaries, Wyoming had 14 total pledged delegates, and California had 415 total pledged delegates. So California had more weight than Wyoming to determine the nomination.

deterministic battle is called a separating battle. There is always one separating battle in any all-pay SMBC.

In the experimental work of SMBC, many studies (Mago et al., 2013; Gelder and Kovenock, 2017; Mago and Sheremeta, 2017, 2019; Mago and Razzolini, 2019; Descamps et al., 2022) found evidence that supports the New Hampshire Effect, where the winner of the first battle gains a considerable advantage and eventually has a higher probability of winning the contest. Nevertheless, the battles are differentiated by the weight in presidential primaries. The weight of New Hampshire (the first battle, or the total number of revealed delegates combined before Super Tuesday) is relatively small compared to Super Tuesday (consider Super Tuesday as the second battle in the presidential primaries with multiple bundled states of sufficient weight). Historically, we have seen iconic comebacks on Super Tuesday after losing New Hampshire (behind in primaries before Super Tuesday), such as Bill Clinton and Joe Biden, in the 1992 [2] and 2020 [3] Democratic Party primaries and George Bush in the 2000 [4] Republican Party primaries. Therefore, it is necessary to consider the battles' weight when studying New Hampshire Effect.

Currently, there is no experimental work in contests considering the weight differences in battles, creating discrepancies between reality and theoretical framework. The contribution of our study is to extend the experiment in the contest with the asymmetrically weighted battles. Further, we study the influence of the order of the weight assignments in SMBC.

[2]In 1992, eleven states (DE, FL, HI, LA, MA, MS, MO, OK, RI, TN, and TX) revealed their primary (caucus) election (Democratic) results on Super Tuesday (March 10th). The total number of delegates revealed is 777, which takes 49.23% amount of the 3329 total delegates. On the other hand, New Hampshire only revealed 18 delegates (0.54% or 17.45% for IA, NH, ME, SD, CO, GA, ID, MD, MN, UT, WA, AZ, SC, and WY, which held caucus or primary before Super Tuesday combined).

[3]In 2020, fifteen states and a territory (AL, AZ, AR, CA, CO, ME, MA, MN, NC, OK, TN, TX, UT, VT, and VA) revealed their primary (caucus) election (Democratic) results on Super Tuesday (March 3rd). The total number of delegates revealed is 1344, which takes 33.78% amount of the 3979 total delegates. On the other hand, New Hampshire only revealed 24 delegates (0.60% or 3.90% for IA, NH, NV, and SC, which held caucus or primary before March combined).

[4]In 2000, thirteen states and a territory (CA, CT, GA, ME, MD, MA, MN, MS, NY, OH, RI, VT, and WA) revealed their primary (caucus) election (Republican) results on Super Tuesday (March 7th). The total number of delegates revealed is 605, which takes 28.38% amount of the 2132 total delegates. On the other hand, New Hampshire only revealed 17 delegates (0.80% or 13.13% for AK, IA, NH, DE, SC, AZ, MI, PR, VA, WA, and ND, which held caucuses or primary before March combined).

2.2 Background

The importance of weight is studied in game theory mainly through the Colonel Blotto game (Borel 1921). Friedman (1958) was the first to explore battles with different weights in the Tullock lottery Colonel Blotto game. Then, the majority rule was introduced to the asymmetrically weighted resource allocating game in Young (1978) and Lake (1979). Duffy and Matros (2015) further generalized the result in Lake (1979) to allow for asymmetric budget constraints with n players. The asymmetrical Blotto games have also been studied in experimental research. Similar to the asymmetrical weight setting, Hart (2008); and Avrahami and Kareev (2009) assigned players with different budgets, and Aveahami et al. (2014) gave a different probability of choosing a specific battle to determine the result of the game. Cinar and Goksel (2012) were the first to consider players have asymmetric resources to allocate. Chowdhury et al., 2013 tested the difference between the all-pay auction and the Tullock lottery with a similar setting in Avrahami and Kareev (2009). A series of papers (Hortala-Vallve and Llorente-Saguer, 2010, 2015; Hortala-Vallve et al., 2013) studied asymmetrically weighted battles with the perspective of the heterogeneous valuation of the battles, the importance of communications, and the effect of complete information. Montero et al. (2016) tested the auction rule in asymmetrically weighted battles, and the theoretical framework is close to Young 1978. Alternatively, Duffy and Matros (2017) examined the asymmetrically weighted battles with the Tullock lottery mechanism (Duffy and Matros, 2015). In the most recent development, Chowdhury et al., 2021 studied the effect of salience in lottery Blotto contests with asymmetric weights across battles.

Asymmetry has long been studied in contests. Baye et al. (1996) assume players have asymmetric valuations of the prize in the all-pay auction. In rent-seeking games: Tullock (1980) studies the asymmetric cost functions for contestants; Wärneryd, K. (2003) introduces asymmetric information to the Tullock game; Cornes and Hartley (2005) construct players with different production functions into the game; Chowdhury and Sheremeta (2011), they study a more general format that players have different abilities to influence the contest success function. As for the tournament, Bull et al. (1987) consider the players have different random shocks when competing for the rank order. Players having different contest objectives have been addressed in Smith and Parker (1976); and Kovenock et al. (2010). However, none of the current work considers asymmetric in the battle of a contest.

We summarize the general findings in light of current research work in equally weighted SMBC. Players are generally discouraged when behind their opponent (Mago et al., 2013; Gelder and Kovenock, 2017; Mago and Sheremeta, 2017, 2019; Mago and Razzolini, 2019). In the Blotto game (simultaneous), players are sensitive about the assigned weight of the battle (Hortala-Vallve and Llorente-Saguer, 2010, 2015; Montero et al., 2016; Duffy and Matros, 2017; Chowdhury et al., 2021). As in the primary election example, many contests in life are sequential, and more importantly, the battle weights are not always equal. The primary election in the U. S. is one example that the first battle has less weight than the second. Through the college admission process in the U.S. and the postgraduate admission process in China, there usually is an interview process after the examination. The test and the interview are held in sequential order, and the test result is typically weighted more than the interview result. Similarly, the qualifying race is held before the race in Formula One, determining the order in which a racer starts a race. The position gained in the qualifying race is very deterministic in the result of any Grand Prix. In all of these examples, the second battle is weighted more than the first battle. There are also many examples of equally-weighted battles in SMBC, such as NBA finals (best of seven) and the general rules in tennis tournaments.

In sequential movement games, the only theory work which assigns different weights to each move (not a battle) is Harris and Vickers (1985). In a more complex model, Schwabe

(2015) proved the candidates might drop out before Super Tuesday due to overspending in the previous battle. However, this work is not a conventional SMBC. In this paper, we conducted a laboratory experiment to study the weights and sequential order of the weights in SMBC. Our research contributes to current research in the contest in the following three ways. First, we extend the weighted component from the simultaneous game to a sequential mechanism. Second, we tested how the order of the sequential weights affects the contest. When studying asymmetrical values in battles, we examine costly effort instead of use-it-or-lose-it in the Blotto game.

2.3 Theoretical Framework

2.3.1 A General Model

There are 2 risk-neutral players in the player set $N = \{1, 2\}$ ($i \in N$) who compete for a commonly known indivisible prize valued V in a series of battles.

The player who reaches state=3 first wins the contest. We call the required state for winning as the winning threshold $T = 3$.

We use a battle node t (battle t) to specify each player i's current state (s_{1t}, s_{2t}), where $t = \{1, 2...m...\}$ and $s_{it} = 0$, 1 *or* 2.

The state of winning battle node t is distinguished by the weight of the battle θ_t. The contest is terminated at battle node m when $s_{im} = T$. We denote the total revealed weight at battle t as $\sum \theta_t = s_{1t} + s_{2t}$, where $5 \geqslant \sum \theta_t \geqslant T$.

At each battle t , player i choose a costly effort $e_{it} \in [0, \infty)$ to exert [5]in order to win the battle node t. Consider the cost function of the effort e_{it} is linear (Eq. (2.1); Klumpp and

Polborn, 2006; Konrad and Kovenock, 2009; Mago et al., 2013; Gelder, 2014; Doğan et al., 2018):

$$c(e_{it}) = e_{it} \tag{2.1}$$

We applied the all-pay auction (Eq. (2.2); Hillman and Samet, 1987; Hirshleifer and Riley, 1978; Nalebuff and Stiglitz, 1983; Dasgupta and Maskin, 1986; Hillman and Riley, 1989; Baye et al., 1996) as the contest success function, which serves as the rule to determine the player i's battle t's winning probability p_{it}:

$$p_{it} = \begin{cases} 1 & \text{if } e_{it} > e_{jt} \\ [0,1] & \text{if } e_{it} = e_{jt} \\ 0 & \text{if } e_{it} < e_{jt} \end{cases} \tag{2.2}$$

The payoff for player i is:

$$\pi_i = \begin{cases} V - \sum_{t=1}^{m} e_{it} & \text{if player } i \text{ wins the contest} \\ -\sum_{t=1}^{m} e_{it} & \text{otherwise} \end{cases} \tag{2.3}$$

2.3.2 Equilibrium Predictions

Table 2.1: Weights in Different Battles

	Battle 1	**Battle 2**	**Battle 3**	**Battle 4**	**Battle 5**
Weight 1:1	$\theta_1 = 1$	$\theta_2 = 1$	$\theta_3 = 1$	$\theta_4 = 1$	$\theta_5 = 1$
Weight 1:2	$\theta_1 = 1$	$\theta_2 = 2$	$\theta_3 = 1$	$\theta_4 = 1$	
Weight 2:1	$\theta_1 = 2$	$\theta_2 = 1$	$\theta_3 = 1$	$\theta_4 = 1$	

[5] In the experiments, participants are allowed to bid a whole number between 0 and 1000. The minimum is 0 tokens, and the maximum is 1000 tokens.

To study how weight (θ) and the order of the weights (θ_t) affect the New Hampshire Effect or an SMBC in general, we conduct a laboratory experiment with one control and two treatments groups (Table 2.1). The control group in the experiment is a regular best-of-five contest (Mago and Razzolini, 2019). We assign $\theta_1 = 2$ to Treatment 1 and $\theta_2 = 2$ to Treatment 2, respectively, to either place the relative larger weighted battle into the first or second positions to reveal the result in a sequential multi-battle contest. Namely, we call the treatment group by their assigned weight for the first two battles: the Weight 1:1, the Weight 1:2, and the Weight 2:1 for the control and treatment 1, and treatment 2, respectively.

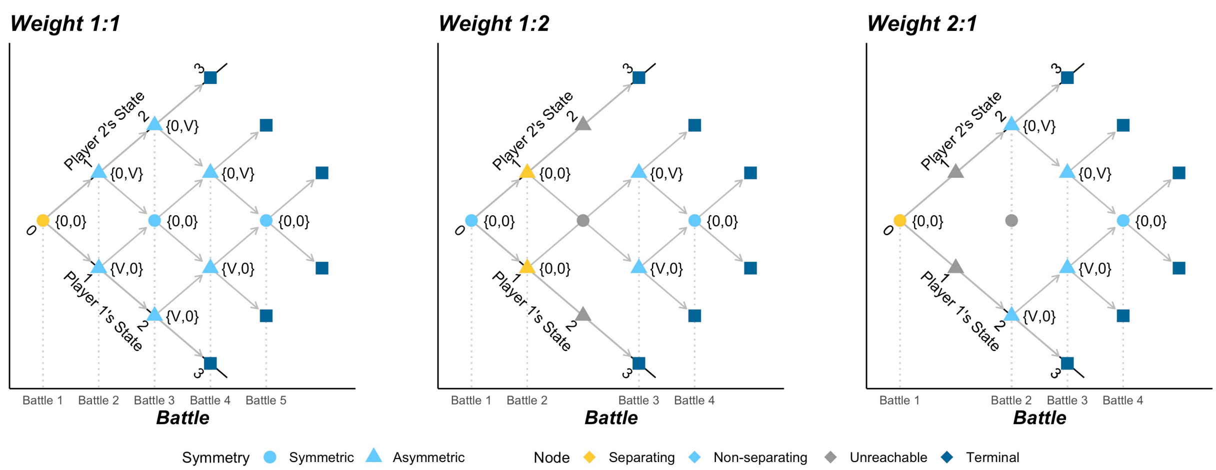

Figure 2.1: Game Tree

At the current battle node $t(s_{it}, s_{jt})$, we define the following four notions: denote z_{it} as the attributed prize of player i to win the game at battle t; denote v_{it} as the expected payoff if the game is at battle t; denote p_{it} as the probability of player i winning battle t; and denote $E(e_{it})$ as the expected effort that player i exerts at battle t. The game tree in Figure 2.1 illustrates the player's value of the prize of winning at each battle node t (including the unreachable node for different groups). In theory, the player will only compete in the separating battle nodes among the first two battles (Cai, 2022). We derived the equilibrium values for the three treatment groups and summarized them in Table 2.2.

Table 2.2: Equilibrium Values

	Weight 1:1	Weight 1:2	Weight 2:1
z_{it}	V, if $s_{it} = s_{jt}$ 0, otherwise	V, if $s_{it} = s_{jt}$ 0, otherwise	V, if $(s_{it}, s_{jt}) = (1,2), (2,1), (2,2)$ 0, otherwise
v_{it}	V, if $s_{it} > s_{jt}$ 0, if $s_{it} < s_{jt}$ 0, if $s_{it} = s_{jt}$	V, if $s_{it} > s_{jt}$ 0, if $s_{it} < s_{jt}$ 0, if $s_{it} = s_{jt}$	V, if $(s_{it}, s_{jt}) = (2,1)$ 0, otherwise
p_{it}	1, if $s_{it} > s_{jt}$ 0, if $s_{it} < s_{jt}$ .5, if $s_{it} = s_{jt}$	1, if $s_{it} > s_{jt}$ 0, if $s_{it} < s_{jt}$ .5, if $s_{it} = s_{jt}$	1, if $(s_{it}, s_{jt}) = (2,1)$ 0, if $(s_{it}, s_{jt}) = (1,2)$ .5, otherwise
$E(e_{it})$	$.5V$, if $s_{it} = s_{jt}$ 0, otherwise	$.5V$, if $s_{it} = s_{jt}$ 0, otherwise	$.5V$, if $(s_{it}, s_{jt}) = (1,2), (2,1), (2,2)$ 0, otherwise

2.4 The Laboratory Experiment

2.4.1 Experimental Hypotheses

In theory, we can show that when the first battle's weight is large enough, increasing more weight to battle 1 will not create more strategic momentum. There will be no strategic momentum when the first battle's weight is relatively small. Based on the theoretical prediction, our research will prioritize the following five main hypotheses:

Hypothesis 1. The efforts in battle 1 in weight 2:1 and weight 1:2 treatments are equal and greater than in the weight 1:2 treatment.

Hypothesis 2. The difference in battle 1 efforts between winner and loser is smaller in weight 1:2 treatment than in weight 1:1 and weight 2:1 treatments.

Hypothesis 3. Players behave equally in all equally weighted symmetric battle nodes. There is no treatment effect for equally weighted symmetric battle nodes.

Hypothesis 4. Players behave equally in all equally weighted asymmetric battle nodes. There is no treatment effect for any identical asymmetric battle node.

Hypothesis 5. There is no treatment effect on total effort.

2.4.2 Experimental Procedures

We conducted an experiment at The Experimental Economics Laboratory of CCBEF (China Center for Behavioral Economics and Finance) at the Southwestern University of Finance and Economics (SWUFE, China) to test the hypotheses and study the weight effect on different treatment groups. We hosted 15 sessions of in-person laboratory experiments. There were 12 subjects for each experimental session; 180 subjects were recruited to participate in the experiment. All of the subjects were undergraduate students from the SWUFE. We fixed the treatment for each experimental session, and the treatment was randomly assigned to the session.

There were two subsections of the program. When all participants arrived at the lab, we started the program simultaneously. In the first subsection, subjects read the instruction manual of the contest. There were follow-up quizzes to help the subjects understand the game's rules. After the quizzes, each subject played 20 contests with a randomly assigned opponent for each game. For each battle, subjects can submit a bid between 0 and 1000 tokens, and the higher bid wins the battle. The contest winner (who reached state=3 first) got 1000 tokens as a reward minus the tokens they bided through the sequential battles. The loser lost all the tokens they bid.

In the second subsection, subjects answered risk aversion and lost aversion questions (Shupp et al., 2013). They chose a preferred choice between A and B for all 16 questions, which would affect their final earnings. A socio-economic survey was also filled out at their own pace.

By the end of the session, we paid the subjects through an electronic payment platform based on the following three aspects: first, 10 RMB is the participation fee; second, two randomly selected results from 20 contests, of which we converted 50 tokens to 1 RMB; third, two randomly chosen question out of thirty-two survey questions (loss avers and loss averse). On average, each subject earned 95.32 RMB from a 50-minute experimental session.

2.5 Experimental Results

2.5.1 General Comparative Statics

Table 2.3: Summary Statistics of Experiment (Nash equilibrium)

	Weight 1:1		**Weight 1:2**		**Weight 2:1**	
Position	Leader	Non-leader	Leader	Non-leader	Leader	Non-leader
State (0,0)		106.32 (500)		122.71 (0)		386.89 (500)
State (1,0)	184.51 (1)	129.25 (0)	506.32 (500)	367.17 (500)		
State (1,1)		207.97 (500)				
State (2,0)	150.92 (1)	121.78 (0)			155.15 (1)	145.08 (0)
State (2,1)	278.24 (1)	242.05 (0)	257.29 (1)	265.01 (0)	232.72 (1)	257.72 (0)
State (2,2)		334.76 (500)		338.70 (500)		302.18 (500)
Total	810.78 (501)	516.10 (500)	903.34 (500.5)	538.68 (500.5)	872.33 (501)	534.85 (500)

We report the average effort for each corresponding battle and state.
The Nash Equilibrium predictions are listed in the parentheses.

Table 2.3 provides an overview of the average efforts in a given State corresponding to each different weight treatment. To compare to the theory benchmark in the parentheses, we find that the battle 1 bids never surpass the predictions in Nash. In later battles in all three weight treatments (excluding battle 2 in the weight 1:2 treatment), the non-leading player was never fully discouraged, which was dictated by the theoretical prediction. Meanwhile, the leading players can never win the contest with the predicted minimum effort. The observations in our experiments are consistent with previously conducted all-pay auction SMBC experiments (Gelder and Kovenock, 2014; Mago and Sheremeta, 2017). The total efforts among the contest's winner and loser are greater than the predictions in theory. The well-known over-dissipation phenomenon (Gneezy and Smorodinsky, 2006) is observed in the data. The efforts are sensitive to the assigned weight of battles when comparing the behaviors in different treatments, and we will address this finding in later discussions.

Table 2.4: The Probabilities of Battle 1 Winner Wins the Game

Treatment	Weight 1:1	Weight 1:2	Weight 2:1
Theory	100.00%	50%	100.00%
Random	68.75%	62.5%	87.50%
Experiment	78.17%	71.83%	89.00%

Jointly in Table 2.4 and Table 2.5, we present the probabilities of the battle 1 winners eventually becoming the winner of the entire contest and the possibilities of the contests ending with a particular Revealed State in each treatment. In Table 2.4, we find the probability of battle 1 winner winning the contest exceeds the winning rate when later battles' winners are randomly assigned, which provides evidence for the momentum effect (Mago et al., 2013). However, the magnitude of the momentum effects is different when comparing the probabilities of the experiments in Table 2.4. The weight 2:1 creates the least amount of this effect, and we can find more evidence comparing the ending probability of the Weight 1:2 (63.00%) and 2:1 (57.17%) treatment at Revealed State=3 (Table 2.5). Table 2.5 shows that the cumulative probability of Weight 1:2 and Weight 2:1 are similar, which are more likely to end with Revealed State=3 or 4 than the Weight 1:1 treatment. When the State (1:1)-reaching contests are excluded from the Weight 1:1 treatment, the three treatments are close in the proportion of "Revealed State ends."

Table 2.5: Contest Ends at Different Revealed States

	Revealed State=3	Revealed State=4	Revealed State=5
Weight 1:1	40.67%	35.00%	24.33%
Weight 1:1 None-State (1:1)	63.54%	21.23%	15.23%
Weight 1:2	63.00%	17.83%	19.17%
Weight 2:1	59.17%	18.83%	22.00%

Contests that went to State (1:1) are excluded from the Weight 1:1 None-State (1:1).

2.5.2 Initiating the Contests

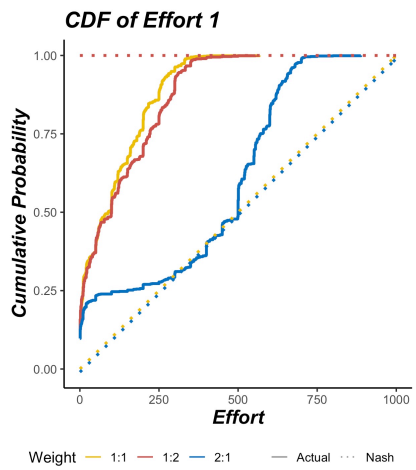

Figure 2.2: Effort in Battle 1

One fundamental research question in any sequential multi-battle contest is how players initiate a contest. In theory, it predicts that a risk-neutral player would focus on the very first battle when there is an equally weighted SMBC. On the other hand, the player will focus on the battle with more assigned weight when the weights are not equally assigned. Additionally, the weight will not affect the effort in the focused battle.

Result 1. Weight 1 (θ_1) and Weight 2 (θ_2) have both positive effects on battle 1 effort.

At the beginning of a contest, the theory predicts the battle 1 efforts in the weight 2:1 treatment and weight 1:1 treatment are equivalent, and this amount is larger than the effort 1 in weight 1:2 treatment. In the experiment, we find the subjects' battle 1 efforts are responsive to the nominal weight of battle 1. As presented in Figure 2.2, the CDF curve of battle 1 effort of weight 2:1 treatment is located on the right side of the other two treatments. Another noticeable result is that the right tail effort 1 distribution in weight 1:2 treatment also surpasses the CDF curve of the weight 1:1 treatment. We examine this result by splitting the data into two subsets with the median of effort 1 as the threshold. We consider the player whose bid 1 is greater or equal to the median of bid 1 as the player with a higher valuation of battle 1. We call this subset the higher battle 1 valuation subset. In contrast, the subset of bid 1 that is less than the median of bid 1 is the lower battle 1 valuation subset. In Table 2.6, we found a statistical significance from the random effects regression that effort 1 of weight 1:2 is greater than in weight 1:1 treatment. This difference is also economically significant; the intercept is 22.87% greater in the 1:2 treatment. For the lower battle 1 valuation player, there is no difference between weight 1:1 and weight 1:2 treatments. Descamps et al. (2022) find evidence against that player using the backward induction method to guide the biding behavior in SMBC.

Table 2.6: Random Effects Panel on Effort 1 and Effort 2

	Effort 1			**Effort 2 (Won Battle 1)**			**Effort 2 (Lost Battle 1)**		
Effort 1	**Full**	**⩾Median**	**<Median**	**Full**	**⩾Median**	**<Median**	**Full**	**⩾Median**	**<Median**
Weight 1:1	136.59 (15.20)	179.74 (15.29)	54.45 (15.78)	148.58 (17.70)	189.31 (18.47)	104.86 (20.07)	199.38 (14.13)	231.91 (11.36)	169.40 (15.84)
Weight 1:2	16.40 (14.44)	41.11** (18.40)	3.77 (6.27)	334.13*** (33.49)	311.82*** (35.51)	381.64*** (54.97)	241.64*** (36.03)	250.00*** (26.15)	241.25*** (32.66)
Weight 2:1	280.57*** (17.04)	379.82*** (14.82)	175.03*** (18.89)	-6.63 (8.27)	-55.17*** (12.94)	101.56*** (18.73)	8.15 (12.05)	-88.20*** 16.67	51.53*** (17.58)
Trend	-2.88*** (0.96)	1.37 (1.36)	-5.23** (2.50)	2.30 (1.60)	1.42 (2.28)	2.11 (3.12)	-5.98*** (1.38)	-7.68*** (1.55)	-7.02*** (2.06)
Observations	3,600	1,821	1779	1,800	1,336	464	1,800	485	1,315

The dependent variable is the effort for each corresponding battle and state.
The Weight 1:1 is the constant in the regression.
We also separate the data into two subsets depending on Effort 1.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

The result from Figure 2.2 and Table 2.6 suggest that contestants' bid 1 is affected by the weight of battle 2 when comparing effort 1 in the Weight 1:1 and 1:2 treatment. It confirms the forward induction conjecture (Descamps et al., 2022). In Weight 1:1 and Weight 2:1 treatment, the underbidding in battle 1 is consistent with previous all-pay auction SMBC studies (Mago and Sheremeta, 2017; Gelder and Kovenock, 2017; Zou and Cai 2022). In

Weight 1:2, players are overbidding, referring to the theoretical prediction. The primary driven factor of battle 1 behavior is the battle 1 weight (θ_1), and the secondary driven factor is the weight of the consecutive battle (θ_2). We reject **Hypothesis 1.** from the finding in **Result 1.**.

2.5.3 Differences in the New Hampshire Effect

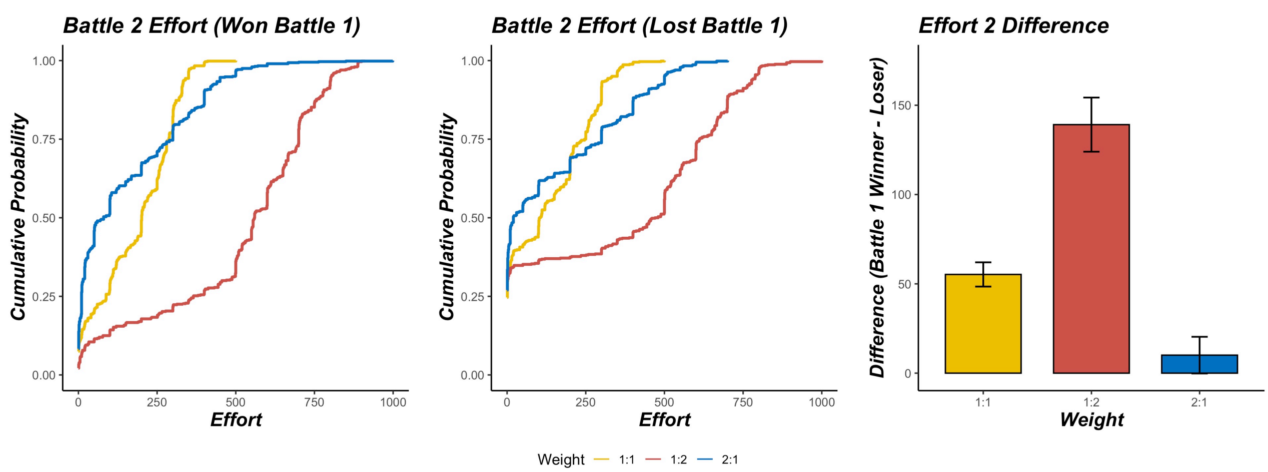

Figure 2.3: Effort in Battle 2

In previous studies, the observed new Hampshire effects are measured by the gaps in effort between the first battle winner and loser (Zizzo, 2002; Mago et al., 2013; Mago and Sheremeta, 2017, 2019; Mago and Razzolini, 2019; Descamps, 2022). The difference in effort between the winners and losers of battle 1 is commonly observed in SMBC. We adhere to focus on the experimental treatment effect on observed gaps between battle 1 winners and losers.

Result 2. Initiating a contest with a greater weight increases the observed discouragement among the losers of battle 1, but does not necessarily increase the difference between the winner and loser's effort.

In Figure 2.3, we find that the behavior of the winner and loser of the battle exhibit a coherent pattern. The median of effort 2 for Weight 1:1 treatment is between Weight 1:2 and Weight 2:1 treatment, regardless of the battle 1 result. When comparing the difference in effort 2 between the winner and loser of battle 1, we see that the weight 1:2 treatment has the highest difference. The difference in weight 1:1 treatment is the second, and the 2:1 treatment has the smallest difference between winner and loser in effort 2 (Figure 2.3 and Figure 2.4).

We also observe a universal left-skewed distribution of effort 2 for all three treatments when losing the first battle in Figure 2.3. The histograms for battle 1 losers' efforts clearly show that the most popular bids are less than 100 tokens for three different weight treatments. The losers of battle 1 are in a non-leading position, so biding relatively small in the allowed effort range means strategically giving up the contest. The proportions of participants who bid less than 100 are 44.83%, 35.50%, and 57.50% for the weight 1:1, 1:2, 2:1 treatment respectfully (Figure A.2 in Appendix A.2). The result suggests that the discouragement difference closely corresponds to the weight ratio between the first and second battles (50% for 1:1, 33.33% for 1:2, and 66.67% for 2:1 treatment).

From the perspective of the winner of battle 1, the most reasonable strategy is to secure the winning position, but with a minimum cost. This explains the observed relationship between battle 1 winner and loser is coherent. When the loser is bidding high, the winner is bidding high, and when the loser is bidding low, the winner is bidding low. The relativity of high and low is consistent with the weight ratio between battle 1 and battle 2 (Figure A.1

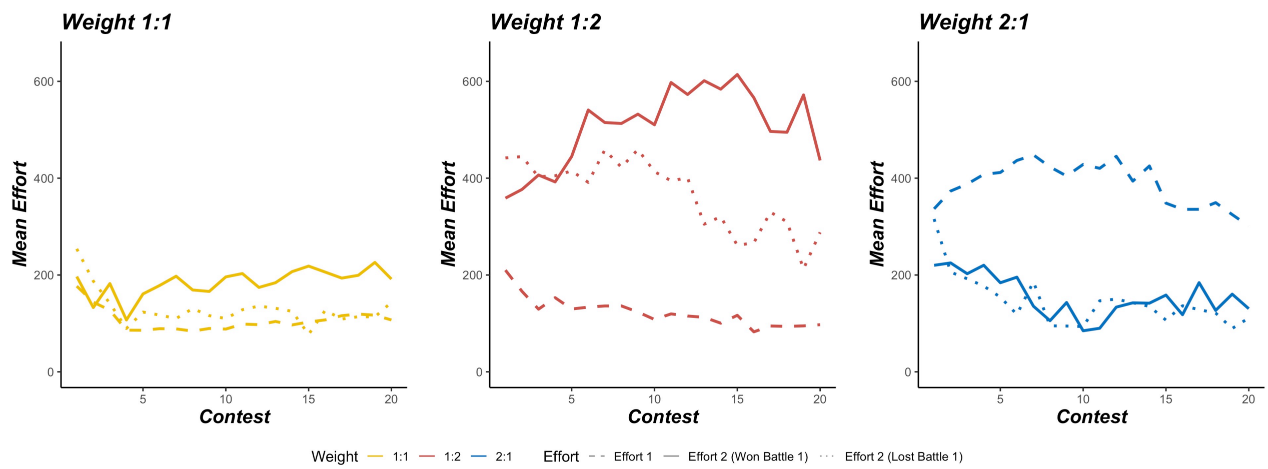

Figure 2.4: Mean Effort 1 and Effort 2 for 20 Contests

in Appendix A.1).

When comparing the difference in the winner and loser of battle 1 bid, the weight 1:2 is the highest, the difference in weight 1:1, and the weight 1:2 treatment has the lowest contrast. One of the reasons to explain the same directional tendency relationship between weight ratio and magnitude of difference between winter and loser of battle 1 is when both winner and loser bid more (weight 1:2) in battle 2, we observe a more severe difference. On the contrary, when both winner and loser bid less in battle 2 (in weight 2:1), the difference between the leader and the non-leader is compressed. The weight ratio in our design partially explains the difference in the observed New Hampshire effect. From Figure 2.3, Table 2.6 and Table 2.7 (Table A.1 and Table A.2 in Appendix A.3), we find the most observable treatment effect comes from the weight difference. Additionally, the observed differences between the winner and the loser of battle 1 mostly come from the contest's current state. We will explore more reasons in the following session. **Hypothesis 2.** is firmly rejected by **Result 2.**, where the difference between the winner and loser of battle 1 is the highest in weight 1:2, and such differences are not the same for the weight 1:1 and weight 2:1 treatment.

Table 2.7: Random Effects Panel on Effort in Battle 2

Treatment	**Weight 1:1**		**Weight 1:2**		**Weight 2:1**	
	(1)	(2)	(1)	(2)	(1)	(2)
Constant	172.12 (17.17)	180.65 (18.33)	511.55 (48.97)	483.86 (51.64)	209.32 (21.44)	201.29 (17.42)
Lost Battle 1	-32.43*** (2.01)	-57.46*** (9.67)	-124.54*** (27.24)	-141.93*** (20.40)	-15.63*** (5.79)	-32.98*** (8.05)
Utility of Winning		-19.00 (12.20)		76.42* (40.54)		21.81 (20.81)
Lost Battle 1 × Utility of Winning		65.90*** (24.00)		47.69*** (47.18)		47.51*** (17.32)
Trend	0.09 (1.42)	0.11 (1.35)	-1.19 (2.27)	-1.14 (2.22)	-4.89*** (1.87)	-4.82** (1.90)
Observations	1,200	1,200	1,200	1,200	1,200	1,200

The dependent variable is the Effort 2 for each corresponding treatment.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the Lost Battle 1 × Utility of Winning is the testing result of the Utility of Winning effect on the Lost Battle 1.
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

2.5.4 Symmetric Battles

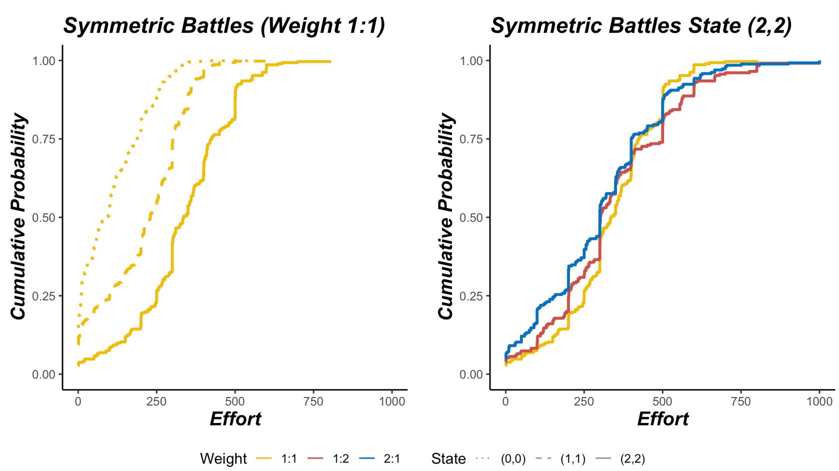

Figure 2.5: Effort in Symmetric Battles

In this section, we analyze the dynamics of the games based on the symmetry and the intensity of the battle nodes. We would like to address the importance of progress in a sequential contest. In previous 2-players sequential multi-battle contest experiments with $T > 2$ (Zizzo, 2002; Gelder and Kovenock, 2017; Mago and Razzolini, 2019), researchers find the players are bidding more and more as the contest goes to later battles. We present our results in this session with a more well-defined notion of the relative position and intensity of the contest. We use the regular best-of-five (Weight 1:1) game as the frame of reference to achieve this purpose.

Result 3. Players bid more aggressively when there are fewer remaining battles in the contest in symmetric battles. No significant evidence supports the treatment effect for equally weighted symmetric battles.

In the symmetric battle nodes (State (0:0), State (1:1), State (2:2)), we see players in the Weight 1:1 treatment acting more and more aggressively as the contest reach later progress (Figure 2.5). Based on the rule of our experiment, a player needs to win three points ($T = 3$) to win the contest. Therefore a player who has 2 points (s=2) in the game reaches the **match point**, which they only need one addition battle won to win the contest. A match point is commonly used in a tennis game to describe the situation in which a player can win the contest by winning the next point (battle). We use the distance from the match point of the players in a symmetric battle node to describe the intensity of the game. We consider the game is more intense when the battle node is closer to the match point. From this perspective, the players' effort increase significantly when the game is more intense (Table 2.3, Figure 2.5 and Table 2.10). When both players are at match point (state (2:2)), we observe no treatment effect regardless of the history through the contest. In Figure 2.5, we discover that the shape of the CDFs for all three treatments is very similar to the best-of-five game when the intensity varies for the given symmetric battle.

We also find that the estimated y-intercept (136.59, 183.28, 260.51 for the state (0,0), state (1,1), and state (2,2), respectively) of random effects regression for weight 1:1 treatment increases significantly when the contest reaches the end in Table 2.10. Additionally, there is no statistical evidence suggesting a treatment effect in Table 2.10 when both players reach the match point. We reject **Hypothesis 3.** with the findings of **Result 3.**.

2.5.5 Asymmetric Battles

Result 4. Players bid according to the intensity of the asymmetric battles.

Another aspect to consider in the SMBC is the asymmetric battles, where players are at different strategic states; in other words, leader and non-leader in a battle. Similar to symmetric battles, the relative positions between players can also distinguish different states in the contest's progress. We divide all the asymmetric battles into the following three scenarios, then present and analyze the experimental results.

Intensity. When the game is in an asymmetric battle, the battle node is defined as the following three intensity assessments given the states of a contest:

Non-intensive: At state (1:0), the leader has not reached the match point and leads by a small margin. The non-leader has made no progress in the contest.

Semi-intensive: At state (2:0), the leader reaches the match point and leads by a large margin. The non-leader has made no progress in the contest.

Full-intensive: At state (2:1), the leader reaches the match point and leads by a small margin. The non-leader has made some progress in the contest.

We discuss the difference in distributional behavior based on the intensity of the state. To observe the behavioral difference when the intensity is different in the dynamic progress, we use the cumulative distribution function of the effort in weight 1:1 treatment. When comparing the intensity of the battle, we use the non-intensive State (1:0) as a reference.

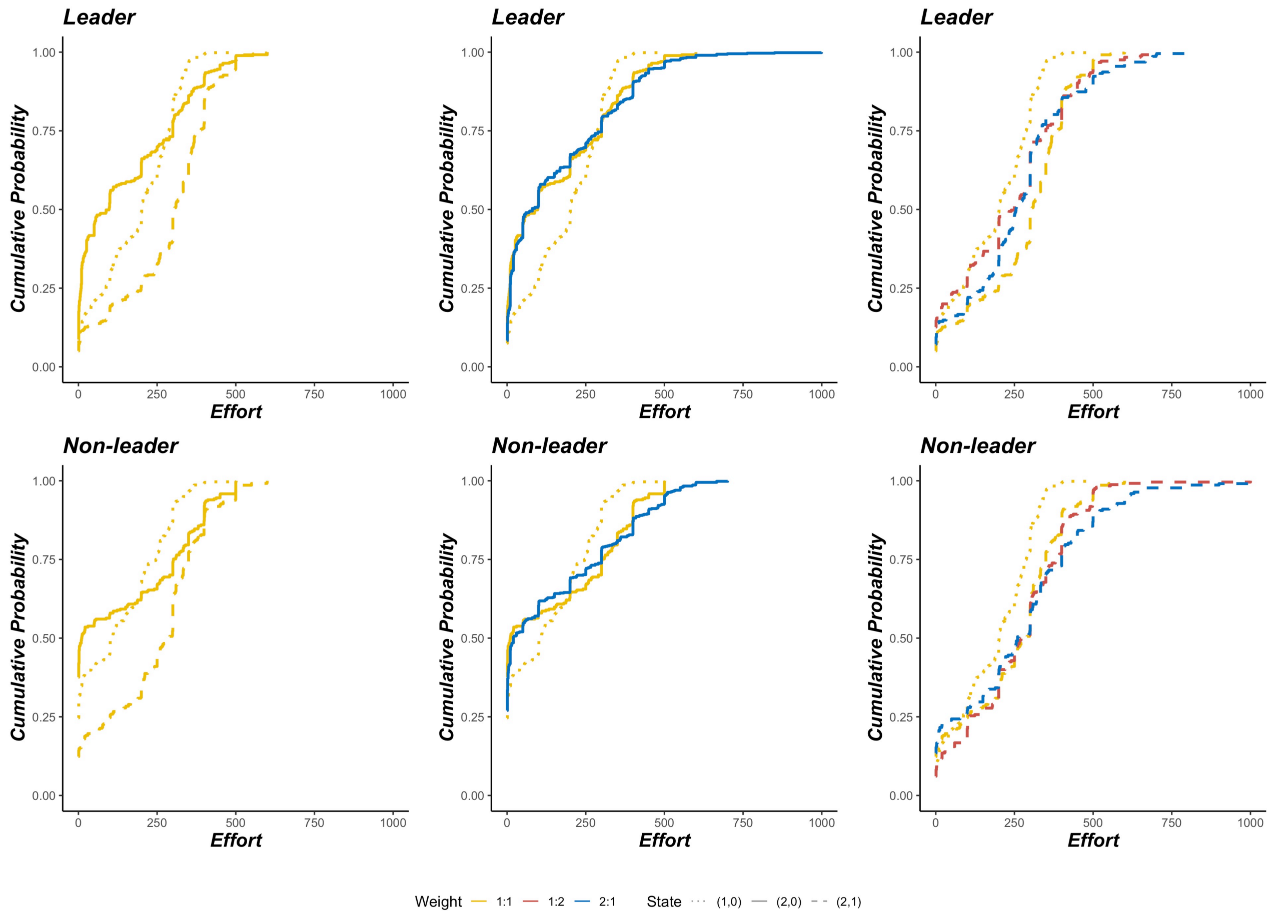

Figure 2.6: Effort in Asymmetric Battles

From the non-intensive State (1:0) to the semi-intensive State (2:0), the only thing that changes is the gap between the leader and non-leaders in the contest. We see that both leaders' and non-leaders' bidding distributions are distorted in the sense that the distribution is leaning toward a U-shape tendency. Specifically, the boundary behavior is the more intense State (2:0) compared to the State (1:0) in the weight 1:1 treatment. We find that the leaders are trying to reduce their bids as non-leaders are discouraged in bidding, suggesting that distributionally leaders are trying to win the contest with a minimum cost. Additionally, the comparative distortion of the boundary behavior in State (2:0) compared to State (1:0) is similar for both Weight 1:1 and Weight 2:1 treatment, where the high bid is higher, and the low bid is lower (Figure 2.6). It explained that the observed New Hampshire effect is smaller in the Weight 2:1 treatment than in the other two treatments, driven by the coherent behavior of leaders and non-leaders at State 2:0. More evidence is provided in Table 2.8, where comparing the dispersion measurements (standard deviation, interquartile range, and mean absolute deviation) between State (1:0) and State (2:0), we see efforts in State (2:0) are relatively more spread. Meanwhile, the effort is spread equally despite the weight treatment difference.

Table 2.8: Measures of Dispersion for State (1:0) and State (2:0)

Position	Leader			Non-leader		
Measurement	SD	IQR	MAD	SD	IQR	MAD
Weight 1:1 & **State (1:0)**	117.83	199.25	143.09	124.49	246	190.15
Weight 1:1 & **State (2:0)**	160.44	291	214.86	172.92	302	212.17
Weight 2:1 & **State (2:0)**	173.84	290	214.75	184.44	300	215.09

Players who dropped out after losing battle 1 are excluded from the Weight 1:1 treatment.

The similarity of behavior in State (2:0) of Weight 1:1 and Weight 2:1 treatments further strengthens our explanation of high and low bids in leaders and non-leaders. In Table 2.8, the observed treatment difference between Weight 1:1 and Weight 2:1 in State (2:0) comes from different historical paths. The leader in Weight 2:1 bid more in previous battles to gain the leading position, and we can see the clear pattern in Figure 2.7. On the other hand, the non-leader in the Weight 1:1 treatment was already determined to drop out since State (1:0). When players who quit the contest in State (1:0) are removed from the CDF (Figure 2.6), we see no vertical distance in the non-leaders effort CDF in State (2:0) between Weight 1:1 and Weight 2:1 treatment.

Then we compare the non-intensive versus the full-intensive State. We consider the leading gap is equal in State (1,0) and State (2,1), but the intensity of the progress of the contest is not equivalent because in leaders in State (2,1) reach the match point. We find consistently that both leaders and the non-leaders in State (2,1) bid more aggressively than

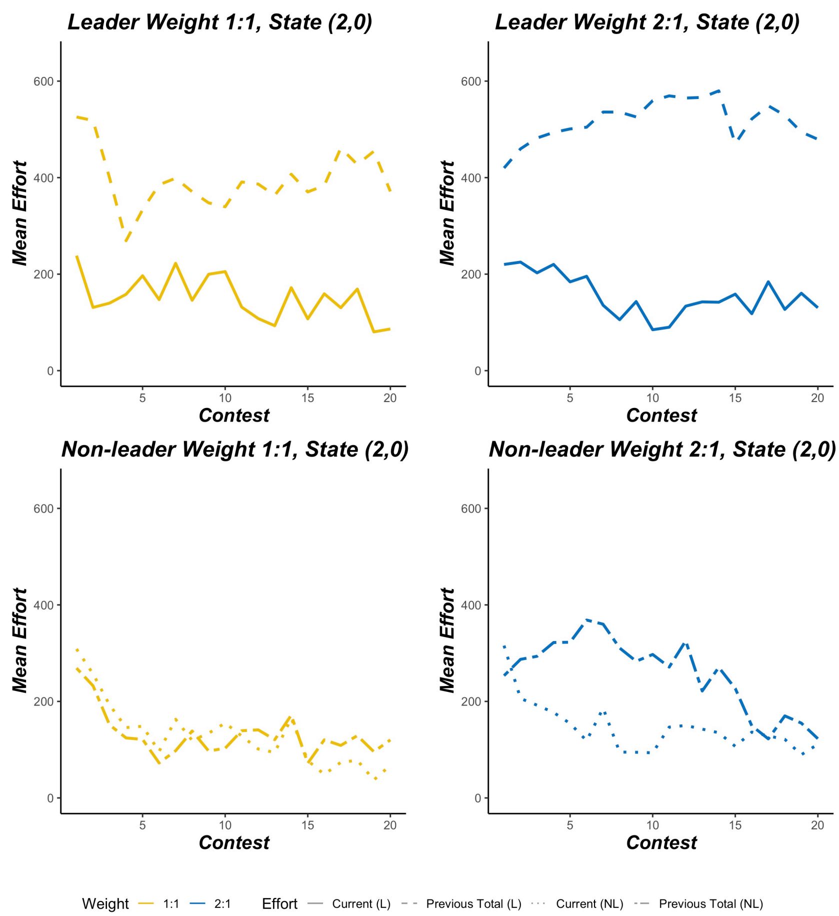

Figure 2.7: Effort and Previous Total Effort at State (2,0)

in State (1:0) from Figure 2.6. Symbolically the leaders in full-intensive States have more incentive to secure the win than in the non-intensive State. On the other hand, the non-leader put more effort, given that the current battle has the potential to be the very last battle if they do not win it. Notably, there is less dropping out behavior for the non-leading player in State (2,1) than State (2,0) because the leading margin is smaller. In the full-intensive State across treatments, we see a minor treatment effect on Weight 2:1 leaders, where they exert less effort with statistical significance at 10% (Table 2.10). One reasonable source of observed bidding less of leaders in the Weight 1:2 and Weight 2:1 treatment referencing Weight 1:1 treatment is that the loser of the previous battle tends to bid more (Table 2.14).

We reject **Hypothesis 4.** with the findings of **Result 4.**.

Table 2.9: Random Effects Panel on Difference between Leader and Non-leader

State	**(1,0)**		**(2,0)**		**(2,1)**		
Weight	**1:1**	**1:2**	**1:1**	**2:1**	**1:1**	**1:2**	**2:1**
Constant	172.12	511.55	218.65	209.32	256.75	227.25	200.16
	(17.17)	(48.97)	(28.75)	(21.44)	(5.81)	(37.78)	(12.17)
Non-leader	-32.43***	-124.54***	-21.92*	-15.63***	-29.33*	-0.15	19.37
	(2.01)	(27.24)	(12.94)	(5.79)	(15.66)	(28.50)	(25.44)
Trend	0.09	-1.19	-10.09***	-4.89***	2.35*	6.26	7.05
	(1.42)	(2.27)	(2.85)	(1.87)	(1.31)	(8.63)	(4.17)
Observations	1200	1200	738	1200	712	444	490

The dependent variable is the effort for each corresponding state and treatment.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

Table 2.9 lists all possible compression of leader and non-leader for all different asymmetric States in the contest. We see the difference between leaders and non-leaders are also responsive to the current battle weight and intensity. In State (1,0), the observed discouraging effect closely reflects the different weights of the battle 2 in Weight 1:1 and Weight 1:2 treatment. Then in State (2,0), there is no severe economic difference between Weight 1:1 and Weight 2:1 treatments. Next in State (2,1), Weight 1:1 has a slightly more significant gap between leader and non-leader than the other two treatments, likely due to the number of battles before the current battle. However, the difference is neither statically nor economically significant. We see trending of decaying in the gaps as the contests go to the later battles.

2.5.6 The Dynamic Decay of Treatment Effects

In Table 2.10 (Table A.3), we summarize the general observation of the progress of a sequential contest. Firstly, players are very responsive to the assigned weight of the current battle. In State (0,0) and State (1,0), the bids are statistically and economically significantly

Table 2.10: Decaying in Treatment Effect

	State (0,0)	State (1,0)		State (1,1)	State (2,0)		State (2,1)		State (2,2)
	Non-leader	Leader	Non-leader	Non-leader	Leader	Non-leader	Leader	Non-leader	Non-leader
Weight 1:1	136.59	133.94	185.74	183.28	183.79	169.72	241.89	224.02	260.51
	(15.20)	(13.73)	(13.78)	(10.57)	(11.39)	(13.92)	(16.40)	(25.66)	(15.70)
Weight 1:2	16.40	334.23***	242.56***				-6.67	20.31	11.18
	(14.44)	(34.07)	(36.28)				(20.54)	(37.34)	(29.19)
Weight 2:1	280.57***				23.95**	29.85*	-35.79*	9.72	-33.71
	(17.04)				(10.57)	(17.60)	(18.66)	(31.61)	(28.00)
Trend	-2.88***	6.74***	-8.58***	5.22***	-7.53***	-9.30***	9.05**	3.92	23.63***
	(0.96)	(2.18)	(2.50)	(1.76)	(2.60)	(2.71)	(3.86)	(3.59)	(4.05)
Observations	3600	1200	1200	462	969	969	823	823	786

The dependent variable is the effort for each corresponding state and position.
The Weight 1:1 is the constant in the regression.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

higher when the given weight of the current battle is 2 compared to 1. The weighting treatment effect of the sequential contest is verified in our experiment. As the contest goes into the later stages, we find a general pattern of decaying in treatment effect, until at State (2,2), all treatment effects are statistically insignificant. In both symmetric and asymmetric battles, players are responsive to the current state's intensity. Overall, the intensity depends on how many remaining battles are left before the leader wins the contest (See the robustness checking on the weight from Figure A.3 in Appendix A.5). In other words, the intensity depends on how close the leader is to the match point. When any player reaches the match point in the contest, the most aggressive contents exert more effort.

2.5.7 Total Effort

This session explores the total effort and compares the treatment differences. We divide the contest into three categories depending on when they end. There are three possibilities of when the contents end, and we distinguish the contests by their corresponding ending revealed State (Score). The winning threshold in the experimental design is $T = 3$; therefore,

Table 2.11: Random Effects Panel on Total Effort

Contest Ends	Revealed Weight=3		Revealed Weight=4		Revealed Weight=5		All	
	Winner	Loser	Winner	Loser	Winner	Loser	Winner	Loser
Weight 1:1	556.33 (12.92)	277.40 (21.88)	809.57 (30.78)	646.43 (29.36)	1059.85 (46.57)	925.01 (53.38)	844.47 (18.83)	691.37 (19.31)
Weight 1:2	223.02*** (43.93)	179.42*** (53.70)	145.17** (64.99)	138.07* (77.26)	223.07*** (43.04)	168.18** (73.29)	94.88* (51.97)	19.99 (65.24)
Weight 2:1	183.11*** (17.28)	197.93*** (29.50)	87.04*** (26.89)	104.53** (43.24)	92.18 (94.49)	-18.55 (36.58)	70.32 (42.70)	13.13 (20.13)
Trend	-7.55*** (1.99)	-18.66*** (2.60)	11.25 (7.80)	-5.98* (3.56)	32.91*** (11.03)	23.57* (13.03)	-4.63*** (1.59)	-13.36*** (1.74)
Observations	977	977	430	430	393	393	1,800	1,800

The dependent variable is the total effort for each corresponding revealed weight and contest result.
The Weight 1:1 is the constant in the regression.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

the contests end at the revealed state $\sum \theta_t$ equal to 3, 4, or 5.

Result 5. Weight 1:2 treatment creates the most long-lasting treatment effect on total effort, followed by the Weight 2:1 treatment.

We present the random effects regression of total effort based on the revealed weight and the final result of the contest. In Table 2.11, when the contests end at revealed weight equals 3 or 4, the total effort in Weight 1:2 and Weight 2:1 are significantly larger than in Weight 1:1 treatment. The total bid in Weight 1:2 being larger than Weight 1:1 is due to the less discouragement effect in battle 2. On the other hand, the total bid in Weight 2:1 being higher than Weight 1:1 is because the player reacting to the Weight 2 battle is more than doubling the Weight 1 battle as battle 1. However, unlike in the Weight 1:2 treatment, the positive effect of the Weight 2 battles in the Weight 2:1 treatment does not last until the revealed weight equals 5.

Generally considering total effort in the contests, we see the Weight effect, especially the effect of weighted 2 battles' positive impact on the total effect. From Figure 2.8, we can find

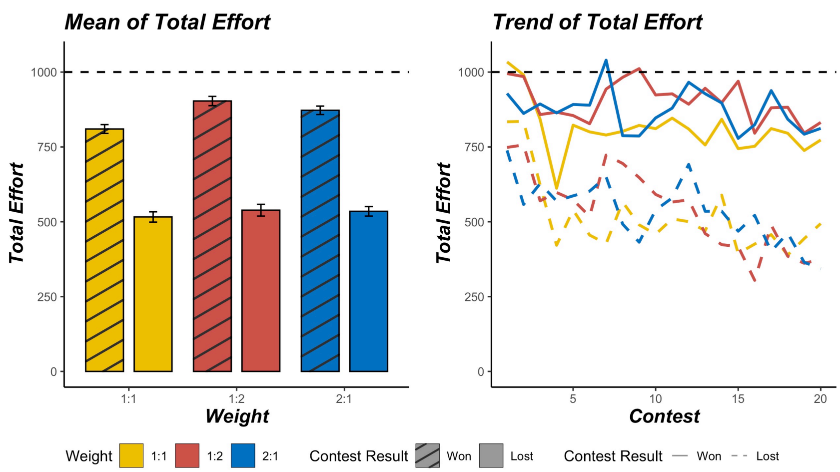

Figure 2.8: Total Effort

that the effect is stronger in the Weight 1:2 treatment than in the Weight 2:1 treatment. Additionally, the effect is stronger in winners than the losers of the contest (Table 2.11). Lastly, we find a solid and consistent downward trend in the total effort among the losers in Figure 2.8. The **Hypothesis 5.** is rejected by the evidence found in **Result 5.** as a consequence of observed weight effects.

2.5.8 Utility of Winning, Loss Aversion, and Physiological Momentum

Cognitive biases are commonly addressed in previous SMBC experiments (Mago et al., 2013; Mago and Razzolini, 2015; Mago and Sheremeta, 2019; Zou and Cai, 2022). We mainly focus on the following three aspects: utility of winning (Schmitt et al., 2004; Sheremeta, 2010; Zizzo, 2010; Price and Sheremeta, 2011, 2015; Brookins and Ryvkin, 2015; Mago et al., 2016; Cason et al., 2020), loss aversion (Kong, 2008; Boosey et al., 2020; Bruner, 2022; Zou and Cai, 2022), and physiological momentum (Mago et al., 2013, Mago and Razzolini 2019) that has been studied in contest experiments in general.

Table 2.12: Random Effects Panel on Leader

	State (1,0)	State (2,0)	State (2,1)
Weight 1:1	103.43	193.15	248.21
	(40.98)	(32.98)	(17.06)
Weight 1:2	332.46***		-6.18
	(35.00)		(20.63)
Weight 2:1		22.81**	-35.86*
		(9.58)	(18.39)
Trend	6.73***	-7.57***	9.06**
	(2.18)	(2.60)	(3.88)
Utility of Winning	25.68	24.38**	-0.55
	(28.17)	(10.76)	(15.32)
Loss Aversion	26.27	-22.40	-7.72
	(34.19)	(34.89)	(13.32)
Observations	1,200	969	823

The dependent variable is the effort for each corresponding state.
The Weight 1:1 is the constant in the regression.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

In the survey section of our experiment, we ask the participant which they prefer the most among the two options, “earning more monetary award” and “winning the contest.” We use their answer to the binary survey question as a dummy variable in the regression reported in Table 2.7, Table 2.12, and Table 2.13. Consider the participants who chose “winning the contest” over ”earning more monetary award” to evaluate winning the contest more than their counterparts. In a leading position, the player with more valuation of winning bids is more in a semi-intense state (Table 2.12). In contrast, people with more utility of winning consistently spend more effort in any non-leading position (Table 2.7 and Table 2.13). We also see that the participants who have less utility in winning are more like drop out in battle 2 in Table 2.15. The utility of winning is robustly revealed that only players are behind in a sequential contest.

To see how heterogeneous players behave in the game, we elicited the subjects’ loss preference with the method used in Shupp et al. (2013). Table 2.13 shows the loss aversion

Table 2.13: Random Effects Panel on Non-leader

	Score (0,0)	Score (1,0)	Score (1,1)	Score (2,0)	Score (2,1)	Score (2,2)
Weight 1:1	132.12 (17.48)	198.53 (26.09)	173.19 (12.77)	186.30 (18.85)	233.31 (32.04)	252.07 (24.03)
Weight 1:2	16.70 (14.43)	243.64*** (35.06)			20.09 (36.51)	10.59 (28.50)
Weight 2:1	279.26*** (17.36)			24.65 (15.96)	6.32 (31.27)	-33.19 (28.09)
Trend	-3.01*** (0.97)	-8.33*** (2.46)	5.21*** (1.81)	-8.97*** (2.10)	3.59 (3.53)	23.86*** (3.98)
Utility of Winning	30.06** (15.01)	73.43*** (25.12)	6.59 (8.80)	51.22** (21.97)	33.86** (15.93)	-5.91 (18.11)
Loss Aversion	-16.03 (10.10)	-47.90* (26.90)	9.49 (15.23)	-41.38** (20.70)	-24.99 (15.46)	12.73 (17.48)
Previous Win	17.59* (10.17)					
Observations	3,600	1,200	462	969	823	786

The dependent variable is the effort for each corresponding state.
The Weight 1:1 is the constant in the regression.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

player [6] bids comparatively less when their current state is 0. We do not observe such a loss aversion effect in the semi-intence State (2,0). The finding that Loss-averse players bid less at the non-leading position is also reported in Zou and Cai (2022).

Lastly, we find minor evidence that suggests physiological momentum [7] in various aspects. In Table 2.14, we find that the previous battle win has a negative effect on the current bid for both leading and non-leading positions. This effect is only exhibited in the asymmetric battle. In other words, in a full-intense State (2,1), the path in history affects the current behavior. The leader bids less when the previous State is (1,1), and the non-leader bid less when the previous State is (2,0). We artificially skipped State (1,1) in the Weight 1:2 and Weight 2:1; the weight effect turns off this observed psychological momentum. Additionally, we

[6] We use the Holt and Laury (2002) method that considers the first switching point before the central question in the table as a more loss aversion subject. We consider the subject who switches from A to B before the central question as loss aversion, which differs from the method used in Shupp et al. 2013. The same approach is adopted in Zou and Cai 2022.

[7] In Mago et al. (2013), players are considered to have psychological momentum when differently in the same current state. The causes of psychological momentum could be either previous battles' or last contests' results.

Table 2.14: Random Effects Panel on Previous Battle Won

State	(1,1)	(2,1)		(2,2)		
Weight (Position)	1:1 (NL)	1:1 (L)	1:1 (NL)	1:1 (NL)	1:2 (NL)	2:1 (NL)
Constant	175.41	251.01	249.45	289.54	251.06	210.41
	(10.91)	(9.14)	(25.78)	(9.25)	(39.45)	(22.71)
Previous Battle Won	21.24	-21.13***	-24.93*	1.15	-4.59	14.56
	(12.82)	(7.98)	(12.94)	(12.14)	(27.04)	(11.18)
Trend	4.80**	9.39***	0.20	14.09***	31.99***	26.89***
	(1.91)	(3.55)	(1.96)	(3.66)	(11.24)	(4.92)
Observations	462	356	356	292	230	264

The dependent variable is the effort for each corresponding state, treatment, and position.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

find some evidence that supports the existence of psychological momentum across contests. Winning the previous contest positively correlates with a higher bid in battle 1 (Table 2.13) and the possibility of entering the contest in battle 1 (Table 2.15).

2.6 Conclusion

Weight or asymmetric values of battles is generally considered in a multi-battle contest. In our paper, we extended the research on the importance of weight in sequential games. We modeled spending a costly effort (all-pay auction) instead of allocating resources in the sequential multi-battle contest. Then we discussed the impact of weight difference and the order difference of the weight in battles in both theoretical and experimental sense.

We find strong economic and statistical significance in the weight effect. When increasing the weight of the current battle, regardless of the battle's position in the multi-battle competition's sequence order, contestants exert sufficiently more effort. If the battles are equally weighted, the current state's intensity dominates players' behaviors. In symmetric battles, players bid more intensely as the contest goes to later battles. In asymmetric battles, the tails of the effort distribution depend on whether the leader reaches the match point. The

Table 2.15: Random Effects Probit on Non-leaders' Behavior (exclude no-enter)

	Effort 1 (Enter and Dormant)		Effort 2 (Stay and Drop)	
	(1)	(2)	(1)	(2)
Weight 1:1	1.03***	0.98***	2.43***	2.64***
	(0.20)	(0.25)	(0.28)	(0.37)
Weight 1:2	0.51**	0.51**	-0.60***	-0.61***
	(0.20)	(0.20)	(0.22)	(0.22)
Weight 2:1	0.29	0.29	-0.72***	-0.77***
	(0.24)	(0.24)	(0.12)	(0.12)
Trend	0.01	0.01	-0.06***	-0.06
	(0.01)	(0.01)	(0.02)	(0.02)
Utility of Winning		-0.01		0.40**
		(0.15)		(0.18)
Loss Aversion		-0.02		-0.39
		(0.13)		(0.31)
Previous Win		0.19***		
		(0.06)		
Observations	3,435	3,435	1,636	1,636

The dependent variable of Effort 1 is 1 for Entry and 0 for Dormant.
The dependent variable of Effort 2 is 1 for Stay and 0 for Dropout.
The Weight 1:1 is the constant in the regression.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

distribution's right tail becomes more intense when any player reaches the match point. On the contrary, there is more of a discouragement effect when the leading gap is enlarged.

We find a general decaying treatment effect when the contest progresses to later battles. In other words, we find a path-independent strategic momentum. The results shown in our work are the first to bring the weight aspect in a sequential game. Despite the order sequence weight, we find a robust but not long-lasting effect. Further developments, such as the asymmetric valuation of players, different contest success functions, number of players, group contest, and elimination mechanism, are also worth discovering.

Chapter 3

Group Size Effects in Sequential Multi-battle Contests

Kevin Zou, Yichuan Cai

Abstract

This paper presents the result of a laboratory experiment that investigates the group size effect in the sequential multi-battle contest. We first generalized Konrad and Kovenock's (2009) result to multi-player ($n \geqslant 2$) games. Then we conduct an n-player ($n = 3$ or 6) experiment to compare the all-pay auction and the Tullock lottery. When increasing the number of contestants, we get the following result: First, in battle 1, the boundary distribution is distorted in the all-pay auction, but only the no-entry rate is increased in the Tullock lottery. Second, the dropout rate is higher when the players are not in the lead. Third, the total number of battles has not increased significantly, but more players have adopted the dormant-reenter strategy. Finally, The winner, maximum battle effort, and total contest effort increase with n. In general, the Tullock provides a more accurate theoretical prediction.

JEL Classification: C72; C73; C91; C92; D44; D72

Keywords: Group Size; Sequential Contest; All-pay Auction; Tullock Lottery; Experiments.

3.1 Introduction

The number of contestants in many sequential multi-battle competitions is not limited to two. For instance, the primary election in the United States, auto racing (Formula One and NASCAR), multisport races (triathlon and biathlon), and combined track and field events (decathlon and heptathlon) are all considered sequential multi-battle contests with more than two players. Since many competitions have more than two contestants, the group size of a game becomes an essential component in research.

It is common to consider a multi-battle contest as dynamic or sequential. In early works, the structure of sequential multi-battle contest (SMBC) is applied to study R&D competition. Fudenberg et al. (1983) study a non-constant sum multi-battle contest between two firms. In Tullock (1980, 2001), two identical players simultaneously expand efforts in a one-shot contest. The winner of the contest is determined probabilistically. In Harris and Vickers (1985), two firms sequentially expend effort to decrease their continuous distance to the finish line. The continuous distance to the finish line becomes a series of battles in Harris and Vickers (1987). Klumpp and Polborn (2006) generalize two-player sequential multi-battle Tullock lottery contest results with different sensitivity parameters ($r \leqslant 1$). The two-players Tullock multi-battle contest's result is extended into a multi-player race ($n > 2$) in Doğan et al., 2018. Regarding another most commonly used contest success function (CSF) of the Colonel Blotto game, Baye et al. (1996) employ an all-pay auction in the one-shot contest with multi-player ($n \geqslant 2$) and asymmetrical valuation. Konrad and Kovenock (2009) develop the all-pay auction (Baye et al., 1996) into a multi-battle contest with two players. Deck and Sheremeta (2012) use a special case in Konrad and Kovenock (2009) to examine a two-player game of the weakest-link system. Gelder (2014) introduces a prenatal structure into the two-player all-pay auction sequential multi-battle contest. In the theory portion of this paper, we generalize the result of the multi-battle auction contest into $n \geqslant 2$ players.

In the current experimental work, we find two features in common. First, all contest experiments that study group size effects focus on a one-shot ($T = 1$) contest (Table 3.1). Second, all sequential multi-battle experiments study the game's dynamic with a two-player ($n = 2$) setting (Table 3.2).

This paper contributes to the existing literature in the following three dimensions. First, we generalize the theory result of sequential all-lay auction multi-battle contest from Konrad and Kovenock 2009 to n ($n \geqslant 2$) players and extend the SMBC experiment into a multi-player ($n > 2$) research. Second, we extend the laboratory contest experiment in group size to a multi-battle setup ($T > 1$). Third, we provide a side-by-side compression between the all-pay auction ($r = \infty$) and the Tullock lottery ($r = 1$) in the sequential multi-battle contest.

3.2 Background

The number of players is critical in contest research. In theory, the group size effect depends on the structure of a contest. For all-pay auctions, the net rent is n-invariant with rational players (Baye et al., 1996) and is decreased in group size with bounded rationality (Anderson et al., 1998). In the Tullock rent-seeking game, the average effort decreases when there are more contestants (Lim and Matros, 2009). Allard (1988), Leininger (2001), and Ryvkin (2007) find that the number of players negatively influences the effort of individuals when players' abilities are not identical. As for rank-order tournaments, Clark and Riis (2001) extended the result into a n-players' game. Gerchak and He (2003) point out that the tournaments' noise distributions affect the expected effort in a rank-order tournament. In the multi-battle individual contest, the player's number changes the entire dynamic of the contest (Doğan et al., 2018). In multi-battle team contests, the total effort is not influenced by the group size (Fu et al., 2015). The group size effect is more negatively influential when the group size is unknown (Boosey et al., 2017, 2019). Similar results can be generalized in the Poisson contests (Kahana and Klunover, 2015). We can find more detailed summaries from Konrad (2009).

Table 3.1: Group Size Experiments

Year	Author(s)	Contest type	Players n	Contests
2003	Harbring and Irlenbusch	All-pay auction	2, 3, 6	20
2004	Orrison, Schotter, and Weigelt	Tournaments	2, 4, 6	20
2006	Gneezy and Smorodinsky	All-pay auction	4, 8, 12	10
2009	Amaldoss and Rapoport	Tournaments.	3, 8	60
2011	Sheremeta	Tullock lottery	2, 4	60
2012	Morgan, Orzen, and Sefton	Tullock lottery	2, 3, 4, 5	50
2013	Ernst and Thöni	All-pay auction	2, 3	10
2014	Lim, Matros, and Turocy	Tullock lottery	2, 4, 9	10
2017	Booseya, Brookinsb, and Ryvkin.	Tullock lottery	3, 6	30
2020	List, Van Soest, Stoop, and Zhou	Tournaments	2, 4, 8	4
2020	Nelson	Stackelberg	2, 3	25
2021	Hudja	Innovation	2, 4	30
2021	Fallucchi, Niederreiter, and Riccaboni	Tullock lottery	3, 5	60
2021	Peeters, Rao, and Wolk	Proportional-prize	2, 3, 4	20

Many studies have been conducted to see how group size affects behavior in experiments. We list some experimental contest research that focuses on the group size effect perspective in Table 3.1. In Tullock contests, Sheremeta (2011), Morgan et al. (2012), and Boosey et al. (2017) find that the expected effort decreases as the number of players increases. However, Lim et al. (2014) find that the expected effort is not sensitive to the group size. Nelson's (2020) Stackelburg version (Linster, 1993; Hinnosaar, 2018) of Tullock contest experiments finds evidence for entry deterrence of leaders when enlarging the group size. Fallucchi et al. (2021) discover that the fraction of zero effort increases as the group size of a contest increases. In all-pay auctions, Gneezy and Smorodinsky (2006) find that when there are more players in the game, the average bids decrease. Nevertheless, Harbring and Irlenbusch (2003) point out that a weak increase in the mean bid increases the group size in the contest. In the tournament contest, Orrison et al. (2004) find that the mean effort is n-invariant if the noise is uniformly distributed. Unlike the finding in Orrison et al. (2004), List et al. (2010) find mean effort decreases when contestant number increases. Amaldoss and Rapoport (2009) identify that players exert more effort on the stage that has a larger group in a two-stage tournament. Ernst and Thöni's analysis (2013) indicates that the participants' behavior comes closer to theory prediction for a larger group size. When considering the proportional-prize rule in the contest, Petters et al. (2021) observe that the mechanism of common knowledge of realization probabilities operates better with larger group size. Lastly, Hudija (2021) detects that increasing player numbers result in more innovations in an innovation contest; meanwhile, individual innovation attempts decrease when enlarging the group size. Generally, all the existing experimental contest studies the group size effect in a static framework because the winning threshold[1]is always $T = 1$ (including Hudija, 2021).

Table 3.2: Multi-battle Experiments

Year	Author(s)	Contest type	Players n	Threshold T	Contests
2002	Zizzo	Tullock lottery	2	10	2
2013	Mago, Sheremeta, and Yates	Tullock lottery	2	2	20
2017	Mago and Sheremeta	Tullock lottery	2	2	12
2017	Gelder and Kovenock	All-pay auction	2	4	20
2019	Mago and Sheremeta	All-pay auction	2	2	12
2019	Mago and Razzolini	Tullock lottery	2	3	15
2022	Descamps, Ke, and Page	Tullock lottery	2	2	1

Table 3.2 presents the laboratory research on sequential multi-battle contests. By far, all current sequential multi-battle experiments study the game with two players' interactions ($n = 2$). Our contribution to the multi-battle contest studies is that we expand the experiments to a multi-player dimension ($n > 2$). The very first experiment to study the sequential Tullock multi-battle contest is Zizzo (2002). When the winning threshold is large ($T = 10$), they find the leaders do not bid significantly more than the follower; additionally, the followers are not necessarily dropped out of the race when the gap increases. Mago et al. (2013) find evidence supporting the strategic momentum, in which the winner of battle 1

[1]The winning threshold T of a contest indicates the required number of battles for a player to claim victory in a game. For example, the winning threshold of a two-player best-of-three game is $T = 2$.

bids more than the loser when $T = 2$. They also discovered that including the intermediate prize and reducing the role of luck (increase r) led to greater Tullock contest effort. Mago and Razzolini (2019) show that women exert more effort when their opponents are women for a $T = 3$ Tullock experiment. Sequential multi-battle contests ($T = 2$) create higher spending when compared to simultaneous multi-battle contests (Mago and Sheremeta, 2019). A limited number of experiments study sequential multi-battle contests using the all-pay auction as the CSF. Gelder and Kovenock (2017) explored the escalation of conflict effort with a losing penalty in a sequential all-pay auction contest ($T = 4$). Mago and Sheremeta (2017) discussed the rurally studied underbidding behavior in the first battle of a sequential all-pay multi-battle contest ($T = 2$). Descamps et al., 2022, identified that the momentum effect is likely not caused by asymmetric positions. The momentum is observed if the leading position is gained through the first battle in a best-of-three contest ($T = 2$).

Lastly, we look at laboratory experiments that directly compare the contest success function. When comparing the all-pay auction to the Tullock lottery in the one-shot contest, Davis and Reilly (1998) and Potters et al. (1998) demonstrate that subjects bid more in the all-pay auction. Mago et al. (2013) illustrate that increasing the sensitivity parameter r results in higher bids in the Tullock contest. Suppose we consider the proportional-prize rule as a variant of the contest success function; Cason et al. (2010) provide evidence suggesting proportional-prize generates more effort than the all-pay auction. Chowdhury et al. (2013) found that players' resource allocating process is more concentrated in an all-pay auction than in Tullock Lottery Colonel Blotto games. The Tullock lottery, in turn, contests generate more effort than the proportional-prize rule Cason et al. (2020).

3.3 Theory and Hypotheses

3.3.1 Theoretical Model

We study sequential multi-battle contests with n players. Consider a set of risk-neutral players $N = \{1, 2...n\}$ (where $n \geqslant 2$) compete for a prize in a series of sequential battles.

Let V denote the prize of winning the contest, and V_i denote the valuation of V for player i, for every $i \in N$. We assume that $V_1 \geqslant V_2... \geqslant V_n$, and V_i is publicly known to all players[2].

To win a contest, the player i has to be the first one in n players who wins T battles ($T \geqslant 2$), and we call T a winning threshold. We take $T = 2$ to meet the minimum requirements of a multi-battle contest in our experimental design.

We use a battle node t to specify each player i's current state $(s_{1t}, s_{2t}...s_{nt})$, where $t = \{1, 2...m...n+1\}$ and $s_{it} = 0 \; or \; 1$. More explicitly, $s_{it} = 1$ denotes player i has won a battle at current battle node t, and $s_{it} = 0$ denotes player i has not won any battle at current battle node t. Therefore $\sum_{i=1}^{n} s_{it} = t - 1$. For simplicity, we call battle node t battle t, and the contest is terminated at battle node m.

Here we denote m as the terminal battle node, and we have $T \leqslant m \leqslant n(T-1) + 1$. Suppose $T = 2$, and we have $2 \leqslant m \leqslant n + 1$.

At each battle t, player i choose a costly effort $e_{it} \in [0, V]$ to expend [3]in order to win the battle node t.

Consider the cost of the effort takes a linear functional form (Eq. (3.1); Klumpp and Polborn, 2006; Konrad and Kovenock, 2009; Mago et al., 2013; Gelder, 2014; Doğan et al., 2018):

[2]For the design purpose of the experiment, we consider $V = V_1 = V_2... = V_n$. In theory, we generalized the multi-battle all-pay auction contest result with asymmetrical valuations.

$$c(e_{it}) = e_{it} \tag{3.1}$$

To determine player i's probability of winning battle t, we applied the following contest success function (CSF):

$$p_{it}(e_{it}^r, e_{-it}^r) = \frac{e_{it}^r}{\sum_{j=1}^{n} e_{jt}^r} \tag{3.2}$$

where r (Eq. (3.2)) is the sensitivity (decisiveness) parameter (Hirshleifer, 1995) that measures how individual i's winning probability is affected by their effort[4]. We consider the two extreme cases: the all-pay auction when $r = \infty$ (Hillman and Samet, 1987; Hirshleifer and Riley, 1978; Nalebuff and Stiglitz, 1983; Dasgupta and Maskin, 1986; Hillman and Riley, 1989; Baye et al., 1996) and the Tullock lottery when $r = 1$ (Tullock 1980, 2001).

The payoff for player i is:

$$\pi_i = \begin{cases} V - \sum_{t=1}^{m} e_{it} & \text{if player } i \text{ wins the contest} \\ -\sum_{t=1}^{m} e_{it} & \text{otherwise} \end{cases} \tag{3.3}$$

[3] In the experiments, participants bid a whole number between 0 and 1000, and the minimum positive bid is 1 token.

[4] When $\sum_{j=1}^{n} e_{jt}^r = 0$, we consider $p_{it} = \frac{1}{n}$.

3.3.2 Theoretical Result

Theorem. Given a well-defined terminal battle node m, any battle node t that arises in a sequential all-pay auction multi-battle contest can be a standard all-pay auction contest.

In a sequential all-pay auction contest, we denote a player i's expected payoff at a battle node t as $v_{it} = v_i(s_{1t}, s_{2t}...s_{nt})$. Let $(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$, for $k \in N$, denote a path from the battle node $(s_{1t}...s_{kt}...s_{nt})$ to the battle node $(s_{1t}...s_{kt} + 1...s_{nt})$. Let $p_t(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$ denote the probability that the path $(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$ is realized. Then $p_t(s_{1t}...s^{\downarrow}_{kt}...s_{nt}) = 0$ implies that the probability of player k winning battle node t is zero. Denote $t_{k\downarrow}$ as the consequence node of player k wins battle node t. For simplicity, we denote $p_t(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$ as $p_{t_{k\downarrow}}$.

Lemma. Given a well-defined terminal battle node m, with only one player having a positive value v_{it}, at most, one player has a positive value v_{it} in each node t.

Proof of Lemma

We assume that there are two players in a battle node $t = (s_{1t}, s_{2t}...s_{nt})$ with positive expected payoffs. We denote these two payers as player i and player j, for $i, j \in N$ and $i \neq j$. Suppose $t_{i\downarrow} = (s_{1t}...s_{it} + 1...s_{nt})$ and $t_{j\downarrow} = (s_{1t}...s_{jt} + 1...s_{nt})$, we assume that $v_{it_{i\downarrow}} > v_{jt_{j\downarrow}}$. In any equilibrium, we have $p_{t_{j\downarrow}} \cdot v_{jt_{j\downarrow}} - e_{jt} = 0$. Therefore at least one path with a positive possibility that leads player j to obtain a positive expected payoff must exist. We assume that path $(s_{1t}...s^{\downarrow}_{kt}...s_{nt})$ satisfies this condition. Then we have $v_{kt_{k\downarrow}} > 0$ and $v_{jt_{k\downarrow}} > 0$. Continuing the process, we have at least two players with positive values in the terminal battle node m, which is a contradiction to the assumption of the terminal battle node. Therefore we have proved the Lemma. Using the **Lemma**, we can prove that the players who lose in a single battle will have a zero expected payoff. Therefore the **Theorem** is proved.

From the **Theorem** of multi-battle auction contest[5], we can see in battle 1; each player bids an expected effort of $\frac{V}{n}$. For battle 2, all players except the winner of battle 1 bid 0. The winner of battle 1 exerts the maximum effort allowed to secure the contest-winning. For any post-battle 2 ($t > 2$), the players who have the current state s=1 will bid an expected effort of $\frac{V}{t-1}$. All other players drop out from the contest when $t > 2$.

As a comparison, in a multi-battle Tullock lottery contest (Doğan et al., 2018)[6], each player bids $\frac{(n-1)(21n^2-24n+6)V}{4n^2(3n-2)^2}$ in battle 1. In battle 2, the winner of battle 1 bids $(2n-1)\frac{(3n-3)V}{4(3n-2)^2}$, and each player, who lost battle 1, bids $\frac{(3n-3)V}{4(3n-2)^2}$. For all t ($t > 2$, post-battle 2), the players, who have a current $s = 1$, bid $\frac{(t-2)V}{(t-1)^2}$; otherwise, players drop out from the contest.

The expected total effort in a multi-battle all-pay auction is V, which is n-invariant. On the contrary, the expected total effort in a multi-battle Tullock lottery contest is $\frac{(12n^3-35n^2+20n-3)V}{2n(3n-2)^2}$, which is increasing as n increases and less than V.

3.3.3 Predictions and Hypotheses

We design an experiment to examine the group size effect in a multi-battle contest. Our objective is to examine how CSFs affect the group size effect in a multi-battle setup. Figure 3.1 displays the theoretical prediction when n takes 3 and 6 in the all-pay auction and the Tullock lottery multi-battle contests. The statistical analysis will test the following six hypotheses based on the theoretical predictions:

Hypothesis 1. In battle 1, the expected effort decreases as player number n increases

[5] In the unique symmetric Nash equilibrium, players' bids follow the CDF of $\frac{e}{V}^{\frac{1}{n-1}}$ in battle 1 and the leading players' bids follow the CDF of $\frac{e}{V}^{\frac{1}{t-2}}$ in post-battle 2.

[6] The results come from Doğan et al. (2018). **Proposition 1** and **Proposition 2** when $T = 2$ and $n \geqslant 2$.

in both all-pay auction and Tullock lottery.

Hypothesis 2. In battle 2, there is no group size effect in the all-pay auction contest. In battle 2 of the Tullock lottery, the effort increases as n increases when $s = 1$ but decreases as n increase when $s = 0$.

Hypothesis 3. In any post-battle 2, there is no group size effect for all-pay auction and Tullock lottery regardless of the current state.

Hypothesis 4. For both CSFs in post-battle 1, players with leading positions bid more.

Hypothesis 5. Only when in battle 1 or post-battle 2, and s=1, players bid more in the all-pay auction than in the Tullock lottery.

Hypothesis 6. To win a contest, the winner of a game exerts more effort when there are fewer contestants. The direction holds for both CSFs.

Hypothesis 7. The expected total effort in the all-pay auction is n-invariant. On the contrary, the expected total effort in the Tullock lottery increases when n increases.

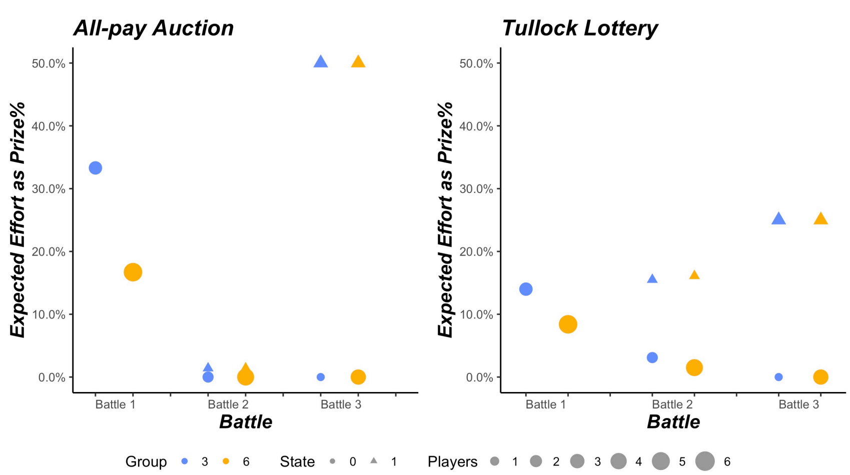

Figure 3.1: Predictions of Effort as Proportion of the Prize, when $n = 3$ and 6.

3.4 Experiment Design and Procedures

We design an experiment to study group size effects in multi-battle contests. The goal of the experiment is to compare strategic behaviors when the group size is different, further, to compare the difference in the size effect when the CSFs are different. We employed a 2 (n=3 and n=6) by 2 (all-pay auction and Tullock lottery) design for the experiment. We recruited 120 subjects to participate in 12 sessions (between 6 to 12 subjects) in the Virginia Tech Econ Lab. All subjects are students, staff, and faculty at Virginia Tech.

For each session, we randomly selected a fixed group size treatment in the contest, where n = 3 or 6. Participants played 12 all-pay auction multi-battle contests and 12 Tullock lottery multi-battle contests as phase one of the experimental session. The groups were anonymously reassigned after each contest, and subjects did not know their opponents in a game. We randomized the order in which the CSF of the games is played first in an experimental session. After reading the instructions for the games, subjects started to play the 24 contests (12 for each CSF). Through out the contest, players decide on an integer bid between 0 and 1000 tokens on a computer through an oTree program (Chen et al. 2016). The player who wins two battles first wins a prize of 1000 tokens for that contest. One token is worth $0.01.

In phase two of the experiment, we elicited participants' preferences toward risk, ambiguity, and loss aversion with a modified survey (Appendix B.1) from Shupp et al. (2013). Participants chose their preferred option between A and B for each lottery. There are a set of 12 lotteries for each of the three preference elicitations. Lastly, subjects fill out a socio-economic survey before reviewing their experiment earnings.

By the end of the experiment, we paid subjects in cash, $7 as compensation, $12 as the endowment of the all-pay contest plus a randomly selected result from 12 contests, another

$12 as the endowment of the Tullock contest plus a randomly selected result from 12 contests, and three randomly chosen earnings from 36 risk, ambiguity, loss aversion lotteries. On average, participants earned $25.75 within approximately 90 minutes of the experiment.

3.5 Experimental Results

3.5.1 General Comparative Statics

Table 3.3 summarizes the participants' mean effort of each battle at the given states and their expected payoff compared to theoretical predictions. We find consistent overbidding in the Tullock lottery contest across all battles and states. However, the overbidding only exists in battle 2 in the all-pay auction treatment. This underbidding behavior is consistent with previous multi-battle all-pay auction contest experimental studies (Mago and Sheremeta, 2017). Comparing the payoff across all treatments suggests a persistent over-dissipation.[7]

Table 3.3: Summary Statistics of Prediction vs. Observation

CSF	**All-pay Auction**				**Tullock Lottery**			
n	$n = 3$		$n = 6$		$n = 3$		$n = 6$	
Expectation	Nash	Actual	Nash	Actual	Nash	Actual	Nash	Actual
Battle 1	334	229.96	167	199.35	140	227.59	84	165.94
Battle 2 ($s = 1$)	1	344.13	1	508.68	154	300.05	162	345.23
Battle 2 ($s = 0$)	0	254.41	0	155.87	31	206.80	15	131.32
Battle 3 ($s = 1$)	500	382.58	500	472.80	250	278.13	250	295.37
Battle 3 ($s = 0$)	0	159.18	0	114.10	0	163.92	0	117.64
Payoff	-0.34	-382.46	-0.17	-453.49	95.24	-282.36	21.65	-325.72

Actual: the average effort for each corresponding battle and state.
Nash: the expected Nash equilibrium prediction for each corresponding battle and state.

Table 3.4 presents the cumulative probability of games ending in each corresponding

[7] We consider a player's behavior as over-dissipation when the total effort surpasses the contest's prize. The definition is adopted from Gneezy and Smorodinsky (2006).

battle. Unlike the theoretical prediction, it shows that an insufficient number of battles end in 2 battles. We observe a general trend that when the group size increases from 3 to 6, the probability of games ending in early battles decreases significantly; however, the difference is not severe when the CSF switches from the all-pay auction to the Tullock lottery. Compared to the multi-battle with 2 players' experimental studies (Mago and Sheremeta, 2017, 2019)[8], the decreasing probability of ending in battle 2 is consistent. Table 3.4 shows no significant support for the theoretical prediction that the all-pay auction contest ends in 2 battles and the Tullock lottery contest ends in 3 battles. The probability of the contest going to battle 3 increases when n increases. The difference in possibilities of games ending in a specific battle is negotiable in different CSFs for the same player number n.

Table 3.4: The Cumulative Probabilities of Games that End in Individual Battle

Treatment	Battle 2	Battle 3	Battle 4	Battle 5	Battle 6	Battle 7
All-pay ($n = 3$)	45.76%	91.53%	100%			
Tullock ($n = 3$)	44.78%	89.26%	100%			
All-pay ($n = 6$)	35.28%	82.08%	97.92%	100%	100%	100%
Tullock ($n = 6$)	36.47%	75.34%	94.45%	99.34%	99.97%	100%

The probabilities in all-pay auction contests are adjusted for the tie bids.

Table 3.5 shows the probabilities of the battle 1 winner becoming the contest winner. The probabilities we find in the all-pay auction are significantly lower than the theoretical prediction of 100% ($n = 3$ or 6). On the other hand, the observed probabilities in the Tullock lottery, 69.06% ($n = 3$) and 63.50% ($n = 6$) are very close to the prediction in theory (Doğan et al., 2018). More generally, the decreasing pattern in the experimental observation when increasing the group size and the minor difference when changing CSFs in the contests is consistent with the comparison between our results and previous research (Mago and Sheremeta, 2017, 2019).[9]We also listed the probability of the battle 1 winner winning the game, suppose the winner of the contest is randomly assigned. We also calculate the probability that battle 1 winner wins the game under the assumption that the contest's

[8]The probabilities of contests ending in battle are 62% and 64%, respectively, for the Mago and Sheremeta (2017 Tullock lottery with 2 players) and Mago and Sheremeta (2019 all-pay auction with 2 players).

winner is randomly assigned. We see that the observed probability in the experiment is sufficiently higher than the randomly assigned winning probability, which indicates evidence of strategic momentum.

Table 3.5: The Probabilities of Battle 1 Winner Wins the Game

Treatment	All-pay ($n = 3$)	Tullock ($n = 3$)	All-pay ($n = 6$)	Tullock ($n = 6$)
Theory	100.00%	65.32%	100.00%	62.95%
Random	62.96%	62.96%	46.24%	46.24%
Experiment	68.33%	69.06%	60.56%	63.50%

The probabilities in all-pay auction contests are adjusted for the tie bids.

Table 3.6 lists the maximum player's effort, the maximum battle effort, and the maximum player's total effort in a contest. Among these three categories, the two total efforts increase as n increases. Some size effect studies demonstrated the phenomenon of total efforts increasing as n increases (Sheremeta 2011; Lim et al., 2014; Fallucchi et al., 2021). Meanwhile, contestants exerted more effort as the r increased from 1 to ∞ consistent with previous research (Davis and Reilly, 1998; Potters et al., 1998; Cason et al., 2020). Especially intriguing, the average player's best battle effort is maximized in the six-player all-pay auction contest. We will provide the potential explanation in later sections. Table B.2 reports the proportions of the corresponding condition that maximize the three categories in Table Table 3.6. It illustrates that players' battle effort is likely to be maximized when s=1 in the all-pay auction but is likely to be maximized at battle 1 in Tullock. A sufficient amount of the battle's total effort is maximized in battle 2. Lastly, more maximum players' total effort comes from the contest winner when decreasing the n and increasing the r.

Table 3.6: Average Maximum Effort

Treatment	All-pay ($n = 3$)	Tullock ($n = 3$)	All-pay ($n = 6$)	Tullock ($n = 6$)
Player's Battle	539.25 (11.20)	429.38 (12.03)	642.25 (17.00)	545.82 (17.91)
Battle's Total	1033.60 (26.20)	826.95 (23.09)	1425.67 (36.90)	1208.19 (32.33)
Player's Total	1073.56 (28.81)	870.53 (27.66)	1362.23 (45.08)	1177.02 (53.21)

Standard errors are reported in the parentheses.

[9] The probabilities of battle 1 winner winning the overall contest are 75% and 80%, respectively, for the Mago and Sheremeta (2017 Tullock lottery with 2 players) and Mago and Sheremeta (2019 all-pay auction with 2 players).

3.5.2 Behaviors in Battles

Result 1. There is an assertive boundary behavior difference in the all-pay auction and a weak mean difference in the Tullock lottery at battle 1.

Table 3.7: Random Effects Panel on Efforts in Different Battles

CSF		**All-pay Auction**			**Tullock Lottery**		
Effort		Battle 1	Battle 2	Post-battle 2	Battle 1	Battle 2	Post-battle 2
Constant	$(n = 3 \ \& \ s = 0)$	289.17 (36.44)	318.59*** (21.34)	223.68*** (44.99)	211.41*** (28.86)	208.88*** (25.67)	177.67*** (27.59)
Group	$(n = 6 \ \& \ s = 0)$	-30.60 (29.65)	-81.88*** (26.01)	-28.44 (39.29)	-61.65* (34.45)	-69.28*** (27.46)	-39.27 (28.96)
State	$(n = 3 \ \& \ s = 1)$		49.94*** (18.00)	205.97*** (29.34)		67.69*** (18.02)	93.35*** (12.93)
Group × State	$(n = 6 \ \& \ s = 1)$		123.35 (32.00)	65.27 (48.28)		57.88 (31.06)	31.29 (22.75)
Trend		-9.11*** (3.35)	-7.83*** (2.76)	-8.51** (3.50)	2.49 (1.52)	0.99 (1.53)	-0.40 (2.06)
Observations		1440	1440	1071	1440	1440	1068

The dependent variable is the effort for each corresponding battle.
The random effects model is structured at the subject level.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the constant is the testing result for overbidding compared to the theoretical prediction.
The reported significance of the Group × State is the testing result of Group ($n = 6$) effect on State ($s = 1$).
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

From the theoretical prediction, we expect to see the group size effects in battle 1 of both all-pay auction and the Tullock lottery games. Instead of observing the significant difference in the average battle 1 bids, we find more boundary bids (0 or 1000) in the all-pay auction with 6 players compared to 3 players' games (Figure 3.2, Figure B.1 in Appendix B.2). In contrast, we find weak evidence of group size effects in mean effort in battle 1 of the Tullock lottery games (Table 3.7). Another noticeable discovery from Figure 3.2 is that the all-pay auction leads to higher dispersion in bids than the Tullock lottery in battle 1, which aligns with the theory that the all-pay auction game has a mixed strategy symmetric equilibrium. The standard deviations for all-pay auction and Tullock lottery are 257.20 and 196.92 in battle 1, respectively.

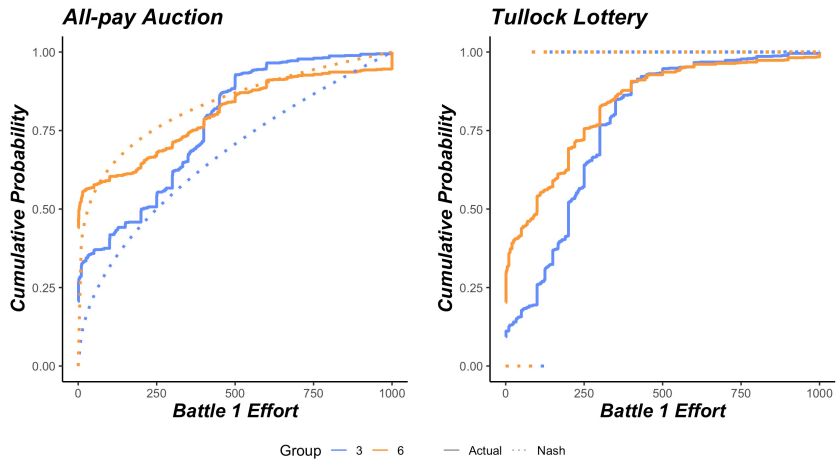

Figure 3.2: The Cumulative Distribution Function of Effort in Battle 1.

The **Result 1** of the all-pay auction challenges the theoretical prediction in **Hypothesis 1** because there is a substantial amount of maximum bid in battle 1 when $n = 6$. As n increases, the theory yields a more accurate prediction when considering the entire CDF in the all-pay auction. The U-shaped distribution of bids can be obtained in an all-pay auction experiment (Gneezy and Smorodinsky, 2006; Klose and Sheremeta, 2012; Dechenaux et al., 2015) but not in a multi-battle contest experiment. We notice that as the number of participants grows, the boundary behaviors on both sides get more intense (Figure B.1). The finding that an increase in n increases the boundary behavior of multi-battle contests can be generalized beyond our experimental design. Compared to Sheremeta and Mago (2017, $n = 2$), the distribution of effort when $n = 3$ is more distorted in two boundaries. When $n = 6$, we observe more efforts in both boundaries compared to $n = 3$. We checked if the distribution of bids greater than 500 in $n = 6$ is shifted to the right compared to $n = 3$ with the Mann–Whitney U test, and the p-value is sufficiently small. Ernst and Thöni (2013) suggested a loss aversion explanation for this bimodal behavior. In our experiment, the U-shaped distribution is extended from battle 1 to battle 2 and $s = 1$ in the six-player all-pay auction contest (Figure B.1). One convincing explanation is that contestants bid more competitively to avoid the contests going to later battles.

Result 2. Average effort changes significantly across group size treatments in battle 2 when current state s=0.

Theory predicts no group size effects in all-pay auction contests in battle 2. On the contrary, there is the group size effect in the Tullock lottery games, and group size effects exist in both current states ($s = 0$ or 1). Our experiment data indicate a significant difference in mean effort when the group size of the contest is different. When $s = 0$ in battle 2, the average effort in 3-player games is significantly higher than 6 players' games (Table 3.7 and Figure B.2 from Appendix B.3). We will explore the source of group size effects in later sessions when decomposing behaviors into different strategies. **Result 2** only provides statistical evidence supporting **Hypothesis 2** in the Tullock contest. We also bootstrapped the probabilities of battle 1's winner winning the second battle for different treatment. The results are 43.21%, 43.84%, 48.04%, 36.58% for All-pay 3, Tullock Lottery 3, All-pay 6, Tullock Lottery 6, respectfully. This indicates that when increasing both n and r, the contest would more likely end in battle 2.

Result 3. Sufficiently decay in group size effects after battle 2.

One advantage of our design is that players can be distinguished by their state in post-battle 2 because $s = 1$ at battle 3 for the two-player contest (Mago and Sheremeta, 2017, 2019). We find in Table 3.7 that all group size effects die out for post-battle 2 as predicted, supporting **Hypothesis 3**. As the contest goes to post-battle 2, the discourage effect becomes more severe, and the high dropout rate for the player with $s = 0$ overrides the group-size effect.

Result 4. Strong strategic momentum is revealed in all post-battle 1 efforts for both CSFs.

In Table 3.7, the regression result shows that the effort is significantly higher when the current state $s = 1$ compares to $s = 0$ for all post-battle 1 battles. This lends credence to **Hypothesis 4** Our finding is consistent across all group size treatments and compatible with previous multi-battle contest experiments (Mago et al., 2013; Gelder and Kovenock, 2017; Mago and Sheremeta, 2017, 2019). Our study affirms that strategic momentum is group size invariant in multi-battle contest experiments.[10]

3.5.3 Contest Success Functions

Result 5. Predominantly, players bid more in the all-pay auction game than the Tullock game in most battle-state.

Comparing the CSFs in different battles and states, we show the regression result in Table 3.8 to quantify the effect of the contest success function on the effort for each given battle and state. As a supplement to Table 3.7, we find a consistent pattern in which participants insert more effort in all-pay than in the Tullock contest. This increase in effort generated by r vanished for post-battle 2 when the $s = 0$. The tendency of more expenditure in the all-pay than the Tullock is robust despite the group size in the contest, which rejects the **Hypothesis 5**. This result extended the discovery from the previous studies of single-battle contests (Davis and Reilly, 1998; Potters et al., 1998; Cason et al., 2020). Apart from the conduct in battle 1, our findings align with Mago and Sheremeta's (2017, 2019) two-player research.[11]

[10] The unreported 6-player games' strategic momentum (State) for the all-pay auction and the Tullock lottery are significant at 1%.

[11] Mago and Sheremeta (2017, 2019) found players in battle 1 bid less in the all-pay (16.7 out of 100) than in the Tullock (22.1 out of 100).

Table 3.8: Random Effects Panel on Contest Success Function

Battle		**Battle 1**	**Battle 2**	**Battle 2**	**Post-battle 2**	**Post-battle 2**
Effort		$s = 0$	$s = 0$	$s = 1$	$s = 0$	$s = 1$
Constant	Tullock & $n = 3$	211.41***	210.37***	273.60***	216.67***	249.34
		(28.86)	(24.35)	(25.87)	(32.10)	(21.66)
CSF	All-pay & $n = 3$	77.77**	107.06***	107.59**	34.92	174.15***
		(36.06)	(21.39)	(42.61)	(44.34)	(45.06)
Group	Tullock & $n = 6$	-61.65*	-69.438**	15.76	-44.23	21.57
		(34.46)	(27.13)	(36.75)	(33.66)	(34.46)
CSF × Group	All-pay & $n = 6$	31.05***	-22.65***	90.12**	-3.41	52.26***
		(27.94)	(25.33)	(54.25)	(37.69)	(32.36)
Trend	Tullock	2.49	0.90	3.68	-5.58**	4.67
		(1.52)	(1.14)	(4.26)	(2.21)	(3.03)
CSF × Trend	All-pay	-11.60***	-8.25***	-10.69	-4.64*	-10.8
		(3.63)	(2.59)	(8.09)	(2.80)	(6.94)
Observations		2,880	2,160	720	1,018	1,121

The dependent variable is the effort for each corresponding battle and state.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the constant is the testing result for overbidding compared to the theoretical prediction.
The reported significance of the CSF × Group is the testing result for CSF (Tullock) effect on the Group ($n = 6$).
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

3.5.4 Total Effort

From the game organizer's perspective, total effort is another essential aspect to explore. In particular, we will focus on the total effort of a game, the total effort of the winner of a game, and the average total effort for all individual players across different group size treatments.

Result 6. The winners spend more total effort in 6 players' games, despite the fact that average individual effort is higher in 3 players' games.

In Figure B.3 (Appendix B.4), we see a pattern that the overall average individual effort exhibits group size effects. Average individual effort is higher for the game with 3 players than the game with 6 players for both CSF treatments. On the other hand, we reach a different conclusion if we consider the player's final state and analyze them separately. The results from Table 3.9 and Figure B.3 show that to win a multi-battle contest, the winner in 6 players' games exerts more effort than the winner in the contest with 3 players. The group size effect on the game-winner average effort is stronger in the all-lay auction game than in the Tullock lottery. **Result 6** provides direct evidence against **Hypothesis 6** from the theoretical prediction.

Table 3.9: Random Effects Panel on Total Efforts for Different Final States

CSF		**All-pay Auction**			**Tullock Lottery**		
Total Effort		Final $s = 2$	Final $s = 1$	Final $s = 0$	Final $s = 2$	Final $s = 1$	Final $s = 0$
Constant	$(n = 3)$	1240.65***	1223.36***	511.12***	759.47***	887.09***	422.59***
		(96.88)	(98.71)	(60.76)	(60.69)	(61.11)	(68.05)
Group	$(n = 6)$	391.23***	383.12***	-53.11	177.24*	111.20	-103.85
		(95.94)	(122.94)	(69.11)	(104.75)	(105.72)	(68.05)
Trend		-33.15***	-32.70**	-16.02***	1.74	-0.52	1.43
		(10.99)	(15.00)	(4.81)	(7.07)	(6.59)	(3.21)
Observations		360	254	826	360	252	828

The dependent variable is the total effort for each corresponding final state.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the constant is the testing result for overbidding compared to the theoretical prediction.
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

The previous section points out that most games end within three battles; moreover, the number of games that end in two battles is similar to the number of games that end in three battles. Therefore, we dig into more details about the group size effect on individual effort. In games that end in two or three battles, the winner (potential winner) spends more total effort in a three-player competition than in a six-player competition. Losers bid more in total in 6 players' games than 3 players' games, regardless of when the game ended (Figure 3.3). The result is inconsistent with the theory in either CSFs. With bigger group sizes, the rise in winner overall effort is a consequence of increased effort in individual battles.

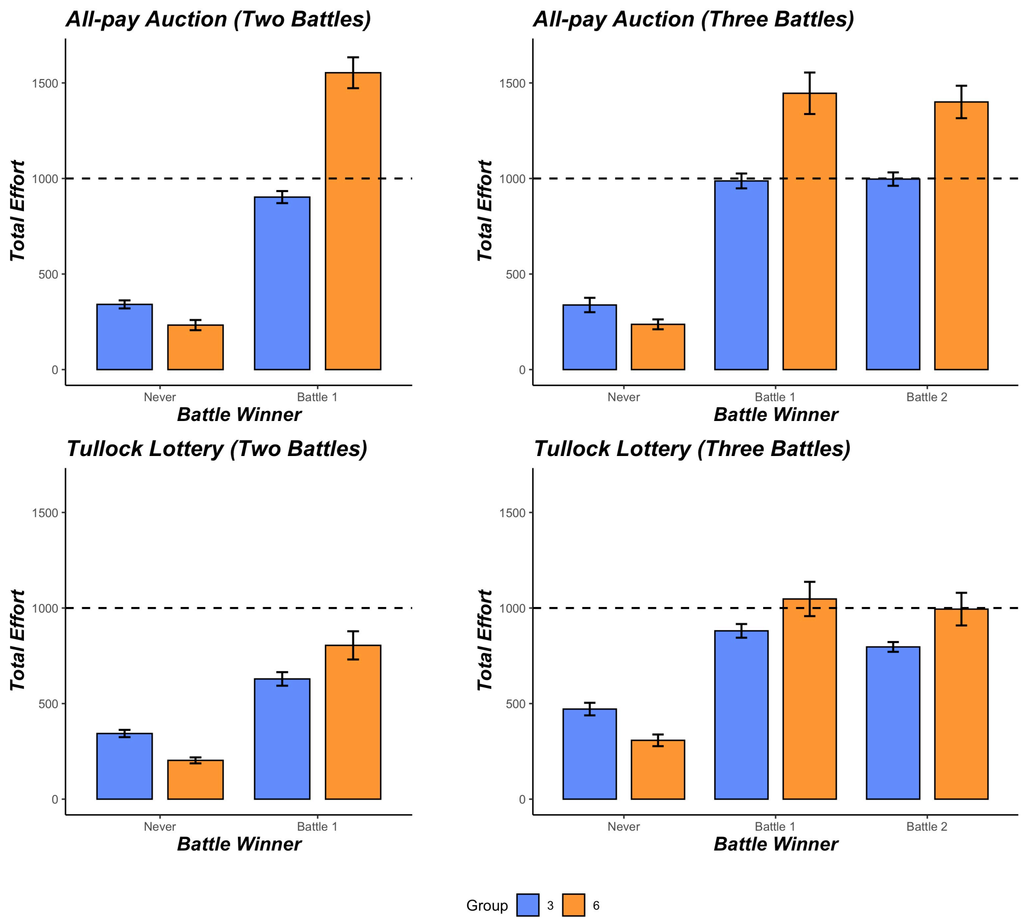

Figure 3.3: Total Effort with the Contests Ending in 2 and 3 Battles

Result 7. The total expected effort in a game is increasing as n increases.

We tested **Hypothesis 7** and present some observations in Figure B.3. When n increases from 3 to 6, we observe the total expected effort increase for all-pay auctions and Tullock lottery contests. In other words, the total expected price decreases as the player number increases in a game. The experimental observations align with theoretical predictions in the Tullock lottery contest (Doğan et al., 2018); however, it does not match the predictions in the all-pay auction where the total expected prize is n-invariant. Another worth mentioning fact is that when the player number doubles from 3 to 6, we see a sufficient increase in battle total effort and contest total effort. Still, this increased total effort is far from being doubled (Figure B.3).

3.5.5 Strategy Types

The previous section discussed the group size effect in battle 2 when the current $s = 0$, but we did not find evidence suggesting distributional shifting among the active players considering both CSFs. We see a considerable number of zero bids in our data. This section will discover the source of the group size effect by classifying different strategy types with the difference in appearances in zero bid behaviors. Further, we will see how different treatments affect the players' types in the contest games.

Classification. A player's bids are categorized into the following six classes. Based on the six bid classes, we classify the player's strategy into three strategy types:

Entry: a player submits a positive bid in battle 1.

Stay: a player submits a positive bid in post-battle 1, given the previous bid is positive.

Reenter: a player submits a positive bid if the previous bid is zero.

No entry: any zero bid in a sequence of zero bids for the entire contest.

Dormant: a zero bid followed by some positive bid in the later battles.

Dropout: a zero bid followed by all zero bids in the later battles.

The Three Strategy Types are:

Strategy 1. Entry-Stay (Dropout).

Strategy 2. Dormant-Reenter (Dropout).

Strategy 3. No Entry.

Table 3.10: Participation Rate

Treatment	All-pay ($n = 3$)	Tullock ($n = 3$)	All-pay ($n = 6$)	Tullock ($n = 6$)
Proportion	89.58%	93.75%	69.02%	85.56%

The participation rates are shown in Table 3.10. We consider the players who are adopting **Strategy 1** and **Strategy 2** as participating in the contest and adopting **Strategy 3** are not participating. Two things stand out. First, the participation rate decreases as group size increases. Second, the participation rate decreases when the contest success function switch from the Tullock lottery to the all-pay auction.

Result 8. An increase in group size leads to an increase in zero-effort rate when the current $s = 0$ for both CSFs. This increasing treads in zero effort are consistent among all three classes.

In Figure 3.4, empirical evidence suggests the zero effort rate is significantly higher in 6-player contests than 3-player contests for battle 1 and post-battle 1 given $s = 0$. When increasing the group size in a game, such increased zero effort rates are consistent with different CSFs. The increase in zero effort rate explains the observed group size effect in the battle 2 bid, given that there is no significant change in the positive bids in the battle (Table B.4 in Appendix B.5). When $s = 1$, The dropout rate differences in different group sizes are not noticeable for both contest success functions. (Figure 3.4).

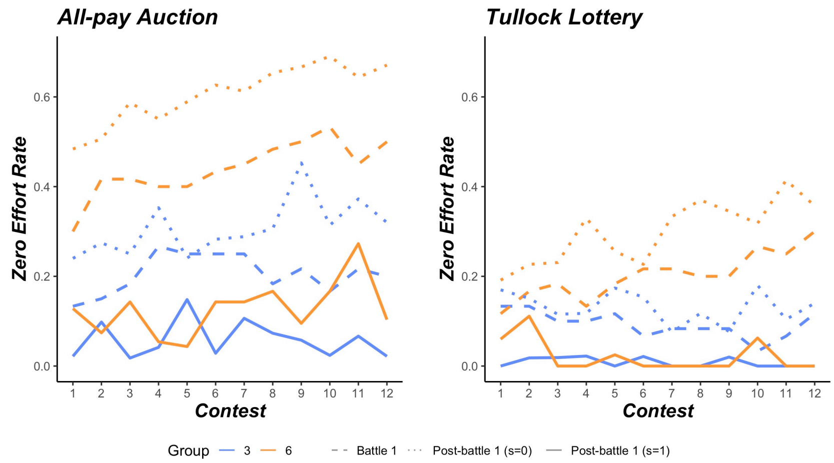

Figure 3.4: The Zero Bid Rate in the Different States

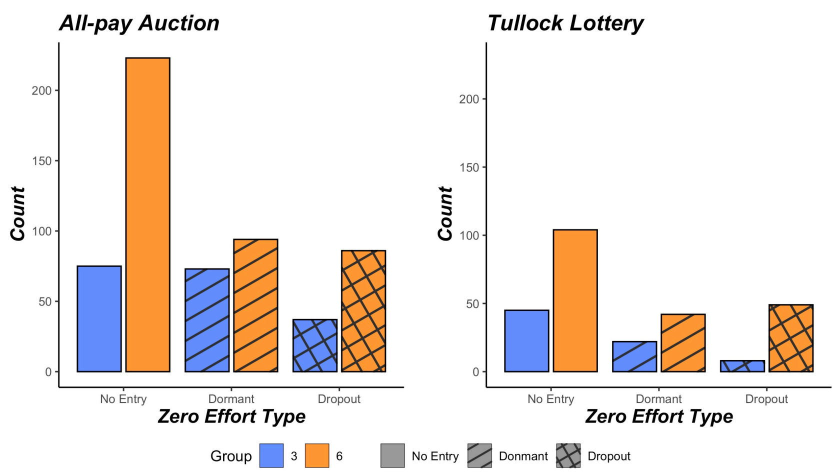

Figure 3.5: The Zero Bid in Different Strategies

Moreover, we discovered that the rising trend in No Entry, Dropout, and Dormant is persistent, while the group size increased from 3 to 6 in auction and lottery games (Figure 3.5). We also persistently detect the more No Enter, Dropout, and Dormant behavior when the group size is increased. Such group size effect is more evident in all-pay the auction compared to the Tullock lottery [12].

[12] We here only consider the considered no enter and dormant from battle 1 and dropout from battle 2 in Figure 3.5. We did not exclude the drop out of battle 1 winners, considering they only happen rarely.

Result 9. Increasing the number of players increases the probability of entering the contest in battle 1 but decreases their probability of staying in the game.

Table 3.11: Random Effects Probit Panel on Strategy Types

CSF		**All-pay Auction**			**Tullock Lottery**		
Strategy (Excluded Dormant & Reenter)		Entry (Battle 1)	Stay (Battle 2)	Stay (Post-battle 2)	Entry (Battle 1)	Stay (Battle 2)	Stay (Post-battle 2)
Constant	($n = 3$ & $s = 0$)	3.11***	2.20***	0.59*	4.05***	4.52***	0.29
		(0.36)	(0.26)	(0.34)	(0.62)	(0.67)	(0.36)
Group	($n = 6$ & $s = 0$)	-1.70***	-1.31***	-0.16	-1.25***	-1.54***	-0.34
		(0.35)	(0.40)	(0.36)	(0.43)	(0.33)	(0.39)
State	($n = 3$ & $s = 1$)		6.96***	2.07***		7.33***	1.56***
			(0.77)	(0.31)		(0.53)	(0.44)
Group × State	($n = 6$ & $s = 1$)		-4.88***	-0.77***		0.46	0.02
			(0.82)	(0.40)		(0.39)	(0.60)
Trend		-0.07***	-0.06***	-0.05*	-0.05	-0.11***	-0.04*
		(0.02)	(0.02)	(0.03)	(0.04)	(0.03)	(0.02)
Observations		1,273	921	763	1,376	1,206	952

The dependent variable of Battle 1 is 1 for Entry and 0 for No entry.
The dependent variable of Battle 2 and Post-battle 2 is 1 for Stay and 0 for Dropout.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the constant is the testing result for overbidding compared to the theoretical prediction.
The reported significance of the Group × State is the testing result of Group ($n = 6$) effect on State ($s = 1$).
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

Comparing the **Strategy 1** and **Strategy 3** subjects in the experiment, we see that increasing the player number significantly decreases the probability of entry for auction and lottery games in battle 1. The probability of staying in the auction is significantly higher for all post-battle 1 in the 6 players' auction games when $s = 1$. Then, when increasing group size in battle 2 for both CSFs, evidence suggests that the probability of staying in the game decreases. Besides the group size effect, the probability of staying is higher when $s = 1$ in all post-battle 1 for both auction and lottery contests, suggesting strategic momentum in the strategy type point of view (Table 3.11).

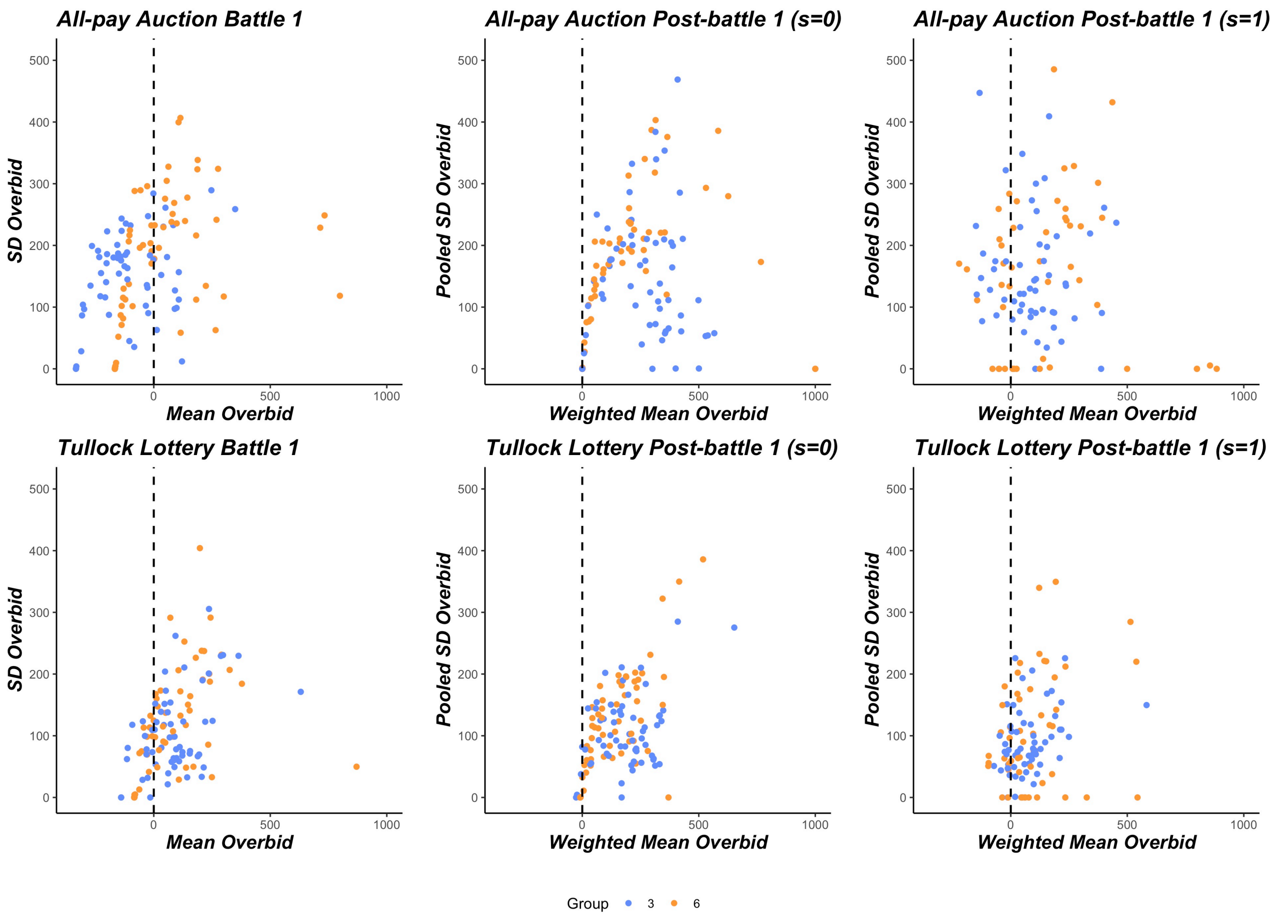

Figure 3.6: Overbidding Relative to Theoretical Predictions

3.5.6 Overbidding

We previously addressed the over-dissipation behavior where the total bid surpasses the prize of a contest. This section will closely examine the origins of over-dissipation in various battles and states. We emphasize that we define overbidding as a player bid higher than the expected prediction in Nash equilibrium at the current battle and state. This is slightly different from the definition of over-dissipation, which compares total effort with contest prizes. Sheremeta (2013) summarized the observed over-dissipation and overbidding behavior in earlier studies. Commonly overbidding is more desired in Tullck experiments than in the all-pay auction.

Table 3.12: Random Effects Panel on Overbidding

CSF		**All-pay Auction**		**Tullock Lottery**	
Post-battle 1 Overbid		$s = 1$	$s = 0$	$s = 1$	$s = 0$
Constant	$(n = 3)$	151.49**	261.71***	42.88	148.94***
		(62.75)	(23.98)	(32.28)	(20.09)
Group	$(n = 6)$	75.76	-77.62***	11.48	-44.50***
		(52.86)	(20.26)	(35.62)	(19.67)
Previous Total Effort		-0.03	0.16***	0.03	0.20***
		(0.08)	(0.05)	(0.04)	(0.04)
Trend		-5.61	-5.55***	3.86	-1.71***
		(5.15)	(1.90)	(2.97)	(1.00)
Observations		917	1594	924	1584

The dependent variable is the overbid relative to theoretical prediction for each corresponding state.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the constant is the testing result for overbidding compared to the theoretical prediction.
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

Overbidding behavior is revealed differently depending on CSFs, group size, battle, and state. We summarize some observations from Figure 3.6. First, the dispersion of overbidding is more severe in all-pay than in the Tullock lottery contest. Second, there is a clear tendency that 6 player auction game exhibits more overbidding than 3 player game in battler 1. Third, when the $s = 0$ in post-battle 1, 3 players game presents more overbidding than 6 players game for both CSFs.

Result 10. Sunk cost fallacy explains the overbidding behavior in both CSFs in post-battle 1 when $s = 0$.

Mago and Sheremeta (2019) discussed that sunk cost fallacy is an explication of overbidding behavior in a lottery multi-battle contest. They conducted a second experiment to confirm that overbidding in battle 2 is directly from a false concern of the sunk cost in battle 1 rather than a player constantly bidding high in both battles. In our experiment, we ruled out this concern for the following reasons. Firstly, we do not observe consistently overbidding as a popular strategy among the participants from Figure 3.6. Secondly, the contest's proportion of contests ends in 2 battles and 3 battles are very similar. We are using the summation of the bids in all previous battles to represent the sunk cost. As shown in Table 3.12, the amount of sunk cost is positively correlated to the amount of overbidding in the current battle when $s = 0$ for both all-pay and Tullock contests. Meanwhile, the overbidding is group size sensitive, as we pointed out from the summary in Figure 3.6.

3.5.7 Psychological and Demographic Heterogeneity

In Mago et al. 2013, psychological momentum exists in a 2 players game if the winner of battle 1 and the winner of battle 2 bid differently in battle 3. Our research extended the number of players to more than 2 in the experiments, so we specify the psychological momentum in a more general case. We consider that for any post-battle 2 if the bid of the previous battle winner is different from other players whose current $s = 1$, then there exists psychological momentum. In Table 3.13, the panel regression shows that the previous battle winner's effort is significantly higher than the other battles' winner for post-battle 2 in the all-pay auction game, which suggests the evidence for psychological momentum defined in Mago et al. (2013).

We also explored the near-win effect (Kassinove and Schare, 2001; Daugherty and MacLin, 2007; Wadhwa and Kimas, 2015) and the psychological explanation for the behavior in the experiments. We characterize $s = 1$ at the end of a contest as a near-win since the winning threshold in our design is $T = 2$. There is a strong negative near-win effect in the lottery contest. In other words, given the $s = 1$ in the current contest, the near-win player of the previous game tends to bid less (Table 3.13). Our finding is opposite from the psychological research that suggests that near-win motivates players in a game even though the estimated

Table 3.13: Random Effects Panel on Psychological Momentum

CSF	**All-pay Auction**		**Tullock Lottery**	
Effort (State)	Post-battle 1 ($s = 1$)	Post-battle 2 ($s = 1$)	Post-battle 1 ($s = 1$)	Post-battle 2 ($s = 1$)
Constant	443.24*** (42.20)	432.62 (48.26)	277.98*** (26.12)	266.00 (23.18)
Previous Battle Winner		47.38** (21.14)		1.81 (12.34)
Near Win Previous Contest	11.32 (11.47)	19.29 (12.48)	-34.25*** (10.49)	-42.38*** (8.26)
Trend	-6.44 (4.69)	-6.20 (5.82)	3.68 (2.95)	4.38 (2.90)
Observations	917	557	924	564

The dependent variable is the effort for each corresponding battle and state.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the constant is the testing result for overbidding compared to the theoretical prediction.
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

parameters are positive in the auction games. Behavior in contests indicates more evidence supporting discouragement than motivation.

Table 3.14: Random Effects Panel on Gender and Loss Aversion

CSF	**All-pay Auction**			**Tullock Lottery**		
Effort (State)	Battle 1	Battle 2 (s=0)	Post-battle 2 (s=0)	Battle 1	Battle 2 (s=0)	Post-battle 2 (s=0)
Constant	302.30 (43.93)	328.70*** (33.26)	314.63*** (41.32)	206.73*** (31.07)	200.12*** (29.56)	254.39*** (33.12)
Woman	126.49*** (36.57)	101.05** (45.82)	80.97* (45.60)	73.27** (35.01)	55.70** (22.22)	12.33 (19.04)
Loss Aversion	-112.54*** (37.08)	-129.99*** (21.26)	-176.75*** (38.91)	-77.97** (35.71)	-65.65** (37.58)	-82.07** (34.46)
Trend	-9.11*** (3.35)	-6.88*** (2.46)	-9.35*** (2.44)	2.49 (1.52)	0.87 (1.08)	-5.56** (2.29)
Observations	1,440	1,080	514	1,440	1,080	504

The dependent variable is the effort for each corresponding battle and state.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the constant is the testing result for overbidding compared to the theoretical prediction.
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

In theory, we assume the players are risk-neutral; this is one of the important reasons for the discrepancy between theoretical prediction and empirical observation. With a slightly modified version of Shupp et al. (2013), the subjects' preferences toward risk, ambiguity, and loss are elicited. The original definition from prospect theory (Tversky and Kahneman 1992) is used to create the dummy variables of the preference in regressions. Take the loss aversion survey as an example (Appendix B.1). Suppose in a subject's revealed choice, the "turning point" is before question 6, where the expected payoff is the same for opinion A and option B; we label the subject as loss averse. The result in Table 3.14 demonstrates that loss-averse participants' bids sufficiently less than any players when $s = 0$ in any battle for both CSFs. This partially explains the observed underbid in battle 1 of the auction game and the sunk cost fallacy behaviors.

From previous studies, a well-known result is that women tended to bid more than men in contest games (Charness and Levin, 2009; Chen et al., 2009; Ham and Kagel, 2006; Price and Sheremeta; 2012, Mago et al., 2013; Lim et al., 2014). Our experiment found that women bid more when the current $s = 0$ for auctions and lottery contests. Additionally, we see a more substantial effect in auction games than in lottery games, and the gender effect declines in later battles (Table 3.14). The gender effect in overbidding is constrained to the current state, and it is not presented when the current $s = 1$; to put it in another way, women bid as much as men when they are in a winning position.

One last important aspect in our data we would like to discuss is the observed trending of the bid through the games. In general, we find descending trends in auction games in terms of efforts in all battles, the total effort for any final state, and entering and staying rates (Table 3.3, Table 3.9, and Table 3.11). On the other hand, we only detect the descending trends of total effort for the winners of the games and the staying rate in Tullock lottery contests (Table 3.9 and Table 3.11). We find statistically significant trading in auction contests but not in lottery contests, which are different from previous studies (Mago and Sheremeta, 2017, 2019; Fallucchi et al., 2021).

3.6 Conclusion

We conducted a laboratory experiment at Virginia Tech Econ Lab to study the group size effect in the sequential multi-battle contest. The theoretical framework is based on the theory of Konrad and Kovenock 2009, and Doğan et al., 2018. We generalized all-pay auction result from $n = 2$ to $n \geqslant 2$. We employed a two by two design, where $n = 3$ or 6, and $r = 1$ or ∞ in a multi-battle contest ($T = 2$).

We find that group size effects exist in the sequential multi-battle contest, which differs across contest success functions. In battle 1, the observed group size effect consists of the Tullock lottery's mean effort. On the other hand, when $r = 1$ increases to $r = \infty$, the distribution of battle 1 effort is severely distorted, which players bid more intensively for both boundaries. As we increased the group size, the proportions of boundary bids (0 or 1000) increased significantly. Dynamically, for the players who lose battle 1, their mean effort in battle 2 dropped significantly when increasing the number of players in the all-pay auction and the Tullock lottery. When we decompose the subject behavior into different strategic profiles, we discover that the mean effect in the battle 2 effort mainly comes from the proportion of people who drop out of the contest. Increasing the group size in a sequential contest led to more battle 1 losers exerting zero-bid in the consecutive battle. Increasing n in the contest increases not only the dropout rate in post-battle 1 but also increases the probability of adopting the strategy of no entry and dormant-reenter in both CSFs. As for the total effort, we find that the average individual effort is higher when there are fewer contestants in the game for both CSFs. On the other hand, increasing the player number generates more effort in the player's battle, battle's total, player's total, winner total, average battle total, and average contest total effort. Further, the group size effect in the total effort is stronger in the all-pay auction compared to the Tullock lottery. Additionally, contestants exert more effort in all-pay games than in Tullock games. Decrease n and r increase the participation rate in a multi-battle contest.

Besides the obtained group size effects, we also confirmed the overbidding and over-dissipation behaviors from the previous experimental contest studies (Davis and Reilly, 1998; Potters et al. 1998, Mago et al., 2013; Mago and Sheremeta, 2017, 2019). The observed overbidding is mostly concentrated in post-battle 1, which is more consistent in the Tullock lottery than in the all-pay auction. As suggested in Mago and Sheremeta (2019), we confirm that observed overbidding can be partially explained by the sunk cost fallacy. Further, the over-dissipation behavior is more likely to be observed in the contest winners when increasing the player. This trend is more robust in the all-pay auction than in the Tullock lottery.

Lastly, we discovered some psychological and demographic heterogeneity in our experiments. Strategically, previous studies (Mago et al., 2013; Mago and Sheremeta, 2017, 2019) and our result reached the consensus that the players expended more effort than their opponents when they gained a leading position in a sequential multi-battle contest. Besides, we find the players bid more when they gain the leading position in the directly previous battle than all the remaining players who have the same leading position in the all-pay auction. Meanwhile, we note that the player who almost won ($s = 1$ at the end of a contest) in the previous Tullock lottery contest tended to bid less in the current game when facing the same situation in a later contest. As for the heterogeneity in the subjects, we detect women exert more effort, and the loss aversion subjects bid less when the current $s = 0$ given both CSFs. Our results are similar to the previous experimental studies (Mago et al., 2013; Lim et al., 2014; Mago and Razzolini, 2019).

There are some directions we could extend our study. First, in Zizzo (2002), the strategic momentum was undetected when the winning threshold was large. One future research direction is discovering if the result can be generalized to n player's contest. Second, we conduct the sequential multi-battle contest with the all-pay auction and the Tullock lottery. We could expand the existing tournament and elimination literature (Parco et al., 2005; Amaldoss and Rapoport, 2009; Altmann et al., 2012) to more than two players in each stage. Third, we assume players are risk-neutral in the contest, which is a conventional setting in SMBC. A sufficient amount of work in one-shot contests assumes risk aversion in players (Skaperdas and Gan, 1995; Konrad and Schlesinger, 1997; Cornes and Hartley, 2003, 2012; Treich, 2010; Fu et al., 2021). There is a potential to extend the SMBC with a risk-averse-player assumption. Lastly, there is some theoretical potential to explore group contest experiments (Abbink et al., 2010) in multi-battle settings.

Chapter 4

A General Beta-delta Discounting Model

Kevin Zou

Abstract

In this paper, we developed the Quasi-hyperbolic Discounting model into the most general form of the General Beta-delta Discounting model (GBDD). The proposed framework of GBDD helps identify individuals' prospective present (how soon is now) and types of dynamic inconsistency (present bias, future bias, and mixed inconsistency). We applied our GBDD model to seven Convex Time Budget experimental data sets (AS12a, AS12b, CMW16, BCG18, LSW18 CJK19, and BHJ20), including 3670 subjects. We find the perceptual present varies substantially across individuals. The empirical evidence supporting the subjective sense of present lasts longer than the immediate present. We also find that 22.75% of subjects' revealed time preference exhibits future bias and mixed inconsistent patterns, which is overlooked in previous studies.

JEL Classification: C91; D01; D11; D12; D15; D71; D90; D91

Keywords: Revealed Time Preference; Quasi-hyperbolic Discounting; Present Bias; Convex Time Budget; Meta-analysis; Experiments

4.1 Introduction

Time is a fundamental concept in decision-making, and understanding the trade-off between present and future consumption is critical to studying economic behavior. To understand how we make decisions along a time horizon, we study time preference. Then, we apply the discoveries of time preference to many aspects such as health, saving, retirement decisions, and investment; (Uzawa, 1969; Fuchs, 1980; Finke and Huston, 2013; Hunter et al., 2018; Ebert et al., 2020). It is crucial for economics research to understand and model time preference accurately. We often find present bias behavior (a tendency that people give more substantial weight to the payoffs closer to the present than the future) in time preference among individuals. To model the inconsistent behaviors (O'Donoghue and Rabin, 1999). The Quasi-Hyperoboslic model became one of the most commonly adopted approaches in modeling present bias behaviors (Thaler, 1981; Frederick et al., 2002).

"How soon is now?" is one of the most actively controversial topics in the Quasi-Hyperbolic Discounting (QHD) model-based time preference literature (Glimcher et al., 2007; DellaVigna, 2018; Ericson and Laibson, 2019; Balakrishnan et al., 2020). To explore the question of "nowness," we link time inconsistency with the perceptual sense of time, then found the threshold that distinguishes the present and the future in the QHD framework. The threshold we find challenges the conventional belief that present bias-driven time inconsistency only applies to a relatively short period of time. Our results show that disregarding the importance of time perception in a revealed preference study leads to misleading conclusions about time inconsistency. With our newly proposed General Quasi-Hyperbolic Discounting (GQHD) approach and its empirical application, we find a link between time perception and dynamic inconsistency, which helps us to give a more precise answer about when time inconsistencies occur.

Conventionally, empirical models set a boundary that distinguishes present and future, an approach that limits our ability to observe present bias to immediate consumption choices. In other words, the subjects are being observational present biased if and only if the present bias happened precisely according to the constraint artificial boundary that specifies their present and future. In lab experiments, the design of immediate pay-off varies for different studies from hours, days, weeks, to years. These studies find disagreement about the existence of the present bias to the dynamic inconsistency related to research questions (Angeletos et al., 2001; McClure et al., 2007; Augenblick et 2015; Augenblick, 2017; White, 2020; Branas-Garza and Prissé, 2021). Other research finds that time preferences also depend on the subjective sense of time (Angeletos, Laibson, Repetto, Tobacman, and Weinberg 2001; dos Santos and Martinez, 2018). Balakrishnan et al. (2020) explore the timing of lab payments and find that present bias only exists when participants are paid at the immediate conclusion of the experimental session. However, a structural model that restricts the notion of the present to the immediate now may lead to errors, such as unobserved dynamic inconsistency in time preference studies (Imai et al., 2021).

We divide our study into the following sections to find a more general definition of the time horizon that better explains the data. First, we develop a new framework that generalizes the QHD model with a more flexible definition of the present sense and its well-defined characterizations (secondary present bias and secondary future bias behavior in time preference). This general format comes with a modified β_τ parameter of the QHD that captures the perceptual present (Connor and Smith, 2019) and its inconsistency. This theoretical work closely follows the Convex Time Budget (CTB) design to find empirical evidence of perceptual dynamic inconsistency from existing experimental data. We also demonstrate that potential ignorance of the perception present in time preference research leads to the imprecise judgment of the types of inconsistency with both derived intuition and simulated data. Next, we applied the GQHD method with a nonlinear least square estimation approach to seven lab experiment data from Andreoni and Sprenger, 2012a (AS12a), 2012b (AS12b); Carvalho et al., 2016 (CMW16); Blumenstock et al., 2018 (BCG18); Luhrmann et al., 2018 (LSW18); Chen et al., 2019 (CJK19); and Balakrishnan et al., 2020 (BHJ20). [1] We mainly focus on reporting the results of CMW16 for this paper with different strengths of restrictions on the utility assumptions, and robustness checks of various functional forms. We find strong evidence to support the existence of a more diversified perspective of the present in the observational data set than the previous assumption that the present means "now." In fact, the average perceptual present lasts for days or even months. Most importantly, when the present is correctly specified in the model, conclusions about the direction of time inconsistency may differ from previous studies.

There is a rich literature study about the errors or violations in revealed preference. There are many different methods for detecting violations of the rationality of revealed preference (GARP in general; Afriat 1967; Varian; 1982), such as the critical cost efficiency index (CCEI; Afriat 1972, 1973), the Houtman Marks index (HMI; Houtman and Maks, 1985), money pump index (MPI; Echenique et al., 2011) and the minimum cost index (MCI; Dean and Martin 2016). Particularly for revealed time preference studies, there are attempts to explain the observed irrationality with nonparametric methods (Millner and Heal, 2018; Echenique et al., 2020; Blow et al., 2021). In this study, we extend the GQHD model with a more general functional form of the GBDD model to explain the observed errors that the GQHD model does not explain. The nonparametric methods are utilized in the parametrial method of GBDD. Moreover, the mixed inconsistent time preference (introduced in Jackson and Yariv, 2014) is also discovered and explained based on the CTB experimental data.

The main contribution of our research is to establish the importance of considering the

[1] Thanks, Dr. Mike Callen and Dr. Pamela Jakiela, for providing their data on BCG18 and BHJ20.

perceptual sense of the present. Failing to do so leads to incorrect analysis of present and future bias behaviors, which undermines policy efforts to improve behaviors related to time discounting. We propose a General Quasi-Hyperbolic Discounting method that addresses both perception of the present and its corresponding dynamic consistency. This method allows us to assess time inconsistency more precisely at both aggregated and individual levels in the perceptual time scale. Furthermore, we extended our method with a more general form of the General Beta-delta Discounting model to further explain the observed irrationality, in which subjects make systematic mistakes (mixed inconsistent) in revealed time preference laboratory experiments.

4.2 Background

Time preference measures how individuals evaluate receiving a good at an earlier time compared to receiving it at a later time. It is essential for economists to understand it to explain how individuals, even the whole population smooth out their consumption. There are many other applications, such as deriving discounted net benefits in benefit-cost analysis and commercial analysis of consumers' behaviors. Historically, we tried to model time preference with many different models. Samuelson started this path with his proposed Exponential Discounting (ED) utility model (Samuelson, 1937; Koopmans, 1960). The ED model assumes that individuals' valuation of consumption depreciates at a constant rate δ over time [2] (Eq. (4.3)). Researchers have shown that behavior is often inconsistent with the ED model (Ainslie, 1975; Thaler, 1981; Frederick et al., 2002). Ainslie (1975) first showed the inconsistency in time preferences and illustrated more significant discounting for short-run consumption than long-run consumption, a phenomenon called present bias. As an example,

[2] In ED model $D(t) = \delta^t$.

suppose we have the following two choices:

Example. A. \$10 today or B. \$11 in a week vs. A'. \$10 in 50 weeks or B'. \$11 in 51 weeks.

Many decision-makers choose A over B and B' over A.' Many well-known theoretical models in economics and psychology studies address this documented observational reversal behavior in time preference (Thaler, 1981; Frederick et al., 2002). Some of the models (Appendix C.2) that specifically study inconsistent time discounting are the proportional discounting model (Ainslie and Herrnstein, 1981), the power discounting model (Harvey 1986), the hyperbolic discounting ($\frac{1}{1+(\kappa t)}$) model (Mazur, 1987; Loewenstein and Prelec, 1992), and the constant sensitivity model (Ebert and Prelec, 2007). Many studies discuss the separability of a utility function (Cherchye et al., 2015), especially in revealed time preference (Ericson et al., 2015; Dragone and Ziebarth, 2015). Among the discounting models, which assume separable discounted utility, Hyperbolic Discounting becomes the most commonly chosen one to model this time-inconsistency due to the accuracy of explaining this reversal behavior.

Laibson (1997) successfully approximated the hyperbolic discounting model with the Quasi-Hyperbolic Discounting function (Eq. (4.1)); Phelps and Pollak, 1968) in a discrete-time preference study. The critical advantage of this approximation is that the quasi-hyperbolic model (QHD, Eq. (4.1)) has a β parameter [3] that represents a departure from Samuelson's exponential discounting model (Eq. (4.3)) and separates time discounting into distinct processes for present and future rewards. Conventionally, we constrain the β parameter to be strictly less than one to indicate present bias in behavior. However, some recent research has found evidence that time preferences exhibit future bias characteristics (Aycinena, Blazsek, Rentschler, and Sandoval 2015; Corbett, 2016; Aycinena, and Rentschler

[3] In QHD model $D(t) = \beta\delta^t$.

2018). Additionally, there are several meta-analyses on time preference (Percoco and Nijkamp, 2006; Cheung et al., 2021; Imai et al., 2021; Weinsztok et al., 2021; Matousek et al., 2022). Among them, Imai et al. (2021) analyzed studies that use the Quasi-Hyperbolic Discounting (QHD) models with the Convex Time Budget (CTB) design and found evidence of over-reporting present bias. Future biased time preference behaviors are not only found in CTB studies but are also discovered in other non-CTB experimental designs (Takeuchi, 2011; Jackson and Yariv, 2014). Thus, previous results suggest that distinguishing present and future bias is essential in revealed time preference studies.

In addition to research on time inconsistency parameters, another stream of research focuses on establishing how one's perspective (subjective) time scale affects the measurement of dynamic inconsistency. The subjective time can be measured by the ability to distinguish the proportional difference between three months, one year, and three years on a physical object or measured by the ability to accurately tell how long a minute is (Zauberman et al., 2009; Bradford, Dolan, Galizzi 2013; Brocas, Carrillo, and Tarrasó 2018). Research shows that the degree of hyperbolic discounting (present bias) is reduced significantly when the dynamic inconsistency is measured by subjective time perception. In other words, people are less inconsistent when we measure time in their perspective time scale (Zauberman et al., 2009). In addition to the inconsistency of time preference, the results can also be extended to the discounting parameter δ estimation. The mental representation of the delays is correlated to our ability to delay consumption to the future. If the delay in time is exaggerated in one mind (for example, if the mental presentation of one day is longer than the physics definition of one day), then we will observe more deprecation of the delayed award in value. The more we shrink the time in our perspective sense, the more we will discount the valuation of future consumption (Brocas, Carrillo, and Tarrasó 2018).

Given that time is a conceptually subjective matter, the notion of present bias should also be attached to such conjecture. The present (future) biased behavior could also occur on a perceptual time scale. Most of the time, the definition of the present is predetermined by the experimental design but not discovered by the empirical work. The definition of the present can vary in length in different studies (Angeletos et al., 2001; McClure et al., 2007; Augenblick et al., 2015; Augenblick, 2017; Strack and Taubinsky, 2021). With the different experimental designs of the "immediate present" and future, studies find the inconsistency decay of discounting can be diverse base on the particular setting. These discussions lead to another open-ended research question: when and where we can observe present bias on the time horizon (DellaVigna, 2018; Ericson and Laibson, 2019). It is debatable about when the present is. Augenblick et al. (2015) found present bias behavior when considering an interval between a few minutes and weeks as the present for exerting effort experiment. On the other hand, another research observed the present bias behavior if considering a few hours as the present for a monetary award in the experiment. (Augenblick 2017; Balakrishnan et al., 2020).

The CTB experiment is a commonly employed design of dynamic inconsistency time preference in laboratory and field economic studies (Andreoni and Sprenger, 2012a, Andreoni et al., 2015; Carvalho et al., 2016; Giné et al., 2018). The CTB experiments ask the subjects to make decisions about dividing their award into two accounts, the sooner and the delayed account. If a certain amount of award is located in the delayed account, the subjects will be paid with that certain amount plus interest. However, there will be no interest for the amount they put in the sooner account when they get paid. Two separate awards will be issued to the subjects accordingly to the "sooner" and the "delay" in the experiment. One of the essential advantages of the CTB design is that the subject can split the reward into two accounts on a continuous scale. Therefore, it allows a structural model to estimate the utility's curvature parameter (elasticity of substitution) more precisely. Thereby we can estimate the discounting parameter more accurately (Eq. (4.4)). Given the fixability of the CTB design, a considerable amount of economics (Atalay et al., 2014; Liu et al., 2014; Cheung, 2015; Alan and Ertac, 2015; Carvalho et al., 2016; Kuhn et al., 2017; Luhrmann et al., 2018; Balakrishnan et al., 2020) and psychology (Yang and Carlsson, 2016; Hoel et al., 2016; Lindner and Rose, 2017) studies combine the Convex Time Budget and Quasi-Hyperbolic Discounting to study revealed time preference. These studies come with different treatment dimensions in the experiments to either suggest an improvement for the design or provide policy applications. If the QHD model does not precisely characterize the time preference, the validities of these policies would be undermined. We will follow the CTB design closely to further develop the QHD model with the perception of the present extension in the **The Theoretical Framework** section to address our concern about the current time preference studies.

4.3 The Theoretical Framework

4.3.1 The General Quasi-Hyperbolic Discounting Model

To study dynamic inconsistency in the revealed time preference, we apply the Quasi-Hyperbolic Discounting (QHD) function to an additive time separable utility function. After the first consumption period, the QHD model has a β parameter (Eq. (4.1)). The β allows the Exponential Discounting (ED) model (Eq. (4.3)) to mimic the feature of Hyperbolic Discounting. It captures the departure of the inconsistent discounting from the ED function (Strotz, 1956; Phelps and Pollak, 1968; Laibson, 1997; O'Donoghue and Rabin, 1999a). The interpretation of this departure is limited by where we place the β parameter. In empirical and experimental studies, we interpret the β as the inconsistency of discounting rate of all future consumption (x_t, where $t \in \{t_1, t_2 ... t_n\}$) compared to the consumption (x_{t_0}) of the soonest observational period (Eq. (4.1)).

$$u(x_{t_0}) + \beta \sum_{t=t_1}^{t_n} \delta^t u(x_t) \tag{4.1}$$

where the time $t \in \{t_1, t_2 ... t_n\}$ depends on when we observe the consumption, and x_{t_0} is the initial consumption level at t_0, and x_ts' is all future consumption. We call β a consistency parameter, which identifies the deviation from the exponential discounting parameter δ (Eq. (4.1)).

To characterize dynamic inconsistency as a perceptual notion, we modified QHD into a more general functional form (Eq. (4.2) (Echenique et al., 2020), and introduce a new parameter $\beta_{\tau_{t_i}}$ (β_τ for simplicity) where t_i can take different values in $\{t_0, t_1 ... t_n\}$ to adjust for an individual's specific perception of the present. We define τ_{t_i} (τ for simplicity) as the Present Threshold that distinguishes a subject's perception of the present and future. We name β_τ as the Perceptual Consistency Parameter. The β_τ not only characterizes the direction of bias (either present bias ($\beta_\tau < 1$) or future bias ($\beta_\tau > 1$)) with its magnitude but also captures the individual's perceptual distinction of the present and the future with the τ. Specifically, the dynamic consistency parameter β_τ starts to appear in Eq. (4.2) if and only if the assigned time of sooner consumption and later consumption crosses the individual perceptual present threshold τ. The functional form in Eq. (4.2) is more general in choosing the value of t_i compared to the QHD model (Eq. (4.1)), and we call it the General Quasi-Hyperbolic Discounting (GQHD).

$$\sum_{t=t_0}^{t_i} \delta^t u(x_t) + \beta_{\tau_{t_i}} \sum_{t=t_i+1}^{t_n} \delta^t u(x_t) \tag{4.2}$$

where $t_i \in \{t_0, t_1, ... t_n\}$, which depends on the experimental design, and $\tau_{t_i} \in \{\tau_{t_0}, \tau_{t_1}, ... \tau_{t_n}\}$,

$t_0 \leqslant \tau_{t_0} < t_1$, $t_1 \leqslant \tau_{t_1} < t_2$, ... $t_n \leqslant \tau_{t_n}$. Each τ_{t_i} (τ for simplicity) can take any arbitrary value within the given interval, and it would not affect the specification of the structural model for the empirical work. If $\tau < t_0$ or $\tau \geqslant t_n$, then the QHD utility reduces to the exponential discounting utility (Eq. (4.3)). Therefore, the estimation of β equals 1 has two possible interpretations. Either the subject has a dynamic consistency utility, or the design failed to trigger the subject's sense that distinguishes his or her perception of the present and future (see **Property 1**).

$$\sum_{t=t_0}^{t_n} \delta^t u(x_t) \tag{4.3}$$

We present some unique GQHD model definitions corresponding to the Convex Time Budget (CTB) experiment.

Definition 1: ***Time*** $t \in [0, t_n] = T$ is a continuous variable that describes the progress domain of a CTB experiment from the very moment when we observe subject's time preference ($t = 0$) until the moment when the very last award is scheduled to be delivered ($t = t_n$) to the subjects.

Definition 2: ***Now*** $t = 0$ is a discrete definition that defines the very moment when we observe the time preference behavior.[4] It doesn't take a long time (minutes) to collect the choices from a CTB experiment, and given a long enough time horizon (months) for the first award to be delivered (Andreoni and Sprenger 2012a); so we define Now as discrete (a point on time horizon) arbitrarily.

Considering the revealed time preference data is collected at Now, we will label the observed utility function given the data set as $U_0(\cdot_t)$ to maintain consistency in notation in the theoretical framework. [5]

[4] If we extend time $t \in \mathbb{R}$, then the ***Past*** can be defined as $t \in (-\infty, 0) = T_h$ (h for history).

[5] Notice that we do not have to assume the utility is time-invariant ($U_t(\cdot_t) \equiv U(\cdot_t), \forall t \in T$) to estimate the parameters for

Definition 3: ***Delivery Date*** $t = t_i \in \{t_0, t_1, ...t_n\}$, (where $i = 0, 1, 2, ...n$) is a discrete variable that identifies the time when the reward is scheduled to be delivered to the subjects.

One important note we need to emphasize is that there exists a small time gap (ϵ) between the preference being revealed and the first reward being delivered ($0 + \epsilon = t_0, \epsilon > 0$). For example, the subjects may need to fill out a questionnaire after the CTB experiment, or it takes some time for the electronic transaction to process. If "how soon is now" type of research is addressing the question about how small ϵ should be, then $U_0(\cdot_t) \equiv U_{0+\epsilon}(\cdot_t)$ (Balakrishnan et al., 2020). Kirby (1997) was concerned about not distinguishing between the instant and the nearly instant awards. In our framework, we also assume $0 < t_0$.

Definition 4: ***Present Threshold*** $\tau \in \{\tau_{t_0}, \tau_{t_1}, \tau_{t_2}, ...\tau_{t_n}\}$ ($t_i < \tau_{t_i} \leqslant t_{i+1}, i \in \{0, 1, 2, ...n-1\}$ or $t_i < \tau_{t_i}, i = n$) is a discrete variable that captures a value of time, which distinguishes the Present and the Future.

The Present Threshold (τ_{t_i}) can take any value between the interval $(t_i, t_{i+1}]$ because consumption level is not observed for any available Delivery Date in T. Taking any value within the interval will not change the estimation strategy of any structural model. Additionally, the Present Threshold can be further decomposed into three subcategories the ***Primary Present Threshold*** (τ_{t_0} for QHD model), the ***Secondary Present Threshold*** (τ_{t_i} $i \in \{1, 2, 3, ...n-1\}$ for GQHD model), and the ***Observational Dynamic Consistency Threshold***. (τ_{t_n} for ED model). The τ_{t_1} model is equivalent to the conventional Quasi-Hyperbolic Discounting model (Phelps and Pollak 1968); τ_{t_n} is equivalent to the exponential discounting model (Koopmans, 1960); the Secondary Present Threshold is unique to our General Quasi-Hyperbolic Discounting model. A subject's time preference is GQHD rationalizable if he or she has a unique and stable τ corresponding to revealing decisions

the empirical work. However, to further extend the GQHD model to O'Donoghue and Rabin's procrastination studies (1999a; 1999b; 2001) or to develop any policy implication of the empirical results, imposing the time-invariant assumption is necessary.

at Now. The subscript τ indicates the actual Time location of the ***Perceptual Dynamic Inconsistency Parameter*** β_τ (Eq. (4.2)).

Definition 5: The ***Present*** $t \in [0, \tau] = T_p$ is an interval of time (t) between Now ($t = 0$) and the Present Threshold (τ). We denote the individual's ***Perceptual Present*** as $T_{\tilde{p}}$.

Definition 6: The ***Future*** $t \in (\tau, t_n] = T_f$ is an interval of time (t) between the ***Present Threshold*** (τ) and the date that the latest award is scheduled to be delivered (t_n). We denote the individual's ***Perceptual Future*** as $T_{\tilde{f}}$[6].

In the framework of the GQHD model, the Now and the Present are different. Now is the lower bound to the Present. On the contrary, the upper bound of the Present is determined by the individual's time perception. Previously, we considered the t_0 as the upper bound of the present in the structural models' estimation strategy. The GQHD model is more general compared to the QHD in the sense that we propose a more flexible upper bound of the Present to detect how long it would last. Simultaneously, the Future is no longer defined as all Delivery Date that is not t_0, but as the Time (t) after the Present Threshold. We define $t \in [0, \tau_0] = T_{p_0}$ as the ***Primary Present***, $t \in [0, \tau_i] = T_{p_i}$ as the ***Secondary Present***, $t \in (\tau_0, t_n] = T_{f_0}$ as the ***Primary Future***, and $t \in (\tau_i, t_n] = T_{f_i}$ as the ***Secondary Future*** (where $i \in \{1, 2, 3, ...n - 1\}$).

4.3.2 The Behavioral Characterizations of the GQHD Model

With a formal extension of the QHD model, the Perceptual Consistency Parameter β_τ differs the Perceptual Present from the Perceptual Future in the GQHD model. As a result of the flexible location of the β_τ, the GQHD model has some unique properties compared to the ED and QHD models. We demonstrate five properties of the GQHD model to characterize

[6] If we extend time $t \in \mathbb{R}$, then the ***Future*** can be defined as $t \in (\tau, \infty) = T_f$.

the behavior in the revealed time preference.

Property 1: The revealed time preference exhibits ***Observational Dynamic Consistent*** if $\beta_\tau = 1$.

Generally, we emphasize that the dynamic consistency is observational because the structural model fits the data in the given experimental time horizon. The inconsistency in time preference might happen beyond the experimental time domain. The locations of β_τ are constrained by the way that we collect data in the experiment. The β_τs exist in the structural model if and only if $t_0 \leqslant \tau < t_n$. The structural model cannot detect β_τ, if the τ exists at a location further than t_n. Similarly, the structural model is not sensitive to the time inconsistency behavior if τ exists at a location that is sooner than t_0. Balakrishnan et al. (2020) address this $\tau < t_0$ issue with a modified design of the CTB. However, the $t_n \leqslant \tau$ question remains to be discovered. $ED \equiv QHD \equiv GQHD$, if $\beta_\tau = \beta = 1$ or $\tau < t_0$ or $t_n \leqslant \tau$.

Property 2: The revealed time preference exhibits ***Primary Present Bias*** if $t_0 \leqslant \tau < t_1$, and $\beta_\tau < 1$.

Property 3: The revealed time preference exhibits ***Primary Future Bias*** if $t_0 \leqslant \tau < t_1$, and $\beta_\tau > 1$.

The primary bias model, either present bias or future bias, is equivalent to the conventional setup (Laibson, 1997; Andreoni and Sprenger 2012a). The additional explanation we provide to this traditional notion is that we abolish the restriction of the sense of threshold that differs present and future at the individual level. We distinguish the present and future, not by a single value in the timeline, say today verse any day that is not today. Instead, we consider any arbitrary $\tau < t_1$ can be interpenetrated as an individual's perception of the present. $QHD \equiv GQHD$, if $t_0 \leqslant \tau < t_1$.

Property 4: The revealed time preference exhibits ***Secondary Present Bias*** if $t_1 \leqslant \tau < t_n$, and $\beta_\tau < 1$.

Property 5: The revealed time preference exhibits ***Secondary Future Bias*** if $t_1 \leqslant \tau < t_n$, and $\beta_\tau > 1$.

The secondary bias model is a general extension of the primary bias model. We allow the τ to take the value greater than t_1, which is the nearest observed consumption. This general format can detect the inconsistent time preference between present and future with a relaxed assumption that individual subjects have different time perceptions (Zauberman et al., 2009). The subjective time perception affects the way we describe time inconsistency severely.

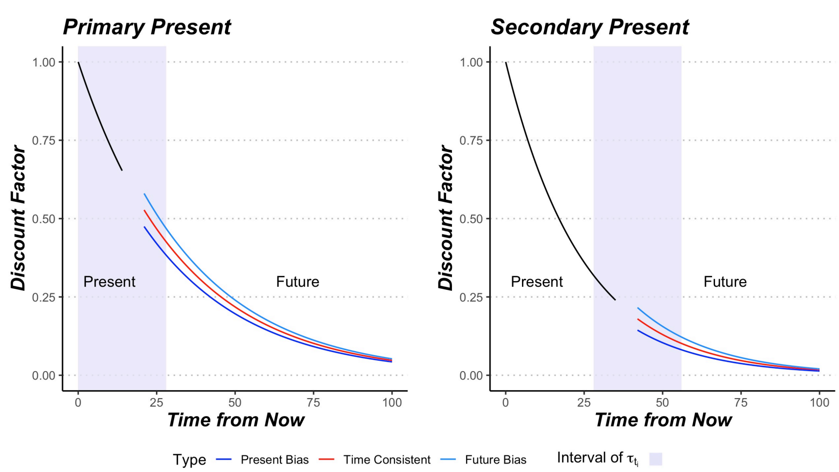

Figure 4.1: Primary Present and Secondary Present

There is a gap between the theoretical framework and the structural empirical model proposed by the CTB protocol. The gap commonly exists for all of the intertemporal choices studies; the simple version (Fisher 1930) or any of its variations (Benhabib, Bisin, and Schotter 2010; Takeuchi, 2011; Jackson, and Yariv 2014; Andreoni and Sprenger, 2012a; Montiel Olea and Strzalecki, 2014; Dziewulski, 2018). The subjects make decisions in discrete-time slots. The consumption in the potential interval of the τ area is unobserved. We use the structural model to fit the observations and then describe the subjects' dynamic time inconsistency. Theoretically, the primary and secondary present models are equally compelling to explain time inconsistency with the assumption that the subjects have an inconsistent time preference within the shaded area. The advantage of the GQHD model is to initiate β_τ at different time spots, then find the most unexpected kinky consumption among all others (Figure 4.1).

4.3.3 Empirical Strategy

Given the general theoretical framework of the convex time budgets (CTB) in time preference studies (Andreoni and Sprenger, 2012), we assume the subjects in the CTB experiment maximize an additively time separable constant relative risk aversion (CRRA) utility function of consumption ($U(x)$) between two Delivery Date t_i, and t_i+k. Such utility function is discounted with a GQHD function. Consider a subject maximizing the GQHD-CRRA utility at Now (Eq. (4.4)), subject to a given time budget constraint in the experiments (Eq. (4.5)).

$$U_0(x_{t_i}, x_{t_i+k}) = \delta^{t_i}\frac{1}{1-\eta}(x_{t_i}+\omega)^{1-\eta} + \beta_\tau^{\mathbb{1}_{\{t_i \leqslant \tau < t_i+k\}}}\delta^{t_i+k}\frac{1}{1-\eta}(x_{t_i+k}+\omega)^{1-\eta} \tag{4.4}$$

Where we denote the one-period discount factor as δ, the curvature parameter as $1-\eta$, and the background consumption as ω, which we assume as constant for every period. Lastly, x_{t_i} and x_{t_i+k} denote the consumption that allocate into the sooner account and the later account correspondingly.

$$x_{t_i} + (1+r)^{-1}x_{t_i+k} = m \tag{4.5}$$

Where we denote the r as the interest rate through k days and m as the total endowment for the current trail. Then the optimal demand function of sooner consumption is:

$$x_{t_i}^* = \begin{cases} \frac{(\delta^k(1+r))^{\eta^{-1}}-1}{1+(1+r)(\delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\delta^k(1+r))^{\eta^{-1}}}{1+(1+r)(\delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } \tau < t_i \text{ or } t_i + k \leqslant \tau \\ \frac{(\beta_\tau\delta^k(1+r))^{\eta^{-1}}-1}{1+(1+r)(\beta_\tau\delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\beta_\tau\delta^k(1+r))^{\eta^{-1}}}{1+(1+r)(\beta_\tau\delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } t_i \leqslant \tau < t_i + k \end{cases} \tag{4.6}$$

To illustrate the strategy of finding the optimal τ, we provide an example under the assumptions of the empirical framework. Consider a subject's three-period discounting function exhibit the following pattern $D_\tau(t_0, t_1, t_2) = (\delta^{t_0}, \delta^{t_1}, \beta\delta^{t_2})$, which is a secondarily bias time preference behavior. Suppose we correctly specified the structure model with a sequence of time-dependent discount factors $\hat{D}_\tau(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\delta}^{t_1}, \hat{\beta}\hat{\delta}^{t_2})$. We can recover the true value of δ by solving the equation between the marginal rate of substitution and the price ratio of x_{t_0} and x_{t_1}. Here we denote the growth interest rate as r_{t_0,t_1} (Eq. (4.7)).

$$\frac{\partial U_0/\partial x_{t_1}}{\partial U_0/\partial x_{t_0}} = \frac{1}{1+r_{t_0,t_1}} \implies \delta^* = (\frac{1}{1+r_{t_0,t_1}})^{\frac{1}{t_1-t_0}}(\frac{x_{t_1}}{x_{t_0}})^{\frac{1-\eta}{t_1-t_0}} \tag{4.7}$$

Similarly, we can recover the true value of β by solving the equation between the marginal rate of substitution and the price ratio of x_{t_1} and x_{t_2}. We denote the growth interest rate as r_{t_1,t_2}. Here notice β^* (or $\beta^{*t_2-t_1}$) is strictly greater than 0 (Eq. (4.8)).

$$\frac{\partial U_0/\partial x_{t_2}}{\partial U_0/\partial x_{t_1}} = \frac{1}{1+r_{t_1,t_2}} \implies \beta^{*t_2-t_1} = (\frac{1}{1+r_{t_1,t_2}})^{\frac{1}{t_2-t_1}}(\frac{x_{t_2}}{x_{t_1}})^{\frac{1-\eta}{t_2-t_1}}(\frac{1}{1+r_{t_0,t_1}})^{\frac{-1}{t_1-t_0}}(\frac{x_{t_1}}{x_{t_0}})^{\frac{\eta-1}{t_1-t_0}} \tag{4.8}$$

Contrarily, suppose we follow the QHD set up and misspecify the structure model with $\hat{D}_{\tau'}(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\beta}\hat{\delta}^{t_1}, \hat{\beta}\hat{\delta}^{t_2})$. Then misleadingly equating the misspecified estimation with the true parameter $D_\tau(.) = \hat{D}_{\tau'}(.)$, we can derive $\hat{\delta}$ in Eq. (4.9). For simplicity, we consider $\delta^{t_0} = 1$.

$$\frac{\hat{\beta}\hat{\delta}^{t_2}}{\hat{\beta}\hat{\delta}^{t_1}} = \frac{\beta\delta^{t_2}}{\delta^{t_1}} \implies \hat{\delta} = \beta^{\frac{1}{t_2-t_1}}\delta \implies \begin{cases} \hat{\delta} > \delta, & \text{if} \quad \beta > 1 \\ \hat{\delta} < \delta, & \text{if} \quad \beta < 1 \end{cases} \tag{4.9}$$

For $\hat{\delta}$, we can use the Eq. (4.8) to quantify the magnitude of overestimation (underestimation). We can derive $\hat{\beta}$ in (with a similar approach in Eq. (4.10)).

$$\frac{\hat{\beta}\hat{\delta}^{t_1}}{\hat{\delta}^{t_0}} = \frac{\delta^{t_1}}{\delta^{t_0}} \implies \hat{\beta} = \frac{\delta^{t_1-t_0}}{\hat{\delta}^{t_1-t_0}} \implies \begin{cases} \hat{\beta} < 1 < \beta, & \text{if} \quad \beta > 1 \\ \hat{\beta} > 1 > \beta, & \text{if} \quad \beta < 1 \end{cases} \tag{4.10}$$

The result above has an intuition that can guide us to initiate the β_τ at an optimal location that provides a more precise inference for the time preference (Eq. (4.9) and Eq. (4.10)). Suppose we misplace a secondary future bias subject's initial β_τ earlier than the true location, then we are likely to interpret this subject's behavior as primary present bias. By pushing τ to the other locations, we would observe a decreasing pattern of $\hat{\delta}$, along with an increasing trend of $\hat{\beta}_\tau$. Ideally, we can continuously push the τ to another location until we observe the $\hat{\beta}_\tau > 1$. Similarly, suppose we place a secondary present biased subject's τ at a closer location. In that case, we should expect an increasing trend of the $\hat{\delta}$ along with a decreasing pattern of the $\hat{\beta}_\tau$ as we push the τ to the later position. The optimal initial location of the β_τ in the summation is obtained when the $\hat{\beta}_\tau < 1$ in this scenario. As many dynamic discrete choice studies demonstrated, the β and δ vary accordingly with a systematic mechanism in QHD (Laibson, Repetto, and Tobacman, 2007; Arcidiacono and Miller, 2008; Mahajan and Tarozzi, 2012; Fang and Wang, 2015; Abbring, Daljord, and Iskhakov 2018). This grand truth also applies to the Convex Time Budget since the choices are still in a dynamic setting. Our intuitive derivation reveals a potential confounding classification of the Primary Present

Bias and the Secondary Future Bias (the Primary Future Bias and the Secondary Present Bias). However, the confusion of mixing different types of behaviors has two directions (Eq. (4.9), Eq. (4.10), Eq. (C.1) and Eq. (C.2) from Appendix C.1). Besides, the trend of changing in β and δ becomes more complicated when dynamic choices are made in many periods. We will provide more detailed findings with the empirical methods to find the models that give a more precise explanation of the behaviors. To estimate the parameters, we will employ the non-linear least square (NLS) strategy (Andreoni and Sprenger, 2012a; Andreoni et al., 2015; See the **Aggregate Level Results** section).

4.4 Simulated and Experimental Data

Table 4.1: The Basic Information of the Included Studies

	Authors	Journal	Year	Region	Type
AS12a	Andreoni, Sprenger	American Economic Review	2012	United States	lab
AS12b	Andreoni, Sprenger	American Economic Review	2012	United States	lab
CMW16	Carvalho, Meier, Wang	American Economic Review	2016	United States	field
BCG18	Blumenstock, Callen, Ghani	American Economic Review	2018	Afghanistan	field
LSW18	Luhrmann, Serra-Garcia, Winter	Economic Policy	2018	German	field
CJK19	Chen, Jiang, Krupka	Experimental Economics	2019	China	field
BHJ20	Balakrishnan, Haushofer, Jakiela	Experimental Economics	2020	Kenya	lab

We acquired seven data sets (Andreoni and Sprenger, 2012a (AS12a), 2012b (AS12b); Carvalho et al., 2016 (CMW16); Blumenstock et al., 2018 (BCG18); Luhrmann et al., 2018 (LSW18); Chen et al., 2019 (CJK19); Balakrishnan et al., 2020[7] (BHJ20)) with the Convex Time Budget design to detect the perception of the present and its corresponding time inconsistency to see the robustness of the primary present model. We summarized the basic information (name of the authors, published journal, year of publication, region and the type of the experiments, etc.) of these seven published papers with the CTB experiment in Table 4.1. Combined, we analyzed 3760 subjects' CTB choices for the seven experiments

[7] In BHJ20, We used the data when the budget equals 600 only because the sets number for budget equals 400 is missing in the data provided.

over the last decade. The experimental design difference among the seven CTB experiments is listed in Table 4.2.

Table 4.2: The Experimental Design of the Included Data Sets

	Subjects	Sets (c_{t_i}, c_{t_i+k})	Time Slots i	Choices	Budget (tokens, m)	Payment
AS12a	97	9	9	45	100	real
AS12b	80	2	3	14	20	real
CMW16	1060	3	4	12	500	real
BCG18	824	2	3	10	250	hypothetical
LSW18	914	3	3	21	6	real
CJK19	201	4	5	24	100	real
BHJ20	494	8	5	48	400 or 600	real

In the following sections, we will mainly focus on the CMW16 data set because it has the largest subject size (1060) with a reach variation of β_τ for specifications compared to the other six remaining data sets. Since we focus on the design and the data set from the CMW16 data set, we also simulated 21 different data sets according to the CTB design to demonstrate the intuition or confirm the findings in our studies.

Table 4.3: The CTB Design in CMW16

choice	t_i	$t_i + k$	r	m
1	28 days	84 days	0%	$500
2	28 days	84 days	0.5%	$500
3	28 days	84 days	1%	$500
4	28 days	84 days	3%	$500
5	28 days	56 days	0%	$500
6	28 days	56 days	0.5%	$500
7	28 days	56 days	1%	$500
8	28 days	56 days	3%	$500
9	today	28 days	0%	$500
10	today	28 days	0.5%	$500
11	today	28 days	1%	$500
12	today	28 days	3%	$500

In Carvalho et al. (2016), the participants are asked to allocate their experimental budget of $500 into two payment accounts. The sooner account (t_i), and its paired later account ($t_i + k$). There are three paired accounts with $(t_i, t_i + k) \in \{(28, 84), (28, 56), (0, 28)\}$[8] . For each paired account, subjects choose how they would like to split 500 dollars into sooner and

[8] In the CTB design of CMW16, the payment dates are labeled as today, 4 weeks, and 8 weeks. We denote 4 weeks as 28 days, 8 weeks and 56 days, and 12 weeks as 84 days to be consistent with our structural model. We also consider t_i for today as $t_0 = 0$ to fit the model.

later accounts. The delayed (later) account has an associated interest rate of 0%, 0.5%, 1%, and 3% on payday. Each subject makes 12 choices (we have 12 observations for each subject). The researchers randomly selected one of twelve choices to issue the reward to the subjects on the paydays. If the choice has a nonzero amount in the later account, they will be paid with the associated interest rate **r** (Table 4.3). They restricted the sample to 1060 subjects that finished all 12 trials in the data set. Our analysis will impose the same restriction on these participants who complete all 12 trials in the experiments. They observed a slight primary present bias for the subjects at the aggregate level from a tow-limit Tobit model with a CARA utility setup (Carvalho et al., 2016).

The CTB design in CMW16 allows us to set the Present Thresholds (τ) at four different time slots in the GQHD specification. Firstly at τ_0, the subject can consider any day between today and 28 days later as their perceptual present. First, we consider the Primary Present (T_{p_0}), which is the QHD model. Second, τ_{28} is when a subject's τ can arbitrarily take any day between 28 days and 56 days. Then, τ_{56} ranges between 56 days and 84 days. These specifications consider the Secondary Present (T_{p_i}), which is the innovation of the GQHD. Third, we consider the τ_{84} model, which indicates the Present Thresholds surpasses 84 days, and the specification is equivalent to the ED model.

We illustrate the systematic changes in the β and the δ parameters in the **Empirical Strategy** section. However, that derived intuition was limited to three-period consumption observations. The design of CMW16 has four periods (today, 28 days, 56 days, and 84 days) instead of three. We will confirm if the systematic changes in the β and the δ parameters remain unchanged from 18 simulated data sets corresponding to the CTB in CMW16. To begin with, we simulated the following 18 data sets to detect the validity of the intuition that the structural model tends to underestimate the β parameter if we assume the perception of the present ends right after the soonest consumption rather than terminates at a later

Table 4.4: Estimations of 18 Simulated Data Sets (CMW16)

		τ_0	τ_{28}	τ_{56}	τ_0	τ_{28}	τ_{56}
$\mathbf{1-\eta}$		$\boldsymbol{\beta_{\tau_{56}} = 1.1\ \delta = 0.9995}$			$\boldsymbol{\beta_{\tau_{56}} = 1.05\ \delta = 0.9995}$		
0.9	$\hat{\beta}$	0.9622	0.9999	1.1000	0.9811	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0013	0.9995	1.0003	1.0003	0.9995
0.85	$\hat{\beta}$	0.9626	1.0007	1.1000	0.9814	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0011	0.9995	1.0003	1.0003	0.9995
0.8	$\hat{\beta}$	0.9624	0.9992	1.1000	0.9815	1.0000	1.0500
	$\hat{\delta}$	1.0012	1.0011	0.9995	1.0003	1.0003	0.9995
$\mathbf{1-\eta}$		$\boldsymbol{\beta_{\tau_{56}} = 1.1\ \delta = 0.9985}$			$\boldsymbol{\beta_{\tau_{56}} = 1.05\ \delta = 0.9985}$		
0.9	$\hat{\beta}$	0.9608	1.0000	1.1000	0.9807	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9999	0.9985	0.9992	0.9991	0.9985
0.85	$\hat{\beta}$	0.9614	1.0000	1.1000	0.9808	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9999	0.9985	0.9992	0.9991	0.9985
0.8	$\hat{\beta}$	0.9617	1.0000	1.1000	0.9809	1.0000	1.0500
	$\hat{\delta}$	1.0000	0.9998	0.9985	0.9992	0.9991	0.9985
$\mathbf{1-\eta}$		$\boldsymbol{\beta_{\tau_{56}} = 1.1\ \delta = 0.9975}$			$\boldsymbol{\beta_{\tau_{56}} = 1.05\ \delta = 0.9975}$		
0.9	$\hat{\beta}$	0.9612	1.0000	1.1000	0.9810	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9986	0.9975	0.9982	0.9980	0.9975
0.85	$\hat{\beta}$	0.9617	1.0000	1.1000	0.9810	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9987	0.9975	0.9982	0.9981	0.9975
0.8	$\hat{\beta}$	0.9618	1.0000	1.1000	0.9811	1.0000	1.0500
	$\hat{\delta}$	0.9989	0.9987	0.9975	0.9982	0.9981	0.9975

Each dataset contains 1000 simulated subjects.

possible locations. Suppose we have two sets of secondary future biased time preference populations with $\beta_{\tau_{56}}$ equals to either 1.1 or 1.05 ($\beta_{\tau_{56}} \in B = \{1.1, 1.05\}$). For each of these two secondary future biased populations with selected curvature ($1-\eta \in H = \{0.9, 0.85, 0.8\}$) parameters and discount parameters ($\delta \in D = \{0.9995, 0.9985, 0.9975\}$), we simulated 1000 subjects' choices on the CMW16 choice set. The annual discount rates $(1/\delta)^{365} - 1$ are 20.03%, 72.96%, and 149.34% for the selected discount parameter 0.9995, 0.9985, and 0.9975 respectively. We simulated 1000 subjects' optimal consumption level $x^*_{t_i}$ at date t_i (Eq. (4.6)) according to the CMW16 design with a random normally distributed error ($\boldsymbol{e} \overset{i.i.d}{\sim} \boldsymbol{N}(0, 0.01)$) for all 18 data sets ($H \times B \times D$). The simulated results are presented in Table 4.4. As a consequence, we observed a strong tendency of the increasing trend in the estimated β and the decreasing trend of the estimated δ as we move the τ to the true location τ_{56} from the

QHD assumed τ_0. Such tendency suggests the degree of present bias can be overestimated if we treat the subjective present unanimously as the Primary Present (T_{p_i}), especially for those who exhibit Secondary Future Bias ($\beta_{\tau_{56}} > 1$) in their time preference (see the **Analysis and Results** section for the detailed statistical approach). The findings coincide with the grand truth in Eq. (4.9) and Eq. (4.10).

4.5 Analysis and Results

4.5.1 Aggregate Level Results

To estimate the parameters, we employed a non-linear least square (NLS) strategy similar to that proposed by Andreoni and Sprenger (2012a). We specified the indicator function of β to identify the perceptual present accordingly. Furthermore, we estimate the model with the assumption of the time-independent $\omega \approx 0$ in Eq. (4.6), considering the decision-maker is not necessarily narrowly bracketed in the experiment (Rabin and Weizsacker, 2009). The differences between restricting $\omega = 0$ or not are statistically negligible, and we present the results of the estimations without such restriction in Table 4.5.

Table 4.5: Aggregated Estimation for All Subjects (CMW16)

Model	τ_0	τ_{28}	τ_{56}	τ_{84}
$\hat{\beta}$	0.9516 (0.006)	1.001 (0.0086)	1.128 (0.0121)	
$\hat{\delta}$	0.9979 (0.0001)	0.9976 (0.0002)	0.9962 (0.0002)	0.9976 (0.0001)
$1 - \hat{\eta}$	0.7367 (0.2107)	0.7227 (0.2314)	0.7563 (0.2201)	0.7227 (0.2314)
AIC			✓	
BIC			✓	
RSS			✓	
Subjects	1060	1060	1060	1060

✓ for the best-performed model based on each criterion.

From Table 4.5, We find the estimation of the β in the primary model is equal to 0.9516, which suggests the subjects exhibits Primary Present Bias, which is consistent with the results in CMW16. However, as we relocate τ into a further location, we find a clear increasing trend in the estimation of β and a decreasing trend in δ estimation. This pattern suggests that more subjects' behaviors follow theSecondary Future Bias instead of the Primary Present Bias. The future bias is observed at τ_{56}. We chose three different criteria to compare the overall performance of other models (Akaike information criterion (AIC), Bayesian Information Criterion (BIC), and Residual Sum of Squares (RSS)). These three criteria also provide evidence that $\beta_{\tau_{56}}$ is the best-performed model to describe the data at the aggregate level. The model of $\beta_{\tau_{56}}$has the lowest AIC, BIC, and RSS, given all of the possible locations of the τ we allowed.

The credibility of our structural model relies heavily on the existence of well-behaved (monotonic, quasi-concave, continuous, and non-satiated) utility functions. We next evaluate individual participants' data for consistency with the requirements of rationality, exclude observations that fail to meet the criteria, repeat the analysis and compare results. A commonly used approach for detecting the existence of well-behaved utility is satisfying the Generalized Axiom of Revealed Preference (GARP) and its non-parametric test (Afriat 1967, Afriat 1972, Varian 1982, and Varian 1991). However, this approach will not work due to the original design of the CTB experiments since the commodities from different trials are not nested, and the time budget lines barely intersect with one other. Therefore, we employed the Law of demand test, which requires that if a trial with a higher interest rate for the later account than the previous trial, we should observe that the consumption level of the later account is at least as much as that of the previous trial. This basic consistency approach has been utilized by earlier studies to detect rationality (Giné et al., 2018; Balakrishnan et al., 2020; Echenique et al., 2020).

Table 4.6: Aggregated Estimation for Law of Demand Subjects (CMW16)

Model	τ_0	τ_{28}	τ_{56}	τ_{84}
$\hat{\beta}$	0.9625 (0.0064)	1.0000 (0.0085)	1.1010 (0.0123)	
$\hat{\delta}$	0.9977 (0.0001)	0.9974 (0.0002)	0.9964 (0.0002)	0.9974 (0.0001)
$1-\hat{\eta}$	0.8450 (0.0826)	0.8392 (0.0848)	0.8534 (0.1129)	0.8392 (0.0848)
AIC			✓	
BIC			✓	
RSS			✓	
Subjects	452	452	452	452

✓ for the best-performed model based on each criterion.

We find that 452 out of 1060 subjects' choice sets strictly follow the law of demand and repeat our analysis with only these subsets Table 4.5. We found these people, whose time preference follows the law of demand, also exhibit present bias when we set β_τ at t_0 on an aggregated level. Additionally, there is an increasing trend of the β estimator and a decreasing trend of the δ estimator as we move τ to later possible locations (τ_{28}, τ_{56}, and τ_{84}). We observe the best performance when τ_{84} according to AIC, BIC, and RSS criteria, which suggests the subjects are more likely to have a secondary future biased time preference than a primary present biased at the aggregate level (Table 4.6).

We consider the concentration of these dynamically inconsistent behaviors at the individual level and how it would affect the aggregate level estimation. It is necessary to sort out these subjects with dynamically consistent time preferences. The Strong Axiom of Revealed Exponentially Discounted Utility (SAR-EDU) suggests that if a finite time revealed time preference data set is EDU rational. Then the time preferences can be rationalized with an exponential discounted well-behaved utility function, where δ can take the value of 1. The additional finding suggests that the test's result is aligned with parametric estimations. If the revealed time preference data set passes the SAR-EDU test, then the empirical estimation of the β_τ is likely close to 1 (Echenique et al., 2020).

Table 4.7: Aggregated Estimation for Law of Demand and Non-EDU Subjects (CMW16)

Model	τ_0	τ_{28}	τ_{56}	τ_{84}
$\hat{\beta}$	0.9560 (0.0067)	1.0020 (0.0086)	1.1190 (0.0151)	
$\hat{\delta}$	0.9974 (0.0001)	0.9970 (0.0002)	0.9959 (0.0003)	0.9970 (0.0001)
$1-\hat{\eta}$	0.8819 (0.0503)	0.8779 (0.0485)	0.8884 (0.1179)	0.8779 (0.0485)
AIC			✓	
BIC			✓	
RSS			✓	
Subjects	230	230	230	230

✓ for the best-performed model based on each criterion.

We rerun the analysis with the 230 subjects whose choice set did not pass the SAR-EDU test within the 452 subjects. These 230 people's revealed time preference choice set is theoretically unrationalizable with dynamic consistency in well-behaved utilities (EU). The trend of β estimation and δ estimations remain unchanged. The model τ_{56} produced the best overall performance among all models (Table 4.7). We will investigate how different restrictions change the concentrations of the different types of dynamically inconsistent behavior and how such concentration could affect the aggregate level conclusion in the **Individual Level Results** section.

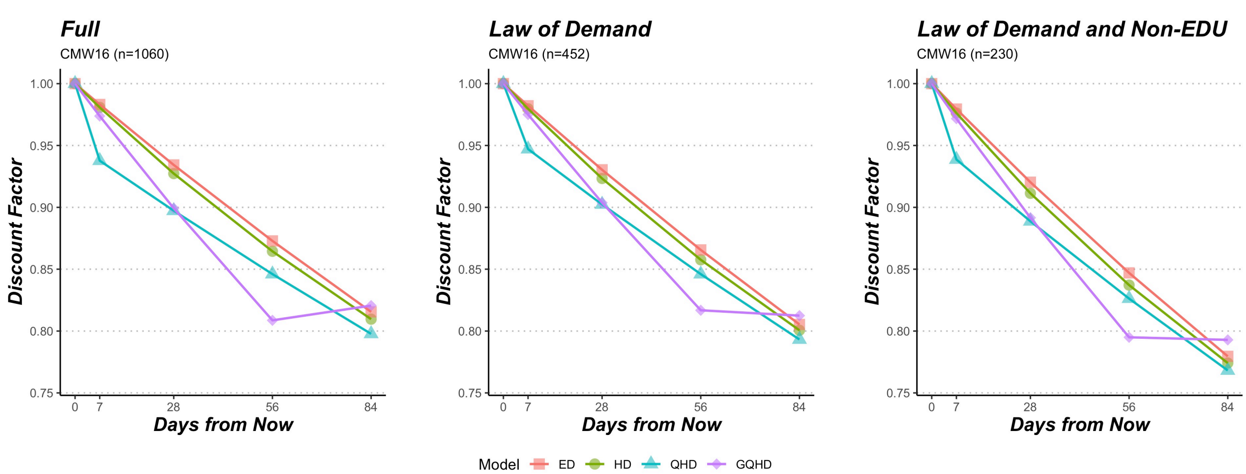

Figure 4.2: Estimated Discount Factors for 3 Datasets (CMW16)

To make the comparisons between the different discounting models, we present the estimated discount factors among the ED, the HD (the Hyperbolic Discounting Model: see the results from Appendix C.2), the QHD, and the GQHD models in Figure 4.2. We see a departure in the initial days of the HD model and the QHD models for all three different data sets. This departure indicates that the initial sharp drop of the QHD model was likely to catch a later seemly inconsistency discount rate caused by the underestimation of the δ, rather than to present the inconsistent discounting rates between the present and the future. The GQHD model indeed captures some unusual behavior in the discount function at the aggregate level for all different data sets. The GQHD model indeed captures some unusual behavior of the curvature in the discount function (Takeuchi, 2012) at the aggregate level for all three different subsets. It has the overall best performance among all four different models.

Table 4.8: Aggregated Estimation for All Seven Data Sets based BIC

	AS12a GBDD (S)	AS12b ED	CMW16 GBDD (S)	BCG18 ED	LSW18 ED	CJK19 GBDD (S)	BHJ20 GBDD (P)
$\hat{\beta}$	0.9627*** (0.0113)		1.1284*** (0.0121)			0.9265*** (0.0066)	1.0692*** (0.0203)
$\hat{\delta}$	0.9991*** (0.0001)	0.9994*** (0.0001)	0.9962*** (0.0002)	0.9938*** (0.0002)	0.9939*** (0.0002)	0.9981*** (0.0001)	0.982*** (0.0010)
$1-\hat{\eta}$	0.8714*** (0.0130)	0.7783*** (0.0555)	0.7563*** (0.2201)	0.7832*** (0.0424)	0.5844*** (0.0770)	0.8212*** (0.0133)	0.5313*** (0.0452)
Subjects	97	80	1060	824	914	201	494

GBDD (P): Primary present specification of the GQHD model.
GBDD (S): Secondary present specification of the GQHD model.

We then performed all possible specifications of the GQHD (GBDD from the later Section) model on all available data sets. In Table 4.8, we summarized the individual results for each data set based on the BIC criteria. We found that three (AS12b, BCG18, LSW18) out of seven experiments subjects exhibit no time inconsistency at the aggregate level. Only one CTB experimental data (BHJ20) has the primary present specification as the best-performed model. However, the subjects are overall future biased instead of present biased. Among the secondary present specification-best-performed experiment, we found the AS12a and CJK19 subjects are present biased, and the CMW16 subjects are future biased. Among the secondary present specification-best-performed experiment, we found the AS12a and CJK19 subjects are present biased, and the CMW16 subjects are future biased. We find no experimental evidence to support the mixed inconsistency revealed time preference behavior at the aggregated level.

Table 4.9: The Proportion of the Experimental Subjects Satisfying Different Axioms

Study	AS12a	AS12b	CMW16	BCG18	LSW18	CJK19	BHJ20	Percentage
Law of Demand and Non-EDU	20	13	230	172	70	80	63	648 (17.66%)
Law of Demand	49	74	452	444	244	108	180	1551(42.26%)
Full	97	80	1060	824	914	201	494	3670(100%)

Worth mentioning that the law of demand and non-EDU data set is the only legit data sets to apply the conventional QHD model parametrically. For future reference, we test the law of demand and SUR-EDU and present the result in Table 4.9. We find that 42.26% of subjects in the seven experiments strictly follow the law of demand. Only 17.66% of subjects satisfy the assumption of the law of demand and non-EDU. An additional investigation at the individual level is necessary to explain what we find at the aggregated level.

4.5.2 Individual Level Results

At the aggregate level, the people with secondary future biased time preference are dominating the time preference behavior in the CMW16 data set ($\beta_{\tau_{56}} > 1$ models have the best overall performance). We can also reach this intuition by unanimously observing the same pattern of trends of estimations of β and δ parameters when relocating τ to later positions. To verify how the concentration of the different behavior changes the pattern of the estimations, we newly simulated three data sets (100 simulated subjects each) with different proportion combinations of primary present biased, time consistent, and secondary future biased subjects. The proportions of primary present biased, time consistent, and secondary future biased subjects for these three simulated data sets are 2:2:6 (secondary future biased dominating), 2:6:2 (time consistent), and 6:2:2 (primary present biased dominating), respectively. For the simulated data, the η takes a uniform distribution between 0.87 and 0.89 ($1 - \eta \in [0.87, 0.89]$), and the δ takes a uniform distribution between 0.9997 and 0.9999 ($(1/\delta)^{365} - 1 \in [20.03\%, 149.34\%]$). We also assumed the background consumption ω is 0 for all the simulated subjects. The biased parameter β are uniformly generated between 0.95 and 0.99 for the primary present biased ($\beta_{\tau_0} \in [0.95, 0.99]$) simulated subjects, and between 1.05 and 1.1 for the secondary future biased ($\beta_{\tau_{56}} \in [1.05, 1.1]$) simulated subjects. The β are equal to 1 for the time consistent time preference ($\beta_{\tau_{28}} = 1$) simulated subjects. Then we estimated the β and δ parameters with the NLS approach for these simulated subjects.

To visualize the relationship between the change of the estimations and the proportion of the different types of time preference, we plotted a heat map of the proportions of increased β for all pairwise comparisons of two different β_τ identified structural models. For example, suppose we estimate the β_{τ_0} as the first model (labeled as before), and we estimate the $\beta_{\tau_{28}}$ as the second model (labeled as after). The pixel for β_{τ_0} (before) and $\beta_{\tau_{28}}$ (after) represents the percentage of the increased β for the second model compare to the first model. We apply the same rule for the percentage of decreased δ estimations. We can recognize the different patterns in the heat maps when comparing plots A and B (secondary future biased dominating) vs. C and D (time consistent) vs. E and F (primary present biased dominating) in Figure 4.3. We will use the simulated heat maps as the reference to trace the hypothetical concentration of different behaviors in CMW16 data.

As a comparison, we plot the heat maps of the NLS estimations of the β and δ parameters for all three CMW 16 subsets (1060, 452, and 230) in the same style as the 300 simulated subjects. For the full sample (n=1060) and the other two sub-samples (n=452, or 230),

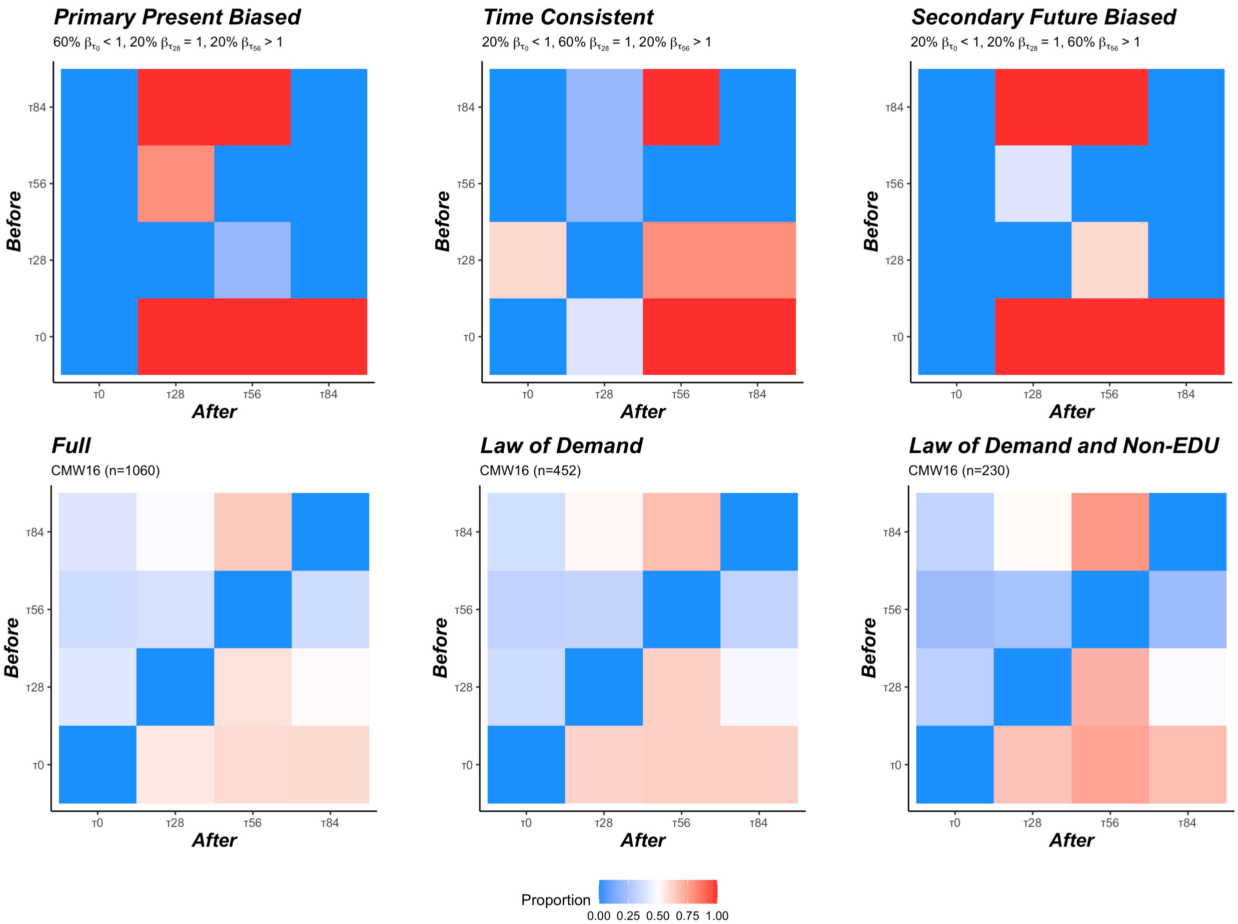

Figure 4.3: The Proportion of Increased β_τ

the observed proportion of the increased β is always surpassing 50 % when comparing the closer τ (before) models (τ_0, τ_0, and τ_{28}) to the further τ (after) models (τ_{28}, τ_{56}, and τ_{56}) respectively. The decreasing trend of the estimated δ is also dominating (surpassing 50%) for all samples, considering we focus on the counterpart of the β pixels. Figure shows this clear pattern in the lower triangular region (bottom right) of the heat maps in Figure . We can also find this coherent similarity of the color pattern in plots A and B from Figure . The similarity in color patterns indicates the subjects in CMW16 are likely to be dominated by the secondary future biased time preference rather than the reported primary present biased behaviors (Carvalho et al., 2016). Our finding explains the reason that there were an additional 68 subjects' time preferences became Future-Biased QHD rational (28.94 percent increase). Still, only 9 more subjects became Present-Biased QHD rational (3.83 percent increase) when the τ was relocated at later positions in CMW16 data (Echenique et al., 2020)[9]. More importantly, we observed denser and denser color pixels as more restricting assumptions were imposed on the CMW16 data. This tendency pinpoints that the failure to separate different behavior with the theoretical framework would lead to misconceptions at the aggregate level.

Table 4.10: Best-performed Model based on Different Criteria for Individual Subject (CMW16)

Criterion	Subjects	τ_0	τ_{28}	τ_{56}	τ_{84}
AIC	1060	156 (14.72%)	105 (9.91%)	418 (39.43%)	381 (35.94%)
	452	182 (40.26%)	167 (36.95%)	63 (13.94%)	40 (8.85%)
	230	116 (50.44%)	59 (25.65%)	33 (14.35%)	22 (9.56%)
AICc	1060	90 (8.49%)	48 (4.53%)	253 (23.87%)	669 (63.11%)
	452	118 (26.11%)	262 (57.96%)	48 (10.62%)	24 (5.31%)
	230	74 (32.17%)	118 (51.31%)	28 (12.17%)	10 (4.35%)
BIC	1060	146 (13.77%)	89 (8.40%)	394 (37.17%)	431 (40.66%)
	452	176 (38.94%)	181 (40.04%)	59 (13.05%)	36 (7.97%)
	230	113 (49.13%)	64 (27.82%)	32 (13.92%)	21 (9.13%)
RSS	1060	241 (22.74%)	189 (17.83%)	577 (54.43%)	53 (5.00%)
	452	245 (54.20%)	36 (7.96%)	101 (22.35%)	70 (15.49%)
	230	135 (58.70%)	7 (3.04%)	52 (22.61%)	36 (15.65%)

The result of the comparison between the simulations and the experimental data indicates that some of the model selection criteria (AIC, BIC, and RSS) are sensitive in terms of detecting the actual location of the Present Threshold (τ). Therefore, the AIC, BIC, and RSS have also been calculated for the individual subject level NLS models. The number and the percentage of the best-performed model based on three different criteria by the given subsets are presented in Table 4.10. The result suggests that τ_{56} has the most robust performance, mainly when we impose stronger assumptions on the samples. Overall, the secondary biased (τ_{28} or τ_{56}) models (either present biased or future biased) are the majority of the best-performed models among all subsets, which is different from the traditional

[9]Echenique et al. (2020) observed 235 strictly QHD rational time preferences among 1060 subjects from the CMW16 when considering τ_{t_0}.

assumption of τ_0.

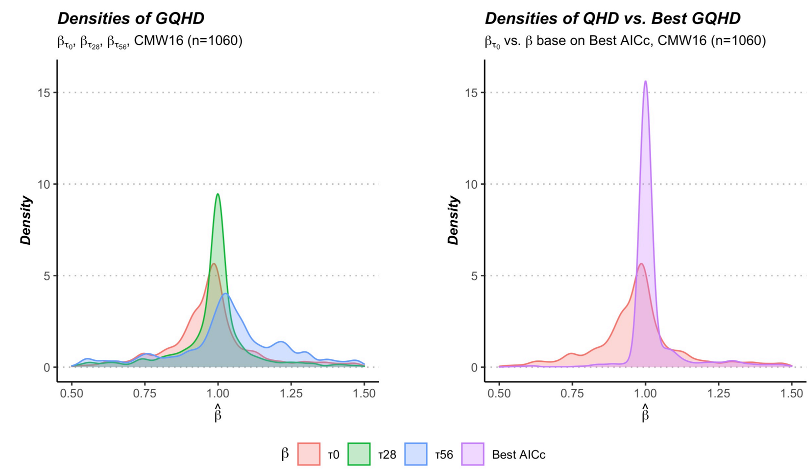

Figure 4.4: β_τs and β base on Best AICc (CMW16)

The best estimation of β_τ can be decomposed based on the best location of the perspective of the present according to the revealed choice of time preference at the subject level. Figure 4.4 (left) depicts the estimated beta distribution at the perspective's different underlying assumptions. The distribution is skewed to the right as the assumptions of the perspective present are pushed to the future. The further we assume the perspective present is, the more likely we consider the subjects as future biased. Then we merged all 1060 estimated β_τ (13.67% of the primary present behaved subject, 46.59% of secondary present behaved subjects, and 38.74% time consistent subjects) based on the BIC (Figure 4.4 (right)). The over distribution is shifted to the right for the best BIC β_τ compare the conventional setup β_{τ_0} with a one-sided Mann–Whitney U test's p-value of 0.0001 (alternative less than).

4.5.3 Robustness Check with the CARA Utility Specification

There are two major concerns of the estimation strategy for the CTB design, along with the non-linear least square approach. First, the NLS could create a bias for parameter estimations due to not adjusting for corner solutions. We have already addressed this concern with the simulations. We find that the tendency of change for both δ and β remains stable with or without the censoring. Another concern is that the CRRA utility functional form can drive the current result. Therefore, we specify the structural model with the constant absolute risk aversion (CARA) utility along with the additive discounting function in Eq. (4.11):

$$U_0(x_{t_i}, x_{t_i+k}) = \delta^t(-exp(-\alpha(x_{t_i} + \omega))) + \beta_\tau^{\mathbb{1}_{\{t_i \leqslant \tau < t_i+k\}}} \delta^{t_i+k}(-exp(-\alpha(x_{t_i+k} + \omega))) \quad (4.11)$$

With the CARA specification, the solution of the optimization takes:

$$x^*_{t_i} = \begin{cases} [\frac{k\ln(\delta)}{-\alpha} + \frac{\ln(1+r)}{-\alpha} + m]\frac{1}{1+(1+r)}, & \text{if } \tau < t_i \text{ or } t_i + k \leqslant \tau \\ [\frac{\ln(\beta_\tau)}{-\alpha} + \frac{k\ln(\delta)}{-\alpha} + \frac{\ln(1+r)}{-\alpha} + m]\frac{1}{1+(1+r)}, & \text{if } t_i \leqslant \tau < t_i + k \end{cases} \quad (4.12)$$

Table 4.11: CARA Estimations of All Three Subsets (CMW16)

CARA Models (Sbs)	τ_0	τ_{28}	τ_{56}	τ_{84}
$\hat{\beta}$	0.9496	1.0000	1.1340	
$\hat{\delta}$	0.9978	0.9975	0.9960	0.9975
$\hat{\alpha}$	0.0012	0.0012	0.0011	0.0012
BIC (1060)			✓	
$\hat{\beta}$	0.9596	0.9999	1.1078	
$\hat{\delta}$	0.9976	0.9973	0.9961	0.9973
$\hat{\alpha}$	0.0007	0.0008	0.0007	0.0008
BIC (452)			✓	
$\hat{\beta}$	0.9507	1.0022	1.1280	
$\hat{\delta}$	0.9973	0.9969	0.9956	0.9969
$\hat{\alpha}$	0.0006	0.0006	0.0005	0.0006
BIC (230)			✓	

✓ for the best-performed model based on BIC.

Andreoni and Sprenger (2012a) suggest that the parameter can be recovered by a two-step tow-limit Tobit (MLE) model (Eq. (4.12)). However, the Tobit model's specification cannot be estimated due to the multicollinearity problem when τ_{56} is assumed with the CMW16 design. Then, we employed the NLS approach again to estimate the CARA specification. We find the robust result of the increasing trend of β and decreasing trend of δ estimations as we push the position of τ from τ_0 to τ_{56} given the CARA utility. The $\beta_{\tau_{56}}$ is also the most desired model among all others for the BIC criterion for all three subsets of CMW16. Our results hold across the different functional forms of the utility (Table 4.11).

4.5.4 The Average of the perceptual Present

To find a revealing description of the central tendency, we calculate the average of the perceptual present ($\bar{T}_{\tilde{p}}$) among all of the subjects (Eq. (4.13)).

$$\bar{T}_{\tilde{p}} = \sum_{i=1}^{n-1} W_{\tau_{t_i}} (\frac{t_i + t_{i+1}}{2}) \tag{4.13}$$

where $W_{\tau_{t_i}}$ is a weight function that proportion of the dynamic inconsistency subject whose perception of the present is likely to be located at τ_{t_i}. We consider the middle point of the interval $[t_i, t_{i+1})$ as the optimal location of τ_{t_i} for simplicity. Notice that the Observational Dynamic Consistent subject has no observed $\bar{T}_{\tilde{p}}$.

We find that $\bar{T}_{\tilde{p}}(CMW16) = 47.59$, $\bar{T}_{\tilde{p}}(AS12a) = 65.44$, $\bar{T}_{\tilde{p}}(AS12b) = 22.4$, $\bar{T}_{\tilde{p}}(BCG18) = 26.91$, $\bar{T}_{\tilde{p}}(BHJ20) = 14.80$, $\bar{T}_{\tilde{p}}(CJK19) = 60.86$, and $\bar{T}_{\tilde{p}}(LSW18) = 21.68$ respectively. The results suggest that the observed kinky point of chosen consumption in these lab experiments happened far away from the conventional assumption that the time preference was discounted inconsistently immediately after the nearest to now award. Our empirical method shows that it is more precise to discover how far could the perception of the present lasts rather than to question how soon it ends (Balakrishnan et al., 2020).

4.5.5 Dynamic Inconsistent in Perceptual Time Scale

Ignorance of the perceptual sense of the present would lead to a misleading conclusion of the directions of the time inconsistency when we evaluate the data in depth. We present the final breakdown of different time preference behaviors based on the AICc criteria in Table 4.12. For the CMW16 experiment, despite the small proportion of dynamic consistency behavior, the percentage of future bias (both primary and secondary) adds up to 20.19%, which is higher than 8.30% of the present bias (both primary and secondary). We conclude that future bias time preference dominates the population in the CMW16, which is different from their study's reported present bias behavior (Carvalho et al., 2016). Similarly, the proportion of the observed perceptual present biased behavior in AS12 is 40.21%, which is higher than the 39.18% of future bias behavior. When time inconsistency is measured on a procreational scale, we also reached a different conclusion from the original discovered slightly future biased behavior in Andreoni and Sprenger (2012a). We applied this approach to all seven experimental data sets and found the total amount of present biased subjects (12.01%) is slightly less than the observed future biased (15.74%) subjects. When time is considered on a perceptual scale and explored in more detail, our results are not consistent with the discovery of the previous studies.

Table 4.12: AICc Classification of GQHD Models

Study	AS12a	AS12b	CMW16	BCG18	LSW18	CJK19	BHJ20	Percentage
Dynamic Consistency	20	45	758	605	790	70	363	72.23%
Primary Present Bias	2	16	47	79	22	49	38	6.89%
Primary Future Bias	6	11	30	39	36	9	20	4.11%
Secondary Present Bias	37	0	41	38	16	22	34	5.12%
Secondary Future Bias	32	8	184	63	50	51	39	11.63%
Subjects	97	80	1060	824	914	201	494	3670(100%)

4.6 Mixed Inconsistency

4.6.1 The General Beta-delta Discounting Model

Jackson and Yariv 2014 developed an alternative method to elicit dynamic inconsistency in time preference. Their specification further distinguishes mixed inconsistency from time consistent, present bias, and future bias. Other time preference research (Schafer, 2016; Brocas et al., 2018; Abbring et al., 2018; Samek et al., 2021) has its unique structure of understanding the unregulated time inconsistency patterns as measurement error (Gillen et al., 2019). We address the mixed inconsistency concern in time by relaxing the assumption that observed time preference from a CTB must follow the equation (Eq. (4.2)). To differentiate time inconsistency from other time preference types, we respecify the GQHD model into the following format:

$$\sum_{t=t_0}^{t_n} \beta^{\mathbb{1}_t} \delta^t u(x_t) \tag{4.14}$$

We call this model the General Beta-delta discounting Model (GBDD) with a relaxed assumption. The $\mathbb{1}_t$ is an indicator function that identifies if β is presented at date t in the summation.

Table 4.13: Discounting Patterns with GBDD

	t_0	t_1	t_2	t_3	t_4	t_5
ED	δ^{t_0}	δ^{t_1}	δ^{t_2}	δ^{t_3}	δ^{t_4}	δ^{t_5}
QHD	δ^{t_0}	$\beta\delta^{t_1}$	$\beta\delta^{t_2}$	$\beta\delta^{t_3}$	$\beta\delta^{t_4}$	$\beta\delta^{t_5}$
GQHD	δ^{t_0}	δ^{t_1}	δ^{t_2}	$\beta\delta^{t_3}$	$\beta\delta^{t_4}$	$\beta\delta^{t_5}$
GBDD	δ^{t_0}	δ^{t_1}	$\beta\delta^{t_2}$	δ^{t_3}	$\beta\delta^{t_4}$	δ^{t_5}

Suppose we have six arbitrarily given time spots t_i ($i \in \{0, 1, 2, 3, 4, 5\}$) where we observe the time preference in the CTB experiment. The Eq. (4.14) can represent any discounting pattern in Table 4.13; therefore, the GBDD model is most general in this cluster of quasi-hyperbolic discounting models. With the discounting function $D(t) = \beta^{\mathbb{1}_t}\delta^t$ defined by GBDD model, the optimal demand function (updated from GQHD) of sooner consumption is:

$$x_t^* = \begin{cases} \frac{(\delta^k(1+r))^{\eta^{-1}}-1}{1+(1+r)(\delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\delta^k(1+r))^{\eta^{-1}}}{1+(1+r)(\delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } \frac{D(t+k)}{D(t)} = \delta^k \\ \frac{(\beta\delta^k(1+r))^{\eta^{-1}}-1}{1+(1+r)(\beta\delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\beta\delta^k(1+r))^{\eta^{-1}}}{1+(1+r)(\beta\delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } \frac{D(t+k)}{D(t)} = \beta\delta^k \\ \frac{(\beta^{-1}\delta^k(1+r))^{\eta^{-1}}-1}{1+(1+r)(\beta^{-1}\delta^k(1+r))^{\eta^{-1}}}(\omega) + \frac{(\beta^{-1}\delta^k(1+r))^{\eta^{-1}}}{1+(1+r)(\beta^{-1}\delta^k(1+r))^{\eta^{-1}}}(m), & \text{if } \frac{D(t+k)}{D(t)} = \beta^{-1}\delta^k \end{cases} \tag{4.15}$$

Compared to Exponential Discounting (ED), Quasi-hyperbolic Discounting, and General Quasi-hyperbolic Discounting, the relaxed version of the General Belta-delta discounting model, can adapt to any possible discounting pattern, even if the laboratory observation of the subject does not fit any existing known rationalization time preference. The purpose of our most general functional form of discounting pattern is not to further explain a rational behavior in time preference choice; instead, it is to sort out all other irregular discounting patterns in the observations. In other words, the GBDD is used to classify the random noise into a new category, i.e., the mixed inconsistent. More generally, Time Consistent is equivalent to exponentially discounting; Present Bias and Future Bias both consider the Primary Present and the Secondary Present; the Mixed Inconsistent represents all other discounting patterns in the GBDD model.

4.6.2 The AICc Classification

We fitted the models with the new specification of the GBDD model (Eq. (4.15)) to all seven data sets at the individual subject level. We used the NLS approach (Andreoni

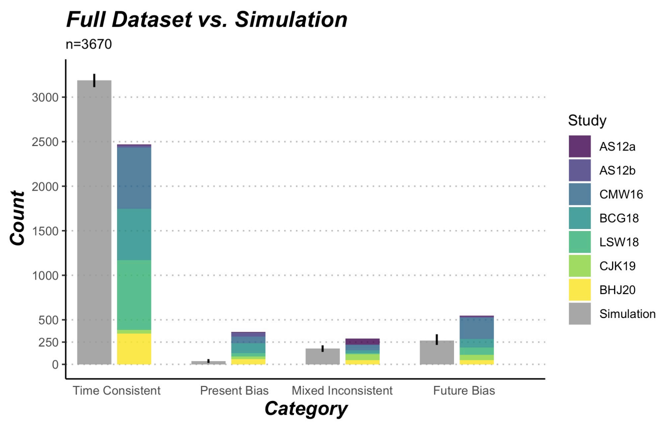

Figure 4.5: Count of Subjects' Time Consistency Category.

and Sprenger (2012a)) and adjusted to the GBDD model for all 3670 participants' choices of CTB experiments. Then we classified the individuals' CTB choices into four categories, Time Consistent, Present Bias, Mixed Inconsistent, and Future Bias. We exhausted all of the possible permutations of specifications of GBDD models for all 3670 subjects' data and classified their revealed time preference based on the AICc. The AICc method has the least mean and variance in classification error (See details in Appendix C.3). To compare the difference between the human subjects and the random selections, we simulated 100,000 subjects (using the relative proportion of each of the seven research) that made uniformly random decisions on the CTB for the seven designs. Next, we used the bootstrapping approach to repeatedly select 3670 out of 100,000 simulated subjects. The bootstrapping mean and range are reported as the gray bars in Figure 4.5 compared to the colored bars as the results of the human subjects. We point out four general observations: first, we confirm human subjects' choices are substantially different from random; second, the time consistent (67.30%) choices are the most common time preference type from the CTB experiments; third, compared to the present bias (9.95%), there are more future bias (14.85%) subjects in the data set; fourth, there are some subjects (7.90%) make mixed inconsistent revealed time preference decisions.

4.6.3 The Departure from the Exponential Discounting

Different from the GQHD model, the specification of the beta parameters in the GBDD does not follow a conventional pattern (Table 4.13). In the previous section, we discussed the GBDD model is fitted, and the best model is selected with AICc at the individual level. So each subject has a unique estimated beta parameter ($\hat{\beta}$) in the General Beta-delta Discounting model.

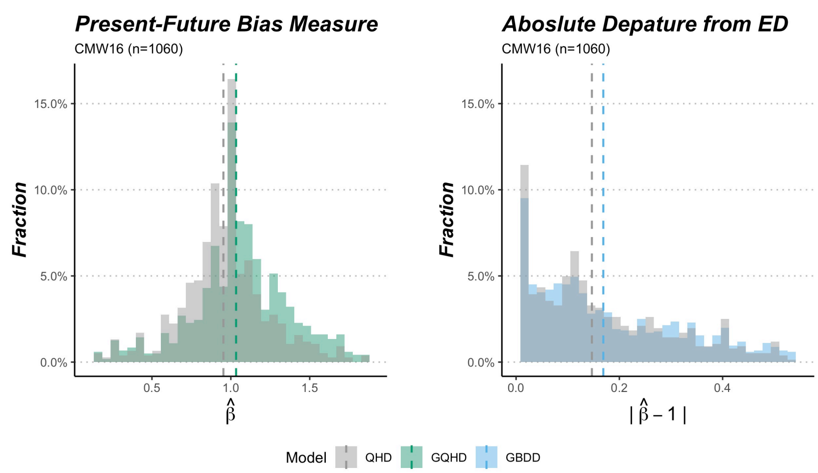

Figure 4.6: The Estimated β of the GQHD and the GBDD Models are based on AICc.

In the previous section, we discussed the GBDD model is fitted, and the best model is selected with AICc at the individual level. So each subject has a unique estimated $\hat{\beta}$ parameter in the General Beta-delta Discounting model. To interpret the magnitude of the beta, we consider the sign of the $\hat{\beta}$ from the GQHD model as the direction of the bias in revealed time preference, i.e., present bias is less than 1 and future bias if greater than 1. However, the sign of $\hat{\beta}$ in the GBDD model has no meaning due to the complexity of the mixed inconsistency behavior. Therefore we took the absolute value of the $\hat{\beta}$ from the GBDD model and then interpreted its magnitude as the absolute departure from the exponential discounting, where $\beta = 1$.

We plot the distribution of the estimated bias parameters in Figure 4.6. When comparing the GQHD model, we find the QHD model generally overestimates the intensity of present bias (Figure 4.6 (left)). The vertical dashed lines are the median values of $\hat{\beta}$. This finding is consistence with Imai et al., 2021. On the other hand, comparing the estimations in GBDD with QHD, we find the QHD model tends to underestimate the departure from the exponential discounting (Figure 4.6 (right)). All combined, this empirical evidence suggests the misspecification problem of the QHD model of ignoring the proception of the present and misinterpreting the mixed inconsistency in the revealed time preference.

4.6.4 Heterogeneity in Subjects

It is common to consider that individual's revealed time preferences are different due to the subjects exhibiting some heterogeneity in characteristics. The related attributes among subjects that affect revealed time preference are age, risk, income, gender, and health. (Lawrance, 1991; Barsky et al., 1997; Becker and Mulligan, 1997; Chesson and Viscusi, 2000; Bishai, 2004; Munasinghe and Sicherman, 2005; Scharff and Viscusi, 2005; Chao et al., 2009; Finke and Huston, 2013; Lawless et al., 2013; Kuhn, 2014; Foltyn and Olsson, 2021).

Given the richness of our acquired dataset, we assess the heterogeneity in revealed time preference among the 3670 subjects of two main aspects: (1) perception of the present and (2) the consistency of time discounting. We measure the heterogeneity in time preference corresponding to the six characteristics available in our acquired data for the studies we included in our research. The six characteristics are age (from AS12a, BHJ20, CJK19, CMW16, LSW18; n=2706), gender (AS12a, BHJ20, CJK19, CMW16, LSW18; n=2734), risk situation (from AS12b; n=480)[10], financial education (from LSW18; n=201), hunger condition (from CJK19; n=201), and payment time (from BHJ20; n=494).

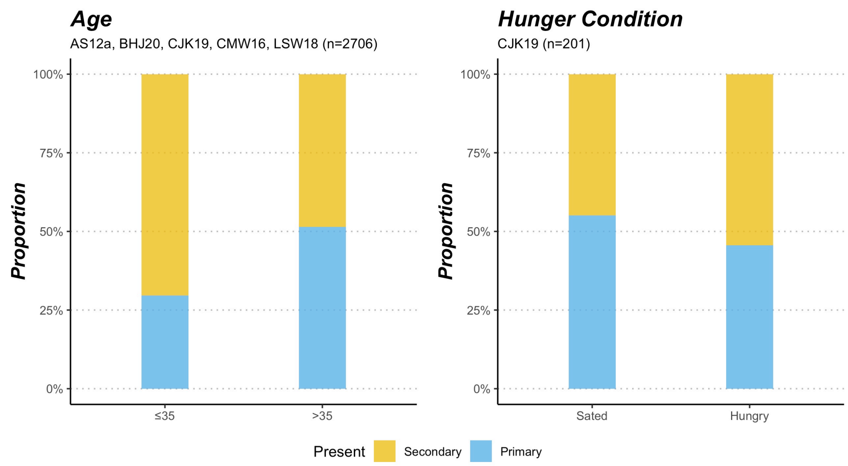

Figure 4.7: Age, Hunger, and Perception of Present.

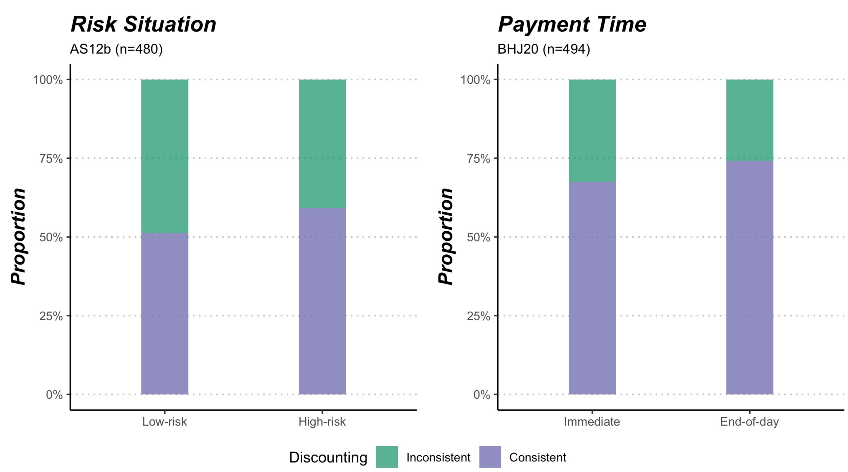

Figure 4.8: Risk, Waiting, and Time Consistency.

Figure 4.7 and Figure 4.8 summarize the most significant observed difference among subjects' heterogeneity. We discuss the perception of the present at the beginning.[11] First, we find that younger adults have a longer perception of nowness, and a more significant

[10]There are 80 subjects' experimental data available, and each subject experienced 6 different risk condiction in AS12b, so we consider 80*6 i.e., n=480.

[11]We only consider the primary and secondary specifications in the GQHD model and report the best-performed model based on AICc.

proportion (70.38%) of the adults aged less or equal to 35 have a present secondary behavior compared to the elderly adults (48.54%; Figure 4.7 (left)). Second, for the 201 subjects from the CJK19, we separate the data by the hunger condition and find out the perception of the present is shortened in a sated condition compared to hunger, where 55.10% of the subjects' time preference fits the primary model in sated condition compare to 45.61% in hunger condition (Figure 4.7 (right)). Then we examine the consistency measurement.[12] Third, we discover that subjects are more likely to make time inconsistency decisions in risky environments. There is 59.17% of subjects make time consistent decisions under high-risk conditions; however, this percentage decreased to 51.25% under low-risk conditions (Figure 4.8 (left)). Lastly, the inconsistent time preferences in the BHJ20 (including present bias, future bias, and mixed inconsistency) tend to be triggered when the payment time is immediate, with a proportion of 32.54%. On the contrary, the inconsistency proportion in end-of-day treatment is 25.79% (Figure 4.8 (right)).

4.7 Conclusion and Discussion

We developed an extension of the QHD model and added the concept of perceptual time scale to the Qausi-Hyperbolic Discounting function. By adjusting for the perceptual present feature in time preference, we developed the QHD into the General Qausi-Hyperbolic Discounting model. With the application of our GQHD model, the results point to two conclusions. First, the perception of the present is a critical component of measuring inconsistency in time preference. Disregarding the importance of the perceptual dimension of time would lead to a misleading conclusion about the time inconsistency. The secondary future bias behavior could be misleadingly treated as the primary present bias behavior. The secondary

[12] We consider the ED specification as time consistent and all other specifications in GBDD as time inconsistent.

future bias behavior could be misleadingly treated as the primary present bias behavior. Simultaneously, the secondary present bias behavior could be misleadingly treated as the primary future bias behavior. Simply assuming time preference discounted inconsistency only after the immediate payoff without any hypothetical test, we either fail to observe the inconsistency or fail to distinguish the present bias from the future bias. The misconception of the direction of the bias could sufficiently undermine the conclusions and challenge the validity of the policy application in time preference studies. If a policy is intended to address the primary present bias issue; however, it is applied to the secondary future bias population; this policy may further intensify future bias behavior.

Secondly, with the introduction of a flexible parameter β_τ in the empirical work, we were able to detect the length of the perception of the present at both aggregate and individual levels. The results indicate the perceptual present reaches far beyond immediate soon (t_0). Constraining the β parameter at its initial place limits the ability of the empirical model to fit the data. Additionally, it constrains the QHD model to explain the time preference behavior more precisely. The GQHD model provides a more insightful interpretation of the time inconsistency and additional information about when it happens.

By adding a perception dimension of time in the Quasi-Hyperbolic Discounting, we provide a more general method to differentiate the observed primary present bias from its quasi-twin secondary future bias in time preference. We answered the research question of "what the present is" and then discussed how the observed bias of the time preference changed toward this discovered perceptual present. Our findings advocate a necessity of consideration of the perceptual present for future dynamic inconsistency in time preference studies. With an even more relaxed assumption in the GBDD model, we further sort out the mixed inconsistent time preference behavior from the laboratory data. We found some discounting pattern that is not rationalizable from any previous model's specifications. The

GBDD model provides the most adoptive parametric method to study the cluster of all variations of quasi-hyperbolic discounting behavior. However, our conclusion has its constraint to a certain extent.

First, the possible locations of the structural model in our identifications heavily rely on the design of AS12a, AS12b, CMW16, BCG18, LSW18 CJK19, and BHJ20. All of these studies focus on time inconsistency for the monetary incentive. To have a more versatile measurement of how late the present could last and even further generalize our discussion beyond monetary incentive, we need an adaptive budget set design to resolve the problems of preventing corner solutions, then detecting the present beyond money(Imai and Camerer, 2018). More importantly, the current literature on time preference in economics is present-biased-focused. The research which finds evidence of the existence of future biased behavior either challenges the design fails to prevail the myopic behavior (Shu, 2008), or provides little explanation of the abnormal finding (Aycinena et al., 2015; Corbett, 2016; Aycinena and Rentschler, 2018). On the other hand, some studies in both economics and phonology emphasize the importance of future bias (Dougherty, 2015; Greene and Sullivan, 2015; Dorsey, 2017). Suppose a new design can successfully control the uncertainty of future income, and the higher interest rates are positively correlated with the probability of triggering perceptual future bias. We can link our GQHD model to early over-saving and retirement research studies to find some additional practical applications (Diamond and Köszegi, 2003; Salanié and Treich, 2006; Zhang, 2013).

Second, we find the optimal duration for the present in the experiment. Still, our finding was not intended to explain if the observed future bias is caused by the risk-averse driven behavior concerning the uncertainty of getting the reward after the experiment (Andreoni and Sprenger, 2012b). Additionally, we find this optimal duration based on our modified version of the QHD specification. We chose the model mainly to keep the beta parameter in a discrete setting, then compare our results with the findings in related literature. Other models can also identify the turning point in the discounting function (for instance, Loewenstein and Prelec 1992). In other models, it is also compelling to see the relationship between the perceptual present and the reversal point of the discounting factor.

Third, the criteria and visualization methods we used successfully identify the optimal location of the present threshold and the concentration of different behaviors in the population. However, we are uncertain how accurate and sensitive these methods are, especially for mild inconsistency in time preference.

Lastly, This study is a complement to “how soon is now” and “now or as soon as possible” (Glimcher et al., 2007; Balakrishnan et al., 2020). It’s necessary to conduct a conurbation of our studies to comprehend if the unobserved present bias is caused by the present being too soon or if we fail to consider present could last longer on a perceptual scale. We will leave these concerns to future work.

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Appendices

Appendix A

Appendix to Chapter 2

A.1 Battle 1 versus Battle 2

We provide more evidence for the treatment effect on discouragement. In Figure A.1, we show the relationship between efforts 1 and 2 for each player on an individual contest level. We take the relative weight of battles 1 and 2 into consideration. We use the weight ratio between battle 1 and battle 2 as the effort ratio of the first two battles. Compared to the other two treatments, the proportion of effort 2 that is larger than the effort time ratio between weight 1 and 2 is sufficiently smaller in the weight 2:1 treatment. Such observation is consistent regardless of the battle 1 result. More specifically, in both Weight 1:1 and Weight 1:2, more than 60% of players have the bid ratio of the first two battles surpass the weight ratio, which is larger than in the Weight 2:1 treatment. This behavior confirms our conjecture that the leader is incentivized to win the contest with a minimum cost given a relative leading gap after the initial battle. The results strengthen the finding of the discouragement effect in Figure 2.3 and Figure A.2 in Appendix A.2.

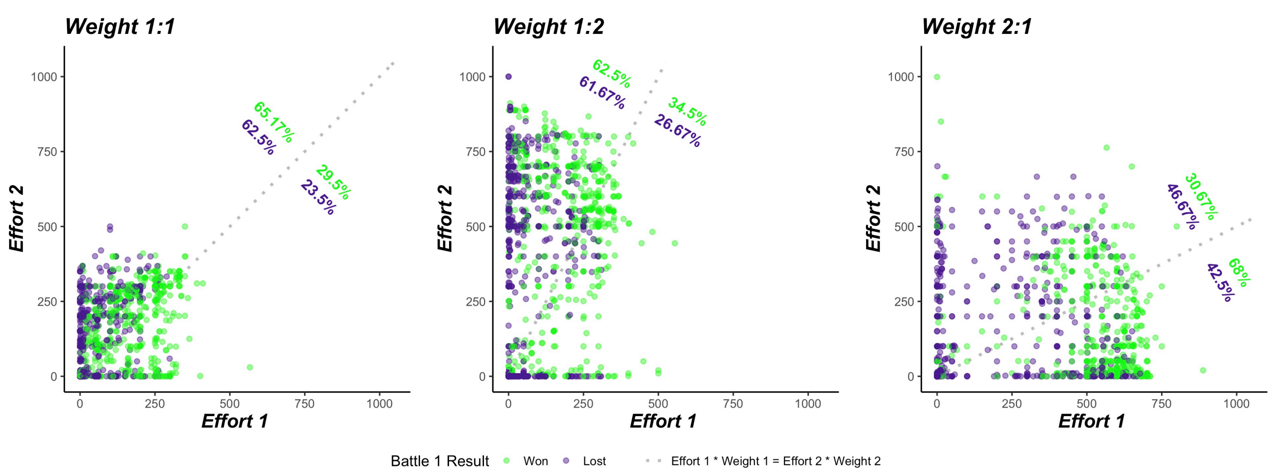

Figure A.1: The Concentration of Battle 1 and Battle 2

A.2 Distribution of Efforts

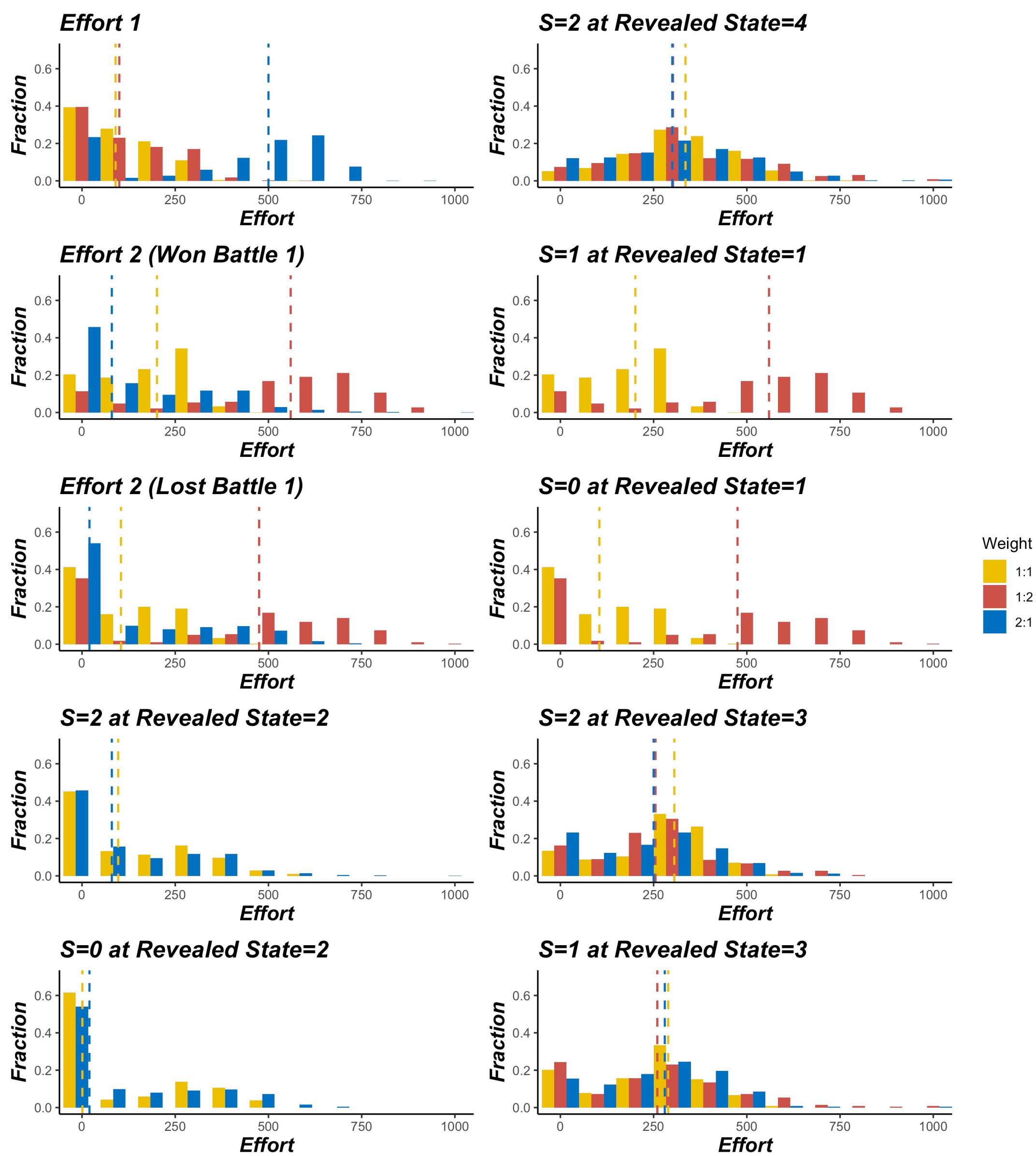

Figure A.2: Distributions of Effort for Given Battle and State.

A.3 Effort 2 based on Effort 1

We report battle 2 behavior (effort 1 based) among treatments. Table A.1 and Table A.2 are robustness checks for Table 2.7.

Table A.1 shows that the discouragement of losing battle 1 mainly comes from the player whose effort 1 surpasses its median. For the player who does not exert sufficient effort, the loss is psychologically expected. In other words, the loss is planned.

Table A.1: Random Effects Panel on Efforts in Battle 2

Treatment	**Weight 1:1**		**Weight 1:2**		**Weight 2:1**	
Effort 1	**⩾Median**	**<Median**	**⩾Median**	**<Median**	**⩾Median**	**<Median**
Constant	207.87 (19.35)	128.30 (12.25)	497.09 (41.89)	526.57 (73.55)	177.07 (-45.75)	246.06 (22.24)
Lost Battle 1	-24.33*** (8.37)	10.49 (7.82)	-91.38*** (28.62)	-131.02*** (32.15)	-45.75*** (12.69)	-36.11 (23.24)
Trend	-0.63 (2.05)	-2.15 (2.10)	2.50 (3.09)	5.22 (4.69)	-5.61** (2.65)	-5.99*** (2.11)
Observations	600	600	604	596	617	583

The dependent variable is the Effort 2 for each corresponding treatment.
We separate the data into two subsets depending on Effort 1.
The random effects model is structured at the subject level.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for p < 0.1 ** for p < 0.05 *** for p < 0.01

Results in Table A.2 confirms three major results. Firstly, it finds the finding in Table A.1, in which the source of strategic momentum is the players who bid seriously in battle 1. Secondly, the utility of winning effect who refused to give up when in a non-leading position in battle 2 mainly comes from the player who bid low in battle 1. Thirdly, the utility of winning positively affects the non-leading position; however, this effect is only strong on the low bidders of battle 1.

Table A.2: Random Effects Panel on Efforts in Battle 2 (Heterogeneity)

Treatment	**Weight 1:1**		**Weight 1:2**		**Weight 2:1**	
Effort 1	**⩾Median**	**<Median**	**⩾Median**	**<Median**	**⩾Median**	**<Median**
Constant	218.02 (18.94)	124.67 (11.18)	495.23 (50.68)	468.73 (79.99)	171.97 (17.25)	224.21 (23.77)
Lost Battle 1	-56.69*** (19.48)	1.92 (6.45)	-109.87** (47.50)	-120.25*** (19.49)	-44.27*** (16.40)	-52.09** (20.70)
Utility of Winning	-32.11*** (6.35)	11.15 (14.22)	8.56 (30.66)	188.96** (73.67)	13.69 (22.19)	62.47*** (23.13)
Lost Battle 1 × Utility of Winning	89.01* (39.95)	22.56** (11.25)	37.88 (79.67)	-42.96*** (76.26)	-3.30 (21.98)	43.56*** (24.76)
Trend	-0.63 (1.88)	-2.04 (2.04)	2.39 (3.13)	-5.12 (4.93)	-5.62** (2.70)	-5.90*** (2.21)
Observations	600	600	604	596	617	583

The dependent variable is the Effort 2 for each corresponding treatment.
We separate the data into two subsets depending on Effort 1.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance of the Lost Battle 1 × Utility of Winning is the testing result of the Utility of Winning effect on the Lost Battle 1.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

A.4 Decaying in Treatment Effect based on Effort 1

We present the robustness check for Table 2.10 when players exert effort in battle 1 differently in Table A.3. Firstly, we find a very strong weight effect in both efforts 1 and 2 despite the differences in battle 1 efforts. Secondly, we find evidence suggesting the path-independent strategic momentum for both high and low effort in battle 1. Lastly, the decaying treatment effects are presented for both high and low data sets; the treatment effects are entirely removed at State (2,2) for all three treatments. A noticeable observation is that non-leaders behavior is distorted more severely in the Weight 2:1 treatment in State (2,0) compared to the Weight 1:1 treatment.

Table A.3: Decaying in Treatment Effect (High and Low Effort 1)

Effort 1	**State (0,0)**	**State (1,0)**		**State (1,1)**	**State (2,0)**		**State (2,1)**		**State (2,2)**
⩾Median	Non-leader	Leader	Non-leader	Non-leader	Leader	Non-leader	Leader	Non-leader	Non-leader
Weight 1:1	179.74	167.73	234.50	220.31	178.06	174.97	268.56	226.62	263.35
	(15.29)	(17.96)	(12.89)	(9.39)	(13.03)	(15.91)	(17.60)	(28.13)	(24.63)
Weight 1:2	41.11**	311.29***	251.54***				3.61	37.57	19.84
	(18.40)	(37.03)	(28.02)				(15.38)	(39.34)	(29.61)
Weight 2:1	379.82***				6.92	-56.24**	-54.06***	-58.77	-7.50
	(14.82)				(13.29)	(24.80)	(17.96)	(35.80)	(29.50)
Trend	1.37	6.24**	-22.03***	1.91	-8.53***	-10.83	4.67	4.16	29.49***
	(1.36)	(2.77)	(6.14)	(4.27)	(2.68)	(7.74)	(5.94)	(4.66)	(7.04)
Observations	1821	881	323	221	736	260	474	370	404

Effort 1	**State (0,0)**	**State (1,0)**		**State (1,1)**	**State (2,0)**		**State (2,1)**		**State (2,2)**
<Median	Non-leader	Leader	Non-leader	Non-leader	Leader	Non-leader	Leader	Non-leader	Non-leader
Weight 1:1	54.45	110.28	160.28	168.67	195.77	171.21	255.27	233.51	273.76
	(15.78)	20.09	(18.98)	(15.55)	(34.50)	(13.25)	(22.49)	(25.09)	(16.56)
Weight 1:2	3.77	384.65***	244.31***				-18.47	-35.98	12.60
	(6.27)	(54.76)	(33.60)				(27.19)	(42.82)	(56.60)
Weight 2:1	175.03***				38.00	55.64***	-2.99	27.17	-42.85
	(18.89)				(29.40)	(18.57)	(37.59)	(34.90)	(37.62)
Trend	-5.23	2.95	-7.64**	7.50***	-7.37	-10.68***	7.70	5.03	23.45***
	(2.50)	(7.60)	(3.63)	2.45	(10.25)	(2.60)	(5.98)	(5.66)	6.20)
Observations	1779	319	877	241	233	709	349	453	382

The dependent variable is the effort for each corresponding state and position.
We separate the data into two subsets depending on Effort 1.
The Weight 1:1 is the constant in the regression.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

A.5 Weight and Intensity

We also investigate if the weight affects the relative behavior when the intensity of a battle changes. The results are presented in Figure A.3. We find the consistency in behavior that the effort tends to have a more U-shaped distribution when the battle gets more intense regardless of the weight assignment of the current battle.

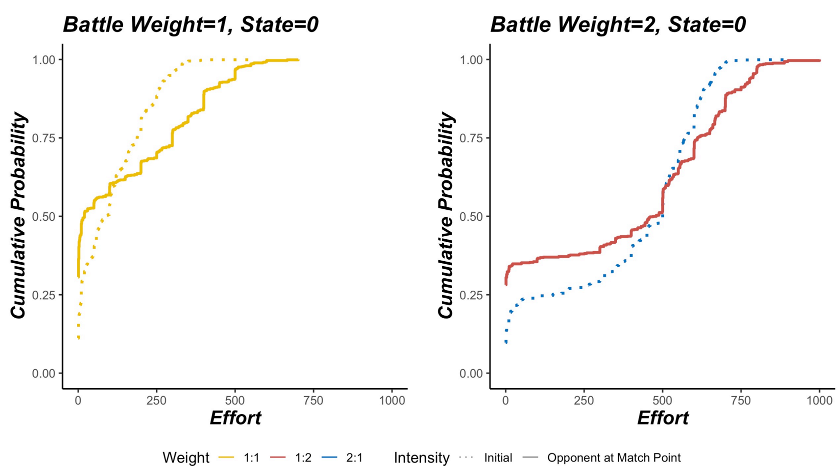

Figure A.3: Weight and Match Point Oriented Intensity.

Compared to the initial battle, the high bids get higher, and the low bids get lower when the opponent reaches the match point. This phenomenon can be observed in both weighted 1 and weighted 2 battles. The intensity-corresponding behavior of weighted 2 battle is not directly observed from one treatment. Therefore we compared the bid in the state (0:0) of Weight 2:1 and state (1:0) of Weight 1:2 treatment.

Appendix B

Appendix to Chapter 3

B.1 Elicitation of Risk, Ambiguity, and Loss Preference

We employed three modified versions of preference surveys from Shupp et al. 2013 to elicit risk, ambiguity, and loss preferences. Here we show the instruction of the loss preference as an example, given the fact that we find the loss aversion partially explained the reduction in the bid for all post-battle 1 when the current $s = 0$ (Table 3.14). Unlike the loss preference survey, the omitted risk and ambiguity surveys' choices are in the gain domain. In expectation, a risk-neutral subject will have an average payoff of \$5.13 from the three surveys. On average, participants' earnings from the three surveys are \$3.81. Therefore, our average representative subject has a risk-seeking preference.

We present a sample of the Loss aversion survey (Table B.1), and for more detailed instructions, please check the Supplementary Material.

Decision

- For each question in the following table, please choose or **B** as your preferred opinion.
- There are 12 questions in total, and please consider each of them as a separate decision.

Earning

- We will **select one out of 12 questions at random** (roll a 12-sided dice) to calculate your earnings at the end of the experiment.
- , For example, suppose question 6 (see below) is selected by the 12-sided dice to calculate your earnings:
  - If your choice is option A (for question 6), there is a 50% of chance we pay you $3.00 and a 50% of chance we deduct $3.00 from your participation reward (flip a coin).
  - If your choice is option B (for question 6), we will pay you $0.00.

Table B.1: A Sample Survey

	Option A	Option B
Question 1	**$3.00** with 50% chance and **-$0.50** with 50% chance.	**$0.00** for sure.
Question 2	**$3.00** with 50% chance and **-$1.00** with 50% chance.	**$0.00** for sure.
Question 3	**$3.00** with 50% chance and **-$1.50** with 50% chance.	**$0.00** for sure.
Question 4	**$3.00** with 50% chance and **-$2.00** with 50% chance.	**$0.00** for sure.
Question 5	**$3.00** with 50% chance and **-$2.50** with 50% chance.	**$0.00** for sure.
Question 6	**$3.00** with 50% chance and **-$3.00** with 50% chance.	**$0.00** for sure.
Question 7	**$3.00** with 50% chance and **-$3.50** with 50% chance.	**$0.00** for sure.
Question 8	**$3.00** with 50% chance and **-$4.00** with 50% chance.	**$0.00** for sure.
Question 9	**$3.00** with 50% chance and **-$4.50** with 50% chance.	**$0.00** for sure.
Question 10	**$3.00** with 50% chance and **-$5.00** with 50% chance.	**$0.00** for sure.
Question 11	**$3.00** with 50% chance and **-$5.50** with 50% chance.	**$0.00** for sure.
Question 12	**$3.00** with 50% chance and **-$6.00** with 50% chance.	**$0.00** for sure.

B.2 Distribution of Efforts

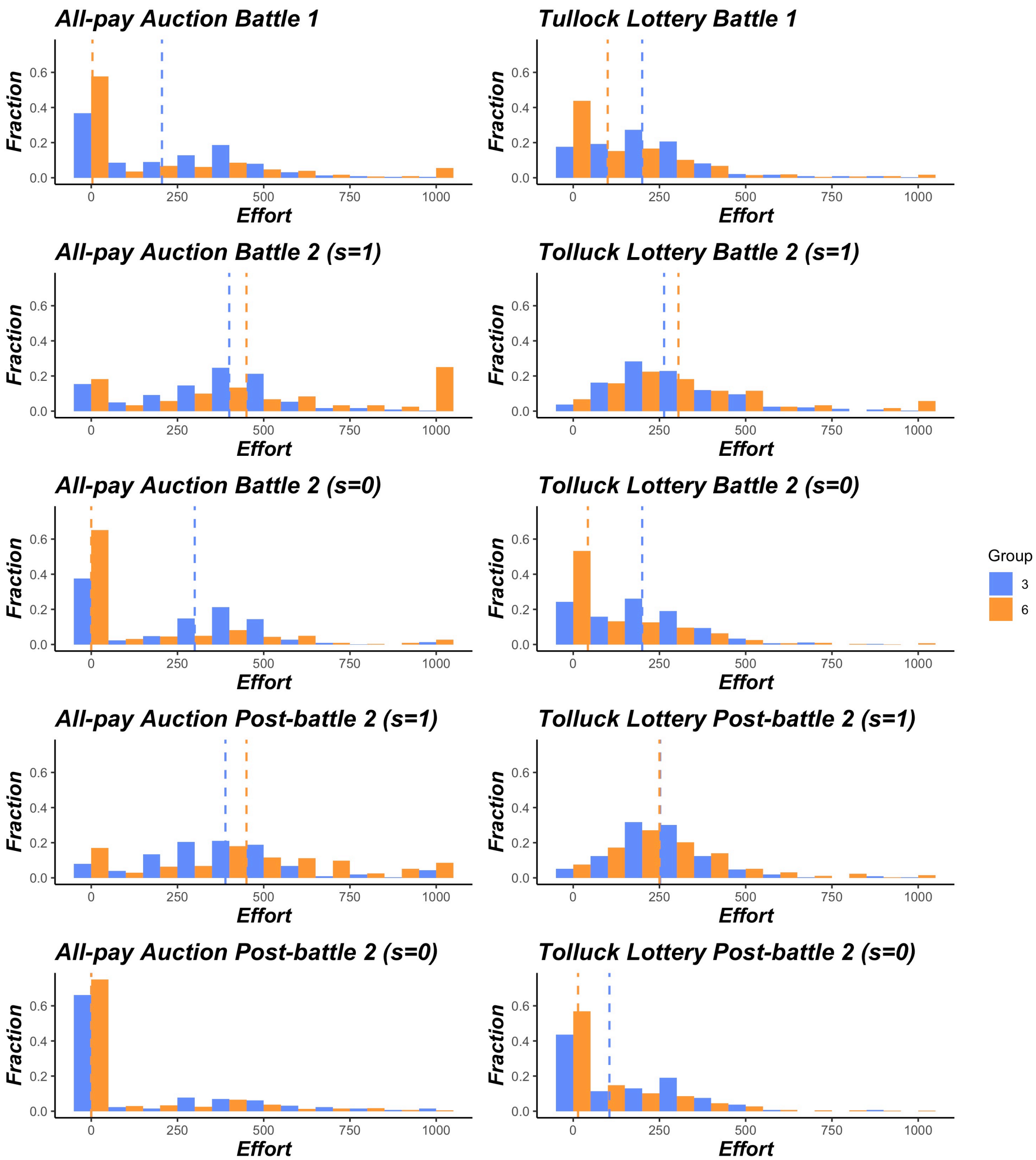

Figure B.1: Distributions of Effort for Given Battle and State.

B.3 Strategic Momentum in Battle 2

In Figure B.2, we present the average difference in effort 2 between the winner and loser of battle 1. This is the supplement to Table 3.3 and Table 3.7 on the treatment difference in the New Hampshire effect. We took all the possible differences between the winner's and losers' bids within the same contest, then calculated the mean value for each treatment. We can observe clearly that the New Hampshire effect is more severe when increasing the player number. This difference is more observable when changing the CSF from the Tullock lottery to the all-pay auction.

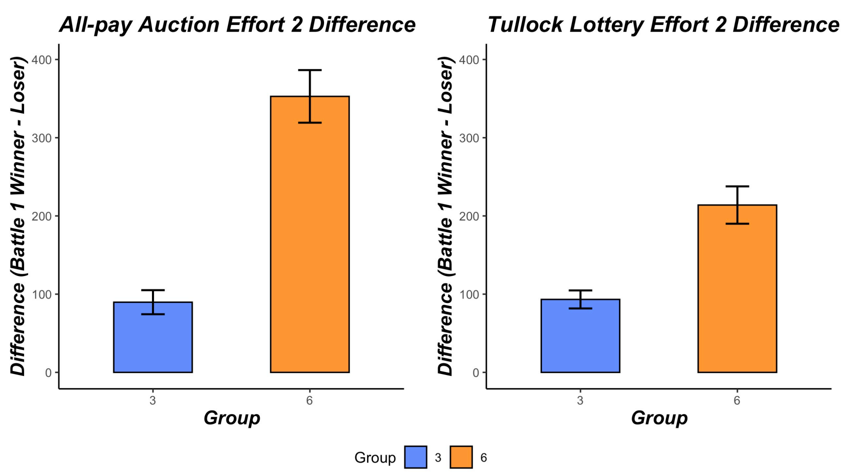

Figure B.2: The Average Differences in Effort 2

B.4 Optimal Mechanism Design

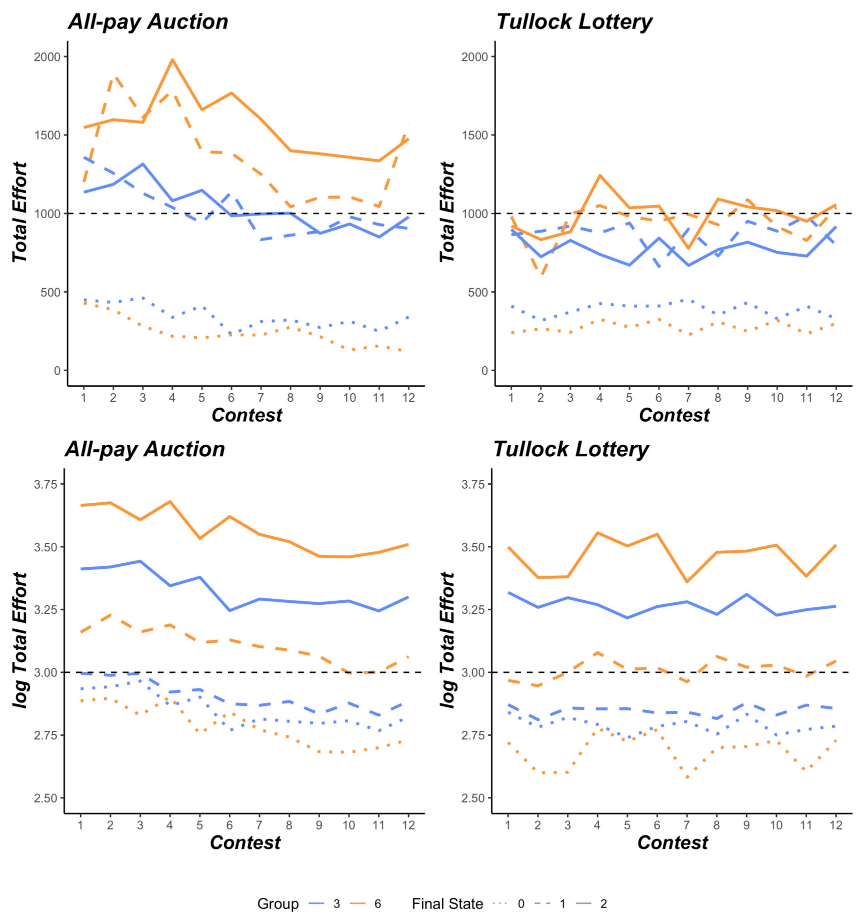

Figure B.3: The Trends of Total Efforts.

In this part, we present additional plots and tables and summarize the general guidance for optimal mechanism designs according to the findings from this study. Firstly, mainly the winner spends the most total effort in a contest (Figure B.3 and Table B.2). Secondly, as the group size increases, the winner and the almost winner's total effort increases; however, it decreases for the player with the final $s = 0$ (Figure B.3). Thirdly, as n grows, the average battle total effort and the average contest total effort rise, while the average player's effort falls.

Table B.2: The Proportion of the Objective Function is Maximized in the (Condition)

Treatment		All-pay ($n = 3$)	Tullock ($n = 3$)	All-pay ($n = 6$)	Tullock ($n = 6$)
Player's Battle	($s = 1$)	53.75%	38.33%	49.17%	31.67%
Player's Battle	(Battle 1)	25.00%	43.75%	24.17%	49.16%
Battle's Total	(Battle 2)	47.08%	43.75%	46.67%	43.33%
Player's Total	(Contest Winner)	81.67%	62.08%	67.50%	58.33%

Additionally, the best-performed player's battle is likely to happen when $s = 1$ in an auction game and when it is battle 1 in a lottery game. Then, the best-performed battle is likely to happen in battle 2 in both CSFs (Table B.2). Overall the payer in individual battles, the individual battle total, and the player in the contest have better performance (more average effort) when increasing both n and r (Table 3.6).

Table B.3: Maximization of Each Objective Function

Treatment	All-pay ($n = 3$)	Tullock ($n = 3$)	All-pay ($n = 6$)	Tullock ($n = 6$)
Player's Battle			✓	
Battle's Total			✓	
Player's Total			✓	
Winner Total			✓	
Participation Rate		✓		
Average Player Total	✓			
Average Battle Total			✓	
Average Contest Total			✓	

✓ for the best-performed CSF-n setting based on each objective function.

All in all, the all-pay with 6 players generates more effort from the players in the following categories: Player's Battle, Battle's Total, Player's Total, Winner Total, Average Battle Total, and Average Contest Total. All-pay with 3 players and Tullock with 3 players performed the best in the perspective of Average Player Total and Participation Rate, respectively.

B.5 Dynamic Difference in Positive Effort

We present the Random Effects Panel regression result to check the dynamic difference in positive bids for subsequent battles Table B.4. The battle 2 positive bids in the auction contest are significantly higher than in battle 1. Consider that most contests end within 4 battles; we use battle 3 and battle 4 to represent post-battle 2 behavior. We find no significant difference between the post-battle positive effort.

Table B.4: Random Effects Panel on Dynamic Comparison of Positive Bid

CSF		**All-pay Auction**		**Tullock Lottery**	
Positive Effort		$n = 3$	$n = 6$	$n = 3$	$n = 6$
Battle 1 vs. Battle 2	$(s = 0)$	65.33***	43.38*	-9.22	-3.70
		(16.00)	(22.59)	10.86)	(10.64)
Battle 2 vs. Post-battle 2	$(s = 0)$	38.21	-21.17	-20.29	-4.95
		(33.93)	(21.92)	(15.58)	(7.80)
	$(s = 1)$	32.44	8.63	-18.86	-41.47**
		(21.28)	(35.40)	(16.65)	(20.16)
Battle 3 vs. Battle 4	$(s = 0)$		-4.24		-7.20
			(18.23)		(19.62)
	$(s = 1)$	24.56	-23.73	1.71	14.03
		(50.91)	49.34	(31.84)	20.48

The dependent variable is the effort for each corresponding battle and state.
We exclude the zero bid from the dataset.
The random effects model is structured at the subject level.
The experimental session-level clustered robust standard errors are reported in the parentheses.
The reported significance reflects if the positive bid of the later battle is different from the former.
* for $p < 0.1$ ** for $p < 0.05$ *** for $p < 0.01$

Appendix C

Appendix to Chapter 4

C.1 Misspecification with the Secondary Models

We also provide some institutions of the consequence of misspecified secondary present models for the primary present behaviors. Consider a subject's three-period discounting function exhibit the following secondarily bias pattern $D_\tau(t_0, t_1, t_2) = (\delta^{t_0}, \beta\delta^{t_1}, \beta\delta^{t_2})$. Nevertheless, we specified a secondary present model with a sequence of time-dependent discount factors $\hat{D}_{\tau'}(t_0, t_1, t_2) = (\hat{\delta}^{t_0}, \hat{\delta}^{t_1}, \hat{\beta}\hat{\delta}^{t_2})$.

With a similar approach in Eq. (4.8) and Eq. (4.9), we can derive $\hat{\delta}$ in Eq. (C.1), and $\hat{\beta}$ in Eq. (C.2). For simplicity, we consider $\delta^{t_0} = 1$.

$$\frac{\hat{\delta}^{t_1}}{\hat{\delta}^{t_0}} = \frac{\beta\delta^{t_1}}{\delta^{t_0}} \implies \hat{\delta} = \beta^{\frac{1}{t_1 - t_0}}\delta \implies \begin{cases} \hat{\delta} > \delta, & \text{if} \quad \beta > 1 \\ \hat{\delta} < \delta, & \text{if} \quad \beta < 1 \end{cases} \tag{C.1}$$

$$\frac{\hat{\beta}\hat{\delta}^{t_2}}{\hat{\delta}^{t_1}} = \frac{\beta\delta^{t_2}}{\beta\delta^{t_1}} \implies \hat{\beta} = \frac{\delta^{t_2 - t_1}}{\hat{\delta}^{t_2 - t_1}} \implies \begin{cases} \hat{\beta} < 1 < \beta, & \text{if} \quad \beta > 1 \\ \hat{\beta} > 1 > \beta, & \text{if} \quad \beta < 1 \end{cases} \tag{C.2}$$

The equations in Eq. (4.8), Eq. (4.9), Eq. (C.1), and Eq. (C.2) show that the ***Primary Present Bias*** and the ***Secondary Future Bias*** create very similar trends (sign) when pushing τ to later positions in the empirical models. The ***Primary Future Bias*** and the ***Secondary Present Bias*** tends to have the same character.

C.2 Other Discounting Models

Consider a Hyperbolic Discounting (HD, Mazur, 1987; Loewenstein and Prelec, 1992) function $D(t_i) = \frac{1}{1+(\kappa t_i)}$, then the CRRA utility of the CTB takes the form in Eq. (C.3). Then the optimal demand function of sooner consumption would be Eq. (C.4) accordingly.

$$U_0(x_{t_i}, x_{t_i+k}) = \frac{1}{1+\kappa(t_i)} \frac{1}{1-\eta} (x_{t_i} + \omega)^{1-\eta} + \frac{1}{1+\kappa(t_i+k)} \frac{1}{1-\eta} (x_{t_i+k} + \omega)^{1-\eta} \tag{C.3}$$

$$x^*_{t_i} = \frac{(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}} - \frac{1}{1+\kappa(t_i)}}{\frac{1}{1+\kappa(t_i)} + (1+r)(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}(\omega) + \frac{(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}{\frac{1}{1+\kappa(t_i)} + (1+r)(\frac{1}{1+\kappa(t_i+k)}(1+r))^{\eta^{-1}}}(m) \tag{C.4}$$

Table C.1: Aggregate Level BIC for Different Models (CMW16)

$\hat{\kappa}$	0.0028	0.0030	0.0035
	(0.0002)	(0.0002)	(0.0002)
$1-\hat{\eta}$	0.7274	0.8416	0.8796
	(0.2317)	(0.0872)	(0.0511)
ED	165492	71316	35779
HD	165471	71299	35753
QHD	165442	71290	35740
GQHD	165346	71233	35667
Subjects	1060	452	230

As a result, the HD model's performance (measured with BIC) is better than the ED model for all three data sets, suggesting the overall behavior favoring the inconsistent dynamic models. The departure between the HD model and the QHD model provides evidence against the immediate present bias behavior (Table C.1).

$$x^*_{t_i} = \frac{(D(t_i+k)(1+r))^{\eta^{-1}} - D(t_i)}{D(t_i) + (1+r)(D(t_i+k)(1+r))^{\eta^{-1}}}(\omega) + \frac{(D(t_i+k)(1+r))^{\eta^{-1}}}{D(t_i) + (1+r)(D(t_i+k)(1+r))^{\eta^{-1}}}(m) \tag{C.5}$$

We can generalize such $x^*_{t_i}$ to other (proportional, Ainslie and Herrnstein, 1981; power, Harvey, 1986; constant sensitivity, Ebert and Prelec, 2007) $D(t_.)$ functions in Eq. (C.5).

C.3 Simulations and Classification Algorithm in GBDD

To validate the performance of the classification algorithm in the General Beta-delta Discounting model (GBDD), we simulated the time preference CTB choices based on the CMW16 design. We first parametrized the parameter space of the 1060 real subjects' curvature $(1-\eta)$, consistency (β), and discounting (δ) parameters by using the Ripley (2002) method.

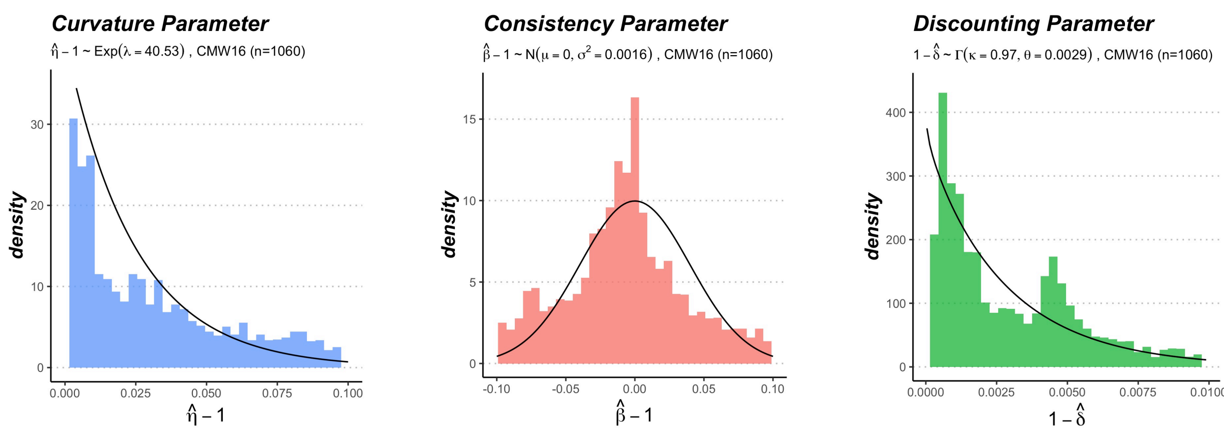

Figure C.1: Fitted Distributions of the Parameters of the GBDD Model for CMW16 Subjects.

The fitted parameter distributions are the exponential distribution $(\lambda = 40.5)$ for $\hat{\eta}-1$, normal distribution $(\mu = 0$ and $\sigma^2 = 0.0016)$ for $\hat{\beta}-1$, and gamma $(\kappa = 0.97$ and $\theta = 0.0029)$ distribution for $1-\hat{\delta}$ (Figure C.1). We assumed that each parameter is independently distributed through the simulation process. Each choice of the simulated subject on the CTB of CMW16 has a random noise that follows a standardized normal distribution.

For each of the four categories of the revealed time preference type (time consistent (TC), present bias (PB), mixed inconsistent (MI), and future bias (FB)), we simulated 25% in the simulated testing data. The confusion matrices of six different classification methods and the testing accuracy rate of each time preference category are reported in Figure C.2. Their" method is we classify time preference type purely based on each of the criteria of AIC, AICc,

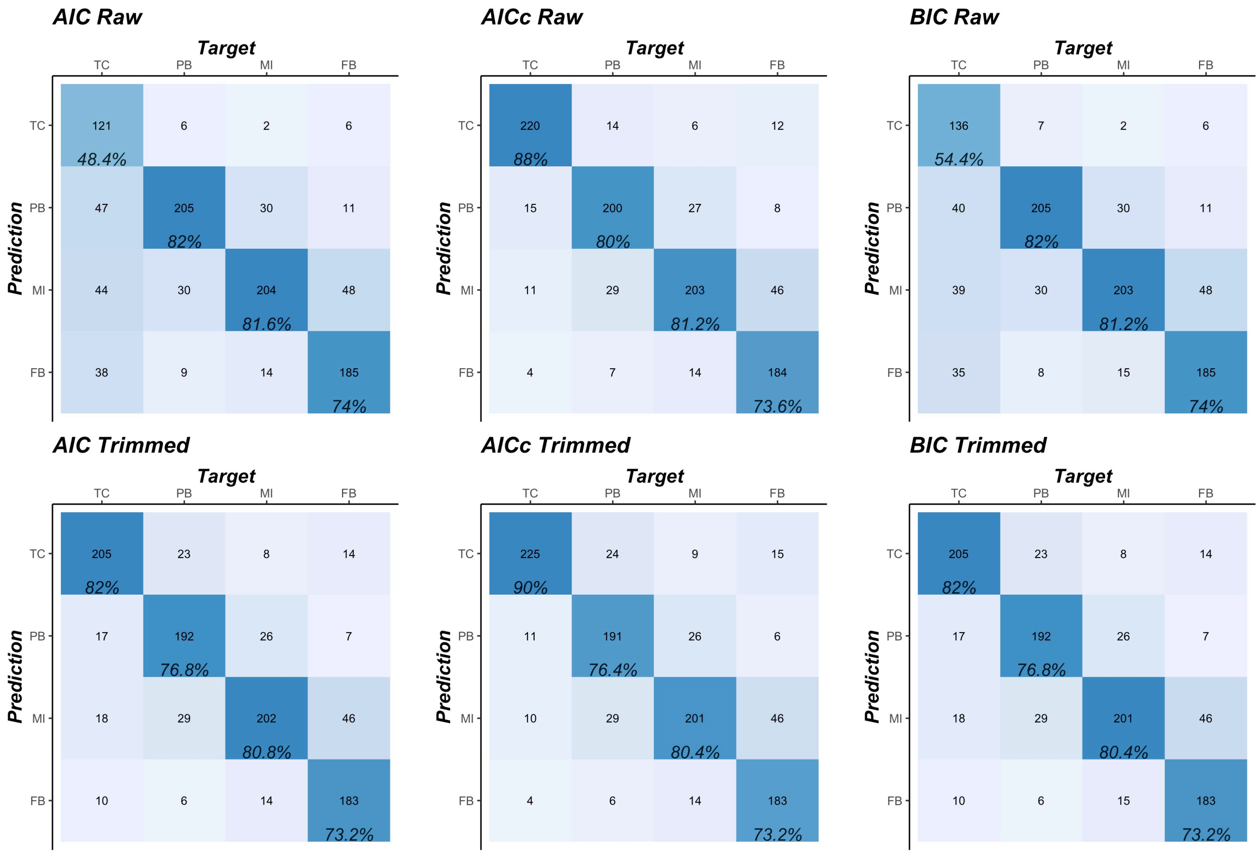

Figure C.2: The Confusion Matrices of Different Classification Algorithms

and BIC. In the "Trimmed" method, we classify the time preference type as time consistent (ED) if the selected model has a 95% confidence interval of $\hat{\beta}$ covering 1. We found the AICc Raw method has the lowest average testing error rate and the lowest variance in testing error rate. Therefore all of the conclusions in **Section 6.** are based on the AICc Raw method.

We then applied the AICc Raw algorithm to all four data sets, the simulated dataset of 100,000 subjects, the full dataset of 3670, the law of demand dataset of 1551 subjects, and the law of demand and non-EDU dataset of 648 subjects. The results are shown in Figure C.3.

In Figure C.3, we can spot some general findings and list them as follows: first, the human subjects' time behaviors are very different from the simulated subjects, where the uniformly random simulations have a high proportion of time consistent decisions; second, as we subsetting to a smaller and smaller subset (from the Full to the Law of Demand, then to the Law of Demand and non-EDU), which is more robust in following the assumption in GQHD model, we see less and less time consistent subjects, which is consistent with the theories (Giné et al., 2018; Echenique et al., 2020) and expectation of our specifications; third, sufficient subjects make the time inconsistent decisions, especially the future bias and the mixed inconsistent choices, which the previous present-biased-oriented literature has overlooked; Forth, the participants of the experiments are heavily influenced by design, i.e., the difficulties of making inconsistent time decisions in a given experiment. We see strong positive correlations between simulations and real subjects' decisions regarding making time consistent preference choices.

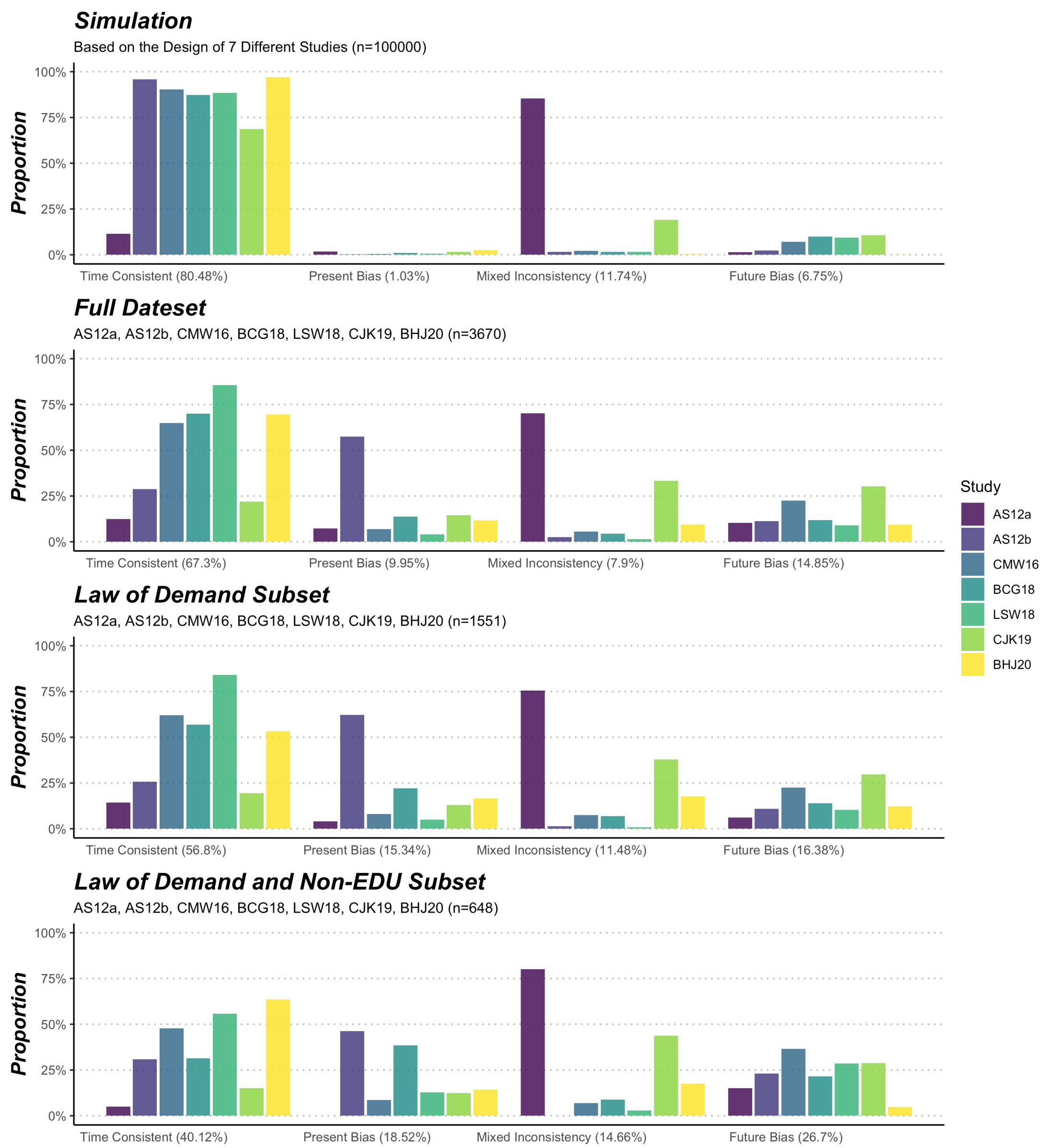

Figure C.3: AICc Raw Classifications for Different Subsets.