# A CRITIQUE OF GOAL PROGRAMMING FOR PUBLIC SECTOR DECISIONS

bу

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#### CHAPTER I

#### INTRODUCTION

Public decision-making in recent years has become increasingly reliant on quantitative techniques for the analysis of complex problems. In line with this expanding dependence on analytical techniques has come an increasing interest in the application of mathematical models to public sector decisions. This heightened interest in part has been a product of the expanding complexity of such decisions, and a fuller understanding of the extent of impacts generated by governmental activities. One type of model which has seldom been applied to decision-making in the public sector is goal programming. Developed by Charnes and Cooper, doal programming is essentially a special type of linear programming that is capable of handling problems with multiple goals and/or a single goal with multiple The discussion which follows is an attempt to subgoals. test the applicability of the goal programming technique to one type of public sector decision; budgeting for a school district. From this specific application, the appropriateness of goal programming as a general analytical tool for public sector decisions will be assessed.

The technique of goal programming in recent years has been widely applied to the analysis of financial problems

in the private sector of the economy. Often the approach has been utilized by private industry to analyze alternatives for the allocation of scarce resources. The purpose of this paper is to analyze and evaluate the applicability of a goal programming model to the type of allocation problems common to local government. To conduct this evaluation, the educational budget of Wise County, Virginia<sup>2</sup> was analyzed through the application of basic goal programming methods. After the completion of this analysis, standards of educational quality developed from the school budget of Arlington County, Virginia<sup>3</sup> were applied to the goal programming model. In addition, the potential for the development and public sector application of a time dynamic goal programming model have been investigated.

To apply the goal programming technique, an APL computer program was specifically developed. This interactive program allows the rapid modification of goals, constraints and priorities necessary for the effective use of the technique. The ability to rapidly determine the effects of altered goals or constraints greatly facilitates the analysis of available tradeoffs, permitting optimal use of the goal programming algorithm.

The choice of a educational budgeting problem for testing the public sector applicability of goal programming was the result of several considerations. Initially, the availability of this type of data was a positive factor. In

addition, a school district budget can be considered as a relatively self-contained problem. Exogenuous variables need not necessarily be included in the analysis. Finally, the problem chosen clearly involves the distribution of scarce resources in accordance with a variety of goals. These considerations contribute to, but do not ensure the appropriateness of a problem for analysis via goal programming.

By using the goal programming technique to analyze the educational budget for Wise County, the intent of the paper is to assess the potential for public sector applications of this technique. Of particular concern are the inherent weaknesses of the technique and the validity of required assumptions. In addition, the lack of quantitative data on the interrelationships and tradeoffs among decision variables may be a major hindrance to meaningful public sector applications. In sum, the paper seeks to determine to what extent, if any, the technique of goal programming may be effectively used for the analysis of public sector financial decisions.

The format of the analysis which follows includes in Chapter II a review of previous public sector applications of the goal programming technique. Chapter III contains a description of the goal programming model, and a discussion of the problems involved in adapting the technique to public sector use. The application of the model to the Wise County

school budget is examined in Chapter IV, together with a detailed description of the goal programming analyses that were conducted. This is followed in Chapter V by a discussion of the potential for and validity of a timedynamic goal programming model. Conclusions are presented in Chapter VI. Finally, in the Appendices are found a listing and discussion of the APL computer program used in the analysis and the computer input and output for each of the goal programming problems discussed in the text.

#### NOTES - CHAPTER I

<sup>1</sup>A. Charnes and W. W. Cooper, <u>Management Models and</u> <u>Industrial Applications of Linear Programming</u>, Vol. I (New York: John Wiley and Sons, Inc., 1961), pp. 215-22.

<sup>2</sup>Wise County School Board, "Proposed General County School Budget, 1976-77." (Xeroxed)

<sup>3</sup>Arlington Public Schools 1976-77 Tentative Budget, Pt. I.

#### CHAPTER II

#### LITERATURE REVIEW

#### Private Sector Applications

Goal programming techniques in recent years have been used principally as aids to private sector decision-making. For this type of application, economic profit is the key to the formulation of an objective function,<sup>1</sup> with the various aspects of profit and external considerations serving as multiple subgoals. This use for goal programming may be seen in the works of Charnes and Cooper<sup>2</sup> and particularly, in the models developed by Ijiri.<sup>3</sup> The goal programming techniques developed and refined by Ijiri are intended primarily for use in managerial accounting in the private sector. The work contains an excellent discussion of the theory of linear and goal programming, but specific suggestions for use of the techniques for public sector decisions are not present. The focus of the book is the technical and mathematical subtleties of the use of goal programming for private sector accounting decisions. Ijiri examines in detail the mathematical representation of cost functions, environmental constraints and feedback generation.

Unlike that of Ijiri, the work of Lee<sup>4</sup> is a general discussion of goal programming techniques and applications.

After a thorough examination of the requirements and assumptions of goal programming, Lee cites applications of the technique in the areas of production planning, financial and marketing decisions, corporate, academic and municipal planning, and the allocation of health services. In general, the examination of these goal programming applications serves to illustrate how the structure of decision problems must often be severely modified for incorporation into a goal programming model. This characteristic of the method is particularly evident for Lee's application of the technique to health services allocation. The decision environments considered by Lee are invariably interpreted as purely economic problems. Social, political, and environmental aspects of health services allocation are considered only to the extent that they can be expressed in economic terms.

## Public Sector Applications

The above criticisms are equally applicable to Lee's approach to goal programming for capital improvements planning in the Town of Blacksburg.<sup>5</sup> Lee and Sevebeck reduce the problem to a purely economic decision based upon six basic goals. These goals are essentially quantitative objectives derived from policy goals of the Town. The problem is thus reduced from one of policy formulation to cost/effectiveness evaluation prior to the application of goal programming. The underlying assumption is that the

efficient completion of street improvements and water storage facility expansion constitutes goal achievement. Given this assumption, the technique employed measures only the extent of goal achievement, not the external impacts of various courses of action. This narrow scope of concern is a major shortcoming of the model, and can potentially be remedied only through subjective consideration of goal programming solutions. However, the method employed by Lee and Sevebeck does allow for the analysis of economic tradeoffs through the iterative revision of the goal priority structure.

Further applications of goal programming to public sector decision-making are examined by Dickey.<sup>6</sup> In this study, a goal programming approach is applied to the analysis of complex transportation problems involving the impact of transit systems on land use and neighborhood socio-economic characteristics. Rather than seeking the solution to a specific problem, the technique is used to estimate the effects of alternative transit system routings on the quantity and quality of physical development, and the extent of socio-economic segregation in the Boston metropolitan area. The importance of Dickey's application of goal programming lies in his recognition that the model is an inadequate representation of reality. He therefore attempts to draw only conclusions which are qualitative rather than quantitative in nature.

A process of goal programming involving continuing interaction with decision-makers is examined by Dyer.<sup>7</sup> Through such interaction, the goals and constraints of the problem are continually revised during the operation of the model. In essence, the Dyer model is a simulation of decision-making rather than a decision tool. It places emphasis on the quantitative subtleties of modeling rather than the qualitative impacts of decision alternatives.

A discussion of goal programming for public accounting by Killough and Souders<sup>8</sup> has relatively little application outside of accounting practices. Shortcomings of the model as noted by the authors conform to the general limitations discussed above.

Perhaps the most recent contribution on the use of goal programming methods in the area of education is the work of Schroeder.<sup>9</sup> In this article, a goal programming decision model is used to analyze a resource allocation problem for university departments. Emphasis is placed upon the importance of sensitivity analysis of goals, priorities and constraints for effective utilization of the technique. In addition, the potential for model modification in order to more accurately represent the decision environment is discussed in terms of the introduction of stochastic variables and a utility-oriented objective function.

# Inherent Shortcomings of the Model

In an application of goal programming to a private sector type of production problem, Goodman<sup>10</sup> has specifically tested the assumption that a decision environment can be adequately represented via a set of linear constraints. In general, he concluded that the accuracy of the goal programming model, and hence the validity of the results, are highly dependent upon the extent to which linear expressions can appropriately represent the decision system under consideration. He suggests that the goal programming model can be effectively applied only to those problems for which the linear expression of constraints is a substantially accurate representation.

The study of public transit assessment through goal programming conducted by Hawthorne<sup>11</sup> again confirms the crucial nature of the linear expressions in the model. In the application, considerations of the environment and community factors could not be adequately represented as linear expressions. These concerns, because of their nonlinear nature, were excluded from the model although they constitute significant decision constraints in this and most other public sector applications.

In general, the literature on the use of goal programming provides a significant number of successful applications of the model. The majority of these successes, however, concern the analysis of private sector decision problems.

Many of these problems are principally concerned with profit maximization.<sup>12</sup> Most importantly, private sector decision-makers often have access to relatively precise information on the relative importance or value of various decision variables. That is to say, crucial interrelationships and potential tradeoffs are defined in quantitative terms. When such data do not exist for private sector problems, quantitative assumptions are used to derive needed information.

For public sector applications, of goal programming, however, adequate data on the interrelationships and tradeoffs among variables are seldom available. Public sector decision-makers are generally reluctant to employ assumptions to derive necessary information. In such cases, the analytical capacity of the goal programming model cannot be effectively utilized. Thus, goal programming approaches to the analysis of public sector decisions have rarely been utilized due primarily to the difficulties existent in adapting a broad range of concerns to the analytical limitations of the technique. Where the approach has been applied in the public sector, an artificial definition of the decision environment has often been used in an attempt to overcome adaptive difficulties. Obviously, the effective utilization of goal programming techniques for public sector decisions lies not in restricting the scope of analysis, but in providing necessary and complete information on all

decision variables. At the present time, however, the most appropriate approach would seem to be limiting the use of the technique to those problems for which it is easily adapted--public sector decisions that involve a relatively narrow scope of concern, and for which there is adequate quantitative data on available tradeoffs among decision variables.

The chapter which follows contains a detailed description of the goal programming model, and a discussion of the requirements for its effective use.

#### NOTES - CHAPTER II

<sup>1</sup>Refer to Chapter III for an explanation of the objective function.

<sup>2</sup>Charnes and Cooper, <u>Management Models and Industrial</u> Applications.

<sup>3</sup>Yugi Ijiri, <u>Management Goals for Accounting and</u> Control (Chicago: Rand McNally, 1965).

<sup>4</sup>Sang M. Lee, <u>Goal Programming for Decision Analysis</u> (Philadelphia: Auerbach Publishers, Inc., 1972).

<sup>5</sup>Sang M. Lee and William R. Sevebeck, "An Aggregative Model for Municipal Economic Planning," <u>Policy Sciences</u>, Vol. 2, No. 2 (June, 1971), pp. 99-115.

<sup>6</sup>John W. Dickey, "Minimizing Economic Segregation Through Transit System Changes," Virginia Polytechnic Institute and State University, December, 1970.

<sup>1</sup>James S. Dyer, "Interactive Goal Programming," <u>Management Science</u>, Vol. 19, No. 1 (September, 1972), pp. 62-69.

<sup>8</sup>Larry N. Killough and Thomas L. Souders, "A Goal Programming Model for Public Accounting Firms," The <u>Accounting Review</u>, Vol. XLVIII, No. 2 (April, 1973), pp. 268-279.

<sup>9</sup>Roger G. Schroeder, "Resource Planning in University Management by Goal Programming," <u>Operations Research</u>, Vol. 22 (1974), pp. 700-710.

<sup>10</sup>David A. Goodman, "A Goal Programming Approach to Aggregate Planning of Production and Work Force," <u>Management Science</u>, Vol. 22, No. 12 (August, 1974), pp. 1569-1575.

<sup>11</sup>Richard C. Hawthorne, Jr., "A Goal Programming Approach to Public Transit Assessment," (unpublished Master's Thesis, Virginia Polytechnic Institute and State University, 1973).

<sup>12</sup>For examples of private sector applications, and assumptions employed, refer to Lee, <u>Goal Programming for</u> Decision Analysis, pp. 39-63.

#### CHAPTER III

THE GOAL PROGRAMMING MODEL

#### Description of the Basic Model

As developed by Charnes and Cooper,<sup>1</sup> goal programming is essentially a special type of linear programming which is capable of analyzing problems with multiple goals, or a single goal with multiple subgoals. To some extent, the rigidity present in traditional linear programming is avoided by the goal programming technique. Significant constraints to general applicability nevertheless remain. To achieve a basic understanding of the goal programming model, some familiarity with the principles of linear programming is required. The discussion below begins with an examination of linear programming concepts, followed by an analysis of the distinctive features of the goal programming model.

Linear programming was first operationalized by George B. Dantzig during the 1940's as a technique for military planning.<sup>2</sup> More recently, the technique has been applied to a wide variety of decision-making problems. The first requirement of linear programming is that the variables involved be of the first order, such as X, 2X, 1/3X, etc.

No higher order variables, such as  $X^2$  or  $X^3$ , are permissable. In addition, a set of constraints must be present. Constraints are linear combinations of variables which represent the limits of the solution to a problem under consideration. To obtain the flexibility necessary to reach a solution, constraints are expressed as inequalities. Generally, they appear in the form:

 $aX + bY \leq or \geq C$ 

For a typical problem, overlapping constraints define an area of feasible solutions. Beyond this area, no real solution exists. A final requirement of linear programming is the existence of a linear objective function, which is an additive combination of key decision variables. For a typical linear programming model, the intent is to optimize the objective function given the system of constraints. Finally, all variables considered in a linear programming problem must be non-negative.

In order to model complex decision processes, slack variables are incorporated into the problem constraints to facilitate iterative solutions. Such variables represent the unused portion of available resources idle due to various other constraints. For optimal solutions achieved through iteration, slack variables are minimized to the point at which any further utilization does not enhance the objective function. Most importantly, linear programming requires that the decision-maker have a single expressed objective. Thus, when this technique is used, the decision-maker must subjectively evaluate the solution in terms of any other criteria not included in the model.

#### Multiple Goals

The goal programming approach is one method by which to alleviate the problem of multiple objectives. Goal programming is basically a modification and extension of linear programming which produces a single solution for a set of multiple objectives. The technique is thus capable of optimizing an objective function with multiple goals, or a single goal with multiple subgoals. The multiple goals employed in a goal programming problem must first be arranged by the decision-maker in a hierarchy of importance. The goals are not assigned priority ratings per se, but each item in the hierarchy of goals has complete pre-emptive priority over those goals which fall below it in the structure. Thus low order goals are considered only after higher order goals have been satisfied to the maximum extent allowable by the constraints. In sum, if goals can be expressed as an ordinal ranking of linear relationships, a solution can be reached via goal programming. The use of the goal programming model assumes that the variables directly involved in a decision process, the

decision variables, can be mathematically arranged in such a way as to accurately represent the problem situation, or decision environment. The mathematical formulation of a goal programming model is thus expressed as follows:<sup>3</sup>

Minimize  $M^{-}Y^{-} + M^{+}Y^{+}$ 

subject to

 $AX+IY - IY^+ = G; X \ge 0, Y^- \ge 0, Y^+ \ge 0$ 

For these expressions,  $M^-$  and  $M^+$  are vectors of goal priorities;  $Y^+$  and  $Y^-$  are the deviational variables representing overachievement and underachievement of goals respectively. A is a matrix of variable coefficients; decision variables are represented by the vector X; I is the identity matrix; and G is an expression of desired variable levels reflected in the goal structure.

Assuming that for the above formulation there are n goals, the objective function may be stated as: $^4$ 

Minimize  $\sum M_i (Y_i^+ + Y_i)$ 

When the goal structure, M, is arranged in order of importance, the model specifies that  $M_1$  has complete preemptive priority over  $M_2$ , and so on for  $M_1$  through  $M_n$ . This ordinal relationship of goals is expressed in the objective function shown above.

For an operational model, goal achievement is obtained by minimizing to zero the goal deviations represented by  $Y^+$  and  $Y^-$ . For purposes of this study, the goal programming formulations will be solved using a variant of the simplex method commonly used for linear programming models.<sup>5</sup> In using the simplex procedure, the value of all decision variables is initially set to zero. Consequently, negative deviational variables ( $Y^-$ ) take on the desired variable levels reflected in the goal structure. Thus, as the value of these deviational variables is decreased, the value of relevant decision variables is increased, thereby approaching goal achievement. The extent to which the value of a deviational variable is reduced from its initial or starting value is reflective of the extent of goal achievement.

#### Requirements for Application

The technique of goal programming is an analytical model which can be particularly useful for examining problems that involve tradeoffs in the allocation of scarce resources. As such, the model appears to be of particular value in the analysis of budgetary decisions, as well as for determining the adequacy of projected future revenues in meeting specified objectives. However, to apply the model effectively, a set of specific requirements must be fully satisfied. Perhaps the most limiting of these requirements is that all goals and constraints be expressed as linear relationships. Obviously, a relatively minor proportion of public decisions requiring an analytical model possess

characteristics expressable as linear functions. Difficulties in applying the goal programming model frequently arise when critical considerations are excluded or inappropriately modified due to this requirement of the model.

A second requirement of the goal programming technique is that decision preferences be expressed within an ordinal goal structure based upon preemptive priorities. Achievement of the first goal in the structure will thus take absolute precedence over the achievement of all other goals. Often, the preferences of decision-makers cannot be expressed in such an absolute fashion.<sup>6</sup>

In attempting to construct a goal programming model for public sector decisions from available data, several additional requirements are immediately apparent. Foremost is the necessity of comprehensive goals defined in a quantitative manner. Public goals expressing desired standards of quality for service provision often defy quantification. When numeric expression of these goals is possible, the lack of relevant data may undermine their legitimacy, necessitating multiple assumptions. In addition, all goals and constraints must be defined in terms of a single unit of measure. Yet, the reduction of all constraints to equations with uniform units of measure can be of great difficulty. A distorted representation of the decision

environment can often result from inappropriate manipulation of constraints in this manner. Difficulties in the public sector application of goal programming are thus often a single abstract quantity. For private sector use, constraints in general are easily converted to relationships of economic units.<sup>7</sup> Yet, the diversity and complexity of variables involved in public sector decision problems is a formidable barrier to the utilization of uniform units of measure. At present, the most feasible solution remains the conversion of all relationships to economic units. This transformation, though necessary for the effective use of the technique, is among the more questionable aspects of its public sector application.

The chapter which follows examines the procedure followed in assessing the applicability of the model to analysis of the Wise County educational budget. In particular, the purpose of this analysis was to determine the consequences of the shortcomings of the goal programming model, as outlined above, in public sector applications.

# NOTES - CHAPTER III

<sup>1</sup>Charnes and Cooper, <u>Management Models and Industrial</u> <u>Applications</u>, pp. 215-221.

<sup>2</sup>Lee, <u>Goal Programming for Decision Analysis</u>, p. 15.

<sup>3</sup>Schroeder, "Resource Planning in University Management by Goal Programming," p. 703.

<sup>4</sup>Ibid.

<sup>5</sup>Lee, <u>Goal Programming for Decision Analysis</u>, pp. 97-106.

<sup>6</sup>For a complete discussion of the characteristics of governmental decision-making, refer to Edward S. Quade, <u>Analysis for Public Decisions</u> (New York: American Elsevier <u>Publishing Co., 1975), pp. 1-21, 89-91, 162-166, 181-186,</u> 269-275.

<sup>7</sup>For examples of private sector applications of goal programming, refer to Lee, <u>Goal Programming for Decision</u> <u>Analysis</u>, chapters 3, 4, and 5.

#### CHAPTER IV

APPLYING THE MODEL

#### The Wise County Decision Environment

In order to be effectively used, the application of goal programming must occur in a decision environment possessive of several specific characteristics. Most importantly, there must be a decision problem to which the goal programming model can be applied. In addition, decision-makers must be capable of identifying specific goals relevant to the problem solution, and must rank these goals in an ordinal fashion. Feasible tradeoffs among various levels of goal achievement must next be determined, allowing quantitative relationships between key decision variables to be established. Finally, decision-makers must be willing to allow for a degree of flexibility in the expression of constraints, and the achievement of goals.

Unlike those situations encountered in the private sector, typical public sector decision environments are seldom characterized by flexibility of goals and constraints, or by adequate quantitative information on available tradeoffs. Governmental decisions, in general, are incremental in nature. Interrelationships among various

decision variables are not often analyzed in a quantitative manner.<sup>1</sup> Non-quantifiable considerations may preclude achievement of an otherwise optimum solution.<sup>2</sup> For such public sector decision environments, a goal programming model cannot be effectively applied.

In order to demonstrate the difficulties common to public sector applications of the goal programming model, it was necessary to isolate a typical governmental decision environment. Information and data concerning the educational budget of Wise County were obtained through the cooperation of the College of Education, V.P.I. and S.U.<sup>3</sup> In addition, personal interviews with officials of the Wise County School Board<sup>4</sup> provided information relevant to model constraints and goal formulation. After an initial assessment, it was determined that the characteristics of the Wise County decision environment were typical of those for most public sector decisions. The educational budgeting decision for Wise County therefore was chosen for analysis via the goal programming model.

Of particular concern to the Wise County School Board was the impact of standards of quality upon the allocations for various budget categories. For this reason, the analysis was concentrated on allocations for teacher salaries and instructional costs. In formulating a basic goal programming model for Wise County, several assumptions were employed.

Initially, it was assumed that expenditures for any specific budget category at a minimum must conform to any relevant state educational standards. In addition, it was assumed for the purpose of practicality that the expenditure for other instructional personnel (administrators) should not exceed eleven percent of the expenditure for teacher salaries. The constraint resulting from this assumption was included to avoid excessive allocation of funds for administration. Finally, it was assumed that the dollar sums designated in the initial analysis should be those which provide the optimum allocation for various budget categories given the funds available. These dollar figures were derived from a summary of the Wise County School Budget,<sup>5</sup> and through consultation with County officials.

As is the case for all goal programming models, the analysis of the Wise County educational budget is based upon a set of linear constraints. As stated previously, one of the difficulties most frequently encountered in applying this type of model is the inadequacy of the linear representation of constraints. Nonlinear relationships frequently occur as expressions of quantitative relationships among decision variables. However, the Wise County analyses contain little information on such interrelationships, since this type of data was generally unavailable. Therefore, no difficulty was encountered in converting the

available data to a series of linear expressions. Those relationships among variables which are expressed in the models are simply additive in nature. Clearly, none of the equations included attempt to approximate a non-linear relationship via a linear expression. Therefore, the requirement of the goal programming model that all constraints be expressed as linear relationships does not in itself create inaccuracies in the analyses of the Wise County School Budget.

In developing appropriate goals and constraints for the analysis of a local public school budget in Virginia, of particular importance are the standards of quality set forth by the State.<sup>6</sup> Wise County currently meets or exceeds all State educational standards related to expenditures. For example, the State of Virginia currently requires that a local school district employ forty-nine professional instructional personnel for each one thousand pupils in average daily membership. Wise County currently employs fifty-three such personnel for each one thousand pupils. The minimum standards of quality prescribed by the State are thus accounted for by the goal structure of the model. Significant problems could develop only in the event of an extremely low level of goal achievement for a specific goal. Since the expenditure for instructional personnel is most affected by State standards of quality, two distinct goals

were created for this budget category. The first seeks to achieve the minimum acceptable expenditure level, while the second is an expression of the desired allocation for this category.

The minimum standards of educational quality prescribed by the State of Virginia had little direct effect upon the goals and constraints included in the Wise County model, since all such standards applicable to the 1976-77 school year are currently met by the locality. Therefore, if the goal programming model was to be used to gauge the impact of various standards of quality upon the distribution of expenditures, a different and more demanding set of such standards are needed. To ensure the continued conformance to State minimum standards of quality, the choice of another local jurisdiction in Virginia seemed most appropriate. Wise County currently has a relatively low per pupil expenditure for education in comparison with other Virginia counties.<sup>7</sup> Assuming that a greater per capita expenditure results in an increase in educational quality, then the school budget of a locality characterized by a relatively high per pupil educational expenditure could presumably be used as an index of quality against which to compare the Wise County allocations.

As a result of the above considerations, the educational budget for Arlington County, Virginia, was chosen

for comparative analysis. Using a budget summary for Arlington County,<sup>8</sup> standards of quality were developed in terms of expenditures per budget category. These measures of educational quality, shown in Tables 1 and 2, are given as both per pupil allocations and percent of total expenditures for each budget category. Modifications to the goal programming model for Wise County were then used to analyze the potential for and impact of meeting these standards.

In contrast to the rural County of Wise, Arlington County is characterized by a predominance of urban and suburban development. Concomitantly, the population density is considerably greater in Arlington than in Wise County. Population growth continues to occur in Arlington.<sup>9</sup> As a result of these characteristic differences, Wise County may be expected to require substantially greater per pupil allocations for transportation. Arlington County, however, may be expected to expend a greater amount per pupil on capital construction necessitated by population increases. In addition, the existence of a subsidized school lunch program in Wise County could result in a greater per pupil expenditure for food services. It is evident, therefore, that the use of Arlington County educational expenditures as surrogate standards of quality for Wise County may

produce results which are generally misleading. Potential tradeoffs revealed by the analysis in actuality may not exist.

#### The Initial Model

The initial use of the goal programming for the analysis of the Wise County school budget was a relatively basic application of the technique. Essentially, this model was a simplistic representation of the decision environment described by the local educational administrators. In conjunction with the concerns of local officials and State standards of quality, the analysis focused upon the allocations for personnel salaries and instructional costs. For this model, all other expenditures were considered as a single allocation.<sup>10</sup>

The allocative goals and constraints shown below are based upon data obtained from local officials and State standards of quality for education for the 1976-77 school year.<sup>11</sup> Based upon these standards, Wise County must employ a minimum of 495 teachers. With an average teacher salary of \$9470,<sup>12</sup> this requires a minimum allocation of \$4,687,650. However, minimum standards of quality dictate that at least twenty-three percent of these teachers possess a graduate degree. Since the average salary for teachers with graduate degrees is \$10,470,<sup>13</sup> this requires the allocation of an additional \$1000 on average for each of 114 teachers, or \$114,000. However, the County desires to exceed these minimum standards by employing a total of 512 teachers, forty percent of which possess graduate degrees. This requires a total allocation for teacher salaries of \$5,053,640, as opposed to the minimum allocation of \$4,801,650.

The Wise County School Board has requested funding for FY 1976-77 of \$9,382,440,<sup>14</sup> an increase of approximately twelve percent over 1975-76. For purposes of the goal programming analysis, it was assumed that only about onehalf or \$501,400 of this desired increase will be allocated. This assumed increase would limit total 1976-77 educational expenditures to \$8,860,000.

The decision variables utilized in this goal programming model were as follows:

- X1: The allocation for all costs other than those for salaries and instructional costs, including administration, transportation, operation and maintenance, special programs, etc.
- X2: The allocation for instructional costs other than salaries.
- X3: The allocation for teacher salaries, exclusive of the additional increment provided for those with graduate degrees.

- X4: The allocation provided to supplement the salaries of teachers with graduate degrees.
- X5: The allocation for instructional personnel other than teachers.

Based upon the proposed school budget for FY 1976-77,<sup>15</sup> the goal constraints utilized in the initial analysis were as follows:

- a) The total school budget should not exceed \$8,860,000.
- b) The allocation for all budget categories other than salaries and instructional costs should not be less than \$2,749,000.
- c) The base allocation for teacher salaries should not be less than \$4,688,000.
- d) The supplemental allocation for salaries of teachers with graduate degrees should not be less than \$114,000.
- e) The allocation for other instructional personnel should not be less than \$646,000.

A goal programming solution is primarily based upon the priority structure of established goal constraints. By varying the priority structure, separate solutions to the problem are produced, allowing the analysis of potential tradeoffs for goal achievement. Three distinct solutions to the school district budgeting problem have been generated, and are summarized below. For each of these solutions, the variables and constraints are as stated above. Only the priority ranking of the goals was altered, as is indicated.

A complete listing of the APL program used in the goal programming analysis can be found in Appendix A. The computer input and output for the solutions shown below is contained in Appendix B.

#### Solution One

## Goal Priorities:

- 1: Limit total expenditures to \$8,860,000.
- 2: Allocate \$2,749,000 for all costs other than salaries and instruction.
- 3: Allocate \$533,000 for instructional costs.
- 4: Allocate at least \$4,688,000 for base teacher salaries.
- 5: Allocate at least \$114,000 to supplement the salaries of teachers with graduate degrees.
- 6: Allocate \$646,000 for the salaries of other instructional personnel.
- 7: Increase the total allocation for teacher salaries by \$252,000.

#### Results:

Goals 1 through 6: Achieved.

Goal 7: 51 Percent Achieved. The allocation for teacher salaries could be increased by only \$130,000; \$122,000 less than desired.

#### Solution Two

#### Goal Priorities:

The total expenditure goal was changed from priority 1 to priority 7. All other goal priorities were decreased by 1 from those shown for Solution One.

#### Results:

Goals 1 through 6: Achieved.

Goal 7: 98 Percent Achieved. The total expenditure limit was exceeded by \$122,000.

#### Solution Three

#### Goal Priorities:

- 1: Allocate \$2,749,000 for all costs other than salaries and instruction.
- 2: Allocate \$646,000 for other instructional personnel.
- 3: Limit total expenditures to \$8,860,000.
- 4: Allocate \$533,000 for instructional costs.
- 5: Allocate \$114,000 to supplement the salaries of teachers with graduate degrees.
- 6: Increase the total allocation for teacher salaries by \$252,000.
- 7: Allocate \$4,688,000 for base teacher salaries.

#### Results:

Goals 1 through 6: Achieved.

Goal 7: 97 Percent Achieved. Only \$4,566,000
was available for base teacher salaries,
\$122,000 less than desired.
A comparison of the three solutions shown above clearly indicates the existence of a conflict between the limit upon total expenditures and the desired levels of funding for the various categories. The sum of the desired allocation exceeds the expenditure limit by \$122,000. Unless partial funding of one or more of the budget categories is deemed appropriate, while still fulfilling minimum standards of quality, total expenditures must be increased by the above amount. More important, however, is that the goal programming solutions shown demonstrate ineffective use of the model. Since quantitative tradeoffs among decision variables are not specified, the goal programming algorithm does little but perform simple arithmetic operations.

### Arlington County as a Standard of Quality

After modifying the general budget categories for Wise County to conform to those of Arlington County, per pupil expenditures were compared, as shown in Table 1. The percentage of total expenditures allocated to each budget category are shown in Table 2.

From the figures in Tables 1 and 2, a comparative analysis was conducted using the goal programming model. No effort was made to analyze the Arlington County budget per se; rather, various aspects of this budget were used in the analysis of educational expenditures in Wise County.

# TABLE 1

### PER PUPIL EXPENDITURES FOR EDUCATION BY BUDGET CATEGORY

Category	Arlington Co.	Wise Co.			
TOTAL	\$2312	\$911			
Instruction	1535	651			
Fixed Costs	409	44			
Operation and Maintenance	233	91			
Transportation and Food Service	s 65	111			
Administration	69	15			

NOTE: Columns do not total due to rounding.

## SOURCES:

Arlington Public Schools Tentative Budget, Pt. I, pp. iv-1 - iv-4.

Wise County School Board, "Proposed General County School Budget," 1976-77, pp. 3-6.

## TABLE 2

# PERCENTAGE OF TOTAL EXPENDITURES BY BUDGET CATEGORY

Category	Arlington Co.	Wise Co.
Instruction	66.41%	71.42%
Fixed Costs	17.73	4.83
Operation and Maintenance	10.06	9.94
Transportation and Food Services	s 2.83	12.18
Administration	2.98	1.63
	100.01	100.00

## SOURCES:

Arlington Public Schools Tentative Budget, Pt. I, pp. iv-1 - iv-4.

Wise County School Board, "Proposed General County School Budget," 1976-77, pp. 3-6.

Using the comparative data for Wise and Arlington Counties, six individual goal programming problems were developed and iteratively analyzed. The intent and result of each of these iterations is summarized below. Detailed listings of computer input and output are shown in Appendix B.

# Iterations A and B

Iterations A and B were used to analyze the effect upon the Wise County budget of increasing per pupil expenditures for instruction to correspond with those of Arlington County. In an attempt to provide funds for such an increase, per pupil expenditures for transportation and food services were reduced to \$65, corresponding to the Arlington County figure.

Results of the analyses indicate that given the existing limit on total expenditures, only forty-five percent of the funds desired for instruction could be allocated. To obtain the additional \$839 per pupil for instruction would require an increase of approximately ninety-three percent in total expenditures.

# Iterations C and D

One type of expenditure included in the category of Fixed Costs is debt service. This expenditure is a significant portion of the annual allocation devoted to the capital investments for education. A desire to expand or replace existing educational facilities in Wise County would thus be reflected in increased expenditures for Fixed Costs.

Iterations C and D examined the potential of Wise County to significantly expand its capital program to a level corresponding to that of Arlington County. Results of the analyses indicate that the specififed increase in the Fixed Costs expenditure would necessitate a reduction of \$366 per pupil from other categories, particularly that of instruction. To avoid such a reduction, an increase in total expenditures of approximately forty-one percent would be required.

### Iteration E

The analysis conducted as Iteration E examined the proportional differences in expenditure categories for the educational budgets of Wise and Arlington Counties. Using the data in Table 2, the percentage allocations shown for Arlington County were taken as goals. Goal achievement for Wise County was then evaluated accordingly. The results of this analysis, shown in Table 3, are indicative of the extent to which Wise County percentage allocations meet or exceed those of Arlington County.

# TABLE 3

# RESULTS OF ITERATION E

Category	Goal	Achievement			
Instruction		108%			
Fixed Costs		278			
Operation and Maintenance		99%			
Transportation and Food Services		4278			
Administration		56%			

## Iteration F

The goal programming formulations utilized for this iteration of the model were in effect the inverse of those used for Iteration E. Initially, it was hypothesized that the percentage allocations for Arlington County, shown in Table 2, were adopted by Wise County. Resulting per capita expenditures for each budget category were then computed. Using existing goals for Wise County, the effective achievement of this new expenditure schedule was evaluated. In essence, the analysis measured the effect upon existing Wise County goals of adopting the percentage allocations characteristic of the Arlington County educational budget. Results of the analysis are found in Table 4.

An examination of the six goal programming analyses discussed above, and shown in Appendix B, reveals that the lack of specified tradeoffs among decision variables has again resulted in underutilization of the model. However, the results do indicate that expenditures for Administration and Fixed Costs are relatively low in Wise County in relation to those of Arlington County, while the allocation for Transportation and Food Services is extremely high in comparison. Given the goal of increasing expenditures for Instruction, it is possible that a reduction in funding for Transportation and Food Services is one alternative by which

# TABLE 4

# RESULTS OF ITERATION F

Category	Goal Achievement
Instruction	93%
Fixed Costs	368%
Operation and Maintenance	101%
Transportation and Food Services	23%
Administration	180%

to obtain the desired funds. But as discussed previously, these descrepancies in levels of funding are a product of the characteristic differences in the two localities under consideration, and are not reflective of genuine alternatives for the distribution of educational expenditures. Further study of the above solutions nevertheless produces additional insight into the quality of education provided in Wise County. Iterations A through D clearly indicate that, in terms of per pupil expenditures, Wise County is far below Arlington County. This finding suggests that the quality of education in Wise County is below that in Arlington. However, the results of Iterations E and F suggest that far more fiscal effort is directed toward instructional costs in Wise County. This conclusion is supported by a comparison of the ratio of per capita educational expenditures to per capita income. This ratio, which is in effect an index of educational effort, is shown below for each of the two counties:<sup>16</sup>

Arlington County0.048Wise County0.143

The analyses indicate that per pupil educational expenditures in Wise County are significantly less than those in Arlington County. However, in Wise County far greater financial effort is directed toward education expenditures in general, and instructional activities in particular.

### NOTES - CHAPTER IV

<sup>1</sup>For a discussion of incrementalism in public decisionmaking, see: Charles E. Lindblom, <u>The Intelligence of</u> <u>Democracy</u> (New York: The Free Press, 1965). A rebuttal to Lindblom's concepts is presented in Yehezkel Dror, <u>Ventures in Policy Sciences: Concepts and Applications</u> (New York: American Elsevier, 1971).

<sup>2</sup>For a complete description of the characteristics of a typical public sector decision environment, refer to Edward S. Quade, <u>Analysis for Public Decisions</u> (New York: American Elsevier Publishing Co., 1975), pp. 1-21, 89-91, 162-166, 181-186, and 269-275.

<sup>3</sup>All data for Wise and Arlington Counties, and information concerning standards of educational quality was furnished by Dr. A. P. Johnston, Assistant Professor, College of Education.

<sup>4</sup>Officials of the Wise County School Board personally contacted were Jim D. Graham, Assistant Superintendent for Instruction, and J. P. Horton, Director of Federal Programs, acting for Harley T. Stallard, Superintendent of Schools.

<sup>5</sup>Wise County School Board, "Proposed General County School Budget, 1976-77," pp. 3-6.

<sup>6</sup>Virginia State Department of Education, "Standards of Quality." (Xeroxed)

<sup>/</sup>Virginia State Department of Education, Division of Research and Statistics, Facing Up: Statistical Data on <u>Virginia's Public Schools</u>, Vol. 10 (1976), Table 7, pp. 56-59.

<sup>8</sup>Arlington Public Schools 1976-77 Tentative Budget, pp. IV-1 - IV-13.

<sup>9</sup>1970 Census of Population, Vol. 1, Pt. 48 (Virginia), Tables 119 and 124.

<sup>10</sup>This consolidation of budget categories into a single decision variable was justified in light of the budgetary process and goal priorities described by local officials.

<sup>11</sup>Virginia State Department of Education, "Standards of Quality."

<sup>12</sup>Data were obtained by telephone directly from Wise County School Board officials. This average figure excludes the additional increment of \$1000 provided to teachers with graduate degrees.

<sup>13</sup>Data were obtained by telephone directly from Wise County School Board Officials.

<sup>14</sup>Wise County School Board, "Proposed General County School Budget, 1976-77," p. 6.

<sup>15</sup>Ibid., pp. 3-6.

<sup>16</sup>Data on the population and per capita income for Wise and Arlington Counties were obtained from the <u>1970 Census</u> <u>of Population</u>, Vol. 1, Pt. 48 (Virginia), Tables <u>119</u> and <u>124</u>.

### CHAPTER V

### THE POTENTIAL FOR TIME-DYNAMIC GOAL PROGRAMMING

For many types of public sector financial decisions, the variable time is not a primary consideration. That is, many such decisions are static in nature, concerned only with a limited block of time for which characteristics of the decision environment will remain relatively uniform. This characteristic is particularly evident for annual budgetary decisions such as those discussed in the previous chapter. In terms of long range budgetary trends, while some expenditures can remain constant over a number of years, many others can vary significantly on an annual basis. Thus, budgetary decisions, in the optimum case, are made on the basis of the most recent information available. Current information on existing and potential needs are of particular importance, since budgetary tradeoffs necessary in preceding years may produce the most definitive needs for present and future funding. Regardless of the direction of past expenditures, however, budgetary priorities can change rapidly over time. Therefore, the considerations involved in any individual budgeting decision are not necessarily valid for a series of such decisions that occur over an extended period of time.

In attempting to develop a goal programming model which is time-dynamic, one difficulty is immediately apparent. The nature of the goal programming technique is such that it can only be used to analyze situations in which characteristics of the decision environment are fixed. At best, this limits application of the model to a series of discreet points over time. Approaching the problem from this perspective, an iterative progression of static solutions could be used for long range analysis. However, this type of approach assumes that goals, constraints, and priorities remain constant over time. As discussed above, this situation seldom prevails. Based upon these considerations, the potential for formulating a time-dynamic goal programming technique lies exclusively in the development of a discreet rather than a continuous model.

The attempted development of a discreet analytical model using goal programming immediately encounters several problems. Most significant is the fact that goals, constraints and priorities cannot be altered within the goal programming algorithm itself. Without additional analytical formulations outside of the goal programming model, the socalled time-dynamic model can be nothing more than a series of static analyses conducted for identical decision environments. Given the possibility of rapid and far-reaching changes in the basis and structure of budgetary decision-

making over even a short period of time, such an approach is clearly invalid, and hence of little value to the decision-maker.

Goal programming is a highly specific analytical technique with a very limited range of potential modifications. From the above considerations, it is apparent that there is no effective approach by which to develop a reliable, self-contained time-dynamic model from the goal programming algorithm used in this study. One alternative is to utilize a technique for recursive optimization. <sup> $\perp$ </sup> A more sophisticated approach would involve incorporating the goal programming technique in a more general model for the purpose of analyzing externally specified goals, constraints and priorities. In other words, the goal programming algorithm, in effect, could be utilized as the static, analytical segment of an otherwise dynamic simulation model. Such a model could assess the relevant characteristics of a decision environment at regular intervals over time, utilizing these data as static input for the goal programming algorithm. To construct such a model at a minimum would require major modification to existing simulation models. However, no revision to the goal programming method would be required. In essence, the simulation model would automatically dictate the types of alternations to the goal programming model performed

interactively for the problems analyzed in Chapter V. The basic goal programming model would thus remain unaltered, becoming time-dynamic only through the external formulation of a series of unique decision environments.

At the present time, this approach to the development of a dynamic goal programming model poses a series of very complex problems. To develop such a model, an existing dynamic simulation model must first be converted to the APL computer language, thereby allowing interface with the goal programming computer program used in this analysis. Only after an adequate dynamic simulation program was available in the APL language could modification be made to allow integration of the goal programming algorithm used in this study. Nevertheless, this approach remains the most promising avenue for further research and application of the goal programming technique.

# NOTES - CHAPTER V

<sup>1</sup>George L. Nemhauser, <u>Introduction to Dynamic</u> <u>Programming</u>, (New York: John Wiley and Sons, Inc., 1966).

### CHAPTER VI

### CONCLUSIONS

The use of a goal programming model for analysis of the educational budget of Wise County was intended primarily to demonstrate the potential for and the difficulties involved in such public sector applications. The analysis however has revealed some possible tradeoffs in expenditures for the budget categories examined. In relation to those of Arlington County, expenditures for Administration and Fixed Costs are relatively low in Wise County, while the allocation for Transportation and Food Services is extremely high in comparison. These relationships conceivably are a product of the stable, rural nature of Wise County, in contrast with the urban characteristics of Arlington County. However, given the goal of increasing expenditures for Instruction, it is possible that a reduction in funding for Transportation and Food Services is one alternative by which to obtain the desired funds. The most pervasive constraint to improving educational quality in Wise County is the limit upon total expenditures. Additional funding for education is clearly necessary if the quality of education available is to be significantly improved. Yet, as indicated by the index of effort, current local

contributions for education are relatively high. Therefore, the most appropriate alternatives may be to attempt to expand the local tax base, or to seek outside sources for additional funding.

Goal programming approaches to public sector decisions have rarely been utilized due primarily to the difficulties existent in adapting a broad range of concerns to the analytical limitations of the technique. Where the approach has been applied in the public sector, an artificial definition of the decision environment has often been used in an attempt to overcome adaptive difficulties. In terms of the model, the most significant limitation is the requirement that goals and constraints be expressed as linear functions. Obviously, a relatively small proportion of the situations requiring a decision model possess characteristics expressable as linear functions. Secondly, the preemptive goal priority structure utilized in goal programming is often an insufficient representation of the preferences of decision-makers, thus detracting from the legitimacy of the method. Goal programming solutions to decision problems can be regarded as valid only to the extent that the decision environment can be accurately represented by a linear-based goal programming model.

Among the potential public sector uses for goal programming are the evaluation of tradeoffs involved in

budgetary decisions, and the determination of the adequacy of projected future revenues for meeting specified objectives. To be used effectively, the model requires that the relative importance of objectives be established, and most importantly, that the critical interrelationships of all variables be quantitatively defined. This type of quantitative information does not exist within a typical public sector decision environment, however. Without such interrelationships, the goal programming model does little but perform simple arithmetic operations, providing no new information to decision-makers.

Even if all requirements of the model could be met, significant probelms of application would nevertheless remain. Initially, the model simulates a static decision environment. However, the considerations relevant to public sector decisions are often dynamic in nature. Interrelations of variables may be in constant flux. While this difficulty can potentially be resolved through the use of a simulation model in conjunction with a goal programming algorithm, this approach is appropriate only when a series of decisions are involved. Such a model would in effect utilize a chronological progression of static analyses. For any single decision, the goal programming model remains static in nature. Secondly, variables and relationships necessary for the use of goal programming are often unavailable. When

obtainable, such data are often based upon projections or estimates of questionable reliability. Thirdly, typical public sector decision environments do not meet the assumptions of the model. Since the goal programming approach is relatively inflexible, characteristics of the decision environment must sometimes be distorted in order to apply the technique. Yet, the greatest single problem in accurately modeling a public sector decision environment is a general inability to quantify crucial interrelationships among the decision variables. Often this difficulty may be traced to the lack of an adequate objective function for public sector expenditures. In other cases, adequate data are simply unavailable. Regardless, the application of goal programming to typical public sector decisions for which information on the interaction of variables cannot be included in the model is of little value to decision-makers.

Based upon the study of the Wise County educational budget, public sector decision-makers lack information essential to the effective use of the goal programming technique. Basic requirements of the model cannot be met for the type of public sector decision investigated. In addition, the characteristics of a typical public sector decision environment do not conform to those characteristics required for effective application of the goal programming technique. As a result, it is concluded

that the application of a goal programming model to a typical public sector decision produces no unique analytical results, and therefore is an inappropriate use of the technique.

From the problems and limitations associated with the use of goal programming for public sector decisions, it is apparent that further study of the approach is needed. In particular, new information or methods for providing data on the interaction of public sector decision variables would be particularly useful. The development of a simple yet accurate dynamic analytical model using goal programming would be a valuable expansion of the technique, but the barriers to public sector application would nevertheless remain.

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# APPENDIX A: APL COMPUTER PROGRAM

Pages 57 through 62 contain the APL computer program used to conduct the goal programming analyses in this study. A brief description of the operation of this program is given following the APL listing.

V PROG; L;C;R;P;NN;RC;T [1] L1: DO YOU WISH TO FORMAT INPUT? (Y/N)' 1 - 1 [2][3]  $\rightarrow FIVP \times iI = !Y!$ [4] SPECIFY INPUT NAME! [5]  $X \leftarrow X \leftarrow \overline{1}$ [6] SIDEV+CH+CM [7] PRINTOUT X 'IS THIS THE CORRECT INPUT DATA? (Y/M) ! [8] [9]  $\rightarrow L 1 \times \iota \eta = 1 N^{\dagger}$ [10]→3£GIN [11]  $FINP: XX \leftarrow X \leftarrow GPINPT$ [12] BEGIN: NN+(pX)[2] [13] $I \leftarrow M + MM$ [14] PH1 [15] STAPT: P-PLEV [16] →0PT×6P=999 [17] $I \vdash (MM + M + 1) - P$ [13]  $RC \leftarrow (P, I) POINT X$ [13]  $\rightarrow OPT1 \times \iota (\rho RC) = 3$ [20]  $R \leftarrow RC[1]$ CGRC[2] [21][22]AA+SIMPLEX X [23] $SIDEV[R] \leftarrow C-1$  $[24] OPT: \rightarrow OPSOL \times \iota I \leftarrow CHKOPT$ [25] $X \leftarrow AA$ [26] →START [27] OPSOL: THE OPTIMAL SOLUTION IS: [28] PRINTOUF AA [2] END:GOALACH [30] RESET: DO YOU WISH TO RERUN PROGRAM WITH FIFFERENT PRIORITIES? (Y/N)[31] --.)×1 T≠ Y \* [32] RESET 2: A+XX[ LM; ] [33]X-SETOF [34]-PRINTOUT X \*ARE CORRECT PRIORITIES AND WEIGHTS SHOWN? (Y/N)\* [35] →RESET×L [=\*N\* [36] [37] → BEGIN  $[ ] 3 ] OPT 1: \rightarrow OPT \times \iota (P \leftarrow P + 1) \leq M$ [3]] → JPS OL

V

```
V PRINTOUT X;Q;SW;SP;R
[1]
        2 101 1
\begin{bmatrix} 2 \end{bmatrix}
        Q \leftarrow ((4, (pPROW))p0), [1]X
[3]
        Q[R+1;]-0, LN
[4]
        Q[2;]←WTROW
        Q[3;]⊢PROW
[5]
[6]
        2←(7,1,((2×N)p3 0)) ▼Q
     OSET:Q[R; (9]+*
[7]
[8]
        \rightarrow QSET \times \iota (R \leftarrow R + 1) \leq 3
[9]
        SV+2 00(((oSIDEWT)+4+MM),1)p0,0,0,0,0,SIDEWT,(MMp0)
[10]
        S#[1 2 3;]-! !
[11]
        SV[2;((pSV)[2])]+'W*
[12]
        S = \{(pSW)[1]\} + 1 - cMM; ] \leftarrow 1
        SP \leftarrow 2 = 0 \bullet (((pSIDEP) + 4 + MM), 1) p 0, 0, 0, 0, 0, SIDEP, ((MM+1) - \iota MM))
[13]
       SP[1 2-3;]-1 1
[14]
        SP[3;((pSP)[2])] \leftarrow P^*
[15]
[16]
        VAR+2 = 0 \bullet ( (\rho SP) [1], 1 ) \rho 0, 0, 0, 0, SIDEV, (MM \rho 0 )
[17]
        VARL1 2 3; ]+1 1
[18]
        VAR[1;]+*VN*
        VAE[ ( oVAR) [ 1 ]+1-iMM; ]+* *
[19]
        Q-VAR;[2](SW,[2](SP,[2]*['),[2]'['),[2]0
[20]
        Q[4;]-*=*
[21]
[22]
        Q
        2 10 1
[23]
     17
```

[21]N-7 CII-N[1] [3]  $N \leftarrow + / N$ [4]. [5] TENTER NUMBER OF GOAL CONSTRAINTS! [6] M←Γ [7] A←(M,V),00 L1: 'ENTER '; M; ' LEFT-HAND-SIDE COFFICIENTS' [8] [9] →L1×i(µLHS+7)≠M I THERE ARE I'N' CONSTRAINT COEFFICIENTS FOR EACH OF ''M'' [[1]] CONSTRAINTS \* ENTER ONE ROW AT A TIME\* [11] [12]1-1 [13] L2: AL I; (N]-7 -→1,2× 1 M≥ I+1+1 [14] A+LHS,A [15]

INPUT MATRIX, WITH OBJECTIVE FUNCTION:

'ENTER NUNBER OF CHOICE AND DEV. VARIABLES AS 2 ELEMENT

```
\nabla x \leftarrow SEIOF; I; PLS; WTS; PS
```

 $[1] = PS \leftarrow ((5,0), ((2 \times N) \rho 3, 0)) = ((2, (N+1)) \rho(0, iN)), [1] A$ 

[2] PS[1; 15]+V/N

 $\forall \forall X \leftarrow GP I \lor P T : T : L \sqcup S$ 

VECTOR!

[1]

[16]

[17]

[18] [19]

 $[3] PS[2;] \leftarrow ! = !$ 

A-SETOF

 $X \leftarrow A$ 

PRINTOUT A

[4] PS

[5] LI: VARIABLE NUMBERS FOP PRIORITY COEFFICIENTS\*

 $[6] \rightarrow L1 \times \iota(\rho CN + 1 + \Box) > N$ 

[7] L2: MENTER PRIORITY LEVELS FOR VARIABLES "; CN-1

 $[8] \rightarrow L2 \times \iota(\rho^2 LS \leftarrow \Box) \neq \rho CN$ 

 $[9] \qquad PROW \leftarrow WT ROW \leftarrow (N+1)\rho 0$ 

 $[10] PROW[CN] \leftarrow PLS$ 

```
[11] L3: PENTER WEIGHTS FOR THESE PRIORITIES!
```

```
[12] \rightarrow L \exists \times \iota (\rho \forall T S \leftarrow \neg) \neq \rho C N
```

```
[13] WTROW[CV]+WTS
```

```
[14] = SIDEP - SIDEWT - Mp0
```

[15] 1-1

```
[16] L4: SIDEP[I] \leftarrow ((Z \leftarrow ((A[I;] \neq 0) \times PROW) \neq 0) / PROW)[1]
```

```
[17] = SIDEWT[I] \leftarrow (Z/WTROW)[1]
```

```
[18] \rightarrow L4 \times \iota(I \leftarrow I+1) \leq M
```

[19] SIDEV-CH+LM

 $[20] \quad X \leftarrow OBFTN$ 

```
V PP+PLEV; C; ZZ
[.1]
         C \leftarrow X [MM + M + 1 - \iota MM; 1]
[2]
         ZZ + X[; 1 + iN]
[3] = STAET: \Rightarrow RO \times C[P] = 0
         \rightarrow RO \times \iota \sim (1 \epsilon \times ZZ[MM+M+1-P;])
[4]
[5]
         PP⊷P
161
         [7]
      RO:→START×L(P←P+1)≤MM
[8]
         PP€993
```

▼ XHOUFTN; I; OA; OA2

 $\rightarrow \neg \exists \forall J \times \iota (I \leftarrow I + 1) \leq MM$ 

0 + ((MM+1), (N+1)) = 0

 $0A2 \leftarrow ((MM+1), (N+1)) \rho 0$ 

 $\rightarrow O H J 2 \times \iota (I \leftarrow I + 1) \leq (N + 1)$ 

OUJ2: 042[ (MM+1)-PROW[ I ]; I]+WTFOW[ I ]

MA- PROW

0A ← 0A − 0A 2

A - A, [1] O A

OA+OA[ &MM; ]

 $I \leftarrow 1$ 

I = 2

 $X \leftarrow A$ 

 $\begin{bmatrix} 1 \end{bmatrix}$ 

[2]

[3] [4]

[3]

[6]

[7]

[8]

[9]

[10]

[11]

[12]

[13]

 $\nabla$ 

 $\nabla$ 

```
60
```

 $OBJ:OA[(MM+1)-SIDEP[I];]+OA[(MM+1)-SIDEP[I];]+SIDEWT[I] \times A[I;]$ 

7 RC+PT POINT X; I;C; R; NOZ; RR; ZZ; CC; TC; P; PC  $P \leftarrow PT[1]$ [1]  $I \leftarrow PT[2]$ [2] C C ← L 0 [3] START: PC-C-(+(NN-1))+1 [4] COLPOINT: C+(([/X[I;C])=X[J;C])/C [5] CC+C,CC [5]  $\rightarrow ROMPOINT \times \iota(\rho C) = 1$ [7] -COLPOINT×L (I-I-1)>MM 131  $\rightarrow NOTRANS \times (\rho C) = 0$ [9] $C \leftarrow CC \leftarrow (PC \in CC) / PC$ [10] [11] CHECKCP: TC-CC[1] $\rightarrow ROWPOINT \times \iota \sim (-1e \times (X[(MM+M+\iota P)-P;7C]))$ [12] [13] $C \leftarrow (C \neq TC) / C$ [14]CC←1 ↓CC [15] $\rightarrow CHECKCP \times (\rho CC) \neq 0$ [16] →COLPOINT [17] ROVPOINT: C+C[1] [18] $NOZ \leftarrow \{(, X[LMM; C]) > 0\} / L(p(, X[LMM; C]))$ [19] = R + ((1/x[NOZ;1] + x[NOZ;C]) = (x[NOZ;1] + x[NOZ;C]))/NOZ[20] $\neg TRANSFORM\times i(pR)=1$ [21] $\rightarrow NOTRANS \times \iota (\rho R) = 0$  $[22] RP3:R \leftarrow (([/SIDEWT[R]) = (SIDEWT[R]))/R$ [23] $\Rightarrow TEANSFORM \times \iota(\rho R) = 1$ . [.24] $I \leftarrow 1$ [25] 23-60  $[26] RP 2:RR \leftarrow (SIDEP[R[I]] = PROW) / LNN$ [27] ZZ+ZZ, RR [28]  $\Rightarrow R P 2 \times \iota (I \leftarrow l+1) \leq \rho R$ [29] $\mathbb{R} \leftarrow \left( \left( \left( \int W T R O W \left[ Z Z \right] \right) = W T R O W \right) / \iota N N \right) \begin{bmatrix} 1 \end{bmatrix}$  $R \leftarrow (PROW[R] = SIDEP) / \iota (\rho SIDEP)$ 1.301 [31] TRANSFORM:  $R \leftarrow R[1]$ [32]KJ+R,C [33] 

[34] NOTRANS: RC+3ρ0

V

 $\nabla AA \leftarrow S IMPLEX X; I$ [1]  $R \rightarrow (A \neq \iota (M + MM)) / \iota (M + MM)$ [2]  $AA \leftarrow (pX) \Rightarrow 0$ [3]  $AA \{R; 1 \leftarrow X[R; ]; X[R; C]$ [4]  $I \leftarrow (pRS)[1]$ [5]  $GO: NR \leftarrow RS[I]$ [6]  $AA [NR; ] \leftarrow X[NR; ] - X[R; ] \times X[NR; C]$ [7]  $\rightarrow GO \times \iota (I \leftarrow I - 1) > 0$ 

```
∇ V ← CHKOPT;Z;I;S;J;C
[1]
          \rightarrow OPT \times \iota \theta = + / Z \leftarrow (\theta \neq AA[MM + M + 1 - \iota MM; 1])
[2]
             1-1
[3]
           RO: \rightarrow COL \times \iota \left( 1 \in S \leftarrow \times (1 \downarrow A \land [MM + M + 1 - (, (Z/\iota MM))[T]; ]) \right)
[1]
          LOOP: \rightarrow RO \times \iota (I \leftarrow I + 1) \leq + / ?
[5]
        .0P1:V ← L
            -+ ()
[6]
[7]
         COL: C \leftarrow (1=S) / 1 + i (DS)
131
             J \leftarrow 1
[9]
           COLCK: \rightarrow END\times\iota \sim ([1e \times AA[MM+M+1-\iota((, (7/\iota \forall M))[I]); C[J]])
[10]
          \rightarrow C \cup L \subset \mathcal{K} \times \iota (J \vdash J + 1) \leq \rho C
            -+L00P
\begin{bmatrix} 11 \end{bmatrix}
[12] ENO:I ← 1+ MM
[13] L2: \rightarrow SETP \times iAA[I;1] > X[I;1]
[ [ ] ]
            \rightarrow 1/2 \times \iota (I \vdash I \vdash 1) \geq M + MM + 1 - P
[15] E3:VHO
[16]
            -> ()
[17] SETP: A \leftarrow X
[18]
            \rightarrow OPT \times \iota P \geq M
[13]
          P \leftarrow P + 1
[ 30]
            +1.3
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4.
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- V GOALACH; I;Z;GA

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[1]
          2 101 1
          'GOAL ACHIEVEMENT EVALUATION: '
[2]
[3]
          1-1
[4]
        GO: \rightarrow L1 \times \iota \left( Z \leftarrow \left( AA[(MM+M+1)-I;1] \right) \right) = 0
          \rightarrow L \Im \times \iota \Im = A [MM + M + 1 - I; 1]
[5]
[6]
         \rightarrow L 2 \times \iota (Z > 0) \wedge (Z < A [(M M + M + 1) - I; 1])
[7]
          -1.3
[8]
        L1: GA \leftarrow ^{\dagger} FULLY ^{\dagger}
[9]
         →L4
[10] L2: GA+(@(l(1-|Z+(A[MM+M+1-T;1]))×100)), * PERCENT *
[11] -14
[12] LJ: GAH NOT !
[13] L4: 'GOAL '; I; ' : '; GA; 'ACRIEVED'
[14] \rightarrow GO \times \iota (I \leftarrow I + 1) \leq MM
          2 Lp * *
[15]
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. 7 The APL computer program used to conduct the goal programming analyses is composed of ten individual functions. Each of these functions performs a specific role in converting input data into a goal programming solution. The operations performed by each function are summarized below. Functions are listed in the order in which they are first called during program operation.

PROG: This is the main function in the APL program. As such, it calls other functions as needed to solve the goal programming problem. It allows for options regarding the formatting of input data, and restructuring priorities.

PRINTOUT: Compiles and formats matrices for visual display. This function is used for both input and output data.

GPINPT: Receives initial data concerning the constraints of the goal programming problem.

SETOF: Receives data on goals, priorities and weights necessary to formulate the objective function.

OBFTN: Constructs the objective function from the input data supplied.

PLEV: Determines the priority level for which to maximize goal achievement during each iteration of the model.

POINT: Determines the row and column used as the pivot point for the matrix transformation accomplished by SIMPLEX.

SIMPLEX: Transforms the goal programming matrix to maximize goal achievement for each priority level. This function is in fact the heart of the model, performing the actual analysis.

CHKOPT: At each iteration, checks the goal programming matrix to determine if an optimum solution has been reached.

GOALACH: Evaluates the extent of goal achievement in percentage terms, and displays the results.

In addition to the functions listed above, the program employs a number of global variables to record priorities, weights, column numbers, matrix size, etc. However, it is more important to note that the program, like the goal programming model, is iterative in nature. As a result, many loops are often required to obtain an optimum solution. The APL program used in the analysis is based upon the simplex method for solving goal programming problems. The first step of the simplex method is to formulate constraint equations by introducing deviational variables.<sup>1</sup> At this phase of the simplex procedure, the value of all decision variables is set to zero. Based upon variable coefficients, the solution base, or objective function of the goal programming matrix is formulated. A series of iterative matrix transformations are then conducted around specifically determined pivot points. The optimum solution is reached when no further transformations can occur without detracting from previously established levels of goal achievement. For a complete explanation of the simplex method of goal programming, refer to Lee, <u>Goal Programming</u> for Decision Analysis.<sup>2</sup>

The APL algorithms shown above comprise an interactive computer program designed for use on high-speed terminals. As such, the program allows for the rapid modification of the goals and constraints for goal programming problems. This potential for rapid comparison of altered goals and constraints greatly facilitates the analysis of available trade-offs. The use of the APL language thus increases the analytical value of the goal programming model by producing optimum solutions far more rapidly than a

FORTRAN-based computer program requiring card input.<sup>3</sup> Obviously, the rapid turn-around time of the APL program facilitates error detection, greatly reducing time spent on non-productive runs.

# NOTES - APPENDIX A

<sup>1</sup>Refer to Chapter III for a description of deviational variables.

<sup>2</sup>Lee, <u>Goal Programming for Decision Analysis</u>, pp. 93-125.

<sup>3</sup>A FORTRAN-based computer program for goal programming can be found in Lee (<u>supra</u>, pp. 140-157).

# APPENDIX B: COMPUTER INPUT AND OUTPUT

Computer input and output for the nine goal programming problems analyzed in Chapter V are contained below. The following listing indicates the page on which data for each problem may be found.

Solution	One	٠	•	•	•	•	•	•	• .	•	•	•	69								
Solution	Two	•	•	•	•	•	•	•		•	•	.•	70								
Solution	Thre	ee	•	•	•	.•	•	•	. •	•	.• .:	•	71								
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Iteration	ъВ	•	•	•	·	•	•	•,	•	•	•	•	73								
Iteration	n C	•	•	•	•	•	•	•	•	•	•	•	74								
Iteratior	ı D	•	•	•	•	•	•	•	•	•	یں۔ •	•	75								
Iteration	ı E	•	•	•	•	•	•	•	•	•	•	•	76								
Iteratior	ı F	•	•	•	•	•	•	•	•	•	•		77								
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GOAL	2	:	FULLY ACUTEVED
COAL	3	:	TULLY ACTIEVID
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COAL	5	:	PULLA ACHERVEL
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## A CRITIQUE OF GOAL PROGRAMMING FOR PUBLIC SECTOR DECISIONS

by

## Timothy Patrick Roberts

## (ABSTRACT)

The purpose of this investigation was to evaluate the applicability of goal programming to public sector types of allocation problems. In the process of this study, the educational budget of Wise County, Virginia was analyzed via goal programming. In addition, the potential for the development of a time-dynamic goal programming model was explored.

Results of the investigation indicate that rigidity of the model and lack of adequate data severely limit potential public sector applications of the goal programming technique. Development of a time-dynamic goal programming model was deemed possible only by incorporating goal programming into a dynamic computer simulation model.

An APL computer program designed to perform the goal programming analysis, and a description of its operation are also included.