

Multiphysics model of hypervelocity impact and impact induced plasma formation

Shafquat T. Islam

Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Aerospace Engineering

Kevin G. Wang, Chair
Charbel Farhat
Gary D. Seidel
Wayne A. Scales

April 3, 2025
Blacksburg, Virginia

Keywords: Multiphysics, hypervelocity impact, plasma formation, material model calibration

Copyright 2025, Shafquat T. Islam

Multiphysics model of hypervelocity impact and impact induced plasma formation

Shafquat T. Islam

(ABSTRACT)

Previous studies on hypervelocity impact (HVI) have primarily focused on the mechanical response of solids, namely the projectile and the target. While several experiments have observed plasma formation and associated electromagnetic (EM) emissions, key questions remain unresolved — including the source of ionization (projectile, target, or ambient medium), the plasma composition, and the spectral radiance of emitted EM waves. Addressing these questions requires expanding current models to incorporate extreme thermodynamics, ionization, and chemical reactions.

Most research in HVI has focused on exoatmospheric applications, where the ambient environment is typically assumed to be vacuum. For example, satellite-chondrite collisions often exceed impact velocities of 7 km/s, justifying this assumption. However, such simplifications are no longer valid for terrestrial or atmospheric applications of HVI. Developing a complete physical model requires expanding the analysis to include the surrounding fluid and its interaction with the solid materials. This dissertation presents a novel computational framework to simulate hypervelocity impacts and the resulting plasma formation within the projectile, target, and ambient fluid. A fully coupled fluid-solid simulation platform has been developed and applied to study metallic rods impacting soda-lime glass (SLG) in a chemically inert environment at hypersonic velocities. The framework addresses challenges such as interface tracking under large deformation and discontinuities in state variables and material models. Each of the three material subdomains — metallic projectile, silicate target, and gaseous environment — is modeled using a distinct equation of state (EOS) to accurately capture their thermodynamic behavior.

In particular, SLG is modeled using an extended Noble-Abel stiffened gas EOS that supports a nonlinear relationship between the Grüneisen parameter and specific volume. Its parameters are calibrated using shock compression experiments conducted in the Lindhurst Laboratory at Caltech with a two-stage light-gas gun, which measured pressures up to 110 GPa and temperatures up to 5300 K. Plasma formation is modeled using the non-ideal Saha equation, solved under constraints of conservation of charge and nuclei. This allows for quantification of plasma temperature, charge density, Debye length, and elemental composition. An extended level-set method with dual signed-distance functions is employed to track interfaces, accommodating large deformation, contact, and separation between materials. The Euler equations are solved using a high-resolution finite volume method. Interfacial fluxes are computed using the FInite Volume method with Exact multi-material Riemann solvers (FIVER). All modular components of the framework have been independently verified or validated against analytical solutions, published numerical results, or experimental data. A series of parametric studies were performed to investigate the effect of impact velocity, projectile radius, and material type on plasma characteristics and thermal response in the solid and fluid domains. Across velocities ranging from 2 to 7 km/s, the results show that ionization in the ambient gas becomes significant above 4 km/s. Under these conditions, plasma density in the fluid is found to be two orders of magnitude greater than in the solids. These results confirm that at moderate impact velocities, the ambient gas is the dominant source of impact-induced plasma. It was also found that within the SLG target, ionization is primarily driven by metallic constituents such as sodium, magnesium, and calcium, which contribute over 99% of free electrons despite comprising less than 15% of the total mass.

This work assumes local thermodynamic equilibrium (LTE), a valid approximation in the early stages of impact when hydrodynamic forces dominate. Electromagnetic forces and plasma self-consistency are not resolved in the present model but can be incorporated by extending the solver to include the Vlasov–Maxwell equations. This would enable modeling of electrodynamic forces and the propagation of EM waves as the plasma expands and transitions into a non-collisional regime.

Multiphysics model of hypervelocity impact and impact induced plasma formation

Shafquat T. Islam

(GENERAL AUDIENCE ABSTRACT)

Hypervelocity impacts occur when a projectile strikes a target at speeds exceeding 2 km/s—nearly six times the speed of sound under standard atmospheric conditions. These high-speed collisions involve rapid transfer and dissipation of energy through mechanical, thermal, chemical, and electromagnetic processes. They generate extremely high temperatures and pressures not only in the solids (i.e., the projectile and target), but also in the surrounding gas. The extreme conditions arise from strong pressure waves—known as shock waves—that travel through the materials during impact. These waves compress and heat the solids and surrounding gas, often reaching temperatures of thousands of Kelvins—similar to the surface of the Sun—and pressures tens of thousands of times greater than atmospheric levels. Under such conditions, atoms can ionize—electrons separate from nuclei—forming a dense, highly energetic state of matter called plasma. Although plasma from impacts has been observed in laboratory experiments, its source, composition, and behavior are still not fully understood.

This dissertation presents a computational approach to simulate hypervelocity impacts in a gas environment and to predict when and where plasma forms, along with its key properties such as temperature, density, and composition. The model focuses on a metallic projectile striking a soda-lime glass (SLG) target surrounded by a chemically non-reactive gas.

Under the extreme pressures of impact, even solid materials behave more like highly compressed fluids. To realistically model this behavior, each material in the simulation—the projectile, target,

and gas—must be modeled to reflect their unique physical behavior. Because no reliable data existed for how SLG behaves under such extreme conditions, a series of experiments were carried out. These tests involved shooting high-speed projectiles at SLG and measuring the resulting pressures and temperatures. The data were then used to guide how SLG is modeled in the simulation.

To predict plasma formation, a model was developed based on these materials' ionization behavior at high temperatures and pressures. This was integrated with the impact simulation to capture the onset and distribution of plasma across all materials.

The model was used to study how changes in impact velocity, projectile size, and material affect plasma formation. Results show that above 4 km/s, the gas surrounding the impact becomes the main contributor to the plasma—contrary to many past studies that ignored its role. In the glass, plasma is mostly generated by a few metallic elements like sodium, magnesium, and calcium, which release over 99% of the free electrons despite making up less than 20% of the material.

While this work does not include electromagnetic forces, the simulation could be extended in the future to study how electric and magnetic fields evolve and interact with the plasma over time.

Dedication

To my parents, without whose tireless efforts and endless sacrifice none of this would be possible.

To my mentors, who I have been lucky to have found at critical junctions of my life, who have shaped my world view and fanned my flame. To my fur babies Leo and Louis, the most loving cats who have forced me to find the motivation to keep going and get out of bed, some days against my will.

Acknowledgments

First and foremost, I would like to thank my advisor, Dr. Kevin G. Wang, for his patience, support, and kindness. His incredible insight and mentorship not only shaped my understanding of computational physics but also life in general. His uncanny knack of presenting complex ideas in an effortlessly uncomplicated manner gave me faith in my own abilities.

I would also like to extend my gratitude to the other members of my graduate committee members. I thank Dr. Charbel Farhat for encouraging me to think more deeply about my research and consider details I would have missed. I thank Dr. Gary Seidel for his rigorous review and insightful feedback which has greatly enhanced the depth of this work. I would also like to thank Dr. Wayne Scales, for not only being an integral member of this committee, but also for inspiring me in his class and solidifying my foundations in plasma physics.

I am also deeply thankful to my collaborators whose invaluable contributions to my work have made me the researcher I am today, and whose work will also be presented in this dissertation. I thank Dr. John G. Michopoulos for being a constant guide throughout my doctoral career and whose expertise in multiphysics approach to impact engineering has enabled my line of research. Furthermore, I would also like to thank Dr. Paul Asimow for his incredible theoretical and experimental insight into material behavior under extreme conditions, which was an integral part of dissertation.

I am also indebted to the team here at the “Multiphysics Modelling and Computation” lab. I thank Dr. Wentao Ma and Dr. Xuning Zhao, who have patiently shown me the ropes when I was a younger doctoral student. I thank Aditya Narkhede for being ever present during long frustrating days of debugging my code. I thank Erick Rivas, for always indulging my requests to procrastinate and being an amazing partner (and opponent) in table tennis, although his skills in chess may require some attention.

I would like to thank my family back home, who have supported me and been my pillars of strength throughout these years. I thank my cats, who have never failed to comfort me when I have needed them. I would like to thank all the friends I have made in Blacksburg, for helping me find a home on the other side of the world. I recognize that my achievements are not only my own but also those of everyone I have named and some I haven't. For all this love, I am truly grateful.

Contents

List of Figures	xii
-----------------	-----

List of Tables	xvii
----------------	------

1	Introduction	1
1.1	Background	1
1.2	Literature Review	3
1.3	Motivation and Objective	7
1.4	Accomplishments and Outline	9
	Bibliography	12
2	Plasma Formation in Ambient Fluid from Hypervelocity Impacts	17
2.1	Introduction	18
2.2	Physical model and computational methods	21
2.3	Results and discussion	29
2.4	Conclusions	37
	Bibliography	39
3	Fluid-Solid Coupled Simulation of Hypervelocity Impact and Plasma Formation	47

3.1	Introduction	49
3.2	Materials and methods	55
3.2.1	Continuum dynamics	55
3.2.2	Thermodynamic models	58
3.2.3	Interface tracking and treatment	60
3.2.4	Ionization model	61
3.3	Solver verification	66
3.3.1	An infinite ideal plasma	66
3.3.2	1D multi-material hypervelocity impact analysis	68
3.4	Fluid-solid coupled impact analysis	73
3.5	Conclusion	80
	Bibliography	81
4	Ionization Induced by Fluid-Solid Interaction During Hypervelocity Impact	88
4.1	Introduction	90
4.2	Computational Model	95
4.2.1	Continuum mechanics	95
4.2.2	Equations of state and temperature laws	98
4.2.3	Interface tracking and treatment	102
4.2.4	Ionization model: Non-ideal multi-species plasma	105

4.3	Hypervelocity shock compression experiments	111
4.3.1	Equation of state experiments	112
4.3.2	Pyrometer experiments	114
4.3.3	Discussion of experimental results	117
4.4	Calibration of Equation of State for Soda-Lime Glass	120
4.5	Trimaterial shock compression simulations	124
4.6	Hypervelocity Impact Simulations	128
4.6.1	Energy transfer and partition	130
4.6.2	Impact-induced heating and plasma formation	135
4.6.3	Ionization of SLG target	138
4.7	Conclusion	141
	Bibliography	143
5	Conclusion	156
5.1	Concluding Remarks	156
5.2	Perspectives for future work	158
	Bibliography	161

List of Figures

2.1	Hypervelocity impact: Problem setup. (a) Hypervelocity impact flash captured in a laboratory experiment [7]. (b) The model problem investigated: A long copper rod impacting on a soda lime glass target in neon gas. The impact velocity, V_0 , is varied between 3 km/s and 6 km/s. (c) An overview of the computational multiphysics model used in this study, which accounts for the dynamic interaction between the solid and fluid (gas) materials and fluid ionization.	22
2.2	Major components of the computational framework. (a) Simulation of the hypersonic fluid flow surrounding the projectile before the collision through a steady-state, body-fitted CFD analysis. (b) Initialization of the fluid-solid coupled impact analysis. The fluid density, velocity, and pressure fields obtained from the CFD analysis are mapped to a new mesh that does not resolve the boundaries of the projectile and the target. (c) Implicit tracking of the three material interfaces between the projectile, the target, and the ambient gas by solving two level set equations that share the same velocity field. (d) Solution of the Saha equations for neon (Ne) at the atmospheric pressure and different temperatures between 0 and 6000 K. The plotted result (α) is the molar fractions of neon atoms and ions with different charge numbers (0 - 8 shown here).	26

2.3	Time evolution of state variables for a hypervelocity impact of copper projectile hitting an SLG target at 5 km/s. From left to right, the pressure, velocity magnitude, temperature, and average charge fields are shown at times of 0.625 μs , 1.25 μs , 2.5 μs , and 2.5 μs (top to bottom). The temperature and average charge fields are computed only in the subdomain occupied by neon, as the present study focuses on the ionization of the ambient fluid.	32
2.4	Formation of a neon plasma pocket shortly after collision. (a) State variable fields at $t = 0.625 \mu s$, showing a pocket of neon compressed by the hypervelocity projectile ($V_0 = 5.0$ km/s) and the target, resulting in high velocity, high temperature, and the formation of a dense plasma. (b) A rudimentary analysis explaining the high fluid velocity in the pocket.	33
2.5	Effects of the impact velocity (V_0) on the thermodynamics and ionization of neon. (a) Location of three virtual sensors (A, B, and C) in neon. (b) Time history of pressure at Sensor A. (c) Peak temperature at each sensor (d) Peak average charge at each sensor	35
3.1	Problem setup. (A) A 3D depiction of the target, projectile, and ambient fluid in the model problem. (B) A 2D cross-section with annotated geometry and probe locations.	54
3.2	Velocity and pressure in ambient fluid for 3D steady state CFD analysis shown on the left. Mapped data on 2D domain after changing frame of reference — initial conditions for fluid-solid coupled impact analysis shown on the right	57
3.3	Contours of the level set functions for a representative impact simulation ($r_p = 5$ mm, $V_0 = 5$ km/s). A) $t = 0.4 \mu s$. B) $t = 1.25 \mu s$. C) $t = 2.5 \mu s$. D) $t = 5 \mu s$. . .	62

3.4	Multiphysics model overview. The role of the different solvers (M2C and AERO-F) is illustrated, along with the details of modular components and their interactions within M2C, i.e. the thermodynamics model (EOS), continuum dynamics model, boundary tracking algorithm, and ionization model.	65
3.5	Molar fraction (α) of ionic species and mean charge (Z_{av}) of ideal argon plasma at 1 atm (101 kPa) pressure, for temperature $T \leq 5 \times 10^4$ K. A) Present work. B) Plot recreated from Zaghloul <i>et al.</i> [51].	68
3.6	Numerical and analytical solution of a one-dimensional hypervelocity impact of a tantalum projectile ($V_0 = 3$ km/s) and SLG target in atmospheric argon environment	71
3.7	Solution snapshots obtained from a representative impact simulation ($V_0 = 5$ km/s).	72
3.8	Time history of pressure and temperature at probe 2 for $V_0 = 5$ km/s	74
3.9	Shock temperature vs. shock pressure values at Probe 1, in comparison with experimental results presented in Kobayashi et al. [28]	75
3.10	Maximum values of temperature (T) and mean charge number (Z_{av}) within the ambient argon gas for $V_0 = 5$ km/s at probe 3.	77
3.11	Mean charge of impact-generated plasma for variable projectile radii.	79
4.1	Setup of the fluid-solid coupled impact problem investigated in this study	92
4.2	Initialization of fluid-solid coupled impact simulations: Fluid conditions obtained by a 3D steady-state computational fluid dynamics (CFD) simulation ($U_0 = 5$ km/s in this example)	97

4.3	Implicit interface tracking by multiple (2) level set functions. (A) ϕ_1 tracks the SLG target subdomain: $\phi_1 < 0$ inside the subdomain and $\phi_1 > 0$ outside the subdomain. (B) ϕ_2 tracks the tantalum projectile subdomain: $\phi_2 < 0$ inside the subdomain and $\phi_2 > 0$ outside the subdomain. (C, D, E) Time evolution of the iso-contours of ϕ_1 and ϕ_2 , reflecting the motion and deformation of each material subdomain.	104
4.4	Overview of the iterative procedure for computing the composition of non-ideal plasma	109
4.5	Verification of the non-ideal Saha equation solver implemented in M2C: Dependence of the composition of a 5 eV non-ideal plasma mixture [He(0.3):Ne(0.6):Ar(0.1)] on density. Reference: Zaghloul [68]	111
4.6	Experimental setup of the equation of state shots	112
4.7	Representative streak record of shot 1117	113
4.8	Experimental setup of the pyrometer shots. Note that this configuration allows pre-heating of the target but the data reported herein all used room-temperature targets .	116
4.9	Schematic timing of shock wave propagation shown on a $x - t$ diagram	117
4.10	Experimentally measured Hugoniot states of SLG in (A) $p - V$ space and (B) $U_s - U_p$ space	119
4.11	Measurement of pressure and temperature of shocked SLG	119
4.12	Shock Hugoniot data predicted by the calibrated EOS, in the $U_s - U_p$ space (A) and the $p - V$ space (B)	121
4.13	Temperature of shocked SLG as a function of pressure	123

4.14	Evolution of the density (A), pressure (B), and temperature (C) fields obtained from the 1D shock compression simulation, for impact velocity $U_0 = 5$ km/s	125
4.15	State of the computational domain at 24 ns and 40 ns during the 1D shock compression simulation with impact velocity $U_0 = 5$ km/s	127
4.16	Comparison of shock Hugoniot data obtained from experiments and simulations. .	128
4.17	Evolution of the pressure field within each material subdomain for impact velocity $U_0 = 5$ km/s. Fluid and solid pressures mapped on different scales due to difference in orders of magnitude. Inset images showing 3D deformation of the solids.	129
4.18	Evolution of kinetic, internal and total energy within each material domain	132
4.19	Comparison of specific energies at $t = 1.25 \mu\text{s}$. (A) Specific internal energy (left) and kinetic energy (right) at $1.25 \mu\text{s}$ (unit: J/kg), and the locations of three sensors. (B) Rise in specific energies in each material, measured at the sensor location. . . .	134
4.20	Evolution of temperature and plasma density (n_e) for impact velocity $U_0 = 5$ km/s .	136
4.21	Maximum temperature behind the shock wave in each material subdomain	137
4.22	Composition of SLG plasma. (A) Plasma density within the SLG at different time instances (B) Time history of $n_{e,j}^{\text{max}}$ along the central axis. (C) $n_{e,j}^{\text{max}}$ at $t = 1 \mu\text{s}$ (D) Molar composition of each element in SLG.	140
4.23	Electronic and molar composition of SLG at $t = 1 \mu\text{s}$. A) Electronic contribution of each element to impact-generated plasma on a semi log scale B) Molar composition.	141
4.24	Temperature probe lines at $t = 2.5 \mu\text{s}$	155

List of Tables

2.1	Parameters of the equations of state for copper, SLG, and neon.	25
2.2	Sample spectroscopic data of neon for $r = 0, 1, 2, 3$ and $n = 1, 2, 3$ [39]. I_r denotes the ionization energy of the r -th charge state ion. $g_{r,n}$ denotes the degeneracy of an electron of the r -th ion at the n -th energy level, and $E_{r,n}$ its corresponding excitation energy.	28
2.3	Typical values of state variables behind the bow shock and in the plasma pocket, at $t = 0.625 \mu\text{s}$	33
3.1	Parameters of the equations of state	58
3.2	Sample spectroscopic data of argon for $r = 0, 1, 2, 3$ and $n = 1, 2, 3$ [36].	67
4.1	Experimental data obtained from the equation of state shots	114
4.2	Experimental data from the thick flyer pyrometer shots	118
4.3	Experimental data from the thin flyer pyrometer shots	118
4.4	Plasma density (n_e), temperature (T), and shock propagation distance (d_{shock}) in SLG and argon, at time $t = 0.9 \mu\text{s}$ ($0.5 \mu\text{s}$ after impact)	139
4.5	Sample spectroscopic and ionization data for SLG [49]	153

Chapter 1

Introduction

1.1 Background

Hypervelocity impacts refer to collisions between objects at velocities that exceed the speed of sound in the impacted materials, typically greater than 2 km/s for most solids. These collisions involve the rapid dissipation of kinetic energy through mechanical, thermal, chemical, and electromagnetic processes. Hypervelocity impacts are highly relevant across a broad range of scientific and engineering disciplines, including planetary science, aerospace engineering, defense applications, and impact physics [1, 2, 16, 20].

In near-Earth space, hypervelocity collisions with micrometeoroids and anthropogenic debris pose serious threats to spacecraft integrity and functionality. The orbital environment is populated by high-speed particles—ranging from tiny dust grains to spent rocket stages—that travel at velocities of up to 70 km/s [31]. Despite their small size, such particles can impart substantial momentum and energy, penetrating shielding and damaging critical components. Impacts have been linked to the failure of subsystems, including electronics and communications [26]. To mitigate this risk, spacecraft are often protected by multilayer Whipple shields [33], designed to fragment and decelerate incoming particles before they reach the main structure.

Hypervelocity impacts also play a central role in shaping planetary surfaces. The collision of comets, asteroids, and meteoroids with planets and moons produces high-energy events capable

of generating impact craters, driving geological transformation, and contributing to planetary evolution [27, 32]. The Chicxulub impact, widely associated with the Cretaceous–Paleogene extinction event, illustrates the catastrophic potential of large-scale hypervelocity events [25]. Understanding the thermomechanical effects of such impacts informs our knowledge of planetary history and supports efforts to develop hazard mitigation strategies for future near-Earth object encounters.

These impacts are characterized by the generation of intense shock waves that propagate through both, the target and projectile, and the surrounding fluid. The resulting thermodynamic conditions—pressures on the order of tens of gigapascals and temperatures of thousands of Kelvins—can induce deformation, phase transitions, vaporization, and ionization. Under sufficiently extreme conditions, the impacted material transitions into a plasma state: a dense, partially ionized mixture of electrons and ions. This ionization occurs when the internal energy overcomes the binding energy of electrons, allowing them to dissociate from nuclei. In such dense plasmas, electron separation is counteracted by neighboring ions, effectively reducing the ionization potential and creating a strongly coupled, non-ideal plasma.

Plasma generated during hypervelocity impact has been observed to emit electromagnetic radiation, with potential implications for spacecraft anomaly events [14]. In particular, impact-induced plasma may interfere with electronic systems via RF emissions, spark discharges, or radiation-driven damage. Additionally, the release of high-velocity charged particles has been hypothesized to produce Cherenkov radiation when traversing optically dense media [18].

Beyond practical implications, the study of impact-induced plasma offers diagnostic opportunities. By analyzing the composition, energy spectrum, and temporal evolution of the plasma plume, researchers can infer information about the impactor, the target, and the ambient environment. However, significant questions remain unresolved. These include the causal origin of the plasma (i.e., whether it primarily arises from the projectile, target, or surrounding gas), how its properties scale with impact velocity, and what governs its electromagnetic emissions [11, 21, 28].

While many studies assume vacuum conditions—appropriate for deep space applications—this simplification may overlook important interactions between the solids and the ambient environment. In terrestrial or atmospheric contexts, a projectile often travels through a gaseous medium prior to impact, generating a bow shock and compressing the fluid. Upon contact, additional shock waves propagate through the gas, which, due to its low density and high compressibility, can reach elevated temperatures and potentially ionize. A closely related phenomenon occurs during atmospheric re-entry, where hypersonic vehicles generate a plasma sheath by ionizing the surrounding atmospheric gases. Although re-entry conditions are far less energetic than impacts, the underlying mechanism of fluid-driven ionization is similar. This analogy suggests that the ambient gas may play a non-negligible role in impact-induced plasma formation. However, the extent and significance of these effects remain largely unexplored in the existing literature [5].

To fully characterize the physics of hypervelocity impact, it is essential to consider not only solid-solid interactions but also the complex dynamics of the surrounding fluid. The interplay of continuum mechanics, shock thermodynamics, ionization kinetics, and radiation transport defines a rich multiphysics problem—one that this dissertation seeks to address. In the following section, the current body of literature will be reviewed, and key deficiencies will be identified. This analysis provides the context for the contributions of this dissertation, highlighting its novelty and the broader implications of its findings. The interplay of continuum mechanics, shock thermodynamics, and ionization kinetics defines a rich multiphysics problem—one that this dissertation seeks to address.

1.2 Literature Review

Hypervelocity impacts have long captivated researchers across disciplines due to their relevance in contexts ranging from spacecraft shielding to planetary geology and high-strain-rate materials test-

ing. Traditionally, these extreme events have been treated through the lens of structural mechanics, where the dominant concerns involve shock wave propagation, dynamic deformation, fracture, and fragmentation[4, 29]. To accurately capture these responses, extensive efforts have been devoted to developing thermodynamic models of shocked materials, informed by both atomistic simulations and macroscopic continuum theories[7, 9]. These models have underpinned the foundational understanding of impact mechanics and informed the design of experimental and computational strategies used to replicate such extreme conditions.

Over the past few decades, this foundational body of work has evolved in parallel with a broader interest in multiphysics problems involving high-energy density environments. As experimental capabilities and diagnostic tools have improved—especially in optical spectroscopy, high-speed imaging, and electromagnetic sensing—the field has broadened to include investigations into plasma formation and electromagnetic emissions triggered by hypervelocity collisions. This interdisciplinary expansion has led to three distinct but overlapping research threads: solid mechanics, high-pressure thermodynamics, and plasma/electromagnetic physics. Although each of these areas has seen individual progress, fully integrated frameworks that simultaneously account for solid deformation, plasma generation, and radiative effects remain scarce. Moreover, many numerical models assume vacuum ambient conditions, a simplification that is often practical but may fail to capture the contributions of the surrounding gas, particularly in atmospheric or near-surface impacts.

A pivotal experimental milestone in this domain was the 1963 work by Friichtenicht and Slattery[13]. Employing an electrostatic accelerator, they launched micron-sized iron and graphite spheres at velocities up to 16 km/s and recorded the emission of electrically charged particles upon impact. This marked one of the earliest demonstrations of impact-induced ionization. Intriguingly, the authors observed that the collected charge was not simply proportional to velocity, but instead scaled nonlinearly with both kinetic energy and atomic weight. They proposed an empirical relation connecting charge production to projectile energy and composition, suggesting complex underlying plasma

formation mechanisms sensitive to both material and dynamic parameters.

The theoretical underpinning of such ionization events has been tackled from various angles. Horning and Kissel developed a shock-compression-based framework leveraging the Thomas-Fermi model to predict volumetric ionization[15]. Their approach linked the degree of ionization to the compression history of the material and highlighted the role of the material's expansion isentrope in governing its post-shock ionization threshold. Complementing this continuum perspective, Krueger introduced the “sudden perturbation ion formation” model, rooted in non-equilibrium thermodynamics[17]. This model emphasized that during hypervelocity collisions, the timescales of particle acceleration and compression may be too short to assume thermodynamic equilibrium. Instead, transient electron dynamics dominate, leading to ion formation that defies predictions based on traditional equilibrium assumptions.

Experimental investigations have shown significant variability in reported plasma temperatures, reflecting differences in measurement techniques and theoretical assumptions. Ratcliff *et al.* utilized a Van de Graaff accelerator to fire iron dust particles at rhodium foils at speeds ranging from 1.2 to 87 km/s[28]. From measurements of ion velocities and inferred energy conservation, they reported plasma temperatures ranging from 20 to 30 eV. These values contrasted sharply with the results of Friichtenicht *et al.*, who used the Saha equation under an assumption of thermal equilibrium to estimate significantly lower temperatures of approximately 0.5 eV[12]. The difference in methods—direct energy balance versus spectroscopic inference—reflects an ongoing divide in the field regarding the appropriate modeling assumptions for impact plasmas.

Later studies sought to clarify these discrepancies by improving experimental controls and introducing more sophisticated diagnostics. For instance, Lee *et al.* employed a 3 MV electrostatic dust accelerator to investigate how electrically biased spacecraft surfaces respond to micrometeoroid impacts[21]. Their findings revealed that plasma temperatures around 2 eV could be observed at velocities between 10 and 30 km/s, and that target bias significantly influenced these values. The

electric potential of the target enhanced plasma expansion and heating, underscoring the importance of electrostatic conditions. Similarly, Estacio *et al.* studied how background gas and dust levels affect plasma behavior[8]. By controlling gas pressure and particulate concentration, they demonstrated that ambient conditions could modulate plasma current density by orders of magnitude, drawing attention to the role of environmental coupling in plasma evolution.

Alongside thermodynamic and kinetic studies, attention has also turned toward the electromagnetic emissions produced during impact events. The earliest reports, such as those by Bianchi *et al.*, noted anomalous RF signals during hypervelocity impacts[3]. Maki *et al.* advanced this line of investigation using railgun-launched polycarbonate projectiles (1.1 g mass) to produce impacts at velocities between 2 and 6.7 km/s[23, 24]. They detected emissions in the 22 GHz microwave range and proposed that the signals originated from microcrack formation and dielectric breakdown in the target material. This interpretation linked mechanical fracture to electromagnetic signatures, offering a coupling between solid mechanics and radiation physics.

Expanding on this, Close *et al.* conducted a suite of experiments using micron-scale iron particles and patch antennas tuned to 315 and 916 MHz[6]. Their data revealed repeatable RF signatures that varied with target material and impact velocity. Follow-up analysis by Fletcher *et al.* attributed these emissions through ambipolar field generation, where local charge separation within the plasma drives collective motion of electrons and ions[11]. These results suggest that plasma composition and expansion geometry directly influence electromagnetic emissions—a topic with both fundamental and applied importance.

In parallel, modeling efforts have advanced significantly. Li *et al.* built SPH simulations of aluminum-on-aluminum impacts, incorporating the Thomas-Fermi ionization model to estimate charge densities and temperature fields during collision[22, 30]. Fletcher *et al.* later improved this framework by integrating non-ideal Saha equations to account for electron-electron and electron-ion interactions[10]. Their subsequent use of a particle-in-cell (PIC) approach enabled the direct sim-

ulation of electromagnetic wave propagation within the post-impact plasma[11]. Most recently, La Spina *et al.* introduced a semi-analytic model combining a spherically symmetric blast wave with the non-ideal Saha framework and the Frank-Tamm formula to predict Cherenkov radiation from shocked dielectric materials—a theoretical milestone linking ionization physics with observable radiation signatures[19].

Despite these advances, many fundamental questions remain unresolved. Which material domain—projectile, target, or surrounding medium—is the dominant source of observed plasma? How does impact velocity shape the plasma’s ionization profile and elemental composition? What role, if any, does the ambient gas play in the impact-induced plasma formation and subsequent electromagnetic emissions? Addressing these questions will require a comprehensive and unified approach that integrates theory, experimentation, and simulation while accounting for non-equilibrium thermodynamics, radiative transfer, and fluid-structure interaction.

1.3 Motivation and Objective

The body of literature reviewed in the previous section highlights the interdisciplinary nature of hypervelocity impact research, encompassing solid mechanics, shock thermodynamics, plasma physics, and electromagnetic phenomena. Despite extensive experimental and computational efforts, our understanding of the coupling between these domains, particularly in environments involving both solids and ambient gases, remains poorly understood. Most studies have focused either on the mechanical response of impacted materials or on post-impact plasma behavior, often assuming idealized conditions such as vacuum environments or simplified material models. This reductionist approach, while practical, limits our ability to interpret the coupled physical processes governing plasma generation and electromagnetic emissions under realistic impact scenarios.

A key gap in current research lies in the treatment of the ambient medium. The majority of com-

putational frameworks and laboratory experiments assume negligible or absent ambient gas, an assumption valid for exoatmospheric impacts but not for atmospheric, terrestrial, or near-surface applications. In many real-world scenarios, a projectile traverses a fluid medium before reaching the target, generating hypersonic flow fields and bow shocks that precondition the gas. Upon impact, additional shock waves propagate through the fluid, potentially heating it to ionization thresholds. This effect is often overlooked, yet it may significantly influence the composition, energy density, and evolution profile of the resulting plasma. Moreover, the ambient fluid may alter the coupling between ionized matter and electromagnetic fields, thereby affecting the intensity and spectral characteristics of radiated emissions.

Another critical gap concerns the origin and composition of the plasma itself. It remains unclear whether the dominant contribution comes from the projectile, the target, or the surrounding gas. Determining the source of ionization, as well as its elemental makeup, is essential for applications ranging from spacecraft diagnostics to planetary impact modeling. Understanding the dependence of plasma generation on impact velocity, material properties, and ambient conditions would also provide predictive capability that is currently lacking in existing models.

The current work aims to develop a robust computational framework to simulate fluid-solid coupling during a hypervelocity impact and accurately predict the resulting thermodynamic states under such extreme loading conditions. The model treats each material subdomain—the projectile, target, and ambient gas—as having its own constitutive behavior, capturing material-specific responses to shock loading. All three regions are considered capable of undergoing ionization, with the degree of ionization determined by their respective thermodynamic conditions. Ionization levels are governed by the local energy density, itself determined by shock compression and material-specific equations of state.

The goals of this work are as follows:

- To develop and validate a numerical method that resolves fluid-solid coupling during hypervelocity impacts, including ambient effects.
- To predict thermodynamic states and deformation in the projectile, target, and gas domains using appropriate equations of state.
- To quantify ionization thresholds and plasma composition based on post-shock temperature and density distributions.
- To determine the dominant source(s) and elemental constituents of impact-induced plasma.
- To evaluate the sensitivity of plasma formation to impact velocity, material properties, and ambient conditions.

Through these objectives, this work aims to advance our fundamental understanding of impact-induced plasma physics and provide a robust and accurate computational framework for future research in aerospace, planetary science, and high-energy-density physics.

1.4 Accomplishments and Outline

To address the objectives outlined above, this dissertation makes several technical and methodological contributions:

- Developed a fluid-structure coupled numerical solver for hypervelocity impacts that incorporates the dynamics of both solid materials (projectile and target) and the surrounding ambient gas. The framework is modular and consists of the following components:
 - A continuum mechanics solver to capture the governing equations for both fluid and solid domains.

- A material interface tracker to resolve the location and evolution of phase boundaries between different materials.
 - Thermodynamic material models that describe the pressure, temperature, and energy behavior.
 - An ionization model to compute plasma states based on local thermodynamic conditions.
- Formulated and implemented a method to compute the analytical solution to a one-dimensional multi-material shock-shock Riemann problem. This setup serves as an analog for fluid-solid interaction during hypervelocity impact in an ambient fluid and was used to verify the accuracy of the numerical solver.
 - Proposed an extension to the Noble-Abel Stiffened Gas equation of state (NASG-EOS) to better capture the temperature response of soda-lime glass under shock loading.
 - Developed a calibration procedure to compute the numerical parameters for the equation of state of the target by casting the problem as a constrained optimization problem using new experimental shock compression data of soda-lime glass.
 - Extended Mie–Grüneisen equation of state for modeling metallic projectiles to ensure numerical stability in both compression and tension regimes by preserving hyperbolicity.
 - Validated the solver’s performance and the fidelity of material models through a series of one-dimensional, tri-material simulations by comparing thermodynamic states of the shocked materials against experimental data.
 - Implemented an ionization model for non-ideal, multi-component plasmas incorporating generalized Saha equations with conservation laws for charge and nuclei. The model was validated against benchmark results for both ideal and non-ideal plasmas.

- Identified the impact velocity threshold for significant ionization when ambient gas is included in the analysis. For velocities exceeding 4 km/s, the ambient gas exhibited notable ionization levels, while the solids contributed only marginally to the total free charge due to their lower temperatures and higher densities.
- Determined that chemical composition is reflected in the plasma formation. Ionization in soda-lime glass is primarily driven by its metallic components, which provide the vast majority of free electrons despite representing a small fraction of the material's overall mass.

This dissertation is structured in the manuscript (multi-paper) format. Chapters 2, 3, and 4 each correspond to journal articles that I have published as first author in *Extreme Mechanics Letter*, *International Journal of Impact Engineering*, and *International Journal of Solids and Structures*, respectively. The core content of these chapters are summarized:

- **Chapter 2** focuses on plasma formation and thermodynamic evolution in the ambient fluid domain. It presents the initial development of the computational framework and characterizes ionization thresholds in the gas.
- **Chapter 3** expands upon the previous work by incorporating more detailed modeling of the solid domains. It introduces temperature laws for simplified solid equations of state and verifies the impact solver against analytic solutions and experimental Hugoniot data.
- **Chapter 4** presents the most recent advancements, including shock compression experiments on soda-lime glass, equation-of-state calibration, and application of the non-ideal ionization model to both gas and solid phases. This chapter offers a comprehensive picture of plasma composition and behavior across all material domains.

Finally, Chapter 5 summarizes the key findings of this research and outlines possible future directions.

Bibliography

- [1] Paul D. Asimow, Chaney Lin, Luca Bindi, Chi Ma, Oliver Tschauner, Lincoln S. Hollister, and Paul J. Steinhardt. Shock synthesis of quasicrystals with implications for their origin in asteroid collisions. *Proceedings of the National Academy of Sciences*, 113(26):7077–7081, 2016. doi: 10.1073/pnas.1600321113.
- [2] Roshdy G Barsoum. Shockwave compression physics of condensed matter lattice boltzmann method with application to glass under hypervelocity impact, 2019.
- [3] R. Bianchi, F. Capaccioni, P. Cerroni, M. Coradini, E. Flamini, P. Hurren, G. Martelli, and P. N. Smith. Radiofrequency emissions observed during macroscopic hypervelocity impact experiments. *Nature*, 308(5962):830–832, 1984. doi: 10.1038/308830a0.
- [4] Andrew J. Birnbaum, John C. Steuben, Athanasios P. Iliopoulos, and John G. Michopoulos. Simulating hypervelocity impact and material failure in glass. *Volume 1B: 38th Computers and Information in Engineering Conference*, 2018. doi: 10.1115/detc2018-85948.
- [5] Iain D. Boyd. *Nonequilibrium Gas Dynamics and Molecular Simulation*. Cambridge University Press, 2007.
- [6] Sigrid Close, Ivan Linscott, Nicolas Lee, Theresa Johnson, David Strauss, Ashish Goel, Alexander Fletcher, David Lauben, Ralf Srama, Anna Mockler, et al. Detection of electromagnetic pulses produced by hypervelocity micro particle impact plasmas. *Physics of Plasmas*, 20(9):092102, 2013.
- [7] Lee Davison and Robert Albert Graham. Shock compression of solids. *Physics Reports*, 55(4):255–379, 1979.
- [8] Benjamin Estacio, Gil Shohet, Sean AQ Young, Isaac Matthews, Nicolas Lee, and Sigrid

- Close. Dust and atmospheric influence on plasma properties observed in light gas gun hypervelocity impact experiments. *International Journal of Impact Engineering*, 151:103833, 2021.
- [9] EP Fahrenthold and BA Horban. Thermodynamics of continuum damage and fragmentation models for hypervelocity impact. *International journal of impact engineering*, 20(1-5):241–252, 1997.
- [10] Alex Fletcher, Sigrid Close, and Donovan Mathias. Simulating plasma production from hypervelocity impacts. *Physics of Plasmas*, 22(9):093504, 2015. doi: 10.1063/1.4930281.
- [11] Alex C. Fletcher and Sigrid Close. Particle-in-cell simulations of an rf emission mechanism associated with hypervelocity impact plasmas. *Physics of Plasmas*, 24(5):053102, 2017. doi: 10.1063/1.4980833.
- [12] JF Friichtenicht, NL Roy, and DG Becker. The cosmic dust analyzer: experimental evaluation of an impact ionization model. In *International Astronomical Union Colloquium*, volume 13, pages 299–310. Cambridge University Press, 1971.
- [13] Joseph Fred Friichtenicht and John Charles Slattery. Ionization associated with hypervelocity impact. Technical report, 1963.
- [14] Ashish Goel and Sigrid Close. Electrical anomalies on spacecraft due to hypervelocity impacts. In *2015 IEEE aerospace conference*, pages 1–7. IEEE, 2015.
- [15] K Hornung and J Kissel. On shock wave impact ionization of dust particles. *Astronomy and Astrophysics (ISSN 0004-6361)*, vol. 291, no. 1, p. 324-336, 291:324–336, 1994.
- [16] Hao Jiang, Valerie Scott, and Bo Li. Modeling and simulations of high and hypervelocity impact of small ice particles. *International Journal of Impact Engineering*, 155:103906, 2021. doi: 10.1016/j.ijimpeng.2021.103906.

- [17] FR Krueger. Thermodynamics of ion formation by fast dissipation of energy at solid surfaces. *Zeitschrift für Naturforschung A*, 38(4):385–394, 1983.
- [18] Andrea La Spina and Jacob Fish. Time-and frequency-domain hybridizable discontinuous galerkin solvers for the calculation of the cherenkov radiation. *Computer Methods in Applied Mechanics and Engineering*, 402:115170, 2022.
- [19] Andrea La Spina, Alex C Fletcher, and Jacob Fish. Semi-analytic model for plasma production and cherenkov radiation emission from hypervelocity impacts on soda–lime glass. *Physics of Plasmas*, 30(1):013902, 2023.
- [20] R.J. Lawrence, W.D. Reinhart, L.C. Chhabildas, and T.F. Thornhill. Spectral measurements of hypervelocity impact flash. *International Journal of Impact Engineering*, 33(1-12):353–363, 2006. doi: 10.1016/j.ijimpeng.2006.09.010.
- [21] Nicolas Lee, Sigrid Close, Ashish Goel, David Lauben, Ivan Linscott, Theresa Johnson, David Strauss, Sebastian Bugiel, Anna Mocker, Ralf Srama, and et al. Theory and experiments characterizing hypervelocity impact plasmas on biased spacecraft materials. *Physics of Plasmas*, 20(3):032901, 2013. doi: 10.1063/1.4794331.
- [22] Jianqiao Li, Weidong Song, and Jianguo Ning. Theoretical and numerical predictions of hypervelocity impact-generated plasma. *Physics of Plasmas*, 21(8):082112, 2014. doi: 10.1063/1.4893310.
- [23] K Maki, T Takano, A Fujiwara, and A Yamori. Radio-wave emission due to hypervelocity impacts in relation to optical observation and projectile speed. *Advances in Space Research*, 34(5):1085–1089, 2004.
- [24] K Maki, E Soma, T Takano, A Fujiwara, and A Yamori. Dependence of microwave emissions

- from hypervelocity impacts on the target material. *Journal of applied physics*, 97(10):104911, 2005.
- [25] Joanna V Morgan, Timothy J Bralower, Julia Brugger, and Kai Wünnemann. The chicxulub impact and its environmental consequences. *Nature Reviews Earth & Environment*, 3(5): 338–354, 2022.
- [26] Andrew Nuttall and Sigrid Close. A thermodynamic analysis of hypervelocity impacts on metals. *International Journal of Impact Engineering*, 144:103645, 2020.
- [27] Gordon R Osinski, Richard AF Grieve, Jacob E Bleacher, Catherine D Neish, Eric A Pilles, and Livio L Tornabene. Igneous rocks formed by hypervelocity impact. *Journal of Volcanology and Geothermal Research*, 353:25–54, 2018.
- [28] Paul R. Ratcliff, Mark J. Burchell, Mike J. Cole, Tom W. Murphy, and Firooz Alladadi. Experimental measurements of hypervelocity impact plasma yield and energetics. *International Journal of Impact Engineering*, 20(6-10):663–674, 1997. doi: 10.1016/s0734-743x(97)87453-2.
- [29] William P Schonberg. Hypervelocity impact response of spaced composite material structures. *International Journal of Impact Engineering*, 10(1-4):509–523, 1990.
- [30] Weidong Song, Jianqiao Li, and Jianguo Ning. Characteristics of plasma generated by hypervelocity impact. *Physics of Plasmas*, 20(9):093501, 2013. doi: 10.1063/1.4819829.
- [31] MJ Starks, DL Cooke, BK Dichter, LC Chhabildas, WD Reinhart, and TF Thornhill Iii. Seeking radio emissions from hypervelocity micrometeoroid impacts: Early experimental results from the ground. *International journal of impact engineering*, 33(1-12):781–787, 2006.
- [32] Wesley Andrés Watters. *Hypervelocity impacts and the evolution of planetary surfaces and interiors*. PhD thesis, Massachusetts Institute of Technology, 2009.

- [33] F.L. Whipple. Meteorites and space travel. *Astronomical Journal*, 52:137, 1947.

Chapter 2

Plasma Formation in Ambient Fluid from Hypervelocity Impacts

(Published in *Extreme Mechanics Letters*, 58, 101927, 2023.)

S. Islam^a, W. Ma^a, John G. Michopoulos^b, K. Wang^a

^a Department of Aerospace and Ocean Engineering, Virginia Tech, Blacksburg, VA, USA

^b US Naval Research Laboratory, Washington DC, USA

Abstract

The generation of plasma from hypervelocity impacts is an active research topic due to its important science and engineering ramifications in various applications. Previous studies have mainly focused on the ionization of the solid materials that constitute the projectile and the target. In this letter, we consider impact events that occur in a fluid (e.g., gas) medium, and present a multiphysics computational modeling approach and associated analysis to predict the behavior of the dynamic fluid-solid interaction that causes the surrounding fluid to ionize. The proposed computational framework is applied to a specific case involving a system of three interacting domains: a copper rod projectile impacting onto a soda lime glass target in a neon gas environment. The impact velocity is varied between 3 km/s and 6 km/s in different simulations. The computational model couples the compressible inviscid Navier-Stokes equations with the Saha ionization equations. The three material interfaces formed among the projectile, the target, and the ambient gas are tracked implicitly by solving two level set equations that share the same velocity field. The mass, momentum, and energy fluxes across the interfaces are computed using the FInite Volume method with Exact two-material Riemann problems (FIVER). The simulation result reveals a region of neon gas with high velocity, temperature, pressure, and mass density, formed in the early stage of the impact mainly due to the hypersonic compression of the fluid between the projectile and the target. For impact velocities higher than 4 km/s, ionization is predicted in this region.

2.1 Introduction

Hypervelocity impact describes the collision of a solid projectile onto a target structure at a velocity that exceeds the speed of sound in the target medium. It features the rapid transport and dissipation of kinetic energy through mechanical, thermal, chemical, and electromagnetic pathways; and

is relevant to many applications in geological and planetary sciences, aerospace engineering, and defense engineering [6, 7, 28, 31]. In the past, extensive research has been conducted to understand and predict the mechanical response and failure of the target and the projectile, including shock waves, deformation, fracture, and fragmentation [9, 15, 34, 49, 55]. It has also been found that the temperature behind an impact-induced shock wave may exceed thousands of Kelvin, causing the material to ionize, thereby producing a complex plasma mixture (Fig. 4.1(a)) [7, 30]. Scientific studies on impact-generated plasma can be traced back to Friichtenicht and Slattery (1963) [23], if not earlier. Since then, various authors have published data that characterize the composition and energy of plasma generated under different impact conditions (e.g., [21, 33, 44]). Electromagnetic waves emitted by impact-generated plasma have also been detected in laboratory experiments. For example, Bianchi *et al.* [8] captured electromagnetic pulses in the radio frequency range from experiments with shaped charges accelerated to 10 km/s.

In the last decade, there has been a growing interest in developing models and numerical tools to predict and characterize impact-generated plasma and electromagnetic waves. This trend has largely aligned with the rising concern of collisions between space vehicles and space debris. In this context, the electromagnetic waves emitted by an impact-generated plasma may pose a threat to the function and safety of on-board electronic devices [32]. For example, Li *et al.* simulated hypervelocity impact of aluminum projectiles and targets for impact velocities between 5 and 10 km/s [35, 51]. They developed a smoothed particle hydrodynamics (SPH) solver to simulate the solid dynamics, and used the Thomas-Fermi model to predict temperature and ionization. Fletcher *et al.* [20, 21] also developed an SPH code to simulate hypervelocity impacts, and used the non-ideal Saha equations to predict ionization of the target material. They tested a wider range of impact velocities, between 10 and 72 km/s. Fletcher *et al.* also developed a particle-in-cell code to investigate the mechanism of electromagnetic emission from the impact-generated plasma [22]. Despite these progresses, the generation of plasma and electromagnetic waves from hyper-

velocity impacts remains an active research area. The determination and explanation of the causal source (i.e. projectile, target, or the surrounding gas) and composition of plasma, the dependence of plasma energy on impact velocity (cf. [33, 44]), and the energy and spectrum of the electromagnetic emissions [22] remain open questions in general.

Previous studies on hypervelocity impact have largely focused on the dynamic response and material behaviors of the projectile and the target, while the ambient environment is usually assumed to be a vacuum [26, 33, 40, 59]. This assumption can be valid when studying collisions that occur in the exoatmospheric space environment. However, for terrestrial and atmospheric applications in which the collision occurs in a fluid medium, the projectile produces a hypersonic fluid flow during its flight, which can easily exceed Mach 10. The collision between the projectile and the target causes a sudden change in the velocity and thermodynamic state on the fluid domain boundary, which leads to the emission of shock waves that propagate through the fluid domain. These impact-induced shock waves disrupt the initial flow field. Depending on the impact velocity and the properties of the fluid material, the energy carried by the shock waves may cause the fluid to ionize, thereby forming a plasma. Understanding the fluid dynamics, thermodynamics, and plasma characteristics is important for developing a complete description of hypervelocity impact events that occur in a fluid medium. The knowledge obtained may also lead to the development of new diagnostic tools for hypervelocity impact detection and characterization.

In this letter, we present a computational framework for analyzing the interaction between the projectile, the target, and the ambient fluid during a hypervelocity impact; focusing on the formation of plasma in the fluid. The impact velocity of the projectile is varied between 3 km/s and 6 km/s in different tests. In each test case, the simulation is performed in two steps, including a steady-state computational fluid dynamics (CFD) analysis that generates the initial fluid flow around the projectile, followed by a fluid-solid coupled analysis of the impact process. In Section 4.2, we present the geometry, model equations, and numerical methods involved in this study. In Section 2.3, we

present and discuss the obtained numerical results, including the time histories of pressure, velocity, temperature, and average charge number. Finally, some concluding remarks are provided in Section 4.7.

2.2 Physical model and computational methods

As a model problem, we consider the collision of an oxygen-free high conductivity (OFHC) copper rod projectile onto a soda lime glass target (SLG) in a neon gas environment. Copper projectiles and SLG targets are frequently used in laboratory experiments (e.g. [4, 29]), and thus have been chosen as the respective materials for this computational study. The choice of SLG as the material of the target is further motivated by its potential use in armor and protective systems [5, 11, 42]. It is inexpensive, transparent, and has relatively high mechanical strength. Neon gas is selected as the ambient fluid because it is monoatomic, chemically inert, and has a density similar to that of air.

The computational model adopted in this study combines the compressible inviscid Navier-Stokes equations with the Saha ionization equations. We solve the Navier-Stokes equations in the Eulerian reference frame using a high-resolution finite volume method. The spatial domain includes three subdomains, occupied by the projectile, the target, and the surrounding fluid, respectively. The boundaries of the projectile and the target are represented implicitly as the 0 level set of two signed distance functions [43, 50]. The motion and deformation of these boundaries are predicted by solving two level set equations that share the same velocity field. This method allows us to keep track of three sharp, interconnected material interfaces (i.e. projectile-fluid, target-fluid, and projectile-target) that undergo large, complex deformations. Across the interfaces between neon gas and the solid materials, the density jump can exceed a factor of 10,000, and the thermodynamic relations (i.e. equations of state) also differ significantly. This type of discontinuities pose a challenge to the computation of fluxes across material interfaces. In this work, the mass, momen-

tum, and energy fluxes across material interfaces are computed using the Finite Volume method based on Exact two-material Riemann problem (FIVER) [18, 19, 27, 38, 53]. By constructing and solving an exact Riemann problem along each edge in the mesh that crosses a material interface, FIVER explicitly accounts for the change of equation of state across the interface. Previously, it has been validated for several shock-dominated multiphase flow and fluid-structure interaction problems in underwater explosion and implosion, pipeline explosion, cavitation erosion, and shock wave lithotripsy [12, 13, 17, 36, 53, 54, 56]. Within the fluid subdomain, we solve the Saha equations coupled with the condition of electroneutrality and mass conservation of nuclei to predict the onset of ionization and the distribution of ionization products. This model assumes local thermodynamic equilibrium, which can be justified for predicting the formation and initial expansion of plasma during hypervelocity impacts [14, 21].

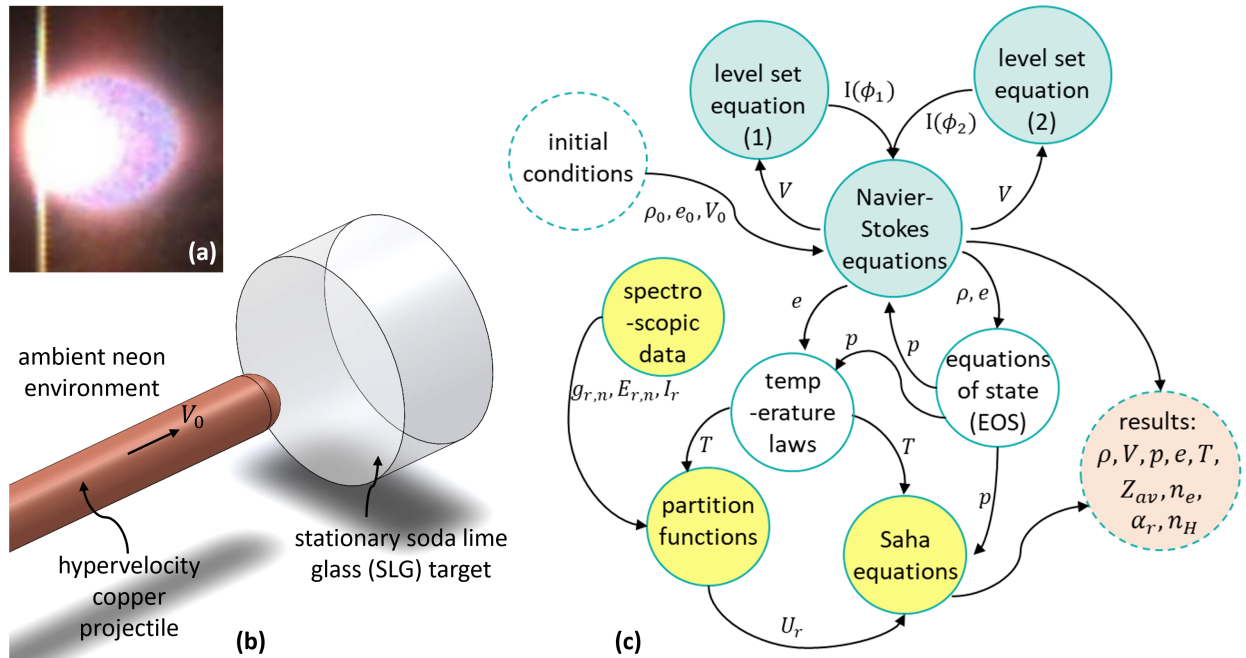


Figure 2.1: Hypervelocity impact: Problem setup. (a) Hypervelocity impact flash captured in a laboratory experiment [7]. (b) The model problem investigated: A long copper rod impacting on a soda lime glass target in neon gas. The impact velocity, V_0 , is varied between 3 km/s and 6 km/s. (c) An overview of the computational multiphysics model used in this study, which accounts for the dynamic interaction between the solid and fluid (gas) materials and fluid ionization.

Figure 4.1(b) presents the setup of the hypervelocity impact problem investigated in this paper. The projectile is a cylindrical copper rod of 5 mm radius, with a semi-spherical leading edge. The target is an SLG cylinder with a radius of 30 mm and a height of 20 mm. The ambient gas is assumed to be neon, which is inert even at high pressures and temperatures. In the far-field, the gas density, pressure, and temperature are fixed at 0.82 kg/m^3 , 100 kPa, and 300 K, respectively. The velocity of the projectile, V_0 , is varied between 3 km/s and 6 km/s in different simulations. To put this into context, the bulk speed of sound in SLG and copper are around 4 km/s and 3.7 km/s at room temperature and atmospheric pressure. The speed of sound in neon is 0.46 km/s.

Because of the high impact velocities, the solid materials involved in this problem, namely copper and SLG, are modeled as compressible fluids. When the impact velocity exceeds 3 km/s, the density of the energy transferred from the projectile to the SLG target is far greater than the strain energy density of SLG. In the past, Kobayashi *et al.* showed that when impacted by steel and tungsten projectiles traveling at 4 km/s to 6 km/s, the maximum pressure inside SLG exceeds 50 GPa [30], which is one order of magnitude higher than the material's Hugoniot elastic limit (3 GPa to 8 GPa) [4, 10, 16, 24, 46, 47].

Therefore, the dynamics of the target, the projectile, and the surrounding gas can be considered to be governed by the three-dimensional (3D) compressible Navier-Stokes equations. In addition, we neglect the effects of viscosity and heat diffusion, which reduces the Navier-Stokes equations to

$$\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho V \\ \rho e_t \end{bmatrix} + \nabla \cdot \begin{bmatrix} \rho V^T \\ \rho V \otimes V + p \mathbf{I} \\ (\rho e_t + p) V^T \end{bmatrix} = 0, \quad (2.1)$$

also known as the Euler equations. Here ρ , V , e_t , p denote density, velocity, total energy per unit mass, and pressure, respectively. $\mathbf{I}$ is the three-dimensional (3D) identity matrix.

At any time $t \geq 0$, the physical domain Ω consists of three material subdomains, occupied by neon,

copper, and SLG, respectively. Across material interfaces, pressure and the normal component of velocity are continuous, whereas density and the tangential components of velocity are generally discontinuous. Equation (3.1) is algebraically closed by a thermodynamic equation of state (EOS) for each material. The monoatomic configuration and the absence of valence electrons entail that neon atoms have only translational degrees of freedom, but not vibrational, rotational, or electronic degrees of freedom. Therefore, the specific heat of neon is independent of temperature [25], which justifies the use of the calorically perfect gas EOS. The copper projectile is modeled using the Mie-Grüneisen EOS [45], i.e.

$$p(\rho, e) = \frac{\rho_0 c_0^2 \eta}{(1 - s\eta)^2} \left(1 - \frac{1}{2} \Gamma_0 \eta \right) + \rho_0 \Gamma_0 e, \quad (2.2)$$

where ρ_0 and c_0 denote the density and bulk speed of sound in the ambient condition. s is the slope of the Hugoniot curve. Γ_0 is the Grüneisen parameter. $\eta = 1 - \rho_0/\rho$. The SLG target is modeled using the stiffened gas EOS [48], i.e.

$$p(\rho, e) = (\gamma - 1)\rho e - \gamma p_c, \quad (2.3)$$

where γ and p_c are empirical model parameters. The SLG material is not modeled using the Mie-Grüneisen EOS, because certain regions in the SLG target experience high tensile stresses during hypervelocity impact, while the Mie-Grüneisen EOS has been primarily used to model materials in compression [45]. On the other hand, the stiffened gas EOS can be calibrated to capture the shock Hugoniot obtained from laboratory experiments [48]. Specifically, we combine Eq. (3.4) with the Rankine-Hugoniot jump conditions, then fit them to the shock Hugoniot, $u_s = c_0 + su_p$, obtained from laboratory impact experiments, where u_s and u_p denote the shock speed and the downstream particle velocity, respectively. The shock Hugoniot data presented in [24] ($c_0 = 2.01$ km/s, $s = 1.7$) is adopted in this work, which gives $\gamma = 3.9$ and $p_c = 2.62$ GPa. All the EOS parameters used in

the simulations are presented in Table 2.1.

Substance	EOS	Parameters			
Copper	Mie-Grüneisen	c_0 (km/s)	s	ρ_0 (kg/m ³)	Γ_0
		3.93 [41]	1.50 [41]	8960	2.12 [37]
Soda lime glass (SLG)	Stiffened gas	γ	p_c (GPa)		
		3.9	2.62		
Neon	Perfect gas	γ	c_p (J/(K · kg))		
		1.667	618.3		

Table 2.1: Parameters of the equations of state for copper, SLG, and neon.

Each simulation is performed in two steps. First, a steady-state computational fluid dynamics (CFD) analysis is performed to capture the hypersonic fluid flow surrounding the projectile in flight, which features a curved bow shock (Fig. 3.2(a)). Next, a fluid-solid coupled impact analysis is performed to predict the dynamic interaction between the projectile, the target, and the surrounding gas, including the possible ionization of the gas. The impact analysis is initialized at a time shortly before the bow shock reaches the target, and the steady-state CFD result is used as the initial condition within the fluid subdomain (Fig. 3.2(b)). In both steps, the Navier-Stokes equations (Eq. (3.1)) are solved using a finite volume method. The steady-state CFD analysis is performed using the AERO-F solver [1] on an unstructured, body-fitted mesh. In the most refined region, the characteristic element size is approximately 0.15 mm. The fluid-solid coupled impact analysis is performed using the M2C solver [2] on a non-body-fitted Cartesian mesh. In the most refined region, the element size is 0.1 mm.

In the impact analysis, the material interfaces are tracked implicitly using the level set method [43, 50]. Specifically, two level set equations of the form

$$\frac{\partial \phi_s}{\partial t} + V \cdot \nabla \phi_s = 0, \quad s = 1, 2 \quad (2.4)$$

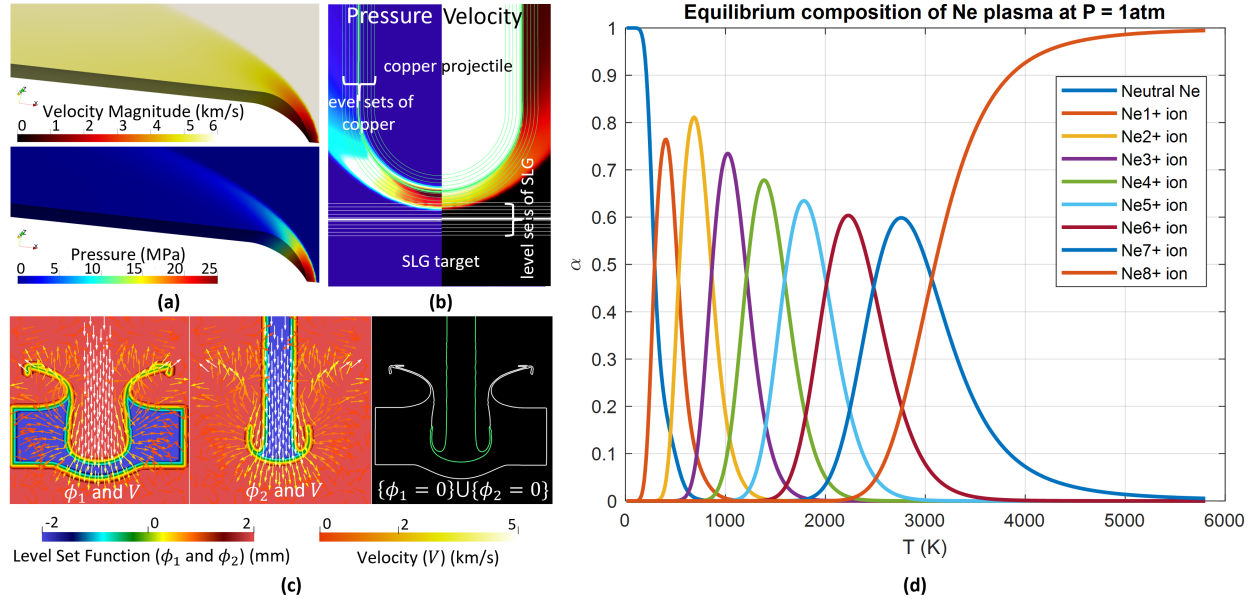


Figure 2.2: Major components of the computational framework. (a) Simulation of the hypersonic fluid flow surrounding the projectile before the collision through a steady-state, body-fitted CFD analysis. (b) Initialization of the fluid-solid coupled impact analysis. The fluid density, velocity, and pressure fields obtained from the CFD analysis are mapped to a new mesh that does not resolve the boundaries of the projectile and the target. (c) Implicit tracking of the three material interfaces between the projectile, the target, and the ambient gas by solving two level set equations that share the same velocity field. (d) Solution of the Saha equations for neon (Ne) at the atmospheric pressure and different temperatures between 0 and 6000 K. The plotted result (α) is the molar fractions of neon atoms and ions with different charge numbers (0 - 8 shown here).

are solved synchronously to track the boundaries of the copper projectile and the SLG target, respectively. The two level set functions, ϕ_1 and ϕ_2 , are initialized to be the signed distance from each point in the computational domain to the surface of the target and the projectile, respectively. Notably, the two level set equations share the same velocity field. As a result, the contact and separation between the projectile and the target are captured naturally, while spurious overlaps are avoided (Fig. 3.2(c)). The mass, momentum, and energy fluxes across material interfaces are computed using the method of FIVER (FInite Volume method based on Exact multiphase Riemann solvers), which has been validated for several multiphase flow problems featuring large density jumps [12, 36, 53]. Following this method, a one-dimensional bimaterial Riemann problem is con-

structed along each edge in the mesh that crosses a material interface. The solution of this exact Riemann problem is used to compute the local fluxes.

In the impact analysis, the onset and extent of ionization in the ambient neon gas, under the assumption of non-interacting electrons and ions, is predicted by the Saha equations, i.e.

$$\frac{n_{r+1}n_e}{n_r} = 2 \frac{U_{r+1}}{U_r} \left[\frac{2\pi m_e k_B T}{h^2} \right]^{3/2} \exp\left(\frac{-I_r}{k_B T}\right), \quad r = 0, 1, \dots, N, \quad (2.5)$$

where n_r is the number density of the r -th charge state ion, n_e is the number density of electrons (also referred to as the plasma density), T is the temperature, h is the Planck constant, k_B is the Boltzmann constant, m_e is the stationary mass of an electron, and I_r the r -th ionization energy. For neon, N is equal to 9.

In this work, we couple the Saha equations with the condition of quasi-neutrality (i.e. conservation of charge) and the conservation of nuclei, which yields the one dimensional transcendental equation [57],

$$Z_{\text{av}} \left(1 + \sum_{r=1}^N \frac{\prod_{m=1}^r f_m}{(Z_{\text{av}} n_H)^r} \right) = \sum_{r=1}^N \left(\frac{r}{(Z_{\text{av}} n_H)^r} \prod_{m=1}^r f_m \right), \quad (2.6)$$

where Z_{av} denotes the average charge number of neon, and n_H the number density of heavy particles. f_m is given by

$$f_m = 2 \frac{U_{m+1}}{U_m} \left(\frac{2\pi m_e k_B T}{h^2} \right)^{3/2} \exp\left(-\frac{I_m}{k_B T}\right). \quad (2.7)$$

U_r is the state-dependent partition function of the r -th charge state ion, given by

$$U_r = \sum_{n=1}^{\infty} g_{r,n} \exp\left(-\frac{E_{r,n}}{k_B T}\right), \quad (2.8)$$

where $g_{r,n}$ and $E_{r,n}$ denote the degeneracy and excitation energy of the r -th ion at the n -th energy level. In this work, the values of I_r , $E_{r,n}$, and $g_{r,n}$ for neon are obtained from the NIST atomic

spectra database [39]. Sample spectroscopic data are shown in Table 3.2 for the atomic neon and the first three ionic stages (i.e. $r = 0, 1, 2, 3$). The ionization energy for each state, along with the excitation energies and their corresponding degeneracies for the first three electronic energy levels (i.e. $n = 1, 2, 3$) are shown in the table. In the simulations, up to 1943 levels are accounted for in the solution of the Saha equations.

r	$I_r(\text{eV})$	n	$g_{r,n}$	$E_{r,n}(\text{eV})$
0	21.565	1	1	0.000
		2	5	16.619
		3	3	16.671
1	40.963	1	4	0.000
		2	2	0.967
		3	2	26.910
2	63.423	1	5	0.000
		2	3	0.080
		3	1	0.114
3	97.190	1	4	0.000
		2	6	5.112
		3	4	5.118

Table 2.2: Sample spectroscopic data of neon for $r = 0, 1, 2, 3$ and $n = 1, 2, 3$ [39]. I_r denotes the ionization energy of the r -th charge state ion. $g_{r,n}$ denotes the degeneracy of an electron of the r -th ion at the n -th energy level, and $E_{r,n}$ its corresponding excitation energy.

In each time step, we solve Eq. (3.9) for Z_{av} using a safeguarded iterative method, TOMS748 [3], at each node within the subdomain occupied by neon. After that, the plasma density and molar fraction of each ion state can be easily calculated following the method described by Zaghloul *et al.* [57].

The molar fraction, α , of the r -th ion is the ratio of the number density of said ion to the total number density of heavy particles. To accelerate the solution process, we tabulate U_r , $r = 0, 1, \dots, 10$ as functions of $\exp(-1/T)$ at the beginning of the impact analysis. In each time step, we calculate the values of U_r using cubic spline interpolation. We have verified our Saha equation solver against the results presented by Zaghoul *et al.* [58]. As an example, Figure 3.2(d) presents the variation of molar fractions of neon ions at a fixed pressure, $p_0 = 101$ kPa, for temperatures between 0 and 6000 K.

In summary, Fig. 4.1(c) presents the schematic of the computational framework adopted in this study. The two solvers used in this study, namely AERO-F [1] and M2C [2], are both publicly available.

2.3 Results and discussion

Figure 2.3 shows (from left to right) the pressure, velocity magnitude, temperature, and average charge number of neon predicted by the M2C solver for a representative test case with impact velocity $V_0 = 5$ km/s. The figure displays the fields at progressive time instances of (from top to bottom) $0.625 \mu\text{s}$, $1.25 \mu\text{s}$, $2.5 \mu\text{s}$, $5 \mu\text{s}$. The collision between the projectile and the target occurs at $0.4 \mu\text{s}$.

The emission of shock waves can be observed from the snapshots taken at $t = 0.625 \mu\text{s}$. These include a forward-propagating shock wave in the target, a backward-propagating shock wave in the projectile, and a shock wave in the fluid that expands from the point of collision. The high rate of energy transfer manifests itself in the strong compression waves. In both the projectile and the target, the pressure behind the shock wave reaches 80 GPa at this time. This pressure is one order of magnitude higher than the Hugoniot elastic limit of SLG, and is of similar magnitude to experimental values reported by Kobayashi *et al.* [30]. This observation justifies modeling SLG

as a fluid, since the elasticity of the material can be neglected in this pressure regime. In the neon fluid, the peak pressure behind the shock front is around 300 MPa. This is one order of magnitude higher than the peak pressure of the bow shock (cf. Fig. 3.2(a)), but two orders of magnitude lower than the peak pressure in the solid materials. As such, the pressure plots in Fig. 2.3 have been scaled to show mainly the variations within the projectile and the target.

At $t = 0.625 \mu\text{s}$, high temperatures can be observed in the fluid subdomain behind the shock wave. This leads to the formation of highly ionized pockets of neon plasma, which expands radially outwards from the point of collision. The mechanism of this plasma formation is investigated in Fig. 2.4. Specifically, Fig. 2.4(a) presents a local view of the pressure, velocity, temperature, and average charge number, rescaled to better visualize their variations in a plasma pocket. The neon fluid in the pocket expands outwards at a typical velocity of 10 km/s, which is twice the impact velocity. Similar ratios are observed for other tested impact velocities between 3 and 6 km/s. Figure 2.4(a) also shows that the average charge number (Z_{av}) is the highest near the point of impact, and then drops off radially. Nonetheless, the plasma density, n_e , given by

$$n_e = Z_{\text{av}} n_H = Z_{\text{av}} \rho N_A / M, \quad (2.9)$$

is fairly constant within the pocket, in the order of 10^{17} m^{-3} . Here, N_A is the Avogadro constant, and M the molar mass of neon. This behavior can be explained by the fact that the mass density of neon (ρ) increases within the pocket in the radial directions. The high velocities in this region can be explained by a rudimentary analysis, shown in Fig. 2.4(b). Consider a fluid particle near the point of collision. Right before collision occurs, this particle moves at a velocity similar to the impact velocity of the projectile. A short time, Δt , after collision, the particle will have traveled roughly a distance of $V_0 \Delta t / \tan(\theta)$, where θ is the instantaneous tangential angle between the projectile and the target. This corresponds to a particle velocity of approximately $V_0 / \tan(\theta)$. For the blunt projectile used in this study, θ is 0 at the initial time of collision, and remains very small shortly thereafter.

Therefore, the velocity of the fluid particle can be far greater than the impact velocity, as observed in Fig. 2.4(b). The impact also rapidly compresses the fluid in this region, resulting in high pressures, which in turn leads to high temperatures and ionization.

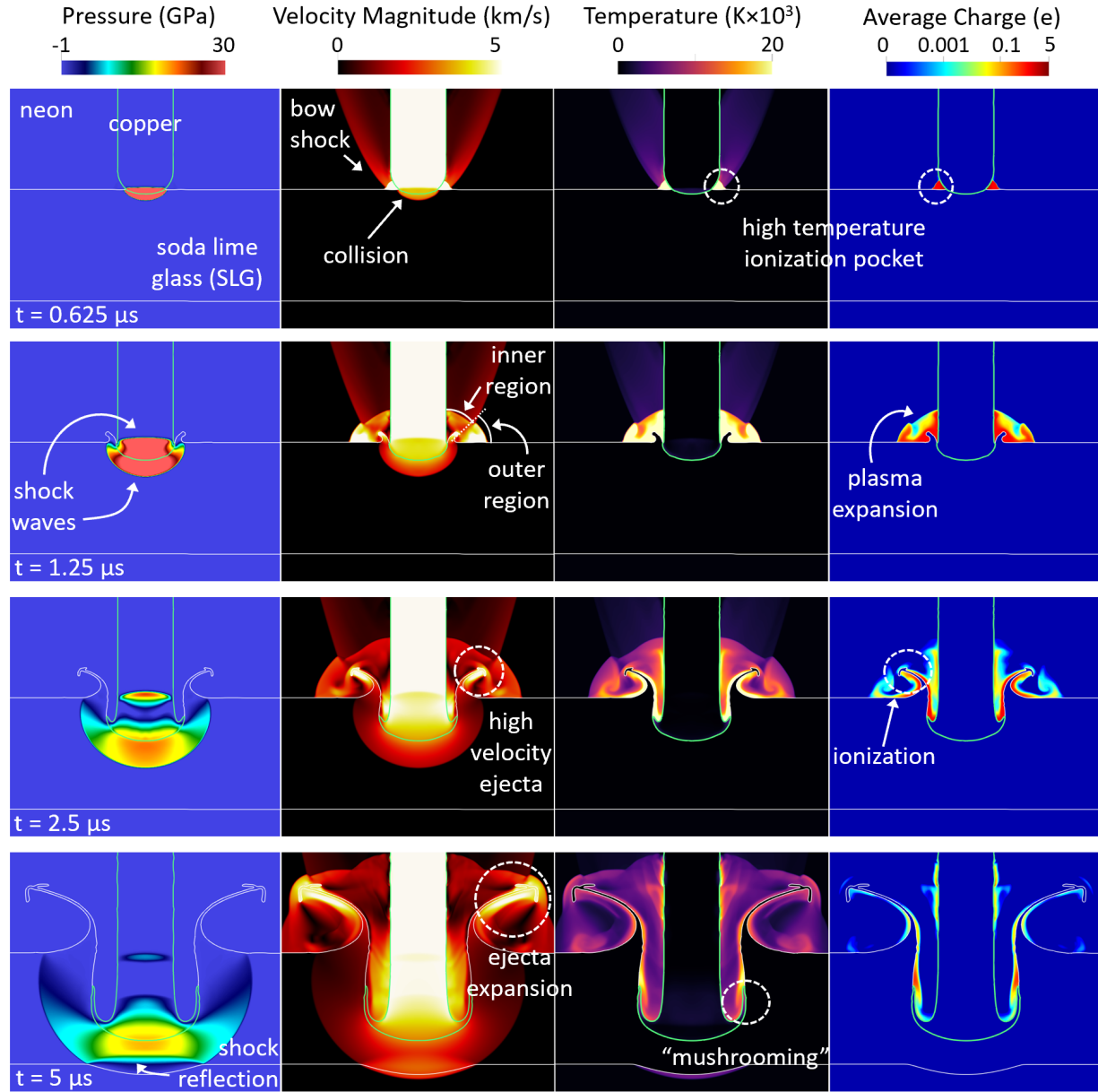


Figure 2.3: Time evolution of state variables for a hypervelocity impact of copper projectile hitting an SLG target at 5 km/s. From left to right, the pressure, velocity magnitude, temperature, and average charge fields are shown at times of 0.625 μ s, 1.25 μ s, 2.5 μ s, and 2.5 μ s (top to bottom). The temperature and average charge fields are computed only in the subdomain occupied by neon, as the present study focuses on the ionization of the ambient fluid.

It is noteworthy that the bow shock generated during the flight of the projectile reaches the target before collision occurs. Remnants of this bow shock can still be observed from the velocity and

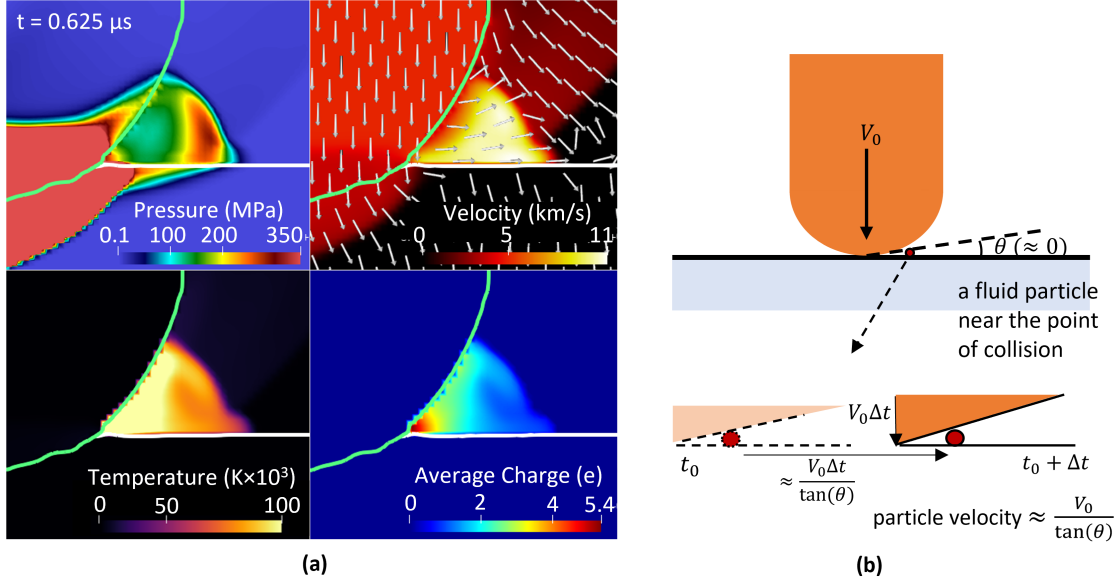


Figure 2.4: Formation of a neon plasma pocket shortly after collision. (a) State variable fields at $t = 0.625 \mu\text{s}$, showing a pocket of neon compressed by the hypervelocity projectile ($V_0 = 5.0 \text{ km/s}$) and the target, resulting in high velocity, high temperature, and the formation of a dense plasma. (b) A rudimentary analysis explaining the high fluid velocity in the pocket.

temperature fields at $t = 0.625 \mu\text{s}$, $1.25 \mu\text{s}$, and $2.5 \mu\text{s}$ (Fig. 2.3). However, a comparison of the state variables behind the bow shock and within the plasma pockets shows that the effect of the bow shock is negligible relative to that of the collision. To this end, typical values of fluid velocity, pressure, and average charge number are shown in Table 2.3. In particular, the bow shock does not cause notable ionization in the neon gas, while the average charge number of neon within the impact-generated plasma pocket is around +1.

	Behind the bow shock	In the plasma pocket
Velocity magnitude $ V $ (km/s)	2.5	10
Pressure p (MPa)	7.5	300
Average charge number Z_{av} (e)	1×10^{-8}	1

Table 2.3: Typical values of state variables behind the bow shock and in the plasma pocket, at $t = 0.625 \mu\text{s}$.

The impact causes the formation of a crater in the target. This process is captured in Fig. 2.3 by the snapshots taken at $t = 1.25 \mu\text{s}$, $2.5 \mu\text{s}$, and $5.0 \mu\text{s}$. During this process, the SLG material from the rim of the crater is ejected into the neon gas in an explosive manner. The tip of the ejecta moves at roughly the impact velocity throughout the simulation, which causes the body of the ejecta to experience high tensile stress. The ejecta divides the fluid subdomain into an inner region and an outer region, as marked in Fig. 2.3. The outer region contains the plasma pocket that had initially formed due to the rapid compression of the neon gas between the projectile and the target. At $t = 1.25 \mu\text{s}$, it can be seen that the plasma pockets are expanding rapidly due to the expanding ejecta. This results in a difference between the velocity of neon in the outer region ($\sim 7 \text{ km/s}$) and that in the inner region ($\sim 1 \text{ km/s}$). The inner region evolves into the crater in the target. Rapid deceleration occurs in the region of the projectile that comes in contact with the initially static target, while the main body of the projectile still moves at the impact velocity. This causes the leading edge of the projectile to morph into a “mushroom head” structure, as seen in the snapshots taken at $t = 5 \mu\text{s}$ (Fig. 2.3). This type of mushrooming deformation has been observed in previous studies [52].

It is evident that the rapid expansion and deformation of the SLG ejecta also causes the surrounding neon gas to ionize. This region of ionization is marked in the average charge plot in Fig. 2.3 at $t = 2.5 \mu\text{s}$. Within this region, neon has high temperature, but low mass density ($\sim 10^{-2} \text{ kg/m}^3$). The plasma density within this region is significantly lower than what was found earlier in the plasma pockets.

Inside the SLG target, the energy density behind the shock front decreases as the shock wave expands. When the shock front reaches the back wall of the target, it causes the wall to deform in the radial directions. At the same time, the compressive shock wave is reflected as a tensile wave, which interferes with the incident shock wave. This behavior is captured in Fig. 2.3, in the snapshots taken at $t = 5 \mu\text{s}$.

In summary, the results obtained from the 5 km/s impact test (Figs. 2.3 and 2.4) show that the collision between the projectile and the target causes the surrounding neon gas to ionize, thereby forming a plasma. The primary mechanism of plasma formation is found to be the rapid compression of the neon gas between the projectile and the target.

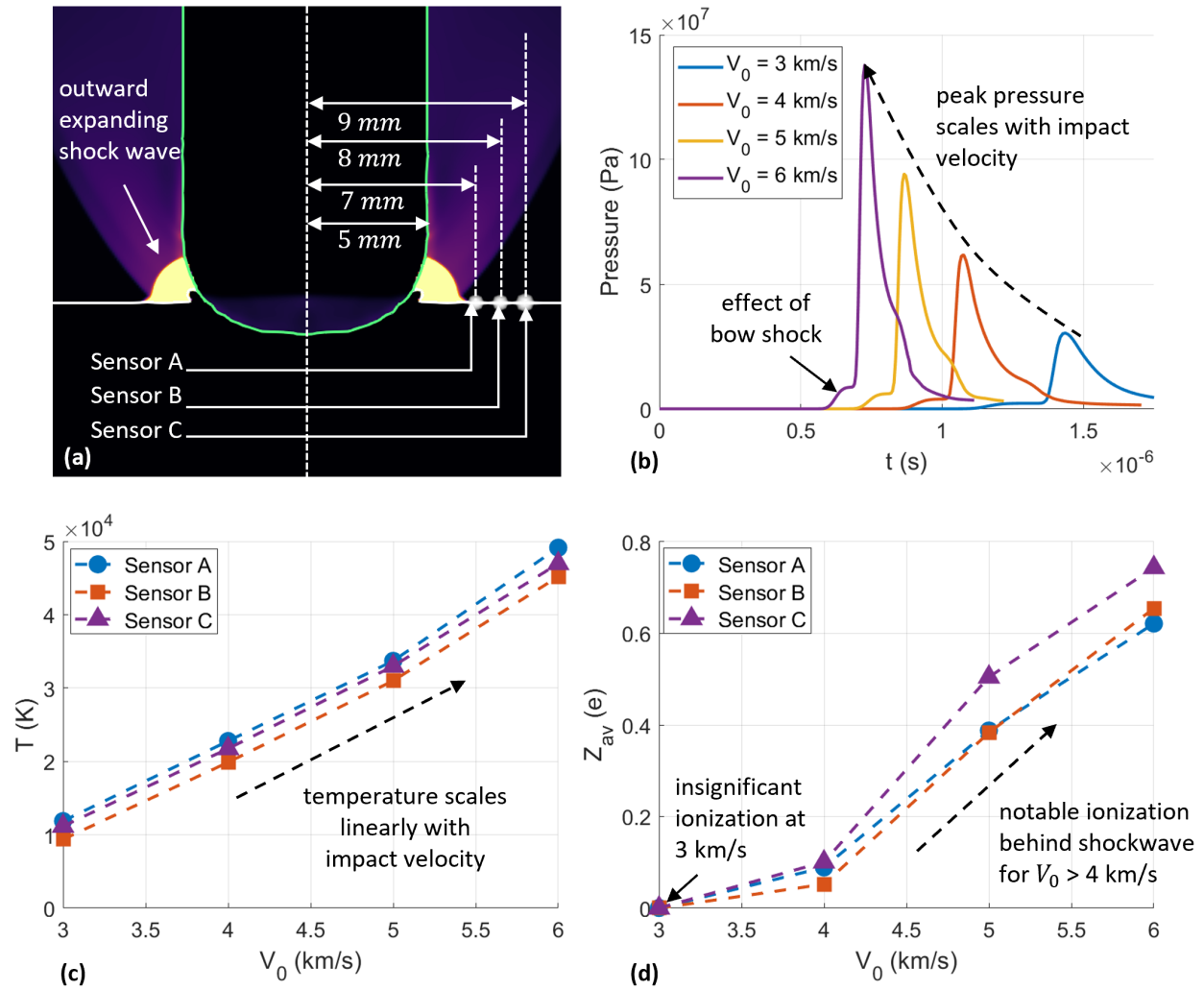


Figure 2.5: Effects of the impact velocity (V_0) on the thermodynamics and ionization of neon. (a) Location of three virtual sensors (A, B, and C) in neon. (b) Time history of pressure at Sensor A. (c) Peak temperature at each sensor (d) Peak average charge at each sensor

To investigate the effects of impact velocity (V_0) on the dynamics, thermodynamics and ionization of neon, additional impact tests are performed for V_0 between 3 and 6 km/s. In each test, three

virtual sensors are positioned 1 mm above the SLG target and at radial distance $r = 7, 8,$ and 9 mm, respectively. The sensors are labeled as A, B, and C, as shown in Fig. 2.5 (a). The time-histories of pressure, temperature, and average charge number are recorded at the sensor locations. Specifically, Fig. 2.5(b) shows the pressure time-histories at Sensor A for $V_0 = 3, 4, 5,$ and 6 km/s. In each case, a sharp pressure pulse is captured, corresponding to the arrival of the shock wave propagating in the neon gas. As V_0 increases, the peak pressure increases monotonically from approximately 30 MPa to 140 MPa. As time progresses, the pressure at the sensor location drops off rapidly. Eventually, the ejecta collides with the sensor, and the reading is terminated at that time instance. Finally, Fig. 2.5(c) and (d) show the maximum values of temperature (T) and average charge number (Z_{av}) of neon at the three sensor locations. As impact velocity increases from 3 to 6 km/s, the maximum temperature increases almost linearly at all the sensors locations. The average charge number also increases with impact velocity, but in a nonlinear fashion. At 3 km/s, the maximum value of Z_{av} is found to be less than 10^{-3} at all the sensor locations, which indicates that ionization is insignificant. As impact velocity exceeds 4 km/s, Z_{av} quickly increases, showing noticeable ionization. At 6 km/s, the maximum value of Z_{av} reaches 0.74 at Sensor C. In this case, the plasma density is of the order of 10^{17} m^{-3} at all the sensor locations.

It is also interesting to note that Fletcher *et al.* [21] showed that Z_{av} in solids is significant at impact velocities > 18 km/s. However, our results indicate that the fluid ionizes at much lower impact velocities. This is due to the much higher temperatures behind the shock waves in the fluids than the solids, which is an order of magnitude greater than the experimental temperatures recorded by Kobayashi *et al.* [30] for shock temperatures in SLG for similar impact velocities (4 to 6 km/s). As such the plasma formed from the ambient fluid for atmospheric hypervelocity impacts may play a critical role in the overall dynamics and characteristics of the impact generated plasma.

2.4 Conclusions

This letter presents a computational analysis of hypervelocity impact events that occur in a fluid medium. The analysis focuses on predicting the formation of plasma in the surrounding fluid, and finding the possible mechanisms that may cause the fluid to ionize. The projectile, target, and ambient fluid materials used in this study are copper, soda lime glass (SLG), and neon, respectively. The projectile is in the shape of a cylindrical rod, with a semi-spherical leading edge. The impact velocity is varied between 3 km/s and 6 km/s in different tests. The computational framework developed for this study couples a compressible Navier-Stokes equations solver with a Saha ionization equations solver. Compared to conventional impact simulators, a notable difference of the current framework is that the computational domain includes the ambient fluid as a material subdomain, and the fluid's dynamics, thermodynamics, and ionization are included in the physical model. Also, the shock-dominated hypersonic flow around the flying projectile is captured through a steady-state computational fluid dynamics (CFD) analysis, and prescribed as an initial condition for the impact analysis.

The inclusion of the ambient fluid leads to two computational challenges. First, the domain includes three internal material interfaces between the projectile, the target, and the ambient fluid. These interfaces are interconnected (in the sense that triple points exist), and undergo large, complex deformation during the impact. In this study, the interfaces are tracked implicitly by solving two level set equations that share the same velocity field. This method maintains sharp (i.e. 0 thickness) interfaces, and naturally avoids spurious overlapping and separation between different material subdomains. Second, the state variables and thermodynamic relations change drastically across the solid-gas interfaces. To account for this type of discontinuities, the recently developed FIVER method is employed. When computing numerical fluxes across a material interface, FIVER constructs and solves an exact bimaterial Riemann problem using the correct equations of state on

both sides of the interface.

The main finding of this study is that for impact velocities greater than 4 km/s, ionization of neon is predicted in the early stage of the impact, adjacent to the point of collision. The ionized region is behind the shock front in the ambient gas, referred to as plasma pockets in the letter. Within this region, neon has high temperature, pressure, velocity, and mass density. Moreover, the plasma density (i.e. electron number density) reaches the order of 10^{17} m^{-3} in the test case with an impact velocity of 6 km/s. The simulation results suggest that the main cause of ionization in this region is the rapid compression of the neon gas between the hypervelocity projectile and the target. The high velocity in the region can also be related to the geometry of the projectile, particularly, the blunt leading edge. As time progresses, the size of the plasma pockets increases, while the intensity decreases. Meanwhile, the SLG material at the rim of the impact crater is ejected into the ambient environment in an explosive manner, which also causes the surrounding neon to ionize. Nonetheless, the effect of this mechanism is insignificant compared to the hypervelocity compression that formed the plasma pockets. The computational analysis also captures the impact of the bow shock in the neon gas on the SLG target, which occurs before the collision. The effect of the bow shock is found to be insignificant compared to that of the collision. Following these findings, additional work is needed to design and perform a validation study, and to investigate the sensitivity of plasma density with respect to geometric and material properties, possibly over a broader range of impact velocities.

Acknowledgment

S.T.I, W.M., and K.W. gratefully acknowledge the support of the Office of Naval Research (ONR) under Award N00014-19-1-2102. J.G.M. gratefully acknowledges the support of the ONR under award N00014-21-WX01554.

Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Bibliography

- [1] Aero-f source code, 2021. URL <https://bitbucket.org/frg/aero-f>.
- [2] M2c source code, 2022. URL <https://github.com/kevinwgy/m2c>.
- [3] GE Alefeld, Florian A Potra, and Yixun Shi. Algorithm 748: Enclosing zeros of continuous functions. *ACM Transactions on Mathematical Software (TOMS)*, 21(3):327–344, 1995.
- [4] C. S. Alexander, L. C. Chhabildas, D. W. Templeton, Mark Elert, Michael D. Furnish, Ricky Chau, Neil Holmes, and Jeffrey Nguyen. The hugoniot elastic limit of soda-lime glass. *AIP Conference Proceedings*, 2007. doi: 10.1063/1.2833222.
- [5] C Scott Alexander, William D Reinhart, Tracy Vogler, Lalit C Chhabildas, TF Thornhill, and DE Grady. Status of glass experiments at sandia national laboratories. Technical report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States); Sandia ..., 2006.
- [6] Paul D. Asimow, Chaney Lin, Luca Bindi, Chi Ma, Oliver Tschauner, Lincoln S. Hollister, and Paul J. Steinhardt. Shock synthesis of quasicrystals with implications for their origin in asteroid collisions. *Proceedings of the National Academy of Sciences*, 113(26):7077–7081, 2016. doi: 10.1073/pnas.1600321113.
- [7] Roshdy G Barsoum. Shockwave compression physics of condensed matter lattice boltzmann method with application to glass under hypervelocity impact, 2019.

- [8] R. Bianchi, F. Capaccioni, P. Cerroni, M. Coradini, E. Flamini, P. Hurren, G. Martelli, and P. N. Smith. Radiofrequency emissions observed during macroscopic hypervelocity impact experiments. *Nature*, 308(5962):830–832, 1984. doi: 10.1038/308830a0.
- [9] Andrew J. Birnbaum, John C. Steuben, Athanasios P. Iliopoulos, and John G. Michopoulos. Simulating hypervelocity impact and material failure in glass. *Volume 1B: 38th Computers and Information in Engineering Conference*, 2018. doi: 10.1115/detc2018-85948.
- [10] N. S. Brar and S. J. Bless. Failure waves in glass under dynamic compression. *High Pressure Research*, 10(5-6):773–784, 1992. doi: 10.1080/08957959208225330.
- [11] Andrew Cachiaras, Luke Gilde, Jeffrey J Swab, Parimal J Patel, and George D Quinn. Soda-lime-silicate float glass: A property comparison. Technical report, US Army Research Laboratory Aberdeen Proving Ground United States, 2017.
- [12] S Cao, G Wang, O Coutier-Delgosha, and K Wang. Shock-induced bubble collapse near solid materials: Effect of acoustic impedance. *Journal of Fluid Mechanics*, 907, 2021.
- [13] Shunxiang Cao, Ying Zhang, Defei Liao, Pei Zhong, and Kevin G Wang. Shock-induced damage and dynamic fracture in cylindrical bodies submerged in liquid. *International journal of solids and structures*, 169:55–71, 2019.
- [14] Francis F. Chen. *Introduction to plasma physics and Controlled Fusion*. Springer, 2018.
- [15] Burton G. Cour-Palais. Hypervelocity impact in metals, glass and composites. *International Journal of Impact Engineering*, 5:221–237, 01 1987. doi: 10.1016/0734-743x(87)90040-6.
- [16] Dattatraya P. Dandekar. Index of refraction and mechanical behavior of soda lime glass under shock and release wave propagations. *Journal of Applied Physics*, 84(12):6614–6622, 1998. doi: 10.1063/1.369035.

- [17] C. Farhat, K.G. Wang, A. Main, S. Kyriakides, L.-H. Lee, K. Ravi-Chandar, and T. Be-lytschko. Dynamic implosion of underwater cylindrical shells: Experiments and computations. *International Journal of Solids and Structures*, 50(19):2943–2961, 2013. doi: 10.1016/j.ijsolstr.2013.05.006.
- [18] Charbel Farhat, Arthur Rallu, and Sriram Shankaran. A higher-order generalized ghost fluid method for the poor for the three-dimensional two-phase flow computation of underwater implosions. *Journal of Computational Physics*, 227(16):7674–7700, 2008. doi: 10.1016/j.jcp.2008.04.032.
- [19] Charbel Farhat, Jean-Frédéric Gerbeau, and Arthur Rallu. Fiver: A finite volume method based on exact two-phase riemann problems and sparse grids for multi-material flows with large density jumps. *Journal of Computational Physics*, 231(19):6360–6379, 2012. doi: 10.1016/j.jcp.2012.05.026.
- [20] Alex Fletcher. *Plasma production and radiation from meteoroid impacts on spacecraft*. PhD thesis, 2015.
- [21] Alex Fletcher, Sigrid Close, and Donovan Mathias. Simulating plasma production from hypervelocity impacts. *Physics of Plasmas*, 22(9):093504, 2015. doi: 10.1063/1.4930281.
- [22] Alex C. Fletcher and Sigrid Close. Particle-in-cell simulations of an rf emission mechanism associated with hypervelocity impact plasmas. *Physics of Plasmas*, 24(5):053102, 2017. doi: 10.1063/1.4980833.
- [23] Joseph Fred Friichtenicht and John Charles Slattery. Ionization associated with hypervelocity impact. Technical report, 1963.
- [24] Dennis E Grady and LC Chhabildas. Shock-wave properties of soda-lime glass. Technical report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 1996.

- [25] E. P. Gyftopoulos and Gian Paolo. Beretta. *Thermodynamics: foundations and applications*. Dover Publications, 2005.
- [26] T. Harano, Y. Machida, S. Fukushige, T. Koura, S. Hosoda, M. Cho, and Y. Akahoshi. Preliminary study on sustained arc due to plasma excited by hypervelocity impact of space debris on the solar array coupon. *International Journal of Impact Engineering*, 33(1-12):326–334, 2006. doi: 10.1016/j.ijimpeng.2006.09.016.
- [27] Jonathan Ho and Charbel Farhat. Discrete embedded boundary method with smooth dependence on the evolution of a fluid-structure interface. *International Journal for Numerical Methods in Engineering*, 122(19):5353–5383, 2021.
- [28] Hao Jiang, Valerie Scott, and Bo Li. Modeling and simulations of high and hypervelocity impact of small ice particles. *International Journal of Impact Engineering*, 155:103906, 2021. doi: 10.1016/j.ijimpeng.2021.103906.
- [29] Akshay Joshi, Suraj Ravindran, Vatsa Gandhi, and Guruswami Ravichandran. Probing the properties and mechanisms of failure waves in soda-lime glass. *Journal of Applied Physics*, 129(18):185902, 2021. doi: 10.1063/5.0047950.
- [30] T. Kobayashi, T. Sekine, O. V. Fat’yanov, E. Takazawa, and Q. Y. Zhu. Radiation temperatures of soda-lime glass in its shock-compressed liquid state. *Journal of Applied Physics*, 83(3): 1711–1716, 1998. doi: 10.1063/1.366889.
- [31] R.J. Lawrence, W.D. Reinhart, L.C. Chhabildas, and T.F. Thornhill. Spectral measurements of hypervelocity impact flash. *International Journal of Impact Engineering*, 33(1-12):353–363, 2006. doi: 10.1016/j.ijimpeng.2006.09.010.
- [32] Nicolas Lee. *Understanding spacecraft electrical anomalies: Theory and experiments characterizing hypervelocity impact plasma dynamics*. Stanford University, 2012.

- [33] Nicolas Lee, Sigrid Close, Ashish Goel, David Lauben, Ivan Linscott, Theresa Johnson, David Strauss, Sebastian Bugiel, Anna Mocker, Ralf Srama, and et al. Theory and experiments characterizing hypervelocity impact plasmas on biased spacecraft materials. *Physics of Plasmas*, 20(3):032901, 2013. doi: 10.1063/1.4794331.
- [34] B Li, A Kidane, G Ravichandran, and M Ortiz. Verification and validation of the optimal transportation meshfree (otm) simulation of terminal ballistics. *International Journal of Impact Engineering*, 42:25–36, 2012.
- [35] Jianqiao Li, Weidong Song, and Jianguo Ning. Theoretical and numerical predictions of hypervelocity impact-generated plasma. *Physics of Plasmas*, 21(8):082112, 2014. doi: 10.1063/1.4893310.
- [36] Wentao Ma, Xuning Zhao, Christine Gilbert, and Kevin Wang. Computational analysis of bubble–structure interactions in near-field underwater explosion. *International Journal of Solids and Structures*, 242:111527, 2022.
- [37] Rosemary A. MacDonald and William M. MacDonald. Thermodynamic properties of fcc metals at high temperatures. *Physical Review B*, 24(4):1715–1724, August 1981. doi: 10.1103/PhysRevB.24.1715. URL <https://link.aps.org/doi/10.1103/PhysRevB.24.1715>.
- [38] Alex Main, Xianyi Zeng, Philip Avery, and Charbel Farhat. An enhanced five method for multi-material flow problems with second-order convergence rate. *Journal of Computational Physics*, 329:141–172, 2017. doi: 10.1016/j.jcp.2016.10.028.
- [39] William Martin, Jeffrey Fuhr, Daniel Kelleher, Arlene Musgrove, J Sugar, Wolfgang Wiese, Peter Mohr, and Karen Olsen. Nist atomic spectra database (version 2.0) (1999), 1999.
- [40] Neil McBride and J. A. McDonnell. Meteoroid impacts on spacecraft:: sporadics, streams,

- and the 1999 leonids. *Planetary and Space Science*, 47(8-9):1005–1013, 1999. doi: 10.1016/S0032-0633(99)00023-9.
- [41] A. C. Mitchell and W. J. Nellis. Shock compression of aluminum, copper, and tantalum. *Journal of Applied Physics*, 52(5):3363–3374, May 1981. ISSN 0021-8979. doi: 10.1063/1.329160. URL <https://aip.scitation.org/doi/10.1063/1.329160>.
- [42] Karina A Monroe. Time-resolved fractoluminescence characterization in soda-lime glass via near hypervelocity kinetic impact fast fracture. Master’s thesis, Monterey, CA; Naval Postgraduate School, 2021.
- [43] Stanley Osher and Ronald P. Fedkiw. *Level set methods and dynamic implicit surfaces*. Springer, 2009.
- [44] Paul R. Ratcliff, Mark J. Burchell, Mike J. Cole, Tom W. Murphy, and Firooz Alladfadi. Experimental measurements of hypervelocity impact plasma yield and energetics. *International Journal of Impact Engineering*, 20(6-10):663–674, 1997. doi: 10.1016/S0734-743X(97)87453-2.
- [45] Allen C Robinson. The mie-gruneisen power equation of state. Technical report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 2019.
- [46] Z. Rosenberg, D. Yaziv, and S. Bless. Spall strength of shock-loaded glass. *Journal of Applied Physics*, 58(8):3249–3251, 1985. doi: 10.1063/1.335781.
- [47] Z Rosenberg, Y Ashuach, E Dekel, and E Bar-On. The split-target technique for measurements of strength under high shock pressures. *Measurement Science and Technology*, 19(4):045706, 2008. doi: 10.1088/0957-0233/19/4/045706.
- [48] Richard Saurel and Rémi Abgrall. A simple method for compressible multifluid flows. *SIAM Journal on Scientific Computing*, 21(3):1115–1145, 1999.

- [49] William P. Schonberg. Hypervelocity impact penetration phenomena in aluminum space structures. *Journal of Aerospace Engineering*, 3:173–185, 07 1990. doi: 10.1061/(asce)0893-1321(1990)3:3(173).
- [50] James Albert Sethian and James Albert Sethian. *Level set methods and fast marching methods: Evolving interfaces in computational geometry, Fluid Mechanics, computer vision, and materials science*. Cambridge University Press, 1999.
- [51] Weidong Song, Jianqiao Li, and Jianguo Ning. Characteristics of plasma generated by hypervelocity impact. *Physics of Plasmas*, 20(9):093501, 2013. doi: 10.1063/1.4819829.
- [52] Yunhou Sun, Cuncheng Shi, Zheng Liu, and Desheng Wen. Theoretical research progress in high-velocity/hypervelocity impact on semi-infinite targets. *Shock and Vibration*, 2015:1–15, 2015. doi: 10.1155/2015/265321.
- [53] K. G. Wang, P. Lea, and C. Farhat. A computational framework for the simulation of high-speed multi-material fluid-structure interaction problems with dynamic fracture. *International Journal for Numerical Methods in Engineering*, 104(7):585–623, 2015. doi: 10.1002/nme.4873.
- [54] Kevin G. Wang. Multiphase fluid-solid coupled analysis of shock-bubble-stone interaction in shockwave lithotripsy. *International Journal for Numerical Methods in Biomedical Engineering*, 33(10), 2017. doi: 10.1002/cnm.2855.
- [55] F.L. Whipple. Meteorites and space travel. *Astronomical Journal*, 52:137, 1947.
- [56] Gaoming Xiang, Xiaojian Ma, Cosima Liang, Hongyang Yu, Defei Liao, Georgy Sankin, Shunxiang Cao, Kevin Wang, and Pei Zhong. Variations of stress field and stone fracture produced at different lateral locations in a shockwave lithotripter field. *The Journal of the Acoustical Society of America*, 150(2):1013–1029, 2021. doi: 10.1121/10.0005823.

- [57] Mofreh R. Zaghloul. Reduced formulation and efficient algorithm for the determination of equilibrium composition and partition functions of ideal and nonideal complex plasma mixtures. *Physical Review E*, 69(2), 2004. doi: 10.1103/physreve.69.026702.
- [58] Mofreh R Zaghloul, Mohamed A Bourham, and J Michael Doster. A simple formulation and solution strategy of the saha equation for ideal and nonideal plasmas. *Journal of Physics D: Applied Physics*, 33(8):977–984, 2000. doi: 10.1088/0022-3727/33/8/314.
- [59] Kai Zhang, Renrong Long, Qingming Zhang, Yijiang Xue, and Yuanyuan Ju. Flash characteristics of plasma induced by hypervelocity impact. *Physics of Plasmas*, 23(8):083519, 2016.

Chapter 3

Fluid-Solid Coupled Simulation of Hypervelocity Impact and Plasma Formation

(Published in *International Journal of Impact Engineering*, 180, 104695, 2023.)

S. Islam^a, W. Ma^a, John G. Michopoulos^b, K. Wang^a

^a Department of Aerospace and Ocean Engineering, Virginia Tech, Blacksburg, VA, USA

^b US Naval Research Laboratory, Washington DC, USA

Abstract

Previous theoretical and computational studies on hypervelocity impact have mainly focused on the dynamic response of the solid materials that constitute the projectile and the target, while the surrounding environment is often assumed to be a vacuum. In this paper, we consider impact events that occur in a fluid (e.g., gas) medium, and present a computational model that includes the dynamics, thermodynamics, and ionization of the surrounding fluid material. The model couples the compressible Navier-Stokes equations with the Saha equations to predict the onset and extent of ionization in the surrounding fluid. An extended level set method with two signed distance functions is employed to track the three material interfaces between the projectile, the target, and the ambient fluid. This method accommodates the large deformation, contact, and separation of the interfaces, while avoiding spurious overlapping of different material subdomains. The advective fluxes across material interfaces are computed by constructing and solving bimaterial Riemann problems, thereby taking into account the discontinuities in both state variables (e.g., density) and thermodynamic relations. The computational model is first verified for an infinite ideal plasma and a one-dimensional three-material impact problem. Next, the model is employed to analyze the impact of a tantalum rod projectile onto a soda lime glass (SLG) target in an argon gas environment. In different analyses, the impact velocity is varied between 3 and 6 km/s, and the radius of the projectile is varied between 2.5 and 10 mm. Each analysis starts with a steady-state fluid dynamics simulation that generates the shock-dominated hypersonic flow around the projectile. This flow field is then used as an initial condition to start the fluid-solid coupled impact simulation. The predicted maximum temperature and pressure within the SLG target agree reasonably well with published experimental data for a similar material (fused quartz). Within the ambient gas, the impact-generated shock wave is found to be stronger than the initial bow shock in front of the projectile. Behind this shock wave, a region of high pressure and temperature forms in the early stage of the impact, mainly due to the hypersonic compression of the fluid between the projectile

and the target. The temperature within this region is significantly higher than the peak temperature in the solid materials. For impact velocities higher than 4 km/s, ionization is predicted. This finding indicates that the ambient gas may be a nontrivial contributor to the plasma formed in terrestrial and atmospheric hypervelocity impact events.

3.1 Introduction

Hypervelocity impact is a challenging multiphysics problem that features the rapid transport and dissipation of kinetic energy through mechanical, thermal, chemical, and electromagnetic pathways. In the past, extensive research has been conducted to understand and predict the mechanical response of the target and the projectile, such as shock waves, deformation, fracture, and fragmentation [8, 45]. It has also been found that behind the shock waves the solid materials can have pressures of the order of tens of gigapascals, and temperatures of thousands of Kelvins. This extreme thermodynamic state may cause the material to ionize and form a plasma [6, 28]. One of the earliest reports of impact-generated plasma was by Friichtenicht and Slattery (1963) [18], in which spherical projectiles made of iron and graphite were accelerated using an electrostatic accelerator to velocities of up to 16 km/s. Since then, various authors have investigated the composition and energy of plasma generated under different impact conditions. For example, Ratcliff *et al.* reported plasma temperatures ranging between 20 eV¹ and 40 eV, when the impact velocity is varied between 1.2 km/s and 87 km/s [40]. However, the correlation between temperature and impact velocity was not discussed. Studies were also performed by Lee *et al.* and Fletcher *et al.* on biased and grounded targets. They reported temperature measurements around 2 eV for impact velocities between 10 km/s and 30 km/s [16, 30]. In some cases, it is found that impact-generated plasma is accompanied with electromagnetic emissions. One of the earliest reports in this regard was made

¹1 eV = 11, 605 K.

by Bianchi *et al.* [7]. Since then, some efforts have been made to characterize and explain these emissions. Maki *et al.* conducted rail gun experiments using polycarbonate projectiles with a mass of 1.1 g, accelerated to 2 – 6.7 km/s [34, 35]. They found electromagnetic emissions in the microwave frequency range (22 GHz-band), and hypothesized that these emissions may come from impact-generated micro-cracks in the target. More recently, Close *et al.* performed impact experiments with electrostatic accelerators, using micro-projectiles (10^{-16} – 10^{-11} g) made of iron and targets made of aluminum and tungsten [11]. At impact velocities between 3 km/s and 66 km/s, they detected emissions in the radio frequency range using patch antennas tuned to 315 MHz and 916 MHz. They attributed the emissions to the coherent motion of charged particles in the plasma due to a self generated ambipolar field [17]. Furthermore, Estacio *et al.* performed a series of hypervelocity experiments on grounded and biased targets to explore the causal source of these radio frequency electromagnetic pulse emissions [12]. They used a two-stage light gas gun and a Van de Graaf accelerator. The projectiles used in the light gas gun were 46.5 mg aluminum spheres with impact velocities of 5 km/s and a vacuum pressure of 93 Pa to 67 Pa. Whereas, the Van de Graaf accelerator launched projectiles made of iron and their mass ranged from 35 pg to 0.15 fg and the impact velocity ranged between 2.8 to 67 km/s with a vacuum pressure of roughly 10^{-4} Pa. In this study they observed that the peak frequency of the electromagnetic pulses corresponded directly with the target configuration. Overall, researchers' knowledge on impact-generated ionization, plasma formation, and electromagnetic emission is still limited. Discrepancies amongst different studies can often be attributed to the lack of uniformity in experimental conditions. For example, the vacuum pressures in light gas gun experiments can range between 10^0 and 10^2 Pa, whereas, for experiments with electrostatic accelerators the pressure can range between 10^{-5} and 10^{-3} Pa. While the latter experiments can attain a better vacuum condition and accelerate projectiles to higher velocities, they can only launch projectiles of a much smaller mass.

Partly due to the high experimental costs and limitations of apparatus, there has been growing inter-

est in developing computational models to predict impact-generated plasma and electromagnetic waves. For example, Li *et al.* [31, 47] simulated hypervelocity impact of aluminum projectile and target for impact velocities between 5 and 10 km/s using both commercial and self-developed codes that couple smoothed particle hydrodynamics (SPH) with the Thomas-Fermi model. Fletcher *et al.* [16] also developed a SPH code to simulate hypervelocity impacts and used the non-ideal Saha equations to predict ionization in the target material. Later, Fletcher *et al.* developed a particle-in-cell (PIC) code to investigate the source of electromagnetic emission from the impact-generated plasma [17]. Recently, La Spina *et al.* combined a spherically symmetric blast wave model with the non-ideal Saha equations and the Frank-Tamm formula to investigate the onset of Cherenkov radiation from glass materials under hypervelocity impact [29]. Despite these progresses, the generation of plasma and electromagnetic waves from hypervelocity impacts remains an active research area. Open questions in this area include the source (i.e. projectile, target, or the surrounding gas) and composition of plasma, the dependence of plasma energy on impact velocity (cf. [30, 40]), and the energy and spectrum of the electromagnetic emissions [17].

Previous theoretical and computational studies on hypervelocity impact usually assume the ambient environment to be a vacuum, thereby neglecting its role in the impact events. This assumption can be valid for impacts that occur on spacecrafts in the absence of an atmosphere. However, the ambient fluid may be a contributor to the impact-generated plasma for terrestrial and atmospheric applications of hypervelocity impact. Before the projectile reaches the target, it is already surrounded by a shock-dominated hypersonic fluid flow. When the projectile impacts on the target, another shock wave forms at the point of contact, and propagates radially through the fluid medium. This shock wave can be much stronger than the bow shock formed during the hypersonic flight of the projectile [24]. It was found that the pressure in the fluid behind the impact-generated shock wave can be approximately 40 times greater than in the fluid behind the bow shock. Compared to the solid materials that constitute the projectile and the target, the ambient gas has much lower

density, and is far more compressible. Therefore, the gaseous material behind the shock wave may reach a temperature that is higher than that in the solids. Hence, the ambient gas may also ionize, thereby contributing to the plasma mixture formed during hypervelocity impact events. It is important to understand the role of the ambient fluid material in order to develop a complete description of hypervelocity impacts.

In this paper, we present the development of a fluid-solid coupled computational model of hypervelocity impact, including the formation of plasma within the ambient fluid. The computational domain is comprised of three non-overlapping subdomains, occupied by the projectile, the target, and the ambient fluid, respectively. The inclusion of the ambient fluid flow is a main feature that distinguishes this work from previous studies (e.g., [9, 19]). The compressible inviscid Navier-Stokes equations are adopted to model the dynamics of all the solid and fluid materials. The fluid flow is dominated by shock waves, and the solid structures exhibit large, complex deformations. Therefore, we solve the governing equations in the Eulerian reference frame, using a high-resolution finite volume method. To track the dynamics of the material interfaces (i.e. subdomain boundaries), we apply the level set method [39, 46, 52]. In particular, the boundaries of the projectile and the target are represented implicitly as the 0 level set of two signed distance functions. In this way, we solve two level set equations to track three material interfaces, namely, projectile-target, projectile-fluid, and target-fluid. Across the solid-fluid interfaces, mass density jumps by several orders of magnitude; the ambient density in solids is usually greater than 1 g/cm^3 whereas the density of the gaseous fluid is of the order of 10^{-3} g/cm^3 . Moreover, the equations of state (EOS) used to model these materials may also differ significantly; the solids are modelled using much stiffer EOS (e.g. Mie Grüneisen) as opposed to softer EOS used for the fluid (e.g. perfect gas). This type of discontinuities poses a challenge to the computation of fluxes across material interfaces. Several numerical schemes which are known to perform well for single-phase flows, develop spurious oscillations near material interfaces [25]. These oscillations may lead to loss of accuracy and numerical

stability. In this work, we compute the mass, momentum, and energy fluxes across material interfaces using the FInite Volume method based on Exact multi-material Riemann problem (FIVER) [14, 48]. By constructing and solving an exact bimaterial Riemann problem along each edge in the mesh that crosses a material interface, FIVER explicitly accounts for the change of equation of state across the interface. Previously, FIVER has been validated for several shock-dominated multiphase flow and fluid-structure interaction problems in underwater explosion and implosion, pipeline explosion, cavitation erosion, and shock wave lithotripsy [10, 13, 32, 48, 49].

To predict ionization in the fluid subdomain, we solve the ideal Saha equation, coupled with the conservation of charge and the conservation of nuclei [50]. These equations are referred to collectively as the Saha equations. In this study the most significant ionization was found in the fluid near the projectile-fluid interface. At these locations, the mass density of the gaseous material is found to be very low — an order of magnitude lower than the ambient ($\sim 2 \times 10^{-4} \text{ g/cm}^3$) — the impact-generated plasma has a large Debye length. This minimizes the effects of non-ideality of the plasma [15]. The Saha equations relate the ionization state of the plasma with its thermodynamic state, i.e. pressure and temperature. This coupling allows us to predict the extent of ionization and compute the distribution of ionization products. This model assumes local thermodynamic equilibrium, which can be justified for predicting the formation and initial expansion of plasma during hypervelocity impacts [16].

We present a verification study that includes two simplified model problems relevant to hypervelocity impact. In the first problem, we assume an infinite ideal plasma at constant pressure, with temperature up to $5 \times 10^4 \text{ K}$. We solve the Saha equations to compute the composition and mean charge of the plasma, and compare the results with reference data provided in Zaghloul *et al.* [51]. The second example is a one-dimensional multi-material impact problem, for which the exact solution can be obtained up to the time that any rarefaction wave reaches a material interface.

Next, we apply the computational model to simulate the impact of a tantalum rod projectile onto

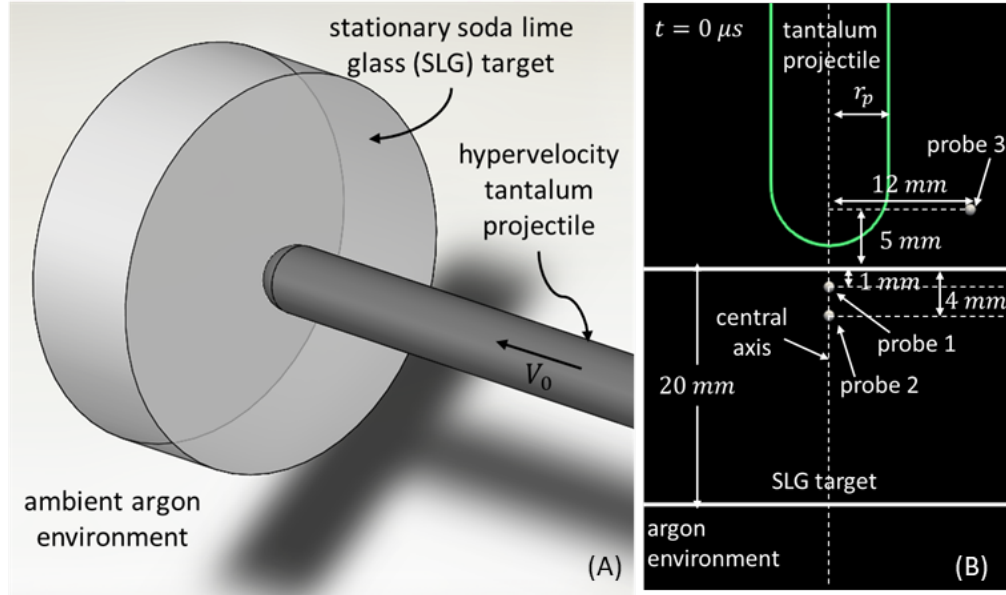


Figure 3.1: Problem setup. (A) A 3D depiction of the target, projectile, and ambient fluid in the model problem. (B) A 2D cross-section with annotated geometry and probe locations.

a target made of soda lime glass (SLG) in an argon gas environment (Fig. 3.1). Tantalum is a hard, refractory metal that is often used in impact experiments and applications. SLG is selected as the target material for its potential application in armor and protective systems [27, 38]. Argon is selected as the ambient fluid because it is monoatomic, and chemically inert even under extreme pressure and temperature conditions. In different simulations, we vary the projectile's impact velocity (V_0) between 3 km/s and 6 km/s, and its radius (r_p) between 2.5 mm and 10 mm. The velocity and thermodynamic state within the solid and fluid materials are investigated and compared. The extent of ionization in the ambient fluid is characterized by the mean charge and the plasma density.

3.2 Materials and methods

3.2.1 Continuum dynamics

Figure 3.1 presents the setup of the hypervelocity impact problem investigated in this paper. Without loss of generality, the projectile is assumed to be a cylindrical tantalum rod with a spherical leading edge. The target is a cylindrical plate made of SLG with a radius of 30 mm and a height of 20 mm. In the far-field, the density, pressure, and temperature of argon are fixed at $1.78 \times 10^{-3} \text{ g/cm}^3$, 100 kPa, and 300 K, respectively. Figure 3.1(B) displays the geometry of the three material subdomains and the location of three virtual probes that are placed within the SLG target and the ambient gas.

For the range of impact velocities studied in this work (3 km/s to 6 km/s), the density of the energy transferred from the projectile to the SLG target is far greater than the strain energy density of SLG. In the past, Kobayashi *et al.* showed that when SLG is impacted by steel and tungsten projectiles traveling at 4 km/s to 6 km/s, the maximum pressure inside it exceeds 50 GPa [28], which is an order of magnitude higher than the material's Hugoniot elastic limit (3 GPa to 8 GPa). Tantalum also has a relatively low Hugoniot elastic limit, around 2 GPa [41]. Therefore, the solid materials involved in this problem, namely tantalum and SLG, are modeled as compressible fluids. Material failure models (e.g., plasticity, fracture) are currently not included in the computational framework.

Therefore, the dynamics of the target, the projectile, and the surrounding gas can be considered to be governed by the three-dimensional (3D) compressible Navier-Stokes equations. In addition, we neglect the effects of viscosity and heat diffusion, which reduces the Navier-Stokes equations to

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot F(\mathbf{q}) = 0, \quad (3.1)$$

In the Cartesian coordinate system,

$$F(\mathbf{q}) = \begin{bmatrix} f(\mathbf{q}) & g(\mathbf{q}) & h(\mathbf{q}) \end{bmatrix}, \quad \text{with}$$

$$\mathbf{q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{bmatrix}, \quad f(\mathbf{q}) = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ \rho Hu \end{bmatrix}, \quad g(\mathbf{q}) = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ \rho Hv \end{bmatrix}, \quad h(\mathbf{q}) = \begin{bmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho v^2 + p \\ \rho Hw \end{bmatrix}.$$

Here, ρ is the mass density. p is the pressure, $\mathbf{V} = \begin{bmatrix} u & v & w \end{bmatrix}^T$ is the velocity vector. E is the total energy per unit volume, given by $E = \rho e + \frac{1}{2}\rho\|\mathbf{V}\|_2^2$; and H is the total enthalpy per unit mass, defined by $H = \frac{1}{\rho}(E + p)$.

In this study we assume that the azimuthal variations in $\mathbf{q}$ are negligible in comparison to its axial and radial components. Thus by leveraging the cylindrical symmetry of this problem (see Fig. 3.1), we solve the 3D problem in a 2D computational domain. The 2D governing equations that account for cylindrical symmetry are given by

$$\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho u_r \\ \rho w \\ E \end{bmatrix} + \frac{\partial}{\partial r} \begin{bmatrix} \rho u_r \\ \rho u_r^2 + p \\ \rho u_r w \\ \rho u_r H \end{bmatrix} + \frac{\partial}{\partial z} \begin{bmatrix} \rho w \\ \rho u_r w \\ \rho w^2 + p \\ \rho w H \end{bmatrix} = -\frac{1}{r} \begin{bmatrix} \rho u_r \\ \rho u_r^2 \\ \rho u_r w \\ \rho u_r H \end{bmatrix}, \quad (3.2)$$

Here, u_r and w are the radial and axial components of velocity, and r and z are the radial and axial directional coordinates.

Before the projectile makes contact with the target, the surrounding fluid flow is already non-trivial.

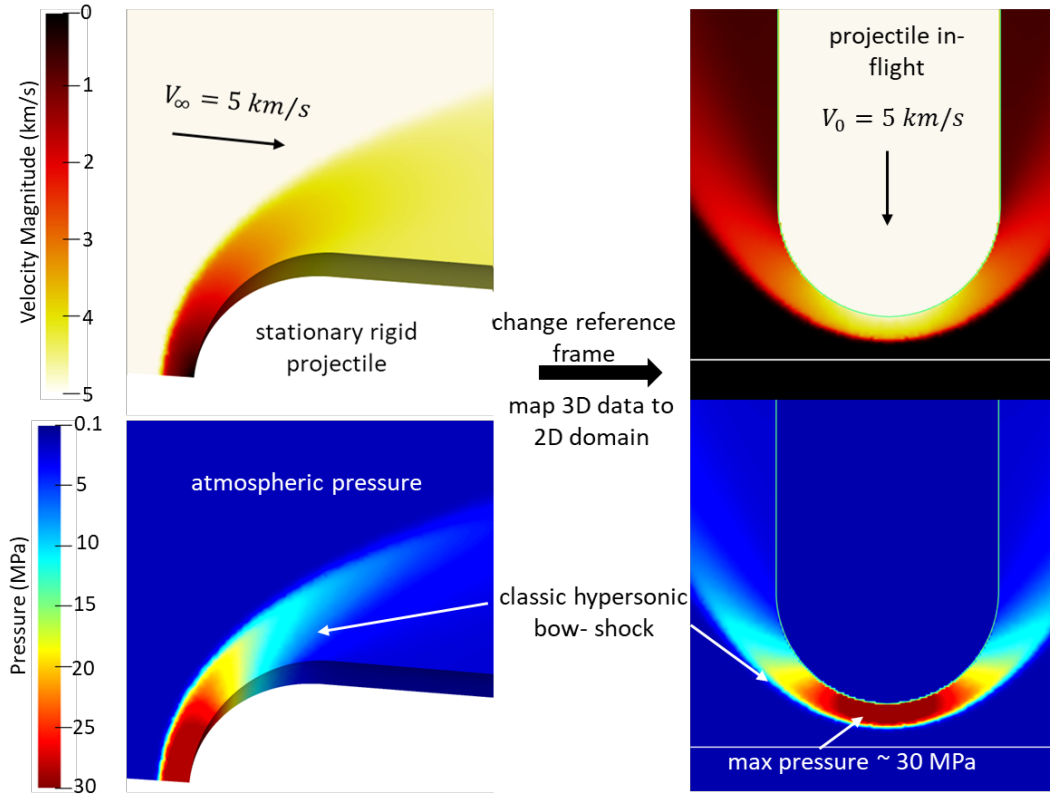


Figure 3.2: Velocity and pressure in ambient fluid for 3D steady state CFD analysis shown on the left. Mapped data on 2D domain after changing frame of reference — initial conditions for fluid-solid coupled impact analysis shown on the right

It is hypersonic, and features a bow shock that reaches the target before the projectile. To account for this flow field, we perform a 3D steady state computational fluid dynamics (CFD) analysis, and use its result as the initial condition of the fluid-solid coupled impact analysis. This 3D CFD analysis is performed using the AERO-F solver [1] on an unstructured, body-fitted mesh [22]. In the most refined region, the element size is approximately 0.15 mm. Since this analysis is performed in the body-fixed reference frame, the far-field fluid velocity V_∞ is given by $V_\infty = -V_0$. As an example, Fig. 3.2(left) shows the velocity and pressure fields obtained from a CFD analysis with $V_0 = 5 \text{ km/s}$ and $r_p = 5 \text{ mm}$. The bow shock ahead of the projectile is captured clearly.

After converting to a stationary laboratory reference frame, the pressure, velocity, and density so-

Substance	EOS	Parameters			
Tantalum	Mie-Grüneisen	c_0 (km/s)	s	ρ_0 (g/cm ³)	Γ_0
		3.293 [37]	1.307 [37]	16.65 [43]	1.64 [26]
Soda lime glass (SLG)	Stiffened gas	γ	p_c (GPa)	c_v (J/(K · kg))	
		3.9	2.62	1156 [23]	
Argon	Perfect gas	γ	c_v (J/(K · kg))		
		1.667	312.2		

Table 3.1: Parameters of the equations of state

lutions of the 3D CFD analysis are extracted and mapped on to a non-body-fitted 2D Cartesian mesh using radial basis functions. In the most refined region, the element size in the 2D domain is 0.1 mm. This mapped data is used as the initial condition for a fluid-solid coupled impact analysis, where the leading edge of the projectile is initialized to be 2 mm away from the target surface. This analysis was performed using the M2C solver [3], the results of which is discussed in details in Sec. 3.4. The two solvers used in this study, namely AERO-F [1] and M2C [3], are both publicly available.

3.2.2 Thermodynamic models

The compressible Navier-Stokes equations (3.2) must be complemented with a thermodynamic equation of state (EOS) to algebraically close the system. In this work, a different EOS is adopted for each material subdomain, to accommodate for their unique features (i.e. silicate target, metallic projectile, and gaseous ambient fluid).

In the literature, the Mie-Grüneisen EOS has been a popular choice to model solid materials in hydrocodes [53]. As such, the tantalum projectile subdomain is modeled using this EOS. It can be

formulated as [42]:

$$p(\rho, e) = \frac{\rho_0 c_0^2 \eta}{(1 - s\eta)^2} \left(1 - \frac{1}{2} \Gamma_0 \eta \right) + \rho_0 \Gamma_0 e, \quad (3.3)$$

where e is the specific internal energy, and ρ_0 and c_0 denote the density and bulk speed of sound in the ambient condition. s is the slope of the Hugoniot curve. Γ_0 is the Grüneisen parameter. η is the volumetric strain, and can be expressed as: $1 - \rho_0/\rho$. Argon is modeled using the perfect gas EOS, with specific heat ratio $\gamma = 1.667$. The SLG target is modeled using the stiffened gas EOS [44], i.e.

$$p(\rho, e) = (\gamma - 1)\rho e - \gamma p_c, \quad (3.4)$$

where γ and p_c are empirical model parameters.

SLG is modeled after the glass commercially known as Starphire®, which has a chemical composition (by weight): 73% SiO₂, 14% Na₂O, 10% CaO, and 3% MgO [2]. Unlike tantalum, SLG is not modeled using the Mie-Grüneisen EOS, because certain regions in the SLG target experience high tensile stresses during the impact event. The Mie-Grüneisen EOS can be an excellent choice to model solids under compression, but might lose hyperbolicity when used to model materials under tension. The stiffened gas EOS, however, can be calibrated to capture the shock Hugoniot obtained from laboratory experiments [44]. Eq. (3.4) has been combined with the Rankine-Hugoniot jump conditions, and fit to the shock Hugoniot: $u_s = c_0 + su_p$. Here u_s and u_p denote the shock speed and the downstream particle velocity, respectively. Using the shock Hugoniot data presented by Grady and Chhabildas [20] ($c_0 = 2.01$ km/s, $s = 1.7$) in the aforementioned procedure, gives $\gamma = 3.9$ and $p_c = 2.62$ GPa.

For all the materials, temperature is assumed to be a function of only specific internal energy (e). A constant specific heat is specified for each material, which yields a linear relation, $T = (e - e_0)/c_v + T_0$, where T denotes temperature and the subscript 0 refers to a reference state. The monoatomic configuration and the absence of valence electrons entail that argon atoms have only translational

degrees of freedom, but not vibrational, rotational, or electronic degrees of freedom. Therefore, the specific heat of argon is independent of temperature, which justifies the use of a constant specific heat [21]. For the solid materials, the specific heat is computed using the Dulong-Petit law, which matches reasonably well with measurements obtained in laboratory experiments. All the material parameters used in the simulations are presented in Table 3.1.

3.2.3 Interface tracking and treatment

At any time $t \geq 0$, the spatial domain of the fluid-solid coupled analysis consists of three material subdomains, occupied by the ambient argon, the tantalum projectile, and the SLG target, respectively. Pressure and normal velocity are continuous across the material interfaces; however, density and the tangential component of velocity may have significant discontinuities. During a hypervelocity impact event, the material subdomains undergo rapid deformation. As such, the motion of the material interfaces are predicted by solving two level set equations that share the same velocity field, $\mathbf{V}$. This method allows us to keep track of three interconnected material interfaces (i.e. projectile-fluid, target-fluid, and projectile-target) that undergo large, complex deformations. In this work, the narrow-band level set method is employed, which means the equations are solved only near material interfaces.

Specifically, the level set equations are given by

$$\frac{\partial \phi_s}{\partial t} + \mathbf{V} \cdot \nabla \phi_s = 0, \quad s = 1, 2. \quad (3.5)$$

Here, ϕ_s is the level set function employed to track the boundary of the target ($s = 1$) or the projectile ($s = 2$). ϕ_s is initialized to be the signed distance from each point in the computational domain to the material subdomain's boundary. Notably, the two level set equations ($s = 1, 2$) share the

same velocity field, and are solved synchronously to track the motion of material interfaces. This method naturally captures the contact and separation between different materials whilst avoiding non-physical subdomain overlaps. As an example, Fig. 3.3 visualizes the two level set functions at four time instances in the impact simulation with $V_0 = 5$ km/s and $r_p = 5$ mm. Compared to numerical methods that diffuse the interface, the current method is able to capture the sharp interfaces between different solid and fluid materials.

Computing the advective fluxes of mass, momentum, and energy at material interfaces is also challenging, as the EOS varies across the interface. In this work, we adopt the FIVER (FInite Volume method based on Exact multiphase Riemann solvers) method, which is based on the construction and solution of exact bimaterial Riemann problems. Specifically, a one-dimensional bimaterial Riemann problem is constructed along each edge in the mesh that intersects a material interface. This exact Riemann problem is solved iteratively, and its solution is used to compute the local fluxes. For additional details about FIVER, the reader is referred to Ref. [32, 33, 48].

3.2.4 Ionization model

The plasma density, the molar fraction of each ionic state, and the mean charge number of the plasma are calculated by solving the ideal Saha equation, i.e.

$$\frac{n_{r+1}n_e}{n_r} = 2 \frac{U_{r+1}}{U_r} \left[\frac{2\pi m_e k_B T}{h^2} \right]^{3/2} \exp\left(\frac{-I_r}{k_B T}\right), \quad r = 0, 1, \dots, N, \quad (3.6)$$

where n_r is the number density of the r -th charge state ion, n_e is the number density of electrons (also referred to as the plasma density), T is the temperature, h is the Planck constant, k_B is the Boltzmann constant, m_e is the stationary mass of an electron, and I_r the r -th ionization energy. $N = Z - 1$, where Z is the atomic number (18 for argon).

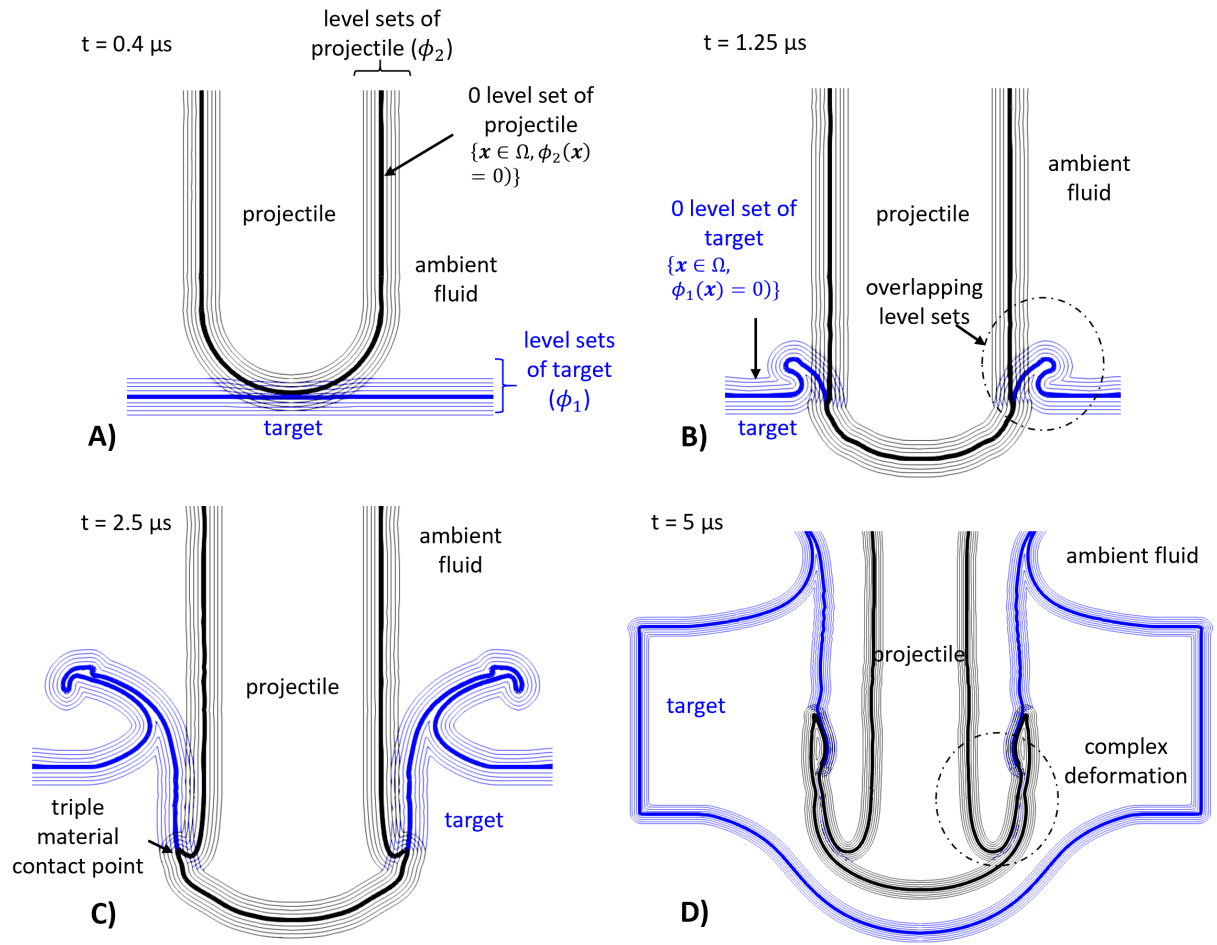


Figure 3.3: Contours of the level set functions for a representative impact simulation ($r_p = 5 \text{ mm}$, $V_0 = 5 \text{ km/s}$). A) $t = 0.4 \mu\text{s}$. B) $t = 1.25 \mu\text{s}$. C) $t = 2.5 \mu\text{s}$. D) $t = 5 \mu\text{s}$.

The Saha equation assumes the condition of local thermodynamic equilibrium. A plasma under this condition must also obey the condition of quasi-neutrality (i.e. conservation of charge; see (3.7)), and from conservation of mass also follows that the plasma must observe conservation of nuclei ((3.8)). Specifically,

$$\sum_{i=1}^Z i n_i = n_e, \quad (3.7)$$

$$\sum_{r=0}^Z n_r = n_H. \quad (3.8)$$

Here, n_H is the number density of “heavy particles”, or nuclei. Following the discussion of Zaghoul *et al.* [50], the combination of Eqs. (3.6, 3.7, 3.8) yields the one-dimensional transcendental equation,

$$Z_{av} = \sum_{j=1}^J c_j \times \left[\left(\sum_{i=1}^{Z_j} \frac{i}{(Z_{av} n_H)^i} \prod_{m=1}^i f_{m,j} \right) / \left(1 + \sum_{i=1}^{Z_j} \frac{\prod_{m=1}^i f_{m,j}}{(Z_{av} n_H)^i} \right) \right] \quad (3.9)$$

where Z_{av} denotes the mean charge in the plasma and c_j the molar fraction of the j -th species. f_m is given by

$$f_m = 2 \frac{U_{m+1}}{U_m} \left(\frac{2\pi m_e k_B T}{h^2} \right)^{\frac{3}{2}} \exp \left(- \frac{I_m}{k_B T} \right). \quad (3.10)$$

U_r is the state-dependent partition function of the r -th charge state ion, given by

$$U_r = \sum_{n=1}^{n_{\max}} g_{r,n} \exp \left(- \frac{E_{r,n}}{k_B T} \right), \quad (3.11)$$

where $g_{r,n}$ and $E_{r,n}$ denote the degeneracy and excitation energy of the r -th ion at the n -th energy level. The statistical weighting is done by the degeneracy, $g_{r,n}$ and is related to the angular

momentum quantum number, $l_{r,n}$, via the following relation,

$$g_{r,n} = 2l_{r,n} + 1 \quad (3.12)$$

The summation in (4.38) is limited to a maximum excitation state at $n = n_{\max}$, where $n_{\max}$ indicates the last element in the $E_{r,n}$ sequence ($n = 1, 2, \dots$) that is smaller than or equal to I_r . The values of I_r , $E_{r,n}$, and $l_{r,n}$ for all elements modelled in this study are obtained from the NIST atomic spectral database [36]. Representative data for the only first three energy levels of the first 4 ions of argon are shown in Table 3.2. However, up to 1,685 energy levels of argon were used in this study.

Eq. (3.9) is solved in each time step at each node occupied by argon. This transcendental equation in Z_{av} is solved using a safeguarded iterative method, TOMS748 [5]. After that, contribution of free electrons liberated from the elemental species j to the average charge per heavy particle ($\bar{Z}_{e,j}$), can be computed by evaluating (4.39):

$$\bar{Z}_{e,j} = c_j \times \left[\left(\sum_{i=1}^{Z_j} \frac{i}{(Z_{av} n_H)^i} \prod_{m=1}^i f_{m,j} \right) / \left(1 + \sum_{i=1}^{Z_j} \frac{\prod_{m=1}^i f_{m,j}}{(Z_{av} n_H)^i} \right) \right] \quad (3.13)$$

Moreover, the molar fraction of the neutral atom and each ion state can also be calculated using Eq. (4.40, 3.15). Let $\alpha_{i,j}$ represent the molar fraction of the i -th ion of the j -th elemental species. Then,

$$\alpha_{0,j} = \frac{\bar{Z}_{e,j}}{\sum_{i=1}^{Z_j} \frac{i}{(Z_{av} n_H)^i} \prod_{m=1}^i f_{m,j}} \quad (3.14)$$

$$\alpha_{r+1,j} = \frac{\alpha_{r,j}}{Z_{av} n_H} f_{r+1}, \quad r = 1, 2, \dots, Z_j. \quad (3.15)$$

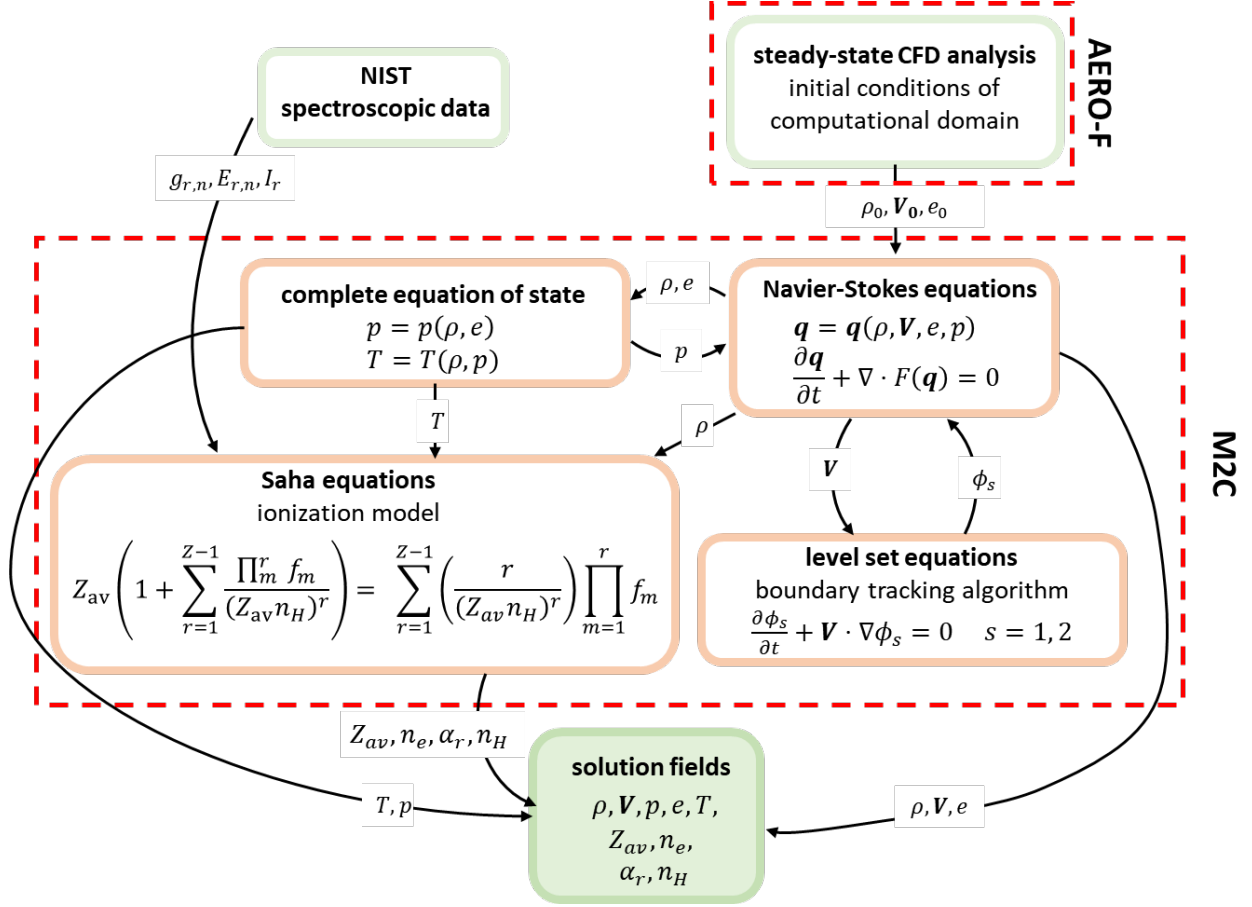


Figure 3.4: Multiphysics model overview. The role of the different solvers (M2C and AERO-F) is illustrated, along with the details of modular components and their interactions within M2C, i.e. the thermodynamics model (EOS), continuum dynamics model, boundary tracking algorithm, and ionization model.

To accelerate the solution process, we tabulate U_r , $r = 0, 1, \dots, 10$ as functions of $\exp(-1/T)$ at the beginning of the impact analysis. In each time step, we calculate the values of U_r using cubic spline interpolation. A stand-alone solver of the Saha equations can be found at [4].

A schematic overview of the computational multiphysics model used has been illustrated in Fig. 3.4.

3.3 Solver verification

3.3.1 An infinite ideal plasma

In the initial stage of the impact, the assumption of local thermodynamic equilibrium is justifiable. The impact dynamics creates the dominant forces, whereas the reciprocal effect of ionization on the dynamics of the materials can be neglected. As such, only the thermodynamic state at each point in the computational domain is required to determine the mean charge, ionic molar fractions, and plasma density.

We solve the Saha equations for an ideal argon plasma at temperatures between 500 K and 50,000 K. 100 analyses are performed, with a temperature step of 500 K. The argon pressure is fixed at 100 kPa in all the analyses. The values of material parameters I_r , $E_{r,n}$, and $l_{r,n}$ are obtained from the NIST spectral database [36]. Some sample parameters are shown in table 3.2.

r	$I_r(\text{eV})$	n	$l_{r,n}$	$E_{r,n}(\text{eV})$
0	15.759	1	0	0.000
		2	2	11.548
		3	1	11.624
1	27.630	1	$3/2$	0.000
		2	$1/2$	0.177
		3	$7/2$	16.407
2	40.735	1	2	0.000
		2	1	0.138
		3	0	0.195
3	59.58	1	$3/2$	0.000
		2	$3/2$	2.615
		3	$5/2$	2.631

Table 3.2: Sample spectroscopic data of argon for $r = 0, 1, 2, 3$ and $n = 1, 2, 3$ [36].

Figure 3.5 shows the composition of the argon plasma as a function of temperature. It can be observed that as temperature increases, the mean charge number increases monotonically, and the higher ionic species get excited. This solution is in good agreement with results by Zaghloul *et al.* [51]. There is some minor discrepancy in the magnitude of the ionic molar fractions in the higher temperature regimes, particularly in the curve for neutral argon. This could be a result of truncation errors or algorithmic accelerations employed in this study. However, the curve for the mean charge shows no such discrepancy and the temperature for onset of each ion is also in excellent agreement with Zaghloul *et al.*

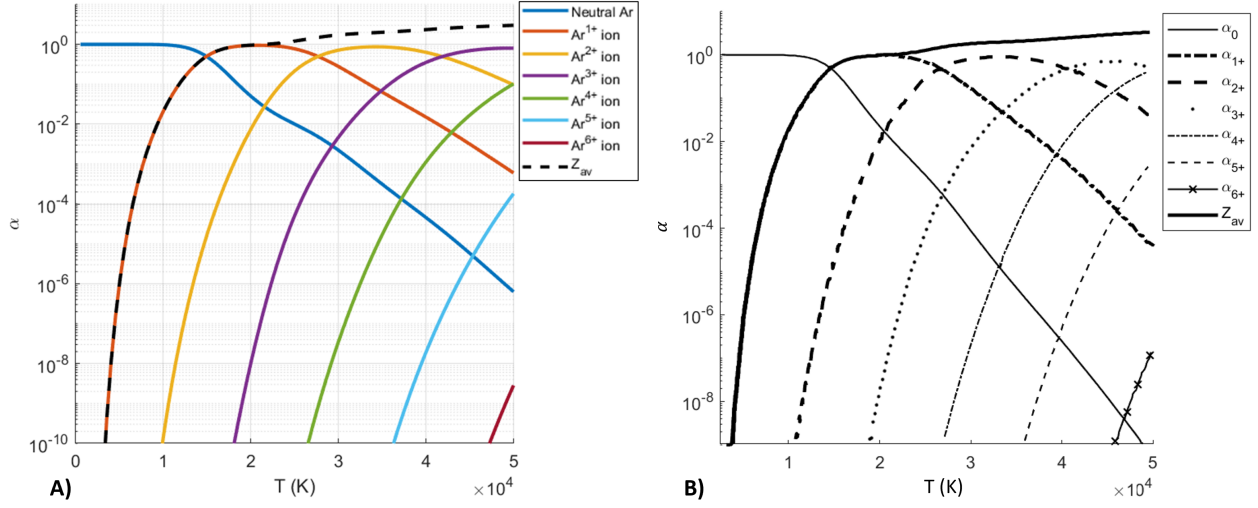


Figure 3.5: Molar fraction (α) of ionic species and mean charge (Z_{av}) of ideal argon plasma at 1 atm (101 kPa) pressure, for temperature $T \leq 5 \times 10^4$ K. A) Present work. B) Plot recreated from Zaghloul *et al.* [51].

3.3.2 1D multi-material hypervelocity impact analysis

We consider a 1D model of the 3D hypervelocity impact problem illustrated in Fig. 3.1. The 1D computational domain is along the axial direction of the 3D impact problem ($0 \leq x \leq 1$ mm). The domain is divided into four separate material subdomains, occupied by argon, tantalum, SLG, and argon again. The initial condition of the 1D model problem is given by

$$\rho(x, 0) = \begin{cases} 1.78 \times 10^{-3} \text{ g/cm}^3 & 0 \text{ mm} < x \leq 0.15 \text{ mm}, \\ 16.65 \text{ g/cm}^3 & 0.15 \text{ mm} < x \leq 0.35 \text{ mm}, \\ 2.204 \text{ g/cm}^3 & 0.35 \text{ mm} < x \leq 0.60 \text{ mm}, \\ 1.78 \times 10^{-3} \text{ g/cm}^3 & 0.60 \text{ mm} < x \leq 1 \text{ mm}, \end{cases} \quad (3.16)$$

$$u(x, 0) = \begin{cases} 3 \text{ km/s} & 0 \text{ mm} < x \leq 0.35 \text{ mm}, \\ 0 \text{ km/s} & 0.35 \text{ mm} < x \leq 1 \text{ mm}, \end{cases} \quad (3.17)$$

and

$$p(x, 0) = 100 \text{ kPa} \quad 0 \text{ mm} \leq x \leq 1 \text{ mm}. \quad (3.18)$$

The initial stage of the impact can be represented by a sequence of classical 1D Riemann problems, for which the exact solution can be computed. However, when the impact-generated shock waves reach the back surfaces of the projectile and the target, they reflect as rarefaction fans. These rarefaction fans propagate towards the projectile-target interface. When either of them reaches the interface, exact solution can no longer be obtained. In this example, the exact solution is available up to 60 ns.

This 1D analysis starts exactly at the time of collision. It is performed up to 60 ns. Figure 3.6 presents the density, velocity, and pressure fields at five time instances: $t = 0$ ns, 24 ns, 36.8 ns, 49.6 ns, and 60 ns. It can be observed that for all the solution variables, the numerical solution is in excellent agreement with the exact solution.

The initial state is shown in the first row of images in Fig. 3.6. The white region denotes the argon domain, the orange color shows the tantalum projectile, and the blue color represents the target SLG. The widths of the solid subdomains were chosen to clearly show the impact dynamics. The projectile and the fluid behind it are initialized at the impact velocity of 3 km/s, whereas the stationary target and the fluid behind it is set to be stationary.

The impact sends a forward propagating shock into the projectile and a backward propagating shock into the target, which can be seen in the snapshots taken at $t = 24$ ns. The shock waves then

propagate through their respective mediums, eventually reaching the solid-fluid material interfaces. The shock wave in the target SLG reaches the SLG-Ar interface first, which sends a rarefaction fan back into the target. The discontinuity in their states at this moment when the shock wave hits the interface can be represented by another Riemann problem, and is shown at $t = 36.8$ ns. At this point, the mass density jumps by 3 orders of magnitude across the SLG-Ar interface (i.e. 3.47 g/cm^3 vs. $1.78 \times 10^{-3} \text{ g/cm}^3$), which challenges the robustness of the solver. Similarly, when the shock wave in the tantalum projectile reaches Ta-Ar interface, the discontinuity can be represented by yet another Riemann problem, which can be seen at the snap shot at $t = 49.6$ ns. This sends a rarefaction fan into the target and a shock into the ambient fluid. However, after $t = 60$ ns the rarefaction fan in the target material reaches the Ta-SLG interface, and beyond this point an analytical solution cannot be obtained, but the system can be numerically solved to simulate the impact further in time.

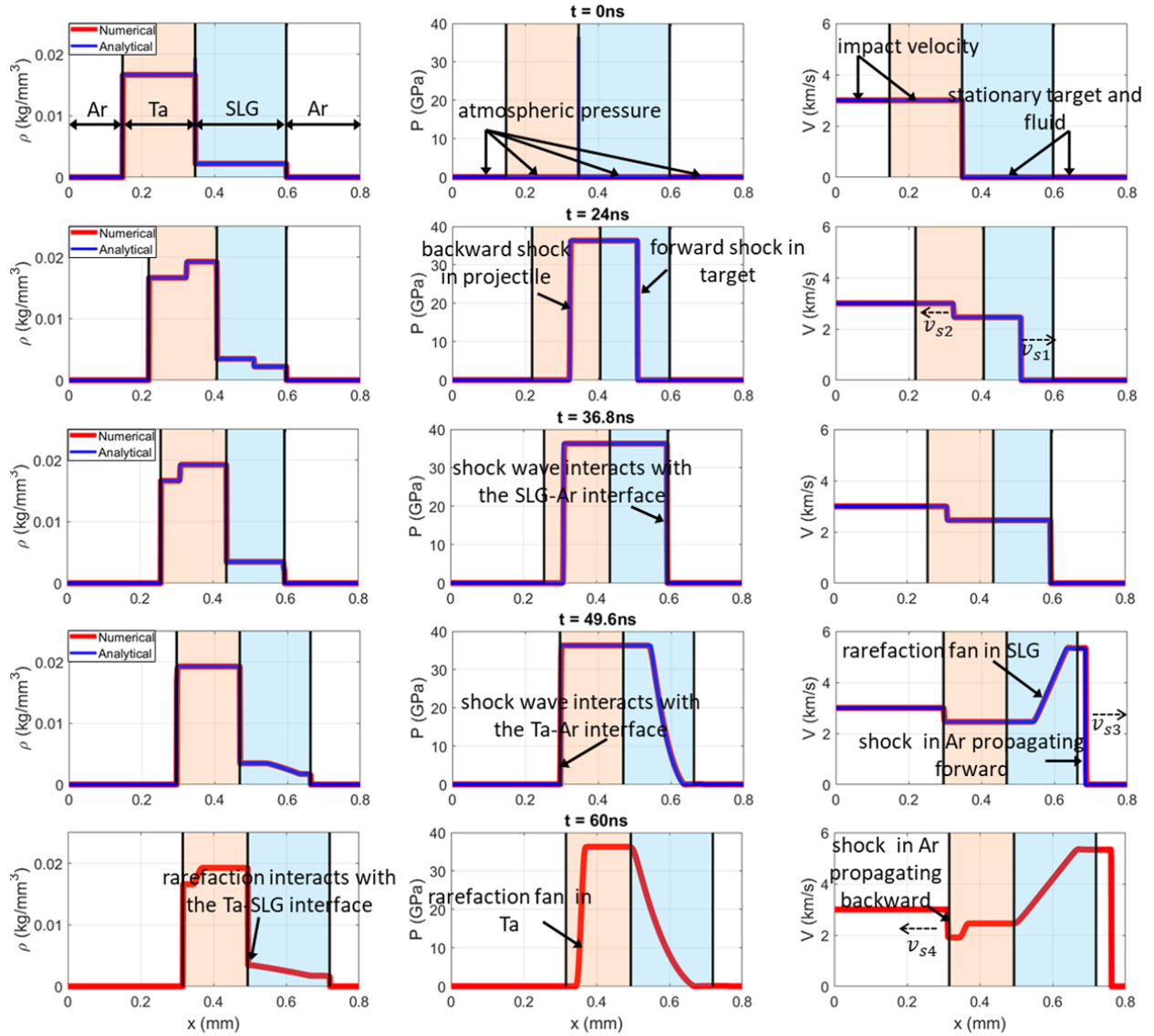


Figure 3.6: Numerical and analytical solution of a one-dimensional hypervelocity impact of a tantalum projectile ($V_0 = 3$ km/s) and SLG target in atmospheric argon environment

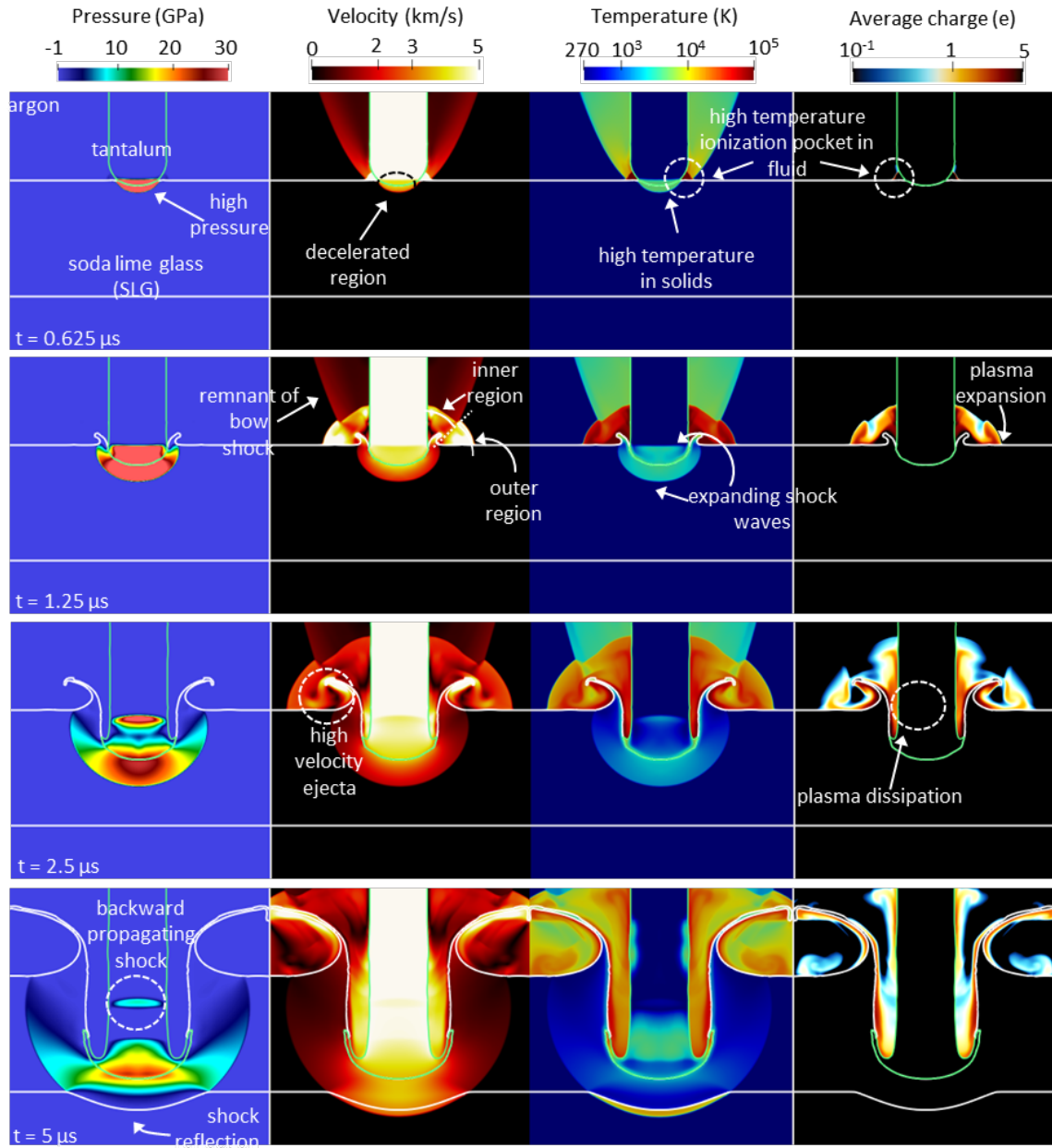


Figure 3.7: Solution snapshots obtained from a representative impact simulation ($V_0 = 5$ km/s).

3.4 Fluid-solid coupled impact analysis

The test case with impact velocity $V_0 = 5$ km/s and projectile radius $r_p = 5$ mm is considered here as a representative case. At time $t = 0$ μ s, the leading edge of the tantalum projectile is 2 mm away from the SLG target. Impact occurs at $t = 0.4$ μ s. Figure 3.7 presents a sequence of snapshots of the simulation results, taken at $t = 0.625$ μ s, 1.25 μ s, 2.5 μ s, and 5 μ s, respectively. The columns in this figure (from left to right) represent the pressure, velocity magnitude, temperature, and mean charge fields. Since the temperature and mean charge values span several orders of magnitude, they have both been plotted on a logarithmic scale.

The sudden deceleration of the tantalum projectile due to the collision imparts a large amount of energy from the projectile into the target and the surrounding fluid. This decelerated region can be seen in the velocity field plotted at $t = 0.625$ μ s. The impact also causes the formation of high pressure and temperature shock waves within all the three material subdomains. The shock wave propagates radially forward in the target, backwards in the projectile, and outwards from the point of impact in the fluid. However, due to the difference in magnitudes of pressure between the solid and fluid subdomains, only the variations in the projectile and target are visible in Fig (3.7). The high temperature and pressure in the fluid causes the argon atoms to ionize, and form a pocket of plasma.

At $t = 1.25$ μ s the projectile has displaced a small volume of the target material, and a crater has begun to form. The high pressure region behind the rim of the crater causes the target material to accelerate outwards at approximately 5 km/s (V_0). The pocket of plasma has expanded, and is divided into an inner and outer region by the tip of the SLG ejecta. The ejecta accelerates the fluid in the outer region and it reaches velocities of ~ 7 km/s, whereas the fluid in the inner region has velocity magnitudes of ~ 1 km/s. The remnants of the bow shock can still be seen at this time instance. However, the magnitudes of the state variables behind the bow shock are much smaller

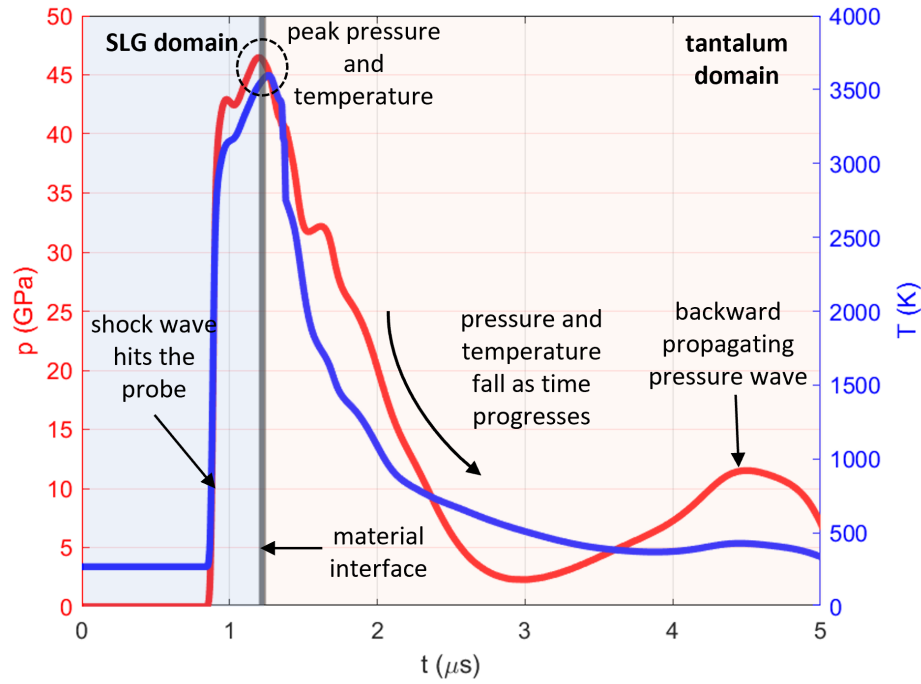


Figure 3.8: Time history of pressure and temperature at probe 2 for $V_0 = 5$ km/s

in comparison to the magnitudes behind the impact-generated shock waves; as such the effects of the bow shock on dynamics of the impact are negligible. The temperature behind the shock wave is high in both the solid materials and the argon gas. However, because the argon gas has low density and specific heat, its temperature is several orders of magnitude higher than those found in the solid materials.

As time progresses, the shock waves expand further and the energy density behind them decreases. This process can be clearly seen in the snapshots at $t = 2.5 \mu\text{s}$ and $5 \mu\text{s}$. The ionized plasma that was pushed by the ejecta into the outer region has dissipated significantly faster than in the inner region, as the mean charge is much higher within the crater. The pressure wave expands within the target and eventually hits the back wall and causes it to deform radially outwards, and then reflects backwards. The reflected wave destructively interferes with the incident wave, and causes the magnitude of the pressure to fall.

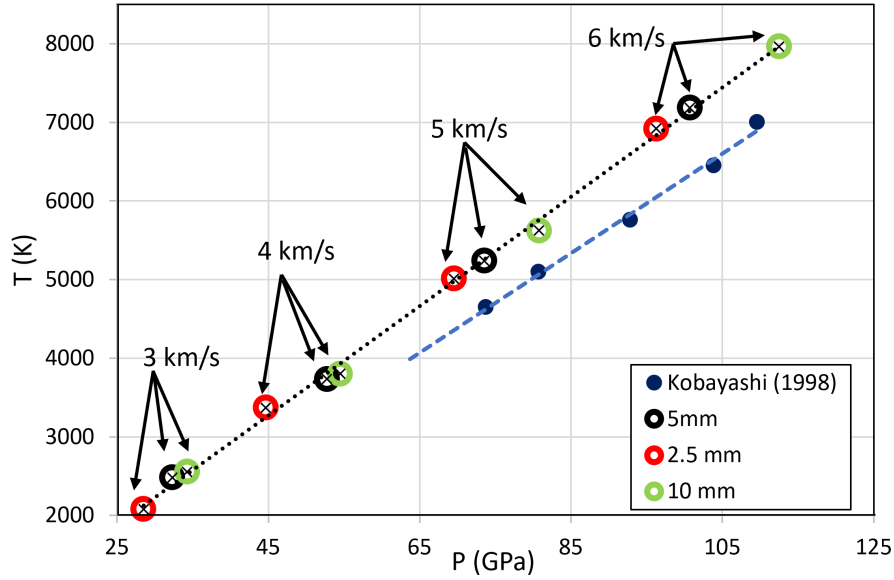


Figure 3.9: Shock temperature vs. shock pressure values at Probe 1, in comparison with experimental results presented in Kobayashi et al. [28]

For this representative case ($V_0 = 5 \text{ km/s}$, $r_p = 5 \text{ mm}$), the time history of pressure and temperature at the Probe 2 are shown in Fig. 3.8. The locations of all the three probes can be found in Fig. 3.1(B). They are fixed in space during the impact process. Probe 2 is initially within the SLG target, 1 mm from the upper surface. It is crossed by the leading edge of the tantalum projectile at $1.225 \mu\text{s}$. The time intervals in which the probe is located within SLG and tantalum are shaded in light blue and light red colors in Fig. 3.8. It can be observed that as the impact-generated shock wave reaches the probe location, both pressure and temperature increase drastically. They keep increasing until the leading edge of the tantalum projectile reaches the probe. Afterwards, both quantities decrease, as the initial forward propagating shock wave propagates and dissipates. A secondary peak is seen in the pressure time history at $t = 4.5 \mu\text{s}$, whereas the temperature does not rise significantly at that time. This occurs as the backward propagating shock wave, which was formed in the projectile at the instance of impact, reaches the probe. The peak pressure and temperature at this probe location are found to be approximately 46 GPa and 3500 K, respectively.

Additional simulations are performed with impact velocity $V_0 = 3$ km/s, 4 km/s, 5 km/s, and 6 km/s and $r_p = 2.5$ mm, 5 mm, and 10 mm. In each case, a new steady state CFD analysis is performed to generate the initial fluid flow around the projectile. This flow field is used to initialize the impact simulation. In each simulation, the maximum pressure and temperature at Probe 1 are extracted. As expected, these peak values are achieved at the time at which the target-projectile interface passes over the probe. It is found that for each impact velocity tested, the projectiles with larger radii generate stronger shocks, that is, with higher magnitude of pressure and temperature behind the shock wave. This behavior is expected, since a greater amount of kinetic energy is deposited from the projectile into the target SLG. Across all the simulations, our computational model predicts a linear dependence of pressure on temperature, as a linear regression of the $P - T$ values yield an R^2 of 0.998. These values are plotted in Fig. 3.9, alongside the experimental results for one-dimensional impact on fused quartz presented by Kobayashi *et al.* [28], for a similar range of impact velocities. Probe 1 was placed near the impact surface in the SLG to minimize the three-dimensional effects and to more accurately capture the initial shock state in the SLG. The computational and experimental results agree reasonably well with each other. The slopes of the linear fit of the data differ by less than 9%. However, there is obvious discrepancy in the intercepts of the linear fits, which may be attributed to the difference in compositions of fused quartz and Starphire SLG, as well as the simple temperature law adopted in this work (Sec. 3.2.2).

Figure 3.10 shows the maximum values of temperature and mean charge number obtained at Probe 3, which is placed within the ambient argon gas. As expected, as impact velocity increases, both quantities increase accordingly. Comparing this figure with Figs. 3.8 and 3.9 shows that the temperature in the ambient gas is significantly higher than that in the solid materials. This is not surprising as temperature is a measure of internal energy per unit mass, and the mass density of the argon gas is much lower than that of the solid materials. Therefore, although the fraction of impact energy (i.e. kinetic energy carried by the projectile) transferred to the surrounding gas is small compared

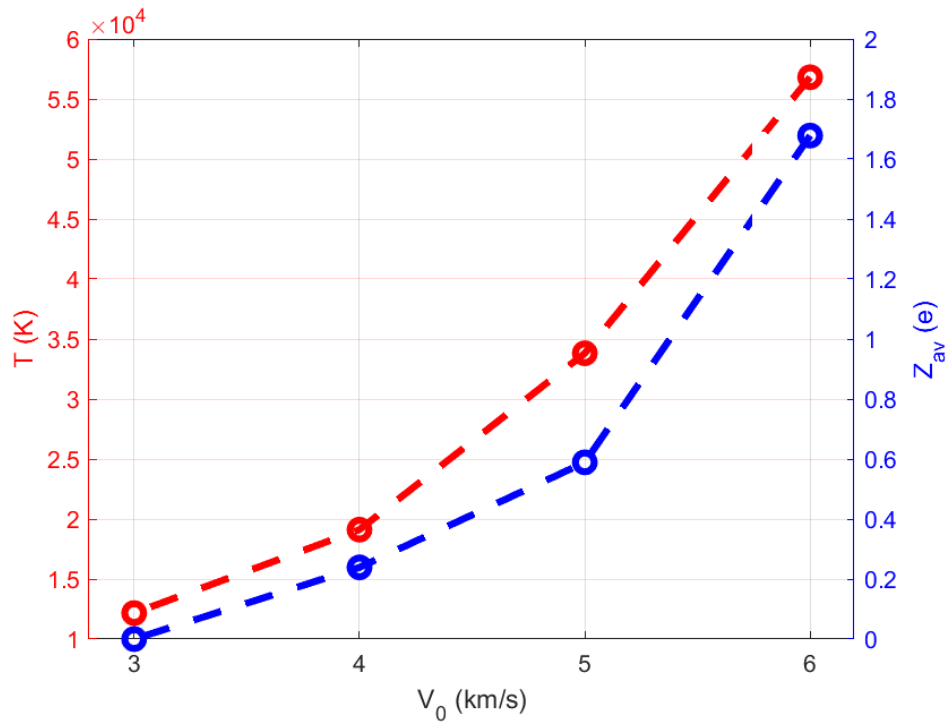


Figure 3.10: Maximum values of temperature (T) and mean charge number (Z_{av}) within the ambient argon gas for $V_0 = 5$ km/s at probe 3.

to that shared between the solid projectile and target, it is enough to cause dramatic temperature increase in the gas. The high temperature causes argon gas to ionize. Figure 3.10 shows that at the probe location, ionization is significant for impact velocities higher than 4 km/s.

Moreover, Fig. 3.11 shows the mean charge, Z_{av} , and the structure of the plasma plume at $t = 1.25 \mu s$, obtained from three projectiles with (from top to bottom) $r_p = 2.5$ mm, 5 mm, and 10 mm, while the impact velocity is fixed at $V_0 = 5$ km/s. The three projectiles generate similar penetration depth of 3.5 mm, at this time instance. However, the smaller the projectile radius the more deformation that can be seen in the projectile itself. The structure of the ejecta also displays significant difference, as the projectile with the larger radius has a greater radial velocity at the ejecta tip, but there is limited variance in the axial velocity. This causes the 2.5 mm projectile to be expelled more in the axial direction and disrupt the shock structure in the fluid. For the larger radii, the shock structure is still undisturbed at this time, and the shock velocity does not depend significantly on the radius. Therefore, the volume of fluid behind the shock, and thus the volume of the plasma plume, are similar for these cases. However, since the 10 mm projectile deposits kinetic energy into the fluid at a faster rate, the extent of ionization (mean charge) is greater than in the 5 mm projectile.

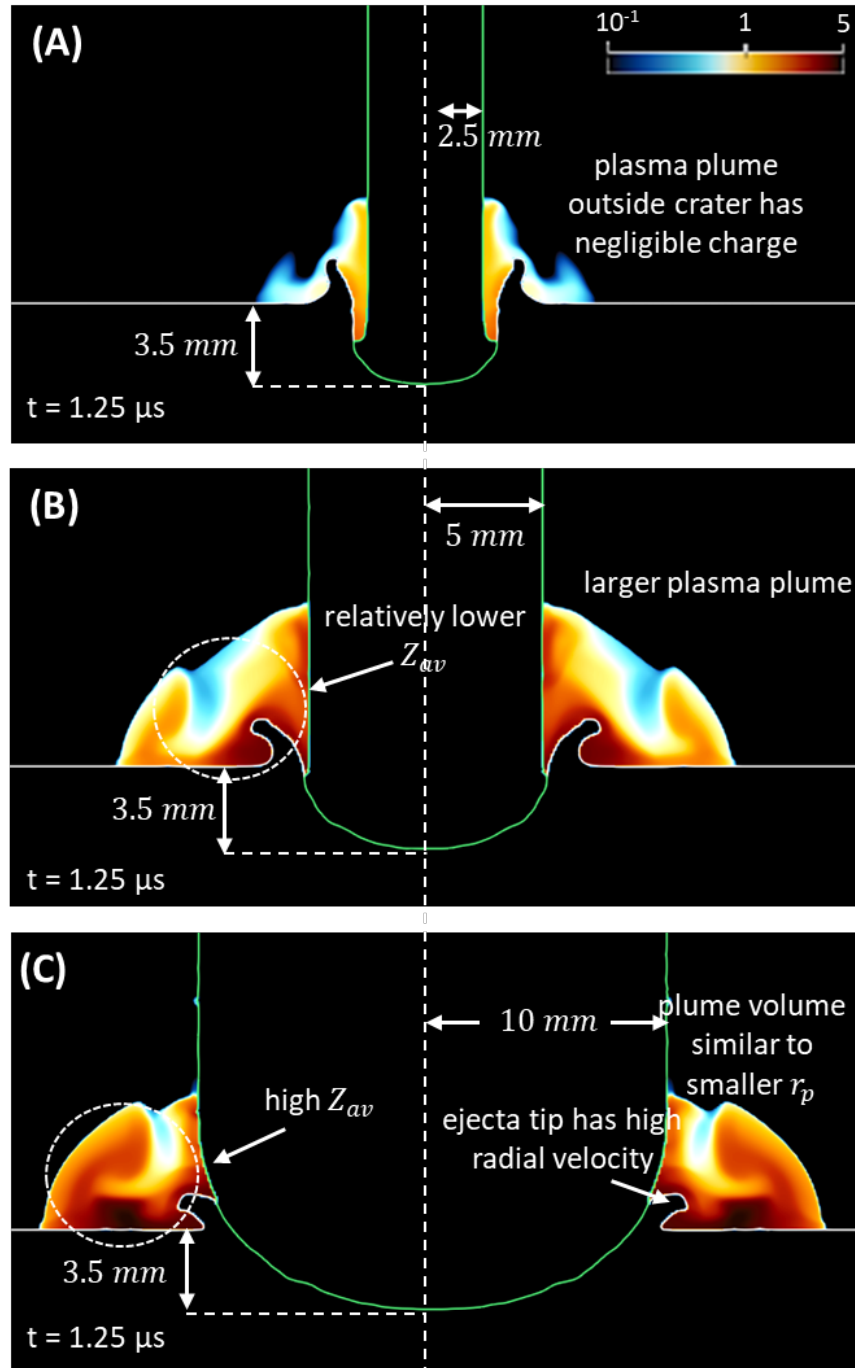


Figure 3.11: Mean charge of impact-generated plasma for variable projectile radii.

3.5 Conclusion

This paper presents a new computational model of hypervelocity impact that accounts for the dynamics, thermodynamics, and ionization of the ambient fluid (gas), as well as the interaction of the fluid flow with the solid projectile and target. The main features of this model include (1) the solution of two level set equations to track the three sharp material interfaces between the projectile, the target, and the ambient fluid, (2) the construction and solution of exact, one-dimensional bimaterial Riemann problems to enforce interface conditions (a method known as “FIVER”), and (3) the solution of Saha equations to predict ionization and plasma density within the ambient fluid. The implementation of this model is first verified using two benchmark problems for which either the exact solution or reference data are available. Next, the computational model is applied to simulate the impact of tantalum projectiles on soda lime glass (SLG) in an argon gas environment. In different simulations, the impact velocity is varied between 3 and 6 km/s, while the radius of the projectile is varied between 2.5 and 10 mm. The predicted maximum temperature and pressure within SLG agree reasonably well with published experimental data for a similar material (fused quartz). The temperature in the surrounding gas is found to be significantly higher (by 1 ~ 2 orders of magnitude) than that in the solid materials. This indicates that for impact events that occur in a fluid environment, the fluid may have a substantial effect on the generation of plasma and the emission of electromagnetic waves. For the test case simulated in this paper, ionization of argon is observed at impact velocities above 4 km/s.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

S.T.I, W.M., and K.W. gratefully acknowledge the support of the Office of Naval Research (ONR) under award N00014-19-1-2102. K.W. also acknowledges the support of the National Science Foundation (NSF) under award CBET-1751487. J.G.M. gratefully acknowledges the support of the ONR under award N00014-21-WX01554.

Bibliography

- [1] Aero-f source code, 2020. URL <https://bitbucket.org/frg/aero-f/src/master/>.
- [2] Starphire technical product data. Technical Report 1.855.887.6457 / 1.855.VTRO.GLS, Vitro Architectural Glass, Pittsburgh, PA, April 2020.
- [3] M2c source code, 2022. URL <https://github.com/kevinwgy/m2c>.
- [4] Saha equation solver source code, 2022. URL <https://github.com/kevinwgy/saha>.
- [5] GE Alefeld, Florian A Potra, and Yixun Shi. Algorithm 748: Enclosing zeros of continuous functions. *ACM Transactions on Mathematical Software (TOMS)*, 21(3):327–344, 1995.
- [6] Roshdy G Barsoum. Shockwave compression physics of condensed matter lattice boltzmann method with application to glass under hypervelocity impact, 2019.
- [7] R. Bianchi, F. Capaccioni, P. Cerroni, M. Coradini, E. Flamini, P. Hurren, G. Martelli, and P. N. Smith. Radiofrequency emissions observed during macroscopic hypervelocity impact experiments. *Nature*, 308(5962):830–832, 1984. doi: 10.1038/308830a0.
- [8] Andrew J. Birnbaum, John C. Steuben, Athanasios P. Iliopoulos, and John G. Michopoulos.

- Simulating hypervelocity impact and material failure in glass. *Volume 1B: 38th Computers and Information in Engineering Conference*, 2018. doi: 10.1115/detc2018-85948.
- [9] Stephanie N.Q. Bouchey and Jeromy T. Hollenshead. Mesoscale modeling of hypervelocity impacts using the cth shock physics code. *International Journal of Impact Engineering*, 137: 103462, 2020. ISSN 0734-743X. doi: <https://doi.org/10.1016/j.ijimpeng.2019.103462>. URL <https://www.sciencedirect.com/science/article/pii/S0734743X19307122>.
- [10] Shunxiang Cao, Ying Zhang, Defei Liao, Pei Zhong, and Kevin G Wang. Shock-induced damage and dynamic fracture in cylindrical bodies submerged in liquid. *International journal of solids and structures*, 169:55–71, 2019.
- [11] Sigrid Close, Ivan Linscott, Nicolas Lee, Theresa Johnson, David Strauss, Ashish Goel, Alexander Fletcher, David Lauben, Ralf Srama, Anna Mocker, et al. Detection of electromagnetic pulses produced by hypervelocity micro particle impact plasmas. *Physics of Plasmas*, 20(9):092102, 2013.
- [12] Benjamin Estacio, Sean A.Q. Young, Nicolas Lee, and Sigrid Elschot. Experimental evidence of rapid target charging electromagnetic pulse from hypervelocity impact. *International Journal of Impact Engineering*, 173:104473, 2023. ISSN 0734-743X. doi: <https://doi.org/10.1016/j.ijimpeng.2022.104473>. URL <https://www.sciencedirect.com/science/article/pii/S0734743X2200313X>.
- [13] C. Farhat, K.G. Wang, A. Main, S. Kyriakides, L.-H. Lee, K. Ravi-Chandar, and T. Be-lytschko. Dynamic implosion of underwater cylindrical shells: Experiments and computations. *International Journal of Solids and Structures*, 50(19):2943–2961, 2013. doi: 10.1016/j.ijsolstr.2013.05.006.
- [14] Charbel Farhat, Jean-Frédéric Gerbeau, and Arthur Rallu. Fiver: A finite volume method based on exact two-phase riemann problems and sparse grids for multi-material flows with

- large density jumps. *Journal of Computational Physics*, 231(19):6360–6379, 2012. doi: 10.1016/j.jcp.2012.05.026.
- [15] Alex Fletcher. *Plasma production and radiation from meteoroid impacts on spacecraft*. PhD thesis, 2015.
- [16] Alex Fletcher, Sigrid Close, and Donovan Mathias. Simulating plasma production from hypervelocity impacts. *Physics of Plasmas*, 22(9):093504, 2015. doi: 10.1063/1.4930281.
- [17] Alex C. Fletcher and Sigrid Close. Particle-in-cell simulations of an rf emission mechanism associated with hypervelocity impact plasmas. *Physics of Plasmas*, 24(5):053102, 2017. doi: 10.1063/1.4980833.
- [18] Joseph Fred Friichtenicht and John Charles Slattery. Ionization associated with hypervelocity impact. Technical report, 1963.
- [19] E Giannaros, A Kotzakolios, V Kostopoulos, and G Campoli. Hypervelocity impact response of cfrp laminates using smoothed particle hydrodynamics method: Implementation and validation. *International Journal of Impact Engineering*, 123:56–69, 2019.
- [20] Dennis E Grady and LC Chhabildas. Shock-wave properties of soda-lime glass. Technical report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 1996.
- [21] E. P. Gyftopoulos and Gian Paolo. Beretta. *Thermodynamics: foundations and applications*. Dover Publications, 2005.
- [22] Daniel Z Huang, Philip Avery, Charbel Farhat, Jason Rabinovitch, Armen Derkevorkian, and Lee D Peterson. Modeling, simulation and validation of supersonic parachute inflation dynamics during mars landing. In *AIAA Scitech 2020 Forum*, page 0313, 2020.
- [23] J Huang and PK Gupta. Temperature dependence of the isostructural heat capacity of a soda lime silicate glass. *Journal of non-crystalline solids*, 139:239–247, 1992.

- [24] Shafquat T Islam, Wentao Ma, John G Michopoulos, and Kevin Wang. Plasma formation in ambient fluid from hypervelocity impacts. *Extreme Mechanics Letters*, 58:101927, 2023.
- [25] Eric Johnsen and Frank Ham. Preventing numerical errors generated by interface-capturing schemes in compressible multi-material flows. *Journal of Computational Physics*, 231(17): 5705–5717, 2012.
- [26] KW Katahara, MH Manghnani, and ES Fisher. Pressure derivatives of the elastic moduli of bcc ti-v-cr, nb-mo and ta-w alloys. *Journal of Physics F: Metal Physics*, 9(5):773, 1979.
- [27] Christopher T Key and C Scott Alexander. Numerical and experimental evaluations of a glass-epoxy composite material under high velocity oblique impacts. *International Journal of Impact Engineering*, 137:103443, 2020.
- [28] T Kobayashi, T Sekine, OV Fat’yanov, E Takazawa, and QY Zhu. Radiation temperatures of soda-lime glass in its shock-compressed liquid state. *Journal of applied physics*, 83(3): 1711–1716, 1998.
- [29] Andrea La Spina, Alex C Fletcher, and Jacob Fish. Semi-analytic model for plasma production and cherenkov radiation emission from hypervelocity impacts on soda–lime glass. *Physics of Plasmas*, 30(1):013902, 2023.
- [30] Nicolas Lee, Sigrid Close, Ashish Goel, David Lauben, Ivan Linscott, Theresa Johnson, David Strauss, Sebastian Bugiel, Anna Mockler, Ralf Srama, and et al. Theory and experiments characterizing hypervelocity impact plasmas on biased spacecraft materials. *Physics of Plasmas*, 20(3):032901, 2013. doi: 10.1063/1.4794331.
- [31] Jianqiao Li, Weidong Song, and Jianguo Ning. Theoretical and numerical predictions of hypervelocity impact-generated plasma. *Physics of Plasmas*, 21(8):082112, 2014. doi: 10.1063/1.4893310.

- [32] Wentao Ma, Xuning Zhao, Christine Gilbert, and Kevin Wang. Computational analysis of bubble–structure interactions in near-field underwater explosion. *International Journal of Solids and Structures*, 242:111527, 2022.
- [33] Alex Main, Xianyi Zeng, Philip Avery, and Charbel Farhat. An enhanced fiver method for multi-material flow problems with second-order convergence rate. *Journal of Computational Physics*, 329:141–172, 2017.
- [34] K Maki, T Takano, A Fujiwara, and A Yamori. Radio-wave emission due to hypervelocity impacts in relation to optical observation and projectile speed. *Advances in Space Research*, 34(5):1085–1089, 2004.
- [35] K Maki, E Soma, T Takano, A Fujiwara, and A Yamori. Dependence of microwave emissions from hypervelocity impacts on the target material. *Journal of applied physics*, 97(10):104911, 2005.
- [36] William Martin, Jeffrey Fuhr, Daniel Kelleher, Arlene Musgrove, J Sugar, Wolfgang Wiese, Peter Mohr, and Karen Olsen. Nist atomic spectra database (version 2.0) (1999), 1999.
- [37] AC Mitchell and WJ Nellis. Shock compression of aluminum, copper, and tantalum. *Journal of Applied Physics*, 52(5):3363–3374, 1981.
- [38] Karina A Monroe. Time-resolved fractoluminescence characterization in soda-lime glass via near hypervelocity kinetic impact fast fracture. Master’s thesis, Monterey, CA; Naval Post-graduate School, 2021.
- [39] Stanley Osher and Ronald P. Fedkiw. *Level set methods and dynamic implicit surfaces*. Springer, 2009.
- [40] Paul R. Ratcliff, Mark J. Burchell, Mike J. Cole, Tom W. Murphy, and Firooz Alladfadi. Experimental measurements of hypervelocity impact plasma yield and energetics. *International*

- Journal of Impact Engineering*, 20(6-10):663–674, 1997. doi: 10.1016/s0734-743x(97)87453-2.
- [41] Sergey V Razorenov, Gennady Garkushin, Gennady I Kanel, and Olga Nikolaevna Ignatova. The spall strength and hugoniot elastic limit of tantalum with various grain size. In *AIP Conference Proceedings*, volume 1426, pages 991–994. American Institute of Physics, 2012.
- [42] Allen C Robinson. The mie-gruneisen power equation of state. Technical report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 2019.
- [43] Alan Russell and Kok Loong Lee. *Structure-property relations in nonferrous metals*. John Wiley & Sons, 2005.
- [44] Richard Saurel and Rémi Abgrall. A simple method for compressible multifluid flows. *SIAM Journal on Scientific Computing*, 21(3):1115–1145, 1999.
- [45] William P Schonberg. Hypervelocity impact response of spaced composite material structures. *International Journal of Impact Engineering*, 10(1-4):509–523, 1990.
- [46] James Albert Sethian and James Albert Sethian. *Level set methods and fast marching methods: Evolving interfaces in computational geometry, Fluid Mechanics, computer vision, and materials science*. Cambridge University Press, 1999.
- [47] Weidong Song, Jianqiao Li, and Jianguo Ning. Characteristics of plasma generated by hypervelocity impact. *Physics of Plasmas*, 20(9):093501, 2013. doi: 10.1063/1.4819829.
- [48] K. G. Wang, P. Lea, and C. Farhat. A computational framework for the simulation of high-speed multi-material fluid-structure interaction problems with dynamic fracture. *International Journal for Numerical Methods in Engineering*, 104(7):585–623, 2015. doi: 10.1002/nme.4873.

- [49] Kevin G. Wang. Multiphase fluid-solid coupled analysis of shock-bubble-stone interaction in shockwave lithotripsy. *International Journal for Numerical Methods in Biomedical Engineering*, 33(10), 2017. doi: 10.1002/cnm.2855.
- [50] Mofreh R. Zaghloul. Reduced formulation and efficient algorithm for the determination of equilibrium composition and partition functions of ideal and nonideal complex plasma mixtures. *Physical Review E*, 69(2), 2004. doi: 10.1103/physreve.69.026702.
- [51] Mofreh R Zaghloul, Mohamed A Bourham, and J Michael Doster. A simple formulation and solution strategy of the saha equation for ideal and nonideal plasmas. *Journal of Physics D: Applied Physics*, 33(8):977–984, 2000. doi: 10.1088/0022-3727/33/8/314.
- [52] Xuning Zhao, Wentao Ma, and Kevin Wang. Simulating laser-fluid coupling and laser-induced cavitation using embedded boundary and level set methods. *Journal of Computational Physics*, 472:111656, 2023.
- [53] Jonas Zukas. *Introduction to hydrocodes*. Elsevier, 2004.

Chapter 4

Ionization Induced by Fluid-Solid Interaction During Hypervelocity Impact

(Published in *International Journal of Solids and Structures*, 113278, 2025.)

S. Islam^a, A. Narkhede^a, P. Asimow^b, J. Michopoulos^c, K. Wang^a

^a Department of Aerospace and Ocean Engineering, Virginia Tech, Blacksburg, VA, USA

^b Division of Geological and Planetary Sciences, California Institute of Technology, Pasadena,
CA, USA

^c US Naval Research Laboratory, Washington DC, USA

Abstract

This paper presents a computational model to represent and investigate hypervelocity impacts that occur in an atmospheric environment, focusing on energy partitions and impact-induced ionization. The computational domain includes the projectile, the target, and the ambient gas. The physical model combines the compressible Navier-Stokes equations, a complete thermodynamic equation of state (EOS) for each material, and a non-ideal, multi-species Saha equation for ionization prediction. Material interfaces are tracked using an extended two-equation level set method, and the interfacial mass, momentum, and energy fluxes are computed by the FIVER (Finite Volume method with Exact multi-material Riemann problems) method. Using this model, the impact of tantalum on soda-lime glass (SLG) within argon gas is analyzed. Shock compression experiments are conducted to capture the thermodynamics of SLG under high pressure and temperature, yielding shock Hugoniot data up to 112 GPa and 5300 K. This data is used to calibrate a Noble-Abel stiffened gas EOS. Impact simulations are performed with initial velocity ranging from 3 km/s to 7 km/s. Time histories of the pressure, temperature, and plasma density fields are compared across the three materials. Less than 1% of the total impact energy is transferred to the ambient gas, yet its specific internal energy is of the same order of magnitude as that of the projectile and target. Argon gas exhibits higher temperature and plasma density than SLG and tantalum. The ionization of SLG is found to be highly selective, with the metallic elements contributing over 99.9% of the plasma's charged particles despite comprising less than 15% of the molar composition. In general, the results suggest that the plasma's density and energy depend on both impact velocity and the material combination, including the ambient gas. The plasma's composition further reflects the properties (e.g., ionization energies) of the chemical elements in each material.

4.1 Introduction

When two solid objects collide at a relative velocity higher than the speed of sound in either material, the phenomenon can be classified as a hypervelocity impact. It typically involves velocities on the order of kilometers per second or higher, relevant in planetary science, aerospace engineering, and defense engineering. As the impact velocity increases, the energy imparted by the collision grows quadratically, while its distribution is limited by the speed of shock waves. When the impact velocity is sufficiently high, the energy cannot be fully absorbed through deformation, fracture, and temperature rise alone. Additional energy dissipation mechanisms are activated, including phase transitions, chemical reactions, and ionization. Impact-induced ionization is particularly important as it generates a plasma that emits electromagnetic waves. In some applications, such emissions may interfere with radio communications or damage electronic devices [14]. More generally, it implies that hypervelocity impacts can be diagnosed using high-speed (optical, X-ray, or gamma ray) images of the plasma plume, if the causality between material properties, impact conditions, and plasma characteristics is established.

Although impact-generated plasma was detected as early as the 1960s [23], there is still no consensus on the mechanism of plasma formation. Hornung and Kissell proposed a continuum mechanism for volume ionization based on the Thomas-Fermi model [28]. In contrast, Krueger's "sudden perturbation ion formation" model arises from a non-equilibrium thermodynamic standpoint [37]. Moreover, the density, composition, and temperature conditions of impact-generated plasma remain unclear. For instance, Ratcliff *et al.* reported plasma temperatures ranging from 20 to 30 eV (232, 100 to 348, 150 K) with no significant effect from impact velocity across a wide range, from 1.2 to 87 km/s [58]. Friichtenicht *et al.* reported plasma temperatures around 0.5 eV (5802.5 K) for impact velocities spanning 17 to 47 km/s [22]. Lee *et al.* reported temperatures around 2 eV (23, 210 K) for impact velocities of 10 to 30 km/s [41]. These variations may be attributed to

different experimental setups (e.g., vacuum conditions), but they may also reflect the range of indirect methods used to infer plasma temperature. Meanwhile, advances in numerical methods and computer hardware have spurred growing interest in developing computational models to simulate hypervelocity impact events and resolve some of these experimental ambiguities. Song *et al.* developed a smoothed-particle hydrodynamics (SPH) code that applies the Thomas-Fermi model to compute thermal ionization and plasma temperature [63]. Its predictions of plasma charge differ significantly from the empirical models of Ratcliff *et al.* [58] and McBride *et al.* [50]. Fletcher *et al.* also developed an SPH code to simulate hypervelocity impacts, using the Saha equation to model plasma ionization [20]. Fletcher’s model, run for impacts in high vacuum, predicts a threshold for significant plasma formation around 18 km/s and higher; yet Close *et al.* experimentally detected ionized material at much lower impact velocities [14]. Recently, La Spina *et al.* developed both semi-analytical and computational models to investigate the emission of Cherenkov radiation, in which fluid dynamics equations are coupled with either the Frank-Tamm formula or Maxwell’s equations [38, 39].

We investigate the energy partitions and ionization during hypervelocity impacts that occur in an atmospheric environment. In this scenario, the collision generates a shock wave that propagates through the ambient gas. Although the gas only receives a small fraction of the total energy involved in the collision, its low density means this fraction may already be enough to significantly raise its temperature and cause ionization. Thus, we include the ambient gas in our study. Figure 4.1 shows the model problem considered in this paper. It features a long cylindrical tantalum (Ta) projectile impacting on a wide soda-lime glass (SLG) target in an argon (Ar) gas environment. The dimensions of the projectile and target are shown in the figure. The impact velocity is varied from 3 km/s to 7 km/s. There are a few considerations behind our material selections. Tantalum is a hard metal widely used in the impact research community. Argon has a density similar to air; but being an inert noble gas, its behavior in extreme mechanical and thermal conditions is easier to model.

SLG is relevant to some impact mitigation applications [5, 12, 52]. Moreover, it contains several alkali and alkaline earth metal elements (e.g., sodium, calcium, magnesium) that ionize easily.

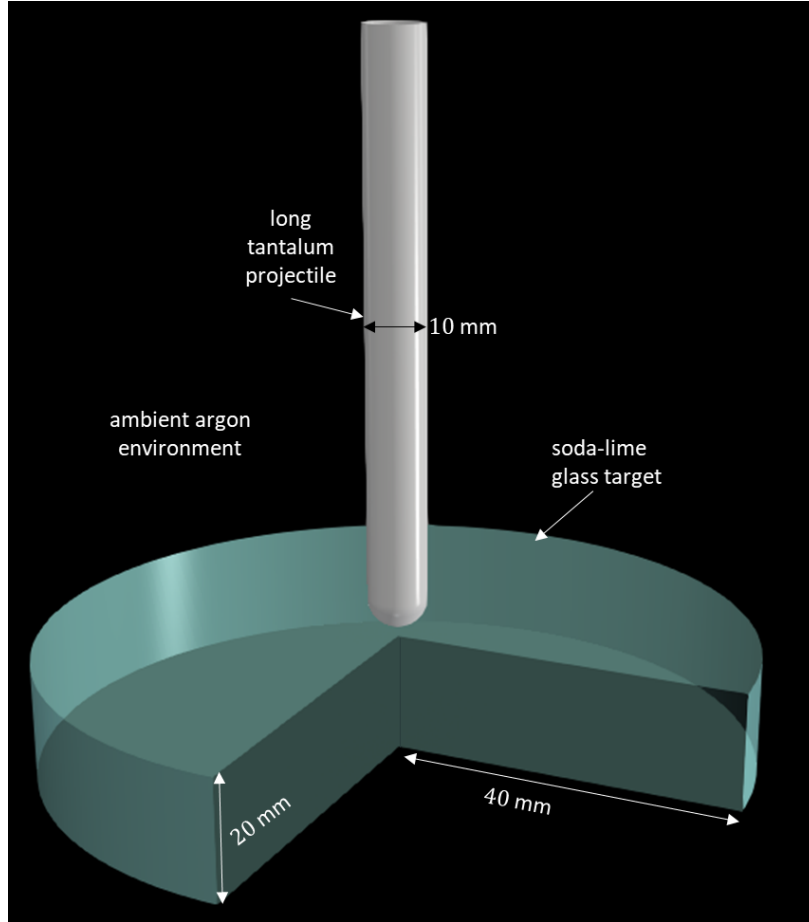


Figure 4.1: Setup of the fluid-solid coupled impact problem investigated in this study

We present a computational model to predict the dynamics, thermodynamics, and ionization of the three materials involved in the problem. The computational domain consists of three non-overlapping subdomains corresponding to the projectile, target, and ambient gas. The physical model integrates three main components: continuum mechanics governing equations, thermodynamic equations of state (EOS), and ionization model equations. For the impact velocities considered, the stresses within the projectile and target exceed their Hugoniot elastic limits by at least one order of magnitude. The specific internal energy in these materials also far exceeds their frac-

ture toughness values. Therefore, we treat all three materials as fluids, and solve the compressible Navier-Stokes equations to simulate their dynamics. Each material is assigned a complete EOS that includes a pressure equation and a temperature law. For argon, we use the calorically perfect gas EOS. For tantalum, we employ an extended version of the Mie-Grüneisen EOS that handles both compression and tension [60].

For SLG, data and models in high pressure, high temperature conditions are scarce in the literature. To overcome this issue, we conduct a series of hypervelocity shock compression experiments in the Lindhurst Laboratory of Experimental Geophysics at Caltech. The impact velocity varies from 1.2 km/s to 6.3 km/s. We measure particle velocity, pressure, density, sound speed, and temperature behind the shock wave. Using this new experimental data, we calibrate a Noble-Abel stiffened gas (NASG) EOS for SLG. Generalized from the Noble-Abel EOS and the stiffened gas EOS, NASG has been used to model condensed matter in both liquid and solid phases [55, 57]. It implies that the Grüneisen parameter is a convex function of specific volume, which is consistent with our finding from the shock compression experiments.

We predict ionization in each material by solving the Saha equation, which enforces quasi-neutrality and particle conservation within the plasma. Notably, impact-generated plasma is relatively cold and initially dense, which depresses the ionization energy of the particles in the plasma. Therefore, we solve the non-ideal Saha equation, using Ebeling’s model to account for the depression [15, 20, 68]. Again, SLG poses a difficulty, as it is a mixture of silica and various metallic silicates. In this work, we neglect the bond energies within these components, and treat SLG as a mixture of five pure elements, namely silicon (Si), oxygen (O), sodium (Na), calcium (Ca), and magnesium (Mg). This simplification can be justified by the fact that the bond energies are much lower than the ionization energies of these pure elements.

Numerically solving the equations in the physical model is nontrivial. The material subdomains undergo large, complex deformations during impact. A new subdomain interface forms when the

projectile reaches in contact with the target. Some state variables (e.g., density, internal energy) jump by 3 to 4 orders of magnitude across subdomain interfaces. The EOS functions also exhibit significant discontinuities across subdomain interfaces. In this work, we solve the compressible Navier-Stokes equations in the Eulerian reference frame using a high-resolution finite volume method. We track subdomain boundaries using an extended level set method featuring two advection equations that share the same velocity field [31, 32, 46]. We compute interfacial fluxes using the Finite Volume method based on Exact multi-material Riemann problems (FIVER)[17, 65]. FIVER naturally accommodates strong discontinuities in thermodynamic states and EOS across material interfaces, and has been validated for a wide range of shock-dominated multiphase flow and fluid-solid interaction problems (e.g., [13, 16, 45, 65, 66]). We also adopt the acceleration techniques recently proposed by Ma *et al.* [46] to reduce the computational cost of this method.

This paper builds on our previous work in the same area [31, 32]. In the earlier papers, we presented a fluid-solid coupled computational model of hypervelocity impact, focusing on the behavior of the ambient gas. Modeling of the solid materials was simplified. For example, temperature was assumed to be a linear function of internal energy. The stiffened gas EOS was used to model SLG, calibrated with limited data. Moreover, ionization in the solid materials was not modeled. These studies captured clear ionization in the ambient argon gas for impact velocities above 4 km/s. However, the energy exchange between the solids and the gas was not discussed. The temperature and potential ionization in the solid materials remained unclear.

This paper is organized as follows. In Sec. 4.2, we present the physical model and computational methods, including a verification of our iterative method for solving the non-ideal Saha equation for multi-species materials. In Sec. 4.3, we present the SLG shock compression experiments, in which states along the shock Hugoniot as well as partial release states are measured up to 112 GPa and 5300 K. In Sec. 4.4, we present the calibration of the NASG EOS using the experimental data. In Sec. 4.5, we employ the computational model to conduct a series of one-dimensional

impact simulations. The results allow us to validate the model against the shock compression experiments, and to capture some salient features of atmospheric hypervelocity impact events. In Sec. 4.6, we present our 3D impact simulations in detail. The one with a 5 km/s impact velocity is used as a representative case. Pressure, temperature, and plasma density results from all material subdomains are presented. Our discussion focuses on three aspects: energy transfer and partition (Sec. 4.6.1), impact-induced heating and plasma formation (Sec. 4.6.2), and ionization of the SLG target (Sec. 4.6.3). Finally, some concluding remarks are provided in Sec. 4.7.

4.2 Computational Model

4.2.1 Continuum mechanics

The collision between the projectile and the target generates shock waves that travel through the projectile, the target, and the surrounding fluid medium. Behind the shock waves, the materials are subjected to intense compression, resulting in high temperature and pressure. For example, in one-dimensional (1D) shock compression experiments, we find that for an impact velocity around 6 km/s, the pressure and temperature within the SLG target exceeds 100 GPa and 5000 K, respectively (Sec. 4.3, Table 4.2). In comparison, the Hugoniot elastic limit of SLG is only around 5 GPa [25], and its melting temperature is around 1000 K (at atmospheric pressure). Therefore, in this work, we model the solid materials as compressible inviscid fluids, with a complete thermodynamic equation of state (EOS) relating pressure, density, internal energy, and temperature. Moreover, the heat diffusion timescale, defined by $t_d = \Delta x^2 / D$ with Δx representing characteristic length and D thermal diffusivity, is greater than the time window of our analysis¹. Hence, heat diffusion is also neglected. Based on these considerations, the dynamics of all three materials is governed by the

¹Setting Δx to our mesh resolution (0.05 mm) and D to the thermal diffusivity of tantalum (22.94 mm²/s [62]) yields $t_d \approx 100 \mu\text{s}$, while our simulation time is around 10 μs .

following Navier-Stokes equations that enforce conservation of mass, momentum, and energy.

$$\frac{\partial \mathbf{q}}{\partial t} + \nabla \cdot F(\mathbf{q}) = 0, \quad \forall \mathbf{x} \in \Omega = \Omega_P(t) \cup \Omega_T(t) \cup \Omega_G(t), \quad t > 0. \quad (4.1)$$

In Cartesian coordinates,

$$F(\mathbf{q}) = \begin{bmatrix} f(\mathbf{q}) & g(\mathbf{q}) & h(\mathbf{q}) \end{bmatrix}, \quad \text{with}$$

$$\mathbf{q} = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ E \end{bmatrix}, \quad f(\mathbf{q}) = \begin{bmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ \rho uw \\ \rho Hu \end{bmatrix}, \quad (4.2)$$

$$g(\mathbf{q}) = \begin{bmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ \rho vw \\ \rho Hv \end{bmatrix}, \quad h(\mathbf{q}) = \begin{bmatrix} \rho w \\ \rho uw \\ \rho vw \\ \rho v^2 + p \\ \rho Hw \end{bmatrix}.$$

Here, ρ denotes mass density. p is the pressure, $\mathbf{U} = \begin{bmatrix} u & v & w \end{bmatrix}^T$ is the velocity vector. E is the total energy per unit volume, given by $E = \rho e + \frac{1}{2} \rho \|\mathbf{U}\|_2^2$. H is the total enthalpy per unit mass, defined by $H = \frac{1}{\rho}(E + p)$.

Due to the large deformation of the solid materials and the involvement of the ambient gas, we have adopted the Eulerian reference frame. $\Omega_P(t)$, $\Omega_T(t)$, and $\Omega_G(t)$ denote, respectively, the subdomains occupied by the projectile, the target, and the ambient gas at time t .

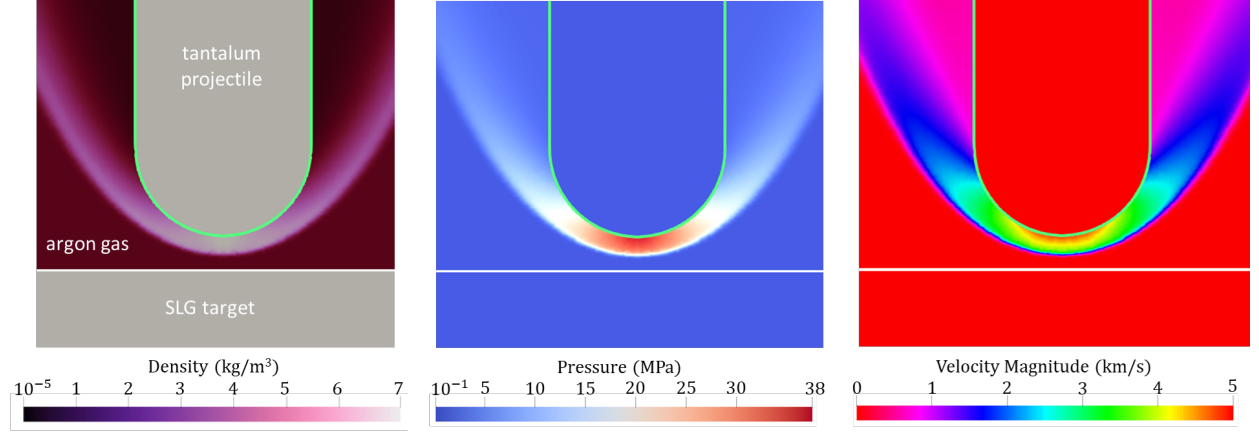


Figure 4.2: Initialization of fluid-solid coupled impact simulations: Fluid conditions obtained by a 3D steady-state computational fluid dynamics (CFD) simulation ($U_0 = 5$ km/s in this example)

We assume that the projectile impacts on the target in the normal direction. Leveraging the cylindrical symmetry about the centerline of the projectile, Eq. (4.1) reduces to

$$\frac{\partial}{\partial t} \begin{bmatrix} \rho \\ \rho u_r \\ \rho w \\ E \end{bmatrix} + \frac{\partial}{\partial r} \begin{bmatrix} \rho u_r \\ \rho u_r^2 + p \\ \rho u_r w \\ \rho u_r H \end{bmatrix} + \frac{\partial}{\partial z} \begin{bmatrix} \rho w \\ \rho u_r w \\ \rho w^2 + p \\ \rho w H \end{bmatrix} = -\frac{1}{r} \begin{bmatrix} \rho u_r \\ \rho u_r^2 \\ \rho u_r w \\ \rho u_r H \end{bmatrix}, \quad (4.3)$$

where u_r and w are the radial and axial components of velocity. r and z denote the radial and axial directional coordinates, respectively.

The projectile disrupts the surrounding gas as it travels, creating a hypersonic flow characterized by shock waves. To account for this phenomenon, we conduct a steady-state fluid dynamics simulation to analyze the flow around the projectile prior to impact, utilizing a body-stationary frame of reference. In this simulation, the far-field flow velocity is set to the inverse of the impact velocity. The governing equations (4.1) are solved within the fluid domain Ω_G using a finite volume method implemented in the AERO-F solver [1, 9, 29]. The steady-state solution is used to initialize the fluid-solid coupled impact simulation (Fig. 4.2). The impact simulation is performed by solving

Eq. (4.3) in the entire domain (Ω) within a ground-stationary frame of reference. For this task, we employ the M2C solver, which utilizes a high-resolution finite volume method with linear reconstruction [30, 69, 70]. In this work, a generalized minmod slope limiter and the local Lax-Friedrichs (LLF) flux function are employed. Both AERO-F and M2C are open-source C++ codes, developed mainly at Stanford University and Virginia Tech, respectively [1, 2].

4.2.2 Equations of state and temperature laws

A thermodynamic equation of state (EOS) is needed to algebraically close the Navier-Stokes equations. In this work, a complete EOS is used to model each material. It includes a pressure equation that relates ρ , p , and e , as well as a temperature law that defines temperature, T .

Specifically, the argon gas is modeled using the calorically perfect gas EOS, given by

$$p(\rho, e) = (\gamma - 1)\rho e \quad (4.4)$$

and

$$T = \frac{1}{c_v}(e - e_0) + T_0, \quad (4.5)$$

where γ denotes the isentropic expansion factor, and c_v is the specific heat at constant volume. Here and after, the subscript 0 indicates the value of a material property in the reference condition. For the argon gas, $\gamma = 1.67$, $c_v = 312.2 \text{ J/kg} \cdot \text{K}$, $T_0 = 270 \text{ K}$, and $e_0 = 0$. The choice of the temperature dependence on e via Eq. (4.5), with a constant c_v can be justified by the fact that argon is an inert monoatomic gas [26]. The absence of valence electrons causes argon atoms to have only translational degrees of freedom, but not vibrational, rotational, or electronic degrees of freedom.

The projectile's tantalum material is modeled using an extended form of the Mie-Grüneisen EOS. The Mie-Grüneisen EOS has been widely used to model metals in impact analysis [72]. The general

form of this EOS can be expressed as

$$p = p_R(\rho) + \rho_0 \Gamma_0 (e - e_R(\rho)), \quad (4.6)$$

$$e = e_R(\rho) + c_v (T - T_R(\rho)), \quad (4.7)$$

where Γ denotes the Grüneisen parameter. p_R , e_R , and T_R represent reference functions that depend only on density. Originally, these functions were developed to model materials under compression. Applying them to materials under tension can cause numerical issues due to the loss of hyperbolicity of the governing equations [67]. In this work, while the tantalum projectile is primarily under compression upon impact, some regions (e.g., the rim) experience high tensile stresses. Therefore, we apply an extended version of the Mie-Grüneisen EOS, in which the reference functions take different forms in three different conditions, namely compression, expansion, and a user-defined pressure cutoff condition [60]. For convenience, we introduce the volumetric strain, $\eta = 1 - \rho_0/\rho$.

For $\eta > 0$ (compression), the reference functions are developed using the Rankine-Hugoniot jump conditions and shock compression experiments. Assuming a linear relation between shock velocity (U_s) and particle velocity (U_p),

$$p_R = \frac{\rho_0 c_0^2 \eta}{(1 - s\eta)^2}, \quad (4.8)$$

$$e_R = \frac{p_R \eta}{2\rho_0} + e_0, \quad (4.9)$$

where s is the slope of the Hugoniot curve in the $U_s - U_p$ space. To obtain a temperature law that satisfies the first law of thermodynamics, we combine the general form of the EOS with Maxwell's relations, which yields

$$T_R = e^{\Gamma_0 \eta} \left(T_0 + \frac{c_0^2 s}{c_v} \int_0^\eta \frac{e^{\Gamma_0 z} z^2}{(1 - sz)^3} dz \right). \quad (4.10)$$

The speed of sound, c , is given by

$$c^2 = \left. \frac{\partial p}{\partial \rho} \right|_e + \frac{p}{\rho^2} \left. \frac{\partial p}{\partial e} \right|_\rho = \frac{1}{\rho^2} \left\{ \frac{\rho_0^2 c_0^2 (1 + \eta s)}{(1 - s\eta)^3} + \rho_0 \Gamma_0 \left(p - \rho_0 \left[\frac{c_0^2 \eta}{(1 - s\eta)^3} \right] \right) \right\}. \quad (4.11)$$

Clearly, c^2 may be negative if the material has negative pressure or low density, which explains the need to define the reference functions differently for the case of tension.

For $\eta_{\min} \leq \eta < 0$ (tension), the reference functions are obtained based on isentropic relations. In this case,

$$p_R = K_0 \eta, \quad (4.12)$$

$$e_R = \frac{K_0 \eta^2}{2\rho_0} + e_0, \quad (4.13)$$

$$T_R = T_0 e^{\Gamma_0 \eta}, \quad (4.14)$$

where $K_0 = \rho_0 c_0^2$ is the adiabatic bulk modulus in the reference condition. The speed of sound is given by

$$c^2 = \left(\frac{\rho_0}{\rho} \right)^2 \left(c_0^2 + \Gamma_0^2 (e - e_R(\eta)) \right). \quad (4.15)$$

Even in this case, ensuring $c^2 > 0$ requires that $p - K_0 \eta > -\frac{\rho c_0^2}{\Gamma_0}$. To avoid a negative c^2 , we introduce a user-defined minimum pressure, $p_{\min} (< 0)$, and for $\eta < \eta_{\min} \equiv p_{\min}/K_0$, we define a continuation of the isentropic reference function, such that additional tensile stress is not sustained.

This gives

$$p_R = p_{\min}, \quad (4.16)$$

$$e_R = \frac{K_0 \eta_{\min}^2}{2\rho_0} + e_0 + \frac{p_{\min}}{\rho_0}(\eta - \eta_{\min}), \quad (4.17)$$

$$T_R = T_0 e^{\Gamma_0 \eta}. \quad (4.18)$$

In our simulations we have used the following parameter values for tantalum: $c_0 = 3.293$ km/s [51], $s = 1.307$ [51], $\rho_0 = 16.65$ g/cm³ [61], $\Gamma_0 = 1.64$ [35], $T_0 = 270$ K, $e_0 = 0$, and $\eta_{\min} = -4$.

The Mie-Grüneisen EOS implies a linear relationship between the Grüneisen parameter, Γ , and the specific volume V ($= 1/\rho$), that is,

$$\Gamma = V \left. \frac{\partial p}{\partial e} \right|_V = \rho_0 \Gamma_0 V. \quad (4.19)$$

However, our experimental result (see Sec. 4.3) shows that for SLG, Γ behaves as a nonlinear convex function of V within the impact velocity range considered in this study. Therefore, we do not use the Mie-Grüneisen EOS to model SLG. Instead, we employ the Noble-Abel stiffened gas EOS, which conforms to this observed behavior. Specifically, the pressure equation is given by [40]

$$p(e, V) = \frac{(e - q)(\gamma - 1)}{V - b} - \gamma p_c, \quad (4.20)$$

where q , b , p_c , and γ are constant parameters. The Noble-Abel stiffened gas EOS has proven effective in modeling condensed matters in both liquid and solid phases [55, 57]. Importantly, the Grüneisen parameter, given by

$$\Gamma = V \left. \frac{\partial p}{\partial e} \right|_V = (\gamma - 1) \frac{V}{V - b}, \quad (4.21)$$

is indeed a convex function of V .

For the SLG material, we employ the temperature law introduced by Le M'etayer and Saurel [40], derived by combining Eq. (4.20) with Maxwell's relations. Specifically,

$$T(p, V) = \left[p + p_c - \left(\frac{C}{V - b} \right)^\gamma p_c \right] \frac{V - b}{(\gamma - 1)c_v}. \quad (4.22)$$

We can also express temperature as a function of specific internal energy and specific volume by substituting Eq. (4.20) into Eq. (4.22), i.e.,

$$T(e, V) = \frac{V - b}{c_v} \left[\frac{e - q}{V - b} - p_c - \left(\frac{C}{V - b} \right)^\gamma \frac{p_c}{\gamma - 1} \right]. \quad (4.23)$$

Here, C is an additional constant material parameter. The values of all the material parameters of SLG relevant to the NASG EOS will be determined in Sec. 4.4 from the experimental results presented in Sec. 4.3.

4.2.3 Interface tracking and treatment

In the impact simulations, the computational domain Ω is the union of three material subdomains (Ω_P , Ω_T , and Ω_G), occupied by the projectile, the target, and the ambient gas, respectively. Across subdomain boundaries (i.e., material interfaces), continuity of pressure and normal velocity is enforced. During a hypervelocity impact event, the material interfaces undergo rapid deformation. In this work, the motion of material interfaces are predicted by solving two level set equations that share the common velocity field [30, 44, 69]. This method allows us to keep track of three sharp, interconnected material interfaces (i.e. projectile-fluid, target-fluid, and projectile-target) that undergo large, complex deformations.

The level set equations are given by

$$\frac{\partial \phi_s}{\partial t} + \mathbf{U} \cdot \nabla \phi_s = 0, \quad s = 1, 2, \quad \forall \mathbf{x} \in \Omega, \quad t > 0, \quad (4.24)$$

where ϕ_s , $s = 1, 2$ represent two different level set functions that track the boundaries of Ω_T and Ω_P , respectively. $\mathbf{U}$ is the same velocity field in (4.3). At any point $\mathbf{x} \in \Omega$, ϕ_1 and ϕ_2 are initialized to be the signed shortest distance from $\mathbf{x}$ to the corresponding subdomain boundary at $t = 0$. At any time $t \geq 0$, the material at $\mathbf{x}$ is within Ω_T if $\phi_1(\mathbf{x}, t) < 0$, within Ω_P if $\phi_2(\mathbf{x}, t) < 0$, and within Ω_G if $\phi_1(\mathbf{x}, t) > 0$ and $\phi_2(\mathbf{x}, t) > 0$ (Fig. 4.3). This status allows the correct EOS to be applied. The three material interfaces are given by

$$\overline{\Omega_T(t)} \cap \overline{\Omega_G(t)} = \{\mathbf{x} \in \Omega, \phi_1(\mathbf{x}, t) = 0, \phi_2(\mathbf{x}, t) > 0\}, \quad (4.25)$$

$$\overline{\Omega_P(t)} \cap \overline{\Omega_G(t)} = \{\mathbf{x} \in \Omega, \phi_1(\mathbf{x}, t) > 0, \phi_2(\mathbf{x}, t) = 0\}, \quad (4.26)$$

$$\overline{\Omega_T(t)} \cap \overline{\Omega_P(t)} = \{\mathbf{x} \in \Omega, \phi_1(\mathbf{x}, t) = 0, \phi_2(\mathbf{x}, t) = 0\}, \quad (4.27)$$

where an overline indicates set closure. Compared to explicit interface tracking and representation methods, this implicit approach naturally captures the contact and separation between the different materials whilst avoiding non-physical subdomain overlaps.

Computing the fluxes of mass, momentum, and energy across material interfaces with different EOS also poses a challenge, given the significant jump in some state variables (e.g., density). In this work, we employ the FIVER (FInite Volume method based on Exact multiphase Riemann solvers) method to compute the interfacial fluxes. Along each edge in the computational mesh that crosses a material interface, we construct a 1D bimaterial Riemann problem, and substitute its exact solution into numerical flux functions. The details of FIVER, including its design, properties, and applications, can be found in Refs. [17, 45, 47, 48, 65].

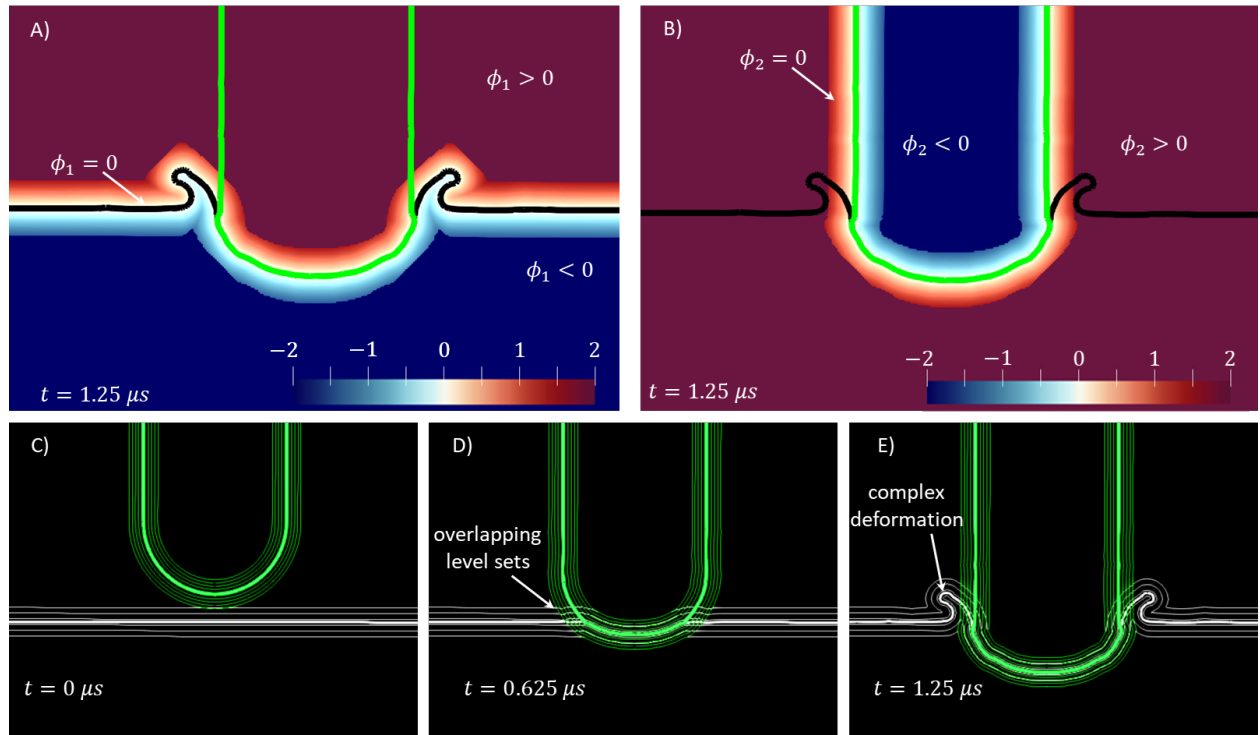


Figure 4.3: Implicit interface tracking by multiple (2) level set functions. (A) ϕ_1 tracks the SLG target subdomain: $\phi_1 < 0$ inside the subdomain and $\phi_1 > 0$ outside the subdomain. (B) ϕ_2 tracks the tantalum projectile subdomain: $\phi_2 < 0$ inside the subdomain and $\phi_2 > 0$ outside the subdomain. (C, D, E) Time evolution of the iso-contours of ϕ_1 and ϕ_2 , reflecting the motion and deformation of each material subdomain.

4.2.4 Ionization model: Non-ideal multi-species plasma

We consider all three material subdomains (target, projectile, and ambient gas) to be subject to ionization during the impact event. The extent of ionization within the computational domain is determined by the generalized (i.e., non-ideal) Saha ionization equation. Tantalum and argon are both treated as single-element substances comprised of Ta and Ar atoms, respectively. Modeling SLG poses an additional challenge, as it is a mixture of several different chemical compounds. In this work, the composition of SLG is determined based on *Starphire*, a commercially available float glass containing 73% SiO₂, 14% Na₂O, 10% CaO, and 3% MgO by mass [24]. As such, upon ionization, the SLG is expected to form a complex plasma mixture. We neglect the bond energies within the compounds. This is a justifiable simplification since the chemical bond energies are much smaller than the ionization energies of the corresponding elements. For example, the bond energy of 1 mole of SiO₂ is 1243.4 kJ [34]. In comparison, the sum of the first five ionization energies of 1 mole of Si and 2 moles of O is 82,966.4 kJ (Table 4.5). Therefore, SLG is treated here as a mixture of 5 pure elements, namely Si, O, Na, Ca, and Mg.

For each element, the generalized Saha ionization equation expresses the number density ratio between the $(r + 1)$ -th ion ($r = 0, 1, 2, \dots$) and the r -th ion as a function of temperature and plasma density (i.e., number density of electrons). Specifically,

$$\frac{n_{r+1,j}n_e}{n_{r,j}} = \frac{2U_{r+1,j}}{U_{r,j}} \left[\frac{2\pi m_e k_B T}{h^2} \right]^{\frac{3}{2}} \exp \left(-\frac{I_{r,j}^{\text{eff}}}{k_B T} \right), \quad r = 0, 1, \dots, Z_j - 1, \quad j = 1, 2, \dots, J, \quad (4.28)$$

where r and j are indices for charge number and element species, respectively. Z_j denotes the atomic number of element j , e.g., 14 for Si. J denotes the number of elements in the plasma, which is 1 for tantalum and argon, and 5 for SLG. $n_{r,j}$ denotes the number density of the r -th ion of element j . T and n_e represent temperature and plasma density, respectively. m_e , k_B , and h represent the physical constants of electron mass ($9.1093837 \times 10^{-31}$ kg), Boltzmann's constant ($1.3806485 \times$

10^{-23} J/K), and Planck's constant ($6.62607015 \times 10^{-34}$ J/Hz), respectively. Furthermore, $U_{r,j}$ is the partition function and $I_{r,j}^{\text{eff}}$ represents the effective ionization energy, to be defined shortly.

The ionization model must also account for the conservation of charge and nuclei, which implies

$$n_e = \sum_{j=1}^J \sum_{r=1}^{Z_j} r n_{r,j}, \quad (4.29)$$

and

$$n_H = \sum_{j=1}^J \sum_{r=0}^{Z_j} n_{r,j}. \quad (4.30)$$

Here, n_H denotes the number density of nuclei. It is determined by the material's mass density and composition, that is,

$$n_H = \rho N_A \sum_{j=1}^J \frac{c_j}{M_j}, \quad (4.31)$$

where ρ denotes mass density, and N_A the Avogadro's constant ($6.02214076 \times 10^{23}$ mol⁻¹). M_j and c_j are the molar mass and molar fraction of element j , respectively.

It can be shown that Eqs. (4.28), (4.29), (4.30), and (4.31) can be combined together, leading to a single transcendental equation [68],

$$Z_{\text{av}} = \sum_{j=1}^J c_j \left[\left(\sum_{r=1}^{Z_j} \frac{r}{(Z_{\text{av}} n_H)^r} \prod_{m=1}^r f_{m,j} \right) \right] / \left(1 + \sum_{r=1}^{Z_j} \frac{\prod_{m=1}^r f_{m,j}}{(Z_{\text{av}} n_H)^r} \right), \quad (4.32)$$

where the unknown variable, Z_{av} , is the mean charge of the plasma defined by $Z_{\text{av}} = n_e/n_H$. $f_{m,j}$ is the Saha coefficient, given by (the right-hand side of Eq. (4.28))

$$f_{r+1,j} = 2 \frac{U_{r+1,j}}{U_{r,j}} \left[\frac{2\pi m_e k_B T}{h^2} \right]^{\frac{3}{2}} \exp \left(-\frac{I_{r,j}^{\text{eff}}}{k_B T} \right). \quad (4.33)$$

The non-ideality of the plasma manifests itself in the depression of the ionization energy introduced

by low temperatures and high densities. Thus, the effective ionization energy, $I_{r,j}^{\text{eff}}$, can be expressed as

$$I_{r,j}^{\text{eff}} = I_{r,j} - \Delta I_r. \quad (4.34)$$

Many models for the depression term (ΔI_r) of varying complexities have been proposed [21]. In this work, we adopt the model introduced by Ebeling *et al.* [15], given by

$$\Delta I_r = \frac{(r+1)\tilde{e}^2}{4\pi\epsilon_0 L}. \quad (4.35)$$

Here, ϵ_0 is the permittivity of free space ($8.8541878 \times 10^{-12}$ F/m). $\tilde{e}$ denotes the elementary charge ($1.6021766 \times 10^{-19}$ C), with the tilde added to distinguish it from internal energy, e . L is a characteristic length, defined by

$$L = \lambda_D + \frac{\Lambda_B}{8}, \quad (4.36)$$

where $\Lambda_B = h(2\pi m_e k_B T)^{-1/2}$ is the de Broglie wavelength and λ_D is the Debye length. For a plasma mixture composed of J elemental species (e.g., $J = 5$ for SLG), the Debye length can be computed as

$$\lambda_D^2 = \frac{\epsilon_0 k_B T}{\tilde{e}^2 (n_e + \sum_{j=1}^J \sum_{r=0}^{Z_j} r^2 n_{r,j})}. \quad (4.37)$$

Lastly, the partition function in Eq. (4.28), $U_{r,j}$, is given by

$$U_{r,j} = \sum_{n=1}^{n_{\max}} g_{r,n,j} \exp\left(-\frac{E_{r,n,j}}{k_B T}\right), \quad (4.38)$$

where the subscript n indicates the n -th electronic energy level. $g_{r,n,j}$ is the degeneracy of the electron at this level, which acts as a statistical weighting term in the summation, and $E_{r,n,j}$ is the excitation energy of the electron. Note that for an isolated atom in an infinite vacuum, the

summation in Eq. (4.38) would continue with $n \rightarrow \infty$. However, due to the interaction of the plasma particles, the summation stops at $n_{\max}$, corresponding to the maximum value of n such that $E_{r,n,j} \leq I_{r,j}^{\text{eff}}$. Therefore the choice of non-ideal plasma model (which often depends on variables other than only temperature) also directly affects the computation of the partition function.

The spectroscopic data required to model the ionization of the materials in this study were obtained from the NIST database [49], namely the degeneracy ($g_{r,n,j}$), the excitation energies ($E_{r,n,j}$), and the ionization energies ($I_{r,j}$). Sample spectroscopic data for the elements in SLG are shown in Table 4.5. Note that this table only shows the first 5 ionic levels ($0 \leq r \leq 4$) and the first 5 electronic levels ($1 \leq n \leq 5$) for each element in SLG. In the actual simulations, we have used all the ionic levels ($0 \leq r \leq Z - 1$) and the electronic energy levels ($\sim 10^2$ energy levels per ion) that are available in the database.

Figure 4.4 outlines our fixed-point iteration procedure for solving the system of model equations. Specifically, at each time step of the impact simulation, the thermodynamic state (ρ, T) at each node on the computational grid is used as input to compute the local plasma characteristics. Once ρ is specified, n_{H} can be calculated using (4.31). We start the iterative procedure with a trivial initial guess, $\Delta I_r^{(0)} = 0, \forall r$. In each iteration, we solve Eq. (4.32) using a safeguarded iterative method, TOMS748 [4]. The resulting Z_{av} is then used to compute the mean electronic contribution of each elemental species in the plasma, $\bar{Z}_{e,j}$. Specifically,

$$\bar{Z}_{e,j} = c_j \left[\left(\sum_{i=1}^{Z_j} \frac{i}{(Z_{\text{av}} n_{\text{H}})^i} \prod_{m=1}^i f_{m,j} \right) \right] / \left(1 + \sum_{i=1}^{Z_j} \frac{\prod_{m=1}^i f_{m,j}}{(Z_{\text{av}} n_{\text{H}})^i} \right). \quad (4.39)$$

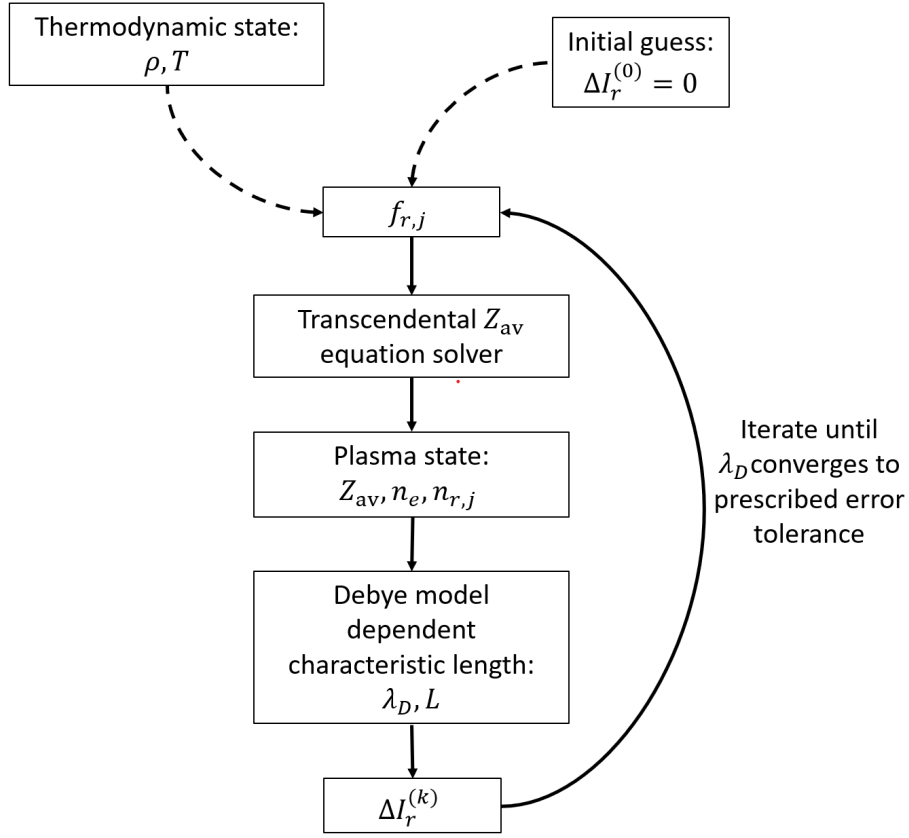


Figure 4.4: Overview of the iterative procedure for computing the composition of non-ideal plasma

Next, the number densities of neutral and charged particles are calculated by

$$n_{0,j} = \frac{\bar{Z}_{e_j} n_H}{\sum_{i=1}^{Z_j} \frac{i}{(Z_{av} n_H)^i} \prod_{m=1}^i f_{m,j}}, \quad (4.40)$$

$$n_{r+1,j} = \frac{n_{r,j}}{Z_{av} n_H} f_{r+1,j}, \quad r = 1, 2, \dots, Z_j - 1. \quad (4.41)$$

Subsequently, the plasma density, n_e , is obtained from Eq. (4.29). All the calculated number densities are then substituted into Eq. (4.37) to calculate the Debye length, which generates a new ΔI_r for use in the next iteration. In our simulations, the iterative procedure converges relatively fast. The Debye length often converges to less than 0.1% deviation within a few iterations.

Figure 4.5 compares the solution of the Saha equation generated by our solver against the results published by Zaghloul [68]. For this verification case, the ionic composition of a non-ideal plasma mixture is computed. This mixture constitutes of helium (He), neon (Ne), and argon (Ar) at a ratio of He : Ne : Ar = 0.3 : 0.6 : 0.1. The ionic composition of the plasma is computed over a density range of $10^{-6} \text{ kg/m}^3 \leq \rho \leq 10^2 \text{ kg/m}^3$ and a temperature of 5 eV (58030 K). As can be seen in Fig. 4.5, our result adheres well to Zaghloul's solution, especially for the Ar and He ions. Some minor discrepancy can be seen for the Ne ions, which could be caused by numerical errors or algorithmic accelerations used in our code. Furthermore, a simpler version of this solver for single-species plasma has also been verified for the solution of the ideal Saha equations, see Ref. [32].

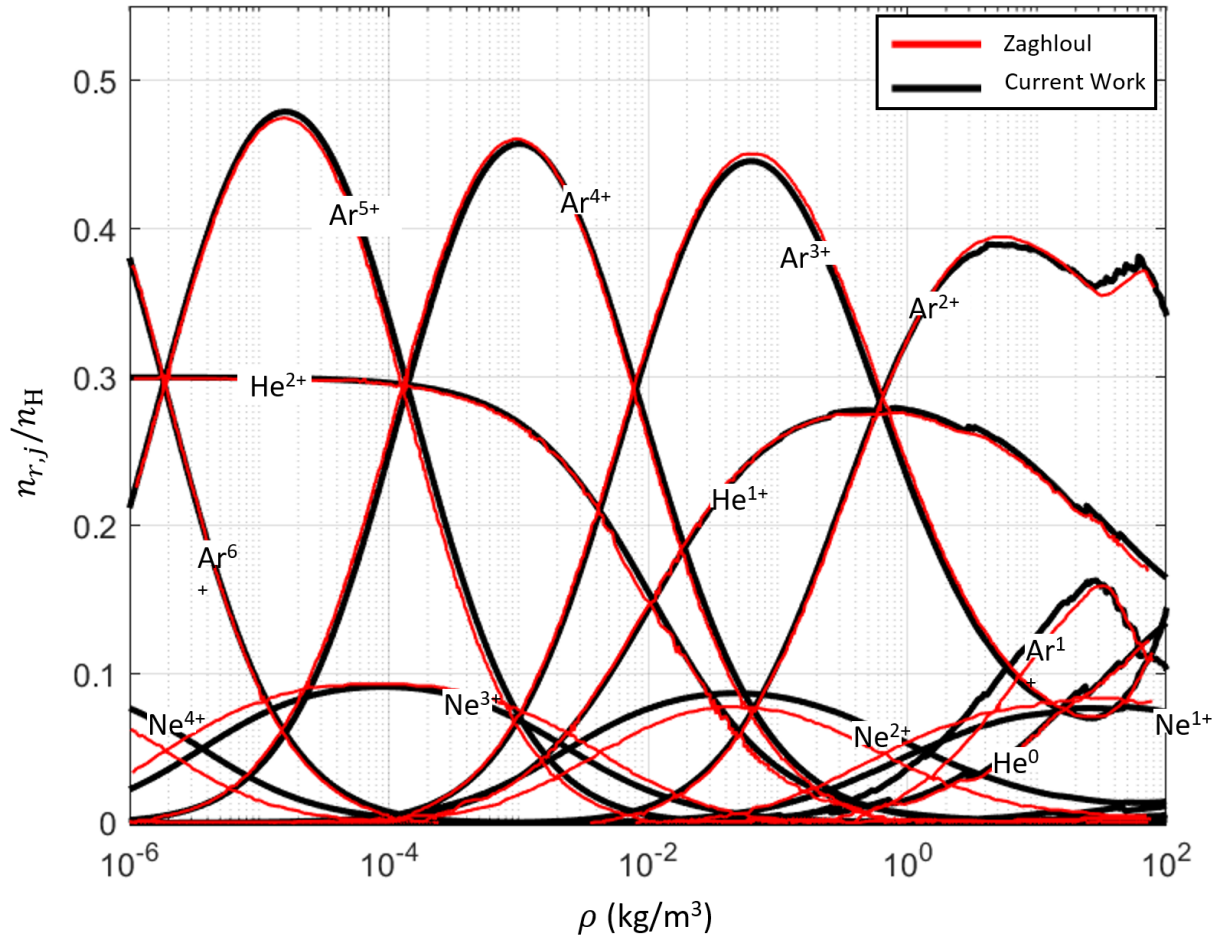


Figure 4.5: Verification of the non-ideal Saha equation solver implemented in M2C: Dependence of the composition of a 5 eV non-ideal plasma mixture [He(0.3):Ne(0.6):Ar(0.1)] on density. Reference: Zaghloul [68]

4.3 Hypervelocity shock compression experiments

To develop a thermodynamic EOS for SLG in the high-pressure regime (60 – 115 GPa), an experimental campaign was carried out in the Lindhurst Laboratory of Experimental Geophysics in Caltech's Division of Geological and Planetary Sciences. During this campaign 13 successful *in situ* diagnostic shots were performed: 5 experiments were designed to measure shock velocity and a partial release state (EOS shots), 4 were designed to measure shock velocity and temperature (thick flyer pyrometer shots), and 4 were designed to measure temperature and sound speed in the shock

state (thin flyer pyrometer shots). The projectiles in the pyrometer shots carry Molybdenum (Mo) flyers, whose thickness was varied between shots to adjust the time delay between shock and release waves and thereby observe and measure different physical phenomena.

4.3.1 Equation of state experiments

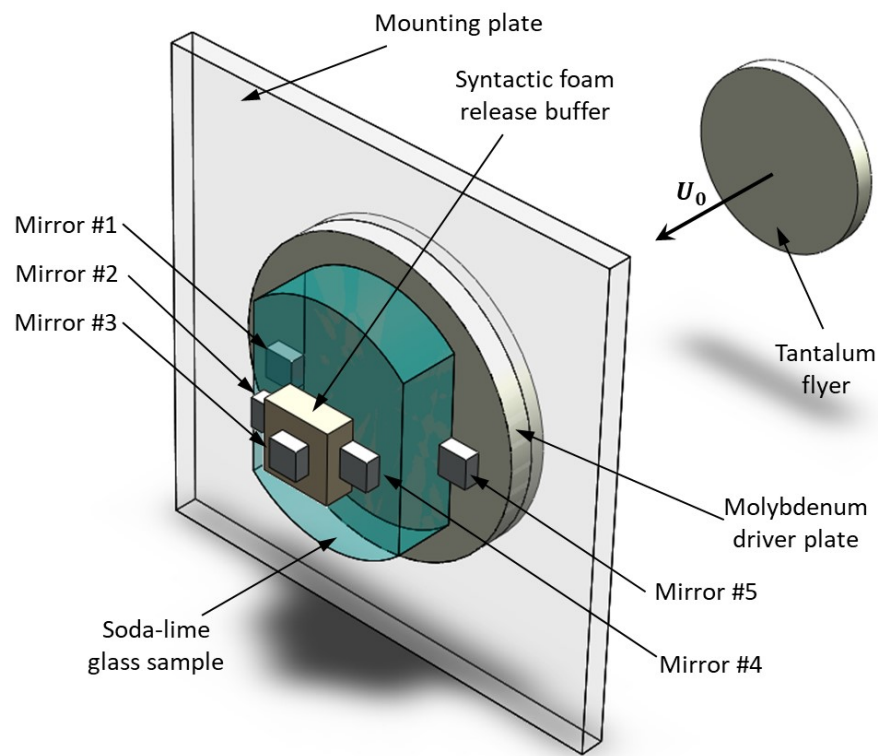


Figure 4.6: Experimental setup of the equation of state shots

For the experimental determination of the EOS, five successful shots were performed on the 40 mm single-stage propellant gun to capture shock velocity and a partial release state in SLG. The setup for these shots is shown in Fig. 4.6. The method is documented in detail in Mosenfelder *et al.* [53]. A Ta driver plate is glued to the front of the SLG sample and a syntactic foam release buffer is glued behind it. There are three sets of front-surface mirrors attached to the targets for streak camera photography, which records the time at which the reflectivity of each mirror changes as

the shock hits the mirror, thereby capturing the moment at which the shock arrives at the mirror location. In Fig. 4.7 the streak record of a representative shot (shot 1117) is shown. Mirrors 1 and 5 are mounted on the driver, 2 and 4 are on the back face of the SLG sample, and 3 is on the back face of the foam. The streak camera is calibrated with a 80 MHz radio signal and the total streak duration is $3.3 \mu\text{s}$. Mirrors 1 and 5 and mirrors 2 and 4 were placed in pairs, so that tilting or bowing of the shock as it enters and leaves the sample can be accounted for. In this case, the shock wave takes $539.3 \pm 1 \text{ ns}$ to travel through the SLG sample of a thickness of $3.155 \pm 0.001 \text{ mm}$ and $660.3 \pm 1 \text{ ns}$ to travel through $3.338 \pm 0.001 \text{ mm}$ of foam.

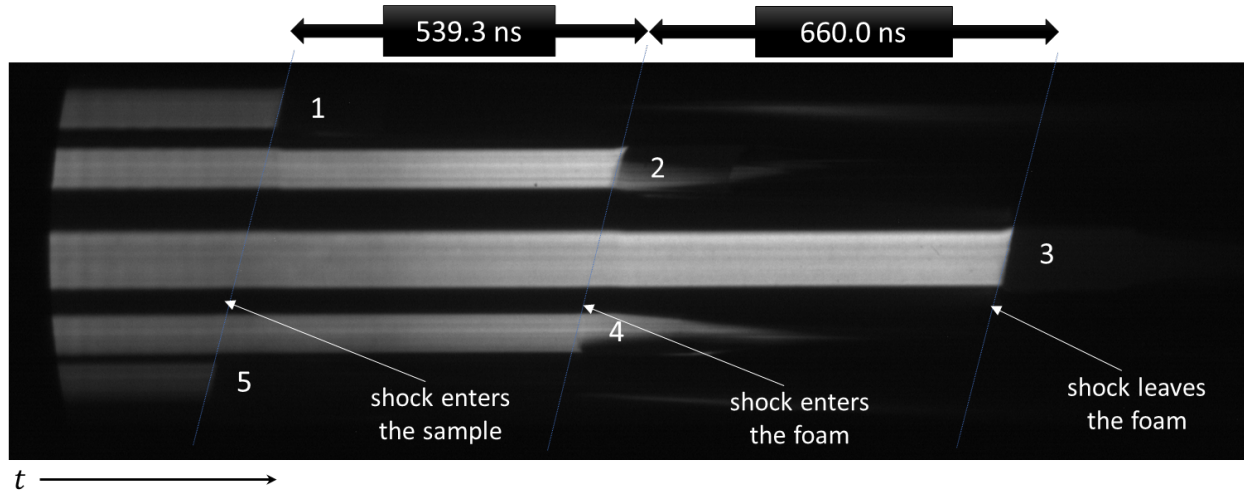


Figure 4.7: Representative streak record of shot 1117

Thus, the measured quantities in this set of experiments are: the velocity of the Ta projectile, the initial density and thickness of sample and foam, and the shock travel time through the sample and foam. Travel time and initial thickness of sample and foam layers immediately yield shock velocity in these two layers. Then impedance matching is used to calculate the particle velocity, shock pressure, and shock-state density in the SLG, using the Ta reference properties from Mitchell and Nellis [51]. Furthermore, since the Hugoniot of the foam is known, the particle velocity of the foam (and hence of the release state in the sample) can be calculated from its shock velocity. Then Riemann integration from the particle velocity in the sample to the particle velocity after the

shock releases into the foam yields the density and pressure of the partial release state in the sample (e.g., Ref. [33]). The shock velocity and the release state data obtained from these experiments are summarized below in Table 4.1. The uncertainties on flyer velocity measurements are typically ± 0.005 km/s, on particle velocity ± 0.01 km/s, on shock velocity ± 0.05 km/s, on pressure ± 1 GPa, and on shock volume ± 0.003 cm³/g.

Shot	u_{fp} (km/s)	U_p (km/s)	U_s (km/s)	P (GPa)	V (cm ³ /g)
1118	1.214	0.926	4.893	11.252	0.326
1114	1.257	1.039	4.872	12.572	0.317
1115	1.969	1.642	4.965	20.255	0.269
1117	2.605	2.136	5.744	30.481	0.253
1116	2.640	2.167	5.791	31.172	0.252
release data					
1118		1.295	2.336	1.734	0.339
1114		1.014	1.869	1.086	0.317
1115		2.447	4.083	5.724	0.301
1117		3.143	5.058	9.107	0.286
1116		3.030	4.902	8.508	0.276

Table 4.1: Experimental data obtained from the equation of state shots

4.3.2 Pyrometer experiments

The pyrometer experiments involved shots that were performed by launching molybdenum (Mo) flyers with a 25 mm two-stage light gas gun into a Mo driver plate attached to an SLG sample. However, in this case the mirrors and foam are not used, since the main objective is to measure the radiation history as the shock and release propagate within the volume of the SLG sample. The radiation is monitored with a six-channel optical pyrometer with ~ 3 ns time resolution. The calibration of this pyrometer using a primary NIST-traceable tungsten ribbon lamp and a secondary tungsten coil lamp is described in detail in Fat'yanov and Asimow [19], and the experimental setup

is shown in Fig. 4.8. These shots are divided into two categories: “thick-flyer” and “thin-flyer” shots. The thick-flyer experiments used a 1.5 mm thick Mo flyer, which delays the rarefaction from the back of the flyer long enough that the shock arrives at the rear free surface of the sample before being overtaken by the rarefaction. The supported shock travel time is measured from the onset of radiation when the shock passes from driver into sample until the sharp drop of brightness when the rear surface of the sample releases into vacuum. Hence these experiments yield both shock temperature and shock velocity. Again, impedance matching is used to obtain the particle velocity, shock pressure, and shock-state density in the SLG sample using accurate Mo Hugoniot properties from Fat’yanov *et al.* [18].

The thin-flyer experiments used an 0.8 mm thick Mo flyer, which yields a release wave following closely behind the shock front that overtakes the shock within the volume of the sample. The radiance history in these experiments yields both the shock temperature and the time when the onset of attenuation of radiance indicates overtaking of the shock by the rarefaction. Knowing the shock and release velocity in Mo flyer and driver and the shock velocity in the SLG from a matching thick-flyer experiment, the overtake time determines the velocity of the leading edge of the release wave and hence the speed of sound in the shock state (e.g., Nguyen *et al.* [54]). The propagation of the waves generated from the hypervelocity impact and their interactions with material surfaces in such a thin-flyer experiment can be shown on an $x - t$ diagram, e.g. Fig. 4.9. In this figure the red lines represent shock fronts, the blue lines represent the leading edges of release (rarefaction) waves, and the black lines show the time-dependent location of the material boundaries in the laboratory reference frame. When the Mo flyer makes contact with the driver, it sends a shock backwards into the flyer, and one forward into the driver. When the shock in the flyer reaches the rear surface of the flyer, it generates a forward-propagating rarefaction wave. In the laboratory reference frame, this release wave travels forward at a velocity given by the sum of the sound-speed in Mo and the particle velocity, which together are faster than the original shock velocity. Hence at some point

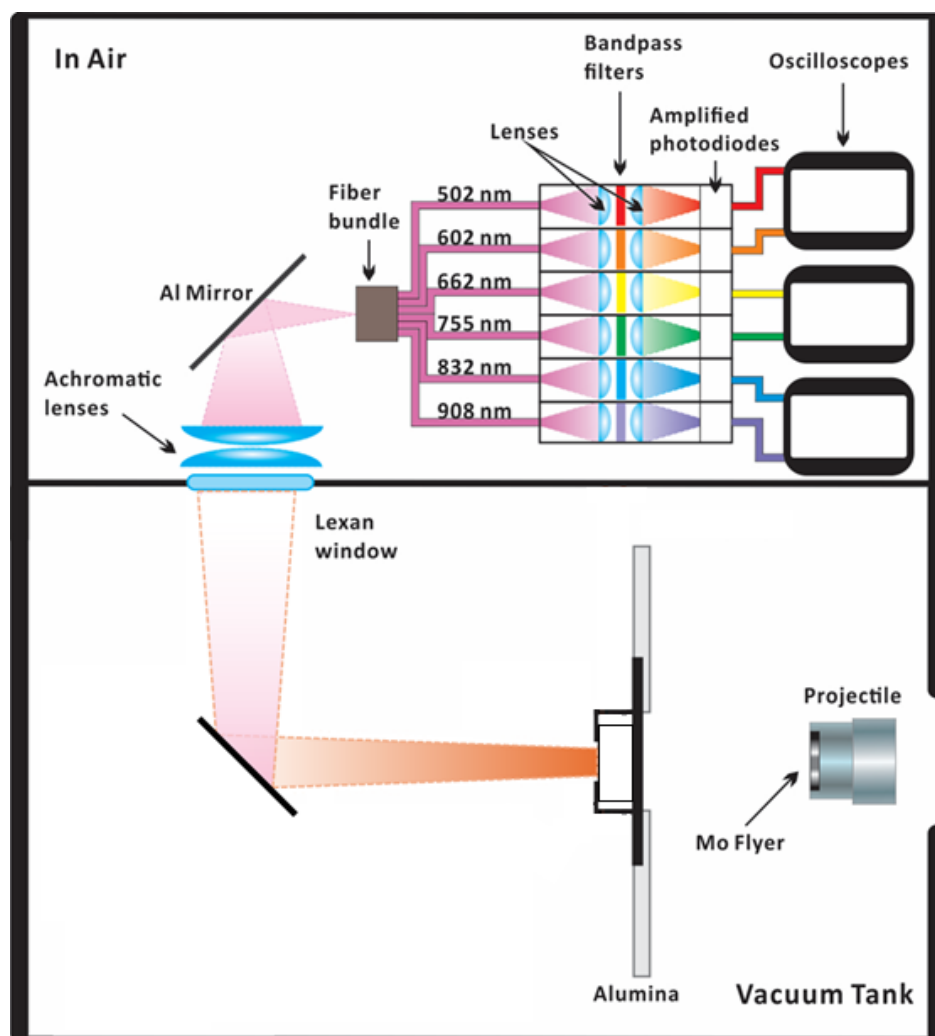


Figure 4.8: Experimental setup of the pyrometer shots. Note that this configuration allows pre-heating of the target but the data reported herein all used room-temperature targets

the release overtakes the shock and begins to attenuate it. The thin-flyer experiment is designed so that this overtake occurs with the volume of the SLG sample. As the shock enters the sample, the onset of radiance is detected by the pyrometer. Typically the onset of attenuation is marked by a sharp shoulder in the radiance record, also detected by the pyrometer. The time difference between these events is sufficient to define speed of sound in the target.

In a liquid medium, where the overtake velocity is a bulk sound speed, the combination of a sound

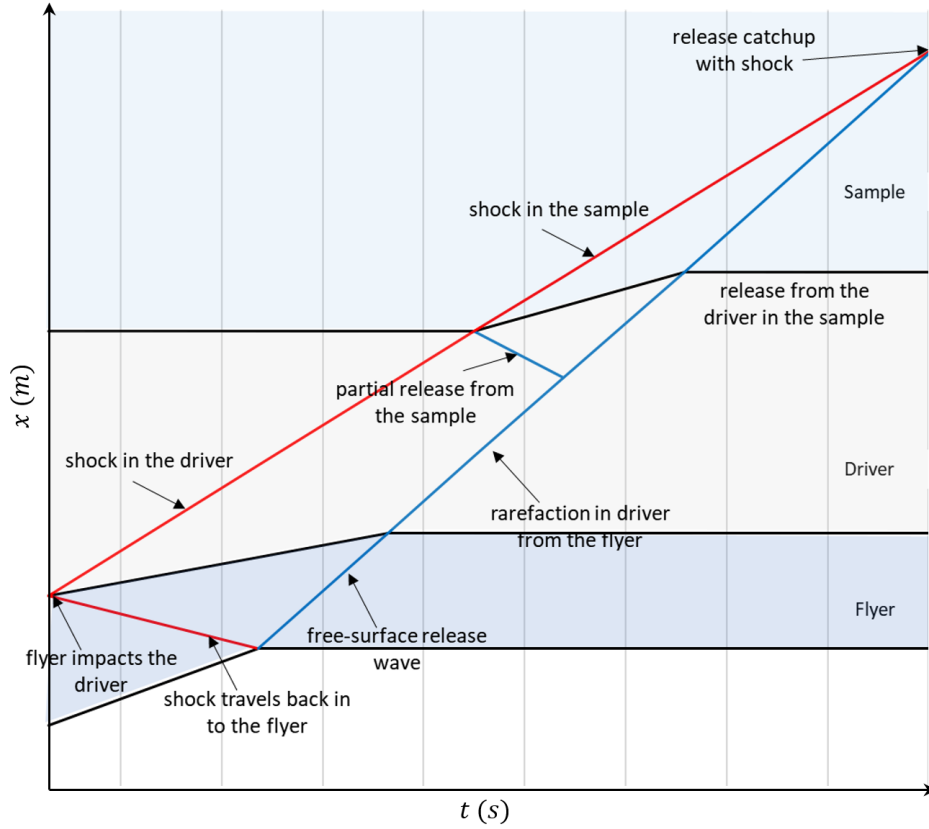


Figure 4.9: Schematic timing of shock wave propagation shown on a $x - t$ diagram

speed measurement and the slope of the Hugoniot (determined by fitting a line to the thick-flyer experiments) can be used to obtain a local measurement of the thermodynamic Grüneisen parameter [11]. For shots above 60 GPa, where we believe the conditions are sufficient to melt soda-lime glass to a liquid, the calculated Grüneisen parameter for each thin-flyer shot is given in Table 4.3.

4.3.3 Discussion of experimental results

The shock velocity data (Fig. 4.10), compared to previous low-pressure results by Alexander *et al.* [6] and Bourne and Rosenberg [10] and previous high-pressure results by Kobayashi *et al.* [36], suggest that our soda-lime glass stays on the extension of the low-pressure (low-density structure) configuration to significantly higher particle velocity than expected (> 3 km/s) and then converges

Shot	u_{fp} (km/s)	U_p (km/s)	U_s (km/s)	P (GPa)	V (cm ³ /g)	T (K)	$\pm T$ (K)
540	4.227	3.283	7.390	60.267	0.2237	3530	248
539	4.795	3.697	7.905	72.600	0.2143	3900	292
562	5.470	4.180	8.560	88.887	0.2060	4060	40
564	6.260	4.720	9.310	109.16	0.1985	5100	50

Table 4.2: Experimental data from the thick flyer pyrometer shots

-1cm

Shot	u_{fp} (km/s)	U_p (km/s)	U_s (km/s)	P (GPa)	V (cm ³ /g)	T (K)	$\pm T$ (K)	c (km/s)	$\pm c$ (km/s)	Γ	$\pm \Gamma$
559	4.240	3.293	7.386	60.421	0.223	3118	27	7.884	0.37	0.93	0.24
547	4.739	3.651	7.866	71.343	0.216	3619	39	7.748	0.24	1.18	0.11
563	5.540	4.250	8.800	92.909	0.208	4640	20	7.57	0.2	1.39	0.08
565	6.327	4.771	9.367	111.74	0.196	5300	100	7.75	0.27	1.47	0.13

Table 4.3: Experimental data from the thin flyer pyrometer shots

with the high-pressure (high-density structure) Hugoniot from Kobayashi *et al.* Our data agree with the new results from Renganathan and Duffy [59] insofar as they indicate a serious problem with the shock velocities from Kobayashi *et al.* in the 50 – 80 GPa range. However, we differ in the slope of the high-pressure Hugoniot; Renganathan and Duffy’s remains stiff and stays more dense than Kobayashi’s at all pressures, whereas ours eventually converges with Kobayashi at around 100 GPa. The release states are all well below the Hugoniot in $p - V$ space, demonstrating major irreversible densification of the material upon shock compression to as little as 11 GPa. This is not unexpected for glassy starting materials, which are known to experience irreversible densification (e.g., Lee *et al.* [42]).

The shock temperature data (Fig. 4.11) are generally consistent with Kobayashi’s experiments around 60 – 80 GPa, except that our results suggest a much lower slope, perhaps because our lower-pressure point probes a different (lower-density) shock state than Kobayashi’s experiment in

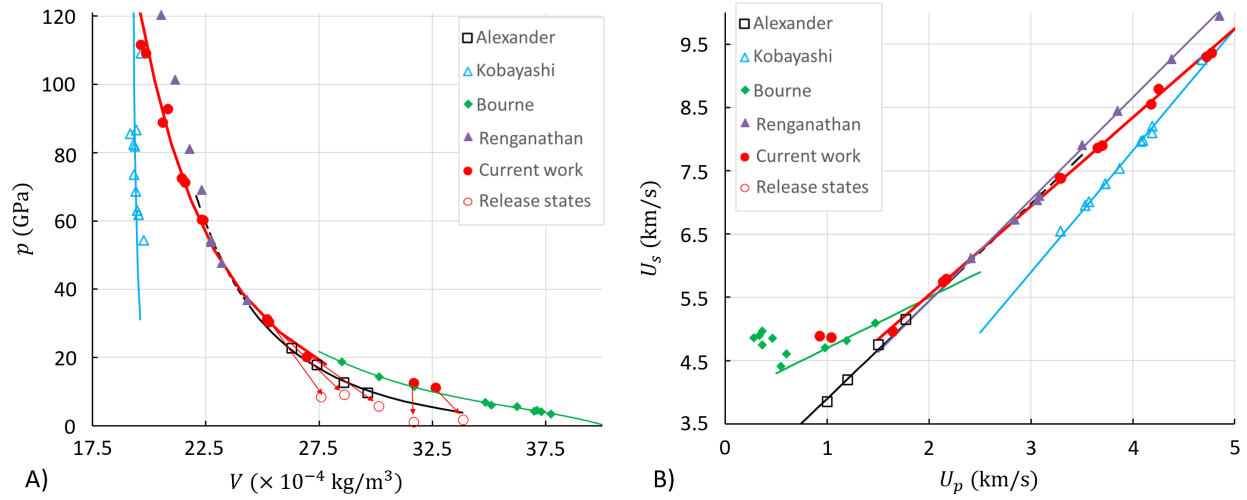


Figure 4.10: Experimentally measured Hugoniot states of SLG in (A) $p - V$ space and (B) $U_s - U_p$ space

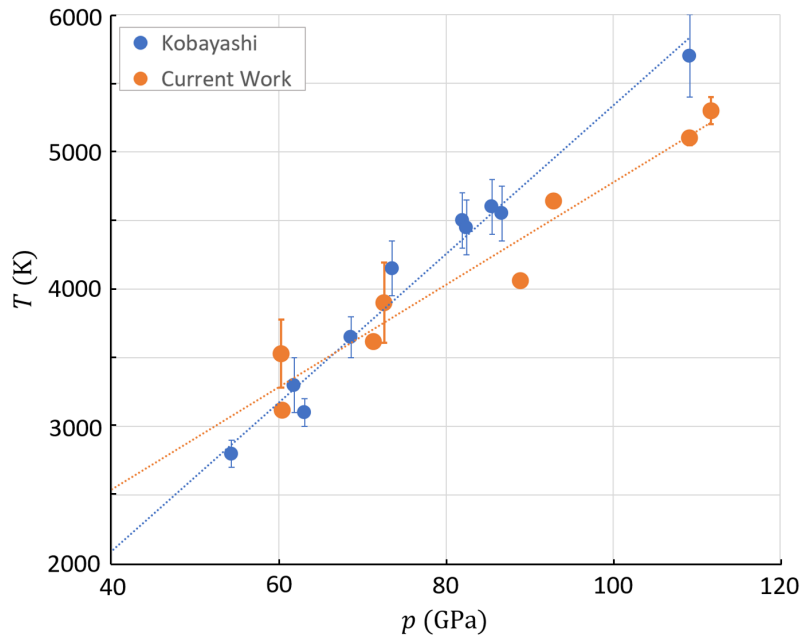


Figure 4.11: Measurement of pressure and temperature of shocked SLG

this pressure range. However, our temperature results sometimes exceed the precision of individual measurements by Kobayashi by a factor of five. Our precise data at higher pressure confirm the low slope of the increase in temperature in the shocked SLG with increasing shock pressure. An attempt to fit these temperature data with a constant heat capacity shows that the data are much lower in

temperature than predicted. The implication is that heat capacity strongly increases upon compression in shocked SLG, exceeding the Dulong-Petit limit of $3 k_B$ per atom. This behavior has been observed previously at much higher temperatures (10000 – 50000 K) in liquid silica [27], where it has been associated with dissociation into an ionic fluid. In this temperature range (3000 – 5000 K) in soda-lime liquid, it is more likely associated with configurational entropy gain due to progressive transformation from a polymerized silicate liquid to a molecular silicate liquid to an atomic liquid.

The sound speed results yield values of the Grüneisen parameter (Γ) that are entirely consistent with the anomalous volume dependence predicted and observed in several silicate liquids under compression [8, 64]. A fit of the sound speed results assuming $\Gamma = \Gamma(V)$ and the simple form

$$\frac{\Gamma}{\Gamma_0} = \left(\frac{V}{V_0} \right)^q \quad (4.42)$$

yields a best-fit value of $q = -2.00$. All the silicate liquids we have examined using the finite offset Hugoniot method, which has larger errors than the rarefaction overtake method, give $-2.5 < q < -1$ [7]. The new sound-speed based result is consistent with this behavior and notably more precise. However, it is unclear that the Mie-Grüneisen approximation is appropriate for silicate liquids (see Ref. [71]), which is one reason to develop the new equation of state formalism in this work.

4.4 Calibration of Equation of State for Soda-Lime Glass

The Noble-Abel stiffened gas EOS described in Sec. 4.2.2 contains six parameters, namely γ , p_c , b , q , c_v , and C , as shown in Eqs. (4.20) and (4.22). We present a two-step procedure to calibrate these parameters using the results of the hypervelocity shock compression experiments presented in Sec. 4.3.

First, we determine the parameters in the pressure equation (4.20), namely γ , p_c , b , and q . Across

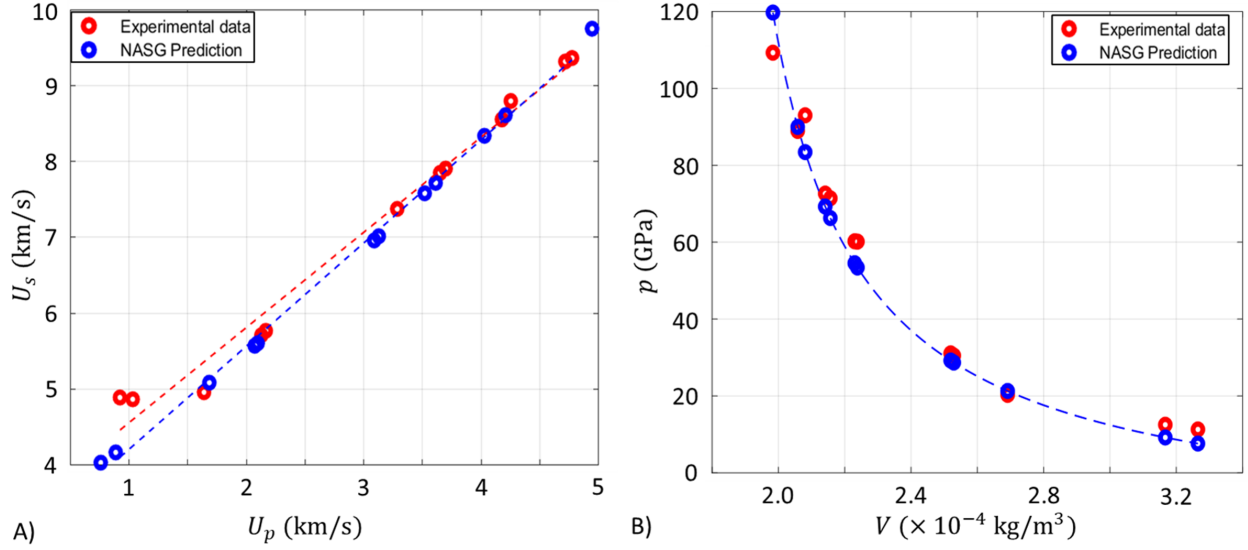


Figure 4.12: Shock Hugoniot data predicted by the calibrated EOS, in the $U_s - U_p$ space (A) and the $p - V$ space (B)

the shock wave in the SLG target, the following Rankine-Hugoniot jump conditions can be derived from the conservation of mass, momentum, and energy.

$$\frac{V}{V_0} = 1 - \frac{U_p}{U_s}, \quad (4.43)$$

$$p - p_0 = \frac{U_s U_p}{V_0}, \quad (4.44)$$

$$e - e_0 = \frac{1}{2}(p + p_0)(V_0 - V), \quad (4.45)$$

where p , e , V , and U_p denote the pressure, specific internal energy, specific volume, and velocity behind the shock wave. U_s denotes the shock propagation speed. The subscript 0 indicates the ambient state in front of the shock wave. Specifically, $p_0 = 101$ kPa and $V_0 = 4.024 \times 10^{-4}$ cm³/g.

Equations (4.43) and (4.44) allow us to convert the Hugoniot states between the p - V space and the

U_s - U_p space, as shown in Fig. 4.10. Coefficient e_0 in (4.45) can be eliminated by enforcing the EOS (4.20) in the ambient state, i.e.,

$$e_0 = \frac{(p_0 + \gamma p_c)(V_0 - b)}{\gamma - 1} + q. \quad (4.46)$$

Combining (4.20), (4.45), and (4.46), we obtain p as a function of V :

$$p = \frac{V(p_0 + K_1 \gamma p_c) - (K_1 \gamma p_c b + p_0 V_0 + K_2)}{K_1 b + V_0 - V(1 + K_1)}, \quad (4.47)$$

with

$$K_1 = \frac{2}{\gamma - 1}, \quad K_2 = K_1(p_0 + \gamma p_c)(V_0 - b). \quad (4.48)$$

Equation (4.47) still contains three free parameters: γ , p_c , and b . They can be related by requiring the EOS to capture the Grüneisen parameter and speed of sound measured at the ambient state, i.e.,

$$\Gamma_0 = (\gamma - 1) \frac{V_0}{V_0 - b} = 0.35, \quad (4.49)$$

$$c_0 = \sqrt{\frac{\gamma V_0^2 (p_0 + p_c)}{V_0 - b}} = 2708 \text{ m/s}. \quad (4.50)$$

Here, c_0 is taken as the U_s intercept of shock Hugoniot in the U_s - U_p space. Using (4.49) and (4.50), γ and p_c can both be expressed as a function of b . Substituting them into (4.47) allows us to eliminate γ and p_c , leaving b as the only free parameter, i.e. $p = p(V; b)$.

Let $\{(V_i^{(\text{exp})}, p_i^{(\text{exp})})\}$, $i = 1, 2, \dots, N$ represent the p - V pairs obtained from the shock compression

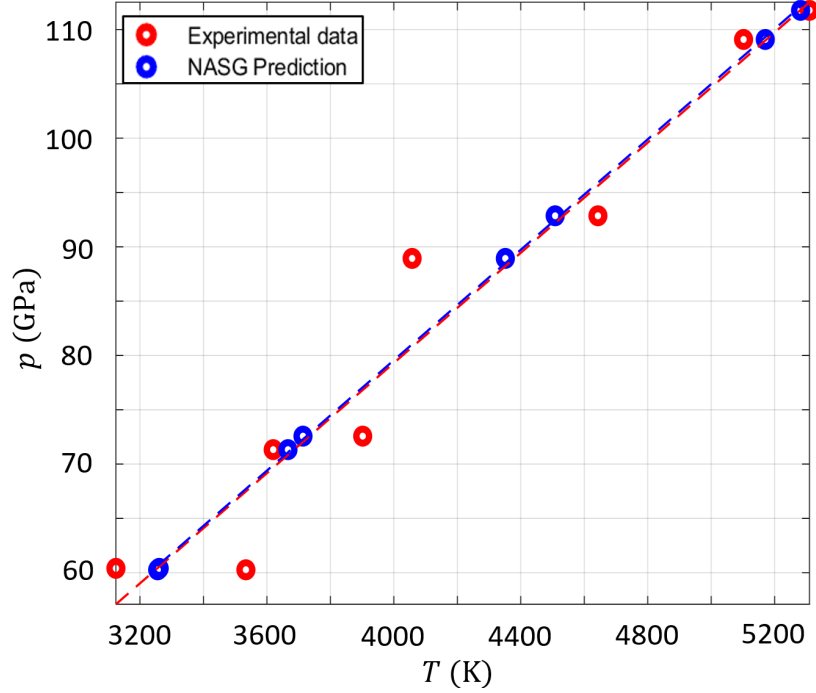


Figure 4.13: Temperature of shocked SLG as a function of pressure

experiments, that is, the filled circles in Fig. 4.10. The free parameter b is determined by solving the one-dimensional optimization problem

$$\min_b \sum_{i=1}^N \left(p(V_i^{(\text{exp})}; b) - p_i^{(\text{exp})} \right)^2, \quad (4.51)$$

which yields $b = 1.46 \times 10^{-4} \text{ m}^3/\text{kg}$. Substituting this value into (4.49) and (4.50) yields $\gamma = 1.22$ and $p_c = 14.14 \text{ GPa}$. From (4.46), it is clear that parameter q determines the value of internal energy at the ambient state. For simplicity, we set $q = 0$. Figure 4.12 shows a comparison between the experimental data and the result obtained from the calibrated EOS.

Next, the parameters c_v and C in the temperature law are determined using the Hugoniot data in the $T - p$ space (Fig. 4.11). First, the experimental data is fitted linearly by

$$T^{(\text{exp})} = \Pi p^{(\text{exp})} + \kappa, \quad (4.52)$$

with $\Pi = 2.5 \times 10^7$ K/Pa and $\kappa = -2.2 \times 10^{10}$ K. Using the Noble-Abel stiffened gas EOS, the $T - p$ relation along the shock Hugoniot is given by (4.22) and (4.47). In particular, V can be obtained by inverting Eq. (4.47). This relation is nearly linear. Therefore, c_v and C are determined by first fitting the $T - p$ relation of the EOS with

$$T = \Pi' p + \kappa', \quad (4.53)$$

followed by matching Π' and κ' to Π and κ . In this way, we obtain $c_v = 2420$ J/kg · K and $C = 1.97 \times 10^{-4}$ m³/kg. Figure 4.13 compares the experimental data with the result obtained from the EOS, together with their linear regressions. Using the EOS, temperatures are calculated at the same pressures in the experiment.

4.5 Trimaterial shock compression simulations

We conduct a series of 1D shock compression simulations to demonstrate the impact solver described in Secs. 4.2.1, 4.2.2, and 4.2.3. The initial velocity of the tantalum projectile is varied between 2.5 km/s and 6.5 km/s in 9 simulations at 0.5 km/s increments. The pressure, specific volume, and temperature results obtained in the SLG target are validated against the shock compression experiments presented in Sec. 4.3.

Each simulation is initialized at the time instant that the tantalum projectile has just reached the SLG target. The subdomains of tantalum and SLG are initialized by $\Omega_P(0) = (-0.2 \text{ mm}, 0 \text{ mm})$ and $\Omega_T(0) = (0 \text{ mm}, 0.25 \text{ mm})$, respectively. The rest of the 1D domain is occupied by argon gas, i.e., $\Omega_G(0) = (-\infty, -0.2 \text{ mm}) \cup (0.25 \text{ mm}, \infty)$. The pressure across the domain is initialized

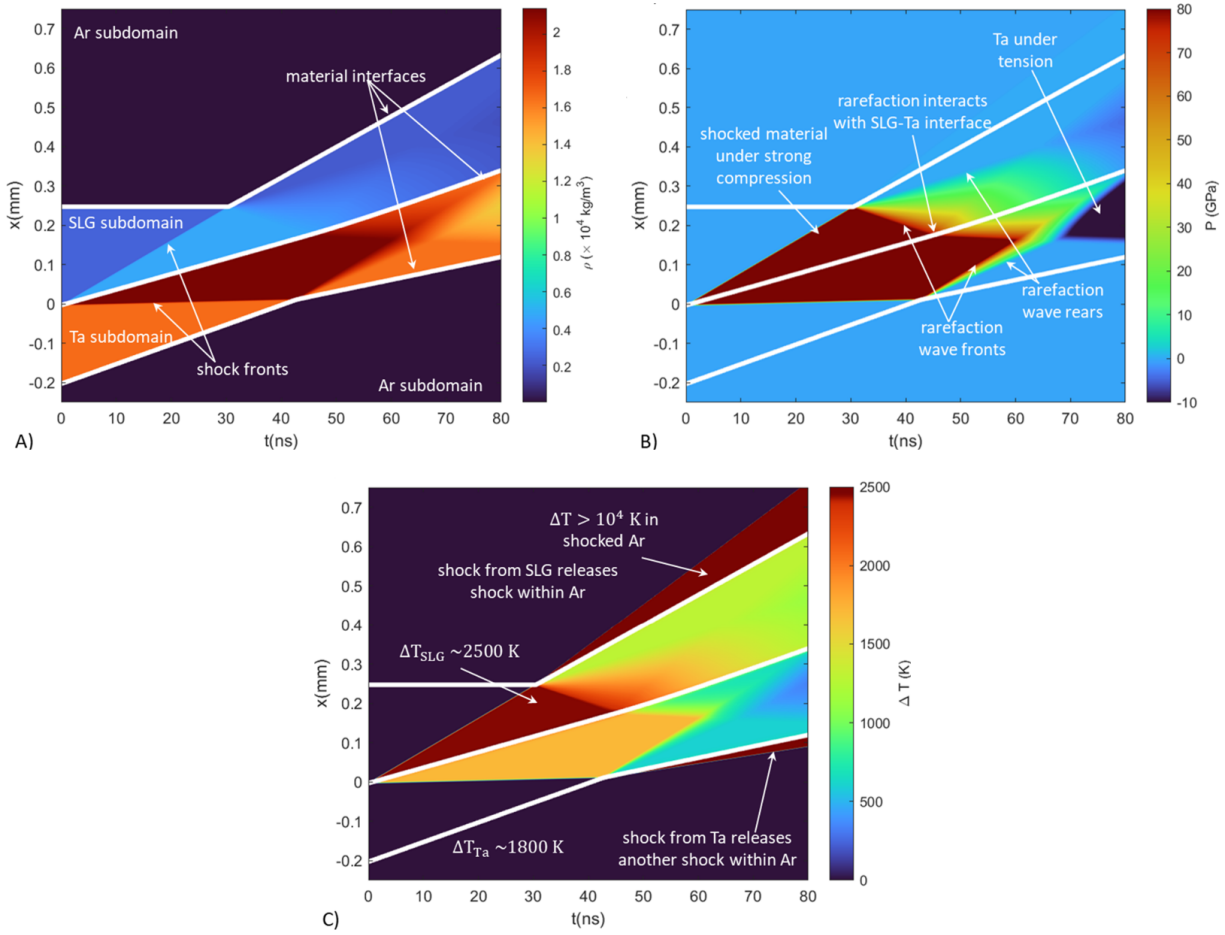


Figure 4.14: Evolution of the density (A), pressure (B), and temperature (C) fields obtained from the 1D shock compression simulation, for impact velocity $U_0 = 5$ km/s

to 100 kPa. The density of tantalum, SLG, and argon are set to 1.665×10^{-2} g/mm³, 2.485×10^{-3} g/mm³, and 1.780×10^{-6} g/mm³, respectively. The velocity of the tantalum projectile and the argon gas behind it is set to the impact velocity, U_0 , varying from 2.5 km/s to 6.5 km/s in different cases. The SLG and the argon gas behind it (i.e., $x > 2.5$ mm) are set to be stationary.

Figure 4.14 shows the simulation results obtained with $U_0 = 5$ km/s. The evolution of the density, pressure, and temperature fields are visualized. Snapshots of the thermodynamic states at $t = 24$ ns and 40 ns are shown in Fig. 4.15 to further highlight the shock states and interesting flow phenomena. As the simulation starts, the impact generates two shock waves that travel backward through

the tantalum projectile and forward through the SLG target. The shock waves are captured in each of the solution fields shown in Fig. 4.14, specifically marked in sub-figure (A). The pressure behind the shock waves is 81 GPa, continuous across the material interface. Although temperature and density also increase in both shocked solids, they are discontinuous across the material interface. The internal energy rise in SLG is an order of magnitude greater than in tantalum. However, the temperature elevation in both materials remains comparable, as tantalum has a much lower specific heat capacity (2420 J/kg · K versus 139 J/(kg · K), see Sec. 4.2.2).

The shock wave travels faster through SLG, reaching the SLG-argon interface ($t = 30$ ns) before the shock in tantalum reaches the tantalum-argon interface ($t = 43$ ns). This interaction is captured in the solution snapshots at $t = 40$ ns. It generates a shock wave that propagates in argon and a rarefaction fan that travels backward in SLG. Once the shock reaches tantalum-argon interface, another shock wave is sent into the argon and a rarefaction fan into the tantalum. The rarefaction fan decompresses and cools the material behind them. The shock wave in argon compresses the gas to 144 MPa while elevating its density nearly fourfold. Its temperature exceeds 100,000 K, much higher than that in the solid materials. Furthermore, this shock accelerates the argon gas to a hypersonic velocity of 7.8 km/s. Eventually the rarefaction fan within the SLG reaches the SLG-tantalum interface around $t = 50$ ns. The exact solution of this 1D problem can be obtained until this point in time. A verification analysis using the exact solution as reference (but simpler EOS) can be found in [32].

The elevated thermodynamic states behind the shock wave are plotted in Fig. 4.16, alongside the experimental data from the shock compression experiments. Within the range of simulated impact velocities, pressure ranges between 28 GPa and 125 GPa and temperature varies between 2200 K and 5600 K. In both p - V and p - T spaces, the simulation results match closely the experimental data.

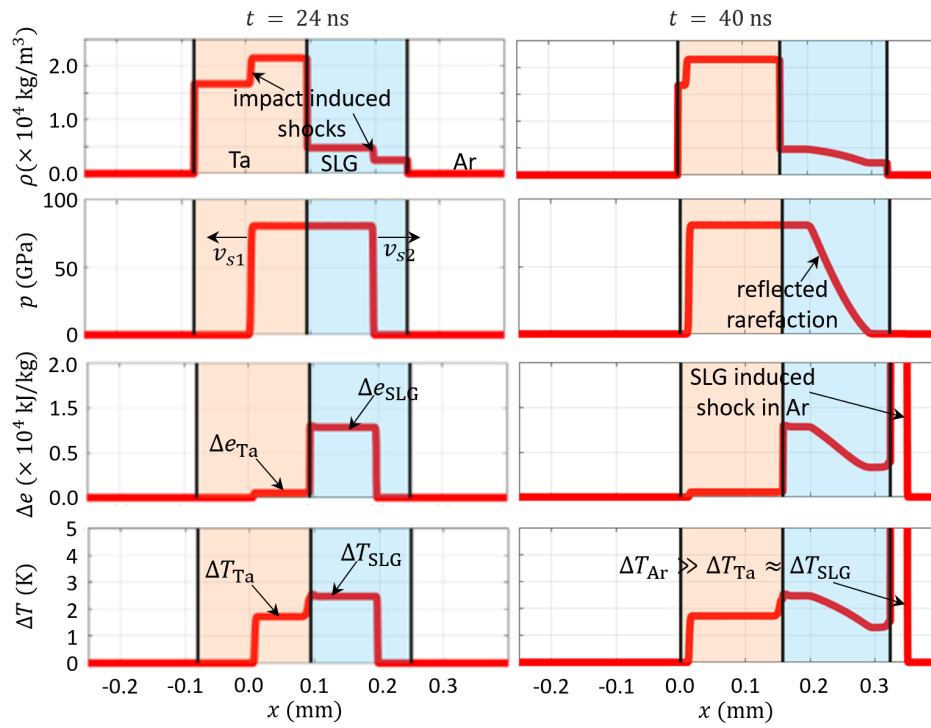


Figure 4.15: State of the computational domain at 24 ns and 40 ns during the 1D shock compression simulation with impact velocity $U_0 = 5$ km/s

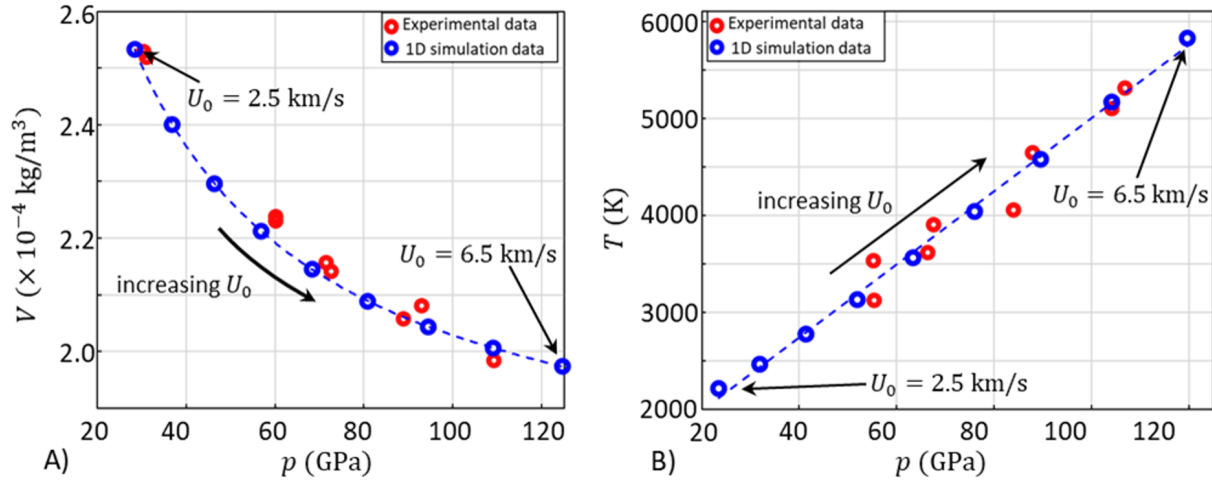


Figure 4.16: Comparison of shock Hugoniot data obtained from experiments and simulations.

4.6 Hypervelocity Impact Simulations

A series of 5 impact simulations are conducted, with impact velocity U_0 varied from 3 km/s to 7 km/s, at 1 km/s increments. The dimensions and initial positions of the projectile and target are depicted in Fig. 4.1. The tantalum projectile is initialized with impact velocity U_0 , density $\rho = 0.01665 \text{ g/mm}^3$, and pressure $p = 100 \text{ kPa}$. The initial temperature is the same as T_0 in the Mie-Grüneisen EOS, i.e., 270 K. The SLG target is stationary, with density $\rho = 0.0025 \text{ g/mm}^3$ and pressure $p = 100 \text{ kPa}$. The initial temperature, derived from the Noble-Abel stiffened gas EOS, is 1497 K. The ambient argon gas is initialized with the result of a steady-state CFD simulation, as described in Sec. 4.2.1. At far-field, it has density $\rho = 1.78 \times 10^{-6} \text{ g/mm}^3$, velocity $U = 0 \text{ km/s}$, pressure $p = 100 \text{ kPa}$, and temperature $T = 270 \text{ K}$. Initially, the leading edge of the projectile is 2 mm away from the target. The bow shock in the ambient gas has not reached the target yet.

The computational domain Ω is 240 mm long in the axial direction and 120 mm wide in the radial direction. It is designed to be sufficiently large such that any wave reflections at the far-field boundaries would not reach the SLG target within the simulation time window. It is discretized by a non-uniform Cartesian grid with approximately two million nodes. Around the projectile and

the target, the element size is 0.05 mm. All the simulations are conducted on the Virginia Tech Tinkercliffs computer cluster [3].

We choose the simulation with $U_0 = 5$ km/s as a representative case. Figure 4.17 visualizes the pressure field at 6 different time instances: $t = 1.0, 2.5, 4.0, 5.0, 7.5$, and $10 \mu\text{s}$. The pressure within the solids and fluids are displayed on separate scales due to their significant difference. Within each sub-figure, an inset figure displays the deformation of the projectile and the target in 3D.

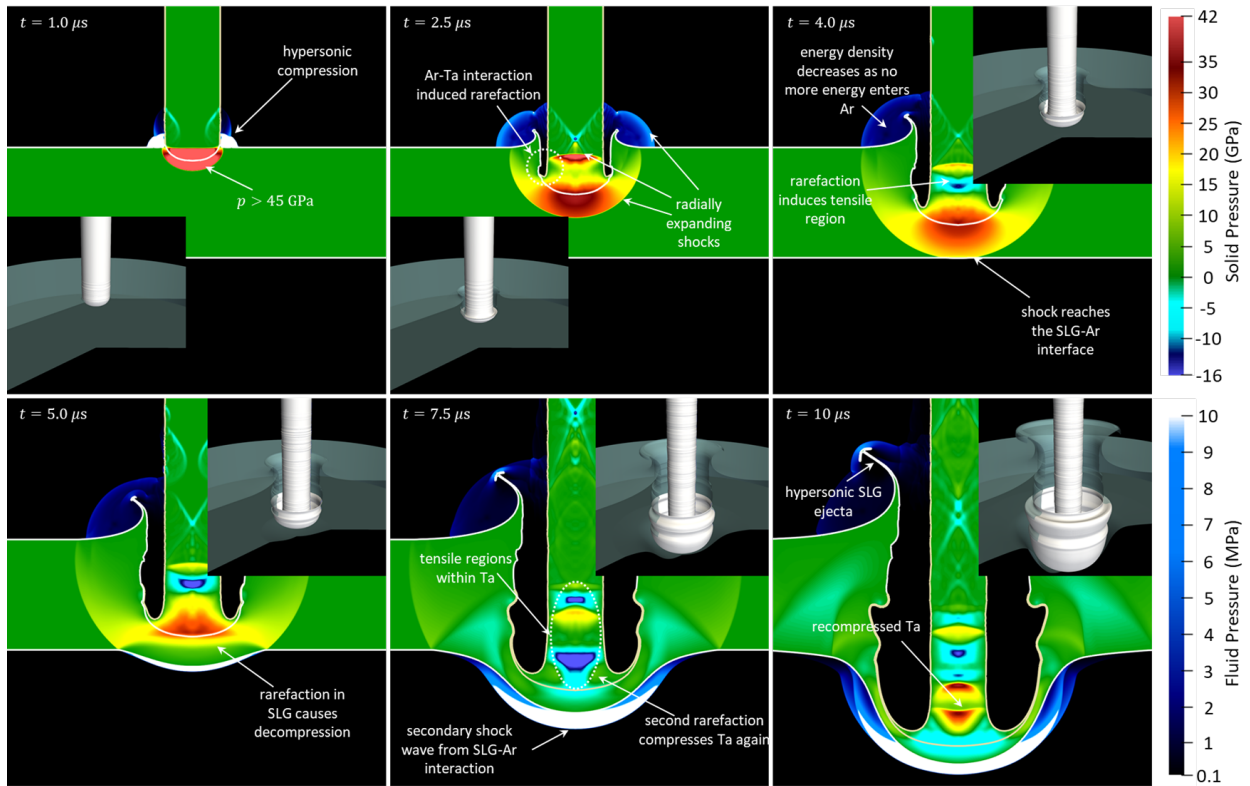


Figure 4.17: Evolution of the pressure field within each material subdomain for impact velocity $U_0 = 5$ km/s. Fluid and solid pressures mapped on different scales due to difference in orders of magnitude. Inset images showing 3D deformation of the solids.

The snapshot at $t = 1 \mu\text{s}$ shows the computational domain shortly ($0.6 \mu\text{s}$) after collision. At this time, the shock waves expand outward from the point of impact in all three material subdomains. The solids undergo intense compression, with pressures behind these shock fronts ex-

ceeding 45 GPa. The fluid also experiences hypersonic compression, forming a pocket of highly compressed and heated argon. This behavior was also captured in our previous analysis with a simplified EOS [31]. At $2.5 \mu\text{s}$, it can be seen that the shock wave in the tantalum projectile is travelling primarily along the axial direction. However, this shock interacts with the argon within the impact-induced crater, releasing a lateral rarefaction within the projectile. This rarefaction initially decompresses the region behind the shock wave within the tantalum, eventually forming a region of tension, as seen in the subsequent snapshots. At $4.0 \mu\text{s}$, the shock wave in SLG has reached the rear surface. Its interaction with the argon gas launches a rarefaction fan back into SLG and another shock wave into argon. This phenomenon is also captured in the 1D shock compression simulations, see Fig. 4.15. The rarefaction fan spreads through the SLG target and decompresses the material, as shown in the snapshot at 7.5 and $10 \mu\text{s}$. The waves within the tantalum projectile cyclically interact with the argon in the crater through the lateral interface and generate alternating regions of tension and compression, as shown in the snapshot at $10 \mu\text{s}$.

Next, we analyze the simulation results quantitatively, focusing on energy partition, temperature increase, ionization, and plasma formation. We will primarily examine the representative case with $U_0 = 5 \text{ km/s}$ to contrast the behaviors of the three materials involved. The effect of impact velocity within the tested range will be discussed briefly.

4.6.1 Energy transfer and partition

We integrate the internal energy $\mathcal{E}^{(int)}$, kinetic energy $\mathcal{E}^{(kin)}$, and total energy $\mathcal{E}^{(tot)}$ within each of the three material subdomains. For example, at any time $t > 0$, the internal energy of the SLG target is calculated by

$$\begin{aligned}
\mathcal{E}_T^{(int)}(t) &= \int_{\Omega_T} \rho(\mathbf{x}, t) e(\mathbf{x}, t) d\mathbf{x} = \int_{\Omega} 1_{\{\mathbf{x} \in \Omega_T(t)\}} \rho e d\mathbf{x} \\
&= \int_{\theta} \int_z \int_r 1_{\{(r, z, \theta) \in \Omega_T(t)\}} \rho e dr dz d\theta
\end{aligned} \tag{4.54}$$

$$\begin{aligned}
&= 2\pi \int_z \int_r 1_{\{(r, z, \theta) \in \Omega_T(t)\}} \rho e r dr dz \\
&\approx 2\pi \sum_{C_i \subset \Omega^{(h)}} 1_{\{i: \phi_{1,i} < 0\}} \rho_i e_i r_i |C_i|,
\end{aligned} \tag{4.55}$$

where 1 denotes the indicator function. $\Omega^{(h)}$ denotes the computational grid. C_i represents an arbitrary control volume in the grid, and r_i the radial coordinate of its center. ϕ_1 is the level set function introduced to track the subdomain of the target, Ω_T (Sec. 4.2.3).

Figure 4.18 shows the energy variations in the projectile, target, and ambient gas. All energy in this impact event originates from the projectile's kinetic energy. At $t_1 = 0.4 \mu\text{s}$, the projectile reaches in contact with the target. Figure 4.18(A) shows that after this time, the projectile's kinetic energy decreases steadily at nearly a constant rate. Meanwhile, the internal energy rises in both the target and the projectile. Notably, the target accumulates internal energy much faster than the projectile. From $0.4 \mu\text{s}$ to $4 \mu\text{s}$, $\mathcal{E}_T^{(int)}$ increases by $2.3 \times 10^4 \text{ J}$, while $\mathcal{E}_P^{(int)}$ increases by only $2.1 \times 10^3 \text{ J}$. This disparity is consistent with the result of the 1D shock compression simulations, specifically, the Δe plot at 24 ns in Fig. 4.15. To explain this trend intuitively, we define shock impedance as the product of shock speed and density, analogous to acoustic impedance. In this way, the shock impedance of tantalum is approximately 3.5 times greater than that of SLG. Applying the linear acoustic theory suggests that most of the energy would be transferred into SLG, which is consistent with our results.

The label $t_2 = 1 \mu\text{s}$ marks the time when the entire spherical leading edge of the projectile contacts the target. Between t_1 and t_2 , the leading edge is compressed, resulting in a higher rate of internal

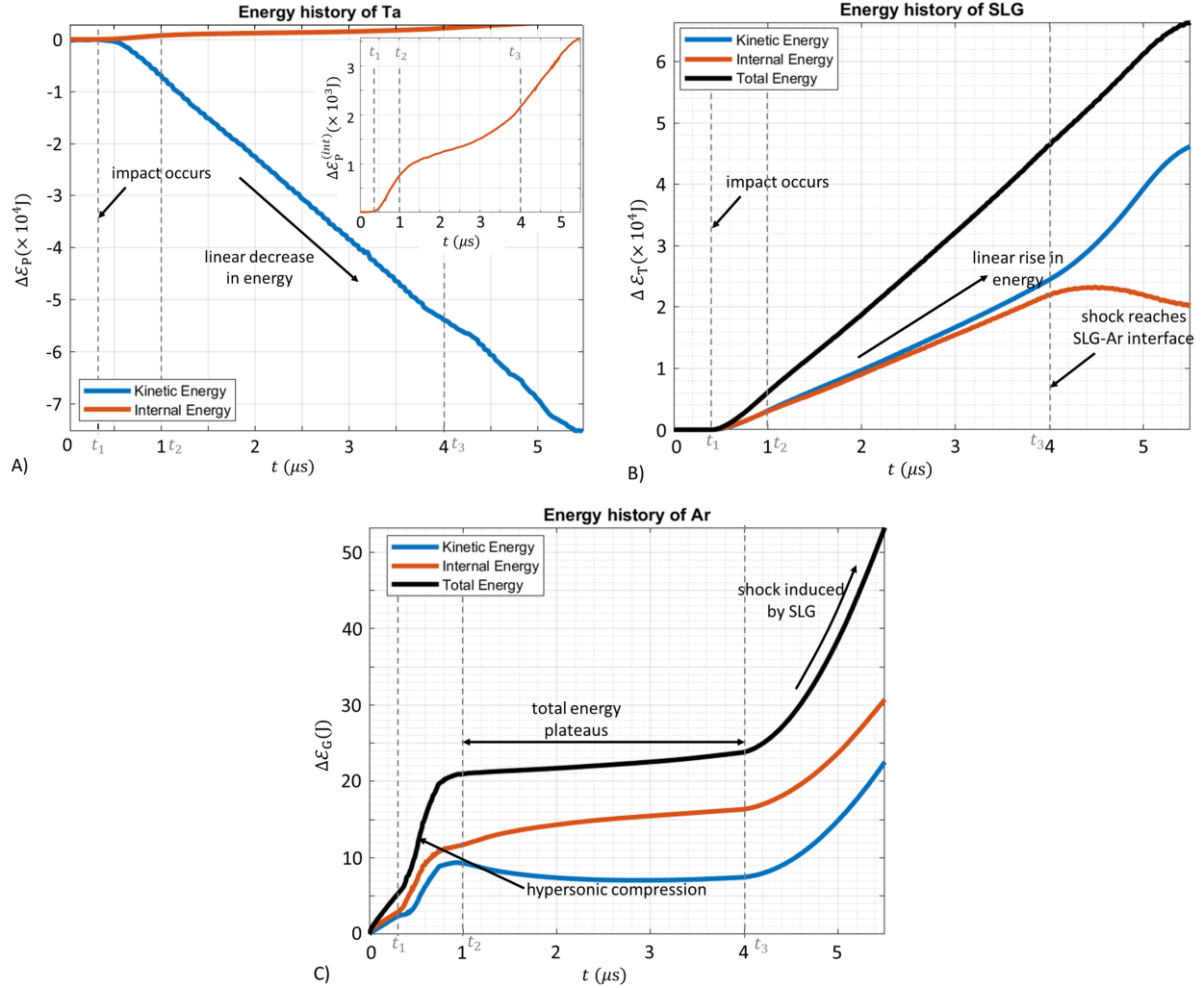


Figure 4.18: Evolution of kinetic, internal and total energy within each material domain

energy increase as shown in Fig. 4.18(A). At $t_3 = 4 \mu\text{s}$, the impact-induced shock wave reaches the target's rear surface. Soon after, the reflected shock wave sweeps through the projectile, causing its leading edge to decelerate and deform into a "mushroom head" shape. Around the edge of the mushroom head, the projectile's internal energy increases rapidly, which explains the upward trend of the internal energy curve (after t_3) in Fig. 4.18(A)).

Figure 4.18(B) shows that from the moment of impact (t_1), the target accumulates both kinetic and internal energy at nearly a constant rate. Moreover, the energy it receives from the projectile is

almost evenly split between kinetic and internal energy. This trend lasts until t_3 , when the two curves begin to diverge. The linear behavior observed between t_1 and t_3 can be explained by the fact that within this time interval, energy transfers in the target mainly through the impact-induced shock wave. This shock wave travels at a constant speed.

At time t_4 , the shock wave reaches the target's rear surface. This event sends a shock wave into the ambient gas behind the target and creates a rarefaction that reflects back within the target. The rarefaction reduces internal energy (and thus temperature), while accelerating the material, increasing its kinetic energy. This phenomenon is also evident in the 1D simulations, specifically, in Fig. 4.15 at $t = 40$ ns. The amount of energy transferred from the target into the gas is less than 1%. It does not make a noticeable difference in the plot of energy variation in the target.

Figure 4.18(C) shows the energy variations within the ambient argon gas. Compared to tantalum and SLG, argon has a much lower density. Therefore, the integrated kinetic, internal, and total energy are two to three orders of magnitude lower than their counterparts for the solid materials. Unlike the projectile and the target, the ambient gas gains energy mainly in two time intervals, namely between t_1 and t_2 and after t_4 . Within the first interval, the gas between the projectile and the target is violently compressed, which forms a high energy pocket of gas around the point of impact. Between t_2 and t_4 , the gas does not gain much energy from the solid materials, as the total energy remains nearly constant. This period features the conversion of kinetic energy to internal energy. In other words, the gas decelerates and heats itself up. At t_4 , a shock wave forms in the gas behind the target, which transfers energy from the target to the gas. Once again, both kinetic and internal energy in the gas start to increase.

While the integrated energy in the argon gas is significantly lower than its counterparts in the solids, the specific energy (i.e., energy per unit mass) follows the opposite trend. Figure 4.19 (A) depicts the specific energy at $t = 1.25$ μ s. The left and right halves of the figure show specific internal energy (e) and specific kinetic energy ($\|\mathbf{U}\|_2^2/2$), respectively. To provide a quantitative description, three

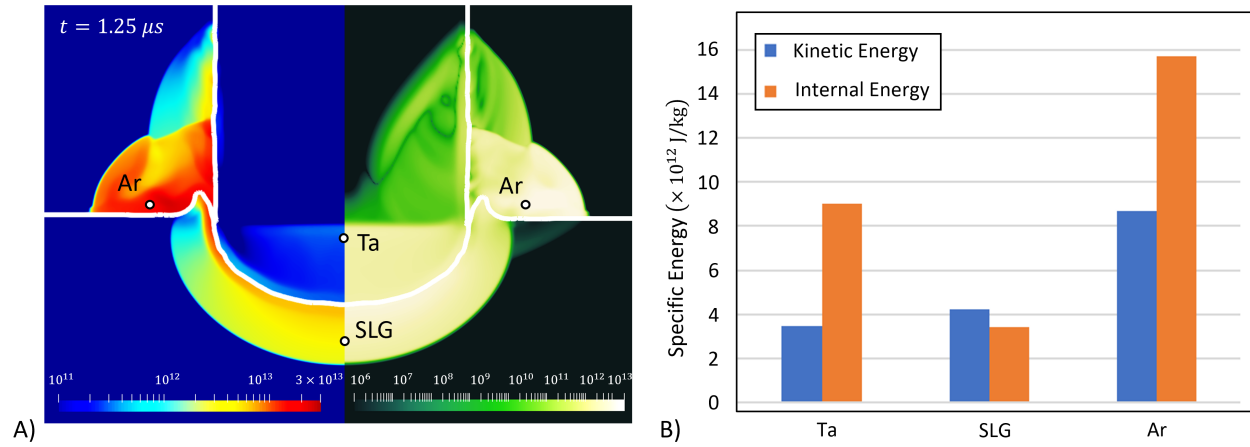


Figure 4.19: Comparison of specific energies at $t = 1.25 \mu s$. (A) Specific internal energy (left) and kinetic energy (right) at $1.25 \mu s$ (unit: J/kg), and the locations of three sensors. (B) Rise in specific energies in each material, measured at the sensor location.

virtual probes are placed within the domain, one in each material. Figure 4.19 (B) shows the energy variations at these sensor locations.

Clearly, the rise in specific internal and kinetic energy are both higher in argon than in the solid materials. In particular, the rise in specific internal energy in argon is about twice that of the tantalum projectile and four times that of the SLG target. Moreover, the specific heat of argon is of the order of 10^{-1} J/kg K, while the solids' range from 10^2 to 10^3 J/kg K. In combination, these observations imply that the temperature within the ambient gas will be much higher than that in the solid materials. Additionally, the extent of ionization may also be greater in the gas than in the solid materials. These aspects will be examined in the next subsection.

The energy results shown in Fig. 4.19 can also justify some of the modeling assumptions mentioned in Sec. 4.2.1 and Sec. 4.2.4. At the sensor within SLG, the specific internal energy exceeds 3×10^6 J/kg. The fracture toughness of SLG, estimated based on data presented in [43], is only 680 J/kg. In tantalum, the specific internal energy is around 9×10^6 J/kg, while its toughness is estimated to be less than or around 10^4 J/kg [56]. Although the toughness of both materials depends highly on loading conditions (e.g., strain rate, temperature, etc.), the disparity in order-of-

magnitude between the simulated internal energy and toughness supports the use of a fluid model for both materials. Furthermore, when modeling the ionization of SLG, our model also neglects the enthalpy of dissociation of the oxides in the material. The bond energy of SiO_2 is 1.24×10^6 J/mol, or 7.47×10^4 J/kg. It is much higher than the material's fracture toughness, but still two orders of magnitude lower than the increase in internal energy obtained from the simulation.

4.6.2 Impact-induced heating and plasma formation

Figure 4.20 shows the rise in temperature (T) and plasma density (n_e) in each material subdomain at the same time instants presented in Fig. 4.17. Since the thermodynamic states are symmetric about the central axis, the left half of each sub-figure represents the plasma density, whereas the right half shows the change in temperature during the impact event. Furthermore, since these states are orders of magnitude different in the solids as opposed to the ambient argon gas, they have been displayed using different scales. Specifically, the plasma density, n_e , spans several orders of magnitude and varies exponentially in space and time, thus a logarithmic scale has been adopted to highlight its variation.

At $t = t_2 = 1.0 \mu\text{s}$, the spherical leading edge of the projectile is fully in contact with the target. The impact-induced shock waves can be observed in all the three material subdomains. The highest temperatures in the solid materials are at the contact surface, specifically, 2400 K in the projectile and 3200 K in the target. At the same time, the peak temperature in the ambient gas is 1.2×10^5 K, that is, over 40 times higher. This high temperature gas ionizes and forms a dense pocket of plasma. The average charge number within this pocket of fluid ranges upwards of 5. Although the solid materials have lower temperatures, they also ionize within the region behind the shock front. Notably, their relatively low temperature and high density leads to the depression of ionization energy. Specifically, the ΔI_r term in Eq. (4.34) approaches the same order of magnitude as I_r . This

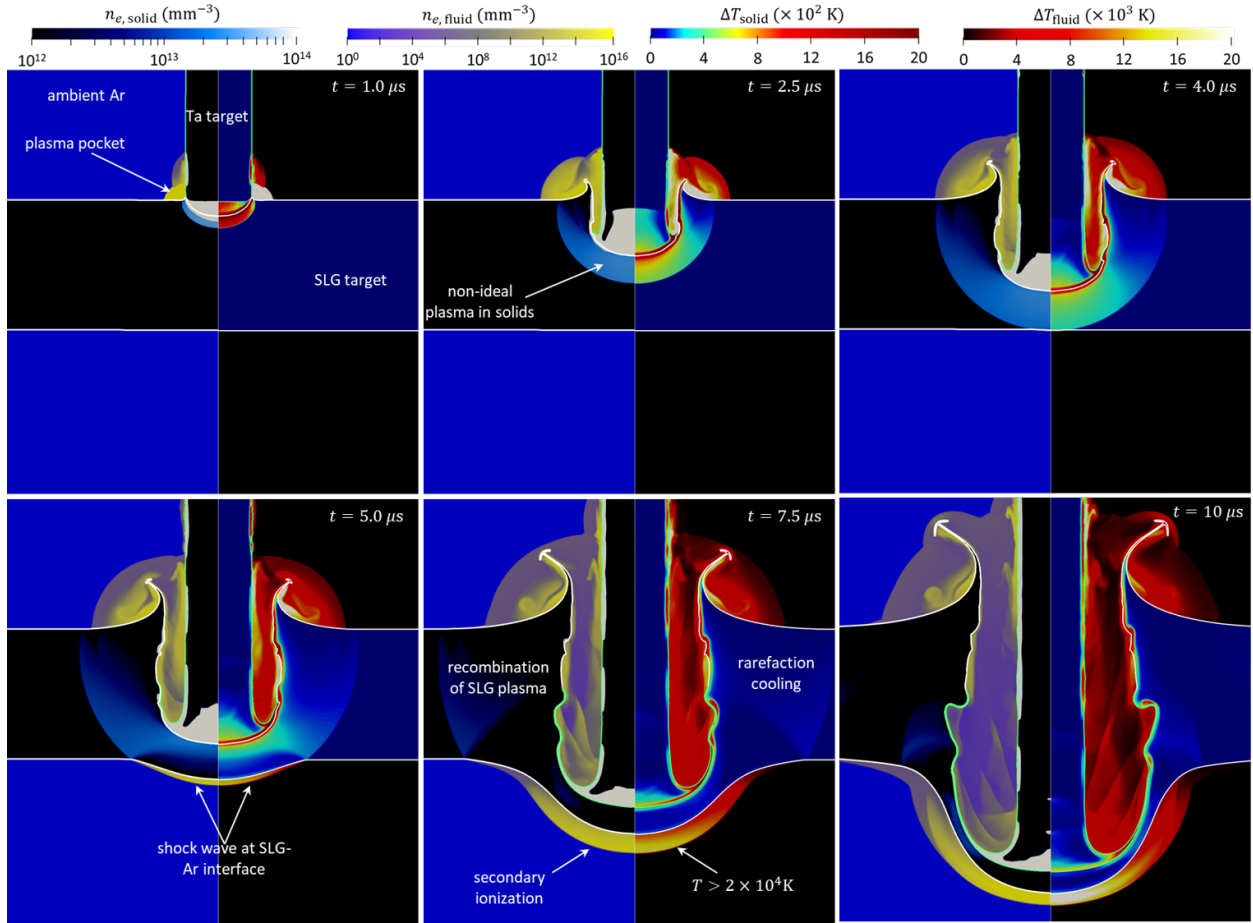


Figure 4.20: Evolution of temperature and plasma density (n_e) for impact velocity $U_0 = 5$ km/s

reduces the effective ionization energy, I_r^{eff} , and also indicates the formation of a non-ideal plasma.

At $t = 2.5 \mu\text{s}$, the impact has formed a crater in the target. While the total internal energy in the ambient gas is increasing, most of this increase stems from its own kinetic energy. The energy contribution from the projectile is minimal (cf. 4.18(C)). Therefore, as the shock wave in the gas expands, the gas temperature decreases rapidly. However, the projectile continues to transfer its kinetic energy to increase the internal energy of both itself and the target. As a result, behind the shock waves in the projectile and target, the temperature decrease rate is much lower than in the gas. Thus, the plasma in the ambient gas recombines quickly, whereas plasma density in the solids remains relatively constant.

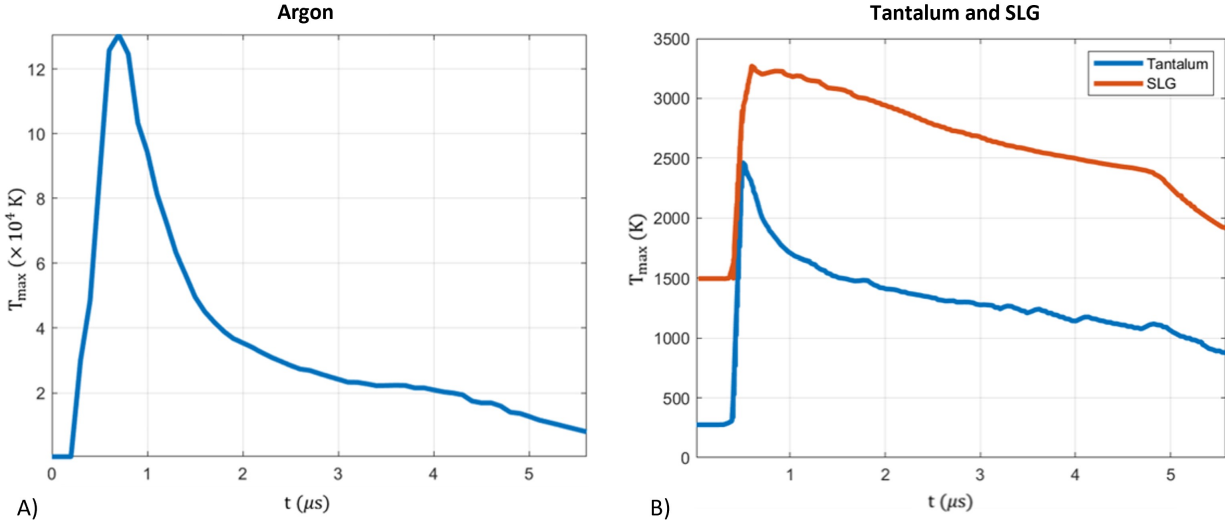


Figure 4.21: Maximum temperature behind the shock wave in each material subdomain

At $t = 5.0 \mu\text{s}$, the rarefaction fan within the SLG can be clearly seen. This rarefaction is the primary source of cooling within the SLG, as both the temperature and the plasma density start decreasing rapidly. The snapshots taken at $7.5 \mu\text{s}$ and $10 \mu\text{s}$ show the propagation of the second shock wave in the argon gas. This effect can also be seen in the increase in energy within the argon for $t > t_3$ in 4.18(C). This shock wave creates a second region of dense argon plasma that escapes at hypersonic velocities. By this time, the argon plasma represents the principal source of impact induced plasma, as the temperatures within the solids cannot sustain further ionization.

Figure 4.21 shows the time evolution of maximum temperature within the three materials. The maximum temperature in argon is one to two orders of magnitude higher compared to the solid materials. Notably, the temperature in argon begins to rise even before impact occurs (at $0.4 \mu\text{s}$). This is because the volume of gas between the projectile and the target is rapidly compressed and heated before impact occurs — by a projectile flying at Mach 15. The global (that is, temporal and spatial) maximum temperature in argon is observed after impact, at time $t = 0.6 \mu\text{s}$. Subsequently, temperature in the gas decreases rapidly as the shock waves expand in radial directions, thereby dispersing the internal energy. The behavior of tantalum and SLG is different. While their global

maximum temperatures are lower compared to the gas, the instantaneous peak temperature remains above half of the global maximum for approximately $5 \mu\text{s}$. This sustained high temperature is due to the continuous conversion of the projectile's kinetic energy into internal energy, deposited in both itself and the target as it penetrates. Furthermore, a sharp temperature decrease in the SLG after $5 \mu\text{s}$ is observed. This phenomenon is due to rarefaction cooling, as previously seen in Fig. 4.17. Over the impact velocities tested in this study, the dynamics of the impact is qualitatively similar. However, the thermodynamic states and plasma characteristics scaled with the impact velocity of the projectile.

The thermodynamic and plasma states behind the initial impact-induced shock waves are compared across the range of impact velocities. Table 4.4 presents the plasma density, temperature variation, and the shock propagation distance within SLG and argon at $t = 0.9 \mu\text{s}$, that is, $0.5 \mu\text{s}$ after impact. As expected, most of these variables scale strongly with the impact velocity. Notably, when impact velocity increases from 5 km/s to 6 km/s, the plasma density in SLG jumps by two orders of magnitude. This sudden jump may be attributed to the significant depression of ionization energies resulting from impact-induced material compression. It can also be observed that when impact velocity exceeds 3 km/s, the temperature within the argon plasma plume (T_{Ar}) does not increase consistently. This may reflect the fact that in the early stage of impact, transfer of energy to the surrounding gas is mainly through hypersonic compression, that is, compression of the gas between the projectile and target (cf. [31]). As impact velocity increases, the gas volume remains constant, and the compression duration decreases.

4.6.3 Ionization of SLG target

Figure 4.22(A) presents the plasma density (n_e) in the SLG target at three time instances, along with the plasma composition over time. Specifically, sub-figure (B) is generated as follows. At

U_0 (km/s)	$n_{e,\text{SLG}}$ (mm ³)	T_{SLG} (K)	$d_{\text{shock, SLG}}$ (mm)	$n_{e,\text{Ar}}$ (mm ³)	T_{Ar} (K)	$d_{\text{shock, Ar}}$ (mm)
7	3.9×10^{15}	3.8×10^3	4.5	3.2×10^{17}	1.4×10^5	6.76
6	2.0×10^{15}	3.6×10^3	4.0	2.6×10^{17}	2.0×10^5	6.8
5	4.0×10^{13}	2.8×10^3	3.7	1.8×10^{17}	1.5×10^5	6.0
4	3.4×10^{13}	2.4×10^3	3.2	2.2×10^{17}	1.6×10^5	4.0
3	2.1×10^{13}	1.9×10^3	2.7	1.3×10^{17}	6.6×10^4	2.0

Table 4.4: Plasma density (n_e), temperature (T), and shock propagation distance (d_{shock}) in SLG and argon, at time $t = 0.9 \mu\text{s}$ ($0.5 \mu\text{s}$ after impact)

each time step, after solving the non-ideal Saha equations at each node on the computational grid (see Fig. 4.4), we use Eq. 4.39 to compute $\bar{Z}_{e,j}$ for $j = 1$ (O), 2 (Si), 3 (Na), 4 (Ca), and 5 (Mg). The electron number density from each element j is given by

$$n_{e,j} = \sum_{r=1}^Z r n_{r,j} = \bar{Z}_{e,j} n_{\text{H}}. \quad (4.56)$$

Next, we find the maximum of $n_{e,j}$ along the central axis, and denote it by $n_{e,j}^{\text{max}}$. The time history of $n_{e,j}^{\text{max}}$ is plotted in Fig. 4.22(B), with $j = 1, \dots, 5$ for each element in SLG. Due to the considerable differences in elemental contributions, this time history is presented on a semi-log scale. Additionally, Fig. 4.23(A) shows $n_{e,j}^{\text{max}}$ at $t = 0.9 \mu\text{s}$ to highlight a typical composition of the impact generated SLG plasma at time. Meanwhile sub-figure(B) shows the molar composition of SLG to highlight the availability of elemental nuclei in the plasma.

The plasma in the SLG is predominantly composed of Na and Ca, despite their relatively low molar fractions of 9.26% and 5.48%, respectively. Being alkali and alkaline earth metals, Na and Ca have low first ionization energies (see Table 4.5), making them highly prone to ionization. This effect is exacerbated by the fact that the magnitude of the ΔI_r approaches the same order of magnitude for $I_{r,j}$ for these elements and has a more significant effect on the effective ionization energy (I_r^{eff}). Thus, the exponential term when computing the Saha coefficients (Eq. (4.33)) dominates the mag-

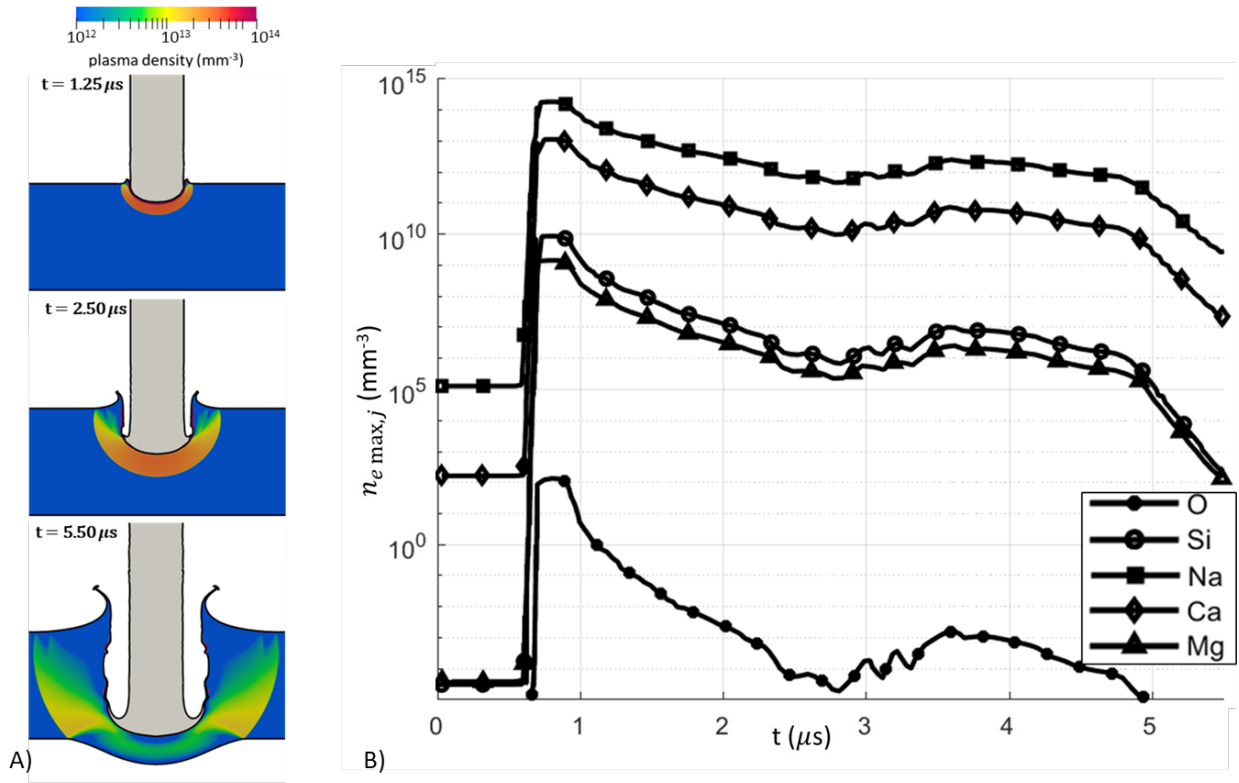


Figure 4.22: Composition of SLG plasma. (A) Plasma density within the SLG at different time instances (B) Time history of $n_{e,j}^{\max}$ along the central axis. (C) $n_{e,j}^{\max}$ at $t = 1 \mu s$ (D) Molar composition of each element in SLG.

nitide of the $\bar{Z}_{e,j}$, which explains the higher electronic contribution of these elements in the SLG plasma. This phenomenon is also realizable for Mg, which is another alkaline earth metal. But its contribution to the plasma is similar to that of Si, which is a group 4 metalloid. This is because Mg constitutes only 0.24% molar fraction of the SLG, whereas Si constitutes 24.89%. Notably, the significant ionization of alkali and alkaline earth metals was also reported by Ratcliff *et al.* [58]. In their experiments, these metals were observed only as a contaminant in their samples, and yet they become the major constituents of the impact-generated plasma. Lastly, oxygen (O) contributes minimally to the plasma, despite comprising 60.13% of SLG. As a non-metal, oxygen atoms' valence electrons are tightly bound to their nuclei, resulting in high ionization energies that hinder their contribution to the plasma.

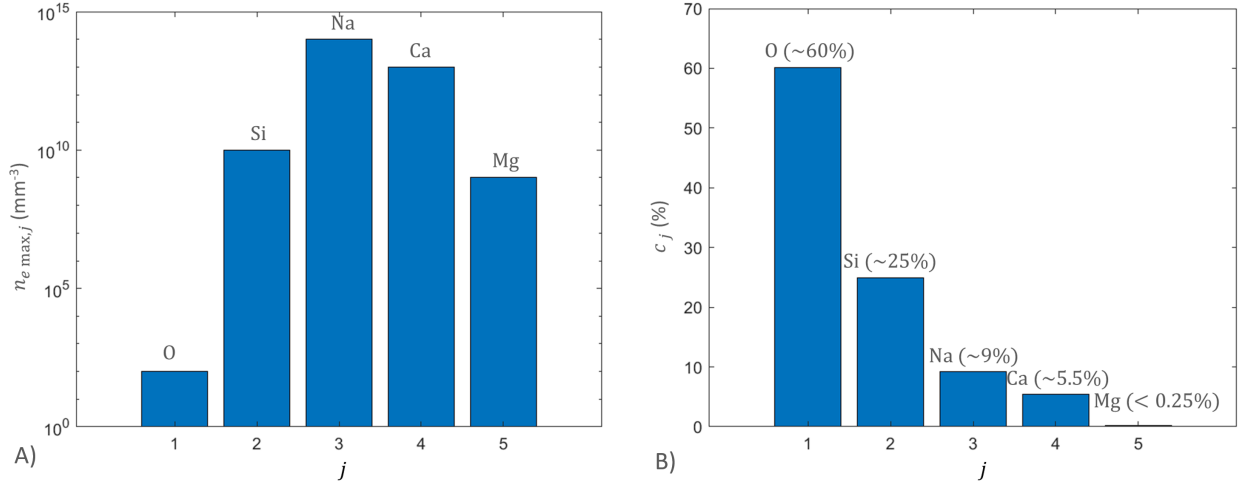


Figure 4.23: Electronic and molar composition of SLG at $t = 1 \mu s$. A) Electronic contribution of each element to impact-generated plasma on a semi log scale B) Molar composition.

4.7 Conclusion

We have developed a computational model of atmospheric hypervelocity impact, focusing on the distribution of energy in various forms across different materials and the properties of impact-induced ionization and the resulting plasma. The software implementation of this model is open-source. Our study centers on a model problem involving a tantalum rod impacting a soda-lime glass (SLG) target in an argon gas environment. Nonetheless, the computational model is broadly applicable to other geometries and material combinations. Generally speaking, this effort has two novelties compared to previous studies on hypervelocity impacts. First, we capture the phenomena of impact-induced ionization and plasma formation, alongside the mechanical and thermal responses. Second, we consider the presence of ambient gas, treating atmospheric hypervelocity impacts as a fluid-solid interaction problem.

Predicting impact-induced ionization necessitates accurate thermodynamic equations of state (EOS) for the participating materials under extreme pressure and temperature conditions. In this regard, relevant data and models for SLG are scarce in the literature. To address this gap, we conducted

an experimental campaign to measure SLG properties under high pressure and high temperature conditions. The obtained data were utilized to calibrate a Noble-Abel stiffened gas EOS. This EOS is utilized in our computational analysis. It can also be employed independently in other theoretical and computational studies.

Using the newly developed computational model, we have conducted simulations with impact velocities ranging from 3 km/s to 7 km/s. During the impact process, the energy transferred to the ambient gas is found to be a very small fraction — less than 1% — of the total energy involved in the collision. The pressure in the gas is also significantly lower than in the projectile and target. This confirms the validity of solid-only impact analyses that exclude the ambient gas, as long as the focus is on the responses of the projectile and target. However, the specific internal energy of the ambient gas is on the same order of magnitude as that of the projectile and target, primarily due to the low density of the gas. This finding will likely hold true for a broad range of material combinations. Additionally, the temperature of the argon gas is found to be higher than that of SLG and tantalum. This trend can be roughly explained by the specific heat capacities of the materials. Our simulations also indicate that the plasma density is higher in the argon gas compared to SLG and tantalum. This variation is influenced by both the energy supply and the ionization energies of each material. A detailed examination of the SLG plasma reveals highly selective ionization. The metallic atoms (Na, Ca, and Mg) contribute over 99.9% of the charged particles while comprising less than 15% of the molar composition. The influence of impact velocity on the thermodynamic and plasma states behind the initial impact-generated shock waves is also investigated. The plasma density within SLG jumps by two orders of magnitude when impact velocity exceeds 5 km/s. This may be due to the significant non-idealities in the SLG plasma at high impact velocities.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgment

S.T.I, A.N., and K.W. gratefully acknowledge the support of the U.S. Office of Naval Research (ONR) under award N00014-19-1-2102, the support of the U.S. Air Force Office of Scientific Research under contract No. FA9550-22-C-001, and the support of the National Science Foundation (NSF) under award CBET-1751487. J.G.M. gratefully acknowledges the support of ONR through U.S. Naval Research Laboratory's core funding. PDA gratefully acknowledges support from ONR under awards N00014-16-1-2751, N00014-17-1-2772, N00014-18-1-2712, and N00014-20-1-2603.

Bibliography

- [1] AERO-F source code, 2020. URL <https://bitbucket.org/frg/aero-f/src/master/>.
- [2] M2C source code, 2022. URL <https://github.com/kevinwgy/m2c>.
- [3] Tinkercliffs resources, 2024. URL <https://www.docs.arc.vt.edu/resources/compute/00tinkercliffs.html>.
- [4] GE Alefeld, Florian A Potra, and Yixun Shi. Algorithm 748: Enclosing zeros of continuous functions. *ACM Transactions on Mathematical Software (TOMS)*, 21(3):327–344, 1995.
- [5] C Scott Alexander, William D Reinhart, Tracy Vogler, Lalit C Chhabildas, TF Thornhill, and

- DE Grady. Status of glass experiments at Sandia national laboratories. Technical report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States); Sandia ..., 2006.
- [6] C Scott Alexander, LC Chhabildas, and DW Templeton. The Hugoniot elastic limit of soda-lime glass. In *AIP Conference Proceedings*, volume 955, pages 733–738. American Institute of Physics, 2007.
- [7] Paul D Asimow. Shock compression of preheated silicate liquids: Apparent universality of increasing grüneisen parameter upon compression. In *AIP Conference Proceedings*, volume 1426, pages 887–890. American Institute of Physics, 2012.
- [8] Paul D Asimow and Thomas J Ahrens. Shock compression of liquid silicates to 125 GPa: The anorthite-diopside join. *Journal of Geophysical Research: Solid Earth*, 115(B10), 2010.
- [9] Philip Avery, Daniel Z Huang, Wanli He, Johanna Ehlers, Armen Derkevorkian, and Charbel Farhat. A computationally tractable framework for nonlinear dynamic multiscale modeling of membrane woven fabrics. *International Journal for Numerical Methods in Engineering*, 122(10):2598–2625, 2021.
- [10] NK Bourne and Z Rosenberg. The dynamic response of soda-lime glass. In *AIP Conference Proceedings*, volume 370, pages 567–572. American Institute of Physics, 1996.
- [11] J Michael Brown, Michael D Furnish, and Robert G Mcqueen. Thermodynamics for (Mg, Fe)₂SiO₄ from the Hugoniot. *High-pressure research in mineral physics: A volume in honor of Syun-iti Akimoto*, 39:373–384, 1987.
- [12] Andrew Cachiaras, Luke Gilde, Jeffrey J Swab, Parimal J Patel, and George D Quinn. Soda-lime-silicate float glass: A property comparison. Technical report, US Army Research Laboratory Aberdeen Proving Ground United States, 2017.

- [13] Shunxiang Cao, Ying Zhang, Defei Liao, Pei Zhong, and Kevin G Wang. Shock-induced damage and dynamic fracture in cylindrical bodies submerged in liquid. *International journal of solids and structures*, 169:55–71, 2019.
- [14] Sigrid Close, Ivan Linscott, Nicolas Lee, Theresa Johnson, David Strauss, Ashish Goel, Alexander Fletcher, David Lauben, Ralf Srama, Anna Mocker, et al. Detection of electromagnetic pulses produced by hypervelocity micro particle impact plasmas. *Physics of Plasmas*, 20(9), 2013.
- [15] Werner Ebeling and Dietrich Kremp. *Theory of bound states and ionization equilibrium in plasmas and solids*, volume 5. Akademie-Verlag, 1976.
- [16] C. Farhat, K.G. Wang, A. Main, S. Kyriakides, L.-H. Lee, K. Ravi-Chandar, and T. Be-lytschko. Dynamic implosion of underwater cylindrical shells: Experiments and computations. *International Journal of Solids and Structures*, 50(19):2943–2961, 2013. doi: 10.1016/j.ijsolstr.2013.05.006.
- [17] Charbel Farhat, Jean-Frédéric Gerbeau, and Arthur Rallu. FIVER: A finite volume method based on exact two-phase Riemann problems and sparse grids for multi-material flows with large density jumps. *Journal of Computational Physics*, 231(19):6360–6379, 2012. doi: 10.1016/j.jcp.2012.05.026.
- [18] OV Fat’yanov and PD Asimow. Equation of state of Mo from shock compression experiments on preheated samples. *Journal of Applied Physics*, 121(11), 2017.
- [19] OV Fat’yanov and PD Asimow. Contributed review: Absolute spectral radiance calibration of fiber-optic shock-temperature pyrometers using a coiled-coil irradiance standard lamp. *Review of Scientific Instruments*, 86(10), 2015.

- [20] Alex Fletcher, Sigrid Close, and Donovan Mathias. Simulating plasma production from hypervelocity impacts. *Physics of Plasmas*, 22(9):093504, 2015. doi: 10.1063/1.4930281.
- [21] Vladimir Fortov, Igor Iakubov, and Alexey Khrapak. *Physics of strongly coupled plasma*, volume 135. OUP Oxford, 2006.
- [22] JF Friichtenicht, NL Roy, and DG Becker. The cosmic dust analyzer: experimental evaluation of an impact ionization model. In *International Astronomical Union Colloquium*, volume 13, pages 299–310. Cambridge University Press, 1971.
- [23] Joseph Fred Friichtenicht and John Charles Slattery. Ionization associated with hypervelocity impact. Technical report, NASA, 1963.
- [24] Vitro Architectural Glass. Starphire technical product data. Technical Report 1.855.887.6457 / 1.855.VTRO.GLS, Vitro Architectural Glass, Pittsburgh, PA, April 2020.
- [25] Dennis E Grady and LC Chhabildas. Shock-wave properties of soda-lime glass. Technical report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 1996.
- [26] E. P. Gyftopoulos and Gian Paolo. Beretta. *Thermodynamics: foundations and applications*. Dover Publications, 2005.
- [27] DG Hicks, TR Boehly, JH Eggert, JE Miller, PM Celliers, and GW Collins. Dissociation of liquid silica at high pressures and temperatures. *Physical Review Letters*, 97(2):025502, 2006.
- [28] K Hornung and J Kissel. On shock wave impact ionization of dust particles. *Astronomy and Astrophysics (ISSN 0004-6361)*, vol. 291, no. 1, p. 324-336, 291:324–336, 1994.
- [29] Daniel Z Huang, Philip Avery, Charbel Farhat, Jason Rabinovitch, Armen Derkevorkian, and Lee D Peterson. Modeling, simulation and validation of supersonic parachute inflation dynamics during mars landing. In *AIAA scitech 2020 forum*, page 0313, 2020.

- [30] Shafquat Islam, Aditya Narkhede, and Kevin Wang. Investigation of energy transport during hypervelocity impacts in an ambient fluid. In *AIAA SCITECH 2024 Forum*, page 0936, 2024.
- [31] Shafquat T Islam, Wentao Ma, John G Michopoulos, and Kevin Wang. Plasma formation in ambient fluid from hypervelocity impacts. *Extreme Mechanics Letters*, 58:101927, 2023.
- [32] Shafquat T Islam, Wentao Ma, John G Michopoulos, and Kevin Wang. Fluid-solid coupled simulation of hypervelocity impact and plasma formation. *International Journal of Impact Engineering*, page 104695, 2023.
- [33] Ian Jackson and Thomas J Ahrens. Shock wave compression of single-crystal forsterite. *Journal of Geophysical Research: Solid Earth*, 84(B6):3039–3048, 1979.
- [34] Peter Jutzi and Ulrich Schubert. *Silicon chemistry*. Wiley Online Library, 2007.
- [35] KW Katahara, MH Manghnani, and ES Fisher. Pressure derivatives of the elastic moduli of BCC Ti-V-Cr, Nb-Mo and Ta-W alloys. *Journal of Physics F: Metal Physics*, 9(5):773, 1979.
- [36] T Kobayashi, T Sekine, OV Fat’yanov, E Takazawa, and QY Zhu. Radiation temperatures of soda-lime glass in its shock-compressed liquid state. *Journal of applied physics*, 83(3): 1711–1716, 1998.
- [37] FR Krueger. Thermodynamics of ion formation by fast dissipation of energy at solid surfaces. *Zeitschrift für Naturforschung A*, 38(4):385–394, 1983.
- [38] Andrea La Spina and Jacob Fish. A hybridizable discontinuous Galerkin formulation for the Euler–Maxwell plasma model. *Journal of Computational Physics*, 496:112535, 2024.
- [39] Andrea La Spina, Alex C Fletcher, and Jacob Fish. Semi-analytic model for plasma production and Cherenkov radiation emission from hypervelocity impacts on soda–lime glass. *Physics of Plasmas*, 30(1), 2023.

- [40] Olivier Le Métayer and Richard Saurel. The Noble-Abel stiffened-gas equation of state. *Physics of Fluids*, 28(4):046102, 2016.
- [41] Nicolas Lee, Sigrid Close, Ashish Goel, David Lauben, Ivan Linscott, Theresa Johnson, David Strauss, Sebastian Bugiel, Anna Mockler, Ralf Srama, and et al. Theory and experiments characterizing hypervelocity impact plasmas on biased spacecraft materials. *Physics of Plasmas*, 20(3):032901, 2013. doi: 10.1063/1.4794331.
- [42] Sung Keun Lee, Kwan Young Mun, Yong-Hyun Kim, Juho Lhee, Takuo Okuchi, and Jung-Fu Lin. Degree of permanent densification in oxide glasses upon extreme compression up to 24 Gpa at room temperature. *The Journal of Physical Chemistry Letters*, 11(8):2917–2924, 2020.
- [43] WD Liu, LC Zhang, and K Mylvaganam. Relaxation oscillation of borosilicate glasses in supercooled liquid region. *Scientific reports*, 7(1):15872, 2017.
- [44] Wentao Ma, Xuning Zhao, Christine Gilbert, and Kevin Wang. Computational analysis of bubble–structure interactions in near-field underwater explosion. *International Journal of Solids and Structures*, 242:111527, 2022. ISSN 0020-7683. doi: <https://doi.org/10.1016/j.ijsolstr.2022.111527>.
- [45] Wentao Ma, Xuning Zhao, Christine Gilbert, and Kevin Wang. Computational analysis of bubble–structure interactions in near-field underwater explosion. *International Journal of Solids and Structures*, 242:111527, 2022.
- [46] Wentao Ma, Xuning Zhao, Shafquat Islam, Aditya Narkhede, and Kevin Wang. Efficient solution of bimaterial Riemann problems for compressible multi-material flow simulations. *Journal of Computational Physics*, 493:112474, 2023. ISSN 0021-9991. doi: <https://doi.org/10.1016/j.jcp.2023.112474>.

- [47] Alex Main, Xianyi Zeng, Philip Avery, and Charbel Farhat. An enhanced FIVER method for multi-material flow problems with second-order convergence rate. *Journal of Computational Physics*, 329:141–172, 2017.
- [48] Alex Main, Xianyi Zeng, Philip Avery, and Charbel Farhat. An enhanced FIVER method for multi-material flow problems with second-order convergence rate. *Journal of Computational Physics*, 329:141–172, 2017. doi: 10.1016/j.jcp.2016.10.028.
- [49] William Martin, Jeffrey Fuhr, Daniel Kelleher, Arlene Musgrove, J Sugar, Wolfgang Wiese, Peter Mohr, and Karen Olsen. NIST atomic spectra database (version 2.0) (1999), 1999.
- [50] Neil McBride and JAM McDonnell. Meteoroid impacts on spacecraft:: sporadics, streams, and the 1999 Leonids. *Planetary and Space Science*, 47(8-9):1005–1013, 1999.
- [51] AC Mitchell and WJ Nellis. Shock compression of aluminum, copper, and tantalum. *Journal of Applied Physics*, 52(5):3363–3374, 1981.
- [52] Karina A Monroe. Time-resolved fractoluminescence characterization in soda-lime glass via near hypervelocity kinetic impact fast fracture. Master’s thesis, Monterey, CA; Naval Post-graduate School, 2021.
- [53] Jed L Mosenfelder, Paul D Asimow, and Thomas J Ahrens. Thermodynamic properties of Mg_2SiO_4 liquid at ultra-high pressures from shock measurements to 200 GPa on forsterite and wadsleyite. *Journal of Geophysical Research: Solid Earth*, 112(B6), 2007.
- [54] Jeffrey H Nguyen, Minta C Akin, Ricky Chau, Dayne E Fratanduono, W Patrick Ambrose, Oleg V Fat’yanov, Paul D Asimow, and Neil C Holmes. Molybdenum sound velocity and shear modulus softening under shock compression. *Physical Review B*, 89(17):174109, 2014.
- [55] SS Prasanna Kumar, K Ramamurthi, and BSV Patnaik. Numerical study of a foam-shock trap based blast mitigation strategy. *Physics of Fluids*, 30(8), 2018.

- [56] Michael B Prime, Athanasios Arsenlis, Ryan A Austin, Nathan R Barton, Corbett C Battaile, Justin L Brown, Leonid Burakovsky, William T Buttler, Shuh-Rong Chen, Dana M Dattelbaum, et al. A broad study of tantalum strength from ambient to extreme conditions. *Acta Materialia*, 231:117875, 2022.
- [57] Matei Ioan Radulescu. Compressible flow in a Noble–Abel stiffened gas fluid. *Physics of Fluids*, 32(5):056101, 2020.
- [58] Paul R Ratcliff, Mark J Burchell, Mike J Cole, Tom W Murphy, and Firooz Alladfadi. Experimental measurements of hypervelocity impact plasma yield and energetics. *International journal of impact engineering*, 20(6-10):663–674, 1997.
- [59] Prithachakaran Renganathan, Thomas S Duffy, and Yogendra M Gupta. Hugoniot states and optical response of soda lime glass shock compressed to 120 gpa. *Journal of Applied Physics*, 127(20), 2020.
- [60] Allen C Robinson. The Mie-Gruneisen power equation of state. Technical report, Sandia National Lab.(SNL-NM), Albuquerque, NM (United States), 2019.
- [61] Alan Russell and Kok Loong Lee. *Structure-property relations in nonferrous metals*. John Wiley & Sons, 2005.
- [62] IV Savchenko and SV Stankus. Thermal conductivity and thermal diffusivity of tantalum in the temperature range from 293 to 1800 k. *Thermophysics and Aeromechanics*, 15(4):679–682, 2008.
- [63] Weidong Song, Yangtao Lv, Jianqiao Li, Cheng Wang, and Jianguo Ning. Influence of impact conditions on plasma generation during hypervelocity impact by aluminum projectile. *Physics of Plasmas*, 23(7), 2016.

- [64] Lars Stixrude, Nico de Koker, Ni Sun, Mainak Mookherjee, and Bijaya B Karki. Thermodynamics of silicate liquids in the deep earth. *Earth and Planetary Science Letters*, 278(3-4): 226–232, 2009.
- [65] K. G. Wang, P. Lea, and C. Farhat. A computational framework for the simulation of high-speed multi-material fluid-structure interaction problems with dynamic fracture. *International Journal for Numerical Methods in Engineering*, 104(7):585–623, 2015. doi: 10.1002/nme.4873.
- [66] Kevin G. Wang. Multiphase fluid-solid coupled analysis of shock-bubble-stone interaction in shockwave lithotripsy. *International Journal for Numerical Methods in Biomedical Engineering*, 33(10), 2017. doi: 10.1002/cnm.2855.
- [67] GM Ward and DI Pullin. A study of planar Richtmyer-Meshkov instability in fluids with Mie-Grüneisen equations of state. *Physics of Fluids*, 23(7), 2011.
- [68] Mofreh R. Zaghloul. Reduced formulation and efficient algorithm for the determination of equilibrium composition and partition functions of ideal and nonideal complex plasma mixtures. *Physical Review E*, 69(2), 2004. doi: 10.1103/physreve.69.026702.
- [69] Xuning Zhao, Wentao Ma, Junqin Chen, Gaoming Xiang, Pei Zhong, and Kevin Wang. Long-pulse laser-induced cavitation: A race between advection and phase transition. *Bulletin of the American Physical Society*, 2023.
- [70] Xuning Zhao, Wentao Ma, and Kevin Wang. Simulating laser-fluid coupling and laser-induced cavitation using embedded boundary and level set methods. *Journal of Computational Physics*, 472:111656, 2023.
- [71] Yacong Brooke Zhou, William A Goddard, and Paul D Asimow. An internal energy-

dependent model for the grüneisen parameter of silicate liquids. *Geochimica et Cosmochimica Acta*, 316:59–68, 2022.

[72] Jonas A. Zukas. *High velocity impact dynamics*. Wiley, 1990.

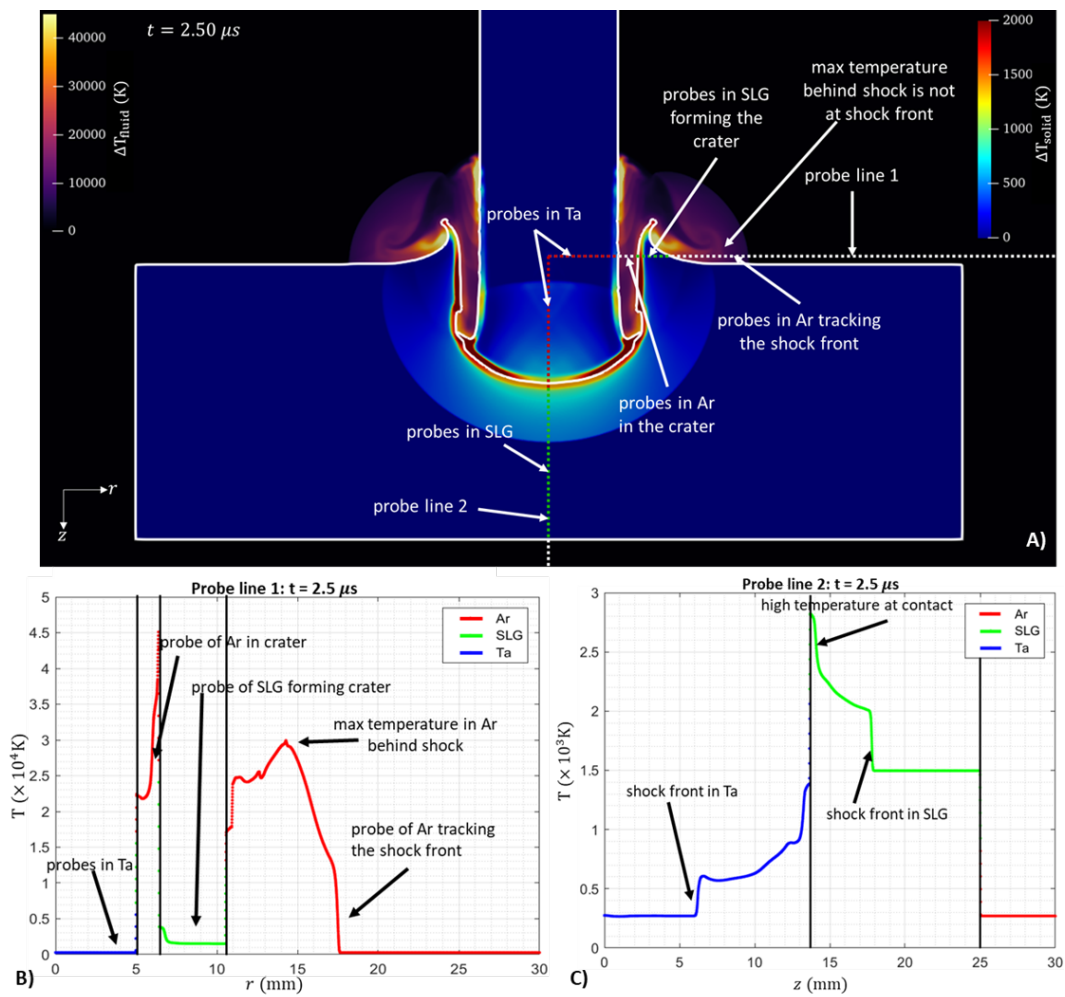
Appendix A: Sample spectroscopic data

Element	O			Si			Na			Ca			Mg			
j	1			2			3			4			5			
c_j	0.6013			0.2489			0.0926			0.0548			0.0024			
r	n	$I_{r,j}$ (eV)	$E_{r,n,j}$ (eV)	$g_{r,n,j}$	$I_{r,j}$ (eV)	$E_{r,n,j}$ (eV)	$g_{r,n,j}$	$I_{r,j}$ (eV)	$E_{r,n,j}$ (eV)	$g_{r,n,j}$	$I_{r,j}$ (eV)	$E_{r,n,j}$ (eV)	$g_{r,n,j}$	$I_{r,j}$ (eV)	$E_{r,n,j}$ (eV)	$g_{r,n,j}$
0	1	13.62	0.000	5	8.15	0.000	1	5.14	0.000	2	6.113	0.000	1	7.65	0.000	1
	2		0.020	3		0.010	3		2.102	2		1.879	1		2.709	1
	3		0.028	1		0.028	5		2.104	4		1.886	3		2.712	3
	4		1.967	5		0.781	5		3.191	2		1.899	5		2.717	5
	5		1.967	1		1.909	1		3.617	6		2.252	3		4.346	3
1	1	35.12	0.000	4	16.35	0.000	2	47.29	0.000	1	11.87	0.000	2	15.04	0.000	2
	2		3.324	6		0.036	4		32.846	5		1.692	4		4.422	2
	3		3.327	4		5.310	2		32.941	3		1.700	6		4.434	4
	4		5.017	4		5.323	4		33.015	1		3.123	2		8.655	2
	5		5.018	2		5.345	6		33.322	3		3.151	4		8.864	6
2	1	54.94	0.000	1	33.49	0.000	1	71.62	0.000	4	50.91	0.000	1	80.14	0.000	1
	2		0.014	3		6.537	1		0.169	2		25.215	1		52.777	5
	3		0.038	5		6.553	3		32.788	2		25.274	3		52.925	3
	4		2.514	1		10.276	5		45.397	6		25.397	5		53.047	1
	5		7.479	5		15.153	3		45.507	4		26.323	9		53.503	3
3	1	77.44	0.000	2	45.14	0.000	2	98.94	0.000	5	67.27	0.000	4	109.3	0.000	4
	2		0.048	4		8.839	2		0.137	3		0.387	2		0.276	2
	3		8.857	2		8.896	4		0.195	1		18.900	2		38.625	2
	4		8.874	4		19.884	6		3.824	5		24.983	8		67.413	6
	5		8.897	6		19.884	4		8.244	1		25.013	6		67.588	4
4	1	113.9	0.000	1	166.8	0.000	1	138.4	0.000	4	84.34	0.000	5	141.3	0.000	5
	2		10.160	1		103.901	5		5.994	6		0.298	3		0.221	3
	3		10.176	3		104.220	3		5.998	4		0.406	1		0.313	1
	4		10.214	5		104.527	1		9.080	2		2.335	5		4.454	5
	5		19.688	3		105.202	3		9.084	4		5.435	1		9.581	1

Table 4.5: Sample spectroscopic and ionization data for SLG [49]

Appendix B: Calculation of maximum temperature in simulations

To quantify the temperature variations within different material subdomains, we extract the time history of temperature along two probe lines in the axial and radial directions, respectively. Figure 4.24(A) shows their locations. While the material subdomains deform in time, the two probe lines are fixed. Their locations are chosen such that at any time during the impact, they run through the high temperature regions in all the three material subdomains. For example, Figure 4.24(B) and (C) show the temperature variations along the two probe lines at time $t = 2.5 \mu s$. In each sub-figure, the vertical black lines mark the boundaries of material subdomains. The maximum temperature within each subdomain is then extracted, and used to create Fig. 4.21 in this paper.

Figure 4.24: Temperature probe lines at $t = 2.5 \mu\text{s}$

Chapter 5

Conclusion

5.1 Concluding Remarks

This dissertation presents a comprehensive model for hypervelocity impact and the resulting formation of plasma. Much of the existing literature assumes such impacts occur in a vacuum—a reasonable simplification for exoatmospheric conditions—but this assumption overlooks the role of the surrounding environment in many terrestrial and atmospheric applications. The model developed here accounts for the dynamics of the projectile, target, and ambient gas, resolving their coupled fluid-solid interaction under extreme loading conditions. Hypervelocity impacts are characterized by rapid deformation, strong discontinuities in thermodynamic variables, and shock-induced energy deposition, which can drive phase changes including ionization and plasma formation. The model is applied to scenarios involving metallic projectiles striking soda-lime glass (SLG) targets in a chemically inert gas at impact velocities ranging from 3 to 7 km/s.

The thermodynamic response of SLG under such extreme loading was not previously well characterized, especially in terms of its Hugoniot temperature. Due to the scarcity of reliable experimental data, the Stiffened Gas equation of state was initially employed. To verify this model, a framework was developed to compute the analytical solution of a one-dimensional multi-material shock-shock Riemann problem. These results showed excellent agreement, as demonstrated in Fig. 3.6.

To overcome limitations in available data, a series of shock compression experiments was conducted

at the Lindhurst Laboratory at Caltech. These experiments provided pressure and temperature data for SLG under shock loading up to 110 GPa—conditions relevant to impact velocities studied here. Based on these results, a temperature law was developed by extending the Noble-Abel stiffened gas equation of state and incorporating Maxwell relations to represent temperature behavior along the Hugoniot. The complete equation of state was then calibrated using a constrained optimization approach to fit the experimental data. This calibrated model was validated in one-dimensional simulations, showing good agreement with experimental Hugoniot data, with less than 5% relative error (see Fig. 4.16).

A major contribution of this work is the model's capability to predict impact-induced plasma formation. This is achieved by coupling the generalized Saha equation with the constraints of charge neutrality and conservation of nuclei. The model can predict fundamental plasma properties such as plasma density, average ionization state, and composition. The ionization solver was first verified by computing the composition of a pure, ideal argon plasma over a temperature range of 0 to 5×10^4 K at atmospheric pressure, and compared against benchmark results published by Zaghloul *et al.* (see Fig. 3.5) [4]. It was then applied to a more complex non-ideal plasma mixture consisting of helium, neon, and argon at a fixed temperature of 5 eV ($\sim 58,000$ K), over a wide density range from 10^{-6} to 10^2 kg/m³. These results also showed excellent agreement with published solutions [3] (see Fig. 4.5). The close agreement in both test cases confirms the model's validity across a broad range of temperatures, pressures, and plasma compositions.

The fully integrated model was then applied to simulate three-dimensional impacts involving long tantalum rods striking cylindrical SLG targets in an argon atmosphere. While the specific material configuration is analyzed in this study, the underlying physical trends are expected to hold more broadly. Energy analysis showed that less than 0.1% of the total impact energy is transferred to the ambient gas, supporting the common simplification of neglecting the gas in structural response studies. However, due to its low density, the gas reaches much higher specific internal energy

and temperature compared to the solids. As a result, the ambient gas undergoes significantly more ionization, with plasma formation occurring at lower impact velocity thresholds. At velocities above 4 km/s, ionization in the gas is substantial, while ionization in the solids remains several orders of magnitude lower.

The model also captures the composition of plasma generated in SLG. It reveals that the dominant contributors to the charge population are metallic atoms such as sodium, calcium, and magnesium, which together account for over 99% of the free electrons despite comprising less than 15% of the material by molar composition, as is shown in Fig. 4.23. This is attributed to their lower first ionization energies, which make them more susceptible to ionization under impact conditions.

Overall, this work advances the understanding of plasma formation during hypervelocity impacts, particularly in the presence of an ambient gas. It introduces a robust computational toolset and experimental foundation that can be extended in future work to incorporate electromagnetic effects and investigate electrodynamic wave propagation following impact-induced ionization events.

5.2 Perspectives for future work

Although the work presented in this dissertation significantly advances the current body of literature, it also serves as a starting point for many promising research directions. While the qualitative trends identified in this study are expected to apply to a broad range of material combinations, geometries, and impact conditions, future work could explore a wider design space through systematic parametric studies and establish a correlation matrix between key variables.

The current model incorporates several assumptions that, while justifiable, may influence the accuracy of predictions—especially in the ionization model. For example, the chemical reaction pathways and fracture mechanics that may dissipate energy during the impact process are currently

neglected. These phenomena generally involve energy scales much lower than ionization energies, but incorporating them could improve model fidelity. In addition, most non-ideal ionization models assume that the depression in ionization potential remains small compared to the element's first ionization energy. However, this is not always valid in practice, and large depressions may lead to numerical instability. Developing alternative ionization models that remain well-posed under strongly non-ideal conditions would be valuable. At present, the selection of an ionization model can significantly affect simulation outcomes. Experimental data on plasma composition and energy density in the early stages of impact would help constrain these models, but such measurements are currently scarce—representing another important area for future research.

The present solver also assumes that hydrodynamic forces dominate over electromagnetic forces within the plasma. This is likely valid during the early stages of impact, when the plasma is highly collisional and non-Maxwellian. However, as the plasma expands and transitions toward a more ideal, collisionless regime, electromagnetic forces become more important. A relative measure of how collisional the plasma is can be quantified by computing the ratio of the plasma frequency (ω_p) and the Coulomb collision frequency (ν) of the plasma.

The plasma frequency can be computed as [1]

$$\omega_p = \sqrt{\frac{n_e e^2}{\epsilon_0 m_e}} \quad (5.1)$$

The collision frequency can be computed from the empirical model as shown in the NRL Plasma Formulary [2]:

$$\nu = \frac{3.38 \times 10^{-2} \Lambda}{T^{1.5}} \quad (5.2)$$

Here, Λ is the Coulomb logarithm, and can be computed as:

$$\Lambda = \ln \left(1 + \frac{\lambda_D}{b_{\min}} \right) \quad (5.3)$$

$b_{\min}$ is the smallest distance at which two charged particles can interact before classical effects dominate and invalidate the basic assumptions of Coulomb scattering. The classical limit comes from equating the Coulomb potential energy to the thermal kinetic energy of an electron in the plasma [1], and can be expressed as:

$$b_{\min} = \frac{Z_{\text{av}} e^2}{4\pi\epsilon_0 k_B T} \quad (5.4)$$

It is evident from these equations that both the plasma frequency and the Coulomb collision frequency depend strongly on temperature, plasma density, and the average ionization state—quantities that can differ by orders of magnitude between the solids and the surrounding fluid. For instance, at 1 μs after the hypervelocity impact of a tantalum projectile on a soda-lime glass target at 5 km/s in argon gas, the computed ratio ω_p/ν behind the shock front in each material is:

	Ar	SLG	Ta
ω_p/ν	6.59	0.47	0.21

As can be seen, at this stage in the plasma evolution during the impact process, the plasma within the solids remains highly collisional, while the fluid plasma may have entered a regime where the collisional assumption no longer holds. To address this, the current solver could be extended to incorporate Maxwell's equations—or more comprehensively, to solve the full Vlasov–Maxwell system. This approach would enable simulations within control volumes that extend beyond the collisional regime, thereby allowing for detailed modeling of electromagnetic wave generation and propagation from the impact-induced plasma.

Ultimately, a more holistic understanding of hypervelocity impacts and plasma formation could support the development of new diagnostic tools. Such tools may be able to infer impact conditions by observing plasma expansion rates or identifying material composition via spectroscopic signatures. These diagnostics could aid in forensic analysis of impact events or serve as onboard tools for spacecraft and planetary probes operating in high-risk environments.

Bibliography

- [1] Francis F. Chen. *Introduction to plasma physics and Controlled Fusion*. Springer, 2018.
- [2] Joseph Donald Huba. *NRL plasma formulary*, volume 6790. Naval Research Laboratory, 1998.
- [3] Mofreh R. Zaghloul. Reduced formulation and efficient algorithm for the determination of equilibrium composition and partition functions of ideal and nonideal complex plasma mixtures. *Physical Review E*, 69(2), 2004. doi: 10.1103/physreve.69.026702.
- [4] Mofreh R Zaghloul, Mohamed A Bourham, and J Michael Doster. A simple formulation and solution strategy of the saha equation for ideal and nonideal plasmas. *Journal of Physics D: Applied Physics*, 33(8):977–984, 2000. doi: 10.1088/0022-3727/33/8/314.