

Piezohydraulic Actuator Design and Modeling Using a Lumped-Parameter Approach

by

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Thesis submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Master of Science

in

Mechanical Engineering

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December 2002

Blacksburg, Virginia

Keywords: Piezohydraulic Actuator with Active Valves, Lumped-parameter Fluid Model,
State-space Model, Piezohydraulic Actuator Design Concepts.

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Virginia Polytechnic Institute and State University, 2002

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ABSTRACT

The concept of piezohydraulic actuation is to transfer the reciprocal small stroke displacement of piezoceramics into unidirectional motion by frequency rectification through a hydraulic fluid. It takes advantage of the high force capabilities that piezoelectric materials have and couples it with very stiff media such as hydraulic fluid to amplify and create this unidirectional motion. Inlet and outlet valves are connected to a pumping chamber where pressure is built by the displacement of the piezoelectric material and released by the opening of the outlet valve, thus achieving a variable flow rate that is used to push a hydraulic cylinder. Loads may be connected to this hydraulic cylinder for measuring/achieving mechanical power.

As part of this research, a benchtop piezohydraulic actuator with active piezohydraulic valves has been developed and the concept of piezohydraulic actuation has been demonstrated. Displacement of a hydraulic cylinder by driving a piezoelectric stack has been achieved while the cylinder was loaded or unloaded. Lumped-parameter state-space models have been developed in order to simulate the dynamics of the active valves and entire actuator system. The model simulates the chamber pressure, displacement of the hydraulic cylinder, and power of the piezohydraulic unit. A four-stage cycle simulation was used to model the pumping operation and dynamic response of the system.

Experimental results demonstrate the importance of fluid compressibility, valve timing, and fluid circuit components in the optimization of the output power of the actuation system. An array of different timing tests run on the inlet and outlet valves shows that their timing is crucial to the performance of the system. Also shown is that the optimal timing

conditions change slightly while under different loads. When operating at higher frequencies (above 140 Hz), it is shown that the hydraulic fluid circuit does not respond quickly enough for the piston to fully extend against the fluid and loaded cylinder. There is not sufficient time when operating at higher frequencies to push all the fluid from the chamber into the hydraulic cylinder, operation is too fast for the dynamics of the fluid circuit.

The four stage lumped-parameter model achieves good approximations of the experimental results when the load inertia was neglected while operating at frequencies below 120 Hz and under loads at or below 12.825 kg. Memory limitations caused the number of elements included in the lumped-parameter model to be limited, and are believed to be the source of the errors for the higher operation frequencies and loads. The model never converged due to the lack of elements, and the simulated system did not respond quickly enough to accurately model the fluid exiting the chamber. When operating at frequencies above the 120 Hz value, this error in modeling the fluid exiting the valves becomes very important. The simulation predicts higher values than the experiment and fails to correlate to the actual results at the higher frequencies and while under the higher loads. The errors at higher loads may also be attributed to the neglected inertia.

The most recent tests on the benchtop set-up were all run with a pre-pressure value of 190 psi, a piston duty cycle of 50%, valve duty cycles of 40% for each, and a 5% outlet valve offset. Slightly better operation performance might be achieved at frequencies higher than 140 Hz by increasing the piston duty cycle and varying the valve parameters. Also, increasing the pre-pressure of the fluid may help by stiffening the system to create a faster response, however this will have an adverse effect also by creating more force against piston motion. Lastly, the hydraulic cylinder was built for high pressures and had considerable friction associated with it. Obtaining a different cylinder with less friction may also help the response time of the fluid circuit.

Acknowledgments

First, I would like to thank my advisor, Dr. Donald J. Leo. His friendship, patience, and support have helped make my time here at Virginia Tech a very rewarding working and learning experience. Also, I would like to extend my thanks to Dr. Daniel J. Inman and Dr. William R. Saunders for their support as members of my advisory committee.

In addition, I want to thank my colleagues in the Center for Intelligent Material Systems and Structures (CIMSS), especially my research partners Nikola Vujic and most notably Honghui Tan. Their helpful ideas and friendship were invaluable toward assisting my research. I also appreciate the support of Oak Ridge National Laboratory who funded this work through contract number 4000007002, under subcontract from the DARPA (Defense Advanced Research Projects Agency).

Finally, I would like to thank my family and friends for their continued support during my years at Virginia Tech. Special thanks go to my parents Lark and Stephen for always supporting me and helping me accomplish my goals; my sisters Bernadette, Rosalie, and Victoria for their understanding, friendship and caring; and lastly my very thoughtful and giving grandparents Loraine, Edwin, Jane, and Donald. A special thanks also goes out to my best friend Verónica for her love, care, support, and friendship during our time together here in Blacksburg.

WILLIAM E. HURST

Virginia Polytechnic Institute and State University

December 2002

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Chapter 1

Introduction

A prototype of a piezohydraulic actuator has been designed and constructed with emphasis being placed on limiting the size and weight of the actuator to achieve maximum power per unit weight, also known as power density. The following chapters are related to the development and analysis of the dynamic model used to simulate the experimental results obtained and to predict actuator performance under various operating conditions.

1.1 Motivation

The motivating theory behind this research comes from hydraulic fluid flow/properties and piezoelectric material properties. The combination of the two systems thus creates our piezohydraulic system, used to achieve mechanical motion and power.

1.1.1 Hydraulics

The history of hydraulic power goes all the way back to mills. It has been around many years to propel boats or turn water wheels. Today, however, the use of hydraulics doesn't usually include these examples. Now it is more traditionally known as the transmission of pressure or force through pressurized fluids. It is very common to use hydraulic mechanisms in cars, trains, robots, and especially airplanes due to its reliability and high power density. Pressure is stored as potential energy rather than kinetic energy in these cases to create mechanical motion or mechanical force.

Hydraulic power as it is known today originated in 1648 when a Frenchman by the name of Pascal published a book on hydrostatics, which included his law that stated pressure

in a fluid is transmitted equally in all directions. He was more interested in studying and developing theory however; and more than a century would be lost before practical fluid power systems were devised by Bramah. (Martin and McCloy, 1980) Bramah demonstrated that a relatively small force on a long stroke, small diameter piston could produce a very large force on a short stroke, large diameter piston. Even though the force amplification was impressive, his *observation* that he had a way to transmit power as well as force was more important. Here was a way to change power from mechanical to hydraulic and then back to mechanical, all done efficiently. In the early days of use, this type of hydraulic power was produced by a hand pump and then converted to mechanical power by using a hydraulic ram. The development grew rapidly, but declined with the entrance of electrical power at the end of the 19th century (Martin and McCloy, 1980).

These early applications of hydraulics only transmitted power from one place to another, and controlling this power soon became essential. Power is by definition the product of force and velocity, or in hydraulic systems pressure and flow rate. Thus, controlling either pressure or flow rate will control the power. Variable restrictors in the form of needle or ball valves were developed to control the flow rate. The pressure could be controlled by a relief or check valve, which has a poppet that opens when the fluid pressure force overcomes the spring force on the opposite end. These methods have become very popular, however they come at the cost of lost power and are not very efficient.

A more efficient way to provide the correct power, flow, or motion is enabled by the use of feedback control and a calibrated system of valve opening to flow rate. Servo-mechanisms are commonly used in the control of machine tools and aircraft control surfaces. Servos are accurate, rugged, have fast responses, and can produce large output forces. Their output tracks the input, while also producing a great force amplification. The valve performs the role of output measurement and error detection as well as supplying the corrective power to the cylinder.

After electric power emerged, the unique advantages of a hydraulic system became more apparent. The high force to weight ratio, known as force density, is a very important feature and is used widely by the aircraft industry or in missiles where weight must be minimized. In cars, hydraulics is used in areas like the braking system to eliminate the mechanical system. Using the fluid system gives the ability to balance the breaking force on all wheels while flexible pipes filled with fluid transfer power more easily and reliably

than moving parts.

Hydraulic pumps are also much smaller than electrical motors with the same power rating. A figure in Martin and McCloy (1980) illustrates this comparison. For a given power, a hydraulic pump can work at high pressures and deliver a small flow rate, thus only requiring a small sized pump. Due to this advantage, hydraulic power has become a large part in the efforts of miniaturization. Martin and McCloy (1980) document that a 0.6 kW capable hydraulic pump may only be 32 mm in diameter and 50 mm in length while its maximum speed is $30,000 \frac{\text{rev}}{\text{min}}$ and its weight is only 0.3 kg. (Martin and McCloy, 1980) Assuming a solid volume of steel for the pump ($\rho_{\text{steel}} = 7850 \frac{\text{kg}}{\text{m}^3}$), that would provide a power density of $1.9 \frac{\text{kW}}{\text{kg}}$.

1.1.2 Piezoelectricity

Piezoelectricity is one of the phenomena of nature that makes possible the changing from one form of energy to another form of energy. It is a fundamental process of electromechanical interaction and is representative of linear coupling in energy conversion by changing from mechanical motion into electric potential. Piezoelectricity is defined as the electricity generated in a material when mechanical pressure is applied to it. The opposite effect also occurs; where the material changes physical shape when an electric charge is applied across it. Pierre Curie first discovered the piezoelectric effect in 1880. (Shields, 1966) Now there is a well-established branch of crystal physics and piezoelectricity is used for a wide range applications in the field of electrical engineering. Piezoelectric materials are used for electromechanical transducers, such as ultrasonic generators, filters, sensors, and actuators (Shields, 1966).

Since the eighteenth century, electrification in a heated crystal of tourmaline has been known. The opposite polarities at opposite ends of the material were noticed, and in 1824 a man named Brewster saw this effect in many different crystals. He thus coined the term “pyroelectricity”. The pyroelectricity effect then became known as the temperature coefficient of this polarization. The pyroelectric effect is now known as the interaction between electrical and thermal systems (Ikeda, 1990). In 1880, Pierre and Jacques Curie discovered the piezoelectric effect in different materials like rochelle salt, quartz, and tourmaline. This effect is the interaction between electrical and mechanical systems. The opposite effect was also noticed, that when an electric charge was applied, the material strained and changed

shape (Ikeda, 1990). Piezoelectricity is a linear interaction between electrical and mechanical systems; where pyroelectricity is an interaction between electrical and thermal systems. All three of these interactions are related and a very descriptive picture of how they are all connected in a triangular fashion is given by Ikeda (1990).

The study of piezoelectricity was limited to lab analysis until 1916 when a Frenchman by the name of Paul Langevin developed the first major application, an ultrasonic submarine detector. This device consisted of a piece of quartz between two metal electrodes. It created short bursts of high-frequency mechanical vibrations that were transmitted through the water when an electrical signal was applied to it. These vibrations then reflected off of objects in the water, like submarines. A second quartz element was then used to detect the reflected signal. The time for the signal to travel and return was then used to detect how far away the objects or submarines were. This technique paved the way for modern day sonar (Shields, 1966).

After this, the resonant properties of the piezoelectric materials soon became well known and this was used to develop highly stable frequency determining elements in radio communication equipment. Bell Labs developed Wave filters for use in multichannel telephony work. (Shields, 1966) In the 1930's and 1940's, a crystal pickup cartridge was used for phonographs. (Shields, 1966) In 1940, the first synthetic piezo substance was created, Barium titanate. This synthetic material had a couple advantages compared to non-synthetic piezo materials. By not being water-soluble it could operate reasonably at high temperatures and it could also be fabricated into almost any shape. Soon many other synthetic materials followed. By 1958, piezoelectric elements were readily available and there was no longer a necessity to rely on natural resources (Shields, 1966).

Today piezoelectric ceramics come in many forms for many applications. It is a quickly moving field and research still continues for developing greater stroke and force capabilities, while also finding new applications for this technology. Advantages of using piezoelectric ceramics include high frequency operation (kHz range), high output forces (several kN), high stiffnesses ($\text{N}/\mu\text{m}$ or higher), fast response or rise time (millisecond or less), and accuracies down into the micrometer or nanometer regions. Disadvantages of piezoelectric ceramics include low pulling forces compared to the pushing forces and they only have short displacements (strains on order of 0.2%) (PI, 1999, 2002).

1.1.3 Piezohydraulics

The concept of piezohydraulic actuation is to transfer the reciprocal small stroke displacement of piezoelectric ceramic material into one directional motion by frequency rectification through the media of hydraulic fluid. Frequency rectification is the process of converting an AC signal into a lower frequency or DC signal. In the piezohydraulic case, the high frequency oscillation is changed into a unidirectional hydraulic fluid flow by using valves to control the process.

A piezohydraulic pump consists of a piezoelectric ceramic material connected to a piston. The material is driven with a voltage to create an oscillatory motion, much like a car engine piston with much smaller displacements. This piston compresses hydraulic fluid inside a pumping chamber and builds pressure. While the piston extends, the outlet valve is opened to push fluid out of the chamber and move a hydraulic cylinder type of device to create mechanical motion. When the piston retracts, the outlet valve is closed and the inlet valve opens, allowing the pumping chamber to fill with low pressure hydraulic fluid. This low pressure fluid is stored in a reservoir or accumulator. The ability to operate at high frequencies with the piezoelectric material and the hydraulic fluid allows for large total one directional motion, despite the piston only moving tens or hundreds of microns each period. For example, if the output of the fluid is able to move the hydraulic cylinder $200\text{ }\mu\text{m/cycle}$; a pump operating at 100 Hz will displace the cylinder 20mm/sec.

The advantages of the piezohydraulic pump come through the high frequency operation, high stiffness, large force production, and quick response. The faster the pump is able to operate, the more fluid displacement, cylinder velocity, and mechanical power may be obtained. The higher the stiffness of the system (stiffness of piezoelectric ceramic material and fluid media), the more displacement per cycle is produced. Also, higher loads are able to be moved mechanically by increasing the force capability of the piezoelectric ceramic material. All of these factors, maximum displacement of the piston per cycle, and speed of valve response times will contribute to how well the pump is able to operate as an actuator and generate mechanical power.

1.2 Literature Review

1.2.1 Piezoelectric Hybrid Pumps and Systems

A piezoelectric hybrid system is a system consisting of one part piezoelectric ceramic material and the other parts consisting of different solid or fluid media. The parts act in some manner to produce the desired motion or output of the system. Many hybrid systems have been found involving piezoelectrics, and some have even been involved in various pumping devices.

Inchworm motors/actuators are unique devices which utilize a step and repeat action similar to the piezohydraulic concept. Instead of using hydraulic fluid however, these systems use multiple piezoelectric actuators, solid materials, and friction to achieve a uni-directional motion. Vaughan (2001) describes the design, fabrication, and modeling of one linear motor. He also provides a very detailed literature review discussing other similar devices. A rotating “inchworm” mechanism is discussed by Bexell et al. (1994). The piezoelectric actuators grip and turn a rotor sequentially to achieve a step and repeat motion. These piezoelectric actuators utilize the bending capability of the ceramics. Then, after bending, some of the actuators grip, and this is followed by the release of the clamped actuators. Thus the rotor spins to the unstressed position of the bent and gripping actuators, and then this process is repeated by the actuators that just released their gripping position. In the end, the system performs a one directional rotation at a lower frequency than the oscillatory motion of the bending and gripping motion of the piezoelectric actuators. A frequency rectified, step and repeat mechanism.

Advantages of the inchworm actuator include high clamping forces and significant displacements/velocities. Disadvantages are displayed by using friction as the primary source to create motion, resulting in loss of energy and deterioration. Other articles on the subject are by Lee and Esashi (1995) who give a detailed design that is able to move 2.167 mm/sec when operated at 1.4 kHz, Bamford (1994) who produces a detailed drawing of an inchworm actuator that is capable of a 30 mm end-to-end stroke, and Frank et al. (1999) who developed a high force motor (150 N dynamic, 275 N holding) reaching speeds of 1 mm/sec when operated at 100 Hz.

Many different styles of pumping devices were found utilizing the piezoelectric ceramic materials, with a large emphasis on miniaturization. In the articles by Li et al.

(2000), Koch et al. (1997), and Smits (1990), the systems were so small that the flow passages and pumping chambers were just micro machined or layered into a material, neglecting the need for pipes and connectors. All of the pumps researched use the piezoelectric material to displace a membrane, diaphragm, plate, or somehow compress a volume of fluid to build pressure and create fluid flow. Check valves, passive valves, and diffuser nozzles are all used to control the volume entering and exiting the pumping chambers, however no true active valves could be found in these pumps. The diffuser nozzles found in the publications by Ullmann (1998) and Gerlach and Wurmus (1995) will give the ability to pump at high frequencies, yet the performance and efficiency will be less than ideal due to back flow into the chamber. A short summary of each pump is given in the following paragraphs.

A MEMS micropump was developed by Li et al. (2000) using different layers of materials with valves and holes built into them for pumping fluid. A small piece of piezoelectric material in the middle of the design is connected to one of the layers of material. This layer bends when the stack translates, compressing the fluid in the chamber, and resulting in fluid moving through the holes and valves.

Koch et al. (1997) discusses a silicon-based micropump that uses passive cantilever valves that open and close after the pumping action builds and releases pressure in the chamber. The pumping action is performed by a piezoelectric stack actuator made of silicon with gold ink used as electrodes. The system was modeled by finite element techniques and computational fluid dynamics. The article talks mostly about the processes of making the valves, making the piezoelectric stack actuator, and other materials and fabrications used.

Ullmann (1998) analyzes single and double acting chamber pumps in series and parallel. This concept is used for micropumps that have small flow rates. The reciprocating motion of a diaphragm, which is actuated by a piezoelectric disk, performs the pumping action. Diffuser nozzles are used in place of conventional valves, so high frequency operation is not a problem. Diffuser nozzles have different flow resistances, depending on the direction the fluid is flowing through them. They allow the high frequency operation, but do not entirely prevent “back flow” in the system. The net flow is in the direction of least resistance, however the system is not the most efficient due to the losses in flow. Also to note, the series connection of the pumping chambers had better results than the parallel for maximizing pump performance.

Gerlach and Wurmus (1995) describe the modeling and building of a micropump

using dynamic passive valves for flux rectification. It is completely structured and mounted by microtechnologies and hybrid technologies based on silicon. It utilizes a piezo-biomorph membrane to push glass and change the volume of the pumping chamber. Once again, conventional valves are absent and the diffuser nozzles are used to achieve the input and output flow to the chamber.

A silicon micropump with piezoelectric chambers was reported by Smits (1990). It pumps liquids or gases to high pressures, and then is relieved by a check valve. The piezo action allows the opening and closing of the chambers electronically, thus they can be controlled to operate at the optimal times. The first chamber opens and sucks in the fluid, then the second chamber opens and stores the fluid just before the first one closes. Then, the third chamber opens and stores the fluid, just before the second chamber closes. This process is repeated to produce a one directional flow through the three chambers.

1.2.2 Piezohydraulic Pumping Actuators and Valves

Piezohydraulic actuators are a very recent concept and technology, however there are several patents on the use of piezoelectric materials for producing fluid motion (Beckman and Blickstein, 1984, 1985; Culp and Nuys, 1993; Sager and Matice, 1998; Ogawa et al., 1999; Bishop et al., 2000). The first use of piezohydraulics for producing 1 - 10 W mechanical power output was probably reported by Mauck and Lynch (1999, 2000a). They describe the design and testing of a reciprocating piezoelectric pump and passive check valve system. They report a speed of 72.5 mm/sec without load and a blocked force of 271 N for a particular value of cylinder bore diameter (Mauck and Lynch, 2000b). Assuming a linear relationship between these parameters, the maximum mechanical power output is estimated to be 5 W. Their first system showed good linear performance up to around 10 Hz, but broke down at higher frequencies. This breakdown was documented to be caused by the “current limit of the power supply, cavitation on the intake stroke, and inadequate response of the check valves” (Mauck and Lynch, 1999). Their second actuator was capable of producing even higher flow rates, however the system could not operate above the 60 Hz limit without incorporating active cooling (Mauck and Lynch, 2000b).

After this group, Nasser et al. (2000) developed a bench top prototype piezohydraulic pump with active servo valves. This system demonstrated 100 N blocked forces with unloaded velocities of 180 $\mu\text{m}/\text{sec}$ when operated at a frequency of 6 - 7 Hz. Due to the time

delay associated with the opening and closing of the servo valves, operating at frequencies above 7 Hz was reported to actually cause a drop in output velocity. This behavior was attributed to an overlap in the valve operation, where the inlet and outlet to the chamber were open simultaneously. The mechanical valves could not respond quickly enough to the electrical signals they received, causing both valves to overlap at frequencies above the 7 Hz limit (Nasser et al., 2000).

Recently, experimental work was performed by Sirohi and Chopra (2001) on a piezo-hydraulic pump utilizing passive check valves, hydraulic accumulators, and control valves within the hydraulic circuit. An accumulator was placed between the piezohydraulic pump and the output hydraulic cylinder to build pressure, while the check valves were used to control the flow in and out of the pumping chamber. The control valves were then used to control the flow from the accumulator to the hydraulic cylinder. No output velocities or blocked force measurements were documented, but rather accumulator pressure increase and volumetric fluid displacement were reported over a 5 min time period. A maximum 180 psi accumulator pressure increase and 100 ml fluid volume displacement were achieved while operating at 150 Hz. Once again, the passive check valves were a limiting factor and accumulator pressures actually decreased when the system was operated above the 150 Hz limit (Sirohi and Chopra, 2001).

A unique piezohydraulic microvalve was designed and modeled by Roberts et al. (2000) for high pressure and high frequency applications. This design is based on the concept of converting the small displacement of the piezoelectric element into a large valve cap stroke through the use of hydraulic fluid, while maintaining high force capability. The construction consists of many layers of materials, resulting in a MEMS-scale high frequency active valve. It utilizes a hydraulic amplification chamber to displace the valve cap up to 40 μm and has the ability to operate up to a frequency of 24 kHz (Roberts et al., 2000).

1.2.3 Modeling

Due to the fact that a piezohydraulic pump is a hybrid device consisting of piezoelectric ceramic materials and hydraulic fluid, modeling is necessary for both parts of the system. The governing equations for the dynamics of a piezoelectric ceramic material have become well known and can be found in many places such as various manufacturers' catalogs and web sites (PI, 1999, 2002; Piezomechanik, 2002), the books by Shields (1966) and Ikeda (1990),

and also The IEEE Standard on Piezoelectricity (IEEE, 1987). Fluid Dynamics is also a very well known field, with some different modeling techniques available. Pressure, velocity, and flowrate equations are probably the most widely used in systems where fluid is only traveling through pipes. Within piezohydraulic hybrid systems however, lumped-parameter models (Nasser et al., 2000) have also been successful tools along with the conventional and finite-element techniques (Roberts et al., 2000, 2001).

The articles by Tang et al. (1997) and Zhou et al. (1999) model the piezohydraulic actuators for control applications. They provide descriptions of their models, however the focus is almost purely on the piezoelectric modeling and neglects the hydraulic modeling. Their applications use the hydraulic fluid only for force/displacement transmission, and thus there is no fluid flow. Only using a closed volume of fluid gives them the ability to neglect the hydraulic component and focus mainly on the piezoelectric dynamics.

Nasser et al. (2000), Roberts et al. (2001), Regelbrugge and Anderson (2001), and Tan (2002) are probably the first to use dynamic models of piezohydraulic systems where the dynamics of the fluid are also included. Their approaches differ however, where Roberts et al. (2001) and Regelbrugge and Anderson (2001) use pressure/volume change models, Nasser et al. (2000) uses a lumped-parameter model of a fluid circuit, and Tan (2002) uses incompressible and compressible fluid models. The model of the microvalve developed by Roberts et al. (2001) consists of a piezoelectric ceramic element that pushes a plate/membrane to compress a volume of fluid. This compressed fluid volume creates motion on the opposite end of the valve's cap/membrane, which is then pressed up against the valve opening to obstruct flow. This obstruction creates a closed valve situation, which then returns to open when the piezoelectric material retracts. Once again there is no fluid flow within the model, only pressure changes, nonlinear material displacements, and wall/cap displacements, all caused by an initial change in volume.

One other contribution from the Roberts et al. (2001) publication was the demonstration of the ability to model the dynamic pressure/force loss through the valve orifice. A few publications including the dynamic pressure/force losses through valves are Johnston et al. (1991); Sirohi and Chopra (2001); Roberts et al. (2001) They provide analysis of fluid flow and pressure changes through various types of valves and orifices. They also detail formulas for, or estimate based on experiments, the flow coefficients of certain orifice dimensions. The flow coefficient is a multiplying factor in the equation for the flow and

pressure loss through an orifice. Through experiments, Johnston et al. (1991) determined that the flow coefficient does not change based on how far a poppet valve is open, rather only on the dimensions and angles of the poppet valve. In a disk valve however, the flow factor does change based on the size of the opening.

Regelbrugge and Anderson (2001) simulated the performance of a piezohydraulic system and highlighted the need for system-level analysis. A theoretical analysis of a piezohydraulic pump, accumulator, and four-way valve fluid circuit was conducted using pressure/flow equations. System parameters were chosen and simulations were run at different pre-pressures, bulk modulus values, valve areas, and using valve types (active and passive). Conclusions found that the entire piezohydraulic actuator system must be designed together, with the piezoelectric material and the fluid properties carefully chosen for optimal performance. Also noted was a need for well designed active valves capable of high frequency operation to rectify the high frequency pressurization of the working fluid (Regelbrugge and Anderson, 2001).

Nasser et al. (2000) was the first to use a lumped-parameter model for a piezohydraulic actuator. They used this model to accurately predict the unidirectional output cylinder displacement of a closed piezohydraulic circuit with active servo valves. A phase delay was introduced into the system model to accurately predict the valve operation effect on the closed circuit dynamics. Hydraulic, mechanical, and electromechanical components were separately modeled and then combined via a coordinate reduction technique to obtain the coupled equations of motion. The model was correlated to experimental data for the piezohydraulic circuit with one valve open and one valve closed (Nasser et al., 2000).

The lumped-parameter model used by Nasser et al. (2000) in their piezohydraulic modeling was first developed by Doebelin (1972). He created individual fluid elements of inertance, resistance, and compliance based on a pipe's properties and the properties of the fluid inside. These fluid elements are similar to the mechanical elements of a mass, damper, and spring, which are used to model mechanical systems. The inertance element is dependent on fluid density, element length, and element area. Fluid resistance is based on the fluid's dynamic viscosity, pipe I.D., and element length. Compliance relies on the fluid bulk modulus, element area, and element length. With the exception of fluid bulk modulus, all of these variables may be measured or determined from the fluid manufacturer's specifications.

Bulk modulus has a manufacturer's value associated with it, however there are many other variables that affect its actual value. An article by Jinghong et al. (1994) discusses the different factors that effect the bulk modulus property. It is assumed to be a combination of the air free manufacturer's bulk modulus rating, the entrained air's bulk modulus, and the bulk modulus of the container holding the fluids (pipes, etc.). The bulk modulus is a very important parameter and its value effects the dynamic characteristics of the system a great deal.

The five factors that were determined to effect the bulk modulus value are the air content of the oil, the oil pressure, the oil temperature, the pipe stiffness, and the oil/air interface. The percentage of air in the fluid substantially reduces the effective bulk modulus from the manufacturer's rating. Jinghong et al. (1994) state that a 1% air to MIL-H-5606 hydraulic fluid volume ratio caused a 25% reduction in the effective bulk modulus from the air free value. Dissolved air in the oil was determined to usually not be a problem, however the entrained air is a serious problem. The percent of air by volume in the system, and thus the effective bulk modulus, was also determined to be effected by the pre-pressurization of the fluid. The higher the pre-pressure, the higher the effective bulk modulus value (Jinghong et al., 1994).

Manring (1997) describes a method for experimentally measuring the bulk modulus in a hydrostatic transmission. Again, the most influencing variable that determined the effective bulk modulus was found to be the volumetric percent of entrained or non-dissolved air in the system. It was shown that 1% volume of entrained air could reduce the effective bulk modulus by 1085 MPa, a 75% decrease from the manufacturer's value (Manring, 1997).

1.3 Overview of Thesis

1.3.1 Objectives

The objectives of this work were to design, build, and determine the dynamic performance (experimentally and theoretically) of a piezohydraulic actuator with active piezohydraulic valves. This was a second prototype of the piezohydraulic pump, with an effort made to remedy the frequency mismatch of the reciprocating pump and the valves used in the first design. A pressure sensor was also to be included inside the pumping chamber to study the valve timing and its effects on the performance. The goals were to study the actuator under

a variety of different parameters including different frequencies, pre-pressures, loads, valve timings, and output cylinders. Using the data from these varying parameters, a conclusion could be drawn about which conditions provide the actuator with optimal performance. Short term goals were to demonstrate a minimum of 5 W mechanical power by the end of the second year, with a final goal of 600 W by the end of the third year. Currently the project is in the second year, and our piezohydraulic actuator was measured at a maximum 2.37 W of mechanical power while operating at 100 Hz.

1.3.2 Contribution

This work focuses on the development of piezohydraulic actuators that incorporate active valves for flow control. The contributions of this work are:

1. Theoretical lumped-parameter models are created to simulate the active valve dynamics along with their effect on the closed fluid circuit and actuator performance.
2. The importance of valve timing and fluid compressibility are clearly seen in the experimental results when looking at the chamber pressure and actuator power output.
3. New results are presented here for a piezohydraulic system utilizing active valves to control fluid flow. Previous work has been limited by the mismatch between the bandwidth of the piezoelectric pump and the electromagnetic active valves/passive check valves.
 - Our current system incorporates piezohydraulic active valves that have a nominal bandwidth on the order of 1 kHz.
 - This bandwidth more closely matches the required operating frequency for the reciprocating piezoelectric pump to achieve the maximum desired output power.
 - Using active valves in the system provides the ability to operate at higher frequencies and thus increase output power, while also providing the ability to actively control pressure and flow.
4. Simulated and experimental results are shown for closed chamber pressure and piston displacement.
5. Simulated and experimental results are shown for operating chamber pressures and overall actuator performance.

1.3.3 Approach

The pump was originally designed around the use of an off-the-shelf piezoceramic stack actuator already in house (Physik Instrumente model P-844.60) to save money, time, and to give a base point for starting the design process. The outer casing, pumping chamber, and piston of the pump were all custom designed and manufactured. The original pumping chamber was designed with three ports, two for connecting check valves while the active valves were being constructed and one more for a pressure sensor. The pressure sensor provides the ability to see how the fluid inside the chamber interacts with the valves while pressure is built and released. The active valves were contracted out to a vendor due to the vendor's expertise in the area along with the ability to custom design the valves to fit our existing pump casing and piston. Also, by having the valves separate from the rest of the pump, either part could be modified or redesigned more easily to accommodate future needs.

Experiments were conducted using a dSPACE system and a Simulink model, which allowed the ability to control the operation frequency along with the variables that controlled the valve timing. Thus, analysis was feasible at multiple frequencies and valve timings. The use of vacuum pumps and hydraulic pumps also provided the ability to remove air from the system and then fill the system with fluid up to a wide range of pre-pressures. Depending on the amount of air in the system and the pre-pressure of the system, the fluid compressibility would change and yield different results.

Three theoretical lumped-parameter fluid models are built for the piezohydraulic valves, the pumping chamber, and the entire fluid circuit including the output cylinder. With the valves opening and closing, there is not a continuous system to analyze, therefore multiple lumped-parameter systems are used for the simulations. The valve state-space lumped-parameter fluid model is used to determine when the valves are open or closed, which then results in simulating a closed chamber, the entire open fluid circuit, or both. A Matlab m-file and multiple Simulink models are used to build the state-space systems and match the experimental conditions of frequency, pre-pressure, air content, load, valve timing, and the fluid circuit dimensions. By viewing the states of these systems, the output cylinder displacement/velocity and piston displacement can be read directly, while pressure can be calculated based on the states inside the chamber. Valve opening, valve pressure loss,

and other parameters may also be easily read or calculated from the states of the system. Model parameters such as piezoceramic properties, fluid properties, and piping dimensions can also be modified to simulate the effects of changing the actuator design.

Chapter 2

Piezohydraulic Systems

2.1 Introduction

Piezoelectric materials are capable of producing large forces over small displacements very rapidly. They also are very efficient, lightweight, and highly durable, while converting electrical voltage into linear motion for hundreds of millions of cycles without wear. By coupling this actuator technology to a very stiff, high force transmission, hydraulic fluid circuit; a new piezohydraulic system is created that can still produce large forces very quickly. The theory behind piezohydraulic actuation is the transfer of a piezoelectric's small stroke oscillatory displacement into one directional fluid motion through frequency rectification. In the piezohydraulic case, the high frequency oscillation is changed or rectified into a unidirectional displacement by using valves to control flow into and out of a chamber. The small displacement problem of the piezoelectric material becomes trivial by operating at very high frequencies and thus producing large fluid flow through the chamber. The advantage is that the piezohydraulic actuator still retains the high force capability and quick response time inherent in the piezoelectric material. Ideally this also provides a piezohydraulic actuator system with higher power density than other conventional actuators, or in other words, a system capable of large power output while remaining lighter than other actuators with the same power rating.

There have been some problems previously however, while trying to build these types of systems. Electronic amplifiers used to drive the piezoelectric materials are a major issue with these devices along with valves that open and close quickly enough to control the one directional flow. Piezoelectric material displacements are very small, thus it is realized

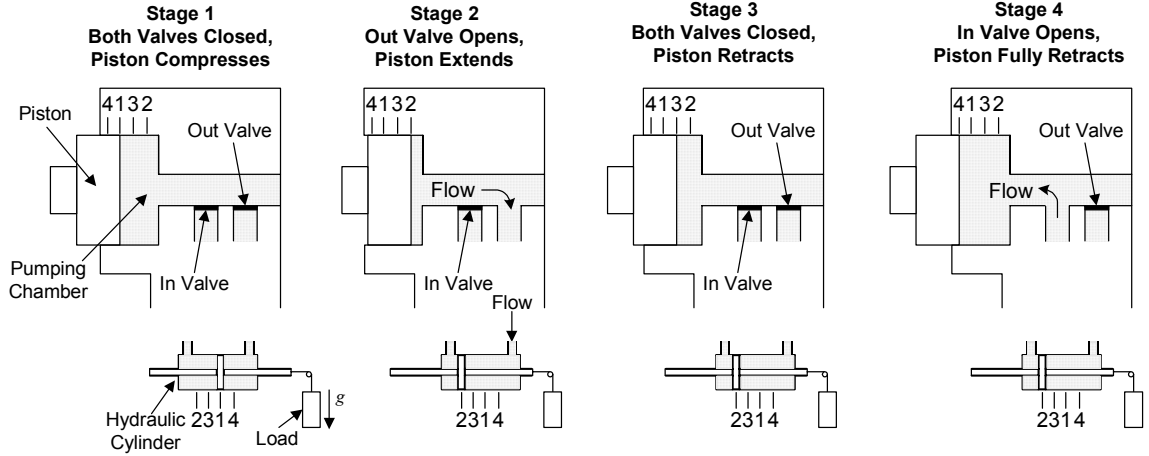


Figure 2.1: Frequency Rectification Concept

that operation must be at very high frequencies (several hundreds of Hz to kHz range) to produce significant fluid flow and mechanical power. This also means the electronics driving the piezoelectric materials must have very fast rise times while producing the very high electrical power the materials require. The issue with the valves is that the one directional flow can only be produced as fast as the valve operation controlling it. Previously, check valves and active solenoid valves did not operate nearly fast enough to produce significant flow and power output.

2.2 Piezoelectric Operating Principle

The basic concept for piezohydraulic actuation can be seen in Figure 2.1. It shows the conversion of the piezoelectric material/piston oscillatory motion into one directional fluid flow. In stage 1 the piston compresses the fluid in the chamber by moving from the fourth position into the first position. In stage 2, the outlet valve opens, the piston continues to extend to the second position, and fluid flows out of the chamber. In stage 3, both valves are closed again, the piston begins its reverse part of the oscillatory motion back to the third position, and pressure drops inside the chamber. For the final stage (stage 4), the inlet valve opens, fluid fills and pressurizes the chamber, and the piston returns to its minimum extension. Thus, with the chamber reset, the piston cycle begins once again to create one directional fluid flow.



Figure 2.2: PI piezoelectric actuator

2.3 Piezoelectric Actuator and Pump Casing Design

When designing the prototype piezohydraulic system, the piezoelectric actuator became the base point of the design. The idea for the design eventually evolved into making the stack as stiff as possible with the highest blocked force and the most tip displacement possible. Some original ideas were to match the piezoelectric impedance to the impedance of the hydraulic system/circuit. While this may work well for an optimal efficiency analysis, this was not the case for achieving the most fluid flow and mechanical power. To push the greatest amount of load at the fastest velocity (actuator power), the maximum fluid displacement and force is desired. Thus, displacing the piezoelectric material greater and greater distances causes more and more fluid to be pushed through the chamber per cycle. Making the piezoelectric actuator stiffer decreases its rise time (higher natural frequency), which then creates motion and force in the connecting fluid faster. Finally, increasing the actuator blocked force will also increase the amount of force capable of being transmitted to the fluid, and thus the amount of load that can be pulled by the hydraulic cylinder.

For our prototype, we chose to use an off-the-shelf piezoelectric actuator manufactured by Physik Instrumente (PI, 1999, 2002). This actuator (model PI 844.60) had previously been purchased and was already at our site. The amount of money and time saved by using this actuator was the main reason for not exploring other alternatives or custom piezoelectric actuator designs. A picture of this actuator attached to the base plate of the piezohydraulic pump can be seen in Figure 2.2.

Another issue in the design was how long to make the pumping chamber. By increasing the length of the chamber you increase the initial volume (V) of fluid inside, and thus decrease the stiffness of the fluid inside also. This increase in initial volume then directly translates into a decrease in the maximum pressure that can be accumulated inside the

chamber before the out valve opens, as seen in equation 2.1

$$\Delta P = - \left(\frac{\beta \Delta V}{V} \right). \quad (2.1)$$

The maximum amount of pressure is desired so that the rest of the fluid in the hydraulic circuit flows as quickly as possible, thus it is also desired to make the pumping chamber length as short as possible. The chamber pressure minus losses in the fluid circuit is also what creates force on the hydraulic cylinder to overcome the friction and load acting upon it, creating acceleration of the cylinder/load. By quickly moving the cylinder/load, this allows for full load displacement each cycle, even while operating at very high frequencies. Thus, high frequency operation will not be limited due to the cylinder response time if the chamber pressure and hydraulic circuit stiffness are large enough. The prototype actuator design actually incorporates spacers so that the chamber length can be adjusted and performance could be analyzed at different lengths. The shorter lengths did enhance performance, and the shortness of the chamber length was actually limited to 1 - 2 mm due to the thickness of the o-ring needed to seal the chamber from leaking fluid.

Pumping chamber diameter/area was the last factor for the pumping chamber design. This factor also directly effects the stiffness of the fluid inside the chamber, however after the out valve opens, it helps determine the volume of fluid displaced from the pumping chamber for a given piston displacement. A large diameter means high volumetric fluid displacement at the expense of a softer fluid and lower chamber pressures. A large diameter also means a larger casing, possibly more weight, and larger overall dimensions of the piezohydraulic actuator. Therefore, a smaller diameter/area is better, while making sure enough fluid is pushed through the chamber per cycle to achieve the desired fluid flow, cylinder velocity, and output power. Our prototype chamber diameter was chosen to be 25 mm, slightly bigger than the piezoelectric actuator diameter and approximately the same size as the hydraulic cylinder diameter.

The casing of the prototype piezohydraulic actuator was made of stainless steel and comes in three pieces. First, there is a base plate that the piezoelectric actuator attaches to. Then, a hollow main body is slid over the piezoelectric actuator and attaches to the base plate also. After attaching a piston to the tip of the piezoelectric actuator, the pumping chamber is glided over the piston and bolted to the main body. The active valves were contracted out to a company who built them around our design and combined them with a

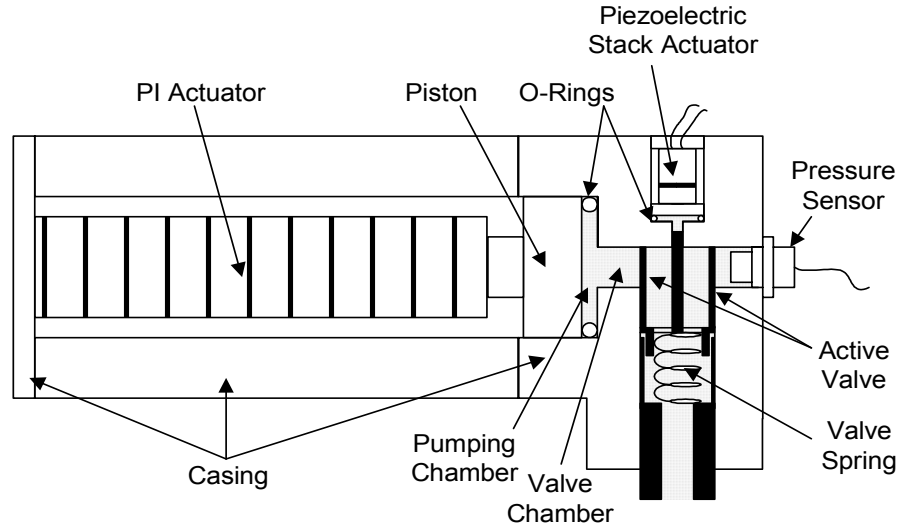


Figure 2.3: Piezohydraulic Pump Design

pumping chamber attachment. Two pumping chamber pieces were actually built as seen in Figure 2.4, one for use with check valves while the active valves were constructed, and the other which was attached along with the active valves. As mentioned previously, o-rings were also used to seal the chamber from leaking hydraulic fluid.

2.4 Hydraulic Circuit Design Ideas

In many engineering applications hydraulics or fluids are considered to be incompressible when developing theory or calculating flow parameters. In the piezohydraulic case however, the piezoelectric material is moving such small distances and pushing only tens of microns of fluid each cycle. Since the ratio of the displaced fluid to the actual amount of fluid in the circuit is so small, this incompressible assumption is not valid here. Isothermal or negligible thermal effects during flow, along with constant fluid density, are two basic and very important assumptions for our analysis however. Since no heat is being added or taken away from the system during operation and fluid density doesn't change significantly with pressure, these two assumptions should be valid.

For fluid flow through pipes, a basic analysis starts with a long tube of fluid. Fluid flow occurs when the pressure at one end of the tube is increased or initially greater than the pressure at the other end. Assuming no resistances or losses in the pipe, fluid flow will automatically begin. In actual pipes there is always some flow resistance, which depends on

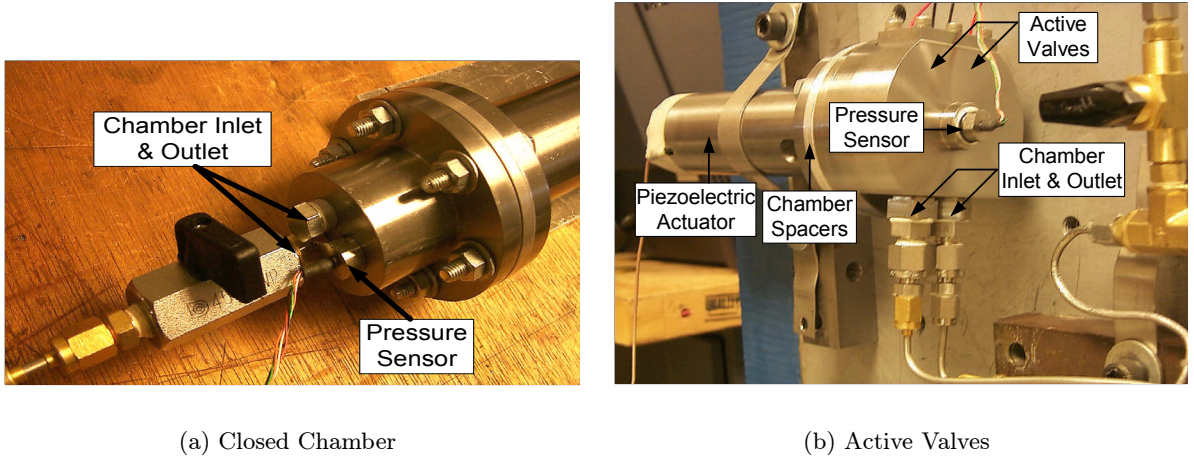


Figure 2.4: Piezohydraulic Pumps

the pipe geometry, length of pipe, the fluid viscosity, and type of flow (laminar or turbulent). The pressure difference between the ends of the pipe must overcome this resistance for fluid to flow from one end to the other. The greater the pressure difference, the faster the flow rate of the fluid through the pipe. Figure 2.5 shows a basic diagram of fluid flow within a pipe section. The flow resistances (R_f) are based on the geometry of the pipe and formulas for their values can be found in the book by Doebelin (1972). The pressure, flow rate, and resistance relationship is described as:

$$P_{n-1} - P_n = R_{f_n} q_n = R_{f_n} A v_n, \quad (2.2)$$

where:

q_n = section flow rate

A = pipe area

Other resistances may also occur in a fluid circuit when connecting multiple pipes and other components. The first type is a flow resistance due to an area change in the fluid circuit. When a fluid undergoes a change in area, the pressure drop or flow resistance is dependent upon a loss factor. This loss factor will vary due to the type of the area change (gradual or sudden change) and the diameters of the pipes before and after the change. Once a loss factor is found, it must be divided by the friction factor to obtain the equivalent

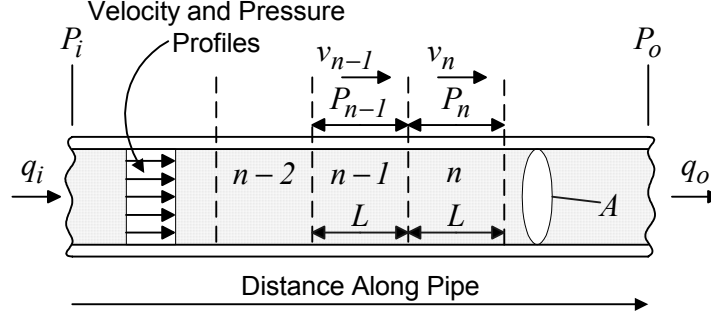


Figure 2.5: Pipe Flow Analysis

length value as seen:

$$\left(\frac{L}{D}\right)_{eq} = \frac{K}{f} \quad (2.3)$$

$$L_{eq} = \frac{K}{f} D \quad (2.4)$$

where:

$$\begin{aligned} K &\triangleq \text{loss factor} \\ f &\triangleq \text{friction factor} \\ L_{eq} &\triangleq \text{equivalent length} \\ D &\triangleq \text{diameter of exit tube} \end{aligned}$$

This equivalent length can then be added to the length of the pipe in the resistance value equation, found in the book by Doebelin (1972). This total length thus yields the total resistance of the pipe plus area change. The friction factor can be found from the Moody chart as seen in Avallone and Baumeister (1996) and depends on the type of flow (laminar or turbulent), the pipe roughness, and the actual value of the Reynolds number (**R**) (Doebelin, 1972; Avallone and Baumeister, 1996).

One last type of resistance or loss we require for our actuator analysis comes from fluid flowing through valves or small orifices. Since the valve passageway area is small compared to the entrance tube and exit tube areas, a pressure differential occurs across the valve. Depending on the type of valve, the size of the opening of the valve, and the flowrate entering the valve, this pressure loss may be calculated. Johnston et al. (1991) give an equation to calculate the pressure drop (ΔP) for poppet and disk valves based on fluid flowrate (q), fluid density (ρ), and valve opening area (A_0)

$$\Delta P = \frac{1}{2} \rho \frac{q^2}{C_d^2 A_0^2}. \quad (2.5)$$

The piezohydraulic valves used with our actuator are very similar to disk valves, so this analysis may be used. The flow loss coefficient (C_d) in this pressure differential equation varies depending on the height of the valve opening and falls between 0.75 and 1.1 for disk valves. These results are based on experimental analysis of valves for different fixed size openings. Roberts et al. (2001) took this equation into account and added another factor, $\zeta_{orifice}$, to dynamically calculate the pressure change based on how far the valve was open at a given time. This new loss factor is based on turbulent flow with very high Reynolds numbers however, and must be combined with the experimentally found factor to obtain the pressure loss for low turbulence or laminar flows. Equation 2.6 shows the formula for the new loss factor and equation 2.7 shows the formula for the total dynamic pressure loss through the valve

$$\zeta_{orifice} = \frac{\Delta P}{\frac{1}{2}\rho\dot{x}_{avg}^2} = \left[\frac{1}{2} \left(1 - \frac{A_0}{A_2} \right)^{\frac{3}{4}} + \left(1 - \frac{A_0}{A_1} \right)^2 \right]. \quad (2.6)$$

A_0 is the area of the valve opening, A_1 is the inlet area, A_2 is the outlet area, and $\dot{x}_{avg} = \frac{q}{A_0}$ is the average velocity through the valve. Combining equation 2.5 with the new loss factor then reduces to:

$$\Delta P = P_1 - P_2 = \frac{1}{2C_d^2} \rho \zeta_{orifice} \frac{q^2}{A_0^2} \quad (2.7)$$

Another area of fluid circuit theory that should be noted for our analysis is the compressibility. Fluid compressibility is the reciprocal of bulk modulus, such that the bulk modulus is defined by the equation:

$$\beta = - \left(\frac{\Delta P}{\Delta V} \frac{V}{P} \right) \quad (2.8)$$

where:

$$\begin{aligned} \Delta V &\triangleq \text{volume change} \\ V &\triangleq \text{initial volume} \\ \Delta P &\triangleq \text{pressure increase due to applied force, } F \\ \beta &\triangleq \text{fluid bulk modulus} \end{aligned}$$

Figure 2.6 is included to help visualize a fluid volume under compression.

Jinghong et al. (1994) discuss the effective bulk modulus of hydraulic oil and the different factors that effect the bulk modulus property. It is assumed to be a combination

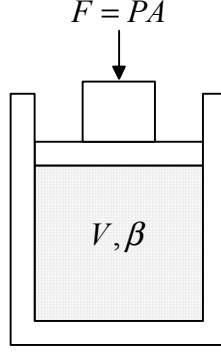


Figure 2.6: Fluid Compression Analysis

of the air free manufacturer's bulk modulus rating, the entrained air's bulk modulus, and the bulk modulus of the container holding the fluids.

Manring (1997) describes a method for experimentally measuring the bulk modulus in a hydrostatic transmission. Again, the most influencing variable that determined the effective bulk modulus was found to be the volumetric % of entrained or non-dissolved air in the system. Here it was shown that 1% volume of entrained air could reduce the effective bulk modulus by 1085 MPa, a 75% decrease from the manufacturer's value.

Doebelin (1972); Martin and McCloy (1980) develop relationships for the fluid (ΔV_{fluid}), air (ΔV_{gas}), and thin-walled tubing (ΔV_{tube}) volumetric changes due to a given pressure change ΔP , within an initial volume V :

$$\Delta V_{fluid} = \frac{(1-x)V}{\beta_{fluid}} \Delta P \quad (2.9)$$

$$\Delta V_{gas} = \frac{xV}{\beta_{gas}} \Delta P \quad (2.10)$$

$$\Delta V_{tube} = \frac{VD}{tE} \Delta P \quad (2.11)$$

where:

$P_0 \triangleq$ initial pressure

$\alpha \triangleq$ ratio of specific heats

$x \triangleq$ percent air content

$t \triangleq$ tube thickness

$D \triangleq$ tube diameter

$E \triangleq$ tube modulus of elasticity

$\beta_{fluid} \triangleq$ manufacturer's fluid bulk modulus $\approx 250,000$ for hydraulic oils

$\beta_{gas} \triangleq$ air bulk modulus $\approx \alpha P_0$, assumed adiabatic compression

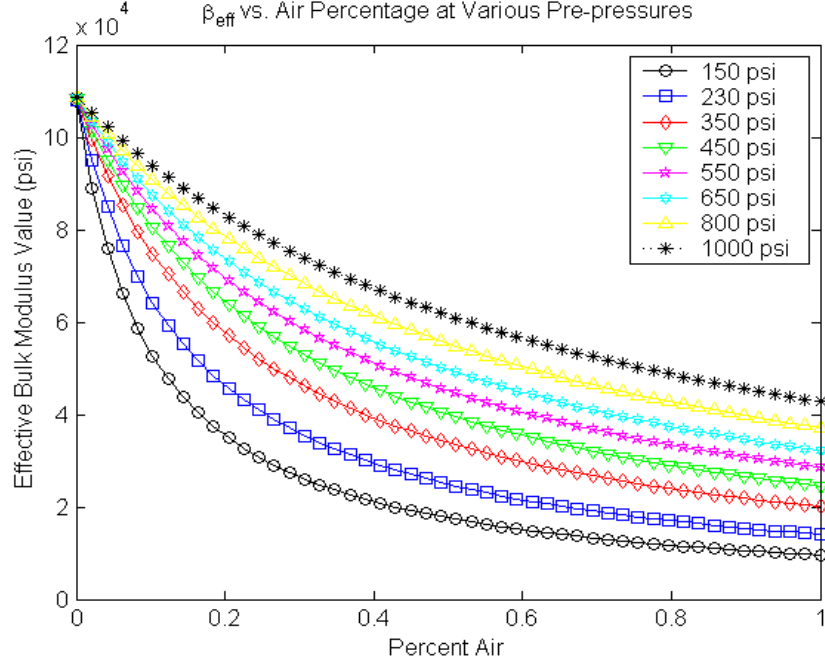


Figure 2.7: Effective Bulk Modulus Analysis

Combining equations 2.9, 2.10, and 2.11 into one total volumetric change thus yields equation 2.12

$$\Delta V_{total} = \left(\frac{(1-x)}{\beta_{fluid}} + \frac{x}{\beta_{air}} + \frac{D}{tE} \right) V \Delta P. \quad (2.12)$$

Once again, by applying the bulk modulus definition of equation 2.8, where ΔV is a negative volumetric change due to the pressure change ΔP , we can now find an equivalent bulk modulus value from equation 2.13

$$\beta_{eff} = \frac{V \Delta P}{\Delta V_{total}} = \frac{1}{\left(\frac{(1-x)}{\beta_{fluid}} + \frac{x}{\beta_{air}} + \frac{D}{tE} \right)}. \quad (2.13)$$

Using equation 2.13 to simulate the effective bulk modulus inside the pumping chamber and valve chamber, the variations with volumetric percent air and pre-pressure are displayed in Figure 2.7. This simulation was done using an estimated liquid bulk modulus (β_{fluid}) value of 250,000 psi, an air bulk modulus (β_{air}) value of $1.4 \times$ pre-pressure in psi, a stainless steel elastic modulus of 190×10^9 Pa, along with the pumping chamber and valve chamber dimensions shown in Table 4.2.

Other than resistances and losses, cavitation is a concern within the pumping chamber part of the hydraulic circuit. Cavitation refers to the formation and collapse of cavities

in the liquid. Cavities may contain air or gas, and if the pressure drops too low, liquid may vaporize and vapor cavities will form. The pressure where vaporization occurs is the vapor pressure of the liquid and depends on the operation temperature also. Vapor pressures of mineral oils are much lower than water, where a typical one is 6×10^{-4} mbar (8.7×10^{-6} psi) at 40 °C. Since the vapor pressure is so low for hydraulic oils, cavitation should not be a problem for our application. Also to note, casing materials with low strain energy such as aluminum are more inclined to have cavitation erosion occur than high strain energy materials such as stainless steel. (Martin and McCloy, 1980)

Thus, to achieve optimal flow and power transmission within the fluid circuit, the maximum amount of air should be removed and the tubing must be made of stiff materials and/or be made very thick. To reduce the friction and resistance losses, the circuit tubing length should be as short as possible, should have the least amount of area changes, and the valves should open as far as possible. Other circuit components effecting the flow and power rating of the piezohydraulic actuator are the hydraulic cylinder & air/hydraulic accumulator. These should have high enough pressure ratings, quick response times, and create the least amount of friction/resistance possible.

2.5 Valve Design Concerns

The frequency rectification concept is the entire basis for the piezohydraulic actuator. If the valves are not able to function rapidly and convert the high frequency operation of the piston into large fluid displacement/flow, then the actuator will never achieve high output power. The actuator power will reach a maximum at the highest operational frequency of the valves if the fluid circuit has a higher bandwidth than the valves. The valves must be designed and built to respond quickly, which means the electronics driving them must be built in a similar fashion also. If piezohydraulic actuator operation is desired at 1 kHz, the valves and their power amplifiers must have time constants at least twice as fast (0.0005 sec.) because one valve must close before the other opens. Ideally, the valves will open and close fully before the piezoelectric actuator driving the piston even changes direction, which means very small time constants are ideal. Realizing there was better expertise than ours in both of these areas, the valve design and power electronic designs were both contracted out to other companies.

Table 2.1: Active Valve Properties

Property	Value
Maximum Operating Frequency	1,000 to 2,000 [Hz]
Average Flow Rate	43 [$\frac{\text{ml}}{\text{sec}}$]
Peak Flow Rate	172 [$\frac{\text{ml}}{\text{sec}}$]
Operating Pressure	up to 2,000 [psi]
Valve Diameter	8.4 [mm]
Maximum Valve Opening (h)	250 [μm]
Maximum Valve Orifice Area (A_o)	6.6×10^{-6} [m^2]

The piezohydraulic valves were built into a new pumping chamber head and operate using a hydraulic amplification to open them. The piezoelectric materials require high voltage signals to drive them, operating over a 0 - 1300 Volt range. The estimated mechanical properties of the active valves provided by the manufacturer are shown in Table 2.1.

A three channel switching amplifier was custom built for us by DSM to drive the piezoelectric materials for the piston and valves. The three channels consisted of one low voltage channel ranging from 50 - 150 Volts and two high voltage channels ranging from 400 - 1200 Volts. Also, all channels were constructed to have very fast rise times (approximately 0.0005 sec. or faster) so that the piezoelectric, mechanical, and fluid components could be driven up to very high frequencies (kHz range).

2.6 Piezohydraulic Design

Taking all the ideas into consideration from all the parts of the piezohydraulic actuator, a compact prototype was built as seen in Figure 2.8.

The piezohydraulic pump is mounted to the bread board along with the hydraulic cylinder. The output tube from the pumping chamber is connected with a very short steel connector to the input of the hydraulic cylinder. This minimal length tube is used to limit friction losses during flow. The input tube to the pumping chamber is connected to the other side of the hydraulic cylinder and then attached to the air/hydraulic accumulator.

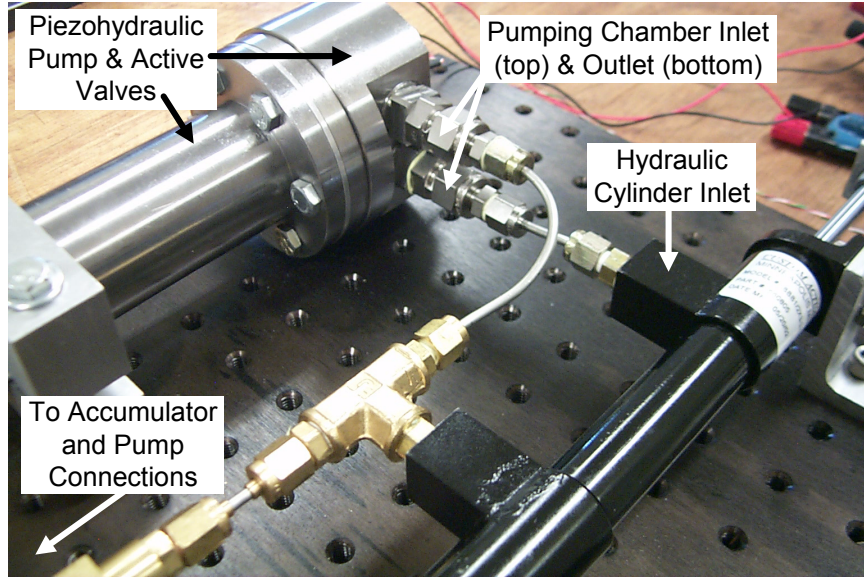


Figure 2.8: Piezohydraulic Actuator

Also connected to this line is the vacuum pump for evacuating air and the hydraulic pump for pressurizing the fluid circuit.

Initially nitrogen at 150 psi was inserted into one end of the accumulator. Then, the hydraulic circuit was built by connecting all the actuator components and pipe lines. When the circuit was completely set-up, the vacuum pump was connected to remove as much air from the system as possible. Immediately after closing the valve to the vacuum pump, hydraulic fluid was pumped into the fluid circuit, accumulator, hydraulic cylinder, and piezohydraulic pump. The amount of fluid fed into the actuator system by this pump thus determined how much the nitrogen in the accumulator was compressed, and therefore the fluid pre-pressurized value. With the system pressurized with hydraulic fluid, the piezohydraulic actuator was then ready for operation.

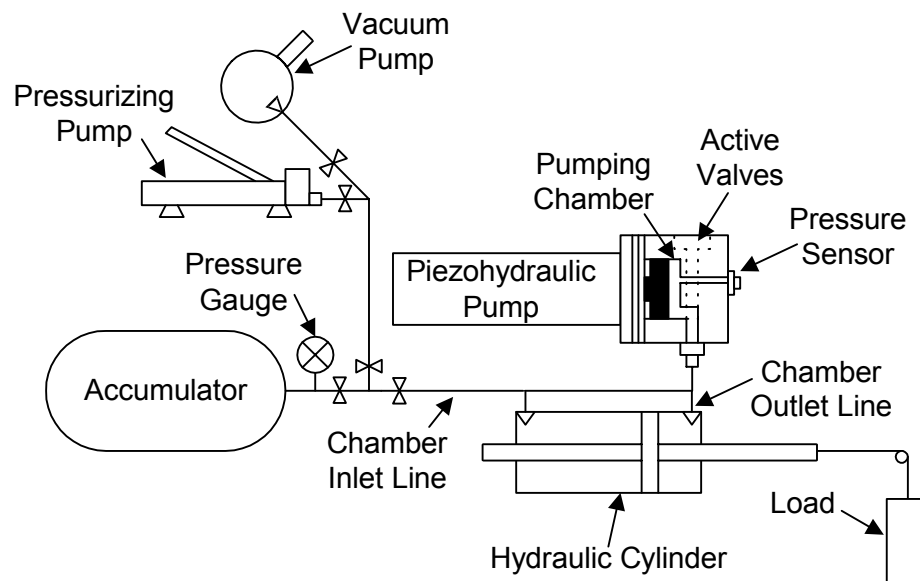


Figure 2.9: Piezohydraulic Actuator Schematic

Chapter 3

Lumped-Parameter Fluid Modeling

3.1 Introduction

Nasser et al. (2001); Nasser (2000) have previously used lumped-parameter models to simulate piezohydraulic actuators. They developed state-space modeling techniques to couple piezoelectric, hydraulic, and mechanical systems. Constraint matrices were also developed for connecting the elements, while linear algebra is used to form the state-space matrices for analysis. The models they developed have been tested and proven to be a good for predicting the dynamics of the piezohydraulic system by comparing the simulated results to actual experimental data. Along with being accurate, the lumped-parameter and state-space techniques are also easily implemented using computers and programs such as Matlab and Simulink.

3.2 Mechanical, Electrical and Fluid Element Analogy

To show how the mechanical, electrical, and hydraulic systems are similar, a brief description of each is presented.

3.2.1 Basic Mechanical System

First, the mechanical system is represented as seen in Figure 3.1 with a spring, mass, and damper. Using Newton's second law, the resulting equation of motion from the free body

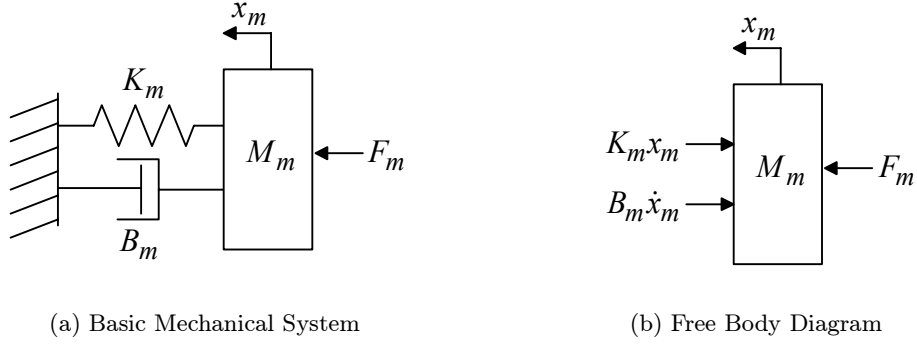


Figure 3.1: Mechanical System Analysis

diagram becomes:

$$\sum F = M_m \ddot{x}_m \quad (3.1)$$

$$M_m \ddot{x}_m = F_m - B_m \dot{x}_m - K_m x_m \quad (3.2)$$

$$F_m = M_m \ddot{x}_m + B_m \dot{x}_m + K_m x_m \quad (3.3)$$

where:

M_m	$\triangleq$	mass	$[\text{kg}]$
B_m	$\triangleq$	damper value	$\left[\frac{\text{N}\cdot\text{s}}{\text{m}}\right]$
K_m	$\triangleq$	spring stiffness	$\left[\frac{\text{N}}{\text{m}}\right]$
F_m	$\triangleq$	external force	$[\text{N}]$
$x_m, \dot{x}_m, \ddot{x}_m$	$\triangleq$	mass displacement, velocity, and acceleration	$[\text{m}], \left[\frac{\text{m}}{\text{s}}\right], \left[\frac{\text{m}}{\text{s}^2}\right]$

Taking the Laplace Transform of equation 3.3 then results in:

$$F_m = \dot{X}_m \left(M_m s + B_m + \frac{K_m}{s} \right) \quad (3.4)$$

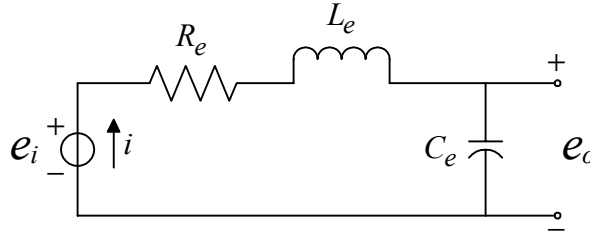
In transfer function form, the force/velocity ratio defines the mechanical impedance, Z_m , as:

$$\frac{F_m}{\dot{X}_m} = Z_m = \left(M_m s + B_m + \frac{K_m}{s} \right) \quad (3.5)$$

Mechanical Impedance is defined as the transfer function in which force is the numerator and velocity is the denominator. This force/velocity relationship seen in equation 3.5 is known as mechanical impedance, and the three terms can be broken-up for the three mechanical elements. Replacing the Laplace operator ($s = j\omega$) with the derivative operator ($D = \frac{d}{dt}$), the three elemental impedances are written as functions of derivatives with respect to time (Doebelin, 1972):

Table 3.1: Mechanical Elemental Impedance

Impedance Type	Definition
Stiffness ($Z_{spring}(D)$)	$\frac{K_m}{D} = \int K_m dt$
Damping ($Z_{damper}(D)$)	B_m
Inertia ($Z_{mass}(D)$)	$DM_m = \frac{dM_m}{dt}$

**Figure 3.2: Electrical System Analysis**

3.2.2 Basic Electrical System

A basic electrical circuit is shown in Figure 3.2 with a resistor, inductor, and capacitor, where:

e_i	$\triangleq$	input voltage	[Volts]
e_o	$\triangleq$	output voltage	[Volts]
R_e	$\triangleq$	electrical resistance	[ohms]
L_e	$\triangleq$	electrical inductance	[henries]
C_e	$\triangleq$	electrical capacitance	[farads]
i	$\triangleq$	electrical current	[amperes]

The electrical analysis is done using Kirchhoff's voltage loop law and Kirchhoff's current node law. The sum of the voltage drops in a closed loop must equal the sum of the voltage rises in the closed loop and the sum of the currents entering a node must equal the sum of the currents exiting a node. With the current traveling from left to right across an electrical element, the relationships are shown for the voltage drop across a resistor, capacitor, and inductor. In an ideal resistor, the voltage drop is equal to the current times the electrical resistance value as seen in Figure 3.3. In a capacitor, the voltage drop is equal to the inverse of the capacitance value times the integral of the current with respect to time as in

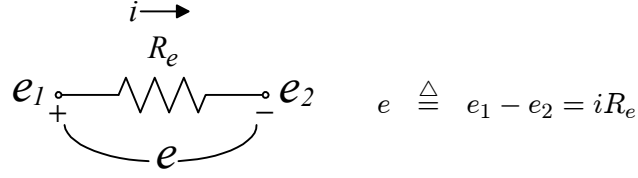


Figure 3.3: Electrical Resistor Voltage Drop

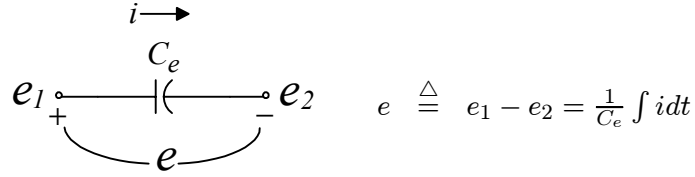


Figure 3.4: Electrical Capacitor Voltage Drop

Figure 3.4. And in an inductor, the voltage drop is equal to the inductance multiplied by the derivative of the current with respect to time, as demonstrated in Figure 3.5.

Using the voltage loop law, the input voltage will be equal to the sum of the individual voltage drops:

$$e_i = \left(iR_e + \frac{1}{C_e} \int idt + L_e \frac{di}{dt} \right) \quad (3.6)$$

Taking the Laplace Transform of equation 3.6 results in:

$$E_i = I \left(R_e + \frac{1}{sC_e} + sL_e \right) \quad (3.7)$$

And in transfer function form, the voltage/current ratio defines the electrical impedance, Z_e :

$$\frac{E_i}{I} = Z_e = \left(R_e + \frac{1}{sC_e} + sL_e \right) \quad (3.8)$$

Electrical Impedance is defined as the transfer function in which voltage is the numerator and current is the denominator. This voltage/current relationship seen in equation 3.8

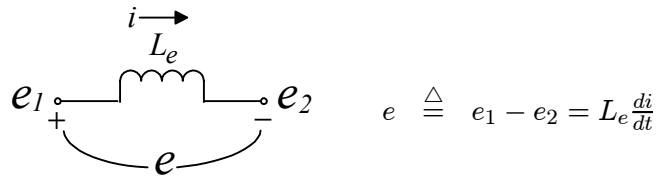


Figure 3.5: Electrical Inductor Voltage Drop

Table 3.2: Electrical Elemental Impedance

Impedance Type	Definition
Capacitance ($Z_{capacitor}(D)$)	$\frac{1}{DC_e} = \int \frac{1}{C_e} dt$
Resistance ($Z_{resistor}(D)$)	R_e
Inductance ($Z_{inductor}(D)$)	$DL_e = \frac{dL_e}{dt}$

is known as the circuit's electrical impedance, determined by the three individual electrical element impedances. Again, replacing the Laplace operator ($s = j\omega$) with the derivative operator ($D = \frac{d}{dt}$), the three elemental impedances are written as functions of derivatives with respect to time (Doebelin, 1972), as shown in Table 3.2.

3.2.3 Basic Fluid System

Finally, a basic fluid system as seen in Figure 3.6(a) is analyzed for a small pipe section, which can represent one of the sections from Figure 2.5. The tube length is broken-up into little sections of fluid and tubing where the pressure and velocity may vary with time, but are assumed uniform over the small volume. This system is a bit more complex, but the three basic element theory still applies and relates closely to the mechanical and electrical elements. Based on the type of flow, type of fluid used, thermal effects, type of tubing, and other variables, this analysis yields the fluid resistance, inertance, and compliance where:

P_{n-1}	$\triangleq$	previous section pressure	$\left[\frac{\text{N}}{\text{m}^2}\right]$
P_n	$\triangleq$	section pressure	$\left[\frac{\text{N}}{\text{m}^2}\right]$
R_f	$\triangleq$	section flow resistance	$\left[\frac{\text{kg}}{\text{m}^4\text{s}}\right]$
L_f	$\triangleq$	section fluid inertance	$\left[\frac{\text{kg}}{\text{m}^4}\right]$
C_f	$\triangleq$	section fluid compliance	$\left[\frac{\text{m}^4\text{s}^2}{\text{kg}}\right]$
q_{n-1}	$\triangleq$	previous section flowrate	$\left[\frac{\text{m}^3}{\text{s}}\right]$
q_n	$\triangleq$	section flowrate	$\left[\frac{\text{m}^3}{\text{s}}\right]$
A	$\triangleq$	section area	$[\text{m}^2]$
L	$\triangleq$	section length	$[\text{m}]$

The assumptions for our fluid analysis are one-dimensional horizontal flow, velocity

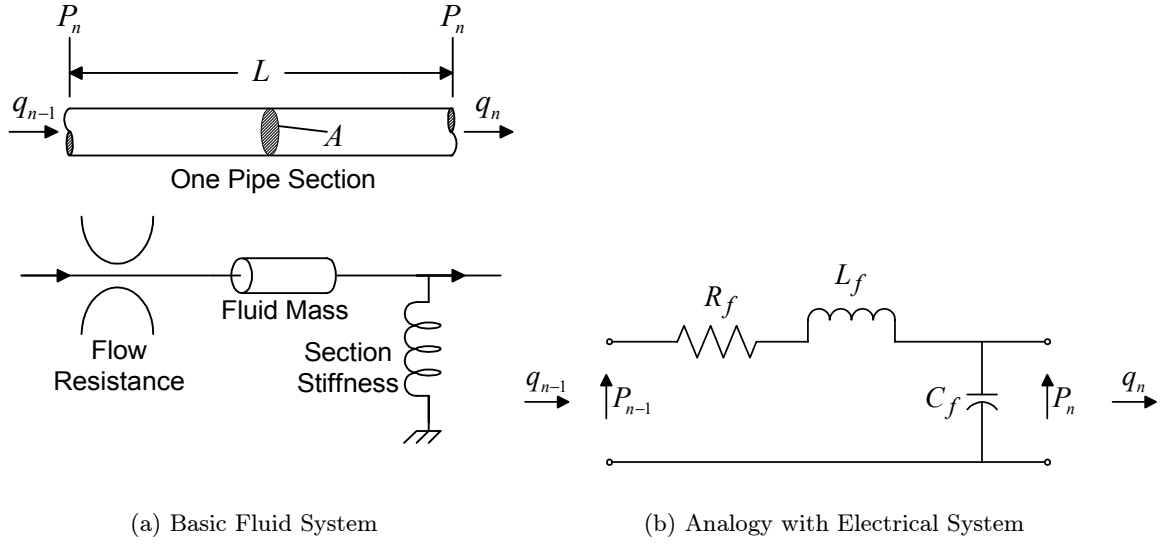


Figure 3.6: Fluid System Analysis

and pressure are uniform over the section area, average velocity is taken as the velocity of the entire pipe section, and section pressure corresponds to the pressure at any point in the cross-section. Also, thermal effects and small density changes with pressure are assumed to be negligible. Using the conservation of mass theory and Newton's second law, the definitions for the fluid impedance elements can be found.

3.3 Fluid Element Theory and Definition

3.3.1 Flow Resistance

Fluid friction is a complex topic and is treated at length in fluid dynamics courses. It is the cause for losses in flowrate and pressure within a pipe, thus it is known as flow resistance. The lumped-parameter flow resistance element is derived from Newton's second law where the sum of the forces acting upon a fluid element's mass is equal to the derivative with respect to time of the element's momentum. Thus, for a fluid section with constant mass, the sum of the forces is equal to the elemental mass times the derivative of velocity, or mass times acceleration. Due to the fluid flow and acceleration being so complex, the actual resistance value is highly dependent on the type of flow, dimensions of the tubing, and the length of travel.

Using Newton's second law for the sum of the forces acting on the fluid element

mass, the resistance element equation of motion is found

$$\begin{aligned}\sum F &= M_f \ddot{x}_f = \rho L \dot{q}_n \\ A(P_{n-1} - P_n - R_f q_n) &= \rho L \dot{q}_n.\end{aligned}$$

Assuming a massless fluid section, the pressure difference is equal to the fluid resistance times the flow rate

$$\Delta P = P_{n-1} - P_n = R_f q_n. \quad (3.9)$$

Now, by referring to the resistance equations from Doebelin (1972), the actual fluid resistance values for laminar flow may be found based on the pipe dimensions. For example, for low frequency laminar flow within a long thin walled pipe of circular cross-section, the flow resistance would be defined as in equation 3.10

$$R_f = \frac{128\mu L}{\pi D^4}. \quad (3.10)$$

where:

$$\begin{aligned}\mu &\triangleq \text{fluid dynamic viscosity} \quad \left[\frac{\text{kg}}{\text{m-s}} \right] \\ D &\triangleq \text{pipe inside diameter} \quad [\text{m}]\end{aligned}$$

For laminar flow conditions, the flow frequency (ω) and Reynolds number ($\mathbf{R}$) are defined by the following relationships, where $\dot{x}_f$ is the average velocity of the fluid section (Doebelin, 1972):

$$\begin{aligned}\omega &< \frac{32\mu}{\rho D^2} \\ \mathbf{R} &\triangleq \frac{\rho D \dot{x}_f}{\mu} < 2000\end{aligned}$$

If the flow frequency and Reynolds number are not under the conditions for laminar flow, then the flow will operate in the transition region or turbulent region. For the transition region, the flow resistance is undefined and laminar or turbulent definitions may be used. For turbulent flow to exist, the flow frequency and Reynolds number are defined as (Doebelin, 1972):

$$\begin{aligned}\omega &> \frac{7200\mu}{\rho D^2} \\ \mathbf{R} &\triangleq \frac{\rho D \dot{x}_f}{\mu} > 4000\end{aligned}$$

Also, for turbulent flow, the pressure/flowrate relationship for a circular pipe now becomes non-linear and is defined by equation 3.11

$$\Delta P = \frac{0.242 L \mu^{0.25} \rho^{0.75}}{D^{4.75}} q_n^{1.75}. \quad (3.11)$$

To linearly approximate this turbulent flow relationship, an expression has been found where the flow resistance value increases proportional to $\sqrt{\omega}$

$$R_f = \frac{8L}{\pi D^3} (2\rho\mu\omega)^{\frac{1}{2}}. \quad (3.12)$$

However, for non-circular pipe cross-sections, the diameter value (D) must be replaced with the equivalent hydraulic diameter, defined as $D_h = \frac{4 \text{ (pipe cross-section area)}}{\text{pipe perimeter}}$.

3.3.2 Fluid Inertance

Fluid inertance is the sum of the kinetic energy stored in a lump or section of fluid. Its derivation comes from the same equation of motion as shown for the flow resistance. This time however, the pipe section is assumed to have no losses or resistance, while also assuming that the section consists of a defined constant mass. Once again, pressure drops across a section of fluid will produce the external forces, which cause acceleration. Assuming all the mass particles within the section have the same velocity, we can treat it as a constant lump of mass with one velocity. Taking the equation of motion and applying the assumptions results in the pressure difference of equation 3.13

$$A(P_{n-1} - P_n - R_f q_n) = \rho L \dot{q}_n$$

$$\Delta P = P_{n-1} - P_n = \frac{\rho L}{A} \frac{dq_n}{dt} = I_f \frac{dq_n}{dt}. \quad (3.13)$$

Thus, the pressure difference (ΔP) and fluid inertance term (I_f) for turbulent flow are defined by equations 3.14 and 3.15

$$\Delta P = I_f \frac{dq_n}{dt} \quad (3.14)$$

$$I_f = \frac{\rho L}{A}. \quad (3.15)$$

where:

$$\omega > \frac{7200\mu}{\rho D^2}$$

$$\mathbf{R} > 4000$$

The fluid inertance term is not always fixed to equation 3.15 however, and is dependent on flow rate, similar to the flow resistance value. The equation above holds for high frequency operation and turbulent flow due to the assumed nearly square velocity profile. If

established laminar flow or low frequency operation is the case; then an effective mass term ($M_{eff} = \frac{4}{3}M_f$) will replace the original section mass. This is due to the actual parabolic velocity profile for laminar pipe flow, where a uniform profile was assumed in the analysis. Thus, equation 3.16 defines the fluid inertance value for laminar flow (Doebelin, 1972)

$$I_f = \frac{4\rho L}{3A}. \quad (3.16)$$

where:

$$\begin{aligned} \omega &< \frac{32\mu}{\rho D^2} \\ \mathbf{R} &< 2000 \end{aligned}$$

3.3.3 Fluid Compliance

Fluid compliance is derived by applying the conservation of mass to the small section of pipe. Referring to Figure 3.6, fluid mass enters the section at a rate of ρq_{n-1} , while it exits at a rate of ρq_n . For incompressible fluids the flow rates are identical, and thus there is no stored energy. Real fluids however, are compressible as seen in the bulk modulus analysis of section 2.4. This compressibility translates into mass storage where the flowrates are unequal on each side of a pipe section. For a section of fluid, the change in mass per time is defined by equation 3.17

$$\frac{dM_f}{dt} = (\rho q_{n-1} - \rho q_n). \quad (3.17)$$

The change in mass may also be defined by the change in volume, related to pressure through the bulk modulus value

$$dM_f = \rho dV_n = \rho \frac{AL}{\beta} dp_n. \quad (3.18)$$

Now, combining equations 3.17 and 3.18 results in the following expressions for pressure:

$$\frac{AL}{\beta} dp_n = (q_{n-1} - q_n) dt \quad (3.19)$$

$$dp_n = \frac{\beta}{AL} (q_{n-1} - q_n) dt \quad (3.20)$$

Taking the integral of equation 3.20 thus yields the equation for fluid compliance where P_n is equal to the pressure drop across the compliance element and $q_{n-1} - q_n$ corresponds to the flowrate passing through the compliance element. This is very similar to the

electrical capacitor analysis as seen in Figure 3.4, where i is the current passing through the capacitor and e is the voltage drop across the capacitor

$$P_n = \frac{\beta}{AL} \int (q_{n-1} - q_n) dt \quad (3.21)$$

$$P_n = \frac{1}{C_f} \int (q_{n-1} - q_n) dt \quad (3.22)$$

$$C_f = \frac{AL}{\beta}. \quad (3.23)$$

Combining this fluid compliance analysis with the effective fluid bulk modulus analysis of section 2.4, the β value in equation 3.23 may be substituted with the β_{eff} value from equation 2.13

$$C_f = AL \left(\frac{(1-x)}{\beta_{fluid}} + \frac{x}{\beta_{air}} + \frac{D}{tE} \right). \quad (3.24)$$

3.4 Single Fluid Lump/Element Model

After defining all the fluid elements, they can be combined in a similar fashion to the electrical circuit analysis where the sum of the voltage drops is equal to the input voltage. Using q as the flow rate passing through each fluid component, the sum of the pressure drops reduces to equation 3.25

$$\Delta P = \left(qR_f + \frac{1}{C_f} \int q dt + L_f \frac{dq}{dt} \right). \quad (3.25)$$

Taking the Laplace Transform of equation 3.25 results in

$$\Delta P = q \left(R_f + \frac{1}{sC_f} + sL_f \right), \quad (3.26)$$

and in transfer function form, the pressure/flow rate ratio defines the fluid impedance, Z_f

$$\frac{\Delta P}{q} = Z_f = \left(R_f + \frac{1}{sC_f} + sL_f \right). \quad (3.27)$$

The fluid section impedance is defined as the change in pressure (ΔP) divided by the flow rate (q). This pressure/flow rate relationship as seen in equation 3.27 is known as the pipe section's fluid impedance, determined by the three individual fluid element impedances. Again, replacing the Laplace operator ($s = j\omega$) with the derivative operator ($D = \frac{d}{dt}$), the three elemental impedances are written as functions of derivatives with respect to time (Doebelin, 1972; Martin and McCloy, 1980). Table 3.3 shows these derivatives.

Table 3.3: Fluid Elemental Impedance for Circular Pipe

Impedance Type	Definition	Impedance Value
Compliance ($Z_{\text{section stiffness}}(D)$)	$\frac{1}{DC_f} = \int \frac{1}{C_f} dt$	$C_f = AL \left(\frac{(1-x)}{\beta_{\text{fluid}}} + \frac{x}{\beta_{\text{air}}} + \frac{D}{tE} \right)$
Laminar Resistance ($Z_{\text{flow resistance}}(D)$)	R_f	$R_f = \frac{128\mu L}{\pi D^4}$
Laminar Inertance ($Z_{\text{fluid mass}}(D)$)	$DL_f = \frac{dL_f}{dt}$	$I_f = \frac{4\rho L}{3A}$
Turbulent Resistance ($Z_{\text{flow resistance}}(D)$)	R_f	$R_f = \frac{8L}{\pi D^3} (2\rho\mu\omega)^{\frac{1}{2}}$
Turbulent Inertance ($Z_{\text{fluid mass}}(D)$)	$DL_f = \frac{dL_f}{dt}$	$I_f = \frac{\rho L}{A}$

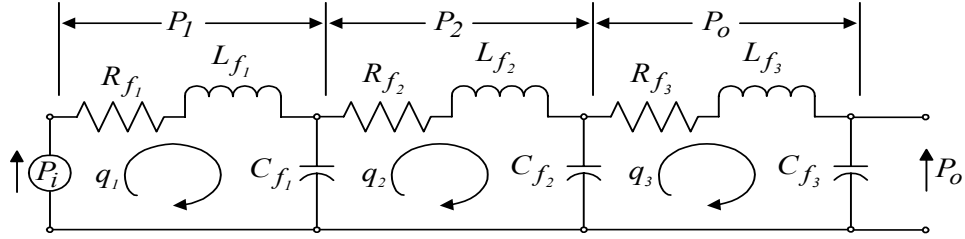


Figure 3.7: Fluid System of Multiple Elements

3.5 Multiple Lump/Element Models

Using the analysis for the one small section of pipe, several sections may be combined to analyze an entire length of pipe. Also, electrical and mechanical systems have well defined techniques for developing dynamic equations based on how their basic elements are connected. Due to the fluid's close correlation with these systems, similar techniques may be used to develop equations for systems with many basic fluid elements also. Figure 3.7 shows multiple lumps or sections of pipe connected for analysis.

Using the definitions of the individual fluid impedances, the following equations are developed based on the three loops and three flow rates defined as q_1 , q_2 , and q_3

$$\begin{aligned}
 P_i - P_1 &= \left(q_1 R_{f1} + L_{f1} \frac{dq_1}{dt} \right), \quad \text{where } P_1 = \frac{1}{C_{f1}} \int (q_1 - q_2) dt \\
 P_1 - P_2 &= \left(q_2 R_{f2} + L_{f2} \frac{dq_2}{dt} \right), \quad \text{where } P_2 = \frac{1}{C_{f2}} \int (q_2 - q_3) dt \\
 P_2 - P_o &= \left(q_3 R_{f3} + L_{f3} \frac{dq_3}{dt} \right), \quad \text{where } P_o = \frac{1}{C_{f3}} \int (q_3) dt.
 \end{aligned}$$

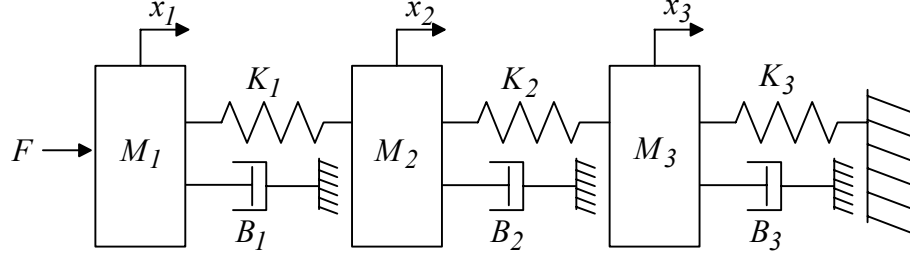


Figure 3.8: Mechanical System of Multiple Elements

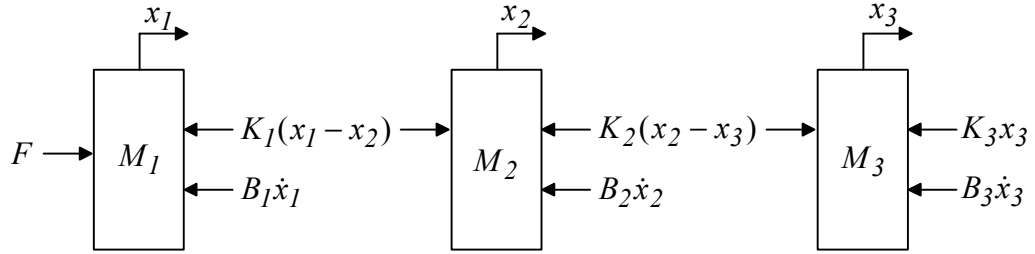


Figure 3.9: Multiple Element Free Body Diagram

Simplifying these equations by substitution then yields

$$P_i = q_1 R_{f1} + L_{f1} \frac{dq_1}{dt} + \frac{1}{C_{f1}} \int (q_1 - q_2) dt \quad (3.28)$$

$$0 = q_2 R_{f2} + L_{f2} \frac{dq_2}{dt} + \frac{1}{C_{f1}} \int (q_2 - q_1) dt + \frac{1}{C_{f2}} \int (q_2 - q_3) dt \quad (3.29)$$

$$0 = q_3 R_{f3} + L_{f3} \frac{dq_3}{dt} + \frac{1}{C_{f2}} \int (q_3 - q_2) dt + \frac{1}{C_{f3}} \int (q_3) dt. \quad (3.30)$$

This analysis is very close to the analysis for a similar electric circuit where the flow rate is replaced by current and pressure is replaced by voltage. It also relates closely to a mechanical system where the flow rate is replaced by velocity and pressure is replaced by force. Analyzing a multiple mechanical element system as seen in Figure 3.8 will show this similarity. The free body diagrams that result from the multiple element system are seen in Figure 3.9.

Using Newton's second law once again, the sum of the forces acting on each body must equal the time rate of change of each body's momentum. Assuming constant mass, the following are the resulting equations of motion for each mass/element:

$$M_1 \ddot{x}_1 = F - K_1(x_1 - x_2) - B_1 \dot{x}_1$$

$$M_2 \ddot{x}_2 = K_1(x_1 - x_2) - K_2(x_2 - x_3) - B_2 \dot{x}_2$$

$$M_3 \ddot{x}_3 = K_2(x_2 - x_3) - K_3 x_3 - B_3 \dot{x}_3$$

Table 3.4: Mechanical/Fluid Analogy

Mechanical Elements	Mechanical Force	Fluid Elements	Fluid Pressure
M_m (Inertia)	$F = M_m \frac{d\dot{x}_m}{dt}$	L_f (Inertance)	$\Delta P = L_f \frac{dq}{dt}$
B_m (Damping)	$F = B_m \dot{x}_m$	R_f (Resistance)	$\Delta P = R_f q$
K_m (Stiffness)	$F = K_m \int (\dot{x}_m) dt$	C_f (Compliance)	$P = \frac{1}{C_f} \int (q) dt$

Simplifying these equations thus leads to:

$$\begin{aligned}
F &= M_1 \ddot{x}_1 + B_1 \dot{x}_1 + K_1 (x_1 - x_2) \\
0 &= M_2 \ddot{x}_2 + B_2 \dot{x}_2 + K_1 (x_2 - x_1) + K_2 (x_2 - x_3) \\
0 &= M_3 \ddot{x}_3 + B_3 \dot{x}_3 + K_2 (x_3 - x_2) + K_3 x_3
\end{aligned}$$

Rewriting the relationships in terms of velocity only (and thus placing them into a form for comparison) results in equations 3.31, 3.32, and 3.33

$$F = M_1 \frac{d\dot{x}_1}{dt} + B_1 \dot{x}_1 + K_1 \int (\dot{x}_1 - \dot{x}_2) dt \quad (3.31)$$

$$0 = M_2 \frac{d\dot{x}_2}{dt} + B_2 \dot{x}_2 + K_1 \int (\dot{x}_2 - \dot{x}_1) dt + K_2 \int (\dot{x}_2 - \dot{x}_3) dt \quad (3.32)$$

$$0 = M_3 \frac{d\dot{x}_3}{dt} + B_3 \dot{x}_3 + K_2 \int (\dot{x}_3 - \dot{x}_2) dt + K_3 \int (\dot{x}_3) dt. \quad (3.33)$$

Now, comparing equations 3.28, 3.29, and 3.30 with equations 3.31, 3.32, and 3.33; the similarity of the systems becomes very clear. The input force to input pressure relationship is seen along with the flow rate to velocity relationship. Also, the locations inside the equations of the fluid elemental values (inertance, resistance, and compliance) is the same for the mechanical elemental values (inertia, damping, and stiffness), relative to the flow variables q and $\dot{x}$. The force/pressure analogy between the mechanical system and the fluid system is shown in Table 3.4.

By realizing that pressure (P) is related to force (F) and flow rate (q) is related to velocity ($\dot{x}$), relationships can be found to equate the mechanical elements to the fluid elements. Force is equal to pressure times area and flow rate is equal to velocity times area. Using these definitions, the force/velocity relationship is found to be equal to the

Table 3.5: Fluid Element Definitions Using Mechanical System Analysis

New Fluid Element Definition	Force Analysis
M_f (Fluid Mass) $= A^2 L_f$	$F = M_f \frac{d\dot{x}_f}{dt}$
B_f (Fluid Damping) $= A^2 R_f$	$F = B_f \dot{x}_f$
K_f (Fluid/Pipe Stiffness) $= \frac{A^2}{C_f}$	$F = K_f \int (\dot{x}_f) dt$

pressure/flow rate relationship multiplied by the area squared

$$F = PA \quad , \quad \dot{x} = \frac{q}{A}$$

$$\frac{F}{\dot{x}} = \frac{PA}{\frac{q}{A}}$$

$$\frac{F}{\dot{x}} = \frac{PA^2}{q}. \quad (3.34)$$

Knowing this, the definition of the mechanical elements can be found in terms of the fluid elements

$$\frac{F}{\dot{x}} = (\text{mechanical element}) = \frac{PA^2}{q} = (\text{fluid element})A^2$$

$$(\text{mechanical element}) = (\text{fluid element})A^2.$$

Thus, the analysis of a fluid system can be defined and analyzed as a mechanical system ($\dot{x}_f$ = average fluid velocity), with the elemental definitions seen in Table 3.5.

Chapter 4

Piezohydraulic System Models

4.1 Introduction

The model developed by Nasser et al. (2001); Nasser (2000) works for one entire piezohydraulic system, where the piezoelectric actuator is the beginning and the closed valve is the end. Two long hydraulic fluid lines connect the piston, hydraulic cylinder, and closed valve. Depending on the motion of the piston and which valve is open, the system would either be pushing fluid or pulling fluid from one of the long fluid lines. In pushing mode, the fluid lines are compressed and the hydraulic cylinder between them is displaced by this compression. In pulling mode, the compressed fluid expands and the cylinder is displaced even more when the valves and piston switch/reverse direction. This dynamic behavior thus resulted in the development of two models to simulate the piezohydraulic circuit, one for pushing and one for pulling. The valve operation used to determine which model to run was simulated by choosing different valve transition times/rise time values.

The model developed for the piezohydraulic system pertaining to this research has more systems/different elements and consists of four operation stages opposed to the two used by Nasser et al. (2001); Nasser (2000). Also, an attempt was made to model the valve dynamics for pressure loss calculations and more accurate stage timing. Other contributions from this model include pumping chamber, output cylinder, and fluid circuit dynamics as well.

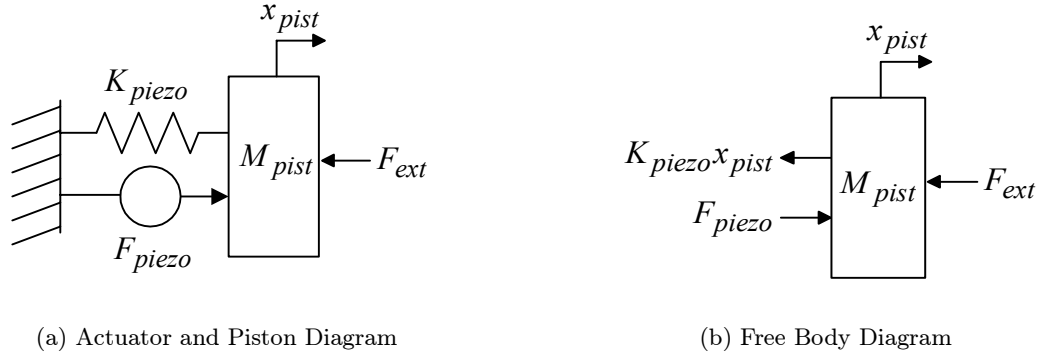


Figure 4.1: Piezoelectric Actuator and Piston Analysis

4.2 Piezoelectric Actuator and Piston Model

The tip displacement of a piezoelectric material consisting of certain dimensions and properties is dependent on applied charge, current, and voltage. Very accurate and complex equations for the dynamics of a piezoelectric material based on material properties and dimensions can be found in Ikeda (1990); Shields (1966); IEEE (1987), while PI (1999, 2002); Piezomechanik (2002) provide only a basic theory. As mentioned before, the dynamics are now very well known, and these more accurate piezoelectric equations even include nonlinear effects such as the hysteresis between applied voltage and tip extension. This is a good beginning, however our concern in piezohydraulics is focused more on the entire piezohydraulic actuator system as opposed to just piezoelectric dynamics. Therefore, only a basic model of a piezoelectric actuator will be discussed and used, with material/actuator properties derived by the manufacturers. The force generation ($F_{piezo} = K_{piezo}x_oV$) and tip extension (x_{piezo}) will thus be determined by the applied amplifier voltage (V), piezoelectric material/actuator stiffness (K_{piezo}), other material/actuator constants (x_o), and the external forces acting on actuator (F_{piezo}).

A one-dimensional linear model of a piezoelectric actuator with a mass/piston at the tip is displayed in Figure 4.1, based on the one used by Nasser and Leo (2000) in their efficiency of frequency-rectified piezohydraulic actuation analysis. The maximum force able to be generated is the manufacturer's blocked force ($F_{blocked}$), while the rated free displacement (x_{free}) is the maximum tip/mass displacement (x_{pist}) of the actuator at the specified applied maximum voltage value (V_{max}). Other important parameters are the actuator's mechanical stiffness value (K_{piezo}) and the displacement to voltage conversion

factor (x_o), defined as $\frac{x_{free}}{V_{max}}$. Analyzing the free body diagram of Figure 4.1 results in the equations of motion

$$\sum F = M_{pist}\ddot{x}_{pist} \quad (4.1)$$

$$M_{pist}\ddot{x}_{pist} = K_{piezo}x_oV - K_{piezo}x_{pist} - F_{ext}. \quad (4.2)$$

Equation 4.3 is used to calculate the actual tip displacement based on the applied voltage and external forces acting on the tip, while equation 4.4 is used to calculate the force generated by the piezoelectric actuator

$$x_{pist} = x_oV - \frac{F_{ext}}{K_{piezo}} \quad (4.3)$$

$$F_{ext} = K_{piezo}x_oV - K_{piezo}x_{pist}. \quad (4.4)$$

Using the equation of motion and removing the piston mass attached, the equations for just the piezoelectric actuator can be defined. When $F_{ext} = 0$, $M_{pist} = 0$, and $x_{pist} = x_{free}$, the free displacement equation becomes

$$x_{free} = x_oV_{max}. \quad (4.5)$$

When $x_{pist} = 0$, $M_{pist} = 0$, and $F_{blocked} = F_{ext}$, the blocked force equation becomes

$$F_{blocked} = K_{piezo}x_oV_{max}. \quad (4.6)$$

When modeling this system, the variables in these equations are replaced by the actual values for the piezoelectric actuator used in the experiments (model no. PI 844.60). Table 4.1 defines the properties for this actuator.

4.3 Closed Pumping Chamber Model

The closed pumping chamber system consists of the piezoelectric actuator, piston, pumping chamber, valve chamber entrance and exit lines, and the closed valves. A detailed view of this interaction is displayed in Figure 2.3, while a schematic diagram for this analysis can be seen in Figure 4.2. The variables in the model are: K_{o-ring} (stiffness of the o-ring), B_{pist} (damping/friction on the piston), B_{o-ring} (damping from the o-ring), x_{pc} (displacement of the fluid in the pumping chamber), x_{pce} (displacement of the fluid exiting the pumping chamber), x_{vc} (displacement of the fluid in the valve chamber), K_{pc} (stiffness of the

Table 4.1: Piezoelectric Actuator Properties (model no. PI 844.60) (PI, 1999, 2002)

Property	Value
Free Displacement, x_{free}	90 [μm]
Blocked Force, $F_{blocked}$	3000 [N]
Max Pull Force, F_{pull}	700 [N]
Maximum Voltage, V_{max}	100 [Volts]
Displacement to Voltage Factor, x_o	$9 \times 10^{-7} [\frac{\text{m}}{\text{Volt}}]$
Actuator Stiffness, K_{piezo}	$38 \times 10^6 \pm 20\% [\frac{\text{N}}{\text{m}}]$

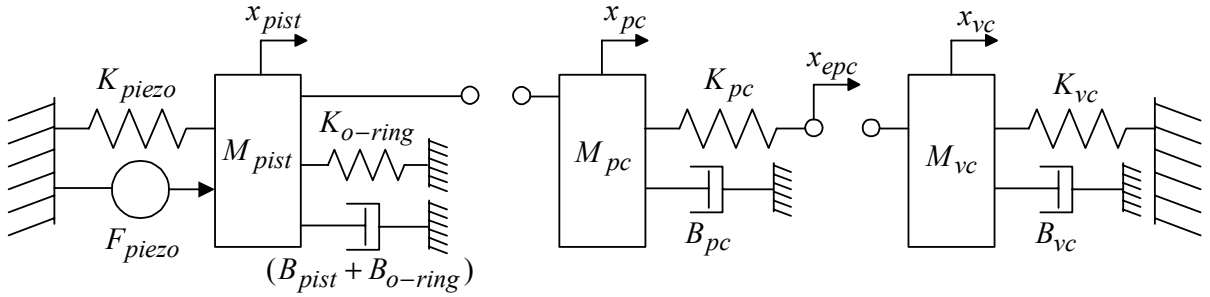


Figure 4.2: Closed Pumping Chamber Diagram

fluid/piping in the pumping chamber), B_{pc} (damping of the fluid in the pumping chamber), K_{vc} (stiffness of the fluid in the valve chamber), and B_{vc} (damping of the fluid in the valve chamber). The piston is assumed to be much stiffer than the fluid and piezoelectric material such that its stiffness is infinite. Thus, the piston and fluid in the pumping chamber have the same displacement, velocity, and acceleration. In this closed chamber system, the piezoceramic material will extend with the voltage applied to it, the fluid elements will be compressed (volume change), and pressure will build until the out valve opens.

The free body diagrams that result from the analysis of this system are seen in Figure 4.3. Using the free body diagram analysis, the resulting equations become:

$$\begin{aligned}
 M_{pist}\ddot{x}_{pist} &= K_{piezo}x_oV - K_{piezo}x_{pist} - K_{o-ring}x_{pist} - B_{pist}\dot{x}_{pist} - B_{o-ring}\dot{x}_{pist} \\
 M_{pc}\ddot{x}_{pc} &= -K_{pc}x_{pc} - B_{pc}\dot{x}_{pc} + K_{pc}x_{epc} \\
 0 &= K_{pc}x_{pc} - K_{pc}x_{epc} \\
 M_{vc}\ddot{x}_{vc} &= -K_{vc}x_{vc} - B_{vc}\dot{x}_{vc}
 \end{aligned}$$

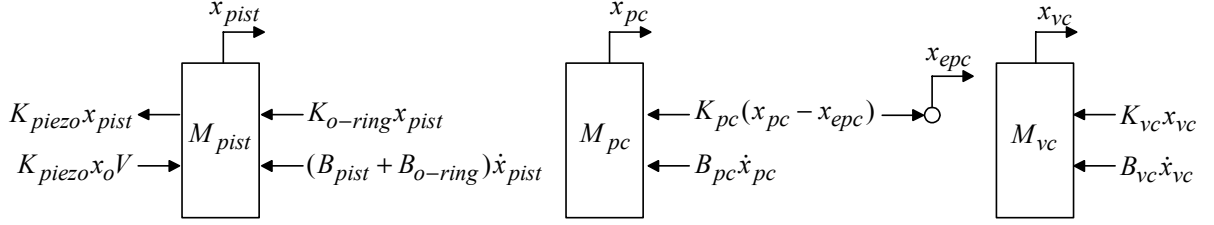


Figure 4.3: Pumping Chamber Free Body Diagrams

Simplifying yields the following relationships:

$$\begin{aligned}
 M_{pist} \ddot{x}_{pist} + (B_{pist} + B_{o-ring}) \dot{x}_{pist} + (K_{piezo} + K_{o-ring}) x_{pist} &= K_{piezo} x_o V \\
 M_{pc} \ddot{x}_{pc} + B_{pc} \dot{x}_{pc} + K_{pc} x_{pc} - K_{pc} x_{epc} &= 0 \\
 -K_{pc} x_{pc} + K_{pc} x_{epc} &= 0 \\
 M_{vc} \ddot{x}_{vc} + B_{vc} \dot{x}_{vc} + K_{vc} x_{vc} &= 0
 \end{aligned}$$

Putting the equations in matrix form, $\mathbf{M}\ddot{\mathbf{X}} + \mathbf{B}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F}$, becomes:

$$\begin{bmatrix} M_{pist} & 0 & 0 & 0 \\ 0 & M_{pc} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{vc} \end{bmatrix} \begin{bmatrix} \ddot{x}_{pist} \\ \ddot{x}_{pc} \\ \ddot{x}_{epc} \\ \ddot{x}_{vc} \end{bmatrix} + \begin{bmatrix} (B_{pist} + B_{o-ring}) & 0 & 0 & 0 \\ 0 & B_{pc} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_{vc} \end{bmatrix} \begin{bmatrix} \dot{x}_{pist} \\ \dot{x}_{pc} \\ \dot{x}_{epc} \\ \dot{x}_{vc} \end{bmatrix} + \begin{bmatrix} (K_{piezo} + K_{o-ring}) & 0 & 0 & 0 \\ 0 & K_{pc} & -K_{pc} & 0 \\ 0 & -K_{pc} & K_{pc} & 0 \\ 0 & 0 & 0 & K_{vc} \end{bmatrix} \begin{bmatrix} x_{pist} \\ x_{pc} \\ x_{epc} \\ x_{vc} \end{bmatrix} = \begin{bmatrix} K_{piezo} x_o V \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The constraint matrix is defined using the conservation of volume, incorporating the change of area from the pumping chamber to the valve chamber. The resulting relationships are thus, $\mathbf{X} = \mathbf{T}\mathbf{X}_{states}$, $\dot{\mathbf{X}} = \mathbf{T}\dot{\mathbf{X}}_{states}$, and $\ddot{\mathbf{X}} = \mathbf{T}\ddot{\mathbf{X}}_{states}$. Also to note, the valve chamber damping value (B_{vc}) is increased by the change of area as discussed in section 2.4. There is a loss factor associated with this area change that can be converted into an equivalent length. This in turn is then added to the actual length of the valve chamber to obtain the increased damping value. The conversion from all the node displacements to only the state

displacements is defined by equation 4.7

$$\begin{bmatrix} x_{pist} \\ x_{pc} \\ x_{epc} \\ x_{vc} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & \frac{A_{vc}}{A_{pc}} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_{pist} \\ x_{vc} \end{bmatrix}. \quad (4.7)$$

Linear algebra then allows us to convert the equations into a state-space system form

$$\begin{aligned} \mathbf{M}\ddot{\mathbf{X}} + \mathbf{B}\dot{\mathbf{X}} + \mathbf{K}\mathbf{X} &= \mathbf{F} \\ \mathbf{MT}\ddot{\mathbf{X}}_{states} + \mathbf{BT}\dot{\mathbf{X}}_{states} + \mathbf{KT}\mathbf{X}_{states} &= \mathbf{F} \end{aligned}$$

$$\mathbf{T}^T \mathbf{MT}\ddot{\mathbf{X}}_{states} + \mathbf{T}^T \mathbf{BT}\dot{\mathbf{X}}_{states} + \mathbf{T}^T \mathbf{KT}\mathbf{X}_{states} = \mathbf{T}^T \mathbf{F}. \quad (4.8)$$

Using equation 4.8, we now define the terms $\mathbf{M}_{trans}$, $\mathbf{B}_{trans}$, and $\mathbf{K}_{trans}$

$$\mathbf{M}_{trans} = \mathbf{T}^T \mathbf{MT} \quad (4.9)$$

$$\mathbf{B}_{trans} = \mathbf{T}^T \mathbf{BT} \quad (4.10)$$

$$\mathbf{K}_{trans} = \mathbf{T}^T \mathbf{KT}. \quad (4.11)$$

Left multiplying by $\mathbf{M}_{trans}^{-1}$ into equation 4.8 and rearranging the terms yields the second order relationship

$$\ddot{\mathbf{X}}_{states} = -\mathbf{M}_{trans}^{-1} \mathbf{B}_{trans} \dot{\mathbf{X}}_{states} - \mathbf{M}_{trans}^{-1} \mathbf{K}_{trans} \mathbf{X}_{states} + \mathbf{M}_{trans}^{-1} \mathbf{T}^T \mathbf{F}.$$

Converting this into the state-space form, $\begin{Bmatrix} \dot{\mathbf{X}}_{states} \\ \ddot{\mathbf{X}}_{states} \end{Bmatrix} = \mathbf{A} \begin{Bmatrix} \mathbf{X}_{states} \\ \dot{\mathbf{X}}_{states} \end{Bmatrix} + \mathbf{BU}$, and defining $\mathbf{U}$ as the input voltage V , results in equation 4.12

$$\begin{bmatrix} \dot{\mathbf{X}}_{states} \\ \ddot{\mathbf{X}}_{states} \end{bmatrix} = \begin{bmatrix} 0 & \mathbf{I} \\ -\mathbf{M}_{trans}^{-1} \mathbf{K}_{trans} & -\mathbf{M}_{trans}^{-1} \mathbf{B}_{trans} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{states} \\ \dot{\mathbf{X}}_{states} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathbf{M}_{trans}^{-1} \mathbf{T}^T K_{piezo} x_o \end{bmatrix} V. \quad (4.12)$$

The state-space output equation, $\mathbf{Y} = \mathbf{C} \begin{Bmatrix} \mathbf{X}_{states} \\ \dot{\mathbf{X}}_{states} \end{Bmatrix} + \mathbf{DU}$, is defined by outputting all the states

$$\mathbf{Y} = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{I} \end{bmatrix} \begin{bmatrix} \mathbf{X}_{states} \\ \dot{\mathbf{X}}_{states} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} V. \quad (4.13)$$

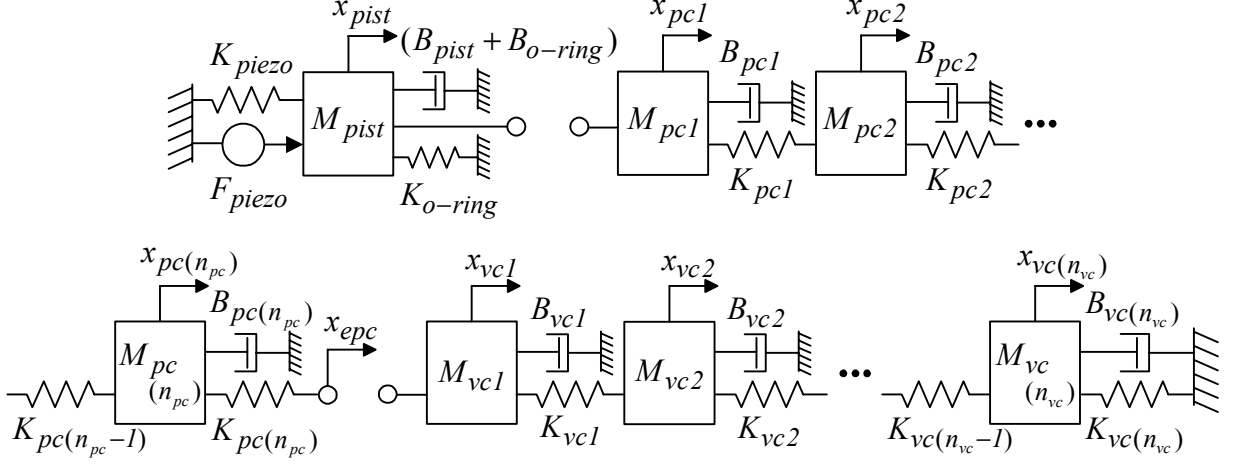


Figure 4.4: Multiple Fluid Element Closed Pumping Chamber Diagram

4.3.1 Multiple Fluid Elements

The derivation shown for the closed chamber model only consisted of one fluid element for the pumping chamber section and one fluid element for the valve chamber section. Due to the differences in area, at least one element is needed for each section in order to model the system. This one element for each section does not, however, guarantee an accurate model. Thus, a diagram is shown in Figure 4.4, depicting multiple fluid elements within each part where fluid is present (pumping chamber and valve chamber).

Once again free body diagrams may be drawn and analyzed for the multiple fluid element system; with the mass (**M**), damping (**B**), and stiffness (**K**) matrices all formed in the same manner as the single element model. The resulting constraint matrix (**T**) is shown in equation 4.14, where n_{pc} is the number of fluid elements in the pumping chamber

and n_{vc} is the number of fluid elements in the valve chamber

$$\begin{bmatrix} x_{pist} \\ x_{pc1} \\ x_{pc2} \\ \vdots \\ x_{pc(n_{pc})} \\ x_{epc} \\ x_{vc1} \\ x_{vc2} \\ \vdots \\ x_{vc(n_{vc})} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & \frac{A_{vc}}{A_{pc}} & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \dots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 & 0 & 0 & \dots & 1 \end{bmatrix} \begin{bmatrix} x_{pist} \\ x_{pc2} \\ \vdots \\ x_{pc(n_{pc})} \\ x_{vc1} \\ x_{vc2} \\ \vdots \\ x_{vc(n_{vc})} \end{bmatrix}. \quad (4.14)$$

4.4 Piezohydraulic Active Valve Model

The Active Valve Model is very similar to the closed chamber model, however the piezoelectric material, fluid areas, and fluid lengths are much smaller in the valves. Also, a spring is used to help the valve return to the closed position, which is not present in the closed chamber model. A piezoelectric translator is used once again to create the piston motion within the valve pumping chamber. Then, a hydraulic displacement/velocity amplification occurs when the fluid passes from the large area of the valve pumping chamber to the small area of the valve amplification section. This fluid then pushes the valve open, which also compresses the return spring. Figure 4.5 describes the parts of the valve where h is the height of the valve opening (amount the spring is compressed dynamically) and q is the flowrate of fluid into the valve.

The valve opening area (A_o) is thus equal to the outer circumference of the cylindrical valve multiplied by the valve opening (h). Defining the area of the pipe section after the valve as the valve exit area (A_{ve}) and applying equations 2.6 and 2.7, the pressure loss through the valve becomes:

$$\zeta_{orifice} = \left[\frac{1}{2} \left(1 - \frac{(\pi D_{valve} h)}{A_{vc}} \right)^{\frac{3}{4}} + \left(1 - \frac{(\pi D_{valve} h)}{A_{ve}} \right)^2 \right] \quad (4.15)$$

$$\Delta P = \frac{1}{2C_d^2} \rho \zeta_{orifice} \frac{(A_{vc} \dot{x}_{vc})^2}{(\pi D_{valve} h)^2} \quad (4.16)$$

Multiplying this pressure loss by the open valve area provides an equivalent force that can

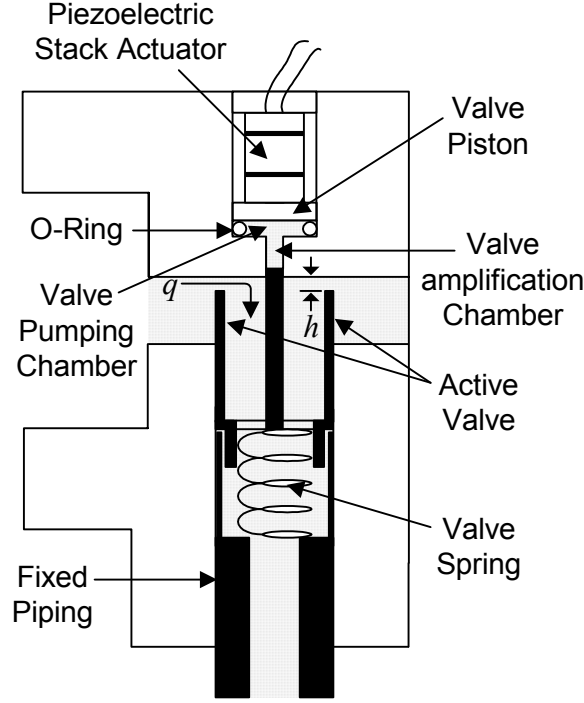


Figure 4.5: Active Valve Detail

be calculated for dynamic analysis. This force can be applied to the mass in the model located on the other side of the valve to simulate the pressure drop in the system

$$F_{valve} = \frac{1}{2C_d^2} \rho \zeta_{orifice} \frac{(A_{vc} \dot{x}_{vc})^2}{(\pi D_{valve} h)}. \quad (4.17)$$

4.5 Modeling After Outlet Valve Opens

The inertia associated with and the force acting upon the hydraulic cylinder while the outlet valve is open may be implemented by a model using a basic dynamic analysis of the hydraulic cylinder rod with a load hanging from it. The fluid entering the hydraulic cylinder applies a force on the rod to cause motion. At the same time, there are friction forces and a tension force acting against the fluid force to restrict motion. A better visualization of these forces can be seen in Figure 4.6(a) and a free body diagram analysis is shown in Figure 4.6(b).

Using the sum of forces analysis where g = acceleration due to gravity, the resulting

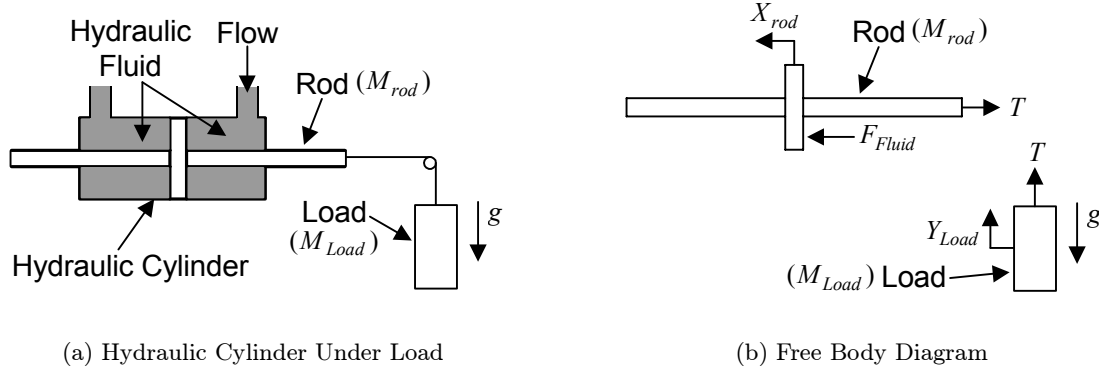


Figure 4.6: Hydraulic Cylinder and Load Analysis

equations of motion for the rod and the load are:

$$M_{rod}\ddot{X}_{rod} = F_{Fluid} - T \quad (4.18)$$

$$M_{Load}\ddot{Y}_{Load} = T - M_{Load}(g) \quad (4.19)$$

Solving for T and combining results in equations 4.20 and 4.21

$$M_{rod}\ddot{X}_{rod} = F_{Fluid} - M_{Load}\ddot{Y}_{Load} - M_{Load}(g) \quad (4.20)$$

$$(M_{rod} + M_{Load})\ddot{X}_{rod} = F_{Fluid} - M_{Load}(g). \quad (4.21)$$

If a rigid cable connects the rod and load, we can assume that they both travel with the same displacement, velocity, and acceleration to further simplify the results ($\ddot{Y}_{Load} = \ddot{X}_{rod}$). For the hydraulic cylinder to accelerate and thus start to move from rest, the fluid force pushing on it must overcome the inertia of the cylinder rod, the load, and the static weight of the load. The rod is assumed to have infinite stiffness such that its displacement, velocity, and acceleration are equal to the motion of the fluid lump directly after it. Also, the fluid force (F_{Fluid}) acting on the rod will be equal to the force transmitted by the fluid element in front, minus the force transmitted against the rod motion by the fluid element after it.

When the outlet valve of the actuator opens, the dynamic system model extends from the piezoceramic actuator pushing the piston, through the outlet valve, up to the hydraulic cylinder, all the way until the pressurized accumulator. Therefore, all of these components and the hydraulic fluid between them must be modeled. This model can be seen in Figure 4.7, where the bold arrows signify a force generation (piezoelectric actuator) or force loss (valves or load) in the system. Also, each different section of pipe in the fluid

circuit must have at least one element associated with it for modeling purposes, so there are many different fluid element definitions. Just as in the closed chamber model, each of these fluid lumps may be broken up into multiple element/lump sections for a more accurate model.

The model seen in Figure 4.7 is of the entire open fluid circuit. The piezoelectric actuator pushing the piston will extend until the force of the circuit (elements, valve, and load) is equal to the pushing force of the stack. The fluid elements are compressed and force is applied to each, causing them to move and eventually force hydraulic cylinder rod motion. The time for this to occur depends on the stiffness of the elements, the inertia (mass) of the elements and load, and the weight of the load that must be overcome. On the other side of the hydraulic cylinder rod, fluid elements are compressed against the gas (nitrogen) inside the accumulator. Finally, the outlet valve closes and cylinder rod displacement settles until eventually stopping when the fluid force pushing the rod is no longer greater than the load and friction inside the hydraulic cylinder.

Lengths, diameters, and tubing thicknesses for each section of the fluid circuit seen in Figure 4.7 were measured or estimated. Once gathered, these dimensions were incorporated into the modeling. Table 4.2 gives dimensions for each of the piping sections filled with fluid. The free body diagrams and equations of motion are developed for this large system in the same manner as the smaller systems. Once again, constraint matrices defined by the conservation of volume are used to couple the individual elements. Matlab code, seen in Appendix B is used to define the mass, damping, stiffness, force, and constraint matrices for each of these systems. Then, linear algebra is used inside the m-file to convert these into a state-space form. Once in state-space form, the A, B, C, and D matrices are used inside Simulink block diagram models to simulate the dynamic response due to voltage inputs and pre-pressure, while under various loads.

4.5.1 Out Valve Closes

When the outlet valve closes, the hydraulic cylinder will continue to move and settle based on how much fluid has passed through the valve into the cylinder side of the valve. The volume of fluid inside the pumping and valve chambers is less than the initial volume while the piezoceramic material attempts to retract to its original position. Thus, two systems are needed in this cycle to model the system dynamics. The closed chamber model of before

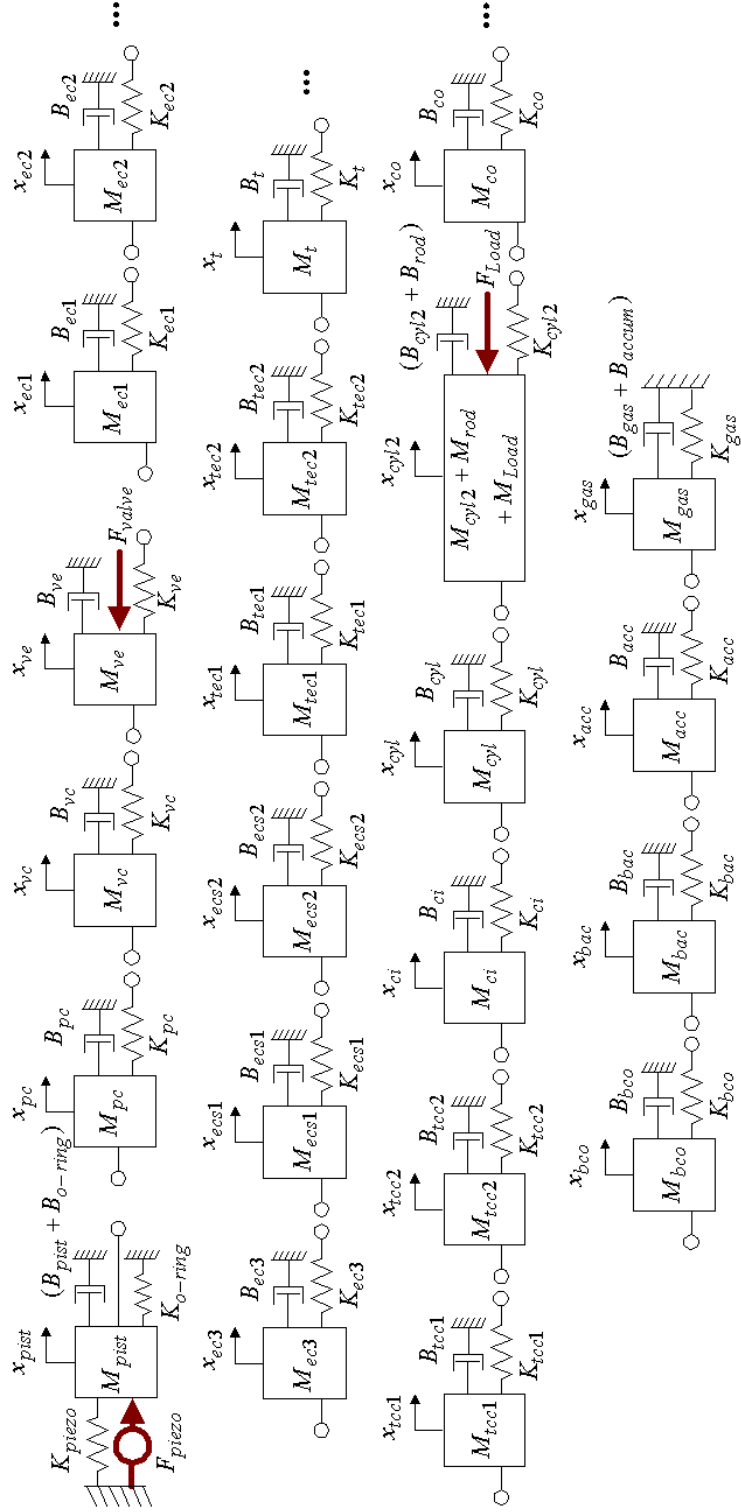


Figure 4.7: Entire Fluid System

Table 4.2: Fluid Section Dimensions

Section	Length [mm]	Inside Diameter [mm]	Tube Thickness [mm]	Material
<i>pc</i>	2.4	25	10	Stainless
<i>vc</i>	22	$H = 2.9, W = 10.5$	10	Stainless
<i>ve</i>	10	8	10.5	Stainless
<i>ec1</i>	7.15	8.5	3.5	Stainless
<i>ec2</i>	11.48	5	5	Stainless
<i>ec3</i>	8.77	11.55	5	Stainless
<i>ecs1</i>	8.6	6.35	2.6	Stainless
<i>ecs2</i>	9.9	6.35	11	Stainless
<i>tec1</i>	17	6.35	0.8	Stainless
<i>tec2</i>	4.5	2.286	4.387	Stainless
<i>t</i>	47	2.286	0.4445	Steel
<i>tcc1</i>	5.6	2.286	4.457	Steel
<i>tcc2</i>	9.652	2.286	3.682	Steel
<i>ci</i>	17.898	9.65	5	Steel
<i>cyl</i>	36.2	12.7	6.45	Steel
<i>cyl2</i>	88.8	12.7	6.45	Steel
<i>co</i>	18.9	9.65	5	Steel
<i>bco</i>	22.6	9.7	5	Brass
<i>bac</i>	280	9.7	5	Brass
<i>acc</i>	$153 - \frac{V_{gas}}{A_{acc}}$	88.9	50	Steel
<i>gas</i>	$\frac{V_{gas}}{A_{acc}}$	88.9	50	Steel

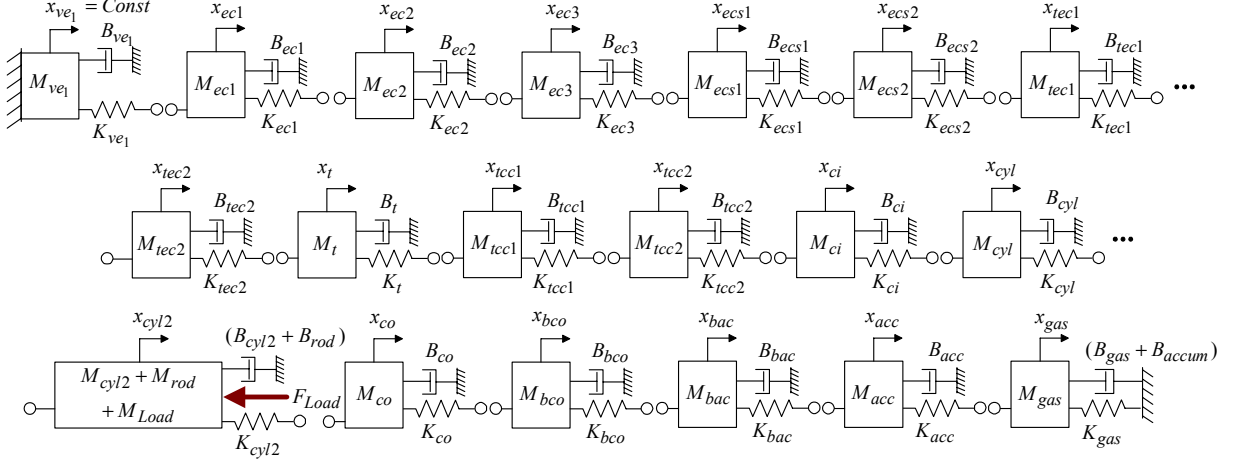


Figure 4.8: System on Cylinder Side of Closed Valves

can be used for the piezoceramic, piston, pumping chamber, and valve chamber dynamics, however the conditions must be changed for the loss of volume and change in pressure. A new system is then built for the cylinder side of the closed valves, utilizing the displacement of the fluid element just after the valves as the input. Figure 4.8 shows this model for the hydraulic cylinder and fluid on the outside of the closed valves.

4.5.2 In Valve Opens

Just after the in valve opens, the models must change again. The major differences here are seen in the pumping chamber and valve chamber sections, now connected to the fluid circuit side of the system which contains the accumulator. Assuming the accumulator pushes the hydraulic fluid through the inlet valve, it would become necessary is to add the accumulator and entire fluid circuit to the closed chamber elements. The pumping and valve chamber elements would be connected to the end of the fluid circuit in the Cylinder Side of Closed Valve System of Figure 4.8, with an added branch. A diagram of the new system where the chamber is refilling can be seen in Figure 4.9. A state-space model based on this diagram can be used until the inlet valve closes. After closing, the entire cycle starts again.

4.6 Four-Stage Cycle Operation

Each of the systems based on the previous diagrams work in stages depending on the simulated inlet and outlet valve dynamics. Since four stages occur per cycle, at least four

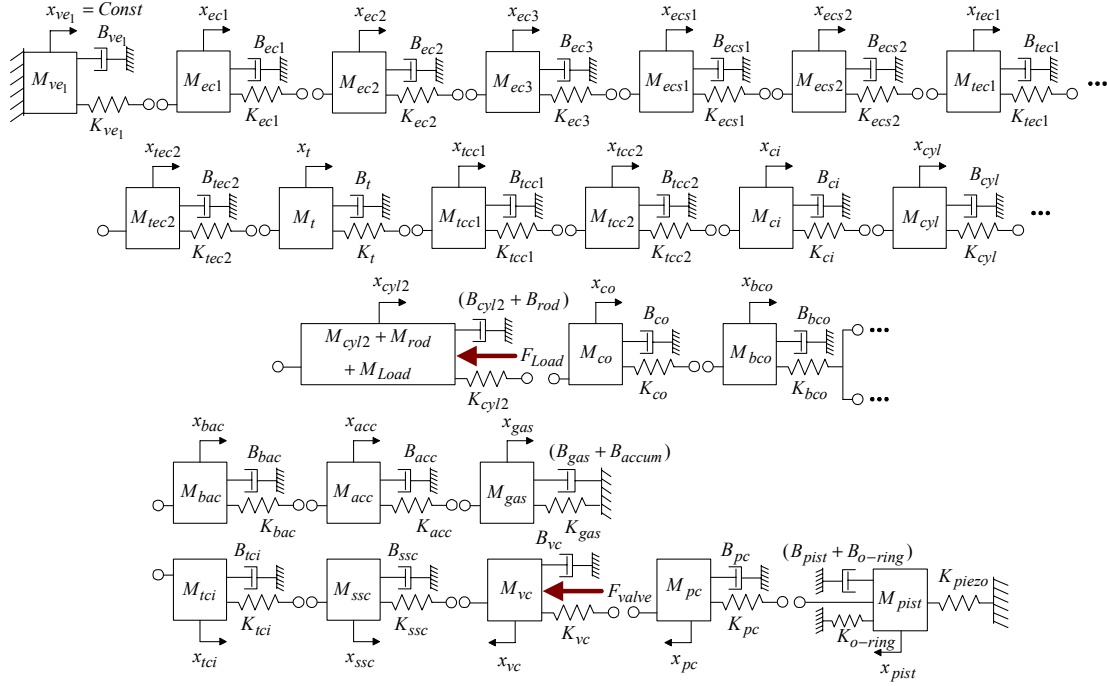


Figure 4.9: Chamber Refilling After Inlet Valve Opens

different block diagram models are needed to simulate their operation. Figure 4.10 shows the flow chart through the different possible Simulink diagrams.

Initially the closed chamber model is used to simulate the system while both valves are closed. At the same time, the outlet valve dynamics are simulated until the time when the valve opens. Then, the simulation changes to a model where the outlet valve is open and the simulation models the entire system until the outlet valve closes. After this, once again the chamber is simulated with both valves closed, while the hydraulic cylinder rod dynamics are simulated with a different system. In this different system, a static displacement equal to the fluid amount displaced from the valve chamber through the out valve is used as the constant input. Finally, the inlet valve is simulated to determine when the fluid will begin to refill inside the valve and pumping chambers using one last model.

An example of a Simulink diagram that is used to perform the simulations on the different models is seen in Figure 4.11. The state-space models of the different systems are built in the m-file environment and fed into the Simulink state-space blocks. One constant frequency block is used to control the operation timing of the entire system. Also, the function generator blocks are masked such that the voltage amplitude and duty cycle variables can be controlled. The dynamic states of the systems are recorded so that the

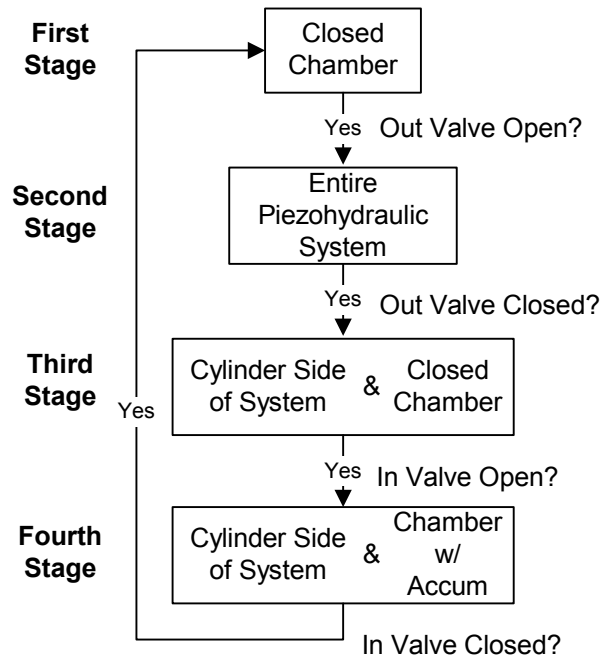


Figure 4.10: Flow Chart Through Simulation Stages for One Operation Cycle

final states can be used as the initial conditions for the next stage. Lastly, a logical check is used to determine if the valve is closed or open, and thus to continue running the simulation or to stop and start the next.

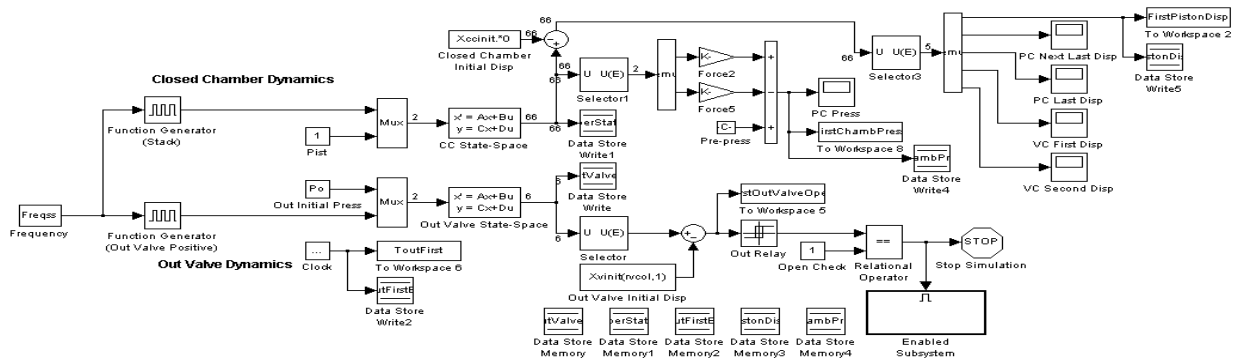


Figure 4.11: Simulink Diagram for First Stage Simulations

Chapter 5

Experimental Setups, Analysis and Simulated Correlation

5.1 Introduction

For the experimental analysis of our prototype piezohydraulic actuator with active valves, two setups were used. The first setup used an off the shelf hydraulic cylinder and an off the shelf accumulator in a wall mounted configuration. There were several drawbacks associated with this arrangement, however. The fluid circuit was not very compact, along with the hydraulic cylinder and accumulator having very low pressure ratings. For the second compact bench top setup configuration, new additions included a custom hydraulic cylinder constructed for us along with the purchase of a high pressure accumulator.

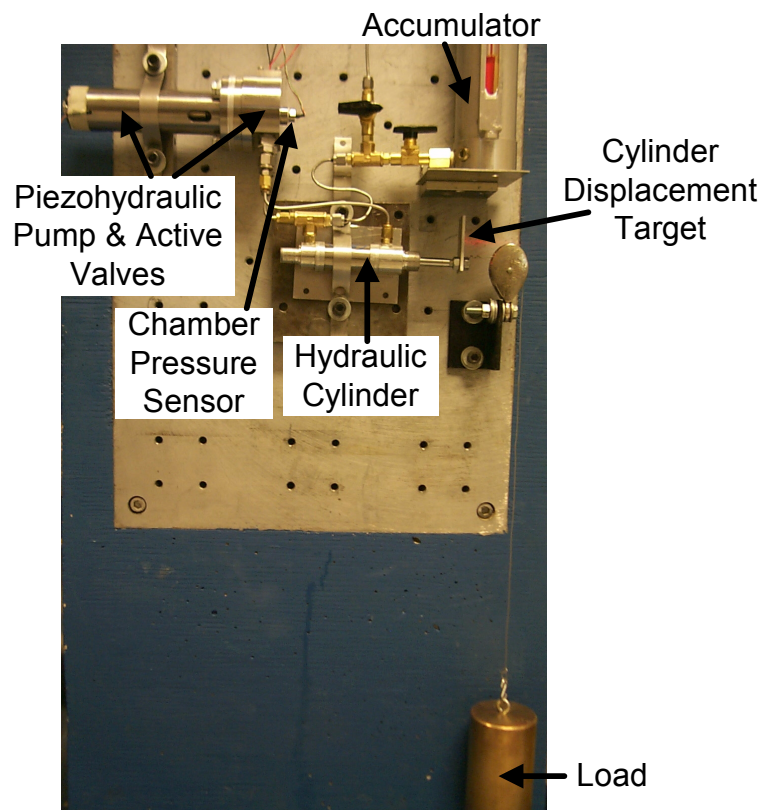
Experiments were conducted on both setups to determine piston displacement, chamber pressure, and hydraulic cylinder displacement/velocity. All of these variables were recorded at various frequencies, timing cycles, and pre-pressures. Simulations were also performed under these varying conditions in an attempt to correlate the experimental and simulated data. Tests were run when the active valves remained closed the entire time and where they were in operation to create fluid flow. This was done to provide a more accurate analysis of the actuator system components and for a more complete correlation of the experimental and simulated data.

5.2 Experimental Setups

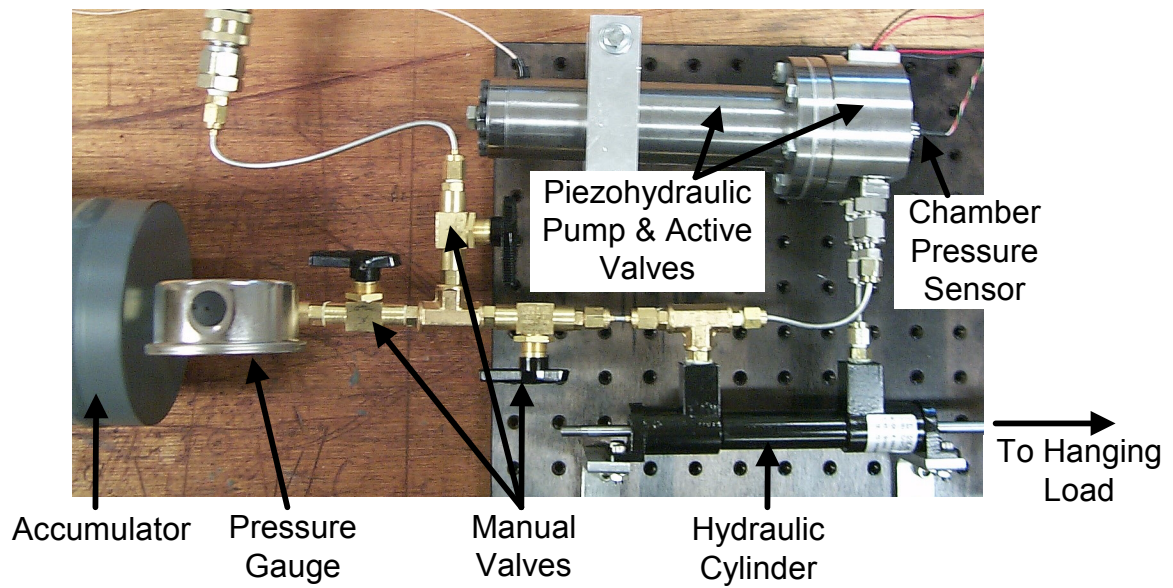
A piezohydraulic pump system consisting of a housing, pumping chamber, and piston was fabricated. The casing components were designed in house, whereas the piezoceramic stack actuator and corresponding flexure were purchased from Physik Instrumente (P-844.60 and P-176.60). These components were assembled together with aluminum plates separating the housing and pumping chamber, which allowed us to vary the volume of fluid inside the pumping chamber. The actual pump part of the actuator system (includes stainless steel casing, piezoceramic material, piston, pumping and valve chambers, and active valves) measured approximately 32 cm in length with an O.D. of almost 8 cm around the pumping chamber. The inside pumping chamber measured 2.5 cm in diameter with approximately a 2.4 mm length for both setups. A Sensotec pressure transducer (model S/1542-11G) was also added to the end of the pumping chamber to allow for monitoring of both static and dynamic pressure inside.

Two experimental setups were used in the testing. Both incorporated active valves to pump fluid and move a hydraulic cylinder, however different tests were conducted on each one, resulting in different sets of experimental data. The experimental setups used can be seen in Figure 5.1, where Figure 5.1(a) shows the wall mounted setup and Figure 5.1(b) shows the more compact bench top setup.

For both setups, the air was evacuated from the system using a vacuum pump and hydraulic fluid was fed in using a pump or accumulator to pre-pressurize the system. The piezoceramic stack actuator and active valves were all driven and controlled by a dSPACE system that also acquired and recorded the data from the sensors used in the experiments. Required amplifiers between the dSPACE and the piezoelectric components (piezoceramic stack actuator and valves) produced the electric power needed for operation. For the wall mounted setup, a DSM 2 Level PZT Driver supplied the 0 - 150 Volt square wave for the stack and a Trek (model 50/750) two channel linear amplifier provided the 0 - 750 Volts for opening and closing of the piezohydraulic valves. For the bench top setup, a custom built three channel DSM amplifier was used to supply the power to the piezoceramic stacks. The three channels consisted of one variable low voltage channel operating from 50 - 150 Volts and two variable high voltage channels operating from 400 - 1200 Volts.



(a) Wall Mounted Setup



(b) Benchtop Setup

Figure 5.1: Experimental Test Setups

5.3 Wall Mounted Valve Timing Analysis

Using the setup as seen in Figure 5.1(a), experiments were conducted to characterize the piston and valve interaction of the actuator system at a fluid pre-pressure of 100 psi using different valve timings while under no load and while under a load of 12.63 kg. The pre-pressure value was limited to 125 psi due to the Lube Devices, Inc. air/oil reservoir used as the accumulator in the setup. This fluid circuit also incorporated a $1\frac{1}{16}$ inch bore hydraulic cylinder from Bimba Manufacturing Company (model H-091-DUZ). The air was evacuated from the system and an accumulator in conjunction with an air compressor was used to fill and pre-pressurize the system. This system pressure was used as the low side or input to the chamber for pumping. During operation, the piezoceramic stack actuator drove a piston inside the pumping chamber while piezohydraulic active valves controlled the fluid flow. Dynamic pressure was built inside the chamber after piston extension and a variable flow rate was achieved based on the system frequency, valve timing, and valve duty cycle. The flow was a result of the step and repeat motion that continuously moved fluid through the valves and pumping chamber using the oscillatory piezoceramic stack actuator motion along with the opening and closing of the valves.

The fluid proceeded from the pumping chamber into one end of the hydraulic cylinder where the increased fluid volume created a pressure imbalance and either raised or lowered a load. The dSPACE model was constructed so that the system could either pull or push the load using the cylinder motion. Throughout the experiments, the displacement of the hydraulic cylinder was recorded over time to calculate the output velocity for each valve timing. A Polytec GmbH laser vibrometer (model OFV 303) and controller (model OFV 3001) were used to measure the motion of a target connected to the end of the hydraulic cylinder. The laser vibrometer measured the cylinder displacement over a finite amount of time while pulling the load. Velocities of the hydraulic cylinder pulling the known or zero load were measured by taking the slope from a cylinder displacement curve over a finite amount of time as seen in Figure 5.2.

The ControlDesk layout built using the dSPACE system software had the capability of controlling the duty cycles and phasing of the piezoceramic stack actuator and active valves. This enabled the ability to see the effects on the actuator system while changing these parameters. An array of test data was taken by measuring the hydraulic cylinder

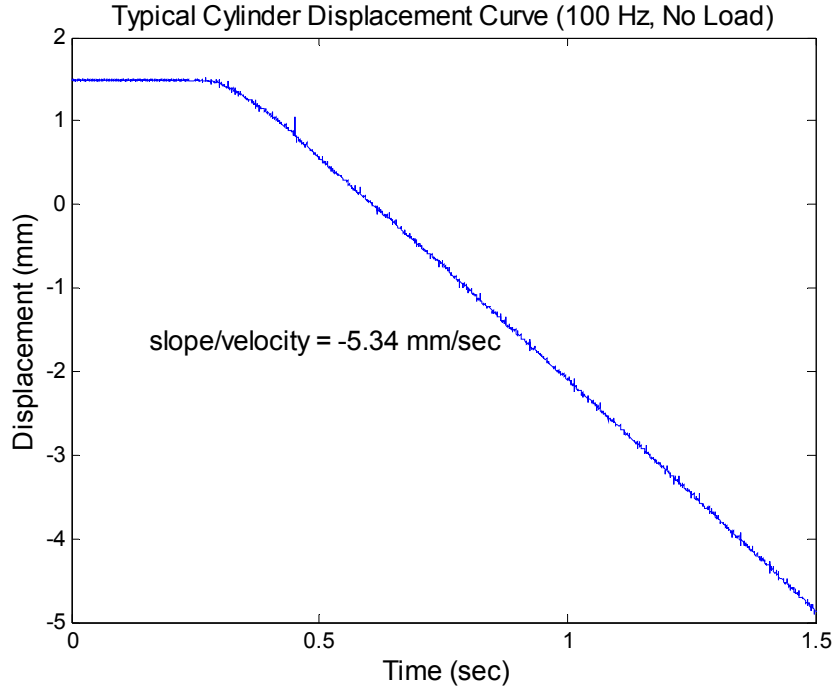
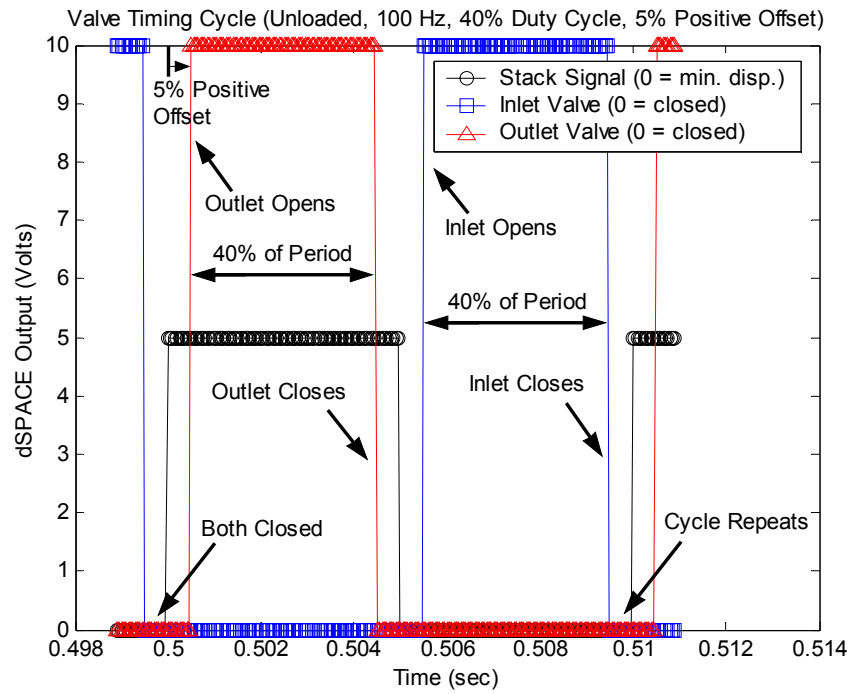


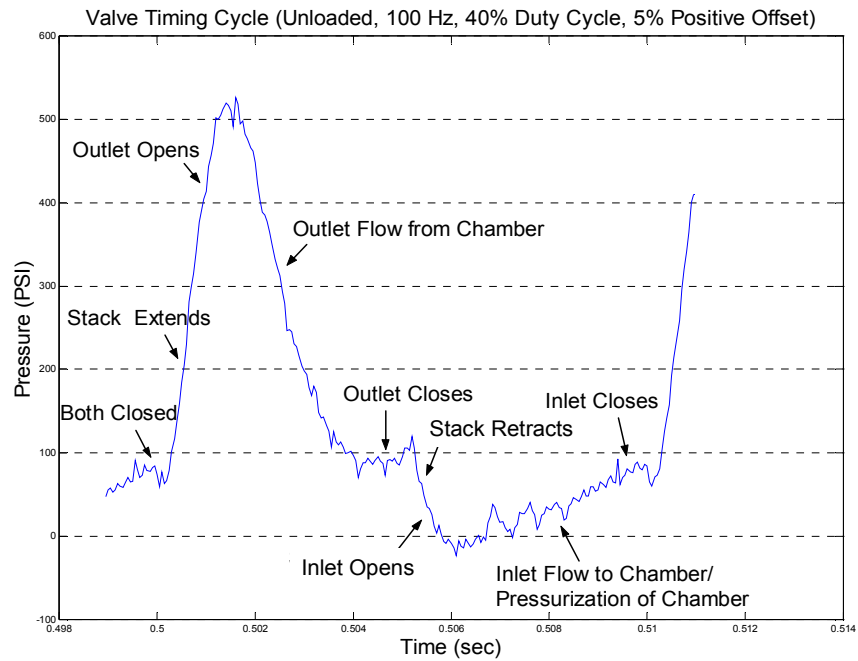
Figure 5.2: Cylinder Displacement Data Recorded by Laser Vibrometer

velocity while under no load and while under a 12.63 kg load. This data was taken for a valve duty cycle ranging from 30% to 70% and an outlet valve offset ranging from -20% to 20% with respect to the piezoceramic stack actuator operation cycle. Both variables had a step size of 5% when taking the data, producing 81 permutations. Throughout these combinations, the piezoceramic stack actuator operation parameters remained constant with a 50% duty cycle and a zero offset. This provided half a period of piezoceramic stack actuator or piston extension and half a period of retraction. The other constant throughout the experiment was that the inlet valve and outlet valve were always offset by 50% of each other, operating 180° out of phase. By only varying the valve conditions, the effects could then be seen clearly. Figure 5.3(a) shows typical timing cycle signals before being sent through amplifiers to the piezoelectric components and Figure 5.3(b) shows the chamber pressure changes during this cycle. These figures show an unloaded cycle at 100 Hz operation with a 40% valve duty cycle and a positive 5% outlet valve offset.

Figure 5.4 and Figure 5.5 show the timing array results for an unloaded system and a system under a 12.63 kg load. In the unloaded case, the maximum flow rate or cylinder velocity of 6.5 mm/sec is achieved with a 40% duty cycle and a positive 5% output valve



(a) Stack Actuator and Valve Timing



(b) Chamber Pressure

Figure 5.3: Typical Pumping Cycle

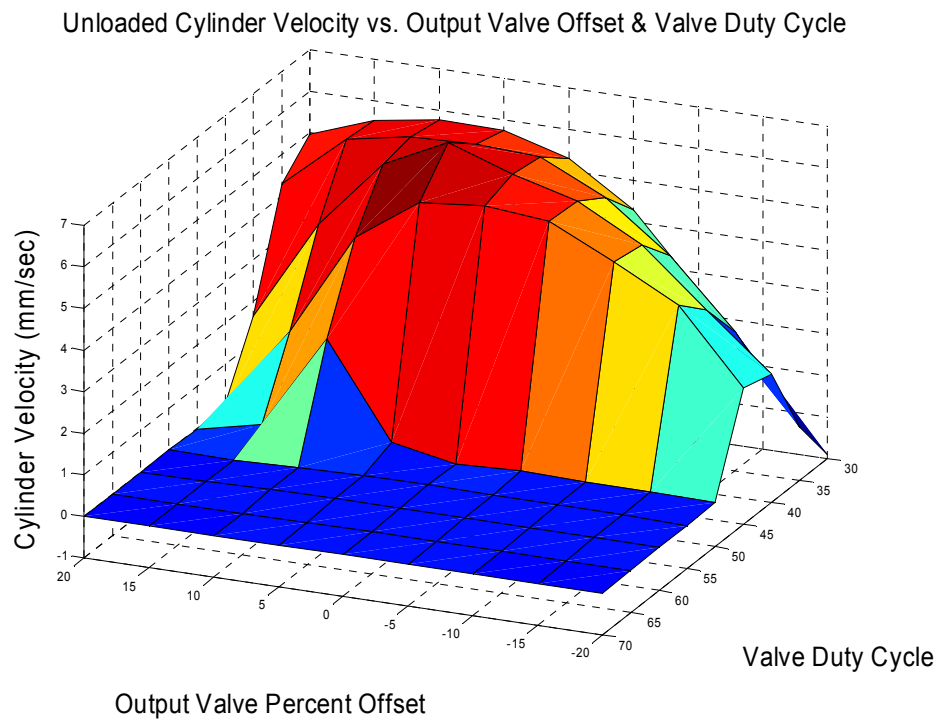


Figure 5.4: Timing Study of an Unloaded System

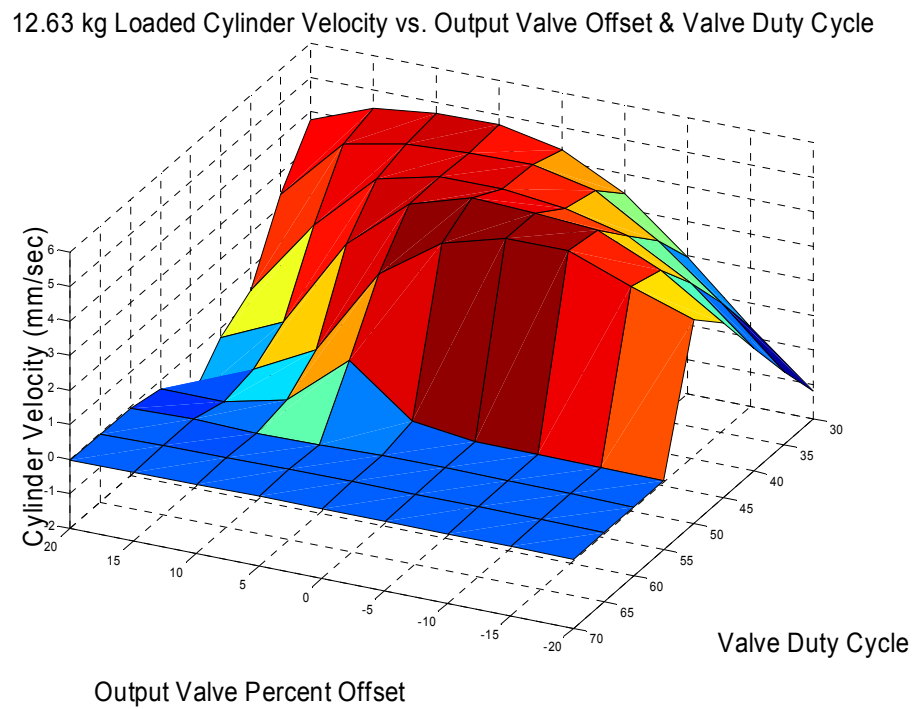


Figure 5.5: Timing Study of a Loaded System

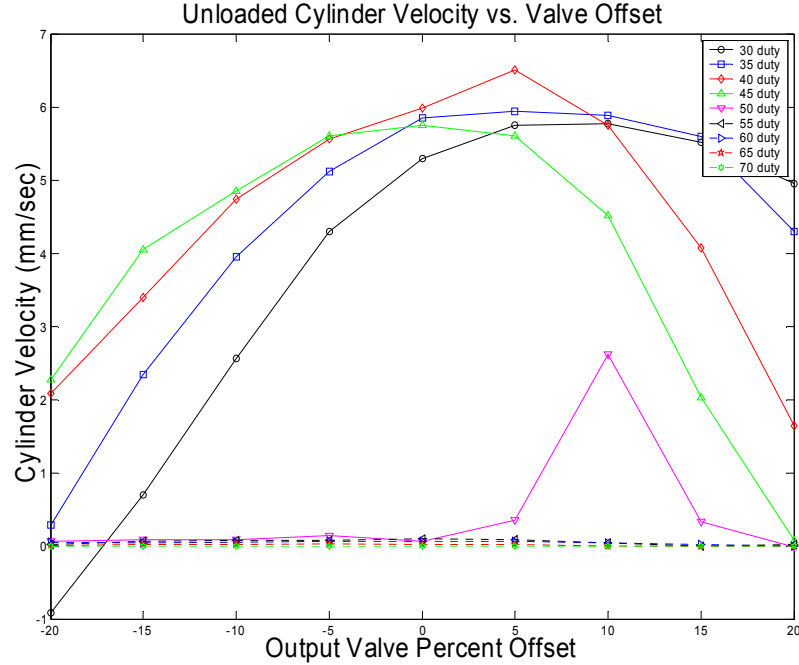


Figure 5.6: Unloaded Timing Results vs. Offset

offset. When the duty cycle is 50% or greater there is overlap between the inlet and outlet valves. This condition proves to result in very low velocities, independent of the offset value. For duty cycles of 45% and under the offset is a factor, and the velocity tends to increase as the offset becomes more positive up to around positive 5% or positive 10%. Beyond the positive 10% value, the velocities then begin to decrease. This effect can be seen more clearly in Figure 5.6.

For the loaded case, the maximum velocity value of 5.23 mm/sec is achieved with a 45% duty cycle and a 0% output valve offset. This is a bit different from the unloaded condition, as are the tendencies dependent on duty cycle. Once again very low velocities are produced independent of the offset value, but this time it is only when the duty cycle is 55% and above. The 50% offset now becomes a very usable condition and even produces the second highest flow rate of 5.19 mm/sec at an offset value of negative 10%. For the range of duty cycles between 30% and 50%, the maximum velocity point varied from between negative 10% offset to positive 10% offset. Also for this range, as the duty cycle increased, the maximum point tended to move toward a more negative offset value. Figure 5.7 demonstrates the tendency more clearly.

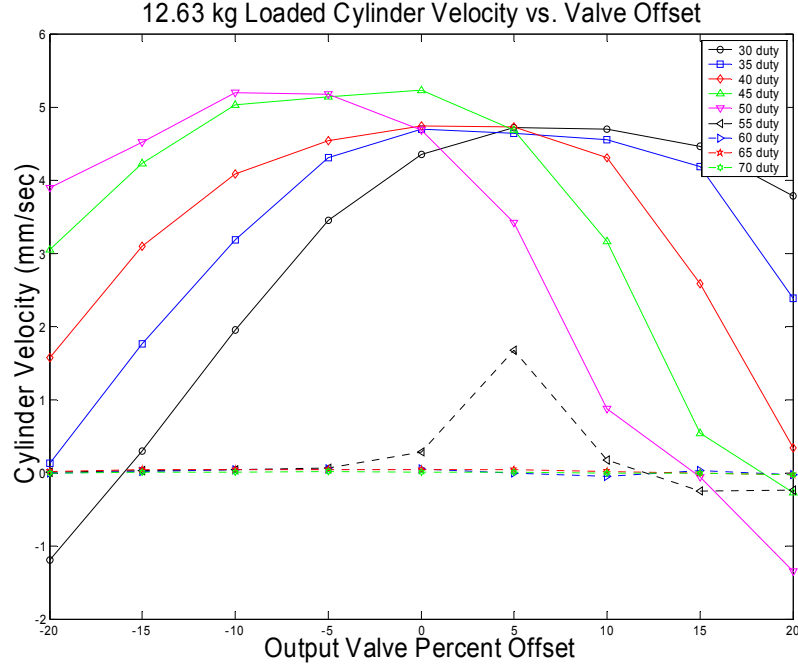


Figure 5.7: Loaded Timing Results vs. Offset

Also to note, the velocities are much lower and the loads are much higher in the wall mounted timing test analysis when compared to the bench top setup analysis. This is due to the differences in the fluid circuit layout, most notably the difference in the output hydraulic cylinder bore diameters. The volume of fluid pushed out of the chamber by the piston and into the hydraulic cylinder is the reason for this. If the area of the bore of the hydraulic cylinder is large, there will be little displacement and a large force is produced. If the area of the bore is small, there will be large displacement and a small force produced.

5.4 Benchtop Piezohydraulic Actuator Performance

Using the setup as seen in Figure 5.1(b), experiments were conducted to characterize the closed valve chamber dynamics as well as the overall actuator performance. This bench top fluid circuit incorporated a $\frac{1}{2}$ inch bore hydraulic cylinder custom made for high pressures by Custom Actuator Products, Inc. (model BBB1/2X3-3-1-1P-1 DE). Also, a high pressure (2,000 psi max) bladder type accumulator manufactured by Tobul Accumulator, Inc. (model EBR20-60) was used to replace the previous pressure limited one. The bladder of the accumulator was filled with nitrogen gas at a pressure of 150 psi. The air was evacuated

from this system in the same manner as the wall mounted system and fluid was fed in using a hydraulic hand pump or some other hydraulic pump for pre-pressurization. Once again, the Polytec GmbH laser vibrometer (model OFV 303) and controller (model OFV 3001) were used to measure the displacement of a target connected to the end of the hydraulic cylinder as well as a target connected to the back of the piston. The same dSPACE model was used for actuator operation and data acquisition, and the cylinder velocity was also calculated in the same manner by taking the average slope of the displacement curve.

5.4.1 Closed Valve Piston and Chamber Pressure Correlation

Experiments were performed to measure the closed chamber dynamics and for model correlation. Recording chamber pressure and piston displacement enabled correlation to a state-space linear lumped-parameter model. Thus, simulated matching of the experimental dynamics inside the closed valve configuration allowed for an accurate estimation of model parameters such as percent air, o-ring damping, and o-ring stiffness. Experimental tests and simulations were performed with the piezoelectric stack being driven by a 0 - 100 Volt square wave.

Fluid leakage through the valves made matching the experimental data difficult, especially at very low frequencies. Therefore, the technique used to obtain the measurements will be described first, for both the experimental and simulated data.

For the experimental piston displacement measurement and the experimental pressure difference measurement, an attempt was made to measure the peak values from the time location where the pressure started to fall off (indicating the start of leakage). This is shown at the location of the max measurement arrow in Figures 5.8, 5.10, 5.12, and 5.14. The location of the min measurement arrow in these same figures was attempted to be at the location just before fluid leakage into the valves. The Matlab function `ginput` was used to pick these values from the graphs and then the differences between the max and min values were computed by hand.

For the simulated piston displacement and pressure difference measurements, an average of the points just before half of the period were used for the max measurement value. These points are labeled in Figures 5.9, 5.11, 5.13, and 5.15. The points averaged to obtain the min measurement value were taken from just before the end of the period. They too can be seen in the same figures as the min measurement points. Matlab was automated

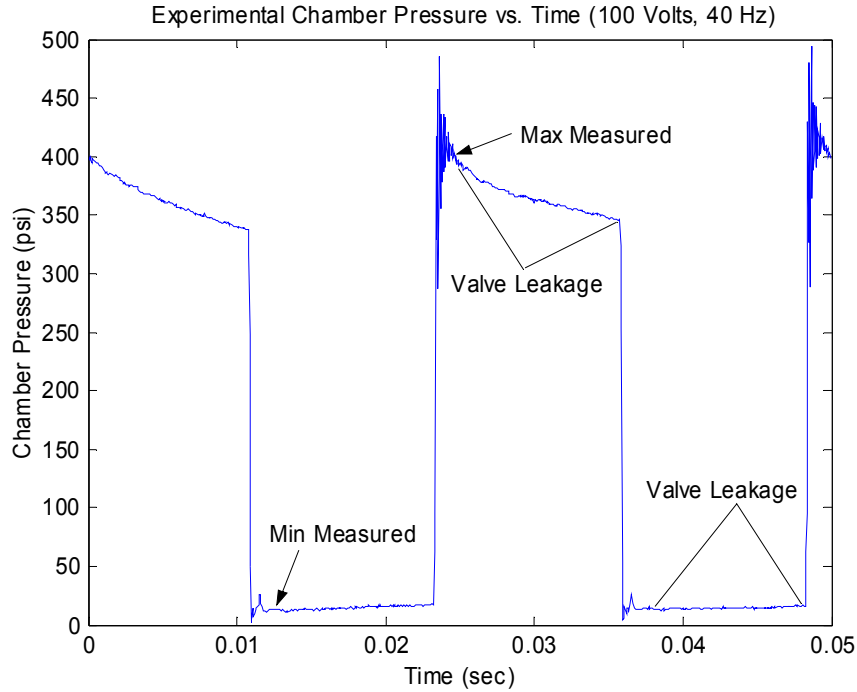


Figure 5.8: Experimental Pressure Difference Measurement at 40 Hz

to select these values, average them, and then subtract the min average value from the max average value to obtain the differences.

Correlation of the closed chamber model was done by comparing these different values as well as the actual waveforms. The actual piston and pressure curves gave a more detailed look at the dynamics, allowing for a finer tuning of the model parameters. The parameters which provided the closest matching of the waveforms, pressure differences, and piston displacements were then used in the model for the entire actuator.

The previously measured experimental and simulated values of piston displacement and pressure difference were compared over a range of frequencies, as shown in Figure 5.16 and Figure 5.17. The difference in the experimental and simulated values may be attributed to both measurement and model error.

The waveform comparisons of experimental and simulated data at 300 Hz and 100 Hz are seen in Figure 5.18 thru Figure 5.21. In the experimental results, the lower part of the square wave for the pressure signal starts very low due to the leakage of fluid from the chamber. At the start of the test, the lower pressure part of the signal is near the system pre-pressure value, but quickly decays due to fluid leaking from the chamber. This loss of

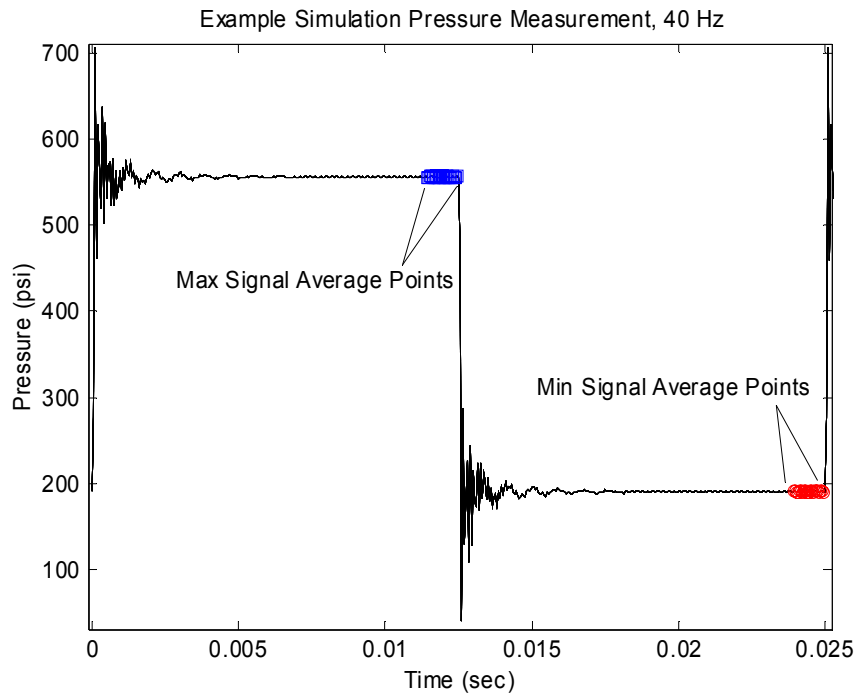


Figure 5.9: Simulated Pressure Difference Measurement at 40 Hz

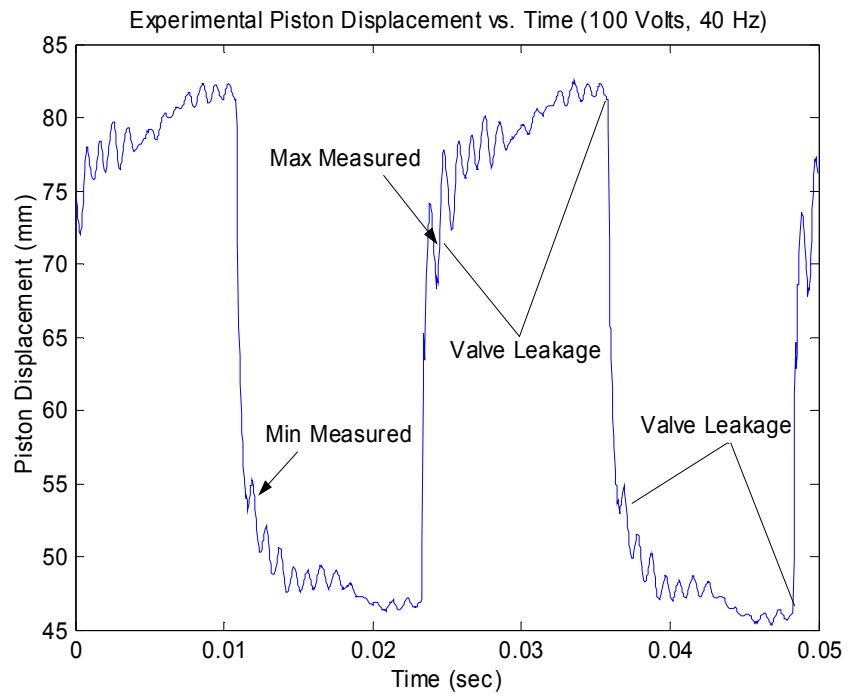


Figure 5.10: Experimental Piston Displacement Measurement at 40 Hz

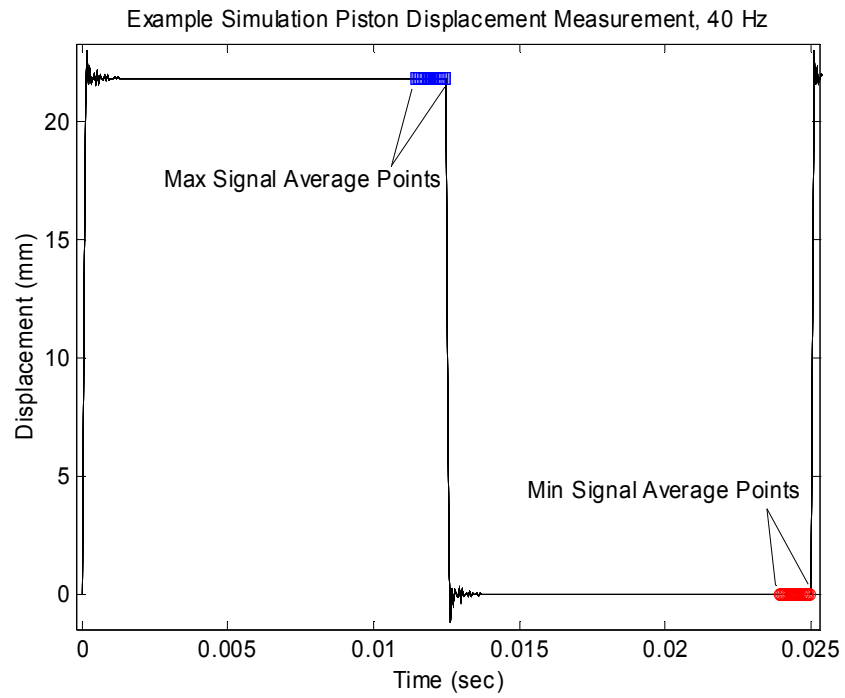


Figure 5.11: Simulated Piston Displacement Measurement at 40 Hz

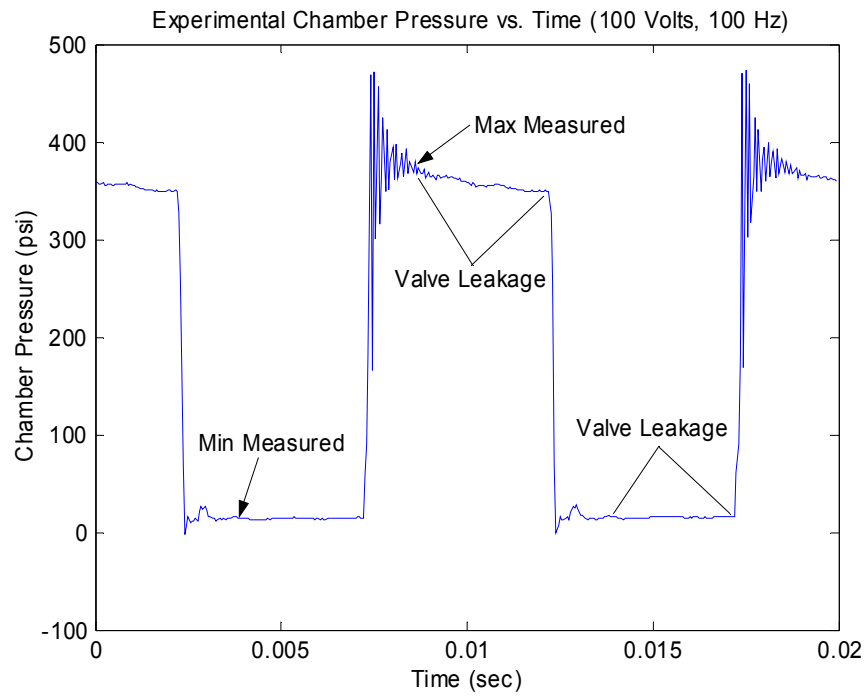


Figure 5.12: Experimental Pressure Difference Measurement at 100 Hz

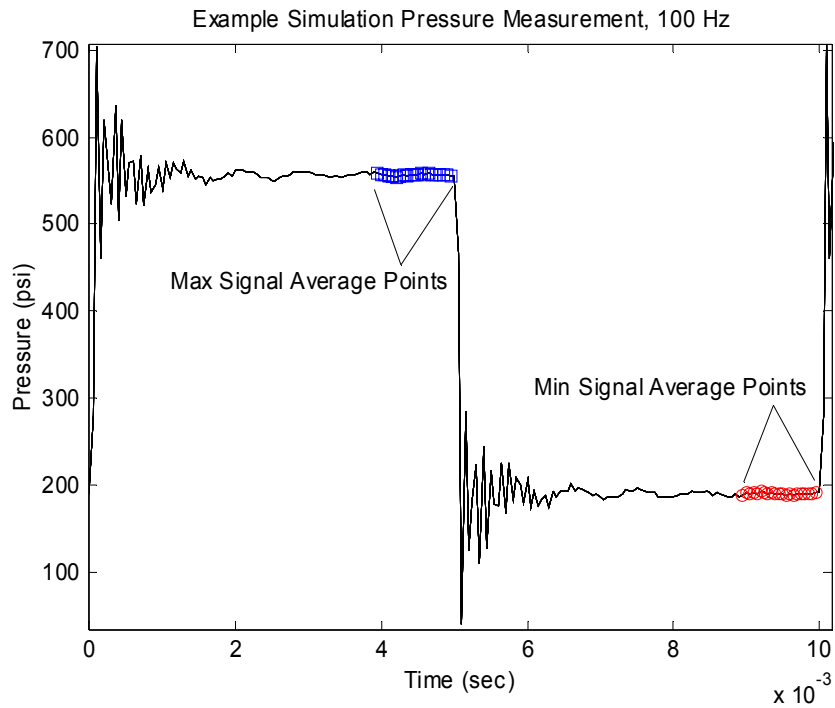


Figure 5.13: Simulated Pressure Difference Measurement at 100 Hz

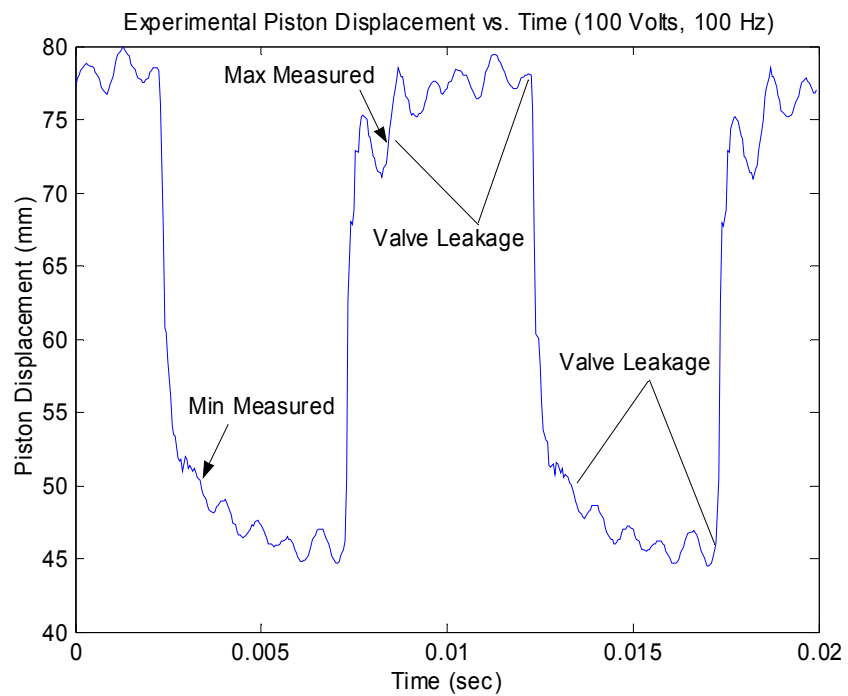


Figure 5.14: Experimental Piston Displacement Measurement at 100 Hz

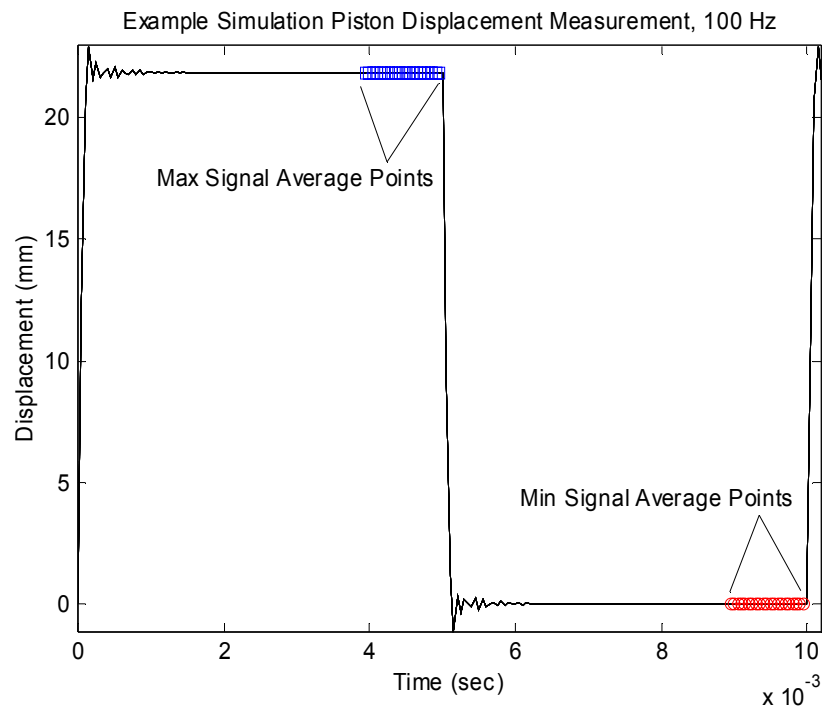


Figure 5.15: Simulated Piston Displacement Measurement at 100 Hz

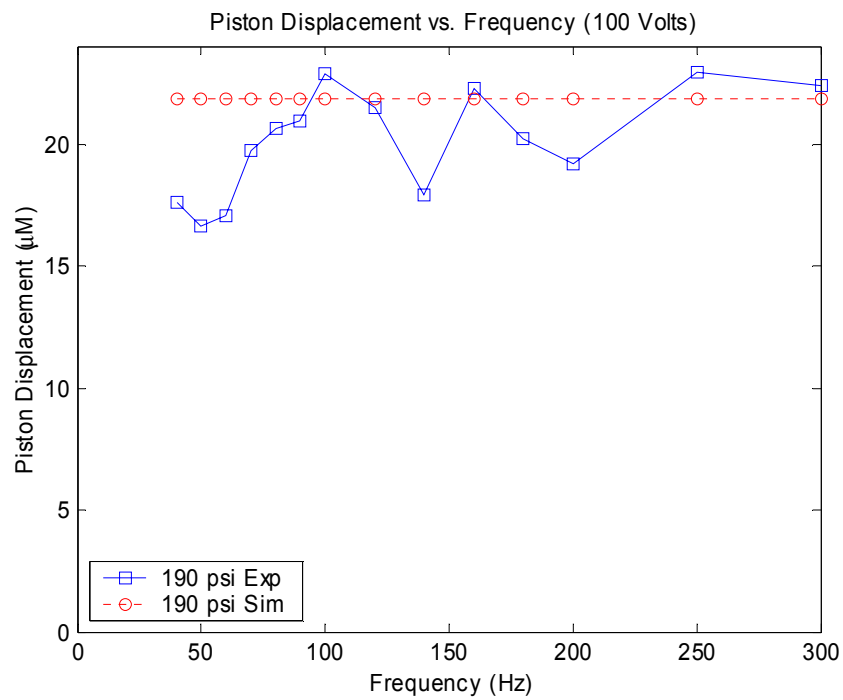


Figure 5.16: Comparison of Piston Displacement Vs. Frequency

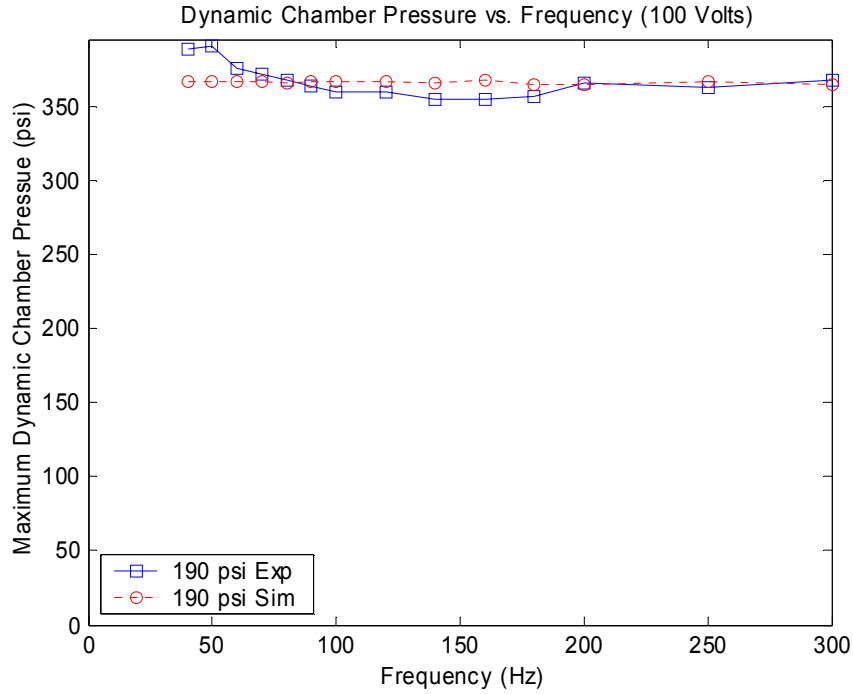


Figure 5.17: Comparison of Chamber Pressure Vs. Frequency

fluid isn't replaced during steady state operation and therefore the piston can not retract fully to its initial position. Also, while retracting, the piston feels a very stiff fluid 'pulling' the other direction due to the smaller fluid volume. This higher stiffness of the smaller fluid volume results in the pressure signal having almost no overshoot or oscillation near the bottom.

The experimental and simulated chamber dynamics for one cycle operation at 300 Hz and 100 Hz are seen in Figure 5.22 thru Figure 5.25 (thick line is pressure, thin line is piston displacement). The piston motion can be directly compared to the pressure in the chamber when looking at the signals in this manner. The leakage effects can also be seen, where the piston motion greatly increases after the volume of fluid is released. Also, the lower frequency oscillation seen on top of the piston displacement is believed to be caused in part by the vibration of the target connected to the piston. That vibration may also be caused in part by the vibration of the table used for mounting the actuator.

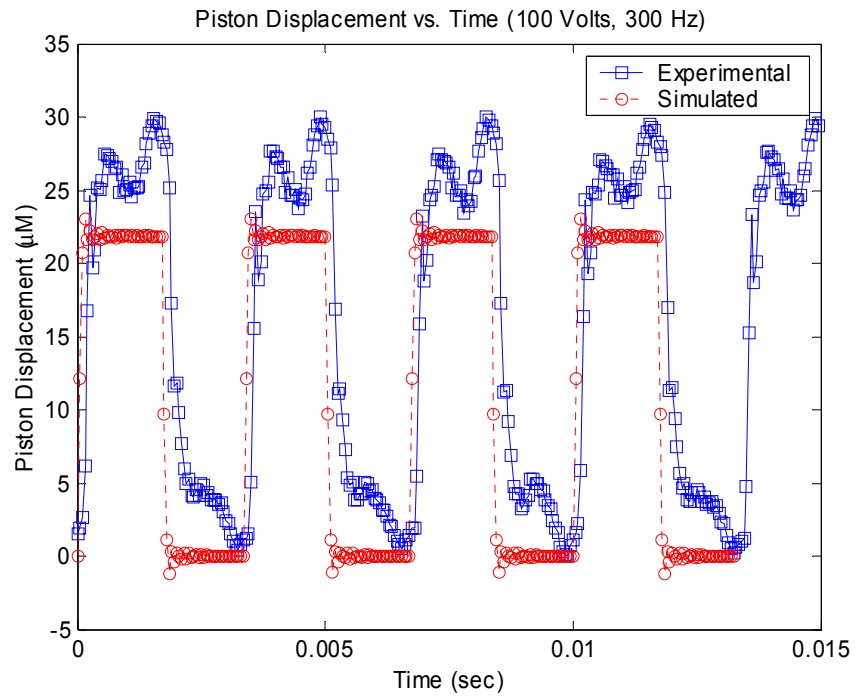


Figure 5.18: 300 Hz Comparison of Piston Displacements

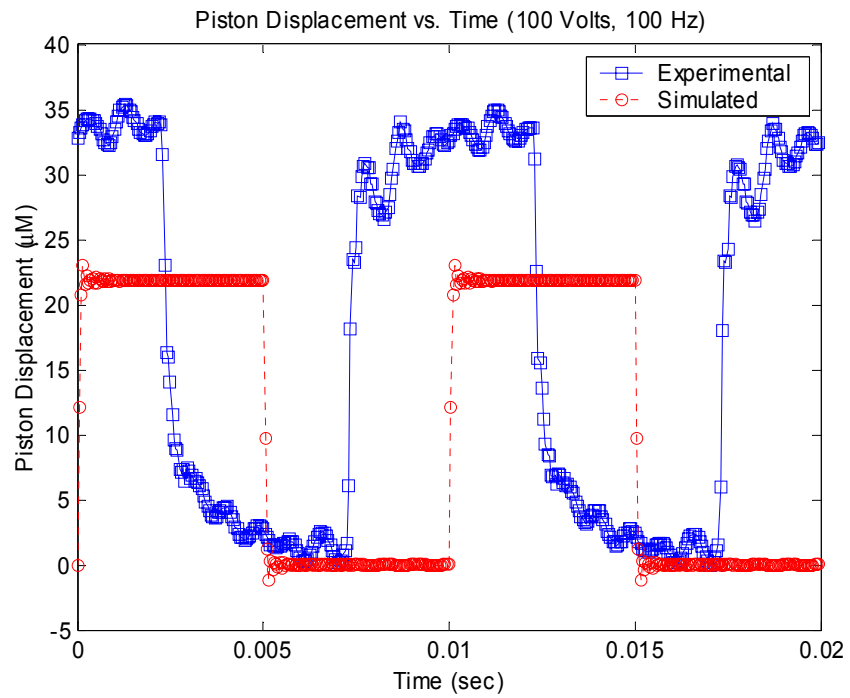


Figure 5.19: 100 Hz Comparison of Piston Displacements

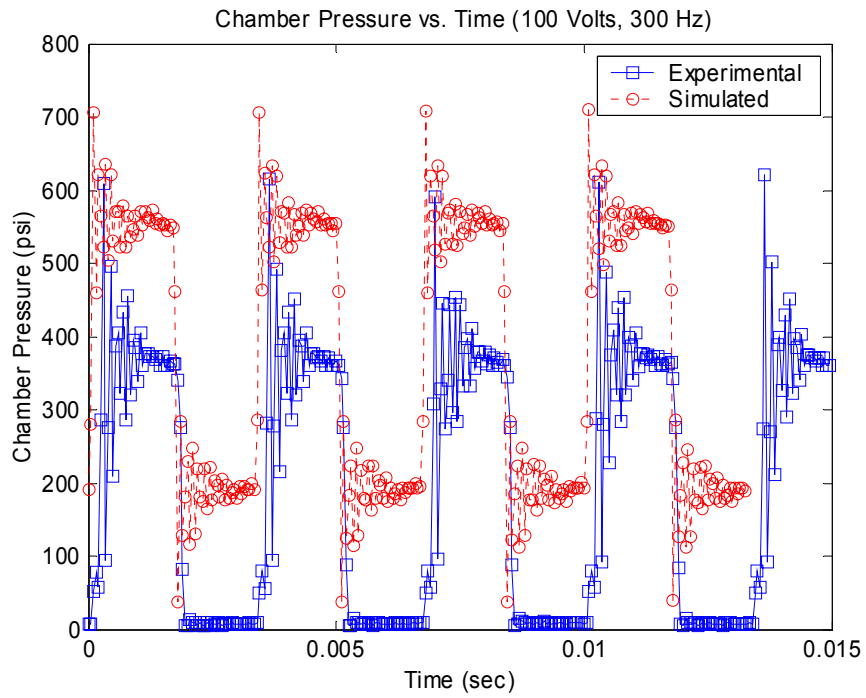


Figure 5.20: 300 Hz Comparison of Chamber Pressures

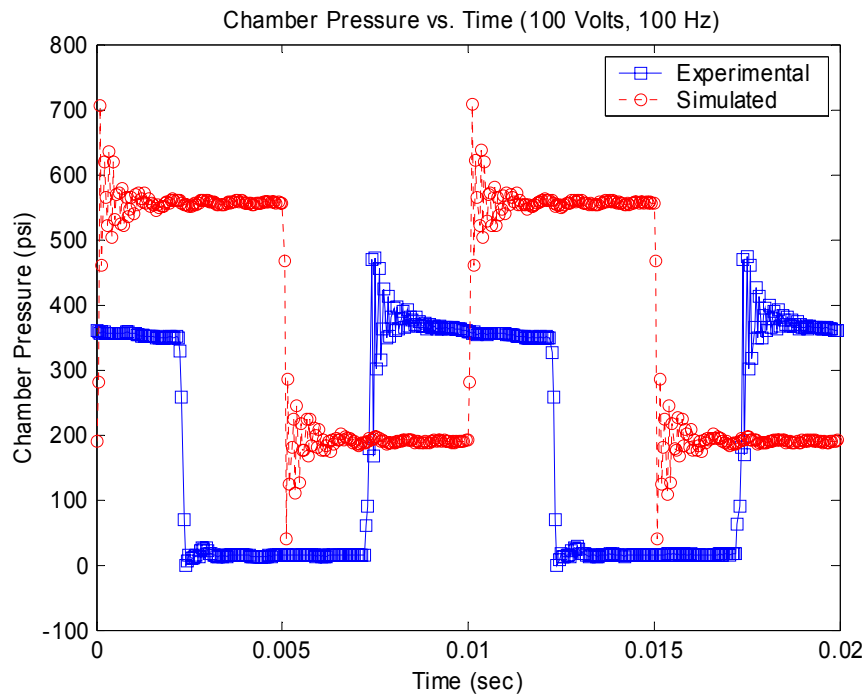


Figure 5.21: 100 Hz Comparison of Chamber Pressures

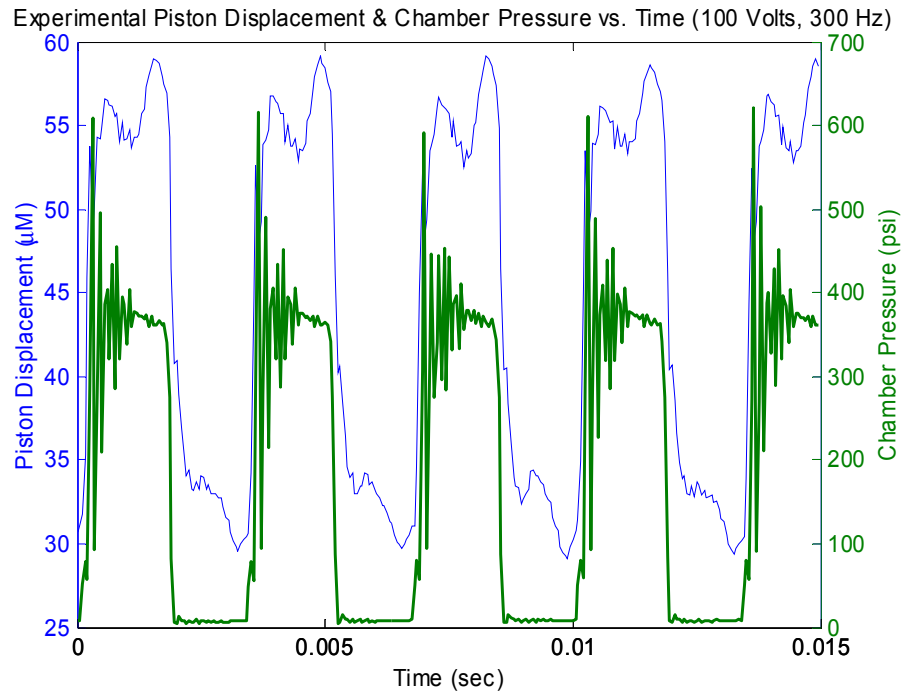


Figure 5.22: 300 Hz Typical Experimental Closed Chamber Cycle (thick line = pressure)

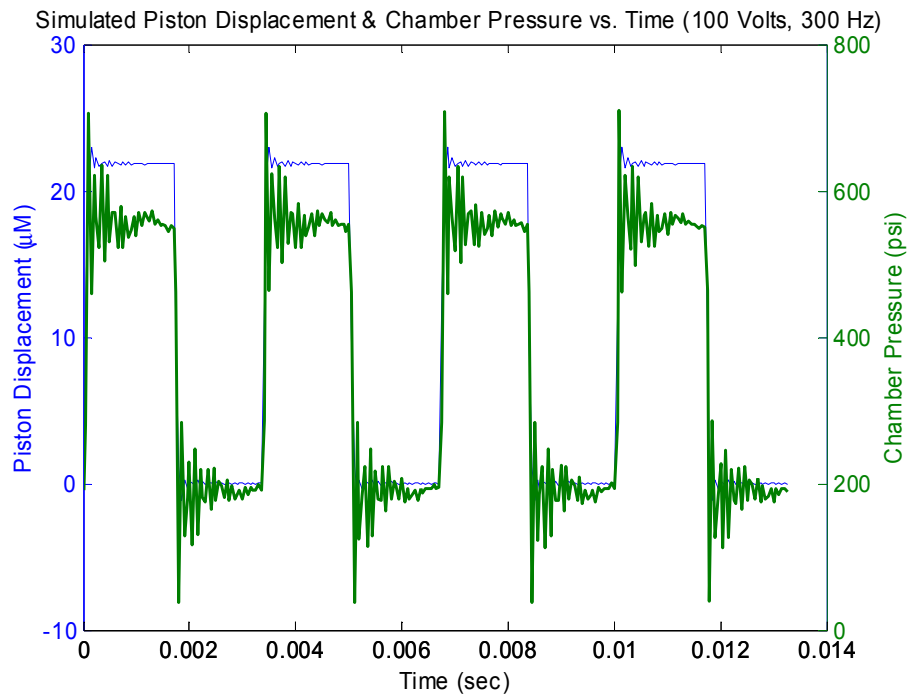


Figure 5.23: 300 Hz Typical Simulated Closed Chamber Cycle (thick line = pressure)

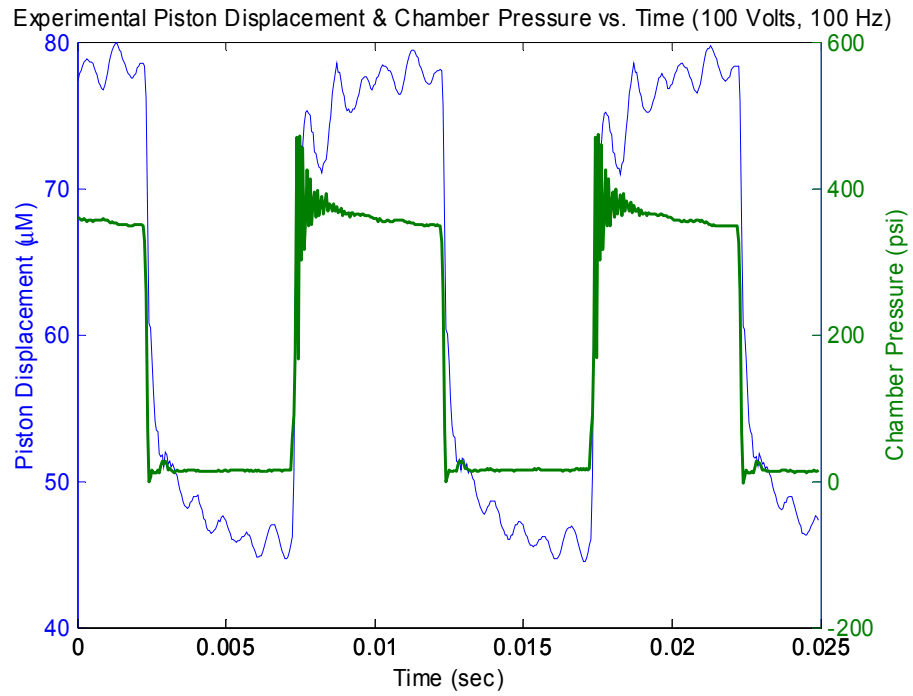


Figure 5.24: 100 Hz Typical Experimental Closed Chamber Cycle (thick line = pressure)

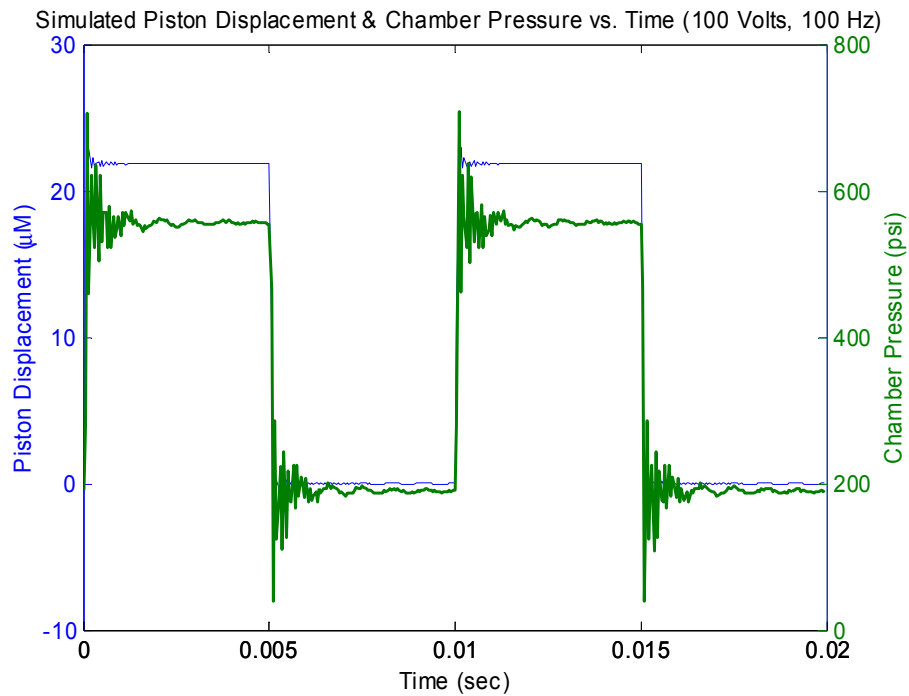


Figure 5.25: 100 Hz Typical Simulated Closed Chamber Cycle (thick line = pressure)

5.4.2 Actuator Performance

The bench top setup configuration was also used for pumping analysis, dynamic chamber pressure analysis, and determining actuator mechanical power output capabilities. Once again, recording chamber pressure and hydraulic cylinder displacement enabled the matching of experimental data to simulated state-space linear lumped-parameter models. Power calculations were also performed to rate actuator performance under various operating conditions.

Current actuator performance was characterized for the bench top piezohydraulic actuator with the valves having 40% duty cycles and a 5% positive offset for the outlet valve. The piezoelectric stack driving the piston was controlled with a 0 - 110 Volt square wave with a 50% duty cycle. The fluid circuit pre-pressure was set to a value of 190 psi, and velocity/output power was recorded/calculated for various frequencies ranging from 50 Hz to 250 Hz while under loads ranging from 0 kg to 22.207 kg. Figure 5.26 thru Figure 5.29 illustrate the current piezohydraulic actuator performance while operating under steady state flow. A maximum actuator power of 3.6 Watts was achieved at a frequency of 140 Hz under a load of 12.825 kg, and a maximum hydraulic cylinder velocity of 39 mm/sec was achieved at a frequency of 120 Hz under no load.

When looking at Figures 5.26 and 5.27 the velocity and thus the power of the actuator appeared to reach a maximum around 120 or 140 Hz for all loads. Both velocity and power increase linearly up to this 120 or 140 Hz value, then begin to decrease. This is believed to be a result of two characteristics of the test, which will be described later by looking at the chamber pressure and hydraulic cylinder displacement curves for one cycle.

In Figure 5.28 the velocity of the output cylinder decreases approximately linearly with increasing load. This is due to the inertia from the added mass and static load due to gravity acting on the hanging mass. With added load, more force is pushing against the motion of the piezoelectric stack driving the hydraulic cylinder, therefore the displacement per cycle is smaller.

There does appear to be an optimal load point when looking at the plot of the power against load in Figure 5.29. This is due to the previously mentioned inverse relationship between the two. To achieve the maximum velocity, the load should be minimized. To achieve the maximum pulling force, the load should be maximized. The power is obtained

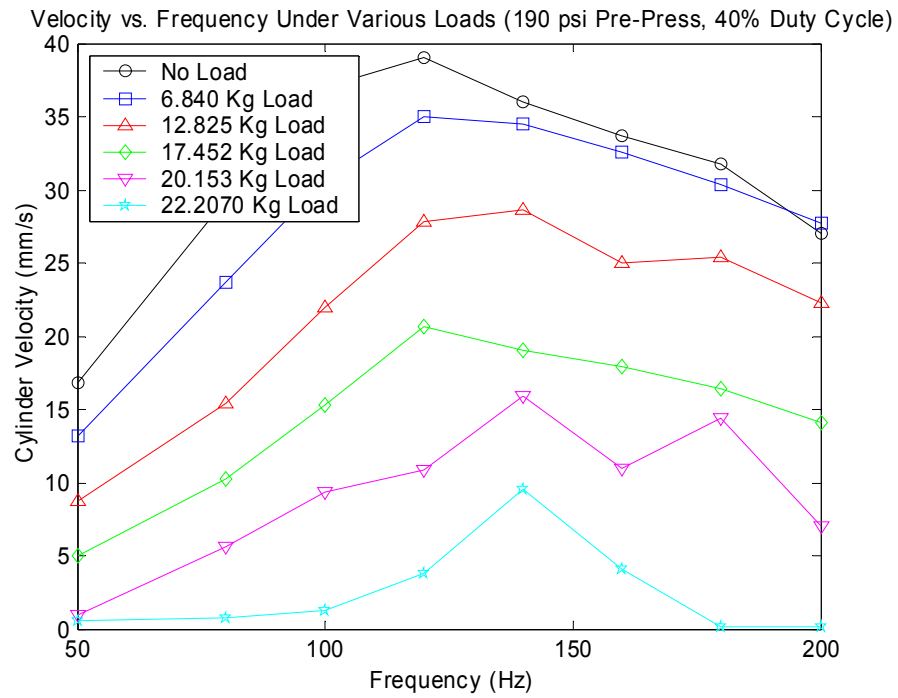


Figure 5.26: Experimental Hydraulic Cylinder/Load Velocity vs. Frequency

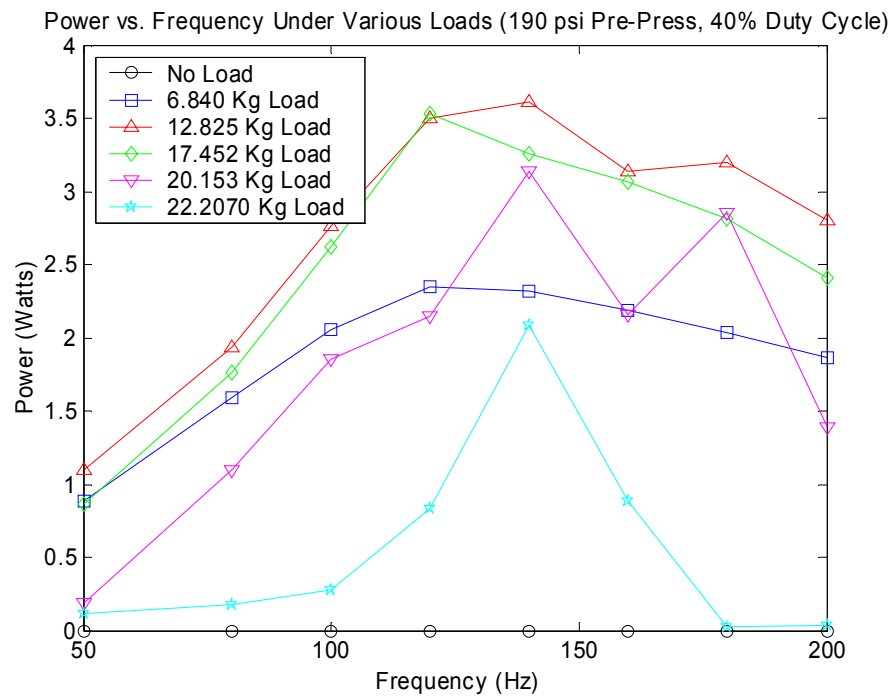


Figure 5.27: Experimental Mechanical Output Power vs. Frequency

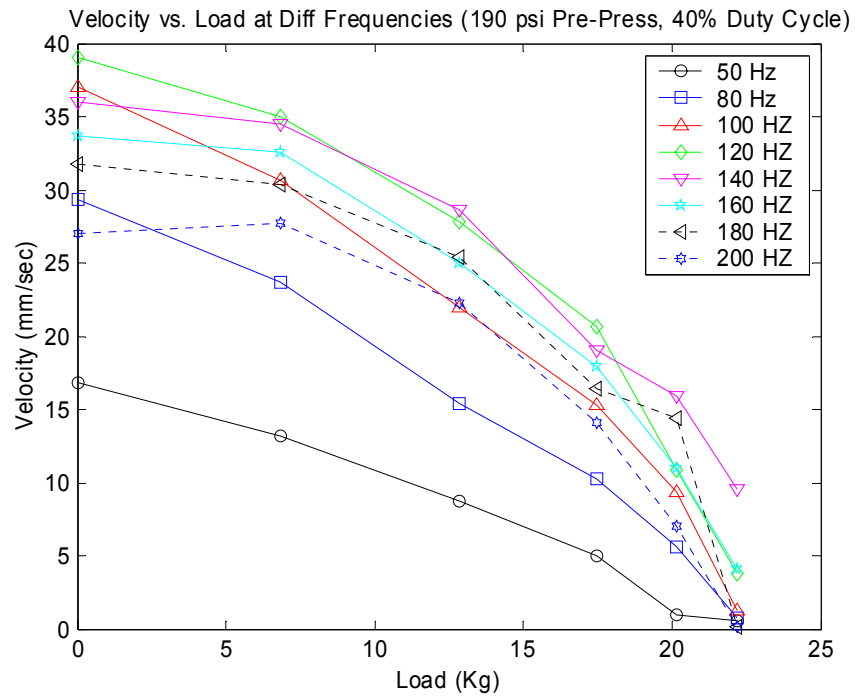


Figure 5.28: Experimental Hydraulic Cylinder/Load Velocity vs. Load

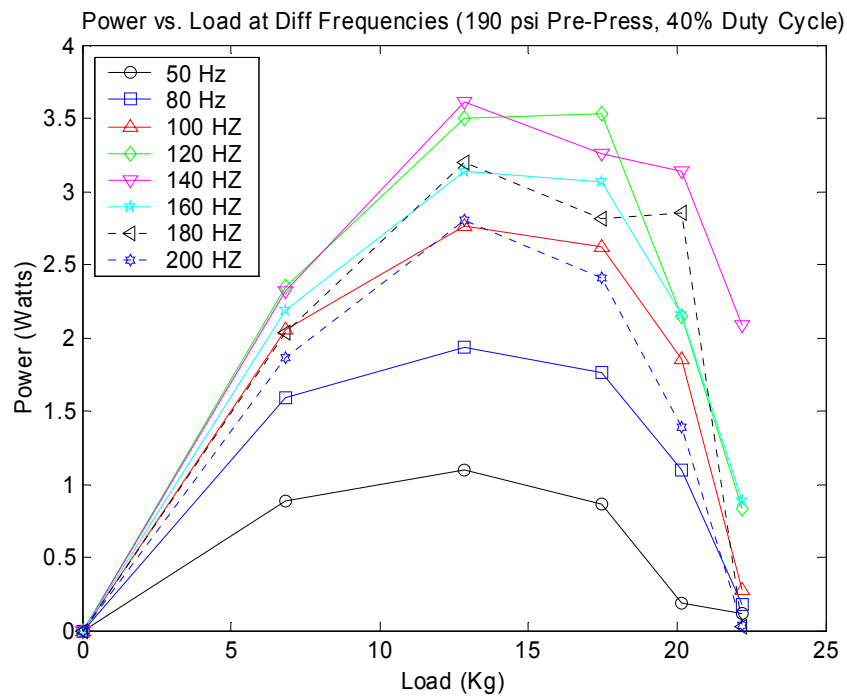


Figure 5.29: Experimental Mechanical Output Power vs. Load

by multiplying the force and velocity, thus both should be maximized. Maximizing both is impossible due to their relationship however, therefore the maximum power is located where both have values somewhere in the middle. The load where this occurs is then the optimal power point, around 12.825 kg for our particular actuator.

Figure 5.30 thru Figure 5.35 compare the chamber pressure curve with the hydraulic cylinder displacement curve at 100 Hz and 200 Hz for one cycle under loads of 0 kg, 6.840 kg, and 20.153 kg (thick line is pressure, thin line is piston displacement). These plots provide detailed knowledge of what is happening during the operation cycle and why the velocity/power tail off after the 120 or 140 Hz frequency.

For no load, one cycle operation at 100 Hz is shown in Figure 5.30 (thick line is pressure, thin line is piston displacement). Notice how the cylinder displacement begins to increase at the same time as the pressure begins to decrease from the 700 psi value, just after the out valve opens. After motion begins, the piston pushing the fluid continues to extend along with the hydraulic cylinder, while the pressure built up continues to drop. This occurs until the piston reaches its maximum displacement and there is no longer any pressure built up for pushing the hydraulic cylinder. The pressure inside the fluid circuit drops to a pressure close to the pre-pressurized value of 190 psi and the hydraulic cylinder settles around its displaced value. Finally, the valves switch (in open, out closed), fluid pressure builds inside the chamber once again, and the hydraulic cylinder continues to settle until the cycle is repeated. The same sequence of events occurs for the systems under 6.840 kg and 20.153 kg loads while operating at 100 Hz, shown in Figures 5.31 and 5.32. The one big difference for the loaded cases however, is that the pressure curves settle out at higher values due to the static pressure generated by the hanging load. These settling pressures are around 315 psi for the 6.840 kg load and 600 psi for the 20.153 kg load.

A major difference in the operation cycle can be seen when comparing the 100 Hz operation with the 200 Hz operation. For no load, one cycle operation at 200 Hz as seen in Figure 5.33, changes in the process are seen right away. Due to the increased frequency, the period is smaller, and thus the valve timing changes. For the 200 Hz cycle a 5% positive offset is only half the time as a 5% positive offset in a 200 Hz cycle. This results in the out valve opening sooner and thus the maximum pressure does not reach the same peak value as in the 100 Hz case. The approximate maximum value for the 200 Hz cycle is only 765 psi, compared to the approximate 900 psi value for the 100 Hz cycle. Also to note, the

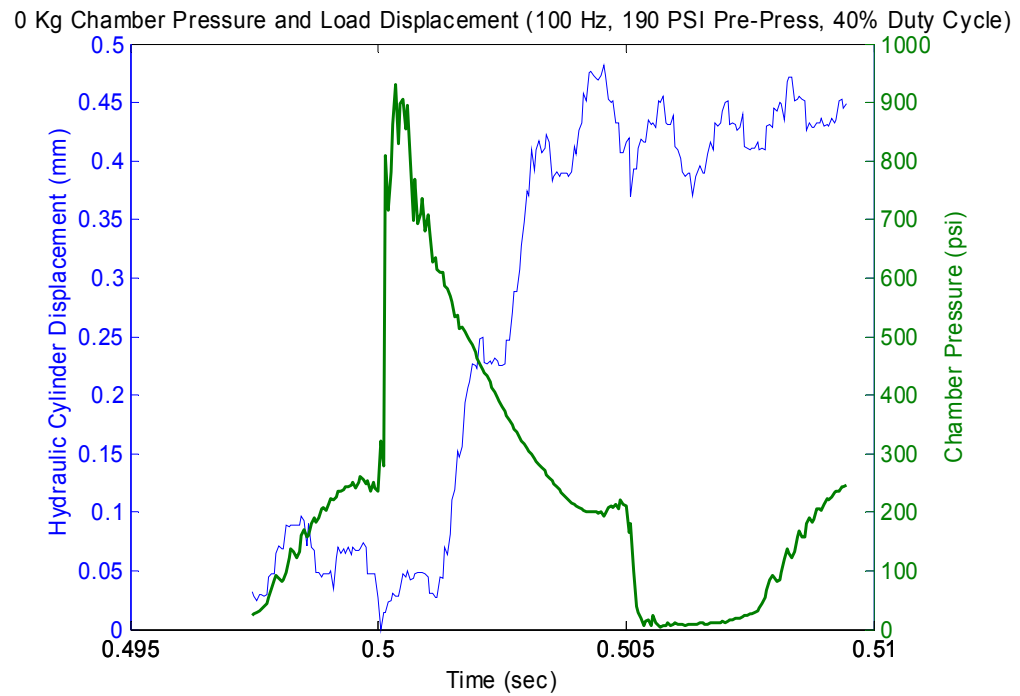


Figure 5.30: 100 Hz Typical Experimental Flow Cycle, 0 kg (thick line = pressure)

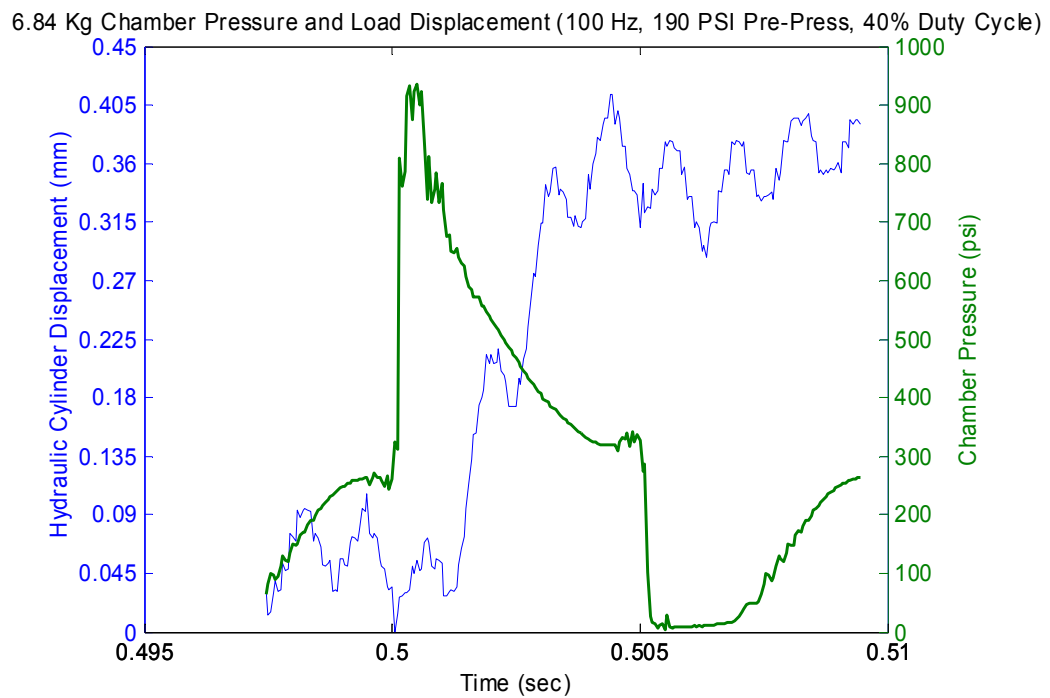


Figure 5.31: 100 Hz Typical Experimental Flow Cycle, 6.840 kg (thick line = pressure)

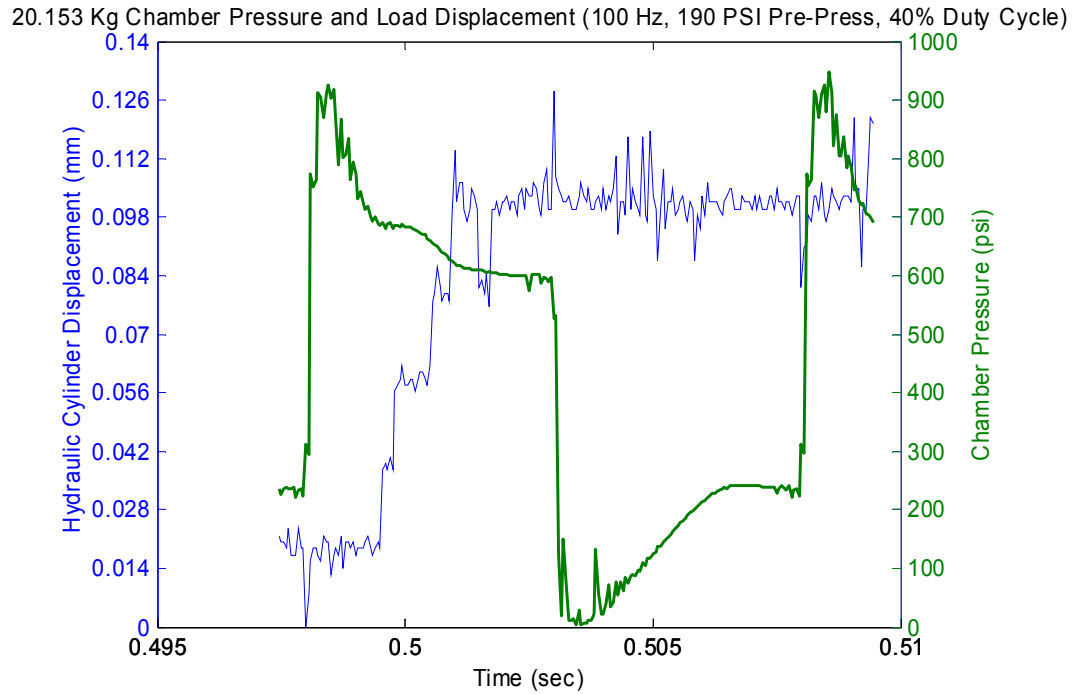


Figure 5.32: 100 Hz Typical Experimental Flow Cycle, 20.153 kg (thick line = pressure)

hydraulic cylinder displacement still begins to increase at the same time as the pressure begins to decrease, but the decrease in pressure is from a lower value now. Instead of the 700 psi value as stated before, the decrease for the 200 Hz cycle begins at around 520 psi. Then once again, the piston pushing the fluid continues to extend along with the hydraulic cylinder, while the pressure built up continues to drop. The extension of the piston in the 200 Hz cycle does not reach it's maximum displacement however, and the pressure before the outlet valve closes does not drop to the pre-pressure/load pressure value. In the no loaded case, the pressure only drops to around 425 psi for the 200 Hz cycle as opposed to the 100 Hz cycle value of 190 psi. There is not enough time in the 200 Hz cycle for the cylinder to be moved and the piston to extend to its maximum displacement. Pressure remains built up inside the fluid circuit when the outlet valve closes. The same is true in the loaded cases of the 200 Hz cycle. Their pressures at the time of valve closure are around 500 psi for the 6.840 kg load and 680 psi for the 20.153 kg load.

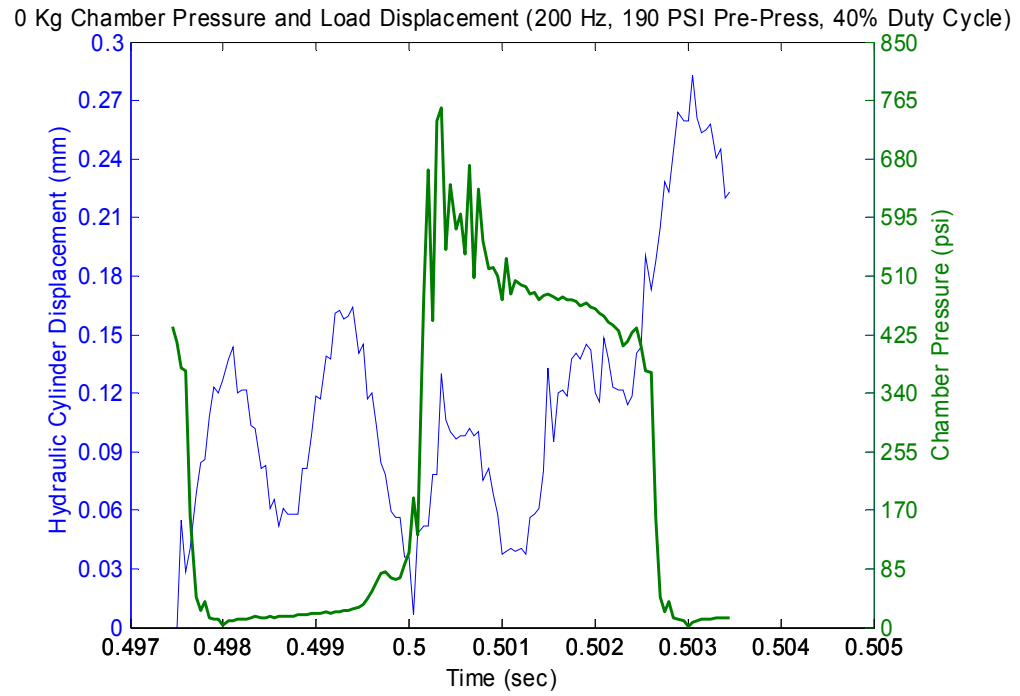


Figure 5.33: 200 Hz Typical Experimental Flow Cycle, 0 kg (thick line = pressure)

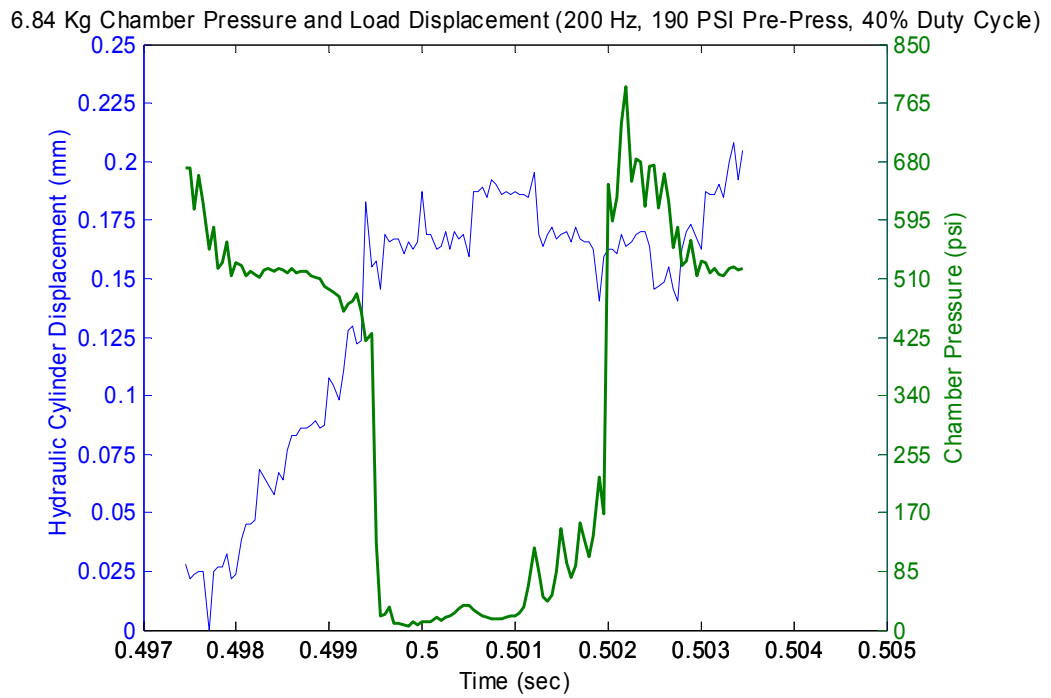


Figure 5.34: 200 Hz Typical Experimental Flow Cycle, 6.840 kg (thick line = pressure)

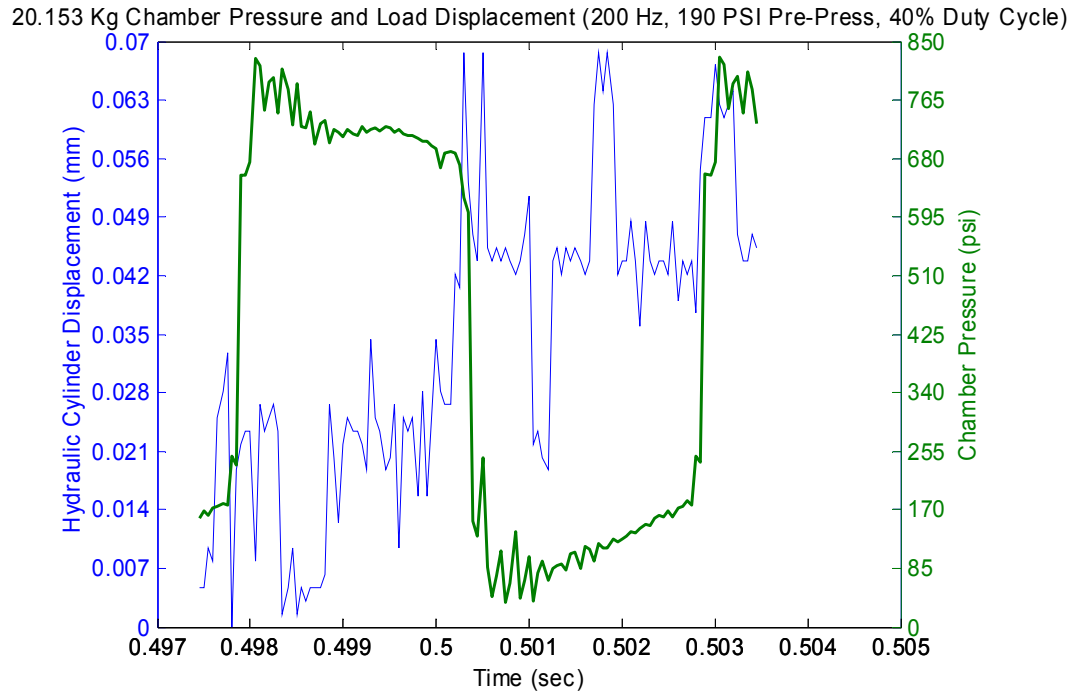


Figure 5.35: 200 Hz Typical Experimental Flow Cycle, 20.153 kg (thick line = pressure)

5.4.3 Actuator System Correlation

One cycle of the hydraulic cylinder/load displacement while operating at 100 Hz is shown in Figure 5.36 for each load. The time taken to reach the settling value each cycle is approximately equal for each load measured (0.003 - 0.004 sec or faster). Thus, it appears that the fluid circuit is very stiff such that the added inertia from the load does not seem to add any significant delay to the displacement response. The chamber pressure for each load is shown in Figure 5.37 and also contributes to understanding the dynamics. For each load, the chamber pressure settles at a value equal to the pre-pressure plus the back pressure acting on the hydraulic cylinder due to the load. The pressure does settle around this value just before the outlet valve closes for the 100 Hz cycle. The negative slope of the pressure from its peak down to the settling value, however, significantly decrease with increasing load. This implies that the load and added inertia do have an effect on the cylinder displacement response time, but still enables the settling of the cylinder at its peak value. This indicates that the piston reached full extension and the maximum amount of fluid was flushed from the chamber prior to the outlet valve closing for a 100 Hz cycle.

Experimental Cylinder Displacement for One Cycle (190 PSI Pre-Press, 40% Duty Cycle)

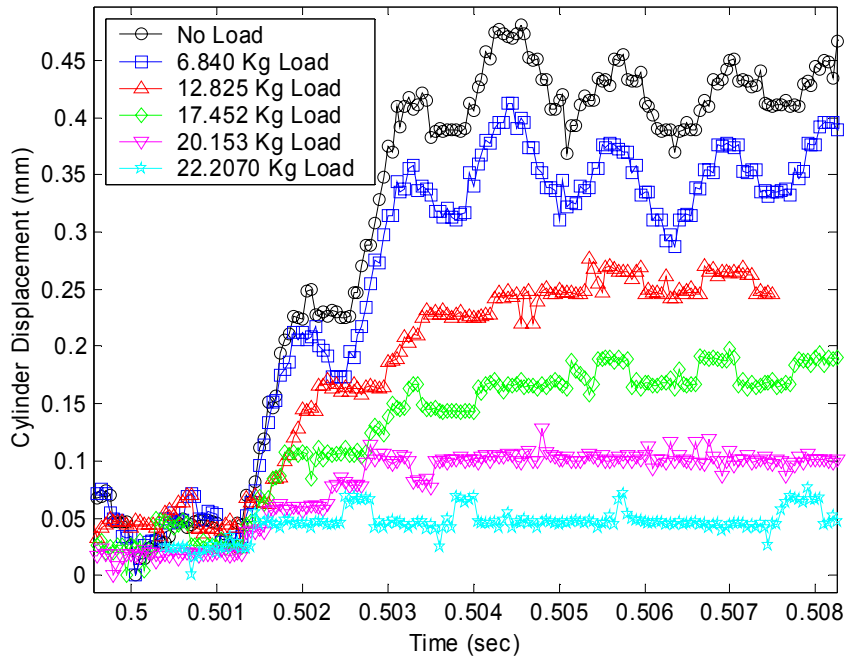


Figure 5.36: Experimental Cylinder/Load Displacement for One 100 Hz Cycle

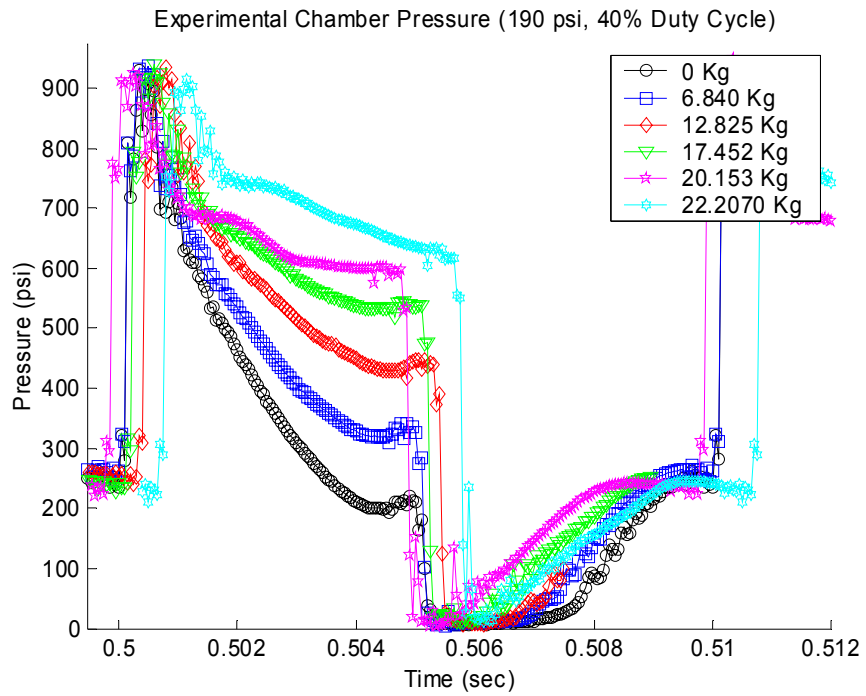


Figure 5.37: Experimental Chamber Pressure for One 100 Hz Cycle

An attempt was made to model these experimental results using the lumped-parameter approach outlined in Chapter 4. Throughout the modeling process, memory errors began to appear. These memory errors occurred due to the large state matrices being built for the entire circuit. Thus, simulations were run using a maximum number of elements, while also neglecting the chamber re-pressurization model of Section 4.5.2. This model would only have a significant effect on the pressure waveform during the fourth cycle, and was modeled using the chamber elements along with one other added element instead. This added element's stiffness, damping, and mass were tuned to obtain a reasonably similar response to the experiment, but was not meant to accurately match experimental pressure values during the fourth stage. Table 5.1 provides a definition of the number of elements (lumps) used for each fluid section and the elemental stiffnesses associated with them. A maximum of 1021 fluid elements (states) could be built into the largest state-space matrix before memory errors appeared. Simulations were performed using Matlab 6.1 and the computer used for simulation was a DELL DIMENSION 8200, with a 1.8 GHz Pentium 4 processor, 512 MB of RAM, and Windows XP.

Using these defined elements and stiffnesses, the state-space models were run while under the same loads as in the experiment. The simulated hydraulic cylinder/load displacements for one cycle while operating at 100 Hz can be seen in Figure 5.38 and the chamber pressures can be seen in Figure 5.39. The load displacements for this cycle appear to be nothing like the experiment, except for the unloaded case. This is attributed to the model's inability to match the experimental fluid stiffness. In the unloaded case, there is no added mass/inertia on the cylinder rod and therefore the elemental stiffnesses are high enough for a fast cylinder displacement rise time, which is similar to the experimental results. The chamber pressure here also drops much further than the experiment just after the outlet valve opens. This too may be attributed to the inadequate stiffness of the individual elements in the fluid circuit model. If the elemental stiffnesses were higher (smaller length of element, more elements per fluid section), the pressure/force would be stored between the elements better and transmitted more quickly to the load. There wouldn't be such a large loss because the elements would be able to store the potential and transmit it through to the next element very quickly.

In the simulated chamber pressure results of Figures 5.39 and 5.41, a negative pressure appears in the third stage of the simulation. This occurs with the lower volume of fluid

Table 5.1: Fluid Section Elements and Stiffnesses

Section	Number of Elements	Elemental Stiffness $\left[\frac{\text{N}}{\text{m}}\right]$
<i>pc</i>	8	$1.3267e^9$
<i>vc</i>	25	$2.8302e^7$
<i>ve</i>	10	$4.0097e^8$
<i>ec1</i>	10	$6.4367e^7$
<i>ec2</i>	25	$3.4892e^7$
<i>ec3</i>	10	$9.6944e^7$
<i>ecs1</i>	20	$5.9729e^7$
<i>ecs2</i>	20	$5.2303e^7$
<i>tec1</i>	25	$3.6904e^7$
<i>tec2</i>	45	$3.3561e^7$
<i>t</i>	200	$1.4011e^7$
<i>tcc1</i>	40	$2.3975e^7$
<i>tcc2</i>	60	$2.0855e^7$
<i>ci</i>	40	$1.3288e^8$
<i>cyl</i>	45	$9.6392e^7$
<i>cyl2</i>	60	$5.2393e^7$
<i>co</i>	25	$7.8648e^7$
<i>bco</i>	25	$6.6004e^7$
<i>bac</i>	100	$2.131e^7$
<i>acc</i>	8	$1.4766e^9$
<i>gas</i>	220	$1.7086e^7$

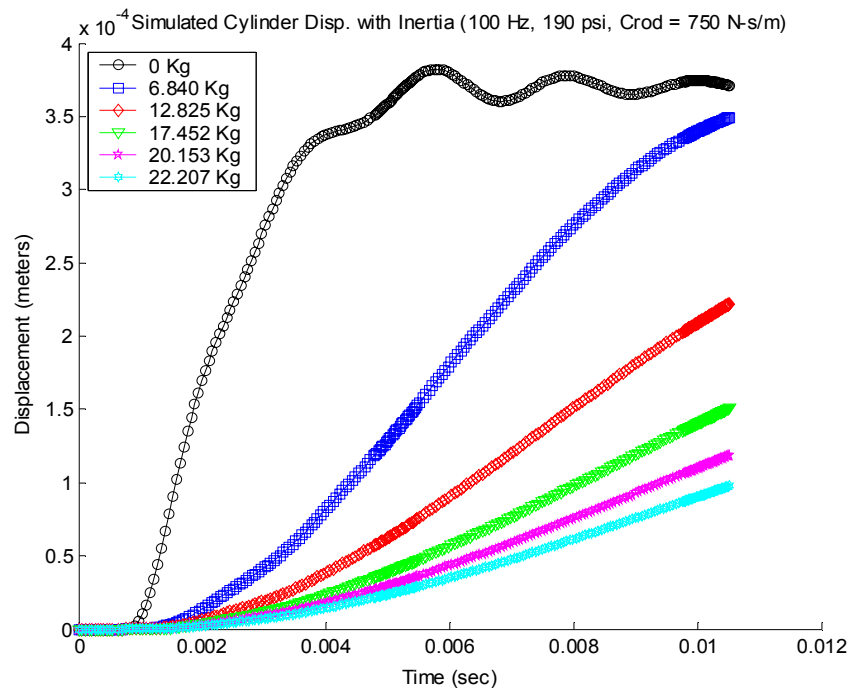


Figure 5.38: Simulated Cylinder/Load Displacement for One 100 Hz Cycle, With Load Inertia

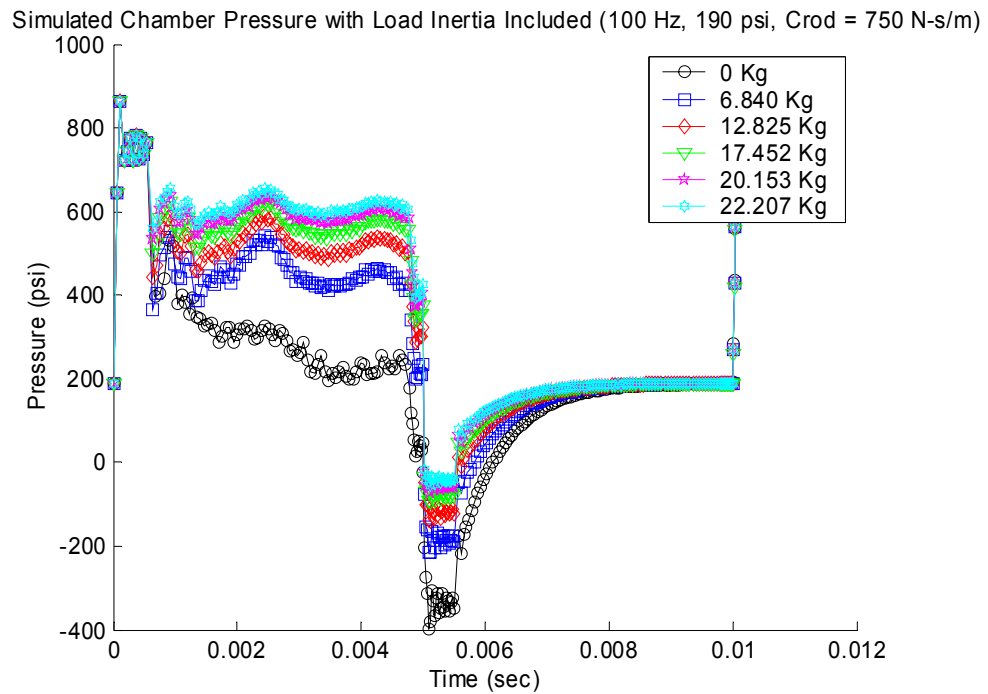


Figure 5.39: Simulated Chamber Pressure for One 100 Hz Cycle, With Load Inertia

in the chamber while the piston retracts. In the experimental chamber, this is the condition where cavitation may occur due to the large volume increase and pressures dropping below the standard atmospheric value. A pressure of zero or below will never occur in the real system due to the nonlinear relationship of an adiabatic closed volume expansion. In the simulated system, the force used to push the fluid from the chamber settles to a certain pressure at the end of the second stage. When the voltage driving the piezoelectric material returns to zero during the third stage, there is a force equal to the product of the piezoelectric material stiffness and the current piston displacement value. This force is simulated within a linear system and is shown as a negative pressure because it is much larger than the pressure where the system had settled to. If it were in a non-linear system, the force could still be large, but the pressure would always remain above the zero value. Thus, linear lumped-parameter fluid models can not be used to predict pressure for a non-linear adiabatic volume expansion using a simple force/area approach. Other non-linear equations must be used instead.

Simulations were also performed without the added mass/inertia of the load on the hydraulic cylinder rod, due to its domination of the system dynamics with the small elemental stiffnesses. The cylinder/load displacements per cycle for this study are shown in Figure 5.40 and the corresponding chamber pressures are displayed in Figure 5.41. The cylinder rod displacements match fairly well with the experimental curves, however at the higher loads the settling values are a bit larger in the simulation. The lack of added mass/inertia allows for a quicker displacement rise time, full piston extension, more fluid volume pushed from the chamber, and thus more cylinder rod displacement per cycle. The pressure curves without the added load inertia still appear similar to the previous simulated results due to the stiffness of the system remaining the same. There is a little downward slope in the second stage of the pressure curves without added mass however, when compared to the large drop and almost constant values of the curves with added load inertia.

Since the results without the added inertia matched the experiment better, simulations were also run for this case over a range of frequencies from 40 to 300 Hz for each load. The cylinder velocities for the simulated case were calculated by taking each settled displacement value per cycle and multiplying that value by the operation frequency. The powers were calculated as the product of these velocities and the load weight. Their comparisons with the experimental values can be seen in Figure 5.42 thru Figure 5.45 by

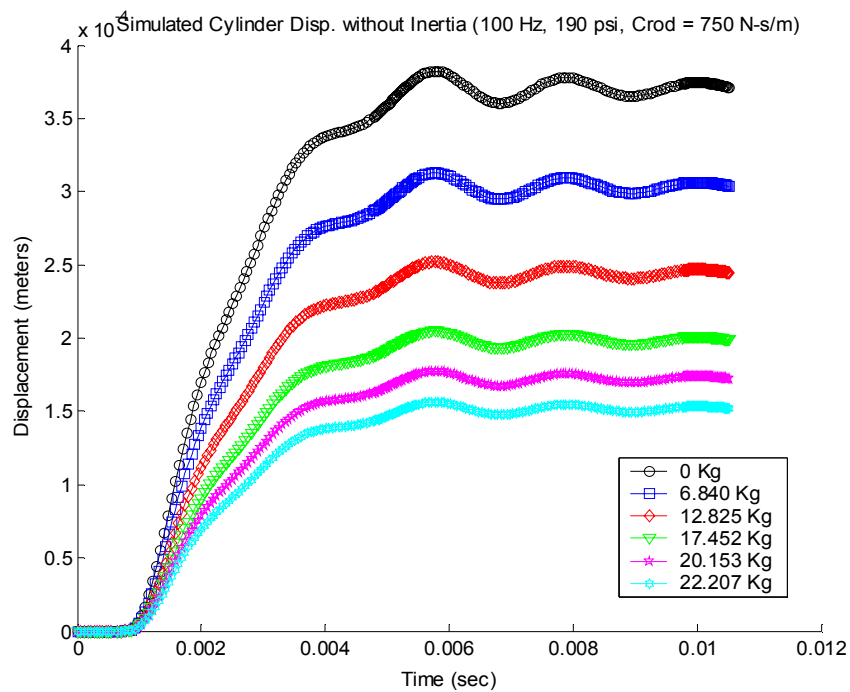


Figure 5.40: Simulated Cylinder/Load Displacement for One 100 Hz Cycle, Without Load Inertia

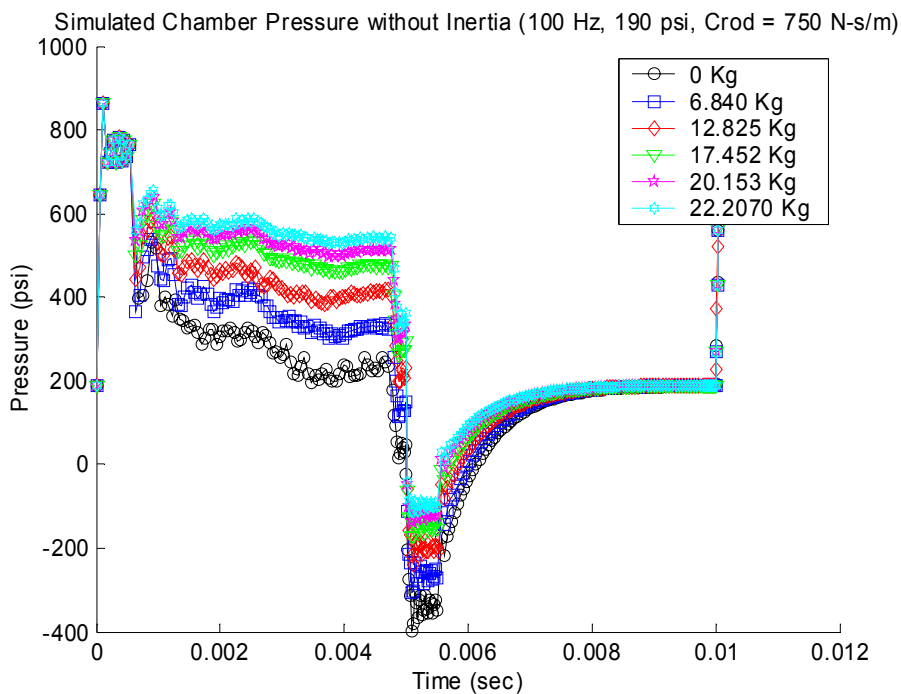


Figure 5.41: Simulated Chamber Pressure for One 100 Hz Cycle, Without Load Inertia

plotting them against frequency and load.

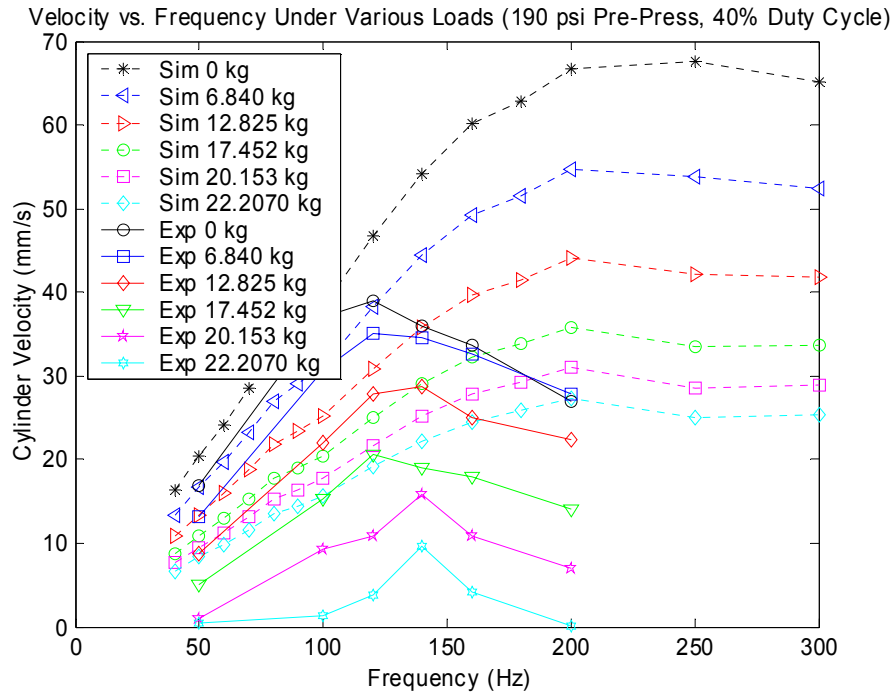


Figure 5.42: Load/Cylinder Velocity vs. Frequency Comparison

The lower frequency and lower load cases track well when comparing experimental with simulated results, as seen in Figure 5.26 and Figure 5.27. Load values above the 12.825 kg condition result in large discrepancies, along with frequencies above the 120 to 140 value. This is definitely a result of the low elemental stiffnesses in the simulation, and may be a result of the excluded inertias as well. As seen in the simulated pressures, the slope from the peak value down to the settling pressure is not even close to the experimental results. The piston displacement, and thus the fluid volume pushed from the chamber follows a curve opposite of the pressure curve. Due to the pressure curve inaccuracies, this means that the fluid volume pushed from the chamber to move the hydraulic cylinder is inaccurate also. Thus, the resulting velocities and powers differ from the experiment.

At the frequencies below 100 Hz and the loads at or under the 12.825 kg value, the settling values are all about the same when looking at the simulated and experimental cylinder displacements or pressures. Once again, the higher loads may also be inaccurate due to the neglected inertia. When the operation frequency increases, the pressure and chamber piston displacement (directly effects fluid volume pushed into hydraulic cylinder)

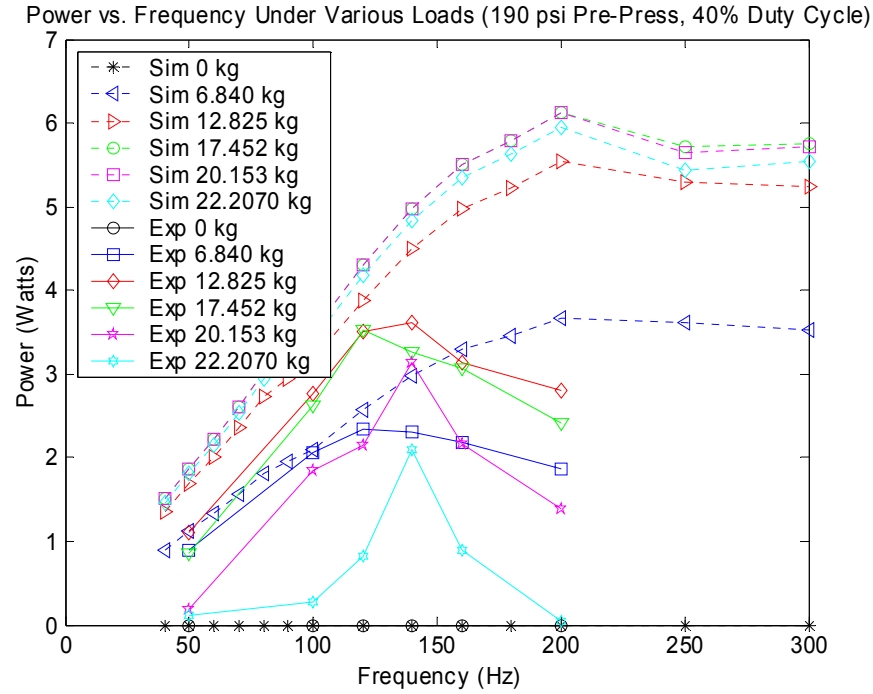


Figure 5.43: Output Power vs. Frequency Comparison

become cut-off and can not drop/rise to their settling values. Thus, the volume of fluid pushing the cylinder drops significantly along with the velocity and overall power of the actuator. Due to the very small slope of the simulated pressure and thus chamber piston displacement, this cut-off at high frequencies doesn't have any effect until you get much closer to the peak/cycle starting values. This is where the drop in the simulated results starts to occur, around the 250 Hz operation frequency. The comparison of the results when plotted against load value are shown in Figure 5.44 and Figure 5.45.

Bode plots help to explain these results also. Figure 5.46 shows the frequency response of the simulated closed chamber pressure. The results shows a second order system with a damped natural frequency around 3250 Hz. There is no lower frequency mode present that would cause the nonlinear velocity changes in the 250 Hz actuator simulation. Figure 5.47 shows the chamber piston, chamber pressure, and hydraulic cylinder displacement frequency responses for the entire actuator system. Here the first damped natural frequency is around 450 Hz, which also should not reflect any changes in the simulated dynamic response when operating at 250 Hz. Lastly, Figure 5.48 shows the hydraulic cylinder frequency response for the system after the outlet valve closes. Once again there is a

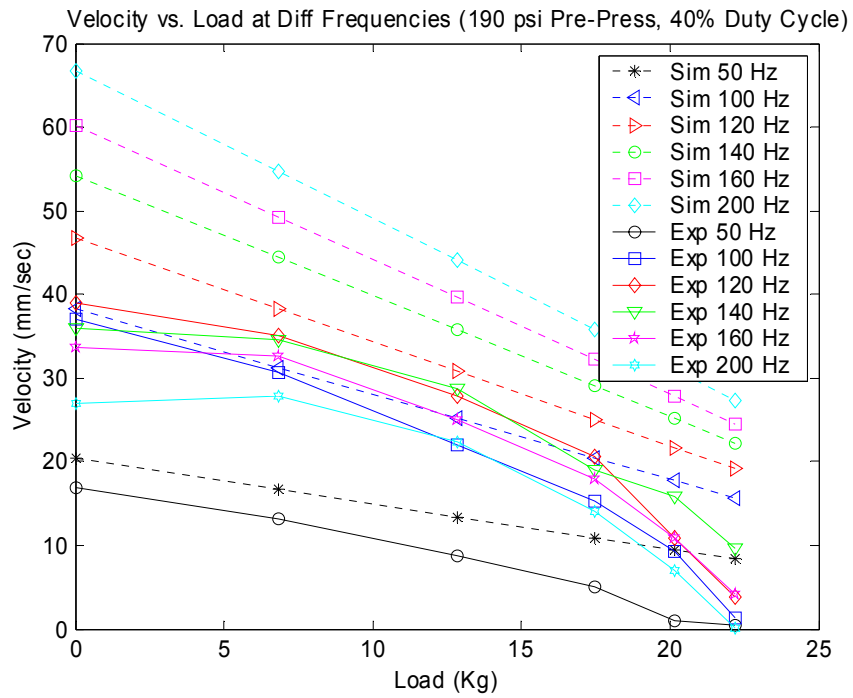


Figure 5.44: Load/Cylinder Velocity vs. Load Comparison

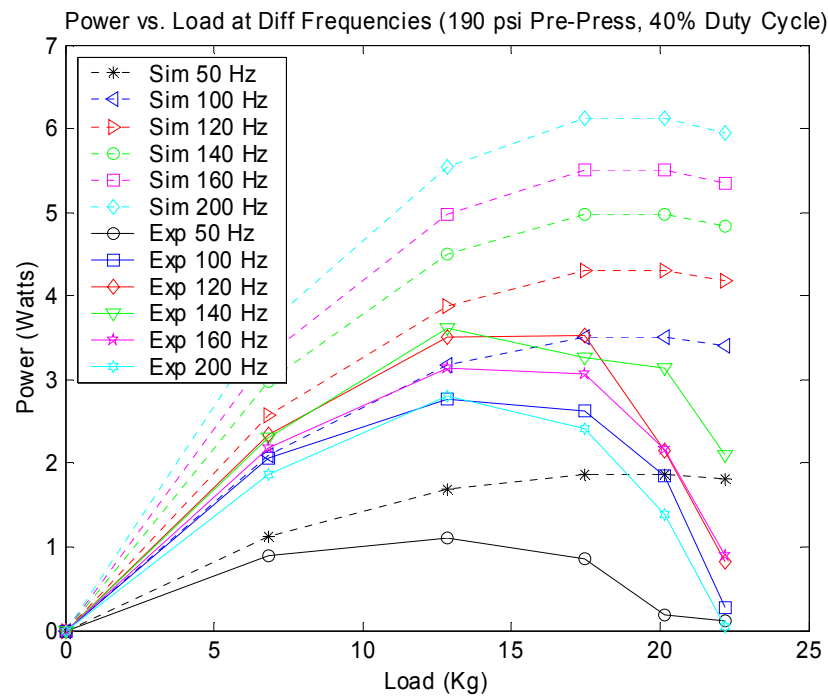


Figure 5.45: Output Power vs. Load Comparison

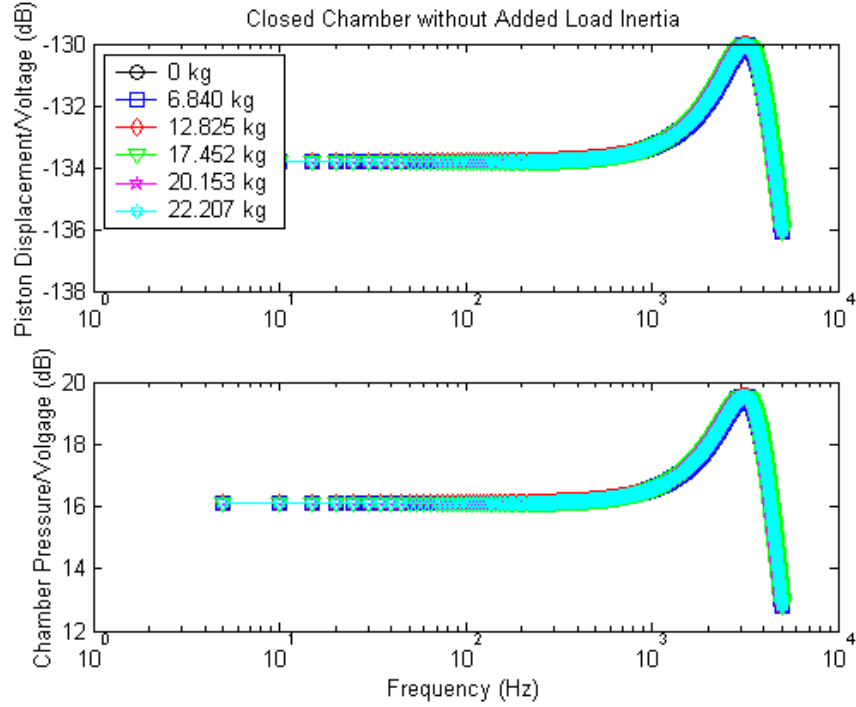


Figure 5.46: Simulated Closed Chamber Bode Plot, Without Load Inertia

resonance around 500 Hz with an antiresonance around 450 Hz. Thus, the dynamics of the simulated system at 250 Hz should not be affected by this system either. These three plots show that the velocity of the hydraulic cylinder is being limited by the closing of the outlet valve before all the fluid is pushed from the chamber, not the dynamics of the system. At the simulated higher frequencies (around 250 Hz), the dynamics of the system do not allow the fluid to be pushed quickly through the valves, and thus the chamber piston can not extend to its maximum length before the outlet valve closes. The inherent system properties do not, however, cause the drops in velocity above the 250 Hz frequency due to resonance or antiresonance.

If more elements could be added to the lumped-parameter model without memory errors, the load inertia could be included in the simulations based on a quicker model response. Model convergence would then be achieved when increasing the number of elements had no greater effect on the dynamics of the system. This convergence would result in closer matching of the experimental chamber piston and pressure curves. Increasing the operation frequency or load would then have a significant effect on the simulated results, as seen in the experiments. Thus, the matching of the pressure and chamber piston curves would then

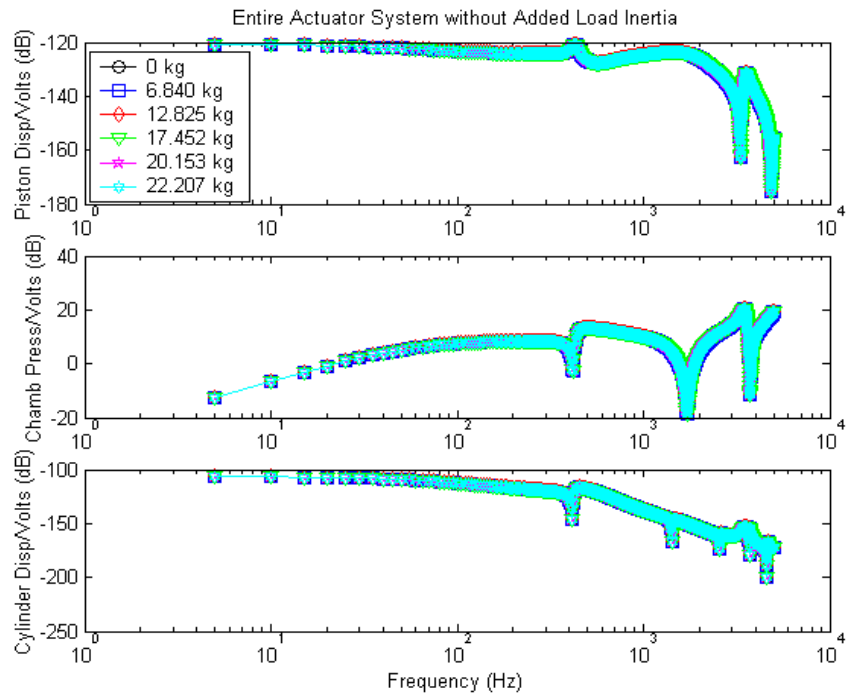


Figure 5.47: Simulated Entire Actuator System, Without Load Inertia

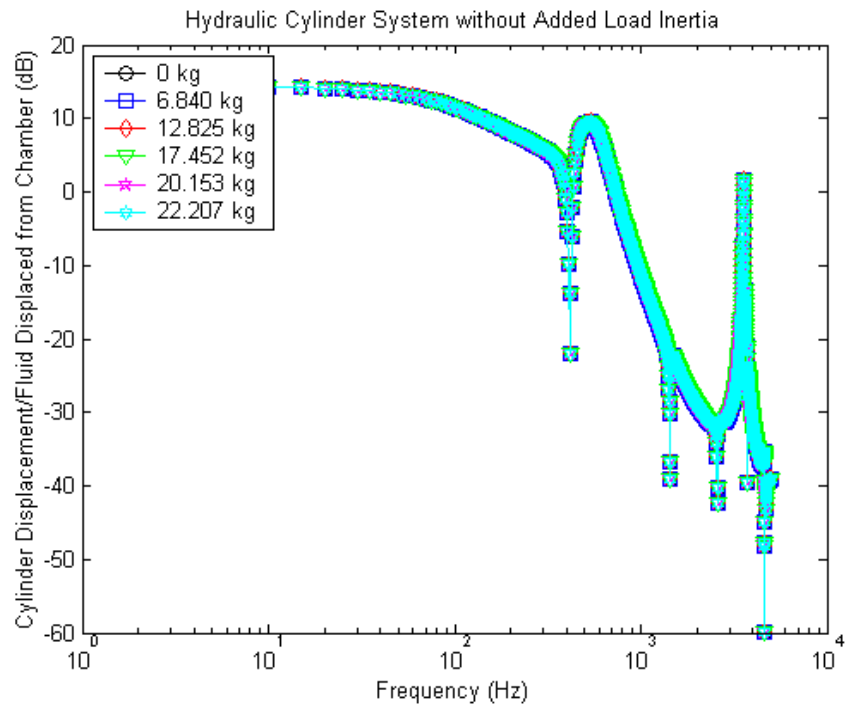


Figure 5.48: Simulated Hydraulic Cylinder After Valve Closing, Without Load Inertia

result in closer matching of the velocity and power vs. frequency curves.

Also, when comparing a 1021 element lumped-parameter model to standard finite-element packages, that would not be enough elements to obtain convergence in most cases. Most finite-element simulations done using a package such as ANSYS require tens of thousands of elements to achieve results that are close to actual values. This lumped-parameter state-space simulation results in a very computationally intensive method, which requires enormous amounts of memory to store the state-space matrices in Matlab.

Chapter 6

Conclusions

A piezohydraulic actuator with active valves was developed and tested. Current maximum actuator power is 3.6 Watts while operating at a frequency of 140 Hz under a load of 12.825 kg, and current maximum hydraulic cylinder velocity is 39 mm/sec while operating at a frequency of 120 Hz under no load.

When designing a piezohydraulic system for optimal flow and power, there are many concerns, which are outlined here:

- The maximum amount of air should be removed fluid circuit and the tubing must be made of stiff materials and/or be made very thick.
- To reduce the friction and resistance losses, the circuit tube lengths should be as short as possible, should have the least amount of area changes, and the valves should open as far as possible. Also, making the tube inside diameters an average size (not too small) will increase the stiffness of the system and lower its response time.
- Other circuit components effecting the flow and power rating of the piezohydraulic actuator are the hydraulic cylinder, air/hydraulic accumulator, and chamber valves. These should have high enough pressure ratings, quick response times, and create the least amount of friction/resistance possible.
- The piezoelectric material driving the piston should be made as stiff as possible, have the maximum amount of pushing force, and extend as far as possible. Thus, the piston will then respond the fastest, create the most pressure, and push the most fluid to move the hydraulic cylinder/load.

- Electronics operating the actuator components should provide enough power and have very quick rise times so that there is negligible time delay associated with them.

A free body diagram analysis of each actuator component enables the development of equations of motion, and by coupling them together state-space models can be formed for the entire piezohydraulic actuator. Piston dynamics can be determined based on the force applied by the piezoelectric material. Converting the fluid sections into mechanical springs, masses, and dampers using lumped-parameter model theory enables their dynamics to be linked to the piston and each other. The hydraulic cylinder and load can be included in this series of elements by combining the masses of the load and cylinder rod along with including a static force due to gravity on the hanging mass. With the equations of motion defined, all the elements may be linked and conservation of mass can be implemented on the fluid sections using a constraint matrix. Once this is done, linear algebra can be used to develop the state-space matrices for simulation.

A four stage cycle can be used to simulate the actuator dynamics based on the state-space matrix models for each cycle. The first stage (both valves closed) simulates the states inside the chamber along with the outlet valve displacement. A logical check is used to determine when the outlet valve opens. Once the outlet valve is opened, the entire actuator dynamics are simulated along with the outlet valve displacement once again, to determine when the valve closes. The beginning chamber states for the second stage (entire actuator system) are initially set to the final chamber state values from the first stage. In the third stage, both valves are closed once again, but there are two dynamic systems in motion along with the inlet valve displacement simulation. One simulation is used to model the chamber dynamics with a lower volume of fluid, another is used to model the fluid circuit and hydraulic cylinder settling, and a third is used to check when the inlet valve opens. In the fourth stage, the fluid circuit and hydraulic cylinder simulation continues to settle and a second basic simulation is used to model the chamber re-pressurization due to memory limitations on the computer. All initial state values for each stage are taken from the final states of the previous stage until the cycle repeats after the fourth stage.

Experimental closed chamber pressure and piston displacement were correlated to simulated results for tuning model parameters and validating the closed chamber state-space model. Due to fluid/pressure leakage through the valves, experimental data was difficult to quantify, especially at lower operation frequencies where there was ample time for

leakage. Experimental and simulated signals did match well when neglecting the differences attributed to leakage, so the tuned parameters were used in future simulations of the entire actuator system.

A wall mounted actuator system was tested with an array of different valve duty cycles and offsets relative to the piezoceramic stack/piston motion. Test results show that the timing of the valves was crucial to the performance of the system and optimal actuator performance is highly dependent on the timing. The results also indicate that the optimal timing will also slightly change under different loads. In the unloaded case, the maximum flow rate or cylinder velocity of 6.5 mm/sec is achieved with a 40% duty cycle and a positive 5% output valve offset. For the loaded case (12.825 kg), the maximum velocity value of 5.23 mm/sec is achieved with a 45% duty cycle and a 0% output valve offset.

A bench top actuator system was tested with the valves having 40% duty cycles and a 5% positive offset for the outlet valve. An array of tests was conducted for various operation frequencies ranging from 50 Hz to 250 Hz while under loads ranging from 0 kg to 22.207 kg. These observations were seen:

- Cylinder velocity and actuator power increased fairly linearly with increasing frequency up to 120 Hz. At that point, the velocities and powers began to decrease with increasing frequency (140 Hz for the two highest loads). This is attributed to the actuator system's inability to push the maximum amount of fluid from the chamber and into the hydraulic cylinder while operating at these higher frequencies. The chamber pressure never drops to the pre-pressure plus loaded pressure value, indicating that the full fluid flow does not exit the chamber prior to the outlet valve closing. There is still dynamic pressure built up between the piston and the cylinder rod when the valve closes.
- Hydraulic cylinder velocity decreased approximately linearly with increasing load for all operation frequencies.
- Actuator power formed a concave down parabola when plotted against load. The maximum power was achieved at an approximate load value of 12.825 kg for all frequencies.

Approximate correlation of cylinder velocity and actuator power was achieved when using a lumped-parameter model which neglected the inertia of the hanging load. It is

believed that the inertia of the load dominated all other inertias and the system dynamics such that the results were not accurate when included. This may have been due to the fact that a maximum of only 1021 elements could be included in the largest state-space model due to memory limitations of Matlab 6.1 and the computer used for simulation (DELL DIMENSION 8200, 1.8 GHz Pentium 4 processor, 512 MB of RAM, Windows XP). Since only this many elements were included in the model, there was a limitation on how stiff each individual element could be defined as. Thus, there was also a limitation on how stiff the system was when modeled, and how quickly it could respond when a very large inertia was included in the dynamics. It is believed that the simulation was performed well away from the convergence of the lumped-parameter system.

The lumped-parameter model approximated the hydraulic cylinder displacement/velocity and actuator power fairly well when the load inertia was neglected at frequencies below 120 Hz and loads at or below 12.825 kg. There were large discrepancies when looking at the chamber pressure comparisons for all cases. The first two stages modeled the pressure fairly well when the piezoelectric actuator was compressing the fluid in the circuit. In the third stage however, the simulations showed negative pressures when the piezoelectric material was trying to return to its original position with a lower volume of fluid inside the chamber. Based on the actual displacement of the piston/piezoelectric material tip at the end of the second stage, there is a force being transmitted to the fluid that can be calculated. When the outlet valve closes and the applied piezoelectric voltage goes to zero in the third stage, that force (equivalent to the piezoelectric material stiffness times the displacement of the piston just prior to the voltage drop) then reverses direction on the piston. In our linear lumped-parameter model, the pressure associated with this force (force/area) is greater than the pressure in the chamber at the time, which results in a negative overall pressure. Thus, this linear model can not accurately model the true nonlinear pressure/volume relationship of the volume trying to expand.

6.1 Recommendations and Future Work

From the experience of this research, the following recommendations on future work are suggested:

- Further analysis of the bench top actuator system should be done to study the effects

of higher pre-pressures on the system. These higher pressures may lead to a stiffer fluid and a quicker response of the hydraulic cylinder. This possibly will create higher velocities and more mechanical power when operated at higher frequencies.

- Further study should also be done on the piston/valve interaction. Increasing the duty cycle of the piston and outlet valve should give more time to push fluid from the chamber and into the hydraulic cylinder while operating at frequencies above 120 Hz. This too, may lead to higher velocities and more mechanical power.
- The four-stage model should be simulated on a super computer with many more lumped-parameter elements for better convergence.
- The four-stage model should be directly coded into a program, bypassing the Simulink models for faster simulations. Currently the simulations are very computationally intense and require many hours, much like other finite-element simulations.
- Simulations should also be studied using different piezoelectric actuator properties to determine the optimal piezoelectric material for performance. This will lead to a better understanding of how to modify the actuator to achieve the powers and velocities desired. Also, many other system dimensions and variables can be changed for simulating the effects on possible actuator redesigns.
- Other non-linear equations and theories should be incorporated into the simulation to better model the pressures and forces. The linear lumped-parameter model might still be used to simulate the velocities and displacements. Then, these values could be taken to calculate volume and flow rate, for inclusion in non-linear pressure equations.

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Appendix A

Matlab Code for Closed Chamber Simulations and Comparisons

Closed chamber state-space simulations were performed using a Matlab m-file (Chamber-StateSpaceCompareSaved.m) and compared to experimental data.

```

clear all
close all
clc

Kact = 31e6; % N/m
Xo = 9.0e-7; % m/V
Mpist = 0.075; % Kg
Cpist = 100;
Coring = 4000;
Koring = 2.3e3; % N/m

% Define Fluid Constants
KinVisc = 3.21e-5; % (m^2)/(s)
%RhoWater = 999.973; % Kg/(m^3)
Rho = 869.9765; % Kg/(m^3)
DynVisc = KinVisc.*Rho; % (Kg/(m-s))
BI = 250000.*6894.75729; % N/(m^2) (250,000 psi)
%x = 0.00131; % Air content in fluid
x = 0.00103; % Air content in fluid
Estainless = 190*10^9; % [Pa] (Elastic Modulus of Stainless Steel)
Renum = 1800; % Reynolds Number
Frict = 64/Renum; % Friction Factor

%Pumping Chamber Dimensions
Dpc = 0.025; % m
tpc = 0.01; % m
Apc = pi*(Dpc^2)/4; % m^2
Lpc = 0.0024; % m

%Valve Chamber Dimensions
Hvc = 0.0029; % m
Wvc = 0.0105; % m
Dvc = 4.*(Wvc.*Hvc)./(2.*Wvc+2.*Hvc); % m
tvc = 0.01; % m
Avc = Wvc.*Hvc; % m^2
Lvc = 0.022; % m
KLvc = 0.5; % Resistance Coefficient

FreqInit = [1 5 10 40 50 60 70 80 90 100 120 140 160 180 200 250 300];
Omega = 2.*pi.*FreqInit;
PressInit = [190];
Volts = [110];
NumPCelem = [15];
NumVCElem = [45];

n = size(PressInit,2);

m = size(FreqInit,2);

q1 = size(NumPCelem,2);
q2 = size(NumVCElem,2);

colors = ['ko-','bs-','rd-','gv-','mp-','ch-','y^-' ; 'k*-' ; 'b<-' ; 'r>-'];

for Num1 = 1:q1 %Num of Pumping Chamb Elements
    for Num2 = 1:q2 %Num of Valve Chamb Elements
        for Press = 1:n
            clear Po Beff %PressInit

```

```

Po = PressInit(1,Press).*6894.75729;
Bg = 1.4.*Po;
Bpc(Press) = 1./((Dpc./(tpc.*Estainless))+((1-x)./BI)+(x./Bg));
Bvc(Press) = 1./((Dvc./(tvc.*Estainless))+((1-x)./BI)+(x./Bg));
Beff = 1./((1./Bpc(Press))+(1./Bvc(Press)));

for Freq = 1:m %FreqInit
    clear FluidSys Acc Bcc Ccc Dcc Mcc CDcc Kcc Tcc Tcc2 Fcc Y X Xcc T1 U Freqss Omegass

    Freqss = FreqInit(1,Freq);
    Omegass = Omega(1,Freq);

    if (Omegass < ((7200.*DynVisc)./(Rho.*(Dpc.^2))))
        Mpc = (4./3).*Rho.*(Lpc./NumPCelem(1,Num1)).*Apc;
        Cpc = 8.*pi.*DynVisc.*(Lpc./NumPCelem(1,Num1));
    else
        Mpc = Rho.*(Lpc./NumPCelem(1,Num1)).*Apc;
        Cpc = 0.5.*pi.*Dpc.*(Lpc./NumPCelem(1,Num1)).*((2.*Rho.*DynVisc.*Omegass)^0.5);
    end
    if (Omegass < ((7200.*DynVisc)./(Rho.*(Dvc.^2))))
        Mvc = (4./3).*Rho.*(Lvc./NumVCElem(1,Num2)).*Avc;
        Rvc1 = (12.*DynVisc.*(Lvc./NumVCElem(1,Num2)))./(Wvc.*(Hvc.^3).*(1-
        (((192.*Hvc)./(pi.^5).*Wvc)).*tan((Hvc.*pi.*Wvc)./(2.*Hvc))));
        Cvc1 = (Avc.^2).*Rvc1;
    else
        Mvc = Rho.*(Lvc./NumVCElem(1,Num2)).*Avc;
        Cvc1 = 0.5.*pi.*Dvc.*(Lvc./NumVCElem(1,Num2)).*((2.*Rho.*DynVisc.*Omegass)^0.5);
    end

    Kpc = Apc.*Beff./(Lpc./NumPCelem(1,Num1));
    Rvc2 = (128.*DynVisc.*(KLvc./Frict))./(pi.*(Dvc.^3));
    Cvc2 = 2.*(Avc.^2).*Rvc2;
    Kvc = Avc.*Beff./(Lvc./NumVCElem(1,Num2));

    for i1 = 1:(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)
        if (i1 == 1)
            Mcc(i1,:) = [Mpist zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
            CDcc(i1,:) = [(Cpist+Coring) zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
            Kcc(i1,:) = [(Kact+Koring) zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
            Tcc(i1,:) = [1 zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2))-i1)];
            Fcc(i1,:) = [(Kact.*Xo)-((Po.*Apc)./Volts)];
        elseif (i1 == 2)
            Mcc(i1,:) = [0 Mpc zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
            CDcc(i1,:) = [0 Cpc zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
            Kcc(i1,:) = [0 Kpc -Kpc zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+1)-i1)];
            Tcc(i1,:) = [1 zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+1)-i1)];
            Fcc(i1,:) = [0];
        elseif ((i1 > 2) & (i1 < (NumPCelem(1,Num1)+2)) & (NumPCelem(1,Num1) ~= 1))
            Mcc(i1,:) = [zeros(1,i1-1) Mpc zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
            CDcc(i1,:) = [zeros(1,i1-1) Cpc zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
            Kcc(i1,:) = [zeros(1,i1-2) -Kpc 2.*Kpc -Kpc zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+1)-i1)];
            Tcc(i1,:) = [zeros(1,i1-2) 1 zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+1)-i1)];
            Fcc(i1,:) = [0];
        elseif ((i1 == 3) & (NumPCelem(1,Num1) == 1)) | ((i1 > 2) & (i1 == (NumPCelem(1,Num1)+2)) &
        (NumPCelem(1,Num1) ~= 1))
            Mcc(i1,:) = [zeros(1,i1-1) 0 zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
            CDcc(i1,:) = [zeros(1,i1-1) 0 zeros(1,(NumPCelem(1,Num1)+NumVCElem(1,Num2)+2)-i1)];
        i1)];

```

```

Kcc(i1,:) = [zeros(1,i1-2) -Kpc Kpc zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)-i1)];
Tcc(i1,:) = [zeros(1,i1-2) (Avc./Apc) zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+1)-i1)];
Fcc(i1,:) = [0];
elseif ((i1 == (NumPCelem(1,Num1)+3)) & (NumVCelem(1,Num2) ~= 1))
Mcc(i1,:) = [zeros(1,i1-1) Mvc zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)-i1)];
CDcc(i1,:) = [zeros(1,i1-1) (Cvc1+Cvc2) zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)-i1)];
Kcc(i1,:) = [zeros(1,i1-1) Kvc -Kvc zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+1)-i1)];
Tcc(i1,:) = [zeros(1,i1-3) 1 zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)-i1)];
Fcc(i1,:) = [0];
elseif ((i1 == (NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)) & (NumVCelem(1,Num2) == 1))
Mcc(i1,:) = [zeros(1,i1-1) Mvc];
CDcc(i1,:) = [zeros(1,i1-1) (Cvc1+Cvc2)];
Kcc(i1,:) = [zeros(1,i1-1) Kvc];
Tcc(i1,:) = [zeros(1,i1-3) 1];
Fcc(i1,:) = [0];
elseif ((i1 == (NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)) & (NumVCelem(1,Num2) ~= 1))
Mcc(i1,:) = [zeros(1,i1-1) Mvc];
CDcc(i1,:) = [zeros(1,i1-1) Cvc1];
Kcc(i1,:) = [zeros(1,i1-2) -Kvc (2.*Kvc)];
Tcc(i1,:) = [zeros(1,i1-3) 1];
Fcc(i1,:) = [0];
elseif ((i1 > (NumPCelem(1,Num1)+3)) & (i1 < (NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)) &
(NumVCelem(1,Num2) ~= 1))
Mcc(i1,:) = [zeros(1,i1-1) Mvc zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)-i1)];
CDcc(i1,:) = [zeros(1,i1-1) Cvc1 zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)-i1)];
Kcc(i1,:) = [zeros(1,i1-2) -Kvc (2.*Kvc) -Kvc zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+1)-
i1)];
Tcc(i1,:) = [zeros(1,i1-3) 1 zeros(1,(NumPCelem(1,Num1)+NumVCelem(1,Num2)+2)-i1)];
Fcc(i1,:) = [0];
end
end

Mcctrans = ((Tcc.')*Mcc*Tcc);
Ccctrans = ((Tcc.')*CDcc*Tcc);
Kcctrans = ((Tcc.')*Kcc*Tcc);
Fcctrans = ((Tcc.')*Fcc);
McctransInv = inv(Mcctrans);

rccol = size(Mcctrans,2);

Acc = [zeros(rccol,rccol) eye(rccol,rccol);-McctransInv*Kcctrans -McctransInv*Ccctrans];
Bcc = [zeros(rccol,1);McctransInv*Fcctrans];
Ccc = [eye(2.*rccol)];
Dcc = [zeros(2.*rccol,1)];

Xccinit = [zeros(2.*rccol,1)];
MyClosedChambSys = ss(Acc,Bcc,Ccc,Dcc);
eig(MyClosedChambSys);

t = 0:5e-5:2;
%U = (Volts - ((Po.*Apc)./(Kact.*Xo))).*(((0.5.*sin(2.*pi.*Freqss.*t))+0.5).');
U = (Volts - ((Po.*Apc)./(Kact.*Xo))).*(((0.5.*square(2.*pi.*Freqss.*t))+0.5).');

[Y,T1,X] = lsim(MyClosedChambSys,U,t,Xccinit);

ChambPress(Num1,Num2,Press,Freq,:) = ((Y(:,rccol).*Kvc./Avc)+Po)./6894.75729;
PistonDisp(Num1,Num2,Press,Freq,:) = (Y(:,1).*(10.^6));
SysInput(Num1,Num2,Press,Freq,:) = U;
Time(Num1,Num2,Press,Freq,:) = T1(:,1);

```

```

BulkEff(Num1,Num2,Press,Freq) = Beff./6894.75729;
    end
end

    eval(['PistonDispMicrons' num2str(Num1) '_' num2str(Num2) ' = PistonDisp(' num2str(Num1) ',' num2str(Num2)
';,,:);'])
    eval(['ChambPressPSI' num2str(Num1) '_' num2str(Num2) ' = ChambPress(' num2str(Num1) ',' num2str(Num2)
';,,:);'])
    eval(['TimeVector' num2str(Num1) '_' num2str(Num2) ' = Time(' num2str(Num1) ',' num2str(Num2) ';,,:);'])
    if ((Num1 == 1) & (Num2 ==1))
        eval(['save ClosedChamberSimulationData PistonDispMicrons' num2str(Num1) '_' num2str(Num2) '
ChambPressPSI' num2str(Num1) '_' num2str(Num2) ' TimeVector' num2str(Num1) '_' num2str(Num2) ' PressInit
FreqInit NumPCelem NumVCElem Volts colors Apc Avc']);
    else
        eval(['save ClosedChamberSimulationData PistonDispMicrons' num2str(Num1) '_' num2str(Num2) '
ChambPressPSI' num2str(Num1) '_' num2str(Num2) ' TimeVector' num2str(Num1) '_' num2str(Num2) ' -APPEND']);
    end
end
end
end

ClsdChambExpPlotsSaved

```

Appendix B

Matlab Code for Simulating Actuator Dynamics

Modeling of the four stage piezohydraulic cycle was done using a Matlab m-file (Final-StateSpaceAccumOnlyVar.m). The various state-space systems were created inside this m-file using the measured and estimated constant parameters, for loops, if then statements, and linear algebra algorithms, all based on the derived equations of motion. Once created, the state-space matrices were fed into Simulink blocks to simulate the dynamics of the system. The same m-file was used to call the correct simulink model and step through the simulation stages. Once simulated, the results were plotted.

```

clear all
close all
clc

alpha = 300;

OutValveDuty = 40;
InValveDuty = 40;
InOffset = 0.55;
OutOffset = 0.05;
StackDuty = 50;

Lfact = 1;

%Define Number of Elements for Each Fluid Section
NumPC = 8;
NumVC = 25;
ClosedChambNums = NumPC+NumVC;

NumVE = 10;
NumEC1 = 10;
NumEC2 = 25;
NumEC3 = 10;
NumSEC1 = 20;
NumSEC2 = 20;
NumTEC1 = 25;
NumTEC2 = 45;
NumT = 200;
NumTCC1 = 40;
NumTCC2 = 60;
NumCI = 40;
NumCYL = 45;
FirstNums =
NumPC+NumVC+NumVE+NumEC1+NumEC2+NumEC3+NumSEC1+NumSEC2+NumTEC1+NumTEC2+NumT+NumTCC1+NumTCC2+NumCI+NumCYL;

NumCYL2 = 60;
NumCO = 25;
NumBCO = 25;
NumBAC = 100;
NumAIR = 220;
NumACCUM = 8;
SecondNums = NumCYL2+NumCO+NumBCO+NumBAC+NumAIR+NumACCUM;

% Define Fluid Constants
%KinVisc = 3.21e-5; % (m^2)/(s)
KinVisc = 6.51e-4; % (m^2)/(s)
%RhoWater = 999.973; % Kg/(m^3)
Rho = 869.9765; % Kg/(m^3)
DynVisc = KinVisc.*Rho; % (Kg/(m-s))
BI = 220000.*6894.75729; % N/(m^2) (250,000 psi)
x = 0.00103; % Air content in fluid
Renum = 1800; % Reynolds Number
Frict = 64/Renum; % Friction Factor
RhoAir = 1.225; % Kg/(m^3)
RhoNit = 1.2506; % Kg/(m^3)
DynViscAir = 1.875e-5; % (Kg/(m-s))
DynViscNit = 1.7481e-5; % (Kg/(m-s))

```

```

%Valve Constants
Kvact = 1e8; % N/m
Xov = 28e-9; % m/V
Mvpist = 0.005; % Kg
Mvmove = 0.0031; % Kg
Kvspring = 6.4e4; % N/m
Cvpist = 550; % Damping [N-s/m]
Cvmove = 50; % Damping [N-s/m]
Dvpc = 0.015; % m
Avpc = pi*(Dvpc^2)/4; % m^2
Lvpc = 0.0005; % m
tvpc = 0.00475; % m
Dvamp = 0.004; % m
Avamp = pi*(Dvamp^2)/4; % m^2
Lvamp = 0.001; % m
tvamp = 0.00911; % m
KLvamp = 0.5; % Resistance Coefficient
Cd = 0.61;

%PZT Actuator Constants
Kact = 31e6; % N/m
Xo = 9.0e-7; % m/V
Mpist = 0.25; % Kg
Cpist = 100;
Coring = 4000;
Koring = 2.3e4; % N/m

%Pumping Chamber Dimensions
Dpc = 0.025; % m
tpc = 0.01; % m
Apc = pi*(Dpc^2)/4; % m^2
Lpc = 0.0024./Lfact; % m
Vpc = Lpc.*Apc;

%Valve Chamber Dimensions
Hvc = 0.0029; % m
Wvc = 0.0105; % m
Dvc = 4.*(Wvc.*Hvc)./(2.*Wvc+2.*Hvc); % m (0.0045)
tvc = 0.01; % m
Avc = Wvc.*Hvc; % m^2
Lvc = 0.022./Lfact; % m
KLvc = 0.5; % Resistance Coefficient
Vvc = Lvc.*Avc;
Vo = ((Lpc.*Apc)+(Lvc.*Avc));

%Valve Exit Dimensions
Dve = 0.008; % m
tve = 0.0105; % m
Ave = pi*(Dpc^2)/4; % m^2
Lve = 0.010./Lfact; % m

%Valve Exit Connector Dimensions
Dec1 = 0.0085; % m
tec1 = 0.0035; % m
Dec2 = 0.005; % m
tec2 = 0.005; % m
Dec3 = 0.01155; % m
tec3 = 0.005; % m
Aec1 = pi*(Dec1^2)/4; % m^2

```

```

Atec1 = pi*(Dtec1^2)/4;          % m^2
Atec2 = pi*(Dtec2^2)/4;          % m^2
Ltec1 = 0.017./Lfact;             % m
Ltec2 = 0.0045./Lfact;           % m
KLtec2 = 0.4111;
%KLtec2 = 0;

%1/8" Steel Tube Dimensions
tt = 0.0004445;                  % m
Dt = 0.002286;                   % m
At = pi*(Dt^2)/4;                % m^2
Lt = 0.047./Lfact;               % m

%Tube and Cylinder Connector Dimensions
Dtcc1 = 0.002286;                % m
ttcc1 = 0.004457;                % m
Dtcc2 = 0.002286;                % m
ttcc2 = 0.003682;                % m
Atcc1 = pi*(Dtcc1^2)/4;          % m^2
Atcc2 = pi*(Dtcc2^2)/4;          % m^2
Ltcc1 = 0.0056./Lfact;           % m
Ltcc2 = 0.009652./Lfact;         % m

%Inlet to Cylinder Dimensions
Dci = 0.00965;                   % m
tci = 0.005;                     % m
Aci = pi*(Dci^2)/4;              % m^2
Lci = 0.017898./Lfact;           % m
KLci = 0.8909;
%KLci = 0;

Aec2 = pi*(Dec2^2)/4;            % m^2
Aec3 = pi*(Dec3^2)/4;            % m^2
Lec1 = 0.00715./Lfact;           % m
Lec2 = 0.01148./Lfact;           % m
Lec3 = 0.00877./Lfact;           % m
KLec1 = 0.1304;
KLec2 = 0.312;
KLec3 = 0.6603;
%KLec1 = 0;
%KLec2 = 0;
%KLec3 = 0;

%Second Valve Exit Connector Dimensions
Dsec1 = 0.00635;                 % m
tsec1 = 0.0026;                  % m
Dsec2 = 0.00635;                 % m
tsec2 = 0.011;                   % m
Asec1 = pi*(Dsec1^2)/4;          % m^2
Asec2 = pi*(Dsec2^2)/4;          % m^2
Lsec1 = 0.0086./Lfact;           % m
Lsec2 = 0.0099./Lfact;           % m
KLsec1 = 0.331;
%KLsec1 = 0;

%Third Valve Exit Connector Dimensions
Dtec1 = 0.00635;                 % m
ttec1 = 0.0008;                  % m
Dtec2 = 0.002286;                % m

```

```

ttec2 = 0.004387; % m

%Cylinder & Rod Dimensions
Dcyl = 0.0127; % m
tcyl = 0.00645; % m
Acyl = pi*(Dcyl^2)/4; % m^2
Lcyl = 0.0362./Lfact; % m
Drod = 0.00635; % m
Arod = pi*(Drod^2)/4; % m^2
Acrod = (Acyl-Arod); % m^2
Mrod = 0.075; % Mass [Kg]
Crod = 0; % Damping [N-s/m]
KLcyl = 0.086;
%KLcyl = 0;

%Accumulator Side of Cylinder Dimensions
Lcyl2 = 0.0888./Lfact; % m

%Accumulator Side Outlet of Cylinder Dimensions
Dco = 0.00965; % m
tco = 0.005; % m
Aco = pi*(Dco^2)/4; % m^2
Lco = 0.0189./Lfact; % m
KLco = 0.25;
%KLco = 0;

%Brass Connector on Outlet of Cylinder Dimensions
Dbco = 0.0097; % m
tbco = 0.005; % m
Abco = pi*(Dbco^2)/4; % m^2
Lbco = 0.0226./Lfact; % m

%Brass Connectors to Accumulator Dimensions
tbac = 0.005; % m
Dbac = 0.0097; % m
Abac = pi*(Dbac^2)/4; % m^2
Lbac = 0.280./Lfact; % m

%Accumulator Nitrogen Dimensions
Length = 0.153; % m
taccum = 0.05;
Dair = 0.0889; % m
Aair = pi*(Dair^2)/4; % m^2
Cbladder = 0;
KLaccum = 0.9596;
%KLaccum = 0;

Ebrass = 106*10^9; % [Pa] (Elastic Modulus of Brass)
Esteel = 195.3*10^9; % [Pa] (" " " Steel)
Estainless = 190*10^9; % [Pa] (" " " Stainless Steel)

Volts = [110];
%PressInit = [190 250 300 350 450 550 650];
PressInit = [190];
FreqInit = [50 80 100 120 140 160 180 200 250 300];
%FreqInit = [100];
Omega = 2.*pi.*FreqInit;
%Myx = [0.00103 0.0005 0.0002 0.0001 0.00005 0.00002 0.00001];
%Myx = [0.000103 0.00005 0.00002 0.00001];

```

```

Myx = [0.00103];
%Myx = x;

Weight = [0 6.840 12.825 17.452 20.153 22.207]; %Kg
%Weight = [12.825]; %Kg

n = size(PressInit);
n = n(1,2);
m = size(FreqInit);
m = m(1,2);

p = size(Weight);
p = p(1,2);

s = size(Myx);
s = s(1,2);

Static = 9.81.*0.*(ones(1,size(Weight,2)));
Force = (9.81.*Weight)+Static;

colors = ['ko-','bs-','rd-','gv-','mp-','ch-','y^','k*','b<','r>','go:','ms:'];

for MyA = 1:s % Percent Air Content Loop
    x = Myx(1,MyA);
    for Load = 1:p % Load Loop
        for Press = 1:n % Pressure Loop
            % Effective Bulk Modulus Definitions
            Po = PressInit(1,Press).*6894.75729;
            Bg = 1.4.*Po;
            Bnit = 1.2.*Po;

            Lair = (((150.*6894.75729)./Po).*((Length*Air)^1.2))^(1/1.2))./Air;
            Laccum = (Length-Lair)./Lfact;
            Lair = Lair./Lfact;

            Bvpc = 1./(((Dvpc./(tvpc.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bvamp = 1./(((Dvamp./(tvamp.*Estainless))+((1-x)./Bl)+(x./Bg)));

            Bpc = 1./(((Dpc./(tpc.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bvc = 1./(((Dvc./(tvc.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bve = 1./(((Dve./(tve.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bec1 = 1./(((Dec1./(tec1.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bec2 = 1./(((Dec2./(tec2.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bec3 = 1./(((Dec3./(tec3.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bsec1 = 1./(((Dsec1./(tsec1.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bsec2 = 1./(((Dsec2./(tsec2.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Btec1 = 1./(((Dtcc1./(ttcc1.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Btec2 = 1./(((Dtcc2./(ttcc2.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bt = 1./(((Dt./(tt.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Btcc1 = 1./(((Dtcc1./(ttcc1.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Btcc2 = 1./(((Dtcc2./(ttcc2.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bci = 1./(((Dci./(tci.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bcylwrod = 1./(((Dcyl-Drod)./(tcyl.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bcylnorod = 1./(((Dcyl)./(tcyl.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bco = 1./(((Dco./(tco.*Estainless))+((1-x)./Bl)+(x./Bg)));
            Bbco = 1./(((Dbco./(tbco.*Ebrass))+((1-x)./Bl)+(x./Bg)));
            Bbac = 1./(((Dbac./(tbac.*Ebrass))+((1-x)./Bl)+(x./Bg)));
            Baccum = 1./(((Dair./taccum.*Estainless))+((1-x)./Bl)+(x./Bg)));

```

```

for Freq = 1:m
clear ChamberPressureTemp FirstChambPress SecondChambPress ThirdChambPress FourthChambPress
FifthChambPress CylinderDispTemp SecondCylinderDisp ThirdCylinderDisp FourthCylinderDisp FifthCylinderDisp
ToutTemp ToutFirst ToutSecond ToutThird ToutFourth ToutFifth
clear PistonDispTemp FirstPistonDisp SecondPistonDisp ThirdPistonDisp FourthPistonDisp FifthPistonDisp
OutValveOpenTemp FirstOutValveOpen SecondOutValveOpen FifthOutValveOpen InValveOpenTemp
ThirdInValveOpen FourthInValveOpen CylinderForceTemp SecondCylinderForce
clear ThirdCylinderForce FourthCylinderForce FifthCylinderForce Tout ChamberPressure CylinderDisp
OutValveOpen InValveOpen PistonDisp CylinderForce

Freqss = FreqInit(1,Freq);
Omegass = Omega(1,Freq);

% Fluid Circuit Elements
Mvpc = Rho.*Lvpc.*Avpc; % Kg

Cvpc = 8.*pi.*DynVisc.*Lvpc; % N/m/s
Kvpc = Avpc.*Bvpc./Lvpc; % N/m

Mvamp = Rho.*Lvamp.*Avamp; % Kg
LDvamp = (KLvamp./Frict);
Cvamp1 = 8.*pi.*DynVisc.*Lvamp; % N/m/s
Cvamp2 = 8.*pi.*DynVisc.*(Dvamp.*LDvamp); % N/m/s
Kvamp = Avamp.*Bvamp./Lvamp; % N/m

if (Omegass < ((7200.*DynVisc)./(Rho.*(Dpc.^2))))
Mpc = (4./3).*Rho.*(Lpc./NumPC).*Apc.*Lfact;
Cpc = 8.*pi.*DynVisc.*(Lpc./NumPC).*Lfact;
else
Mpc = Rho.*(Lpc./NumPC).*Apc.*Lfact;
Cpc = 0.5.*pi.*Dpc.*(Lpc./NumPC).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dvc.^2))))
Mvc = (4./3).*Rho.*(Lvc./NumVC).*Avc.*Lfact;
Rvca = (12.*DynVisc.*(Lvc./NumVC))./(Wvc.*(Hvc.^3).*(1-
(((192.*Hvc)./(pi.^5).*Wvc)).tan((Hvc.*pi.*Wvc)./(2.*Hvc)))));
Cvca = (Avc.^2).*Rvca.*Lfact;
else
Mvc = Rho.*(Lvc./NumVC).*Avc.*Lfact;
Cvca = 0.5.*pi.*Dvc.*(Lvc./NumVC).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dve.^2))))
Mve = (4./3).*Rho.*(Lve./NumVE).*Ave.*Lfact;
Cve = 8.*pi.*DynVisc.*(Lve./NumVE).*Lfact;
else
Mve = Rho.*(Lve./NumVE).*Ave.*Lfact;
Cve = 0.5.*pi.*Dve.*(Lve./NumVE).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dec1.^2))))
Mec1 = (4./3).*Rho.*(Lec1./NumEC1).*Aec1.*Lfact;
Cec1a = 8.*pi.*DynVisc.*(Lec1./NumEC1).*Lfact;
else
Mec1 = Rho.*(Lec1./NumEC1).*Aec1.*Lfact;
Cec1a = 0.5.*pi.*Dec1.*(Lec1./NumEC1).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dec2.^2))))
Mec2 = (4./3).*Rho.*(Lec2./NumEC2).*Aec2.*Lfact;
Cec2a = 8.*pi.*DynVisc.*(Lec2./NumEC2).*Lfact;
else

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```

Mec2 = Rho.*(Lec2./NumEC2).*Aec2.*Lfact;
    Cec2a = 0.5.*pi.*Dec2.*(Lec2./NumEC2).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dec3.^2))))
    Mec3 = (4./3).*Rho.*(Lec3./NumEC3).*Aec3.*Lfact;
    Cec3a = 8.*pi.*DynVisc.*(Lec3./NumEC3).*Lfact;
else
Mec3 = Rho.*(Lec3./NumEC3).*Aec3.*Lfact;
    Cec3a = 0.5.*pi.*Dec3.*(Lec3./NumEC3).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dsec1.^2))))
    Msec1 = (4./3).*Rho.*(Lsec1./NumSEC1).*Asec1.*Lfact;
    Csec1a = 8.*pi.*DynVisc.*(Lsec1./NumSEC1).*Lfact;
else
    Msec1 = Rho.*(Lsec1./NumSEC1).*Asec1.*Lfact;
    Csec1a = 0.5.*pi.*Dsec1.*(Lsec1./NumSEC1).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dsec2.^2))))
    Msec2 = (4./3).*Rho.*(Lsec2./NumSEC2).*Asec2.*Lfact;
    Csec2 = 8.*pi.*DynVisc.*(Lsec2./NumSEC2).*Lfact;
else
    Msec2 = Rho.*(Lsec2./NumSEC2).*Asec2.*Lfact;
    Csec2 = 0.5.*pi.*Dsec2.*(Lsec2./NumSEC2).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dtec1.^2))))
    Mtec1 = (4./3).*Rho.*(Ltec1./NumTEC1).*Atec1.*Lfact;
    Ctec1 = 8.*pi.*DynVisc.*(Ltec1./NumTEC1).*Lfact;
else
    Mtec1 = Rho.*(Ltec1./NumTEC1).*Atec1.*Lfact;
    Ctec1 = 0.5.*pi.*Dtec1.*(Ltec1./NumTEC1).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dtec2.^2))))
    Mtec2 = (4./3).*Rho.*(Ltec2./NumTEC2).*Atec2.*Lfact;
    Ctec2a = 8.*pi.*DynVisc.*(Ltec2./NumTEC2).*Lfact;
else
    Mtec2 = Rho.*(Ltec2./NumTEC2).*Atec2.*Lfact;
Ctec2a = 0.5.*pi.*Dtec2.*(Ltec2./NumTEC2).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dt.^2))))
    Mt = (4./3).*Rho.*(Lt./NumT).*At.*Lfact;
    Ct = 8.*pi.*DynVisc.*(Lt./NumT).*Lfact;
else
    Mt = Rho.*(Lt./NumT).*At.*Lfact;
    Ct = 0.5.*pi.*Dt.*(Lt./NumT).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dtcc1.^2))))
    Mtcc1 = (4./3).*Rho.*(Ltcc1./NumTCC1).*Atcc1.*Lfact;
    Ctcc1 = 8.*pi.*DynVisc.*(Ltcc1./NumTCC1).*Lfact;
else
    Mtcc1 = Rho.*(Ltcc1./NumTCC1).*Atcc1.*Lfact;
    Ctcc1 = 0.5.*pi.*Dtcc1.*(Ltcc1./NumTCC1).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dtcc2.^2))))
    Mtcc2 = (4./3).*Rho.*(Ltcc2./NumTCC2).*Atcc2.*Lfact;
    Ctcc2 = 8.*pi.*DynVisc.*(Ltcc2./NumTCC2).*Lfact;
else
    Mtcc2 = Rho.*(Ltcc2./NumTCC2).*Atcc2.*Lfact;
    Ctcc2 = 0.5.*pi.*Dtcc2.*(Ltcc2./NumTCC2).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;

```

```

end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dci.^2))))
    Mci = (4./3).*Rho.*(Lci./NumCI).*Aci.*Lfact;
    Ccia = 8.*pi.*DynVisc.*(Lci./NumCI).*Lfact;
else
Mci = Rho.*(Lci./NumCI).*Aci.*Lfact;
    Ccia = 0.5.*pi.*Dci.*(Lci./NumCI).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dcyl.^2))))
    Mcyl = (4./3).*Rho.*(Lcyl./NumCYL).*Acrod.*Lfact;
    Ccyla = 8.*pi.*DynVisc.*(Lcyl./NumCYL).*Lfact;
    Mcyl2 = (4./3).*Rho.*(Lcyl2./NumCYL2).*Acrod.*Lfact;
    Ccyl2a = 8.*pi.*DynVisc.*(Lcyl2./NumCYL2).*Lfact;
else
    Mcyl = Rho.*(Lcyl./NumCYL).*Acrod.*Lfact;
    Ccyla = 0.5.*pi.*Dcyl.*(Lcyl./NumCYL).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
    Mcyl2 = Rho.*(Lcyl2./NumCYL2).*Acrod.*Lfact;
    Ccyl2a = 0.5.*pi.*Dcyl.*(Lcyl2./NumCYL2).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dco.^2))))
    Mco = (4./3).*Rho.*(Lco./NumCO).*Aco.*Lfact;
    Ccoa = 8.*pi.*DynVisc.*(Lco./NumCO).*Lfact;
else
    Mco = Rho.*(Lco./NumCO).*Aco.*Lfact;
    Ccoa = 0.5.*pi.*Dco.*(Lco./NumCO).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dbco.^2))))
Mbco = (4./3).*Rho.*(Lbco./NumBCO).*Abco.*Lfact;
    Cbcoa = 8.*pi.*DynVisc.*(Lbco./NumBCO).*Lfact;
else
    Mbco = Rho.*(Lbco./NumBCO).*Abco.*Lfact;
    Cbcoa = 0.5.*pi.*Dbco.*(Lbco./NumBCO).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dbac.^2))))
    Mbac = (4./3).*Rho.*(Lbac./NumBAC).*Abac.*Lfact;
    Cbaca = 8.*pi.*DynVisc.*(Lbac./NumBAC).*Lfact;
else
    Mbac = Rho.*(Lbac./NumBAC).*Abac.*Lfact;
    Cbaca = 0.5.*pi.*Dbac.*(Lbac./NumBAC).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynViscNit)./(RhoNit.*(Dair.^2))))
    Mair = (4./3).*RhoNit.*(Lair./NumAIR).*Aair.*Lfact;
    Cair = 8.*pi.*DynViscNit.*(Lair./NumAIR).*Lfact;
else
    Mair = RhoNit.*(Lair./NumAIR).*Aair.*Lfact;
    Cair = 0.5.*pi.*Dair.*(Lair./NumAIR).*((2.*RhoNit.*DynViscNit.*Omegass)^0.5).*Lfact;
end
if (Omegass < ((7200.*DynVisc)./(Rho.*(Dair.^2))))
    Maccum = (4./3).*Rho.*(Laccum./NumACCUM).*Aair.*Lfact;
    Caccuma = 8.*pi.*DynVisc.*(Laccum./NumACCUM).*Lfact;
else
    Maccum = Rho.*(Laccum./NumACCUM).*Aair.*Lfact;
    Caccuma = 0.5.*pi.*Dair.*(Laccum./NumACCUM).*((2.*Rho.*DynVisc.*Omegass)^0.5).*Lfact;
end
Kpc = (Apc.*Bpc./(Lpc./NumPC));
Rvcb = (128.*DynVisc.*(KLvc./Frict))./(pi.*(Dvc.^3));
Cvcb = (Avc.^2).*Rvcb;
Kvc = (Avc.*Bvc./(Lvc./NumVC));

```

```

Kve = (Ave.*Bve./(Lve./NumVE));
LDec1 = (KLec1./Frict);
Cec1b = 8.*pi.*DynVisc.*(Dec1.*LDec1);
Kec1 = (Aec1.*Bec1./(Lec1./NumEC1));
LDec2 = (KLec2./Frict);
Cec2b = 8.*pi.*DynVisc.*(Dec2.*LDec2);
Kec2 = (Aec2.*Bec2./(Lec2./NumEC2));
LDec3 = (KLec3./Frict);
Cec3b = 8.*pi.*DynVisc.*(Dec3.*LDec3);
Kec3 = (Aec3.*Bec3./(Lec3./NumEC3));
LDsec1 = (KLsec1./Frict);
Csec1b = 8.*pi.*DynVisc.*(Dsec1.*LDsec1);
Ksec1 = (Asec1.*Bsec1./(Lsec1./NumSEC1));
Ksec2 = (Asec2.*Bsec2./(Lsec2./NumSEC2));
Ktec1 = (Atec1.*Btec1./(Ltec1./NumTEC1));
LDtec2 = (KLtec2./Frict);
Ctec2b = 8.*pi.*DynVisc.*(Dtec2.*LDtec2);
Ktec2 = (Atec2.*Btec2./(Ltec2./NumTEC2));
Kt = (At.*Bt./(Lt./NumT));
Ktcc1 = (Atcc1.*Btcc1./(Ltcc1./NumTCC1));
Ktcc2 = (Atcc2.*Btcc2./(Ltcc2./NumTCC2));
LDci = (KLci./Frict);
Ccib = 8.*pi.*DynVisc.*(Dci.*LDci);
Kci = (Aci.*Bci./(Lci./NumCI));
LDcyl = (KLcyl./Frict);
Ccylb = 8.*pi.*DynVisc.*(Dcyl.*LDcyl);
Kcyl = (Acrod.*Bcylwrod./(Lcyl./NumCYL));
Ccyl2b = 0;
Kcyl2 = (Acrod.*Bcylwrod./(Lcyl2./NumCYL2));
LDco = (KLco./Frict);
Ccob = 8.*pi.*DynVisc.*(Dco.*LDco);
Kco = (Aco.*Bco./(Lco./NumCO));
Cbcob = 0;
Kbco = (Abco.*Bbco./(Lbco./NumBCO));
Cbacb = 0;
Kbac = (Abac.*Bbac./(Lbac./NumBAC));
Kair = (Air.*Brit./(Lair./NumAIR));
LDaccum = (KLaccum./Frict);
Caccumb = 8.*pi.*DynVisc.*(Dair.*LDaccum);
Kaccum = (Air.*Baccum./(Laccum./NumACCUM));

```

```

[MyStiff] = sprintf('Kpc = %0.5g\nKvc = %0.5g\nKve = %0.5g\nKec1 = %0.5g\nKec2 = %0.5g\nKec3 = %0.5g\nKsec1 = %0.5g\nKsec2 = %0.5g\nKtec1 = %0.5g\nKtec2 = %0.5g\nKt = %0.5g\nKtcc1 = %0.5g\nKtcc2 = %0.5g\nKci = %0.5g\nKcyl = %0.5g\nKcyl2 = %0.5g\nKco = %0.5g\nKbco = %0.5g\nKbac = %0.5g\nKair = %0.5g\nKaccum = %0.5g\n',Kpc,Kvc,Kve,Kec1,Kec2,Kec3,Ksec1,Ksec2,Ktec1,Ktec2,Kt,Ktcc1,Ktcc2,Kci,Kcyl,Kcyl2,Kco,Kbco,Kbac,Kair,Kaccum)

```

```

for i1 = 1:(FirstNums+SecondNums+21)
    if (i1 == 1)
        Mv(i1,:) = [Mvpist zeros(1,5)];
        CDv(i1,:) = [Cvpist zeros(1,5)];
        Kv(i1,:) = [Kvact zeros(1,5)];
        Tv(i1,:) = [1 zeros(1,2)];
        Fv(i1,:) = [-Avpc (Kvact.*Xov)];

        Mcc(i1,:) = [Mpist zeros(1,ClosedChambNums+2-i1)];
        CDcc(i1,:) = [(Cpist+Coring) zeros(1,ClosedChambNums+2-i1)];
        Kcc(i1,:) = [(Kact+Koring) zeros(1,ClosedChambNums+2-i1)];
        Tcc(i1,:) = [1 zeros(1,ClosedChambNums-i1)];
    end
end

```

```

Fcc(i1,:) = [(Kact.*Xo) (-Po.*Apc)];
Mcc2(i1,:) = [Mpist zeros(1,ClosedChambNums+4-i1)];
CDcc2(i1,:) = [(Cpist+Coring) zeros(1,ClosedChambNums+4-i1)];
Kcc2(i1,:) = [(Kact+Koring) zeros(1,ClosedChambNums+4-i1)];
Tcc2(i1,:) = [1 zeros(1,ClosedChambNums+1-i1)];
Fcc2(i1,:) = [(Kact.*Xo) 0];

Mcc3(i1,:) = [Mpist zeros(1,ClosedChambNums+1-i1)];
CDcc3(i1,:) = [(Cpist+Coring) zeros(1,ClosedChambNums+1-i1)];
Kcc3(i1,:) = [0 zeros(1,ClosedChambNums+1-i1)];
Tcc3(i1,:) = [1 zeros(1,ClosedChambNums-1-i1)];
Fcc3(i1,:) = [(Kact.*Xo) -1 -1];

Mac(i1,:) = [Mpist zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [(Cpist+Coring) zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [(Kact+Koring) zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac(i1,:) = [1 zeros(1,(FirstNums+SecondNums)-i1)];
Fac(i1,:) = [(Kact.*Xo) 0 0 (-Po.*Apc)];
elseif (i1 == 2)
    Mv(i1,:) = [zeros(1,i1-1) Mvpc zeros(1,4)];
    CDv(i1,:) = [zeros(1,i1-1) Cvpc zeros(1,4)];
    Kv(i1,:) = [zeros(1,i1-1) Kvpc -Kvpc zeros(1,3)];
    Tv(i1,:) = [1 zeros(1,2)];
    Fv(i1,:) = [0 0];

    Mcc(i1,:) = [0 Mpc zeros(1,ClosedChambNums+2-i1)];
    CDcc(i1,:) = [0 Cpc zeros(1,ClosedChambNums+2-i1)];
    Kcc(i1,:) = [0 Kpc -Kpc zeros(1,ClosedChambNums+1-i1)];
    Tcc(i1,:) = [1 zeros(1,ClosedChambNums+1-i1)];
    Fcc(i1,:) = [0 0];

    Mcc2(i1,:) = [zeros(1,i1-1) Mpc zeros(1,ClosedChambNums+4-i1)];
    CDcc2(i1,:) = [zeros(1,i1-1) Cpc zeros(1,ClosedChambNums+4-i1)];
    Kcc2(i1,:) = [0 Kpc -Kpc zeros(1,ClosedChambNums+3-i1)];
    Tcc2(i1,:) = [1 zeros(1,ClosedChambNums+2-i1)];
    Fcc2(i1,:) = [0 0];

    Mcc3(i1,:) = [zeros(1,i1-1) Mpc zeros(1,ClosedChambNums+1-i1)];
    CDcc3(i1,:) = [zeros(1,i1-1) Cpc zeros(1,ClosedChambNums+1-i1)];
    Kcc3(i1,:) = [0 Kpc -Kpc zeros(1,ClosedChambNums-i1)];
    Tcc3(i1,:) = [1 zeros(1,ClosedChambNums-i1)];
    Fcc3(i1,:) = [0 0 0];

    Mac(i1,:) = [zeros(1,i1-1) Mpc zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Cpc zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-1) Kpc -Kpc zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [1 zeros(1,(FirstNums+SecondNums+1)-i1)];
    Fac(i1,:) = [0 0 0 0];
elseif ((i1 > 2) & (i1 < (NumPC+2)))
    Mcc(i1,:) = [zeros(1,i1-1) Mpc zeros(1,ClosedChambNums+2-i1)];
    CDcc(i1,:) = [zeros(1,i1-1) Cpc zeros(1,ClosedChambNums+2-i1)];
    Kcc(i1,:) = [zeros(1,i1-2) -Kpc 2.*Kpc -Kpc zeros(1,ClosedChambNums+1-i1)];
    Tcc(i1,:) = [zeros(1,i1-2) 1 zeros(1,ClosedChambNums+1-i1)];
    Fcc(i1,:) = [0 0];
Mcc2(i1,:) = [zeros(1,i1-1) Mpc zeros(1,ClosedChambNums+4-i1)];
CDcc2(i1,:) = [zeros(1,i1-1) Cpc zeros(1,ClosedChambNums+4-i1)];
Kcc2(i1,:) = [zeros(1,i1-2) -Kpc 2.*Kpc -Kpc zeros(1,ClosedChambNums+3-i1)];
Tcc2(i1,:) = [zeros(1,i1-2) 1 zeros(1,ClosedChambNums+2-i1)];
Fcc2(i1,:) = [0 0];

```

```

Mcc3(i1,:) = [zeros(1,i1-1) Mpc zeros(1,ClosedChambNums+1-i1)];
CDcc3(i1,:) = [zeros(1,i1-1) Cpc zeros(1,ClosedChambNums+1-i1)];
Kcc3(i1,:) = [zeros(1,i1-2) -Kpc 2.*Kpc -Kpc zeros(1,ClosedChambNums-i1)];
Tcc3(i1,:) = [zeros(1,i1-2) 1 zeros(1,ClosedChambNums-i1)];
Fcc3(i1,:) = [0 0 0];

Mac(i1,:) = [zeros(1,i1-1) Mpc zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) Cpc zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kpc 2.*Kpc -Kpc zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-2) 1 zeros(1,(FirstNums+SecondNums+1)-i1)];
Fac(i1,:) = [0 0 0 0];
if (i1 == 3)
    Mv(i1,:) = [zeros(1,i1-1) 0 zeros(1,3)];
    CDv(i1,:) = [zeros(1,i1-1) 0 zeros(1,3)];
    Kv(i1,:) = [zeros(1,i1-2) -Kvpc Kvpc zeros(1,3)];
    Tv(i1,:) = [zeros(1,i1-2) (Avamp./Avpc) zeros(1,1)];
    Fv(i1,:) = [0 0];
elseif (i1 == 4)
    Mv(i1,:) = [zeros(1,i1-1) Mvamp zeros(1,2)];
    CDv(i1,:) = [zeros(1,i1-1) (Cvamp1+Cvamp2) zeros(1,2)];
    Kv(i1,:) = [zeros(1,i1-1) Kvamp -Kvamp zeros(1,1)];
    Tv(i1,:) = [zeros(1,i1-3) 1 zeros(1,1)];
    Fv(i1,:) = [0 0];
elseif (i1 == 5)
    Mv(i1,:) = [zeros(1,i1-1) 0 zeros(1,1)];
    CDv(i1,:) = [zeros(1,i1-1) 0 zeros(1,1)];
    Kv(i1,:) = [zeros(1,i1-2) -Kvamp Kvamp zeros(1,1)];
    Tv(i1,:) = [zeros(1,i1-3) 1];
    Fv(i1,:) = [0 0];
elseif (i1 == 6)
    Mv(i1,:) = [zeros(1,i1-1) Mvmove];
    CDv(i1,:) = [zeros(1,i1-1) Cvmove];
    Kv(i1,:) = [zeros(1,i1-1) Kvspring];
    Tv(i1,:) = [zeros(1,i1-4) 1];
    Fv(i1,:) = [-64./Po 0];
end
elseif (i1 == (NumPC+2))
    Mcc(i1,:) = [zeros(1,i1-1) 0 zeros(1,ClosedChambNums+2-i1)];
    CDcc(i1,:) = [zeros(1,i1-1) 0 zeros(1,ClosedChambNums+2-i1)];
    Kcc(i1,:) = [zeros(1,i1-2) -Kpc Kpc zeros(1,ClosedChambNums+2-i1)];
    Tcc(i1,:) = [zeros(1,i1-2) (Avc./Apc) zeros(1,ClosedChambNums+1-i1)];
    Fcc(i1,:) = [0 0];
Mcc2(i1,:) = [zeros(1,i1-1) 0 zeros(1,ClosedChambNums+4-i1)];
CDcc2(i1,:) = [zeros(1,i1-1) 0 zeros(1,ClosedChambNums+4-i1)];
Kcc2(i1,:) = [zeros(1,i1-2) -Kpc Kpc zeros(1,ClosedChambNums+4-i1)];
Tcc2(i1,:) = [zeros(1,i1-2) (Avc./Apc) zeros(1,ClosedChambNums+2-i1)];
Fcc2(i1,:) = [0 0];

Mcc3(i1,:) = [zeros(1,i1-1) 0 zeros(1,ClosedChambNums+1-i1)];
CDcc3(i1,:) = [zeros(1,i1-1) 0 zeros(1,ClosedChambNums+1-i1)];
Kcc3(i1,:) = [zeros(1,i1-2) -Kpc Kpc zeros(1,ClosedChambNums+1-i1)];
Tcc3(i1,:) = [zeros(1,i1-2) (Avc./Apc) zeros(1,ClosedChambNums-i1)];
Fcc3(i1,:) = [0 0 0];

Mac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kpc Kpc zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac(i1,:) = [zeros(1,i1-2) (Avc./Apc) zeros(1,(FirstNums+SecondNums+1)-i1)];

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    Fac(i1,:) = [0 0 0 0];
elseif (i1 == (NumPC+3))
    Mcc(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+2-i1)];
    CDcc(i1,:) = [zeros(1,i1-1) (Cvca+Cvcb) zeros(1,ClosedChambNums+2-i1)];
    Kcc(i1,:) = [zeros(1,i1-1) Kvc -Kvc zeros(1,ClosedChambNums+1-i1)];
    Tcc(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+2-i1)];
    Fcc(i1,:) = [0 0];

    Mcc2(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+4-i1)];
    CDcc2(i1,:) = [zeros(1,i1-1) (Cvca+Cvcb) zeros(1,ClosedChambNums+4-i1)];
    Kcc2(i1,:) = [zeros(1,i1-1) Kvc -Kvc zeros(1,ClosedChambNums+3-i1)];
    Tcc2(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+3-i1)];
    Fcc2(i1,:) = [0 0];

    Mcc3(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+1-i1)];
    CDcc3(i1,:) = [zeros(1,i1-1) (Cvca+Cvcb) zeros(1,ClosedChambNums+1-i1)];
    Kcc3(i1,:) = [zeros(1,i1-1) Kvc -Kvc zeros(1,ClosedChambNums-i1)];
    Tcc3(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+1-i1)];
    Fcc3(i1,:) = [0 0 0];

    Mac(i1,:) = [zeros(1,i1-1) Mvc zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) (Cvca+Cvcb) zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-1) Kvc -Kvc zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-3) 1 zeros(1,(FirstNums+SecondNums+2)-i1)];
    Fac(i1,:) = [0 0 0 0];
elseif ((i1 >= (NumPC+4)) & (i1 <= ClosedChambNums))
    Mcc(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+2-i1)];
    CDcc(i1,:) = [zeros(1,i1-1) Cvca zeros(1,ClosedChambNums+2-i1)];
    Kcc(i1,:) = [zeros(1,i1-2) -Kvc (2.*Kvc) -Kvc zeros(1,ClosedChambNums+1-i1)];
    Tcc(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+2-i1)];
    Fcc(i1,:) = [0 0];

    Mcc2(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+4-i1)];
    CDcc2(i1,:) = [zeros(1,i1-1) Cvca zeros(1,ClosedChambNums+4-i1)];
    Kcc2(i1,:) = [zeros(1,i1-2) -Kvc 2.*Kvc -Kvc zeros(1,ClosedChambNums+3-i1)];
    Tcc2(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+3-i1)];
    Fcc2(i1,:) = [0 0];

    Mcc3(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+1-i1)];
    CDcc3(i1,:) = [zeros(1,i1-1) Cvca zeros(1,ClosedChambNums+1-i1)];
    Kcc3(i1,:) = [zeros(1,i1-2) -Kvc 2.*Kvc -Kvc zeros(1,ClosedChambNums-i1)];
    Tcc3(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+1-i1)];
    Fcc3(i1,:) = [0 0 0];

    Mac(i1,:) = [zeros(1,i1-1) Mvc zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Cvca zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kvc 2.*Kvc -Kvc zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-3) 1 zeros(1,(FirstNums+SecondNums+2)-i1)];
    Fac(i1,:) = [0 0 0 0];
elseif (i1 == (ClosedChambNums+1))
    Mcc(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+2-i1)];
    CDcc(i1,:) = [zeros(1,i1-1) Cvca zeros(1,ClosedChambNums+2-i1)];
    Kcc(i1,:) = [zeros(1,i1-2) -Kvc (2.*Kvc) -Kvc zeros(1,ClosedChambNums+1-i1)];
    Tcc(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+2-i1)];
    Fcc(i1,:) = [0 0];

    Mcc2(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+4-i1)];
    CDcc2(i1,:) = [zeros(1,i1-1) Cvca zeros(1,ClosedChambNums+4-i1)];
    Kcc2(i1,:) = [zeros(1,i1-2) -Kvc 2.*Kvc -Kvc zeros(1,ClosedChambNums+3-i1)];

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Tcc2(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+3-i1)];
Fcc2(i1,:) = [0 0];

Mcc3(i1,:) = [zeros(1,i1-1) Mvc];
CDcc3(i1,:) = [zeros(1,i1-1) Cvca];
Kcc3(i1,:) = [zeros(1,i1-2) -Kvc 2.*Kvc];
Tcc3(i1,:) = [zeros(1,i1-3) 1];
Fcc3(i1,:) = [0 0 0];

Mac(i1,:) = [zeros(1,i1-1) Mvc zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) Cvca zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kvc 2.*Kvc -Kvc zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-3) 1 zeros(1,(FirstNums+SecondNums+2)-i1)];
Fac(i1,:) = [0 0 0 0];
elseif (i1 == (ClosedChambNums+2))
    Mcc(i1,:) = [zeros(1,i1-1) Mvc];
    CDcc(i1,:) = [zeros(1,i1-1) Cvca];
    Kcc(i1,:) = [zeros(1,i1-2) -Kvc (2.*Kvc)];
    Tcc(i1,:) = [zeros(1,i1-3) 1];
    Fcc(i1,:) = [0 0];

    Mcc2(i1,:) = [zeros(1,i1-1) Mvc zeros(1,ClosedChambNums+4-i1)];
    CDcc2(i1,:) = [zeros(1,i1-1) Cvca zeros(1,ClosedChambNums+4-i1)];
    Kcc2(i1,:) = [zeros(1,i1-2) -Kvc 2.*Kvc -Kvc zeros(1,ClosedChambNums+3-i1)];
    Tcc2(i1,:) = [zeros(1,i1-3) 1 zeros(1,ClosedChambNums+3-i1)];
    Fcc2(i1,:) = [0 0];

    Mac(i1,:) = [zeros(1,i1-1) Mvc zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Cvca zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kvc 2.*Kvc -Kvc zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-3) 1 zeros(1,(FirstNums+SecondNums+2)-i1)];
    Fac(i1,:) = [0 0 0 0];
elseif (i1 == (ClosedChambNums+3))
    Mcc2(i1,:) = [zeros(1,i1-1) 0 zeros(1,ClosedChambNums+4-i1)];
    CDcc2(i1,:) = [zeros(1,i1-1) 0 zeros(1,ClosedChambNums+4-i1)];
    Kcc2(i1,:) = [zeros(1,i1-2) -Kvc Kvc zeros(1,ClosedChambNums+4-i1)];
    Tcc2(i1,:) = [zeros(1,i1-3) (Ave./Avc)];
    Fcc2(i1,:) = [0 0];

    Mac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kvc Kvc zeros(1,(FirstNums+SecondNums+21)-i1)];
    Tac(i1,:) = [zeros(1,i1-3) (Ave./Avc) zeros(1,(FirstNums+SecondNums+2)-i1)];
    Fac(i1,:) = [0 0 0 0];
elseif (i1 == (ClosedChambNums+4))
    Mcc2(i1,:) = [zeros(1,i1-1) Mve.*NumVE];
    CDcc2(i1,:) = [zeros(1,i1-1) (Cve+15000)];
    Kcc2(i1,:) = [zeros(1,i1-1) Kair./(NumAIR.*1000)];
    Tcc2(i1,:) = [zeros(1,i1-4) 1];
    Fcc2(i1,:) = [0 -1];

    Mac(i1,:) = [zeros(1,i1-1) Mve zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Cve zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-1) Kve -Kve zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-4) 1 zeros(1,(FirstNums+SecondNums+3)-i1)];
    Fac(i1,:) = [0 -1 0 0];
elseif (i1 == (ClosedChambNums+5))
    Mac(i1,:) = [zeros(1,i1-1) Mve zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Cve zeros(1,(FirstNums+SecondNums+21)-i1)];

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Kac(i1,:) = [zeros(1,i1-2) -Kve 2.*Kve -Kve zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-4) 1 zeros(1,(FirstNums+SecondNums+3)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [Mve zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [Cve zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [2.*Kve -Kve zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [1 zeros(1,(FirstNums+SecondNums+3)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [Kve 0];
elseif ((i1 >= (ClosedChambNums+6)) & (i1 <= (ClosedChambNums+NumVE+3)))
Mac(i1,:) = [zeros(1,i1-1) Mve zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) Cve zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kve 2.*Kve -Kve zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-4) 1 zeros(1,(FirstNums+SecondNums+3)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mve
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Cve
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kve 2.*Kve -Kve
zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 1
zeros(1,(FirstNums+SecondNums+3)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (ClosedChambNums+NumVE+4))
Mac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kve Kve zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac(i1,:) = [zeros(1,i1-4) (Aec1./Ave) zeros(1,(FirstNums+SecondNums+3)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kve Kve
zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) (Aec1./Ave)
zeros(1,(FirstNums+SecondNums+3)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (ClosedChambNums+NumVE+5))
Mac(i1,:) = [zeros(1,i1-1) Mec1 zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) (Cec1a+Cec1b) zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-1) Kec1 -Kec1 zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-5) 1 zeros(1,(FirstNums+SecondNums+4)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mec1
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) (Cec1a+Cec1b)
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Kec1 -Kec1
zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) 1 zeros(1,(FirstNums+SecondNums+4)-
i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];

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elseif ((i1 >= (ClosedChambNums+NumVE+6)) & (i1 <= (ClosedChambNums+NumVE+NumEC1+4)))
    Mac(i1,:) = [zeros(1,i1-1) Mec1 zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Cec1a zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kec1 2.*Kec1 -Kec1 zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-5) 1 zeros(1,(FirstNums+SecondNums+4)-i1)];
    Fac(i1,:) = [0 0 0 0];

    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mec1
zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Cec1a
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kec1 2.*Kec1 -Kec1
zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) 1
zeros(1,(FirstNums+SecondNums+4)-i1)];
    Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (ClosedChambNums+NumVE+NumEC1+5))
    Mac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kec1 Kec1 zeros(1,(FirstNums+SecondNums+21)-i1)];
    Tac(i1,:) = [zeros(1,i1-5) (Aec2./Aec1) zeros(1,(FirstNums+SecondNums+4)-i1)];
    Fac(i1,:) = [0 0 0 0];

    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kec1 Kec1
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) (Aec2./Aec1)
zeros(1,(FirstNums+SecondNums+4)-i1)];
    Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (ClosedChambNums+NumVE+NumEC1+6))
    Mac(i1,:) = [zeros(1,i1-1) Mec2 zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) (Cec2a+Cec2b) zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-1) Kec2 -Kec2 zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-6) 1 zeros(1,(FirstNums+SecondNums+5)-i1)];
    Fac(i1,:) = [0 0 0 0];

    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mec2
zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) (Cec2a+Cec2b)
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Kec2 -Kec2
zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+7)) 1
zeros(1,(FirstNums+SecondNums+5)-i1)];
    Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif ((i1 >= (ClosedChambNums+NumVE+NumEC1+7)) & (i1 <=
(ClosedChambNums+NumVE+NumEC1+NumEC2+5)))
    Mac(i1,:) = [zeros(1,i1-1) Mec2 zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Cec2a zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kec2 2.*Kec2 -Kec2 zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-6) 1 zeros(1,(FirstNums+SecondNums+5)-i1)];
    Fac(i1,:) = [0 0 0 0];

    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mec2
zeros(1,(FirstNums+SecondNums+21)-i1)];

```

```

CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Cec2a
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kec2 2.*Kec2 -Kec2
zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+7)) 1
zeros(1,(FirstNums+SecondNums+5)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (ClosedChambNums+NumVE+NumEC1+NumEC2+6))
Mac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kec2 Kec2 zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac(i1,:) = [zeros(1,i1-6) (Aec3./Aec2) zeros(1,(FirstNums+SecondNums+5)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kec2 Kec2
zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+7)) (Aec3./Aec2)
zeros(1,(FirstNums+SecondNums+5)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (ClosedChambNums+NumVE+NumEC1+NumEC2+7))
Mac(i1,:) = [zeros(1,i1-1) Mec3 zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) (Cec3a+Cec3b) zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-1) Kec3 -Kec3 zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-7) 1 zeros(1,(FirstNums+SecondNums+6)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mec3
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) (Cec3a+Cec3b)
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Kec3 -Kec3
zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+8)) 1
zeros(1,(FirstNums+SecondNums+6)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif ((i1 >= (ClosedChambNums+NumVE+NumEC1+NumEC2+8)) & (i1 <=
(ClosedChambNums+NumVE+NumEC1+NumEC2+NumEC3+6)))
Mac(i1,:) = [zeros(1,i1-1) Mec3 zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) Cec3a zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kec3 2.*Kec3 -Kec3 zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-7) 1 zeros(1,(FirstNums+SecondNums+6)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mec3
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Cec3a
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kec3 2.*Kec3 -Kec3
zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+8)) 1
zeros(1,(FirstNums+SecondNums+6)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (ClosedChambNums+NumVE+NumEC1+NumEC2+NumEC3+7))
Mac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];

```

```

Kac(i1,:) = [zeros(1,i1-2) -Kec3 Kec3 zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac(i1,:) = [zeros(1,i1-7) (Asec1./Aec3) zeros(1,(FirstNums+SecondNums+6)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kec3 Kec3
zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+8)) (Asec1./Aec3)
zeros(1,(FirstNums+SecondNums+6)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];

```

CODE CONTINUES IN SAME MANNER FOR ELEMENTS: sec1, sec2, tec1, tec2, t, tcc1, tcc2, ci, cyl.

```

elseif (i1 == (FirstNums+17))
    %Mac(i1,:) = [zeros(1,i1-1) (Mrod+Mcy12+Weight(1,Load)) zeros(1,(FirstNums+SecondNums+21)-i1)];
    Mac(i1,:) = [zeros(1,i1-1) (Mrod+Mcy12) zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) (Ccyl2a+Ccyl2b+Crod) zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-1) Kcyl2 -Kcyl2 zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-17) 1 zeros(1,(FirstNums+SecondNums+16)-i1)];
    Fac(i1,:) = [0 0 -Force(1,Load) 0];

    %Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5))
(Mrod+Mcy12+Weight(1,Load)) zeros(1,(FirstNums+SecondNums+21)-i1)];
    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) (Mrod+Mcy12)
zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) (Ccyl2a+Ccyl2b+Crod)
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Kcyl2 -Kcyl2
zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+18)) 1
zeros(1,(FirstNums+SecondNums+16)-i1)];
    Fac2(i1-(ClosedChambNums+4),:) = [0 -Force(1,Load)];
elseif (i1 == (FirstNums+18))
    Mac(i1,:) = [zeros(1,i1-1) Mcyl2 zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Ccyl2a zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kcyl2 2.*Kcyl2 -Kcyl2 zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-17) 1 zeros(1,(FirstNums+SecondNums+16)-i1)];
    Fac(i1,:) = [0 0 0 0];
    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mcyl2
zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Ccyl2a
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kcyl2 2.*Kcyl2 -Kcyl2
zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+18)) 1
zeros(1,(FirstNums+SecondNums+16)-i1)];
    Fac2(i1-(ClosedChambNums+4),:) = [0 0];

```

```

elseif ((i1 > (FirstNums+18)) & (i1 < (FirstNums+NumCYL2+17)))
    Mac(i1,:) = [zeros(1,i1-1) Mcyl2 zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) Ccyl2a zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kcyl2 2.*Kcyl2 -Kcyl2 zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-17) 1 zeros(1,(FirstNums+SecondNums+16)-i1)];
    Fac(i1,:) = [0 0 0 0];

    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mcyl2
zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Ccyl2a
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kcyl2 2.*Kcyl2 -Kcyl2
zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+18)) 1
zeros(1,(FirstNums+SecondNums+16)-i1)];
    Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (FirstNums+NumCYL2+17))
    Mac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-2) -Kcyl2 Kcyl2 zeros(1,(FirstNums+SecondNums+21)-i1)];
    Tac(i1,:) = [zeros(1,i1-17) (Aco./Acyl) zeros(1,(FirstNums+SecondNums+16)-i1)];
    Fac(i1,:) = [0 0 0 0];

    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kcyl2 Kcyl2
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+18)) (Aco./Acyl)
zeros(1,(FirstNums+SecondNums+16)-i1)];
    Fac2(i1-(ClosedChambNums+4),:) = [0 0];

```

CODE CONTINUES IN SAME MANNER FOR ELEMENTS: co, bco, bac.

```

elseif (i1 == (FirstNums+NumCYL2+NumCO+NumBCO+NumBAC+21))
    Mac(i1,:) = [zeros(1,i1-1) Maccum zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac(i1,:) = [zeros(1,i1-1) (Caccuma+Caccumb) zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac(i1,:) = [zeros(1,i1-1) Kaccum -Kaccum zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac(i1,:) = [zeros(1,i1-21) 1 zeros(1,(FirstNums+SecondNums+20)-i1)];
    Fac(i1,:) = [0 0 0 0];

    Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Maccum
zeros(1,(FirstNums+SecondNums+21)-i1)];
    CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) (Caccuma+Caccumb)
zeros(1,(FirstNums+SecondNums+21)-i1)];
    Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Kaccum -Kaccum
zeros(1,(FirstNums+SecondNums+20)-i1)];
    Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+22)) 1
zeros(1,(FirstNums+SecondNums+20)-i1)];
    Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif ((i1 > (FirstNums+NumCYL2+NumCO+NumBCO+NumBAC+21)) & (i1 <
(FirstNums+NumCYL2+NumCO+NumBCO+NumBAC+NumACCUM+21)))

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```

Mac(i1,:) = [zeros(1,i1-1) Maccum zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) Caccuma zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kaccum 2.*Kaccum -Kaccum zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-21) 1 zeros(1,(FirstNums+SecondNums+20)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Maccum
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Caccuma
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kaccum 2.*Kaccum -
Kaccum zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+22)) 1
zeros(1,(FirstNums+SecondNums+20)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (FirstNums+NumCYL2+NumCO+NumBCO+NumBAC+NumACCUM+21))
Mac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) 0 zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kaccum Kaccum zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac(i1,:) = [zeros(1,i1-21) 1 zeros(1,(FirstNums+SecondNums+20)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) 0
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kaccum
Kaccum zeros(1,(FirstNums+SecondNums+21)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+22)) 1
zeros(1,(FirstNums+SecondNums+20)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif (i1 == (FirstNums+NumCYL2+NumCO+NumBCO+NumBAC+NumACCUM+22))
Mac(i1,:) = [zeros(1,i1-1) Mair zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) Cair zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-1) Kair -Kair zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-22) 1 zeros(1,(FirstNums+SecondNums+21)-i1)];
Fac(i1,:) = [0 0 0 0];

Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mair
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Cair
zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Kair -Kair
zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+23)) 1
zeros(1,(FirstNums+SecondNums+21)-i1)];
Fac2(i1-(ClosedChambNums+4),:) = [0 0];
elseif ((i1 > (FirstNums+NumCYL2+NumCO+NumBCO+NumBAC+NumACCUM+22)) & (i1 <
(FirstNums+SecondNums+21)))
Mac(i1,:) = [zeros(1,i1-1) Mair zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac(i1,:) = [zeros(1,i1-1) Cair zeros(1,(FirstNums+SecondNums+21)-i1)];
Kac(i1,:) = [zeros(1,i1-2) -Kair 2.*Kair -Kair zeros(1,(FirstNums+SecondNums+20)-i1)];
Tac(i1,:) = [zeros(1,i1-22) 1 zeros(1,(FirstNums+SecondNums+21)-i1)];
Fac(i1,:) = [0 0 0 0];
Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mair
zeros(1,(FirstNums+SecondNums+21)-i1)];
CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Cair
zeros(1,(FirstNums+SecondNums+21)-i1)];

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        Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kair 2.*Kair -Kair
zeros(1,(FirstNums+SecondNums+20)-i1)];
        Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+23)) 1
zeros(1,(FirstNums+SecondNums+21)-i1)];
        Fac2(i1-(ClosedChambNums+4),:) = [0 0];
        elseif (i1 == (FirstNums+SecondNums+21))
            Mac(i1,:) = [zeros(1,i1-1) Mair];
            CDac(i1,:) = [zeros(1,i1-1) (Cair+Cbladder)];
            Kac(i1,:) = [zeros(1,i1-2) -Kair 2.*Kair];
            Tac(i1,:) = [zeros(1,i1-22) 1];
            Fac(i1,:) = [0 0 0 0];

            Mac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) Mair];
            CDac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+5)) (Cair+Cbladder)];
            Kac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+6)) -Kair 2.*Kair];
            Tac2(i1-(ClosedChambNums+4),:) = [zeros(1,i1-(ClosedChambNums+23)) 1];
            Fac2(i1-(ClosedChambNums+4),:) = [0 0];
        end
    end

    Mvtrans = ((Tv.)*Mv*Tv);
    Cvtrans = ((Tv.)*CDv*Tv);
    Kvtrans = ((Tv.)*Kv*Tv);
    Fvtrans = ((Tv.)*Fv);
    MvtransInv = inv(Mvtrans);
    rvc = size(Mvtrans,2);
    Av = [zeros(rvc,rvc) eye(rvc,rvc);-MvtransInv*Kvtrans -MvtransInv*Cvtrans];
    Bv = [zeros(rvc,2);MvtransInv*Fvtrans];
    Cv = [eye(2.*rvc)];
    Dv = [zeros(2.*rvc,2)];
    Avinit = Kvtrans;
    Bvinit = Fvtrans(:,1).*Po;
    Xvinit = [Avinit\Bvinit;zeros(rvc,1)]; %Inv(A)*B
    % MyValveSys = ss(Av,Bv,Cv,Dv);
    ToutSecondNoDec = zeros(1,2.*rvc);

    Mcctrans = ((Tcc.)*Mcc*Tcc);
    Ccctrans = ((Tcc.)*CDcc*Tcc);
    Kcctrans = ((Tcc.)*Kcc*Tcc);
    Fcctrans = ((Tcc.)*Fcc);
    McctransInv = inv(Mcctrans);
    rcc = size(Mcctrans,2);
    Acc = [zeros(rcc,rcc) eye(rcc,rcc);-McctransInv*Kcctrans -McctransInv*Ccctrans];
    Bcc = [zeros(rcc,2);McctransInv*Fcctrans];
    Ccc = [eye(2.*rcc)];
    Dcc = [zeros(2.*rcc,2)];
    Xccinit = [Kcctrans\Fcctrans(:,2);zeros(rcc,1)]; %Inv(K)*F
    %Xccinit = [zeros(2.*rcc,1)];
    % MyClosedChambSys = ss(Acc,Bcc,Ccc,Dcc);

    Mcctrans2 = ((Tcc2.)*Mcc2*Tcc2);
    Ccctrans2 = ((Tcc2.)*CDcc2*Tcc2);
    Kcctrans2 = ((Tcc2.)*Kcc2*Tcc2);
    Fcctrans2 = ((Tcc2.)*Fcc2);
    McctransInv2 = inv(Mcctrans2);
    rcc2 = size(Mcctrans2,2);
    Acc2 = [zeros(rcc2,rcc2) eye(rcc2,rcc2);-McctransInv2*Kcctrans2 -McctransInv2*Ccctrans2];
    Bcc2 = [zeros(rcc2,2);McctransInv2*Fcctrans2];
    Ccc2 = [eye(2.*rcc2)];

```

```

Dcc2 = [zeros(2.*rccol2,2)];
Xccinit2Pist = [Kcctrans2\((Po.*Ave.*Fcctrans2(:,2));zeros(rccol2,1)];
Xccinit2 = [zeros(2.*rccol2,1)];

Mcctrans3 = ((Tcc3.').*Mcc3*Tcc3);
Ccctrans3 = ((Tcc3.').*CDcc3*Tcc3);
Kcctrans3 = ((Tcc3.').*Kcc3*Tcc3);
Fcctrans3 = ((Tcc3.').*Fcc3);
McctransInv3 = inv(Mcctrans3);
rccol3 = size(Mcctrans3,2);
Acc3 = [zeros(rccol3,rccol3) eye(rccol3,rccol3);-McctransInv3*Kcctrans3 -McctransInv3*Ccctrans3];
Bcc3 = [zeros(rccol3,3);McctransInv3*Fcctrans3];
Ccc3 = [eye(2.*rccol3)];
Dcc3 = [zeros(2.*rccol3,3)];
%          Xccinit3Volts = [Kcctrans3\Fcctrans3(:,1);zeros(rccol3,1)];          %Inv(A)*B
Xccinit3Pist = [Kcctrans3\((Po.*Apc.*Fcctrans3(:,2));zeros(rccol3,1)];
%          Xccinit3Extra = [Kcctrans3\Fcctrans3(:,3);zeros(rccol3,1)];
%          Xccinit3 = Xccinit3Volts+Xccinit3Pist+Xccinit3Extra;
Xccinit3 = [zeros(2.*rccol3,1)];
%MyClosedChambSys3 = ss(Acc3,Bcc3,Ccc3,Dcc3);

Mtrans = ((Tac.').*Mac*Tac);
Ctrans = ((Tac.').*CDac*Tac);
Ktrans = ((Tac.').*Kac*Tac);
Ftrans = ((Tac.').*Fac);
MtransInv = inv(Mtrans);
rcol = size(Mtrans,2);
Aac = [zeros(rcol,rcol) eye(rcol,rcol);-MtransInv*Ktrans -MtransInv*Ctrans];
Bac = [zeros(rcol,4);MtransInv*Ftrans];
Cac = [eye(2.*rcol)];
Dac = [zeros(2.*rcol,4)];
XacinitLoad = [Ktrans\Ftrans(:,3);zeros(rcol,1)];          %Inv(K)*F
XacinitPress = [Ktrans\Ftrans(:,4);zeros(rcol,1)];
Xacinit = XacinitLoad+XacinitPress;
%Xacinit = [zeros(2.*rcol,1)];
%MyActuatorSys = ss(Aac,Bac,Cac,Dac);
%SecondSystemStates = zeros(1,2.*rcol);
%Xacinit = [Ktrans\Ftrans(:,3);zeros(rcol,1)];

%          [Xccinit(1:rccol,1);0;Xacinit2Load(1:rcol2,1);Xccinit(rccol+1:2.*rccol);0;Xacinit2Load(1+rcol2:2.*rcol2,1)]

Mtrans2 = ((Tac2.').*Mac2*Tac2);
Ctrans2 = ((Tac2.').*CDac2*Tac2);
Ktrans2 = ((Tac2.').*Kac2*Tac2);
Ftrans2 = ((Tac2.').*Fac2);
MtransInv2 = inv(Mtrans2);
rcol2 = size(Mtrans2,2);
Aac2 = [zeros(rcol2,rcol2) eye(rcol2,rcol2);-MtransInv2*Ktrans2 -MtransInv2*Ctrans2];
Bac2 = [zeros(rcol2,2);MtransInv2*Ftrans2];
Cac2 = [eye(2.*rcol2)];
Dac2 = [zeros(2.*rcol2,2)];
%Xacinit2 = [zeros(2.*rcol2,1)];
Xacinit2Load = [Ktrans2\Ftrans2(:,2);zeros(rcol2,1)];          %Inv(A)*B
Xacinit2 = Xacinit2Load;
%MyCylinderActuatorSys = ss(Aac2,Bac2,Cac2,Dac2);

rcol3 = FirstNums+1;

clear Aac5 Bac5 Cac5 Dac5 Xacinit5

```

```

clear Tac Mac CDac Kac Fac Mtrans Ctrans Ktrans Ftrans MtransInv Tac2 Mac2 CDac2 Kac2 Fac2 Mtrans2
Ctrans2 Ktrans2 Ftrans2 MtransInv2 Tac3 Mac3 CDac3 Kac3 Fac3 Mtrans3 Ctrans3 Ktrans3 Ftrans3 MtransInv3 Tac4
Mac4 CDac4 Kac4 Fac4 Mtrans4 Ctrans4 Ktrans4 Ftrans4 MtransInv4 Tac5 Mac5 CDac5 Kac5 Fac5 Mtrans5 Ctrans5
Ktrans5 Ftrans5 MtransInv5 Tcc Mcc CDcc Kcc Fcc Mcctrans Ccctrans Kcctrans Fcctrans McctransInv Tcc2 Mcc2
CDcc2 Kcc2 Fcc2 Mcctrans2 Ccctrans2 Kcctrans2 Fcctrans2 McctransInv2 Tcc3 Mcc3 CDcc3 Kcc3 Fcc3 Mcctrans3
Ccctrans3 Kcctrans3 Fcctrans3 McctransInv3 Tv Mv CDv Kv Fv Mvtrans Cvtrans Kvtrans Fvtrans MvtransInv

cwd = pwd;
cd(tempdir);
pack
cd(cwd)

if ((x == 0.00103) & (Freqss == 100))
for Cycles = 1:1
clear CylinderDispTemp ToutTemp
if (Cycles == 1)
Name2 = ['SecondStageRobustVariable'];
Name3 = ['ThirdStageRobustVariable'];
Name4 = ['FourthStageRobustVariable'];
Name5 = ['FifthStageRobustVariable'];
sim('FirstStageRobustVariable',[0 inf]);
sfirst = size(ToutFirst,1);
sim(Name2,[ToutFirstEnd inf]);
ssecond = size(ToutSecond,1);
sim(Name3,[ToutSecondEnd inf]);
sthird = size(ToutThird,1);
sim(Name4,[ToutThirdEnd inf]);
sfourth = size(ToutFourth,1);
sim(Name5,[ToutFourthEnd inf]);
sfifth = size(ToutFifth,1);
ChamberPressureTemp =
[FirstChambPress;SecondChambPress(2:end,1);ThirdChambPress(2:end,1);FourthChambPress(2:end,1);FifthChamb
Press(2:end,1)];
CylinderDispTemp =
[zeros(sfirst,1);SecondCylinderDisp(2:end,1);ThirdCylinderDisp(2:end,1);FourthCylinderDisp(2:end,1);FifthCylinderDisp
(2:end,1)];
ToutTemp = [ToutFirst;ToutSecond(2:end,1);ToutThird(2:end,1);ToutFourth(2:end,1);ToutFifth(2:end,1)];
PistonDispTemp =
[FirstPistonDisp(:,1);SecondPistonDisp(2:end,1);ThirdPistonDisp(2:end,1);FourthPistonDisp(2:end,1);FifthPistonDisp(2:
end,1)];
OutValveOpenTemp = [FirstOutValveOpen;SecondOutValveOpen(2:end,1);zeros(sthird-
1,1);zeros(sfourth-1,1);FifthOutValveOpen(2:end,1)];
InValveOpenTemp = [zeros(sfirst,1);zeros(ssecond-
1,1);ThirdInValveOpen(2:end,1);FourthInValveOpen(2:end,1);zeros(sfifth-1,1)];
CylinderForceTemp =
[zeros(sfirst,1);SecondCylinderForce(2:end,1);ThirdCylinderForce(2:end,1);FourthCylinderForce(2:end,1);FifthCylinder
Force(2:end,1)];
Tout = ToutTemp;
ChamberPressure = ChamberPressureTemp;
CylinderDisp = CylinderDispTemp;
OutValveOpen = OutValveOpenTemp;
InValveOpen = InValveOpenTemp;
PistonDisp = PistonDispTemp;
CylinderForce = CylinderForceTemp;
else
Name3 = ['ThirdStageRobustVariable'];
Name4 = ['FourthStageRobustVariable'];
Name5 = ['FifthStageRobustVariable'];
Name6 = ['SixthStageRobustVariable'];

```

```

sim(Name6,[ToutFifthEnd inf]);
    ssixth = size(ToutSecond,1);
    HeldDisp = SecondSystemStates(end,rccol2);
    sim(Name3,[ToutSecondEnd inf]);
    sthird = size(ToutThird,1);
    sim(Name4,[ToutThirdEnd inf]);
    sfourth = size(ToutFourth,1);
    sim(Name5,[ToutFourthEnd inf]);
    sfifth = size(ToutFifth,1);
    ChamberPressureTemp =
[SecondChambPress(2:end,1);ThirdChambPress(2:end,1);FourthChambPress(2:end,1);FifthChambPress(2:end,1)];
    CylinderDispTemp =
[SecondCylinderDisp(2:end,1);ThirdCylinderDisp(2:end,1);FourthCylinderDisp(2:end,1);FifthCylinderDisp(2:end,1)];
    CylinderForceTemp =
[SecondCylinderForce(2:end,1);ThirdCylinderForce(2:end,1);FourthCylinderForce(2:end,1);FifthCylinderForce(2:end,1)
];
    PistonDispTemp =
[SecondPistonDisp(2:end,1);ThirdPistonDisp(2:end,1);FourthPistonDisp(2:end,1);FifthPistonDisp(2:end,1)];
    ToutTemp = [ToutSecond(2:end,1);ToutThird(2:end,1);ToutFourth(2:end,1);ToutFifth(2:end,1)];
    Tout = [Tout;ToutTemp];
    ChamberPressure = [ChamberPressure;ChamberPressureTemp];
    CylinderDisp = [CylinderDisp;CylinderDispTemp];
    PistonDisp = [PistonDisp;PistonDispTemp];
    CylinderForce = [CylinderForce;CylinderForceTemp];
end
end
figure(154)
    hold on
    plot(Tout,CylinderDisp,colors(Load,:))
    legend('0 Kg','6.840 Kg','12.825 Kg','17.452 Kg','20.153 Kg','22.207 Kg')
    Mytitle10 = ['Double Ended Cylinder Displacement (x = 0.00103, Freq = 100 Hz, Press = 190 psi, Crod = '
num2str(Crod) ')'];
    title(Mytitle10)
    xlabel('Time (sec)')
    ylabel('Displacement (meters)')

    figure(155)
    hold on
    plot(Tout,ChamberPressure,colors(Load,:))
    legend('0 Kg','6.840 Kg','12.825 Kg','17.452 Kg','20.153 Kg','22.207 Kg')
    Mytitle10 = ['Double Ended Chamber Pressure (x = 0.00103, Freq = 100 Hz, Press = 190 psi, Crod = '
num2str(Crod) ')'];
    title(Mytitle10)
    xlabel('Time (sec)')
    ylabel('Pressure (psi)')

    figure(160)
    plotyy(Tout,OutValveOpen,Tout,CylinderDisp)

    figure(161)
    plotyy(Tout,OutValveOpen,Tout,ChamberPressure)

    figure(162)
    plotyy(Tout,CylinderDisp,Tout,ChamberPressure)

else
    for Cycles = 1:1
        clear CylinderDispTemp ToutTemp

```

```

if (Cycles == 1)
    Name2 = ['SecondStageRobustVariable'];
    Name3 = ['ThirdStageRobustVariable'];
    Name4 = ['FourthStageRobustVariable'];
    Name5 = ['FifthStageRobustVariable'];
    sim('FirstStageRobustVariable',[0 inf]);
    sfirst = size(ToutFirst,1);
    sim(Name2,[ToutFirstEnd inf]);
    ssecond = size(ToutSecond,1);
    sim(Name3,[ToutSecondEnd inf]);
    sthird = size(ToutThird,1);
    sim(Name4,[ToutThirdEnd inf]);
    sfourth = size(ToutFourth,1);
    sim(Name5,[ToutFourthEnd inf]);
    sfifth = size(ToutFifth,1);
    ChamberPressureTemp =
[FirstChambPress;SecondChambPress(2:end,1);ThirdChambPress(2:end,1);FourthChambPress(2:end,1);FifthChamb
Press(2:end,1)];
    CylinderDispTemp =
[zeros(sfirst,1);SecondCylinderDisp(2:end,1);ThirdCylinderDisp(2:end,1);FourthCylinderDisp(2:end,1);FifthCylinderDisp
(2:end,1)];
    ToutTemp =
[ToutFirst;ToutSecond(2:end,1);ToutThird(2:end,1);ToutFourth(2:end,1);ToutFifth(2:end,1)];
    PistonDispTemp =
[FirstPistonDisp(:,1);SecondPistonDisp(2:end,1);ThirdPistonDisp(2:end,1);FourthPistonDisp(2:end,1);FifthPistonDisp(2:
end,1)];
    OutValveOpenTemp = [FirstOutValveOpen;SecondOutValveOpen(2:end,1);zeros(sthird-
1,1);zeros(sfourth-1,1);FifthOutValveOpen(2:end,1)];
    InValveOpenTemp = [zeros(sfirst,1);zeros(ssecond-
1,1);ThirdInValveOpen(2:end,1);FourthInValveOpen(2:end,1);zeros(sfifth-1,1)];
    CylinderForceTemp =
[zeros(sfirst,1);SecondCylinderForce(2:end,1);ThirdCylinderForce(2:end,1);FourthCylinderForce(2:end,1);FifthCylinder
Force(2:end,1)];
    Tout = ToutTemp;
    ChamberPressure = ChamberPressureTemp;
    CylinderDisp = CylinderDispTemp;
    OutValveOpen = OutValveOpenTemp;
    InValveOpen = InValveOpenTemp;
    PistonDisp = PistonDispTemp;
    CylinderForce = CylinderForceTemp;
else
    Name3 = ['ThirdStageRobustVariable'];
    Name4 = ['FourthStageRobustVariable'];
    Name5 = ['FifthStageRobustVariable'];
    Name6 = ['SixthStageRobustVariable'];
    sim(Name5,[ToutFourthEnd inf]);
    sfifth = size(ToutFifth,1);
    sim(Name6,[ToutFifthEnd inf]);
    ssixth = size(ToutSecond,1);
    sim(Name3,[ToutSecondEnd inf]);
    sthird = size(ToutThird,1);
    sim(Name4,[ToutThirdEnd inf]);
    sfourth = size(ToutFourth,1);
    ChamberPressureTemp =
[FifthChambPress(2:end,1);SecondChambPress(2:end,1);ThirdChambPress(2:end,1);FourthChambPress(2:end,1)];
    CylinderDispTemp =
[FifthCylinderDisp(2:end,1);SecondCylinderDisp(2:end,1);ThirdCylinderDisp(2:end,1);FourthCylinderDisp(2:end,1)];

```

```

CylinderForceTemp =
[FifthCylinderForce(2:end,1);SecondCylinderForce(2:end,1);ThirdCylinderForce(2:end,1);FourthCylinderForce(2:end,1)
];
    ToutTemp = [ToutFifth(2:end,1);ToutSecond(2:end,1);ToutThird(2:end,1);ToutFourth(2:end,1)];
    Tout = [Tout;ToutTemp];
    ChamberPressure = [ChamberPressure;ChamberPressureTemp];
    CylinderDisp = [CylinderDisp;CylinderDispTemp];
    PistonDisp = [PistonDisp;PistonDispTemp];
    CylinderForce = [CylinderForce;CylinderForceTemp];
    end
end
end
    AllCylinderVeloc(Load,Press,Freq) = (CylinderDisp(end,1)-min(CylinderDisp)).*Freqss.*(10^3);    %
mm/sec
    AllPistonDisp(Load,Press,Freq) = (max(PistonDisp)-min(PistonDisp)).*(10^6);    % micrometers
    end
end
    AllCylinderPower(Load,,:) = AllCylinderVeloc(Load,,:).*Force(1,Load)./1000;
    save TempNoMassSimulatedData110Volts.mat AllCylinderVeloc AllPistonDisp AllCylinderPower FreqInit Weight
MyStiff
    if (x == 0.00103)
        figure(150)
        plot(FreqInit,squeeze(AllCylinderVeloc(Load,1,:)),colors(Load,:))
        hold on

        figure(151)
        plot(FreqInit,squeeze(AllPistonDisp(Load,1,:)),colors(Load,:))
        hold on

        figure(149)
        plot(FreqInit,squeeze(AllCylinderPower(Load,:)),colors(Load,:))
        hold on
    end
end
for Freq = 1:m
    if (x == 0.00103)
        figure(152)
        plot(Weight,squeeze(AllCylinderVeloc(:,Freq)),colors(Freq,:))
        hold on

        figure(153)
        plot(Weight,squeeze(AllCylinderPower(:,Freq)),colors(Freq,:))
        hold on
    end
end
end
end

```

Appendix C

Simulink Diagrams Used to Step Through the Four Stage Cycle

M-file code called the different simulink diagrams for simulation. The final states for each stage were saved using the enabled subsystem, triggered by the inlet or outlet valve opening value, which also stopped the simulation. These final state values were then used as the beginning state values for the next stage simulation.

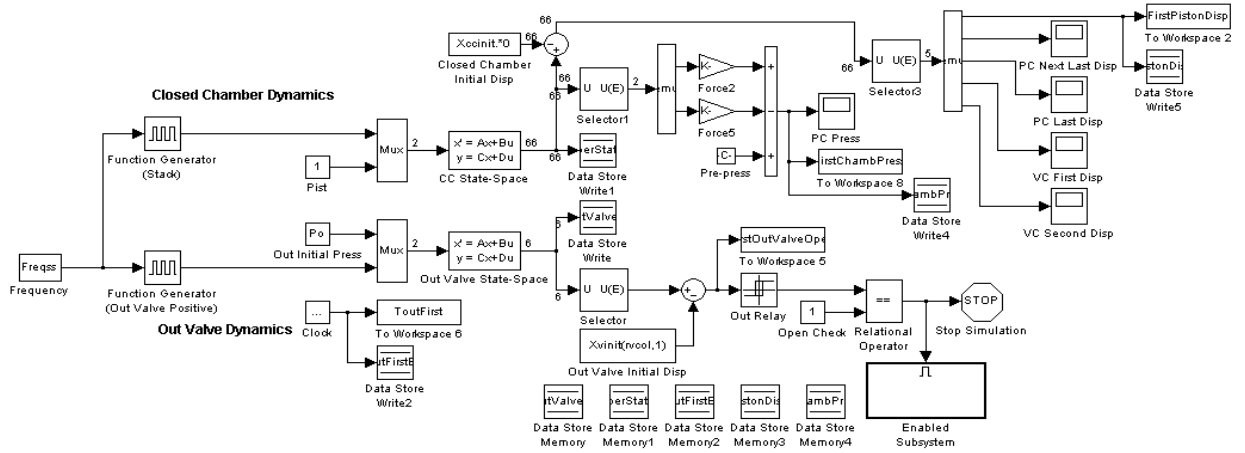


Figure C.1: First Stage Simulink Diagram

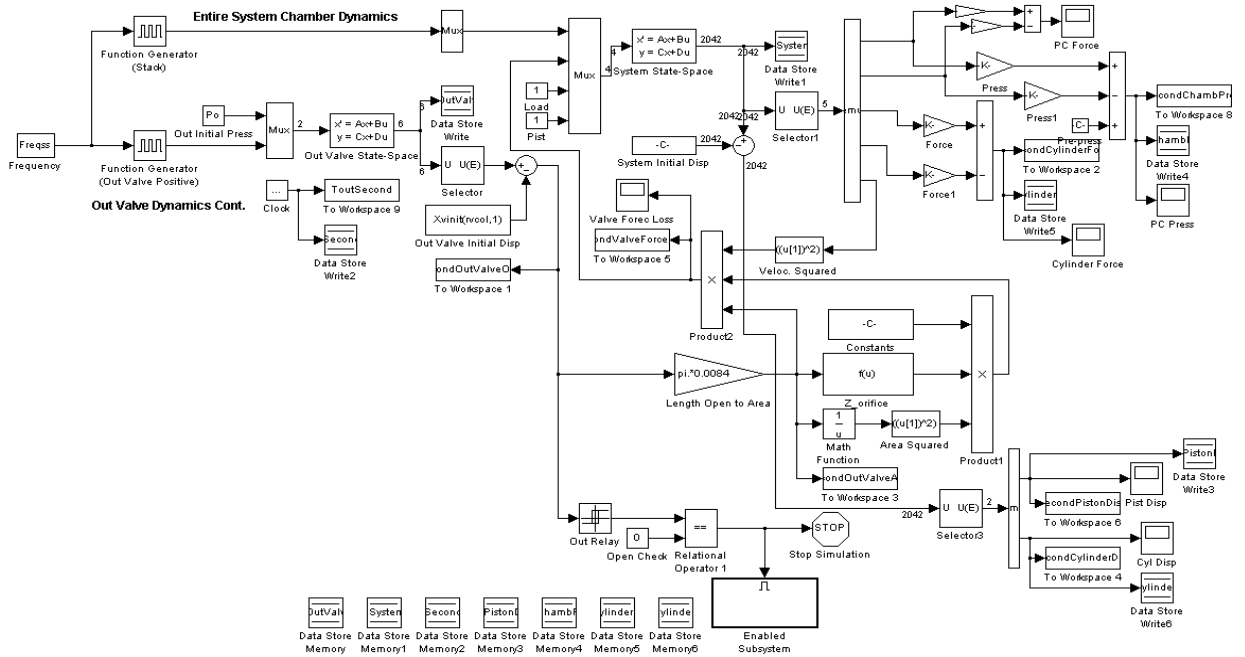


Figure C.2: Second Stage Simulink Diagram

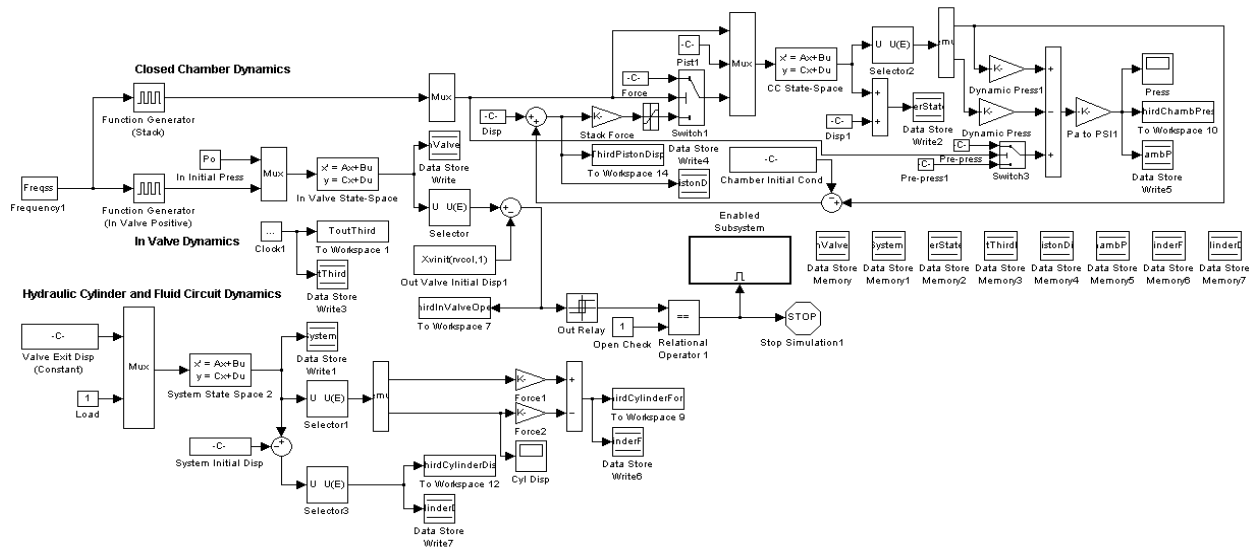


Figure C.3: Third Stage Simulink Diagram

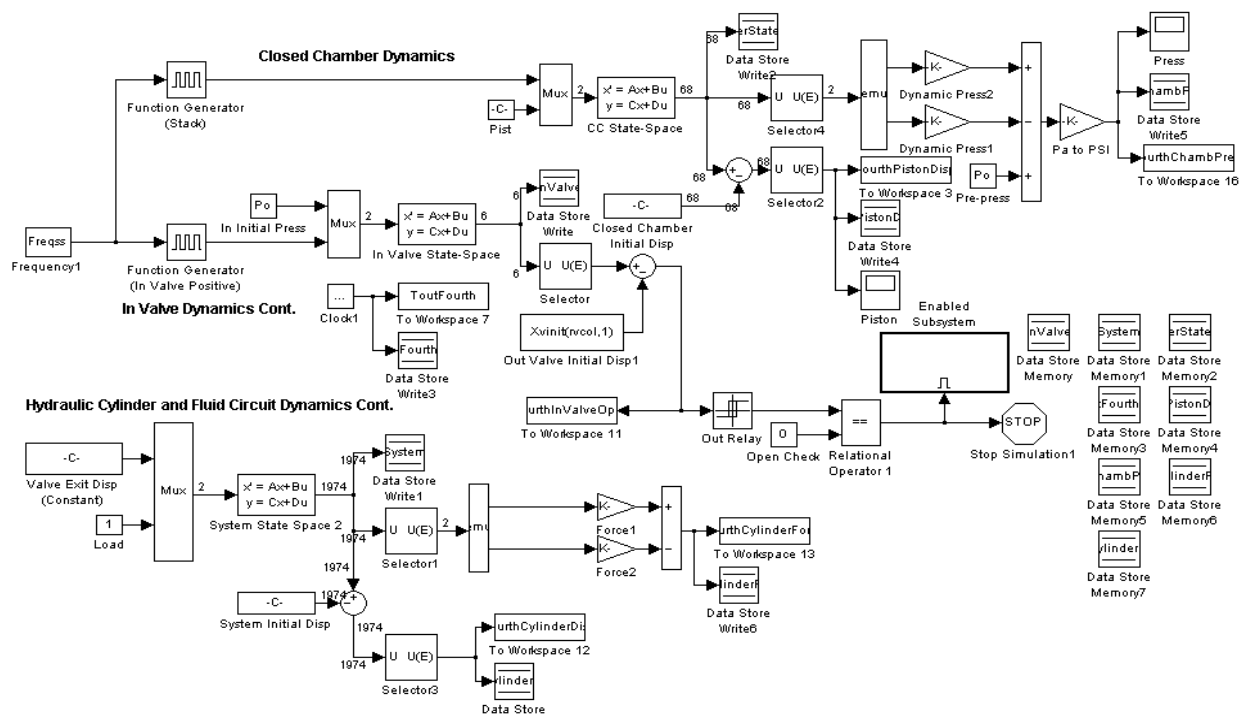


Figure C.4: Fourth Stage Simulink Diagram

Vita

William Edwin Hurst, son of Stephen and Lark Hurst, was born on May 19, 1977, in Fort Sill, Oklahoma. A few years later his family moved back home where he grew up in the eastern suburbs of Cincinnati, OH. After graduating from Glen Este High School in May 1995, he began his undergraduate degree at the University of Cincinnati. There he was able to earn valuable work experience at Cincinnati Thermal Spray, Inc. and Sheet Dynamics, Ltd. through the Engineering coop program. In June 2000 William graduated *Cum Laude* from UC with a B.S. in Mechanical Engineering. The following fall, he was enrolled for graduate studies at Virginia Tech. After one semester working for the Mechanical Engineering Computer Department, he started working for Dr. Donald Leo in the Center for Intelligent Material Systems and Structures (CIMSS). His graduate research involved designing and modeling a piezohydraulic actuator with active valves, while his course work focused on controls, vibration, and signal processing.

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This thesis was typeset with L^AT_EX 2_ε¹ by the author.

¹L^AT_EX 2_ε is an extension of L^AT_EX. L^AT_EX is a collection of macros for T_EX. T_EX is a trademark of the American Mathematical Society. The macros used in formatting this thesis were written by Greg Walker, Department of Mechanical Engineering, Virginia Tech.