HIERARCHICAL STRUCTURES COMPUTATIONAL DESIGN AND DIGITAL 3D PRINTING

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ABSTRACT

Current advances in construction automation, especially in large-scale additive manufacturing, highlight the vast potential for robots in architecture. Robotic construction is unique in its potential to reproduce highly complex structures. To advance the question of how rapid prototyping techniques are adopted in large-scale 3D printing of forms and structures, this paper presents computational methods for the design and robotic construction of cellular membranes. This research presents a comprehensive morphological model of structurally differentiated cellular membranes based on the theoretical biology model of hierarchical structures found in natural cellular solids, and, more specifically, in trabecular bone. The morphological model originates from a system of forces in equilibrium; therefore, it presents the geometric homology of a static tensional system. This research offers a methodology for the design and manufacture of meso- to large-scale triangulated geometric configurations by discrete design methods that are suitable for the robotic fused deposition of lattices and their architectural implementation in the automated manufacturing of shell structures. First, this paper explores how a form can be digitally created by geometrically emulating a given static system of forces in space. Second, inspired by the complex mechanical behavior of cancellous bone, we apply hierarchical principles found in bone remodeling to characterize discrete units that conform to continuous trabecular-like lattices. We study the geometry, limitations, opportunities for optimization, and mechanical characteristics of the lattice. The computational design methods and additive manufacturing techniques are tested in the design and construction hierarchical structures.

Keywords: *hierarchical structures, robotic fabrication, additive manufacturing, structural optimization, lightweight shells, lattice, cellular solids, biomimicry*

1. INTRODUCTION

Current advances in computational design and in large scale additive manufacturing (AM) have increased the productivity and performance of building shell structures. Robotic manufacturing methods make way for interdisciplinary research fields around computational design and engineering. Identifying construction elements susceptible to automation is key in these fields.

At the architectural scale, construction automation and AM present a novel production scenario [1], [2], [3] of a promising increase in productivity when building complex surfaces [4]. Moreover, AM offers an unparalleled design freedom to build forms of highly complex mechanical behavior.

Merging stress data, and material differentiations to gradually integrate various functions, we explored stress-influenced design methods and their inherent manufacturing constraints for 3D printing.

This research offers an architectural application of the underlying material design strategies of naturally formed patterns found in bone structure. Cancellous bone structure continually adapts to the stress trajectories finding the optimal dissipation of stress.

In this study we elucidate a general design and fabrication method for lightweight hierarchical structures capable of being manufactured by robots continuously on-site or by prefabricated segments assembled into a mechanically continuous complex surface.

1.1. Membrane design and spatial 3D printing

AM generally involves the creation of a series of coordinated, automatic, electromechanic operations and demands the configuration of specialized deposition toolpaths. Spatial 3D printing, a Fused Deposition Modeling (FDM) extrusion-based technology, offers exceptional characteristics for creating complex cellular structures. Spatial 3D printing techniques are increasingly being successfully implemented in the construction of large-scale assemblies [5] demonstrating the vast potential of robots in construction. Recently, Virginia Tech and Nagami Design created a 3D printed lattice for the CaixaForum Valencia central pavilion, an effort that overcame the challenges of applying the technique in a public space (Figure 1).

Figure 1: *Architectural application of the spatial 3D printing technique: 3D printed lattice designed by Virginia Tech and manufactured by Nagami Design*

Automated design and fabrication methods are progressively being applied in the design and construction of complex shapes. This research builds on current studies in spatial 3D printing methods and presents novel algorithmic methods and geometric generalization to robotically fabricate triangulated tessellations that conform complex surfaces. The particularization of models for creating the voxelized approximation of a shape is one method [6], [7], [8] that has allowed designers to defy the layerwise stacking principle of standard AM and allowed the creation of complex spatial arrangements (Figure 2).

Figure 2: *Spatial 3D printing of a lattice with multiple materials and differentiated polyhedral arrangements*

Digital tools in design for fabrication integrate geometric logic of constructability and design goals [9]. The system of particlizing involves a computational discretization of a shape where each particle represents an individually indexed container and value, ordered in space. Sequencing the particles and their geometric information allows a robotic system to fabricate the designs in a linear, additive fashion. This study employs generative and analytical digital tools that create and evaluate triangulated tessellations inspired in the theoretical biology model of cancellous bone reformation.

A key concern we aimed to develop is the criteria by which we can 3D print a large-scale stable shape. We approximated the bone reformation model using static analysis results of a given form in equilibrium to create FEM influenced 3D printed structures.

The methodological process involves characterizing the set of complex surfaces, determined by Bézier curves or the spaces determined by said surfaces. In order to determine stable behavior and define spaces and surfaces of continuous structural behavior, we can configure the geometry by aggregating cellular elements that make up a stable tessellation or lattice in a continuous path of 3D printing (Figure 3).

(a)

(b)

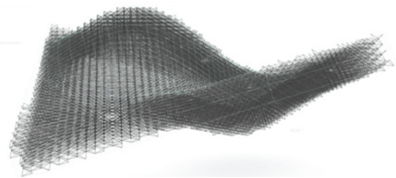

Figure 3: *(a) Reference surface and (b) triangulated lattice derived from stress trajectories and additive manufacturing parameters*

2. LITERATURE REVIEW

We studied stabilization by triangulation and by application of forces to balance a digital cellular membrane. The paper examines the following:

- Configuration of complex geometric arrangements based on biomechanics
- Digital stereotomy by discrete design
- Cellular membrane design, optimization, and hierarchical mechanical characterization
- Spatial 3D printing and robotic AM methods

2.1. Trabecular membrane force to shape morphological principles applied to large-scale additive manufacturing

Cellular solids (of a volume fraction below 30%) and hierarchical structures (arrangements differentiated by stress concentration and trajectories) are often found in natural fibrous composites remarkably illustrating R. Le Ricolais' words "the art of structure is where to put the holes". This generally results in ultralightweight structural members. The interest in configuring complex cellular membranes demands the development of a comprehensive simulation-fabrication morphological method where different skills and disciplines converge to successfully present a model of hierarchically engineered continuous surfaces that follow a system of endogenic and exogenic forces in equilibrium while obeying the principles of 3D printing.

We evaluate the following considerations for the development of an abstraction model that links form configuration to a comprehensive workflow of design-simulation-3D printing:

- Current large-scale AM techniques demand the configuration of custom layerwise and spatial deposition instruments.
- Digitally configured membranes are based on the logic of computation. This logic is fundamentally different from predigital models emphasizing the necessity of creating explicit and implicit models of varying levels of abstraction [10], that result in a digital stereotomy.

This evaluation allows us to create a methodological framework for digital 3D printing (3D printing based on particlizing a form, enhancing the performance by inserting additional information, and concatenating commands of deposition) that can be applied to the stress-informed hierarchical organizations.

Biomimicry involves innovation inspired by nature. As T. Mak and L. Shu argue "biomimetic design examines biological analogies to solve engineering problems" [11]. Morphogenetic and theoretical biology strategies help us create algorithmic principles of complex form generation [12].

Nature provides fruitful examples of algorithmic complexity, manifest in the wide interest and influence of current digital design [13], [14], [15] and automatic construction practices [16], [17], [18]. Nature´s cellular solid materials display geometric arrangements of exceptional mechanical properties [19], [20]. The evolutive association of growth and force in natural systems suggests an optimal response to load conditions. Certain bone microstructures exhibit an ultra-lightweight structure, high stiffness, damage tolerance, controlled energy absorption, and functional robustness [21], properties that are critical to critical toward architectural applications.

The particular interest of this research is drawn from the structural lattices found in cancellous bone [22]. Geometry is the most influential aspect of strength found in very lightweight trabecular architectures found in biological composites [23, 24]. AM technologies have allowed designers to develop novel material arrangements that effectively expand the material property space [25], and design infills patterns that conform very complex structures that are influenced by stress trajectories [26].

Although 3D printing technologies, due to their high precision, open new opportunities to design and fabricate complex structures [27], these studies are generally tested at a small scale. At the architectural scale, the use of natural cellular arrangements as structural components presents design and manufacture challenges of scale and production time. The implementation model capable of following the trajectorial hypothesis of stress in complex cellular arrangements demands the assessment of geometries that follow the principles of digital 3D printing.

2.2. Digital configuration of a cellular membrane

Using discrete methods, we design for additive manufacturing incrementally building complex tessellations of differentiated architectured properties (such as stiffness, strength, and density) configured by the forces in equilibrium that define it.

The analysis, identification and generalization of geometries and methods to produce hierarchical fibrous arrangements that comply with the several limiting constraints of a robotic technique of 3D printing demanded the following:

First, the development of algorithms that bridge mechanical analysis information with discrete data structures.

Second, algorithms that bridge the discrete data structures and geometrical configurations capable of originating a mechanically continuous membrane through design and optimization with nonlinear mechanical characterization.

2.2.1. Discrete data structures

The membrane configuration process involves computing mechanical data, interpreting it in an intelligible order and using the information to populate a digitized shape. To illustrate this process, below we describe the analysis a 23 m x 13 m x 7 m ellipsoid-like shell fixed at the equator, with a variable wall thickness that spans 30 cm at top and bottom boundaries to 50 cm at the equator regions (Figure 4 and 5).

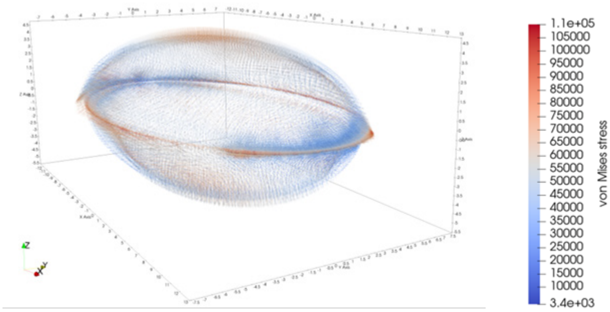

Figure 4: *ParaView visualization of a point cloud showing displacement vectors and von Mises stress values expressed in pascals*

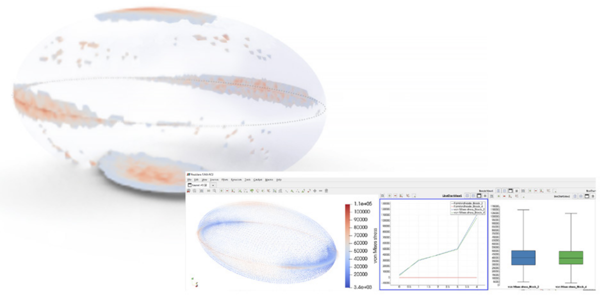

Figure 5: *Rhino 3D visualization of a stress region information entered into Grasshopper, the static analysis data is calculated using SimScale and discretized into quantiles using ParaView*

Made of a typical polymer 3D printed material, with a 30% volume fraction, we estimate its young modulus (E) to be 80MPa and Poisson ratio (v) to be 0.38 based on experimental tests [28], [29]. The nonlinear static analysis performed in SimScale [30] and processed in ParaView [31] shows the reaction to its dead load and 600 KN vertical load. In this case, von Mises stress values range from 3.4 to 110 kPa and vectorial displacement values, from 0 to 15 mm. The values are organized as an isostatic point cloud, a large set of values the sum of which define a shape in equilibrium.

Point cloud values are interpreted in Rhino 3D and Grasshopper [32] as stress regions to create a stress-informed shape.

Geometry and force are linked in a structure design with this method. The discrete design method of multiresolution [33] allows us to define volume out of indexed data containers to create a digitized equivalent model, in the digital version we bridge design, simulation, and fabrication as illustrated by Figure 6.

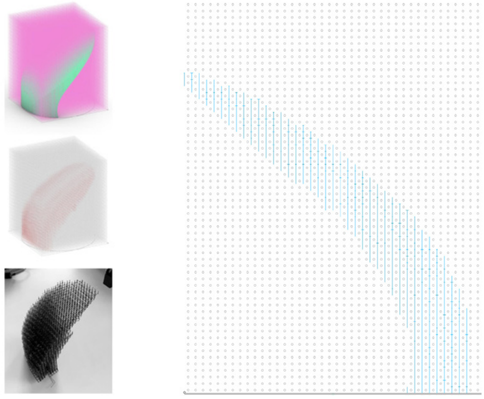

Figure 6: *Structured infill configuration process: discretization of the ellipsoid membrane segment.*

2.2.2. Space filling polyhedra

The method allows us to determine morphogenetic principles to approximate complex forms, populate space-filling regular tessellations, and incorporate the capacity of 3D printing.

An advantage of using space grids such as regular space filling polyhedral arrays is that each member contributes to the general load carrying capacity and that the spatial arrays can be configured to work under simple stress rather than bending [34].

In other structures, each structural member, such as a beam, must be capable of dissipating concentrated stress when subject to a given concentrated load. Instead, for spatial structures, concentrated loads will be distributed along and throughout the lattice, distributing the stress more efficiently, therefore utilizing less material to carry the same load.

The studies of tetrahedral, semi octahedral and rhombic dodecahedral space filling arrangements in this research (Figure 7) are largely influenced by R. B. Fuller studies on “cubic packing” structures [35].

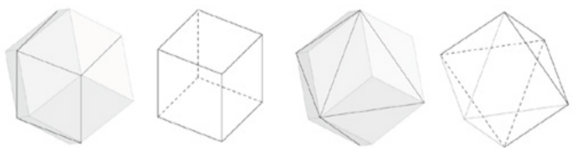

Figure 7: *Different regular polyhedral arrangements inscribed in an orthonormal grid exhibit geometrical relationships*

Certain space filling polyhedral, such as octet trusses, can be used to create tessellations (Figure 8) that populate the orthonormal grid of a discretized membrane to be individually and incrementally 3D printed to constitute a continuous spatial structure.

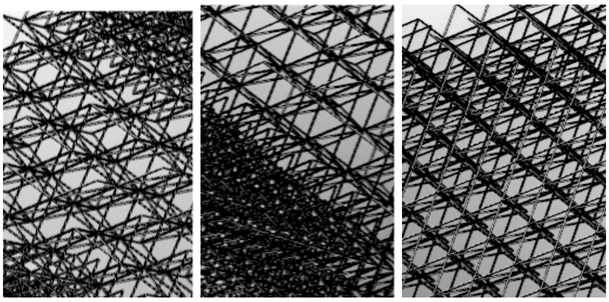

Figure 8: *Details of different polyhedral spatial arrays inscribed in an orthonormal grid*

These arrays create an optimal organization of struts for stress distribution. A. Edmonson [35] succinctly describes the enormous structural potential of geometry when describing Fuller's octet-truss "we can now appreciate the difference between diamond and graphite. Their organization can be thought of as a double octet truss, two intersecting matrices with the vertices of one overlapping the cells of the other. Stabilized by the high number of bonds between neighboring atoms, which also allow forces to be distributed in many directions at once, the configuration is supremely invulnerable. In contrast, carbon atoms in graphite are organized into planar layers of hexagons, triangulated and stable in themselves, but not rigidly connected to other layers. The comparison provides a spectacular example of synergy: rearrangement of identical constituents produces two vastly different systems".

2.2.3. Hierarchical differentiation of fibrous arrangements

3D printing is usually divided in two unrelated parts an external perimeter and an internal structure. Structural optimization methods adapting theoretical biology models are increasingly being applied to tailor the internal structure of the 3D print (called infill) by varying material allocation to create foam or dense tessellations [36] [37].

The proposed methodology allowed us to test membranes to work only under simple stress, 3D print cellular membranes with individual characterization of struts, determine their local density, material and shape optimization in segments and as whole membrane structures. Strong yet light, cellular solids, or foams, found in nature exhibit very complex, mechanical performance, with strategic variations they are capable of transitioning from fully dense and highly stiff bodies to very light, energy absorbing configurations (Figure 9).

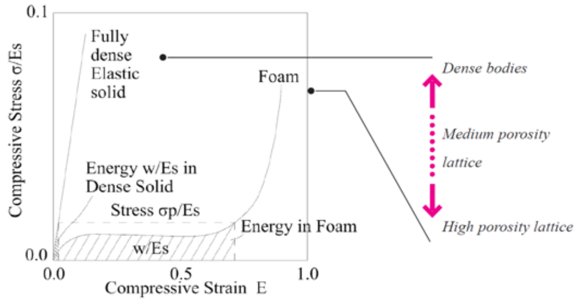

Figure 9: *Mechanical behavior of foams under compression graph, adapted from [18]*

Inspired in the bone reformation model we distinguished variable mechanical properties depending on density, trabecular architecture, and material. Bone structure presents an optimal porous distribution of mass that is intimately coordinated with the stress to which it is subject. Substantial research effort has been dedicated to comprehending the robust [24], [2], [38] morphogenetic configuration and hierarchical functional adaptation of bone under mechanical loading, since J. Wolff in [39] [40] presented the concept of biomechanical adaptation interpreting the static graphics method by K. Culmann [41], [42].

Bone transformation is a highly coordinated adaptation of the bone tissue, J. Wolff's stress trajectory hypothesis has been corroborated under relatively simple load FEA studies and is debated as there is currently a lack of consensus on the precise interpretation of stress trajectories in trabecular and cortical bone in complex load scenarios, as discussed by J. Skedros [43]. We are only beginning to understand the mechanical work of bone due to advances in computational models such as analytical discrete methods [44].

Although J. Wolff's intuition remains a matter of debate, the literature review finds consensus in geometry as the most influential aspect of bone strength, while density, strut orientation and strut material provide the main mechanical characteristics, in that order of importance [23].

The efforts of this research focus on presenting a method to shape a membrane by configuring the forces in equilibrium that define it. Cellular morphologies of different scales exhibit unparalleled freedom to engineer anisotropy and introduce hierarchical mechanical grading [45]. Optimizing the material distribution by controlling levels of porosity, creating more efficient stress distribution, and inducing stiffer cellular arrangements is key to achieving larger scale constructions as it reduces the weight of the structure and reduces material waste.

Although more limited than other high-resolution printing techniques commonly applied to lattice 3D printing, FDM provides an efficient platform of fabrication, particularly for meso- to large-scale arrangements.

3. METHODS

We explored geometrical potential to spatially vary lattice parameters to accommodate complex loading conditions to localize reinforcement material in regions where tensile loads are projected, thus optimizing material use. First, we perform a nonlinear static analysis simulation of a shell using SimScale and export the results as a point cloud. The scalar and vectorial load scenario results are linked to each sample point. We then process the large amount of unstructured information using ParaView to differentiate stress regions. Finally we import stress regions into Rhino3D and Grasshopper software, where we algorithmically populate the different stress regions with structured hierarchical fibrous arrangements.

The resulting design reconfigures the input solid into a hierarchical membrane emulating the stress trajectories in a force-geometry homology model.

3.1. Force-determined hierarchical membranes

In this research, lattices were designed to extend periodically and by transposition of the geometric arrangements, optimized for minimal deformation (improved stiffness) by allocating the material where it responds to stress and strain. With SimScale and ParaView we produced and postprocess implicit informed data structures. The information included vector and scalar values per sampling point of an unstructured mesh of tetrahedral VTK cells (Figure 10). VTK cells are three dimensional meshes. VTK is an abstract class that specifies the interfaces for data cells and provides data visualization for a variety of grid types. Data cells are simple topological elements such as points, lines, polygons, and tetrahedra of which visualization datasets are composed. Visualization datasets may compose cells explicitly (e.g., vtkUnstructuredGrid), or implicitly (e.g., vtkStructuredPoints).

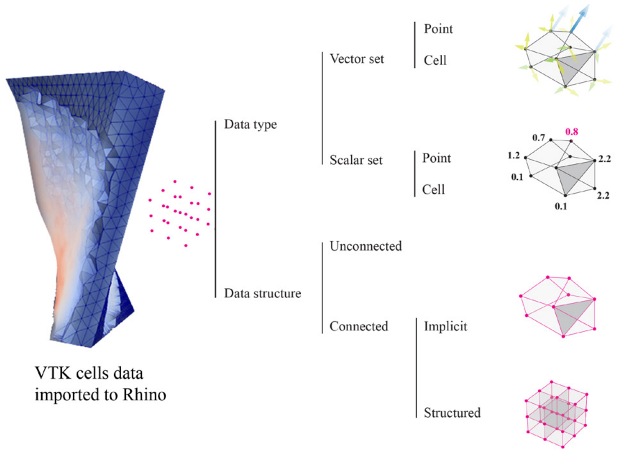

Figure 10: *VTK cells: the static analysis scalar and vector information sets are embedded in an implicit grid*

Cellular membrane engineering with this method involved discretizing the surfaces into ordered, indexed particles. To create sequential 3D printing commands, the particles were organized in a linear list of rows, columns, and levels [33].

Indexing allowed us computational access to the particle level, and offered a tool to fabricate a highly intricate, customized structure (Figure 11). Each particle that composes a digital mockup can host an indeterminate amount of information allowing us to individually allocate properties to create complex geometric arrangements inspired on biomechanics.

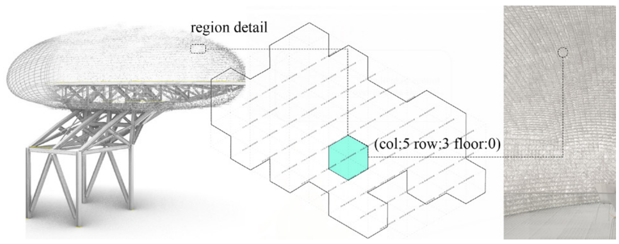

Figure 11: *Design of a shell structure by discrete methods: to form a shell, a reference surface is discretized into populated indexed coordinates at [x-axis, y-axis, z-axis]*

We differentiated each particle according to the stress regions they belong to and populated the indexes with lightweight polygonal arrays that improve stress distribution (Figure 12).

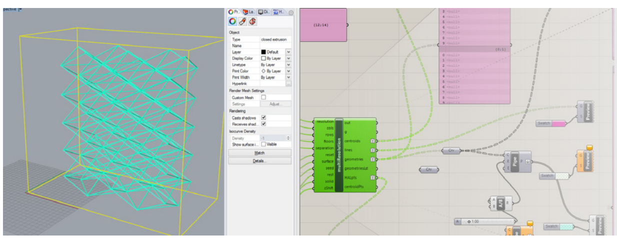

Figure 12: *Lattice configuration in Grasshopper*

The process of design transfers a data point cloud stress field to geometry, transferring differentiated characteristics from the whole to the level of the part. The geometric definition of each cell or particle depends on the general conditions of equilibrium. The key aspects of the workflow proposed is its capacity of AM generalization and its potential to automate the geometric configuration mimic bone formation applying:

- Variable volume fraction properties
- Variable strut architecture properties
- Variable material properties

Each segment of the lattice must comply with the constraints of the robotic 3D printing process [46]. This characteristic proves their capacity to be fabricated with large-scale 3D printers or robotic mechanisms, on-site or by prefabrication.

3.2. Characterization workflow summary and implementation

The toolpath creation method was principally aimed at being able to access every portion of the linear additive process to be able to examine and determine its characteristics individually.

The difference from other methods of force influenced form configuration is that this method accessed and interpreted the information of a point cloud, three dimensionally, and discretely throughout the solid.

We computed the stress values using SimScale, the software discretizes a solid into tetrahedral VTK cell samples for static analysis; the results were exported to OpenFoam format [47]. Other software such as Elmer FEM [48] capable of calling ParaView or exporting cloud point can be used. With ParaView we postprocessed the vector and scalar information data sets and to exported a simplified set to Rhino3D-Grasshopper in the form of lists and brep stress regions. We then interpreted the cloud of forces in the 3D modeling software using parametric tools, finding a particularization resolution that balanced shape fidelity, and computational power.

Figure 13 illustrates the intensification of trabecular arrangements in two levels of variable resolution and variable thickness. The computation of a stress field and the hierarchical configuration of lattices operated at two levels: the general voxel resolution by the incremental subdivision of cells and the intensification of areas according to stress.

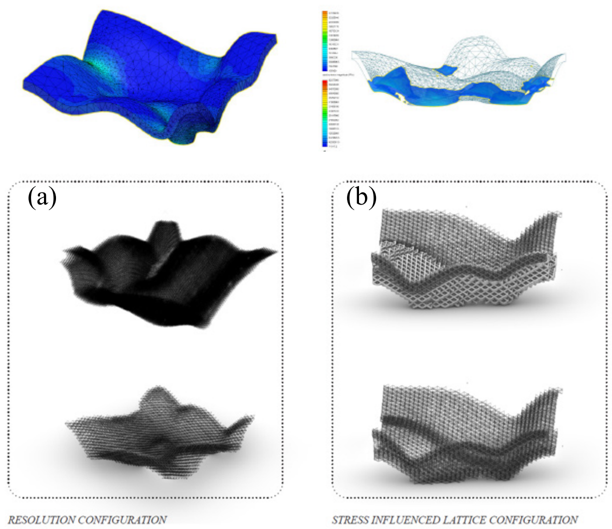

Figure 13: *Hierarchical configuration of an open membrane: (a), resolution; (b) trabeculae intensification*

Figure 14 illustrates the four steps we performed to characterize lattices: (a) an isostatic cloud that contains scalar and vector stress information is computed; (b), discrete units of geometry linearly aggregate into a continuous cellular membrane influenced by the stress field, density variation, trabecular architecture, and material configuration create a mechanically complex fibrous arrangement; (c) the characteristics of individual struts is determined by differentiated electromechanical commands of spatial 3D printing, to vary its specific architecture; and (d) a hierarchical membrane shape is incrementally built.

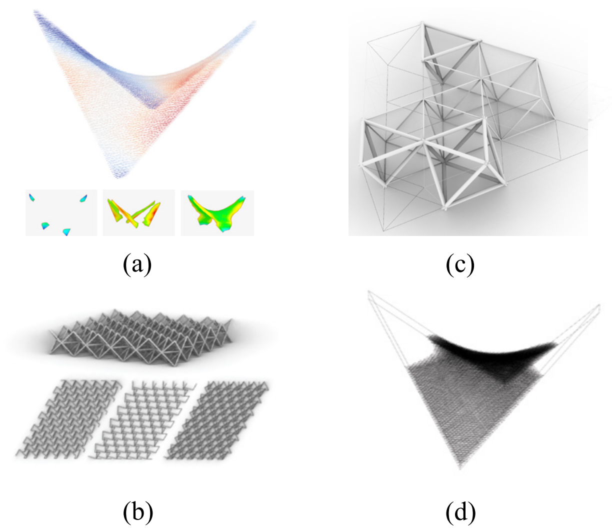

Figure 14: *Hierarchical characterization components: (a) stress regions; (b) space-filling tessellations; (c) individual variation of struts; (d) incremental construction.*

3.3. Generalization of triangulated lattices for digital 3D printing

We defined the principles of 3D printing triangulated lattices and cellular structures with this method. We created parametric algorithms able to fabricate continuous complex forms serialized in 3D printable structural components.

3.3.1. Digital 3D printing

Digital 3D printing infers the existence of a digitized equivalent model to which the fabrication technique is applied. It infers the existence of intelligible data in the model to be printed and presents a continuous method of analytical data processing from raw data into usable form into instruction [49].

In a regular 3D printing processes with standard 3D printers, the fabrication file is first generated by converting the model into an STL file format (an abbreviation of stereolithography). This process describes a given geometry in a raw, unstructured triangulated surface by the unit normal and vertices of the triangles using a three-dimensional Cartesian coordinate system. A 3D printing generator, called slicer, takes a given STL file and generates a G-Code file (i.e., the instruction language used by 3D printers) to produce the 3D modeled object.

Instead, digital 3D printing consists first, of the digitization process that structures a model into sequenced indexes with information, and second, the definition of the commands definition to materialize each index. The fabrication toolpath will process and actuate commands linearly for the z, x, and y indexes of each coordinate, in that order.

Using digital 3D printing we fabricated the fibrous arrangements that approximate a discretized membrane within an orthonormal hexahedral reference grid. The fabrication instrument form factor limited the geometric freedom of the fibrous arrangements. We used a custom a nozzle to test the 3D printing of the fibrous arrangements. Compared to a standard nozzle design, the elongated custom nozzle maximized the design freedom (Figure 15). The lattices were generated using two inputs: the solid membrane to be converted onto a hierarchical structure and the point cloud to extract information from. Different geometrical arrangements yield very different mechanical behavior effectively emulating the characteristics of foams and dense solids; density variations in cellular solids gradually modify elastic moduli to display increased stiffness or sponginess to control energy absorption as desired [28].

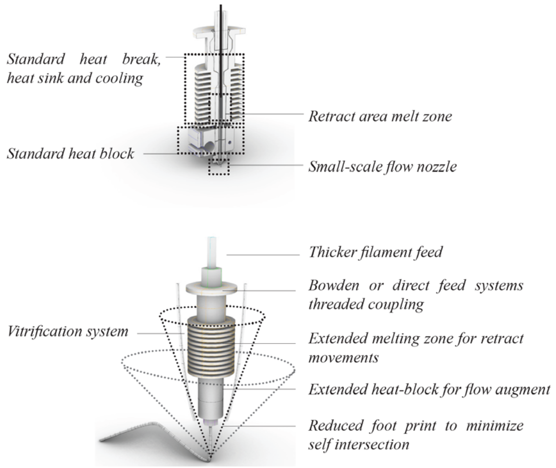

Figure 15: *3D print instrument general components and adjustment, nozzle characteristics*

Translating polylines first to sequential target points and then to fabrication codes, we extracted the coordinates that create polygonal arrangements and sorted these into types referring to their requirements of deposition. This helped us determine the correct speed, flow, and cooling deposition commands. We exported the information as G-Code and Rapid code using a python script that applied the conditions of 3D to the spatial lattice generalization [50]. Four kinds of points, each with a unique set of electromechanics commands, were identified to describe any given tessellation to be 3D printed with this method (Figure 16).

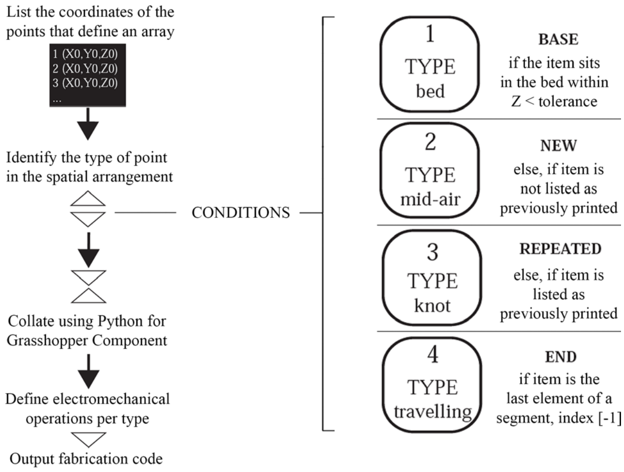

Figure 16: *Categorization of points in polygonal lattice arrangements for spatial 3D printing*

The foundation of the lattice required increased flow and no cooling system to ensure a strong contact with the base; points in mid-air that had not been created priorly required cooling systems to vitrify the segment in space; repeated points that had been created before highlight the creation of a knot and require slow approximation motions, higher flow and no cooling to ensure bonding; and lastly points at the end of a segment require to remove the nozzle out of the lattice to avoid collisions with priorly deposited material, as illustrated by Figure 17.

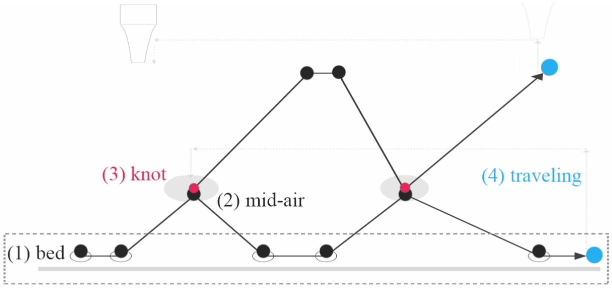

Figure 17: *Side view diagram displaying types of polylineal lattice for spatial 3D printing*

3.3.2. Triangulated spatial filling arrays

The polyhedral arrays are configured to approximate the mechanical behavior found in the theoretical biology model and stress trajectory theory of bone formation. According to J. Chilton [34] "the stability of rigid jointed space frames depends on the bending resistance of the joints for its structural integrity. However, space truss structures depend on their geometrical configuration to ensure stability.

To form a stable truss structure composed of nodes interconnected by axially loaded bars only, a fully triangulated structure must be formed." Maxwell's equation of equilibrium in a three-dimensional space structure determines tetrahedra as the minimal unit of a lattice. Additionally, if a structure is not fully triangulated, from the equation follows that stability can be achieved by additional external struts.

Tetrahedral lattices and other spatial tessellations in certain types of filling arrangements can follow a square pyramidal segmentation (Figure 18).

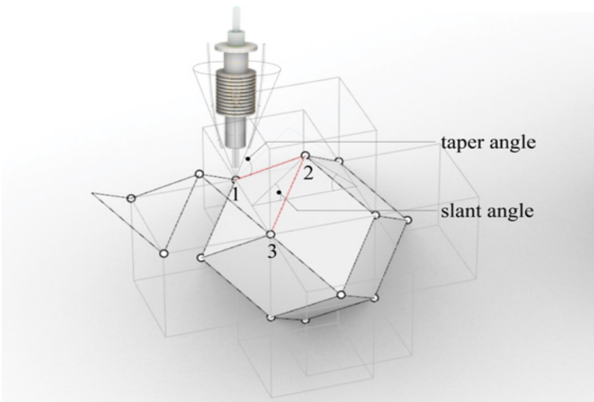

Figure 18: *Polygonal lattice printability based on the square pyramid rule*

This characteristic determined that we can always print a segment of a face of the polyhedral array with three target points that create a triangle, base in the bottom, apex on the top, to assemble a face of the pyramid segmentation.

Tetrahedral octahedral 3D infills are among the most extensively studied spatial arrangements due to their mechanical properties. Studies are more commonly found as small-scale lattice structures, built at the meso scale with layerwise systems, and out of planar aggregated planar segments with snap fit designs achieving architectured material properties [51].

In this research we studied orienting the direction of the fiber and deposition along each strut member to test the generalization method proposed and extend the technique to large-scale assemblies in continuous structural surfaces configured as hierarchical cellular membranes using regular polyhedra.

The spatial arrangements we studied for digital 3D printing include tetrahedra, octahedra, rhombic dodecahedra, truncated cuboctahedra, and combined polyhedral arrays.

The lattices were created starting from one end of the lattice defined as base and incrementally growing by attaching struts. Preferably the first line to deposit is a horizontal foundation to create greater grip for the subsequent targets that create a triangle in space. The omni symmetrical qualities of and uniform dispersion of points certain regular tessellations allow for a geometrical arrangement of a single type of triangle to conform to all faces of the polyhedral array simply by a mirror transformation along row and column order, always shifting one unit in the next shifting floor order (Figure 19).

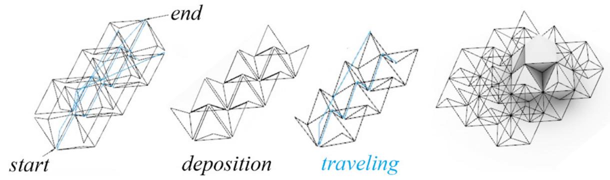

Figure 19: *Tetrahedral octahedral toolpath configuration describing deposition segments in black, and traveling segments in blue*

The main aspect to ensure printability when choosing and configuring the tessellation is the capacity to build it out of a concatenation of square pyramids with apex at the top.

The square pyramid principle allows the 3D print tool to navigate the space without intersecting with previously deposited material and to incrementally build a membrane by deposition. The triangular generalization of lateral faces of polyhedral arrays creates a continuously additive process of square base pyramid formations. Open polygons create very weak links and the vitrification time they require to form makes them difficult to print with enough accuracy.

The lattice is built by recreating triangular faces of the intercalating tetrahedra, which leave an open volume area to reconfigure the path of deposition depending on the position of the triangle in space. The triangulation needs any of two base points to begin the construction form as depicted in the diagrams. This feature allows us to create overhanging structures. To ensure freedom of movement of the nozzle, the square pyramids must be compliant with the tape angle of the deposition system. For example, for a 45-degrees nozzle taper, the square pyramid must simply be composed of congruent isosceles triangle angles. Square pyramid characteristics determine the manufacturability of a given regular polyhedral lattice.

Location in quadrants will determine its direction of print. The quadrant differentiation produced by the row, column, level order of print is bypassed by printing lateral faces of pyramids left to right, or top to bottom, depending on the quadrant in which the unit is located. Otherwise, the first initial point of a print segment of deposition of the lateral triangle would be freely located in space, without a structure to attach to.

In summary the main characteristics that determine the manufacturability of tessellations configured by the square pyramid segmentation method were as follows:

- Slant angle of the pyramid < taper angle
- The total height of a segment must be free from nozzle and other 3D print system obstructions
- For small taper angles, scaling the tessellation in height to clear slant angles will allow nozzle systems adequately 3D print in space

4. RESULTS AND DISCUSSION

In the workflow studied, the principles behind the morphological configuration of forms relate not only to stress and load as in topology optimization methods, but it also to a homology concept that continuously characterizes the geometric constraints, and the forces that shape them.

According to the concepts presented, any change in the system of forces that determines a stable form automatically and necessarily reshapes the fibrous arrangement. The methodology allowed us to determine the efficient material distribution of cellular solids and their manufacturability by additive methods. In the theoretical biology model density variations along principal stress lines optimally distribute tensions throughout the cellular solid. The hierarchical structures of nature remain homologic forms of the tension that shape them as geometrical expressions of force. Akin to the process of remodeling found in bone tissue, we used the stress trajectories that determine hierarchical designs to propose a homology model of line of force and geometrical arrangements. By rearranging polyhedral arrays and differentiating each segment we can configure functionally graded solids with improved mechanical qualities.

In terms of manufacturability, the hypothesis demands a technical configuration of force influenced membrane design that proves the capacity to compute variable strength modulus and elastic moduli solely with variations in density, orientation, and material, using the electromechanical means of deposition (numerical positioning, extrusion rate, extrusion angle). Tetrahedral-octahedral and rhombic-dodecahedron tessellations were found to provide very stable organizations for FDM spatial 3D printing.

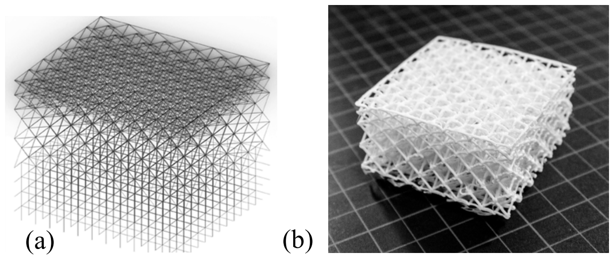

Figure 20: (a) *Polyhedral array model and (b) FDM 3D print test exhibiting geometric and density variations*

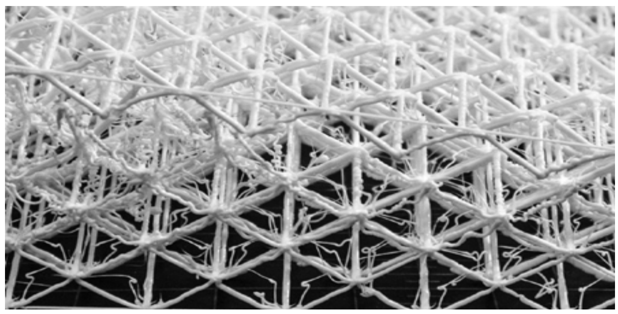

Figure 21: *Meso-scale FDM 3D print test of a rhombic-dodecahedron tessellation*

The triangulation of the cellular arrangement distributes applied forces efficiently across a gridshell. We tested printing different polyhedral arrays with commercial FDM printers and robot arms equipped with custom extrusion mechanisms (Figure 20 and 21). We controlled individual strut thickness by incrementally depositing more material repeating a segment or using unique electromechanic commands, changing speed-flow correlation.

In terms of design, this research processes the information in a force field, making it explicit as geometry instantiations. The design methods presented in this paper were tested in a large-scale study of a potential application of the resulting structures (Figure 22). Employing the methodology to an alternative design of "La Nube", we controlled every part of the material network so that the individual unit's value is related to the value of the whole. The hierarchical structure was digitally assembled discretizing the ellipsoid onto 16,800 indexes, ordered in a 3D printable sequence. Indexed voxels were populated with polygonal shapes according the stress quartile they belong to.

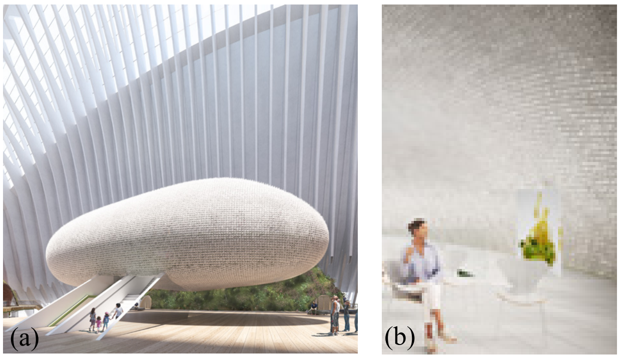

Figure 22: *Hierarchical structure concept applied to the computational design of a 3D printed shell, proposal for La Nube: (a) exterior view; (b) interior view*

The distribution and orientation of fibers were algorithmically chosen to vary volume fraction and architecture of the shell to populate low to high stress regions. 174 km of robotic toolpaths build the shape, of which traveling sequences are one-third and material deposition sequences are two-thirds. Using PETG as feedstock, the total weight of the structure would be approximately 9,620 kg.

Hierarchical membrane designs display a very high mechanical performance with a very low weight. Form-function hierarchical relationship in the examples presented are derived and prioritized from stress trajectories, resembling the adaptability and biomechanics of cancellous bone.

The methods presented require high computational power. Due to computational demand limitations, the design studies were restricted to a simplified interpretation of the model based on 4 stress regions.

Future work involves the creation of algorithms that can efficiently compute force and geometry homology models in a higher resolution potentially creating unique architecture, volume fraction and material solutions for each particle of the force field.

Although this research studies the application of spatial 3D printing and FDM, the methods described here are not limited to 3D printing with thermoplastic polymers. AM applications in construction are increasingly advancing towards larger scale applications and high strength materials. The computational design workflow automates the generation of cellular design following strict geometric rules. Current developments in AM setups cable robotics [52] and wire arc additive manufacturing [53] encourage us to visualize the large-scale application of hierarchical design methods with higher strength materials in the future.

5. CONCLUSION

The paper discusses the application of digital 3D printing to the production of complex geometries capable of assuming a continuous shell structural behavior, whose definition and computational design is based on a trabecular system. We propose a comprehensive morphological model of hierarchical fibrous structures. Functionally graded cellular structures found in nature display high mechanical capacity, stiffness, damage tolerance and ultra light weight. We make use of the additive manufacturing's robotic precision advantages to design and build complex cellular arrangements.

We approximated the biomechanical characteristics of cancellous bone remodeling following its morphogenetic logics in response to respond to external loading conditions to define hierarchical structures. The morphological principle was integrated into a comprehensive workflow of computational design and multicriteria optimization. First, an isostatic cloud that contains scalar and vector stress information was computed and categorized into quartiles. Second, discrete units of geometry are mechanically characterized and linearly aggregated into a continuous cellular membrane influenced by the stress field. Third, each lattice segment is categorized for 3D printing using unique electromechanical commands.

The methodology developed allowed us to study the characterization of a hierarchically organized cellular membrane, and to establish the general design characteristics that ensure the digital manufacturing of hierarchical free- form structure.

The studies center the definition and computational design of complex surfaces as well as the respective construction of these shell structures by means of robotically printed three-dimensional tessellations, capable of configuring mechanically continuous complex surfaces, based on a comprehensive morphological workflow, which yield extraordinary ultralightweight structures. The method proposed offers a force - geometry homology model.

We proposed a method to characterize trabeculae networks of fibers configured by a static system of internal forces. We studied the principal aspect that influences the strength of cellular structures in trabecular arrangements and geometry. We applied and verified the mechanical gradation capacity of the system. We modified the trabecular architecture reallocation mass strategically. We increased the density where stress is concentrated and along tension trajectories to improve the mechanical characteristics of the cellular structure. We presented a systematic organization method of very large amounts of data that merge within a form to create a cellular membrane. We defined hierarchical design as the process of structuring geometric arrangements into continuous structural surfaces as a function of force trajectories and stress values.

This research offers a general method for designing and 3D printing hierarchical structures. We tested and evaluated the serialized fabrication of different space filling polyhedral arrays to assess their potential to conform hierarchical structures. Tetrahedral, octahedral, and rhombic-dodecahedron tessellations were found to provide the most appropriate geometric organizations and mechanical qualities for a lightweight FDM technology.

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DATA AVAILABILITY

Some or all data, models or codes that support the finding of this study are available from the corresponding author upon reasonable request.

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