DESIGN OF A

FRICTIONLESS HYDRAULIC TRANSMISSION DYNAMOMETER

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It is well known that when two pieces slide on one another, friction is created, so that, as a matter of fact, a machine with sliding parts cannot be frictionless. Since, then, the main element of the dynamometer with which this thesis deals rotates in bearings, there must be friction; therefore, first of all, an explanation of the claim that the dynamometer is frictionless has to be made.

By the term "Frictionless", as we use it in this discussion, we mean to imply that friction does not interfere with the perfect operation of those parts of the mechanism which transmit and record the torque that the dynamometer measures. Since the dynamometer is designed to register the torque transmitted to it, it may be well considered frictionless if this torque is transmitted and recorded without the interference of friction. How this is effected will be understood from the following description of the machine, and a study of the drawings.

The Principle of the Dynamometer

Two flexibly connected, coaxial, rotative elements, (2) and (3), capable of undergoing a small angular displacement relative to each other, constitute the principal member of the dynamometer. The first of these elements we shall call the casing-element; the second, the floating spider-element.

To the free end of the casing-element is directly coupled the driving motor; to the free end of the floating spider-element is directly connected the machine to be tested. Thus from one of these elements to the other passes the torque which the driving motor exerts on the machine under test. The flexible element which unites the spider-element and the casing-element consists of the following parts: (a) three fluid-tight fluid-chambers, (13), completely filled with fluid and rigidly fastened to the casing; (b) three flexible, radially-placed metal diaphragms (14), which are bolted at their rims to the fluid-chambers (13) and at their centres to the three radial arms of spider (5); (c) two sets of radially-placed, metal torque strips (21) and (24). The fluid-chambers communicate by piping (51) with each other and also with the axially-placed, fluid-tight chamber (29), which forms a part of the transmission device, and which is firmly attached to casing (4) at its left-hand end. The right-hand end of this fluid-chamber (29) consists of a flexible metal bellows (26), fastened rigidly at its right-hand end to the three-armed spider (27). This spider, in turn, is rigidly connected by three rods (28) to transmission sleeve (7) and this sleeve is rotatively connected by ball-bearings to yoke (6) of the recording device. Two of these ball bearings are designed to take axial thrust and the third, radial load. Yoke (6) of the recording device is bolted firmly to the center of each of two flexible, metal diaphragms (16), and each of these diaphragms is bolted to a stationary, fluid-tight fluid-chamber (10). These two fluid-chambers, which are placed one on each side of the main axis of the dynamometer, are connected to each other and to a fine-bore mercury manometer (41), by flexible piping, (40). The flexibility of this piping allows tube (41) to be placed at any angle to the horizontal.

Thus it will be seen that when a torque is transmitted from the casing-element to the spider-element the diaphragms (14) are cupped or deflected and some of the fluid in piping (51) is driven into fluid-chamber (29). As a result, the metal bellows (26) of chamber (29) is expanded and spider (27) is moved to the right. Then, through rods (28), sleeve (7), ball bearings (34), and yoke (6), this small linear displacement, along with its corresponding load, is transmitted to the diaphragms of the stationary recording device. The pressure thus put upon these recording-device diaphragms forces the mercury to rise in tube (41) to a corresponding head; in other words, this pressure forces the mercury to rise in the tube (41) to a head which is exactly proportional to the torque transmitted from the motor to the machine under test.

From the nature of the connection between the casing-element and the spider-element it will, we think, be obvious that the small angular displacement between these two elements takes place unaffected by friction.

It will be obvious also, we think, that the corresponding axial, linear displacement of transmission-spider (27) is transmitted to yoke (6) of the recording device uninfluenced by friction. Therefore, as we have said, it follows that the head of the mercury in tube (41) is exactly proportional to the torque transmitted from the casing-element to the spider-element; that is to say, therefore, to the torque transmitted to the machine under test.

Description of the Dynamometer

A transmission dynamometer may be used to measure either the torsional output of, or the torsional input to, a machine. If the dynamometer described in this thesis is to be used to measure the torsional output of a machine, then the spider-element of the dynamometer would be connected to the machine to be tested, and the casing-element of the dynamometer to the machine which utilizes the energy supplied by the machine to be tested. If the dynamometer is to be used to measure the torsional power-input to a machine - for example, the power necessary to drive a lathe - then the spider-element of the dynamometer would be connected to the machine to be tested and the casing-element to the driving mechanism. The essential point here is that the spider-element of the dynamometer should be connected to the machine to be tested. In this description we shall deal only with the case where the dynamometer is to be used for measuring the torsional input to a machine. The theory of operation of the dynamometer is not, however, altered when it is used to measure the torsional output of a machine.

It will be noticed that this dynamometer is of the "Direct-Drive" type, i.e. the machine whose torsional input is to be measured, the dynamometer, and the driving mechanism are all three coaxial.

The machine to be tested is connected to the dynamometer through the frictionless, flexible coupling (23), which is keyed to the shaft (22). This shaft (22) passes through casing (3) without touching it, being finally keyed to spider (5).

Spider (5) has three arms, equally spaced around the main axis and hollowed out to admit three radially placed fluid-chambers (13) and six flexible metal diaphragms (14). To ensure stiffness the spider is ribbed. The fluid-chambers (13) are rigidly fastened to the casing parts (2) and (3), by tightly fitting trunnions, integral with the fluid chambers. Each diaphragm (14) is fastened to the fluid-chambers (13) by ten stud-bolts, and a fluid-tight joint is effected by compressing a thin copper gasket (held in the recess in the fluid-chambers by the lip on the diaphragm) to a greater pressure than would ever be exerted by the fluid on the gasket.

To the centre of each metal diaphragm (14) is riveted an adjusting screw (15), on which are threaded two lock-nuts. This screw (15) passes through a reamed hole in spider (5). Adjustment of the diaphragm is made by loosening one lock-nut and tightening the other until the required deflection of the diaphragm is obtained. Adjusting screw (15), of course, is not threaded to the spider. The two parts of the casing (2) and (3), are rigidly fastened together by stud-bolts, and a reversed lip is provided at this junction to counteract, to some extent, the centrifugal force which at high speeds tends to distort part (2).

Rotative part (2) is rigidly fastened to coaxial rotative part (4), to which is keyed flexible coupling (37) connecting the dynamometer to the driving mechanism.

The rotative casing-element is supported by ball bearings (36) and (18), one of which, (36), is designed to carry radial loads only, while the other, (18), is designed to carry both radial and axial loads. The ball-bearing (18) is housed in support (8) and is protected from dirt by seals (20). Any wear which occurs in this ball bearing (18) is taken up by adjusting-ring (19), which, after being adjusted, is held firmly in place by the headless set screw (21). Support (8) also has a slight allowance for adjustment. Ball-bearing (36) is housed in support (9) and it also is protected from dirt by seals. The inner race of this ball bearing is tightly clamped to rotative part (4) while the outer race can move slightly, axially, in the support (9). Connecting the casing-element to the spider-element are two sets of radially-placed metal torque-strips, (21) and (24), three in each set. The torque-strips in each set are equally spaced around the axis of rotation. Thus the casing-element and the spider-element are capable of a very small angular displacement relative to each other. The dynamometer is so designed that the maximum angular displacement of the spider-element and the casing-element relative to each other is less than .00003 radians, or less than 6 seconds.

The transmission-device, consisting of fluid-chambers (25) and (29), spider (27), rods (28), sleeve (7) and yoke (6) is coaxial with the centre line of the dynamometer. The transmission fluid-chambers (29) and (25) are connected by piping (51) to the load side and counterbalancing side, respectively, of the fluid-chambers (13), so that the fluid-chambers (29), (25) and (13), and the pipes connecting them form a hermetically sealed system. The two metal bellows (26) connect the transmission fluid-chambers (29) and (25) to the transmission-spider (27), which is thus capable of axial motion relative to the fluid-chambers. The transmission-spider (27) is rigidly fastened to the transmission sleeve (7) by three equally-spaced transmission rods (28); and sleeve (7) is rotatively connected by ball-bearings (34) and (33) to yoke (6).

Ball-bearing (33), designed to carry radial load, is housed in yoke (6) and supports the rotative transmission-sleeve (7). The two thrust bearings (34), between the sleeve and the yoke, afford the means for transmitting axial motion from the rotative sleeve to the non-rotative yoke, or vice versa. These ball-bearings, it will be seen, are protected from dirt by seals (35).

Screws (17) are riveted to the centres of the recording diaphragms (16), and these latter are fastened to the supports (10) and (11) in the same manner as the diaphragms (14) are fastened to the fluid-chambers (13), already explained. A flexible pipe (40) connects support (10) with a mercury reservoir (50) which can be rotated on its axis in bearings on support (12), so as to make any required angle with the horizontal. The base of the mercury reservoir (50) is a flexible metal diaphragm which has, integral with it, a needle (47). When this metal diaphragm is in its neutral position, the needle will be just clear of the opening in the reservoir cap (44), and when the diaphragm is deflected upwards by adjusting screw (46) the needle will tend to close the opening in the reservoir cap, thus cutting off the connection between the mercury tube and the mercury reservoir. By this means it is possible to dampen out the vibrations of the mercury column. To protect the glass tube (41) against breakage, a steel guard (42), slotted to permit the reading of the manometer, is slipped over the tube, and is screwed to the reservoir cap (44). The glass tube extends into the hole in the reservoir cap, and leakage of mercury is prevented by packing (43), held in position by guard (42). Axial adjustment of transmission fluid-chamber (29) is effected by the adjusting bolt (49), which is locked in place by locknut (38).

The three transmission rods (28) are small in diameter, and at high rotative speeds would tend to buckle, so a disc, (30), is slipped over them to prevent this buckling to some extent.

Safety guards (39) and (31) are added to obviate danger to the operator from the stud-bolts when the main element of the dynamometer is rotating.

Operation of the Dynamometer

The machine to be tested is directly connected to the spider-element of the dynamometer by coupling (23), and the driving motor is directly coupled to the casing-element of the dynamometer by coupling (37). When the driving motor applies torque to the casing-element the latter revolves and is displaced through a small angle relative to the spider-element.

As a consequence the diaphragms (14) are deflected and some of the fluid in piping (51) is driven into fluid-chamber (29). Whereupon metal bellows (26) of fluid-chamber (29) is expanded and transmission spider (27) forced to the right.

Now, as we have already explained, this motion of the spider (27) is transmitted to the diaphragms of the stationary recording device by means of the transmission device. The pressure thus put upon these diaphragms forces the mercury to rise in tube (41) to a corresponding head.

The fluid-chamber (13), it should be noticed, is not located on the transverse axis of the dynamometer, so that when the main element of the dynamometer rotates fluid pressure due to centrifugal force is created in the fluid of the fluid-chamber (13). In a minor degree fluid pressure due to centrifugal force is also created in the fluid of chamber (29), so that as the angular velocity of the mechanism varies the transmission-spider (27) would have varying fluid pressures exerted on it, even though the torque transmitted were to remain constant. To counteract this centrifugal effect of the fluid, counterbalancing fluid-chambers are added to the load fluid-chambers (13). These two opposed sets of fluid-chambers are, it will be seen, at the same radial distance from the main axis of the dynamometer. Another counterbalancing fluid-chamber (25) is attached to spider (27) to neutralize the centrifugal effect of the fluid in the transmission fluid-chamber (29). This counterbalancing fluid system, consisting of fluid-chambers (13), piping (51) and fluid-chamber (25) is completely filled with fluid and is then hermetically sealed.

It will be obvious that the increased fluid pressure in fluid-chamber (29), due to rotation of the dynamometer, will be completely counteracted by the increased fluid pressure in fluid-chamber (25) due to this same rotation, so that the net force on the transmission-spider (27) pulling it to the right will be only that due to torque transmitted by the spider-element.

When the torque transmitted by the spider-element is reduced, or removed, the head of mercury in the manometer forces the diaphragms (16) back towards their neutral positions. This motion of the diaphragms is transmitted through the transmission device to spider (27) which will move to the left, thus forcing fluid out of fluid-chamber (29) into piping (51), and thence into fluid-chamber (13). The increased quantity of fluid in chambers (13) forces the diaphragms (14) towards their neutral positions. This movement of the diaphragms (14) will force the excess of fluid in the counterbalancing fluid-chambers(13) into the fluid-chamber (25), filling the vacuum created there by the motion of the spider (27).

The Problem of Filling the Fluid Systems of the Machine.

Since the diaphragms are deflected only a fraction of one thousandth of an inch maximum, a very small amount of air in the fluid system would destroy the sensitiveness of the dynamometer in recording torque. Therefore it is essential that all air be expelled from the fluid systems. How this is done we shall now explain.

The spider (5), fluid-chambers (13), diaphragms (14), studs (15), piping (51), fluid-chambers (25) and (29) and transmission-spider (27) are assembled as one element. The axis of this element is placed in a vertical position, with the transmission fluid-chamber (29) at the bottom; the air vents in transmission-spider (27) and fluid-chamber (25) are opened; the two vents in each fluid-chamber (13) are also opened, and oil is poured in thru one of the openings until only oil issues from the air vent in fluid-chamber (29). This vent is then closed, and oil is still poured in thru the same openings, the air now escaping through the companion hole in the fluid-chamber (13). When, finally, only oil emerges from the holes in chamber (13) these openings are closed. The process is now repeated for the counterbalancing oil system. The whole element is then immersed in a bath of oil, the air vents in all fluid-chambers are opened, and the bath of oil is heated and boiled until all of the air is driven out of both systems. And then, while in this position, the whole apparatus is allowed to cool, so that the vacuum created by the contraction of the oil in the system will be immediately filled with oil from the bath. All holes are then completely and carefully sealed to avoid any possibility of air again entering the system.

Adjustment of the Dynamometer

If no torque is being exerted on the spider-element of the dynamometer by the machine to be tested the diaphragms (14) will be unflexed and the transmission-spider (27) will be in its neutral position: the recording diaphragms (16) will also be in their neutral positions, so that the mercury manometer will read zero. One arm of the spider (5) would be placed in a vertical position and the four lock nuts on the adjusting screws (15) of this arm would be loosened. If the mercury column in the mercury manometer (41) is above the zero mark, (which is that position occupied by the mercury meniscus when the transmission-device is uncoupled from the recording diaphragms), the load diaphragm (14) would be pulled away from the fluid chamber-back by screwing the external lock nut further on to the load adjusting screw (15) until the meniscus of the mercury column reaches the zero mark. When this has been done all four nuts on this vertical arm of the spider would be tightened. The adjustment of the other two arms would be accomplished in a similar manner.

The Effect of Heating of the Oil or Mercury

The heating of the oil during the test (or due to heat from external sources) would not influence the readings obtained from the mercury manometer, as the effect would be merely to increase the oil pressure on each side of the transmission-spider (27), and therefore neither motion nor pressure would be transmitted to the recording diaphragms through the effect of heating of the oil.

The heating of the mercury (as, for example, that which would occur on a warm day), would, it is true, tend to cause the mercury column to rise; but this increased height would create an increased pressure on the recording diaphragms (16), forcing their centres to the left, or away from supports (10) and (11). Through the medium of the transmission yoke, thrust bearings, sleeve and rods, this motion of the centre of the recording diaphragms would be transferred to the transmission-spider (27). The motion of spider (27) to the left would force some of the oil out of piping (51) into the load fluid-chamber (13), flexing the diaphragm (14) to make room for it. Since the distance between the centres of the diaphragms (14) would remain a constant, deflection of the load diaphragm would cause an equal deflection of the counterbalancing diaphragm and some of the oil in piping (51) leading from the counterbalancing fluid-chamber would be discharged into fluid-chamber (25), filling the partial vacuum created there by the motion of spider (27). Thus it will be seen that immediately the mercury column in the manometer begins to rise the diaphragms (14) will deflect, bringing the meniscus of the mercury back to its zero or neutral position.

Since a deflection of diaphragms (14) of .0002" is sufficient to cause the mercury in the manometer to rise 30", (as will be shown later) a slight increase in the volume of the mercury due to heating would deflect the diaphragms a very small amount, and would cause only a negligible amount of stress in them.

The Effect of Capillary Action on the Readings of the Manometer.

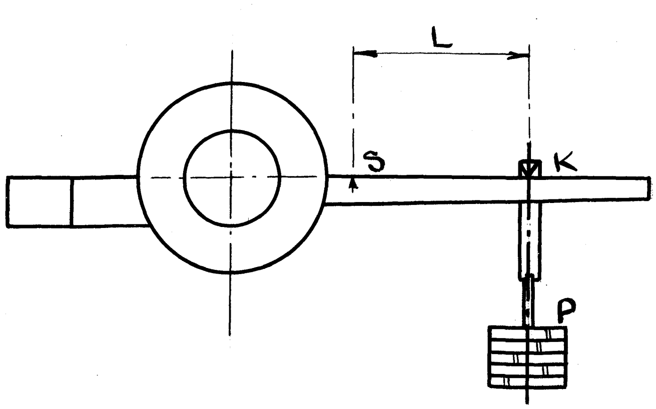
The bore of the glass tube of the mercury manometer is very small (.0147" diameter) so that the mercury would rise in the tube due to capillary action between the two. By having the scale beside the manometer adjustable it could be easily adjusted to allow for this capillary attraction.

Calibration of the Dynamometer

After the dynamometer has been assembled it is necessary that it be calibrated for a range of torque from 0 to 2,625 lbs. ft. A graph would be plotted showing the true value of the torque transmitted by the spider-element of the dynamometer against the actual reading of the length of the mercury column in the manometer. This calibration would have to be very carefully performed.

It has been shown before that rotation of the main element of the dynamometer does not affect the height of the mercury column in the manometer, so that it is unnecessary to calibrate the dynamometer while it is rotating - on the contrary, it may be calibrated while the main element of the dynamometer is stationary.

One method of calibrating the dynamometer would be to apply to the spider-element a known torque, in the form of a couple, noting the height of the mercury column; the process would be repeated for a great many values between the minimum and maximum torques to be transmitted by the dynamometer.



Another method of calibrating the dynamometer would be to attach a lever, balanced about the coupling centre, to the coupling or shaft of the spider-element and apply a known weight P to this lever. The distance between the knife edge K and the geometrical center of the shaft, multiplied by the weight P, might not give with sufficient accuracy the desired torque transmitted to the spider-element, since the geometrical centre of the shaft might not be the centre of rotation. This difficulty could be got over by applying weight P at a known point S on the lever arm, and noting the height, h_s, of the mercury column in the manometer. Let the distance of S from the centre of rotation be denoted by a. Therefore when P is at S, the torque exerted on the shaft is Pa. The weight P would then be moved to the right to point K, and the height of the mercury column, h_k, again noted. Let the distance L, between S and K, be accurately determined. When P is at K, the torque on the shaft is P(L+a). Therefore the difference in torque on the shaft when P is at S and when P is at K is $P(L+a-a) = PL$, and the height of the mercury column corresponding to this difference is $(h_k - h_s)$.

Now if $h_k = 2h_s$, then, obviously, $L = a$,
for $h_s = bPa$, and $h_k = bP(L+a)$, where b is constant;
and if $h_k = 2h_s$, $bP(L+a) = 2bPa$, or $L = a$.
Thus the centre of rotation can be found and, therefore, also, the torque on the shaft.

Various other points of application of P on the lever arm are taken, and the corresponding height of the mercury column noted. The calibration is completed when a graph has been drawn showing heights of mercury columns against torque on the shaft.

<u>The Influence of Variable Speed on Calibration</u>

If a constant horsepower is being transmitted by the dynamometer, a reduction in speed of the driving motor to one half of its original value would double the torque on the spider element, and thus the force on the recording diaphragms would also be doubled, causing the mercury column to rise to twice its original height. Since, however, the increased fluid pressure in fluid-chamber (29), due to rotation of the dynamometer will be completely counteracted by the increased fluid pressure in fluid-chamber (25), due to this same rotation, the net force on the transmission-spider (27) pulling it to the right will be only that due to torque transmitted by the spider-element, no matter what the speed may be.

General

The intricacy of the dynamometer described in this thesis and the difficulty of manufacturing it, prohibit its use for ordinary purposes where only an approximate value of the torque transmitted by the dynamometer is sufficient. Where very accurate work is necessary, however, as in testing the efficiency of gearing, this dynamometer would give any degree of accuracy required, and the reliability of the results obtained might warrant the additional cost of this machine over that of a simpler one.

It will be noticed that this dynamometer is of the "Direct-Coupled Type" - it would, however, be an easy matter to convert it into one of the "Geared" Types.

If this dynamometer is to be used to measure the Horse Power transmitted to a machine, the instrument for measuring the angular velocity of the rotative element of the dynamometer must, for accurate work, be equally as accurate as the dynamometer itself in recording torque. It would obviously be foolish to determine the torque transmitted by the dynamometer to an accuracy of .1 of 1% of the maximum torque, for example, if the corresponding speed can be determined accurately to only 1%.

CALCULATIONS

The Horse Power (H.P.) which a shaft transmits is obtained from the following formula:—

$H.P. = \frac{2\pi NT}{33,000}$, where N = Revolutions of the shaft per minute, and T = torque exerted on the shaft in pounds feet (lbs.ft.).

Let us assume for our illustrative design that the dynamometer has to transmit 100 H.P. at speeds varying from 200 to 1200 Revolutions per minute.

For constant H.P. of 100,

$$T = \frac{33,000 \times 100}{2\pi \times N} = \frac{525,000}{N} \text{ lbs. feet}$$

(a) Let $N_1 = 200$ R.P.M. $\therefore$ $T_1 = 2625$ lbs. ft.

(b) " $N_2 = 1200$ R.P.M. $\therefore$ $T_2 = 438$ " " (approx.)

Diameter of Shaft (22) Let us consider case (a).

The torque transmitted by the shaft is a maximum at this minimum operating speed, and the shaft, feathers, etc., must be designed for this value.

Let d = diameter of the shaft (22)

$$\therefore \frac{\pi}{16} d^3 f_s = 2,625 \times 12 \text{ lbs. inches, where}$$

f_s = the allowable shearing stress of the shaft in lbs./sq.in.

$$\therefore \quad d^3 f_s = 160,500 \text{ lbs. inches.}$$

Let the maximum stress in the shaft be 6,000 lbs. per square inch in shear

$$\therefore d^3 = \frac{160,500}{6,000} = 26.8 \text{ cubic inches.}$$

$\therefore$ d = 3 inches. $\therefore$ Make Shaft (22) 3" in diameter

The Feather in the Shaft (22)

Let the feather be of standard width and height, i.e. $\frac{3}{4}$" square, and let it be subjected to a maximum stress of 5,000 lbs. per square inch in shear.

$\therefore$ 2,625 x 12 = $\frac{3}{4}$ x L x 5,000 x $1\frac{1}{2}$, where

L = length of the feather in inches.

$\therefore$ L = 6" long.

The Force exerted on the Diaphragms (14) by the Spider (5) when the maximum torque is being transmitted by the dynamometer.

The Spider has three arms, and if each takes one third of the load, then the maximum torque exerted by each arm is $\frac{2,625}{3}$ or 875 lbs. feet. The distance of the centre of the fluid-chambers from the axis of the dynamometer is 7".

$\therefore$ The force exerted by adjusting screw (15) on diaphragm (14) when the maximum torque is applied to the spider-element is $\frac{875 \times 12}{7}$ = 1,500 lbs. The size of screw necessary to carry this load successfully would be about $\frac{3}{4}$" diameter.

Deflection of Diaphragms. Let the diaphragms (14) be 4" in diameter. Let these diaphragms (14) be deflected .0002" when the maximum torque is applied to the shaft (22). Thus, approximately, the volume of oil displaced when the diaphragms (14) are deflected .0002" is $\frac{1}{3}$ x .0002 x $\frac{\pi}{4}$ x 16 = .00084 cubic inches.

The diameter of the transmission fluid-chamber (29) is 2".

$\therefore$ The axial motion of the transmission device relative to the casing at this maximum torque $= \frac{.00084}{\frac{\pi}{4}x4} = \frac{.00027"}{}$.

Bore of the mercury manometer tube.

Let the diameter of the recording diaphragms be 6".

Let the bore of the mercury tube be d_1.

$\therefore 2 \times \frac{1}{3} \times .00027 \times \frac{\pi}{4} \times 36 = \frac{\pi}{4} d_1^2 \times 30$, assuming that the mercury column has a maximum length of 30". $\therefore d_1 =$ .0147" dia. $\therefore$ Bore of glass tube = .0147" dia.

Fluid Pressures Thru the Mechanism.

The force on each diaphragm (14) was found to be 1,500 lbs. due to torque alone, when the machine is revolving at 200 R.P.M. and transmitting 100 H.P. It will be shown later that at this speed the fluid pressure in the transmission fluid-chamber (29) due to rotation is only about 1 lb. per square inch, and can be neglected for the following calculation.

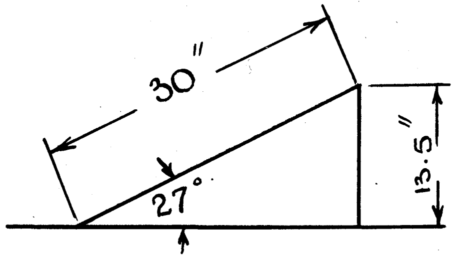
The unit fluid pressure in the fluid-chamber (13) is $\frac{1500}{\frac{\pi}{4}x16} = 120$ lbs. per square inch, due to torque alone, at 200 R.P.M. and 100 H.P.

This unit pressure is transmitted to the transmission-spider (27). $\therefore$ The horizontal or axial force exerted on the transmission device is $120 \times \frac{\pi}{4} \times 4 = 377$ lbs.

$\therefore$ Each mercury diaphragm has a force of 188.5 lbs. acting on it. $\therefore$ The pressure exerted inside the mercury diaphragm-chamber (10) is $\frac{188.5}{\frac{\pi}{4}x36} = 6.67$ lbs. per sq. inch.

Now if a column of mercury is to exert a pressure of 6.67 lbs. per square inch it is necessary for the column to be 13.5 inches in vertical height.

$\therefore$ The mercury column must be inclined at an angle to the horizontal whose sine is $\frac{13.5}{30}$ or .45, or an angle of 27° to the horizontal.



Obviously if the torque on the spider-element is decreased the pressure on the mercury diaphragms is lowered and the mercury column would be shorter. Now, if a column of mercury 13.5 inches in length is all that is required, it could be vertical for the conditions given, but the bore of the glass tube might be increased, if it is desired to keep the deflection of the diaphragms (14) the same.

In this case, $2 \times \frac{1}{3} \times .00027 \times \frac{\pi}{4} \times 36 = \frac{\pi}{4} \times d_2^2 \times 13.5$

where d_2 is the new bore of the glass tube of the manometer

$\therefore d_2 = .022''$.

Thus it is evident that the angle of the mercury column to the horizontal can be varied to suit the conditions imposed. If great accuracy be required the angle can be reduced and the size of the mercury diaphragms increased until satisfactory results are obtained.

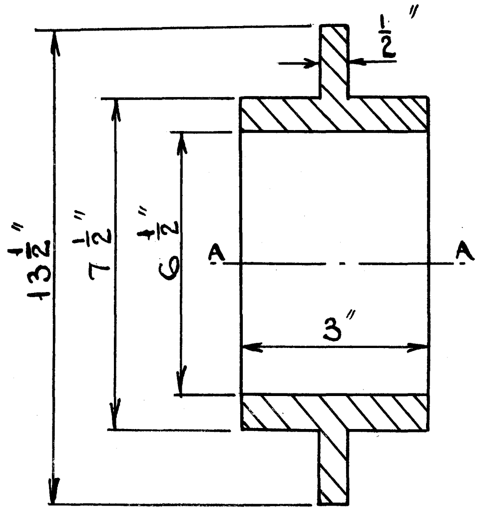
Strength of Spider (5)

The maximum torque transmitted by the dynamometer is 2,625 lbs. feet, exerting a force of 1,500 lbs. on each arm of the spider at the centre of the fluid-chambers(13).

The tension and compression sides of the spider arm are rigidly connected; therefore they act as a single beam in resisting this force.

Bending moment at worst position on the arm =

1,500 x $3\frac{1}{2}$ = 5,250 lbs. inches.



The two ribs of $\frac{1}{4}''$ thickness on each side may be combined as one of $\frac{1}{2}''$ thickness on each side for calculating the moment of inertia of the arm at this weakest point about AA, the neutral axis of the section.

Moment of Inertia (M.I.) about axis AA

$$= \frac{3}{12}\left[7\frac{1}{2}^3 - 6\frac{1}{2}^3\right] + \frac{1}{24}\left[13\frac{1}{2}^3 - 7\frac{1}{2}^3\right]$$

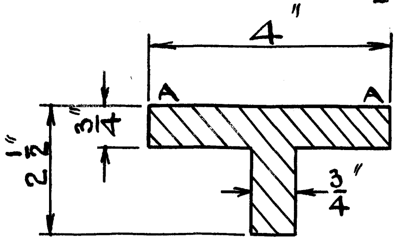
$$= \frac{1}{4}\left[422 - 274\right] + \frac{1}{24}\left[2460 - 422\right]$$

$$= 122 \text{ (inches)}^4$$

Greatest tensile stress at this section = $p = \frac{5,250}{122}$ x 6.75 = 291 lbs. per sq. in. Since it is highly desirable to prevent much distortion of the spider while in rotation or under torque, this value for stress would be suitable. Ordinarily, of course, it would be too low, even for cast iron.

Strength of Arms on Transmission Yoke (6)

Each arm carries approximately a force of 190 lbs. The Bending Moment at the worst part of the arm is 190 x $6\frac{1}{2}$ = 1,230 lbs. inches.



$$\text{M.I.}_{AA} = \frac{1}{3} \times 3\frac{1}{4} \times \left(\frac{3}{4}\right)^3 + \frac{1}{3} \times \frac{3}{4} \times \left(2\frac{1}{2}\right)^3$$

$$= \frac{13 \times 9}{256} + \frac{125}{32} = .457 + 3.91 \doteq 4.37 \text{(in.)}^4$$

Area of section = $\left(3\frac{1}{4} \times \frac{3}{4}\right) + \left(\frac{3}{4} \times 2\frac{1}{2}\right)$ = 2.44 + 1.88 = 4.32 sq. inches.

Area of each part of the section x its distance from AA

$= (2.44 \times \frac{3}{8}) + (1.88 \times 1\frac{1}{4}) = .915 + 2.35 = 3.27(\text{inches})^3$.

$\therefore$ Distance of centre of gravity of section from AA $= \frac{3.27}{4.32} =$ .757 inches.

$\therefore I = 4.37 - 3.27 \times .757 = 4.37 - 2.47 = 1.9(\text{inches})^4$

where I is the moment of inertia of the section about its neutral axis.

$\therefore$ Maximum tensile stress $= \frac{1,230}{1.9} \times .757 = 490$ lbs.per sq.in.

Strength of Housing for Transmission Device.

Maximum Torque = 2,625 x 12 lbs. inches = 31,500 lbs.inches.

Let D = external diameter and d the internal diameter of the cast iron cylinder, and f the maximum torsional stress in lbs. per square inch. $\therefore \frac{\pi}{16}(\frac{D^4 - d^4}{D})f = 31,500$. $\therefore (\frac{D^4 - d^4}{D})f = 160,300$. Now $d = 8''$ $\therefore d^4 = 4,100$. Let $D = 9\frac{1}{2}''$ $\therefore D^4 = 8,150$

$\therefore \frac{4,050}{9.5}f = 160,300$ $\therefore f = 376$ lbs. per square inch.

$\therefore$ This part is quite safe.

Strength of Supported Shaft (4)

Torque = 31,500 lbs. inches. Use the same symbols as in the last calculation. Let $d = 1\frac{1}{2}''$ $\therefore d^4 = 5.06$ Let $D = 3\frac{3}{4}''$

$\therefore D^4 = 197.6$. $\therefore \frac{192.5}{3.75} \times f = 160,300$. $\therefore f = 3,120$ lbs. per sq. inch.

Under this condition this shaft must be made of steel, though not necessarily of very high grade.

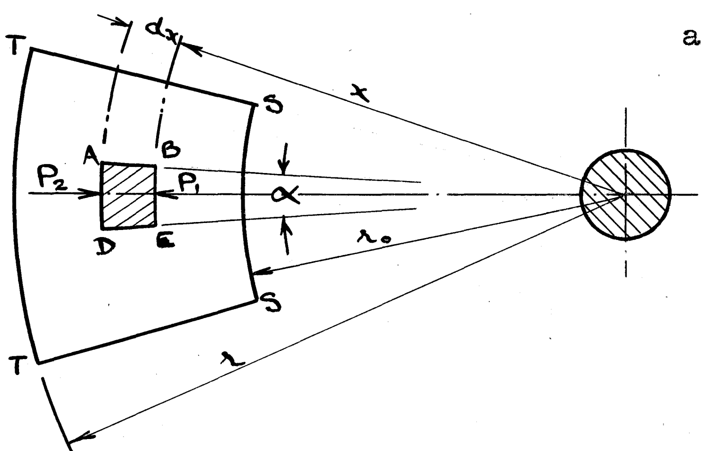
Strength of the Three Transmission Rods (28)

The force exerted in tension on the three rods is 377 lbs. maximum. $\therefore$ 126 lb. force acts on each rod. Assume maximum tensile stress of 3,000 lbs. per sq. inch in each rod.

$\therefore \frac{\pi}{4} d^2 \times 3000 = 126. \quad \therefore d = \frac{1}{4}'' =$ dia. of rods.

Since a rod of this diameter would tend to buckle under moderate rotative speeds, the diameter of the rods was increased to $\frac{1}{2}$ inch.

Fluid Pressures due to rotation. Consider a vessel containing fluid to rotate about the shaft as centre.



The volume of the elementary section BADE is $V = x\alpha\, dx$ if the axial length is unity. Since the force is purely a radial one the length need not be considered as a variable.

$\therefore$ weight of V is $G = k x \alpha\, dx$, where $k =$ the density of the mass.

Let pressure p act on inner surface BE per square unit.

" " (p+dp) act on outer surface AD per square unit.

The centrifugal face $dC = \frac{G}{g}\omega^2 x$, where ω is the angular velocity of the elementary section.

$\therefore dC = \frac{k}{g} x\alpha\, \omega^2 x\, dx$

$\therefore (p+dp)\, x\alpha = p x\alpha + \frac{k}{g} x\alpha\, \omega^2 x\, dx$

$\therefore p + dp = p + \frac{k}{g}\omega^2 x\, dx.$

$\therefore dp = \frac{k}{g}\omega^2 x\, dx.$

Now integrate between the limits of the fluid radii, r and r_0 , and the pressure per square unit at the outer radius r is

$$P = \int_{r_o}^{r} dp = \frac{k}{g}\,\omega^2\left(\frac{r^2 - ro^2}{2}\right) = \frac{k}{2g}(r^2 - ro^2)\frac{4\pi^2 n^2}{3,600}$$

where n = revolutions per minute of the vessel.

$\therefore P = .00017\,k n^2 (r^2 - ro^2).$

If oil, with a specific gravity of .92, is used,

$P = .00017 \times .92 \times 62.4\, n^2 (r^2 - r_o^2)$

$\therefore P = .00978\, n^2 (r^2 - ro^2)$

Consider fluid-chamber (13)

For this particular design of dynamometer, r = .75 ft. and r_o = .417 ft.

$\therefore P = .00379\, n^2$

<u>at 200 R.P.M.</u>

P = .00379 x 40,000 = 151.5 lbs. per square foot.

$\therefore$ P = 1.05 lbs. per square inch.

<u>at 400 R.P.M.</u>

$P = .00379 \times \frac{160,000}{144} = 4.2$ lbs. per square inch

<u>at 200 R.P.M.</u>

The pressure due to torque	=	120	lbs. per sq.in.
" " " " motion	=	1	" " " "
$\therefore$ Total pressure	=	121	" " " "

<u>at 400 R.P.M.</u>

The pressure due to torque	$= \frac{120}{2}$ =	60	lbs. per sq.in.
" " " " motion	=	4.2	" " " "
$\therefore$ Total pressure	=	64.2	" " " "

For a fuller discussion of forces in centrifugal machines see volume 14 of the A.S.M.E. Transactions. The article by Gustav Herrmann was the basis of the theory of centrifugal forces worked out in this thesis.

From values obtained in this manner the following table has been made.

R.P.M.	Pressure Due To Torque	Pressure Due To Motion	Total Pressure
200	120 Lbs./Sq.In.	1 Lb./Sq.In.	121 Lbs./Sq.In.
400	60 " " "	4.2" " "	64.2 " " "
600	40 " " "	9.5" " "	49.5 " " "
800	30 " " "	16.8" " "	46.8 " " "
1,000	24 " " "	26.3" " "	50.3 " " "
1,200	20 " " "	38 " " "	58.0 " " "

It can be seen that as the speed increases the fluid pressure in the fluid-chambers(13) due to rotation increases while that due to torque decreases. The minimum total presure could be obtained from a graph showing total pressure against R.P.M., but since at present only the maximum pressure interest us this need not be done, for obviously the maximum pressure exists at 200 R.P.M. The fluid-chambers therefore must be designed to withstand that pressure.At this low speed the stress in the chambers due to rotation is very small, and is neglected.

<u>Strength of Fluid-Chambers (13)</u>

Let f = maximum tensile stress in the material in lbs. per sq. inch.

" d = internal diameter of chambers in inches.

" t = thickness of metal in chambers in inches

$\therefore 2\,t \times f = 121 \times d = 121 \times 4 \qquad \therefore t = \frac{242}{f}$

Let f = 800 lbs. per square inch.

$$\therefore\ t = \frac{242}{800} = .303 \qquad \text{Make } t = \frac{3}{8}''$$

It will be obvious that the fluid pressure due to torque is transmitted from the fluid-chambers (13) to the fluid-chambers (29) and (25) but the fluid pressure due to rotation is not thus transmitted, since the distance of the fluid from the axis of rotation has decreased, and the fluid pressure due to rotation varies as the square of this distance.