

**Characterization and Development of
General Material Models
for use in Modeling Structures
Bonded with Ductile Adhesives
by
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Thesis submitted to the faculty of
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements towards a degree of
Master of Science in Mechanical Engineering

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April 22nd, 2005

Blacksburg, Virginia

Keywords: Adhesives, Finite Element Analysis, Nonlinear Material Models, Fracture

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Abstract

Structural adhesives are materials that are capable of bearing significant loads in shear, and sometimes tension, over a range of strains and strain rates. Adhesively bonded structures can dissipate large amounts of mechanical energy and can be lighter and more efficient than many bolted or vibration welded parts. The largest barrier to using structural adhesives in more applications is the many challenges engineers are presented with when designing and analyzing adhesively bonded structures. This study develops, characterizes and compares several material models for use in finite element analysis of adhesively bonded structures, in general, and a bonded tongue and groove (TNG) joint in particular. The results indicate that it is possible to develop a general material model for ductile adhesives used in structural applications under quasi-static conditions. Furthermore, the results also show that it is also possible to take bulk material data and apply it to an adhesively bonded specimen provided that the mode of failure of the bulk test specimen closely approximates the mode of failure of the bonded joint.

Author's Dedication

This work is dedicated to my wife Theresa. None of my accomplishments at Virginia Tech would have been possible or worthwhile without her. I owe her everything.

Author's Acknowledgements

I would like to thank some of the many people that made this study possible. My advisors, Dr. Bob West and Dr. Eric Johnson for having faith in my ability to learn finite element software without a prior background in numerical analysis. Their experience and insight got me through many rough patches during this project. Dr. Johnson was especially kind in checking my work and teaching me aspects of mechanics that I never really understood before. I am indebted to Dr. David Dillard and Dr. Larry Mitchell for sharing their experience in designing with adhesives and industry standards. Dr. Vinay Goyal, for helping me to understand ABAQUS syntax and teaching me FORTRAN. John Hennage, Dr. Soo Jae Park and Josh Grohs for providing all of the machining, material data and experimental support for my models. Dave, Daryl and the guys in the ESM shop for giving me use of the machines to do work when I needed to. Kris Mount for helping me to learn ABAQUS CAE. Brian Painter for the many hours of bouncing ideas around and camaraderie in our noisy, smelly and cold, concrete box of an office. The basement compressor for keeping me awake at 3 AM when I was finishing my literature review. My wonderful daughter for being so understanding when Daddy had “just one more paragraph to finish” or “one more line of code to check”. And last, but not least, DOW Automotive for supporting this research.

And finally, as a gift to Dr. West's future graduate students, here is a list of stupid things and mistakes that Kris, Brian and I have compiled over the last four years. Please learn from these mistakes so that you don't make the same ones we did when modeling glue.

Do not:

- try to fix the network when the computer is not connected to it.
- put data into the wrong spreadsheet and wonder why the numbers don't work.
- number nodes so that the elements have negative area.
- forget to read the ABAQUS manual and notes before using a new material model.
- forget to convert units before using different units systems.
- delete journal files before converting CAE files to a new version.
- make models so detailed that it's impossible to understand them.
- fail to “partition the Hell” out of a 3D model before running it.

Good luck working for a great advisor.

Nomenclature

Physical, Material and Geometrical Constants

ν : Poisson's Ratio

E : Young's Modulus

G : Shear Modulus

r : Radius

t : thickness

T : Absolute Temperature

c : half-length of adhesive bond

l : thickness of annular section in torsion

A : Cross-sectional area

J_p : Polar Moment of Inertia

k : Boltzmann Constant

v_c : Crack Speed

S : Ratio of the Compressive Yield Strength to the Tensile Yield Strength

Y_t : Tensile Yield Strength

Y_s : Shear Yield Strength

Y_c : Compressive Yield Strength

n_c : Number of cross-links in a polymer

n_s : Number of slip-links in a polymer

Engineering and Mechanics Terms

σ : Axial Stress

τ : Shear Stress

ϵ : Axial Strain

γ : Shear Strain

ϵ_{fail} : Axial failure strain

γ_{fail} : Shear failure strain

σ_{af} : Average axial failure stress

τ_{af} : Average shear failure stress

$\sigma_1, \sigma_2, \sigma_3$: Principal Stresses

q : Equivalent stress

w : Energy per unit volume

w_d : Dissipation energy of an adhesive

p : Hydrostatic or Confining Pressure

P : Load Vector

p_i : Load

Δ : Displacement vector

δ : displacement

Φ : Rotation vector

ϕ : Rotation

J_I : First Invariant of the Stress Tensor

J_{2D} : Second Invariant of the Stress Tensor

λ : Principal stretch or extension ratio

K : Stiffness matrix
 I, II, III: Subscripts that refer to fracture failure modes. I, is the opening mode-In tension.
 II, is the sliding mode-In shear. III, is the tearing mode-In shear.
 G : Adherence Energy
 G_c : Critical Strain Energy Release Rate
 G_a : Crack Arrest Strain Energy Release Rate
 T_i : Mode Specific Traction
 Δ_i : Mode Specific Displacement Jumps
 T_c : Critical Traction
 δ_c : Critical Opening Displacement
 Φ : Yield Function

Abbreviations and Acronyms

ADCB: Asymmetric Double Cantilever Beam
 ASTM: American Society of Testing Materials
 BJ: Butt Joint
 CAD: Computer Aided Design
 CZM: Cohesive Zone Model
 DCB: Double Cantilever Beam
 DLJ: Double-Lap Joint
 FE: Finite Element
 FEA: Finite Element Analysis
 FEM: Finite Element Method
 IDM: Interfacial Damage Mechanics
 IEL: Interface Element
 ISO: International Standards Organization
 MATT: Multi-Axial Time Temperature
 MOPP: Modified Oung Porous Plasticity Model
 MSLM: Modified Shear Lag Model
 NDER: Nonlinear Decoupled Elastic Raghava Law
 NHVP: Norton-Hoff Viscoplastic Law
 NR: Napkin Ring
 NSED: Normalized Strain Energy Density
 OPP: Oung Porous Plasticity Model
 RO: Ramberg-Osgood Model
 SLJ: Single-Lap Joint
 TAST: Thick Adherend Shear Test
 TGA: Thermal Graphic Analysis
 TNG: Tongue and Groove Joint
 UEL: User Defined Element
 UMAT: User Defined Material
 VCCT: Virtual Crack Closure Technique
 WLF: Williams-Landel-Ferry

Model Specific Terms and Parameters

κ : Nondimensional interpenetration parameter in Goyal model
 φ : Nondimensional Brittleness factor in Goyal model
 θ_i : Mode specific failure parameters in Goyal model
 λ : Mode Mixity Parameter in Goyal model
 μ_i : Normalized mode specific coupled displacement jumps in Goyal model
 f, f^* : Void Volume Fraction in Gurson-Tveergard Porous Plasticity model
 q_1, q_2, q_3 : Fit parameters in Gurson-Tveergard Porous Plasticity model
 α : Nonlinear axial fit parameter, in Ramberg-Osgood and NDER models
 n : Nonlinear axial strain hardening exponent, in Ramberg-Osgood, NHVP and NDER models
 β : Nonlinear shear fit parameter in NDER model
 m : Nonlinear shear strain hardening exponent in NDER model, damping exponent in NHVP
 $f(\bar{\sigma})$: Normalized equivalent stress function in NDER
 C_n : Linear compliance matrix in NDER
 C_m : Nonlinear compliance matrix in NDER
 f : Ratio of the total moduli in NHVP
 H' : Apparent plastic modulus in NHVP
 $\dot{\varepsilon}_0$: Strain rate in NHVP
 K_V, K_P, K : Elastic modulus parameters in NHVP
 A : Material parameter in NHVP
 F : Fracture toughness in Rahulkumar model
 ψ : Material dilation angle in Raghava model
 p_t : Hardening parameter in Raghava model
 ν_p : Plastic Poisson's ratio in Raghava model
 $\underline{P}$: Scaling Factor in Richardson's MATT model
 L : Williams-Landel-Ferry Time Temperature Shift Factor

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Chapter 1

Introduction to the problem

The technology for adhesively bonding dissimilar materials has been around for a long time. The benefits of using adhesives in structures have also been known for many years but use of structural adhesives has become widely accepted in only a few industries. Structural adhesives are materials that are capable of bearing significant loads in shear, and sometimes tension, over a range of strains and strain rates. Adhesively bonded structures can dissipate large amounts of mechanical energy and are lighter and more durable than many comparably sized bolted or vibration welded parts. The largest barrier to using structural adhesives in more applications is the many challenges engineers are presented with when designing and analyzing adhesively bonded structures.

Some of these difficulties include accurately characterizing the material behavior of the adhesive in a bond (in-situ), accounting for fracture and failure in the adhesive and developing simple forms of analysis that allow engineers to refine their designs quickly.

Design issues with adhesive structures

There is a need for material models that support structural analysis under multi-axial conditions. Many parts and structures designed today are created by computer aided design (CAD) and verified by finite element analysis (FEA) both of which require materials that can support the flexibility of those design tools. Many of the adhesive material models developed from research are not simple to use and are difficult to characterize for general multi-axial loading conditions. This is especially true for general part configurations that are not similar to the butt or lap joints used in characterization tests, such as tongue and groove (TNG) joints or joints with self-aligning features such as the ones shown in Figure 1-1.



Figure 1-1: Two pictures of “Non-Standard”, but typical, joint configurations. Shown here, without adhesive applied, a flanged lip on a pressure vessel (left) and a TNG test coupon.

The TNG is a useful design feature because it requires a minimal amount of area to join the two halves of the pressure vessel. It would be ideal if there were performance-based standards for these kinds of joints and structures. An example of a performance-based guideline is ‘the bearing capacity of the joint in tension must exceed 100 MPa.’ In this case, the goal or main design requirement is given, but the methods and materials are left to the discretion of the designer. The strength of performance-based design is the greatly improved flexibility, allowing the engineer to optimize materials and methods used to generate an acceptable result. Unfortunately, performance-based standards also have a drawback because they require detailed knowledge and confidence of the system

and materials used in the design. This is possible in many cases because of the knowledge base regarding the materials involved in the design, for instance using steel to build an automobile chassis. An engineer must verify that a design meets the required guidelines when using novel materials in new configurations. In many cases this is done by producing a prototype of the product which can then be tested under operating conditions. However, prototyping is expensive and time consuming, which discourages many new and potentially more efficient designs using adhesives. The example below explores a simple method of analyzing an adhesively bonded part and highlights some of the shortcomings of the traditional solution.

Design example using a simple material model

Imagine a client asks for a mechanical analysis of an adhesively bonded part. The client describes that the part is essentially a tubular butt joint which will be loaded in shear and tension and is bonded with an acrylic adhesive that has the properties shown in Table 1-1. Imagine also that the adhesive behaves differently in-situ than in bulk and that the “effective” average shear modulus is 790 MPa but with a range from 700 MPa to 850 MPa, with an average of 790 MPa. The client believes this difference occurs because of cracks and voids in the bond.

Table 1-1: Material Properties of Acrylic Adhesive

Adhesive		
E	2290	MPa
ν	0.380	
γ_{fail}	0.60	
ε_{fail}	0.07	
Aluminum Substrate		
E	70000	MPa
ν	0.330	

A simple model of the part is shown in Figure 1-2.

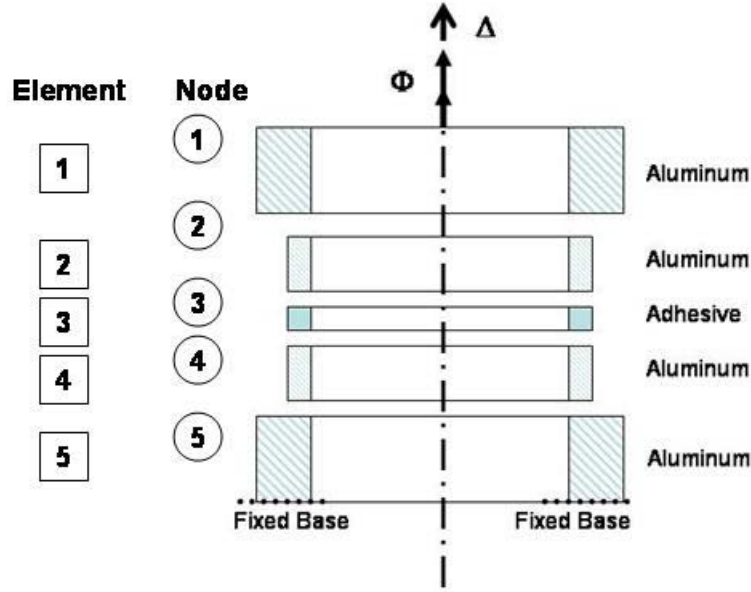


Figure 1-2: Simple Napkin Ring Model

The part has an inner radius of 38.1 mm. The outer radius in elements 1 and 5 is 48.26 mm. The outer radius of elements 2, 3 and 4 is 41.91 mm. Elements 1 and 5 are 25.4 mm thick. Elements 2 and 4 are 18 mm thick. Element 3 is 0.5 mm thick. The physical constants for the part, such as the polar moment of area for each region, are summarized in Table 1-2.

Table 1-2: Physical Dimensions and Properties of Sample Part by Element

Variable	Reference	Material	Value	Unit
$A_1=A_5$	A_1	Aluminum	2756	mm^2
$A_2=A_3=A_4$	A_2	varies	958	mm^2
$E_1=E_2=E_4=E_5$	E_1	Aluminum	70000	MPa
E_3	E_2	Adhesive	2290	MPa
$G_1=G_2=G_4=G_5$	G_1	Aluminum	26300	MPa
G_3	G_2	Adhesive	varies	MPa
$L_1=L_5$	L_1	Aluminum	25.40	mm
$L_2=L_4$	L_2	Aluminum	18.0	mm
L_3	L_3	Adhesive	0.50	mm
$J_1=J_5$	J_1	Aluminum	1536000	mm^4
$J_2=J_4$	J_2	Aluminum	1436000	mm^4
J_3	J_3	Adhesive	1336000	mm^4

The value of the adhesive shear modulus in the model (G_2) is either 700, 790 or 850 MPa. The model structure has a positive axial displacement, Δ , uniformly applied to it and a rotation, Φ , applied in the center of the top surface. The bottom surface is fixed and cannot rotate or translate. Assuming a linear relationship between the shear and axial strain in the adhesive produces the failure criteria defined in Figure 1-3 below.

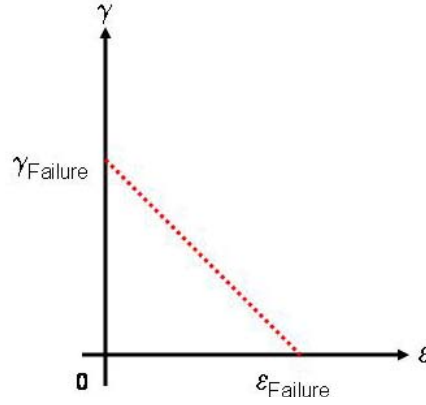


Figure 1-3: Failure Criteria for Linear Relationship between Shear and Axial Strain.

Using the values of $\gamma_{fail}=0.60$ and $\epsilon_{fail}=0.07$ for the criteria gives

$$\gamma = -8.57\epsilon + 0.60 \quad (1-1)$$

If the problem is formulated in terms of the axial spring stiffness in each element then the stiffness matrix is

$$K = \begin{bmatrix} \frac{A_2 E_1}{L_2} + \frac{A_1 E_1}{L_1} & \frac{-A_2 E_1}{L_2} & 0 & 0 \\ \frac{-A_2 E_1}{L_2} & \frac{A_2 E_2}{L_3} + \frac{A_2 E_1}{L_2} & \frac{-A_2 E_2}{L_3} & 0 \\ 0 & \frac{-A_2 E_2}{L_3} & \frac{A_2 E_1}{L_2} + \frac{A_2 E_2}{L_3} & \frac{-A_2 E_1}{L_2} \\ 0 & 0 & \frac{-A_2 E_1}{L_2} & \frac{A_2 E_1}{L_2} + \frac{A_1 E_1}{L_1} \end{bmatrix} \quad (1-2)$$

$$K = \begin{bmatrix} 13358946 & -10719678 & 0 & 0 \\ -10719678 & 23344396 & -12624718 & 0 \\ 0 & -12624718 & 16349018 & -10719678 \\ 0 & 0 & -10719678 & 13358946 \end{bmatrix} [N / mm]$$

The displacements and loads in the model are related by

$$P = K\Delta \quad (1-3)$$

where P is a vector of external forces given by, $P = [(\Delta_1 - \Delta_2)A_1E_1/L_1 \ 0 \ 0 \ 0]^T$ and Δ is a vector of displacements defined as, $\Delta = [\Delta_2 \ \Delta_3 \ \Delta_4 \ \Delta_5]^T$. Rearranging (1-3) to solve for the set of displacements and substitution of the given values into P yields,

$$\Delta = K^{-1}P = \begin{bmatrix} 0.879(\Delta_1 - \Delta_2) \\ 0.816(\Delta_1 - \Delta_2) \\ 0.179(\Delta_1 - \Delta_2) \\ 0.118(\Delta_1 - \Delta_2) \end{bmatrix} \quad (1-4)$$

so that all displacements are written in terms of the axial displacement of the uppermost section in the model. Taking the axial failure strain as the change in the length of a fiber of the adhesive relative to the initial length of the fibers and assuming a linear Hookean constitutive law for the acrylic, the axial strain can be written in terms of axial displacement. In this case, an applied axial displacement of $\Delta_1 - \Delta_2 = 0.035$ mm will result in a failure strain of 0.07 mm/mm in the adhesive layer. The failure stress in the adhesive layer is

$$\sigma_{af} = \frac{A_2 E_2}{L_3} (\Delta_3 - \Delta_4) = 82.7 \text{ MPa}$$

Repeating the procedure for the shear case, but allowing the shear modulus to vary within a reported range creates a stiffness matrix that is dependent on the shear modulus of the adhesive. The resulting stiffness matrix in terms of the shear modulus, polar moment of inertia and element thickness is

$$K = \begin{bmatrix} \frac{G_1 J_1}{L_1} + \frac{G_1 J_2}{L_2} & -\frac{G_1 J_2}{L_2} & 0 & 0 \\ -\frac{G_1 J_2}{L_2} & \frac{G_1 J_2}{L_2} + \frac{G_2 J_3}{L_3} & -\frac{G_2 J_3}{L_3} & 0 \\ 0 & -\frac{G_2 J_3}{L_3} & \frac{G_2 J_3}{L_3} + \frac{G_1 J_2}{L_2} & -\frac{G_1 J_2}{L_2} \\ 0 & 0 & -\frac{G_1 J_2}{L_2} & \frac{G_1 J_2}{L_2} + \frac{G_1 J_1}{L_1} \end{bmatrix} \quad (1-5)$$

$$K = \begin{bmatrix} 3.39 \cdot 10^9 & -2.10 \cdot 10^9 & 0 & 0 \\ -2.10 \cdot 10^9 & 2.10 \cdot 10^9 + 3.07 \cdot 10^6 G_2 & -3.07 \cdot 10^6 G_2 & 0 \\ 0 & -3.07 \cdot 10^6 G_2 & 2.10 \cdot 10^9 + 3.07 \cdot 10^6 G_2 & -2.10 \cdot 10^9 \\ 0 & 0 & -2.10 \cdot 10^9 & 3.39 \cdot 10^9 \end{bmatrix} [N - mm]$$

The rotations and torques in the model are related by

$$T = K\Phi \quad (1-6)$$

where T is a vector of torques given by, $T = [G_1 J_1 (\Phi_1 - \Phi_2) / t_1 \ 0 \ 0 \ 0]^T$ and Φ is a vector of external rotations defined as, $\Phi = [\Phi_2 \ \Phi_3 \ \Phi_4 \ \Phi_5]^T$. Rearranging (1-6) to solve for the set of rotations and substitution of the given values into T , for $G_2 = 700$ MPa, yields,

$$\Phi = K^{-1}T = \begin{bmatrix} 0.7345(\Phi_1 - \Phi_2) \\ 0.5766(\Phi_1 - \Phi_2) \\ 0.4209(\Phi_1 - \Phi_2) \\ 0.2631(\Phi_1 - \Phi_2) \end{bmatrix} \quad (1-7)$$

In this case, the shear strain at failure is $\gamma_{fail} = 0.60$ which corresponds to an applied rotation of 0.007158 radians, and Φ_1 is therefore 0.0124 radians. The shear stress in the adhesive layer is equal to

$$\tau_3 = \frac{r_1 (\Phi_3 - \Phi_4) G_2}{L_3} \quad (1-8)$$

For $\Phi_1 = 0.0124$ radians the rotation in the adhesive layer associated with a failure shear strain, Φ_{af} , is 0.0052 radians. The failure shear stress in the adhesive layer, τ_{af} , is 242.16 MPa. Repeating the above procedure for G_2 equal to 790 MPa and 850 MPa gives values of $\Phi_{af} = 0.007125$ and 0.007133 radians respectively. The shear stress at failure in the adhesive layer for each case is then $\tau_{af} = 471.8$ and 508.2 MPa respectively. The results of this simple exercise are summarized in Figure 1-4.

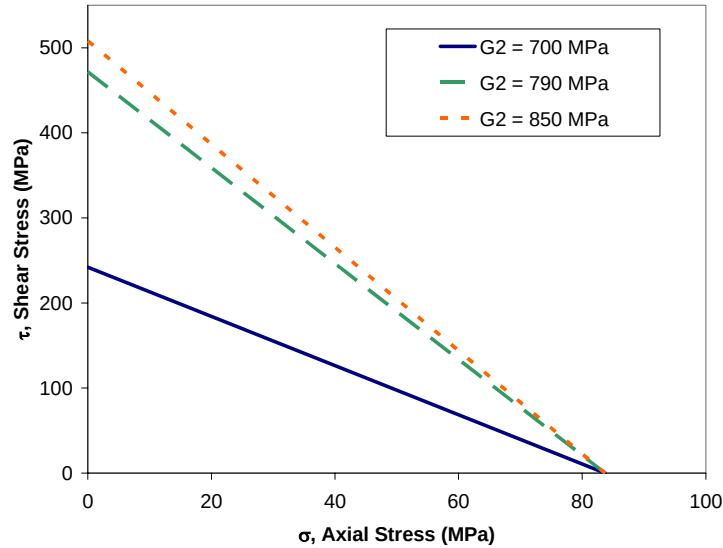


Figure 1-4: Plot of Shear and Axial stresses at Failure as a function of Shear Modulus.

The analysis reveals that the response of the part is very sensitive to variations in the shear modulus of the adhesive. Even though the failure envelope would change based on the value of the shear modulus the relationship between the shear modulus and the failure stress is clearly not linear or simple. Yet many classical methods of analysis lead engineers to proceed making those very assumptions. If the assembly was bonded with a nonlinear material the resulting analysis would be more complex and the response of the part would be equally inconsistent to small variations in test data. This analysis also ignores the potential for fracture, voids or other complications that would affect the response of the adhesive.

The classical method of analysis is capable of determining limits to the stresses but it leaves questions unanswered. For instance, how should an engineer interrogate this simple model? How does assuming a nonlinear material behavior affect the failure surface used in this method? How sensitive is the torque at failure in the part with respect to other variations in material data? Unlike many other methods of joining materials there are no existing guides or standards that can readily answer these questions for adhesives. It is imperative that engineers and analysts have access to tools that can aid them in making sound decisions based on their engineering judgment and experience. Simple tools, like the linear analysis in the example, should be available so that a designer can quickly determine what parameters to change in order to get the desired performance

from an adhesively bonded structure with a ductile nonlinear adhesive. However, in cases where simple approximations are not sufficient, detailed models should also be available to determine if the state of stress in the adhesive is acceptable.

Proposed solution

There are several options that may be able to address the concerns mentioned above. These models are proposed as tools for designers and analysts to use in quasi-static finite element analysis of adhesively bonded structures. Dynamic effects and rate dependency are not covered in this thesis. The material models include a viscoplastic¹ model, a porous plasticity model, an incremental plasticity model, an interfacial damage law and a new nonlinear elastic law. The goals of this thesis are:

1. To provide examples of how to characterize and implement nonlinear material models of adhesives using bulk material data.
2. To use these models to predict the state of stress in adhesively bonded parts due to multi-axial loading.
3. To estimate failure limits in a complex, but realistic, joint geometry using the same material models.

This work is intended for two audiences: the analyst who is very familiar with finite element modeling but is not familiar with adhesives and the designer who is familiar with adhesives but not finite element analysis. Chapter 2 of the thesis reviews recent developments in adhesive material models. Chapter 3 discusses the material laws available in the ABAQUS Standard library that are useful for modeling the quasi-static behavior of adhesive materials. Chapter 4 goes into the details of the nonlinear elastic law and discusses how to implement the model. Chapter 5 compares the predicted results of the four material laws against test data from bulk uniaxial tension specimens and bulk shear tests. Chapter 6 uses the previous results to examine how the material laws perform when modeling more complex multi-axial load states in stiff adherend Napkin Ring (NR) and Tongue-and-Groove (TNG) specimens. Chapter 7 is presented as a conclusion to the thesis and summarizes the significant results of Chapters 5 and 6.

¹ It is important to note that the Viscoplastic model described in this thesis uses the VISCOUS and PLASTIC options in ABAQUS but is implemented at a single, effectively quasi-static, rate. However, the model does require data from multiple rate of loading tests to characterize it. See Chapter 3 for details.

Chapter 2

A Review of Thermomechanical Constitutive laws

The body of knowledge in adhesion science has grown considerably since 1900 and especially in the last forty years [1]. One of the earlier, and perhaps most well known, attempts to model adhesive behavior is Volkersen's [2] analysis in 1938. Volkersen's model assumes linear properties for both the adhesive and the adherend for a double-lap joint. This led to the development of the "shear lag" concept and the classical assumption that under an applied load the adherend would deform in extension only and the adhesive layer would deform via shear.

Volkersen's work inspired two other investigations into the mechanical behavior of adhesives. Goland and Reissner [3] developed a model similar to Volkersen's with the added assumptions that both shear and peel strains were important in the deformation of the adhesive layer. Their final solution closely resembled the equations that define the response of a beam on an elastic foundation, which were originally developed by Winkler [4]. At nearly the same time Plantema [5] published a solution that included shear and axial deformation in the adherends as well. This was different from Goland and Reissner because their work only accounted for axial extension in the adherends. The fundamental assumptions and models from these analyses are still in use today.

Early adoption of adhesives by the aircraft industry encouraged the development of engineering models of adhesive behavior. Perhaps one of the better known collaborations between the aircraft industry and adhesive science is Hart-Smith's work with NASA and McDonnell-Douglas. Hart-Smith was one of the more vocal engineers in breaking with the classical and convenient assumption of linear material behavior in both the adhesive and the adherend. In fact, Hart-Smith states in a contractor summary report to NASA in 1977 [6] that:

“The key to being able to predict failure of adhesive-bonded joints is the use of a non-linear adhesive characterization, rather than the linear one used in classical analyses.”

Around this time more detailed constitutive models involving polymer systems and failure were also being investigated. Shapery [7] proposed extending Eyring's model of viscoelasticity [8] with an integral approach to account for nonlinear rate-dependent viscoelasticity, and applied it to polymer systems under different load conditions. Later, Shapery further extended his own work to include modeling fracture in viscoelastic materials [9]. Kinloch [10] also applied such ideas to the fracture of adhesively bonded materials. The original nature of the work that proceeded in the area of adhesive mechanics and characterization from Volkersen's early work in 1938 to Shapery's important constitutive relationships in 1989 set the stage for growth in this research area for the next 15 years.

Material Models of Polymers and Bulk Adhesives

In general, the mechanics of stress analysis in polymeric materials is little different from other materials. Those interested in general mechanics of polymers are directed to Williams' book on the Stress Analysis of Polymers [11]. However, the analysis of adhesive systems is fundamentally different from the general analysis presented in many textbooks because they are inherently inhomogeneous multi-material systems. There are two main categories in adhesive material model research: phenomenological and empirical. Those that approach material modeling of adhesives and polymers from a phenomenological point of view seek to characterize behavior based on such things as the free volume in the polymer, surface energy, the percentage of cross-linking, chain entanglements, etc. Those that approach material modeling from an empirical point of view seek to characterize behavior based on assumed damage accumulation functions, failure envelopes, specific assembly geometries, etc.

Many of the fundamental phenomenological models are based on the 1936 Eyring model of viscous flow [8]. For instance, Arruda and Boyce [12] and Arruda, Boyce and Jaychandran [13] have developed some important phenomenological models based on the initial approach Eyring used in his study of plasticity, viscosity and diffusion in polymer networks. Arruda and Boyce's models consider the probabilistic realignment of individual chains in a polymer network under an applied load. Arruda, Boyce and Jaychadran considered multi-axial loading via true stress and strain, coupled with thermal effects on a cube-like test specimen and then developed material models to predict the behavior of the specimen. The foundation for many of their assumptions was a series of previous research studies exploring the birefringence of polymethylmethacrylate (PMMA) and elastomeric networks [14]. Even though these models specify values of shear rate they can be used for constant rate and rate-independent situations as well. Arruda and Boyce incorporate temperature-dependent strain hardening into their model by using a relationship between the thermally evolving chain density and a non-disassociating link in a chain network. In their 1995 paper they reported impressive results even though the implementation of their finite element model was odd. The oddity was that even though their test specimen was cubical they chose to use axisymmetric elements and a fairly coarse mesh to represent it so they could retain the use of

cylindrical coordinates. The close agreement they were able to achieve suggested that had they refined their models they would have seen even better agreement, which has been proven and supported by several other researchers and most recently Bardella [15] and Bergstrom and Boyce [16].

Another interesting statement to come out of the work of Arruda and Boyce was the form that the model approximating the inelastic viscous behavior of a polymeric material should take. Bardella concluded from Arruda and Boyce's work that when developing models based on the Eyring equation for multi-axial conditions the hyperbolic sine function is necessary so that the model is easier to implement in commercial finite element code and better captures the behavior of viscous materials with instantaneously applied and cyclic loads. Bardella goes on to say that if the hyperbolic sine term is neglected or approximated using a simple exponential term then Eyring's equation loses the thermoreversible nature of the effect that the loading process has on the molecules in the polymer and gives a physically meaningless creep behavior. The distinction that Bardella makes between them comes from two similar models of a viscoelastic material.

Bergstrom and Boyce [16] later used their result to predict the hysteretic behavior of rubbers and epoxies although Bardella cautions that these results should not be used for syntactic foams and filler modified epoxies. Their work was, however, an important improvement over earlier work, by Haward and Thackeray [17] for instance, because many previous studies had used a more convenient exponential term to account for the inelastic properties. However, in nearly quasi-static loading this behavior is not as significant and either form may be appropriate.

Drozdov [18] develops similar ideas for glassy polymers at a constant temperature but subject to both small and large deformations. Drozdov's model is based on a network of adaptive links and is formulated using functions that define the strain energy of an elastoplastic link. Although Drozdov simplifies the model by neglecting higher order terms, the equations developed for the strain energy of a temporary network based on the displacement equations agree very closely with the presented test data. The examples in the paper include a comparison of the model predictions for a linear, isotropic, incompressible, viscoelastic material to relaxation test data from polycarbonate specimens at different temperatures.

An interesting phenomenological model that can be applied to adhesive materials is the two-layer viscoplastic model developed by Norton and Hoff [19]. The work of Hoff [20] and later Norton and Hoff [21] focused on models that describe the viscoplastic behavior of materials. These models can be easily characterized by simple mechanical tests and accurately describe the behavior of polymers with significant viscous and creep response due to loads. The phenomena that is used to develop these models is known as Hoff's Analogy, which states that the same form of solution can be used to model both the plastic and creep response of the material provided a power law plasticity relationship accurately describes the response of the material. This model can be extended for use in fracture analyses as well as general loading conditions. The Norton-Hoff model is best applied to polymeric materials that are tough and highly rate sensitive so that the model will produce physically relevant results.

The difficulty in directly applying phenomenological material models to adhesive systems is their range of applicability. Questions such as 'will the model be physically meaningful for a bonded structure at large strains or high rates of strain' are not necessarily addressed by detailed modeling of bulk material behavior. Empirical models seek to address this problem by modeling behavior specifically tied to a general condition or configuration rather than a specific material or constitutive law. A good example of an empirical model is the work by Ganghoffer and Schultz [22], which deals with modeling geometrically nonlinear contact problems involving thin elastic layers. They developed a mechanical model to achieve a solution that is applicable to a wide range of materials. Another interesting example of an empirical approach is a paper by Andruet et al. [23] which discusses a class of elements, called AD2DH, for the finite element software ABAQUS that are tailored to adhesives used in nonlinear geometries.

While few phenomenological researchers give general statements on features all models should address, Yu et al. [24] venture to state what should be taken into account when modeling adhesives based on their experimental observation and experience. These are:

- 1.) Logarithmic Strain Rate Dependence
- 2.) Hydrostatic Stress Sensitivity
- 3.) Volume Increasing Nonlinear Deformation

- 4.) Largely Recoverable Nonlinear Deformation
- 5.) Temperature Sensitivity
- 6.) Variable Instantaneous Modulus

In Yu's opinion, many of these criteria are addressed by the Drucker-Prager [27] or von Mises models, both of which can be specialized for rate dependency but commonly deal with constant strain rate only. In fact, the modified von Mises law used by both Crocombe and Zgoul [25] and Adams and Harris [26] is in itself a specialized version of a more general yield surface known as the Raghava criterion. The yield criterion that Raghava proposed is

$$2Y_c Y_t = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + 2(Y_c - Y_t)(\sigma_1 + \sigma_2 + \sigma_3) \quad (2-1)$$

where σ_1 , σ_2 and σ_3 are the principal stress components and Y_c and Y_t are the values of the uniaxial compressive and tensile yield stress [27]. Let the first invariant of the stress tensor, denoted by J_1 , the second invariant of the deviatoric stress tensor, denoted by J_{2D} , and let the ratio of the compressive and tensile yield stresses be denoted by S ; i.e.,

$$\begin{aligned} J_1 &= \sigma_1 + \sigma_2 + \sigma_3 \\ J_{2D} &= ((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2)/6 \\ S &= Y_c/Y_t \end{aligned} \quad (2-2)$$

then the Raghava yield surface governing the plastic flow can be defined. The equations describing the onset of failure and the equivalent stress are

$$F(\sigma) = q - Y_t = 0 \quad (2-3)$$

$$[J_1(S-1) + (J_1^2(S-1)^2 + 12J_{2D}S)^{1/2}]/2S - q = 0 \quad (2-4)$$

with q equal to the Raghava equivalent stress. This model has been shown to accurately predict the behavior of epoxy resins and other adhesives in-situ [28].

Yu et al. also address work performed by Chiu and Jones [29] that compares both classical viscoplastic behavior and a unified constitutive model. However, neither model is recommended for structural analysis of adhesives in multi-axial loading or thermal analyses. Chiu focused on the shear behavior of adhesives using a unified approach that collects all factors influencing the elastic and inelastic, or plastic, deformation into two separate terms, δ_{elastic} and $\delta_{\text{inelastic}}$.

Crocombe et al. [30] presents a unified viscoplastic model with both uniaxial and multi-axial formulations based on five independent parameters. While the material law

incorporates the strain rate, a similar model could be derived for time-independent cases as well by assuming either a constant inelastic strain rate or a different relationship for the inelastic strain. The constitutive equations for Crocombe et al.'s basic uniaxial model are

$$\varepsilon = \varepsilon^e + \varepsilon^{in} \quad (2-6)$$

$$\varepsilon^e = \sigma / E \quad (2-7)$$

$$d\varepsilon^{in}/dt = A(\sigma - \alpha) \exp(k|\sigma - \alpha|) \quad (2-8)$$

$$\alpha = A_1 \tanh(k_1 \varepsilon) \quad (2-9)$$

where A , A_1 and k , k_1 are constant material parameters, σ is the applied stress, α is defined as the back stress. Equation (2-6) calculates the total strain as the sum of the elastic (ε^e) and inelastic (ε^{in}) strains. Crocombe et al. applied the model to double-lap joints and produced fairly accurate predictions of failure loads.

A useful paper that discusses a multi-axial, time and temperature (MATT) model for epoxies is one by Richardson et al. [31]. Richardson et al.'s model is based on an ellipsoidal failure envelope and is fairly accurate for temperatures between 70°F to 115°F for a wide range of normal and shear load combinations provided that the assembly is loaded at a constant rate. The model equation is

$$AP^2 J_{2D} + BPJ_1 = 1 \quad (2-10)$$

where A and B are shape parameters, P is a scaling factor, J_{2D} is the second deviatoric stress invariant and J_1 is the first stress invariant. Note that for a constant value of P , the model is equivalent to the Tsai-Wu model used in composites and the modified Drucker-Prager failure model [31]. The model assumed a linear function for damage accumulation for both the tensile and shear adhesion properties of the epoxy. Richardson et al. states that in the data used to characterize the model that the ratio between the tensile failure and shear failure in the adhesive is constant and independent of time and temperature. Therefore this model may not be applicable to adhesives that exhibit large plastic deformations in shear compared to small axial deformations.

Sweeney and Ward [32] looked at the same sort of chain extension models that Arruda and Boyce [12] based their work on, but instead focused on the observed behavior of polymers at high temperatures. Sweeney and Ward modified the model originally used by Ball et al. [33] for the rate-independent necking and strain-hardening behavior of

polymers at high temperatures. Ball et al. assume a hyperelastic model for polypropylene given by

$$\sigma_{ii} = \lambda_i \frac{\partial W}{\partial \lambda_i} - p, (i=1,2,3). \quad (2-11)$$

where σ_{ii} are the true normal stresses, λ_i are the principal extension ratios, W is the energy per unit volume and p is the hydrostatic pressure. Sweeney and Ward modified the model so that it is no longer hyperelastic and is weighted by the number of potential “slip-links” and successful “cross-links” that they determine by experiment. The new model they proposed was

$$\frac{\sigma_{ii}}{kT} = N_c \lambda_i^2 + N_s (I_1) \lambda_i b'(\lambda_i) - p, (i=1,2,3). \quad (2-12)$$

where k is the Boltzmann constant, T is the absolute temperature, N_c is the number of cross-links in the material, N_s is the number of slip links in the material for $N_c \gg N_s$, I_1 is the first strain invariant given by $I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$, and $b'(\lambda_i)$ is a derivative of some function of the principal extension ratios with respect to the principal strain ratios which is determined later. This sort of model has proven useful in modeling residual stresses in the adhesive layer due to heat curing in bonded structures.

Models of In-situ Adhesives and Adhesive Systems

The mechanical properties of bulk polymers can be very different from the behavior observed in a bonded assembly. Early work by Baun [34] examined the role of the adherend surface and the bulk adhesive properties in the failure of adhesive joints. His work emphasized that even though the boundary between an adhesive and an adherend is conveniently represented mathematically as a straight line the interaction of the two materials in reality is not so simple. Baun clearly developed and presented the idea that the bond surface is best thought of as an “interphase”. The term “interphase” is meant to describe a complex region where the properties of the adherend and the adhesive are not completely indistinguishable from one another and the distributions of both materials are not necessarily consistent. Baun highlights a few of the different potential failure locations possible along the interphase between an adherend and adhesive and various spectrographic methods of verifying the failure surface.

Adams and Harris [35] examined how local geometry could influence the strength of adhesive joints using finite element modeling. They were primarily interested in the effect of spew and the machining specimen corners on the presence of stress concentrations in single-lap joint configurations. Adams and Harris chose several machining options for their specimens: square corners, filleted corners and rounded corners. Their results predicted a much greater failure load for specimens that had rounded corners than unrelieved corners left square and experimental results closely agreed with their models.

Crocombe et al. [36] also modeled the effect of local geometry on adhesive bonds but eschewed specific material behavior of in-situ adhesives for a general elastic-plastic material law. They used the displacement field at the corners of cleavage and compression shear fixtures near the numerical singularity as a way to predict the ultimate failure load in the material. Crocombe et al. compared their finite element results to nonlinear numerical routines they developed using this method. In all cases the nonlinear numerical routines provided better estimates of the test data than the finite element models.

Van der Berg et al. [37] were more interested in the effect of specimen geometry on the measured yield stresses of the epoxy adhesive joint assemblies. They compared the results of a thick adherend shear test (TAST), a modified Arcan specimen and a double-lap joint with stiff adherends. Van der Berg et al. reported significant differences in the yield stress of the assemblies based on temperature and the specimen configurations. They concluded that when testing adhesives in-situ it is crucial to be aware of all important parameters in the test and control them strictly in order to obtain repeatability.

Tsai and Morton [38] published a paper even further removed from detailed material modeling of bulk adhesive specimens. They modified the Volkersen and Goland and Reissner solutions for stress and strain in single-lap and double-lap beams using elongation and shear deformation parameters for linear materials with 1-D bars and beams. The modified form of the Volkersen equation that they presented is

$$\tau_c = A \sinh(\beta x) + B \cosh(\beta x) \quad (2-13)$$

where

$$A = \frac{\beta c \tau_{avg}}{\cosh(\beta c)} \left[\frac{1 - \frac{E_2 t_2}{E_1 t_1}}{1 + \frac{E_2 t_2}{E_1 t_1}} \right] \quad (2-14)$$

$$B = \frac{\beta c \tau_{avg}}{\sinh(\beta c)} \quad (2-15)$$

and

$$\beta = \alpha^2 \lambda^2 \quad (2-16)$$

where α and λ are the shear deformation and elongation parameters, respectively, given by

$$\alpha^2 = \left[1 + \frac{G_c}{\eta} \left(\frac{t_1}{3G_1} + \frac{t_2}{3G_2} \right) \right]^{-1} \quad (2-17)$$

$$\lambda^2 = \frac{G_c}{\eta} \left(\frac{1}{E_1 t_1} + \frac{1}{E_2 t_2} \right) \quad (2-18)$$

In the above equations, c is the half the length of the overlap, and τ_{avg} is equal to $T/2c$ where T is the average applied force per unit width on the adherends. Parameters E_1 , E_2 , E_c and G_1 , G_2 , G_c refer to the Young's modulus and shear modulus of the upper adherend, lower adherend and adhesive, respectively. Parameter η is the thickness of the adhesive layer. When the effect of shear deformation is negligible, α is equal to one, and the modified form yields the original Volkersen equation. The modified form of the Goland and Reissner equation is

$$\frac{\tau_c}{\tau_{avg}} = \frac{1}{4} \left[\frac{\beta c}{t} (1 + 3k) \frac{\cosh\left(\frac{\beta c}{t} \frac{x}{c}\right)}{\sinh \frac{\beta c}{t}} + 3(1 - k) \right] \quad (2-19)$$

where $k = 2M_o/Tt$ and M_o is the applied moment. The equation for β is the same as given by (2-16), but the equations for α and λ have changed. Parameters α and λ are now written as

$$\alpha^2 = \frac{1}{1 + \frac{2G_c t}{3\eta G}} \quad (2-20)$$

$$\lambda^2 = \frac{8G_c t}{E\eta} \quad (2-21)$$

where G and G_c refer to the shear modulus of the adherend and adhesive, respectively. The original Goland and Reissner solution is recovered when $2G_c t / 2\eta G \ll 1$, and α is equal to one. Tsai et al. verified their modified versions of the classic equations using both finite element models and Moiré interferometry. Their results showed that modifying the Volkersen and Goland and Reissner equations to account for shear, moment and axial elongation produced stress distributions that compared well with the experimental contours and finite element predictions.

Sheppard et al. [39] proposed a method for defining the critical region and load in an adhesive assembly rather than a specific material model or equation to model a joint assembly. Sheppard outlines the procedure to predict the critical load in a bonded joint via a damage zone that can be determined using whichever material properties or analysis framework best fits the case at hand. Sheppard et al. chose to use the Mises strain criterion for finite element models of single-lap and double strap joints with composite and aluminum adherends. Using experimental data they estimated the size of the damage model and scaled the finite element mesh so that they could approximate a representative area using anywhere from 10 to 100 elements. The load that the elements in the damage zone could carry when the modeled damage zone was the size observed at failure compared very well to the ultimate load of both specimens with either adherend material.

While Sheppard et al.[39], Crocombe et al.[36] and others are interested in using FE to predict load and failure behavior other researchers applied FE to verify standard test properties. Alfano and Rosati [40] applied FE to study specimens because of the extremely brittle nature of the materials in tension. Their models were designed to handle "no-tension" materials where accurate materials characterization is very difficult. Alfano and Rosati developed robust algorithms for modeling the failure of these materials with a high degree of sensitivity. Ochsner and Gegner [41] used FE models to verify adhesive test results for important material properties using ASTM and ISO standards. However, both of the models and methods used in the two preceding articles were highly specific to the geometry and materials used.

Lazzarin et al. [42] developed a much more general approach for predicting the stress distributions in bonded joints of different geometries. Although the concepts in the article are very similar to the ideas published by Williams [43] and Boggy, [44] Lazzarin et al. extend those ideas based on recent work by Lefebvre and Dillard [45] so that the eigenvalue solutions become more tractable for an adhesive joint configuration. They apply their model to butt-joints, symmetric double-lap joints and single-lap joints. The numerical models developed using this approach agree very well with finite element results for the stress distribution along the interface of the adhesive and the adherend, however, no experimental data is provided for verification.

Rosettos [46] also developed a general model that could be applied to single-lap joints to predict the thermal peel, warpage and interfacial stresses. Rosettos takes a similar approach to Volkersen and Hart-Smith and develops a modified shear lag model (MSLM) that includes bending and extension in the adherends and extension and shear strains in the adhesive. While the materials used in the examples given in the paper are for aluminum adherends and an epoxy adhesive any material properties could be applied to the model. The stresses developed in the adhesive are due solely to the thermal mismatch between the adherends and the adhesive. The MSLM predicts oscillations in the shear and peel stresses along the bond line and a sinusoidal distribution of displacements in the adherend which indicates warping of the material.

Guillemenet et al. [47] studied the affect of viscous behavior on the performance of an adhesively bonded DCB assembly via a wedge test. They used a general relationship between the adherence energy and the dissipation energy in an adhesive bond to determine the strength of the adhesive bond. The relationship is given by

$$G - W = \Phi(\alpha_T v) \quad (2-22)$$

where G is the adherence, or strain energy of the adhesive and W is the dissipation energy of the adhesive and Φ is a function of the Williams-Landel-Ferry [48] time-temperature shift factor, α_T , and v is the crack speed. An interesting result to come from the work of Guillemenet et al. is that the stress at the crack-tip of an adhesive is independent of the wedge introduction speed.

Yan et al. [50] performed a thorough study on the effect of the substrate material on the fracture properties of an adhesive. They based their conclusions on both finite

element analysis and experimental data from DCB specimens made from different materials. Yan et al. found that if the adhesive were modeled as an elastic material the opening stress ahead of the crack-tip would increase with a reduction in the Young's modulus for the same applied load. Their models also indicated that the small-scale yielding criteria for homogeneous materials seemed unsuitable for an adhesive joint. Finally, they also noticed that the critical strain energy release rate and the critical J-integral for a DCB joint with the same adhesive changed with bond thickness and adherend material.

Models of Adhesive Systems that account for Damage and Failure

Models that can accurately predict loads for in-situ or bulk cases, but cannot address the development of damage and degrading of materials are of limited use. Engineers and scientists have developed several different ways of modeling damage in real materials. Early studies by Irwin [51] and Griffith [52] led to the concept of fracture mechanics, which used energy-based criteria to predict failure load in both brittle and ductile materials. Irwin and Griffith provided the foundation for both phenomenological and empirical modeling of adhesive systems, such as the more recent work of Swadener et al. [53] and Yu et al. [54].

Later, the concept of damage mechanics was applied to engineering materials. In damage mechanics the material properties of a continuum are degraded according to an equation of state until the material has failed. Kachanov's [56] text is an excellent introduction to the subject of damage mechanics. While the fundamental concepts of fracture mechanics and damage mechanics appear similar, they are different in several respects. Fracture mechanics can only be applied given an initial crack at a known location in a continuum and is best used to describe the separation of two surfaces. Damage mechanics can include distributed, or "smeared", cracks so that the location of discontinuities is not important and is best used to describe the growth of micro-cracks and how their presence affects the continuum without any prior knowledge of the size or location of the cracks. To date, only a few researchers have attempted to reconcile the differences between the two theories into a unified form. Mazars et al. [57], Bazant [58], Pijadier-Cabot and Benallal [59] have all published studies on such unified theories.

Mazars et al.'s work in particular shows promise because it divides the behavior of a material into two regimes, an initial one where damage mechanics applies and a later one where fracture mechanics governs, and unites the two approaches using thermodynamics.

An interesting smeared damage model is the Gurson-Tvergaard model. Gurson [60] and Tvergaard [61] used each others ideas to develop another approach to considering damage evolution in materials. By considering a unit cell of a material with a known volume of voids and generally specifying the constitutive relationship of the voids as they coalesce within the solid matrix, a good approximation of a damaged material is possible. However, this method is limited to materials that have less than 10% voids by volume initially for the results to be physically meaningful. The Gurson-Tvergaard model is generally written as

$$\Phi = \left(\frac{q}{Y_t} \right)^2 + 2q_1 f \cosh \left(\frac{-3q_2 p}{Y_t} \right) - (1 + q_3 f^2) = 0 \quad (2-23)$$

where q is the effective stress, Y_t is the yield stress, f is the volumetric void ratio in the material, p is the hydrostatic stress and the parameters q_1 , q_2 , q_3 are used to calibrate the model based on the response of the material to the voids coalescing [62]. The original form of the model is recovered when q_1 , q_2 and q_3 are set to a value of one. Kim et al. [63] recently applied the model to describe the behavior of imperfect ductile solids under triaxial loading. Kim et al. calibrated the model using results from a study by Faleskog et al. [64] on using a unit cell and fracture tests to develop master curves for the fit parameters q_1 , q_2 and q_3 . Although the works of Kim et al. and Faleskog et al. never refer to polymeric materials the Gurson-Tveergard model is not limited to metals. Lee and Oung [65] developed a similar form of the Gurson model exclusively for polymers since the failure of glassy polymers is also affected by the nucleation, growth and coalescence of voids. The model Lee and Oung developed, henceforth referred to as the Oung Porous Plasticity (OPP) model, is written as

$$\Phi = q^2 + 2Y_c Y_t f \cosh \left(\frac{J_1}{2\sqrt{Y_c Y_t}} \right) + (Y_c - Y_t)(1 - f)J_1 - (1 + f^2)Y_c Y_t = 0 \quad (2-24)$$

where Y_c and Y_t are the yield strength of the material in compression and tension respectively. Note that for $Y_c = Y_t$ the original Gurson model for spherical voids is

recovered. Oung achieved excellent results when the model was used to describe the failure of polycarbonate under a variety of loading conditions [66].

A number of hybrid failure modeling methods have also been developed. The virtual crack closure technique (VCCT), popularly used by NASA and summarized in a NASA report by Krueger [67], assumes that fracture mechanics can be used in any material based on the location and length of a "virtual crack." This method incorporates the crack length and location requirements into the model and is useful for designing against "worst case" scenarios but it is computationally expensive.

Another popular method is the concept of interfacial damage mechanics. Using this concept, the properties of the interface between two materials are modeled using fracture mechanics and the progression of damage is controlled by degrading all properties of the material according to a damage parameter. This method is relatively easy to implement using finite element analysis (FEA) and has been used to solve many different complicated problems. The benefit to using this approach is that no initial crack is required and multiple crack paths are possible. Those interested in the various issues of implementing these sorts of models should review the work of de Borst [68]. However, many of the methods used today have similar roots in work done by parties investigating elasticity, damage and fracture mechanics.

Williams [43] and Bogy [44], for instance, initially investigated eigenvalue problems developed from elastic isotropic wedges that could be used to model sharp edged cracks. Both authors also pursued the effect of bi-material interfaces on their homogeneous models. Dugdale [69] and Barenblatt [70] proposed that a cohesive zone formed ahead of crack-tips in rate-independent materials that would allow the continuum at a crack tip to degrade in a number of ways. Williams [71], Knauss [72] and Shapery [73] extended these ideas and predicted the behavior of macroscopic crack propagation in linear viscoelastic continuums. Rice [74] published results of a study reminiscent of the works of Williams and Bogy, but applied his results to dealing with cracks at interfaces between two elastic continuums. Rice added parameters in his model to account for contact between two surfaces to prevent interpenetration after failure. These contact restrictions were a significant improvement over the works of Williams and Bogy and made the model valid at the crack-tip. The development of these ideas led to the concept

of a cohesive zone model (CZM) that could be used in finite element analysis. Hillerborg et al. [75] developed the first CZM damage model for FEA and applied the model to his study of concrete. Needleman [76] and Chen and Mai [77] used Hillerborg et al.'s concept to propose a cubic relationship based on the separating force required for two atomic particles. More recently Goyal et al. [78] and Blackman et al. [79] have implemented this model in the finite element software ABAQUS.

The strength of the CZM model is that it can capture the fracture properties of the adhesive, such as the critical energy release rates (G_c) and the critical strength of the material (T_c), using elements with an irreversible interfacial damage law. The combination of strength and fracture properties in the constitutive laws accounts for failure using the bulk properties of the adhesive and crack growth. In Blackman's work, only one mode of failure is considered. In the work of Goyal et al., multiple modes of failure are considered and once a crack begins to propagate subsequent crack growth is considered a mixed-mode problem using mode-I and mode-II strain energy release rates as well as the peel and shear strength of the adhesive. However, Goyal's model can also be extended to three dimensions. In Goyal's work the Ramberg-Osgood model is the constitutive law for the bulk adhesive. The Ramberg-Osgood model assumes that the nonlinear part of the effective stress and effective plastic strain are related by

$$\varepsilon^p = \frac{\alpha}{E} Y_t \left(\frac{\sigma}{Y_t} \right)^n \quad (2-25)$$

where Y_t is the yield strength, n is the strain hardening parameter, and α is a dimensionless material constant. The formation of cracks in the adhesive layer is governed by relationships between the interfacial tractions and displacement jumps. These equations mathematically describe the mechanics of the crack growth as a process involving three distinct phases: initiation of the crack, the evolution of the material degradation zone and crack growth. The mixed mode constitutive equations are

$$\begin{Bmatrix} \overline{T}_1 \\ \overline{T}_2 \end{Bmatrix} = \begin{Bmatrix} \overline{\Delta}_1 \Theta_1 \\ \overline{\Delta}_2 \Theta_2 \end{Bmatrix} - \begin{Bmatrix} 0 \\ \langle -\overline{\Delta}_2 \rangle \end{Bmatrix} \exp \left(\frac{1 + \kappa |\overline{\Delta}_2|^\beta}{\beta} \right) \quad (2-26)$$

where $\bar{T}_1$, $\bar{T}_2$, $\bar{\Delta}_1$ and $\bar{\Delta}_2$ are the normalized interfacial tractions and the corresponding normalized displacement jumps at the interface, respectively. The variables $\bar{T}_1$, $\bar{T}_2$, $\bar{\Delta}_1$ and $\bar{\Delta}_2$ are dimensionless and are normalized by the respective critical values, for example $\bar{T}_1 = T_1/T_{1c}$ where T_{1c} is the yield strength in tension of the material. See Reference [78] for more detail on the development of the equations for the interfacial damage law. The operator $|x|$ is the absolute value of x and $\langle x \rangle$ is the Heaviside step operator. The variables κ and β are non-dimensional interpenetration and brittleness factors, respectively.

In order to implement the interfacial law in a finite element code κ should be set relatively high, on the order of 10^5 . The brittleness factor, β , should be set high for a brittle fracture and low for a ductile fracture. The failure parameters Θ_1 and Θ_2 are functions that couple the mixed-mode parameters and the normalized displacement jumps for the opening (mode-I) and sliding (mode-II) modes. Θ_1 and Θ_2 are given by

$$\Theta_j = \exp\left(\frac{2 - (\mu_j^\beta)(d_j) - d_j}{\beta}\right), j = 1, 2 \quad (2-27)$$

where μ_1 and μ_2 couple the normalized displacement jumps for the different failure modes. The equations for μ_j are

$$\mu_j = |\bar{\Delta}_j|^{1 - \frac{1}{\lambda_j}} (\bar{\Delta}_1^2 + \langle \bar{\Delta}_2 \rangle^2)^{\frac{1}{2\lambda_j}}, j = 1, 2 \quad (2-28)$$

$$\bar{\Delta}_i = \Delta_i / \Delta_{ic}$$

and d_j are damage variables and Δ_{1c} and Δ_{2c} are empirically based on the critical energy release rates, G_{Ic} and G_{IIc} , and the maximum interfacial strengths, S_I and S_{II} . λ_j are material parameters that determine the failure locus for the initiation and progression of cracks under mixed-mode conditions.

Blackman et al. [79] use a different equation for their model of the cubic relationship between the tractions and displacements in a material, which is given by

$$\sigma = \frac{27}{4} \sigma_{\max} \frac{\delta}{\delta_c} \left(1 - \frac{\delta}{\delta_c}\right)^2 \quad (2-29)$$

$$G_c = \int_0^{\delta_c} \sigma d\delta = \frac{9}{16} \delta_c \sigma_{\max} \quad (2-30)$$

where $\sigma_{\max}$ is equivalent to T_c and δ_c is the critical value of the relative displacement between two surfaces at the onset of fracture. Blackman et al. specialized their model to peel configurations and did not include a mixed-mode constitutive relationship. None of the models proposed by Needleman, Goyal et al. or Blackman et al. address the problem of thermal loading. In fact, when using their CZM the only way to model the effect of thermal shrinkage or residual stresses from curing is to include continuum elements with some thermal expansion law in the finite element model.

Rahulkumar et al. [80] proposed a model similar to that of Goyal et al. that also includes temperature dependence. Rahulkumar et al.'s CZM model is formulated using the experimentally measured fracture toughness, Γ . The empirical relation that describes this formulation is

$$\Gamma = \Gamma_0 [1 + f(a_T v, T_i)] \quad (2-31)$$

where Γ_0 is the intrinsic fracture energy of the surface, a_T is the WLF (Williams-Landel-Ferry) shift function for a polymer [49], v is the peel rate of the material and T_i is the reference temperature. The function $f(a_T v, T_i)$ represents the irreversible loss of energy due to viscous dissipation of the adhesive. Rahulkumar specialized the model to deal with peel only but it could be expanded to deal with multi-axial cases.

A significant advantage to using the CZM is the applicability of the model to many different specimen configurations regardless of load-rate and whether the adhesive is modeled as a bulk or in-situ specimen. Some researchers have suggested that a rate-dependent CZM model is actually necessary, but at the current time the existing rate-independent models presented here closely agree with experimental test data. Recent work by Georgiou et al. [81] suggests that it is the fracture properties of the model, namely G_c and T_c , that are more important than the form of the relationship between the relative surface displacements and the tractions at the surface or even if the model is rate-dependent. While other researchers have developed different models using CZM theory the three discussed in detail here are representative of most of them.

Observations on Future Research Needs

The industries that currently use adhesive technology are expanding. In the automotive industry for instance, the current Mercedes-Benz models have 90 linear meters of adhesive bonds [82]. While the airline industry has used adhesives for nearly one hundred years it is still seeking better ways to account for failure of adhesives. In a recent technical report the Federal Aviation Administration (FAA) [83] summarized the applicability of several different failure criteria to adhesively bonded composite structures only to conclude that some of the basic assumptions in their analysis still need to be verified. A glaring omission in this body of work is the method to put these models into practice. Few authors state which tests should be performed to obtain the data necessary to implement the models. Fewer still give recommendations as to how practicing engineers should use their results. In short, there is a lot of work yet to be done.

Chapter 3

Implementing and Characterizing Adhesive Material Models

There are many constitutive laws that can be used for modeling adhesive systems. This work will focus on using three commonly available models and one new model in the finite element program ABAQUS. The current Chapter discusses the means and methods for implementing, characterizing and fitting the common models to test data. The next Chapter focuses on the development and characterization of the new user defined model. The models discussed in this Chapter are: a two-layer viscoplastic model based on the Norton-Hoff rate law, an incremental plasticity model that uses the Raghava yield criterion and a general Gurson-Tvergaard model with fit parameters chosen to

reflect polymer properties based on the work of Lee and Oung. An interfacial element (IEL) used to model discrete crack growth, which can be interspersed between bulk elements in any of the constitutive models discussed here, is also described.

The Norton-Hoff model in ABAQUS

The Norton-Hoff viscoplastic (NHVP) model in ABAQUS is comprised of several different material options with the required parameters chosen according to specific criteria. This model is considered a good choice for materials in which significant plasticity is observed and has been shown to provide good results for thermomechanical loading [19]. The benefit to using the NHVP is that one can obtain good estimates of material behavior at large strain magnitudes for a complex rate-dependent material with much less effort than fitting a fully rate-dependent model. The NHVP model can also be extended to accommodate rate-dependency once the model parameters are known.

The primary assumption of the model is that if a relationship between the viscous behavior and the plastic behavior can be defined that relationship can also be used to define the creep behavior of the material. This relationship is the Norton-Hoff rate law and is defined, for one dimension, as

$$\dot{\varepsilon}_v^v = A\sigma_v^n \quad (3-1)$$

where $\dot{\varepsilon}_v^v$ is the volumetric strain rate of the viscous material, σ_v is the inelastic stress in the viscous material, A is the slope of the stress-strain curve of viscous material and n is an exponential fit parameter. The Norton-Hoff rate law assumes that the total volumetric strain in the material can be separated into elastic, plastic and viscous components such that

$$\varepsilon = \varepsilon^{el} + (1 - f)\varepsilon^{pl} + f\varepsilon^v \quad (3-2)$$

where ε^{el} , ε^{pl} and ε^v are the elastic, plastic and viscous strains in the material respectively. The constant f is a ratio of elastic and inelastic moduli that determines the contribution of the plastic and viscous behavior [85] in the total volumetric strain.

The ABAQUS ELASTIC, PLASTIC and VISCOUS² options are used to create an elastic-plastic network with an elastic-viscous network in parallel. In the post-yield region the creep and plasticity are in series, coupling the plastic and viscous networks, instead. The NHVP model is intended for modeling materials under a wide range of temperatures, or fluctuating loads, which exhibit time-dependent behavior. It is included here to contrast the behavior of the other models and to demonstrate that a quasi-static model can be extended to handle time-dependent behavior too. Unfortunately, the model cannot be used to predict softening behavior in a material. The NHVP model uses the von Mises yield criterion.

Fitting the Norton-Hoff viscoplastic model in ABAQUS

The NHVP is ideally separated into three different parts: elastic, plastic and viscous [19]. The required inputs to the model are: the slope of the initial elastic modulus (E), the average slope of the plastic response (H'), the yield stress (Y_t), the viscous modulus (A), the ratio of the moduli (f) and two exponential fit parameters n and m that affect the viscous and creep response of the model. Note that because H' is an average and may be determined from many data sets it may not be equal to the more common notation E' or E'' .

It is important to note that any ELASTIC option in the ABAQUS standard library can be used to define the initial elastic response of the material. Those interested in different methods of modeling the elastic response should refer to the ABAQUS Analysis User's manual [86]. The PLASTIC option requires plastic stress versus strain data input as a table with the first value at the yield stress and zero plastic strain.

Several sets of uniaxial tension data at different load rates define each of the material constants in the model. Using the tension data in this fashion makes use of two important assumptions: first, that the overall material behavior can be reduced to a simple one-dimensional model, and second, that enough data exists to define the parameters at a steady-state. In this case, steady-state refers to the variables A and H' changing very little between different applied uniaxial strain rates in the tension tests.

² The capitalized terms "ELASTIC", "PLASTIC", "VISCOUS", etc., refer to cards in the ABAQUS/Standard input file that call specific functions from the Standard material library.

The ABAQUS User's Manual recommends that the slowest load rate should be several orders of magnitude slower than the fastest tested load rate. This data is used to define the ratio, f , of the elastic modulus of the elastic-viscous network to the instantaneous modulus. The f ratio is defined in the ABAQUS Theory [19] manual as

$$f = \frac{K_V}{K_P + K_V} \quad (3-3)$$

where K_P is the elastic modulus at the slowest load rate and K_V is defined by

$$K_V = K - K_P \quad (3-4)$$

where K is the elastic modulus of the material at the fastest load rate. The average slope of the plastic response is defined as the average slope of the stress-strain curve from the beginning of the deviation from elastic response to the yield stress between the different load rates. See Figure 3-1 for a graphical explanation of H' . In general, H' is assumed to be much less than K_P .

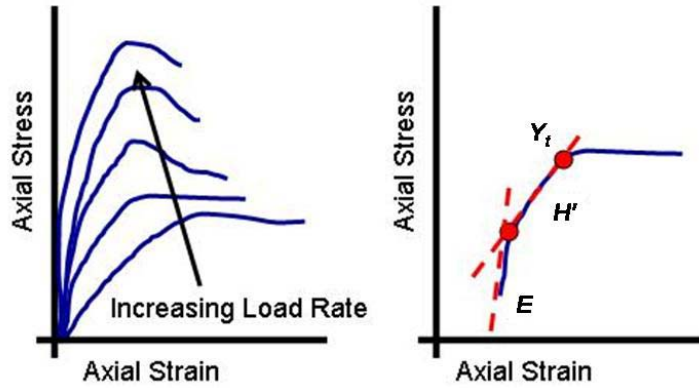


Figure 3-1: Typical series of stress-strain curves used to define H' .

The variables A and n are determined once the values for f and H' are known using the data for the different stress-strain curves in the Equation

$$\sigma = A^{-1/n} \dot{\varepsilon}_0^{1/n} + Y_t + H' \varepsilon \quad (3-5)$$

where σ and ε are the recorded values of stress and strain of the material at a known strain rate $\dot{\varepsilon}_0$. The fit parameter m cannot be solved for directly using these assumptions and it is best to simply try different values of m in the model until the desired creep response is achieved. Solving for the model parameters in this fashion will give a conservative error in the predicted stress-strain at lower strains but a more accurate

estimate of the plastic behavior of the material. See Figure 3-2 for a comparison of the typical stress-strain curve of the model described above to data.

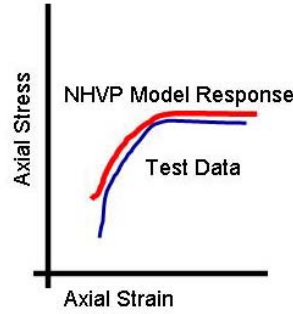


Figure 3-2: Typical Stress-Strain response of Norton-Hoff model.

The Raghava Yield Criterion in ABAQUS

The Raghava yield criterion can be implemented in ABAQUS as a quadratic form of the exponential Drucker-Prager model [27]. The exponential form of the Drucker-Prager model provides the most general yield criteria available in the library of ABAQUS polymer behavior models. This yield function is defined in the ABAQUS Theory [86] manual as

$$F = aq^b - p_h - p_t = 0 \quad (3-6)$$

Where $a = a(\theta, f^\alpha)$ and $b = b(\theta, f^\alpha)$, are material parameters with θ as temperature and f^α ($\alpha = 1, 2, 3 \dots$) equal to other predefined field parameters [87]. In the yield function, p_t is a hardening parameter, p_h is the equivalent hydrostatic stress and q is the Von Mises stress. If $b = 2$ and the Equations for hydrostatic stress and Von Mises equivalent stress are substituted into Equation (5) the result is Equation (8) shown below.

$$p_h = -\sum_{i=1}^3 \frac{\sigma_i}{3} \quad (3-7)$$

$$q = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \quad (3-8)$$

$$F = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 + \frac{2}{3a}(\sigma_1 + \sigma_2 + \sigma_3) - \frac{2}{a}p_t = 0 \quad (3-9)$$

Comparing Equation (3-8) with the yield criterion that Raghava proposed (see Equation 2-1), it is clear that setting the parameter $b = 2$ forces the exponential Drucker-Prager criterion to revert to Raghava's criterion, with the parameters a and p_t equal to

$$a = \frac{1}{3(Y_c - Y_t)} \quad (3-10)$$

$$p_t = aY_cY_t = \frac{Y_cY_t}{3(Y_c - Y_t)} \quad (3-11)$$

if the tension or compression hardening is selected under the Drucker-Prager Hardening option. Note that in ABAQUS, p_t is defined as

$$p_t = a\sigma_t^b + \sigma_t/3$$

which after substitution and multiplying by a common denominator gives the result shown as Equation (3-11).

The Drucker-Prager hardening option is designed to accommodate general plastic response for hardening or softening materials. The benefits to using the Drucker-Prager/Raghava model are that the model parameters are simple to identify and the predicted response of the material in both bulk and in-situ applications can be very accurate over a wide range of strains. The model cannot be directly extended to accommodate rate-dependent behavior [87].

Fitting the Raghava Yield Criterion in ABAQUS

The Raghava criterion requires several input parameters and a set of uniaxial plastic stress-strain data. The tension hardening criteria should be chosen to model the adhesive. The required data for that option are the values of plastic strain and axial stress. The first data point in that record must be zero plastic strain at the yield stress of the material. The data record from a bulk tensile specimen is the easiest source for this information.

The fixed input values to the model are the elastic modulus E , Poisson's Ratio ν , the ratio of the compressive yield strength to the tensile yield strength, S , and the angle of dilation ψ . Since the yield criterion imposed by the Drucker-Prager and Raghava models are very specific, the fit of the models to the test data must fall within certain guidelines. For instance, the Raghava criterion form of the exponential Drucker-Prager model should only be used when the compressive yield stress is more than 1.05 times the tensile yield stress because the value for a (see Equation 3-10) varies hyperbolically with a decreasing

ratio of compressive to tensile yield stress. See Figure 3-3 for a graph of the value of a as a function of the ratio S , which is the ratio of the compressive to the tensile yield stress.

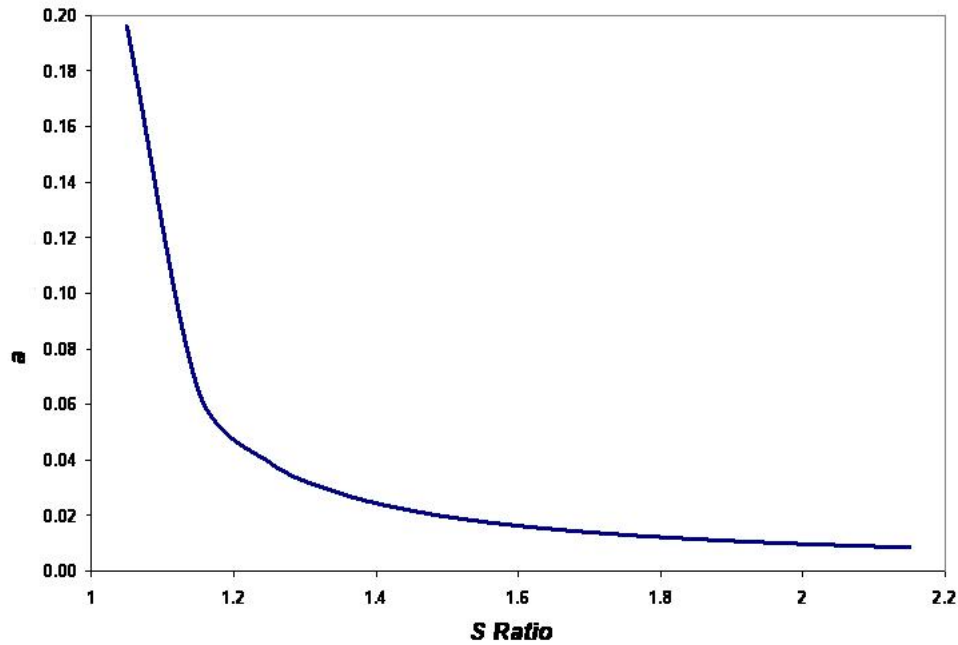


Figure 3-3: Plot of a versus S

As the compressive yield strength increases the tensile strength must decrease. See Figure 3-4 for a graph of tensile yield stress as a function of increasing values of S . Determining the S ratio with bulk material data is possible via a four-point bend test of a bulk specimen [88]. The S ratio can also be determined provided the shear yield strength of the material is known. The relationship between the shear and tensile yield strengths in adhesive materials can be defined in a different fashion from the standard von Mises form. For materials that are sensitive to the hydrostatic state of stress the following relationship can be derived using Equation (3-9) and assuming a state of pure torsion

$$Y_s = \sqrt{\frac{SY_t^2}{3}} \quad (3-12)$$

where Y_t is the tensile yield strength and Y_s is the shear yield strength. Rearranging the Equation to solve for S gives

$$S = \frac{3Y_s^2}{Y_t^2} \quad (3-13)$$

The results of (3-12) and (3-13) are discussed in more detail in Chapter 4. The data required to use these equations is easily obtained from either a bulk notched beam Iosipescu test (ASTM D 5379/5379M-98) or a pure shear Arcan test [93].

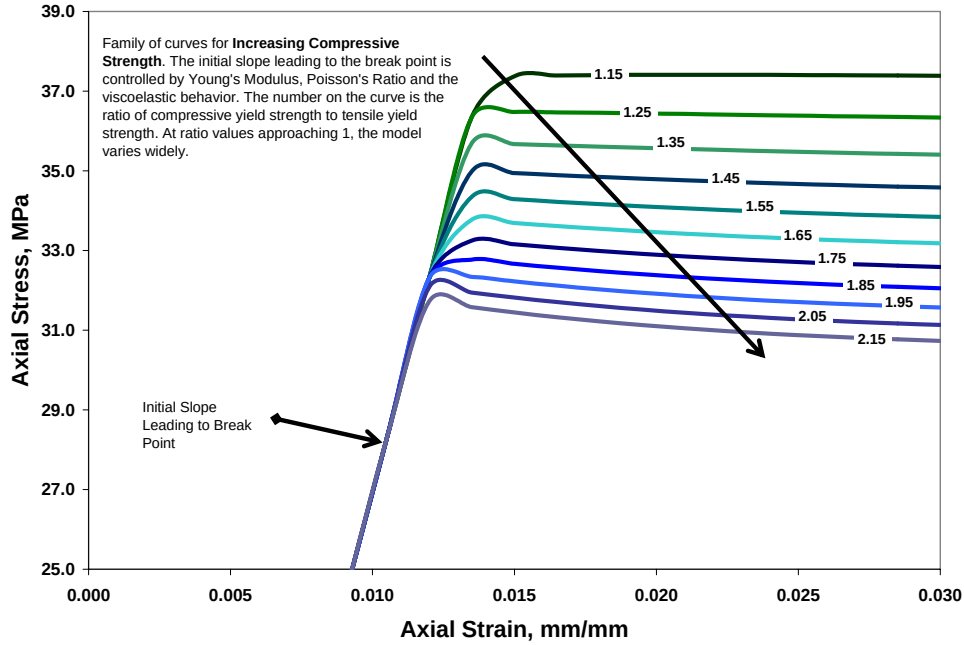


Figure 3-4: Yield Stress as a function of S .

The ABAQUS theory manual gives a detailed discussion of how ψ can be determined using tri-axial stress-strain data. However, two simpler methods are presented here. Simply put, ψ is the slope of the asymptotic line that the general exponent form of the Drucker-Prager model approaches when the effective stress, q (see Equation 3-8), is plotted as a function of p (see Equation 3-7), the equivalent pressure stress. See Figure 3-5 for an illustration of the p - q plane. Here the effective stress is defined as the Mises stress, given as $\sqrt{\frac{3}{2} \sigma_D : \sigma_D}$, where σ_D is the stress deviator tensor, and the equivalent pressure stress is defined as the hydrostatic pressure, given by $-\frac{1}{3} tr \sigma$, where σ is the stress tensor. By assuming an initial eccentricity (or x-axis intercept) to the exponential curve, an estimated value of ψ can be determined given two points $\underline{a}$ and p_t , where

$$p_t = a Y_c Y_t = a S Y_t^2 \quad (3-14)$$

$$\underline{a} = (p_t / a)^{1/b} \quad (3-15)$$

For example, using $Y_t = 34.5$ MPa, $S = 1.75$, $b=2$, and $a = 0.012882$, $p_t = 26.83$ MPa and $\underline{a} = 45.64$ MPa. Assuming that the projected intercept on the p -axis will be less than or equal to 10% of a , two points can be put on a graph and the slope of the line that connects them will be ψ . See Figure 3-5 for the graph related to the data given in the example. Since this method is based on simple geometric intercepts it is important to verify these assumptions using plots of q as a function of p .

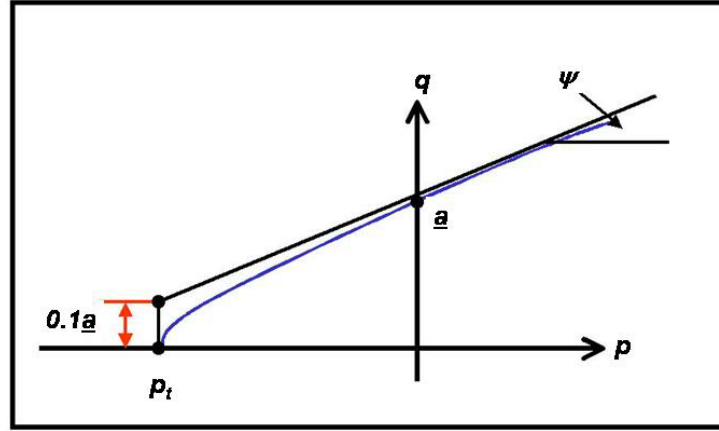


Figure 3-5: Sketch of Initial Estimate of ψ for LESA.

Zgoul and Crocombe define the tangent of ψ in terms of the plastic Poisson's ratio [25]. Since the tangent of ψ is the ratio of the hydrostatic and deviatoric strain invariants, the angle ψ is really the orientation of the plastic flow vector. In terms of the plastic Poisson's ratio the relationship is given by

$$\tan \psi = \frac{3(1 - 2\nu_p)}{2(1 + \nu_p)} \quad (3-16)$$

where ν_p is given by the ratio of the transverse to the axial strains immediately after the material has yielded. Equation (3-16) is the preferred method for obtaining ψ provided test data is available.

The Modified Oung Porous Plasticity (MOPP) Model in ABAQUS

The porous plasticity model in the ABAQUS Standard library is based on the general form of the Gurson-Tvergaard model and uses the ELASTIC, PLASTIC and POUROUS options. The porous model is based on an ideal unit cell of a material with a spherical void that can grow with increasing pressure and rules that govern how the voids

from multiple cells can coalesce. The model has been shown to predict hardening and softening behavior very well in some metals [60], but has only recently been applied to a small range of other materials. The work of Lee and Oung is similar to Gurson except that they use special yield functions and flow rules for glassy polymers [65]. The criterion that they chose to use is the modified von Mises form instead of the general von Mises law and assumes that the material is stronger in compression than tension (i.e. $Y_c > Y_t$). This kind of material law is very useful for the analysis of adhesive structures because it directly addresses the presence of voids. Voids occur in an adhesive layer for a variety of reasons, including improper surface preparation in the bond, temperature discontinuities during curing, residual stresses due to curing, out gassing, etc. It is often possible to estimate the average void volume fraction when designing with adhesives but it is nearly impossible to estimate where and how large an individual void may be. The porous plasticity law effectively “smears” the voids throughout the material which allows the use of the average void volume fraction in an analysis.

The constitutive law described by Oung is not available in the ABAQUS Standard library. However, the general form of the Gurson model has three variables that can be chosen such that the yield function becomes very similar to the one described in Oung. Simplicity and post-yield accuracy are the main benefits to using the model because it easily accommodates damage in the material and requires few tests to characterize the model. This modified form of the POROUS model cannot be directly extended for rate dependency and will not give reasonable results for materials with initial void volume fraction greater than 10% of the total volume [62]. The Gurson and modified Oung models in ABAQUS are formulated so that they can only be used with plane-strain elements.

Fitting the Modified Oung Porous Plasticity (MOPP) Model in ABAQUS

The Oung Porous Plasticity (OPP) model is a specialized form of the general Gurson model derived to account for hydrostatic sensitivity. The MOPP model is a specialized form of the general Gurson porous plasticity model available in ABAQUS/Standard with the model parameters chosen in a very specific fashion. The MOPP model is similar to the OPP model, but they are not identical. The discussion

below describes how to obtain and fit the modified form of the model available in ABAQUS/Standard.

The ABAQUS/Standard MOPP model is comprised of several material input options: ELASTIC, PLASTIC and POROUS. Two bulk material tests are required to identify the model parameters: a uniaxial tensile test and an ARCAN or Iosipescu shear test. The ELASTIC option requires the elastic modulus and Poisson's ratio while the PLASTIC option uses the set of plastic stress versus strain (as described in the Norton-Hoff and Raghava models). The final parameters for the POUROUS option are selected based on the results published by Oung [66]. The general form of the Gurson model is

$$\Phi = \left(\frac{q}{Y_t} \right)^2 + 2q_1 f \cosh \left(\frac{-3q_2 p}{Y_t} \right) - (1 + q_3 f^2) = 0 \quad (3-17)$$

while the form of the OPP yield function is

$$\Phi = q^2 + 2Y_c Y_t f \cosh \left(\frac{J_1}{2\sqrt{Y_c Y_t}} \right) + (Y_c - Y_t)(1 - f)p - (1 + f^2)Y_c Y_t = 0 \quad (3-18)$$

where J_1 is the first stress invariant. See Chapter 2 for more details on these equations.

The Gurson yield function in the ABAQUS Standard library is

$$\Phi = \left(\frac{q}{Y_t} \right)^2 + 2q_1 f^* \cosh \left(\frac{-3q_2 p}{2Y_t} \right) - (1 + q_3 f^{*2}) = 0 \quad (3-19)$$

where f^* is a function of the void volume fraction. The variables q_1 , q_2 and q_3 can be determined by comparing the general Gurson model to the Oung model, replacing Y_c with SY_t and using the relationship between q_1 and q_3 defined in the ABAQUS Theory manual [62]. The relationship between q_1 and q_3 is given as

$$\bar{f}_F = \frac{q_1 + \sqrt{q_1^2 - q_3}}{q_3} \quad (3-20)$$

where $\bar{f}_F$ is the void volume fraction at failure. The values q_1 , q_2 and q_3 become,

assuming $\bar{f}_F = 1$ at failure,

$$\begin{aligned} q_1 &= S \\ q_2 &= \frac{1}{\sqrt{S}} \\ q_3 &= 2S - 1 \end{aligned}$$

Using these values in ABAQUS results in the MOPP yield function

$$\Phi = \left(\frac{q}{Y_t}\right)^2 + 2Sf^* \cosh\left(\frac{J_1}{2Y_t\sqrt{S}}\right) + [f^{*2} - 2Sf^{*2} - 1] = 0 \quad (3-21)$$

Rearranging Oung's yield function, dividing by Y_t^2 , and collecting like terms gives the equation

$$\Phi = \left(\frac{q}{Y_t}\right)^2 + 2Sf \cosh\left(\frac{J_1}{2Y_t\sqrt{S}}\right) + \left(\frac{S}{Y_t} - \frac{1}{Y_t}\right)(f - 1)J_1 - S(f^2 + 1) = 0 \quad (3-22)$$

The third term in (3-22) goes to zero as f approaches one. The fourth term in (3-22) is quadratic in nature and differs only by a constant factor from the third quadratic term in (3-21). Comparing (3-22) to (3-21) shows that the equations become very similar as the material fails and f approaches a value of 1.

The difference between the two yield functions is that the ABAQUS approximation tends to overestimate the axial and shear stresses in the material post yield. The ABAQUS version of the Oung model is also not as sensitive to variations in the yield strength ratio. Choosing q_1 , q_2 and q_3 in this fashion is simple but should be considered an *approximation* based on recently published results [65] unless axial, shear and in-situ test data are available to confirm that choosing the parameters based on Oung does not lead to large errors. A more rigorous approach for determining q_1 , q_2 and q_3 is detailed in Faleskog et al. [64]. Although Faleskog et al. describe the results of experiments on high modulus, low-void-fraction materials, the method used to relate void growth to stress states could be applied to glassy polymers too.

Interfacial Elements in ABAQUS

The concept of interfacial damage mechanics (IDM) and interfacial elements (IELs) are extremely useful in finite element modeling for two reasons. First, the assumptions required to use IDM reduces the complex phenomena that occur in the region between bonded materials to a clean boundary with known properties that is easily handled in FEA. Second, IDM eliminates numerical singularities that can occur at boundary regions due to nonlinear materials or geometric effects because the maximum displacement the elements can withstand before separating is defined. The IELs used in

this analysis were taken from Goyal and Johnson [78] and implemented in ABAQUS Standard as User Defined Elements (UELs). See Chapter 2 for more detail on the constitutive law that governs the IELs mentioned here. In this work the IELs are used to model the adhesive layer joining adherends in a simple fashion to obtain estimates on crack growth in complex joints geometries. IELs are also used to provide extremely detailed analysis of failure mechanisms in the TNG fixtures discussed in Chapter 6 as a component in a cohesive zone model (CZM). The main benefit to designers in using these elements in either a stand alone or complementary fashion is to assess the useful range of the nonlinear constitutive laws mentioned previously and predict joint failure mechanisms in the TNG. Figure 3-6 illustrates a possible 8 element configuration, 4 continuum elements and 4 interface elements, for a CZM.

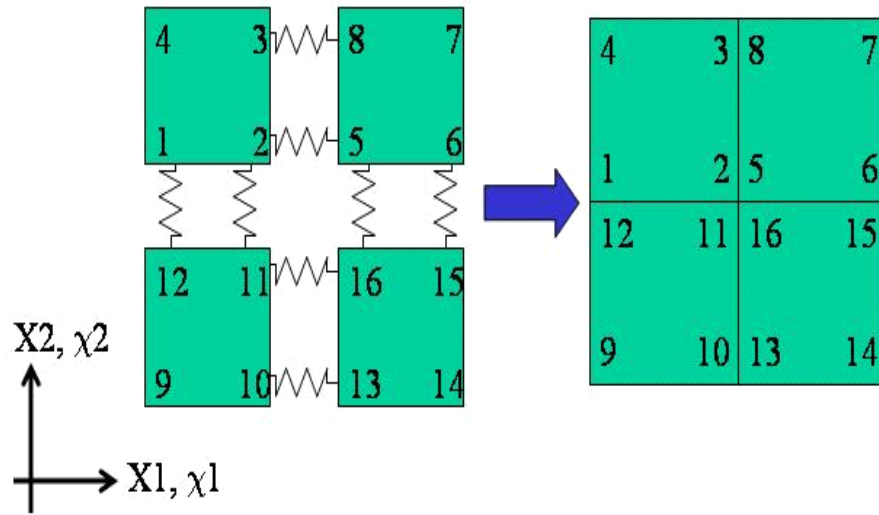


Figure 3-6: 8 Element CZM Configuration.

Four material parameters are required to use the IELs in ABAQUS: G_{Ic} , G_{IIc} , S_I and S_{II} . The critical strain energy release rates in the first and second modes are usually determined by double cantilever beam (DCB) specimens. See Kinloch et al. [10] and Blackman et al. [79] for more details on those test methods. In the Goyal model all other parameters can be chosen based on convenience and numerical efficiency. The strength parameters, S_I and S_{II} , are taken from a uniaxial tension test and a shear test as the values for the tensile and shear yield strengths respectively. The mode mixity exponents, λ , are determined from ADCB tests.

Comparing the Different Yield Surfaces

There are important differences between each of the material models described in this Chapter. The most significant differences from the perspective of an engineer deal with how each model predicts yield. In each material model a different yield criterion is used that will result in different yield strengths. Figure 3-7 illustrates the difference between the yield criteria by plotting the von Mises, Raghava and MOPP yield surfaces in principal space, assuming $\sigma_1 > \sigma_2 > \sigma_3$.

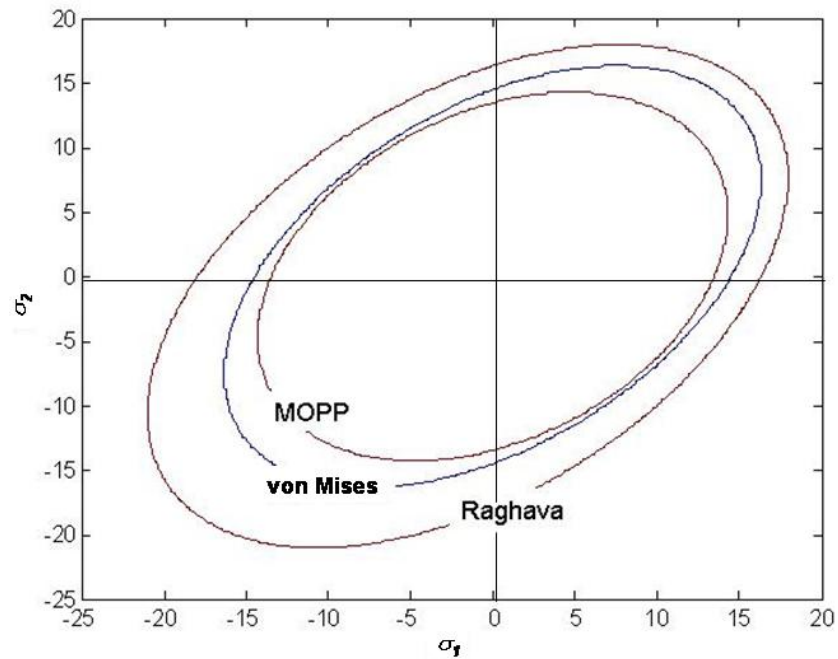


Figure 3-7: Plot of Mises, Raghava and MOPP yield surfaces. $\sigma_3=0$ MPa, $S=1.1$, $f=0.1$, $Y_t=16.5$ MPa.

The differences between the yield criteria are obvious from the figure above. The Raghava criterion predicts greater yield strength in compression than in tension so it has more space available in the 3rd quadrant. The MOPP criterion predicts larger yield strength in tension than the Raghava criterion and smaller shear yield strength for a nearly perfect material (i.e. $f = 0.001$). The MOPP criteria also produces a larger yield surface than the von Mises yield criteria which is the most generous in tension and the least generous in compression. However, for a realistic level of voids in a material that will still give physically meaningful results in the model (i.e. $f = 0.1$), the MOPP yield surface is much smaller than either the von Mises or Raghava models.

Each material model, and the corresponding yield criteria, is sensitive to variations in material parameters. The yield criteria are also sensitive to different material parameters. The von Mises criteria is sensitive only to changes in Y_t only, whereas the other criteria are sensitive to changes in the yield strength ratio, S , and the void volume fraction, f . For instance, at low values of S , the Raghava yield criterion predicts yield strengths close to the von Mises criterion. For larger values of S , for example $S=3.6$, the Raghava criterion shifts the failure envelope to the second and third quadrants. This effectively reduces the predicted equivalent stress in pure tension so that the Raghava criterion predicts that failure in tension will occur at half of the value predicted by the von Mises yield surface. In compression, the Raghava yield criterion at $S=3.6$ will predict failure at nearly seven times the limit of the von Mises yield criterion. Figure 3-8 illustrates the differences between the von Mises and Raghava yield criterion for the same Y_t and three different values of S .

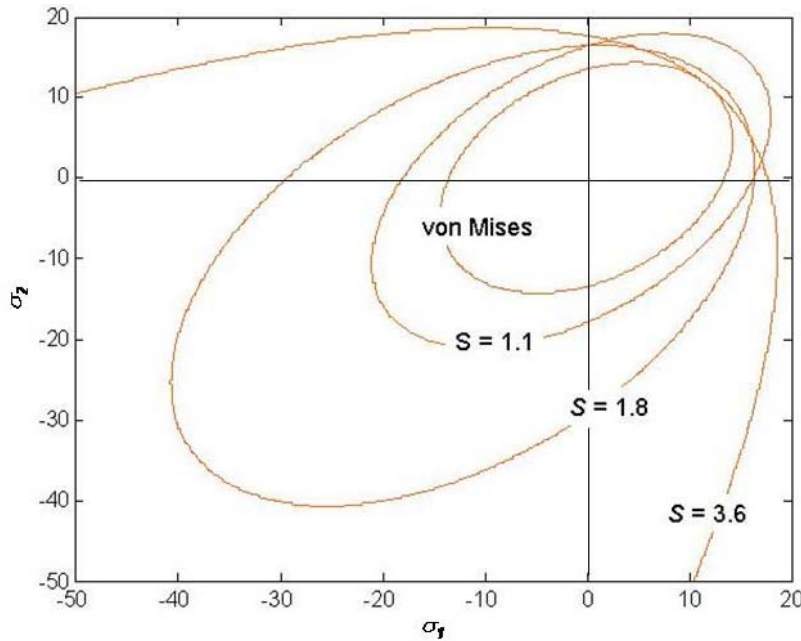


Figure 3-8: Raghava and von Mises yield surfaces for $\sigma_3=0$ MPa, $Y_t=16.5$ MPa and $S = 1.1, 1.8$ and 3.6 .

Comparing the Raghava and MOPP criteria shows that they are very similar for low values of S and f . However, for either highly porous materials, or high values of S the yield surface generated by the two criteria become very different. Figure 3-9 illustrates

how the Raghava and MOPP criteria change with respect to S . Figure 3-10 shows how the Raghava and MOPP criteria change with respect to increasing f .

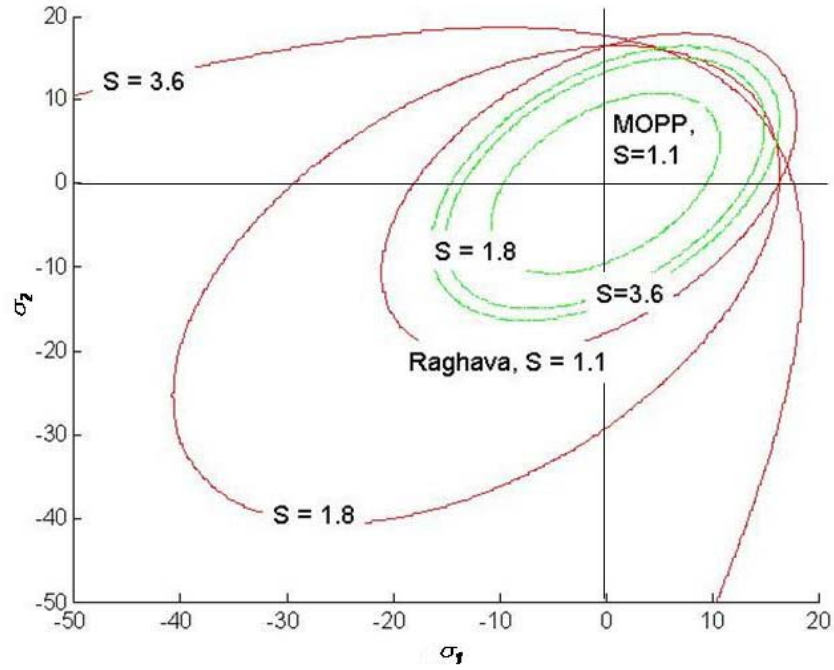


Figure 3-9: MOPP and Raghava surfaces plotted for $S = 1.1, 1.8$ and 3.6 . $\sigma_3 = 0$ MPa, $Y_t = 16.5$ MPa, $f = 0.1$.

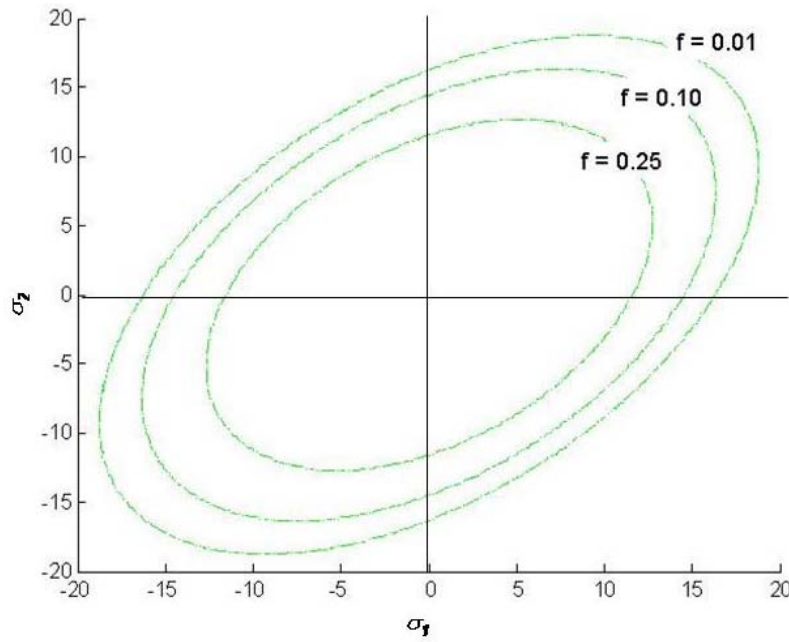


Figure 3-10: MOPP yield surface plotted for $f = 0.01, 0.10$ and 0.25 . $\sigma_3 = 0$ MPa, $Y_t = 16.5$ MPa, $S = 1.1$.

Understanding the behavior of the yield criteria each model uses is important because they help determine the strengths and weaknesses of a constitutive law based on that model. For instance, if it is necessary to estimate the stresses in an adhesively bonded part loaded primarily in tension, the von Mises law may provide better estimates for yielding and failure. However, if the loading on the part was due to compression or shear the von Mises model may be too conservative. In such a case, the Raghava or MOPP models maybe more appropriate. It is important to understand the ramifications of the choice of material model relative to the kind of loading expected on a part before selecting a constitutive law for a finite element model. There are other factors to consider when these models are implemented in FEA routines. The choice of element, mesh refinement and boundary conditions can have just as much of an impact in determining when the model will predict yield in a given part.

Revisiting the simple design example

The example in Chapter 1, using the assumed linear failure surface, leaves many questions unanswered. How do the more complicated models presented here perform in a similar, but more realistic, situation? Imagine that the napkin ring described in Chapter 1 is bonded with a new adhesive that has the properties described in Table 3-1 below.

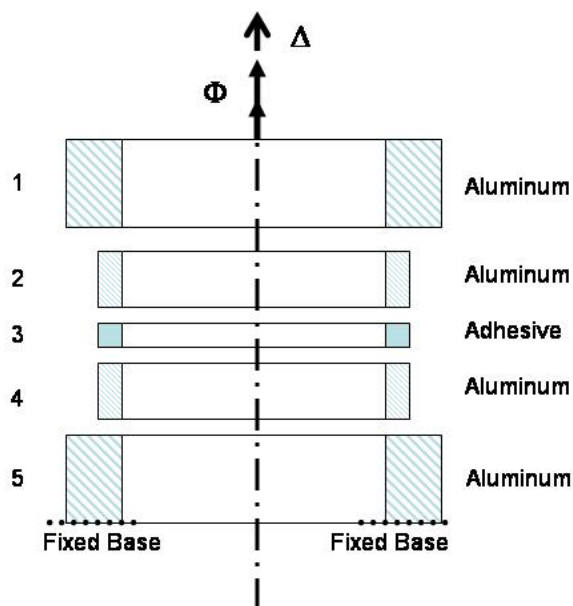


Figure 3-11: Adhesively Bonded Test Specimen

Table 3-1: Bulk Acrylic Adhesive Properties

Y_t	16.50	MPa
Y_s	10.00	MPa
E	1069.00	MPa
G	343.00	MPa
ν	0.433	
S	1.10	
f	0.10	

Imagine also that the part manufacturer has also provided some prototype test data with a new failure surface shown in Figure 3-11. The first example assumed that the relationship between shear and tension stresses in the material allowed path independent loading. For example, if the material was loaded in shear so that the shear strength of the material was not exceeded, and then loaded in tension so that the tensile strength was not exceeded, the stress state on the material would be the same as if both the shear and the tension had incrementally increased to the same levels. In reality materials and material failures are path dependent. Adhesive materials tend to be weaker in tension than in shear and loading an adhesive in tension will reduce the range of shear stress the material can handle before failure.

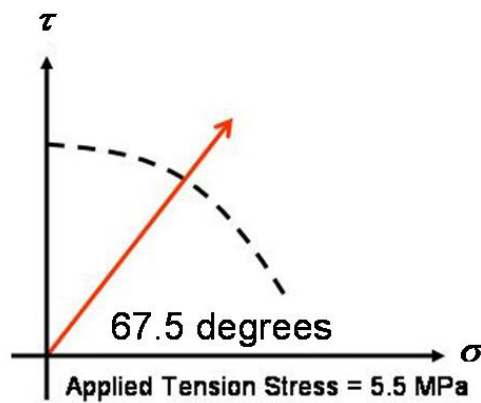


Figure 3-12: Failure Surface for Combined Loading

The prototype testing revealed that the worst load case occurs when the part is loaded in a combined state of shear and tension due to an axial displacement and a rotation. The specific combination of tension and shear that caused the most failures occurred when a load ray of 67.5° was applied to the part, at a measured tensile stress of

5.5 MPa.³ The part manufacturer wants to know which yield criterion to use when calculating the factor of safety for the part when it is loaded in this way.

The adhesive bond is designed to be 1 mm thick. The state of stress for a bulk element at the centerline of the bond can be determined given the load ray angle and the failure surface shown in Figure 3-11. The applied tension and shear stress are related by

$$\tau = \sigma_{APP} \tan \theta \quad (3-23)$$

where σ_{APP} is equivalent to the measured tensile stress at failure (5.5 MPa). The other stresses in the stress tensor for this element can be determined assuming some geometric constraints and assuming that the applied shear, τ , will be much greater than the shear stresses in the other directions. For instance, that the strain in the radial and circumferential directions, ε_r and ε_θ , are generated by Poisson effects because the bond substrate is much stiffer than the adhesive material.⁴ The strains in these directions can be calculated using the following equations, assuming a perfect bond,

$$\varepsilon_r = \frac{-\nu_s \sigma_{APP}}{E_s} \quad (3-24)$$

$$\varepsilon_\theta = \frac{-\nu_s \sigma_{APP}}{E_s} \quad (3-25)$$

where ν_s is the Poisson's ratio of the substrate and E_s is the Young's modulus of the substrate.

The part configuration will generate a bending stress in the adhesive layer. The tensile stress applied to the top of the part is not what the adhesive in the bond will see. Furthermore, assume that the manufacturer specified an average volume of voids caused by imperfections in the casting process, so the cross sectional area of the adhesive must be reduced. Timoshenko [89] analyzed similar bending problems and using his results the bending in this problem can be accounted for by increasing the applied axial stress in the adhesive layer by a factor K_{22} . The reduced cross sectional area can be calculated assuming an even distribution of voids throughout the adhesive so that the reduced area is equal to

³: Note that the load ray angle in this example is not the same as an equivalent Arcan angle.

⁴: Much of the stress analysis of the Napkin Ring reported here was taken from the research of Brian Painter, a graduate student in the Department of Mechanical Engineering at Virginia Tech.

$$A_{red} = A_{TOT} - fhV \quad (3-26)$$

where f is the void volume fraction, h is the bond thickness, V is the volume of adhesive in the bond assuming no voids and A_{TOT} is the cross sectional area for an adhesive without voiding. The applied axial stress for a bulk adhesive element at the centerline of the bond is then

$$(\sigma_L)_a = \sigma_L = \frac{K_{22}\sigma_{APP}}{A_{red}} = \frac{K_{22}\sigma_{APP}}{1 - \frac{fh}{A_{TOT}}} \quad (3-27)$$

Note that the applied axial stress in the adhesive layer will not be equal to the applied axial stress in the substrate. The radial and circumferential strains in the material can then be written using radial, circumferential and applied loads in the adhesive layer. It should also be noted that the radial and circumferential strains will not be equal to the applied strains from the substrate either. However, it is acceptable to use this approximation for the simple analysis explained here. In this case, ε_r and ε_θ are given by

$$\varepsilon_r = \frac{1}{E_a} [\sigma_r - \nu_a (\sigma_\theta + \sigma_L)] \quad (3-28)$$

$$\varepsilon_\theta = \frac{1}{E_a} [\sigma_\theta - \nu_a (\sigma_r + \sigma_L)] \quad (3-29)$$

where E_a and ν_a are the Young's modulus and Poisson's ratio of the adhesive and the subscripts r and θ denote the radial and circumferential stress terms. Substituting Equations (3-24) and (3-25) into Equations (3-28) and (3-29) and solving for σ_r and σ_θ gives

$$\sigma_r = \nu_a (\sigma_\theta + \sigma_L) - \frac{\nu_s \sigma_L E_a}{E_s} \quad (3-30)$$

$$\sigma_\theta = \nu_a (\sigma_r + \sigma_L) - \frac{\nu_s \sigma_L E_a}{E_s} \quad (3-31)$$

Substituting (3-31) into (3-30) yields the equation for σ_r in terms of σ_L and material constants. Collecting terms to use Equations (3-24) and (3-25) to simplify the end result yields

$$\sigma_r = \frac{\varepsilon_r E_a + \nu_a \sigma_L + \nu_a \varepsilon_\theta E_a + \nu_a^2 \sigma_L}{1 - \nu_a^2} \quad (3-32)$$

$$\sigma_{\theta} = \frac{\varepsilon_{\theta} E_a + \nu_a \sigma_L + \nu_a \varepsilon_r E_a + \nu_a^2 \sigma_L}{1 - \nu_a^2} \quad (3-33)$$

The full stress tensor generated using these assumptions is

$$\sigma = \begin{bmatrix} \sigma_r & \tau & 0 \\ \tau & \sigma_L & 0 \\ 0 & 0 & \sigma_{\theta} \end{bmatrix} \quad (3-34)$$

The eigenvalues of the matrix in (3-34) are the principal stresses for the adhesive element at the centerline of the bond. However, the stresses determined in (3-34) are only estimates based on geometric constraints and a perfect bond, which will tend to be an upper limit on the state of stress in the material. Assuming that the adhesive is not tied to the substrate in the radial direction negates the radial strain which makes σ_r equal to zero. This assumption is a lower bound for the state of stress and will generate a different estimate for the principal stresses. Arranging the principal stresses such that $\sigma_1 > \sigma_2 > \sigma_3$ and substituting them into Equations (3-8), (3-9) and (3-21) yields the plots shown in Figures 3-12 and 3-13. In each figure, the yield surfaces are plotted assuming a constant value for the third principal stress, so that the yield surfaces are on the same plane as the calculated principal stress combination taken from the prototype test data.

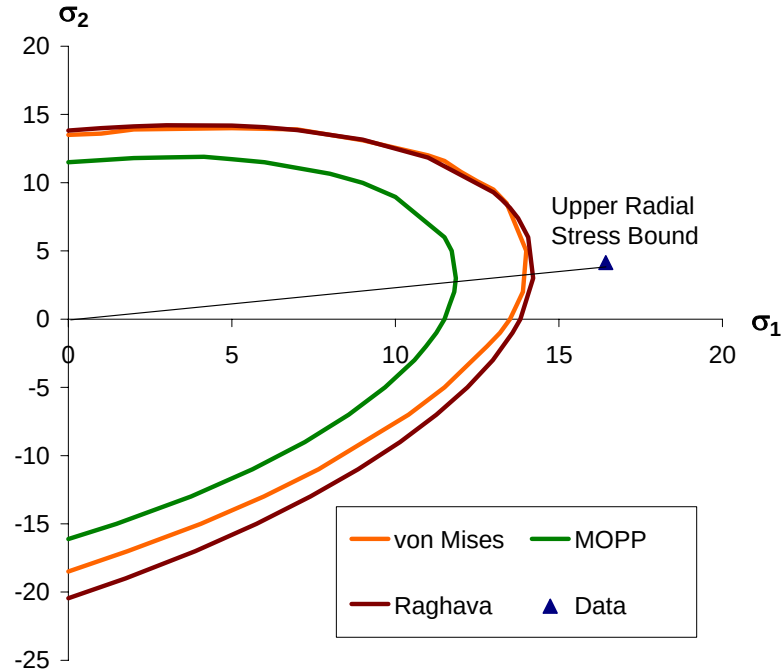


Figure 3-13: Plot of Yield Surfaces for $Y_t=16.5$ MPa, $S=1.1$, $f=0.1$, $\sigma_3=-7.5$ MPa.

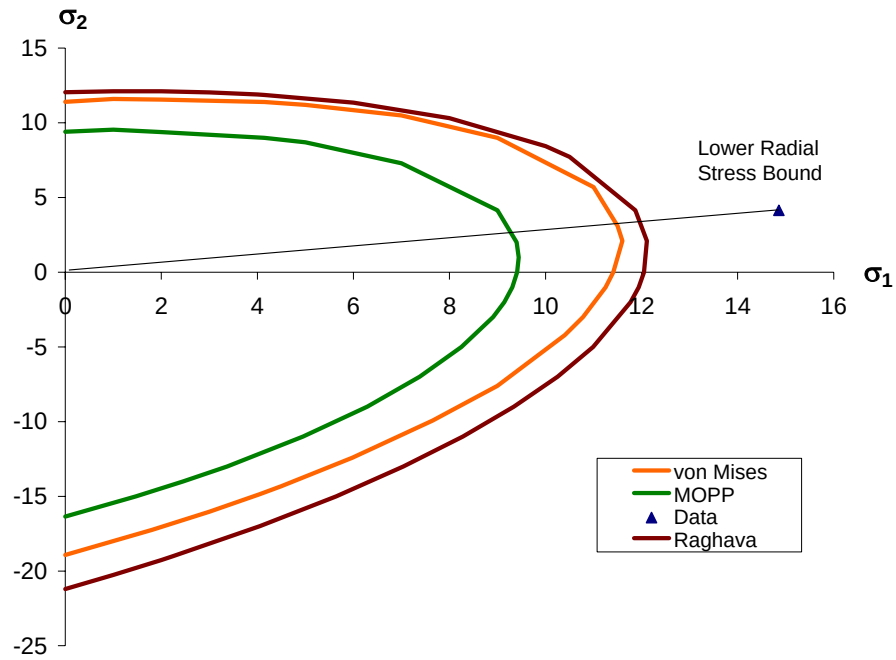


Figure 3-14: Plot of Yield Surfaces for $Y_t=16.5$ MPa, $S=1.1$, $f=0.1$, $\sigma_3=-5$ MPa.

Drawing a line from the origin in Figures 3-12 and 3-13 to the calculated data point shows the principal stress values that each yield criteria predicts the napkin ring adhesive will fail. The results for Figure 3-12 are summarized in Table 3-2 below. The results for Figure 3-13 are summarized in Table 3-3.

Table 3-2: Principal Stress Summary at Failure for Upper-Bound Case

	σ_1	σ_2	σ_3
Raghava	14.21	3	-5
von Mises	14.1	2.9	-5
MOPP	11.8	2	-5
Calculated	16.43	4.15	-5

Table 3-3: Principal Stress Summary at Failure for Lower-Bound Case

	σ_1	σ_2	σ_3
Raghava	11.9	4	-7.5
von Mises	11.5	3.1	-7.5
MOPP	9.4	2	-7.5
Calculated	14.85	4.15	-7.5

In each case, for each yield criteria, the distance from the origin to the edge of the yield surface defines the predicted strength limit of the adhesive. The length of the vector

from the origin to the edge of the yield surface is the available strength of the material as predicted by each yield criterion. The length of the vector is calculated using the equation

$$R = \sqrt{(\sigma_1)^2 + (\sigma_2)^2 + (\sigma_3)^2} \quad (3-35)$$

where R is the Residual, or Reserve Strength, of the material given a set of principal stress coordinates and a defined yield criteria. The ratio of the Reserve Strength to the Actual Strength defines the factor of safety for a material. The equation for the factor of safety in terms of R is

$$n = \frac{R_{DATA}}{R_{MODEL}} \quad (3-36)$$

where R_{DATA} is the length of the vector from the origin to the calculated failure point and R_{MODEL} is the length of the vector from the origin to the edge of a yield surface. The lengths of the vector R for the upper bound case for the test data, Raghava, von Mises and MOPP are 17.67, 15.36, 15.24 and 12.97, respectively. The lengths of the vector R for the lower bound case for the test data, Raghava, von Mises and MOPP are 17.15, 14.62, 14.08 and 12.19, respectively. The factor of safety, n , is defined so that $n > 1$ when a yield criteria predicts failure before the observed failure point. The factor of safety for each yield criterion is summarized in Table 3-4.

Table 3-4: Factor of Safety Summary

	Upper	Lower
Raghava	1.15	1.17
von Mises	1.16	1.22
MOPP	1.36	1.41

The lower radial stress bound predicts a higher factor of safety for each yield criteria. This analysis reveals that the Raghava yield criterion is the most conservative and the MOPP model is the least conservative. Unlike the previous example, this analysis gives the part manufacturer a variety of options. This analysis also gives the designer a simple way to compare different design options. The Raghava criterion is the best option if there is not a lot of confidence in the reported material data or the manufacturer wants to use the minimum amount of adhesive on the part. The MOPP model is the best choice if there is a lot of variance in the test data or if the manufacturer wants to design the part so that it will not fail in the adhesive layer. The von Mises criterion is not the most

accurate nor is it the most conservative. In this particular situation the von Mises criterion should be used only if the manufacturer wants the part to have a slightly higher safety factor than the Raghava criterion. If this analysis procedure were automated the factor of safety at each point in a load deflection record of the prototype could be evaluated.

Chapter 4

Development of a Nonlinear Elastic Law

The preceding Chapters were concerned with what existing analysis tools could be applied to the problem of modeling adhesive bonds. This Chapter distills the pros and cons of those approaches into a simple nonlinear elastic law and gives details on how to implement that law as either a non-FEA model or as a UMAT in ABAQUS. The concept of a nonlinear elastic law is not new. Nonlinear elastic laws were initially used to describe the behavior of many materials and also in incremental plasticity where each deformation step is modeled using a nonlinear curve [85]. The work of Ramberg and Osgood [86] proved that a nonlinear elastic relationship described by an exponential fit of

the strain in terms of stress accurately predicted the behavior of metals that exhibited shear flow at finite strains such as Aluminum. Besides modeling metals, nonlinear elastic laws are commonly used to model nonlinear springs or to provide a uniform smooth curve for fracture analyses in plastically deforming materials under monotonic loads [86].

General foundation for a nonlinear decoupled elastic Raghava law

The nonlinear decoupled elastic Raghava law (NDER) described here is specifically tailored to model ductile materials that have largely recoverable nonlinear deformations and where the tensile yield strength and the shear yield strength are not necessarily related by the von Mises relationship

$$Y_s = \frac{Y_t}{\sqrt{3}} \quad (4-1)$$

where Y_s is the shear yield strength and Y_t is the tensile yield strength. The relationship defined by (4-1) is not always true for highly ductile amorphous cross-linked polymers [97]. Assuming (4-1) is valid can add to the error in modeling such materials when they are loaded in shear. This error can become a problem because many structural adhesives are used in joints that are designed to carry the majority of the load in shear.

The law is unique because it effectively decouples the nonlinear behavior of the material under uniaxial loads and for the case of pure shear. But in the case of multi-axial loading the Raghava criterion couples the loads so that the yield surface changes to reflect the multi-axial state of stress. The general form of the law is

$$\tilde{\varepsilon} = (C_m + C_n f(\tilde{\sigma}))\tilde{\sigma} \quad (4-2)$$

where C_m is the compliance matrix for a linear material written in compressed notation so that it is only a 6 x 6 matrix and C_n is the nonlinear compliance matrix. C_n and C_m are written as

$$C_m = \begin{bmatrix} \frac{1}{E_m} & -\frac{\nu}{E_m} & -\frac{\nu}{E_m} & 0 & 0 & 0 \\ -\frac{\nu}{E_m} & \frac{1}{E_m} & -\frac{\nu}{E_m} & 0 & 0 & 0 \\ -\frac{\nu}{E_m} & -\frac{\nu}{E_m} & \frac{1}{E_m} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_m} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_m} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_m} \end{bmatrix} \quad (4-3)$$

$$C_n = \begin{bmatrix} \frac{\alpha f(\tilde{\sigma})^{n-1}}{E_m} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{\alpha f(\tilde{\sigma})^{n-1}}{E_m} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\alpha f(\tilde{\sigma})^{n-1}}{E_m} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\beta f(\tilde{\sigma})^{m-1}}{G_m} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\beta f(\tilde{\sigma})^{m-1}}{G_m} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{\beta f(\tilde{\sigma})^{m-1}}{G_m} \end{bmatrix} \quad (4-4)$$

where α , β , n and m are the Ramberg-Osgood curve fitting parameters. The function $f(\tilde{\sigma})$ is similar to the Raghava equivalent stress, defined in terms of the S ratio and the stress invariants, as

$$q = [J_1(S-1) + (J_1^2(S-1)^2 + 12J_{2D}S)^{1/2}]/2S \quad (4-5)$$

Which is equivalent to the Raghava equivalent stress described in Chapter 2 (see Equation 2-4), divided by the tensile yield strength, Y_t . The variables E_m , G_m and ν are the linear material properties of the bulk material: the bulk tensile modulus, the bulk material shear modulus and the bulk material Poisson's Ratio.

$\tilde{\varepsilon} = [\varepsilon_{xx} \quad \varepsilon_{yy} \quad \varepsilon_{zz} \quad \varepsilon_{yz} \quad \varepsilon_{zx} \quad \varepsilon_{xy}]^T$ and $\tilde{\sigma} = [\sigma_{xx} \quad \sigma_{yy} \quad \sigma_{zz} \quad \sigma_{yz} \quad \sigma_{zx} \quad \sigma_{xy}]^T$ are the strain and stress vectors, written in terms of Cartesian coordinates, respectively, for an element in the coordinate system shown in Figure 4-1 below.

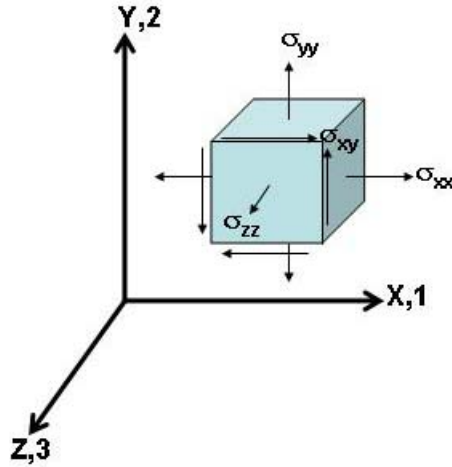


Figure 4-1: Orientation of reference axis and applied stresses on a bulk adhesive element.

This formulation directly addresses several of Yu's criteria [24] for effective empirical modeling of adhesive systems. The decoupled law is:

1. Sensitive to the hydrostatic stress
2. Allows for volume increasing nonlinear deformation
3. Allows for a variable instantaneous modulus under multi-axial loads.

It does not have any innate strain-rate variability but it is possible to address that concern by fitting multiple curves for the behavior of the material at different rates of strain using Marlow and Isabel's approach [90]. One assumption that does bear mentioning is that the nonlinear deformation is entirely recoverable. This means that even though there may be finite strains and significant nonlinear deformation there is no plasticity in the material and no energy is lost due to loading and unloading the material. Therefore NDER cannot be used to predict yield in the traditional sense and cannot be used to define a yield surface. For the purposes of modeling adhesive materials simply this assumption is permissible if the results of the model are checked against known experimental data for the range of stress and strain applied in the analysis.

In the event that the simple model predicts a stress or strain outside of the failure limits of the material then the simple model results should be checked against a more complex model that accurately accounts for nonlinear deformation and plasticity in the traditional sense.

An additional benefit of this model is the ability to independently adjust parameters according variations in data with respect to either tension or shear. The model can be fit to the results of a bulk tension test at different strain rates without directly affecting the shear behavior. Likewise, if a wide variation in the shear behavior of the model is shown in test results that can be accounted for by fitting different curves too. Updating one set of data does not affect the other set of data but it will affect the “yield surface” and change where the ultimate limit of the nonlinear curve. The ability to independently tune parameters to test results is particularly ideal for adhesives because many different formulations of a given adhesive are often developed based on particular goals for performance.

Characterizing NDER

The model can be characterized for any material with a largely nonlinear elastic response. Two bulk material tests are required to generate the nine constants that fit the model to the desired data. The two bulk tests are composed of a tension specimen test performed at a rate that is representative of the loading the material will see and a notched beam shear or Iosipescu specimen tests also tested under appropriate conditions. The bulk material tests supply five of the constants in the model: E_m , G_m , ν , S and Y_t . The remaining four parameters (α , β , n , m) are chosen based on the best fit of the nonlinear curves to each of the stress-strain curves in pure tension and pure shear. Table 4-1 summarizes the required inputs for the model and the pertinent ASTM standards that govern each test.

Table 4-1: Decoupled Law Input Parameters.

Test	Data	Reference
Bulk Tension Test	E_m, ν, Y_t	ASTM D 638
Bulk Shear Test	G_m, Y_s	ASTM D 5379/5379M-98
Hydrostatic Stress Sensitivity Test	S	J.Adhesion, 1993, Vol. 42, pp. 209-223

The parameters S and Y_t are important because they define the bounds of the model and are necessary because the Raghava equivalent stress is used in the model. Crocombe et al. [88] have provided a method to determine S for a bulk adhesive using four-point bend specimens. Another method for determining S comes from the use of the Raghava

equivalent stress in the model for $f(\tilde{\sigma})$. Consider the case of pure shear on a bulk adhesive element so that the principal stresses on the element are

$$\sigma_1 = \tau, \sigma_2 = -\tau, \sigma_3 = 0 \quad (4-6)$$

The first stress invariant, J_1 , is then equal to

$$J_1 = \tau + (-\tau) + 0 = 0 \quad (4-7)$$

and the second deviatoric stress invariant, J_{2D} , becomes

$$J_{2D} = [(\tau - (-\tau))^2 + (-\tau - 0)^2 + (0 - \tau)^2] / 6 = \tau^2 \quad (4-8)$$

Substitution of J_1 and J_{2D} into Equation (2-4) gives

$$\sigma_{eq} = \frac{1}{2S} [0 \cdot (S - 1) + [0^2 \cdot (S - 1)^2 + 12\tau^2 S]^{1/2}] = \tau \sqrt{\frac{3}{S}} \quad (4-9)$$

The quotient of the equivalent stress divided by the yield strength is equal to one at yield. Using the result from Equation (4-8) and setting the quotient equal to one defines the relationship between the shear yield strength and the tensile yield strength for this model.

$$\frac{\sigma_{eq}}{Y_t} = \frac{\tau}{Y_t} \sqrt{\frac{3}{S}} = \frac{\tau \sqrt{3}}{\sqrt{Y_c Y_t}} = \frac{\tau}{\sqrt{\frac{1}{3} Y_c Y_t}} \rightarrow \frac{Y_s}{\sqrt{\frac{1}{3} Y_c Y_t}} = 1 \rightarrow Y_s = \frac{\sqrt{Y_c Y_t}}{\sqrt{3}}$$

Using S in the relationship instead of Y_c gives a more familiar result

$$Y_s = \sqrt{\frac{S Y_t^2}{3}} \quad (4-10)$$

which is equal to the von Mises relationship described in Equation (4-1) for S equal to one. Five of the nine required constants can then be easily determined directly by two simple bulk material tests. The fit parameters are left to be found using any method available to the user, such as a multiple regression least squares fit.

Applying NDER to the case of Uniaxial Tension and Compression

The model clearly gives a reasonable result for the case of pure shear. It remains to check that the model gives a reasonable result for the cases of uniaxial tension and uniaxial compression. For the case of a uniaxial stress, $\sigma_1 = \sigma$, applied in the positive direction on the face of a bulk material element J_1 and J_{2D} become

$$J_1 = \sigma_1 + \sigma_2 + \sigma_3 = \sigma + 0 + 0 = \sigma$$

$$J_{2D} = [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] / 6 = [(\sigma - 0)^2 + (0 - 0)^2 + (0 - \sigma)^2] / 6 = \frac{\sigma^2}{3}$$

Substituting J_1 and J_{2D} into Equation (2-4) gives the result

$$\sigma_{eq} = \frac{1}{2S} \left[\sigma \cdot (S - 1) + [\sigma^2 \cdot (S - 1)^2 + 12 \frac{\sigma^2}{3} S]^{1/2} \right] = \sigma \quad (11)$$

Dividing (4-11) by Y_t and shows that the model predicts a value of one when the applied stress is equal to the yield strength in tension. Performing the same series of calculations for $\sigma_1 = -\sigma$ yields

$$J_1 = -\sigma \quad (12)$$

$$J_{2D} = \sigma^2 / 3 \quad (13)$$

$$\sigma_{eq} = \sigma / S \quad (14)$$

Again, dividing the equivalent stress by Y_t gives

$$\frac{\sigma_{eq}}{Y_t} = \frac{\sigma}{Y_t \left(\frac{Y_c}{Y_t} \right)} = \frac{\sigma}{Y_c} \quad (15)$$

This is reasonable because the model predicts a value of one when the applied compression stress is equal to the compressive yield strength of the material.

Testing NDER against von Mises for Multi-axial loading

The cases of pure tension, compression and shear are important but it is how the model behaves under multi-axial conditions that ultimately determine if it is useful. It is equally important to compare the new model's behavior against established mechanical models. These questions must be addressed before developing the model for use in a commercial finite element software package. This was accomplished by programming Equation (4-2) into Mathematica™ and testing the model on a bulk material element similar to the one shown in Figure 4-1. This approach is consistent with the methods used by Lee and Oung [65] in their work to verify the constitutive model they developed.

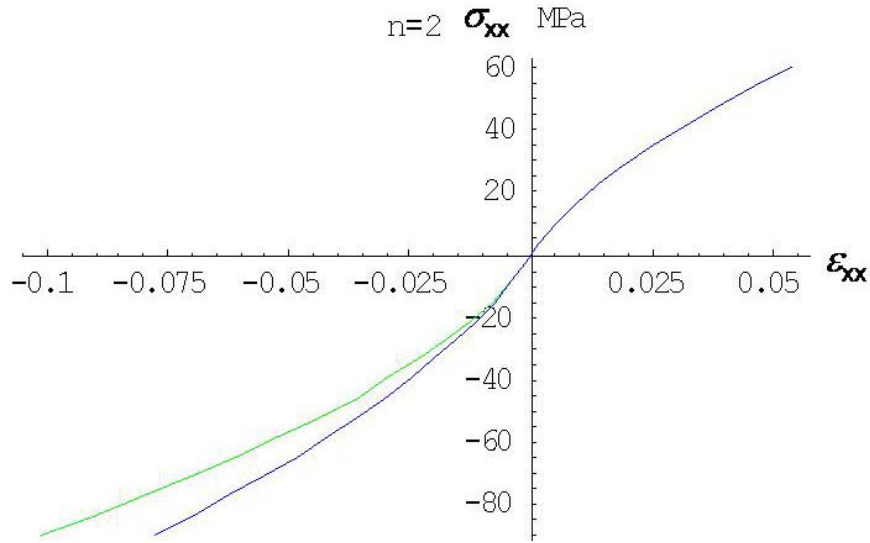
In order to test the new model assumptions against established mechanical models, two different functions were used for $f(\tilde{\sigma})$. In the first case the $f(\tilde{\sigma})$ function was the von Mises equivalent stress divide by the yield strength in tension, Y_t . In the

second case the $f(\tilde{\sigma})$ function was the Raghava effective stress divided by the yield strength in tension, Y_t . Applying the first choice of $f(\tilde{\sigma})$ to Equation (4-2) describes a general multi-axial version of the Ramberg-Osgood model [87]. Applying the second choice to Equation (4-2) describes the NDER model previously discussed. Table 4-2 summarizes the values of the constants input to the NDER and Ramberg-Osgood models in MathematicaTM.

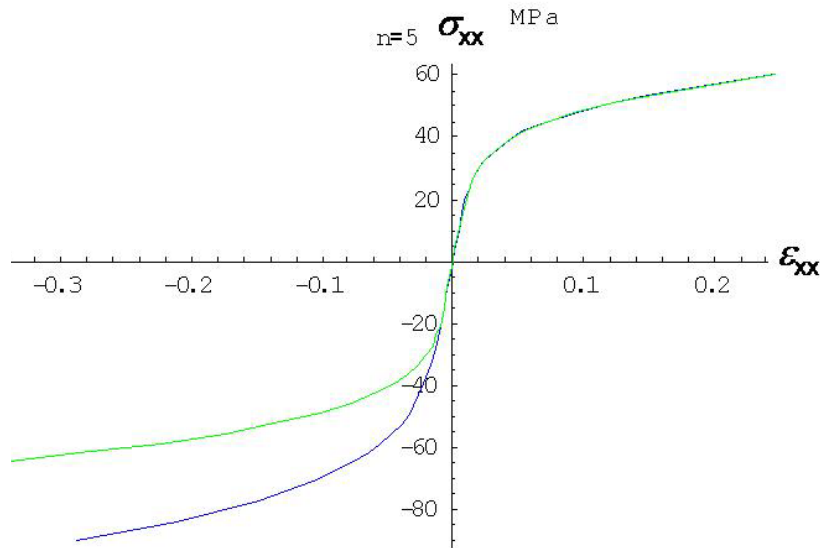
Table 4-2: Input Summary for NDER Test Model.

E_m	2290.00	MPa
G_m	790.00	MPa
ν	0.3840	
α	0.5267	
β	0.7955	
Y_t	30.00	MPa
Y_c	48.40	MPa
S	1.613	

The test model results for uniaxial tension, compression and pure shear are summarized in Figures 4-2 and 4-3.



(a)

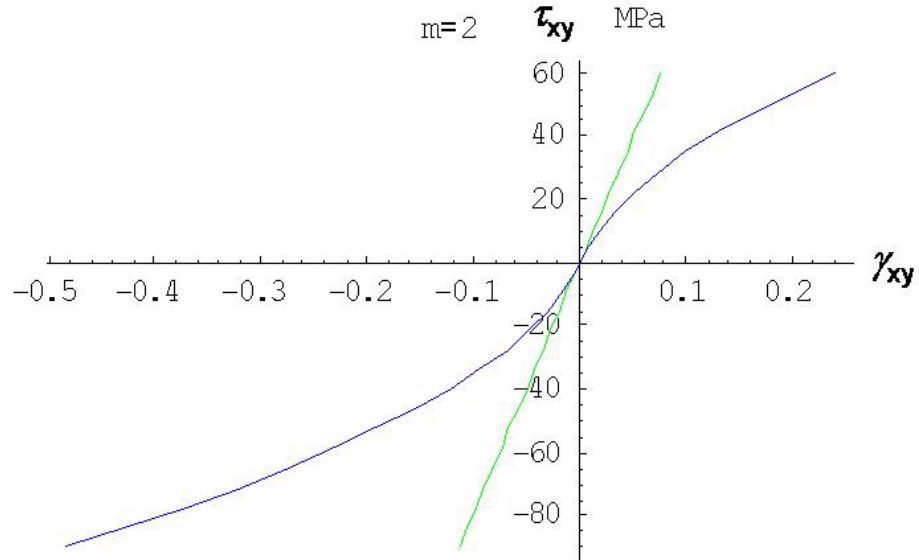


(b)

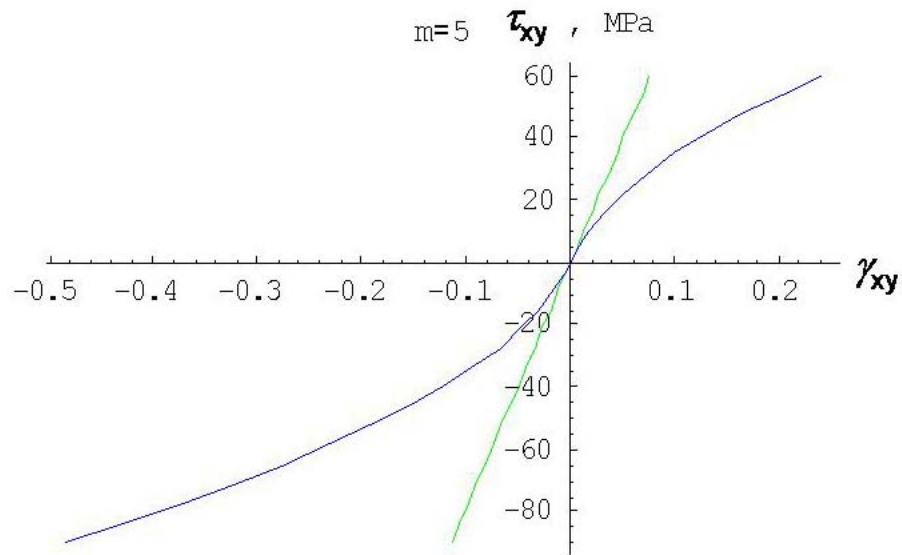
Figure 4-2: Case of Uniaxial Tension and Compression, values of σ_{xx} versus ϵ_{xx} plotted. (a) $n=2$, (b) $n=5$ with the Ramberg-Osgood result plotted in green and the NDER result plotted in blue.

Figure 4-2 illustrates the first consequence of using the Raghava equivalent stress rather than the von Mises equivalent stress. Given an applied compressive stress the NDER model predicts a larger stress than the Ramberg-Osgood model. This behavior is acceptable because many polymeric and adhesive materials are stronger in compression than in tension. The nonlinear portion of the stress response is governed by the n

parameter, which the plots show for n equal to 2 and 5. In either the Ramberg-Osgood model or NDER the n parameter is a constant integer greater than 1. A high value of n will result in a very nonlinear response whereas a low value of n will result in a roughly linear response. Figure 4-3 compares the different model predictions for an applied shear stress.



(a)



(b)

Figure 4-3: Case of Pure Shear, values of σ_{xy} versus ϵ_{xy} plotted. (a) $m=2$, (b) $m=5$ with the Ramberg-Osgood result plotted in green and the NDER result plotted in blue.

Figure 4-3 illustrates the most significant difference between the two models. At any value of m and β the Ramberg-Osgood model will predict a much greater shear stress in the material than the NDER model. In many cases this means that the Ramberg-Osgood model over-estimates the shear response of the material. It is important to note that in the NDER model the shear response can be fit directly to the observed material behavior regardless of the tension response.

Figure 4-4 illustrates how the NDER model behaves under an applied axial and shear stress. The result is a plot of the axial stress, σ_{xx} , versus the axial strain, ϵ_{xx} , for an applied shear stress ranging from one-third to four times the tensile yield strength.

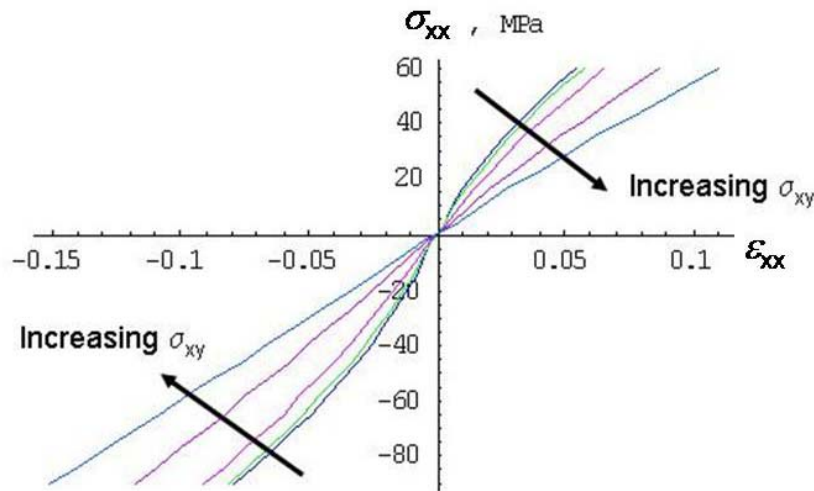


Figure 4-4: Multi-axial applied loads. Plot of σ_{xx} versus ϵ_{xx} for values of $\sigma_{xy} = 10, 20, 40, 80, 120$ MPa with $n=2$ and $m=5$.

An interesting result of the combined load effect is that the applied shear stress decreases the nonlinear response in the axial direction. The initial slope of the stress-strain response varies significantly with the applied shear stress. The variable slope demonstrates that the model is affected by a multi-axial state of stress. Figure 4-5 demonstrates how the NDER model behaves given two applied normal stresses, σ_{xx} and σ_{zz} , and an applied shear stress, σ_{xy} . The applied normal stress, σ_{zz} , ranges from one-third to four times the tensile yield strength.

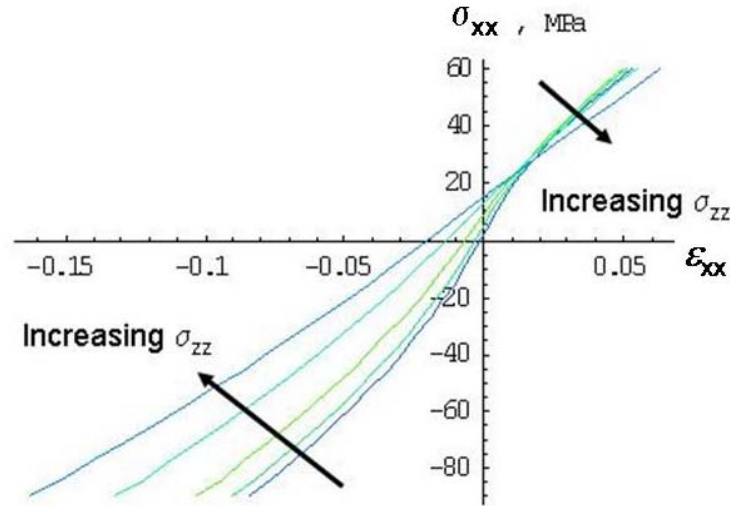


Figure 4-5: Multi-axial applied loads. Plot of σ_{xx} versus ϵ_{xx} for values of $\sigma_{xy}=10$ MPa and $\sigma_{zz} = 10, 20, 40, 80, 120$ MPa with $n=2$ and $m=5$.

In this case the NDER model has shifted the stress strain curves into the upper right quadrant of the σ_{xx} - ϵ_{xx} plot. Again the state of multi-axial stress on the element decreases the nonlinear response of the model as the out of plane stress increases. The test model comparison indicates that the NDER model behaves similarly to other mechanical laws. However, it also shows that the NDER model is different in ways that make it advantageous to use when predicting the nonlinear behavior of adhesive materials described in Chapters 2 and 3.

Implementing NDER in ABAQUS

It is possible to implement any material model not already in the ABAQUS material library in the form of a user-defined material law, or UMAT [84]. The code for a UMAT should be written in FORTRAN to ensure that it is fully compatible with the ABAQUS compiler. The code for a UMAT must define two things: the stress tensor and how it changes, *DSTRES*, as the analysis progresses and the material Jacobian, *DDSDDE*, in terms of the change in the change in stress per change in strain as the analysis progresses. The UMAT can also define how the state variables are updated during the analysis and the strain energy in the material. However those functions do not affect the solution and if they are neglected in the UMAT code ABAQUS will simply ignore them during the analysis.

It is at this step of the problem that model must pay the price for using the compliance form of the constitutive law rather than the stiffness form. The compliance matrix of Equation (4-2) must be inverted to solve for the stresses in terms of the strains and the Jacobian in terms of the change in stress per change in strain. This requires a code that can invert a matrix that may be singular, is fully populated and cannot be inverted symbolically. Fortunately, years of research by numerical analysts has produced a wealth of numerical code capable of doing matrix inversions under those conditions [91]. The following steps must then be taken to complete the required code for the ABAQUS UMAT.

1. Calculate the stress terms by taking the numerical inverse of the compliance matrix in (4-2) and formulating the evolution of the stress terms as a function of the change in the strain terms using a stable operator. In this case the central difference operator was chosen.
2. Again apply a stable, efficient operator to Equation (4-2) so that the way the stress and strain evolve as the analysis progresses is defined. Then, take the derivative of compliance matrix in (4-2) with respect to each of the stresses and invert the matrix numerically.
3. Finally, use (4-2) to calculate the complementary strain energy because the typical linear Equation [89] does not apply to a nonlinear stress-strain curve.

Most of these operations can also be developed in MathematicaTM, which has a feature to export the symbolic code to a text file as FORTRAN code.

Applying NDER to the Design Example

How would an engineer use a nonlinear elastic law to model adhesives? What useful information can NDER provide? NDER cannot be used to predict yield but the model can be used to estimate stresses and strains due to applied displacements and loads. The results of the finite element analysis can then be used in any yield criteria desired.

Suppose that an engineer wanted to get a simple estimate of the stresses in the adhesively bond part described in Chapter 1. The same prototype data is available as in

the Chapter 3 example and the part manufacturer simply wants estimates of the axial and shear strains assuming that the material is nonlinear for the worst load case at 67.5 degrees and a 5.5 MPa tension stress applied to the entire fixture. Table 4-3 summarizes the model inputs from the bulk material data. Table 4-4 summarizes the model inputs based on fitting the NDER model to the uniaxial tension and bulk shear test stress-strain curves.

Table 4-3: Summary of Bulk Material Properties

Y_t	16.50	MPa
Y_s	10.00	MPa
E	1069	MPa
G	343	MPa
ν	0.433	
S	1.100	

Table 4-4: Summary of NDER Fit Parameters

α	0.5
β	10
n	125
m	55

Substitution of the bulk material properties into Equations (4-3) and (4-4) gives

$$C_m = \begin{bmatrix} 9.35 & -4.05 & -4.05 & 0 & 0 & 0 \\ -4.05 & 9.35 & -4.05 & 0 & 0 & 0 \\ -4.05 & -4.05 & 9.35 & 0 & 0 & 0 \\ 0 & 0 & 0 & 29.15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 29.15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 29.15 \end{bmatrix} \cdot 10^{-4} \quad (4-16)$$

$$C_n = \begin{bmatrix} 4.68 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4.68 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4.68 & 0 & 0 & 0 \\ 0 & 0 & 0 & 29.15 & 0 & 0 \\ 0 & 0 & 0 & 0 & 29.15 & 0 \\ 0 & 0 & 0 & 0 & 0 & 29.15 \end{bmatrix} \begin{Bmatrix} f(\tilde{\sigma})^{124} \\ f(\tilde{\sigma})^{124} \\ f(\tilde{\sigma})^{124} \\ f(\tilde{\sigma})^{54} \\ f(\tilde{\sigma})^{54} \\ f(\tilde{\sigma})^{54} \end{Bmatrix} \cdot 10^{-4} \quad (4-17)$$

The highest stresses and strains will come from using the upper bound limit on the radial stresses in the adhesive. Calculating $f(\tilde{\sigma})$ by substituting $\sigma_1=16.43$ MPa, $\sigma_2=4.15$ MPa and $\sigma_3=-5$ MPa into Equation (4-11) and then dividing the result by Y_t yields

$$f(\tilde{\sigma}) = \frac{\sigma_{eq}}{Y_t} = \frac{18.48}{16.50} = 1.12 \quad (4-18)$$

Assuming the same load state as in the previous example and using the results from Equations (4-16), (4-17) and (4-18) in Equation (4-2) gives

$$\begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{xy} \end{bmatrix} = \begin{bmatrix} 602.313 & -4.05 & -4.05 & 0 \\ -4.05 & 602.313 & -4.05 & 0 \\ -4.05 & -4.05 & 602.313 & 0 \\ 0 & 0 & 0 & 13285.1 \end{bmatrix} \begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{xy} \end{bmatrix} \cdot 10^{-4} \quad (4-19)$$

Solving for the strains at failure for the part using the assumed load state reveals that the NDER model estimates that the adhesive is failing at an axial strain of 6.06%. The shear strain at failure is estimated to be

$$\varepsilon_{xy} = \frac{\Delta l}{h} \cong 1.33$$

$$\gamma = \tan^{-1}\left(\frac{\Delta l}{h}\right) \cong 0.926$$

or roughly 93%. Note that it is not uncommon for ductile adhesive materials to fail at large shear strains and yet be fairly brittle in tension. The ability to characterize the response in the decoupled manner illustrated here is useful in such cases.

This example shows that NDER is useful for obtaining “first cut” estimates of the stress and strain in an adhesive material in a multi-axial load state. Using the NDER model provides a quick way to get model results and insight that will be useful when implementing more complex nonlinear material models. If one of the material models discussed in Chapter 3 is not capable of reliably predicting results over the range of shear strains estimated by the NDER model then it can be quickly discarded. NDER is not intended for use as the sole material model in a detailed analysis of an adhesive material.

Chapter 5

Bulk Material Behavior in Tension and Shear

A model is only as good as the assumptions in it and the errors those assumptions create. The current Chapter compares the predicted response of the material models discussed in Chapters 3 and 4 to test data and discusses the difference between the models and the data. DOW Automotive developed the acrylic adhesive used for the tests described in this Chapter. The adhesive is a novel acrylic formulation designed to adhere to low-surface-energy materials and will be referred to as LESA. A uniaxial tension test and a thick bond Arcan shear test are compared against FEA results. Both of these specimens test the bulk response of the LESA to uniaxial tension and shear. The next

Chapter discusses the behavior and predictions of the models for in-situ materials and more complex load conditions.

Bulk Material Characterization Issues

It is impossible to make anything more than an educated guess about material behavior without data. Likewise, it is equally impossible to validate material models without using correct input and comparing the finite element results to the test results correctly. It is useful for analysts to understand test related issues and sources of error in data. The following is a brief review of the types of test specimens, geometries and conditions used to obtain bulk material data. The reader interested in more information should consult the appropriate ASTM standard.

Uniaxial Tension Test

This test procedure is described in ASTM D-638 and details the test method and preparation for a waisted, or “dog bone”, tension specimen machined out of a single kind of material. Figure 5-1 shows a typical uniaxial test specimen. Adhesive specimens for bulk uniaxial tension tests may be machined out of a platen of cured material or they could be cured in a mold. An analyst is concerned about two things regardless of the method used to procure the specimens: discontinuities in the test section and inconsistencies in the test method. It is unreasonable to expect finite element results to closely agree with tension test data obtained from specimens with large voids in the test section or incomplete curing. Likewise, it is unreasonable to expect finite element results to agree with tension test data if the reported displacement or load rate was not used, if the specimen dimensions were not reported correctly or if the specimen was not placed in the testing machine grips in a consistent fashion. For these reasons it is best if the analyst is also the person performing the material characterization tests. However, if time or project constraints do not allow for that level of involvement a brief list of concerns that may skew test results are presented here.

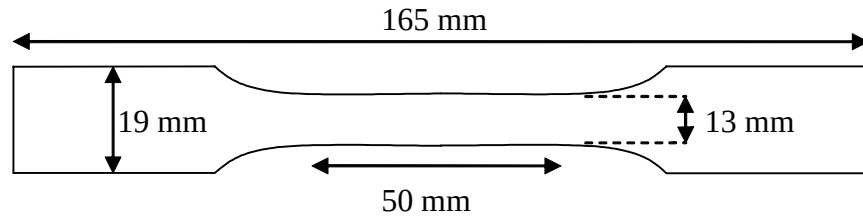


Figure 5-1: Schematic of a typical bulk uniaxial test specimen.

Quasi-Static testing: In order for a load test to be considered quasi-static the load must be applied slow enough that rate-dependent effects do not dominate the test results. This may require testing specimens at significantly slower rates than expected. By way of example, some of the test results used in the models for this thesis came from specimens loaded at a rate of 0.05 mm/min, which meant that a single specimen often took more than an hour to test to failure.

Rate-dependent testing: Although this subject is not covered in the current work it bears mentioning. Care should be taken to match the proper rate to the correct FEA model. The analyst should also realize that the displacement rate of the test grips is not necessarily the displacement rate of the material in the test section. Experimental results should be questioned if no strain gages or extensometers were used to obtain rate-dependent test results for rate-dependent materials.

Gage Creep: Materials that are highly viscous exhibit creep phenomena. This is true regardless of the displacement rate used and may affect the quality of the data from the test. The creep behavior may be important if the gage pins, knife edge, or extensometer attachments excessively deform the test section before the test begins. In some cases, the gage pins may sink deep into the test section before the test starts which will skew any recorded strain data. A possible remedy for this problem is to attach a strain gauge or extensometer to the test specimen and wait until the gage has reached a steady state of penetration. If gage creep is a problem then the test set-up may have to be reconsidered.

Failure Outside of Test Section: In uniaxial tension tests the specimen is supposed to fail within the waisted test section. Voids, microfractures or cracks due to gage pins may lead to failure outside of the test section. Data from such tests should be discarded.

Strain Gage Saturation: The limit of biaxial strain gages may be reached or exceeded by uncured or very ductile adhesives. In some cases it may be sufficient to use the global or crosshead displacement data for strain calculations once this has occurred. If the plastic Poisson's ratio is not required for material model identification then the biaxial strain data from the initial linear response is all that is required. However, if the plastic Poisson's Ratio is required, or if a plot of the lateral strain to the axial strain is not linear in the small strain regime, the strain data should be questioned. Calculating the plastic Poisson's Ratio should be done using post-yield strain data, however, a good estimate can be obtained provided the test section has experienced some plastic deformation. Furthermore, a reported value of Poisson's Ratio, or the plastic Poisson's Ratio, greater than 0.5 should be considered suspect.

Bulk Shear Test

It is difficult to obtain consistent and accurate bulk shear data for some adhesive materials. The two test methods used throughout the course of this work have different sources of error. The bulk notched beam, or Iosipescu specimen, is taken from ASTM D 5379 which was originally intended for use on composite materials. The thick bond Arcan specimen is a slight modification of a popular test used for many different kinds of materials. Figures 5-2 and 5-3 show typical specimens from each test. The most important considerations in comparing shear test results to finite element analysis are proper boundary conditions and mesh quality.

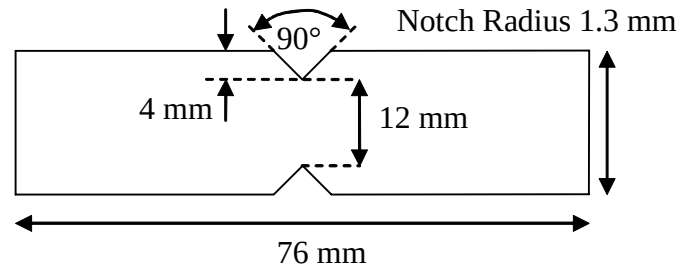


Figure 5-2: Sketch of a Typical Iosipescu Bulk Shear Specimen.

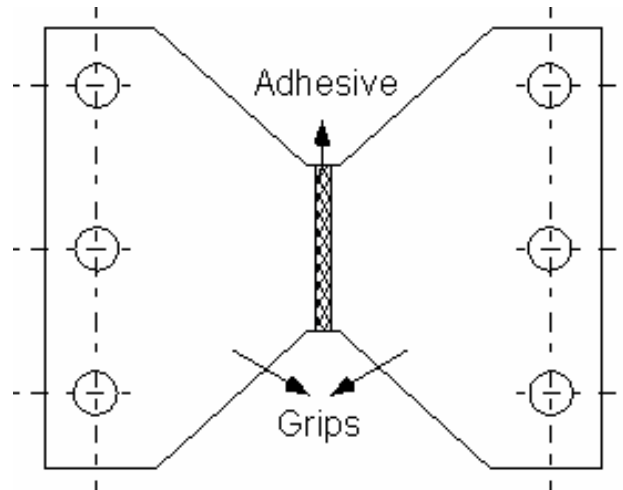


Figure 5-3: Sketch of a typical Arcan specimen.

In the case of the Iosipescu specimen it is difficult to model the test without either multiple contact conditions or constraining several degrees of freedom at the edges. The Arcan specimen is very large compared to the test section and a fine higher order mesh is required to model the test section without distorting the elements in the test section. Several other sources of finite element or test error are listed below.

Shear-Strain Gage Delamination: In the Iosipescu specimen the strain gage may delaminate early in the test. This is especially problematic for adhesive materials because failure in shear may not occur until 100%, or larger, shear strain. If the strain gage delaminated post-yield, the data may be acceptable and the crosshead displacement can be used for shear strain values later in the stress-strain history. The data should be discarded if the gage delaminated pre-yield.

Bending: In both the Iosipescu and Arcan test configurations specimen bending is a concern. Ideally the bending in either should be limited to 10% or less of the total stress contribution. A mechanical analysis will reveal if the test specimen has too large of a moment arm.

Specimen Constraints: How thick should an Arcan bond be to mitigate edge effects in the center of the test section? How long does an Iosipescu beam need to be to eliminate edge effects? The answer to both of those questions will vary for each type of material tested. If the specimen is not sized properly the material in the test section will be too constrained and will not experience a load state representative of pure shear.

Out-of-Plane Alignment: The Iosipescu specimen can be positioned at an angle in the test grip. Likewise, the Arcan specimen can be bonded out of alignment. Each of these conditions will produce warping and may lead to the specimen buckling. Care should be taken so that the specimens are prepared and placed in the same fashion each time. Data from failures due to buckling instead of shear should be discarded.

It is always helpful to understand the test procedure and realize potential sources of error in the data. The people involved with obtaining and analysis of test data should ideally have a good working relationship.

Bulk Tension Results

Uniaxial tension specimens were prepared according to ASTM D-638 by machining the specimens from a cured platen of LESA. The LESA was allowed to air cure for 24 hours, the specimens were then heat cured in an oven following a pre-determined curing cycle. The cure cycle was determined from curing modulus tests performed using thermal graphic analysis (TGA). The cycle ramped the temperature from room temperature to 80 C at a rate of 2.4 C/min and then back again to room temperature over a period of 30 hours. The specimen dimensions were 80 mm long by 20 mm wide by 1 mm thick with a 50 mm long by 4 mm wide by 1 mm thick test section. The specimens were tested on Instron 4505 load frame under displacement control with a 5 kN load cell

at a nominal rate of 0.05 mm/min. The displacement rate was monitored so that the displacement experienced by the test section was applied at a constant rate. Figure 5-4 shows the results of the experiment.

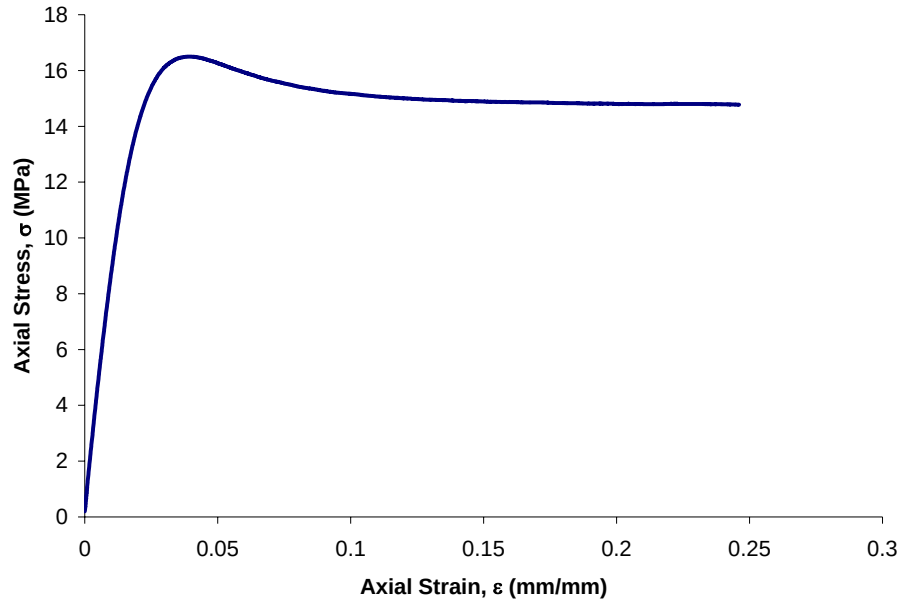


Figure 5-4: Plot of tension test data at 0.05 mm/min for quasi-static comparison.

Yield is considered to have occurred at the first point where the measured strain increases without a related increase in the calculated stress. The yield strength of LESA, at this load rate, is 16.5 MPa. The specimens yielded at 4% nominal strain and ruptured at 24% nominal strain. The specimens all failed within the waisted test section and there appeared to be few voids in the material. The biaxial strain data was accurate to nearly 3% nominal strain but after that the lateral strain gage readings became saturated. The elastic Poisson's ratio of the cured LESA was 0.433. The plastic Poisson's ratio of the cured LESA, at a nominal strain of 3% and an estimated nominal plastic strain of 1.5%, was 0.401. The average Young's Modulus was 1069 MPa.

Bulk Tension Comparison

The bulk tension results were compared to four different models. A modified form of the Gurson porous plasticity law (MOPP), a viscoplastic law (NHVP), an incremental plasticity law (Raghava) and a nonlinear decoupled elastic law (NDER)

implemented as model in Mathematica™. Each model used CPS8H elements (hybrid quadratic plane stress elements) with full integration, except for the MOPP model which used CPE8H elements (hybrid quadratic plane strain elements). Please review Chapters 3 and 4 for detailed explanations of the behavior of these models. See Appendix A for details on the ABAQUS Standard model input parameters used to achieve the results described in this Chapter.

A simple mesh and boundary conditions were used to represent the load state on an element of adhesive material in the test section. Figure 5-5 shows the boundary conditions and the mesh used for all of the tension models. Multiple elements were used to model the test section even though a single element would have been sufficient to approximate the nature of the uniaxial tension test. However, it would have been more difficult to accurately represent the boundary conditions or discover any preliminary convergence issues if only a single element had been used.

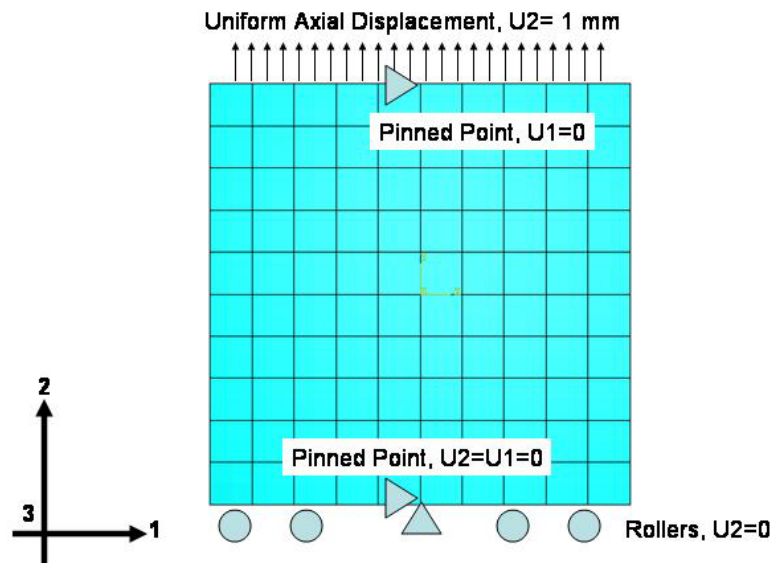


Figure 5-5: Model boundary conditions and mesh.

The nominal axial stress and strain were determined by operating on the history output of the ABAQUS data file. The axial stress was calculated by summing the reaction forces in the vertical direction along the top boundary edge of the model and dividing by the cross sectional area. The engineering axial strain was determined by dividing the change in length by the original, undeformed length of the model. Equations (5-1) and

(5-2) describe how these calculations were performed using ABAQUS and the history output function in ABAQUS CAE.

$$\sigma = \frac{\sum RF2_{TOP}}{A} \quad (5-1)$$

$$\varepsilon = \frac{\Delta U2_{TOP}}{h} \quad (5-2)$$

In Equation (5-1) the $\sum RF2_{TOP}$ term refers to the sum of the reaction forces in the axial or 2 direction, according to the global coordinate system, taken at the top edge of the model where the uniform displacement is applied. In Equation (5-2) the $\Delta U2_{TOP}$ term refers to the difference between the displacement of the top edge in the axial or 2 direction, according to the global coordinate system, with respect to the initial undeformed configuration. Equations (5-1) and (5-2) are implemented in a post-processing routine using the ABAQUS/CAE time history data functions.

The FEA model results approached the estimated yield strengths predicted by the yield criteria. It should be noted that voiding was an issue during the specimen preparation. The voiding in the specimens worsened when the LESA was highly constrained during casting. In order to minimize the voids in the test specimens all of the tension specimens were cast so that the LESA adhesive was free to expand or contract between two plates with lubricated surfaces. The specimens were then machined from the cured LESA platen so that there were no visible voids on the exterior of the tension specimens. Therefore, all models assumed a nearly perfect cross section with 0.1% voids by volume ($f=0.001$). See Table 5-1 for a summary of the yield strength comparison for each yield criteria and Table 5-2 for a summary of the yield strengths based on the FEA results.

Table 5-1: Percent Difference between Predicted and Observed Yield Strength.

	Est. Yield	% Difference
von Mises	16.50	0.00
MOPP	16.49	0.00
Raghava	16.50	0.00

Table 5-2: Percent difference between FEM and Observed Yield Strength.

	Model Data	% Difference
NHVP	16.36	-0.85
MOPP	19.03	15.33
Raghava	16.48	-0.12
NDER	15.68	-4.97

The results show that the three different yield criteria accurately predict the yield strength in tension. The finite element results indicate that there are slight differences between the assumptions in the yield criteria and the material models implemented in ABAQUS. The differences in the model results versus the observed data were generally less than 5%. However, the MOPP model over estimated the yield strength by nearly 15% because the form of the law in the ABAQUS Standard library cannot be used in a plane-stress analysis. Figure 5-6 compares the axial stress-strain plots predicted by the finite element models to a 6th-order polynomial fit of the uniaxial tension data. All of the model stress-strain curves capture the character of the test data but two are clearly more accurate. The Raghava criterion and NHVP models come very close to approximating the test data. The NDER model reproduces the important characteristics of the test data, but at low strains it overestimates the bulk tension response.

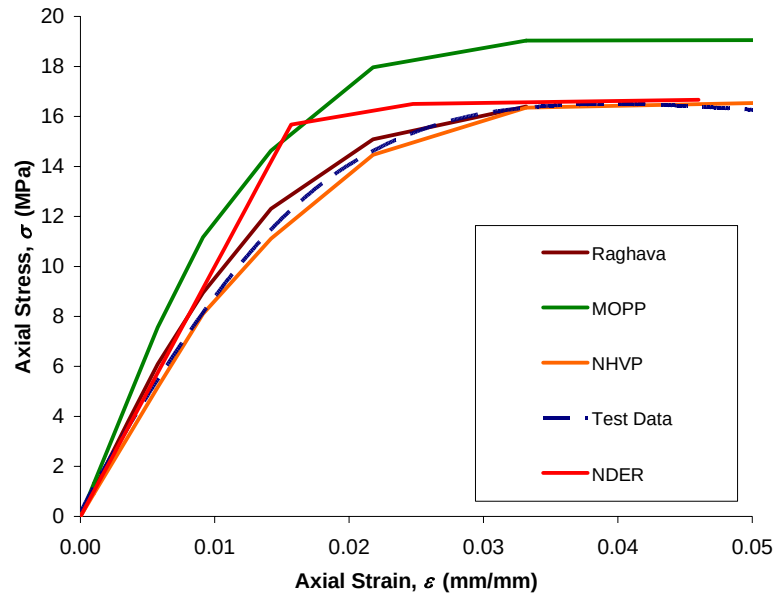


Figure 5-6: Plot of material model predictions and test data.

Three different error measures describe the accuracy and range of applicability for each model: average total error, average point error and normalized strain energy difference. The average total error describes the average difference between the material model and a 6th-order polynomial fit of the test data with respect to the maximum observed test value over the entire observed strain range. Equation (5-3) specifies how to calculate the average total error. This error measure best describes the average difference between the test data and the model results overall.

$$\bar{E}_T = \frac{\sqrt{\sum_{i=1}^N \left(\frac{Model_i - Data_i}{MAX(Data)} \right)^2}}{N-1} \quad (5-3)$$

In Equation (5-3) the term $MAX(Data)$ refers to the maximum value of the test data over the strain range of interest. Table 5-3 summarizes the average total error for each material model.

Table 5-3: Summary of Average Total Error for Bulk Models in Tension.

Average % Total Error	
Raghava	1.05
NHVP	1.52
NDER	1.97
MOPP	11.10

The average point error evaluates the average difference between the model data and a 6th-order polynomial fit of the test data for each value of stress over the entire observed strain range. Equation (5-4) specifies how to calculate the average point error. This error measure best describes the average difference between the test data and the model results at any given point.

$$\bar{E}_P = \frac{\sqrt{\sum_{i=1}^N \left(\frac{Model_i - Data_i}{Data_i} \right)^2}}{N-1} \quad (5-4)$$

Table 5-4 summarizes the average point error for each material model.

Table 5-4: Summary of Average Point Error for Bulk Models in Tension.

Average % Point Error	
Raghava	2.29
NHVP	2.84
NDER	3.17
MOPP	7.23

The normalized strain energy difference (NSED) calculates the difference in strain energy between the model stress-strain results and a 6th-order polynomial fit of test data as a function of strain. Equation (5-5) specifies how to calculate the NSED. The NSED illustrates over which ranges of strain each model is most accurate compared to the test data. This error measure is best used to determine what model to use for predicting stress in a material over a given range of strain. Figure 5-7 shows the NSED results for each material model.

$$NSED_i = \left(\frac{\int \sigma_{MODEL} d\varepsilon - \int \sigma_{DATA} d\varepsilon}{\int \sigma_{DATA} d\varepsilon} \right)_i \quad (5-5)$$

The Raghava criterion model had the least overall and point-by-point error. The MOPP model had the greatest overall and point error although both error measures were still reasonable. The NDER model did not perform significantly worse than any of the more complex material models.

The average accuracy of each model is only part of the story. It may be the case that an engineer is more concerned about the range over which the model will perform than its average accuracy. It might also be the case that an analyst is especially concerned with the accuracy of a model in a very specific strain range. This is especially true if a designer wants to use a single model to analyze a part over the range of anticipated loads and displacements. Plotting the NSED as a function of strain reveals that the MOPP model has the largest useful range. By contrast, the Raghava criterion model has the smallest applicable range. The NDER model can cover an essentially infinite range of strains but is limited in accuracy, for this particular set of fit parameters, to a range slightly larger than the Raghava criterion.

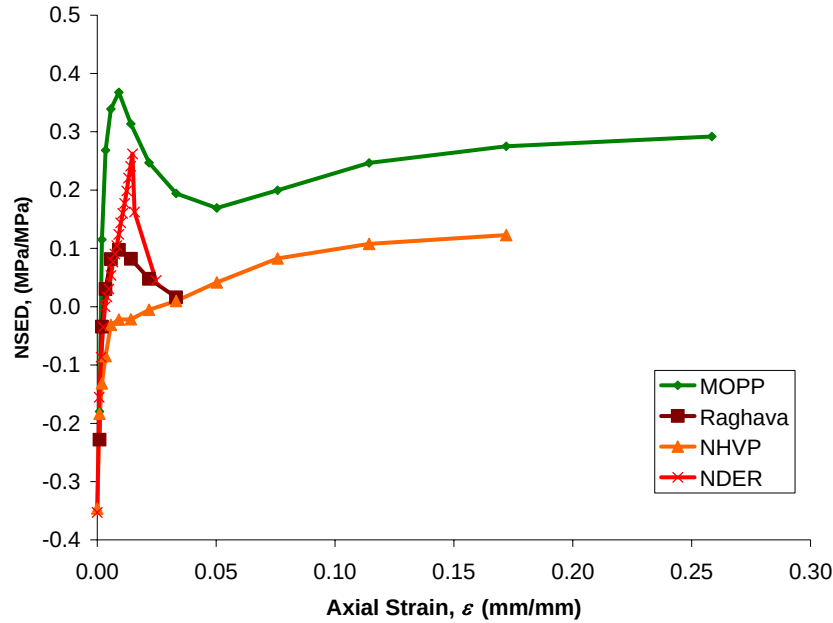


Figure 5-7: Plot of Normalized Strain Energy Difference in Tension.

The results of all three error measures serve as guides to understanding how each material model is best applied to modeling the bulk tensile behavior of an adhesive. For instance, the Raghava criterion is best used to provide very accurate estimates over a small range of strain values. By contrast, the MOPP model is best used to provide a reasonable estimate over a large range of strains. The best model to use for predicting the bulk tensile response is the NHVP model. The NDER model compares favorably with the NHVP model in terms of accuracy and strain range and it is much simpler to characterize than the NHVP model. The NHVP model performed the best overall in modeling the bulk tensile response of LESA.

Bulk Shear Results

Arcan specimens with 2 mm thick bonds were prepared according to Arcan et al. [93]. The Arcan specimens were bonded and then air cured at room temperature for 24 hours before finishing the curing process in the oven. The Arcan specimens were cured using the same curing cycle for the tension specimens. The specimen had a 39 mm long by 5 mm wide by 2 mm thick bonded test section. The Arcan specimen with a thick bond was used to determine the shear properties of LESA because it was simpler to control the

displacement rate in the Arcan test. The Arcan specimen bond was more than twice as thick as the intended in-situ applications that would use LESA as the adhesive.

The specimens were tested in Instron Universal Testing machine under displacement control with a 5 kN load cell at a nominal rate of 0.00001 rads/sec. The displacement rate at the test specimen was monitored so that the displacement experienced by the test section was applied at a nearly constant rate. Figure 5-8 shows the results of the experiment. Yield is considered to have occurred at the first point where the measured strain increases without a related increase in the calculated stress. The yield strength of LESA, at this load rate, is 10.01 MPa. The specimens yielded at 5% nominal strain and ruptured at 46% nominal strain. The average Shear Modulus was 343 MPa.

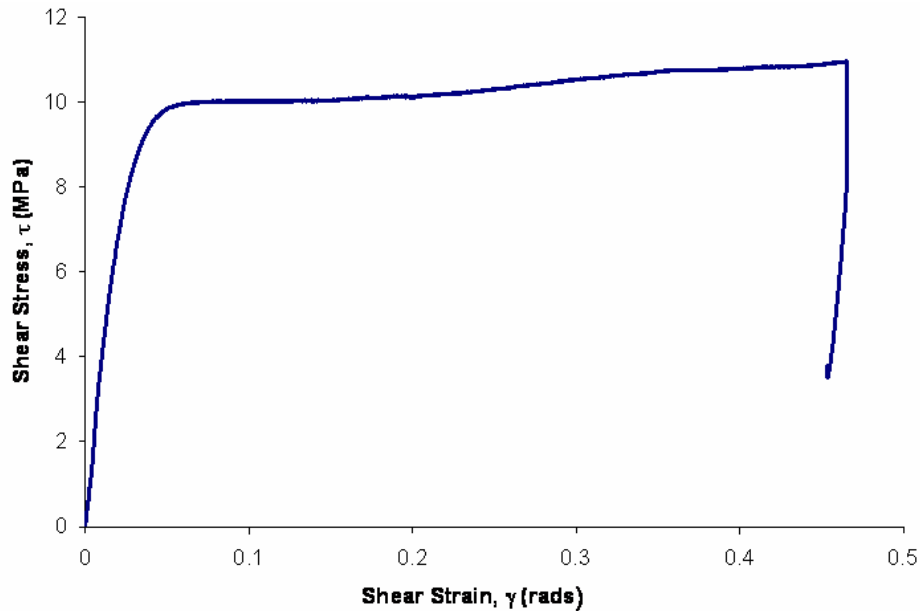


Figure 5-8: Plot of Arcan test data for quasi-static comparison.

Bulk Shear Comparison

The Arcan results were compared to the same material models used for the tension comparison. Each model used CPS8H elements (hybrid quadratic plane-stress elements) with full integration in the adhesive layer, except for the MOPP model which used CPE8H elements (hybrid quadratic plane-strain elements). The aluminum Arcan fixture was modeled using CPS8 elements. The model was converged within 3% for the reaction forces at the boundaries. The size of the elements in the fixture was limited by distortion issues in the mesh that occurred as the inner angles of the quadrilaterals

became smaller than 30 degrees. The average element area for the converged model in the bond region was 0.016 mm². Figure 5-9 illustrates the mesh used for the Arcan model and the boundary conditions for the analysis.

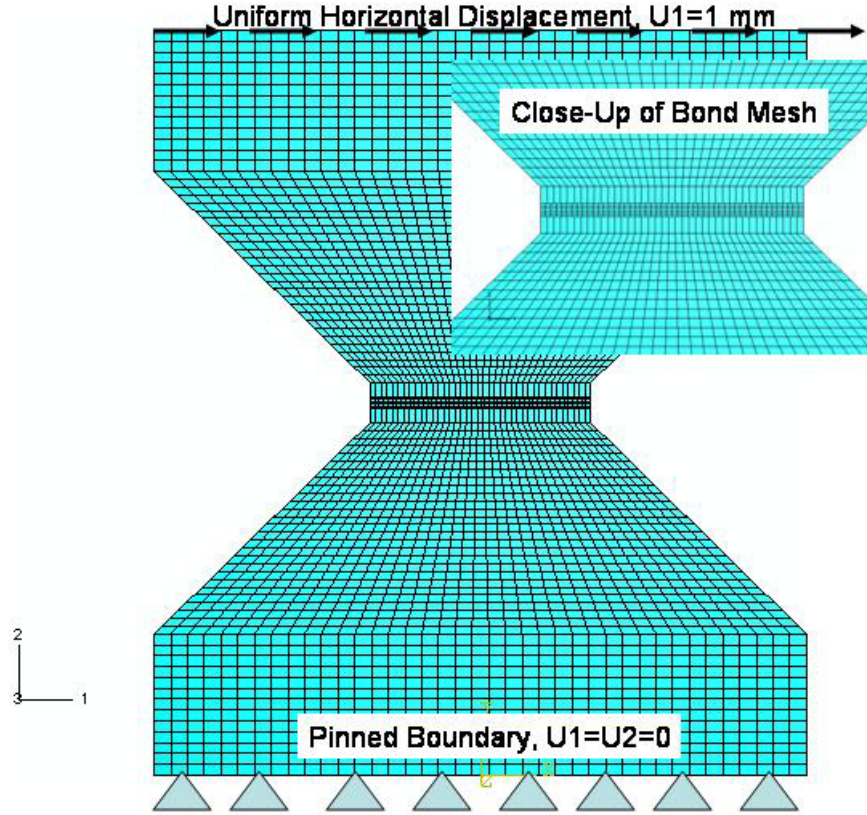


Figure 5-9: Model boundary conditions and mesh.

The nominal shear stress and strain were determined by operating on the history output of the ABAQUS data file. The shear stress was calculated by summing the reaction forces in the horizontal direction along the top boundary edge of the model and dividing by the cross sectional bond area. The shear strain was determined by taking the inverse tangent of the difference in horizontal displacement between the upper and lower specimens at the left side of the bond divided by the bond thickness. Equations (5-6) and (5-7) describe how these calculations were performed using ABAQUS and the history output function in ABAQUS CAE.

$$\tau = \frac{\sum RF1_{TOP}}{A} \quad (5-6)$$

$$\gamma = \tan^{-1} \left(\frac{U1_{TOP} - U1_{BOTTOM}}{h} \right) \quad (5-7)$$

In Equation (5-6) the $\Sigma RF1_{TOP}$ term refers to the sum of the reaction forces in the horizontal or 1 direction, according to the global coordinate system, taken at the top edge of the model where horizontal displacement was applied. In Equation (5-7) the terms $U1_{TOP}$ and $U1_{BOTTOM}$ refer to the horizontal displacements of the left hand edge of the specimen on the top Arcan fixture and the bottom Arcan fixture.

The FEA model results approached the estimated yield strengths predicted by the yield criteria. All models assumed a nearly perfect cross section with 0.1% voids by volume because the precautions were taken to give the LESA freedom to expand or contract from the sides, thereby reducing voids, during the cure. See Table 5-5 for a summary of the yield strength comparison for each yield criteria and Table 5-6 for a summary of the yield strengths based on the FEA results.

Table 5-5: Percent Difference between Predicted and Observed Yield Strength.

	Est. Yield	% Difference
von Mises	9.50	-5.00
MOPP	9.52	-4.80
Raghava	9.99	-0.10

Table 5-6: Percent Difference between FEM and Observed Yield Strength.

	Model Data	% Difference
NHVP	9.42	-5.80
MOPP	9.45	-5.50
Raghava	9.53	-4.70
NDER	9.07	-9.30

Comparing the test results to the yield predictions shows that the von Mises and MOPP yield functions underestimate the shear strength of the adhesive. The Raghava criterion accurately predicts the shear yield strength within 0.10%. The finite element results show that there is a small difference between the yield criteria predictions and the numerical implementation of the material models. For example, the von Mises yield criterion underestimates the shear yield strength by 5% and the NHVP model which uses the von Mises criterion underestimates the shear yield strength by 5.80%. The Raghava model is the closest to the observed test value but underestimates the shear yield strength more than the yield prediction. The MOPP model is more accurate than the NHVP and

NDER models. The NDER model reproduces the important characteristics of the test data but at low strains underestimates the shear response at yield. Figure 5-10 compares the shear stress-strain plots predicted by the finite element models to a 6th- order polynomial fit of the shear data.

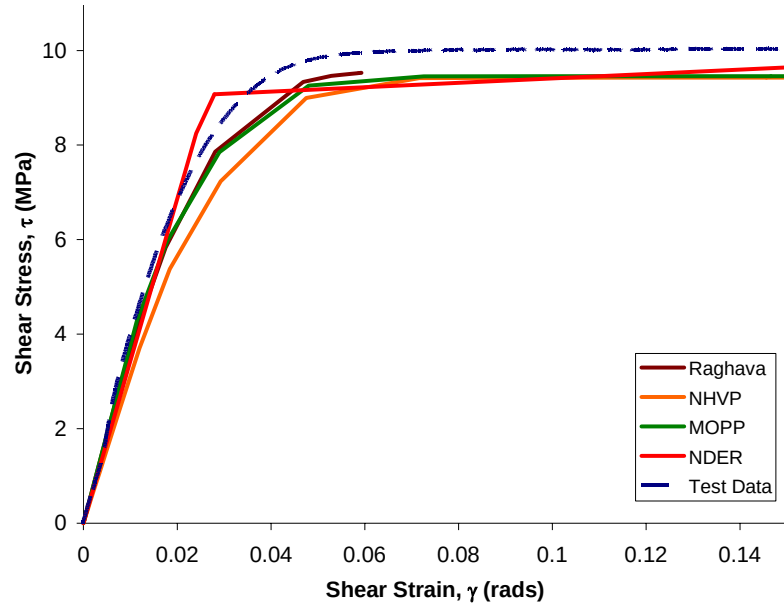


Figure 5-10: Plot of material model predictions and test data.

The same error measures used to compare the tension data were applied to the shear model results. Table 5-7 summarizes the average total error results and Table 5-8 summarizes the average point error results calculated using Equations (5-3) and (5-4). The Raghava criterion model had the least overall and point-by-point error. The NDER model had the greatest overall and point error although both error measures were still reasonable. The NDER model did not perform significantly worse than any of the more complex material models and in both cases was less than 6% different from the Raghava, MOPP and NHVP models overall.

Table 5-7: Summary of Average Total Error for Bulk Models in Shear.

Average % Total Error	
Raghava	0.50
MOPP	0.95
NHVP	1.38
NDER	5.56

Table 5-8: Summary of Average Point Error for Bulk Models in Shear.

Average % Point Error	
Raghava	0.78
MOPP	1.88
NHVP	3.99
NDER	6.26

Plotting the NSED, using Equation (5-5), reveals that the MOPP model has the largest useful range. The Raghava model has the smallest applicable range. Because the NDER model is nonlinear elastic it can cover an infinite range of strains but is limited in accuracy, for this particular set of fit parameters, to a range slightly less than the Raghava criterion.

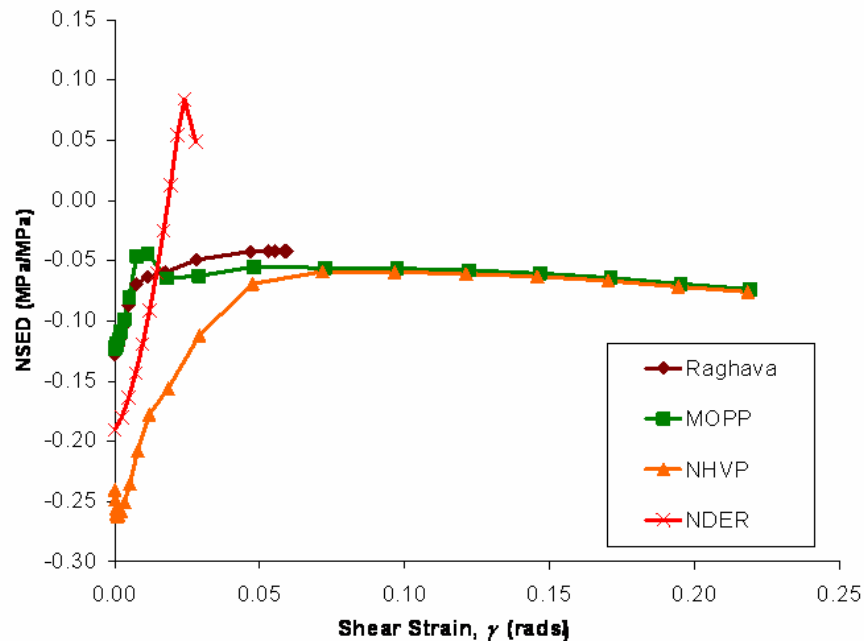


Figure 5-11: Plot of Normalized Strain Energy Difference in Shear.

Comparing the results of all three error measures shows how each material model is best applied to modeling the shear behavior of the adhesive. The Raghava criterion can be used to provide very accurate estimates over a small range of strain values. By contrast, the MOPP model is best used to provide a reasonable estimate over a large range of strains. The best model to use for predicting the bulk tensile response is the NHVP model. The NDER model compares favorably with the MOPP model in terms of accuracy and strain range and it is much simpler to characterize than the NHVP or MOPP

models. However, for the purposes of predicting the shear response of LESA the MOPP model performed the best.

Model Sensitivity to Variations in Test Data

Adhesive material properties vary depending on the environment in which the samples are tested, the sample preparation method and the test method. Test data is normally given as an average value with a measure of the error in the average reported as well, for example $E=2100 \text{ MPa} \pm 213 \text{ MPa}$. The “plus or minus” term is usually the standard deviation of the data and gives the reported data a valid range of values for the variable in question based on a statistical analysis of the data. The test data for LESA reported a standard deviation in Young’s Modulus of $\pm 5.2\%$, in Poisson’s Ratio of $\pm 0.77\%$ and in shear modulus of $\pm 4.8\%$. But how does this variation affect the material models used in finite element analysis?

In cases where there is a vast difference between the material properties of the adhesive and the adherend, for example steel plates bonded with an epoxy, a small amount of variance in the test data may not be significant. But adhesives are often used to bond relatively flexible adherends together that are not as stiff or strong as steel. In the case that the stiffness of the adherend and adhesive are close a small variation in the Young’s modulus or Poisson’s ratio may cause the models to predict entirely different failure results since one may yield before the other. It is also important to understand which test data or variable is required to obtain accurate model results. For example, if it is important to be able to predict yield in an adhesively bonded structure within $\pm 5\%$ using a model sensitive to variations in Young’s modulus, it is necessary to know what level of accuracy in the data for Young’s modulus is required so that the model is capable of predicting yield within the desired error tolerance.

It is possible to estimate how variance in a material property can affect the different yield criteria on a single element of bulk material. Assuming a state of generalized plane strain on an element of a linear material and then solving for the stresses on that element results in a stress tensor that is a function of the strains, E and ν . Solving for the eigenvalues of the stress tensor and assuming a unit value for each of the strains gives the principal stresses in terms of E and ν . Now the effect of varying E and ν

for this particular load state on the principal stresses can be directly examined. Substitution of the principal stresses into the von Mises, MOPP and Raghava yield criteria produces yield surfaces as functions of E and ν . The NHVP model uses the von Mises yield criterion, so any characteristic response to the variation in input should be similar to the von Mises yield criterion. The NDER is a nonlinear elastic law, which means it is meaningless to discuss how the variation in input data affects the way the model predicts yield in the material. However, because the NDER model is based on the Raghava criterion it is reasonable to assume that the model should respond to variations in input similarly.

The effect of varying E , ν and G can be examined using the relationship for a linear material between E , ν and G . The percent difference between the calculated equivalent stress using the extreme value and the calculated equivalent stress using the average value is determined using Equation (5-8).

$$\%diff = \frac{q'(extreme) - q'(average)}{q'(average)} \cdot 100 \quad (5-8)$$

In Equation 5-8 q' denotes the equivalent stress calculated using the appropriate yield criteria (i.e. von Mises, Raghava or MOPP). Table 5-9 summarizes the percent difference between the equivalent stress calculated using the average test data value and the extreme values in the reported data range.

Table 5-9: Percent difference in q with reported variation in test data.

	E	G	ν
von Mises	3.17	19.70	33.30
Raghava	0.49	2.52	0.24
MOPP	5.10	7.10	3.10

The results of this analysis indicate that varying Poisson's ratio will result in the largest difference in the calculated equivalent stress. The von Mises equivalent stress in particular is very sensitive to changes in Poisson's ratio. The variance in the Raghava equivalent stress is much less than the reported test range. The MOPP equivalent stress is less sensitive to variations in the test data than the von Mises equivalent stress and is generally on the order of the reported range of test data. The conclusion from this simple analysis is that it is very important to know the Poisson's ratio for an adhesive material

very accurately. It is also very important to check model results for the entire range of test data.

The Raghava and MOPP yield criteria are sensitive to two other variables as well, the yield strength ratio S and the void volume fraction f . A parametric sensitivity analysis of the equivalent stress for each yield criteria illustrates how varying the model inputs will affect the yield criteria. Figure 5-12 is a plot of the von Mises (see Equation 3-8), Raghava (see Equation 2-4) and MOPP (see Equation 3-21) equivalent stresses as a function of S . It is assumed that $f = 0.01$ in each case so that the MOPP model can be compared to the von Mises and Raghava criteria which are not sensitive to the void volume fraction, normalized by the equivalent stress calculated using the average material properties (e.g. using $E = 1069$ MPa rather than 1069 ± 53 MPa).

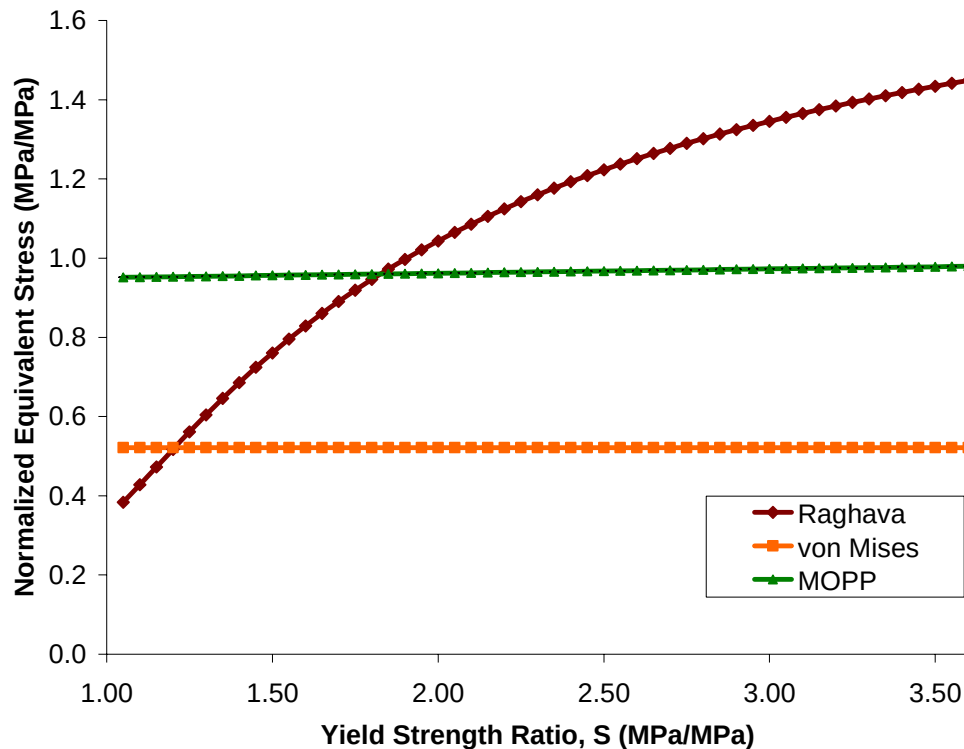


Figure 5-12: Plot of Normalized Equivalent Stress as a Function of S .

The von Mises equivalent stress is not sensitive to variations in S and is constant over the entire range of S . The Raghava equivalent stress is highly sensitive to changes in S and for the bulk material element described above varies more than 100%. The MOPP equivalent stress is slightly sensitive to variations in S with a difference of 4% over the

range of S values. The MOPP equivalent stress is also sensitive to changes in the void volume fraction. Figure 5-13 is a plot of the MOPP equivalent stress as a function S for different values of f from a nearly perfect material ($f = 0.01$) to the limit of physically reasonable behavior ($f = 0.50$) for the model. The figure clearly shows that the for the same principal stress state the slope of the MOPP equivalent stress as function of S increases with increasing f . This means that the MOPP yield criterion will predict yield at lower stresses as f increases which is reasonable because as f increases the amount of material available to bear the load decreases.

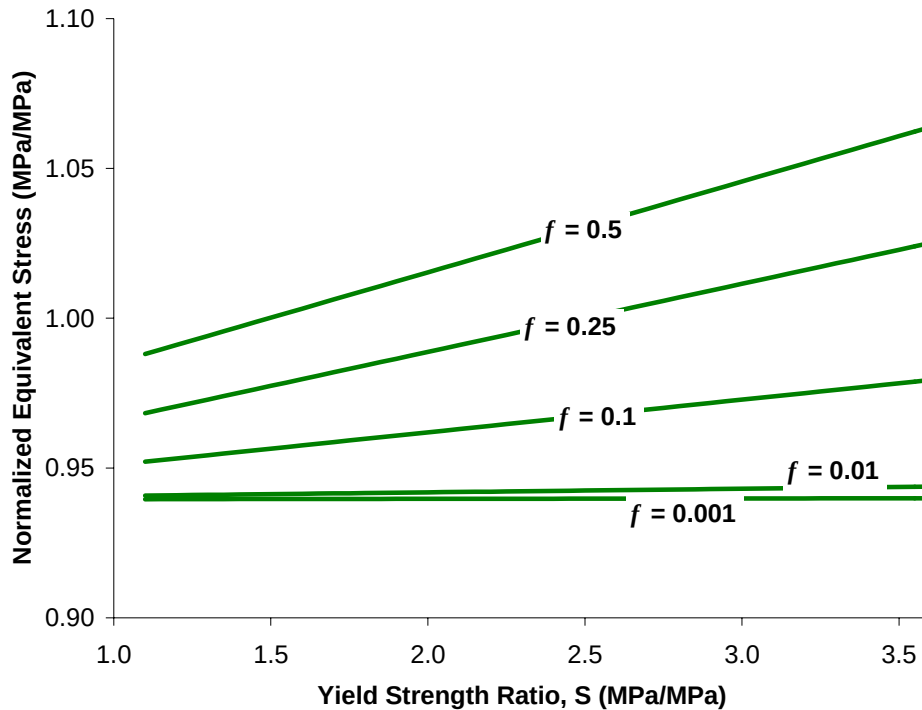


Figure 5-13: Plot of MOPP Normalized Equivalent Stress as a Function of S .

One important conclusion from Figures 5-12 and 5-13 is that small variations in the S -ratio can drastically affect the model results for the Raghava criterion and the MOPP model.

Chapter 6

In-situ Behavior and Multi-axial Loading

Material models are of little use if they are only accurate when modeling bulk adhesives. Because adhesives are designed to bond materials together the applied, or in-situ, material predictions are more important to engineers than predictions of bulk behavior. The current Chapter focuses on how the material models discussed in Chapters 3 and 4 behave perform when modeling bonded structures under multi-axial load states.

Two part configurations are examined: a Napkin Ring (NR) and a Tongue and Groove (TNG) joint.⁵

In-situ Modeling Issues

There are many different issues that can skew model results for adhesive in-situ applications. First and foremost is the lack of well defined, simple analytical comparisons to obtain bounds on the behavior of the model or part configuration. Except for the cases of double-lap and single-lap joints there are few other closed form analytical solutions for the stresses at the bondline. There have been many different approaches to numerically modeling the behavior of a variety of joint configurations. Some of these are listed in Chapter 2. However, it would be ideal if an engineer did not have to resort to developing a complete series of numerical and finite element models to predict the stresses in an adhesively joined part. The list below describes some of the common issues that an analyst or engineer may encounter when designing finite element models of adhesively bonded parts.

Scale: In many cases the dimensions of the bondline are significantly smaller than the greatest dimension of the part. Designing a mesh based on the geometry of the bond may produce a large and inefficient model. Ignoring the geometry at the bond may produce a mesh that will not provide accurate results at the bondline. The best solution lies in between these two extremes and usually requires a global/local model approach or a severely biased mesh that seeds more nodes towards the bondline. Any mesh that is used for a detailed finite element model of an in-situ part must be sufficiently refined to capture the important behavior in the model. It is highly recommended to consult the literature for critical material length scales related to the dominate forms of loading before designing a mesh for an adhesively bonded part.

Convergence: What criteria should be used to verify convergence? What value should a model approach in order to be considered “converged”? These questions are not simple to

⁵ The NR specimen was designed, built and tested by John Hennage, a graduate student in the Department of Mechanical Engineering at Virginia Tec. Mr. Hennage also helped to design the TNG specimen and machined the fixtures used to obtain the results reported in Chapter 6.

answer models of adhesively bonded joints. In displacement based FEA the strain energy or energy norm is often used to assess convergence but in this case such a standard may not be as meaningful as how well the model result compares to test data. Some elements or material models require a large number of nodes before the results appear mesh independent, for example the IEL used by Goyal. The impossible ideal for the IEL models is an infinite number of nodes. It is useful to define “global” and “local” convergence criteria and use an acceptable error tolerance with respect to available test data to minimize unnecessary mesh refinement. An example of global convergence criteria might be to use the load versus displacement measured at the boundary of the part and sufficiently far away from the adhesive layer as a convergence check. An example of local convergence criteria might be the shear stresses along the centerline of the bond. Developing accurate finite element models of adhesively bonded joints without predetermined error and convergence criteria is very difficult.

Time: How quickly is an answer needed? Is it important to know the stresses at the center of the bond or at a highly refined corner with a sharp edge? The size and complexity of an in-situ model are often determined by the question of how much information is desired and a project timeline. Simple models with small numbers of nodes should always be developed prior to a very complex model regardless of how much time is available to develop, refine and run a model of an in-situ joint. In the best circumstance, these simple models will provide insight that will be useful when developing the highly detailed models desired by the design engineers. In the worst case, simple models can provide estimates that may help a design team meet a required deadline.

Computing Power: How much computer power is available to run the model? If an analyst has access to a supercomputing facility many of these concerns become trivial. However it is quite possible to create a model using commercially available FEA software packages that stretch the limits of most desktop systems. It is very important that simple models be tested before developing highly refined models.

Benefits versus Cost: The amount of time required to develop a model is directly proportional to the complexity of the model. The overall goals of the project should always be considered before allocating the effort and resources necessary to develop a very complex model. The worst case scenario for an analyst is to spend many hours identifying parameters and creating a model that takes a significant amount of time to generate an answer that cannot be easily communicated to other people and is difficult to use. Einstein once said that, “Everything should be as simple as possible, but not any simpler than that.” An analyst should keep that concept in mind when deciding how much will be “enough” for a model.

Napkin Ring Comparison

The Napkin Ring is an ideal specimen to study the effects of combined loading on an adhesive material. The NR is an excellent specimen to study because it provides a continuous bondline, is simple to test in either shear or tension or any combination of the shear and tension and can be analyzed using the methods described in the previous Chapters. In practice, the specimen has fallen out of favor due to Arcan tests and other specimens. Recent efforts to identify issues with the specimen may lead to a new ASTM test standard [94]. But how do the material models used in the previous design examples and bulk characterization studies perform in this case? The details of such a comparison will tell if it is feasible to verify characterization data from bulk specimens in bulk FEM and use the same models for in-situ FEA predictions.

Napkin Ring Test Data

Bonded LESA NR specimens were tested on a 100 kN capacity Instron load frame with two degrees of freedom. The load frame applied a rotation and an axial displacement to the NR specimen on the top substrate while the bottom substrate was effectively fixed to the Instron base plate. See Figure 6-1 for a picture of the NR assembly. The torque, axial displacement, rotation and axial load were recorded during the test. The axial displacement and rotation were linearly ramped at a predetermined rate so that the specimen would be loaded along a load ray at an equivalent τ - σ angle of 67.5 degrees. Additional strain gages were used to see if there was bending in the adherend close to the bonded region.

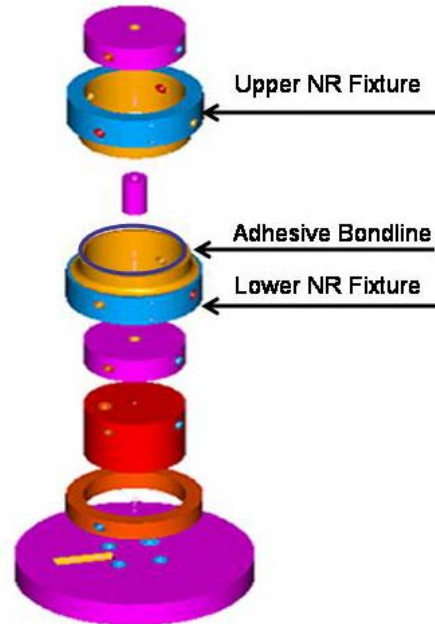


Figure 6-1: Napkin Ring Assembly

The specimen was cleaned using acetone and ScotchBrite™ to produce a rough and clean surface for bonding. The center pieces shown in Figure 6-1 act as a spacers during casting so that the exact bond thickness can be set without placing spacers in the bond. The bond was 1.016 mm thick and the assembly was cured using the same procedure described in Chapter 5.

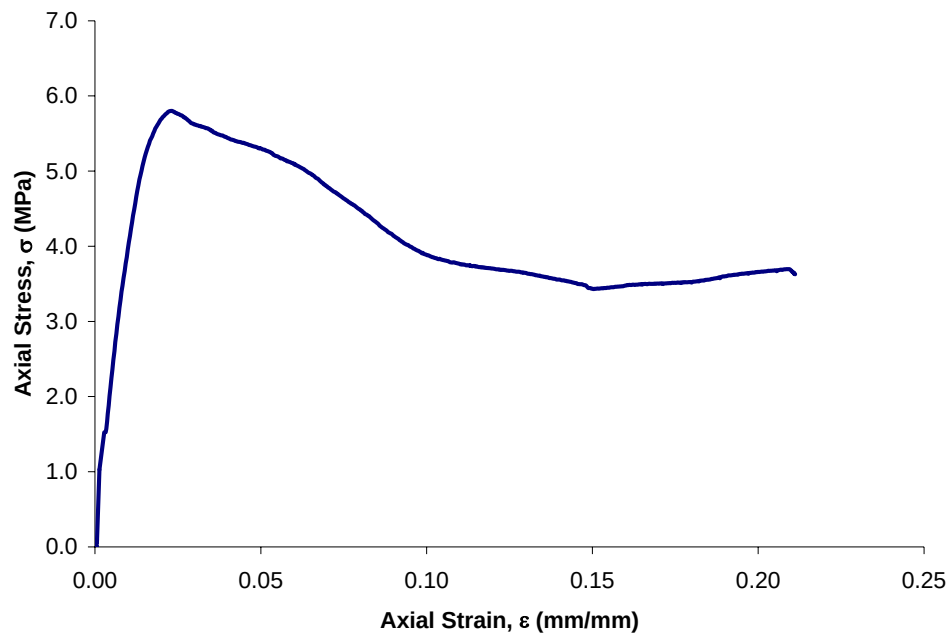


Figure 6-2: Axial Stress-Strain Plot for 67.5 Degrees Loading in the NR Test.

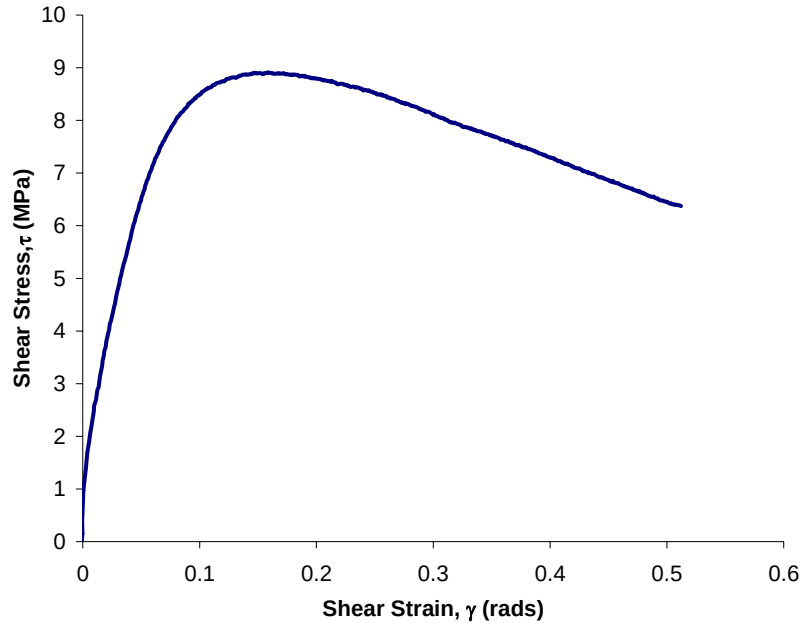


Figure 6-3: Shear Stress-Strain Plot for 67.5 Degrees Loading in the NR test.

The peak tension stress in Figure 6-2 (5.80 MPa) and peak shear stress in Figure 6-3 (8.89 MPa) are lower than the bulk yield strengths in tension (16.5 MPa) or shear (10.01 MPa). The large difference between the bulk and in-situ response two suggests that there is a significant volume of voids in the adhesive layer. The practical value of this observation is that any finite element model using continuum elements is likely to overestimate the tensile stresses that the adhesive layer can tolerate in this specimen. However, the shear response may also be affected depending on the shape and distribution of the cracks in the adhesive layer.

NR FEA Model Details and Results

The Napkin Ring model used general, linear, hybrid four node axisymmetric elements with twist (CGAX4H) tied to a reference axis on the line of symmetry and a rigid line connecting the reference axis to the outer fiber of the top adherend. The displacements were applied to a reference point on the reference axis so that both the axial displacement and rotation were applied linearly to the fixture model. The model used 3470 elements, with an average element area of 0.612 mm^2 in the substrate near the bond and an average area of 0.543 mm^2 based in the adhesive on the width of the adhesive bond, with 5 elements through the bond thickness. The mesh was not uniformly

spaced so that the outer edges of the part were coarsely meshed, but a band of fine elements ran through the substrate into the bond area. See Figure 6-4 for details on the model configuration and mesh. The model was considered converged when the peak reaction force at the boundary in the upper substrate changed by less than 3% upon subsequent mesh refinement. See Appendix A for details of the inputs and commands required to recreate the results for each NR FE model.

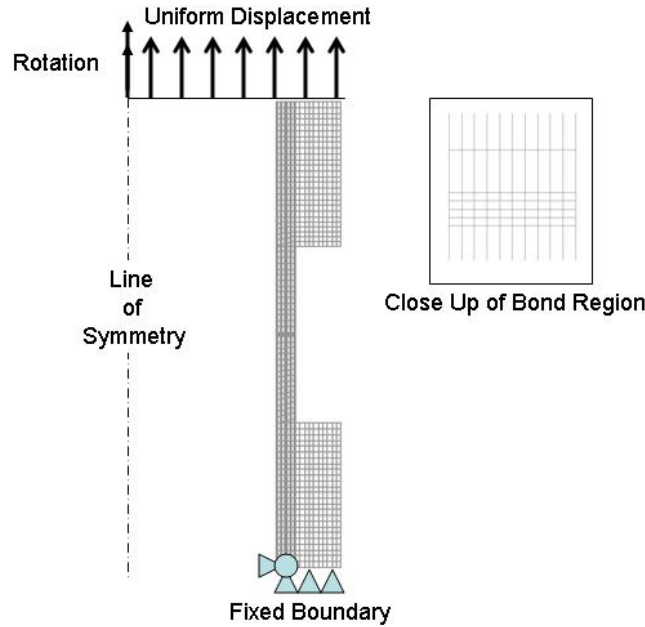


Figure 6-4: Picture of Napkin Ring Mesh and Boundary Conditions.

The reactive loads from the finite element models were taken from a node on the same reference point where the rotation was applied, see Figure 6-4. The nominal axial stress and strain in the NR bond layer is calculated using Equations (6-1) and (6-2) below.

$$\sigma = \frac{RF2}{A_o} \quad (6-1)$$

$$\varepsilon = \frac{U2_{top} - U2_{bot}}{L_o} \quad (6-2)$$

In Equation (6-1), $RF2$ is the axial reactive force at the reference point node located on the axis of symmetry and A_o is the original cross-sectional area of the bond. In these tests, A_o is equal to 934.72 mm^2 and L_o is equal to 1.016 mm . In Equation (6-2), $U2_{top}$ is the axial displacement of the node at the top edge of the bond on the outermost edge and

$U2_{bot}$ is the axial displacement of the node at the bottom edge of the bond on the outermost edge of the bond. The nominal shear stress and strain in the bond layer are calculated using Equations (6-3) and (6-4) below.

$$\tau = RM2 \frac{c}{J} \quad (6-3)$$

$$\gamma = UR2 \frac{L_o}{c} \quad (6-4)$$

In Equation (6-3), $RM2$ is the reactive moment taken at the reference point node located on the axis of symmetry, c is the radius to the outer most edge of the NR and J is the polar moment of area for the cross section of the bond layer. In Equation (6-4), $UR2$ is the specified rotation taken at the reference point node located on the axis of symmetry and L_o is the thickness of the adhesive bond layer. For these tests, c is equal to 65.38 mm and J is equal to $1.346 \cdot 10^5 \text{ mm}^4$. The method for sampling the finite element model and calculating the nominal stresses and strains is as similar to the actual test procedure as possible.

The FEA models predicted values for the axial stress in the adhesive layer that were very different from the test data shown in Figure 6-2. All of the models except for the Raghava criterion overestimated the axial stress due to the combined load state. See Table 6-1 for a summary of the predicted peak axial stress.

Table 6-1: Percent difference between the peak tension stress model results and the peak tension stress in Figure 6-2 for the 67.5 Degree NR Test Data

Peak Stress % Difference	
NHVP	324.09
MOPP	321.80
Raghava	55.17
NDER	-0.44

The material models compared much more favorably to a “perfect” adhesive bond (e.g. $f = 0.001$) without any voids. See Table 6-2 for a comparison of the test data and the estimated tension yield data for the NR specimen using the Raghava, von Mises and MOPP yield criteria.

Table 6-2: Comparison of peak tension stress in Figure 6-2 to the estimated peak tension stress from the yield criteria.

Est. Peak Stress % Difference	
Raghava	-49.9
von Mises	173.7
MOPP	20.4

The strain range predicted by each model was consistent with the test data for the applied displacements. Similar to the results for the bulk material models, the Raghava criterion had the least strain range. See Figure 6-5 for a plot of the model results and a 6th order fit of the axial stress-strain test data. See Figure 6-6 for a plot of the model results and a 6th order fit of the shear stress-strain test data.

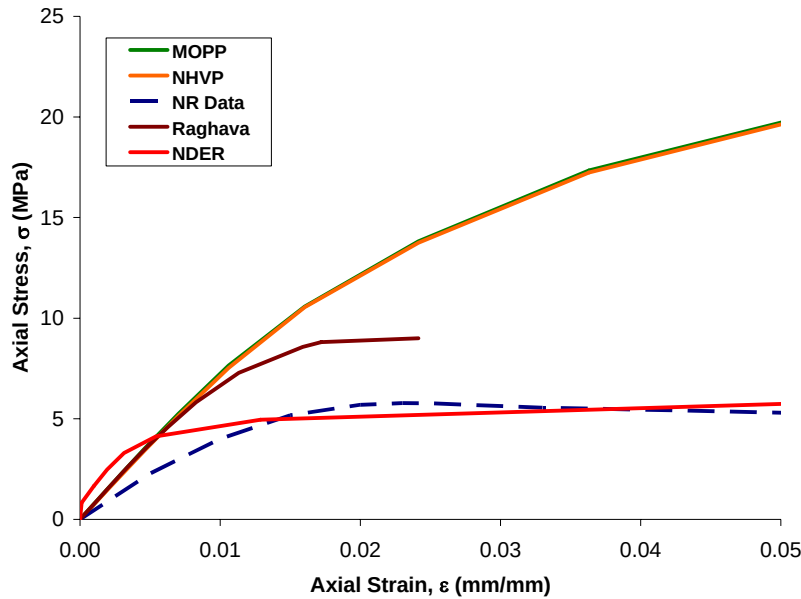


Figure 6-5: Plot of Model Results and Test Data for Axial Stress-Strain at 67.5 Degrees in the NR Test.

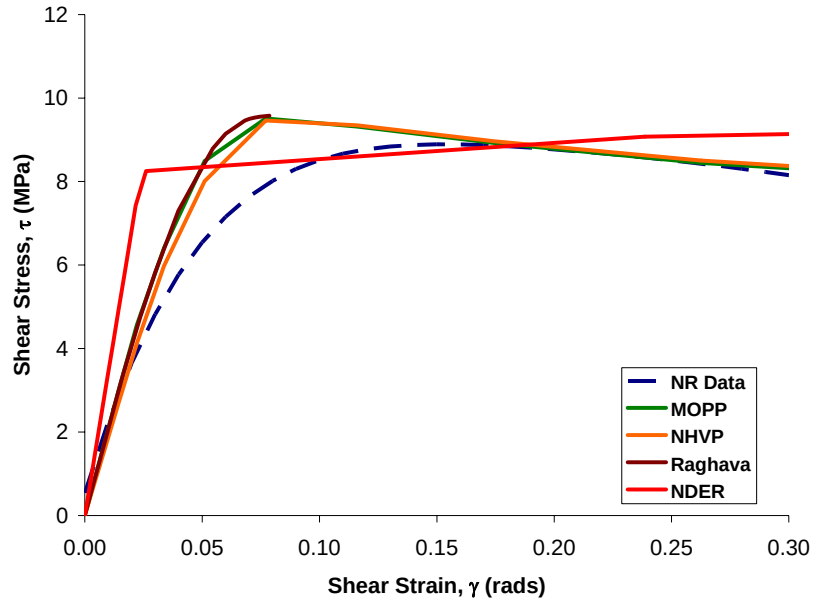


Figure 6-6: Plot of Model Results and Test Data for Shear Stress-Strain at 67.5 Degrees in the NR Test.

The average total error, average point error and NSED for each NR model were calculated using the same procedure in chapter 5. See Tables 6-3 and 6-5 for summaries of the Average Total Error and Average Point Error of the axial stress results. See Tables 6-4 and 6-6 for summaries of the Average Total Error and Average Point Error of the shear stress results. Figure 6-7 shows a plot of the NSED as a function of axial strain for the different NR models. Figure 6-8 shows a plot of the NSED as a function of shear strain for the different NR models.

Table 6-3: Average Percent Total Error in Tension.

Average % Total Error	
NHVP	24.5
MOPP	24.4
Raghava	3.9
NDER	40.3

Table 6-4: Average Percent Total Error in Shear.

Average % Total Error	
NDER	31.4
MOPP	12.2
NHVP	1.7
Raghava	2.3

Table 6-5: Average Percent Point Error in Tension.

Average % Point Error	
NHVP	76.6
MOPP	76.5
Raghava	5.2
NDER	116.7

Table 6-6: Average Percent Point Error in Shear.

Average % Point Error	
NDER	23.8
MOPP	14.4
NHVP	13.4
Raghava	1.9

The material models used in the FEM of the NR performed similarly to material models used in the FEM of the bulk specimens. The Raghava criterion is the most accurate representation of the state of stress in the material due to the multi-axial load state caused by the combined axial displacement and rotation. However, the Raghava criterion has the smallest useful range. The MOPP and NHVP models overestimate the axial stress in the adhesive but capture the shear behavior fairly well. The NDER model performed poorly in tension and shear but gave reasonable values for the stress in the material and captured some of the important characteristics in the axial and shear stress curves.

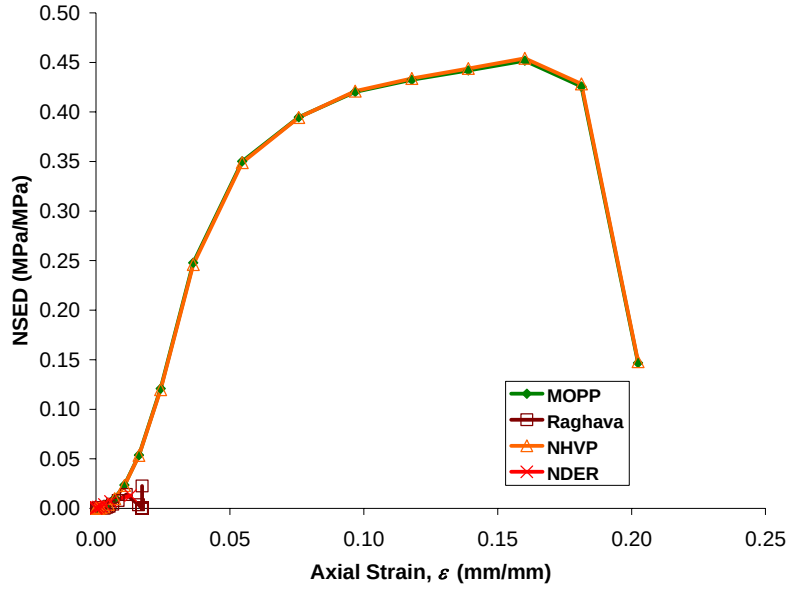


Figure 6-7: NSED as a function of Axial Stress at 67.5 degrees in the NR Test.

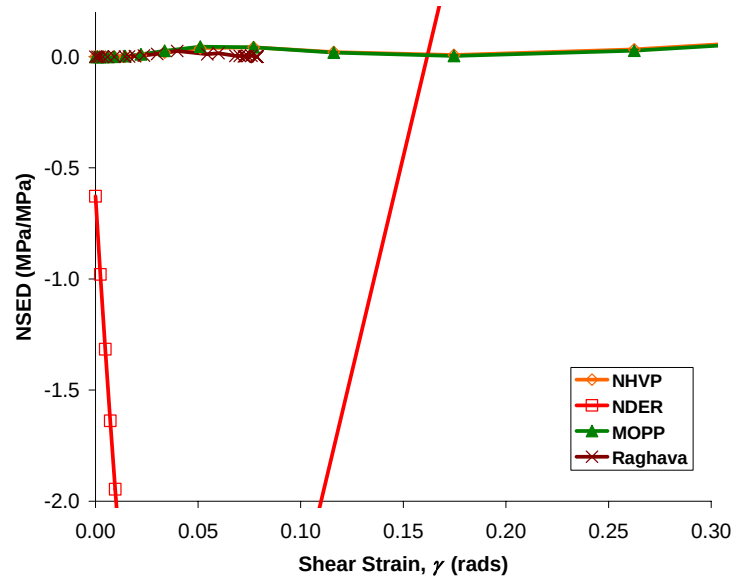


Figure 6-8: NSED as a function of Shear Strain at 67.5 degrees in the NR Test.

The models that use the von Mises (MOPP, NHVP) criterion overestimate the axial stresses in the adhesive by a large margin. The presence of voids in the bondline was confirmed upon examination of the failure surface after the specimen was tested. The disparities between these models and the data should decrease if the adhesive in the fixture maintains a greater stress before yielding or yields at a larger displacement as the amount of voids in the material decreases. It is also highly likely that the difference

between the estimated axial stress and the tested axial strength would be closer for a specimen with weaker adherends (e.g. polypropylene). These results show that in-situ FEA models based on bulk characterization data cannot be relied upon to predict yield in tension in the specimen. This is because the mode of failure in tension in the NR is different from the mode of failure in the uniaxial tension specimen. However, it is possible to use the material models characterized using the bulk material data for predicting the shear behavior of the NR over a wide range of finite strains. This is because the slow, gradual yielding of the adhesive bond with respect to shear is very similar to the response of the adhesive in the thick bond Arcan specimen.

It is unreasonable to expect the continuum material models described in Chapter 3 to accommodate the many variables that can affect the tension strength of the bond. There are simply too many details in the casting, curing, surface preparation, location of voids, etc. that simple continuum element models cannot accommodate. The in-situ models developed from bulk characterization data can be used to obtain conservative estimates of the behavior in the joint provided test data is available. For example, using the equivalent damaged zone approach described by Sheppard [38] the axial failure stress in the NR specimen occurs at roughly 2.5% strain which corresponds to an estimated axial strength based on the FEA results of 5 MPa. Using the FEA results in this manner will not provide detailed information on the state of stress in the bond but it can give a designer a useful tool to use when verifying if a given bond geometry and load state will function within acceptable limits.

Tongue and Groove Comparison

The Napkin Ring examples discussed throughout this work are not representative of many adhesively bonded structures. For example, the halves of the pressure vessel shown originally in Figure 1-1, and again below in Figure 6-9, have a long and narrow tongue that inserts to a narrow groove in the other half.

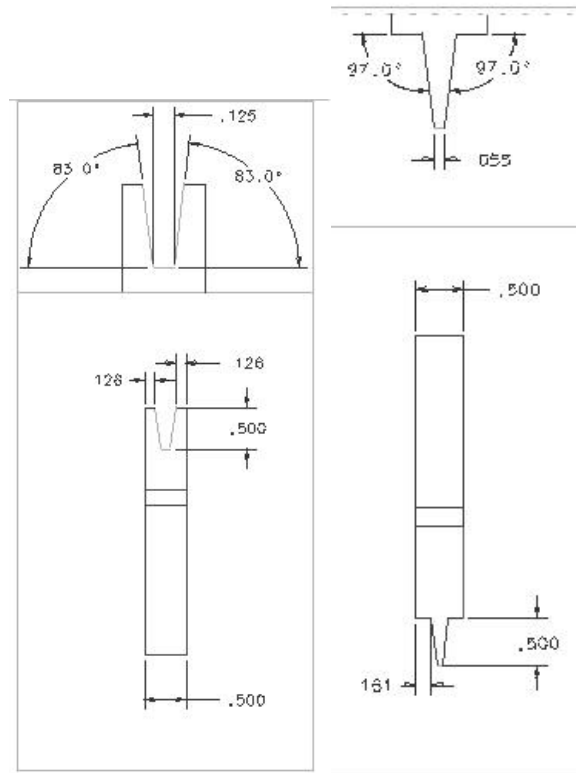


Figure 6-9: Picture of TNG joint used on an elliptical pressure vessel and a Nylon TNG test coupon.

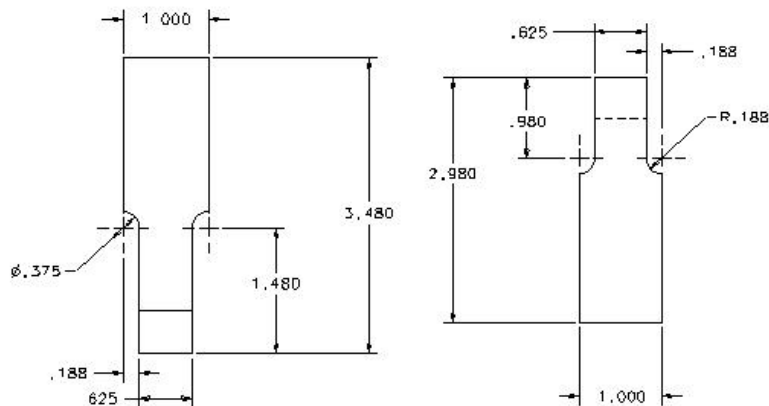
How should a designer or an analyst model this part? Which of the previous results can be applied to this new geometry? The results of the Napkin Ring comparison indicate that the continuum element material models perform poorly when the specimen fails due to the presence of voids or cracks in the adhesive. However, the results of the Napkin Ring comparison show that the continuum models can accommodate multi-axial load states and predict shear stresses in the material fairly well. The analysis results below combines the insight from the continuum element models with fracture based failure behavior.

Tongue and Groove Test Data

A TNG fixture based on the joint shown in Figure 6-8 was machined from T6061 aluminum alloy, with a 7° draft angle and a bond overlap of 12.7 mm. See figure 6-10 for a detailed sketch of the fixture. This geometry presents several issues for both testing and any subsequent modeling of the results. First, misalignment of the part during curing or testing will result in out-of-plane bending stresses. The presence of voids in the bond or spew on the fixture will also skew the test results and make it difficult to account for the joint behavior using FEA. The geometry of the part makes it easy to have voided adhesive in the bond because there are multiple sharp corners in the tongue and groove pieces. The voiding problem can be somewhat mitigated by preparing a rough surface on the adherend using acetone and ScotchBriteTM. Finally, the displacement rate used to load the TNG specimen is very important. Several different displacement rates were used in this study to test how slowly the TNG specimens could be loaded without experiencing stick-slip conditions in the bond. The displacement rate that could still compare well to a quasi-static model and not experience stick-slip in the bond layer was 0.5 mm/min.



(a)



(b)

Figure 6-10: Sketch of TNG Specimen in side (a) and plan (b) elevations.

Dimensions given in decimal inches.

It should be noted that the TNG test specimens are not examples of good design. Rather, the TNG is an example of designing a specimen to accommodate FEA analysis. The sharp corner transitions in the TNG specimen cause large stress concentrations in the

adhesive that would most likely not appear if large radius fillets were used instead. However, the specimen configuration shown in Figure 6-10 is easy to approximate with a finite element mesh, whereas the rounded fillets would not be as simple to model.

Reported TNG results

Five TNG specimens were tested in an Instron 4504 load frame with a 5 kN load cell. The failure surface of each TNG specimen did have voids. The adhesive had voids in the corners at the bottom landing of the groove piece in 75% of the specimens tested. A representative axial stress-strain curve, based on the cross head displacement of the load frame, is shown in Figure 6-11.

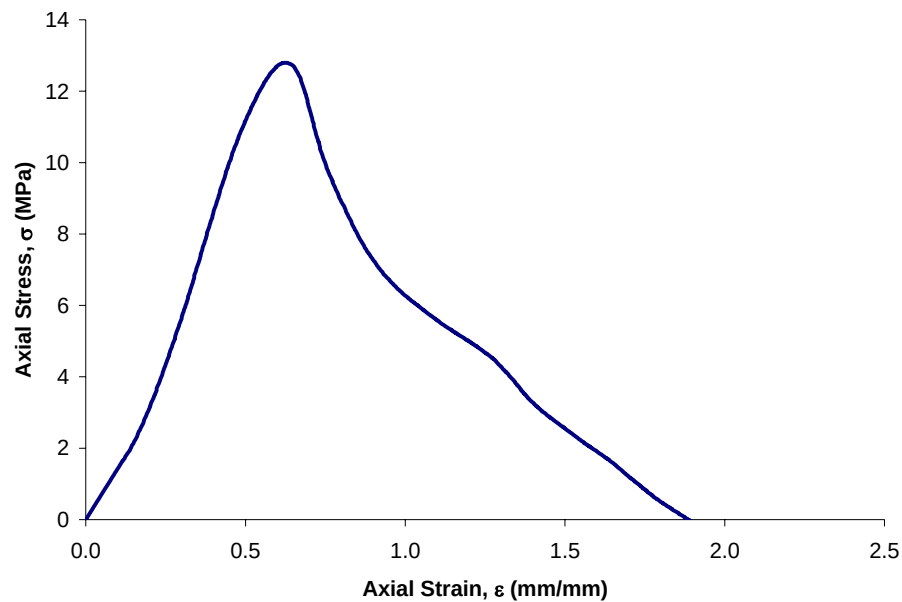


Figure 6-11: Plot of 0.5 mm/min TNG test data.

The average peak axial stress in the fixture, at failure, was 10.26 MPa at an axial strain of 63%. The post-yield behavior of the stress-strain curve varied significantly, but the curve shown in Figure 6-11 is representative of the pre and post-yield behavior in the five specimens. There were two different trends in the small strain regime that appeared to be governed by the presence of voids in the landings of the groove piece. See Figure 6-12 for a plot of the two trends. Fracture data is also important in analysis of the TNG specimen.

Additional tests were performed to obtain the mode dependent critical strain energy release rates, G_{Ic} and G_{IIc} . The data for the first mode was obtained from a double

cantilever beam (DCB) specimen and the data for the second mode was obtained from an asymmetric double cantilever beam (ADCB) specimen. The critical strain energy release rates for LESA are $G_{Ic} = 4$ N/mm and $G_{IIc} = 14$ N/mm. The mode mixity ratios, λ_i , for fracture between the tongue and groove pieces were approximated from ADCB test results and taken to be 0.9.⁶ The comparison between the ADCB test and mixed mode failure in the TNG specimen is reasonable for the range of results where the ratio of the thickness of the ADCB adherends is similar to the ratio of the thickness of the tongue and groove pieces.

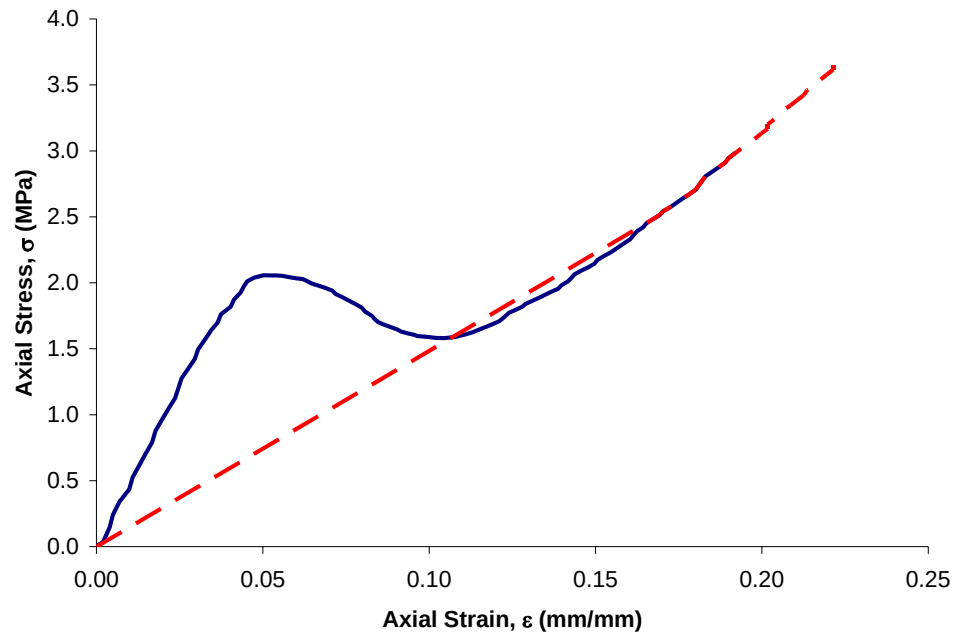


Figure 6-12: Plot of Initial 0.5 mm/min TNG test data showing landing failure.

When the top landings had few voids the stress-strain curve had a much stiffer response, shown as the solid curve in figure 6-12. When the landings had many voids the stress-strain curve had a much softer response, shown as the dashed curve in figure 6-12. The adhesive in the top landings failed at average axial stress of 2.02 MPa and an average axial strain of 5.2%. The slope of the stress-strain curve became much softer after the adhesive in the landings failed and approached the dashed curve shown in Figure 6-11. In general, the adhesive failed in the horizontal landings before failing in the tapered region.

⁶: Dr. Soo Jae Park obtained the fracture data reported here. Dr. Park was a post-doctoral fellow in the Department of Engineering Science and Mechanics at Virginia Tech during the time the work for this thesis was performed.

TNG FEA Model Details and Results

The TNG specimen is a very complex structure. The behavior shown in the test data indicates that there are several important characteristics that must be accommodated by any analysis. These features are: initial joint stiffness, initial and post yield behavior, strain range and peak failure stress. No single model previously discussed can accommodate all of these issues. But several of the modeling tools can address some of the important issues. Three sets of models were developed to address these issues: a continuum element model for the initial joint response, an IEL model to capture the peak load and a CZM model to address the strain range and failure mechanisms. The continuum element models used quadratic, plane strain, hybrid elements. The adhesive elements were modeled using the four different material models discussed in Chapter 3. See Figure 6-13 for a picture of the continuum element mesh and the boundary conditions in the model. See Appendix A for details of the inputs and commands required to recreate the results for each TNG FE model.

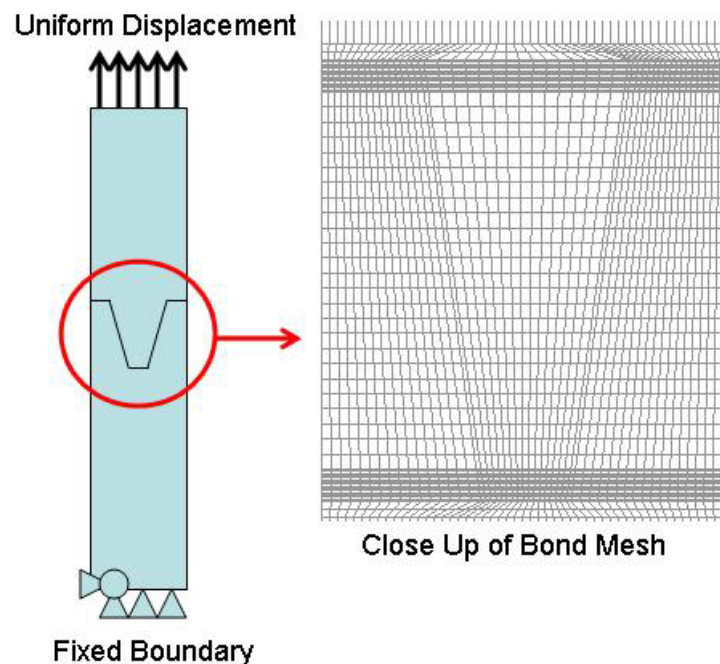


Figure 6-13: TNG Continuum Element Model Mesh and Boundary Conditions.

The IEL and CZM models used linear plane strain elements for the adherend and bulk adhesive behavior. The IEL models used the interface elements described by Goyal

[78] to account for the fracture behavior in the adhesive bond. In the CZM models the adherend and bulk adhesive behavior was modeled using a linear elastic homogeneous isotropic material law. The interfacial and adhesive bulk fracture behavior was modeled using the interface elements too. The CZM and IEL meshes were changed from the specimen dimensions to simplify modeling and prevent early failure in the models. The IEL models had chamfers added to the mesh to relieve the stress at the sharp corners in the bond. See figure 6-14 for a picture of the IEL model boundary conditions and mesh. Only half of the specimen width was modeled for both the IEL and CZM models using symmetry conditions to reduce the number of elements in each model.

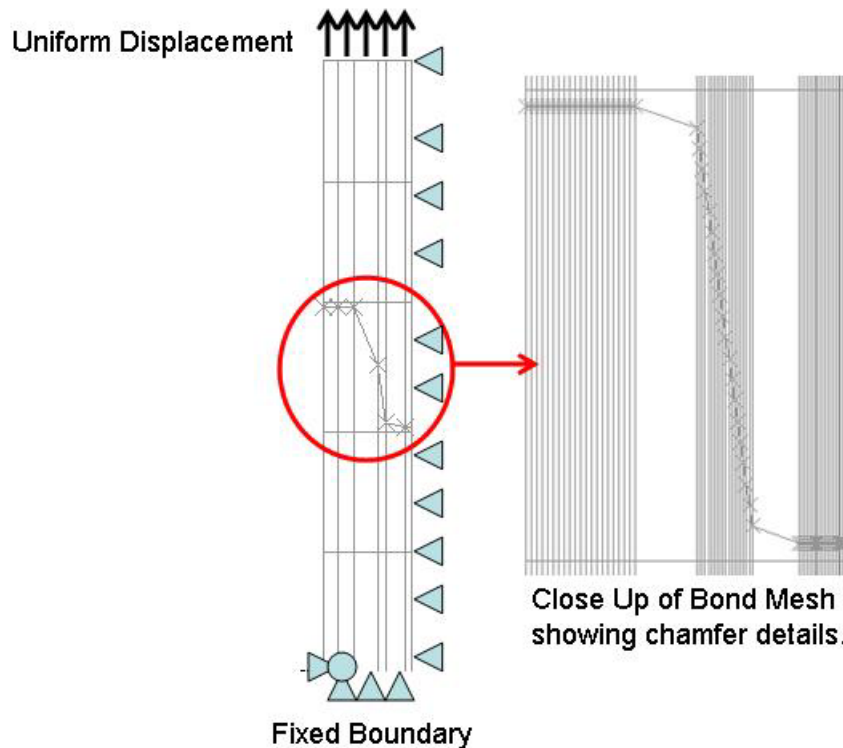


Figure 6-14: Coarse IEL TNG Mesh and Boundary Conditions.

The continuum elements in the adhesive bond region absorb most of the stress that would initiate an early failure in an IEL only model. However, the model specimen geometry was altered so that the tongue and groove pieces had the same draft angle. See Figure 6-15 for a picture of the CZM model boundary conditions and mesh.

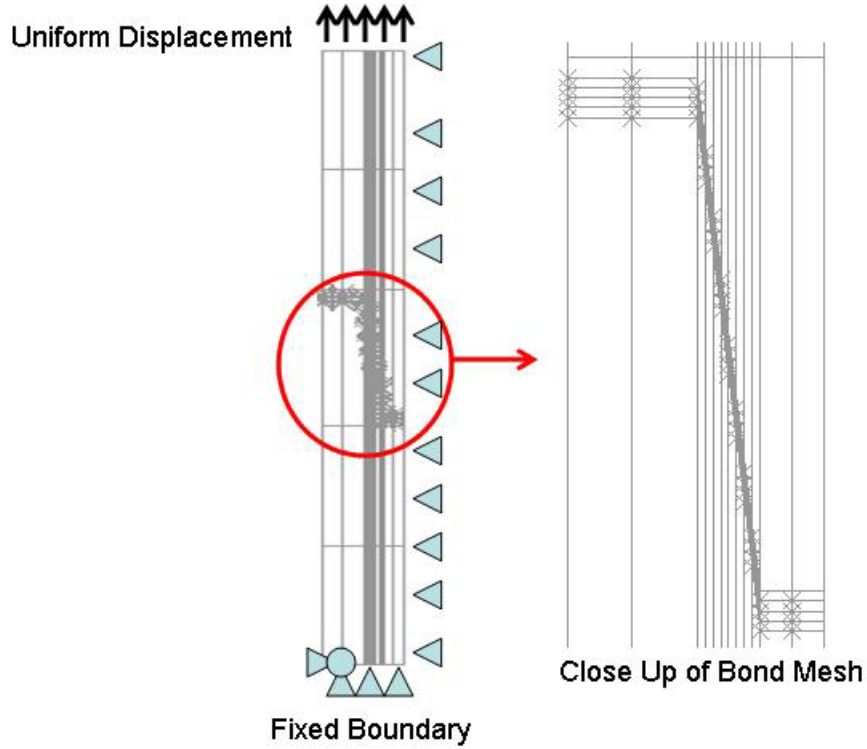


Figure 6-15: CZM TNG Mesh and Boundary Conditions.

All models were converged using the peak axial stress at failure so that there was less than a 3% difference upon further mesh refinement. The length scale of the elements in the cohesive zone or interface elements is very important. If the elements are too large they will not capture the desired behavior. If they are too small the analysis will require too long to complete. The initial size of the adhesive elements for these models was based upon the shear and tension yield strength and the critical strain energy release rates. Equation (6-1) relates these properties to a length scale that provides an estimate of the required element size for the models to function properly based on empirical evidence. The length scale of these adhesive elements, λ_c , is given by

$$\lambda_c = \frac{h}{2} \sqrt{\frac{S}{3D^2}} \quad (6-5)$$

where h is the bond thickness, S is the yield strength ratio and D is a nondimensional parameter based on the fracture properties of the adhesive. The equation for D is

$$D = \frac{G_{IC}}{G_{IIC}} \sin \alpha + \frac{G_{IIC}}{G_{IC}} \cos \alpha \quad (6-6)$$

where G_{IC} and G_{IIC} are the mode-I and mode-II critical strain energy release rates and α is the draft angle measured from the vertical to the plane of the top groove landings. The estimated length for the elements in the bond region of the CZM and IEL models is 0.09 mm for $\alpha = 7^\circ$, $G_{IC} = 4$ N/mm, $G_{IIC} = 14$ N/mm, $S = 1.1$ and $h = 1.016$ mm. Equation (6-5) is similar to the length scale used by de Borst [95] for determining the length of the cohesive elements in their study of cracks in three-point bend specimens.

The continuum, IEL and CZM model results were all compared against the results from a similar mesh. The average element area in, or near, the adhesive bond layer was approximately 0.013 mm^2 . The results of all of the models were compared against a 6th-order polynomial fit of the stress-strain curve shown in Figure 6-11. The stress is calculated from the model data by dividing the axial load at the boundary of the tongue adherend by the width of the adherend 70 mm from the bond region (12.7 mm). The axial strain is calculated using the undeformed bond thickness in the shear region as the reference length. The nominal axial stress is calculated from the model data by dividing the axial load at the boundary of the tongue adherend by the width of the adherend 70 mm from the bond region (12.7 mm). The equation used to calculate the nominal axial stress in the TNG is:

$$\sigma = \frac{\sum RF2}{A_{throat}} \quad (6-7)$$

where $\sum RF2$ is the sum of the reactive nodal forces at the top edge of the tongue specimen where the uniform displacement is applied to the model and A_{throat} is the cross-sectional area of the TNG specimen at the 15.875 mm wide throat, after it narrows past the fillets, but before it reaches the draft angle region of the specimen (see Figure 6-10b). In these tests, A_{throat} is equal to 201.165 mm^2 . The nominal axial strain is calculated using the undeformed bond thickness in the shear region as the reference length. The equation used to calculate the nominal axial strain in the TNG is:

$$\varepsilon = \frac{U2_{top} - U2_{bot}}{L_o} \quad (6-8)$$

where $U2_{top}$ is the axial displacement of the top edge of the tongue specimen, where the uniform displacement was applied, $U2_{bot}$ is the axial displacement of the top edge of the

groove landing and L_o is the undeformed bond thickness. In these tests, L_o was 1.016 mm (0.040 inches).

Continuum Material Model Results

The continuum material model analysis performed as expected based on the NR results. All of the material models predicted a stiffer initial slope than observed in the test data. None of the material models predicted a “roll-over” or inflection of the axial stress-strain curve at a peak axial stress within the tested displacement range. See Figure 6-16 for a plot of the axial stress contours plotted on the undeformed model geometry.

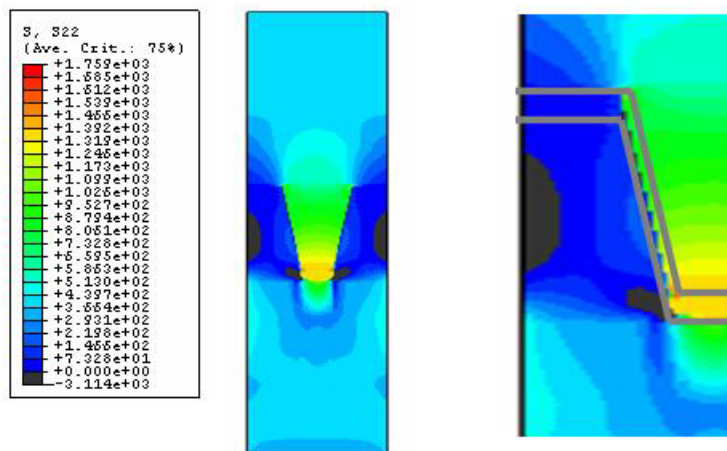


Figure 6-16: Axial Stress Contours for MOPP model. Dark Blue = 0.0 MPa, Bright Red = 175.9 MPa.

The grey lines in Figure 6-16 were added to highlight the adhesive element region in the model. The contours in Figure 6-16 are colored so that dark blue is zero and red is the maximum stress value. Comparing the predicted axial stress-strain curve to the polynomial fit of the test data shows that the MOPP and NHVP models perform the best over the observed strain range pre-yield. See Figure 6-17 for a plot comparing the four material models to a fit of the test data.

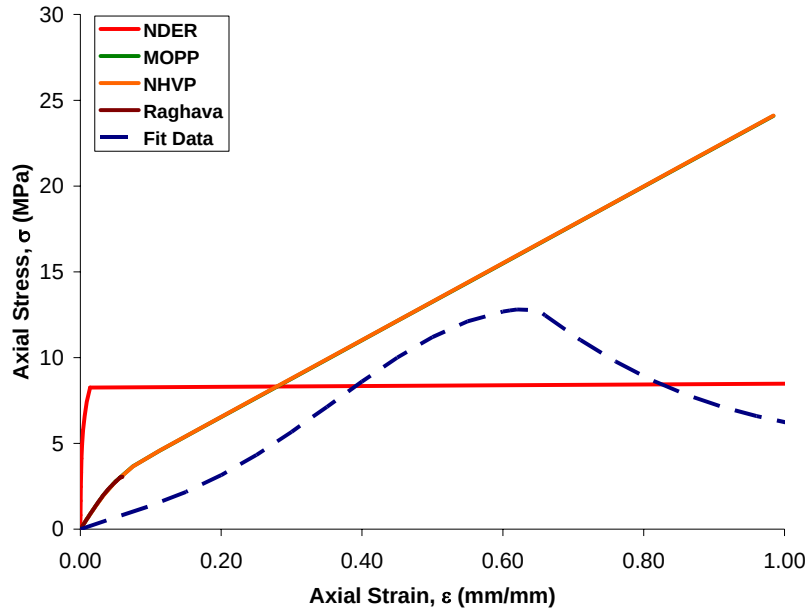


Figure 6-17: Plot of Continuum Material Model Results compared to test data.

The Raghava criterion does not have the strain range necessary to account for the observed behavior and the nonlinear elastic NDER law cannot accommodate the complex yield behavior in the joint. The MOPP and NHVP models do not predict a maximum peak stress in the observed strain range. It appears that the continuum material models are not capable of capturing the complex behavior in the TNG specimen. However, it does appear that the MOPP and NHVP models are capable of predicting an upper bound on the nominal axial stress.

IEL Model Results

The IEL models predicted a stiffer structure than observed as well. The initial stiffness of the elements in the bond can be adjusted by varying the mode-I strength (S_1). Decreasing the mode-I strength is also a way to represent a weak bond due to voids. See Figure 6-18 for a plot of the Mises stress contours on the undeformed model geometry. The contours in Figure 6-18 are colored so that dark blue is zero and red is the maximum stress value. The Mises stress is a useful indicator of the presence of local stress concentrations that are usually correlated with the presence of cracks in the bond layer using the IEL.

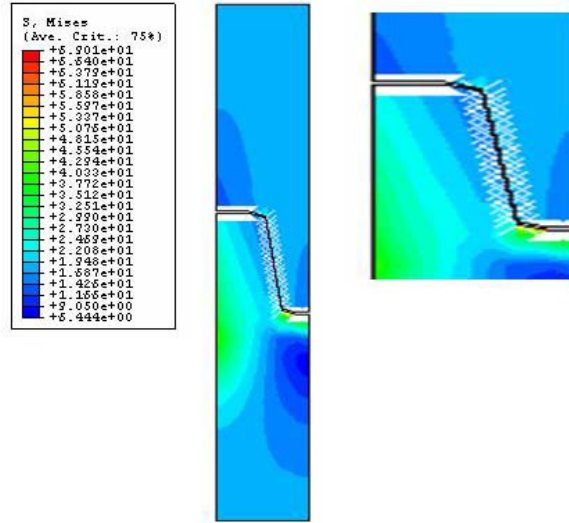


Figure 6-18: Mises Stress contours plotted on undeformed model.

Dark Blue = 0.0 MPa. Bright Red = 69.01 MPa.

The contours in Figure 6-18 show that there are no stress concentrations at the top landings but there still is a region of increased stress near the corner at the lower landing. See Figure 6-19 for a plot of the axial stress predicted by an IEL model with a mode-I strength equal to Y_t and a model with a mode-I strength equal to 50% of Y_t . The critical strain energy release rates for mode-I and mode-II were unmodified from the reported test values, $G_{IC} = 4 \text{ N/mm}$ and $G_{IIC} = 14 \text{ N/mm}$ ⁷. The values for S_1 , S_2 , λ_1 , λ_2 are given in Appendix A.

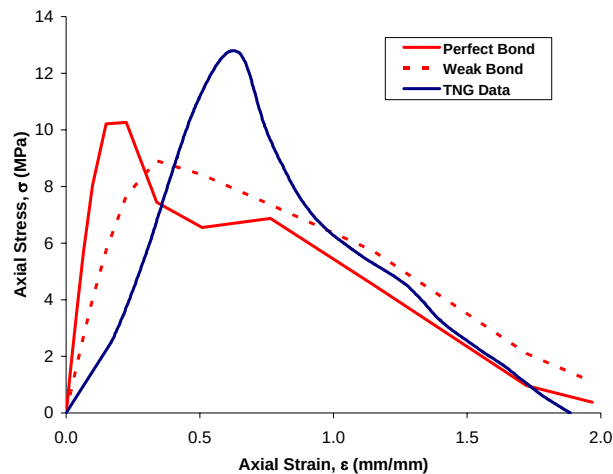


Figure 6-19: Plot of FEA IEL model results compared to test data for $S_1 = Y_t$ and $S_1 = 0.5 Y_t$.

⁷ The units for G_{IC} and G_{IIC} required by the user-defined elements are N/mm. These units are dimensionally consistent with the standard units of J/m^2 .

The IEL model with $S_1=Y_t$ predicts two distinct peaks. The first peak corresponds to the mode-I strength (Y_t) and the second peak corresponds to the mode-II strength (Y_s). The first peak disappears as the mode-I strength is reduced. See Figure 6-20 for a plot of the predicted axial stress-strain curve as a function of the mode-I strength.

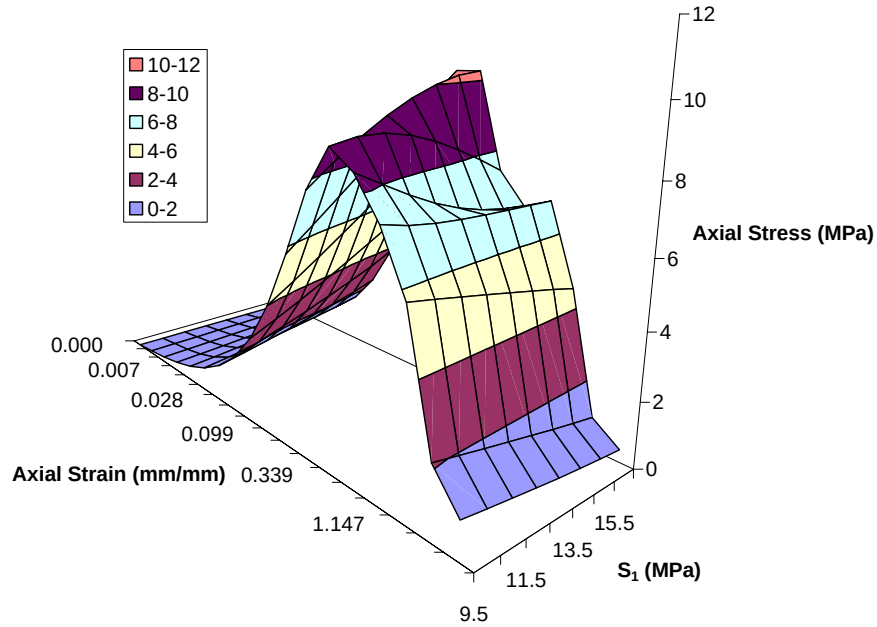


Figure 6-20: Surface Plot of FEA IEL model predictions with S_1 varying from Y_t to $0.5 Y_t$. The IEL models capture the pre-yield and post yield behavior of the test data better than the continuum element material models.

CZM Results

The CZM predictions compared very well to the initial and post yield test data. The failure mechanisms that the model predicted were also corroborated by the test data. See Figure 6-21 for a plot of Mises stress contours on the deformed model geometry.

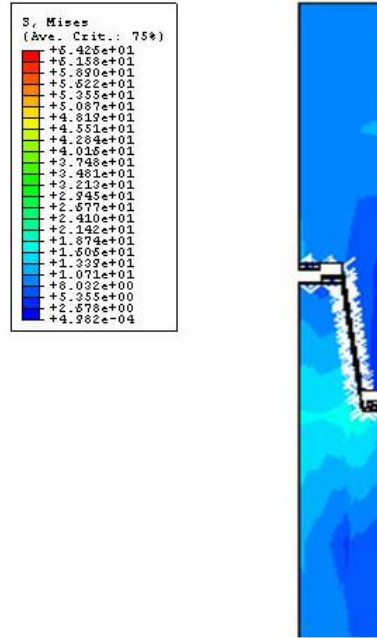


Figure 6-21: Plot of Mises Stress contours on deformed geometry at failure. Dark Blue = 0.0 MPa, Bright Red = 54.25 MPa.

The contours in Figure 6-21 are colored so that dark blue is zero and red is the maximum stress value. The Mises stress is a useful indicator of the presence of local stress concentrations that are usually correlated with the presence of cracks in the bond layer using the IEL. The CZM can also estimate which parts of the bond will fail first. Using the unmodified test values as inputs for λ_1 , λ_2 , S_1 , S_2 , G_{IC} and G_{IIC} the adhesive in the bottom landing failed first, followed by the adhesive at the top landings before the shear region finally let go of the lower adherend.

The CZM predictions were very accurate over the initial strain range, 0 – 25%, and the later post yield range, 100 – 110%, but did not capture the peak failure stress. See Figure 6-22 for plot of the axial stress-strain curve. The CZM also predicted the failure in the top landings at 2.01 MPa. See Figure 6-23 for a plot of the axial stress contours on the deformed mesh geometry focusing on the bottom landing.

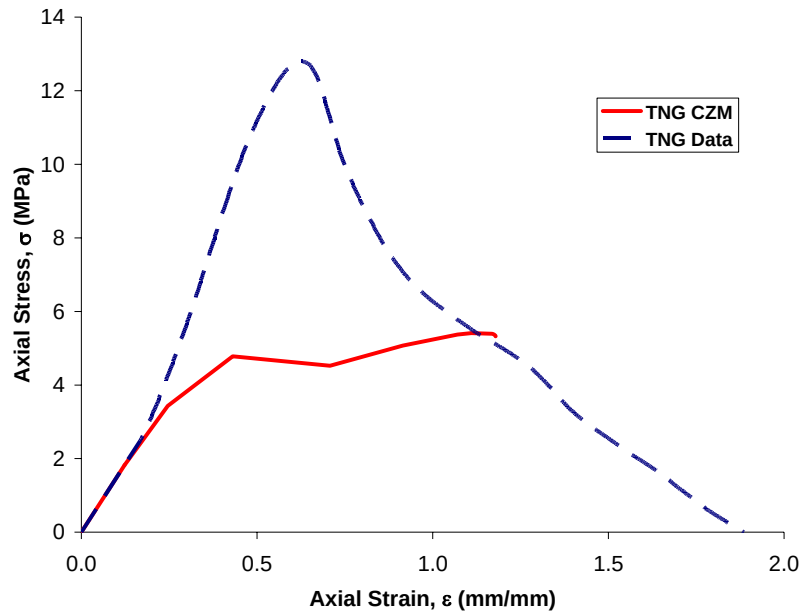


Figure 6-22: Plot of FEA CZM results compared to test data.

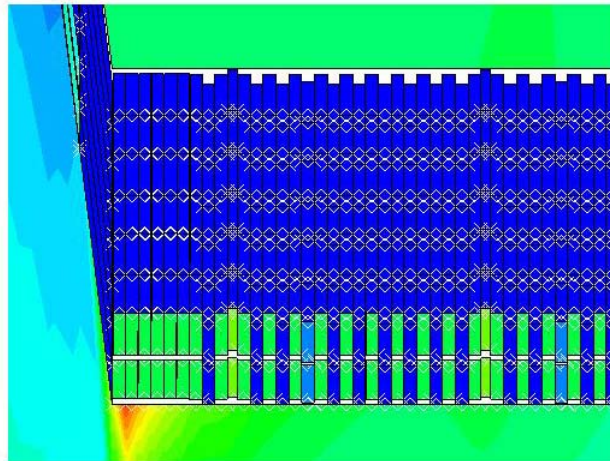


Figure 6-23: Plot of Axial Stress contours on deformed geometry in lower landings.

Dark Blue contours = 0.0 MPa, Bright Red =19.0 MPa.

The white space shown in Figure 6-23 indicates the presence of cracks that have formed in the adhesive due to loading. It is clear from Figure 6-23 that most of the adhesive material in the bottom landing has failed at 2 MPa.

Comparison of TNG Model Error

Each type of model performed differently over the observed strain range. The continuum material model results were less accurate than the more complex IEL and

CZM model results. See Tables 6-7 and 6-8 for summaries of the average total error of the model data compared to the fit of the test data. The NDER model performed the worst over the observed strain range. The CZM and IEL models had less than 8% total error over the entire strain range.

Table 6-7: Average Percent Total Error for Continuum Material Models.

Average % Total Error	
NDER	806.5
MOPP	11.2
NHVP	11.2
Raghava	16.2

Table 6-8: Average Percent Total Error for CZM and IEL Models.

Average % Total Error	
Perfect IEL	7.8
Weak IEL	4.1
CZM	4.7

But the IEL models were not significantly more accurate than the continuum material models on a point error basis. See Tables 6-9 and 6-10 for summaries of the average point error of the model data compared to the test data fit.

Table 6-9: Average Percent Point Error for Continuum Material Models.

Average % Point Error	
NDER	14764.7
MOPP	57.9
NHVP	57.0
Raghava	20.4

Table 6-10: Average Percent Error for CZM and IEL Models.

Average % Point Error	
Perfect IEL	100.3
Weak IEL	39.1
CZM	5.4

The continuum material models had less of an applicable strain range than the IEL and CZM models because the continuum laws could not fail in the same manner as the IEL

based models. See Figure 6-24 for a plot of the NSED over the applicable strain range for each material model. The difference between what a continuum model can predict and the test data suggests that these models should not be relied upon to estimate failure behavior in the TNG specimen.

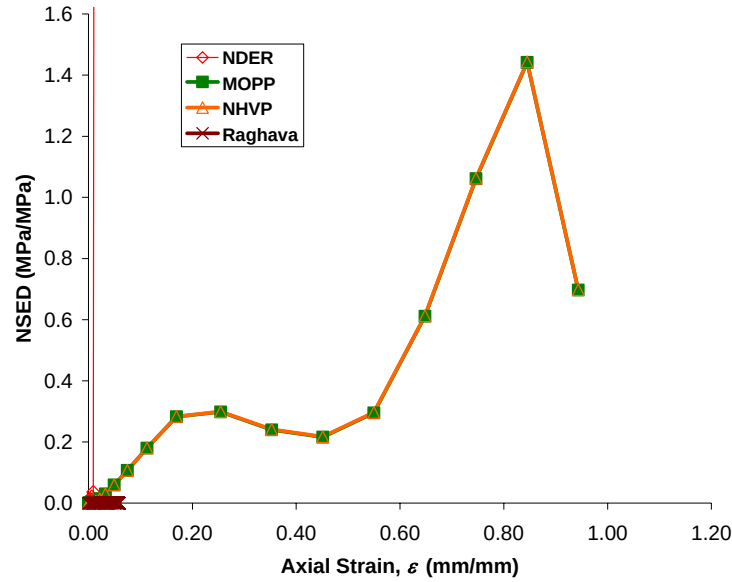


Figure 6-24: NSED as a function of Axial Strain for the Continuum Models.

The continuum material models tended to overestimate the stress in the structure by anywhere from 3 to 150%. The IEL and CZM models tended to underestimate the stress in the structure compared to the test data at yield. See Figure 6-25 for a plot of the NSED over the applicable strain range for each model.

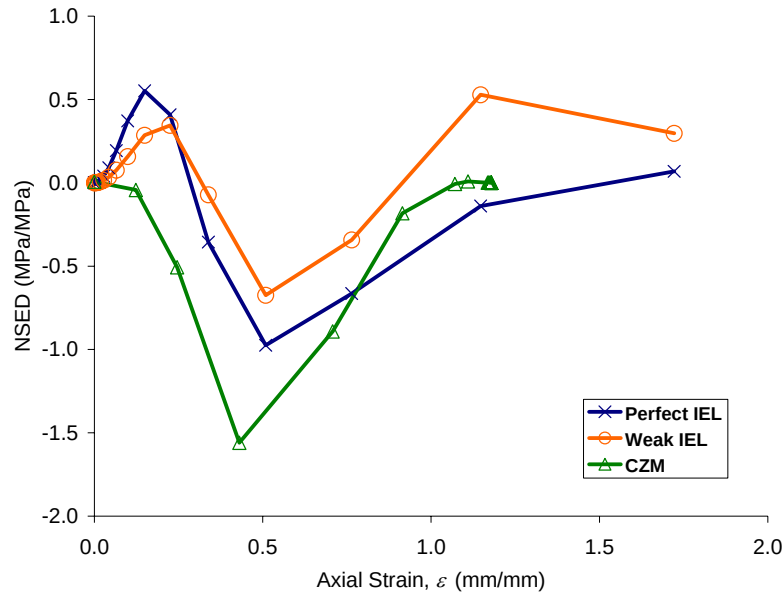


Figure 6-25: Plot of NSED as a function of Axial Strain for the CZM and IEL Models.

The CZM and IEL models estimated lower stresses over most of the applicable strain range and provided more conservative estimates of the TNG joint behavior. The results of the error analysis suggest that the TNG structure is best modeled using CZM in the bond but the joint behavior can also be reasonably captured by a less complex IEL model.

Comparison of Initial and Post Yield Results

The IEL and CZM models performed best in the small strain regime. Comparing the model results to the alternate curve in Figure 6-12 for strain less than 1% shows that the Weak IEL model closely approximates the stiffer initial behavior of the TNG until the landings begin to fail. See Figure 6-26 for a plot of the model results compared against the alternate data curve.

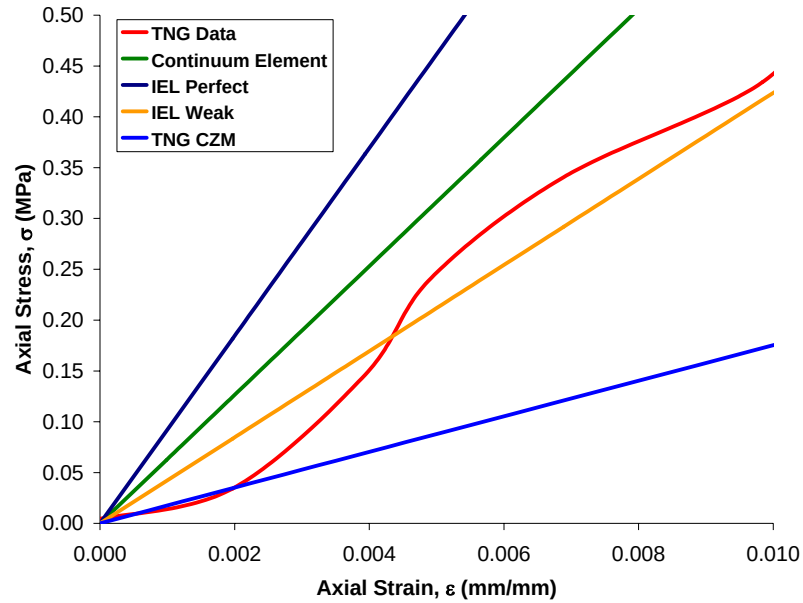


Figure 6-26: Model results compared to test data for small strains.

Figure 6-26 shows that for the small strain regime the slope of the TNG data is always less than the continuum material models and roughly equal to the CZM and Weak IEL models. Figure 6-27 compares the model results against the test data over the entire strain range.

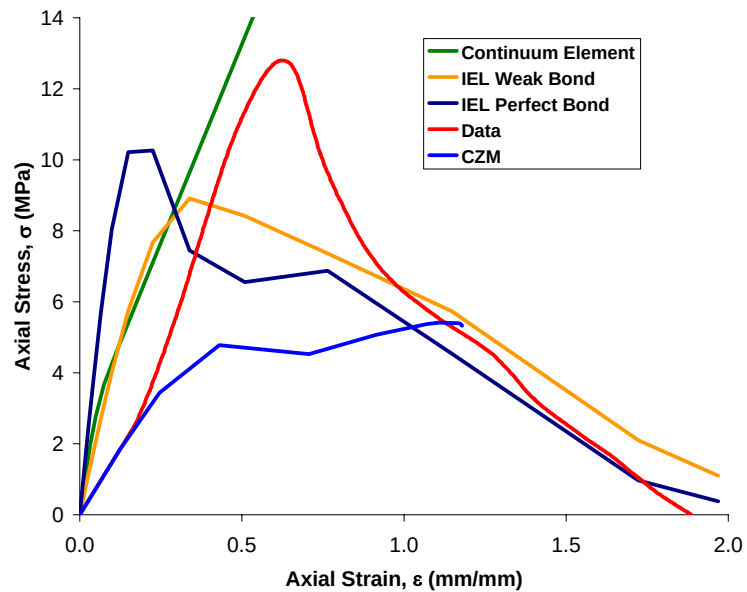


Figure 6-27: Comparison of all model results against test data.

None of the models captured the peak stress accurately using the unmodified bulk material and fracture characterization data. The Perfect IEL and continuum material

models come close to approximating the TNG until yield. The CZM and Weak IEL models come close to approximating the post-yield behavior of the TNG. Figure 6-28 shows how the test data compares to the extreme model results.

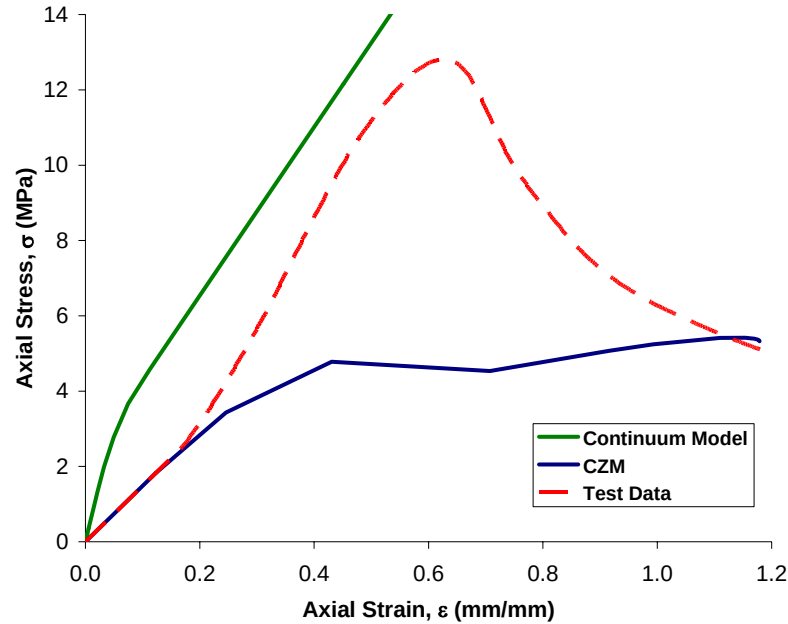


Figure 6-28: Estimated Bounds of TNG Axial Stress-Strain Behavior.

The model results suggest that the TNG behavior will be bounded over the entire strain range by the continuum models as an upper limit and the CZM as a lower limit. The limits of this envelope can be used to predict the failure stresses and strains in the joint and to obtain a factor of safety based on a specific design configuration.

Benefits of Modeling Complex Behavior

What is gained by doing the complex modeling involved with these different analyses? A lot is gained by including the complexity in the models because the most simple and relevant analysis methods that do not involve the same level of detail are not accurate in the case of the TNG. For example, consider the classical model of a double lap joint (DLJ) by Volkersen. An analytical solution for the load and displacement of the center adherend can be derived assuming the adhesive has only shear stiffness and that the adherends can only extend axially. It is reasonable to assume that for some geometric configuration the TNG might behave like a double lap joint. Also consider a CZM model

of a comparably sized Butt Joint (BJ) as a more complex, but still less complicated, approximation of the TNG behavior. It is reasonable to assume that a similarly sized butt joint would represent an upper limit on the stiffness and failure stress in the TNG. Comparing the test data against these models shows that neither model accurately bounds the behavior of the TNG. See Figure 6-29 for plot of the results of the BJ CZM and DLJ analysis compared to the test data.

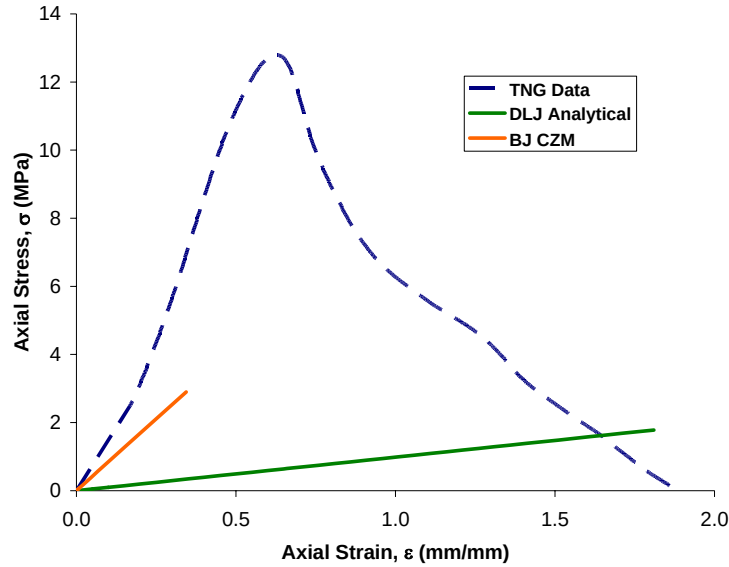


Figure 6-29: Plot of center adherend DLJ result, BJ CZM and TNG data.

Earlier work [92] had shown that the BJ CZM and DLJ could bound the TNG for configurations where there was enough bond area to act like a BJ in tension and enough of a lapped area to behave like a DLJ in shear. Those results used a TNG with a total bonded lapped area equal to roughly 70% of the tension area. But the TNG configurations tested and modeled had a lapped area equal to 230% of the tension area. The ratio of the bond area in shear to the bond area in tension is too high for the TNG to be considered a butt joint. However, there is still enough of the adhesive in tension that the lower limit of the DLJ is not valid either. The draft angle of the tongue piece makes the DLJ and the TNG even more different. Because most of the TNG configurations designed include a larger amount of bonded lap area with some kind of draft angle the simple DLJ analysis or the more complex BJ CZM will not provide useful limits on the behavior of the joint.

If the TNG cannot be modeled as a simpler joint configuration does that also mean the TNG behavior cannot be approximated by a simpler model? No, in fact lower order CZM and IEL models with less mesh refinement are capable of estimating the load deflection behavior of the TNG. Figure 6-30 shows the results for an IEL model with 56 elements (13 IEL) and a model with 1463 elements (431 IEL). The primary differences between the two curves occur after the peak stress.

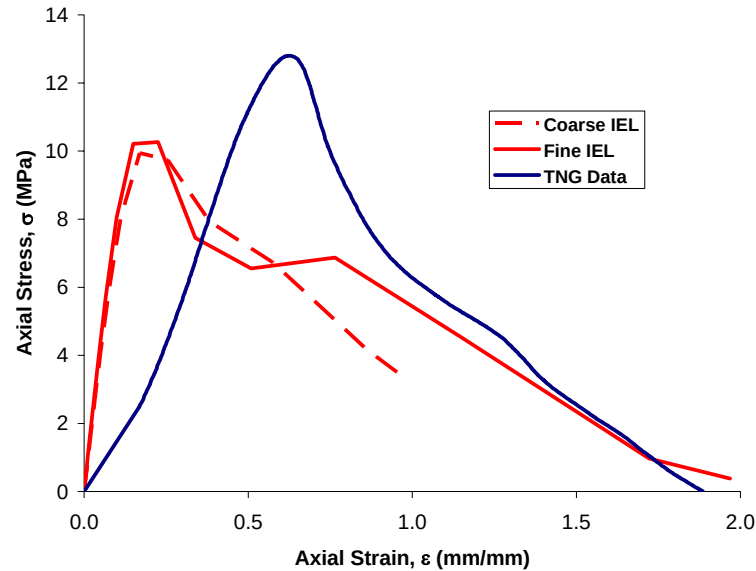


Figure 6-30: Plot of Coarse and Fine Mesh TNG IEL Model Results.

The larger IEL model estimates the post failure behavior better than the smaller, less refined IEL model. But both models behave similarly up to the peak stress in the joint. If a designer was only concerned with the TNG behavior up to 25% axial strain, the smaller IEL model would perform almost as well as the larger model. There is a smaller difference between the predicted behavior of a larger CZM and a smaller CZM TNG. Figure 6-31 compares the results from a small CZM model with 244 elements (108 IEL) and a large model with 1463 elements (1034 IEL) to the test data. It is possible that the refinement in the CZM model prevents it from converging to the observed result. For example, the coarse IEL result closely resembles the test data because the test data represents an average stress over the throat of the TNG. The CZM result represents a finer approximation of the TNG and is highly dependent on local effects. Furthermore, in the CZM model the crack path is not confined as in the IEL only model. The differences between the two approaches may be why the CZM is capable of capturing the initial

slope and landing failures but the IEL models predict a peak stress that is closer to the observed value.

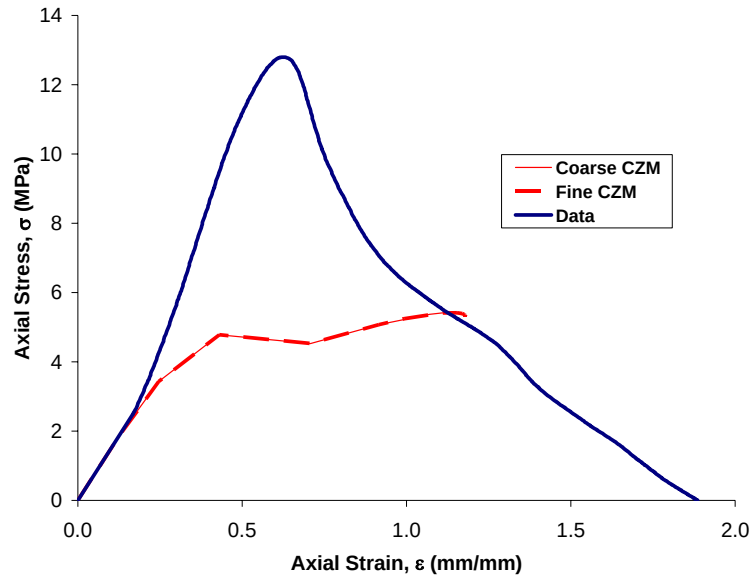


Figure 6-31: Plot of Coarse and Fine Mesh CZM Results.

The differences between the two models in Figure 6-31 are very slight and can only be noticed near a peak. In general, the CZM with more elements will predict a greater peak axial stress value than the smaller CZM. A designer may not be concerned with the best estimate of the nominal peak axial stress when a reasonable estimate that is obtained quickly can suffice. Table 6-11 lists the number of elements and time required to obtain the results in Figures 6-17, 6-30 and 6-31. Each kind of model was run on a Pentium 4 2.2 GHz 512 MB desktop computer and all models were converged to the same level for the nominal peak axial stress.

Table 6-11: Computing Details of Different TNG models.

	Number of Elements -	Avg. Elem. Area mm^2	Time to Finish <i>Minutes</i>
Cont. Elem.	3300	0.0165	6
IEL Only	42	2.49	2
	1463	0.037	12
	2926	0.01248	34
CZM	244	0.056	6
	1385	0.01246	18
	18300	0.006231	56

The pre and post processing time for all of these models was negligible because routines were developed to create, mesh, apply boundary conditions and specify how the output data would be processed for each kind of model. ABAQUS/CAE was used in the case of the TNG continuum element models and user defined routines were developed for the CZM and IEL models. If those routines had not been available it would have been necessary to add a considerable amount of time to the ‘time to finish’ column in table 6-11 to represent the development effort necessary for the CZM and IEL models. Table 6-11 illustrates that a designer could obtain the failure envelope shown in Figure 6-28 in only 12 minutes using the smaller CZM model and a continuum material model of the TNG. This means that more complex TNG models are not only capable of estimating the TNG behavior better than a simple DLJ analysis but just as quickly. Table 6-11 also demonstrates that developing more detailed models of adhesive specimens rewards the analyst with detailed data that can be obtained as quickly as the simpler FE models.

But how much detail is required in the TNG model? Is the draft angle detail really necessary in the model? Yes and no. If it is necessary to know the applicable strain range for a TNG the draft angle must be included because it is the detail of the draft angle that provides the most accurate approximation of the post-failure behavior. If only the peak stress is required then the draft angle detail can be neglected. Figure 6-32 compares the data to results from IEL models with and without a draft angle.

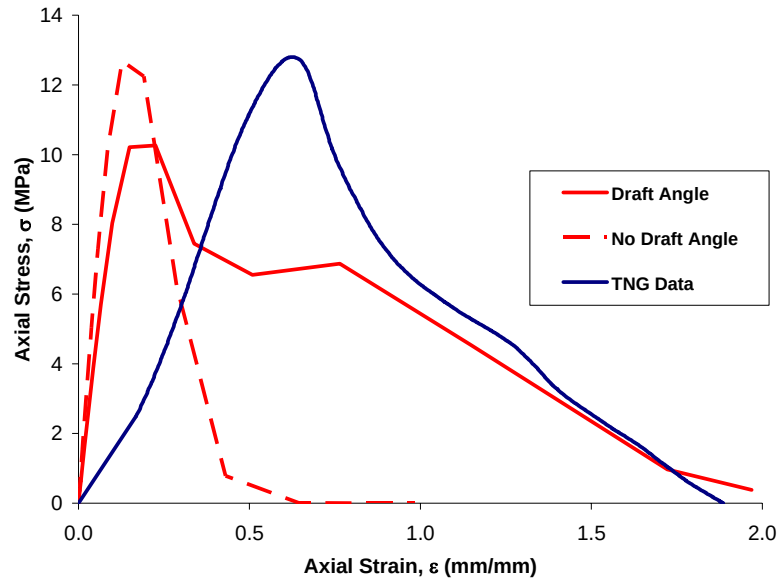


Figure 6-32: Plot of TNG IEL Model Results with and without a draft angle.

However, neglecting the draft angle requires a much finer mesh to capture TNG behavior accurately. In Figure 6-32, both models were converged to within a 3% difference over the previous model in the nominal peak axial stress. The model without a draft angle required 19262 elements and 1.5 hours to get the results shown in Figure 6-32. The model with a draft angle took only 12 minutes to finish and used only 1463 elements. The IEL model without a draft angle is not as useful in estimating the stresses in the adherend or verifying the design geometry. But if that information is not necessary the draft angle detail may be neglected in the IEL models.

The draft angle cannot be neglected in the CZM of the TNG. Figure 6-33 shows that the CZM without a draft angle performs very poorly compared to the test data.

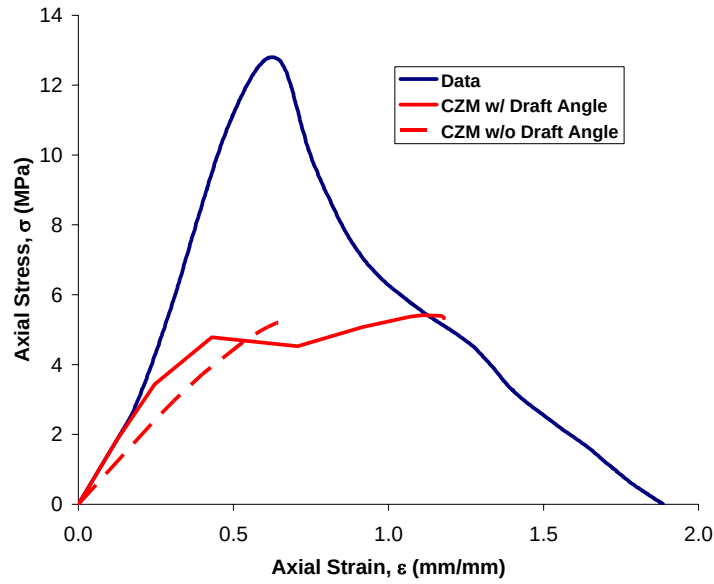


Figure 6-33: Plot of CZM Results with and without a draft angle.

The CZM without a draft also requires a mesh that is several orders of magnitude larger than the CZM with a draft angle. The CZM without a draft angle required 70812 elements (20212 IEL) and took 26.4 hours to complete. The motives behind developing a CZM model are to capture the fine details in a structure, such as bond thickness, draft angle, stress in the adherend, etc. In this case it makes little sense to develop a model that not only requires more time and effort to complete than a smaller model but also provides less accurate information that is not as useful. It is possible to vary a wide variety of parameters in a CZM to get the models to match the test data as closely as possible. For instance, the critical strain energy release rates, failure mode mixity, fracture strengths, bulk adherend and bulk adhesive properties can all be varied from the average values used in the model results reported here. In that case a higher order dimensional model can be developed based on a response surface that defines, for example, how the mode-I strength and G_{IC} affect the overall behavior of the model. However, fitting a response surface based on the bulk and fracture characterization results is outside of the scope of this work.

Tables 6-12 and 6-13 summarize the difference between the peak stress and strain predicted by each of the modeling approaches.

**Table 6-12: Average Percent Difference in Nominal Peak Stress
in the models with respect to the test data.**

Average % Peak Stress Difference	
Perfect IEL Draft	-19.7
Weak IEL Draft	-30.3
IEL No Draft	-0.8
Continuum	88.5
CZM	-57.6

**Table 6-13: Average Percent Difference in Nominal Axial Strain at Peak Stress
in the models with respect to the test data.**

Average % Strain Difference at Peak	
Perfect IEL Draft	-64.2
Weak IEL Draft	-46.2
IEL No Draft	-79.8
Continuum	56.5
CZM	76.5

These results, combined with the previously discussed details, indicate that the best way to estimate the initial, peak stress and post yield behavior of a TNG is to use IEL models and vary the fracture strengths to fit the test data. However, the continuum models developed using the bulk material characteristics are capable of providing limits on the behavior of the TNG up to the nominal peak axial stress in the specimen. The CZM and IEL models developed using both the bulk materials characterization and fracture data can be used to identify the possible failure modes in the TNG specimen.

But what do the results mean in context with the goal of developing a general material model? What do the model results for the TNG and NR specimens demonstrate about the feasibility of characterizing a material model with bulk material properties and then using that same model to represent the adhesive layer in a completely different configuration? The conclusion from this study as a whole is that it is possible to identify, develop and characterize material models based on bulk material characteristics and apply those same models to different in-situ part geometries. The results of that approach will vary based on the specimen, however, and should be validated against test data. The results of the continuum models demonstrate that in situations where the failure mode of the part is similar to the failure mode of the characterization test, such as in the thick bond Arcan and NR specimen in shear, the models give very reasonable results. In cases where the behavior of the part is defined by fracture, or another mode of failure than

observed in the bulk characterization specimens, the continuum models do not give reasonable results. The distinction between the two cases means that it is very important to have the option to include fracture properties in the models of complex specimen geometries. The CZM and IEL models represent two potential methods of addressing these concerns. The ideal method to address the differences between the bulk characterization tests and the in-situ configurations would be to identify the bulk and cohesive properties via the tests and then be able to specialize the models for the type of failure modes observed in a specimen.

The critical step in applying general material models to complex joints such as the TNG is defining a model that can accommodate both the phenomenological nature (e.g. work of adhesion, fracture properties, void coalescence, etc.), and empirical behavior (e.g. failure mode mixity, details of the geometry, etc.) of the part. In other words, for a model to truly be effective the constitutive law must balance how the loads are applied to the adhesive layer against the mechanical response of the adhesive to those loads. In the case of the TNG, the models that were the most useful for capturing the behavior of the specimen and providing useful limits for design had the bulk material properties of the adhesive combined with mode specific failure data. If a material model does not include information about failure modes of the adhesive it will not give an accurate estimate of the state of stress in the part as the adhesive layer approaches yielding and ultimate failure. If the material model is too dependent on fracture properties and does not properly account for the cohesive strength of the adhesive layer it will predict failure prematurely. Regardless of how refined the material model is, or what level of data can be used as an input, the model will not be able to give accurate estimates of the state of stress in the part without including the important features of the part geometry in the finite element model that uses the material law. While it may be convenient to exclude features such as draft angles or bond thickness doing so will limit the accuracy of the model results and the ability of the model to give useful insight to the failure modes in the specimen.

Chapter 7

Conclusions

The previous Chapters presented a problem and possible solutions. The current Chapter summarizes the results of Chapters 5 and 6 to assess which of the models are best used to predict the behavior of the TNG specifically and ductile adhesives in general. Possible directions for future work are also discussed.

Summary of Bulk and In-situ Material Model Results

The continuum element material models discussed in Chapters 3, 4, 5 and 6 all have different advantages and disadvantages. There are several important criteria to judge the performance of each material model, such as: accuracy, sensitivity to variations in test data, range of applicability and ease of use. The Modified Oung Porous Plasticity (MOPP) and Norton Hoff Viscoplastic (NHVP) models performed the best overall.

However, the MOPP model cannot be applied to plane-stress configurations. Therefore, the NHVP model is the best choice for modeling adhesives such as LESA in either bulk or in-situ conditions. The NHVP model is the best model to use because it is fairly accurate in either tension or shear and has the largest range of applicability in-situ without requiring a great amount of material characterization tests. The NHVP model may significantly under or over estimate the state of stress in the adhesive, especially in fracture-controlled failures or cases where both the shear and tensile loads are significant. The Nonlinear Decoupled Elastic Raghava (NDER) law could be used in those cases as an independent check on the values of stress and strain that the NHVP model predicts.

An added benefit to using the Norton Hoff model is that it can be simply extended to accommodate rate dependency. Figure 7-1 shows the NHVP and NDER predictions for the case of uniaxial tension compared to the test data. Figure 7-2 shows the NHVP and NDER predictions for the shear stress in a thick bond Arcan specimen compared to the test data. Figures 7-3 and 7-4 show the results of the NHVP and NDER models for the case of multi-axial loading in a Napkin Ring specimen at 67.5 °.

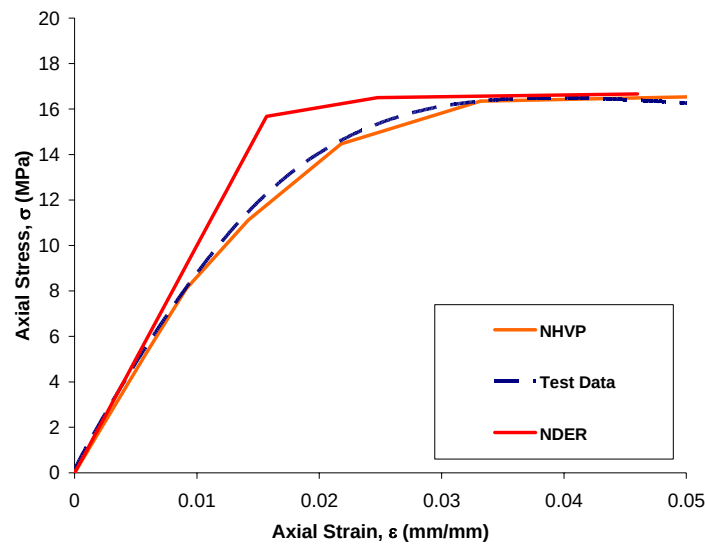


Figure 7-1: Plot of NHVP and NDER model results compared to uniaxial test data.

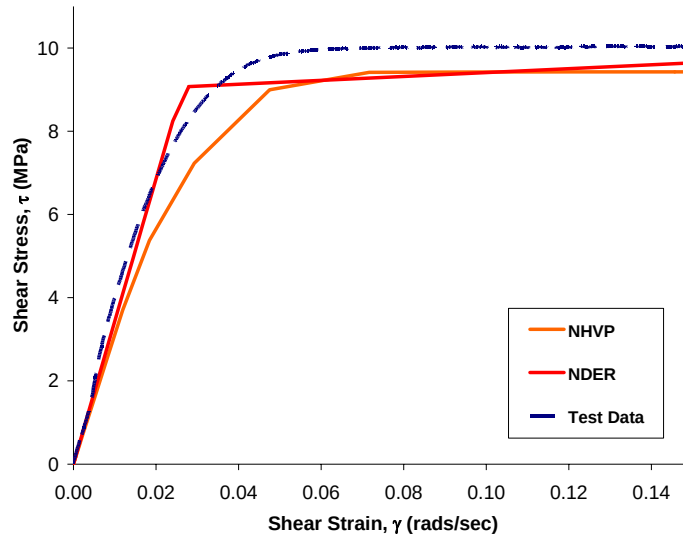


Figure 7-2: Plot of NHVP and NDER model results compared to data from Arcan shear specimen.

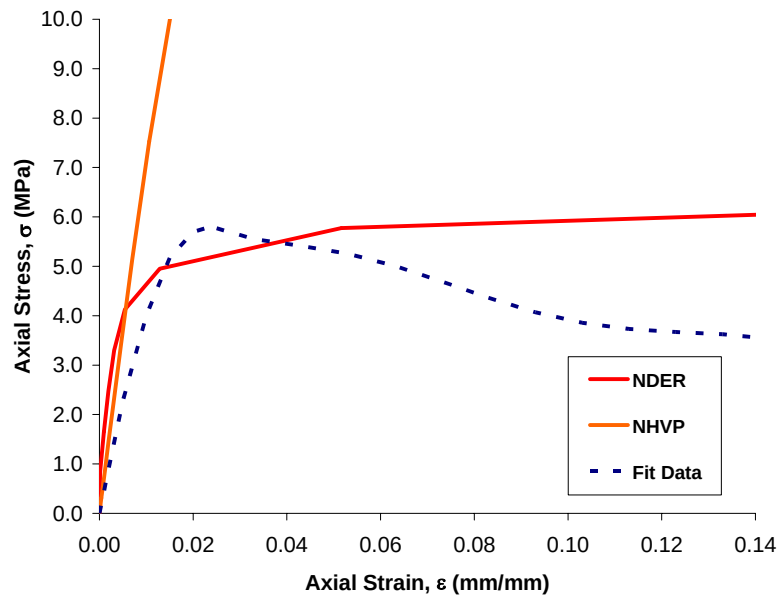


Figure 7-3: NHVP and NDER model results for axial stress in NR specimen loaded at 67.5°.

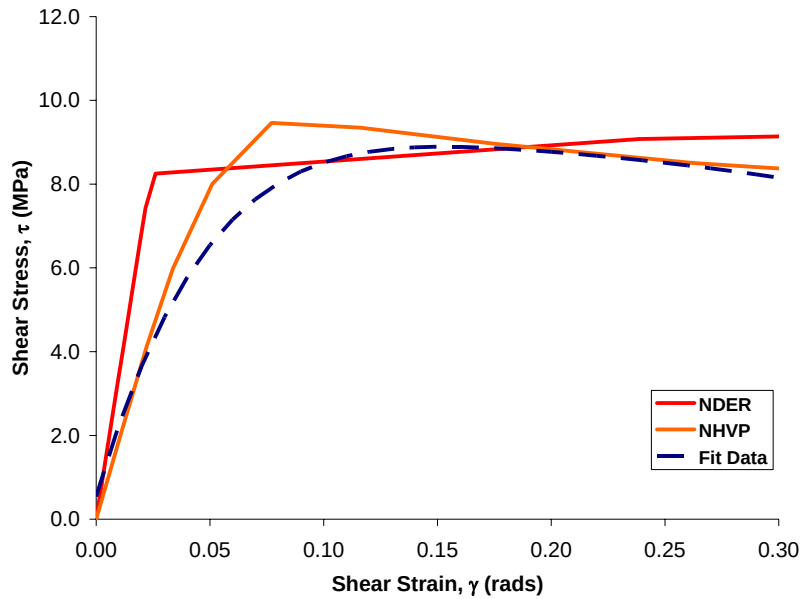


Figure 7-4: NHVP and NDER model results for shear stress in NR specimen loaded at 67.5° .

Summary of Tongue and Groove Modeling Results

The tongue and groove (TNG) specimen is a commonly used, but a complex joint geometry. The failure in the TNG is dominated by the fracture behavior of LESA. The NHVP and NDER continuum material models do not have the ability to predict fracture. However, the models are useful for determining the upper limits of the TNG axial stress and strain before the joint begins to fail. Although the NHVP model will over estimate the axial stress in the joint the NDER model predicts a much lower stress that can bound the model estimate. Figure 7-5 shows the NHVP and NDER results compared to the TNG test data.

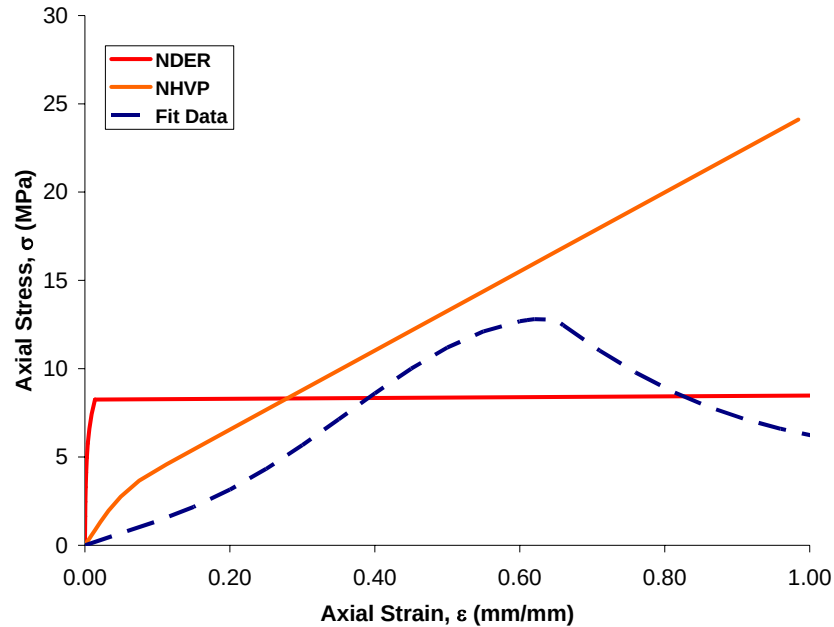


Figure 7-5: NHVP and NDER model predictions for axial stress in the TNG.

In this case, the NDER model provides a much better estimate of the stress in the TNG after the joint begins to fail. Other finite element models are necessary to predict the fracture-dependent behavior of the TNG joint. The IEL and CZM models with a coarse mesh, but a representative geometry that includes as many details as possible, accurately predict the failure limits in the joint due to fracture. Figure 7-6 shows the model results for a CZM and IEL model with input parameters determined from both fracture and bulk characterization tests.

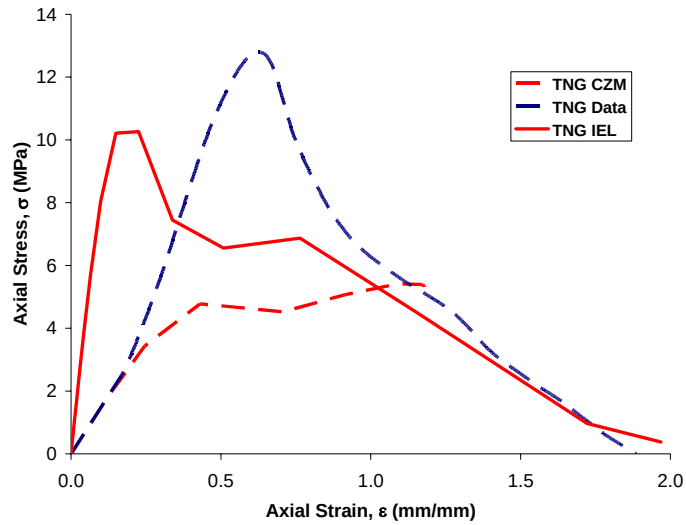


Figure 7-6: CZM and IEL results for the TNG using unmodified test input.

The CZM and IEL models have a large degree of flexibility that allow a variety of analysis options and can explore worst case conditions. The lower strength limit of the TNG is determined by fracture and the upper strength limit is determined by a perfect bond that does not fracture. The CZM and NHVP models bound the behavior of the TNG and can be used to verify failure limits in designs. Figure 7-7 shows how the CZM and NHVP models bound the axial stress data for the TNG.

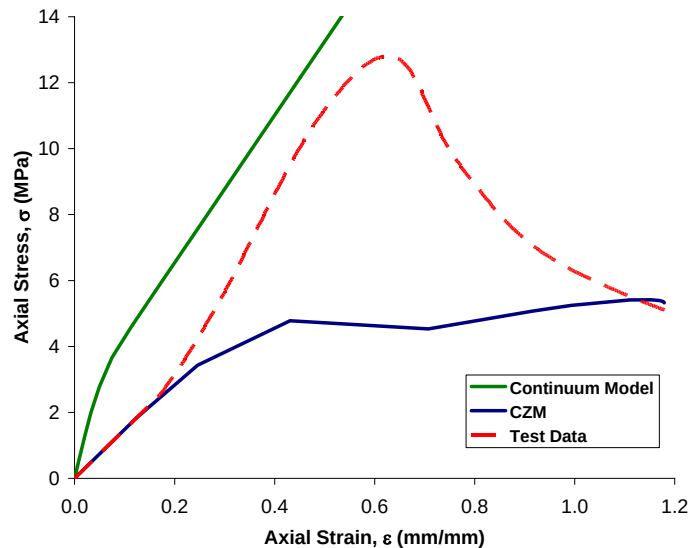


Figure 7-7: TNG failure limits defined by the CZM and NHVP models.

The conclusion of this thesis is summed up in Figure 7-7 where the results of two different models characterized using general test specimens are compared to the response of a realistic adhesively bonded structure. The fact that the envelop described by the CZM and NHVP curves bound the behavior of the TNG reasonably well indicates that it is possible to characterize material models using bulk material data and apply them in a general fashion to a wide variety of geometries. However, the accuracy of the general model on a given configuration is dependent on how closely the failure modes of the test specimens compare to the given geometry. In some cases, such as the TNG, more detailed data is required to obtain more than a wide envelope. It is often very difficult to define a specific critical element or section in the case of a complex specimen like the TNG. But it may be easier to use the two simple quasi-static finite element models shown in Figure 7-7 to check a potential design than to develop a variety of hand solutions to estimate the state of stress in the fixture.

Although the best possible use of the models presented here is to guide the selection of design methods used on a particular geometry. For instance, if the CZM or IEL models closely approximate the behavior of the joint, the designer should then use limits based on the cohesive or interfacial failure of a material. If the continuum models accurately predict the failure of a given specimen then a strength based failure criterion may be the best choice to use when designing the part. Integrating the capabilities of the CZM, IEL and continuum models gives an engineer a powerful tool to both verify designs and examine the effects of “what if” scenarios on potential designs. Such tools make it much easier to use adhesives in complicated designs and help to determine which prototypes may be necessary to validate a candidate design.

Future Work

This work is limited in several respects. None of the models are extended to predict the rate-dependent behavior of the adhesive. The relationship between the bulk and fracture properties, and how they interact in-situ, was not explored. The MOPP and NDER models could also be refined so that they are more useful in FEA using ABAQUS. Exploring those areas would provide engineers with much better tools to verify potential part designs. Extending the models to accommodate rate-dependent behavior would also

give engineers a range of options to use when analyzing the state of stress in an adhesive bond. The work of Liechti [97] and Park [98] in using nonlinear viscoplastic models to approximate adhesive behavior are viable directions for future research. It is also very important to develop statistical methods for predicting the influence of a poor bond, voids or variance in the cohesive strength of the adhesive. The CZM tools described here present a possible choice for exploring the combined effects of those variables on the models. The more recent work of Moës and Belytschko [99] on the extended finite element method (XFEM) could also be used to analyze the behavior of joints such as the TNG.

Regardless of the direction future work takes the lessons learned from this thesis and the results presented here indicate that it is possible to develop a general material model for ductile adhesives used in structural applications under quasi-static conditions. Furthermore, it is also possible to take bulk material data and apply it to an adhesively bonded specimen. The results shown here indicate that it is possible to obtain good results using bulk material data to characterize material models for in-situ configurations provided that the mode of failure of the bulk material specimens closely approximates the mode of failure of the bonded joint.

While none of the models presented in this work perfectly matched the data under all circumstances each model highlights several key features that an ideal general material model should have. Some of the features that a general quasi-static material model should have are:

- (1) Nonlinear stress-strain behavior
- (2) Strength based failure criteria
- (3) Fracture based failure criteria
- (4) The option to control the limit between strength based and fracture based failure
- (5) A strain softening post yield behavior in tension
- (6) The ability to be characterized over a wide range of finite strain magnitudes
- (7) A way to scale the shear strength of the material by the S-ratio
- (8) A way to accommodate the presence of voids in the adhesive

Each of the criteria mentioned above contributed to the performance of the material models presented in this work. Items (4) and (8) may prove the most difficult to incorporate into future work. The work of Richardson et al. [31], in which the relationship between tensile and shear failure is assumed constant and independent of time and temperature, may be a useful guide in determining the relationship between strength and fracture-based failure in adhesive layers. Unfortunately there is little in the way of guidance available for engineers concerned about large voids and cavities developing in the adhesive. However, voiding must be addressed especially when dealing with adhesively bonded structures that have continuous bondlines which do not allow for visual inspection of the bond. Developing a constitutive law that takes all of the criteria into account will give engineers a powerful tool to use when analyzing adhesively bonded parts.

If structural adhesives are to become widely accepted any analysis tools that are developed should be brought to a level where they can be used by the engineering community as whole. Developing modeling tools and analysis methods that cannot be easily implemented by anyone other than a few elite researchers will prevent many engineers from using adhesives in their designs.

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Appendix A: ABAQUS Model Inputs

Each of the ABAQUS/Standard material model options and choices that produced the results discussed in this work are listed here. Readers that are interested in specific details of the UEL code and other options or inputs are directed to the literature in the Bibliography and the ABAQUS documentation.

Raghava Criterion Model

Elastic, Linear Material: $E = 1069 \text{ MPa}$, $\nu=0.433$

Drucker-Prager, Exponential Model: $a = 0.0258$, $b = 2$, $\psi = 12^\circ$.

Drucker-Prager Hardening

Exponential Model

Tension Hardening

Plastic Strain	Stress
0.0000	5.7810
0.0004	8.0661
0.0013	10.2946
0.0026	12.2471
0.0047	13.9234
0.0064	14.6991
0.0091	15.5208
0.0125	16.0928
0.0154	16.3229
0.0215	16.5003
0.0216	16.4800
0.0217	16.4400
0.0219	16.5000
0.0230	16.4000
0.0250	16.3000

Modified Oung Porous Plasticity Model (MOPP)

Elastic, Linear Material: $E = 1069 \text{ MPa}$, $\nu=0.433$

Porous Plasticity Option: $q_1 = 1.10$, $q_2 = 0.953$, $q_3 = 1.20$, Initial Density = 0.95.

Plastic Option

Plastic Strain	Stress
0.000	5.781
0.0001	8.066
0.001	10.295
0.003	12.247
0.005	13.923
0.006	14.699
0.009	15.521
0.013	16.093
0.015	16.323
0.022	16.500

Norton-Hoff Viscoplastic Model (NHVP)

Elastic, Linear Material: $E = 1069 \text{ MPa}$, $\nu = 0.433$

Viscous: $A = 10 \text{ MPa}$, $n = 3$, $m = -0.99$, $f = 0.16817$

Plastic Option

Plastic Strain	Stress
0.000	5.781
0.0001	8.066
0.001	10.295
0.003	12.247
0.005	13.923
0.006	14.699
0.009	15.521
0.013	16.093
0.015	16.323
0.022	16.500

Nonlinear Decoupled Elastic Raghava Law (NDER)

$E = 1069 \text{ MPa}$, $G = 343 \text{ MPa}$, $S = 1.10$, $n = 0.433$, $Y_t = 16.5 \text{ MPa}$, $Y_s = 10 \text{ MPa}$, $\alpha = 0.5$,

$\beta = 10$, $n = 125$, $m = 55$

Goyal and Johnson Interface Element Parameters (IEL)

$G_{IC} = 4 \text{ N/mm}$, $G_{IIC} = 14 \text{ N/mm}$, $S_I = 16.5 \text{ MPa}$, $S_{II} = 10 \text{ MPa}$, Interpenetration Constant, $KNP = 5.0 \times 10^6$, Mixed Mode Parameters, $\lambda_I = \lambda_{II} = 0.9$, Integration Scheme = 3.0, Number of Integration Points per Node = 1.0

Bulk Adhesive Elements for the Cohesive Zone Model (CZM)

Elastic, Linear Material: $E = 1069 \text{ MPa}$, $\nu = 0.433$

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Vita

Chris Cassino was born in 1977. In 1996 he was part of the first class of civil and structural engineering students at Rowan University as part of the PRIDE 2000 Engineering Scholarship. Chris has published research on adhesives, the failure mechanisms of composite shell structures, the testing of civil engineering materials, nondestructive evaluation and finite element modeling of nonlinear materials.

Chris Cassino holds Master's degrees from both the Engineering Mechanics and Mechanical Engineering Departments of Virginia Polytechnic Institute and State University which are, despite the similar names that appear on the degrees, vastly different fields of study. Chris is currently employed by Westinghouse Electric Company as a forensic engineer and design analyst and hopes to be a licensed professional engineer soon. In his spare time he is an Instructional Associate teaching Mechanical Vibrations for Laureate University in their accredited Engineering and Applied Science program. Chris is certain that several of his previous professors will have apoplexies upon finding that out.

He resides in Pittsburgh with his wife and daughter and has promised both of them that he will never go back to school again. Unless a really interesting class is offered at a local college. Or perhaps to pursue a degree in law...