

**SCALING A PRISMATIC REVOLUTE JOINT (PR) MANIPULATOR USING
SIMILITUDE AND BUCKINGHAM PI TECHNIQUES**

by

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(ABSTRACT)

This thesis presents scaling methods for sizing a prototype micro prismatic revolute (PR) manipulator actuated by permanent magnet (PM) direct current (d.c.) gearmotors. Dimensional analysis was the principle tool used in this investigation, and addressed the problems of scaling a trajectory planner, control law, and gearmotors that exhibit internal nonlinear friction. Similitude methods were used to develop a scaleable two degree-of-freedom trajectory planner from a third order polynomial. Scaling laws were developed from Buckingham's Pi theorem to facilitate the selection process of gearmotors. Nondimensional, nonlinear, differential equations were developed to describe viscous, Coulomb and static friction in comparative PM d.c. motors. From the insights gained through dimensional analysis, a scaleable controller based on the computed torque method was developed and implemented with a cubic trajectory planner. Model and prototype PR manipulator systems were simulated using a hybrid Matlab/Simulink simulation scheme. Experimental systems were constructed with dissimilar model and prototype motors. Control was provided by an AT class PC equipped with 12-bit A/D, D/A cards operating at a sample rate of 100 Hz. The control algorithm was written in Borland 3.1 C for DOS. Results from the experimental testing showed excellent agreement between the test and simulated data and verified the viability of the scaling laws. The techniques presented in this thesis are expected to be applicable to any application that involves scaling PM d.c. micro gearmotors that have significant internal friction terms. These simple, practical tools should be especially beneficial to designers of micro robotic systems.

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