Investigation of the Forces Produced by Impact

by

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Investigation of Forces Produced by Impact

I. Introduction:

An important problem confronting the designer of machines and structures which may be subjected to suddenly applied loads is the determination of the actual forces which the members must withstand.

This investigation has been confined to a study of impact forces on beams in simple flexure.

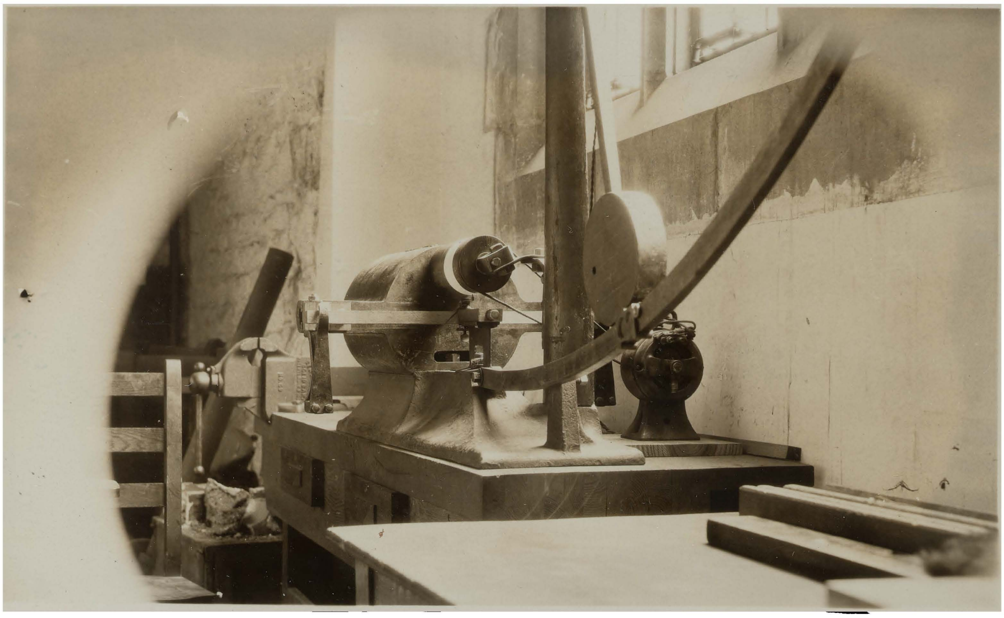
An interesting case in which impact confronts the designer is that of the airplane. The airplane, because of its high landing speed, is subjected to enormous forces caused by impact when it lands. The landing gear which is a necessary evil is made as light as possible, and in many cases it is crushed by the impact force at landing.

If a method could be devised for measuring directly the maximum force occurring during impact, many of the designer's problems would be overcome. Some work has been done with this object in view, but no satisfactory results have been obtained.

Margerum (American Society of Testing Materials, 1922) suggested a method whereby the maximum load might be found by obtaining indentations on a calibration bar attached to the moving head of the impact machine, and suggested using this in conjunction with an autographic stress strain diagram.

Tests have been made upon both notched and unnotched bars, but the results were not satisfactory.

It has been proposed that an accelerometer be designed with which the acceleration could be measured directly. After the acceleration is known the force producing the acceleration can be calculated. The acceleration of a point can be determined by a double differentiation of a space-time diagram. This method is being employed in this thesis.



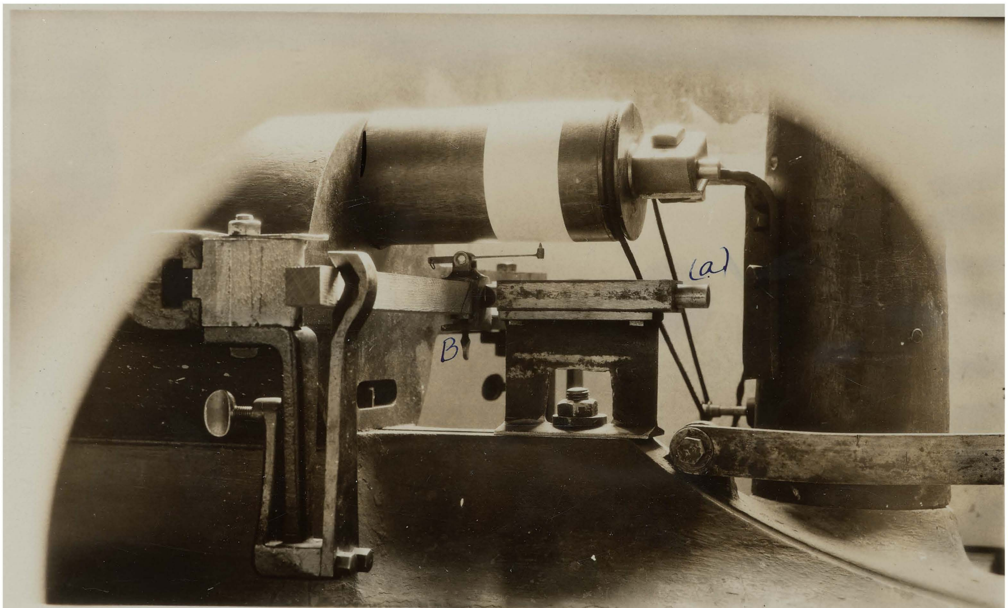


Fig-A

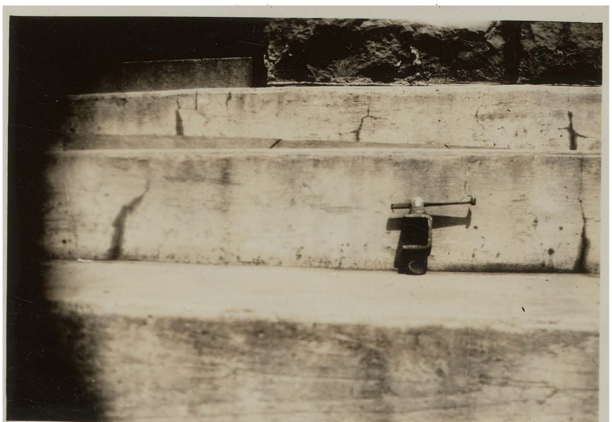


Fig-B

II. Description of Apparatus:

The machine used for these experiments was originally used for notched bar tests and had to be redesigned.

The machine, as shown in figure A, consists of a body with two supports which are designed to clamp a horizontal beam at both ends. A plunger (a) is located horizontally and at right angles to the beam. The hammer on this machine is of the pendulum type, weighs 25 pounds, and strikes the plunger longitudinally.

The pencil attachment, as shown in figure B, was designed and made in the laboratory. It consisted of a U clamp which was attached to the center of the beam, the pencil was attached to the top of the clamp so that the deflection of the pencil would be in the line of deflection of the center of the beam. This was a very important feature since it minimized vibrations and the effects of inertia of the pencil.

The rotating drum in this case was made of wood. The drum was located directly above the pencil point with its axis parallel to the line of action of the plunger. The drum was rotated by means of a small motor.

A tuning fork of 119 vibrations per second was used for determining the speed of the drum and the time of deflection of the beam. It was operated on 110 volts alternating current. The tuning fork was mounted so that the pointer was at right angles to the drum.

The object of having a machine of this type is to obtain a space-time curve from which the velocity and acceleration of a point can be determined at any instant after impact. The (s-t) curves of these experiments are shown in figures 1 to 14 inclusive.

III. Procedure of Tests Performed:

In these experiments oak beams 1" x 1" x 25" were investigated. The center of the beams were determined and 12" laid off on each side from the center. The pencil was attached to the center of the beam and the beam placed in the machine so that the distance between supports was 24", the center of the beam being in line with the plunger. A strip of chronograph paper was placed upon the drum.

The motor was started and the tuning fork set vibrating. The hammer was raised and let fall from a certain height. The time of deflection was taken by means of the tuning fork.

Four tests were made upon one beam with a 4" fall so that the results could be compared, figures 1, 2, 3 and 4. Four tests were made upon another beam, two with a 4" fall and two with a 5" fall, figures 5, 6, 7, and 8. Six tests were made upon another beam with the height of fall varying from 2" up to 9", figures 9, 10, 11, 12, 13 and 14.

IV. Sample Calculation:

If a point moves in such a way that the distance s cannot be expressed algebraically in terms of time t, the speed may be found graphically from a distance-time (s-t) curve. Such a curve is shown in Figure 1, time in seconds being measured horizontally and distance in inches being measured vertically.

Since the slope of the (s-t) curve is represented by $\frac{ds}{dt}$ and since velocity (v) = $\frac{ds}{dt}$, then the slope at any point of the (s-t) curve represents, to some scale, the speed at the corresponding instant.

Take for an example point 1 of figure 1, the slope of the curve at point 1 = $\frac{BC}{AC} = \frac{1}{1} = 1$, but 1 inch vertically represents 0.167 inches and 1 inch horizontally represents 0.0032 seconds, Therefore,

$$\text{Unit slope} = \frac{0.167}{0.0032 \times 12} = 4.34 \text{ ft./sec.}$$

Therefore the velocity at that instant = slope at point 1 multiplied by the unit slope.

$$\text{Velocity} = 1.000 \times 4.34 = 4.34 \text{ ft./sec.}$$

Now knowing the velocity at any instant a velocity-time curve can be plotted.

Since the acceleration of a point is equal to the derivative of the velocity, acceleration = $\frac{d^2s}{dt^2} = \frac{dv}{dt}$ and since the slope of the (v-t) curve = $\frac{dv}{dt}$, the acceleration at any instant can be determined graphically.

Solution:

Slope of (v-t) curve at point A' = $\frac{A'B}{BC'} = \frac{4.34}{.77} = 5.57$ but 1 inch vertically represents 1 ft/sec, 1 inch horizontally represents 0.0032 sec., Therefore,

$$\text{Unit slope} = \frac{1}{0.0032} = 312 \text{ ft/sec}^2$$

Acceleration = Slope x Unit slope = 5.57 x 312 = 1736 ft/sec^2

From Newton's law of gravitation,

Force = Mass x Acceleration

$$F = \frac{W}{g} \times a,$$

we can find the force producing the acceleration (a).

W = 25 lbs. weight of hammer

a = 1736 ft/sec^2

g = 32.2 ft/sec^2

$$F = \frac{25}{32.2} \times 1736 = 1350 \text{ lbs.}$$

Calculations for other points were made in the same manner.

Fig. 1.

Point	Slope of Curve	Velocity ft/sec
1	1.000	4.34
2	.890	3.89
3	.867	3.78
4	.696	3.01
5	.893	3.88
6	.880	3.82
7	.895	3.88
8	.842	3.66
9	.878	3.82
10	.800	3.47
11	.763	3.32
12	.698	3.03
13	.637	2.68
14	.572	2.48
15	.521	2.26
16	.474	2.06
17	.443	1.92
18	.414	1.88
19	.369	1.61
20	.302	1.32
21	.250	1.09
22	.191	.825
23	.144	.625
24	.088	.055

Scale

1" horizontally $= \frac{4.24}{119 \times 11.55} =$ .0032 sec.

1" vertically = 0.167"

Unit slope $= \frac{0.167}{.0032 \times 12} = 4.34$ ft/sec

<u>Calculations</u>.(Point 1)
Slope of curve x Unit slope = Velocity

1.000 x 4.34 = 4.34 ft/sec

For the acceleration:
Scale

1" horizontally = .0032 sec.

1" vertically = 1 ft/sec

Unit slope $= \frac{1}{.0032} = 312$ ft/sec^2

Slope of velocity curve at point 1 = 5.57

Acceleration = 312 x 5.57 = 1736 ft/sec^2

Force $= \frac{W}{g} \times a$

$= \frac{25}{32.2} \times 1736 = 1350$ lbs.

Fig. 2.

Point	Slope of Curve	Velocity ft/sec
1	1.000	4.34
2	.960	4.17
3	.850	3.70
4	.840	3.65
5	.816	3.55
6	.857	3.72
7	.924	4.01
8	.844	3.66
9	.800	3.47
10	.778	3.38
11	.741	3.20
12	.698	3.03
13	.667	2.90
14	.606	2.64
15	.567	2.46
16	.507	2.20
17	.445	1.93
18	.410	1.79
19	.378	1.64
20	.345	1.50
21	.260	1.13
22	.189	.817
23	.132	.577
24	.087	.037

<u>Calculations</u> (Point 1)

Slope of curve x unit slope = velocity

1.000 x 4.34 = 4.34 ft/sec

For the acceleration:

Unit slope = 312 ft/sec^2

Slope of velocity curve at point 1 = 5.64

Acceleration = 312 x 5.64 = 1760 ft/sec^2

Force = $\frac{25}{32.2}$ x 1760 = 1370 lbs

Fig. 3.

Point	Slope of curve	Velocity ft/sec
1	1.000	4.34
2	.818	3.57
3	.750	3.27
4	.850	3.70
5	.881	3.82
6	.881	3.82
7	.842	3.66
8	.845	3.68
9	.818	3.56
10	.756	3.29
11	.726	3.15
12	.684	2.95
13	.606	2.64
14	.573	2.49
15	.548	2.38
16	.549	2.38
17	.520	2.26
18	.480	2.09

Calculations (Point 1)

Unit slope = 4.34 ft/sec

Velocity = 1.000 x 4.34 = 4.34 ft/sec

For the acceleration:

Unit slope = 312 ft/sec^2

Slope of velocity curve at point 1 = 4.98

Acceleration = 312 x 4.98 = 1560 ft/sec^2

Force = $\frac{25}{32.2}$ x 1560 = 1210 lbs.

Fig. 4.

Point	Slope of curve	Velocity ft/sec
1	1.000	4.34
2	.957	4.16
3	.848	3.69
4	.762	3.31
5	.745	3.23
6	.802	3.48
7	.789	3.43
8	.785	3.41
9	.778	3.38
10	.735	3.19
11	.717	3.12
12	.678	2.95
13	.664	2.88
14	.632	2.75
15	.570	2.48
16	.542	2.36
17	.500	2.17
18	.442	1.93

Calculations (Point 1)

Unit slope = 4.34 ft/sec

Velocity = 1.000 x 4.34 = 4.34 ft/sec

For the acceleration:

Unit slope = 312 ft/sec^2

Acceleration = 312 x 6.19 = 1930 ft/sec^2

Force $= \frac{25}{32.2} \times 1930 = 1500$ lbs.

Fig. 5.

Point	Slope of Curve	Velocity ft/sec
1	1.000	4.61
2	.882	4.07
3	.887	4.08
4	.847	3.90
5	.865	3.98
6	.828	3.81
7	.803	3.70
8	.782	3.60
9	.774	3.56
10	.757	3.49
11	.725	3.34
12	.714	3.29
13	.677	3.12
14	.638	2.94
15	.596	2.74
16	.555	2.56
17	.532	2.45
18	.502	2.31
19	.473	2.18
20	.442	2.04
21	.403	1.86
22	.363	1.67
23	.322	1.48
24	.285	1.31

Calculations (Point 1)

Unit slope = 4.61 ft/sec

Velocity = 1.000 x 4.61 = 4.61 ft/sec

For the acceleration:

Unit slope = 331 ft/sec^2

Acceleration = $331 \times \frac{4.61}{.83}$ =

1840 ft/sec^2

Force = $\frac{25}{32.2} \times 1840$ = 1428 lbs.

Fig. 6.

Point	Slope of Curve	Velocity ft/sec
1	1.000	4.57
2	.921	4.22
3	.891	4.08
4	.863	3.95
5	.857	3.92
6	.832	3.81
7	.821	3.76
8	.792	3.62
9	.777	3.56
10	.756	3.46
11	.718	3.29
12	.700	3.20
13	.663	3.03
14	.622	2.85
15	.588	2.69
16	.555	2.54
17	.526	2.41
18	.495	2.27
19	.457	2.09
20	.424	1.94
21	.387	1.77
22	.363	1.67

Calculations (Point 1)

Unit slope = 4.57 ft/sec

Velocity = 1.000 x 4.57 = 4.57 ft/sec

For the acceleration:

Unit slope = 329 ft/sec^2

Acceleration = $329 \times \frac{4.57}{.83}$ = 1805 ft/sec

Force = $\frac{25}{32.2} \times 1805$ = 1450 lb.

Fig. 7.

Point	Slope of Curve	Velocity ft/sec
1	1.100	5.03
2	.948	4.33
3	.928	4.24
4	.922	4.21
5	.915	4.18
6	.901	4.12
7	.867	3.96
8	.850	3.88
9	.821	3.75
10	.804	3.67
11	.780	3.57
12	.767	3.51
13	.751	3.43
14	.728	3.32
15	.697	3.19
16	.673	3.10
17	.650	2.97
18	.618	2.83
19	.568	2.60
20	.522	2.39

Calculations (Point 1)

Unit slope = 4.57 ft/sec

Velocity = 4.57 x 1.100 = 5.03 ft/sec

Unit acceleration = 329 ft/sec^2

Acceleration = 3.29 x $\frac{5.03}{.67}$ = 2460 ft/sec^2

Force = $\frac{25}{32.2}$ x 2460 = 1915 lbs.

Fig. 8.

Point	Slope of curve	Velocity ft/sec
1	1.060	4.80
2	.942	4.26
3	.930	4.20
4	.934	4.22
5	.916	4.13
6	.894	4.03
7	.864	3.90
8	.863	3.89
9	.842	3.80
10	.813	3.67
11	.804	3.63
12	.790	3.57
13	.760	3.43
14	.743	3.36
15	.719	3.23
16	.693	3.13
17	.657	2.97
18	.623	2.82
19	.578	2.61
20	.540	2.46
21	.485	2 .19
22	.441	1.99

Calculations (Point 1)

Unit slope = 4.52 ft/sec

Velocity = 4.52 x 1.060 = 480 $\frac{ft}{sec}$

For the acceleration:

Unit slope = 325 ft/sec^2

Acceleration = 325 x $\frac{4.80}{.72}$ = 2170 ft/sec^2

Force = $\frac{25}{32.2}$ x 2170 = 1700 lbs.

Fig. 9.

Point	Slope of curve	Velocity ft/sec
1	.615	2.71
2	.565	2.39
3	.588	2.59
4	.523	2.30
5	.535	2.35
6	.522	2.29
7	.506	2.23
8	.489	2.15
9	.479	2.11
10	.462	2.03
11	.452	1.99
12	.447	1.97
13	.432	1.90
14	.418	1.84
15	.405	1.78

Calculations (Point 1)

Unit slope = 4.40 ft/sec

Velocity = 4.40 x .615 = 2.71 ft/sec

Unit acceleration = 316 ft/sec^2

Acceleration = 316 x $\frac{2.70}{1.0}$ = 854 ft/sec^2

Force = $\frac{25}{32.2}$ x 854 = 662 lbs.

Fig. 10.

Point	Slope of curve	Velocity ft/sec
1	.928	3.86
2	.742	3.09
3	.760	3.16
4	.725	3.02
5	.723	3.01
6	.700	2.92
7	.683	2.82
8	.659	2.72
9	.631	2.61
10	.601	2.50
11	.572	2.38
12	.548	2.28
13	.525	2.19
14	.497	2.07
15	.462	1.92

Calculations (Point 1)

Unit slope = 4.17 ft/sec

Velocity = 4.17 x .928 =

3.86 ft/sec

Unit acceleration = 299 ft/sec^2

Acceleration = 299 x $\frac{3.85}{.63}$ =

1825 ft/sec^2

Force = $\frac{25}{32.2}$ x 1825 = 1415 lbs.

Fig. 11.

Point	Slope of curve	Velocity ft/sec
1	1.000	4.51
2	.973	4.38
3	.890	4.02
4	.890	4.02
5	.880	3.98
6	.862	3.89
7	.832	3.78
8	.803	3.62
9	.780	3.52
10	.759	3.42
11	.720	3.25
12	.675	3.04
13	.632	2.85
14	.585	2.64
15	.540	2.44

Calculation(Point 1)

Unit slope = 451 ft

Velocity = 4.51 x 1.000 = 4.51 ft/sec

Unit acceleration = 322 ft/sec^2

Acceleration = $322 \times \frac{4.50}{1.52}$ = 955 ft/sec^2

Force = $\frac{25}{32.2} \times 955$ = 740 lbs.

Fig. 12.

Point	Slope of curve	Velocity ft/sec
1	1.086	4.90
2	1.070	4.83
3	1.045	4.72
4	1.011	4.56
5	1.000	4.51
6	.960	4.63
7	.938	4.32
8	.902	4.16
9	.873	4.02
10	.844	3.89
11	.820	3.78
12	.798	3.68
13	.760	3.50
14	.740	3.41
15	.708	3.26

Calculation (Point 1)

Unit slope = 4.51 ft/sec

Velocity = 4.51 x 1.086 = 4.90 ft/sec

Fig. 13.

Point	Slope of curve	Velocity ft/sec
1	1.320	5.82
2	1.300	5.72
3	1.265	5.57
4	1.220	5.37
5	1.195	5.26
6	1.180	5.19
7	1.120	4.93
8	1.087	4.78
9	1.046	4.60
10	1.012	4.45
11	.978	4.31
12	.933	4.11
13	.916	4.03
14	.882	3.88
15	.838	3.69

Calculations (Point 1)

Unit slope = 4.40 ft/sec

Velocity = 4.40 x 1.320 = 5.82 ft/sec

Fig. 14.

Point	Slope of curve	Velocity ft/sec
1	1.359	6.32
2	1.345	6.26
3	1.325	6.17
4	1.292	6.02
5	1.260	5.86
6	1.222	5.68
7	1.179	5.48
8	1.153	5.37
9	1.133	5.27
10	1.090	5.11
11	1.055	4.91
12	1.041	4.85
13	.990	4.61
14	.940	4.37
15	.889	4.13

Calculations (Point 1)

Unit slope = 4.66 ft/sec

Velocity = 4.66 x 1.359 =

6.32 ft/sec

Applying the equation of work and energy to the beam, the energy stored in the beam is equal to the work done.

$$Wh = \frac{1}{2} Pd$$

where

W = weight of hammer (lbs)

h = height of fall (in.)

P = force produced (lbs)

d = deflection of beam (in.)

Using the above formula the force produced when the deflection was a maximum was calculated.

Weight of hammer 25 lbs.

From:

Fig. 1 P = 400 lbs.

Fig. 2 P = 400 lbs.

Fig 3 P = 417 lbs.

Fig. 4 P = 408 lbs.

Fig. 5 P = 344 lbs.

Fig. 6 P = 350 lbs.

Fig. 7 P = 373 lbs.

Fig. 8 P = 386 lbs.

Fig. 9 P = 278 lbs.

Fig. 10 P = 334 lbs.

Fig. 11 P = 378 lbs.

Fig. 12 P = 455 lbs.

Fig. 13 P = 515 lbs.

Fig. 14 P = 550 lbs.

These results do not check with the differentation method.

V. Discussion:

From these experiments no definite conclusion could be reached, however several interesting points developed which would be worthy of further investigation. These will be offered as suggestions in case the investigation is continued.

The results of eleven out of fourteen tests were decidedly similar. The velocity-time curves showed the same general characteristics and the maximum force was found to check within reasonable limits.

The last three experiments showed very erratic results as compared with the first eleven. The velocity-time curves show a gradual decrease in velocity from a maximum to zero where the deflection is a maximum, whereas in the first eleven experiments the velocity dropped suddenly from a maximum, and then decreased gradually to zero where the deflection was a maximum. The acceleration and maximum force were much lower as compared with the first experiments. It appears that a portion of the velocity-time curves of the first eleven experiments are incorrect as compared with the last three since there is no force acting on the beam after the instant of impact except that due to impact, the inertia of the beam, and the resistance of the fibers, which could cause the velocity to decrease suddenly. This point should be further investigated before any conclusion could be reached.

Another point of interest which was noted was the vibration of the beam after impact. These vibrations appear in the form of a sine wave and gradually decrease to zero.

These vibrations were obtained by attaching the pencil directly to the center of the beam. This method should give more accurate results than that of attaching the pencil to the falling weight, which has been used in some experiments. With the pencil attached to the hammer one cannot get the vibrations of the beam. The maximum deflection would not be obtained unless the falling hammer is in contact with the beam throughout the deflection.

In order to obtain more accurate results a machine would have to be carefully designed. The rotating drum should be made of aluminum, if possible, so that it will be very light, and should rotate on conical points to insure perfect alignment. The surface should be very smooth. The pencil attached to the center of the beam should be made as light as possible in order to reduce the inertia effect which is introduced.

Another factor which must have careful attention is that of determining the slope of the space-time and velocity-time curves. In these experiments a mirror was used for drawing the normals to the curves by reflecting a portion of the curve through 180°. This is the best known method where a number of points are involved, however, it was found that the slightest error in drawing the normals to the curves will cause a large error in the results since there is a rapid change in velocity.

The tuning fork used in these experiments was calibrated and found to produce the same number of vibrations per second by using either alternating or direct current. Alternating current was used in these experiments. It is hoped that these suggestions will be helpful in case the investigation is continued.

In this discussion the assumption was made that the acceleration of the center of the beam was the same as that of the hammer while the plunger and hammer were in contact. This acceleration was used in determining the approximate force of impact on the beams.

The force (F) was calculated from the formula

$$F = \frac{W}{g} \times a$$

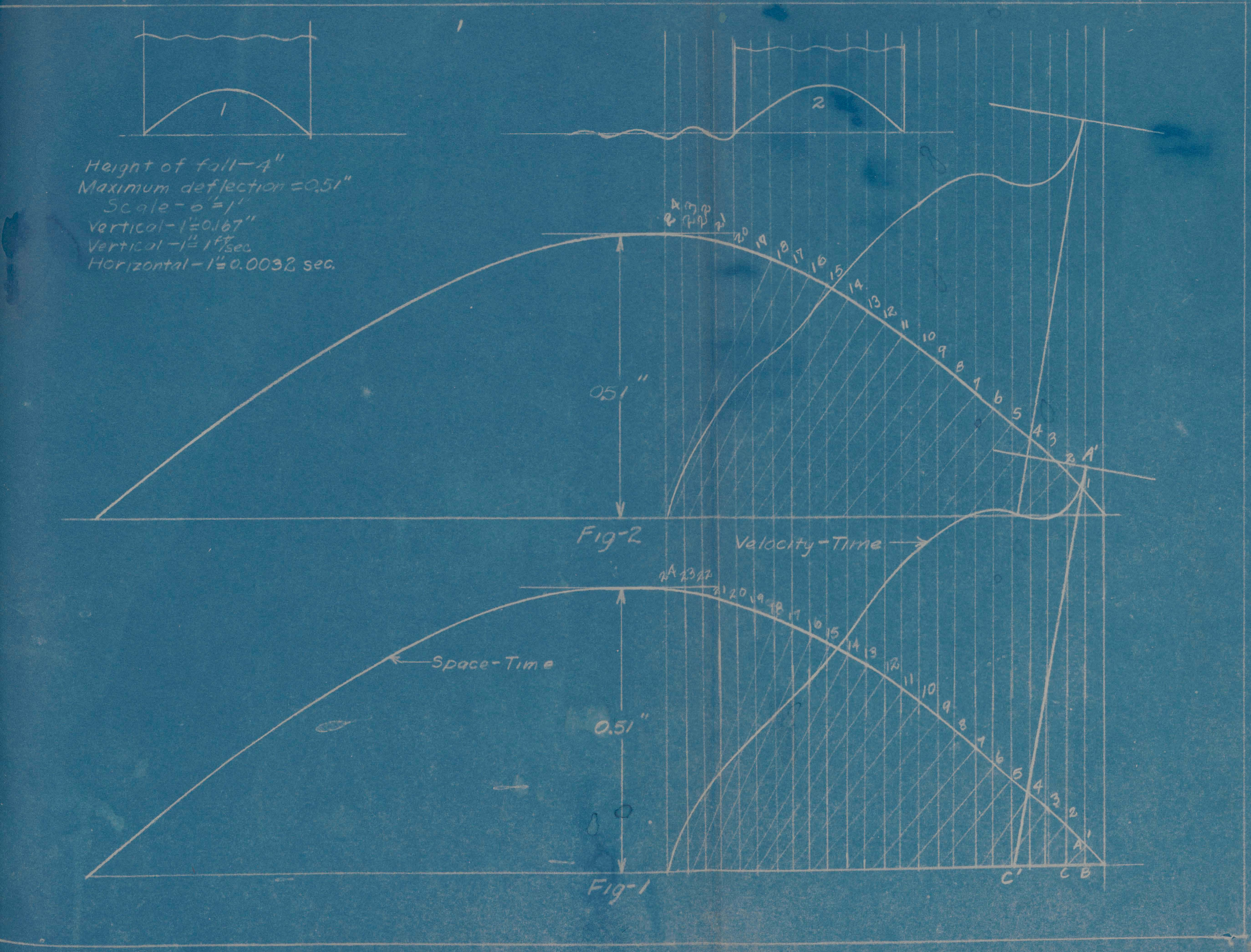
where W = weight of hammer (lbs)

a = acceleration of the center of the beam (ft/sec^2)

g = acceleration due to gravity (ft/sec^2)

In this formula the acceleration (a) should be that of the hammer, since the weight of the hammer (W) is used.

PLATE-1



Height of fall-4"
Maximum deflection=0.51"
Scale-6"=1'
Vertical-1"=0.167"
Vertical-1"=1 ft/sec
Horizontal-1"=0.0032 sec.
0.51"
Fig-2
Velocity-Time
Space-Time
0.51"
Fig-1
A'
C'
C
B
A

Plate-2

Height of fall - 4"
Maximum deflection fig-3 = 0.483"
" " fig 4 = 0.493"
Scale - vertical - 1" = 0.167"
" " - 1" = 1 ft/sec
" horizontal 1" = 0.0032 sec.

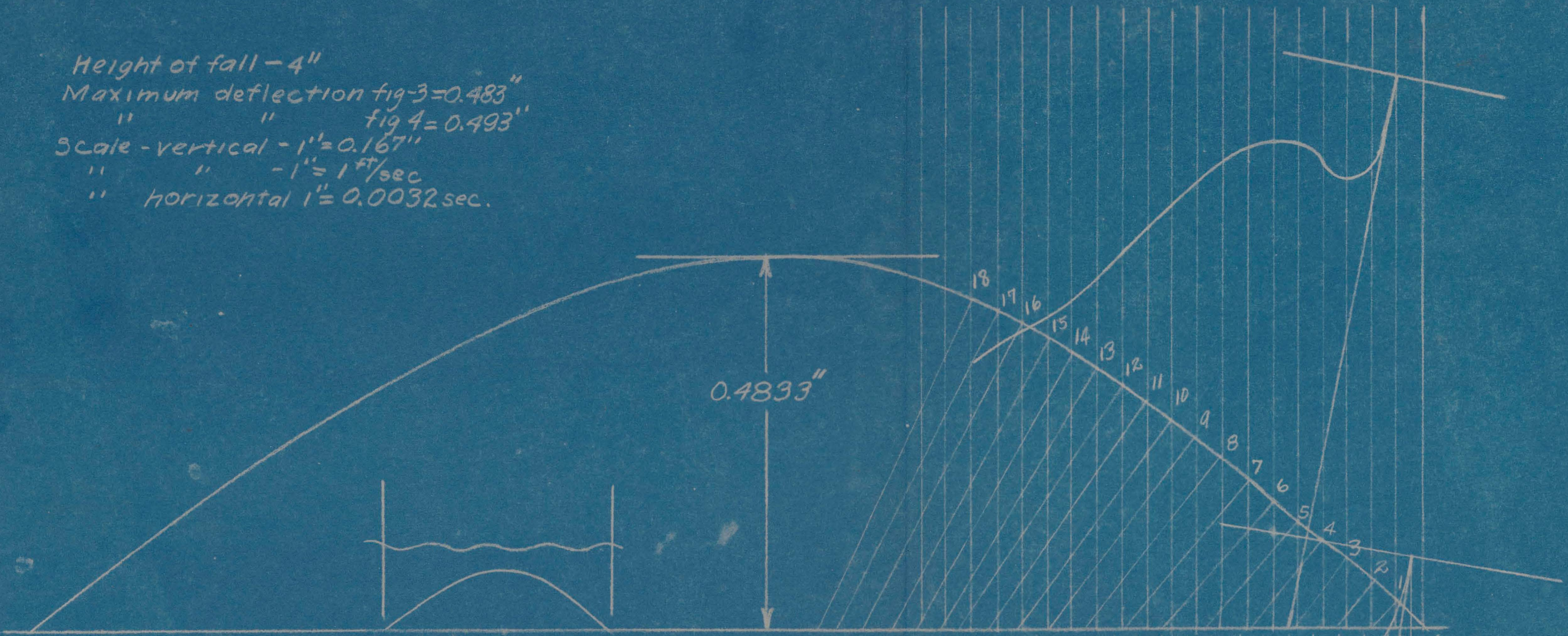


Fig-3

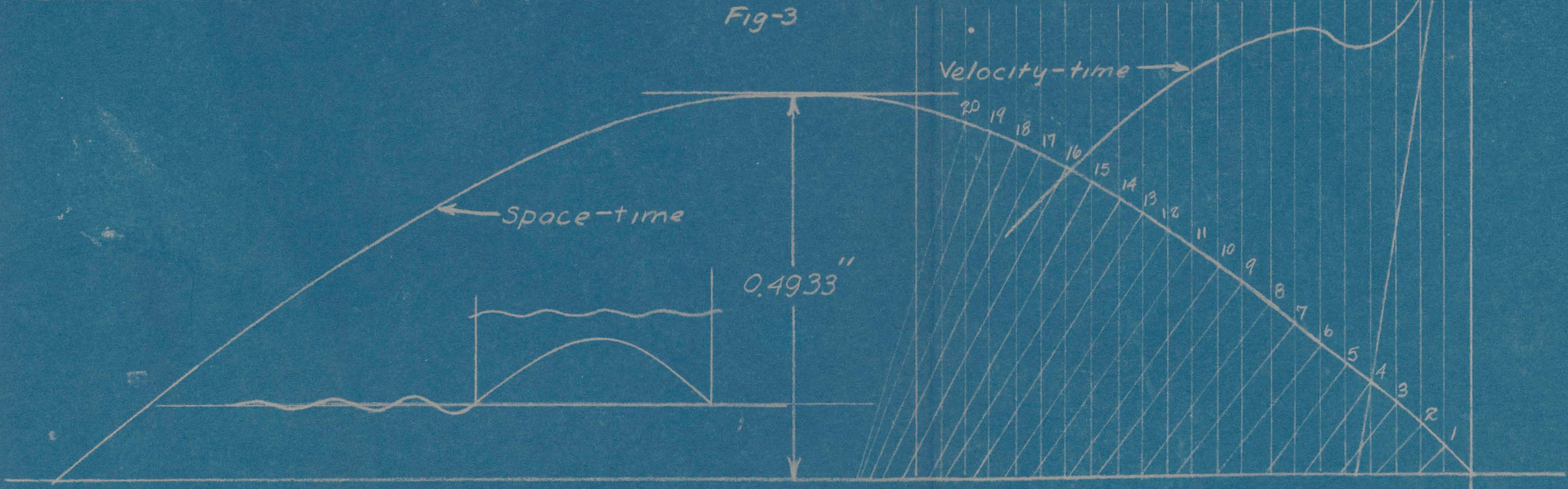


Fig-4

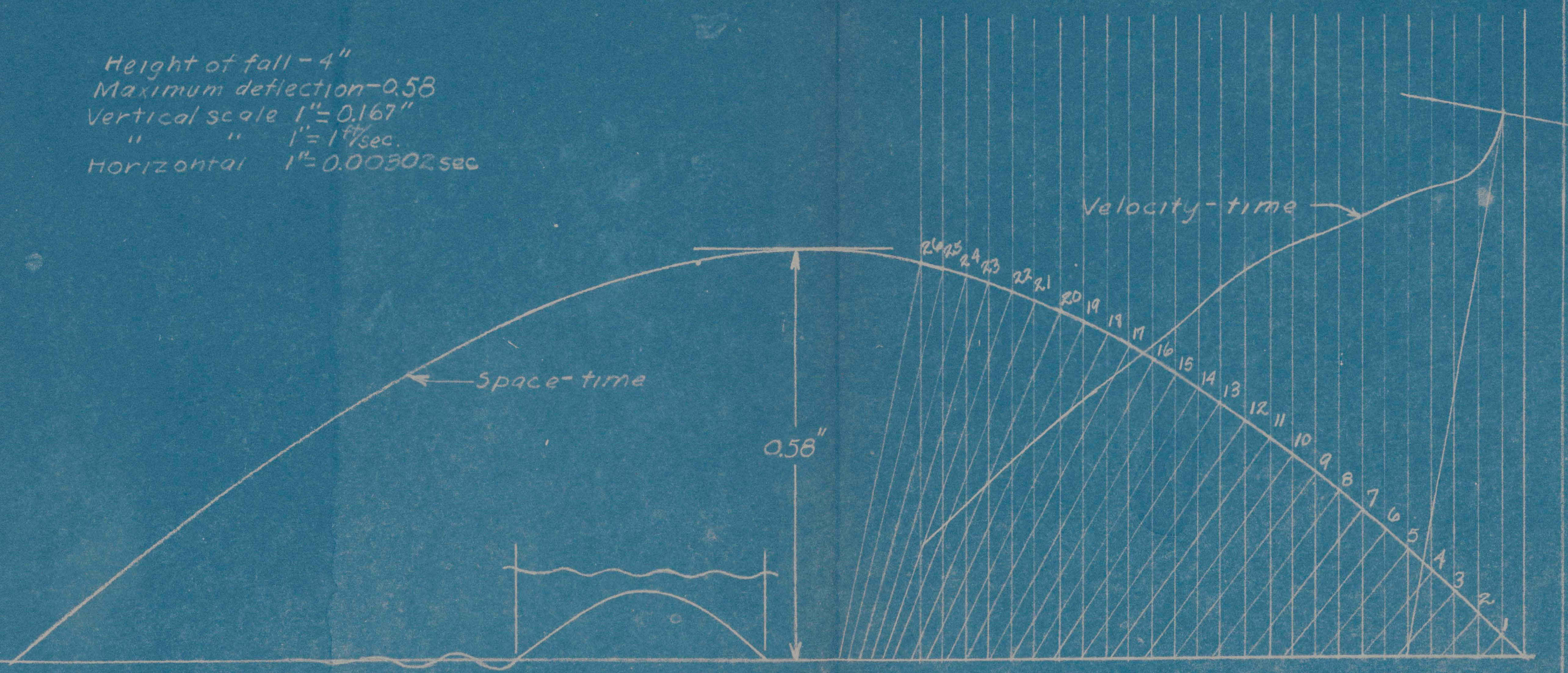
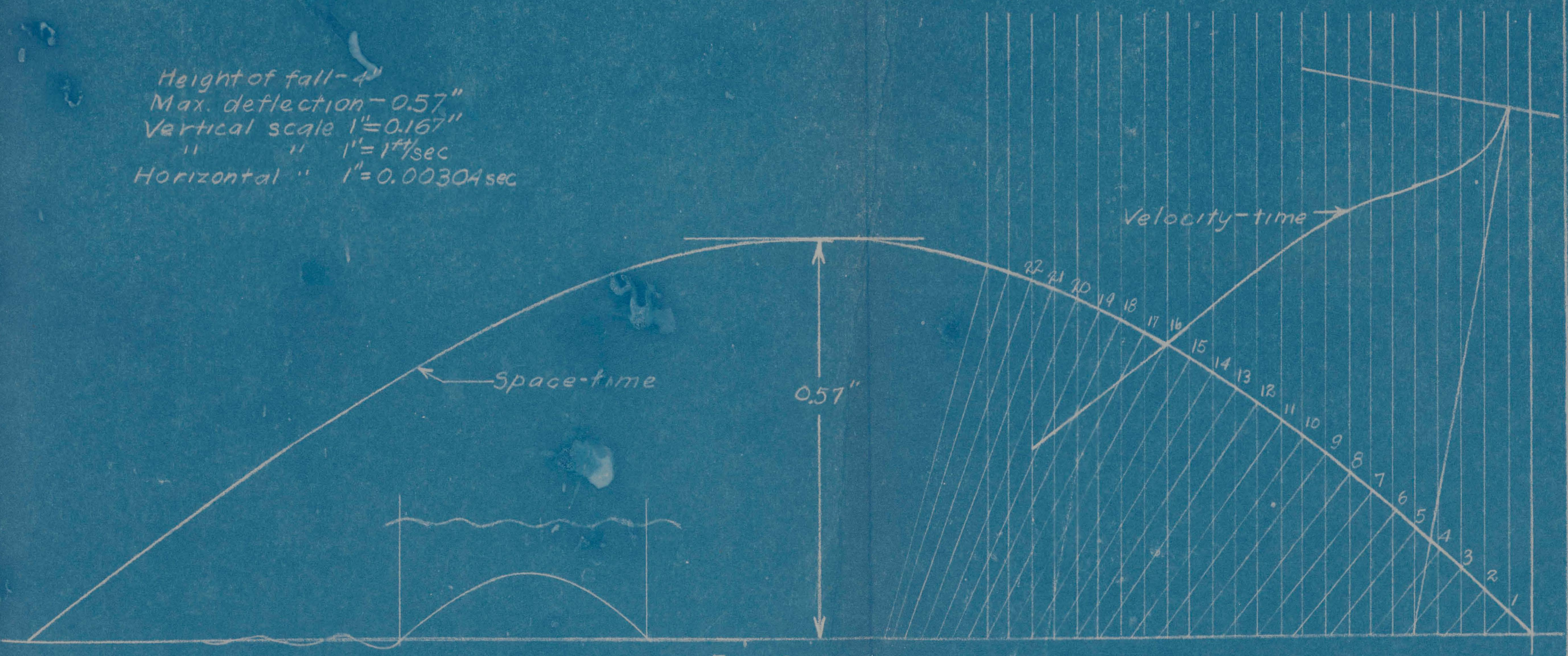

Height of fall - 4"
Maximum deflection - 0.58
Vertical scale 1" = 0.167"
" " 1" = 1 ft/sec.
Horizontal 1" = 0.00302 sec
Velocity - time
Space - time
0.58"
26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 5 4 3 2 1

Fig-5



Height of fall - 4"
Max. deflection - 0.57"
Vertical scale 1"=0.167"
" " 1"=1 ft/sec
Horizontal " 1"=0.00304 sec
Velocity-time
Space-time
0.57"
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

Fig-6

Plate-5

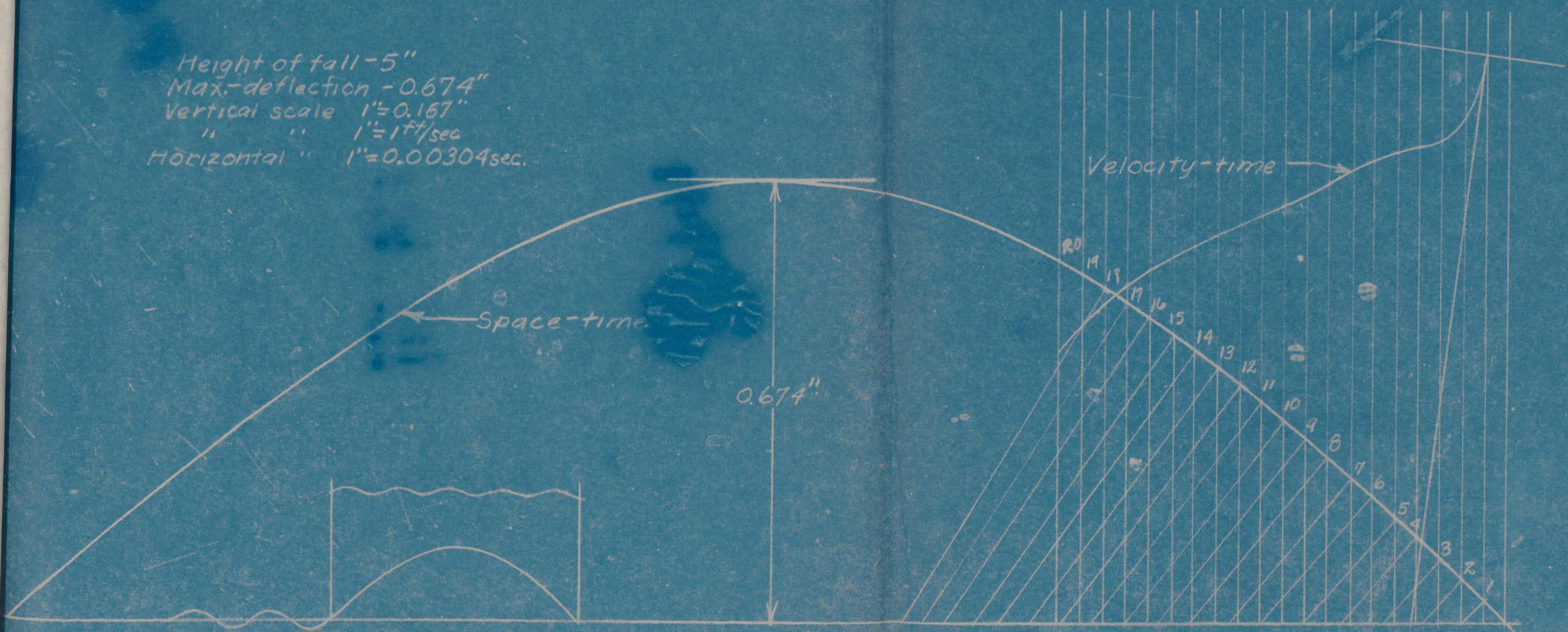

Height of fall-5"
Max.-deflection -0.674"
Vertical scale 1"=0.167"
" " 1"=1ft/sec
Horizontal " 1"=0.00304sec.
Velocity-time
Space-time
0.674"
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

Fig-7

Plate-6

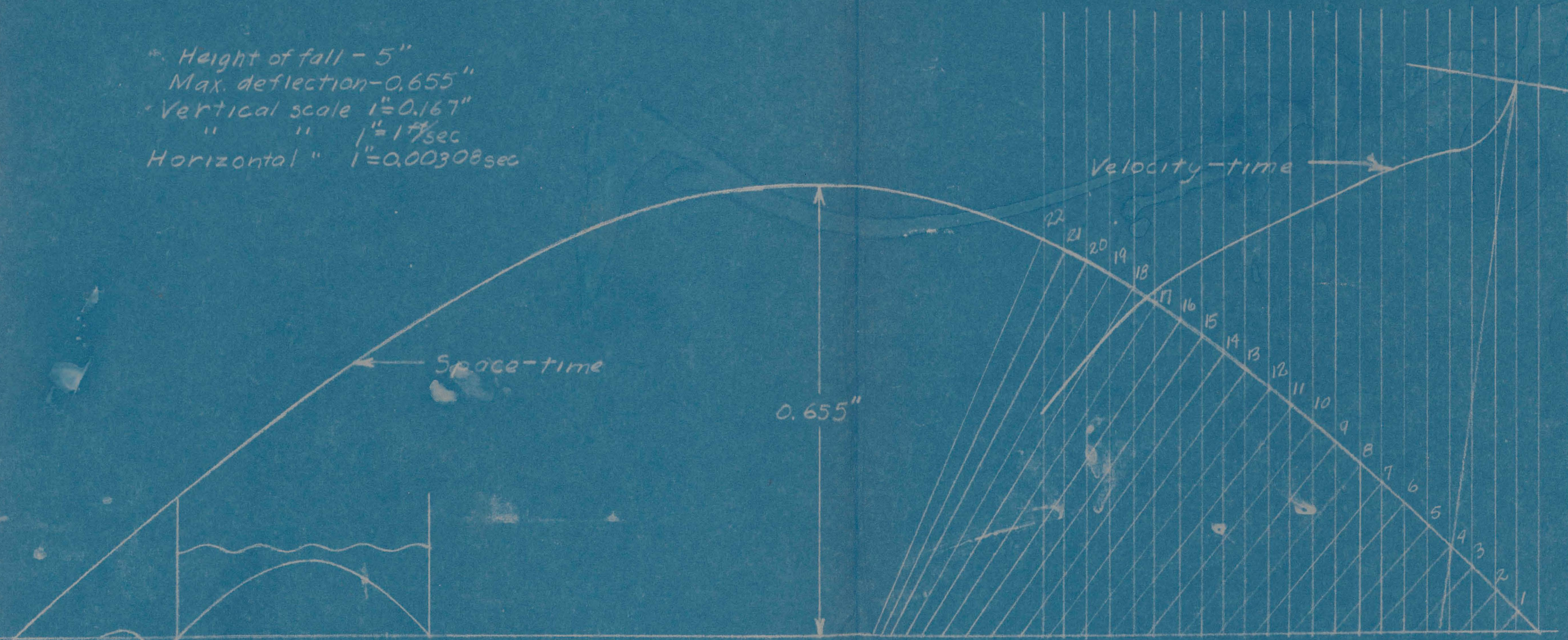

Height of fall - 5"
Max. deflection - 0.655"
Vertical scale 1" = 0.167"
" " 1" = 1 ft/sec
Horizontal " 1" = 0.00308 sec
Velocity-time
Space-time
0.655"
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

Fig 8

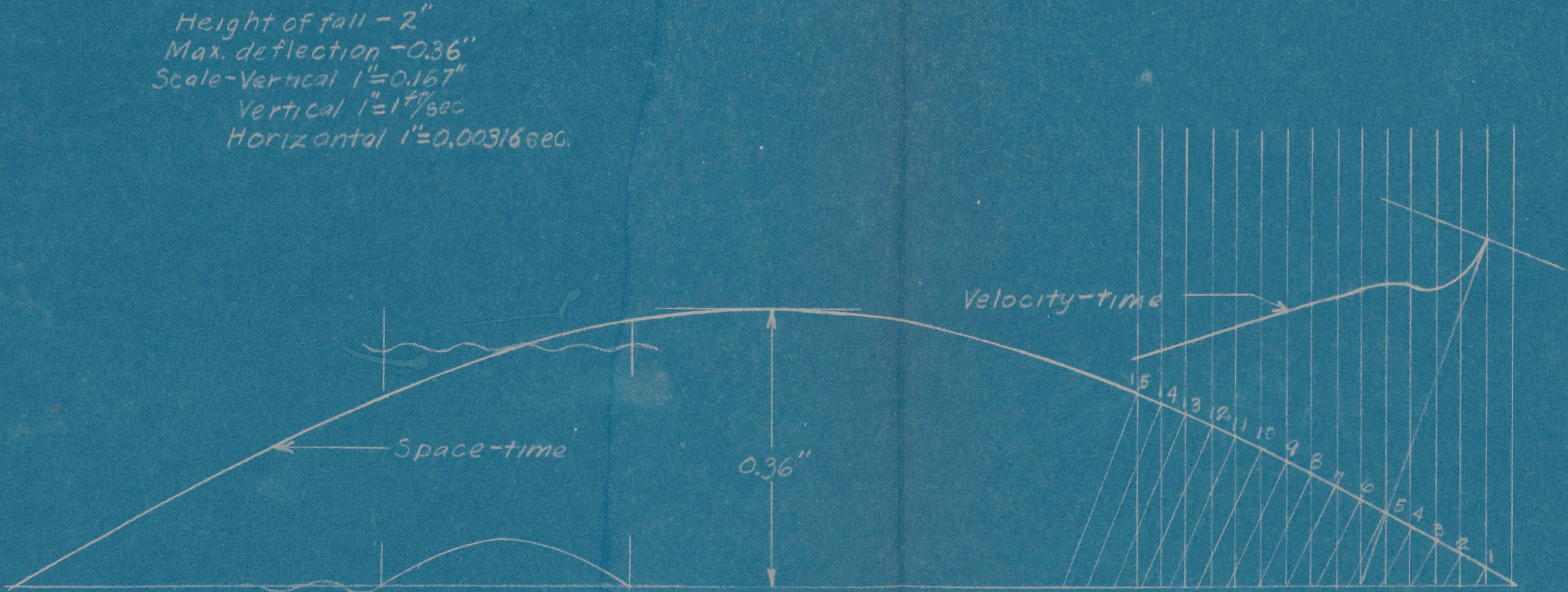

Height of fall - 2"
Max. deflection - 0.36"
Scale-Vertical 1" = 0.167"
Vertical 1" = 1 ft/sec
Horizontal 1" = 0.00316 sec.
Velocity-time
Space-time
0.36"
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

Fig 9

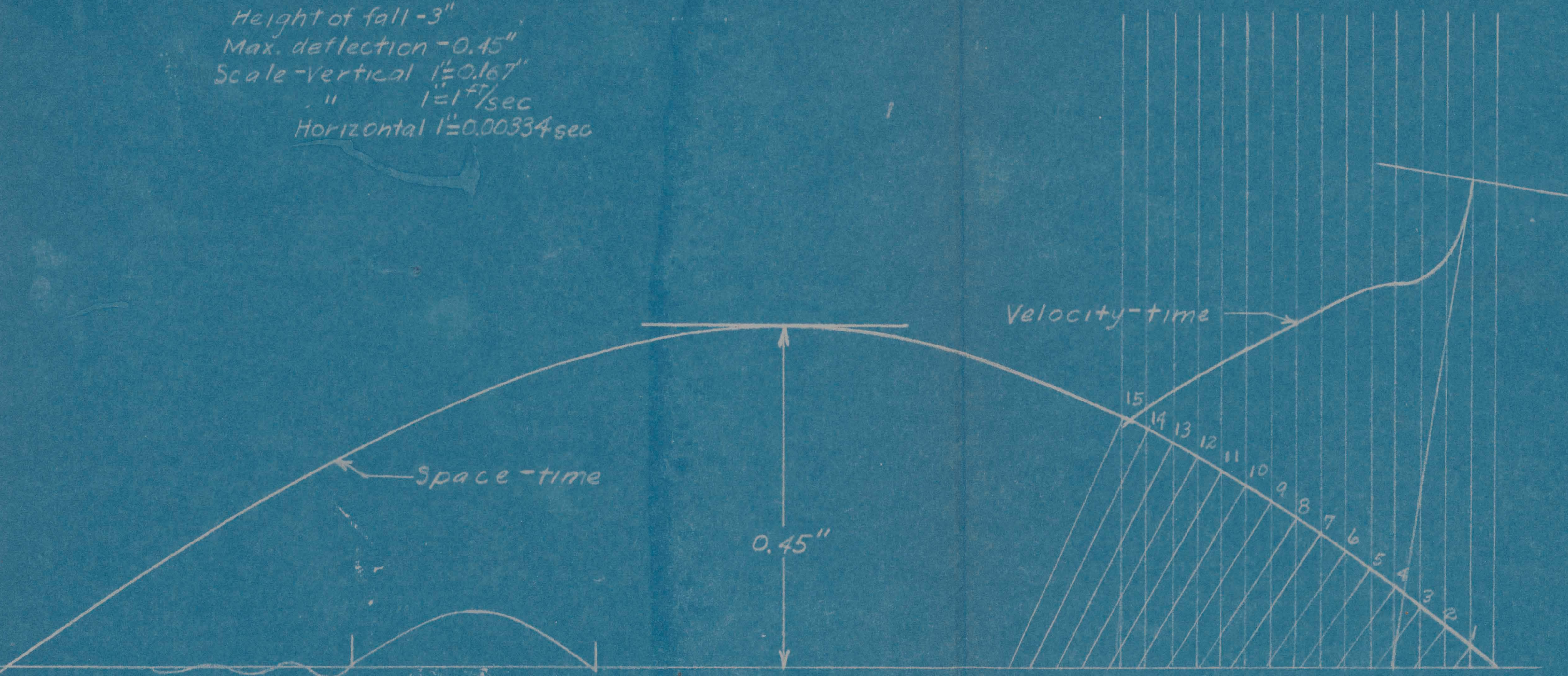
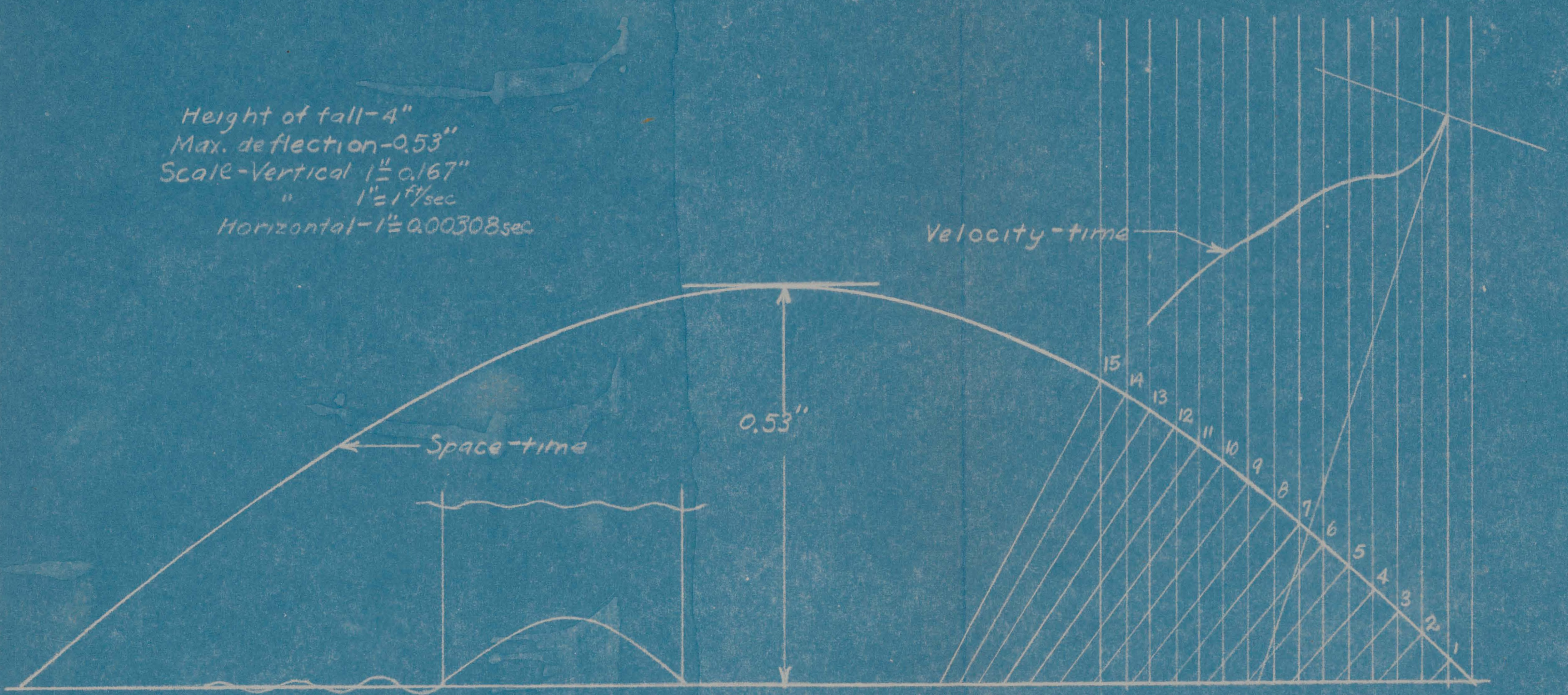

Height of fall - 3"
Max. deflection - 0.45"
Scale - Vertical 1" = 0.167"
" 1" = 1 ft/sec
Horizontal 1" = 0.00334 sec
Velocity-time
Space-time
0.45"
15
14
13
12
11
10
9
8
7
6
5
4
3
2

Fig-10



Height of fall-4"
Max. deflection-0.53"
Scale-Vertical 1"= 0.167"
" 1"=1 ft/sec
Horizontal-1"= 0.00308 sec
Velocity-time
Space-time
0.53"
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

Fig-11

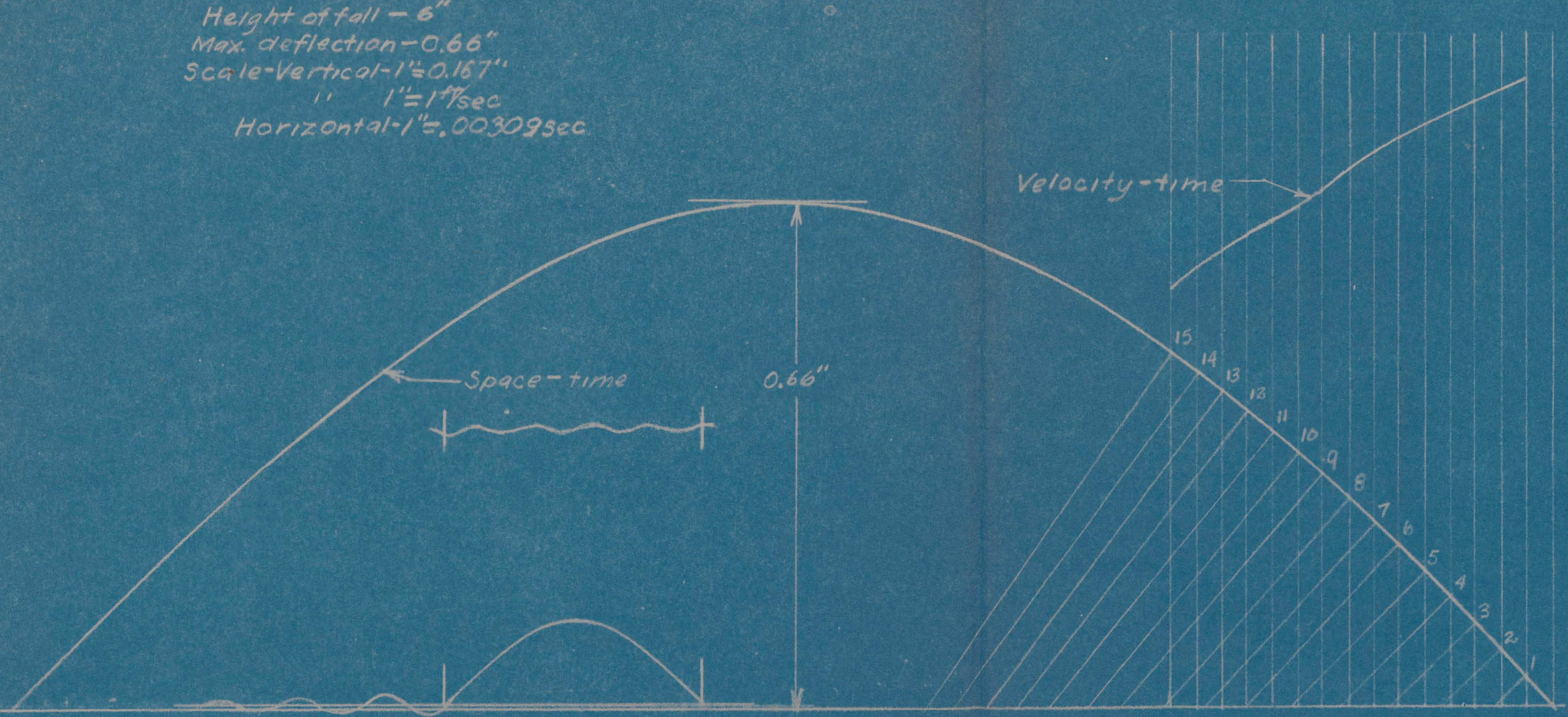

Height of fall – 6"
Max. deflection – 0.66"
Scale-Vertical-1" = 0.167"
" 1" = 1 ft/sec
Horizontal-1" = .00309 sec
Velocity-time
Space-time
0.66"
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1

Fig 12

Plate 11

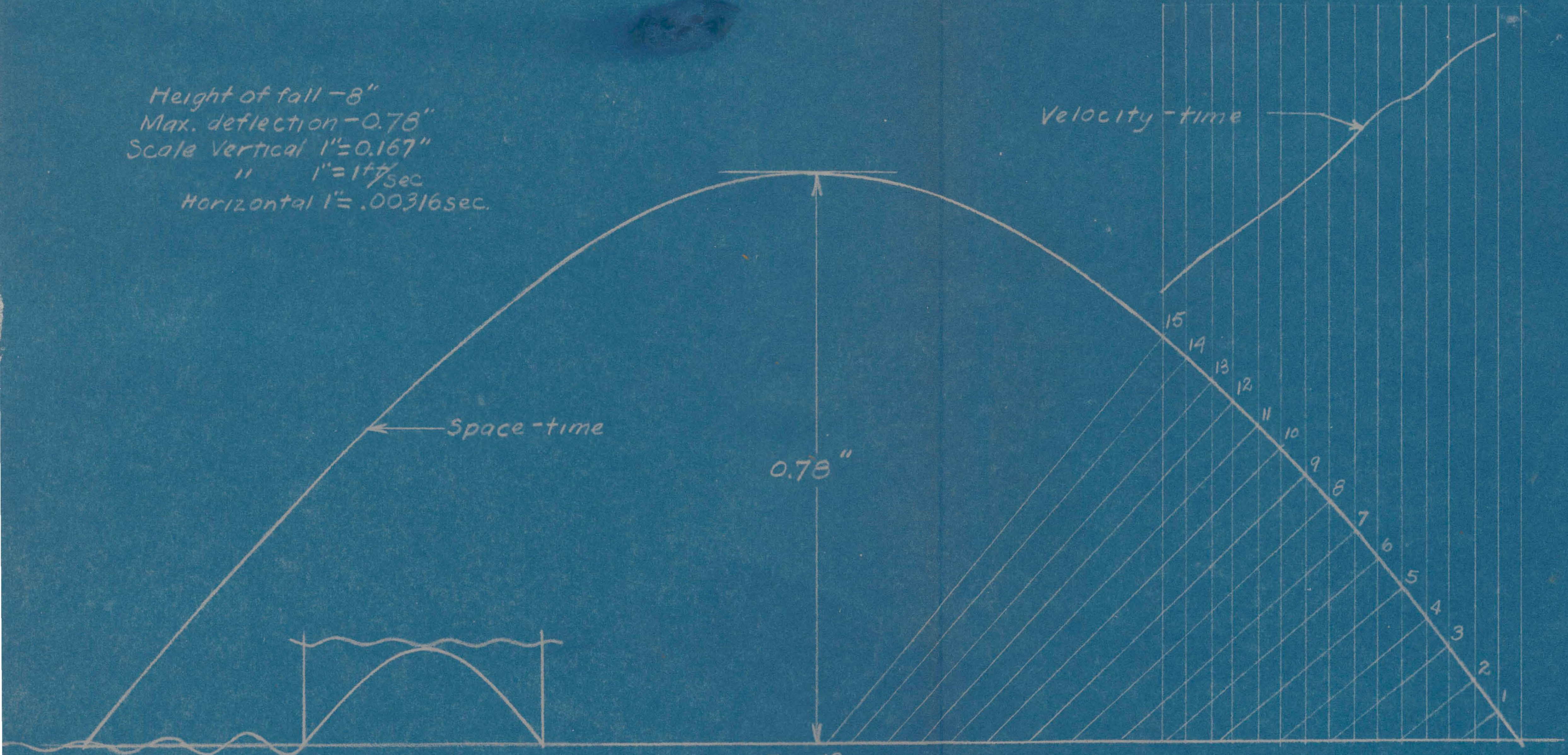


Fig - 13

Plate-12

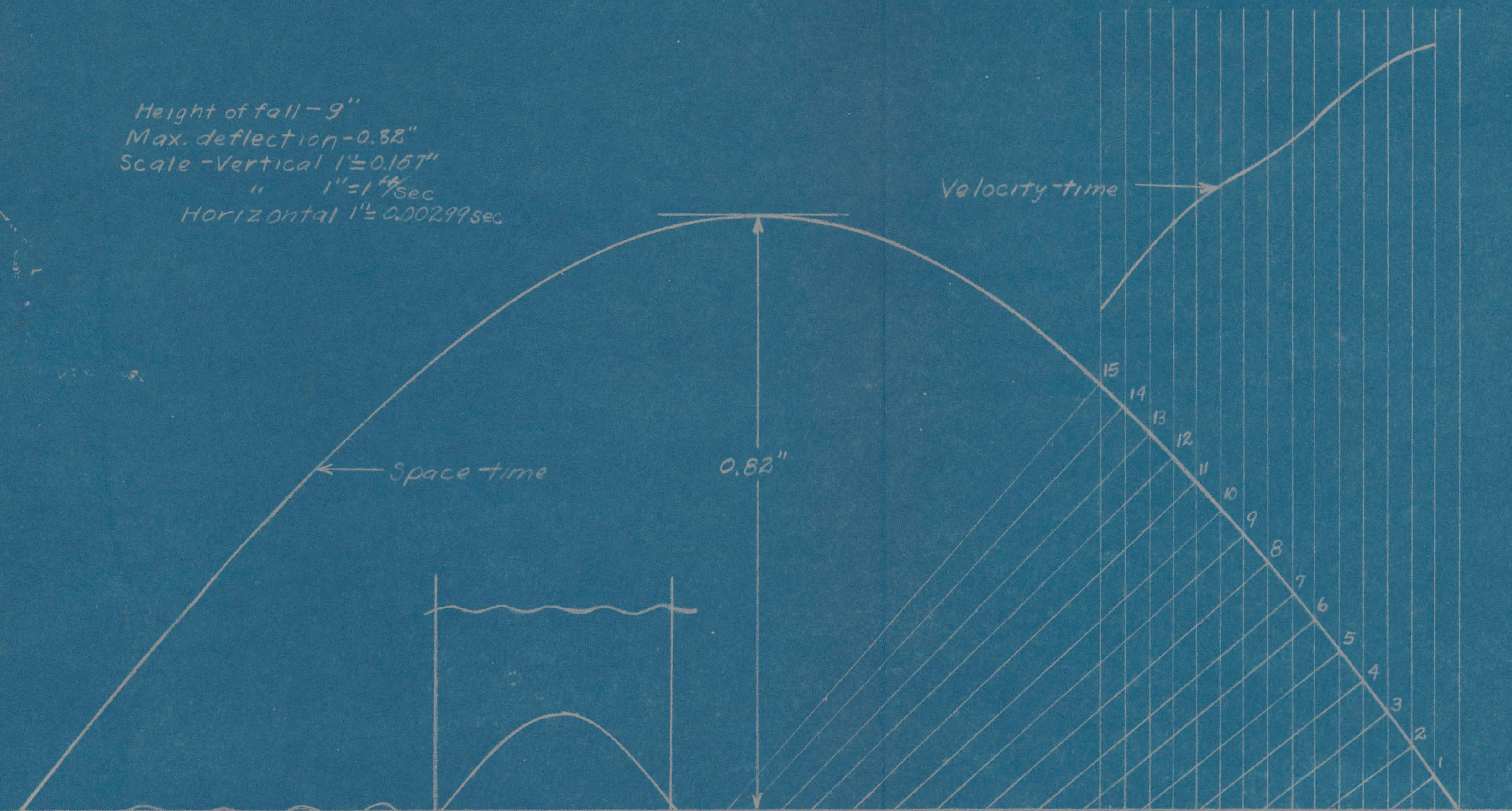


Fig-14