

A STUDY OF DYNAMICS AND STABILITY OF TWO-CRAFT COULOMB TETHER FORMATIONS

by

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Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy
in
Aerospace Engineering

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April 26, 2007

Blacksburg, Virginia

Keywords: Coulomb Tether, Spacecraft Formation Flying

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Abstract

In this dissertation the linearized dynamics and stability of a two-craft Coulomb tether formation are investigated. With a Coulomb tether the relative distance between two satellites is controlled using electrostatic Coulomb forces. A charge feedback law is introduced to stabilize the relative distance between the satellites to a constant value. Compared to previous Coulomb thrusting research, this is the first feedback control law that stabilizes a particular formation shape. The two craft are connected by an electrostatic virtual tether that essentially acts as a long, slender near-rigid body. Inter-spacecraft Coulomb forces cannot influence the inertial angular momentum of this formation. However, the differential gravitational attraction can be exploited to stabilize the attitude of this Coulomb tether formation about an orbit nadir direction. Stabilizing the separation distance will also stabilize the in-plane rotation angle, while the out-of-plane rotational motion remains unaffected. The other two relative equilibriums of the charged 2-craft problem are along the orbit-normal and the along-track direction. Unlike the charged 2-craft formation scenario aligned along the orbit radial direction, a feedback control law using inter-spacecraft electrostatic Coulomb forces and the differential gravitational accelerations is not sufficient to stabilize the Coulomb tether length and the formation attitude. Therefore, hybrid feedback control laws are presented which combine conventional thrusters and Coulomb forces. The Coulomb force feedback requires measurements of separation distance error and error rate, while the thruster feedback is in terms of Euler angles and their rates. This hybrid feedback control is designed to asymptotically stabilize the satellite formation shape and attitude while avoiding plume impingement issues. The relative distance between the two

satellites can be increased or decreased using electrostatic Coulomb forces. The linear dynamics and stability analysis of such reconfiguration are studied for all the three equilibrium. The Coulomb tether expansion and contraction rates affect the stability of the structure and limits on these rates are discussed using the linearized time-varying dynamical models. These limits allow the reference length time histories to be designed while ensuring linear stability of the virtual structure. Throughout this dissertation the Coulomb tether is modeled as a massless, elastic component and, a point charge model is used to describe the charged craft.

To my parents

Acknowledgements

I would like to thank Dr. Hanspeter Schaub for introducing me to this wonderful field of Astrodynamics. This dissertation would not have been possible without his guidance and support. I would also like to acknowledge and thank him for the financial support during the past two years of my doctoral program. Dr. Christopher D. Hall, Dr. Craig A. Woolsey and Dr. Scott L Hendricks were the other professors in the research committee and I am grateful for their valuable suggestions. The Aerospace and Ocean Engineering Department offered me financial assistantship during my first year. I thank them for this support. I am indebted to all the past and present members of Dr. Schaub's research team for their constructive comments and suggestions during the research meetings and general discussions.

There are a lot of people who made my stay in Blacksburg very memorable and enjoyable. A special thanks to Scott and Emily Kowalchuk for all their nice parties and impromptu dinner invitations. Some of the nicest times I spent in Blacksburg were in their company. I would like to also thank my gang of Indian friends - Rajesh, Parthiban, Vidya, Pradeep, Siva, Ashvin, Ajit and Anu. They made me feel at home and our weekend get-togethers helped me to unwind and relax. Finally, I would like to thank my parents Rajam and Natarajan, and my twin brother Diwakar for their lifelong love, encouragement and support.

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1 Introduction

1.1 Coulomb Formation Flying

Formation flying of spacecraft using Coulomb forces is a new and emerging field of study. Here, the electrostatic (Coulomb) charge of spacecraft is varied by active emission of either negative electric charges (electrons) or positive electric charges (ions). The resulting changes in inter-spacecraft Coulomb forces are used to control the relative motion of the spacecraft as illustrated by Figure 1.1. This novel concept of propellantless relative navigation control has many advantages over conventional thrusters like ion engines. For example, this method of propulsion has been shown to require essentially no consumables (fuel efficiencies ranging up to 10^{13} seconds), require very little electric power to operate (often less than 1 Watt), and can be controlled with a high bandwidth (zero to maximum charge transition times are of the order of milli-seconds). Thus, this propulsion concept could enable high precision formation flying with separation distances ranging between 10–100 meters. It is also a clean method of propulsion compared to ion engines, thereby avoiding the thruster plume contamination issue with neighboring satellites. For this range of separation distances, the plume-impingement problem of high-efficiency ion engines would be severe.

Proposed uses of the Coulomb propulsion concept include high-accuracy, wide-field-of-view optical interferometry missions with geostationary orbits(GEO),¹ controlling clusters of spacecraft to maintain a bounded shape,² as well as the use of drone-worker concepts

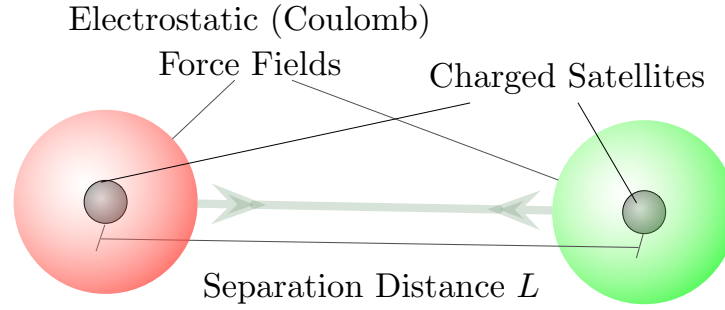


Figure 1.1: The Coulomb force interaction between two charged craft.

where dedicated craft place a sensor in space using Coulomb forces.³ The cluster Coulomb formation flying concept is illustrated in Figure 1.2. In this type of formation the Coulomb forces not only keep the satellites bounded, but also keep them from colliding with each other. The drone-worker concept or the “Gluon” Coulomb formation concept is shown in Figure 1.3. Here, the gluon satellite in the center acts like the mother satellite that is capable of carrying a large charge, and the satellites in the periphery, or daughter satellites, carry only small charges. The net force experienced by a daughter satellite will be high as it will depend on the product of gluon charge and daughter satellite charge. Thus, this concept allows a daughter satellite to carry sophisticated instruments that might be affected by high charging. Also, in this type of formation the force interactions between the daughter satellites are negligible and therefore, effectively decouples the complex system. Figure 1.4 shows another potential application for a two-craft Coulomb formation setup. A camera or a probe can be deployed from a satellite and retrieved back using the Coulomb forces.

While the Coulomb propulsion concept has many exciting advantages and potential applications, it does come at the price of greatly increased coupling and nonlinearity of the charged spacecraft equations of motion. The relative motion of all other neighboring charged craft will be affected by changing the charge of a single craft. Further, with the Coulomb

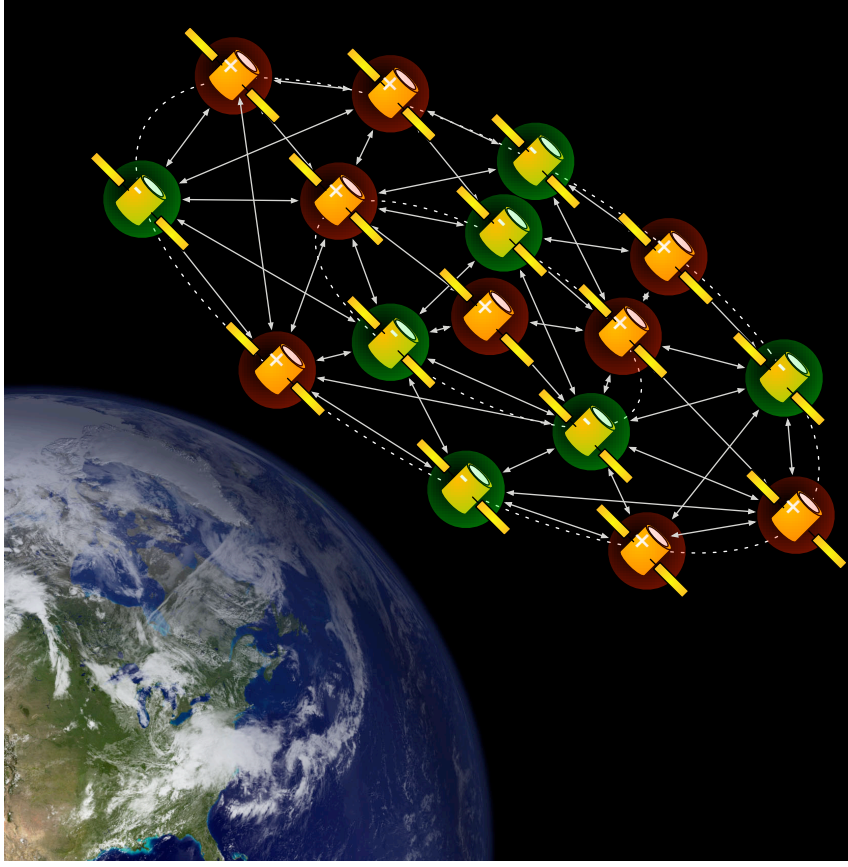


Figure 1.2: The cluster Coulomb formation flying concept.

forces being formation-internal forces, some constraints are applicable to all feasible charged spacecraft motions. In particular, Coulomb forces cannot be used to change the total inertial formation angular momentum vector.^{4,5} As a result, the reorientation of a formation as a whole to a new orientation must satisfy this momentum constraint.

When charging spacecraft to control relative motion, differential charging across the spacecraft components must be minimized to avoid arcing. However, note that the control charge levels required for the Coulomb formation are similar to the naturally occurring charge levels of GEO spacecraft during periods of high solar activity. The technology to

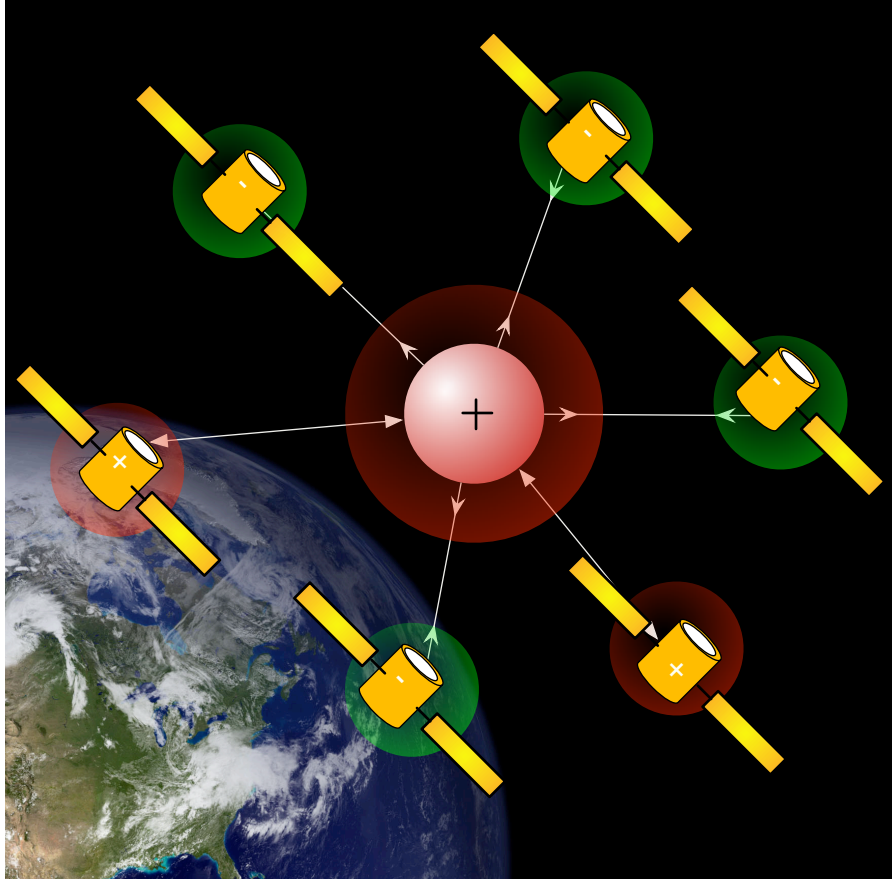


Figure 1.3: The gluon Coulomb formation flying concept.

control the charge involves high-speed ion and electron emitters, and is similar to what is currently flying on the CLUSTERS mission⁶ or to that flown on the SCATHA mission.⁷ On the CLUSTERS mission the spacecraft charge is actively controlled to neutralize its potential relative to the space plasma environment. Because of the high fuel efficiency of the Coulomb thrusting concept,^{1,8} where relative motion I_{sp} values can range between 10^{10} – 10^{13} seconds, the change in momentum and plasma environment due to the expelled charges is negligible.

The study of electrostatic charging data of SCATHA spacecraft⁷ in GEO has shown

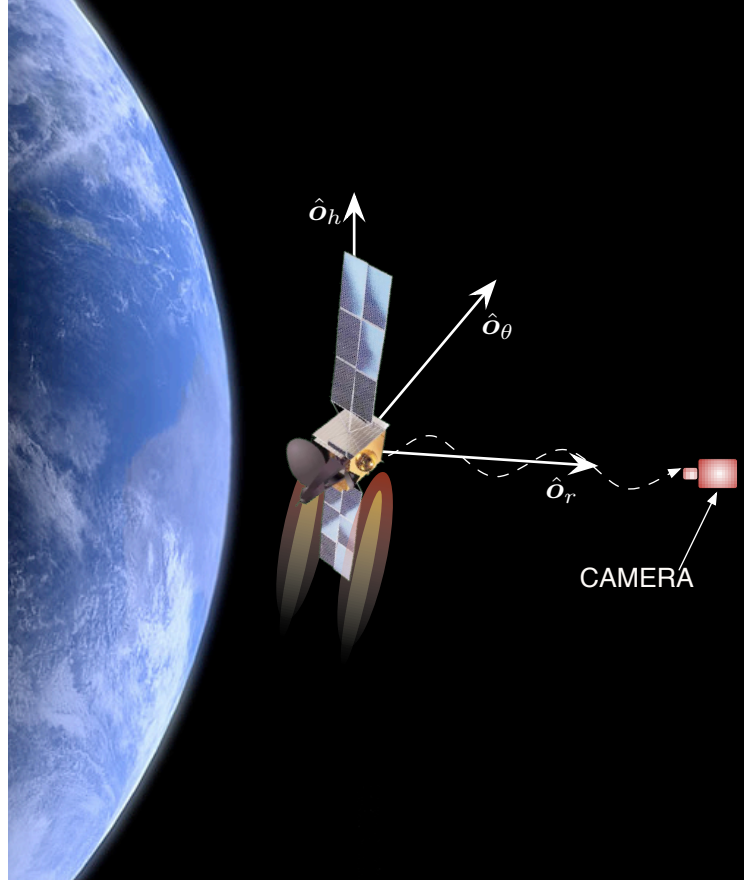


Figure 1.4: The release of a camera - a potential application of Coulomb formation flying.

that the spacecraft can naturally charge to high voltages in low plasma environments such as at GEO. The level of natural charge depends on the current solar activity. Further, this mission demonstrated that the spacecraft charge could be actively controlled. The Coulomb propulsion has its own set of limitations, however. The magnitude of Coulomb electrostatic force is inversely proportional to the square of separation distance, which makes this method effective only for close formations of the order of 10-100 m, depending on the maximum allowable level of spacecraft charge. Moreover, in the presence charged plasma

particles, the effectiveness of Coulomb force is diminished with the electric field dropping off exponentially. The severity of this drop is measured using the Debye length.^{9,10} For low Earth orbits (LEO), the Debye length is of the order of centimeters, making the Coulomb formation flying concept impractical. At geostationary orbits (GEO) or higher, where the plasma environment is less dense, the Debye length is about 100-1400 meters. The Coulomb formation flying concepts can be comfortably applied at this altitude.

1.2 Related Work

The concept of formation flying using electrostatic propulsion was introduced in References 1,2 and 8. These pioneering works discussed the static Coulomb satellite formations and the associated equilibrium charges, but did not address the stabilization of these formations. The NIAC report by King et al.⁸ found analytical solutions for Hill-frame invariant Coulomb formations. Here spacecraft were placed at specific locations in the rotating Hill frame with specific electrostatic charges. As a result the Coulomb forces perfectly cancel all Keplerian relative orbit accelerations, causing the satellites to remain fixed or static as seen by the constantly rotating Hill frame. The analytical solutions were found for simple geometries involving 3 to 7 satellites using formation symmetry. In all these formations one satellite is located at the center of mass of the satellite formation. The equations of motion representing these Coulomb formations in the Hill frame are highly coupled and complex non-linear equations. With multiple craft, complex static formations other than the simple symmetric formations found in Reference 8 are also possible. However, these complex static formations are non-intuitive and a numerical approach is needed to find the constant Hill frame position and charge that result in a static formation. One such numerical approach using a genetic algorithm is given in Reference 11. In Reference 11, Coulomb formation shapes involving up to 9 craft were discussed. The necessary conditions for achieving such

static Coulomb formations were determined in Reference 12 using Hamiltonian formulations of the Coulomb formation dynamics. These hamiltonian formulations are analogous to the study of equilibrium conditions of rigid bodies in orbit. The analytical solution for the static charge and their feasibility for different shapes in two-craft and three-craft formations were discussed in detail in Reference 13. Romanelli et al.¹⁴ showed that Coulomb forces can be used to cancel the differential drag due to solar radiation, J_2 effect and atmospheric drag, experienced by craft in a static formation. Note that the charge is held constant in the above mentioned open-loop static Coulomb formations studies. The discovered open-loop static Coulomb formations were all found to be unstable.

Reference 2 discussed these static Coulomb satellite formations and a nonlinear control law that was capable of bounding the relative motion between two close craft. This charge feedback control can also be used to control general orbit element differences with guaranteed stability, but not necessarily with asymptotic convergence. Reference 15 presents an open loop stable spinning two craft Coulomb tether. The reconfiguration of this spinning Coulomb tether in deep space was also discussed in that paper. A Lyapunov-based control law for stabilizing a 1D-restricted three-craft Coulomb structure was shown in Reference 16. The Lyapunov-based control law identified the required charge products and real implementable charge values for each craft were extracted by studying the null space of the charge product matrix. A control law for avoiding collision between spacecraft in a cluster in free space was proposed in Reference 17. The proposed control law required only the separation distance between the craft and its rate for determining the charge feedback. The control law ensured that no two craft were within each other's safety spherical zones of fixed radius.

Another potential area of application for Coulomb forces is to change the position or orientation of a space structure. A charge control law was developed in Reference 3 to reposition a charged body using three charged drones. The control law assumed that the

Coulomb attraction is the only dominant force acting on the system and neglects orbital mechanics. Another deployment technique where a chief satellite deploys a drone craft to a specific location was developed in Reference 18. Here, the chief has small charged spheres attached to it and the charge of each small sphere was actively controlled to achieve the desired deployment. Izzo and Pettazzi¹⁹ proposed using Coulomb forces for aiding the self-assembling of large structures in space. Here, the use of Coulomb forces reduce the propellant consumption significantly.

Formation flying using electromagnetic fields for propulsion is an area of research where lot of work is being published.²⁰⁻²² Unlike Coulomb forces, the electromagnetic fields are not constrained by the Debye shielding effects and therefore, are feasible in low Earth orbits (LEO). But, the $1/r^3$ decay of magnetic field strength over distance is greater than the $1/r^2$ decay for Coulomb forces, making the range of deployment very small. The Lorentz Augmented Orbit (LAO) system discussed in Reference 23 also used a type of electromagnetic propulsion. Here, the propulsion was generated by the interaction of the Earth's magnetic field and the static charge present in the satellite. A limitation of this propulsion technique was that the high charge required for this system to work can not be currently achieved in practice with any existing technology.

1.3 Dissertation Overview

This dissertation is organized as follows. First, the force experienced by two charged craft is studied in the plasma environment. The feasibility of using the point charge model for describing these craft is discussed. In the next chapter, the idea of using Coulomb forces for reconfiguration of satellites is explored by studying two-craft Coulomb formations in free space. The charges or voltages required for such a maneuver are studied and the case for exploring the Coulomb formation in orbit is established. Next, the study of a two-craft

Coulomb virtual tether that is aligned along the orbit-radial (nadir) direction and in a Keplerian Geo-synchronous orbit, is presented. Feedback laws for stabilizing this formation along the orbit radial axis for a particular shape are derived. In the following chapter, the two-craft formation stability along the orbit-normal direction and along-track direction are studied and hybrid feedback control laws are derived for stabilizing the formation. After developing control laws for stabilizing the formation along all three axes, the reconfiguration maneuver for craft aligned along the orbit-radial direction is presented. Next, analytical solutions using Bessel functions are established for the decoupled out-of-plane motion when the reconfiguration is taking place at a constant rate. In the succeeding chapter, the instantaneous changes in the reconfiguration rate is smoothed to ease the excessive oscillations about the ideal prescribed trajectory for reconfiguration. Finally, the reconfiguration about the orbit-normal and along-track equilibrium is also presented.

2 Discretized Model for Coulomb Forces Under Plasma Screening

2.1 Introduction

Gauss's law^{24,25} states that the net electric flux coming out of a closed surface is directly proportional to the net charge enclosed by the surface. Using this law it can be easily shown that the electric field at any point outside a uniformly charged conducting sphere (solid or shell) is the same as if the net charge were concentrated at the center of the sphere (like a point charge). This same concept can be extended while calculating the force between two charged spheres. The charged spheres can be assumed as point charges at the center of each sphere, provided the separation distance between them is sufficiently large. At small separation distances the surface charge distribution will not be even due to the induced charge effects and the point charge model is not valid.

The behavior of a charge particle is significantly different in a plasma environment than its behavior in vacuum. If a test charge (positive) is placed in a uniform plasma, it will attract electrons and repel the positive ions in the plasma. This will result in the gathering of electrons around the test charge, which will act as a shielding cloud that cancels the effect of the test charge. Thus the electric field or potential decays much faster in plasma environment than in vacuum. This phenomenon is called Debye shielding^{9,10} and is illus-

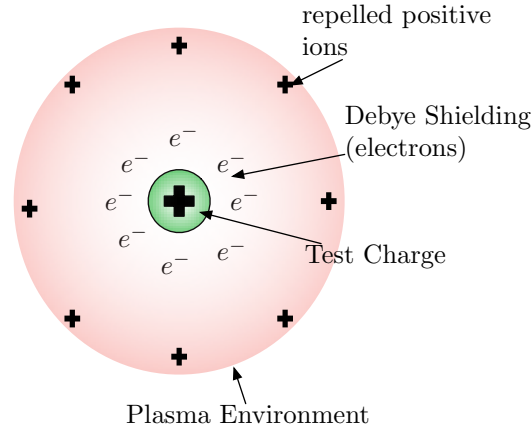


Figure 2.1: The Debye shielding.

trated in Figure 2.1. In References 9 and 10, an expression for the electric field due to a point charge in plasma is given and a new parameter called Debye length, which has the dimensions of length, is introduced. At distances that are greater than the Debye length the electric field or potential decays exponentially, and at distances that are much smaller than the Debye length the electric field is close to the point charge field in vacuum.

In MEO (Medium Earth orbit) and LEO (Low Earth orbit), the Debye lengths are small (order of few centimeters) and one possible application of Coulomb force is the docking of two satellites. The satellites can be modeled as two spheres and we study the Coulomb force between two such spheres in close quarters in a plasma environment. The spheres are hollow shells with a large surface area (similar to Gluon), and are porous so that the plasma can seep inside the spheres. Thus the environment inside and outside the spheres is the same. When the two charged spheres are in a close formation, the effectiveness of the charge on the surfaces that are further away might be reduced because of Debye shielding. Hence, point charge modeling of spheres based on Gauss's law might not give the accurate Coulomb force acting between the spheres that are in plasma environment. It should be

noted that this problem is not very acute when the Debye lengths are very large or when the separation distance between the spheres is large.

In this chapter, we discretize the surface of the spheres to small elemental areas and the elemental charges in these discretized areas are considered to be point charges. The resultant Coulomb force between the two spheres is found by adding, vectorially, the forces due to each elemental charge that make up the spheres. While calculating the forces due to elemental charge the effect of Debye shielding is taken into account. The net force obtained from the discretized model is compared with the point charge model, and situations in which point charge model will fail are identified. As discussed in the beginning of this chapter, two spheres in close quarters might introduce induced charge separation in each other. For instance, a positively charged sphere will attract more electrons in the other sphere to move to the surface facing the positively charged sphere as shown in Figure 2.2. This induced charging will result in nonuniform surface charge distribution and coupled with Debye shielding effects might drastically change the net effective Coulomb force. But, for the time being we neglect this induced charge effect and assume the spheres to have a uniform surface charge density.

This chapter is organized as follows. Initially, the effective choice of the discretization mesh size is identified. Next, the net Coulomb force acting between the two spheres is calculated using the discretized model. Finally, the forces calculated using the discretized model and the point charge model are compared to identify the situations when the point charge models fail.

2.2 Choice of Discretization Mesh

Consider a small elemental area on the surface of a sphere as shown in Figure 2.3. The polar coordinates of the elemental area are given by radius r and angles ψ and θ . The expression

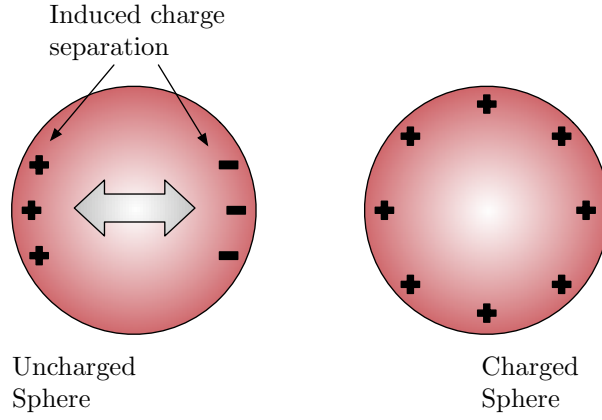


Figure 2.2: The induced charge in a neutral sphere in the vicinity of a charged sphere.

for the area of this element is given by

$$dA = r \cos \psi \, d\theta d\psi \quad (2.1)$$

The elemental charge dq on the elemental surface area dA can be written as

$$dq = \sigma dA = \sigma r \cos \psi \, d\theta d\psi \quad (2.2)$$

where the charge density on the surface of the sphere is σ . Rewriting Eq. (2.2) in terms of the total charge q carried by the sphere, we get

$$dq = \frac{q}{4\pi r^2} r \cos \psi \, d\theta d\psi \quad (2.3)$$

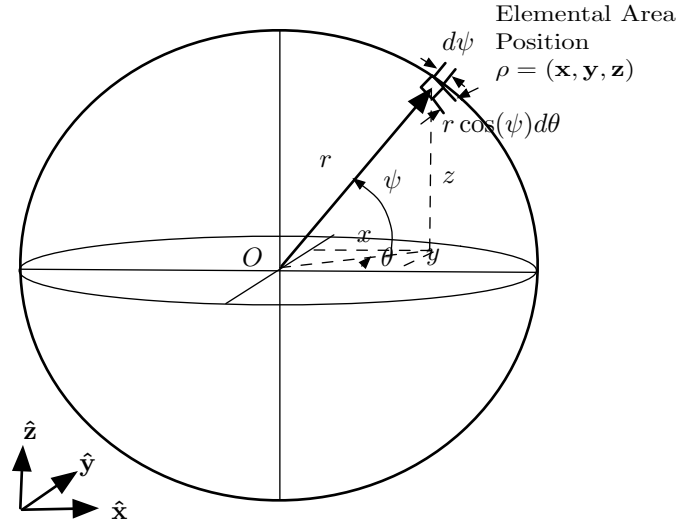


Figure 2.3: A simple spherical surface with the illustration of an elemental surface.

This elemental charge dq can be considered as a point charge and its position in terms of the cartesian coordinates (with origin at the center of the sphere) can be written as

$$\boldsymbol{\rho} = [x, y, z]^T = [r \cos(\psi) \cos(\theta), r \cos(\psi) \sin(\theta), r \sin(\psi)]^T \quad (2.4)$$

Similarly, by varying the polar angles ψ and θ from $-\pi/2$ to $\pi/2$ and 0 to 2π , respectively, all the discretized elemental charges and their position can be identified.

Now, consider two such discretized spheres with n elemental areas in each. The net Coulomb force acting between the spheres is the vector sum of the interaction of each individual elemental charge. The Coulomb force using the discretized model can be written

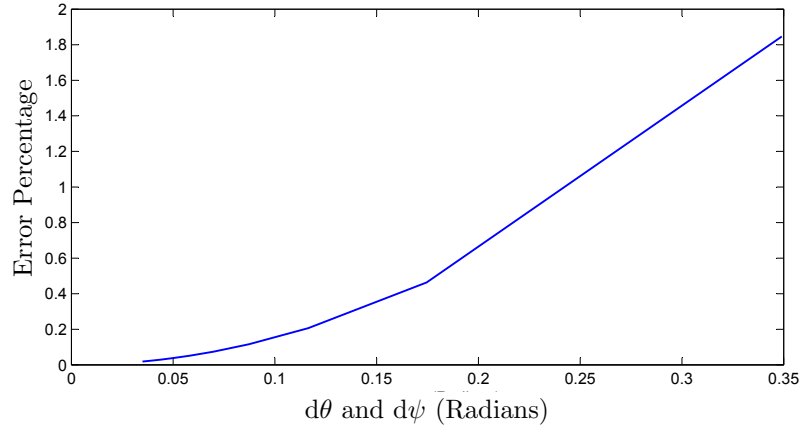


Figure 2.4: The percentage error in the Coulomb force calculated using the point charge model and the discretized surface model for two spheres for different discretization mesh size.

as

$$\mathbf{F} = \sum_{i=1}^n \sum_{j=1}^n k_c \frac{dq_i dq_j}{|\boldsymbol{\rho}_j - \boldsymbol{\rho}_i|^3} (\boldsymbol{\rho}_j - \boldsymbol{\rho}_i) \quad (2.5)$$

where k_c is the Coulomb constant, dq_i and $\boldsymbol{\rho}_i$ are the i^{th} elemental charge and its cartesian position vector on sphere one, and similarly, dq_j and $\boldsymbol{\rho}_j$ are the j^{th} elemental charge and its cartesian position vector on sphere two.

The force between two spheres based on the point charge model is given by

$$\mathbf{F} = k_c \frac{q_1 q_2}{d^2} \quad (2.6)$$

where q_1 and q_2 are the respective total charges on sphere one and two, and d is the center-to-center separation distance.

In order to establish acceptable mesh size (i.e. $d\psi$ and $d\theta$ values), the Coulomb force between two test spheres based on the discretized model was calculated for various $d\psi$ and

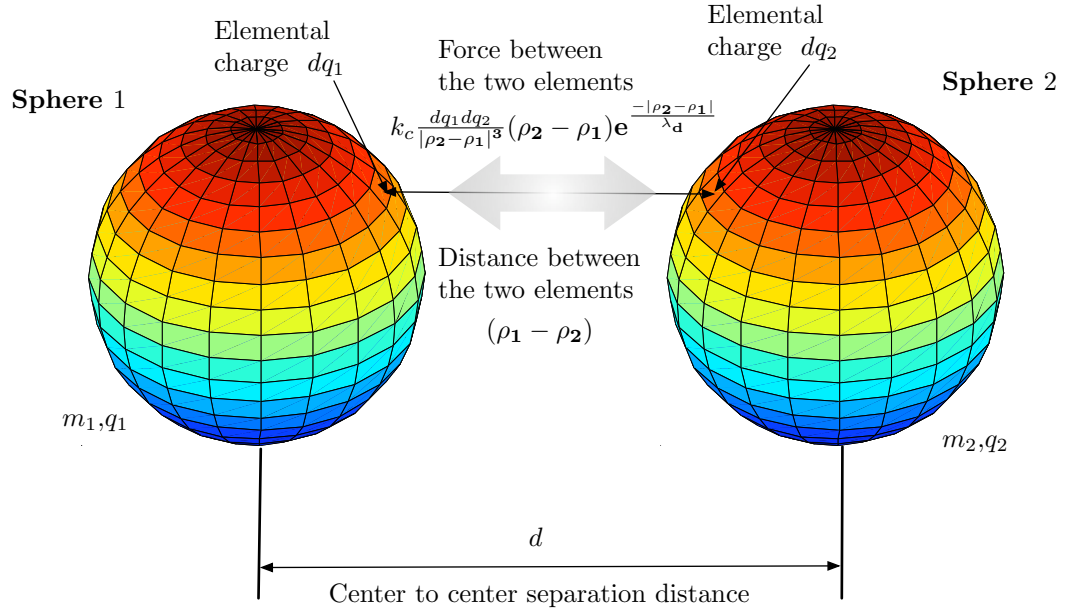


Figure 2.5: Two spheres with discretized surfaces resulting in discretized surface charges and their interaction.

$d\theta$ values. It should be recalled that when in vacuum (i.e. neglecting plasma effect) and when the induced charge redistribution effects are neglected the point charge models of the spheres are valid. Hence, the force calculated from the discretized model is compared with the point charge model and the percentage error is plotted for various $d\psi$ and $d\theta$ values as shown in Figure 2.4. A $d\psi$ and $d\theta$ value that results in a percentage error of less than 1% is an acceptable discretization. By using the Figure 2.4, $d\psi$ and $d\theta$ values are chosen as 10° or 0.17 radians that result in a percentage error of 0.46% (well within 1%).

2.3 Discretized Model in Plasma Environment

The Coulomb force experienced by a test charge dq that is at a distance d from a point charge q in a plasma environment is given by

$$\mathbf{F} = k_c \frac{q dq}{d^2} e^{\frac{-d}{\lambda_d}} \quad (2.7)$$

where λ_d is the Debye length. It can be inferred from Eq. (2.7) that as the separation distance d increases, the Coulomb force exponentially decays, and the exponential decay is more severe when d is greater than the Debye Length λ_d .

A charged sphere in a plasma environment modeled as an equivalent point charge at the center of the sphere will not give accurate results. At small Debye lengths, the charge on the surfaces that are facing each other might be within the Debye length and have a greater interaction than the charge on the opposite surfaces of the spheres because their effect might be canceled due to Debye shielding. For calculating the Coulomb force, we have modeled the spheres as porous shells and assume that the plasma can seep through. This assumption might lead us to believe that the point charge approximations will be well within acceptable limits. However, this is not true for all situations. When the separation distances are comparable to the radius of the spheres and at Debye lengths that are less than the separation distance, the Coulomb force by replacing the spheres with equivalent point charges at the center would have acutely decayed. But, in reality the charges on the surface of the spheres that are closest to each other will be well within the Debye length and their interaction might result in a net Coulomb force that is significantly higher than the force calculated using the point charge model. Thus, our main aim of this study is to identify those regions of separation distance and Debye lengths where the point charge model fails and a discretized surface model should be used instead.

The Coulomb force between two spheres in plasma environment is given using the dis-

cretized model as

$$\mathbf{F}_d = \sum_{i=1}^n \sum_{j=1}^n k_c \frac{dq_i dq_j}{|\boldsymbol{\rho}_j - \boldsymbol{\rho}_i|^3} (\boldsymbol{\rho}_j - \boldsymbol{\rho}_i) e^{\frac{-|\boldsymbol{\rho}_j - \boldsymbol{\rho}_i|}{\lambda_d}} \quad (2.8)$$

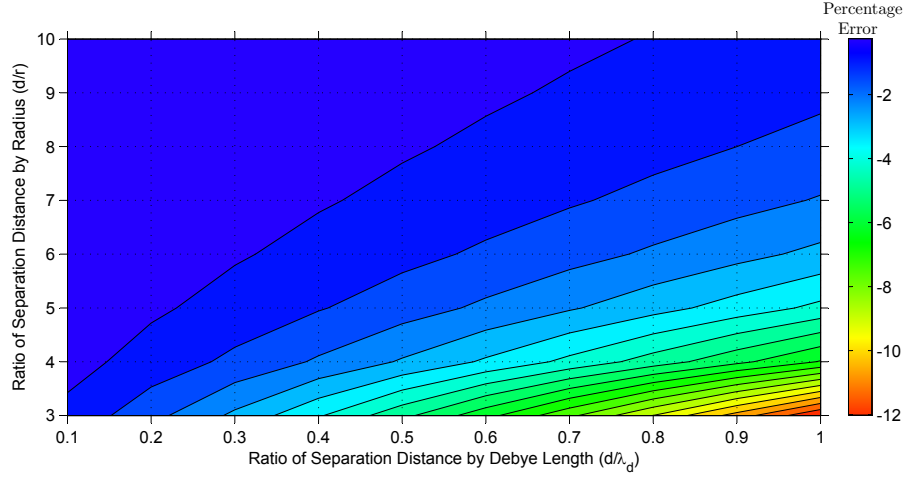
where the definitions of dq_i , $\boldsymbol{\rho}_i$, dq_j and $\boldsymbol{\rho}_j$ are the same as in Eq. (2.5). This process is illustrated in Figure 2.5. The force between two spheres in plasma environment based on the point charge model is given by

$$\mathbf{F}_p = k_c \frac{q_1 q_2}{|\mathbf{d}|^3} \mathbf{d} e^{\frac{-|\mathbf{d}|}{\lambda_d}} \quad (2.9)$$

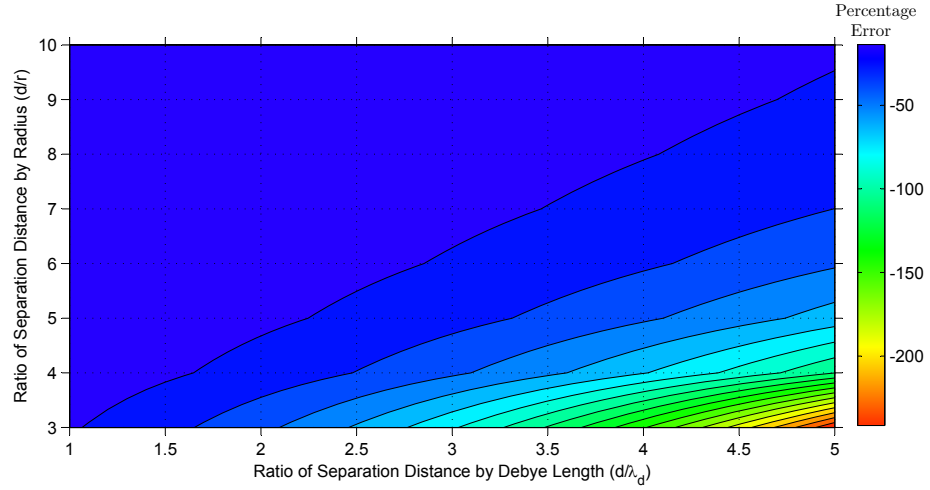
where q_1 and q_2 are the respective total charges on sphere 1 and 2, and $|\mathbf{d}|$ is the center to center separation distance. The percentage error between the two methods is calculated as

$$\text{Error} = \frac{|\mathbf{F}_p| - |\mathbf{F}_d|}{|\mathbf{F}_p|} 100 \quad (2.10)$$

The Coulomb forces using the point charge model and discretized model, and the percentage error are calculated for various combinations of radii-separation-distances ratios and separation-distance-Debye-length ratios. Figure 2.6 shows the contour plots for the percentage error in the Coulomb force calculated using the point charge model and the discretized surface model for two spheres under plasma screening. In Figure 2.6(a), the separation distance-Debye length ratio (d/λ_d) is varied from 0.1 to 1. It can be observed from this plot that at high separation distance-radius ratio (d/r), the difference between the point charge model and discretized model is insignificant even when the Debye lengths are comparable to the separation distance (i.e. at high d/λ_d ratios). There is considerable difference when separation distance-Debye length ratio (d/λ_d) is low and the Debye lengths are comparable to the separation distance, and in this region the Coulomb force due to the discretized model is higher than that due to the point charge model. This phenomenon



(a) Separation distance by Debye length ratio varies from 0.1 to 1



(b) Separation distance by Debye length ratio varies from 1 to 5

Figure 2.6: Contour plots showing the percentage error in the Coulomb force calculated using the point charge model and the discretized surface model for two spheres under plasma screening.

is further illustrated in Figure 2.6(b) in which the separation distance-Debye length ratio (d/λ_d) varies from 0.1 to 5. At separation distances that are much greater than the Debye

lengths and at low separation distance-radius ratios ($\frac{d}{r}$) the difference between the models is more than 100%.

2.4 Summary

In this chapter we demonstrate that for two spheres that are in the plasma environment, the difference between the force calculated using the point charge model and the force calculated using the discretized model is negligible, provided the separation distance is large compared to the radius of the sphere and the Debye length is large compared to the separation distance. The point charge model only fails at separation distances that are close to the radius of the sphere and when the Debye lengths are comparable to or greater than the separation distance. In these situations the discretized model gives results that are closer to the actual value. In this dissertation, a point charge model is used for charged spacecraft.

3 Reconfiguration of a Two-Craft System in Free-Space Using Coulomb Forces

3.1 Introduction

The study of electrostatic charging data of the SCATHA spacecraft⁷ verified that spacecraft can charge to high voltages in low plasma environments such as GEO, and the electric power requirement is typically less than 1 Watt. The charged spacecraft can produce electrostatic Coulomb forces that can be used to increase or decrease the relative distance between the two craft. Henceforth, this kind of reconfiguration using Coulomb forces is referred as Coulomb reconfiguration. This novel propellantless reconfiguration concept has many advantages over conventional thrusters like ion engines. Coulomb propulsion effectively uses no consumables and is also a clean method of propulsion compared to ion engines, thereby avoiding the thruster plume contamination issue with neighboring crafts. However, this Coulomb reconfiguration also has its own set of limitations. The Coulomb electrostatic force magnitude is inversely proportional to the square of the separation distance. Additionally, Coulomb force effectiveness is diminished in a space plasma environment due to the presence of charged plasma particles. The electric field strength drops off exponentially with increasing separation distance. The severity of this drop is characterized using the Debye length.^{9,10} For low earth orbits (LEO), the Debye length is of the order of millimeters

to centimeters, making the Coulomb reconfiguration concept impractical at these low orbit altitudes. At geostationary orbit (GEO) altitudes or higher, which has a hotter and less dense plasma environment, the Debye length can vary between 100-1000 meters depending on the solar activity cycles. The Coulomb reconfiguration concept appears to be feasible at this altitude.

In this chapter, we study the feasibility and the charge requirements for Coulomb reconfiguration. The craft are assumed to be in free-space. A bang-bang type of charging is used to achieve this reconfiguration. Initially, the craft are given a fixed charge of same polarity and they accelerate away from each other. After a fixed time the polarity is reversed. The resulting attraction between the craft decelerates their motion and brings them to a complete stop at the required separation distance. For various desired final separation distances and maneuver times, the fixed charge and the time (switch time) at which their polarity has to be reversed to achieve the bang-bang reconfiguration are determined. Two scenarios are investigated when 1) the craft have equal masses and 2) the craft have unequal masses model as Gluon-Deputy. The effects of the Debye length are included. This chapter is organized as follows. First, the reconfiguration dynamics are discussed and this is followed by the bang-bang charging process. Finally, the simulation results and conclusion are given.

3.2 Free-Space Reconfiguration Dynamics

Consider a two-satellite arrangement as shown in Figure 3.1. Let the origin be located at the center of mass. The distances of craft 1 and craft 2 from the origin are denoted x_1 and x_2 , respectively. The separation distance between the two craft is d , given by

$$d = x_1 - x_2 \tag{3.1}$$

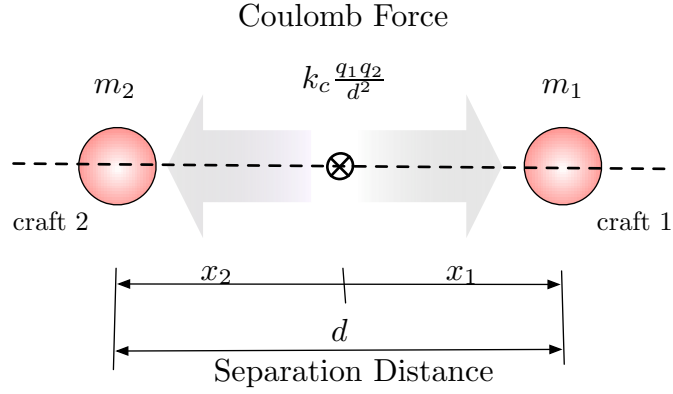


Figure 3.1: The two-craft arrangement in free-space.

Since, the origin is located at the center of mass, the center of mass condition dictates that

$$m_1 x_1 + m_2 x_2 = 0 \quad (3.2)$$

Therefore, once we know the motion of one craft, the motion of the other craft can be determined using the above condition. From Eq. (3.1) and Eq. (3.2), the position of the craft 1 can be written as

$$x_1 = d \left(1 + \frac{m_1}{m_2} \right)^{-1} \quad (3.3)$$

The craft are assumed to be in free space and therefore, no external force is acting on the formation. The initial internal forces acting on the deputy are the gravitational force of attraction between the two craft and Coulomb force. The former is given by

$$F_g = G \frac{m_1 m_2}{d^2} \quad (3.4)$$

where G is the universal gravity constant and, m_1 and m_2 are the masses of craft 1 and craft 2, respectively. The Coulomb force is given by

$$F_c = k_c \frac{q_1 q_2}{d^2} \quad (3.5)$$

where k_c is the Coulomb constant, and q_1 and q_2 are the charges of craft 1 and craft 2, respectively. The force expression in Eq. (3.5) is rewritten using the voltages produced due to the charges as

$$F_c = \frac{r_1 r_2}{k_c} \frac{V_1 V_2}{d^2} e^{\left(\frac{-d}{\lambda_d}\right)} \quad (3.6)$$

where V_i and r_i are the voltage and radius of craft i . The exponential decay term is due to the Debye shielding where λ_d is the Debye length.

Using Newton's second law, the equation of motion of craft 1 (m_1) is written as

$$m_1 \ddot{x}_1 = -G \frac{m_1 m_2}{d^2} + \frac{r_1 r_2}{k_c} \frac{V_1 V_2}{d^2} e^{\left(\frac{-d}{\lambda_d}\right)} \quad (3.7)$$

Using Eq. (3.3), separation distance d in Eq. (3.7) can be substituted with an expression in terms of x_1 . The gravitational force acting on the satellites is small compared to the Coulomb force, so we can neglect the former, and the equation of motion of craft 1 (m_1) can be rewritten as

$$m_1 \ddot{x}_1 = \frac{r_1 r_2}{k_c} \frac{V_1 V_2}{x_1^2 \left(1 + \frac{m_1}{m_2}\right)^2} e^{\left(\frac{-x_1 \left(1 + \frac{m_1}{m_2}\right)}{\lambda_d}\right)} \quad (3.8)$$

Thus, Eq. (3.8) gives the equation of motion for two charged craft in free space that are one dimensionally restricted.

3.3 Bang-Bang Charging

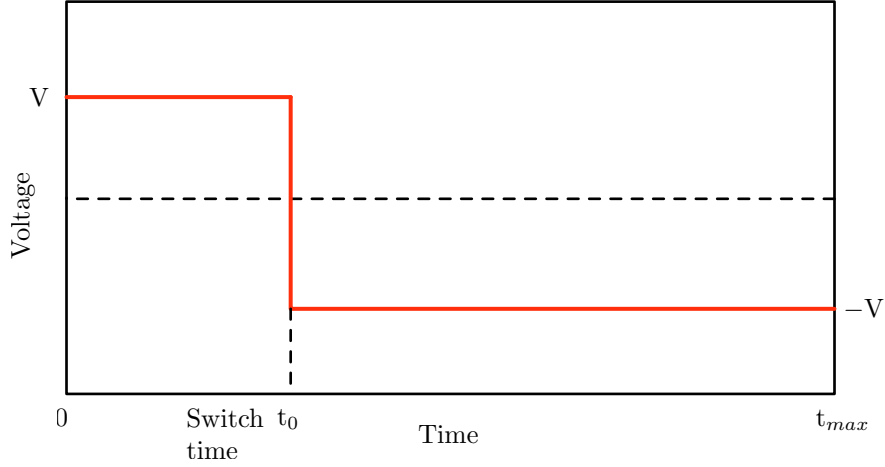
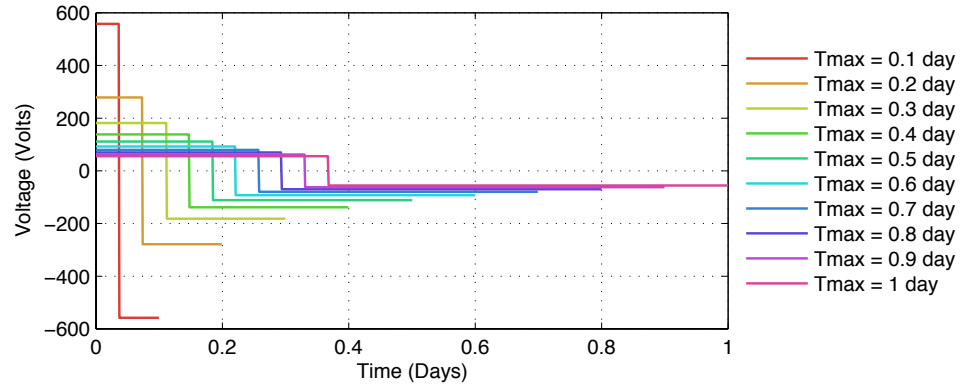


Figure 3.2: Schematic representation of the bang-bang charging.

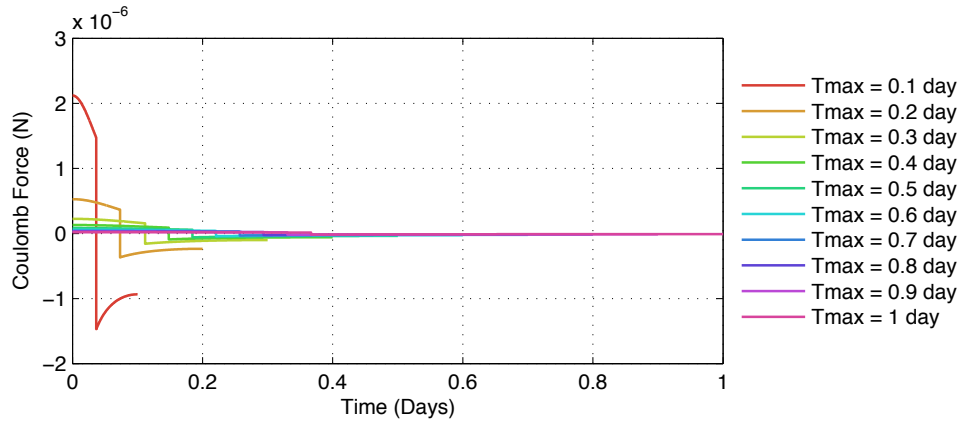
The equation of motion for the two-craft reconfiguration in free space was developed in the previous section and in this section we discuss the type of charging that will be used. The main idea behind bang-bang charging is to use a fixed voltage for accelerating the craft, and reversing the polarity of the fixed voltage after the switch time to decelerate the craft. Figure 3.2 shows a schematic representation of this kind of charging, where the voltage is maintained at a constant value V until it reaches the switching time t_0 , after which the voltage polarity is reversed.

$$\text{voltage} = \begin{cases} V & 0 < t \leq t_0 \\ -V & t_0 < t < t_{max} \end{cases} \quad (3.9)$$

For a given separation distance and time (t_{max}) for achieving this distance, a particular voltage V and switch time t_0 must be found. We start with an initial guess and use the shooting method to converge to this solution. The initial guess for the switch time is given



(a) Voltage time histories



(b) Coulomb force time histories

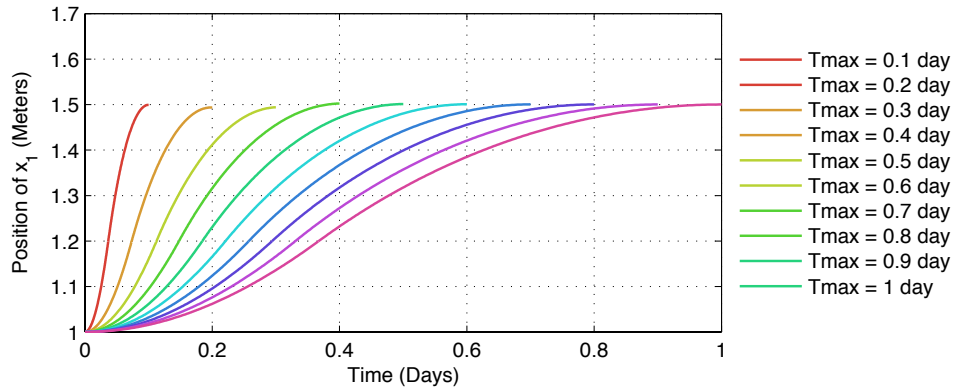

(c) Time histories of the position of mass m_1

Figure 3.3: Simulation results for increasing the separation between two satellites of equal masses from 2m to 3m in different maneuver time using bang-bang charging process.

by

$$t_0 = t_{max}/2 \quad (3.10)$$

Let $\bar{d}$ be the final desired separation and $\bar{x}_1$ correspond to the position of mass m_1 at this separation distance. Then the initial guess for the product of voltages is given by

$$V_1 V_2 = 2(\bar{x}_1 - x_1) t_{max}^2 \frac{k_c \bar{d}^2}{r_1 r_2 e^{\left(\frac{-\bar{d}}{\lambda_d}\right)}} \quad (3.11)$$

From Eq. (3.8), it is clear that we have a $1/r^2$ type of decay for the force acting between the craft as the separation distance increases. In addition to this decay, there is also an exponential decay term due to the Debye shielding effect. For these reasons, the initial conditions in Eq. (3.10) and Eq. (3.11) will not work well when trying to increase the separation distance by large values. In order to overcome this difficulty, a technique called homotopy²⁶ is used. In this technique the distances are increased in small incremental steps and the converged solution (using shooting method) for the previous step is used as the current initial guess.

Figure 3.3 illustrates the bang-bang charging process for sample maneuvers where the separation distance between two craft of equal masses is increased from 2 m to 3 m. Figures 3.3(a), 3.3(b) and 3.3(c), give the voltage, Coulomb force experienced, and the position time histories of mass m_1 . It can be seen from the graphs that as the total maneuver time is increased from 0.1 day to 1 day, the voltage required falls rapidly and the switch time increases.

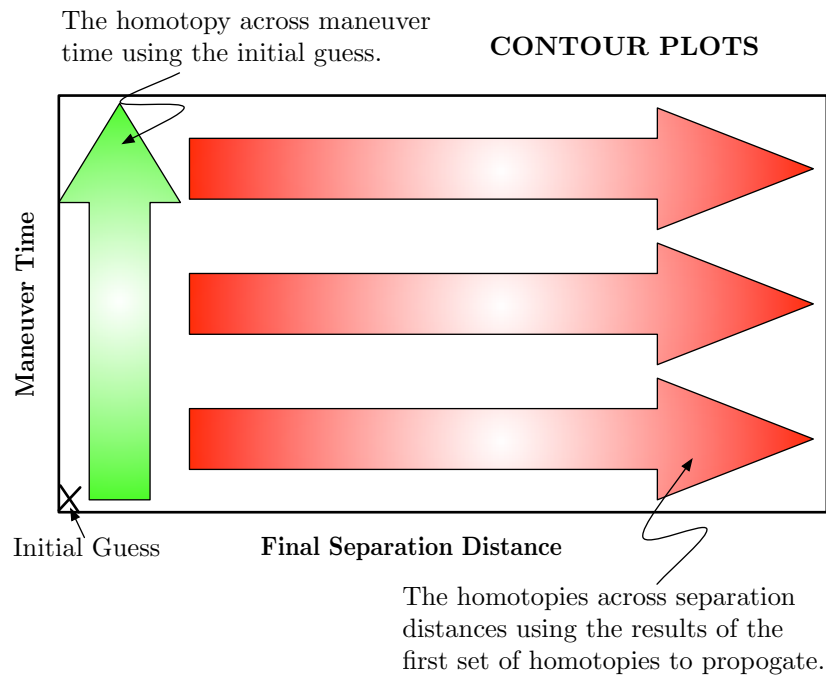


Figure 3.4: Schematic representation of the homotopies used in generating the contour plots.

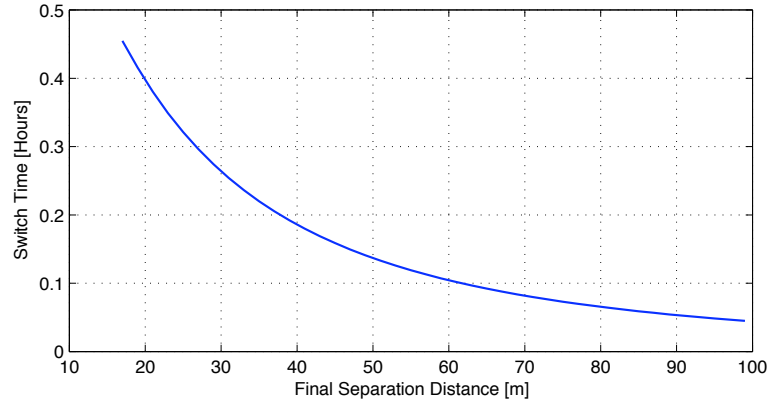
3.4 Simulation Results

The simulations are carried out for two different scenarios. In the first scenario, the two craft are of equal mass and size. In the second scenario the gluon-deputy arrangement is used where the gluon is much heavier and larger in size than the deputy craft. Due to its enormous mass, the distance moved by the gluon will be small and we can consider it to be stationary. In this case, the total separation distance will be roughly equal to the distance moved by deputy. Whereas in the equal mass scenario, the two craft will move equal distances and the total separation distance will be twice the distance moved by any one of the craft. The various input parameters are given in Table 3.1.

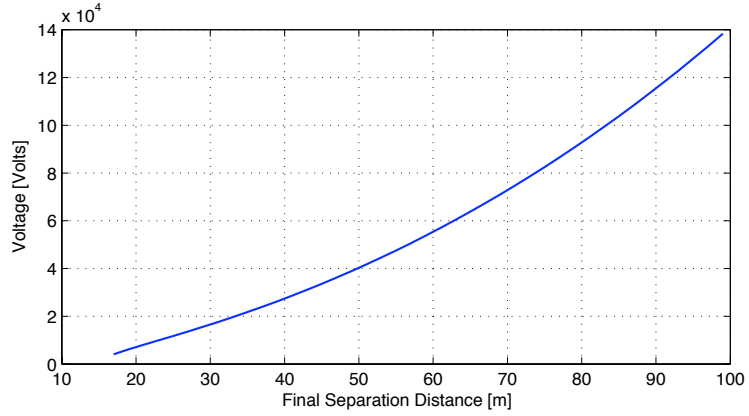
Table 3.1: Input parameters used in simulation of free-space reconfiguration.

Parameter	Equal Mass Crafts	Gluon-Deputy Crafts
Craft 1 Mass, m_1	50 kg	50 kg
Craft 2 Mass, m_2	50 kg	1000 kg
Craft 1 Radius, r_1	0.5 m	0.5 m
Craft 2 Radius, r_2	0.5 m	10 m
Initial Separation distance, d	2 m	15 m

It is mentioned in the previous section that the shooting method and homotopy method are used in calculating the voltage and switch time for various separation distances and maneuver times. Before we present the simulated results, let us discuss these methods in a more detailed fashion and explain how exactly they are used here. Our aim is to generate the contour plots of the fixed voltage required to carry out these bang-bang maneuvers for different final separation distance and maneuver time. Figure 3.4 gives the schematic representation of the homotopies used in generating the contour plots. Initially, for a small increase in the separation distance in a small time, we use the initial guess in the shooting method to find the switch time and the fixed voltage. Using this result, the first set of homotopies are carried out across the maneuver time keeping the separation distance

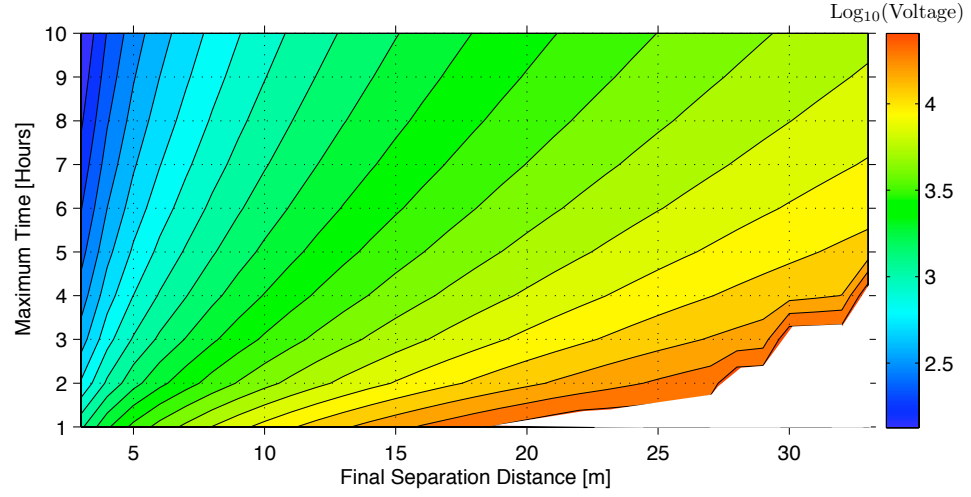


(a) Switch Time Vs Separation Distance

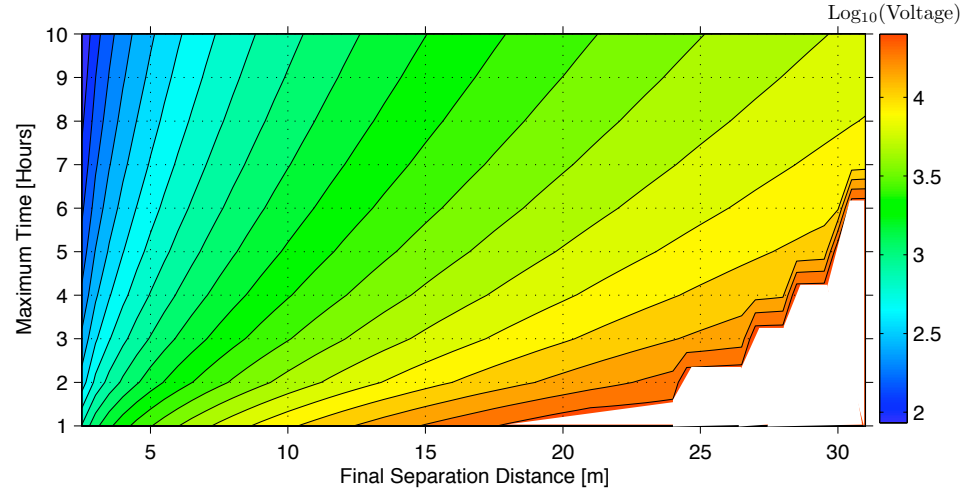


(b) Voltage Vs Separation Distance

Figure 3.5: The switch time and voltage as functions of increase in separation distance in a constant maneuver time. These voltages and switch times correspond to the gluon-deputy arrangement with a constant maneuver time of 1.0 hour and initial separation distance of 15 m.

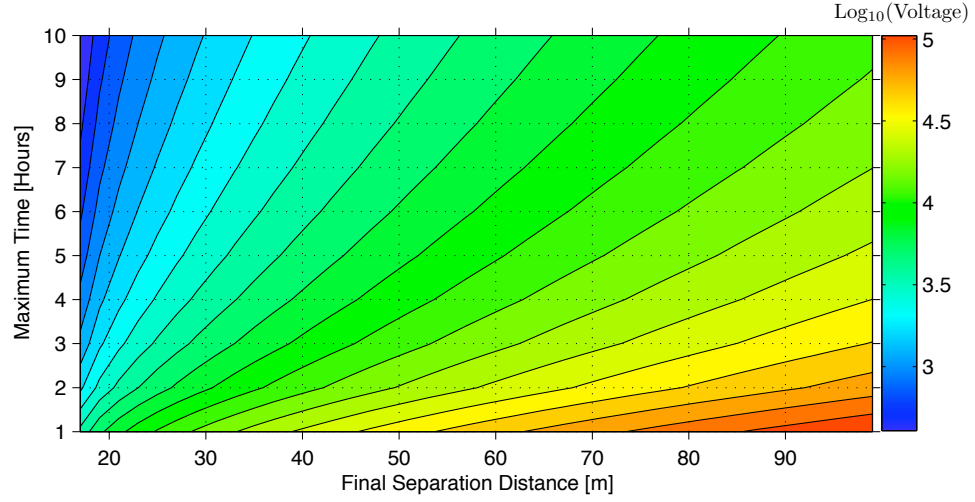


(a) Debye length of 100 m

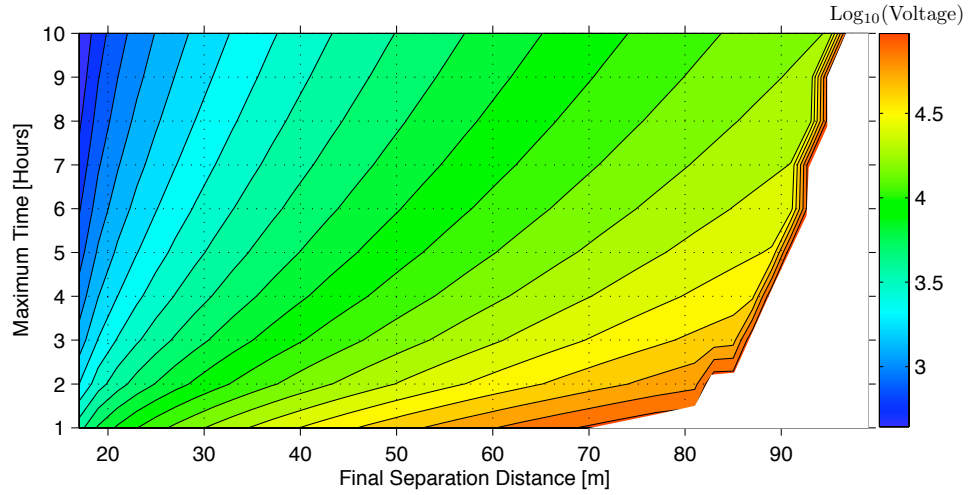


(b) Debye length of 50 m

Figure 3.6: The contour plot of the voltage (log) needed to increase the separation distance of two craft of equal mass, from 2 m to several meters in different maneuver times, using bang-bang charging.

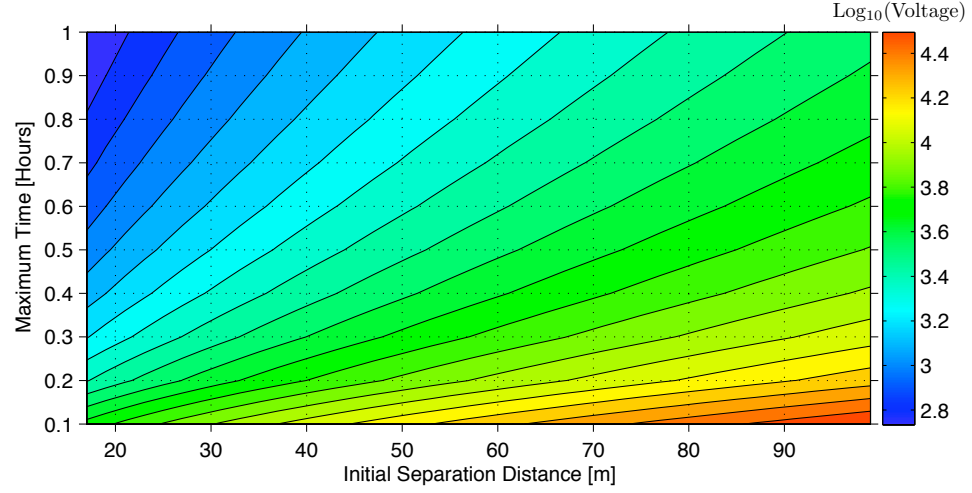


(a) Debye length of 100 m

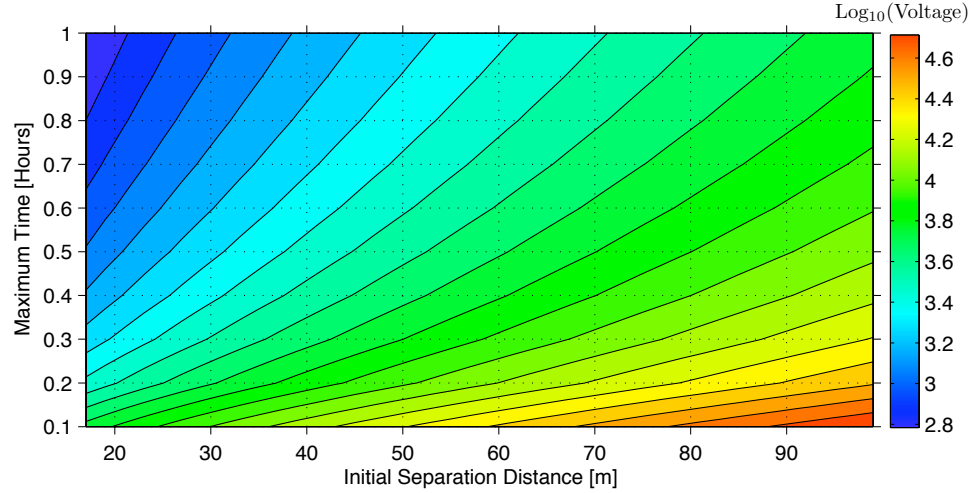


(b) Debye length of 50 m

Figure 3.7: The contour plot of the voltage (log) needed for a gluon-deputy satellite arrangement to increase the separation distance from 15 m to several meters in different maneuver times, using bang-bang charging.



(a) Debye length of 100 m



(b) Debye length of 50 m

Figure 3.8: The contour plot of the voltage (log) needed for a gluon-deputy satellite arrangement to increase the separation distance by 10 m for various initial separation distances in different maneuver times, using bang-bang charging.

constant. It should be noted that the homotopy means using the converged results in the previous case as the initial guess for the current problem and shooting method is the one that performs the convergence. The other homotopies are performed across the final separation distance using the results of the first set of homotopies as initial conditions.

In order to understand how the voltage and switch time vary across the separation distance for a fixed maneuver time, let us study the Figure 3.5. For a fixed maneuver time of 1 hour, Figures 3.5(a) and 3.5(b) give the switch time and voltage, respectively, for the gluon-deputy arrangement to increase their separation distance from 15 m to various lengths. It can be observed that the increase in voltage with increase in separation distance is more or less linear, whereas the switch time t_0 decreases exponentially with increase in separation distance. This exponential decrease is due to the $1/r^2$ term in the Coulomb force, and without this term the switch time would be exactly half the total maneuver time. We can also observe in Figures 3.5(a) that for small separation distances the switch time approaches 0.5 hours, which is half of the 1.0 hour maneuver time. As the final separation distance increases the switch time approaches zero (i.e. the initial voltage becomes a pulse).

Figure 3.6 shows the contour plots of the voltage required to increase the separation distance between two craft to several meters in different maneuver times. The craft have equal mass and the initial separation distance is 2 m. The contour plots are shown for two different Debye lengths of 100m and 50m. The white patches in the contour plots represent regions where we are unable to find a converged solution. In the shooting method, we use the MATLAB function ode45 with absolute and relative tolerances both set to 10^{-12} . As the final separation distance increases, even though we use homotopy, for certain points the integration tolerance exceeds the allowed limit. Hence, for these points we are not able to find converged solutions. Further investigation need to be carried out to find these solutions. However, the contour plots give the general trend in the voltage requirements. For example, in Figure 3.6(a) the voltage required to increase the separation distance in 4

hours from 2 m to 5 m is roughly $10^{2.75}$ volts, and to increase 10 m it is $10^{3.5}$ volts.

The second set of simulations is for the gluon-deputy arrangement. The specifications of the gluon and deputy are given in Table 3.1. The initial center-to-center separation distance between the gluon and the deputy is 15 m. Figure 3.7 gives the simulation result for this arrangement. The voltage appears similar to the previous case. But, it should be noted that the gluon due to its large radius is carrying a greater charge for the same voltage levels. The final simulation is carried out using the same gluon deputy arrangement as given in Table 3.1. The maneuvers involve increasing the separation distance by 10 m for various initial separation distance. Figure 3.8 gives the contour plots of the voltage needed for these maneuvers. Again, the simulation is carried out for two different Debye lengths (100 m and 50 m).

3.5 Summary

The reconfiguration of a two-craft formation in free-space using Coulomb forces is studied. The bang-bang charging sequence is successfully employed to increase the separation distances between the craft. The simulation results from both the gluon-deputy arrangement and equal-mass -equal-size arrangement show that the voltages required to carry out these maneuvers are quite low. These voltages are realizable in practice. This gives us the impetus to extend this study when the craft are in orbit.

4 Two-Craft Coulomb Tether Formation Along Orbit-Radial Direction

4.1 Introduction

A new application of the Coulomb propulsion concept is to use the electrostatic force to control the separation distance between two physically unconnected craft. Due to the similarities with using a tether cable to connect two craft, this concept is called a Coulomb tether formation. Note that contrary to traditional tethers, the Coulomb tether is capable of receiving both tensile and compressive forces. Further, the stiffness of the satellite connection can be controlled through feedback control laws. This will allow for the Coulomb tether stiffness to be varied with changing mission requirements. Scenarios with two spacecraft flying only dozens of meters apart are investigated in this chapter. Potential applications include releasing a sensor or camera unit from the primary spacecraft and holding it at fixed distance above or below the spacecraft. From this non-Keplerian orbit, the sensor craft could monitor the spacecraft itself, or perform other scientific measurements.

Coulomb forces cannot be used to change the total inertial formation angular momentum vector.^{4,5} As a result, these spacecraft charges cannot be used to reorient a formation as a whole to a new orientation. An external influence must be used or generated through thrusters to reorient a Coulomb formation. Spacecraft are not subjected to the same grav-

itational pull throughout the formation. The sections which are closer to the Earth are attracted more strongly than those that are further away. This force or gravity gradient²⁷ has been used in stabilizing some satellites. To guarantee linear stability of rigid body attitudes in orbit, the principal inertias of the body must satisfy well-known constraints. Typically gravity-gradient stabilized satellites are tall and slender, and aligned with the local nadir direction. The same concept of stabilization can be extended to the two spacecraft Coulomb tether concept where the craft are assumed to be flying apart by a few dozen meters. A charge feedback law is employed to stabilize the spacecraft separation distance (making the formation act as a rigid, slender rod), while the gravity gradient torque is exploited to assist in stabilizing the formation attitude.

King et. al.⁸ found analytical solutions for Hill-frame invariant Coulomb formations. Here spacecraft are placed at specific locations in the rotating Hill frame with specific electrostatic charges. As a result the Coulomb forces perfectly cancel all natural orbital accelerations, causing the satellites to remain fixed or static as seen by the Hill frame. However, the charge was held constant in their analysis. The discovered open-loop static Coulomb formations were all found to be unstable. References 1, 2, 8 discuss the static Coulomb satellite formations and the associated equilibrium charges, but do not address the stabilization of these formations. In this chapter, stabilization of a simple static Coulomb structure is discussed for the first time. An active charge feedback control is presented to stabilize the static 2-craft formation shape and orientation. In order to achieve this goal we use known stability characteristics of orbital rigid body motion under a gravity gradient field and examine its applicability to a Coulomb tethered two-spacecraft system. To avoid the very small plasma Debye lengths found at LEO, the Coulomb tether formation studied is at GEO. The formation center of mass or chief motion is assumed to be circular. In formation flying the chief is the reference location about which all other deputy satellites are flying. The two body Coulomb tether problem considered here can be viewed as a

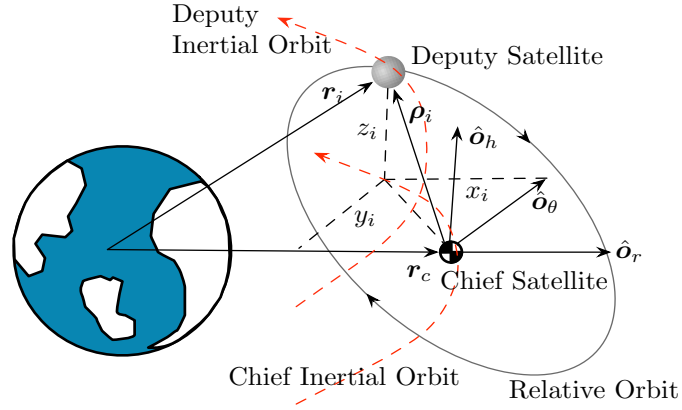


Figure 4.1: Rotating Hill coordinate system used to describe the relative position of the satellites

sub-problem of the multi-satellite formation flying problem. In future work, attempts will be made to extend the feedback control discussed here to multi-satellite formations. The chapter is organized as follows. After discussing the charged spacecraft equations of motion, the equations are rewritten using spherical coordinates and linearized for small departure angles relative to an equilibrium attitude. A feedback charge control law is introduced to stabilize the separation distance, followed by a combined attitude and separation distance linear stability analysis. A numerical simulation illustrates the results and compares the linearized performance predictions to the actual nonlinear system response.

4.2 Static (Rigid) Formation Dynamics

To start with, the equations of motion of a cluster of charged spacecraft are briefly reviewed. The Clohessy-Wiltshire-Hill's equations^{28,29} are commonly used for spacecraft formation studies. These equations express the linearized motion of one satellite relative to a circularly orbiting reference point or chief location. Note that this chief location does not have to be actually occupied by a satellite. For the present discussion, the formation chief location

is set to be equal to the formation center of mass. The various satellites in a formation are called the deputy satellites. The system of Cartesian coordinates used to describe the relative motion of a satellite with respect to the chief location is defined in the rotating Hill orbit frame $\mathcal{O} : \{\hat{\boldsymbol{o}}_r, \hat{\boldsymbol{o}}_\theta, \hat{\boldsymbol{o}}_h\}$ as shown in Figure 4.1. The origin of the coordinate system is chosen to be the formation center of mass or chief location. The Cartesian x , y and z coordinates are the vector components of the relative position vector

$$\boldsymbol{\rho} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}^{\mathcal{O}} \quad (4.1)$$

along the directions of orbit radial $\hat{\boldsymbol{o}}_r$ (outward), the orbital velocity vector $\hat{\boldsymbol{o}}_\theta$, and the normal vector $\hat{\boldsymbol{o}}_h$ with respect to the orbit plane. Assuming that the Coulomb formation contains N satellites, the CW equations of the i^{th} deputy with respect to the chief are expressed as

$$\ddot{x}_i - 2\Omega\dot{y}_i - 3\Omega^2 x_i = \frac{k_c}{m_i} \sum_{j=1}^N \frac{(x_i - x_j)}{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^3} q_i q_j e^{-|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|/\lambda_d} \quad j \neq i \quad (4.2a)$$

$$\ddot{y}_i + 2\Omega\dot{x}_i = \frac{k_c}{m_i} \sum_{j=1}^N \frac{(y_i - y_j)}{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^3} q_i q_j e^{-|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|/\lambda_d} \quad j \neq i \quad (4.2b)$$

$$\ddot{z}_i + \Omega^2 z_i = \frac{k_c}{m_i} \sum_{j=1}^N \frac{(z_i - z_j)}{|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|^3} q_i q_j e^{-|\boldsymbol{\rho}_i - \boldsymbol{\rho}_j|/\lambda_d} \quad j \neq i \quad (4.2c)$$

where $\boldsymbol{\rho}_i = (x_i, y_i, z_i)^T$ is the position vector of the i^{th} satellite in Hill frame components, m_i is the satellite mass, and q_i is the satellite charge. The chief position vector $\boldsymbol{r}_c$ is assumed to have a constant orbital rate of $\Omega = \sqrt{GM_e/r_c^3}$, where G is the gravity constant and M_e is the Earth's mass. The parameter $k_c = 8.99 \cdot 10^9 \text{ Nm}^2/\text{C}^2$ is the Coulomb's constant, while the parameter λ_d is the Debye length. Because the Coulomb tether formations are

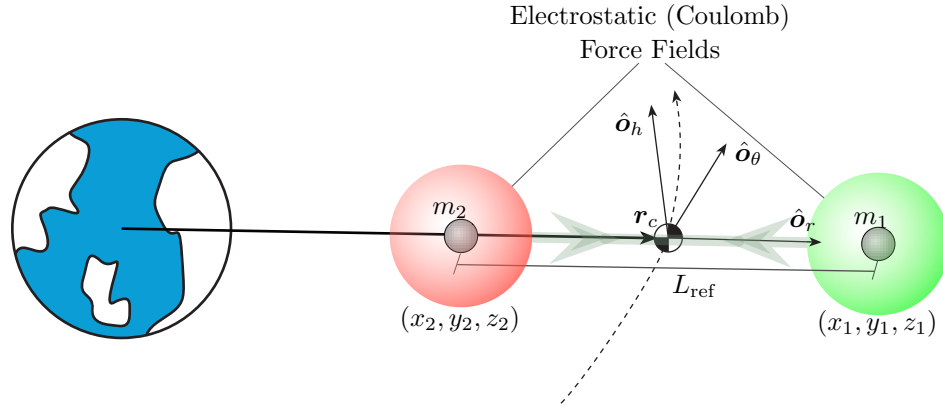


Figure 4.2: Coulomb tethered two satellite formation with the satellites aligned along the orbit nadir direction

assumed to be at GEO where the Debye length is much larger than the typical Coulomb tether length, the Debye length influence is ignored as a higher order term for the remainder of this chapter. Note that these relative equations of motion of a charged spacecraft contain linearized orbital dynamics, while retaining the full nonlinear Coulomb force expression. In fact, it is this nonlinear Coulomb force term that causes the strong and complex coupling between the spacecraft motions.

The formation geometry of the ideal two-craft Coulomb tether formation is shown in Figure 4.2. As is shown later in this section, there exists a two-craft static Coulomb formation solution where both masses must be aligned equal distances away from the chief along the nadir direction. The ideal separation distance is called L_{ref} . If each craft has a certain charge, then the resulting Coulomb forces will perfectly cancel the linearized orbital accelerations in the Hill frame. As a result, the two craft would each remain aligned in the chief nadir direction and perform non-Keplerian motions. To an external observer the two physically unconnected craft would appear to both be performing perfectly circular motions, but with a non-Keplerian orbit period for their individual altitudes. The invisible

Coulomb tether is applied to get the required inter-spacecraft force, similar to how a cable tether could provide the required tension between the craft to maintain such non-Keplerian orbits.

Since the Coulomb tether formation considered has only two spacecraft, the CW equations in Eq. (4.2) for satellite 1 can be simplified as

$$\ddot{x}_1 - 2\Omega\dot{y}_1 - 3\Omega^2 x_1 = \frac{k_c}{m_1} \frac{(x_1 - x_2)}{L^3} q_1 q_2 \quad (4.3a)$$

$$\ddot{y}_1 + 2\Omega\dot{x}_1 = \frac{k_c}{m_1} \frac{(y_1 - y_2)}{L^3} q_1 q_2 \quad (4.3b)$$

$$\ddot{z}_1 + \Omega^2 z_1 = \frac{k_c}{m_1} \frac{(z_1 - z_2)}{L^3} q_1 q_2 \quad (4.3c)$$

where L is the distance between the satellites 1 and 2. As the Hill frame $\mathcal{O}$ origin is assumed to be identical to the formation center of mass, the center of mass condition dictates that^{4,5}

$$m_1 \boldsymbol{\rho}_1 + m_2 \boldsymbol{\rho}_2 = 0 \quad (4.4)$$

Thus, by controlling the motion of satellite 1, the motion of the second satellite is also determined implicitly through the center of mass constraint.

In order for this top-down spacecraft formation to remain statically fixed relative to the rotating orbit frame $\mathcal{O}$, the CW equations in Eq. (4.3) must be satisfied with zero initial velocity and acceleration for each vehicle

$$\dot{x}_i = \ddot{x}_i = \dot{y}_i = \ddot{y}_i = \dot{z}_i = \ddot{z}_i = 0$$

For a two-craft Coulomb formation, this is possible if the relative positions are expressed

through:

$$m_1 x_1 + m_2 x_2 = 0 \quad (4.5a)$$

$$x_1 - x_2 = L \quad (4.5b)$$

$$x_1 = \frac{m_2}{m_1 + m_2} L \quad (4.5c)$$

$$x_2 = -\frac{m_1}{m_1 + m_2} L \quad (4.5d)$$

$$y_1 = y_2 = z_1 = z_2 = 0 \quad (4.5e)$$

Substituting the above conditions and constraints in Eq. (4.3), one obtains the following two spacecraft charge conditions for a static nadir-aligned formation.

$$\frac{k_c}{m_1} \frac{q_1 q_2}{L^2} + 3\Omega^2 \frac{m_2 L}{m_1 + m_2} = 0 \Rightarrow q_1 q_2 = -3\Omega^2 \frac{L^3}{k_c} \frac{m_1 m_2}{m_1 + m_2} \quad (4.6a)$$

$$\frac{k_c}{m_2} \frac{q_1 q_2}{L^2} + 3\Omega^2 \frac{m_1 L}{m_1 + m_2} = 0 \Rightarrow q_1 q_2 = -3\Omega^2 \frac{L^3}{k_c} \frac{m_1 m_2}{m_1 + m_2} \quad (4.6b)$$

The ideal product of charges Q_{ref} needed to achieve this static Coulomb formation is

$$Q_{\text{ref}} = q_1 q_2 = -3\Omega^2 \frac{L^3}{k_c} \frac{m_1 m_2}{m_1 + m_2} \quad (4.7)$$

Thus, if the satellites are placed at the locations shown in Eq. (4.5), and have the charges q_1 and q_2 satisfying Eq. (6.5), then the satellites will appear to be frozen or fixed as seen by the rotating frame $\mathcal{O}$. Note that this reference charge product term will be negative! This dictates that the spacecraft charges q_1 and q_2 will have opposite charge signs. However, there is an infinite number of charge pairs which satisfy $Q_{\text{ref}} = q_1 q_2$. When implementing charge control strategies in this study, the charge magnitudes are set equal. If one craft is capable of higher charge levels, it is possible to have unequal charges as long as their product satisfies the required $Q = q_1 q_2$ value.

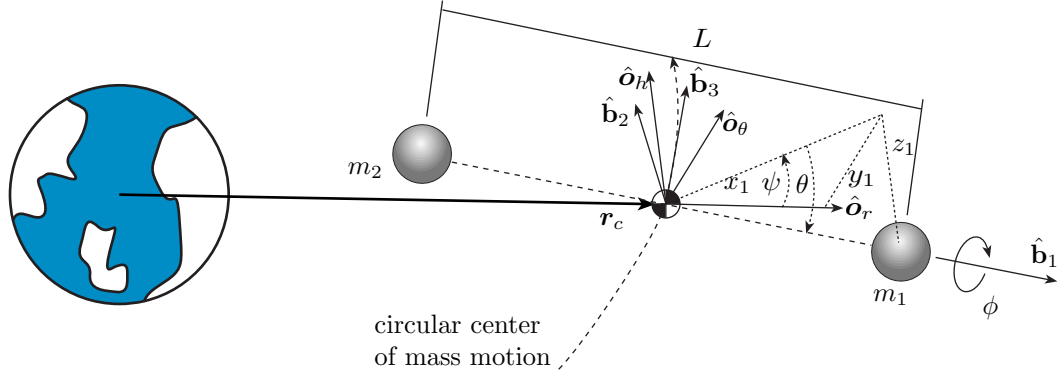


Figure 4.3: Euler angles representing the attitude of Coulomb tether with respect to the orbit frame

4.3 Linearized Orbital Perturbation

The constant charge computed in accordance with Eq. (6.5) will only result in the static nadir formation if there are no position or velocity errors, and no perturbations are present. Otherwise, the relative separation will become unstable and the satellites will separate. This problem can be overcome by allowing a suitable variation of charges. In this section, a relationship between these position and charge states is established by considering small perturbations about the established reference states.

Let the two-craft formation be treated as if it were a rigid body. Accordingly, consider a body-fixed coordinate frame $\mathcal{B} : \{\hat{b}_1, \hat{b}_2, \hat{b}_3\}$ where $\hat{b}_1$ is aligned with the relative position vector $\boldsymbol{\rho}_1$. Note that if the body is at the ideal Coulomb tether orientation where the masses are aligned exactly along the orbit nadir direction $\hat{o}_r$, then the $\mathcal{O}$ and $\mathcal{B}$ frame orientation vectors are identical. The relative position vector of mass m_1 in body fixed axes is given by

$$\boldsymbol{\rho}_1 = \frac{m_2}{m_1 + m_2} L \hat{b}_1 + 0 \hat{b}_2 + 0 \hat{b}_3 \quad (4.8)$$

Let the 3-2-1 Euler angles (ψ, θ, ϕ) represent the Coulomb tether $\mathcal{B}$ frame attitude relative to the orbit frame $\mathcal{O}$ for small angular perturbations as shown in Figure 4.3. Because point masses are being considered, the rotation about $\hat{\mathbf{b}}_1$ (angle ϕ) can be neglected. The direction cosine matrix $[BO(\psi, \theta)]$, which relates the $\mathcal{O}$ frame to $\mathcal{B}$ frame, is given by

$$[BO] = \begin{bmatrix} \cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\ -\sin \psi & \cos \psi & 0 \\ \sin \theta \cos \psi & \sin \theta \sin \psi & \cos \theta \end{bmatrix} \quad (4.9)$$

Using small angle approximations for the trigonometric functions, the position vector of mass m_1 in $\mathcal{O}$ frame can be written as

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = [BO]^T \begin{pmatrix} \frac{m_2}{m_1+m_2} L \\ 0 \\ 0 \end{pmatrix} \approx \begin{pmatrix} \frac{m_2}{m_1+m_2} L \\ \psi \frac{m_2}{m_1+m_2} L \\ -\theta \frac{m_2}{m_1+m_2} L \end{pmatrix} \quad (4.10)$$

Taking the derivative of this expression, the linearized Hill frame relative velocity coordinates are found to be

$$\begin{pmatrix} \dot{x}_1 \\ \dot{y}_1 \\ \dot{z}_1 \end{pmatrix} \approx \frac{m_2}{m_1+m_2} \begin{pmatrix} \dot{L} \\ \psi \dot{L} + \dot{\psi} L \\ -\theta \dot{L} - \dot{\theta} L \end{pmatrix} \quad (4.11)$$

The distance L between the two masses m_1 and m_2 is given by

$$L^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 \quad (4.12)$$

Using the center of mass condition in Eq. (4.4), this can be simplified to

$$L^2 = \left(\frac{m_1 + m_2}{m_2} \right)^2 (x_1^2 + y_1^2 + z_1^2) \quad (4.13)$$

Differentiating Eq. (4.13) twice and substituting Eq. (4.3) into the resulting expression yields,

$$\begin{aligned} \dot{L}^2 + L\ddot{L} = & \left(\frac{m_1 + m_2}{m_2} \right)^2 \left(\dot{x}_1^2 + x_1 \left(2\Omega\dot{y}_1 + 3\Omega^2 x_1 + \frac{k_c}{m_1} \frac{(x_1 - x_2)}{L^3} Q \right) + \dot{y}_1^2 + y_1 \left(-2\Omega\dot{x}_1 \right. \right. \\ & \left. \left. + \frac{k_c}{m_1} \frac{(y_1 - y_2)}{L^3} Q \right) + \dot{z}_1^2 + z_1 \left(-\Omega^2 z_1 + \frac{k_c}{m_1} \frac{(z_1 - z_2)}{L^3} Q \right) \right) \end{aligned} \quad (4.14)$$

Transforming the Cartesian coordinates (x_1, y_1, z_1) to spherical coordinates (L, ψ, θ) using Eq. (4.10) and Eq. (4.11), while neglecting higher order terms in ψ and θ , we get the linearized differential equation of the separation distance L .

$$\ddot{L} = (2\Omega\dot{\psi} + 3\Omega^2)L + \frac{k_c}{m_1} Q \frac{1}{L^2} \frac{m_1 + m_2}{m_2} \quad (4.15)$$

Note the following special case. Assume that the charge product term Q is zero (i.e. classical Keplerian motion), and that the satellites are initially at rest with $\dot{\psi} = 0$. In this case the separation distance equations of motion simplify to

$$\ddot{L} - 3\Omega^2 L = 0$$

This unstable oscillator equation demonstrates that without any Coulomb force active, this formation could not remain at the specific nadir locations.

Next the separation distance equations of motion are linearized about small variations in length δL and small variations in the product charge term δQ . The reference separation length L_{ref} is determined by the mission requirement. The reference charge product term

is determined through the L_{ref} choice and the constraint in Eq. (6.5).

$$L = L_{\text{ref}} + \delta L \quad (4.16a)$$

$$Q = Q_{\text{ref}} + \delta Q \quad (4.16b)$$

Substituting these L and Q definitions into Eq. (5.2) and linearizing leads to

$$\delta \ddot{L} = (2\Omega L_{\text{ref}})\dot{\psi} + (9\Omega^2)\delta L + \left(\frac{k_c}{m_1} \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2} \right) \delta Q \quad (4.17)$$

This equation establishes the desired relationship between the additional charge product δQ required and the change in relative separation of the satellites. It is observed that this relation is coupled to the body frame yaw rate $\dot{\psi}$. The Coulomb tether attitude differential equations will be developed later using angular momentum expressions.

To develop a feedback law to control the separation distance using the Coulomb forces, the small charge product variation δQ is treated as a control variable. Because the charge of each craft causes a force along the relative position vector, the Coulomb charges can be used to control the spacecraft separation distance. By defining

$$\delta Q = \frac{m_1 m_2 L_{\text{ref}}^2}{(m_1 + m_2) k_c} (-C_1 \delta L - C_2 \delta \dot{L}) \quad (4.18)$$

the closed-loop separation distance dynamics become

$$\delta \ddot{L} + (C_1 - 9\Omega^2)\delta L + C_2 \delta \dot{L} - (2\Omega L_{\text{ref}})\dot{\psi} = 0 \quad (4.19)$$

This control law provides both proportional and derivative feedback of δL . Because the δL differential equation does not contain a damping term $\delta \dot{L}$, the inclusion of the derivative feedback is essential to ensure asymptotic convergence. Note that in the absence of the yaw

rate term $\dot{\psi}$, these closed-loop dynamics would be stable if $C_1 > 9\Omega^2$ and $C_2 > 0$. However, due to the coupling with the yaw (in-orbit-plane) rotation, the complete Coulomb tether motion must be analyzed for stability.

To implement this charge feedback control law, the spacecraft charges q_1 and q_2 must be determined. The value of Q_{ref} is determined through Eq. (6.5), while the value of δQ is given by the feedback law expression in Eq. (6.9). Thus, the spacecraft charges q_1 and q_2 must satisfy

$$q_1 q_2 = Q_{\text{ref}} + \delta Q \quad (4.20)$$

There are an infinite number of solutions to the above constraint. To keep the charges equal in magnitude across the craft, the following implementation was used.

$$q_1 = \sqrt{|Q_{\text{ref}} + \delta Q|} \quad (4.21)$$

$$q_2 = -q_1 \quad (4.22)$$

Note that here $Q_{\text{ref}} + \delta Q < 0$ because $\delta Q \ll Q_{\text{ref}}$ and $Q_{\text{ref}} < 0$. With this charging convention we find $q_1 > 0$ and $q_2 < 0$.

4.4 Stability Analysis Using the Gravity Gradient Torque

In this section the stability of both the Coulomb tether attitude (ψ, θ) and the separation distance L is analysed. The gravity gradient torque is included to exert an external torque onto the Coulomb tether. Let the orbit angular velocity vector relative to the inertial frame $\mathcal{N}$ be given by

$$\boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} = \Omega \hat{\boldsymbol{o}}_h \quad (4.23)$$

To develop the tether attitude differential equations of motion, the 2-craft formation is treated as a continuous body. This is motivated by the stable Coulomb tether formation acting as a rigid dumbbell spacecraft. The formation inertia matrix is expressed as²⁷

$$[I] = -m_1[\tilde{\boldsymbol{\rho}}_1][\tilde{\boldsymbol{\rho}}_1] - m_2[\tilde{\boldsymbol{\rho}}_2][\tilde{\boldsymbol{\rho}}_2] \quad (4.24)$$

where $[\tilde{\boldsymbol{\rho}}_1]$ is a skew-symmetric matrix that is equivalent to the vector cross product operator $\mathbf{a} \times \mathbf{b} \simeq [\tilde{\mathbf{a}}]\mathbf{b}$. For the 2-craft Coulomb tether formation, using the center of mass definition in Eq. (4.4), the inertia matrix is trivially given in the body frame $\mathcal{B}$ as

$${}^{\mathcal{B}}[I] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \quad (4.25)$$

where

$$I = \frac{m_1 m_2}{m_1 + m_2} L^2 \quad (4.26)$$

Note that these moments of inertia vary with time due to their dependence on the variable formation length L . The $\mathcal{B}$ -frame derivative of the inertia matrix is

$${}^{\mathcal{B}} \frac{d}{dt} [I] = {}^{\mathcal{B}} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \dot{I} & 0 \\ 0 & 0 & \dot{I} \end{bmatrix} \quad (4.27)$$

where

$$\dot{I} = 2 \frac{m_1 m_2}{m_1 + m_2} L \dot{L} = 2 \frac{m_1 m_2}{m_1 + m_2} (L_{\text{ref}} + \delta L) \delta \dot{L} \quad (4.28)$$

because $L_{\text{ref}} = \text{constant}$.

To develop the attitude differential equations, the total inertial angular momentum of the 2-craft formation is

$$\mathbf{H} = [I](\boldsymbol{\omega}_{\mathcal{B}/\mathcal{O}} + \boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}}) \quad (4.29)$$

Because the Coulomb control forces are formation internal forces, one finds that the inertial derivative of $\mathbf{H}$ is equal to the total external torque acting on the system. Euler's rotational equation of motion with a time-varying inertia matrix $[I]$ and gravity gradient torque vector $\mathbf{L}_G$ is given in body frame $\mathcal{B}$ components by

$${}^{\mathcal{B}}[I] {}^{\mathcal{B}}\dot{\boldsymbol{\omega}} + {}^{\mathcal{B}}[\dot{I}] {}^{\mathcal{B}}\boldsymbol{\omega} + {}^{\mathcal{B}}[\tilde{\boldsymbol{\omega}}] {}^{\mathcal{B}}[I] {}^{\mathcal{B}}\boldsymbol{\omega} = {}^{\mathcal{B}}\mathbf{L}_G \quad (4.30)$$

where ${}^{\mathcal{B}}\boldsymbol{\omega} = {}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$ and the notation $[\tilde{\boldsymbol{\omega}}]\mathbf{x} \equiv \boldsymbol{\omega} \times \mathbf{x}$ is used. Using the direction cosine matrix definition in Eq. (4.9), the orbit angular velocity vector can be written as

$${}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} = [BO] {}^{\mathcal{O}}\boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} = \begin{bmatrix} -\Omega \sin \theta \\ 0 \\ \Omega \cos \theta \end{bmatrix} \quad (4.31)$$

The yaw and pitch rates of the Coulomb tether body frame $\mathcal{B}$ relative to the orbit $\mathcal{O}$ frame yield

$${}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{O}} = \begin{bmatrix} -\sin \theta & 0 \\ 0 & 1 \\ \cos \theta & 0 \end{bmatrix} \begin{bmatrix} \dot{\psi} \\ \dot{\theta} \end{bmatrix} \quad (4.32)$$

The Coulomb tether body frame angular velocity vector relative to the inertial frame $\mathcal{N}$ is

$${}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} = {}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{O}} + {}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{O}/\mathcal{N}} = \begin{bmatrix} {}^{\mathcal{B}}[-\sin\theta\dot{\psi} - \Omega\sin\theta] \\ \dot{\theta} \\ \cos\theta\dot{\psi} + \Omega\cos\theta \end{bmatrix} \quad (4.33)$$

Linearizing Eq. (4.33) about small yaw and pitch angles, we get

$${}^{\mathcal{B}}\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}} \approx \begin{bmatrix} {}^{\mathcal{B}}[-\Omega\theta] \\ \dot{\theta} \\ \dot{\psi} + \Omega \end{bmatrix} \quad (4.34)$$

Taking the inertial derivative of this vector and noting that Ω is constant in this application, the $\mathcal{B}$ frame angular acceleration is

$${}^{\mathcal{B}}\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}} \approx \begin{bmatrix} {}^{\mathcal{B}}[-\Omega\dot{\theta}] \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} \quad (4.35)$$

The gravity gradient torque $\mathbf{L}_G$ also has to be expressed using the tether coordinates. The center of mass position vector $\mathbf{r}_c$, given in $\mathcal{O}$ frame components as

$$\mathbf{r}_c = \begin{bmatrix} {}^{\mathcal{O}}(r_c) \\ 0 \\ 0 \end{bmatrix} \quad (4.36)$$

is transformed to the $\mathcal{B}$ frame as

$$\mathbf{r}_c = \begin{pmatrix} r_{c1} \\ r_{c2} \\ r_{c3} \end{pmatrix}^{\mathcal{B}} = \begin{pmatrix} \cos \theta \cos \psi \\ -\sin \psi \\ \sin \theta \cos \psi \end{pmatrix}^{\mathcal{B}} r_c \quad (4.37)$$

Reference 27 provides the following expression for the gravity gradient torque:

$$\begin{bmatrix} L_{G1} \\ L_{G2} \\ L_{G3} \end{bmatrix}^{\mathcal{B}} = \frac{3GM_e}{r_c^5} \begin{bmatrix} r_{c2}r_{c3}(I_{33} - I_{22}) \\ r_{c1}r_{c3}(I_{11} - I_{33}) \\ r_{c1}r_{c2}(I_{22} - I_{11}) \end{bmatrix} \quad (4.38)$$

After substituting for r_{c_i} from Eq. (4.37) and using the known value of Ω from Kepler's equation, namely,

$$\frac{GM_e}{r_c^3} = \Omega^2 \quad (4.39)$$

the linearized gravity gradient torque vector acting on the Coulomb tether body frame is written as

$${}^{\mathcal{B}}\mathbf{L}_G \cong 3\Omega^2 \begin{bmatrix} 0 \\ -I\theta \\ -I\psi \end{bmatrix} \quad (4.40)$$

Substituting these results for $\mathbf{L}_G$, ${}^{\mathcal{B}}[\dot{I}]$, ${}^{\mathcal{B}}[I]$, $\boldsymbol{\omega}_{\mathcal{B}/\mathcal{N}}$ and $\dot{\boldsymbol{\omega}}_{\mathcal{B}/\mathcal{N}}$ back into Euler's rotational equations of motion in Eq. (4.30) and after simplifying the algebra, the resulting linearized attitude dynamics of the Coulomb tether body frame $\mathcal{B}$ are written along with the separation

distance differential equation as:

$$\ddot{\theta} + 4\Omega^2\theta = 0 \quad (4.41a)$$

$$\ddot{\psi} + \frac{2\Omega}{L_{\text{ref}}}\delta\dot{L} + 3\Omega^2\psi = 0 \quad (4.41b)$$

$$\delta\ddot{L} + C_2\delta\dot{L} - (2\Omega L_{\text{ref}})\dot{\psi} + (C_1 - 9\Omega^2)\delta L = 0 \quad (4.41c)$$

Thus, Eqs. (4.41a) – (4.41c) are the linearized equations of motion of the Coulomb tether body about the static nadir reference configuration. It should be noted that only the linearized δL differential equation was obtained using the Clohessy-Wiltshire-Hill equations, while the linearized differential equations of ψ and θ were derived from the full formation angular momentum expression along with Euler's equation. These equations have terms that depend on orbital rate Ω which happens to be a small value at GEO. In order to avoid numerical issues while carrying out numerical integrations, it is desired to have these equations be independent of Ω . This can be achieved by using the following transformation.

$$d\tau = \Omega dt \quad (4.42a)$$

$$(*)' = \frac{d(*)}{d\tau} = \frac{1}{\Omega} \frac{d(*)}{dt} \quad (4.42b)$$

By carrying out the above transformation in Eqs. (4.41a) – (5.5c), the orbit rate Ω independent linearized equations of motion of the Coulomb tether body are given by

$$\theta'' + 4\theta = 0 \quad (4.43a)$$

$$\psi'' + \frac{2}{L_{\text{ref}}}\delta L' + 3\psi = 0 \quad (4.43b)$$

$$\delta L'' + \tilde{C}_2\delta L' - (2L_{\text{ref}})\psi' + (\tilde{C}_1 - 9)\delta L = 0 \quad (4.43c)$$

where $\tilde{C}_2 = (C_2/\Omega)$ and $\tilde{C}_1 = (C_1/\Omega^2)$ are non-dimensionalized feedback gains. It can be observed from these equations that the out-of-plane motion $\theta(t)$ is decoupled and its equation is that of a simple oscillator. This decoupling is analogous to what occurs with the linearized rigid body attitude dynamics subject to a gravity gradient torque. Because the $\theta(t)$ motion is not coupled to the tether charge product term δQ , or the separation distance variation δL , it is not possible to control the pitch motion θ with the Coulomb charge in this linearized analysis. The yaw motion $\psi(t)$ is coupled with the $\delta L(t)$ motion in the form of a driving force, which may make it amenable to asymptotic stabilization by controlling the charge.

The values of gain $\tilde{C}_1$ and $\tilde{C}_2$ can be tuned to meet the stability requirements using Routh-Hurwitz stability criterion. The characteristic equation for the coupled δL and ψ equations is

$$\lambda^4 + \tilde{C}_2\lambda^3 + (\tilde{C}_1 - 2)\lambda^2 + 3\tilde{C}_2\lambda + 3(\tilde{C}_1 - 9) = 0 \quad (4.44)$$

While the linearized closed-loop dynamics do depend on the Coulomb tether reference length L_{ref} , note that the characteristic equation does not. To ensure asymptotic stability, roots of this equation should have negative real parts. The constraints on the gains $\tilde{C}_1$ and $\tilde{C}_2$ for meeting this condition are identified by constructing a Routh table and are found to be

$$\tilde{C}_1 > 9 \quad (4.45a)$$

$$\tilde{C}_2 > 0 \quad (4.45b)$$

Incidentally, these constraints also ensure the stability of δL equation ignoring the ψ' term.

The stability criterion imposes constraints on the choice of the feedback gains $\tilde{C}_1$, $\tilde{C}_2$ but is not enough to actually decide their values. One needs to look for alternate criteria

for fixing them. One satisfying way would be to fix the gains by demanding conditions of critical or near critical damping. For ease of discussion, let the feedback gains be expressed in terms of scaling factor n and α , both taken as positive and real. The gains can be rewritten as

$$\tilde{C}_1 = n > 9 \quad (4.46)$$

and

$$\tilde{C}_2 = \alpha\sqrt{n-9} \quad (4.47)$$

The natural frequency of the ψ equation is $\sqrt{3}$ and is not affected by the choice of $\tilde{C}_1$ and $\tilde{C}_2$, whereas the natural frequency for δL equation is $\sqrt{(n-9)}$. The value of $n = 12$ will match these frequencies making the ψ' coupling term in δL equation serve as defacto damping term. A similar remark applies to the ψ equation. In Eq. (4.47), $\alpha = 2$ ensures that the δL equation without the ψ' term is critically damped. For effective damping with the inclusion of ψ' term, the value of α and n need to be modified. However, one expects the value of α to be in the vicinity of $\alpha = 2$ and n to be around 12. Hence, root locus plots for the coupled δL and ψ equations are studied with a range of α values in the vicinity of $\alpha = 2$ with n varying between 9 and 20. Figure 4.4 shows the root locus plots for two different α values. Studying the root locus plots, it can be observed that as the n value increases beyond 12 the rate of convergence of one of the modes increases and the other decreases. Therefore, $n = 12$ is ideal for ensuring good rates of convergence for both the modes. It is also noted that $\alpha = 2.28$ resulted in effective damping for the modes.

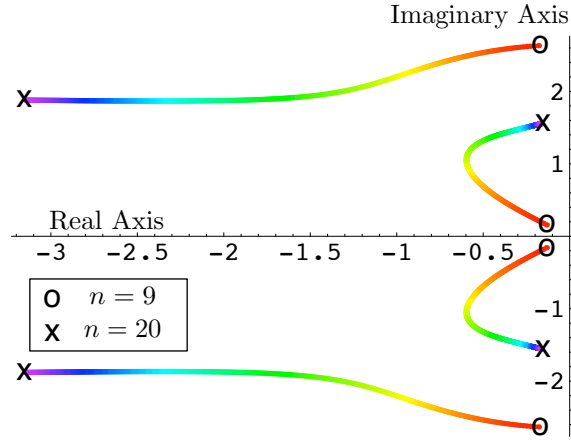
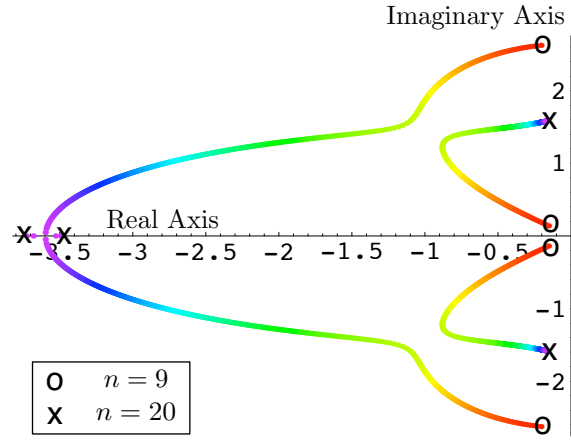
(a) Gain $\alpha = 2.0$ (b) Gain $\alpha = 2.28$

Figure 4.4: Root-locus plot of the linearized spherical coordinate differential equations for different gain α values.

4.5 Numerical Simulation

A numerical simulation is presented to illustrate the performance and stability of a 25 meter Coulomb tether formation. The simulation parameters are listed in Table 4.1. The initial

attitude values are set to $\psi = 0.1$ radians and $\theta = 0.1$ radians. The separation length error (Coulomb tether length error) is $\delta L = 0.5$ meters. All initial rates are set to zero through $\dot{\psi} = \delta \dot{L} = \dot{\theta} = 0$.

The choice of values for the gains $\tilde{C}_1$ and $\tilde{C}_2$ should not only satisfy the stability criterion mentioned in Eq. (4.45) but also should be such as to lead to near-ideal damping. Studying the root locus plots where the parameters n and α are varied, the values $n = 12$ and $\alpha = 2.28$ were chosen. Hence, using Eq. (4.47) the gain $\tilde{C}_2$ was found to be $2.28\sqrt{3}$.

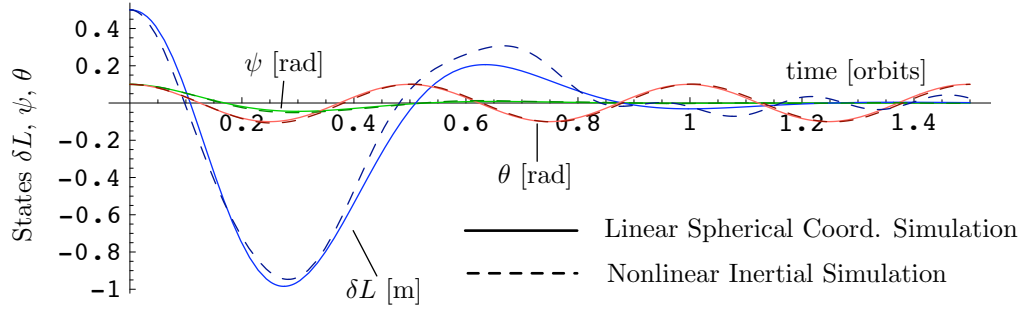
Table 4.1: Input parameters used in orbit-radial simulation

Parameter	Value	Units
m_1	150	kg
m_2	150	kg
L_{ref}	25	m
k_c	8.99×10^9	$\frac{\text{Nm}^2}{\text{C}^2}$
Q_{ref}	-2.07911	μC^2
Ω	7.2915×10^{-5}	rad/sec
n	12	
α	2.28	
$\delta L(0)$	0.5	m
$\psi(0)$	0.1	rad
$\theta(0)$	0.1	rad

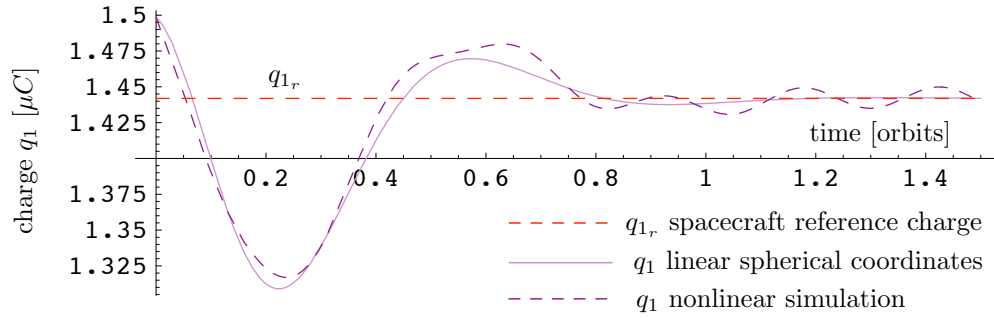
The Coulomb tether performance is simulated in two different manner. First the linearized spherical coordinate differential equations are integrated. This simulation illustrates the linear performance of the charge control. Second, the linearized results are compared with those obtained from the exact nonlinear equation of motion of the deputy satellites given by

$$\ddot{\mathbf{r}}_1 + \frac{\mu}{r_1^3} \mathbf{r}_1 = \frac{k_c}{m_1} \frac{Q}{L^3} (\mathbf{r}_1 - \mathbf{r}_2) \quad (4.48a)$$

$$\ddot{\mathbf{r}}_2 + \frac{\mu}{r_2^3} \mathbf{r}_2 = \frac{k_c}{m_2} \frac{Q}{L^3} (\mathbf{r}_2 - \mathbf{r}_1) \quad (4.48b)$$



(a) Time histories of length variations δL , in-plane rotation angle ψ , and out-of-plane rotation angle θ .



(b) Spacecraft charge time histories

Figure 4.5: Simulation results of integrating either the linearized spherical coordinates differential equations (solid lines) or the nonlinear inertial coordinate differential equations (dashed lines).

where $\mathbf{r}_1 = \mathbf{r}_c + \boldsymbol{\rho}_1$ and $\mathbf{r}_2 = \mathbf{r}_c + \boldsymbol{\rho}_2$ are the inertial position vectors of the masses m_1 and m_2 , while $L = \sqrt{(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{r}_2 - \mathbf{r}_1)}$. The gravitational coefficient μ is defined as $\mu \approx GM_e$. After integrating the motion using inertial Cartesian coordinates, the separation distance L , as well as the in-plane and out-of-plane angles ψ and θ , are computed in post-processing using the exact kinematic transformation. The Debye length is kept at zero during this simulation to study in detail the effects of the relative motion linearization.

Figure 4.5(a) shows the Coulomb tether motion in the linearized spherical coordinates

$(\psi, \theta, \delta L)$, along with the full nonlinear spherical coordinates shown as dashed lines. With the presented charge feedback law, both the yaw motion ψ and the separation distance deviation δL converged to zero. By stabilizing the δL state to zero, the in-plane rotation $\psi(t)$ also converges to zero. For the set of initial conditions used in this simulation, the δL and ψ states have converged after about 0.9 orbits. As expected, the pitch motion $\theta(t)$ is a stable sinusoidal motion. Further, Figure 4.5(a) shows that the nonlinear simulation closely follows the linearized simulation. However, there is one notable difference. The δL states converge to zero asymptotically in the linearized simulation, while they achieve a steady-state oscillation in the nonlinear simulation. This difference in behaviour occurs because the same reference charge product Q_{ref} (computed using Eq. (6.5)) is used in both simulations. This charge will achieve a static formation in the linearized CW equations. However, this charge value will not achieve a static formation in the nonlinear problem. Thus, the charge feedback control is not actually operating about a proper steady-state charge of the nonlinear problem. As the δL and ψ tracking errors go to zero, the orbital dynamics will perturb the system and cause these states to grow again. This persistent disturbance results in the final steady-state oscillations shown. To implement such a control strategy for an actual mission, the Q_{ref} value would be recomputed numerically for the nonlinear problem. Even with this deviation, the nonlinear and linear performance predictions compare very well, thus verifying the presented linearization results.

Figure 4.5(b) shows the spacecraft control charge q_1 for both the linearized and full nonlinear simulation models. Both converge to the reference value pertaining to the static equilibrium. As defined, the control charge q_2 is just the negative of q_1 . Note that the deviation from the value of reference charges is small, justifying the linearization assumptions used. The magnitude of the control charges is in the order of micro-Coulomb which is easily realizable in practice using charge emission devices.

4.6 Summary

The concept of a Coulomb (electrostatic) tether is introduced to bind two satellites in a near-rigid formation. While the Coulomb force cannot directly stabilize the attitude, the gravity gradient torque is exploited to stabilize the Coulomb tether formation about the orbit radial direction. The formulation allows for unequal masses. The analysis is based on a linearized dynamics and charge behavior model whose validity is also shown. It was observed that a linear charge feedback law in terms of separation distance errors and separation rate is adequate for stabilizing the separation distance and in-plane angular motion. The control charges needed are small in the order of micro-Coulombs and realizable in practice.

5 Orbit Normal and Along-Track Two-Craft Coulomb Tethers

5.1 Introduction

The previous chapter introduces the concept of a Coulomb tether.^{30,31} Here a conventional mechanical tether cable connecting two craft is replaced by an electrostatic force which acts as a virtual tether. Conventional tethers are limited to tensile forces whereas Coulomb tethers allow both tensile and compressive forces. However, while traditional spacecraft tether missions consider very large separation distances of multiple kilometers, the Coulomb tether concept is only viable for separation distances up to about 100 meters because of the electrical field strength drop off. The previous chapter studies the stabilization of the simple nadir-aligned static 2-craft Coulomb tether structure. Compared to the previous works on static Coulomb structures,^{2,8,11,13} Reference 30 is the first study to introduce a charge feedback law to stabilize a charged spacecraft cluster to a specific shape and orientation. Coulomb forces are inter-spacecraft forces and cannot control the inertial angular momentum of the formation. Hence, stability characteristics of orbital rigid body motion under a differential gravity field are applied to a Coulomb tethered two-spacecraft system to develop an active charge feedback control. With this control the spacecraft separation distance is maintained at a fixed value, while the coupled formation gravity gradient torque is exploited

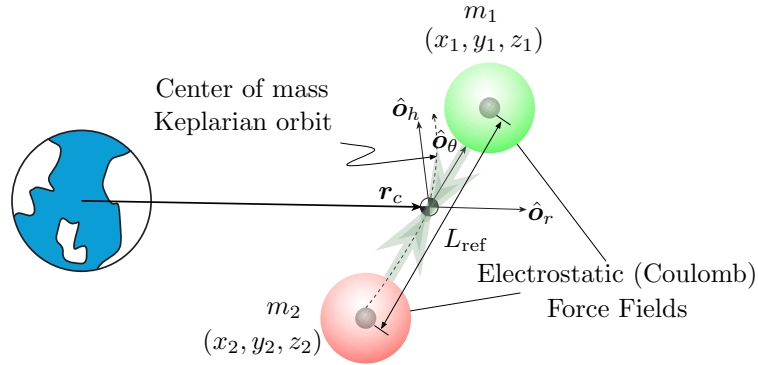


Figure 5.1: Static coulomb tether formation aligned with along-track direction.

to stabilize the tether attitude about the orbit radial direction. Gravity gradient rigid satellites or conventional tethers have only bounded stability along the orbit radial direction.²⁷ In comparison, the feedback control laws for the Coulomb tether regulation³⁰ problem in the previous chapter guarantee asymptotic stability for separation distance and in-plane angle. This asymptotic stability is achieved by exploiting the charged relative motion of the spacecraft and varying the separation distance (virtual tether length).

Similar to the study of rigid axially symmetric body under the influence of the gravity gradient torque, we know that there are two other relative equilibriums of the charged 2-craft problem other than the orbit radial or nadir direction. These equilibriums are along the orbit normal direction, and the along-track direction¹¹ shown in Figure 5.1. In particular, zero tension is required between the two crafts aligned with the along-track direction to maintain the static unperturbed formation. On the other hand, repulsive forces are required to maintaining the cluster along the orbit normal direction. It is worth noting that both zero tension and compression cases considered are not possible with conventional cable tethers.

This chapter studies the stability of a two-craft formation about along-track and orbit-normal relative equilibrium configurations. A feedback control law is introduced to asymp-

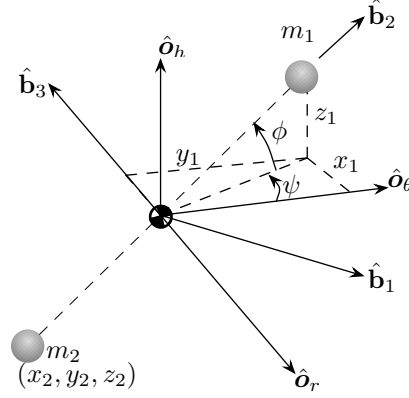


Figure 5.2: (3-1) Euler angles describing the Coulomb tether orientation for the along-track relative equilibria

totically stabilize both the shape and orientation of this cluster. While the charged two-craft formation aligned along the orbit radial direction could stabilize the cluster using only Coulomb forces, this study investigates a hybrid feedback control strategy where both conventional thrusters and Coulomb forces are used. The goal is to use the thrusters as little as possible and make the Coulomb forces provide the bulk of the actuation requirement. However, to employ small-force thrusters like ion-engines in close proximity to other spacecraft, great care must be taken that the thruster exhaust plume does not impinge on the neighboring craft. These plumes can be caustic and cause damage to on-board sensors. The control strategy must be designed such that the thruster is never directed at the 2nd craft.

The formation is studied at GEO where the Debye lengths are large enough to consider Coulomb spacecraft missions. Reference 14 establishes that the differential solar drag is the largest disturbance acting on a Coulomb formation at GEO. Therefore, the effects of differential solar drag on the formation and the ability of the controller to withstand this disturbance are also studied.

5.2 Charged Relative Equations of Motion

5.2.1 Along-Track Configuration

This section derives the equations of motion of a 2-craft Coulomb tether that is nominally aligned with the along-track direction $\hat{\mathbf{o}}_\theta$ of the orbit or Hill frame $\mathcal{O} : \{\hat{\mathbf{o}}_r, \hat{\mathbf{o}}_\theta, \hat{\mathbf{o}}_h, \}$ shown in Figure 5.1. This derivation closely follows the derivation of the equations of motion for craft aligned along the orbit radial direction, which is given in detail in Reference 30. Figure 5.1 illustrates a static two-craft formation in the orbit velocity direction with a separation distance of L_{ref} . Let $Q = q_1 q_2$ be the charge product of the spacecraft charges q_i . The reference charge product Q_{ref} required to maintain this static formation can be computed using the Clohessy-Wiltshire-Hill's equations^{27–29} for charged spacecraft. The analytical expression of Q_{ref} for the along-track equilibrium is written as¹³

$$Q_{\text{ref}} = 0 \quad (5.1)$$

The required relative equilibrium charge is zero because this Coulomb tether configuration is equivalent to a lead-follower spacecraft formation. As a consequence the necessary Coulomb tether tension is zero. However, this static equilibrium is unstable, similar to a rigid rod being unstable if aligned with $\hat{\mathbf{o}}_\theta$. The separation distance instability can be stabilized by continuously varying the charges and generating positive or negative tension within the Coulomb tether.

Of interest are the coupled separation distance dynamics and the orientation of the Coulomb tether. Consider the perturbed satellite 1 position (x_1, y_1, z_1) relative to the equilibrium position. The Coulomb tether is only a 1-dimensional structure and thus only requires the $(3 - 1)$ Euler angles (ψ, ϕ) to define its orientation relative to the orbit frame $\mathcal{O}$ (Hill frame). The virtual Coulomb structure body frame $\mathcal{B} : \{\hat{\mathbf{b}}_1, \hat{\mathbf{b}}_2, \hat{\mathbf{b}}_3, \}$ is defined such

that $\mathcal{B} = \mathcal{O}$ for zero ψ and ϕ angles, while $\hat{\mathbf{b}}_2$ tracks the tether heading. Rotations about $\hat{\mathbf{b}}_2$ (θ) can be neglected with point mass assumption of the craft. The Euler angles are illustrated in Figure 5.2. Following the same steps as in Reference 30, the differential equation of motion for the charged separation distance is given by

$$\ddot{L} = 2\Omega\dot{\psi}L + \frac{k_c}{m_1}Q\frac{1}{L^2}\frac{m_1 + m_2}{m_2} \quad (5.2)$$

Next the separation distance equations of motion are linearized about small variations in length δL and small variations in the product charge term δQ . The fixed reference separation length L_{ref} is determined by the mission requirement. The reference charge product term for this along-track configuration is known to be zero from Eq. (5.1). The separation distance L and charge product Q are given by

$$L = L_{\text{ref}} + \delta L \quad (5.3a)$$

$$Q = Q_{\text{ref}} + \delta Q \quad (5.3b)$$

Note that these developments treat the required changes in the charge product δQ as the control variable. Substituting these definitions of L and Q into Eq. (5.2) and linearizing leads to

$$\delta\ddot{L} = (2\Omega L_{\text{ref}})\dot{\psi} + \left(\frac{k_c}{m_1} \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2} \right) \delta Q \quad (5.4)$$

Note that this relationship is coupled to the angular in-orbit-plane rate $\dot{\psi}$. In order to obtain an expression for this rate, a stability analysis using the gravity gradient is employed. The derivation of the expression for angular perturbation closely follows the derivation given in Reference 30 for the orbit radially aligned Coulomb tether. The linearized attitude dynamics of the Coulomb tether body frame are written along with the separation distance equation

as:

$$\ddot{\phi} + \Omega^2 \phi = 0 \quad (5.5a)$$

$$\ddot{\psi} + 2\frac{\Omega}{L_{\text{ref}}}\delta\dot{L} - 3\Omega^2\psi = 0 \quad (5.5b)$$

$$\delta\ddot{L} - (2\Omega L_{\text{ref}})\dot{\psi} - \left(\frac{k_c}{m_1}\frac{1}{L_{\text{ref}}^2}\frac{m_1 + m_2}{m_2}\right)\delta Q = 0 \quad (5.5c)$$

Note that the out-of-plane angle ϕ is decoupled from the separation distance error δL and in-plane angle ψ . Further, the linearized ϕ motion is that of a marginally stable linear oscillator.

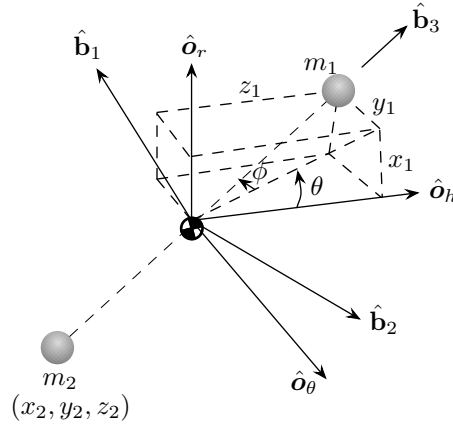


Figure 5.3: (2-1) Euler angles describing the Coulomb tether orientation for the orbit normal relative equilibria

5.2.2 Orbit Normal Configuration

The derivation of the equations of motion for a two-craft Coulomb tether along orbit normal direction follows the same steps as those of the along-track equilibrium. The analytical

expression for the orbit normal relative equilibria charge product Q_{ref} is written as¹³

$$Q_{\text{ref}} = q_1 q_2 = \Omega^2 \frac{L_{\text{ref}}^3}{k_c} \frac{m_1 m_2}{m_1 + m_2} \quad (5.6)$$

Note that $Q_{\text{ref}} > 0$, which requires a repulsive Coulomb force to establish this charged equilibrium. A physical structure in this orientation must compensate for compressive forces, a task conventional tethers are incapable of achieving.

Again, consider small deviations about the equilibrium position and let the $(2 - 1)$ Euler angles (θ, ϕ) represent the tether body frame $\mathcal{B}$ attitude with respect to the orbit frame $\mathcal{O}$. Here the axis $\hat{\mathbf{b}}_3$ tracks the orientation of the orbit-normal tether configuration. The Euler angles are illustrated in Figure 5.3. Note these angle definitions reflect rotations about the same body axes $\hat{\mathbf{b}}_i$ as in the along-track description. However, their zero values are offset by 90 degrees to reflect the different nominal tether orientation.

The differential equation for the separation distance is given by

$$\ddot{L} = -\Omega^2 L + \frac{k_c}{m_1} Q \frac{1}{L^2} \frac{m_1 + m_2}{m_2} \quad (5.7)$$

We can observe that the separation distance differential equation in Eq. (5.7) is decoupled from both the orientation angles θ and ϕ . The above equation can be further linearized using Eqs. (5.3) and the Q_{ref} definition in Eq. (5.6) to

$$\delta \ddot{L} = -(3\Omega^2) \delta L + \left(\frac{k_c}{m_1} \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2} \right) \delta Q \quad (5.8)$$

The differential equation for Euler angles can be obtained similar to the along-track development. The linearized attitude dynamics of the Coulomb tether are written along with

the separation distance equation as:

$$\ddot{\phi} - \Omega^2 \phi - 2\Omega \dot{\theta} = 0 \quad (5.9a)$$

$$\ddot{\theta} - 4\Omega^2 \theta + 2\Omega \dot{\phi} = 0 \quad (5.9b)$$

$$\delta \ddot{L} + (3\Omega^2) \delta L - \left(\frac{m_1 + m_2}{m_1 m_2} \frac{k_c}{L_{\text{ref}}^2} \right) \delta Q = 0 \quad (5.9c)$$

Note both the out-of-plane angles θ and ϕ are coupled, while the charged separation distance error dynamics is uncoupled in this linearized formulation. Also, one can observe from Eq. (5.9c) that the separation distance error (δL) is already marginally stable even without any feedback control through the charge product error term (δQ).

5.3 Hybrid Feedback Control Development

5.3.1 Along-Track Configuration

In this section, we investigate the stability of the linearized along-track equations of motion given by Eq. (5.5) and develop a hybrid feedback control law that stabilizes the system. Reading Eq. (5.5) it is clear that the out-of-plane angle ϕ is fully decoupled from the in-plane angle ψ and separation distance error δL . The equation of motion for the out-of-plane angle ϕ represents a stable simple harmonic oscillator. Next, consider the coupled in-plane angle ψ and separation distance error δL equations of motion given in Eqs. (5.5b)–(5.5c). The charges on the craft can be used to control the separation distance since they cause an electrostatic force along the relative position vector. The charge product variation δQ is treated as the control variable and the feedback control law is defined as

$$\delta Q = \frac{m_1 m_2 L_{\text{ref}}^2}{(m_1 + m_2) k_c} (-C_1 \delta L - C_2 \delta \dot{L}) \quad (5.10)$$

Here C_1 and C_2 are the position and velocity gains, respectively. Thus, the closed loop equations of motion for the coupled ψ and δL system are written as

$$\ddot{\psi} + 2\frac{\Omega}{L_{\text{ref}}}\delta\dot{L} - 3\Omega^2\psi = 0 \quad (5.11a)$$

$$\delta\ddot{L} - (2\Omega L_{\text{ref}})\dot{\psi} + C_1\delta L + C_2\delta\dot{L} = 0 \quad (5.11b)$$

The in-plane angle ψ is coupled with the δL in the form of a driving force ($2\frac{\Omega}{L_{\text{ref}}}\delta\dot{L}$). Hence we select the gains C_1 and C_2 using the Routh-Hurwitz stability criterion to asymptotically stabilize both δL and ψ . The characteristic equation for the equations given in Eq. (5.11) is

$$\lambda^4 + C_2\lambda^3 + (C_1 + \Omega^2)\lambda^2 + (-3C_2\Omega^2)\lambda + (-3C_1\Omega^2) = 0 \quad (5.12)$$

In order to ensure asymptotic stability, the real parts of the roots of this characteristic polynomial should be negative definite. The constraints on the gains that will guarantee negative definite roots can be identified by constructing a Routh table and are found to be

$$C_2 > 0 \quad (5.13a)$$

$$C_1 + 4\Omega^2 > 0 \quad (5.13b)$$

$$\frac{-12C_2\Omega^4}{C_1 + 4\Omega^2} > 0 \quad (5.13c)$$

There are no real values for gain C_1 and C_2 that will satisfy all three conditions given in Eq. (5.13). Hence, the coupled system can not be stabilized with only the Coulomb forces. In addition to the Coulomb forces, we require some thrust forces acting on both satellites along the $\hat{b}_1$ axis that stabilize the in-plane angle ψ . These thrust forces can be modeled as equal and opposite forces with magnitude F_1 . The thrust force magnitude is the second

control variable with in-plane angle ψ feedback and it is defined as

$$F_1 = \frac{m_1 m_2}{m_1 + m_2} L_{\text{ref}}(K_1 \psi) \quad (5.14)$$

where K_1 is the in-plane angle feedback gain. These forces introduce a net torque in the ψ equation and the modified coupled equations of motion are written as

$$\ddot{\psi} + 2\frac{\Omega}{L_{\text{ref}}}\delta\dot{L} + (K_1 - 3\Omega^2)\psi = 0 \quad (5.15a)$$

$$\delta\ddot{L} - (2\Omega L_{\text{ref}})\dot{\psi} + C_1\delta L + C_2\delta\dot{L} = 0 \quad (5.15b)$$

The characteristic equation for the equations given in Eq. (5.15) is

$$\lambda^4 + C_2\lambda^3 + (C_1 + K_1 + \Omega^2)\lambda^2 + (C_2K_1 - 3C_2\Omega^2)\lambda + (C_1K_1 - 3C_1\Omega^2) = 0 \quad (5.16)$$

The constraints on the gains to ensure asymptotic stability are found using the Routh table to be

$$C_2 > 0 \quad (5.17a)$$

$$C_1 > -4\Omega^2 \quad (5.17b)$$

$$K_1 > 3\Omega^2 \quad (5.17c)$$

The constraints given in Eq. (5.17) guarantee asymptotic stability, but we need other criteria for fixing their values to yield a satisfactory performance. One way of looking at the problem is to consider the δL equation without the $\dot{\psi}$ term. For ease of discussion, let us rewrite the

position and velocity gains in terms of scaling factors n_1 and α_1 as

$$C_1 = n_1 \Omega^2 > -4\Omega^2 \quad (5.18)$$

$$C_2 = \alpha_1 \sqrt{n_1} \Omega \quad (5.19)$$

The δL equation without the $\dot{\psi}$ term is critically damped with $\alpha_1 = 2$. The value of α_1 needs to be altered for achieving near critical damping for the complete δL equation with the $\dot{\psi}$ term. The in-plane angle gain is also rewritten in terms of a scaling factor n_2 as

$$K_1 = n_2 \Omega^2 > 3\Omega^2 \quad (5.20)$$

The natural frequency of the ψ and δL equations are $\sqrt{n_2 - 3}\Omega$ and $\sqrt{n_1}\Omega$, respectively. If n_1 and n_2 are chosen in such a way that these frequencies match, then the $\delta \dot{L}$ term in the ψ equation will act as a defacto damping term, and $\dot{\psi}$ term will damp the δL equation. The value of n_2 is chosen as 6, and this results in a settling time of about 1 day (1 cycle). For this fixed value of n_2 , the root locus for the coupled δL and ψ equations is studied for a range of α_1 values in the vicinity of $\alpha_1 = 2$, with n_1 varying from 0.1 to 20. Based on visual observation of the root locus plots the scaling factors are chosen to be $\alpha_1 = 2.3$ and $n_1 = 2.97$. Figure 5.4 shows the root locus plot for $n_2 = 6$ and $\alpha_1 = 2.3$, with n_1 varying from 0.1 to 20.

As discussed earlier the equation of motion for the out-of-plane angle ϕ represents a simple harmonic oscillator. This out-of-plane angle can be asymptotically stabilized by using an equal and opposite thrust force on both the satellites along the $\hat{b}_3$ axis. The thrust force magnitude F_3 is the third control variable with $\dot{\phi}$ feedback and it is defined as

$$F_3 = \frac{m_1 m_2}{m_1 + m_2} L_{\text{ref}}(K_2 \dot{\phi}) \quad (5.21)$$

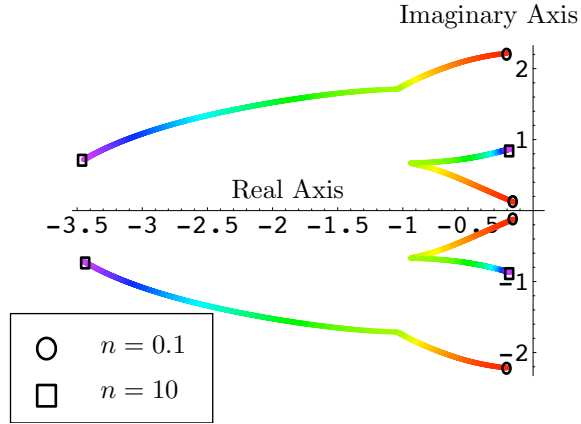


Figure 5.4: Root Locus Plot for Along-Track Configuration with $n_2 = 6$ and $\alpha_1 = 2.3$.

where K_2 is the out-of-plane angle feedback gain. These forces introduce a net torque in the ϕ equation and the modified equation of motion are written as

$$\ddot{\phi} + \Omega^2 \phi + K_2 \dot{\phi} = 0 \quad (5.22)$$

Critical damping is achieved with $K_2 = 2\Omega$. Figure 5.5 illustrates the thrusters in action along the $\hat{b}_1$ and $\hat{b}_3$ axes for the along-track configuration. The thrusting force F_1 is acting along the positive $\hat{b}_1$ direction and the force F_3 is acting along the negative $\hat{b}_3$ direction for the satellite 1. The direction of these forces are in reverse for the satellite 2. Note all thruster forces are directed in orthogonal directions to the cluster line of sight vector ($\hat{b}_2$) and thereby avoid any potential plume exhaust impingement issues.

5.3.2 Orbit Normal Configuration

Unlike the along-track configuration, the equation of motion of the separation distance error δL are decoupled from the angles in the orbit normal configuration. The equations of

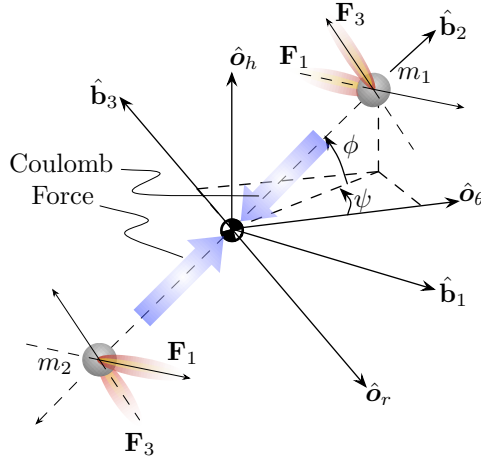


Figure 5.5: Figure Illustrating the Thrusters Along $\hat{b}_1$ and $\hat{b}_3$ Axes for Along-Track Configuration.

motion of the two out-of-plane angles θ and ϕ are coupled instead. Therefore, the linearized Coulomb forces can be used to stabilize only the separation distance and some thrust force is needed to stabilize the angles. From Eq. (5.9c), it is clear that without the charge product variation (δQ) term the δL equation of motion about the charged orbit-normal equilibrium represents a stable simple harmonic oscillator. In order to make δL equation of motion asymptotically stable a separation distance error rate ($\delta \dot{L}$) feedback through the control variable δQ is sufficient. But here we also introduce a separation distance error (δL) feedback which enables us to control the natural frequency and thereby the settling time. The feedback control law is given as

$$\delta Q = \frac{m_1 m_2 L_{\text{ref}}^2}{(m_1 + m_2) k_c} (-C_1 \delta L - C_2 \delta \dot{L}) \quad (5.23)$$

where $C_1 > -3\Omega^2$ and $C_2 > 0$ are the position and velocity feedback gain, respectively. Now, the closed loop separation distance error equation is written as

$$\delta\ddot{L} + (3\Omega^2 + C_1)\delta\dot{L} + C_2\delta L = 0 \quad (5.24)$$

Fixing $C_2 = 2\sqrt{3\Omega^2 + C_1}$ makes the separation distance equation critically damped.

The coupled out-of-plane angles can be stabilized by using thrust forces on both the satellites. One set of equal and opposite forces with magnitude F_1 acts along the $\hat{b}_1$ axis. The other set of forces with magnitude F_2 acts along the $\hat{b}_2$ axis. The feedback control laws for the thrust force magnitudes are defined as

$$F_1 = \frac{m_1 m_2}{m_1 + m_2} L_{\text{ref}}(K_2 \theta) \quad (5.25)$$

$$F_2 = \frac{m_1 m_2}{m_1 + m_2} L_{\text{ref}}(K_1 \phi + K_3 \dot{\phi}) \quad (5.26)$$

where K_1 and K_3 are the angle and angle rate gains for ϕ , and K_2 is the angle gain for θ . It should be noted that the thrust forces F_1 and F_2 stabilize the out-of-plane angles θ and ϕ , respectively. Further, these forces too only act orthogonal to the line of sight vector of the 2 craft, thus avoiding plume impingement issues. These forces introduce torque into the angular equations of motion and the augmented coupled closed loop equations are

$$\ddot{\phi} - 2\Omega\dot{\theta} + (K_1 - \Omega^2)\phi + K_3\dot{\phi} = 0 \quad (5.27a)$$

$$\ddot{\theta} + (K_2 - 4\Omega^2)\theta + 2\Omega\dot{\phi} = 0 \quad (5.27b)$$

The characteristic equation of the coupled equations of motion given in Eq. (5.27) is

$$\begin{aligned} \lambda^4 + K_3\lambda^3 + (K_1 + K_2 - \Omega^2)\lambda^2 + (K_2K_3 - 4K_3\Omega^2)\lambda \\ + (K_1K_2 - 4K_1\Omega^2 - K_2\Omega^2 + 4\Omega^2) = 0 \end{aligned} \quad (5.28)$$

The characteristic equation should have roots with negative real parts to guarantee asymptotic stability. The Routh-Hurwitz criterion can be used to establish the constraints on the gains that will result in the characteristic equation given in Eq. (5.28) to have negative define roots. The constraints on the gains are

$$K_1 > \Omega^2 \quad (5.29a)$$

$$K_2 > 4\Omega^2 \quad (5.29b)$$

$$K_3 > 0 \quad (5.29c)$$

Before we proceed to establish the value of the gains, it is important to note that without the $\dot{\phi}$ feedback the characteristic equation would have been

$$\lambda^4 + (K_1 + K_2 - 2\Omega^2)\lambda^2 + (K_1K_2 - 4K_1\Omega^2 - K_2\Omega^2 + 4\Omega^2) = 0 \quad (5.30)$$

and one can come up with gains that will only guarantee marginal stability, but not convergence. This justifies the use of angle rate ($\dot{\phi}$) feedback for achieving asymptotic stability.

The gain values are fixed in such a way that they guarantee near critical damping. The gains K_1 and K_3 are rewritten in terms of scaling factors n and α as

$$K_1 = n\Omega^2 > \Omega^2 \quad (5.31)$$

$$K_3 = \alpha\sqrt{(n-1)}\Omega \quad (5.32)$$

In the ϕ equation of motion, $\alpha = 2$ guarantees critical damping if one ignores the $\dot{\theta}$ term. For fixed values of $K_2 > 4\Omega^2$, the root locus for the coupled θ and ϕ equations is studied for a range of α values in the vicinity of $\alpha = 2$ with n varying from 1.1 to 10. Based on visual observation of the root locus plots the gain K_2 is chosen to be $5\Omega^2$ and the scaling factors

are chosen to be $\alpha = 2.5$ and $n = 2.7$. Figure. 5.6 shows the root locus plot for $K_2 = 5\Omega^2$ and $\alpha = 2.5$, with n varying from 1.1 to 10.

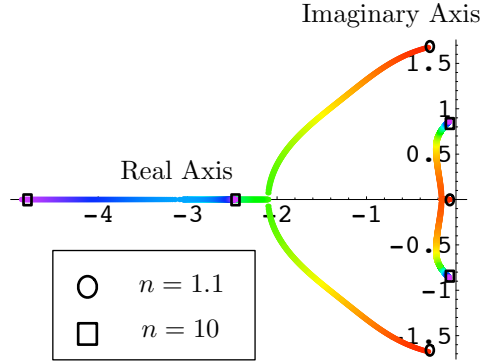


Figure 5.6: Root Locus Plot for Orbit Normal Configuration with $K_2 = 5\Omega^2$ and $\alpha = 2.5$

5.4 Numerical Simulation

This section presents numerical simulations of the along-track and orbit normal Coulomb tether formations to illustrate the performance and stability of the presented hybrid feedback control strategy. The Coulomb tether performance is simulated in two different manners. First the linearized spherical coordinate differential equations are integrated. This simulation illustrates the linear performance of the charge control. Second, the linearized results are compared with those obtained from the exact nonlinear equation of motion of the deputy satellites given by

$$\ddot{\mathbf{r}}_1 + \frac{\mu}{r_1^3} \mathbf{r}_1 = \frac{k_c}{m_1} \frac{Q}{L^3} (\mathbf{r}_1 - \mathbf{r}_2) \quad (5.33a)$$

$$\ddot{\mathbf{r}}_2 + \frac{\mu}{r_2^3} \mathbf{r}_2 = \frac{k_c}{m_2} \frac{Q}{L^3} (\mathbf{r}_2 - \mathbf{r}_1) \quad (5.33b)$$

where $\mathbf{r}_1 = \mathbf{r}_c + \boldsymbol{\rho}_1$ and $\mathbf{r}_2 = \mathbf{r}_c + \boldsymbol{\rho}_2$ are the inertial position vectors of the masses m_1 and m_2 , while $L = \sqrt{(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{r}_2 - \mathbf{r}_1)}$. The gravitational coefficient μ is defined as $\mu \approx GM_e$. After integrating the motion using inertial Cartesian coordinates, the separation distance L , as well as the corresponding angles are computed in post-processing using the exact kinematic transformation. Finally, the robustness of the control laws is illustrated in the presence of differential solar perturbation. For all cases the cluster center of mass is assumed to be a GEO orbit.

5.4.1 Along-Track Configuration

The along-track Coulomb tether with a separation distance of 25 meter is simulated first. The input parameters are given in Table 5.1. The initial separation distance error (δL) is set to 0.5 meter and the Euler angles are set to $\psi = 0.1$ radians and $\phi = 0.1$ radians. All initial rates are set to zero through $\dot{\psi} = \delta \dot{L} = \dot{\phi} = 0$. As discussed in the previous section, the gain values are chosen, based on studying the root locus plot, to be $C_1 = 2.97\Omega^2$, $C_2 = 3.9637\Omega$, $K_1 = 6\Omega^2$ and $K_2 = 2\Omega$.

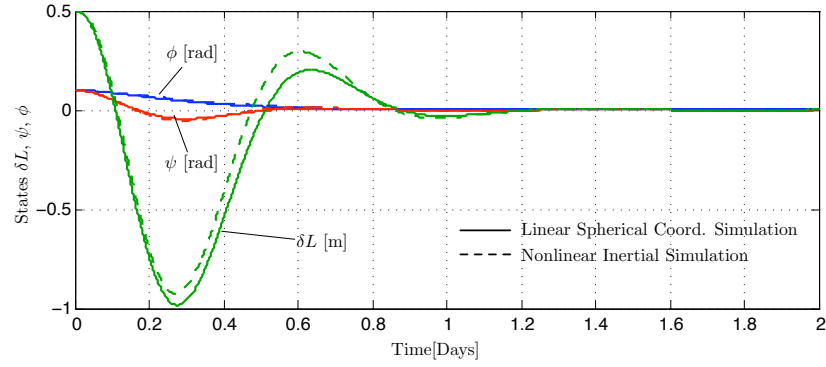
Table 5.1: Input parameters used in along-track simulation

Parameter	Value	Units
m_1	150	kg
m_2	150	kg
L_{ref}	25	m
k_c	8.99×10^9	$\frac{\text{Nm}^2}{\text{C}^2}$
Q_{ref}	0	μC^2
Ω	7.2915×10^{-5}	rad/sec
C_1	$2.97\Omega^2$	
C_2	3.9637Ω	
K_1	$6\Omega^2$	
K_2	2Ω	
$\delta L(0)$	0.5	m
$\psi(0)$	0.1	rad
$\phi(0)$	0.1	rad

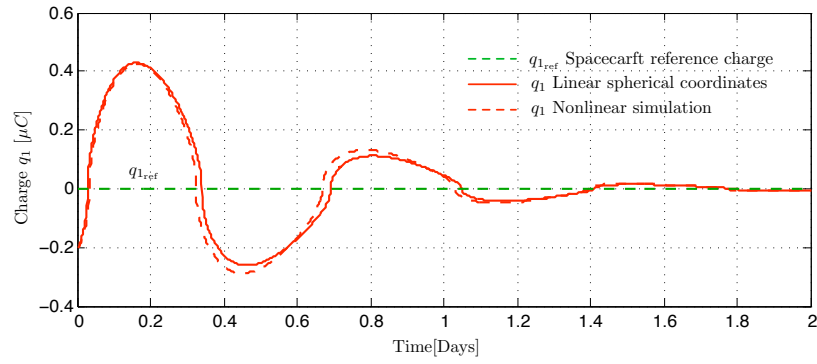
Figure 5.7(a) shows the Coulomb tether motion in both linearized spherical coordinates δL , ψ and ϕ (continuous line), and the full nonlinear spherical coordinates (dashed lines). It shows that the nonlinear simulation closely follows the linear simulation, validating the linearizing assumptions. The charge feedback law augmented with the thrust forces (using angle and angle rate feedback) ensures the convergence of all states to zero. Figure 5.7(b) illustrates the control charge on a single spacecraft for both linearized and full nonlinear simulation models. The reference charge pertaining to static equilibrium for along-track formation is zero and control charges are converging to this value. Note that the deviation from the value of reference charges is small, justifying the charge linearization assumptions used. The magnitude of the control charges is in the order of micro-Coulomb, which is easily realizable in practice using charge emission devices. Figure 5.7(c) gives the thrusting force that is required to stabilize the angles. Again, the dashed lines represent the full nonlinear model and the continuous lines represent the linearized model. The thrust forces can be generated using conventional thrusters. In the body fixed coordinates, the craft are aligned along the $\hat{b}_2$ axis and the thrust forces F_1 and F_2 are acting along the $\hat{b}_1$ and $\hat{b}_3$ directions, respectively. Thus, the thrusting always takes place perpendicular to the craft orientation, thereby avoiding plume impingement issues.

5.4.2 Orbit Normal Configuration

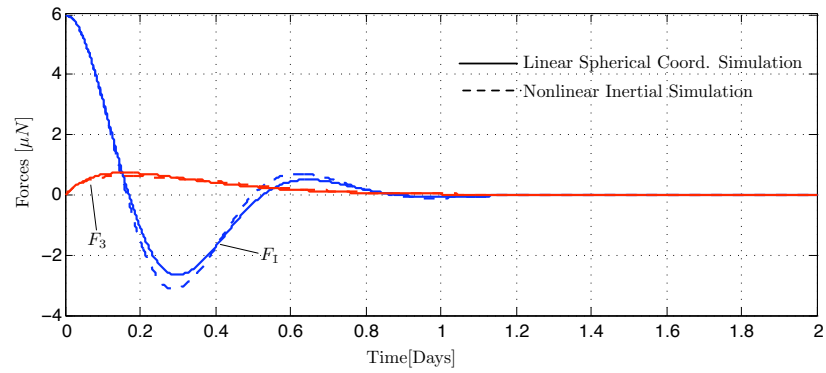
The orbit normal Coulomb tether is also simulated with a separation distance of 25 meter like the along-track configuration. The same spacecraft parameters and nominal separation distance are used as in Table 5.1. The initial separation distance error, initial Euler angles and gains are given in Table 5.2. Figures 5.8(a) , 5.8(b), 5.8(c) show the tether motion (spherical coordinates), charge on a single craft and thrust forces, respectively. Again, the dashed lines depicting the full nonlinear model closely follow continuous lines depicting the linearized model. It can be observed from Figure 5.8(a) that the separation distance error is



(a) Time histories of length variations δL , in-plane rotation angle ψ , and out-of-plane rotation angle ϕ .



(b) Spacecraft charge time histories



(c) Spacecraft force time histories

Figure 5.7: Simulation results for two craft aligned along the along-track direction with a separation distance of 25m.

critically damped and the out-of-plane angles ϕ and θ asymptotically go to zero. The thrust forces F_1 and F_2 are acting in the $\hat{b}_1$ and $\hat{b}_2$ direction with the Coulomb tether aligned along the $\hat{b}_3$ direction. Thus, plume impingement problems are avoided.

Table 5.2: Input parameters used in orbit normal simulation

Parameter	Value	Units
Q_{ref}	6.9304×10^{-13}	μC^2
C_2	$2\sqrt{3}\Omega$	
K_1	$2.7\Omega^2$	
K_3	3.2596Ω	
K_2	$5\Omega^2$	
$\delta L(0)$	0.5	m
$\theta(0)$	0.06	rad
$\phi(0)$	0.04	rad

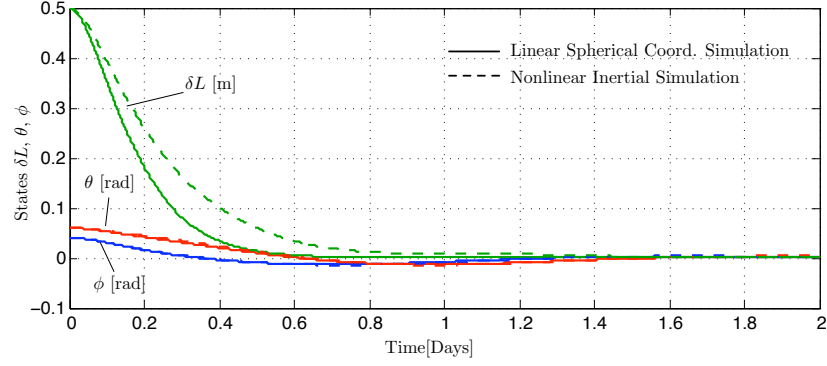
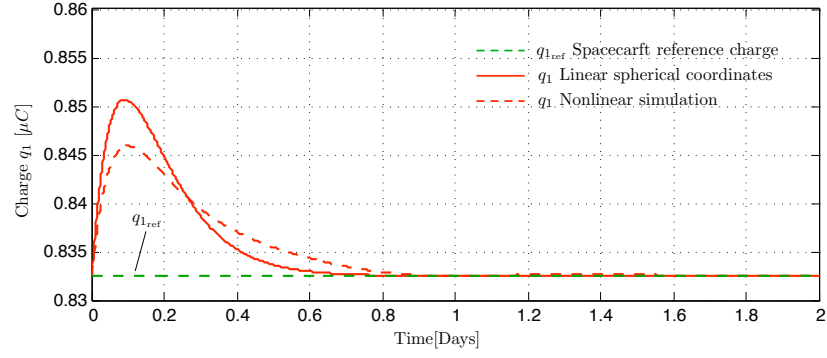
5.4.3 Differential Solar Perturbation

At GEO, differential solar drag is the largest disturbance acting on the Coulomb formation. Hence, full nonlinear model simulation for both along-track and orbit normal configuration are carried out including the effects of solar drag to study the ability of the controller to withstand this disturbance. The inertial acceleration vector $\mathbf{r}_s$ due to the effects of solar radiation pressure is given as

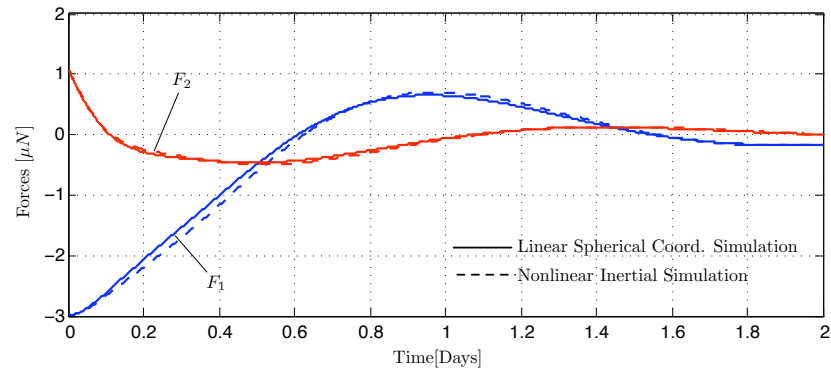
$$\mathbf{r}_s = \frac{-C_r A F}{mc} \frac{\mathbf{r}}{\|\mathbf{r}\|^3} \quad (5.34)$$

where $\mathbf{r}$ is the position vector from the sun to the orbiting planet in AU, m is the mass of the spacecraft in kg, A is the cross section area of the spacecraft that is facing the sun in m^2 . The constant $F = 1372.5398 \text{ Watts/m}^2$ is the solar radiation flux, $c = 2.997 \times 10^8 \text{ m/s}$ is the speed of light, and $C_r = 1.3$ is the radiation pressure coefficient.

The simulation is carried out over a period of 3 days and the Sun's position is assumed


 (a) Time histories of length variations δL , out-of-plane rotation angles θ and ϕ .


(b) Spacecraft charge time histories



(c) Spacecraft force time histories

Figure 5.8: Simulation results for two craft aligned along the orbit normal direction with a separation distance of 25m.

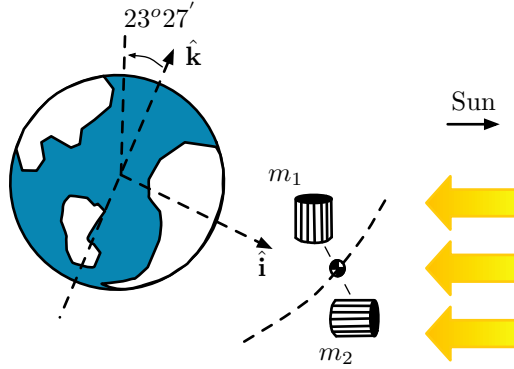
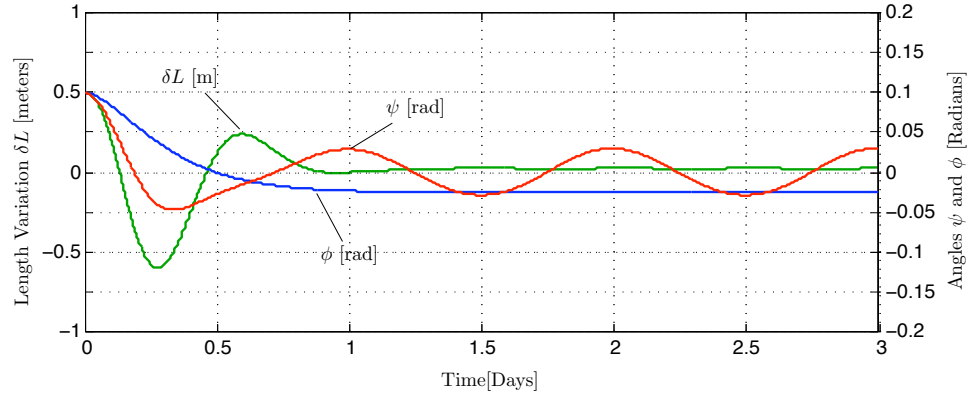
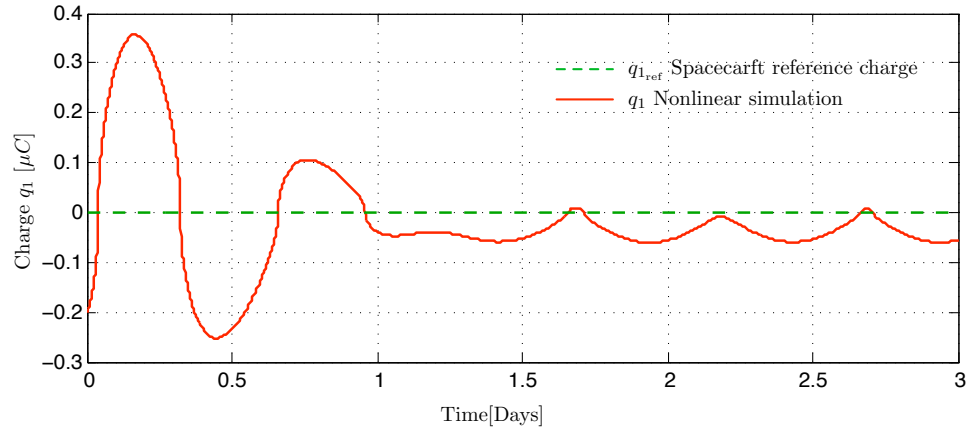


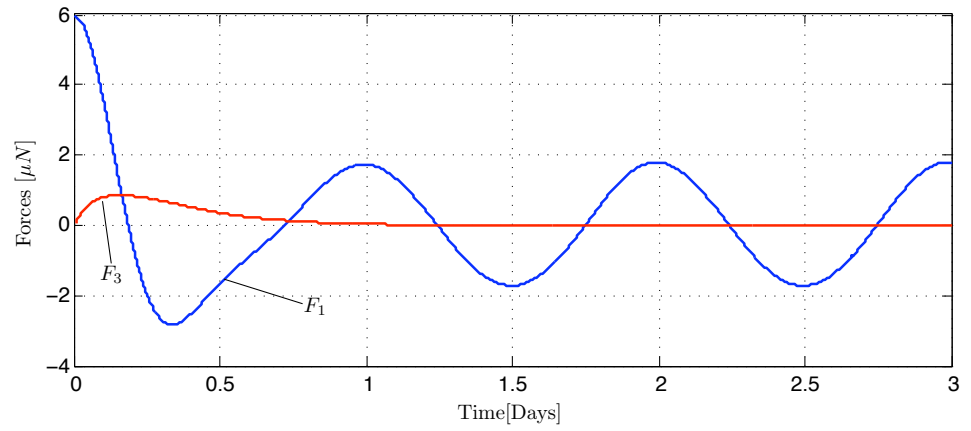
Figure 5.9: The Orientation of the Cylindrical Craft and the Sun's Position

to be fixed with respect to the Earth fixed inertial coordinates. As shown in Figure 5.9, the solar rays are assumed to be making an angle of $23^\circ 27'$ with respect to the earth's equatorial plane to account for the earth's axial tilt. The craft are modeled as cylinders with radius of 0.5 m, height of 1 m and mass of 150 kg. For craft 1, the cylindrical surface is constantly facing the sun resulting in a square cross section area of 1 m^2 , whereas for craft 2, it is the circular cross section ($0.25\pi \text{ m}^2$) of the top of the cylinder that is facing the sun.

Figure 5.10(a) shows the time histories of the spherical coordinates δL , ψ and ϕ for along-track Coulomb tether formation with differential solar drag. The coupled states δL and ψ no longer asymptotically converge to zero, but they are still bounded. The in-plane angle ψ oscillates with maximum amplitude of ± 0.05 radians and the separation distance error δL oscillations are negligible. The out-of-plane motion ϕ settles with a constant steady state offset. This offset can be explained by looking at the linearized ϕ equation of motion. The ϕ equation is decoupled and with a constant external torque due to the differential solar drag, will result in a steady state offset. Let the constant inertial acceleration vector along the $\hat{o}_h$ direction due to solar drag for satellites one and two be $\mathbf{r}_{s1}(3, 1)$ and $\mathbf{r}_{s2}(3, 1)$, respectively.

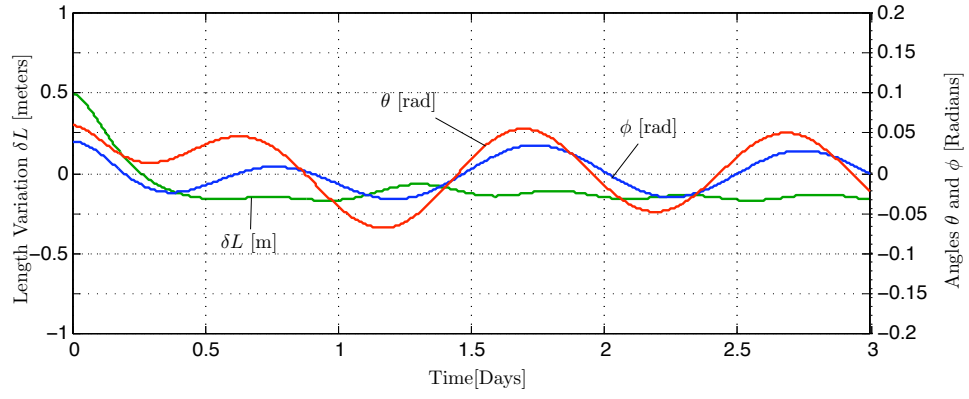
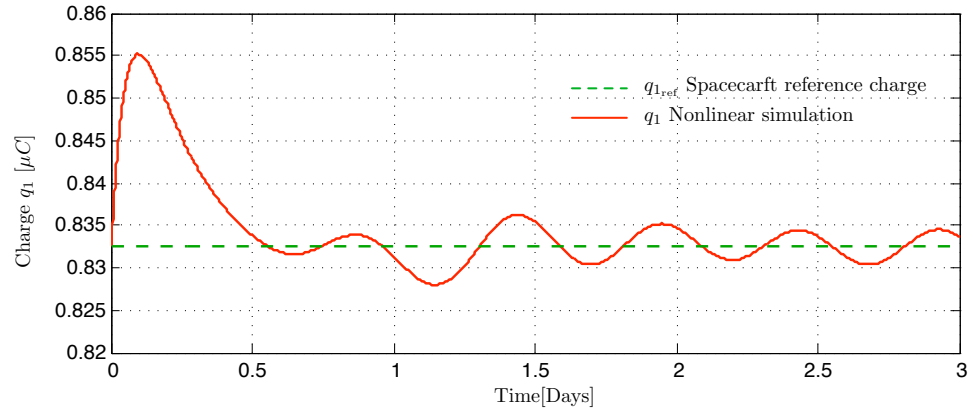

 (a) Time histories of length variations δL , out-of-plane rotation angles θ and ϕ .


(b) Spacecraft charge time histories

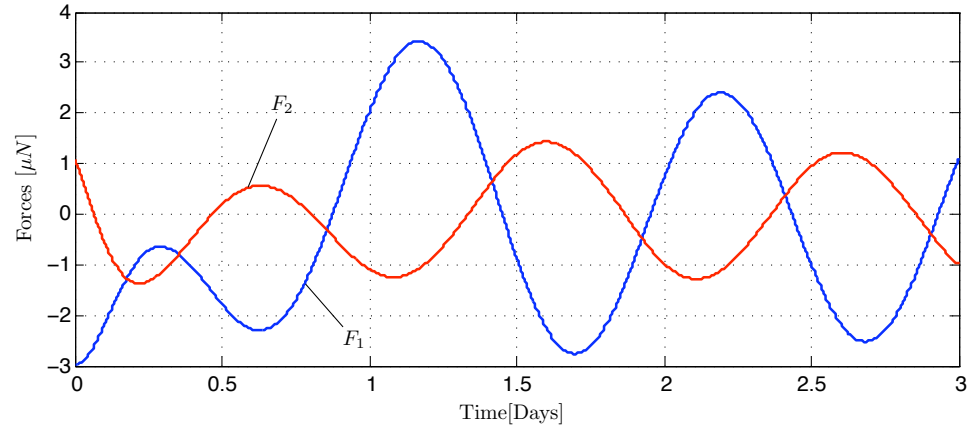


(c) Spacecraft force time histories

Figure 5.10: Simulation results for two craft aligned along the along-track direction with constant differential solar perturbation.


 (a) Time histories of length variations δL , out-of-plane rotation angles θ and ϕ .


(b) Spacecraft charge time histories



(c) Spacecraft force time histories

Figure 5.11: Simulation results for two craft aligned along the orbit normal direction with constant differential solar perturbation.

The total constant force acting on the satellite formation along the $\hat{o}_h$ direction is

$$F_s = m_1 \mathbf{r}_{s1}(3, 1) + m_2 \mathbf{r}_{s2}(3, 1)$$

The resulting torque due to this force is given by

$$T_s = \frac{m_1}{m_1 + m_2} L(m_1 \mathbf{r}_{s1}(3, 1)) - \frac{m_2}{m_1 + m_2} L(m_2 \mathbf{r}_{s2}(3, 1)) \quad (5.35)$$

The linearized ϕ equation for along track configuration (Eq. (5.5a)) can be modified to incorporate the constant torque given in Eq. (5.35) as

$$\ddot{\phi} + \Omega^2 \phi = \frac{\frac{1}{m_1 + m_2} L(m_1^2 \mathbf{r}_{s1}(3, 1) - m_2^2 \mathbf{r}_{s2}(3, 1))}{\frac{m_1 m_2}{m_1 + m_2} L^2} \quad (5.36)$$

From Eq. (5.36), the analytical expression for steady state offset in the presence of differential solar drag can be written as

$$\phi = \frac{(m_1/m_2 \mathbf{r}_{s1}(3, 1) - m_2/m_1 \mathbf{r}_{s2}(3, 1))}{L\Omega^2} \quad (5.37)$$

For the linearized model the offset was calculated to be -0.0255 radians and it is very close to the offset observed for the full nonlinear model. Figures 5.10(b) and 5.10(c) give the spacecraft charge and thrust force time histories, respectively.

Figure 5.11(a) shows the performance of orbit normal Coulomb tether in the presence of differential solar drag. Again, it can be observed that the states are bounded. On close observation of the figure one can come to the conclusion that the separation distance error (δL) is oscillating about an offset at steady state. The linearized separation distance error (δL) is decoupled from the angles and constant differential solar drag acting on the formation results in a steady state offset for δL . The analytical expression for this steady state δL

offset can be derived for the linearized model as

$$\delta L = \frac{(m_1 \mathbf{r}_{s1}(3, 1) - m_2 \mathbf{r}_{s2}(3, 1))}{3m_1 \Omega^2} \quad (5.38)$$

Thus, the linearized model offset for δL is -0.2125 m. The observed steady state offset in the figure is close to this value and the oscillations can be explained due to the second order coupling of the separation distance error (δL) with the angles. The oscillations in the δL result in the oscillations of the spacecraft charge value around the reference charge value, as seen in Figures 5.11(b). Figures 5.11(c) shows the thrust force time histories.

5.5 Summary

A 2-craft Coulomb tethered structure aligned along the orbit normal or along-track direction cannot be stabilized with only a charge feedback law. But, both Coulomb tether configurations can be stabilized with a hybrid control of Coulomb forces and conventional thrusters that stabilize the separation distance and orientation respectively. The control charges needed are small in the order of micro-Coulombs and realizable in practice. The thrusting forces required are in the order of micro-Newtons and the thrusting is always done orthogonal to the Coulomb tether axis, thus avoiding plume exhaust impingement problems. For the along-track configuration the separation distance and in-plan angle are coupled and unstable without feedback. An interesting result is that for the orbit-normal configuration the separation distance is decoupled and marginally stable even without charge feedback, while the orientation has to be feedback stabilized. Numerical simulations of the full nonlinear motion are carried out to illustrate the results and compare the linearized performance predictions to the actual nonlinear system response. Finally, the robustness of the controller to withstand differential solar drag is illustrated through simulations.

6 Reconfiguration of a Nadir-Pointing 2-Craft Coulomb Tether

6.1 Introduction

In chapter 4, the stabilization of a simple static 2-craft Coulomb tether structure along the orbit radial direction is studied.³⁰ Compared to the previous works on static Coulomb structures, this is the first study to introduce a charge feedback law to stabilize a simple Coulomb structure to a specific shape and orientation. Coulomb forces are inter-spacecraft forces and can not control the inertial angular momentum of the formation. Hence, stability characteristics of orbital rigid body motion under a gravity gradient field was applied to a Coulomb tethered two-spacecraft system to develop an active charge feedback control. With this control the spacecraft separation distance can be maintained at a fixed value, while the coupled gravity gradient torque is exploited to stabilize the formation attitude about the orbit nadir vector. Further, as the separation distance converged to the desired value, the in-plane rotation angle is shown to converge to zero as well. The out-of-plane angle is shown to be decoupled from the other modes and not influenced (to first order) by the spacecraft charges.

This chapter extends this earlier work by investigating how to reconfigure the 2-craft Coulomb tether formation by forcing the craft to move apart or come closer using the

Coulomb force and again using the gravity gradient to stabilize the formation. An active charge feedback law is introduced and the linear stability of the coupled separation distance and attitude is evaluated for this time-variant system. Based on this analysis, stability regions for expanding and contracting the two-craft formation are established. In References 32 and 33, the dynamics of a traditional two-craft tether is studied where they develop length rate laws that guarantee stability. The attitude stability that is achieved is only a bounded stability. In the current work, with an electrostatic virtual tether replacing the actual tether, the feedback law attempts to asymptotically stabilize the separation distance and the in-plane oscillations. The asymptotic stability is achievable due to the virtual tether which allows both compression and tension and a flexible tether length. The formation is studied in GEO and the Debye lengths are assumed to be sufficiently large so that the effects of Debye shielding can be neglected. Finally, numerical simulations illustrate the analytical stability predictions.

6.2 Satellite Reconfiguration Dynamics

A 2-satellite formation is considered as shown in the Figure 6.1. The center of mass is assumed to maintain a circular Keplerian orbit and the two satellites are nominally aligned along the orbit radial direction. In essence, these two charged spacecraft will behave like a conventional 2-craft tether system, with the exception that this electrostatic tether is capable both of attractive and repulsive forces. Chapter 4 shows that the relative distance between the two satellites can be controlled using electrostatic Coulomb forces. A charge feedback law is used to maintain the relative distance at a constant value. As a result, the two satellites behave like a long slender nearly rigid body and the differential gravitational attraction is used to stabilize the attitude of this formation about the orbit radial direction. From this point onwards, this will be referred to as the Coulomb tether regulation problem.

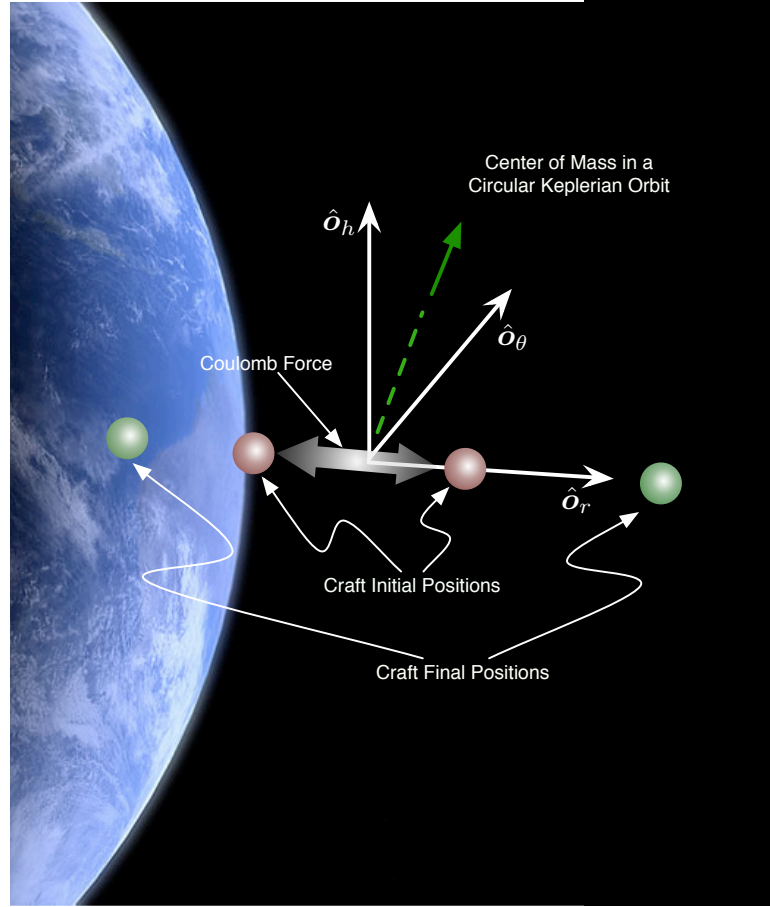


Figure 6.1: A simple Coulomb tracking illustration.

These concepts are extended for the time varying Coulomb tether length tracking problem. The main aim in the tracking (reconfiguration) problem is to increase or decrease the relative distance between the satellites by forcing them to move relative to each other along a prescribed path. This static Coulomb structure reconfiguration is to be accomplished without losing altitude stability.

The Clohessy-Wiltshire-Hill's equations^{27–29} for one of the spacecraft in the 2-craft Coulomb

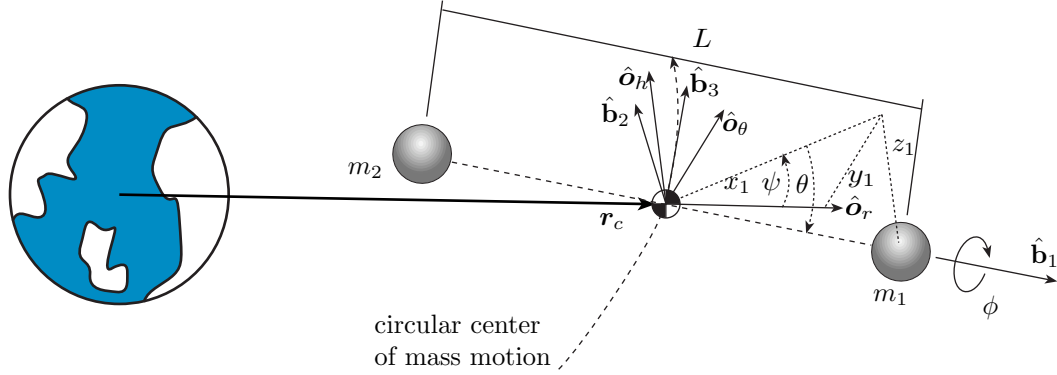


Figure 6.2: Coulomb Tethered Two Satellite Formation with the Satellites Aligned Along the Orbit Nadir Direction

tether formation as shown in Figure 6.2 is given by

$$\ddot{x}_1 - 2\Omega\dot{y}_1 - 3\Omega^2x_1 = \frac{k_c}{m_1} \frac{(x_1 - x_2)}{L^3} q_1q_2 \quad (6.1a)$$

$$\ddot{y}_1 + 2\Omega\dot{x}_1 = \frac{k_c}{m_1} \frac{(y_1 - y_2)}{L^3} q_1q_2 \quad (6.1b)$$

$$\ddot{z}_1 + \Omega^2z_1 = \frac{k_c}{m_1} \frac{(z_1 - z_2)}{L^3} q_1q_2 \quad (6.1c)$$

where $(x_i, y_i, z_i)^T$ is the position vector of the i^{th} satellite in Hill frame components, m_1 and q_1 are the mass and charge of satellite 1, and L is the distance between the satellites 1 and 2. The constant chief orbital rate is given by $\Omega = \sqrt{\mu/r_c^3}$, where μ is the gravitational coefficient and r_c is center of mass position vector. The parameter $k_c = 8.99 \cdot 10^9 \text{ Nm}^2/\text{C}^2$ is the Coulomb constant. As the Hill frame origin is set to be identical to the formation center of mass, the motion of the 2nd craft can be found by noting that the center of mass vector is constant due to conservation of linear momentum. This yields^{4,5}

$$m_1\boldsymbol{\rho}_1 + m_2\boldsymbol{\rho}_2 = 0 \quad (6.2)$$

The differential equation of the separation distance L , between the two satellites is given by³⁰

$$\ddot{L} = (2\Omega\dot{\psi} + 3\Omega^2)L + \frac{k_c}{m_1}Q \frac{1}{L^2} \frac{m_1 + m_2}{m_2} \quad (6.3)$$

For the Coulomb tether regulation problem, L is the sum of a constant reference length L_{ref} and a small varying length δL . Similarly, let Q be the sum of Q_{ref} , which is the ideal constant charge needed to maintain the satellites in a rigid formation of length L_{ref} , and a small charge variation δQ .

$$L(t) = L_{\text{ref}} + \delta L(t) \quad (6.4a)$$

$$Q(t) = Q_{\text{ref}} + \delta Q(t) \quad (6.4b)$$

The reference charge Q_{ref} is a function of L_{ref} and is computed analytically from the linearized Hill frame equations. The analytical expression for Q_{ref} is written as^{13,30}

$$Q_{\text{ref}} = -3\Omega^2 \frac{L^3}{k_c} \frac{m_1 m_2}{m_1 + m_2} \quad (6.5)$$

It should be noted that in the Coulomb tether regulation problem L_{ref} is constant and the differential equation given in Eq. (6.3) is linearized by assuming a small δL separation distance error. This can be slightly modified to accommodate the Coulomb tracking problem. The reference Coulomb structure length $L_{\text{ref}}(t)$ is made time varying, but the separation distance errors $\delta L(t)$ are still assumed to be small.

$$L(t) = L_{\text{ref}}(t) + \delta L(t) \quad (6.6a)$$

$$Q(t) = Q_{\text{ref}}(t) + \delta Q(t) \quad (6.6b)$$

Here $L_{\text{ref}}(t)$ is the time varying reference separation distance and $Q_{\text{ref}}(t)$ is the corresponding reference charge which can be calculated using Eq. (6.5). Substituting the assumptions in Eq. (6.6) into Eq. (6.3) and linearizing assuming small δL yields

$$\delta \ddot{L} = -\ddot{L}_{\text{ref}} + 2\Omega L_{\text{ref}} \dot{\psi} + 9\Omega^2 \delta L + \frac{k_c}{m_1} \delta Q \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2} \quad (6.7)$$

This equation establishes the relation between the additional charge δQ required and the change in relative separation of the satellites. Note that this relation is coupled to the angular in-plane perturbation rate $\dot{\psi}$. In order to obtain an expression for this, a stability analysis using the gravity gradient is employed. The derivation of the expression for angular perturbation closely follows the derivation given in Ref. 30 for the Coulomb regulation problem. The linearized attitude dynamics of the Coulomb tether body frame are written along with the separation distance equation as:

$$\ddot{\theta} + \frac{2\dot{L}_{\text{ref}}}{L_{\text{ref}}} \dot{\theta} + 4\Omega^2 \theta = 0 \quad (6.8a)$$

$$\ddot{\psi} + \frac{2\dot{L}_{\text{ref}}}{L_{\text{ref}}} \dot{\psi} + \frac{2\Omega}{L_{\text{ref}}} \delta \dot{L} - \frac{2\dot{L}_{\text{ref}}}{L_{\text{ref}}^2} \Omega \delta L + \frac{2\dot{L}_{\text{ref}}}{L_{\text{ref}}} \Omega + 3\Omega^2 \psi = 0 \quad (6.8b)$$

$$\delta \ddot{L} + \ddot{L}_{\text{ref}} - 2\Omega L_{\text{ref}} \dot{\psi} - 9\Omega^2 \delta L - \frac{k_c}{m_1} \delta Q \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2} = 0 \quad (6.8c)$$

Thus, Eq. (6.8a) – (6.8c) are the linearized equations of motion of the Coulomb tracking about the static nadir reference configuration. Only the linearized δL differential equation was obtained using the Clohessy-Wiltshire-Hill equations, while the linearized differential equations of ψ and θ were derived from the full formation angular momentum expression along with Euler's equation. Compared to the regulation problem, these differential equations are non-autonomous and depend explicitly on time through $L_{\text{ref}}(t)$. This greatly complicates the stability analysis of any feedback control law.

Let the charge product variation δQ be the control signal. The Coulomb regulation

feedback control is then modified to incorporate a time-varying $L_{\text{ref}}(t)$ term.

$$\delta Q = \frac{m_1 m_2 L_{\text{ref}}^2(t)}{(m_1 + m_2) k_c} (-C_1 \delta L - C_2 \delta \dot{L}) \quad (6.9)$$

The constants C_1 and C_2 are the position and velocity feedback gains. Incorporating this feedback law in to the δL differential equation in Eq. (6.8c) yields the following closed-loop separation distance dynamics:

$$\delta \ddot{L} + \ddot{L}_{\text{ref}} - 2\Omega L_{\text{ref}} \dot{\psi} + (C_1 - 9\Omega^2) \delta L + C_2 \delta \dot{L} = 0 \quad (6.10)$$

It can be observed that the linearized equations in Eq. (6.8a) – (6.8c) depend on the mean orbit rate n which has a very small value at GEO. In order to eliminate the numerical issues that might arise while integrating due to the small n value, the following normalization transformation is employed to make these equations independent of n .

$$d\tau = \Omega dt \quad (6.11a)$$

$$(*)' = \frac{d(*)}{d\tau} = \frac{1}{\Omega} \frac{d(*)}{dt} \quad (6.11b)$$

The orbit rate independent form of the linearized equations in Eq. (6.8a) – (6.8c) are written as

$$\theta'' + \frac{2L'_{\text{ref}}}{L_{\text{ref}}} \theta' + 4\theta = 0 \quad (6.12a)$$

$$\psi'' + \frac{2L'_{\text{ref}}}{L_{\text{ref}}} \psi' + \frac{2}{L_{\text{ref}}} \delta L' - \frac{2L'_{\text{ref}}}{L_{\text{ref}}^2} \delta L + \frac{2L'_{\text{ref}}}{L_{\text{ref}}} + 3\psi = 0 \quad (6.12b)$$

$$\delta L'' + L''_{\text{ref}} - 2L_{\text{ref}} \psi' + (\tilde{C}_1 - 9) \delta L + \tilde{C}_2 \delta L' = 0 \quad (6.12c)$$

where $\tilde{C}_2 = (C_2/\Omega)$ and $\tilde{C}_1 = (C_1/\Omega^2)$ are non-dimensionalized feedback gains. These

equations show that the out-of-plane motion $\theta(t)$ is decoupled from the charge product term δQ and separation distance variation $\delta L(t)$. Therefore, it is not possible to control the out-of-plane motion using charge control in this linearized analysis. However, the in-plane motion $\psi(t)$ is coupled to the $\delta L(t)$ motion in the form of a driving force and hence, requiring a coupled in-plane attitude and separation distance stability analysis.

6.3 Stability Analysis

With time varying $L_{\text{ref}}(t)$, the equations of motion are linear and time dependent. Rosenbrock³⁴ shows that the linear time-dependent system given by $\dot{\mathbf{x}} = A(t)\mathbf{x}$ is asymptotically stable if the frozen system for each t is stable and the rate of change of $A(t)$ is very small. Reference 34 also establishes a bound for $A'(t)$ when $A(t)$ is in the control canonical form. The stability of the 2-craft Coulomb tether formation with varying reference length is analyzed using this method. The coupled δL and ψ equations in Eq. (6.12b) – (6.12c) are written in the state space form as

$$\begin{pmatrix} \psi' \\ \psi'' \\ \delta L' \\ \delta L'' \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -3 & -\frac{2L'_{\text{ref}}}{L_{\text{ref}}} & \frac{2L'_{\text{ref}}}{L_{\text{ref}}^2} & -\frac{2}{L_{\text{ref}}} \\ 0 & 0 & 0 & 1 \\ 0 & 2L_{\text{ref}} & 9 - \tilde{C}_1 & -\tilde{C}_2 \end{bmatrix}}_{A(t)} \begin{pmatrix} \psi \\ \psi' \\ \delta L \\ \delta L' \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ -\frac{2L'_{\text{ref}}}{L_{\text{ref}}} \\ 0 \\ -L''_{\text{ref}} \end{pmatrix}}_{d(t)} \quad (6.13)$$

The square matrix in the above equation is $A(t)$ and the time dependency in this matrix is due to the terms L_{ref} and L'_{ref} . The stability of the system greatly depends on the rate at which L_{ref} is varied. The rate of change of reference length L'_{ref} , can be chosen according to the mission requirement or design. Of interest is how large L'_{ref} can be while still guaranteeing stability. From Eq. (6.13), it can be observed that there is a state independent

term $d(t)$ which only depends on the specified rate of change of reference length (L'_{ref}). This term in the equation of motion will lead to a steady state offset as long as L_{ref} is time varying. The analytical expression for the steady state offset is given as follows

$$\begin{pmatrix} \psi_{\text{offset}} \\ \delta L_{\text{offset}} \end{pmatrix} = \begin{pmatrix} -\frac{2L'_{\text{ref}}}{3L_{\text{ref}}} + \frac{2L'_{\text{ref}}L''_{\text{ref}}}{3(-9+3\tilde{C}_1)L_{\text{ref}}^2} \\ \frac{L''_{\text{ref}}}{(-9+3\tilde{C}_1)} \end{pmatrix} = \begin{pmatrix} -\frac{2\dot{L}_{\text{ref}}}{3\Omega L_{\text{ref}}} + \frac{2\dot{L}_{\text{ref}}\ddot{L}_{\text{ref}}}{3\Omega(-9\Omega^2+3C_1)L_{\text{ref}}^2} \\ \frac{\ddot{L}_{\text{ref}}}{(-9\Omega^2+3C_1)} \end{pmatrix} \quad (6.14)$$

Before fixing the limits for L'_{ref} , the values for gains are chosen such that the $A(t)$ matrix is Hurwitz at any given time t . This does not guarantee stability for a time varying system, but this is a necessary step for the Rosenbrock stability conditions. In the regulation problem the feedback gains were expressed in terms of scaling factor c and α . Since this work is an extension of the regulation problem, the same scaling factor for the gains are chosen. They can be written as

$$\tilde{C}_1 = c \quad (6.15)$$

and

$$\tilde{C}_2 = \alpha\sqrt{c-9} \quad (6.16)$$

The characteristic equation of the $A(t)$ matrix is given by

$$\begin{aligned} \lambda^4 + (\tilde{C}_2 + 2\frac{L'_{\text{ref}}}{L_{\text{ref}}})\lambda^3 + (\tilde{C}_1 + 2\tilde{C}_2\frac{L'_{\text{ref}}}{L_{\text{ref}}} - 2)\lambda^2 + (3\tilde{C}_2 - 22\frac{L'_{\text{ref}}}{L_{\text{ref}}} + 2\tilde{C}_1\frac{L'_{\text{ref}}}{L_{\text{ref}}})\lambda \\ + 3(\tilde{C}_1 - 9) = 0 \end{aligned} \quad (6.17)$$

Let $k = L'_{\text{ref}}/L_{\text{ref}}$ be a time varying coefficient which is determined through the chosen reference separation time history $L_{\text{ref}}(t)$. With this simplification the characteristic equation

of $A(t)$ becomes

$$\lambda^4 + (\tilde{C}_2 + 2k)\lambda^3 + (\tilde{C}_1 + 2\tilde{C}_2k - 2)\lambda^2 + (3\tilde{C}_2 - 22k + 2\tilde{C}_1k)\lambda + 3(\tilde{C}_1 - 9) = 0 \quad (6.18)$$

To ensure stability, roots of the characteristic equation should have negative real parts (Hurwitz matrix). This requirement is satisfied using the Routh-Hurwitz stability criterion. Based on this criterion it is established that $\tilde{C}_1$ should have a value greater than 9 and the range of possible values for k and α for certain fixed $\tilde{C}_1$ is shown in Figure 6.3. The shaded region illustrates the possible values of k and α that guarantee that roots of the characteristic equation (i.e. the eigenvalues of the matrix $A(t)$) have negative real parts. It can be observed from Figure 6.3 that for $\tilde{C}_1 > 10$ there is no bounds on k when we are expanding the separation distance. But, for contracting or decreasing the separation distance (i.e. $-k$) we have a tight limit on k . The α value is fixed such that we have a maximum range of k . From Figures 6.3(b) and 6.3(c), the values of α are taken as 1.4 and 0.9 for the $\tilde{C}_1$ values of 12 and 14, respectively.

By satisfying the Routh-Hurwitz criterion, the eigenvalues of $A(t)$ at any fixed time t will always be in the left half of the plane. This is not sufficient to guarantee stability of the system. The sufficient condition is that rate of change of $A(t)$ be very small. Rosenbrock³⁴ established bounds for this rate of change and stated it as a theorem when $A(t)$ is in the control canonical form ($A_c(t)$). For the sake of continuity the theorem is stated here, but the reader should refer to Reference 34 for the detailed derivation of the theorem. Let the matrix R be defined as

$$R = SA_c^T + A_cS - S' + \eta I < 0 \quad (6.19)$$

$$(S_{ij}) = \sum_{k=1}^n \lambda_k^{i-1} \bar{\lambda}_k^{j-1} \quad (6.20)$$

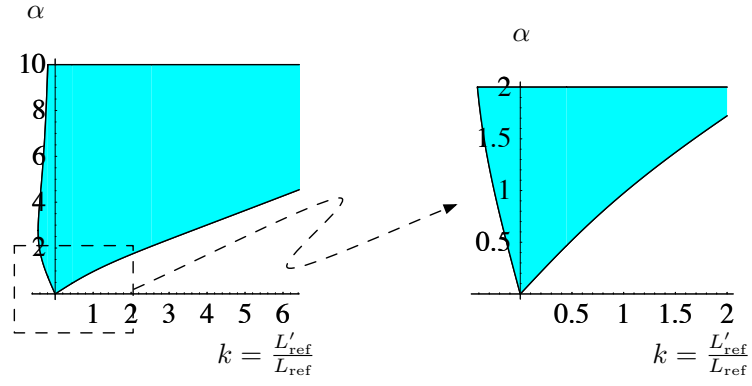
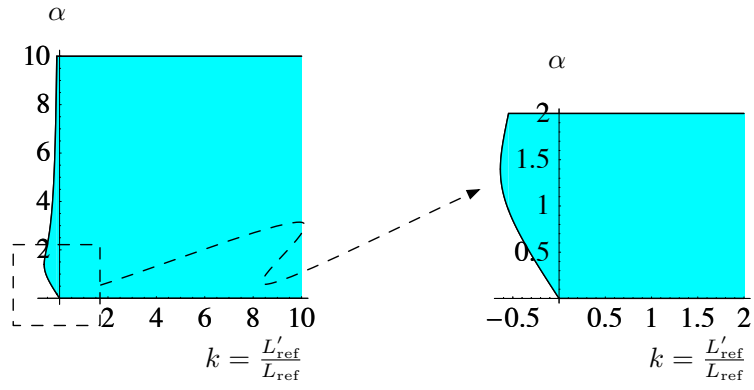
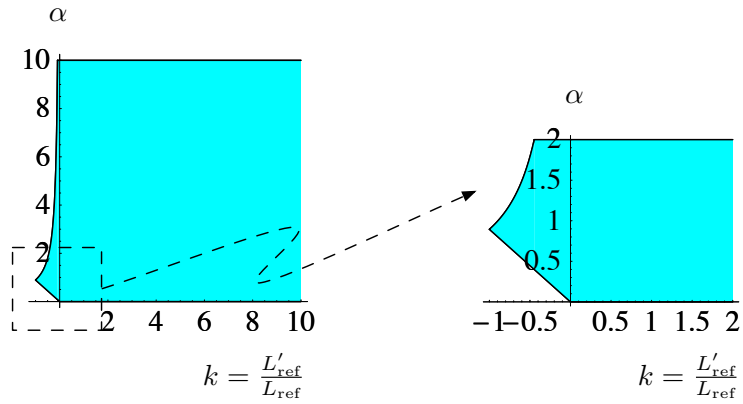

 (a) For $\tilde{C}_1 = 10$

 (b) For $\tilde{C}_1 = 12$

 (c) For $\tilde{C}_1 = 14$

Figure 6.3: Plots showing the regions that satisfy the Routh Hurwitz stability criterion.

where S_{ij} are the elements of the S matrix, λ_k and $\bar{\lambda}_k$ are the eigenvalues and its conjugate, S' is the derivative of S and $\eta > 0$ is some arbitrary constant. When all the eigen values of A_c are distinct and in the left half of the plane at any given instant of time, and R is negative definite throughout the maneuver, the system is asymptotically stable about $\mathbf{x} = 0$. For the 2-craft Coulomb tether problem, this requires the time varying reference separation distance $L_{\text{ref}}(t)$ to be carefully chosen so that the R is negative definite at all times. This theorem is based on the fact that for a matrix in the control canonical form, the eigenvalues are uniquely related to the elements of the matrix and hence, the bounds on the rate of change of the matrix can be replaced by bounds on the rate of change of the eigenvalues. Some more details about the S matrix are given in the following equation.

$$S = HH^* \quad (6.21)$$

where H is the is the eigenvector matrix and H^* is the transposed complex conjugate of H . The matrix H is defined as

$$H = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_n^2 \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_1^{n-1} & \lambda_2^{n-1} & \cdots & \lambda_n^{n-1} \end{bmatrix} \quad (6.22)$$

Studying the characteristic equation in Eq. (6.18), note that if $L'_{\text{ref}}(t)$ is chosen such that the coefficient $k = L'_{\text{ref}}/L_{\text{ref}}$ is constant, then the eigenvalues of $A_c(t)$ are also constant. For this special case the Rosenbrock stability conditions on the rate of change of $A(t)$ are trivially satisfied, and the overall stability is determined through the Routh-Hurwitz stability conditions. However, having a constant k coefficient is not a practical maneuver

because it requires exponential expansion or contraction.

The $A(t)$ matrix in Eq. (6.13) is not in the control canonical form, but it can be transformed in a control canonical form using a similarity transformation $\boldsymbol{\xi} = T\mathbf{x}$ which yields the differential vector equation

$$\boldsymbol{\xi}' = A_c(t)\boldsymbol{\xi} \quad (6.23)$$

It should be noted that the characteristic equation of the transformed matrix $A_c(t)$ is the same as the original matrix $A(t)$. Hence, the values of gains chosen earlier will keep the eigenvalues in the left half plane. For this transformed matrix we can establish the bounds on L_{ref} and L'_{ref} which guarantee that the matrix R remains negative definite. The transformed states $\boldsymbol{\xi}$ are linear combinations of the original states $\mathbf{x}$. Therefore, if the transformed states are stable then the original states are also stable. The control canonical form of the matrix ($A_c(t)$) for the given matrix $A(t)$ can be easily written by observing the characteristic equation. It is given by

$$A_c(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3(\tilde{C}_1 - 9) & -(3\tilde{C}_2 - 22k + 2\tilde{C}_1k) & -(\tilde{C}_1 + 2\tilde{C}_2k - 2) & -(\tilde{C}_2 + 2k) \end{bmatrix} \quad (6.24)$$

Because $A_c(t)$ is a 4×4 matrix, analytically finding the expression for eigenvalues and using them in the inequality in Eq. 6.19 is very challenging. The resulting expressions are too complex to be insightful. Instead the feasible values of L_{ref} and L'_{ref} that satisfies the inequality in Eq. 6.19 for the chosen values of $\tilde{C}_1$ and α are identified numerically. These feasible values are shown in Figure 6.4. The plots can be used to specify the reference trajectory $L_{\text{ref}}(t)$. Kulla³⁵ has developed a critical limit for the ratio $L'(t)/L(t)$ which

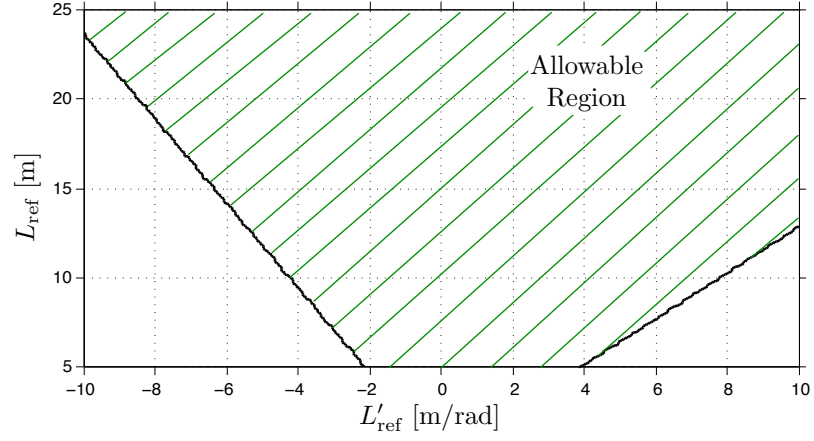
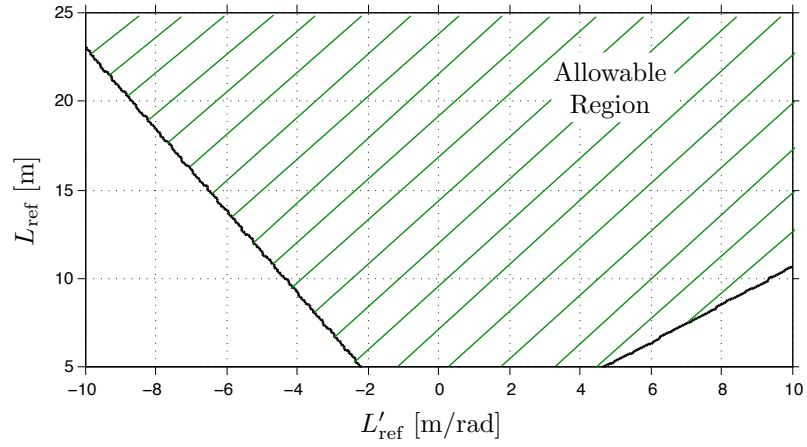

 (a) For $\tilde{C}_1 = 12$ and $\alpha = 1.4$

 (b) For $\tilde{C}_1 = 14$ and $\alpha = 0.9$

Figure 6.4: Plots showing the regions that satisfy the Routh Hurwitz stability criterion and Rosenbrock bounds.

guarantees stability for a traditional tethered two-craft system. This critical limit is given as

$$L'(t)/L(t) = \dot{L}(t)/(\Omega L(t)) \leq 0.75 \quad (6.25)$$

This limit comes from a trigonometric constraint while balancing the Coriolis forces by the gravity gradient forces. The identified feasible values of L_{ref} and L'_{ref} for the current two craft virtual tether problem have linear constraint boundaries similar to the Kulla critical limit. The Coulomb tether problem is significantly different as a virtual tether allows both tension and compression, and the stability depends on the feedback gains. In comparison, the classical nadir-pointing tether reconfiguration problem requires tension at all times and only depends on the length rate $\dot{L}$.

6.4 Numerical Simulation

To illustrate the performance and stability of Coulomb tether reconfiguration maneuvers, the following numerical simulations are performed. The simulation parameters that used are listed in Table 6.1. The initial attitude values are set to $\psi = 0.1$ radians and $\theta = 0.1$ rad. The separation length error (Coulomb tether length error) is $\delta L = 0.5$ meters. All initial rates are set to zero through $\dot{\psi} = \delta \dot{L} = \dot{\theta} = 0$. Two sets of maneuvers, expanding the Coulomb tether formation from 25m to 35m in 1.8 days and contracting the formation from a separation distance of 25m to 15m, are shown.

The Coulomb tether performance is simulated in two different manners. First the linearized spherical coordinate differential equations are integrated. This simulation illustrates the charge control performance operating on the linearized dynamical system. Second, the exact nonlinear equations of motion of the deputy satellites are solved using the same charge feedback control, and compared to the performance of the linearized dynamical system. The

Table 6.1: Input parameters used in orbit-radial reconfiguration simulation

Parameter	Value	Units
m_1	150	kg
m_2	150	kg
k_c	8.99×10^9	$\frac{\text{Nm}^2}{\text{C}^2}$
Ω	7.2915×10^{-5}	rad/sec
$\delta L(0)$	0.5	m
$\psi(0)$	0.1	rad
$\theta(0)$	0.1	rad

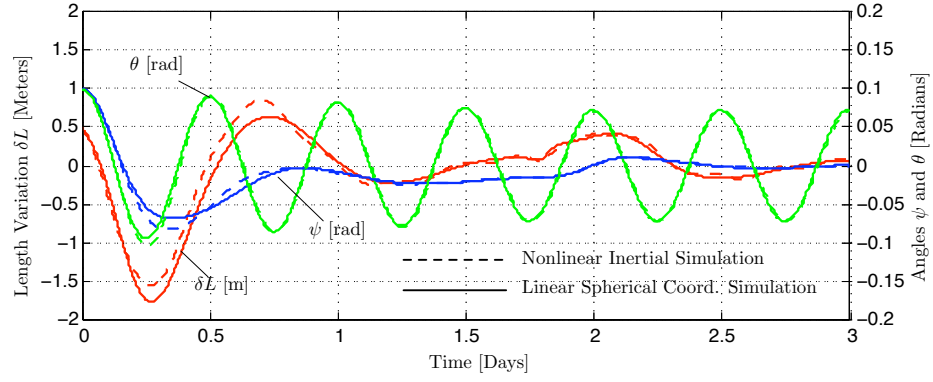
nonlinear deputy equations are given through Cowell's equations

$$\ddot{\mathbf{r}}_1 + \frac{\mu}{r_1^3} \mathbf{r}_1 = \frac{k_c}{m_1} \frac{Q}{L(t)^3} (\mathbf{r}_1 - \mathbf{r}_2) \quad (6.26a)$$

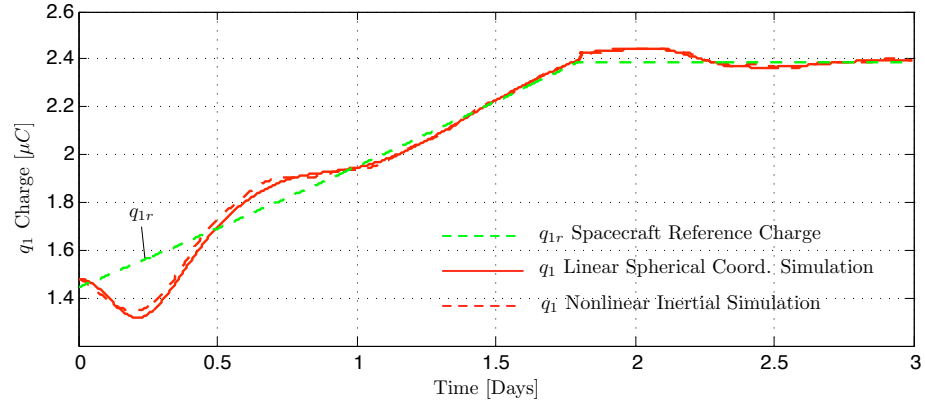
$$\ddot{\mathbf{r}}_2 + \frac{\mu}{r_2^3} \mathbf{r}_2 = \frac{k_c}{m_2} \frac{Q}{L(t)^3} (\mathbf{r}_2 - \mathbf{r}_1) \quad (6.26b)$$

where $\mathbf{r}_1 = \mathbf{r}_c + \boldsymbol{\rho}_1$ and $\mathbf{r}_2 = \mathbf{r}_c + \boldsymbol{\rho}_2$ are the inertial position vectors of the masses m_1 and m_2 , while $L = \sqrt{(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{r}_2 - \mathbf{r}_1)}$. The vector $\mathbf{r}_c$ denotes the position of the formation center of mass or chief location. The gravitational coefficient μ is defined as $\mu \approx GM_e$. After integrating the motion using inertial Cartesian coordinates, the separation distance L , as well as the in-plane and out-of-plane angles ψ and θ , are computed in post-processing using the exact kinematic transformation.

Figure 6.5(a) shows the Coulomb tether motion for increasing the separation distance from 25m to 35m in the linearized spherical coordinates $(\psi, \theta, \delta L)$, along with the full nonlinear spherical coordinates shown as continuous lines. The expansion is done in 1.8 days and this corresponds to a constant L'_{ref} of 0.88. After 1.8 days, the L'_{ref} is zero and the formation is allowed to stabilize about the final separation distance. The feedback gains are $\tilde{C}_1 = 12$ and $\alpha = 1.4$. With the presented charge feedback law, both the yaw motion ψ and the separation distance deviation δL converge to zero. By stabilizing the

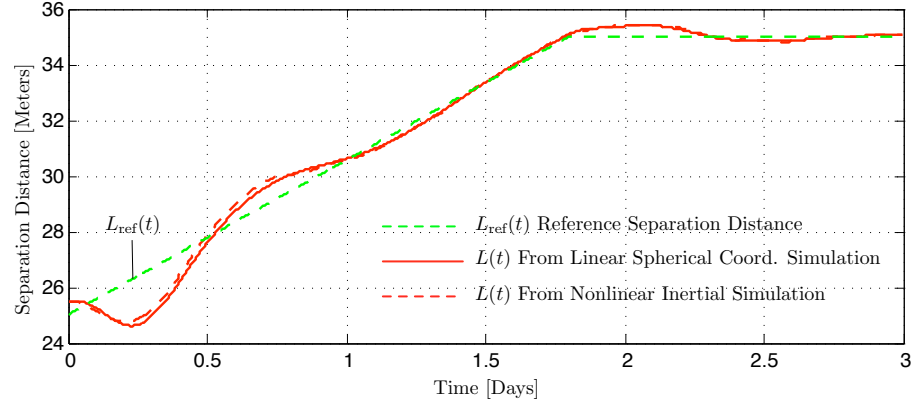


(a) Time histories of length variation δL , in-plane rotation angle ψ , and out-of-plane rotation angle θ .

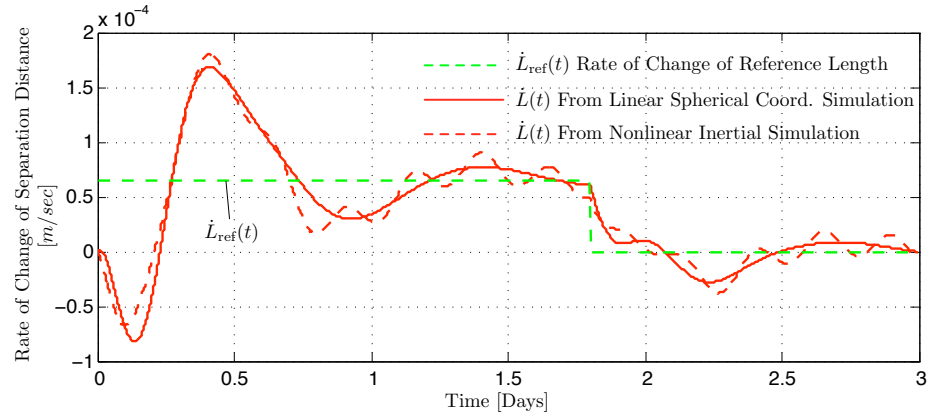


(b) Spacecraft charge time histories

Figure 6.5: Simulation results for expanding the spacecraft separation distance from 25 m to 35 m in 1.8 days. The feedback gains are $\tilde{C}_1 = 12$ and $\alpha = 1.4$.

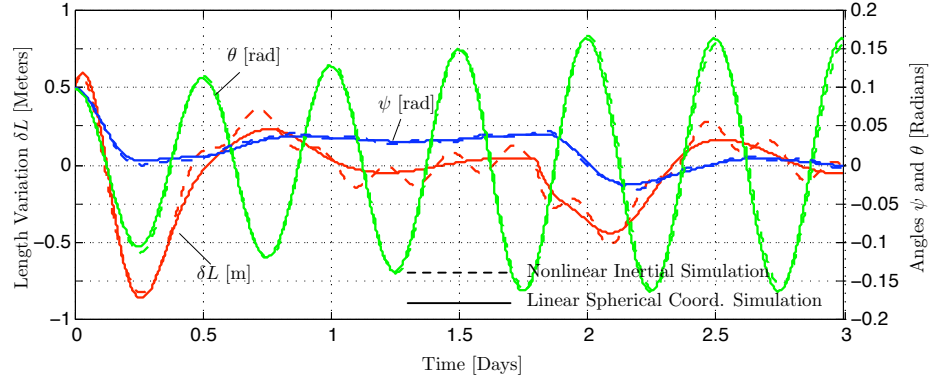


(a) Time histories of separation distance.

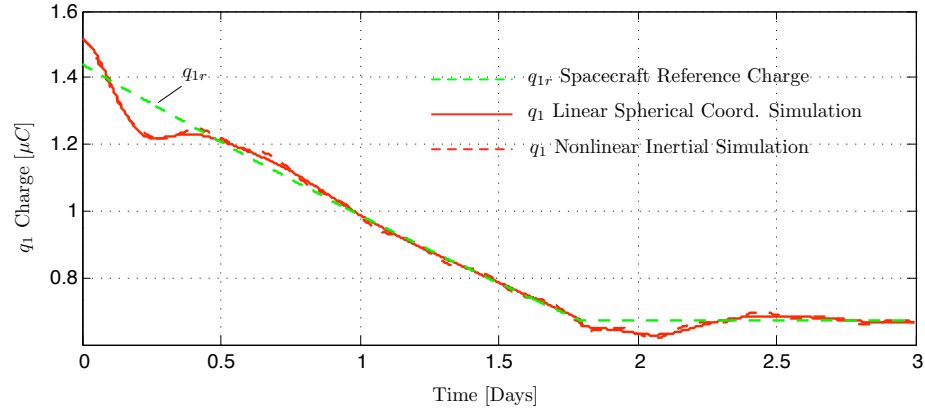


(b) Time histories of rate of change of separation distance

Figure 6.6: Simulation results for expanding the spacecraft separation distance from 25 m to 35 m in 1.8 days. The feedback gains are $\hat{C}_1 = 12$ and $\alpha = 1.4$.



(a) Time histories of length variation δL , in-plane rotation angle ψ , and out-of-plane rotation angle θ .



(b) Spacecraft charge time histories

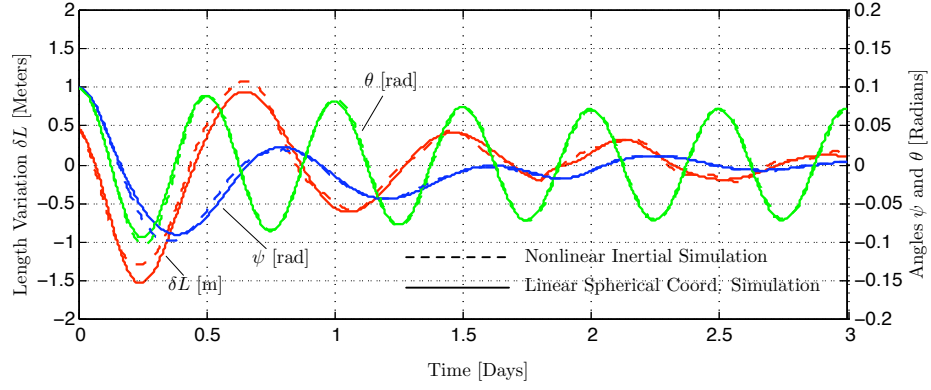
Figure 6.7: Simulation results for contracting the spacecraft separation distance from 25 m to 15 m in 1.8 days. The feedback gains are $\tilde{C}_1 = 12$ and $\alpha = 1.4$.

δL state to zero, the in-plane rotation $\psi(t)$ also converges to zero. As expected, the pitch motion $\theta(t)$ was a stable sinusoidal motion, decoupled from the controlled in-plane orbital motion. Further, Figure 6.5(a) shows that the nonlinear simulation closely follows the linearized simulation, validating the linearizing assumption and illustrating robustness to the unmodelled dynamics. Since L'_{ref} is constant, there is no steady state offset for δL and the offset for ψ is very small (order of 10^{-2} rad) and hence, not visible in the graph.

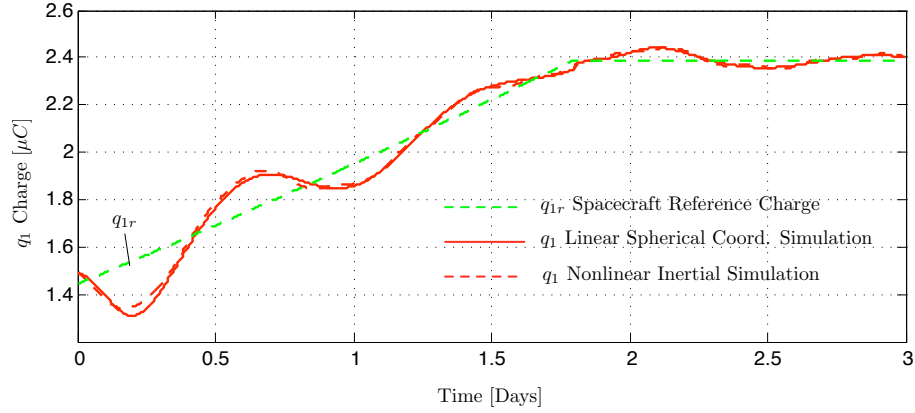
Figure 6.5(b) shows the spacecraft control charge q_1 (on craft 1) for both the linearized and full nonlinear simulation models. Both are converging to the reference value pertaining to the static equilibrium at each instant of time. Note that the deviation from the value of reference charges is small, justifying the linearization assumptions used. The magnitude of the control charges is in the order of micro-Coulomb which is easily realizable in practice using charge emission devices. The charge on craft 2 will be equal and opposite to the charge on craft 1.

In order to illustrate how well the system is tracking the prescribed reference trajectory $L_{\text{ref}}(t)$, the time histories of separation distance $L(t)$ and the time histories of rate of change of separation distance $\dot{L}(t)$ are shown in Figure 6.6(a) and Figure 6.6(b), respectively. Figure 6.6(a) shows that the reference separation distance ($L_{\text{ref}}(t)$) increases linearly until 1.8 days before settling to a constant value and both the linear and inertial nonlinear simulations track the reference separation distance closely. Figure 6.6(b) illustrates that the rate of change of the reference separation distance ($\dot{L}_{\text{ref}}(t)$) is a discrete step change. In the linear and inertial nonlinear simulations the formation is assumed to be static to begin with and hence, their rate of change of separation distance ($\dot{L}(t)$) are zero initially. But they converge with the reference rate $\dot{L}_{\text{ref}}(t)$ within 1.2 days. A faster convergence can be achieved by replacing the sharp corners of the reference rate (infinite reference acceleration) with a smooth polynomial function or spline (finite reference acceleration).

Figure 6.7(a) and Figure 6.7(b) show Coulomb tether motion and charge time histories

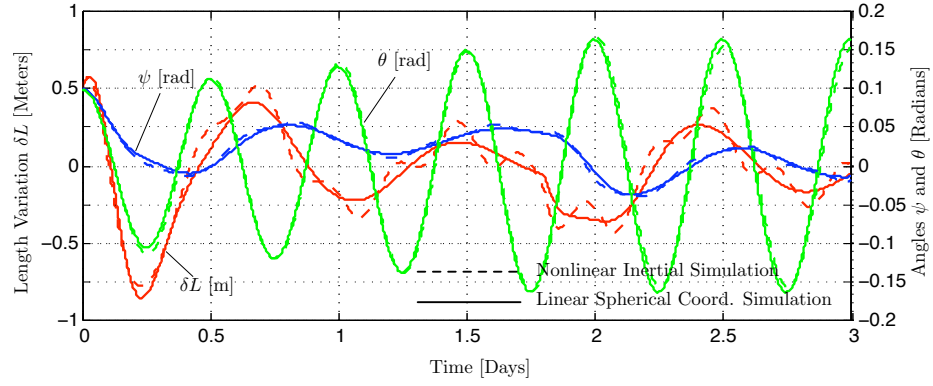


(a) Time histories of length variation δL , in-plane rotation angle ψ , and out-of-plane rotation angle θ .

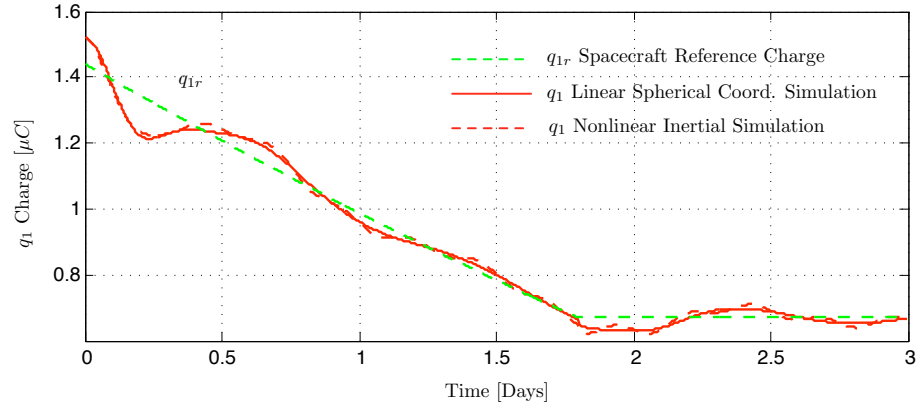


(b) Spacecraft charge time histories

Figure 6.8: Simulation results for expanding the spacecraft separation distance from 25 m to 35 m in 1.8 days. The feedback gains are $\hat{C}_1 = 14$ and $\alpha = 0.9$.



(a) Time histories of length variation δL , in-plane rotation angle ψ , and out-of-plane rotation angle θ .



(b) Spacecraft charge time histories

Figure 6.9: Simulation results for contracting the spacecraft separation distance from 25 m to 15 m in 1.8 days. The feedback gains are $\tilde{C}_1 = 14$ and $\alpha = 0.9$.

for decreasing the separation distance from 25m to 15m. Contractions are more challenging because the angular momentum will cause to destabilize the in-plane attitude motion. The maneuvers must be performed slow enough to allow the gravity gradient to maintain stability. Again the maneuver is done in 1.8 days which means L'_{ref} is -0.88 and the gains are same as above expansion maneuver. These two sets of maneuvers are repeated for the gain values $\tilde{C}_1 = 14$ and $\alpha = 0.9$ and, Figure 6.8 and Figure 6.9 illustrate their time histories. It can be observed from these two graphs that even though the system is stable, the performance could potentially be improved by tuning the feedback gains.

6.5 Summary

A charge feedback control law for reconfiguring a 2-craft Coulomb tether formation with time varying length is given. The 2-craft system forms a simple virtual Coulomb structure where the electrostatic force replaces the conventional tether cable. Previous work only considered stabilizing a static structure with a fixed length. This paper discusses an expanded feedback control law which allows for the Coulomb tether length to vary with time. During these maneuvers care is taken to ensure that the gravity gradient torque is still sufficient to stabilize the in-plane attitude of the nadir pointing formation. The stability regions for expanding and contracting the formation are established through linearization of the motion and by applying criteria developed by Rosenbrock for linear time-varying systems. Contracting the virtual structure is more difficult to perform while guaranteeing stability. The system angular momentum will cause any in-plane angular motion to increase with decreasing tether length. The magnitude of the local gravity gradient limits the rate at which the separation distance can be reduced. In contrast, expanding the virtual structure length is easier because the angular momentum helps contain in-plane rotation. The out-of-plane motion of the craft is decoupled from the in-plane motion with the linearized dynamics,

and not controllable with the Coulomb forces. Numerical simulations of the full nonlinear motion are carried out to illustrate the results and compare the linearized performance predictions to the actual nonlinear system response.

7 Analytical Solution for Out-of-Plane Motion Using Bessel Functions

7.1 Introduction

In the previous chapter, the reconfiguration of a two craft Coulomb tether aligned along the orbit radial direction is presented in detail. The linearized equations of motion (EOM) for this configuration reveals that the out-of-plane angle (θ) is decoupled from the in-plane angle (ψ) and the separation distance error (δL). Hence, out-of-plane angle can not be actively controlled by the Coulomb force. This issue with the out-of-plane angle is also true for the regulation problem. But, for the regulation problem the out-of-plane angle EOM is a time-invariant second order differential equation that results in a stable simple harmonic oscillation motion for θ . Whereas, for the reconfiguration problem the EOM is a second order differential equation with time dependent terms. The time dependent terms are prescribed the reference length (L_{ref}) and reference length rate ($\dot{L}_{\text{ref}}$). Depending on whether one is expanding (positive $\dot{L}_{\text{ref}}$) or contracting (negative $\dot{L}_{\text{ref}}$) the two craft formation, the initial out-of-plane angle oscillations will decrease or increase, respectively. We are keenly interested in studying the contracting operation which has potential applications in space structure's docking operations. These docking operations might require the final angular oscillations to be within certain limits. The in-plane angle can be asymptotically

controlled by using the Coulomb force. This now leaves the out-of-plane angle and we are interested in developing certain bounds on the initial out-of-plane oscillation such that the final oscillation will be within the prescribed limits.

In this chapter, we develop analytical solutions for out-of-plane angle EOM when the reference length rate ($\dot{L}_{\text{ref}}$) is a constant. This analytical solution is derived using the Bessel function³⁶ and the final general solution does depend on the initial conditions. Based on this solution the bounds on the initial conditions (initial out-of-plane angle) have been established to keep the oscillations, at the end of the operation, within the prescribed limits. This chapter is organized as follows. First, the analytical solution for the linearized out-of-plane angle EOM is derived and this followed by establishing the bounds on the initial out-of-plane angle. Finally, numeric simulations are carried out to illustrate the results.

7.2 Analytical Solution Derivation

The equation of motion for the out-of-plane angular motion is given by

$$\ddot{\theta} + 2\frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}}\dot{\theta} + 4\Omega^2\theta = 0 \quad (7.1)$$

For a constant reference length rate ($\dot{L}_{\text{ref}}$), the reference length can be written as

$$L_{\text{ref}}(t) = L_0 + \dot{L}_{\text{ref}} t \quad (7.2)$$

where L_0 is the initial reference separation distance and t is the time. Substituting Eq. (7.2) back into the out-of-plane equation of motion given in Eq. (7.1) results in

$$\ddot{\theta} + 2\frac{\dot{L}_{\text{ref}}}{L_0 + \dot{L}_{\text{ref}} t}\dot{\theta} + 4\Omega^2\theta = 0 \quad (7.3)$$

An analytical solution for the equation of motion given in Eq. (7.3) can be obtained by transforming the equation to the Bessel equation.³⁶ Define a new variable z as follows

$$z = L_0 + \dot{L}_{\text{ref}} t \quad (7.4)$$

$$dz = \dot{L}_{\text{ref}} dt \quad (7.5)$$

$$\frac{dz}{dt} = \dot{L}_{\text{ref}} \quad (7.6)$$

Substituting this new variable given in Eq. (7.4) into Eq. (7.3), and changing the derivatives with respect to t (time) to derivatives with respect to z results in

$$\begin{aligned} \frac{d^2\theta}{dz^2} \dot{L}_{\text{ref}}^2 + 2 \frac{\dot{L}_{\text{ref}}}{z} \frac{d\theta}{dz} \dot{L}_{\text{ref}} + 4\Omega^2\theta &= 0 \\ \frac{d^2\theta}{dz^2} + \frac{2}{z} \frac{d\theta}{dz} + \frac{4\Omega^2}{\dot{L}_{\text{ref}}^2} \theta &= 0 \\ \frac{d^2\theta}{dz^2} + \frac{2}{z} \frac{d\theta}{dz} + k^2\theta &= 0 \end{aligned} \quad (7.7)$$

where $k = \frac{2\Omega}{\dot{L}_{\text{ref}}}$. The transformed equation in Eq. (7.7) is still not in the standard Bessel equation form and needs one more transformation. Assume, the out-of-plane angle θ to be of the form

$$\theta = z^{-\frac{1}{2}} y(z) \quad (7.8)$$

Now, the first and second derivative of θ with respect to z can be written as

$$\frac{d\theta}{dz} = z^{-\frac{1}{2}} \frac{dy}{dz} - \frac{1}{2} y z^{-\frac{3}{2}} \quad (7.9)$$

$$\frac{d^2\theta}{dz^2} = z^{-\frac{1}{2}} \frac{d^2y}{dz^2} - z^{-\frac{3}{2}} \frac{dy}{dz} + \frac{3}{4} y z^{-\frac{5}{2}} \quad (7.10)$$

Using Eq. (7.8), Eq. (7.9) and Eq. (7.10), the θ equation of motion given in Eq. (7.7) can be transformed as

$$\frac{d^2 y}{dz^2} + \frac{1}{z} \frac{dy}{dz} + \left(k^2 - \frac{(1/2)^2}{z^2} \right) y = 0 \quad (7.11)$$

The standard form of the Bessel equation given in reference [36] is as follows

$$\frac{d^2 y}{dz^2} + \frac{1}{z} \frac{dy}{dz} + \left(k^2 - \frac{v^2}{z^2} \right) y = 0 \quad (7.12)$$

and the complete solution for Eq. (7.12) when v is non-integral, is given by

$$y = AJ_v(kz) + BJ_{-v}(kz) \quad (7.13)$$

where A and B are constants whose values can be determined using initial conditions. The Bessel functions $J_v(kz)$ and $J_{-v}(kz)$ are given by

$$J_v = \sum_{r=0}^{\infty} \left((-1)^r \frac{\left(\frac{1}{2}kz\right)^{v+2r}}{r! \Gamma(v+r+1)} \right) \quad (7.14)$$

$$J_{-v} = \sum_{r=0}^{\infty} \left((-1)^r \frac{\left(\frac{1}{2}kz\right)^{-v+2r}}{r! \Gamma(-v+r+1)} \right) \quad (7.15)$$

where $\Gamma(*)$ is the Gamma function defined as

$$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt \quad (7.16)$$

By comparing the equation of motion (EOM) given in Eq. (7.11) and the standard Bessel equation given in Eq. (7.12), the analytical solution for the EOM can be written as

$$y = AJ_{1/2}(kz) + BJ_{-1/2}(kz) \quad (7.17)$$

The Bessel function $J_{1/2}(kz)$ can be written using Eq. (7.14) as

$$J_{1/2}(kz) = \frac{\left(\frac{1}{2}kz\right)^{1/2}}{\Gamma(3/2)} \left\{ 1 - \frac{(kz)^2}{2.3} + \frac{(kz)^4}{2.3.4.5} - \dots \right\} \quad (7.18)$$

The value of the function $\Gamma(3/2)$ is calculated to be $\frac{1}{2}\pi^{\frac{1}{2}}$. Using this value and rearranging Eq. (7.18), one arrives at

$$\begin{aligned} J_{1/2}(kz) &= \left(\frac{2}{\pi kz}\right)^{1/2} \left\{ kz - \frac{(kz)^3}{2.3} + \frac{(kz)^5}{2.3.4.5} - \dots \right\} \\ J_{1/2}(kz) &= \left(\frac{2}{\pi kz}\right)^{1/2} \sin(kz) \end{aligned} \quad (7.19)$$

Similarly, the expression for the $J_{-1/2}(kz)$ boils down to

$$J_{-1/2}(kz) = \left(\frac{2}{\pi kz}\right)^{1/2} \cos(kz) \quad (7.20)$$

The analytical solution for the out-of-plane angular motion θ can be written by combining Eq. (7.8), Eq. (7.17), Eq. (7.19) and Eq. (7.20) as

$$\theta = z^{-\frac{1}{2}} \left(A \left(\frac{2}{\pi kz}\right)^{1/2} \sin(kz) + B \left(\frac{2}{\pi kz}\right)^{1/2} \cos(kz) \right) \quad (7.21)$$

Substituting back the definitions of z and k in Eq. (7.21), one arrives at the expression for θ as a function of time t , which is written as

$$\theta(t) = \left(\frac{\dot{L}_{\text{ref}}}{\pi\Omega}\right)^{1/2} \frac{1}{L_0 + \dot{L}_{\text{ref}} t} \left(A \sin\left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t\right) + B \cos\left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t\right) \right) \quad (7.22)$$

The term $\left(\frac{\dot{L}_{\text{ref}}}{\pi\Omega}\right)^{1/2}$ is a constant and can be absorbed in to the arbitrary constants A and

B. Therefore, Eq. (7.22) can be rewritten as

$$\theta(t) = \frac{1}{L_0 + \dot{L}_{\text{ref}} t} \left(A \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t \right) + B \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t \right) \right) \quad (7.23)$$

The arbitrary constants A and B can be evaluated using the initial conditions and the analytical solution given in Eq. (7.23) can be further simplified using trigonometric identities.

7.3 Initial Conditions

Let the out-of-plane angle (θ) and its rate ($\dot{\theta}$) at $t = 0$ be

$$\theta(0) = \theta_0 \quad (7.24)$$

$$\dot{\theta}(0) = \dot{\theta}_0 \quad (7.25)$$

From Eq. (7.23), the expression for θ at $t = 0$ can be written as

$$\theta(0) = \frac{1}{L_0} \left(A \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) + B \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) \right) \quad (7.26)$$

Comparing Eq. (7.24) and Eq. (7.26), one can write

$$A \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) + B \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) = L_0 \theta_0 \quad (7.27)$$

Taking the time derivative of Eq. (7.23), one gets the expression for angle rate ($\dot{\theta}$) as

$$\begin{aligned} \dot{\theta}(t) = & \frac{-\dot{L}_{\text{ref}}}{(L_0 + \dot{L}_{\text{ref}} t)^2} \left[A \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t \right) + B \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t \right) \right] \\ & + \frac{2\Omega}{L_0 + \dot{L}_{\text{ref}} t} \left[A \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t \right) - B \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t \right) \right] \end{aligned} \quad (7.28)$$

The expression for $\dot{\theta}(t)$ at $t = 0$ can be derived from Eq. (7.28) as

$$\begin{aligned}\dot{\theta}(0) = & \frac{-\dot{L}_{\text{ref}}}{L_0^2} \left[A \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) + B \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) \right] \\ & + \frac{2\Omega}{L_0} \left[A \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) - B \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) \right]\end{aligned}\quad (7.29)$$

Again, by comparing Eq. (7.29) and the initial condition given in Eq. (7.25), one can write

$$\begin{aligned}A \left[\frac{-\dot{L}_{\text{ref}}}{L_0^2} \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) + \frac{2\Omega}{L_0} \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) \right] + B \left[\frac{-\dot{L}_{\text{ref}}}{L_0^2} \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) - \frac{2\Omega}{L_0} \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) \right] \\ = \dot{\theta}_0\end{aligned}\quad (7.30)$$

Solving equations Eq. (7.27) and Eq. (7.30) for A and B gives

$$\begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) & \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) \\ \frac{-\dot{L}_{\text{ref}}}{L_0^2} \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) + \frac{2\Omega}{L_0} \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) & \frac{-\dot{L}_{\text{ref}}}{L_0^2} \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) - \frac{2\Omega}{L_0} \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) \end{bmatrix}^{-1} \begin{bmatrix} L_0 \theta_0 \\ \dot{\theta}_0 \end{bmatrix}\quad (7.31)$$

$$\Rightarrow A = \left(\frac{\dot{L}_{\text{ref}} \theta_0 + L_0 \dot{\theta}_0}{2\Omega} \right) \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) + L_0 \theta_0 \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right)\quad (7.32)$$

$$\Rightarrow B = - \left(\frac{\dot{L}_{\text{ref}} \theta_0 + L_0 \dot{\theta}_0}{2\Omega} \right) \sin \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right) + L_0 \theta_0 \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} \right)\quad (7.33)$$

Thus, Eq. (7.32) and Eq. (7.33) give the expressions for the arbitrary constants A and B .

The analytical solution given in Eq. (7.23) can be further simplified into a single trigono-

metric function with a phase angle and amplitude. Recall the trigonometric identity

$$A \sin \theta + B \cos \theta = \sqrt{A^2 + B^2} \cos \left(\theta - \tan^{-1} \left(\frac{A}{B} \right) \right) \quad (7.34)$$

One can easily prove this identity by multiplying and dividing the left hand side (LHS) of the equation by $\sqrt{A^2 + B^2}$. The proof is given below.

$$\begin{aligned} A \sin \theta + B \cos \theta &= \frac{\sqrt{A^2 + B^2}}{\sqrt{A^2 + B^2}} (A \sin \theta + B \cos \theta) \\ &= \sqrt{A^2 + B^2} \left(\frac{A}{\sqrt{A^2 + B^2}} \sin \theta + \frac{B}{\sqrt{A^2 + B^2}} \cos \theta \right) \\ &= \sqrt{A^2 + B^2} (\sin \phi \sin \theta + \cos \phi \cos \theta) \end{aligned} \quad (7.35)$$

where ϕ is given by

$$\phi = \tan^{-1} \left(\frac{A}{B} \right)$$

Using the trigonometric identity $\cos(x - y) = \cos x \cos y + \sin x \sin y$ in Eq. (7.35), one gets

$$\begin{aligned} A \sin \theta + B \cos \theta &= \sqrt{A^2 + B^2} \cos(\theta - \phi) \\ &= \sqrt{A^2 + B^2} \cos \left(\theta - \tan^{-1} \left(\frac{A}{B} \right) \right) \end{aligned}$$

Now, using the trigonometric identity given in Eq. (7.34), the analytical expression for $\theta(t)$ given in Eq. (7.23) can be rewritten as

$$\theta(t) = \frac{1}{L_0 + \dot{L}_{\text{ref}} t} \sqrt{A^2 + B^2} \cos \left(\frac{2\Omega L_0}{\dot{L}_{\text{ref}}} + 2\Omega t - \tan^{-1} \left(\frac{A}{B} \right) \right) \quad (7.36)$$

Therefore, Eq. (7.36) gives the final form of the analytical solution with constants A and B defined in Eq. (7.32) and Eq. (7.33).

7.4 Bounds on Initial Out-Of-Plane Angle

By studying Eq. (7.36), one can write the time varying amplitude of the out-of-plane angle θ as

$$[\theta(t)]_{\text{amp}} = \frac{1}{L_0 + \dot{L}_{\text{ref}} t} \sqrt{A^2 + B^2} \quad (7.37)$$

The amplitude expression in Eq. (7.37) can be further simplified by substituting the expressions for A and B from Eq. (7.32) and Eq. (7.33). The simplified amplitude expression in terms of initial out-of-plane angle θ_0 and initial rate $\dot{\theta}_0$ is given by

$$[\theta(t)]_{\text{amp}} = \frac{1}{L_0 + \dot{L}_{\text{ref}} t} \left[\left(\frac{\dot{L}_{\text{ref}} \theta_0 + L_0 \dot{\theta}_0}{2\Omega} \right)^2 + (L_0 \theta_0)^2 \right]^{\frac{1}{2}} \quad (7.38)$$

Note, all that we have derived till now in this chapter holds good for any general reconfiguration problem in the orbit radial direction. That is, the analytical solution using Bessel functions and the time-varying amplitude expression holds good for both expansion and contraction of the separation distance between the two satellites. It was shown in the previous chapter that during expansion the out-of-plane oscillations are reduced due to angular momentum conservation. Therefore, regardless of the initial out-of-plane angle at the beginning of the expansion, one is guaranteed to have a smaller out-of-plane oscillation at the end of the operation. The vise-versa is true for contracting the 2-craft Coulomb tether. The initial out-of-plane oscillation will increase as the satellites are brought closer due to conservation of angular momentum. We are interested in establishing a bound on the initial oscillation so that the final oscillations at the end of the contraction will be within certain limits. Let the maximum initial out-of-plane oscillation be $\theta_{0_{\text{max}}}$ and at this angle value the angular rate will be zero. Using this information the amplitude expression in Eq. (7.38) can

be rewritten as

$$[\theta(t)]_{\text{amp}} = \frac{\theta_{0\text{max}}}{L_0 + \dot{L}_{\text{ref}} t} \left[\left(\frac{\dot{L}_{\text{ref}}}{2\Omega} \right)^2 + (L_0)^2 \right]^{\frac{1}{2}} \quad (7.39)$$

For the given initial and final separation distances, and the constant rate of change of separation distance ($\dot{L}_{\text{ref}}$), the total time involved for the contraction operation can be determined. Let this time is given by t_{max} . The desired bound on the out-of-plane oscillation amplitude at the end of the time t_{max} is given by $[\theta(t_{\text{max}})]_{\text{amp}}$. Inserting these values in Eq. (7.39) results in

$$[\theta(t_{\text{max}})]_{\text{amp}} \geq \frac{\theta_{0\text{max}}}{L_0 + \dot{L}_{\text{ref}} t_{\text{max}}} \left[\left(\frac{\dot{L}_{\text{ref}}}{2\Omega} \right)^2 + (L_0)^2 \right]^{\frac{1}{2}} \quad (7.40)$$

Rearranging Eq. (7.40) by keeping $\theta_{0\text{max}}$ on one side and taking the remaining terms to the other side gives

$$\theta_{0\text{max}} \leq [\theta(t_{\text{max}})]_{\text{amp}} \frac{(L_0 + \dot{L}_{\text{ref}} t_{\text{max}})}{\sqrt{\left(\frac{\dot{L}_{\text{ref}}}{2\Omega} \right)^2 + (L_0)^2}} \quad (7.41)$$

The inequality given in Eq. (7.41) establishes the bounds on the initial out-of-plane angle value and satisfying this inequality will lead to an final out-of-plane oscillation that is less than the permissible value of $[\theta(t_{\text{max}})]_{\text{amp}}$.

7.5 Numerical Simulations

In this section, numeric simulations are carried out to illustrate the analytical solution and to verify the final out-of-plane oscillation predictions. We will concentrate on the maneuver involving the contraction of the separation distance between the two satellites. The initial

out-of-plane angle and angle rate are set to $\theta = 0.1$ radians and $\dot{\theta} = 0$ rad/sec, respectively. The satellites are assumed to be in GEO and hence, the mean orbit rate value is taken to be $\Omega = 7.2915 \times 10^{-5}$ rad/sec. The satellite formation is contracted from an initial separation distance of 25 m to 15 m in 1.8 days. The out-of-plane angle (θ) time history and amplitude of the oscillation (shown as bounds), found using the equations Eq. (7.23) and Eq. (7.38), are illustrated in Figure 7.1.

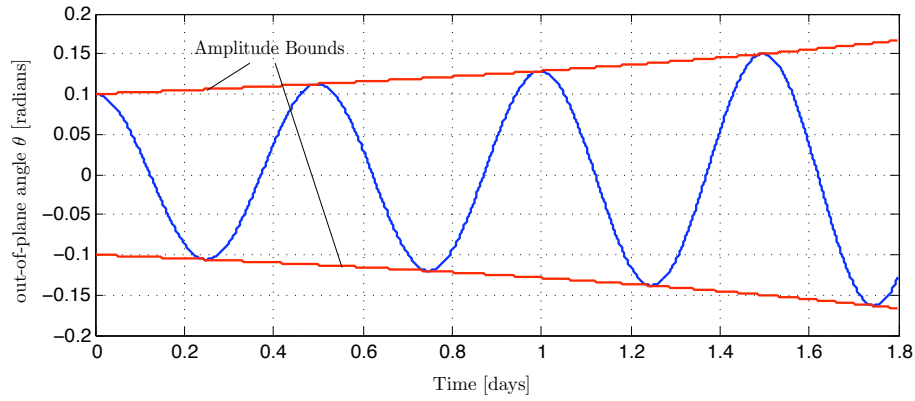


Figure 7.1: The time histories of the out-of-plane angular motion (θ) using analytical solution with the amplitude as bounds.

The out-of-plane angle equation of motion shown in Eq. (7.1) is the linearized decoupled equation. As in the previous chapter, we compare its performance with the full nonlinear equation given by the Cowell's equation. The initial conditions and gains used are same as in previous chapter and are given in Table. 6.1. Again, the satellite formation is contracted from an initial separation distance of 25 m to 15 m in 1.8 days. The results of this simulation are illustrated in the Figure 7.2. It can be observed from the figure that the analytical solution of the linearized equation for the out-of-plane angle (θ) closely follows the actual out-of-plane angle time history obtained by simulating the full nonlinear equation. Therefore, the analytical solution for the linearized equation can be used to set bounds on the

initial condition ($\theta(0)$ and $\dot{\theta}(0)$) so that the final out-of-plane angle is within prescribed limits.

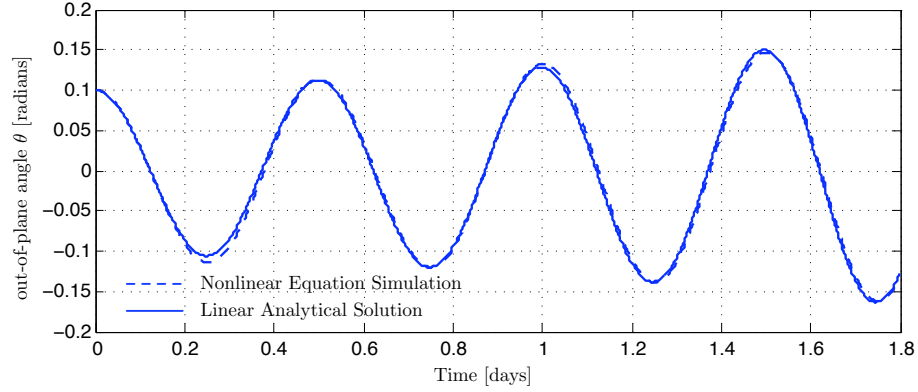


Figure 7.2: The time histories of the out-of-plane angular motion (θ) using the analytical solution and by simulating the full nonlinear equation.

From Figure 7.2, one can conclude that the final oscillation amplitude or peak out-of-plane angle after the completion of the contraction will be around 0.17 rad. We now calculate the bound on the initial θ value so that the final oscillation will not exceed 0.1 rad. Using Eq. (7.38) and substituting the corresponding values gives

$$\begin{aligned}
 \theta_{0_{\max}} &\leq [\theta(t_{\max})]_{\text{amp}} \frac{(L_0 + \dot{L}_{\text{ref}} t_{\max})}{\sqrt{\left(\frac{\dot{L}_{\text{ref}}}{2\Omega}\right)^2 + (L_0)^2}} \\
 &\leq (0.1) \left(\frac{25 - 6.43 \times 10^{-5} \times 1.56 \times 10^5}{\sqrt{\left(\frac{-6.43 \times 10^{-5}}{2 \times 7.2915 \times 10^{-5}}\right)^2 + (25)^2}} \right) \\
 &\leq 0.06 \text{ rad}
 \end{aligned} \tag{7.42}$$

Now, the simulation of the maneuver carried out previously and illustrated in Figure 7.2, is again carried out with initial conditions $\theta(0) = 0.06$ rad and $\dot{\theta}(0) = 0$ rad/sec. The

simulation results are illustrated in Figure 7.3. It can be seen from the figure that the final out-of-plane angular oscillation amplitude is 0.1 radians as expected.

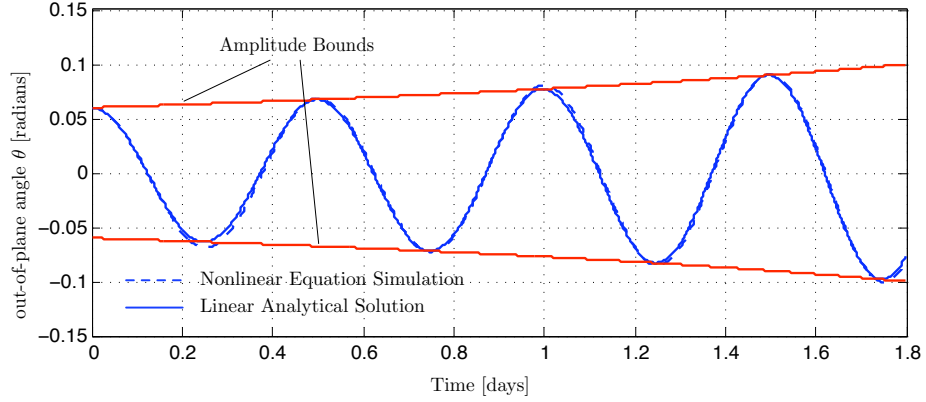


Figure 7.3: The time histories of the out-of-plane angular motion (θ) using the analytical solution and by simulating the full nonlinear equation. The initial θ value is 0.06 radians, resulting in the final out-of-plane angular oscillation amplitude of 0.1 radians.

By observing the expression given for the final amplitude in Eq. (7.39), one can see that it also depends on the initial separation distance L_0 and the constant rate of change of reference length $\dot{L}_{\text{ref}}$. The changes in the final oscillation amplitude for different $\dot{L}_{\text{ref}}$ shown in figure 7.4. The contraction maneuver performed with different $\dot{L}_{\text{ref}}$ involved contracting an initial separation distance of 25 m to 10 m and the initial out-of-plane angle error is 0.1 radians. From the figure it can be seen that the final amplitude increases with the contraction rate, as one would expect. Figure 7.5 illustrates behavior of the final oscillation amplitude for different initial separation distances. In all the cases, the formation is contracted by 10 m in 2 days and the initial out-of-plane angle error is 0.1 radians. The figure shows that the final oscillation amplitude is high for small initial separation distances and drops of significantly for large initial separation distances.

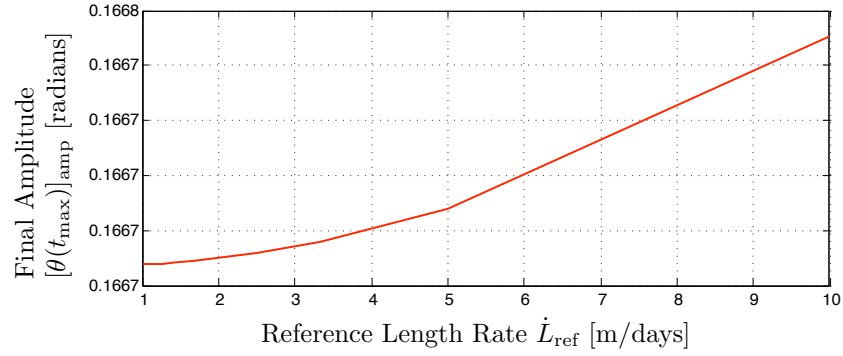


Figure 7.4: The final out-of-plane amplitude for different rate of contraction. In all the cases, the formation with initial separation distance of 25 m is contracted by 10 m and the initial out-of-plane angle error is 0.1 radians.

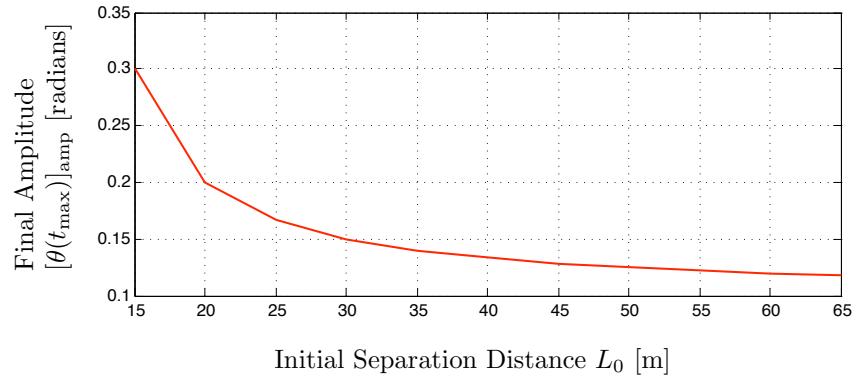


Figure 7.5: The final out-of-plane amplitude for different initial separation distance. In all the cases, the formation is contracted by 10 m in 2 days and the initial out-of-plane angle error is 0.1 radians.

7.6 Summary

The analytical solution for the linearized out-of-plane angle (θ) equation of motion for the reconfiguration problem is successfully developed using the Bessel functions. This solution

is used to come up with bounds on the initial out-of-plane angle so that the out-of-plane angular oscillations is within the prescribed limit at the end of the contraction operation. Numerical simulations of the full nonlinear motion are carried out to illustrate the results and compare the linearized performance predictions to the actual nonlinear system response.

8 Smooth Transition of Discontinuity in Reference Length Rate

8.1 Introduction

In Chapter 5, the reconfiguration of a Coulomb tether aligned along the orbit radial direction is studied in detail. The reconfiguration is an extension of the regulation problem studied in chapter 4, and the key difference being that the reference length (L_{ref}) is made time varying. The reference length rate ($\dot{L}_{\text{ref}}$) is user prescribed and bounds on it for maintaining stability has been identified using linear stability analysis. In the simulations that follow, a constant reference length rate ($\dot{L}_{\text{ref}}$) is used for expanding or contracting the formation. A Dirac delta function in the beginning of the maneuver facilitates the reference length rate to reach a constant value from zero instantaneously, and a similar function in the end of the maneuver facilitates the converse. The Dirac delta function results in an infinite acceleration at the point of transition which the finite Coulomb forces will not be able to produce. That is, one can not have infinite charge to produce the required infinite Coulomb force. This infinite acceleration results in noticeable oscillations in separation distance error (δL) which is asymptotically stabilized by the feedback term in the due course of time.

Our goal is to eliminate or reduce this oscillation of δL at the points of transition of reference length rate. In the absence of these oscillations the separation distance between

the two craft will closely follow the prescribed reference length time histories. The reduction in oscillation is achieved by using a smooth transition function for the reference length rate instead of the dirac delta function and by adding the finite acceleration of the reference length ($\ddot{L}_{\text{ref}}$) to the reference charge product Q_{ref} . This chapter is organized as follows. First, the smooth transition function is defined, followed by adapting this function for the reference length rate problem. Finally, numerical simulations illustrate the results.

8.2 Smooth Transition Function

Let us assume the function $F(t)$ jumps from zero to one at the point $t = t_0$. The expression for the smoothed representation of the jump discontinuity is discussed in this section. This function is based on the hyperbolic tangent function $\tanh(x)$. To represent a value jumping up at $t = t_0$, use

$$F(t) = \frac{1}{2} + \frac{1}{2} \tanh\left(\frac{t - t_0}{\sigma}\right) \quad (8.1)$$

The hyperbolic tangent function used in Eq. (8.1) is a monotonically growing one with the value approaching -1 as $t \rightarrow -\infty$ and approaching $+1$ as $t \rightarrow \infty$. A plot of this function is shown in Figure 8.1 and the plot shows that it is symmetric about the point $t = t_0$. An examination of the plot also shows that appreciable change in the value of the hyperbolic tangent function occurs while t lies in the interval $(t_0 - 16\sigma)$ and $(t_0 + 16\sigma)$. In fact, because of the finite precision involved in computing, the value of hyperbolic tangent function will get rounded off to $+1$ for values of t beyond $(t_0 + 16\sigma)$, and to -1 for values of t below $(t_0 - 16\sigma)$.

Now consider the smooth transition function given in Eq. (8.1). Figure 8.2 illustrates the function behavior over the time interval $(t_0 - 16\sigma)$ and $(t_0 + 16\sigma)$. Note, this time interval is chosen since the smooth transition function depends on the hyperbolic tangent

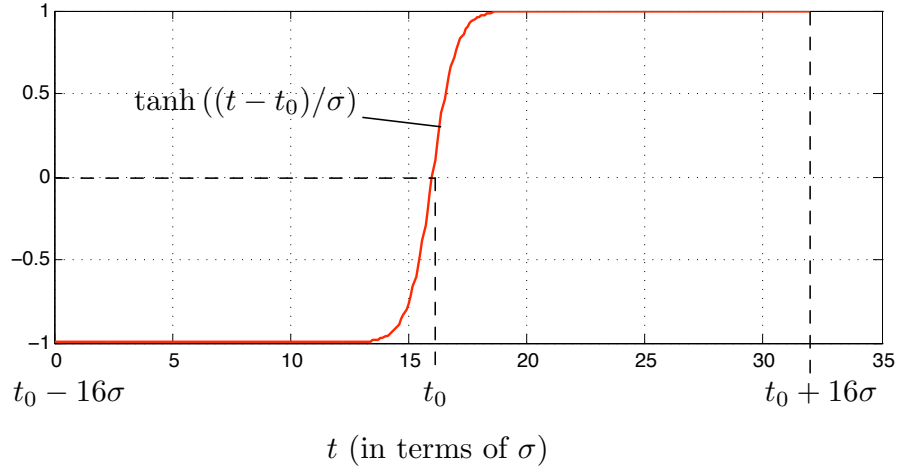


Figure 8.1: Time history of the hyperbolic tangent function.

and appreciable change in hyperbolic tangent is in this interval. It is clear from the plot that $F(t)$ would vary between 0 and 1 over the interval $(t_0 - 16\sigma)$ and $(t_0 + 16\sigma)$. The interval can be made as small as desired by choosing an appropriate value of σ , thus enabling the smoothed representation of function jumping from 0 to 1 at $t = t_0$. Moreover, $t = t_0$ is a point of inflection with the value of the function equal to $1/2$. The function $1.0 - F(t)$ can be used to represent sudden fall from 1 to 0.

Function $F(t)$ is differentiable to any order, and the derivatives of all orders would tend to zero practically at $(t_0 \pm 16\sigma)$. This matches near ideally the characteristics of the constancy function before and after the jump. The time derivative of function $F(t)$ is obtained using the identity $\text{sech}^2(x) = 1.0 - \tanh^2(x)$, as

$$\dot{F}(t) = \frac{1}{2\sigma} \text{sech}^2\left(\frac{t - t_0}{\sigma}\right) \quad (8.2)$$

The derivative of $F(t)$ given in Eq. (8.2) is plotted in Figure 8.3. It can be seen from the

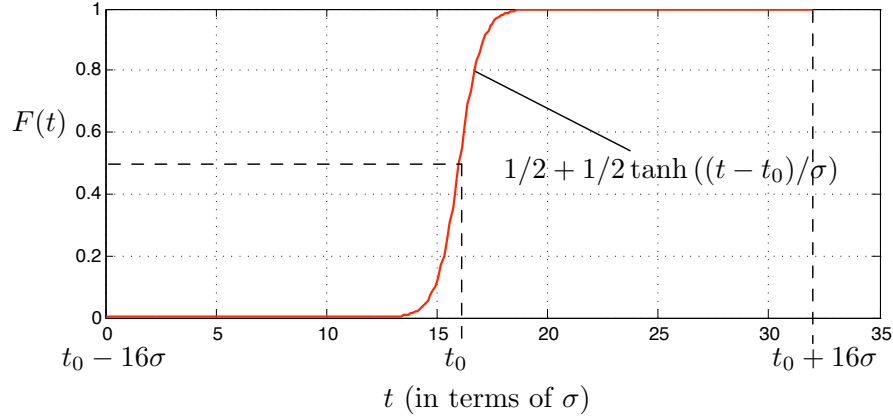


Figure 8.2: Time history of the smooth transition function $F(t)$.

plot that the $\dot{F}(t)$ reaches its maximum value at $t = t_0$.

8.3 Reference Length Rate Transition Function

In this section, we adapt the smooth transition function defined in the previous section, to fit the reference length rate transition problem. The time interval needed for making this transition is identified using the maximum charge available in the crafts. Let the constant reference length rate be given by K_L . The function representing the smooth transition of the reference length rate from zero to K_L is given by

$$\dot{L}_{\text{ref}}(t) = K_L \left(\frac{1}{2} + \frac{1}{2} \tanh \left(\frac{t - t_0}{\sigma} \right) \right) \quad (8.3)$$

where t_0 is the point of transition and this smooth transition takes place between the time interval $(t_0 - 16\sigma)$ to $(t_0 + 16\sigma)$. The reference length acceleration at any given point of

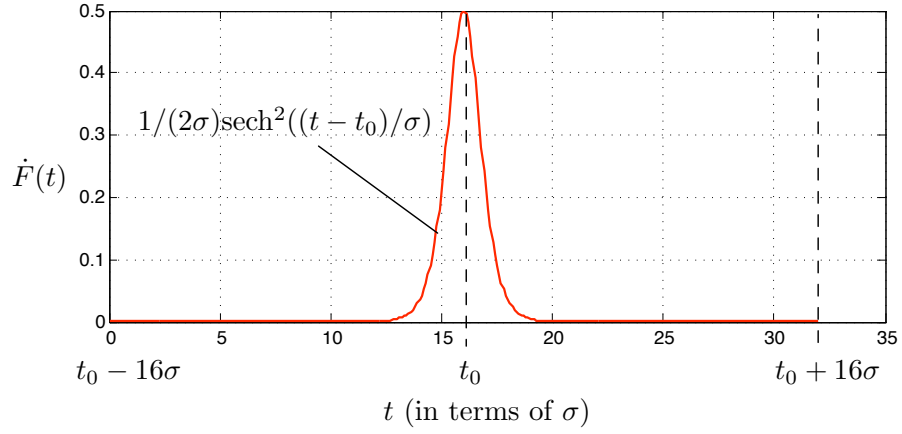


Figure 8.3: Time history of the time derivative of the smooth transition function ($\dot{F}(t)$).

time is given by

$$\ddot{L}_{\text{ref}}(t) = \frac{K_L}{2\sigma} \text{sech}^2\left(\frac{t - t_0}{\sigma}\right) \quad (8.4)$$

The maximum acceleration will be at $t = t_0$ and is written as

$$\ddot{L}_{\text{ref}}(t) = \frac{K_L}{2\sigma} \quad (8.5)$$

The change in reference length during this smooth transition can be found out by integrating Eq. (8.3) over the time period $(t_0 - 16\sigma)$ to $(t_0 + 16\sigma)$. The expression for this change in reference length is given as

$$L_{\text{ref}} = K_L \left[\frac{1}{2}t + \frac{1}{2} \ln \left(\cosh \left(\frac{t - t_0}{\sigma} \right) \right) \right]_{t_0 - 16\sigma}^{t_0 + 16\sigma} \quad (8.6)$$

Now, one has to identify the time period over which this transition should take place. In

other words, one has to identify the value of the quantity σ . The limiting factor for sigma will be the maximum available charge. Consider the equation of motion of the separation distance between two craft given below

$$\ddot{L} = (2n\dot{\psi} + 3\Omega^2)L + \frac{k_c}{m_1}Q \frac{1}{L^2} \frac{m_1 + m_2}{m_2} \quad (8.7)$$

where L is the separation distance, Q is the charge product, Ω is the mean orbit rate, k_c is the Coulomb constant and m_i is the mass of each craft. For an ideal equilibrium case, the in-plane angle rate ($\dot{\phi}$) will be zero and separation distance L will follow L_{ref} . Implementing this ideal scenario in Eq. (8.7) results in

$$\ddot{L}_{\text{ref}} = 3\Omega^2 L_{\text{ref}} + \frac{k_c}{m_1}Q \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2} \quad (8.8)$$

Substituting the maximum reference length acceleration from Eq. (8.5) in to Eq. (8.8) and by approximating L_{ref} as initial separation length L_0 , gives

$$\frac{K_L}{2\sigma} = 3\Omega^2 L_0 + \frac{k_c}{m_1}Q \frac{1}{L_0^2} \frac{m_1 + m_2}{m_2} \quad (8.9)$$

Note, the reference length ($L_{\text{ref}}(t_0)$) at maximum reference acceleration ($t = t_0$) is ideally found by integrating the reference length rate ($\dot{L}_{\text{ref}}$). To avoid the complex integral during the computation of σ , we are using the initial separation distance L_0 . Similarly, while finding the σ needed at the end of the maneuver, the final separation distance (L_f) will be used an approximation. Let the maximum available charge product be Q_{max} . Using this information in Eq. (8.9) and rearranging the equation gives the minimum σ required as

$$\sigma_{\text{min}} = \frac{K_L}{6\Omega^2 L_0 + 2\frac{k_c}{m_1}Q \frac{1}{L_0^2} \frac{m_1 + m_2}{m_2}} \quad (8.10)$$

In all our simulations, we will use at least twice this minimum σ , so that our charge requirement is well within the limit at any given time.

The reference charge product (Q_{ref}) used here is also slightly different from the one used in chapter 5. Usually, the reference charge product Q_{ref} is calculated by setting the left hand side of the equation Eq. (8.8) to zero. That is, the reference length acceleration $\ddot{L}_{\text{ref}}$ is set to zero. Instead, in this chapter we will incorporate it in to the reference charge product. The new reference charge product is given as

$$Q_{\text{ref}} = \frac{L_{\text{ref}}^2}{k_c} \frac{m_1 m_2}{m_1 + m_2} (\ddot{L}_{\text{ref}} - 3\Omega^2 L_{\text{ref}}) \quad (8.11)$$

This reference charge product could not be used in chapter 5 as the reference length acceleration ($\ddot{L}_{\text{ref}}$) shot to infinity due to the dirac delta function.

8.4 Numerical Simulation

In this section, the performance of the reference length rate transition function and the new reference charge product is illustrated using simulations. Consider a maneuver similar to chapter 5 simulations, of expanding the Coulomb tether formation from 25m to 35m in 1.8 days. The simulation parameters that used are listed in Table 8.1. The initial attitude values ($\psi(0)$, $\theta(0)$) and the separation length error ($\delta L(0)$) value are set to zero. All initial rates are also set to zero through $\dot{\psi} = \dot{\delta L} = \dot{\theta} = 0$. In the reconfiguration simulations in chapter 5, the oscillations are due to both initial condition errors and the sudden jumps in reference length (L_{ref}) rate due to the dirac delta function. In order to isolate and show that the oscillations due to the latter have been eliminated, the initial conditions are all taken to be zero.

Before going to the simulation, one has calculate the necessary σ values. Initially, at the start of the reconfiguration the reference length is 25 m and the maximum available charge

Table 8.1: Input parameters used in orbit-radial reconfiguration simulation with smooth transition

Parameter	Value	Units
m_1	150	kg
m_2	150	kg
k_c	8.99×10^9	Nm ² /C ²
Ω	7.2915×10^{-5}	rad/sec
$\delta L(0)$	0.0	m
$\psi(0)$	0.0	rad
$\theta(0)$	0.0	rad
$\tilde{C}_1$	12	
$\tilde{C}_2$	2.4249	

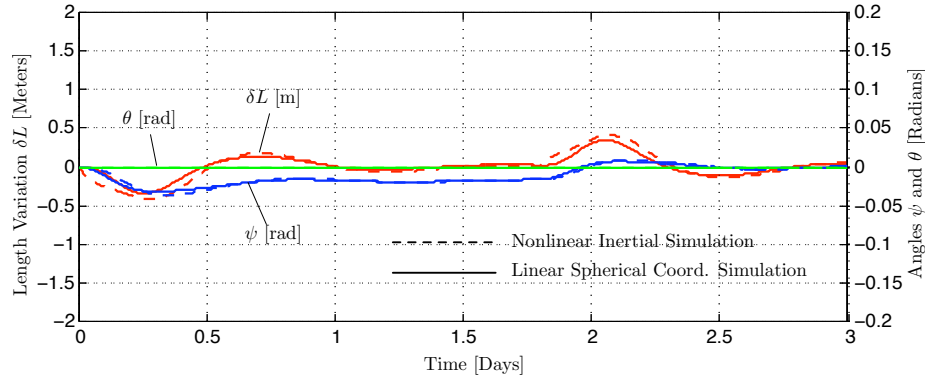
on each craft be $5 \mu C$. The minimum required σ can be calculated using Eq. (8.10) as

$$\sigma_{1_{\min}} = 6.1905$$

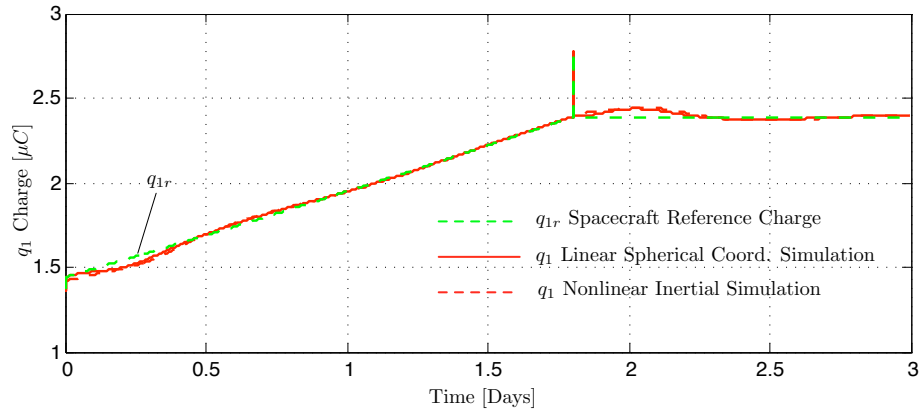
Similarly, the minimum σ value needed at the end of reconfiguration can be calculated using the final separation distance of 35 m as $\sigma_{2_{\min}} = 10.70$. We will take $\sigma_1 = 15$ and $\sigma_2 = 25$ which are more than twice the minimum required.

As in chapter 5 the Coulomb tether performance is simulated in two different manners. First the linearized spherical coordinate differential equations are integrated. Second, the exact nonlinear equations of motion of the deputy satellites are solved using the same charge feedback control, and compared to the performance of the linearized dynamical system. The nonlinear deputy equations are given through Cowell's equations and the spherical coordinates are computed back in post-processing using the exact kinematic transformation.

Figure 8.4(a) shows the Coulomb tether motion for increasing the separation distance from 25m to 35m in the linearized spherical coordinates $(\psi, \theta, \delta L)$, along with the full nonlinear spherical coordinates shown as dotted lines. The expansion is done in 1.8 days.



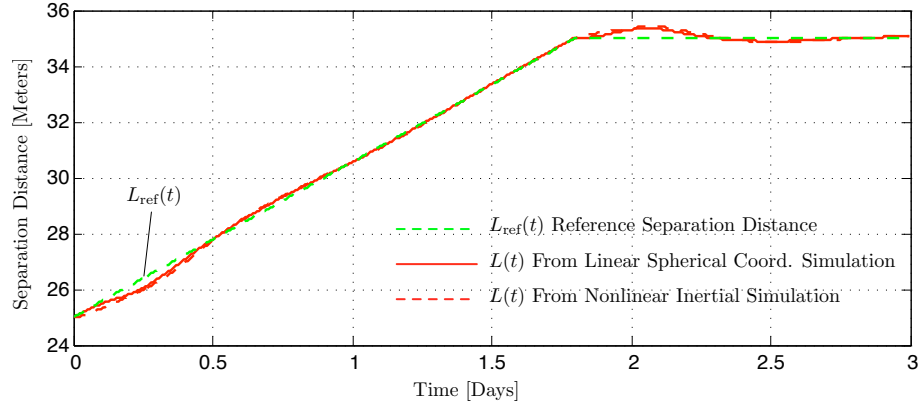
(a) Time histories of length variation δL , in-plane rotation angle ψ , and out-of-plane rotation angle θ .



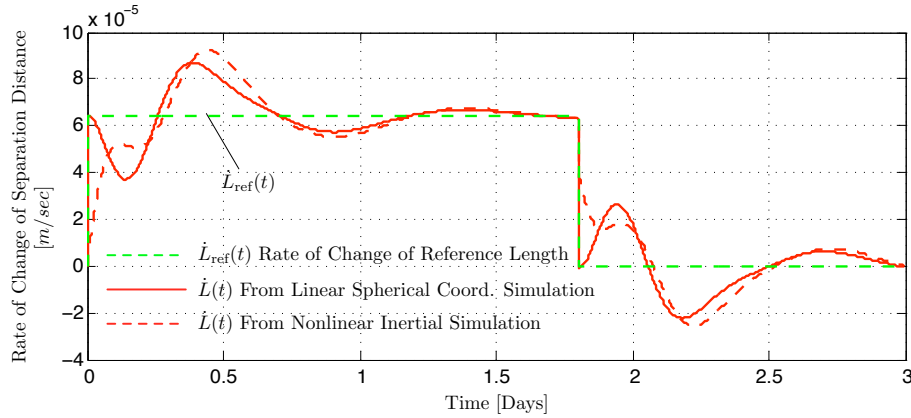
(b) Spacecraft charge time histories

Figure 8.4: Simulation results for expanding the spacecraft separation distance from 25 m to 35 m in 1.8 days. The feedback gains are $\hat{C}_1 = 12$ and $\hat{C}_2 = 2.4249$.

It can be seen from the figure that the oscillations in the in-plane angle (ψ) and separation distance error (δL) are significantly reduced. The in-plane angle is coupled to the separation distance error equation through the in-plane angle rate ($\dot{\psi}$), which is not modeled in to the reference charge. This is the reason for the initial oscillation even though the initial error in the states are zero. The out-of-plane angle (θ) is constantly zero since it is decoupled from



(a) Time histories of separation distance.



(b) Time histories of rate of change of separation distance

Figure 8.5: Simulation results for expanding the spacecraft separation distance from 25 m to 35 m in 1.8 days. The feedback gains are $\hat{C}_1 = 12$ and $\hat{C}_2 = 2.4249$.

the other two states and its initial states are zero to begin with. Figure 8.4(b) shows the spacecraft control charge q_1 (on craft 1) for both the linearized and full nonlinear simulation models. Both are nearly on top of the reference value pertaining to the static equilibrium at each instant of time. The spikes in control charge observed in the graph are due to the finite reference length accelerations during the smooth reference length rate transition.

In order to illustrate how well the system is tracking the prescribed reference trajectory $L_{\text{ref}}(t)$, the time histories of separation distance $L(t)$ and the time histories of rate of change of separation distance $\dot{L}(t)$ are shown in Figure 8.5(a) and Figure 8.5(b) , respectively. Figure 8.5(a) shows that the reference separation distance ($L_{\text{ref}}(t)$) increases linearly until 1.8 days before settling to a constant value and both the linear and inertial nonlinear simulations track the reference separation distance perfectly. Figure 8.5(b) illustrates that the rate of change of the reference separation distance ($\dot{L}_{\text{ref}}(t)$) is a smoothed representation of a discrete step change.

8.5 Summary

A smooth transition function is used in the beginning and end of the prescribed reference length rate. This eliminates the abrupt increase or decrease of the reference length rate resulting in a finite acceleration at the points of transition. This finite reference length acceleration has been added to the reference charge product term to achieve near perfect tracking. Simulation results in the end validate the claim.

9 Reconfiguration Along Orbit-Normal and Along-Track Equilibrium

9.1 Introduction

The reconfiguration of a two-craft formation aligned along the orbit radial direction is presented in detail in chapter 6. Analogously, the reconfiguration of two-craft Coulomb formation aligned along the orbit-normal and along-track direction is studied in this chapter. With this study, the two-craft Coulomb structure regulation and reconfiguration study along all three equilibria will be complete. Similar to the orbit radial direction reconfiguration, this study is also an extension of the along-track and orbit-normal regulation problem studied in Chapter 5. An hybrid control strategy using both Coulomb forces and conventional thrusters is introduced and the linear stability of the time variant system is studied based on the method devised by Rosenbrock.³⁴ Based on this analysis, stability regions for expanding and contracting the two-craft formations are established. As in the previous studies, the formation is studied in GEO and the Debye lengths are assumed to be sufficiently large so that the effects of Debye shielding can be neglected. Finally, numerical simulations illustrate the analytical stability predictions.

9.2 Reconfiguration Dynamics

9.2.1 Along-Track Configuration

This section derives the equations of motion for reconfiguring a 2-craft Coulomb tether which is nominally aligned with the along-track direction $\hat{\mathbf{o}}_\theta$ of the orbit or Hill frame $\mathcal{O} : \{\hat{\mathbf{o}}_r, \hat{\mathbf{o}}_\theta, \hat{\mathbf{o}}_h, \}$. This derivation closely follows the derivation of the equations of motion for along-track formation given in Chapter 5. The differential equation of motion for the separation distance L between the craft is given by

$$\ddot{L} = 2\Omega\dot{\psi}L + \frac{k_c}{m_1}Q\frac{1}{L^2}\frac{m_1 + m_2}{m_2} \quad (9.1)$$

where $Q = q_1q_2$ is the charge product of the spacecraft charges and $\dot{\psi}$ in-plane angular rate. Next the separation distance equations of motion are linearized about small variations in length δL and small variations in the product charge term δQ . Unlike the regulation problem the reference separation length L_{ref} is not constant, but is made time varying. The reference charge product term for this along-track configuration is known to be zero.

$$L(t) = L_{\text{ref}}(t) + \delta L(t) \quad (9.2a)$$

$$Q(t) = Q_{\text{ref}}(t) + \delta Q(t) = \delta Q(t) \quad (9.2b)$$

Substituting these L and Q definitions into Eq. (9.1) and linearizing leads to

$$\delta\ddot{L} = (2\Omega L_{\text{ref}})\dot{\psi} + \left(\frac{k_c}{m_1} \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2} \right) \delta Q - \ddot{L}_{\text{ref}} \quad (9.3)$$

Note that this relationship is coupled to the angular in-orbit-plane rate $\dot{\psi}$. In order to obtain an expression for this, a stability analysis using the gravity gradient is employed. The linearized attitude dynamics of the Coulomb tether body frame are written along with

the separation distance equation as:

$$\ddot{\phi} + 2\frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}}\dot{\phi} + \Omega^2\phi = 0 \quad (9.4a)$$

$$\ddot{\psi} + 2\frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}}\Omega + 2\frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}}\dot{\psi} - 2\frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}}\Omega\delta L + 2\frac{\Omega}{L_{\text{ref}}}\delta\dot{L} - 3\Omega^2\psi = 0 \quad (9.4b)$$

$$\delta\ddot{L} + \ddot{L}_{\text{ref}} - (2\Omega L_{\text{ref}})\dot{\psi} - \left(\frac{k_c}{m_1} \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2}\right)\delta Q = 0 \quad (9.4c)$$

Note that the out-of-plane angle ϕ is decoupled from the separation distance error δL and in-plane angle ψ . The charge product variation δQ is treated as the control variable and the feedback control law is defined as

$$\delta Q = \frac{m_1 m_2 L_{\text{ref}}^2}{(m_1 + m_2)k_c}(-C_1\delta L - C_2\delta\dot{L}) \quad (9.5)$$

where C_1 and C_2 are the position and velocity feedback gain. Recall from chapter 5 that Coulomb forces alone are not sufficient to guarantee asymptotic stability for this formation and one requires thrusting along the $\hat{b}_1$ and $\hat{b}_3$ axes. The thrust force feedback law are given by

$$F_1 = \frac{m_1 m_2}{m_1 + m_2} L_{\text{ref}}(K_1\psi) \quad (9.6)$$

$$F_3 = \frac{m_1 m_2}{m_1 + m_2} L_{\text{ref}}(K_2\dot{\phi}) \quad (9.7)$$

where K_1 and K_2 are the in-plane and out-of-plane angle feedback gain.. By introducing these forces and charge control law back in to the equations given in Eq. (9.4), the augmented

equations of motion are written as

$$\ddot{\phi} + \left(2 \frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}} + K_2 \right) \dot{\phi} + \Omega^2 \phi = 0 \quad (9.8a)$$

$$\ddot{\psi} + 2 \frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}} \Omega + 2 \frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}} \dot{\psi} - 2 \frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}} \Omega \delta L + 2 \frac{\Omega}{L_{\text{ref}}} \delta \dot{L} + (K_1 - 3\Omega^2) \psi = 0 \quad (9.8b)$$

$$\delta \ddot{L} + \ddot{L}_{\text{ref}} - (2\Omega L_{\text{ref}}) \dot{\psi} + C_1 \delta L + C_2 \delta \dot{L} = 0 \quad (9.8c)$$

The following normalization transformation is employed to make these equations independent of Ω .

$$d\tau = \Omega dt \quad (9.9a)$$

$$(*)' = \frac{d(*)}{d\tau} = \frac{1}{\Omega} \frac{d(*)}{dt} \quad (9.9b)$$

The orbit rate independent form of the linearized equations in Eq. (9.8a) – (9.8c) are written as

$$\phi'' + \left(2 \frac{L'_{\text{ref}}}{L_{\text{ref}}} + \tilde{K}_2 \right) \phi' + \phi = 0 \quad (9.10a)$$

$$\psi'' + 2 \frac{L'_{\text{ref}}}{L_{\text{ref}}} + 2 \frac{L'_{\text{ref}}}{L_{\text{ref}}} \psi' - 2 \frac{L'_{\text{ref}}}{L_{\text{ref}}} \delta L + \frac{2}{L_{\text{ref}}} \delta L' + (\tilde{K}_1 - 3) \psi = 0 \quad (9.10b)$$

$$\delta L'' + L''_{\text{ref}} - (2L_{\text{ref}}) \psi' + \tilde{C}_1 \delta L + \tilde{C}_2 \delta L' = 0 \quad (9.10c)$$

where $\tilde{C}_1 = C_1/\Omega^2$, $\tilde{C}_2 = C_2/\Omega$, $\tilde{K}_1 = K_1/\Omega^2$ and $\tilde{K}_2 = K_2/\Omega$ are the non-dimensionalized gains.

9.2.2 Orbit Normal Configuration

The derivation of the equations of motion for reconfiguration of 2-craft Coulomb tether along orbit normal direction follows the same steps as those of the along-track equilibrium.

One key difference is that the analytical expression for the orbit normal reference charge product Q_{ref} is not zero and is given by¹³

$$Q_{\text{ref}}(t) = q_1 q_2 = \Omega^2 \frac{L_{\text{ref}}^3(t)}{k_c} \frac{m_1 m_2}{m_1 + m_2} \quad (9.11)$$

The differential equation for the separation distance is given by

$$\ddot{L} = -\Omega^2 L + \frac{k_c}{m_1} Q \frac{1}{L^2} \frac{m_1 + m_2}{m_2} \quad (9.12)$$

The above equation can be further linearized using Eqs. (9.2) and the Q_{ref} definition in Eq. (9.11) to

$$\delta \ddot{L} = -(3\Omega^2) \delta L + \left(\frac{k_c}{m_1} \frac{1}{L_{\text{ref}}^2} \frac{m_1 + m_2}{m_2} \right) \delta Q - \ddot{L}_{\text{ref}} \quad (9.13)$$

The spherical coordinate representation of this formation involves two Euler angles θ and ϕ , which are decoupled from the separation distance differential equation. The differential equation for Euler angles can be obtained similar to the along-track development. The linearized attitude dynamics of the Coulomb tether are written along with the separation distance equation as:

$$\ddot{\phi} - \Omega^2 \phi - 2\Omega \dot{\theta} + 2 \frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}} (\dot{\phi} - \Omega \theta) = 0 \quad (9.14a)$$

$$\ddot{\theta} - 4\Omega^2 \theta + 2\Omega \dot{\phi} + 2 \frac{\dot{L}_{\text{ref}}}{L_{\text{ref}}} (\dot{\theta} + \Omega \phi) = 0 \quad (9.14b)$$

$$\delta \ddot{L} + \ddot{L}_{\text{ref}} + (3\Omega^2) \delta L - \left(\frac{m_1 + m_2}{m_1 m_2} \frac{k_c}{L_{\text{ref}}^2} \right) \delta Q = 0 \quad (9.14c)$$

As shown in chapter 5, δQ will be one of the control signals and we will introduce two thrust forces along the $\hat{b}_1$ and $\hat{b}_2$ axes that will stabilize the angles. The feedback control

laws for the charge product error term and thrust forces are given by

$$\delta Q = \frac{m_1 m_2 L_{\text{ref}}^2}{(m_1 + m_2) k_c} (-C_1 \delta L - C_2 \delta \dot{L}) \quad (9.15)$$

$$F_1 = \frac{m_1 m_2}{m_1 + m_2} L_{\text{ref}} (K_2 \theta) \quad (9.16)$$

$$F_2 = \frac{m_1 m_2}{m_1 + m_2} L_{\text{ref}} (K_1 \phi + K_3 \dot{\phi}) \quad (9.17)$$

where C_1 and C_2 are the position and velocity feedback gain, K_1 and K_2 are the out-of-plane angle feedback gain, and K_3 is the angle rate feedback gain. By introducing these feedback laws in Eqs. (9.14a)–(9.14c) and using the transformation given in Eq. (9.9), one can get the final non-dimensional form of the equation of motion as

$$\phi'' + (\tilde{K}_1 - 1)\phi - 2\theta' + \tilde{K}_3\phi' + 2\frac{L'_{\text{ref}}}{L_{\text{ref}}}(\phi' - \theta) = 0 \quad (9.18a)$$

$$\theta'' + (\tilde{K}_2 - 4)\theta + 2\phi' + 2\frac{L'_{\text{ref}}}{L_{\text{ref}}}(\theta' + \phi) = 0 \quad (9.18b)$$

$$\delta L'' + L''_{\text{ref}} + (3 + \tilde{C}_1)\delta L + \tilde{C}_2\delta L' = 0 \quad (9.18c)$$

where $\tilde{C}_1 = C_1/\Omega^2$, $\tilde{C}_2 = C_2/\Omega$, $\tilde{K}_1 = K_1/\Omega^2$, $\tilde{K}_2 = K_2/\Omega^2$ and $\tilde{K}_3 = K_3/\Omega$ are the non-dimensionalized gains.

9.3 Stability Analysis

The stability analysis for the time varying equations of motion derived in the previous section will be carried out using the method put forward by Rosenbrock.³⁴ This method is discussed in detail in chapter 6 and is explained here for the sake of continuity. A linear time-dependent system given by $\dot{\mathbf{x}} = A(t)\mathbf{x}$ is asymptotically stable if the frozen system for each t is stable and the rate of change of $A(t)$ is very small. Rosenbrock³⁴ has established bounds on this rate of change of $A(t)$ when $A(t)$ is in the control canonical form.

9.3.1 Along-Track Formation

The coupled δL and ψ equations in Eq. (9.10b)–(9.10c) are written in the state space form as

$$\begin{pmatrix} \psi' \\ \psi'' \\ \delta L' \\ \delta L'' \end{pmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 3 - \tilde{K}_1 & -2\frac{L'_{\text{ref}}}{L_{\text{ref}}} & 2\frac{L'_{\text{ref}}}{L_{\text{ref}}^2} & -\frac{2}{L_{\text{ref}}} \\ 0 & 0 & 0 & 1 \\ 0 & 2L_{\text{ref}} & -\tilde{C}_1 & -\tilde{C}_2 \end{bmatrix}}_{A(t)} \begin{pmatrix} \psi \\ \psi' \\ \delta L \\ \delta L' \end{pmatrix} + \underbrace{\begin{pmatrix} 0 \\ -\frac{2L'_{\text{ref}}}{L_{\text{ref}}} \\ 0 \\ -L''_{\text{ref}} \end{pmatrix}}_{d(t)} \quad (9.19)$$

The square matrix in the above equation is $A(t)$ and the time dependency in this matrix is due to the terms L_{ref} and L'_{ref} . The stability of the system greatly depends on the rate at which L_{ref} is varied. The limits on how large L'_{ref} can be while still guaranteeing stability, will be established later in this section. From Eq. (9.19), it can be observed that there is a state independent term $d(t)$ which will lead to a steady state offset as long as L_{ref} is time varying. The analytical expression for the steady state offset is given as follows

$$\begin{pmatrix} \psi_{\text{offset}} \\ \delta L_{\text{offset}} \end{pmatrix} = \begin{pmatrix} -\frac{2L'_{\text{ref}}}{(\tilde{K}_1 - 3)L_{\text{ref}}} + \frac{2L'_{\text{ref}}L''_{\text{ref}}}{(\tilde{K}_1 - 3)\tilde{C}_1 L_{\text{ref}}^2} \\ \frac{L''_{\text{ref}}}{\tilde{C}_1} \end{pmatrix} = \begin{pmatrix} -\frac{2\Omega\dot{L}_{\text{ref}}}{(K_1 - 3\Omega^2)L_{\text{ref}}} + \frac{2\Omega\dot{L}_{\text{ref}}\ddot{L}_{\text{ref}}}{(K_1 - 3\Omega^2)\tilde{C}_1 L_{\text{ref}}^2} \\ \frac{\ddot{L}_{\text{ref}}}{\tilde{C}_1} \end{pmatrix} \quad (9.20)$$

Initially, the characteristic equation of $A(t)$ matrix is studied to identify the gains that will make the matrix Hurwitz. The characteristic equation is given by

$$\begin{aligned} \lambda^4 + (\tilde{C}_2 + 2k)\lambda^3 + (\tilde{C}_1 + \tilde{K}_1 + 2k\tilde{C}_2 + 1)\lambda^2 + (\tilde{C}_2\tilde{K}_1 - 3\tilde{C}_2 + 2k\tilde{C}_1 - 4k)\lambda \\ + (-3\tilde{C}_1 + \tilde{C}_1\tilde{K}_1) = 0 \end{aligned} \quad (9.21)$$

where $k = \frac{L'_{\text{ref}}}{L_{\text{ref}}^2}$ is a time varying coefficient. In the along-track regulation problem given in

chapter 5, the gain values are determined to be $\tilde{K}_1 = 6$ and $\tilde{C}_1 = 2.97$. Since, this work is an extension of the regulation problem, the same gains can be used here. The velocity gain $\tilde{C}_2$ is defined using a scaling factor as shown below.

$$\tilde{C}_2 = \alpha \sqrt{\tilde{C}_1}$$

Substituting these gains back in to Eq. (9.21) results in a characteristic equation whose coefficients are a function of k and α . The range of values of k and α that guarantee negative definite roots for the characteristic equation can be found out using Routh-Hurwitz stability criterion. The shaded region in Figure 9.1 shows the possible values of k and α for the fixed gain values of $\tilde{K}_1 = 6$ and $\tilde{C}_1 = 2.97$. It can be seen from the figure that the positive k indicating expansion is always stable, whereas negative k (contraction) has a tight bound. The value of α is chosen as 1.8 since k has maximum range at this α value.

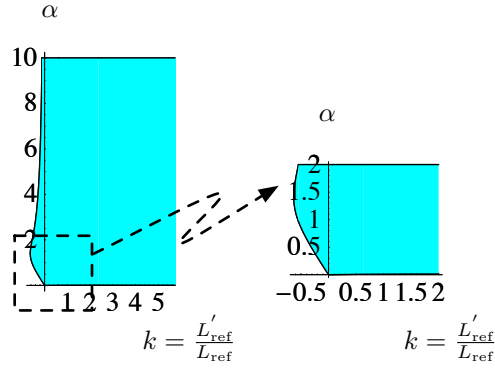


Figure 9.1: Plot showing the regions that satisfy the Routh Hurwitz stability criterion for along-track formation. The gain values are $\tilde{K}_1 = 6$ and $\tilde{C}_1 = 2.97$.

By satisfying the Routh-Hurwitz criterion, the eigenvalues of $A(t)$ at any fixed time t will always be in the left half of the plane. This is not sufficient to guarantee stability of the system. The sufficient condition is that rate of change of $A(t)$ be very small. Rosenbrock³⁴

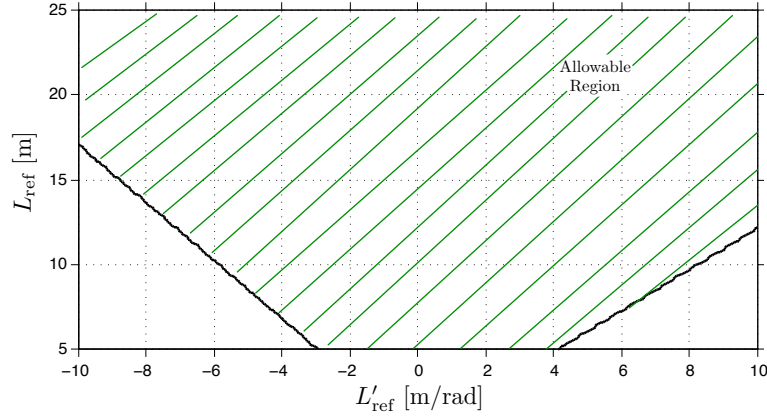


Figure 9.2: Plot showing the regions that satisfy the Routh Hurwitz stability criterion and Rosenbrock bounds for along-track formation.

established bounds for this rate of change and stated it as a theorem when $A(t)$ is in the control canonical form ($A_c(t)$). The reader is referred to chapter 6 for details of the theorem. The $A(t)$ matrix in Eq. (9.19) is not in the control canonical form, but it can be transformed in a control canonical form using a similarity transformation. Using this transformed matrix the feasible values of L_{ref} and L'_{ref} that satisfies the Rosenbrock theorem, for the chosen values of $\tilde{K}_1$, $\tilde{C}_1$ and α are identified numerically. These feasible values are shown in Figure 9.2. This plot can be used to specify the reference trajectory $L_{\text{ref}}(t)$.

The out-of-plane angle (ϕ) is decoupled from the other two states. By studying the equation of motion given in Eq. (9.10a), one can conclude that in the absence of angle rate feedback the out-of-plane angle will have a sinusoidal motion. The amplitude of oscillation will increase or decreases depending whether the satellite formation is contracted or expanded. For the regulation problem, the feedback gain value is chosen as $\tilde{K}_2 = 2$, and this guaranteed asymptotic convergence. We will retain the same gain value for the current reconfiguration problem.

9.3.2 Orbit-Normal Formation

The linearized equations of motion for the orbit-normal configuration given in Eq. (9.18a)–(9.18c) show that the separation distance error (δL) equation is decoupled from the two out-of-plane equations of motion. It can be seen from Eq. (9.18c) that the separation distance error (δL) will have a steady state offset due to the reference length acceleration (L''_{ref}). Other than this offset, reconfiguration will not have any significant effect on the stability of the separation distance. Therefore, we will retain the same gain values determined in the regulation problem given in chapter 5. The values for position and velocity gains are taken as $\tilde{C}_1 = 0$ and $\tilde{C}_2 = 2\sqrt{3}$.

However, the stability of the coupled out-of-plane angles (ϕ and θ) will depend on the rate of expansion or contraction of the formation. Their stability is studied in this section using the Rosenbrock technique and this study closely follows along-track configuration stability analysis. The equations of motion of the coupled out-of-plane angles given in Eq. (9.18a)–(9.18b) can be written in the matrix form as

$$\begin{pmatrix} \phi' \\ \phi'' \\ \theta' \\ \theta'' \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (1 - \tilde{K}_1) & -\left(\tilde{K}_3 + 2\frac{L'_{\text{ref}}}{L_{\text{ref}}}\right) & 2\frac{L'_{\text{ref}}}{L_{\text{ref}}^2} & 2 \\ 0 & 0 & 0 & 1 \\ -2\frac{L'_{\text{ref}}}{L_{\text{ref}}} & -2 & (4 - \tilde{K}_2) & -2\frac{L'_{\text{ref}}}{L_{\text{ref}}} \end{bmatrix} \begin{pmatrix} \phi \\ \phi' \\ \theta \\ \theta' \end{pmatrix} \quad (9.22)$$

The corresponding characteristic equation for the matrix given in Eq. (9.22) can be written as

$$\begin{aligned} \lambda^4 + (\tilde{K}_3 + 4k)\lambda^3 + (\tilde{K}_1 + \tilde{K}_2 + 2k\tilde{K}_3 + 4k^2 - 1)\lambda^2 + (-4\tilde{K}_3 + \tilde{K}_2\tilde{K}_3 + 2k\tilde{K}_2 + 2k\tilde{K}_1 - 2k)\lambda \\ + (4 - 4\tilde{K}_1 + \tilde{K}_1\tilde{K}_2 - \tilde{K}_2) = 0 \end{aligned} \quad (9.23)$$

where $k = \frac{L'_{\text{ref}}}{L_{\text{ref}}^2}$ is a time varying coefficient. Again, we will retain the gain values from the regulation problem and their values are given by $\tilde{K}_1 = 2.7$ and $\tilde{K}_2 = 5$. The angle rate gain $\tilde{K}_3$ is defined using a scaling factor α as

$$\tilde{K}_3 = \alpha \sqrt{\tilde{K}_1}$$

Substituting these gains back in to Eq. (9.23) results in a much simplified characteristic equation. The the real parts of the roots of this characteristic equation should be negative for the system to be Hurwitz. The range of values of k and α that guarantee negative roots for the characteristic equation can be found out using Routh-Hurwitz stability criterion. The shaded region in Figure 9.3 shows the possible values of k and α for the fixed gain values of $\tilde{K}_1 = 2.7$ and $\tilde{K}_2 = 5$. The value of α is chosen as 3.6 since k has maximum range both in the positive and negative direction at this α value.

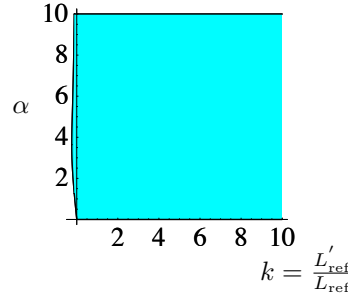


Figure 9.3: Plot showing the regions that satisfy the Routh Hurwitz stability criterion for orbit-normal formation. The gain values are $\tilde{K}_1 = 2.7$ and $\tilde{K}_2 = 5$.

Similar to the along-track configuration the feasible values of L_{ref} and L'_{ref} that satisfies the Rosenbrock theorem, for the chosen values of $\tilde{K}_1$, $\tilde{K}_2$ and α are identified numerically. These feasible values are shown in Figure 9.4. This plot can be used to specify the reference trajectory $L_{\text{ref}}(t)$.

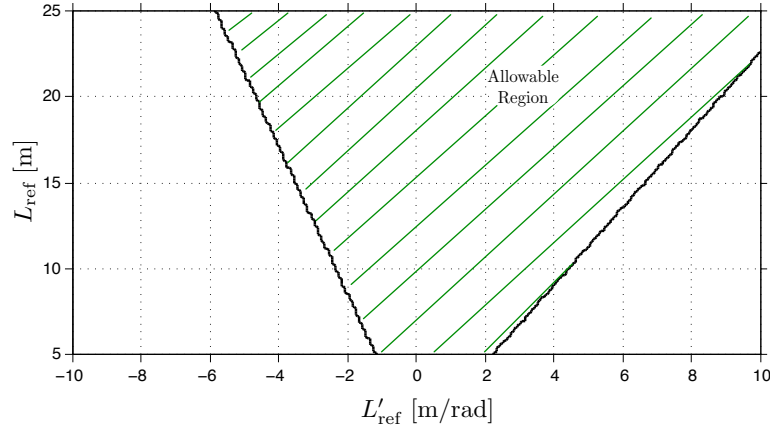


Figure 9.4: Plot showing the regions that satisfy the Routh Hurwitz stability criterion and Rosenbrock bounds for orbit-normal formation.

9.4 Numerical Simulations

Numerical simulations illustrating the reconfiguration maneuvers of the along-track and orbit normal Coulomb tether formations are presented in this section. These simulations serve to validate the performance and stability of the feedback control strategy when the reconfiguration rate are with in the established bounds. Analogous to all the other simulations in this dissertation, the Coulomb tether performance is simulated in two different manners. First the linearized spherical coordinate differential equations are integrated. This simulation illustrates the linear performance of the charge control. Second, the linearized results are compared with those obtained from the exact nonlinear equation of motion of the deputy satellites given by

$$\ddot{\mathbf{r}}_1 + \frac{\mu}{r_1^3} \mathbf{r}_1 = \frac{k_c}{m_1} \frac{Q}{L^3} (\mathbf{r}_1 - \mathbf{r}_2) \quad (9.24a)$$

$$\ddot{\mathbf{r}}_2 + \frac{\mu}{r_2^3} \mathbf{r}_2 = \frac{k_c}{m_2} \frac{Q}{L^3} (\mathbf{r}_2 - \mathbf{r}_1) \quad (9.24b)$$

where $\mathbf{r}_1 = \mathbf{r}_c + \boldsymbol{\rho}_1$ and $\mathbf{r}_2 = \mathbf{r}_c + \boldsymbol{\rho}_2$ are the inertial position vectors of the masses m_1 and m_2 , while $L = \sqrt{(\mathbf{r}_2 - \mathbf{r}_1) \cdot (\mathbf{r}_2 - \mathbf{r}_1)}$. The gravitational coefficient μ is defined as $\mu \approx GM_e$. After integrating the motion using inertial Cartesian coordinates, the separation distance L , as well as the corresponding angles are computed in post-processing using the exact kinematic transformation. For all cases the cluster center of mass is assumed to be a GEO orbit.

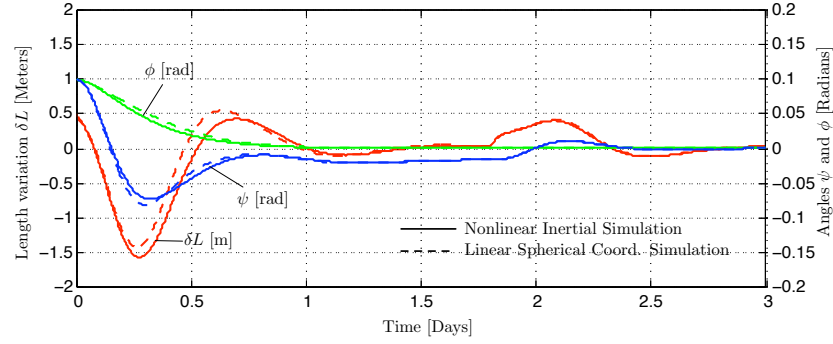
9.4.1 Along-Track Configuration

The simulation parameters that used are listed in Table 9.1. The initial attitude values are set to $\psi = 0.1$ radians and $\phi = 0.1$ rad. The separation length error (Coulomb tether length error) is $\delta L = 0.5$ meters. All initial rates are set to zero through $\dot{\psi} = \delta \dot{L} = \dot{\phi} = 0$. Two sets of maneuvers, expanding the Coulomb tether formation from 25m to 35m in 1.8 days and contracting the formation from a separation distance of 25m to 15m, are shown.

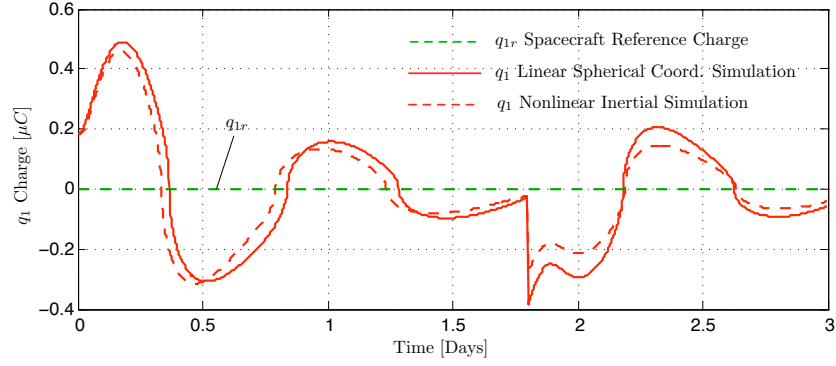
Table 9.1: Input parameters used in along-track reconfiguration simulation

Parameter	Value	Units
m_1	150	kg
m_2	150	kg
L_{ref}	25	m
k_c	8.99×10^9	$\frac{\text{Nm}^2}{\text{C}^2}$
Q_{ref}	0	μC^2
Ω	7.2915×10^{-5}	rad/sec
C_1	$2.97\Omega^2$	
C_2	3.9637Ω	
K_1	$6\Omega^2$	
K_2	2Ω	
$\delta L(0)$	0.5	m
$\psi(0)$	0.1	rad
$\phi(0)$	0.1	rad

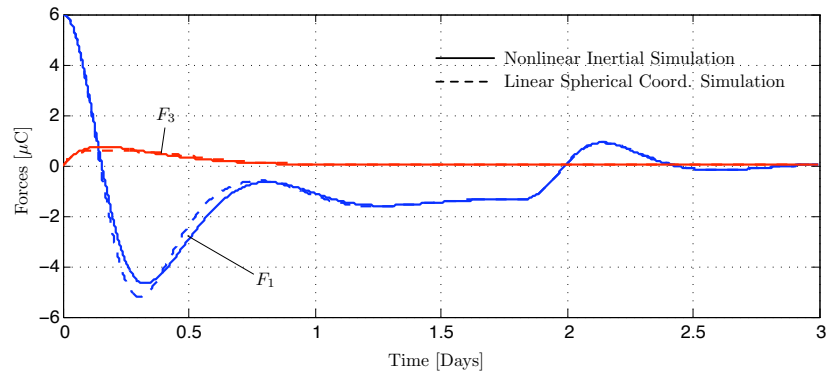
The Coulomb tether motion for increasing the separation distance from 25m to 35m is



(a) Time histories of length variations δL , in-plane rotation angle ψ and out-of-plane rotation angle ϕ

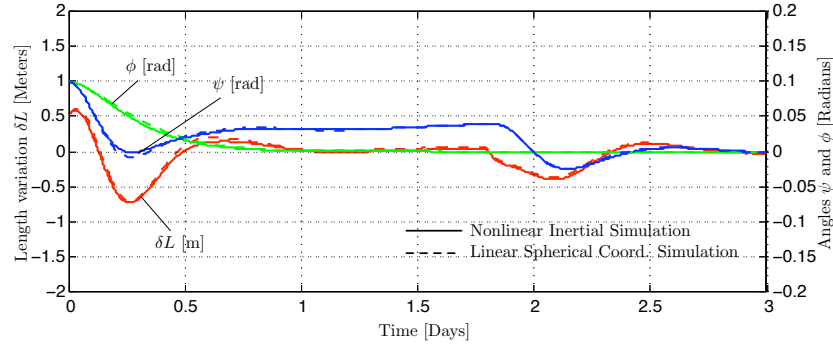


(b) Spacecraft charge time histories

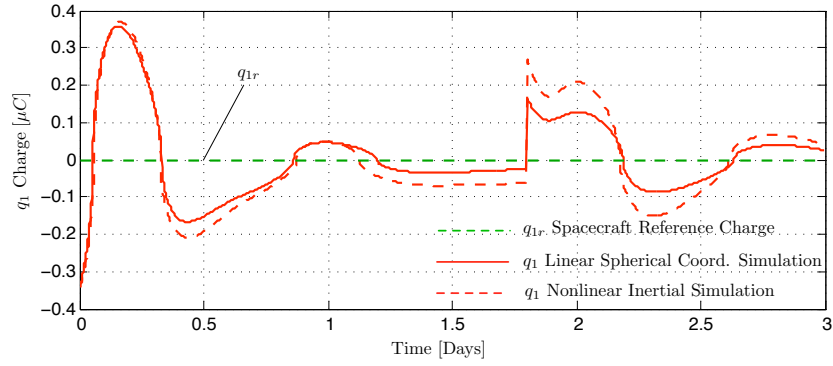


(c) Spacecraft force time histories

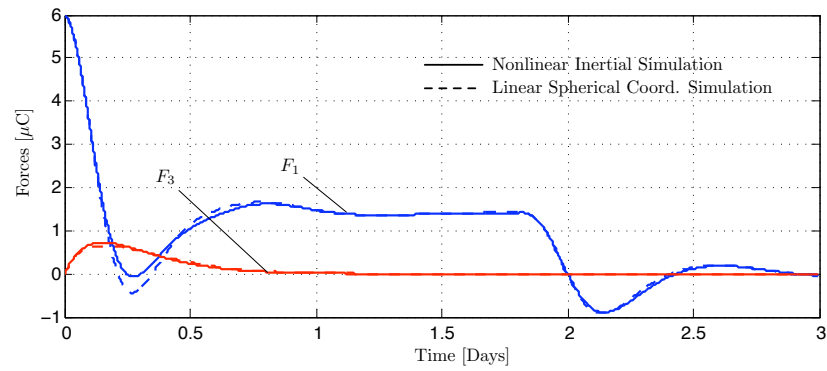
Figure 9.5: Simulation results for expanding the separation distance of an along-track formation from 25 m to 35 m. The feedback gains are $\tilde{C}_1 = 2.97$, $\tilde{C}_2 = 3.10$, $\tilde{K}_1 = 6$ and $\tilde{K}_2 = 2$.



(a) Time histories of length variations δL , in-plane rotation angle ψ and out-of-plane rotation angle ϕ



(b) Spacecraft charge time histories



(c) Spacecraft force time histories

Figure 9.6: Simulation results for contracting the separation distance of an along-track formation from 25 m to 15 m. The feedback gains are $\tilde{C}_1 = 2.97$, $\tilde{C}_2 = 3.10$, $\tilde{K}_1 = 6$ and $\tilde{K}_2 = 2$.

shown in Figure 9.5(a). The continuous lines represent the states in linearized spherical coordinates $(\psi, \theta, \delta L)$, and the states in full nonlinear spherical coordinates are shown as dotted lines. The expansion is done in 1.8 days and this corresponds to a constant L'_{ref} of 0.88. After 1.8 days, the L'_{ref} is zero and the formation is allowed to stabilize about the final separation distance. The feedback gains are $\tilde{C}_1 = 2.97$, $\tilde{C}_2 = 3.10$, $\tilde{K}_1 = 6$ and $\tilde{K}_2 = 2$. With the presented charge feedback law, the states all converge to zero. At the end of 1.8 days, the constant reference length rate $\dot{L}_{\text{ref}}$ abruptly goes to zero and this results in noticeable oscillations which converge with time. Further, the figure shows that the nonlinear simulation closely follows the linearized simulation, validating the linearizing assumption and illustrating robustness to the unmodelled dynamics. Figure 9.5(b) shows the spacecraft control charge q_1 (on craft 1) for both the linearized and full nonlinear simulation models. Both are converging to the reference value pertaining to the static equilibrium at each instant of time. Note that the deviation from the value of reference charges is small, justifying the linearization assumptions used. A spike in the charge required at the end of 1.8 days is due to the abrupt drop of constant reference length rate to zero. The magnitude of the control charges is in the order of micro-Coulomb which is easily realizable in practice using charge emission devices. Figure 9.5(c) shows the thrusting force required along the $\hat{b}_1$ and $\hat{b}_3$ direction. The force required for both the linearized and nonlinear model are nearly identical.

Figure 9.6 shows the Coulomb tether motion, charge and thrust force time histories for decreasing the separation distance from 25m to 15m. Contractions are more challenging because the angular motions will increase due to conservation of angular momentum. Again the maneuver is done in 1.8 days which means L'_{ref} is -0.88 and the gains are same as above expansion maneuver. The figure illustrates that the system is stable and the reconfiguration goals are successfully met.

9.4.2 Orbit-Normal Formation

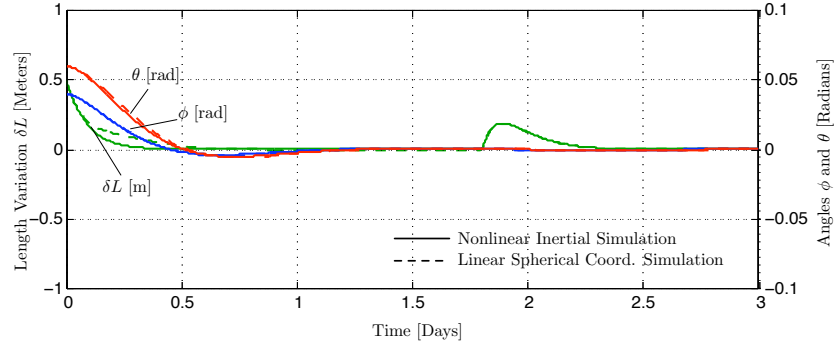
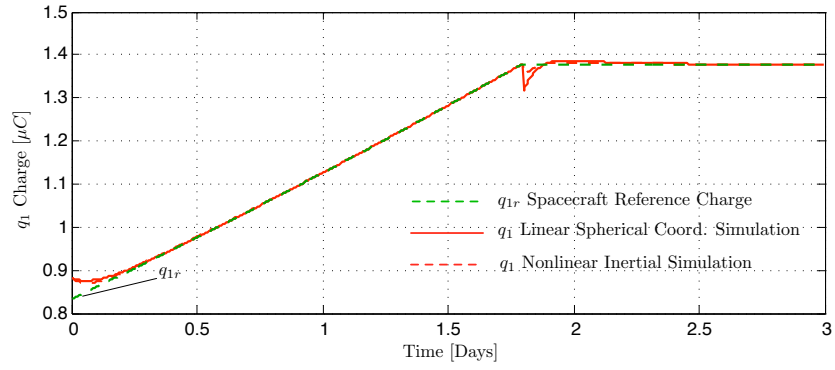
Table 9.2: Input parameters used in orbit normal reconfiguration simulation

Parameter	Value	Units
Q_{ref}	6.9304×10^{-13}	μC^2
C_2	$2\sqrt{3}\Omega$	
K_1	$2.7\Omega^2$	
K_3	3.2596Ω	
K_2	$5\Omega^2$	
$\delta L(0)$	0.5	m
$\theta(0)$	0.06	rad
$\phi(0)$	0.04	rad

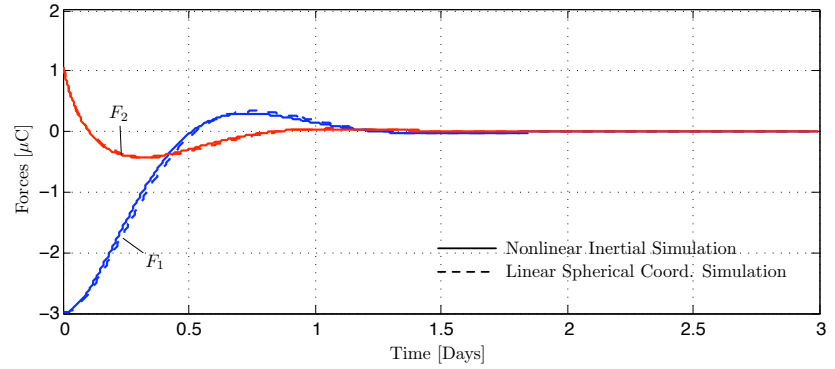
The same set of expansion and contraction maneuver are performed for the orbit-normal configuration. The simulation parameters that used are listed in Table 9.2. The initial attitude values are set to $\theta = 0.06$ radians and $\phi = 0.04$ rad. The separation length error (Coulomb tether length error) is $\delta L = 0.5$ meters. All initial rates are set to zero through $\dot{\phi} = \delta\dot{L} = \dot{\theta} = 0$. Figure 9.7 and Figure 9.8 give the system response for expanding the Coulomb tether formation from 25m to 35m in 1.8 days, and contracting the formation from a separation distance of 25m to 15m, respectively. In both the simulations, the states do converge asymptotically to zero. The nonlinear system response closely follows the linearized system, thus validating the linearization assumptions.

9.5 Summary

A charge feedback control law for reconfiguring a 2-craft Coulomb tether formation along the orbit-normal and along-track direction is given. With this work, the study of reconfiguration of 2-craft Coulomb tether along all three equilibrium is complete. The control law used is here is a hybrid of Coulomb forces and conventional thrust forces, and is similar to the one developed for the regulation problem. Like in orbit-radial direction reconfiguration, the

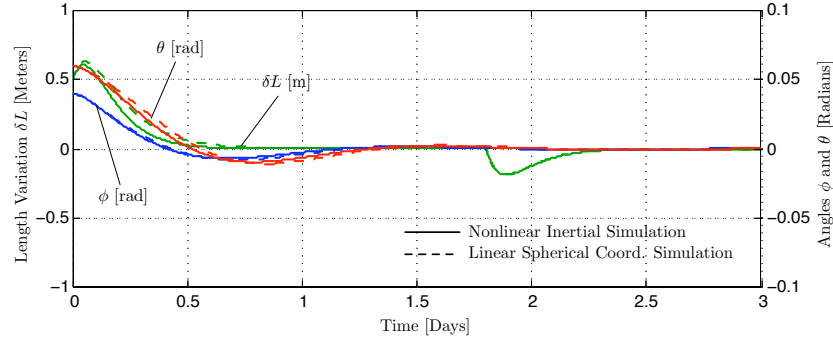
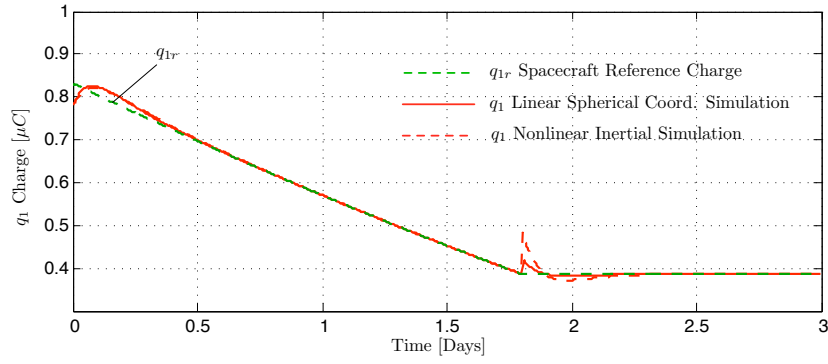
(a) Time histories of length variations δL , out-of-plane rotation angles ϕ and θ 

(b) Spacecraft charge time histories

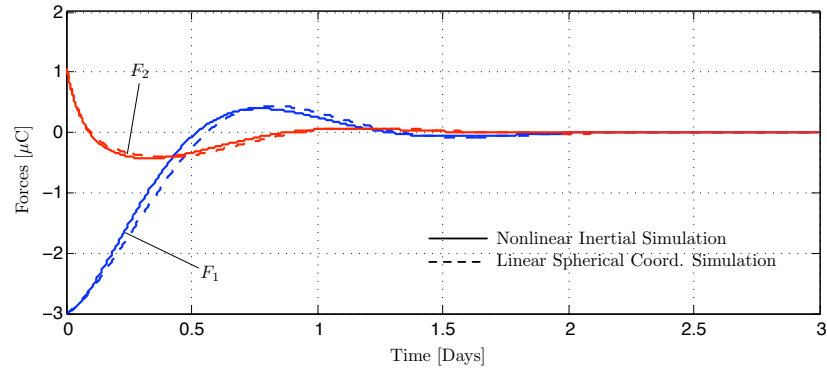


(c) Spacecraft force time histories

Figure 9.7: Simulation results for expanding the separation distance of an orbit-normal formation from 25 m to 35 m. The feedback gains are $\tilde{C}_1 = 0$, $\tilde{C}_2 = 3.4641$, $\tilde{K}_1 = 2.7$, $\tilde{K}_2 = 5$ and $\tilde{K}_3 = 4.6938$.

(a) Time histories of length variations δL , out-of-plane rotation angles ϕ and θ 

(b) Spacecraft charge time histories



(c) Spacecraft force time histories

Figure 9.8: Simulation results for contracting the separation distance of an orbit-normal formation from 25 m to 15 m. The feedback gains are $\tilde{C}_1 = 0$, $\tilde{C}_2 = 3.4641$, $\tilde{K}_1 = 2.7$, $\tilde{K}_2 = 5$ and $\tilde{K}_3 = 4.6938$.

contraction maneuver is found to be more challenging since the angular motions increase due to the conservation of angular momentum. Numerical simulations of the full nonlinear motion are carried out to illustrate the results and compare the linearized performance predictions to the actual nonlinear system response.

10 Conclusion

The concept of a Coulomb (electrostatic) tether is introduced to bind two satellites in a near-rigid formation. It is shown through numerical analysis that the point charge model for charged spheres can be used for calculating the Coulomb force, provided the separation distances are large compared to the radius of the sphere and the Debye length is very large compared to the separation distance. Thus validating the point charge model for charged sphere used in the entire dissertation. First, the stability of a 2-craft Coulomb tether along the orbit-radial (orbit-nadir) direction is analyzed based on a linearized dynamics and charge behavior model whose validity is also shown. While the Coulomb force cannot directly stabilize the attitude, the gravity gradient torque is exploited to stabilize the Coulomb tether formation about the orbit radial direction. It is observed that a linear charge feedback law in terms of separation distance errors and separation rate is adequate for stabilizing the separation distance and in-plane angular motion. The control charges needed are small in the order of micro-Coulombs and realizable in practice

Unlike the orbit-radial configuration, a 2-craft Coulomb tethered structure aligned along the orbit normal or along-track direction cannot be stabilized with only a charge feedback law. But, both Coulomb tether configurations can be stabilized with a hybrid control of Coulomb forces and conventional thrusters that stabilize the separation distance and orientation respectively. The control charges needed are small in the order of micro-Coulombs and realizable in practice. The thrusting forces required are in the order of micro-Newtons

and the thrusting is always done orthogonal to the Coulomb tether axis, thus avoiding plume exhaust impingement problems. For the along-track configuration the separation distance and in-plan angle are coupled and unstable without feedback. An interesting result is that for the orbit-normal configuration the separation distance is decoupled and marginally stable even without charge feedback, while the orientation has to be feedback stabilized.

The study of reconfiguration of 2-craft in free space using Coulomb force showed that the charge required are small and realizable in practice. Thus, providing the impetus to study the reconfiguration of 2-craft Coulomb formation in orbit. A charge feedback control law for reconfiguring a 2-craft Coulomb tether formation aligned along orbit-radial direction is successfully derived. During these maneuvers care is taken to ensure that the gravity gradient torque is still sufficient to stabilize the in-plane attitude of the nadir pointing formation. The stability regions for expanding and contracting the formation are established through linearization of the motion and by applying criteria developed by Rosenbrock for linear time-varying systems. Contracting the virtual structure is more difficult to perform while guaranteeing stability. The system angular momentum will cause any in-plane angular motion to increase with decreasing tether length. The magnitude of the local gravity gradient limits the rate at which the separation distance can be reduced. In contrast, expanding the virtual structure length is easier because the angular momentum helps contain in-plane rotation. The out-of-plane motion of the craft is decoupled from the in-plane motion with the linearized dynamics, and not controllable with the Coulomb forces. However, the analytical solution for the out-of-plane angle equation of motion when the reference length rate is constant, is successfully developed using the Bessel functions. This solution is used to come up with bounds on the initial out-of-plane angle so that the out-of-plane angular oscillations is within the prescribed limit at the end of the contraction operation. The jump discontinuity in the reference length rate has been smoothed using a transition function.

A hybrid feedback control for reconfiguration of 2-craft formation in the along-track and orbit-normal direction has also been developed. Numerical simulations of the full nonlinear motion are carried out for all control laws to illustrate the results and compare the linearized performance predictions to the actual nonlinear system response.

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