THE CONSTRUCTION OF INFLUENCE LINES WITH A MECHANICAL INTERFEROMETER

by

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VI. INTRODUCTION

In the design of both statically determinate and indeterminate structures it is necessary for the designer to predict the effects on the structure of static and moving loads. He must place the movable loads on the structure in such a manner that their effects--shears, moments, thrusts, reactions--have maximum values. There are several methods of placing the loads for the desired effects. The designer is forced by economic considerations to choose a method which is both rapid and accurate. In addition, he is encouraged by his own human nature to devise an easy approach to his problem.

The influence line has been recognized as a useful tool for the analysis of structures acted upon by moving loads. This thesis is the report of an attempt to develop a method whereby the influence line may be obtained more easily than was heretofore possible. The newness of the method is in the means employed for measuring the deflection of a structural model. The method and the results obtained are described in greater detail in the following pages.

VII. INFLUENCE LINES

An influence line for a structure is a curve for which the ordinate at a point is some linear function--such as shear, moment, thrust or reaction--of a unit load on the structure at that point [1]. The influence diagram for moment, for example, differs from the regular moment diagram in that it gives a moment at one point for any position of the load. The moment diagram, on the other hand, gives the moment at any point due to a fixed position of the load. Given an influence line for some position on a structural member, the effect of a concentrated load of any magnitude at any point may be determined by multiplying the ordinate of the influence line at that point by the applied load. The effect of a uniformly distributed load may be obtained by multiplying the intensity of the load by the area under the influence diagram for those parts subjected to the load.

Influence ordinates may be calculated by using several theoretical methods. However, as structures become more complex, mathematical solutions become exceedingly laborious. In many cases, solutions depend upon certain simplifying assumptions. Structural models offer the designer an opportunity to check the results of his calculations. At times, model analysis is the only practical method of justifying the assumptions necessary to the theoretical analysis.

VIII. MODEL ANALYSIS

A. Types of Models

Structural models are of two general types--loaded and unloaded. The loaded models may be either full-sized or scaled-down reproductions of some prototype. The load may be either the full intended load or some scaled-down load. The effects of loads in terms of stress are generally determined by measuring values of strain or deflection which may be converted to stress.

The unloaded model is subjected to a known deformation at some point and is allowed to deflect. According to the Müller-Breslau Principle (1), the ordinates to the influence line for a structure are proportional to the ordinates of the deflected centerline of the structure. The designer may obtain the influence line for the structure by causing a structural model to deflect under the action of some known deformation. He must, however, be careful to satisfy the principles of similitude.

The unloaded model is used in many different methods of structural model analysis and was used in this investigation.

B. Model Similarity

The principles of similitude require that the model be geometrically similar to the prototype if the effects of shear and thrust deformation are to be considered. However, if the major part of the strain energy is provided by bending--whether due to reaction, shear, moment or thrust--the model will be sufficient if the width of the member is proportional to the cube root of the moment of inertia for the prototype. The length of the model must be proportional to the length of the corresponding member of the prototype. The material from which the model is made must be homogeneous, elastic and of constant thickness [2].

Ordinarily the direct and shear strains will not be large enough to consider. They may not be neglected in deep beams of short span.

C. Methods of Model Analysis

Beggs [3] developed the most widely known application of the method of analysis using unloaded models. He devised gages which he called deformeters. The gages are fitted with plugs for applying deformations to models. The deflections produced by the plugs are so small that they must be measured with micrometer microscopes. The procedure is tedious, the equipment is very expensive, there is no permanent record of the deflections which can be used to check the work and the models are affected by temperature and creep.

Other people [4] have employed apparatus based on the same principles for model analysis. In general, their methods meet with many of the same objections.

Eney [5] devised a deformeter apparatus which gives large deflections which may be measured with an engineers scale. The method does not provide a permanent record and there is the possibility that the material will be overstressed.

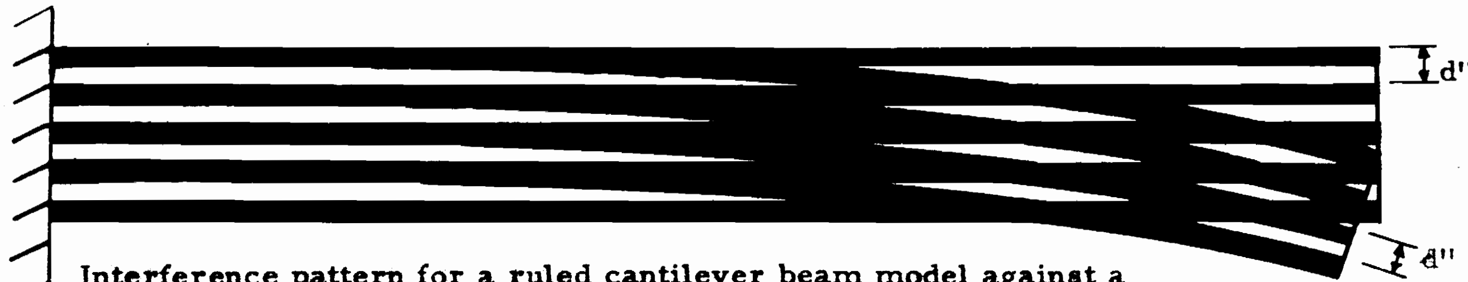
Rocha [6] recently published an account of a method for the determination of displacements by taking double-exposure photographs of a target attached to a celluloid model. The deflections are large and must be scaled from a photograph. The photographic equipment is expensive and the lights necessary for good photographic detail produce heat which tends to distort the model.

The technique of photoelasticity offers a possible approach to the analysis of structures. The method is time-consuming and it requires rather expensive equipment.

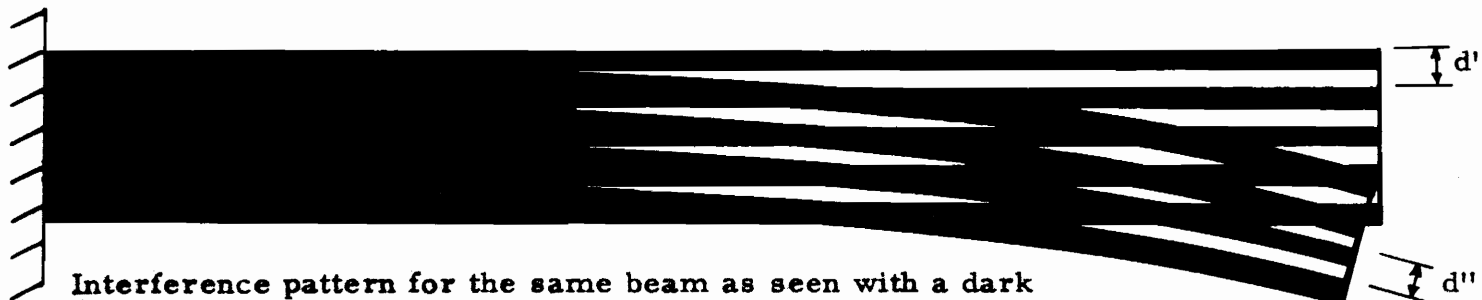
IX. MECHANICAL INTERFEROMETRY

Weller and Shepard [7] showed that two transparent plates, each ruled with equally spaced dark lines, will exhibit a fringe pattern similar in appearance to the photoelastic stress pattern if one of the plates is moved relative to the other. The fringes which appear are the result of a mechanical interference by the dark lines to the passage of light through the transparent material. If the spacing of the lines is known, it is an easy matter to measure deflections by observing the pattern. The ruled lines act as a vernier. Each fringe produced represents a motion perpendicular to the ruled lines of a magnitude equal to the line spacing. The method may be used to measure the expansion or contraction of a member subjected to an axial load, the rotation of a member under a bending load and, by the use of two sets of ruled lines, it may measure displacements in two directions. Figure 1 demonstrates the mechanics of the method as applied to a beam.

Okie [8] tried to use mechanical interferometry to measure strains attempting thereby to obtain values of stress in a beam. He deemed the method impractical, however, when he discovered that it was necessary to correct for the rotation of the member. The fringe in the interference pattern represents the total motion of a point in a particular direction. Therefore, since each displacement is a function of both the rotation and translation of a member, it is neither necessary nor desirable to introduce a rotation correction for



Interference pattern for a ruled cantilever beam model against a ruled background with a light field at the point of support. Each suceeding light field or fringe represents the location of a point which has deflected d inches from the preceding fringe.



Interference pattern for the same beam as seen with a dark field at the point of support. Each dark fringe represents a deflection of d inches from the preceding fringe.

Mechanical Interferometry Applied to Model Analysis

Figure 1

measuring deflections.

X. THE INVESTIGATION

A. Statement of the Problem

It has long been established that there are relatively accurate experimental methods of obtaining influence ordinates for structures through the use of models. As stated previously, there are certain disadvantages attached to their use by the ordinary design office. The present investigation is an attempt to provide an easier and more economical method for measuring the deformation of a structural model. The method employs a mechanical interferometer to determine the desired deflections.

B. The Models

The three models used in this investigation were a fixed-ended beam, a two-span continuous beam and a symmetrically haunched arch. They were cut from sheets of cast Lucite to which a ruling of one hundred lines per inch had been applied by a commercial lithographer. The width of each model was proportional to the cube root of the cross-sectional moment of inertia of the prototype. The average thickness of the material was 0.083 $\pm$0.005 inches. The variation did not seem to affect the results of the experiments adversely.

C. The Deflection Apparatus

In planning the experiment, it was intended that a Beggs-type deformeter [3] would be used to produce the model deflections.

Tests showed that the deflections thus produced were not large enough to give a good interferometer fringe pattern. A deflection apparatus, similar in some respects to the Eney Deformeter [5], was devised by the author. The device consists of a plate to which the model may be fixed and which may be attached to a base plate by inserting pins in appropriately placed holes. To induce a deformation in the model, either a translation or a rotation, it is only necessary to move the pins to new positions.

The deflection apparatus and the method of using it to produce distortions are shown in Figure 2.

The rest of the apparatus consisted of cover plates of the ruled plastic, supports for the models, a light source, photographic copy paper and a drawing board to which the supports and the base plate of the deflection apparatus were attached. The cover sheets were cut to size so that they would fit between model supports. The model supports were gages from the deformeter. The models were clamped to the gages for a fixed end effect or were pinned to them for a pin joint.

The photographic paper was "Contura Contact Orthochromatic Reflex Paper". This is a high contrast paper which may be safely handled in subdued light and may be developed as ordinary photographic film. The light source used to expose the paper was a "Strobelume" electronic flash. A photoflash bulb or an ordinary incandescent bulb would serve the same purpose.

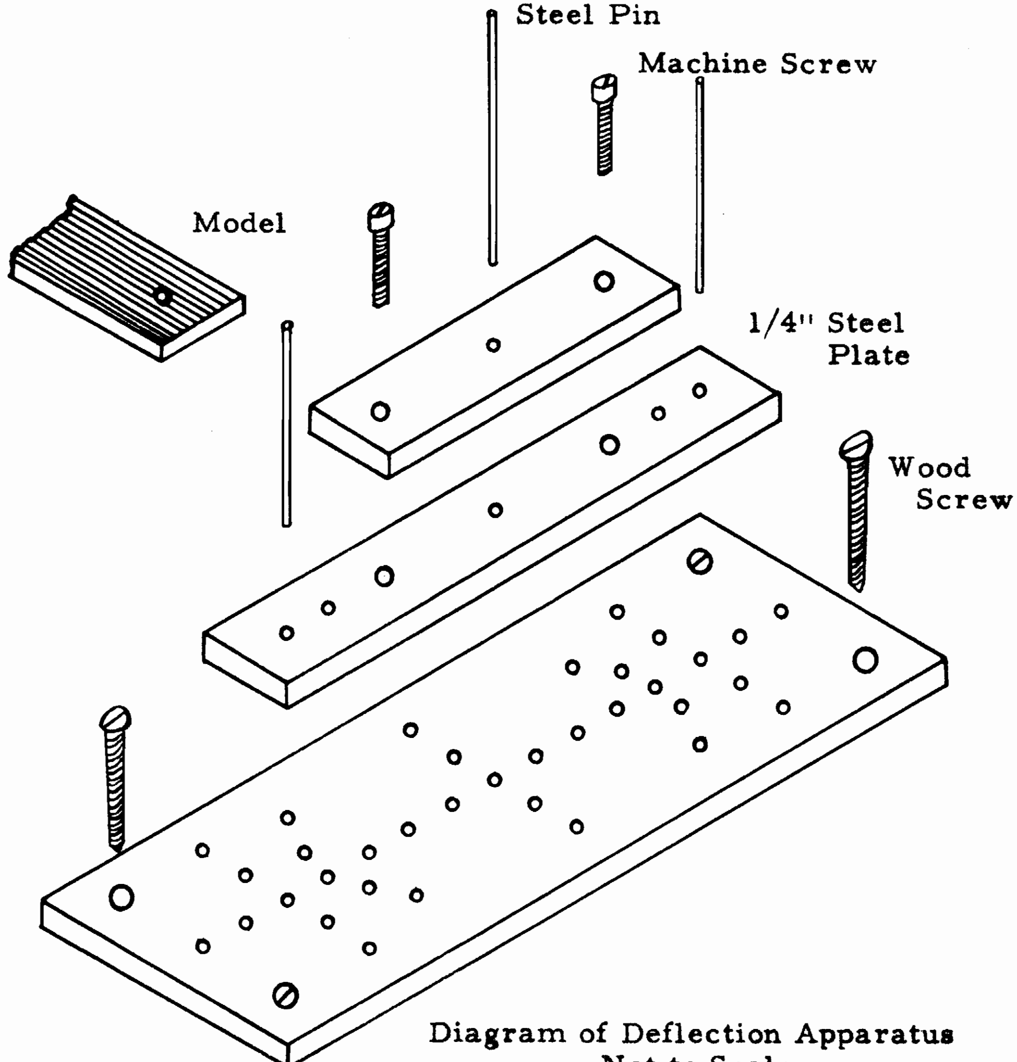
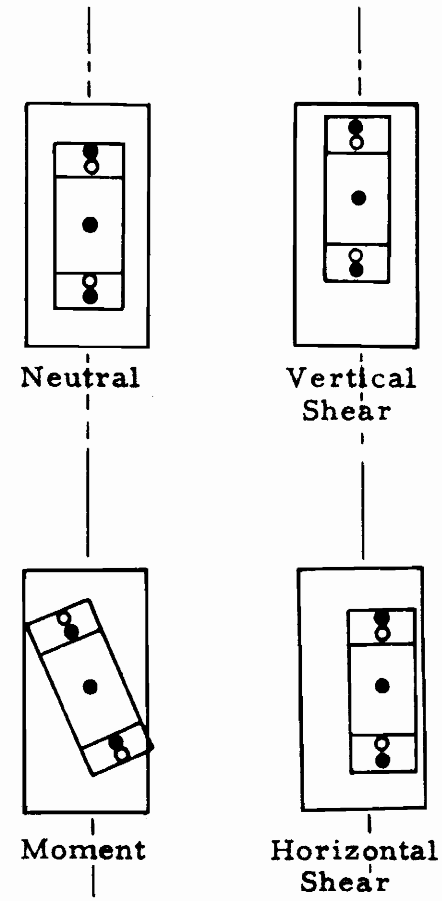


Diagram of Deflection Apparatus
Not to Scale



Method of Applying Deflections
● Denotes Pin Position

Figure 2

D. Procedure

Each of the three models was tested for moment and for vertical shear at the left support due to a vertical load on the member. In addition, the arch was checked for horizontal thrust due to a vertical load.

For each test the copy paper was placed, emulsion side up, on the model supports. The model was clamped in place, free from initial deformation, over the paper. The cover plate was then put into position over the model so that it gave either a uniform light or dark field and was fixed to the drawing board so that it would not move during the test. To minimize the effect of parallax, the ruled side of the cover sheet was placed next to the ruled side of the model. With the model and cover plate in position, the model was deformed by an appropriate movement of the deflection apparatus. The motion of the model with respect to the cover sheet produced an interference pattern which was contact-printed on the photographic copy paper by a single flash of the "Strobelume". In an effort to further reduce the effect of parallax, the light was held directly over the model at a distance of approximately five feet. After the paper was exposed, the model was unloaded and the light field checked to see that there were no residual deformations and that the cover plate had not moved during the test. The paper was then removed and developed in a photographic darkroom. The pattern appeared on the print as a photographic negative. A positive pattern was obtained by contact-printing the original on another sheet of copy paper.

The photographic reproduction of the fringe pattern is not an essential part of the method. The deflections could be easily measured on the deformed model. The photographs provide a permanent record; they allow simultaneous readings, thus eliminating creep and temperature effects by removing time as a factor; they can be interpreted at leisure; and each can be compared with the deflected model as a check on the accuracy of the data.

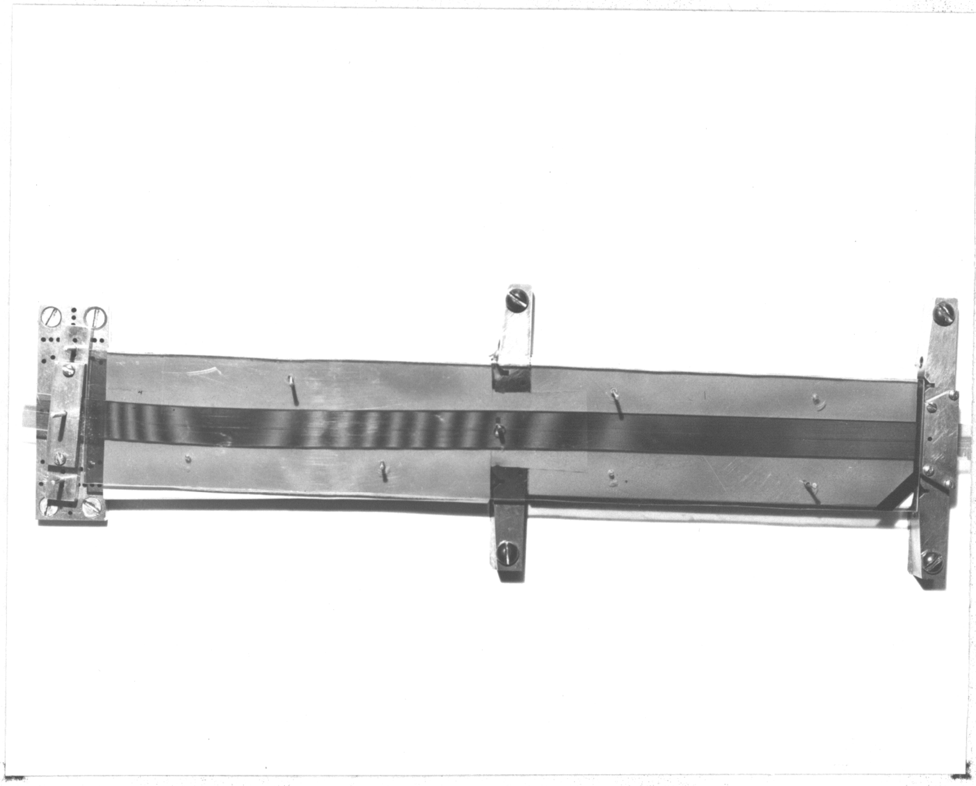
Figure 3 is a photograph of the two-span beam in its deflected position. It shows the fringe pattern due to a moment applied at the left end. Figure 4 shows the fringes on the arch due to a horizontal thrust. Both figures show the method of supporting the models.

Figures 5 through 11 show the fringe patterns obtained from the tests reported herein.

E. Intrepretation of Data

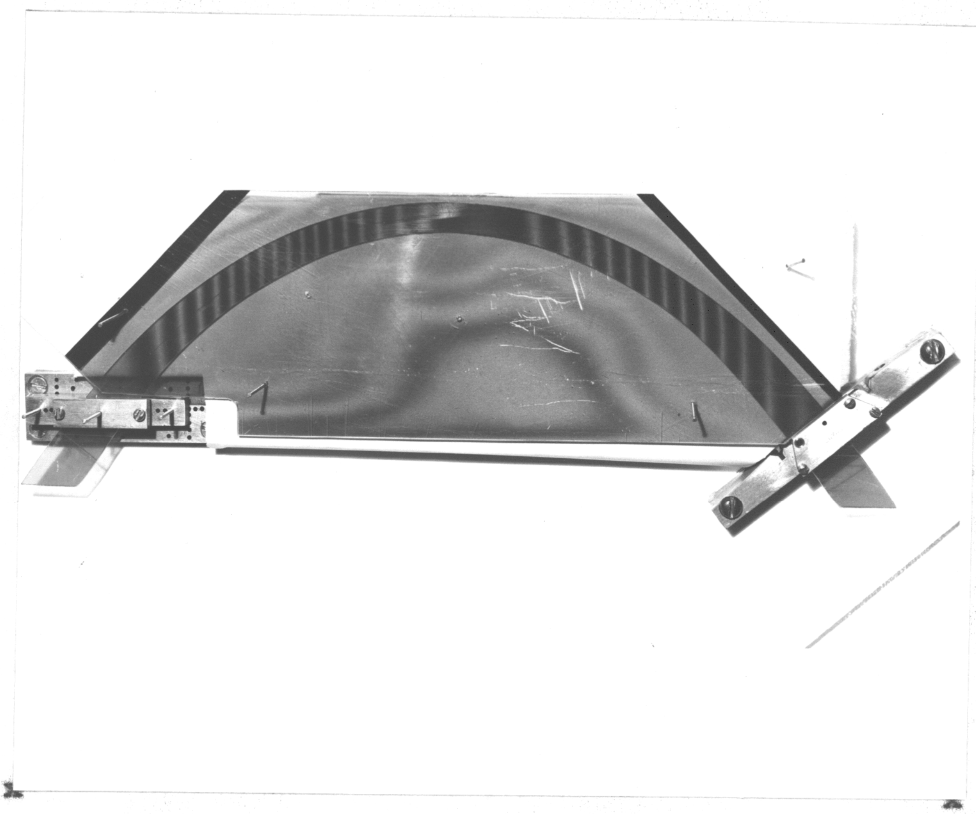
The interference patterns were used to provide data for the construction of influence lines due to a unit vertical load. There was some definite point of zero deflection from which to count the fringes in each test. For moments and thrusts, there was zero vertical deflection at each point of support. For shear, there was no movement at a minimum of one support.

Maxwell's Law [3] was used to calculate the influence ordinates from the measured deflections. In determining the ordinate to the influence line for thrust or shear, it was only necessary to measure the model deflection and to know the applied deformation. For



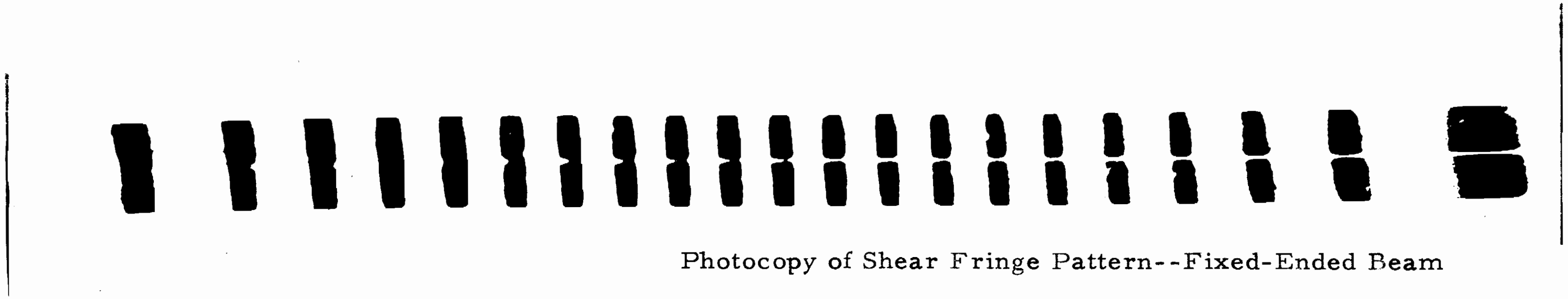
Photograph of Two-Span Beam--Moment Fringe Pattern

Figure 3



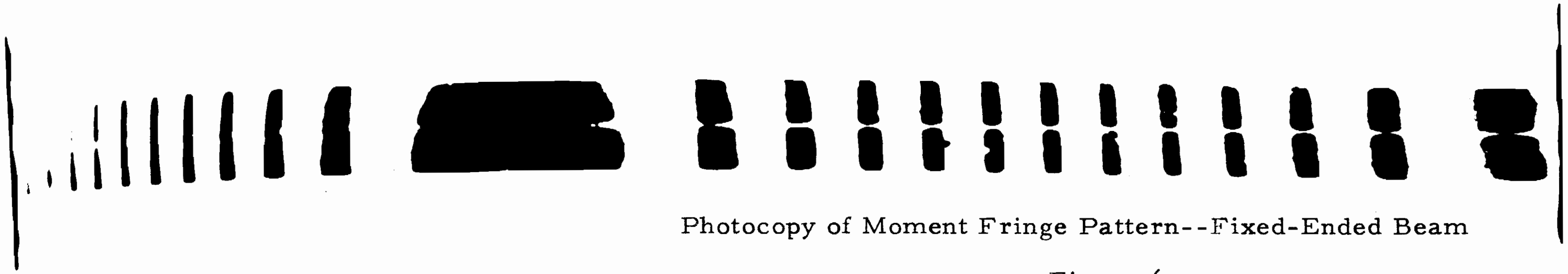
Photograph of Arch--Thrust Fringe Pattern

Figure 4



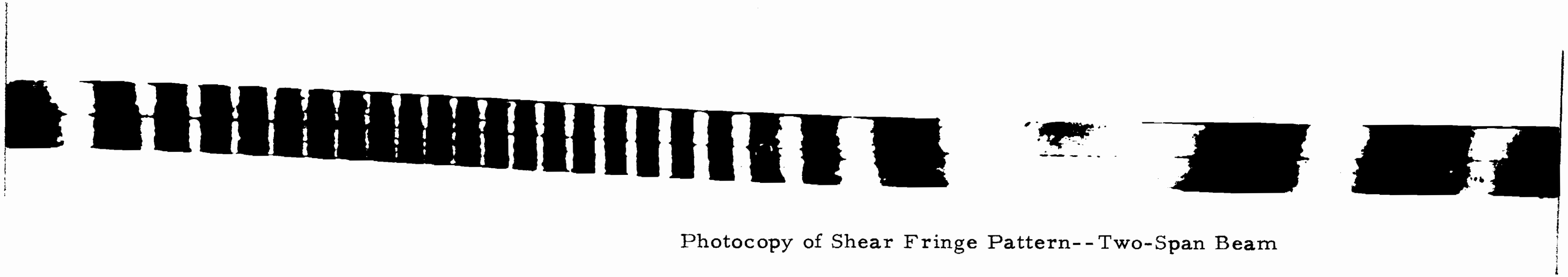
Photocopy of Shear Fringe Pattern--Fixed-Ended Beam

Figure 5



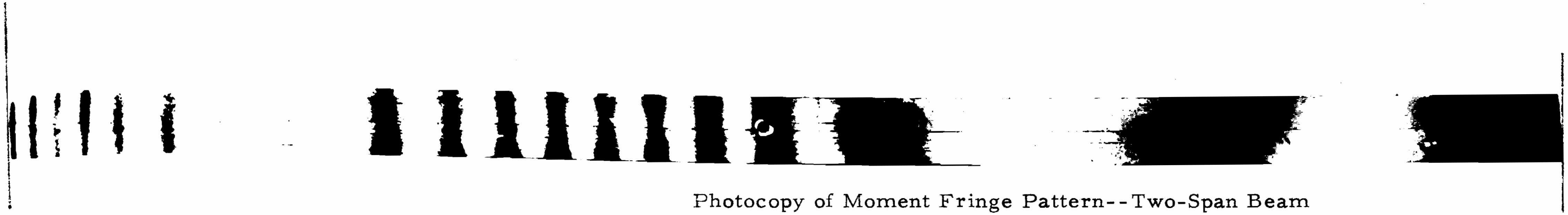
Photocopy of Moment Fringe Pattern--Fixed-Ended Beam

Figure 6



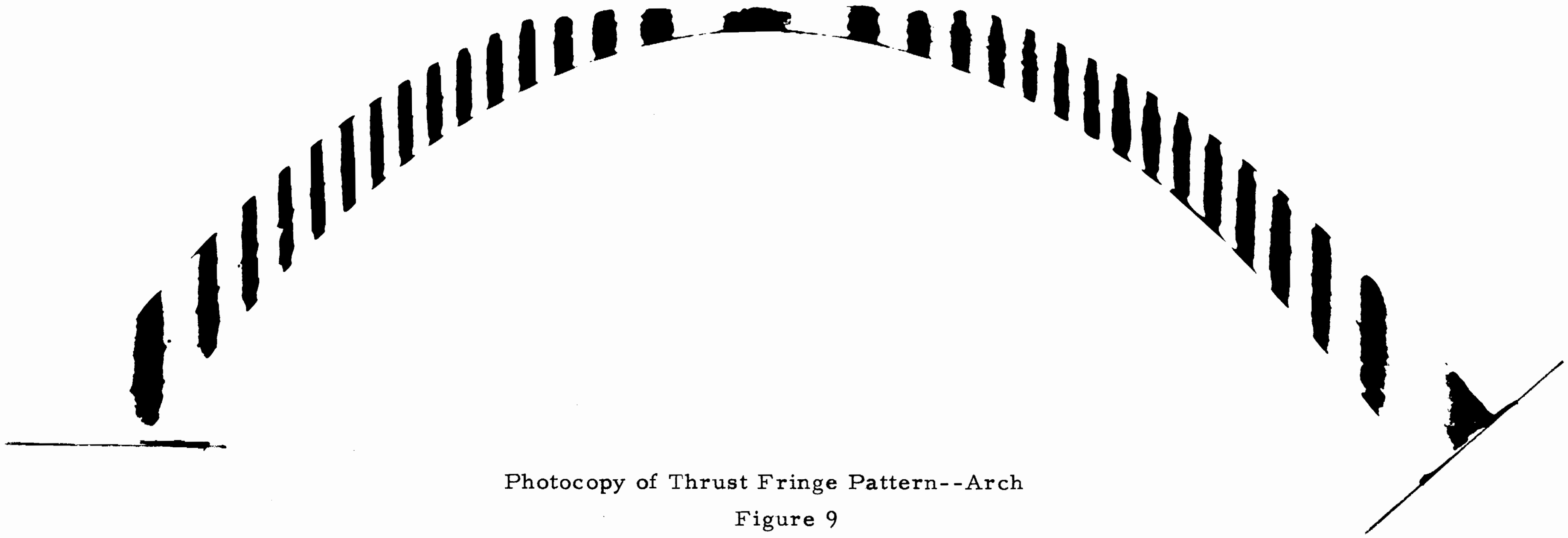
Photocopy of Shear Fringe Pattern--Two-Span Beam

Figure 7

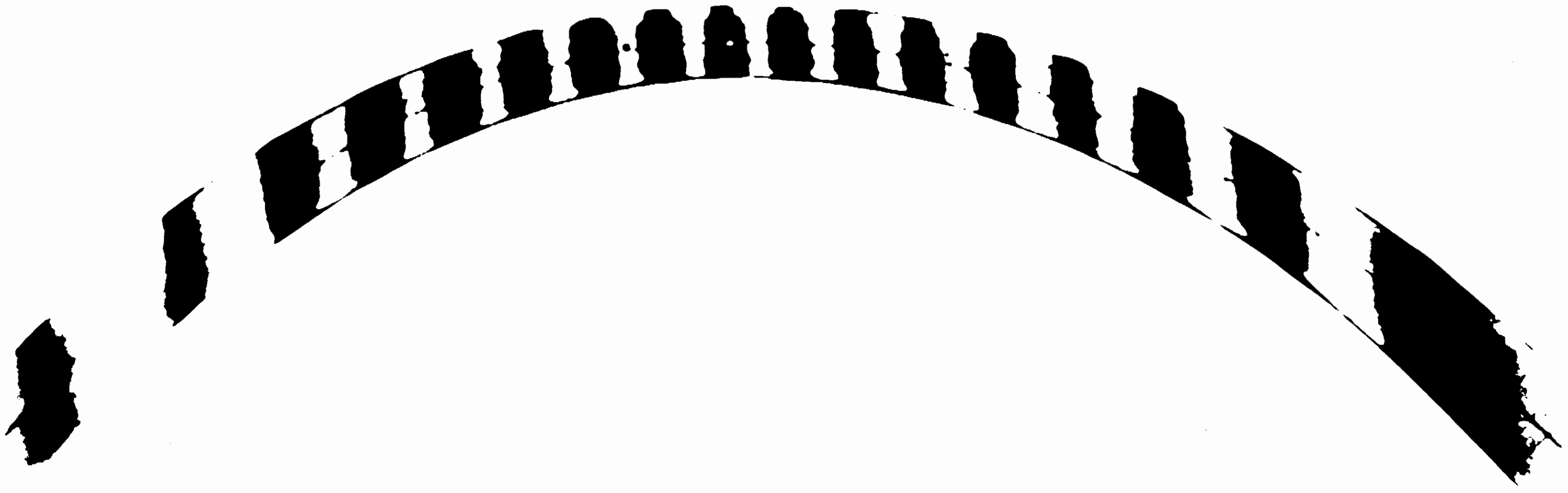


Photocopy of Moment Fringe Pattern--Two-Span Beam

Figure 8

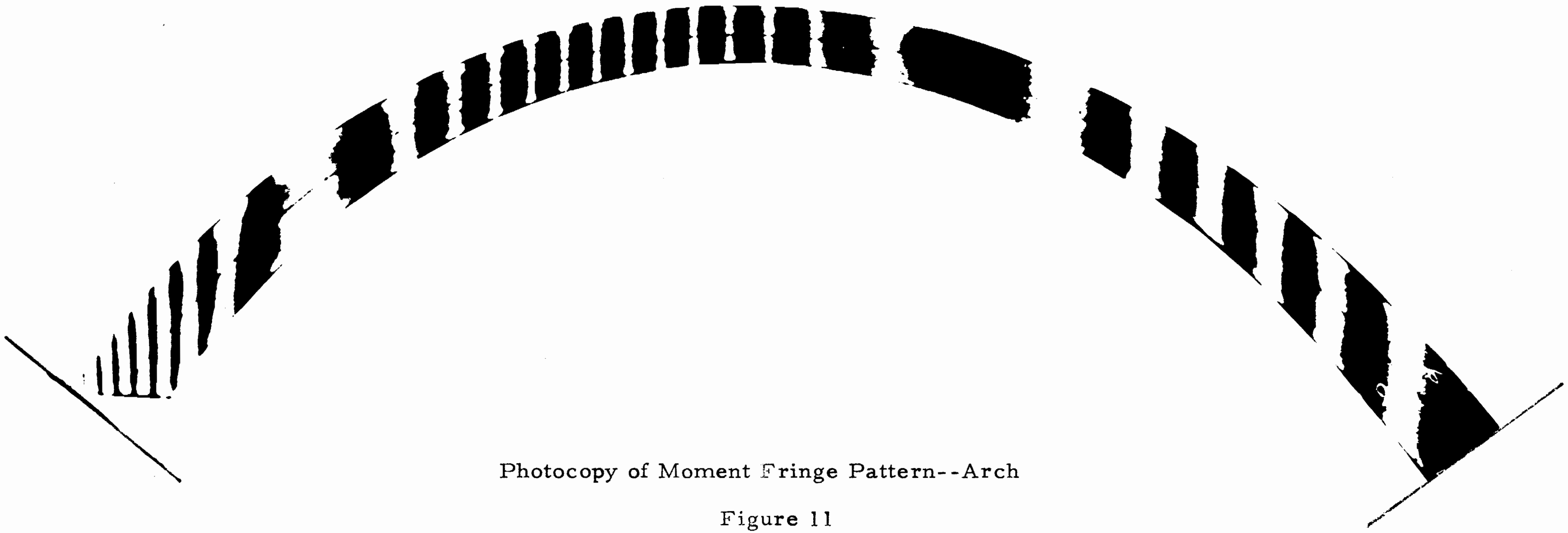


Photocopy of Thrust Fringe Pattern--Arch
Figure 9



Photocopy of Shear Fringe Pattern--Arch

Figure 10



Photocopy of Moment Fringe Pattern--Arch

Figure 11

example, the equation for shear ordinate is

$$V = P \frac{d}{y}$$

when V is the shear force, P is an applied load (in this case a unit load), d is the measured vertical deflection of the model and y is the applied distortion in the direction of the shear force. The similar equation for the thrust ordinate is

$$H = P \frac{d}{y}$$

when H is the horizontal thrust due to a vertical load.

The equation for the moment ordinate

$$M = P \frac{d}{\theta}$$

contains an angular distortion term, θ, which is the angle through which the support was turned to produce the influence line for moment. In order to determine θ, it was necessary to calibrate the apparatus by mounting a strip of the ruled plastic and rotating it through a unit angle. The measured deflection divided by the length of the strip gave the magnitude of the angle, in radians. The M in the moment equation is the moment in a structure of the same scale as the model. Therefore, it was necessary to multiply the value of M from the equation by the length scale factor of the model to obtain the influence ordinate.

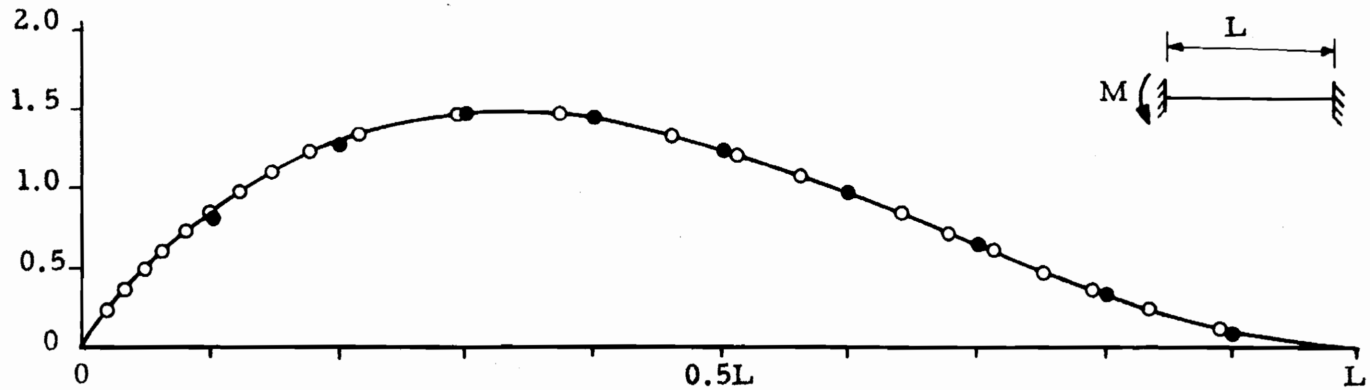
The values of shear and thrust do not require a correction for the scale of the model.

XI. DISCUSSION OF RESULTS

The experimentally determined influence lines for the two beams were compared with theoretical curves from Hool and Johnson's Concrete Engineer's Handbook [9]. Except for the right span of the continuous beam, the experimental influence lines agreed very closely with those obtained by theoretical means. The deflection of the right span was very small and the experiment did not give enough points for a good curve. The disagreement between the curves is not serious, however, because a load anywhere on the right span would have such a comparatively small effect on the left support that the error could safely be neglected.

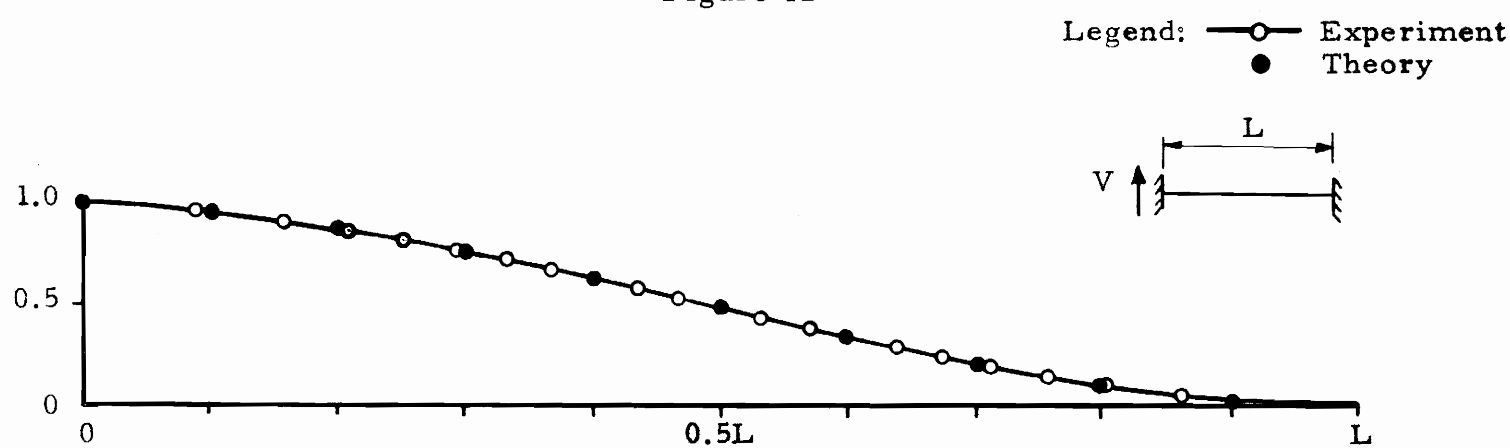
The comparison curves for the symmetrically haunched arch were obtained by solving for influence ordinates using the Column Analogy [10]. Again the experimental curves agreed very closely with those obtained theoretically.

Figures 12 through 18 are the influence lines for the models. Tables I through VII are the tabulated influence ordinates.



Influence Line for Moment at the Left End of a Fixed-Ended Beam

Figure 12



Influence Line for Shear at the Left End of a Fixed-Ended Beam

Figure 13

Table I

Influence Ordinates for Moment at the Left End of a Fixed-Ended Beam

(L' = Span = 10 feet; L = Model Span = 9 . 7 units)

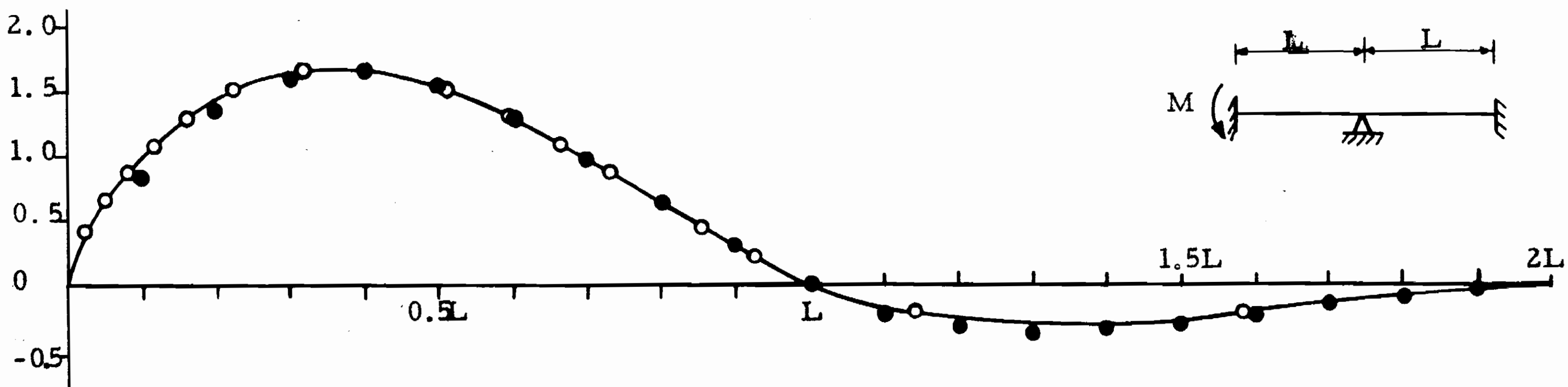
Position $\frac{x}{L}$	Fringe Value d	Moment on Beam $M' = P\frac{d}{\theta} = 1\frac{d}{10}$	Experimental Moment $M = M'\frac{L'}{L} = 10.3M'$	Theoretical Moment
0	0	0	0	0
0.001	2	0.200	0.206	-
0.003	3	0.300	0.309	-
0.034	4	0.400	0.412	-
0.057	5	0.500	0.516	-
0.074	6	0.600	0.618	-
0.095	7	0.700	0.722	-
0.100	-	-	-	0.810
0.115	8	0.800	0.825	-
0.141	9	0.900	0.928	-
0.173	10	1.000	1.032	-
0.200	-	-	-	1.280
0.210	11	1.100	1.133	-
0.286	12	1.200	1.238	-
0.300	-	-	-	1.470
0.328	12.5	1.250	1.290	-
0.366	12	1.200	1.238	-
0.400	-	-	-	1.440
0.457	11	1.100	1.133	-
0.500	-	-	-	1.250
0.512	10	1.000	1.032	-
0.555	9	0.900	0.928	-
0.597	8	0.800	0.825	-
0.600	-	-	-	0.960
0.635	7	0.700	0.722	-
0.672	6	0.600	0.618	-
0.700	-	-	-	0.630
0.712	5	0.500	0.516	-
0.748	4	0.400	0.412	-
0.790	3	0.300	0.309	-
0.800	-	-	-	0.320
0.835	2	0.200	0.206	-
0.900	-	-	-	0.090
0.950	1	0.100	0.103	-
1.000	0	0	0	0

Table II

Influence Ordinates for Shear at the Left End of a Fixed-Ended Beam

(L = Span = 10 feet)

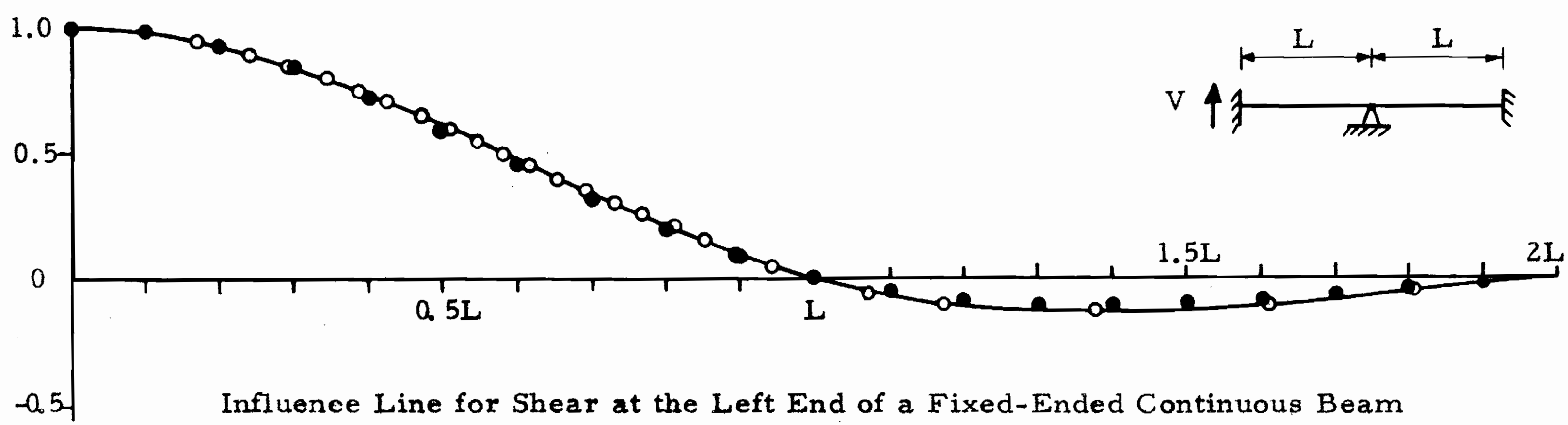
Position $\frac{x}{L}$	Fringe Value d	Experimental Shear $V = P\frac{d}{y} = 1\frac{d}{20.5}$	Theoretical Shear
0	20.5	1.000	1.000
0.086	20	0.977	-
0.100	-	-	0.972
0.155	19	0.926	-
0.200	-	-	0.896
0.207	18	0.879	-
0.252	17	0.830	-
0.294	16	0.780	-
0.300	-	-	0.784
0.331	15	0.732	-
0.367	14	0.683	-
0.400	13	0.635	0.648
0.435	12	0.586	-
0.469	11	0.537	-
0.500	10	0.488	0.500
0.535	9	0.439	-
0.569	8	0.390	-
0.600	-	-	0.352
0.603	7	0.342	-
0.638	6	0.293	-
0.675	5	0.244	-
0.700	-	-	0.216
0.714	4	0.196	-
0.755	3	0.146	-
0.800	-	-	0.104
0.804	2	0.098	-
0.863	1	0.049	-
0.900	-	-	0.028
1.000	0	0	0



Influence Line for Moment at the Left End of a Fixed-Ended Continuous Beam

Figure 14

Legend: —o— Experiment
● Theory



Influence Line for Shear at the Left End of a Fixed-Ended Continuous Beam

Figure 15

Table III

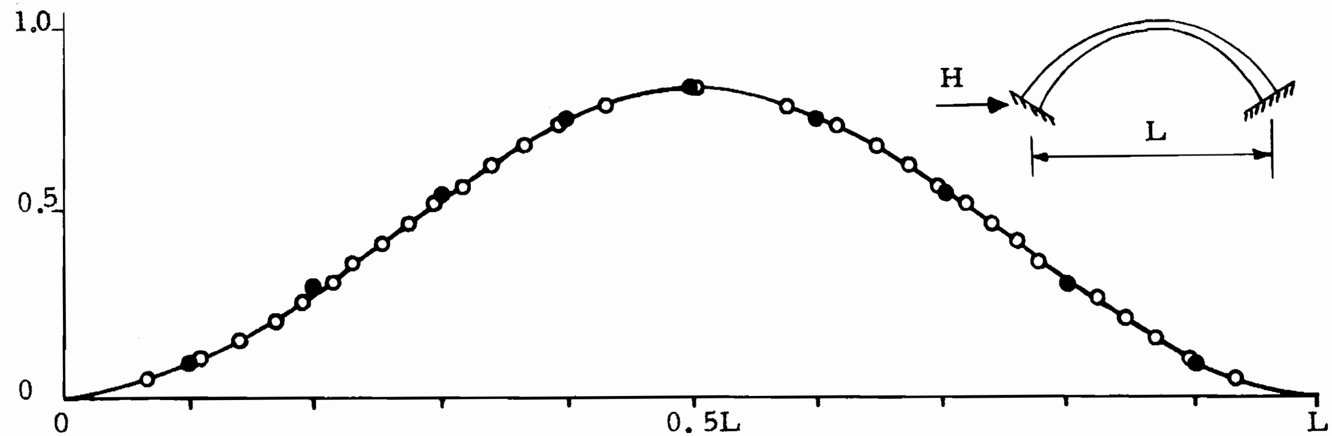
Influence Ordinates for Moment at the Left End of a Two-Span Continuous Beam (L' = Span = 10 feet each; L = Model Span = 6 units

Position $\frac{x}{L}$	Fringe Value d	Moment on Beam $M' = P\frac{d}{\theta} = 1\frac{d}{10}$	Experimental Moment $M = M'\frac{L'}{L} = 10.3M'$	Theoretical Moment
0	0	0	0	0
0.013	2	0.270	0.450	-
0.039	3	0.405	0.675	-
0.070	4	0.541	0.902	-
0.100	-	-	-	0.832
0.107	5	0.675	1.127	-
0.157	6	0.812	1.355	-
0.200	-	-	-	1.360
0.217	7	0.946	1.5	-
0.300	-	-	-	1.627
0.348	8	1.082	1.802	-
0.400	-	-	-	1.680
0.500	-	-	-	1.562
0.508	7	0.946	1.578	-
0.592	6	0.812	1.355	-
0.600	-	-	-	1.320
0.663	5	0.675	1.127	-
0.700	-	-	-	0.997
0.728	4	0.541	0.902	-
0.793	3	0.405	0.675	-
0.800	-	-	-	0.640
0.857	2	0.270	0.450	-
0.900	-	-	-	0.292
0.928	1	0.135	0.225	-
1.000	0	0	0	0
1.100	-	-	-	-0.202
1.135	-1	-0.135	-0.225	-
1.200	-	-	-	-0.320
1.300	-	-	-	-0.367
1.400	-	-	-	-0.360
1.500	-	-	-	-0.312
1.570	-1	-0.135	-0.225	-
1.600	-	-	-	-0.240
1.700	-	-	-	-0.157
1.800	-	-	-	-0.080
1.900	-	-	-	-0.023
2.000	0	0	0	0

Table IV

Influence Ordinates for Shear at the Left End of a Two-Span Continuous Beam (L = Span = 10 feet, each span)

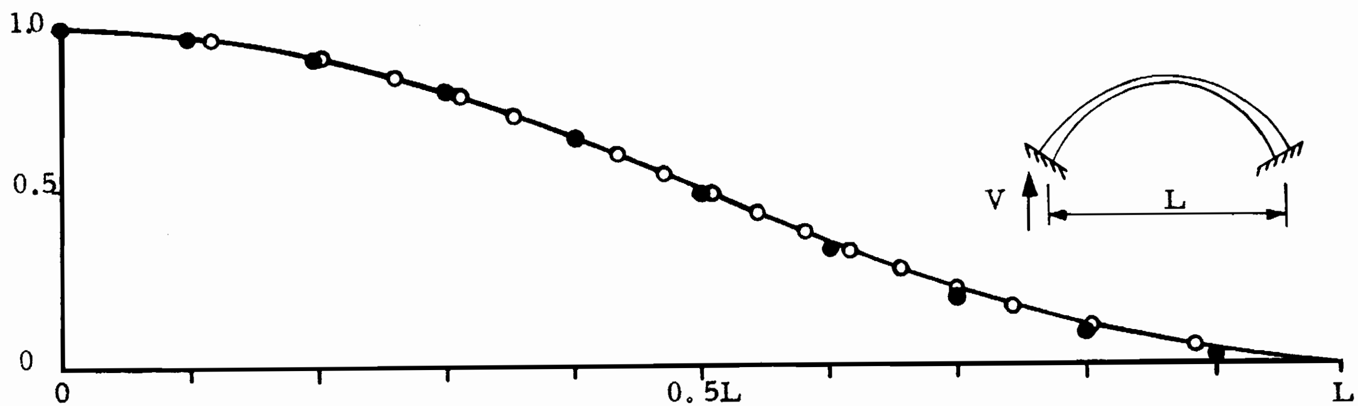
Position $\frac{x}{L}$	Fringe Value d	Experimental Shear $V = P\frac{d}{y} = 1\frac{d}{20}$	Theoretical Shear
0	20	1.000	1.000
0.100	-	-	0.979
0.161	19	0.950	-
0.200	-	-	0.920
0.230	18	0.900	-
0.285	17	0.850	-
0.300	-	-	0.831
0.336	16	0.800	-
0.382	15	0.750	-
0.400	-	-	0.720
0.423	14	0.700	-
0.465	13	0.650	-
0.500	-	-	0.594
0.504	12	0.600	-
0.541	11	0.550	-
0.578	10	0.500	-
0.600	-	-	0.460
0.617	9	0.450	-
0.652	8	0.400	-
0.690	7	0.350	-
0.700	-	-	0.326
0.728	6	0.300	-
0.765	5	0.250	-
0.800	-	-	0.200
0.808	4	0.200	-
0.850	3	0.150	-
0.900	2	0.100	0.089
0.943	1	0.050	-
1.000	0	0	0
1.075	- 1	- 0.050	-
1.100	-	-	- 0.061
1.175	- 2	- 0.100	-
1.200	-	-	- 0.094
1.300	-	-	- 0.110
1.375	- 2.5	- 0.125	-
1.400	-	-	- 0.108
1.600	-	-	- 0.072
1.613	- 2	- 0.100	-
1.800	-	-	- 0.024
1.805	- 1	- 0.050	-
2.000	0	0	0



Influence Line for Horizontal Thrust for a Symmetrically Haunched Arch

Figure 16

Legend: —o— Experiment
● Theory



Influence Line For Shear at the Left End of a Symmetrically Haunched Arch

Figure 17

Table V

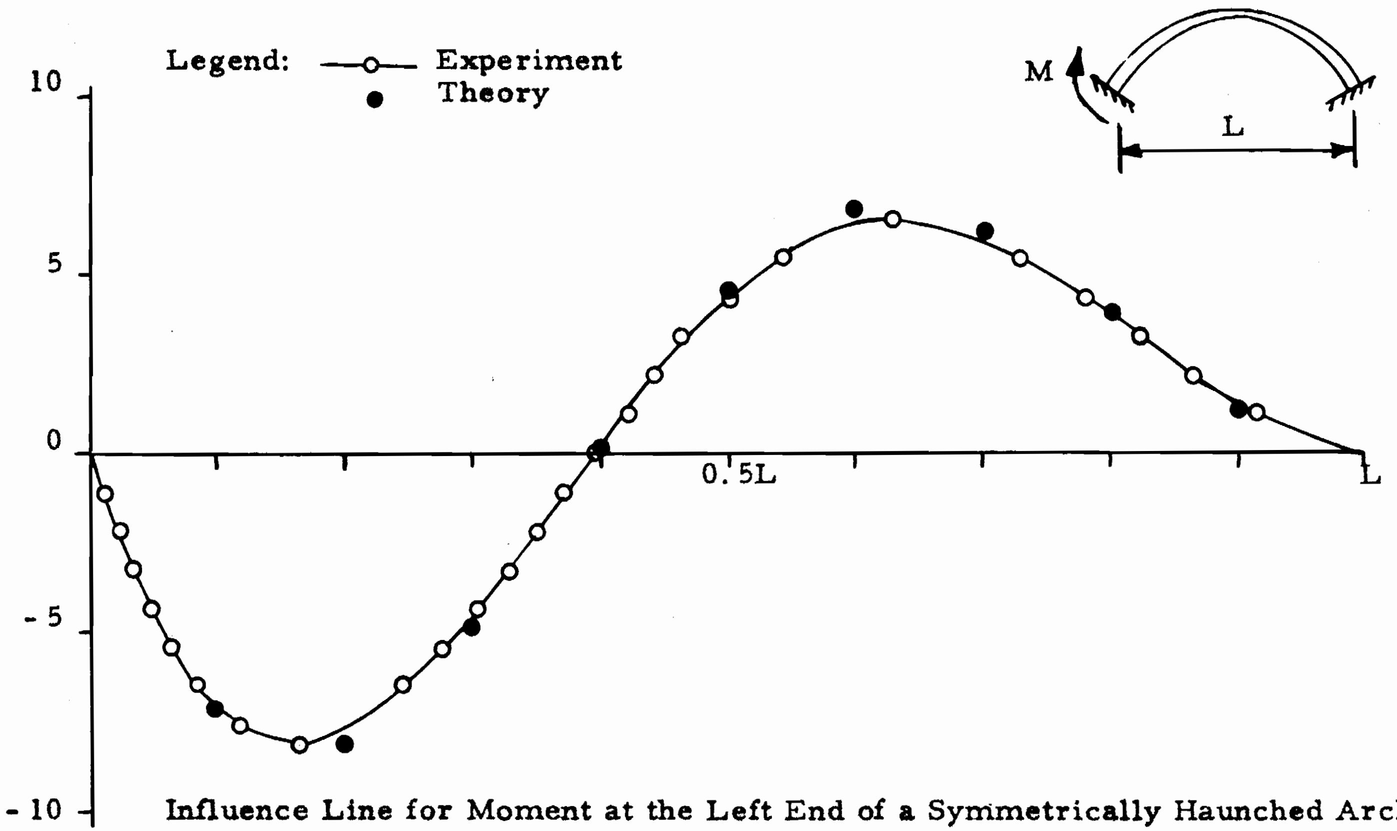
Influence Ordinates for Horizontal Thrust on a Symmetrically Haunched Arch (L = Span = 100 feet)

Position $\frac{x}{L}$	Fringe Value d	Experimental Thrust $H = P\frac{d}{y} = 1\frac{d}{20}$	Theoretical Thrust
0	0	0	0
0.064	1	0.050	-
0.100	-	-	0.086
0.108	2	0.100	-
0.138	3	0.150	-
0.167	4	0.200	-
0.190	5	0.250	-
0.200	-	-	0.298
0.211	6	0.300	-
0.231	7	0.350	-
0.254	8	0.400	-
0.274	9	0.450	-
0.295	10	0.500	-
0.300	-	-	0.547
0.318	11	0.550	-
0.341	12	0.600	-
0.367	13	0.650	-
0.395	14	0.700	-
0.400	-	-	0.748
0.433	15	0.750	-
0.500	-	-	0.825
0.505	16	0.800	-
0.580	15	0.750	-
0.600	-	-	0.748
0.618	14	0.700	-
0.649	13	0.650	-
0.675	12	0.600	-
0.697	11	0.550	-
0.700	-	-	0.547
0.720	10	0.500	-
0.742	9	0.450	-
0.761	8	0.400	-
0.780	7	0.350	-
0.800	-	-	0.298
0.803	6	0.300	-
0.825	5	0.250	-
0.849	4	0.200	-
0.872	3	0.150	-
0.900	2	0.100	0.086
0.935	1	0.050	-
1.000	0	0	0

Table VI

Influence Ordinates for Shear at the Left End of a Symmetrically Haunched Arch (L = Span = 100 feet)

Position $\frac{x}{L}$	Fringe Value d	Experimental Shear $V = P\frac{d}{y} = 1\frac{d}{18}$	Theoretical Shear
0	18	1.000	1.000
0.100	-	-	0.978
0.118	17	0.944	-
0.200	-	-	0.912
0.202	16	0.888	-
0.260	15	0.833	-
0.300	-	-	0.803
0.311	14	0.777	-
0.355	13	0.722	-
0.398	12	0.666	-
0.400	-	-	0.662
0.435	11	0.610	-
0.473	10	0.555	-
0.500	-	-	0.500
0.507	9	0.500	-
0.542	8	0.444	-
0.580	7	0.389	-
0.600	-	-	0.338
0.615	6	0.333	-
0.655	5	0.278	-
0.697	4	0.222	-
0.700	-	-	0.197
0.742	3	0.165	-
0.800	-	-	0.088
0.803	2	0.111	-
0.885	1	0.056	-
0.900	-	-	0.022
1.000	0	0	0



Influence Line for Moment at the Left End of a Symmetrically Haunched Arch

Figure 18

Table VII

Influence Ordinates for Moment at the Left End of a Symmetrically Haunched Arch (L' = Span = 100 feet ; L = Model Span = 10 units)

Position $\frac{x}{L}$	Fringe Value d	Moment on Beam $M' = P\frac{d}{\theta} = 1\frac{d}{10}$	Experimental Moment $M = M'\frac{L'}{L} = 10.3M'$	Theoretical Moment
0	0	0	0	0
0.013	-1	-0.100	-1.000	-
0.025	-2	-0.200	-2.000	-
0.036	-3	-0.300	-3.000	-
0.050	-4	-0.400	-4.000	-
0.066	-5	-0.500	-5.000	-
0.087	-6	-0.600	-6.000	-
0.100	-	-	-	-6.65
0.120	-7	-0.700	-7.000	-
0.167	-7.5	-0.750	-7.500	-
0.200	-	-	-	-7.50
0.202	-7	-0.700	-7.000	-
0.247	-6	-0.600	-6.000	-
0.280	-5	-0.500	-5.000	-
0.300	-	-	-	-4.50
0.305	-4	-0.400	-4.000	-
0.330	-3	-0.300	-3.000	-
0.352	-2	-0.200	-2.000	-
0.375	-1	-0.100	-1.000	-
0.395	0	0	0	-
0.400	-	-	-	0.13
0.422	1	0.100	1.000	-
0.445	2	0.200	2.000	-
0.470	3	0.300	3.000	-
0.500	-	-	-	4.27
0.502	4	0.400	4.000	-
0.542	5	0.500	5.000	-
0.600	-	-	-	6.33
0.626	6	0.600	6.000	-
0.700	-	-	-	5.84
0.730	5	0.500	5.000	-
0.780	4	0.400	4.000	-
0.800	-	-	-	3.66
0.824	3	0.300	3.000	-
0.867	2	0.200	2.000	-
0.900	-	-	-	1.16
0.915	1	0.100	1.000	-
1.000	0	0	0	0

XII. CONCLUSIONS AND RECOMMENDATIONS

The close agreement of the results of the experimental investigation with theory supports the original premise that mechanical interferometry could be successfully used for the determination of influence lines. The method offers the advantages of inexpensive apparatus and materials, accuracy and permanence of record over some of the methods in present use. For speed and the ease with which deflections may be read, the method is unexcelled.

The method should, of course, be subjected to further tests in order to extend its usefulness. It should be used with models of other types of structures to obtain influence lines for positions other than points of support.

It should prove interesting to use the ruled plastic models with other types of deformeter apparatus, particularly the Eney Deformeter. With a finer ruled pattern--more lines per inch--it might be possible to use the Beggs Deformeter.

The method would be much more useful if models could be built up using splines cemented together. Several such models should be tested to see how they compare with one-piece models.

An attempt should be made to reproduce the fringe pattern by some method such as Ozalid or blueprint, to further increase the utility of the method. Every effort should be made to enable the prospective user to work with the equipment which he has. The commercial possibilities of the method should be investigated.

XIII. SUMMARY

The experimental investigation of the application of mechanical interferometry to the construction of influence lines has proven successful. The method is similar in principle to other methods of model analysis. Mechanical interferometry offers the advantages of speed and permanence of record over the other methods in present use. The idea seems to offer commercial possibilities.

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XVI. VITA

James Howard Sword was born in Derby, Wise County, Virginia, on January 1, 1924. He attended grammar school in Derby, high school through the junior year in Appalachia, Virginia, and was graduated from Woodrow Wilson High School in Portsmouth, Virginia, in June, 1942.

He worked as a store manager for Colonial Stores, Inc., in Portsmouth until his induction into the Army in March 1943. After approximately nineteen months training he was sent to the Pacific Theater where he served with the 511th Parachute Infantry of the 11th Airborne Division. He was discharged in February 1946 with the rank of Sergeant.

He entered Virginia Polytechnic Institute in September 1946 and was graduated, with Honors, in June 1950 with the Bachelor of Science Degree in Civil Engineering. He held a teaching fellowship in the Civil Engineering Department at V. P. I. for six months and was an instructor for six months. In September 1951 he was made Instructor in the Department of Applied Mechanics at V. P. I. where he is now an Assistant Professor.

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