

**Application of Localized Hybrid Methods of Stress Analysis to Some
Problems in the Mechanics of Composites**

by

Ming-Yi Tsai

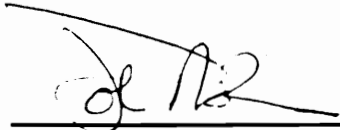
Dissertation submitted to the Faculty of the
Virginia Polytechnic Institute and State University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

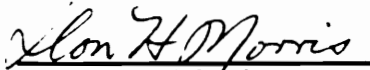
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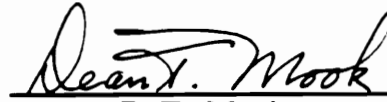
Engineering Mechanics

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(ABSTRACT)

A new method of stress analysis which combines an experimental technique –moire interferometry, and a numerical method – finite element analysis, is presented. In this localized hybrid method, the displacement fields which the moire experiments provide in some local regions of interest are used as input data for finite element stress analyses.

Two important and controversial problems in the mechanics of composites are investigated using the localized hybrid method. One is a thermally loaded bimetal plate, and the other involves the Iosipescu shear specimen popularly used to determine the shear modulus and strength of a fiber reinforced composite. Before applying the localized hybrid analysis, the mechanics of the problems are discussed individually, through a numerical study and strength of materials analysis. Based on these fundamental studies, the localized hybrid method is applied to stress or strain analyses of moire experimental results, and special techniques of this method are developed to handle these data .

For the thermally loaded plate, several finite element models, simulating 2-D and 3-D mechanics, are used to assess the stress state at the interface near the free surface, and identify a boundary layer. It is shown that high gradients and stress turnaround are documented in stress component normal to the interface, along the surface line crossing the interface, and a boundary layer is identified in the small region near the interface around the free surfaces. These observations are also confirmed by the hybrid analysis of moire experimental results. However, additional variations of the localized hybrid method were needed to capture the three-dimensional nature of the problem. A comparison of numerical

results with experimental data resolved an apparent anomaly between experiments and mechanics principles.

For the Iosipescu shear specimen, the 3-D mechanics associated with twisting is proposed for accounting for the inconsistent and variable results in the literatures. The results from the localized hybrid analysis indicate that uniformity and purity of shear stress state in the test section cannot be accomplished for each fiber orientation specimens; the 0° specimens suffer from a load proximity effect, the 90° specimens are affected by apparent in- and out-plane bending, and the $0^\circ/90^\circ$ specimens are midway between those.

Several variations of the localized hybrid method of stress analysis have been presented for three-dimensional problems in the mechanics of solids. It is showed that the approach developed not only provides a powerful and efficient technique for the reduction of experimental data, but also gives a good insight into the mechanics of the experimental observations.

ACKNOWLEDGEMENTS

I would like to express great thanks to Dr. John Morton, the chairman of the committee, for his knowledge, guidance, encouragement, and patience throughout Ph. D. program. Also, I am very thankful to Dr. R. Czarnek and his students, Joosik Lee and Shih-Yung Lin, for their helps in the moire experiments, and my graduate committees, Dr. J. P. Wightman, Dr. D. H. Morris, Dr. D.T. Mook, and Dr. S. Kellas for their comments and review of this dissertation. And the sponsorships from the DARPA/BTI program under Grant ONR/N00014-88-K-0741, US Army Aerostructures Directorate under NASA Research Grant NAG-1-1053 and the NSF Science and Technology Center at Virginia Polytechnic Institute and State University are very appreciated. In addition, thanks :

- To my group fellow students, Henjen Ho, Todd Wieland, Kuen Tat Teh, David Johnson, Haryanto Budiman and Catherine Traffanstedt, for their cooperation and good friendship.
- To Chari Morton and her sons, John and Michael, whose great friendship is appreciated.

Finally, I would like to thank my family (especially my mother) and friends for always keeping me in good spirit through my academic career.

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Chapter 1 Introduction

1.1 Introduction

Moire interferometry is an optical method of full-field displacement measurement, which features very high sensitivity and resolution [1, 2, 26]. Accurate in-plane displacement components of a loaded body can be easily determined by this method. In recent years, it has been successfully used in the investigation of mechanics phenomena and the determination of material properties in composite materials or structure problems: For example, free-edge effects of composite and thermal strains in composite laminates, contact problems in composite components, joint problems in composite structures, strain concentrations in notched specimens, the nonuniformities and properties of composite materials, etc.

Although moire interferometry can provide highly accurate, full-field and in-plane displacement data on the surface of the loaded specimen, strain and stress data are needed in practical engineering analyses. For the commonly used manual reduction of displacement data, the strains are obtained by differentiating the displacements in the immediate vicinity of each point of interest. This requires the extraction of displacements at several locations near each point of interest, followed by effective curve fitting and slope determination. This point-by-point manual differentiation is extremely tedious when the strain data at many points are required, and inaccurate if the region of interest has high gradients of strain. Such an approach might have merit in determining strains at a few discrete points, but otherwise the localized hybrid method [3] is more attractive. The complete stress, strain and displacement fields can be evaluated by the hybrid method for any region of interest. Meanwhile, the examination of the mechanics of the loaded specimen can be made during or after implementation of the hybrid method. That is, essential checks of equilibrium can be made in a routine manner.

The localized hybrid method was first developed for the stress (or strain) analysis of highly sensitive moiré (moiré interferometry) experimental data [3-5]. In reference [3], the localized hybrid method involving the specification of moiré displacement data around the boundary of the region of interest was developed and demonstrated in an experimental stress analysis of a pin-loaded aluminum plate problem. The accuracy and sensitivity of this method were tested by means of introducing the random-displacement errors and different mesh geometries. It was concluded that the mesh geometry used in the localized hybrid analysis is not a critical factor in the two-dimensional linear analysis, and the accuracy of this method can be accepted only in the application to high sensitive moiré displacement data (sensitivity = $0.417\text{ }\mu\text{m}$) but not to geometric moiré data (sensitivity = $4.17\text{ }\mu\text{m}$ or less). In addition, a pin-loaded composite plate problem has been also analyzed by a localized hybrid method [4]. Since this method can be applied to any arbitrary moiré region, the local failure regions were avoided and regions which were linearly elastic were analyzed. As a result, the contact stress distributions have been determined successfully. It has been shown, therefore, that the localized hybrid method is an efficient technique for extracting strains and stresses from experimentally determined surface displacement fields, and provides a convenient means of examining the mechanics of the loaded bodies.

1.2 Objectives

The main purposes of this study are to develop and apply the localized hybrid method in the stress analyses of composite materials and also to assess the suitability and stability of this method in the three-dimensional regions. Since the previous analyses were limited in two-dimensional plane problems, the present study will focus on the development of a localized hybrid method to deal with three-dimensional or mixed two-dimensional and three-dimensional problems. The application of the localized hybrid method is described through the investigation of two case studies which are of great interest in the mechanics of composites. One is a bimetal plate subjected to a uniform change of temperature, and the other is the evaluation of the composite Iosipescu shear specimen tested in a modified Wyoming fixture. The results indicate that the localized hybrid method is essential for providing an efficient means of data reduction from the displacement fields

resulting from the two-dimensional and three-dimensional natures of the mechanics, and for checking and interpreting the complex mechanics of the analyzed problems.

1.3 Dissertation Outline

In the following descriptions, the theoretically based techniques are introduced first in Chapter 2. The principles of high sensitive moire interferometry are addressed briefly and the new ideas of the localized hybrid method are proposed and developed. Two important and controversial problems in the mechanics of composites, a thermally loaded bimetal plate and the Iosipescu shear specimen, are investigated and discussed in terms of the mechanics of the problems, and the solutions of the localized hybrid analyses, in Chapters 3 and 4 respectively. Finally, conclusions and recommendations are presented in Chapter 5.

Chapter 2 Background

2.1 Introduction

The localized hybrid method is a stress (or strain) analysis method of combining highly sensitive moire interferometry and numerical finite element analyses. The principles of moire interferometry are briefly reviewed first. Then some ideas on the localized hybrid method are proposed through the basic formulations of the finite element technique. With these concepts, three possible schemes of the localized hybrid method will be summarized and discussed individually.

2.2 Moire interferometry

Moire interferometry is a highly sensitive full-field method of deformation analysis which uses the interference of two coherent light beams to measure the in-plane surface displacement components (recorded as fringe patterns for the horizontal and vertical components, u and v , respectively). The method provides subwavelength sensitivity and high spatial resolution [1, 2, 26]. For this technique, a crossed-line diffraction grating (usually, 30,480 lines/in [1200 lines/mm]) is replicated on the surface of specimen and deforms together with it. The principles are shown schematically in Figure 1. Two coherent beams, A and B, illuminate the specimen grating and are diffracted by it. The first orders diffracted beams (1, -1) are focused on a screen where interference takes place to form the N_x fringe pattern which represents contours of constant horizontal displacement u . An additional set of two beams (not shown) illuminate in the vertical plane and form a fringe pattern called v -field displacement. This interference is governed by the equation;

$$f = \frac{2}{\lambda} \sin \alpha \quad (2.1)$$

where α is the incident angle, λ is the wavelength of the incident light, and f represents the frequency of a virtual reference grating. These resulting displacement component contours or fringes (u and v) can be expressed in terms of f ;

$$u = \frac{1}{f} N_x \quad \text{and} \quad v = \frac{1}{f} N_y \quad (2.2)$$

where N_x and N_y denote the fringe order in u - and v -field respectively. In the arrangement used in this investigation, $f = 60,960$ lines/in (2400 lines/mm), so one fringe order difference corresponds to a relative displacement of 16.4×10^{-6} in. ($0.417 \mu\text{m}$). The strain components can be determined by

$$\begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} = \frac{1}{f} \frac{\partial N_x}{\partial x} \\ \epsilon_y &= \frac{\partial v}{\partial y} = \frac{1}{f} \frac{\partial N_y}{\partial y} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{1}{f} \left(\frac{\partial N_x}{\partial y} + \frac{\partial N_y}{\partial x} \right) \end{aligned} \quad (2.3)$$

2.3 Localized hybrid method

The conventional finite element method is a powerful method of numerical stress analysis. In the finite element analyses, structural components are modelled as a collection of elements. The displacements in each element are assumed to follow given functional relations (e.g. linear, quadratic, etc). The stiffness of each element is calculated in terms of the coefficients of the displacement function and the material constitutive law. The global stiffness of the structural component is assembled by the requirements of satisfying the compatibility and equilibrium at the nodes of all elements. This technique is a "stand-

alone" method of stress analysis. However, when it is applied, two problems usually occur. One difficulty is the need to apply the correct boundary conditions in the stress analysis of real engineering problems. Another problem is that a very large number of elements are often required to model the structural component even though the region of prime interest is quite small.

The localized hybrid method is a strain (or stress) analysis method which combines moire interferometry and finite element analysis in some local, arbitrary regions of a structural component. Moire interferometry provides full-field displacement data from the experiment. Finite element analysis is used as a convenient tool for differentiating the resulting displacement (fringe) data, or for computing the strain field using boundary displacements and the global stiffness. Then stresses may be determined by introducing the appropriate material constitutive laws.

Three methods of localized hybrid method are presented in terms of fundamental finite element theory. In the finite element formulation, the global stiffness matrix is assembled with all element stiffness matrices by incorporating the requirement of nodal compatibility and equilibrium. A convenient, partitioned form is

$$\begin{Bmatrix} \{P_1\} \\ \{P_2\} \end{Bmatrix} = \begin{bmatrix} [K_{11}] & [K_{12}] \\ [K_{21}] & [K_{22}] \end{bmatrix} \begin{Bmatrix} \{\Delta_1\} \\ \{\Delta_2\} \end{Bmatrix} \quad (2.4)$$

where $\{\Delta_1\}$ and $\{P_1\}$ denote the boundary displacements and forces respectively, $\{\Delta_2\}$ and $\{P_2\}$ represent the internal nodal displacements and forces. Once Equation (2.4) has been solved, the element strains can be calculated by

$$\{\epsilon\}_e = [B]_e \{\Delta\}_e \quad (2.5)$$

where $[B]_e$ is a matrix expressed in terms of the interpolation function and $\{\Delta\}_e$ is the element displacement vector.

Method I (boundary-node method): The boundaries of the arbitrary region of moire pattern are defined and the boundary-node displacements applied as input to the finite

element calculation. Let $\{\Delta_1\}$ be the displacements of nodes around the boundary of the region for which a pair of moire fringe patterns, i. e. maps of u and v , is available. $\{\Delta_1\}$ are known displacements, but $\{P_1\}$ are unknown forces at boundary nodes. The internal nodal forces $\{P_2\}$ should be zero if no external force acts at the internal nodes. The internal nodal displacements $\{\Delta_2\}$ are regarded as unknown. Then, the simultaneous equations can be solved. $\{\Delta_2\}$ and $\{P_1\}$ are given by

$$\begin{aligned}\{\Delta_2\} &= -[K_{22}]^{-1}[K_{21}]\{\Delta_1\} \\ \text{and} \\ \{P_1\} &= ([K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}])\{\Delta_1\}\end{aligned}\quad (2.6)$$

Note that the application of this method requires the correct material constitutive laws to satisfy the equilibrium condition.

Method II (full-node method): Instead of using the boundary displacements only, the displacements at each node (including the internal nodes) are specified through the experimental moire data. The strains can be determined directly using Equation (2.5) (involving only the chosen interpolation function) without any material constitutive laws involved. Of course, the stress determination still requires the material constitutive equation. This approach has its advantages and limitations. It features the differentiation of moire fringe, region by region, through the given appropriate interpolation function even in nonlinear regions (such as plastic deformation, local failure zones, etc.) and three-dimensional effect regions. But it requires a large number of boundary data determined from the moire experiment to be used as input to finite element calculation.

Method III (mixed-boundary method): The mixed force and displacement boundary conditions can be applied in a localized hybrid analyses. If, in Equation (2.4), any of $\{\Delta\}$, $\{P\}$, or part of $\{\Delta\}$ and the rest of $\{P\}$ are known, this equation is soluble and its solution is unique except for the trivial solution. That is, when some of the nodal forces are zero (e.g. the internal nodal forces, free boundary nodal forces, etc.), if no external forces act on elements or nodes, only specification of the displacements at the rest of nodes, at which nodal forces are not zero, is required. If some of the nodal forces have been given values (non-zero, if external forces applied), mixed force (non-zero known values) and displacement boundaries need to be given in the finite element analysis. In this approach,

the correct material constitutive law is needed so that the equilibrium conditions can be satisfied exactly.

These three methods of the localized hybrid method will be demonstrated in case studies. Their applications, and advantages and disadvantages will also be discussed individually.

Chapter 3 Case Study I—Thermally Loaded Bimetal Problem

3.1 Introduction

A bimetal plate subjected to a uniform thermal loading is investigated in the present study. Attention is focused on the understanding of the stress state in the bimaterial interface region, in which the mismatch of thermal and mechanical properties and specimen geometry play an important role. In this study, a simple geometry, a rectangular plate, is adopted to restrict the influence of geometric factors. Of particular interest is the nature of a thermally loaded steel/brass bimetal plate, which have different thermal expansion coefficients and mechanical properties.

The bimaterial interface has practical application in thermostats in which the mismatch of material properties is exploited. Bimaterial interfaces are also common in the semiconductor industry where the mismatch of thermal and mechanical properties can cause severe manufacturing, service and durability problems. Parallels can also be drawn in composite materials technology. It is well known that laminate composites develop free-edge stresses which are not predicted by classical lamination theory but which are required for equilibrium, and which can be the cause of damage and failure in composites. On an even smaller scale, the fiber/matrix interface at a free surface would suffer from a stress system, as a result of the fabrication process or under service thermal loading, which has the same large differences in the normal stresses as the interface was approached from the fiber or the matrix.

In the following section, the theoretical analyses relevant to the bimaterial problem are surveyed from the literatures. The present problem, which was motivated by apparently anomalous moire data reported by Wood *et al.* [6], is defined, and the objective of this study is then described. To confirm and interpret Wood's experimental results, a simple mechanics analysis with free body diagrams, based on the published theoretical

arguments, is presented first in Section 3.3. Further numerical analyses are carried out, and the resulting solutions are then discussed in Section 3.4. Instead of manually differentiating moire fringes, a convenient, accurate localized hybrid method is applied to the strain (or stress) analysis of moire experimental results and is presented in Section 3.5. Meanwhile, several approaches to the application of the localized hybrid method are tested in terms of validity, accuracy, and efficiency.

3.2 Literature review and definition of the problem

A stress analysis of the heated bimetallic strip shown in Figure 2 was first performed by Timoshenko [7] who used a plane elasticity approach to predict the radius of curvature of such a strip. This model was successfully applied to the design of thermostats. Timoshenko's analysis predicted that the strip would develop a constant radius of curvature along its length and, away from the ends of the strip, generate two linear axial stress distributions through its thickness, defined by

Strip 1:

$$\frac{\sigma_{x_1}}{\sigma_T} = \frac{3mn(1+m)(y_1/c_1) + (m^3n+1)/m}{A\sqrt{n}}$$

$$\sigma_{y_1} = \tau_{x_1y_1} = 0$$

Strip 2:

$$\frac{\sigma_{x_2}}{\sigma_T} = \frac{3(1+m)(y_2/c_2) - (m^3n+1)}{A\sqrt{n}}$$

$$\sigma_{y_2} = \tau_{x_2y_2} = 0 \quad (3.1)$$

$$\text{where } m = \frac{c_1}{c_2} = \frac{t_1}{t_2}$$

$$n = E_1/E_2$$

$$A = 3(1+m)^2 + (1+mn)(m^2+1/mn)$$

$$\sigma_T = (\alpha_2 - \alpha_1) \sqrt{E_1 E_2} \Delta T$$

At the ends of the strip, Timoshenko's solution could not account for the zero axial stress condition. Hess [8] examined the stress distributions near the ends of a bimaterial interface where the shear stresses at the interface were significant. Hess showed that Timoshenko's solution was accurate provided that the length of the bimetal plate was more than twice the total depth and that the stresses were calculated no closer than at a distance equal to the total depth from the free ends. Recently Wood *et al.* [6] performed a detailed experimental (moiré interferometry) analysis of the in-plane surface displacement fields in the interface region of a thermally loaded square steel/brass bimetal plate which had the dimensions and material properties shown in Figure 3. The strains were obtained by taking the appropriate derivatives, and the stress components determined from the Hooke's law for plane stress conditions. The result of this was that the normal stress component perpendicular to the interface was large and tensile in the steel, and large and compressive in the brass, when the bimetal plate was cooled through 240°F. It appeared that the conditions of equilibrium were violated across the bimaterial interface [9].

The present study first examines the mechanics of the thermally loaded bimaterial interface (for Wood's specimen) in detail by considering a number of simple bimaterial numerical cases which can be analyzed as two- and three-dimensional problems which reflect the nature of the phenomenon. Then, the localized hybrid method is employed in a further study of the moiré experimental results. Thus strain and stress fields can be determined from moiré displacement field. The mechanics of the thermally loaded bimetal plate is illustrated in the process of applying the localized hybrid technique.

3.3 Mechanics of the bimetal problem

Timoshenko's solution for the stresses in the bimetallic strip predicts two linear axial stress distributions across the strip. The normal stress perpendicular to the interface is zero, in this one-dimensional approach. This cannot be applied to the geometry in Figure 3 which represents a square bimetal plate, which is outside the (two-to-one) condition

determined by Hess [8]. It might be argued that Hess's two-dimensional representation would be appropriate to this problem, in which the stresses are independent of the coordinate z . However, further application of Hess's own arguments for the free body diagrams for the steel/brass plate under thermal loading suggests that the problem is three-dimensional, as shown in Figure 4. The plane elasticity approach yields the free body diagrams in Figure 4a, essentially those used by Hess to illustrate the need for the normal and shear stress system (σ_y, τ_{xy}) for equilibrium at the free ends. However, precisely the same arguments must be applied across the thickness of the plate, as shown in Figure 4b. Thus there will be a skin or boundary layer near the interface in each material. This interface effect region is a zone around the interface, intersecting the free surfaces, and extending some distance within the material. In this region a three-dimensional stress system occurs, and special characteristics will be observed at an edge, where two such regions intersect. This implies that the two-dimensional analysis of Hess may be applicable to the mid-plane of the bimetal plate, but not very close to the ends. However, no matter how thin the plate Hess's solution will not predict the stresses on the surface of the plate close to the bimaterial interface. An essentially similar effect is observed in the finite element analysis of the free-edge effect stresses at a $\pm\theta$ interface in a composite laminate [10].

The implication of this is that the experiment of Wood *et al* .[6] will be capturing this free-edge effect which is a three-dimensional rather than two-dimensional problem. The central issue is then how can the experimental observation, that there are large normal stresses perpendicular to the interface on either side of it, be rationalized in terms of classical equilibrium considerations? A further question concerns the size or extent of the interface effect region; what scale must be considered in order to capture it? It is clear that even in the bimetallic strip there will be an interface effect region which does not influence the curvature of the strip when thermally loaded, but which may suffer from highly localized stresses which could ultimately cause failure.

If the concept of the interfacial boundary region is correct, and Wood's experimental observations are also correct, then this region must be very small and very large stress gradients must be present within this region if equilibrium conditions were to be satisfied. Thus any analysis of the problem must have sufficient resolution to capture these gradients within this region. Treatments of the free-edge effects in composites

suggest that singular stresses will occur [11, 12], so numerical models will have to have very fine meshes.

3.4 Numerical analyses (finite element models)

Two-dimensional finite element models of Wood's bimetal plate are attractive since they can be performed with modest computational resources even for very dense finite meshes. Several such models are presented below in order to illustrate the limitations of two-dimensional stress analyses and the nature of the stress systems near a bimaterial interface. A two-dimensional plane stress model of the steel/ brass plate is described below as the "2-D plate model", for the x-y plane in Figure 3. Another approach is to consider the y-z plane through the middle of the plate in Figure 3, defined by $x=2.20$ in.. It might be argued that a plane strain model could be used to investigate the stresses in this slice, near the interface, but the thermal loading precludes this. Instead, a plane stress analysis is presented below, and denoted the "2-D slice model". Rather than consider two plates, two steel/brass cylinders can be analyzed as a two-dimensional axisymmetric problem without the uncertainties of the plane stress/plane strain arguments. A bimetal cylinder is discussed below under the heading the "2-D axisymmetric model". The same mesh as the 2-D slice model can be used and the two sets of results compared with each other. In terms of the experimental data available for the steel/brass plate, the normal stresses parallel to the interface can be examined to see if the trends in the numerical models reflect the gradients needed to satisfy equilibrium, while validating the experimental observations.

The plate-like geometry of Figure 3 is not amenable to full three-dimensional analysis without recourse to substantial computational resources. To illustrate the three-dimensional nature of the bimaterial interface at an edge, two cubes with the properties of steel and brass are modelled. The model is described below under the heading the "3-D model".

All of the finite element calculations were performed on an Apollo DN 4000 workstation using the FINEL and ABAQUS computer codes. The analyses were linear, and used the material properties and the temperature change shown in Figure 3.

2-D Plate Model

A 2-D plate model of finite element analysis is adopted to simulate the mechanics of Wood's specimen in the x-y plane. The geometry of this model is shown in Figure 5a. Exploiting symmetry, half the length of the plate, was analyzed using four-node constant strain elements. These calculations yielded the deformed mesh shown in Figure 5b. This deformed shape is similar to that used in the free body diagrams developed in Figure 4a. Contour maps of the normal stress σ_x are shown in Figure 5c. It is clear that at the center of the plate the stresses are almost independent of x, in agreement with Hess's observation. The contour maps of σ_y are shown in Figure 5d. Except near the free surface, these stresses are approximately zero. A large gradient in σ_y is observed at the free surface near the interface. Along the interface shear stresses are observed, as shown in Figure 5e, but at the middle of the plate these approach zero. Thus the two-dimensional finite element solution has all the features suggested by Hess, and even Timoshenko's assumptions are satisfied to a reasonable degree, on a line perpendicular to the interface, through the middle of the plate. The normal stress σ_x distributions from Timoshenko's, moire, and 2-D finite element models are plotted along the line AA (midway between the edge and the center of the plate), in Figure 6. It is apparent that σ_x from moire and 2-D finite element are in close agreement, while that from Timoshenko's prediction is in reasonable agreement. In Figure 7, the solution of this model (all three stress components) is also compared with data from Wood's moire experiment along a line AA. Results from the two studies are in good agreement for the σ_x and τ_{xy} stress distributions, but not for σ_y . The mechanics arguments, made above, however, suggest that this model will not provide an accurate assessment of the surface stresses at the interface of a real plate, even near the middle.

2-D Slice Model

The model used is shown in Figure 8a. The symmetry of the problem was exploited to produce a strip of width equal to half the thickness of the plate in Figure 3. Eight-node quadrilateral elements were employed. Note the very fine scale of the mesh in the vicinity of the bimaterial interface, especially near the free surface.

The results of the finite element computations are presented as stress distributions along four key lines defined in Figure 8b. These consist of the line AA which corresponds to a line on the surface on either side of the interface, the lines BB and DD which are parallel and near the interface in the steel and brass respectively, and the line CC which lies

along the interface. The stress components along AA are shown in Figure 8c in which it is seen that the normal stresses σ_y on either side of the interface are indeed of opposite sign as the interface is approached, and very close to the interface the stress in the brass undergoes a very rapid reversal. The normal stress σ_z and the shear stress τ_{yz} are essentially zero, as expected from the boundary conditions, except very close to the interface; this is a finite element modelling phenomenon which will be discussed later. The stress distributions on either side of the interface, Figures 8d and f, confirm the differences in sign in σ_y and show the very rapid gradient in σ_y as the surface is approached. The stresses along the interface line CC are shown in Figure 8e in which it is apparent that the stress σ_y is tensile at the interface on the free surface. Note that the stresses σ_z are unreliable along CC since these are discontinuous along the interface, and the calculation takes the average of these values.

2-D Axisymmetric Model

The 2-D axisymmetric model, with a two-dimensional scheme but a three-dimensional nature, represents two solid cylinders of steel and brass, as shown in Figure 9a, for simulating the y-z plane through the middle of the Wood's bimetal plate. The same mesh as in the 2-D model was used but the calculation was performed with the axisymmetric option. Note that this needed a hole of small diameter to be modelled along the y-axis.

The stresses computed along the line AA in this model are shown in Figure 9b. The distributions are similar in form to those of the plane stress 2-D slice case study, but the peak magnitudes of σ_y are somewhat greater. The effect of the mesh size on the stress distributions was examined by refining the mesh to yield minimum distances between nodes of a half and then a third of that in the initial mesh. The results of this convergence study are shown in Figures 9c-e for a region very close to the interface along AA, and detailed in Table 1. The results for σ_y are presented in Figure 9c which shows that the greatest (compressive) stress in the brass is well captured in all meshes and occurs at a distance of about 0.009 in.. (or, about 0.1 l_r , where r is the radius of the cylinders) from the interface. As the mesh is refined the stress σ_y in the steel increases, indicating that, either the mesh was not fine enough to capture a turn around in the steel similar to that in the brass but much closer to the interface, or that σ_y does not converge, but is singular. If such a stress turn around in the steel were to occur this would have to take place at a

distance of less than 0.0015 in.. (about 0.019r) from the interface. The corresponding normal stress σ_z and the shear stress τ_{yz} are plotted in Figures 9d and e, respectively. These should be zero everywhere; the non-zero values define regions in which the computations are not reliable. As the mesh is refined this zone becomes smaller so that, for the finest mesh, this represents a region of less than 0.0015 in.. on either side of the interface. Within this region none of the stress component values can be used with confidence.

3-D Model

The steel and brass cubes, in Figure 10a, were modelled first with 8-node, and then 20-node brick elements. The line AA, in Figure 10b, represents an edge which crosses the interface. The stresses near the interface will then be affected by two interface effect zones. The normal stress σ_y along this line is shown in Figure 10c for the 8- and 20-node element models. The peak stress values are similar to those in the cylinder case and the (compressive) stress in the brass cube also reaches its greatest value. However, the 8-node element model is not sufficient to define the location with confidence, and the 20-node element model reached the practical limits of the Apollo DN4000 workstation used in the calculations when run times of 72 hours were needed. This limitation was circumvented using a two stage global-local approach [13] to show that the peak stress value and its location in the brass were reliable. The location of the peak stress in the brass occurred at about 0.004 in. (or, about $L/20$, where L is the cube side) from the interface. The three stress components, σ_x , σ_y and τ_{xy} , are presented in Figure 10d for the 20-node element model. Again there is a region of uncertainty on either side of the interface where the boundary conditions are not satisfied by the computed stresses. The stresses computed along the line BB, a surface line crossing the interface 0.017 in. from the edge, are shown in Figure 10e. The distribution of the normal stress σ_y has a similar form to that along AA, but the peak values are lower. Note that along BB the stresses σ_x and τ_{xy} do not have to be zero everywhere. The line CC, in Figure 10b, lies within the cubes and crosses the interface. The stresses σ_x , σ_y , τ_{xy} along this line are shown in Figure 10f. Again, the normal stress σ_y in the steel and brass are of opposite sign near the interface but apparently continuous at the interface, as expected by considering the satisfaction of equilibrium.

Three-dimensional representations of the distribution of the normal stress σ_y can also be made. For a small region near the edge, three-dimensional plots are presented on planes

0.004 in. from the interface, in the brass, at the interface, and 0.004 in. from the interface, in the steel, in Figures 11a-c, respectively. Compressive stresses σ_y are observed in the brass on a plane 0.004 in. from the interface, as shown in Figure 11a. These have a value of about 8 ksi away from the edge, but reach a peak (compressive) value of about 12 ksi at the edge. The interface effect zone extends within the material about 0.004 in. to 0.008 in. (or, about $L/20$ to $L/10$). The normal stress σ_y on the interface plane is shown in Figure 11b. The stress at the free surface is tensile with a value of about 7 ksi, rising to 12 ksi at the edge. It is important to recall the limitation of the model in which the mesh is not fine enough to provide an unequivocal value of stress on the interface at the free surface. If the stress field were indeed singular, then finer meshes will simply produce larger stresses at the free surface. Very large tensile stresses are documented in Figure 11c. These are about 18 ksi rising to 28 ksi at the edge.

3.5 Discussions and Summary of numerical analyses

The original motivation for this study was the apparent conflict between analytical predictions of the stresses at the bimaterial interface and those from an experiment which seemed not to satisfy the equilibrium conditions. The 2-D plate model presented above reflected all the features of Hess's analytical solution. This finite element solution is compared with Wood's experimental data in Figure 7, in which the stress components along a line AA midway between the edge and the center of the plate are presented. It is seen that the stresses σ_x and τ_{xy} are in good agreement, but the normal stress σ_y is quite different in the analysis and experiment. The lack of agreement in σ_y strongly suggests the presence of shear stress gradients normal to the plane of the plate, and the presence of three-dimensional effects.

A comparison of the Timoshenko, moire and 2-D plate models for σ_x is shown in Figure 6, for the same line AA. It is clear that the experiment is successful in determining σ_x so it would seem reasonable to believe that σ_y was also correct. Note also that Timoshenko's solution gives acceptable values even along this line. The equilibrium condition violation seems to be resolved in the 2-D slice and 2- axisymmetric models. Both of these predict compressive stresses σ_y in the brass and tensile in the steel, close to the interface. This agrees with Wood's experimental data. However, the numerical analyses

also predict that the stress in the brass reaches a definite peak value at some location near the interface before becoming less compressive or even tensile. Both finite element models provide similar distributions of σ_y on the surface near the interface, but the peak values differ slightly, as shown in Figure 12.

The 3-D model illustrates a phenomenon not present in the other numerical analyses, that of an edge crossing the interface. In such a region two interacting free surface effects occur, and very large normal stresses are produced. These may be singular, but in practice plastic deformation or local delamination might occur. In terms of Wood's experiment, the normal stress σ_y would be distributed qualitatively as shown in Figure 13. In this case, an experiment which measured surface parameters (displacements, strains, stresses) near the interface would document three-dimensional phenomena, no matter how thin or large the plate was. The turn around in the compressive stress in the brass seems to have occurred at a distance which was too close to the interface to be detected in Wood's experiment.

The occurrence of this interface effect region is important and well-appreciated in the mechanics of laminated composites as the free edge effect. The nature of the stress gradients and stress signs of the normal stresses across such interfaces do not, however, seem to have been documented previously. The scale on which these stresses occur can have great practical significance in laminated fiber composites since these interface (free-edge) stresses may act over a distances comparable to the fiber diameter, so the continuum approach would be invalid. Important consequences are also apparent in the design and analysis of bimaterial components of homogeneous elements. Thermal loading can produce very large normal stresses which can only be determined with three-dimensional finite element analyses with extremely fine meshes in the interface region.

3.6 Application of the localized hybrid method

The moire fringe patterns [9], u and v fields, depicting the stress-induced displacements due to a thermal load ($\Delta T=240^\circ\text{F}$) are shown in Figure 14. Instead of differentiating fringes manually [9], the localized hybrid technique was employed to determine the strain or stress fields from the displacement fields of moire results and further

confirm the nature of mechanics. Boundary-node, full-node, partial-node, and mixed-node methods will be adopted in sequence to perform trial analyses and validate the methods themselves.

Boundary-node method

The first-test region, shown in Figure 15a, is a region covering the entire moire grating area of steel. The mesh used in this region contains 392 constant-strain elements. The smaller elements were placed on the regions close to the interface and the free edge. Then the displacement values, obtained from moire data, at nodes around the boundary of this region were specified and input into 2-D plane stress finite element calculation. In order to assess the validity of the calculated results, the displacement fields reproduced from localized hybrid analysis are compared with those from the moire fringes in detail. The displacement components, u and v , are respectively plotted along the lines AA and BB (the center of the moire fringe area in steel) from both results, as shown in Figure 15b. It appears that very good agreement is found in u displacement components along the lines AA and BB, and v displacement components along the line BB, but not in v displacement components along the line AA. That is, the derivatives $\delta u/\delta x$, $\delta u/\delta y$ and $\delta v/\delta x$ are reproducible but $\delta v/\delta y$ is not. In other words, only ϵ_y could not be captured using the two-dimensional finite element model. The question then is; does this region belong to a 3-D region, or a mixed 2-D and 3-D region?

The second test was conducted by considering the moire fringe area of the bimetal plate as a mixed 2-D and 3-D region. Thus, based on the numerical analysis above, this area has been divided into four regions; Regions R(I) and R(II) in the steel, and Regions R(IV) and R(III) in the brass, representing 2-D and 3-D zones respectively, shown in Figure 16a. As in the first test, a localized hybrid method with the 2-D plane stress model associated with the boundary-node displacement condition was applied to these four regions individually. Reproduced displacement fields were examined and compared with the moire fringe data in detail. Comparisons of displacement components in Region R(I) of the steel are shown in Figure 16b and those in Region R(II) of the steel in Figure 16c. It is clearly shown that the reproduced displacement fields in Region R(I) are exactly the same as those from the moire fringe patterns, while those in Region R(II) still have trouble with ϵ_y . This suggests that Region R(I) in the steel reveals the nature of 2-D mechanics under the thermal loading, $\Delta T = -240^\circ\text{F}$. Similarly, in Regions R(III) and R(IV) of the brass, the

results (shown in Appendix A-1,2) suggest that 2-D mechanics works successfully in Region R(IV) but not in Region R(III). Since the displacement fields in Regions R(I) and R(IV) were exactly reproduced through the hybrid analysis, the correct, resulting stress fields representative of those in Region R(I) of the steel and Region R(IV) of the brass were thus obtained and are shown in Figures 17 and 18. Detailed examination shows that these stress contours reveal the same features as proposed above from 2-D plate model's analysis and Hess's analytical solutions. Regarding the other two regions, it is expected that 2-D hybrid analyses of Regions R(II) and R(III) cannot be performed successfully in the regions with the presence of 3-D effects. The question to be asked is how to deal with these regions involving 3-D mechanics (if the strain [or stress] fields are needed in these regions).

Full-node method

A full-node method of a localized hybrid technique (called Method II previously) was introduced to differentiate the moire data mathematically in a 3-D region or a mixed 3-D and 2-D region without any requirements of material constitutive law and the equilibrium condition. (However, this method automatically satisfies the compatibility condition.) Four new regions (Regions R(V) and R(VII) in the steel, and Regions R(VIII) and R(VI) in the brass), in Figure 19a, are employed to test the full-node method and, meanwhile, to determine strain and stress data from the moire experimental results. In Figure 19b, the meshes with 72 four-node elements (0.03"x0.03") were used in Region R(V) and R(VI), while finer meshes with 128 four-node elements (0.015"x0.015") in Region R(VII) and R(VIII) which are superimposed with Regions R(V) and R(VI) in some local areas. Rather than specifying the boundary-node displacements only, the displacement in every node of each mesh was used in a localized hybrid analysis. In Figure 20, the fringe patterns, u and v , reproduced from the hybrid analysis are shown and compared with those from the moire experiment. These results show that reproduced hybrid fringes are identical to the moire fringes. Thus, the calculated strain data can be totally representative of those in the analyzed moire regions. Once the strain field has been determined, the 2-D material constitutive law was applied to calculate the stress values (since the 2-D plane stress material law is valid on the free surface). The resulting stress distributions along the line aa (located at 1/4 plate width from the free edge) are shown in Figure 21. It is shown that high gradient of σ_y in the zone close to the interface, which represents the 3-D effect, has been captured. It is also pointed out that the use of the finer mesh produces the higher peak stress σ_y in the

steel but no change in peak stress σ_y in the brass (the same phenomenon as in the purely numerical analysis). Furthermore, the whole-field stress distributions in Regions R(I), R(VI), R(VII) and R(VIII) are respectively shown in Figures 22-25 which indicate that the stress distributions are virtually independent of x in these small, local regions, in good correlation with those along the line aa, shown in Figure 21.

The stress component σ_y from the full node method is compared with that from the 2-D axisymmetric finite element model and the moire results of manual differentiation [9] in Figure 26. The results indicate that the peak value of σ_y in the steel and the turn around value of σ_y in the brass are closer to those obtained from the 2-D axisymmetric numerical solution than those determined by manually differentiating the moire results. Moreover, of interest is the variations of σ_x and τ_{xy} from the hybrid analysis and manual differentiation, as shown in Figure 27. It is apparent that the τ_{xy} distributions from both methods are in good agreement, but the σ_x 's are almost coincident except in the region near the interface. The slight deviation from coincidence in σ_x is caused by the inaccuracy of manually differentiating fringes in the region where only a few fringes occur.

Even though the full-node method can be successfully applied in the 3-D or mixed 2-D and 3-D regions, a shortcoming of this method is the requirement of inputting every nodal displacement data in the finite element calculation. In this method, the process of data reduction is not as efficient as that in the boundary-node method. In order to ameliorate the inefficiency of the full-node method, a partial-node method will be suggested in the following sub-section.

Partial-node method

A partial-node method which combines boundary-node and full-node methods is proposed for dealing with the mixed 2-D and 3-D region accurately and efficiently. As mentioned before, the full node method works very well in the 3-D region or the mixed 2-D and 3-D region, but this method has an inherent problem; a huge amount of displacement data at all nodes need to be determined from the moire data prior to the finite element calculation. Unlike the boundary-node method, the full-node method requires more moire fringe data which have to be determined manually. To overcome this disadvantage, the partial-node method has been developed.

In Figure 28, the region with 2-D and 3-D effects in the steel was analyzed using full-node, boundary-node and partial-node methods individually. In the mesh used in the partial-node method, a part of the region near a steel/brass interface was taken to be a 3-D effect zone. Then the displacement data at all nodes in this 3-D zone and the boundary displacement data at the rest of the region were specified in a two-dimensional calculation. The resulting stress distributions along the line bb, as shown in Figure 29, are compared with those from full-node and boundary-node methods. It is shown that the stress distributions in the 3-D zone ($y=0''$ to $0.18''$) are exactly the same as those calculated by the full-node method, and these stress distributions in 2-D zone ($y=0.18''$ to $0.55''$) are also in very good agreement with those from boundary-node and full-node methods (even the full-node results have fairly small fluctuations due to the fact that the equilibrium conditions are not imposed). That is, the partial-node method can provide as good data reduction in the mixed 2-D and 3-D region as the full-node method, without the shortcomings of full-node method (the need of numerous data input).

Mixed-boundary method

The fourth method, different from boundary-node and full-node methods, is called mixed-boundary method (defined as Method III previously), in which the boundary conditions are not only specified with displacement values but also with force values (if the external forces are known). The evaluation of this method applied in the localized hybrid analysis was performed in the two-dimensional region, R(I), with one free edge, shown in Figure 30. This test was carried out by specifying the displacement values only on the three constrained edges of the region and keeping the nodes free on the free edge, on which the nodal force values were automatically defined as zero. The resulting displacement fields are, in detail, compared with those from moire fringe patterns. The comparisons, shown in Figure 31, indicated very good agreement between each other. It can be concluded that the mixed-boundary method can be applied in hybrid analysis of data reduction of moire fringes, provided that the correct material constitutive laws are applied.

3.7 Discussions and summary of hybrid techniques

The four different methods of a localized hybrid technique have been tested in the data reduction of the stress-induced moire fringes in a thermally loaded bimetal plate. A

boundary-node method, which is commonly used and has been previously demonstrated for good data reduction of 2-D plane moire fringes, was first employed in the mixed 2-D and 3-D zone. As a result, data reduction could not be achieved successfully in this region. According to the numerical solutions, the 2-D regions and the 3-D regions were approximately extracted from the whole moire area regarded as the mixed 2-D and 3-D region. Again, the boundary-node method was applied to these four regions, however, results were accurate in the two 2-D regions but not in the 3-D regions. That is, the two-dimensional hybrid analysis has confirmed the presence of some local regions with 2-D and 3-D effects. For differentiating the fringes in the 3-D regions, the full-node method of involving the interpolation functions, without the material constitutive laws, was developed and implemented. The results showed that the full-node method can be applied to the mixed 2-D and 3-D region and even the 3-D region but not quite as efficiently as the boundary-node method. The strain distributions along the line aa, calculated from the boundary-node and full-node methods in the mixed 2-D and 3-D region are shown in Figure 32. It appears that the strain data in the 2-D regions ($y=-0.55''$ to $-0.13''$ of the brass and $y=0.13''$ to $0.55''$ of the steel) are virtually the same from both methods, while the strain ϵ_y values from the boundary-node method, in the 3-D effect region (approximately $y=-0.09''$ to $0.09''$) are very different from those of the full-node method, but the strains ϵ_x and γ_{xy} values were not. It is thought that the large values of ϵ_y near the interface resulted from the equilibrium of the free end in the y-z plane and, of course, belong to 3-D phenomenon, so that the 2-D analysis can not capture those up, but the ϵ_x and γ_{xy} still have the 2-D nature such that they can be well reproduced. Furthermore, the partial-node method which combines boundary-node and full-node methods was developed for improving the efficiency of the full-node method in the mixed 2-D and 3-D regions. The results showed that the partial-node method is more effective than the boundary-node method, and more efficient than the full-node method. Finally, the mixed-boundary method with the mixed displacement and force boundary conditions was also evaluated in a hybrid analysis. The results showed that the mixed-boundary method can be used in hybrid analysis, provided that the correct material constitutive laws are applied.

Why is the hybrid analysis better than the manual differentiation in data reduction of the moire fringe patterns? Improved efficiency and accuracy is the answer. As far as the efficiency is concerned, the hybrid analysis is applied area by area and the strain and stress fields can be obtained immediately, while, when the manual differentiation is used point by

point, a huge amount of strain (stress) data need to be manipulated to obtain a strain (stress) field. For accuracy, the hybrid analysis features a two-dimensional linear or nonlinear interpolation (dependent on the given interpolation function) and the satisfaction of the equilibrium and compatibility at each nodal point; on the other hand, manual reduction faces a one-dimensional linear interpolation and only compatibility. Due to limitations of manual reduction, this method suffers from the potentially great large errors in regions with high gradients of strains or with only few fringes occurring. In addition to the efficiency and accuracy, one significant advantage in the hybrid analysis rather than manual reduction is that the essential check of mechanics of the loaded specimen can be accomplished during and after the application of hybrid analysis.

Chapter 4 Case Study II— Iosipescu Shear Problem

4.1 Introduction

The in-plane shear modulus G_{12} of a unidirectional composite material is an essential parameter for the elastic characterization of composite laminates. The Iosipescu specimen [14] has been widely used for the shear modulus and strength determination of a range of composite material systems. When tested in a modified Wyoming fixture [15], the specimen geometry, as refined by Adams and Walrath, forms part of one of the most popular shear test methods for composite materials [16].

Despite its popularity, this test method has been the topic of much discussion [17-21], primarily due to inconsistent shear strengths obtained with this technique. Shear modulus values determined from this test method must also be interpreted with great care. For example, the shear modulus for unidirectional AS4/3501-6 graphite-epoxy has been measured by a number of investigators using the Iosipescu specimen geometry and the modified Wyoming test fixture. It is apparent that the shear moduli obtained for the 0° and 90° fiber orientations are different, even though elementary mechanics principles require them to be the same. Also there is a very large degree of scatter associated with the data. Correction factors have been proposed to account for non-uniform stress fields in the specimen [22,23]. However, these correction factors depend upon the material orthotropy (E_{11}/E_{22}) and the fiber orientation. When applied to the data from the test, the correction factors do bring the 0° and 90° specimen data closer together.

In the present study, an experimental analysis of the test section of the Iosipescu specimen is presented. Moire interferometry is used to determine the in-plane displacement fields of unidirectional and cross-ply graphite-epoxy specimens. The displacement data are reduced using a localized hybrid technique. The full-field strain data are obtained in the test section. The non-uniformity of strain (or stress) distributions and

the experimentally determined correction factors for 0°, 90° and 0°/90° orientation are evaluated.

4.2 Mechanics of Iosipescu specimen

The one-, two- and three-dimensional aspects of the mechanics of Iosipescu specimen are examined to account for the large variation in the published shear modulus data.

The geometry used in the present study and a schematic of the modified Wyoming test fixture are shown in Figure 33. Based on a strength of materials analysis, the one-dimensional approach, a beam loaded anti-symmetrically has the shear force and bending moment diagrams depicted in Figure 34. It is shown that a constant applied shear force, $-P$, is generated between the inner loads, and zero bending moment at middle of the beam. It was supposed that this would produce a state of uniform and pure shear between the notches. Iosipescu [14] performed photoelastic experiments to demonstrate the uniformity of the stress field for isotropic material, and then postulated that the strength of materials approach was an over simplification, and that an elasticity solution would show non-uniform stress distributions due to the load proximity to the test section.

A two-dimensional analysis is needed to investigate the non-uniformity and purity of shear stress field in Iosipescu specimen loaded in the modified Wyoming fixture. There have been many finite element analyses of notched shear specimens [19,22-24] which show that the uniformity of the stress field depends strongly upon the orthotropy ratio of the material. The strain gage data which are usually employed to determine shear modulus must then be corrected to allow for the non-uniformity of the stress field. Correction factors for the strain gage data have been defined and determined from the linear finite element analyses by normalizing the shear strain (stress) at the center of the specimen with the average shear strain (stress) calculated along the line, between the two notches [22, 23]. That is,

$$\gamma_{avg} = \frac{1}{h} \int_{-\frac{h}{2}}^{\frac{h}{2}} \gamma_{xy} dy \quad (4.1)$$

where h is the distance between the notches. The correction factor is defined by

$$C F = \frac{\gamma_{xy}^{0-FEM}}{\gamma_{avg}} \quad (4.2)$$

where γ_{xy}^{0-FEM} denotes the shear strain at the center of the specimen between the notches. Analogous expressions can also be given in terms of the shear stresses. The corrected shear modulus is then obtained from

$$G_{xy} = CF G_{xy}^{SG} \quad (4.3)$$

where G_{xy}^{SG} is the shear modulus determined from the strain gage data which represents the shear strains at the center of the specimen between the notches. Actually, the major difficulty with this method is that the strain gages do not measure strains at a point, such that factors involving the average strains measured in the gage-size region in a non-uniform strain field, and whether the gage rosettes are stacked or not, should be taken into account .

When the Iosipescu specimens are tested in the modified Wyoming fixture, different shear strains are recorded on the opposite faces of the 90° and $0^\circ/90^\circ$ specimens but not on those of 0° [21]. This effect cannot be explained in a two-dimensional model. A three-dimensional representation of the free body diagram associated with the twist effect is shown in Figure 35. The resultant forces F and Q acting on the upper and lower contact surfaces at the right hand side of the specimen, may in general, have a certain degree of eccentricity with respect to the mid-plane of the specimen. On the y - z plane, this effect results in a twist, M_x , which generates the shear flow contours, shown qualitatively in Figure 35(b). The question arises as to why the 90° fiber orientation is much more prone to twisting than the 0° specimen which is almost free from twisting effects, and the $0^\circ/90^\circ$ specimen which is about midway of that two. It is proposed that the small degrees of load eccentricity would produce twisting effects which result in the large, equal and opposite-sign shear stresses on the opposite surfaces of the specimen. The cause of eccentricity could be due to local hard spots on the contact surfaces between fixture and specimen with higher stiffness in the loading direction. This could be much more likely to occur in the case of the 90° specimen rather than the 0° specimen. In the 0° case, the matrix material acts as a soft layer which promotes self-alignment. This hypothesis has been validated through

the experiments in which the twisting in the 90° and $0^\circ/90^\circ$ specimens was reduced by means of inserting a soft layer between the fixture and specimen [21].

4.3 Moire experiments and results

Specimen instrumentation

Graphite-epoxy (AS4/3501-6) specimens were cut from 20-ply unidirectional panels with the 90° notch angle and 0.05 in. notch radius, and dimensions shown in Figure 33a, which are suggested by a draft ASTM standard. Specimens were also cut from a cross-ply panel of the same material but which had 21 plies rather than 20. Two specimens of each fiber orientation with ground surfaces of contact were instrumented with a stacked three-gage rosette (Micro-measurements WK-06-060WR-350) at the center of the test section; the two gages being at $\pm 45^\circ$ (SG1,SG2) and the third parallel to the specimen longitudinal axis, as shown in Figure 33a. A crossed-line diffraction grating (30,480 lines/in. [2400 lines/mm]) was applied to the face opposite the strain gage rosette, and covered an area across the width of the specimen, and extended 0.5 in. on either side of the notch.

Loading fixture and Optical system

The modified Wyoming fixture, shown in Figure 33b, mainly consists of two robust brackets; one is firmly fixed to the base plate, and the other can be moved vertically with a linear roller bearing guided by one rod attached to the base plate. The specimen is inserted into the fixture, centered with the alignment pin and clamped with two adjustable wedges by finger-tightening the screws. The compressive load P is applied from the test machine to the adaptor which is connected to the moving bracket and aligned with the center of the specimen. This loading system of transferring the applied load P by two reaction forces on the contact surfaces of the right-sided specimen, in principle, creates a pure shear along the center of the specimen.

The optical system of the moire interferometer used in the present study is shown schematically in Figure 36. A coherent light beam from a He-Ne laser illuminates the flat mirror, reflects to the parabolic mirror through the beam expander which is used for forming a spatial beam which becomes collimated after reflection at parabolic mirror. The

collimated beam is partitioned by a beam selector into four portions; the beams A, B, C and D. The partial beams A and B reflecting from one flat mirror illuminate the specimen directly and by reflection from the horizontal mirror of the three-mirror system, respectively, and generate the u-field moire fringe pattern on the screen behind the camera lens; the partial beams C and D also illuminate the specimen upward and downward by reflection from the two vertical mirrors of the three-mirror system, and create the v-field fringe pattern. All optical elements are mounted on the base plate which is fixed on the stage of the testing machine for eliminating relative vibration effects. The whole optical system has an adjustable rotation-controlled screw to allow the introduction of the relative rotation to the specimen in the process of adding or subtracting the carrier fringes.

Specimen testing

The instrumented specimens were placed in the modified Wyoming test fixture in which the specimen is centered using the alignment pin and lightly clamped with the adjustable wedges, shown in Figure 33b. The optical system with the three-mirror interferometer was used to interrogate the in-plane deformation of the grating surface of the specimen loaded in the test fixture by a Tinius-Olsen testing machine.

Before loading, the interferometer was tuned to give no-load (null) fringe patterns. These consisted of one fringe or less across the field of view. The specimen was then loaded to some predetermined value, and, under constant loading, u and v fringe patterns were recorded. A set of patterns were also obtained with rigid body rotation introduced to the u field. The shear strain γ_r due to rigid body rotation is given by

$$\gamma_r = \frac{\partial v_r}{\partial x} + \frac{\partial u_r}{\partial y} = 0 \quad (4.4)$$

so the rotation could be adjusted to cancel out the cross derivative term, $\partial u/\partial y$, of the u field fringe pattern, and the v field (with rotation) would contain all the shear information (in the uniform region of the test section). This technique is known as applying a carrier pattern of rotation. In this experiment, it is a convenience for fringe pattern interpretation and data reduction. This procedure was repeated for other load increments until the specimen failed. While performing the moire experiment, the individual elements of the

strain gage rosette were monitored and logged on a Macintosh SE computer with a Strawberry Tree Inc. data acquisition system.

Moire fringe patterns

Typical fringe patterns from the 0° , 90° , and $0^\circ/90^\circ$ specimens are shown in Figures 37-39. The normal and shear strains at any location in the field of view can be determined by taking the appropriate derivatives. The normal strains are readily interpreted in terms of the gradients of the fringe contours, in the u field patterns in the x -direction (horizontal), $\partial u/\partial x$, and, in the v field patterns in the y -direction (vertical), $\partial v/\partial y$. The shear strains require the cross derivatives in both fringe patterns.

The high flexural stiffness of the 0° and $0^\circ/90^\circ$ specimens is apparent in Figures 37a and 39a, in which the normal strain ϵ_x is very small everywhere except at the notch roots. In contrast, the 90° specimen has a much lower flexural stiffness (due to a low E_x relative to E_y) so that a significant normal strain ϵ_x develops across the test section, between the notches.

The v -field for the 0° specimens consists of dense, almost vertical fringes in the region between the notches, as shown in Figure 37b. Since the vertical gradient of the fringe contours in the u field (Figure 37a) is small in comparison with the horizontal gradient in the v -field, most of the shear strain is seen in Figure 37b as the term representing $\partial v/\partial x$. This is especially the case near the center of the specimens; that region normally instrumented with strain gages. Near the notches, however, the term $\partial u/\partial y$ becomes significant, and is required to satisfy the stress-free condition there. Close examination of the fringes in the v -field for the 0° specimen reveals that they are not perfectly straight, but S-shaped. Thus there is a vertical gradient corresponding to a normal strain ϵ_y . This indicates that the proximity of the applied load to the test section is detected by the moire experiment on this specimen.

The v -field for the 90° specimen, shown in Figure 38b, consists of almost perfectly straight vertical fringes. Thus, along the line between the notches, and in the region normally occupied by the strain gages, the normal strain ϵ_y is zero. This specimen does not show any proximity effect of the applied load in the test section. This is essentially the case, also, in the $0^\circ/90^\circ$ specimen, as shown in Figure 39b. Close examination of the v -

field reveals that, near the central region, the gradients of fringes in the horizontal direction vary; that is, the shear strain distribution has some fluctuations. The fringes in the u-field of the 0°/90° specimen, shown in Figure 39a, are more zigzag than those of the unidirectional specimen.

4.4 Shear strains from manual reduction

The calculation of shear strains

The shear strains for the gage rosettes were calculated from the normal strains in the $\pm 45^\circ$ gages by

$$\bar{\gamma}_{xy} = \epsilon_{45} - \epsilon_{-45} \quad (4.5)$$

This equation is valid, whether the gage region is in a state of pure shear or not. However, when this equation is applied to the non-pure shear region, the normal strains, ϵ_x and ϵ_y have to be uniform in this gage-measurement region. Otherwise, the measured average shear strains calculated from this equation have errors caused by the effect of non-uniform ϵ_x and ϵ_y to the normal strains ϵ_{45} and ϵ_{-45} in gage directions. On the other hand, for the moire fringe patterns, the average shear strain over an area equivalent to that covered by the strain gages can be determined manually by Equation (2.3). The shear stress-strain data for the moire and strain gage rosettes can then be compared directly. In Figure 40, the average shear stress (the applied load P divided by notch section area A) is plotted against the shear strains from the moire and strain gage rosettes. Recall that these represent strains on opposite faces of the same specimen. Note that, with the coordinate system employed, all the shear strains, and stresses, are negative. The values presented here exclude sign.

The shear stress-strain data for the 0° specimen, shown in Figure 40a, are almost coincident. However, the data for the 90° and 0°/90° specimens are quite different for the moire and strain gage rosette, as shown in Figures 40b and c. This has been discussed earlier and ascribed to twisting of the 90° and 0°/90° specimens.

The observations of uniformity

One way of estimating the uniformity of the shear stress or strain field is to examine the distribution along a line joining the notch tips. The shear strains, normalized with

respect to the average shear strain γ_{avg} , are shown in Figure 41a-c for the 0° , 90° , and $0^\circ/90^\circ$ specimens respectively. Results are shown for several load levels.

The 0° specimen, Figure 41a, has the largest shear strains near the notch tips and, at the center, the shear strain is about 90% of the average value across the section. It is also shown that the distribution, particularly near the notch tips, depends upon the magnitude of the applied load, and takes on a more symmetric form under the higher load level. This implies that the specimen was not loaded linearly from the level 150 lb to 300 lb. The 90° specimen has the peak shear strain at the center, as shown in Figure 41b. This varies with load, from about 29% to 18% above the average value as the load increases. The distribution for the $0^\circ/90^\circ$ specimen, Figure 41c, is similar to that of the 90° specimen but more fluctuating. The peak shear strain near the center is about 10 to 18% above the average value across the section.

The manual differentiation of moire fringes has provided strain data at particular points, but full-field strain data and mechanics checking are required in further analysis. An efficient and accurate way of applying the localized hybrid method will be adopted to achieve such an analysis.

4.5 Application of the localized hybrid method

Implementation of the localized hybrid analysis

The localized hybrid analysis of combining the moire and finite element methods was applied for data reductions of the moire displacement fringes. A rectangular region of interest was chosen for each fiber orientation in the Iosipescu specimen. This region, covering most of the zone between the notches, is divided into 384 rectangular constant-strain elements for 0° specimen and 192 for 90° and $0^\circ/90^\circ$ specimens. The u and v displacements at the nodes on the boundary of this region were determined from the moire results. The finite element calculation with these displacement boundary conditions was then implemented on an Apollo workstation with the ABAQUS package. The displacements, strains, and stresses everywhere within this region were determined. The reproduced displacement fields within the region were compared with those from the

moire. If these agreed in fringe gradient and shape, then the obtained strains could be used as representatives of those in this region of the specimen.

As mentioned before, the 90° and $0^\circ/90^\circ$ specimens rather than the 0° specimen suffer the twisting effect which can cause a gross error in shear modulus measurement, if neglected. Twisting is one of three-dimensional effects which two-dimensional analysis can not simulate, but it has the special feature of only generating pure shear states on the planes parallel to the twisting axis. If local regions in these planes are under uniform shear, planar self-equilibrium conditions are always satisfied for these regions. That is, a two-dimensional hybrid analysis can be applied on the region with a uniform pure shear due to three-dimensional twisting effect. Based on this ideal ration, the shear stresses due to the twist can be assumed to be approximately uniform in the analyzed surface region, and thus the surface displacement fields in this region can be considered as the superposition of the displacements from the in-plane loaded deformation and the constant shear deformation due to the twist. Therefore, the two-dimensional hybrid analysis can be applied to this region. However, in this analyzed region, the shear stress due to the twist is, actually, not quite uniform (about 50% increase from the right or left side to the center, but nearly constant along each vertical line except near the free edges), resulting from the variation of the cross-section area from the notch wing to tip. Fortunately, the shear strains due to the twist are only about 20% of the shear strains caused by in-plane shear loading. So the two-dimensional hybrid analysis still can be attempted but the results should be further confirmed by the other approaches.

Results of data reduction

The boundary-node method of the localized hybrid analysis, which used the displacements obtained from moire data as the boundary conditions in the two-dimensional finite element analysis of a region of interest, works very well in the 0° specimen. As shown in Figure 42, the reproduced displacement fields are very much the same as the moire fringes. Only the local non-uniformity in the u-field cannot be replicated in the hybrid patterns, Figure 42b. Due to the density of the fringes in v-field, only every 5th fringe from the localized hybrid analysis is presented to render clearer visualization, as shown in Figure 42d. For the 90° case, the reproduced u-field is two or three fringes away from the moire data, Figure 43c. The reproduced v-field (showing every 3rd fringe) in Figure 43d is more or less similar to the moire data except that the fringes close to the

right edge of the boundary are slightly inclined. For the $0^\circ/90^\circ$ specimen shown in Figure 44, the reproduced u-field looks very similar to moire fringe data except the non-uniform zigzag fringes due to the non-uniformity of the material itself and the sub-surface layer effect. The dense v-field fringe pattern is plotted with every third fringe and compared with the moire pattern, as shown in Figure 44b,d. It appears that no difference can be seen between the two v-field patterns.

The resulting normal strains, ϵ_x , ϵ_y , and the shear strains, γ_{xy} , together with the deformed shape, obtained from the localized hybrid analysis, are shown in Figures 45-47 for 0° , 90° and $0^\circ/90^\circ$ specimens, respectively. More confidence is placed in the results for the 0° specimen in which no obvious twisting effect occurs and the reproduced fringes match the original moire patterns very well. On the other hand, the strain data determined from the two-dimensional hybrid analysis for the 90° and $0^\circ/90^\circ$ specimens require further verification because of the twisting effect, even though the reproduced fringes showed reasonable agreement.

Check of validity

The reproduced u-field pattern of the 90° specimen in the analyzed region is slightly different by two or three fringes compared to the moire pattern, but those of the 0° and $0^\circ/90^\circ$ are not. Detailed examination of moire fringes shows that the analyzed region of the u-field fringe pattern at the initial state (at no load, but still clamped with wedges) has only half fringe in the 0° specimen, one fringe in the $0^\circ/90^\circ$ but two fringes in the 90° . This can be ascribed to the lower stiffness of the 90° specimen in the x-axis (u) direction. It is possible that the initial fringes (not due to applied load) contribute somewhat to the mismatch between the moire and reproduced hybrid fringes. Another possibility is that the twisting effect may have to be taken into consideration.

No matter which factor affects the mismatch of the fringe patterns, of interest is how much error the mismatch in the u-field fringe pattern would cause. The 90° specimen with a severe twisting effect was chosen for checking the validity of the results of the two-dimensional hybrid analysis. First, from an examination of the fringe patterns for the 90° specimen obtained from the moire and hybrid analysis, it is founded that the gradients of fringes (corresponding to the strains) in both results remain similar, even though a two or three fringe difference appears in the u-field. Furthermore, the full-node method, which

works well in the three-dimensional effect region, is applied to the narrow region across the notches for checking the validity of the results of the boundary-node method analysis. The resulting strain fields and deformed mesh are shown in Figure 48 . It is apparent that the strain ϵ_x and γ_{xy} distributions (in Figure 48b,d) and deformed shape (in Figure 48a) are textremely close to those obtained from the boundary-node method in the corresponding region, but the ϵ_y distribution (in Figure 48c) is not, due to very small values. The strain components along the line across the notches are plotted and compared with the resulting data from the boundary-node method, as shown in Figure 49. It is indicated that the strain data calculated from the full-node and boundary-node methods are almost coincident except for the slight inconsistency of γ_{xy} near the notch tips. That is, the strain data determined by the boundary-node method are accurate enough to represent the strain fields of the moire results for the 90° specimen which suffers from the severe twisting effect.

The results of the localized hybrid analysis for each specimen type are further validated by comparing them with those from the commonly used manual reduction technique. Comparisons of shear strain data along the line across the notches ($x=0$) for the 0° , 90° and $0^\circ/90^\circ$ specimens are shown in Figure 50. In Figure 50a and b for the 0° and 90° specimens, the strain distributions between the notches, calculated from the localized hybrid analysis and manual data reduction, are in good agreement, but for the $0^\circ/90^\circ$ specimen, the strain distribution from the hybrid analysis is more uniform than that from the manual reduction which captured the non-uniformity of the strains near the middle, due to the local non-uniformity of material on this surface region (which was not found in the other $0^\circ/90^\circ$ specimen.) Moreover, comparisons of shear strain data along the x-axis ($y=0$) near the center of the specimen are shown in Figure 51. It is obvious that the strain distributions from the hybrid analysis, along the x-axis for the 0° and 90° specimens, are also in good agreement with those from the manual reduction, and for the $0^\circ/90^\circ$ specimen a slight inconsistency occurs near the middle.

4.6 Discussions and conclusions of the Iosipescu shear study

0° specimen

The results of the localized hybrid analysis for 0° specimen are shown in Figure 42b,d. The reproduced displacement fields are very similar to the moire fringes. Only the

local non-uniformity in the u field cannot be replicated in the hybrid patterns, Figure 42b. The jagged nature represents local non-uniformity of the displacement field which could be caused by groups or bundles of fibers being sheared relative to each other. The effect of this non-uniform displacement to the shear strain can be negligible. Due to the high density of the fringes in the v field, only every 5th fringe from the localized hybrid analysis is presented to render clearer visualization, shown in Figure 42d. The contours of the same S-shape as in the moire fringe patterns, show the presence of the apparent compressive strain ϵ_y .

The deformed mesh, in Figure 45a, indicates that the deformation corresponds to uniform shear over most of the analyzed region. However, the corresponding strain fields, in Figure 45b-d, reveal that a significant compressive strain ϵ_y (about 13% of the shear strain at the center) resulting from the load proximity to the test section, and the strain ϵ_x is small everywhere across the test section, with a value of about 2% of the shear strain at the center. This observation suggests that the test zone in the 0° specimen is not subjected to pure shear, and that $\pm 45^\circ$ strain gages used in the traditional Iosipescu test would not read equal and opposite strains.

90° specimen

The 90° specimen has the inherent problem that twisting is generated due to the higher stiffness in the loading direction, and results in the different measurements of shear strains on the opposite surfaces, when this specimen is thought to be loaded with in-plane shear in the modified Wyoming fixture. This twisting is a three-dimensional effect, and produces equal and opposite-sign values of the shear strains on the opposite faces of this specimen, such that the two-dimensional hybrid analysis applied to the reduction of moire data of this specimen is questionable. However, the boundary-node method of the two-dimensional hybrid analysis can be applied by assuming a uniform shear stress in the analyzed region, resulting from the twisting. The results indicated that the reproduced fringe patterns are slightly inconsistent with those of the moire, especially for the u -field fringe patterns, but the strains are not. The validity of the strain fields determined from the boundary-node method is further confirmed by the applications of the full-node method and manual reduction technique. It was shown that the strain fields obtained from the boundary-node method can be almost representative of those in the analyzed moire region.

The deformed mesh determined from the localized hybrid analysis of the 90° specimen, Figure 46a, shows that significant in-plane bending takes place in the test section, due to the lower stiffness E_x in the x-direction. This bending effect is also shown in the contour map for ϵ_x with a combination of in-plane and out-plane bending. A feature of this distribution is that the axial strain due to bending is small (about 0.03%) near the center of the test section, but large close to the notches. This is attributed to the lower stiffness, E_x , in the horizontal direction. The strain ϵ_y is also small over most of the test section and about 0.02% at the center. This is a result of the larger stiffness, E_y , in the loading direction for this specimen. Both normal strains are relatively small compared to the shear strain value of about 0.61% near the center of the specimen. This is in agreement with the observation that the 90° specimen does give equal and opposite strains in $\pm 45^\circ$ gages. The implication of this is that the 90° specimen is insensitive to the proximity of the load to the test section.

0°/90° specimen

Although, as mentioned earlier, this specimen suffers less twisting than the 90° specimen, non-uniformity of shear strain distribution was found near the central region of this specimen. The boundary-node method of the hybrid analysis could not reproduce the non-uniformity of shear strains which was corresponding to the local material non-uniformity (such as lack of fiber straightness or local resin rich region as a result of the prepreg and laminate manufacturing procedures). The variation of this non-uniformity is about 10% of the uniform shear strain. Thus, the strain data from the boundary-node method still reasonably represent those of the results in this analyzed region.

The strain data and deformed mesh for the 0°/90° specimen, as shown in Figure 47, are essentially a combination of those from the 0° and 90° specimens. The strains ϵ_x across the section and the strain ϵ_y near the center are very small about 0.01% ~ 0.02%, while the shear strain at the center is 0.74%. This specimen will also give nearly equal and opposite strains in the $\pm 45^\circ$ strain gages, as shown by Morton et al.[21].

Correction factors

The results of finite element linear analysis [21] showed that the correction factors for the 90° specimen depend upon material orthotropic ratio only, but those for the 0° specimen are affected by material orthotropic ratio and load modelling (loading

distribution). However, in the finite element analysis it is difficult to determine the actual variation of the correction factor for the 0° specimen tested in the modified Wyoming fixture, since the real contact load distributions are unknown. The full-field method of the moire interferometry is capable of determining the correction factors from the real experiments, which can be used to confirm the numerical predictions.

From the manual reduction of moire data, the results of normalized shear strain distributions, in Figure 41, showed that the uniformity of shear strains could not be achieved in each fiber orientation specimen at different load levels, and the general trends of the strain distributions are in good agreement with those predicted by the finite element analysis [21]. The gage-determined shear modulus is required to be modified by a correction factor. From Figure 41, the correction factors are determined by the average values of the gage region covering the $y/y_{\max}$ from about -0.075 to 0.075. The results showed that the correction factor is about 0.91~ 0.92 for the 0° specimen, about 1.18 ~ 1.24 for the 90° specimen, and 1.12 ~ 1.15 for the $0^\circ/90^\circ$ specimen. Also, It is apparent that the correction factor seems to be independent of load levels for the 0° specimen (even though a quite non-symmetric shear strain distribution occurs at the low load level in this specimen), dependent of load levels for the 90° and $0^\circ/90^\circ$ specimens. But, as mentioned before, the 90° and $0^\circ/90^\circ$ specimens suffer from the twisting effect which affects the shear strain values on the measured surfaces, such that the correction factors in these two specimens could not be determined accurately, unless the twisting were eliminated. Note that the correction factors are only valid in the linear elastic analysis and used to correct the linear shear moduli which are usually determined at low strain state, such as the strain from 0.3% to 0.5% (approximately in the linear range) corresponding to loading level 100 lb to 150 lb for the present specimens. The accuracy and uniqueness of correction factors for the 0° specimen need to be further investigated by recording more moire data at low load levels.

Conclusions

Moire interferometry and the commonly used strain-gage method were used to evaluate the Iosipescu shear test method which has been widely employed to determine the shear modulus and strength of composite material systems. The moire fringe patterns on front-face region of interest and the corresponding strain-gage data on the back-face central point were recorded for each specimen type under different load levels. The shear stress-

strain curves from the moire and gage results were plotted for each specimen. It was found that, in the graphite-epoxy (AS4/3501-6) material system, the shear moduli determined from the front-side moire and the back-side strain gage measurements are coincident for the 0° specimens but not for the 90° and $0^\circ/90^\circ$ specimens. This fact has been illustrated by Morton et al [21]; the 90° and $0^\circ/90^\circ$ specimens are subjected to the twisting due to the higher stiffness in the loading direction, but the 0° specimens are not.

The boundary-node method of two-dimensional hybrid analysis was applied to the strain analysis of moire fringes for the 0° , 90° and $0^\circ/90^\circ$ specimens. This analysis appeared to be adequate for the 0° specimen. However, in the case of the 90° and $0^\circ/90^\circ$ specimens, this was unsuccessful, because of the mismatched fringes between the u-fields, affected by the twisting. The full-node method which works very well in 3-D region was then employed for validating the boundary-node results of the 90° specimens which suffered mostly from twisting. Furthermore, the shear strain data obtained from the manual reduction were compared with those from the boundary-node method along the cross central lines. The results showed that the strain fields calculated from the boundary-node method can be representative of those of the moire regions for the 0° , 90° and $0^\circ/90^\circ$ specimen except for a small variation in shear strain thought to be due to material non-uniformity at the test region of the $0^\circ/90^\circ$ specimen.

Full-field strain data on the region of interest for each specimen type were determined by the application of the localized hybrid method. The results indicated that the uniformity of the shear strain distributions in the test section could not be achieved for each specimen type, such that the shear modulus determined from strain gage measurements, at the central small region, required adjustments by a correction factor. For the 0° specimen, a state of pure shear cannot be achieved in the test section. That is, a large ϵ_y , resulting from the load proximity effect, was found in the test section of this specimen with the lower stiffness in the loading direction. This effect can account for the non-symmetric reading of the $\pm 45^\circ$ gage rosette at the central point. For the 90° specimen, a large ϵ_x , caused by the apparent in- and out-of-plane bending in the material with low stiffnesses in the directions perpendicular to the fiber direction, was shown in the test section near the notches, but over most of the test section the value of ϵ_x remained small and could not obviously affect the symmetric reading of the $\pm 45^\circ$ gage rosette in the gage region. For the $0^\circ/90^\circ$ specimen, the apparent zigzag fringes shown in the u-field merely affected the shear strain distribution.

The fluctuations in the shear strain distribution observed in the test section may result from material non-uniformity. The strain fields for this specimen are approximately midway between those for the 0° and 90° specimens.

Chapter 5 Conclusions and Recommendations

5.1 Conclusions

The localized hybrid method which combines the experimental method of the high-sensitivity moire interferometry and numerical method of the finite element analysis, has been applied to two important problems in the mechanics of composites; a thermally loaded bimetal plate and the Iosipescu shear test. Several techniques have been proposed for improving the accuracy and efficiency of the application to 2- and 3-dimensional effect regions. The results of these two representative problems and evaluation of the new techniques of the localized hybrid analysis are concluded as follows:

The thermally loaded bimetal problem

A numerical analysis has been performed for understanding the mechanics of this problem. It was concluded that the thermally loaded bimaterial interface suffers from a three-dimensional stress system in the vicinity of the free surface. That is, in this stress system, the surface stresses normal to the interface are of opposite sign in each material but very close to the interface high stress gradients occur and the stress in one of the materials, at least, will change sign to satisfy the equilibrium conditions at the interface. This boundary or skin effect will occur near the free surface and around the interface in all bimaterial problems no matter what the relative dimensions; so that in the case of the bimaterial strip model of a thermostat, for example, the one-dimensional (Timoshenko) approach would be successful in predicting the radius of curvature, and Hess's analysis would indicate the extent over which the two-dimensional shear stress system operated near the free ends of the strips, but the three-dimensional free surface interface effects must be incorporated in an evaluation of the largest stresses occurring in this and other

related problems. The edges which cross the interface will be subjected to the largest normal stresses.

The localized hybrid analysis was employed for data reduction of the moire experimental results, and checking and interpreting the mechanics of this problem. The stress fields with two-dimensional natures in the moire grating region were determined and showed a very good agreement with the two-dimensional numerical results. A local surface region across the interface, away from the free end was analyzed by a special application of the localized hybrid method. As a result, the stress fields in this region showed the presence of the three-dimensional nature which could not be captured in the two-dimensional analysis. The high gradients of stress σ_y , normal to the interface, which are due to three-dimensional effects, were well captured in the hybrid analysis with the special technique. A comparison showed that this captured σ_y distribution is closer to that of the numerical prediction than that from the manual reduction.

The Iosipescu shear problem

The Iosipescu shear test method of determining the in-plane shear property of composite materials was evaluated using the moire interferometry associated with the strain gage measurement. The front-side moire displacement fields and the corresponding back-side strain gage data were determined at different shear load levels for the 0° , 90° and $0^\circ/90^\circ$ specimens. The shear stress-strain curves were, respectively, obtained from the strain gage data and the moire results calculated in the corresponding gage region. The results showed that the shear moduli of the graphite-epoxy (AS4/3501-6) determined from the front-side moire and back-side strain gage are consistent for the 0° specimens, but not for the 90° and $0^\circ/90^\circ$ specimens. The reason for this, was explained by Morton *et al.* [21] who noted that the 90° and $0^\circ/90^\circ$ specimens suffer from twisting effects due to their higher stiffness in the loading direction, whereas the 0° specimens do not.

The whole-field strain data in the region of interest for the 0° , 90° and $0^\circ/90^\circ$ specimens were obtained by the application of the localized hybrid method, and the uniformity and purity of shear strain (or stress) distributions in the test section were revealed in this analysis. The results showed that the uniformity of shear strain distribution in the test section could not be accomplished for each specimen type, so that

correction factors are needed to allow for the determination of the shear modulus using strain gage measurements. However, the correction factors could not be accurately determined in this experiments for each specimen type, as a result of the twisting effects for the 90° and $0^\circ/90^\circ$ specimens and the lack of low-load moire data for the 0° specimens. As far as the purity of the shear stress state in the test section is concerned, for the 0° specimen, a large ϵ_y resulting from the load proximity effect was observed in the test section due to the lower stiffness of this specimen in the loading direction. This can account for the non-symmetric reading of the $\pm 45^\circ$ gage rosette measuring at the central region, but not affect the shear strain measurement; for the 90° specimen, the large ϵ_x , due to the presence of in- and out-of-plane bending in this specimen with lower stiffness in the direction vertical to the loading direction, was examined in the test section close to the notches, but the small ϵ_x dominated over most of the test section and did not, apparently, affect the symmetric reading of the $\pm 45^\circ$ gage rosette in the gage region; for the $0^\circ/90^\circ$ specimen, the strain fields generally behaved in a manner midway between the 0° and 90° specimens, and the obvious variation of shear strain distribution, maybe resulting from the local material non-uniformity, was found in the test section, but the zigzag fringes observed in the u-field merely affected the shear strain distribution.

The localized hybrid method

The localized hybrid method is more efficient and more accurate than the manual differentiation in data reduction of the moire fringe patterns. For efficiency, the hybrid analysis features an area by area approach; on the other hand, the manual differentiation is a point by point method. For accuracy, the hybrid analysis features a two-dimensionally linear or non-linear interpolation and the satisfaction of the equilibrium and compatibility at each nodal point, whereas manual reduction is involved in a one-dimensional linear interpolation and compatibility. As a result of the limitation of its one-dimensional nature, the manual differentiation suffers from potentially large errors in regions with high gradients of strains or with only few fringes occurring. In addition to the efficiency and accuracy, one significant advantage of the hybrid analysis is that the essential check of mechanics of the loaded body can be implemented during and after the application of the hybrid analysis.

Instead of the commonly used boundary-node method, three techniques (the full-node, partial-node, and mixed-boundary methods) of the localized hybrid method were

developed, and their accuracy and efficiency were evaluated in the present study. The boundary-node has been successfully used for data reduction of moire fringes in the two-dimensional planar regions, but problematically in the three-dimension effect regions. The full-node method which features no material constitutive law involved and only the use of the chosen interpolation function, was proposed and tested successfully for data reduction in the mixed 2-D and 3-D regions and the purely 3-D effect regions. However, this method is not as efficient as the boundary-node method, since it requires a considerable amount of displacement data determined from the moire fringes, to be used as input to the finite element calculation. The combination of the full-node and boundary-node methods, called partial-node method, was used for improving the efficiency of the full-node method in the analysis of the mixed 2-D and 3-D regions. It has been showed that the partial-node method is more effective than the boundary-node method, and more efficient than the full-node method for data reduction in the mixed 2-D and 3-D regions. In addition, the mixed-boundary method which consists of the displacement and force boundaries was illustrated in the hybrid analysis. It was shown that the mixed-boundary method can be used in the hybrid analysis, provided that the correct material constitutive laws are applied.

5.2 Recommendations

Some suggestions related to the thermally loaded bimetal problem, Iosipescu shear problem, and the localized hybrid technique are made for future work. These suggestions are described as follows:

- (1) In the bimetal problem, one free-surface effect behavior has been documented in the hybrid analysis. Also, of interest is the double free-surface effect which is located at the edge and results in the largest normal stress σ_y . This double free-surface effect probably dominating the bimaterial failure should be analyzed using the localized hybrid method.
- (2) From the present bimetal problem, the interface near the straight free surface suffers from the high normal stress gradients. Ways of reducing the high normal stress

gradients are needed. The wedge-shape free-surface effect will be of interest. Parallels can be drawn in the study of bimaterial interfacial cracks.

- (3) In the Iosipescu shear problem, the experimental correction factors can be accurately determined in full-field moire experiments, provided that the specimens with a soft shim are used for eliminating twisting, and adequate moire data are recorded at low load levels.
- (4) In the moire experiment of Iosipescu shear test, the carrier rotation was introduced to reduce the $\partial u/\partial y$ component and enhance the $\partial v/\partial x$ component, in order to make the manual reduction of moire data easy and efficient. As a result, the densities of the resulting u- and v-filed fringe patterns are extremely different. This did not improve the efficiency of the hybrid analysis of moire data. This may degrade the accuracy of the hybrid analysis, since the input displacement data came from adjudgment of u and v fringe orders with quite different accuracy. Also, this makes the comparison of reproduced fringes and moire fringes difficult. Similar density of each component fringe pattern should be attempted to test whether the accuracy of the hybrid analysis is improved or not.
- (5) In the hybrid analysis, a localized mesh was taken to calculate the strain (or stress) data by the requirement of the equilibrium and compatibility. The nodal strain (or stress) data were determined by averaging those data calculated at common nodal points of adjacent elements. Based on this concept, the calculated strain (or stress) data at the boundary nodes may be less accurate than those at the internal nodes, especially in the high strain gradient regions (such as the regions close to the contact interface, interface with a free-surface effect, etc.). This is a potential problem in the finite element calculation. A way of improving the accuracy of the boundary data is, if these data are of interest, to make finer meshes in the region close to the boundary and check the variation of the strain (or stress) data between the boundary node and the next interior node, until the needed accuracy is obtained.
- (6) In addition to the finite element method associated with the moire displacement data, the hybrid analysis of combining the boundary element method and moire interferometry will be worth developing.

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TABLE

Table 1. Convergence study for 2-D axisymmetric model.

Minimum nodal spacing (in.)	Zone of uncertainty, $\sigma_z=0, \tau_{yz}=0$. (in.)	Peak stress σ_y in steel		Peak stress σ_y in brass	
		value (ksi)	location (y, in.)	value (ksi)	location (y, in.)
0.0011	$y=-0.0044\sim0.0044$	9.4	0.0044	-8.8	0.0078
0.00055	$y=-0.0022\sim0.0022$	22.2	0.0022	-8.85	0.0094
0.00037	$y=-0.0015\sim0.0015$	23.8	0.0015	-8.85	0.0093

FIGURES

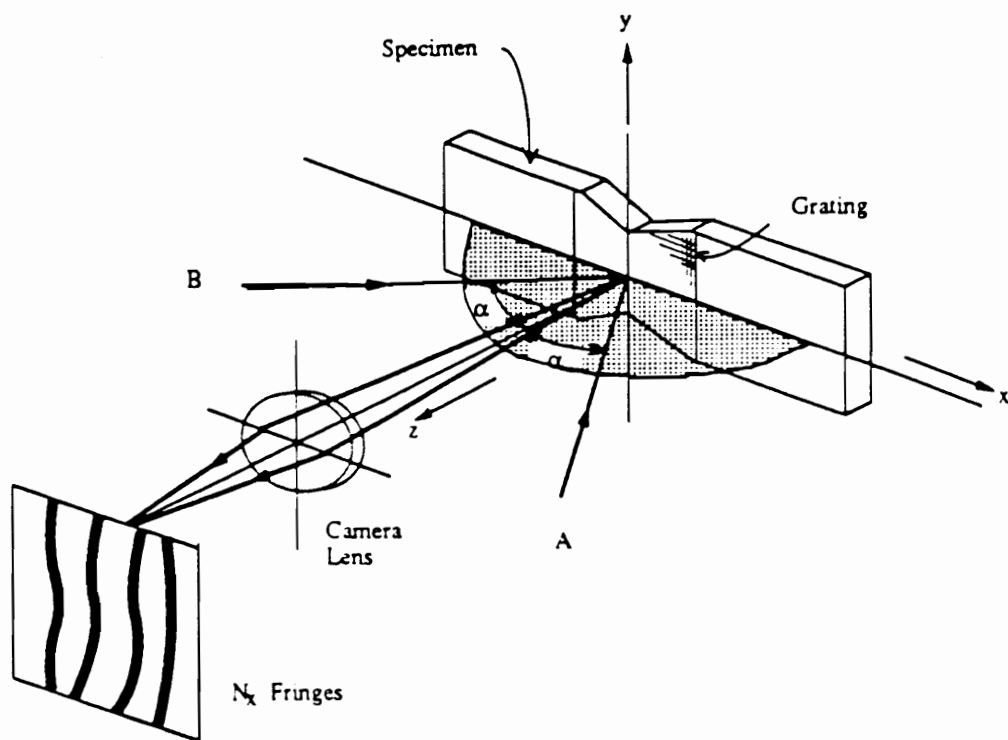


Fig. 1. Two-beam interferometry.

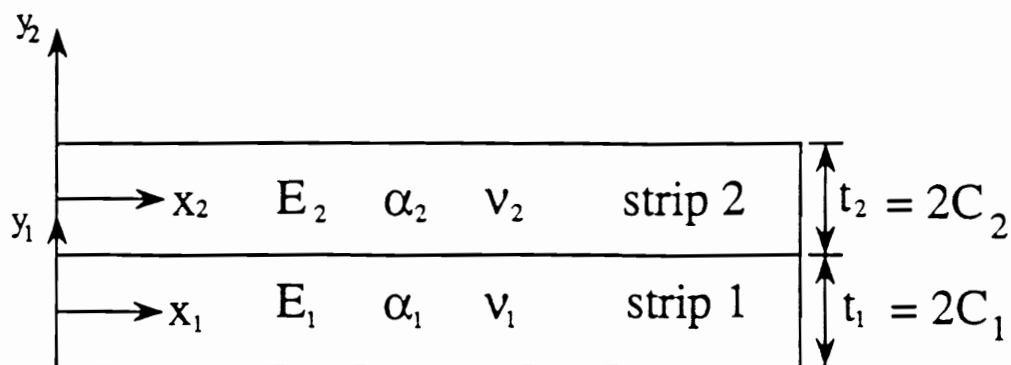
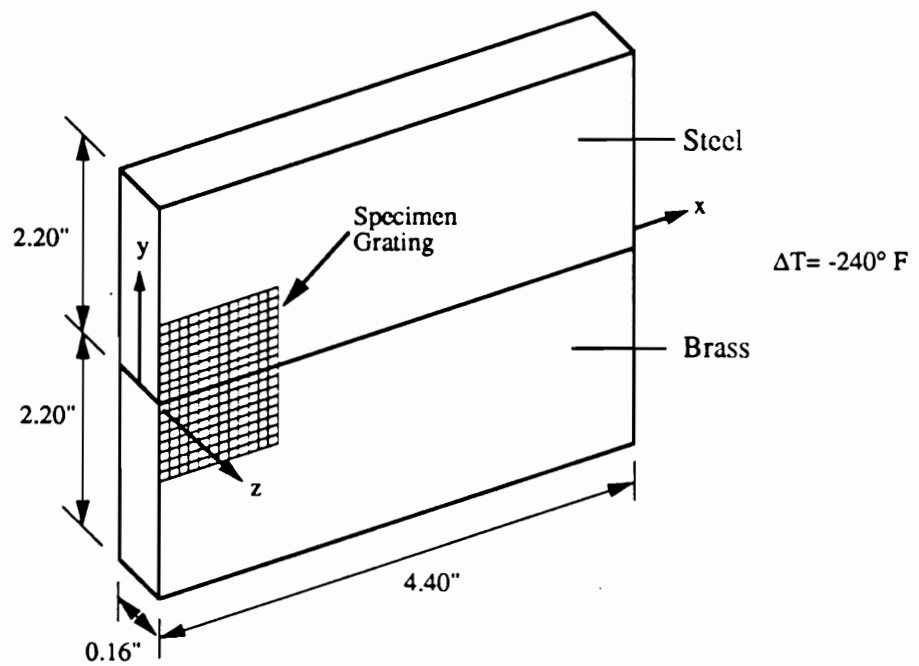


Fig. 2. General geometry of the bimetallic strip for Timoshenko's [7] plane elasticity analysis.



property material	E (Msi)	ν	α ($\mu/\text{°F}$)
Steel	29.5	0.29	6.9
Brass	15.9	0.33	10.8

Fig. 3. Plate geometry and material properties used in Wood's [6] moiré experiment.

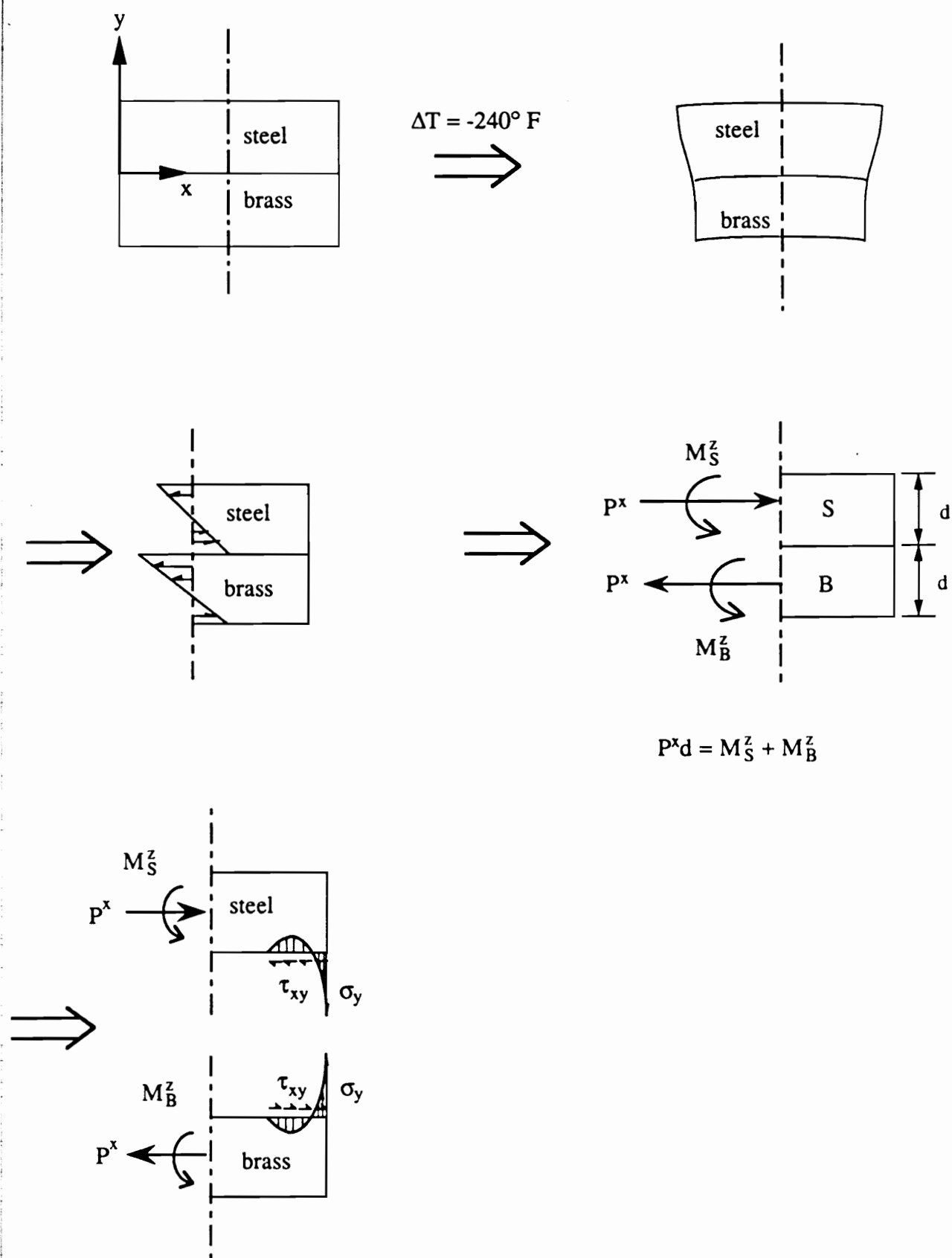


Fig. 4a. Deformation, free body diagrams and stress systems of the thermally loaded bimetal plate in the x-y plane.

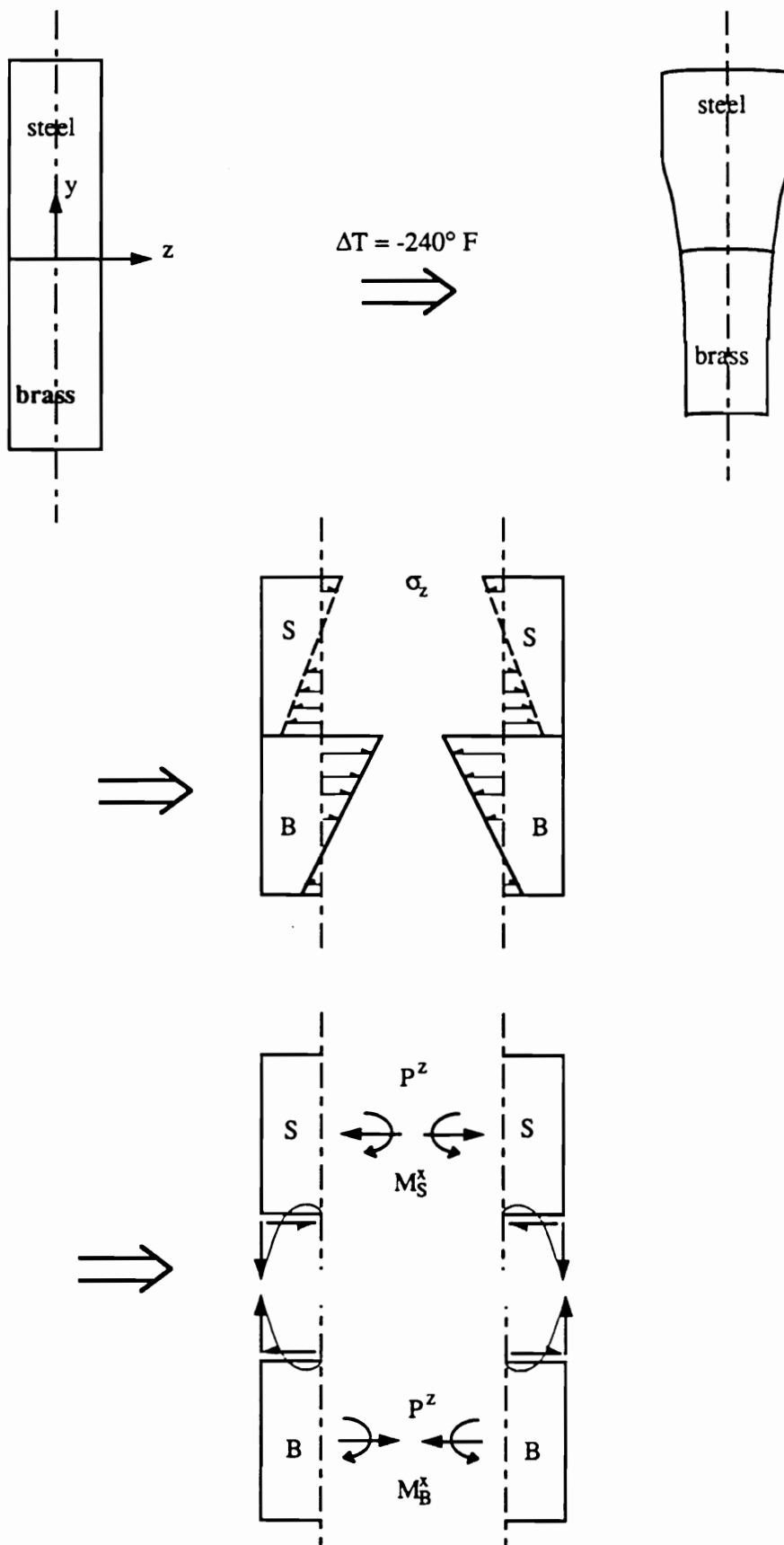


Fig. 4b. Deformation, free body diagrams and stress systems of the thermally loaded bimetal plate in the y - z plane.

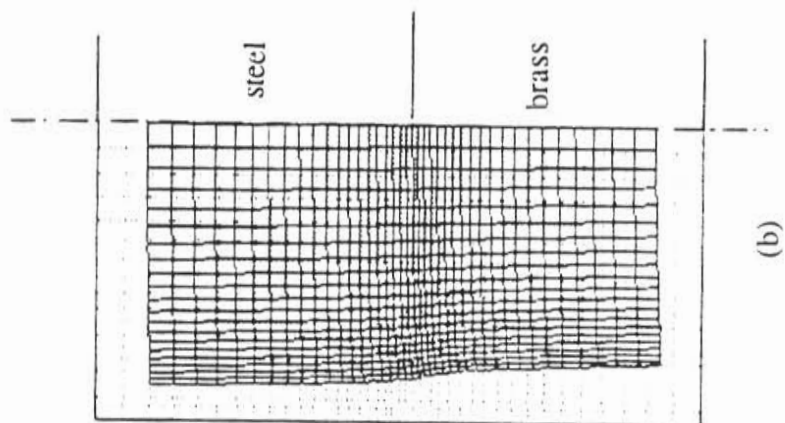
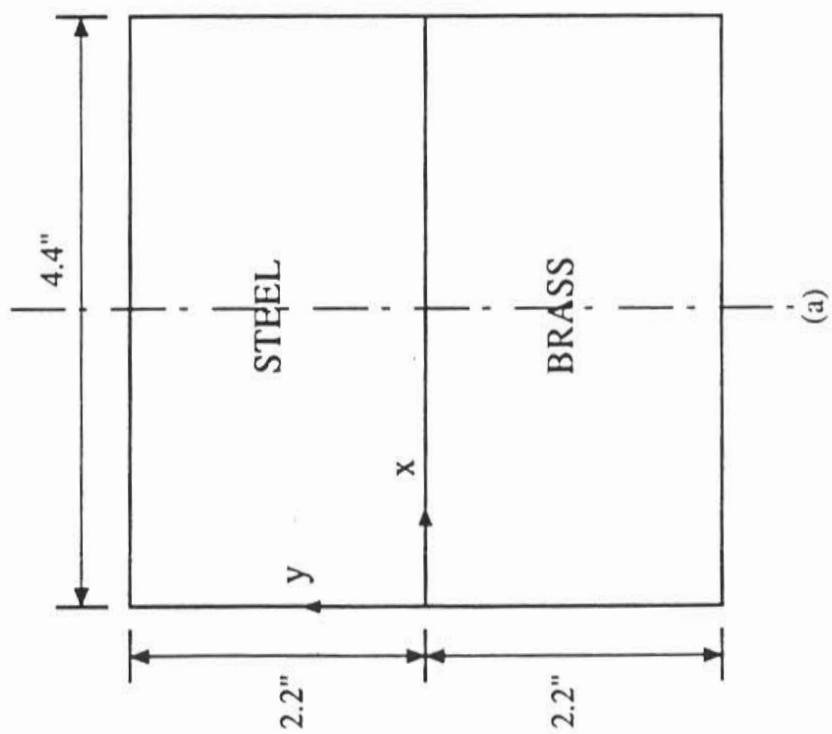


Fig. 5a,b. (a) Geometry, (b) deformed mesh in a 2-D plate analysis of Wood's [6] specimen.

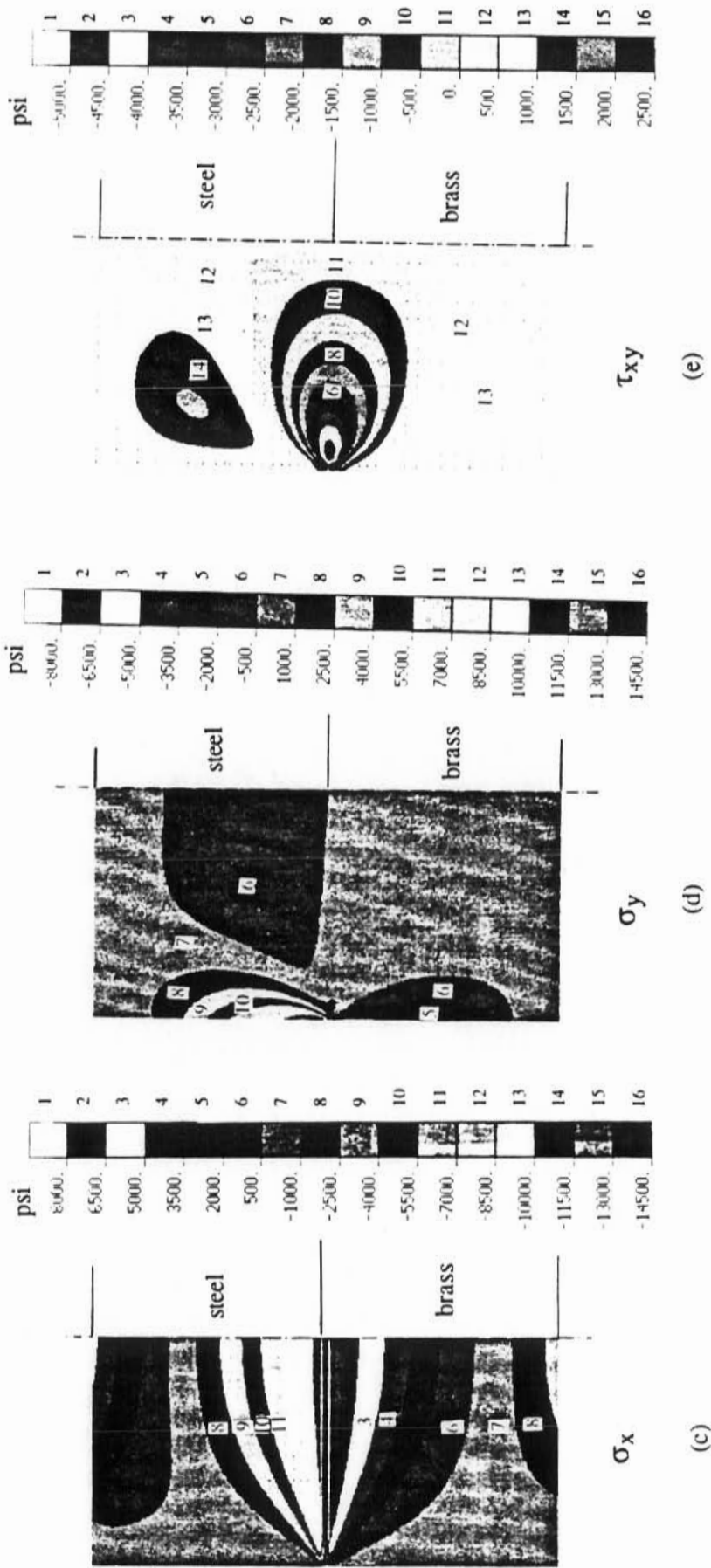


Fig. 5c-e. (c) σ_x , (d) σ_y , (e) τ_{xy} contours in a 2-D plate analysis of Wood's [6] specimen.

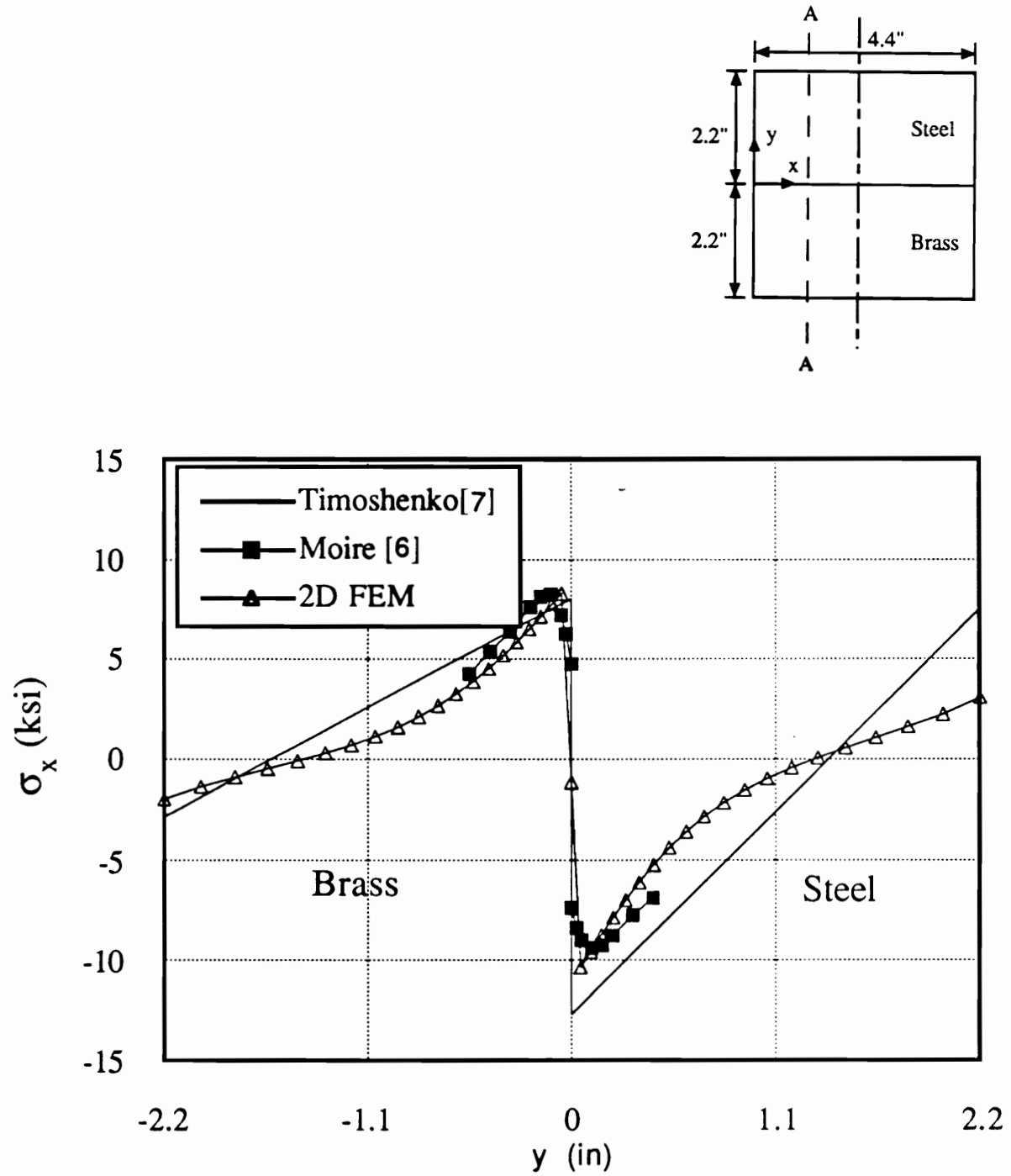


Fig. 6. A comparison of Timoshenko [7], moire [6], and 2-D plate models for σ_x along the line AA (midway between the edge and the center of the plate).

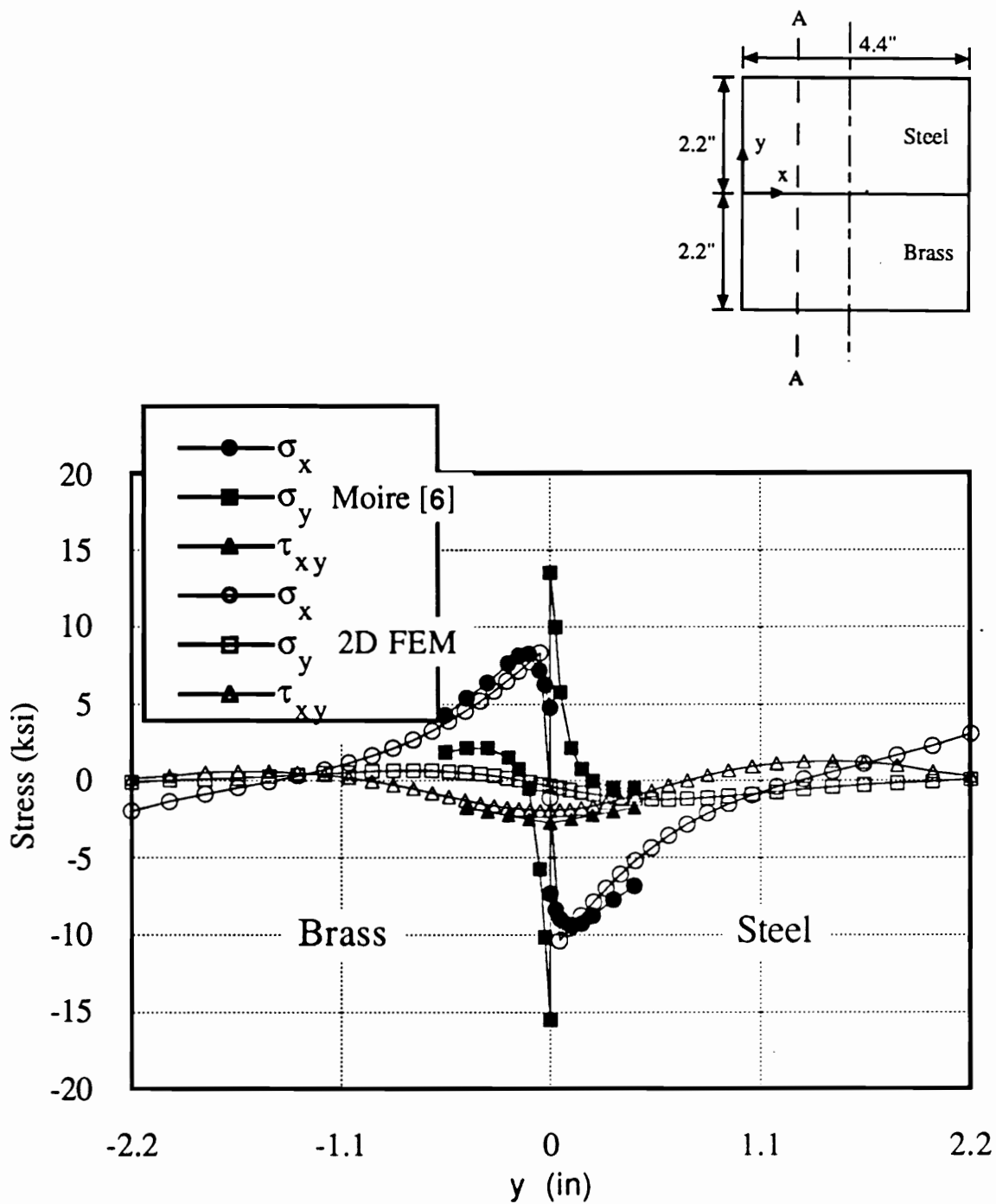


Fig. 7. A comparison of 2-D plate model and Wood's moire [6] results along a line AA midway between the edge and the center of the plate.

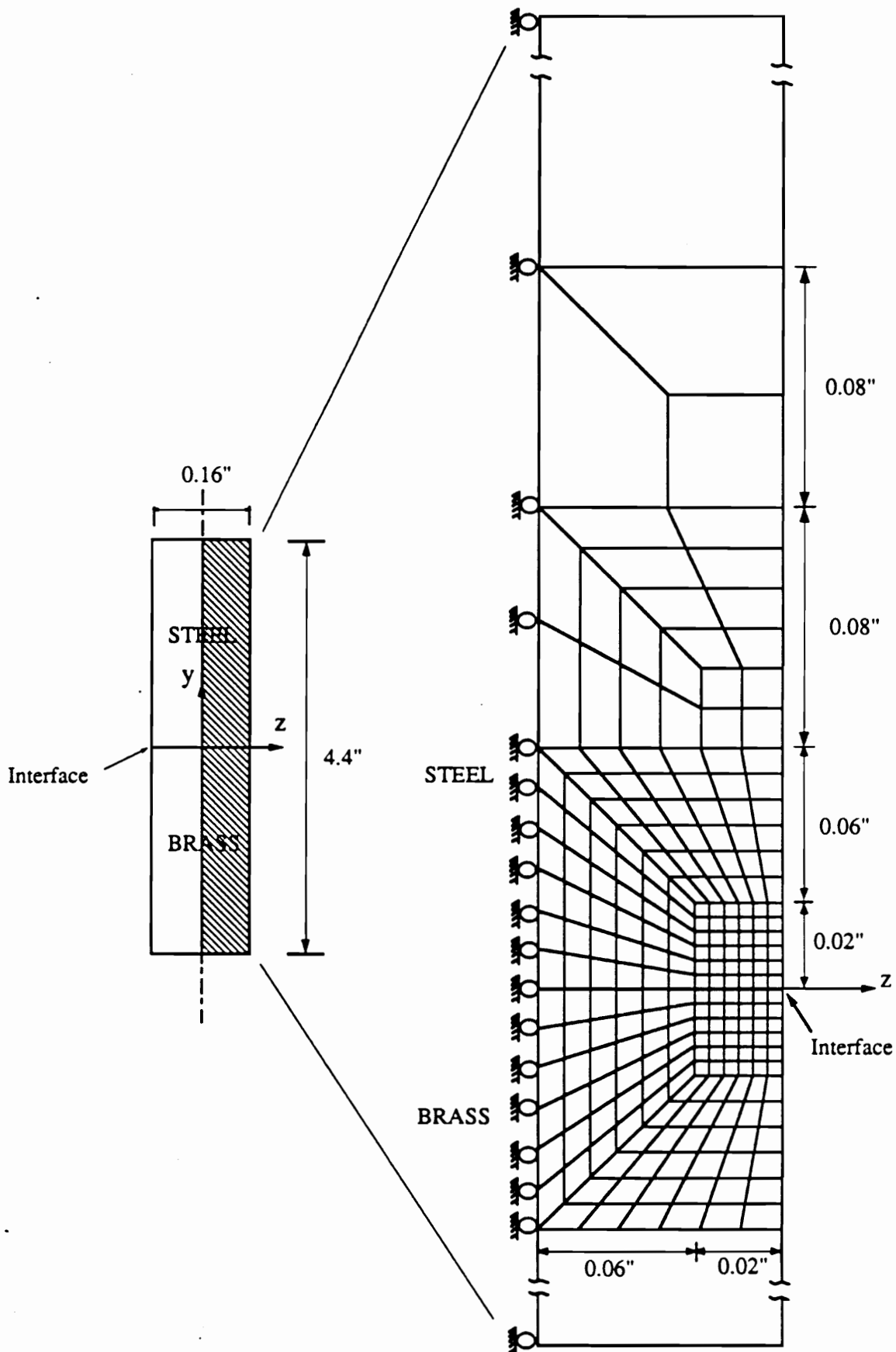


Fig. 8a. Geometry and finite element mesh used in a 2-D slice analysis for Wood's [6] specimen.

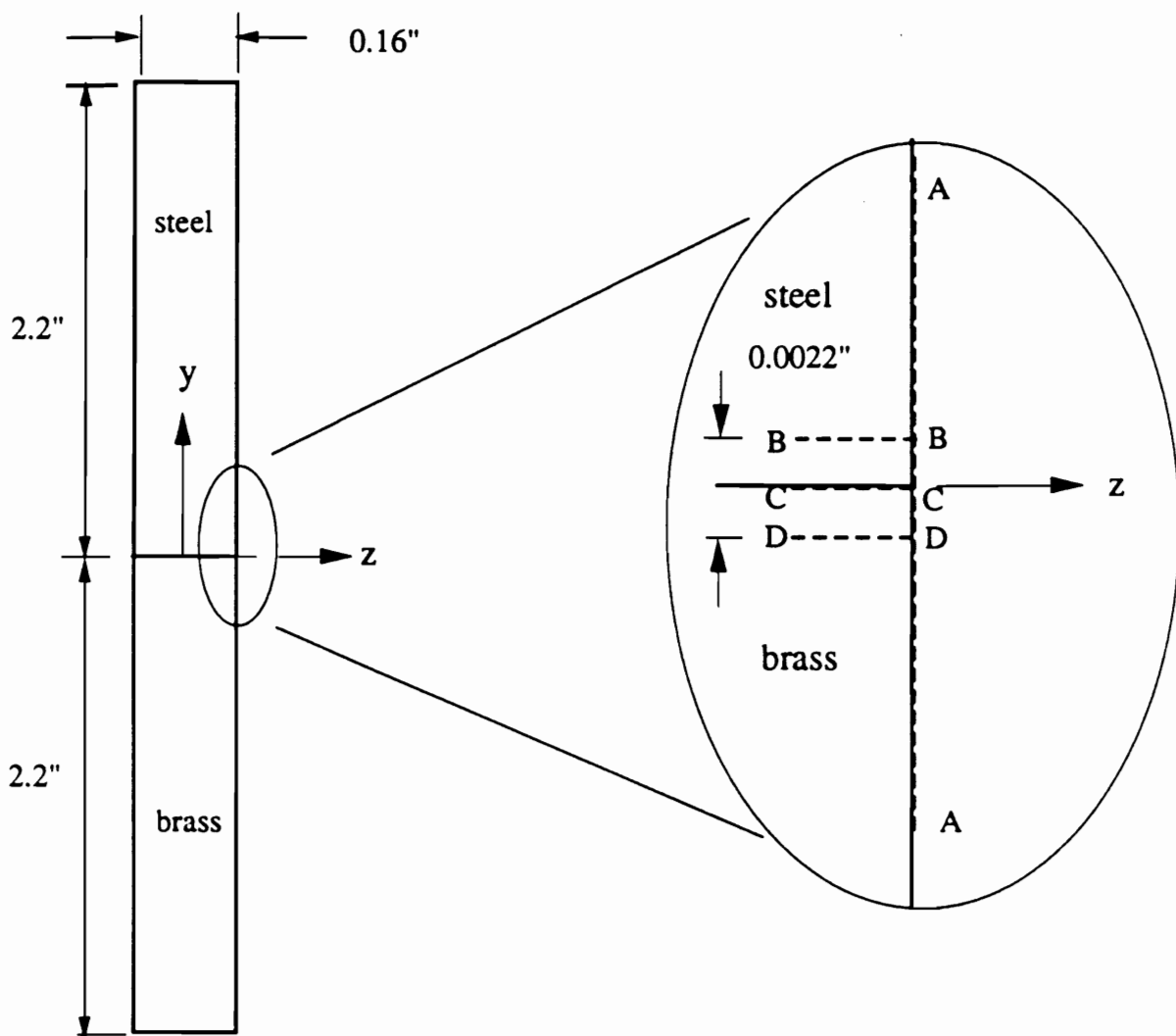


Fig. 8b. Co-ordinate system and location of lines for which data are presented in the 2-D slice analysis.

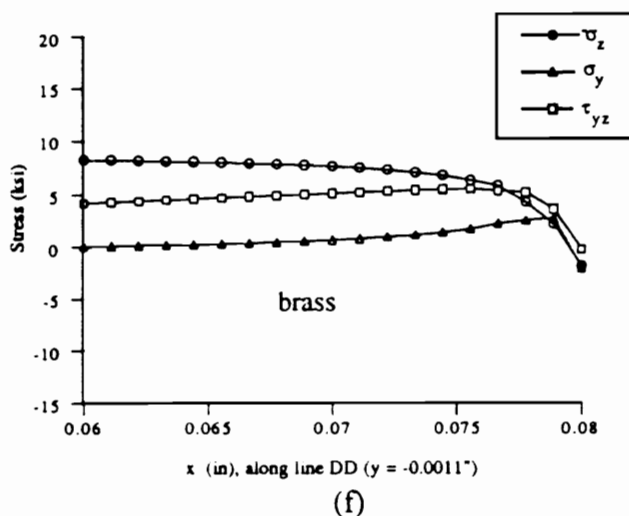
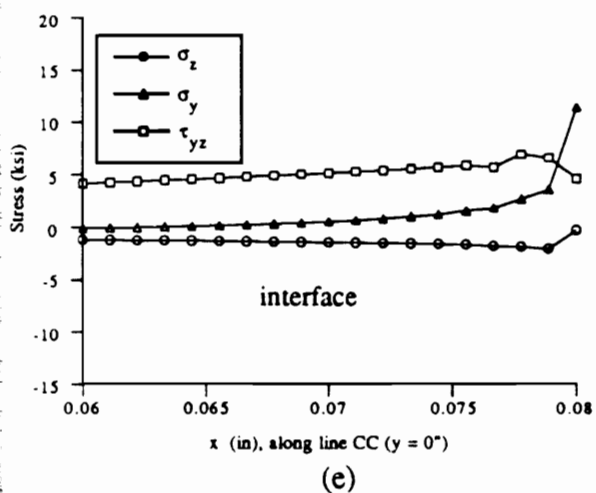
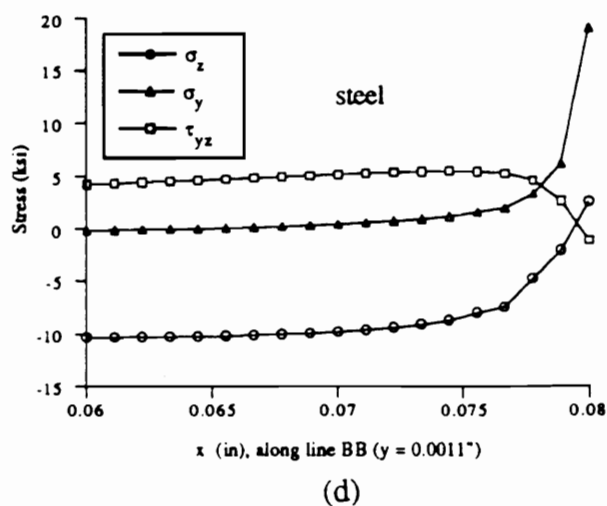
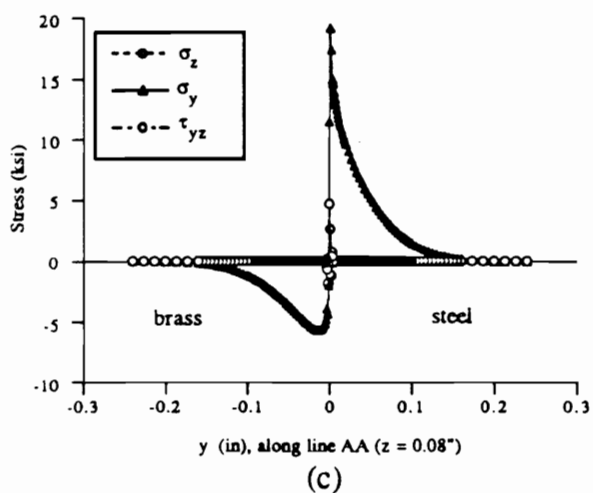


Fig. 8c-f . The stress distributions along (c) line AA, (d) line BB, (e) line CC, (f) line DD for the 2-D slice analysis.

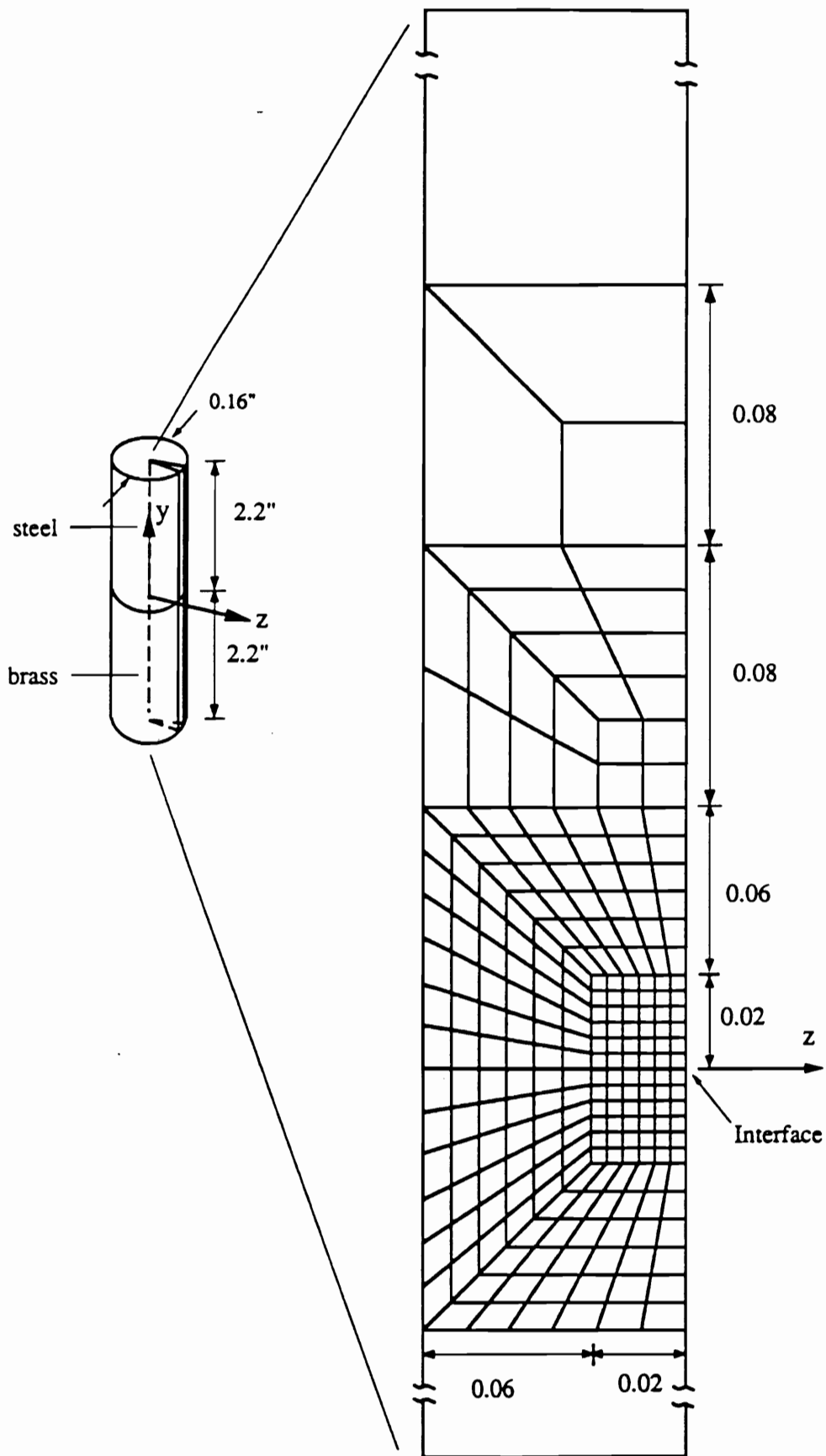
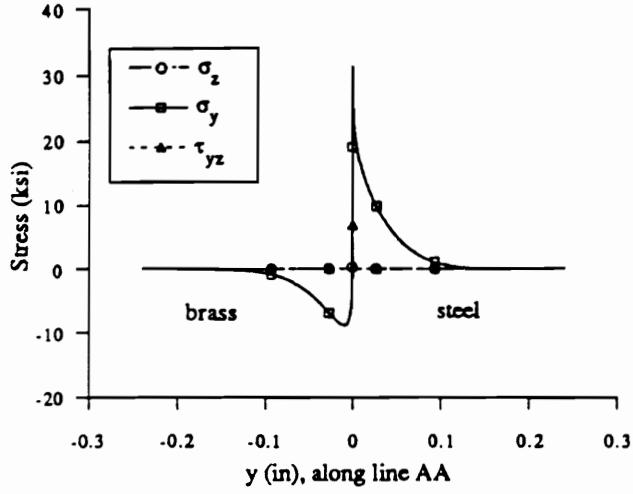
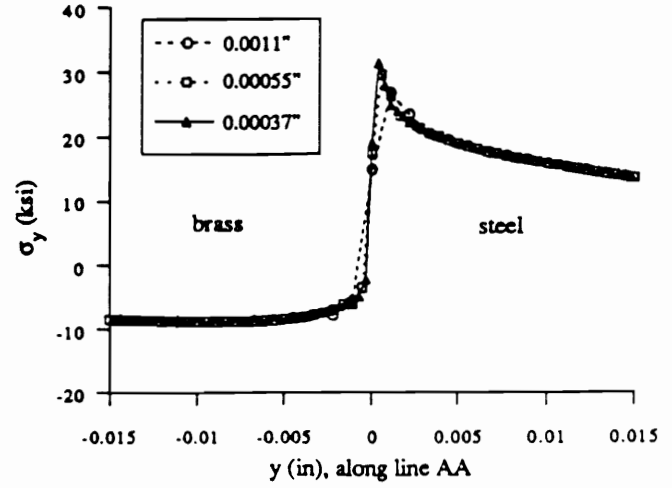


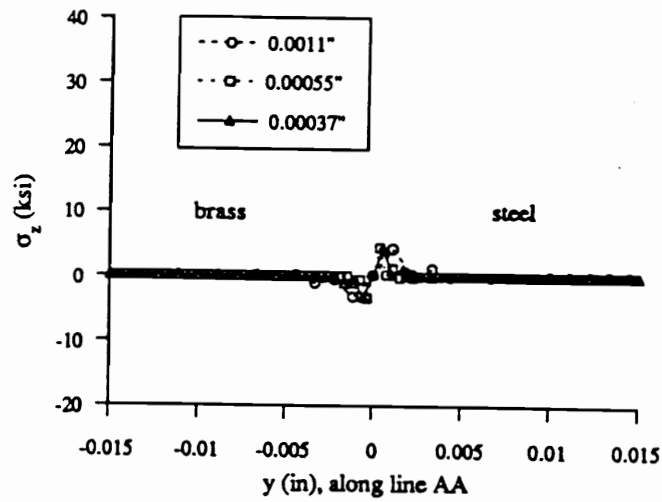
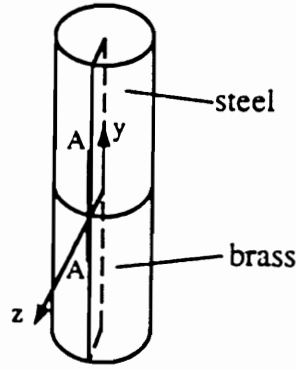
Fig. 9a. Geometry and finite element mesh used in 2-D axisymmetric model analysis.



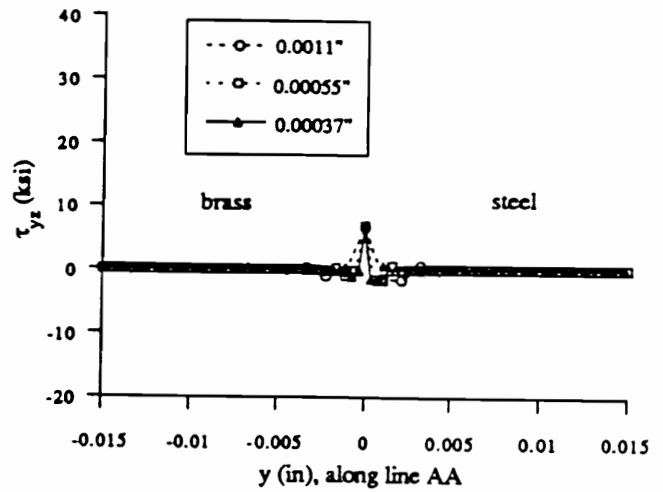
(b)



(c)



(d)



(e)

Fig. 9b-e. (b) The stress distributions along line AA, (c) the normal stress σ_y , (d) the normal stress σ_z , (e) the shear stress τ_{yz} , along line AA with different meshes.

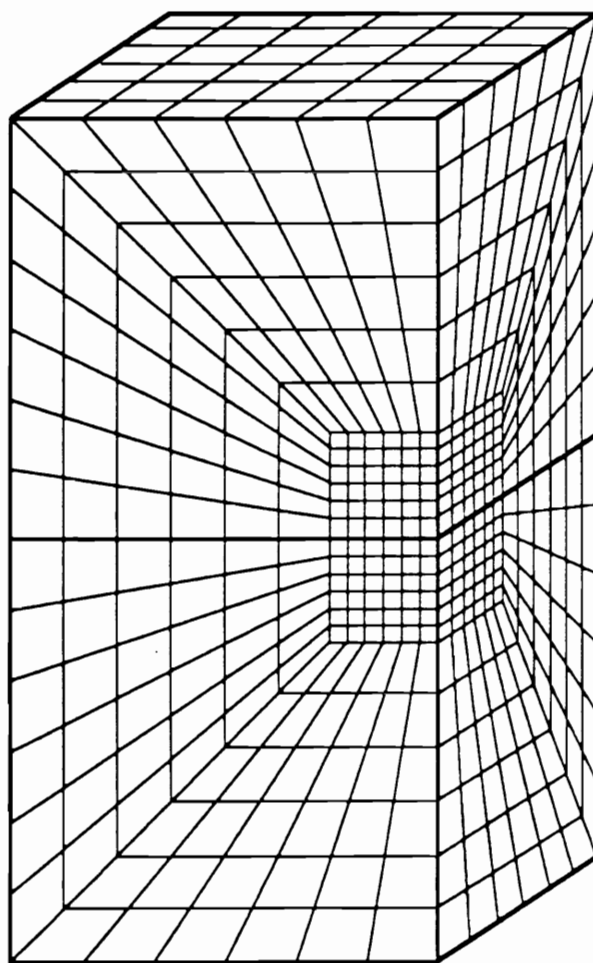
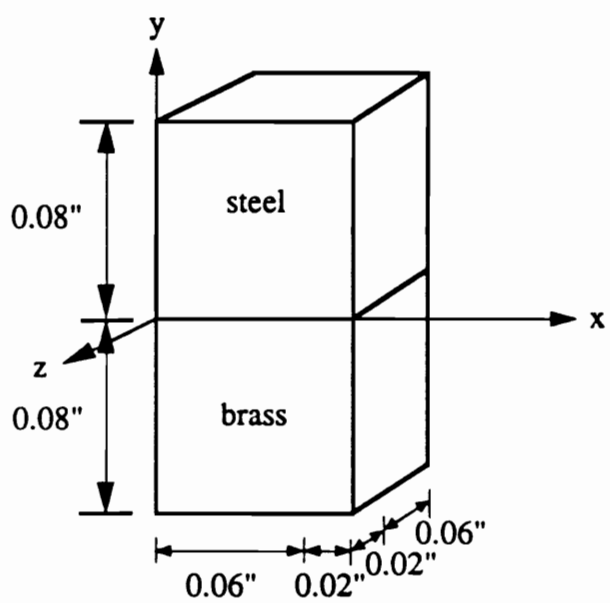


Fig. 10a. Geometry and FEM mesh in a 3-D cube analysis.

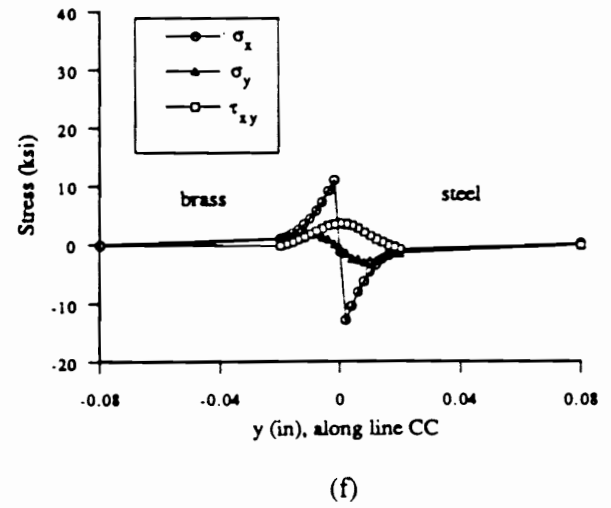
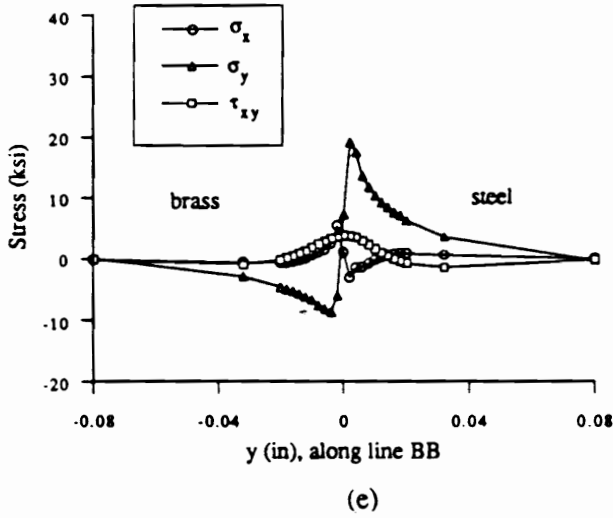
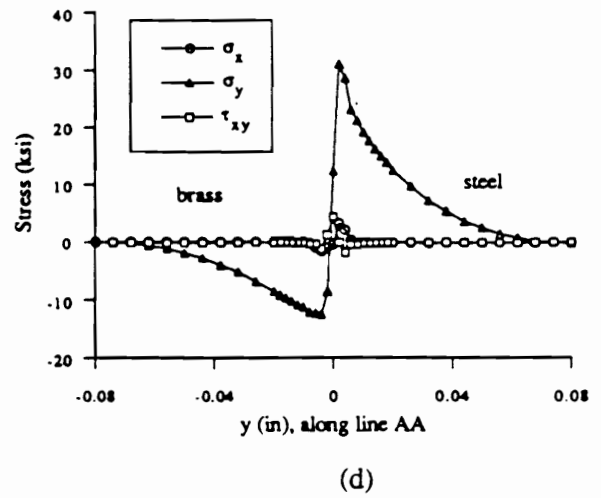
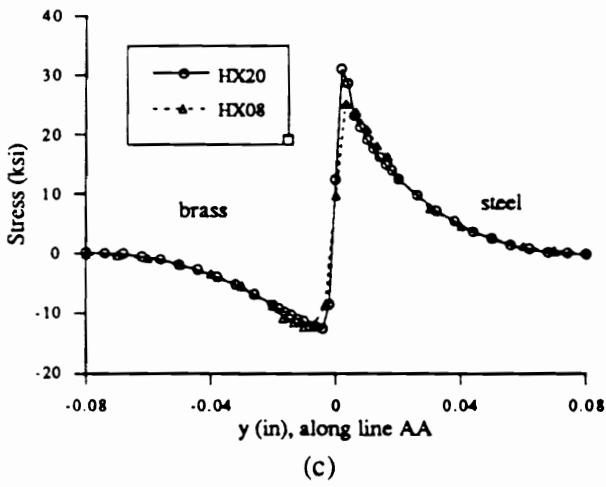
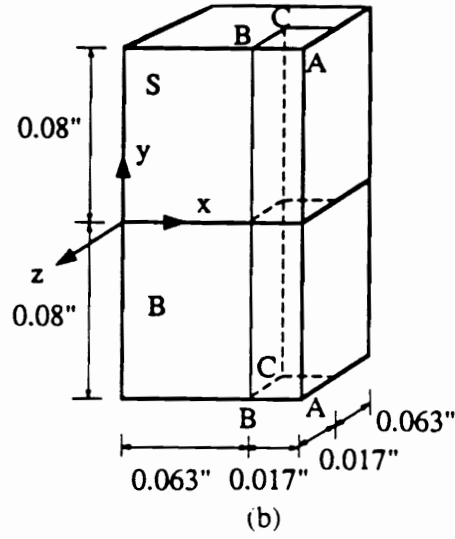
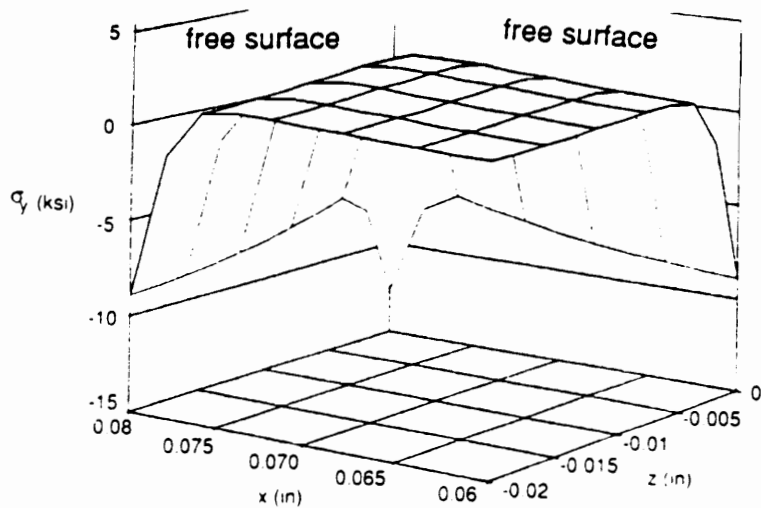
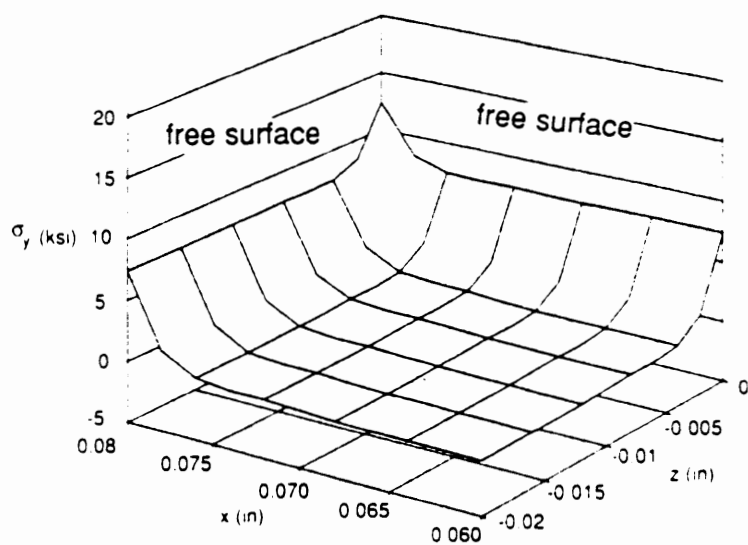


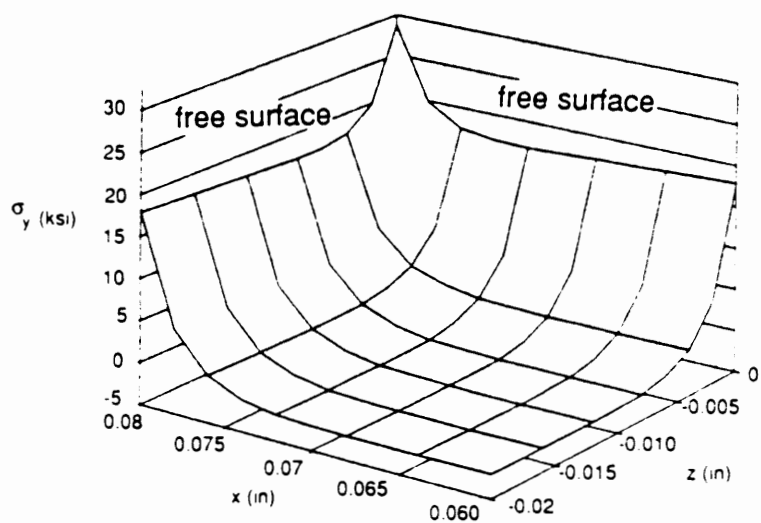
Fig. 10b-f. (b) The locations of the lines AA, BB and CC. (c) The normal stress σ_y along line AA using coarse and refined meshes. The stress distributions along (d) line AA, (e) line BB, (f) line CC with refined meshes.



(a) σ_y on the brass surface near the interface ($y = -0.004''$)



(b) σ_y on the interface of the steel and brass ($y = 0''$)



(c) σ_y on the steel surface near the interface ($y = 0.004''$)

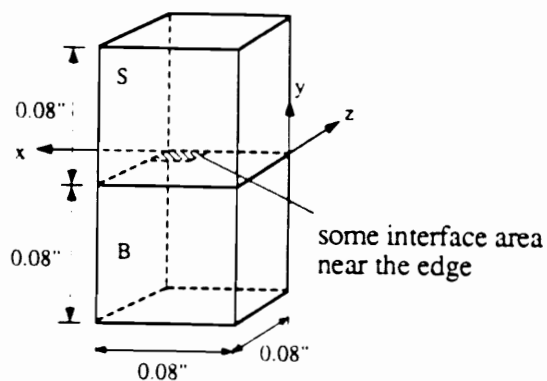


Fig. 11. The normal stress σ_y near the edge
(a) in the brass on the plane, $y = -0.004''$,
(b) on the interface plane, $y = 0$,
(c) in the steel on the plane, $y = 0.004''$.

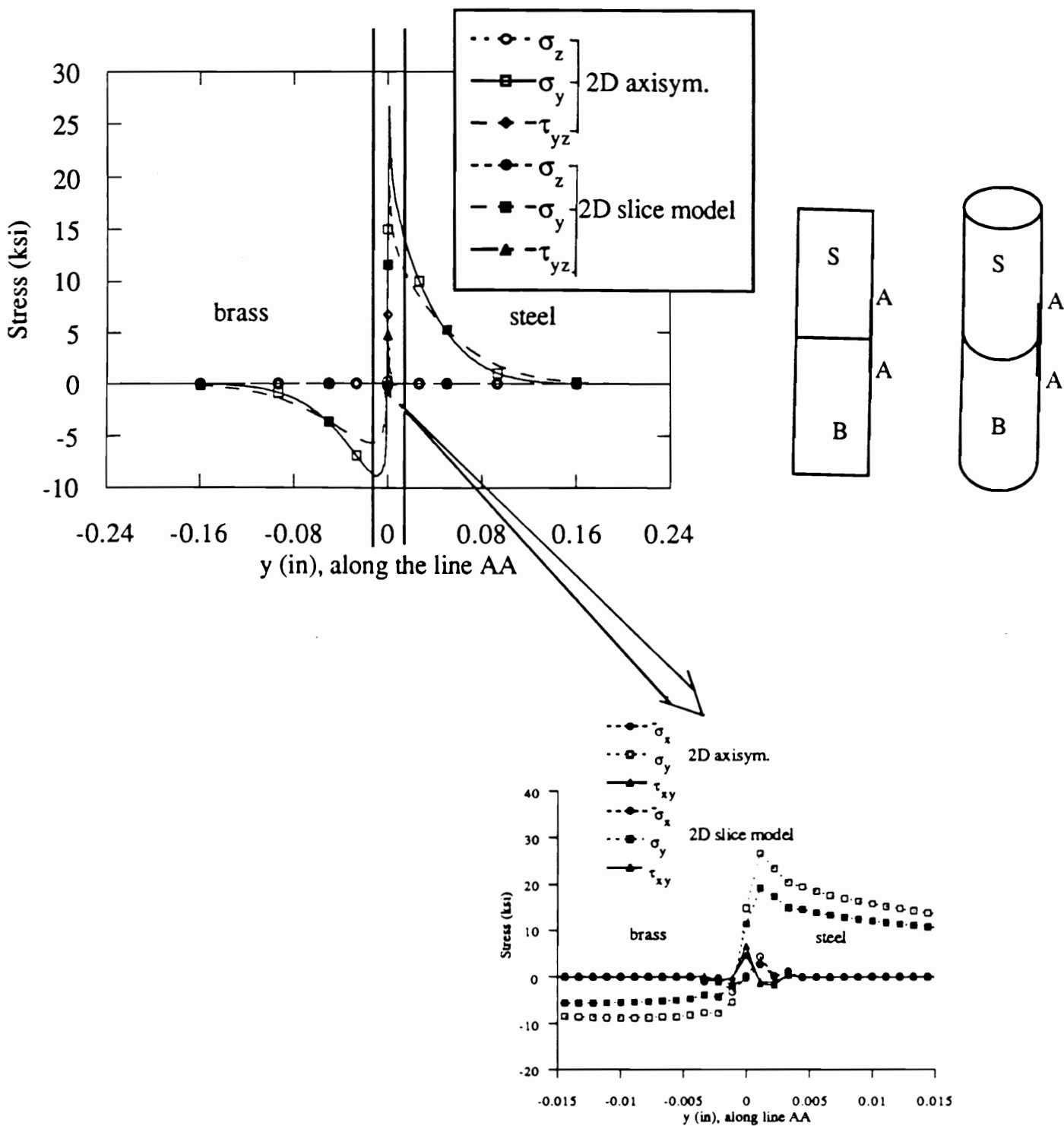


Fig. 12. A comparison of stress distributions for 2-D slice and axisymmetric models

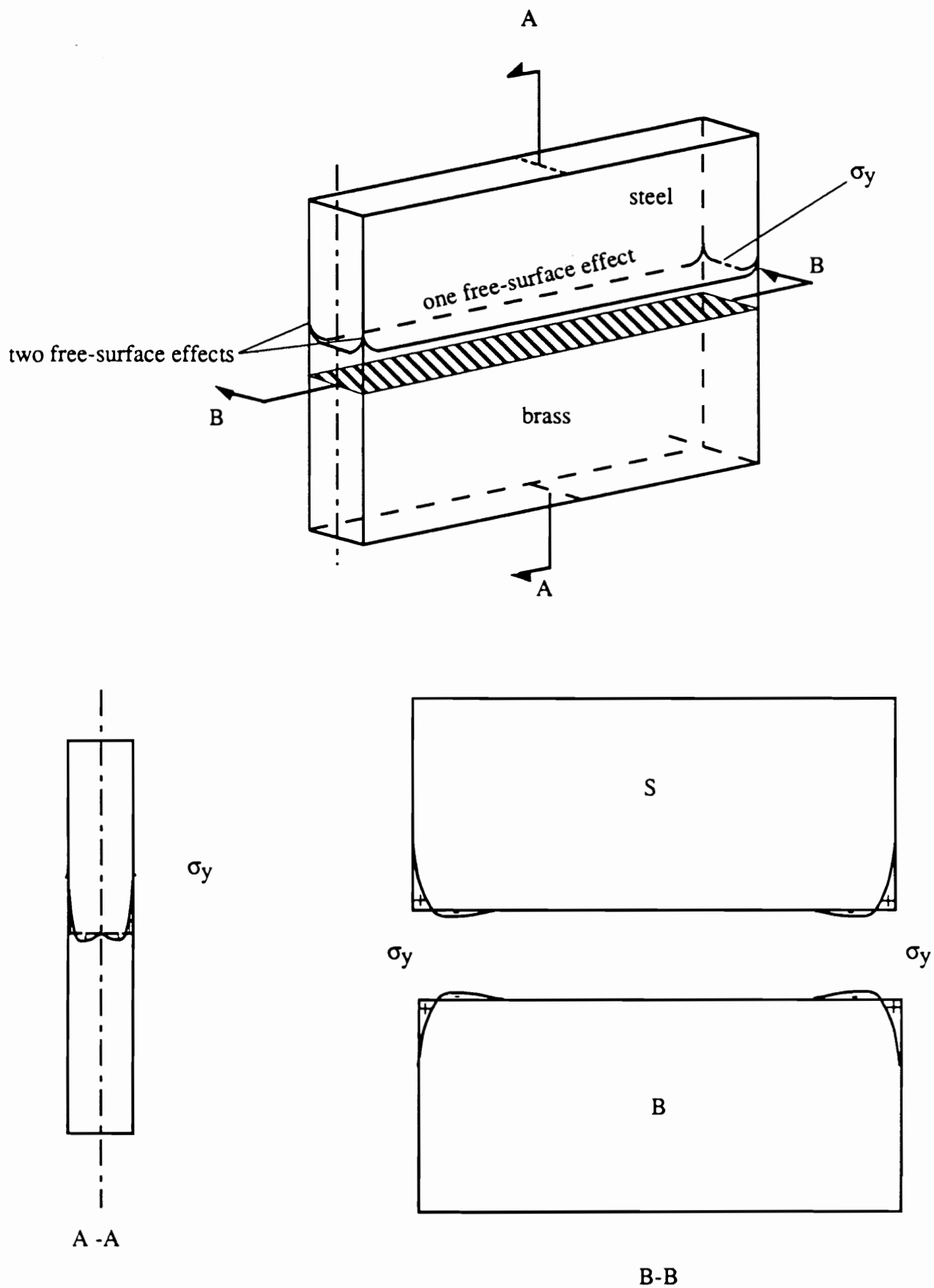


Fig. 13. Qualitative stress distribution of σ_y on the interface (a) for plate geometry, (b) for cross-section AA, (c) for cross-section BB.

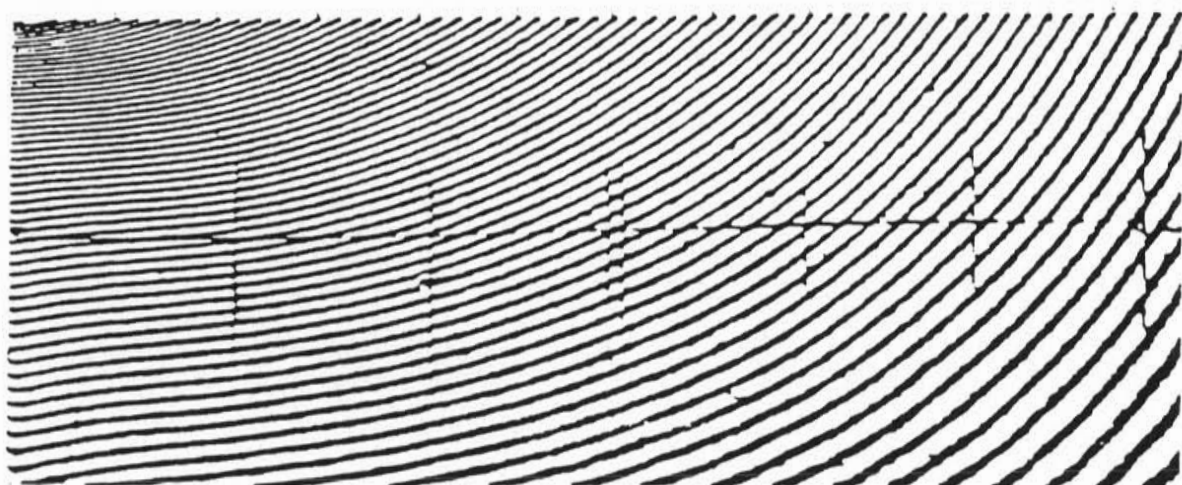
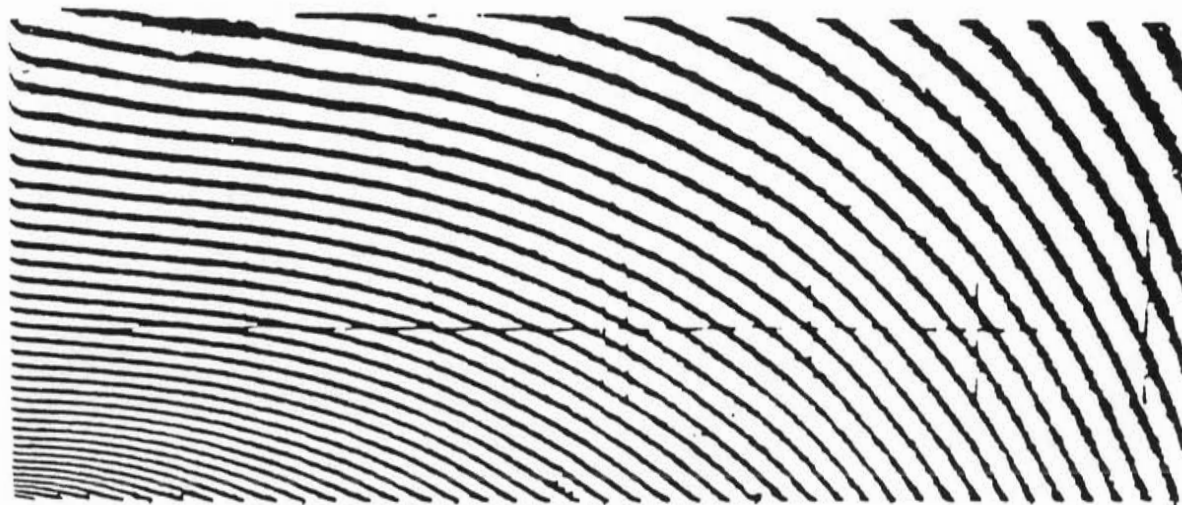
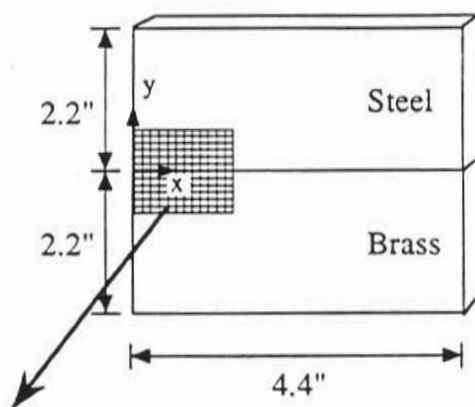
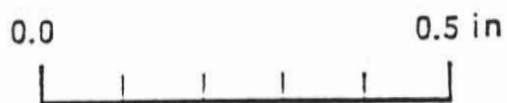
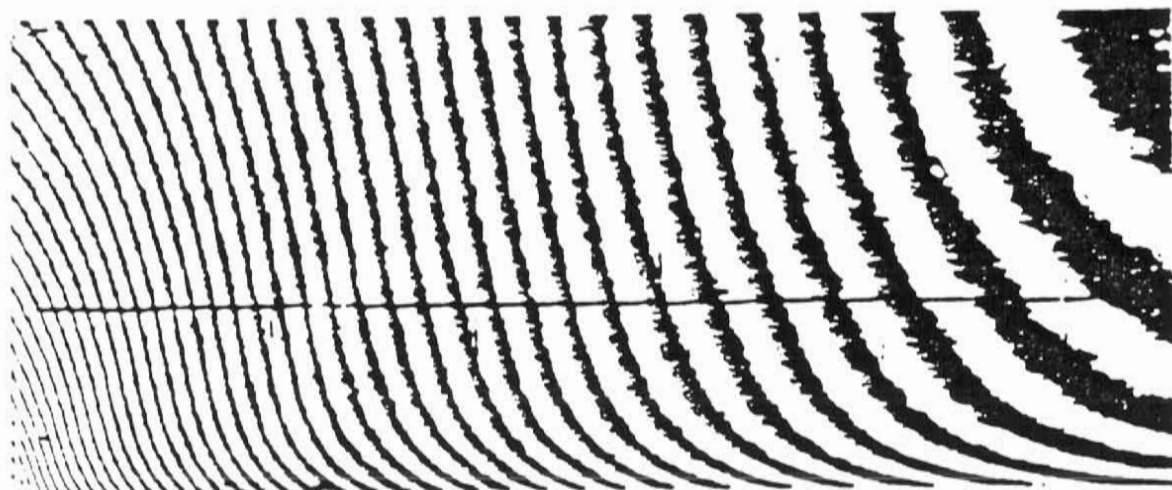
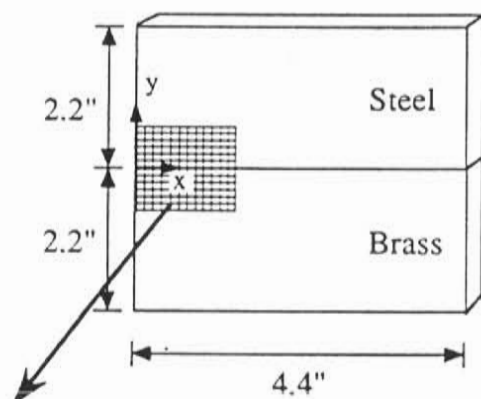
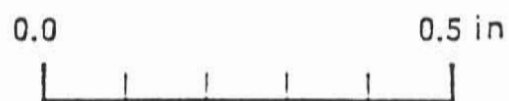
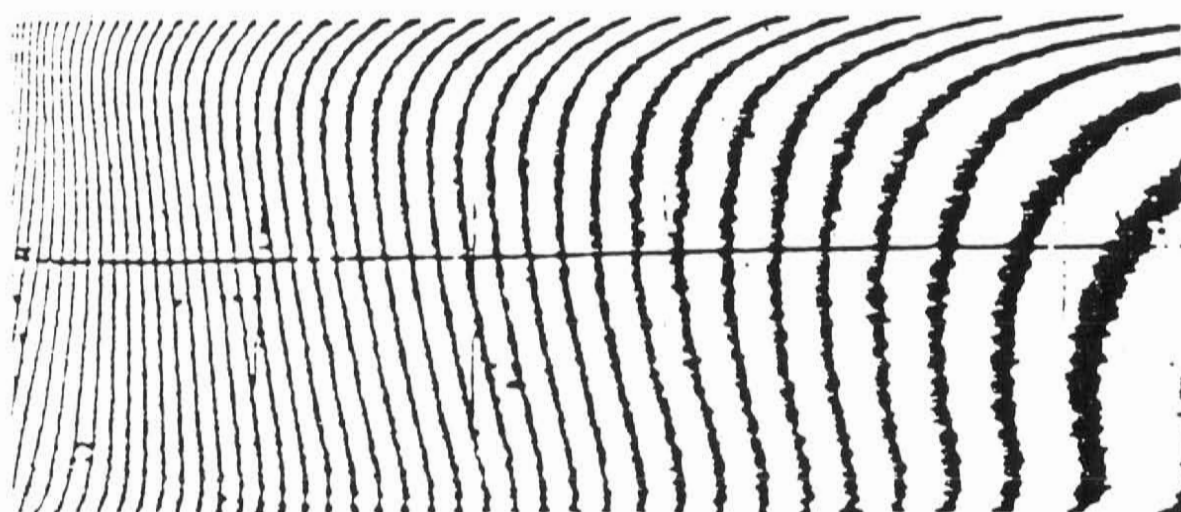


Fig. 14a. Stress-induced horizontal displacement (u) contours [9] due to thermal load ($\Delta T = -240^\circ \text{ F}$).



Steel



Brass

Fig.14b. Stress-induced vertical displacement (v) contours [9] due to thermal load ($\Delta T = -240^\circ \text{F}$).

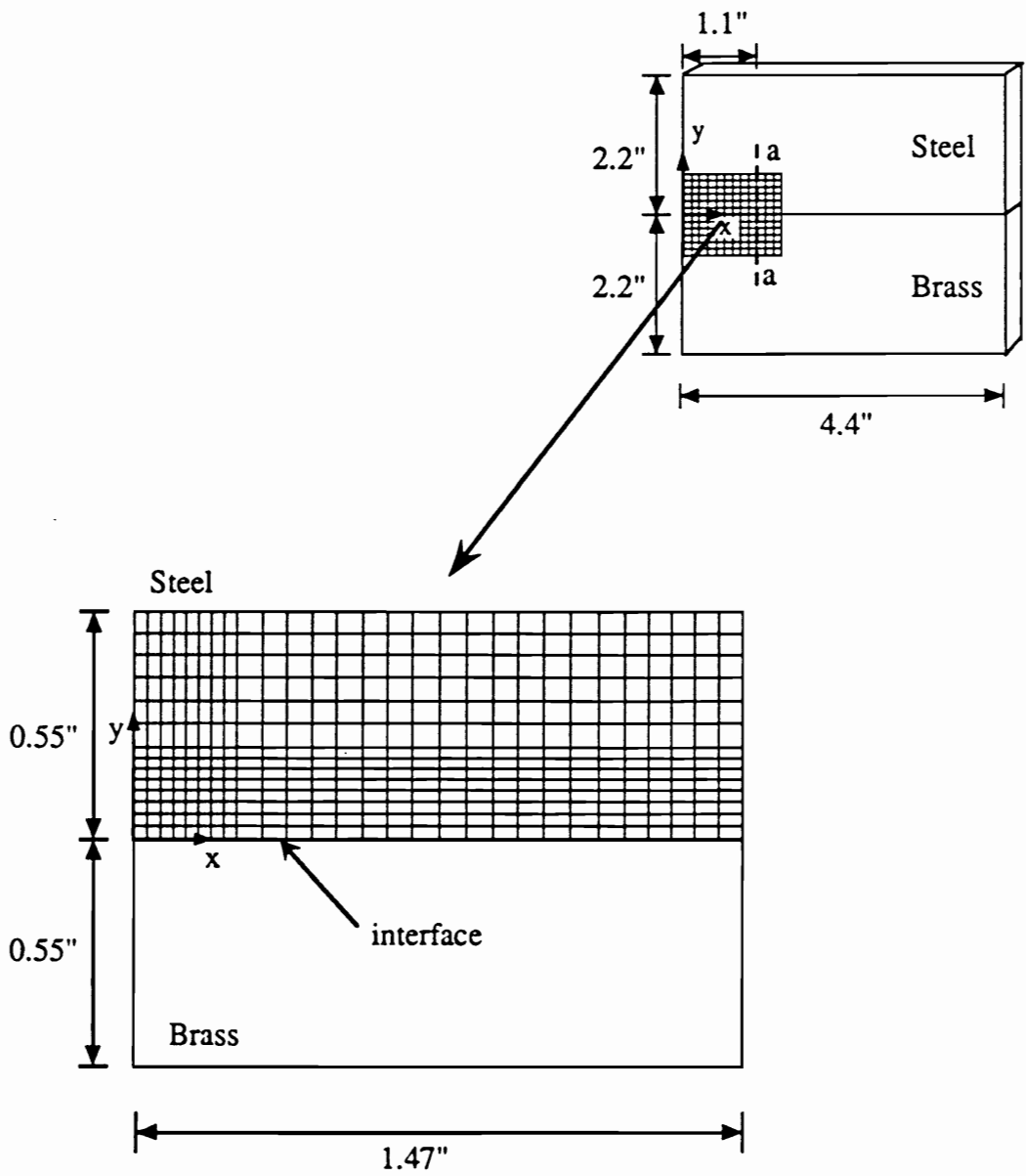


Fig.15a. Finite element mesh in steel in localized hybrid analysis.

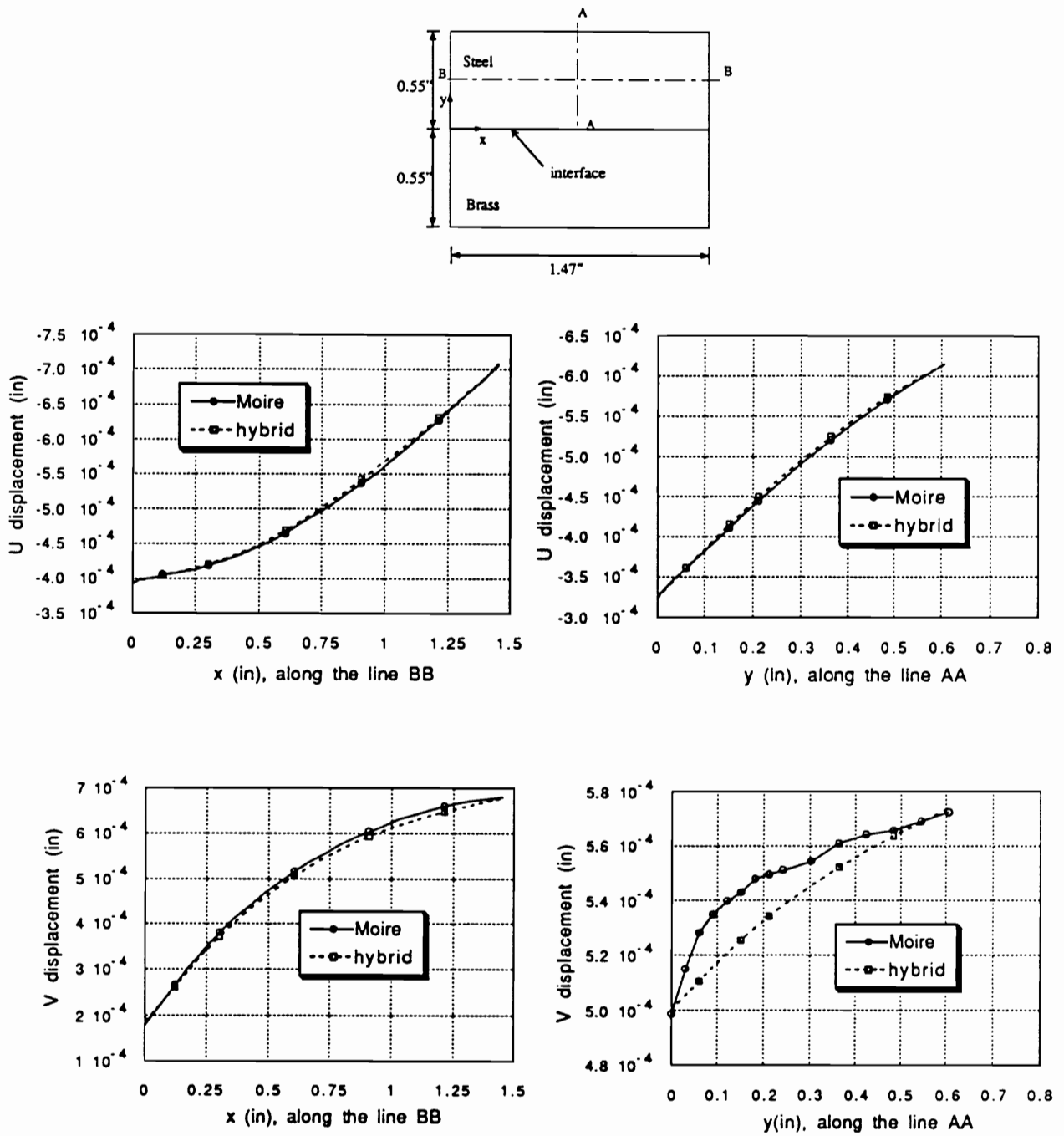


Fig.15b. Comparisons of displacement components in moiré and localized hybrid results along two central lines (lines AA and BB) in steel region.

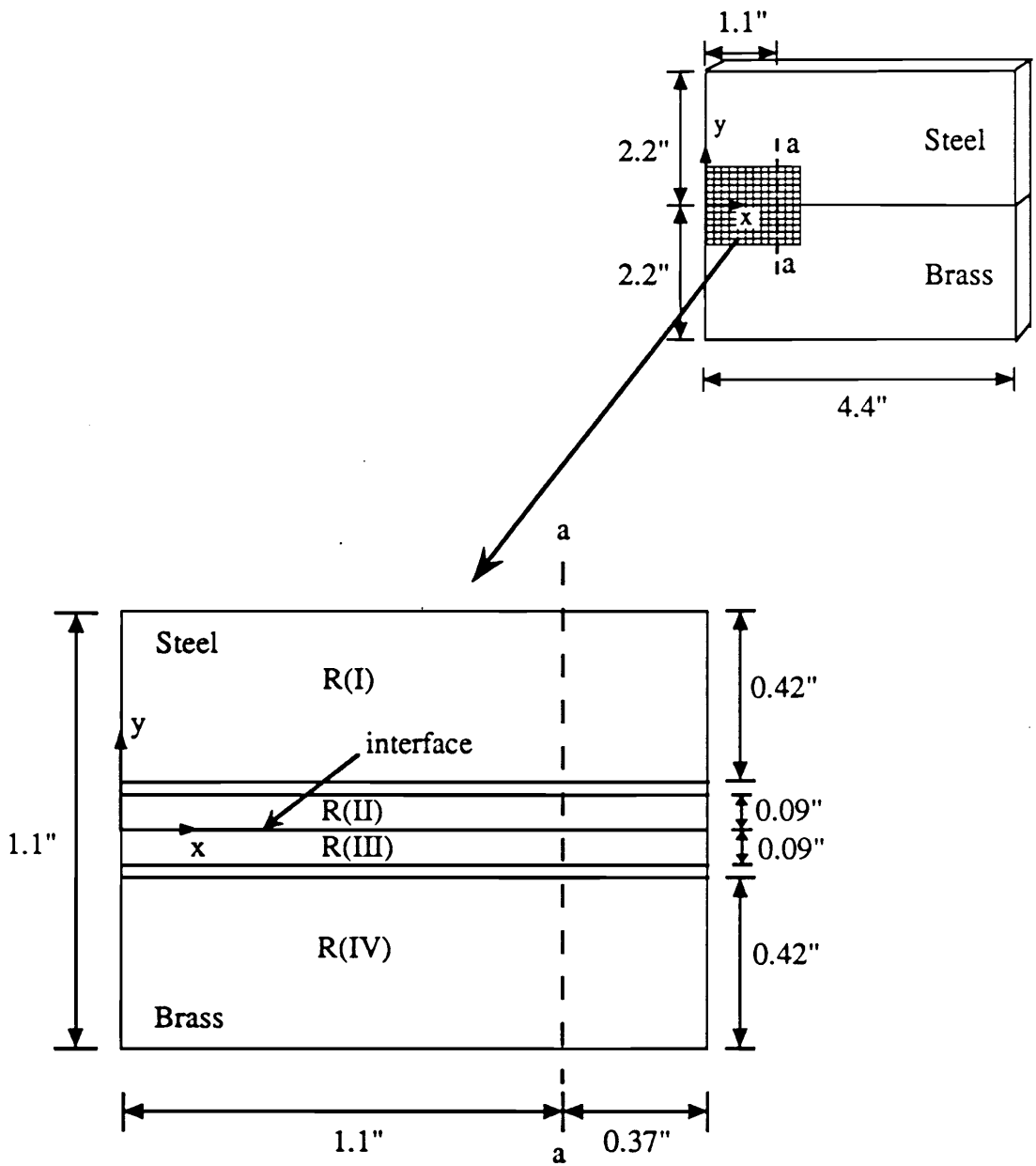


Fig.16a. Regions R(I) and R(II) in steel (R(III) and R(IV) in brass), in which localized hybrid method is applied, represents 2D and 3D zones approximately.

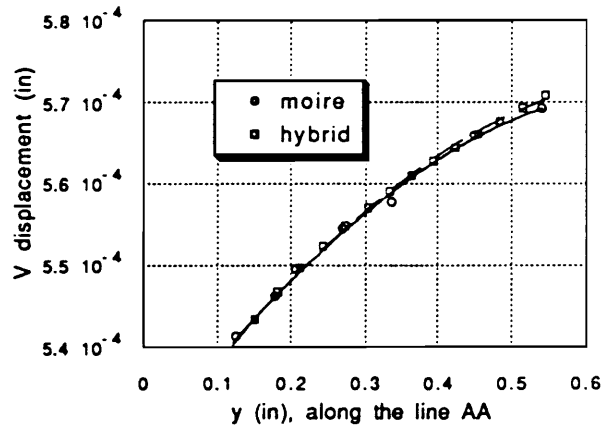
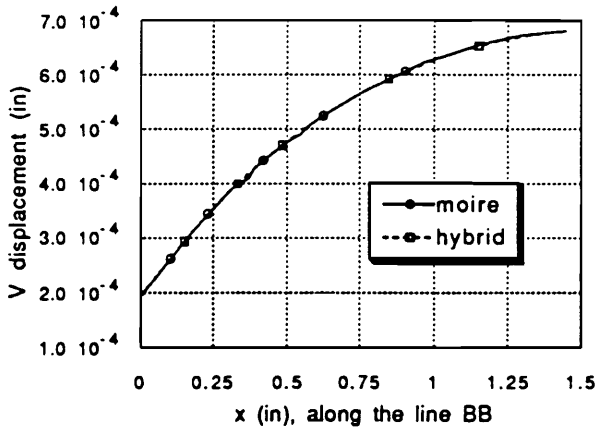
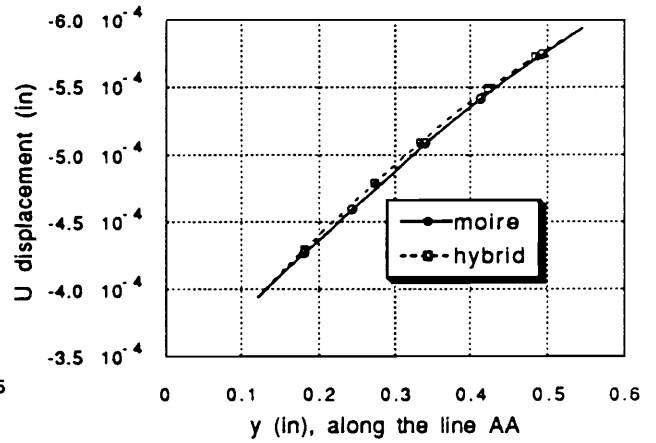
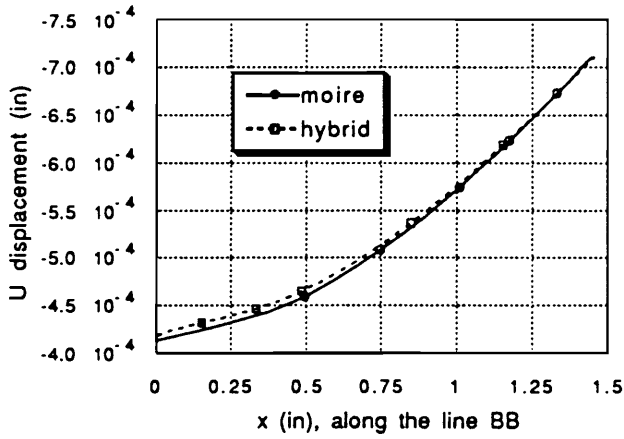
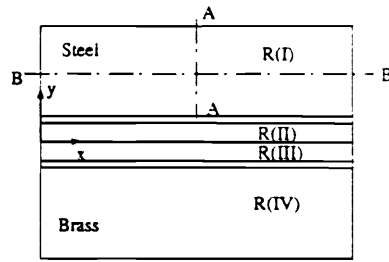


Fig.16b. Comparisons of displacement components in moire and localized hybrid results along two central lines (lines AA and BB) in Region R(I), steel.

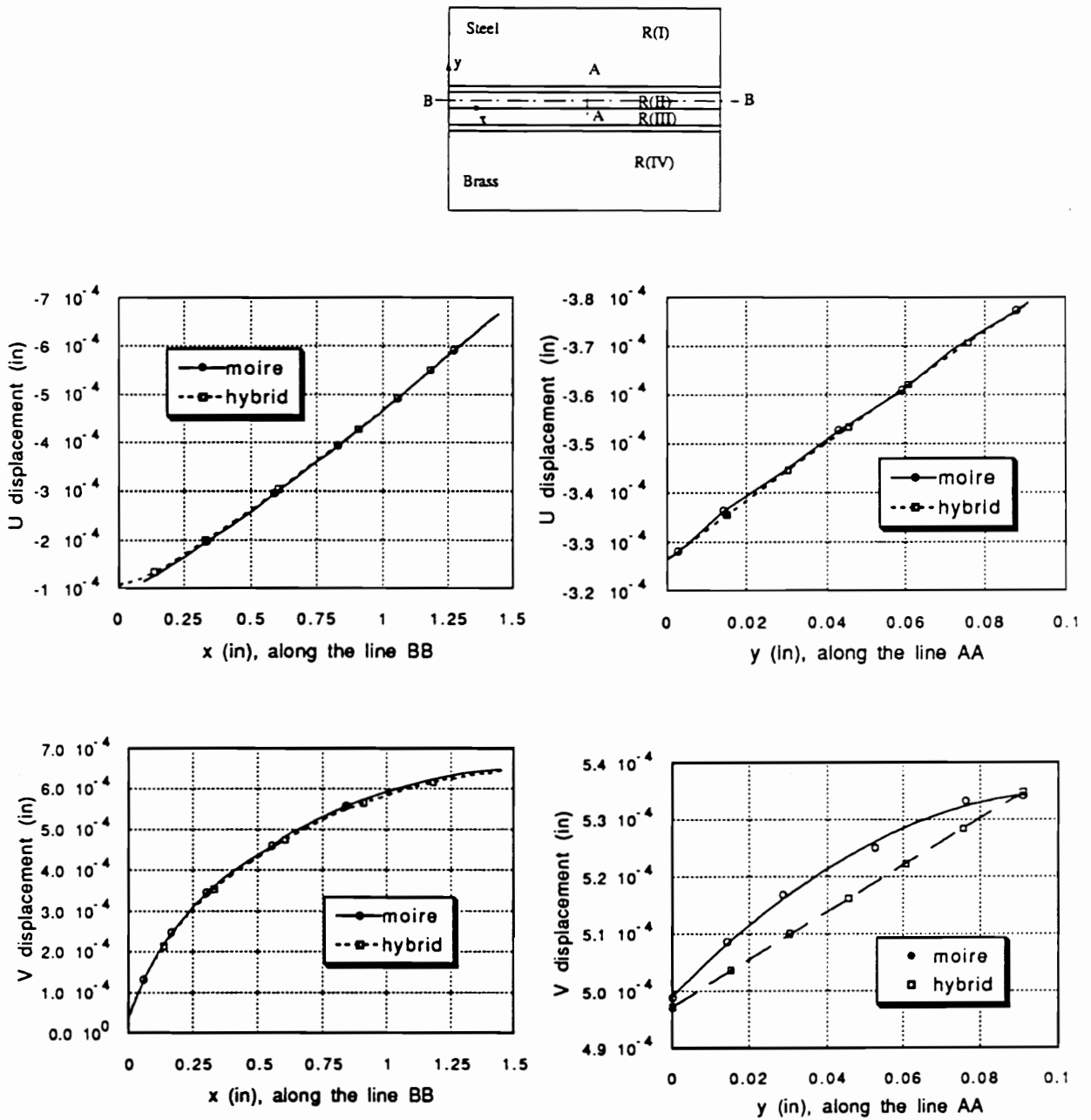


Fig. 16c. Comparisons of displacement components in moiré and localized hybrid results along two central lines (lines AA and BB) in Region R(II), steel.

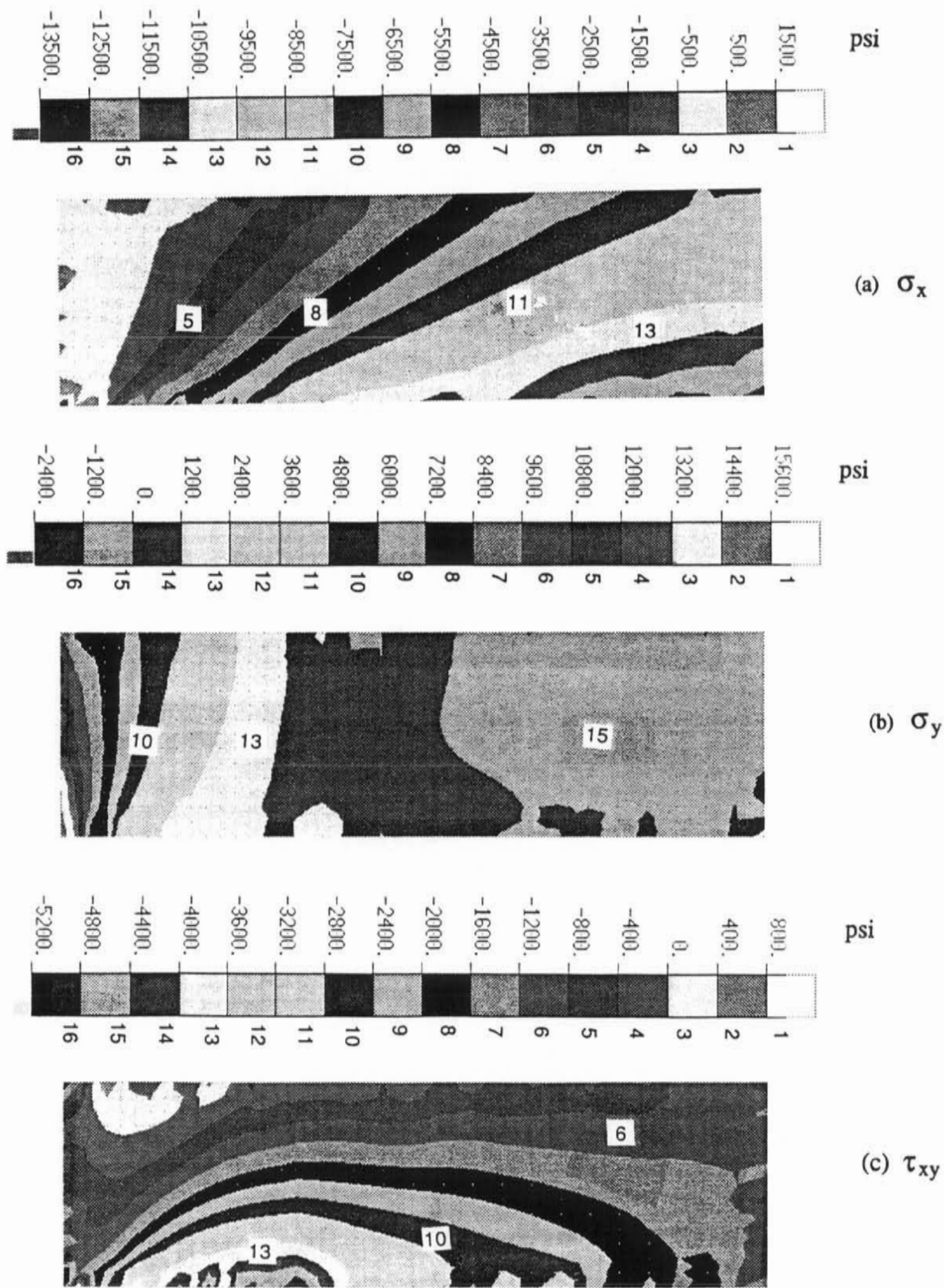


Fig. 17. Stress component contours in Region R(I), steel; (a) the axial stress, σ_x (b) the normal stress, σ_y (c) the shear stress, τ_{xy} .

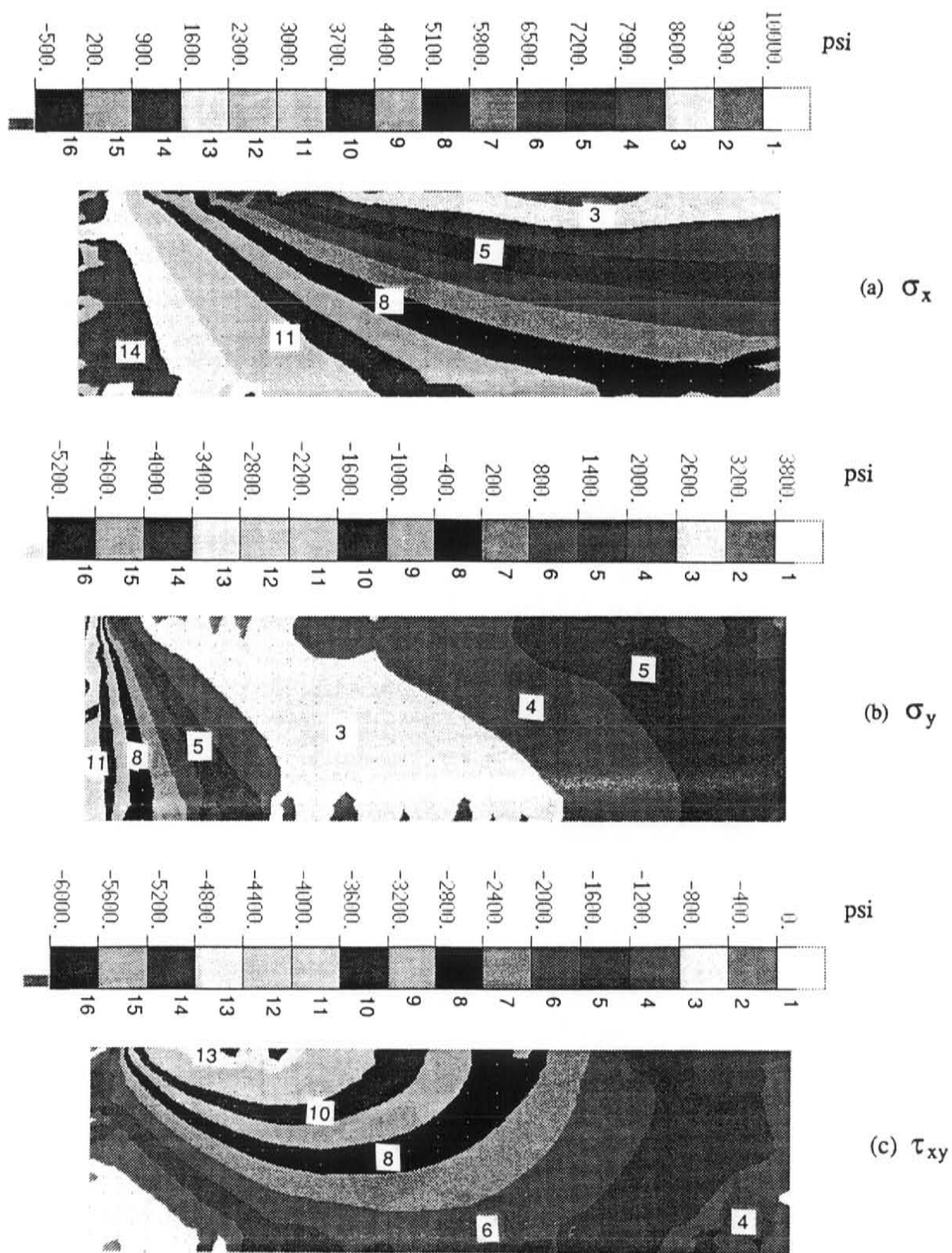


Fig. 18. Stress component contours in Region R(IV), brass; (a) the axial stress, σ_x (b) the normal stress, σ_y (c) the shear stress, τ_{xy} .

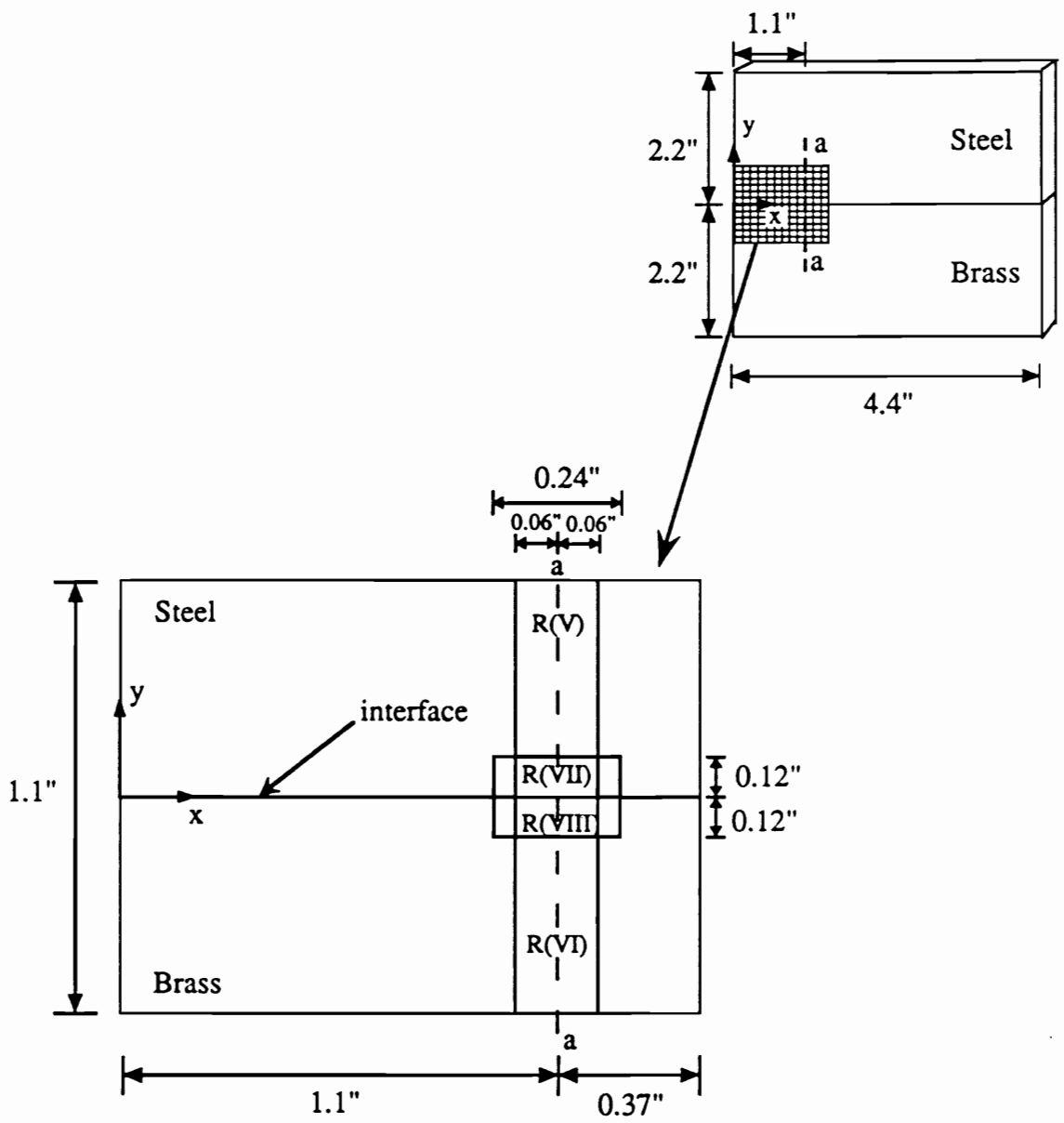


Fig. 19a. Regions R(V) and R(VII) in steel (R(VI) and R(VIII) in brass), in which the localized hybrid method is applied.

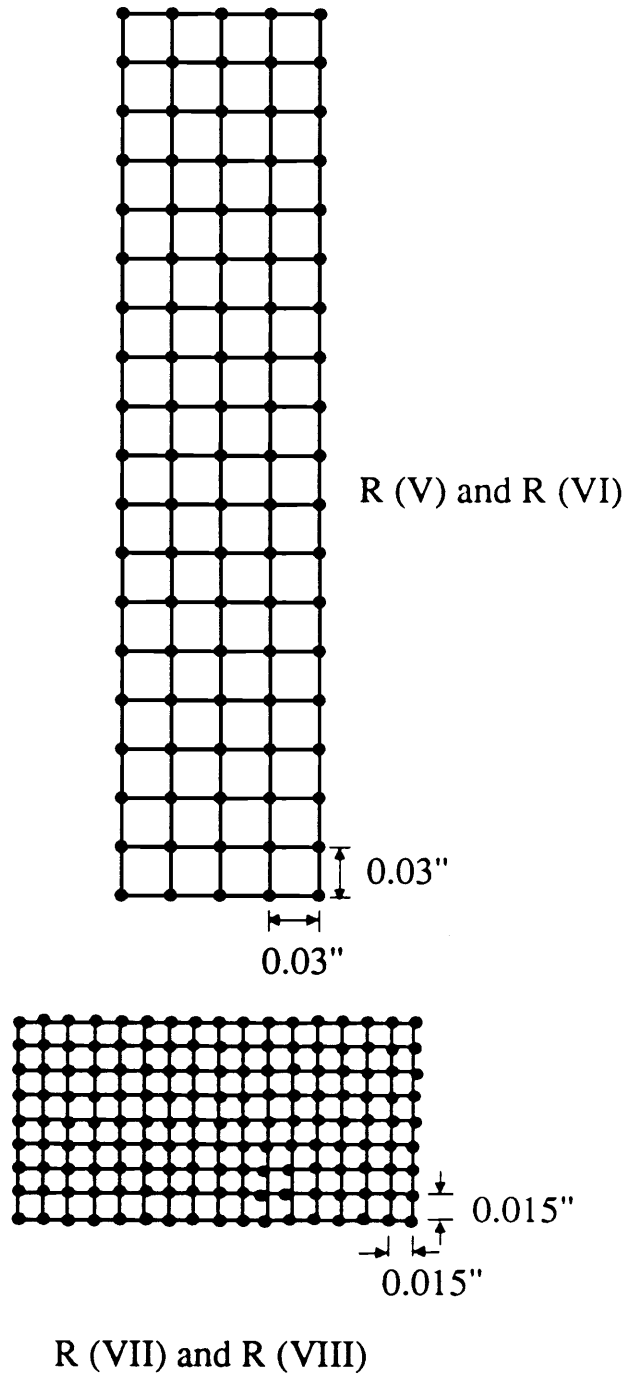


Fig. 19b. Meshes of Regions R(V) and R(VI), and finer meshes of Regions R(VII) and R(VIII) in localized hybrid analysis. In these meshes, all node points were specified displacement values obtained from moire data.

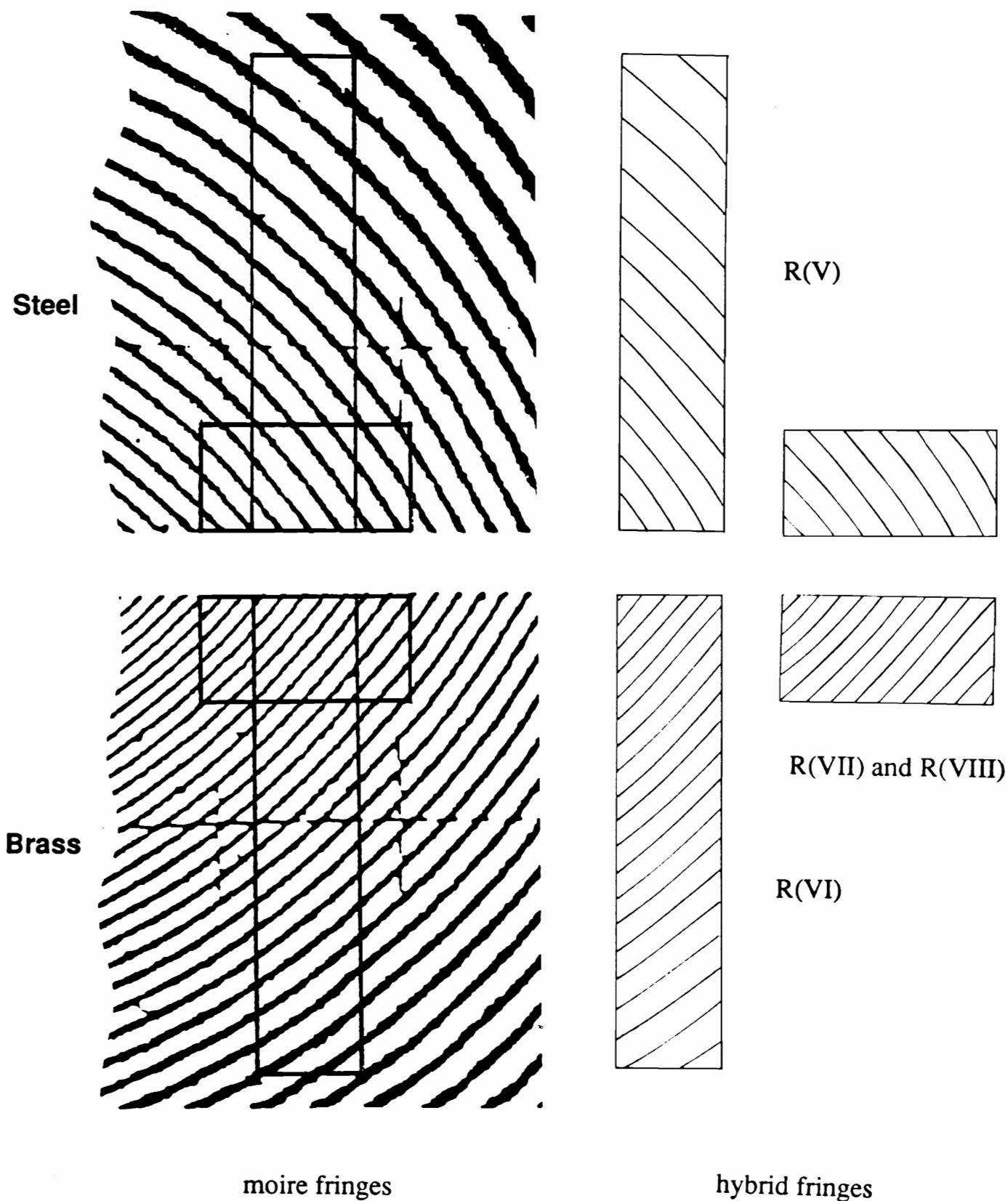


Fig. 20a Comparisons of u -field fringe patterns obtained from the moire [9] and hybrid analysis in the Regions R(V), R(VI), R(VII) and R(VIII).

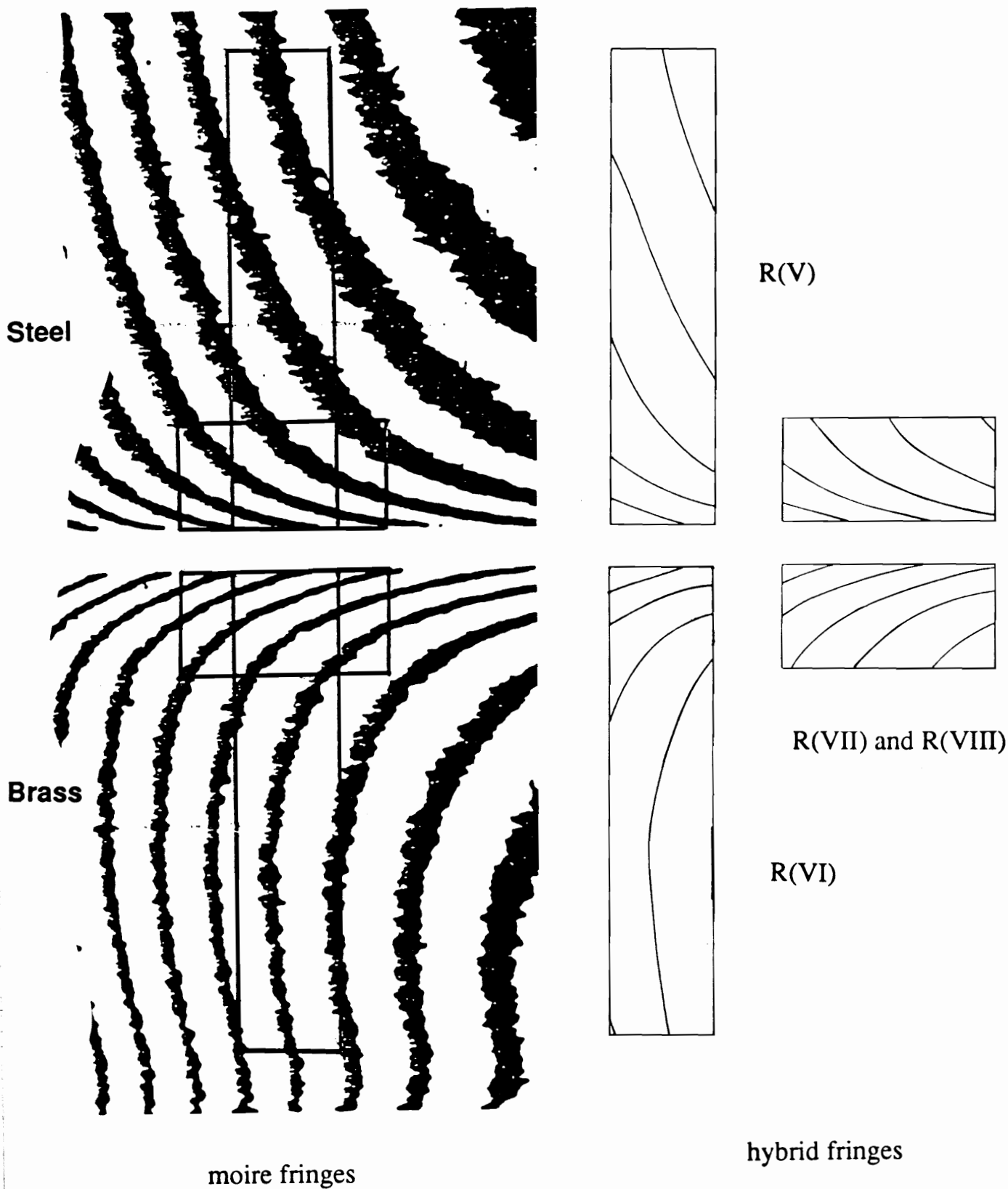


Fig. 20b Comparisons of v-field fringe patterns obtained from the moire [9] and hybrid analysis in the Regions R(V), R(VI), R(VII) and R(VIII).

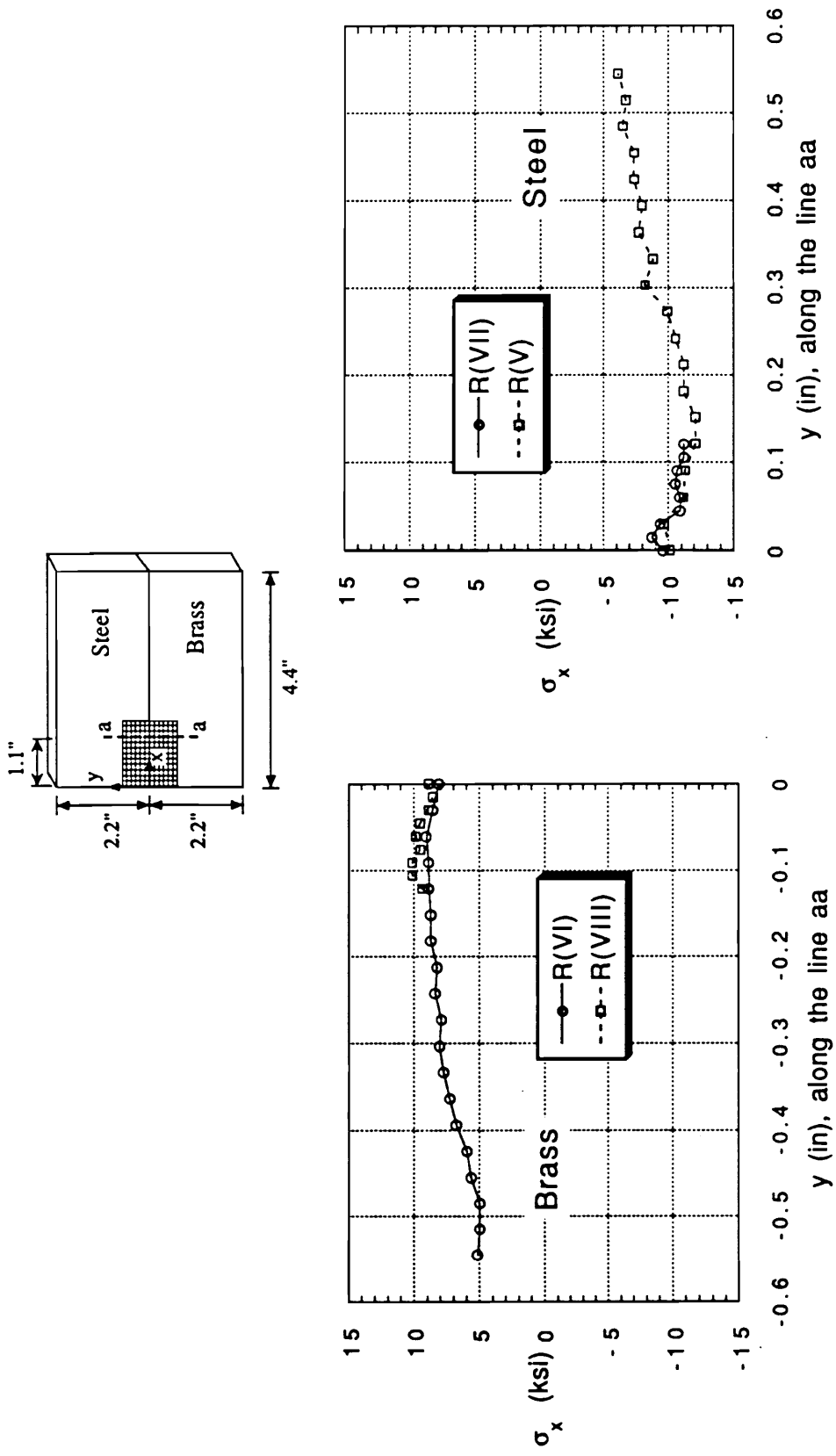


Fig. 21a. σ_x stress distributions along the line aa from localized hybrid analysis in Regions R(V), R(VI), R(VII) and R(VIII).

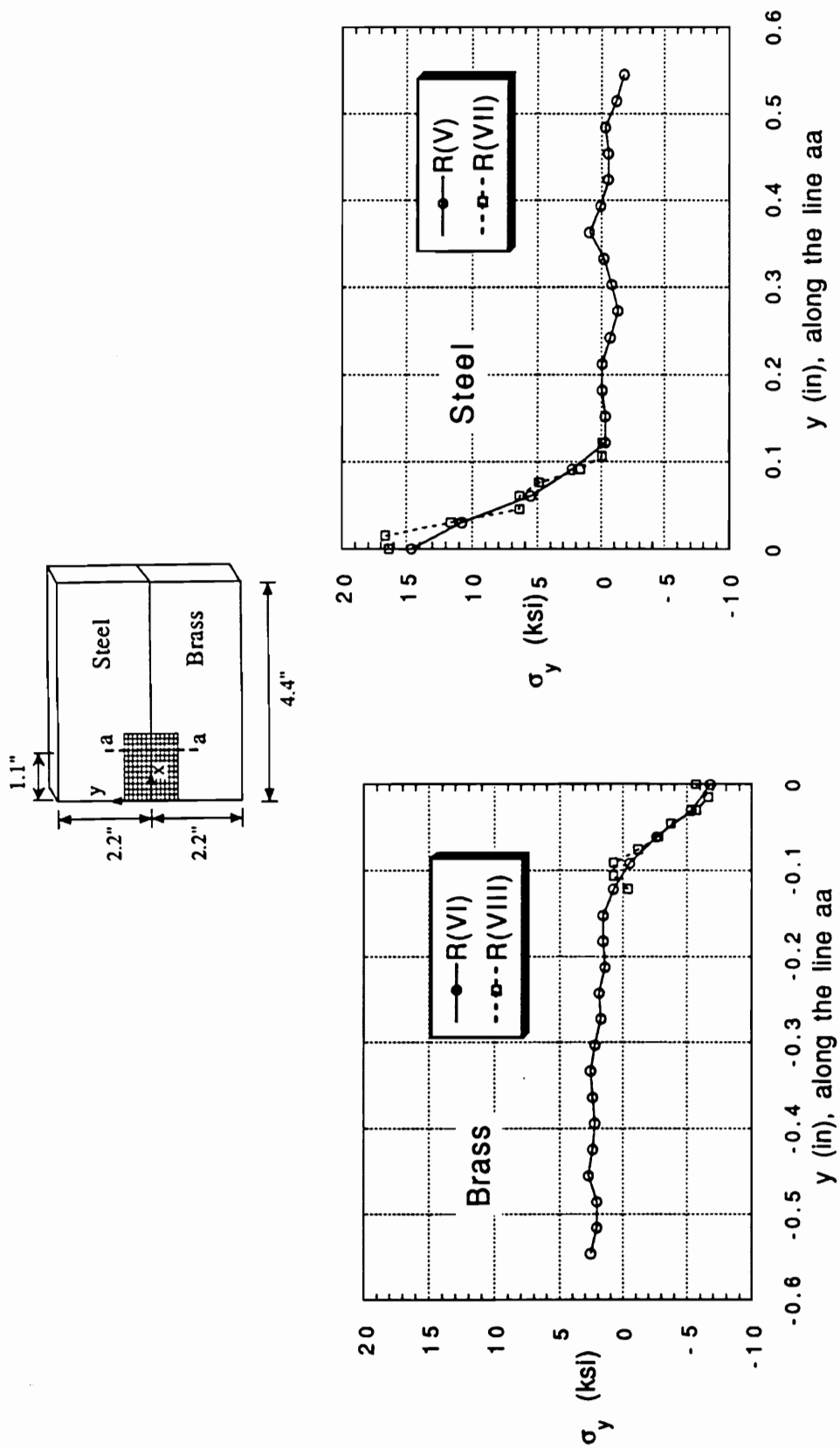


Fig. 21b. σ_y stress distributions along the line aa from localized hybrid analysis in Regions R(V), R(VI), R(VII) and R(VIII).

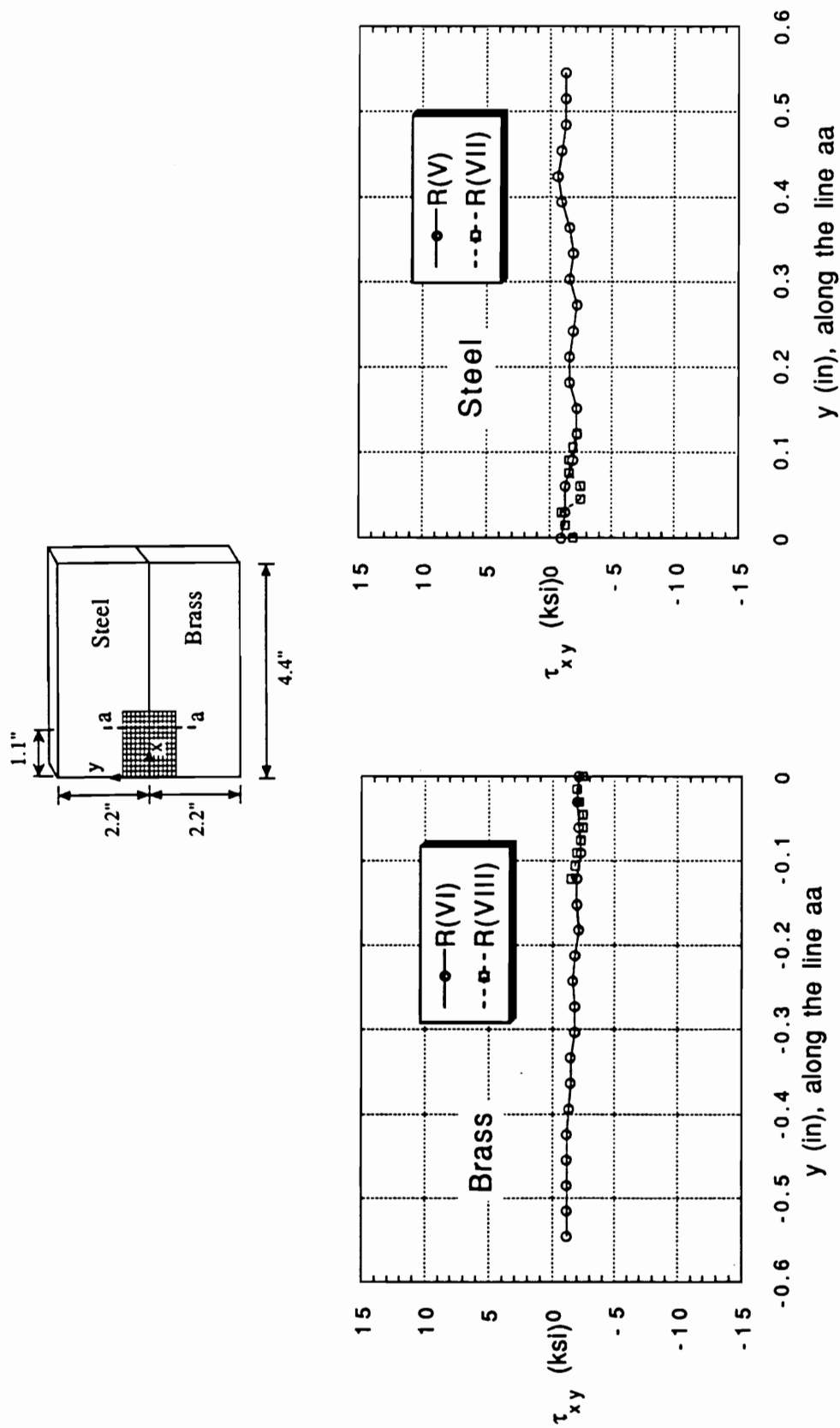


Fig. 21c. τ_{xy} stress distributions along the line aa from localized hybrid analysis in Regions R(V), R(VI), R(VII) and R(VIII).

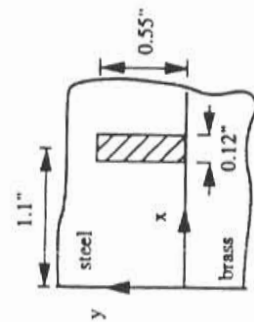
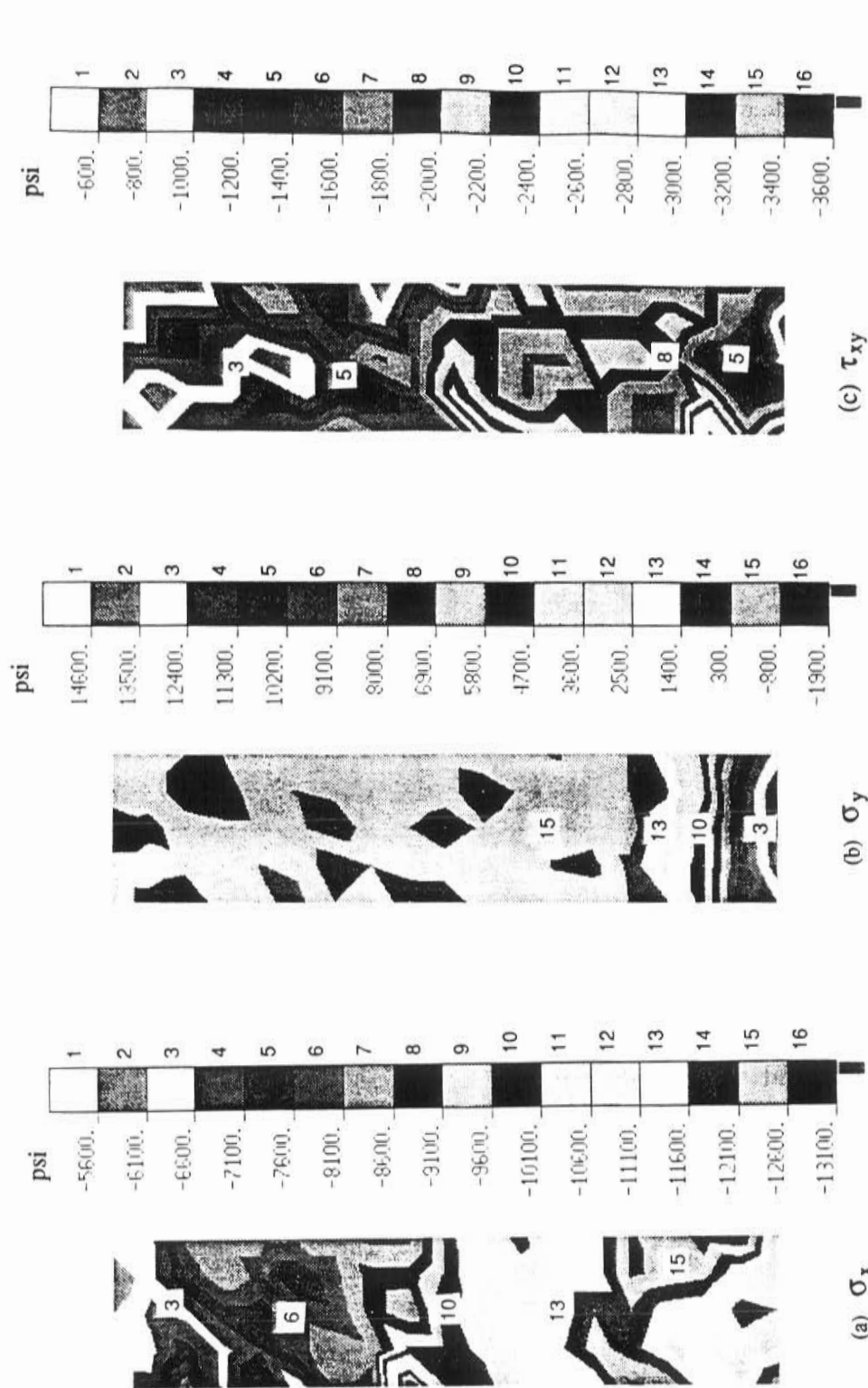


Fig. 22. Stress component contours in Region R(V), steel; (a) the axial stress, σ_x (b) the normal stress, σ_y (c) the shear stress, τ_{xy} .

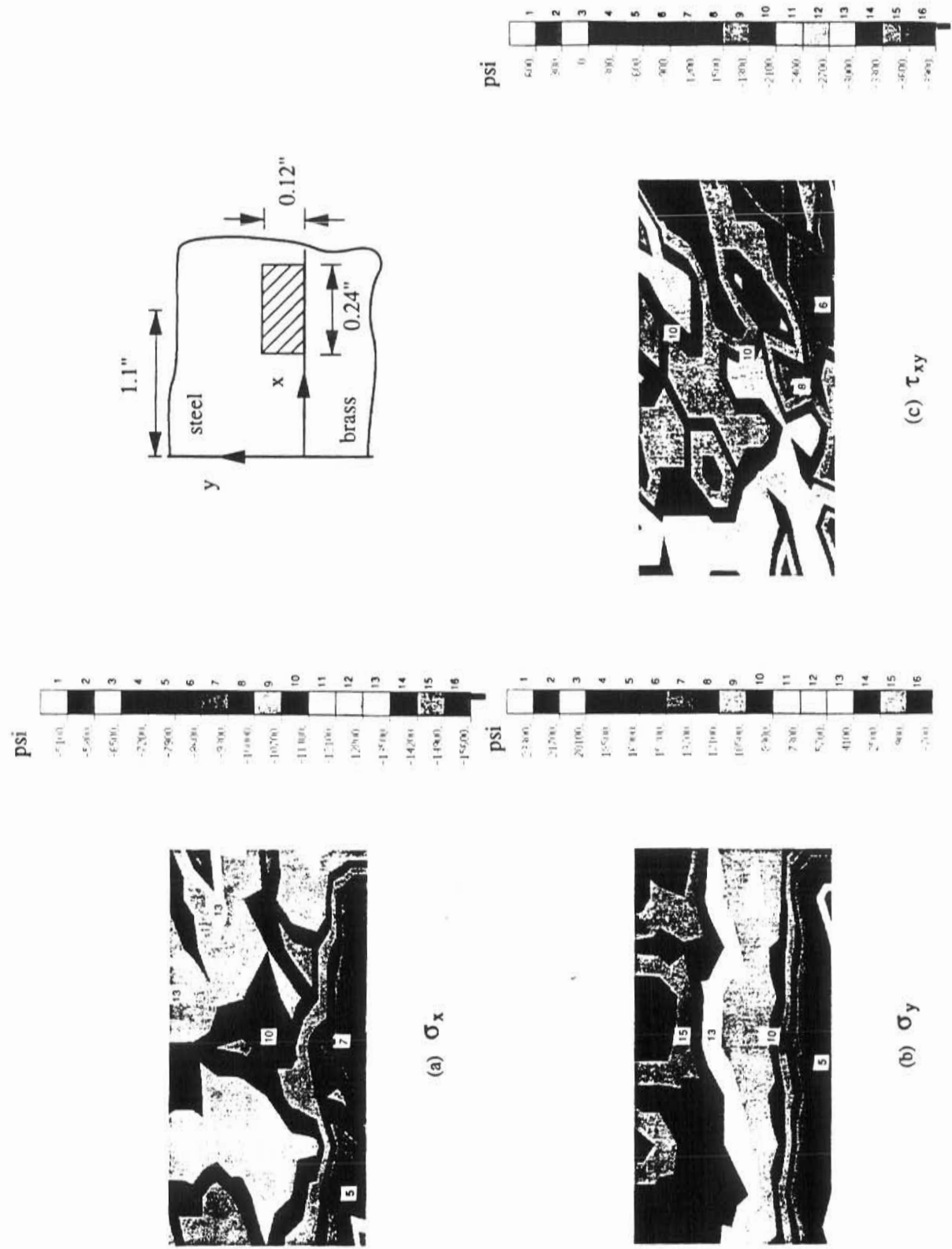


Fig. 23. Stress component contours in Region R(VII), steel; (a) the axial stress, σ_x (b) the normal stress, σ_y (c) the shear stress, τ_{xy} .

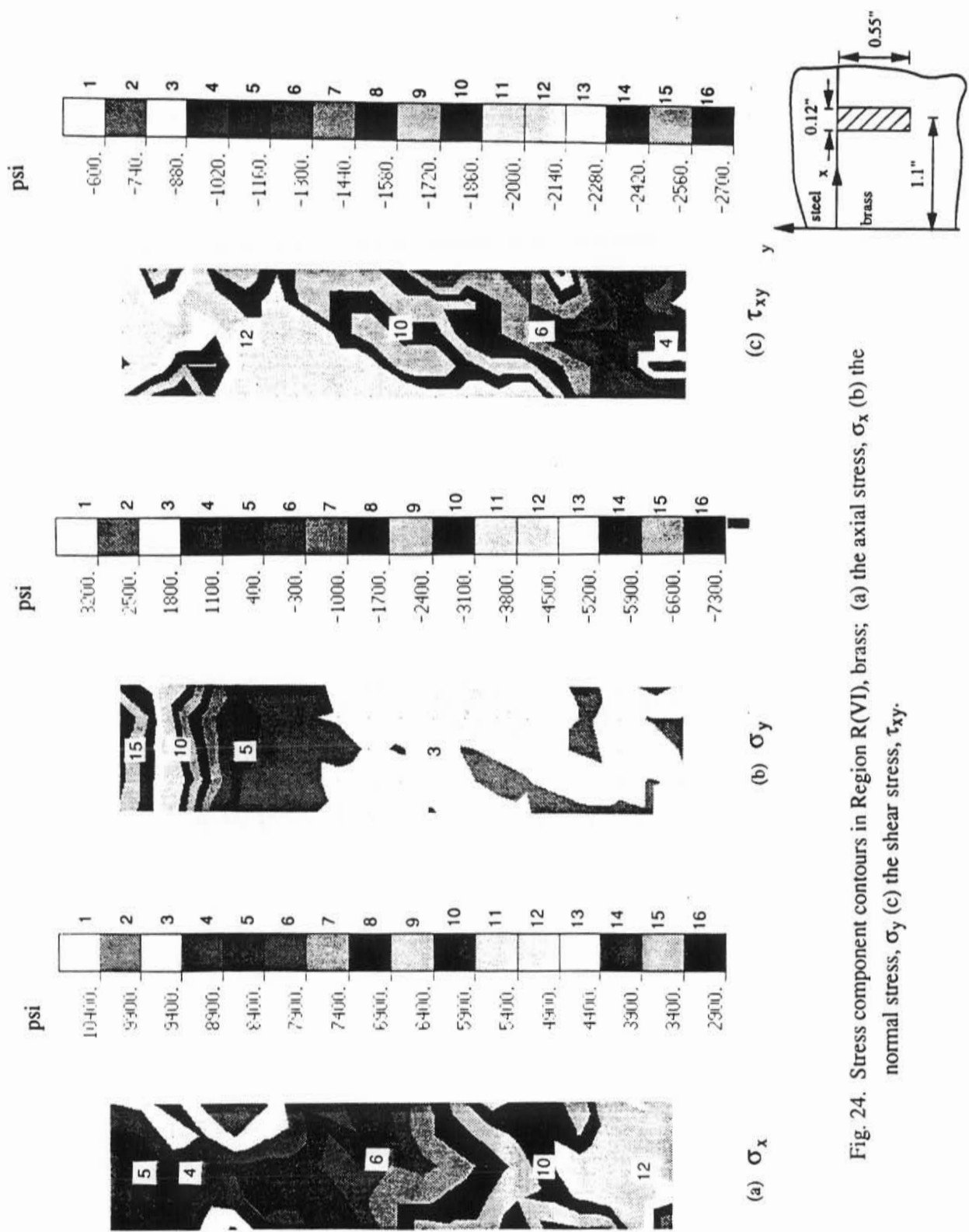


Fig. 24. Stress component contours in Region R(VI), brass; (a) the axial stress, σ_x (b) the normal stress, σ_y (c) the shear stress, τ_{xy} .

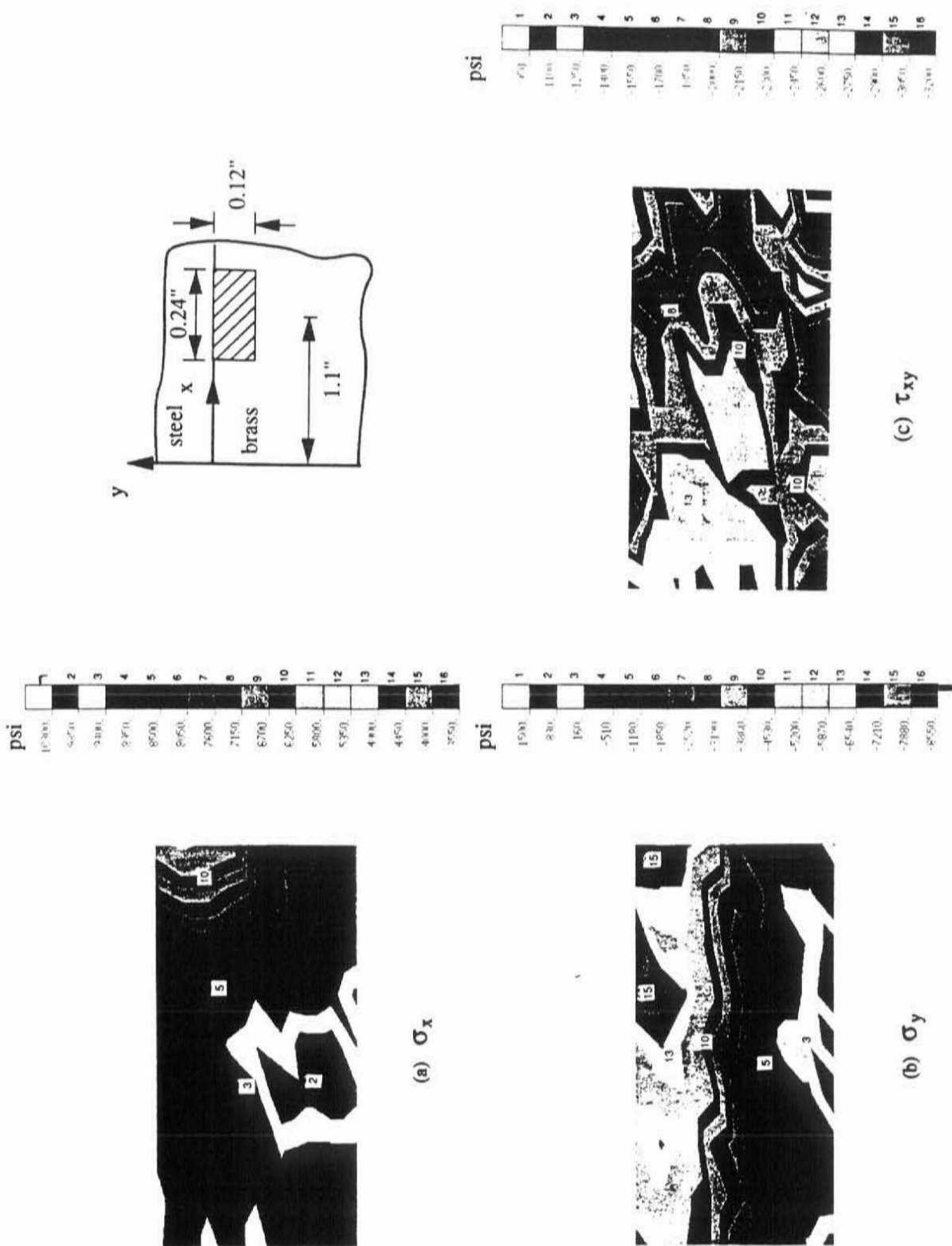
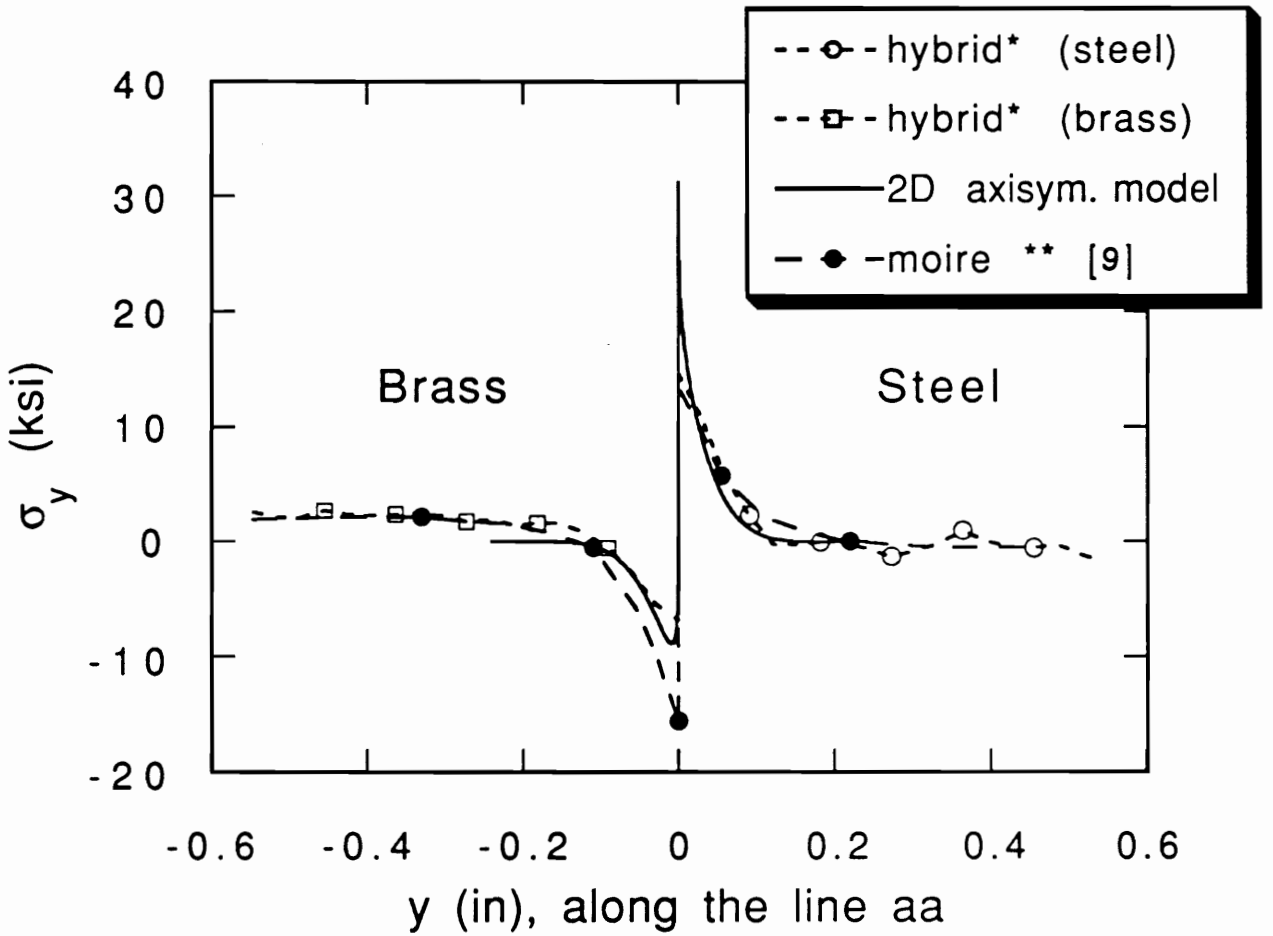
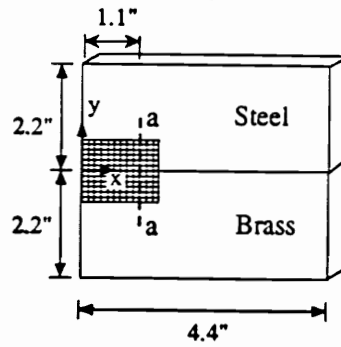


Fig. 25. Stress component contours in Region R(VIII), brass; (a) the axial stress, σ_x (b) the normal stress, σ_y (c) the shear stress, τ_{xy} .



* Using full-node method
 ** Using manual differentiation

Fig. 26. Comparisons of σ_y stress distribution along the line aa in localized hybrid analysis (full-node method), finite element analysis (2D axisymmetric model), and moire results (manual differentiation).

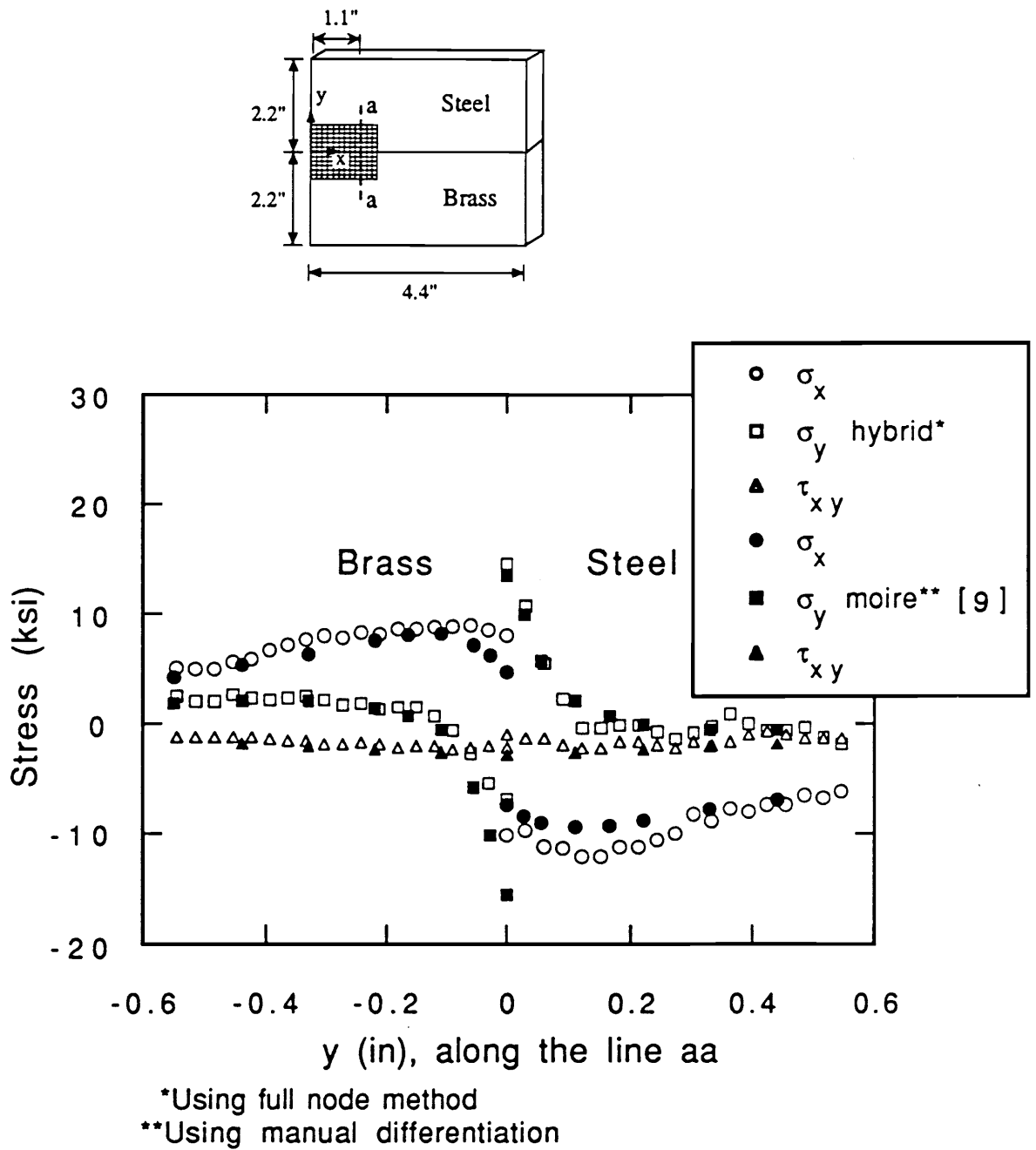


Fig. 27. Stress distributions along the line aa, obtained from the localized hybrid analysis (full-node method), are compared with those obtained from the manual differentiation of moiré fringes.

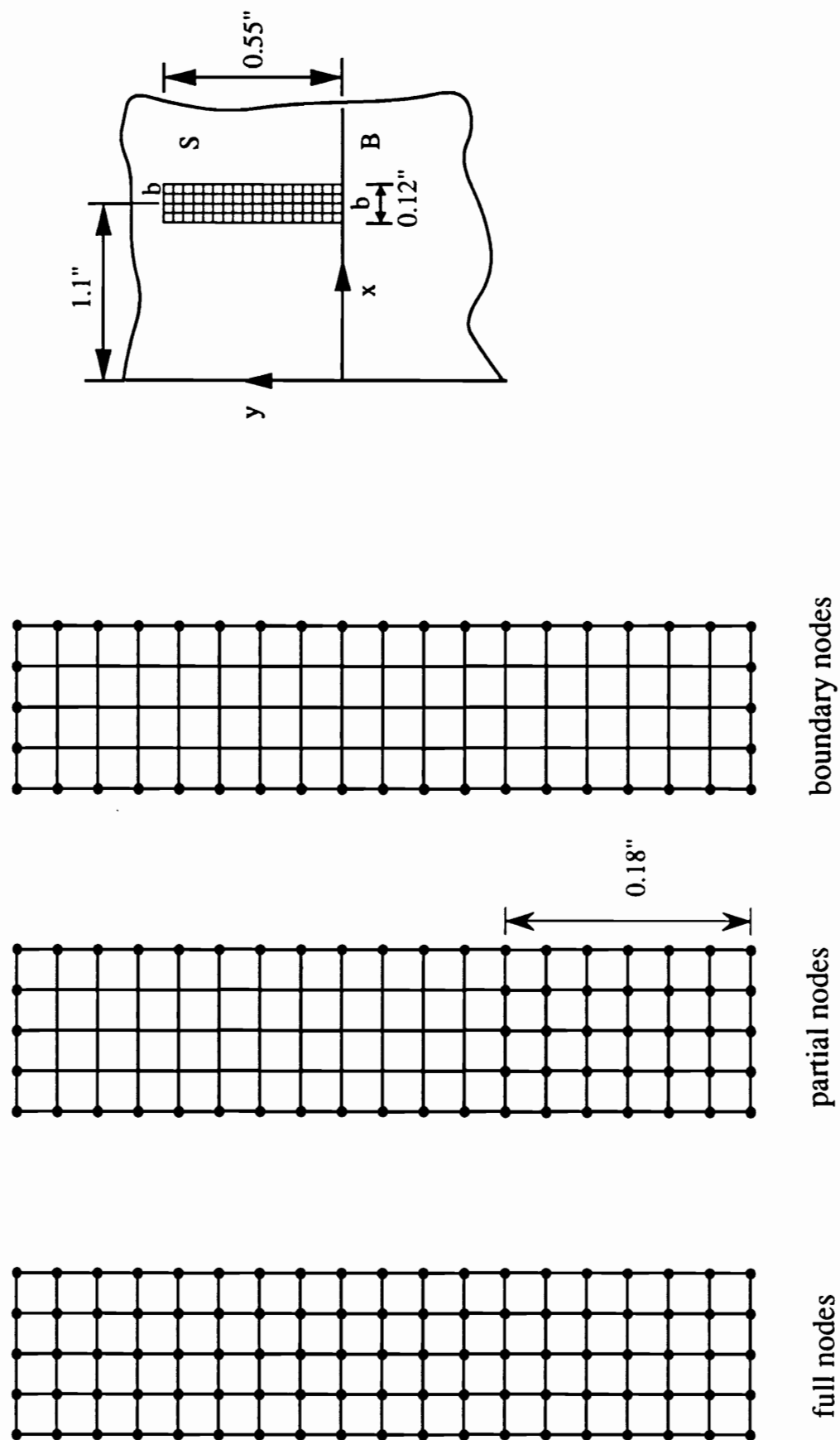


Fig. 28. Localized hybrid meshes with full nodes, partial nodes and boundary nodes which are specified displacement values obtained from moiré data.

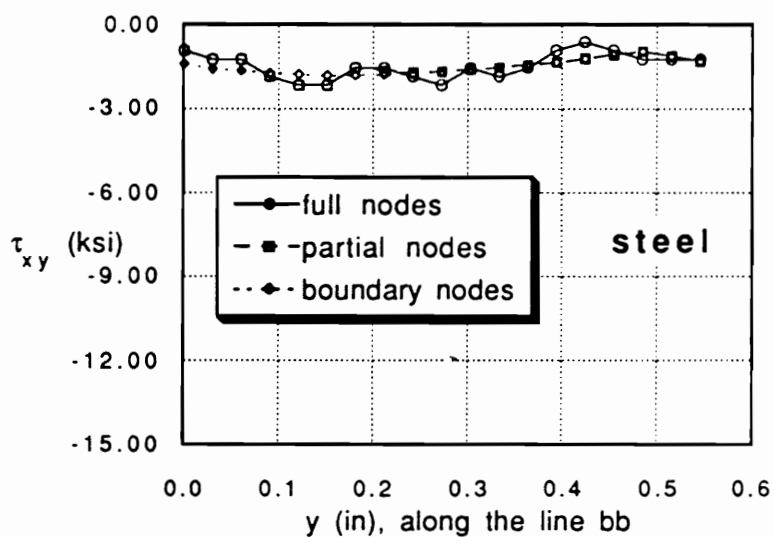
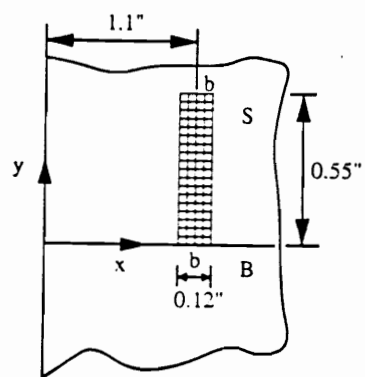
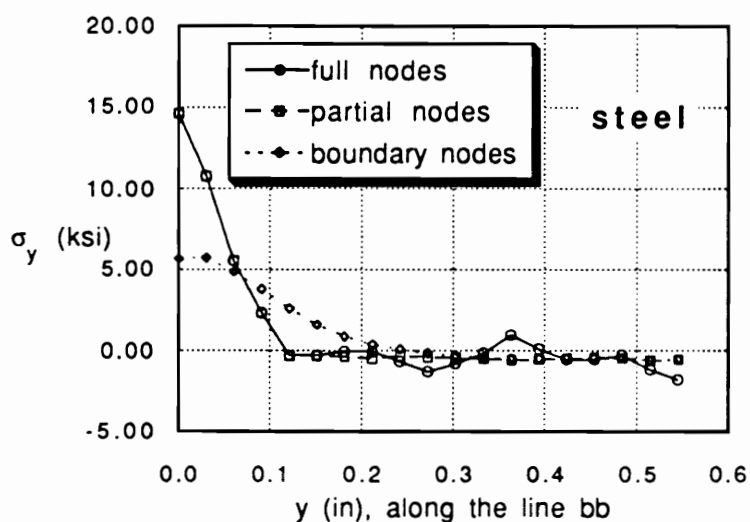
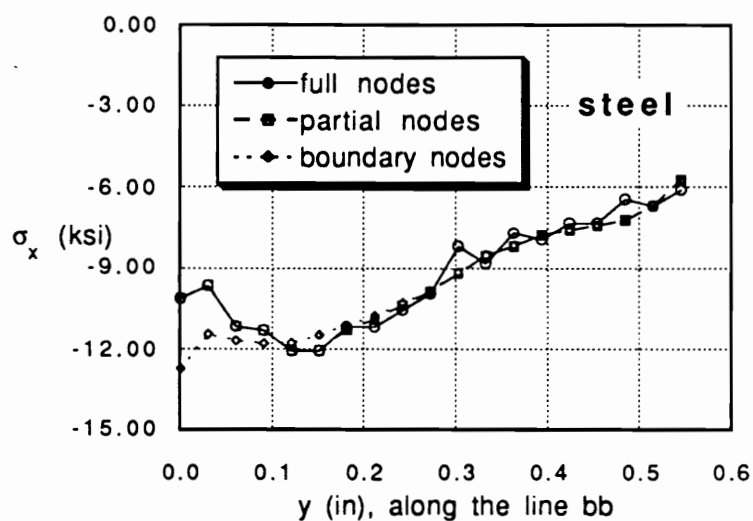


Fig. 29. Comparisons of stress components along the line bb calculated from full-node method, boundary-node method and partial-node method.

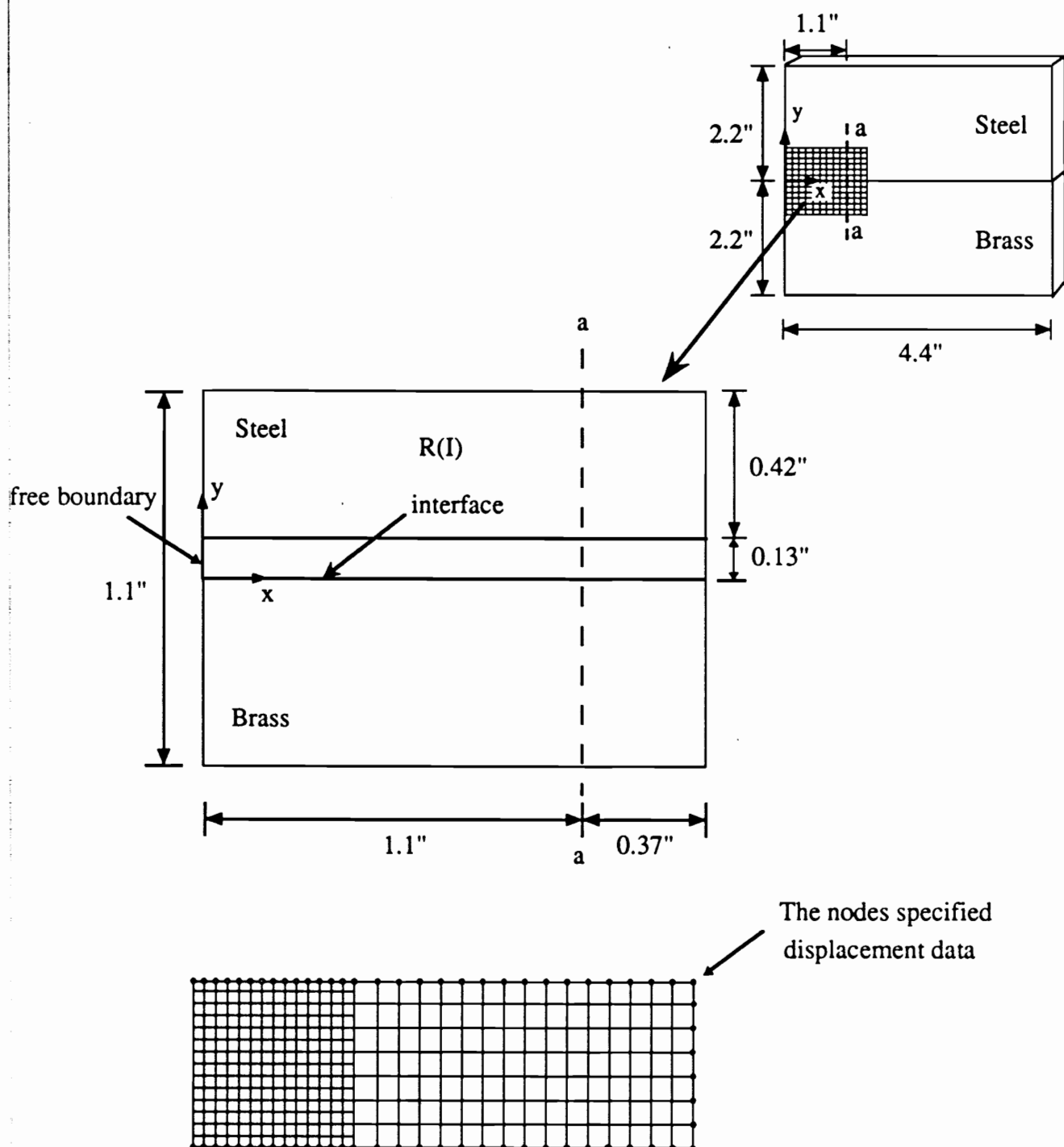


Fig. 30. Mixed-boundary method is tested in Region R(I), steel, and the used mesh is associated with the boundaries specified displacement data except a free boundary.

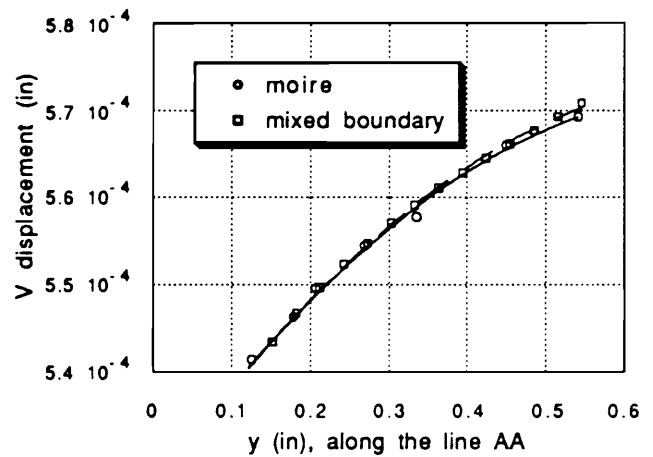
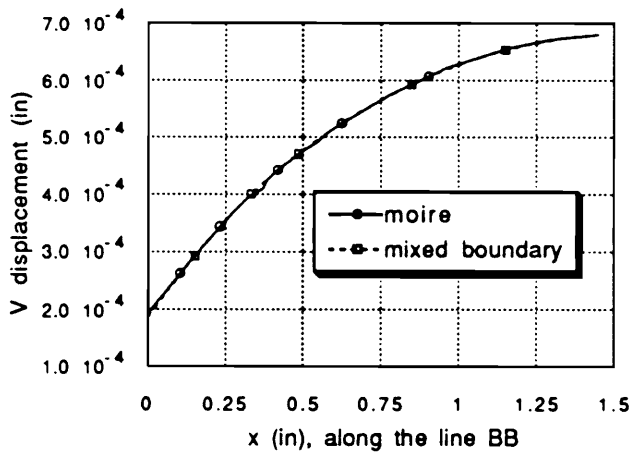
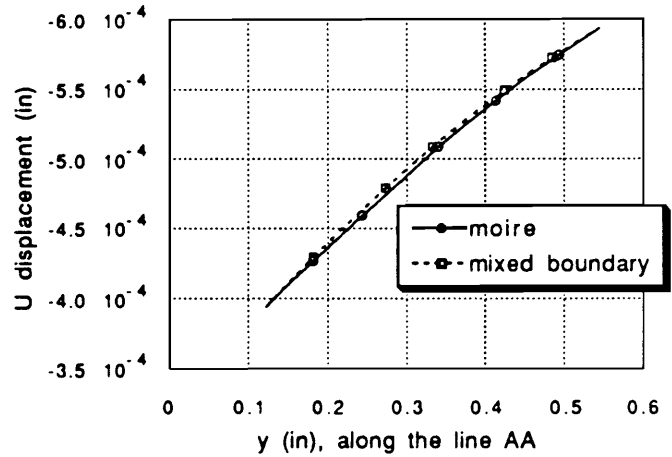
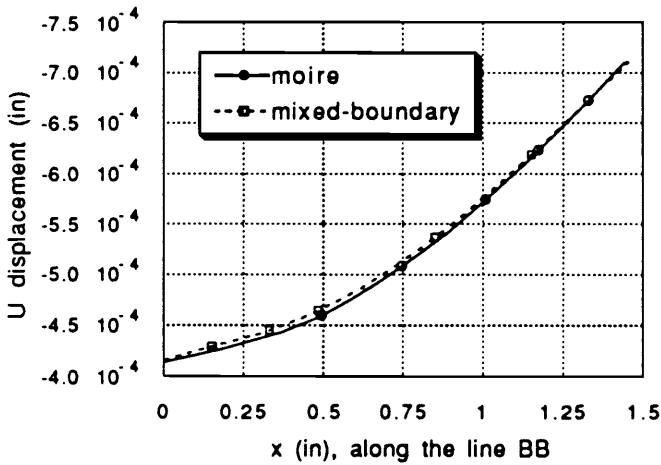
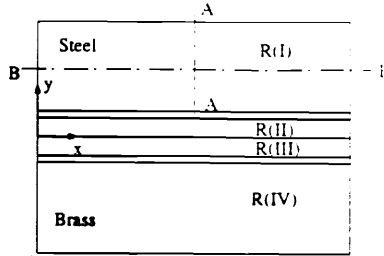
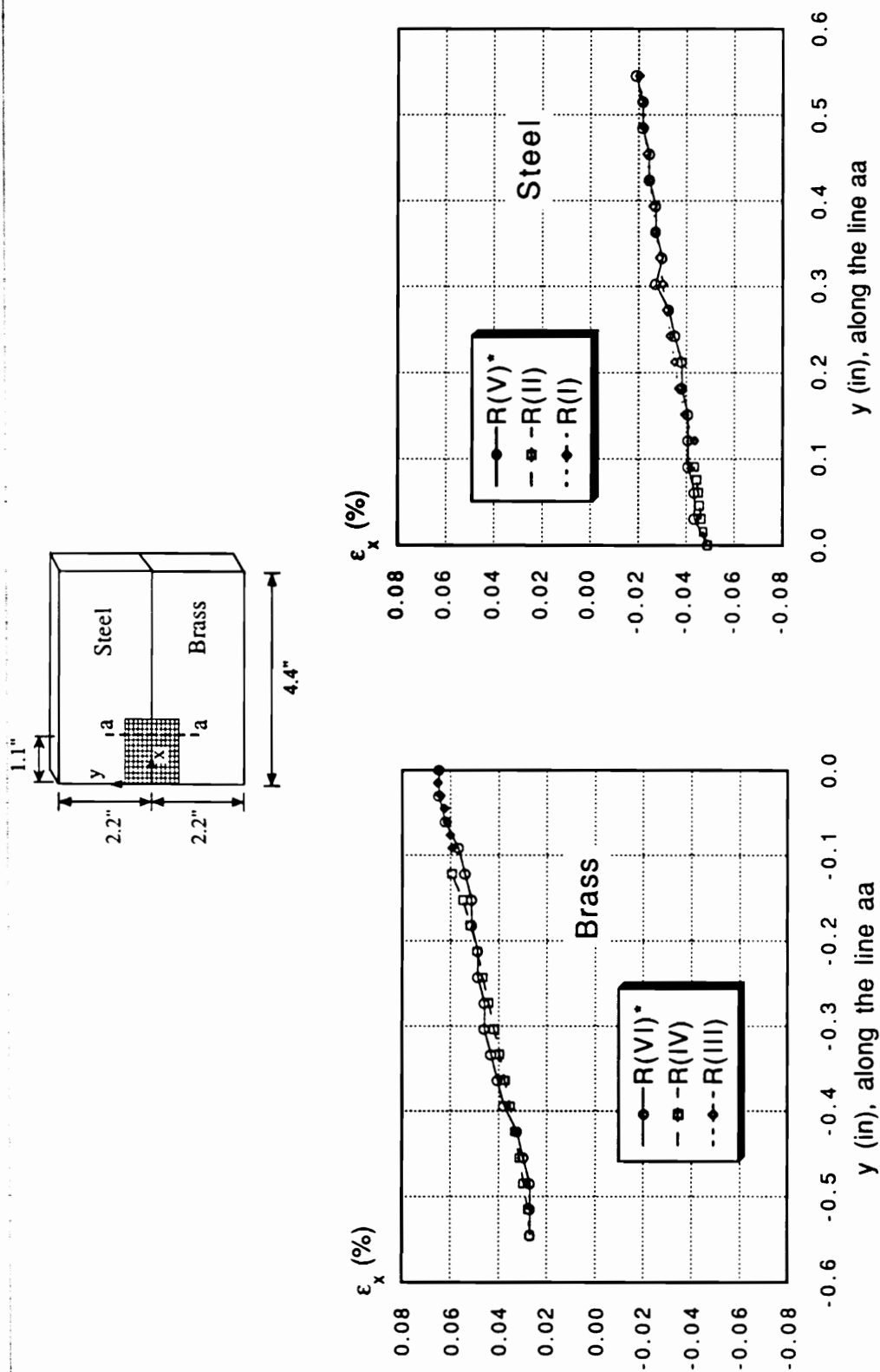
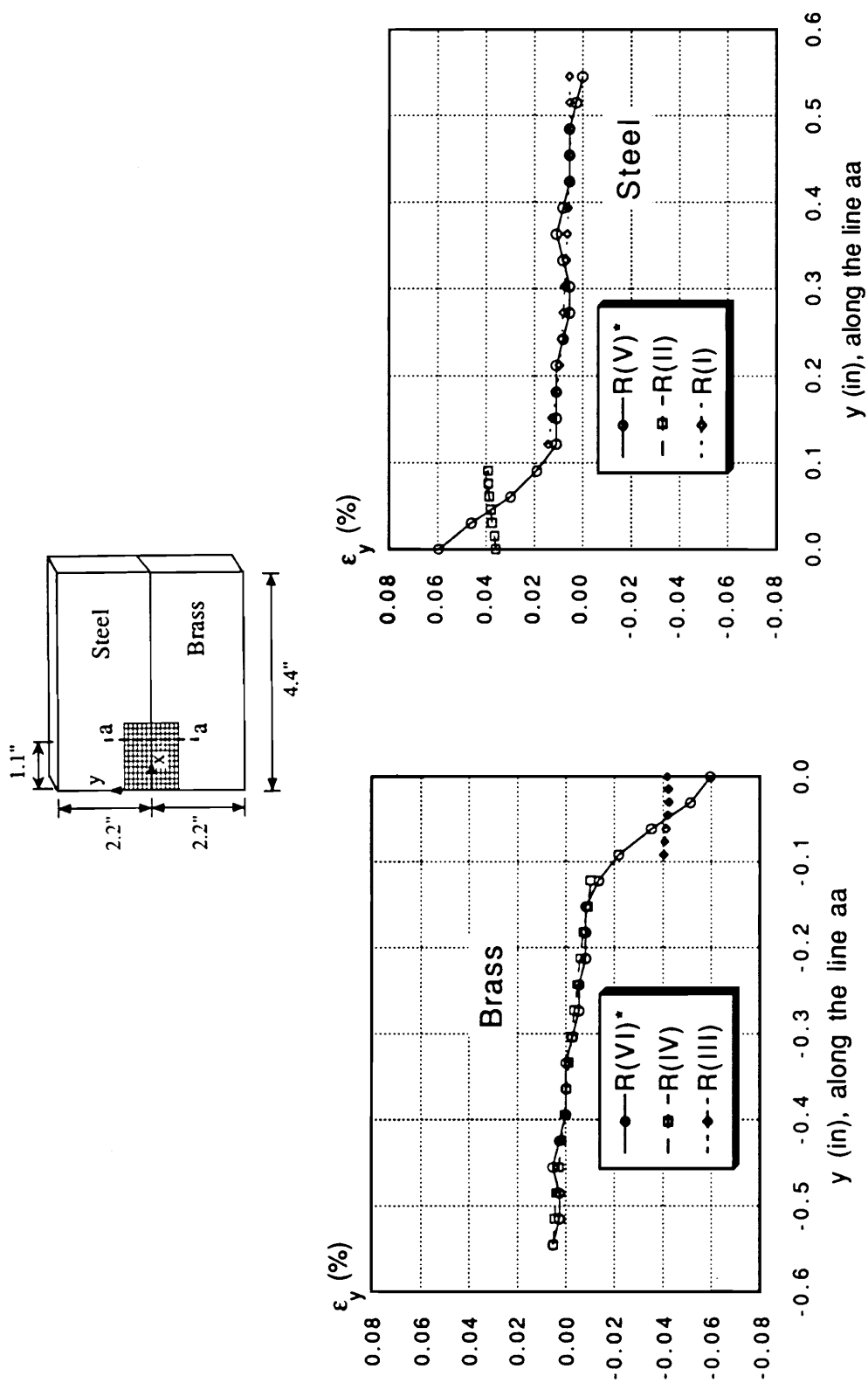


Fig. 31. Comparisons of displacement components in moire and localized hybrid(mixed-boundary method) results along two central lines (lines AA and BB) in Region R(I), steel.



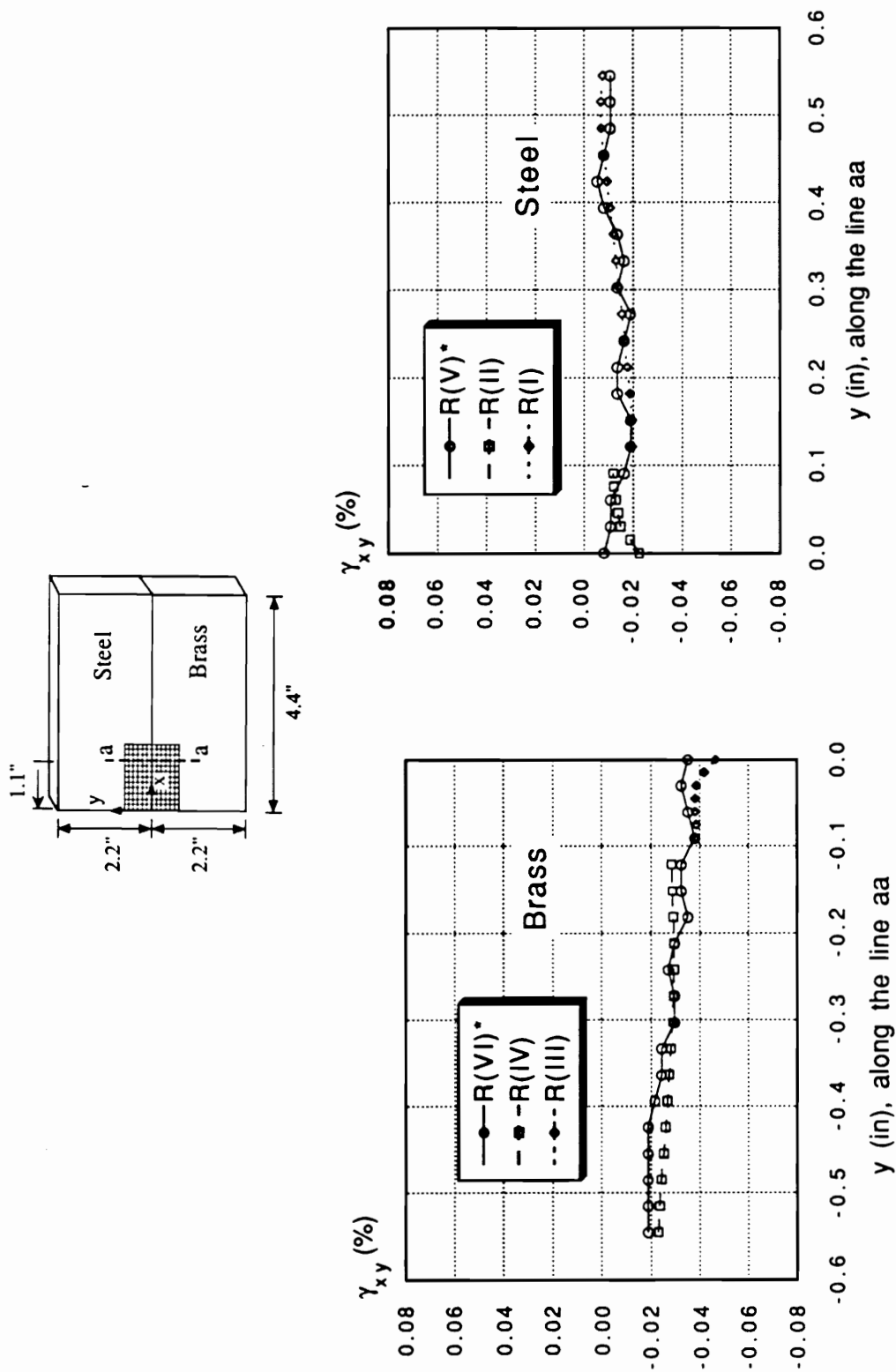
* using the full-node method

Fig. 32a. Comparisons of the axial strain, ϵ_x , distributions along the line aa, calculated from the full-node method and boundary-node method.



* using the full-node method

Fig. 32b. Comparisons of the normal strain, ϵ_y , distributions along the line aa, calculated from the full-node method and boundary-node method.



* using the full-node method

Fig. 32c. Comparisons of the shear strain, γ_{xy} , distributions along the line aa, calculated from the full-node method and boundary-node method.

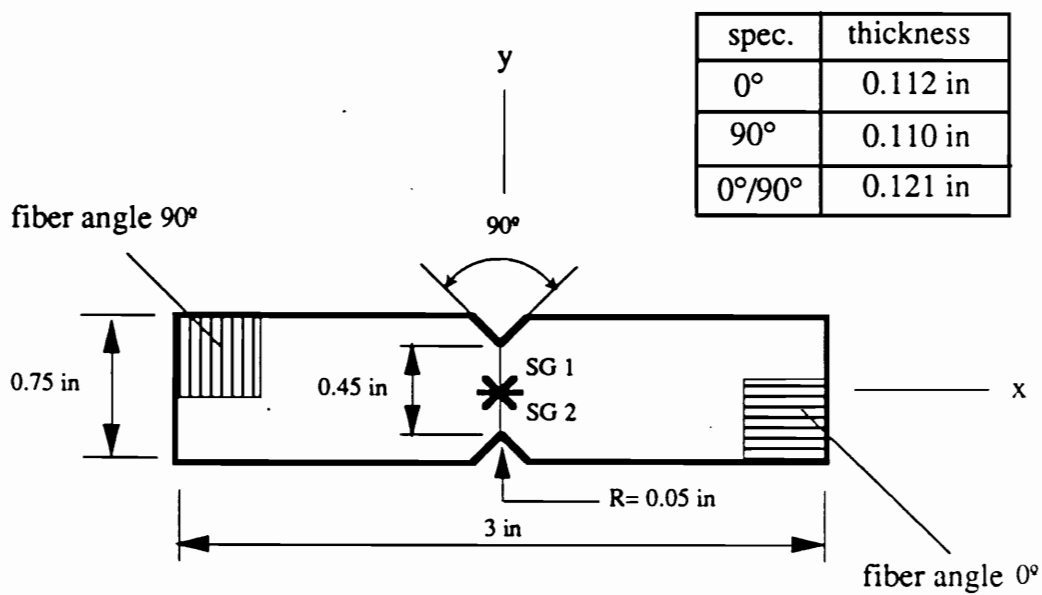


Fig. 33a. Configuration of Iosipescu specimen.

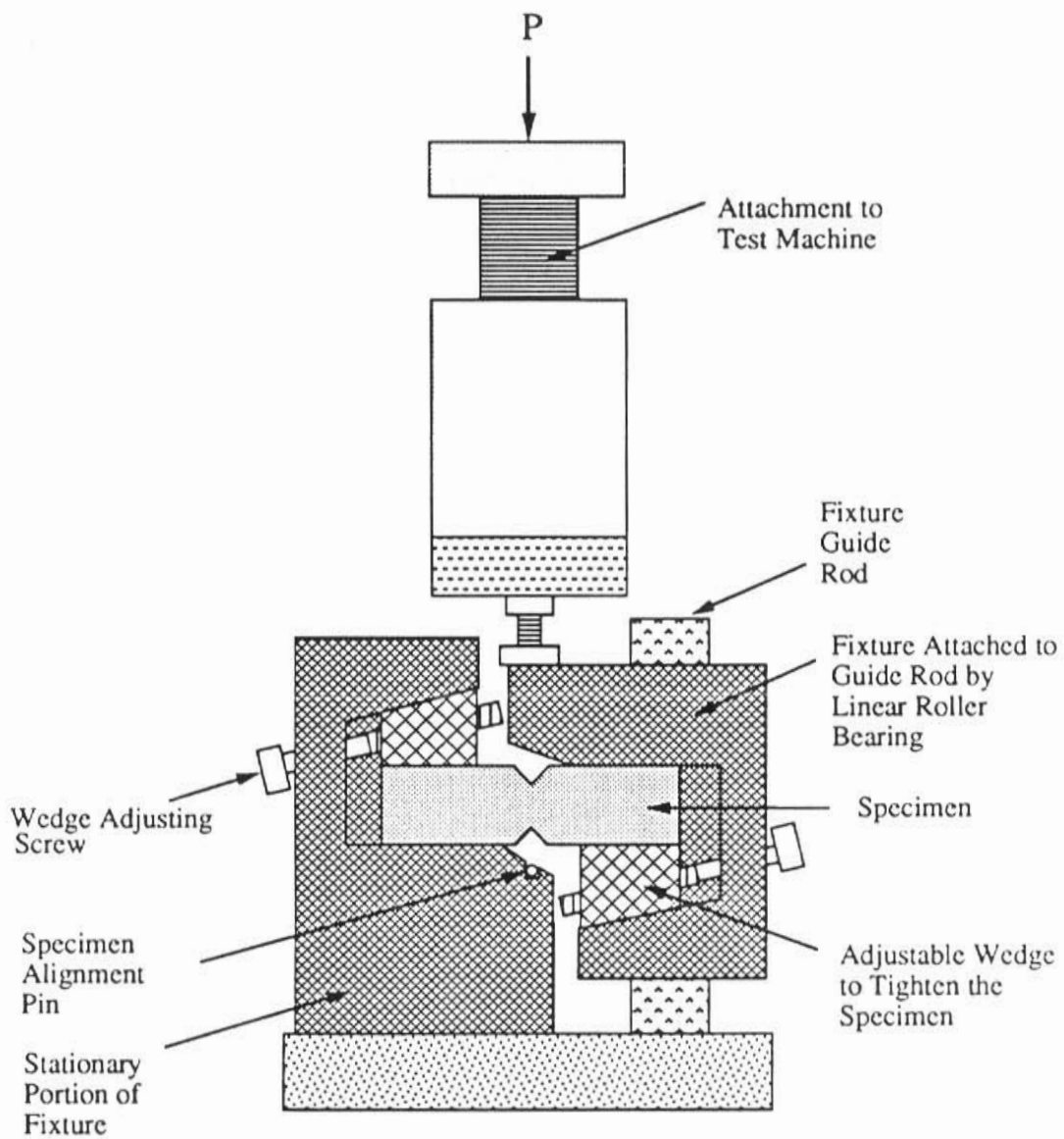


Fig. 33b. Modified Wyoming test fixture.

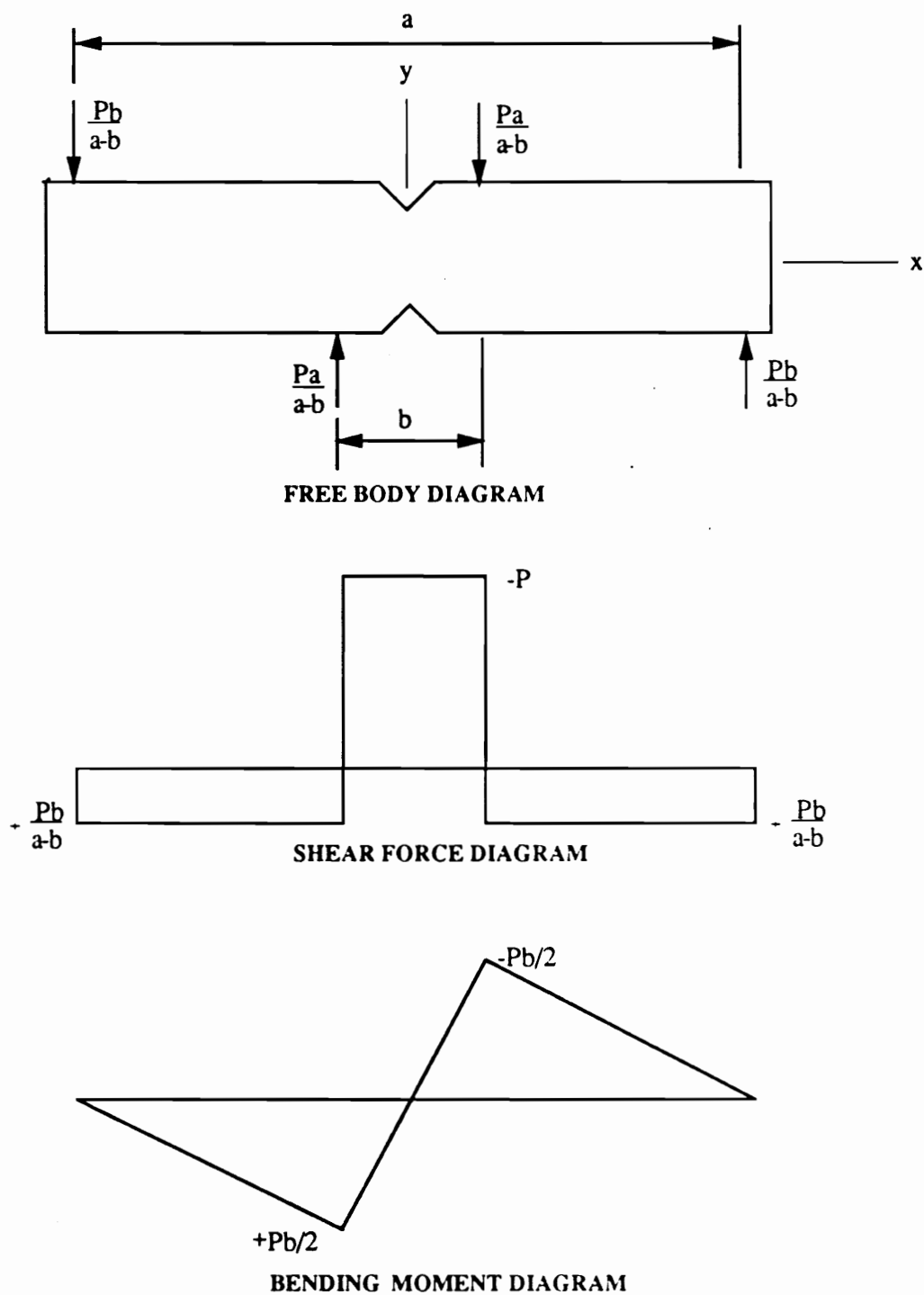
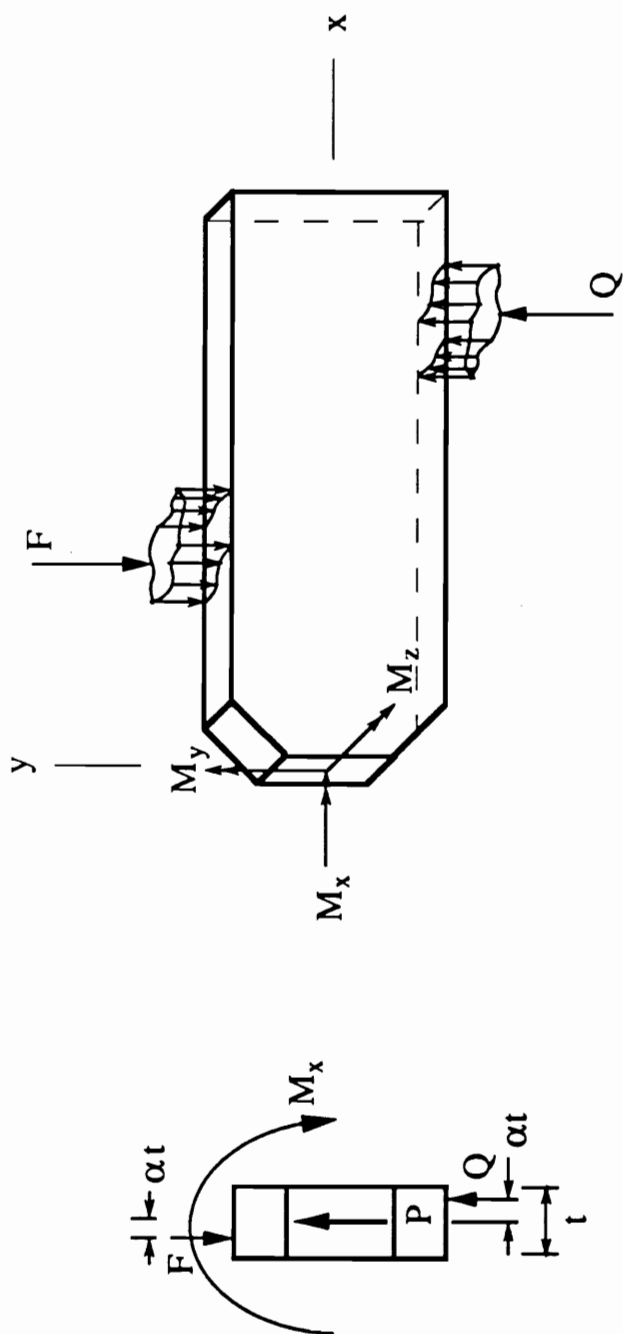
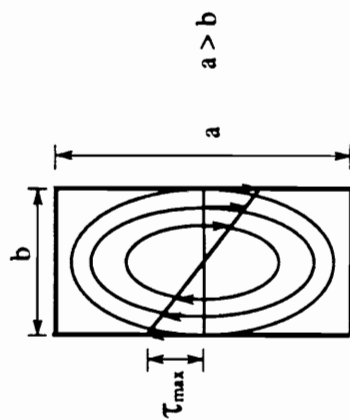


Fig. 34. Force and moment equilibrium models of the Iosipescu specimen--one dimensional model.



(a) Three-dimensional free body diagram of Iosipescu specimen



(b) The qualitative shear flow contours on the y-z plane, due to the twist M_x

Fig. 35. (a) Three-dimensional representation of the free body diagram of the Iosipescu specimen. (b) The qualitative shear flow contours on the y-z plane, due to the twist M_x

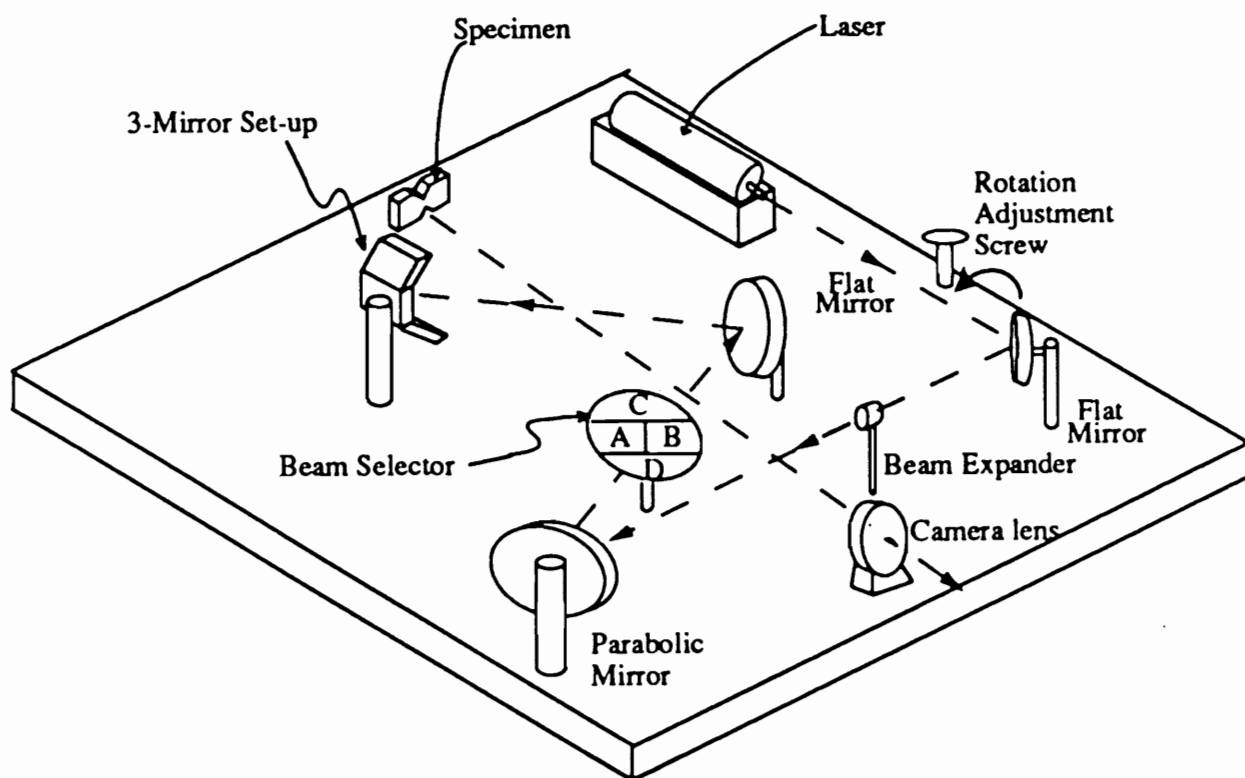


Fig. 36. Optical system used in this study.



Fig. 37a. u-field moiré fringe patterns for a 0° graphite-epoxy Iosipescu specimen at an applied load of 150 lb.

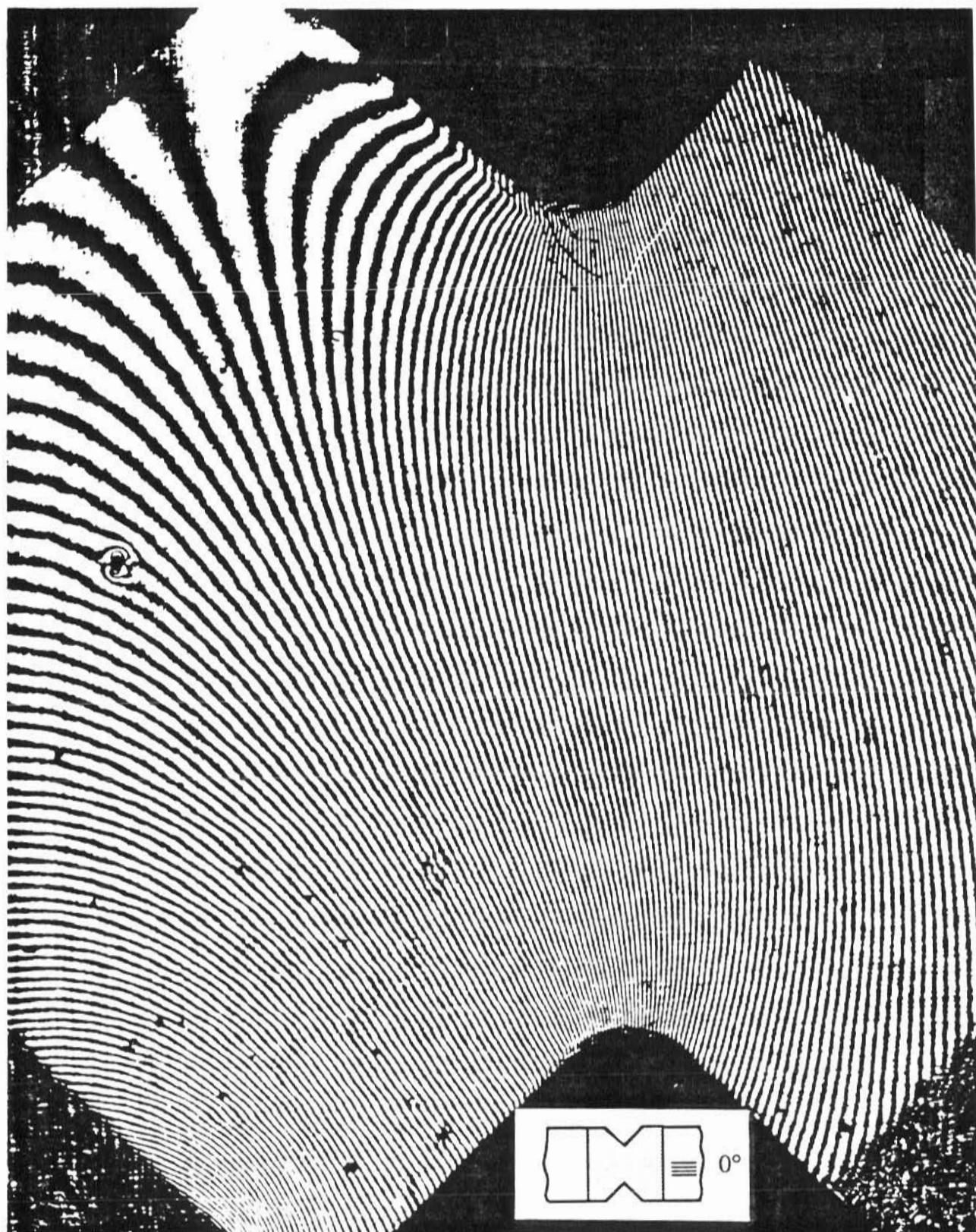


Fig. 37b. v-field moiré fringe patterns for a 0° graphite-epoxy Iosipescu specimen at an applied load of 150 lb.

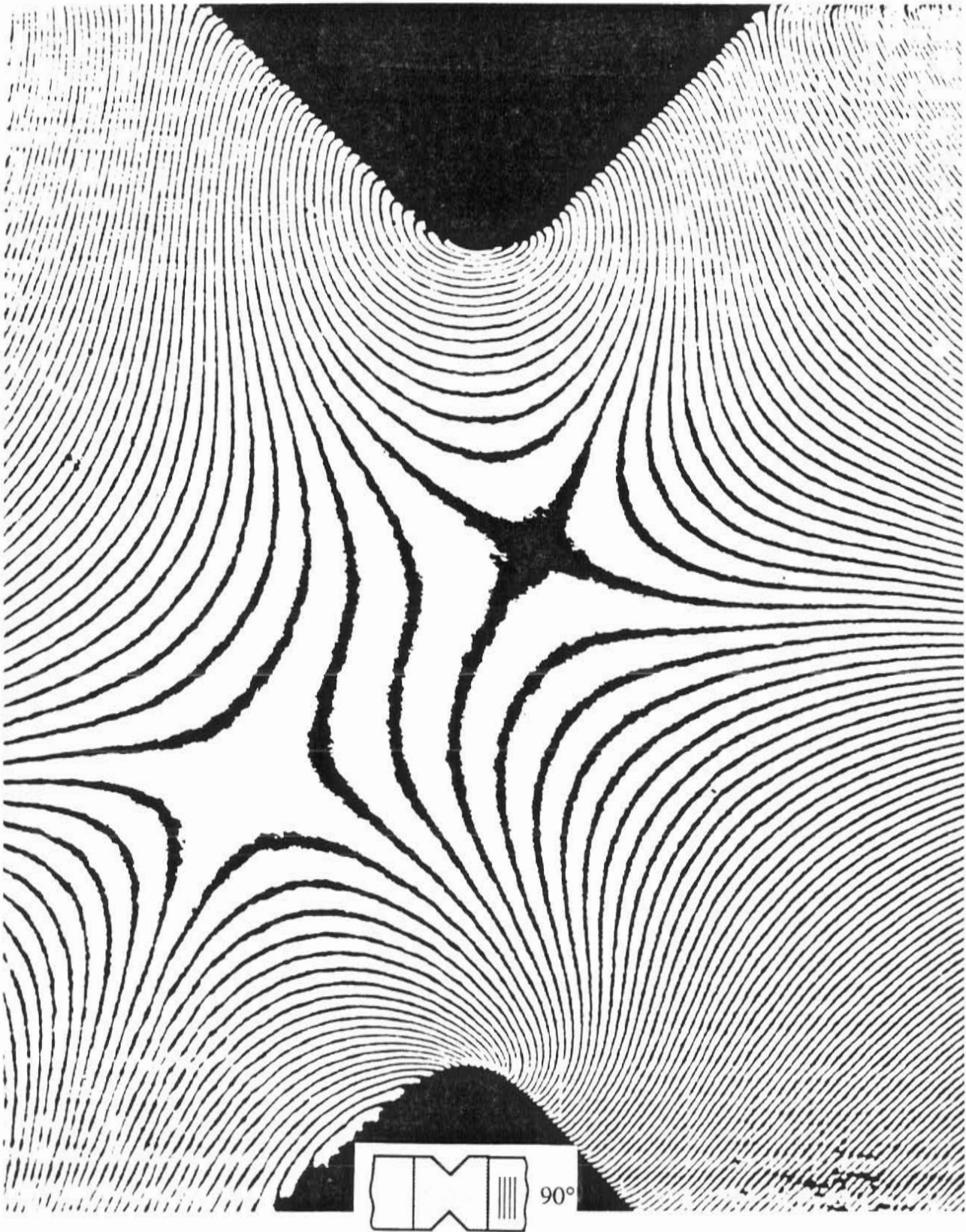


Fig. 38a. u-field moire fringe patterns for a 90° graphite-epoxy Iosipescu specimen at an applied load of 250 lb.

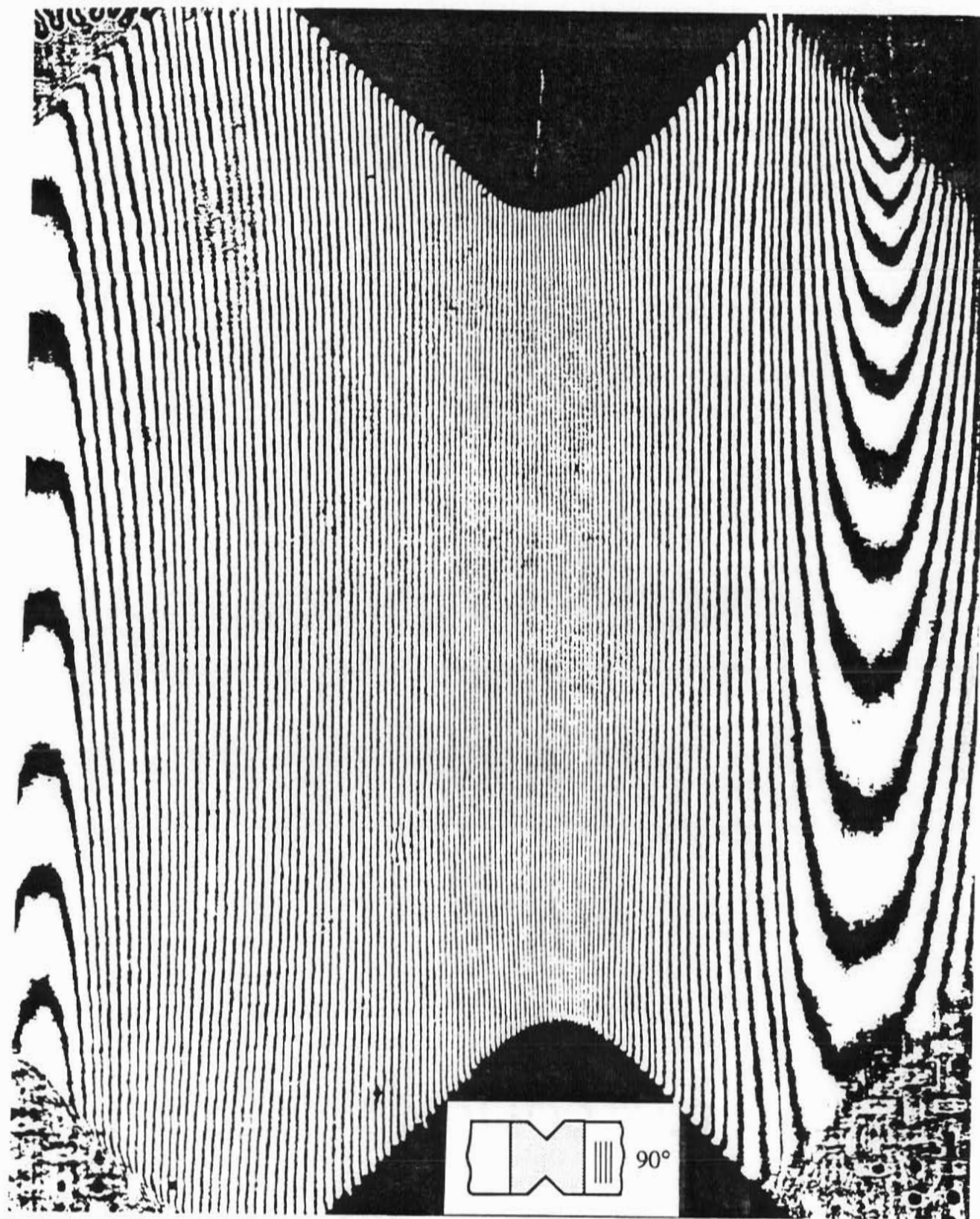


Fig. 38b. v-field moire fringe patterns for a 90° graphite-epoxy Iosipescu specimen at an applied load of 250 lb.



Fig. 39a. u-field moire fringe patterns for a $0^{\circ}/90^{\circ}$ graphite-epoxy Iosipescu specimen at an applied load of 300 lb.

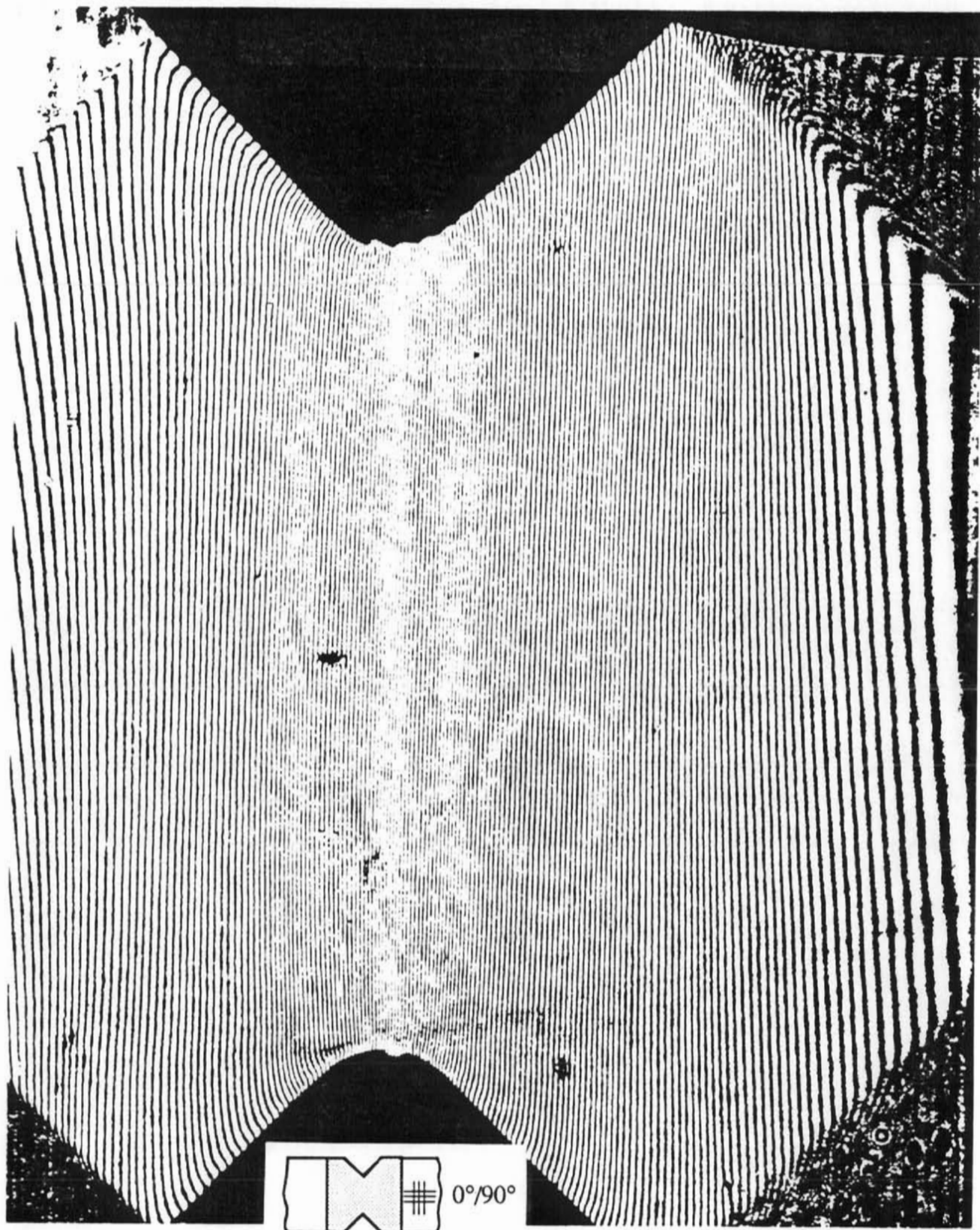
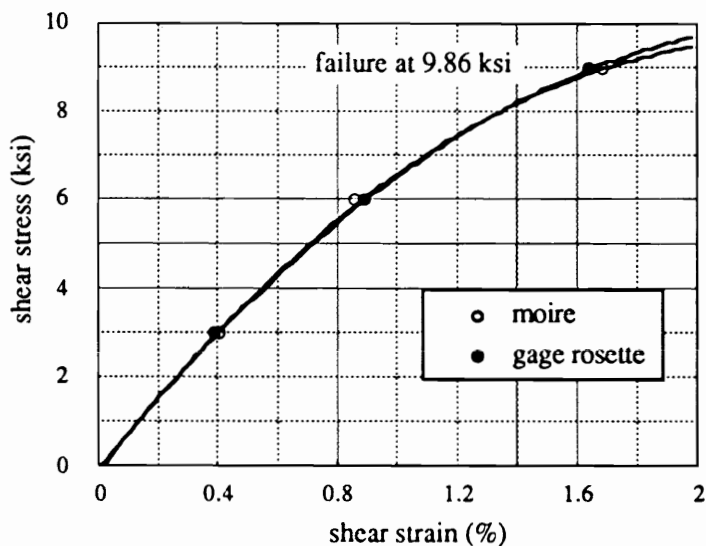
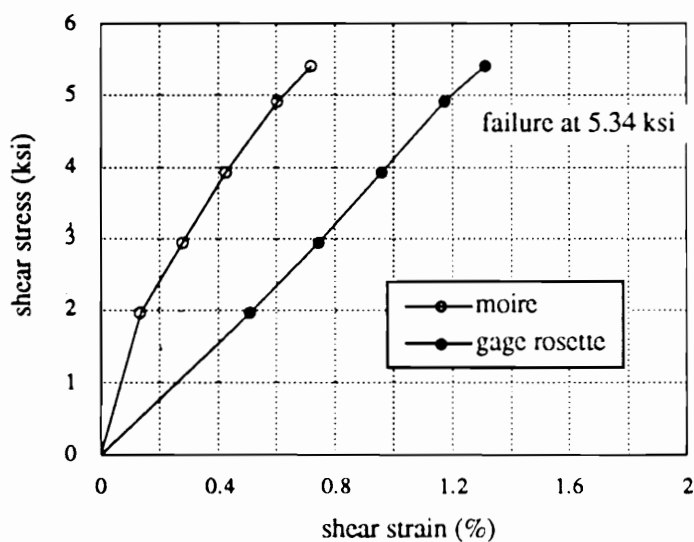


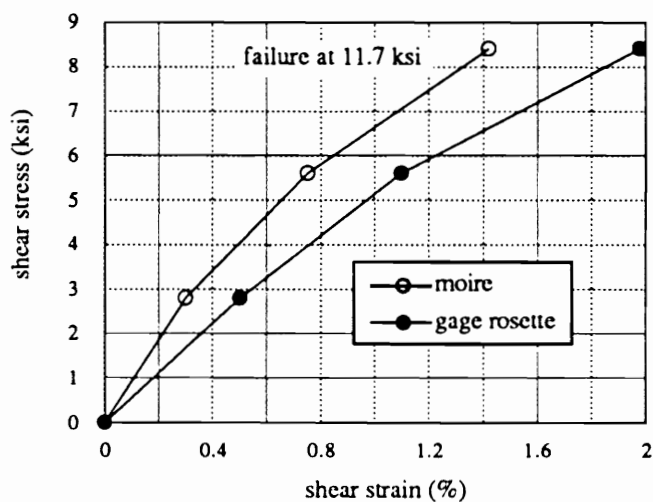
Fig. 39b. v-field moire fringe patterns for a $0^{\circ}/90^{\circ}$ graphite-epoxy Iosipescu specimen at an applied load of 300 lb.



(a)

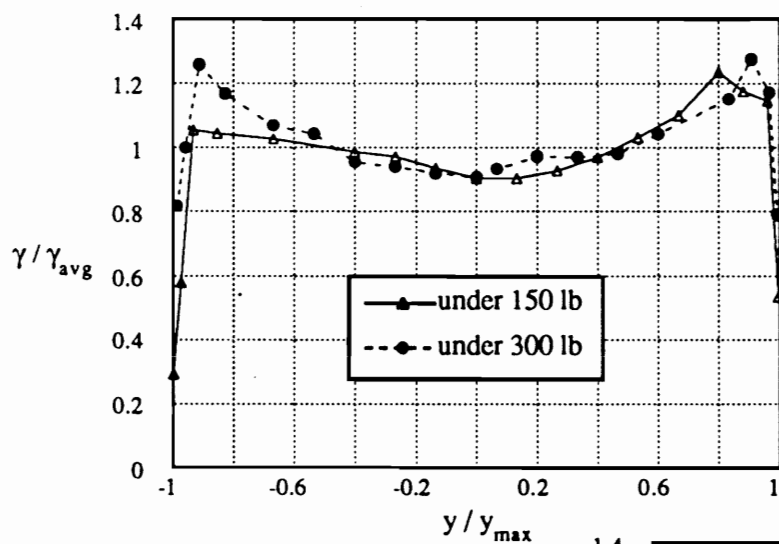


(b)

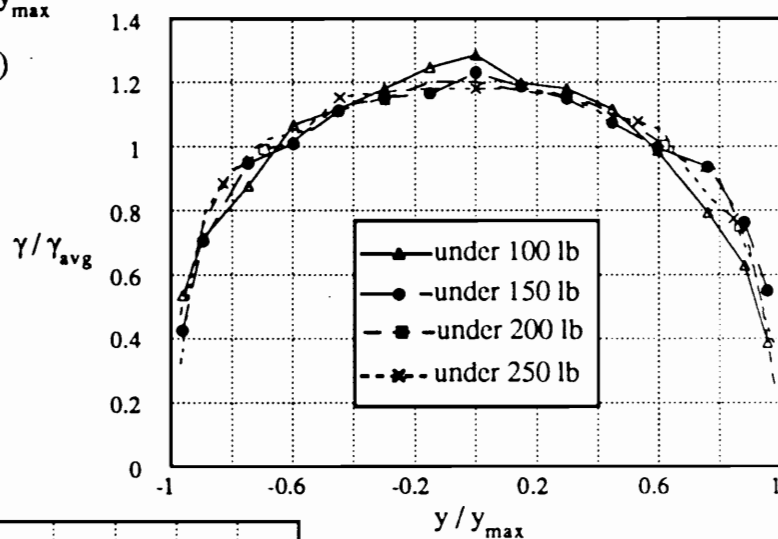


(c)

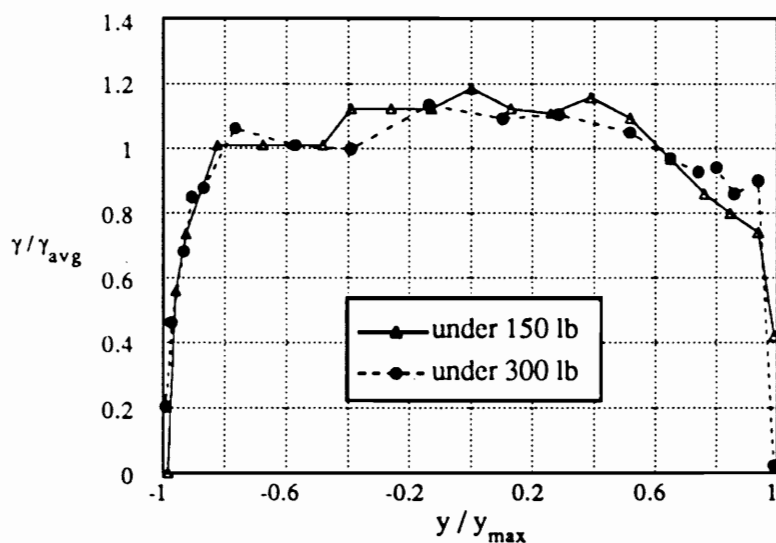
Fig. 40. Shear stress-strain data obtained from moire (front face) and gages (back face) for (a) 0° , (b) 90° , (c) $0^\circ/90^\circ$ graphite-epoxy Iosipescu specimens.



(a)



(b)



(c)

Fig. 41. Normalized shear strain distributions along the line connecting the notch tips, obtained from manual data reduction of moire fringe patterns for (a) 0°, (b) 90°, (c) 0°/90° graphite-epoxy Iosipescu specimens.

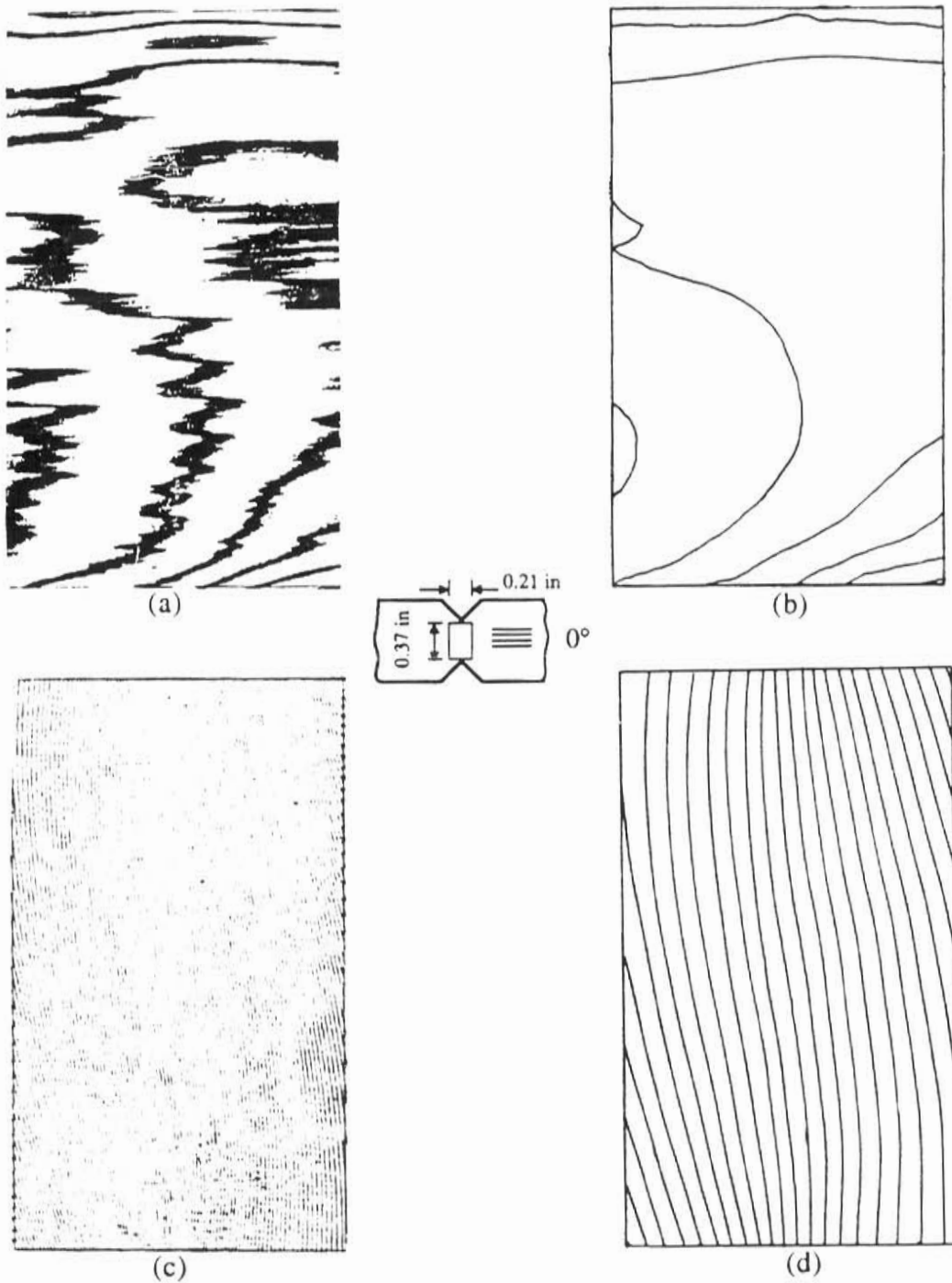


Fig. 42. (a) u-field moiré fringe pattern, (b) reproduced u-field from the localized hybrid analysis, (c) v-field moiré fringe pattern, (d) reproduced v-field (showing every 5th fringe) from the localized hybrid analysis of a 0° specimen at an applied load of 300 lb.

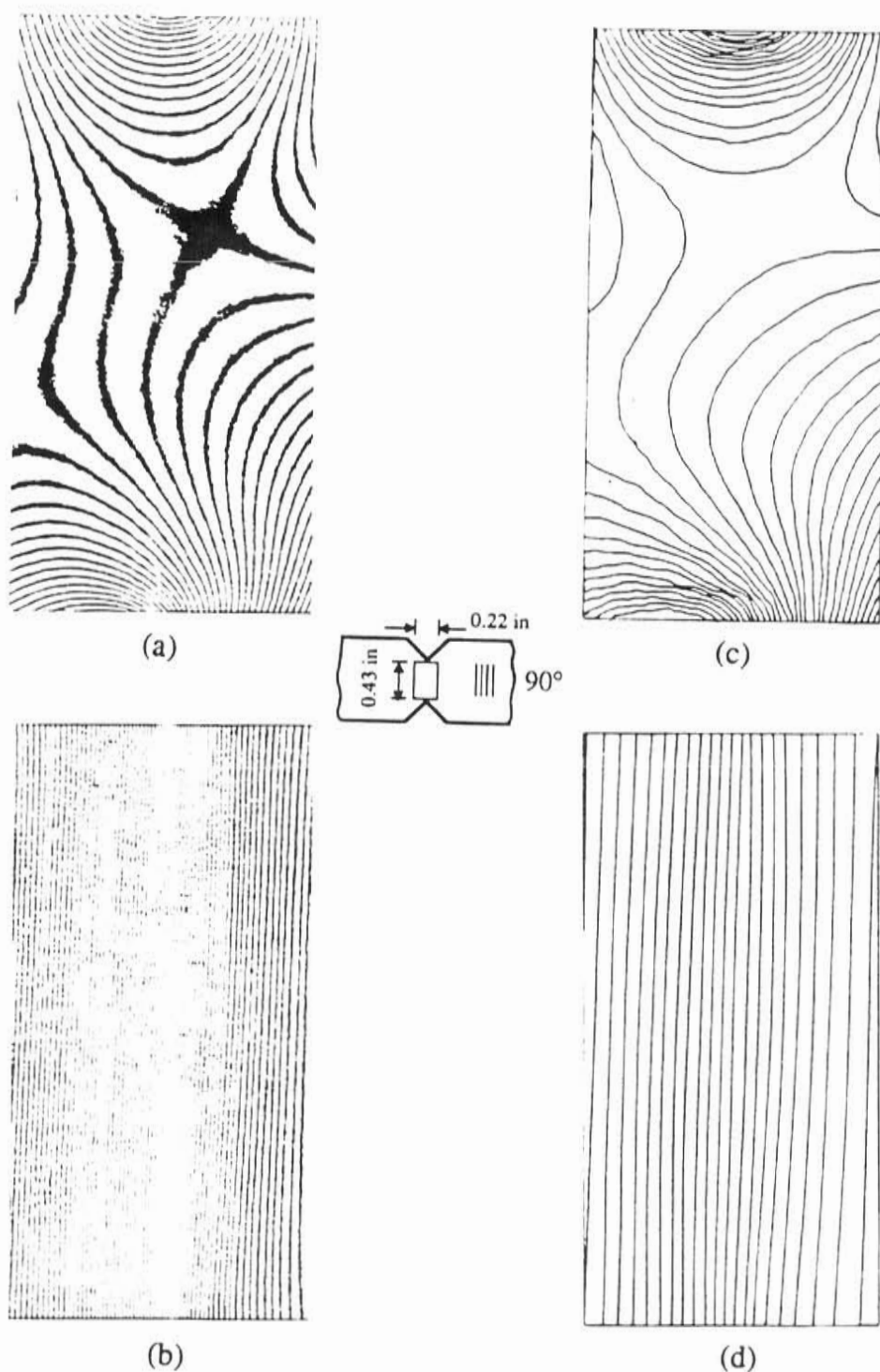


Fig. 43. Moiré fringe patterns for (a) u-field, (b) v-field. Reproduced fringe patterns for (c) u-field, (d) v-field (showing every third fringe) from the localized hybrid analysis. The data correspond to the 90° specimen at an applied load of 250 lb.

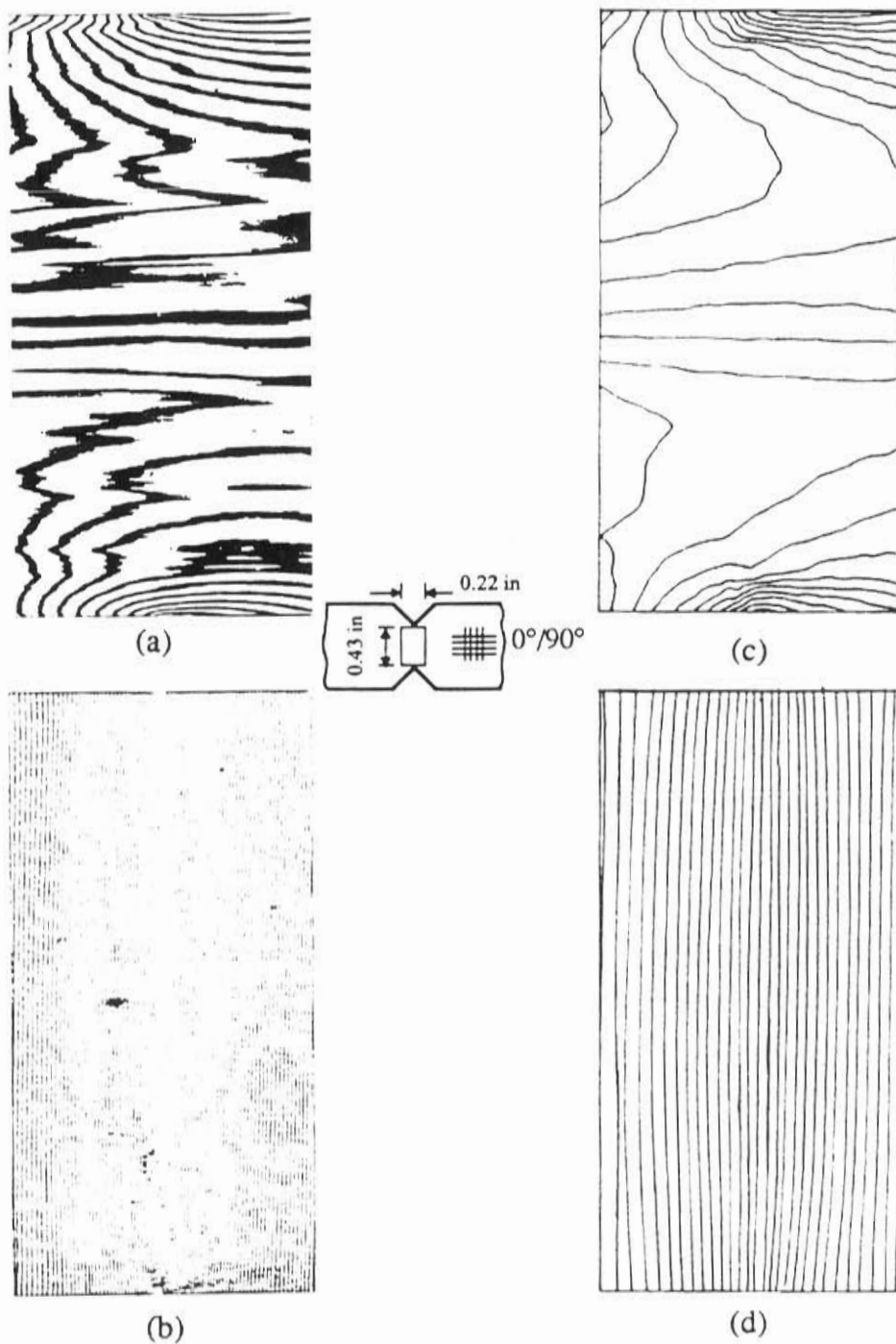


Fig.44. Moiré fringe patterns for (a) u-field, (b) v-field. Reproduced fringe patterns for (c) u-field, (d) v-field (showing every third fringe) from the localized hybrid analysis. The data correspond to the $0^\circ/90^\circ$ specimen at an applied load of 300 lb.

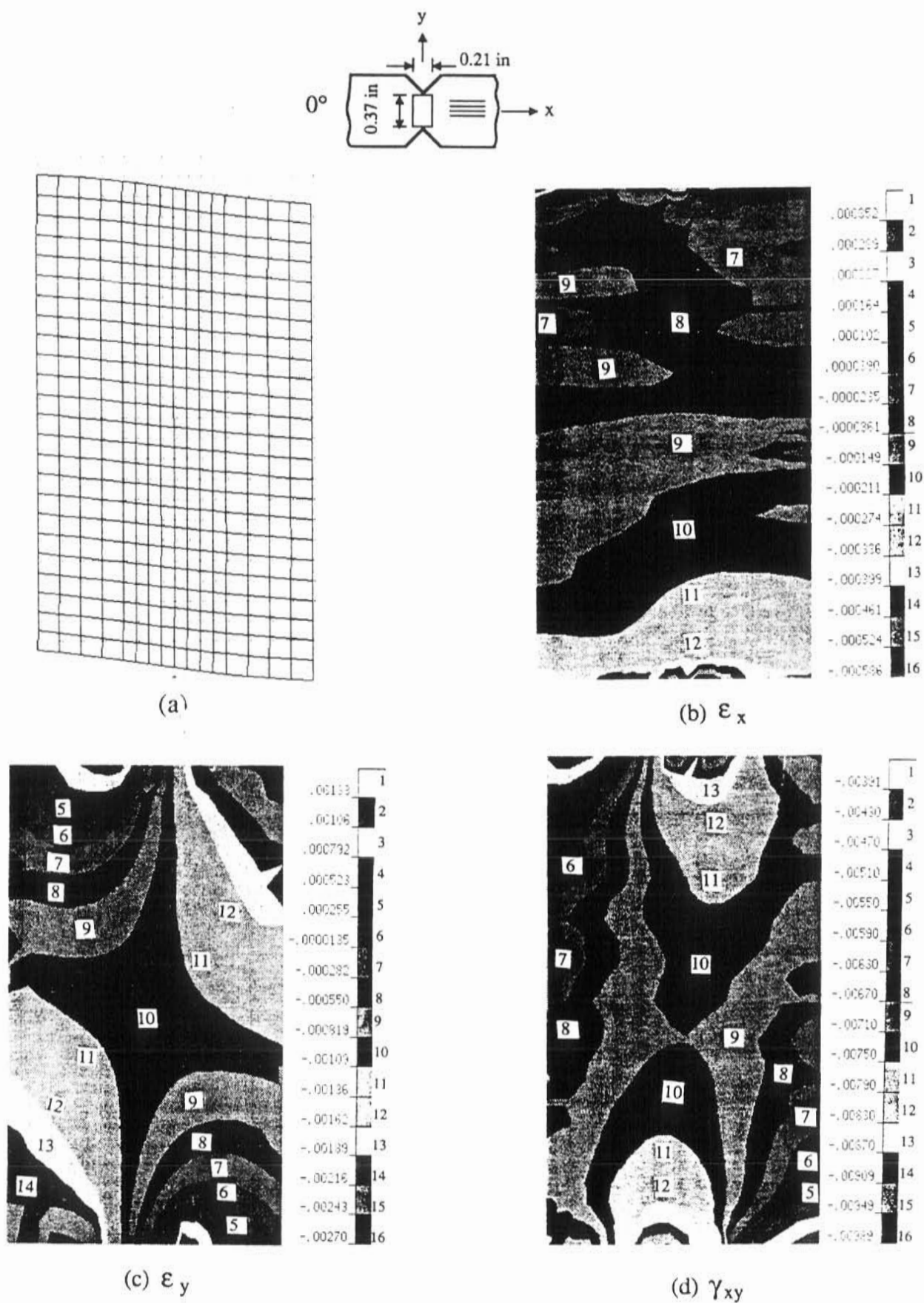


Fig.45. (a) Deformed shape, contours of (b) ϵ_x , (c) ϵ_y , (d) γ_{xy} obtained from a localized hybrid analysis of a 0° specimen at an applied load of 300 lb.

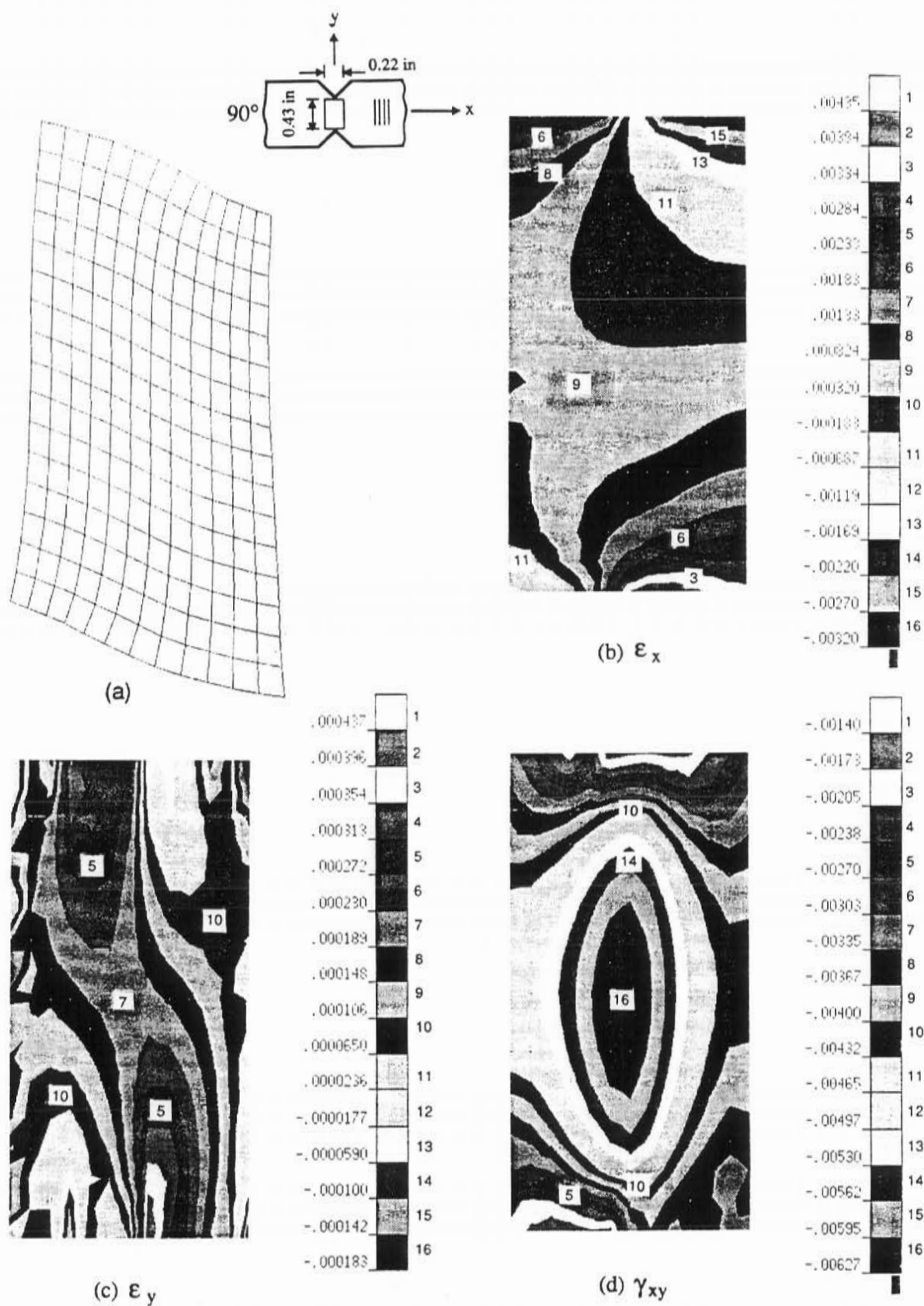


Fig.46. (a) Deformed shape, contours of (b) ϵ_x , (c) ϵ_y , (d) γ_{xy} obtained from a localized hybrid analysis of a 90° specimen at an applied load of 250 lb

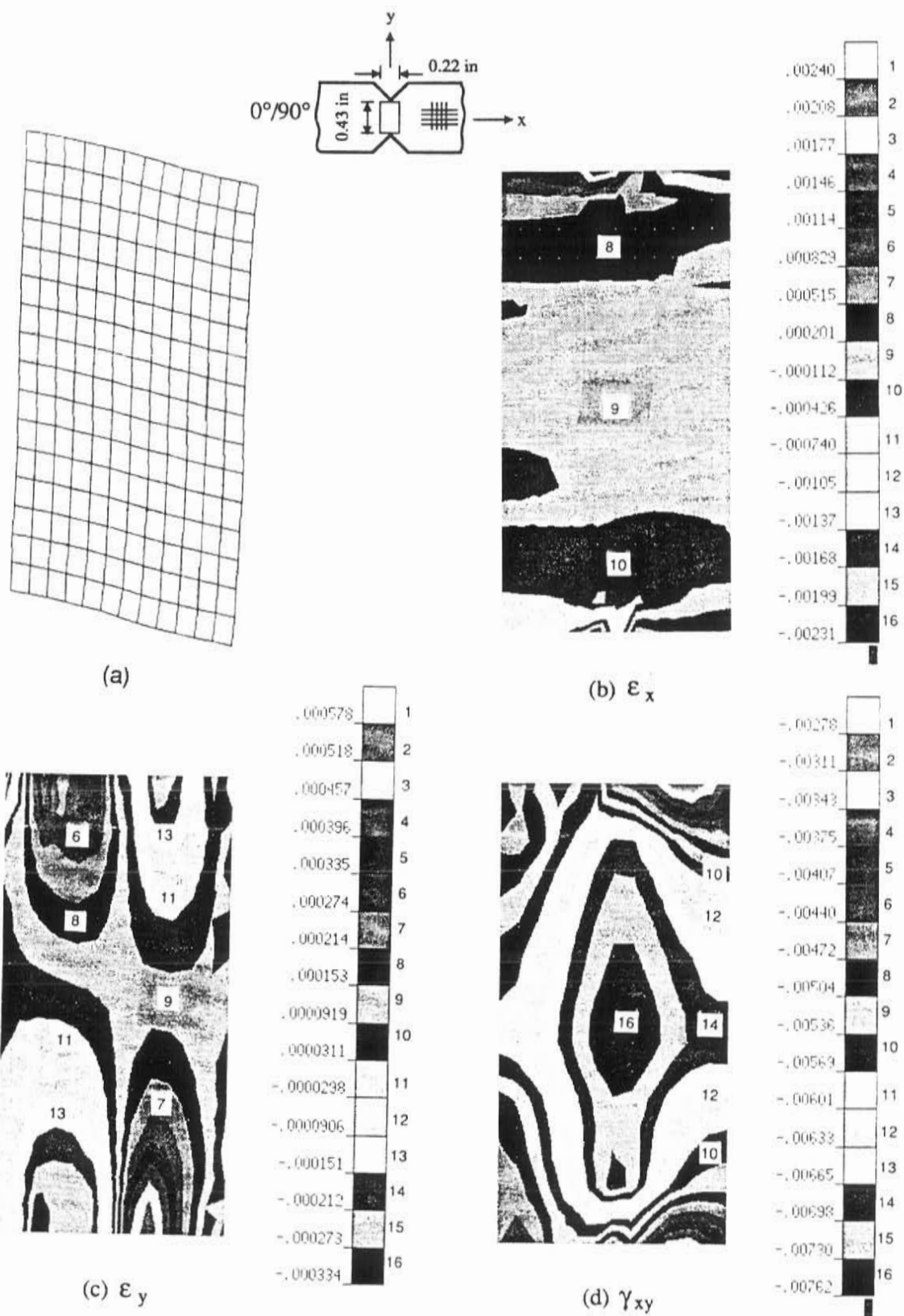


Fig.47. (a) Deformed shape, contours of (b) ϵ_x , (c) ϵ_y , (d) γ_{xy} obtained from a localized hybrid analysis of a 0°/90° specimen at an applied load of 300 lb.

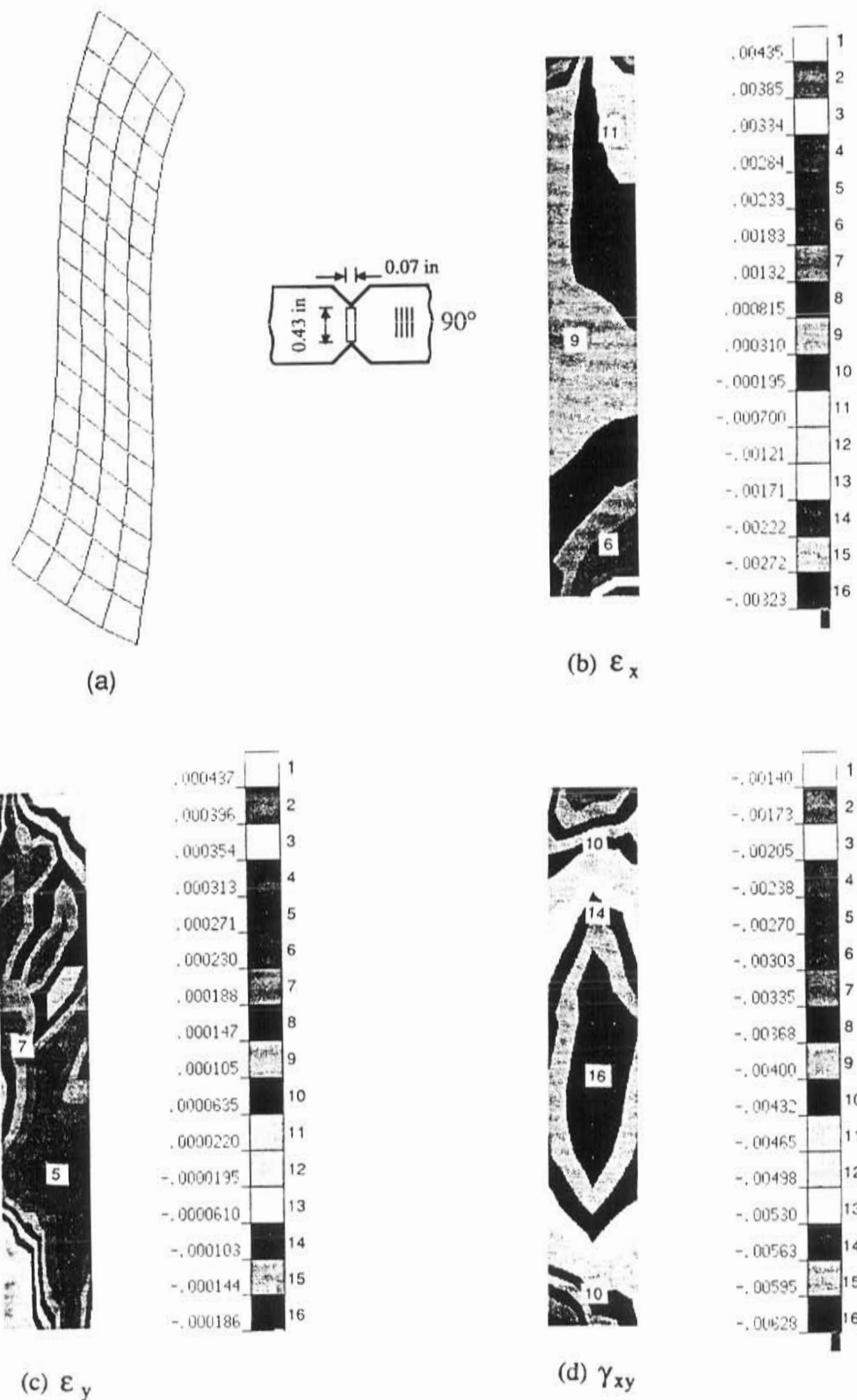


Fig.48. (a) Deformed shape, contours of (b) ϵ_x , (c) ϵ_y , (d) γ_{xy} obtained from the full-node method of a localized hybrid analysis of a 90° specimen at an applied load of 250 lb.

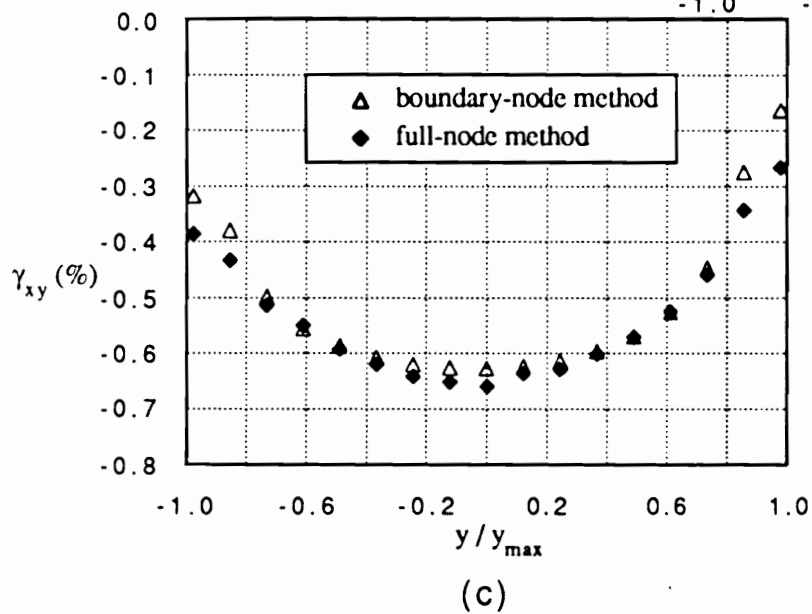
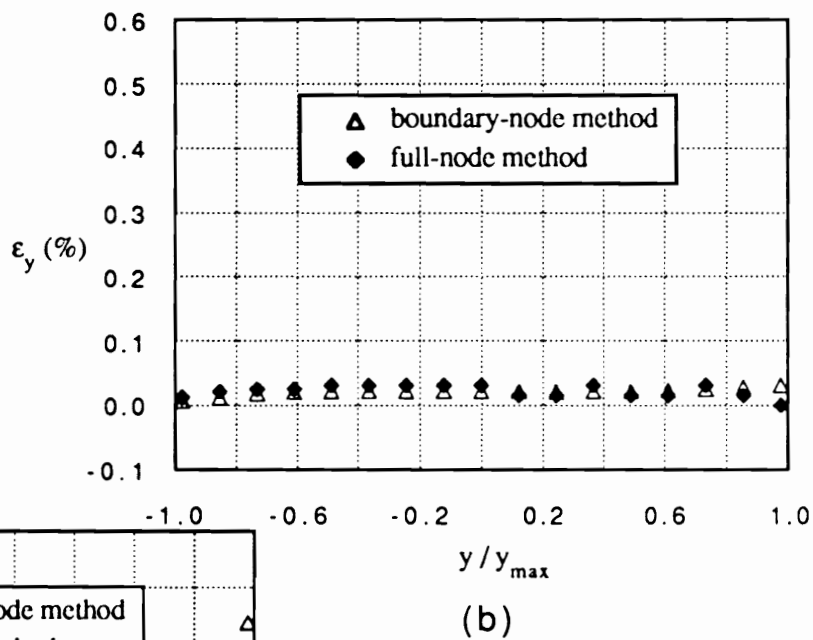
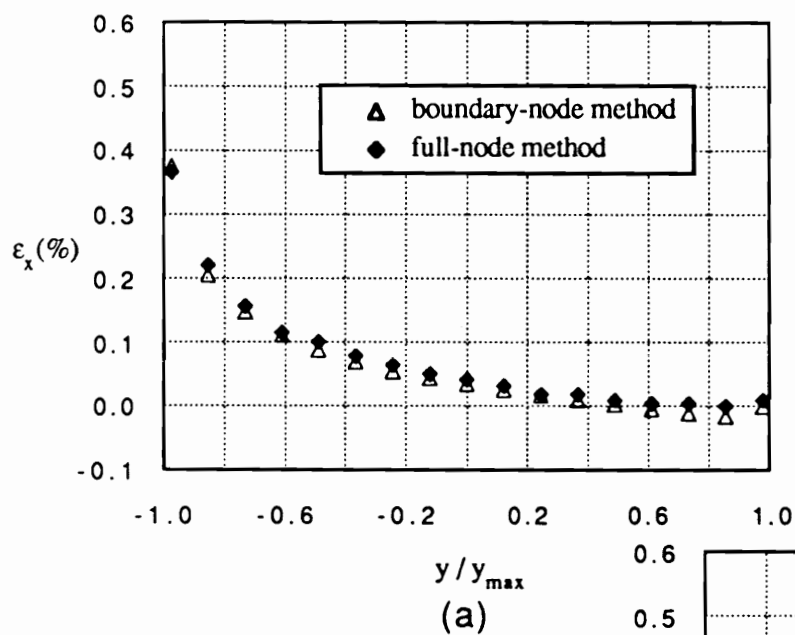


Fig.49. Comparisons of the strain (a) ϵ_x , (b) ϵ_y , (c) γ_{xy} data along the line connecting the notch tips, obtained from the full-node and boundary-node methods.

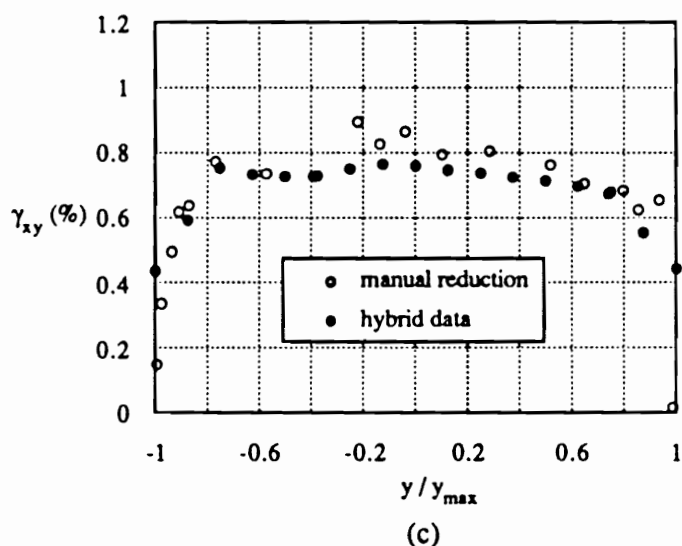
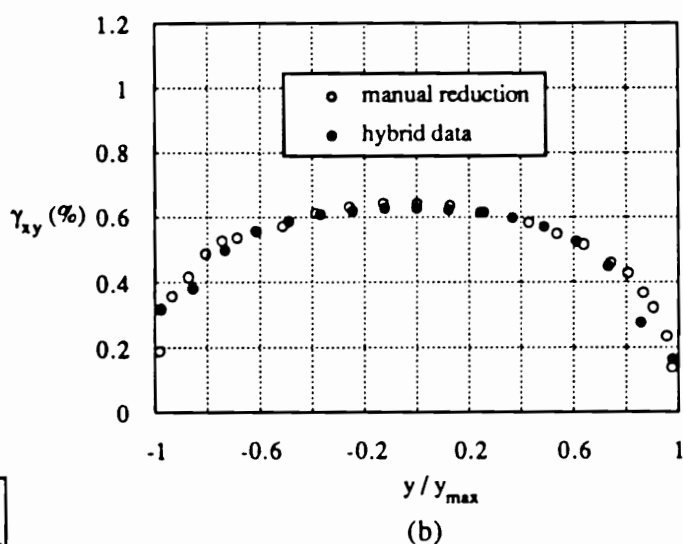
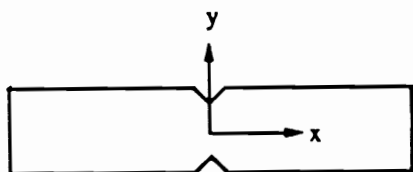
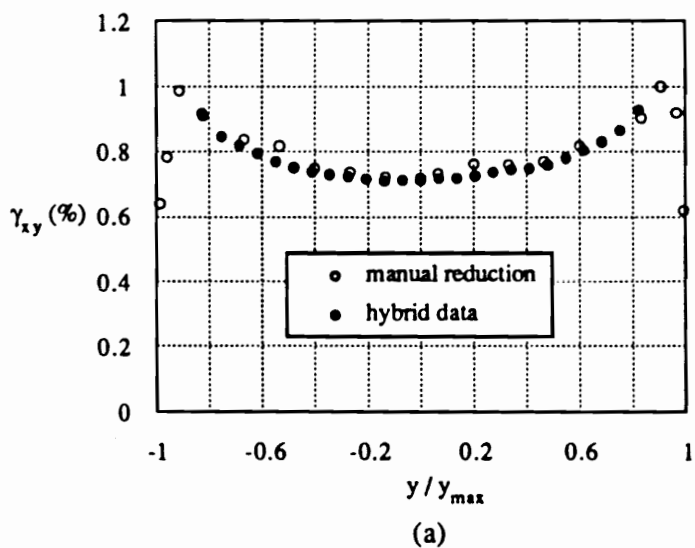


Fig.50. Comparisons of shear strain data along the line connecting the notch tips, obtained from manual data reduction and the localized hybrid analysis for (a) 0° (at an applied load of 300 lb), (b) 90° (at an applied load of 250 lb), (c) $0^\circ/90^\circ$ (at an applied load of 300 lb) graphite-epoxy Iosipescu specimens.

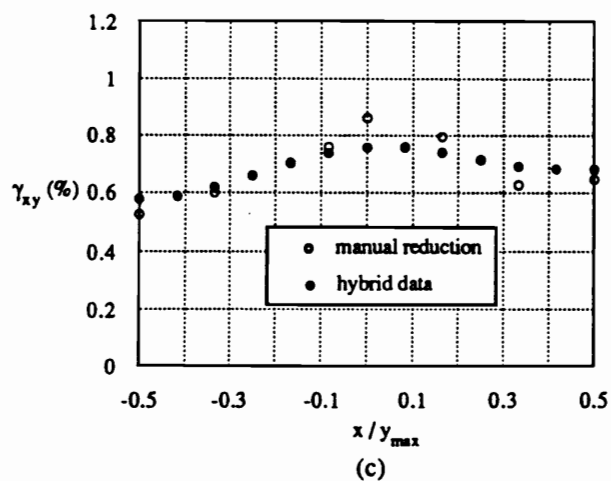
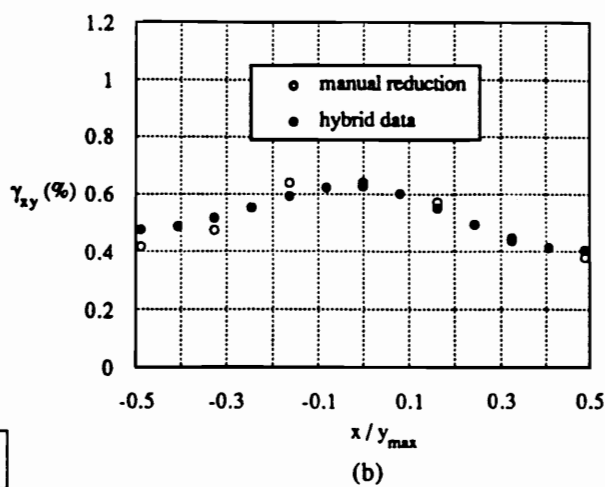
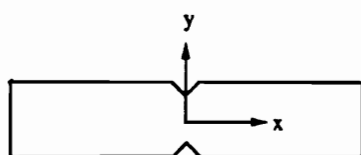
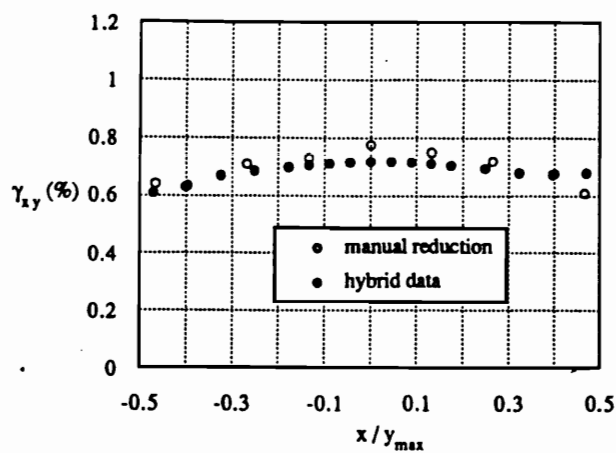
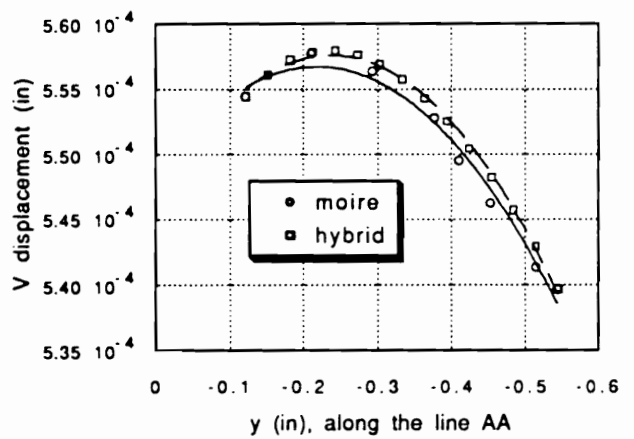
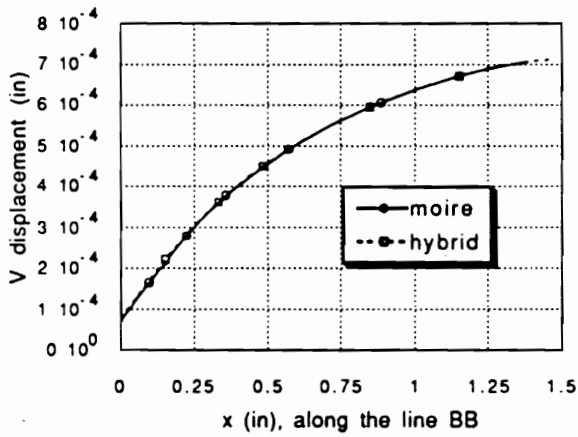
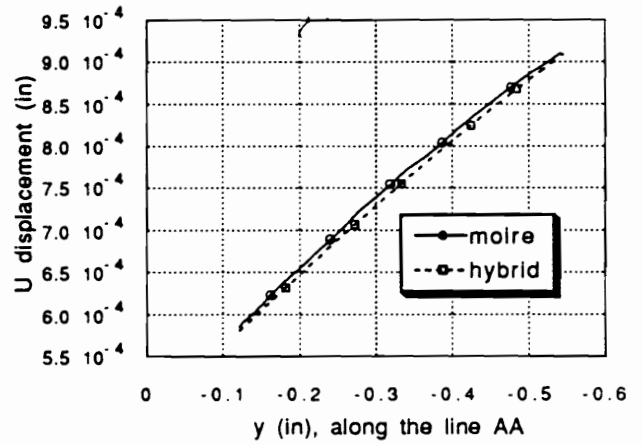
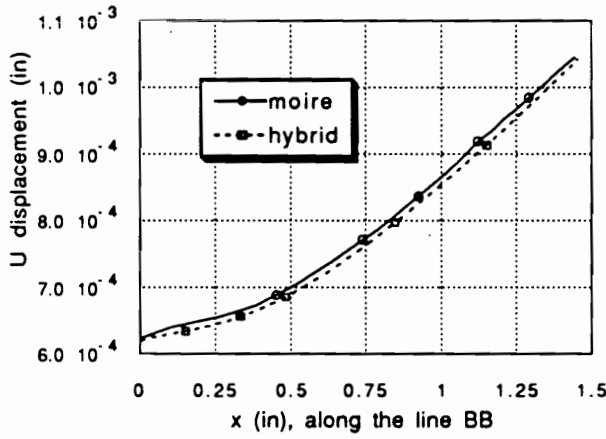
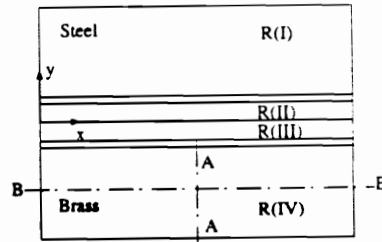
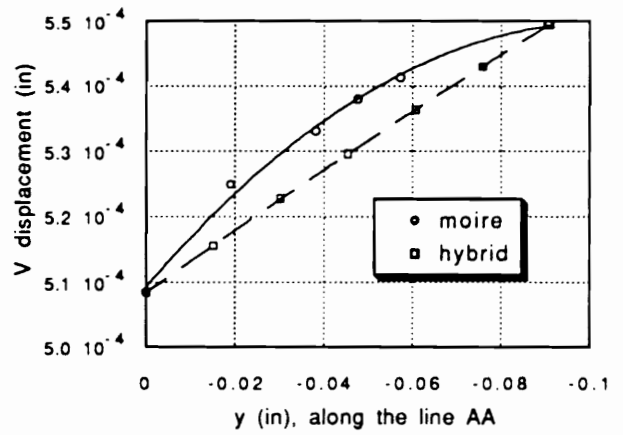
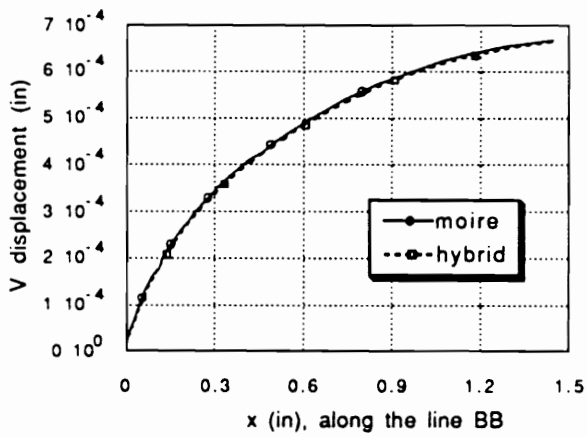
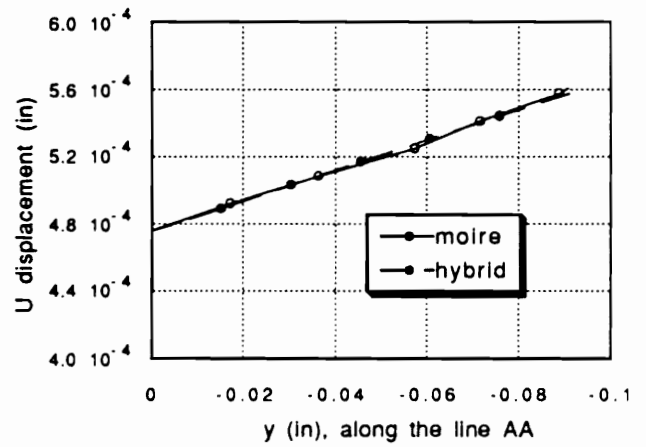
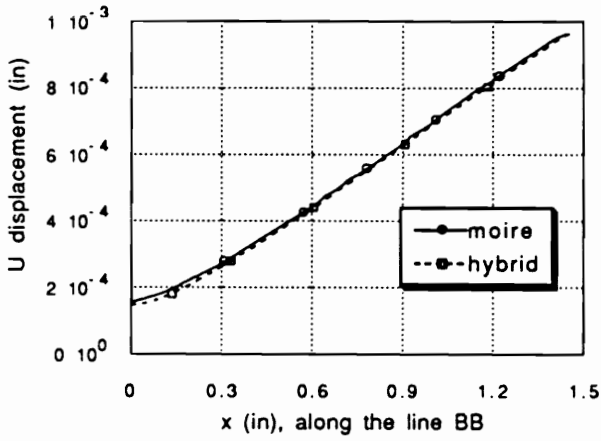
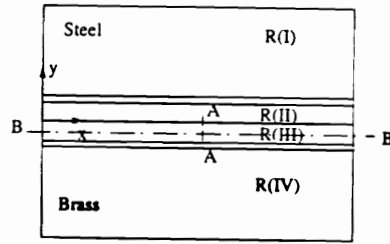


Fig.51. Comparisons of shear strain data along the x-axis near the center of specimen, obtained from manual data reduction and the localized hybrid analysis for (a) 0° (at an applied load of 300 lb), (b) 90° (at an applied load of 250 lb), (c) $0^\circ/90^\circ$ (at an applied load of 300 lb) graphite-epoxy Iosipescu specimens.

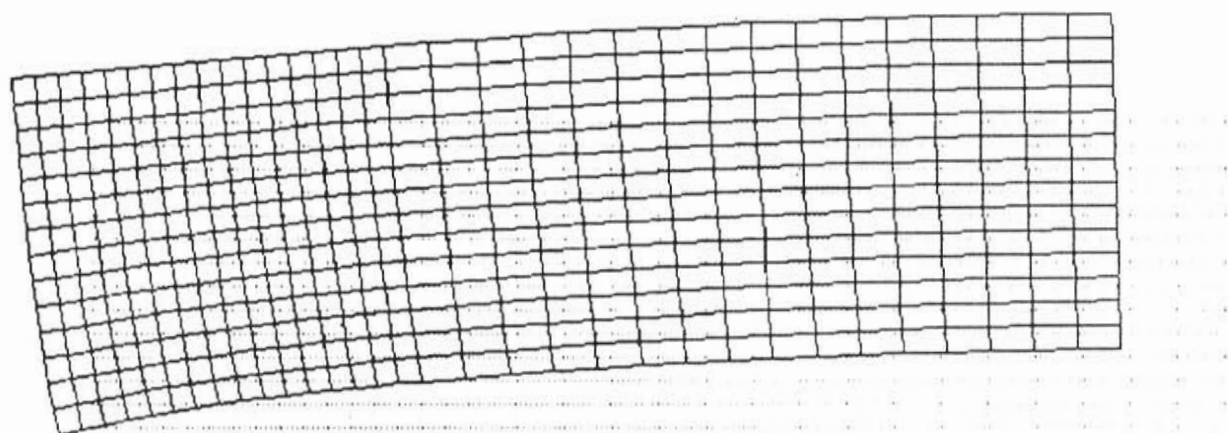
APPENDIX



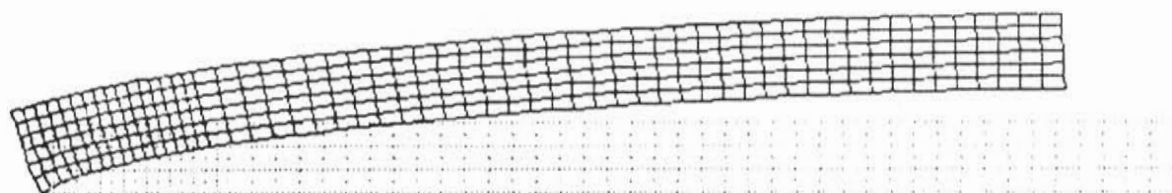
A-1. Comparisons of displacement components in moire and localized hybrid results along two central lines (lines AA and BB) in Region R(IV), steel.



A-2. Comparisons of displacement components in moire and localized hybrid results along two central lines (lines AA and BB) in Region R(III), steel.

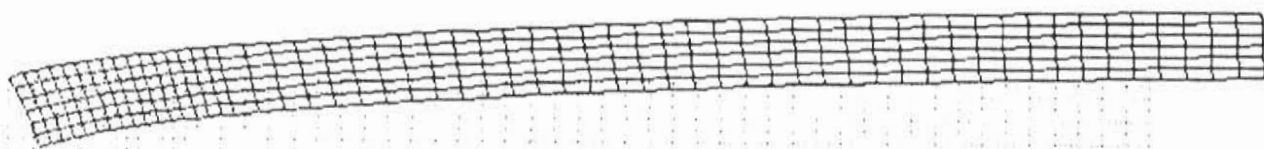


R(I)

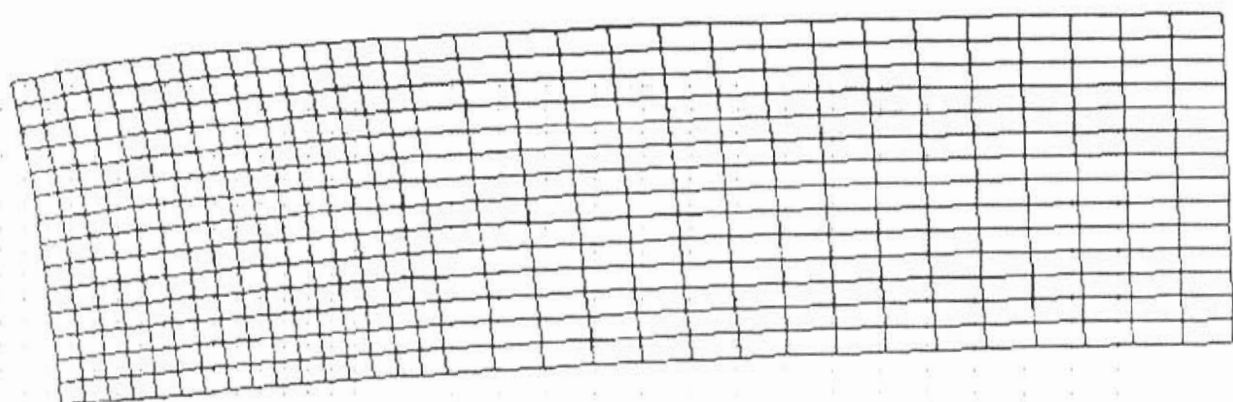


R(II)

A-3. Mesh and Deformed shape in Region R(I) and Region R(II) of the steel under the thermal load, $\Delta T = -240^\circ \text{ F}$.



R(III)



R(IV)

A-4. Mesh and Deformed shape in Region R(III) and Region R(IV) of the brass under the thermal load, $\Delta T = -240^\circ \text{ F}$.

VITA

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