Exploring the Impacts of Aspect Ratios on Visual Perception in Scatterplots

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(ABSTRACT)

This thesis investigates the effect of aspect ratio on visual perception in scatterplots. Four tasks explored how the aspect ratio affects participants' perception of distance, amount, and correlation in scatterplots. The results showed that square aspect ratio scatterplots are more suitable for detecting length and number, while rectangular aspect ratio scatterplots are better for detecting correlation. In addition, the JND (Just Noticeable Difference) was used in evaluating the visual perception of scatterplots in this experiment. The findings of this study have important implications for the design of scatterplots in data visualization, as well as for future research on visual perception in data visualization.

Exploring the Impacts of Aspect Ratios on Visual Perception in Scatterplots

Zhen Shen

(GENERAL AUDIENCE ABSTRACT)

This thesis focuses on understanding how the aspect ratio of scatterplots affects how people perceive data. Scatterplots are graphs that display data points on two variables, allowing researchers to identify relationships between variables visually. The aspect ratio of a scatterplot refers to the percentage of its width to height. This study found that people are better at detecting distances and amounts in scatterplots with a square aspect ratio. In contrast, they better see scatterplot correlations with a rectangular aspect ratio. The results suggest that the aspect ratio of a scatterplot plays a critical role in how people perceive and interpret data. These findings are essential for researchers, analysts, and designers who want to create compelling and accurate data visualizations.

Dedication

To my family.

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List of Abbreviations

σ The total mass of angels per unit area

ANOVA Analysis of Variance

JND Just Noticeable Difference

JND is a psychophysical concept that refers to the slightest detectable difference between two stimuli.

σ is the eighteenth letter of the Greek alphabet, and carries the 's' sound. In the system of Greek numerals, it has a value of 200.

ANOVA is a statistical technique used to assess the differences among group means by analyzing the variability within and between groups.

Chapter 1

Introduction

Scatterplots are ubiquitous in data visualization for displaying the relationship between two variables [1]. The choice of aspect ratio, defined as the ratio of the width to height of the plot, can affect how the viewer perceives and interprets the data. However, the optimal aspect ratio for a given dataset needs to be better understood, especially in human-computer interaction [2].

Scatterplots are a widely used visualization technique for representing bivariate data. In a scatterplot, each point represents a pair of values from two variables, which can be plotted along the x- and y-axes. Scatterplots provide a powerful way to explore the relationship between two variables, such as correlation, trend, and outliers [3, 4].

In medical research, scatterplots investigate correlations between variables such as patient demographics, disease progression, and treatment outcomes [5]. These plots enable researchers to identify potential risk factors, understand the impact of interventions, and explore associations between clinical parameters, leading to advancements in healthcare practices and personalized medicine.

Also, scatterplots could be used to understand consumer behavior, market trends, and customer preferences. Marketers can use scatterplots to visualize product attributes, customer satisfaction ratings, and purchase behavior data [6, 7]. By analyzing the plotted data, businesses can identify target segments, tailor marketing strategies, and make data-driven decisions to enhance customer experiences.

One key aspect of scatterplot design is the aspect ratio, which determines the proportion between the width and height of the plot [8]. The aspect ratio can significantly impact the perception and interpretation of the data in a scatterplot. A square aspect ratio, for example, can make the scatterplot appear more symmetrical and balanced. In contrast, a wide or tall aspect ratio can emphasize the range of values along one axis and compress the other [9].

Recent research has explored the impact of aspect ratio on scatterplot perception and comprehension. For example, a study by Heer and Bostock [10] used crowdsourcing to evaluate the accuracy and efficiency of visual judgments in scatterplots with different aspect ratios. They found that the optimal aspect ratio varied depending on the task and the nature of the data. Another study by Fink proposed a method for determining the optimal aspect ratio of a scatter plot based on the Delaunay triangulation of the data points [11]. The Delaunay triangulation is a mathematical technique that divides the scatter plot into triangles. The paper suggests that the aspect ratio of the property should be chosen to maximize the minimum angle of these triangles [12].

Despite these efforts, more research is needed to fully understand the effects of aspect ratios on scatterplot perception. In the previous papers, the researchers researched how to generate suitable scatterplots using Machine Learning technologies [13] or explored various other features that may influence the perception of scatterplots [14, 15, 16, 17]. Specifically, previous research has primarily focused on exploring the effects of individual visual properties, such as color, size, and shape, on perception and cognition. However, the influence of aspect ratios, which determine scatterplot width and height proportions, has received relatively less attention. This thesis contributes to this body of knowledge by presenting a user study that examines the impact of aspect ratio on scatterplot perception. We aim to investigate how different aspect ratios affect the accuracy, efficiency, and preference of user judgments in scatterplots and explore how users percept scatterplots with different aspect ratios. Specifically, we explored how the aspect ratio may affect user perception and scatterplots, such as determining distance, amount, and correlation.

Moreover, while previous studies have often utilized complex stimuli or explored high-dimensional scatterplots [18, 19, 20], we specifically focus on 2D scatterplots to ensure simplicity and accessibility for participants. This allows us to investigate the fundamental aspects of visual perception in scatterplots without the added complexity of additional dimensions or intricate visual designs.

1.1 Problem Statement

The central research question of this thesis paper is: What is the impact of aspect ratios on visual perception in scatterplots? Specifically, we aim to investigate how different aspect ratios affect the accuracy, efficiency, and preference of users' judgments and decisions based on scatterplots and how users percept with scatterplots of varying aspect ratios. We hypothesize that specific aspect ratios may enhance or hinder scatterplot perception depending on the characteristics of the data and the tasks involved. By addressing this research question, we aim to contribute to understanding the role of aspect ratio in scatterplot design and visualization practice.

1.2 Objectives

The objectives of this paper are as follows:

- To design a user study that investigates the effects of aspect ratios on scatterplot perception and interaction using a variety of datasets and tasks.
- To evaluate and discuss the impacts of aspect ratios on the accuracy, efficiency, and preference of user judgments and decisions in scatterplots, as well as on users' interaction patterns and behaviors.
- To identify the implications and limitations of the findings.

1.3 Contributions

Our study contributes to the field of human-computer interaction by providing empirical evidence and insights on the design and use of scatterplots with different aspect ratios. It also has implications for data visualization, statistical analysis, and decision-making in various domains, such as finance, healthcare, and education. By exploring the impacts of aspect ratios on visual perception in scatterplots, we hope to enhance the effectiveness and efficiency of visual communication and analysis in the digital age.

Chapter 2

Related Work

2.1 Scatterplots

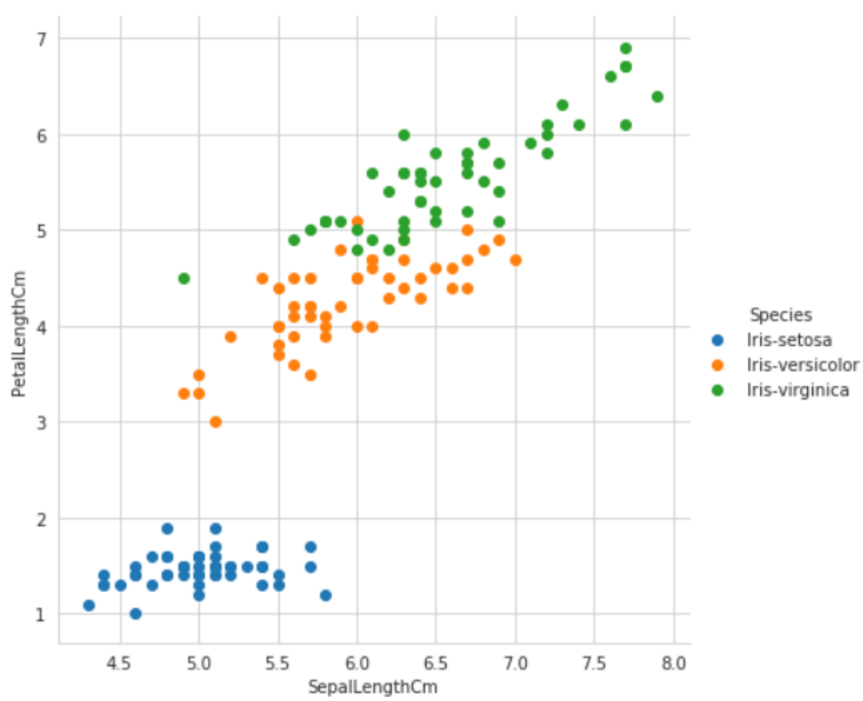

Figure 2.1: Scatterplots With the Iconic Iris Dataset

Scatterplots are one of the most widely used visualization techniques for exploring relationships between two variables. They provide a way to represent the distribution and pattern of data points visually and can help users identify correlations, trends, and outliers [21].

Using the iconic Iris dataset [22], the scatterplot (Figure 2.1) showcases the relationships between sepal length and width, as well as petal length and width. Each data point represents an individual iris flower, and the scatterplot reveals distinct clusters that correspond to different species within the dataset. The sepal measurements are displayed along the x-axis, while the petal measurements are depicted along the y-axis. The scatterplot visually demonstrates that the setosa species tend to have smaller sepal and petal dimensions than the versicolor and virginica species, which exhibit a wider range of sizes. This plot clearly visualizes the dataset's patterns, allowing for further exploration and analysis of the iris flowers' morphological characteristics.

In Human-Computer Interaction, scatterplots were utilized to explore user behavior and interaction with mobile devices. By incorporating interactive animations, the study aims to enhance user engagement and facilitate adequate data comprehension [23]. Piringer [24] explored a novel approach to data visualization by integrating focus plus context techniques with linked 2D/3D scatterplots. The study aims to enhance data exploration and analysis by allowing users to navigate between detailed views and overall context seamlessly.

Research on scatterplots has focused on several areas, including their effectiveness for data exploration and analysis, design and aesthetics, and interaction and usability [25].

In terms of effectiveness, several studies have investigated using scatterplots for data analysis and decision-making. For example, Kehrer and Hauser [26] examined the effectiveness of different types of scatterplot visualizations, such as jittered and stacked scatterplots. They found that they can improve the readability and interpretability of the data. Raidou [27] proposed a pixel-based mapping approach, where each pixel in the plot is assigned a color based on the density of data points within its corresponding region. The technique aims to reduce the visual clutter and improve the legibility of scatter plots by representing the data distribution more continuously and smoothly.

In terms of design and aesthetics, several studies have explored the impact of visual features on scatterplot perception and comprehension. Cleveland and McGill [28] proposed a model of human perceptual tasks in graphical displays, including scatterplots, and identified critical principles for effective visualization. Heer and Bostock [10] investigated the impact of aspect ratio on scatterplot perception and found that the optimal aspect ratio varies depending on the task and the data.

Regarding interaction and usability, several studies have examined the impact of different interaction techniques on scatterplot exploration and analysis. Bezerianos et al. [29] investigated the effect of brushing and linking on scatterplot analysis and found that they can improve user performance and insight. Liao [30] suggests a cluster-based approach, where data points are grouped into clusters based on similarity and represented by a single visual element, such as a glyph or a marker. The clusters are arranged based on similarity, and the resulting layout creates an abstract scatterplot.

Our research contributes to the existing body of scatterplot studies by focusing on the impact of aspect ratio on visual perception in scatterplots. While previous research has explored various aspects of scatterplots in applications, generations, interactions, etc., our study delves into the relationship between aspect ratio and visual perception. We conducted a user study using different aspect ratios and assessed participants' abilities to detect outliers, compare amounts and identify correlations. By investigating the influence of aspect ratio on these perceptual tasks, we provide unique insights into how the geometric properties of scatterplots can affect users' cognitive processes and decision-making. Our findings shed light on the importance of aspect ratio as a critical design factor in creating effective scatterplots.

2.2 Aspect Ratio

Aspect ratio is a critical factor in designing and interpreting visual displays, such as charts, graphs, and images. Aspect ratio refers to the proportional relationship between the width and height of a visual display [31]. It can significantly impact the perception, comprehension, and aesthetics of graphical displays.

Research on aspect ratios has focused on several areas, including their impact on visual perception and cognition, their relationship with other design variables, and their use in specific types of graphical displays [32].

In terms of perception and cognition, several studies have investigated the impact of aspect ratios on visual processing and attention. Cleveland and McGill [28] conducted a seminal study on the graphical perception of data, identifying the importance of aspect ratio in determining the accuracy of interpreting data in scatterplots. Heer and Bostock [10] explored the influence of aspect ratio on the perception of correlation in scatterplots, highlighting that aspect ratios can impact the perceived strength of correlation between variables.

Regarding design variables, several studies have explored the relationship between aspect ratio and other visual variables, such as color, size, and shape. Tufte [33] emphasized the importance of the data-ink balance in data visualization, suggesting that the aspect ratio should be chosen to maximize the amount of data displayed and minimize non-data elements.

Regarding specific types of visual displays, several studies have examined the use of aspect ratios in charts, graphs, and images. Wickham and Hofmann [34] discussed the role of aspect ratio in bar charts, emphasizing that aspect ratios can impact the perception of the relative sizes of bars, potentially leading to misinterpretation of the data. Wang [35] investigates the effect of aspect ratio on the readability and accuracy of line charts. In the research, the authors examine several techniques for selecting the aspect ratio of a line chart.

Shneiderman and Wattenberg [36] investigated the impact of aspect ratios on the readability of treemaps, suggesting that maintaining low aspect ratios in the layout of treemaps can enhance readability and comprehension.

Unlike bar graphs or line charts that primarily depict categorical or sequential data, scatterplots showcase the relationships between two continuous variables [37]. They enable the simultaneous representation of data points and reveal patterns, trends, clusters, and correlations [38] that might not be immediately apparent in other chart types. Scatterplots provide a comprehensive and holistic view of data, allowing users to explore and analyze complex datasets visually and intuitively. This research focuses on understanding the impact of aspect ratio on visual perception in scatterplots, contributing valuable insights into optimizing their design and enhancing users' ability to interpret and derive meaningful insights from scatterplot visualizations.

2.3 Visual Perception

Visual perception is interpreting and making sense of visual stimuli, such as images, objects, and scenes. It involves a complex interplay between the eyes, brain, and environment and can have significant implications for various domains, including human-computer interaction, cognitive psychology, and neuroscience.

Research on visual perception has focused on several areas, including the mechanisms and processes involved in visual processing, the factors that influence visual perception, and the applications and implications of visual perception for various domains [39].

In terms of mechanisms and processes, several studies have investigated the neural and cognitive mechanisms involved in visual processing, such as the roles of attention, memory, and perception in visual perception. Cleveland and McGill [28] conducted a foundational study on the graphical perception of data, examining how different visual encodings impact the accuracy and speed of data interpretation.

Regarding factors that influence visual perception, several studies have examined the impact of various observable variables, such as color, contrast, and spatial frequency, on visual perception. For example, Palmer and Rock [40] found that the perceived shape of an object can be influenced by the surrounding context and the object's orientation. Tversky and Hemenway [41] found that the perceived distance between two things can be affected by their relative size and orientation.

In terms of applications and implications, several studies have explored visual perception in various domains, such as human-computer interaction, marketing, and education. For example, Norman [42] proposed the concept of affordances in design, which suggests that objects' perceived functions and properties can influence their use and interpretation.

These studies have provided insights into how individuals perceive and process visual cues related to data points, patterns, and relationships. Building upon this existing research, this study focuses on the aspect ratio of scatterplots and their impact on visual perception. This will help expand the understanding of how aspect ratio influences users' perception of scatterplot visualizations, contributing to developing guidelines and best practices for optimizing scatterplot designs and improving the effectiveness of data communication through visual representations.

2.4 Just Noticeable Difference(JND)

Just noticeable difference (JND) is a psychophysical concept that refers to the slightest detectable difference between two stimuli. It has been widely used in perceptual psychology and psychophysics to measure the sensitivity of human perception to changes in stimuli. It has applications in various domains, including human-computer interaction, sensory evaluation, and marketing.

Research on JND has focused on several areas, including its theoretical foundations and measurement methods, applications in various domains, and relationship with other perceptual and cognitive processes [14].

Several studies have investigated JND's mathematical and statistical models and the factors that affect its measurement and interpretation regarding theoretical foundations and measurement methods. For example, Weber proposed the first quantitative law of sensory discrimination, which suggests that the JND is proportional to the magnitude of the stimulus [43]. Stevens [44] proposed the power law of sensation, which indicates that the JND varies as a power function of the stimulus magnitude.

In terms of applications, several studies have explored the use of JND in various domains, such as human-computer interaction, sensory evaluation, and marketing. Lawless and Heymann [45] discussed using JND in sensory evaluation and identified its advantages in measuring sensory discrimination and detecting product differences. Yang [46] suggested a motion-compensated residue preprocessing technique that uses a JND profile to guide the filtering of the residual signal.

The research used the JND to quantify participants' perceptual sensitivity in distinguishing differences between scatterplots with different aspect ratios. The use of JND in our study was motivated by its ability to capture the minor detectable difference in stimuli, providing a fine-grained measure of participants' perceptual acuity. By employing JND, we were able to investigate the threshold at which participants could discern differences in scatterplot aspect ratios, shedding light on the perceptual boundaries and sensitivity of individuals when perceiving and interpreting visualizations [47]. While other measures, such as accuracy rates or response times, can provide valuable insights, JND offers a unique perspective by focusing on the perceptual discrimination abilities of participants [48], enabling us to uncover subtle differences in scatterplot aspect ratios that other measures may not capture.

Chapter 3

Methodology

The methodology section describes the user study conducted to explore the impacts of aspect ratios on visual perception in scatterplots. We recruited participants to complete four tasks using a website we developed for the study. Each task involved comparing and analyzing scatterplots with different aspect ratios, and the participants' responses were measured in terms of accuracy and response time. For tasks 1 to 3, we used a staircase procedure to control the difficulty of the tasks and ensure accurate measurements of just noticeable differences (JND). The data collected from the study were analyzed using statistical methods to test the hypotheses and research questions. The user study was conducted on a 27-inch 4K monitor to ensure consistency and accuracy in the display of the scatterplots and other interface elements. The methodology section describes the procedures and tools used in the user study, including the generation of scatterplot data and the implementation of the staircase procedure [49]. The findings from the study contribute to understanding the effects of aspect ratios on visual perception in scatterplots. They can inform the design of visualization tools in human-computer interaction.

3.1 Tasks and Data

3.1.1 Task Selection

In designing our user study, we drew inspiration from previous scatterplot research [11, 50, 51, 52], particularly Sarikaya's work [25] (Figure 3.1). The article covers the primary tasks that scatterplots can help accomplish, such as identifying trends, outliers, and clusters in data. It also discusses the data types that can be effectively represented using scatterplots, emphasizing the importance of choosing suitable variables to convey meaningful information.

	#	Task	Description
object-centric	1	Identify object	Identify the referent from the representation
	2	Locate object	Find a particular object in its new spatialization
	3	Verify object	Reconcile attribute of an object with its spatialization (or other encoding)
	4	Object comparison	Do objects have similar attributes? Are these objects similar in some way?
browsing	5	Explore neighborhood	Explore the properties of objects in a neighborhood
	6	Search for known motif	Find a particular known pattern (cluster, correlation)
	7	Explore data	Look for things that look unusual, global trends
aggregate-level	8	Characterize distribution	Do objects cluster? Part of a manifold? Range of values?
	9	Identify anomalies	Find objects that do not match the 'modal' distribution
	10	Identify correlation	Determine level of correlation
	11	Numerosity comparison	Compare the numerosity/density in different regions of the graph
	12	Understand distances	Understanding a given spatialization (e.g. relative distances)

Figure 3.1: The list of abstracted analysis tasks that are performed with scatterplots

This paper provided a comprehensive overview of various tasks that can be performed using scatterplots, including tasks initially designed for 3D scatterplots [18, 19, 20]. Although our research focused explicitly on 2D scatterplots, we found the tasks designed for 3D scatterplots informative and relevant to our study.

To ensure simplicity and ease of comprehension for the participants, we aimed to keep the tasks as straightforward as possible. We deliberately avoided including complex tasks [13] requiring participants to learn new concepts or techniques before engaging in the study. By selecting relatively simple and intuitive tasks [2], we aimed to minimize any potential learning effects and allow participants to focus primarily on their visual perception of the scatterplots.

Furthermore, we specifically focused on tasks [27] that involved multiple properties of scatterplots, as opposed to tasks that solely explored a single aspect, such as outliers or correlation. By including tasks that covered a range of properties, such as distance, quantity, and correlation, we aimed to provide a more comprehensive assessment of participants' visual perception abilities and their ability to discriminate between different aspects of scatterplots.

In addition, we combined and modified some tasks from the original set to reduce repetition and overlap, ensuring more efficient use of participants' time and attention. This approach allowed us to cover a broader range of scatterplot properties and variations within a manageable set of tasks.

Overall, our task selection process was driven by investigating visual perception in the context of scatterplots, drawing inspiration from previous research while tailoring the tasks to be accessible, comprehensive, and engaging for participants.

3.1.2 Task 1: Outlier Selection

Participants were presented with a scatterplot containing a cluster of points and two outliers (Figure 3.2). They were instructed to select the outlier that had a further distance from the cluster. The scatterplot had different aspect ratios, ranging from 1:2 to 2:1. There were two groups of tasks, each consisting of a version with a different aspect ratio. In task 1, the outliers are positioned in specific settings to create different scenarios for participants, and they would be put in opposite directions. The cluster had a radius of R, and the default distance of one outlier from the center of the cluster was set at 1.5R. The distance of the second outlier from the cluster's center varied based on the JND principle, ranging from 1.525R to 1.725R.

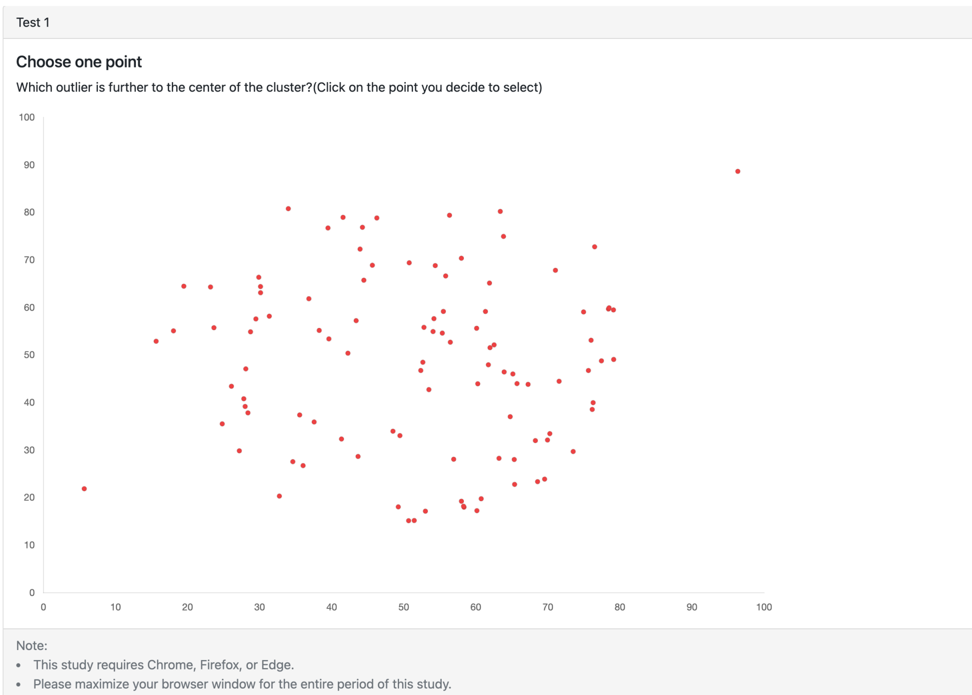

Figure 3.2: Task 1 Example Implemented on the Website

3.1.3 Task 2: Point Number Comparison

Participants were presented with two scatterplots containing a different number of points, and they were instructed to select the scatterplot with more points (Figure 3.3). The scatterplot had different aspect ratios, ranging from 1:2 to 2:1. There were two groups of tasks, each consisting of a version with a different aspect ratio. The default scatterplot had 100 points, and the number of points in the other scatterplot varied based on the JND principle.

The number of points in the other scatterplot ranged from 82 to 118.

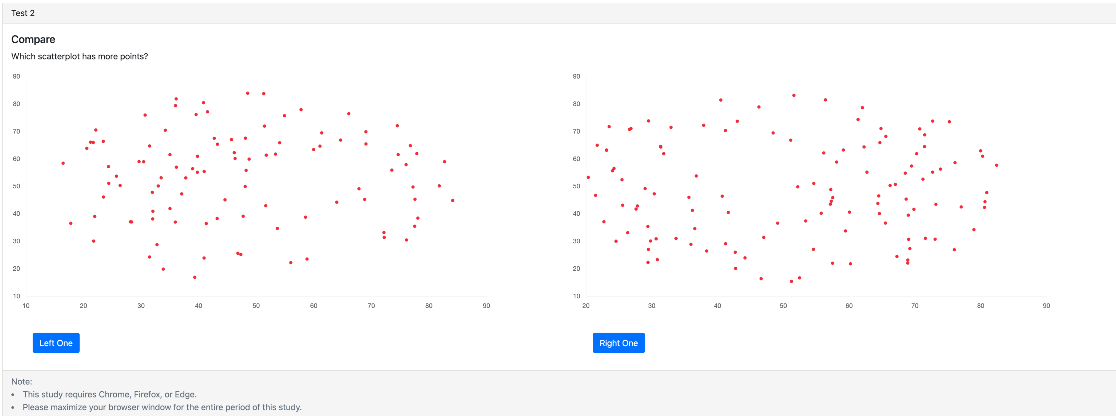

Figure 3.3: Task 2 Example Implemented on the Website

3.1.4 Task 3: Correlation Comparison

Participants were presented with two scatterplots containing points with different correlations. They were instructed to select the scatterplot with a stronger correlation between the points (Figure 3.4). The scatterplot had different aspect ratios, ranging from 1:2 to 2:1. There were two groups of tasks, each consisting of a version with a different aspect ratio. The default scatterplot had a correlation of 0.75, and the correlation in the other scatterplot varied based on the JND principle. The correlation in the other scatterplot ranged from -0.84 to -0.66, or 0.66 to 0.84.

3.1.5 Task 4: Correlation Input

Participants were presented with a scatterplot and were asked to estimate the correlation between the points (Figure 3.5). The scatterplot had different aspect ratios, ranging from 1:2 to 2:1. There were 30 trials in total, with the correlation between the points varying across trials. The correlation in the scatterplot ranged from -1 to -0.45, or 0.45 to 1.

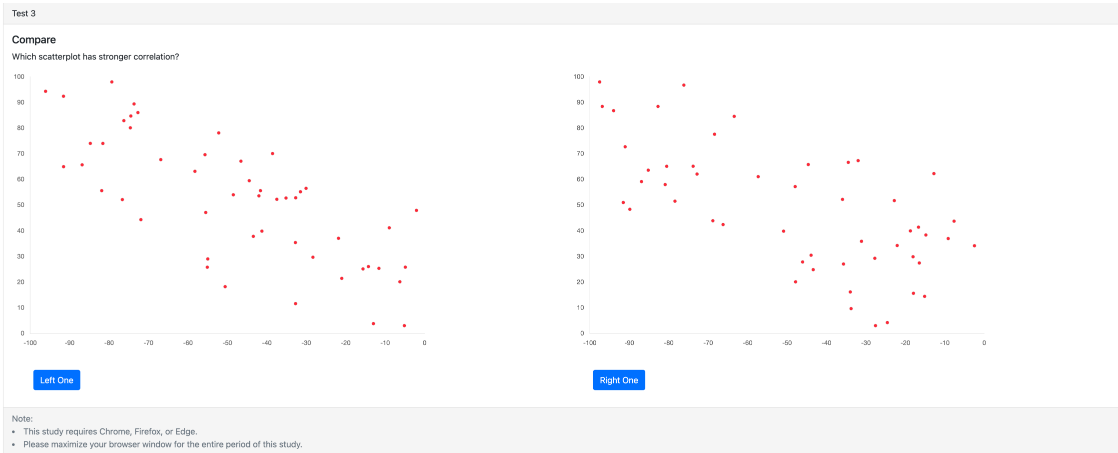

Figure 3.4: Task 3 Example Implemented on the Website

3.1.6 Data

To generate the data for Task 1, we first set the center position and radius of the cluster. The center position of the group in both scatterplots was appointed to a random position within the range of 45 - 55 for both the x-axis and y-axis, and the radius of the cluster was set to a random value within the scope of 33 - 37. We then used a margin to set the minimum and maximum values for the x-axis and y-axis to ensure that points were not drawn on the axes. Using our custom-built tool, we generated 100 points within the 0 - 100 for both the x-axis and y-axis.

Next, we used a Python script to generate two outliers based on the center position and radius of the cluster. The further outliers were positioned at a JND distance from the center of the cluster, ranging from 1.525R to 1.725R (Table 3.1).

To generate the data for Task 2, we first set the cluster's center position and radius, then used a margin to set the minimum and maximum values for the x-axis and y-axis to ensure that points were not drawn on the axes. The condition of the center position and radius of the cluster is the same in Task 1.

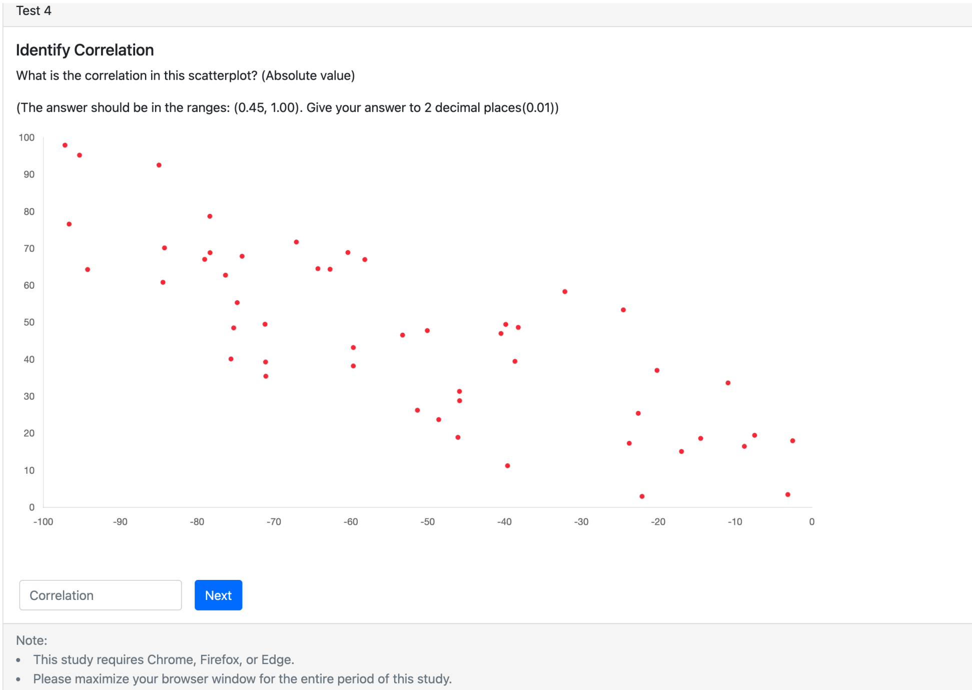

Figure 3.5: Task 4 Example Implemented on the Website

Using our custom-built tool, we generated 100 points within the range of 0-100 for both the x-axis and y-axis for the default scatterplot. For the other scatterplot, we used the JND principle to generate a different number of points, with the number of points ranging from 82 to 118 (Table 3.2).

For Task 3 and Task 4, we generated the scatterplots using a combination of random number generation and correlation calculation. For each scatterplot, we used a margin to set the minimum and maximum values for the x-axis and y-axis to ensure that points were not drawn on the axes.

For Task 3, we randomly generated a scatterplot with 50 points and a slope of 1 or -1. We then used a for loop to calculate the correlation of the scatterplot, and if the correlation did not meet the requirement, the program generated a new scatterplot until the need was met. Once the scatterplot met the condition, we modified the scale to have a range of 0-100 for

Level	Distance
1	1.725R
2	1.7R
3	1.675R
4	1.65R
5	1.625R
6	1.6R
7	1.575R
8	1.5R
9	1.525R

Table 3.1: Difficulty level for Task 1, R is the radius of the cluster in the scatterplot

Level	Number
1	82/118
2	84/116
3	86/114
4	88/112
5	90/110
6	92/108
7	94/106
8	96/104
9	98/102

Table 3.2: Difficulty level for Task 2

the y-axis and 0 - 100 or -100 - 0 for the x-axis (Table 3.3).

For Task 4, we used the same approach as Task 3 to generate scatterplot data. We randomly generated scatterplots data, then modified the scale to have a range of 0-100 for the y-axis and 0 - 100 or -100 - 0 for the x-axis, and the correlation ranges would be -1 to -0.45 and 0.45 to 1, and the difference between close levels in the field is 0.05.

Level	Correlation
1	0.66/0.84/-0.66/-0.84
2	0.67/0.83/-0.67/-0.83
3	0.68/0.82/-0.68/-0.82
4	0.69/0.81/-0.69/-0.81
5	0.70/0.80/-0.70/-0.80
6	0.71/0.79/-0.71/-0.79
7	0.72/0.78/-0.72/-0.78
8	0.73/0.77/-0.73/-0.77
9	0.74/0.76/-0.74/-0.76

Table 3.3: Difficulty level for Task 3

3.2 Participants

In this study, we recruited 20 participants to complete a user study on the relationship between aspect ratio and visual perception in scatterplots. Of the 20 participants, six were female, and fourteen were male. All participants were over 18 and had normal or corrected-to-normal vision. They were recruited from various majors and educational levels to ensure diverse perspectives. The participants were provided with a website that allowed them to complete tasks related to outlier detection, point counting, and correlation perception in scatterplots of different aspect ratios.

3.3 Procedures

The study was conducted using a custom-designed website presenting the scatterplot tasks and collecting participants' responses and performance data.

To ensure consistency and accuracy in our user study, we used a 27-inch 4K monitor to display the scatterplots and other interface elements. The monitor was calibrated to ensure the color and brightness were accurate and consistent across all participants. We also set the monitor's resolution to the native 4K resolution to ensure that the scatterplots and other interface elements were displayed clearly and accurately.

By using the same hardware for all participants, we aimed to eliminate any potential biases or confounding variables that could affect the participants' visual perception or performance in the tasks. This allowed us to more accurately evaluate the impacts of aspect ratios on visual perception in scatterplots.

The study consisted of Tasks 1 to 3, with two versions with different aspect ratios. The aspect ratios tested were: 2:1, 3:2, 1:1, 2:3 and 1:2. The tasks were presented in random order for each participant, completing two versions of each task with different aspect ratios. A Latin square design determined the order of the aspect ratios and tasks (Table 3.4). For Task 4, the participants will be presented with on scatterplot with different aspect ratios.

Participant Order	Task 1 - 1	Task 1 - 2	Task 2 - 1	Task 2 - 2	Task 3 - 1	Task 3 - 2
1	1:1	2:1	2:3	1:2	3:2	1:1
2	2:1	2:3	1:2	3:2	1:1	2:1
3	2:3	1:2	3:2	1:1	2:1	2:3
4	1:2	3:2	1:1	2:1	2:3	1:2
5	3:2	1:1	2:1	2:3	1:2	3:2
6	1:1	2:1	2:3	1:2	3:2	1:1
7	2:1	2:3	1:2	3:2	1:1	2:1
8	2:3	1:2	3:2	1:1	2:1	2:3
9	1:2	3:2	1:1	2:1	2:3	1:2
10	3:2	1:1	2:1	2:3	1:2	3:2

Table 3.4: Latin square design for Task 1 to Task 3

In the study, the researchers ensured that the participants saw scatterplots with the same area for each task. For tasks 1 and 4, there was only one scatterplot on each page, and the aspect ratios varied between a square, a landscape rectangle, and a portrait rectangle. For tasks 3 and 4, there were two scatterplots on each page, and the aspect ratios varied the same way for tasks 1 and 4. This standardization allowed for a fair comparison of the participants' performances on each task, and the areas of scatterplots in each tasks are the same.

Before each group of tasks, participants were given two example questions to familiarize themselves with the task and the interface. These example questions were not included in the analysis. After participants answered each example question, they received immediate feedback on whether their answer was correct or incorrect.

Before starting the official test for each group of tasks, participants were notified that they were about to begin the trial. They could take a break after each group of tasks if needed.

For tasks 1 to 3, the JND principle was used to set the distance between the outliers and the center of the cluster, the number of points in the scatterplots, and the correlation between the points. The staircase procedure was used for each group of tasks to determine the JND threshold for each aspect ratio. The staircase procedure began with a setting value increment, where the difficulty level decreased every time the participant gave an incorrect answer. Conversely, the difficulty level increased after three consecutive correct answers. The step size was reduced after every two reversals, starting with a more significant step and then reducing it to smaller steps. The staircase procedure ended when the step size reached the minimum grade, or the participant completed a maximum of 50 trials for each group. After collecting all the data, we computed the mean JND for each task. Before the start of task 3, the participants were required to complete a test for matching the strength of the correlations of four scatterplots. This ensured that the participants were familiar with the concept of correlation.

The participants were instructed to complete each task as quickly and accurately as possible and to provide feedback on the difficulty and clarity of the tasks. After completing each trial, the participants were asked to rate their confidence level in their answer on a scale from 1 to 5, where 1 indicated low confidence, and 5 showed high confidence. The rating scale was defined as (Figure 3.6).

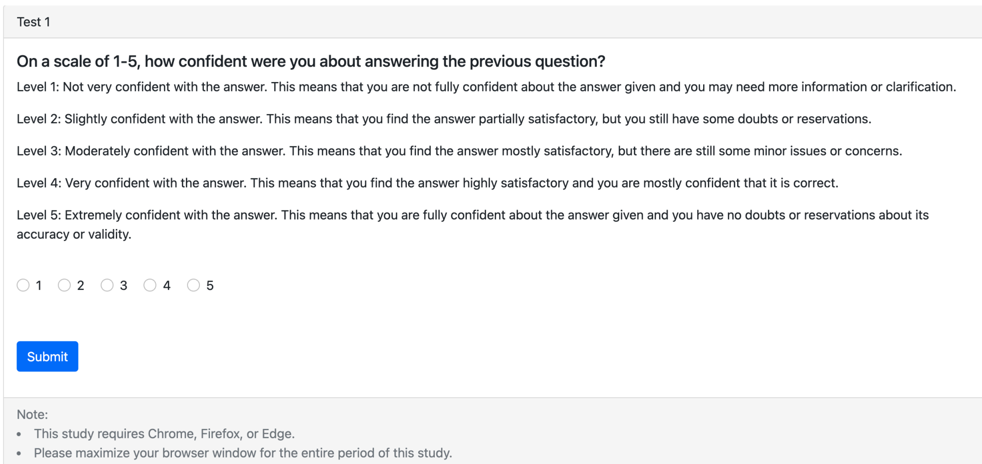

Figure 3.6: Confident Level Page Screenshot

The data collected from the study were stored in Firebase, a cloud-based database service. Firebase provides a secure and scalable platform for storing and managing data, with automatic backups and real-time synchronization features. The data were stored in a private database accessible only to the research team, and all data were kept confidential and anonymous.

3.4 Measures

In this study, we collected various data to investigate the participants' performance and perception of the tasks. The collected data encompassed both objective performance measures and subjective ratings provided by the participants. The measures provided a comprehensive understanding of participants' responses and experiences during the study.

3.4.1 Objective Performance Measures

For each task, we recorded the accuracy rate, which indicated the proportion of correct responses the participants gave. This measure allowed us to accurately assess the participants' performance in completing the assigned tasks.

In addition to the accuracy rate, we recorded each task's response time. Response time provided insight into the speed at which participants processed and made decisions during the tasks. This measure helped us understand the cognitive effort required and the time participants took to complete each task.

3.4.2 Subjective Ratings

We included several rating scales to capture participants' subjective experiences and perceptions. One such scale was the confident rating scale, where participants rated their confidence level in their responses or interpretations for each task. This scale enabled us to explore participants' perceived certainty in their judgments and provided insights into their confidence in their decisions.

Furthermore, we utilized a survey to collect qualitative feedback from participants. Participants were asked open-ended questions about their observations, interpretations, and experiences during the tasks. This qualitative feedback allowed us to gain deeper insights into participants' subjective perspectives, preferences, and the reasoning behind their decision-making process.

3.4.3 Data Analysis

The collected data were analyzed using various statistical techniques to derive meaningful insights and draw conclusions. Descriptive statistics were employed to summarize the objective performance measures and participants' subjective ratings. Inferential statistics, such as Kruskal-Wallis tests, were used to assess the significance of differences among conditions or tasks.

By combining objective performance measures and subjective ratings, we comprehensively understood participants' performance, confidence, and personal experiences about the different tasks and conditions. Analyzing these measures enabled us to gain valuable insights into participants' visual perception and decision-making processes within the context of scatterplots.

3.5 Tasks Hypotheses

In this study, we formulated specific hypotheses for each task based on previous studies and our informed assumptions. These hypotheses aimed to guide our investigation and provide a focused direction for analyzing the participants' performance and perception in each task.

3.5.1 Task 1 Hypotheses

For Task 1, which involved identifying an outlier in a scatterplot, we hypothesized that participants could detect outliers more accurately when presented with scatterplots having a square aspect ratio. This hypothesis was derived from previous research indicating that square aspect ratios provide a more balanced and evenly distributed visual representation, allowing for better differentiation and identification of outliers [2]. We hypothesized that participants better identify outliers in scatterplots with square aspect ratios than rectangular aspect ratios.

3.5.2 Task 2 Hypotheses

For Task 2, which required participants to choose the scatterplot with more points from two options, we hypothesized that participants would have a higher accuracy rate when the scatterplots had a square aspect ratio. This hypothesis was based on studies suggesting that square aspect ratios provide an equal distribution of points across the scatterplot area, making it easier for individuals to estimate the number of points [16]. Thus, we hypothesized that participants better determine the scatterplot with more points when presented with square aspect ratios.

3.5.3 Task 3 Hypotheses

Task 3 focused on selecting the scatterplot with a stronger correlation from two options. Here, we hypothesized that participants would exhibit higher accuracy rates when the scatterplots had a rectangular aspect ratio. This hypothesis was grounded in the guess that rectangular aspect ratios can enhance the perception of correlation strength by elongating the scatterplot's shape and making the correlation pattern more visually prominent.

3.5.4 Task 4 Hypotheses

In Task 4, participants were asked to input a correlation value after viewing scatterplots with different aspect ratios. Given the exploratory nature of this task and the absence of previous studies specifically investigating the effect of aspect ratio on participants' ability to estimate correlation, we formulated an open hypothesis without a specific directional prediction. We aimed to explore participants' correlation estimation accuracy and identify potential patterns or tendencies in their responses across different aspect ratios.

3.6 Research Questions

Research Question 1: What is the relationship between aspect ratio and visual perception in scatterplots?

This research question investigates the perceptual and cognitive processes involved in visual data analysis and how aspect ratio affects these processes. The answer to this question can provide insights into the mechanisms and principles of scatterplot design and evaluation.

Research Question 2: What is the optimal aspect ratio for scatterplot design and evaluation based on perceptual and cognitive principles?

This research question aims to identify the optimal aspect ratio for scatterplot design and assessment based on visual perception and cognition principles. The answer to this question can provide guidelines and recommendations for scatterplot designers and evaluators.

Chapter 4

Results

4.1 Analytical Methods

To analyze the data collected in this study, we employed several statistical methods to examine the relationships and differences between variables. The specific methods used depended on the nature of the data and the assumptions associated with each analysis.

First, we assessed whether the data obtained from each task followed a normal distribution. We employed one-way ANOVA (Analysis of Variance) [53] for normally distributed data as our first analysis measure. One-way ANOVA allowed us to assess whether there were significant differences in the means of multiple groups (i.e., different aspect ratios) for each task. We performed Tukey's post hoc test [54] to determine which groups significantly differed. This test enabled pairwise comparisons and helped identify specific aspect ratios that showed significant differences.

If the data did not follow a normal distribution, we attempted to transform the data using mathematical functions such as logarithmic or square root transformations [55]. Transformations can help normalize the data and meet the assumptions of parametric tests. If victorious, we proceeded with the normality-transformed data and repeated the initial normality check.

However, if the data remained non-normally distributed even after transformation attempts, we utilized non-parametric statistical tests. In our case, we employed the Kruskal-Wallis test

[56], a non-parametric alternative to one-way ANOVA. This test allowed us to compare the medians of multiple groups when the data did not meet the assumptions of normality. We employed Dunn's post hoc test [57] to perform pairwise comparisons and identify significant differences between groups.

By employing these analytical methods, we aimed to appropriately analyze the data collected in each task while considering the distributional characteristics of the variables. This comprehensive approach allowed us to uncover significant differences and relationships among aspect ratios in the participants' performance and perception.

4.2 Quantitative Analysis

The user study calculated the accuracy rate for each task and aspect ratio combination. A total of 20 participants completed the study; for Task 1 to Task 3, the accuracy rate and response time with 95% confidence of different visualizations were presented in Figure 4.1, 4.2, 4.3. One-way ANOVA tests were conducted to determine if there were statistically significant differences in accuracy rate between the different aspect ratios for each task. Post-hoc Tukey's HSD tests were used to determine which aspect ratio pairs were significantly different from each other.

For Task 1, the mean accuracy rates for each group were as follows: group 1:1 = 0.8675 (CI = 0.02), group 2:1 = 0.7475 (CI = 0.09), group 2:3 = 0.8325 (CI = 0.06), group 1:2 = 0.82 (CI = 0.05), group 3:2 = 0.87 (CI = 0.02). The results of the one-way ANOVA test revealed a significant disparity in mean accuracy rates among the groups (p=0.005). Further analysis with the Tukey HSD post hoc test indicated significant differences between group 1:1 and group 2:1 (p=0.008) and between group 2:1 and group 3:2 (p=0.007). All other comparisons were not meaningful. Therefore, there is a significant difference in accuracy rates between

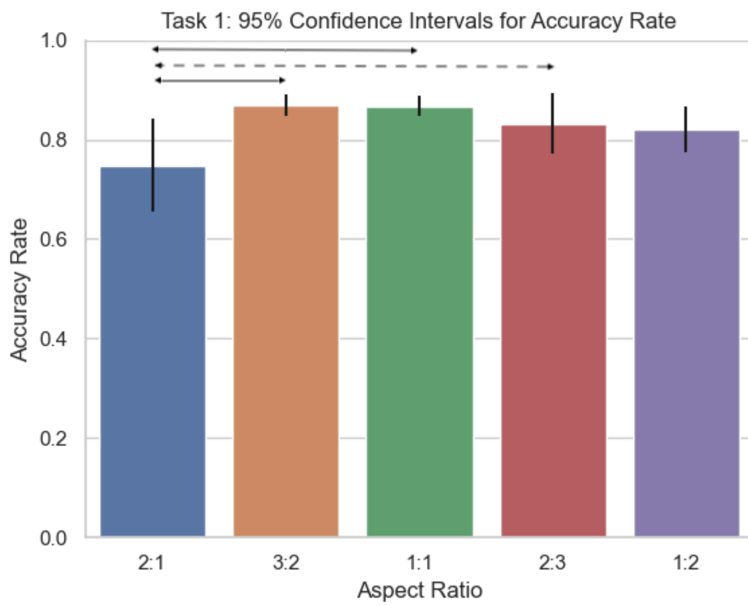

Figure 4.1: Task 1's 95% Confidence Intervals for Accuracy Rate. A dashed line indicates statistical significance for $0.05 < p < .1$, and a solid line indicates statistical significance for $p < .05$.

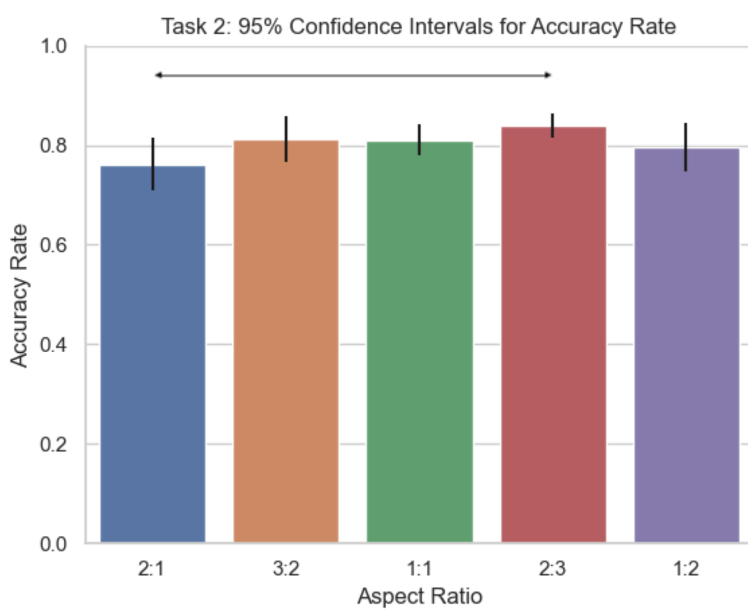

Figure 4.2: Task 2's 95% Confidence Intervals for Accuracy Rate. A solid line indicates statistical significance for $p < .05$.

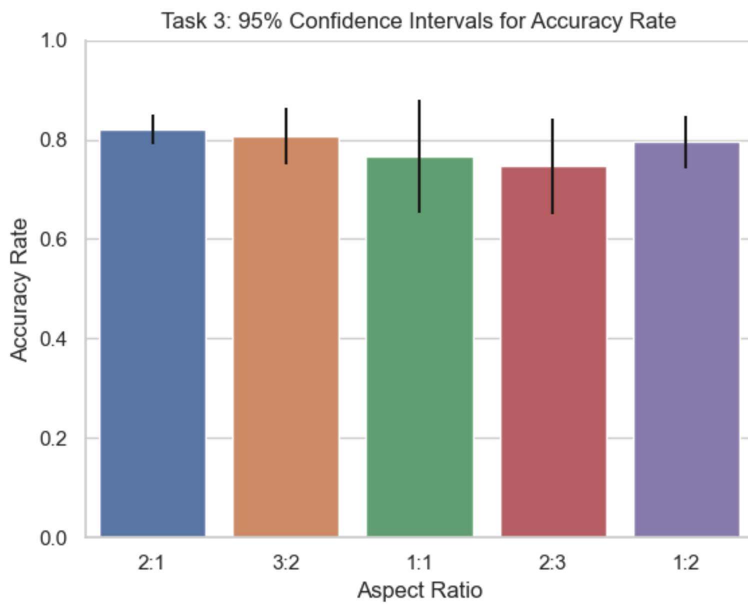

Figure 4.3: Task 3's 95% Confidence Intervals for Accuracy Rate

some groups but not all. Group 2:1 had the lowest mean accuracy rate, while group 3:2 had the highest.

In Task 2, the accuracy rate of participants was analyzed using ANOVA, which showed a non-significant effect of different scatterplot configurations on the accuracy rate (p=0.054). The Tukey HSD post-hoc test did not find any significant differences between the mean accuracy rates of the different scatterplot configurations. Therefore, the scatterplot configuration did not substantially affect the accuracy rate of participants in Task 2. In Task 3, we did not find any significant differences between the mean accuracy rates of the different scatterplot configurations.

Regarding the differences between groups for Task 4 (Figure 4.4), the Kruskal-Wallis test did not find evidence of a statistically significant difference ($p = 0.511$). Therefore, it is not possible to conclude that there is a substantial difference between the groups.

For Task 1 response time (Figure 4.5), the Kruskal-Wallis test was significant ($p < 0.001$),

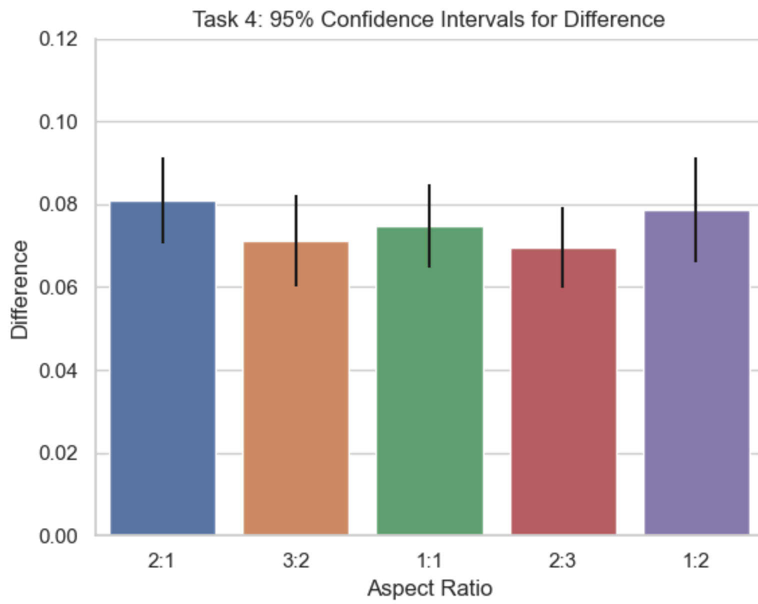

Figure 4.4: Task 4's 95% Confidence Intervals for Difference

indicating that there was a substantial difference in response times across conditions. To determine which conditions were significantly different from each other, post-hoc pairwise comparisons were conducted using Dunn's test with Bonferroni correction. Based on Dunn's test with Bonferroni correction, most conditions were significantly different from each other ($p < 0.05$).

For Task 2 (Figure 4.6), the Kruskal-Wallis test yielded a significant result ($p = 0.03$), indicating differences in response times among the five conditions. Upon conducting a post hoc examination using Dunn's test, a notable distinction was observed between condition 1 and condition 3 ($p < 0.05$). However, no other pairwise comparisons were meaningful after adjusting for multiple comparisons. Participants were slowest in condition 3 (2:3) and fastest in condition 1(1:1).

For Task 3 (Figure 4.7), the Kruskal-Wallis test showed a statistically significant difference in response time across the five interface variations ($p < 0.001$). To further explore this

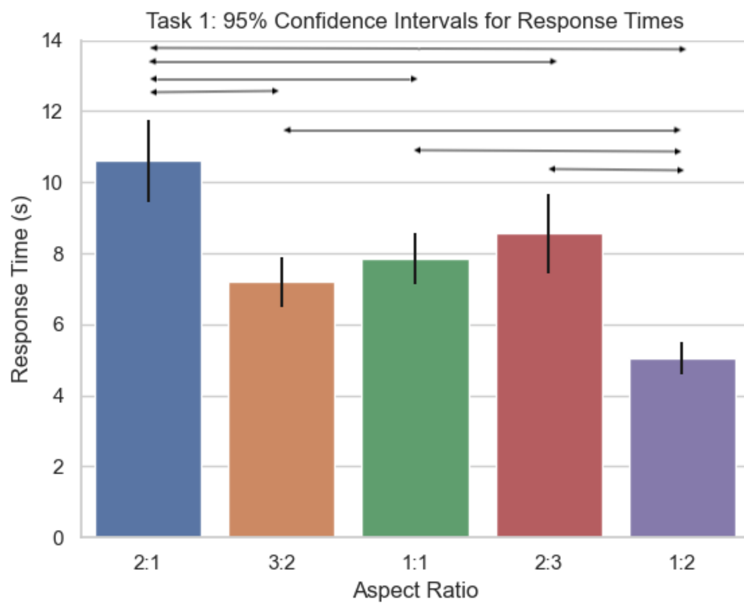

Figure 4.5: Task 1's 95% Confidence Intervals for Response Times. A solid line indicates statistical significance for $p < .05$.

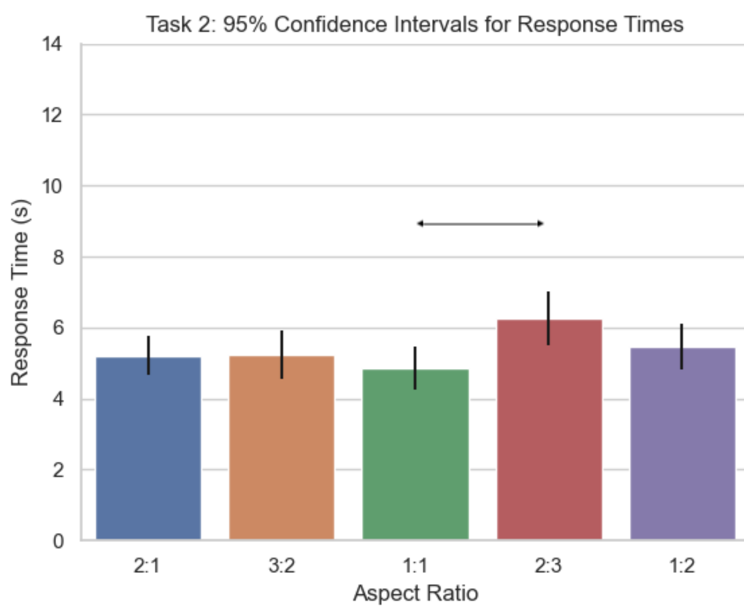

Figure 4.6: Task 2's 95% Confidence Intervals for Response Times. A solid line indicates statistical significance for $p < .05$.

difference, post-hoc pairwise comparisons using Dunn's test with Bonferroni correction were conducted. The results showed that there was a significant difference in response time between interface condition 1 (1:1) and condition 2 (2:1) ($p = 0.034$), condition 1 (1:1) and condition 4 (1:2) ($p = 0.006$), condition 2 (2:1) and condition 5 (3:2) ($p = 0.01$), condition 3 (2:3) and condition 4 (1:2) ($p = 0.034$) and condition 4 (1:2) and condition 5 (3:2) ($p = 0.002$).

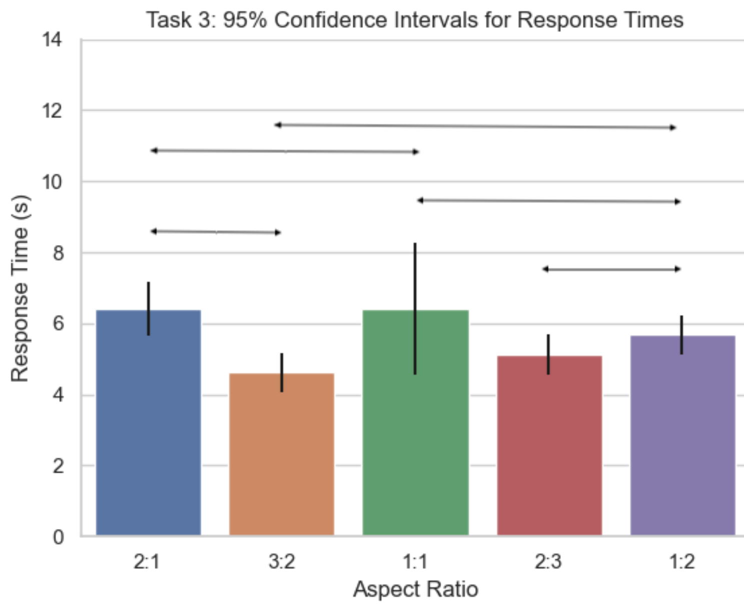

Figure 4.7: Task 3's 95% Confidence Intervals for Response Times. A solid line indicates statistical significance for $p < .05$.

For Task 4 (Figure 4.8), the Kruskal-Wallis test showed a marginally significant difference in response times across the different conditions ($p = 0.049$). Pairwise comparisons with Dunn's post-hoc test did not reveal any significant differences.

The Kruskal-Wallis test was conducted to determine if there were any significant differences in confident ratings between the five conditions in Task 1 (Figure 4.9). The test revealed a statistically significant difference between the groups ($p < 0.001$). A post hoc Dunn's test with Bonferroni correction was conducted to compare the pairs of conditions. The results

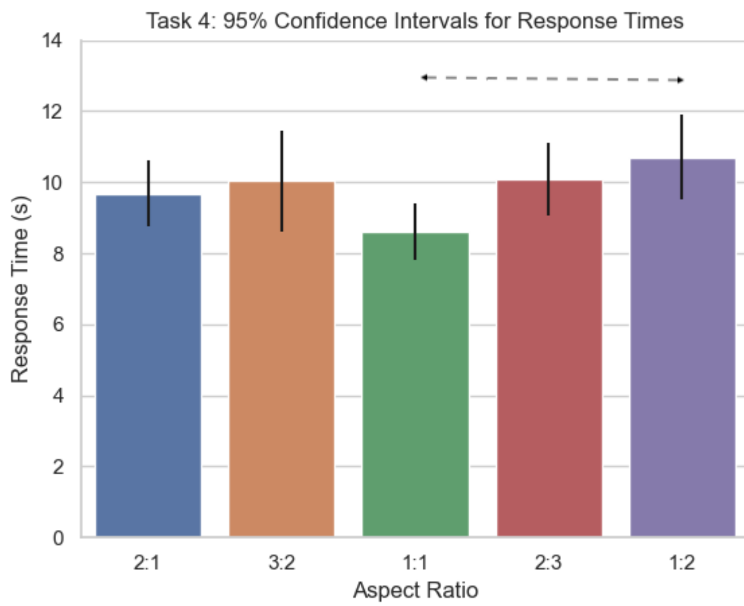

Figure 4.8: Task 4's 95% Confidence Intervals for Response Times. A dashed line indicates statistical significance for $0.05 < p < .1$.

showed significant differences between all teams of conditions except for conditions 1 (1:1) and 2 (2:1).

For the Confident rating part of Task 2 (Figure 4.10), the Kruskal-Wallis H test resulted in a statistically significant difference in confident ratings across the five conditions ($p < 0.001$). Post-hoc analysis using Dunn's test showed significant differences between condition 1 (1:1) and condition 4 (1:2) ($p = 0.003$), and condition 4 (1:2) and condition 5 (3:2) ($p < 0.001$).

For the confident rating part of Task 3 (Figure 4.11), the Kruskal-Wallis test showed a statistically significant difference between the groups ($p = 0.0027$), indicating that the groups rated their confidence levels differently. Post-hoc analysis could be conducted to determine which groups differed significantly from each other: condition 1 (1:1) and condition 5 (3:2) ($p= 0.002$).

For Task 4 (Figure 4.12), there were no significant differences between the confident ratings

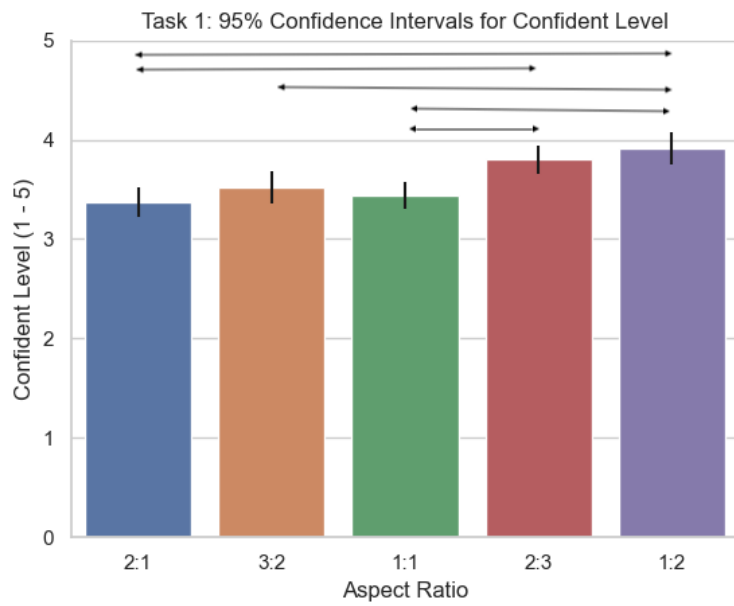

Figure 4.9: Task 1's 95% Confidence Intervals for Confident Level. A solid line indicates statistical significance for p < .05.

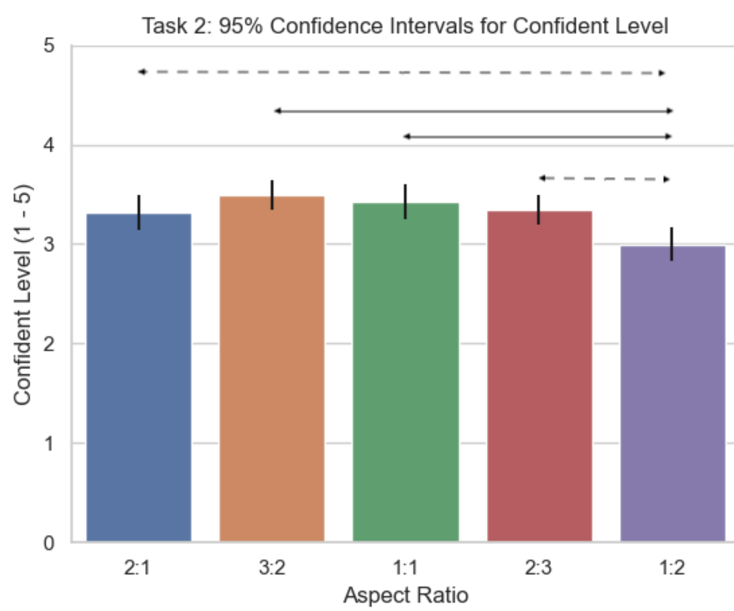

Figure 4.10: Task 2's 95% Confidence Intervals for Confident Level. A dashed line indicates statistical significance for 0.05 < p < .1, and a solid line indicates statistical significance for p < .05.

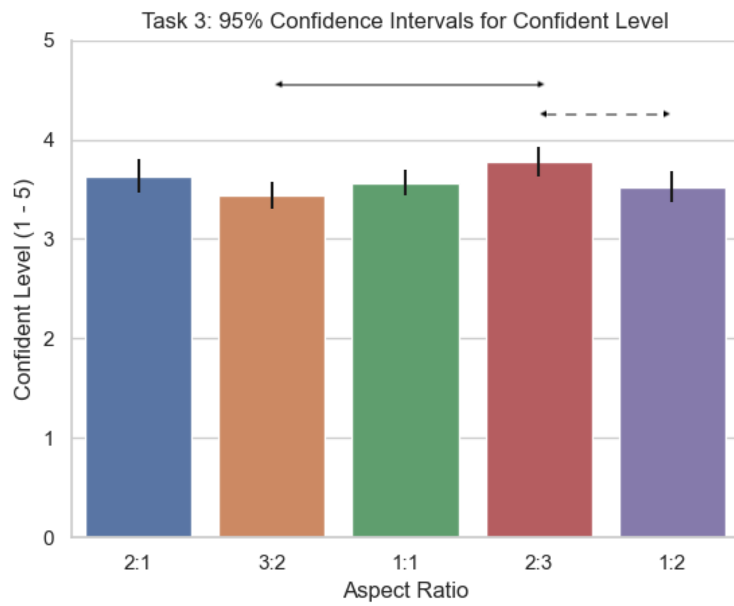

Figure 4.11: Task 3's 95% Confidence Intervals for Confident Level. A dashed line indicates statistical significance for $0.05 < p < .1$, and a solid line indicates statistical significance for $p < .05$.

of the five conditions.

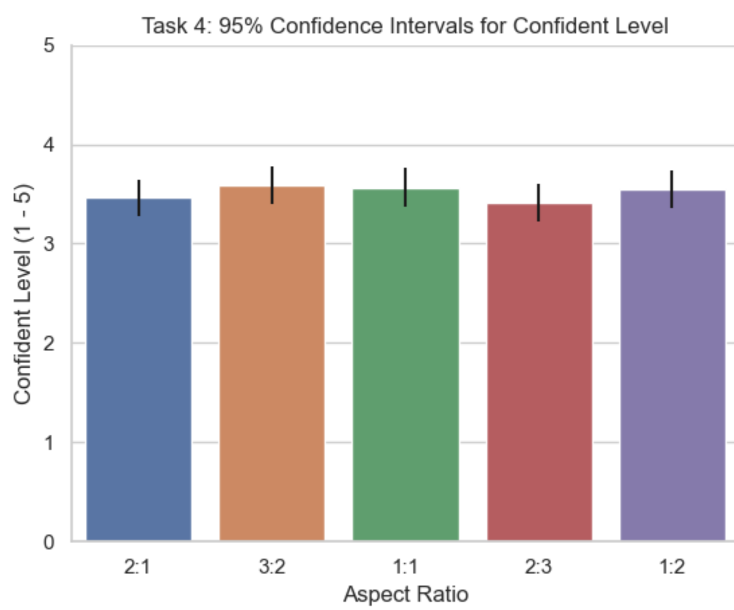

Figure 4.12: Task 4's 95% Confidence Intervals for Confident Level.

4.3 Qualitative Feedback

Based on the participants' feedback, they had mixed experiences interpreting scatterplots based on their aspect ratios. Some participants found it difficult to perceive the correlation between points in rectangular scatterplots at first glance, while others found it easier to find outliers. Some participants reported that the aspect ratio impacted their perception of the quantity of dots and their distance from one another. Participants also had varied opinions on whether square or rectangular ratios were easier to perceive, with some stating that square ratios were more manageable while others preferred rectangular proportions. Additionally, some participants reported that wider scatterplots were harder to identify correlations with, while others were fine with this. Overall, participants said that aspect ratio could impact their decision in the tasks, but the exact pattern of how it affected their decisions was unclear.

Most participants believe that the aspect ratios of scatterplots impact visual perception. Many participants noted that the rectangular aspect ratio can make it easier to spot outliers and make correlations more challenging to interpret. Some participants pointed out that the stretched distances between dots (depending on the ratio) can make it difficult to diagnose relationships. Other participants pointed out that the density of the data can be affected by the aspect ratio, making it easier or harder to find differences between points. Some participants needed clarification on whether aspect ratios impacted visual perception, and a few believed that it was more about personal preference or the density of the data rather than the aspect ratio. Overall, the responses suggest that aspect ratios can impact visual perception, which may vary depending on the specific task or data being presented.

Chapter 5

Discussion

The present study investigated the impact of aspect ratio on the visual perception of scatterplots in four different tasks: detecting further outliers, comparing the number of points, judging correlation strength, and estimating the correlation. We found that aspect ratios significantly impact visual perception and can affect participants' ability to complete these tasks accurately.

5.1 User Study Settings

The aspect ratio in the context of scatterplots refers to the ratio of the width of the plot to its height. It is essential to clarify that the aspect ratio is determined by the unit on the axis, not the range of the axis or the pixel values. The aspect ratio does not depend on the specific numerical values of the plotted data but rather on the relative scaling of the x-axis and y-axis.

In the context of the thesis, the aspect ratios were designed to maintain a consistent shape and relative scaling of the scatterplots, regardless of the specific data values. This means that the aspect ratios were applied to the range of the data values rather than pixel values. Therefore, normalization of the data was unnecessary since the specific numerical values or ranges of the data did not influence the aspect ratio. In this thesis, the aspect ratio was based on the unit of the axis and aimed to maintain consistent visual characteristics across different scatterplots.

Crowdsourcing platforms are a potential data collection method in this thesis. Crowdsourcing platforms offer the advantage of accessing a large and diverse pool of participants, which can enhance the generalizability of the findings [58]. Additionally, they provide a cost-effective and efficient way to collect data.

However, there are several reasons to refrain from using crowdsourcing platforms. Firstly, the thesis required participants to engage in tasks that required a certain level of familiarity and understanding of scatterplots. Using crowdsourcing platforms may introduce variability in participants' knowledge and experience with scatterplots, potentially impacting the quality and reliability of the data.

Secondly, ensure a controlled and standardized environment for data collection. Conducting the study in a controlled setting with recruited participants provided clear instructions, monitored the participants' progress, and promptly addressed any questions or concerns. This helped to minimize potential confounding factors and ensure the accuracy and consistency of the data.

Additionally, this thesis involved tasks that required a certain level of cognitive effort and concentration. Conducting these tasks on crowdsourcing platforms where participants may be distracted, or multitasking could compromise the validity of the results.

While crowdsourcing platforms have their benefits and are widely used in the study, conducting this study with recruited participants in a controlled environment allowed maintain the necessary level of rigor and control over the experimental conditions.

In task 1, this thesis focused on examining the effects of aspect ratios on visual perception and did not specifically investigate the correlation between aspect ratios and perceived distances. However, the changes in aspect ratios, which alter the scaling and proportions of the scatterplots, could impact the perceived distances between data points.

5.2 Accuracy Rates for Task 1 to 3

Based on the accuracy rates for Task 1, we can observe differences in participants' performance across different aspect ratios. The overall mean accuracy rate is 82.75%. Among the five aspect ratios, 3:2 has the highest mean accuracy rate of 87%(CI = 2%), followed by 1:1 (86.75%, CI = 2%), 2:3 (83.25%, CI = 6%), 1:2 (82%, CI = 5%), and 2:1 (74.75%, CI = 9%).

There is a noticeable variation in participants' performance when identifying outliers in scatterplots with different aspect ratios. The overall mean accuracy rate for Task 1 is 82.75%, indicating that participants were generally able to identify outliers successfully. Among the five aspect ratios, the 3:2 aspect ratio yielded the highest mean accuracy rate of 87%, followed by the 1:1 aspect ratio at 86.75%. These results suggest that participants found it relatively easier to identify outliers in scatterplots with these aspect ratios. In contrast, the 2:1 aspect ratio had the lowest mean accuracy rate at 74.75%, indicating that participants found it most challenging to identify outliers in scatterplots with a wider width-to-height ratio.

The findings highlight the importance of considering aspect ratio in designing scatterplots and interpreting the results. It is also important to note that individual differences in cognitive abilities and prior experience with different aspect ratios may affect participants' performance in these tasks.

Based on the accuracy rates for Task 2, the aspect ratio of scatterplots impacted participants' ability to complete the task accurately. The overall average accuracy rate was highest for the 2:3 aspect ratio and lowest for the 2:1 aspect ratio, and this is the same with Task 1.

However, the differences in accuracy rates between the aspect ratios were minor, suggesting that the impact of the aspect ratio may not be a significant factor in completing this task.

Interestingly, the 2:1 aspect ratio consistently had the lowest accuracy rate across all participants. This may indicate that this aspect ratio was particularly challenging for participants, and further research could explore why this is the case. Additionally, the 2:3 aspect ratio had the highest accuracy rate on average, which suggests that this aspect ratio may have specific benefits for visual perception and interpretation.

While the aspect ratio of scatterplots does not significantly impact accuracy rates for Task 2, it may still be a factor to consider in creating compelling visualizations for this type of task. Further research could investigate the specific visual perception mechanisms that are impacted by aspect ratio and how they affect task performance.

The accuracy rate for Task 3 is generally lower than for Task 1 and 2. This is expected as Task 3 requires participants to identify the stronger correlation between two scatterplots, which can be more challenging than simply identifying a specific pattern or outlier.

The aspect ratio of the scatterplots does not significantly impact the accuracy rate in this task, as the results across different aspect ratios are mixed. However, the highest accuracy rates are generally seen in the 2:1 and 1:2 aspect ratios, suggesting that these ratios may be slightly more favorable for this task. Opposite to Task 1 and 2, participants did better on 2:1 and 1:2 aspect ratios than on 2:3 and 3:2 aspect ratios.

The lower accuracy rate in Task 3 highlights the importance of careful consideration when designing scatterplots for specific tasks. In this case, identifying a stronger correlation may have been more difficult than anticipated and could benefit from additional training or instruction.

For all tasks, the accuracy rates between the 1:2 and 2:1 aspect ratios were similar, indicating that participants performed similarly in accuracy when comparing these two aspect ratios. Similarly, the accuracy rates between the 2:3 and 3:2 aspect ratios were also similar, suggesting that participants achieved comparable levels of accuracy when interpreting scatterplots with these aspect ratios.

5.3 Difference for Task 4

Based on the difference results for Task 4, participants did not perceive significant differences in scatterplots based on their aspect ratios. The difference rate values are relatively small and consistent across all aspect ratios. This finding contrasts Task 3, where participants were asked to input the correlation for scatterplots given to them. The results showed that aspect ratios did impact participants' perception of scatterplot correlations.

One possible explanation for this difference could be that the range of correlations in Task 4 was much more extensive, allowing participants to have a clearer sense of different levels of correlations regardless of aspect ratio. Additionally, Task 4 was focused on comparing scatterplots with varying numbers of correlations, which may have made the aspect ratio less salient compared to Task 3, where participants were asked to evaluate correlations directly.

5.4 Response Time

From the response time data, we can draw some conclusions about the efficiency and difficulty of each task.

For task 1, the average response time ranged from 2.58 seconds to 21.48 seconds across different aspect ratios. The average response time is 8.08s. The fastest response time was for the 1:2 aspect ratio, and the slowest was for the 2:1 aspect ratio. This indicates that task 1 was the most difficult for the participants to complete, especially for the 2:1 aspect ratio.

For task 2, the average response time ranged from 2.14 seconds to 10.55 seconds across different aspect ratios. The average response time is 5.43s. The fastest response time was for the 1:1 aspect ratio, and the slowest was for the 2:3 aspect ratio. This indicates that task 2 was relatively easy for the participants to complete, especially for the 1:1 aspect ratio.

For task 3, the average response time ranged from 2.6 seconds to 24.72 seconds across different aspect ratios. The average response time is 5.68s. The fastest response time was for the 2:3 aspect ratio, and the slowest was for the 1:1 aspect ratio. This indicates that task 3 was moderately challenging for the participants, with a high difficulty level for the 1:1 aspect ratio.

For task 4, the average response time ranged from 8.5 to 10.7 seconds across different aspect ratios. The response time was consistent across all aspect ratios. This indicates that task 4 was the most straightforward task for the participants, but it required more effort than the other tasks because they had to input numbers instead of just clicking.

Overall, the aspect ratio of scatterplots can significantly impact user performance and efficiency for different tasks. Tasks that require participants to identify outliers or correlations are more difficult for scatterplots with square aspect ratios. In contrast, tasks that require participants to choose scatterplots with more points or input correlation values are less affected by aspect ratio. These findings can help design more effective and efficient data visualization tools and interfaces.

5.5 Confident Level

Based on the confident level data, participants were more confident in their responses in Task 1 compared to other tasks. The scatterplot in Task 1 had only two outliers, and participants were asked to choose the one with a further distance from the cluster. As the scatterplot was less complex, participants could locate the outlier more efficiently and confidently.

For Task 2, participants had to choose a scatterplot with more points from two scatterplots. The scatterplots in this task were of similar complexity, and participants showed the least confidence in their responses. The scatterplots in this task had a more significant number of points compared to the scatterplots in Task 1, which made the task more complex.

Task 3 required participants to choose a scatterplot with a stronger correlation from two scatterplots. Participants were moderately confident in their responses for this task, compared to task 2. The participants in this task would only need to observe the correlations between two scatterplots, and they did not need an overall insight into them.

For Task 4, participants were required to input a correlation value after they saw the scatterplot with different aspect ratios. Participants mentioned they were less confident in this task than the previous ones, possibly because they had to input a numerical value rather than click on a visual element. However, the difference in correlation values for different aspect ratios was more minor than 0.085, and the confident rating is at a moderate level.

Overall, the results suggest that the complexity of the scatterplot and the type of task (clicking vs. inputting values) did not affect the participant's confidence in their responses much.

5.6 JND

The JND results for tasks 1-3 were provided (Table 5.1, 5.2, 5.3), indicating the minimum difference in aspect ratio that participants could detect between two scatterplots for each task.

For task 1, the JND values were consistent across all aspect ratios, with an average JND of 4. This suggests that participants could detect differences in outlier distances with similar accuracy across all aspect ratios.

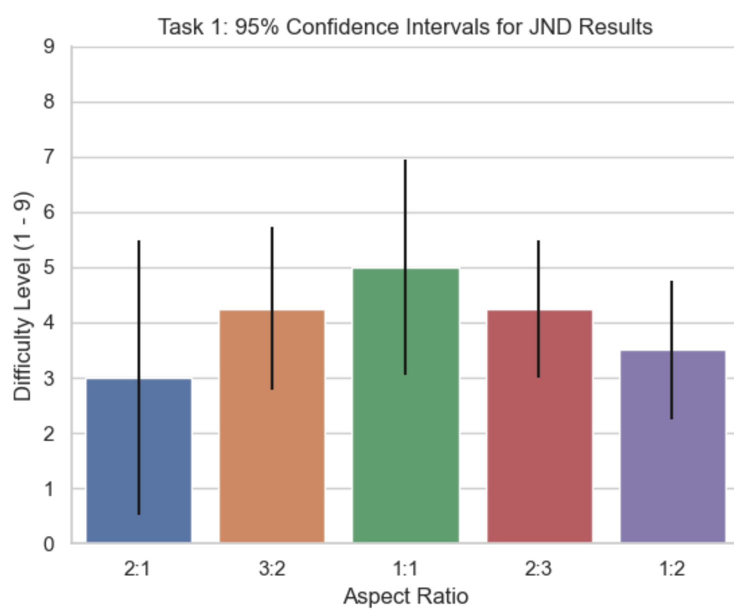

Figure 5.1: Task 1's 95% Confidence Intervals for JND Results.

For task 2, the JND values were consistent across all aspect ratios, with an average JND of 2.7. For task 3, the JND values varied across different aspect ratios, with an average JND of around 2.28. The participants did worse in those two tasks than in task 1.

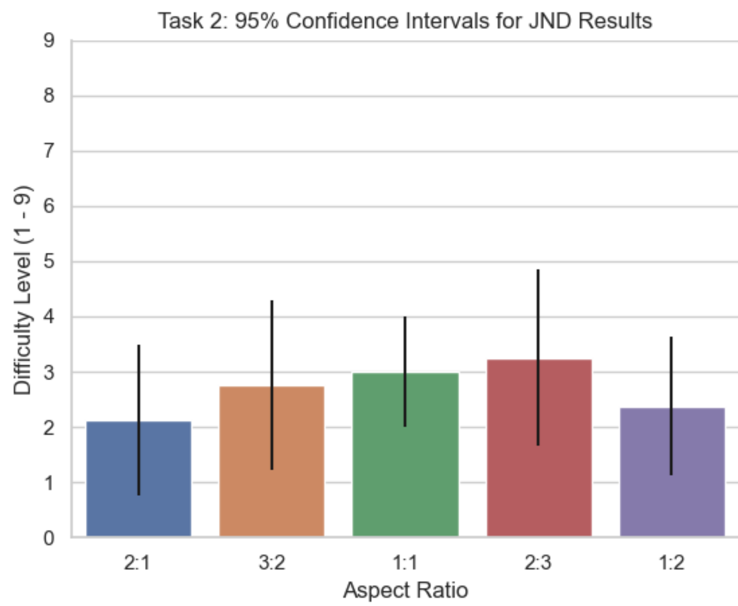

Figure 5.2: Task 2's 95% Confidence Intervals for JND Results.

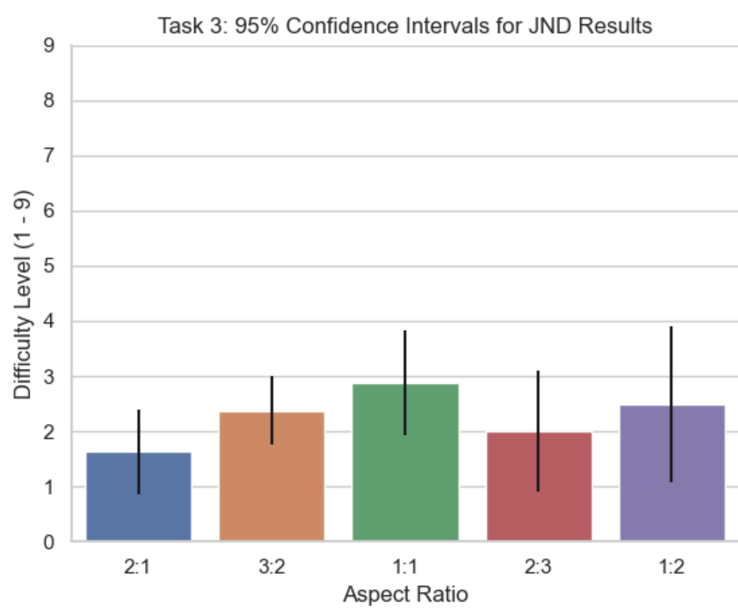

Figure 5.3: Task 3's 95% Confidence Intervals for JND Results.

5.7 Findings

Participants achieved a higher accuracy rate and spent less time on the 1:2 aspect ratio compared to the 2:1 aspect ratio in task 1(Figure 4.1, 4.5, 4.9). The 1:2 aspect ratio, with a narrower width and extended height, may provide a more elongated and vertically oriented scatterplot. This vertical orientation could make it easier for participants to distinguish the outliers further from the cluster visually. The elongated shape may enhance the vertical separation between the outliers and the cluster, making them more salient and easier to identify. On the other hand, the 2:1 aspect ratio, with a broader width and a shorter height, could compress the scatterplot horizontally. This compression may decrease the visual separation between the outliers and the cluster, making it more challenging for participants to detect the outliers accurately. Furthermore, the participants' reduced response time on the 1:2 scatterplots suggests they could make quicker and more confident judgments in identifying the outliers. The elongated shape of the 1:2 scatterplots facilitated rapid visual scanning and processing of the data, allowing participants to make accurate judgments more efficiently.

Challenging observation of outliers and amounts of points with rectangular aspect ratios. In task 1 and task 2, the rectangular aspect ratios (2:1 and 1:2) made it more challenging for participants to observe outliers and amounts of points in the data. On the other hand, the square aspect ratio (1:1) and moderate aspect ratios (2:3 and 3:2) facilitated easier identification of outliers and amounts of points. This finding aligns with previous research that suggests that square or moderately rectangular scatterplots provide a more balanced and perceptually accurate representation of the data [9]. The elongated aspect ratios in rectangular scatterplots may distort the relative distances between data points, leading to difficulty in detecting outliers and assessing correlations.

Better observation of correlations with rectangular aspect ratios. The accuracy rate for task 3, which involved participants choosing a scatterplot with a stronger correlation, varied depending on the aspect ratios. Participants performed better when the scatterplot had a rectangular aspect ratio (Figure 4.3). The elongated shape of the scatterplot might make it easier to visually identify the trend or pattern in the data, resulting in more accurate judgments of correlation strength.

More time cost with rectangular aspect ratios. With the data, which has a statistically significant difference in response time (Figure 4.5, 4.7), The average response time they were varied depending on the aspect ratios of the scatterplots. Participants spent more time when the scatterplot had a rectangular aspect ratio. This finding suggests that the elongated shape of rectangular scatterplots may increase the cognitive load and require additional mental effort to process the data accurately. The participants might need to mentally adjust the proportions of the scatterplot to compensate for the distorted distances between data points. This additional cognitive processing time can lead to longer response times in rectangular scatterplot tasks.

The complexity of the scatterplot and the type of task had a limited impact on participants' confidence in their responses. Regardless of the task complexity or the number of objects to observe, participants generally expressed a consistent level of confidence in their responses (Figure 4.9, 4.10, 4.11, 4.12). However, when participants were required to observe a larger number of objects or make more comprehensive judgments, their confidence levels tended to be slightly lower. Conversely, when the tasks involved observing a smaller number of data points or making simpler judgments, participants tended to express slightly higher confidence in their responses. These findings suggest that task complexity and the number of objects can subtly influence participants' confidence levels but do not have a substantial impact on their overall confidence ratings.

Positive correlation between JND and accuracy rates. The results from the JND measures and accuracy rates for tasks 1 to 3 exhibited a higher positive correlation [59]. This suggests that the JND results, which reflect participants' ability to detect differences in visual stimuli, align well with the accuracy rate, which measures the correctness of participants' judgments. The higher positive correlation between these measures indicates that the JND results effectively reflect participants' accuracy in performing the tasks. These findings suggest that users may not be benefit from using a rectangular aspect ratio when they need to identify distances and amounts in scatterplots. In contrast, when users need to detect correlations between variables in the scatterplots, using scatterplots with rectangular aspect ratios may be more preferred. These results underscore the importance of considering the aspect ratio of scatterplots in visualization design, as it can significantly impact users' visual perception and accuracy in interpreting visual information.

There is a lack of significant results in the JND results for task 3. The quality of the data collected for the JND task could have influenced the results. The data did not capture enough variation or nuance in participants' perceptual judgments, leading to less reliable or discernible differences between scatterplots. Also, individual differences in perception and cognitive processing among participants may contribute to the lack of significant results in the JND analysis. Participants may vary in their sensitivity to perceiving differences in correlation strength, with some being more attuned to subtle variations while others requiring more pronounced differences to detect changes. This variability in participants' perceptual acuity could reduce the statistical power of the JND analysis.

Chapter 6

Generalizations, Limitations and Future Work

6.1 Generalizations

The findings of this research paper provide valuable insights into the relationship between aspect ratios and visual perception in scatterplots. While the study focused on specific tasks and a particular participant group, the results can be generalized to broader contexts and populations with caution.

Firstly, the findings suggest that aspect ratios are crucial in perceiving scatterplots. The observed differences in accuracy rates, JND values, and confident ratings across different aspect ratios highlight the impact of visual design on task performance and interpretation. These findings can be applied to various domains where scatterplots are utilized, such as data visualization, scientific research, and decision-making processes. Secondly, staircase procedures and psychophysical measurements, such as JND [59], provide a practical methodology for investigating visual perception in scatterplots. Integrating quantitative measures and subjective ratings enhances our understanding of participants' perceptual sensitivity and preferences.

Furthermore, the selection of tasks in this research paper showcases the importance of considering different perceptual aspects when designing and evaluating scatterplots. The tasks focused on outlier detection, quantity estimation, correlation judgment, and aspect ratio perception, representing various cognitive processes in interpreting scatterplots. These task categories can guide future research and aid in developing standardized evaluation protocols for scatterplot designs.

However, it is essential to acknowledge the limitations of generalizing the findings. The study employed a specific set of aspect ratios and tasks, which may need to capture the range of real-world scenarios and user contexts fully. Additionally, the participant sample consisted of a specific demographic, and the findings may not directly apply to diverse populations or specialized user groups. Future research should replicate and extend these findings with more extensive and varied participant samples, considering variations in expertise, cultural backgrounds, and specific application domains.

In conclusion, this thesis contributes to understanding the relationship between aspect ratios and visual perception in scatterplots. The generalizations drawn from the study provide valuable insights for researchers, designers, and practitioners involved in data visualization, cognitive psychology, and human-computer interaction. By considering the impact of aspect ratios on task performance and perceptual sensitivity, future visual design and data visualization efforts can optimize scatterplot representations to enhance user understanding, decision-making, and overall user experience.

6.2 Limitations

There are several limitations to this thesis:

Firstly, the study only focused on four specific tasks related to scatterplots, which may only represent some types of visual tasks that individuals encounter daily. Different tasks may have different JND values and different aspect ratio preferences.

Secondly, the study only recruited participants from one specific population: college students. This may limit the generalizability of the results to other people, such as individuals from different age groups or cultural backgrounds.

Finally, the study used a relatively small sample size, which may limit the statistical power of the analysis. A larger sample size may be necessary to assess the relationship between aspect ratio and visual perception accurately.

6.3 Future Work

While this thesis sheds light on the relationship between aspect ratios and visual perception in scatterplots, several avenues for future work can further advance our understanding in this field.

6.3.1 Expansion of Aspect Ratio Range

This study focused on a specific range of aspect ratios for scatterplots. Future research could explore a broader range of aspect ratios, including extreme ratios [60], to examine how these variations impact visual perception and task performance. Additionally, investigating the effects of dynamic aspect ratios that adapt to user interactions [61] or data characteristics could provide insights into real-time visualization scenarios [62].

6.3.2 User-Centric Design

Considering user preferences and individual differences in visual perception is crucial for effective data visualization. Future studies could incorporate user-centered design [63] principles to evaluate the impact of aspect ratios on different user groups, including individuals with varying levels of expertise, cultural backgrounds, and cognitive abilities. Understanding how other users perceive and interact with scatterplots can inform the development of personalized visualization techniques [64].

6.3.3 Task Complexity and Contextual Factors

This thesis focused on specific tasks related to outlier detection, quantity estimation, correlation judgment, and aspect ratio perception. Future work can explore additional task types and their relationship to aspect ratios, such as trend identification, pattern recognition [65], and multivariate data analysis [66, 67]. Moreover, investigating the influence of contextual factors [68], such as data complexity [69], domain-specific requirements [70], and task goals, can provide valuable insights into the subtle effects of aspect ratios in different visualization contexts.

By addressing these future directions, researchers can further refine our understanding of the impact of aspect ratios on visual perception in scatterplots. The insights gained from such endeavors can inform the development of more effective and user-centered data visualization techniques, enhancing decision-making processes, knowledge discovery, and communication of complex data.

Chapter 7

Conclusions and Summary

In conclusion, this thesis investigated the relationship between aspect ratio and visual perception in scatterplots. The study included subjective and objective measurements to understand how the aspect ratio of a scatterplot affects users' abilities to perceive different visual properties, such as distance, amount, and correlation. The study results showed that aspect ratio plays a significant role in users' perception of scatterplots, with square (1:1) or moderate (2:3 or 3:2) aspect ratios being more effective for detecting distance and amount and rectangular (1:2 or 2:1) aspect ratios being more effective for detecting correlation. This could help readers avoid deceptive visualizations [71]. These findings provide valuable insights into the design and use of scatterplots and implications for data visualization practices in various fields.

The JND showed that smaller aspect ratios had a lower JND, indicating that participants were less able to detect differences in distance, amount, and correlation. The subjective measure, accuracy rate, also showed that participants performed better with square or moderate aspect ratios for detecting distance and amount in scatterplots and with rectangular aspect ratios for detecting correlation in scatterplots. Also, we may improve our JND approach in the future [72, 73].

Overall, the results of this study suggest that the aspect ratio of a scatterplot should be chosen based on the visual property that needs to be emphasized. In addition, the results of this study provide valuable information for designers and researchers interested in optimizing scatterplot design for better data visualization and analysis [13, 74, 75, 76, 77]. However, it is essential to note that this study has some limitations, such as the use of artificial stimuli and the limited sample size. Therefore, further research is needed to fully explore the impact of aspect ratio on visual perception in scatterplots with other features [50, 78].

Bibliography

[1] Michael Friendly and Daniel Denis. The early origins and development of the scatterplot. *Journal of the History of the Behavioral Sciences*, 41(2):103–130, 2005.

[2] Luana Micallef, Gregorio Palmas, Antti Oulasvirta, and Tino Weinkauf. Towards perceptual optimization of the visual design of scatterplots. *IEEE transactions on visualization and computer graphics*, 23(6):1588–1599, 2017.

[3] Thomas W Lauer and Gerald V Post. Density in scatterplots and the estimation of correlation. *Behaviour & Information Technology*, 8(3):235–244, 1989.

[4] Mark Last and Abraham Kandel. Automated detection of outliers in real-world data. In *Proceedings of the second international conference on intelligent technologies*, pages 292–301. InTech, 2001.

[5] Gabriel Humpire-Mamani, Agma Juci Machado Traina, and Caetano Traina. Fscoms: Feature selection of medical images based on compactness measure from scatterplots. In *2012 IEEE Second International Conference on Healthcare Informatics, Imaging and Systems Biology*, pages 86–95. IEEE, 2012.

[6] Derek D Rucker and Adam D Galinsky. Conspicuous consumption versus utilitarian ideals: How different levels of power shape consumer behavior. *Journal of Experimental Social Psychology*, 45(3):549–555, 2009.

[7] Jyoti Bala. Contribution of spss in social sciences research. *International Journal of Advanced Research in Computer Science*, 7(6), 2016.

[8] Yunhai Wang, Zeyu Wang, Chi-Wing Fu, Hansjörq Schmauder, Oliver Deussen, and Daniel Weiskopf. Image-based aspect ratio selection. *IEEE Transactions on Visualization and Computer Graphics*, 25(1):840–849, 2018.

[9] William S Cleveland. A model for studying display methods of statistical graphics. *Journal of Computational and Graphical Statistics*, 2(4):323–343, 1993.

[10] Jeffrey Heer and Michael Bostock. Crowdsourcing graphical perception: using mechanical turk to assess visualization design. In *Proceedings of the SIGCHI conference on human factors in computing systems*, pages 203–212, 2010.

[11] Martin Fink, Jan-Henrik Haunert, Joachim Spoerhase, and Alexander Wolff. Selecting the aspect ratio of a scatter plot based on its delaunay triangulation. *IEEE transactions on visualization and computer graphics*, 19(12):2326–2335, 2013.

[12] Oleg R Musin. Properties of the delaunay triangulation. In *Proceedings of the thirteenth annual symposium on Computational geometry*, pages 424–426, 1997.

[13] Yunhai Wang, Xin Chen, Tong Ge, Chen Bao, Michael Sedlmair, Chi-Wing Fu, Oliver Deussen, and Baoquan Chen. Optimizing color assignment for perception of class separability in multiclass scatterplots. *IEEE transactions on visualization and computer graphics*, 25(1):820–829, 2018.

[14] Ronald A Rensink and Gideon Baldridge. The perception of correlation in scatterplots. In *Computer Graphics Forum*, volume 29, pages 1203–1210. Wiley Online Library, 2010.

[15] Ghulam Jilani Quadri and Paul Rosen. Modeling the influence of visual density on cluster perception in scatterplots using topology. *IEEE Transactions on Visualization and Computer Graphics*, 27(2):1829–1839, 2020.

[16] Michael Gleicher, Michael Correll, Christine Nothelfer, and Steven Franconeri. Perception of average value in multiclass scatterplots. *IEEE transactions on visualization and computer graphics*, 19(12):2316–2325, 2013.

[17] Gabriel Strain, Andrew J Stewart, Paul Warren, and Caroline Jay. The effects of contrast on correlation perception in scatterplots. *International Journal of Human-Computer Studies*, page 103040, 2023.

[18] Jorge A Wagner Filho, Carla Maria Dal Sasso Freitas, and Luciana Nedel. Virtualdesk: A comfortable and efficient immersive information visualization approach. In *Computer Graphics Forum*, volume 37, pages 415–426. Wiley Online Library, 2018.

[19] Yalong Yang, Maxime Cordeil, Johanna Beyer, Tim Dwyer, Kim Marriott, and Hanspeter Pfister. Embodied navigation in immersive abstract data visualization: Is overview+ detail or zooming better for 3d scatterplots? *IEEE Transactions on Visualization and Computer Graphics*, 27(2):1214–1224, 2020.

[20] Jorge A Wagner Filho, Marina F Rey, Carla MDS Freitas, and Luciana Nedel. Immersive visualization of abstract information: An evaluation on dimensionally-reduced data scatterplots. In *2018 IEEE Conference on Virtual Reality and 3D User Interfaces (VR)*, pages 483–490. IEEE, 2018.

[21] Tamara Munzner. *Visualization analysis and design.* CRC press, 2014.

[22] Patrick S Hoey. Statistical analysis of the iris flower dataset. *University of Massachusetts At Lowell, Massachusetts*, 2004.

[23] Paul Craig. Interactive animated mobile information visualisation. In *SIGGRAPH Asia 2015 Mobile Graphics and Interactive Applications*, pages 1–6. 2015.

[24] Harald Piringer, Robert Kosara, and Helwig Hauser. Interactive focus+ context visualization with linked 2d/3d scatterplots. In *Proceedings. Second International Conference on Coordinated and Multiple Views in Exploratory Visualization, 2004.*, pages 49–60. IEEE, 2004.

[25] Alper Sarikaya and Michael Gleicher. Scatterplots: Tasks, data, and designs. *IEEE transactions on visualization and computer graphics*, 24(1):402–412, 2017.

[26] Johannes Kehrer and Helwig Hauser. Visualization and visual analysis of multifaceted scientific data: A survey. *IEEE transactions on visualization and computer graphics*, 19(3):495–513, 2012.

[27] Renata G Raidou, M Eduard Gröller, and Martin Eisemann. Relaxing dense scatter plots with pixel-based mappings. *IEEE transactions on visualization and computer graphics*, 25(6):2205–2216, 2019.

[28] William S Cleveland and Robert McGill. Graphical perception: Theory, experimentation, and application to the development of graphical methods. *Journal of the American statistical association*, 79(387):531–554, 1984.

[29] Anastasia Bezerianos, Fanny Chevalier, Pierre Dragicevic, Niklas Elmqvist, and Jean-Daniel Fekete. Graphdice: A system for exploring multivariate social networks. In *Computer graphics forum*, volume 29, pages 863–872. Wiley Online Library, 2010.

[30] Hongsen Liao, Yingcai Wu, Li Chen, and Wei Chen. Cluster-based visual abstraction for multivariate scatterplots. *IEEE transactions on visualization and computer graphics*, 24(9):2531–2545, 2017.

[31] William S Cleveland. *Visualizing data.* Hobart press, 1993.

[32] Yunhai Wang, Fubo Han, Lifeng Zhu, Oliver Deussen, and Baoquan Chen. Line graph or scatter plot? automatic selection of methods for visualizing trends in time series. *IEEE transactions on visualization and computer graphics*, 24(2):1141–1154, 2017.

[33] Edward R Tufte. The visual display of quantitative information. *The Journal for Healthcare Quality (JHQ)*, 7(3):15, 1985.

[34] Hadley Wickham and Heike Hofmann. Product plots. *IEEE Transactions on Visualization and Computer Graphics*, 17(12):2223–2230, 2011.

[35] Yunhai Wang, Zeyu Wang, Lifeng Zhu, Jian Zhang, Chi-Wing Fu, Zhanglin Cheng, Changhe Tu, and Baoquan Chen. Is there a robust technique for selecting aspect ratios in line charts? *IEEE Transactions on Visualization and Computer Graphics*, 24(12):3096–3110, 2017.

[36] Ben Shneiderman and Martin Wattenberg. Ordered treemap layouts. In *IEEE Symposium on Information Visualization, 2001. INFOVIS 2001.*, pages 73–78. IEEE, 2001.

[37] Sven Bachthaler and Daniel Weiskopf. Continuous scatterplots. *IEEE transactions on visualization and computer graphics*, 14(6):1428–1435, 2008.

[38] Eser Kandogan. Just-in-time annotation of clusters, outliers, and trends in point-based data visualizations. In *2012 IEEE Conference on Visual Analytics Science and Technology (VAST)*, pages 73–82. IEEE, 2012.

[39] Christopher G Healey and James T Enns. Large datasets at a glance: Combining textures and colors in scientific visualization. *IEEE transactions on visualization and computer graphics*, 5(2):145–167, 1999.

[40] Stephen Palmer and Irvin Rock. Rethinking perceptual organization: The role of uniform connectedness. *Psychonomic bulletin & review*, 1(1):29–55, 1994.

[41] Barbara Tversky and Kathleen Hemenway. Categories of environmental scenes. *Cognitive psychology*, 15(1):121–149, 1983.

[42] Donald A Norman. Affordance, conventions, and design. *interactions*, 6(3):38–43, 1999.

[43] Sergio Cesare Masin, Verena Zudini, and Mauro Antonelli. Early alternative derivations of fechner's law. *Journal of the History of the Behavioral Sciences*, 45(1):56–65, 2009.

[44] Stanley S Stevens. On the psychophysical law. *Psychological review*, 64(3):153, 1957.

[45] Harry T Lawless, Hildegarde Heymann, Harry T Lawless, and Hildegarde Heymann. Physiological and psychological foundations of sensory function. *Sensory evaluation of food: principles and practices*, pages 19–56, 2010.

[46] Xiaokang Yang, Weisi Lin, Zhongkhang Lu, EePing Ong, and Susu Yao. Motion-compensated residue preprocessing in video coding based on just-noticeable-distortion profile. *IEEE Transactions on Circuits and Systems for Video Technology*, 15(6):742–752, 2005.

[47] Nikil Jayant, James Johnston, and Robert Safranek. Signal compression based on models of human perception. *Proceedings of the IEEE*, 81(10):1385–1422, 1993.

[48] Madison A Elliott, Christine Nothelfer, Cindy Xiong, and Danielle Albers Szafir. A design space of vision science methods for visualization research. *IEEE Transactions on Visualization and Computer Graphics*, 27(2):1117–1127, 2020.

[49] GB Wetherill and H Levitt. Sequential estimation of points on a psychometric function. *British Journal of Mathematical and Statistical Psychology*, 18(1):1–10, 1965.

[50] Haidong Chen, Wei Chen, Honghui Mei, Zhiqi Liu, Kun Zhou, Weifeng Chen, Wentao Gu, and Kwan-Liu Ma. Visual abstraction and exploration of multi-class scatterplots. *IEEE Transactions on Visualization and Computer Graphics*, 20(12):1683–1692, 2014.

[51] Min Lu, Shuaiqi Wang, Joel Lanir, Noa Fish, Yang Yue, Daniel Cohen-Or, and Hui Huang. Winglets: Visualizing association with uncertainty in multi-class scatterplots. *IEEE transactions on visualization and computer graphics*, 26(1):770–779, 2019.

[52] Rassadarie Kanjanabose, Alfie Abdul-Rahman, and Min Chen. A multi-task comparative study on scatter plots and parallel coordinates plots. In *Computer Graphics Forum*, volume 34, pages 261–270. Wiley Online Library, 2015.

[53] Andy Field, Jeremy Miles, and Zoë Field. *Discovering statistics using R*. Sage publications, 2012.

[54] Russell V Lenth. Least-squares means: the r package lsmeans. *Journal of statistical software*, 69:1–33, 2016.

[55] Maurice S Bartlett. The square root transformation in analysis of variance. *Supplement to the Journal of the Royal Statistical Society*, 3(1):68–78, 1936.

[56] Patrick E McKight and Julius Najab. Kruskal-wallis test. *The corsini encyclopedia of psychology*, pages 1–1, 2010.

[57] Dulce G Pereira, Anabela Afonso, and Fátima Melo Medeiros. Overview of friedman's test and post-hoc analysis. *Communications in Statistics-Simulation and Computation*, 44(10):2636–2653, 2015.

[58] Mokter Hossain. Users' motivation to participate in online crowdsourcing platforms. In *2012 International Conference on Innovation Management and Technology Research*, pages 310–315. IEEE, 2012.

[59] Varshita Sher, Karen G Bemis, Ilaria Liccardi, and Min Chen. An empirical study on the reliability of perceiving correlation indices using scatterplots. In *Computer Graphics Forum*, volume 36, pages 61–72. Wiley Online Library, 2017.

[60] Michael J Feldman. Spiked singular values and vectors under extreme aspect ratios. *Journal of Multivariate Analysis*, page 105187, 2023.

[61] Tom Horak, Andreas Mathisen, Clemens N Klokmose, Raimund Dachselt, and Niklas Elmqvist. Vistribute: Distributing interactive visualizations in dynamic multi-device setups. In *Proceedings of the 2019 CHI conference on human factors in computing systems*, pages 1–13, 2019.

[62] Yalong Yang, Tim Dwyer, Kim Marriott, Bernhard Jenny, and Sarah Goodwin. Tilt map: Interactive transitions between choropleth map, prism map and bar chart in immersive environments. *IEEE Transactions on Visualization and Computer Graphics*, 27(12):4507–4519, 2020.

[63] Chadia Abras, Diane Maloney-Krichmar, Jenny Preece, et al. User-centered design. *Bainbridge, W. Encyclopedia of Human-Computer Interaction. Thousand Oaks: Sage Publications*, 37(4):445–456, 2004.

[64] Xi Chen, Zibin Zheng, Xudong Liu, Zicheng Huang, and Hailong Sun. Personalized qos-aware web service recommendation and visualization. *IEEE Transactions on Services Computing*, 6(1):35–47, 2011.

[65] Tuan Nhon Dang and Leland Wilkinson. Scagexplorer: Exploring scatterplots by their scagnostics. In *2014 IEEE Pacific visualization symposium*, pages 73–80. IEEE, 2014.

[66] Joseph F Hair. Multivariate data analysis. 2009.

[67] Brian S Everitt, Graham Dunn, et al. *Applied multivariate data analysis*, volume 2. Wiley Online Library, 2001.

[68] David Borland, Wenyuan Wang, and David Gotz. Contextual visualization. *IEEE Computer Graphics and Applications*, 38(6):17–23, 2018.

[69] David Gotz and David Borland. Data-driven healthcare: challenges and opportunities for interactive visualization. *IEEE computer graphics and applications*, 36(3):90–96, 2016.

[70] Matthew Brehmer and Tamara Munzner. A multi-level typology of abstract visualization tasks. *IEEE transactions on visualization and computer graphics*, 19(12):2376–2385, 2013.

[71] Anshul Vikram Pandey, Katharina Rall, Margaret L Satterthwaite, Oded Nov, and Enrico Bertini. How deceptive are deceptive visualizations? an empirical analysis of common distortion techniques. In *Proceedings of the 33rd annual acm conference on human factors in computing systems*, pages 1469–1478, 2015.

[72] Qiuping Jiang, Zhentao Liu, Shiqi Wang, Feng Shao, and Weisi Lin. Toward top-down just noticeable difference estimation of natural images. *IEEE Transactions on Image Processing*, 31:3697–3712, 2022.

[73] Stephen Smart and Danielle Albers Szafir. Measuring the separability of shape, size, and color in scatterplots. In *Proceedings of the 2019 CHI Conference on Human Factors in Computing Systems*, pages 1–14, 2019.

[74] Connor C Gramazio, Karen B Schloss, and David H Laidlaw. The relation between visualization size, grouping, and user performance. *IEEE transactions on visualization and computer graphics*, 20(12):1953–1962, 2014.

[75] Xin Chen, Tong Ge, Jian Zhang, Baoquan Chen, Chi-Wing Fu, Oliver Deussen, and Yunhai Wang. A recursive subdivision technique for sampling multi-class scatterplots. *IEEE transactions on visualization and computer graphics*, 26(1):729–738, 2019.

[76] Florian Heimerl, Chih-Ching Chang, Alper Sarikaya, and Michael Gleicher. Visual designs for binned aggregation of multi-class scatterplots. *arXiv preprint arXiv:1810.02445*, 2018.

[77] Deokgun Park, Sung-Hee Kim, and Niklas Elmqvist. Gatherplots: generalized scatterplots for nominal data. *arXiv preprint arXiv:1708.08033*, 2017.

[78] Adrian Mayorga and Michael Gleicher. Splatterplots: Overcoming overdraw in scatter plots. *IEEE transactions on visualization and computer graphics*, 19(9):1526–1538, 2013.