THE APPLICATION OF DIMENSIONAL AND STATISTICAL ANALYSIS TO FLUIDIZATION STUDIES

by

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I. INTRODUCTION

The application of the principles of fluidization to the cracking of petroleum hydrocarbons has been one of the most remarkable developments in the petroleum industry in recent years. By use of fluidization, whereby solid particles are maintained in a turbulent or pseudo-liquid state by the moving fluid, the national production and quality of gasoline and other petroleum products was greatly increased during World War II when their demand was at its peak.

The wise use and application of fluidization today has made further study on this unit operation necessary. The important physical variables in fluidization are the characteristics of the fluidized solid, fluidizing medium, and the retaining vessel. In spite of the effect of these physical variables in fluidization some important information pertaining to fundamental relationships between these variables and the operational variables of the system is still lacking.

The principal developments in fluidization in recent years have been directed primarily along studies of fluid flow through the fluidized bed. At the same time, however, the influence of mass velocity on the pressure drop through the system has led to quantitative research on this phase of fluidization. In many of these investigations the application of dimensional and statistical analysis have proved effective in the correlation of the results.

The purpose of the investigation was to correlate the pressure drop across a fluidized bed of ottawa sand with the variables of bed height, bed diameter, and particle size by dimensional and statistical means.

II. LITERATURE REVIEW

Fluidization as a chemical process is rather recent, dating back only to 1942 when the application of fluidized-solids techniques in the petroleum industry paved the way for its use in many related processes. Ever since the application of fluidized-solids techniques proved effective in the production of high-octane gasoline, further application and research into the fundamentals of fluidized-solid techniques has been, and is being, investigated. The following review of the literature attempts to familiarize the reader with the more important principles and variables of fluidized systems and the relating of these principles and variables through the application of dimensional analysis and statistical analysis.

Industrial Applications of Fluidization

Fluidization of solids has been termed the intermediate operation between the flow of solids through fluids and the flow of fluids through solids(4). The use of a fluidized system for the conveying of solids(4,19) is perhaps the oldest physical use of fluidized-solids techniques. Yet, with the increasing complexity of many chemical processes the application of fluidized-solids techniques has proved to be very satisfactory in increasing industrial yields, lowering existing operational costs, and providing for the applicability of continuous operational systems. The following is a list of some of the more important applications of fluidized-solid techniques in chemical industry: (1) distillation of hardwood, (2) catalytic cracking of hydrocarbons[40,53,60], (3) manufacture of activated carbon[16,18], (4) production of phthalic anhydride, (5) calcination of limestone[19], (6) gasification of coal, (7) evaporation of solids[58], (8) synthesis of hydrocarbons from natural gas[53], (9) fractionation of gases using solid adsorbents[58], and (10) drying of gases by hot fluidized-solids[58].

Fluidization Terminology

The increasing application of fluidized-solids techniques, both in industry and research, has resulted in a wide and diversified number of definitions of the phases and characteristics of fluidized systems. Recently, however, attempts[14] have been made to standardize the nomenclature commonly employed in descriptions of fluidized systems. The following list of nomenclature on fluidization is presented so as to provide the reader with the foundation for the proper understanding of the terms used throughout the course of this investigation.

Fluidization. Fluidization has been defined as the unit operation in which a mass of finely divided solids is maintained in a turbulent dense state by being dispersed in an upward moving gas stream[24]. Nicholson and Moise[47] define fluidization as the state which exists when a gas separates and supports the particles of a bed and provides mobility and fluidity for the mass of suspended particles. According to Wilhelm and Kwauk[64] fluidization may be divided into two modes of fluidization, namely particulate and aggregative.

Fluidized Mass. A fluidized mass[14] of solid particles is one which exhibits the mobility and hydrostatic pressure characteristics of a fluid. Investigators[17] studying the various phases and states of a fluidized mass of solid particles have drawn analogies between the states of a fluidized mass and the three physical states of matter.

Fluidized Bed. A fluidized bed[14] is a mass of solid particles which exhibit the liquid-like characteristics of mobility, hydrostatic pressure, and an observable upper free surface or boundary zone across which a marked change in the concentration of the particles occurs. A fluidized bed is characterized by the presence of two distinct phases[13,60],

(1) a continuous phase, and (2) a discontinuous phase. The continuous phase is characterized by the uniform distribution of the solid particles in the supporting gas stream, while the discontinuous phase is represented by the formation of bubbles, slugs, or pockets of gas moving upward through the bed of solids. Thomas and Hoekstra[59] in similar investigations state that a fluidized bed may be produced merely by the passage of gases and vapors upward through powdered solids.

Incipient Fluidization. Incipient fluidization occurs when the particles of the bed are just suspended in the upward gas stream of the fluidizing gas[60].

Quiescent Fluidized Bed. A quiescent fluidized bed is a dense fluidized bed which exhibits little or no mixing of the solid particles of the bed[14].

Dispersed Suspension. A dispersed suspension[14] is a mass of solid particles or aggregates suspended in a current of liquid or gas rising past the particles. This suspension which differs from a fluidized bed in that an upper level or interface is not formed under conditions of continuous solids entrainment and uniform superficial velocity.

Channeling. Channeling[14] is the establishment of flow paths in a bed of solid particles through which a disproportionate quantity of the introduced fluid passes. Channeling is likely to occur most frequently at low fluid mass velocities[22].

Slugging. Slugging[14] is a condition in which pockets or bubbles of the supporting fluid grow to the diameter of the containing vessel and the mass of particles trapped between adjacent pockets move upward in a pistonlike fashion. Slugging is especially common in retaining vessels of small diameter-to-length ratios[22].

Principles of Fluidization

Fluidization occurs when the fluidizing gas or medium separates and supports the particles, and provides mobility and fluidity for the mass of suspended particles[47]. It has been observed, however, that the passage of a gas through a bed of solids produces certain physical changes in the bed, these changes being referred to as stages in fluidization.

Stages in Fluidization. It has been observed[43] that a definite relationship exists between the mass velocity of the fluidizing medium and the pressure drop through the fluidized bed. Studies have revealed that the pressure drop through a fluidized bed is related to the various stages that occur when the fluid velocity through the bed is increased and then decreased. Miller and Logwinuk[43] investigated the relationship between pressure drop and mass velocity on silicon carbide, sand, aluminum oxide, and silica gel. Referring to Figure 1, page 9, it is observed that as the fluid velocity is increased slightly a corresponding pressure drop occurs through and over the bed of solids; the pressure drop increasing linearly with gas flow between points 1 and 2. Point 2 corresponds to a pressure drop equal to the weight of the solids divided by the cross-sectional area of the column. From point 2 to point 3 the bed begins to expand, until at point 3 the bed has expanded sufficiently to establish fluidization. From point 3 to point 4 a small increase in the mass velocity results in a decrease in the pressure drop for the system. A further increase in the mass velocity of the fluid causes an increase in the pressure drop through the bed until point 5 is reached, corresponding to the pressure drop at which fluidization begins. When the mass velocity is increased further, past point 5, the bed becomes more vigorously fluidized. It has been shown that the pressure drop-velocity relationship would follow curve 5-6 or 5-8 for small particles. As the mass velocity is gradually decreased the pressure drop-velocity curve follows points 5-2A-1A. The curve 1-2 cannot be reproduced; the displacement, d, depending on the initial packing of the bed. The mass velocity required for fluidization to begin is shown at the intersection of O of the lines extending from curves

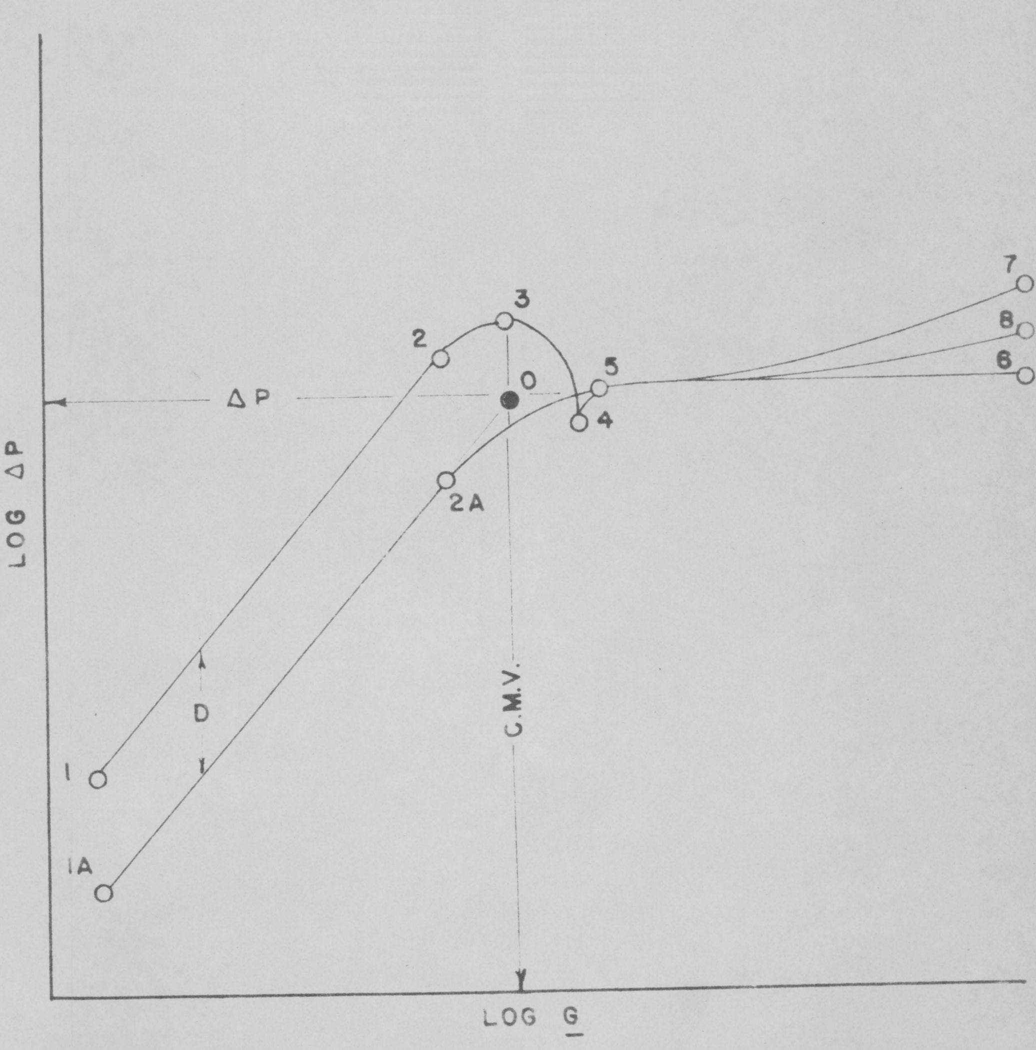


FIGURE I. VARIATION OF PRESSURE DROP ACROSS BEDS OF SOLID PARTICLES WITH CHANGES IN FLUID MASS VELOCITY

MILLER, C.O. AND A.K. LOGWINUK: FLUIDIZATION STUDIES OF SOLID PARTICLES, IND. ENG. CHEM., 43, 1220 (1951).

5-6 and 1A-2A. The mass velocity required for fluidization to begin, or the critical mass velocity, is important, for below this velocity no fluidization of the solid particles of the bed will occur.

Behavior of Bed of Solids. Inasmuch as the critical mass velocity is the boundary condition for the occurrence of fluidization in beds of solid particles, it is necessary to investigate the behavior of the bed at mass velocities less than, equal to, and greater than the critical mass velocity.

Mass Velocity Less than Critical Mass Velocity. Leva[25] stated that if the fluidizing medium is admitted to the bed of solids at a very low rate a small pressure drop will occur, increasing linearly with increasing mass velocity until the critical mass velocity of the fluidizing medium is obtained. In a similar investigation, Resnick and White[54] found that with low air velocities there is no observable motion of the bed. Wilhelm and Kwauk[62] report that increasing the velocity of the fluidizing medium leads to a continually expanding bed, until, at the ultimate limit, particles are so widely separated that they behave individually rather than as a part of the bed, or in short, they are fluidized. All of these investigations indicate that at gas velocities less than the critical mass velocity the gas merely percolates upward through the bed without agitating the individual particles of the bed and the pressure drop through the bed is less than the equivalent weight of the solids that compose the bed.

Mass Velocity Equal to Critical Mass Velocity. In investigations on round and sharp sands, Leva[26] reported that as the fluid mass velocity increased the bed expanded until the critical mass velocity was attained. At this velocity, the bed expanded until the individual particles became disengaged sufficiently from each other to permit internal motion of the bed. Wilhelm and Kwauk[62] state that at the critical mass velocity the particles of the bed are fully supported by the fluidizing medium. In their investigation it was observed that an increase in the mass velocity led to a continually expanding bed, until the particles were so widely separated that they behaved individually rather than as a unified bed.

Mass Velocity Greater than Critical Mass Velocity. At very high velocity rates[26], large bubbles usually force their way upward through beds of solid particles. Miller and Logwinuk[43] state that during velocity increase, bubbles of gas coalesced at times and bubbled through the fluidized material, analogous to a gas bubble passing through a liquid.

At still greater gas flow, these bubbles extended from one wall of the column to the other. As the velocity became more violent[26,44,54] the slugs or streams of powdered material were lifted from the bed. Leva[26] reported that for a slugging bed the pressure drop through the bed fluctuated between wide limits, but in normally fluidized beds the pressure drop remained essentially constant.

Characteristics of Fluidized Beds

In the following paragraphs will be found a resume of some of the more important characteristics of fluidized beds. Included in the review will be the individual effect of each of the variables on the character of a fluidized bed and the mathematical relationship between the variables and the fluidized state.

Bed Expansion. Leva[33] has indicated that a certain amount of expansion of the bed of solids is necessary before fluidization can exist. The maximum expansion of a bed of solids without slugging occurs at approximately the critical mass velocity. Ergun, Sabri, and Orning[12] state that a definite relationship exists between bed expansion and fraction voids under conditions of bed expansion at high fluid mass velocity. According to Leva[31,41], to expand a bed of solids requires a certain amount of energy which is less than that required to expand and fluidize the bed. Also, in expanded beds the particles do not change position with respect to each other but do require additional energy to maintain the internal motion of the particles in the bed. At the same time, it has been shown[30] that mere expansion was insufficient in many cases to permit fluidization of the solids of the bed. However, tests have indicated that the expansion of a bed depends entirely on the original static bed density.

Solids Entrainment. Entrainment of the solid particles of a bed of solids at high fluid mass velocities has been studied by several investigators. Sittig[56] confirms the fact that high fluid mass velocities are required for appreciable solid entrainment in that there is very little solid entrainment initially in fluidizing a bed of solid particles. According to Parent, Yagol, and Steiner[48], the entrainment of solids increases with increasing air flow rate and when the air velocity becomes very high the boundary layer disappears and heavy entrainment becomes apparent.

Bed Height. Very little information is available relative to the effect of the bed height on the other variables encountered in fluidized systems. Miller and Logwinuk[44], Morse and Ballou[45], and Wilhelm and Kwauk[63] all state that the height of the solids in the bed does not have any appreciable effect on such variables as mass velocity, heat transfer, and pressure drop. Resnick and

White[55] agree that the height of the bed has little or no effect upon the heat transfer through the solids. According to Leva[32] and Parent, Yagol, and Steiner[49], the only major effect of bed height is in relation to slugging through the bed, the greater the height of the bed of solids, the greater the chance of slugging.

Slugging. Slugging in fluidized bed occurs in tall, small diameter beds[22]. Leva[26] reports that in small diameter beds, functioning at high gas rates, bubbles of the gas coalesce and form a gas slug which usually extends over the entire cross-section of the retaining vessel. Under these conditions, it has been observed that the pressure manometers in the system fluctuate over wide limits. In further investigations[29], Leva also reports that for most investigations the operation of a fluidized system under slugging conditions is not practical. Yet, in the case of heat transfer, it was observed that the heat transfer coefficients obtained under conditions of slugging did not vary appreciably from those obtained under conditions of smooth fluidization.

Channeling. Leva[26] states that channeling can be shown to depend mainly on four factors: (1) moisture in the bed, (2) diameter of bed, (3) rate of gas flow, and (4) diameter of particles in bed. Tests have shown that channeling is restricted to low mass velocity flow in which case the gas ebbs upward through the spaces around and between the particles of the bed. Thomas and Hoekstra[59] found that with initial slow increase in the mass velocity of the fluidizing medium small channels appear in the bed of solids, accompanied by a slight bed expansion. At high mass velocities the agitation of the bed is more intense than at low velocities and as a result the agitation destroys the channels as soon as they are formed in the bed[27]. Although the reason for increased channeling through fine particles is not clearly understood, investigations have shown that an increase in channeling with decrease in particle diameter does take place.

Pressure Drop. The pressure drop through a packed bed is caused chiefly by the expansion and contraction of the fluids passing into and through the voids of the bed[38]. Belden and Kassel[2] report that the pressure drop through a bed of solids is due to two factors, namely, a static and a friction factor which account for the entire pressure drop through the bed. These two investigations and many others[5,20,21,34,57] have related and correlated pressure drop with many other variables such as surface roughness, particle diameter, fraction voids, and fluid mass velocity. Leva[36] has led the investigations on the effect of the many variables on pressure drop. He has reported that the total pressure drop through a bed is a result of the pressure drop through the packing plus the pressure drop caused by the presence of the retaining wall of the vessel. In an interesting investigation, Parent, Yagol, and Steiner[48] reported that the pressure drop through fixed-bed systems is equivalent to the suspension head opposing the flow regardless of the nature of the gas or solids or the flow rate, provided smooth fluidization is obtained. Studies on tube diameter and fraction voids has resulted in Fujita and Uchida[15] reporting that for a constant vessel diameter and free-fraction void content, a decrease in the diameter of the vessel tends to increase the pressure drop across the length of the vessel. Toomey and Johnstone, in their studies on the gaseous fluidization of solid particles[60], found that a considerable amount of energy is dissipated as a result of the collisions of the particles of the bed in their fluidized state and that this energy is recorded as pressure drop through and across the system. Chilton, Thomas, and Colburn[6], and Valle[61], investigated the influence of particle size on the pressure drop through a fluidized bed of varied solids. Valle restricted his investigation to the relationship between the total surface area of the particles composing the bed and the pressure drop, correlating the data but not developing any empirical equation. Leva[35] was one of the first to investigate the effect of fluid mass velocity on the pressure drop through a bed of solid particles. He reported that the pressure drop could be expressed in terms of two dimensionless equations, one for laminar flow and one for turbulent flow of the fluidizing medium through the bed of solids. Leva also reported that there are two primary reasons for the occurrence of pressure drop through fluidized beds, namely (1) the contraction caused by the shape of the voids into and through which the gas must pass, and (2) the friction between the gas stream and the particles of the bed. The shape and the roughness of the particles of the bed have been studied[39]. In these studies it has been shown, that other things remaining constant, the pressure drop through very rough particles, such as aloxite granules, is more than twice that obtained through smooth particles such as porcelain or glass. Many attempts have been made to correlate the pressure drop through fluidized beds with one or more other variables of these systems such as mass velocity, fraction voids, particle diameter, particle density, and many others. Two of the more important of these attempts are presented in the following review.

Leva[37] has shown that the pressure drop through a fluidized bed is dependent upon the mass velocity, fluid viscosity, particle density, fraction voids, and the particle shape factor for smooth particles. The mathematical correlation for these variables is

$$\Delta p = \frac{0.0243\, G^{1.9} \mu^{0.1} \lambda^{1.1} (1 - \delta)}{D_p^{1.1}\, g \rho \delta^3} \qquad (1)$$

where:

G = fluid mass velocity through bed, lb/hr-sq ft

μ = viscosity of the fluidizing medium, lb/ft-sec

λ = particle shape factor, dimensionless

δ = fraction voids of particles composing bed, dimensionless

D_p = particle diameter of bed particles, ft

g = acceleration due to gravity, ft/sec/sec

ρ = density of particles of bed, lb/cu ft

Δp = pressure drop through bed of solids, lb/sq ft.

Chilton, Thomas, and Colburn[6,7] investigated the influence of fluid viscosity, wall effect, length of packed tube, and other variables on the pressure drop through a bed of fluidized solids. In their investigation the flow of the fluidizing medium through the bed was considered as the flow through parallel conduits of irregular cross-section. The correlation for the pressure drop obtained is

$$\Delta P = \frac{2.27\, A_f\, Z^{0.15}\, L\, G_o^{\,0.85}}{\rho\, D_p^{\,1.15}} \qquad (2)$$

where:

- A_f = wall effect factor, dimensionless
- Z = viscosity of fluidizing medium, lb/ft-sec
- L = length of packed tube, ft
- G_o = mass velocity based on cross-sectional area of empty tube, lb/sq ft-sec
- ρ = average density of fluidizing medium, lb/cu ft
- D_p = average particle diameter, ft
- ΔP = pressure drop through bed of solids, lb/sq ft.

Characteristics of Fluidized Solids

In the following paragraphs some of the more important characteristics of fluidized solids are reviewed. Included in the review are the individual effects of each of the variables on the character of a fluidized system.

Type of Solids. In the many studies of fluidized systems there have been a countless number of different solids employed. Some of the materials are products such as sand, wheat, clover seed, and lead shot which have been transported by the use of fluidized systems[4]. In other investigations the type of solid employed has had some physical property such as roughness, shape, diameter, or density which was to be studied in relationship to other variables encountered in mathematical correlations.

Particle Diameter. The diameter of the particles used in fluidized beds has a marked influence on many of the other variables of the system such as pressure drop and mass velocity. In connection with mass velocity Leva[27] has found that channeling will increase with a decrease in the diameter of the particles. Parallel investigations[29] have shown that the diameter of the particles of the bed has a marked effect on the heat transfer through the bed and also on the fluidization efficiency of the fluidized system. Miller and Logwinuk[44] in their investigation calculated the particle diameter by the geometric opening of two sieves and expressed the mean particle diameter for a mixture of particles as

$$D_p = \sum_{y=1}^{y=y} X \, d_{pgm} \tag{3}$$

where:

D_p = particle diameter, ft

X = weight fraction of closely screened material, dimensionless

d_{pgm} = geometric mean diameter of fraction, ft

y = number of fractions, dimensionless.

<u>Density of Particles</u>. Very little experimentation has been performed in an effort to correlate density of material used in a fluidized system directly to pressure drop or other variables of the system. According to Sittig (58), an increase in the bulk density of the material in the bed will result in a greater pressure drop over the bed as shown in studies on socony bead catalyst. Leva (29) has studied the relationship between particle density and has reported that for all practical purposes the heat transfer through a bed of particles is not significantly influenced by the density of the particles of the bed.

Characteristics of Fluidizing Medium

Some of the characteristics of the fluidizing medium, such as viscosity, density, and mass velocity, and their effect on the character of a fluidized system is presented in the following paragraphs:

Fluidizing Medium. Both gaseous and liquid fluidizing media have been employed in systems of fluidized solids. In many cases the conditions of operation dictate the type and manner of fluidization. The use of a gas medium to fluidize a bed of solids is by far the most widely used, the fluidized cracking of petroleum hydrocarbons being perhaps the most extensive application.

Fluid Viscosity. According to Kiddoo[22], the viscosity of the fluidizing medium is relatively unimportant in its effect upon the other variables of the system. Leva[42] disagrees and states that for a wide range of operating conditions the kinematic viscosity of the fluidizing medium largely determines the fluidization efficiency.

Fluid Mass Velocity. Kiddoo[23] has found that the fluid mass velocity is the most important variable to be encountered in fluidized systems with the exception of the particle size of the solids composing the bed. In his studies on heat transfer Kiddoo states that the effect of fluid mass velocity on the heat transfer through beds of sand and other materials is uncertain[23]. However, for most commercial applications the fluid mass velocity is normally limited to the range of from 0.5 to 2.0 feet per second. Parent, Yagol, and Steiner[50] reported that under proper operational conditions it is possible to have boiling or aeration of the bed of solids at rates which are less than that required to sweep even the finest particles out of the bed. Furthermore, the actual fluidization of the particles of the bed starts in the upper region of the retaining vessel and moves downward through the bed with incremental increases in the mass velocity of the fluidizing medium. Several attempts have been made to correlate the mass velocity with the other variables of the system with the following empirical equation being one of the better ones:

$$G = \frac{0.005\, D_p^{\,2} \delta_{mf}^{\,3} (\rho_s - \rho) \rho g}{\lambda^2 (1 - \delta_{mf}) \mu} \qquad (4)$$

where:

D_p = average particle diameter, ft

δ_{mf} = minimum fluid voidages, dimensionless

ρ_s = density of the solid particles, lb/cu ft

ρ = density of the fluidizing medium, lb/cu ft

g = gravitational constant, ft/sec/sec

λ = shape factor of particles, dimensionless

μ = viscosity of fluidizing medium, lb/ft-sec

G = fluid mass velocity, lb/ft-sec.

Fluid Density. Parent, Ygaol, and Steiner(51) and Kiddoo(23) report that in most cases the effect of the fluid density on pressure drop and other variables of the system is negligible. The reason for this condition is that in most cases the density of the solid particles composing the bed is far greater than the density of even the heavy gases that may be used for fluidization. In the case of fluidization with liquids the effect of fluid density will, of course, be more pronounced.

Characteristics of Retaining Vessel

The characteristics of vessel length, distribution plate, and tube diameter of the retaining vessel are presented in this section.

Vessel Length. Miller and Logwinuk[44] report that the height of the bed of solids has no appreciable effect upon the mass velocity of the fluidizing medium. In their investigation it was found that an increase of three hundred per cent in the weight or height of the bed caused no pronounced change in the critical mass velocity. Other investigations[28,55] have shown that the height of the bed of solids has little or no effect upon the rate of heat transfer through the bed. Wilhelm and Kwauk[63] have reported that the height of the bed of solids has very little effect except on the pressure drop through the bed. In their investigation it was determined that the pressure drop through the bed of solids is directly proportional to the pressure drop per unit initial height of the bed.

Distribution Plate. Very little work has been performed as to the effect of the shape of the distribution plate on the fluidization of the solids of the bed. Investigation[8] has shown that a change in the shape of the distribution plate from concave upward to concave downward, has resulted in the solids of the bed moving upward at the wall and downward at the center of the column as compared with the conventional manner of particle circulation.

<u>Vessel Diameter</u>. It has been reported[32] that the diameter of the vessel has a marked influence upon the advent of slugging with increased fluid mass velocity through the bed of solids. The smaller the diameter, the greater is the tendency for the solids to "slug". In other investigations[48,50] it was observed that if the diameter of the vessel were small the solids would cause a head loss as a result of the drag against the sides of the retaining vessel. This influence of the retaining wall on the fluidization is known as "wall effect". Parent, Yagol, and Steiner[51] have observed that the influence of wall effect can be all but eliminated if the diameter of the vessel is greater than six inches. Still other investigations[29] have indicated that the diameter of the vessel has little or no effect upon the heat transfer through the bed since the advent of nearly perfect mixing causes uniform temperature distribution throughout the bed.

Application of Dimensional Analysis to Fluidization Studies

In recent years the application of the principles of dimensional analysis to the study of fluidization has been very successful. Through the use of dimensional analysis the many dimensional equations developed have proved extremely important in the study and correlation of many of the related variables of fluidized systems. Dimensional analysis is a means of obtaining partial information about relations between variables which must hold true for definite physical systems. The primary advantage of the use of dimensional analysis lies in the fact that it may be applied to a physical system when only a partial knowledge of the relationship of the variables of the system is available. The information obtained through the use of dimensional analysis can then be used to great advantage in limiting the experimental measurements necessary to determine complete information as to the relationship of the variables of the physical system. In general, dimensional analysis is applicable to physical systems only when an understanding of the system is sufficiently complete to explain its behavior.

Fundamental Units. The employment of the principles of dimensional analysis has resulted in the standardization of the fundamental units used throughout fluidization studies. Fundamental units are those units which are derived independently of all other units[46]. The fundamental units of length, mass, time, and temperature are (1) the meter, (2) the gram, (3) the mean solar second, and (4) the degree centigrade, respectively. These fundamental units vary from the metric to the English systems, but are derived independently of all other units in both systems. In many applications the proper choice of fundamental units and the substitution of the correct consistent dimensions for these units are extremely important if the desired results are to be obtained.

Derived Units. The derived units in dimensional analysis are those units which are expressed in terms of the fundamental units[46]. In all cases the derived units of any quantity are dependent upon the fundamental units of mass, length, time, or temperature. The dimensions of the derived units are defined as the powers to which the fundamental units vary in proportion to the derived units.

Correlation of Variables. As stated before, the application of dimensional analysis has permitted the correlation of many of the variables encountered in fluidization studies. In order to obtain the exponents and the dimensional constant for the determined dimensionless equation it is necessary to evaluate experimentally the magnitude of the various dimensionless groups in the equation. Once the magnitude and the variation of the individual groups of the equation are evaluated, the numerical value of the exponents of the groups and the dimensional constant may be determined. A graphical determination of the equation unknowns is possible if the character of the individual dimensionless groups permits the variation of one group at a time while holding the remaining groups constant. In this manner the slope of the line obtained is a measure of the exponent, and the numerical value of the intercept of the line with the ordinate at zero value for the group being varied is a measure of the dimensional constant. This is accomplished when the logarithm of the group being varied is plotted against the logarithm of the remaining groups on rectangular coordinates. In many cases it is not possible to determine a numerical value of zero for the logarithm of the group being varied, in which case the line must be extrapolated in order to obtain an intersection with the ordinate. As stated above, the graphical solution of the unknowns in the dimensionless equation requires that each group of the equation be varied independently of the other groups of the equation. Where this is not possible an alternate means of correlation of the variables is necessary. The use of statistical regression for this purpose is such an alternate means.

Application of Statistical Analysis to Fluidization Studies

In recent years the use of statistical analysis for the determination of the significant variables of a system as well as the correlation of the variables has become prominent. In the following pages will be found a discussion of the application of the principles of analysis of variance and regression as applied to experimental systems.

Analysis of Variance. An analysis of variance is a means of testing the significance or importance of the variables of a system[10]. In many investigations the determination of the important variable or variables can be a time-consuming operation. Prior to the application of the principles of analysis of variance the determination of the independence of one variable of the system upon the other may have required a large number of experimental determinations whereby one variable was tested against the remaining variables by holding the remaining variables constant. In many cases it is not possible to hold the important variable of a system constant. This makes the application of a statistical procedure, which takes into account the variation of the variables, extremely valuable. Also, there is the possibility that holding a variable constant, at some fixed value for the investigation, will cause a different result than would have been obtained in the investigation for other values of the variables. Through the application of the principles of analysis of variance these difficulties can be overcome and an accurate evaluation of the dependence of each variable upon the system determined.

Hypothesis and Assumptions. The hypotheses for the many forms employed in tests of analysis of variance are dependent upon the particular form of analysis used, while the assumptions employed are general and valid for any of the forms employed. Therefore, the test of the significance in an analysis of variance is valid only if the observations are from a normal distribution and the variance of each group of data is the same. This implies that the observations, when plotted, will take on the form of a symmetrical bell-shaped curve which extends infinitely far in both a positive and a negative direction. Also, there should be a homogeneous spread of the observations in each group from the average of the group. In an analysis of variance the hypothesis is stated that there is no effect in the variation of the experimental observations as a result of the variables of the system. The procedure employed is merely to test the variables of the system both independently and together to determine whether any of the variables are significant or important in causing the variation of the experimental data. When the test of significance shows that the variable or variables of the system are significant the hypothesis can be rejected for that variable.

<u>Variables of Classification</u>. The form of the analysis of variance is dependent upon the number of variables that exist in the experimental system. Many different forms[10] are available for the analysis of variance of from one to five separate and independent variables with the following forms being the more important: (1) randomized-block, (2) factorial analysis, (3) split plot, (4) incomplete block, and (5) latin square. Each of the above forms of analysis must be handled according to detailed procedures which apply only to that form. At the same time, there are certain considerations which must be taken into account regardless of the experimental analysis form under consideration. In all forms of an analysis of variance an attempt is made to study the several variables in the most efficient manner. In each form the sum of squares for the individual variables, the sum of squares for the interaction or the residual, and the sum of squares for the total is computed. The mean sum of squares, which serves as the estimate of the variance for the variable or for the

interaction, is always computed by dividing the sum of squares by the degree of freedom for the particular quantity. The total degree of freedom is one less than the number of experimental observations, while the degree of freedom for the quantity under consideration is one less than the number of separate variables involved in the quantity.

Significance. The objective of the analysis of variance is to determine which, if any, of the independent variables of the experimental system are significant or important in causing the variation in the experimental data as a result of the variation of all of the variables taken together. The variation of the observations in the experimental data is caused not only by variance in the population and the variation as a result of experimental error, but also as a result of changes in the variables of the system. The statistic employed in the test for significance is the F statistic. This statistic is the ratio of the mean sum of squares of the quantity under consideration to the mean sum of squares for interaction. In the test for significance a level of significance must be selected to be used in connection with the F test. The level of significance determines the chance that will be permitted for a variable of the system testing significant when it is really not significant. The value of *F* for each of the variables of the system is determined and compared with the value of *F* for the proper number of degrees of freedom at the chosen level of significance. If the experimental value of *F* exceeds the value from computed tables, the variation of the experimental observations is caused more by that particular variable than would be expected by chance at the level of significance. Under these conditions it is possible to have any number of variables significant, as well as significant interaction. The fact that the interaction is significant means that two or more variables are causing effects on the experimental observations when taken together that they would not produce when taken separately. In connection with the significance of the variables of a system it should be pointed out that if two variables of a system are significant, the *F* value of one being twice that of the other, there is no assurance that the first variable is causing twice the variation in the experimental observations than is the second. It should be also pointed out that if there are several variables involved in an experimental system and only one of those variables is significant, it does not mean that only that significant variable is causing the variation in the observed values, but, instead, that all of the variables may be causing variation in the experimental observations to which the significant variable is producing the major variation.

Regression and Correlation. Duncan[11] states that in most cases a dependent variable can be estimated with greater precision if the estimate of the variable is based upon several independent variables rather than only one. This situation is the type that exists in many investigations, particularly in fluidization studies where as many as seven or eight independent variables may be considered in relation to the dependent variable. There has been considerable controversy in recent years as to the best, and simplest, definition of the term "regression". According to Duncan[11], "regression", in statistics, means simply the average relationship between variables. Therefore, the regression of X_1 on X_2 would mean the relationship between the average of the values of X_1 for a given value of X_2. In a similar manner the regression of X_2 on X_1 means the average of the values of X_2 for a given value of X_1. As stated above, in many investigations the relationship between the variables of the system is determined by controlled experiments. However, in many instances it is not physically possible to control the mode of experimentation and in these cases the relationships between the variables of the system must be determined by analyzing the data as it is obtained. The means of achieving the relationships is then by employment of the principles of modern regression and correlation analysis.

Variables of Classification. Regression between the variables of a system can apply to as few as two independent variables. In the case of two-variable regression the time and difficulty of computation is considerably less than that required in multiple regression. When the number of independent variables of the experimental system exceed two, special means must be employed in order to determine the numerical value of the unknown quantites.

Normal Equations. The determination of the unknown quantities for multiple regression involves the solving of a set of simultaneous equations equal to one more equation than the number of independent variables of the system. Since the solving of the simultaneous equations would become extremely difficult if the number of independent variables exceeded two or three, a simplified computational procedure known as the abbreviated Doolittle method[11] has been devised for the solution of the simultaneous equations. In order to solve the equations for the unknown quantities it is first necessary to evaluate the matrix for the experimental data. The matrix consists of a square block of numbers, each number representing either a sum of squares or a sum of products for the experimental observations for each variable taken independently and as a product of the other variables. The matrix, therefore, represents the sum of squares and the sum of products for every possible combination of the experimental observations for each of the independent and dependent variables of the system. The normal equations consist simply of the rows and columns of the matrix in their proper order with the corresponding symbol for the unknown substituted into the proper column of the matrix.

Determination of Unknown Quantities. The actual solution of the simultaneous equations or the normal equations is accomplished by means of the abbreviated Doolittle method[11]. This procedure for the solution of simultaneous equations has two modes of solution, the forward and the backward solutions. The forward solution involves interrelations between the terms of the matrix of the normal equations, while the backward solution involves interrelations between the inverse of the matrix of the normal equations. In both solutions strict methods of computational procedure must be adhered to in order to obtain the desired numerical values for the unknowns of the normal equations. The principal advantage of these methods of solution of simultaneous equations lies, not only in the fact that it is considerably less time consuming, but also that it provides methods for checking both the mathematics and the statistics involved as the solution of the normal equations progresses. At the completion of the computational procedure the numerical values of the unknowns in the normal equations are computed from the statistical relations existing between the intermediate terms computed during the solution of the normal equations by the Doolittle procedure.

Tests of Significance. Tests of significance in multiple regression are provided just as they were in analysis of variance of independent variables. The test of significance in multiple regression involves the standard *F* test whereby the experimental *F* value is compared with the *F* value from computed tables at the chosen level of significance. The experimental *F* value is obtained by dividing the mean sum of squares for regression, where the degree of freedom is equal to the number of independent variables x, by the mean sum of squares for residual, where the degree of freedom is equal to the total number of observations n for the system minus x + 1. If the experimental value of F exceeds the value from the tables at the chosen level of significance it indicates that the regression due to all of the variables of the system was significant. Should the experimental F value not be significant, it indicates that one or more of the independent variables of the system are insignificant and can be eliminated from the normal equations. This means that one variable of the equations must be eliminated, the unknowns re-evaluated, and the significance of the regression re-determined. In many procedures, the significance of the regression is tested prior to the determination of the numerical values of the unknowns in the normal equations, whereby the computation of the unknowns is not repeated every time the regression should prove to be insignificant. If an idea of the relative importance of the independent variables is known prior to the use of the regression computations, the normal equations can be set up with the supposed least significant variable in the last column so that it can be dropped from further computations with the least amount of difficulty should the regression prove to be insignificant for all of the variables.

III. EXPERIMENTAL

The experimental section in the investigation of the application of dimensional and statistical analysis to fluidization studies includes the purpose of the investigation, the plan of experimentation, the materials and apparatus used, the method of procedure, the data and results, and the sample calculations.

Purpose of Investigation

The purpose of the investigation was to correlate the pressure drop across a fluidized bed of ottawa sand with the variables of bed height, bed diameter, and particle size by dimensional and statistical means.

Plan of Investigation

The plan of the investigation consisted of a survey of the literature, the modification of the fluidization equipment, orifice calibration, analytical tests, operational tests, and the analysis of results.

Literature Review. A survey of the available literature pertaining to the pressure drop encountered through a fluidized bed of solid particles was made so as to gain a knowledge of previous work performed by other investigators. In addition, particular attention was devoted to the literature relating to both dimensional and statistical analysis.

Modification of Fluidization Equipment. The apparatus used in this investigation consisted of copper fluidization columns which were used in conjunction with air driers designed and constructed by Breckon(3). The fluidization columns were designed and constructed to permit the determination of the pressure drop encountered throughout the bed of solids at six-inch increments of bed height.

Orifice Calibration. A check was made on the calibration curve for the 3/16-inch orifice plate in the air manometer of the fluidization column as originally calibrated by Dickerson(9). The insufficient capacity of available commercial wet test meters did not permit the calibration of the entire range of the manometer differentials used in this investigation. Therefore, it was necessary to extrapolate the curve of Dickerson(9) to cover operational ranges.

<u>Analytical Tests</u>. Analytical tests were performed with the assistance of Luttrell[69] to determine the absolute density and mean particle diameter of the ottawa sand, the fraction voids of the three ranges of sand, and the humidity of the fluidizing air used in the pressure drop determinations.

<u>Operational Tests</u>. Experimental tests were made using three ranges of ottawa sand, two column diameters, and seven bed heights increasing by six-inch increments. The tests determined the static pressure drop at each combination of the above variables. In addition, tests were conducted to determine the bed expansion at each bed height using the three ranges of ottawa sand and the two column diameters.

<u>Analysis of Results</u>. The data obtained from the operational tests were used to evaluate the significant variables of the fluidization system and to determine the numerical value of the unknown constants in the dimensionless equation.

Materials

The following materials were used in the investigation.

Air. Compressed air, humidity 0.006 pound of water per pound of dry air. Supplied by Nash hytor compressor, Department of Chemical Engineering, Virginia Polytechnic Institute, Blacksburg, Va. Used as fluidizing medium in performance of fluidization tests.

Carbon Tetrachloride. Technical grade. Obtained from Fisher Scientific Co., Silver Spring, Md. Used as manometer fluid for pressure drop manometers of fluidization system.

Drierite. Six-mesh (U.S. Standard); bulk density, 54 to 60 pounds per cubic foot; anhydrous calcium sulfate coated with C.P. cobalt chloride. Obtained from W. A. Hammond Drierite Co., Xenia, Ohio. Used as drying agent for removing moisture from fluidizing medium.

Dye. 1,4-Bis (amylamino) anthraquinone, R-2135-60-F. Obtained from American Cyanamid Co., Bound Brook, N. J. Used to color carbon tetrachloride manometer fluid.

Mercury. Obtained from Fisher Scientific Co., Silver Spring, Md. Used as manometer fluid for reference pressure manometer and air manometer in the fluidization system.

Ottawa Sand. Standard testing sand; 20 to 30- and 30 to 50-mesh U.S. Standard; mean particle diameters of 0.0266 and 0.0180 inch, respectively; absolute density, 166.6 pounds per cubic foot. Obtained from Ottawa Sand Co., Ottawa, Ill. Used as fluidized solid.

Ottawa Sand. Standard testing sand; 50 to 70-mesh U.S. Standard: mean particle diameter 0.0103 inch; absolute density, 166.6 pounds per cubic foot. Obtained from American Graded Sands Co., Chicago 13, Ill. Used as fluidized solid.

Permatex. Obtained from Permated Co., Inc., Brooklyn 35, N. Y. Used to form gasket around air drier windows.

Water, Distilled. Obtained from laboratory still in the Department of Chemical Engineering, Virginia Polytechnic Institute, Blacksburg, Va. Used in absolute density determinations of ottawa sand.

Apparatus

The following apparatus and equipment were used in the course of the investigation.

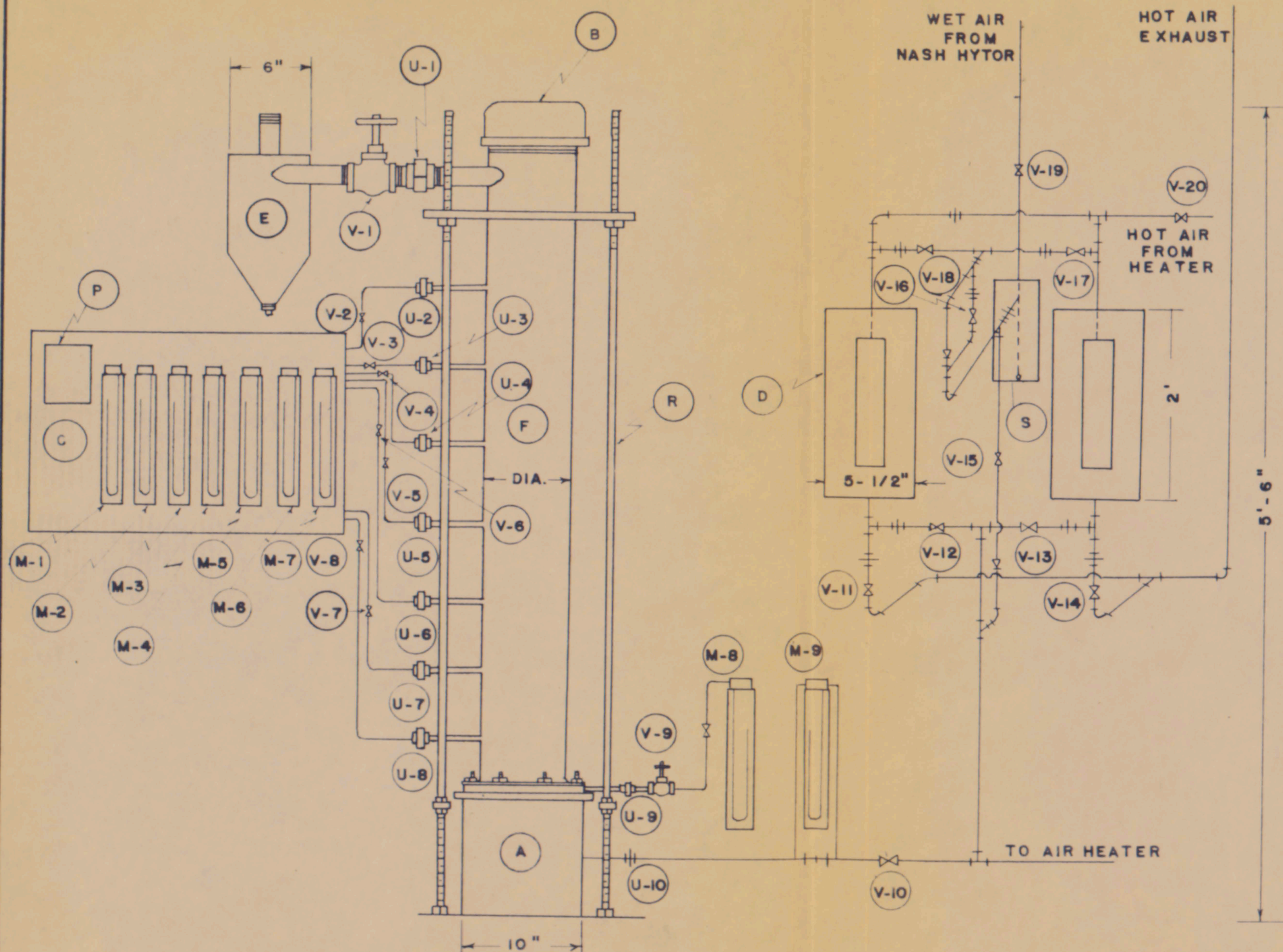
Balance, Analytical. Chain-o-matic, 250 gram capacity, sensitivity 1/20 milligram. Manufactured by Seederer-Kohlbusch, Inc., Jersey City, N. J. Used for weighing sand for density determinations.

Balance, Beam. Twenty-kilogram capacity, one-gram increments. Obtained from Fisher Scientific Co., Silver Spring, Md. Used for weighing sand samples and testing screens.

Fluidization System. A drawing of the fluidization column and accessory equipment showing the relationship of the major components of the system is shown in Figure 2, page 47. A detail drawing of the fluidization column is shown in Figure 3, page 48, with the bill of materials in Table I, page 49.

Furnace. Electric, 3.4 kw, 220 v, maximum temperature 1850 °F. Manufactured by Cooley Electric Manufacturing Co., Indianapolis, Ind. Used to dry sand samples prior to density determinations.

Figure 2. Diagrammatic Sketch of Fluidization Column and Accessories



FRONT ELEVATION

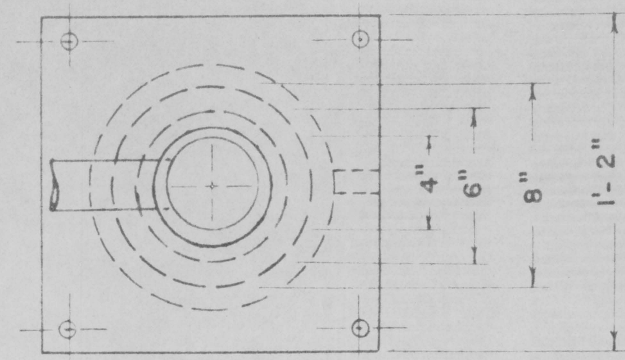
LEGEND

- (A) AIR CALMING CHAMBER
- (B) BLACK IRON CAP
- (C) ELECTRIC CLOCK
- (D) AIR DRIERS PACKED WITH DRIERITE
- (E) CYCLONE SEPARATOR
- (F) FLUIDIZATION COLUMN
- (M-1) - (M-8) PRESSURE DROP MANOMETERS
- (M-9) AIR VELOCITY MANOMETER
- (P) SWITCH BOX
- (R) SUPPORT ROD
- (S) WATER SEPARATOR
- (V-1) CYCLONE CONNECTING VALVE
- (V-2) - (V-9) PRESSURE DROP REGULATING VALVES
- (V-10) AIR CONTROL VALVE
- (V-11) - (V-20) VALVES OF AIR DRYING SYSTEM

DEPARTMENT OF CHEMICAL ENGINEERING
VIRGINIA POLYTECHNIC INSTITUTE
BLACKSBURG, VIRGINIA

DIAGRAMMATIC SKETCH OF FLUIDIZATION COLUMN AND ACCESSORIES

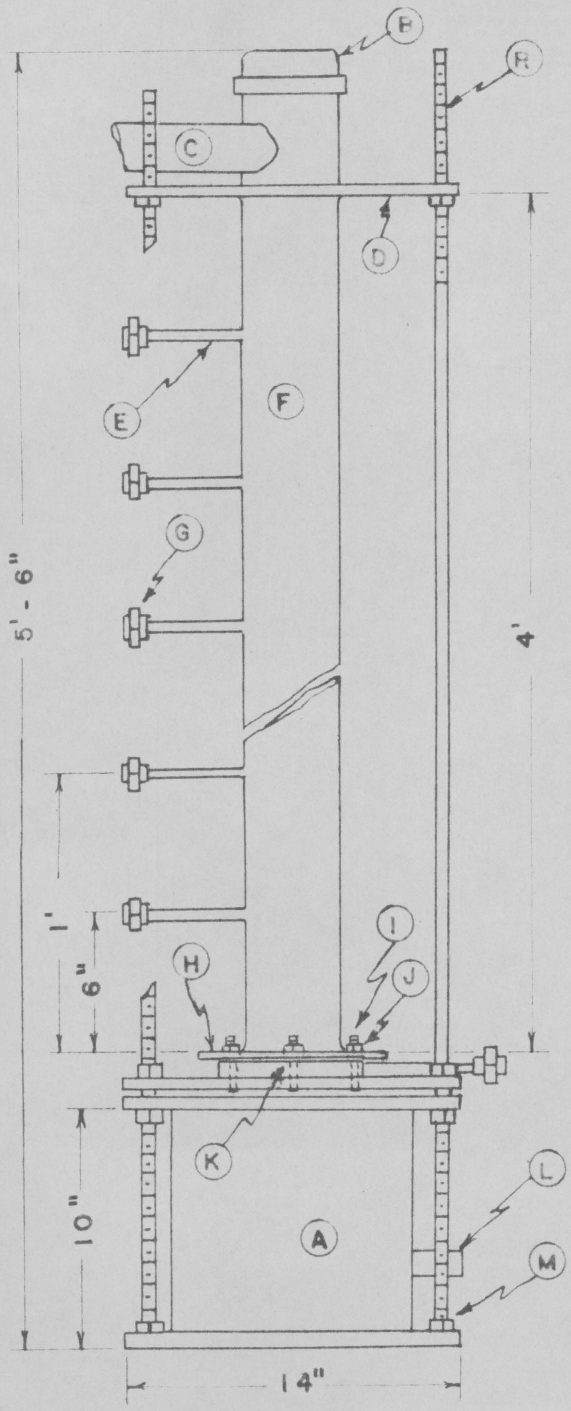
SCALE:	NONE	DATE	CASE NO: 54
DRAWN BY:	ECM	4-14-55	FILE NO: 599
CHECKED BY:	ECM	4-14-55	FIGURE NO: 2
APPROVED BY:	FWB	4-15-55	SHEET NO: 1



LEGEND

- (A) 10-IN. GALVANIZED IRON PIPE
- (B) 2-AND 4-IN. BLACK IRON CAP
- (C) 2-IN. BLACK IRON PIPE
- (D) 1/4-AND 1/8-IN. FLAT IRON
- (E) 1/8-IN. BLACK IRON PIPE
- (F) 2-AND 4-IN. DIA. FLUIDIZATION COLUMNS
- (G) 1/8-IN. BLACK IRON UNION
- (H) 1/4-AND 1/8-IN. FLAT IRON
- (I) 2-1/2 X 1/4-IN. SQUARE-HEADED STEEL BOLTS
- (J) 1/4-IN., SQUARE-HEADED, STEEL NUT
- (K) 9/16-IN. BASE PLATE
- (L) 1-IN. BLACK IRON COUPLE
- (M) 1/2-IN., SQUARE-HEADED, STEEL NUT
- (R) 1/2-IN. STEEL ROD

NOTE: ALL PERMANENT JOINTS ARE WELDED



FRONT ELEVATION

DEPARTMENT OF CHEMICAL ENGINEERING
VIRGINIA POLYTECHNIC INSTITUTE
BLACKSBURG, VIRGINIA

DETAIL DRAWING OF FLUIDIZATION COLUMN

SCALE: 1/8" = 1"	DATE	CASE NO: 54	
DRAWN BY: ECM	4-13-55	FILE NO: 599	
CHECKED BY: ECM	4-13-55	FIGURE NO: 3	
APPROVED BY: FWB	4-15-55	SHEET NO: 1	

TABLE I

Bill of Materials for Fluidization Columns

TABLE I

Bill of Materials for Fluidiation Columns

Item	Quantity	Description	Supplier[a]
Plywood	1	4 ft x 3 ft x 3/4-in.	G. & W. Builder's Supply, Blacksburg, Va.
Manometers	8	18-in. cast iron mountings	Meriam Manometer Co., Cleveland, Ohio
Screws	18	No 10, steel	Blacksburg Hardware Co., Blacksburg, Va.
Screen	6 sq in.	200-mesh, Tyler standard	W. S. Tyler Co., Cleveland, Ohio
Tubing	4 ft	4-in. diameter, copper	
Tubing	4 ft	2-in. diameter, copper	
Tubing	40 ft	1/8-in. diameter, copper	
Pipe	7	4-in. length, 1/8-in. diameter, black iron	
Pipe	1	4-in. length, 1/4-in. diameter, black iron	
Pipe	1	5-in. length, 2-in. diameter, black iron	
Pipe	1	4-in. length, 2-in. diameter, black iron	
Union	8	1/8-in. black iron	
Union	1	2-in. black iron	
Union	4	1/8-in. brass	
Valve	1	globe, 1/8-in., brass	
Valve	7	needle, 1/8-in., brass	
Valve	1	glove, 2-in., brass	
Flat iron	2	1/4-in. mild steel plate	
Flat iron	2	1/8-in. mild steel plate	
Cap	1	4-in. black iron	
Cap	1	2-in. black iron	
Angle iron	2 ft	1-1/2 x 1-1/2 x 1/4-in. mild steel	
Angle iron	1 ft	1 x 1 x 3/16-in. mild steel	
Adapter	7	1/8-in. tubing to pipe, brass Parker	
Adapter	7	1/8-in. tubing to pipe, brass compression-type	
Reducer	1	1/4 x 1/8-in. black iron	
Ell	2	1/8-in. brass, Parker	
Bolts	4	2-1/2 x 1/4-in. mild steel	
Bolts	16	1-1/2 x 1/8-in. mild steel	
Nipple	1	long, 2-in. black iron	
Nipple	2	short, 1/8-in. black iron	

[a] Unless otherwise stated, the supplier is Noland Co., Roanoke, Va.

Glassware. Miscellaneous laboratory glassware used in the investigation included beakers, flasks, and pipets. Manufactured by Corning Glassware Co., Corning, N. Y. Used for miscellaneous test purposes.

Hot Plate. Autemp heater, 115 v, ac, 450 w. Obtained from Fisher Scientific Co., Silver Spring, Md. Used in ottawa sand density determinations.

Meter, Gas. Number 4354740, graduated in cubic feet and tenths of cubic feet. Manufactured by Precision Scientific Co., Chicago, Ill. Used to measure air flow for orifice calibration.

Pans. Steel, four required, 0.1 cubic foot capacity. Obtained from Fisher Scientific Co., Pittsburgh, Pa. Used with sample splitter for quartering samples for screen analysis.

Sieves. Tyler standard screen scale. Seven required; 16-, 20-, 30-, 40-, 50-, 70-, and 100-mesh. Obtained from W. S. Tyler Co., Cleveland, Ohio. Used for screen analysis of ottawa sand.

Riddle, Combs Gyratory. Obtained from Great Western Manufacturing Co., Leavenworth, Kan. Used for shaking test sieves in ottawa sand screen analysis.

<u>Specific Gravity Bottle</u>. Hogarth, A.S.T.M.; 100-ml, pyrex glass. Obtained from Fisher Scientific Co., Silver Spring, Md. Used for determining absolute density of ottawa sand.

<u>Thermometers</u>. Mercury in glass, range 0 to 300 °F, two degree Fahrenheit increments. Obtained from Fisher Scientific Co., Silver Spring, Md. Used for determining absolute density of ottawa sand.

<u>Timer</u>. Electric, 115 v, 60 cy, readings in minutes and hundredths of a minute. Obtained from Fisher Scientific Co., Silver Spring, Md. Used to time shaking of test screens.

<u>Vacuum Cleaner</u>. Series 36686, 110 v, 60 cy. Obtained from Westinghouse Electrical Corp., Mansfield Works, Mansfield, Ohio. Used to withdraw ottawa sand from fluidization columns.

Method of Procedure

The method of procedure followed in this investigation is presented under the following headings: (1) modification of fluidization equipment, (2) orifice calibration, (3) analytical tests, and (4) pressure drop determinations.

Modification of Fluidization Equipment. The fluidization equipment shown in Figure 2, page 47, required several modifications before the pressure drop determinations could be made.

Fluidization Columns. This investigation required that the static pressure drop be determined at six-inch increments along the vertical axis of the fluidization columns. Efforts were made to obtain two- and four-inch glass pyrex columns with the appropriate manometer leads located at six-inch increments along the vertical axis of the columns. However, the cost of such columns was prohibitive and it was decided to construct columns of copper tubing instead. In the construction of the copper fluidization columns, Figure 3, page 48, two- and four-inch copper tubing was cut to a length of exactly four feet. Two pieces of 1/4-inch flat iron were obtained and cut to form a flange at the bottom of the 4-inch column and a support at the top of the column. The bottom flange was formed by cutting a hole in the center of the flat iron plate, slipping it over the outside edge of the 4-inch diameter copper tubing, and welding in place. A similar procedure was employed for the 2-inch column using 1/8-inch flat iron plate. Holes were drilled in the flange sections to correspond to the stationary bolts on the base plate of the column support. The flat iron plate, fourteen inches square, was welded to the top of the copper tubing. A 5/8-inch hole was drilled in each corner of the flat iron plate to accommodate the 1/2-inch support rods. A 4-inch section of 4-inch black iron pipe was threaded at one end and welded to the top of the support flange. A 2-inch hole was burned in the center of the section and a one-foot length of 2-inch black iron pipe welded into the hole. This section was connected to the cyclone separator by means of a 2-inch union. The 2-inch fluidization column was prepared in a similar manner. A black iron cap was provided to close the short sections welded to the top of the columns. With this portion of the construction completed, seven 1/2-inch holes were drilled along the vertical axis of the columns, the holes spaced at six-inch crements. Into each hole was welded a 4-inch length of 1/8-inch black iron pipe threaded at the free end. These short lengths of 1/8-inch pipe served as the leads to the pressure drop manometers. In order to prevent clogging of the manometer lines with fine particles of ottawa sand 1/2-inch diameter sections of 200-mesh screen were cut and placed in the 1/8-inch unions connected to the ends of the manometer leads.

Cyclone. A 6-inch diameter cyclone, constructed from an 18-inch section of black iron pipe, was used to collect the entrained solids from the top of the fluidization columns.

Mounting Board. A 4-foot x 3-foot x 3/4-inch section of plywood was used as a mounting board for the pressure drop manometers of the fluidization system. Seven manometers were mounted to the board, a hole being provided for the leads to the manometers from the back side of the board. One-eighth-inch copper tubing was used to connect the leads from the fluidization columns to the manometers. At the column end of the copper tubing, 1/8-inch Parker fittings were provided for the connection to the unions which served as the filters in the system. At the manometer end of the copper tubing, 1/8-inch compression-type fittings were used to connect the 1/8-inch copper pressure lines to the manometers.

Thermometer Wells. Holes were drilled into the air calming chamber and the air drying chambers of the fluidization system to accommodate thermometers.

Analytical Procedure. The analytical procedure for the investigation consisted of the determination of the ottawa sand density, the mean particle diameter of the ottawa sand, the fraction voids of the three ranges of sand, and the humidity of the fluidising air.

Density Determinations. The absolute density of the 20 to 30-, 30 to 50-, and 50 to 70-mesh ranges of the ottawa sand were determined by A.S.T.M. standard procedure[65].

Particle Diameter by Screen Analysis. In the investigation it was necessary to determine the mean particle diameter for the three ranges of ottawa sand. Representative 100-gram samples of each of the three ranges of sand were used in the screen analysis. Each screen in a nest of seven standard testing sieves consisting of the 16-, 20-, 30-, 40-, 50-, 70-, and 100-mesh screens was tared and placed in the gyratory riddle. The 100-gram sample of the 20 to 30-mesh sand was placed on the 16-mesh screen, the lid placed on the top of the screens, and the gyratory riddle started. After seven minutes of shaking, the riddle was turned off and the weight of sand retained on each of the testing screens was determined. This procedure was repeated for the 100-gram sample of the 30 to 50- and the 50 to 70-mesh ranges of the ottawa sand with the numerical value for the mean particle diameter of each range calculated[44].

Particle Diameter by Photographic Analysis. The mean particle diameter for the three ranges of the ottawa sand was also determined by the use of photographic analysis. A 400-gram sample of each of the three ranges of sand was obtained and quartered until only a 100-gram sample remained. A photographic enlargement of the particles in the final quartered sample was made at an enlargement ratio of 29.5. From the photographic enlargement and the measurement reference scale, which was photographed with the sand sample, it was possible to obtain the mean particle diameter of the sand. Effort was made to select particles for measurement from all portions of the sample and to select approximately the same number of large and small particles in the diameter determination. Once the diameters of the selected particles were measured, the mean particle diameter for that particular range of sand was determined by averaging the values for the individual measurements.

Fraction Voids Determination. In the investigation it was necessary to determine the fraction voids for the 20 to 30-, 30 to 50-, and 50 to 70-mesh ranges of the ottawa sand at the bed expansion corresponding to the critical mass velocity. Since the copper fluidization columns did not permit visual observations for the measurement of the height of the expanded beds of sand it was necessary to determine the fraction voids of the three ranges of sand by other means.

For this test 2- and 4-inch diameter pyrex glass columns were substituted for the copper columns and observations for bed expansion were made at 6-inch bed heights at the critical mass velocity. From measurements made in these columns as to bed expansion and known weights of sand for each 6-inch increment the fraction voids were determined.

Humidity of Fluidizing Air. The humidity of the fluidizing air was determined by obtaining the wet and dry bulb temperature of the air stream from the drying chambers.

Orifice Calibration. The manometer in the air line to the air calming chamber of the fluidizing system was calibrated to determine the velocity of the fluidizing air for corresponding differentials on the mercury manometer. The calibration was performed by measuring gas flow volumes for five-minute intervals with a wet test meter as a function of the pressure drop recorded for the particular flow rate.

Performance of Pressure Drop Determinations. The pressure drop determinations for the investigation were made using the 2- and 4-inch copper fluidization columns. The 4-inch copper column was installed above the air calming section A by fastening the base flange of the column to the base plate of the section with six 2-1/2 x 1/4-inch square-headed steel bolts. The pressure drop lines were connected to the unions U-2 to U-8 and the connection to the cyclone separator was made through U-1 and valve V-1. The reference manometer was connected to the base of the fluidization column by means of valve V-9 and union U-9. With these connections securely fastened, the cap at the top of the column was removed and sufficient 20 to 30-mesh ottawa sand poured into the column to fill it to a height of six inches. Measurements of the actual bed height were made with a 1/4-inch diameter probe with a flat disc welded on the bottom to prevent the probe from penetrating below the surface of the sand layer. Once the level of the sand was determined to be six inches the measuring probe was removed from the column and the cap replaced. Valves V-1, V-8, V-9, V-12, V-18, and V-19 were opened and the air velocity to the fluidization column was increased by opening V-10 until the sand in the fluidization column was slugging. Then, valve V-10 was closed. This procedure attempted to compensate for any difference in the packing of the bed of sand at the different bed heights by starting each pressure drop determination with the bed of sand under conditions of loose packing. Valve V-10 was again opened and the air velocity to the fluidization column increased until the maximum differential was obtained on reference manometer M-8. At this point the differential on the reference manometer and bed height manometer M-1 were recorded as well as that on the air line manometer M-9. Three separate readings were taken for the reference, height, and air line manometers for the 6-inch bed of 20 to 30-mesh sand. With the entering air velocity set at some intermediate velocity between zero and the slugging velocity, the cap on the top of the column was removed and enough of the 20 to 30-mesh sand added to the column to produce an overall bed height of one foot.

This procedure allowed the "fines" to remain on the top of the bed of sand as the column was filled, thereby preventing upward surges of the bed as the critical mass velocity was approached. The cap was replaced, valve V-8 closed and valve V-7 opened. Pressure drop determinations were again made for the one foot bed height as described for the 6-inch bed height. This procedure was repeated for bed heights of 1-1/2-, 2-, 2-1/2-, 3-, and 3-1/2-feet with the differentials being recorded for the air, reference, and height manometers for three tests at each height. In all tests the lines to the bed height manometers were closed with the exception of the line corresponding to the particular height of sand under consideration. At the conclusion of the third test for each of the bed heights the temperature of the fluidizing air was recorded. After the pressure drop over the various bed heights had been obtained increasing the level of sand in the column, the same procedure was repeated decreasing the level by increments of six inches. Three additional values were again recorded for the pressure drop at each of the seven bed heights. The decrease in sand level was accomplished by sucking out the 20 to 30-mesh sand with a vacuum cleaner and a five-foot section of 1-inch diameter plastic tubing. After the pressure drops for the

6-inch bed height were obtained, the remainder of the 20 to 30-mesh sand was sucked out of the column. This concluded the pressure drop determination for the 20 to 30-mesh sand using the 4-inch diameter fluidization column. Identical tests were performed on both the 30 to 50- and the 50 to 70-mesh ranges of the ottawa sand with the 4-inch diameter column. At the conclusion of the tests on the 4-inch diameter column, identical tests were performed on the same three ranges of sand using the 2-inch diameter fluidization column.

Data and Results

The data and results of this investigation are presented on the following pages.

Orifice Calibration. The calibration data for the 3/16-inch orifice plate in the air line to the fluidization column is presented in Table II, page 63. Air flow rates and manometer readings are plotted in Figure 4, page 64.

Density of Ottawa Sand. The data for the determination of the absolute density of the ottawa sand is presented in Table III, page 65.

Particle Diameter. The data for the determination of the particle diameter of the ottawa sand by screen analysis and by photographic analysis is presented in Tables IV and V, pages 66 and 67.

Fraction Voids. The experimental data for the determination of the fraction voids of the three ranges of ottawa sand is presented in Table VI, page 68.

Operational Data. The operational data for the tests on the 2- and 4-inch fluidization columns is presented in Tables VII and VIII, pages 69 and 70.

Operational Variables. Values for the air velocity, pressure drop, and fraction voids for the three ranges of

TABLE II

Calibration Data for 3/16-inch Orifice Plate in Air Line to Fluidization Column

Test No	Wet-test Meter Reading, cu ft	Time, sec	Manometer Reading, in. Hg	Mass Flow Rate lb/hr
1	1.0	63.4	0.25	4.15
2	2.0	41.9	0.50	6.28[a]
3	3.0	63.8	1.00	8.25[a]
4	4.0	46.1	2.00	11.40
5	5.0	36.4	3.00	14.40
6	6.0	46.9	4.00	16.80
7	7.0	54.8	5.00	19.20
8	8.0	47.6	6.00	22.10

a Indicates values confirmed in this investigation

Dickerson, W. H.: Evaluation of the Coefficient of Heat Transfer at the Heater-Wall Boundary of an Internally Heated Air-Fluidized Bed of Aerocat Cracking Catalyst. Unpublished B. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1953).

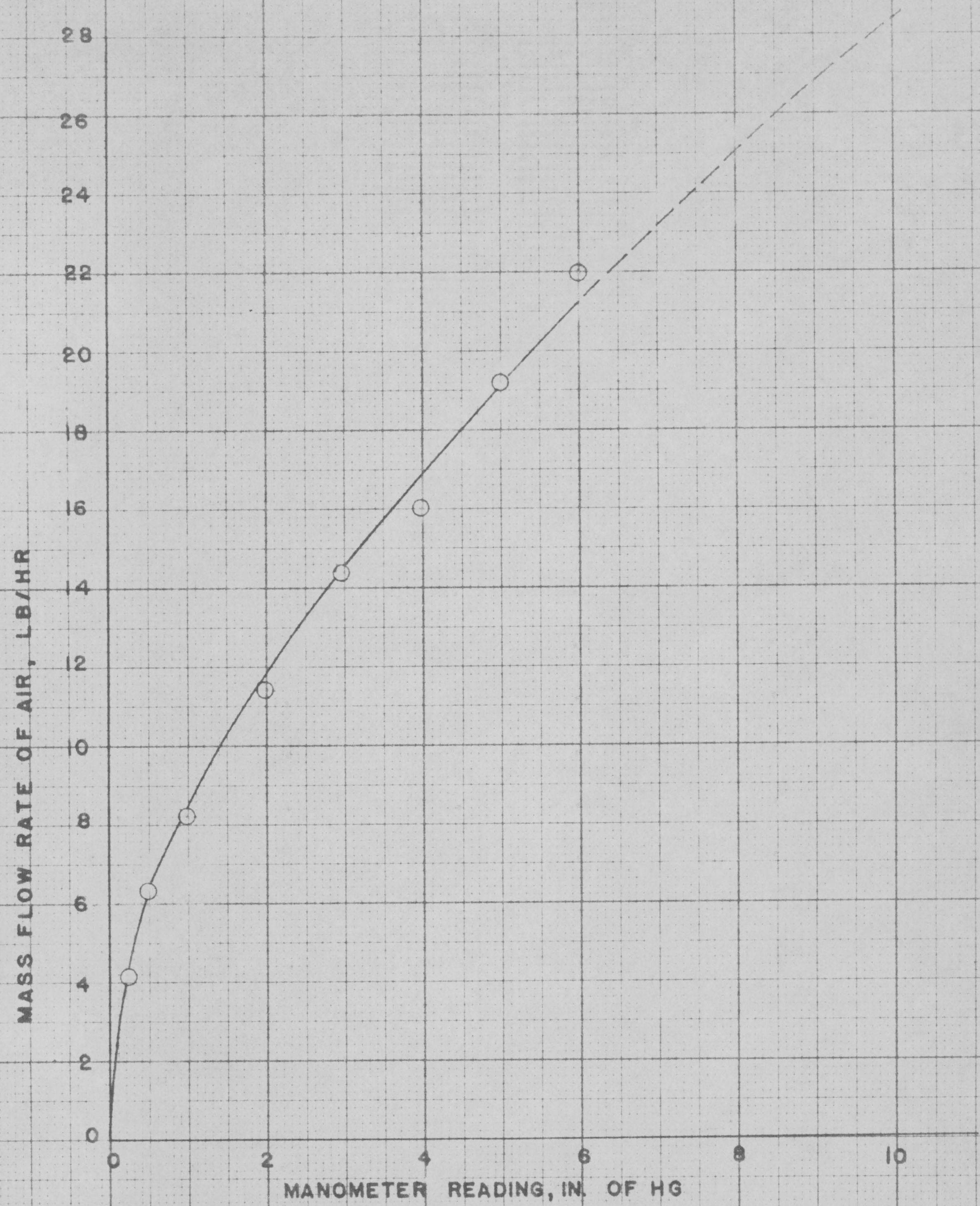


FIGURE 4. CALIBRATION CURVE FOR 3/16-IN. ORIFICE PLATE IN AIR LINE TO FLUIDIZATION COLUMN

DICKERSON, W.H.: EVALUATION OF THE COEFFICIENT OF HEAT TRANSFER AT THE HEATER-WALL BOUNDARY OF AN INTERNALLY HEATED AIR-FLUIDIZED BED OF AEROCAT CRACKING CATALYST. UNPUBLISHED B. SC. THESIS, LIBRARY, VA. POLY. INST., BLACKSBURG, VA. (1953).

TABLE III

Density of Ottawa Sand

Sand Size,	Weight of Bottle[a],	Weight of Bottle and Distilled Water,	Weight of Sand Sample,	Weight of Bottle and Sand	Temperatures		Absolute Density,
					Distilled Water	Sand and Water	
mesh	gm	gm	gm	gm	°F	°F	lb/cu ft
20 to 30	41.4546	157.5913	10.0077	163.8557	75	72	166.1
20 to 30	41.4546	157.5913	10.0002	163.8187	75	74	169.1
20 to 30	41.4546	157.5913	10.0009	163.8101	75	74	164.2
30 to 50	41.4546	157.5913	10.0018	163.8426	72	71	166.0
30 to 50	41.4546	157.5913	10.0044	163.8152	71	73	165.0
30 to 50	41.4546	157.5913	10.0023	163.8944	70	68	168.0
50 to 70	41.4546	157.5913	10.0023	163.8651	71	69	166.8
50 to 70	41.4546	157.5913	10.0010	163.8744	70	70	167.5

a Hogarth specific gravity bottle

Luttrell, R. S.: The Determination of the Mean Particle Diameter, Density, and Fraction Voids of Ottawa Sand. Unpublished B. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1955).

TABLE IV

Screen Analyses of Ottawa Sand for Determination of Particle Diameter

TABLE IV

Screen Analyses of Ottawa Sand for Determination of Particle Diameter

Test No	Sand Size, mesh	Sieve Size[a], mesh	Sieve Aperture, in.	Sand Retained[b], wt %	$\sqrt{d_1 d_2}$ [c] in.	d_p^d in.	D_p^e in.
1	20 to 30	16	0.0464	0.00	———	———	0.02527
		20	0.0328	1.75	0.03800	0.00044	
		30	0.0232	78.65	0.02760	0.02172	
		40	0.0164	10.20	0.01950	0.00200	
		50	0.0116	5.56	0.01380	0.00073	
		70	0.0082	2.74	0.00975	0.00028	
		100	0.0058	1.10	0.00689	0.00010	
2	30 to 50	16	0.0464	0.00	———	———	0.01696
		20	0.0328	0.00	0.03800	———	
		30	0.0232	1.01	0.02760	0.00028	
		40	0.0164	54.18	0.01950	0.01059	
		50	0.0116	41.39	0.01380	0.00579	
		70	0.0082	2.42	0.00975	0.00030	
		100	0.0058	0.00	0.00689	———	
3	50 to 70	16	0.0464	0.00	———	———	0.00980
		20	0.0328	0.00	0.03800	———	
		30	0.0232	0.00	0.02760	———	
		40	0.0164	0.00	0.01950	———	
		50	0.0116	3.42	0.01380	0.00047	
		70	0.0082	93.66	0.00975	0.00913	
		100	0.0058	2.92	0.00689	0.00020	

a Tyler standard

b Represents average of three readings

c $\sqrt{d_1 d_2}$ is the geometric mean of adjacent sieve apertures

d d_p is the product of $\sqrt{d_1 d_2}$ and the percentage of material retained between adjacent sieves

e D_p is the weighted, geometric mean, particle diameter, equal to Σd_p

Luttrell, R. S.: The Determination of the Mean Particle Diameter, Density, and Fraction Voids of Ottawa Sand. Unpublished B. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1955).

TABLE V

Average Particle Diameter of Ottawa Sands from Photographic Analysis

Particle Size, mesh	Particle Diameter[a], in.
20 to 30	0.0278
30 to 50	0.0191
50 to 70	0.0108

a Represents average of 20 values

Luttrell, R. S.: The Determination of the Mean Particle Diameter, Density, and Fraction Voids of Ottawa Sand. Unpublished B. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1955).

TABLE VI

Fraction Voids of Ottawa Sand

Sand Size, mesh	Tube Diameter in.	Bed Height, ft	Solids Weight, lb	Cross-sectional Area of Vessel, sq ft	Fraction Voids[a], %
20 to 30	2	0.5	1.14	0.0218	37.1
	4	0.5	4.49	0.0872	38.2
30 to 50	2	0.5	1.12	0.0218	38.7
	4	0.5	4.31	0.0872	40.7
50 to 70	2	0.5	1.07	0.0218	41.3
	4	0.5	4.16	0.0872	42.7

a Represents average of three tests using 2- and 4-inch diameter columns

Luttrell, R. S.: The Determination of the Mean Particle Diameter, Density, and Fraction Voids of Ottawa Sand. Unpublished B. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1955).

TABLE VII

Operational Data for Fluidization of Ottawa

Sand in Two-inch Diameter

Fluidization Column

TABLE VII

Operational Data for Fluidization of Ottawa Sand in

Two-inch Diameter Fluidization Column

Sand Size, mesh	Bed Height, ft	Manometer Differentials[a] Air, in. Hg	Reference, in. Hg	Bed Height, in. CCl_4	Bed Expansion, in.	Air Temperature, °F
20 to 30	0.5	0.75	0.84	0.38	1.00	68
	1.0	0.71	1.50	1.00	1.00	68
	1.5	0.79	2.25	1.17	1.25	68
	2.0	0.97	3.38	1.44	2.38	68
	2.5	1.46	4.92	2.02	2.50	68
	3.0	1.75	6.73	2.00	3.50	69
	3.5	1.96	7.87	2.19	3.75	69
30 to 50	0.5	0.25	0.75	0.25	0.50	70
	1.0	0.25	1.43	0.84	0.75	70
	1.5	0.33	2.24	1.17	1.00	70
	2.0	0.50	3.00	1.28	1.38	70
	2.5	0.52	3.83	1.47	2.25	70
	3.0	0.63	4.50	1.71	3.13	71
	3.5	0.80	6.08	1.79	4.00	71
50 to 70	0.5	0.21	0.61	0.22	0.50	74
	1.0	0.25	1.38	0.53	1.00	74
	1.5	0.33	2.05	0.96	2.25	74
	2.0	0.40	2.73	1.29	2.75	74
	2.5	0.50	3.48	1.34	3.38	74
	3.0	0.55	4.25	1.48	4.00	74
	3.5	0.74	5.35	1.76	4.50	74

a Represents average of six separate readings. Average wet bulb temperature 56 °F.

TABLE VIII

Operational Data for Fluidization of Ottawa Sand in Four-inch Diameter Fluidization Column

TABLE VIII

Operational Data for Fluidization of Ottawa Sand in

Four-inch Diameter Fluidization Column

Sand Size,	Bed Height,	Manometer Differentials[a]			Bed Expansion,	Air Temperature,
		Air,	Reference,	Bed Height,		
mesh	ft	in. Hg	in. Hg	in. CCl_4	in.	°F
20 to 30	0.5	7.19	1.00	0.25	0.31	75
	1.0	7.63	1.50	0.50	0.31	76
	1.5	8.14	2.25	0.50	0.63	77
	2.0	9.09	3.21	0.65	1.25	77
	2.5	9.58	3.75	0.75	1.50	77
	3.0	10.42	4.46	0.73	1.50	77
	3.5	10.54	5.28	0.82	1.50	77
30 to 50	0.5	2.69	0.75	0.25	0.63	74
	1.0	2.35	1.50	0.63	0.63	74
	1.5	2.95	2.23	0.58	0.75	74
	2.0	2.87	2.92	0.52	1.50	74
	2.5	3.04	3.63	0.57	1.75	74
	3.0	3.21	4.25	0.60	1.86	74
	3.5	3.73	5.00	0.78	2.00	74
50 to 70	0.5	0.75	0.58	0.13	0.25	72
	1.0	0.79	1.38	0.58	0.38	72
	1.5	0.88	2.00	0.88	0.86	72
	2.0	1.00	2.75	1.00	1.00	72
	2.5	1.22	3.40	1.23	1.23	72
	3.0	1.70	4.00	1.45	1.45	72
	3.5	2.00	4.75	1.50	1.50	72

a Represents average of six separate readings. Average wet bulb temperature 56 °F.

ottawa sand using the 2- and 4-inch fluidization columns are presented in Tables IX and X, pages 72 and 73.

Dimensionless Groups. The numerical values for the groups of the dimensionless equation are presented in Table XI, page 74.

TABLE IX

Pressure Drop, Fraction Voids, and Air Velocity
for the Fluidization of Ottawa Sand
in Two-inch Fluidization Column

Sand Size, mesh	Bed Height, ft	Air Velocity, ft/sec	Pressure Drop, lb/sq ft	Fraction Voids, dimensionless
20 to 30	0.5	1.37	56.1	0.391
	1.0	1.35	97.6	0.387
	1.5	1.39	149.2	0.378
	2.0	1.52	226.8	0.372
	2.5	1.78	330.4	0.371
	3.0	1.94	458.4	0.370
	3.5	2.06	537.8	0.373
30 to 50	0.5	0.79	50.8	0.396
	1.0	0.79	94.0	0.380
	1.5	0.93	148.4	0.394
	2.0	1.15	200.3	0.383
	2.5	1.18	258.6	0.398
	3.0	1.27	303.7	0.382
	3.5	1.39	414.1	0.398
50 to 70	0.5	0.73	41.3	0.426
	1.0	0.79	93.1	0.426
	1.5	0.93	136.7	0.418
	2.0	1.03	182.2	0.419
	2.5	1.15	234.7	0.419
	3.0	1.17	287.7	0.419
	3.5	1.35	365.3	0.418

TABLE X

<u>Pressure Drop, Fraction Voids, and Air Velocity</u>

<u>for the Fluidization of Ottawa Sand</u>

<u>in Four-inch Fluidization Column</u>

Sand Size, mesh	Bed Height, ft	Air Velocity, ft/sec	Pressure Drop, lb/sq ft	Fraction Voids, dimensionless
20 to 30	0.5	1.16	67.8	0.379
	1.0	1.21	101.7	0.383
	1.5	1.25	154.6	0.388
	2.0	1.39	221.5	0.385
	2.5	1.45	258.6	0.385
	3.0	1.56	308.4	0.384
	3.5	1.57	366.2	0.385
30 to 50	0.5	0.61	50.9	0.406
	1.0	0.56	100.7	0.410
	1.5	0.64	152.7	0.411
	2.0	0.63	201.8	0.412
	2.5	0.66	251.4	0.411
	3.0	0.67	295.0	0.412
	3.5	0.74	346.5	0.412
50 to 70	0.5	0.34	39.8	0.437
	1.0	0.35	92.7	0.430
	1.5	0.37	133.9	0.429
	2.0	0.38	185.9	0.431
	2.5	0.42	229.9	0.433
	3.0	0.48	270.3	0.432
	3.5	0.52	322.6	0.432

TABLE XI

Numerical Value for Dimensionless Groups

of Pressure Drop Equation

TABLE XI

Numerical Value for Dimensionless Groups of Pressure Drop Equation

Sand Size, mesh	Bed Height, ft	Tube Diameter, in.	$\Delta p/\rho_f D_t$, *	$U_f/g^{\frac{1}{2}}D_t^{\frac{1}{2}}$, *	D_p/D_t, *	ρ_s/ρ_f, *	L/D_t, *	$\mu_f/\rho_f g^{\frac{1}{2}}D_t^{\frac{3}{2}}$, *	ϵ, in.
20 to 30	0.5	2	4840	0.590	0.01331	2400.6	3.19	0.000459	0.391
	1.0	2	8425	0.572	0.01331	2400.6	6.21	0.000459	0.387
	1.5	2	12860	0.598	0.01331	2400.6	9.34	0.000459	0.378
	2.0	2	19480	0.654	0.01331	2400.6	12.65	0.000459	0.372
	2.5	2	28480	0.766	0.01331	2400.6	15.82	0.000459	0.371
	3.0	2	39550	0.845	0.01331	2400.6	18.85	0.000459	0.370
	3.5	2	46310	0.888	0.01331	2400.6	21.75	0.000459	0.373
30 to 50	0.5	2	4385	0.341	0.00899	2400.6	3.31	0.000459	0.396
	1.0	2	8100	0.341	0.00899	2400.6	6.33	0.000459	0.380
	1.5	2	12765	0.401	0.00899	2400.6	9.39	0.000459	0.394
	2.0	2	17280	0.496	0.00899	2400.6	12.82	0.000459	0.383
	2.5	2	22290	0.508	0.00899	2400.6	15.95	0.000459	0.398
	3.0	2	26130	0.546	0.00899	2400.6	19.05	0.000459	0.382
	3.5	2	35730	0.598	0.00899	2400.6	22.10	0.000459	0.398
50 to 70	0.5	2	3559	0.315	0.00515	2400.6	3.13	0.000459	0.426
	1.0	2	8030	0.341	0.00515	2400.6	6.21	0.000459	0.426
	1.5	2	11780	0.401	0.00515	2400.6	9.47	0.000459	0.418
	2.0	2	15720	0.444	0.00515	2400.6	12.59	0.000459	0.419
	2.5	2	20270	0.496	0.00515	2400.6	15.68	0.000459	0.419
	3.0	2	24780	0.504	0.00515	2400.6	18.85	0.000459	0.419
	3.5	2	31500	0.581	0.00515	2400.6	22.35	0.000459	0.418
20 to 30	0.5	4	2935	0.355	0.00667	2400.6	1.59	0.000159	0.379
	1.0	4	4398	0.370	0.00667	2400.6	3.09	0.000159	0.383
	1.5	4	6695	0.383	0.00667	2400.6	4.66	0.000159	0.388
	2.0	4	9590	0.425	0.00667	2400.6	6.31	0.000159	0.385
	2.5	4	11210	0.444	0.00667	2400.6	7.89	0.000159	0.385
	3.0	4	13360	0.478	0.00667	2400.6	9.40	0.000159	0.384
	3.5	4	15880	0.481	0.00667	2400.6	10.90	0.000159	0.385
30 to 50	0.5	4	2208	0.187	0.00450	2400.6	1.65	0.000159	0.406
	1.0	4	4370	0.172	0.00450	2400.6	3.16	0.000159	0.410
	1.5	4	6610	0.196	0.00450	2400.6	4.69	0.000159	0.411
	2.0	4	8725	0.193	0.00450	2400.6	6.39	0.000159	0.412
	2.5	4	10890	0.202	0.00450	2400.6	7.96	0.000159	0.411
	3.0	4	12780	0.205	0.00450	2400.6	9.48	0.000159	0.412
	3.5	4	15000	0.227	0.00450	2400.6	11.02	0.000159	0.412
50 to 70	0.5	4	1724	0.104	0.00258	2400.6	1.56	0.000159	0.437
	1.0	4	4020	0.107	0.00258	2400.6	3.09	0.000159	0.430
	1.5	4	5798	0.113	0.00258	2400.6	4.72	0.000159	0.429
	2.0	4	8045	0.116	0.00258	2400.6	6.28	0.000159	0.431
	2.5	4	9970	0.129	0.00258	2400.6	7.83	0.000159	0.433
	3.0	4	11710	0.147	0.00258	2400.6	9.30	0.000159	0.432
	3.5	4	13980	0.159	0.00258	2400.6	11.15	0.000159	0.432

* Dimensionless

Sample Calculations

A sample of each calculation made in this investigation is presented on the following pages.

Air Humidity. The humidity of the fluidizing medium was determined from the wet and dry bulb temperatures of the air at atmospheric pressure by means of a humidity chart(1). To calculate the humidity, an adiabatic curve was followed from the point at which the wet bulb isotherm and saturation curve intersected, to a point on the adiabatic curve corresponding to the dry bulb temperature. A horizontal line from this point intersected the humidity axis at the humidity of the fluidizing air. The maximum humidity for the tests of this investigation occurred at a wet bulb temperature of 56 °F and a dry bulb temperature of 70 °F. From the humidity chart the humidity of the air was found to be 0.006 pound of water vapor per pound of dry air.

Fluid Density. The value for the fluid density used in this investigation was calculated from the perfect gas law at 712 millimeters pressure and 72 °F:

$$\rho_f = \frac{(P)(M)}{(R)(T)}$$

$$\rho_f = \frac{(\text{lb/sq in.})(28.8)}{(\text{lb mol/°R})(\text{°R})}$$

where:

ρ_f = density, lb/cu ft

P = barometric pressure, lb/sq in.

M = molecular weight, 28.8

R = gas constant, 10.73 lb mol/°R

T = absolute temperature, °R

$$D = \frac{(13.75)(28.8)}{(10.73)(532)}$$

$$D = 0.0694 \text{ lb/cu ft.}$$

Fluid Velocity. The values of fluid velocity used in this investigation were obtained through the use of Figure 4, page 64. For a bed height of 2.0 feet using the 2-inch diameter fluidization column the air velocity through the bed of 20 to 30-mesh sand was:

$$U_f = \frac{W}{(\rho_f)(A)(3600)}$$

$$U_f = \frac{(\text{lb/hr})}{(\text{lb/cu ft})(\text{sq ft})(\text{sec/hr})}$$

where:

U_f = fluid velocity, ft/sec

W = weight flow of fluid, lb/hr

ρ_f = density of fluid, lb/cu ft

A = area of tube, sq ft

$$U_f = \frac{(8.28)}{(0.0694)(0.02185)(3600)}$$

$$U_f = 1.52 \text{ ft/sec.}$$

<u>Fluid Viscosity</u>. The fluid viscosity was calculated from handbook values[52] for various gases at one atmosphere pressure and various temperatures. Perry[52] states that minor deviations from standard pressure have little effect upon the viscosity of gases and so any effects encountered on the viscosity of the air in this investigation as a result of lower operational pressures were neglected. Thus, the value of fluid viscosity used in this investigation was obtained for the average of the fluid temperatures of Table VII, page 69, and Table VIII, page 70, and the use of the line coordinate chart from Perry's Handbook[52]. The fluid viscosity of air at 72 °F is as follows:

$$\mu_{f_m} = 0.0179 \text{ centipoise}$$

$$\mu_{f_e} = \frac{\mu_{f_m}}{c}$$

$$\mu_{f_e} = \frac{(\text{centipoise})}{(\text{centipoise/lb/ft-sec})}$$

where:

μ_{f_m} = fluid viscosity in metric units, centipoise

μ_{f_e} = fluid viscosity in English units, lb/ft-sec

c = conversion factor from metric to English units, 1488 centipoise/lb/ft-sec

$$\mu_{f_e} = \frac{(0.0179)}{1488} = 0.000012 \text{ lb/ft-sec.}$$

<u>Particle Density</u>. The absolute density of the solid particles studied in this investigation was determined by water displacement with a Hogarth specific gravity bottle. The calculation of the particle density for the ottawa sand from Table III, page 65, is presented below.

$$\rho_p = \frac{W_1 \times s_1 \times d_1}{W_2 - W_3(s_1/s_2)}$$

$$\rho_p = \frac{(gm) \times (lb/cu\ ft \times cu\ ft/gm) \times (lb/cu\ ft)}{(gm) - (gm) \times (lb/cu\ ft \times cu\ ft/lb)}$$

where:

ρ_p = absolute density of solid particles, lb/cu ft

W_1 = dry weight of solids, gm

W_2 = weight of water to fill gravity bottle at $\underline{T_1}$, gm

W_3 = weight of water to fill gravity bottle at temperature T_2 minus weight of water displaced by solid particles at $\underline{T_2}$, gm

T_1 = 75 °F (0.99736). Specific gravity, s_1, taken at T_1.

T_2 = 72 °F (0.99774). Specific gravity, s_2, taken at T_2.

$$\rho_p = \frac{(10.0077)(0.99736)(62.4)}{(116.1367 - 112.3934 \times 0.99774/0.99736)}$$

$$\rho_p = 166.1 \text{ lb/cu ft.}$$

Particle Diameter. The weighted, geometric mean, particle diameter was evaluated from screen analyses of the various solid particles in the investigation. The particle diameter for the 50 to 70-mesh range of ottawa sand from Table IV, page 66, is illustrated below.

$$D_p = \sum_{y=1}^{y=y} X \, d_{pgm}$$

$$D_p = \sum_{y=1}^{y=y} (\text{lb/lb}) \ (\text{in.})$$

where:

D_p = weighted, geometric mean particle diameter, in.

y = number of sieved components in the solids mixture

$d_{pgm} = \sqrt{d_1 \times d_2}$ = geometric mean diameter of component retained on adjacent sieves having apertures $\underline{d_1}$ and $\underline{d_2}$, in.

X = weight fraction of closely screened material retained between adjacent sieves of a square root-of-two series of screens

$$D_p = (3.42)(0.0164 \times 0.0116)^{\frac{1}{2}} + (93.66)(0.0116 \times 0.0082)^{\frac{1}{2}} + (2.92)(0.0082 \times 0.0058)^{\frac{1}{2}}$$

$$D_p = 0.00980 \text{ in.}$$

<u>Particle Diameter by Photographic Analysis</u>. The average particle size was also determined by photographic analysis on the three ranges of ottawa sand using an enlargement ratio of 29.5. From Table V, page 67, the particle diameter of the 50 to 70-mesh range of ottawa sand is presented.

$$D_p = \sqrt{\frac{(4)(C)(B)(L)}{\pi}}$$

$$D_p = \sqrt{\frac{(d)(0.76)(in.)(in.)}{(3.1416)}}$$

where:

D_p = average particle diameter, in.

B = average breadth or width of the particles perpendicular to the longest axis of the particles, in.

L = average length of the particles along the longest axis of the particles, in.

C = constant, 0.76

$$D_p = \sqrt{\frac{(4)(0.76)(0.100)(0.0120)}{3.1416}}$$

$$D_p = 0.0108 \text{ in.}$$

Pressure Drop. The pressure drop across the fluidized bed of 20 to 30-mesh ottawa sand for a bed height of 2.0 feet in the 4-inch fluidization column as shown in Table VIII, page 70, was calculated as follows:

$$\Delta p = \frac{(62.3)(13.6\ H_1 - 1.595\ H_2)}{12}$$

$$\Delta p = \frac{(\text{lb/cu ft}) \times (\text{in.} - \text{in.})}{(\text{in./ft})}$$

where:

Δp = pressure drop across fluidized bed, lb/sq ft

H_1 = reference manometer differential, in. Hg

H_2 = bed height manometer differential, in. CCl_4

$$\Delta p = \frac{62.3 \times (13.6 \times 3.21) - (1.595 \times 0.65)}{12}$$

$$\Delta p = 221.5 \text{ lb/sq ft.}$$

<u>Fraction Voids</u>. The fraction voids of a fluidized bed of solid particles at critical mass velocity was calculated as shown below for the 20 to 30-mesh range of ottawa sand in the 2-inch fluidization column, Table VI, page 68.

$$\epsilon = \frac{L - (W/\rho_f A)}{L} \times 100$$

$$\epsilon = \frac{(ft) - (lb/lb\text{-}cu\ ft\text{-}sq\ ft)}{(ft)}$$

where:

ϵ = fraction voids, volume per cent

L = bed depth, ft

W = weight of solid particles in bed, lb

ρ_f = absolute particle density, lb/cu ft

A = cross-sectional area of column, sq ft

$$\epsilon = \frac{(0.5) - (1.14/166.6 \times 0.0218)}{(0.5)} \times 100$$

$$\epsilon = 37.1 \text{ per cent.}$$

<u>Derivation of Correlation Equation</u>. To determine the effects of particle size, tube diameter, and bed height on the pressure drop in a fluidized system a dimensionless equation was derived by dimensional analysis. The variables incorporated in the equation were:

$$\Delta P,\ U_f,\ D_p,\ \rho_f,\ \rho_s,\ \mu_f,\ L,\ g,\ D_t,\ \text{and}\ \epsilon$$

where:

ΔP = pressure drop across fluidized bed, lb/ft-sec/sec

U_f = fluid velocity, ft/sec

D_p = mean particle diameter, ft

ρ_f = fluid density, lb/cu ft

ρ_s = absolute particle density, lb/cu ft

μ_f = fluid viscosity, lb/ft-sec

L = bed height, ft

g = gravitational constant, 32.18 ft/sec/sec

D_t = tube diameter, ft

ϵ = fraction voids, dimensionless.

Equating the variables of the system in exponential form to an unknown dimensionless number, A,

$$A = (\Delta P)^a (U_f)^b (D_p)^c (\rho_f)^d (\rho_s)^e (\mu_f)^f (L)^g (g)^h (D_t)^i \quad (5)$$

where:

a, b, c, d, e, f, g, h, and i = exponents, dimensionless.

Writing the equation in dimensionless form and substituting fundamental units,

$$0 = (ML^{-1}\theta^{-2})^a (L\theta^{-1})^b (L)^c (ML^{-3})^d (ML^{-3})^e (ML^{-1}\theta^{-1})^f (L)^g (L\theta^{-2})^h (L)^i$$

The sum of the exponents for each dimension are equated for each side of the dimensionless equation,

$$M = 0 = a + d + e + f$$

$$L = 0 = -a + b + c - 3d - 3e - f + g + h + i$$

$$\theta = 0 = -2a - b - f - 2h$$

Since there are nine variables having dimensions, and there are three dimensions, the exponents of nine minus three, or six, variables of the equation will be selected for solving the remaining three exponents. Hence, solving in terms of a, b, c, e, f, and g, the relationships of d, h, and i are

$$d = -a - e - f$$

$$h = -a - b/2 - f/2$$

$$i = -a - b/2 - c - 3f/2 - g$$

Substituting the values for the exponents of the dimensional variables into equation (5), page 85,

$$A = (\Delta P)^{a} (U_f)^{b} (D_p)^{c} (\rho_f)^{-a-e-f} (\rho_s)^{e} (\mu_f)^{f} (L)^{g} (g)^{-a-b/2-f/2} (D_t)^{-a-b/2-c-3f/2-g}$$

The variables of like exponents may be grouped in the following manner:

$$A = (\Delta P/\rho_f g D_t)^{a} (U_f/g^{\frac{1}{2}} D g^{\frac{1}{2}})^{b} (D_p/D_t)^{c} (\rho_s/\rho_f)^{e} (\mu_f/\rho_f g^{\frac{1}{2}} D_t^{3/2})^{f} (L/D_t)^{g} \in^{s} \qquad (6)$$

Since the fraction voids, $\in$, are dimensionless, this variable may be added to the other variables of equation (6) at this stage. Inasmuch as $\underline{A}$ has been designated as some unknown, equation (6), page 86, may be rewritten as follows:

$$\left(\frac{\Delta p}{\rho_f D_t}\right) = k\left(\frac{U_f}{\sqrt{g D_t}}\right)^m \left(\frac{D_p}{D_t}\right)^n \left(\frac{\rho_s}{\rho_f}\right)^o \left(\frac{\mu_f}{\rho_f \sqrt{g}\; D_t^{3/2}}\right)^p \left(\frac{L}{D_t}\right)^r \in^s \quad (7)$$

where:

Δp = $\Delta P/g$, lb/sq ft

k = constant factor of proportionality, dimensionless

m = b/a

n = c/a

o = e/a

p = f/a

r = g/a

<u>Analysis of Variance</u>. The form used for the analysis of variance of the experimental observations in this investigation was the <u>split-plot</u> analysis. The algebraic solution of this method of analysis is presented in the accompanying table.

TABLE XII

Algebraic Solution of Split-plot Analysis

TABLE XII

Algebraic Solution of Split-plot Analysis

		A														
		A_1				...	A_i				...	A_a				
		Replications				...	Replications				...	Replications				
		R_1 ·	R_j ·	R_r		...	R_1 ·	R_j ·	R_r		...	R_1 ·	R_j ·	R_r		
	B_1	x_{111} ·	x_{1j1} ·	x_{1r1}	$X_{1.1}$	...	x_{i11} ·	x_{ij1} ·	x_{ir1}	$X_{i.1}$	...	x_{a11} ·	x_{aj1} ·	x_{ar1}	$X_{a.1}$	$X_{..1}$
	.	.	. .	. .	.	...	.	. .	. .	.	...	.	. .	. .	.	.
B	B_k	x_{11k} ·	x_{1jk} ·	x_{1rk}	$X_{1.k}$	...	x_{i1k} ·	x_{ijk} ·	x_{irk}	$X_{i.k}$	...	x_{a1k} ·	x_{ajk} ·	x_{ark}	$X_{a.k}$	$X_{..k}$
	.	.	. .	. .	.	...	.	. .	. .	.	...	.	. .	. .	.	.
	B_b	x_{11b} ·	x_{1jb} ·	x_{1rb}	$X_{1.b}$	...	x_{i1b} ·	x_{ijb} ·	x_{irb}	$X_{i.b}$	...	x_{a1b} ·	x_{ajb} ·	x_{arb}	$X_{a.b}$	$X_{..b}$
		$X_{11.}$ ·	$X_{1j.}$ ·	$X_{1r.}$	$X_{1..}$	...	$X_{i1.}$ ·	$X_{ij.}$ ·	$X_{ir.}$	$X_{i..}$	...	$X_{a1.}$ ·	$X_{aj.}$ ·	$X_{ar.}$	$X_{a..}$	$X_{...}$

R_1	$X_{.1.}$
.	.
R_j	$X_{.j.}$
.	.
R_r	$X_{.r.}$
	$X_{...}$

Where (1) $A_1, \ldots, A_a$ are 'a' classifications

(2) $R_1, \ldots, R_r$ are 'r' replications

(3) $B_1, \ldots, B_b$ are 'b' classifications

(4) x_{ijk} is the observation in the i^{th} A classification, j^{th} replication, k^{th} B classification

$i=1,\ldots,a, \quad j=1,\ldots,r, \quad k=1,\ldots,b$

(5) $$X_{i.k} = \sum_{j=1}^{r} x_{ijk}$$

(6) $$X_{ij.} = \sum_{k=1}^{b} x_{ijk}$$

(7) $$X_{i..} = \sum_{k=1}^{b} X_{i.k} = \sum_{j=1}^{r} X_{ij.}$$

(8) $$X_{..k} = \sum_{i=1}^{a} X_{i.k}$$

(9) $$X_{.j.} = \sum_{i=1}^{a} X_{ij.}$$

(10) $$X_{...} = \sum_{i=1}^{a} X_{i..} = \sum_{j=1}^{r} X_{.j.} = \sum_{k=1}^{b} X_{..k}$$

Kramer, C. Y.: Simplified Computations for Multiple Regression, Va. Poly. Inst., Blacksburg, Va., Agri. Expt. Sta. Report (1954).

In Table XII, any number of variables of classification, A, any number of variables of classification, B, and any number of "repeats" or replications, R, may be employed in the analysis by extending the nomenclature to cover the desired number of variables. When the tables are set up as shown and the sums of the various rows and columns are calculated, the following ten steps are required to calculate the sums of squares to be used in the analysis of variance.

(1) $c = \frac{X^2_{...}}{arb}$

(2) Between A classification = $S_A = \frac{\sum_{i=1}^{a} X^2_{i..}}{rb} - c$

(3) Between Replications = $S_R = \frac{\sum_{j=1}^{r} X^2_{.j.}}{ab} - c$

(4) Total (a) = $S_{T.A.} = \frac{\sum_{i=1}^{a} \sum_{j=1}^{r} X^2_{ij.}}{b} - c$

(5) Error (a) = $S_{E.A.} = S_{T.A.} - S_A - S_R$

(6) Between B classifications = $S_B = \frac{\sum_{k=1}^{b} X^2_{..k}}{ar} - c$

(7) Total (AxB) = $S_{T.A.B.} = \frac{\sum_{i=1}^{a} \sum_{k=1}^{b} X^2_{i.k}}{r} - c$

(8) A x B = $S_{AB} = S_{T.A.B.} - S_A - S_B$

(9) Total = $S_T = \sum_{i=1}^{a} \sum_{j=1}^{r} \sum_{k=1}^{b} x^2_{ijk} - c$

(10) Error (b) = $S_{E.B.} = S_T - S_{T.A.} - S_B - S_{AB}$

When the sums of squares are calculated, the following analysis of variance table is set up for the determination of the experimental F values.

TABLE XIII

Analysis of Variance

Source	df	S.S	M.S.	F
Between A	a-1	S_A	$s_A^2=S_A/(a-1)$	s_A^2/s_a^2
Between Reps	r-1	S_R	$s_R^2=S_R/(r-1)$	s_R^2/s_a^2
Error (a)	(a-1)(r-1)	$S_{E.A.}$	$s_a^2=S_{E.A.}/(a-1)(r-1)$	
Between B	(b-1)	S_B	$s_B^2=S_B/(b-1)$	s_B^2/s_b^2
A x B	(a-1)(b-1)	S_{AB}	$s_{AB}^2=S_{AB}/(a-1)(b-1)$	s_{AB}^2/s_b^2
Error (b)	a(b-1)(r-1)	$S_{E.B.}$	$s_b^2=S_{E.B.}/a(b-1)(r-1)$	
Total	arb - 1	S_T		

Kramer, C. Y.: Simplified Computations for Multiple Regression, Va. Poly. Inst., Blacksburg, Va., Agri. Expt. Sta. Report (1954).

The final step in the procedure is the comparison of the experimental F values with the values from the computed tables at the chosen level of significance for the appropriate number of degrees of freedom.

In this investigation the tube size was designated as the A variable of classification, the bed height was designated as the B variable of classification, and the sand size was chosen as the replications. With the variables in this arrangement the split-plot tables were computed.

TABLE XIV

Split-plot Arrangement of Experimental Pressure Drops

Bed Height	Two-inch Diameter Tube: Sand Size 20-30	Two-inch: 30-50	Two-inch: 50-70	Two-inch: Total	Four-inch Diameter Tube: Sand Size 20-30	Four-inch: 30-50	Four-inch: 50-70	Four-inch: Total	Grand Total
0.5	56.1	50.8	41.3	148.2	67.8	50.9	39.8	158.5	306.7
1.0	97.6	94.0	93.1	284.7	101.7	100.7	92.7	295.1	579.8
1.5	149.2	148.4	136.7	434.3	154.6	152.7	133.9	441.2	875.5
2.0	226.8	200.3	182.2	609.3	221.5	201.8	185.9	609.2	1218.5
2.5	330.4	258.6	234.7	823.7	258.6	251.4	229.9	739.9	1563.6
3.0	458.4	303.7	287.7	1049.8	308.4	295.0	270.3	873.7	1923.5
3.5	537.8	414.1	365.3	1317.2	366.2	346.5	322.6	1035.3	2352.5
Total	1856.3	1469.9	1341.0	4667.2	1478.8	1399.0	1275.1	4152.9	
									8820.1

where, from Table XIV:

$X_{1..} = 4667.8$ $\qquad a = 2$ $\qquad X_{.1.} = 3335.1$

$X_{i..} = 4152.9$ $\qquad r = 3$ $\qquad X_{.j.} = 2868.9$

$X_{...} = 8820.1$ $\qquad b = 7$ $\qquad X_{.r.} = 2616.1$

With the totals computed the sum of squares for the classifications are calculated as follows:

$$c = \frac{X^2_{...}}{arb} = \frac{(8820.1)^2}{(2)(3)(7)} = 1,852,242.00$$

$$S_A = \frac{\sum_{i=1}^{a} X^2_{i..}}{rb} - c = \frac{(4667.2)^2}{(3)(7)} + \frac{(4152.9)^2}{(3)(7)} - 1,852,242.00$$

$$S_A = 1,037,274.087 + 821,265.638 - 1,852,242.00 = 6297.15$$

$$S_R = \frac{\sum_{j=1}^{r} X^2_{.j.}}{ab} - c = \frac{(3335.1)^2}{(2)(7)} + \frac{(2868.9)^2}{(2)(7)} + \frac{(2616.1)^2}{(2)(7)} + 1,852,242.00$$

$$S_R = 794,492.286 + 587,899.086 + 488,855.657 - 1,852,242.00 = 19,005.02$$

$$S_{T.A.} = \frac{\sum_{i=1}^{a}\sum_{j=1}^{r} X^2_{ij.}}{b} - c = \frac{(1856.3)^2}{7} + \frac{(1469.9)^2}{7} + \frac{(1341.0)^2}{7} + \frac{(1478.8)^2}{7} + \frac{(1399.0)^2}{7}$$

$$+ \frac{(1275.1)^2}{7} - 1,852,242.00 = 492,264.24 + 308,658.00 + 256,897.28 + 312,407.06$$

$$+ 279,600.14 + 232,268.57 - 1,852,242.00 = 29,853.29$$

$$S_{E.A.} = S_{T.A.} - S_A - S_R = 29,853.29 - 629.715 - 19,005.02 = 10,218.55$$

$$S_B = \frac{\sum_{k=1}^{b} X^2_{..k}}{ar} - c = \frac{(306.7)^2}{(2)(3)} + \frac{(579.8)^2}{(2)(3)} + \frac{(875.5)^2}{(2)(3)} + \frac{(1218.5)^2}{(2)(3)} + \frac{(1563.6)^2}{(2)(3)} + \frac{(1923.5)^2}{(2)(3)}$$

$$+ \frac{(2352.5)^2}{(2)(3)} - 1,852,242.00 = 2,393,404.82 - 1,852,242.00 = 541,162.00$$

$$S_{T.A.B.} = \frac{\sum_{i=1}^{a}\sum_{k=1}^{b} S^2_{i.k}}{r} - c = \frac{(148.2)^2}{3} + \frac{(284.7)^2}{3} + \ldots \frac{(1317.2)^2}{3} + \frac{(158.5)^2}{3} + \frac{(295.1)^2}{3}$$

$$+ \ldots \frac{(1035.3)^2}{3} - 1,852,242.00 = 2,413,032.00 - 1,852,242.00 = 560,790.00$$

$$S_{AB} = S_{T.A.B.} - S_A - S_B = 560,790.00 - 6,297.15 - 541,162.8 = 13,330.0$$

$$S_T = \frac{\sum_{i=1}^{a}\sum_{j=1}^{r}\sum_{k=1}^{b} x^2_{ijk}}{} - c = (56.1)^2 + (97.6)^2 + \ldots (537.8)^2 + (50.8)^2 + \ldots$$

$$(414.1)^2 + (41.3)^2 + \ldots (365.3)^2 + (67.8)^2 + \ldots (346.5)^2 + (39.8)^2 + \ldots$$

$$(322.6)^2 - 1,852,242.00 = 2,456,301.01 - 1,852,242.00 = 604,059.01$$

$$S_{E.B.} = S_T - S_{T.A.} - S_B - S_{AB} = 604,059.01 - 29,853.29 - 541,162.8 - 13,330.00$$

$$= 19,712.9$$

TABLE XV

Analysis of Variance for Experimental Pressure Drops

Source	df	Sum of Squares	Mean Sum of Squares	*F*
Between A	1	6,297.2	6,297.2	2.768
Between Reps	2	19,005.0	9,502.5	4.176
Error (a)	2	4,550.6	2,275.3	-
Between B	6	541,162.8	90,193.8	109.810
A x B	6	13,330.0	2,221.6	2.705
Error (b)	24	19,712.9	821.3	-
Total	41	604,059.0	-	-

The level of significance chosen in the investigation was α equal to 0.05. Therefore, in the following table the experimental *F* values are compared with the *F* values from computed tables.

TABLE XVI

Comparison of Experimental and Computed "F" Values

Variable	Experimental F Value	$F_{0.95}$ from Computed Tables	Remarks
Tube Diameter	2.768	$F_{0.95}(1,2) = 18.5$	Not Significant
Sand Size	4.176	$F_{0.95}(2,2) = 19.0$	Not Significant
Bed Height	109.810	$F_{0.95}(6,24) = 2.51$	Significant
A x B Interaction	2.705	$F_{0.95}(6,24) = 2.51$	Significant

Evaluation of Equation Unknowns. Prior to the evaluation of the numerical values for the exponents of the groups of the dimensionless equation it was necessary to place the dimensionless equation (7) in the following general form:

$$Y = b_o + b_1X_1 + b_2X_2 + B_3X_3 \tag{8}$$

where:

Y = dependent variable

b_o = numerical constant

b_1, b_2, b_3 = unknown coefficients

X_1, X_2, X_3 = experimental quantities.

Therefore, equation (8) is the general form for a system of one dependent variable, $\underline{Y}$, and three independent variables, $\underline{X_1}$, $\underline{X_2}$, and $\underline{X_3}$.

The matrix of equation (8) is calculated by obtaining the sums of squares and sums of products of the experimental quantities and arranging the matrix in the following form:

$$
\begin{array}{ccccccc}
a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{1y} & a_{1c} \\
 & a_{22} & a_{23} & a_{24} & a_{25} & a_{2y} & a_{2c} \\
 & & a_{33} & a_{34} & a_{35} & a_{3y} & a_{3c} \\
 & & & a_{44} & a_{45} & a_{4y} & a_{4c} \\
 & & & & a_{55} & a_{5y} & a_{5c} \\
 & & & & & a_{yy} & a_{yc}
\end{array}
\qquad (9)
$$

where:

$$
a_{ii} = \sum_{\ell=1}^{n} x_{i\ell}^2 - \frac{\left(\sum_{\ell=1}^{n} x_{i\ell}\right)^2}{n} , \qquad a_{ij} = \sum_{\ell=1}^{n} x_{i\ell} x_{j\ell} - \frac{\left(\sum_{\ell=1}^{n} x_{i\ell}\right)\left(\sum_{\ell=1}^{n} x_{j\ell}\right)}{n}
$$

$$
a_{iy} = \sum_{\ell=1}^{n} x_{i\ell} y_{\ell} - \frac{\left(\sum_{\ell=1}^{n} x_{i\ell}\right)\left(\sum_{\ell=1}^{n} y_{\ell}\right)}{n} , \qquad a_{yy} = \sum_{\ell=1}^{n} y_{\ell}^2 - \frac{\left(\sum_{\ell=1}^{n} y_{\ell}\right)^2}{n} ,
$$

$i = j = 1, 2, 3, 4, 5$, and n is the number of observations.

In the above nomenclature the term, $\sum_{\ell=1}^{n}$, indicates that the quantity under consideration is summed from the first to the n^{th} observation. The term, $\underline{x_i}$, indicates the $\underline{\ell}^{th}$ observation of the $\underline{i}^{th}$ variable.

With the matrix completed the forward solution of the abbreviated Doolittle method is computed from (9).

$$
\begin{array}{ccccccc|l}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{1y} & A_{1c} & \text{Check Column} \\
B_{11} & B_{12} & B_{13} & B_{14} & B_{15} & B_{1y} & B_{1c} & B_{1j}\ (j = 1, 2, 3, 4, 5, y) \\
\hline
 & A_{22} & A_{23} & A_{24} & A_{25} & A_{2y} & A_{2c} & A_{2j}\ (j = 2, 3, 4, 5, y) \\
 & B_{22} & B_{23} & B_{24} & B_{25} & B_{2y} & B_{2c} & B_{2j}\ (j = 2, 3, 4, 5, y) \\
\hline
 & & A_{33} & A_{34} & A_{35} & A_{3y} & A_{3c} & A_{3j}\ (j = 3, 4, 5, y) \\
 & & B_{33} & B_{34} & B_{35} & B_{3y} & B_{3c} & B_{3j}\ (j = 3, 4, 5, y) \\
\hline
 & & & A_{44} & A_{45} & A_{4y} & A_{4c} & A_{4j}\ (j = 4, 5, y) \\
 & & & B_{44} & B_{45} & B_{4y} & B_{4c} & B_{4j}\ (j = 4, 5, y) \\
\hline
 & & & & A_{55} & A_{5y} & A_{5c} & A_{5j}\ (j = 5, y) \\
 & & & & B_{55} & B_{5y} & B_{5c} & B_{5j}\ (j = 5, y) \\
\hline
 & & & & & A_{yy} & A_{yc} & A_{yy}
\end{array}
\qquad (10)
$$

where:

$$
A_{1j} = a_{1j} , \quad B_{1j} = A_{1j}/A_{11} , \quad A_{2j} = a_{2j} - A_{12}B_{1j} , \quad B_{2j} = A_{2j}/A_{22} ,
$$

$$
A_{3j} = a_{3j} - A_{13}B_{1j} - A_{23}B_{2j} , \quad B_{3j} = A_{3j}/A_{33} , \quad A_{4j} = a_{4j} - A_{14}B_{1j} - A_{24}B_{2j} - A_{34}B_{3j} ,
$$

$$
B_{4j} = A_{4j}/A_{44} , \quad A_{yj} = a_{yj} - A_{1y}B_{1j} - A_{2y}B_{2j} - A_{3y}B_{3j} - A_{4y}B_{4j} .
$$

After completing the calculations for each row of the Doolittle procedure[10], the term indicated in the check column is computed and compared with the $\underline{A_{ic}}$ or $\underline{B_{ic}}$ term associated with it. If these terms do not agree within one of the last two decimal places, a mistake has been made and the last row should be checked before proceeding.

The test for regression due to all the variables of the system is computed with the aid of the following table of analysis:

TABLE XVII

<u>Analysis of Variance for Regression</u>

Source	Degree of Freedom	Sum of Squares	Mean Sum of Squares	F
Regression(R)	5	$a_{yy} - A_{yy}$	$\frac{a_{yy} - A_{yy}}{5}$	$\frac{M.S\ (R)}{M.S\ (S)}$
Residual(S)	n - 5	A_{yy}	$\frac{A_{yy}}{n - 5}$	
Total	n - 1	a_{yy}		

Kramer, C. Y.: Simplified Computations for Multiple Regression, Va. Poly. Inst., Blacksburg, Va., Agri. Expt. Sta. Report (1954).

If $\underline{F}$ is not significant, one independent variable of the system must be eliminated from the matrix (9) and the $\underline{F}$ re-evaluated. If the $\underline{F}$ value is significant for (5, n-1) degrees of freedom, the unknown coefficients may be computed:

$$b_5 = B_{5y}, \qquad b_4 = B_{4y} - b_5 B_{45}, \qquad b_3 = B_{3y} - b_4 B_{34} - b_5 B_{35},$$

$$b_2 = B_{2y} - b_3 B_{23} - b_4 B_{24} - b_5 B_{25}, \qquad b_1 = B_{1y} - b_2 B_{12} - b_3 B_{13} - b_4 B_{14} - b_b B_{15},$$

$$b_o = \bar{y} - b_1 \bar{x}_1 - b_2 \bar{x}_2 - b_3 \bar{x}_3 - b_4 \bar{x}_4 - b_5 \bar{x}_5$$

The form of the dimensionless equation (7) can be placed into the form of equation (8) by taking the logarithm of equation (7). The density group cannot be included in the expression since the group does not vary over the $\underline{n}$ observations. Table XVIII contains the logarithmic values for the groups of the dimensionless equation where $\underline{Y}$ is the values for the pressure drop group, $\underline{X_1}$ the values for the velocity group, $\underline{X_2}$ the values for the particle diameter group, $\underline{X_3}$ the values for the bed height group, $\underline{X_4}$ the values for the fraction voids group, and $\underline{X_5}$ the values for the viscosity group.

TABLE XVIII

__Logarithmic Values for Dimensionless Groups__

TABLE XVIII

Logarithmic Values for Dimensionless Groups

n	Y	X_1	X_2	X_3	X_4	X_5
1	3.684	-1.771	-2.124	0.503	-1.591	-4.661
2	3.925	-1.756	-2.124	0.793	-1.587	-4.661
3	4.108	-1.777	-2.124	0.969	-1.577	-4.661
4	4.289	-1.815	-2.124	1.102	-1.570	-4.661
5	4.454	-1.884	-2.124	1.199	-1.569	-4.661
6	4.596	-1.926	-2.124	1.275	-1.567	-4.661
7	4.665	-1.948	-2.124	1.339	-1.571	-4.661
8	3.641	-1.532	-3.953	0.519	-1.597	-4.661
9	3.908	-1.532	-3.953	0.801	-1.579	-4.661
10	4.105	-1.602	-3.953	0.972	-1.595	-4.661
11	4.237	-1.695	-3.953	1.108	-1.583	-4.661
12	4.347	-1.706	-3.953	1.202	-1.599	-4.661
13	4.416	-1.737	-3.953	1.279	-1.581	-4.661
14	4.553	-1.777	-3.953	1.344	-1.599	-4.661
15	3.551	-1.957	-3.712	0.495	-1.629	-4.661
16	3.904	-1.532	-3.712	0.793	-1.629	-4.661
17	4.070	-1.602	-3.712	0.976	-1.621	-4.661
18	4.196	-1.647	-3.712	1.099	-1.622	-4.661
19	4.306	-1.695	-4.712	1.195	-1.622	-4.661
20	4.393	-1.702	-3.712	1.275	-1.622	-4.661
21	4.497	-1.764	-3.712	1.349	-1.621	-4.661
22	3.467	-1.549	-3.823	0.201	-1.578	-4.201
23	3.643	-1.549	-3.823	0.489	-1.583	-4.201
24	3.825	-1.583	-3.823	0.668	-1.589	-4.201
25	3.981	-1.628	-3.823	0.800	-1.585	-4.201
26	4.050	-1.647	-3.823	0.897	-1.585	-4.201
27	4.125	-1.679	-3.823	0.972	-1.584	-4.201
28	4.200	-1.682	-3.823	1.037	-1.585	-4.201
29	3.343	-1.221	-3.653	0.217	-1.608	-4.201
30	3.640	-1.235	-3.653	0.499	-1.612	-4.201
31	3.820	-1.292	-3.653	0.670	-1.613	-4.201
32	3.940	-1.285	-3.653	0.805	-1.614	-4.201
33	4.036	-1.305	-3.653	0.900	-1.613	-4.201
34	4.106	-1.311	-3.653	0.976	-1.614	-4.201
35	4.176	-1.356	-3.653	1.042	-1.614	-4.201
36	3.237	-1.017	-3.411	0.193	-1.640	-4.201
37	3.604	-1.029	-3.411	0.489	-1.633	-4.201
38	3.763	-1.053	-3.411	0.673	-1.632	-4.201
39	3.905	-1.064	-3.411	0.797	-1.634	-4.201
40	3.998	-1.111	-3.411	0.893	-1.636	-4.201
41	4.069	-1.167	-3.411	0.972	-1.635	-4.201
42	4.145	-1.201	-3.411	1.047	-1.635	-4.201

From Table XVIII the values for the sums of squares and sums of products for the variables of the system were determined to be as follows:

Y	168.918	YX_1	-260.855,236
X_1	-64.340	YX_2	-580.255,236
X_2	-144.732	YX_3	152.417,141
X_3	36.824	YX_4	-270.789,408
X_4	-67.354	YX_5	-750.033,218
X_5	-186.102	X_1X_2	219.599,184
X_2^2	514.578,316	X_1X_3	-58.158,510
X_3^2	36.404,276	X_1X_4	103.007,836
X_4^2	108.034,022	X_1X_5	287.016,560
X_5^2	826.839,762	X_2X_3	-125.766,482
X_3X_4	-58.979,559	X_2X_4	232.300,639
X_3X_5	-164.627,644	X_2X_5	639.539,712
X_4X_5	298.378,874	Y^2	684.072,638
		X_1^2	101.626,544

The terms of the matrix are calculated using the values for the sums of squares and sums of products of the experimental quantities.

$$a_{11} = \sum_{\alpha=1}^{n} x_{1\alpha}^2 - \frac{\left(\sum_{\alpha=1}^{n} x_{1\alpha}\right)^2}{n} = 101.626,544 - \frac{(-64.340)^2}{42} = 3.063,792$$

$$a_{12} = \sum_{\alpha=1}^{n} x_{1\alpha} x_{j\alpha} - \frac{\left(\sum_{\alpha=1}^{n} x_{1\alpha}\right)\left(\sum_{\alpha=1}^{n} x_{j\alpha}\right)}{n} =$$

$$219.599,184 - \frac{(-64.340)(-144.732)}{42} = -2.116,456$$

The remaining **a** terms of the matrix are calculated from the sums of squares and sums of products and the employment of the equations of page 99.

a_{14}	-0.172,077	a_{44}	0.020,658
a_{15}	1.926,020	a_{45}	-0.066,699
a_{22}	15.831,845	a_{55}	2.221,801
a_{23}	1.129,021	a_{1y}	-2.088,948
a_{24}	0.198,755	a_{2y}	1.836,191
a_{25}	-1.767,779	a_{3y}	4.316,274
a_{33}	4.118,296	a_{4y}	0.098,757
a_{34}	0.073,862	a_{5y}	-1.557,561
a_{35}	-1.460,500	a_{yy}	4.708,574
a_{13}	-1.747,650		

Substituting the terms of the matrix in their proper position and obtaining the sum of each row including the terms omitted due to symmetry of the matrix, the following calculations are obtained:

3,063,792	-2.116,456	-1.747,650	-0.172,077	1.926,020	-2.088,948	-1.135,319	
	15.831,845	1.129,021	0.198,755	-1.767,779	11.836,191	15.111,577	
		4.118,396	0.073,862	-1.460,500	4.316,274	6.429,403	
			0.020,658	-0.066,699	0.098,757	0.153,256	
				2.221,801	-1.557,561	-0.704,718	
					4.708,574	7.313,287	Check Column
3.063,792	-2.116,456	-1.747,650	-0.172,077	1.027,020	-2.088,948	-1.135,319	-1.135,319
1.000,000	-0.690,796	-0.570,420	-0.056,164	0.628,539	-0.681,817	-0.370,560	-0.370,558
	14.369,805	-0.078,247	0.079,886	-0.437,292	0.393,155	14.327,303	14.327,307
	1.000,000	-0.005,445	0.005,559	-0.030,431	0.027,359	0.997,042	0.997,042
		3.121,075	-0.023,858	-0.364,240	3.126,837	5.859,809	5.859,814
		1.000,000	-0.007,644	-0.116,703	1.001,846	1.877,497	1.877,499
			0.010,367	0.041,122	0.003,148	0.054,634	0.054,634
			1.000,000	3.966,624	0.303,655	5.269,991	5.270,279
				0.792,099	0.120,021	0.912,133	0.912,120
				1.000,000	0.151,522	1.151,539	1.151,522
					0.121,786	0.121,788	0.121,786
					1.000,000	1.000,016	1.000,000

TABLE XIX

Test for Significance of Regression Variables

Source	df	Sum of Squares	Mean Sum of Squares	F
Regression	5	4.586,788	0.917,357	271,247
Residual	36	0.121,786	0.003,382	
Total	41			

Since $F_{0.95}$ (5,36) is only 2.49, the regression due to all variables is significant and the unknown coefficients may be calculated as follows:

$b_5 = 0.151,522$

$b_4 = 0.303,655-(0.151,522)(3.966,624) = -0.297,494$

$b_3 = 1.001,846-(-0.297,494)(-0.007,644)-(0.151,522)(-0.116,703) = 1.017,255$

$b_2 = 0.027,359-(1.017,255)(-0.005,445)-(-0.297,494)(0.005,559)-(0.151,522)(-0.030,431)$
$= 0.039,162$

$b_1 = -0.681,817-(0.039,162)(-0.690,796)-(1.017,255)(-0.570,420)-(-0.297,494)(-0.056,164)$
$-(0.151,522)(0.628,639) = -0.186,462$

$b_o = 4.021,857-(-0.186,462)(-1.531,904)-(0.039,152)(-3.446,000)-(1.017,255)(0.876,761)$
$-(-0.297,494)(-1.603,666)-(0.151,522)(-4.431,000) = 3.173,590$

where $\bar{y} = \sum Y/42$, $\bar{x}_1 = \sum X_1/42$, $\bar{x}_2 = \sum X_2/42$, ... $\bar{x}_5 = \sum X_5/42$.

With the unknown values of the dimensionless equation (7) evaluated, the final form of the equation becomes:

$$Y = 3.174 - 0.186X_1 + 0.039X_2 + 1.017X_3 - 0.297X_4 + 0.151X_5$$

where:

$Y = \log \Delta p/\rho_f D_t$, dimensionless

$X_1 = \log U_f/\sqrt{g\,D_t}$, dimensionless

$X_2 = \log D_p/D_t$, dimensionless

$X_3 = \log L/D_t$, dimensionless

$X_4 = \log \in$, dimensionless

$X_5 = \log \mu_f/\rho_f g^{1/2} D_t^{3/2}$, dimensionless.

IV. DISCUSSION

The results obtained during the performance of this investigation are discussed, recommendations for future work are presented, and the limitations imposed upon the investigation are stated in this section.

Discussion of Results

The discussion of results deals with the fluidization column construction, analytical procedure, orifice calibration, experimental procedure, qualitative observations, quantitative observations, dimensionless pressure drop equation, analysis of variance, statistical regression, and application of developed dimensionless equation.

Fluidization Column Construction. The fluidization columns used in this investigation were constructed of 2- and 4-inch diameter copper tubing. The cost of commercial glass pyrex columns made their use prohibitive for the purposes of this investigation. The copper fluidization columns proved to be very satisfactory for the pressure drop determinations at the seven bed heights. The use of copper construction for the columns was believed to be advantageous from the assembly standpoint. However, the disadvantage of the use of copper construction was in making visual observations and expansion measurements. Measurements of bed height in the columns were made with a 1/4-inch rod probe. The use of the probe to measure the bed heights of sand in the column was believed to be both efficient and accurate for the purposes of the investigation. Had glass construction been employed, the measurement of bed expansion could have been made concurrent with the pressure drop measurements at the critical mass velocity. As it was, glass fluidization columns were substituted for the copper columns for these measurements. At the same time, the qualitative observations of channeling and slugging were obtained on the glass columns.

Analytical Procedure. The method of procedure used for the determination of the absolute density of the solid particles, the mean particle diameter, the fraction voids, and the humidity of the fluidizing medium are considered in the following paragraphs.

Absolute Density of Solid Particles. The absolute densities of the three ranges of ottawa sand were determined by water displacement using a Hogarth specific gravity bottle[65]. Prior to the measurement of the water displaced, the sand was placed in a half-filled bottle of water and boiled for one hour to expel air from the pores of the sand. Duplicate tests were made on the three ranges of sand and the absolute density taken as the average of all individual values. The numerical value for the absolute density from this investigation was found to vary only 0.4 per cent from the value calculated by Lastovica(66).

<u>Particle Size</u>. The numerical value for the diameter of the ottawa sand used in this investigtion was determined from the average of the sand diameters by screen and photographic analyses. The sand diameters for the 20 to 30-, 30 to 50-, and the 50 to 70-mesh ranges of ottawa sand as determined by the weighted, geometric mean, procedure were found to be approximately 9.9, 16.2, and 9.9 per cent, respectively, lower than the values from the photographic analysis. Reference is made to Tables IV and V, pages 66 and 67, for the comparison of the particle diameters by the two methods. It is noted that the values for the sand diameters by the weighted, geometric mean, procedure are dependent primarily upon the sieve aperture and sieve ratio, while the photographic analysis procedure includes the physical

shape of the sand particles. Lastovica[66] has determined the shape factor of ottawa sand to be 1.37 based on a sphere having a shape factor of 1.00. Therefore, it is believed that the physical shape of the ottawa sand is the primary reason for the deviations of the two procedures. However, no quantitative figures are available for the exact deviations of the two procedures for particle diameter as a result of particle shape factor.

Fraction Voids. Determinations of fraction voids were made on the three ranges of ottawa sand in 2- and 4-inch columns. Results, Tables IX and X, pages 72 and 73, show that the fraction voids increased 5.0 per cent with increase in column diameter and 6.0 per cent with decrease in particle size. This was in agreement with the work of Lastovica[67]. The increase in the fraction voids with decrease in particle diameter is due to the ease with which finer particles fluidize. Fluidization has been shown to occur first at the upper surface of a bed of solids and along the wall of the column[36]. Therefore, the increase in the fraction voids with increase in vessel diameter may be attributed to the influence of increased "wall effect", or surface area, on fluidization. This would indicate that an increase in wall surface area would increase the fraction voids. Since only three ranges of sand were employed in the investigation, no trends as to fraction voids could be predicted. However, observations on the ranges used showed erratic deviations from larger to smaller particle size and smaller to larger tube diameter. The maximum deviation in particle size and tube diameter occurred between the 30 to 50-mesh and the 50 to 70-mesh ranges of sand and was evaluated to be 2.6 per cent.

Air Humidity. The maximum humidity of the air in the tests of this investigation was determined to be 0.006 pound of water vapor per pound of dry air. Prior to passing the air into the fluidization column, it was dried in a chamber filled with indicating drierite, Figure 2, page 47. Since the color of the drierite did not change completely from blue to red during the experimental tests, no attempt was made to regenerate the drying agent. Visual observations at the conclusion of the pressure drop determinations showed no signs of particle agglomeration as a result of moisture content. Therefore, the value for the humidity of the fluidizing air used in this investigation was determined at the conclusion of the experimental tests and represented the maximum humidity at any time in the investigation.

Orifice Calibration. The 3/16-inch orifice plate in the air line to the fluidization system was calibrated originally by Dickerson[9]. A large capacity wet test meter was used for this calibration with a range of zero to six inches of mercury covered. In this investigation, however, the range of available wet test meters was only from zero to one inch of mercury. Therefore, the curve of Dickerson[9] was used in this investigation by extrapolation of the curve to cover the higher ranges employed. It is believed that only slight error was introduced into the velocity calculations as a result of extrapolating the calibration curve since the curve formed a straight line above manometer differentials of two inches of mercury. Flow rates of 1/2- and 1-inch of mercury were used in this investigation to confirm the curve of Dickerson[9]. The flow rates calculated at these two values agreed exactly with the corresponding values from the calibration curve.

Experimental Procedure. Consideration of the experimental procedure includes the critical mass velocity and the method of solids charging.

Critical Mass Velocity. All experimental tests in this investigation were conducted at the critical mass velocity of the fluidizing air. The primary reason for operation at the critical mass velocity was that at this velocity the maximum pressure drop is obtained over the various bed heights. It was believed that if a dimensionless equation were to be developed correlating pressure drop with the other variables of a fluidized system it would be most effective if the pressure drop values correlated were the maximum values obtained. Hence, it was decided to operate all tests at the critical mass velocity rather than at some other velocity. The second consideration in relation to critical mass velocity is its variation with the other physical variables of the system. After the critical mass velocity is reached, it becomes extremely difficult to determine accurately the pressure drop over the system due to violent fluctuation of the pressure drop manometers. Therefore, operation at velocities above the critical mass velocity was ruled out. The critical mass velocity is not a constant value, but varies with variation in particle diameter(43). Miller and Logwinuk(43) further state that the critical mass velocity would have to fulfil certain requirements. The selected velocity would have to be high enough to cause bed expansion at large bed heights, and low enough to prevent slugging at low bed heights. In a similar manner, the pre-determined constant velocity would have to be some value which would not cause slugging with fine particles and insufficient bed expansion with large particles. Likewise, similar variation of tube diameter would also have to be considered. It was believed that the selection of a constant velocity to fulfil these conditions was unlikely, and, therefore, all tests were conducted at critical mass velocity. This selection meant that the critical mass velocity would not be a constant in the system, but, instead, a dependent variable. However, operation at the critical mass velocity would further mean that exact replication of the experimental tests could be achieved if desired. This is true since the critical mass velocity would be constant for a particular bed height, a particular particle size, and a particular tube diameter provided the fluidized solid and fluidizing medium remained constant.

Method of Solids Charging. In this investigation the level of sand in the fluidization column was changed by adding or removing sand from the top of the column. In order to prevent any deviation in pressure drop over a bed of sand as a result of particle packing, all bed heights of sand were "slugged" and allowed to settle to a loose packed bed prior to pressure drop determination. Therefore, any effects on bed density as a result of the distance the charged sand fell from the top of the column as it was poured into it were compensated for by slugging each bed of sand. The level of the loosely packed bed of sand was, however, slightly greater than the level before slugging in all cases. This introduced an error of the height of the beds of sand before each pressure drop determination was made. However, these errors due to expansion were eliminated when the critical mass velocity was obtained, for at that velocity maximum bed expansion occurred.

Qualitative Observations. The qualitative observations made in this investigation deal with the slugging of solid particles, the channeling through the beds of solid particles, and the solids entrainment.

Slugging. Since all experimental observations were performed at the critical mass velocity, no slugging occurred during the performance of the pressure drop determinations. Had slugging occurred during the performance of the tests, wide fluctuation would have developed on the pressure drop manometers.

Channeling. Since all beds of sand were "slugged" prior to the pressure drop determinations to produce loosely packed beds, the formation of channels through the bed of solids was quite unlikely. Visual observations of bed expansion at critical mass velocity failed to show any evidence of channeling for the three sand sizes and two tube diameters at the various bed heights.

Solids Entrainment. The experimental tests were conducted with no appreciable solids entrainment. At the conclusion of the tests for each particle size and each vessel diameter, the solids collected in the cyclone separator were removed. In all cases, less than 0.1 per cent of the "fines" of each particle range entrained. However, since the bed height was measured prior to each pressure drop determination, the loss of solids was compensated for when the height of the bed of sand was increased.

Quantitative Observations. The quantitative observations made in this investigation deal with the pressure drop, fraction voids, bed expansion, and mass velocity.

Pressure Drop. Wilhelm and Kwauk[63] have reported that the height of the bed of solids has very little effect on any variable except the pressure drop through the bed. In their investigation it was determined that the pressure drop through the bed of solids was directly proportional to the pressure drop per unit initial height of the bed. In this investigation the pressure drop per unit initial height of sand was found to deviate approximately 10 per cent for the 20 to 30- and 50 to 70-mesh range of the ottawa sand. However, good agreement with the work of Wilhelm and Kwauk was obtained with the 30 to 50-mesh range of sand for the two vessel diameters used. The static pressure drop was found to increase as a whole number multiple of the pressure drop for the six-inch bed of sand. In the case of the 20 to 30-mesh range the deviations were low, whereas with the 50 to 70-mesh range the deviations were approximately 10 per cent high. Consideration of the effects of particle diameter on the pressure drop through the system will be discussed later.

Hariu and Molstab[21] concluded that the total pressure drop across fluidized beds was the sum of a pressure drop due to gas flow and a solids pressure drop consisting of a solids static head, a solids fraction loss due to contact between the solids and tube wall, and an acceleration pressure drop. In this investigation, the total pressure drop at critical mass velocity was found to be essentially equal to the solids static head, or the weight of solid particles in the fluidization vessel divided by the cross-sectional area. The actual pressure drop values were found to be from five to ten per cent higher than the solids static head for all three ranges of material studied. Lastovica[63] in his investigation also found that positive deviations of approximately five per cent were obtained depending on the material under consideration. In this investigation the pressure drops for the 2-inch fluidization column were found to exceed those for the 4-inch column. Leva[37] states that the pressure drop through packed beds was believed to be caused chiefly by expansion and contraction of the fluid passing through the bed voids. Since the fraction voids in the 2-inch diameter bed was found to be less than that of the

4-inch diameter bed, the fluid passing through the smaller bed would encounter more resistance than in the larger bed, hence, a greater pressure drop. The maximum pressure drop for the two columns occurred over the 20 to 30-mesh range of sand with proportional decreases as the 30 to 50- and the 50 to 70-mesh ranges of sand were employed.

<u>Fraction Voids</u>. Leva[40] states that the fraction voids increase with decreasing particle size. Tests of this investigation confirmed this fact, but failed to show any definite relationship between the increase in voidage with decrease in particle size. The increase in fraction voids with decrease in particle size can be attributed to the ease with which fluidization is obtained as the size and weight of the individual particles decreases. Tests on the fraction voids for loose-packed beds and expanded beds at critical mass velocity showed that the fraction voids increased as the vessel diameter was increased for a constant bed height. The reason for this, "wall effect", has previously been discussed. This increase in voidage confirms the results of Lastovica[67] for experimental determinations using ottawa sand as the fluidized solid and air as the fluidizing medium.

Bed Expansion. Tests of this investigation failed to establish any definite relationship between the degree of bed expansion at critical mass velocity. The values for bed expansion for all bed heights in 6-inch increments, Tables IX and X, pages 72 and 73, showed no definite multiple increase for the expansion of the reference six-inch bed. For smaller bed heights the expansion was constant or showed very little increase. As greater bed heights were used, the expansion of beds increased up to 200 per cent in excess of expected values as a result of "wall effect". The influence of "wall effect" was noted further in that the expansion for beds of sand in the 2-inch column was, in all cases, greater than the expansion for the corresponding bed heights using the 4-inch column. One source of experimental error in the expansion measurements was due to the wide fluctuations of the upper surface of the beds of sand. Even at the critical mass velocity, with no evidence of slugging in the beds, the upper surface of the sand beds was difficult to measure accurately as a result of the pulsations of the finer sand particles. As a result, errors of from 1/4- to 1/2-inch were possible in recording the bed expansions.

<u>Critical Mass Velocity</u>. The values for the critical mass velocity were observed to be considerably higher for the pressure drop determinations on the 2-inch diameter column than on the 4-inch column. This would be necessary to overcome the greater resistance of the smaller diameter tube. Tables IX and X, pages 72 and 73, show that the ratio of the mass velocities for the 2- and 4-inch diameter columns increase as the sand size and bed height is increased. Miller and Logwinuk[44] state that a 300 per cent increase in the height of the bed of solids did not appreciably affect the critical mass velocity. Examination of Tables IX and X, pages 72 and 73, show this to be essentially true for bed heights up to 200 per cent increase, but above 200 per cent increase in bed height the critical mass velocity shows a marked increase with increase in bed height. Moreover, Miller and Logwinuk[44] in their investigation failed to include the tube diameter as a possible variable in the fluidization system. Results of this investigation clearly show that for a constant particle size and constant bed height, the critical mass velocity increases as the tube diameter is decreased from 4 to 2 inches.

<u>Dimensionless Pressure Drop Equation</u>. Pressure drop was correlated with the important variables in fluidization by means of equation (7), page 87, which was developed by dimensional analysis. The final form of the dimensionless equation contained the following dimensionless groups:

$$\frac{\Delta p}{\rho_f D_t}, \quad \frac{U_f}{\sqrt{g D_t}}, \quad \frac{D_p}{D_t}, \quad \frac{L}{D_t}, \quad \frac{\mu_f}{\rho_f \sqrt{g}\, D_t^{3/2}}, \text{ and } \epsilon \qquad (11)$$

The unknown exponents and dimensionless constant of equation (7) were evaluated through procedures of statistical regression. Convenience of calculation and evaluation of the unknown quantities required the conversion of the above groups (11) to the following general form:

$$Y = b_o + b_1X_1 + b_2X_2 + b_3X_3 + \cdot \cdot \cdot \qquad (12)$$

This logarithmic form of equation (7) was the convenient form for calculation since the magnitude of the <u>Y</u> and <u>X</u> terms was between $\pm$ 5. With the evaluated <u>b</u> terms substituted into

$$Y = 3.173 - 0.186X_1 + 0.039X_2 + 1.017X_3$$

$$- 0.297X_4 + 0.151X_5 \qquad (13)$$

No attempt was made in the development of the dimensionless equation (7) to combine or prearrange any terms incorporated into the equation. Several attempts were required to obtain the dimensionless grouping of variables. In the final arrangement it was necessary to solve in terms of the viscosity exponent in order to prevent the values of the viscosity group from being of the magnitude of 10^8. In the statistical procedure for the evaluation of the equation unknowns it was necessary to eliminate the ρ_s/ρ_f group from the final equation for reasons to be discussed later. Since the fraction voids term, $\underline{\epsilon}$, is dimensionless it was not necessary to include it in the preliminary steps of the equation development. Had it been included originally the dimensions of the term, $\underline{L^3/L^3}$, would have canceled when the sum of the exponents for each dimension were equated. For this reason, it was only necessary to include the fraction voids term when the other terms of the equation were correlated. The observed and calculated values for the dimensionless groups were checked by means of equation (13). Using equation (13) the average deviation of observed from calculated values was found to be 0.5 per cent for the $\underline{Y}$ values. This resulted in an average deviation of 6.0 per cent for the values of pressure drop throughout the range of operation. Equation (13) was used to check the pressure drops observed by Lastovica[67] over beds of ottawa sand in 2- and 4-inch columns at variable mass velocity. Computed pressure drop values were 19 and 20 per cent higher for the 2- and 4-inch columns, respectively. However, it is believed that such disagreement is justified in as much as the particle size used by Lastovica was not within the limits of this investigation.

Analysis of Variance. The phases of the analysis of variance considered are the variables of classification, the level of significance, the significant variables of the system, and the signifidant interaction.

Variables of Classification. The variables of classification considered were the bed heights, tube diameters, and the particle sizes. These were the independent variables of the fluidized system. No attempt was made to include the critical mass velocity in the analysis of variance since it was a dependent variable similar to the pressure drop. Had a fourth variable, such as solids density, been investigated, the split-plot method of analysis would have been unsatisfactory for the four variable system. A four by four latin square analysis would have been satisfactory in this instance. With the use of the split-plot analysis, Table XII, page 88, any number of observations is possible for each variable of classification. However, through the application of statistical analysis the number of observations required to predict the significant variables of the system is usually reduced considerably.

Level of Significance. In an analysis of variance it is usually stated that the hypothesis (of equal mean values among groups of a system) will be rejected if a value of a statistic occurs which, if the hypothesis were true, would be expected only rarely[10]. In short, the chance of a variable being considered significant when it is really not significant can be determined prior to the analysis of variance. The level of significance, α, is a measure, therefore, of the chance that will be permitted for rejection of a hypothesis when it is really true. An alpha of 0.05 was used in this investigation. This means that the chance permitted for a variable proving significant when it is not significant is five per cent. In the analysis of variance, Table XV, page 95, the experimental F value for bed height was 109.81, while the value for second order interaction was 2.705. The fact that both values were significant, the first value being considerably larger than the second, does not necessarily mean that the chance of error in the smaller value is greater than that in the larger value. Actually, there are five chances in one hundred that both terms are really not significant. As the numerical value of alpha decreases, as from 0.05 to 0.01, the value of F from the computed tables increases, as from 4.28 to 8.47 for (6,6) degrees of freedom. This would, therefore, reduce the chance of an experimental F value exceeding $F_{0.99}$ from the computed tables if it were not really significant.

Significant Variables of Systems. Bed height was determined to be the only significant independent variable of the fluidization system. The fact that bed height proved significant means that it causes the major variation in the pressure drop observations for the system. This agrees with the work of Wilhelm and Kwauk[63] who state that bed height is the only experimental variable having any great effect upon the pressure drop over the system. This does not imply that variation in bed diameter or particle size does not produce variation in the pressure drop. Dixon and Massey[10] state that observation variance is caused not only by differences in variables of classification, but also by the population variance and by experimental errors. No inference can be drawn from the numerical F values for particle diameter or bed diameter. Since these two variables did not prove significant it may only be concluded that the pressure drop variation due to variation of these variables is not as great as that due to changes in bed height.

Significant Interaction. The fact that the interaction between bed height and tube diameter was significant implies that the mean values of the groups were more disperse than was expected when the effects due to bed height and tube diameter were removed separately. Dixon and Massey[10] list several factors that may cause the significant interaction: (1) there is no interaction but a value has been obtained and declared significant, (2) the two variables are producing effects together which they would not produce alone, and (3) another uncontrolled factor is of sufficient importance to include in the experiments. Since an alpha of 0.05 was selected, the chance of number (1) happening is five per cent of the time. The use of alpha as 0.01 would have produced no interaction as the $F_{0.99}$ value from the computed tables is

3.68 as compared with the experimental F value of 2.705. If the bed height and tube diameter were producing effects together that they would not produce separately, there is no assurance that the interaction is due entirely to this factor. As for number (3) above, it is believed that this factor is the least likely to be producing the interaction in as much as nearly all possible variables that could be included in the fluidized system were included. Of course, it is entirely possible that effects from one or more of these reasons are combining to produce the interaction between the variables.

Statistical Regression. The phases of the statistical regression considered in this investigation are the variables of classification, computational procedure, significance of system variables, and the evaluation of the equation unknowns.

Variables of Classification. Five independent variables of classification were considered in this investigation. The algebraic set-up of the statistical regression procedure permits the correlation of any number of variables. However, computational procedures become extremely time-consuming when the number of independent variables exceeds six.

Computational Procedure. The determination of the unknown quantities through statistical regression depends upon the variation of the experimental observations from the mean of the observations. Since the density group of the original equation was a constant for all tests, it could not be included in the statistical regression as a constant has no variance. To evaluate the exponent of the density group, the group would have to be varied by employing two or more solid materials or two or more fluidizing media.

The Doolittle computational procedure handled the five variable system of this investigation quite well. Satisfactory checks were obtained for each line of the computational procedure as it progressed. Extreme care was necessary in computing the sums of squares and sums of products of the observations of the experimental data. Mistakes encountered in the evaluation of the matrix terms usually show up in the computations of each line of the Doolittle procedure. The diagonal terms of the calculation represent a sum of squares and therefore can never contain a negative quantity.

Significant Variables of System. The analysis of variance for regression, Table XIX, page 108, shows that regression due to all the variables of the system was significant. This means that all groups involved in the developed equation were important enough to be included. The fact that the experimental F value exceeded greatly the value of $F_{0.95}$ (5,36) from the computed tables does not mean that the equation is highly significant, but only that the variables are significant.

Evaluation of Equation Unknowns. The unknown quantities of the equation were evaluated by the relationships of page 109. The numerical value for the b terms were evaluated to six decimal places. However, since slide rule, rather than automatic calculator will normally be used for computations, the values for the b terms were rounded off to three decimal places.

Application of Developed Equation. Equation (13) was applied to pressure drop observations of Lastovica[67] for 2- and 4-inch diameter beds of 7 to 30-mesh ottawa sand. Positive pressure drop deviations of 19 and 20 per cent were obtained for the 2- and 4-inch diameter tubes, respectively. Two reasons are possible for the disagreement:

first, the 7 to 30-mesh range of ottawa sand used by Lastovica was larger than the limiting range of this investigation; secondly, the pressure drops observed by Lastovica were in the fluidization range proper and were, therefore, lower than the values at critical mass velocity. For proper operation, equation (13) must be applied at the critical mass velocity for tube ranges from 2 to 4 inches for particle sizes of ottawa sand from 20 to 70-mesh. Attempts to apply the developed equation at vessel diameters below 2 inches, specifically 1 inch and 7/16-inch, resulted in positive deviations of 346 and 438 per cent, respectively. These calculations, though inconclusive, seem to indicate that as the tube diameter approaches 2 inches the pressure drop deviations become less pronounced.

Recommendations

The following recommendations are presented for further study on the pressure drop through fluidized systems.

Polyethylene Columns. Since the construction of the copper fluidization columns did not permit the visual observation of bed expansion, columns of transparent polyethylene should be constructed prior to future pressure drop determinations. Such construction would permit the use of side leads for determining the pressure drop at increments along the column axis.

Investigation of Other Materials. Additional tests should be made using different materials in the particle range from 20 to 70-mesh. Such tests would give further indication as to the validity of equation (13). It is recommended that the selection of other materials be such as to cause wide variation in the ρ_s/ρ_f group of the dimensionless equation. In this way, the variables of the system could be changed to include particle density instead of bed height which has already been shown to be significant. Such materials, with their absolute densities expressed in pounds per cubic foot, are: sovabead fines, 139.2; tubular alumina, 238.8; and superbrite glass beads, 179.5.

Bed Height. Since disagreement was obtained with the results of Miller and Logwinuk[44] in relation to the effects of bed height on critical mass velocity, tests should be conducted to estimate the relationship of bed height on the critical mass velocity for incremental increases of 500 to 1000 per cent. Such tests need only be concerned with the variable of bed height.

Limitations

The limitations imposed upon this investigation are presented in the following paragraphs.

Type of Fluidization Performed. The fluidization tests performed in this investigation were the batch type, in which the material fluidized remained in the fluidization vessel throughout each test.

Material Fluidized. The fluidized material used in this investigation was ottawa sand with an absolute density of 166.6 pounds per cubic foot.

Sizes of Ottawa Sand. Three ranges of ottawa sand were employed with the particle size determined by screen and photographic analysis. The particle diameters by photographic analysis for the three ranges of sand in order of decreasing magnitude were: 20 to 30-mesh, 0.0278 inch; 30 to 50-mesh, 0.0191 inch; 50 to 70-mesh, 0.0108 inch. The particle diameters by screen analysis were: 20 to 30-mesh, 0.0253 inch; 30 to 50-mesh, 0.0169 inch; 50 to 70-mesh, 0.0098 inch.

Vessel Diameter. Fluidization columns of 2- and 4-inch diameter were constructed of copper and employed in the investigation.

Fluidizing Medium. Air was used as the fluidizing medium in the investigation. The air was passed through a bed of 8-mesh indicating drierite prior to introduction into the columns. The air temperature varied from 68 to 76 degrees Fahrenheit and contained a maximum humidity of 0.006 pound of water vapor per pound of dry air.

Air Velocity. The air velocity was held at the critical mass velocity for the tests of this investigation.

V. CONCLUSIONS

Fluidization of ottawa sand of particle diameters, 0.0266 inch, 0.0180 inch, and 0.0103 inch was performed in 2- and 4-inch diameter columns. The static pressure drop was determined at the critical mass velocity for bed heights of 1/2-, 1-, 1-1/2-, 2-, 2-1/2-, 3-, and 3-1/2-feet. Air, with maximum humidity of 0.006 pound of water vapor per pound of dry air, varying from 68 to 76 degrees Fahrenheit was used as the fluidizing medium. The results of the investigation led to the following conclusions:

1. The pressure drop across the fluidized system decreased as the particle diameter decreased. A maximum decrease of 70 per cent occurred for the 6-inch bed height and 4-inch diameter column as the particle size decreased from 0.0266 inch to 0.0103 inch.

2. The pressure drop across the fluidized system increased as the tube diameter decreased. A maximum increase of 49 per cent occurred for the 3-foot bed height and the 20 to 30-mesh range of sand as the tube diameter decreased from 4 to 2 inches.

3. The pressure drop across the fluidized system increased as the bed height increased. A maximum increase

of ten fold occurred for the 2-inch diameter column and the 20 to 30-mesh range of sand as the bed height increased from 1/2-foot to 3-1/2-feet.

4. The pressure drop across the fluidized system increased thirteen fold as the sand size increased from 0.0103 inch to 0.0266 inch, the tube size decreased from 4 to 2 inches, and the bed height increased from 1/2-foot to 3-1/2-feet. An equation was developed correlating the pressure drop with the other factors of the fluidized system. The equation was as follows:

$$Y = 3.173 - 0.186X_1 + 0.039X_2 + 1.017X_3 - 0.297X_4 + 0.151X_5$$

where:

$Y = \log \Delta p / \rho_f D_t$, dimensionless

$X_1 = \log U_f / \sqrt{g D_t}$, dimensionless

$X_2 = \log D_p / D_t$, dimensionless

$X_3 = \log L / D_t$, dimensionless

$X_4 = \log \epsilon$, dimensionless

$X_5 = \log \mu_f / \rho_f \sqrt{g}\, D_t^{3/2}$, dimensionless.

5. The bed height was determined to be the only significant variable when subjected to analysis of variance. Other variables included in the analysis were tube diameter and sand size.

6. The groups, $\Delta p/\rho_f D_t$, $U_f/\sqrt{g D_t}$, D_p/D_t, L/D_t, ϵ, and $\mu_f/\rho_f \sqrt{g} D_t^{3/2}$, of the dimensionless equation were significant as determined by an analysis of variance for regression.

VI. SUMMARY

The use of fluidized solids techniques has been very prominent in recent years, especially in the petroleum industry. With the increasing use of fluidized systems the need for study of the relationships existing between the variables of such systems became more important. The application of the principles of dimensional and statistical analysis to such studies have proved very effective.

The purpose of this investigation was to correlate the pressure drop across a fluidized bed of ottawa sand with the variables of bed height, bed diameter, and particle size by dimensional and statistical means.

In the investigation the effects of bed height, particle size, and vessel diameter on the pressure drop through the fluidized system were studied. Copper fluidization columns were used having internal diameters of 2 and 4 inches. Standard testing grades of ottawa sand were employed as the solid. The sand ranges studied were 20 to 30-, 30 to 50-, and 50 to 70-mesh (Tyler standard) with an absolute density of 166.6 pounds per cubic foot. Seven bed heights of 1/2-, 1-, 1-1/2-, 2-, 2-1/2-, 3-, and 3-1/2-feet were used in the investigation with the static

pressure drop determined at the critical mass velocity at each bed height. Air, varying in temperature from 68 to 76 degrees Fahrenheit and having a maximum humidity of 0.006 pound of water vapor per pound of dry air, was employed as the fluidizing medium.

By means of dimensional and statistical analysis, an empirical equation was developed and the exponents relating pressure drop to the other properties of fluidized systems were evaluated. The equation applies only to velocities of the fluid at the critical mass velocity and is as follows:

$$Y = 3.173 - 0.186X_1 + 0.039X_2 + 1.017X_3 - 0.297X_4 + 0.151X_5$$

where:

$Y = \log \Delta p / \rho_f D_t$, dimensionless

$X_1 = \log U_f / \sqrt{gD_t}$, dimensionless

$X_2 = \log D_p / D_t$, dimensionless

$X_3 = \log L / D_t$, dimensionless

$X_4 = \log \in$, dimensionless

$X_5 = \log \mu_f / \rho_f \sqrt{g}\, D_t^{3/2}$, dimensionless.

The pressure drop over the fluidized system was determined to be dependent primarily on the bed height of sand employed. The application of the principles of multiple regression showed that all the dimensionless groups correlated with pressure drop were significant. The pressure drop was shown to decrease as particle diameter decreased, increase as the tube diameter decreased, and increase as the bed height increased.

VII. BIBLIOGRAPHY

1. Badger, W. L. and W. L. McCabe: "Elements of Chemical Engineering," p. 661. McGraw-Hill Book Co., Inc., New York, N. Y., 1936. 2 ed.

2. Belden, D. H. and L. S. Kassel: Pressure Drops, Ind. and Engr. Chem., 41, 1174 (1949).

3. Breckon, H. C.: Heat Transfer in an Externally Heated Fluidized Bed of Sova Beads, p. 29. Unpublished M. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1950).

4. Brown, G. G. and Associates: "Unit Operations," pp. 269-74. John Wiley and Sons, Inc., New York, N. Y., 1950.

5. Burke, S. P. and W. B. Plummer: Gas Flow Through Packed Columns, Inc. and Engr. Chem., 20, 1196 (1928).

6. Chilton, T. H. and A. P. Colburn: Pressure Drop in Packed Tubes, Trans. Am. Inst. Chem. Engrs., 26, 178 (1931).

7. ibid, p. 189.

8. Conference on Fluidization Technology, Chem. and Ind., 1015-1018 (1953).

9. Dickerson, W. H.: Evaluation of the Coefficient of Heat Transfer at the Heater-Wall Boundary of an Internally Heated Air-Fluidized Bed of Aerocat Cracking Catalyst, pp. 76-77. Unpublished B. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1953).

10. Dixon, W. J. and F. J. Massey, Jr.: "Introduction to Statistical Analysis," pp. 119-144. McGraw-Hill Book Co., Inc., New York, N. Y., 1951.

11. Duncan, A. J.: "Quality Control and Industrial Statistics," pp. 512-544. Richard D. Irwin, Inc., Chicago, Ill., 1952.

12. Ergaun, E., P. Sabri, and A. A. Orning: Fluid Flow Through Randomly Packed Columns and Fluidized Beds, Ind. and Engr. Chem., 41, 1182 (1942).

13. ibid, p. 1183.

14. Fluidization Nomenclature and Symbols, Chem. Engr., 56, pp. 230-234 (1949).

15. Fujita, S. and S. Uchida: Pressure Drop Through Dry Packed Towers, British Chem. Abs., B, 177 (1935).

16. Godel, A.: Fluidization Used in Making Activated Carbon, Chem. Engr., 55, 110 (1948).

17. Gunness, R. C.: Fluidized Solids Technique in the Petroleum Industry, Chem. Engr. Prog., 49, 113 (1953).

18. ibid, p. 114.

19. ibid, p. 118.

20. Happel, J.: Pressure Drop Due to Vapor Flow Through Moving Beds, Ind. and Engr. Chem., 41, 1161 (1949).

21. Hariu, O. H. and M. C. Molstab: Pressure Drop in Vertical Tubes in Transport of Solids by Gases, Ind. and Engr. Chem., 41, 1150 (1949).

22. Kiddoo, G.: Flow in Reacting Systems, Chem. Engr., 56, 113 (1949).

23. ibid, p. 114.

24. Kite, R. P. and E. J. Roberts: Fluidization in Non-Catalytic Operations, Chem. Engr., 54, 112 (1947).

25. Leva, M., M. Grummer, M. Weintraub, and M. Pollchik: Introduction to Fluidization, Chem. Engr. Prog., 44, 511 (1948).

26. ibid, p. 512.

27. ibid, p. 513.

28. Leva, M., M. Grummer, and M. Weintraub: Heat Transmission Through Fluidized Beds of Fine Particles, Chem. Engr. Prog., 45, 568 (1949).

29. ibid, p. 569.

30. ______________________________, and M. Pollchik: Fluidization of Solid Non-Vesicular Particles, Chem. Engr. Prog., 44, 619 (1948).

31. ibid, p. 622.

32. ibid, p. 623.

33. ibid, p. 624.

34. ibid, p. 625.

35. ________: Fluid Flow Through Packed Beds, Chem. Engr., 56, 116 (1949).

36. ________: Pressure Drop Through Packed Tubes, Chem. Engr. Prog., 43, 714 (1947).

37. ________________: Pressure Drop Through Packed Tubes, Chem. Engr. Prog., 43, 633 (1947).

38. ibid, p. 637.

39. ibid, p. 638.

40. ______________________________, and H. H. Storch: A Study of Fluidization of an Iron Fisher-Tropsch Catalyst, Chem. Engr. Prog., 44, 707 (1948).

41. ibid, p. 708.

42. ibid, p. 711.

43. Miller, C. O. and A. K. Logwinuk: Fluidization Studies of Solid Particles, Ind. and Engr. Chem., 43, 1221 (1951).

44. ibid, p. 1222.

45. Morse, R. D. and C. O. Ballou: The Uniformity of Fluidization - its Measurement and Use, Chem. Engr. Prog., 47, 204 (1951).

46. Murphy, N. F.: Dimensional Analysis, Va. Poly. Inst., Blacksburg, Va., Engr. Expt. Sta. Bull., No 73, pp. 5-8 (1949).

47. Nicholson, E. W. and J. E. Moise: Fluidized Solids Pilot Plants, Ind. and Engr. Chem., 40, 2033 (1948).

48. Parent, J. D., N. Yagol, and C. S. Steiner: Fluidizing Processes, Chem. Engr. Prog., 43, 429 (1947).

49. ibid, p. 431.

50. ibid, p. 433.

51. ibid, p. 434.

52. Perry, J. H. et al: Fluids in Motion, "Chemical Engineer's Handbook" (J. H. Perry, Editor), pp. 369-371. McGraw-Hill Book Co., Inc., New York, N. Y., 1950. 3 ed.

53. Resnick, W. and R. R. White: Mass Transfer in Systems of Gas and Fluidized Solids, Chem. Engr. Prog., 45, 377 (1949).

54. ibid, p. 386.

55. ibid, p. 387.

56. Sittig, M.: Fluidized Solids, Chem. Engr., 60, 220 (1953).

57. ibid, p. 222.

58. ibid, p. 227.

59. Thomas, C. L. and J. Hoekstra: Fluidized Fixed Beds, Ind. and Engr. Chem., 37, 332 (1945).

60. Toomey, R. D. and H. F. Johnstone: Gaseous Fluidization of Solid Particles, Chem. Engr. Prog., 48, 220 (1952).

61. Valle, J. M.: Surface Area in Packed Columns, Chem. and Met. Engr., 45, 688 (1938).

62. Wilhelm, R. H. and M. Kwauk: Fluidization of Solid Particles, Chem. Engr. Prog., 44, 203 (1948).

63. ibid, p. 204.

64. ibid, p. 209.

Addenda

65. American Society for Testing Materials: Volume of Cell Space of Lump Coke. D167-24, Non-metallic Materials, Part III-A, pp. 87-88 (1946).

66. Lastovica, J. E., Jr.: The Relation of Vessel Diameter to Several Properties of Fluidized Beds of Solid Particles, p. 99. Unpublished M. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1953).

67. ibid, p. 166.

68. ibid, p. 225.

69. Luttrell, R. S.: The Determination of the Mean Particle Diameter, Density, and Fraction Voids of Ottawa Sand, pp. 27-30. Unpublished B. Sc. Thesis, Library, Va. Poly. Inst., Blacksburg, Va. (1955).

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