Two Essays on Asset Pricing

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ABSTRACT

This dissertation consists of two chapters. The first chapter shows that the measurement errors in betas for stocks induce corresponding measurement errors in alphas and a spurious negative covariance between the estimated betas and alphas across stocks. This negative covariance between the estimated betas and alphas results in a violation of the independence assumption between the independent variable (betas) and error terms in the Fama-MacBeth regressions of tests of the CAPM, thereby creating a downward bias in the estimated market risk premiums. The procedure of using portfolio returns and betas does not necessarily eliminate this bias. Depending upon the grouping variable used to form portfolios, the negative covariance between estimated betas and alphas can be increased, decreased, and can even be made positive. This paper proposes two methods for correcting the downward bias in the estimated market risk premium. The estimated market risk premiums are consistent with the CAPM after the proposed corrections.

The second chapter provides evidence that when the ex-post market risk premium is positive (up markets), the relation between returns and betas is positive, significant, and consistent with the CAPM. However, when the ex-post market risk premium is negative (down markets), the negative relation between betas and returns is significant, but stronger than what is implied by the CAPM. This strong negative relation offsets the positive relation, resulting in an insignificant relation between returns and betas for the overall period. The negative relation between size and returns, after controlling for beta differences, is present only when the ex-post market risk premium is negative, and is responsible for the negative relation for the overall period. This paper decomposes the negative relation between size and returns after controlling for beta differences into the intercept size effect (relation between alphas of stocks and their size) and the residual size effect (relation between residuals of stocks and their size). The asymmetrical size effect between up and down market is being driven by the residual size effect. Long term mean reversion in returns explains, in part, the negative relation between size and returns during down markets.

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Chapter 1

Beta May Not Be Dead After All: A New Framework for Measuring and Correcting the Bias in Cross-Sectional Tests of the CAPM

1.1 Introduction

Cross-sectional studies of the CAPM show that after controlling for size and the book-to-market ratio, beta has no power in explaining the cross sectional differences in stock returns (Fama and French (1992, 1996)). One of the recognized problems in tests of the CAPM is that the measurement error in the estimation of the betas creates a downward bias in the estimated market risk premiums (Miller and Scholes (1972)), and therefore, explains why we do not observe a significant relation between betas and returns. Moreover, this downward bias is picked up by any variable that may be related to the beta (Miller and Scholes (1972), McCallum (1972), Aigner (1974), Kim (1995)). It has been implicitly assumed since Black, Jensen, and Scholes (1972) that the use of portfolio betas sufficiently corrects the bias in the estimated market risk premium in tests of the CAPM.

This paper develops a new framework for measuring and correcting the bias in cross-sectional tests of the CAPM resulting from beta measurement errors and from portfolio grouping procedures. With the proposed corrections, the returns of stocks and stock portfolios are significantly positively related to their betas, even after controlling for size. This new framework also allows us to demonstrate that despite the use of portfolio betas, and in some cases because of the choice of portfolio grouping variables, there is a significant remaining bias in cross-sectional tests of the CAPM[1], and that the lack of any significant relation between betas and returns reported in prior studies is a result of this bias.

Specifically, this new framework allows us to do the following. (1) The new framework allows us to present an alternative characterization of the bias in terms of variables that can be measured, and therefore allows us to assess directly the magnitude of the bias. We show that the bias is equal to the covariance between the estimated betas and alphas divided by the variance of the estimated betas. We also demonstrate that our characterization of the bias is algebraically equivalent to the conventional characterization of the bias in terms of the variance of the measurement error as described in econometrics text books, and as discussed in Miller and Scholes (1972). (2) With this alternative characterization, we are able to demonstrate why the use of portfolio betas does not sufficiently reduce the bias, and how it introduces a new source of bias which depends upon the grouping variable used to form portfolios. This also allows us to explain why different grouping procedures yield surprisingly different results. For example, it allows us to explain why when 100 size portfolios are used in tests of the CAPM, beta has significant power in explaining the cross section of returns, and size has little or no power after controlling for beta differences. And when, as in Fama and French (1996), 100 size-beta portfolios are used, the results are reversed. (3) This alternative characterization of the bias also allows us to propose two procedures for correcting the bias for stocks and portfolios, respectively, and demonstrate that the returns for stocks

[1] Lo and MacKinaly (1990) point out that if grouping is based on either a variable that is empirically correlated with returns or a variable measured within the sample, the test contains a data-snooping bias. However, while their grouping bias is about time series test of the CAPM, the grouping bias in this paper is in cross sectional tests of the CAPM.

and portfolios are significantly and positively related to their betas, even after controlling for size differences.[2]

We demonstrate that the measurement errors in the estimated betas induce corresponding measurement errors in the estimated alphas and a spurious negative covariance between the estimated betas and alphas across stocks.[3] This negative covariance between the estimated betas and alphas creates a violation of the independence assumption between the independent variable (the estimated betas) and the error term in the Fama-MacBeth cross-sectional regressions in tests of the CAPM, and therefore, results in a downward bias in the estimated market risk premiums. The procedure of using portfolio returns and betas does not necessarily eliminate this bias. Depending upon the grouping variable used to form portfolios, the negative covariance between the estimated betas and alphas can be increased, decreased, and can even be made positive, thereby changing the magnitude and the direction of the bias in the estimated market risk premiums.

Several alternatives have been previously suggested to correct for this bias. Litzenberger and Ramaswamy (1979) suggest a correction for the measurement errors that uses the weighted least squares method. However, their corrected estimator can be obtained under the assumption that the security residual variances are known. Kim (1995) proposes a maximum likelihood estimation that explicitly accounts for the measurement error. Brennan, Chordia, and Subrahmanyam (hereafter BCS; 1998) suggest a regression of the risk-adjusted returns of individual securities on the missing factors. Their risk-adjusted returns are the sum of the intercept and the residuals from the 1st-pass time series

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[2] In unreported results when we control for the book-to-market ratio, the results are not affected.

[3] Malkiel and Xu (2000) find that the intercepts and the betas estimated from the CAPM model are negatively correlated and that this reduces the overall explanatory power of β in cross-sectional tests.

regression. They argue that this approach avoids the data-snooping biases in the portfolio-based approach (Lo and MacKinlay(1990)) and also the errors-in-variables bias. However, the linear relation between the CAPM beta and the expected returns cannot be tested by this approach.

Kothari, Shanken, and Sloan (1995) show that the beta has cross-sectional explanatory power using annual returns and a variety of portfolio aggregation procedures; (i) grouping on beta alone, (ii) grouping on size alone, (iii) grouping first on beta and then on size within each beta group, and (iv) grouping first on size and then on beta as in Fama and French (1992, 1996). Lo and MacKinaly (1990) point out that if grouping is based on either a variable that is empirically correlated with returns or a variable measured within the sample, the test contains a data-snooping bias. Berk (2000) shows that the sorting and grouping procedure can introduce a bias in favor of rejecting the model under consideration by picking enough groups to sort into.

In some studies, beta is estimated for stocks (or portfolios) for the entire period (hereafter fixed betas) and then period by period cross sectional regressions are estimated (Davis (1994), Kothari, Shanken, and Sloan (1995), Fama and French (1996), Gomes, Kogan, and Zhang (2003), Ferguson and Shockley (2003), Teo and Woo (2004)). In other studies, betas are estimated over the earlier periods (hereafter rolling betas) and the cross sectional regressions are estimated over non-overlapping later periods (Ferson and Harvey (1991, 1999), Chung, Johnson, and Schill (2006)). The results of our paper show that the bias is marginally bigger for studies that use rolling betas.

The rest of the paper is organized as follows. Section I presents two propositions that (1) demonstrate why beta measurement errors create a spurious negative covariance between the estimated betas and alphas, and (2) how the bias in the cross-sectional tests of the CAPM can be expressed in terms of the covariance between the estimated alphas and betas. Section II describes the data, presents the regression results for individual stocks, and proposes a 3-pass methodology for individual stocks that corrects for the bias resulting from the measurement error induced spurious negative covariance between the estimated betas and alphas. Section III presents the regression results for portfolios, demonstrates that there is significant remaining bias for portfolios, and proposes a portfolio grouping procedure for reducing the bias. Section 5 concludes the paper.

1.2 The covariance between the estimated betas and alphas and the bias in cross-sectional tests of the CAPM

1.2.1 Beta measurement errors and the covariance between the estimated betas and alphas

In cross-sectional tests of the CAPM, betas are estimated for stocks (portfolios) from the time-series regression (generally referred to as 1^{st} pass regressions) of the stock's (portfolio's) excess returns on the market excess returns. This estimation equation can be expressed as

$$R_{it} = a_i + b_i R_{Mt} + e_{it}\text{, for each i = 1, 2, …, N, t = 1, 2, …,T} \qquad (1)$$

where R_{it}'s and R_{Mt}'s are the time-series of observed monthly excess returns of security i and the market portfolio, respectively, and a_i, b_i, and e_{it}'s in (1) are the estimated intercept, the estimated slope, and the time-series of estimated errors, respectively. The true return generating model that underlies (1) can be expressed as

$$R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}, \text{ for each } i = 1, 2, \ldots, N, t = 1, 2, \ldots, T \quad (2)$$

where α_i, β_i, and ε_{it}'s in (2) are the true intercept, the true slope, and the time-series of true errors, respectively. Proposition I below formally proves the argument that measurement errors in betas of individual stocks induce a spurious negative covariance between the estimates of betas and alphas for the cross-section of stocks.

The reason why beta measurement errors induce a spurious negative covariance between the estimates of betas and alphas for stocks is obvious. When individual stock betas are estimated in (1), any underestimation of beta will result in an overestimation of alpha, and vice versa.

Proposition I. Measurement errors in the estimation of betas induce corresponding measurement errors in the opposite direction in the estimates of alphas, resulting in a spurious negative covariance between the two estimates.

Proof of Proposition I

We assume, without loss of generality, that

$$b_i = \beta_i + v_i, \; v_i \sim N(0, \sigma_{v_i}^2) \quad (3)$$

where v_i is the measurement errors in β_i, and $\sigma_{\beta_i v_i} = 0$.

From (1), (2), and (3),

$$a_i = \bar{R}_i - b_i \bar{R}_M$$

$$= \bar{R}_i - \beta_i \bar{R}_M - b_i \bar{R}_M + \beta_i \bar{R}_M$$

$$= \alpha_i - (b_i - \beta_i)\ \bar{R}_M$$

$$= \alpha_i - v_i \bar{R}_M \qquad (4)$$

From (3) and (4), it can be shown that $cov(a_i, b_i) = cov(\alpha_i, \beta_i) - \bar{R}_M\ \sigma^2_{v_i}$.[4] Assuming that there is no relation between the true alphas and betas ($cov(\alpha_i, \beta_i) = 0$) for individual stocks, $cov(a_i, b_i) = -\bar{R}_M\ \sigma^2_{v_i} \neq 0$ (unless $\bar{R}_M = 0$ or $\sigma^2_{v_i} = 0$).[5] Thus, the measurement errors in the estimation of beta induce a spurious negative covariance between the estimates of betas and alphas. The empirical and the simulation results presented in the paper are consistent with this proposition.

In case of portfolios, $cov(\alpha_i, \beta_i)$ may not be zero. This is because the portfolio grouping procedure can group stocks into portfolios with systematic patterns in their betas and alphas. We will present evidence later in the paper, suggesting that the upward bias of the estimated market risk premium observed for some portfolios (e.g. portfolios formed on the basis of size only) is a result of positive covariance between their alphas and betas.

1.2.2 The covariance between the estimated betas and alphas and the bias in the estimated market risk premium

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[4] The above proof makes the assumption that $\alpha_i = \bar{R}_i - \beta_i \bar{R}_M$. However, $\alpha_i = E(R_i) - \beta_i E(R_M)$. The difference is because of sampling error. Since the sampling error will be independent of measurement error, the proposition will continue to be true.

[5] If the CAPM is the true return generating model, then the expected value of alpha is equal to zero, and there should be no relation between the true alphas and betas. Even if there are factors other than the CAPM beta that affect the cross-section of returns, there is no ex-ante reason to believe that the true betas and the true alphas of individual stocks would be systematically related.

In the Fama-MacBeth cross-sectional tests of the CAPM (generally referred to as the 2nd pass regressions), the stock (or portfolio) returns are regressed on their estimated betas for each time period (generally for each month) as follows.

$$R_{it} = \gamma_{0t} + \gamma_{1t} b_i + \eta_{it} \quad \text{for each } t=1, 2, \ldots, T, \quad i=1, 2, \ldots, N \tag{5}$$

The cross-sectional variability of each stock's (portfolio's) return that is not related to the market variability should be captured by the error term (η_{it}) in these cross-sectional regressions. Therefore, the η_{it} in (5) should capture not only the e_{it} from (1), it should also capture the variability in the a_i's. This is because the variability in the a_i's represents the part of the stock's (portfolio's) variability that is not related to the market return in (1). Thus, the η_{it} can be expressed as follows.

$$\eta_{it} = e_{it} + (a_i - \bar{a}) \tag{6}$$

Since the a_i's are correlated with the b_i's (as shown in Proposition I) and are included in η_{it}'s, this results in a violation of the critical regression assumption of independence between the independent variable (the estimated betas) and the error terms in (5). Thus, $\hat{\gamma}_{1t}$, the estimated co-efficient of the beta in (5) is biased. We will show in Proposition II that this bias can be expressed as follows.

$$bias = \frac{\text{cov}(a_i, b_i)}{\text{var}(b_i)} \tag{7}$$

Proposition II. Since b_i is the independent variable in the Fama-MacBeth cross-sectional regressions and the variability of a_i is incorporated in the error term, any covariance between the a_i's and b_i's violates the critical regression assumption of independence between the independent variable and the error term, and leads to a biased estimate of

γ_{1t}, the estimated market risk premium.[6] The expected bias in the estimated market risk premium in the Fama-MacBeth cross sectional regression is equal to the covariance between the estimated betas and alphas divided by the variance of the estimated betas, which is the average slope coefficient of the regression of a_i on b_i.

Proof of Proposition II

$$\hat{\gamma}_{1t} = \frac{\text{cov}(R_{it}, b_i)}{\text{var}(b_i)} = \frac{\text{cov}(a_i + b_i R_{mt} + e_{it}, b_i)}{\text{var}(b_i)}$$

$$= \frac{\text{cov}(a_i, b_i)}{\text{var}(b_i)} + R_{mt} \frac{\text{cov}(b_i, b_i)}{\text{var}(b_i)} + \frac{\text{cov}(e_{it}, b_i)}{\text{var}(b_i)}$$

Since the expected value of $\text{cov}(e_{it}, b_i)$ is equal to zero, the above can be written as

$$\hat{\gamma}_{1t} = \frac{\text{cov}(a_i, b_i)}{\text{var}(b_i)} + R_{mt} \qquad (8)$$

An alternative proof of Proposition II using vector notation is provided in the Appendix. The proof in the Appendix directly demonstrates the violation of the assumption of independence between the independent variable and the error term in the regression.

If there is no measurement error induced negative covariance between the estimated betas and alphas for individual stocks, then $\hat{\gamma}_{1t}$ would have an expected value equal to the market risk premium for the month. However, we know from Proposition I that beta measurement errors induce negative covariance between the estimated betas and alphas for individual stocks and, therefore, $\hat{\gamma}_{1t}$ will be biased downward. In the next section, we show the algebraic equivalence of our characterization of the bias with the conventional

[6] This argument is also valid for cross-sectional regression of average excess returns on the estimated beta as in Miller and Scholes(1972).

characterization when there is beta measurement error, and the relative advantages of our characterization.

1.2.3 The equivalence of our characterization of the bias with the conventional characterization

Proposition II shows that the bias in the estimated average market risk premium ($\hat{\gamma}_{1t}$) from (8) can be expressed as[7]

$$\frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{1t} = \frac{1}{T}\sum_{t=1}^{T}\gamma_{1t} + \frac{\operatorname{cov}(a_i, b_i)}{\operatorname{var}(b_i)} \tag{9}$$

$$= \frac{1}{T}\sum_{t=1}^{T}\gamma_{1t} + [\frac{\operatorname{cov}(\alpha_i, \beta_i)}{\operatorname{var}(b_i)} - \frac{\sigma_{v_i}^2 \overline{R}_M}{\operatorname{var}(b_i)}] \tag{10}$$

The conventional characterization of the bias caused by beta measurement errors, as provided in Miller and Scholes (1972), shows that the estimated average market risk premium is

$$\frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_{1t} = \frac{1}{T}\sum_{t=1}^{T}[\gamma_{1t}\frac{\operatorname{var}(\beta_i)}{\operatorname{var}(b_i)}] \tag{11}$$

where $\operatorname{var}(b_i) = \operatorname{var}(\beta_i) + \sigma_{v_i}^2$. Since $\gamma_{1t} = R_{Mt}$, it can be shown that if $\operatorname{cov}(\alpha_i, \beta_i) = 0$, then (10) becomes

$$\frac{1}{T}\sum_{t=1}^{T}[\gamma_{1t}(1 - \frac{\sigma_{v_i}^2}{\operatorname{var}(b_i)})] = \frac{1}{T}\sum_{t=1}^{T}[\gamma_{1t}\frac{\operatorname{var}(\beta_i)}{\operatorname{var}(b_i)}]. \tag{12}$$

Therefore, the conventional characterization of the bias as expressed in (11) and our characterization as expressed in (10) become algebraically equivalent.

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7 See the proof in the appendix for the detailed derivation.

However, there are two advantages of our characterization over the conventional characterization. First, the conventional characterization (11) implies that there will be only a downward bias in the estimated market risk premium when the CAPM β is estimated with errors. However, we find that portfolios formed on the basis of only size exhibit an upward bias in the estimated market risk premium. Our characterization shows that the estimated market risk premium can be upward or downward biased, depending on the covariance between the alphas and betas for portfolios as shown in (10). Second, since a_i and b_i are observable, our characterization allows us to directly measure and correct for the bias created by the measurement error for individual stocks without the need for measuring the unobservable $\sigma_{v_i}^2$ [8] Third, this new characterization allows us to suggest procedures for correcting the bias directly.

1.3 Data and empirical analyses of individual stocks

1.3.1 Summary statistics of alphas, betas, and covariances

We use the sample of all NYSE and AMEX stocks (hereafter NYAM) over the period July 1931 – June 2005. We divide the sample period into fifteen 5-year periods, with the last period being a 4-year period from July 2001 to June 2005. For each period, we estimate the 1st-pass regression for each stock as described in (1) using the times-series of the stock's and market's excess returns, and obtain the estimates of the alphas (a_i), betas

[8] Miller and Scholes (1972) use the average standard error of the estimated betas as a proxy for the measurement error.

(b_i), and the time-series of error terms (e_{it}) for each stock.[9] For each of the fifteen periods, we calculate the mean and the standard deviations of the estimated alphas and the betas, as well as the cross sectional covariance between the estimated betas and alphas.

Panel A of Table 1.1 presents the average values of the means and the standard deviations of the estimated alphas and betas, as well as the average of the covariances between them. The average of the cross-sectional mean values of the estimated alphas for the fifteen periods is 0.02% and the corresponding average of the estimated betas is 1.04. Consistent with the predictions of Proposition I, the average of the covariances between the estimated alphas and betas is -0.21. Since there is no ex-ante reason for a negative covariance between the true alphas and betas across the individual stocks in the market, we infer that the negative covariance between the estimated alphas and betas is a result of the measurement errors in estimation of betas, as derived in Proposition I.

To further investigate this, we divide our sample firms into ten size deciles for each of the fifteen periods. We calculate the covariances between the estimated betas and alphas separately for firms within each size decile and for each period. Panel B of Table 1.1 presents the average of covariances between the estimated betas and alphas for each size decile. The covariance between the estimated betas and alphas is negative for each of the ten size deciles, consistent with the predictions of Proposition I.

1.3.2 Simulations

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[9] We use the constant betas for each 5-years period since it creates smaller downward bias than the rolling betas. However, using rolling betas does not change the implications or the conclusions. Shanken and Zhou (2006) use constant betas and state that using rolling betas further complicates the econometric analysis.

In order to demonstrate the effect of measurement errors on the covariance between the estimated betas and alphas, we simulate returns for our sample of stocks using alphas and betas that are orthogonal, and show that the estimated betas and alphas for these simulated returns exhibit a negative covariance. The simulation uses the actual values of R_{mt}. The betas for the simulation are randomly drawn from the distribution of the estimated betas from the actual data. Since the variance of the estimated betas from the simulated returns will be greater than the variance of the betas used for simulation because of measurement errors, the distributions of beta is shrunk around a mean value of 1.00, such that the variance of estimated betas from the simulated data is similar to the variance of estimated betas using actual returns. The alphas and error terms for the simulation are randomly drawn from normal distributions. The means and variances of these normal distributions are chosen such that the means and variances of the estimated parameters from the simulated data are similar to their counterparts from the actual data.

Table 1.2 reports the summary statistics of the estimated parameters from the actual returns (Panel A) and the averages of the estimated parameters from 50 simulations (Panel B). Comparing the values in Panels A and B, we can see that the means and the standard deviations of the excess returns and the residuals are very similar for the actual returns and the simulated returns. Moreover, the means and the standard deviations of the estimated betas and alphas for actual and simulated returns are also very similar. These comparisons suggest that our simulations replicate the means and the standard deviations of the actual data very closely. The covariance and the correlation between the true alphas and betas in panel B used for simulation are zero by the design of simulation. However, the covariance (correlation) between the estimated alphas and betas for the simulated returns in Panel B is -0.12 (-0.12). These simulation results clearly demonstrate that measurement errors induce a spurious negative covariance between the estimated betas and alphas. The covariance (correlation) between the estimated alphas and betas is -0.21 (-0.20) in Panel A when actual returns are used. The higher values for covariance and correlation for the actual data as compared to the simulated data can be possibly explained by the fact that the error terms in the simulated data are normally distributed with no serial correlation, while the residuals from the actual data exhibit both skewness and serial correlation. Consequently, the measurement errors and the resulting correlation and covariance between the estimated betas and alphas are likely to be higher for the actual data. Nevertheless, the predictions of Proposition I that measurement errors will induce a spurious negative covariance between the estimated betas and alphas are confirmed by the simulation results.

1.3.3 Regression results

To demonstrate the bias created by the measurement error induced negative covariance between the estimated betas and alphas for individual stocks, we estimate the following Fama-MacBeth cross-sectional regression for each month.

$$R_{it} = \gamma_{0t} + \gamma_{1t} b_i + \gamma_{2t} S_{it-1} + \varepsilon_{it}$$

where $S_{i,t-1}$ is the log of the market capitalization of the firm in the month of June and is used from July to June in the following year.

The results are presented in Panel A of Table 1.3. The average value of $\hat{\gamma}_1$ is only 0.54% as compared to its expected value of 1.07%, the average market risk premium for this sample period. However, when we take into account the expected bias of -0.51% (Panel B) caused by the negative covariance between the estimated betas and alphas, the bias corrected estimate of $\hat{\gamma}_1$ is 1.05%. When we control for size, the average value of $\hat{\gamma}_1$ drops to an insignificant value of 0.37%. These results, which suggest that beta has no significant power in explaining the cross-section of returns after controlling for size differences, are consistent with the results of prior studies. However, they are biased by the measurement error induced negative covariance between the estimated betas and alphas, as derived in Proposition II. In the next section, we propose a 3-pass methodology that purges the negative covariance between the estimated betas and alphas and corrects for this bias.

1.3.4 3-pass Methodology

In order to correct the bias in the estimated market risk premium, we propose a three-pass methodology for individual stocks that purges the negative covariance between the estimated betas and alphas. In a manner identical to the two-pass methodology, alphas (a_i's), betas (b_i's), and the error terms (e_{it}'s) are estimated for each stock in the 1st-pass regression using the time series of R_{it}'s and R_{mt}'s as described in (1) for each 5-year period. In order to purge the negative covariance between the estimated betas and alphas across stocks, a cross sectional regression is estimated in the 2nd-pass with the estimated betas (b_i's) from the 1st-pass as the dependent variable and the estimated alphas (a_i's) from the 1st-pass regression as the independent variable as follows.

$$b_i = \theta_0 + \theta_1 a_i + \xi_i \qquad i=1, 2, \ldots N \tag{13}$$

A purged beta (b_i^*) is calculated for each stock by subtracting from b_i the part of a_i that covaries with b_i, as follows

$$b_i^* = b_i - \hat{\theta}_1 a_i \qquad (14)$$

In the 3rd-pass regression, the purged betas (b_i^*'s) are used in the Fama MacBeth cross-sectional regressions to correct for the violation of the independence between the independent variable and the error terms.

$$R_{it} = \gamma_{0t} + \gamma_{1t} b_i^* + \eta_{it} \quad i = 1, 2, .., N \text{ for each } t = 1,\ldots,T. \qquad (15)$$

Since there is no covariance between the purged betas as an independent variable and the estimated alphas that are included in the error terms in the 3rd-pass regression, the average estimates of $\hat{\gamma}_1$ obtained in the 3rd-pass are unbiased.

To compare the purged betas from the 2nd-pass regressions with the unpurged betas from the 1st-pass regressions, we divide the sample of stocks into 100 size-beta portfolios every June, and calculate the averages of the purged and unpurged betas for each portfolio. Panel A of Table 1.4 presents the percentiles of the differences between the unpurged betas and the purged betas for individual stocks. For 50% of the stocks, the differences between unpurged and purged betas is between -0.08 and 0.06, and for 90% of the stocks, the difference is between -0.21 and 0.23. These differences are relatively small compared to the average estimated beta 1.04. The averages of the unpurged and the purged betas for the 100 size-beta portfolios are presented in Panels B and C of Table 1.4, respectively. The average unpurged and purged betas exhibit very similar patterns across size and beta portfolios. It appears that on average the purging process reduces the betas of high beta firms by 0.01 for S1 and 0.04 for B10, and increases the beta of the low beta firms by 0.02 for S10 and 0.05 for B1. Thus, the average change in the betas as a result of the purging in the 2nd-pass appears to be small. Panel D of Table 1.4 presents the average correlations between the unpurged betas, purged betas, and size. The average correlation between the purged and unpurged betas is very high at 0.97, and the average correlation between size and unpurged betas of -0.30 remains unaltered by the purging process. Other than removing the negative covariance between the estimated betas and alphas, the results of Table 1.4 suggest that (1) the average purged betas are not materially different from the average unpurged betas across size and beta portfolios, (2) the purged betas are highly correlated with the unpurged betas, and (3) the purged betas retain the relation between the unpurged beta and size. Therefore, the purged beta becomes an instrumental variable, which is highly correlated with the unpurged beta, and which does not have any correlation with the error terms in the regression.

In Table 1.5, we present the average values of estimates of $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ for the monthly Fama-Macbeth cross-sectional regressions of R_{it}'s on the purged betas and size. This is done in a manner similar to the results presented in Table 1.3, where the unpurged betas from the 1st-pass regressions were used. The average value of $\hat{\gamma}_1$ is significant at 1.03% as compared to its expected value of 1.07%, the average market risk premium for the period. The use of the purged beta almost doubles the average value of $\hat{\gamma}_1$ from 0.54% in Table 1.3 to 1.03% in Table 1.5. This increase in $\hat{\gamma}_1$ is a result of the correction of the bias introduced by the negative covariance between the independent variable and the error term due to measurement error in these regressions when unpurged betas are used.[10] When size is included as an additional explanatory variable in the regression, the average

[10] Bansal, Dittmar, and Lundblad (2005) show that the cash flow beta has significant cross sectional explanatory power and account for more than 60% of the cross-sectional differences in risk premim across 30 portfolios comprised of 10 size, 10 momentum, and 10 book-to-market portfolios.

$\hat{\gamma}_1$ drops to 0.89%, but remains statistically significant, and only 0.18% below the average market risk premium of 1.07%. Thus, beta has significant power in explaining the cross-section of individual stock returns even after controlling for size differences. Since the purged betas have the same correlation with size as the unpurged betas, the significance of the purged betas is not being driven by any reduction in the correlation between betas and size (panel D of Table 1.4). However, size continues to be significantly negatively related to returns even after controlling for the purged betas with an average $\hat{\gamma}_2$ of -0.09, as compared to -0.13 when the unpurged betas are used. This reduction in the average value of $\hat{\gamma}_2$ by about 30% when the purged betas are used suggests that a part of the observed size effect in Table 1.3 (and previous studies) is a result of downward bias in the estimated market risk premium because of measurement errors.[11]

1.4 Empirical analysis of portfolios

In this section, we empirically demonstrate the bias introduced by the covariance between the estimated betas and alphas on the estimated market risk premium when portfolio betas and returns are used. We also propose a methodology for correcting this bias for studies that use portfolio returns and betas.

1.4.1. Regression results

[11] Chan, Chen, and Hsieh (1985) use an alternative approach to test the firm size effect. They obtaine the estimated residuals from the Fama-MacBeth cross sectional regressions and conduct tests to determine if these residuals are related with firm size.

The first set of portfolios we examine are the portfolios based on size and betas. Following Fama and French (1992, 1996), from June 1931 until June 2004, we restrict the sample to the stocks that have at least 24 months returns in the previous five years as of each June and a valid return in July. Ten portfolios are formed on size (market capitalization) each June and within each size decile, ten portfolios are formed on ranking betas, the slope coefficients from the regression of monthly excess returns on contemporaneous monthly equal weighted market excess returns. The ranking betas use 60 months (at least 24 months) of past returns (as available) each June. We refer to these portfolios as 100 size-beta portfolios.

In Table 1.6, we present the average values of estimates of $\hat{\gamma}_0$, $\hat{\gamma}_1$, and $\hat{\gamma}_2$ for the monthly Fama-Macbeth cross-sectional regressions of R_{it}'s on the portfolio betas and size for the 100 size-beta portfolios. This is done in a manner similar to the results presented in Table 1.3, where individual stocks were used. The average value of $\hat{\gamma}_1$ is 0.80%, when beta is the only independent variable, as compared to its expected value of 1.07%, the average market risk premium for the period. Panel B of the Table 1.6 shows that the average bias for 100 size-beta portfolios is -0.27%, as compared to the average bias of -0.51% for individual stocks in Panel B of Table 1.3. Thus, the bias for 100 size-beta portfolios, although smaller than for individual stocks, continues to be large relative to the expected value of the market risk premium. When size is included as an additional explanatory variable in the regressions, the average $\hat{\gamma}_1$ drops to an insignificant value of 0.37%. These results are consistent with Fama and French (1996) evidence that the CAPM beta has no significant power in explaining cross section of return.

1.4.2. The effect of portfolio formation procedure on the bias

In order to demonstrate the effect of the portfolio formation procedure on the covariance between the portfolios betas and alphas, and the resulting bias in the estimated market risk premium, we form three additional sets of 100 portfolios. The first set of 100 portfolios is formed each June on the basis of size as of end of June (hereafter size portfolios). The second set of 100 portfolios is formed each June on the basis of ranking beta which is estimated over the 60 months (at least 24 months) period ending in June (hereafter beta portfolios). The final set of 100 portfolios is formed each June by first forming 10 portfolios on the basis of ranking beta, and then within each beta portfolio, 10 portfolios are formed based on size (hereafter beta-size portfolios).

We estimate the CAPM beta and alpha for each 5 year period for each portfolio. We first focus on the average values of the bias for the three sets of portfolios which are presented in Panel B of Table 1.7. The average bias (estimated as the covariance between the estimated betas and alphas divided by variance of betas) for the 100 size portfolios is a 0.39%. Since (as per Proposition II) this is equal to the bias in the estimated market risk premium, we would expect the estimated risk premium to be higher by approximately this amount. The corresponding values of this bias are -0.53% and -0.28% for the beta portfolios and beta-size portfolios in Panel B of Table 1.7, respectively.

In Panel A of Table 1.7, we present the results of the regressions of the month-by-month regressions of the portfolio returns on the portfolio beta only, size only and beta and size for each set of portfolios. For the size portfolios, the average value of $\hat{\gamma}_1$ is 1.45%, when beta is the only independent variable, as compared to its expected value of

1.07%, the average market risk premium for the period. Consistent with the predictions of Proposition II, the positive covariance between the estimated portfolio betas and alphas creates a positive bias in the estimated market risk premium, and explains why we observe a higher average value of $\hat{\gamma}_1$ for these portfolios, as compared to the actual market risk premium for the period. This upward bias of the estimated market risk premium cannot be explained by the conventional characterization of the bias of the estimated market risk premium due to measurement error as described in (11). For these portfolios, the positive bias and the significance of the CAPM beta remain even when size is added as an additional explanatory variable. This result demonstrates that the estimated risk premium for portfolios in the Fama MacBeth cross-sectional regressions in the two pass methodology depends upon the covariance between the estimated betas and alphas created by the portfolio formation procedure.

The coefficient for size ($\hat{\gamma}_2$), -0.20% when size is the only explanatory variable is identical to its value for individual stocks in Table 1.3, and very similar to its value of -0.19% for size-beta portfolios in Table 1.6. However, when portfolio beta is included as an explanatory variable for the size portfolios, unlike for individual stocks and size-beta portfolios, $\hat{\gamma}_2$ drops to an insignificant -0.04%. This is because the upward bias in $\hat{\gamma}_1$ (created by the positive covariance between the portfolio betas and alphas) is creating an upward bias in $\hat{\gamma}_2$ since size and beta are negatively related. The observation that beta has significant power in explaining the cross section of returns and size has no power for the 100 size portfolios is the opposite of the results for the 100 size-beta portfolios where beta has no power after controlling for size. Both of these results are driven by the biases created by portfolio formation procedure and are, therefore, incorrect.

The results for the beta portfolios and the beta-size portfolios provide additional evidence on the effect of the covariance between portfolio betas and alphas on the estimates of $\hat{\gamma}_1$ and $\hat{\gamma}_2$. When beta is the only independent variable, the average values of $\hat{\gamma}_1$ in Panel A of Table 1.7 are 0.54% and 0.79% for the beta portfolios and beta–size portfolios, respectively. The biases of -.0.53% and -0.28% (values from Panel B), respectively, for these two portfolios, explain the differences between the estimated market risk premiums and the average true market risk premium of 1.07% for this period. The results of this Table are consistent with the predictions of Proposition II and clearly demonstrate that the estimated risk premiums for portfolios vary widely with the portfolio formation procedures, and that this variability is a result of the varying levels of the covariance between the estimated portfolio betas and alphas. It also questions the use of different portfolio grouping procedures to test the implications of the CAPM.

1.4.3 A portfolio formation procedure for reducing this bias

In this section, we present a portfolio formation procedure that attempts to correct for this bias. The basic approach is to form portfolios independently on the estimated betas and alphas within each size decile or portfolio, and then pick the intersection of these independently created alpha and beta portfolios. We refer to these portfolios as the independent alpha-beta portfolios. Such an approach reduces the dependence between the average alphas and betas of these portfolios. The greater the number of independent alpha and beta portfolios, the greater will be the independence between them. However, as the number of portfolios becomes larger, the number of stocks in each portfolio will become smaller, thereby increasing the effect of the beta measurement error. To balance both concerns, we create 3 by 3, 4 by 4, and 5 by 5 independent alpha-beta portfolios within each size decile, giving us 90, 160, and 250 portfolios, respectively. The alphas and betas that are used to form the independent portfolios are estimated using two approaches. In the first approach, the alphas and betas that are used to form portfolios are estimated over the prior 60 month (at least 24 months) period. In the second approach, concurrent alphas and betas (estimated over the concurrent 60 month test period) are used to form portfolios. The first approach avoids any look ahead bias. However, since the purpose of these portfolios is not to test any trading strategy, but instead to test the asset pricing model, the concern regarding any look ahead bias in the second approach may not be relevant.

The results for the independent alpha-beta portfolios using prior period betas and alphas are presented in Table 1.8. Focusing first on the average bias in panel B of the table, we find that the biases are -0.18%, -0.17%, and -0.22% for the 3 by 3, 4 by 4, and 5 by 5 alpha beta portfolios, respectively. These are considerably lower than the bias of -0.27% for the 100 size-beta portfolios. Our suggested portfolio grouping procedure does appear to reduce the bias, without altering the returns or the betas. Consequently, the corresponding values of $\hat{\gamma}_1$ are higher and more significant than the corresponding values in Table 1.6 for the 100 size-beta portfolios. Moreover, for the 4 by 4 alpha-beta portfolios, beta retains significant power in explaining return even after controlling for size.

In Table 1.9, when we use the concurrent alphas and betas to form portfolios, the bias is reduced further and the results for beta are, consequently, stronger. For example, the bias for the 4 by 4 alpha beta portfolios is now only -0.06%, and when beta is the only independent variable, $\hat{\gamma}_1$ has a value of 1.00%, as compared to the average market risk premium of 1.07% for the period. Moreover, $\hat{\gamma}_1$ continues to be significant even after controlling for size.

1.5 Conclusions

This paper develops a new framework for measuring and correcting the bias in cross-sectional tests of the CAPM resulting from beta measurement errors and from portfolio grouping procedures. With the proposed corrections, the returns of stocks and stock portfolios are significantly positively related to their betas, even after controlling for size.

We demonstrate that the measurement errors in the estimated betas induce corresponding measurement errors in the estimated alphas and a spurious negative covariance between the estimated betas and alphas across stocks. This negative covariance between the estimated betas and alphas creates a violation of the independence assumption between the independent variable (the estimated betas) and the error term in the Fama-MacBeth cross-sectional regressions in tests of the CAPM, and therefore, results in a downward bias in the estimated market risk premiums. The procedure of using portfolio returns and betas does not necessarily eliminate this bias. Depending upon the grouping variable used to form portfolios, the negative covariance between the estimated betas and alphas can be increased, decreased, and can even be made positive, thereby changing the magnitude and the direction of the bias in the estimated market risk premiums. The bias can be directly calculated as the covariance between the estimated alphas and betas divided by the variance of the estimated betas.

This paper proposes a 3-pass methodology for the individual stocks to correct for this problem. In the 1st-pass (time-series) regressions, alphas and betas are estimated for individual stocks. In the 2nd-pass (cross-sectional) regressions, the estimated betas from the 1st-pass regressions are purged of the component that is negatively correlated with the estimated alphas, eliminating the spurious negative covariance between the estimated betas and the alphas. And finally, in the 3rd-pass, the purged betas are used in the Fama-MacBeth cross sectional regressions to correct for the lack of independence between the independent variable and the error terms. We find that there is a significantly positive relation between the purged betas and the returns even after controlling for firm size differences. Moreover, the magnitude of the size effect is reduced by about 30%.

The paper also proposes a second methodology for correcting this bias that can be implemented for studies that use portfolio betas and returns. The portfolio formation procedure, in addition to grouping firms by size, also groups firms independently by the estimated alphas and betas within each size decile such that the covariance between estimated betas and alphas is reduced. For these portfolios, we find that the CAPM beta has significant power in explaining the cross section of return even after controlling for size.

While there may be potential criticisms of our proposed methods for correcting the bias in cross sectional test of the CAPM, an important contribution of this paper is to demonstrate that there continue to be significant biases in cross sectional test of the

CAPM both because of beta measurement errors and portfolio grouping procedures. Consequently, the results and the conclusions of prior studies like Fama and French (1992, 1996) continue to be significantly biased and, therefore, may not be valid. With our proposed corrections for the bias, beta appears to have significant power in explaining the cross section of returns even after controlling for size – Beta may not be dead after all.

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Appendix

Proof of Proposition II

Equation (5) can be written as

$$R_t = X\,\gamma_t + \eta_t \tag{A1}$$

where $R_t = [\ R_{1t,}\ R_{2t,\ldots,}\ R_{N_t t}\ ]'$ is N_t by 1 column vector of monthly excess returns of individual or portfolio securities at time t, X = $[\ 1_{N_t,}\ b_t\ ]$ is N_t by 2 vector of ones and the estimated betas, $\gamma_t = [\ \gamma_{1t}, \gamma_{2t}\ ]'$ is 2 by 1 column vectors of parameters to be estimated, and $\eta_t = [\ a_t + e_t\ ]$ is N_t by 1 column vector of sum of the estimated intercepts and the estimated residuals from the 1st-pass time-series regression.

$$\text{OLS } \hat{\gamma}_t = \begin{bmatrix} \hat{\gamma}_{0t} \\ \hat{\gamma}_{1t} \end{bmatrix} = (X'X)^{-1}(X'R_t) \tag{A2}$$

$$= \gamma_t + (X'X)^{-1}X'\eta_t \tag{A3}$$

$$= \begin{bmatrix} \gamma_{0t} \\ \gamma_{1t} \end{bmatrix} + (X'X)^{-1}X'a_t + (X'X)^{-1}X'e_t \tag{A4}$$

where $\gamma_{0t} = 0$, $\gamma_{1t} = R_{Mt} - R_{ft}$ under the CAPM.

$$E\begin{bmatrix} \hat{\gamma}_{0t} \\ \hat{\gamma}_{1t} \end{bmatrix} = \begin{bmatrix} \gamma_{0t} \\ \gamma_{1t} \end{bmatrix} + E[\ (X'X)^{-1}X'a_t] + E[\ (X'X)^{-1}X'e_t] \qquad \text{(A5)}^{12}$$

Unless X is independent of a_t and e_t, OLS $\hat{\gamma}_t$ is biased. The test of FM CSRs of H(0) : mean(γ_t) = 0 is

[12] Differentiating conditional and unconditional expectation doesn't make any change.

$$\frac{1}{T}\sum_{t=1}^{T}\hat{\gamma}_t = \frac{1}{T}\sum_{t=1}^{T}\gamma_t + \frac{1}{T}\sum_{t=1}^{T} (X'X)^{-1}X'a_i + \frac{1}{T}\sum_{t=1}^{T} (X'X)^{-1}X'e_t \qquad (A6)$$

The 1st term on the right hand side is the average market excess return, i.e. $\frac{1}{T}\sum_{t=1}^{T}(R_{Mt} - R_{ft})$. Since the estimated beta is orthogonal to e_t, the 3rd term is zero. For each t, the 2nd term is

$$(X'X)^{-1} X'a_t = \begin{bmatrix} \bar{a}_i - \dfrac{\text{cov}(a_i, b_i)}{\text{var}(b_i)} \\ \dfrac{\text{cov}(a_i, b_i)}{\text{var}(b_i)} \end{bmatrix} \qquad (A7)^{13}$$

According to Proposition I, $\text{cov}(a_i, b_i) \neq 0$ because of measurement errors. Thus, the bias in the estimated market risk premium in FM CSRs is $(X'X)^{-1}X'a_t$, the slope coefficients of the regression of a_t on X.

[13] If the market index does not include all of the stocks in the test or does include more than the stocks in the test, then $\bar{a}$ is not zero.

Table 1.1
Summary Statistics of 1st-pass Time-series Regressions

Every five years, the monthly excess returns of the individual stocks that are listed on NYSE and AMEX (NYAM) are regressed on the monthly equal weighted market excess returns of NYAM. The first 5 years are from July 1931 to June 1936, the second 5 years are from July 1936 to June 1941, and so on. Since the total number of years is not a multiple of 5 years, the last period is from July 2001 to June 2005. Alpha is the intercept and beta is the slope coefficient of the regression. To be included in the sample, a stock should have at least 24 months of returns for each period. For each 5 year period, mean, standard deviation, and covariance are computed using all alphas and betas. Mean, standard deviation, and covariance in panel A are the average of 15 means, standard deviations, and covariances over the 15 periods, respectively. Ten portfolios are formed on size (market capitalization) each June. Panel B shows the average covariance between alpha and beta for each size deciles. S1 is the smallest size decile and S10 is the largest size decile. For comparison with panel A, covariance between alpha and beta in panel B is computed in the first month of each period of 5 years.

Regression : $R_{it} = \alpha_i + \beta_i R_{Mt} + \varepsilon_{it}$

Panel A : Summary Statistics

	Mean	Standard Deviation	Covariance
Alpha	0.02	1.58	-0.21
Beta	1.04	0.58	

Panel B : Size Rank Covariance

Size Rank	
S1	-0.22
S2	-0.24
S3	-0.19
S4	-0.14
S5	-0.18
S6	-0.17
S7	-0.15
S8	-0.14
S9	-0.11
S10	-0.07

Table 1.2
Summary Statistics of Simulations

The 1st-pass time series regression of excess returns of individual firms on equal-weighted market excess return is estimated every 5 years with all stocks of NYSE and AMEX (NYAM) that have at least 24 months returns in each 5 year period. The first 5 years are from July 1931 to June 1936, the second 5 years are from July 1936 to 1941, and so on. Since the total number of years is not a multiple of 5 years, the last 5 years are from July 2001 to June 2005. The numbers in panel A are the average of 15 numbers from 15 periods. The numbers in panel B are the average of 50 iterations and each iteration has 15 periods from July 1931 to June 2005. In panel B, excess returns of individual firms are generated as follows. $R_{it}=\alpha_i+\beta_i R_{Mt}+\varepsilon_{it}$ where R_{Mt} is the equal-weighted NYAM monthly market excess return. In panel B, α_i and ε_{it} are generated as standard normal distributions. After the generation of α_i, the cross sectional variance of α_i is adjusted each period so that it is a smaller than the cross sectional variance of the estimated alphas each period. After the generation of ε_{it}, the time-series variance of ε_{it} for each individual stock is adjusted to match the variance of actual estimated residuals for each firm each period. β_i is the estimated CAPM β from the 1st-pass time series regression for firm i for each 5 years period. The cross sectional variance of the CAPM β is also reduced each period so that it is smaller than the cross sectional variance of actual β. Since α is randomly generated, the true alphas and true betas are orthogonal across stocks each period. Std. Dev. is the standard deviation.

	Excess Return	Estimated Residuals	True Alpha	True Beta	Estimated Alpha	Estimated Beta
			Panel A : Actual Returns			
Mean	1.13	0.0			0.02	1.04
Std.Dev.	14.8	12.2			1.58	0.58
Covariance					-0.21	
Correlation					-0.20	
			Panel B : Returns with Simulated Alphas and Residuals			
Mean	1.10	0.0	0.0	1.03	0.0	1.03
Std.Dev.	14.5	12.1	1.50	0.44	1.56	0.55
Covariance			0.00		-0.12	
Correlation			0.00		-0.12	

Table 1.3
Fama-MacBeth Cross-sectional Regressions with Individual Stocks

This table shows the results of the Fama-MacBeth month by month cross-sectional regressions with individual stocks that are listed on NYSE and AMEX (NYAM). The estimated beta (b_i) is the slope coefficient of the regression of monthly excess returns of individual stock on the monthly equal weighted market excess returns of NYAM for each five year period. The first 5 years are from July 1931 to June 1936, the second 5 years are from July 1936 to June 1941, and so on. Since the total number of years is not a multiple of 5 years, the last period is from July 2001 to June 2005. To be included in the sample, a stock should have at least 24 months of returns for each period. S_{it-1} is log(market capitalization) each June and is used from July to June in the following year. In panel B, the average bias is the average of the bias for the 15 periods. The average equal-weighted monthly market excess return is 1.07% in the sample period. R^2 is the average of the adjusted R-square. The numbers in parenthesis are t-statistics.

	Panel A : $R_{it} = \gamma_{0t} + \gamma_{1t}b_i + \gamma_{2t}S_{it-1} + \eta_{it}$		
γ_0	0.57 (4.26)	3.08 (4.78)	2.05 (5.80)
γ_1	0.54 (1.95)		0.37 (1.36)
γ_2		-0.20 (-4.24)	-0.13 (-4.11)
R^2	0.06	0.02	0.08

	Panel B : Average bias of the estimated market risk premium
$\frac{\text{cov}(a_i, b_i)}{\text{var}(b_i)}$	-0.51

Table 1.4

Average Purged β and its Correlations of Individual Stocks

The CAPM beta is estimated every 5 years for all stocks that are listed on NYSE and AMEX (NYAM) if they have at least 24 months of returns in the 5 year period. The first 5 years are from July 1931 to June 1936, the second 5 years are from July 1936 to June 1941, and so on. Since the total number of years is not a multiple of 5 years, the last period is from July 2001 to June 2005. The purged beta for each period, $b_i^* = b_i - \hat{\theta}_1 a_i$ and $\hat{\theta}_1$ is from $b_i = \theta_0 + \theta_1 a_i + \eta_i$ for i=1,2, … N, where a_i and b_i are the estimated intercept and slope coefficient of time-series regression of monthly excess return of a stock on the equal-weighted monthly market excess return of NYAM. The 100 size-beta portfolios are formed in a manner similar to Fama and French (1996). To determine the size and beta rank, stocks are selected from NYSE and AMEX each June if they have at least 24 months of returns over the past 5 years and have a valid return in July. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on ranking beta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYAM using the previous 5 years of return each June. Size is the market capitalization in June and is used from July to June in the following year. Panel A shows the percentiles of the difference between the unpurged beta and the purged beta. For example, P1 is 1st percentile, P5 is 5th percentile, and so on. The correlations are averages of 888 monthly correlations.

Panel A : Percentiles of $b - b^*$

P1	P5	P10	P25	P50	P75	P90	P95	P99
-0.35	-0.21	-0.15	-0.08	-0.01	0.06	0.16	0.23	0.40

Panel B : Average Betas of Individual Firms for 100 Size-Beta Portfolios

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	AVG
S1	0.66	0.90	1.00	1.12	1.23	1.32	1.47	1.57	1.75	2.05	1.31
S2	0.61	0.79	0.90	1.00	1.07	1.22	1.28	1.44	1.59	1.94	1.18
S3	0.54	0.74	0.85	0.96	1.04	1.16	1.24	1.39	1.51	1.80	1.12
S4	0.50	0.71	0.84	0.93	1.03	1.13	1.22	1.32	1.47	1.75	1.09
S5	0.50	0.71	0.83	0.93	1.01	1.07	1.19	1.30	1.43	1.70	1.07
S6	0.45	0.64	0.79	0.87	0.95	1.04	1.13	1.23	1.37	1.63	1.01
S7	0.39	0.56	0.69	0.81	0.91	0.99	1.08	1.17	1.27	1.54	0.94
S8	0.38	0.53	0.65	0.76	0.86	0.92	0.98	1.10	1.23	1.50	0.89
S9	0.36	0.51	0.62	0.71	0.78	0.84	0.91	0.99	1.10	1.36	0.82
S10	0.35	0.45	0.53	0.60	0.67	0.71	0.80	0.88	0.99	1.18	0.72
AVG	0.47	0.65	0.77	0.87	0.95	1.04	1.13	1.24	1.37	1.64	1.01

Panel C : Average Purged Betas of Individual Firms for 100 Size-Beta Portfolios

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	AVG
S1	0.71	0.92	1.02	1.14	1.24	1.30	1.44	1.53	1.71	2.00	1.30
S2	0.65	0.82	0.92	1.02	1.08	1.22	1.25	1.40	1.55	1.86	1.18
S3	0.59	0.77	0.88	0.97	1.05	1.16	1.23	1.36	1.46	1.73	1.12
S4	0.55	0.74	0.87	0.95	1.04	1.13	1.21	1.29	1.43	1.69	1.09
S5	0.54	0.74	0.86	0.94	1.01	1.07	1.19	1.29	1.39	1.65	1.07
S6	0.50	0.68	0.81	0.88	0.96	1.05	1.12	1.21	1.34	1.58	1.01
S7	0.44	0.60	0.73	0.83	0.92	1.00	1.08	1.16	1.27	1.49	0.95
S8	0.42	0.57	0.69	0.79	0.88	0.93	0.98	1.09	1.21	1.47	0.90
S9	0.40	0.54	0.65	0.73	0.80	0.85	0.92	1.00	1.10	1.34	0.83
S10	0.39	0.49	0.57	0.63	0.68	0.73	0.80	0.87	0.97	1.17	0.73
AVG	0.52	0.69	0.80	0.89	0.97	1.04	1.12	1.22	1.34	1.60	1.02

Panel D : Correlations

Corr(b, b^*)	0.97
Corr(b, Size)	-0.30
Corr(b^*, Size)	-0.30

Table 1.5

Fama-MacBeth Regression with the Purged Betas of Individual Stocks

This table shows the results of the Fama-MacBeth month by month cross-sectional regressions for individual stocks that are listed on NYSE and AMEX (NYAM). The estimated beta (b_i) is the slope coefficient of the regression of monthly excess returns of individual stock on the monthly equal weighted market excess returns of NYAM every five years. The first 5 years are from July 1931 to June 1936, the second 5 years are from July 1936 to June 1941, and so on. Since the total number of years is not a multiple of 5 years, the last period is from July 2001 to June 2005. To be included in the sample, a stock should have at least 24 months of returns for each period. The purged beta for each period, $b_i^* = b_i - \hat{\theta}_1 a_i$ and $\hat{\theta}_1$ is from $b_i = \theta_0 + \theta_1 a_i + \eta_i$ for i=1,2, … N for each period. S_{it-1} is log(market capitalization) each June and is used from July to June in the following year. In panel B, the average bias is the average of the bias for the 15 periods. The average equal-weighted monthly market excess return is 1.07% in the sample period. R^2 is the average of the adjusted R-square. The numbers in parenthesis are t-statistics.

	Panel A : $R_{it} = \gamma_{0t} + \gamma_{1t} b_i^* + \gamma_{2t} S_{it-1} + \eta_{it}$		
γ_0	0.05 (0.41)	3.08 (4.78)	1.11 (3.07)
γ_1	1.03 (3.75)		0.89 (3.25)
γ_2		-0.20 (-4.24)	-0.09 (-2.88)
R^2	0.06	0.02	0.07

	Panel B : Average bias of the estimated market risk premium
$\frac{\text{cov}(a_i, b_i)}{\text{var}(b_i)}$	-0.01

Table 1.6

Fama-MacBeth Cross-sectional Regressions with 100 Portfolios

This table shows the results of the Fama-MacBeth month by month cross-sectional regressions for 100 size-beta portfolios. The estimated beta (b_i) is the slope coefficient of the regression of monthly excess return of 100 size-beta portfolio on the monthly equal weighted market excess returns of NYSE and AMEX (NYAM) every five years. The first 5 years are from July 1931 to June 1936, the second 5 years are from July 1936 to June 1941, and so on. Since the total number of years is not a multiple of 5 years, the last period is from July 2001 to June 2005. S_{it-1} is log (market capitalization) each June and is used from July to June in the following year. From June 1931 until June 2004, stocks are selected from NYSE and AMEX every June if they have at least 24 months of returns over the past 5 years and have a valid return in July. Ten portfolios are formed on size (market capitalization) each June and within each size decile, 10 portfolios are formed on ranking beta, the slope coefficient from regression of monthly excess returns on monthly equal-weighted market excess returns of NYAM. Equal-weighted monthly return for each portfolio is computed from July to June in the following year, generating a time-series of 888 monthly returns from July 1931 to June 2005. The ranking beta uses 24 to 60 months of past returns (as available). In panel B, the average bias is the average of the bias for the 15 periods. The average equal-weighted monthly market excess return is 1.07% in its sample period. R^2 is the average of the adjusted R-square. The numbers in parenthesis are t-statistics.

	Panel A : $R_{it} = \gamma_{0t} + \gamma_{1t} b_i + \gamma_{2t} S_{it-1} + \eta_{it}$		
γ_0	0.34 (2.30)	3.00 (4.70)	2.11 (5.42)
γ_1	0.80 (2.79)		0.37 (1.33)
γ_2		-0.19 (-4.10)	-0.13 (-3.88)
R^2	0.20	0.13	0.27

	Panel B : Average bias of the estimated market risk premium
$\frac{\text{cov}(a_i, b_i)}{\text{var}(b_i)}$	-0.27

Table 1.7

Fama-MacBeth Cross-sectional Regressions with Alternate Portfolios

This table shows the results of the Fama-MacBeth month by month cross-sectional regressions for three different sets of 100 portfolios. From June 1931 until June 2004, stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in July. Ranking beta is estimated each June using the past 5 years of returns (at least 24 months). The ranking beta is the slope coefficient from regression of monthly excess returns on monthly equal-weighted market excess returns of NYAM. 100 size portfolios are formed based on size (market capitalization) each June. 100 beta portfolios are formed on the ranking beta each June. 100 beta-size portfolios are constructed by forming 10 beta portfolios based on the ranking betas each June and then forming 10 size portfolios within each beta portfolio each June. Equal-weighted monthly return for each portfolio is computed from July to June in the following year, generating a time-series of 888 monthly returns from July 1931 to June 2005. The CAPM beta (b_i) is estimated every five years. The first 5 years are from July 1931 to June 1936, the second 5 years are from July 1936 to June 1941, and so on. Since the total number of years is not a multiple of 5 years, the last period is from July 2001 to June 2005. S_{it-1} is log(market capitalization) each June and is used from July to June in the following year. In panel B, the average bias is the average of the bias for the 15 periods. The average equal-weighted monthly market excess return is 1.07% in the sample period. R^2 is the average of the adjusted R-square. The numbers in parenthesis are t-statistics.

	100 Size Portfolios			100 Beta Portfolios			100 Beta-Size Portfolios		
	Panel A : $R_{it} = \gamma_{0t}+\gamma_{1t}b_i+ \gamma_{2t}S_{it-1} + \eta_{it}$								
γ_0	-0.32	3.15	0.25	0.60	2.61	1.48	0.36	3.07	2.19
	(-1.14)	(4.85)	(0.49)	(4.79)	(3.01)	(4.41)	(2.35)	(4.76)	(5.58)
γ_1	1.45		1.25	0.54		0.36	0.79		0.34
	(3.95)		(3.58)	(1.96)		(1.35)	(2.71)		(1.23)
γ_2		-0.20	-0.04		-0.14	-0.06		-0.20	-0.14
		(-4.26)	(-1.07)		(-2.38)	(-2.57)		(-4.16)	(-4.03)
R^2	0.15	0.16	0.19	0.20	0.12	0.21	0.19	0.14	0.27
	Panel B : Average bias of the estimated market risk premium								
$\frac{\text{cov}(a_i,b_i)}{\text{var}(b_i)}$		0.39			-0.53			-0.28	

Table 1.8

Fama-MacBeth Regressions with Independent Alpha and Beta Portfolios Within Each Size Portfolio (Prior Period Beta and Alpha)

From June 1931 until June 2004, stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in July. Ten portfolios are formed based on size (market capitalization) each June. In panel A (B, C), 3 by 3 (4 by 4, 5 by 5) beta and alpha portfolios are formed independently within each size decile each June. Beta and alpha are the slope and intercept coefficients from regression of monthly excess returns on monthly equal-weighted market excess returns of NYAM stocks using the past 60 months returns (at least 24 months), respectively. Equal-weighted monthly portfolio returns are computed from July to June of the following year, generating a time-series of 888 monthly returns from July 1931 to June 2005. S_{it-1} is log(market capitalization) each June and used from July to June in the following year. In panel B, the average bias is the average of the bias for the 15 periods. The average equal-weighted monthly market excess return is 1.07%. R^2 is the average of the adjusted R-square. The numbers in parenthesis are t-statistics.

	3 beta X 3 alpha portfolios			4 beta X 4 alpha portfolios			5 beta X 5 alpha portfolios		
	Panel A : $R_{it} = \gamma_{0t}+\gamma_{1t}b_i+ \gamma_{2t}S_{it-1} + \eta_{it}$								
γ_0	0.25	3.00	1.84	0.25	3.10	1.81	0.29	3.00	1.77
	(1.54)	(4.69)	(5.12)	(1.61)	(4.80)	(5.10)	(1.85)	(4.66)	(5.15)
γ_1	0.89		0.51	0.90		0.55	0.85		0.54
	(3.03)		(1.83)	(3.10)		(2.02)	(2.98)		(1.99)
γ_2		-0.19	-0.12		-0.20	-0.12		-0.19	-0.12
		(-4.08)	(-3.75)		(-4.21)	(-3.78)		(-4.04)	(-3.79)
R^2	0.17	0.13	0.24	0.13	0.09	0.18	0.10	0.07	0.14
	Panel B : Average bias of the estimated market risk premium								
$\frac{\text{cov}(a_i,b_i)}{\text{var}(b_i)}$		-0.18			-0.17			-0.22	

Table 1.9

Fama-MacBeth Regressions with Independent Alpha and Beta Portfolios Within Each Size Portfolio (Concurrent Beta and Alpha)

From June 1931 until June 2004, ten portfolios are formed based on size (market capitalization) each June. In panel A (B, C), 3 by 3 (4 by 4, 5 by 5) beta and alpha portfolios are formed independently within each size decile each June. Beta and alpha are the slope and intercept coefficients from regression of monthly excess returns on monthly equal-weighted market excess returns of NYSE and AMEX stocks using each 5 year period (at least 24 months), respectively. The first 5 years are from July 1931 to June 1936, the second 5 years are from July 1936 to June 1941, and so on. Since the total number of years is not a multiple of 5 years, the last period is from July 2001 to June 2005. Stocks are selected from NYSE and AMEX if they have at least 24 months of returns for each 5 years period. Equal-weighted monthly portfolio returns are computed from July to June of the following year, generating a time-series of 888 monthly returns from July 1931 to June 2005. S_{it-1} is log(market capitalization) each June and used from July to June in the following year. In panel B, the average bias is the average of the bias for the 15 periods. The average equal-weighted monthly market excess return is 1.07%. R^2 is the average of the adjusted R-square. The numbers in parenthesis are t-statistics.

	3 beta X 3 alpha portfolios			4 beta X 4 alpha portfolios			5 beta X 5 alpha portfolios		
	Panel A : $R_{it} = \gamma_{0t}+\gamma_{1t}b_i+ \gamma_{2t}S_{it-1} + \eta_{it}$								
γ_0	0.14	2.97	1.35	0.13	2.91	1.28	0.14	2.93	1.26
	(1.05)	(4.66)	(4.02)	(0.96)	(4.56)	(3.71)	(1.07)	(4.59)	(3.70)
γ_1	0.99		0.77	1.00		0.80	0.99		0.80
	(3.56)		(2.86)	(3.62)		(2.96)	(3.59)		(2.98)
γ_2		-0.19	-0.10		-0.18	-0.10		-0.18	-0.09
		(-4.07)	(-3.29)		(-3.93)	(-3.10)		(-3.95)	(-3.09)
R^2	0.23	0.11	0.30	0.18	0.08	0.23	0.15	0.06	0.18
	Panel B : Average bias of the estimated market risk premium								
$\frac{\text{cov}(a_i,b_i)}{\text{var}(b_i)}$		-0.07			-0.06			-0.07	

Chapter 2

The Ex-Post Market Risk Premium and the Relationship between Beta, Size, and Returns

2.1. Introduction

Beginning with Banz (1981) and Reinganum (1982), several researchers have documented the inverse relation between firm size and realized returns. Roll (1981) argues that the size effect is an artifact of underestimation of beta for small firms, while Reinganum (1982) finds that the estimation error in beta cannot explain the return difference between small and large firms. Fama and French (1992, 1996) provide evidence that firm size and book to market ratio explain the cross section of realized returns, and that after controlling for differences in size, beta has no significant relation with returns. These results are a violation of the Capital Asset Pricing Model of Sharpe (1964) and Lintner (1965), and have led to claims by some that beta is dead. It is now common for researchers to do a three factor adjustment (for size, beta, and book-to-market) or just a size adjustment to calculate abnormal returns around events.

The CAPM predicts that there will be a positive relation between returns and betas across all market conditions. However, it also implies that there will be a positive relation between observed returns and betas in periods when the ex-post market risk premium is positive (ex-post market return is greater than the risk free return), and a negative relation between observed returns and betas in periods when, contrary to expectations, the ex-post market risk premium is negative (ex-post market return is less than the risk free return).

For sake of convenience, I refer to the periods in which the ex-post market return is greater than the risk free return as up markets, and to the periods in which the ex-post market return is less than the risk free return as down markets.

Similar to Pettengill, Sundaram, and Mathur (PSM) (1995), this paper finds evidence on the significant and economically meaningful relation between betas and observed returns that is consistent with the CAPM under each of the two market conditions, even after controlling for differences in size[14]. However, PSM do not analyze why there is no significant relation between beta and the expected returns across both market conditions. Howton and Peterson (1998) demonstrate that book-to-market ratio and size are important factors in bear markets. Peres-Quiros and Timmermann (2000) find small firms display the highest degree of asymmetry in their conditional return distributions across recession and expansion states. As recessions deepen, small firms rapidly lose collateral and their assets become more risky, causing investors to require a higher premium for holding their shares. These findings suggest that asymmetry plays an important role in the relation between return, beta, and firm characteristics.

It is important to note that the purpose of this paper is to investigate the reasons for the insignificant relation between beta and returns after controlling for size differences and for the asymmetric size effect in up and down markets.

For the periods in which the ex-post market risk premium is positive (which make up 60% of the sample months), the results are strikingly consistent with the implications of the CAPM. First, the highest beta portfolios earn more than twice the average returns of the lowest beta portfolios during these periods, and the differences in their returns can be

[14] The results are robust to different definitions of up and down market conditions such as up market if ex-post market return is positive and down market otherwise.

explained by the differences in their betas. Second, the differences in average returns for the size portfolios for these periods can also be explained by the differences in their betas. Third, graphs for these periods show that the average returns of the beta portfolios have a positive and linear relation with their betas, and the portfolios plot very close to the ex-post Security Market Line as implied by the CAPM. Fourth, Fama-MacBeth regressions for these periods show a significant positive relation between the returns and betas with and without controlling for differences in size. And finally, the regressions show no significant relation between size and returns after controlling for beta differences for these periods. The only piece of evidence that is not entirely consistent with the implications of the CAPM is that the estimated average market risk premium (average coefficient of the estimated beta) for these periods in the Fama-MacBeth regressions is 4.15%, which is lower than the average observed market risk premium of 4.90%. Nevertheless, the linear, positive, significant, and economically meaningful relation between betas and returns in periods when the ex-post market risk premium is positive, and the lack of any relation between size and returns after controlling for beta differences, suggest that beta has explanatory power in explaining the cross-section of returns for these periods in a manner that is quite consistent with the implications of the CAPM.

For the remaining 40% of the sample months, the results are not as consistent with the implications of the CAPM. First, consistent with the implications of the CAPM, the average returns of the highest beta portfolios are more negative as compared to those of the lowest beta portfolios. However, the return differences are larger than what can be explained by the differences in their betas. Second, as implied by the CAPM, graphs for these periods show that the average returns of the beta portfolios have a negative and linear relation with their betas. However, the slope is more negative than what is implied by the CAPM. Third, consistent with implications of the CAPM, Fama-MacBeth regressions for these periods show a significant negative relation between betas and returns with and without controlling for differences in size. However, the average of the estimated coefficients of beta of -5.46% (after controlling for size) is significantly lower than the average ex-post market risk premium of -4.47% for these periods. The observation that higher beta firms lose more than what is implied by their betas in down markets, and the consequent stronger than the CAPM implied negative relation between betas and returns for these periods, offsets the CAPM consistent positive relation between the two in up markets. It results in an insignificant relation between the two on average across both market conditions. Finally, the average returns of small firm portfolios are not as negative as what is implied by their betas for down markets. This results in a negative relation between size and returns after controlling for beta differences. However, despite these inconsistencies with implications of the CAPM for the periods in which the ex-post risk premium is negative, the observed linear, negative, significant, and economically meaningful relation between betas and returns for these periods suggests that beta has explanatory power in explaining the cross section of returns in a manner that is not entirely inconsistent with the implications of the CAPM.

The evidence in the paper also sheds new light on the nature of the negative relation between firm size and observed returns. The evidence suggests that small firms earn no more than what is implied by their betas for up markets. The size effect is observed on average across both market conditions only because small firms do not lose as much as what is implied by their betas for down markets. This suggests that size may not be a proxy for a risk factor. It also questions the use of making only size adjustment for calculating abnormal returns, especially in up market months (60% of all sample months) when there is no size effect after controlling for beta differences.

The paper also finds that small firm portfolios have relatively poor performance in the period leading up to the portfolio formation month, as compared to large firm portfolios. Thus small firms are small, in part, precisely because of their relatively poor performance as compared to large firms. The paper argues that the negative relation between size and observed returns in down markets is, in part, related to the long term mean reversion in returns.

The results are robust to the choice of the sample period, market index, beta estimation method, and whether I use constant or changing portfolio betas. Moreover, the results hold whether or not I include NASDAQ stocks in the sample. The evidence and conclusions of this paper are distinct from those of Jagannathan and Wang (1996) who present a conditional CAPM with time varying betas and risk premiums. While they show that allowing for betas and risk premiums to vary over time provides support for the conditional CAPM, this paper demonstrates that beta and observed returns are related in a manner consistent with the static CAPM in periods when, the ex-post market return is greater than the risk free return. Further, the negative relation between beta and returns is stronger than what is implied by the static CAPM in periods when, contrary to expectations, the ex-post market return is lower than the risk free return.

The remainder of the paper is organized as follows. Section 2 describes the data. Section 3 presents the summary statistics, the graphs of the relations between portfolio returns, size, and estimated betas, and the results of the Fama-MacBeth regressions with the robustness checks. Section 4 decomposes the size effect into intercept size effect and residual size effect and investigates the residual size effect on the asymmetrical size effect between up and down market. Section 5 examines the effect of long-term mean reversion in returns on the size effect. Section 6 concludes the paper.

2.2 Data

The sample includes all NYSE and AMEX (hereafter NYAM) stocks during the period July 1931 - December 2004. In subsequent analysis, I also include NASDAQ stocks during June 1975 – December 2004. Following Fama and French (1992, 1996), from June 1931 until June 2004, I restrict the sample to the stocks that have at least 24 months return in the previous five years as of each June and a valid return in June of the year when the portfolio is formed. Similar to Fama and MacBeth (1972), Black, Jensen and Scholes (1972), Blume and Friend (1973), and Fama and French (1996), I use portfolios of stocks instead of individual securities to minimize any effect of beta measurement error on the relation between betas and returns. Each June, ten portfolios are formed based on size (market capitalization) and within each size portfolio, ten portfolios are formed on betas that are estimated over 60 months (at least 24 months) prior to the formation of the portfolios[15]. I refer to this beta as the PREBETA and estimate it as the slope coefficient from regression of monthly excess returns on the current equal-weighted

[15] NYSE decile break points are used for size and PREBETA ranks

monthly market excess returns.[16] Equal-weighted monthly returns on 100 size-beta portfolios are computed from July to June of the following year. Thus, for each size-beta portfolio, there is a time-series of 882 monthly returns from July 1931 to December 2004, and the composition of the portfolio changes every year in July. The market is defined as an up market if the equal-weighted market monthly excess return is positive, and negative otherwise.[17] In the sample, there are 522 up market months with a mean monthly excess return of 4.90%, and there are 360 down market months with a mean monthly excess return of -4.47%. The frequency weighted mean of monthly excess returns over the two market conditions (which is also the mean of monthly excess returns for the entire period) is 1.07%.[18]

Shanken (1992) shows that the measurement error in the estimated beta declines as the number of periods used for beta estimation is increased, and therefore, the downward bias of the estimated market risk premium is smaller when constant betas are used instead of changing betas. Thus, similar to Fama and French (1996) and Kothari, Shanken and Sloan (1995), I use constant betas for the portfolios estimated using the entire 882 month period. To test for robustness of the results, I also repeat the analysis with changing betas and find similar results. I use the equally-weighted market excess return as the market portfolio for the analysis. To check for the robustness of the results, I repeat the analysis with the value-weighted market index.

[16] Similar results are obtained when the sum of the slope coefficient of current and lagged values of the index is used.

[17] The different definitions of up and down markets do not change the results. The advantage of this definition is that the risk premium (based on the CAPM) of beta in Fama-MacBeth regression is positive for up market months and negative for down market months.

[18] The mean monthly market excess return for the overall period is $1.07\% = \frac{522}{882}4.90\% + \frac{360}{882}(-4.47\%)$

2.3 Results

2.3.1 Portfolio betas, size, and excess returns

Panel A of Table 2.1 presents the average monthly excess returns for the 100 size-beta portfolios for the entire 882 month period from July 1931 to December 2004. Portfolio S1 (S10) includes the decile of the smallest (largest) firms, and portfolio B1 (B10) includes the smallest (largest) beta portfolios within each size portfolio. The results are very similar to those of Fama-French (1996). The portfolio excess returns are much more widely spread across size portfolios than across beta portfolios. The largest beta portfolio and the smallest beta portfolio earn average monthly excess returns of 1.09% and 0.96%, respectively,[19] a difference of only 0.13%. However, the smallest size portfolio earns on average a monthly excess return of 1.91%, while the largest size portfolio earns a monthly excess return of only 0.64%, a difference of 1.27% per month.

In Panel B of Table 2.1, I present the average betas of the 100 size-beta portfolios. The smallest and largest beta portfolios have average betas of 0.57 and 1.34, respectively. While the average beta of the largest beta portfolios is more than twice the average beta of the smallest beta portfolios, the excess returns, as noted above, are not correspondingly different. The smallest size portfolios have an average beta of 1.40, which is little more than twice the average beta of 0.64 for the largest size portfolios, while its average monthly excess return of 1.91% is almost three times the average excess return of 0.64% for the largest size portfolios. These results are consistent with Fama and French (1992,

[19] The beta portfolios are formed on the basis of beta within size deciles. Therefore, the highest beta portfolios are not necessarily comprised of the highest beta firms.

1996), suggesting that firm size has cross-sectional explanatory power in explaining average returns, but the estimated beta has little or no cross-sectional explanatory power. As I will demonstrate in Table 2.2, the results are very different when I examine up and down markets separately. Panel C of Table 2.1 presents the average values of the natural log of the market capitalization for each portfolio, and the results are very similar to those of Fama and French (1996).

In Panels A and B of Table 2.2, I present the average monthly excess returns separately for up and down market months. For the 522 up market months, the smallest size portfolio and the largest size portfolio earn monthly excess returns of 6.66% and 3.36%, respectively, showing that the decile of the smallest firms earn almost twice as much as the decile of the largest firms in up markets, which is consistent with the notion that small firms earn higher returns than large firms in up markets. However, as can be seen in Panel B of Table 2.1, the decile of smallest firms has an average beta of 1.40 which is more than twice the average beta of 0.64 for the decile of largest firms. It appears that the extra returns for the smallest firms in up markets can be adequately explained by the differences in their betas.

Examining the average returns and the betas of 10 beta portfolios in up markets, I observe that average returns increase monotonically as I move from portfolio B1 to B10. The differences are economically large. The smallest beta portfolios earn an average monthly excess return of 3.07% in up markets (Table 2.2 Panel A), while the largest beta portfolios earn an average monthly excess return of 6.45%, which is more than twice the average monthly excess return of the smallest beta portfolios. These results are strikingly different from those of Panel A in Table 2.1 and appear to be consistent with the CAPM given that the average betas of the smallest and the largest beta portfolios are 0.57 and 1.34, respectively (Panel B Table 2.1). The conclusion from the overall period results that beta does not affect the cross section of returns on average is certainly not valid for up markets. Betas of 10 beta portfolios appear to have a positive, monotonic, and economically meaningful relation with monthly excess returns in the months when the ex-post market risk premium is positive, 60% of all months.

Panel B of Table 2.2 presents the average monthly excess returns of 100 portfolios for 360 down market months. The smallest and the largest size portfolios have average monthly excess returns of -4.97% and -3.31%, respectively, in down markets. The larger losses of smaller firms suggest that size may be a risk factor. If small size is a proxy for a risk factor, then smaller (and therefore riskier) firms should be expected to lose more under bad market conditions. However, based on its beta of 1.40 and the average ex-post market risk premium of -4.47% for down market months, the smallest size portfolio should have lost 6.26%, on average for this period. The results suggest that small firms are not losing as much as what is implied by their betas when the ex-post market risk premium is negative.

As implied by the CAPM, the relation between average portfolio betas and average monthly excess returns observed in up markets is reversed for down markets. The average monthly excess returns decrease monotonically as average beta increases. The largest and the smallest beta portfolios have average monthly excess returns of -6.67% and -2.10%, respectively in down markets. The larger losses for the larger beta portfolios and the consequent negative relation between average betas and average monthly excess returns in down markets are consistent with the CAPM. However, the magnitude of the differences in average monthly excess returns is larger than what can be explained by the differences in their average betas. While the average monthly excess return of the largest beta portfolios is more than three times as negative as the average monthly excess return of the smallest beta portfolios, their average beta is less than three times as large. The large negative returns of high beta portfolios in down markets, which appear to be more negative than what is implied by the CAPM, offset to a larger extent the CAPM-consistent higher positive returns of such firms in up markets. This explains the results in Table 2.1 that the average monthly excess returns are not meaningfully different for 10 beta portfolios.

2.3.2 Plots of Security Market Line and the relation between beta, size and its return

Figure 1 illustrates the relation between average ex-post returns and betas for 10 beta portfolios for all months, up market months, and down market months. I also draw a line connecting the origin with the equally weighted market portfolio. This line is essentially the ex-post Security Market Line (SML) implied by the CAPM with the risk free return subtracted from the returns. To be consistent with the CAPM, the average ex-post returns should plot on the ex-post SML. For the entire 882 month period, the relation between returns and beta is weak. The small beta portfolios plot above the (risk free return adjusted) ex-post SML, and the large beta portfolios plot below the ex-post SML. However, for 522 up market months, the relation between betas and returns for these portfolios is positive and linear. Moreover, as implied by the CAPM, the portfolios plot very close to the ex-post SML. For the remaining 360 down market months, there is a linear and inverse relation between returns and betas, as implied by the CAPM. Similar to the overall period, small beta portfolios plot above the ex-post SML for down market months, and the large beta portfolios plot below the ex-post SML, resulting in an observed slope that is greater than the slope of the ex-post SML. Overall, these plots illustrate that the positive relation between betas and returns in up market months is consistent with the CAPM, and that the negative relation between returns and betas in down market months is stronger than what is implied by the CAPM.

Figure 2 illustrates the relation between returns and betas for the ten size portfolios. For the overall period, while portfolios S2 through S10 plot very close to the ex-post SML, S1, the smallest size portfolio plots 0.41% above the SML. As per CAPM, the average excess return for S1 should have been 1.50%, which is the product of the portfolio beta of 1.40 and the market excess return of 1.07% for this period. However, it earns an average excess return of 1.91%, which is 0.41% higher than the value predicted by the CAPM. The second plot shows that this extra return is not being earned in the 522 up market months. Portfolio S1 plots 0.2% below the ex-post SML for the up market months. It appears that small firms do not earn more than what is implied by their betas in the 522 up market months. However, in 360 down market months, the smallest size portfolio plots above the SML. Their monthly excess return of -4.97% in down market months is 1.29% more than the -6.26% return implied by CAPM. It appears that this lower losses of the smallest firm portfolio than what is implied by CAPM in down market months are responsible for the higher average returns for this portfolio in the overall period.[20]

[20] Dif all (0.41%) = 1.91% - 1.40 × 1.07%, Dif up(-0.2%) = 6.66% - 1.40 × 4.90%, Dif down(1.29%) = -4.97% - 1.40 × (-4.47%), where 1.07%, 4.90%, and -4.47% are average of equal weighed market excess return for the entire period, 522 up months, and 360 down months, respectively. Also, Dif all (0.41%) = (522/882)×Dif up(-0.2%) + (360/882)×Dif Down(1.29%).

Figure 3 illustrates the relation between returns and size for the ten size portfolios without controlling for differences in betas. I observe a negative relation between size and returns for the overall period and the up market months, and a positive relation for the down market months. These results suggest that small size may be a proxy for a risk factor, since I would expect riskier firms to perform better than less risky firms in favorable market conditions, and perform worse under unfavorable market conditions. However, this conclusion does not hold up when I control for differences in beta in Figure 4. After controlling for beta differences, there is no relation between returns and size in the 522 up market months, suggesting that there is no size effect for up market months. The negative relation between returns and size in the overall period is driven by the negative relation between the two in 360 down market months[21].

2.3.3 Regression results

Table 2.3 presents regression results of the monthly excess returns for 100 size-beta portfolios of NYSE and AMEX stocks using the Fama-MacBeth methodology. For the overall period, monthly excess returns are positively related to beta, and negatively related to size in simple regressions. However, in multiple regressions, beta loses its explanatory power, while size continues to be negatively related to returns. Thus, small firms on average appear to earn more than large firms even after controlling for beta for the entire 882 month period. These results are consistent with Fama and French (1992, 1996) results.

[21] The slope coefficient in the overall period is a frequency weighted average of the two slope coefficients in the 522 up market and 360 down market conditions.

When I examine 522 up market months only, the coefficient for the estimated market beta is significant and positive in both simple regression and after controlling for size. The estimated coefficients of beta in simple regression and after controlling for size are 4.15% and 4.14%, respectively. Surprisingly, the inclusion of size does not reduce the magnitude or the significance of the estimated coefficients for beta. Moreover, the estimated coefficients for beta in simple regression is not significantly different from 4.90%, the average value of the market excess return for these up market months and the value implied by the CAPM. The values in square brackets under the estimated coefficients for beta are the t-statistics for the test of difference between the estimated value and the value implied by the CAPM. In terms of economic significance, the estimated coefficient of 4.15% (4.14%) for beta in simple (multiple) regression suggests that an increase of 0.25 in beta leads to a 1.04% (1.04%) higher average monthly excess return in up markets. While the coefficient of size is negative and significant in simple regression in up market months, it becomes insignificant after controlling for beta differences. These results are consistent with those presented in Tables 2.1 and 2.2 and Figures 3 and 4. It appears that there is a significant, positive, and economically meaningful relation between average beta and average monthly excess returns in up market months, with or without controlling for differences in size. Furthermore, there is no relation between monthly excess returns and size after controlling for differences in beta in these up market months. Thus for up market months, there is no size effect, and the results for beta are consistent with the CAPM.

For 360 down market months, the coefficients of estimated beta are negative and significant with values of -4.08% in simple regression, and -5.46% after controlling for size. The observation that high beta firms lose significantly more than low beta firms in down markets is consistent with the CAPM. The estimated coefficient for beta of -4.08% in simple regression for down markets is not significantly different from -4.47%, the value implied by the CAPM. However, after controlling for size in multiple regressions, the estimated coefficient for beta is significantly lower than the CAPM-implied value of -4.47%. This difference suggests that after controlling for size, high beta firms lose more than what is implied by the CAPM in down market months. As suggested in the previous section, these greater losses for high beta firms in down markets may offset their higher returns in up markets, and may explain why on average high beta firms do not earn a premium across both market conditions after controlling for differences in size.

The coefficient for size is significantly positive in down markets without controlling for beta, consistent with the evidence that smaller firms lose more in down markets without taking into account differences in beta (Panel B of Table 2.2). However, after taking into account the differences in beta, the coefficient for size reverses and becomes significantly negative. This implies that while smaller firms lose more than large firms, they lose relatively less than what is implied by their higher betas. This is also consistent with the observations in Table 2.1 and 2.2 that while the smallest size decile portfolios lose more than the largest size decile portfolios in down markets, they do not lose as much as their beta would imply. When there is no control for beta, average monthly excess returns have a significantly negative relation with size in the overall period and in up markets, and a significantly positive relation in down markets, suggesting that size may be a proxy for a risk factor. However, after controlling for beta, the significantly negative relation between size and average monthly excess returns in the overall period is driven entirely by the negative relation in down market,[22] and there is no relation between average monthly excess returns and size in up markets after controlling for differences in beta. This suggests that after taking into beta risk, size may not be a proxy for a risk factor.

Panels B and C of Table 2.3 present the results for two sub-periods of July 1931 to June 1963 and July 1963 to December 2004, and the results for both sub-periods are similar to the overall period. The choice of the two sub-periods is based on the findings of prior papers that suggest a difference between pre-1963 results and post-1963 results for the CAPM (Loughran (1997), Ang and Chen (2005), and Fama and French (2006)).[23]

2.3.4 Robustness check

I repeat the analysis for the July 1975 to December 2004 period to include NASDAQ firms. The choice of period to include NASDAQ firms was dictated by the fact that the data for NASDAQ firms is available on CRSP starting 1973, and I need at least 24 months of returns to form portfolios. Table 2.4 presents the results for this extended sample. The results are qualitatively similar, and the conclusions remain unchanged.

I also repeat the analysis with the value weighted index as the choice for market return, and with changing portfolio betas, re-estimated each year instead of constant portfolio betas for the entire period. The results, presented in Panels A and B of Table 2.5, are

[22] The estimated coefficients for the overall period are simply the frequency weighted average of the corresponding coefficients for the two market conditions. For example, the size coefficient of -0.16 in the multiple regression is the frequency weighted (522/882 and 360/882) average of the coefficients of 0.00 and -0.39 for the up and down market months, respectively.

[23] Loughran (1997) argues that there is no value premium among large stocks. Ang and Chen (2005) show that the CAPM captures the value premium for the pre 1963 period and also argue that when the tests allow for time-varying market betas, even the post-1963 period produces no evidence against the CAPM. However, Fama and French (2006) find that Loughran's evidence for a weak value premium among large firms is specific to 1963 to 1995 and Ang and Chen's evidence that the CAPM can explain U.S. value premiums is specific to pre 1963.

similar to those with the equally weighted index with constant betas. The overall conclusion, that beta is significantly related to returns under both market conditions in a manner consistent with CAPM, continues to hold. That there is no size effect in up markets continues to hold for value weighted index. However, although in case of the changing betas, size effect is observed both in up and down markets, about 60% of size effect is from down markets.

Finally, I repeat the analysis with Scholes-Williams betas (Scholes and Wiilliams (1977) and Dimson(1979)), estimated as the sum of the slope coefficients of current and lagged market excess return, instead of market model betas. I do this with both equal and value-weighted market excess return. The results presented in Table 2.6 are similar to the earlier results except for the finding that size is positively related to returns after controlling for beta differences in the up markets. However, the magnitude of this positive relation between size and returns after controlling for beta differences in up markets is small relative to the magnitude of the negative relation in the down markets, and consequently, I continue to observe a negative relation between the two on average across both market conditions.

The first chapter of this dissertation shows that independent grouping of firms on their alphas and betas within each size portfolio reduces the relation between the alphas and betas of the portfolios, thereby reducing the bias in the Fama-MacBeth cross-sectional tests of the CAPM. In unreported results, I find that the independent grouping of firms on their alphas and betas within each size portfolio does not change the empirical findings of this chapter. The negative relation between size and returns, after controlling for beta differences, is present only when the ex-post market risk premium is negative, and is responsible for the negative relation between them for the overall period.

2.4 The intercept size effect or the residual size effect?

The Fama and MacBeth cross sectional regressions in section 3 suggest that size is positively related with returns without controlling for beta differences, but negatively related with returns after controlling for beta differences in down markets. Moreover, this negative relation in down markets after controlling for the CAPM β is responsible for the size effect after controlling for the CAPM β for the overall markets. This suggests that the part of the return that is not explained by the CAPM in down markets can be negatively related to size. This reasoning is consistent with Brennan, Chordia, and Subrahmanyam (BCS) (1998)'s risk-adjusted return (sum of intercept and residuals from the regression of excess returns of individual firm on market excess return) approach. BCS suggest regressing risk-adjusted returns on individual firm's characteristics.[24] Table 2.7 presents the proportion of returns that are unexplained by the CAPM for all markets, up markets, and down markets. The unexplained returns are the sum of intercept and residuals from the time series regression of portfolio monthly excess returns on equal weighted market monthly excess returns. Panel A of Table 2.7 shows that there is a negative relation between size and the unexplained return by the CAPM β. However, there is a positive relation between size and return in up markets in panel B and a negative relation between them in down markets in panel C. It suggests that the negative relation between size and

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[24] BCS methodology is adopted in papers such as Chordia, T, Subrahmanyam, A., and Anshuman, R. (2001), Chordia and Shivakumar (2005), Gebhardt, Hvidkjaer, and Swaminathan (2005), Chordia, Goyal, Sadka, Sadka, and Shivakumar (2006).

the unexplained return is completely from the down markets. The unexplained return, 0.40% of the smallest size portfolio is the frequency weighted average of 1.31% from down markets and -0.22% from up markets.

Hur and Kumar (2006) show that firm size effect after controlling for the CAPM β consists of the intercept size effect and the residual size effect (Proposition III in Hur and Kumar (2006)). The Proposition is as follows. To test the implication that the CAPM β is a complete measure of the risk of a security, the estimated equation is

$$R_{it}=\lambda_{0t}+\lambda_{1t}b_i+\lambda_{2t}f_i+\eta_{it}$$

where f_i is an additional factor. The estimated risk premium of the additional factor using ordinary least squares, λ_{2t} is same as the coefficient of f_i from the regression of the sum of alpha and the estimated residuals on b_i and f_i.

Table 2.8 reports the results of the Fama-MacBeth regression of the intercept and the estimated residuals from the 1st-pass regression on the CAPM β and size. While panel A is the results of the regression of the sum of the estimated intercept and the residuals from the 1st-pass regression, panel B and C report the results of the regression of the estimated intercept and residuals separately. The coefficients in panel A is the sum of the corresponding coefficients in panel B and C. Comparing panel A's of Table 2.3 and Table 2.8 confirms proposition III in Hur and Kumar (2006). The size premium, -0.16% after controlling for the CAPM β across all markets is same between panel A's of Table 2.3 and Table 2.8.

As suggested by Proposition III, I can decompose size premium, -0.16%, after controlling for β into two components, intercept size effect (panel B) and residual size effect (panel C). In case of all 882 months, the 1st row of panel B and panel C shows that

-0.16% of size premium after controlling for β is completely from the intercept. However, when I consider up and down markets separately, the results are very different. As it should, intercept size effect is same between up and down markets in panel B. However, the residual size effect after controlling for β is 0.16% and -0.23% for up and down markets in panel C, respectively. The positive residual size premium, 0.16% in up market in panel C causes size to lose explanatory power after controlling for the CAPM β in up markets in panel A. The size premium at down markets, -0.39% after controlling for the CAPM β in panel A is the sum of intercept size effect, -0.16% in panel B and residual size effect, -0.21% in panel C. Therefore, the residual size effect can explain the asymmetrical size effect in Howton and Peterson (1998) such that size is an important factor only in bear markets and down markets, respectively.

2.5. Long Term Mean Reversion in Returns and the Size Effect

In this section, I examine another possible explanation for the size effect in down markets. Prior studies (Chopra, Lakonishok, and Ritter (1992), De Bondt and Thaler (1985, 1987), Lakonishok, Shleifer, and Vishny (1994)) have documented long term mean reversion in returns over a 3 to 5 year period. Since size portfolios are based on the market value of equity, small firm portfolios are more likely to include firms with lower returns in the period before the portfolio formation month, compared to large firm portfolios. Long term mean reversion would suggest that these firms with lower past returns would perform better than the firms with higher past returns.

Panel A of Table 2.9 presents the average buy and hold returns for 100 size-beta portfolios for the pre 60 months as of each July. As suggested above there are large differences between the past returns across size portfolios. The average pre 60 month buy and hold return for the smallest size portfolio is 41.03%, which is about one-third of the corresponding return for the largest size portfolio, 126.97%. The highest average pre 60 month buy and hold return for any beta decile portfolio within the smallest size portfolio is 57.29%, while the corresponding average return for all portfolios is 103.30%. Moreover, all beta decile portfolios within the smallest size portfolio earn lower pre 60 month buy and hold returns than any of other 90 size-beta portfolios. It appears that the small size portfolios include firms that have performed considerably poorly relative to other firms in the past 60 month period, and these firms are small, at least in part, because of their poor market performance.

In order to examine the effect of long term mean reversion on the size effect, I include the log of the pre 60 month buy and hold return as an additional explanatory variable in the Fama-MacBeth regressions. The results, presented in Panel B of Table 2.9 show that the monthly excess returns are significantly negatively related to the pre 60 month buy and hold return for the overall period, and for down market months. A possible explanation for the negative relation between pre 60 month buy and hold returns and the monthly excess returns in the down markets maybe that small size portfolios include firms with the lowest returns over the pre 60 months, and in down market months these firms do not lose as much as implied by their betas. Including pre 60 month buy and hold return reduces the magnitude of the size effect. The estimated coefficients for size variables change from -0.16 to -0.12 in the overall period. However, the estimated coefficient for beta in up and down market months continues to be significant and in the direction implied by CAPM. It appears that at least a part of the size effect can be attributed to the long term mean reversion in returns.

2.6. Conclusion

The CAPM implies a positive relation between betas and ex-post returns when the market risk premium is positive (ex-post market return is greater than the risk free return) and a negative relation between betas and ex-post returns when, contrary to expectations, the ex-post market risk premium is negative (ex-post market return is less than the risk free return). This paper examines the relation between betas, size, and observed returns under these two market conditions separately, and finds strong and robust evidence that is consistent with the implications of the CAPM for each of the two different market conditions. Specifically, returns exhibit a significant, positive, linear, and economically meaningful relation with the betas in periods when the ex-post market risk premium is positive, and a significant, negative, linear, and economically meaningful relation with the betas in periods when the ex-post market risk premium is negative. The observed negative relation between betas and returns in periods when the ex-post market risk premium is negative is stronger than what is implied by the CAPM. It offsets the positive relation in periods when the ex-post risk premium is positive, resulting in an insignificant relation between the two across both market conditions. Nevertheless, the significant and economically meaningful relations between returns and betas with the CAPM-consistent signs under each of the two market conditions, refutes the suggestion that beta has no explanatory power in explaining the cross-section of observed returns.

The paper also provides new evidence on the nature of the negative relation between firm size and observed returns. The evidence demonstrates that the small firms do not earn more than what is implied by their betas when the ex-post market risk premium is positive, and the size effect is observed because they do not lose as much as what is implied by their betas when the ex-post market risk premium is negative. This suggests that size may not be a proxy for a risk factor. It also questions the use of making only size adjustment for calculating abnormal returns, especially when the ex-post market risk premium is positive and there is no size effect after controlling for beta differences. This asymmetrical size effect can be explained by the residual size effects.

The paper also finds that the small firm portfolios have relatively poor performance in the period leading up to the portfolio formation month, as compared to the large firm portfolios. Thus small firms are small, in part, precisely because of their relatively poor performance as compared to large firms. The paper shows that the negative relation between size and observed returns in down markets is, in part, related to the long term mean reversion in returns

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Table 2.1

Average Returns, Betas, and Size of 100 Size-Beta Portfolios

From June 1931 until June 2004 stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYAM. NYSE size declie and prebeta decile breakpoints are used. Equal-weighted monthly returns for each of 100 portfolios are computed from July to June in the following year, generating a time-series of 882 monthly returns from July 1931 to Dec 2004. The estimated portfolio beta is the slope coefficient of the time-series regression of the monthly excess portfolio returns on the monthly equal-weighted NYAM market excess returns.

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	AVG
Panel A : Average monthly excess returns											
S1	1.64	1.52	1.93	1.98	1.93	2.05	1.92	1.96	2.19	1.99	1.91
S2	1.24	1.22	1.15	1.49	1.53	1.49	1.42	1.53	1.18	1.34	1.36
S3	1.19	1.23	1.28	1.23	1.29	1.39	1.21	1.46	1.09	1.19	1.25
S4	1.04	1.10	1.28	1.03	1.33	1.34	1.23	1.10	1.08	0.96	1.15
S5	0.83	1.13	1.20	1.10	0.96	1.12	1.29	1.19	1.05	0.99	1.09
S6	0.91	0.98	0.95	1.10	1.03	1.07	0.94	0.94	1.11	1.02	1.00
S7	0.75	0.98	0.91	1.02	0.95	0.89	1.14	0.99	0.89	1.01	0.95
S8	0.86	0.80	0.86	0.88	0.99	0.99	0.94	0.83	0.91	0.89	0.90
S9	0.58	0.70	0.77	0.82	0.85	0.92	0.84	0.82	0.83	0.93	0.81
S10	0.55	0.58	0.70	0.71	0.59	0.77	0.64	0.63	0.60	0.59	0.64
AVG	0.96	1.02	1.10	1.14	1.15	1.20	1.16	1.15	1.09	1.09	1.11
Panel B : Estimated Betas											
S1	0.87	1.18	1.33	1.35	1.25	1.51	1.71	1.47	1.70	1.67	1.40
S2	0.78	1.08	1.14	1.16	1.23	1.28	1.32	1.40	1.39	1.60	1.24
S3	0.79	0.88	0.91	1.03	1.03	1.21	1.20	1.37	1.36	1.52	1.13
S4	0.62	0.76	0.95	0.90	1.08	1.12	1.18	1.23	1.35	1.41	1.06
S5	0.52	0.78	0.87	0.83	0.97	0.97	1.13	1.20	1.26	1.41	0.99
S6	0.51	0.63	0.76	0.83	0.87	0.94	1.03	1.17	1.20	1.29	0.92
S7	0.38	0.59	0.67	0.76	0.83	0.92	1.01	1.09	1.10	1.33	0.87
S8	0.43	0.49	0.60	0.73	0.78	0.81	0.93	1.02	1.07	1.10	0.80
S9	0.38	0.46	0.64	0.67	0.78	0.76	0.78	0.87	0.96	1.13	0.74
S10	0.39	0.45	0.50	0.57	0.59	0.65	0.67	0.73	0.89	0.98	0.64
AVG	0.57	0.73	0.84	0.88	0.94	1.02	1.10	1.16	1.23	1.34	0.98
Panel C : Average of Ln(Market Capitalization)											
S1	8.77	8.79	8.78	8.77	8.74	8.77	8.72	8.72	8.69	8.64	8.74
S2	9.78	9.78	9.79	9.79	9.80	9.78	9.79	9.78	9.77	9.78	9.78
S3	10.34	10.32	10.33	10.33	10.33	10.32	10.33	10.33	10.31	10.32	10.33
S4	10.78	10.78	10.77	10.78	10.78	10.77	10.78	10.78	10.77	10.77	10.78
S5	11.20	11.19	11.20	11.21	11.21	11.21	11.19	11.20	11.19	11.18	11.20
S6	11.60	11.62	11.61	11.61	11.60	11.61	11.61	11.60	11.60	11.60	11.61
S7	12.05	12.05	12.06	12.06	12.05	12.04	12.04	12.04	12.04	12.04	12.05
S8	12.58	12.57	12.57	12.57	12.58	12.56	12.56	12.56	12.56	12.54	12.56
S9	13.21	13.22	13.22	13.22	13.23	13.21	13.19	13.21	13.19	13.18	13.21
S10	14.79	14.77	14.76	14.80	14.79	14.82	14.63	14.49	14.34	14.24	14.64
AVG	11.51	11.51	11.51	11.51	11.51	11.51	11.48	11.47	11.45	11.43	11.49

Table 2.2

Average Returns of 100 Size-Beta Portfolios in Up and Down Markets

From June 1931 until June 2004 stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYAM. NYSE size declie and prebeta decile breakpoints are used. Equal-weighted monthly returns for each of 100 portfolios are computed from July to June in the following year, generating a time-series of 882 monthly returns from July 1931 to Dec 2004. Portfolio betas are estimated using the regression of the entire 882 month time series of portfolio excess returns on the equal weighted market excess returns. The market is defined as an Up Market if R_{mt} - R_{ft} > 0, and Down Market otherwise, where R_{mt} is equal-weighted monthly market return and R_{ft} is 30 day T-bill return. There are 522 Up Market Months and 360 Down Market months in the 882 month sample period.

Panel A : Average monthly excess returns in Up markets											
	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	AVG
S1	4.80	5.37	6.02	6.44	6.38	7.05	7.34	7.29	7.97	7.94	6.66
S2	3.94	4.88	4.99	5.60	5.80	6.13	6.22	6.73	6.46	7.40	5.81
S3	3.81	4.36	4.72	5.02	5.09	5.70	5.78	6.43	6.31	7.02	5.42
S4	3.26	3.87	4.69	4.58	5.41	5.53	5.69	5.87	6.31	6.80	5.20
S5	2.67	3.86	4.35	4.48	4.68	5.12	5.52	5.74	5.88	6.59	4.89
S6	2.79	3.33	3.92	4.44	4.59	4.81	4.93	5.43	5.92	6.34	4.65
S7	2.40	3.29	3.75	4.13	4.31	4.68	5.10	5.37	5.40	6.28	4.47
S8	2.66	2.90	3.44	3.77	4.22	4.34	4.60	4.91	5.33	5.75	4.19
S9	2.12	2.80	3.36	3.59	3.92	4.10	4.14	4.51	4.72	5.54	3.88
S10	2.26	2.48	2.88	2.97	3.16	3.52	3.53	3.80	4.18	4.80	3.36
AVG	3.07	3.71	4.21	4.50	4.76	5.10	5.28	5.61	5.85	6.45	4.85
Panel B : Average monthly excess returns in Down markets											
S1	-2.93	-4.07	-4.01	-4.48	-4.50	-5.19	-5.93	-5.76	-6.19	-6.65	-4.97
S2	-2.66	-4.08	-4.41	-4.47	-4.64	-5.25	-5.54	-6.01	-6.48	-7.44	-5.10
S3	-2.61	-3.32	-3.72	-4.27	-4.21	-4.85	-5.42	-5.74	-6.49	-7.27	-4.79
S4	-2.18	-2.92	-3.67	-4.12	-4.58	-4.73	-5.23	-5.83	-6.49	-7.52	-4.73
S5	-1.84	-2.82	-3.36	-3.80	-4.43	-4.68	-4.84	-5.40	-5.96	-7.13	-4.43
S6	-1.80	-2.43	-3.36	-3.75	-4.14	-4.35	-4.84	-5.56	-5.87	-6.70	-4.28
S7	-1.66	-2.35	-3.20	-3.49	-3.93	-4.61	-4.61	-5.37	-5.65	-6.62	-4.15
S8	-1.74	-2.24	-2.89	-3.31	-3.69	-3.87	-4.39	-5.08	-5.48	-6.17	-3.89
S9	-1.65	-2.35	-2.98	-3.19	-3.60	-3.69	-3.95	-4.51	-4.81	-5.74	-3.65
S10	-1.93	-2.18	-2.47	-2.56	-3.13	-3.22	-3.55	-3.97	-4.59	-5.50	-3.31
AVG	-2.10	-2.88	-3.41	-3.75	-4.08	-4.44	-4.83	-5.32	-5.80	-6.67	-4.33

Table 2.3

Fama-MacBeth Cross-Sectional Regressions with NYSE and AMEX

From June 1931 until June 2004 stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYAM. NYSE size declie and prebeta decile breakpoints are used. Equal-weighted monthly returns for each of 100 portfolios are computed from July to June in the following year, generating a time-series of 882 monthly returns from July 1931 to Dec. 2004. The market is defined as Up Market if R_{mt} - R_{ft} > 0, and Down Market otherwise, where R_{mt} is the equal-weighted monthly market return and R_{ft} is 30 day T-bill return. There are 522 Up market months and 360 Down market months in 882 month sample period. Panel A presents the Fama-MacBeth regression results of monthly portfolio returns (R_{it}) on portfolio betas (β_i) and log of market capitalization (S_{it-1}) for July 1931 to December 2004 period. Portfolio betas are estimated using the regression of the entire 882 month time series of portfolio excess returns on the equal weighted NYAM market excess returns. Panels B and C present the regression results for two sub-periods. For the pre 1963 sub-period, there are 236 up market and 148 down market months. For the post 1963 sub-period, there are 286 up market and 212 down market months.

		$R_{it} = \lambda_0 + \lambda_1 \beta_i + e_{it}$		$R_{it} = \lambda_0 + \lambda_1 S_{it-1} + e_{it}$		$R_{it} = \lambda_0 + \lambda_1 \beta_i + \lambda_2 S_{it-1} + e_{it}$		
	$\overline{R_{Mt}-R_{ft}}$	λ_0	λ_1	λ_0	λ_1	λ_0	λ_1	λ_2
Panel A : July 1931 ~ Dec. 2004								
All (882)	1.07	0.33 (2.04)	0.79 (2.73) [-0.95]	2.88 (4.59)	-0.18 (-3.98)	2.54 (5.54)	0.22 (0.78) [-2.96]	-0.16 (-4.28)
Up (522)	4.90	0.78 (3.42)	4.15 (10.31) [-1.85]	10.40 (12.43)	-0.53 (-8.06)	0.49 (0.76)	4.14 (11.62) [-2.13]	0.00 (0.08)
Down (360)	-4.47	-0.33 (-1.60)	-4.08 (-17.16) [1.65]	-8.03 (-14.05)	0.34 (8.31)	5.51 (9.48)	-5.46 (-19.92) [-3.60]	-0.39 (-9.03)
Panel B : July 1931 ~ June 1963								
All (384)	1.49	0.21 (0.80)	1.30 (2.41) [-0.35]	3.85 (3.37)	-0.27 (-3.08)	2.66 (3.78)	0.60 (1.14) [-1.70]	-0.19 (-2.93)
Up (236)	5.84	0.99 (2.71)	4.86 (6.42) [-1.23]	12.00 (7.76)	-0.68 (-5.40)	1.11 (1.10)	4.70 (6.94) [-1.68]	-0.02 (-0.22)
Down (148)	-5.45	-1.02 (-2.80)	-4.38 (-11.13) [2.73]	-9.16 (-10.16)	0.39 (5.76)	5.14 (6.24)	-5.95 (-12.80) [-1.08]	-0.46 (-6.14)
Panel C : July 1963 ~ Dec. 2004								
All (498)	0.76	0.45 (2.38)	0.36 (1.20) [-1.32]	2.13 (3.15)	-0.11 (-2.57)	2.27 (4.16)	-0.03 (-0.09) [-2.61]	-0.12 (-3.11)
Up (286)	4.13	0.49 (1.79)	3.80 (10.66) [-0.92]	9.08 (10.89)	-0.41 (-6.81)	1.01 (1.31)	3.69 (11.31) [-1.36]	-0.04 (-0.73)
Down (212)	-3.80	0.39 (1.62)	-4.28 (-14.08) [-1.58]	-7.24 (-9.84)	0.29 (6.00)	3.95 (5.42)	-5.04 (-15.78) [-3.87]	-0.23 (-4.61)

Table 2.4

Fama-MacBeth Cross-Sectional Regressions including NASDAQ Firms

From June 1975 until June 2004 stocks are selected from NYSE, AMEX, and NASDAQ every June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYSE, AMEX, and NASDAQ stocks. NYSE size declie and prebeta decile breakpoints are used. Equal-weighted monthly returns for each of 100 portfolios are computed from July to June in the following year, generating a time-series of 354 monthly returns from July 1975 to Dec. 2004. The market is defined as an Up market if R_{mt} - R_{ft} > 0, and Down market otherwise, where R_{mt} is the equal-weighted monthly market return and R_{ft} is 30 day T-bill return. There are 212 Up Market Months and 142 Down Market months in the 354 month sample period. The table presents Fama-MacBeth cross sectional regression results of monthly portfolio returns (R_{it}) on portfolio betas (β_i) and log of market capitalization (S_{it-1}) for the July 1975 to December 2004 period. Portfolio betas are estimated using the regression of the entire 354 month time series of portfolio excess returns on the equal weighted NYSE, AMEX, and NASDAQ market excess returns.

		$R_{it} = \lambda_0 + \lambda_1 \beta_i + e_{it}$		$R_{it} = \lambda_0 + \lambda_1 S_{it-1} + e_{it}$		$R_{it} = \lambda_0 + \lambda_1 \beta_i + \lambda_2 S_{it-1} + e_{it}$		
	$\overline{R_{Mt} - R_{ft}}$	λ_0	λ_1	λ_0	λ_1	λ_0	λ_1	λ_2
All (354)	0.95	0.78 (3.55)	0.19 (0.55) [-2.23]	2.27 (3.23)	-0.11 (-2.38)	2.52 (4.08)	-0.11 (-0.32) [-2.99]	-0.12 (-2.82)
Up (212)	4.22	0.84 (2.94)	3.51 (9.51) [-1.92]	8.34 (10.64)	-0.36 (-6.14)	1.69 (2.14)	3.36 (9.01) [-2.32]	-0.06 (-1.09)
Down (142)	-3.94	0.68 (1.99)	-4.77 (-13.22) [-2.30]	-6.79 (-7.90)	0.27 (4.74)	3.76 (3.82)	-5.29 (-13.15) [-3.36]	-0.20 (-3.32)

Table 2.5

Fama-MacBeth Cross-Sectional Regressions with

Value Weighted Market Excess Returns and with Changing Betas

From June 1931 until June 2004 stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current value-weighted (equal-weighted for Panel B) market excess returns of NYAM stocks. NYSE size declie and prebeta decile breakpoints are used. Equal-weighted monthly returns for each of 100 portfolios are computed from July to June in the following year, generating a time-series of 882 monthly returns from July 1931 to Dec. 2004. The market is defined as an Up market if R_{mt} - $R_{ft} > 0$, and Down market otherwise, where R_{mt} is the value-weighted (equal-weighted for Panel B) monthly market return and R_{ft} is 30 day T-bill return. Panel A presents Fama-MacBeth regression results of monthly portfolio returns (R_{it}) on portfolio betas (β_i) and log of market capitalization (S_{it-1}) for the July 1931 to December 2004 period. There are 528 Up market months and 360 Down market months using the value-weighted market excess return in 882 month sample period. Portfolio betas are estimated using the regression of the entire 882 month time series of portfolio excess returns on the value weighted NYAM market excess returns. Panel B presents Fama-MacBeth regression results of monthly portfolio returns (R_{it}) on portfolio betas (β_i) and log of market capitalization (S_{it-1}) for the July 1936 to December 2004 period using the equal weighted index and changing betas. There are 487 Up market months and 335 Down market months using the equal-weighted market excess return in 822 month sample period. Portfolio betas are re-estimated every June from June 1936 to June 2004 using past 5 years of monthly portfolio returns and the equal-weighted NYAM market excess returns.

		$R_{it} = \lambda_0 + \lambda_1 \beta_i + e_{it}$		$R_{it} = \lambda_0 + \lambda_1 S_{it-1} + e_{it}$		$R_{it} = \lambda_0 + \lambda_1 \beta_i + \lambda_2 S_{it-1} + e_{it}$		
	$\overline{R_{Mt} - R_{ft}}$	λ_0	λ_1	λ_0	λ_1	λ_0	λ_1	λ_2
Panel A : Value Weighted Market Excess Return								
All (882)	0.68	0.43 (2.61)	0.54 (2.09) [-0.56]	2.88 (4.59)	-0.18 (-3.98)	2.70 (6.11)	0.13 (0.54) [-2.47]	-0.17 (-4.54)
Up (528)	3.69	0.58 (2.53)	3.25 (9.62) [-1.29]	8.23 (9.80)	-0.37 (-5.60)	1.44 (2.31)	3.02 (11.11) [-2.47]	-0.09 (-1.65)
Down (354)	-3.82	0.21 (0.93)	-3.51 (-12.54) [1.10]	-5.11 (-6.80)	0.10 (2.02)	4.57 (8.018)	-4.19 (-16.72) [-1.51]	-0.29 (-6.69)
Panel B : Changing Betas								
All (822)	0.92	0.55 (4.05)	0.41 (1.79) [-2.25]	2.32 (4.24)	-0.13 (-3.38)	2.20 (5.68)	0.05 (0.22) [-4.04]	-0.12 (-3.81)
Up (487)	4.33	0.95 (5.07)	3.45 (12.51) [-3.21]	9.09 (13.28)	-0.44 (-8.15)	2.30 (4.18)	3.18 (13.37) [-4.82]	-0.10 (-2.21)
Down (335)	-4.04	-0.03 (-0.13)	-4.01 (-17.32) [0.13]	-7.52 (-13.05)	0.32 (8.25)	2.06 (3.97)	-4.51 (-18.99) [-1.98]	-0.14 (-3.77)

Table 2.6

Fama-MacBeth Cross-Sectional Regression with Scholes and Williams Betas

From June 1931 until June 2004 stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYAM stocks. NYSE size declie and prebeta decile breakpoints are used. Equal-weighted monthly returns for each of 100 portfolios are computed from July to June in the following year, generating a time-series of 882 monthly returns from July 1931 to Dec. 2004. The market is defined as an Up market if R_{mt} - R_{ft} > 0, and Down market otherwise, where R_{mt} is the equal-weighted monthly market return and R_{ft} is 30 day T-bill return. There are 522 Up market months and 360 Down market months in the 882 month sample period. Panel A (B) presents Fama-MacBeth regression results of monthly portfolio returns (R_{it}) on Scholes-Williams portfolio betas (β_i) calculated using the equal (value) weighted market excess returns and log of market capitalization (S_{it-1}) for the July 1931 to December 2004 period. Scholes and Williams portfolio betas are estimated using the regression of the entire 882 month time series of portfolio excess returns on the current and lagged equal (value) weighted NYAM market excess returns as the sum of the slope coefficients of current and lagged values of the index. There are 522(528) Up market months and 360(354) Down market months based on equal (value) weighted market excess return.

		$R_{it} = \lambda_0 + \lambda_1 \beta_i + e_{it}$		$R_{it} = \lambda_0 + \lambda_1 S_{it-1} + e_{it}$		$R_{it} = \lambda_0 + \lambda_1 \beta_i + \lambda_2 S_{it-1} + e_{it}$		
	$\overline{R_{Mt}-R_{ft}}$	λ_0	λ_1	λ_0	λ_1	λ_0	λ_1	λ_2
Panel A : Scholes-Williams Betas with Equal-Weighted Market Excess Returns								
All	1.07	0.29	0.84	2.88	-0.18	2.33	0.31	-0.14
(882)		(1.73)	(3.03)	(4.59)	(-3.98)	(4.65)	(0.99)	(-3.66)
			[-0.81]				[-2.49]	
Up	4.90	1.13	3.84	10.40	-0.53	-1.40	4.42	0.16
(522)		(4.94)	(9.71)	(12.43)	(-8.06)	(-2.09)	(11.40)	(3.02)
			[-2.69]				[-1.23]	
Down	-4.47	-0.94	-3.49	-8.03	0.34	7.74	-5.67	-0.57
(360)		(-4.33)	(-15.67)	(-14.05)	(8.31)	(11.90)	(-19.76)	(-12.07)
			[4.38]				[-4.17]	
Panel B : Scholes-Williams Betas with Value-Weighted Market Excess Returns								
All	0.68	0.23	0.64	2.88	-0.18	2.51	0.18	-0.16
(882)		(1.24)	(2.72)	(4.59)	(-3.98)	(5.42)	(0.79)	(-4.33)
			[-0.17]				[-2.21]	
Up	3.69	0.90	2.75	8.23	-0.37	-1.01	3.03	0.10
(528)		(3.55)	(8.44)	(9.80)	(-5.60)	(-1.64)	(10.88)	(2.02)
			[-2.90]				[-2.38]	
Down	-3.82	-0.77	-2.50	-5.11	0.10	7.76	-4.07	-0.55
(354)		(-3.18)	(-9.91)	(-6.80)	(2.02)	(12.79)	(16.36)	(-12.68)
			[5.23]				[-1.00]	

Table 2.7

The Unexplained Returns by the CAPM in Up and Down Markets

From June 1931 until June 2004 stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYAM. NYSE size declie and prebeta decile breakpoints are used. Equal-weighted monthly returns for each of the 100 portfolios are computed from July to June in the following year, generating a time-series of 882 monthly returns from July 1931 to Dec 2004. From time-series regression of portfolio monthly excess return on equal weighted NYAM market excess return, the sum of the intercept and residuals are defined as the unexplained return by the CAPM. The market is defined as an Up market if R_{mt} - R_{ft} > 0, and Down market otherwise, where R_{mt} is equal-weighted monthly market return and R_{ft} is 30 day T-bill return. There are 522 Up market months and 360 Down market months in the 882 month sample period. R_{it}^{A} is the unexplained returns by the CAPM, i.e. the sum of intercept and residuals from time series regression.

Panel A : The Unexplained Returns in All Markets

S1	0.71	0.25	0.50	0.53	0.59	0.43	0.08	0.38	0.36	0.20	0.40
S2	0.41	0.06	-0.08	0.24	0.21	0.11	0.00	0.02	-0.31	-0.38	0.03
S3	0.34	0.28	0.30	0.12	0.18	0.09	-0.08	-0.02	-0.37	-0.44	0.04
S4	0.38	0.29	0.26	0.07	0.17	0.14	-0.04	-0.23	-0.37	-0.56	0.01
S5	0.27	0.29	0.26	0.21	-0.08	0.07	0.08	-0.10	-0.30	-0.53	0.02
S6	0.36	0.30	0.13	0.20	0.10	0.06	-0.17	-0.32	-0.18	-0.37	0.01
S7	0.34	0.35	0.19	0.21	0.06	-0.10	0.05	-0.19	-0.30	-0.41	0.02
S8	0.40	0.28	0.21	0.10	0.16	0.12	-0.07	-0.26	-0.23	-0.30	0.04
S9	0.18	0.20	0.08	0.10	0.02	0.11	0.00	-0.12	-0.20	-0.28	0.01
S10	0.14	0.10	0.15	0.10	-0.05	0.07	-0.09	-0.16	-0.35	-0.47	-0.06
AVG	0.35	0.24	0.20	0.19	0.14	0.11	-0.02	-0.10	-0.23	-0.35	0.05

Panel B : The Unexplained Returns in Up Markets

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	AVG
S1	0.54	-0.42	-0.48	-0.18	0.25	-0.37	-1.06	0.06	-0.35	-0.23	-0.22
S2	0.13	-0.43	-0.61	-0.11	-0.23	-0.14	-0.26	-0.14	-0.34	-0.44	-0.26
S3	-0.05	0.04	0.27	-0.04	0.02	-0.22	-0.12	-0.30	-0.33	-0.42	-0.12
S4	0.22	0.16	0.05	0.18	0.12	0.03	-0.12	-0.18	-0.33	-0.12	0.00
S5	0.13	0.02	0.06	0.42	-0.08	0.35	-0.03	-0.15	-0.28	-0.34	0.01
S6	0.27	0.23	0.20	0.37	0.34	0.19	-0.14	-0.33	0.03	-0.01	0.11
S7	0.54	0.40	0.46	0.42	0.26	0.18	0.13	0.01	-0.02	-0.22	0.22
S8	0.56	0.51	0.48	0.18	0.42	0.39	0.03	-0.07	0.10	0.34	0.29
S9	0.27	0.52	0.20	0.31	0.11	0.40	0.31	0.22	0.03	0.01	0.24
S10	0.36	0.28	0.40	0.17	0.24	0.35	0.22	0.20	-0.17	-0.03	0.20
AVG	0.30	0.13	0.10	0.17	0.15	0.11	-0.10	-0.07	-0.17	-0.15	0.05

Panel C : The Unexplained Returns in Down Markets

S1	0.95	1.22	1.93	1.56	1.09	1.59	1.74	0.84	1.39	0.81	1.31
S2	0.82	0.77	0.70	0.74	0.86	0.48	0.38	0.26	-0.27	-0.29	0.44
S3	0.91	0.63	0.34	0.35	0.42	0.55	-0.03	0.40	-0.42	-0.48	0.27
S4	0.60	0.48	0.56	-0.11	0.25	0.30	0.07	-0.30	-0.43	-1.19	0.02
S5	0.48	0.68	0.55	-0.10	-0.09	-0.33	0.23	-0.03	-0.34	-0.80	0.02
S6	0.50	0.39	0.04	-0.03	-0.25	-0.12	-0.22	-0.31	-0.49	-0.90	-0.14
S7	0.05	0.29	-0.20	-0.10	-0.23	-0.49	-0.07	-0.48	-0.70	-0.69	-0.26
S8	0.17	-0.06	-0.19	-0.03	-0.22	-0.27	-0.21	-0.53	-0.71	-1.23	-0.33
S9	0.04	-0.27	-0.09	-0.21	-0.12	-0.31	-0.45	-0.60	-0.53	-0.70	-0.32
S10	-0.19	-0.18	-0.21	0.00	-0.47	-0.33	-0.53	-0.69	-0.62	-1.10	-0.43
AVG	0.43	0.40	0.34	0.21	0.12	0.11	0.09	-0.14	-0.31	-0.66	0.06

Table 2.8

Intercept Size Effect and Residual Size Effect between Up and Down Markets

From June 1931 until June 2004, stocks are selected from NYSE and AMEX (NYAM) each June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYAM. Equal-weighted monthly returns for each of 100 portfolios are computed from July to June in the following year, generating a time-series of 882 monthly returns from July 1931 to Dec. 2004. The market is defined as Up Market if $R_{mt} - R_{ft} > 0$, and Down Market otherwise, where R_{mt} is the equal-weighted monthly market return and R_{ft} is 30 day T-bill return. There are 522 Up market months and 360 Down market months in 882 month sample period. Panel A presents Fama-MacBeth regression of monthly portfolio returns (R_{it}) on β_i and S_{it-1} for all 882 months, 522 up market months, and 360 down market months. Portfolio betas are estimated using the regression of the entire 882 month time series of portfolio excess returns on the equal weighted NYAM market excess returns. S_{it-1} is log(market capitalization) each June and is used from July to June in the following year. Panels B (C) presents Fama-MacBeth regression of intercept (residuals) from 1st-pass regression on β_i and S_{it-1} for all 882 months, 522 up market months, and 360 down market months. a_i, b_i ,and e_{it} are the estimated intercept, the estimated β, and residuals from 1st pass regression of monthly excess return of security i on equal-weighted monthly market excess return.

Panel A : Sum of intercept and residual size effect								
		$a_i + e_{it} = \lambda_0 + \lambda_1 \beta_i + e_{it}$		$a_i + e_{it} = \lambda_0 + \lambda_1 S_{it-1} + e_{it}$		$a_i + e_{it} = \lambda_0 + \lambda_1 \beta_i + \lambda_2 S_{it-1} + e_{it}$		
	$\overline{R_{Mt}-R_{ft}}$	λ_0	λ_1	λ_0	λ_1	λ_0	λ_1	λ_2
All	1.08	0.33	-0.28	0.35	-0.04	2.54	-0.85	-0.16
(882)		(2.04)	(-1.80)	(1.00)	(-1.14)	(5.54)	(-5.76)	(-4.28)
Up	4.90	0.78	-0.75	-1.49	0.12	0.49	-0.76	0.00
(522)		(3.42)	(-3.38)	(-3.00)	(2.38)	(0.76)	(-3.63)	(0.08)
Down	-4.47	-0.33	0.40	3.01	-0.26	5.51	-0.98	-0.39
(360)		(-1.60)	(2.01)	(7.12)	(-6.53)	(9.48)	(-4.99)	(-9.03)
Panel B : Intercept size effect								
		$a_i = \lambda_0 + \lambda_1 \beta_i + e_{it}$		$a_i = \lambda_0 + \lambda_1 S_{it-1} + e_{it}$		$a_i = \lambda_0 + \lambda_1 \beta_i + \lambda_2 S_{it-1} + e_{it}$		
All (882)	1.08	0.33	-0.28	0.59	-0.05	2.73	-0.84	-0.16
Up (522)	4.90	0.33	-0.28	0.59	-0.05	2.74	-0.84	-0.16
Down (360)	-4.47	0.33	-0.28	0.59	-0.05	2.72	-0.84	-0.16
Panel C : Residual size effect								
		$e_{it} = \lambda_0 + \lambda_1 \beta_i + e_{it}$		$e_{it} = \lambda_0 + \lambda_1 S_{it-1} + e_{it}$		$e_{it} = \lambda_0 + \lambda_1 \beta_i + \lambda_2 S_{it-1} + e_{it}$		
All	1.08	0.00	0.00	-0.24	0.01	-0.19	-0.01	0.00
(882)		(0.00)	(0.00)	(-0.70)	(0.26)	(-0.41)	(-0.08)	(0.10)
Up	4.90	0.45	-0.47	-2.08	0.16	-2.23	0.08	0.16
(522)		(1.99)	(-2.11)	(-4.19)	(3.34)	(-3.43)	(0.37)	(3.09)
Down	-4.47	-0.66	0.68	2.42	-0.21	2.77	-0.14	-0.23
(360)		(-3.18)	(3.42)	(5.72)	(-5.35)	(4.78)	(-0.71)	(-5.29)

Table 2.9

Pre 60 Month Buy and Hold Returns and Fama-MacBeth Cross-Sectional Regressions

From June 1931 until June 2004 stocks are selected from NYSE and AMEX (NYAM) every June if they have at least 24 months of returns over the past 5 years and have a valid return in June. Ten portfolios are formed based on size (market capitalization) each June and within each size decile, 10 portfolios are formed on prebeta, the slope coefficient from regression of monthly excess returns on monthly current equal-weighted market excess returns of NYAM stocks. NYSE size declie and prebeta decile breakpoints are used. Every July, the average buy-and-hold return (BH) in the past 60 months is calculated for each portfolio. Panel A presents the average pre 60 month buy-and-hold return for 100 size beta portfolios. Panel B presents Fama-MacBeth regression results of monthly portfolio returns (R_{it}) on portfolio betas (β_i), the log of market capitalization (S_{it-1}), and the log of the pre 60 month buy-and-hold return (BH) for July 1931 to Dec. 2004 period. Monthly portfolio returns are computed for 100 portfolios from July to June in the following year as the equal weighted average of all stocks in the portfolio, generating a time-series of 882 monthly returns from July 1931 to Dec. 2004. Portfolio betas are estimated using the regression of the entire 882 month time series of portfolio excess returns on the equal weighted NYAM market excess returns. The market is defined as an Up market if $R_{mt} - R_{ft} > 0$, and Down market otherwise, where R_{mt} is the equal-weighted monthly market return and R_{ft} is 30 day T-bill return. There are 522 Up market months and 360 Down market months in the 882 month sample period.

Panel A : Average Buy and Hold Returns for Pre 60 Months as of Each July

	B1	B2	B3	B4	B5	B6	B7	B8	B9	B10	AVG
S1	41.92	36.10	47.57	45.27	40.75	41.59	30.98	39.78	29.10	57.29	41.03
S2	74.72	63.18	66.79	76.11	74.31	73.74	86.44	71.51	64.76	95.43	74.70
S3	70.26	77.51	74.23	78.16	77.44	85.39	86.03	85.29	109.06	145.20	88.86
S4	72.94	91.14	91.40	82.86	104.15	96.72	101.95	103.56	115.18	146.37	100.63
S5	78.24	92.27	98.36	103.60	111.70	114.62	109.98	122.38	120.14	173.48	112.48
S6	71.53	87.58	99.45	104.23	107.94	106.89	112.31	132.10	140.94	175.53	113.85
S7	81.29	85.91	101.94	102.67	111.05	123.75	137.46	132.20	159.59	192.33	122.82
S8	85.77	91.01	106.44	104.91	119.11	118.65	118.67	124.81	152.75	219.47	124.16
S9	92.57	100.02	105.18	100.04	117.90	119.72	133.58	135.50	146.15	223.89	127.46
S10	91.37	93.73	107.49	104.79	113.44	116.76	126.96	134.63	154.21	226.33	126.97
AVG	76.06	81.85	89.89	90.26	97.78	99.78	104.44	108.18	119.19	165.53	103.30

Panel B : Fama-MacBeth Regressions

		$R_{it} = \lambda_0 + \lambda_1 \beta_i + \lambda_2 S_{it} + \lambda_3 \log(BH)_{it} + e_{it}$			
	$\overline{R_{Mt} - R_{ft}}$	λ_0	λ_1	λ_2	λ_3
All (882)	1.07	2.19 (5.35)	0.16 (0.60) [-3.44]	-0.12 (-4.00)	-0.23 (-2.39)
Up (522)	4.90	0.45 (0.79)	3.70 (11.29) [-3.67]	0.02 (0.38)	-0.14 (-1.00)
Down (360)	-4.47	4.72 (8.91)	-4.97 (-18.71) [-1.84]	-0.33 (-8.36)	-0.36 (-3.15)

Figure 2.1. The Relationship between Return and Beta of 10 Beta Portfolios

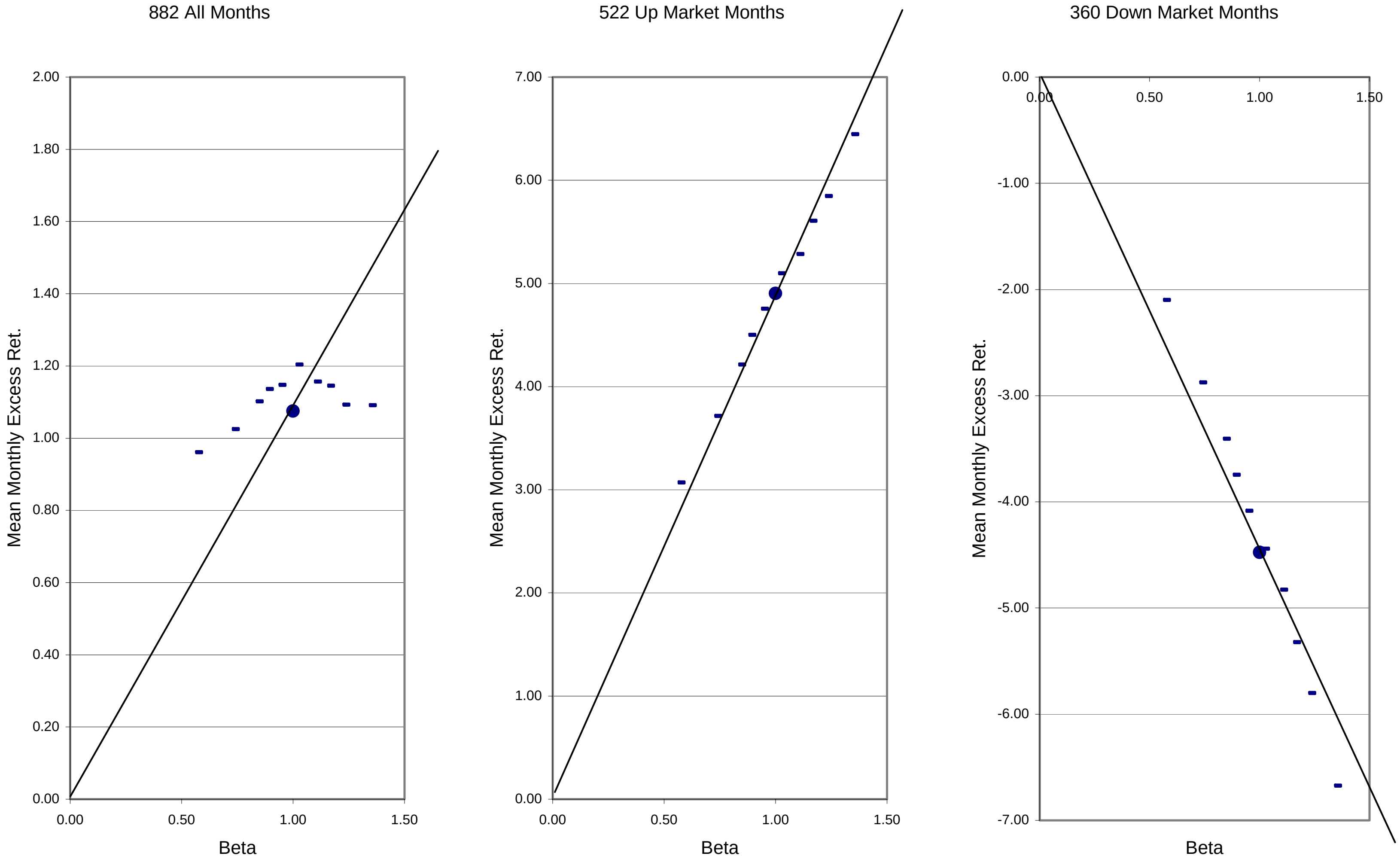

Figure 2.2 The Relationship between Return and Beta of 10 Size Portfolios

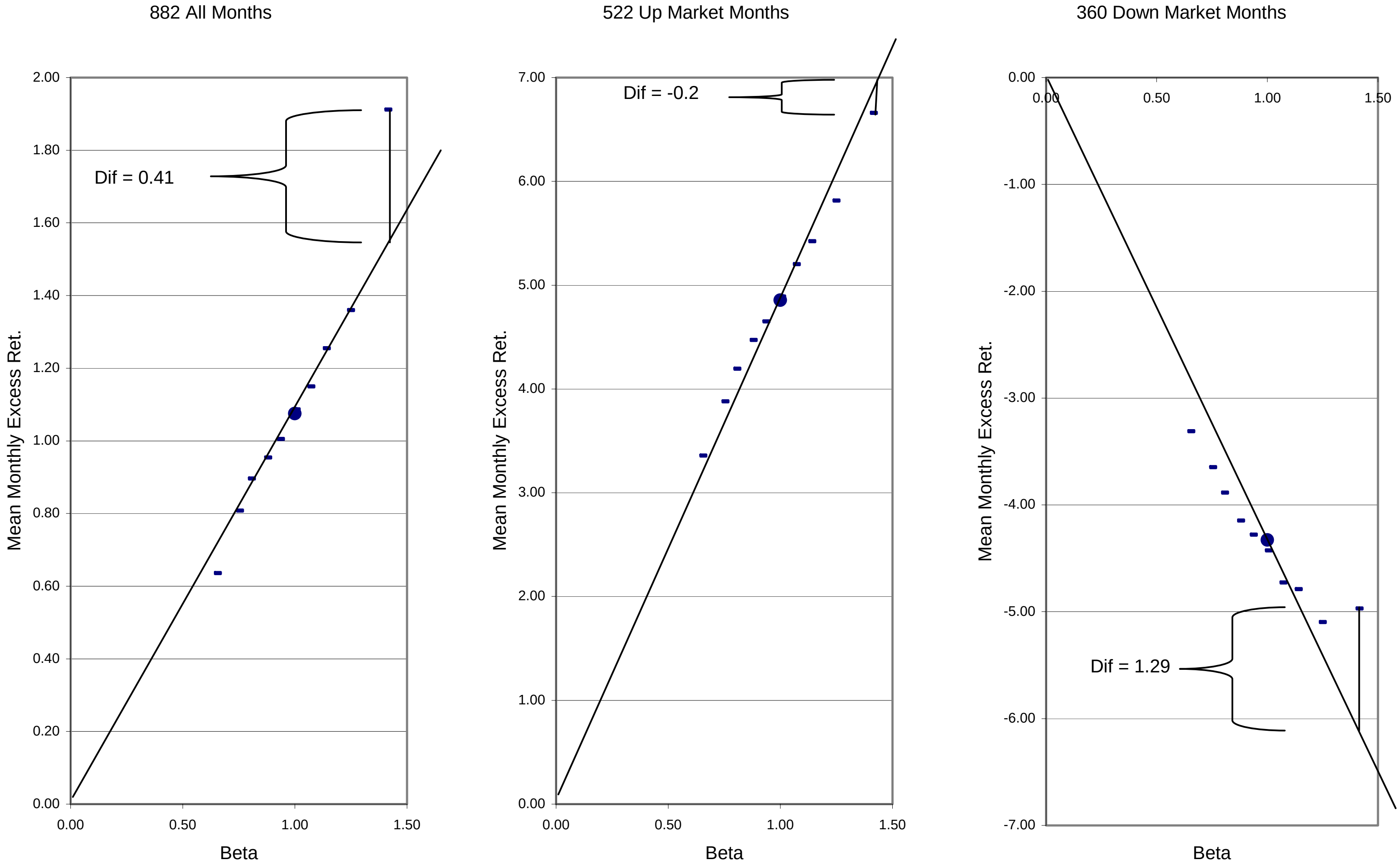

Figure 2.3. The Relationship between Return and Size of 10 Size Portfolios Without Controlling Beta

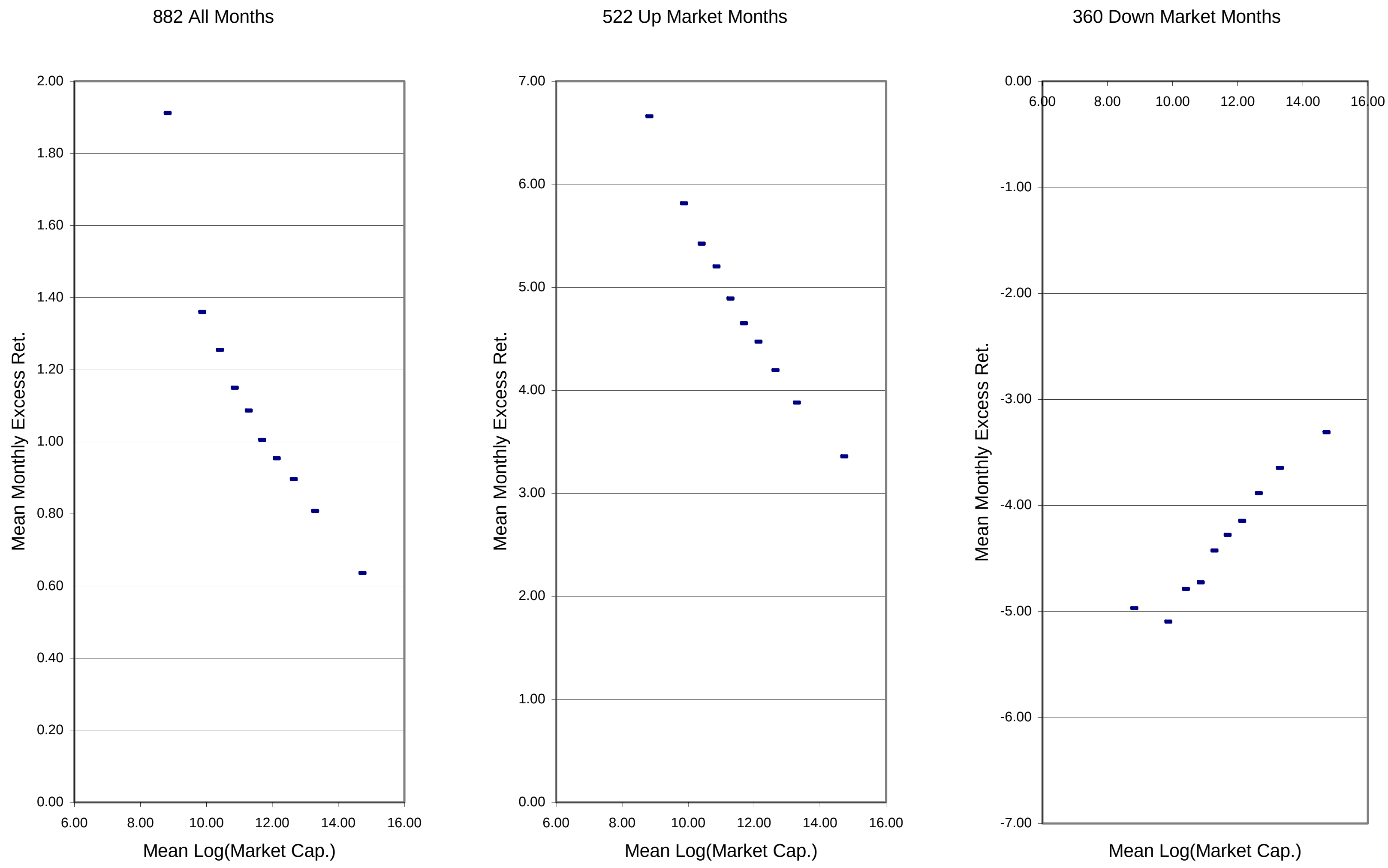

Figure 2.4. The Relationship between Return and Size of 10 Size Portfolios After Controlling Beta