Chapter 4

Description of the Static and Dynamic Tests

4.1 Static Data Analysis

Since a known load is applied to the rope, the only data that varies during the static tests is the displacement. The displacement data is negative in this sign convention because the plate is moving downwards, but in order to simplify the analysis, the negative sign was discarded. Figure 4.1.1 is a plot of the load vs. absolute displacement for one of the loading sequences. This continuous curve forms what are known as hysteresis loops and the data for each static cycle is referred to as a *Static Hysteresis Loop*.

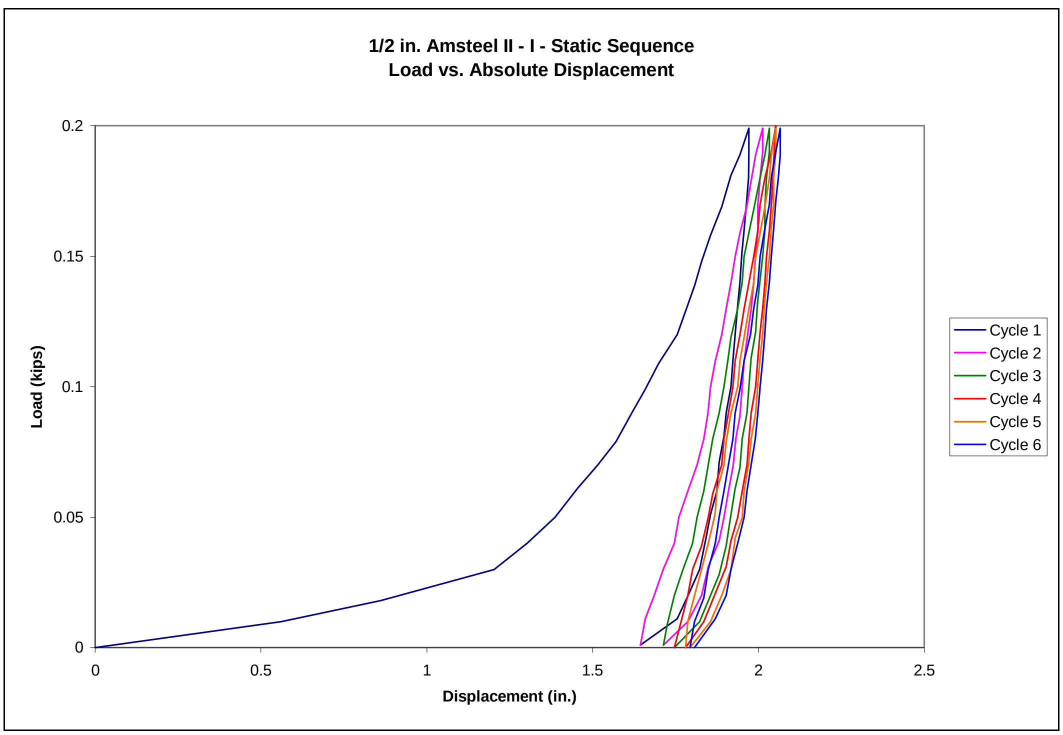

Figure 4.1.1: Load vs. Absolute Displacement Hysteresis Loops for a Static Sequence

The area bound by the load vs. displacement plot for each cycle was determined by using the trapezoidal rule in the form of equation (4-1).

$$A = \Sigma\left[\left(\frac{f_2 + f_1}{2}\right)(\Delta_2 - \Delta_1)\right] \tag{4-1}$$

The variables in this equation represent the area under the curve (A), the force (f), and the displacement (Δ). The subscript 1 represents the load on the left hand side of the interval between data points and the subscript 2 represents the load on the right hand side. For each increase in load, the average of the two forces was multiplied by the change in displacement to find the area under the curve for that particular interval. The areas for all of the intervals up to the maximum displacement are positive, but when the rope begins to rebound from its maximum displacement, the area term becomes negative. All of the areas were then added together to give the total area for one Static Hysteresis loop, known as the *Static Area.*

The *Maximum Displacement* for each cycle and the *Static Rope Elongation* during the precycling process are also of interest. The Maximum Displacement is simply the largest displacement value that was recorded during each cycle. This value increased throughout a sequence as more cycles were run. The elongation of the rope is equal to the last displacement reading that was recorded at the end of the sixth cycle. This point corresponds to the rope being completely unloaded at the end of a sequence, and that displacement value represents how much the rope has elongated from its original length. The amount the rope lengthened during each cycle can also be obtained by calculating the difference between the displacement readings at the zero load points at the beginning and end of each cycle.

The last value of interest taken from the static tests is the *Stiffness* of the rope. This was calculated by using a linear-fit trend line to find the slope of the nearly linear section of the load vs. absolute displacement curve for each cycle. The details of this process are explained in the dynamic test analysis section for the similar calculations of the dynamic slope.

The slope of the curves increases as more cycles are run, but the majority of that increase occurs between the first and second cycles. After the second cycle, the slope values increase slightly, but the change is small compared to that of the first two cycles. Therefore, the stiffness value of each rope was determined by averaging the individual slope values for cycles 2 through 6. This gives a good estimate of the stiffness value to which the rope is converging.

4.2 Dynamic Data Analysis

The types of data that were recorded during the drop tests were the force that was applied to the load cell through the rope, the acceleration of the drop plate, and the time step between each reading. Each test produced a data series that included several thousand data points over four to six cycles. An example of this raw data can be seen in Figure 4.2.1.

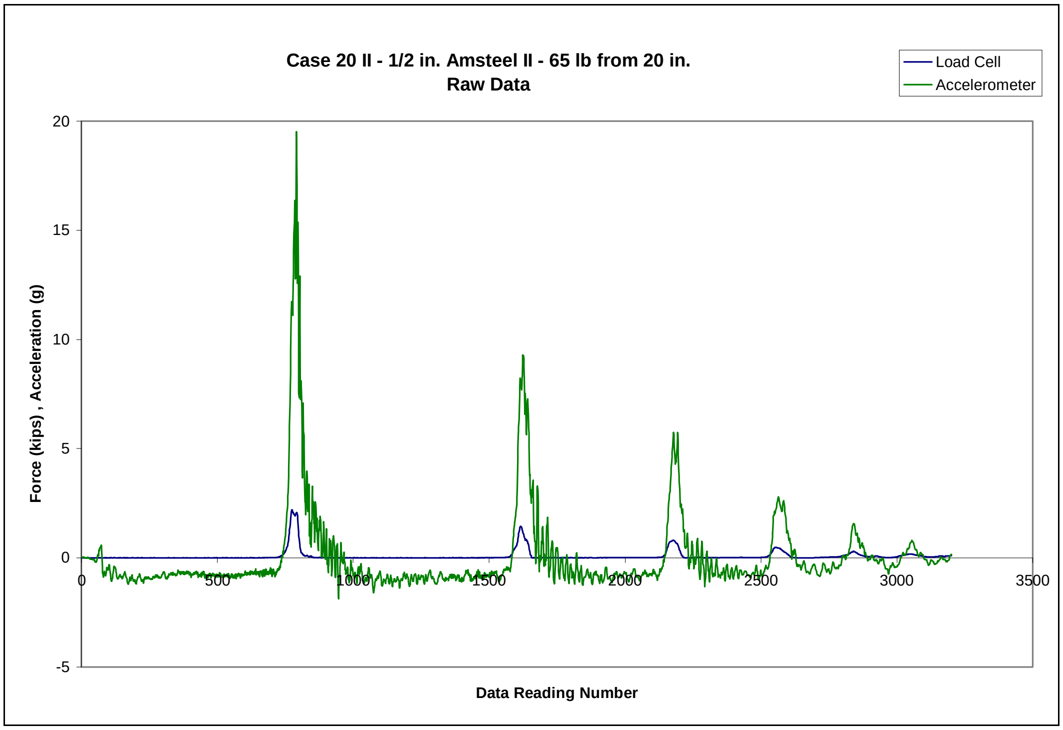

Figure 4.2.1: Dynamic Data Series recorded from a Drop Test

A *Cycle* is defined as the data range in which one snap load has occurred. Six cycles can be seen in the above figure. Since only the first cycle was of interest for this research, most of the other data was discarded in order to save space on the computers. A small amount of data was cut from the beginning of the data series because there was a short delay between the time when the test readings began and when the drop plate was released from the angle seat. In addition, the data was cut off towards the end of the second cycle to ensure the completeness of the data from the first cycle. An example of the cropped data can be found in Figure 4.2.2.

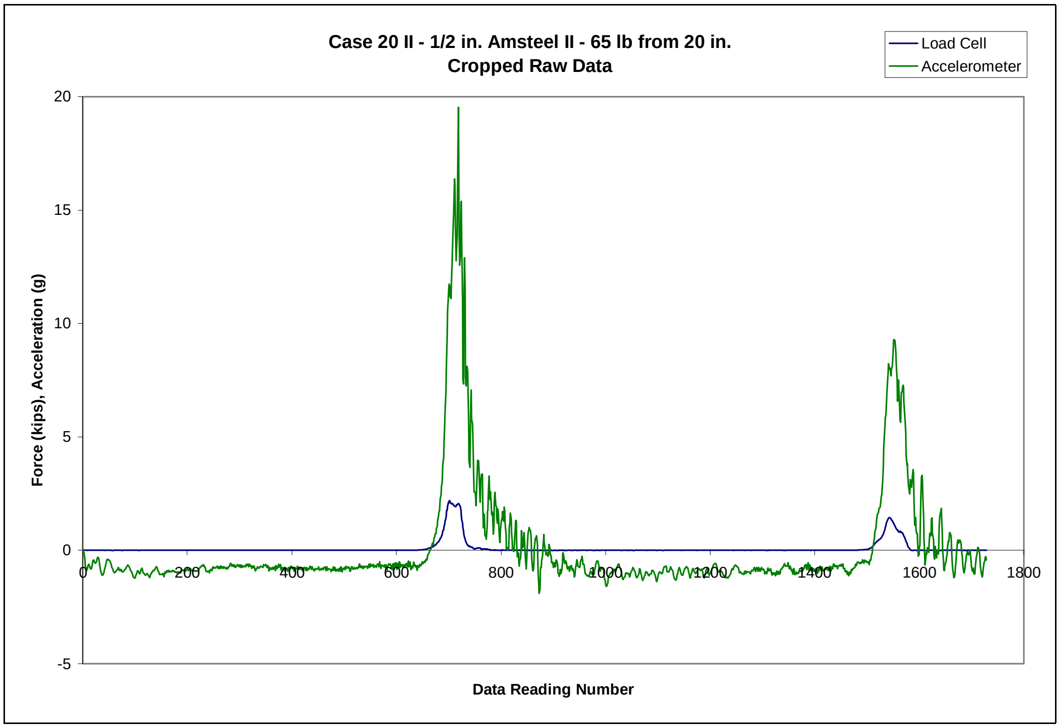

Figure 4.2.2: Cropped Dynamic Data Series

The dynamic tests were analyzed from the point where the drop plate was released to the transition point between the first and second cycles. This transition point will be defined later in this chapter. The trapezoidal rule in the form of equation (4-2) was used to integrate the acceleration data to obtain the velocity values throughout the drop test. This data can be found in Figure 4.2.3.

$$v_1 = v_0 + \left(\frac{a_0 + a_1}{2}\right)(t_1 - t_0) \qquad (4\text{-}2)$$

The variables in the above equation represent the velocity (v), the acceleration (a), and the time (t). The subscript 1 represents the point at which the velocity is being calculated and the subscript 0 represents the point at the beginning of the time step that precedes it.

From the load, acceleration, and velocity data, it was then possible to divide the cycles into three distinct phases and assign several points of significance. The first phase is called the *Free-Fall Phase*. The Free-Fall Phase occurs when the plate is falling and the rope is slack. The acceleration has a negative value during this phase because of the previously mentioned sign convention. For the first cycle, this begins when the drop plate is released from its designated seat on the angle. During this first free-fall, the acceleration essentially reaches a steady-state value, with a magnitude less than 1 g (due to friction in the bearings and the air resistance of the plate) until the plate begins to decelerate due to the tightening of the rope. At this point, the load cell begins to read the first significant force values. Just before the rope begins to tighten, the Free-Fall Phase ends and the Taut Phase begins.

The *Taut Phase* is of primary interest in this research and is the focus of most of the analyses that are performed. This phase is when the snapping of the rope occurs, causing the values of both the load and the acceleration to spike (also referred to as pulses). This in turn causes the velocity to go from its maximum negative value to its maximum positive value in a very short amount of time. The Taut Phase begins when the force readings begin to increase, corresponding to the free-fall acceleration going from its negative value back towards zero. The phase ends when the force value goes to zero at the end of the force spike. This occurs when the rope becomes slack again as the drop plate is rebounding from its maximum displacement.

The last phase is the *Rebound Phase*. This phase begins when the force has gone to zero and the rope becomes slack while the plate continues moving upward. It ends when the velocity goes back to zero (when the drop plate is at the top of its rebound), just before the beginning of the second cycle. These same three phases occur for every cycle, but they were not analyzed beyond the first cycle for any of the tests. Figure 4.2.3 illustrates the three phases.

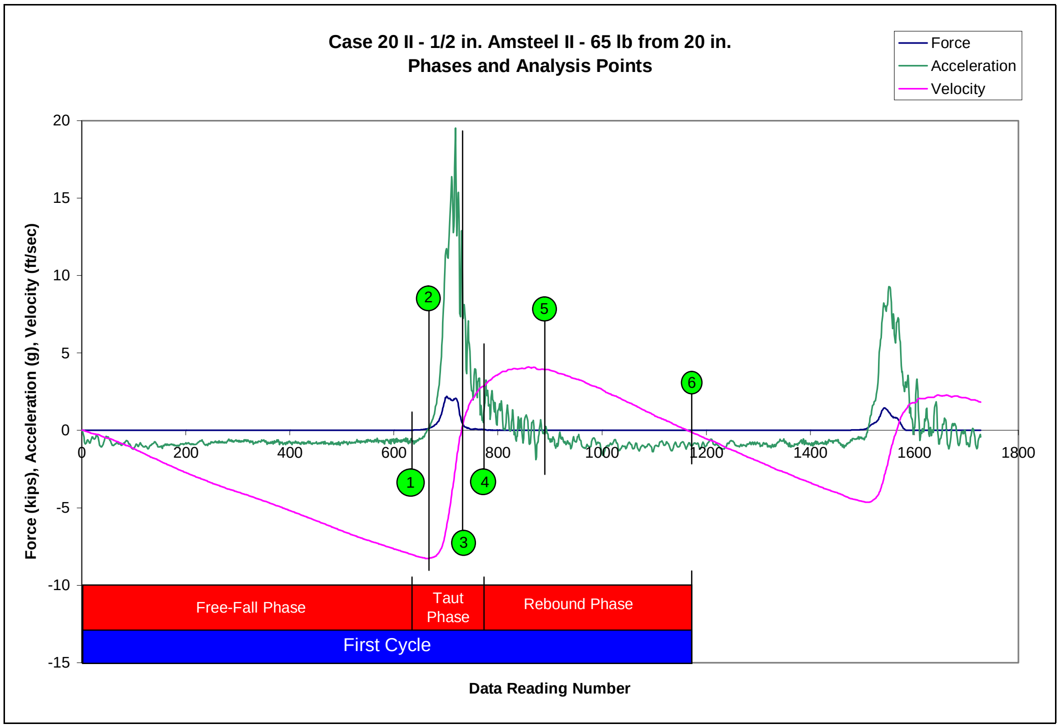

Figure 4.2.3: Force, Acceleration, and Velocity Data and Illustration of Phases and Analysis Points

The points that are identified by green dots are called analysis points. These points are used to identify the beginning and end of the phases as well as to divide the phases up for analysis purposes.

The beginning of the Taut Phase is designated *Point 1*. This is where the readings from the load cell begin to increase and is the first point in the force pulse. The plate begins to decelerate at this point as the acceleration readings start to go from the steady state value back towards zero.

Point 2 occurs when the acceleration reaches zero at the beginning of the acceleration pulse, corresponding to the maximum negative velocity. This point was established as the datum for the purpose of calculating the energy dissipated by the snapping of the rope.

Point 3 occurs when the velocity reaches zero, when the plate reaches its maximum displacement at the bottom of its fall.

Point 4 separates the Taut Phase from the Rebound Phase and occurs when the force goes to zero at the end of the force pulse. This occurs after the plate has begun to move upward, at the point when the snapping of the rope no longer contributes to the upward motion.

Point 5 occurs while the plate is rebounding, when it reaches the datum that was established by Point 2. The exact location of this point in the data series has to be established through the comparison of displacement values, but it occurs between the point of maximum positive velocity and where the acceleration goes to zero at the end of its pulse.

The last analysis point, *Point 6*, occurs when the plate reaches the top of its rebound and the upward velocity goes to zero at the end of the first cycle. This signifies the transition point that separates the Rebound Phase of the first cycle from the Free-Fall Phase of the second cycle.

Even though the analysis points are based on certain force, acceleration, and displacement readings that happen during every dynamic test, the points occur at different locations in each data series and have different intervals between them. This method of defining phases and points was used because it eliminates the effects of the permanent (plastic) elongation of the rope that occurs due to the repeated loadings in the static and dynamic tests. As each testing sequence progresses, the length of the rope increases because the force that the drop plate transfers to the rope causes the fibers to tighten together. If the analyses of the individual ropes had been based on analysis points that were uniform throughout a testing sequence, there would have been a residual error due to this plastic elongation. However, since the analysis points change from test to test based on the behavior of the rope, it is possible to accurately compare all of the tests no matter where the points occur in the testing sequence.

The analysis of the Taut Phase begins at Point 1. The time of the Taut Phase is set to zero and all subsequent times are in reference to this point. The displacement is set to zero as well. The trapezoidal rule in the form of equation (4-3) is used to integrate the velocity data (equivalent to a double integration of the acceleration) to obtain the displacement values throughout the Taut Phase. Figure 4.2.4 contains all of the aforementioned data from the Taut Phase.

$$\Delta_1 = \Delta_0 + \left(\frac{v_0 + v_1}{2}\right)(t_1 - t_0) \qquad (4\text{-}3)$$

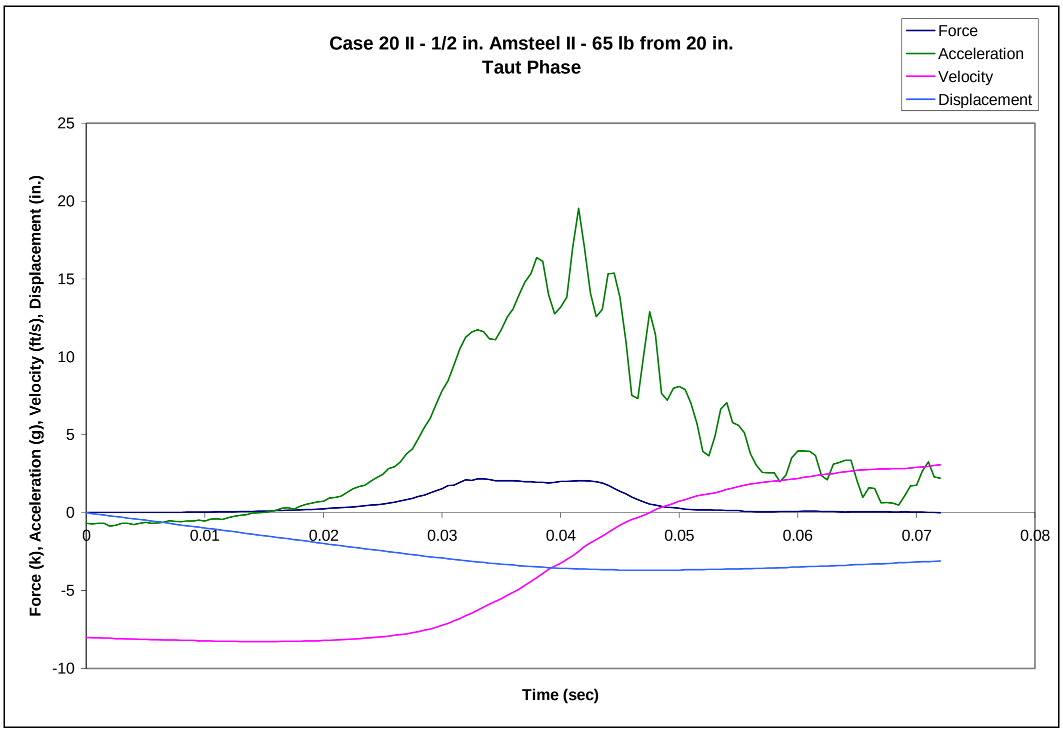

Figure 4.2.4: Force, Acceleration, Velocity, and Displacement Data for the Taut Phase

The displacement (Δ) increases from zero up to its maximum when the plate has reached Point 3 at the bottom of the fall. From there the plate rebounds past the point when the rope became taut until the velocity of the plate reaches zero. The recorded displacement at the previously mentioned datum (Point 2) is noted and when the displacement reaches that point again, that data point is designated Point 5.

4.3 Dynamic Data Analytical Values

The data from the Taut Phase forms the basis for all of the data comparisons and for the mathematical model. Some of the quantities that are of interest can be obtained from the data analyses that have been conducted thus far and are described below.

The *Pulse Duration* (or the length of the Taut Phase) is defined as the amount of time (in seconds) that it takes for the force spike to occur. As stated earlier, the force pulse begins at Point 1 and ends at Point 4, but since the time is zeroed at Point 1, the Pulse Duration is the same as the elapsed time at Point 4. It is denoted τ.

The *Maximum Force* is simply the largest reading (in kips) that is obtained by the load cell during the first cycle.

The *Maximum Acceleration* is the largest reading (in g) that the accelerometer records during the first cycle.

The *Maximum Displacement* is the largest displacement (in inches) that is calculated during the first cycle. This occurs at Point 3.

The *Impact Velocity* is the velocity reading (in ft/sec) at Point 1. This is the greatest velocity that is achieved before the plate begins to decelerate due to the tightening of the rope. From here, the velocity increases until Point 2 where it reaches its maximum negative value. The difference between this value and the impact velocity is very small, and since it is more accurate to compare theoretical velocities to the impact velocity, the maximum velocity value is not of importance.

The remaining analytical quantities can not be taken directly from the preceding work and therefore can only be obtained through either separate calculations or another analytical process.

The first quantity that is computed is known as the *Impulse* (*J*). The impulse is defined as the area beneath the force vs. time curve during the force pulse and is in units of kip-seconds. It is computed using equation (4-4).

$$J = \left(\frac{\tau}{2n}\right)\left(f_0 + 2f_1 + 2f_2 + ... + f_n\right) \qquad \text{(4-4)}$$

In this equation, τ represents the Pulse Duration, *n* represents the number of data points in the series minus one, and *f* represents the force reading at a particular time. Trends in the Impulse will be compared to those of other quantities to see how the behavior of a rope changes during its testing sequence.

The next quantity that is calculated is the *Stiffness* (*k*) of the rope. The stiffness is defined as the slope of a portion of the force vs. displacement curve during the Taut Phase and is in units of kips/inch. That is, the stiffness is the amount of force required to lengthen the rope by one inch. This value is obtained by manipulating the force vs. displacement graph, and the process of doing so is detailed as follows. An un-manipulated plot of this data can be found in Figure 4.3.1.

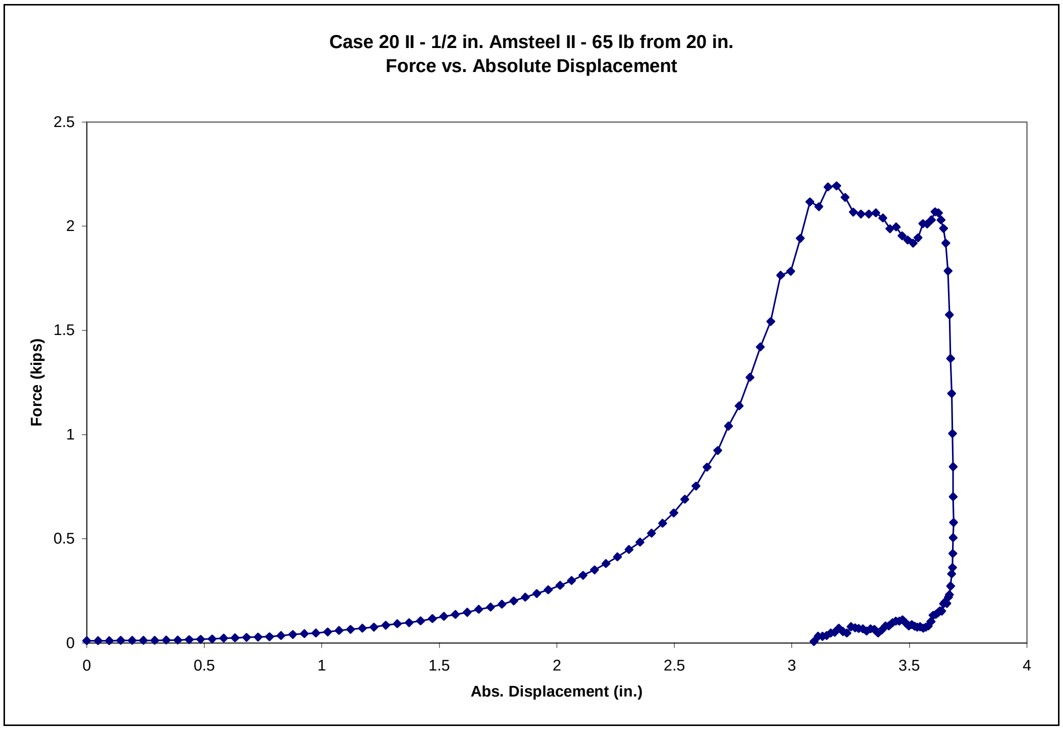

Figure 4.3.1: Unaltered Force vs. Absolute Displacement Plot

The blue diamonds represent individual data points and the lines connecting these points help to show the trend that the data follows. At the beginning of the Taut Phase, the rope begins to displace before a large force is present in the rope. This is when the rope is straightening out due to the falling plate, but before the fibers have completely tightened together. When the force starts to increase significantly, the slope of the curve increases and follows a nearly linear path until the maximum force occurs. The force remains at a high value for a short time and then decreases significantly as the rope reaches its maximum displacement and then begins to rebound from the fall.

The slope of the nearly linear region increases as each testing sequence progresses, indicating that the fibers in the rope tighten up more each time the rope is tested. To document how the stiffness changes throughout a sequence, the slope value is calculated for each test. This is done by creating another data series using only the data points that fall in the nearly linear region. A best-fit, linear trend line is created for that data and the equation of that line is displayed on the graph. The equation is in the form of $y = mx + b$, where m represents the slope value. This can be seen in the Figure 4.3.2.

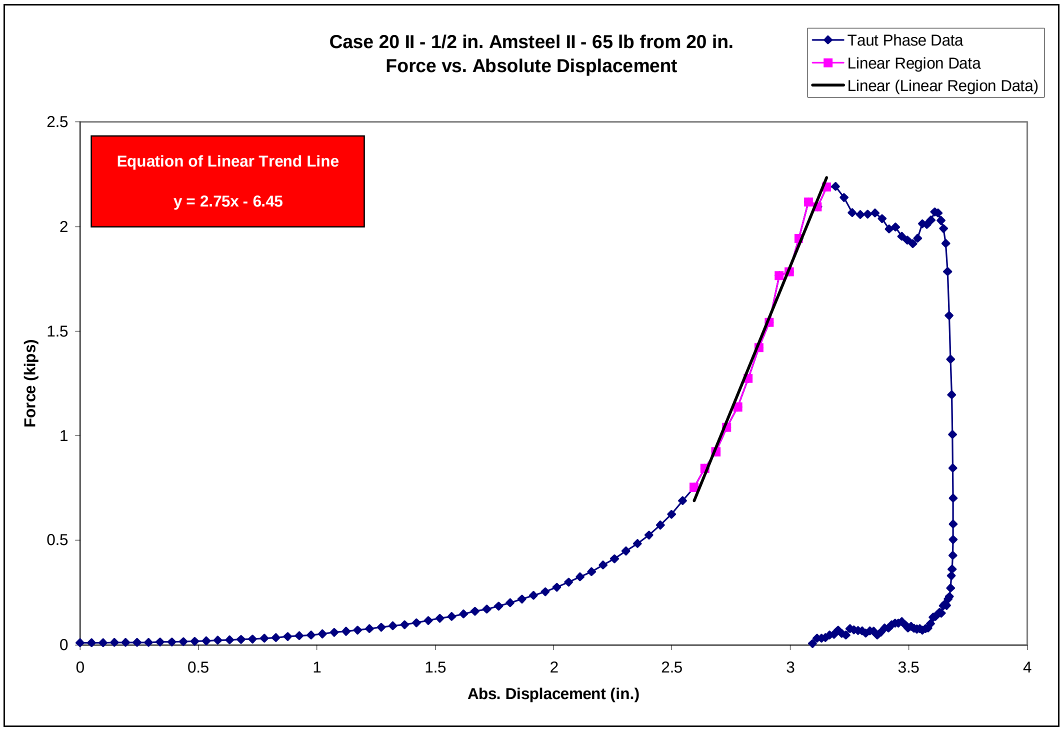

Figure 4.3.2: Force vs. Absolute Displacement Plot with Slope Data

The next analytical quantity is the *Actual Drop Height*. For classification purposes, the ropes were designated with approximate drop heights that correspond to the height of the angle seats they were released from to the bottom of the drop angle. These numbers represent an overestimate of the actual height that the plates fell from, since the ropes became taut before the plate reached the bottom of the drop angle. This does not change the accuracy of the recorded data, but to obtain other analytical quantities and compare the experimental results to theoretical equations, the actual drop heights needed to be calculated. This was done by subtracting the length between the two ends of the ropes while the plate was on its respective seat (called the slack length) from the measured length of the rope before the dynamic tests began. Figure 4.3.3 shows this graphically.

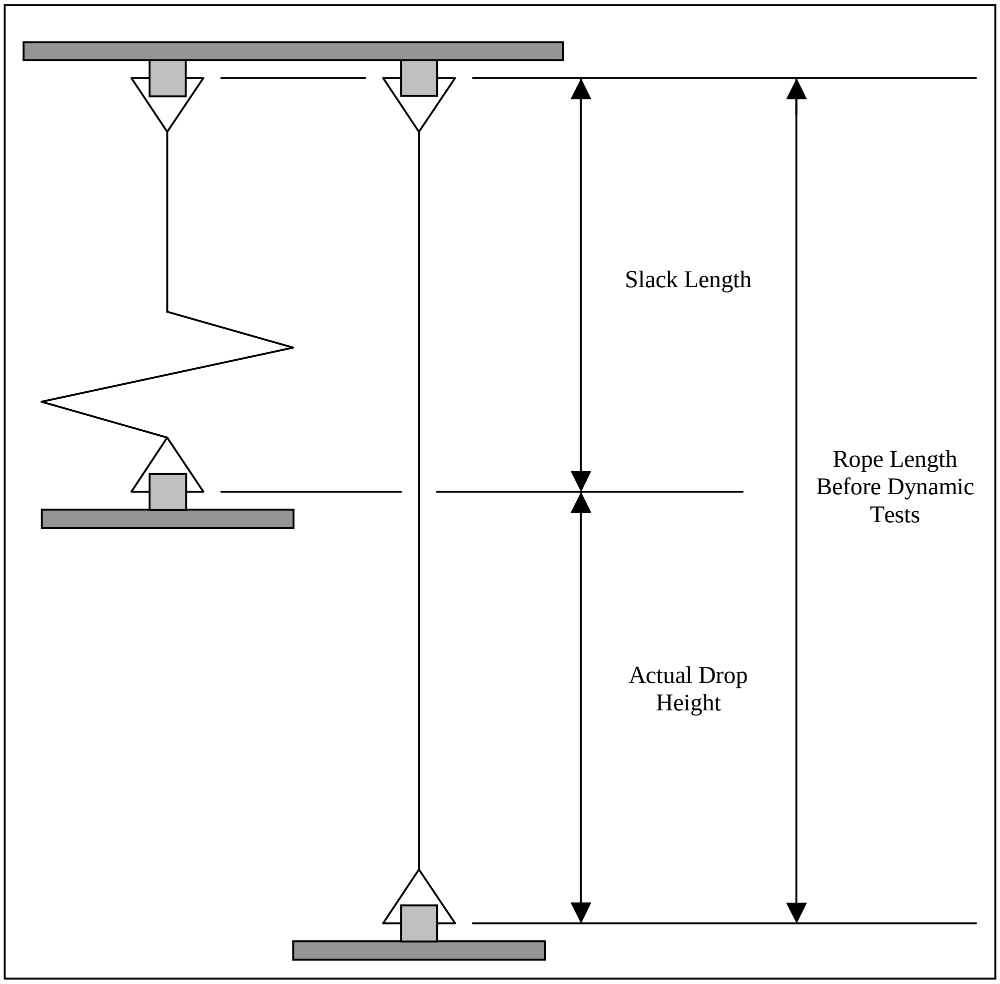

Figure 4.3.3: Illustration of Actual Drop Height

The actual drop height increases as the sequence progresses, but for simplification purposes, the initial drop height for a given sequence was used in most of the subsequent calculations. However, in order to determine the *Dynamic Rope Elongation*, these drop heights had to be calculated. This was done by applying equation (4-3) to the Free-fall Phase of the first cycle with the initial displacement (Δ_o) set to the approximate drop height of the respective angle seat. When the plate reaches Point 1, the corresponding displacement from the angle seat is subtracted from Δ_o and that value equals the actual drop height for the test. The Dynamic Rope Elongation is equal to the Slack Length plus the actual drop height for the last test in the sequence minus the Rope Length before the

Dynamic Tests. The calculated Static and Dynamic Rope Elongation values were then compared to the rope length measurements that were taken. Each rope was measured to the nearest ½-inch with a tape measure when the rope was new, then after it had been statically tested, and finally after it was dynamically tested. This comparison was accurate for most of the ropes, but since errors were made that caused the actual first cycle of some of the static tests and the first drop test of some dynamic sequences to go unrecorded, there were discrepancies for some of the ropes.

4.4 Energy Dissipation

For the dynamic tests, the method used to determine the amount of energy dissipated by the snapping action of the rope was to calculate the energy of the plate at two points and compare the values. The Kinetic Energy (KE) of the plate is given by equation (4-5).

$$KE = \frac{1}{2}mv^2 \qquad (4\text{-}5)$$

The kinetic energy of an object is a property that is associated with its mass (*m*) and its velocity and is in units of kip-inches. When the plate is released from its angle seat, gravity causes the plate to fall and therefore gain kinetic energy. When the plate reaches the bottom of its fall it loses all of the kinetic energy it had acquired, but because the taut rope causes the plate to rebound, it will gain some of that kinetic energy back. Figure 4.4.1 shows this process and the locations of the drop plate at certain points during the Taut and Rebound Phases.

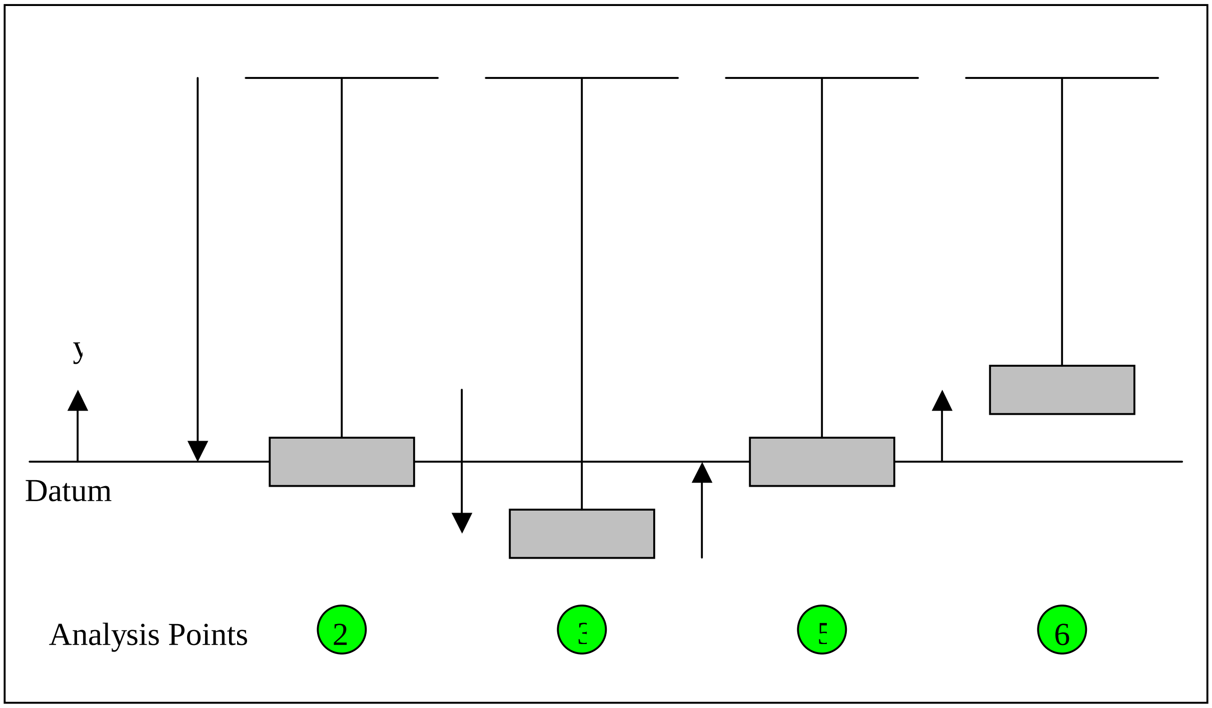

Figure 4.4.1: Illustration of Drop Plate Locations and the Established Datum

The two points that were compared in this analysis were Points 2 and 5, which represent the datum that was previously set. The kinetic energy of the plate at Point 2 is that which was amassed during the fall of the plate and the kinetic energy at Point 5 is that which was regained during the rebound of the plate. Since the mass is constant for the tests, the velocities at these points are the only variables. The kinetic energy was calculated at both points and the difference between the two was defined as the *Energy Loss* (ΔE) for the cycle.

The Total Energy (TE) of the system is calculated by adding the kinetic and potential energies of the plate to the kinetic, potential, and elastic energies of the rope. This is represented by equation (4-6) if k is constant.

$$TE = \Sigma\left(\frac{1}{2}mv^2\right) + \Sigma(mgy) + \frac{1}{2}ky^2 \qquad (4\text{-}6)$$

The first term represents the kinetic energy, the second represents the potential energy due to gravity, and the third represents the elastic energy. The above equation involves the acceleration due to gravity (g), the displacement from the datum (y), and the stiffness coefficient of the rope (k). If the energy were calculated at every data point that was obtained in the dynamic tests, this is the equation that would have to be used. However, since the energy values were only calculated at the two points on the datum, many of the variables can be eliminated.

The kinetic energy of the rope can be ignored because the mass of the rope is considerably less than that of the drop plate. Also, the elastic energy of the rope is not applicable at points 2 and 5 because they are located at the beginning and end of the snapping action when that energy is equal to zero. The potential energy of the plate and rope can also be neglected because the two points are on the datum and the displacement term is equal to zero. This leaves only the kinetic energy of the plate.

Another common way to calculate the loss of energy for a system undergoing periodic motion (unlike the present system) is to calculate the area beneath the load vs. absolute displacement plot (also known as a Dynamic Hysteresis loop). The loss of energy associated with this is calculated by using the same equation that was used for the static tests. This area is referred to as the *Dynamic Area* (*DA*) and is in units of kip-inches. This area was calculated for every test and was compared to the Impulse and Energy Loss as well as the Static Area.

4.5 Longitudinal Stress Waves

During the time when the rope is in the Taut Phase, a large amount of force is transferred from the bottom of the rope up to the top where it is being supported. This force is a result of the snapping action of the rope and it travels in the form of *Longitudinal Stress Waves* which move up and down the rope (Meirovitch 1967). These waves travel at a very high rate of speed, and their velocity and travel time are calculated as follows. The diameter of one precycled rope of each type was measured at the middle, just below the eye-splice, and in the eye-splice using calipers. The diameters at the two locations within the length of the chord of the rope were then averaged because the diameter changes along the length from the eye-splice to the middle of the rope. The average cross-sectional area of the chord and the cross-sectional area of the eye were then calculated. Next, the volumes of the two parts of the rope were calculated. Since there is a 3-inch splice at each end of the rope, the length of the chord is equal to the length of the rope minus 6 inches and the length of the eye-splices is 12 inches since each splice has two 3-inch sides. The sum of the two individual volumes is equal to the total volume of the rope.

The specific weights of the ropes were then calculated by dividing the measured weight of the rope by the total volume. The density (ρ) is then obtained by dividing the specific weight by the acceleration due to gravity. The next value that is calculated is the modulus of elasticity (E). It is found using equation (4-7), assuming E is constant.

$$E = \frac{F_{\max}}{A} \frac{L}{\Delta L} \tag{4-7}$$

The modulus of elasticity is calculated from data that was obtained from the static tests. Therefore, F_{max} is equal to 200-pounds and A is the cross-sectional area of the chord of the precycled ropes. This L is defined as the original length of the rope plus the amount the rope lengthened during the first five static cycles. The ΔL is the amount the rope displaced from the end of the fifth cycle to the maximum point of the sixth cycle due to F_{max}.

The speed of the longitudinal stress waves (c) is then calculated using equation (4-8). It is in units of ft/sec.

$$c = \sqrt{\frac{E}{\rho}} \tag{4-8}$$

The amount of time it takes for one wave to travel up the length of the rope is then found by dividing the length of the rope by the wave speed. This is called the *Stress Wave Duration* (t_w). The number of stress waves that occur during the Taut Phase can also be obtained by dividing the average Pulse Duration for a given rope by the Stress Wave Duration.

4.6 Theoretical Values

In order to validate the data that was obtained analytically from the drop tests, it was desirable to compare it to several theoretical equations. The equations were taken from Lalanne (2002).

The equation for the *Theoretical Stiffness* (k_t) of the rope was derived from what is referred to as the shock duration, but is called the pulse duration in this research. By assuming that the drop plate and the rope constitute a linear mass-spring system with one degree of freedom, the shock duration can be calculated using equation (4-9).

$$\tau = \pi \sqrt{\frac{m}{k}} \tag{4-9}$$

According to this equation, the shock duration is independent of the impact velocity and since the mass and the duration of the taut phase are known, the theoretical stiffness of the rope can be obtained by rearranging equation (4-9) to get equation (4-10).

$$k_t = m\left(\frac{\pi}{\tau}\right)^2 \tag{4-10}$$

The *Theoretical Impact Velocity* (v_{it}) was calculated using a free-fall equation that is based upon the actual drop height (*H*). By simplifying equation (4-11), the Theoretical Impact Velocity can be obtained through equation (4-12).

$$mgH = \frac{1}{2} m v_{it}^2 \tag{4-11}$$

This reduces to

$$v_{it} = \sqrt{2gH} \tag{4-12}$$

Equations (4-11) and (4-12) assume that there is no friction acting upon the falling mass, but since the drop plate is subject to friction and air resistance, a slight difference between the experimental values and the theoretical values should be expected.

The equation for the *Theoretical Maximum Displacement* (Δ_t) is obtained by setting the loss of kinetic energy during the tensioning of the rope equal to the deformation energy. This is represented by equation (4-13).

$$\frac{1}{2}m(v_i)^2 = \frac{1}{2}k(\Delta_t)^2 \qquad (4\text{-}13)$$

By rearranging, the theoretical maximum displacement equation is obtained. This is equation (4-14).

$$\Delta_t = v_i\sqrt{\frac{m}{k}} \qquad (4\text{-}14)$$

The experimental impact velocity and the stiffness obtained from the dynamic slope were used in this equation, rather than their theoretical equivalents, because they produced results that more closely matched the recorded displacements.

The Theoretical Force (F_t) was calculated by using Newton's gravitational force laws. This is represented by equation (4-15).

$$F_t = ma + mg \qquad (4\text{-}15)$$

The theoretical force was calculated for each data point during the Taut Phase. The experimental acceleration at each point is represented by *a* and the mass of the plate is represented by m. It should be noted that this theoretical force is that which is experienced at the bottom of the rope.

The results of the comparisons between the analytical data and the theoretical values are found in the next chapter.