LATERAL RESTRAINT FORCES IN QUARTER-POINT AND THIRD-POINT PLUS SUPPORT BRACED Z-PURLIN SUPPORTED ROOF SYSTEMS SUBJECT TO GRAVITY LOAD

by

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LATERAL RESTRAINT FORCES IN QUARTER-POINT AND THIRD-POINT PLUS SUPPORT BRACED Z-PURLIN SUPPORTED ROOF SYSTEMS UNDER GRAVITY LOAD

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(ABSTRACT)

The objective of this study was to develop design equations that predict lateral restraint forces in two commonly used Z-purlin supported roof systems. These are quarter point bracing and third point plus support bracing. To that end, a stiffness model used in the past has been reintroduced. This model has been modified slightly to better represent roof system behavior. The updated stiffness model was then used to estimate lateral restraint forces for a number of roof systems with a varying cross sectional dimensions of the purlin, number of purlin lines, number of spans, and span length. A regression analysis was then performed on the data to obtain empirical design equations similar to those found in the 1996 Edition of the American Iron and Steel Institute's *Specification for the Design of Cold-Formed Steel Members*, Section D.3.2.1.

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CHAPTER I

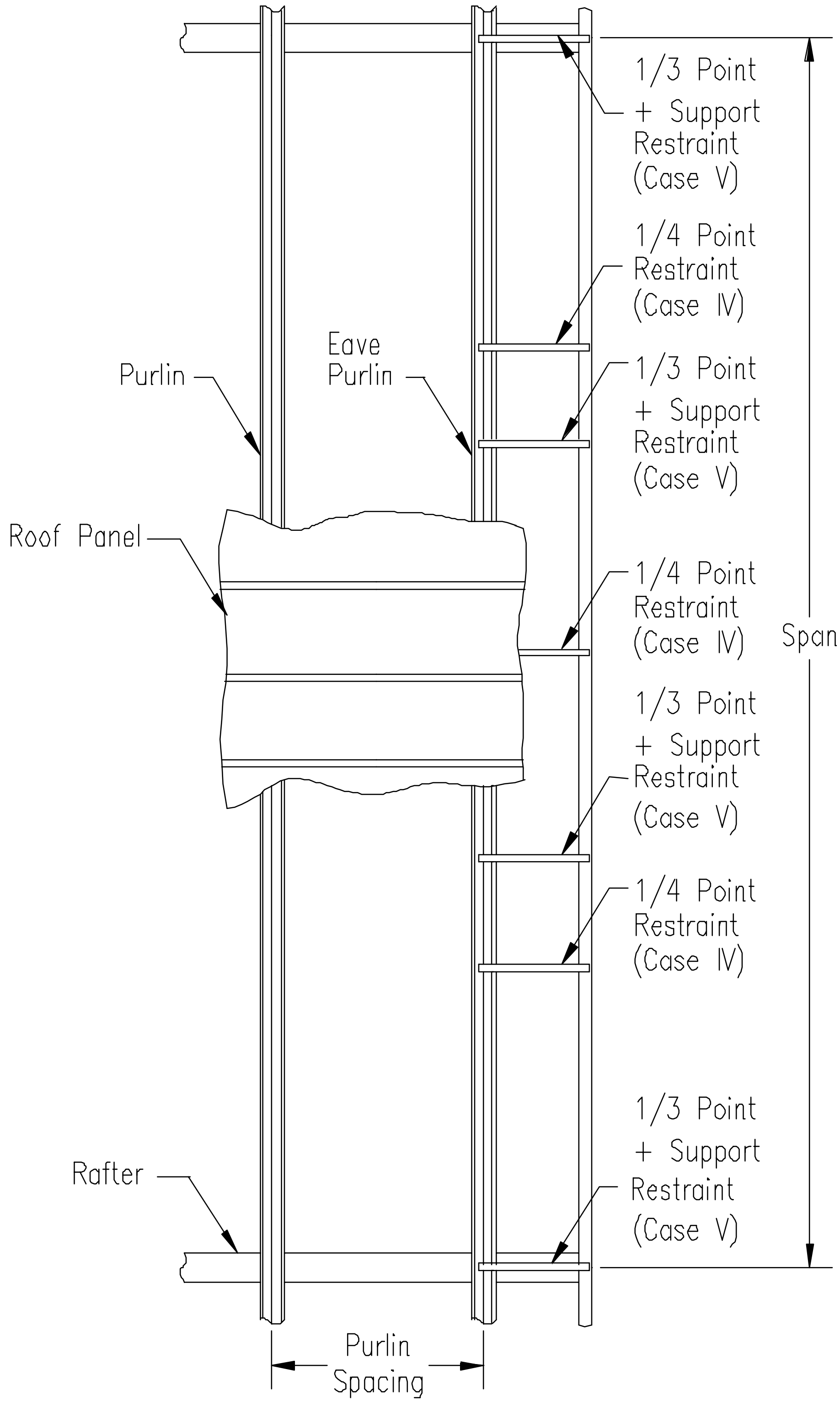
INTRODUCTION

1.1 OBJECTIVE

The research conducted herein is an analytical study to develop design equations that predict lateral restraint forces in quarter point and third point plus support bracing systems for Z-purlin supported roof systems subjected to gravity load, as shown in Figure 1.1-(a). Currently, the American Iron and Steel Institute's *Specification for the Design of Cold-Formed Steel Members* (AISI, 1996), Section D.3.2.1, provides design equations for estimating brace forces for support restraint, mid-span restraint, and third point restraint only, shown in Figure 1.1-(b). This research develops similar design expressions for two commonly used bracing configurations.

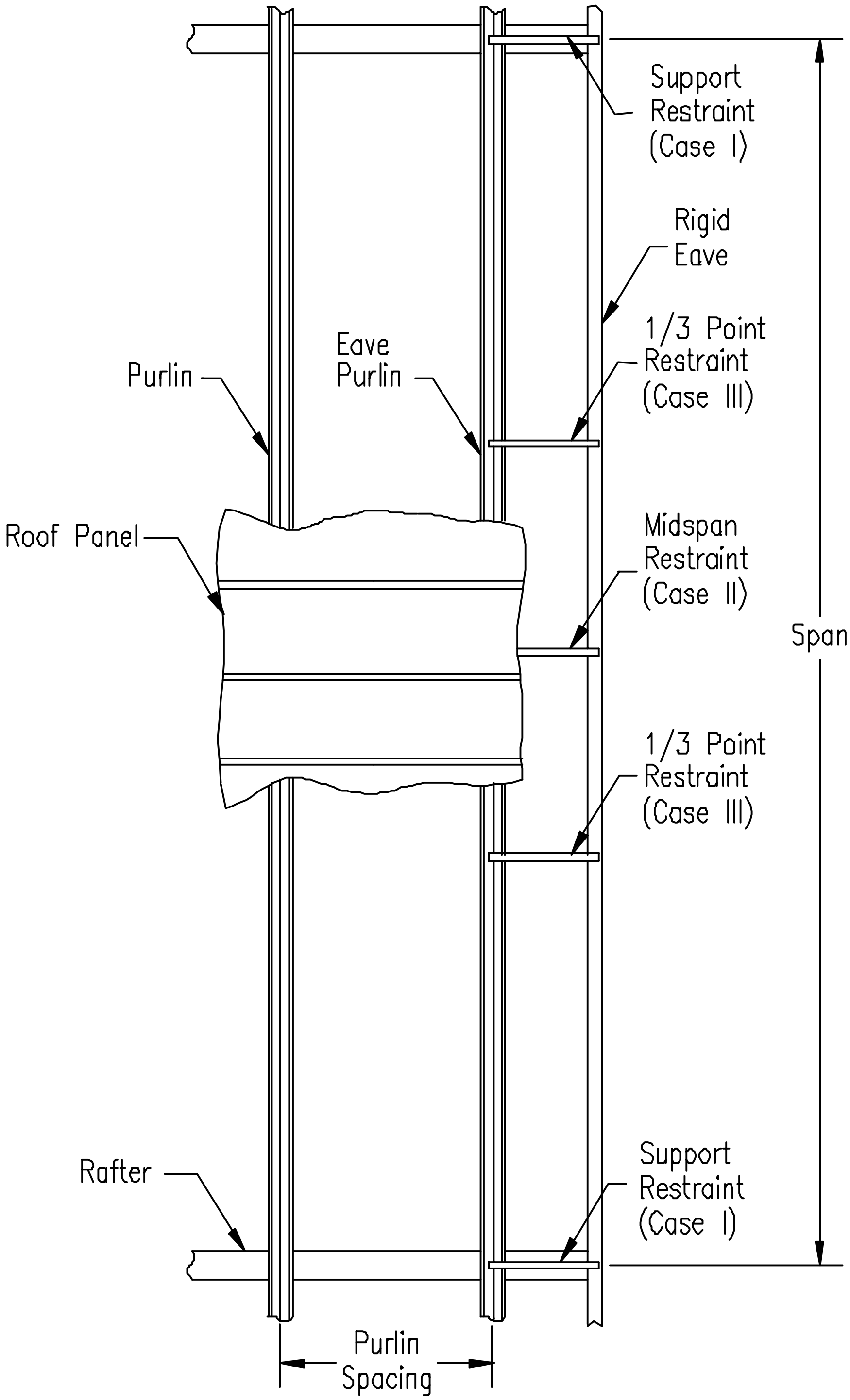
1.2 BACKGROUND

Cold-formed Z-purlins are thin, light weight steel sections (see Figure 1.2). They are commonly used by the metal building industry as secondary structural members in roofing systems. This is due to their ease of production,



(a.) Proposed Design Provisions

Figure 1.1 Commonly Used Bracing Configurations



(b.) Current Design Provisions

Figure 1.1 Cont. Commonly Used Bracing Configurations

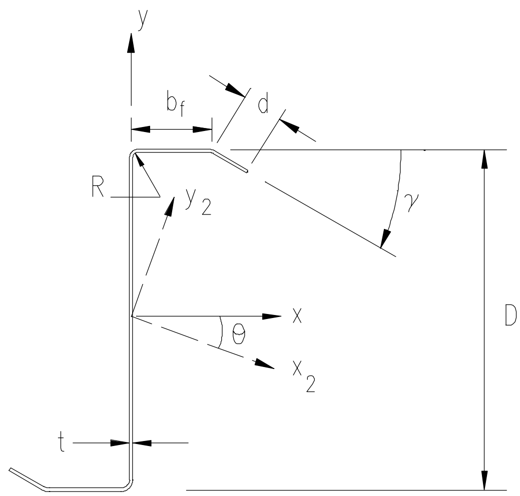


Figure 1.2 Purlin Cross–Section & Geometric Parameters

transportation, handling, and erection. Cold-formed Z-purlins typically span from rafter-to-rafter support steel roof deck attached to the top flange. The deck may be either of the through-fastened or standing seam variety (see Figure 1.3).

While cold-formed Z-purlins (hereafter referred to as Z-purlins) are easy to fabricate and erect, they present a unique problem to the structural engineer during the design process. Because the line of action of the supported load is not parallel to the principal axes, purlins tend to twist and move laterally as load is increased. If this movement is left unrestrained, the strength of the purlin is greatly reduced. Thus, to fully develop the strength of the purlins, lateral-torsional restraint must be provided.

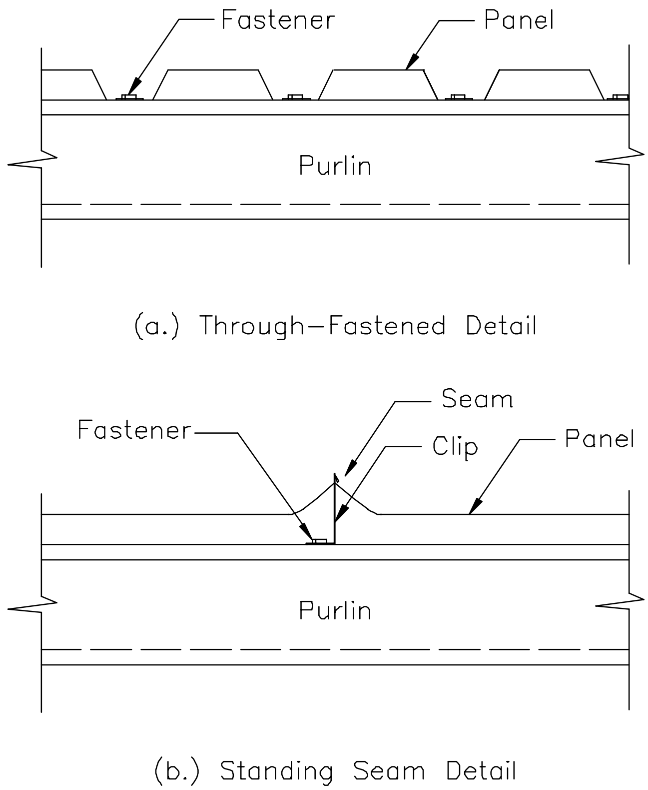


Figure 1.3 Typical Through-Fastened & Standing Seam Roof Deck

This restraint is provided to purlin roofing systems in two ways. First, attaching the top flange of the purlin to roof sheathing—be it through-fastened or standing seam—provides a significant amount of lateral restraint to the purlins. This restraint is usually adequate to prevent relative lateral movement between adjacent purlin lines. Second, to prevent the roof system as a whole from displacing laterally, external restraint must be provided. This is usually accomplished by supplying braces at discrete locations. Braces are generally pinned at each end and carry only the axial load induced by restraining the roof system laterally.

Therefore, knowing the magnitude of the restraint forces in the lateral bracing is a necessary part of designing a roof system. Currently, design equations are available for predicting lateral restraint forces in support (Case I), mid-span (Case II) and third-point (Case III) bracing configurations, as shown in Figure 1.1-(b). The purpose of this research is to develop design equations that predict lateral restraint forces in quarter point and third point plus support bracing configurations as shown in Figure 1.1- (a).

1.3 LITERATURE REVIEW

Zetlin and Winter (1955) considered a simply supported beam loaded obliquely with respect to its principle axis with lateral bracing at intermediate points along its length. They assumed that the twisting of the beam induced by the oblique load was small and could be neglected. Zetlin and Winter considered deflection only in the vertical and longitudinal directions. For this situation they derived a simple, straight forward expression for the total lateral restraint force, BF_x:

$$BF_x = \left(I_{xy} / I_x \right) W_y \tag{1.1}$$

where:

I_{xy} = Moment product of inertia of the purlin cross-section about the axes parallel to and perpendicular to the web,

I_x = Moment of inertia of the purlin cross-section perpendicular to the web,

W_y = Applied gravity load.

Needham (1981) studied the behavior of an obliquely loaded Z-purlin attached at its top flange to a roof panel. As mentioned above, these panels supply significant lateral and torsional restraint to purlins. Figure 1.4 shows the actual system and an idealized model. The idealized model shows that some restraint is provided to the purlin by the panels, but not complete restraint. Needham developed a mathematical model to quantify just how much restraint is supplied. He limited his investigation to simply supported Z-purlins with no discrete lateral bracing. He assumed the roof to be a diaphragm with infinite rigidity and that the panels can not move laterally with respect to the purlins.

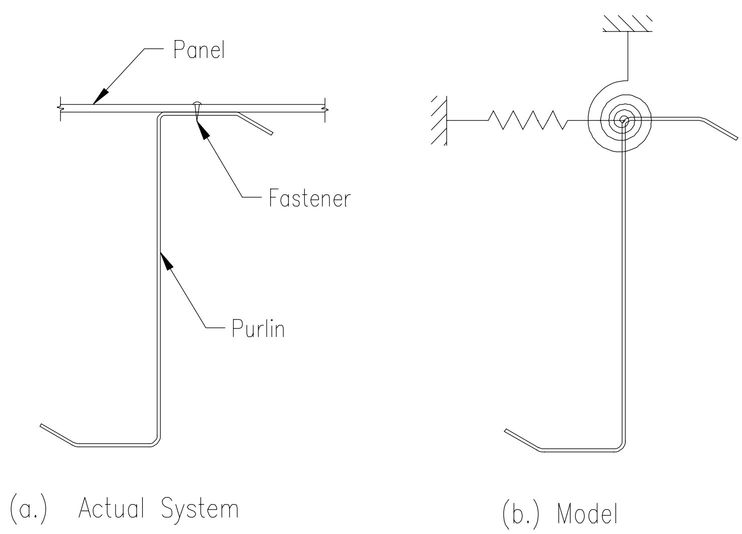


Figure 1.4 Partial Restraint of Purlins

Figure 1.5-(a) shows the loads acting on a typical gravity loaded Z-purlin, where W is the gravity load. This load is actually distributed in some unknown

manner across the purlin top flange. Needham assumed the resultant of this distribution acted at a distance one sixth of the flange width from the web. This gravity load causes lateral movement which in turn causes a resisting force W_p to be generated in the roof panel. This force acts at a vertical distance of D/2 from the centroid of the purlin. To simplify the analysis, Needham transformed the gravity load W acting at an eccentricity e into a torque T_w acting at the purlin centroid. Likewise, the restraining forces W_p acting at a distance D/2 generates a torque T_p . The resulting forces of this transformation are shown in Figure 1.5-(b). The total torque acting on the cross-section is then:

$$T = T_W + T_p = W^*e - W_p (D / 2) \tag{1.2}$$

When substituting $e = b_f / 6$ becomes:

$$T = W (b_f / 6) - W_p (D / 2) \tag{1.3}$$

Using Equation 1.1, Needham set $W_p = W(I_{xy}/I_x)$. The total torque becomes:

$$T = W [(b_f / 6) - (I_{xy}/I_x) (D / 2)] \tag{1.4}$$

Note that the panel torque T_p is less than the gravity load generated torque T_w, since $b_f / 6$ is always less than D/2. This difference in torque T_S must be somehow resisted to satisfy equilibrium. The torque T_S is the reason additional lateral restraint bracing is required. Needham resolved this torque into a force acting horizontally at the purlin top flange, where an additional brace would be placed. This is at a distance D/2 from the centroid. The corresponding force, W_S equals:

$$W_S = T / (D/2) = W [(b_f / 3D) - (I_{xy}/I_x)] \tag{1.5}$$

The total net force acting at the panel, which can be thought of as the total required lateral restraint force, is then:

$$W_{net} = W_p + W_s = W\,[\,I_{xy}/I_x + b_f / 3D - I_{xy}/I_x\,] = \qquad (1.6\text{-a})$$

$$W_{net} = W\,(\,b_f / 3D) \qquad (1.6\text{-b})$$

The above equation is valid for horizontal roofs. If a roof is sloping, the following equation is valid, where θ is the angle of the roof slope with the horizontal:

$$W_{net} = W\,[\,(I_{xy}/I_x) \cos\theta - \sin\theta + b_f / 3D - I_{xy}/I_x\,] \qquad (1.7)$$

Needham checked the validity of his model against three different 20 ft simple span tests. He found that the accuracy of his model depended on the value chosen for the eccentricity of the vertical load.

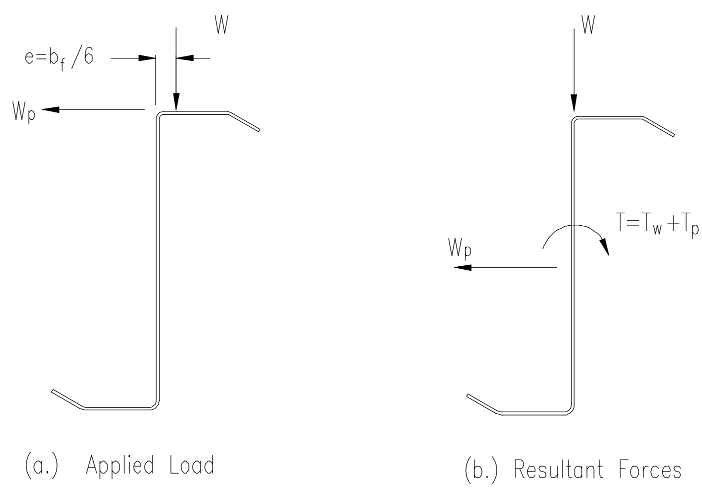


Figure 1.5 Needham's Mathematical Model
(Needham 1981)

Ghazanfari and Murray (1983-b) conducted a study that investigated the effects that roof panels and additional lateral braces have on a single Z-purlin. They developed an analytical model that predicted the magnitude of forces in torsional and intermediate braces for a single span, simply supported, gravity loaded Z-purlin. These results were then compared to full scale tests. Their mathematical model is shown in Figure 1.6. In it they assumed:

1.) No slip between purlin and panel at the location of the fastener.

2.) The line of action of the vertical uniform load W_v is located at one third of the flange from the plane of the web.

3.) The lateral force (W_h) is uniformly distributed.

4.) The load w_h acts at the connection of the web to the compression flange in a plane perpendicular to the web.

5.) Intermediate lateral braces are connected to a rigid eave.

6.) There is no elongation of braces.

The forces W_v and W_h cause the purlin to deflect laterally and vertically. Because these forces are not applied at the centroid of the section, they will produce a torque that increases the lateral movement of the top flange. It is important to note that this torque will be reduced by the deformation of the roof panel. Since the panel deformation can't be determined unless the lateral force W_h is known, and W_h can not be determined unless the total torque is known, an iterative procedure is required to solve the problem. Ghazanfari and Murray developed a computer program that determines the lateral force W_h including second order effects. The procedure is as follows:

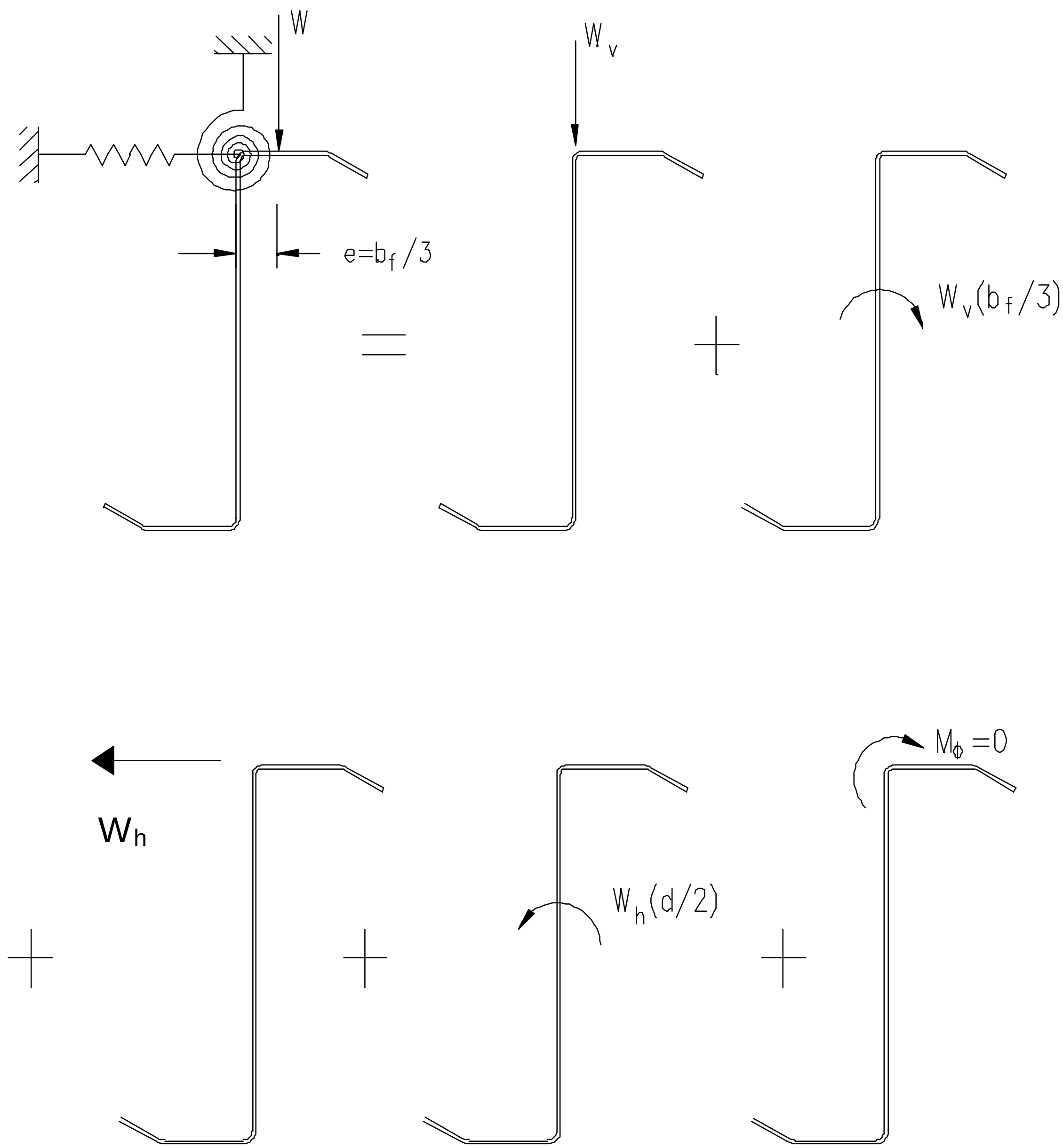


Figure 1.6 Ghazanfari's Mathematical Model

(Ghazanfari and Murray 1983-b)

Step 1 Assume zero second order effects.

Step 2 Calculate the lateral force W_h .

Step 3 Calculate the diaphragm deflection.

Step 4 Revise the values of the torque by introducing secondary effects due to the panel deformation.

Step 5 Recalculate W_h .

Step 6 Compare with previous cycle and repeat steps 3 through 5 until convergence is attained.

Full-scale testing was done to check the adequacy of the proposed analytical method (Ghazanfari and Murray, 1983-a). The test setup included a single, simple span purlin attached to both roof sheathing and additional intermediate lateral bracing. It was found that the third assumption—the horizontal force is uniformly distributed—is not always valid. However, the effect was small. Overall, Ghazanfari and Murray found that the analytical and experimental results were in good agreement. The total brace forces varied from 14% to 29% of the total applied gravity load. A parametric study showed that panel stiffness, span, assumed eccentricity of applied vertical load, and angle of principle axis had the greatest influence on the magnitude of brace forces.

Curtis and Murray (1983) studied what effect that increasing purlin lines has on lateral restraint forces. They tried to show that the method proposed by Ghazanfari and Murray (1983-b) for calculating brace force in a single purlin line could be extended to multiple purlin lines. Twenty gravity loaded tests were conducted with varying brace configurations with both through-fastened and standing seam roof panels. The number of purlin lines were varied from two through six. Curtis and Murray found that the ratio of the lateral forces to the total applied vertical load ranged from 26.6% for two purlin lines to 4% for six purlin lines. That is a decrease of more than 85% with an increase in number of purlin lines of four.

This conclusion was verified by tests conducted by Seshappa and Murray (1985). They used cold-formed, quarter- size Z-purlins to study lateral restraint forces in Z-purlin roof systems. Twenty eight quarter scale tests were conducted using single span and three continuous span configurations with two and six purlin lines and differing brace configurations. The effect of increasing purlin lines produces asymptotic behavior as shown in Figure 1.7. Therefore, it was shown that the method proposed by Ghazanfari and Murray (1983-b) if extended to multiple purlin line roof systems would be overly conservative.

Elhouar and Murray (1985) developed design equations that account for the effect that increasing number of purlin lines have on lateral restraint force for roof systems with through-fastened roof panels. They developed a mathematical stiffness model that represented the roof system. It consisted of the three main components of a roof system: purlin, roof panel, and lateral brace as shown in Figure 1.8. This model will be discussed in greater detail in Chapter II. The stiffness model was loaded according to the model proposed by Ghazanfari and Murray (1983-b) and solved using commercial stiffness analysis computer software. The results were checked against laboratory tests. Satisfied with the

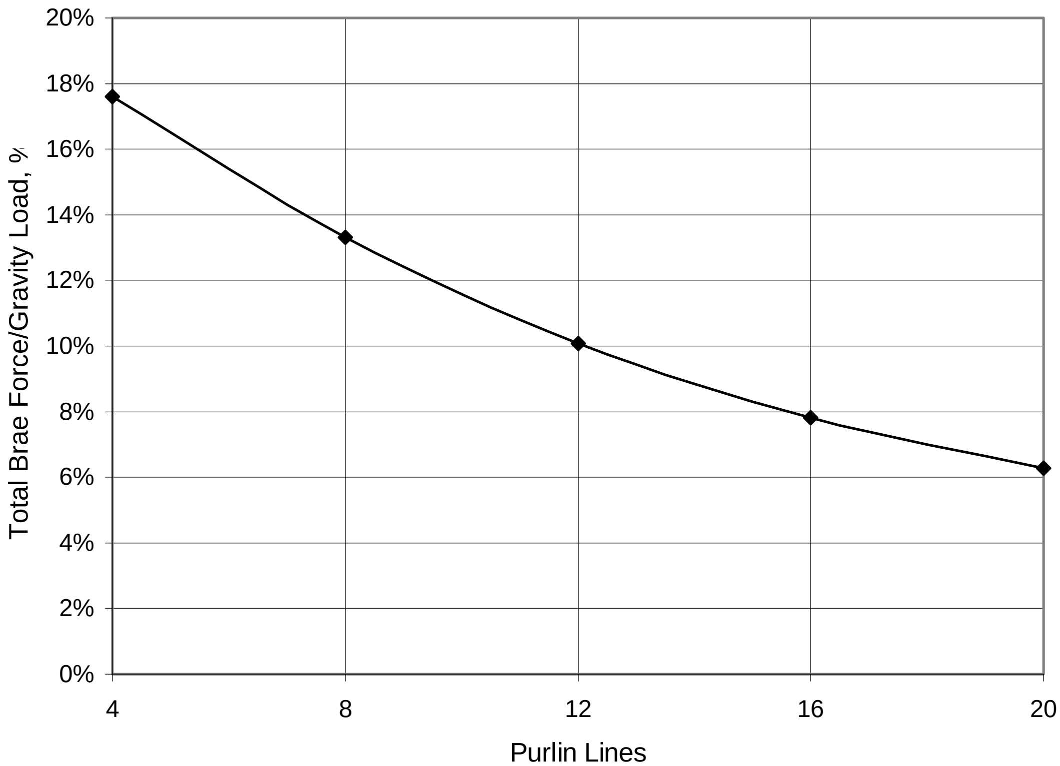


Figure 1.7 Effect of Increasing Purlin Lines on Brace Force
(Seshappa and Murray 1985)

performance of the model, Elhouar and Murray (1985) built hundreds of these models with varying cross-sectional geometry, span lengths, number of purlin lines, and bracing configurations. Furthermore, they considered both single span and multiple span roof systems. The brace force results from each model were recorded and a regression analysis was then performed on the data to develop a single equation that could be used to predict lateral restraint forces for three commonly used brace configurations. The resulting equations are:

(1) Single-Span System with Restraint at the Supports:

$$P_L = 0.5\left[\frac{0.220\, b^{1.50}}{n_p^{0.72}\, d^{0.90}\, t^{0.60}}\right] W \qquad (1.8)$$

(2) Single-Span System with Third-Point Restraints:

$$P_L = 0.5\left[\frac{0.474\, b^{1.22}}{n_p^{0.57}\, d^{0.89}\, t^{0.33}}\right] W \qquad (1.9)$$

(3) Single-Span System with Midspan Restraint:

$$P_L = \left[\frac{0.474\, b^{1.32}}{n_p^{0.65}\, d^{0.83}\, t^{0.50}}\right] W \qquad (1.10)$$

(4) Multiple-Span System with Restraints at the Supports:

$$P_L = C_{tr}\left[\frac{0.053\, b^{1.88} L^{0.13}}{n_p^{0.95}\, d^{1.07}\, t^{0.94}}\right] W \qquad (1.11)$$

with C_{tr} = 0.63 for braces at end supports of multiple-span systems

C_{tr} = 0.87 for brace at the first interior supports

C_{tr} = 0.81 for all other braces

(5) Multiple-Span System with Third-Point Restraints:

$$P_L = C_{th}\left[\frac{0.181\, b^{1.15} L^{0.25}}{n_p^{0.54}\, d^{1.11}\, t^{0.29}}\right] W \qquad (1.12)$$

with C_{th} = 0.57 for outer braces in exterior spans

C_{th} = 0.48 for all other braces

(6) Multiple-Span System with Midspan Restraints:

$$P_L = C_{ms}\left[\frac{0.116\, b^{1.32} L^{0.18}}{n_p^{0.54}\, d\, t^{0.50}}\right] W \qquad (1.13)$$

with C_{ms} = 1.05 for braces in exterior spans

C_{ms} = 0.90 for all other braces

where

P_L = Force in brace of interest,

b = Flange width,

d = Depth of section,

t = Thickness,

L = Span Length,

n_p = Number of parallel purlin lines, and

W = Total load supported by the purlin lines between adjacent supports.

For systems with less than four purlin lines, the brace force is determined by taking 1.1 times the force predicted by Equations 1.8 through 1.13, with n_p = 4. For systems with more than 20 purlin lines, the brace force shall be determined from Equations 1.8 through 1.13, with n_p = 20 and W based on the total number of purlins (Elhouar and Murray, 1985).

The above equations were adopted into the lateral restraint provisions for roof systems under gravity load with top flange connected to sheathing in the American Iron and Steel Institute's *Specification for the Design of Cold-Formed Steel Members* (AISI, 1996). They first appeared in the 1986 edition of the AISI ASD *Specification* with use restricted to through-fastened systems only.

Rivard and Murray (1986) conducted seven single span tests and six multiple span tests using standing seam roof sheathing. One of the goals of this investigation was to determine if Equations 1.8 through 1.13 could be used to predict lateral restraint forces for standing seam roof sheathing. “Good to excellent correlation was found between brace force predictions and experimental data using the Elhouar and Murray Stiffness Model… AISI Brace Force Prediction Equations (section D.3.2.1) developed by Elhouar and Murray (would be adequate for standing seam roof systems” (Rivard and Murray, 1986). As a result of this study, AISI extended the use of Equations 1.8 though 1.13 to standing seam roof systems in addition to through-fastened roof systems.

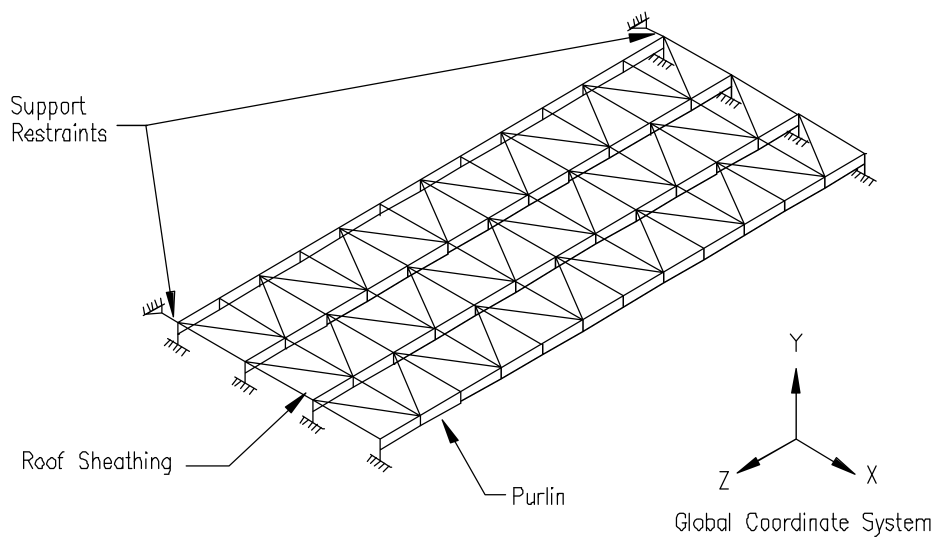


Figure 1.8 Elhouar's Stiffness Model
4 Purlin Line, Single Span, Support Restraints Shown
(Elhouar and Murray 1985)

1.4 SCOPE OF WORK

The purpose of this research is to develop design equations that predict lateral restraint forces in quarter point and third point plus support braced Z-purlin supported roof systems subject to gravity load. The stiffness model used by Elhouar and Murray (1985) is used here but modified slightly to better represent roof system behavior. The updated stiffness model is then used to estimate lateral restraint forces for a number of roof systems with varying cross sectional dimensions of the purlin, number of purlin lines, number of spans, and span length. A regression analysis is performed to obtain empirical design equations similar to those found in American Iron and Steel Institute's *Specification for the Design of Cold-Formed Steel Members,* Section D.3.2.1. (AISI, 1996). These design equations can be used by the practicing design engineer to estimate lateral restraint forces in Z-purlin roof system braces.

CHAPTER II

MATHEMATICAL MODEL

2.1 INTRODUCTION TO MATHEMATICAL MODELING

The first step in the solution of any structural analysis problem is the formulation of a mathematical model that adequately represents the real system. An acceptable mathematical model is one that satisfies equilibrium and compatibility with adherence to material properties. "These three requirements form the basis for all structural analysis, regardless of the level of complexity" (Barker and Puckett, 1997).

Once a mathematical model is developed, a method of solution must be chosen. This method of solution is called the numerical model. Examples of numerical models include direct integration, moment-area, slope-deflection, matrix stiffness analysis, and moment distribution. The selection of the numerical model depends on several factors including availability, ease of application, accuracy, computational efficiency, and structural response required (Barker and Puckett, 1997).

2.2 CHAPTER OBJECTIVE

The objective of this chapter is three fold. First, the stiffness model developed by Elhouar and Murray (1985) will be presented. Second, the model will be modified slightly to better represent true behavior. The modified mathematical model, called the Modified Elhouar and Murray Stiffness Model, is presented. Finally, the accuracy of the modified model is examined versus previously conducted laboratory tests.

2.3 PREVIOUS MATHEMATICAL MODEL AND MODIFICATIONS

2.3.1 Selection of Model

Elhouar and Murray (1985) developed a mathematical model for predicting lateral restraint forces in multiple purlin line, Z-purlin supported roof systems. The model is a combination of a space frame and a space truss. It consists of the three main components of a roof system—purlin, panel, and brace—developed separately then assembled to create the roof system model.

Elhouar and Murray (1985) selected the combination space frame and space truss model because of its computational efficiency and modeling simplicity. A finite element model was considered as an alternative method for modeling the roof system but was rejected. Such a model would have required inordinate computer storage space and computational time, especially for the large twenty purlin line, three span roof systems. Also, the advantage of finite element analysis—finding internal stresses at many locations in a particular member—was not necessary. Elhouar and Murray were concerned not with finding stresses at any point in a roof system but rather with predicting forces in attached braces. Therefore a finite element analysis would have proven excessive and inappropriate. Instead, a combination space frame and space truss stiffness model that could be solved quickly using readily available computer software was clearly a better choice.

2.3.2 Applying Load to the Model

As discussed in Chapter I, gravity load on a purlin supported roof system causes torque in purlins. This occurs because the gravity load does not occur in the line of action of the purlin web but instead is distributed in some unknown fashion across the top flange of the purlin. As shown in Figure 2.1, Elhouar and Murray (1985) assumed the force distribution to be triangular, the resultant of which acts with an eccentricity of one third the flange width. So, in addition to gravity load acting at the purlin web there is also a torque T acting at the purlin centroid:

$$T = W\,(b_f / 3) \qquad (2.1)$$

where W is the gravity load per unit length and b_f is the purlin top flange width. The gravity load and torque are uniformly distributed along the length of the purlin. This force distribution is consistent with the model assumed by Ghazanfari and Murray (1983-b) in previous research. It was proven acceptable by extensive laboratory testing as discussed in Chapter I.

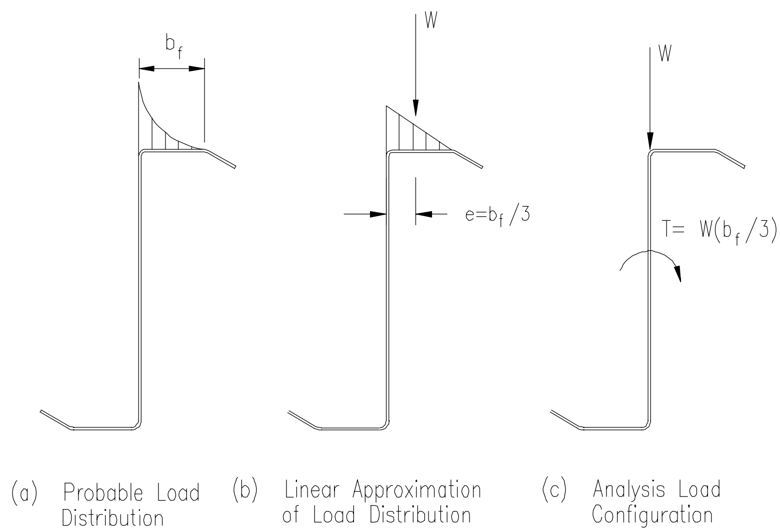


Figure 2.1 Applied Purlin Load

2.3.3 Axes Orientation

To understand the properties discussed below, the reader must be familiar with the axes orientation utilized herein. Figure 2.2 shows the orientation of the global and local axes used for the model.

2.3.4 Modeling of the Purlin

Elhouar and Murray (1985) modeled the purlin using a combination of three space frame line elements: member types A, B, and C, as shown in Figure 2.3. The purlin is divided into twelve equal length segments. This was done so that when braces are attached, all three possible bracing configurations (support, mid-span and third-point) will meet at a purlin joint (1985).

Member type A is a beam element representing the purlin. It has properties equal to those of the purlin itself with one notable exception. The torsional constant, J, is set equal to 10 in^4 for all purlin cross-sections. This value is large compared to the true purlin J values that range from about 0.016 in^4 to as low as 0.0005 in^4. Without this high torsional resistance, the torque applied to member type A would cause it to rotate as shown in Figure 2.4-(a). This extreme deformation clearly is not how a purlin behaves. Using J equal to 10 in^4 stops this rotation and better approximates true purlin behavior as shown in Figure 2.4-(b).

The remaining two members of the purlin are member types B and C. Both are beam elements lying in the global Y direction. Member type B provides compatibility between the purlin and the roof panel and member type C between the purlin and the supports. Each of these vertical lines can be thought of as representing a rectangular area that extends horizontally half the distance to next vertical line and vertically half the depth of the purlin. These areas are shown by dashed lines in Figure 2.3. The beam elements have in-plane properties consistent with dimensions of the dashed area. Out of plane

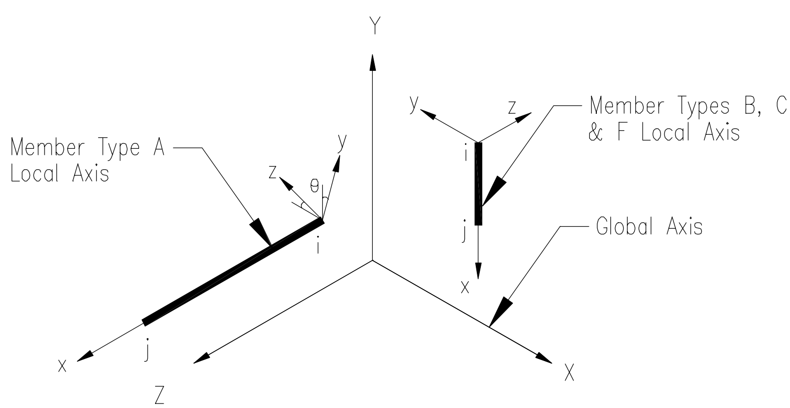


Figure 2.2 Local and Global Axes

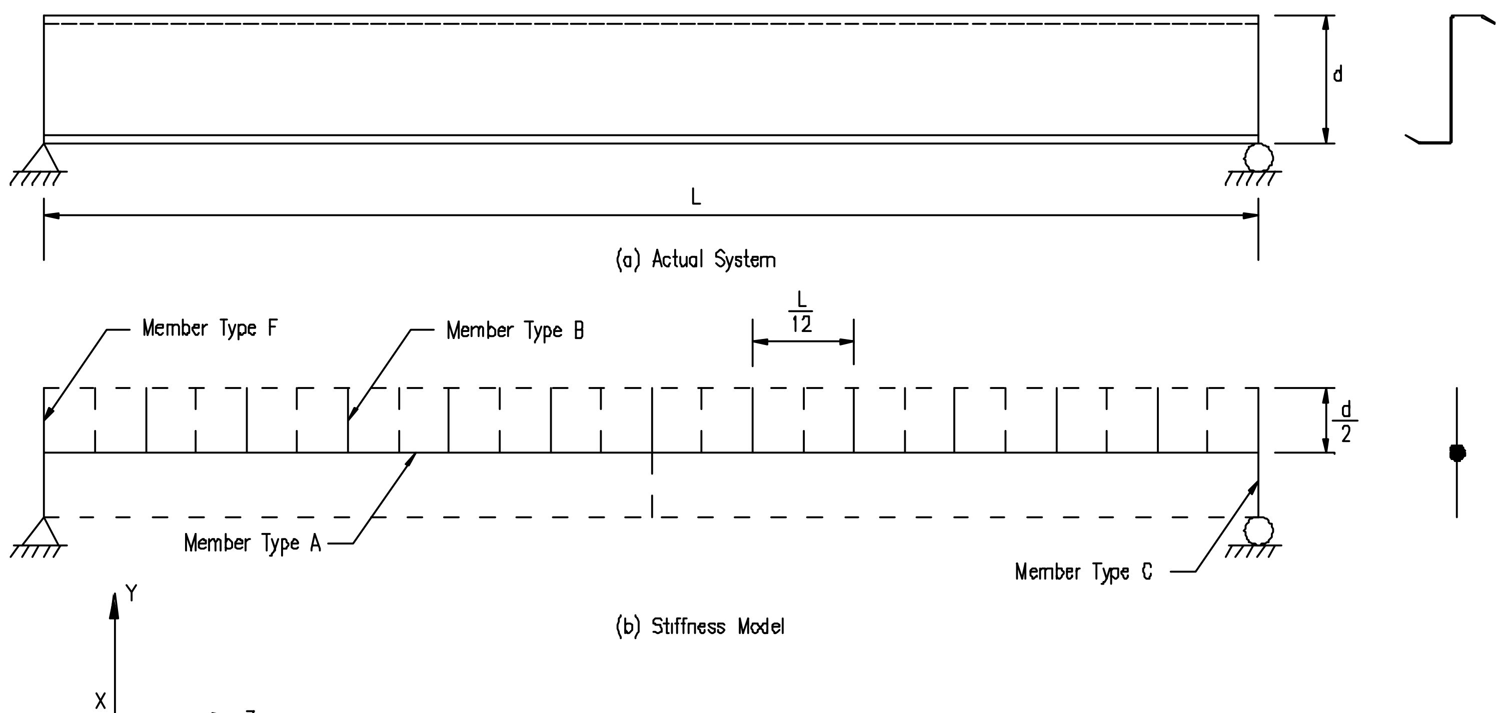


Figure 2.3 Purlin Stiffness Model
(Elhouar and Murray 1985)

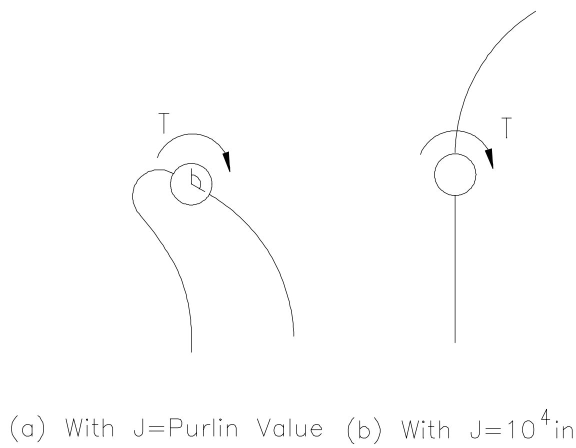


Figure 2.4 Effect of Torsional Constant on Member Type A

properties are consistent with the properties of a portion of the purlin itself. For member type B:

$$A = L / 12 * t \tag{2.2}$$

$$I_z = (L/12) / 12 * t^3 \tag{2.3}$$

$$J = I_y \text{ of the purlin} \tag{2.4}$$

$$I_y = J \text{ of the purlin} \tag{2.5}$$

and for member type C:

$$A = L / 2 * t \tag{2.6}$$

$$I_z = (L/2) / 12 * t^3 \tag{2.7}$$

$$J = I_y \text{ of the purlin} \tag{2.8}$$

$$I_y = J \text{ of the purlin} \tag{2.9}$$

where A equals area, L equals the span of the purlin, and t is equal to the thickness of the purlin, all in inches. These properties are in terms of their local axis.

Elhouar and Murray (1985) found that the model produced using the above properties simulates true purlin behavior fairly well with one exception. Member type C undergoes large amounts of bending as shown in Figure 2.5-(b). This problem is similar to the one described above for member type A. Torque in the purlin cross-section causes bending of member type C inconsistent with true behavior. To correct this problem, Elhouar and Murray modified the expression for I_z to:

$$I_z = (L/2) * t^3 \qquad (2.7\text{-a})$$

Elhouar and Murray were satisfied with this modification, the results of which are shown schematically in Figure 2.5-(c).

2.3.5 Modification to the Previous Purlin Model

The problem of unrealistically large bending in member type C was revisited as part of this research. While Equation 2.7-(a) better represents true purlin behavior, it still allows for large deformation of member type C. This deformation actually reduces the axial force in attached bracing because it allows displacement toward the axial brace. Purlins do not deflect in this manner. Instead, the lower portion of a purlin stays largely undeformed while in the elastic load range. Therefore the moment of inertia about the local z axis, I_z, is modified again to:

$$I_z = 10 \text{ in}^4 \qquad (2.7\text{-b})$$

This modification produced behavior as shown in Figure 2.4-(d) and is consistent with known purlin behavior.

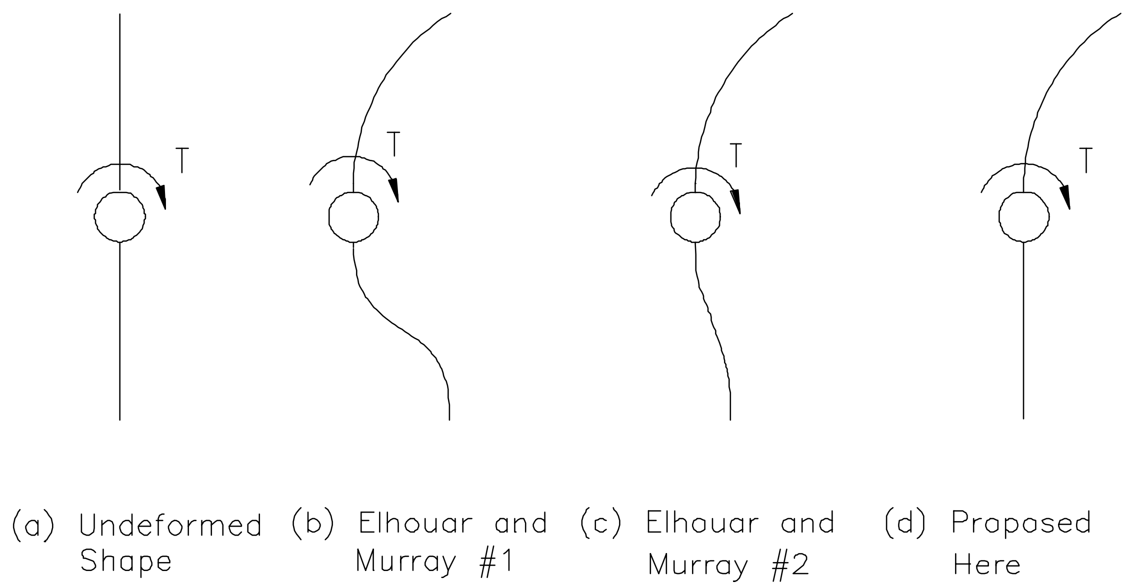


Figure 2.5 Effect of Torsional Constant on Member Type C

The modifications made above result in large torsional stiffness of member type A and large bending stiffness of member type C. Reducing the bending stiffness of member type B was considered. Doing this would cause higher brace forces. This idea was rejected because comparison with previous laboratory tests show the model in its current form predicts brace forces well. Previous laboratory testing and the performance of the model will be discussed later in this chapter.

The final modification to the Elhouar and Murray Stiffness Model requires the introduction of a new member type, F. Close examination of Figure 2.3 reveals that the B type members located at the purlin ends represent tributary widths half that of the other B members. This was an oversight in the Elhouar and Murray Stiffness Model and is corrected by introducing member type F. The moment of inertia about the z-axis for member type F is:

$$I_z = (L/12) / 24 * t^3 \quad (2.10)$$

All other properties for member type F are the same as those of member type B.

2.3.6 Modeling of the Roof Panel

Through-fastened and standing seam roof sheathing are attached to the top flange of Z-purlins using self-tapping screws in the former case or clips in the later case. Both types of connections offer little in the way of rotational restraint to the purlins and consequently can be modeled as simple connections. Therefore, the panel bending stiffness can be disregarded and only the panel shear stiffness need be considered. Elhouar and Murray (1985) represented the roof panel with a plane truss as shown in Figure 2.6. They used the panel shear stiffness to find the cross-sectional area of the plane truss members in the following manner: For a known shear stiffness G', the deflection of a shear panel in the direction of the load P in Figure 2.6-(a) can be determined from:

$$\Delta = (P\ L)/\ (4\ G'\ a) \quad (2.11)$$

where L and a are the dimensions of the panel. By applying the same load P to the truss as shown in Figure 2.6-(b), one can determine the truss member area, A, such that the displacement of the truss equals that calculated from Equation 2.8. Consequently, the truss stiffness will equal that of the roof panel (Elhouar and Murray, 1985).

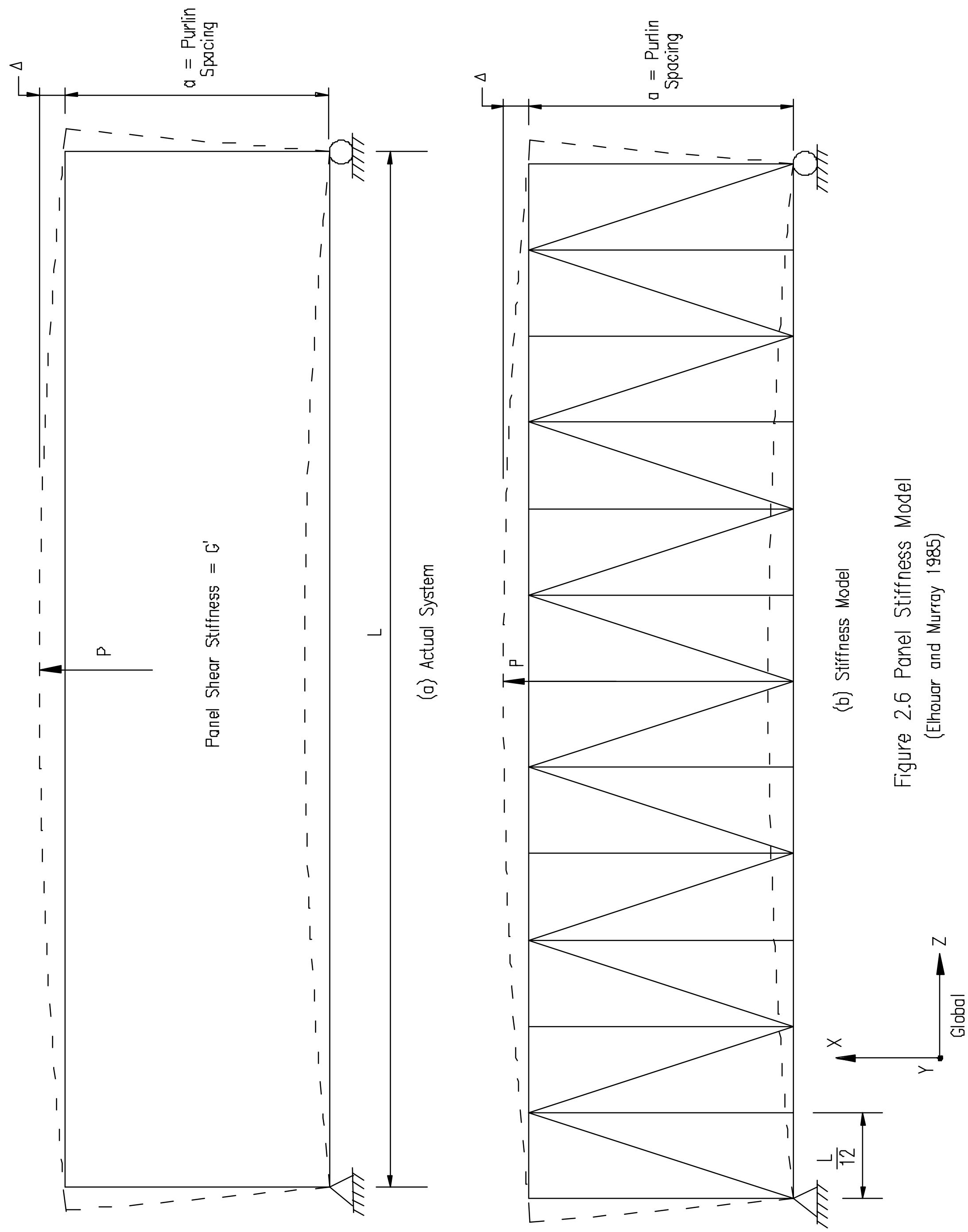


Figure 2.6 Panel Stiffness Model
(Elhouar and Murray 1985)

Elhouar and Murray (1985) chose a constant panel shear stiffness of 2500 lb./in for their mathematical model and using the above equation found the truss member (member type E) area to be equal to 0.03106 in^2. They chose to use a constant shear stiffness for all mathematical models based on a study conducted by Ghazanfari and Murray (1983-a). Ghazanfari and Murray found that lateral brace force increases linearly as panel shear stiffness reaches 1500 lb/in, but remains nearly constant as panel shear stiffness increases from 1500 lb/in. This relationship is shown in Figure 2.7 and means that all panels with stiffness greater than 1500 lb./in produce essentially the same brace force. Therefore, one value for shear stiffness can represent all panels with shear stiffness greater than 1500 lb/in. The selection of 2500 lb/in for the constant value is based on a study by Curtis and Murray (1983). They found that manufactured roof panel shear stiffness vary from about 1000 lb/in to 3000 lb/in with the majority above 2000 lb/in. Therefore, Elhouar and Murray chose a constant panel shear stiffness of 2500 lb/in as a mean value (Elhouar and Murray, 1985).

2.3.7 Modifications to the Roof Panel Model

The relationship between truss member area and roof panel shear stiffness in an effort to verify the use of 0.03106 in^2 is revisited here. The simple expression for stiffness, $K = P/\Delta$, was utilized with the same loading configuration shown in Figure 2.6. Using an area of 0.03106 in^2 with P set to

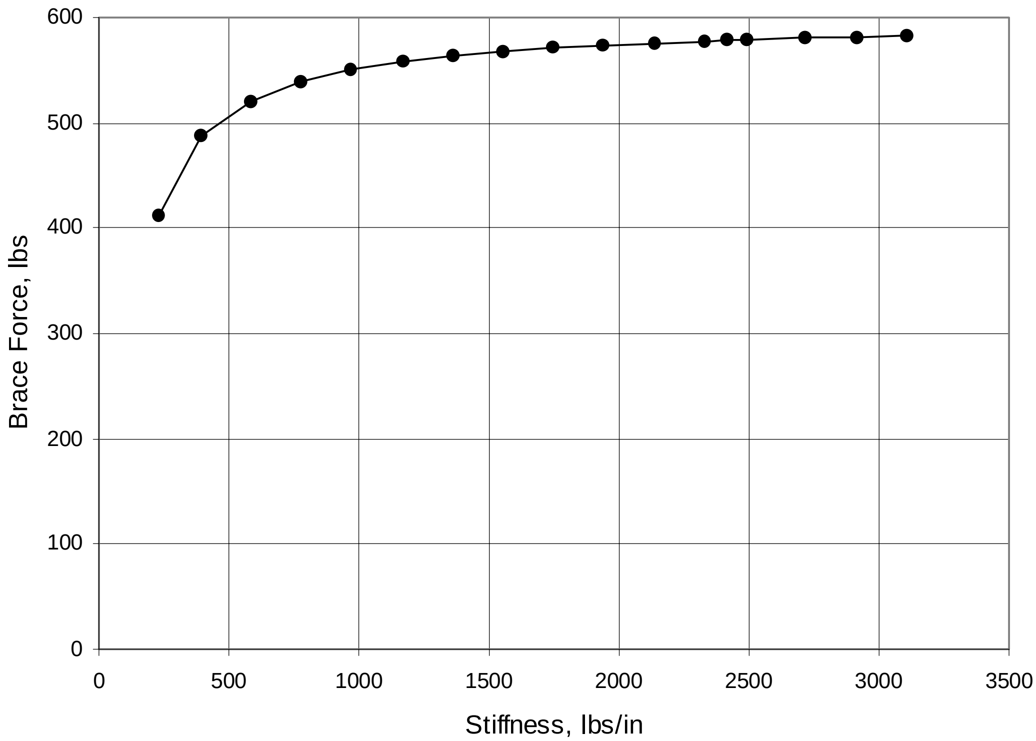


Figure 2.7 Deck Stiffness vs. Brace Force
(Ghazanfari and Murray, 1983)

1 Kip yields a deflection of 0.414 inches thereby producing a stiffness of 2415.5 lb/in. This value is close to the target panel shear stiffness of 2500 lb/in (within 96.6%), but better accuracy can be attained. Using an area of 0.0321 in^2 produces a stiffness of 2493.8 lb/in, which is within 99.75% of the target value of 2500 lb/in. This small discrepancy occurs because Elhouar and Murray (1985) considered a different span length than that used here when finding the truss The area of member type E is revised from 0.03106 in^2 used in the Elhouar and Murray Stiffness Model to 0.0321 in^2 for the Modified Elhouar and Murray

Stiffness Model. Ghazanfari and Murray (1983-a) have shown that this difference will not have much effect on brace force, but the change is made nonetheless.

2.3.8 Modeling of the Braces

External bracing members are quite simple to model. Because they carry only the axial load induced by restraining the lateral movement of the purlins, they can be represented by pin-ended truss elements lying in the global X-Z plane. These members can be seen in Figure 1.8 and are called member type E. The cross-sectional area and lengths of these members are the same as the braces they represent. For the sake of simplicity, an area equal to 0.333 in^2 is used and a length of 8 inches for all bracing configurations is used. The selection of these values are based on Elhouar and Murray's (1985) research.

2.3.9 Summary of Modified Elhouar and Murray Stiffness Model

The modified Elhouar and Murray Stiffness Model consists of six different member types. Member type A, B, C, and F represent the purlin with properties summarized in Table 2.1. Type D members make up the plane truss that represents the roof sheathing. These members carry only axial load and have a cross-sectional area of 0.0321 in^2. Member type E represent the lateral restraint braces, carry only axial load, and have a cross-sectional area set to 0.333 in^2. The connection of the purlin to rafter allows for rotation about the global X and Y axis, but fixes the remaining rotations and translations. The connection of brace to eave is pinned in all directions.

Table 2.1 Modified Elhouar and Murray Stiffness Model Member Properties

Member Type	Area in^2	I_y in^4	I_z in^4	J (I_x) in^4
A	purlin area	I_y of purlin	I_z of purlin	10
B	L / 12 * t	J of purlin	$(L/12) / 12 * t^3$	I_y of purlin
C	L / 2 * t	J of purlin	10	I_y of purlin
D	0.0321	0	0	0
E	0.333	0	0	0
F	L / 12 * t	J of purlin	$(L/12) / 24 * t^3$	I_y of purlin

2.4 METHOD OF SOLUTION

The Modified Elhouar and Murray Stiffness Model was solved using a computer software package called RISA-3D, Rapid Interactive Structural Analysis - 3 Dimensional (RISA-3D, 1998). This software utilizes the matrix stiffness method. Computation time using a Pentium 100 MHz personal computer ranged from about 10 seconds for small systems to as long as 1 hour for 20 purlin lines, 3 span systems. Sample input data and results are given in Appendix A.

2.5 VERIFICATION OF THE MODIFIED MODEL VERSUS PREVIOUSLY CONDUCTED LABORATORY TESTS

Elhouar and Murray (1985) tested their model against a total of eighteen laboratory tests. Five of these were full scale tests conducted by Curtis and Murray (1983) and thirteen were quarter scale tests conducted by Seshappa and

Murray (1985). Elhouar and Murray found good agreement between the stiffness model they developed and the laboratory tests. They concluded that the stiffness model they developed adequately represented Z-purlin supported roof systems for the purpose of finding brace forces. The reader is referred to Elhouar and Murray (1985) for a more detailed discussion regarding this matter.

Since the Elhouar and Murray Stiffness Model predicts brace forces fairly well and the modifications made to it herein are small, new laboratory testing to confirm the accuracy of the Modified Elhouar and Murray Stiffness Model was not deemed necessary. Instead, the results using the new model are compared to a select number of the same laboratory tests used originally by Elhouar and Murray (1985). It is found that in general, the Modified Elhouar and Murray Stiffness Model does indeed predict brace forces more accurately.

Five tests were selected for in depth comparison: one full scale test conducted by Curtis and Murray (1983) and four quarter scale tests by Seshappa and Murray (1985). All tests were single span. Other pertinent test data is summarized in Table 2.2 below. For additional information, see Elhouar and Murray (1985).

Table 2.2 Select Laboratory Test Data

Test Name	Scale	Bracing Config.	Depth in.	Purlin Lines	Span ft
B/2-1-A	Full	Support	8	2	22.3
C/2-1	Quarter	Support	2	2	5
C/6-1	Quarter	Support	2	6	5
C/2-15	Quarter	Third Pt.	2	2	5
C/6-2	Quarter	Third Pt.	2	6	5

Table 2.3 compares the results of previous laboratory testing to the previous model and the modified model. Comparisons are made in terms of the brace force ratio, β. It is defined as total brace force of the system, B_T, divided by gravity load in one span, W. Stated algebraically:

$$\beta = B_T / W \tag{2.12}$$

In all cases, the modified model yields a larger brace force than does the previous model. For tests B/2-1-A, C/2-1, and, C/6-1 the modified model is more accurate than the previous model, producing brace force ratios closer to the observed laboratory test data. However, for tests C/2-15 and C/6-2, the previous model is more accurate than the modified model.

In general, the Modified Elhouar and Murray Stiffness Model produces a larger brace force than the previous model. An examination of the eighteen laboratory tests mentioned above reveals that the previous model underestimated brace forces in eleven of these by about 1-5%. Therefore, one can conclude that in most cases the Modified Elhouar and Murray Stiffness Model will more accurately predict brace forces roof systems than the previous model. Furthermore, if the model does err, it errs consistently on the conservative side. For these reasons the it is concluded that the Modified Elhouar and Murray Stiffness Model more closely approximates true roof system behavior than does the previous model.

Table 2.3 Comparison of Brace Force Ratio Results

Test Name	Laboratory Test	Elhouar and Murray Stiffness Model (1985)	Modified Model
B/2-1-A	0.22	0.21	0.213
C/2-1	0.26	0.23	0.288
C/6-1	0.19	0.17	0.18
C/2-15	0.14	0.22	0.265
C/6-2	0.13	0.17	0.215

CHAPTER III

DEVELOPMENT OF DESIGN EQUATIONS

3.1 INTRODUCTION

The mathematical model developed in Chapter II has been shown to accurately predict brace forces in Z-purlin supported roof systems. It is the objective of this chapter to use the model to develop design equations that predict brace forces in two commonly used bracing configurations: quarter-point and third-point plus support, as shown in Figure 1.1-(a). This can be done by analyzing many different roof systems until sufficient data is collected, then performing a regression analysis to yield a single expression in terms of the varying parameters. Some of the important parameters include cross sectional dimensions of the purlin, number of purlin lines, number of spans, and span length. These empirical equations are similar in form to those developed by Elhouar and Murray (1985) for three other bracing configurations: restraint at the supports, at midspan, and at the third points. These equations are shown in Chapter I and can be found in American Iron and Steel Institute's *Specification for the Design of Cold-Formed Steel Members*, Section D.3.2.1 (AISI, 1996).

3.2 REGRESSION ANALYSIS AND CAUSALITY

Kleinbaum and Kupper (1978) define a regression analysis as a "statistical tool for evaluating the relationship of one or more independent variables , x_1, x_2,… x_k, to a single continuous dependent variable Y." One of the most common uses of a regression analysis is the development of a "quantitative formula or equation to describe the dependent variable as a function of the independent variables."

Kleinbaum and Kupper (1978) warn "It is important to be cautious about the results obtained from a regression analysis. A strong relationship found between variables does not necessarily prove or even imply that the independent variables are causes of the dependent variables. In order to make such an inference addition analysis is required."

Therefore, before performing a regression analysis, an investigation to determine causality needs to be conducted. One needs to identify the important parameters and how they effect brace force. This can be done by analyzing roof systems and examining the results in an effort to identify trends. Such an analysis will be referred to henceforth as a "system behavior analysis." Response of the system to the varied parameter can then be grouped into three categories. These are:

1. There is a direct and identifiable relationship between the varied parameter and the result; in this case brace force. When such a relationship occurs one can conclude that the parameter causes the result. As such, the parameter

can be considered an independent variable and a regression equation can be written in terms of it to predict the dependent variable.

2. The varied parameter has little or no effect on brace force, and its effect can be neglected. Here one can conclude a regression analysis need not consider the effects of this parameter.

3. There is no observable relationship between the varied parameter and brace force. In this case one concludes that causality does not exist. While a regression equation could possibly be written in terms of the parameter, it would be erroneous and unrelated to the actual behavior of the system since it is based on statistical considerations only (Elhouar and Murray, 1985).

Elhouar and Murray (1985) performed a system behavior analysis, the results of which are summarized in the next section. In all cases, the parameter of interest is compared to the brace force ratio, β. It is defined as total brace force of the system, B_T, divided by gravity load in <u>one span</u>, W. This term may also be referred to as percent brace force, in which case it is multiplied by 100%. Stated algebraically:

$$\beta = B_T / W \tag{3.1}$$

3.3 PREVIOUS SYSTEM BEHAVIOR ANALYSIS

Elhouar and Murray (1985) examined the effects of the following parameters on percent brace forces: number of purlin lines; purlin span length; cross-sectional dimensions of the purlins including depth, thickness, and flange width; number of spans; and bracing configuration. All parameters fall into one of the three categories mentioned above.

3.3.1 Bracing Configuration

It was found that “Lateral restraint forces can not be mathematically related to the bracing configuration used and therefore each configuration must be consider separately” (Elhouar and Murray, 1985). This parameter falls into category three, and means that a separate regression equation must be written for every bracing configuration considered.

3.3.2 Number of Spans

In addition to single span systems, Z-purlins are often designed as continuous beams over many spans. They can be easily lapped to create a moment connection. Elhouar and Murray (1985) found that percent brace force decreases by 12% to 30% as the number of spans increases from one to three, then decreases only slightly as the number of spans continues to increase to infinity. This means that percent brace force of the three span roof system can be conservatively used to approximate percent brace force for continuous systems having more than three spans. So, in addition to a regression equation for single span systems, Elhouar and Murray developed a regression equation for multiple span systems based on a three span model. Consequently, every bracing configuration considered needs two separate regression equations: one for single span systems, and one for multiple span systems. Since Elhouar and Murray considered three bracing configurations, they developed six regression equations. Since two bracing configurations are being considering here, four equations must be developed.

3.3.3 Number of Purlin Lines

Research by Ghazanfari and Murray (1983-b) showed that percent brace force decreases as the number of purlin lines increases. Elhouar and Murray confirmed (1985) this relationship for all bracing configurations and for both single and multiple span systems. They found the reduction can be as high as 70%. Therefore, a direct and identifiable relationship of the type described in category one exists. As such, number of purlin lines can be considered an independent variable in a regression equation to predict percent brace force.

3.3.4 Purlin Span Length

Elhouar and Murray (1985) found that purlin span length did not have much of an effect on percent brace force for single span systems. This response is of type two explained above, consequently a regression analysis need not consider purlin span length for single span systems. However, a relationship was found between length and percent brace force for multiple span systems. As length increases, percent brace force increases. This is a category one response. In summary, length can be considered an independent variable in a regression equation for multiple spans, but need not be considered for single spans.

3.3.5 Purlin Cross-Sectional Properties

Elhouar and Murray (1985) found the following purlin cross section properties had a notable effect on percent brace force:

1. Increasing purlin depth results in decreasing lateral restraint forces.
2. Increasing purlin flange width increases lateral restraint forces.
3. As purlin thickness increases, percent brace force decreases slightly.

Each of these parameters are category one responses. As such, purlin depth, purlin flange width, and purlin thickness can be considered independent variables in a regression equation to predict percent brace force. Other purlin properties were investigated by Elhouar and Murray but were found to have a no significant effect on percent brace force; a category two response.

3.3.6 Summary of Previous System Behavior Results

Elhouar and Murray (1985) found that two separate regression analyses are needed for each bracing configuration considered: one for single span systems and one for multiple span systems. For single span systems, percent brace force is a function of number of purlin lines n, purlin depth D, purlin flange width b_f, and purlin thickness t:

$$\beta = f\,(n, D, b_f, t) \tag{3.2}$$

For multiple span systems, purlin span length L is also included:

$$\beta = f\,(n, L, D, b_f, t) \tag{3.3}$$

3.4 VERIFYING AND MODIFYING PREVIOUS SYSTEM BEHAVIOR ANALYSIS

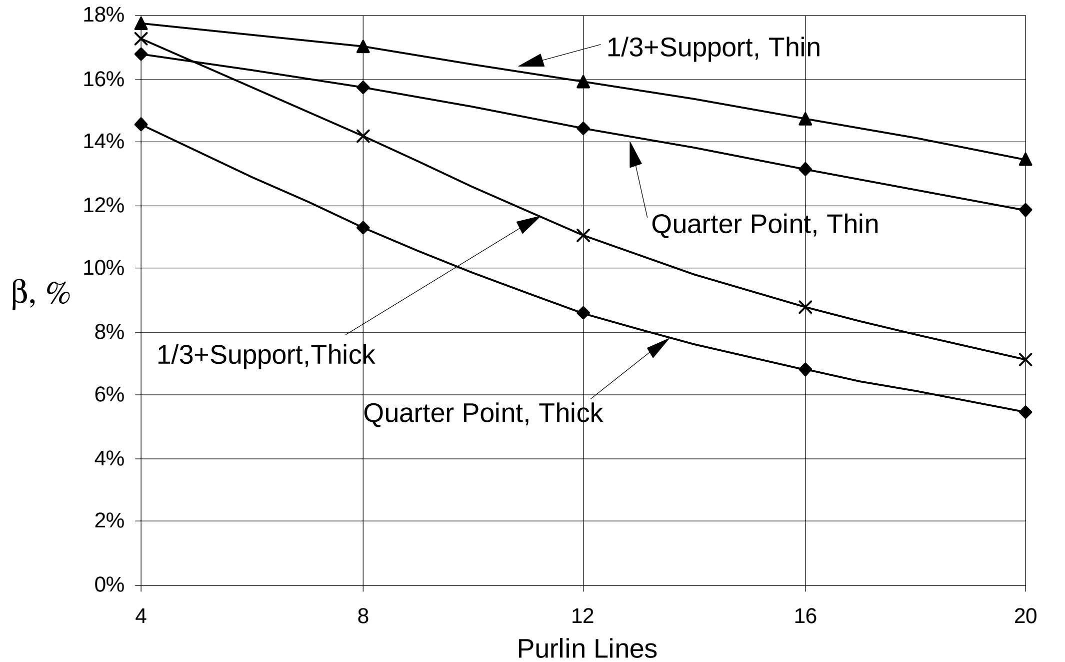
Each of the parameters examined by Elhouar and Murray (1985) are re-examined here. Since the bracing configurations considered herein differ from those considered by Elhouar and Murray, it is possible that different relationships exist between the varied parameters and percent brace force.

3.4.1 Number of Purlin Lines

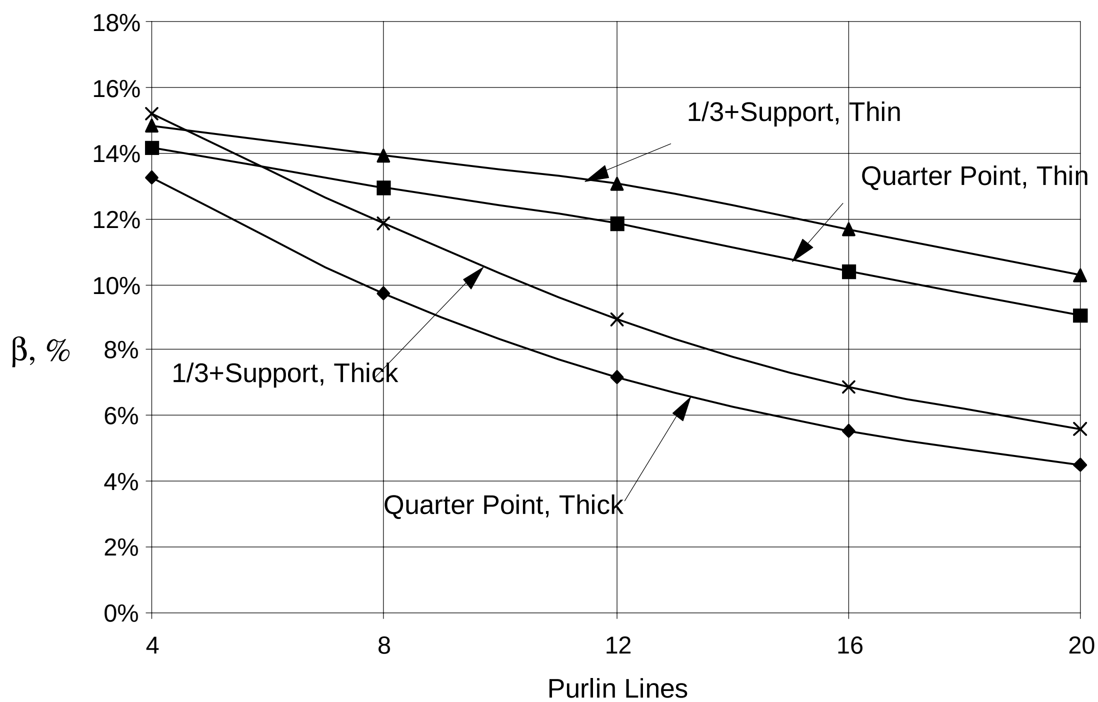
Typical graphs of percent brace force versus number of purlin lines is shown in Figure 3.1. Typical single span systems are shown in 3.1-(a) and typical multiple spans in 3.1-(b). It can be seen that similar response is obtained regardless of the number of spans. Both graphs show significant reduction in percent brace force as number of purlin lines increase; behavior that agrees with prior research by both Elhouar and Murray (1985) and Ghazanfari and Murray (1983-b). Therefore, it is concluded that number of purlin lines produce category one response.

3.4.2 Purlin Span Length

Typical purlin span length effects are shown in Figures 3.2-(a) and (b) for single span systems and multiple span systems, respectively. The graphs show percent brace force versus number of purlin lines where purlin span length is varied. If the curves were to lie directly on top of each other, one can conclude effects of the parameter are negligible; category two response. Figure 3.2-(a) shows the curves for 20 ft and 25 ft span lengths to be close to one another and getting closer as number of purlin lines increases for a single span. For multiple span systems, the plotted curves in Figure 3.2-(b) are further away from one another and remain parallel. Such behavior is identical to that observed by Elhouar and Murray (1985). Therefore, it is concluded that purlin span length need not be included in single span systems but should be included for multiple span systems. However, the former will later be found incorrect. Including length

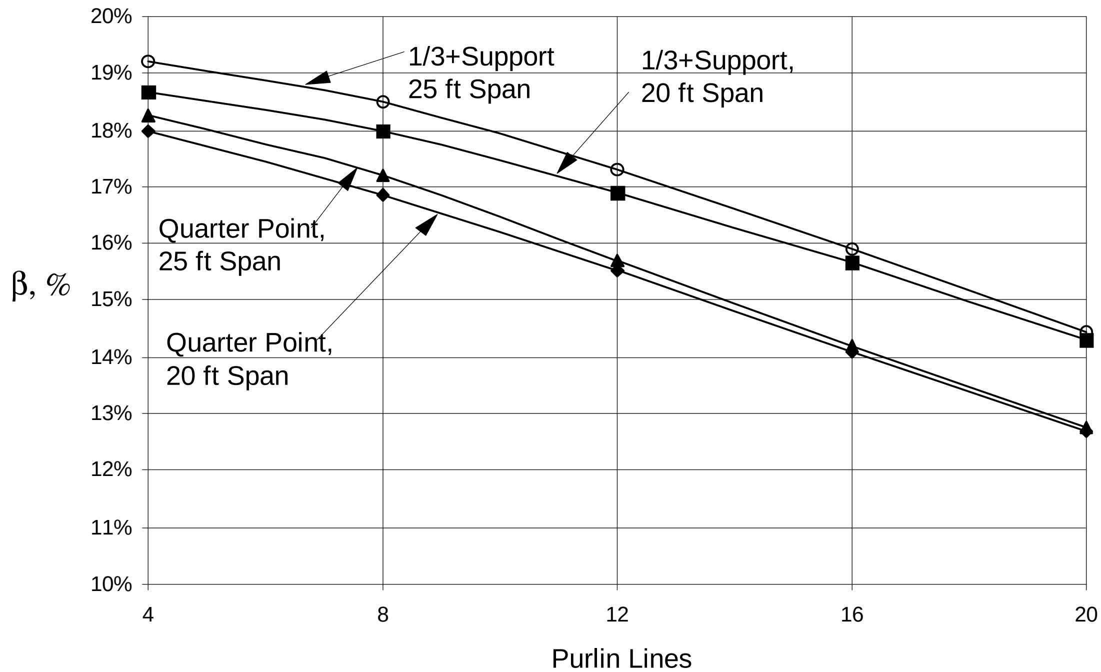


(a) Single Span

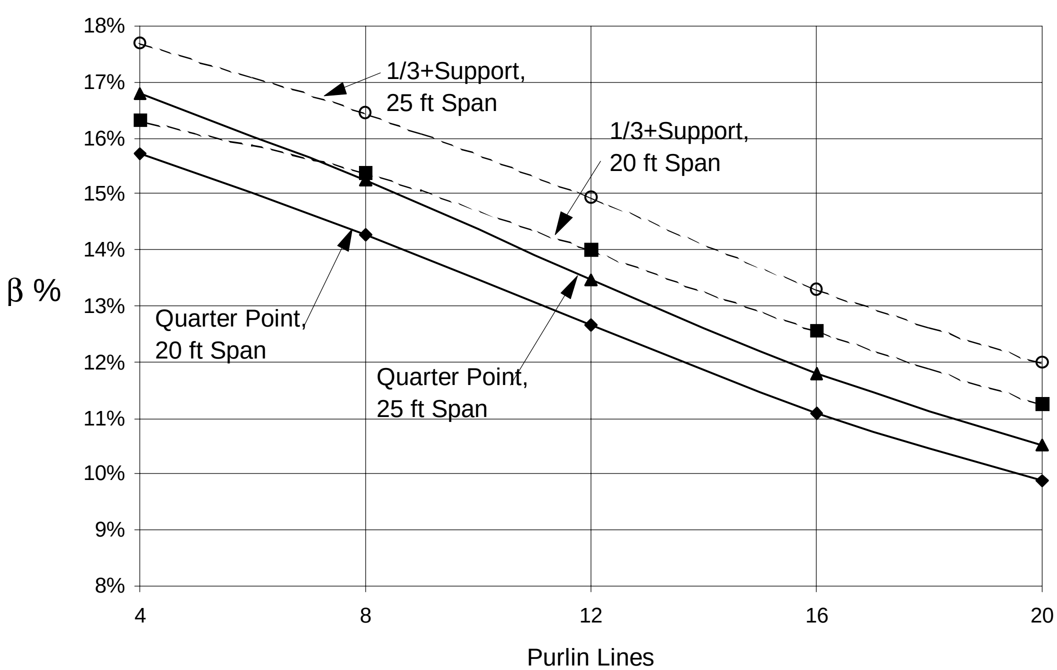


(b) Multiple Span

Figure 3.1 Percent Brace Force vs. Number of Purlin



(a) Single Span



(b) Multiple Span

Figure 3.2 Effect of Span Length on Percent Brace Force in a regression equation for single span systems is necessary.

3.4.3 Purlin Cross-Sectional Properties

Purlin cross-sectional properties were re-examined here. Purlin depth and purlin flange width were found to be in good agreement with Elhouar and Murray's (1985) previous findings. However, purlin thickness was found to contribute greatly to percent brace force. This can be seen in Figure 3.1. Purlins that are relatively thick display a smaller brace force than do the thinnest purlins. The difference is found to be as much as 20%. This behavior is quite different than that observed by Elhouar and Murray.

3.5 DEVELOPMENT OF TEST MATRIX

Once the important parameters are identified, a test matrix that varies each parameter must be developed. The Modified Elhouar and Murray Stiffness model is used to solve each test in the series. The test matrix developed herein considers two bracing configurations: quarter-point restraint and third-point plus support restraint. As discussed earlier, separately analyses are required for single span systems and multiple span systems.

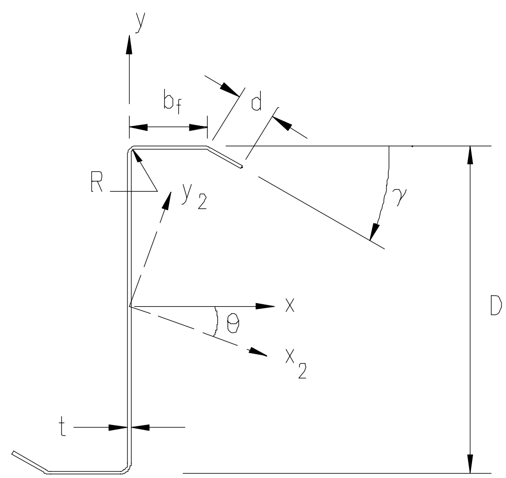
It is impossible to include every possible combination of every parameter in the test matrix. Such a matrix would simply be too large to analyze. However, there are many techniques that can be employed to substantially reduce the size of the test matrix to a more manageable one. For example, the number of purlin lines has been limited to between four and twenty. Clearly, most roof systems fall between these limits. For those that do not, approximations will be offered.

Another way to reduce the size of the test matrix is to simply skip some values. Instead of analyzing all 17 models from 4 to 20 purlin lines, only every fourth will be analyzed (4, 8, 12, 16 and 20 lines) thus reducing the number of tests to five. This technique assumes that the three skipped values lie on a straight line between the computed values.

In addition to number of purlin lines, cross-sectional dimensions of the purlin must also be varied in the test matrix. It has been shown that purlin depth, flange width, and thickness all contribute to percent brace force and therefore need to be represented in the test matrix. The Z-purlins considered are shown in Table 3.1, and are selected from Table I-3 of The American Iron and Steel Institute's *Cold Formed Steel Design Manual* (AISI, 1996). Purlin depth has been limited to 8, 10, and 12 inches because these are most commonly used for roof systems. For each depth examined, two thicknesses were considered. One is the thickest value shown in AISI Table I-3 (AISI, 1996) and the other the thinnest. By considering only the maximum and minimum values, a response envelope is generated. It is assumed that all other thicknesses will yield results that fall within the envelope. Finally, it should be noted that the test matrix does not include the case of varied purlin flange width independent of depth. This is because every purlins shown in AISI Table I-3 (AISI, 1996) has a constant flange width for a given depth.

Table 3.1
Purlin Cross Section Geometry & Properties

ID	Dimensions								Properties of Full Section												
									Axis x-x			Axis y-y									
	D	b_f	t	d	γ	R	Area	wt/ft	I_x	S_x	r_x	I_y	S_y	r_y	I_{xy}	I_{x2}	I_{y2}	r_{min}	θ	J	C_w
	in.	in.	in.	in.	deg	in.	in.^{2}	lb	in.^{4}	in.^{3}	in.	in.^{4}	in.^{3}	in.	in.^{4}	in.^{4}	in.^{4}	in.	deg.	in.^{4}	in.^{6}
S1	12	3.25	0.135	0.75	50	0.188	2.613	8.88	53.7	8.96	4.54	5	1.36	1.38	11.6	56.4	2.38	0.955	12.7	0.0159	130
S2	12	3.25	0.060	0.75	50	0.188	1.177	4.00	24.6	4.1	4.57	2.35	0.64	1.41	5.38	25.9	1.12	0.975	12.9	0.0014	61.6
S3	10	3.00	0.135	0.75	50	0.188	2.275	7.74	33.2	6.65	3.82	4.07	1.19	1.34	8.37	35.5	1.83	0.898	14.9	0.0138	71.7
S4	10	3.00	0.060	0.75	50	0.188	1.027	3.49	15.3	3.06	3.86	1.92	0.56	1.37	3.9	16.4	0.86	0.918	15.1	0.0012	34.3
S5	8	2.50	0.105	0.75	50	0.188	1.466	4.98	13.9	3.47	3.08	2.06	0.7	1.18	3.89	15.1	0.89	0.78	16.7	0.0054	23.2
S6	8	2.50	0.048	0.75	50	0.188	0.631	2.15	5.8	1.45	3.03	0.58	0.24	0.96	1.3	6.11	0.28	0.661	13.3	0.0005	7



Note that the cross-sections used by Elhouar and Murray (1985) in their test matrix do not consider different thicknesses for each depth. Their system behavior analysis showed that the contribution of thickness to brace force was small, so it was decided not to vary thickness independent of depth. Table 3.2 shows the cross-sections Elhouar and Murray considered. This difference will be shown to significantly effect the form of the regression equations developed below.

Table 3.2 Elhouar and Murray (1985) Cross Sections Considered

Name	Depth, D in.	Flange Width, b_f in.	Thickness, t in.
S1	6	2.5	0.075
S2	8	3	0.075
S3	10	3.5	0.105
S4	12	3.5	0.135

The final variable in the test matrix is purlin span length. Two span lengths were considered per cross-section in an effort to represent a span length envelope. The maximum span length for a cross section was found by multiplying the depth of the section by 3.125 ft/in. The minimum length is found by multiplying the depth of the purlin by 2.5 ft/in. These conversion factors represent the current general limitations for span length versus depth. These are summarized in Table 3.3 below.

In order to simplify the analysis procedure, the minimum span length for 12 in. deep purlins was changed to 31.25 ft, and is reflected in Table 3.3. This

Table 3.3 Span Lengths Considered

Purlin Depth in.	Minimum Span Length ft	Maximum Span Length ft
12	31.25	37.5
10	25.0	31.25
8	20.0	25.0

change does not adversely effect the test matrix as it will produce slightly more conservative results.

In summary, a total of 240 tests make up the test matrix. Three purlin depths are considered (D), each with two thickness (t), two span lengths (L), five purlin lines (n), two bracing configurations (B), and two spans (S). Stated numerically:

$$[3\ \mathrm{D}] \times [2\ \mathrm{t}] \times [2\ \mathrm{L}] \times [5\ \mathrm{n}] \times [2\ \mathrm{B}] \times [2\ \mathrm{S}] = 240\ \text{tests} \qquad (3.4)$$

Since four regression equations are required, each will be based on 60 tests. The test matrix is summarized in Table 3.4.

3.6 SOLVING THE TEST MATRIX

Once the test matrix was developed, the Modified Elhouar and Murray Stiffness Model was used to solve all 240 tests in the matrix. Recall that the computer software package RISA 3D (1998) was utilized to solve the models. A sample input is shown in Appendix A while a complete summary of all computer input including cross-sectional properties and model loading is given in Appendix B.

Table 3.4
Test Matrix

Cross Section ID	Brace Type	Spans #	Span ft	Purlin Lines #				
12ZS3.25x135 S1	1/4 pt	1	37.5	4	8	12	16	20
			31.25	4	8	12	16	20
		3	37.5	4	8	12	16	20
			31.25	4	8	12	16	20
	1/3+Support	1	37.5	4	8	12	16	20
			31.25	4	8	12	16	20
		3	37.5	4	8	12	16	20
			31.25	4	8	12	16	20
12ZS3.25x060 S2	1/4 pt	1	37.5	4	8	12	16	20
			31.25	4	8	12	16	20
		3	37.5	4	8	12	16	20
			31.25	4	8	12	16	20
	1/3+Support	1	37.5	4	8	12	16	20
			31.25	4	8	12	16	20
		3	37.5	4	8	12	16	20
			31.25	4	8	12	16	20
10ZS3x135 S3	1/4 pt	1	31.25	4	8	12	16	20
			25	4	8	12	16	20
		3	31.25	4	8	12	16	20
			25	4	8	12	16	20
	1/3+Support	1	31.25	4	8	12	16	20
			25	4	8	12	16	20
		3	31.25	4	8	12	16	20
			25	4	8	12	16	20
10ZS3x060 S4	1/4 pt	1	31.25	4	8	12	16	20
			25	4	8	12	16	20
		3	31.25	4	8	12	16	20
			25	4	8	12	16	20
	1/3+Support	1	31.25	4	8	12	16	20
			25	4	8	12	16	20
		3	31.25	4	8	12	16	20
			25	4	8	12	16	20
8ZS2.5x105 S5	1/4 pt	1	25	4	8	12	16	20
			20	4	8	12	16	20
		3	25	4	8	12	16	20
			20	4	8	12	16	20
	1/3+Support	1	25	4	8	12	16	20
			20	4	8	12	16	20
		3	25	4	8	12	16	20
			20	4	8	12	16	20

Table 3.4, Cont.
Test Matrix

Cross Section ID	Brace Type	Spans #	Span ft	Purlin Lines #				
8ZS2.5x048 S6	1/4 pt	1	25	4	8	12	16	20
			20	4	8	12	16	20
		3	25	4	8	12	16	20
			20	4	8	12	16	20
	1/3+Support	1	25	4	8	12	16	20
			20	4	8	12	16	20
		3	25	4	8	12	16	20
			20	4	8	12	16	20

Tables 3.5 through 3.8 summarize all test parameters and results. Results shown include the forces in each brace, the total brace force, and the brace force ratio. The brace forces in each test are symmetric about the center of the purlin spans and only half the brace forces are shown. Note that brace number one represents the brace at or closest to an end support.

Once all the tests in the test matrix were solved and all results recorded, a regression analysis was performed on the data in order to develop design equations.

3.7 REGRESSION ANALYSIS

Kleinbaum and Kupper (1978) state that there are two basic questions that need to be answered in any regression analysis:

1. What is the most appropriate mathematical model to use? In other words, should one use a straight line, a parabola…or what?

Table 3.5
Single Span Parameters and Results
Quarter-Point Bracing Configuration

File Name	Purlin Span ft.	Purlin Depth in.	Purlin Thickness in.	Flange Width in.	Purlin Lines	Gravity Load kips	Brace Forces 1 kips	Brace Forces 2 kips	Total Force kips	Force Ratio
S1374201	37.5	12	0.135	3.25	20	71.25	1.02	1.87	3.91	0.05488
S1374161	37.5	12	0.135	3.25	16	56.25	1	1.82	3.82	0.06791
S1374121	37.5	12	0.135	3.25	12	41.25	0.93	1.7	3.56	0.08630
S1374081	37.5	12	0.135	3.25	8	26.25	0.78	1.4	2.96	0.11276
S1374041	37.5	12	0.135	3.25	4	11.25	0.46	0.72	1.64	0.14578
S2374201	37.5	12	0.06	3.25	20	71.25	2.18	4.09	8.45	0.11860
S2374161	37.5	12	0.06	3.25	16	56.25	1.91	3.57	7.39	0.13138
S2374121	37.5	12	0.06	3.25	12	41.25	1.54	2.88	5.96	0.14448
S2374081	37.5	12	0.06	3.25	8	26.25	1.07	1.99	4.13	0.15733
S2374041	37.5	12	0.06	3.25	4	11.25	0.52	0.85	1.89	0.16800
S1314201	31.25	12	0.135	3.25	20	59.375	0.8	1.65	3.25	0.05474
S1314161	31.25	12	0.135	3.25	16	46.875	0.78	1.61	3.17	0.06763
S1314121	31.25	12	0.135	3.25	12	34.375	0.73	1.5	2.96	0.08611
S1314081	31.25	12	0.135	3.25	8	21.875	0.62	1.11	2.35	0.10743
S1314041	31.25	12	0.135	3.25	4	9.375	0.35	0.65	1.35	0.14400
S2314201	31.25	12	0.06	3.25	20	59.375	1.67	3.41	6.75	0.11368
S2314161	31.25	12	0.06	3.25	16	46.875	1.47	3	5.94	0.12672
S2314121	31.25	12	0.06	3.25	12	34.375	1.19	2.44	4.82	0.14022
S2314081	31.25	12	0.06	3.25	8	21.875	0.83	1.71	3.37	0.15406
S2314041	31.25	12	0.06	3.25	4	9.375	0.39	0.76	1.54	0.16427
S3314201	31.25	10	0.135	3	20	59.375	0.89	1.78	3.56	0.05996
S3314161	31.25	10	0.135	3	16	46.875	0.87	1.75	3.49	0.07445
S3314121	31.25	10	0.135	3	12	34.375	0.83	1.66	3.32	0.09658
S3314081	31.25	10	0.135	3	8	21.875	0.71	1.41	2.83	0.12937
S3314041	31.25	10	0.135	3	4	9.375	0.43	0.77	1.63	0.17387
S4314201	31.25	10	0.06	3	20	59.375	1.93	3.97	7.83	0.13187
S4314161	31.25	10	0.06	3	16	46.875	1.51	2.72	5.74	0.12245
S4314121	31.25	10	0.06	3	12	34.375	1.38	2.75	5.51	0.16029
S4314081	31.25	10	0.06	3	8	21.875	1.01	2.08	4.1	0.18743
S4314041	31.25	10	0.06	3	4	9.375	0.48	0.87	1.83	0.19520

Table 3.5, Cont.
Single Span Parameters and Results
Quarter-Point Bracing Configuration

File Name	Purlin Span ft.	Purlin Depth in.	Purlin Thickness in.	Flange Width in.	Purlin Lines	Gravity Load kips	Brace Forces 1 kips	Brace Forces 2 kips	Total Force kips	Force Ratio
S3254201	25	10	0.135	3	20	47.5	0.66	1.48	2.8	0.05895
S3254161	25	10	0.135	3	16	37.5	0.66	1.49	2.81	0.07493
S3254121	25	10	0.135	3	12	27.5	0.62	1.41	2.65	0.09636
S3254081	25	10	0.135	3	8	17.5	0.53	1.2	2.26	0.12914
S3254041	25	10	0.135	3	4	7.5	0.31	0.67	1.29	0.17200
S4254201	25	10	0.06	3	20	47.5	1.41	3.09	5.91	0.12442
S4254161	25	10	0.06	3	16	37.5	1.25	2.76	5.26	0.14027
S4254121	25	10	0.06	3	12	27.5	1.03	2.27	4.33	0.15745
S4254081	25	10	0.06	3	8	17.5	0.73	1.61	3.07	0.17543
S4254041	25	10	0.06	3	4	7.5	0.35	0.73	1.43	0.19067
S5254201	25	8	0.105	2.5	20	47.5	0.73	1.52	2.98	0.06274
S5254161	25	8	0.105	2.5	16	37.5	0.72	1.49	2.93	0.07813
S5254121	25	8	0.105	2.5	12	27.5	0.68	1.41	2.77	0.10073
S5254081	25	8	0.105	2.5	8	17.5	0.58	1.17	2.33	0.13314
S5254041	25	8	0.105	2.5	4	7.5	0.34	0.64	1.32	0.17600
S6254201	25	8	0.048	2.5	20	47.5	1.48	3.1	6.06	0.12758
S6254161	25	8	0.048	2.5	16	37.5	1.3	2.72	5.32	0.14187
S6254121	25	8	0.048	2.5	12	27.5	1.06	2.2	4.32	0.15709
S6254081	25	8	0.048	2.5	8	17.5	0.74	1.53	3.01	0.17200
S6254041	25	8	0.048	2.5	4	7.5	0.35	0.67	1.37	0.18267
S5204201	20	8	0.105	2.5	20	38	0.56	1.31	2.43	0.06395
S5204161	20	8	0.105	2.5	16	30	0.55	1.28	2.38	0.07933
S5204121	20	8	0.105	2.5	12	22	0.52	1.21	2.25	0.10227
S5204081	20	8	0.105	2.5	8	14	0.44	1.01	1.89	0.13500
S5204041	20	8	0.105	2.5	4	6	0.25	0.55	1.05	0.17500
S6204201	20	8	0.048	2.5	20	38	1.13	2.57	4.83	0.12711
S6204161	20	8	0.048	2.5	16	30	0.99	2.25	4.23	0.14100
S6204121	20	8	0.048	2.5	12	22	0.8	1.82	3.42	0.15545
S6204081	20	8	0.048	2.5	8	14	0.55	1.26	2.36	0.16857
S6204041	20	8	0.048	2.5	4	6	0.26	0.56	1.08	0.18000

Table 3.6
Single Span Parameters and Results
Third Point + Support Bracing Configuration

File Name	Purlin Span ft.	Purlin Depth in.	Purlin Thickness in.	Flange Width in.	Purlin Lines	Gravity Load kips	Brace Forces 1 kips	Brace Forces 2 kips	Total Force kips	Force Ratio
S1375201	37.5	12	0.135	3.25	20	71.25	0.83	1.71	5.08	0.07130
S1375161	37.5	12	0.135	3.25	16	56.25	0.81	1.66	4.94	0.08782
S1375121	37.5	12	0.135	3.25	12	41.25	0.75	1.53	4.56	0.11055
S1375081	37.5	12	0.135	3.25	8	26.25	0.61	1.25	3.72	0.14171
S1375041	37.5	12	0.135	3.25	4	11.25	0.28	0.69	1.94	0.17244
S2375201	37.5	12	0.06	3.25	20	71.25	1.68	3.11	9.58	0.13446
S2375161	37.5	12	0.06	3.25	16	56.25	1.44	2.7	8.28	0.14720
S2375121	37.5	12	0.06	3.25	12	41.25	1.12	2.16	6.56	0.15903
S2375081	37.5	12	0.06	3.25	8	26.25	0.72	1.51	4.46	0.16990
S2375041	37.5	12	0.06	3.25	4	11.25	0.23	0.77	2	0.17778
S1315201	31.25	12	0.135	3.25	20	59.375	0.68	1.43	4.22	0.07107
S1315161	31.25	12	0.135	3.25	16	46.875	0.66	1.39	4.1	0.08747
S1315121	31.25	12	0.135	3.25	12	34.375	0.61	1.29	3.8	0.11055
S1315081	31.25	12	0.135	3.25	8	21.875	0.57	0.93	3	0.13714
S1315041	31.25	12	0.135	3.25	4	9.375	0.22	0.58	1.6	0.17067
S2315201	31.25	12	0.06	3.25	20	59.375	1.34	2.5	7.68	0.12935
S2315161	31.25	12	0.06	3.25	16	46.875	1.15	2.18	6.66	0.14208
S2315121	31.25	12	0.06	3.25	12	34.375	0.9	1.76	5.32	0.15476
S2315081	31.25	12	0.06	3.25	8	21.875	0.58	1.23	3.62	0.16549
S2315041	31.25	12	0.06	3.25	4	9.375	0.19	0.62	1.62	0.17280
S3315201	31.25	10	0.135	3	20	59.375	0.67	1.62	4.58	0.07714
S3315161	31.25	10	0.135	3	16	46.875	0.65	1.59	4.48	0.09557
S3315121	31.25	10	0.135	3	12	34.375	0.62	1.51	4.26	0.12393
S3315081	31.25	10	0.135	3	8	21.875	0.51	1.27	3.56	0.16274
S3315041	31.25	10	0.135	3	4	9.375	0.24	0.72	1.92	0.20480
S4315201	31.25	10	0.06	3	20	59.375	1.56	2.97	9.06	0.15259
S4315161	31.25	10	0.06	3	16	46.875	1.16	2.29	6.9	0.14720
S4315121	31.25	10	0.06	3	12	34.375	1.08	2.02	6.2	0.18036
S4315081	31.25	10	0.06	3	8	21.875	0.71	1.53	4.48	0.20480
S4315041	31.25	10	0.06	3	4	9.375	0.24	0.73	1.94	0.20693

Table 3.6, Cont.
Single Span Parameters and Results
Third Point + Support Bracing Configuration

File Name	Purlin Span ft.	Purlin Depth in.	Purlin Thickness in.	Flange Width in.	Purlin Lines	Gravity Load kips	Brace Forces 1 kips	Brace Forces 2 kips	Total Force kips	Force Ratio
S3255201	25	10	0.135	3	20	47.5	0.52	1.29	3.62	0.07621
S3255161	25	10	0.135	3	16	37.5	0.49	1.3	3.58	0.09547
S3255121	25	10	0.135	3	12	27.5	0.46	1.23	3.38	0.12291
S3255081	25	10	0.135	3	8	17.5	0.37	1.04	2.82	0.16114
S3255041	25	10	0.135	3	4	7.5	0.17	0.59	1.52	0.20267
S4255201	25	10	0.06	3	20	47.5	1.2	2.19	6.78	0.14274
S4255161	25	10	0.06	3	16	37.5	1.05	1.94	5.98	0.15947
S4255121	25	10	0.06	3	12	27.5	0.83	1.58	4.82	0.17527
S4255081	25	10	0.06	3	8	17.5	0.55	1.12	3.34	0.19086
S4255041	25	10	0.06	3	4	7.5	0.19	0.56	1.5	0.20000
S5255201	25	8	0.105	2.5	20	47.5	0.66	1.3	3.92	0.08253
S5255161	25	8	0.105	2.5	16	37.5	0.65	1.27	3.84	0.10240
S5255121	25	8	0.105	2.5	12	27.5	0.61	1.19	3.6	0.13091
S5255081	25	8	0.105	2.5	8	17.5	0.5	0.98	2.96	0.16914
S5255041	25	8	0.105	2.5	4	7.5	0.24	0.55	1.58	0.21067
S6255201	25	8	0.048	2.5	20	47.5	1.26	2.17	6.86	0.14442
S6255161	25	8	0.048	2.5	16	37.5	1.09	1.89	5.96	0.15893
S6255121	25	8	0.048	2.5	12	27.5	0.87	1.51	4.76	0.17309
S6255081	25	8	0.048	2.5	8	17.5	0.57	1.05	3.24	0.18514
S6255041	25	8	0.048	2.5	4	7.5	0.2	0.52	1.44	0.19200
S5205201	20	8	0.105	2.5	20	38	0.51	1.08	3.18	0.08368
S5205161	20	8	0.105	2.5	16	30	0.5	1.05	3.1	0.10333
S5205121	20	8	0.105	2.5	12	22	0.46	0.98	2.88	0.13091
S5205081	20	8	0.105	2.5	8	14	0.37	0.8	2.34	0.16714
S5205041	20	8	0.105	2.5	4	6	0.17	0.45	1.24	0.20667
S6205201	20	8	0.048	2.5	20	38	0.99	1.73	5.44	0.14316
S6205161	20	8	0.048	2.5	16	30	0.85	1.5	4.7	0.15667
S6205121	20	8	0.048	2.5	12	22	0.66	1.2	3.72	0.16909
S6205081	20	8	0.048	2.5	8	14	0.43	0.83	2.52	0.18000
S6205041	20	8	0.048	2.5	4	6	0.15	0.41	1.12	0.18667

Table 3.7
Three Span Parameters and Results
Quarter Point Bracing Configuration

File Name	Purlin Span ft	Purlin Depth in.	Purlin Thick. in.	Flange Width in.	Purlin Lines	Gravity Load kips	Brace Forces (Symmetric) 1 kips	2 kips	3 kips	4 kips	5 kips	Total Force kips	Force Ratio per Span	Total Force Ratio
S1374203	37.5	12	0.135	3.25	20	71.25	0.95	1.74	0.77	0.74	1.67	10.07	0.1413	0.04711
S1374163	37.5	12	0.135	3.25	16	56.25	0.94	1.71	0.76	0.73	1.64	9.92	0.1764	0.05879
S1374123	37.5	12	0.135	3.25	12	41.25	0.89	1.62	0.72	0.7	1.56	9.42	0.2284	0.07612
S1374083	37.5	12	0.135	3.25	8	26.25	0.75	1.37	0.62	0.6	1.34	8.02	0.3055	0.10184
S1374043	37.5	12	0.135	3.25	4	11.25	0.45	0.74	0.37	0.36	0.76	4.6	0.4089	0.13630
S2374203	37.5	12	0.06	3.25	20	71.25	1.94	3.51	1.6	1.55	3.23	20.43	0.2867	0.09558
S2374163	37.5	12	0.06	3.25	16	56.25	1.43	2.59	1.17	1.13	2.4	15.04	0.2674	0.08913
S2374123	37.5	12	0.06	3.25	12	41.25	1.35	2.12	0.89	0.81	1.76	12.1	0.2933	0.09778
S2374083	37.5	12	0.06	3.25	8	26.25	1.03	1.86	0.83	0.8	1.74	10.78	0.4107	0.13689
S2374043	37.5	12	0.06	3.25	4	11.25	0.53	0.87	0.37	0.38	0.82	5.12	0.4551	0.15170
S1314203	31.25	12	0.135	3.25	20	59.375	0.74	1.47	0.57	0.54	1.34	7.98	0.1344	0.04480
S1314163	31.25	12	0.135	3.25	16	46.875	0.72	1.43	0.56	0.53	1.31	7.79	0.1662	0.05540
S1314123	31.25	12	0.135	3.25	12	34.375	0.68	1.36	0.53	0.51	1.26	7.42	0.2159	0.07195
S1314083	31.25	12	0.135	3.25	8	21.875	0.59	1.16	0.46	0.44	1.09	6.39	0.2921	0.09737
S1314043	31.25	12	0.135	3.25	4	9.375	0.35	0.64	0.28	0.27	0.64	3.72	0.3968	0.13227
S2314203	31.25	12	0.06	3.25	20	59.375	1.49	2.88	1.22	1.17	2.6	16.12	0.2715	0.09050
S2314163	31.25	12	0.06	3.25	16	46.875	1.35	2.64	1.1	1.06	2.36	14.66	0.3127	0.10425
S2314123	31.25	12	0.06	3.25	12	34.375	1.13	2.21	0.91	0.87	1.97	12.21	0.3552	0.11840
S2314083	31.25	12	0.06	3.25	8	21.875	0.79	1.54	0.62	0.6	1.39	8.49	0.3881	0.12937
S2314043	31.25	12	0.06	3.25	4	9.375	0.4	0.71	0.28	0.27	0.66	3.98	0.4245	0.14151
S3314203	31.25	10	0.135	3	20	59.375	0.8	1.54	0.65	0.63	1.48	8.72	0.1469	0.04895
S3314163	31.25	10	0.135	3	16	46.875	0.78	1.51	0.64	0.62	1.45	8.55	0.1824	0.06080
S3314123	31.25	10	0.135	3	12	34.375	0.75	1.45	0.62	0.6	1.4	8.24	0.2397	0.07990
S3314083	31.25	10	0.135	3	8	21.875	0.66	1.27	0.55	0.53	1.25	7.27	0.3323	0.11078
S3314043	31.25	10	0.135	3	4	9.375	0.4	0.72	0.34	0.33	0.73	4.31	0.4597	0.15324
S4314203	31.25	10	0.06	3	20	59.375	1.64	3.16	1.34	1.29	1.64	16.5	0.2779	0.09263
S4314163	31.25	10	0.06	3	16	46.875	1.48	2.85	1.21	1.17	2.63	16.05	0.3424	0.11413
S4314123	31.25	10	0.06	3	12	34.375	1.26	2.44	1.03	1	2.26	13.72	0.3991	0.13304
S4371083	31.25	10	0.06	3	8	21.875	0.93	1.81	0.76	0.73	1.69	10.15	0.4640	0.15467
S4371043	31.25	10	0.06	3	4	9.375	0.48	0.85	0.36	0.35	0.82	4.9	0.5227	0.17422

Table 3.7, Cont.
Three Span Parameters and Results
Quarter Point Bracing Configuration

File Name	Purlin Span ft	Purlin Depth in.	Purlin Thick. in.	Flange Width in.	Purlin Lines	Gravity Load kips	Brace Forces (Symmetric) 1 kips	2 kips	3 kips	4 kips	5 kips	Total Force kips	Force Ratio per Span	Total Force Ratio
S3254203	25	10	0.135	3	20	47.5	0.58	1.24	0.45	0.42	1.12	6.5	0.1368	0.04561
S3254163	25	10	0.135	3	16	37.5	0.57	1.21	0.44	0.42	1.1	6.38	0.1701	0.05671
S3254123	25	10	0.135	3	12	27.5	0.55	1.17	0.42	0.4	1.07	6.15	0.2236	0.07455
S3254083	25	10	0.135	3	8	17.5	0.49	1.03	0.38	0.36	0.96	5.48	0.3131	0.10438
S3254043	25	10	0.135	3	4	7.5	0.3	0.6	0.24	0.23	0.59	3.33	0.4440	0.14800
S4254203	25	10	0.06	3	20	47.5	1.19	2.46	0.96	0.92	2.19	13.25	0.2789	0.09298
S4254163	25	10	0.06	3	16	37.5	1.07	2.22	0.86	0.83	1.98	11.94	0.3184	0.10613
S4254123	25	10	0.06	3	12	27.5	0.92	1.91	0.74	0.7	1.71	10.25	0.3727	0.12424
S4254083	25	10	0.06	3	8	17.5	0.68	1.42	0.53	0.51	1.27	7.55	0.4314	0.14381
S4254043	25	10	0.06	3	4	7.5	0.35	0.68	0.25	0.24	0.62	3.66	0.4880	0.16267
S5254203	25	8	0.105	2.5	20	47.5	0.69	1.41	0.56	0.54	1.34	7.74	0.1629	0.05432
S5254163	25	8	0.105	2.5	16	37.5	0.67	1.37	0.55	0.53	1.31	7.55	0.2013	0.06711
S5254123	25	8	0.105	2.5	12	27.5	0.65	1.32	0.52	0.51	1.26	7.26	0.2640	0.08800
S5254083	25	8	0.105	2.5	8	17.5	0.56	1.14	0.46	0.44	1.11	6.31	0.3606	0.12019
S5254043	25	8	0.105	2.5	4	7.5	0.34	0.65	0.28	0.28	0.65	3.75	0.5000	0.16667
S6254203	25	8	0.048	2.5	20	47.5	1.32	2.69	1.12	1.09	2.53	14.97	0.3152	0.10505
S6254163	25	8	0.048	2.5	16	37.5	1.17	2.38	0.99	0.97	2.24	13.26	0.3536	0.11787
S6254123	25	8	0.048	2.5	12	27.5	0.98	1.99	0.83	0.81	1.89	11.11	0.4040	0.13467
S6254083	25	8	0.048	2.5	8	17.5	0.71	1.44	0.59	0.58	1.36	8	0.4571	0.15238
S6254043	25	8	0.048	2.5	4	7.5	0.36	0.66	0.28	0.27	0.64	3.78	0.5040	0.16800
S5204203	20	8	0.105	2.5	20	38	0.51	1.14	0.39	0.37	1.02	5.84	0.1537	0.05123
S5204163	20	8	0.105	2.5	16	30	0.5	1.11	0.38	0.36	1	5.7	0.1900	0.06333
S5204123	20	8	0.105	2.5	12	22	0.48	1.06	0.37	0.35	0.96	5.48	0.2491	0.08303
S5204083	20	8	0.105	2.5	8	14	0.42	0.93	0.32	0.31	0.85	4.81	0.3436	0.11452
S5204043	20	8	0.105	2.5	4	6	0.25	0.53	0.2	0.19	0.52	2.86	0.4767	0.15889
S6204203	20	8	0.048	2.5	20	38	0.97	2.1	0.82	0.79	1.92	11.28	0.2968	0.09895
S6204163	20	8	0.048	2.5	16	30	0.86	1.86	0.72	0.7	1.71	9.99	0.3330	0.11100
S6204123	20	8	0.048	2.5	12	22	0.72	1.56	0.6	0.58	1.43	8.35	0.3795	0.12652
S6204083	20	8	0.048	2.5	8	14	0.52	1.13	0.42	0.41	1.03	5.99	0.4279	0.14262
S6204043	20	8	0.048	2.5	4	6	0.26	0.53	0.19	0.19	0.49	2.83	0.4717	0.15722

Table 3.8
Three Span Parameters and Results
Third-Point + Support Bracing Configuration

File Name	Purlin Span ft.	Purlin Depth in.	Purlin Thick. in.	Flange Width in.	Purlin Lines	Gravity Load kips	Brace Forces (Symmetric) 1 kips	2 kips	3 kips	4 kips	5 kips	Total Force kips	Force Ratio	Total Force Ratio
S1375203	37.5	12	0.135	3.25	20	71.25	0.75	1.66	1.56	0.82	1.53	12.64	0.1774	0.05913
S1375163	37.5	12	0.135	3.25	16	56.25	0.72	1.61	1.51	0.8	1.49	12.26	0.2180	0.07265
S1375123	37.5	12	0.135	3.25	12	41.25	0.68	1.51	1.43	0.76	1.42	11.6	0.2812	0.09374
S1375083	37.5	12	0.135	3.25	8	26.25	0.55	1.26	1.21	0.64	1.21	9.74	0.3710	0.12368
S1375043	37.5	12	0.135	3.25	4	11.25	0.26	0.71	0.69	0.34	0.69	5.38	0.4782	0.15941
S2375203	37.5	12	0.06	3.25	20	71.25	1.51	2.86	2.67	2.07	2.57	23.36	0.3279	0.10929
S2375163	37.5	12	0.06	3.25	16	56.25	1.03	2.07	1.93	1.43	1.88	16.68	0.2965	0.09884
S2375123	37.5	12	0.06	3.25	12	41.25	0.99	2	1.74	0.9	1.6	14.46	0.3505	0.11685
S2375083	37.5	12	0.06	3.25	8	26.25	0.67	1.48	1.39	0.94	1.36	11.68	0.4450	0.14832
21375043	37.5	12	0.06	3.25	4	11.25	0.2	0.79	0.7	0.32	0.68	5.38	0.4782	0.15941
S1315203	31.25	12	0.135	3.25	20	59.375	0.6	1.35	1.23	0.64	1.17	9.98	0.1681	0.05603
S1315163	31.25	12	0.135	3.25	16	46.875	0.58	1.31	1.19	0.62	1.15	9.7	0.2069	0.06898
S1315123	31.25	12	0.135	3.25	12	34.375	0.55	1.23	1.13	0.59	1.1	9.2	0.2676	0.08921
S1315083	31.25	12	0.135	3.25	8	21.875	0.45	1.04	0.96	0.5	0.95	7.8	0.3566	0.11886
S1315043	31.25	12	0.135	3.25	4	9.375	0.2	0.59	0.55	0.25	0.55	4.28	0.4565	0.15218
S2315203	31.25	12	0.06	3.25	20	59.375	1.2	2.26	2.09	1.63	1.98	18.32	0.3085	0.10285
S2315163	31.25	12	0.06	3.25	16	46.875	1.05	2.06	1.89	1.43	1.78	16.42	0.3503	0.11676
S2315123	31.25	12	0.06	3.25	12	34.375	0.84	1.71	1.56	1.14	1.48	13.46	0.3916	0.13052
S2315083	31.25	12	0.06	3.25	8	21.875	0.54	1.18	1.08	0.73	1.04	9.14	0.4178	0.13928
S2315043	31.25	12	0.06	3.25	4	9.375	0.18	0.61	0.53	0.25	0.52	4.18	0.4459	0.14862
S3315203	31.25	10	0.135	3	20	59.375	0.62	1.44	1.36	0.67	1.34	10.86	0.1829	0.06097
S3315163	31.25	10	0.135	3	16	46.875	0.61	1.41	1.33	0.66	1.31	10.64	0.2270	0.07566
S3315123	31.25	10	0.135	3	12	34.375	0.58	1.35	1.28	0.63	1.27	10.22	0.2973	0.09910
S3315083	31.25	10	0.135	3	8	21.875	0.5	1.16	1.12	0.56	1.12	8.92	0.4078	0.13592
S3315043	31.25	10	0.135	3	4	9.375	0.25	0.67	0.66	0.32	0.66	5.12	0.5461	0.18204
S4315203	31.25	10	0.06	3	20	59.375	1.36	2.55	2.37	1.81	2.28	20.74	0.3493	0.11644
S4315163	31.25	10	0.06	3	16	46.875	1.2	2.28	2.12	1.6	2.05	18.5	0.3947	0.13156
S4315123	31.25	10	0.06	3	12	34.375	0.99	1.93	1.8	1.32	1.75	15.58	0.4532	0.15108
S4315083	31.25	10	0.06	3	8	21.875	0.67	1.41	1.32	0.91	1.3	11.22	0.5129	0.17097
S4315043	31.25	10	0.06	3	4	9.375	0.23	0.72	0.66	0.33	0.66	5.2	0.5547	0.18489

2. Given a specific model, how do we determine the best-fitting model for the data? In other words, if our model is a straight line, how does one find the best-fitting line?

To answer question number one, the Elhouar and Murray (1985) equation form was used, then was adjusted by trial and error until an acceptable form of the equation was attained.

In response to question number two, the computer software program entitled *SigmaPlot 4.0* (1997) was used. There are several independent variables that produce nonlinear response so a multivariable nonlinear regression analysis was necessary. This was a difficult task greatly simplified by the use of computer software.

The dependent variable for all systems is brace force ratio, β. It is defined as total brace force of the system divided by gravity load in <u>one</u> <u>span</u>. This value is per span so that the equations developed for three span systems can also apply to multiple span systems in general. Once the regression equations have been developed to predict brace force ratio, they will be modified to predict the force in each brace.

3.7.1 Initial Trial

The form of the regression equations shown below for the first trial are identical to those developed by Elhouar and Murray (1985). These are shown below in a unit-less form. All variables must be defined in inches. For single span systems:

$$\beta = x_1 \left(\frac{b}{d}\right)^{x_2} \left(\frac{b}{t}\right)^{x_3} n^{x_4} \tag{3.5}$$

and for multiple spans, where L is also in inches:

$$\beta = x_1 \left(\frac{b}{d}\right)^{x_2} \left(\frac{b}{t}\right)^{x_3} \left(\frac{L}{b}\right)^{x_5} n^{x_4} \tag{3.6}$$

The adequacy of regression equations can be measured in a number of ways. The simplest method is to examine the value of R^2, which is the square of the coefficient of correlation between the real and predicted values of the brace force ratio. For the purposes of this study, R^2 values greater than 0.9 are considered acceptable (1 being optimal). The R^2 values for Equations 3.10 and 3.11 were unacceptable, ranging from 0.5 for single span systems to 0.6 for three span systems.

The poor performance of this model is due in part to the independent variable purlin thickness. Because this research considered two purlin thicknesses per depth, two very different brace force behaviors result as Figure 3.1 shows. However, Elhouar and Murray (1985), by considering only one thickness per depth, have only one brace force behavior curve to fit. So it is possible for the equations above to satisfactorily describe Elhouar and Murray's test matrix but not the test matrix described herein. This hypothesis was tested by performing a regression analysis on only the "thick purlin." This analysis produced regression equations whose R^2 values were about 0.95: comparable to those observed by Elhouar and Murray. Therefore, it has been concluded that a different form of equation is necessary to adequately describe the test matrix of this research.

Note that the R^2 values for single span systems are noticeable lower than for three span systems. It has been found that the cause of this additional poor performance is the exclusion of the independent variable L: purlin span length. The system behavior analysis conducted earlier showed that the effects of purlin span length were small and could be disregarded. However, this conclusion must have been incorrect. When L is included, R^2 values for single span systems become comparable to those of multiple span systems.

3.7.2 Final Trial

Equations 3.10 and 3.11 were modified by trial and error until R^2 values were found to be acceptable. The general, unit-less from of the equation found to best describe both single and multiple span systems is:

$$\beta = x_1 \left(\frac{b}{d}\right)^{x_2} \left(\frac{b}{t}\right)^{x_3} n^{x_4} + \left(\frac{t}{L}\right)^{x_5} \tag{3.7}$$

The resulting regression equations for bracing schemes considered are :

For single span, quarter-point restraint:

$$\beta. = \left[\left(\frac{t}{L}\right)^{0.16} - \frac{0.407\, t^{0.75}\, d^{0.50} n^{0.39}}{b^{1.25}} \right] \tag{3.8}$$

($R^2 = 0.955$)

For single span, third-point plus support restraint:

$$\beta. = \left[\left(\frac{t}{L}\right)^{0.17} - \frac{0.223\, t^{1.02}\, d^{0.85}\, n^{0.65}}{b^{1.87}} \right] \tag{3.9}$$

($R^2 = 0.942$)

For three span, quarter-point restraint:

$$\beta = \left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\, t^{0.39}\, d^{0.65}\, n^{0.39}}{b^{1.03}} \right] \tag{3.10}$$

$(R^2 = 0.957)$

For three span, third-point plus support restraint:

$$\beta = \left[\left(\frac{t}{L}\right)^{0.05} - \frac{0.112\, t^{0.57}\, d^{1.03}\, n^{0.67}}{b^{1.60}} \right] \tag{3.11}$$

$(R^2 = 0.933)$

Each of the equations above yield R^2 values well above the minimum limit of 0.9. Complete regression analysis reports can be found in Appendix C. Close examination of these reports show excellent agreement between the brace force generated by these regression equations and those found using the Modified Elhouar and Murray Stiffness Model. Therefore, the regressions equations shown here adequately predict brace force ratio for Z-purlin supported roof systems.

3.7.3 Modifying the Regression Equations

These equations must be modified in order to predict to the force expected in a particular brace. This was done by multiplying β, the brace force ratio predicted by the regression equation, times the gravity load in one span, W, times a factor based on the average force in the brace of interest, $\bar{F}_B$, as observed in the test matrix. Stated algebraically:

$$P_L = \bar{F}_B \left[\beta\right] W \tag{3.12}$$

Where: $$\bar{F}_B = \frac{\sum B_i / B_T}{n_s} \qquad (3.13)$$

B_i = force in brace of interest,

B_T = total brace force of roof system, and

n_s = number of tests in matrix considered, 60.

Table 3.8-(a) lists values of $\bar{F}_B$—the average brace factor—generated in this manner. This table shows five different factors are needed for three span systems: one for two braces. To reduce the number of factors, the coefficient C was introduced. C is the nominal brace factor and its values are shown in Table 3.8-(b). For single span systems, C equals $\bar{F}_B$ to two significant digits, but for three span systems, C has been chosen as shown in Table 3.8-(b) in order to reduce the number of factors from five to three. Equation 3.12 has modified to include the C factor:

$$P_L = C\,[\beta]\,W \qquad (3.12\text{-b})$$

Table 3.9
Average and Nominal Brace Factors

(a) Average Brace Factors, F_B

	1	2	3	4	5	6	7	8	9	10	Sum
Single Span, 1/4 Point	0.2478	0.5045	0.2478								1
Single Span, 1/3+Support	0.1582	0.3418	0.3418	0.1582							1
Three Span, 1/4 Point	0.0919	0.1807	0.0731	0.0705	0.1675	0.0705	0.0731	0.1807	0.0919		1
Three Span, 1/3+Support	0.0585	0.1303	0.1210	0.0720	0.1182	0.1182	0.0720	0.1210	0.1303	0.0585	1

Table 3.9, Cont.
Average and Nominal Brace Factors

(b) Nominal Brace Factors, C

	1	2	3	4	5	6	7	8	9	10	Sum
Single Span 1/4 Point	0.25	0.50	0.25								1
Single Span 1/3+Support	0.16	0.34	0.34	0.16							1
Three Span 1/4 Point	0.09	0.18	0.07	0.07	0.18	0.07	0.07	0.18	0.09		1
Three Span 1/3+Support	0.06	0.125	0.125	0.07	0.125	0.125	0.07	0.125	0.125	0.06	1.01

One other modification must be made for the multiple span system. Since the gravity load, W was is only for one span, the equation must be multiplied by a factor of three to get total gravity load. The C factors for multiple span systems shown in the following equations have been multiplied by three.

The final form of the design equations to predict lateral restraint brace forces are summarized below, and are shown in Figure 3.3.

For single span systems with quarter-point restraint:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.16} - \frac{0.407\, t^{0.75}\, d^{0.50} n^{0.39}}{b^{1.25}}\right]W \tag{3.14}$$

where: $C = 0.25$ for braces near supports

$C = 0.50$ for brace at mid-span

For single span systems with third-point plus support restraint:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.17} - \frac{0.223\, t^{1.02}\, d^{0.85}\, n^{0.65}}{b^{1.87}}\right]W \tag{3.15}$$

where: C = 0.16 for braces at supports

C = 0.34 for braces near mid-span

For multiple-span systems with quarter-point restraint:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\, t^{0.39}\, d^{0.65}\, n^{0.39}}{b^{1.03}}\right]W \qquad (3.16)$$

where: C = 0.27 for braces nearest end supports of multiple-span systems

C = 0.54 for braces at mid-span of multiple-span systems

C = 0.21 for all other braces

For multiple span systems with third-point plus support restraint:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.05} - \frac{0.112\, t^{0.57}\, d^{1.03}\, n^{0.67}}{b^{1.60}}\right]W \qquad (3.17)$$

where: C = 0.18 for braces at end supports of multiple-span systems

C = 0.21 for brace at other supports of multiple-span systems

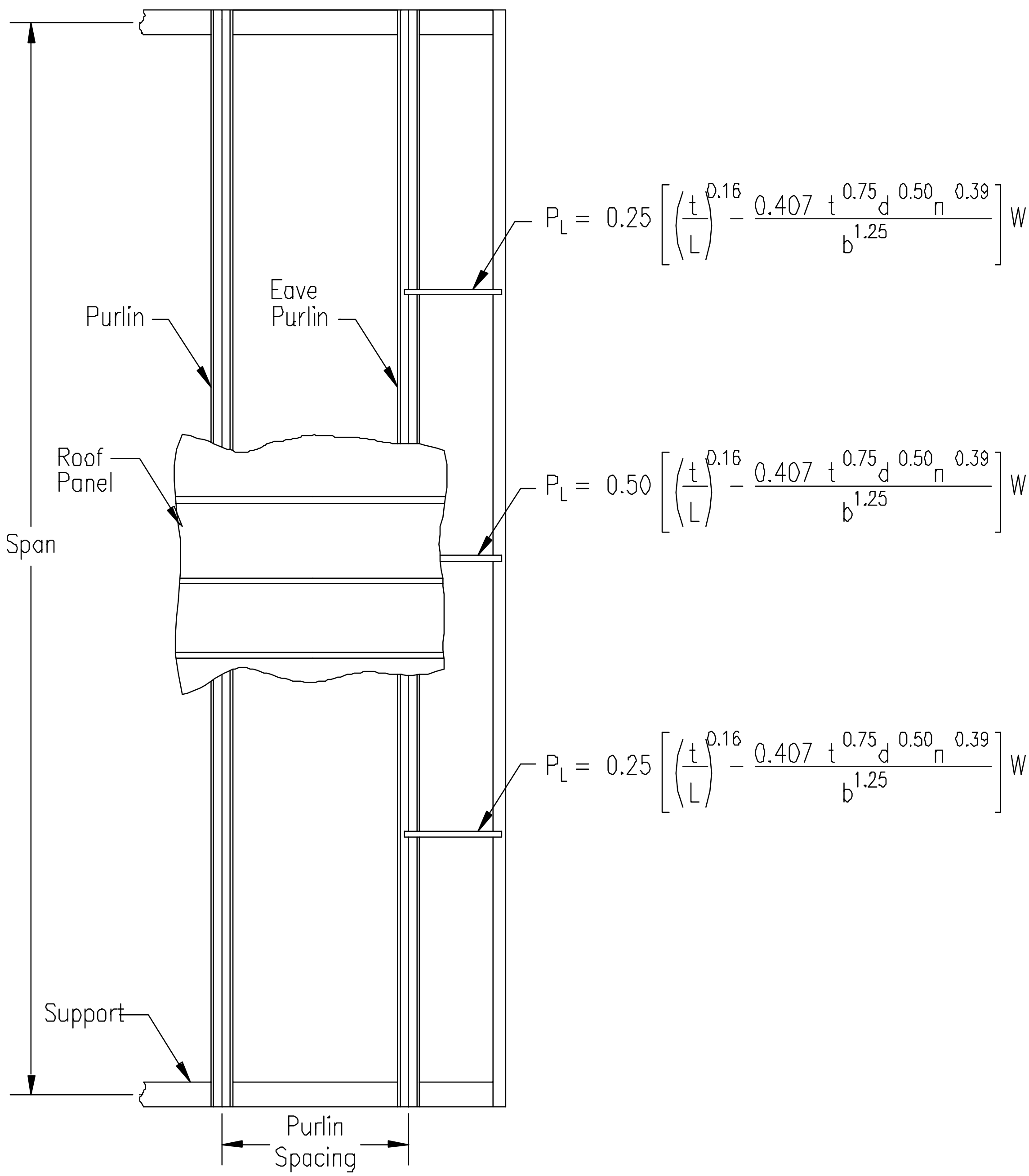
C = 0.375 for all other braces

3.7.4 Limits of Use

The equations developed above are limited to roof systems having four to twenty Z-purlin lines with all top flanges facing the same direction. Restraint braces are limited to quarter-point arrangement, and third-point plus support arrangement. The equations are valid for flat roofs only. Modifications to these equations to account for roof slope are currently being undertaken in a another project.

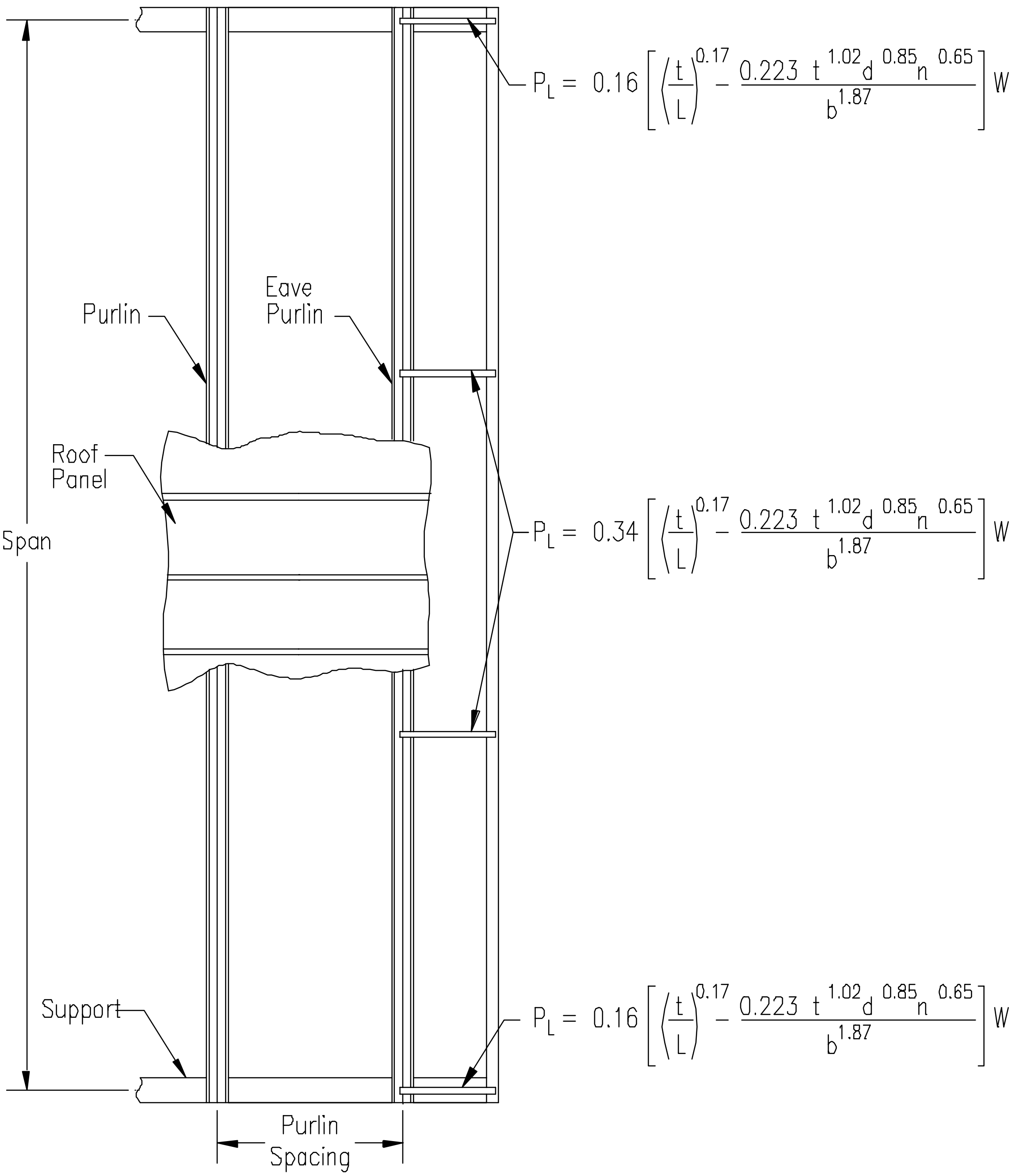
For systems having less than four purlin lines, the brace force shall be determined by taking 1.1 times the force found from Equations 3.14 through 3.17 with n

= 4. For systems having more than twenty purlin lines, the brace force shall be determined by Equations 3.14 through 3.17 with n = 20 and W based on total number of purlins (AISI, 1996). These were first developed by Elhouar and Murray (1985) are consistent, with current design procedures (AISI, 1996), and were found to be valid for Equations 3.14 through 3.17.



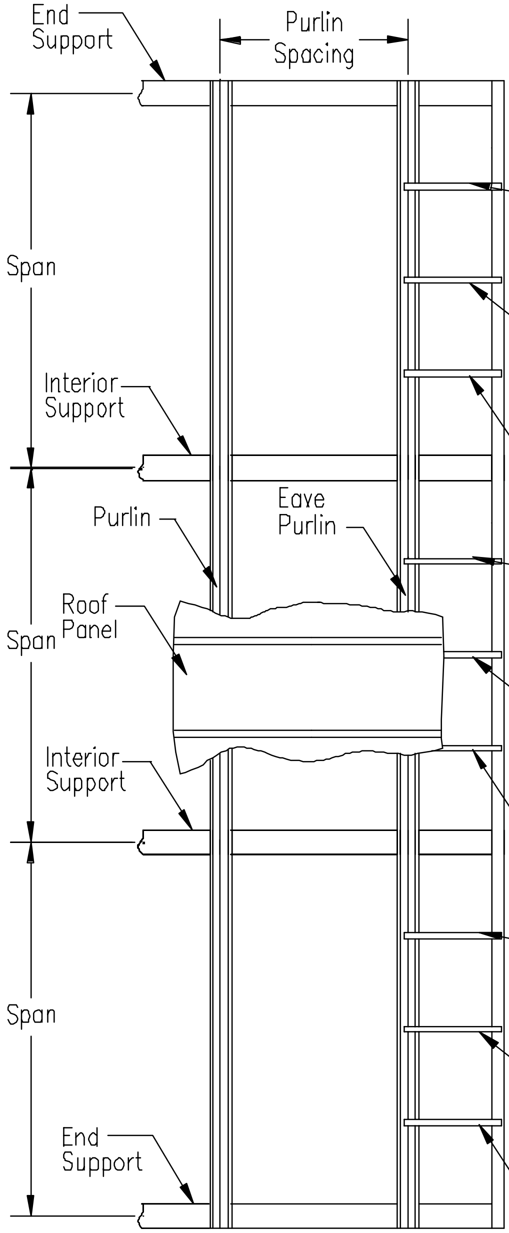
(a) Quarter Point Restraint,
Single Span System

Figure 3.3 Design Equations



(b)Third Point + Supports Restraint,
Single Span System

Figure 3..3 Design Equations, Cont.



$$P_L = 3(0.09)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

$$P_L = 3(0.18)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

$$P_L = 3(0.07)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

$$P_L = 3(0.07)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

$$P_L = 3(0.18)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

$$P_L = 3(0.07)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

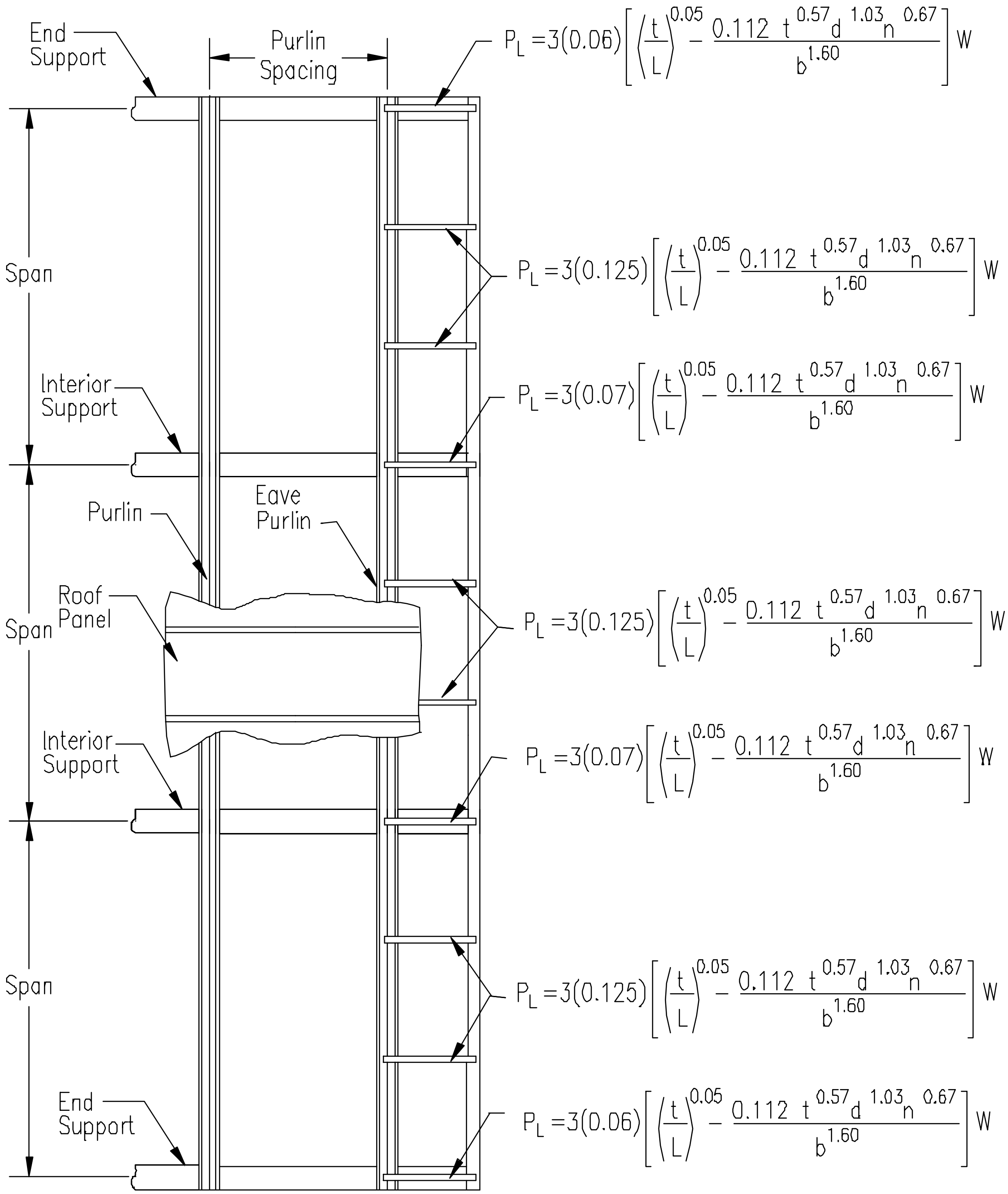
$$P_L = 3(0.07)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

$$P_L = 3(0.18)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

$$P_L = 3(0.09)\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\ t^{0.39} d^{0.65} n^{0.39}}{b^{1.03}}\right] W$$

(c) Quarter Point Restraint,
Multiple Span System

Figure 3..3 Design Equations, Cont.



(d) Third Point + Supports Restraint, Multiple Span System

Figure 3..3 Design Equations, Cont.

CHAPTER IV

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

4.1 SUMMARY

The purpose of this research has been to develop design equations that predict lateral restraint forces in quarter point and third point plus support braced Z-purlin supported roof systems subjected to gravity load. The stiffness model used by Elhouar and Murray (1985) was used here but modified slightly to better represent roof system behavior. The updated stiffness model was then used to estimate lateral restraint forces for a number of roof systems with a varying cross sectional dimensions of the purlin, number of purlin lines, number of spans, and span length. A regression analysis was performed on the data to obtain empirical design equations similar to those found in the American Iron and Steel Institute *Specification for the Design of Cold-Formed Steel Members,* (Section D.3.2.1., AISI, 1996). These design equations are Equations 3.14 through 3.17 and can be used by the practicing design engineer to predict lateral restraint forces in Z-purlin roof system braces.

The following design examples are included to help illustrate the use of Equations 3.14 through 3.17.

4.1.1 Design Examples

Example 1: Find the axial force in lateral restraint braces arranged in a quarter point pattern for a 12 purlin line, 25 ft single span roof system. The roof is subjected to a gravity load of 20 psf, and the Z-purlins have a thickness of 0.06 in, a depth of 10 in, and a purlin flange width of 3 in. Purlin spacing is 5 ft.

Solution: Since bracing is quarter point and system is single span, Equation 3.14 applies:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.16} - \frac{0.407\, t^{0.75}\, d^{0.50} n^{0.39}}{b^{1.25}}\right] W$$

where: C = 0.25 for braces near supports

C = 0.50 for brace at mid-span

The gravity load, W equals:

20 psf x 5 ft x 25 ft x 10 lines + 20 psf x 2.5 ft x 25 ft x 2 lines = 27.5 Kips

Equation 3.14 yields:

$$P_L = C\left[\left(\frac{0.060}{25 \times 12}\right)^{0.16} - \frac{0.407\,(0.060^{0.75})\, 10^{0.50} 12^{0.39}}{3.0^{1.25}}\right] W$$

$$= C\,[\ 0.1014\ ]\ 27.5 \ = \ 2.7885\ C,\ \text{Kips}$$

$$= \ 2.7885 \times 0.25 \ = \ 0.70 \text{ Kips for braces near supports}$$

$$= \ 2.7885 \times 0.50 \ = \ 1.40 \text{ Kips for the brace at mid-span}$$

These values agree well with those predicted by Modified Elhouar and Murray Stiffness Model, 0.62 kips and 1.41 kips respectively. They are shown in Figure 4.1 below.

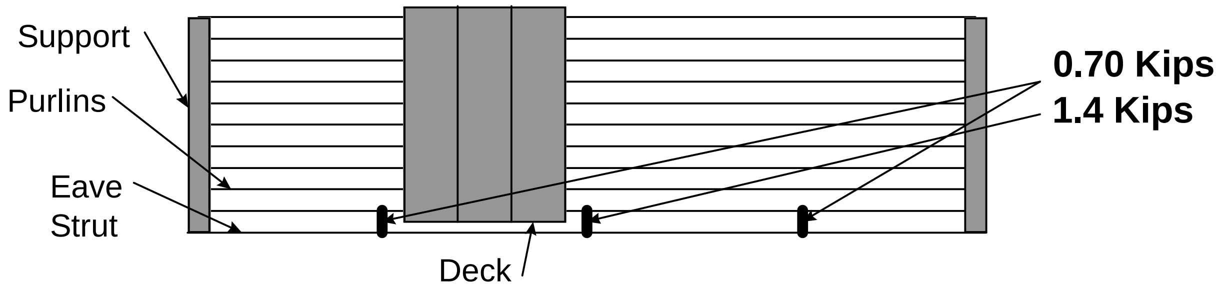


Figure 4.1 Example 1: Brace Force Diagram

Example 2: Find the axial force in lateral restraint braces arranged in a third point plus support pattern for a 5 purlin line, 35 foot 4-span roof system. The roof is subjected to a gravity load of 25 psf, and the Z-purlins have a thickness of 0.06 in, a depth of 12 in, and a purlin flange width of 3.25 in. Purlin spacing is 5 ft.

Solution: Since bracing is 1/3+Support and the system is multiple span, Equation 3.22 applies:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.05} - \frac{0.112\, t^{057}\, d^{1.03}\, n^{0.67}}{b^{1.60}}\right]W$$

Where: C = 0.18 for braces at end supports of multiple-span systems

C = 0.21 for brace at other supports of multiple-span systems

C = 0.3725 for all other braces

The gravity load, W in one span equals:

25 psf x 5 ft x 35 ft x 3 lines + 25 psf x 2.5 ft x 35 ft x 2 lines = 17.5 Kips

Equation 3.22 yields:

$$P_L = C\left[\left(\frac{0.06}{35\times 12}\right)^{0.05} - \frac{0.112\times 0.06^{057}\ 12^{1.03}\ 5^{0.67}}{3.25^{1.60}}\right]W$$

$$= C\,[0.09223]\ 17.5 \text{ Kips} = 1.61\ C,\ \text{Kips}$$

$$= 1.61\times 0.18 = 0.29 \text{ Kips for braces at end supports}$$

$$= 1.61\times 0.21 = 0.34 \text{ Kips for braces at other supports}$$

$$= 1.61\times 0.375 = 0.61 \text{ Kips for all other braces}$$

Figure 4.1 below shows the distribution of brace forces along the four span roof system. Note that this example can not be compared to the numbers obtained from the stiffness model because this case was not considered in the test matrix.

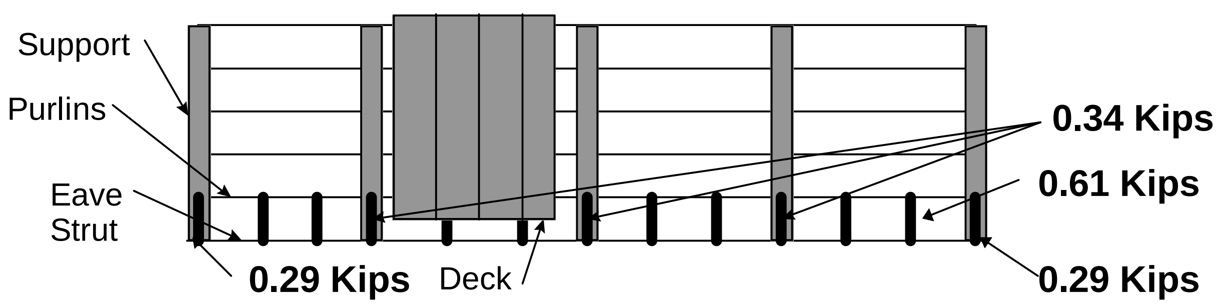


Figure 4.2 Example 2: Brace Force Diagram

4.2 CONCLUSIONS

Equations 3.14 through 3.19, also shown below, adequately represent the brace forces that can be expected in quarter point and 1/3+Support restrained Z-purlin supported roof system, both single span and multiple span systems. A stiffness model that has been shown to agree with experimental test results, and a regression analysis that has been shown to agree with the stiffness model. These equations are:

For single span systems with quarter-point restraint:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.16} - \frac{0.407\, t^{0.75}\, d^{0.50} n^{0.39}}{b^{1.25}}\right] W \qquad (4.1)$$

where: C = 0.25 for braces near supports

C = 0.50 for brace at mid-span

For single span systems with third-point restraint plus restraint at the supports:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.17} - \frac{0.223\, t^{1.02}\, d^{0.85}\, n^{0.65}}{b^{1.87}}\right] W \qquad (4.2)$$

where: C = 0.16 for braces at supports

C = 0.34 for braces near mid-span

For multiple-span systems with quarter—point restraint:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.03} - \frac{0.297\, t^{0.39}\, d^{0.65}\, n^{0.39}}{b^{1.03}}\right] W \qquad (4.3)$$

where: C = 0.27 for braces nearest end supports of multiple-span systems

C = 0.54 for braces at mid-span of multiple-span systems

C = 0.21 for all other braces

For multiple span systems with third-point restraint plus restraint at the Supports:

$$P_L = C\left[\left(\frac{t}{L}\right)^{0.05} - \frac{0.112\, t^{0.57}\, d^{1.03}\, n^{0.67}}{b^{1.60}}\right]W \tag{4.4}$$

where: C = 0.18 for braces at end supports of multiple-span systems

C = 0.21 for brace at other supports of multiple-span systems

C = 0.375 for all other brace

The equations developed above are limited to roof systems having four to twenty Z-purlin lines with all top flanges facing the same direction. Restraint braces are limited to the quarter-point arrangement, and the third-point plus support arrangement. The equations are valid for flat roofs only. Modifications to these equations to account for roof slope are currently being undertaken in a related project.

For systems having less than four purlin lines, the brace force shall be determined by taking 1.1 times the force found from Equations 4.1 through 4.4 with n = 4. For systems having more than twenty purlin lines, the brace force shall be determined by Equations 4.1 through 4.4 with n = 20 and W based on total number of purlins (AISI, 1996). These were first developed by Elhouar and Murray (1985), are consistent with current design procedures (AISI, 1996), and were found to be valid for Equations 4.1 through 4.4.

4.3 RECOMMENDATIONS

A goal of this research was to produce design equations as similar as possible in form to those developed by Elhouar and Murray (1985), shown in the American Iron and Steel Institute's *Specification for the Design of Cold-Formed Steel Members,* (Section D.3.2.1.1996). Identical form equations could not be developed primarily because Elhouar and Murray (1985) did not vary purlin thickness independently of other purlin cross-sectional properties. This topic is discussed in detail in Chapter III. Therefore, in order to unify the form of the equations, it is recommended that the three bracing configurations considered by Elhouar and Murray (supports only, midpoint, and third point) be re-examined using the same procedures utilized in this research. Most importantly, purlin thickness should be varied while other purlin properties are held constant. Doing so will result in new equations for the Elhouar and Murray bracing cases identical in form to Equations 4.1 through 4.4, thus unifying the equation forms.

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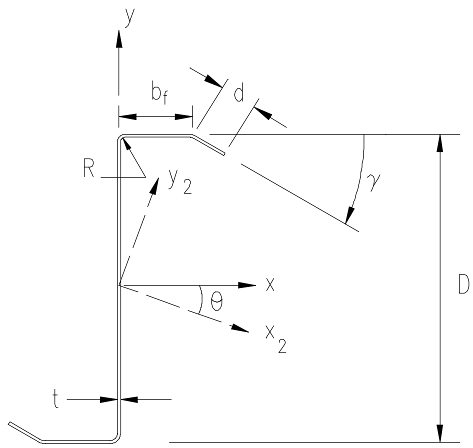
APPENDIX A: SAMPLE MODEL INPUT AND RESULTS



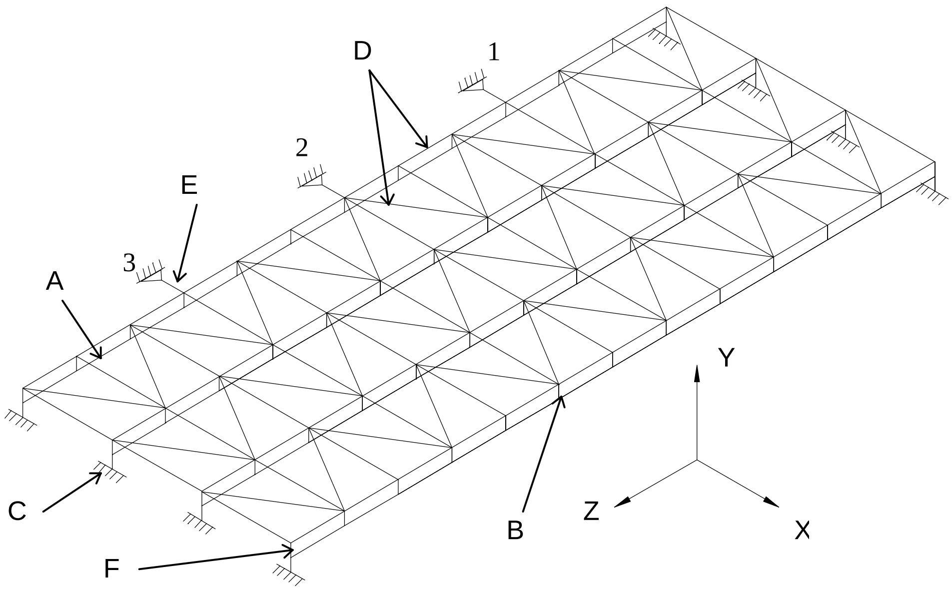
A. Purlin Cross-Sectional Geometry

ID	Dimensions								
	D in.	b_f in.	t in.	d in.	γ deg	R in.	Area in.^{2}	wt/ft lb	θ deg.
8ZS2.5x105	8.00	2.50	0.105	0.75	50	0.1875	1.466	4.98	16.7

B. Sample Test Parameters

File Name	Purlin Lines	Gravity Load K	Purlin Span ft.	Purlin Spacing ft.	Number of Spans	Restraint Type
S5204041	4	6	20	5	1	¼ pt

Figure A.1: Cross-Section and Parameters



Cross Section ID & Span	Mem ID	Area	I_y	I_z	J
		in^2	in^4	in^4	in^4
S5 25 Span	A	1.466	0.893	15.06	10
	B	2.625	0.0054	0.0024	0.893
	C	15.75	0.0054	1	0.893
	D	0.0321	0	0	0
	E	0.333	0	0	0
	F	2.625	0.0054	0.0012	0.893

Note: Properties defined about local axes, see Ch. 3.

Figure A.2 Member Numbering and Properties

Applied Load:

25 psf gravity load = 100 plf on internal purlins, 50 plf on external purlins

For model loading, member type A internal purlins:

Local y Direction, 100 plf x cos 16.7 = 95.8 plf

Local z Direction, 100 plf x sin 16.7 = 28.7 plf

Uniform Torque, 1/3 x 2.5 x 100/12 = 6.944 in-lb/in

Likewise, for external purlins:

Local y Direction, 50 plf x cos 16.7 = 47.9 plf

Local z Direction, 50 plf x sin 16.7 = 14.3 plf

Uniform Torque, 1/3 x 2.5 x 50/12 = 3.4722 in-lb/in

<u>Analysis Results:</u>

$BF_1 = BF_3 =$ 0.25 kips $\quad$ $BF_2 =$ 0.55 kips

Total Brace Force = 1.05 Kips

Brace Force Ratio = 1.05/ 6 = 0.175

APPENDIX B: SUMMARY OF MODEL LOADING

AND MODEL MEMBER PROPERTIES

Table B.1
Summary of Model Loading (RISA Input)

Name	Flng. Width	θ	θ	θ Mem. Input	Loads				Torque									
					Inside Dist. Load		Out. Dist. Load		Inside Dist.	Out. Dist.	Inside Spans				Outside Spans			
					Z	Y	Z	Y			37.5	31.25	25	20	37.5	31.25	25	20
	in.	deg.	rad.	deg.	K/ft	K/ft	K/ft	K/ft	in-lb/in	in-lb/in	K-ft	K-ft	K-ft	K-ft	K-ft	K-ft	K-ft	K-ft
S1	3.25	12.71	0.222	347.29	-0.0220	-0.0975	-0.0110	-0.0488	9.028	4.5139	-0.028	-0.024			-0.014	-0.012		
S2	3.25	12.9	0.225	347.1	-0.0223	-0.0975	-0.0112	-0.0487	9.028	4.5139	-0.028	-0.024			-0.014	-0.012		
S3	3	14.93	0.261	345.07	-0.0258	-0.0966	-0.0129	-0.0483	8.333	4.1667		-0.022	-0.017			-0.011	-0.009	
S4	3	15.14	0.264	344.86	-0.0261	-0.0965	-0.0131	-0.0483	8.333	4.1667		-0.022	-0.017			-0.011	-0.009	
S5	2.5	16.65	0.291	343.35	-0.0287	-0.0958	-0.0143	-0.0479	6.944	3.4722			-0.014	-0.012			-0.007	-0.006
S6	2.5	13.27	0.232	346.73	-0.0230	-0.0973	-0.0115	-0.0487	6.944	3.4722			-0.014	-0.012			-0.007	-0.006

Note: Input torque at purlin ends 1/2 of values shown

Table B.2
Summary of Cross Section Properties (RISA Input)

Name & Span	Mem ID	Area	I_y	I_z	J	Name & Span	Mem ID	Area	I_y	I_z	J
		in^2	in^4	in^4	in^4			in^2	in^4	in^4	in^4
S1 37.5 ft Span	A	2.613	2.38	56.36	10	S1 31.25 ft Span	A	2.613	2.38	56.36	10
	B	5.0625	0.0159	0.0077	2.38		B	4.21875	0.0159	0.0064	2.38
	C	30.375	0.0159	1	2.38		C	25.3125	0.0159	1	2.38
	D	0.0321	0	0	0		D	0.0321	0	0	0
	E	0.333	0	0	0		E	0.333	0	0	0
	F	5.0625	0.0159	0.0038	2.38		F	4.21875	0.0159	0.0032	2.38
S2 37.5 ft Span	A	1.177	1.12	25.85	10	S2 31.25 ft Span	A	1.177	1.12	25.85	10
	B	2.25	0.0014	0.00068	1.12		B	1.875	0.0014	0.00056	1.12
	C	13.5	0.0014	1	1.12		C	11.25	0.0014	1	1.12
	D	0.0321	0	0	0		D	0.0321	0	0	0
	E	0.333	0	0	0		E	0.333	0	0	0
	F	2.25	0.0014	0.0003	1.12		F	1.875	0.0014	0.0003	1.12
S3 31.25 ft Span	A	2.275	1.83	35.47	10	S3 25 ft Span	A	2.275	1.83	35.47	10
	B	4.2188	0.0138	0.00641	1.83		B	3.375	0.0138	0.00513	1.83
	C	25.313	0.0138	1	1.83		C	20.25	0.0138	1	1.83
	D	0.0321	0	0	0		D	0.0321	0	0	0
	E	0.333	0	0	0		E	0.333	0	0	0
	F	4.2188	0.0138	0.0032	1.83		F	3.375	0.0138	0.0026	1.83
S4 31.25 ft Span	A	1.027	0.86	16.35	10	S4 25 ft Span	A	1.027	0.86	16.35	10
	B	1.875	0.0012	0.00056	0.86		B	1.5	0.0012	0.00045	0.86
	C	11.25	0.0012	1	0.86		C	9	0.0012	1	0.86
	D	0.0321	0	0	0		D	0.0321	0	0	0
	E	0.333	0	0	0		E	0.333	0	0	0
	F	1.875	0.0012	0.0003	0.86		F	1.5	0.0012	0.0002	0.86
S5 25 ft Span	A	1.466	0.893	15.06	10	S5 20 ft Span	A	1.466	0.893	15.06	10
	B	2.625	0.0054	0.0024	0.893		B	2.1	0.0054	0.0019	0.893
	C	15.75	0.0054	1	0.893		C	12.6	0.0054	1	0.893
	D	0.0321	0	0	0		D	0.0321	0	0	0
	E	0.333	0	0	0		E	0.333	0	0	0
	F	2.625	0.0054	0.0012	0.893		F	2.1	0.0054	0.001	0.893
S6 25 ft Span	A	0.631	0.276	6.11	10	S6 20 ft Span	A	0.631	0.276	6.11	10
	B	1.2	0.0005	0.0002	0.276		B	0.96	0.0005	0.0002	0.276
	C	7.2	0.0005	1	0.276		C	5.76	0.0005	1	0.276
	D	0.0321	0	0	0		D	0.0321	0	0	0
	E	0.333	0	0	0		E	0.333	0	0	0
	F	1.2	0.0005	0.0001	0.276		F	0.96	0.0005	9E-05	0.276

APPENDIX C: REGRESSION ANALYSIS REPORTS

This appendix contains four regression analysis reports generated by the computer program *SigmaPlot 4.0*, the software utilized to perform the regression analyses discussed herein. Each report is automatically generated by the software package and contain statistical results that show how well the regression equation matches the sample data. A description of each parameter follows. These descriptions are taken from the *SigmaPlot 4.0* User's Manual (1997).

"*R and* R^2 The multiple ***correlation coefficient***, and R^2, the ***coefficient of determination***, are both measures of how well the regression model describes the data. R values near 1 indicate that the equation is a good description of the relation between the independent and dependent variables.

R equals 0 when the values of the independent variable does not allow any prediction of the dependent variables, and equals 1 when you can perfectly predict the dependent variables from the dependent variables.

Adjusted R^2 The adjusted R^2, R^2_{adj}, is also a measure of how well the regression model describes the data, but takes into account the number of independent variables, which reflects the degrees of freedom. Larger R^2_{adj} values (nearer to 1) indicate that the equation describes well the relation independent and dependent variables.

Standard Error of the Estimate $S_{y|x}$ The standard error of the estimate, $S_{y|x}$, is a measure of the actual variability about the regression plane of the underlying population. The underlying population generally falls within about two standard errors of the observed sample.

Statistical Summary Table The standard error, t and P values are approximations based on the final iteration of the regression.

Estimate The value for the constant and coefficients for the independent variables for the regression model listed.

Standard Error The standard errors are estimates of the uncertainties in the estimates of the regression coefficients (analogous to the standard error of the mean). The true regression coefficients of the underlying population generally fall within about two standard errors of the observed sample coefficients. Large standard errors may indicate multicollinearity.

***t* statistic** The t statistic test the null hypothesis that the coefficient of the independent variable is zero, that is, the independent variable does not contribute to predicting the dependent variable. t is the ratio of the regression coefficient to its standard error, or

$$t = \frac{\textit{regression coefficient}}{\textit{standard error of regression coefficient}}$$

You can conclude from "large" *t* values that the independent variable can be used to predict the dependent variable (i.e., that the coefficient is not zero).

***P* value** *P* is the *P* value calculated for *t*. The *P* value is the probability of being wrong in concluding that the coefficient is not zero (i.e., the probability of falsely rejecting the null hypothesis). The smaller the *P* value, the greater the probability that the coefficient is not zero.

Traditionally, you can conclude that the independent variable can be used to predict the dependent variable when $P < 0.05$.

Analysis of Variance (ANOVA) Table

The ANOVA (analysis of variance) table lists the ANOVA statistics for the regression and the corresponding F value for each step.

SS (Sum of Squares) The sum of the squares measures the variability of the dependent variable

- ♦ The sum of the squares due to the regression measures the difference of the regression plane from the mean of the dependent variable.
- ♦ the residual sum of the squares is a measure of the size of residuals, which are the differences between the observed values of the dependent variable and the values predicted by the regression model.

DF (Degrees of Freedom) Degrees of freedom represent the number of observations and variables in the regression equation.

- The regression degrees of freedom is a measure of the number of independent variables
- The residual degrees of freedom is a measure of the number of observations less the number of terms in the equations

MS (Mean Square) The mean square provides two estimates of the population variance. Comparing these variance estimates is the basis of analysis of variance.

The mean square regression is a measure of the variation of the regression from the mean of the dependent variable, or

$$\frac{\textit{sum of the squares due to regression}}{\textit{regression degrees of freedom}} = \frac{SS_{reg}}{DF_{reg}} = MS_{reg}$$

The residual mean square is a measure of the variation of the residuals about the regression plane, or

$$\frac{\textit{residual sum of squares}}{\textit{residual degrees of freedom}} = \frac{SS_{res}}{DF_{res}} = Ms_{res}$$

The residual mean square is also equal to $S^2_{y|x}$.

***F* statistic** The *F* test statistic gauges the contribution of the independent variables in predicting the dependent variable. It is:

$$\frac{\textit{regression variation from the dependent variable mean}}{\textit{residual variation about the regression}} = \frac{MS_{reg}}{MS_{res}} = F$$

If *F* is a large number, you can conclude that the independent variables contribute to the prediction of the dependent variable (i.e., at least one of coefficients is different from zero, and the "unexplained variability" is smaller than what is expected from the random sampling variability of the dependent variable about its mean). If the F ratio is around 1, you can conclude that there is an association between the variables (i.e., the data is consistent with the null hypothesis that all the samples are just randomly distributed).

***P* value** The *P* value is the probability of being wrong in concluding that there is an association between the dependent and independent variables (i.e., the probability of falsely rejecting the null hypothesis). The smaller the *P* value, the greater the probability that there is an association.

Traditionally, you can conclude that the independent variable can be used to predict the dependent variable when $P < 0.05$.

PRESS Statistic PRESS, the ***Predicted Residual Error Sum of Squares*,** is a gauge of how well a regression model predicts new data. The smaller the PRESS statistic, the better the predictive ability of the model.

. The PRESS statistic is computed by summing the squares of the prediction errors (the difference between predicted and observed values) for each observation, with that point deleted from the computation of the regression equation.

Durbin-Watson Statistic	The Durbin-Watson Statistic is a measure of the correlation between the residuals. If the residuals are not correlated, the Durbin-Watson statistic will be 2; the more this value differs from 2, the greater the likelihood that the residuals are correlated. Regression assumes that the residuals are independent of each other; the Durbin-Watson statistic checks this assumption. If the Durbin-Watson value deviates from 2 by more than 0.50, a warning appears in the report, i.e., if the Durbin-Watson Statistic is below 1.50 or above 2.50.
Normality Test	The normality test results display whether or not the data passes or fails the assumption that the source population is normally distributed around the regression, and the P value calculated by the test. All regressions require a source population to be normally distributed about the regression line. When this assumption is violated, a warning appears in the report. Failure of the normality test may indicate the presence of outlying influential points or an incorrect regression model.
Constant Variance Test	The constant variance test results display whether or not the data passed or failed the test of the assumption that the variance of the dependent variable in the source population is constant

regardless of the value of the independent variable, and the P value calculated by the test. When the constant variance assumption may be violated, a warning appears in the report.

If you receive a warning, you should consider trying a different model (i.e., one that more closely follows the pattern of data) using a weighted regression, or transforming the independent variables to stabilize the variance and obtain more accurate estimates of the parameters in the regression equation.

Power The power, or sensitivity, of a regression is the probability that the model correctly describes the relationship of the variables, if there is a relationship.

Regression power is affected by the number of observations, the chance of erroneously reporting a difference a, and the slope of the regression.

Alpha (α) Alpha (α) is the acceptable probability of incorrectly concluding that the model is correct. An α error is also called a ***Type I error*** (a Type I error is when you reject the hypothesis of no association when the hypothesis is in fact true).

The α value is set in the Options dialog; the suggested value is α=0.05 which indicates that a one in twenty chance of error is acceptable. Smaller values of α result in stricter requirements before concluding the model is correct, but greater possibility of concluding the model is bad when it is really correct (a ***Type II error***). Larger values of α make it easier to conclude that the model is correct, but also increase the risk of accepting a bad model (a Type I error).

Regression Diagnostics The regression diagnostic results display the value for the predicted values, residuals, and other diagnostic results. All results that qualify as outlying values are flagged with a < symbol.

Value This is the value of the observation.

Predicted Value This is the value of the dependent variable predicted by the regression model for each observation

Residuals These are the unweighted raw residuals, the difference between the predicted and observed values for the dependent variables.

Standardized Residuals The standardized residual is the raw residual divided by the standard error of the estimate $S_{y|x}$.

If the residuals are normally distributed about the regression, about 66% of the standardized residuals have values between -1 and +1, and about 95% of the standardized residuals have values between -2 and +2. A larger standardized residual indicates the point is far from the regression; the suggested value flagged as an outlier is 2.5.

Studentized Residuals The studentized residual is a standardized residual that also takes into account the greater confidence of the predicted values of the dependent variable in the “middle” of the data set. By weighting the values of the residuals of the extreme data points (those with the lowest ;and highest independent variable values), the studentized residual is more sensitive than the standardized residual in detecting outlier.

Both studentized and studentized deleted residuals that lie outside a specified confidence interval for the regression are flagged as outlying points; the suggested confidence value is 95%.

This residual is also known as the internally Studentized residual, because the standard error of the estimate is computed using all data.

Studentized Deleted Residuals The studentized deleted residuals, or externally studentized residual, is a studentized residual which uses the standard error of the estimate $S_{y|x(-i)}$, computed after deleting the data point associated with the residual. This reflects the greater effect of the outlying points by deleting the data from the variance computation.

Both studentized and studentized deleted residuals that lie outside a specified confidence interval for the regression are flagged as outlying points; the suggested confidence value is 95%.

The studentized deleted residual is more sensitive than the studentized residual in detecting outliers, since the studentized deleted residual results in much larger values for outliers than the studentized residual.

Influence Diagnostics

Value This is the value of the observation.

Cook's Distance Cooks' distance is a measure of how great an effect each point has on the estimate of the parameters in the regression equation. It is a measure of how much the regression coefficient would change if that point is deleted from the analysis. Values above 1 indicate that a point is possibly influential. Cook's distances exceeding 4 indicate that the point has a major effect on the values of the parameter estimates. Points with a Cook's distance greater than 4 are flagged as influential.

Leverage Leverage values identify potentially influential points. Observations with leverages a specified factor greater than the expected leverages are flagged as potentially influential points;

the suggested value is 2.0 times the expected leverage.

The expected leverage of a data point is *p/n*, where there are *p* parameters and *n* data sets.

Because leverage is calculated using only the dependent variable, high leverage points tend to be at the extremes of the independent variables (large and small values), where small changes in the independent variables can have large3 effects on the predicted values of the dependent variable.

DFFITS The DFFITS statistic is a measure of the influence of a data point on regression prediction. It is the number of estimated standard errors the predicted value for a data point changes when the observed value is removed from the data set before computing the regression coefficients.

Predicted values that change by more than the specified number of standard errors when the data point is removed are flagged as influential; the suggested value is 2.0 standard errors.

95% Confidence Intervals

If the confidence interval does not include zero, you can conclude hat the coefficient is different than zero with the level of confidence specified. This can also be described as $P > \alpha$, where α is the acceptable probability of incorrectly concluding that the coefficient is different from zero, and the confidence interval is

100(1-α).

The confidence level for both intervals is fixed at 95% (a=0.05)

Value This is the value of the observation.

Predicted Value This is the value for the dependent variable predicted by the regression model for each observation.

Regression The confidence interval for the regression gives the range of variable values computed for the region containing the true relationship between the dependent and independent variables, for the specified level of confidence.

Population The confidence interval for the population gives the range of variable values computed for the region containing the population from which the observations were drawn, for the specified level of confidence (SigmaPlot User's Manual, 1997)."

Nonlinear Regression: ¼ Point Bracing, 1 Span

[Variables]
d/b
n
y
t/b
t/L
[Parameters]
c=0
g=0
e=0
a=0
h=0
[Equations]
y=a*(t/b)^c*(d/b)^e*n^g+(t/L)^h
[Constraints]
[Options]
tolerance=0.000100
stepsize=100
iterations=100

R = 0.97701183 Rsqr = 0.95455211 Adj Rsqr = 0.95124681

Standard Error of Estimate = 0.0090

	Coefficient	**Std. Error**	**t**	**P**
c	0.7458	0.1049	7.1114	<0.0001
g	0.3875	0.0570	6.7956	<0.0001
e	0.5024	0.1483	3.3869	0.0013
a	-0.4074	0.0723	-5.6373	<0.0001
h	0.1585	0.0090	17.6022	<0.0001

Analysis of Variance:

	DF	**SS**	**MS**	**F**	**P**
Regression	4	0.0935	0.0234	288.7943	<0.0001
Residual	55	0.0045	0.0001		
Total	59	0.0979	0.0017		

PRESS = 0.0053

Durbin-Watson Statistic = 0.9085

Normality Test: Passed (P = 0.7756)

Constant Variance Test: Passed (P = 0.2487)

Power of performed test with alpha = 0.0500: 1.0000

Regression Diagnostics:

Value	Predicted	Residual	Std. Res.	Stud. Res.	Stud. Del. Res.
0.0549	0.0426	0.0122	1.3610	1.5023	1.5201
0.0679	0.0620	0.0059	0.6575	0.6973	0.6940
0.0863	0.0846	0.0017	0.1884	0.1972	0.1954
0.1128	0.1125	0.0003	0.0309	0.0325	0.0322
0.1458	0.1511	-0.0053	-0.5889	-0.6257	-0.6222
0.1186	0.1154	0.0032	0.3601	0.3808	0.3778
0.1314	0.1259	0.0054	0.6055	0.6303	0.6268
0.1445	0.1383	0.0062	0.6890	0.7081	0.7048
0.1573	0.1535	0.0038	0.4249	0.4350	0.4318
0.1680	0.1746	-0.0066	-0.7324	-0.7783	-0.7754
0.0547	0.0507	0.0040	0.4444	0.4919	0.4885
0.0676	0.0701	-0.0025	-0.2746	-0.2914	-0.2890
0.0861	0.0927	-0.0066	-0.7338	-0.7671	-0.7642
0.1074	0.1206	-0.0132	-1.4627	-1.5361	-1.5559
0.1440	0.1592	-0.0152	-1.6879	-1.7909	-1.8286
0.1137	0.1225	-0.0088	-0.9793	-1.0329	-1.0335
0.1267	0.1331	-0.0063	-0.7050	-0.7324	-0.7293
0.1402	0.1454	-0.0052	-0.5770	-0.5925	-0.5890
0.1541	0.1606	-0.0066	-0.7310	-0.7493	-0.7463
0.1643	0.1817	-0.0174	-1.9394	-2.0727	-2.1390
0.0600	0.0488	0.0112	1.2418	1.3309	1.3405
0.0745	0.0683	0.0061	0.6826	0.7059	0.7026
0.0966	0.0911	0.0055	0.6083	0.6232	0.6197
0.1294	0.1192	0.0102	1.1286	1.1684	1.1724
0.1739	0.1581	0.0157	1.7492	1.8417	1.8839
0.1319	0.1214	0.0104	1.1615	1.2093	1.2145
0.1225	0.1321	-0.0096	-1.0709	-1.1008	-1.1029
0.1603	0.1445	0.0158	1.7515	1.7831	1.8202
0.1874	0.1599	0.0275	3.0620	3.1164	3.4029
0.1952	0.1811	0.0141	1.5627	1.6589	1.6865
0.0589	0.0590	-0.0001	-0.0093	-0.0100	-0.0099
0.0749	0.0786	-0.0036	-0.4028	-0.4170	-0.4138
0.0964	0.1014	-0.0050	-0.5550	-0.5682	-0.5647
0.1291	0.1295	-0.0003	-0.0358	-0.0370	-0.0366
0.1720	0.1684	0.0036	0.4025	0.4232	0.4200
0.1244	0.1304	-0.0060	-0.6681	-0.6937	-0.6904
0.1403	0.1411	-0.0008	-0.0913	-0.0936	-0.0928
0.1575	0.1535	0.0039	0.4344	0.4419	0.4387
0.1754	0.1689	0.0065	0.7265	0.7406	0.7375
0.1907	0.1902	0.0005	0.0577	0.0616	0.0611
0.0627	0.0639	-0.0011	-0.1274	-0.1356	-0.1344
0.0781	0.0821	-0.0039	-0.4358	-0.4533	-0.4500
0.1007	0.1033	-0.0025	-0.2819	-0.2914	-0.2890
0.1331	0.1294	0.0037	0.4132	0.4293	0.4261
0.1760	0.1656	0.0104	1.1519	1.2065	1.2116
0.1276	0.1278	-0.0002	-0.0263	-0.0274	-0.0272
0.1419	0.1379	0.0039	0.4361	0.4495	0.4462
0.1571	0.1498	0.0073	0.8126	0.8300	0.8276
0.1720	0.1644	0.0076	0.8482	0.8685	0.8665
0.1827	0.1846	-0.0019	-0.2113	-0.2279	-0.2259
0.0639	0.0741	-0.0101	-1.1266	-1.2010	-1.2060
0.0793	0.0922	-0.0129	-1.4362	-1.4935	-1.5108
0.1023	0.1135	-0.0112	-1.2445	-1.2846	-1.2925
0.1350	0.1396	-0.0046	-0.5138	-0.5328	-0.5293

Regression Diagnostics:

Value	Predicted	Residual	Std. Res.	Stud. Res.	Stud. Del. Res.
0.1750	0.1758	-0.0008	-0.0931	-0.0974	-0.0965
0.1271	0.1368	-0.0097	-1.0799	-1.1253	-1.1281
0.1410	0.1470	-0.0060	-0.6621	-0.6815	-0.6781
0.1555	0.1588	-0.0033	-0.3712	-0.3792	-0.3762
0.1686	0.1734	-0.0048	-0.5346	-0.5488	-0.5453
0.1800	0.1936	-0.0136	-1.5096	-1.6415	-1.6679

Influence Diagnostics:

Value	Cook's D.	Leverage	DFFITS
0.0549	0.0985	0.1792	0.7102
0.0679	0.0121	0.1107	0.2448
0.0863	0.0007	0.0868	0.0602
0.1128	0.0000	0.0968	0.0106
0.1458	0.0101	0.1142	-0.2234
0.1186	0.0034	0.1060	0.1301
0.1314	0.0066	0.0772	0.1813
0.1445	0.0056	0.0533	0.1672
0.1573	0.0018	0.0459	0.0947
0.1680	0.0157	0.1144	-0.2787
0.0547	0.0109	0.1836	0.2317
0.0676	0.0021	0.1119	-0.1026
0.0861	0.0110	0.0851	-0.2332
0.1074	0.0486	0.0934	-0.4993
0.1440	0.0807	0.1117	-0.6486
0.1137	0.0240	0.1012	-0.3468
0.1267	0.0085	0.0736	-0.2056
0.1402	0.0038	0.0518	-0.1377
0.1541	0.0057	0.0482	-0.1680
0.1643	0.1221	0.1245	-0.8065
0.0600	0.0526	0.1294	0.5167
0.0745	0.0069	0.0648	0.1849
0.0966	0.0039	0.0473	0.1381
0.1294	0.0196	0.0669	0.3139
0.1739	0.0736	0.0979	0.6205
0.1319	0.0245	0.0774	0.3517
0.1225	0.0137	0.0534	-0.2621
0.1603	0.0232	0.0351	0.3473
0.1874	0.0696	0.0346	0.6443
0.1952	0.0698	0.1126	0.6008
0.0589	0.0000	0.1349	-0.0039
0.0749	0.0025	0.0666	-0.1106
0.0964	0.0031	0.0459	-0.1239
0.1291	0.0000	0.0634	-0.0095
0.1720	0.0038	0.0954	0.1364
0.1244	0.0075	0.0723	-0.1927
0.1403	0.0001	0.0498	-0.0212
0.1575	0.0014	0.0339	0.0822
0.1754	0.0043	0.0378	0.1461
0.1907	0.0001	0.1247	0.0231
0.0627	0.0005	0.1176	-0.0491
0.0781	0.0034	0.0757	-0.1288
0.1007	0.0012	0.0642	-0.0757
0.1331	0.0029	0.0739	0.1203
0.1760	0.0283	0.0885	0.3776

Influence Diagnostics:

Row	Cook's D.	Leverage	DFFITS
0.1276	0.0000	0.0833	-0.0082
0.1419	0.0025	0.0587	0.1114
0.1571	0.0060	0.0415	0.1722
0.1720	0.0073	0.0463	0.1909
0.1827	0.0017	0.1403	-0.0913
0.0639	0.0394	0.1201	-0.4455
0.0793	0.0363	0.0753	-0.4311
0.1023	0.0217	0.0616	-0.3310
0.1350	0.0043	0.0701	-0.1454
0.1750	0.0002	0.0870	-0.0298
0.1271	0.0217	0.0791	-0.3306
0.1410	0.0055	0.0561	-0.1654
0.1555	0.0013	0.0417	-0.0784
0.1686	0.0032	0.0511	-0.1265
0.1800	0.0983	0.1543	-0.7123

95% Confidence

Value	Predicted	Regres. 5%	Regres. 95%	Population 5%	Population 95%
0.0549	0.0426	0.0350	0.0503	0.0231	0.0622
0.0679	0.0620	0.0560	0.0680	0.0430	0.0810
0.0863	0.0846	0.0793	0.0899	0.0658	0.1034
0.1128	0.1125	0.1069	0.1181	0.0936	0.1314
0.1458	0.1511	0.1450	0.1572	0.1320	0.1701
0.1186	0.1154	0.1095	0.1212	0.0964	0.1343
0.1314	0.1259	0.1209	0.1309	0.1072	0.1446
0.1445	0.1383	0.1341	0.1424	0.1198	0.1568
0.1573	0.1535	0.1496	0.1574	0.1351	0.1719
0.1680	0.1746	0.1685	0.1807	0.1556	0.1936
0.0547	0.0507	0.0430	0.0585	0.0311	0.0704
0.0676	0.0701	0.0641	0.0761	0.0511	0.0891
0.0861	0.0927	0.0875	0.0980	0.0739	0.1115
0.1074	0.1206	0.1151	0.1261	0.1017	0.1394
0.1440	0.1592	0.1532	0.1652	0.1402	0.1782
0.1137	0.1225	0.1168	0.1282	0.1036	0.1414
0.1267	0.1331	0.1282	0.1380	0.1144	0.1517
0.1402	0.1454	0.1413	0.1495	0.1269	0.1639
0.1541	0.1606	0.1567	0.1646	0.1422	0.1791
0.1643	0.1817	0.1754	0.1881	0.1626	0.2008
0.0600	0.0488	0.0423	0.0553	0.0296	0.0679
0.0745	0.0683	0.0637	0.0729	0.0497	0.0869
0.0966	0.0911	0.0872	0.0950	0.0727	0.1096
0.1294	0.1192	0.1146	0.1239	0.1006	0.1378
0.1739	0.1581	0.1525	0.1638	0.1392	0.1770
0.1319	0.1214	0.1164	0.1264	0.1027	0.1401
0.1225	0.1321	0.1279	0.1363	0.1136	0.1506
0.1603	0.1445	0.1412	0.1479	0.1262	0.1629
0.1874	0.1599	0.1565	0.1632	0.1416	0.1782
0.1952	0.1811	0.1751	0.1872	0.1621	0.2002
0.0589	0.0590	0.0524	0.0657	0.0398	0.0782
0.0749	0.0786	0.0739	0.0832	0.0599	0.0972
0.0964	0.1014	0.0975	0.1052	0.0829	0.1198
0.1291	0.1295	0.1249	0.1340	0.1109	0.1481
0.1720	0.1684	0.1628	0.1739	0.1495	0.1872
0.1244	0.1304	0.1256	0.1353	0.1118	0.1491

95% Confidence

Value	Predicted	Regres. 5%	Regres. 95%	Population 5%	Population 95%
0.1403	0.1411	0.1371	0.1451	0.1226	0.1596
0.1575	0.1535	0.1502	0.1569	0.1352	0.1719
0.1754	0.1689	0.1654	0.1724	0.1505	0.1873
0.1907	0.1902	0.1838	0.1965	0.1710	0.2093
0.0627	0.0639	0.0577	0.0701	0.0448	0.0829
0.0781	0.0821	0.0771	0.0870	0.0634	0.1007
0.1007	0.1033	0.0987	0.1078	0.0847	0.1219
0.1331	0.1294	0.1245	0.1343	0.1107	0.1481
0.1760	0.1656	0.1603	0.1710	0.1468	0.1844
0.1276	0.1278	0.1226	0.1330	0.1091	0.1466
0.1419	0.1379	0.1336	0.1423	0.1194	0.1565
0.1571	0.1498	0.1461	0.1535	0.1314	0.1682
0.1720	0.1644	0.1605	0.1682	0.1459	0.1828
0.1827	0.1846	0.1778	0.1913	0.1653	0.2038
0.0639	0.0741	0.0678	0.0803	0.0550	0.0932
0.0793	0.0922	0.0873	0.0972	0.0736	0.1109
0.1023	0.1135	0.1090	0.1179	0.0949	0.1320
0.1350	0.1396	0.1348	0.1444	0.1210	0.1583
0.1750	0.1758	0.1705	0.1812	0.1570	0.1946
0.1271	0.1368	0.1318	0.1419	0.1181	0.1555
0.1410	0.1470	0.1427	0.1512	0.1284	0.1655
0.1555	0.1588	0.1551	0.1625	0.1404	0.1772
0.1686	0.1734	0.1693	0.1775	0.1549	0.1919
0.1800	0.1936	0.1865	0.2007	0.1742	0.2129

Nonlinear Regression 1/3 Point Bracing, 1 Span

[Variables]
d/b
n
y
t/b
t/L
[Parameters]
c=0
g=0
e=0
a=0
h=0
[Equations]
y=a*(t/b)^c*(d/b)^e*n^g+(t/L)^h
[Constraints]
[Options]
tolerance=0.000100
stepsize=100
iterations=100

R = 0.97059394 Rsqr = 0.94205260 Adj Rsqr = 0.93783824

Standard Error of Estimate = 0.0099

	Coefficient	Std. Error	t	P
c	1.0163	0.1189	8.5482	<0.0001
g	0.6517	0.0794	8.2131	<0.0001
e	0.8535	0.2250	3.7937	0.0004
a	-0.2228	0.0654	-3.4088	0.0012
h	0.1728	0.0054	32.2604	<0.0001

Analysis of Variance:

	DF	SS	MS	F	P
Regression	4	0.0876	0.0219	223.5341	<0.0001
Residual	55	0.0054	0.0001		
Total	59	0.0930	0.0016		

PRESS = 0.0064

Durbin-Watson Statistic = 0.8224

Normality Test: Passed (P = 0.5167)

Constant Variance Test: Passed (P = 0.1119)

Power of performed test with alpha = 0.0500: 1.0000

Regression Diagnostics:

Value	Predicted	Residual	Std. Res.	Stud. Res.	Stud. Del. Res.
0.0713	0.0575	0.0138	1.3979	1.5811	1.6036
0.0878	0.0830	0.0048	0.4856	0.5180	0.5145
0.1105	0.1109	-0.0004	-0.0373	-0.0392	-0.0389
0.1417	0.1423	-0.0006	-0.0641	-0.0676	-0.0670
0.1724	0.1801	-0.0077	-0.7748	-0.8115	-0.8089
0.1345	0.1313	0.0032	0.3240	0.3436	0.3409
0.1472	0.1425	0.0047	0.4789	0.4975	0.4941
0.1590	0.1547	0.0043	0.4375	0.4479	0.4446
0.1699	0.1685	0.0014	0.1432	0.1462	0.1449
0.1778	0.1850	-0.0073	-0.7339	-0.7744	-0.7715
0.0711	0.0653	0.0057	0.5787	0.6558	0.6524
0.0875	0.0909	-0.0034	-0.3457	-0.3688	-0.3659
0.1105	0.1188	-0.0082	-0.8333	-0.8755	-0.8736
0.1371	0.1502	-0.0131	-1.3217	-1.3921	-1.4044
0.1707	0.1880	-0.0173	-1.7495	-1.8318	-1.8731
0.1293	0.1381	-0.0088	-0.8841	-0.9361	-0.9351
0.1421	0.1493	-0.0072	-0.7302	-0.7579	-0.7549
0.1548	0.1615	-0.0068	-0.6858	-0.7019	-0.6987
0.1655	0.1753	-0.0098	-0.9941	-1.0155	-1.0158
0.1728	0.1919	-0.0191	-1.9289	-2.0425	-2.1052
0.0771	0.0665	0.0107	1.0789	1.1682	1.1722
0.0956	0.0919	0.0037	0.3748	0.3878	0.3848
0.1239	0.1196	0.0043	0.4373	0.4499	0.4466
0.1627	0.1508	0.0119	1.2020	1.2493	1.2559
0.2048	0.1884	0.0164	1.6584	1.7268	1.7593
0.1526	0.1386	0.0140	1.4139	1.4750	1.4913
0.1472	0.1497	-0.0025	-0.2559	-0.2627	-0.2605
0.1804	0.1619	0.0185	1.8646	1.8954	1.9426
0.2048	0.1756	0.0292	2.9493	3.0021	3.2531
0.2069	0.1921	0.0149	1.5013	1.5889	1.6118
0.0762	0.0764	-0.0002	-0.0239	-0.0259	-0.0257
0.0955	0.1018	-0.0064	-0.6442	-0.6666	-0.6632
0.1229	0.1296	-0.0067	-0.6746	-0.6934	-0.6901
0.1611	0.1608	0.0003	0.0315	0.0327	0.0324
0.2027	0.1984	0.0043	0.4343	0.4522	0.4489
0.1427	0.1473	-0.0045	-0.4581	-0.4771	-0.4738
0.1595	0.1584	0.0011	0.1065	0.1093	0.1083
0.1753	0.1706	0.0047	0.4735	0.4813	0.4779
0.1909	0.1843	0.0066	0.6642	0.6771	0.6737
0.2000	0.2007	-0.0007	-0.0757	-0.0805	-0.0798
0.0825	0.0839	-0.0014	-0.1408	-0.1515	-0.1501
0.1024	0.1068	-0.0044	-0.4437	-0.4629	-0.4596
0.1309	0.1318	-0.0009	-0.0869	-0.0901	-0.0892
0.1691	0.1599	0.0092	0.9336	0.9685	0.9679
0.2107	0.1937	0.0170	1.7143	1.7752	1.8117
0.1444	0.1446	-0.0002	-0.0224	-0.0233	-0.0231
0.1589	0.1550	0.0040	0.4007	0.4119	0.4088
0.1731	0.1662	0.0069	0.6923	0.7053	0.7020
0.1851	0.1789	0.0062	0.6272	0.6416	0.6381
0.1920	0.1942	-0.0022	-0.2208	-0.2364	-0.2343
0.0837	0.0939	-0.0102	-1.0286	-1.1080	-1.1103
0.1033	0.1167	-0.0134	-1.3538	-1.4117	-1.4248
0.1309	0.1417	-0.0108	-1.0909	-1.1290	-1.1318
0.1671	0.1698	-0.0027	-0.2724	-0.2822	-0.2798

Regression Diagnostics:

Value	Predicted	Residual	Std. Res.	Stud. Res.	Stud. Del. Res.
0.2067	0.2036	0.0030	0.3063	0.3172	0.3146
0.1432	0.1533	-0.0102	-1.0266	-1.0708	-1.0723
0.1567	0.1636	-0.0070	-0.7046	-0.7239	-0.7208
0.1691	0.1749	-0.0058	-0.5888	-0.6000	-0.5965
0.1800	0.1876	-0.0076	-0.7690	-0.7882	-0.7854
0.1867	0.2029	-0.0162	-1.6362	-1.7602	-1.7954

Influence Diagnostics:

Value	Cook's D.	Leverage	DFFITS
0.0713	0.1397	0.2183	0.8475
0.0878	0.0074	0.1211	0.1909
0.1105	0.0000	0.0960	-0.0127
0.1417	0.0001	0.1011	-0.0225
0.1724	0.0128	0.0884	-0.2519
0.1345	0.0029	0.1110	0.1204
0.1472	0.0039	0.0736	0.1393
0.1590	0.0019	0.0460	0.0977
0.1699	0.0002	0.0397	0.0295
0.1778	0.0136	0.1017	-0.2597
0.0711	0.0244	0.2213	0.3477
0.0875	0.0038	0.1213	-0.1359
0.1105	0.0159	0.0942	-0.2817
0.1371	0.0424	0.0986	-0.4645
0.1707	0.0646	0.0878	-0.5813
0.1293	0.0213	0.1082	-0.3257
0.1421	0.0089	0.0717	-0.2098
0.1548	0.0047	0.0456	-0.1527
0.1655	0.0090	0.0417	-0.2119
0.1728	0.1012	0.1082	-0.7332
0.0771	0.0470	0.1470	0.4866
0.0956	0.0021	0.0658	0.1021
0.1239	0.0024	0.0552	0.1079
0.1627	0.0251	0.0743	0.3558
0.2048	0.0502	0.0776	0.5104
0.1526	0.0384	0.0811	0.4431
0.1472	0.0008	0.0516	-0.0608
0.1804	0.0240	0.0323	0.3547
0.2048	0.0650	0.0348	0.6180
0.2069	0.0607	0.1073	0.5588
0.0762	0.0000	0.1506	-0.0108
0.0955	0.0063	0.0663	-0.1767
0.1229	0.0054	0.0534	-0.1640
0.1611	0.0000	0.0718	0.0090
0.2027	0.0034	0.0773	0.1300
0.1427	0.0039	0.0782	-0.1380
0.1595	0.0001	0.0498	0.0248
0.1753	0.0015	0.0322	0.0871
0.1909	0.0036	0.0377	0.1333
0.2000	0.0002	0.1153	-0.0288
0.0825	0.0007	0.1368	-0.0598
0.1024	0.0038	0.0811	-0.1366
0.1309	0.0001	0.0685	-0.0242
0.1691	0.0143	0.0707	0.2669
0.2107	0.0456	0.0675	0.4873

Influence Diagnostics:

Value	Cook's D.	Leverage	DFFITS
0.1444	0.0000	0.0831	-0.0070
0.1589	0.0019	0.0538	0.0975
0.1731	0.0038	0.0364	0.1365
0.1851	0.0038	0.0443	0.1374
0.1920	0.0016	0.1270	-0.0894
0.0837	0.0394	0.1382	-0.4446
0.1033	0.0348	0.0803	-0.4210
0.1309	0.0181	0.0662	-0.3014
0.1671	0.0012	0.0684	-0.0758
0.2067	0.0015	0.0681	0.0851
0.1432	0.0202	0.0808	-0.3178
0.1567	0.0058	0.0527	-0.1700
0.1691	0.0028	0.0372	-0.1173
0.1800	0.0063	0.0481	-0.1765
0.1867	0.0974	0.1359	-0.7120

95% Confidence

Value	Predicted	Regres. 5%	Regres. 95%	Population 5%	Population 95%
0.0713	0.0575	0.0482	0.0667	0.0356	0.0794
0.0878	0.0830	0.0761	0.0899	0.0620	0.1040
0.1105	0.1109	0.1048	0.1171	0.0901	0.1317
0.1417	0.1423	0.1360	0.1487	0.1215	0.1632
0.1724	0.1801	0.1742	0.1860	0.1594	0.2008
0.1345	0.1313	0.1246	0.1379	0.1103	0.1522
0.1472	0.1425	0.1371	0.1478	0.1219	0.1630
0.1590	0.1547	0.1504	0.1590	0.1344	0.1750
0.1699	0.1685	0.1645	0.1724	0.1483	0.1887
0.1778	0.1850	0.1787	0.1914	0.1642	0.2059
0.0711	0.0653	0.0560	0.0747	0.0434	0.0873
0.0875	0.0909	0.0840	0.0978	0.0699	0.1119
0.1105	0.1188	0.1127	0.1249	0.0980	0.1396
0.1371	0.1502	0.1440	0.1565	0.1294	0.1710
0.1707	0.1880	0.1821	0.1939	0.1673	0.2087
0.1293	0.1381	0.1316	0.1446	0.1172	0.1590
0.1421	0.1493	0.1440	0.1546	0.1288	0.1698
0.1548	0.1615	0.1573	0.1658	0.1413	0.1818
0.1655	0.1753	0.1713	0.1794	0.1551	0.1956
0.1728	0.1919	0.1854	0.1984	0.1710	0.2128
0.0771	0.0665	0.0589	0.0741	0.0452	0.0877
0.0956	0.0919	0.0868	0.0969	0.0714	0.1123
0.1239	0.1196	0.1149	0.1243	0.0992	0.1400
0.1627	0.1508	0.1454	0.1562	0.1303	0.1714
0.2048	0.1884	0.1829	0.1939	0.1678	0.2090
0.1526	0.1386	0.1329	0.1442	0.1180	0.1592
0.1472	0.1497	0.1452	0.1542	0.1294	0.1701
0.1804	0.1619	0.1583	0.1655	0.1417	0.1821
0.2048	0.1756	0.1719	0.1793	0.1554	0.1958
0.2069	0.1921	0.1856	0.1986	0.1712	0.2129
0.0762	0.0764	0.0687	0.0841	0.0552	0.0977
0.0955	0.1018	0.0967	0.1070	0.0814	0.1223
0.1229	0.1296	0.1250	0.1342	0.1092	0.1500
0.1611	0.1608	0.1555	0.1661	0.1403	0.1814
0.2027	0.1984	0.1929	0.2039	0.1778	0.2190
0.1427	0.1473	0.1417	0.1528	0.1267	0.1679

95% Confidence

Value	Predicted	Regres. 5%	Regres. 95%	Population 5%	Population 95%
0.1595	0.1584	0.1540	0.1628	0.1381	0.1787
0.1753	0.1706	0.1670	0.1741	0.1504	0.1907
0.1909	0.1843	0.1804	0.1881	0.1641	0.2045
0.2000	0.2007	0.1940	0.2075	0.1798	0.2217
0.0825	0.0839	0.0766	0.0913	0.0628	0.1051
0.1024	0.1068	0.1011	0.1124	0.0862	0.1274
0.1309	0.1318	0.1266	0.1370	0.1113	0.1523
0.1691	0.1599	0.1546	0.1652	0.1394	0.1804
0.2107	0.1937	0.1885	0.1989	0.1732	0.2142
0.1444	0.1446	0.1389	0.1504	0.1240	0.1653
0.1589	0.1550	0.1504	0.1596	0.1346	0.1753
0.1731	0.1662	0.1624	0.1700	0.1460	0.1864
0.1851	0.1789	0.1748	0.1831	0.1587	0.1992
0.1920	0.1942	0.1871	0.2013	0.1731	0.2152
0.0837	0.0939	0.0865	0.1012	0.0727	0.1150
0.1033	0.1167	0.1111	0.1224	0.0961	0.1374
0.1309	0.1417	0.1366	0.1468	0.1212	0.1622
0.1671	0.1698	0.1646	0.1750	0.1493	0.1903
0.2067	0.2036	0.1985	0.2088	0.1831	0.2241
0.1432	0.1533	0.1477	0.1590	0.1327	0.1739
0.1567	0.1636	0.1591	0.1682	0.1433	0.1840
0.1691	0.1749	0.1711	0.1787	0.1547	0.1951
0.1800	0.1876	0.1833	0.1920	0.1673	0.2079
0.1867	0.2029	0.1956	0.2102	0.1817	0.2240

Nonlinear Regression: ¼ Point Bracing, 3 Spans

[Variables]
d/b
n
y
t/b
t/L
[Parameters]
c=0
g=0
e=0
a=0
h=0
[Equations]
y=a*(t/b)^c*(d/b)^e*n^g+(t/L)^h
[Constraints]
[Options]
tolerance=0.000100
stepsize=100
iterations=100

R = 0.97839712 Rsqr = 0.95726092 Adj Rsqr = 0.95415262

Standard Error of Estimate = 0.0236

	Coefficient	**Std. Error**	**t**	**P**
c	0.3855	0.0758	5.0870	<0.0001
g	0.3890	0.0661	5.8821	<0.0001
e	0.6479	0.1395	4.6437	<0.0001
a	-0.2967	0.0667	-4.4483	<0.0001
h	0.0343	0.0112	3.0757	0.0033

Analysis of Variance:

	DF	**SS**	**MS**	**F**	**P**
Regression	4	0.6883	0.1721	307.9696	<0.0001
Residual	55	0.0307	0.0006		
Total	59	0.7190	0.0122		

PRESS = 0.0370

Durbin-Watson Statistic = 1.2439

Normality Test: Passed (P = 0.7788)

Constant Variance Test: Passed (P = 0.1081)

Power of performed test with alpha = 0.0500: 1.0000

Regression Diagnostics:

Value	Predicted	Residual	Std. Res.	Stud. Res.	Stud. Del. Res.
0.1413	0.1062	0.0351	1.4847	1.6542	1.6815
0.1764	0.1604	0.0160	0.6772	0.7168	0.7136
0.2284	0.2235	0.0048	0.2040	0.2125	0.2106
0.3055	0.3014	0.0041	0.1744	0.1826	0.1810
0.4089	0.4091	-0.0002	-0.0091	-0.0096	-0.0095
0.2867	0.2602	0.0265	1.1226	1.1891	1.1936
0.2674	0.2998	-0.0324	-1.3712	-1.4314	-1.4455
0.2933	0.3460	-0.0527	-2.2287	-2.3007	-2.3980
0.4107	0.4030	0.0077	0.3258	0.3351	0.3324
0.4551	0.4818	-0.0266	-1.1273	-1.2119	-1.2172
0.1344	0.1110	0.0234	0.9906	1.0906	1.0925
0.1662	0.1651	0.0011	0.0460	0.0485	0.0481
0.2159	0.2283	-0.0124	-0.5262	-0.5497	-0.5461
0.2921	0.3061	-0.0140	-0.5939	-0.6264	-0.6229
0.3968	0.4139	-0.0171	-0.7215	-0.7645	-0.7615
0.2715	0.2648	0.0067	0.2820	0.3000	0.2975
0.3127	0.3044	0.0083	0.3527	0.3699	0.3670
0.3552	0.3506	0.0046	0.1932	0.2001	0.1984
0.3881	0.4076	-0.0195	-0.8240	-0.8468	-0.8445
0.4245	0.4864	-0.0618	-2.6165	-2.7767	-2.9671
0.1469	0.1336	0.0132	0.5598	0.6012	0.5977
0.1824	0.1859	-0.0035	-0.1465	-0.1514	-0.1501
0.2397	0.2468	-0.0071	-0.3021	-0.3089	-0.3064
0.3323	0.3220	0.0103	0.4374	0.4509	0.4476
0.4597	0.4260	0.0338	1.4286	1.4884	1.5054
0.2779	0.2814	-0.0035	-0.1480	-0.1532	-0.1518
0.3424	0.3196	0.0228	0.9646	0.9880	0.9878
0.3991	0.3642	0.0349	1.4772	1.5028	1.5206
0.4640	0.4192	0.0448	1.8958	1.9327	1.9836
0.5227	0.4952	0.0274	1.1607	1.2496	1.2561
0.1368	0.1395	-0.0026	-0.1118	-0.1186	-0.1176
0.1701	0.1917	-0.0216	-0.9133	-0.9411	-0.9401
0.2236	0.2527	-0.0291	-1.2296	-1.2631	-1.2701
0.3131	0.3279	-0.0147	-0.6226	-0.6476	-0.6441
0.4440	0.4318	0.0122	0.5155	0.5399	0.5364
0.2789	0.2871	-0.0081	-0.3440	-0.3577	-0.3548
0.3184	0.3253	-0.0069	-0.2916	-0.3000	-0.2975
0.3727	0.3699	0.0028	0.1195	0.1219	0.1208
0.4314	0.4249	0.0066	0.2771	0.2818	0.2795
0.4880	0.5009	-0.0129	-0.5469	-0.5786	-0.5751
0.1629	0.1653	-0.0024	-0.1015	-0.1082	-0.1072
0.2013	0.2149	-0.0136	-0.5735	-0.5947	-0.5912
0.2640	0.2727	-0.0087	-0.3689	-0.3796	-0.3766
0.3606	0.3440	0.0166	0.7014	0.7244	0.7212
0.5000	0.4426	0.0574	2.4292	2.5248	2.6607
0.3152	0.3004	0.0148	0.6256	0.6533	0.6498
0.3536	0.3370	0.0166	0.7021	0.7243	0.7211
0.4040	0.3798	0.0242	1.0249	1.0492	1.0501
0.4571	0.4325	0.0247	1.0434	1.0722	1.0737
0.5040	0.5054	-0.0014	-0.0586	-0.0644	-0.0638
0.1537	0.1712	-0.0175	-0.7411	-0.7846	-0.7818
0.1900	0.2207	-0.0307	-1.3002	-1.3486	-1.3589
0.2491	0.2786	-0.0295	-1.2471	-1.2904	-1.2984
0.3436	0.3498	-0.0063	-0.2652	-0.2761	-0.2738

Regression Diagnostics:

Value	Predicted	Residual	Std. Res.	Stud. Res.	Stud. Del. Res.
0.4767	0.4484	0.0282	1.1948	1.2451	1.2514
0.2968	0.3061	-0.0092	-0.3903	-0.4092	-0.4061
0.3330	0.3427	-0.0097	-0.4103	-0.4247	-0.4215
0.3795	0.3855	-0.0059	-0.2503	-0.2565	-0.2543
0.4279	0.4382	-0.0103	-0.4362	-0.4462	-0.4429
0.4717	0.5111	-0.0394	-1.6671	-1.7940	-1.8320

Influence Diagnostics:

Value	Cook's D.	Leverage	DFFITS
0.1413	0.1321	0.1945	0.8262
0.1764	0.0124	0.1075	0.2477
0.2284	0.0008	0.0776	0.0611
0.3055	0.0006	0.0874	0.0560
0.4089	0.0000	0.1000	-0.0032
0.2867	0.0345	0.1086	0.4166
0.2674	0.0368	0.0824	-0.4331
0.2933	0.0695	0.0616	-0.6146
0.4107	0.0013	0.0546	0.0799
0.4551	0.0457	0.1347	-0.4802
0.1344	0.0504	0.1749	0.5031
0.1662	0.0001	0.1017	0.0162
0.2159	0.0055	0.0837	-0.1651
0.2921	0.0088	0.1012	-0.2090
0.3968	0.0143	0.1092	-0.2667
0.2715	0.0024	0.1164	0.1080
0.3127	0.0027	0.0906	0.1158
0.3552	0.0006	0.0676	0.0534
0.3881	0.0080	0.0529	-0.1997
0.4245	0.1946	0.1121	-1.0540
0.1469	0.0111	0.1331	0.2342
0.1824	0.0003	0.0634	-0.0390
0.2397	0.0009	0.0440	-0.0657
0.3323	0.0026	0.0592	0.1122
0.4597	0.0379	0.0787	0.4401
0.2779	0.0003	0.0666	-0.0406
0.3424	0.0096	0.0468	0.2190
0.3991	0.0158	0.0338	0.2843
0.4640	0.0294	0.0378	0.3932
0.5227	0.0497	0.1373	0.5011
0.1368	0.0004	0.1121	-0.0418
0.1701	0.0110	0.0582	-0.2338
0.2236	0.0176	0.0523	-0.2984
0.3131	0.0069	0.0757	-0.1844
0.4440	0.0057	0.0886	0.1672
0.2789	0.0021	0.0750	-0.1011
0.3184	0.0011	0.0554	-0.0720
0.3727	0.0001	0.0390	0.0243
0.4314	0.0005	0.0332	0.0518
0.4880	0.0080	0.1066	-0.1987
0.1629	0.0003	0.1205	-0.0397
0.2013	0.0053	0.0702	-0.1625
0.2640	0.0017	0.0554	-0.0912
0.3606	0.0070	0.0624	0.1860
0.5000	0.1024	0.0743	0.7539

Influence Diagnostics:

Value	**Cook's D.**	**Leverage**	**DFFITS**
0.3152	0.0077	0.0829	0.1953
0.3536	0.0068	0.0604	0.1829
0.4040	0.0106	0.0457	0.2299
0.4571	0.0129	0.0531	0.2542
0.5040	0.0002	0.1725	-0.0291
0.1537	0.0149	0.1078	-0.2718
0.1900	0.0276	0.0704	-0.3740
0.2491	0.0235	0.0659	-0.3450
0.3436	0.0013	0.0777	-0.0795
0.4767	0.0266	0.0791	0.3667
0.2968	0.0033	0.0903	-0.1279
0.3330	0.0026	0.0669	-0.1129
0.3795	0.0007	0.0478	-0.0570
0.4279	0.0018	0.0441	-0.0951
0.4717	0.1017	0.1364	-0.7280

95% Confidence

Value	**Predicted**	**Regres. 5%**	**Regres. 95%**	**Population 5%**	**Population 95%**
0.1413	0.1062	0.0853	0.1271	0.0545	0.1580
0.1764	0.1604	0.1448	0.1759	0.1105	0.2102
0.2284	0.2235	0.2103	0.2367	0.1744	0.2727
0.3055	0.3014	0.2874	0.3154	0.2520	0.3508
0.4089	0.4091	0.3941	0.4241	0.3594	0.4588
0.2867	0.2602	0.2446	0.2758	0.2103	0.3101
0.2674	0.2998	0.2862	0.3134	0.2505	0.3491
0.2933	0.3460	0.3343	0.3578	0.2972	0.3948
0.4107	0.4030	0.3919	0.4140	0.3543	0.4516
0.4551	0.4818	0.4644	0.4991	0.4313	0.5322
0.1344	0.1110	0.0912	0.1308	0.0596	0.1623
0.1662	0.1651	0.1500	0.1802	0.1154	0.2148
0.2159	0.2283	0.2146	0.2420	0.1790	0.2776
0.2921	0.3061	0.2911	0.3212	0.2564	0.3559
0.3968	0.4139	0.3982	0.4295	0.3640	0.4637
0.2715	0.2648	0.2487	0.2810	0.2148	0.3149
0.3127	0.3044	0.2902	0.3187	0.2549	0.3539
0.3552	0.3506	0.3383	0.3629	0.3017	0.3996
0.3881	0.4076	0.3967	0.4185	0.3590	0.4562
0.4245	0.4864	0.4705	0.5022	0.4364	0.5363
0.1469	0.1336	0.1163	0.1509	0.0832	0.1841
0.1824	0.1859	0.1739	0.1978	0.1370	0.2347
0.2397	0.2468	0.2369	0.2568	0.1984	0.2953
0.3323	0.3220	0.3105	0.3335	0.2732	0.3708
0.4597	0.4260	0.4127	0.4393	0.3768	0.4752
0.2779	0.2814	0.2692	0.2936	0.2325	0.3303
0.3424	0.3196	0.3093	0.3299	0.2711	0.3681
0.3991	0.3642	0.3555	0.3729	0.3160	0.4124
0.4640	0.4192	0.4100	0.4284	0.3709	0.4674
0.5227	0.4952	0.4777	0.5128	0.4447	0.5458
0.1368	0.1395	0.1236	0.1553	0.0895	0.1894
0.1701	0.1917	0.1803	0.2032	0.1430	0.2404
0.2236	0.2527	0.2419	0.2635	0.2041	0.3013
0.3131	0.3279	0.3148	0.3409	0.2787	0.3770
0.4440	0.4318	0.4177	0.4459	0.3824	0.4812
0.2789	0.2871	0.2741	0.3001	0.2380	0.3362

95% Confidence

Value	Predicted	Regres. 5%	Regres. 95%	Population 5%	Population 95%
0.3184	0.3253	0.3141	0.3364	0.2766	0.3740
0.3727	0.3699	0.3605	0.3793	0.3216	0.4182
0.4314	0.4249	0.4162	0.4335	0.3767	0.4730
0.4880	0.5009	0.4855	0.5164	0.4511	0.5508
0.1629	0.1653	0.1489	0.1818	0.1152	0.2155
0.2013	0.2149	0.2023	0.2274	0.1659	0.2639
0.2640	0.2727	0.2616	0.2839	0.2241	0.3214
0.3606	0.3440	0.3322	0.3558	0.2952	0.3928
0.5000	0.4426	0.4297	0.4555	0.3935	0.4917
0.3152	0.3004	0.2867	0.3140	0.2511	0.3497
0.3536	0.3370	0.3254	0.3487	0.2882	0.3858
0.4040	0.3798	0.3696	0.3899	0.3313	0.4282
0.4571	0.4325	0.4216	0.4434	0.3839	0.4811
0.5040	0.5054	0.4857	0.5251	0.4541	0.5567
0.1537	0.1712	0.1556	0.1868	0.1213	0.2211
0.1900	0.2207	0.2082	0.2333	0.1717	0.2697
0.2491	0.2786	0.2664	0.2907	0.2297	0.3275
0.3436	0.3498	0.3366	0.3630	0.3007	0.3990
0.4767	0.4484	0.4351	0.4617	0.3992	0.4976
0.2968	0.3061	0.2918	0.3203	0.2566	0.3555
0.3330	0.3427	0.3304	0.3550	0.2938	0.3916
0.3795	0.3855	0.3751	0.3958	0.3370	0.4340
0.4279	0.4382	0.4282	0.4481	0.3898	0.4866
0.4717	0.5111	0.4936	0.5286	0.4606	0.5616

Nonlinear Regression: 1/3 Point + Ends Bracing, 3 Spans

[Variables]
d/b
n
y
t/b
t/L
[Parameters]
c=0
g=0
e=0
a=0
h=0
[Equations]
y=a*(t/b)^c*(d/b)^e*n^g+(t/L)^h
[Constraints]
[Options]
tolerance=0.000100
stepsize=100
iterations=100

R = 0.96599375 Rsqr = 0.93314392 Adj Rsqr = 0.92828166

Standard Error of Estimate = 0.0302

	Coefficient	**Std. Error**	**t**	**P**
c	0.5655	0.0918	6.1574	<0.0001
g	0.6653	0.1055	6.3086	<0.0001
e	1.0298	0.2445	4.2123	<0.0001
a	-0.1120	0.0432	-2.5939	0.0121
h	0.0524	0.0074	7.0432	<0.0001

Analysis of Variance:

	DF	**SS**	**MS**	**F**	**P**
Regression	4	0.7006	0.1752	191.9157	<0.0001
Residual	55	0.0502	0.0009		
Total	59	0.7508	0.0127		

PRESS = 0.0602

Durbin-Watson Statistic = 1.1173

Normality Test: Passed (P = 0.0891)

Constant Variance Test: Passed (P = 0.0701)

Power of performed test with alpha = 0.0500: 1.0000

Regression Diagnostics:

Value	Predicted	Residual	Std. Res.	Stud. Res.	Stud. Del. Res.
0.1774	0.1321	0.0453	1.4999	1.7062	1.7373
0.2180	0.2041	0.0139	0.4599	0.4882	0.4848
0.2812	0.2824	-0.0012	-0.0398	-0.0417	-0.0413
0.3710	0.3702	0.0008	0.0272	0.0287	0.0284
0.4782	0.4750	0.0032	0.1065	0.1109	0.1099
0.3279	0.2968	0.0310	1.0275	1.0969	1.0990
0.2965	0.3423	-0.0458	-1.5157	-1.5843	-1.6069
0.3505	0.3918	-0.0413	-1.3670	-1.4080	-1.4210
0.4450	0.4474	-0.0024	-0.0798	-0.0817	-0.0810
0.4782	0.5136	-0.0354	-1.1710	-1.2428	-1.2492
0.1681	0.1384	0.0297	0.9837	1.1140	1.1165
0.2069	0.2103	-0.0034	-0.1129	-0.1198	-0.1187
0.2676	0.2887	-0.0210	-0.6966	-0.7319	-0.7287
0.3566	0.3765	-0.0199	-0.6597	-0.6970	-0.6937
0.4565	0.4813	-0.0247	-0.8191	-0.8539	-0.8518
0.3085	0.3028	0.0057	0.1893	0.2023	0.2006
0.3503	0.3483	0.0020	0.0647	0.0678	0.0671
0.3916	0.3979	-0.0063	-0.2086	-0.2150	-0.2131
0.4178	0.4534	-0.0355	-1.1765	-1.2036	-1.2087
0.4459	0.5196	-0.0737	-2.4408	-2.5788	-2.7253
0.1829	0.1688	0.0141	0.4678	0.5045	0.5011
0.2270	0.2366	-0.0096	-0.3168	-0.3270	-0.3243
0.2973	0.3103	-0.0130	-0.4312	-0.4434	-0.4402
0.4078	0.3930	0.0147	0.4878	0.5054	0.5020
0.5461	0.4917	0.0544	1.8016	1.8611	1.9050
0.3493	0.3221	0.0273	0.9020	0.9380	0.9370
0.3947	0.3649	0.0298	0.9851	1.0101	1.0103
0.4532	0.4115	0.0417	1.3800	1.4032	1.4160
0.5129	0.4638	0.0491	1.6247	1.6542	1.6814
0.5547	0.5262	0.0285	0.9423	1.0058	1.0059
0.1718	0.1765	-0.0047	-0.1571	-0.1687	-0.1673
0.2139	0.2443	-0.0305	-1.0079	-1.0403	-1.0411
0.2800	0.3181	-0.0381	-1.2611	-1.3000	-1.3084
0.3851	0.4008	-0.0157	-0.5182	-0.5387	-0.5352
0.5253	0.4995	0.0259	0.8562	0.8846	0.8828
0.3234	0.3295	-0.0061	-0.2028	-0.2111	-0.2093
0.3653	0.3723	-0.0070	-0.2323	-0.2385	-0.2364
0.4204	0.4190	0.0014	0.0455	0.0463	0.0458
0.4743	0.4713	0.0030	0.1002	0.1018	0.1009
0.5173	0.5336	-0.0163	-0.5398	-0.5725	-0.5690
0.2059	0.2061	-0.0002	-0.0056	-0.0060	-0.0059
0.2544	0.2686	-0.0142	-0.4688	-0.4872	-0.4838
0.3309	0.3366	-0.0057	-0.1883	-0.1945	-0.1928
0.4457	0.4129	0.0329	1.0875	1.1237	1.1264
0.5893	0.5038	0.0855	2.8297	2.9180	3.1451
0.3600	0.3416	0.0184	0.6077	0.6366	0.6331
0.3989	0.3818	0.0171	0.5673	0.5848	0.5812
0.4487	0.4255	0.0232	0.7693	0.7853	0.7825
0.4937	0.4745	0.0192	0.6368	0.6520	0.6485
0.5307	0.5329	-0.0022	-0.0743	-0.0804	-0.0797
0.1947	0.2138	-0.0191	-0.6311	-0.6763	-0.6729
0.2400	0.2763	-0.0363	-1.2020	-1.2501	-1.2567
0.3136	0.3443	-0.0307	-1.0164	-1.0527	-1.0538
0.4200	0.4206	-0.0006	-0.0200	-0.0207	-0.0206

Regression Diagnostics:

Value	Predicted	Residual	Std. Res.	Stud. Res.	Stud. Del. Res.
0.5633	0.5116	0.0517	1.7126	1.7644	1.7999
0.3374	0.3491	-0.0117	-0.3876	-0.4065	-0.4034
0.3767	0.3892	-0.0126	-0.4157	-0.4289	-0.4257
0.4200	0.4329	-0.0129	-0.4279	-0.4368	-0.4336
0.4614	0.4819	-0.0205	-0.6779	-0.6925	-0.6892
0.4900	0.5404	-0.0504	-1.6667	-1.7912	-1.8289

Influence Diagnostics:

Value	Cook's D.	Leverage	DFFITS
0.1774	0.1712	0.2272	0.9421
0.2180	0.0061	0.1129	0.1729
0.2812	0.0000	0.0902	-0.0130
0.3710	0.0000	0.0986	0.0094
0.4782	0.0002	0.0786	0.0321
0.3279	0.0336	0.1225	0.4106
0.2965	0.0465	0.0847	-0.4889
0.3505	0.0242	0.0574	-0.3507
0.4450	0.0001	0.0456	-0.0177
0.4782	0.0391	0.1123	-0.4442
0.1681	0.0701	0.2202	0.5933
0.2069	0.0004	0.1121	-0.0422
0.2676	0.0111	0.0940	-0.2347
0.3566	0.0113	0.1041	-0.2365
0.4565	0.0126	0.0798	-0.2508
0.3085	0.0012	0.1247	0.0757
0.3503	0.0001	0.0873	0.0208
0.3916	0.0006	0.0592	-0.0535
0.4178	0.0135	0.0445	-0.2608
0.4459	0.1546	0.1041	-0.9291
0.1829	0.0083	0.1401	0.2023
0.2270	0.0014	0.0613	-0.0829
0.2973	0.0023	0.0543	-0.1054
0.4078	0.0038	0.0687	0.1363
0.5461	0.0465	0.0629	0.4936
0.3493	0.0143	0.0753	0.2673
0.3947	0.0105	0.0489	0.2290
0.4532	0.0134	0.0328	0.2609
0.5129	0.0200	0.0353	0.3216
0.5547	0.0282	0.1222	0.3754
0.1718	0.0009	0.1331	-0.0655
0.2139	0.0141	0.0612	-0.2658
0.2800	0.0212	0.0590	-0.3276
0.3851	0.0047	0.0748	-0.1522
0.5253	0.0106	0.0632	0.2293
0.3234	0.0007	0.0776	-0.0607
0.3653	0.0006	0.0513	-0.0550
0.4204	0.0000	0.0340	0.0086
0.4743	0.0001	0.0327	0.0185
0.5173	0.0082	0.1109	-0.2009
0.2059	0.0000	0.1331	-0.0023
0.2544	0.0038	0.0741	-0.1368
0.3309	0.0005	0.0630	-0.0500
0.4457	0.0171	0.0634	0.2930
0.5893	0.1080	0.0596	0.7919

Influence Diagnostics:

Row	Cook's D.	Leverage	DFFITS
0.3600	0.0079	0.0889	0.1977
0.3989	0.0043	0.0588	0.1453
0.4487	0.0052	0.0404	0.1606
0.4937	0.0041	0.0460	0.1423
0.5307	0.0002	0.1471	-0.0331
0.1947	0.0136	0.1291	-0.2591
0.2400	0.0255	0.0755	-0.3590
0.3136	0.0161	0.0678	-0.2842
0.4200	0.0000	0.0683	-0.0056
0.5633	0.0382	0.0578	0.4460
0.3374	0.0033	0.0908	-0.1275
0.3767	0.0024	0.0605	-0.1080
0.4200	0.0016	0.0404	-0.0890
0.4614	0.0042	0.0419	-0.1441
0.4900	0.0994	0.1342	-0.7200

95% Confidence

Value	Predicted	Regres. 5%	Regres. 95%	Population 5%	Population 95%
0.1774	0.1321	0.1032	0.1609	0.0650	0.1992
0.2180	0.2041	0.1837	0.2244	0.1402	0.2679
0.2812	0.2824	0.2642	0.3006	0.2192	0.3456
0.3710	0.3702	0.3512	0.3892	0.3068	0.4337
0.4782	0.4750	0.4580	0.4920	0.4121	0.5379
0.3279	0.2968	0.2756	0.3180	0.2327	0.3610
0.2965	0.3423	0.3247	0.3599	0.2793	0.4054
0.3505	0.3918	0.3773	0.4064	0.3296	0.4541
0.4450	0.4474	0.4344	0.4603	0.3855	0.5093
0.4782	0.5136	0.4933	0.5339	0.4497	0.5774
0.1681	0.1384	0.1099	0.1668	0.0715	0.2052
0.2069	0.2103	0.1901	0.2306	0.1465	0.2742
0.2676	0.2887	0.2701	0.3072	0.2254	0.3520
0.3566	0.3765	0.3570	0.3960	0.3129	0.4401
0.4565	0.4813	0.4642	0.4984	0.4184	0.5442
0.3085	0.3028	0.2815	0.3242	0.2386	0.3670
0.3503	0.3483	0.3304	0.3662	0.2852	0.4115
0.3916	0.3979	0.3831	0.4126	0.3356	0.4602
0.4178	0.4534	0.4406	0.4661	0.3915	0.5153
0.4459	0.5196	0.5001	0.5391	0.4560	0.5832
0.1829	0.1688	0.1461	0.1914	0.1041	0.2334
0.2270	0.2366	0.2216	0.2516	0.1742	0.2989
0.2973	0.3103	0.2962	0.3244	0.2482	0.3725
0.4078	0.3930	0.3772	0.4089	0.3304	0.4556
0.5461	0.4917	0.4765	0.5069	0.4293	0.5541
0.3493	0.3221	0.3054	0.3387	0.2593	0.3848
0.3947	0.3649	0.3515	0.3783	0.3029	0.4269
0.4532	0.4115	0.4006	0.4225	0.3500	0.4731
0.5129	0.4638	0.4525	0.4752	0.4022	0.5254
0.5547	0.5262	0.5050	0.5474	0.4621	0.5903
0.1718	0.1765	0.1545	0.1986	0.1121	0.2410
0.2139	0.2443	0.2293	0.2593	0.1820	0.3067
0.2800	0.3181	0.3034	0.3328	0.2558	0.3804
0.3851	0.4008	0.3842	0.4174	0.3380	0.4636
0.5253	0.4995	0.4842	0.5147	0.4370	0.5619
0.3234	0.3295	0.3126	0.3464	0.2666	0.3923

95% Confidence

Value	Predicted	Regres. 5%	Regres. 95%	Population 5%	Population 95%
0.3653	0.3723	0.3586	0.3861	0.3103	0.4344
0.4204	0.4190	0.4078	0.4301	0.3574	0.4806
0.4743	0.4713	0.4603	0.4822	0.4097	0.5328
0.5173	0.5336	0.5135	0.5538	0.4698	0.5975
0.2059	0.2061	0.1840	0.2281	0.1416	0.2705
0.2544	0.2686	0.2521	0.2850	0.2058	0.3313
0.3309	0.3366	0.3214	0.3518	0.2742	0.3990
0.4457	0.4129	0.3976	0.4281	0.3504	0.4753
0.5893	0.5038	0.4891	0.5186	0.4415	0.5662
0.3600	0.3416	0.3236	0.3597	0.2785	0.4048
0.3989	0.3818	0.3671	0.3965	0.3195	0.4441
0.4487	0.4255	0.4133	0.4377	0.3637	0.4872
0.4937	0.4745	0.4615	0.4875	0.4126	0.5364
0.5307	0.5329	0.5097	0.5561	0.4681	0.5978
0.1947	0.2138	0.1921	0.2356	0.1495	0.2781
0.2400	0.2763	0.2597	0.2929	0.2135	0.3391
0.3136	0.3443	0.3286	0.3601	0.2818	0.4069
0.4200	0.4206	0.4048	0.4364	0.3580	0.4832
0.5633	0.5116	0.4970	0.5262	0.4493	0.5739
0.3374	0.3491	0.3308	0.3673	0.2858	0.4123
0.3767	0.3892	0.3743	0.4041	0.3269	0.4516
0.4200	0.4329	0.4208	0.4451	0.3712	0.4947
0.4614	0.4819	0.4695	0.4943	0.4201	0.5437
0.4900	0.5404	0.5182	0.5625	0.4759	0.6048