A STOCHASTIC MODEL OF COMPONENT FAILURE MECHANISMS

by

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(ABSTRACT)

The progress of a unimolecular chemical degradation reaction is used in representing a component failure mechanism. The component is said to fail when the concentration of reaction product accumulates beyond an acceptable level. The process of accumulating reaction product is modelled as a Markov pure birth process which, in turn, is used in developing the failure time distribution. The model is analyzed under the assumption that the reaction rate is constant. Also, the initial state and the final state of the degradation process are assumed to be Poisson variables. Based on numerical examples, it is found that the failure model can be described as a three-parameter or two-parameter Weibull distribution.

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I. INTRODUCTION

In general, there are many failure mechanisms which can initiate different failure modes in a component. These modes, in turn, usually have different effects on the overall system. Consequently, a detailed study of each failure mechanism or at least the most prevalent ones is appropriate and essential.

In this thesis, a component failure mechanism corresponding to the progress of a unimolecular chemical degradation reaction is investigated. An example of such a mechanism is the oxidation of gallium arsenide in LCD's. Another example is the oxidation of the copper "lands" in integrated circuits. In general, this degradation progress occurs as an initial reactive species population is converted into a reaction product. Usually, component failure does not require a complete reaction. Instead, the accumulation of a sufficient concentration of reaction product exceeds an acceptable threshold level and causes failure.

The accumulation of reaction product is modelled as a stochastic process. In particular, a Markov process model with discrete state space and continuous time is used to describe the degradation process. The most appropriate Markov model is the pure birth process since the unimolecular reaction is assumed to be irreversible.

Based on the pure birth process model, the failure time distribution is obtained. The detailed description of the development of the failure distribution is included in Chapter III of this thesis. It is essential to mention that two important assumptions are made during the analysis.

In general, it is reasonable to assume that at time=0, there is an initial number of reaction product molecules. This initial quantity and also the failure threshold are assumed to be random variables. Hence, the distance between these two quantities (or the distance between these two states of the process) is an adequate representation of their randomness. Consequently, the component failure pattern for a particular case of random distance would be different from others. In the current analysis, the initial quantity and the failure threshold are assumed to be Poisson variables.

Also in general, the reaction rate depends both on the time and the state in which the process is located. Different assumptions concerning the reation rate would lead to quite different models and results. A constant reaction rate that is both time and state independent will be examined in the pure birth process model described here.

The second chapter consists of literature review of the papers and articles pertinent to the formulation of the failure distribution. Finally, as the failure time distribution is established, two specific examples are used in the study of the component failure behavior. This analysis is included in Chapter IV.

II. LITERATURE REVIEW

This section descibes the literature related to different stochastic techniques in dealing with chemical kinetics and reactions, physical analysis of stress testing components failure and different models in representing the probability of the difference of two Poisson variables.

Montroll and Shuler (1958) apply the stochastic processes concept to the unimolecular reaction rate theory. They characterize the transitions of the particles between energy levels as a one-dimensional random walk with an absorbing barrier. The theory is developed in terms of the properties of the stochastic matrix whose elements are the transitions probabilities. The reactant molecules are initially in a Maxwell-Boltzmann distribution. The molecules become activated and move to a higher energy level as they collide with each other. As the reactant molecule reaches the (N+1)st energy level, it is removed from the system. This process is being defined as the chemical reaction. The reaction rate is solved in term of the mean first passage time , which is the average time until a molecule passes level N and is removed from the system. One main deficiency of this theory is the restriction to transition between neighboring energy levels. This article is an extension of a Brownian motion model that is developed by Kramers in 1940.

Bartholomay (1958) is one of many individuals who first derives a stochastic model for the basic unimolecular reaction. However, his model is easier to analyze than those of others. The model which is developed for this reaction process is the pure death stochastic process.

Bartholomay (1959) devotes an entire article to the unimolecular reaction rate of the stochastic model that he had developed in 1958. He derives a simple formula for calculating the rate constants of the reaction. This formula is then applied to data from literature. The results are almost identical to the estimates that are obtained by other methods.

McQuarrie has put together a book entitled Stochastic Approach to Chemical Kinetics in 1967. He reviews numerous readily solvable models for both unimolecular reactions and bimolecular reactions. As the mathematics become more difficult and involved for complex stochastic models, several approximate methods for the solution are introduced. McQuarrie also describes in detail some possible applications of these probability models. The book provides an excellent reference source for literature on applying stochastic methods to chemical processes.

Kooi (June 1965) uses the unimolecular reaction law to represent the failure process of a component. The model he develops is the one that motivated this thesis. An important difference between the work here and that of Kooi is that Kooi assumes fixed initial and final states as do all others articles. Based on this model, Kooi derives the cumulative distribution functions for the component failures. The failure rate is assumed to be the Bose-Einstein equations with one stress only. One conclusion that Kooi makes is that "...these cumulative distributions functions are identical to each other if the random variables are certain functions of time, temperature, or voltage, thus showing the equivalence of time, temperature, and voltages as stresses.". Kooi also concludes that the cumulative distribution function in time is the log-normal function. Even though this last statement is very plausible, he does not have any proof to justify his statement. In this article, Kooi has also missed the fact that he represents only one failure mechanism.

Lemoine and Wenocur (1985) use stochastic models in developing failure time distributions. They suggest two mathematical models to be used in modelling system state, or wear and tear. These models are the diffusion processes and the shot-noise processes. The failure assumption is similar to the one discussed in this thesis. Lemoine and Wenocur suggest that the system failure occurs when either wear or tear reaches beyond a certain critical threshold. Several subclasses of the diffusion and the shot-noise approaches in failure modeling are discussed.

For the following literature review, X and Y are assumed to be Poisson variables with means λ_1 and λ_2, respectively.

Irwin (1937) deals with the case where both Poisson variables have the same mean (λ). He shows that the distribution of the difference between X and Y can be expressed as

$$P[X-Y=z] = e^{-2\lambda} I_z(2\lambda)$$

where $I_z(2\lambda)$ is a Bessel's modified fuction of the first kind of order (z) and argument (2λ).

Skellam (1946) considers the case where X and Y have different means. He obtains the distribution of X-Y in the form

$$P[X-Y=z] = e^{-(\lambda_1+\lambda_2)} (\lambda_2/\lambda_1)^{z/2} I_z[2(\lambda_1\lambda_2)^{1/2}] \quad , \qquad (1)$$

the domain of (z) being all the integers.

In 1952, Prékopa also obtains equation (1). In addition, he expresses the same distribution as a sum of series which is used in developing the failure distribution in this thesis. The equations are described on page 18 of Chapter III.

Johnson (1959) shows that the tails of equation (1) are approximately equal to the tails of a noncentral chi-square distribution, thus confirming the Poisson and chi-square relationship. The approximate equations for the distribution of the difference between a Poisson variable and a negative binomial variable, between a negative binomial and a Poisson variable and between a negative binomial and a negative binomial are also discussed.

III. MODEL DEVELOPMENT

This chapter of the thesis lays the foundation for the development of the failure time distribution. The unimolecular degradation process is described and the failure time distribution is formulated. The following chapter discusses the behavior of the failure process on the basis of results for two specific examples.

A. PHYSICAL FAILURE PROCESS

The physical failure process of the component is assumed to be in the form of a unimolecular chemical reaction

$$A \xrightarrow{H} B.$$

[A] represents the initial concentration of the reactant A.

[B] represents the formation concentration of product B.

H represents the reaction rate that converts the concentration in A irreversibly to B [1].

Another way to interpret the model above is to picture A and B as two potential wells which are separated from each other by a potential hill. Figure 1 is an illustration of this view.

Initially, there are N particles in well A, and i particles in well B. Due to a certain reaction process with reaction rate H, the particles in A will one by one move to well B. Since well B is assumed to be very deep, no particles in B can return to A. As a result, the reaction is

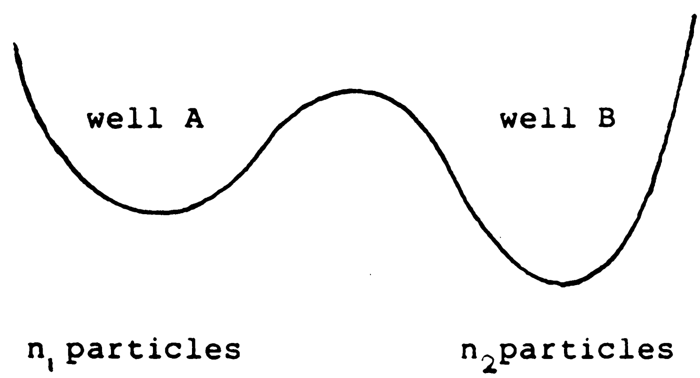


Figure 1 - Double potential well as a model for a degradation process. At $t=0$, $n_1 = N$, $n_2 = i$.

irreversible. After period of time t,there are n particles in well B (which include i particles in B initially). If the number of particles n in B exceeds a threshold number j, the component will fail [9].

As mentioned before, different assumptions concerning the reaction rate would lead to quite different models and results. The reaction rate, H, which converts the concentration in A irreversibly to B can be specified in four different forms.

1. $H = h \ \forall i,t$ for $h > 0$. The reaction rate of the process is a constant number. This reaction rate is then both time invariant and independent of the state i in which the process is located.
2. $H = (h*i) \ \forall i,t$ for $h > 0$. The reaction rate is proportional to the state of the system. However, it is time invariant.
3. $H = m*h*t^{m-1} \ \forall i,t$ for $h > 0$ and $m \geq 1$. The reaction rate is state independent but time dependent. If m=1, case 1 occurs.
4. $H = (h*i)mt^{m-1} \ \forall i,t$ for $h > 0$ and $m \geq 1$. This is the most general case of reaction rate since it is both state and time independent.

It is important to mention that the constant number h is a function of environmental variables such as temperature and voltage. The Arrhenius model is an example of this function. For many years, it has been one of the most useful models in relating the component lifetime and the temperature. The rationale for this model is that the degradation process leading to component failure is governed by chemical and physical processes with a reaction rate

$$h = A\, e^{-(E_A / kT)} ,$$

where A is a constant, E_A is the activation energy (eV), k is Boltzmann's constant = 8.617*10 (eV/ °Kelvin) and T is the absolute temperature (°K) [6].Hence,it is possible to investigate the impact of the environmental conditions in the component failure process. For the current discussion, form 3 will be used to obtain a general solution. For the final analysis, m will be assumed equal to 1.

B. MARKOV PROCESS MODEL: PURE BIRTH PROCESS

It is of interest to note the fact that the stochastic treatment of the unimolecular reaction above should lead to a pure birth process is to be expected on physical grounds, since the number of particles (concentration) in B can only increase from its initial value i to the threshold number j.

Before constructing the Markov model for the unimolecular reaction , some definitions are needed.

1. The state variable for the present application is the

number of reaction product particles that exists in well B.

2. $Y(t)$ represents the state variable at any time t so that $Y(t)=n$ represents the event that state n is occupied at time t.

3. $Y(0) = i$ denotes an initial state and $Y(t) = j$ denotes the final state or the threshold number where the component would fail at time t.

4. $P\{Y(t)=n|Y(0)=i\} = p(i,n,t)$ denotes the transition probability of making a transition from state i to state n in well B in the time interval $(0,t)$.

On the basis of the physical nature of the unimolecular reaction, the following assumptions can be made.

a. $P\{Y(0)=i|Y(0)=i\}=p(i,i,0)=1$. The probability of making no transition in the time interval $(0,0)$ is 1.

b. $P\{Y(0)=n|Y(0)=i]=p(i,n,0)=0$ for $n \neq i$. The probability of making a transition from state i to a state different from i in the interval $(0,0)$ is 0.

Since the form of the reaction rate that will be used in the discussion is $H = m*h*t^{m-1}$, the generator matrix A of the process is given by

$$A(i,i) = -m*h*t^{m-1} \quad \text{for } i \neq j$$

$$A(i,n) = +m*h*t^{m-1} \quad \text{for } n=i+1, i= 0,1,2,..,j-1$$

$$\text{and } A(i,n) = 0 \quad \text{otherwise}$$

or in the matrix form ,

$$A(t) = \begin{array}{c|ccccccc|}
 & 0 & 1 & 2 & \cdot & \cdot & j-1 & j \\
\hline
0 & -mht^{m-1} & mht^{m-1} & 0 & 0 & 0 & & 0 \\
1 & 0 & -mht^{m-1} & mht^{m-1} & 0 & 0 & & 0 \\
2 & 0 & 0 & -mht^{m-1} & mht^{m-1} & 0 & & 0 \\
\cdot & & & & \cdot & \cdot & & \\
\cdot & 0 & 0 & 0 & \cdot & \cdot & & 0 \\
\cdot & & & & & \cdot & \cdot & \\
\cdot & & & & & & & \\
j-1 & 0 & 0 & 0 & 0 & 0 & -mht^{m-1} & mht^{m-1} \\
j & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}$$

It is of interest to note that state j is an absorbing state. In other words, when the number of particles n in B reaches the threshold number j, then the degradation process stops - the component fails. Hence, there is no specification of the reaction rate for state j in the generator matrix A.

Therefore, the Kolmogorov equations for the birth process are given by $P'(t) = P(t) * A(t)$,

or $$p'(i,i,t) = -mht^{m-1} * p(i,i,t) \quad \text{for } i \neq j \tag{1}$$

$$p'(i,n,t) = +mht^{m-1} * p(i,n-1,t) - mht^{m-1} * p(i,n,t) \tag{2}$$

for $n \neq i$ and $n \neq j$.

$$p'(i,j,t) = +mht^{m-1} * p(i,j-1,t). \tag{3}$$ [3]

The details of the solutions for the transition probabilities are described in Appendix A. According to these solutions, the general forms to describe the transition probabilities with the reaction rate $H = mht^{m-1}$ are

$$p(i,n,t) = (ht^{m})^{n-i} * e^{-ht^{m}} \quad \text{for } n \neq j,\ n \geq i \tag{4}$$

$$p(i,j,t) = 1 - \sum_{v=0}^{j-i-1} \frac{[(ht^{m})^{v} * e^{-ht^{m}}]}{v!},\ j > i \tag{5}$$

C. FIRST PASSAGE TIME OF THE BIRTH PROCESS

A conclusion can be reached from looking at the sample path of the birth process (Figure 2).

Since this process is a pure birth process, transition from a state i can only lead to a state which is one step higher (i+1). Consequently, the failure time is the first passage time from state i to state j. Therefore, the first passage time will have the same expression as the transition probability from state i tothe final state j. Hence, it can be expressed as

$$F(t|i,j) = p(i,j,t) = 1 - \sum_{v=0}^{j-i-1} \frac{[(ht^{m})^{v} * e^{-ht^{m}}]}{v!},\ j > i \tag{6}$$

For the current discussion, m is set equal to 1 which corresponds to a constant reaction rate. For many unimolecular reactions, the rate of progress is constant. This is the simplest realization of the above model. The first passage time distribution for this case is obtained by substituting the value of 1 for m. The resulting equation is:

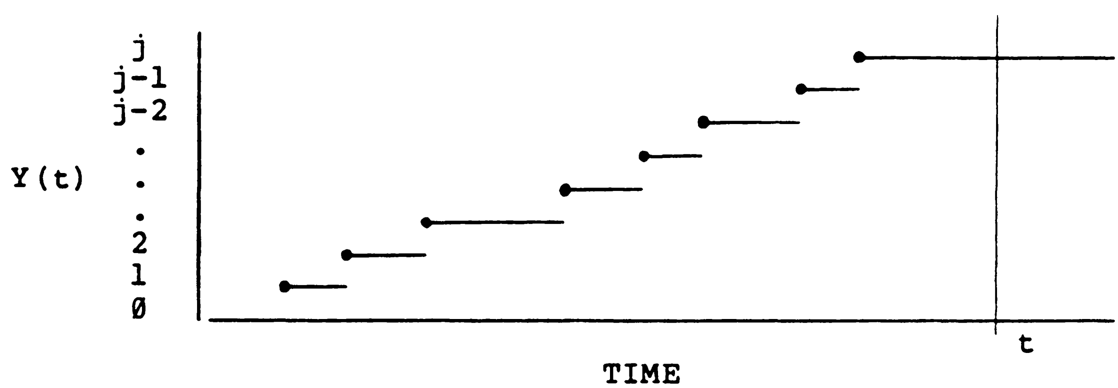


Figure 2 - Sample path of the birth process

$$F(t|i,j) = 1 - \sum_{v=\emptyset}^{j-i-1} \frac{[ht^{v} * e^{-ht}]}{v!} , \; j > i \qquad (7)$$

This is the equation for the gamma distribution.

D. FAILURE TIME DISTRIBUTION

It is worthwhile to point out several facts about the initial state i and the final state j.

1. The initial state i is a consequence of the component (or material) production process.

2. Production process variability or trace impurities in input material yield the initial concentration i.

3. Thus, in some sense, i = the number of defects per unit and such a variable has traditionally been traced as Poisson variable.

4. For some components, the failure threshold j may not be random but for many it is likely that it is random. This may be due to component geometry effects by the reaction process. For the case where the final state j is random, the Poisson is a plausible model.

Recall that the initial state (i) and the final state (j) are assumed to be Poisson variables. Consequently, the distance between these two quantities (j-i=k) where k > Ø is an adequate representation of their randomness.

As λ_1 is the mean of the initial state and λ_2 is the mean of the final state, the moment generating function of the difference of these two Poisson states can be expressed as

$$M_{j-i}(\theta) = M_j(\theta) * M_{-i}(\theta)$$
$$= M_j(\theta) * M_i(-\theta)$$
$$= e^{-\lambda_2(1-e^{\theta})} \; e^{-\lambda_1(1-e^{-\theta})}$$

or $$M_{j-i}(\theta) = e^{-\lambda_2(1-e^{\theta})-\lambda_1(1-e^{-\theta})} \tag{8}$$

In 1952, Prékopa shows that the distribution of (j-i) is

$$P[j-i=k] = \lambda_2^k \, e^{-(\lambda_1+\lambda_2)} \sum_{n=\emptyset}^{\infty} \frac{(\lambda_1\lambda_2)^n}{n!\,(n+k)!} \quad ,k=\emptyset,1,2,... \tag{9}$$

$$P[j-i=-k] = \lambda_1^k \, e^{-(\lambda_1+\lambda_2)} \sum_{n=\emptyset}^{\infty} \frac{(\lambda_1\lambda_2)^n}{n!\,(n+k)!} \quad ,k=\emptyset,1,2,... \tag{1\emptyset}$$

However, for the current discussion, only equation (9) will be considered. The justification for utilizing only the first equation is that the component is considered to be defective or non-functional when $(j - i < \emptyset)$. If a component is defective, the degradation process cannot begin. Equivalently, $j < i$ may be interpreted as corresponding to the case in which the initial status of the component is that it is failed. Thus, there is no failure process for such a component. Also, for practical application, the failure threshold j is always considered to be greater the initial state i. Given the focus on the failure process, λ_1 and λ_2 are assumed to have values so that equation (9) represents nearly all of the probability mass for j-i. As a result, the mass represented by equation (1Ø) is negligible and is ignored.

Recall that the failure time distribution given the initial state and the final state are found to be

$$F(t|i,j) = 1 - \sum_{v=0}^{j-i-1} \frac{[ht^{v} e^{-ht}]}{v!}, \quad t \geq 0, \; j > i \tag{11}$$

or

$$f(t|i,j) = \frac{h^{(j-i)} t^{(j-i-1)} e^{-ht}}{(j-i-1)!}, \quad t \geq 0, \; j > i \tag{12}$$

The failure time distribution can be found by computing the expected value of $f(t|i,j)$ with respect to the distance distribution.

$$E[F(t|i,j)] = F(t) = \sum_{k=1}^{\infty} F(t|i,j) \; P(j-i=k) \tag{13}$$

or

$$E[f(t|i,j)] = f(t) = \sum_{k=1}^{\infty} f(t|i,j) \; P(j-i=k) \tag{14}$$

Substituting equations (11) and (9) into (13) gives the final expression for the failure cumulative distribution function.

$$F(t) = \sum_{k=1}^{\infty} \left\{ \left[1 - \sum_{v=0}^{k-1} \frac{[ht^{v} e^{-ht}]}{v!} \right] \left[\lambda_2^{k} e^{-(\lambda_1+\lambda_2)} \sum_{n=0}^{\infty} \frac{(\lambda_1\lambda_2)^{n}}{n!(n+k)!} \right] \right\}$$

where $t \geq 0$. (15)

Substituting equations (12) and (9) into (14) gives the final expression for the failure time density funtion.

$$f(t) = \sum_{k=1}^{\infty} \left\{ \left[\frac{h^{k} t^{(k-1)} e^{-ht}}{(k-1)!} \right] * \left[\lambda_2^{k} e^{-(\lambda_1+\lambda_2)} \sum_{n=0}^{\infty} \frac{(\lambda_1\lambda_2)^{n}}{n! \; (n+k)!} \right] \right\}$$

where $t \geq 0$. (16)

It is found that the failure density function $f(t)$ is easier to analyze compared to the cumulative distribution function F(t).

IV. MODEL SOLUTION

Recall that the failure time density function is found to be

$$f(t) = \sum_{k=1}^{\infty} \left\{ \left[\frac{h^k \, t^{(k-1)} \, e^{-(ht)}}{(k-1)!} \right] \left[\lambda_2^k \, e^{-(\lambda_1+\lambda_2)} \sum_{n=0}^{\infty} \frac{(\lambda_1 \lambda_2)^n}{n! \, (n+k)!} \right] \right\} \quad (1)$$

The behavior of the failure function is not obvious by looking at equation (1). Hence, data analysis of specific problems are used. Two examples are examined as a basis for drawing general conclusions about the failure model. In general, the solutions of these examples are done in six basic steps.

1. Specific values are assigned for the mean of the initial state and of the final state. Appropriate values for the random distance and reaction rate are also specified.

2. Using these values, the failure probabilities over a range of time can be determined. A computer program for calculating the failure density f(t) is used. The choice of using f(t) instead of F(t) is due to the fact that f(t) requires fewer calculations. The program is included in Appendix B.

3. Data plots are particularly useful for this analysis. In both examples, these plots suggest that the failure model resembles the behavior of the Weibull distribution.

4. The Weibull model is fitted to the data. Linear

regression is used to obtained estimated parameters of the fitted model. A computer program is also used in this section. This program is included in Appendix C.

5. The validity of the model is checked. If there is a need to change the model, steps 4 and 5 are repeated.

6. The solutions provided by the fitted Weibull model are interpreted to provide a basis for drawing conclusions and making decisions for the general failure problem [12].

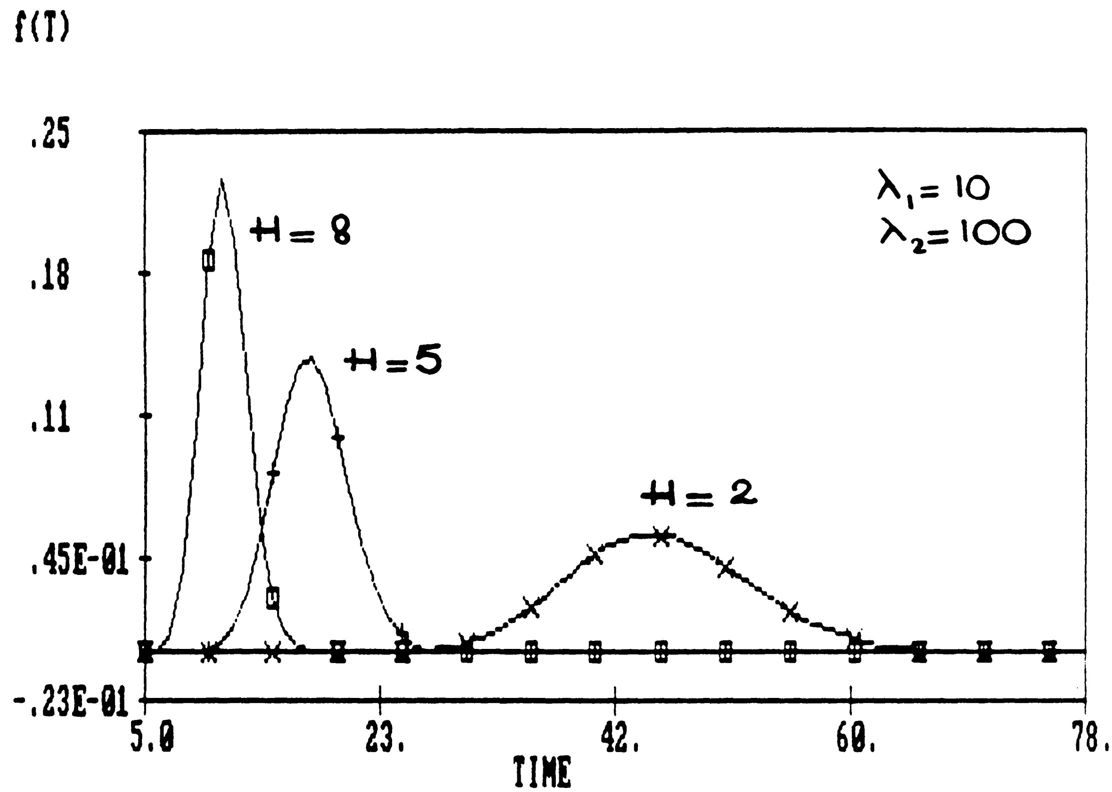
The following examples are chosen for the analysis of the behavior of the failure function. The values that are assumed for these examples are computable and reflect a scale transformation from reality that is reasonable. It is assumed for the first example that

λ_1 = the mean of the initial state (i) = 10

λ_2 = the mean of the failure threshold (j) = 100

Also, three different values of reaction rate, H, are considered. They are H = 2, H = 5, and H = 8.

For each value of the reaction rate, H, the failure density function is calculated and plotted. The plots of these failure curves are included in Graph 1. According to all three data plots, the failure curves are somewhat bell-shaped and resemble the normal curve. However, they are all skewed to the left. Note that as the reaction rate increases, the probability mass moves to the left as would expected. Based on this finding, the Weibull distribution is chosen



Graph 1 - Failure curves of example 1

to be the most suitable model in representing the failure behavior. The validity of the Weibull model is checked below.

The failure data is used in this step to calculate the estimating parameters of the Weibull model. Before solving for these values, the Weibull distribution is defined. The Weibull cumulative distribution function (CDF), with parameters α and β is given by

$$F(t) = 1 - e^{-\alpha t^{\beta}} \quad , t > 0 \qquad (2)$$

$$= 0 \quad \text{elsewhere,}$$

$$\text{where } \alpha > 0 \text{ and } \beta > 0.$$

α is defined as the time scale parameter and β as the shape parameter.

Equation (2) is rewritten as follows.

$$F(t) = 1 - e^{-\alpha t^{\beta}}$$

$$1 - F(t) = e^{-\alpha t^{\beta}}$$

$$-\ln[\ 1-F(t)\ ] = \alpha t^{\beta}$$

$$\ln\{\ -\ln[1-F(t)]\ \} = \ln\ (\alpha t^{\beta})$$

or
$$\ln\{\ -\ln[1-F(t)]\ \} = \beta \ln t + \ln \alpha \qquad (3)$$

This is a linear relationship between $\ln\{ -\ln[1-F(t)] \}$ and $\ln t$. For each t_i of the failure data where i =1,2,...,n, set

$$y_i = \ln\{\ -\ln[1-F(t_i)]\ \}$$

and
$$x_i = \ln t_i \quad .$$

Then, for each t_i, there is an equation in the form

$$y_i = \hat{\beta} x_i + \ln \hat{\alpha} \qquad (4)$$

where $\hat{\alpha}$ = estimate value of α

and $\hat{\beta}$ = estimate value of β .

All the points (x_i, y_i) are plotted. From the plots of all three reaction rate values (Graphs 2,3), it is recognized that the lines are not straight as expected. They look more like a log-curve. From this finding, it is suspected that the Weibull model has a third parameter. With the third parameter, the model is defined to be a shifted Weibull distribution where

$$F(t) = 1 - e^{-\alpha(t-\delta)^{\beta}} \quad \text{for } t > \delta \qquad (5)$$

$$= 0 \quad \text{for } t \leq \delta .$$

As before, equation (5) is rewritten into a natural log form as

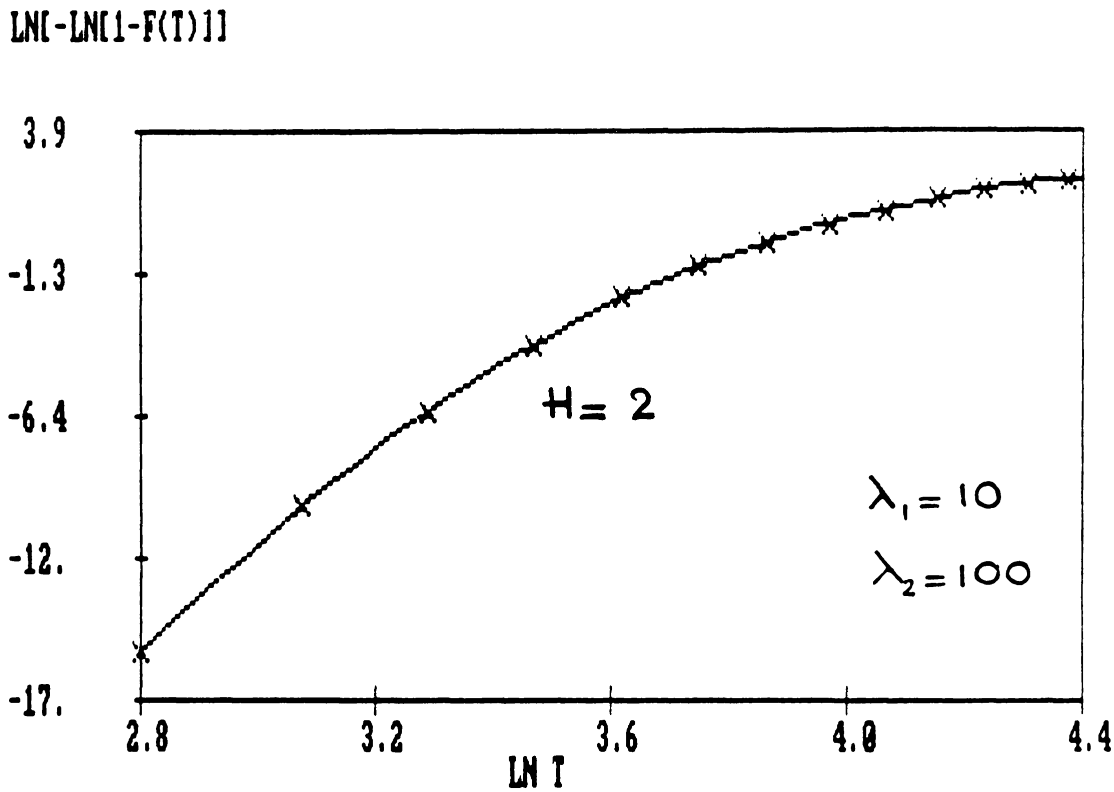
$$\ln\{ -\ln[1-F(t)] \} = \beta \ln[t - \delta] + \ln \alpha \qquad (6)$$

Once again, for a specific value of δ, set

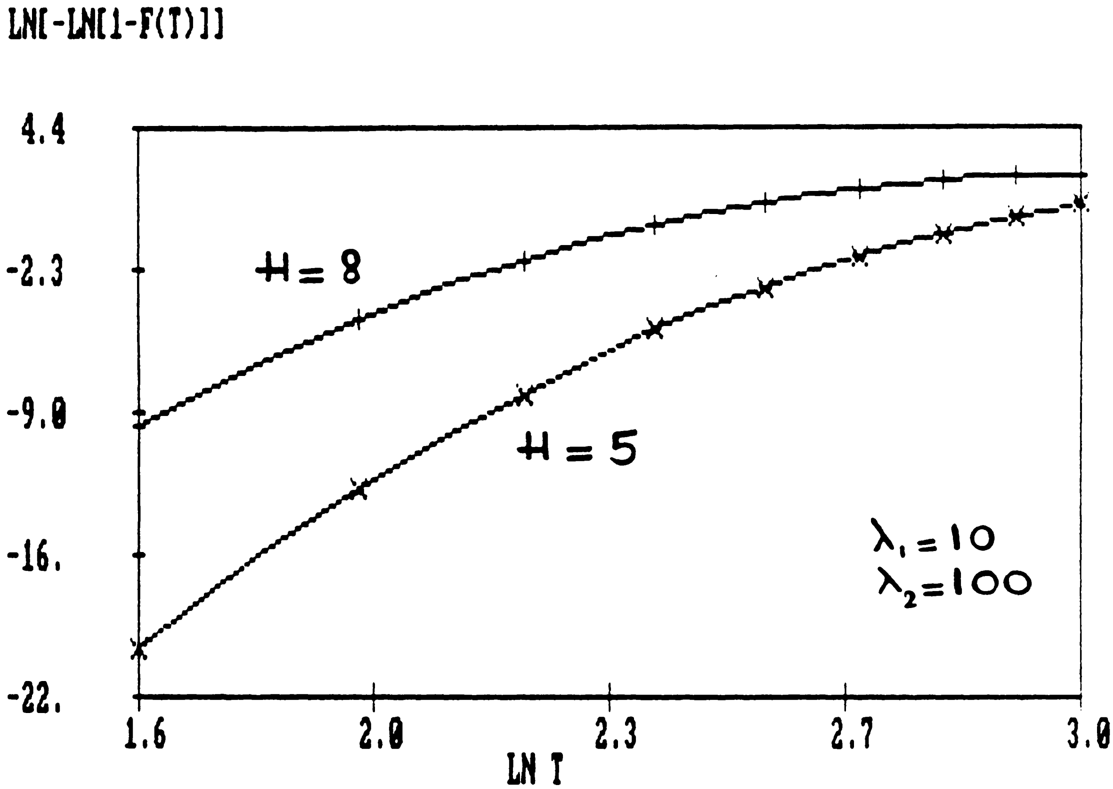
$$y_i = \ln\{ -\ln[1-F(t_i)] \}$$

$$x_i = \ln[t_i - \delta] \qquad \text{for } i = 1,2,\ldots,n.$$

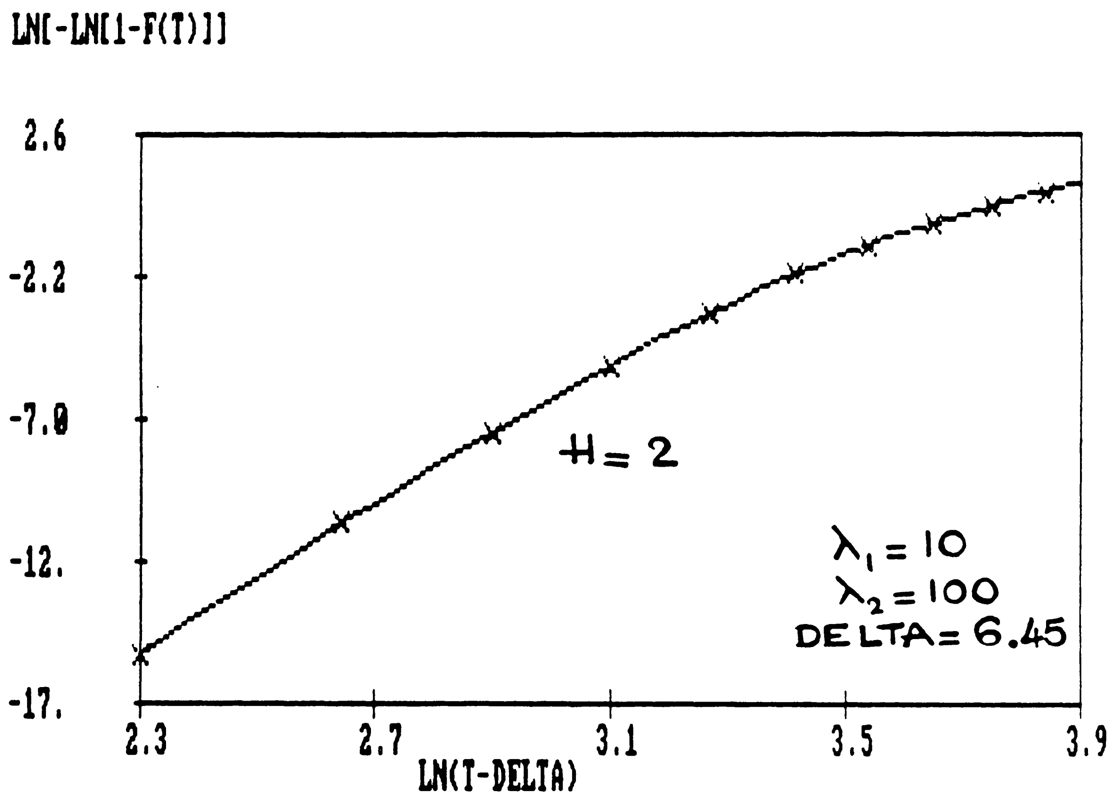
When all the points (x_i,y_i) are plotted they form a graph that looks like a straight line (Graphs 4-6) which is evidence that the three-parameter Weibull distribution is a satisfactorily model. However, it is the regression analysis and R^2 values that confirm the fact that the Weibull is appropriate. The value δ is obtained through trial and error until the graph of $(x_i$, y_i) forms a straight line. Note that the time value δ is not the time where the degradation



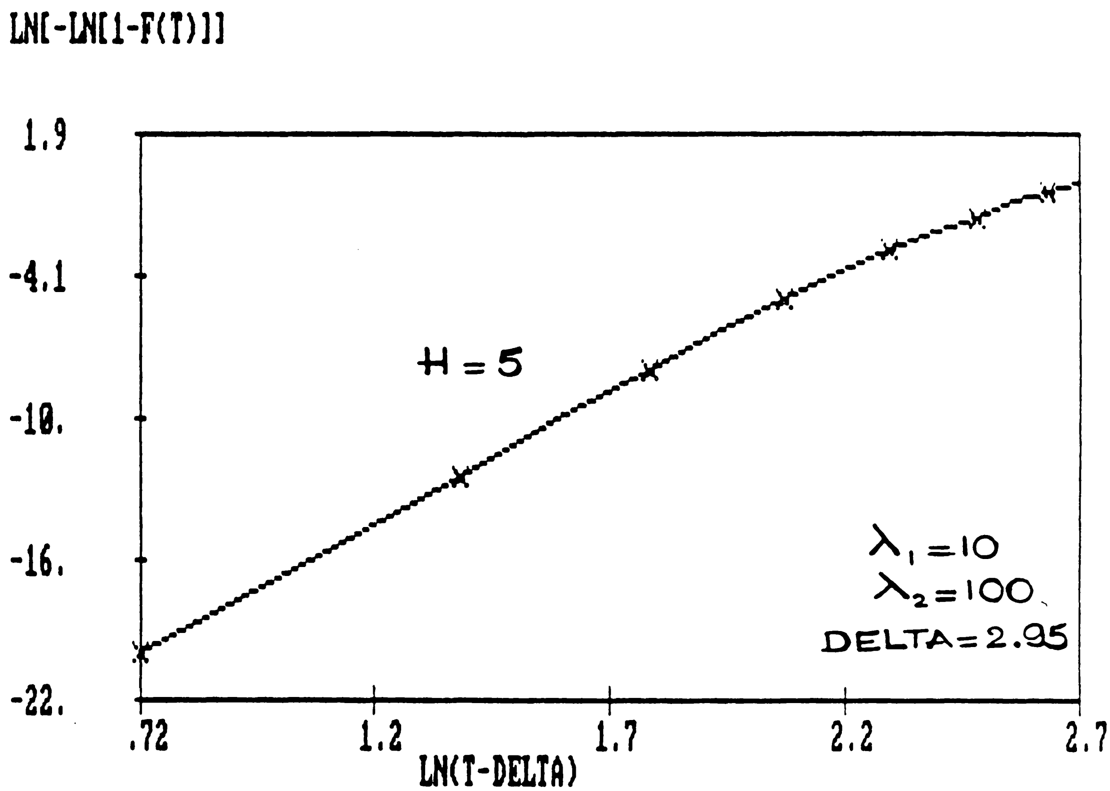
Graph 2 - Example 1-The plot of ln[-ln(1-F(t))] vs. ln(t) of the two-parameter Weibull distribution where H=2.



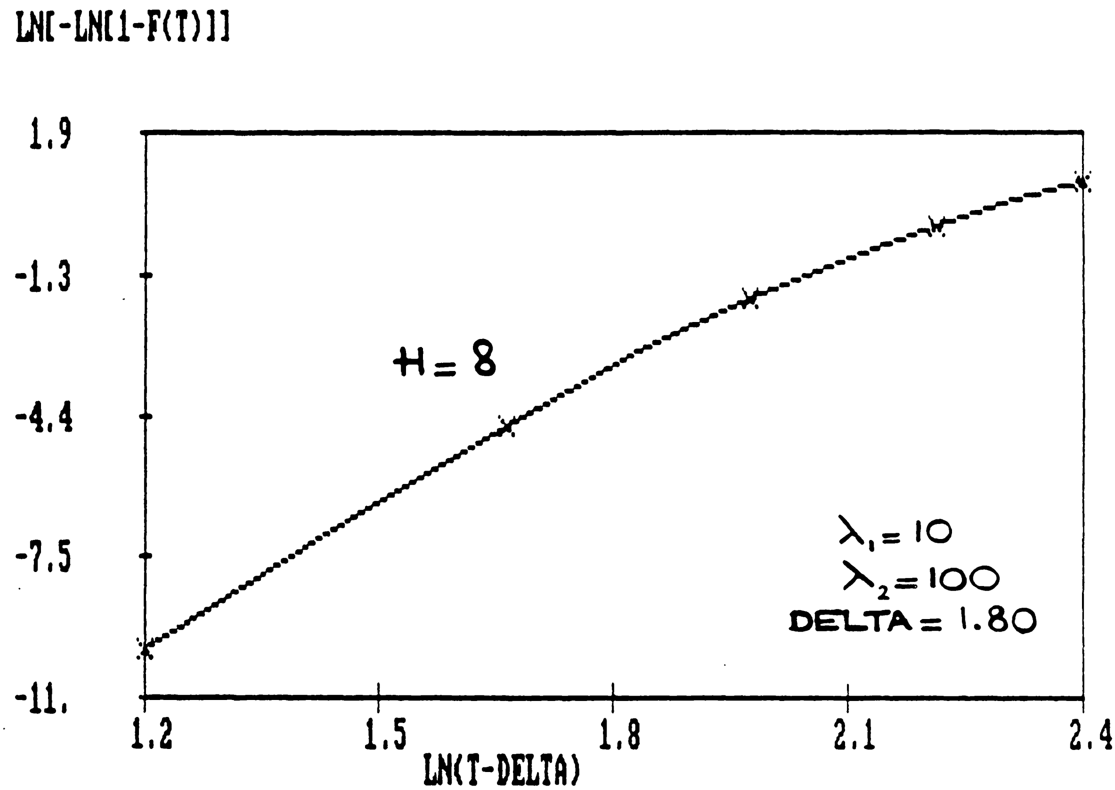
Graph 3 - Example 1-The plots of ln[-ln(1-F(t))] vs. ln(t) of the two-parameter Wibull distribution where H= 5 and 8.



Graph 4 - Example 1-The plot of ln[-ln(1-F(t))] vs. ln(t) of the three-parameter Weibull distribution where H=2.



Graph 5 - Example 1-The plot of ln[-ln(1-F(t))] vs. ln(t) of the three-parameter Weibull distribution where H=5.



Graph 6 - Example 1-The plot of ln[-ln(1-F(t))] vs. ln(t) of the three-parameter Weibull distribution where H=8.

process begins. When $t < \delta$, the probability of failure is so small that it is considered to be negligible.

In order to estimate the two parameters $\hat{\alpha}$ and $\hat{\beta}$, standard regression equations are used. These are:

$$S_{xx} = \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i \right)^2 / n \qquad (7)$$

$$S_{yy} = \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i \right)^2 / n \qquad (8)$$

$$S_{xy} = \sum_{i=1}^{n} (x_i\, y_i) - \left(\sum_{i=1}^{n} x_i\right) \left(\sum_{i=1}^{n} y_i \right) / n \qquad (9)$$

and $\quad \bar{x} = \left(\sum_{i=1}^{n} x_i \right) / n, \quad \bar{y} = \left(\sum_{i=1}^{n} y_i\right) / n$

Then, $\quad \hat{\beta} = S_{xy} / S_{xx}, \qquad (11)$

$$\ln \hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$

or $\quad \hat{\alpha} = e^{(\bar{y} - \hat{\beta}\bar{x})}$ [15]. $\qquad (12)$

To check the validity of the model, $\hat{\alpha}$ and $\hat{\beta}$ are substituted into equation (5) for α and β and $F(t)$ are solved for all t_i where $i = 1,2,...,n$. The comparison between these values and the failure data shows that they are almost alike. It is important mention again that the value R^2 plays an important role in the analysis. The value δ and the estimating parameters $\hat{\alpha}$ and $\hat{\beta}$ change as R^2 changes. Several R^2 values are tried until the desired results are obtained. Hence, the validity of the three-parameter Weibull model is checked. The results of this example are included in Table 1 on page 35.

In the second example,

λ_1 = the mean of the initial state i = 10

λ_2 = the mean of the final state j = 70

The three values of reaction rate, H , that are used are 2,5,8.

Using the same method as in example 1, the failure density function of each reaction rate is calculated and plotted (Graph 7). The failure curves also are bell-shaped and skewed to the left. Hence, the Weibull distribution is used once again as a model representing the failure behavior.

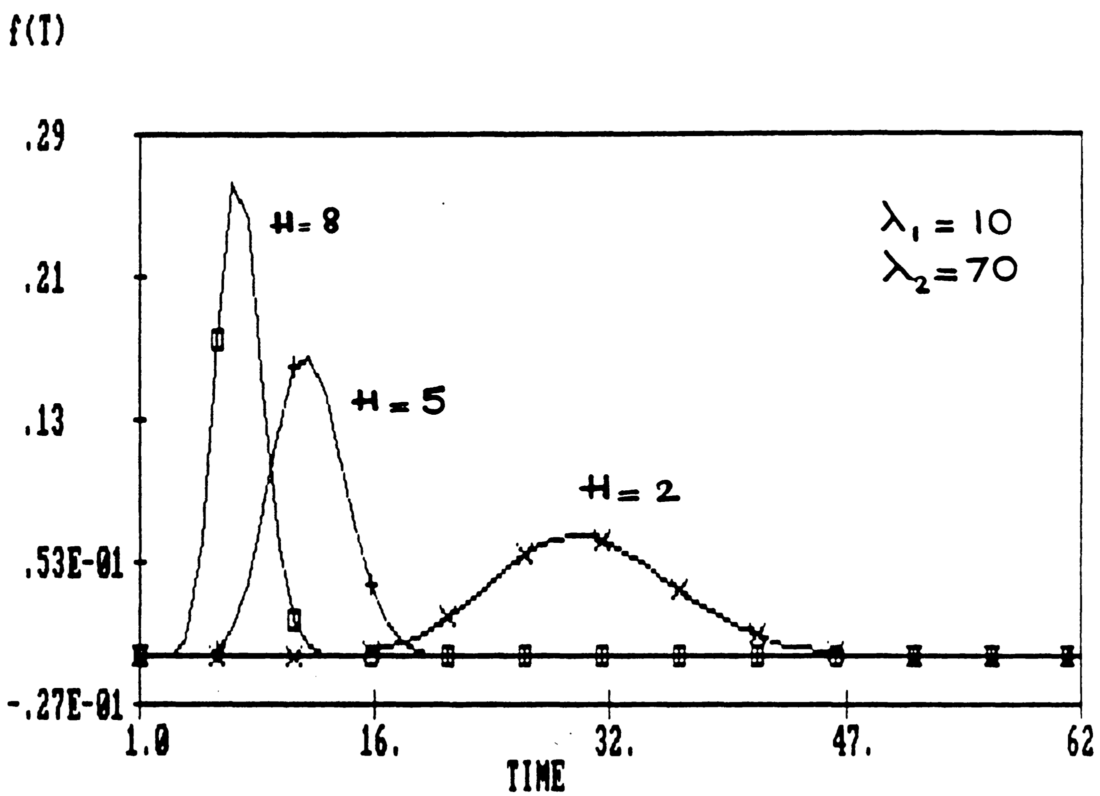
Each t_i and $F(t_i)$ of the failure data is used to calculated

$$x_i = \ln t_i$$

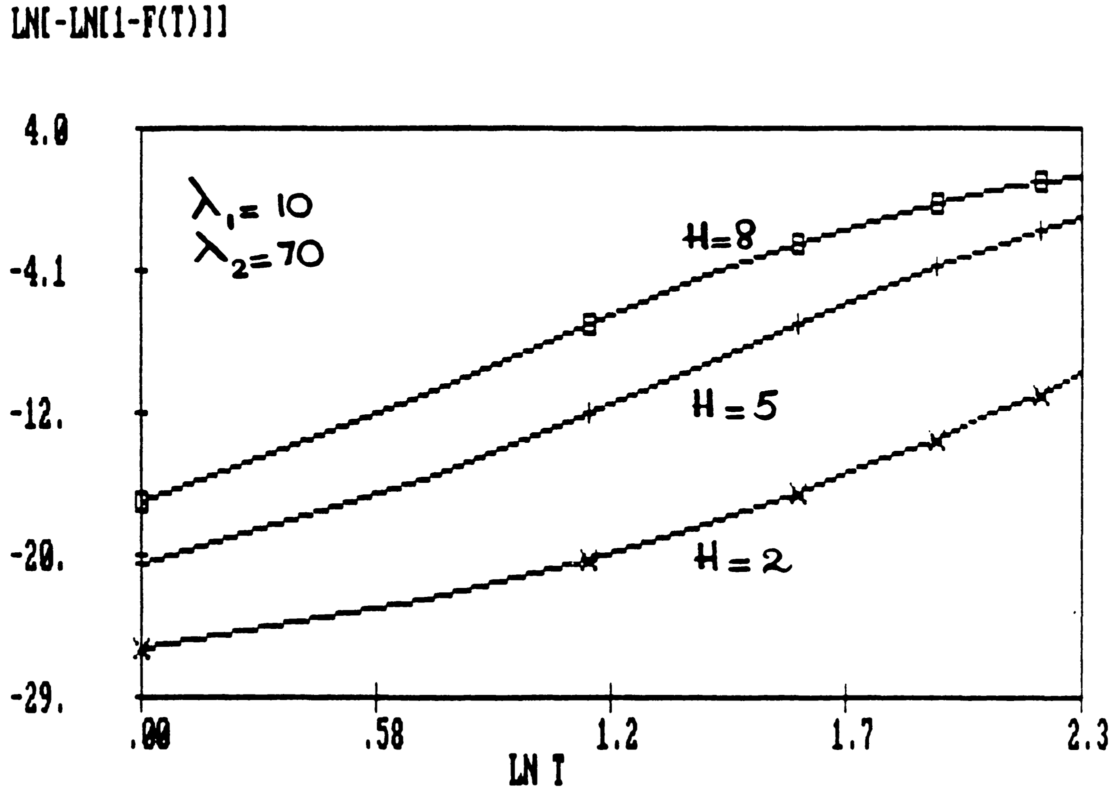
$$y_i = \ln\{ -\ln[1-F(t_i)] \}$$

for all i of equation (4). When these points (x_i, y_i) are plotted, they form a straight line (Graph 8). The linear relationship between x and y is satisfied. Hence, the two-parameter Weibull distribution is a suitable failure model in this problem.

The estimating parameters $\hat{\alpha}$ and $\hat{\beta}$ are calculated using equations (7)-(12). Then, $\hat{\alpha}$ and $\hat{\beta}$ are substituted into equation (2) to find F(t) for all t_i where i=1,2,...,n. These values are approximately equal to the failure data. Hence, the validity of the model is checked. The results for different values of reaction rate are included in Table 1 on page 35.



Graph 7 - Failure curves of example 2



Graph 8 - Example 2-The plots of ln[-ln(1-F(t))] vs. ln(t) of the two-parameter Weibull distribution where H= 2,5 and 8.

Table 1 - The results of the two example problems

Problem	H	α (time units)	β	δ (time units)	R^2
I.					
λ_1 = 10	2	$0.63894*10^{-13}$	8.00	6.45	0.95
λ_2 = 100	5	$0.10589*10^{-09}$	8.00	2.95	0.95
	8	$0.43626*10^{-08}$	8.00	1.80	0.95
II.					
λ_1 = 10	2	$0.75540*10^{-11}$	6.94	0.00	0.95
λ_2 = 70	5	$0.67433*10^{-08}$	6.94	0.00	0.95
	8	$0.25434*10^{-06}$	6.94	0.00	0.95

V. CONCLUSIONS

The progress of an unimolecular chemical degradation reaction is used in representing a component failure mechanism. The component is said to fail when the concentration of reaction product accumulates beyond an acceptable level. The process of accumulating reaction product is modelled as a Markov pure birth process which, in turn, is used in developing the failure time distribution. The model is analyzed under the assumption that the reaction rate is constant. Also the initial state and the final state of the degradation process are assumed to be Poisson variables.

Based upon two specific examples, several conclusions about the behavior of the failure model are reached.The results show that the component failure distribution can be described as a three-parameter Weibull distribution. In other words, when the time t is less than the time value δ , the probability of component failure is very small that it can be ignored. This process only occurs when the failure threshold are significantly different from the initial state. Otherwise,the failure model is a two-parameter Weibull distribution. The results also show that as the reaction rate increases (or decreases), the time scale parameter (α) also increases (or decreases). However the shape parameter (β) of the failure model remains constant. This is a very important result. Many authors have argued that the constant shape parameter should occur regardless of the change in the time scale. However, they have not been able to justify this statement.

Based on these findings, it appears that the degradation model can be used to investigate the impact of environmental conditions in the component failure process for at least some failure processes. The proposed model also provides a physically plausible and analytically tractable framework for investigating other failure processes.

VI. BIBLIOGRAPHY

1. Anthony F. Bartholomay, "Stochastic Models for Chemical Reactions: I. Theory of the Unimolecular Reaction Process", Bulletin of Mathematical Biophysics, vol. 20, 1958, pp. 175-190.

2. Anthony F. Bartholomay, " Stochatic Models for Chemical Reactions: II. The Unimolecular Rate Constant", Bulletin of Mathematical Biophysics, vol. 21, 1959, pp. 363-373.

3. A. T. Bharucha-Reid, Elements of the theory of Markov Processes and their Applications, McGraw-Hill, 1960, pp. 77-82.

4. William E. Boyce and Richard C. Diprima, Elementary Differential Equations and Boundary Values Problems, John Wiley & Sons, 1977, pp. 11-17.

5. J. O. Irwin, "The Frequency Distribution of the Difference Between Two Independent Variates following the same Poisson Distribution", Journal of the Royal Statistical Society, vol. 100, 1937, pp. 415-416.

6. Finn Jensen and Niels Erik Peterson, Burn-In, John Wiley & Sons, 1982, pp.75-81.

7. N. L. Johnson, "On Extension of the Connexion between Poisson and Distributions", Biometrika, vol. 46, 1956, pp. 352-363.

8. H. A. Kramers, "Brownian Motion in a Field of Force and the Diffusion Model of Chemical Reactions", Physica, vol. 7, pp.284-304.

9. Clarence F. Kooi,"Physical Analysis of Stress Testing for Failure of Electronic Components", IEEE Transactions on Reliability, vol R-17, no. 2, June 1965, pp. 80-84.

10. Austin J. Lemoine and Michael L. Wenocur, "On Failure Modeling", Naval Research Logistics Quarterly, vol. 32, 1985, pp. 498-508.

11. Donald A. McQuarrie, Stochastic Approach to Chemical Kinetics, Applied Probability Trust, 1967.

12. Wayne Nelson, Applied Life Data Analysis, John Wiley & Sons, 1982, pp. 5-13.

13. Andra Prekopa, "On Composed Poisson Distribution IV", _Acta Mathematica Academia Scientarium Hungaricae_, vol. 3, 1953, pp. 317-325.

14. J. G. Skellam, "The Frequency Distribution of the Difference Betwwen Two Poisson Variates Belonging to Different Populations", _Journal of the Royal Statistical Society_, vol. 109, 1946, p. 296.

15. Ronald E. Walpole and Raymond H. Meyers, _Probability and Statistics for Engineers and Scientists_, Macmillan Publishing Co., Inc., 1978, pp. 282-308.

APPENDIX A

SOLUTIONS FOR THE TRANSITION PROBABILITIES

SOLUTION FOR THE TRANSITION PROBABILITIES

From chapter II, the Kolmogorov equations for the birth process are given by $P'(t) = P(t) * A(t)$,

or specifically,

$$p'(i,i,t) = -mht^{m-1} * p(i,i,t) \quad \text{for } i \neq j \tag{1}$$

$$p'(i,n,t) = mht^{m-1} * p(i,n-1,t) - mht^{m-1} * p(i,n,t) \tag{2}$$

for $n > i$, $n \neq j$.

$$p'(i,j,t) = mht^{m-1} * p(i,j-1,t) \ , \ j > i \tag{3}$$

1. For equation (1),

$$p'(i,i,t) = -mht^{m-1} * p(i,i,t) \tag{4}$$

By rearranging the terms and integrating both sides,

$$\int \frac{dp(i,i,t)}{p(i,i,t)} = \int_{\emptyset}^{t} -mht^{m-1} \, dt$$

The solution of equation (1) will be

$$\ln \ p(i,i,t) = -ht^{m}$$

$$\boxed{p(i,i,t) = e^{-ht^{m}}} \quad . \tag{5}$$

2. The form of equation (2) is

$$p'(i,n,t) = mht^{m-1} * p(i,n-1,t) - mht^{m-1} * p(i,n,t) . \tag{6}$$

Since the equation is recognized to be the first order differential equation, the transition probability can be solved by using the integrating factor method [4].

Let $e^{G(t)}$ be the integrating factor and $g(t) = mht^{m-1}$.

Then, $e^{G(t)} = e^{\int mht^{m-1}dt} = e^{ht^m}$.

a. For the specific equation,

$$p'(i,i+1,t) = mht^{m-1} * p(i,i,t) - mht^{m-1} * p(i,i+1,t) \qquad (7)$$

After rearranging the term and substituting equation (5) into equation (7),

$$p'(i,i+1,t) + mht^{m-1} * p(i,i+1,t) = mht^{m-1} e^{ht^m} \qquad (8)$$

Also,

$$\frac{d}{dt}\{ e^{ht^m} * p(i,i+1,t)\} = e^{ht^m} * p'(i,i+1,t) + mht^{m-1} * e^{ht^m} * p(i,i+1,t)$$

$$= e^{ht^m} [p'(i,i+1,t) + mht^{m-1} * p(i,i+1,t)]$$

The term in the bracket [ ] is recognized to be equal to $mht^{m-1} * e^{ht^m}$ from equation (8) above.

Therefore,

$$\frac{d}{dt}\{ e^{ht^m} * p(i,i+1,t) \} = e^{ht^m} [mht^{m-1} * e^{-ht^m}]$$

$$= mht^{m-1}$$

By integrating both sides,

$$e^{ht^m} * p(i,i+1,t) = \int_{0}^{t} mht^{m-1} dt$$

$$= ht^m .$$

Therefore,

$$\boxed{p(i,i+1,t) = ht^{m} * e^{-ht^{m}}} \tag{9}$$

b. For the specific equation,

$$p'(i,i+2,t) = mht^{m-1} * p(i,i+1,t) - mht^{m-1} * p(i,i+2,t) \tag{10}$$

By rearranging terms and substituting in the solution for $p(i,i+1,t)$, then

$$p'(i,i+2,t) + mht^{m-1} * p(i,i+2,t) = mht^{m-1} * ht^{m} * e^{-ht^{m}} .$$

And by using the same method above,

$$\boxed{p(i,i+2,t) = (ht^{m})^{2} * e^{-ht^{m}} / 2} \quad . \tag{11}$$

And similarly,

$$\boxed{p(i,i+3,t) = (ht^{m})^{3} * e^{-ht^{m}} / 3!} \quad . \tag{12}$$

Based on these specific equations, the general equation of the transition probability can be written as

$$\boxed{p(i,i+k,t) = (ht^{m})^{k} * e^{-ht^{m}} / k!} \tag{13}$$

or equivalently,

$$\boxed{p(i,n,t) = (ht^{m})^{n-i} * e^{-ht^{m}} / (n-i)!} \quad \text{for } n \neq j \tag{14}$$

$n > i$

3. The equation (3) is

$$p'(i,j,t) = mht^{m-1} * p(i,j-1,t) \tag{15}$$

or equivalently,

$$p'(i,j,t) = mht^{m-1} \frac{[\,(ht^{m})^{j-i-1} * e^{-ht^{m}}\,]}{(j-i-1)!} .$$

After simplifying,

$$p'(i,j,t) = \frac{mh^{(j-i)} * t^{(j-i)m-1} * e^{-ht^{m}}}{(j-i-1)!} . \tag{16}$$

By integrating both sides,

$$p(i,j,t) = \int_{\emptyset}^{t} \frac{mh^{(j-i)} * t^{(j-i)m - 1} * e^{-ht^{m}}}{(j-i-1)!}\, dt \tag{17}$$

The integral on the right hand side can be integrated by parts with

$$\int u\, dw = uw - \int w\, du \tag{18}$$

Let $u = e^{-hs^{m}}$, then $du = -mhs^{m-1} * e^{-hs^{m}}\, ds$

Let $dw = \frac{mh^{(j-i)} * s^{(j-i)m-1}}{(j-i-1)!}\, ds$, then $w = \frac{h^{(j-i)} * s^{(j-i)m}}{(j-i)!}$

Substituting these expressions into equation (18),

$$p(i,j,t) = \frac{(ht^{m})^{(j-i)} * e^{-ht^{m}}}{(j-i)!} + \int_{\emptyset}^{t} \frac{mh^{(j-i+1)} * s^{(j-i+1)m-1} * e^{-hs^{m}}}{(j-i)!}\, ds$$

If the integral on the right hand side is being integrated again, the result would have the same pattern as the integral that has been calculated above. Then,

$$p(i,j,t) = \frac{(ht^{m})^{(j-i)} * e^{-ht^{m}}}{(j-i)!} + \frac{(ht^{m})^{(j-i+1)} * e^{-ht^{m}}}{(j-i+1)!}$$

$$+ \int_{\emptyset}^{t} \frac{mh^{(j-i+2)} * s^{(j-i+2)m-1} * e^{-hs^{m}}}{(j-i+1)!} \, ds$$

Therefore,

$$p(i,j,t) = \sum_{k=\emptyset}^{\infty} \frac{(ht^{m})^{(j-i+k)} * e^{-ht^{m}}}{(j-i+k)!} \tag{19}$$

or equivalently,

$$\boxed{p(i,j,t) = 1 - \sum_{v=\emptyset}^{j-i-1} \frac{(ht^{m})^{v} * e^{-ht^{m}}}{v!}} \tag{2\emptyset}$$

APPENDIX B

A FORTRAN COMPUTER PROGRAM FOR SOLVING THE FAILURE DENSITY FUNCTION AND THE FAILURE CUMULATIVE DISTRIBUTION

```
C
C
C     THIS  PROGRAM  IS USED TO  CALCULATE  THE  FAILURE
C     DENSITY FUNCTION AND THE FAILURE CUMULATIVE
C     DISTRIBUTION FUNCTION
C
C
C
C     DEFINITION OF VARIABLES:
C
C       F(T)    = THE FAILURE DENSITY FUNCTION
C       FF(T)   = THE CUMULATIVE DISTRIBUTION FUNCTION
C       H       = THE REACTION RATE
C       K1,K2   = K1 < K < K2, WHERE K IS THE RANDOM
C                 DISTANCE
C       PROB    = THE PROBABILITY OF K, P[J-I=K]
C       RLAM1   = THE MEAN OF THE INITIAL STATE I
C       RLAM2   = THE MEAN OF THE FINAL STATE J
C       T1,T2   = T1 < T < T2, THE RANGE OF TIME F(T) AND
C                 FF(T) ARE CALCULATED
C       TOTAL   = F(T|I,J) FOR A SPECIFIC (J-I=K)
C
C
C
      DOUBLE PRECISION RLAM1,RLAM2,PROB,H,F(1ØØ),FF(1ØØ),
      DOUBLE PRECISION F1, F2, TOTAL
      INTEGER T,T1,T2,K1,K2,N,C
C
C     OPEN FILES 'F.OUT' TO STORE F(T), AND 'FF.OUT' TO
C     STORE FF(T)
C
        OPEN(6,FILE='F.OUT')
        OPEN(7,FILE='FF.OUT')
C
C     INITIALIZE VARIABLES
C
        RLAM1=1Ø.DØ
        RLAM2=1ØØ.DØ
        H=8.DØ
        T1=1
        T2=24
        K1=5Ø
        K2=13Ø
C
C     THIS DO LOOP IS USED TO CALCULATE F(T) AND FF(T)
C
        IF(T1.EQ.1) GOTO 5
          FF(T1-1)=Ø.DØ
      5 DO 1Ø T=T1,T2
          F(T)=Ø.DØ
```

```
         F1=1.D0
         C=T*INT(H)
         DO 11 I=1,C
           F1=F1*EXP(-1.0)
    11   CONTINUE
C
         F2=1.D0
         DO 20 KK=1,(K1-2)
           F2=F2*H*DFLOAT(T)/DFLOAT(KK)
    20   CONTINUE
         F2=F2*H
C
         DO 30 K=K1,K2
           F2=F2*H*DFLOAT(T)/DFLOAT(K-1)
           CALL SOLVE(RLAM1,RLAM2,K,PROB)
           TOTAL=PROB*F1*F2
           F(T)=F(T)+TOTAL
    30   CONTINUE

         IF (T.GT.1) GOTO 100
           FF(T)=F(T)
           GOTO 200
   100   FF(T)=FF(T-1)+F(T)
   200   WRITE(6,*)T,F(T)
         WRITE(7,*)T,FF(T)
    10 CONTINUE
       STOP
       END


C
C
C    THIS SUBROUTINE IS USED TO CALCULATE P[J-I=K]=PROB
C
C
C    DEFINITION OF VARIABLES:
C
C      FO   = THE FIRST TERM IN THE SERIES TO CALCULATE
C             P[J-I=K] WHERE N=O
C      F(N) = THE Nth TERM IN THE SERIES TO CALCULATE
C             P[J-I=K]
C      K    = THE RANDOM DISTANCE (J-I)
C      M    = 0 < N < M , THE FINAL Nth TERM IN THE SERIES
C      PROB = THE PROBABILITY OF THE RANDOM DISTANCE
C             P[J-I=K]
C
C
C
       SUBROUTINE SOLVE(RLAM1,RLAM2,K,PROB)
       DOUBLE PRECISION RLAM1,RLAM2,PROB,F(100),FO,FF
```

```
      INTEGER K,M
C
C        INITIALIZE THE LAST TERM IN THE SERIES
           M=3Ø
C
           FF=1.DØ
           DO 5 L=1,K
             FF=FF*RLAM1*EXP(-2.Ø)/DFLOAT(L)
         5 CONTINUE
C
C        THIS DO LOOP IS USED TO CALCULATE P[J-I=K]
C
           FO= EXP(-RLAM1+DFLOAT(K))*EXP(-RLAM2+DFLOAT(K))*FF
           F(1)=FO*RLAM1*RLAM2/FLOAT(K+1)
           PROB=FO+F1
           DO 1Ø N=2,M
            F(N)=F(N-1)*RLAM1*RLAM2/FLOAT(N*(K+N))
            PROB=PROB+F(N)
        1Ø CONTINUE
           RETURN
           END
```

APPPENDIX C

A FORTRAN COMPUTER PROGRAM FOR SOLVING THE WEIBULL PARAMETERS AND CHECKING THE VALIDITY OF THE MODEL

```
C
C
C      THIS  PROGRAM IS USED TO CALCULATED THE PARAMETERS
C            OF THE WEIBULL DISTRIBUTION.
C
C
C
C      DEFINITION OF VARIABLES:
C
C           H          = THE REACTION RATE
C           ALPHA      = THE TIME SCALE PARAMETER
C           BETA       = THE SHAPE PARAMETER
C           CORR       = COEFFICIENT OF CORRELATION
C           DELTA      = THE SHIFTED TIME
C           FT         = THE FAILURE CUMULATIVE DISTRIBUTION
C                        FUNCTION AT TIME T
C           T1,T2      = T1 < T < T2, THE TIME INTERVAL WHERE
C                        THE FAILURE DENSITY FUNCTION IS
C                        CALCULATED
C           X2         = THE SUM OF ALL (X**2), WHERE X=LN(T)
C           XS         = THE SUM OF ALL X
C           XU         = THE AVERAGE OF ALL X
C           Y2         = THE SUM OF ALL (Y**2),
C                        WHERE Y=LN[-LN{1-F(T)}]
C           YS         = THE SUMM OF ALL Y
C           YU         = THE AVERAGE OF ALL Y
C
C
C
           DOUBLE PRECISION T,FT,X2,XS,ALPHA,BETA,XY,YS,Y2
           DOUBLE PRECISION LT(100),LF(100),YU,XU,WF(100)
           DOUBLE PRECISION SX,SY,SXY,CORR,DIFF,H,DELTA
           INTEGER TT,T1,T2,D
C
C      OPEN FILE 'FF.OUT' TO READ THE FAILURE CDF
C      OPEN FILE 'WEIBULL.OUT' TO STORE THE RESULTS
C

           OPEN(5,FILE='FF.OUT')
           OPEN(6,FILE='WEIBULL.OUT')
C
C      INITIALIZE VARIABLES
C
           H=8.0
           T1=1
           T2=21
           DELTA=1.81
C
           X2=0.D0
           XS=0.D0
```

```
      XY=Ø.DØ
      Y2=Ø.DØ
      YS=Ø.DØ
C
C     THIS DO LOOP IS USED TO CALCULATE X2,XS,Y2,YS
C
      DO 1Ø T=T1,T2
        READ(5,*) T,FT
        F(T)=FT
        LT(I)=DLOG(T-DELTA)
        LF(I)=DLOG(-DLOG(1.Ø-FT))
        X2=X2+LT(I)**2
        XS=XS+LT(I)
        Y2=Y2+LF(I)**2
        YS=YS+LF(I)
        XY=XY+LT(I)*LF(I)
   1Ø CONTINUE
C
C     COEFFICIENT OF CORRELATION IS SOLVED IN THIS SECTION
C
      SX=X2-(XS**2)/DFLOAT(N)
      SY=Y2-(YS**2)/DFLOAT(N)
      SXY=XY-(XS*YS/DFLOAT(N))
      CORR=(SXY**2)/(SX*SY)
C
C     WRITE THE HEADINGS
C
      WRITE(6,2ØØ)H
  2ØØ FORMAT(32X,'THE REACTION RATE H =',F4.1///)
      WRITE(6,3ØØ) CORR,SX,SY,SXY
  3ØØ FORMAT(15X,'R**2 =',D15.1Ø//,15X,'SXX =',D15.1Ø/,
     $       15X, 'SYY =', D15.1Ø,/,15X,'SXY =',
     $       D15.1Ø/)
C
C
C     CALCULATE ALPHA AND BETA
C

      YU=YS/FLOAT(N)
      XU=XS/FLOAT(N)
      BETA=SXY/SX
      ALPHA=DEXP(YU-BETA*XU)
      WRITE(6,4ØØ)ALPHA,BETA,DELTA
  4ØØ FORMAT(15X,'ALPHA =',D2Ø.1Ø,/,15X,'BETA  =',
     $       F2Ø.1Ø,/, 15X,'DELTA =',F2Ø.1Ø)
C
C     WRITE THE HEADINGS
C
      WRITE(6,5ØØ)
  5ØØ FORMAT(//7X,'T',16X,'F(T)',17X,'WEIBULL WF(T)',
```

```
     $         5X, 'F(T)-WF(T)'/)
C
C     THE CDF OF THE WEIBULL IS CALCULATED AND COMPARED
C     TO THE FAILURE CDF
C

        DO 20 TT=T1,T2
          WF(T)=1.D0-EXP(-ALPHA*(FLOAT(T)**BETA))
          DIF=F(T)-WF(T)
          WRITE(6,600)TT,F(T),WF(T),DIF
  600   FORMAT(5X,I3,5X,D20.10,5X,D20.10,3X,F10.5)
   20 CONTINUE
      STOP
      END
```

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