

**FOUR-BOLT EXTENDED UNSTIFFENED
MOMENT END-PLATE CONNECTIONS**

by

Mary Sue M. Abel

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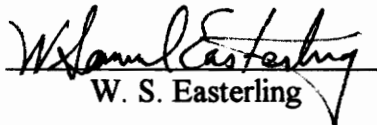
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APPROVED:


T. M. Murray, Chairman


W. S. Easterling


S. M. Holzer

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(ABSTRACT)

Design procedures for moment end-plate connections, which include prying action effects, have been verified for five end-plate configurations. The design procedures include the use of yield-line theory for determining the end-plate thickness and the so-called modified Kennedy method for determining bolt forces.

This investigation extends these design procedures to include the four-bolt extended unstiffened moment end-plate connection. The results of four experimental tests are evaluated and compared to the predicted results obtained from yield-line theory and the modified Kennedy method. Using the finite element method, analyses to determine the stress in the beam at a cross-section near to the connection are also performed. The results from the finite element analyses are compared to experimental results. In addition, the type of construction suitable for this connection is assessed.

Modifications to the analytical predictions are made based on the above comparisons. Results indicate that the flange force is distributed equally to the inner and outer end-plate. Also, prying action is included for the inner bolts at all times, and for the outer bolts when failure is controlled by the end-plates. Design recommendations and an example using the modified design procedure are presented in this thesis.

ACKNOWLEDGMENTS

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CHAPTER I

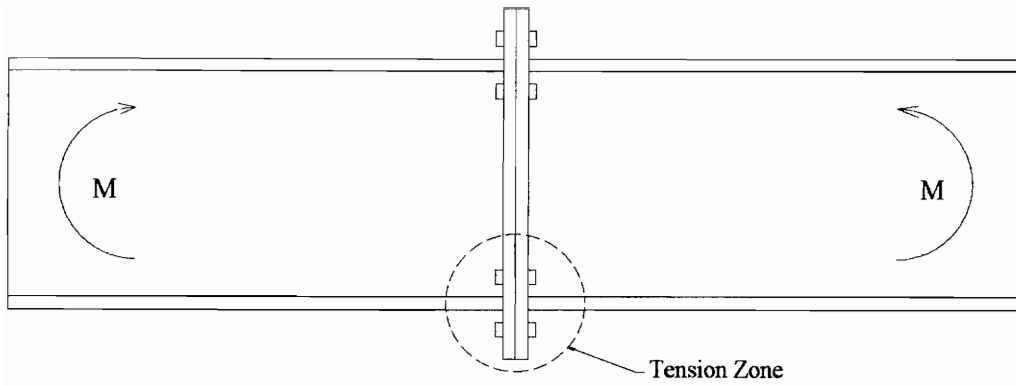
INTRODUCTION

1.1 BACKGROUND

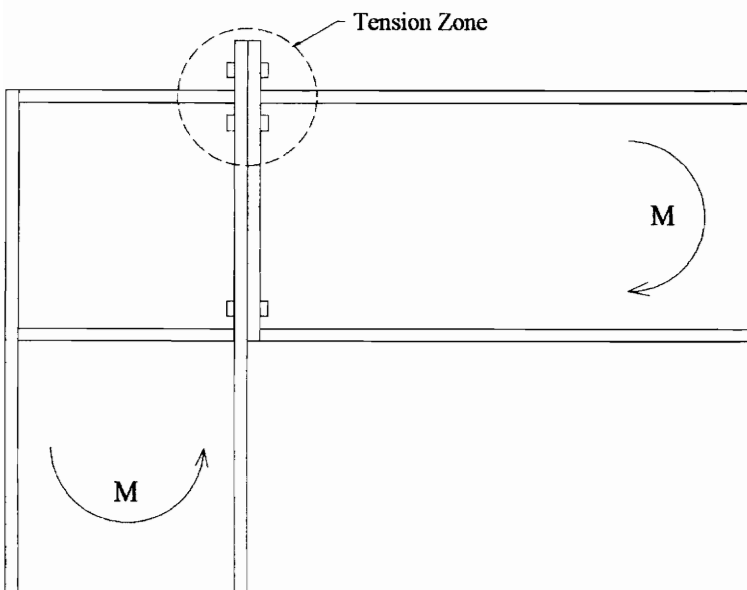
Moment end-plate connections are used in the United States primarily in pre-engineered, industrial buildings. They are commonly used in steel portal frame construction as bolted moment-resistant connections. Their use in multi-story, moment-resistant frame construction is becoming more common because of advancements in design methods and fabrication techniques. The moment end-plate is typically used to connect a beam to a beam, often referred to as a "splice-plate connection", Figure 1.1a, or to connect a beam to a column, Figure 1.1b.

Several design procedures for various moment end-plate configurations have been suggested to determine end-plate thickness and bolt diameter using the finite-element method, yield-line theory, and experimental test data. However, the variation between the different methods is large, especially when predicting the bolt forces. The main reason for large differences when predicting bolt forces is that some methods assume prying action is negligible, while other methods assume prying action is significant and contributes substantially to bolt force.

Hendrick, Kukreti, and Murray (1985) finalized a unification of design procedures for four configurations of the flush type moment end-plate connection. Two of these flush type connections are unstiffened: the two-bolt unstiffened, Figure 1.2(a), and the four-

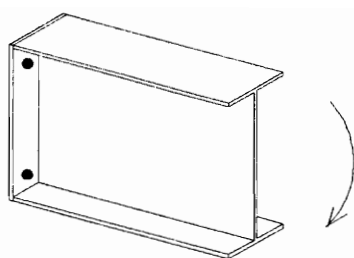


(a) Beam-to-Beam Connection

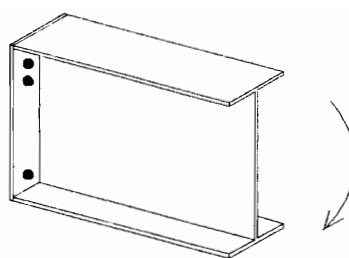


(b) Beam-to-Column Connection

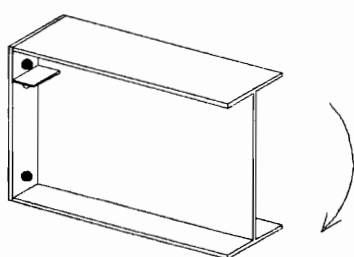
Figure 1.1 Typical Uses of Moment End-Plate Connections



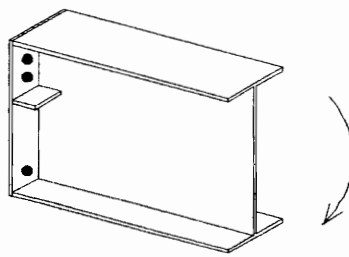
(a) Two-Bolt Unstiffened



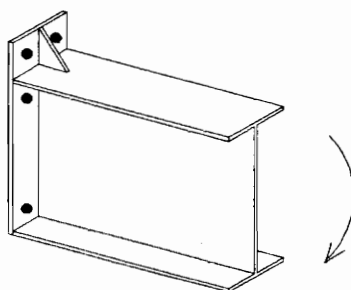
(b) Four-Bolt Unstiffened



(c) Four-Bolt Stiffened with
Web Gussett Plate Between
the Tension Bolt Rows



(d) Four-Bolt Stiffened with
Web Gussett Plate Outside
the Tension Bolt Rows



(e) Four-Bolt Extended Stiffened

Figure 1.2 Five Configurations of Moment End-Plate Connections
Morrison *et al* (1985)

bolt unstiffened, Figure 1.2(b). The other two flush type connections are stiffened: the four-bolt stiffened with web gusset plate between the two tension bolt rows, Figure 1.2(c), and the four-bolt stiffened with web gusset plate outside the two tension bolt rows, Figure 1.2(d). The gusset plates for each of the flush stiffened connections are symmetrical about the beam web and are welded to the end-plate and the beam web. The unification of design procedures for a fifth configuration of moment end-plate connection was done by Morrison, Astanyeh and Murray (1985). This configuration is the four-bolt extended stiffened moment end-plate connection, Figure 1.2(e).

This thesis continues the unification of design procedures for moment end-plate connections established by Hendrick *et al* (1985) for another configuration of moment end-plate. This sixth configuration is the four-bolt extended unstiffened shown in Figure 1.3. In this connection, the four bolts in the tension region are placed one row of two bolts on each side of the beam tension flange. The unified design procedures include determination of end-plate thickness and prediction of bolt forces.

1.2 LITERATURE REVIEW

Research on the four-bolt extended, unstiffened moment end-plate connection was performed by Krishnamurthy. Murray (1987) summarizes these results and notes that the failure modes of Krishnamurthy's tests are associated with the beam and not the end-plate, bolts, or welds. To accurately verify design procedures for the end-plate, bolts, and welds, the failure modes of the tests must be controlled by these factors. Further testing of this configuration of end-plate connection has been requested to develop a design procedure that accurately predicts the behavior of this connection.

An extensive review of end-plate connection literature was reported by Srouji, Kukreti and Murray (1983). They presented the design procedures of various authors and

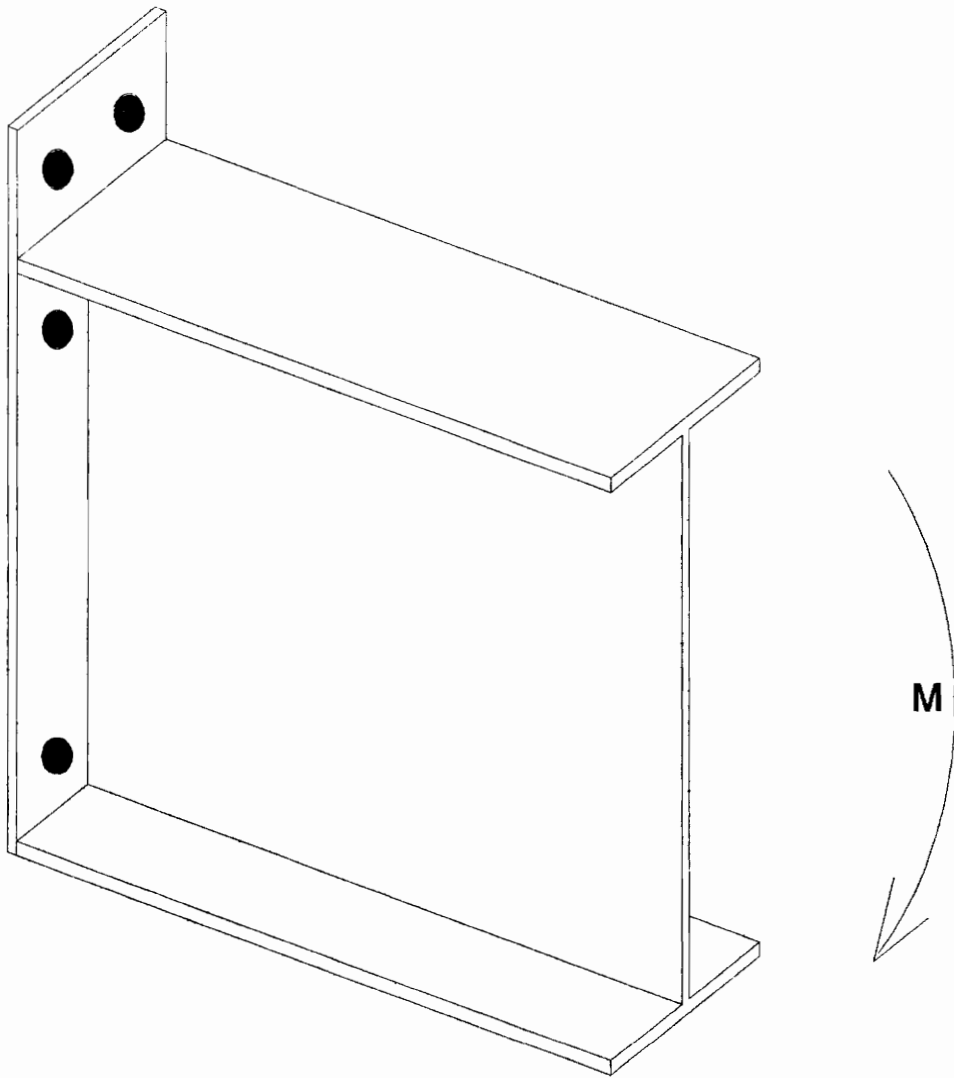


Figure 1.3 Four-Bolt Extended Unstiffened Moment End-Plate Connection

made comparisons of end-plate thickness based on those recommendations. Based on the review, they selected the yield-line method for end-plate analysis and the Kennedy, Vinnakota and Sherbourne (1981) method for bolt force prediction. The Kennedy method includes prying action when determining the bolt force. Following Srouji *et al* (1983), these approaches have been used by Morrison *et al* (1985) and Hendrick *et al* (1985) for different end-plate configurations. A summary of the recommended design procedures for the tests performed by Srouji and Hendrick are discussed by Murray (1987). In each of these studies, successful correlation between the predicted and experimental test results for both end-plate strength and bolt force magnitude was accomplished. For this reason, the same approach was used for the present study of the four-bolt extended, unstiffened moment end-plate connection.

1.3 SCOPE OF RESEARCH

The purpose of this study is to develop design procedures, consistent with those of the Hendrick *et al* (1985) unification, for the four-bolt extended unstiffened moment end-plate connection. More specifically, the design procedures are to provide:

1. Determination of end-plate thickness by yield-line theory given end-plate geometry, beam geometry, and material yield stress; a strength criterion.
2. Determination of bolt forces by a modified Kennedy method given end-plate geometry, bolt diameter, and bolt type; a bolt force criterion.
3. An assessment of construction type for which the connection is suitable; a stiffness criterion.

The objectives of the study were accomplished by developing end-plate strength prediction and bolt force prediction equations. Four tests of full size end-plate configurations were then conducted to verify these analytical prediction equations. Figure 1.4 presents the various parameters that define the end-plate geometry.

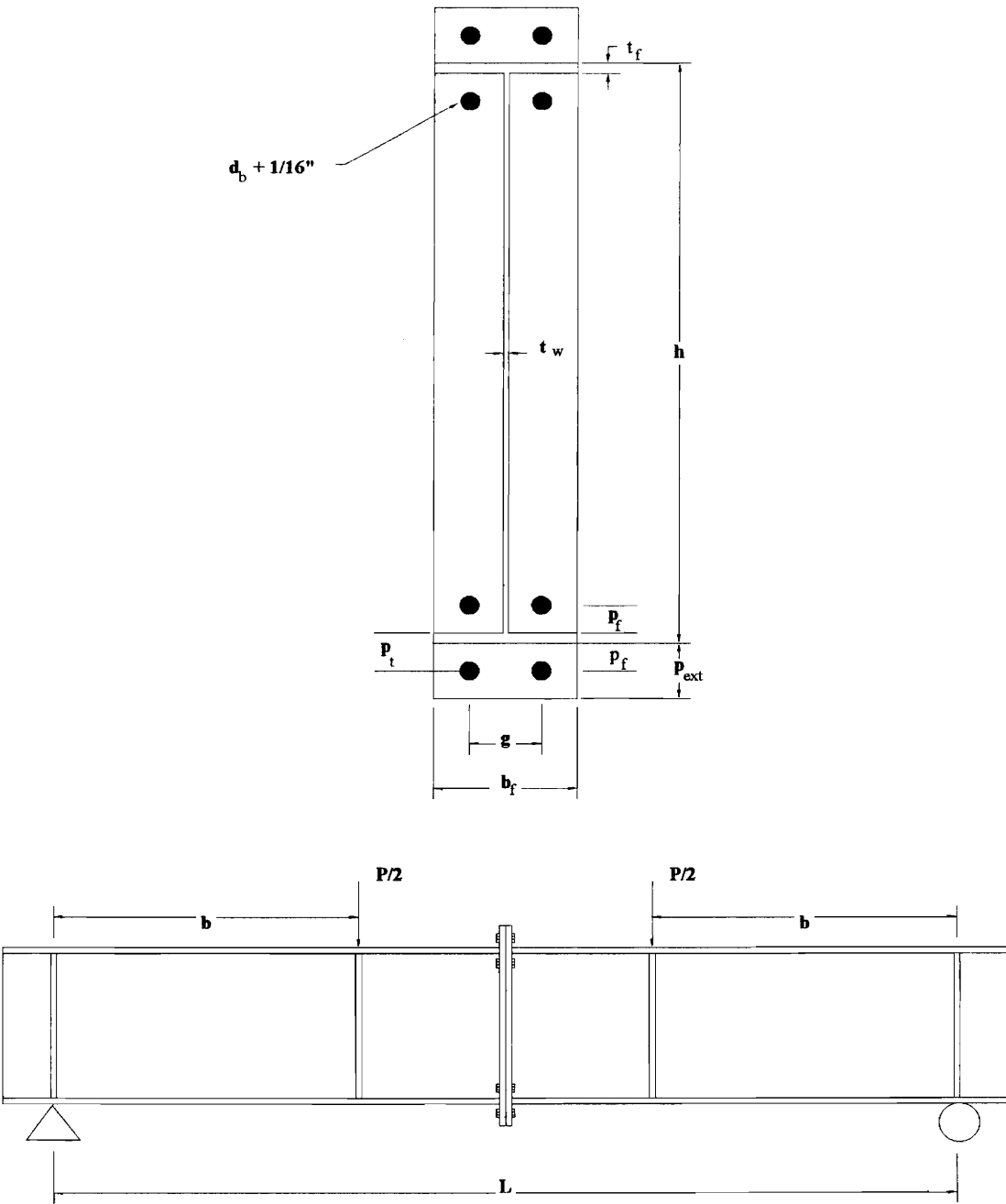


Figure 1.4 Definition of Geometric Parameters

CHAPTER II

ANALYTICAL STUDY

2.1 END-PLATE STRENGTH USING YIELD-LINE THEORY

The procedure to determine an end-plate plastic moment capacity, or limit state, using the yield-line method failure load, is to first select possible yield-line mechanisms. Next, the external and internal work for an virtual displacement are set equal, thereby establishing the relationship between the applied load and the nominal strength of the end-plate. This equation is then solved for either the unknown load or the required nominal strength. By comparing the values obtained from the arbitrarily selected mechanisms, the appropriate yield-line mechanism is that with the largest required nominal strength or the smallest failure load. A more detailed description is presented by Hendrick *et al* (1985).

The yield-line mechanism shown in Figure 2.1 is the controlling yield-line pattern for the four-bolt extended unstiffened moment end-plate. The external, W_e , and internal, W_i , work expressions for this yield-line pattern are:

$$W_e = M_u * \theta = M_u * l/h \quad (2.1)$$

$$W_i = \frac{4m_p}{h} \left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right] \quad (2.2)$$

The nominal strength of the end-plate is found by equating the external and internal work.

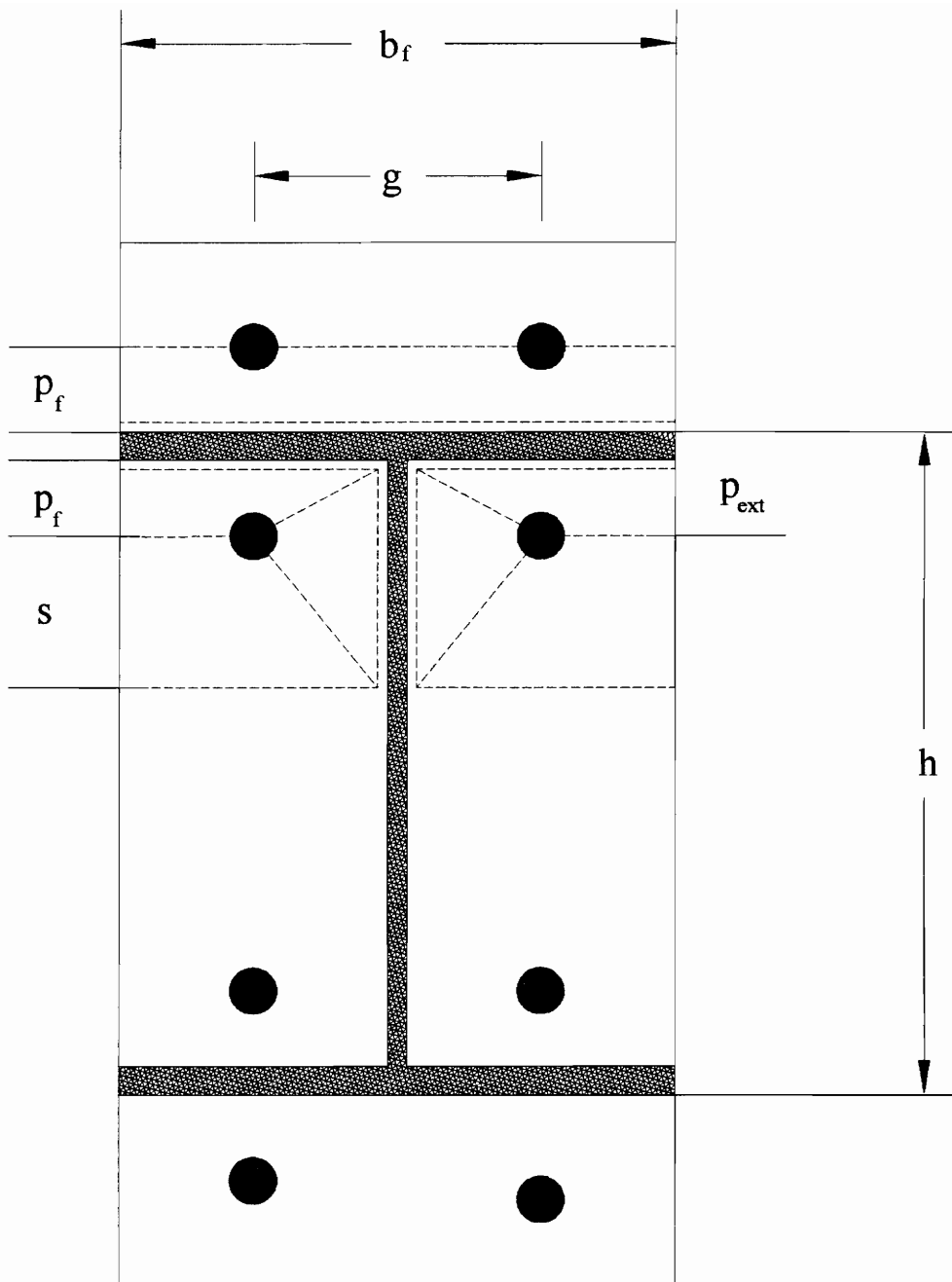


Figure 2.1 Controlling Yield-Line Pattern for the Four-Bolt Extended, Unstiffened Moment End-Plate

$$M_u = 4m_p \left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right] \quad (2.3)$$

where:

$$m_p = \frac{F_{py} t_p^2}{4}$$

F_{py} = yield stress of the end-plate material and t_p = end-plate thickness. The remaining variables are defined in Figure 2.1.

Rearranging M_u results in the required end-plate thickness for a desired nominal strength:

$$t_p = \sqrt{\frac{(M_u / F_{py})}{\left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right]}} \quad (2.4)$$

The unknown dimension "s" in Figure 2.1 is found by differentiating the internal work expression, Equation 2.2, with respect to s and equating to zero. The resulting expression for s is:

$$s = \frac{1}{2} \sqrt{b_f g} \quad (2.5)$$

2.2 BOLT FORCE PREDICTIONS USING THE MODIFIED KENNEDY METHOD

Yield-line theory does not produce bolt force predictions including prying action forces. Because experimental results indicate that prying action behavior is present in end-plate connections, a method suggested by Kennedy *et al* (1981) was adopted by Srouji *et al* (1983) to predict bolt forces as a function of applied flange force.

The Kennedy method is based on the split-tee analogy and three stages of plate behavior: thick plate, intermediate plate and thin plate. Consider a split-tee model, Figure 2.2, consisting of a flange bolted to a rigid support and attached to a web through which a

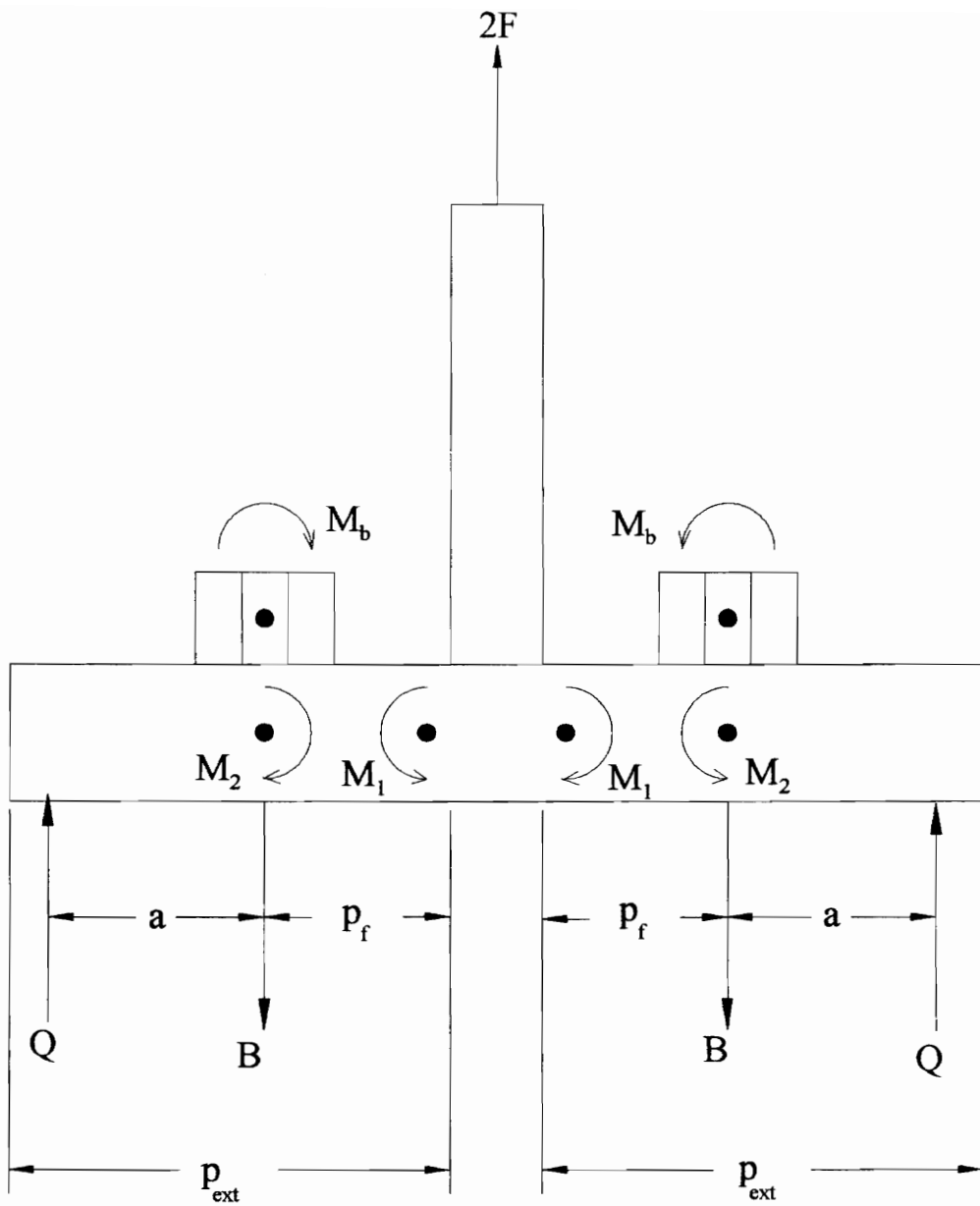


Figure 2.2 Kennedy Method Split-Tee Model
Morrison *et al* (1985)

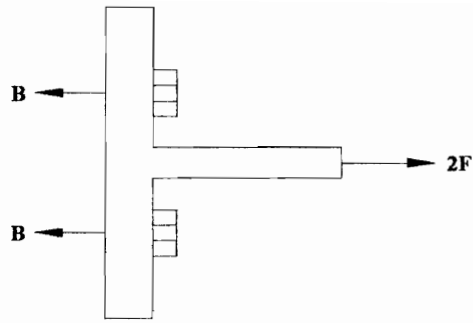
tension load is applied. In Figure 2.2, $2F$ is the applied tension force, Q is a prying action force, and B is the total resulting force, applied plus prying action, in a bolt. At the lower levels of applied load, the flange behavior is termed thick plate behavior as plastic hinges have not formed in the split-tee flange, Figure 2.3(a). As the applied load is increased, two plastic hinges form at the centerline of the flange and each web face intersection, Figure 2.3(b). This yielding marks the "thick plate limit" and indicates the second stage of plate behavior termed intermediate plate behavior. At a greater applied load level, two additional plastic hinges form at the centerline of the flange and each bolt, Figure 2.3(c). The formation of this second set of plastic hinges marks the "thin plate limit" and indicates the third stage of plate behavior termed thin plate behavior.

For all stages of plate behavior, the Kennedy method predicts a bolt force as the sum of a portion of the applied force and a prying force. The portion of the applied force depends on the applied load, while the magnitude of the prying force depends on the stage of plate behavior. For the first stage of behavior, or thick plate behavior, the prying force is assumed to be zero. For the second stage of behavior, or intermediate plate behavior, the prying force increases from zero at the thick plate limit to a maximum at the thin plate limit. For the third stage of behavior, or thin plate behavior, the prying force is maximum and constant. The distance " a " between the point of prying force application and the centerline of bolt (see Figure 2.2) has been determined empirically by Hendrick *et al* (1985), as a function of t_p/d_b and is given by:

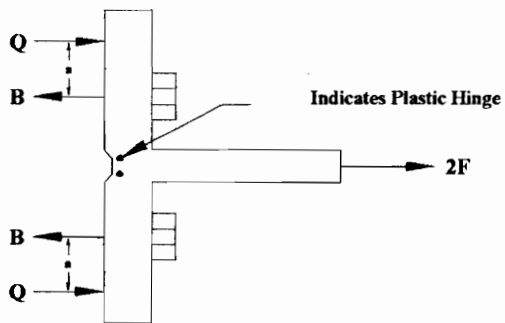
$$a = 3.682(t_p/d_b)^3 - 0.085 \quad (2.6)$$

but not to exceed $(p_{ext}-p_f)$ for the outer portion of the end-plate (see Figure 2.2).

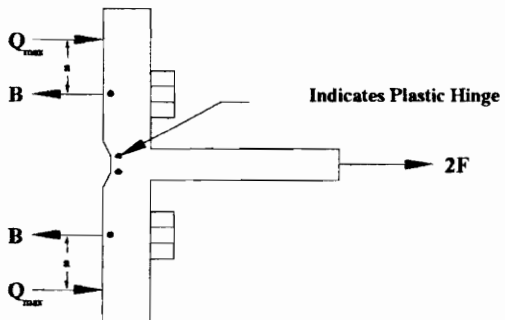
Modifications of the Kennedy method are necessary for application to the four-bolt extended unstiffened moment end-plate connection. First, the connection is idealized in



a) First Stage – Thick Plate Behavior



b) Second Stage – Intermediate Plate Behavior



c) Third Stage – Thin Plate Behavior

Figure 2.3 Kennedy Method Split-Tee Behavior
Morrison *et al* (1985)

two parts: the "outer end-plate" and the "inner end-plate", Figure 2.4. The outer end-plate consists of the end-plate extension outside the beam tension flange and a portion of the beam tension flange. The inner end-plate consists of the end-plate within the beam flanges and the remaining portion of the beam tension flange. Second, two factors, α and β , are introduced. These factors proportion the tension flange force to the outer end-plate and the inner end-plate, respectively. The factors α and β are determined experimentally (see Chapter III) and for this configuration are found to be equal and satisfy:

$$\alpha + \beta = 1.0 \quad (2.7)$$

Therefore, $\alpha = \beta = 0.5$

To determine the magnitude of the prying force, and hence, the inner and outer end-plate bolt forces, B_i and B_o , the stages of inner and outer end-plate behavior must first be ascertained. The end-plate behavior is established by comparing the end-plate force with the thick plate limit force, F_1 , and the thin plate limit force, F_{11} . The thick plate limit force is:

$$F_1 = \frac{b_f t_p^2 F_{py}}{4p_f \sqrt{1 + (3t_p^2 / 16p_f^2)}} \quad (2.8)$$

The thin plate limit force, F_{11} , is:

$$F_{11} = \frac{t_p^2 F_{py} [0.85(b_f / 2) + 0.80w'] + [(\pi d_b^3 F_{yb}) / 8]}{2p_f} \quad (2.9)$$

Equations 2.8 and 2.9 are from Kennedy *et al* (1981).

If the inner or outer end-plate force, βF_f or αF_p , is less than the thick plate limit force, F_1 , the respective portion of the end-plate behaves as a thick plate and the prying

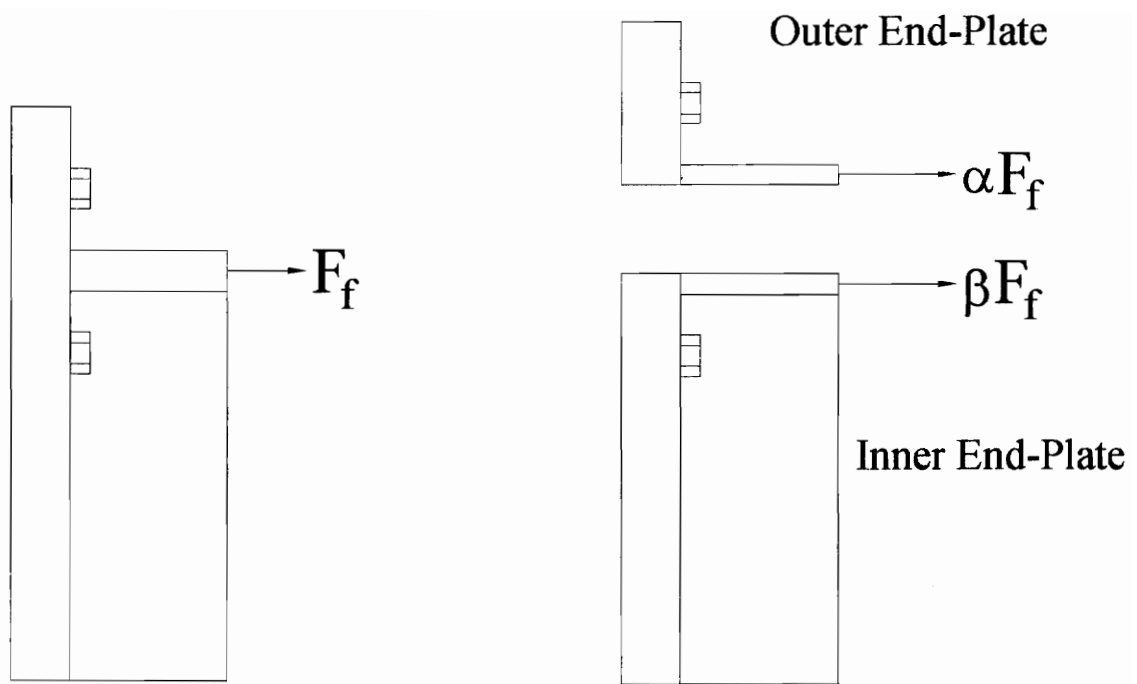


Figure 2.4 Modified Kennedy Method Idealized for the Four-Bolt Extended Unstiffened Moment End-Plate Connection

force in that portion is zero. Hence, the bolt forces, B_i and B_E , for thick plate behavior are βF_f or αF_f divided by the number of bolts in that portion of the end-plate, 2:

$$\begin{aligned} B_i &= \beta F_f / 2 & \text{if} & & \beta F_f < F_1 \\ B_E &= \alpha F_f / 2 & \text{if} & & \alpha F_f < F_1 \end{aligned} \quad (2.10)$$

If the inner or outer end-plate force, βF_f or αF_f , is greater than or equal to the thick plate limit force, F_1 , and less than or equal to the thin plate limit force, F_{11} , the end-plate behavior is intermediate and the prying force, Q , from Kennedy *et al* (1981) is:

$$Q = \frac{\gamma F_f P_f}{2a} - \frac{\pi d_b^3 F_{yb}}{32a} - \frac{b_f t_p^2}{8a} \sqrt{F_{py}^2 - 3(\gamma F_f / b_f t_p)^2} \quad (2.11)$$

where $\gamma = \alpha$ or β depending on which portion of the end-plate being analyzed. If Q is negative, no prying action is added to the bolt force. The bolt forces B_i or B_E for intermediate end-plate behavior are βF_f or αF_f divided by the number of bolts at that portion of the end-plate, 2, plus the prying force, Q :

$$\begin{aligned} B_i &= \beta F_f / 2 + Q & \text{when} & & F_1 \leq \beta F_f \leq F_{11} \\ B_E &= \alpha F_f / 2 + Q & \text{when} & & F_1 \leq \alpha F_f \leq F_{11} \end{aligned} \quad (2.12)$$

Finally, if the inner or outer end-plate force, βF_f or αF_f , is greater than the thin plate limit force, F_{11} , the end-plate behavior is thin and the prying force is at a maximum. The prying force, $Q_{\max}$, from Kennedy *et al* (1981) is:

$$Q_{\max} = \frac{w' t_p^2}{4a} \sqrt{F_{py}^2 - 3(F' / w' t_p)^2} \quad (2.13)$$

where $F' = F_{11}/2$ and $w' =$ end-plate width per bolt hole less the bolt hole diameter. Hence, the bolt forces B_1 or B_E for thin end-plate behavior are βF_f or αF_p , divided by the number of bolts at the portion of the end-plate, 2, plus the prying force, $Q_{\max}$:

$$\begin{aligned} B_1 &= \beta F_f / 2 + Q_{\max} & \text{when } \beta F_f > F_{11} \\ B_E &= \alpha F_p / 2 + Q_{\max} & \text{when } \alpha F_p > F_{11} \end{aligned} \quad (2.14)$$

The reader is cautioned that the quantities under the radicals in Eqs. 2.11 and 2.13 can be negative. A negative value for these terms indicates that the end-plate yielded locally in shear before the bolt prying action force could be developed, thus the connection is not adequate for the applied load.

2.3 WEB STRESS PREDICTIONS USING FINITE ELEMENT ANALYSES

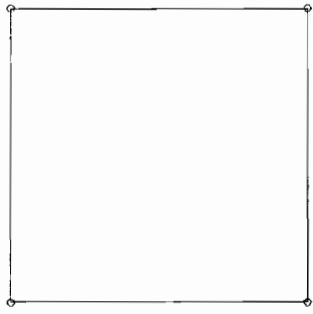
The stress in the web of the beam is needed to determine the weld size between the web and the end-plate. At locations near the connection, there have been questions raised about whether or not classical beam theory, My/I , is an accurate prediction of the stress in this region. Experimental results were verified using the finite element method. Separate analyses were performed for each of the four test configurations. The strains obtained from the experimental tests and finite element analyses are compared in Section 4.3.

ANSYS, SASI (1989), was used to perform the finite element analyses. Nonlinear material properties were entered for the bolts, beam, and end-plate. The nonlinear material properties were obtained from actual testing of the bolts, beam, and end-plate, as discussed in Section 3.5. In addition, nonlinearity was present in the interface elements along the end-plate, where the stress was zero in the tension region.

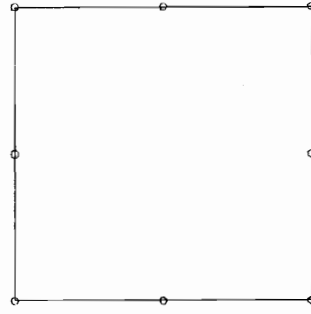
Several modifications were made to the model leading to the final model utilized in the experimental comparisons. First, to eliminate boundary effects at the plane of load

application, the length of the model was increased from one times the depth of the beam to two times the depth of the beam. Next, interface elements were introduced to more accurately model the actual behavior of the connection. Because the face of the end-plate can only transfer compressive forces to the connecting member, the nodes along this plane must not transfer tension. The interface element acts in this capacity, alleviating the iterative procedure required to release the correct nodes from transferring tension. Finally, two models of test E50-3/4-5/8-18, one using 4-node quadrilateral elements, Figure 2.5(a), and one using 8-node quadrilateral elements, Figure 2.5(b), were compared. The strains along the cross-section two inches from the connection are similar, as shown in Figure 2.6. For this reason, a 4-node quadrilateral element was used for simplicity.

The final models utilized in the analyses are 2-dimensional and use three different element types. The first element, STIF42, is a 2-d isoparametric solid. This element was used to model the beam web, beam flange, and end-plate. STIF42 is a quadrilateral element with four nodes located at the corners, and degrees of freedom in the X and Y directions. The plane stress analysis option with the thickness of each component being entered separately was used. The mesh for the analysis representing test E50-3/4-5/8-18 is shown in Figure 2.7. This is representative of the mesh used in all four analyses. In all of the models, the length of the beam mesh is twice the depth. The web, flange, and end-plate are divided into elements which are approximately 2 in x 2 in, 2 in x 1/2 in and 1/2 in x 2 in, respectively. Element STIF39 was used to model the tension bolts, Figure 2.5(c). This element acts as a spring and is allowed to move along the nodal X axes only. To represent the bolts accurately during the finite element analyses, a force versus displacement curve was entered for each bolt. (The data for these curves was obtained from testing performed by Butler Research, Grandview, Missouri.) The third element was STIF12, Figure 2.5(d). This element is an interface element and provides resistance in



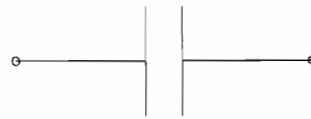
a) 4-Node Quadrilateral Element
STIF42



b) 8-Node Quadrilateral Element
STIF82



c) Spring Element STIF39



d) Interface Element STIF12

Figure 2.5 Stick Figures of Elements

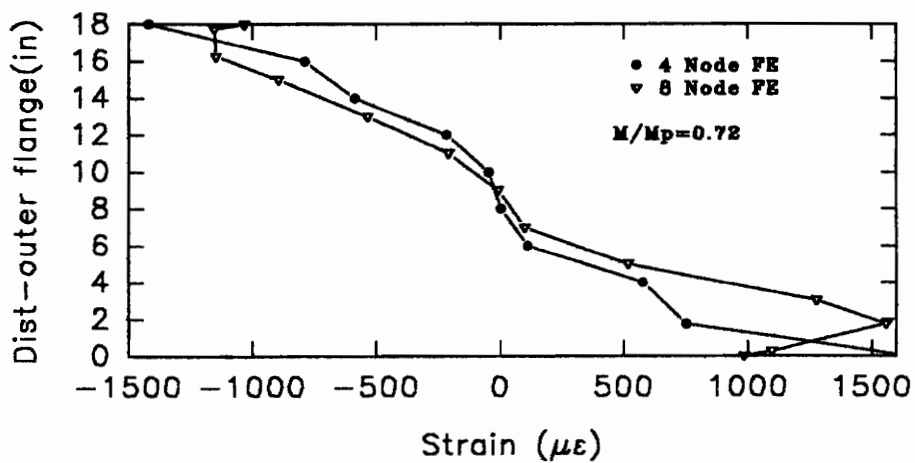
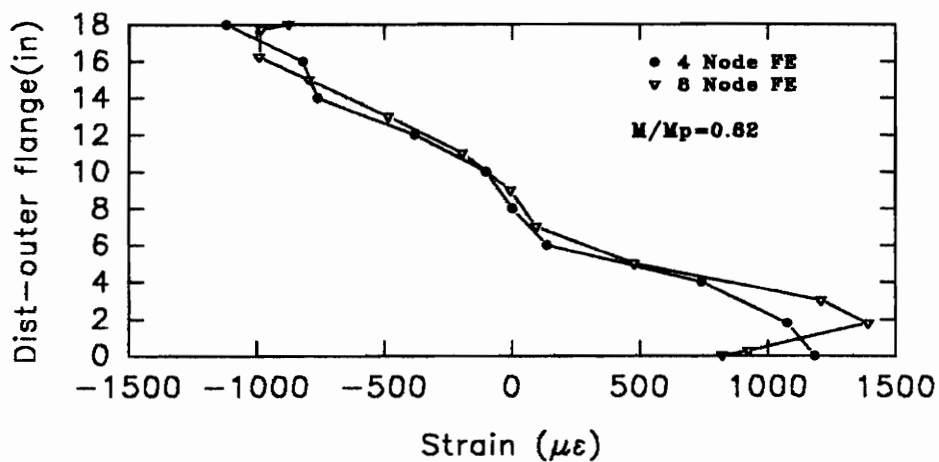
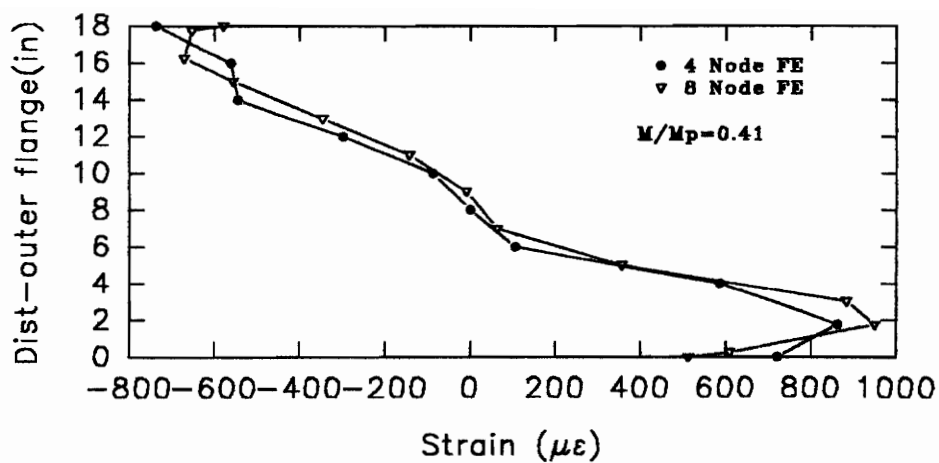


Figure 2.6 4-Node and 8-Node Finite Element Comparisons for Test E50-3/4-5/8-18

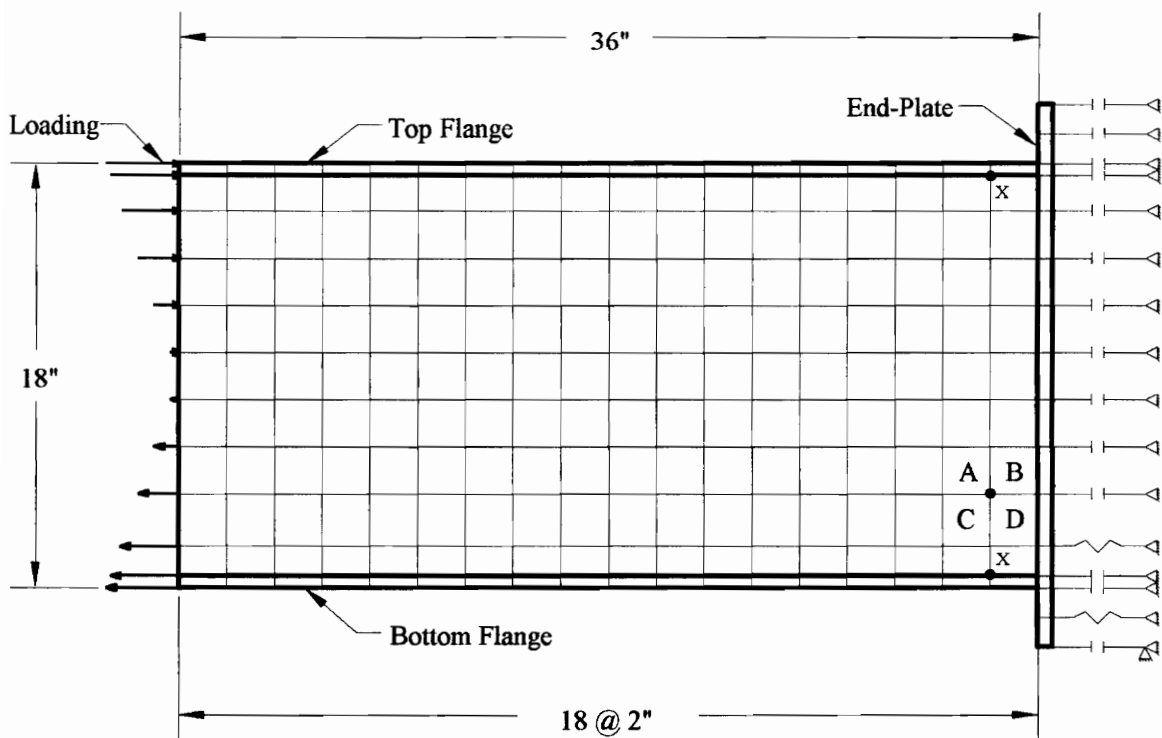


Figure 2.7 Finite Element Model for E50-3/4-5/8-18

compression only. When the node is in tension, it is free to displace. STIF12 was used at the nodes along the end-plate in order to offer resistance which is provided by the opposite end-plate during actual testing. Both STIF39 and STIF12 have two nodes, i and j, which are coincident. The j node of each of these elements was restrained in the X direction and the bottom interface element j node was restrained for equilibrium in the global Y direction.

To compare the analytical and experimental results, three applied moments for which experimental data was available were used in the finite element analyses. The moments in the finite element model were created by applying a horizontal load to each node along the end opposite the connection. This is illustrated in Figure 2.7.

The common method to compute the stress at a cross-section of a beam is classical beam theory, My/I . Using the same theory, My/EI can be used to compute the strain. The classical beam theory was used to verify the results obtained in the finite element analyses. The strain was checked at a point 1.5 times the depth away from the connection to insure that the strain distribution in the model is not affected by the method of load application. Confidence in the validity of the results was derived from the similarity observed between the finite element analysis and classical beam theory, as illustrated in Figure 2.8.

Because experimental data was available at a section 2 in from the connection, the strains from the finite element analyses were also computed at this location. To compute the strain at any node in the section, the nodal strain from each of the elements common to that node were averaged. For example, in Figure 2.7, elements A, B, C and D all have a node in common, and their strains are averaged to compute the strain at this location. The strain for the nodes marked X in Figure 2.7 were not calculated due to the difference in the material thickness between the beam flange and web.

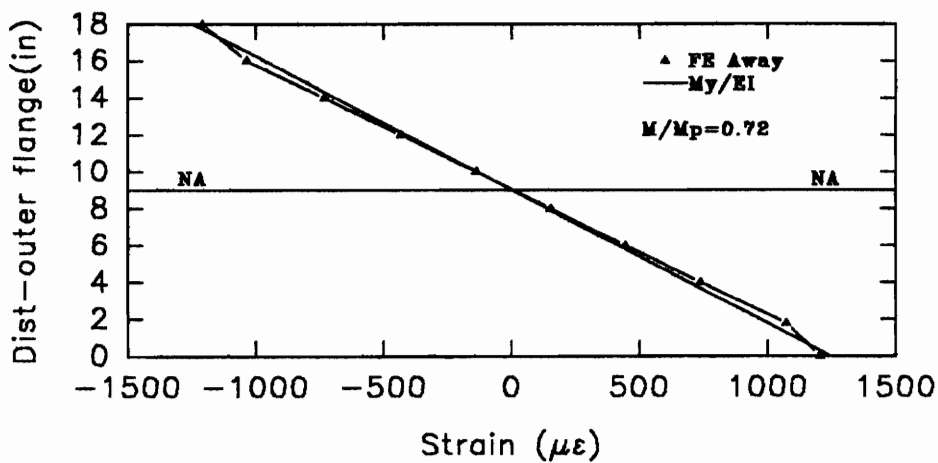
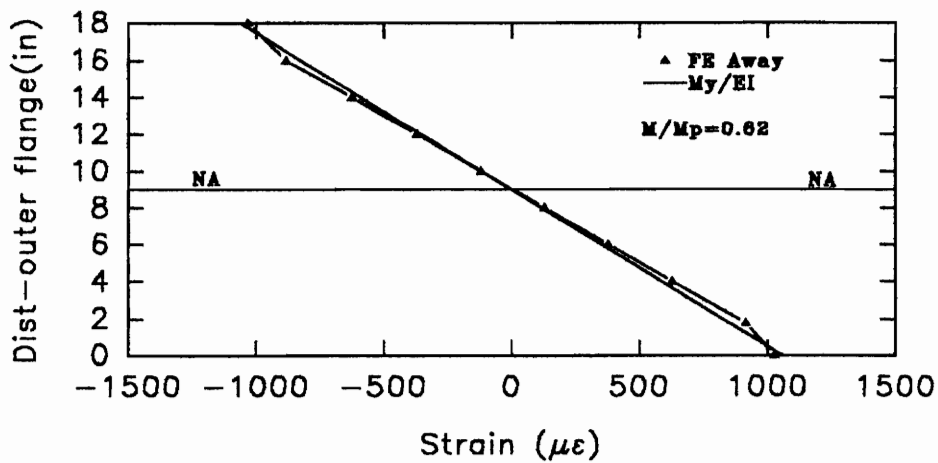
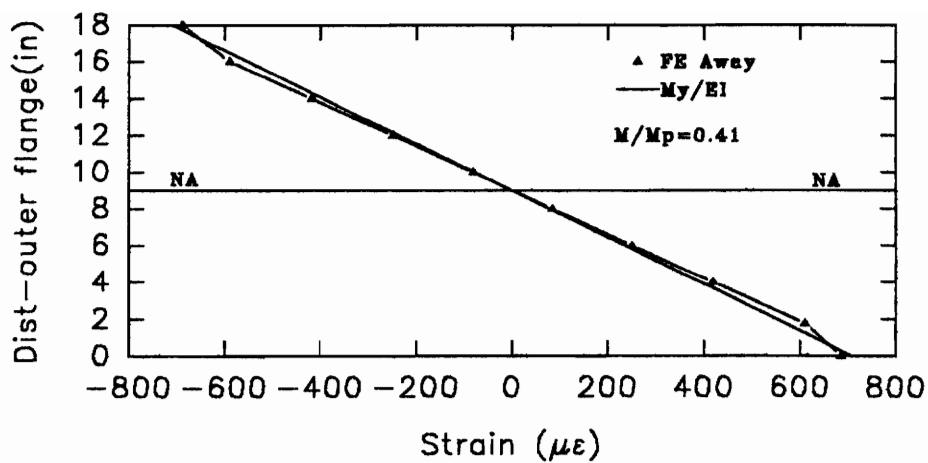


Figure 2.8 Comparisons of Classical Beam Theory and Finite Element Results Away from the Connection

The results of the finite element analyses are shown in the appendices for each of the four tests performed. The finite element results are plotted with the experimental results for comparative purposes.

2.4 MOMENT-ROTATION RELATIONSHIPS

Connection stiffness is the rotational resistance of a connection to applied moment. This connection characteristic is often described with a moment versus rotation or $M-\phi$ diagram. The slope of the $M-\phi$ curve, typically obtained from experimental test data, is an indication of the rotational stiffness of the connection. The greater the slope of the curve, the greater the stiffness of the connection.

This stiffness is reflected in the three types of construction defined by the AISC ASD Specification (AISC 1989): Type I, Type II, and Type III. Type I Construction, or rigid framing, assumes that the connections have sufficient rigidity to fully resist rotation at joints. Type II Construction, or simple framing, assumes that the connections are free to rotate under gravity load and that beams are connected for shear only. Finally, Type III Construction, or semi-rigid framing, assumes that connections have a dependable and known moment capacity as a function of rotation between that of Type I and Type II Construction. The AISC LRFD Specification (AISC 1986) defines two types of construction: FR and PR. Fully restrained or FR construction is the same as ASD Type I. Partially restrained or PR construction encompasses ASD Types II and III.

Idealized $M-\phi$ curves for three typical connections representing the three AISC types of construction are shown in Figure 2.9. Note that the $M-\phi$ curve for an ideally fixed connection is one which traces the ordinate of the $M-\phi$ diagram, whereas the $M-\phi$ curve for an ideally simple connection is one which traces the abscissa of the $M-\phi$ diagram.

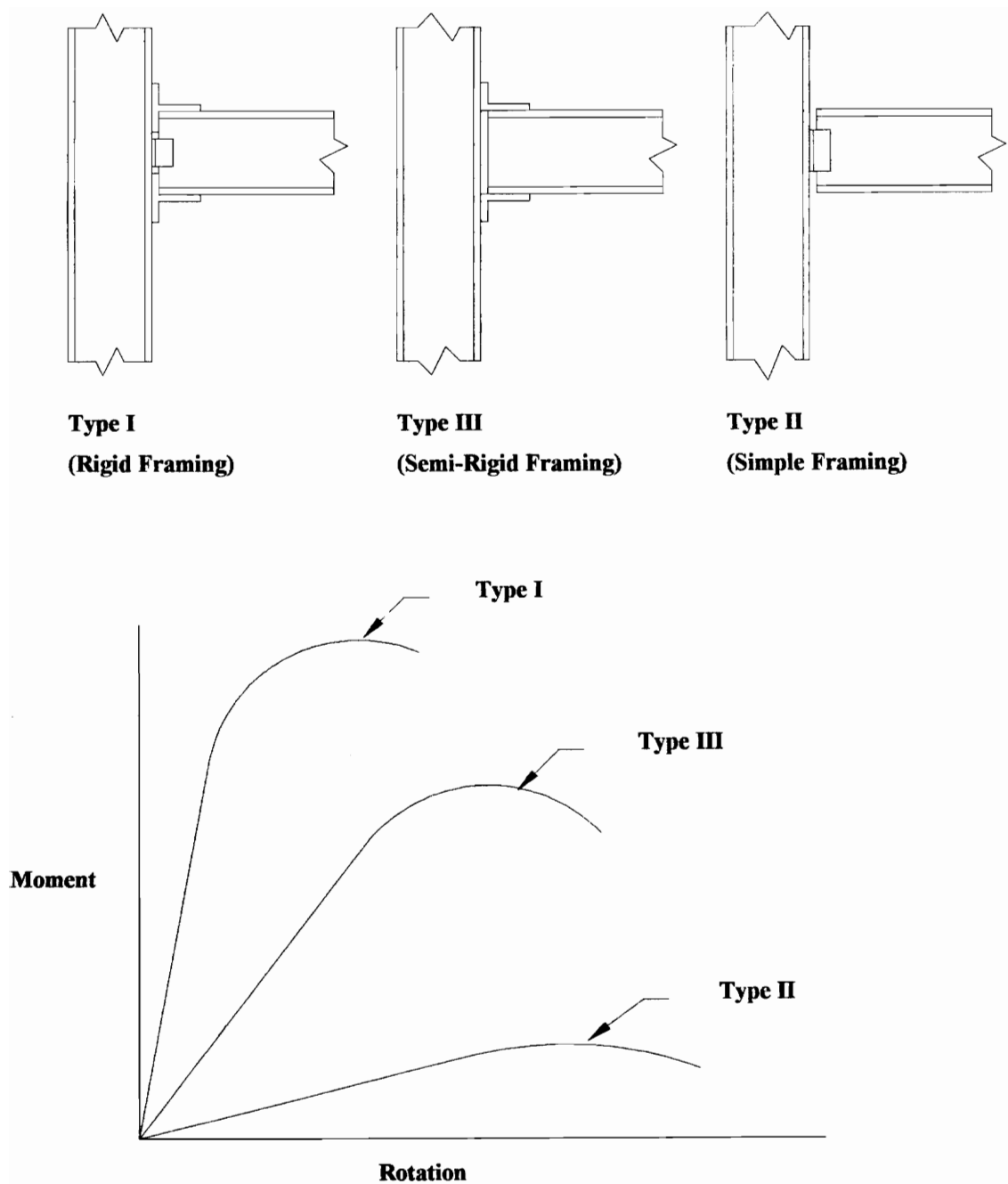


Figure 2.9 Idealized $M-\Phi$ Curves for Typical Connections
Morrison *et al* (1985)

For beams, guidelines have been suggested (Salmon and Johnson 1980) to correlate $M-\phi$ connection behavior and AISC Construction Type. A Type I or FR connection should carry an end moment greater than or equal to 90% of the full fixity end moment and not rotate more than 10% of the simple span rotation. A Type II connection should resist an end moment less than or equal to 20% of the full fixity end moment and rotate at least 80% of the simple span beam end rotation. A Type III connection lies between the limits of the Type I and Type II connections. A PR connection is any connection that does not satisfy the FR requirements.

The simple span beam end rotation for any loading is given by:

$$\theta_s = M_F L / 2EI \quad (2.15)$$

where M_F = fixed end moment for the loading.

Setting M_F equal to the yield moment of the beam, SF_y , and with $I/S = h/2$:

$$\theta_s = F_y L / Eh \quad (2.16)$$

These relationships are used in Section 4.3 to determine the type of construction suitable for four-bolt, extended unstiffened end-plate configuration.

CHAPTER III

EXPERIMENTAL INVESTIGATION

3.1 TEST SPECIMENS

A series of four tests were performed to verify the yield-line theory and modified Kennedy method predictions for the four-bolt extended unstiffened moment end-plate connection. The test specimens consisted of end-plates welded to two beam sections which were in turn bolted together in the beam-to-beam connection configuration shown in Figure 3.1. Four tests were performed for the four-bolt extended unstiffened moment end-plate connection. The beams and end-plate for two of the tests were built-up sections using Grade 50 material. The beams for the other two tests were hot-rolled W16x67 sections. The end-plate material was A36 steel. All bolts were A325.

Each test is designated by a specific code, for example, E50-3/4-5/8-18. The E signifies a four-bolt extended unstiffened moment end-plate configuration, 50 is the yield stress of the end-plate, 3/4 is the bolt diameter in inches, 5/8 is the nominal end-plate thickness in inches, and 18 represents the nominal beam depth in inches. In summary:

E50-3/4-5/8-18 corresponds to $E F_{py} - d_b - t_p - h$

The actual geometric parameters for each test were measured and recorded. Table 3.1 summarizes this data.

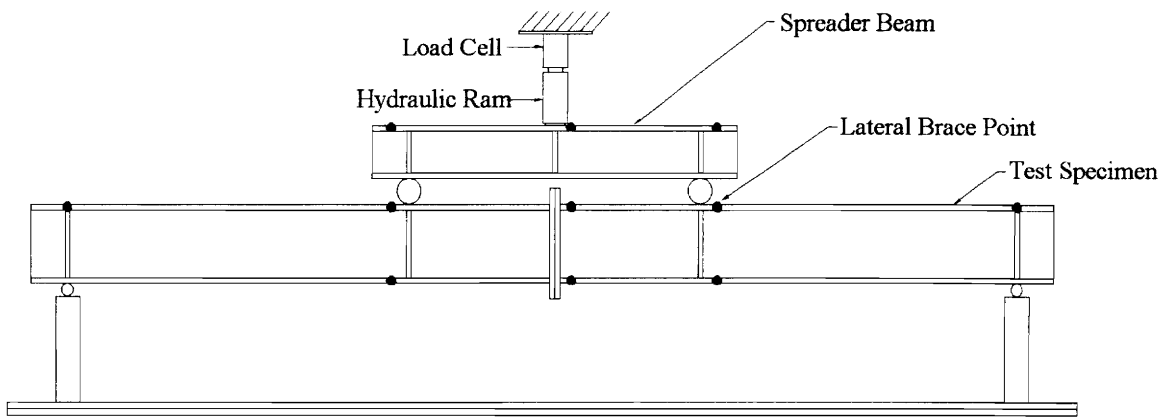


Figure 3.1 Longitudinal Elevation of Test Set-up

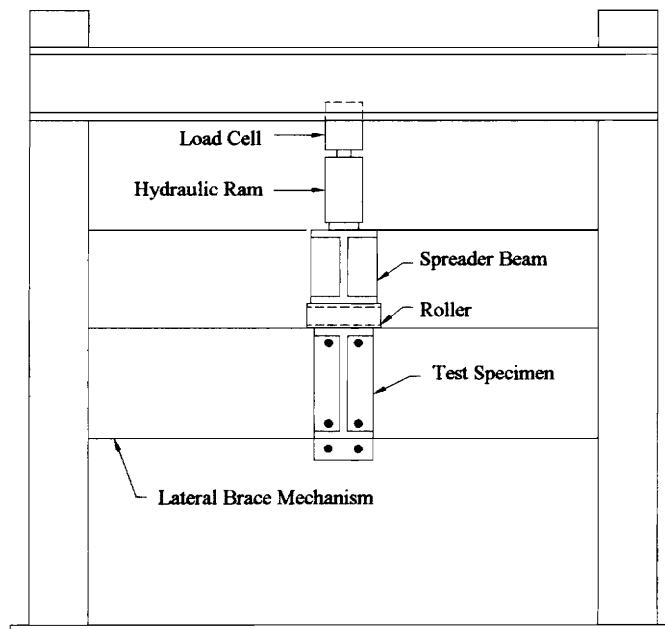
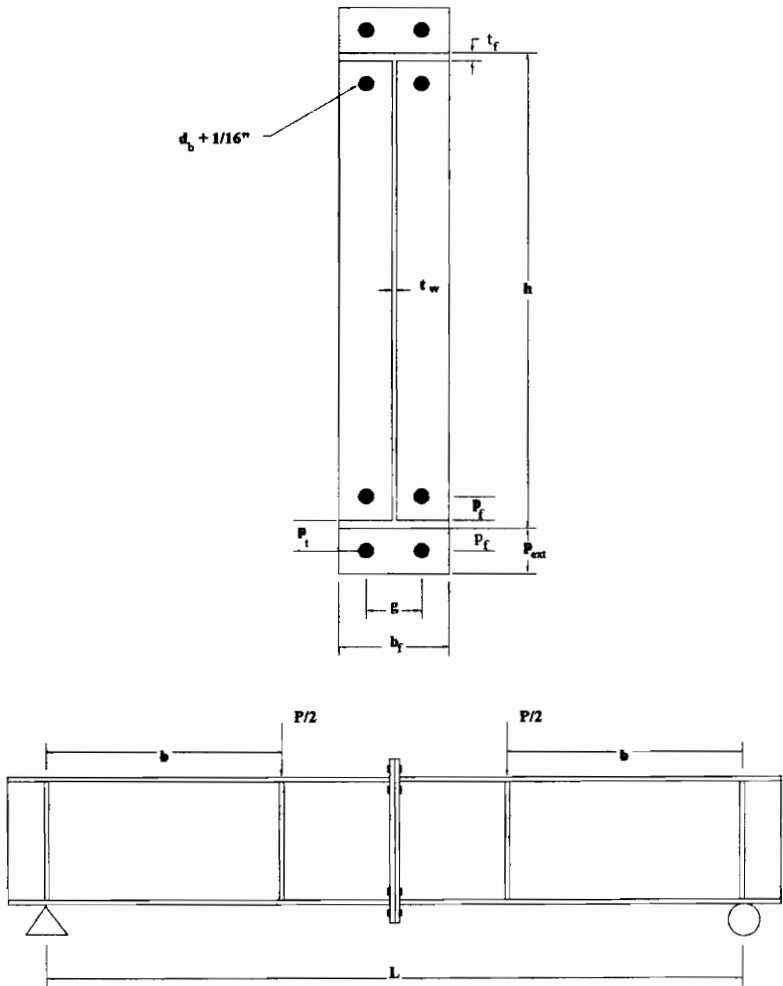


Figure 3.2 Cross-Section of Test Set-up

Table 3.1 Actual Geometric Parameters

Test Designation	t_p (in)	d_b (in)	P_{ext} (in)	P_f (in)	g (in)	h (in)	b_f (in)	t_w (in)	t_f (in)	F_{py} (ksi)	b (ft)	L (ft)
E50-3/4-5/8-18	0.633	0.75	2.438	1.25	2.986	18.0	8.0	0.25	0.512	66.9	7.0	20.0
E50-3/4-3/4-18	0.788	0.75	2.50	1.25	2.991	18.0	8.0	0.25	0.512	57.4	7.0	20.0
E36-1 1/8-7/8-16	0.886	1.125	4.0	2.5	7.0	16.21	10.25	0.42	0.657	46.5	7.0	20.0
E36-1 1/4-7/8-16	0.886	1.25	3.375	2.0	6.0	16.21	10.25	0.42	0.657	46.5	7.0	20.0



3.2 TEST SETUP

Load was applied to a test specimen by a hydraulic ram via a load cell, swivel head, and spreader beam, as shown in Figure 3.2. The end-plates were subjected to pure moment as the test beam was simply supported and loaded with two equal concentrated loads symmetrically placed. Lateral support for both the test specimen and the spreader beam was provided by lateral brace mechanisms bolted to three steel wide flange frames anchored to the reaction floor of the laboratory.

Each test setup was instrumented with a load cell, six displacement transducers, two instrumented bolts, and fourteen strain gauges. Two gauged calipers were used on two tests, E36-1 1/8-7/8-16 and E36-1 1/4-7/8-16, to measure plate separation. Data was collected, processed, and recorded with a pc-based data acquisition/control unit. Plots of selected data were made simultaneous with experimental testing, permitting effective monitoring of the tests.

The load applied by the hydraulic ram to the spreader beam was measured with the load cell. Deflections were measured with displacement transducers. One transducer was located near the test specimen centerline to measure vertical deflection, four transducers were located at the supports, two at each support, to measure the support settlement. The remaining transducer was located near the test specimen centerline to monitor the lateral deflection of the test beam.

End-plate separation in two tests, E36-1 1/8-7/8-16 and E36-1 1/4-7/8-16, was measured with gauged calipers. The separation was measured at two points: "inner" and "outer". Inner plate separation was measured between the edges of adjacent end-plates at the exterior part of the end-plate directly under the beam web as near to the beam tension flange as possible. Outer plate separation was measured between the edges of adjacent end-plates at the exterior part of the end-plate as near as possible to both the test specimen

beam tension flanges and the tension flange tips. The locations of this instrumentation are shown in Figure 3.3.

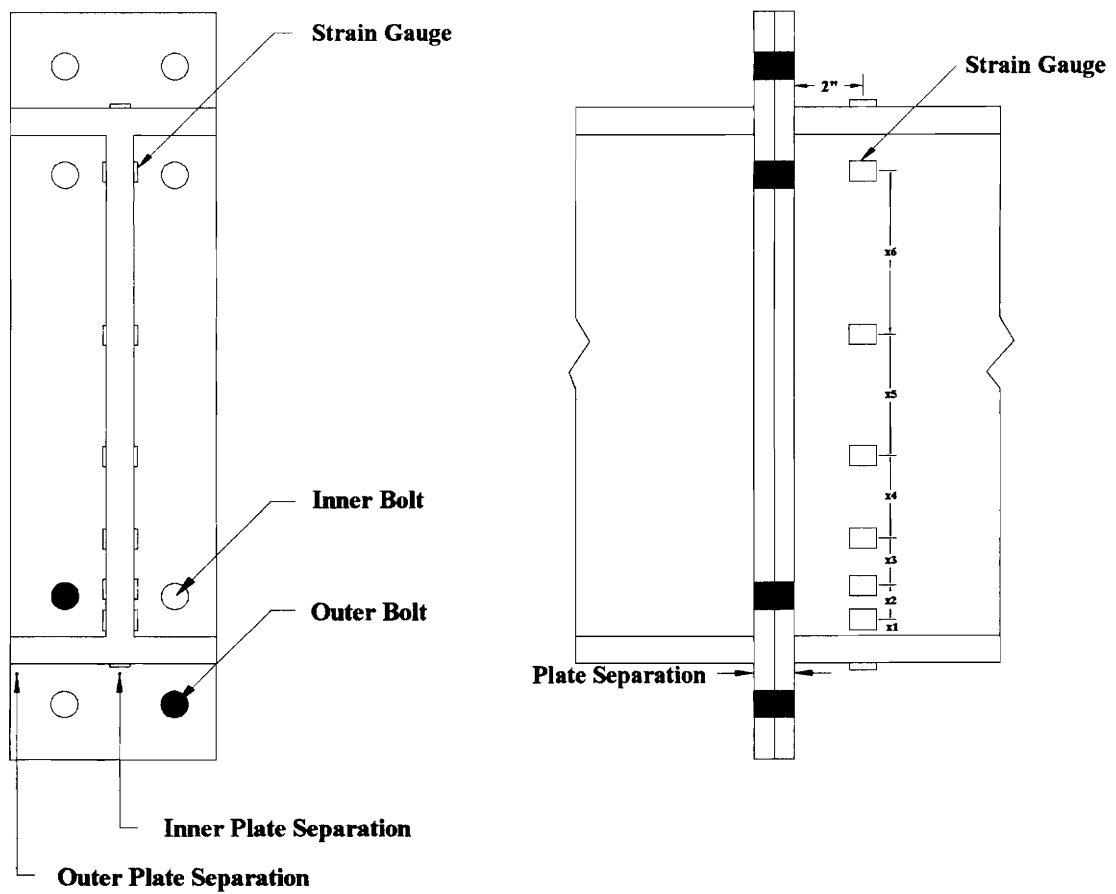
Instrumented bolts were used to measure bolt force at two tension bolt locations: "inner" and "outer". An inner bolt is located inside the beam tension flange, within the inner end-plate. An outer bolt is located outside the beam tension flange, within the outer end-plate. The locations of the instrumented bolts are shown in Figure 3.3.

To instrument a bolt, a small hole (2 mm or 3.5 mm) is drilled through the bolt head and into the unthreaded portion of the bolt shank. A special strain gauge, known as a "bolt gauge" is then glued into the hole. The bolt gauge is positioned so that its gauge length is between the bolt head and the threaded portion of the bolt shank, when possible. The bolt is then calibrated using a special fixture in a universal testing machine.

Fourteen strain gauges were located on one of the beam sections adjacent the end-plate. Gauges were placed along the beam web and at the outside of the beam flanges. These gauges measured strain, and hence, the stress at each gauge location could be calculated. Strain gauge locations are shown in Figure 3.3. Table 3.2 lists the values of the coordinate x_i in Figure 3.3, for each test. Figure 3.4 shows two photographs of the test setup.

Table 3.2 Strain Gauge Locations

Coordinate	E50 Tests	E36 Tests
x_1	1 1/4 in	1 1/4 in
x_2	1/2 in	3/4 in
x_3	1 in	1 in
x_4	2 in	2 in
x_5	4 in	3 in
x_6	6 in	4 in



Note: Instrumented bolts shown shaded

Figure 3.3 Instrumentation Location

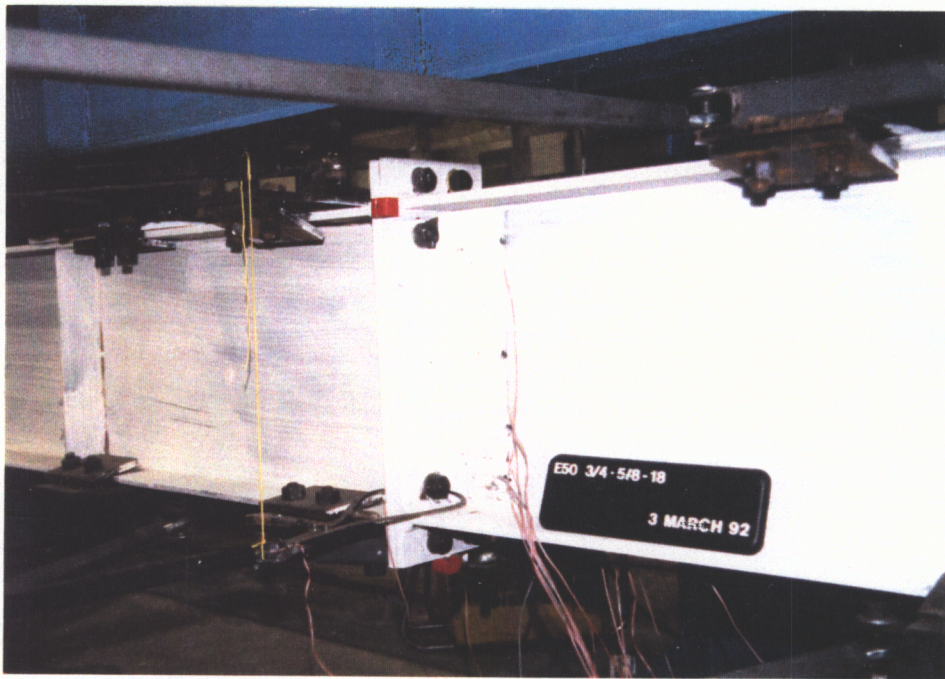
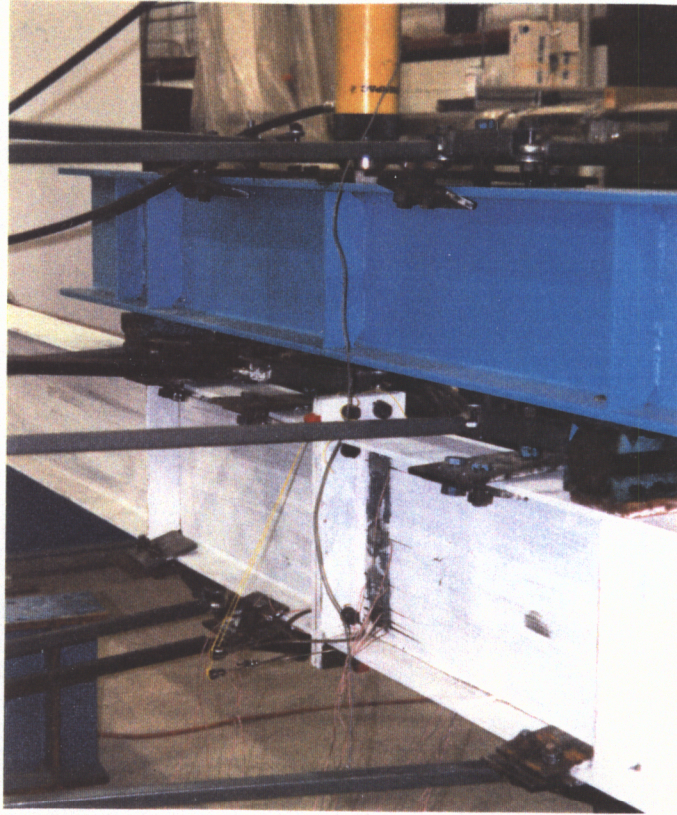


Figure 3.4 Photographs of Test Setup

3.3 TESTING PROCEDURE

Following an initial loading to approximately 20% of the predicted capacity of the end-plate connection, applied to seat the specimen and check instrumentation, the test specimen was loaded and unloaded in two increasing stages until failure. The first unload point was at 67% of the predicted capacity and the second unload point was at the predicted capacity of the end-plate connection. The test was then loaded to failure. Failure is defined as either beam failure, end-plate failure, or bolt failure. Beam failure occurs when a yield plateau is reached on an applied moment versus centerline deflection plot. Similarly, end-plate failure occurs when a yield plateau is reached on an applied moment versus an inner or outer end-plate separation plot. Bolt failure occurs at the applied moment at which an inner or outer bolt force reached its proof load.

3.4 EXPERIMENTAL RESULTS

Results for the four four-bolt extended unstiffened moment end-plate tests are found in Appendices B, C, D and E.

The test results for E50-3/4-5/8-18 and E50-3/4-3/4-18 include a Test Synopsis and seven plots. These plots are: centerline vertical deflection versus applied moment, applied moment versus inner bolt force, applied moment versus outer bolt force, applied moment versus rotation and three plots of the cross-sectional strain.

The test results for E36-1 1/8-7/8-16 and E36-1 1/4-7/8-16 include a Test Synopsis and nine plots. These plots are: centerline vertical deflection versus applied moment, inner end-plate separation versus applied moment, inner end-plate separation versus applied moment, outer end-plate separation versus applied moment, applied moment versus exterior bolt force, applied moment versus interior bolt force and applied moment versus rotation and three plots of the cross-sectional strain.

The centerline vertical deflection versus applied moment plots show both predicted and experimental results. The prediction curve is the elastic centerline deflection for a simply supported beam with two equal concentrated loads symmetrically placed:

$$\Delta_{\text{pred}} = \frac{(P/2)b}{24EI} (3L^2 - 4b^2) \quad (3.1)$$

where b is defined in Figure 1.1. The predicted failure moment is shown on these plots.

The outer end-plate separation versus applied moment plots include experimental results using calipers and feeler gauges. Both the inner and outer end-plate separation plots show the predicted failure moment.

The exterior and interior bolt force versus applied moment plots include both the modified Kennedy method prediction and the experimental test results. For tests E50-3/4-5/8-18 and E50-3/4-3/4-18 the thick and thin plate limits are also shown. The predicted curves are plotted only for values less than or equal to the ultimate tensile load. The measured bolt forces were determined from experimentally developed force versus strain relationships as described in Section 3.5.

The applied moment versus rotation plots are developed by solving the following for the connection rotation, ϕ :

$$\Delta_{\text{test}} = \Delta_{\text{pred}} + \phi L/2 \quad (3.2)$$

where Δ_{pred} is given in Equation 3.1.

The cross-sectional strain plots include the finite element analysis prediction, classical beam theory, My/EI , and the experimental results at three different load levels.

3.5 SUPPLEMENTARY TESTS

Standard ASTM E8 18 in. tensile test coupons were cut from the same plate used to fabricate the test specimen end-plates. Test coupons were also cut from the beam web

and flange after experimental testing was completed. Coupon tests were then performed using a universal testing machine. Results are found in Table 3.3.

One bolt of each diameter used in the four tests was instrumented with a bolt gauge and loaded to failure in a universal testing machine. Bolt gauge strain was recorded at load increments and a bolt force versus strain relationship developed. Figure 3.5 shows the relationships. (This phase of the study was conducted by Butler Research, Grandview, Missouri.)

The ultimate tensile load is shown on each plot in Figure 3.5. The bolt force versus strain relationship was used to obtain a relationship between strain in the elastic region and strain in the plastic region. A relationship for each bolt diameter was obtained:

$$R = \frac{\left[\frac{\text{strain}}{\text{load}} (\text{elastic}) \right]}{\left[\frac{\text{strain}}{\text{load}} (\text{plastic}) \right]} \quad (3.3)$$

As discussed in Section 3.2, each instrumented bolt used in experimental testing was first calibrated to obtain a relationship between the recorded bolt strain and the actual bolt force. However, this relationship is only valid in the elastic region and must be modified for the plastic region. Once yielding of the bolt occurs, bolt yield force was determined from tests done by Butler Research, the ratio in Equation 3.3 is then applied. The bolt force in the plastic region is then:

$$BF_{\text{plastic}} = P_y + (\epsilon_{\text{plastic}} - \epsilon_y) * (m_{\text{elastic}} * R) \quad (3.4)$$

where P_y is the bolt material yield stress, $m_{\text{elastic}} = \frac{P_y}{\epsilon_y}$ and R is defined in Eq. 3.3.

Table 3.3 Tensile Coupon Test Results

Coupon	Yield Stress (ksi)	Tensile Stress (ksi)
E50-3/4-5/8-18		
End-Plate	66.9	92.5
Web	60.3	72.0
Tension Flange	55.4	77.7
Comp Flange	55.5	77.1
E50-3/4-3/4-18		
End-Plate	57.4	81.6
Web	60.3	72.0
Tension Flange	55.4	77.7
Comp Flange	55.5	77.1
E36-1 1/8-7/8-16		
End-Plate	46.5	67.9
Web	60.2	78.8
Tension Flange	60.9	80.0
Comp Flange	60.3	80.5
E36-1 1/4-7/8-16		
End-Plate	46.5	67.9
Web	60.4	79.4
Tension Flange	58.7	78.7
Comp Flange	59.5	80.3

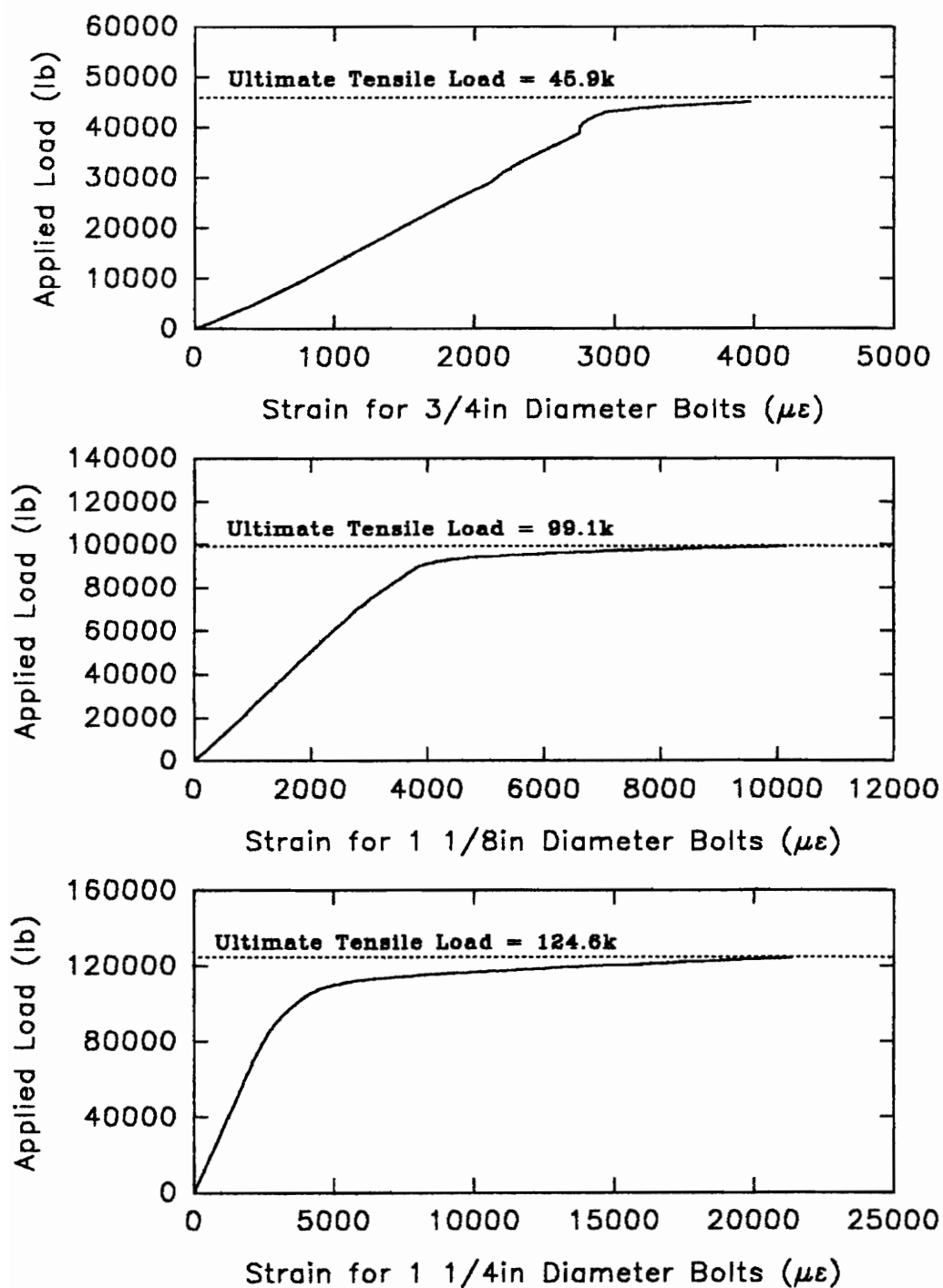


Figure 3.5 Bolt Force versus Strain Relationships

CHAPTER IV

COMPARISON OF EXPERIMENTAL TEST RESULTS AND PREDICTIONS

4.1 END-PLATE STRENGTH COMPARISONS

Tests E36-1 1/8-7/8-16 and E36-1 1/4-7/8-16 were designed to be controlled by end-plate failure. The nominal moment strength for each test was calculated using Equation 2.4 and the measured yield stress in Table 3.2. The maximum applied moment, predicted nominal moment strength, and the ratio of predicted-to-applied moment for each test is shown in Table 4.1. The predicted-to-applied moment ratios were 0.74 and 0.93, both conservative values.

**Table 4.1 Predicted and Experimental Test Results
for E36-1 1/8-7/8-16 and E36-1 1/4-7/8-16**

Test	Maximum Applied Moment	Predicted Nominal Moment Strength for End-Plates	Predicted/Applied
E36-1 1/8-7/8-16	337.8 k-ft	313.8 k-ft	0.93
E36-1 1/4-7/8-16	502.9 k-ft	374.0 k-ft	0.74

From the moment versus end-plate separation plots in Appendix D, the predicted failure moment corresponds very closely to the yield plateau for tests E36-1 1/8-7/8-16. For Test E36-1 1/4-7/8-36, initial separation occurs very close to the predicted strength.

See plot in Appendix E. However, the actual strength continued to increase until the test was terminated.

In both tests, measured bolt forces closely followed the predicted bolt force curves, see plots in Appendices D and E. The ultimate tensile load of the bolts was not reached in either test. For these tests, which were controlled by end-plate failure, prying action was present in both the interior and exterior bolts.

4.2 BOLT FORCE COMPARISONS

Tests E50-3/4-5/8-18 and E50-3/4-3/4-18 were designed to be controlled by bolt failure. Table 4.2 shows the applied and predicted moments at which the ultimate tensile load was reached in the inner and outer bolts for these tests. The ultimate tensile load, determined as described in Section 3.4, is the load which causes tensile failure of the bolt. For the 3/4 in diameter bolt used in both tests, this load is 45.9 kips. The ultimate tensile load is shown on the moment versus bolt force plots in Appendices B and C.

**Table 4.2 Predicted and Experimental Test Results
for E50-3/4-5/8-18 and E50-3/4-3/4-18**

Test	Applied Moment at Ultimate Tensile Load		Predicted Moment at Ultimate Tensile Load		Predicted/Applied	
	Inner Bolt	Outer Bolt	Inner Bolt	Outer Bolt	Inner Bolt	Outer Bolt
E50-3/4-5/8-18	232.5 k-ft	253.8 k-ft	215.4 k-ft	267.6 k-ft	0.91	1.05
E50-3/4-3/4-18	278.1 k-ft	276.9 k-ft	245.7 k-ft	267.6 k-ft	0.88	0.97

The predicted moments were obtained by determining values for the outer and inner end-plate factors, α and β , which are found in Equations 2.7, 2.10, 2.12 and 2.14. These factors proportion the beam tension flange force to the outer and inner end-plates,

respectively. The terms α and β , were empirically determined from the experimental test data to be equal, 0.5.

The predicted-to-applied moment ratios for the inner bolts are 0.88 and 0.91 which is slightly conservative. The ratios for the outer bolts are 0.97 and 1.05.

For test E50-3/4-5/8-18, the inner and outer bolts reached the ultimate tensile load at approximately the same point. At the end of test E50-3/4-3/4-18, the bolt forces for the inner and outer bolts were approximately the same. For this reason, α and β were set equal to 0.5 to best represent the experimental test data. In addition, for these tests, which were controlled by bolt failure, prying action was only present in the inner bolts.

4.3 WEB STRESS COMPARISONS

For test E50-3/4-5/8-18, the finite element analyses results differ from the experimental results by approximately 10% in the tension region at higher load levels. Classical beam theory, My/EI , is a good indication of the cross-sectional strain obtained for this test. See Figure B.4. The finite element analyses results are approximately 20% less than the experimental results at lower load levels for test E50-3/4-3/4-18. At higher load levels, the finite element results are approximately 50% less than the experimental results. Classical beam theory and finite element analysis results are similar at all load levels. See Figure C.4.

For tests E36-1 1/8-7/8-16 and E36-1 1/4-16, the finite element analyses and experimental results differ at all load levels. The main difference occurs in the tension region. For E36-1 1/8-7/8-16, the finite element analyses results are approximately 70% greater than the experimental results in the tension region. For E36-1 1/4-7/8-16, the finite element results are approximately 200% greater than the experimental results. See Figures D.4 and E.4.

The largest difference between the finite element analyses and the experimental results occurs in the tension region, where the tension bolts are located. A couple possible solutions to improve the results of the finite element model in future research include refining the mesh in the tension region, and investigating different elements to use in the mesh. Until future research is performed to improve the finite element analyses, to design the web to end-plate weld, classical beam theory should be used to predict the behavior of the beam at locations near the end-plate connection.

4.4 MOMENT-ROTATION COMPARISONS

For tests E50-3/4-5/8-18 and E50-3/4-3/4-18, L/h is equal to 13.33, F_y is 55 ksi and E is 29,000 ksi. Using Eq. 2.19, this gives a simple span rotation, θ_s , of 0.013 radians. For tests E36-1 1/8-7/8-16 and E36-1 1/4-7/8-16, L/h equals 14.81, F_y is approximately 50 ksi and E is 29,000 ksi. This gives a simple span rotation of 0.015 radians.

For Type I or FR construction, as developed in Section 2.3, the rotation for the E50 tests must be less than 0.0013 radians, 0.1s at 90% of the nominal strength. For the E36 tests, the rotation must be less than 0.0015 radians at 90% of the nominal strength.

The applied moment versus rotation plots show that the Type I or FR criterion is satisfied for all tests except E36-1 1/8-7/8-16 where the criterion is nearly satisfied. Thus, the four bolt, extended, unstiffened end-plate tested are judged satisfactory for Type I or FR Construction.

CHAPTER V

DESIGN RECOMMENDATIONS AND EXAMPLE

5.1 DESIGN RECOMMENDATIONS

This study extends the unification of design procedures for moment end-plate connections by Hendrick *et al* (1985) and Morrison *et al* (1985) to include a sixth configuration, the four-bolt extended unstiffened moment end-plate connection. This unification provides consistent analytical procedures: end-plate strength criterion by yield-line theory and bolt force prediction by a modified Kennedy method. Further, an assessment of the connection rotational stiffness via $M-\Phi$ diagrams is required. These analytical procedures are verified with adequate experimental testing.

The recommended ASD design procedure follows:

1. Compute the allowable beam end moment, M_w , and the ultimate moment using:

$$M_u = \frac{M_w}{0.6} \quad (5.1)$$

2. Establish values to define the end-plate geometry: b_p , g , p_p , p_b , h and t_w .
3. Calculate the distance from the bolt centerline to the outermost yield-line

$$s = \frac{1}{2} \sqrt{b_f g} \quad (5.2)$$

4. With the known yield stress, F_{py} , determine the required end-plate thickness, t_p

$$t_p = \sqrt{\frac{\left(\frac{M_u}{F_{py}} \right)}{\left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right]}} \quad (5.3)$$

5. Compute the moment capacity of the end-plate for the selected end-plate thickness

$$M_{plate} = F_{py} t_p^2 \left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right]$$

6. Select a trial bolt diameter and determine the ultimate tensile load, P_u , which is two times the allowable tension stress times the area of the bolt. The allowable tension stress is found in Table J3.2 of the 1989 ASD Specifications, AISC (1989). Use the flowchart in Figure 5.1 to determine the critical flange force, F_f , which will occur in the inner bolt, at P_t . Compute the bolt moment capacity, M_{bolt} :

$$M_{bolt} = F_f (h - t_f)$$

7. M_{crit} equals the minimum of M_{plate} and M_{bolt} .

If M_{crit} equals M_{plate} then the structure will fail by end-plate failure before bolt failure and prying action is included for both the interior and exterior bolts. Compute the inner bolt force using the flowchart in Figure 5.2 and the outer bolt force using the flowchart in Figure 5.3.

If M_{crit} equals M_{bolt} then the structure will fail by bolt failure before end-plate failure and prying action is not included for the exterior bolts. Compute the inner bolt force using the flowchart in Figure 5.2 and the outer bolt force, B_o , is equal to $\alpha F_t/2$.

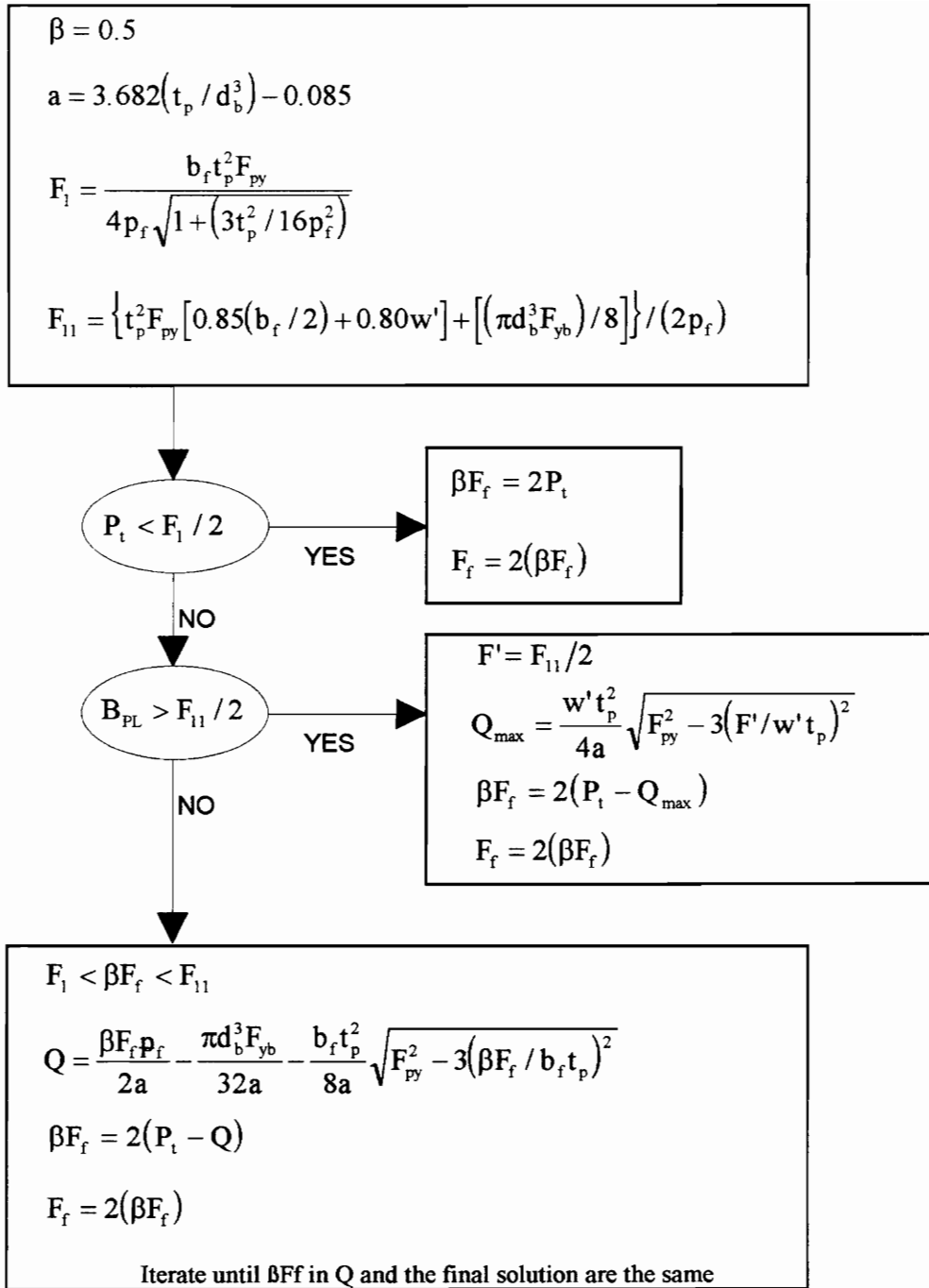


Figure 5.1 Flowchart to Determine Critical Flange Force for Inner Bolt

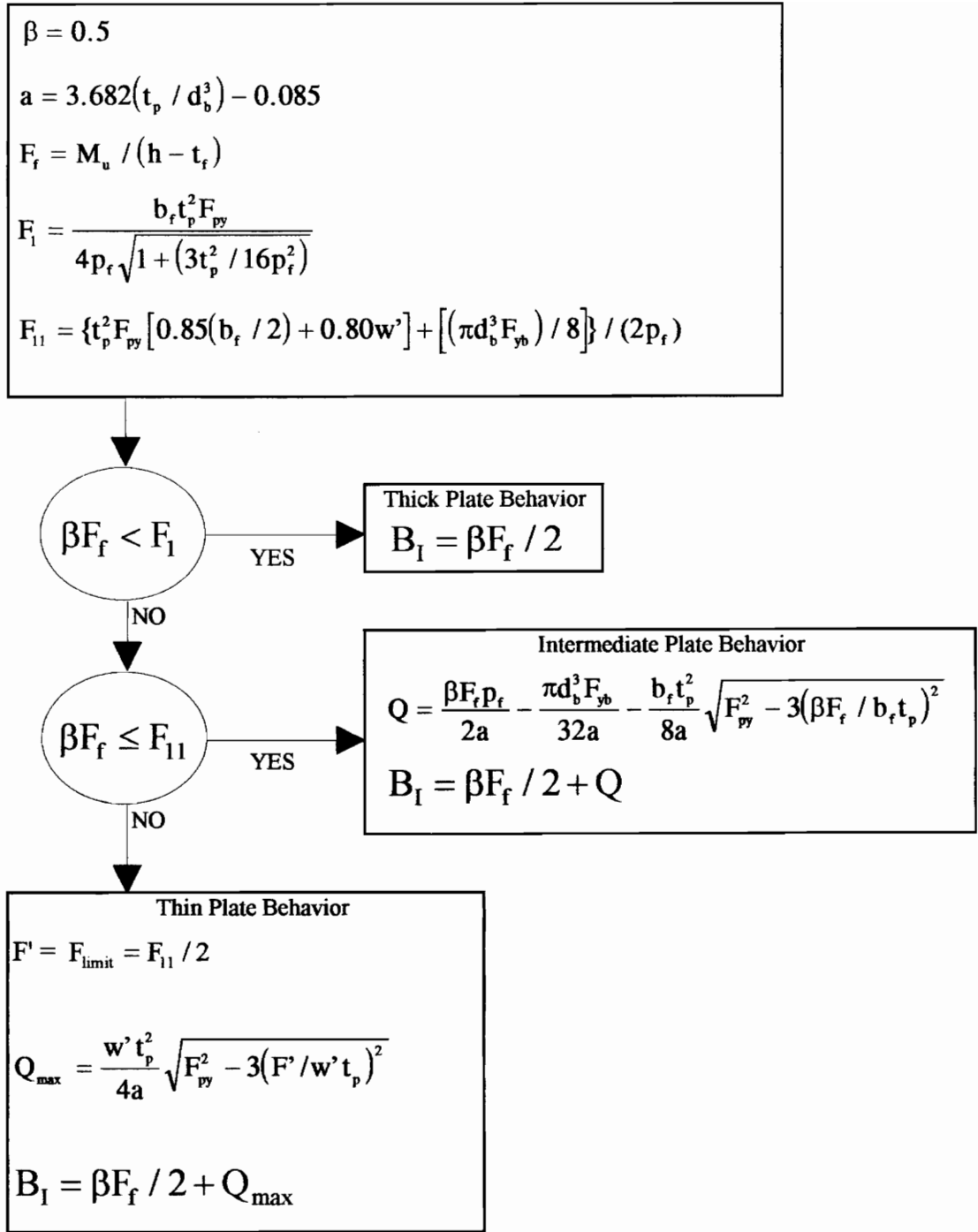


Figure 5.2 Flowchart to Determine Inner Bolt Force

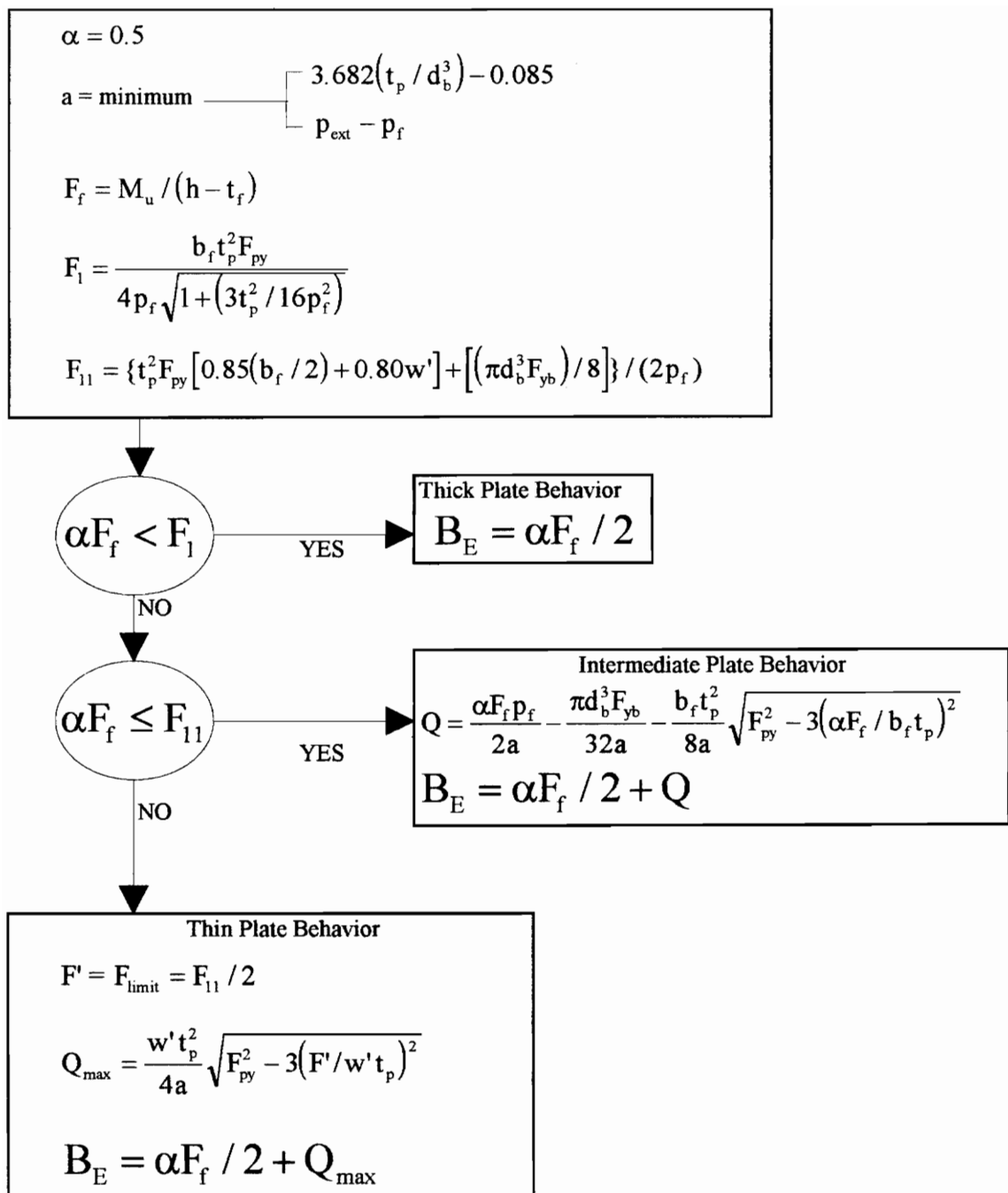


Figure 5.3 Flowchart to Determine Outer Bolt Force including Prying Action

8. Compare B_e and B_t . Choose the maximum value to determine the required bolt diameter using F_t , the bolt allowable tension stress.

$$d_b = \sqrt{\frac{2B_{\max}}{\pi F_t}} \quad (5.4)$$

In the AISC ASD Specification (1989), the allowable bolt tension stress for A325 bolt material is specified as 44 ksi with a factor of safety against yielding of 2.0. Since $B_{\max}$ corresponds to the factored load, the factor of safety is removed in Equation 5.4.

5.2 DESIGN EXAMPLE

Determine the required end-plate thickness and bolt size for a four-bolt extended unstiffened moment end-plate connection given the following:

Beam Data:

A572 Gr 50 steel	$F_y = 50$ ksi
Depth of Beam	$h = 24$ in
Flange Width	$b_f = 8$ in
Web Thickness	$t_w = 1/4$ in
Flange Thickness	$t_f = 1/2$ in

End-Plate Data:

A572 Gr 50 Material	$F_{py} = 50$ ksi
Extension outside beam flange	$p_{\text{ext}} = 3-1/4$ in
Pitch to bolt from beam flange	$p_f = 1-5/8$ in
Gage	$g = 3-1/4$ in

Bolt Data:

A325	$F_t = 44$ ksi
------	----------------

Other Data:

Working Moment	$M_w = 135$ k-ft
Construction Type	Type I

Step 1. Determine M_u , using Eq. 5.1

$$M_u = M_w / 0.60 = 135 / 0.60 = 225 \text{ k-ft}$$

Step 2. Calculate s and required end-plate thickness

$$s = \frac{1}{2} \sqrt{b_f g} = \left(\frac{1}{2} \right) \sqrt{8(3.25)} = 2.55 \text{ in.}$$

$$\begin{aligned} t_p &= \sqrt{\frac{\left(\frac{M_u}{F_{py}} \right)}{\left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right]}} \\ &= \sqrt{\frac{(225(12) / 50)}{\left[\left(\frac{8}{2} \left(\frac{1}{1.625} + \frac{1}{2.55} \right) + (1.625 + 2.55) \left(\frac{2}{3.25} \right) \right) (24 - 2.125) + \frac{8}{2} \left(\frac{1}{2} + \frac{24}{1.625} \right) \right]}} \\ &= 0.51 \text{ in} \quad \text{Use } t_p = 5/8 \text{ in} \end{aligned}$$

Step 3. Determine M_{crit} where M_{plate} and M_{bolt} are calculated as follows:

End-Plate Capacity

$$\begin{aligned} M_{plate} &= \frac{F_{py} t_p^2}{12} \left[\left(\frac{b_f}{2} \left(\frac{1}{p_f} + \frac{1}{s} \right) + (p_f + s) \left(\frac{2}{g} \right) \right) (h - p_t) + \frac{b_f}{2} \left(\frac{1}{2} + \frac{h}{p_f} \right) \right] \\ &= \frac{50(0.625)^2}{12} \left[\left(\frac{8}{2} \left(\frac{1}{1.625} + \frac{1}{2.55} \right) + (1.625 + 2.55) \left(\frac{2}{3.25} \right) \right) (24 - 2.125) + \right. \\ &\quad \left. \frac{8}{2} \left(\frac{1}{2} + \frac{24}{1.625} \right) \right] \\ &= 334.4 \text{ k-ft} \end{aligned}$$

Bolt Capacity

Try 1-1/4 in diameter bolts

$$P_t = 2 \times 44 \text{ ksi} \times 1.227 \text{ in}^2 = 108.0 \text{ kips}$$

$$F_1 = \frac{b_f t_p^2 F_{py}}{4 p_f \sqrt{1 + (3 t_p^2 / 16 p_f^2)}} = \frac{8(0.625^2)50}{4(1.625)\sqrt{1 + (3(0.625^2) / (16 \times 1.625^2))}} = 23.7 \text{ kips}$$

$$w' = \frac{b_f}{2} - \left(d_b + \frac{1}{16}\right) = \frac{8}{2} - \left(1.25 + \frac{1}{16}\right) = 2.6875 \text{ in}$$

$$\begin{aligned} F_{11} &= \{t_p^2 F_{py} [0.85(b_f / 2) + 0.80 w'] + [(\pi d_b^3 F_{yb}) / 8]\} / (2 p_f) \\ &= \{0.625^2 (50) [0.85(8 / 2) + (0.80) 2.6875] + [(\pi 1.25^3 (81)) / 8]\} / (2(1.625)) \\ &= 52.5 \text{ kips} \end{aligned}$$

$P_t > F_{11} / 2$ -- thin plate behavior

$$a = 3.682(t_p / d_b^3) - 0.085 = 3.682(0.625 / 1.25^3) - 0.085 = 1.1 \text{ in}$$

$$F' = F_{11} / 2 = 26.2 \text{ kips}$$

$$\begin{aligned} Q_{\max} &= \frac{w' t_p^2}{4a} \sqrt{F_{py}^2 - 3(F' / w' t_p)^2} = \frac{2.6875(0.625)^2}{4(1.1)} \sqrt{50^2 - 3(26.2 / (2.6875 \times 0.625))^2} \\ &= 10.0 \text{ kips} \end{aligned}$$

$$\beta F_f = 2(P_t - Q_{\max}) = 2(108 - 10) = 196 \text{ kips}$$

$$F_f = 2\beta F_f = 2(196) = 392 \text{ kips}$$

$$M_{\text{bolt}} = F_f (h - t_f) = \frac{392}{12} (24 - 0.5) = 767.7 \text{ k-ft}$$

$$M_{\text{crit}} = M_{\text{plate}} = 334.4 \text{ k-ft}$$

Use Figures 5.2 and 5.3 to determine the bolt forces.

Step 4. Determine flange force

$$F_f = M_u / (h - t_f) = 225(12) / (24 - 0.5) = 114.9 \text{ kips}$$

Step 5. Determine inner end-plate behavior

$$F_1 = \frac{b_f t_p^2 F_{py}}{4 p_f \sqrt{1 + (3 t_p^2 / 16 p_f^2)}} = \frac{8(0.625^2)50}{4(1.625) \sqrt{1 + (3(0.625^2) / (16 \times 1.625^2))}} = 23.7 \text{ kips}$$

Try 1-1/4 in. diameter bolts

$$w' = \frac{b_f}{2} - (d_b + 1/16) = \frac{8}{2} - (1.25 + 1/16) = 2.6875 \text{ in}$$

$$\begin{aligned} F_{11} &= \{t_p^2 F_{py} [0.85(b_f / 2) + 0.80w'] + [\pi d_b^3 F_{yb} / 8]\} / (2 p_f) \\ &= \{0.625^2 (50) [0.85(8 / 2) + (0.80)2.6875] + [\pi (1.25^3) (81)] / 8\} / (2(1.625)) \\ &= 52.5 \text{ kips} \end{aligned}$$

Since $\beta F_f = 0.5(114.9) = 57.5 \text{ kips} > F_{11} = 52.5 \text{ kips}$, inner end-plate behavior is thin plate behavior.

Step 6. Determine inner bolt force

$$a = 3.682(t_p / d_b^3) - 0.085 = 3.682(0.625 / 1.25^3) - 0.085 = 1.1 \text{ in}$$

$$\begin{aligned} Q_{\max} &= \frac{w' t_p^2}{4a} \sqrt{F_{py}^2 - 3(F' / w' t_p)^2} = \frac{2.6875(0.625)^2}{4(1.1)} \sqrt{50^2 - 3(26.2 / (2.6875 \times 0.625))^2} \\ &= 10.0 \text{ kips} \end{aligned}$$

$$B_I = \beta F_f / 2 + Q_{\max} = 0.5(114.9)/2 + 10.0 = 38.8 \text{ kips}$$

Step 7. Determine outer end-plate behavior

$$F_1 = \frac{b_f t_p^2 F_{py}}{4 p_f \sqrt{1 + (3 t_p^2 / 16 p_f^2)}} = \frac{8(0.625^2) 50}{4(1.625) \sqrt{1 + (3(0.625^2) / (16 \times 1.625^2))}} = 23.7 \text{ kips}$$

Try 1-1/4 in. diameter bolts

$$w' = \frac{b_f}{2} - \left(d_b + \frac{1}{16}\right) = \frac{8}{2} - \left(1.25 + \frac{1}{16}\right) = 2.6875 \text{ in}$$

$$\begin{aligned} F_{11} &= \{t_p^2 F_{py} [0.85(b_f / 2) + 0.80w'] + [(\pi d_b^3 F_{yb}) / 8]\} / (2 p_f) \\ &= \{0.625^2 (50) [0.85(8 / 2) + (0.80)2.6875] + [(\pi 1.25^3 (81)) / 8]\} / (2(1.625)) \\ &= 52.5 \text{ kips} \end{aligned}$$

Since $\alpha F_f = 0.5(114.9) = 57.5 \text{ kips} > F_{11} = 52.5$, outer end-plate behavior is thin plate behavior.

Step 8. Determine outer bolt force

$$\begin{aligned} a &= \min \left\{ \begin{array}{l} 3.682(t_p / d_b^3) - 0.085 = 3.682(0.625 / 1.25^3) - 0.085 = 1.1 \text{ in} \\ p_{\text{ext}} - p_f = 3.25 - 1.625 = 1.625 \text{ in} \end{array} \right\} \\ &= 1.1 \text{ in.} \end{aligned}$$

Same as interior bolt

$$B_e = \alpha F_f / 2 + Q_{\max} = 0.5(114.9)/2 + 10.0 = 38.8 \text{ kips}$$

$$B_{\max} = 38.8 \text{ kips}$$

Step 9. Determine bolt diameter

$$d_b = \sqrt{\frac{2B_{\max}}{\pi F_t}}$$

$$= \sqrt{\frac{2(38.8)}{\pi(44)}} = 0.75 \text{ in} \quad \text{Try } 3/4 \text{ in diameter bolt}$$

Repeat Steps 3 through 9 until trial bolt diameter is the same as the required bolt diameter.

Summary: For materials, geometry, and given loading, use A572 Gr 50 end-plate material with 5/8 in. thickness and 3/4 in. diameter bolts.

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APPENDIX A
NOMENCLATURE

NOMENCLATURE

a	=	distance from bolt centerline to prying force for plate
B	=	bolt force
B_I	=	inner bolt force
B_E	=	outer bolt force
B_{max}	=	maximum bolt force of the inner and outer bolts
$BF_{plastic}$	=	bolt force in plastic region
b	=	distance from concentrated load to support for test specimen
b_f	=	beam flange width
d_b	=	bolt diameter
E	=	Young's modulus of elasticity
F	=	force
F_a	=	bolt material allowable stress
F_f	=	flange force
	=	$M / (h - t_f)$
F_{limit}	=	possible flange force per bolt at the thin plate limit
F_{py}	=	end-plate material yield stress
F_t	=	tension strength of bolt material

F_y	=	yield stress
F_{yb}	=	bolt material yield stress
	=	bolt proof which is twice the allowable tension capacity per AISC Specification
F'	=	flange force per bolt at the thin plate limit
F_1	=	flange force at the thick plate limit
F_{11}	=	flange force at the thin plate limit
g	=	end-plate bolt gage
h	=	beam depth
I	=	beam moment of inertia
L	=	distance between test specimen supports
M	=	moment
M_b	=	bolt moment capacity when bolt force is at bolt proof which is twice the allowable tension capacity per AISC Specification
M_{bolt}	=	bolt failure moment
M_{crit}	=	critical moment
M_F	=	fixed end moment; yield moment
M_p	=	predicted failure moment
M_{plate}	=	plate failure moment
M_u	=	end-plate ultimate moment capacity
M_{yb}	=	bolt moment capacity when bolt force is at bolt proof which is twice the allowable tension capacity per AISC Specification
M_w	=	working moment

M_1	=	plastic moment at first hinge line to form
M_2	=	plastic moment at second hinge line to form
m_{elastic}	=	slope of load versus strain in elastic region for bolt material
	=	$\frac{P_y}{\epsilon_y}$
m_p	=	plastic moment capacity of plate per unit length
	=	$(F_{py} t_p^2)/4$
P	=	load applied to test specimen by hydraulic ram
P_t	=	bolt material ultimate tensile load capacity
P_y	=	bolt material yield load
p_{ext}	=	end-plate extension outside beam flange
	=	$p_f + d_e$
p_f	=	distance from bolt centerline to near face of beam flange
p_t	=	distance from bolt centerline to far face of beam flange
	=	$p_f + t_f$
Q	=	prying force
Q_{max}	=	maximum prying force
R	=	bolt relationship
S	=	section modulus
s	=	distance from bolt centerline to outermost yield-line
t_f	=	beam flange thickness
t_p	=	end-plate thickness

t_w	=	beam web thickness; stiffener thickness
t_l	=	plate thickness at thick plate limit
t_{ll}	=	plate thickness at thin plate limit
W_e	=	external work
W_i	=	internal work
w'	=	end-plate width per bolt less the bolt hole diameter (at bolt line)
x	=	distance
α	=	outer end-plate factor
β	=	inner end-plate factor
$\epsilon_{plastic}$	=	bolt material strain in plastic region
ϵ_y	=	bolt material yield strain
γ	=	α or β
Δ_{pred}	=	predicted strength of materials centerline deflection for test specimen
Δ_{test}	=	experimental test centerline deflection for test specimen
θ	=	rotation of end-plate
θ_s	=	simple span end rotation for any loading
π	=	pi
ϕ	=	rotation

APPENDIX B
E50-3/4-5/8-18 TEST RESULTS

TEST SYNOPSIS

PROJECT:	End-Plate
TEST:	E50-3/4-5/8-18
TEST DATE:	03-03-92
CONNECTION DESCRIPTION:	Four-bolt extended unstiffened moment end-plate with a single row of two bolts on each side of the beam tension flange

BEAM DATA (measured):

Depth	h	(in)	=	18.00
Flange width	b _f	(in)	=	8.00
Web thickness	t _w	(in)	=	0.255
Flange thickness	t _f	(in)	=	0.512
Moment of inertia	I	(in ⁴)	=	728.44

END-PLATE DATA (measured):

Thickness	t _p	(in)	=	0.633
Extension outside beam flange	p _{ext}	(in)	=	2.438
Pitch to bolt from flange	p _f	(in)	=	1.25
Gage	g	(in)	=	2.986
Steel yield stress	F _{py}	(ksi)	=	66.9

BOLT DATA:

Type			=	A325
Diameter	d _b	(in)	=	0.75
Pretension force	T _b	(k)	=	28.0
Ultimate tensile load (measured)	P _t	(k)	=	45.9

PREDICTIONS:

End-plate moment strength	M _u	(k-ft)	=	398.2
Interior bolt moment strength	M _{yb}	(k-ft)	=	215.4
Exterior bolt moment strength	M _{yb}	(k-ft)	=	267.6
Beam moment strength		(k-ft)	=	337.2

EXPERIMENTAL:

Maximum applied moment		(k-ft)	=	259.7
Moment at interior bolt ultimate tensile load		(k-ft)	=	237.5
Moment at exterior bolt ultimate tensile load		(k-ft)	=	253.8
Maximum vertical centerline deflection		(in)	=	1.13

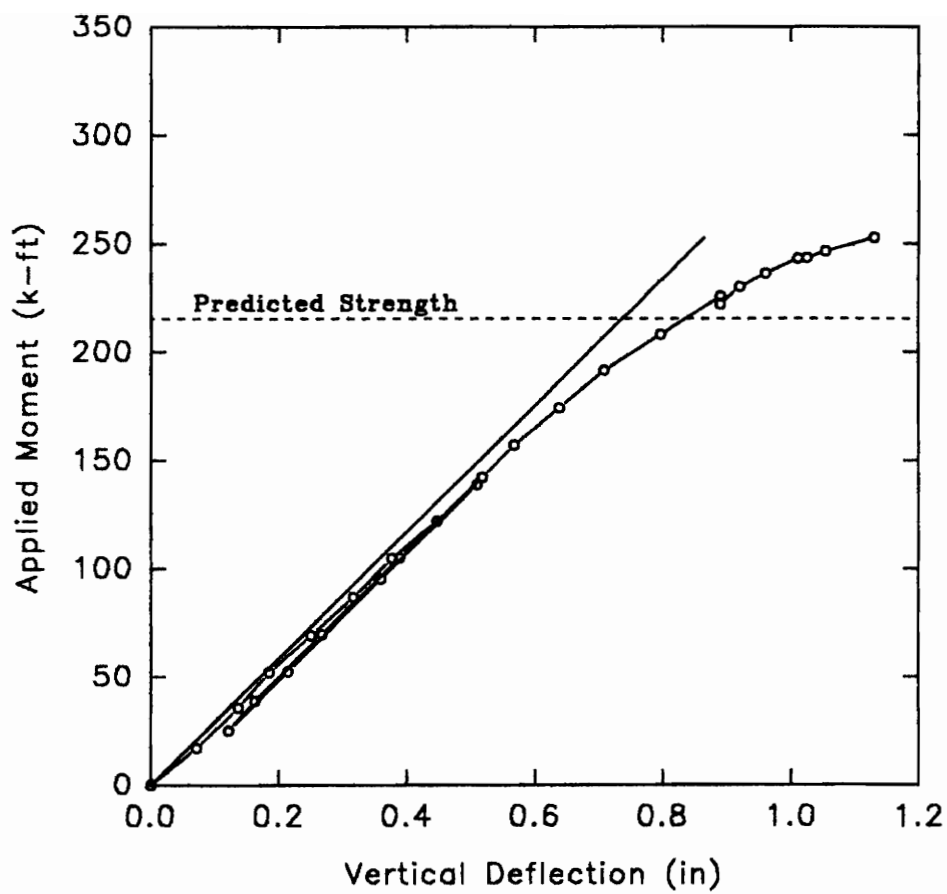


Figure B.1 Applied Moment Versus Vertical Deflection for Test E50-3/4-5/8-18

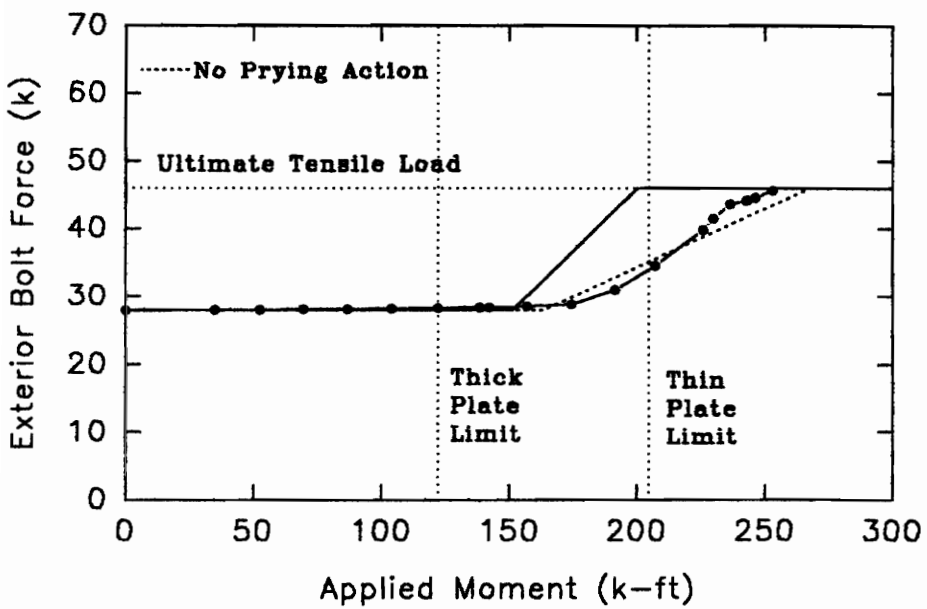
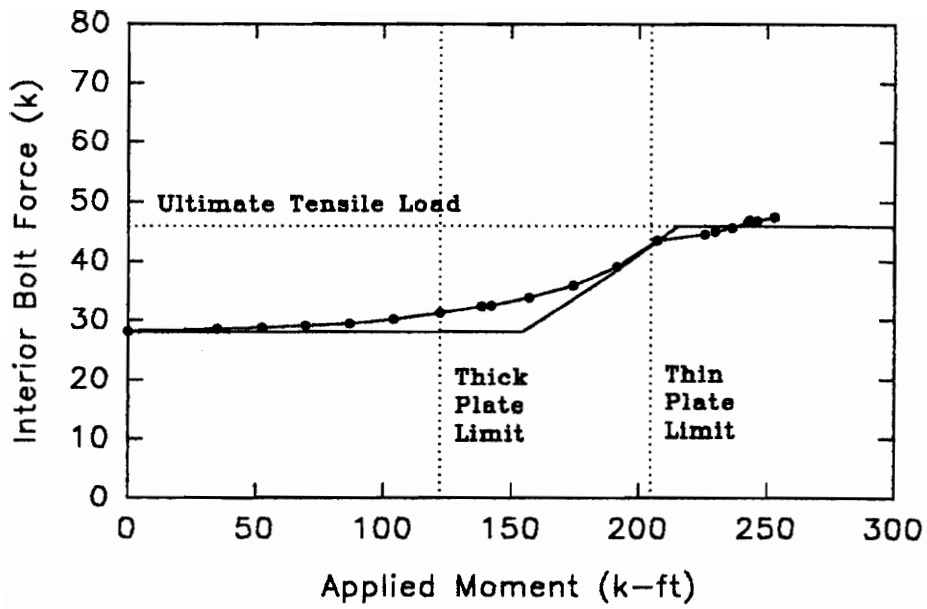


Figure B.2 Bolt Force Versus Applied Moment for Test E50-3/4-5/8-18

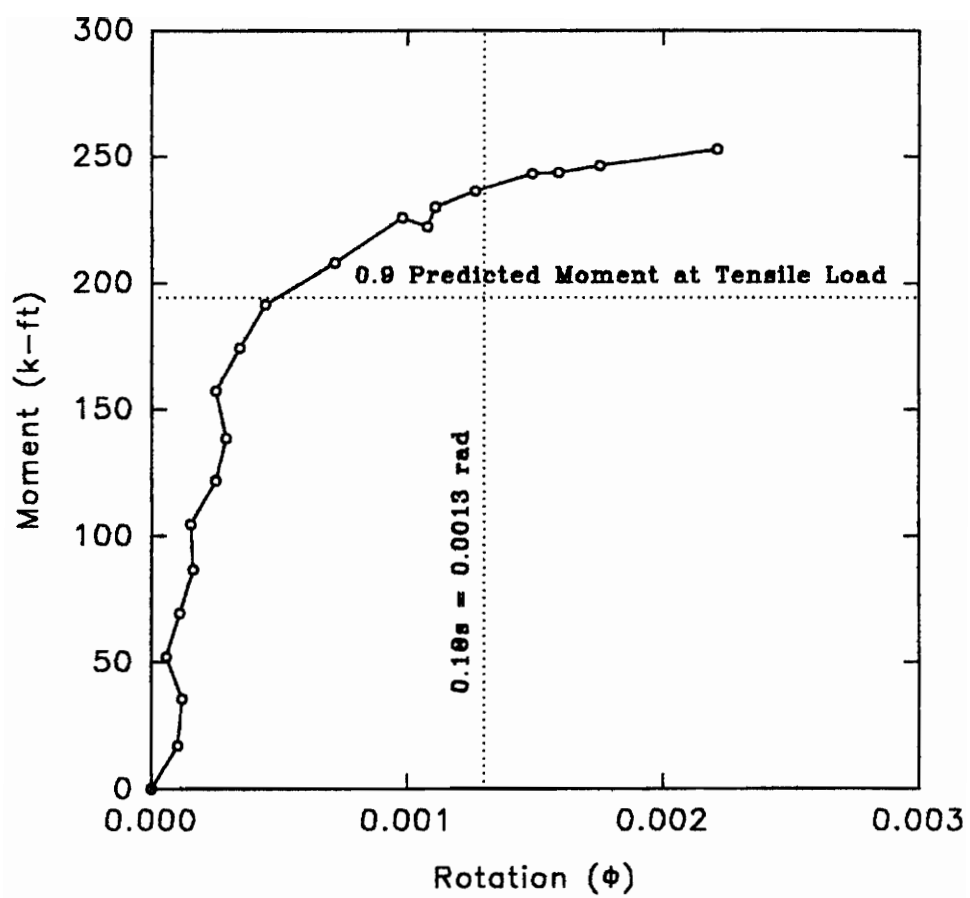


Figure B.3 Applied Moment Versus Rotation for Test E50-3/4-5/8-18

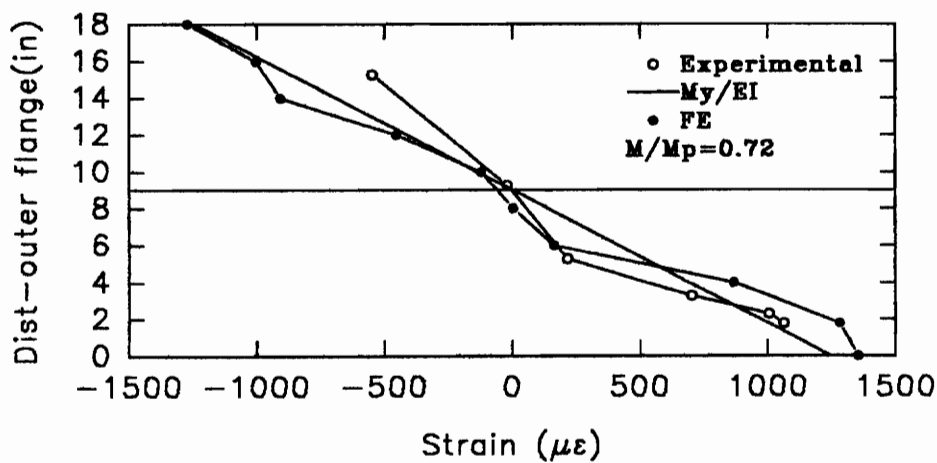
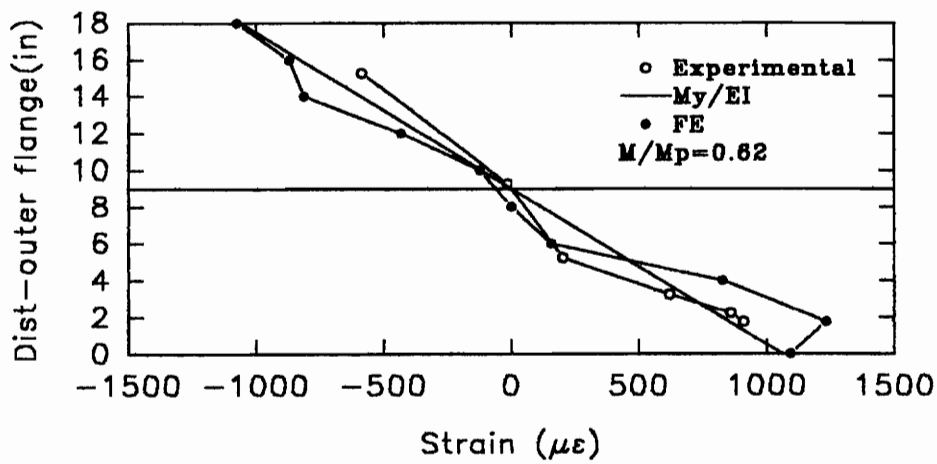
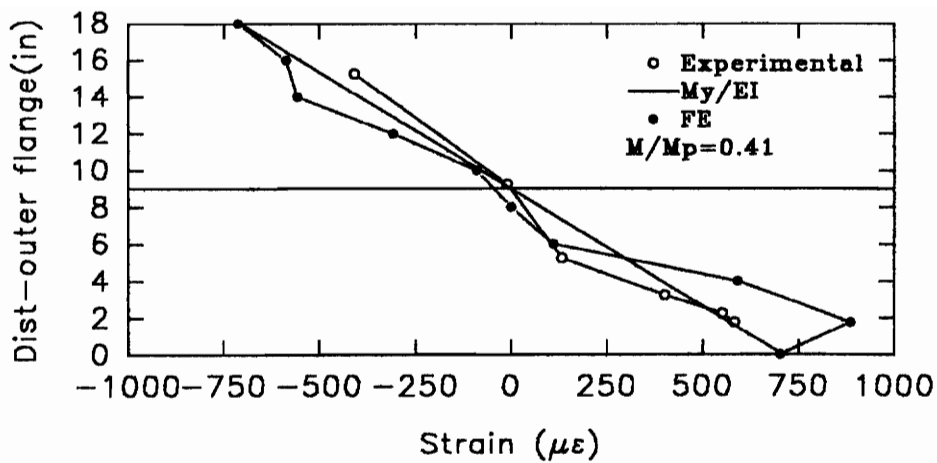


Figure B.4 Finite Element Analyses Comparisons for Test E50-3/4-5/8-18

APPENDIX C
E50-3/4-3/4-18 TEST RESULTS

TEST SYNOPSIS

PROJECT:	End-Plate
TEST:	E50-3/4-3/4-18
TEST DATE:	08-03-92
CONNECTION DESCRIPTION:	Four-bolt extended unstiffened moment end-plate with a single row of two bolts on each side of the beam tension flange

BEAM DATA (measured):

Depth	h	(in)	=	18.00
Flange width	b _f	(in)	=	8.00
Web thickness	t _w	(in)	=	0.25
Flange thickness	t _f	(in)	=	0.512
Moment of inertia	I	(in ⁴)	=	728.44

END-PLATE DATA (measured):

Thickness	t _p	(in)	=	0.788
Extension outside beam flange	p _{ext}	(in)	=	2.50
Pitch to bolt from flange	p _f	(in)	=	1.25
Gage	g	(in)	=	2.991
Steel yield stress	F _{py}	(ksi)	=	57.4

BOLT DATA:

Type			=	A325
Diameter	d _b	(in)	=	0.75
Pretension force	T _b	(k)	=	28.0
Ultimate tensile load (measured)	P _t	(k)	=	45.9

PREDICTIONS:

End-plate moment strength	M _u	(k-ft)	=	538.0
Interior bolt moment strength	M _{yib}	(k-ft)	=	245.7
Exterior bolt moment strength	M _{yeb}	(k-ft)	=	267.6
Beam moment strength		(k-ft)	=	337.2

EXPERIMENTAL:

Maximum applied moment		(k-ft)	=	275.1
Moment at interior bolt ultimate tensile load		(k-ft)	=	278.1
Moment at exterior bolt ultimate tensile load		(k-ft)	=	276.9
Maximum vertical centerline deflection		(in)	=	1.13

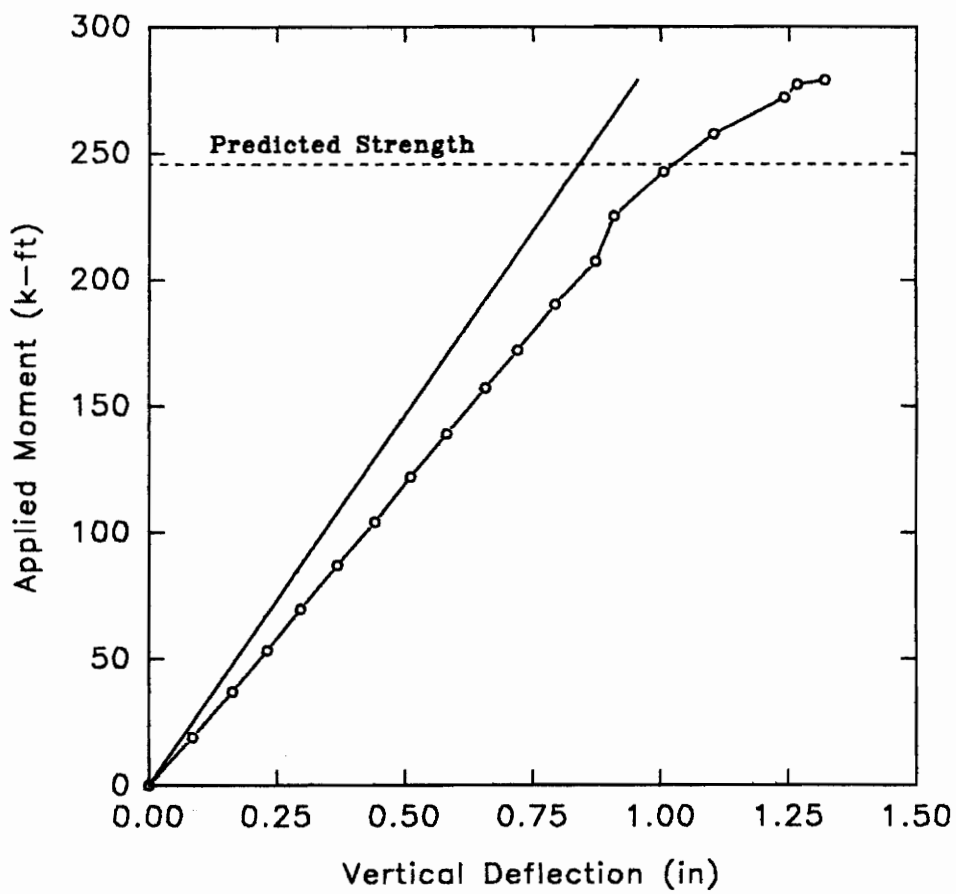


Figure C.1 Applied Moment Versus Vertical Deflection for Test E50-3/4-3/4-18

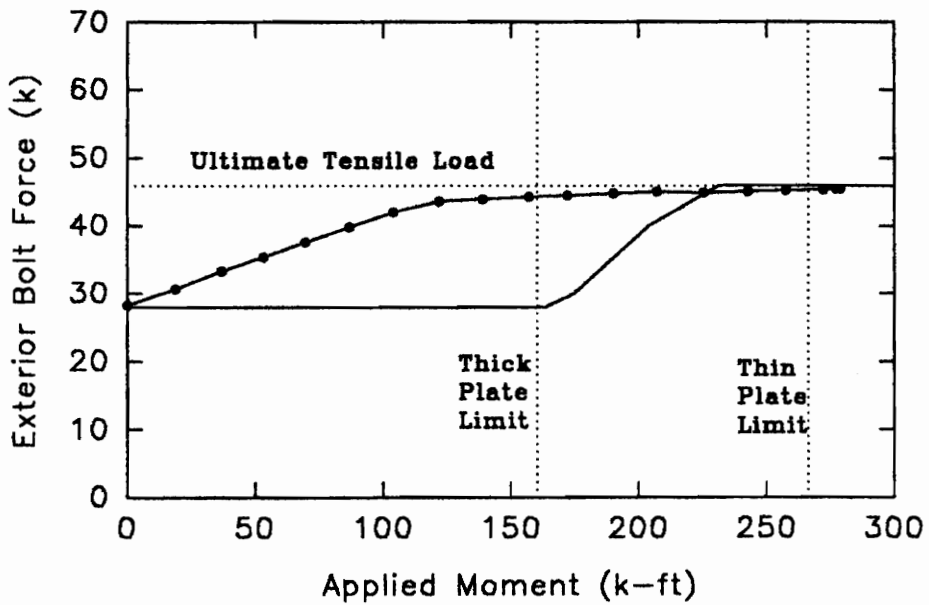
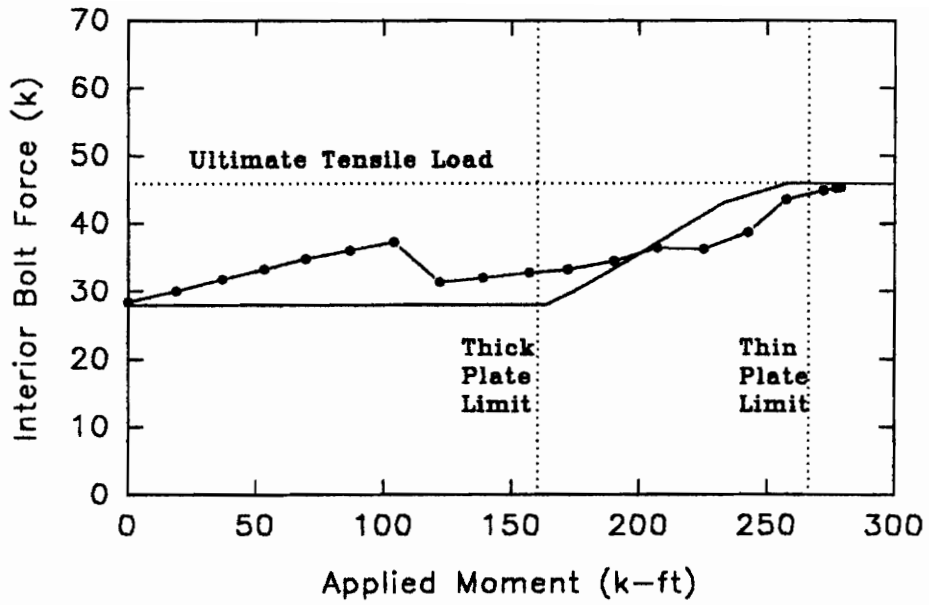


Figure C.2 Bolt Force Versus Applied Moment for Test E50-3/4-3/4-18

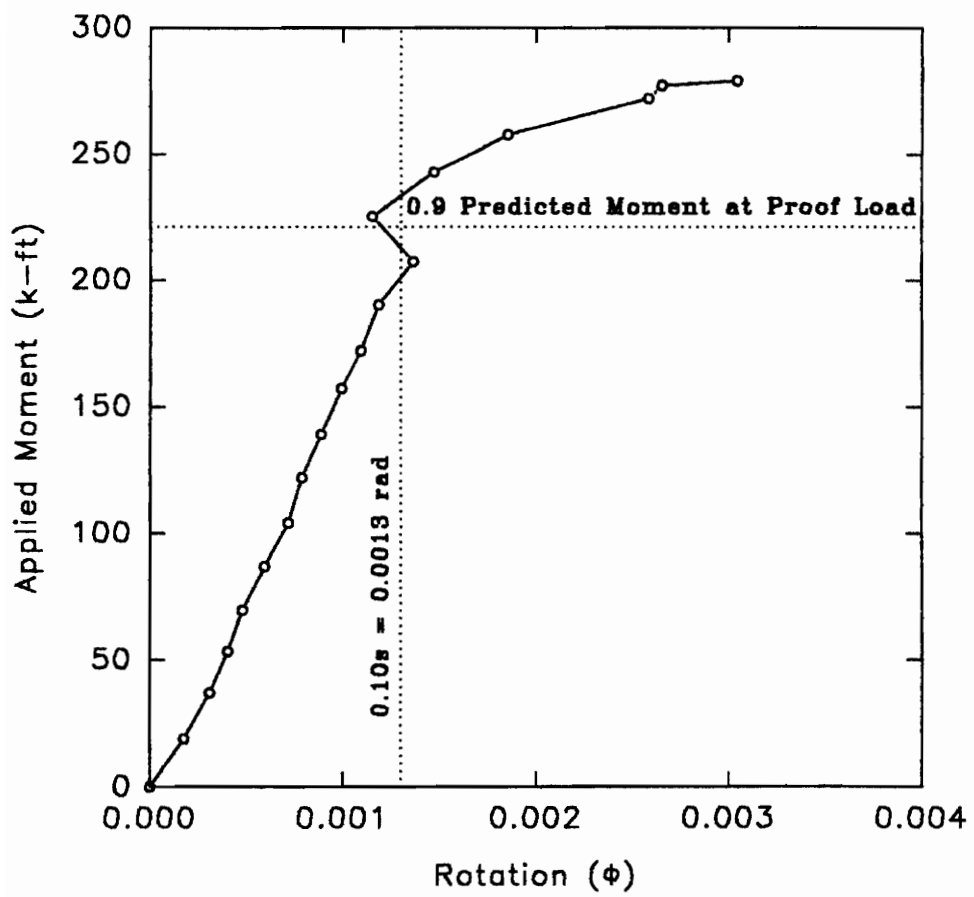


Figure C.3 Applied Moment Versus Rotation for Test E50-3/4-3/4-18

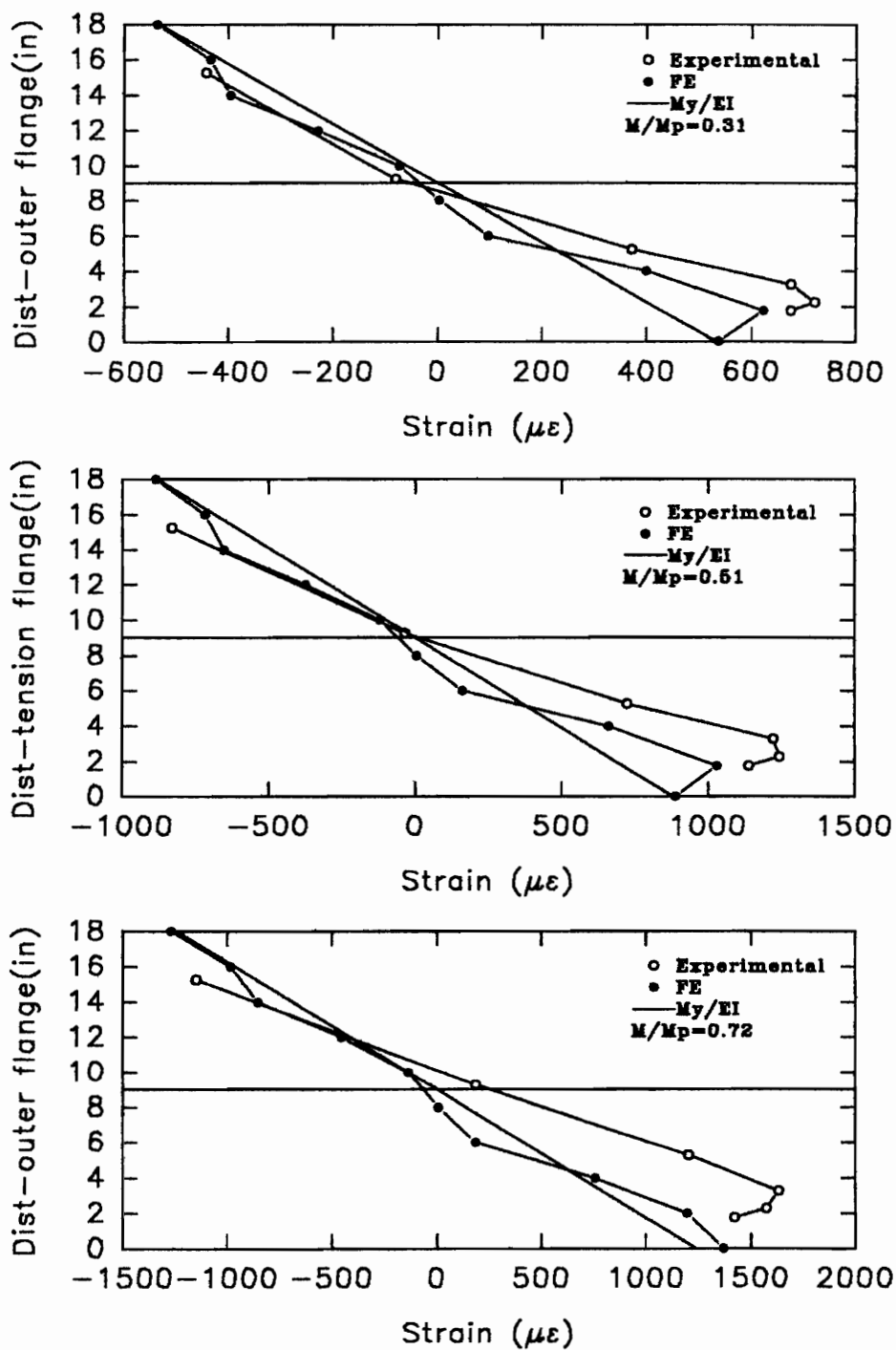


Figure C.4 Finite Element Analyses Comparisons for Test E50-3/4-3/4-18

APPENDIX D

E36-1 1/8-7/8-16 TEST RESULTS

TEST SYNOPSIS

PROJECT:	End-Plate
TEST:	E36-1 1/8-7/8-16
TEST DATE:	08-28-92
CONNECTION DESCRIPTION:	Four-bolt extended unstiffened moment end-plate with a single row of two bolts on each side of the beam tension flange

BEAM DATA (measured)

Depth	h	(in)	=	16.21
Flange width	b _f	(in)	=	10.25
Web thickness	t _w	(in)	=	0.42
Flange thickness	t _f	(in)	=	0.657
Moment of inertia	I	(in ⁴)	=	932.67

END-PLATE DATA (measured)

Thickness	t _p	(in)	=	0.886
Extension outside beam flange	p _{ext}	(in)	=	4.0
Pitch to bolt from flange	p _f	(in)	=	2.5
Gage	g	(in)	=	7.0
Steel yield stress	F _{py}	(ksi)	=	46.5

BOLT DATA:

Type:			=	A325
Diameter	d _b	(in)	=	1.25
Pretension force	T _b	(k)	=	56.0
Ultimate tensile load (measured)	P _t	(k)	=	99.1

PREDICTIONS:

End-plate moment strength	M _u	(k-ft)	=	313.8
Interior bolt moment strength	M _{yb}	(k-ft)	=	411.6
Exterior bolt moment strength	M _{yb}	(k-ft)	=	397.1
Beam moment strength		(k-ft)	=	345.2

EXPERIMENTAL:

Maximum applied moment		(k-ft)	=	337.8
Moment at interior bolt ultimate tensile load		(k-ft)	=	328.2
Moment at exterior bolt ultimate tensile load		(k-ft)	=	334.5
Maximum vertical centerline deflection		(in)	=	1.74
Maximum outer end-plate separation		(in)	=	0.11
Maximum inner end-plate separation		(in)	=	0.12

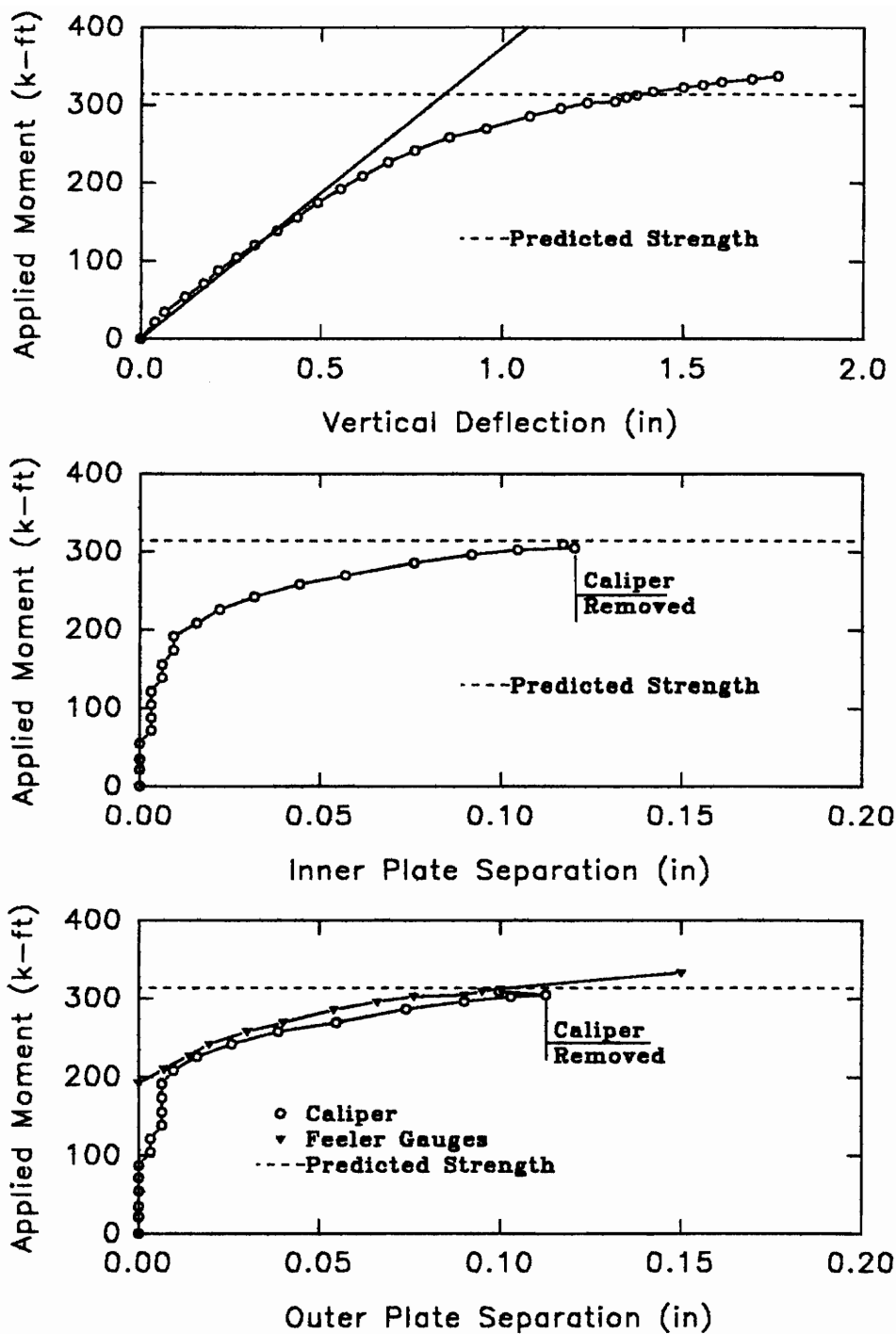


Figure D.1 Applied Moment Versus Vertical Deflection and Plate Separation for Test E36-1 1/8-7/8-16

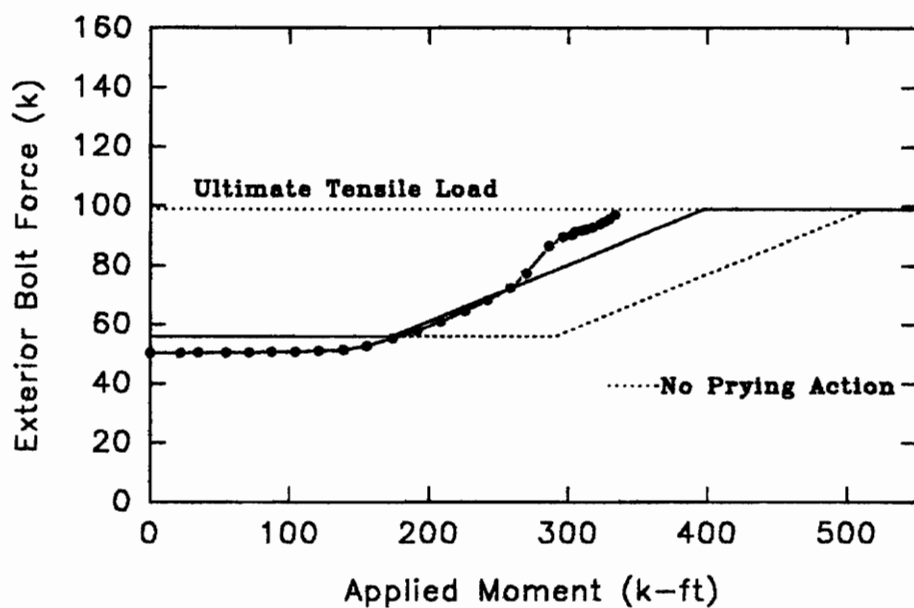
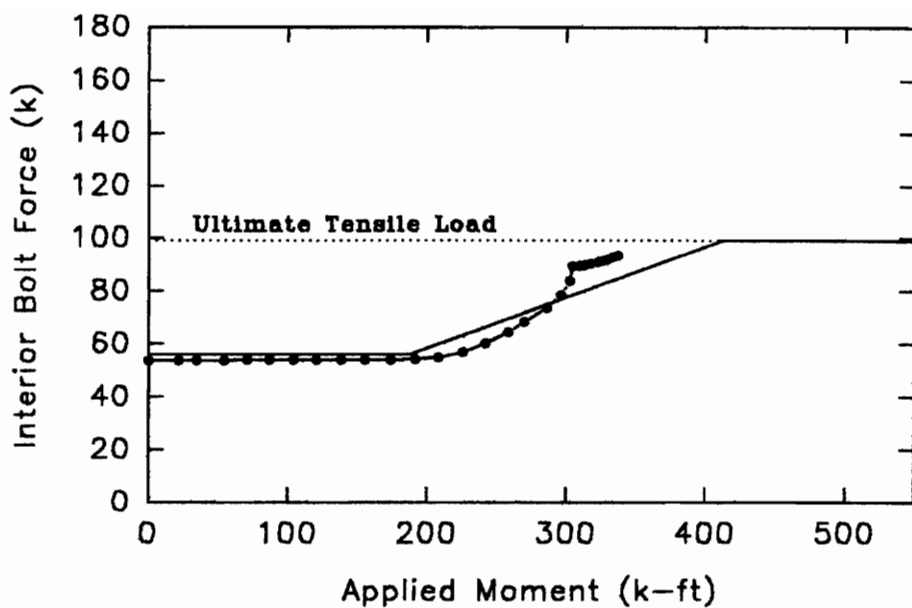


Figure D.2 Bolt Force Versus Applied Moment for Test E36-1 1/8-7/8-16

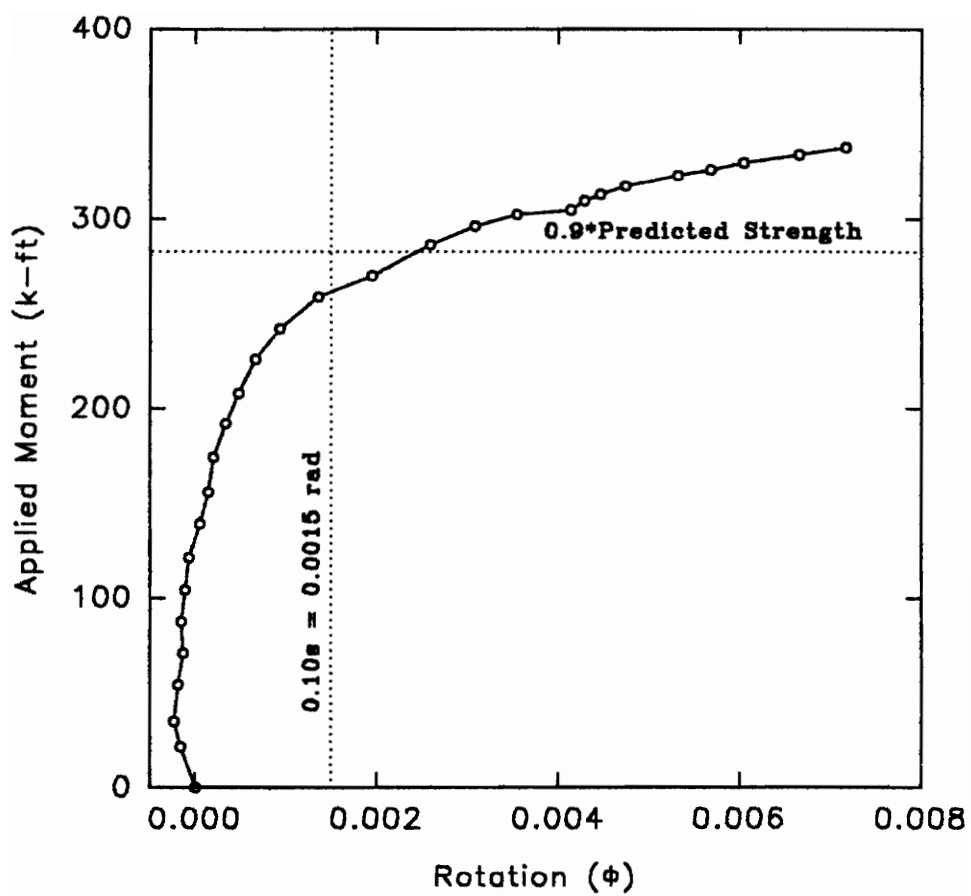


Figure D.3 Applied Moment Versus Rotation for Test E36-1 1/8-7/8-16

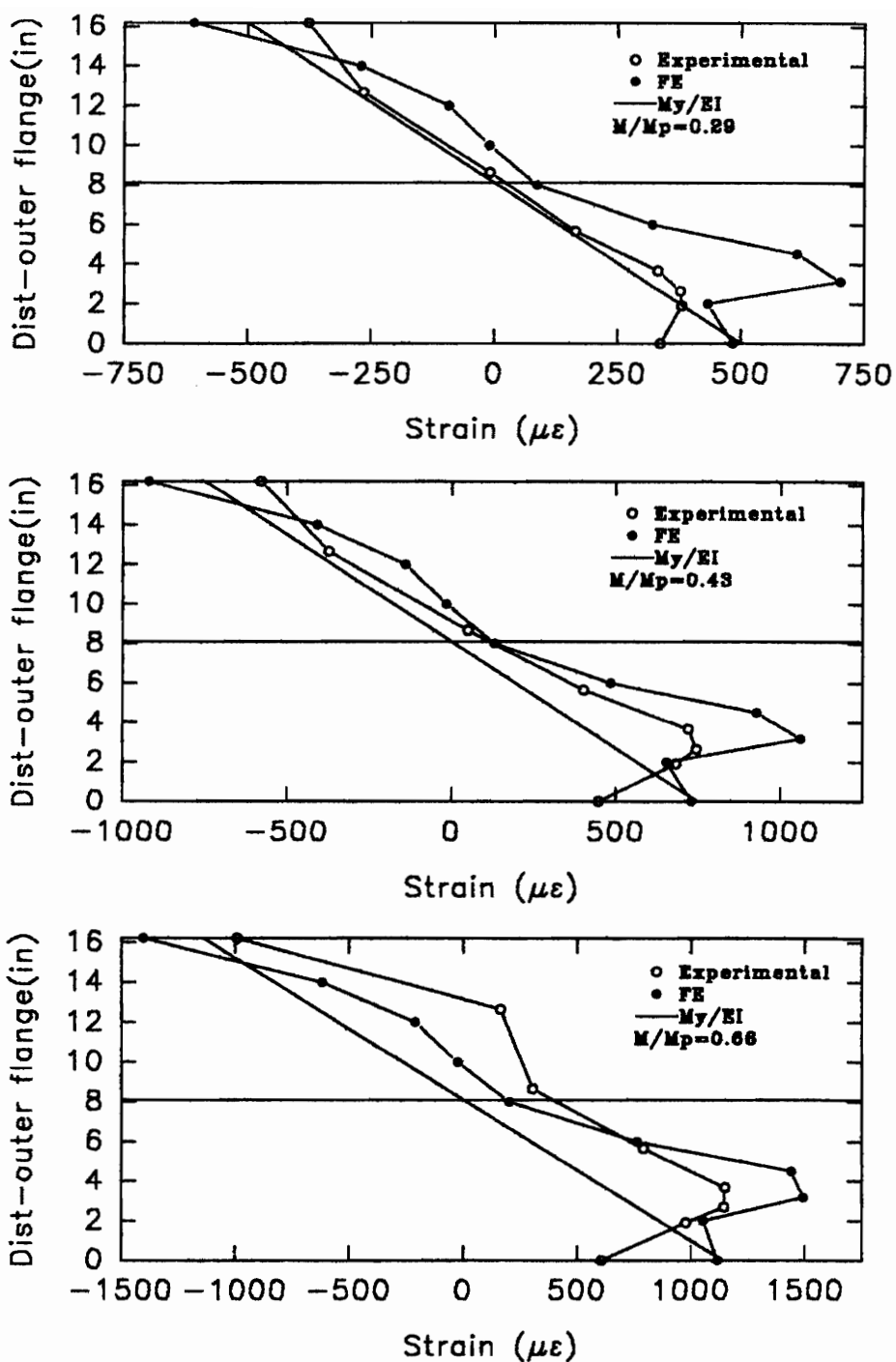


Figure D.4 Finite Element Analyses Comparisons for Test E36-1 1/8-7/8-16

APPENDIX E

E36-1 1/4-7/8-16 TEST RESULTS

TEST SYNOPSIS

PROJECT:	End-Plate
TEST:	E36-1 1/4-7/8-16
TEST DATE:	08-28-92
CONNECTION DESCRIPTION:	Four-bolt extended unstiffened moment end-plate with a single row of two bolts on each side of the beam tension flange

BEAM DATA (measured)

Depth	h	(in)	=	16.21
Flange width	b _f	(in)	=	10.25
Web thickness	t _w	(in)	=	0.42
Flange thickness	t _f	(in)	=	0.657
Moment of inertia	I	(in ⁴)	=	932.67

END-PLATE DATA (measured)

Thickness	t _p	(in)	=	0.886
Extension outside beam flange	p _{ext}	(in)	=	3.375
Pitch to bolt from flange	p _f	(in)	=	2.0
Gage	g	(in)	=	6.0
Steel yield stress	F _{py}	(ksi)	=	46.5

BOLT DATA:

Type:			=	A325
Diameter	d _b	(in)	=	1.25
Pretension force	T _b	(k)	=	71.0
Ultimate tensile load (measured)	P _t	(k)	=	124.6

PREDICTIONS:

End-plate moment strength	M _u	(k-ft)	=	374.0
Interior bolt failure moment	M _{y_b}	(k-ft)	=	515.3
Exterior bolt failure moment	M _{y_b}	(k-ft)	=	515.3
Beam failure moment		(k-ft)	=	345.2

EXPERIMENTAL:

Maximum applied moment		(k-ft)	=	503.0
Moment at interior bolt ultimate tensile load		(k-ft)	=	495.7
Moment at exterior bolt ultimate tensile load		(k-ft)	=	495.7
Maximum vertical centerline deflection		(in)	=	2.40
Maximum outer end-plate separation		(in)	=	0.134
Maximum inner end-plate separation		(in)	=	0.165

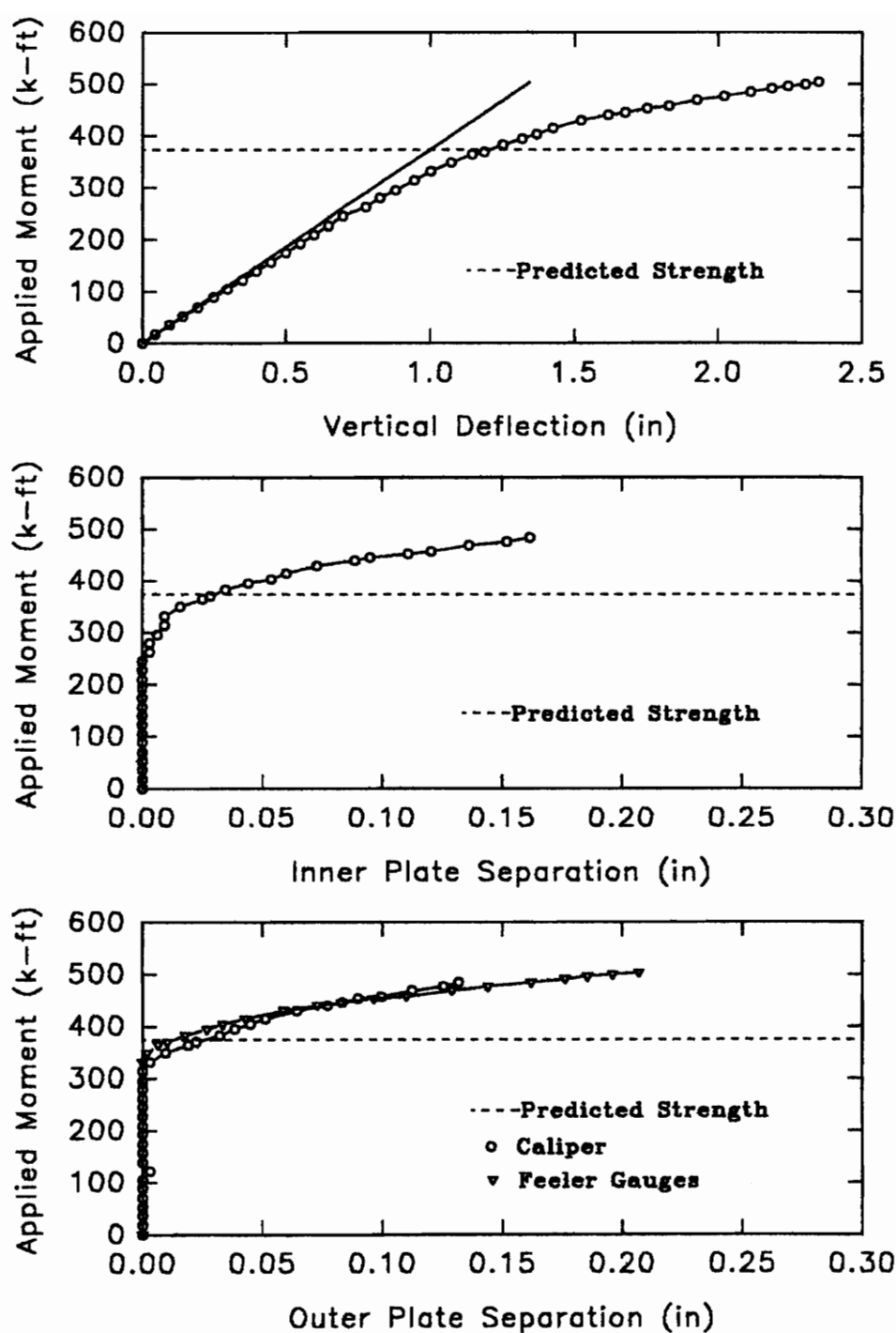


Figure E.1 Applied Moment Versus Vertical Deflection and Plate Separation for Test E36-1 1/4-7/8-16

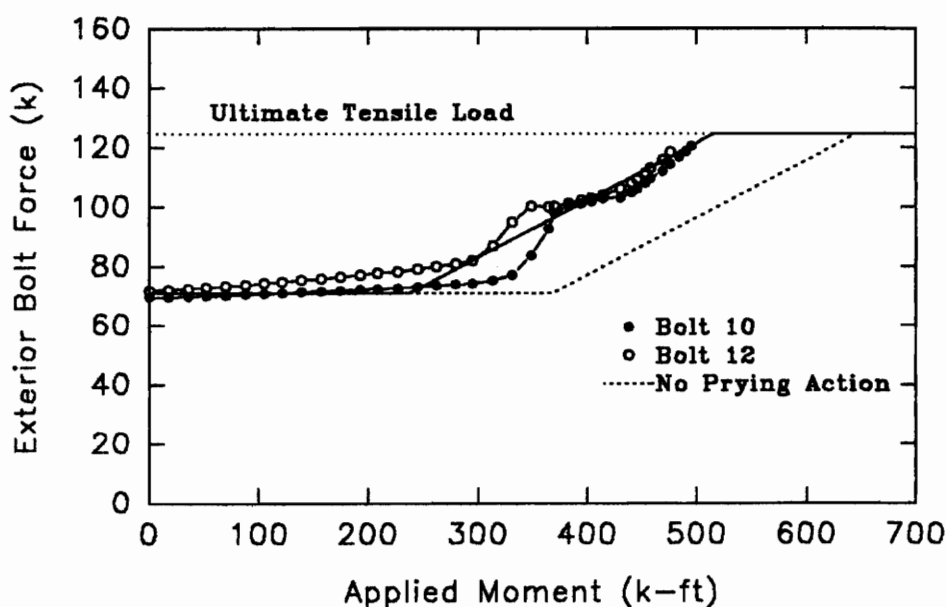
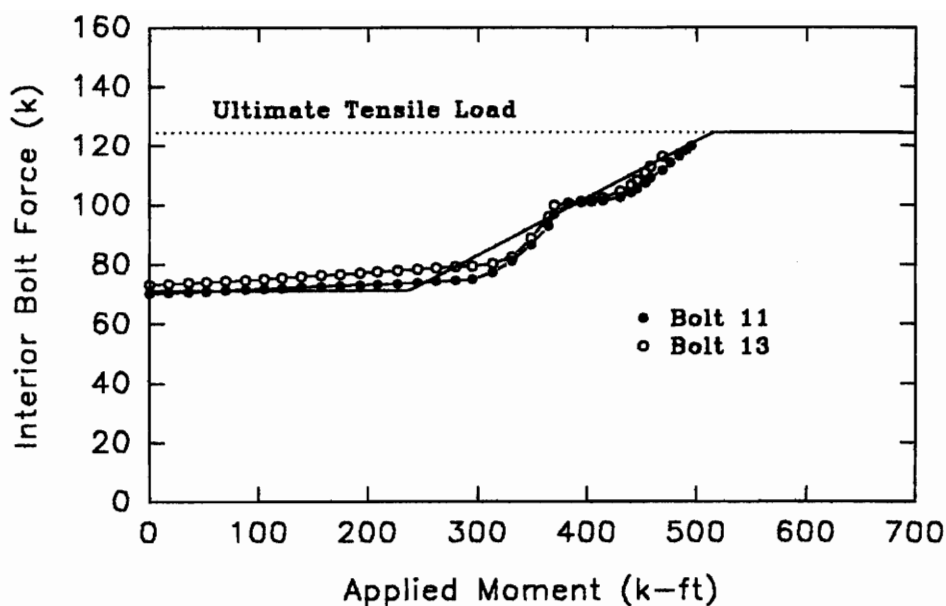


Figure E.2 Bolt Force Versus Applied Moment for Test E36-1 1/4-7/8-16

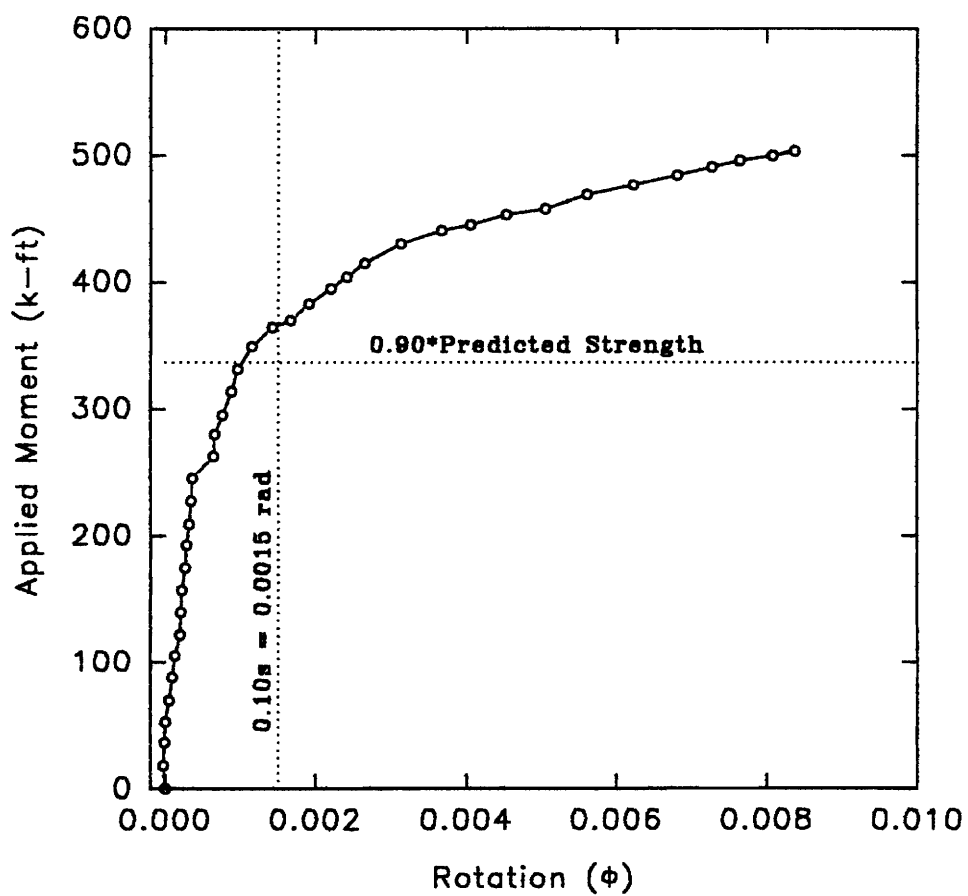


Figure E.3 Applied Moment Versus Rotation for Test E36-1 1/4-7/8-16

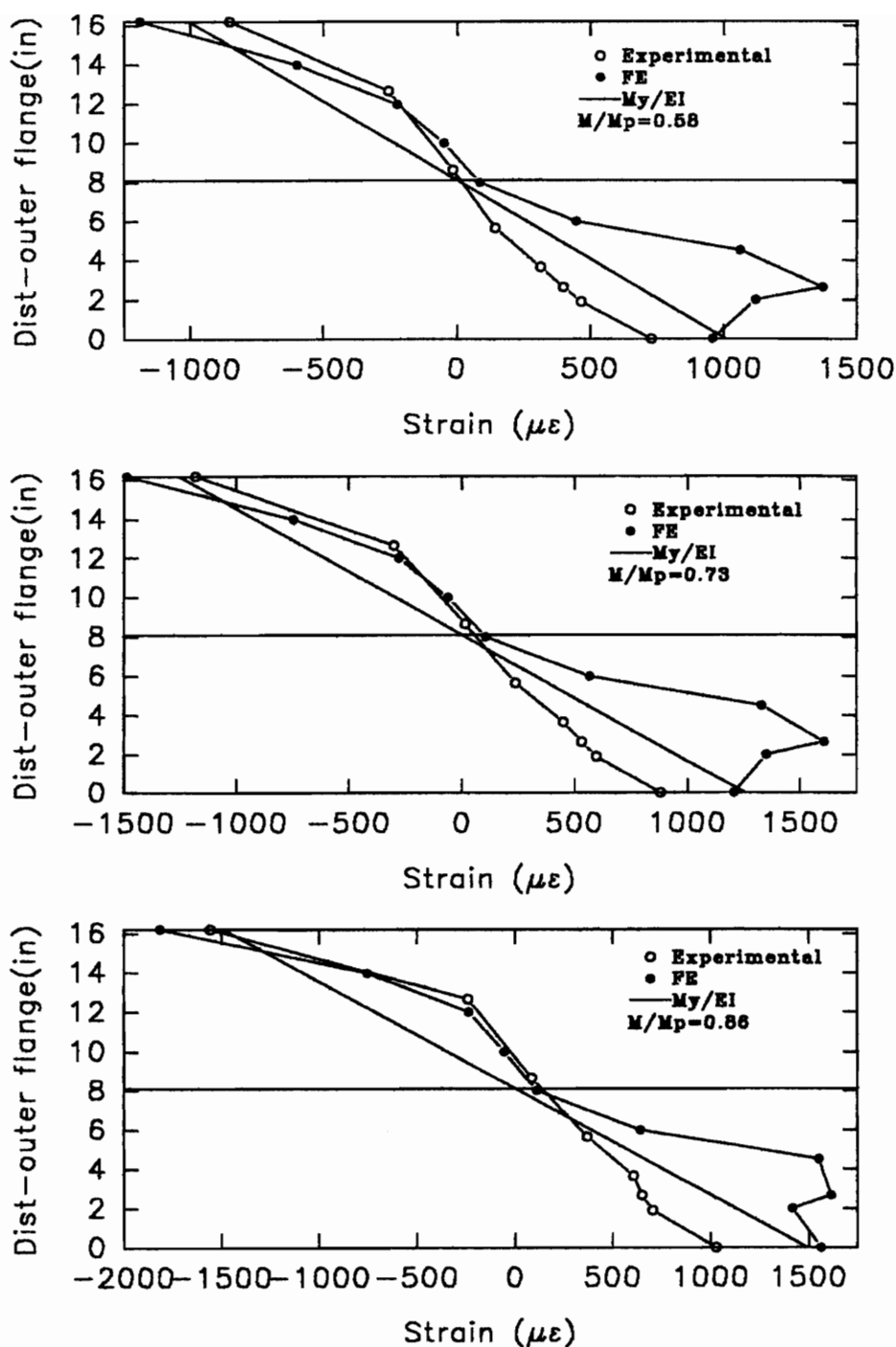


Figure E.4 Finite Element Analyses Comparisons for Test E36-1 1/4-7/8-16

VITA

Mary Sue Mouchka Abel was born in LaCrosse, Wisconsin on September 27, 1969. She graduated from Central High School in LaCrosse. In August, 1991, she received her Bachelor of Science degree in Civil Engineering from Michigan Technological University in Houghton, Michigan. She enrolled in the graduate program at Virginia Polytechnic Institute and State University in August, 1991.


Mary Sue Mouchka Abel