Economies with Public Projects:
Theory and Experimental Evidence

by

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(ABSTRACT)

This dissertation concerns economies with *public projects*. Public projects are a special case of public goods which size is not necessarily measured by *units* and which are either built at some fixed cost or not built at all. Our theoretical studies of public projects are based on personalized prices for *access* to the public project rather than personalized prices for the *units* consumed of public goods as in the literature on Lindahl equilibrium. Furthermore, the private provision of public projects is experimentally investigated with a double oral auction market for assets that are required in order to produce public projects.

The first paper (Chapter 2), "Economies with Multiple Public Projects" (joint work with Robert P. Gilles), discusses an economy with multiple public projects each separately produced by a distinct provider operating under a different cost function. In an economy with the non-Euclidean representation of multiple public projects space we show that the two welfare theorems hold for valuation equilibria in which a public project is financed through a (non-linear) system of taxes or subsidies, called a valuation system, and that the core allocations are equivalent to the set of valuation equilibria with a nonnegative valuation system. Furthermore, if a Euclidean space is used to describe the public projects specified to the standard case of public goods, every Pareto efficient allocation is supported as an affine valuation equilibrium which is characterized with a price per unit of public good and a lump sum tax or subsidy.

The second paper (Chapter 3), "Market Provision of Public Projects: Some Experimental Results" (joint work with Sheryl B. Ball), presents experimental evi-

dence on the provision of a public project which is produced by a coalition of economic agents in the population. A double oral auction asset market is employed as the trading institution for assets that are required in order to produce the public project. The experimental environments differ by rules about who can produce the project, information about the benefits to the other agents of the project, and parameters which include the symmetry and size of individual valuations of the assets and the magnitude of social benefits from the project. We find that individually rational efficient outcomes which are identified by a theoretical analysis based on Chapter 2, are more likely in some environments than others, and suggest that these findings may have implications for the usefulness of this mechanism for public project provision.

The third paper (Chapter 4), "Economies with Costly Trade Links," is an application of the model of economies with public projects to the case of an economy with endogenous formation of costly trade links between industries in different sectors of the economy. The trade links reduce transaction costs, but inevitably incur set-up costs. We prove that the two welfare theorems hold for trade equilibria in which each trade link is separately financed with budget neutrality as well as profit maximization. Furthermore, we introduce an industry-wise efficiency concept which requires that no industry can insure itself a better outcome for the industry itself by changing the industry's trade structure, and show that Pareto efficiency strictly implies industry-wise efficiency.

The fourth paper (Chapter 5), "Efficiency and Egalitarian-Equivalence in Economies with a Public Project" (joint work with Robert P. Gilles), is an application of the model of economies with public projects to the equity concept of egalitarian-equivalence. An allocation is egalitarian-equivalent if there exists a fixed commodity bundle (the same for each agent) that is considered by each agent to be indifferent to the bundle that he/she actually gets in the allocation under consideration. A public project is also produced by a coalition of economic agents as in Chapter 3. We prove that there exist efficient egalitarian-equivalent allocations, which are not equivalent to the set of valuation equilibria and also may not be in the core.

To My Parents

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Contents

Chapter 1

Economies with Public Projects: A Prelude

1.1 Introduction

Consider *public goods* such as national defense, police and fire protection, highways, television and radio broadcasts, national parks, pollution abatement programs, and so on. These goods are classically characterized by *nonexcludability* and *nonrivalry*. A good is considered nonexcludable if people cannot be excluded from consuming it. A good satisfies the nonrival property if "each individual's consumption of such a good leads to no subtraction from any other individual's consumption" (Samuelson, 1954, p.387). These properties of public goods contrast sharply with those of private goods whose consumption only affects a single economic agent. Ordinary private goods are excludable and demonstrate rivalry.[1]

Consequently, resource allocation problems involving public goods turn out to be quite different from resource allocation problems involving private goods. In a private goods economy the concept of a Walras (or competitive) equilibrium is used as a basic benchmark: Walras equilibrium allocations are efficient, and Pareto efficient allocations are usually supported as Walras equilibria. When public goods are

[1] According to Gilles and Ruys (1994), economic goods are characterized by two bases, the domain of valuation, i.e., the desired *attributes*, and the domain of resources, i.e., the *carriers* of these attributes. A pure public commodity is defined by a public carrier with non-interactive valuation, while a pure private commodity is by a private carrier with non-interactive valuation.

present, however, using a notion of Walras equilibria is inappropriate, since public goods are not actually traded in competitive markets. Therefore, a direct application of a Walras equilibrium concept to a public goods economy may lead to the nonexistence of equilibria, or if they exist, they are very likely not Pareto efficient. It is desirable, however, to focus on generalizations of a Walrasian equilibrium that attain Pareto efficiency. Since the publication of Lindahl (1919), such generalizations have been formulated: Lindahl equilibrium (Johansen, 1963; Foley, 1970a), ratio equilibrium (Kaneko, 1977), and cost share equilibrium (Mas-Colell and Silvestre, 1989). Furthermore, based on Foley (1967), and Pazner and Schmeidler (1974), the concept of equity is also introduced in economies with public goods (Varian, 1974; Daniel, 1978; Moulin, 1987; Thomson, 1990; Diamantaras, 1992).

From the viewpoint of competitive markets, these equilibrium approaches look very suspicious, however. There is the famous *free rider* problem to overcome, since each consumer is responsible for determining his/her own demand for public goods, and has a consistent incentive to underreport it. This problem was resulted in theoretical and experimental studies of economies with public goods. On the one hand, this problem is, theoretically, solved by appropriately designing noncooperative games whose equilibria coincide with the equilibria of the economies with public goods (the implementation problem). On the other hand, based on incentive compatible mechanism (Groves and Ledyard, 1977), several variants of experimental designs such as the *auction mechanisms* for the provision of public goods have been proposed and executed (Smith, 1979a, 1979b, 1980). Also, in reaction to the social-psychological approaches to public goods, the *voluntary contribution mechanisms* have been systematically studied (Kim and Walker, 1984; Isaac et al., 1984, 1985, 1988a, 1988b, 1989, 1994).

This chapter is organized as follows. Section 2 focuses on the concepts of efficiency and equity in economies with public goods, section 3 reviews the experimental studies of public goods involving both auction mechanism and voluntary contribution mechanism. Finally, section 4 addresses economies with public projects as an introduction to later chapters.

1.2 Theoretical studies of a public goods economy

The Samuelson Model

The aim of the Samuelson (1954) model is to derive conditions for optimal resource allocations in an economy in which there are two types of goods, private and public. The nature of the two types of goods are defined by the equation which gives the relationship between individual and aggregate consumption. For private goods the total quantity consumed is equal to the sum of the quantities consumed by the individuals, so that

$$f_i = \sum_{a \in A} f_i(a), \quad \text{for } i \in L,$$

where A is a set of economic agents, $a \in A$, and L is a set of private goods, $i \in L = \{1, \ldots, \ell\}$. For public goods the corresponding relationship is one of equality between individual and total consumption, namely, for every $a \in A$,

$$y_t = y_t(a), \quad \text{for } t \in K,$$

where K is a set of public goods, $t \in K = \{1, \ldots, k\}$. Individual preferences, represented by utility functions, are then defined over the quantities consumed of private and public goods, so that we can write the utility function of agent $a \in A$ as

$$\begin{aligned} U_a &= U_a(f_1(a), \ldots, f_\ell(a), y_1(a), \ldots, y_k(a)) \\ &= U_a(f_1(a), \ldots, f_\ell(a), y_1, \ldots, y_k). \end{aligned}$$

It is assumed that the conditions for efficient production are satisfied, so that the production possibilities for the economy can be summarized in the transformation (or production) possibility equation,

$$\overline{F}(f_1, \ldots, f_\ell, y_1, \ldots, y_k) = 0.$$

The problem of Pareto efficiency is formulated as follows: of all allocations satisfying the production possibility equation, find those allocations which maximize utility for an agent $a \in A$, given arbitrary but feasible utility levels for all other agents. The solution can be given by

$$\frac{\partial U_a / \partial f_i(a)}{\partial U_a / \partial f_1(a)} = \frac{\partial F / \partial f_i}{\partial F / \partial f_1}, \quad \text{for } a \in A \text{ and } i \in L,$$

$$\sum_{a\in A} \frac{\partial U_a/\partial y_k}{\partial U_a/\partial f_1(a)} = \frac{\partial F/\partial y_k}{\partial F/\partial f_1}, \text{ for } k \in K.$$

This means that for any two private goods the marginal rate of substitution should be equal to the marginal rate of transformation, while for public goods the sum of the marginal rates of substitution between a private and a public good should be equal to the marginal rate of transformation between them. Or, in the particular case where the private good may be taken as a numeraire commodity ($i = 1$), for the public goods the sum of the marginal willingness to pay for the public good should be equal to the marginal cost of production. The intuition should be clear: an extra unit of supply of the public good benefits all agents simultaneously and to find the total marginal benefit we have to take the sum of the marginal benefits accruing to all agents.

In fact, Samuelson gives the first modern study of economies with public goods. These "Samuelson conditions" have since become one of the fundamental tools for understanding public goods economies. Recently, however, several limitations are indicated. In particular, there exist corner allocations in which at least one private good is not consumed at all by at least one agent, and the sum of marginal rates of substitution may be less than the marginal rate of substitution. Campbell and Truchon (1988) point out that there are cases where some efficient allocations violate the Samuelson conditions, and provide a different specification of the Samuelson conditions which are shown to be necessary and sufficient conditions for efficiency in economies with one private good and a finite number of public goods. Conley and Diamantaras (1994) provide generalized efficiency conditions for economies with a finite number of private and public goods and show the existence of fully supporting prices at any Pareto efficient allocation, for all agents who are allowed a cheaper point by the Samuelson prices corresponding to the allocation.

"Samuelson conditions" for efficiency are actually calculus-based conditions, and public goods spaces are restricted to Euclidean spaces. This restriction leads to the choice of measurements of public goods, or requires the invariance of units of measurements of public goods. Public goods can be, however, measured in several ways based on Euclidean spaces, and do not have any commodity accepted physical units, because public goods are not actually traded in markets. Ito and Kaneko (1981), in particular, focus on this invariance measurement, and require linearity of cost functions.[2]

Equilibria, Efficiency and the Core

The next step in the Samuelson model is to consider that an equilibrium allocation would follow from particular institutional arrangements in a public goods economy, and this is compared with the optimality conditions such as Pareto efficiency and the core which are formally defined as follow:

Definition 1.2.1 *A feasible allocation* (f, y) *is* **Pareto efficient** *if there exists no other feasible allocation* (g, z) *such that*

(i) *for every* $a \in A$, $U_a(g(a), z) \geqq U_a(f(a), y)$,

(ii) *there exists at least one* $b \in A$ *such that* $U_b(g(b), z) > U_b(f(b), y)$,

where, for simplicity, private goods are represented by $f: A \to \mathbb{R}^\ell_+$, and public goods by $y \in \mathbb{R}^k_+$.

Definition 1.2.2 *A feasible allocation* (f, y) *belongs to the* **core** *if there exists no coalition* $E \subset A$ *and no other allocation* (g, z) *such that*

(i) (g, z) *is feasible for* E, *i.e.*, $\sum_{a \in E} g(a) + c(z) \leqq \sum_{a \in E} w(a)$, *and*

(ii) *for every* $a \in E$, $U_a(g, z) > U_a(f, y)$,

where $w: A \to \mathbb{R}^\ell_+$ is an initial endowment of private good of the agents in A, and a cost function for public goods is represented by $c: \mathbb{R}^k \to \mathbb{R}^\ell_+$ instead of the production possibility set F.

The first clear formulation of a theory of public goods which can be given a positive interpretation is presented by Lindahl (1919); an important modern exposition is presented by Johansen (1963), and Foley (1970a). An allocation is a Lindahl equilibrium if one can find a price vector for the private goods and personalized price vectors for the public goods (one per consumer) so that every consumer maximizes his/her preferences, and every firm maximizes profits.

[2]This restriction of Euclidean spaces of public goods leads to an idea of *public projects* characterized by a non-Euclidean representation of public goods as Mas-Colell (1980).

Definition 1.2.3 *A* **Lindahl equilibrium** *with respect to* $w = \{w(a)\}_{a\in A}$ *is a feasible allocation* (f, y) *and a price system* (p_f, p_y) *with* $p_f \in \mathbb{R}_+^\ell$ *and* $p_y: A \to \mathbb{R}_+^k$ *such that*

(i) *for all* $(g, z) \in F$,
$(p_f, \sum_{a\in A} p_y(a)) \cdot [\sum_{a\in A}(f(a) - w(a)), y] \geqq (p_f, \sum_{a\in A} p_y(a)) \cdot (g, z)$

(ii) *if* $U_a(g(a), z) > U_a(f(a), y)$ *then*
$p_f \cdot g(a) + p_y(a) \cdot z > p_f \cdot f(a) + p_y(a) \cdot y = p_f \cdot w(a)$,

where F is a production possibility set satisfying certain regularity conditions.

In particular, Foley (1970a) proposes a generalization of Lindahl's equilibrium solution, a *public competitive equilibrium* in an economy with an arbitrary but finite number of private goods, and proves an existence theorem for it as well as the two welfare theorems: public competitive equilibria are Pareto efficient and Pareto efficient allocations can be supported as public competitive equilibria. He also shows that there exist Lindahl equilibria and that any Lindahl equilibrium is in the core. Good surveys of the results obtained in this line of work can be found in Milleron (1972) and Roberts (1974).

The dominance of Lindahl equilibrium in the theoretic analysis of economies with private and public goods is broken first by Kaneko (1977). He notices that Lindahl equilibria need not belong to the core of the economy (using the concept of the core proposed in Foley (1970a)) if the technology for the production of public goods exhibits decreasing returns to scale (this problem does not arise with constant returns to scale). Kaneko proposes a formal embodiment of Lindahl's original idea that is different from the one first proposed by Johansen (1963), which is the idea of using personalized prices for public goods. Kaneko's proposal is called a *ratio equilibrium*. An allocation is a ratio equilibrium if a price vector for the private goods and a vector of "ratio" for each public good exists which make every consumer maximize preferences on a budget set and each production unit minimize cost:

Definition 1.2.4 *In the presence of a single private good,* (r, f, y) *is a* **ratio equilibrium** *if* $r = \{r_t(a)\}_{t\in K; a\in A} \in \mathbb{R}^{k\times|A|}$ *with* $\sum_{a\in A} r_t(a) = 1$ *for all* $t \in K$, *and if for every* $a \in A$,

(i) $\sum_{t=1}^{k} r_t(a)c_t(y_t(a)) + f(a) \leqq w(a)$,

(ii) *for all* $(g,z) \in \mathbb{R}_+^{1+k}$ *such that* $\sum_{t=1}^{k} r_t(a)c_t(z_t(a)) + g(a) \leqq w(a)$, $U_a(f,y) \geqq U_a(g,z)$

(iii) *for all* $t \in K$, $y_t(a) = y_t$,

where an additively separable cost function is defined by $c(y) := \sum_{t=1}^{k} c_t(y_t)$ for $y \in \mathbb{R}_+^k$, where $c_t: \mathbb{R}_+ \to \mathbb{R}_+$ for $t \in K$.

In particular, there exists a single private good, and the "ratios" sum to 1 across agents for each public good. The production of the public goods must be decentralized (one production unit per public good) for a ratio equilibrium to make sense. Kaneko also presents a voting game in which a level of the public good to be produced is decided. It is proved that the core of the voting game and the ratio equilibria exist simultaneously and that they coincide. But Kaneko studies ratio equilibrium in a context where only one private good is present; this is essentially in a partial equilibrium context. Generalizations to many private goods context have been made by Diamantaras and Wilkie (1994) and Tian and Li (1990).

A further reformulation of Lindahl's idea is offered by Mas-Colell and Silvestre (1989), in which the concept of a *cost share equilibrium*[3] is introduced. Each agent is assigned a monotonic share function, and given these not necessarily linear shares, there is unanimity on the desired level of the public goods:

Definition 1.2.5 *In the presence of a single private good, a* **cost share equilibrium** *is a pair formed by a feasible allocation* (f,y) *and a cost share system* $v: A \times \mathbb{R}_+^k \to \mathbb{R}$, *with* $v(a,0) = 0$ *and* $\sum_{a \in A} v(a,y) = c(y)$ *for all* $y \in \mathbb{R}_+^k$ *such that, for every* $a \in A$,

(i) $f(a) = w(a) - v(a,y)$, *and*

(ii) *for all* $z \in \mathbb{R}_+^k$, $U_a(f(a),y) \geqq U_a(w(a) - v(a,z), z)$.

The allocation is, therefore, designed in a decentralized way, and yields efficient outcomes. This guarantees that individual contributions are in line with individual

[3]The concept of a cost share equilibrium is a special case of a *valuation equilibrium* introduced in Mas-Colell (1980). The notion of a valuation equilibrium is a basic equilibrium concept in economies with public projects, and is separately discussed in Section 4.

benefits. Mas-Colell and Silvestre focus their analysis on a "linear" cost share system and the notions of *linear cost share equilibrium* and *balanced linear cost share equilibrium*, which allow a generalization of the cost sharing paradigm to any number of public goods, even in the increasing returns case. Mas-Colell and Silvestre show that in the convex case the linear cost share equilibria are in one-to-one correspondence with the Lindahl-Foley equilibria. The correspondence is established by varying the profit share parameters which characterize the Lindahl-Foley equilibria. Furthermore, in a more specified model with one private good and one public good, Weber and Wiesmeth (1991) show that an allocation belongs to the core if and only if it is a cost share equilibrium. They also characterize the set of core allocations which can be supported by a linear cost share system. Hirokawa (1992) shows the equivalence between the set of cost share equilibria and the core of the voting game with compensation.

Equity and Efficiency

The notion of efficiency is typically formulated in the sense of Pareto, defined formally as before, i.e., an allocation is said to be Pareto efficient if there does not exist another allocation which is unanimously preferred to it. On the other hand, equity has been subject to considerable discussion. While equity issues have been considered through the formulation of a social welfare function, it was not until Foley (1967) that an equity concept was proposed which is ordinal and free of interpersonal utility comparisons. Foley defines an allocation to be *envy-free* if no agent prefers someone else's commodity bundle to his own. It is natural to study conditions under which there exist allocations which satisfy both the equity and the efficiency criteria.[4] It is possible to show that in a private goods economy without production a competitive equilibrium (Walras equilibrium) from equal endowment is necessarily an envy-free and efficient allocation. It is efficient by the First Theorem of Welfare Economics, and the envy-free property follows from the fact that equal endowment guarantees that all agents will have the same wealth. Varian (1974) formally proves the existence of envy-free and efficient allocations in a private goods economy without production.

[4] An envy-free allocation is sometimes referred to as an equitable allocation. An envy-free and Pareto efficient allocation is often called a 'fair' allocation.

Pazner and Schmeidler (1974), however, showed that a private goods economy with production may not have an envy-free and efficient allocation. Subsequently, Pazner and Schmeidler (1978) proposed the concept of egalitarian-equivalence: An allocation is said to be *egalitarian-equivalent* if there exists a fixed commodity bundle (the same for each agent) that is considered by each agent to be indifferent to the bundle that he actually gets in the allocation under consideration. The notion of egalitarian equivalence can be seen as a modification of the notion of envy-free based on Pareto indifference. However, it is possible that the set of envy-free and efficient allocation is non-empty and disjointed from the set of egalitarian equivalent allocations. In other words, egalitarian-equivalence and the notion of envy-freeness and efficient allocations are formally non-comparable; neither implies the other. In particular, Daniel (1978) mentions that Postlewaite shows that there are well-behaved exchange economies where all efficient and egalitarian-equivalent allocations violate the envy-free condition. Furthermore, Thomson (1990) shows that there may be no allocations that are both egalitarian-equivalent and envy-free in economies with indivisible goods and one divisible good.

A public good economy has similar problems as a private good economy with production: envy-free and efficient allocations may not exist. In particular, the existence of envy-free and efficient allocations cannot be obtained as a direct extension of Lindahl equilibrium with equal endowment. Following Foley (1967), Suzumura and Sato (1985) note that, however, one can use the public competitive equilibria from equal endowments and under proportional taxation, to obtain envy-free and efficient allocations in public good economies. Furthermore, Diamantaras (1992) derives existence theorems for efficient and envy-free allocations for economies with an arbitrary but finite number of private and public goods by adapting the existence proof of Svensson (1983) for private good economies. Diamantaras requires the Pareto efficient subset to be contractible[5] instead of the convexity assumptions on preferences, the production technology, and the Pareto efficient subset.

The notion of egalitarian-equivalence is well defined for the public goods economies, and egalitarian-equivalent and efficient allocations exist under standard con-

[5]Let X be a topological space. A subset of A of X is *contractible* if it can be continuously deformed to a point. For example, every convex subset of a Euclidean space is contractible; every half-circle in the plane is contractible; every circle in the plane is not contractible.

ditions. Sato (1985) considers the case when there is only one private good and obtains efficient and egalitarian-equivalent allocations obtained by requiring the reference bundle to be unit vector in the private good direction. This choice provides a natural interpretation of the reference bundle as measuring the agents' willingness to pay for the public good with the private good. Similarly, Moulin (1987) devises the concepts of egalitarian-equivalent cost sharing mechanism in producing a public good and of egalitarian-equivalent allocation for an economy with one private good and one public good. He shows that an egalitarian-equivalent cost sharing allocation is in the core of the economy and, conversely, any cost sharing method under some conditions must select an egalitarian-equivalent allocation in every economy. Furthermore, Moulin (1992) develops an equal ratio equivalent solution in the former setting. In particular, Yen (1989) argues that the unique egalitarian-equivalent allocation of an economy with one public good is a constrained maximum of an appropriately defined social welfare index.

1.3 Experimental studies of a public goods economy

Auction mechanism

Based on incentive-compatible mechanisms for public goods provision, Smith (1979a, 1979b, 1980) proposes several variants of what he calls the *auction mechanism* for acquiring public goods. In the auction mechanism the agents submit bids indicating their desired quantity of a single public good and the cost shares they would accept. If all the agents agree on the quantity to be provided and on their cost shares, then the public good is provided at the indicated level and with the accepted cost shares. Otherwise, the public good is not produced. The auction mechanism is characterized by collective excludability, unanimity, and budget balance. It exploits the fact that prior to the actual provision of a pure public good, a collective, and therefore each of its members, can be excluded from the good by not providing it. Therefore, the stopping rule requires that (i) the sum of individuals bids (contributions) cover the cost of the proposed public good, and, if this occurs, (ii) the collective then agree unanimously to accept this result. So Smith's auction mechanism, with its requirement that agents unanimously agree to the public good quantity and cost shares, provides a partial remedy for the incentive to underreport. Even if everyone else agrees on a quantity and a distribution of cost shares, the failure of just one agent to agree causes no public good to be provided.

Smith (1979b) reports experiments on some processes[6] – the Groves-Ledyard mechanism, the Lindahl pricing mechanism, and the auction mechanism – for environments with one private good and a single public good. Smith finds that in the absence of income effects, the auction mechanism typically provided a quantity of the public good which is consistent with an efficient allocation (although there is usually a revenue surplus), and that agents' bidding behavior could be accounted for by a model predicting that on the final round each agent bids the Lindahlian personalized price for the public good.

Smith (1980) examines the auction mechanism in an environment in which the choice problem is to select a quantity and cost share for one public good and in which preferences exhibit income effects. In these experiments the quantity of public good provided is approximately that which corresponds to a Lindahl equilibrium, but the allocation of private and public goods departed significantly from the Pareto efficient set.

Follow-up studies to Smith's research can be found in Hastard and Marrese (1981), Ferejohn, Forsythe, Noil and Palfrey (1982), and Coursey and Smith (1984). Ferejohn et al. examine the auction mechanism with multiple discrete public goods. Hastard and Marrese point out the incompleteness of mechanism design – the unspecification of the process, an information logic by which an allocative decision is reached – and replicate the Groves-Ledyard mechanism[7] and Smith's auction mechanism. Hastard and Marrese, however, get some results that more failure to agree are found with the auction mechanism, and neither set of experiments supports the

[6]While in the Groves-Ledyard mechanism based on incentive compatible tax rule the subject agents bid only their increment to the quantity of the public good, in the Lindahl mechanism they bid only their share of the cost for the public good. Furthermore, Smith (1979a) proposes two more variants of auction mechanism: a free-rider mechanism and a quasi free-rider mechanism. A free-rider mechanism is basically the same as the Lindahl mechanism, while a quasi free-rider mechanism is auction mechanism with a rebate of any overbid costs.

[7]Hastard and Marrese emphasize that the Grove-Ledyard incentive-compatible tax rule does not require to balance the budget, and call this mechanism the *incentive-compatible deficit mechanism*.

outcomes approximating the Pareto-efficient allocations. Coursey and Smith propose a decentralized mechanism, called EXTERN, which contains a pure private good and a pure public good mechanism as special cases of an externality mechanism. Their experiments consist of three parts: a private good sealed bid auction experiment, followed by a public good experiment, and finally an experiment in which a private good exhibits external effects. Coursey and Smith find that measures of demand revelation tend to be lower for the private good than the public good, and that this tendency carries over into the private and public components of the externality experiments which lower allocative efficiency.

A more recent work is found in Banks, Plott and Porter (1988), which report an experimental investigation of four methods of allocating public goods. The two basic processes are voluntary contribution mechanism and auction mechanism. Both of these processes are studied with and without an additional unanimity feature. Banks et al. find that the auction mechanism outperforms voluntary contribution mechanism, and that the effect of unanimity is to decrease the efficiency of both processes.

Voluntary contribution mechanism

In experimental studies with public goods the voluntary contribution mechanism has been the most commonly used as an institutional framework. In the *voluntary contribution mechanism* agents voluntarily pay money into a fund to purchase the public good, and the total return on the public good is divided equally among the members of the group independent of their individual contributions. This setting is essentially a prisoner's dilemma in which the dominant strategy is to contribute zero. Initial laboratory examinations of public goods generate widely divergent results. On the one hand, Marwell and Ames (1979, 1980, 1981) and Schneider and Pommerhene (1981) report a positive level of contributions to the public good in a single period situation even with a wide variety of treatment conditions. On the other hand, Kim and Walker (1984) and Isaac, McCue and Plott (1985) find that this optimism is unwarranted in repeated situations. In other words, contributions quickly erode to low levels after two or so periods.

One of the first systematic studies truly designed to figure out the reasons for the range of seemingly divergent experimental results is that of Isaac, Walker and Thomson (1984). They find that the total contributions are strongly dependent on the marginal payoff for contributing. But even with a careful design they conclude that "free riding is neither absolutely all pervasive nor always nonexistent. . . . The extremes of strong free riding and near-Lindahl optimal behavior can and do occur." (p. 140.)

Next, Isaac and Walker (1988a) examine the relationship between variations in group size and free riding behavior in the voluntary provision of public good. They obtain the results that increasing group size leads to a reduction in allocative efficiency when accompanied by a decrease in marginal per capita return (MPCR) to contributions to the public good. Isaac and Walker (1988b) also examine experimentally the role of active communication as a mechanism for improving economic efficiency in a voluntary contribution mechanism. With no communication the voluntary contribution mechanism is shown to induce significant suboptimality, approaching zero contributions with iteration of the decision environment. On the other hand, communication is shown to improve group optimality significantly.

Some variations of voluntary contribution mechanisms are more successful in generating efficient allocations in the laboratory.[8] One of those mechanisms is the voluntary contributions mechanism with provision points: If the number contributing meets or exceeds an announced threshold, then the public good is provided to the group, and all the members of the group receive a prescribed share of good. Otherwise, the public good is not provided.[9]

Palfrey and Rosenthal (1984), both theoretically and experimentally, analyze such games and find that they are capable of generating both efficient and inefficient outcomes. Palfrey and Rosenthal (1988) extend their former analysis to the case in which the cost of contributions is private information, and interpret the results as altruism. Palfrey and Rosenthal (1991) allow communication[10] in their setup, and show that experimental data strongly support the optimal allocations. Similarly, Bagnoli

[8]In social psychology Robyn Dawes and several colleagues (van de Kragt et al., 1983; Dawes et al., 1986) explore 'minimum contributing set' regimes in which each of the members of the group decide whether to contribute (a fixed amount) or not.

[9]There is one more variant that allows for contributions to be refunded when the public good is not provided.

[10]That is cheap talk treatment that agents may send binary messages prior to the decision.

and Lipman (1989) investigate two contribution games in which the individuals decide whether to contribute and the level of their own contributions. Both contribution games involve posting voluntary contributions under the provision rule that the good (or the next unit of the public good) will be provided when the total group contributions meet or exceed a threshold — otherwise the contributions are refunded. In the first game, the group must decide whether or not to provide a public good. In the second, they decide how much (the number of units) as well as whether to provide. Bagnoli and Lipman theoretically show that their first game fully implements the core in undominated perfect equilibria, while the second game fully implements the core in successively undominated strictly perfect equilibria. For the simple problem of whether or not to provide the public good, their predictions are supported by the experimental results reported by Bagnoli and McKee (1991). For the more complicated problem of the level of provision, Bagnoli, Ben-David, and Mckee (1992) show that the experimental results did not rely on the proposed mechanism by Bagnoli and Lipman.

1.4 Economies with Public Projects: A pure approach to a public goods economy

The first mathematically general approach to the pure theory of public goods is introduced by Mas-Colell (1980). He studies a model with one private good and an abstract set of *public projects* without any mathematical structure. In particular, it is not assumed that it is part of a linear space or that there is any ordering at all among public projects based on the Samuelson model.[11] In that paper he introduces a nonlinear system of taxes or subsidies, called a valuation system, for an abstract set of public projects, and defines the *valuation equilibrium*: An allocation is a valuation equilibrium if it can be supported by a valuation system under which every consumer maximizes preferences on a budget set and where each public project

[11]Even though Mas-Colell did not explicitly mention the importance and the applicability of a notion of *public project*, and introduced the terminology *pure* which may follow Samuelson (1954), it is a *pure* theory of public goods with respect that any kind of public goods can be represented by a public project concept such as a *trade center* in Gilles, Diamantaras and Ruys (1995), and *trade links* in Chapter 4.

provider maximizes profit:

Definition 1.4.1 *An allocation* (f, y) *is a* **valuation equilibrium** *if can be supported by a valuation system* $V: A \times \mathcal{Y} \to [-\infty, \infty)$ *such that for some* $\Pi = \{\Pi(a)\}_{a \in A} \in \mathbb{R}^{|A|}$ *with* $\sum_{a \in A} \Pi(a) = \sum_{a \in A} V(a, y) - c(y)$*,*

(i) *for every* $a \in A$*,* (f, y) *maximizes* U_a *on*

$$\{(g, z) \mid v(a, z) + g(a) = w(a) + \Pi(a)\},$$

(ii) y *maximizes* $\sum_{a \in A} v(a, z) - c(z)$ *on* $\mathcal{Y}$*,*

where there exists one private good, $f: A \to \mathbb{R}_+$, $\mathcal{Y}$ is an unstructured set of potential levels of public project, and Π is lump-sum transfer system of profits and losses.

This approach to economies with public projects is based on personalized prices for *access* to the public good ("valuation price") rather than personalized prices for *units* consumed of the public good, as in the literature on Lindahl equilibrium. With Lindahl prices it is assumed that the quantity of each public good is represented by a real number, and the equilibrium establishes what quantity will be supplied. In a valuation equilibrium the supplier chooses from a set of integral public projects, and there is no need to order projects by magnitude or any other criterion. The valuation prices faced by a consumer are different for different public projects, and equilibrium has the property that at these prices no consumer would prefer a different selection from the set of potential projects than the one offered. In addition, no other public project would be profitable. In relation to the generalization of Lindahl equilibrium concept to the case of an economy with an abstract set of public projects, Mas-Colell introduces the appropriate extensions of the standard normative notions of Pareto efficiency and the core, and shows that valuation equilibria decentralize Pareto efficient allocations and the set of valuation equilibria with a nonnegative valuation system is equivalent to the core. As mentioned before, Mas-Colell and Silvestre (1989) further study the valuation system as a cost sharing scheme, and introduce linear cost share equilibrium and balanced cost share equilibrium in a model with one private good and a finite number of public goods as in the literature on Lindahl equilibrium. The equivalence of the set of the cost share equilibria with the core by Weber and Wiesmeth (1991) is also in line with Mas-Colell and Silvestre.

Diamantaras and Gilles (1994) extend the results of Mas-Colell (1980) on valuation equilibria and the relationship with the core in a context of economies with any arbitrary finite number of private goods and a set of public projects without any mathematical structure. Diamantaras and Gilles prove the two welfare theorems for valuation equilibrium, the inclusion of the set of valuation equilibria with a nonnegative valuation system in the core, and the nonequivalence of these two sets. In particular, Diamantaras and Gilles (1993) discuss similar results in setting where a public project can only be established by a certain collection of coalitions of agents. Gilles, Diamantaras and Ruys (1995) study a model in which trade infrastructures are treated as public projects. A public infrastructure has a real cost but it reduces transaction costs for agents making more trading possible. The model in the paper has a continuum of agents, and techniques are used which are also similar to some of the ones applied in Diamantaras and Gilles (1994).

Based on the above studies of economies with public projects, the following chapters provide extensions of the theoretical results and experimental evidence.

Chapter 2, "Economies with Multiple Public Projects," discusses a general equilibrium model of an economy with multiple public projects with additively separable cost functions. We develop the appropriate notion of valuation equilibrium in which a public project is financed through a (non-linear) system of taxes and subsidies, called a valuation system. We show the decentralization of Pareto efficient allocations by valuation equilibria and the equivalence of the core and the set of valuation equilibria with a nonnegative valuation system, extending the result by Mas-Colell (1980). Furthermore, if an Euclidean space is used to describe the public projects, every Pareto efficient allocation is supported as an affine valuation equilibrium which is characterized with a price per unit of public good and a lump sum transfer system.

Chapter 3, "Market Provision of Public Projects: Some Experimental Results," presents experimental evidence of the provision of a public project which is identified with a coalition of economic agents in the population following Diamantaras and Gilles (1993). Establishment of a public project can create both gains from asset trades and benefits from the public project. A double oral auction asset market is employed as the trading institution for environments which differ by rules, information conditions, and parameters. We focus on three groups of issues: (1) whether individually rational efficient outcomes are achieved, (2) whether the coalition forming the public project benefits more or less than the rest of the population, and (3) how free riding and cheap riding occur. We find that individually rational efficient outcomes are more likely in some environments than others, and suggest that these findings may have implications for the usefulness of this mechanism for public project provision.

Chapter 4, "Economies with Costly Trade Links," is an application of the model of economies with public projects to the case of an economy with endogenous formation of costly trade links. The economy consists of a finite number of sectors and industries, and the trade links are built by the same industries from different sectors. The establishment of trade links can reduce the transaction costs, but inevitably incurs set-up costs. We prove that the two welfare theorems in the traditional sense apply, i.e, trade equilibria are Pareto efficient and Pareto efficient allocations can be supported by trade equilibria. Furthermore, we introduce the notion of industry-wise efficiency and an industry-wise trade equilibrium concept, and clarify the relationship between the Pareto efficiency and the industry-wise efficiency concepts: Pareto efficiency implies industry-wise efficiency.

Chapter 5, " Efficiency and Egalitarian-Equivalence in Economies with a Public Project," is an application of the model of economies with any finite number of private goods and one public project to the equity concept of egalitarian-equivalence. An allocation is egalitarian-equivalent if there exists a fixed commodity bundle (the same for each agent) that is considered by each agent to be indifferent to the bundle that he/she actually gets in the allocation under consideration. A public project is identified with a coalition of economic agents which can attain a public project only if a coalition has sufficient resources as well as a subcoalition that is able to execute that project. We prove that there exist efficient egalitarian-equivalent allocations, which are not not equal to the set of valuation equilibria and also may not be in the core.

Chapter 2

Economies with Multiple Public Projects*

2.1 Introduction

We consider economies with multiple public projects each separately produced by a distinct provider operating under a different cost function. Each public project is assumed to be non-Samuelsonian, i.e., there is no common scale in which the chosen level of provision can be expressed as a number. Following Mas-Colell (1980) and Diamantaras and Gilles (1994), technically we represent these provision levels therefore by some unstructured, abstract set rather than a Euclidean space. Under the standard hypothesis of perfect competition among providers and consumers, the natural equilibrium concept is a generalization of Lindahl equilibrium where there is profit and utility maximization given a non-linear tax-subsidy system, called a *valuation system*. In such a *valuation equilibrium* each public project is financed separately through the valuation system. Clearly, this formulation is based on personalized prices for *access* to each public project rather than personalized prices for the *units* consumed of a public good, as in the literature on the Lindahl equilibrium concept (Cf. Foley (1970)).

Our focus is on decentralization issues. The decentralization question is im-

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portant since it shows what type of price systems is necessary to obtain full optimality or first best. We show equivalence of Pareto efficient allocations and the valuation equilibria as well as the equivalence of the core and the valuation equilibrium with nonnegative valuation systems (Theorem 2.2.4).[1]

A personalized access price to some public project as described by a valuation system is based on the provision levels of *all* public projects in the economy rather than on the provision level of the project under consideration. This is due to the spillover effects — or complementarities — among the different public projects and the private good. However, it would be worthwhile to investigate under which conditions these personalized access prices are only based on the provision level of the public project under consideration. This is the subject of Section 3. We show that in the standard case of Samuelsonian public goods, an affine valuation system, which is characterized with a price per unit of public good and a lump sum transfer, is defined for each public project separately. We show that in this case every Pareto efficient allocation can be supported by a valuation equilibrium with an affine valuation system. Furthermore, we show the equivalence of affine valuation equilibria and modified Lindahl-Foley equilibria.

Our main decentralization result (Theorem 2.2.4) is a generalization of Mas-Colell's (1980) results to the case of multiple public projects with separate cost functions. Another generalization of Mas-Colell's (1980) results is given by Mas-Colell and Silvestre (1989), who discuss the case of multiple public projects with a single, total cost function, assuming that the public projects space is the product of the non-negative real-half lines.[2] They focus their analysis on linear cost share systems and show that the linear cost share equilibria are in one-to-one correspondence to the Lindahl equilibria. Following Mas-Colell (1980), Weber and Wiesmeth (1991) obtain the equivalence between the core and cost share equilibria in a more restricted model with one private good and one public good, assuming that the public good space is also the non-negative real half-line. Furthermore, Diamantaras and Gilles (1994) consider a generalization of the model of Mas-Colell (1980) with any arbitrary

[1] A nonnegative valuation system is a tax system only. In the absence of subsidies such a system could also be referred to as a *cost share system*. (Cf. Mas-Colell (1980), Diamantaras and Gilles (1994), and Mas-Colell and Silvestre (1989).)

[2] If non-negative real-half lines are used to represent public projects, we call them *public goods* in distinction from public projects defined through an abstract space.

finite number of private goods allowed. They obtain the first and second welfare theorems for valuation equilibrium, and show the nonequivalence of the set of cost share equilibria and the core.

We note that our decentralization result for Pareto optima is proved under the assumption that the private good can be inessential, i.e., if for certain individual agents utility losses due to choice of alternative public projects cannot be compensated by transfers of the private good. We, however, limit ourselves to the case of essentiality of the private good to show the equivalence of the core and the set of nonnegative valuation equilibria.

Finally, with respect to a price system and lump sum transfers the notion of an affine valuation equilibrium is similar to Foley's (1967 and 1970) notion of *public competitive equilibrium*. On the other hand, Foley's public competitive equilibrium requires the distribution of profit shares to decentralize the public production without the assumption of constant returns to scale. In particular, we emphasize that the affine valuation function is not linear with respect to the cost such as linear cost sharing or with respect to the amount of the public good such as in a Lindahl equilibrium. Our notion of affine valuation equilibrium in this respect forms a bridge between the work based on personalized prices for access to the public projects as discussed by Mas-Colell (1980) and personalized prices for units consumed of public goods as discussed by Foley (1967 and 1970).

2.2 Public projects and valuation equilibria

We consider an economy with multiple public projects and one private good. There is a finite set of identifiable public projects, $K = \{1, \ldots, k\}$, and a certain level of public project $t \in K$ is given by some indicator $y_t \in \mathcal{Y}_t$, where $\mathcal{Y}_t$ is an unstructured set of potential provision levels of the public project t. Now the public projects space can be introduced as the Cartesian product

$$\mathcal{Y} := \Pi_{t=1}^{k} \mathcal{Y}_t.$$

Hence, $y \in \mathcal{Y}$ represents an ordered k-tuple of certain provision levels for each of the k public projects. In particular, in Section 3 we will use non-negative real half-lines as provision spaces of the public projects, i.e,. $\mathcal{Y}_t = \mathbb{R}_+$, for every $t = 1, \ldots, k$. In this case we arrive at the classical public goods model and we call these public projects *public goods* in distinction from abstract case defined with the use of unstructured sets.

To describe the cost of the public projects at a certain provision level, as represented by $y = (y_1, \ldots, y_k) \in \mathcal{Y}$, we assume that each public project t is viewed as being produced by an institutionally distinct organization with cost function $c_t\colon \mathcal{Y}_t \to \mathbb{R}_+$, where $c_t(y_t) \geqq 0$, for $t = 1, \ldots, k$, is the cost of providing public project t at level $y_t \in \mathcal{Y}_t$. The total cost function can now be written in the additively separable form $c = \sum_{t \in K} c_t$.

Each agent $a \in A$ has preferences defined on $\mathbb{R}_+ \times \mathcal{Y}$, which are represented by a real-valued function $U_a\colon \mathbb{R}_+ \times \mathcal{Y} \to \mathbb{R}$. An agent $a \in A$ consumes the unique private good and multiple public projects. This allows for *spillovers* among public projects. Indeed, an agent may contribute to one public project, while he is subsidized regarding another public project. In the sequel some terminology regarding the utility function are introduced: A utility function U_a is *monotone* if for all $f, g \in \mathbb{R}_+$ and all $y \in \mathcal{Y}$ with $f > g$, $U_a(f, y) \geqq U_a(g, y)$. Furthermore, U_a is *strictly monotone* if for all $f, g \in \mathbb{R}_+$ and all $y \in \mathcal{Y}$ with $f > g$, $U_a(f, y) > U_a(g, y)$. We denote by the function $w\colon A \to \mathbb{R}_{++}$ the initial endowment of private good of the agents in A, where it is assumed that $\overline{w} = \sum_{a \in A} w(a) > c(y)$ for all $y \in \mathcal{Y}$.

An **allocation** for an economy is a pair (f, y) where $f\colon A \to \mathbb{R}_+$ and $y = (y_1, \ldots, y_k) \in \mathcal{Y}$. An allocation (f, y) is **feasible** if

$$\sum_{a \in A} f(a) + \sum_{t=1}^{k} c_t(y_t) = \sum_{a \in A} w(a).$$

We denote the set of feasible allocations by Φ. Next we define Pareto efficiency allocations and the core in an economy with multiple public projects.

Definition 2.2.1 *A feasible allocation $(f, y) \in \Phi$ is* **Pareto efficient** *if there exists no other feasible allocation $(g, z) \in \Phi$ such that*

(i) *for every $a \in A$, $U_a(g(a), z) \geqq U_a(f(a), y)$ and*

(ii) *there exists at least one agent $b \in A$ such that $U_b(g(b), z) > U_b(f(b), y)$.*

A feasible allocation $(f, y) \in \Phi$ is a **core allocation** *if there exists no coalition $E \subset A$, public projects $z = (z_1, \ldots, z_k) \in \mathcal{Y}$, and private good allocation $g\colon E \to \mathbb{R}_+$ with*

(i) (g, z) *is feasible for* E, *i.e.,*

$$\sum_{a \in E} g(a) + \sum_{t=1}^{k} c_t(z_t) \leqq \sum_{a \in E} w(a) \quad \textit{and}$$

(ii) *for every* $a \in E$: $U_a(g(a), z) > U_a(f(a), y)$.

We now introduce our valuation equilibrium concept.

Definition 2.2.2 *A feasible allocation* $(f, y) \in \Phi$ *is a* **valuation equilibrium** *if there exist a valuation system* $V_t: A \times \mathcal{Y} \to \mathbb{R}$, $t = 1, \ldots, k$ *such that*

(i) *there is budget neutrality as well as profit maximization for each public project separately, i.e., for every* $t \in K$: $\sum_{a \in A} V_t(a, y) = c_t(y_t)$ *and for every* $z \in \mathcal{Y}$ *and* $t \in K$:

$$\sum_{a \in A} V_t(a, z) - c_t(z_t) \leqq \sum_{a \in A} V_t(a, y) - c_t(y_t), \quad \textit{and}$$

(ii) *for every* $a \in A$, *the pair* $(f(a), y)$ *maximizes* U_a *in the budget set*

$$\left\{ (g, z) \in \mathbb{R}_+ \times \mathcal{Y} \;\middle|\; g + \sum_{t=1}^{k} V_t(a, z) \leqq w(a) \right\}, \quad \textit{and}$$

A feasible allocation $(f, y) \in \Phi$ *is a* **nonnegative valuation equilibrium** *if there exist a nonnegative valuation system* $V_t: A \times \mathcal{Y} \to \mathbb{R}_+$, $t = 1, \ldots, k$ *for which it is a valuation equilibrium.*

Note that a valuation system V_t, $t = 1, \ldots, k$, is defined on $A \times \mathcal{Y}$. This reflects the assumption that an agent evaluates each public project in the context of all of the economy's k public projects. This also allows for spillovers among the multiple public projects in the same way as the formation of utility functions. We define $V(a, y) = (V_1(a, y), \ldots, V_k(a, y)) \in \mathbb{R}^k$. Concerning a model of single public project such as Mas-Colell (1980) and Diamantaras and Gilles (1994), they define a valuation system $V: A \times \Pi_{t=1}^{k} \mathcal{Y}_t \to \mathbb{R}$, and require a single budget neutrality condition. In particular, Mas-Colell (1980) indirectly uses a vector of distribution of profits or losses rather than directly requiring the budget neutrality condition, and notes that at a valuation equilibrium there also exist valuation functions with zero total profit. Here, we require in condition (i) that each public project is provided for through competition among potential providers. Thus, profits for the providers are maximized, and this maximum profit is assumed to be break-even. This incorporates the standard profit maximization assumptiuon as made in the Lindahl equilibrium concept. Furthermore, condition (ii) imposes individual optimality of the assigned tuple $(f(a), y)$, given the valuations of the public projects, i.e., the tax-subsidy system is taken as given by the agents.

Before we state a generalization of these decentralization results to our setting, we introduce the assumption that the private good is essential, i.e., agents can be compensated by private good quantities for utility losses related to the public projects. In other words, we require that each agent receives a fixed minimum utility level if he consumes no private goods, irrespective of the public projects.

Definition 2.2.3 *The economy satisfies the* **essentiality condition** *if for every agent $a \in A$, every $f \in \mathbb{R}_+$, and all potential public project tuples $y, z \in \mathcal{Y}$, there exists a quantity $g \in \mathbb{R}_+$ such that $U_a(g, z) > U_a(f, y)$.*

The essentiality condition has been introduced in a stronger form by Mas-Colell (1980) to prove his main results. Diamantaras and Gilles (1994) modify the condition to show that in case of essentiality of the private goods each Pareto efficient allocation can be supported as a valuation equilibrium with *bounded* valuations, i.e., agents are never taxed so heavily that their budget set becomes empty. We remark that in principle our equilibrium concept is based on unboundedness of the taxes and thus possible emptiness of the budget sets.

Following Diamantaras, Gilles, and Scotchmer (1994), we show the second welfare theorem for valuation equilibria with multiple public projects without the essentiality assumption. However, under the essentiality condition we can prove that every core allocation can be supported as a nonnegative valuation equilibrium.

Theorem 2.2.4 *Let the utility function U_a be monotone on $\mathbb{R}_+$ for every $a \in A$.*

(i) *Every valuation equilibrium is Pareto efficient and every nonnegative valuation equilibrium is a core allocation.*

(ii) *If preferences are continuous and strictly monotone in the private good, then every Pareto efficient allocation can be supported as a valuation equilibrium.*

(iii) *Assume that the essentiality condition is satisfied. If preferences are continuous and strictly monotone in the private good, then every core allocation can be supported as a nonnegative valuation equilibrium.*

The proofs are modifications of Mas-Colell (1980) and Diamantaras, Gilles and Scotchmer (1994), and therefore relegated to the Appendix.

As Diamantaras, Gilles, and Scotchmer (1994) mention, the second welfare theorem for valuation equilibrium differs from that for competitive equilibrium in exchange economies in that we do not need to choose different endowments in order to support different Pareto efficient allocations as equilibria. Since the valuation system can serve the purpose of transferring endowments among agents, the same endowments can be used for different Pareto optima. Thus there is little distinction between proving the second welfare theorem and proving that such equilibria exist. This is summarized in the following corollary.

Corollary 2.2.5 *Consider an economy such that for every agent $a \in A$ the utility function U_a is continuous and strictly monotone on $\mathbb{R}_+$, for all $t \in K$ $\mathcal{Y}_t$ is a finite set, and $w(a) > 0$ for all $a \in A$. Then there exists a valuation equilibrium.*

PROOF

By the second welfare theorem every Pareto efficient allocation can be supported as a valuation equilibrium. Thus it suffices to show that there exists a Pareto efficient allocation. But this follows immediately because utility functions are continuous, the aggregate endowment is finite, and the number of agents and potential public projects are finite. □

Furthermore, we arrive at the equivalence of the core and the set of valuation equilibria with a nonnegative valuation system in an economy with multiple public projects. As pointed out by Diamantaras and Gilles (1994), it is worthwhile to mention that the core equivalence results in Mas-Colell (1980) as well as our setting depend crucially on the assumption that there is only one private good.

2.3 Public goods and affine valuation equilibria

Remark that Theorem 2.2.4 shows that in order to obtain first best decentralization a valuation based on the *total provision* of all public projects jointly has to be taken into account. In an economy with multiple public projects this seems a rather strong condition, since in the standard Lindahl equilibrium concept complete separation in pricing between public goods is obtained. In this section we show that under standard representation of public goods using Euclidean spaces rather than unstructured sets, such separation indeed can be achieved. Hence, here we use nonnegative real-half lines as spaces of public projects, i.e., for each $t \in K$: $\mathcal{Y}_t = \mathbb{R}_+$. As mentioned before, the public projects defined on nonnegative real-half lines are called *public goods* to distinguish this case from the arbitrary situation with abstract public projects spaces.

In the sequel we develop a specification of the concept of valuation equilibrium as introduced above using an *affine valuation system*. In this case a valuation system $V_t: A \times \mathcal{Y}_t \to \mathbb{R}$, $t \in K$, is specified such that for every $y_t \in \mathcal{Y}_t = \mathbb{R}_+$ and for every $a \in A$: $V_t(a, y_t) := p_t(a)\, y_t + \alpha_t(a)$, where $p_t(a) \in \mathbb{R}$ and $\alpha_t(a) \in \mathbb{R}$. The affine valuation system is interpreted as follows: $p_t(a)$ is a personalized price per unit of public project t while the $\alpha_t(a)$, which can be positive or negative, is a lump sum tax or subsidy. We call a valuation equilibrium with an affine valuation system an *affine valuation equilibrium*. It is clear that in an affine valuation equilibrium complete separation of the public goods in pricing is achieved.

Definition 2.3.1 *A feasible allocation* $(f, y) \in \Phi$ *is an* **affine valuation equilibrium** *if there exist a price system* $p: A \to \mathbb{R}^k$ *and a lump sum transfer system* $\alpha: A \to \mathbb{R}^k$ *such that*

(i) *for every* $t \in K$ *there is perfect competition in provision:*

$$\sum_{a \in A} p_t(a)\, y_t + \sum_{a \in A} \alpha_t(a) = c_t(y_t), \text{ and}$$

$$\sum_{a \in A} p_t(a) z_t - c_t(z_t) \leqq \sum_{a \in A} p_t(a) y_t - c_t(y_t), \qquad z \in \mathbb{R}^k_+;$$

(ii) *for every $a \in A$, the pair $(f(a), y)$ maximizes U_a on the budget set*

$$\left\{ (g, z) \in \mathbb{R}_+ \times \mathbb{R}^k_+ \;\middle|\; g + p(a) \cdot z + \sum_{t=1}^{k} \alpha_t(a) \leqq w(a) \right\}.$$

Note that this formulation is a modification of Definition 2.2.2 of a valuation equilibrium with non-separating valuation functions $V_t: A \times \mathcal{Y} \to \mathbb{R}$, $t \in K$. An affine valuation equilibrium requires a separating and affine form of valuation functions $V_t: A \times \mathcal{Y}_t \to \mathbb{R}$, $t \in K$. Moreover, the definition of affine valuation equilibria uses a separated deficit minimization per public good. In particular, the personalized prices per unit of public good can be positive or negative in an affine valuation equilibrium, even though we have a lump sum transfer system.[3]

In a model of multiple public goods, Mas-Colell and Silvestre (1989) modify the notion of Mas-Colell's (1980) valuation equilibrium with linear cost share system with respect to total cost, and show that linear cost share equilibria are in a one-to-one correspondence with the Lindahl equilibria. Our notion of affine valuation equilibrium is another extension of Mas-Colell's (1980) valuation equilibrium in a Lindahlian way.

Before we show that under these assumptions every Pareto efficient allocation can be supported as an affine valuation equilibrium, we have to assume that the cost functions $c_t: \mathcal{Y}_t \to \mathbb{R}_+$, for $t \in K$ are continuous and increasing on $\mathcal{Y}_t = \mathbb{R}_+$, and that $c_t(0) = 0$.[4] We need the following condition, which is an extension of the original indispensability condition of Mas-Colell (1980).

Definition 2.3.2 *The economy satisfies the* **strong essentiality condition** *if the essentiality condition is satisfied, and for every $a \in A$, every $g > 0$, and all $z \in \mathcal{Y} = \mathbb{R}^k_+$, $U_a(g, z) > U_a(0, z) = 0$.*

Furthermore, a utility function U_a is *strictly quasi-concave* if $U_a((1-s)(g(a), z) + s(f(a), y)) > U_a(g(a), z)$ whenever $s \in (0, 1)$ and $U_a(f(a), y) \geqq U_a(g(a), z)$. A utility function U_a is *non-satiated* in the public goods if for every $f > 0$ and $y \in \mathcal{Y} = \mathbb{R}^k_+$,

[3]This formulation is directly related to the monotonicity of preferences with respect to the public good, i.e., whether or not the case of public "bads" is allowed. Almost all authors such as Foley (1970) and Weber and Wiesmeth (1991) require it.

[4]Under these assumptions it is immediate from the definition that in an affine valuation equilibrium it holds that for every public good $t \in K$: $\sum_{a \in A} \alpha_t(a) \leqq 0$.

there exists $z \in \mathcal{Y} = \mathbb{R}^k_+$ with $U_a(f, z) > U_a(f, y)$. We emphasize that, as in Section 2, the monotonicity assumption on the utility function is limited to the private good.

Theorem 2.3.3 *Assume that the strong essentiality condition is satisfied. Let for $t = 1, \ldots, k$ the cost function c_t be convex, and for every agent $a \in A$ the utility function U_a be continuous, strictly quasi-concave, strictly monotone in the private good, and non-satiated in the public goods. Then every Pareto efficient allocation (f, y) can be supported as an affine valuation equilibrium with $p(a) \neq 0$ for $a \in A$ with $f(a) > 0$.*

The proof can be found in the Appendix.

Finally, we show the equivalence of affine valuation equilibria and modified Lindahl-Foley equilibria. To define a modified Lindahl-Foley equilibrium, it is necessary to specify profit shares $\{\theta_t(a)\}_{t \in K; a \in A}$ with $\sum_{a \in A} \theta_t(a) = 1$ and $\theta_t(a) \geq 0$ for every $t \in K$ and $a \in A$. One then has the following notion of equilibrium:

Definition 2.3.4 *A feasible allocation $(f, y) \in \Phi$ is a* **modified Lindahl-Foley equilibrium** *with respect to the profit shares $\{\theta_t(a)\}_{t \in K; a \in A}$ and the system of personalized prices $(\pi_1, \ldots, \pi_k) \in \mathbb{R}^{|A| \times k}$ if*

(i) *for every $a \in A$, $(f(a), y)$ maximizes U_a on the budget set:*

$$\left\{ (g, z) \in \mathbb{R}_+ \times \mathbb{R}^k_+ \,\middle|\, g + \sum_{t=1}^{k} \pi_t(a) z_t = w(a) + \sum_{t=1}^{k} \theta_t(a) (\sum_{b \in A} \pi_t(b) y_t - c_t(y_t)) \right\};$$

(ii) *for all $t \in K$ and for every $z_t \in \mathbb{R}_+$, y_t maximizes $\sum_{a \in A} \pi_t(a) z_t - c_t(z_t)$.*

This modification of Lindahl equilibrium comes from Mas-Colell and Silvestre (1989). On the other hand, they do not impose additively separable cost functions, but a total cost function.

Theorem 2.3.5 *If (f, y) is a modified Lindahl-Foley equilibrium with respect to profit shares θ and personalized prices π, then it arises from an affine valuation equilibrium for the prices $p = \pi$ and for lump sum transfers $\alpha_t(a)$ satisfying $\alpha_t(a) = \theta_t(a)(c_t(y_t) - \sum_{b \in A} \pi_t(b) y_t)$.*

PROOF

By assumption, for all $z \in \mathbb{R}^k_+$,

$$\begin{array}{l} U_a(w(a) - \sum_{t=1}^k \pi_t(a)y_t + \sum_{t=1}^k \theta_t(a)(\sum_{b\in A} \pi_t(b)y_t - c_t(y_t)), y) \quad \geqq \\ U_a(w(a) - \sum_{t=1}^k \pi_t(a)z_t + \sum_{t=1}^k \theta_t(a)(\sum_{b\in A} \pi_t(b)y_t - c_t(y_t)), z). \end{array}$$

Writing $\pi_t(a) = p_t(a)$ and $\theta_t(a)(\sum_{b\in A} \pi_t(b)y_t - c_t(y_t)) = -\theta_t(a)\sum_{b\in A} \alpha_t(b) = -\alpha_t(a)$, we get

$$U_a(w(a) - \sum_{t=1}^k [p_t(a)y_t + \alpha_t(a)], y) \geqq U_a(w(a) - \sum_{t=1}^k [p_t(a)z_t + \alpha_t(a)], z).$$

This is condition (ii) of Definition 2.3.1. Since, again by assumption, y_t maximizes profits for all $t \in K$, we have for all $z \in \mathbb{R}^k_+$:

$$\sum_{a\in A} \pi_t(a)z_t - c_t(z_t) \geqq \sum_{a\in A} \pi_t(a)y_t - c_t(y_t).$$

So, writing $\pi_t(a) = p_t(a)$ and for all $t \in K$, we get

$$\sum_{a\in A} p_t(a)z_t - c_t(z_t) \geqq \sum_{a\in A} p_t(a)y_t - c_t(y_t).$$

This shows condition (i). □

Furthermore, the converse is established as follows:

Theorem 2.3.6 *If (f, y) is an affine valuation equilibrium for the prices p and for lump sum transfers $\alpha_t(a)$, then it arises from a modified Lindahl-Foley equilibrium with respect to personalized prices $\pi = p$ and profit shares such that $\theta_t(a) = \frac{\alpha_t(a)}{\sum_{b\in A}\alpha_t(b)}$, if $\sum_{a\in A}\alpha_t(a) \neq 0$, and $\theta_t(a) = \frac{1}{|A|}$, otherwise.*

The proof is simply a reverse of that of Theorem 2.3.5.

Appendix

The proof of Theorem 2.2.4 (i) is a straightforward extension of the proofs of the first welfare theorems as given in Mas-Colell (1980) and Diamantaras and Gilles (1994) and is therefore omitted.

Proof of Theorem 2.2.4 (ii)

Let (f, y) be a Pareto efficient allocation, and let $z \in \mathcal{Y}$ be arbitrary. We define

$$\begin{aligned} A^\star(z) &:= \{a \in A \mid U_a(g,z) < U_a(f(a),y) \text{ for every } g \in \mathbb{R}_+\}, \\ A^{\star\star}(z) &:= \{a \in A \mid U_a(0,z) > U_a(f(a),y)\}, \\ F(a,z) &:= \{g \in \mathbb{R}_{++} \mid U_a(g,z) > U_a(f(a),y)\}, \quad a \in A \setminus A^\star(z). \end{aligned}$$

By the assumptions on U_a, for each $a \in A \setminus [A^\star(z) \cup A^{\star\star}(z)]$, $F(a,z)$ is a nonempty open interval bounded from below, i.e., $F(a,z) = (g(a,z), \infty)$.

For $z \neq y$ we choose values $x(a,z)$ as follows:

$$x(a,z) = \begin{cases} \inf F(a,z) = g(a,z) & \text{for } a \in A \setminus [A^\star(z) \cup A^{\star\star}(z)] \\ 0 & \text{for } a \in A^\star(z) \cup A^{\star\star}(z) \end{cases}$$

Furthermore, for $z = y$, let $x(a,z) = x(a,y) = f(a)$, for all $a \in A$.

Next we construct a valuation system. For this we let

$$G(z) := \sum_{a \in A} x(a,z) + \sum_{t=1}^{k} c_t(z_t) - \overline{w}.$$

When $A^{\star\star}(z) \neq \emptyset$ we define a parameter $\delta(z) > 0$ as follows: If $G(z) > 0$, let $\delta(z) < (|A^{\star\star}(z)|)^{-1}\ G(z)$. If $G(z) \leqq 0$, then $A^\star(z) \neq \emptyset$, and let $\delta(z) < (|A^{\star\star}(z)|)^{-1}$ $\sum_{a \in A^\star(z)} w(a)$. The latter bound is positive because we have assumed that $w(a) > 0$ for all $a \in A$. Let $V_t(\cdot, z)$ for $t \in K$ be defined by

$$V_t(a,z) := \begin{cases} \sigma_t(z)(w(a) - x(a,z)) & \text{if } a \in A \setminus [A^\star(z) \cup A^{\star\star}(z)], \\ \sigma_t(z)(w(a) - x(a,z) + \delta(z)) & \text{if } a \in A^{\star\star}(z), \\ \min\{0, (|A^\star(z)|)^{-1} \sigma_t(z) G(z)\} & \text{if } a \in A^\star(z), \end{cases}$$

where if $\sum_{s=1}^k c_s(z_s) > 0$,

$$\sigma_t(z) = \frac{c_t(z_t)}{\sum_{s=1}^k c_s(z_s)} > 0,$$

and if $\sum_{s=1}^{k} c_s(z_s) = 0$, $\sigma_t(z) = 0$, $t \in K$. We now check the requirements of (f, y) with V_t, $t \in K$, to be a valuation equilibrium.

CONDITION (i)

By definition, $A^\star(y) = A^{\star\star}(y) = \emptyset$ and so $V_t(a, y) = \sigma_t(y)(w(a) - f(a))$ for $t \in K$. Hence, by feasibility of (f, y),

$$\begin{aligned}\textstyle\sum_{a\in A} V_t(a,y) - c_t(y_i) &= \textstyle\sum_{a\in A} \sigma_t(y)(w(a) - f(a)) - c_t(y_t)\\ &= \textstyle\sigma_t(y)\sum_{a\in A}(w(a) - f(a)) - c_t(y_t)\\ &= \textstyle\sigma_t(y)\sum_{t\in K} c_t(y_t) - c_t(y_t) = 0.\end{aligned}$$

Let $z \in \mathcal{Y}$. Note that $x(a, z) = 0$ if $a \in A^\star(z) \cup A^{\star\star}(z)$. Hence,

$$\begin{aligned}&\textstyle\sum_{a\in A} V_t(a,z) - c_t(z_t)\\ =\ &\textstyle\sum_{a\in A\setminus A^\star(z)} \sigma_t(z)w(a) - \sum_{a\in A}\sigma_t(z)x(a,z) + |A^{\star\star}(z)|\sigma_t(z)\delta(z) +\\ &\min\{0, \sigma_t(z)G(z)\} - c_t(z_t)\\ =\ &\textstyle\sum_{a\in A} \sigma_t(z)(w(a) - x(a,z)) - \sum_{a\in A^\star(z)}\sigma_t(z)w(a) + |A^{\star\star}(z)|\sigma_t(z)\delta(z) +\\ &\min\{0, \sigma_t(z)G(z)\} - \sigma_t(z)c(z)\\ =\ &\textstyle -\sigma_t(z)G(z) - \sum_{a\in A^\star(z)}\sigma_t(z)w(a) + |A^{\star\star}(z)|\sigma_t(z)\delta(z) +\\ &\min\{0, \sigma_t(z)G(z)\} < 0,\end{aligned}$$

by the definition of $\delta(z)$.

CONDITION (ii)

For $a \in A^\star(z)$, there is no (g, z) preferred to $(f(a), y)$, so the condition is satisfied trivially. For $a \in A^{\star\star}(z)$, any $g \geqq 0$ satisfies $U_a(g, z) > U_a(f(a), y)$. Then, since $x(a, z) = 0$, $\sum_{t\in K} \sigma_t(z) = 1$:

$$\begin{aligned}&\textstyle g + \sum_{t=1}^{k} V_t(a,z) = g + \sum_{t\in K}\sigma_t(z)(w(a) - x(a,z) + \delta(z))\\ =\ &g + w(a) + \delta(z) > w(a),\end{aligned}$$

wsince $\delta(z) > 0$ if $A^{\star\star}(z) \neq \emptyset$. For $a \in A \setminus [A^\star(z) \cup A^{\star\star}(z)]$, suppose that there exists $g \geqq 0$ such that $U_a(g, z) > U_a(f(a), y)$. By $a \notin A^{\star\star}(z)$, $U_a(0, z) \leqq U_a(f(a), y) < U_a(g, z)$, hence $g > 0$. By the continuity of $U_a(\cdot, z)$, there exists $\epsilon > 0$ such that $U_a(g - \epsilon, z) > U_a(f(a), y)$, implying that $g > g - \epsilon \geqq x(a, z)$. But then

$$\begin{aligned}&\textstyle g + \sum_{t=1}^{k} V_t(a,z) = g + \sum_{t\in K}\sigma_t(z)(w(a) - x(a,z))\\ =\ &g + w(a) - x(a,z) > w(a),\end{aligned}$$

which proves the required condition. □

Proof of Theorem 2.2.4 (iii)

Let (f, y) be a core allocation, and let $a \in A$ and $z \in \mathcal{Y}$ be arbitrary. Using the notation as introduced above we note first that the essentiality hypothesis implies that $A^*(z) = \emptyset$. Hence, for any $a \in A$ we may define $g(a, z) := \inf\ F(a, z)$. Now for any $E \subset A$ let

$$F'(z, E) := \sum_{a' \in E} F(a', z) + \left\{ \sum_{t=1}^{k} c_t(z_t) - \sum_{a' \in E} w(a') \right\}.$$

$F'(z, E)$ is also a nonempty interval with

$$\inf F'(z, E) = \sum_{a' \in E} g(a', z) + \sum_{t=1}^{k} c_t(z_t) - \sum_{a' \in E} w(a').$$

Next define a valuation system $V_t \colon A \times \mathcal{Y} \to \mathbb{R}$, $t \in K$, by

$$V_t(a, z) := \begin{cases} \sigma_t(z)(w(a) - g(a, z)) & \text{if } z \neq y, \\ \sigma_t(y)(w(a) - f(a)) & \text{if } z = y, \end{cases}$$

where $\sigma_t(z)$ is as defined in the proof of Theorem 2.2.4 (ii).
We show that (f, y) can be supported as a valuation equilibrium with the nonnegative valuation system given by $V_t^+(a, z) = \max\{0, V_t(a, z)\}$ for $a \in A$, $z \in \mathcal{Y}$, and $t \in K$.

CONDITION (i)
By the definition of the core, we have $\inf F'(z, E) \geqq 0$ for all $E \subset A$. Thus,

$$\sum_{a' \in E} g(a', z) + \sum_{t=1}^{k} c_t(z_t) \geqq \sum_{a' \in E} w(a')$$

or, by the definition of the functions V_t, $t \in K$,

$$c_t(z_t) = \sigma_t(z) \sum_{t=1}^{k} c_t(z_t) \geqq \sigma_t(z) \sum_{a' \in E} (w(a') - g(a', z)) = \sum_{a' \in E} V_t(a', z). \tag{2.1}$$

Let

$$C_t \ := \{a \in A \mid V_t(a, z) \geqq 0\}, \quad \text{for } t = 1, \ldots, k,$$

$$C \ := \{a \in A \mid V_t(a, z) \geqq 0 \text{ for all } t = 1, \ldots, k\} = \textstyle\bigcap_{t=1}^{k} C_t.$$

It is trivial to see that $C_t = C$ for $t \in K$. Thus, from the definition of V_t^+

$$\sum_{a' \in A} V_t^+(a', z) = \begin{cases} \sum_{a' \in C} V_t(a', z) & \text{if } C \neq \emptyset, \\ 0 & \text{if } C = \emptyset. \end{cases}$$

Furthermore, by (2.1) applied to the coalition C,

$$\sum_{a' \in A} V_t^+(a', z) = \sum_{a' \in C} V_t(a', z) \leqq c_t(z_t) \tag{2.2}$$

But, from $V_t^+(a', z) \geqq V_t(a', z)$ for $a' \in A$, $z \in \mathcal{Y}$ and the feasibility of (f, y), we derive

$$\sum_{a' \in A} V_t^+(a', y) \geqq \sum_{a' \in A} V_t(a', y) = \sigma_t(y)(\overline{w} - \sum_{a' \in A} f(a)) = c_t(y_t).$$

So, with (2.2) this implies that

$$\sum_{a' \in A} V_t^+(a', y) = c_t(y_t).$$

This shows condition (i) of the assertion.

CONDITION (ii)

By continuity and strict monotonicity of preferences, it follows that $U_a(g(a, z), z) \geqq U_a(f(a), y)$. For any $g \geqq 0$ with $U_a(g, z) > U_a(g(a, z), z) \geqq U_a(f(a), y)$, we have

$$g + \sum_{t=1}^{k} V_t^+(a, z) \geqq g + \sum_{t=1}^{k} V_t(a, z) = g + w(a) - g(a, z) > w(a),$$

since $g > g(a, z)$ by the definition of $g(a, z)$. This shows condition (ii). □

Proof of Theorem 2.3.3

Define $A = \{1, \cdots, n\}$ as the set of agents, where $n = |A|$. Let (f, y) be a Pareto efficient allocation. Let $\overline{f} = \sum_{a \in A} f(a)$. We define two sets F_1 and F_2 as follows:

$$F_1 := \left\{ (\overline{g}, z(1), \ldots, z(n)) \in \mathbb{R} \times \mathbb{R}_+^{kn} \;\middle|\; \begin{array}{l} \text{there is } z \in \mathbb{R}_+^k \text{ with } z(a) = z \\ \text{for all } a \in A \text{ and } \overline{g} + \sum_{t=1}^k c_t(z_t) \leqq \overline{w} \end{array} \right\}.$$

F_1 is closed and convex. Furthermore, since $0 \in F_1$, F_1 is nonempty. The Pareto efficient allocation (f, y) or $(\overline{f}, y, \ldots, y)$ is in F_1, i.e., $(\overline{f}, y, \ldots, y) \in F_1$.

$$F_2 := \left\{ (\overline{g}, z(1), \ldots, z(n)) \in \mathbb{R}_+ \times \mathbb{R}_+^{kn} \;\middle|\; \begin{array}{l} \text{there is } g : A \to \mathbb{R}_+ \text{ such that} \\ \overline{g} = \sum_{a=1}^n g(a) \text{ and} \\ U_a(g(a), z(a)) > U_a(f(a), y) \end{array} \right\}.$$

F_2 is open and convex by the continuity and strict monotonicity of the utility functions in the private good. Furthermore, F_2 is nonempty by the essentiality condition. Given the allocation (f, y), $(\overline{f}, y, \ldots, y)$ is in the closure of F_2, denoted by $\overline{F}_2$. Furthermore, since (f, y) is Pareto efficient, $F_1 \cap F_2 = \emptyset$. Now by Minkowski's separating hyperplane theorem, there exists a nonzero vector $(\lambda, \mu(1), \ldots, \mu(n)) \in \mathbb{R}^{1+kn}$ such that

(a) $\lambda\overline{g} + \sum_{a=1}^{n} \mu(a) \cdot z(a) \leqq \lambda\overline{f} + \sum_{a=1}^{n} \mu(a) \cdot y$, for all $(\overline{g}, z(1), \ldots, z(n)) \in F_1$, and

(b) $\lambda\overline{g} + \sum_{a=1}^{n} \mu(a) \cdot z(a) \geqq \lambda\overline{f} + \sum_{a=1}^{n} \mu(a) \cdot y$, for all $(\overline{g}, z(1), \ldots, z(n)) \in \overline{F}_2$, or
$\lambda\overline{g} + \sum_{a=1}^{n} \mu(a) \cdot z(a) > \lambda\overline{f} + \sum_{a=1}^{n} \mu(a) \cdot y$, for all $(\overline{g}, z(1), \ldots, z(n)) \in F_2$.

Since U_a is strictly monotone in the private good for every $a \in A$, $\lambda \geqq 0$ holds. We show that $\lambda > 0$. Suppose to the contrary that $\lambda = 0$. Let $0 < \epsilon < 1$. Then by strong essentiality there is $g(a) > 0$ for every $a \in A$ with $U_a(g(a), (1-\epsilon)y) > U_a(f(a), y)$. So, $(\overline{g}, (1-\epsilon)y, \ldots, (1-\epsilon)y) \in F_2$, but

$$\lambda\overline{g} + \sum_{a=1}^{n} \mu(a) \cdot (1-\epsilon)y = \sum_{a=1}^{n} \mu(a) \cdot (1-\epsilon)y = (1-\epsilon) \sum_{a=1}^{n} \mu(a) \cdot y < \sum_{a=1}^{n} \mu(a) \cdot y.$$

This contradicts (b). Therefore, $\lambda > 0$.

Next, we show that $\mu(a) \neq 0$ for $a \in A$ with $f(a) > 0$. Suppose to the contrary that $\mu(a) = 0$. Then by the non-satiation of U_a in the public goods, we have $z \in \mathbb{R}_+^k$ with $U_a(f(a), z) > U_a(f(a), y)$. By the continuity in the private good and $f(a) > 0$, there exists $\epsilon > 0$ such that $U_a(f(a) - \epsilon, z) > U_a(f(a), y)$. Define

$$z(b) = \begin{cases} z & \text{if } b = a \\ y & \text{if } b \neq a \end{cases} \qquad \text{and} \qquad g(b) = \begin{cases} f(b) - \epsilon & \text{if } b = a \\ f(b) + \frac{\epsilon}{n-1} & \text{if } b \neq a. \end{cases}$$

Note that $\overline{g} = \sum_{b=1}^{n} g(b) = \sum_{b=1}^{n} f(b) = \overline{f}$. By the monotonicity of the utility function we have for every $b \neq a$, $U_b(g(b), y) = U_b(f(b) + \frac{\epsilon}{n-1}, y) > U_b(f(b), y)$. Thus, $(\overline{g}, z(1), \ldots, z(n)) \in F_2$. So,

$$\lambda\overline{g} + \sum_{b=1}^{n} \mu(b) \cdot z(b) > \lambda\overline{f} + \sum_{b=1}^{n} \mu(b) \cdot y,$$

By $\overline{g} = \overline{f}$ we conclude that $\mu(a) \cdot z > \mu(a) \cdot y$. This is a contradiction.

Let $p(a) := (1/\lambda)\mu(a) \in \mathbb{R}^k$, for every $a \in A$, and let $\{\alpha_t(a)\}_{t\in K; a\in A}$ be determined by

$$\sum_{a\in A} \alpha_t(a) := c_t(y_t) - \sum_{a\in A} p_t(a)y_t, \text{ for } t = 1,\ldots,k. \tag{2.3}$$

This defines an affine valuation system $V_t: A \times \mathcal{Y}_t \to \mathbb{R}$, for $t \in K$. We now check the requirements of Definition 2.3.1.

CONDITION (i)

For every $a \in A$ and for $t \in K$, by (2.3):

$$\begin{aligned}\sum_{a\in A} V_t(a, y_t) &= \sum_{a\in A} p_t(a)y_t + \sum_{a\in A} \alpha_t(a)\\ &= \sum_{a\in A} p_t(a)y_t + c_t(y_t) - \sum_{a\in A} p_t(a)y_t\\ &= c_t(y_t)\end{aligned}$$

Let $z \in \mathbb{R}^k_+$. Define $\overline{g} := \sum_{a\in A} w(a) - \sum_{t=1}^k c_t(z_t)$. Then $(\overline{g}, z, \ldots, z) \in F_1$. Since $(\overline{f}, y, \ldots, y) \in \overline{F}_2$, it follows that

$$\begin{aligned}&\overline{f} + \sum_{a\in A} p(a)\cdot y \geqq \overline{g} + \sum_{a\in A} p(a)\cdot z, \quad \text{or}\\ &\overline{f} + \sum_{a\in A} p(a)\cdot y + \sum_{a\in A}\sum_{t=1}^k \alpha_t(a) \geqq \overline{g} + \sum_{a\in A} p(a)\cdot z + \sum_{a\in A}\sum_{t=1}^k \alpha_t(a).\end{aligned} \tag{2.4}$$

By feasibility condition and the definition of $\overline{g}$,

$$\overline{f} - \overline{g} = \left[\overline{w} - \sum_{t=1}^k c_t(y_t)\right] - \left[\overline{w} - \sum_{t=1}^k c_t(z_t)\right] = \sum_{t=1}^k c_t(z_t) - \sum_{t=1}^k c_t(y_t).$$

Thus, (2.4) implies that

$$\begin{aligned}&\sum_{a\in A} p(a)\cdot y - \sum_{t=1}^k c_t(y_t) \geqq \sum_{a\in A} p(a)\cdot z - \sum_{t=1}^k c_t(z_t), \quad \text{or}\\ &\sum_{t=1}^k \sum_{a\in A} p_t(a)(y_t - z_t) \geqq \sum_{t=1}^k (c_t(y_t) - c_t(z_t)).\end{aligned}$$

Furthermore, let z be such that $z_s = y_s$ for all $s \neq t$ and $z_t \neq y_t$. Then, by the assumption that c_t, $t \in K$, is continuous and increasing, we have for $t \in K$

$$\begin{aligned}&\sum_{a\in A} p_t(a)(y_t - z_t) \geqq (c_t(y_t) - c_t(z_t)), \quad \text{or}\\ &\sum_{a\in A} p_t(a)y_t - c_t(y_t) \geqq \sum_{a\in A} p_t(a)z_t - c_t(z_t).\end{aligned}$$

Thus, condition (i) is established.

CONDITION (ii)

Let $a \in A$ and let (g, z) be such that $U_a(g, z) > U_a(f(a), y)$. By the strong essentiality condition we have $g > 0$, and by the continuity of the utility function there exists $\epsilon > 0$ such that $U_a(g - \epsilon, z) > U_a(f(a), y)$.
Define $g(a) = g - \epsilon$ and $g(b) = f(b) + \frac{\epsilon}{n-1}$, $b \neq a$. Then $(\overline{g}, z, y, \ldots, y) \in F_2$. Thus

$$\sum_{a \in A} g(a) + p(a) \cdot z + \sum_{b \neq a} p(b) \cdot y > \sum_{a \in A} f(a) + \sum_{a \in A} p(a) \cdot y. \tag{2.5}$$

Note that $\sum_{a \in A} g(a) = (g - \epsilon) + \sum_{b \neq a}[f(b) + \frac{\epsilon}{n-1}] = g + \sum_{b \neq a} f(b)$. Hence, from (2.5):

$$g + p(a) \cdot z > f(a) + p(a) \cdot y, \text{ or}$$
$$g + p(a) \cdot z + \textstyle\sum_{t=1}^{k} \alpha_t(a) > f(a) + p(a) \cdot y + \sum_{t=1}^{k} \alpha_t(a).$$

This implies condition (ii). □

Chapter 3

Market Provision of Public Projects: Some Experimental Results*

3.1 Introduction

In this chapter we conduct an experimental study of the provision of a public project.[1] The project is built by a coalition of agents which is a subset of the population. The mechanism is based on an asset exchange market for private goods. This economy can be described as follows: There are n agents who each own 1 of n marketable assets. Each unit is worth a different amount to its current holders. Suppose that a coalition wants to execute a public project, i.e., build a community center. The cost of the project equals the m ($\leq n$) assets. In order to execute the public project the coalition must purchase the m assets from the other members of the population.

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[1]As mentioned before, a theoretical basis for public projects is on Mas-Colell (1980). He postulates the existence of a set of public projects without any mathematical structure such as a linear space or an ordering among projects, and proposes the notion of *valuation equilibrium*, a generalization of Lindahl equilibrium concept for public projects, in which a public project is financed through a nonlinear system of taxes and subsidies, called *valuations*. He proves the two welfare theorems in case of an economy with an abstract set of public projects. Diamantaras and Gilles (1993, 1994) extend Mas-Colell to the case of multiple private goods, and Chapter 2 to the case of multiple public projects.

In general there may be more than one coalition of agents who, upon collecting the m assets, could execute the public project. Each coalition could choose to create a different public project. Once the public project is completed, then every agent in the population, not only the member of the coalition who executed the project, benefits from the public project. These benefits may depend on which coalition executes the public project. If any individual agent sells his asset for more than his private value, he will profit regardless of whether the public project is successfully completed by a coalition. A member of the coalition may lose if, in an attempt to complete the public project, he buys an asset for more than its private value and the public project is not successfully executed. It is also possible for an agent to lose money if he sells his asset for less than his private value and no coalition is successful in completing the project.

Our experimental environments in 24 markets, each with 21 periods, differs by rules about who can produce a public project, information about the benefits to the other agents of the project, and parameters which include the symmetry and size of individual valuation of the assets and the magnitude of social benefits from the project. Our experimental results allows an investigation of three basic issues: (1) whether individually rational efficient outcomes are achieved, (2) whether the coalition forming the public project benefits more or less than the rest of the population, and (3) how free riding or cheap riding phenomena occur.

Compared with recent experimental studies with public goods such as auction mechanism (Smith, 1979a, 1979b, 1980) and voluntary contribution mechanism (Isaac and Walker, 1988a, 1988b, 1994),[2] our experimental study is characterized as follows:

First, as an important driving principle in generating contributions towards public goods, Smith's auction mechanism depends on *unanimity*, and voluntary contribution mechanism is based on *altruism* or *natural cooperation*. Our experimental design of public projects works by *coalitional rationality*.

Second, recent experimental studies are mainly focused on continuous and/or divisible public goods. This work studies a model of public projects which are discrete and indivisible.

Third, in Smith's auction mechanism or voluntary contribution mechanism,

[2]Ledyard (1995) gives a nice survey of experimental studies, particularly voluntary contribution mechanism, with public goods.

public goods are *exogenously* given and the provision of public goods is decided by agents' decisions about whether to contribute or not. Public projects are *endogenously* established by a coalition of economic agents that have sufficient resources and are able to execute that project.

Fourth, our experimental setting of the provision of a public project is organized through an asset exchange market. The asset exchange mechanism works via trade of private goods for individual gain as well as investment in public projects for mutual (social) benefit. On the other hand, experimental studies of public goods regard private goods (generally, one private good) as only a means of contribution or investment to public goods.

Finally, a model of public projects introduces widespread externalities concerning the way in which that project is executed. Therefore, our experimental design explicitly deals with tax (or subsidy) schemes and externalities from public projects by a simple experimental treatment — information controls.

The chapter is organized as follows. Section 2 presents the theoretical model and experimental design for a study of simple public projects, section 3 analyzes the model theoretically, and section 4 offers results of a series of experiments. We conclude with a discussion of private provision of public projects.

3.2 Model and design

We consider a model of a public project which is identified with a coalition of agents that can undertake it. Here we introduce an environment which is a simple special case of public project provision.

There are three agents, $I = \{1, 2, 3\}$. Agents in I are endowed with initial bundle of two private commodities, tokens (T) and assets (A), such that $w_i = (w_{T_i}, w_{A_i})$ for all $i \in I$. We denote by $\mathcal{P} \subset 2^I$ the collection of all coalitions of economic agents that are able to execute a uniquely defined public project. We let $\mathcal{P} = \{\emptyset, \{1\}, \{2\}, \{3\}\}$, that is, any agent can build the public project alone. This means that the public project can not be executed by any group of agents, but established only by one member. $\emptyset \in \mathcal{P}$ means the public project is not executed.

We may summarize the use of private commodities, tokens and assets, to execute the public project with a cost function $c(S) = (c_{T_i}, c_{A_i})$ for the coalition

$S \in \mathcal{P}$, where $c(\emptyset) = (0,0)$.

Every agent evaluates bundles of private commodities, tokens and assets, alongside public projects according to the utility function as follows:

$$u_i(T_i, A_i, S) = T_i + \alpha_i A_i + E_i(S) \text{ for all } i \in I \text{ and all } S \in \mathcal{P},$$

where α_i is an asset return or dividend for agent i, and $E_i(S)$ is a private return from a public project. $E_i : \mathcal{P} \to \mathbb{R}$ defined for $i \in I$ depends on externalities concerning the way in which the public project is executed.

So, every agent meets the following utility maximization problem under his budget constraint:

$$\begin{array}{ll} \text{Maximize} & u_i(T_i, A_i, S) = T_i + \alpha_i A_i + E_i(S) \\ \text{subject to} & T_i + p(S)A_i + V_i(S) \leq w_{Ti} + p(S)w_{A_i}. \end{array}$$

where $p(S)$, an asset price, is a set-valued function on $S \in \mathcal{P}$ assigning every public project $S \in \mathcal{P}$ a normalized price,[3] and $V_i(S)$ is also a set-valued function on $i \in I$ and $S \in \mathcal{P}$. $V_i : \mathcal{P} \to \mathbb{R}$ defined for $i \in I$ is a valuation which represents a tax or a subsidy, and depends on the way in which the public project is executed. Note that the case $S = \emptyset$ is that $(A_i, A_j, A_k) = (1,1,1)$ or $(0,1,2)$, and the case $S \neq \emptyset$ is that $(A_i, A_j, A_k) = (3,0,0)$, for $i, j, k \in I$ and $i \neq j \neq k$.

Based on the above simple model, we introduce an experimental design of economies with a public project as follows:

Markets are composed of 3 agents, each endowed with 100 tokens publicly valued and a single privately valued asset at the beginning of each three-minute period. Sequential trading of assets is allowed during the period. The asset exchange mechanism used in the experiment is the *double oral auction institution*, which allows traders to sell and/or buy assets constrained by their token and asset endowments.[4] When a transaction occurs, all bids and offers are removed. At the end of period,

[3]We remark that widespread externalities, the different use of resources in each public project — as given in the cost function c — as well as the multi-dimensionality of the private goods space induce price changes in the private sector in case a different public project is executed. This is reflected through a price function p from the collection of public projects $\mathcal{P}$ into the price simplex Δ rather than a unique price system $p \in \Delta$. See Diamantaras and Gilles (1993) for a comprehensive discussion for it.

[4]A bid/ask spread reduction rule (new bids have to be higher than previous outstanding bid and offers have to be lower than current market offer) is also in place.

each token pays 1 cent to token holders and each asset pays a liquidating dividend of α_i, $i \in I$, to asset holders. If one of three agents holds the three assets, every agent receives a bonus B which represents a valuation $V_i(S)$ or externalities $E_i(S)$ for $i \in I$ and $S \in \mathcal{P}$.

The 24 markets, each with 21 periods, are organized into the following 2×3 experimental designs as Table 1. In the public bonus markets (APUB, HPUB, LPUB), the value of bonus, B, is publicly announced. In the private bonus markets (APRB, HPRB, LPRB), the value of bonus, B, is private information. In any coordinator markets (APUB, APRB), a bonus is paid if any one of the three traders holds all assets at the end of trading period. In one assigned coordinator markets, either the highest dividend holder (HPUB, HPRB) or the lowest dividend holder (LPUB, LPRB), can be a coordinator. In the latter cases if either of the other traders hold all 3 assets at the end of a period, no bonus is paid.

We use 7 different sets of parameter choices (designated as IA through IVB) for dividends α_i, $i \in I$, and a bonus B as shown at Table 2. We apply the Latin square randomization procedure rather than the complete randomization method in each set of parameter choices. Within these parameter choices, permutations (marked 1, 2, and 3) are presented in a different random order in each market.[5] Among the IA, III, and IVA periods the same bonus, $B = 60$, is given with the different values of dividends, i.e, $(\alpha_i, \alpha_j, \alpha_k) = (20, 25, 30)$, $(26, 28, 30)$, or $(30, 30, 30)$. In IA and IB periods, or in IVA and IVB periods, the same dividend values are given with two different bonus values, $B = 60$ or 10. Furthermore, in IIA and IIB periods, the dividends and bonus values are the same as those in IA period, but the bonus depends on who is a coordinator with different dividend values.

Subjects were undergraduate students at Virginia Polytechnic Institute & State University recruited for Economics and Accounting courses. Experiments lasted approximately 2 hours. Subjects were paid a \$ 3 show up fee and earned between \$10 and \$16 totally.

[5]The random orders for 21 permutations in each market were selected in advance of the experiment using a random number table listed on Kirk (1982).

3.3 Theoretical analysis

Based on the above theoretical model and experimental setting, we identify the individually rational efficient allocations in economies with a public project. The standard efficiency concepts are defined:

An allocation $((T_{n1}, A_{n1}), (T_{n2}, A_{n2}), (T_{n3}, A_{n3}) : S)$, for $n \in \mathbb{N}$ and $S \in \mathcal{P}$, is *individually rational* if $u_i((T_{ni}, A_{ni}) : S) \geq u_i((100, 1) : \emptyset)$ for all $i \in I$.

A feasible allocation $((T_{n1}, A_{n1}), (T_{n2}, A_{n2}), (T_{n3}, A_{n3}) : S)$, for $n \in \mathbb{N}$ and $S \in \mathcal{P}$, is *Pareto efficient* if there is no other feasible allocation $((T_{m1}, A_{m1}), (T_{m2}, A_{m2}), (T_{m3}, A_{m3}) : S')$ for $m \in \mathbb{N}$ such that

(i) for all $i \in I, u_i((T_{mi}, A_{mi}) : S') \geq u_i((T_{ni}, A_{ni}) : S)$, and

(ii) there exists $j \in I$ such that $u_j((T_{mj}, A_{mj}) : S') > u_j((T_{nj}, A_{nj}) : S)$.

Note that the Pareto efficiency does not fully capture a set of interesting and mutually advantage outcomes in this setting. We, therefore, define *coalitional efficiency* as follows:

An allocation $((T_{n1}, A_{n1}), (T_{n2}, A_{n2}), (T_{n3}, A_{n3}) : S)$, for $n \in \mathbb{N}$ and $S \in \mathcal{P}$, is *coalitional efficient* if there is no allocation of private commodities $((T_{m1}, A_{m1}), (T_{m2}, A_{m2}), (T_{m3}, A_{m3}))$ for $m \in \mathbb{N}$ such that

(i) for all $i \in I, u_i((T_{mi}, A_{mi}) : S) \geq u_i((T_{ni}, A_{ni}) : S)$, and

(ii) there exists $j \in I$ such that $u_j((T_{mj}, A_{mj}) : S) > u_j((T_{nj}, A_{nj}) : S)$, and

(iii) $((T_{m1}, A_{m1}), (T_{m2}, A_{m2}), (T_{m3}, A_{m3}))$ is feasible with respect to the given public project S, i.e., $\sum_{i \in I}(T_{mi}, A_{mi}) + c(S) = (300, 3)$.

The Pareto efficiency demands that an outcome be efficient relative to the entire set of feasible outcomes. The coalitional efficiency only requires that an outcome be efficient with respect to outcomes that are feasible given the coalition which created the public project. As such, there are outcomes that are coalitionally efficient which is not Pareto efficient and outcomes which are Pareto efficient which are not feasible given the coalition which created the public project.

Our theoretical analysis is focused on the market with Public Bonus, i.e., the market in which a public project can be established if any agent (in the APUB market) or a designated agent (in the HPUB or LPUB market) of three agents holds 3 assets at the end of a period under the publicly announced bonus. If the values of dividends and a bonus are given as the IA periods in the APUB market, i.e., $\alpha_1 = 20$, $\alpha_2 = 25$, $\alpha_3 = 30$, and $B = 60$, then we have

$$\mathcal{P} = \{\emptyset, \{1\}, \{2\}, \{3\}\},$$

$$c(S) = \begin{cases} (-180, 0) & \text{for } S \in \{\{1\}, \{2\}, \{3\}\}, \\ (0, 0) & \text{for } S = \emptyset, \end{cases}$$

$$E_i(S) = 0 \ \text{ for every } i \in I \text{ and for every } S \in \mathcal{P}, \text{ and}$$

$$V_i(S) = \begin{cases} -60 & \text{for every } i \in I \text{ and for } S \in \{\{1\}, \{2\}, \{3\}\}, \\ 0 & \text{for every } i \in I \text{ and for } S = \emptyset. \end{cases}$$

Note that in our experimental settings there is no physical use of resources, and every agent benefits from creating a public project. Therefore, the cost function, $c(\cdot)$, has a negative value, and the bonus represents a negative valuation or a subsidy.

We specify all possible allocations,

$$L_n = ((T_{n1}, A_{n1}), (T_{n2}, A_{n2}), (T_{n3}, A_{n3}) : S), \ \text{for } n \in \mathbb{N} \text{ and } S \in \mathcal{P},$$

as follows:

$$\begin{aligned}
L_1 &= ((100, 1), (100, 1), (100, 1) : \emptyset), \\
L_2 &= ((100 - p(\emptyset), 2), (100 + p(\emptyset), 0), (100, 1) : \emptyset), \\
L_3 &= ((100 - p(\emptyset), 2), (100, 1), (100 + p(\emptyset), 0) : \emptyset), \\
L_4 &= ((100 + p(\emptyset), 0), (100 - p(\emptyset), 2), (100, 1) : \emptyset), \\
L_5 &= ((100, 1), (100 - p(\emptyset), 2), (100 + p(\emptyset), 0) : \emptyset), \\
L_6 &= ((100 + p(\emptyset), 0), (100, 1), (100 - p(\emptyset), 2) : \emptyset), \\
L_7 &= ((100, 1), (100 + p(\emptyset), 0), (100 - p(\emptyset), 2) : \emptyset), \\
L_8 &= ((100 - 2p(\{1\}), 3), (100 + p(\{1\}), 0), (100 + p(\{1\}), 0) : \{1\}). \\
L_9 &= ((100 + p(\{2\}), 0), (100 - 2p(\{2\}), 3), (100 + p(\{2\}), 0) : \{2\}), \\
L_{10} &= ((100 + p(\{3\}), 0), (100 + p(\{3\}), 0), (100 - 2p(\{3\}), 3) : \{3\}).
\end{aligned}$$

where $p(\emptyset)$is the asset price with no public projects, and $p(S)$ is the asset price for the public project $S \in \{\{1\},\{2\},\{3\}\}$.

Agents' utility levels are calculated as follow:

$$\begin{aligned}
(u_1, u_2, u_3 : L_1) &= (120, 125, 130),\\
(u_1, u_2, u_3 : L_2) &= (140 - p(\emptyset), 100 + p(\emptyset), 130),\\
(u_1, u_2, u_3 : L_3) &= (140 - p(\emptyset), 125, 100 + p(\emptyset)),\\
(u_1, u_2, u_3 : L_4) &= (100 + p(\emptyset), 150 - p(\emptyset), 130),\\
(u_1, u_2, u_3 : L_5) &= (120, 150 - p(\emptyset), 100 + p(\emptyset)),\\
(u_1, u_2, u_3 : L_6) &= (100 + p(\emptyset), 125, 160 - p(\emptyset)),\\
(u_1, u_2, u_3 : L_7) &= (120, 100 + p(\emptyset), 160 - p(\emptyset)),\\
(u_1, u_2, u_3 : L_8) &= (220 - 2p(\{1\}), 160 + p(\{1\}), 160 + p(\{1\})),\\
(u_1, u_2, u_3 : L_9) &= (160 + p(\{2\}), 235 - 2p(\{2\}), 160 + p(\{2\})),\\
(u_1, u_2, u_3 : L_{10}) &= (160 + p(\{3\}), 160 + p(\{3\}), 250 - 2p(\{3\})).
\end{aligned}$$

First, in order to find the individually rational allocations, we check whether the following inequalities hold for every allocations,

$$(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_n), \text{ for } n = 2, \ldots, 10.$$

Then,

$$\begin{aligned}
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_2) &\Rightarrow p(\emptyset) \leq 20, p(\emptyset) \geq 25,\\
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_3) &\Rightarrow p(\emptyset) \leq 20, p(\emptyset) \geq 30,\\
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_4) &\Rightarrow 20 \leq p(\emptyset) \leq 25,\\
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_5) &\Rightarrow p(\emptyset) \leq 25, p(\emptyset) \geq 30,\\
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_6) &\Rightarrow 20 \leq p(\emptyset) \leq 30,\\
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_7) &\Rightarrow 25 \leq p(\emptyset) \leq 30,\\
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_8) &\Rightarrow 0 \leq p(\{1\}) \leq 50,\\
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_9) &\Rightarrow 0 \leq p(\{2\}) \leq 55,\\
(u_1, u_2, u_3 : L_1) \leq (u_1, u_2, u_3 : L_{10}) &\Rightarrow 0 \leq p(\{3\}) \leq 60.
\end{aligned}$$

So, allocations L_2, L_3, and L_5 are not individually rational, because there are not prices $p(\emptyset)$ satisfying given conditions. Furthermore, allocations L_4, L_6, and L_7 can be individually rational, but they are not efficient, because each agent increases his/her utility level by establishing a public project and receiving a bonus. Now we have two different sets of individually rational efficient allocations which depend on the two efficiency concepts, i.e, the Pareto efficiency and the coalitional efficiency. While the individually rational Pareto efficient (IRPE) allocation is given as

$L_{10},\ 0 \leq p(\{3\}) \leq 60,$

at which the agent with the highest dividend value holds 3 assets with the $0 \leq p(\{3\}) \leq 60$, the individually rational coalitional efficient (IRCE) allocations depend on a given public project, and they are identified as

$L_8, \quad 0 \leq p \leq 50,$
$L_9, \quad 0 \leq p \leq 55,$
$L_{10}, \quad 0 \leq p \leq 60,$

where the price system p needs not depend on the public project because the public project is given, and it does not take into account the (global) optimality of the public project. In L_8 agent 1 holds 3 assets, and creates a public project with the asset prices $0 \leq p \leq 50$. In L_9, agent 2 holds 3 assets, and creates a public project with $0 \leq p \leq 55$. In L_{10}, agent 3 also holds 3 assets, and creates a public project with $0 \leq p \leq 60$. These allocations are IRCE in the APUB market given the parameters as the period IA.[6]

In the APUB market these results in the IA periods are similarly extended to those in the III or IVA periods except the dividend values for agents. Table 3 and Table 4 show IRPE and IRCE allocations for each period in the APUB market.

Next, we analyze one assigned coordinator markets, the HPUB and LPUB markets. In these markets we restrict $\mathcal{P}$, the collection of all coalitions of economic agents that are able to execute a uniquely defined public project, with $\mathcal{P} = \{\emptyset, \{i\}\}$, where $\{i\} = S \in \mathcal{P},\ i \in I$, is the agent who can holds 3 assets with the highest dividend value in the HPUB market or the lowest dividend value in the LPUB market among 3 agents. Then we have Tables 5 and 6 showing the individually rational efficient (IRE) allocations in the HPUB and LPUB markets. Note that in one assigned coordinator markets, the IRPE allocations are the same as the IRCE allocations.

In the IA and IIA periods of HPUB markets or in IA and IIB periods of LPUB markets we have the same IRE allocations, and in the IVA and IVB periods of HPUB

[6]Since in APUB markets IRCE allocations are *ex post* identified, those allocations can be different from IRPE allocations. However, in one assigned coordinator markets a possible public project is given in advance, and there can be no difference between IRCE and IRPE allocations.

markets or LPUB markets the IRE allocations are the same even with a different experimental treatment of assigned coordinators. In particular, we have two different IRE allocations in the IB periods of LPUB markets, and the one, S_{20}, is an allocation with the establishment of public project, while the other, S_{30}, is an allocation without it.

Now we apply the same analysis for the market with Private Bonus, i.e., the market in which a public project can be established if any one (in the APRB market) or a designated one (in the HPUB or LPUB market) of 3 agents holds 3 assets at the end of a period under the privately informed bonus, we give the values of dividends and bonus for IA periods for the APRB market as follows:

$$\alpha_1 = 20, \alpha_2 = 25, \alpha_3 = 30, \quad B = 60.$$

$$\mathcal{P} = \{\emptyset, \{1\}, \{2\}, \{3\}\},$$

$$c(S) = \begin{cases} (-180, 0) & \text{for } S \in \{\{1\}, \{2\}, \{3\}\}, \\ (0, 0) & \text{for } S = \emptyset, \end{cases}$$

$$V_i(S) = 0 \text{ for every } i \in I \text{ and for every } S \in \mathcal{P}, \text{ and}$$

$$E_i(S) = \begin{cases} 60 & \text{for every } i \in I \text{ and for } S \in \{\{1\}, \{2\}, \{3\}\}, \\ 0 & \text{for every } i \in I \text{ and for } S = \emptyset. \end{cases}$$

This parameterization represents the market in which any group of agents can execute the public project under privately informed payoffs (valuations). But, we get the same IRPE or IRCE allocations (and also the following asset price patterns issue) for each period as the former parameterization in the APUB market (See the Tables 3 and 4). The economic implication of this parameterization, however, is different from that of the former. This parameterization in the APRB market represents the effect of externalities without valuations, while the former one in the APUB market does the effect of valuations (subsidy or tax) without externalities from the public project. Moreover, we have also the same results in HPRB or LPRB markets as in HPUB or LPUB markets (See the Tables 5 and 6).

3.4 Experimental results

Based on the experimental results, we focus on three basic issues: (1) whether individually rational efficient outcomes are achieved, (2) whether the coalition forming the public project benefits more or less than the rest of the population, and (3) how free riding or cheap riding phenomena occur.

We summarize the frequency of IRE allocations at Tables 7 (IRPE) and 8 (IRCE). Among 504 periods we have the cases of establishing a public project with 351 periods (69.6%), the cases of IRPE allocations with 291 periods (57.7%), and the cases of IRCE allocations with 324 periods (64.3%).[7]

Several hypotheses for the frequency of IRE allocations (denoted as f) are constructed with respect to rules, information conditions, and parameterizations.

Hypothesis 1. $f_{PUB} = f_{PRB}$.

The first hypothesis means that there is no difference in the frequency of IRE allocations with respect to information conditions, i.e, the emergence of IRE allocations is the same in the PUB markets as in the the PRB markets. Following the theoretical analyses, the IRE allocations are identical even with a public bonus or a private bonus. However, this hypothesis is rejected significantly as shown at Table 9. This implies that information conditions significantly affect the achievement of IRE allocations with establishing a public project.

Next, we test whether the rules of creating public projects affect the frequency of IRE allocations, i.e., whether the emergence of IRE allocations is the same in any coordinator markets as in one assigned coordinator markets.

Hypothesis 2. $f_{ANY} = f_{ASSIGNED}$.

From the theoretical analyses, we have different IRE allocations between any coordinator markets and one assigned coordinator markets with the lowest dividend values (LPUB and LPRB markets). However, in any coordinator markets and in one assigned coordinator markets with the highest dividend values (in HPUB and HPRB markets) the IRPE allocations are identical except in an IIB parameterization case.

[7]Appendix 1 contains the frequency of establishing a public project.

Table 10 shows the results of the test of Hypothesis 2. With respect to the frequency of IRPE allocations, any coordinator markets are significantly different from the one assigned coordinator markets. But, Hypothesis 2 is not rejected in case of the frequency of IRCE allocations. This represents that in one assigned coordinator markets the IRCE allocations are identical to the IRPE allocations, and even in any coordinator market the IRCE allocations are *ex post* identified, which is the same as assigning in advance a possible public project in one assigned coordinator markets.

In cases of 7 different parameterizations of dividends and a bonus, i.e., IA to IVB periods, we separately analyze the effects of dividends and a bonus if the experimental treatments such as rules and information conditions are ignored.

Hypothesis 3. $f_{IA} = f_{IB} = f_{IIA} = f_{IIB}$.

Hypothesis 4. $f_{IA} = f_{III} = f_{IVA}$.

With Hypothesis 3 we test whether the difference of bonuses affects the frequency of IRE allocations, and with Hypothesis 4 whether the the difference of dividends is a significant factor to achieve IRE allocations in economies with public projects. Table 11 shows the test results for Hypotheses 3 and 4. While the effect of bonuses is significant in the frequency of IRE allocations, that of dividends is not. We easily expect the similar results for the case of IVA and IVB, i.e, different bonuses with even same dividends.

From the test results of Hypotheses 1 to 4, the frequency of IRE allocations are affected by rules, information conditions, and parameterizations. To analyze in detail the effect of various institutional and environmental factors on performance, the relationship between both of the efficiency concepts and the various experimental controls is estimated using the probit model (Maddala (1983)). This method is appropriate because the achievement of IRE allocation is represented by a dummy dependent variable. We assume that there is an achievement of IRE allocation $y_t^\star$, $t = 1, \ldots, 504$, defined by the regression relationship as follows: for $t = 1, \ldots, 504$,

$$y_t^\star = \beta \cdot (1, D_t, B_t, DB_t, I_t, R_{1t}, R_{2t}, B_{1t}, B_{2t}, O_t) + u_t,$$

where β is the coefficient vector, i.e., $\beta = (\beta_0, \beta_1, \ldots, \beta_9)$, D_t is the difference of dividend values defined as for $i, j, k \in I = \{1, 2, 3\}$ and $i \neq j \neq k$,

$$D_t = \begin{cases} 5 & \text{if } (\alpha_i, \alpha_j, \alpha_k) = (20, 25, 30), \\ 2 & \text{if } (\alpha_i, \alpha_j, \alpha_k) = (26, 28, 30), \\ 0 & \text{if } (\alpha_i, \alpha_j, \alpha_k) = (30, 30, 30). \end{cases}$$

B_t is the minimum of bonus, i.e., $B_t \in \{10, 40, 60\}$, and $DB_t = D_t \times B_t$. I_t is a dummy variable for information conditions, i.e.,

$$I_t = \begin{cases} 1 & \text{if private bonus}, \\ 0 & \text{if public bonus}. \end{cases}$$

R_{1t} and R_{2t} are dummy variables for the rules of creating public projects, i.e.,

$$R_{1t} = \begin{cases} 1 & \text{if one assigned coordinator with the highest dividend}, \\ 0 & \text{otherwise}. \end{cases}$$

$$R_{2t} = \begin{cases} 1 & \text{if one assigned coordinator with the lowest dividend}, \\ 0 & \text{otherwise}. \end{cases}$$

B_{1t} and B_{2t} are also dummy variables representing the dependence of bonuses upon who is a coordinator, i.e.,

$$B_{1t} = \begin{cases} 1 & \text{if IIA periods in APUB and APRB markets}, \\ 0 & \text{otherwise}. \end{cases}$$

$$B_{2t} = \begin{cases} 1 & \text{if IIB periods in APUB and APRB markets}, \\ 0 & \text{otherwise}. \end{cases}$$

O_t is the log transformation of the randomized order of periods, $\{1, \ldots, 21\}$. u_t represents the errors.

However, $y_t^\star$, $t = 1, \ldots, 504$, is not observed in practice, and what we observe is a dummy variable y_t defined by

$$y_t = \begin{cases} 1 & \text{if an IRE allocation is achieved, i.e., } y_t^\star = 1, \\ 0 & \text{otherwise}. \end{cases}$$

In a probit model, assuming that the errors u_t follow a normal distribution, the probability of achieving an IRE allocations, P_t, is given by

$$P_t = \text{Prob}(y_t = 1) = F(X_t'\beta),$$

where F is the cumulative distribution function of the error u, X is the vector of observed values of the independent variables defined as above.

Table 12 presents the results of the probit models for IRPE allocations and IRCE allocation. With the pooled data we have some significant coefficients such as the bonus structure, B, information conditions, I, the assignment of highest dividend holders as a coordinator, R_1, and the randomized order of experimental periods, O. With the data from the markets with private bonus, we have similar results as those with the pooled data. However, the randomized order of experimental periods is not significant with the data from the market with public bonus. In particular, in the data set of any coordinator markets the dividends are significant, while with the data from the lowest assigned coordinator markets the coefficient of dividends is negative even though it is not significant. Concerning the effect of dividends to the IRE allocations, the gains from trades represented by the dividends are important factors as well as the bonus, i.e., benefits from the public project.

As mentioned before, in order to maintain the independence of experimental settings, the experimental periods are randomized. However, the randomized order of experimental periods is one of significant factors of the IRE allocations.[8] Table 13 modifies the probit model to the cases of two different sets with experimental periods. In the first half of periods the coefficient of the experimental periods is significant, while it is not in the second half of periods. This implies that there exists learning at least in the first half of periods.

We similarly extend our analysis to the asset prices.[9] Table 14 shows the asset prices at IRE allocations.

We also estimate linear regression models for the asset prices at the IRE allocations with the same independent variables, and the results of them are presented at Table 15. These regression results are similar to those of probit models. The coefficients of the bonus structure, B, information conditions, I, the assignment of highest dividend holders as a coordinator, R_1, are significant as those in probit models. In particular, the coefficient of dividends is negatively significant, which means that the

[8]Appendix 2 contains the frequency of IRE allocations by experimental periods.

[9]In order to establish a public project, we have at least two asset trades. A coordinator should also purchase two assets. Therefore, we mainly focus on the 1st and 2nd asset purchasing prices of the coordinator at IRPE allocations.

dividend is an important factor for determining the asset prices. The randomized order of experimental periods also affects to the asset prices negative significantly. This implies that even though the random orders are introduced in the experiments, every agent learns the experimental setup, and speculative transactions decrease as the experimental periods go on.

Next, we are interested in the profit distribution among agents. Our experimental designs of public projects are organized through asset exchange markets, and we allow two profit sources: gains from trades and benefits from establishing public projects. As mentioned before, the asset price systems depend on which public project is established, or who is a coordinator of public project. This dependence restricts the asset price at IRPE allocations, rather than at IRCE allocations. If we assume that every agent shares the profit equally, then asset prices are determined by the asset reservation values, i.e., dividends of the coordinators'. The following hypothesis can be constructed:

Hypothesis 5. At IRE allocations every agent has the same profit.

According to Table 7, the asset prices are higher than the coordinators' dividend values in any markets and any periods. This means that the coordinators pay more than their reservation asset values, and they sacrifice their utilities more than non-coordinators. Therefore, non-coordinators have some advantage from trades as well as public projects. According to Table 16 Hypothesis 5 cannot be supported. The coordinators have some disadvantage from public project. However, in the HPRB markets public projects are more advantageous to the coordinators than any other markets. This implies that in this market the coordinators have private information about their dividend values as well as the bonus, and they might control asset trading prices.

The above unequal profit distribution results can be interpreted as *cheap riding* in our experimental settings. According to Isaac et al. (1989), cheap riding problems is related to individual incentives to attempt to obtain an equilibrium outcome with *unequal* distribution of contributions. In other words, there are multiple equilibria supporting the provision point in voluntary contribution mechanism. In our experimental settings it is also observed that when a public project is established, an asset trade occurs outside of bargaining zone which is between seller's and buyer's reservation dividend values. Table 17 presents the results.

Finally, we analyze the cases that a public project is not established. Among them we have *free riding* which means that there were asset prices posted such that (i) one subject rationally (based only on dividend values) accept a posted bid or offer, and (ii) this acceptance would have resulted in creation of the public project, but the subject didn't do it. Other cases are classified by "ran out of time", no individually rational trades possible, voted to close market, and no public project with individually rational trades. In particular, time was being used strategically — "ran out of time", however, means used it up in unfortunately strategic way such that (i) based on the posted prices, there might have been a bid or an offer which was rational for one agent to accept (based on dividend values) which would not have resulted in creating a public project, i.e., coordination failure, (ii) the prices which were posted for the first trade would not allow either agents to make a rational trade, even though they tried to make the first trade, and (iii) no prices were posted as the period ended. No individually rational trade possible means that one trade has already taken place, and posted prices do not allow an individually rational trade to take place. No public project with individually rational trades occurs only in the LPUB and LPRB markets such that two individually rational trades took place but the holder of the 3 asset wasn't the designated trader so that no project was generated. There are 25 incidents of free riding among 153 cases that no project is created (i.e., 16.3%), or among 504 total periods (5.0%). Other cases are shown at Table 18.

3.5 Conclusion

This research investigates how markets can be used to privately provide public projects. 24 markets, each with 21 periods, are organized into 2 experimental designs. In the first set of markets, any group of agents can execute the public project under publicly announced payoffs from the public projects.The second set of markets differs from the first one only by having agents privately informed payoffs from the public project. In the third two sets of markets, only one group of agents (who hold an asset with the highest or the lowest dividend value) have the ability to execute the public project under publicly announced payoffs. The final two sets of markets are like the third two sets except that agents have private information about their own payoffs from the public project. Among 504 periods we have the cases of establishing a public project with 351 periods (69.9allocations with 291 periods (57.7%). The achievement of IRPE allocations mainly depends on the payoffs from public projects, information conditions, and the randomized order of experimental periods as we expected. In particular, among IRPE allocations "cheap riding" is observed with 68.4% because of unequal gains from asset trades even with the same benefits from the public project. Furthermore, there is 5.0% of free riding among 504 total periods.

This mechanism is at work in formal settings. Imagine 3 houses arranged on a cul-de-sac in a suburban neighborhood. The residents of each house have two children ages 3 and 5. The land which surrounds the residents' property is owned by a neighborhood association and can be used by the residents for approved, non-exclusive purpose. Each of three residents owns one unit of building materials (sand, wood, etc.) for a playground that requires 3 units to build, and each may have different marginal valuations for the same unit. Suppose that neighbor A wishes to build a playground. If she can buy a unit from neighbor B and neighbor C then she will build a public playground on the community land. The benefit to each of the three neighbors can be calculated. All three neighbors benefit equally from having a convenient playground for their children. Neighbor B and neighbor C each earn a total benefit equal to the playground's benefit plus the revenue from the sale of their unit minus the marginal value of their unit. Each of these neighbors might choose to sell their unit for more their marginal value in order to earn more money, or for less than their marginal value if they want to aid neighbor A in building the playground. Since neighbor A builds the playground on the land behind her house, she benefits by having her small children within view of her home office. (Because of the layout of the cul-de-sac, it is impossible for neighbors B and C to see the playground.) The value of being able to see the children from the house equals the cost of three units if buys them at a hardware store. This means that the total benefit to neighbor A if the playground is built equals the playground's benefit plus the benefit being able to see the children from the house minus the marginal value of neighbor A's unit and the cost paid to neighbors B and C.

We envision many avenues for future research. In this paper we restrict ourselves to the case where each public project has the same value to every agent in the population. This restriction can be relaxed so that agents have differential valuations for the public project. We can also examine cases where the coalition required to execute the public project is greater than 1 and where not all agents in the population must sell their asset for the public project to be executed.

Tables and Appendices

Table 1. Experimental Designs

	Any coordinator	One assigned coordinator
Public Bonus	APUB–1,2,3,4	HPUB–1,2,3,4, LPUB–1,2,3,4
Private Bonus	APRB–1,2,3,4	HPRB–1,2,3,4, LPRB–1,2,3,4

Table 2. The Dividends and Bonus of 21 periods[10]

Parameter sets		Dividends α_1	α_2	α_3	Bonus (B)
IA	1	20	25	30	If one of 3 agents holds 3 assets, then $B = 60$.
	2	30	20	25	
	3	25	30	20	
IB	4	30	25	20	If one of 3 agents holds 3 assets, then $B = 10$.
	5	20	30	25	
	6	25	20	30	
IIA	7	20	25	30	The bonus B depends on who is a coordinator. If a coordinator with $\alpha = 30, (25, 20)$, respectively, then $B = 60, (50, 40)$.
	8	30	20	25	
	9	25	30	20	
IIB	10	20	25	30	The bonus B depends on who is a coordinator. If a coordinator with $\alpha = 30, (25, 20)$, respectively, then $B = 40, (50, 60)$.
	11	20	25	30	
	12	25	30	20	
III	13	26	28	30	If one of 3 agents holds 3 assets, then $B = 60$.
	14	30	26	28	
	15	28	30	26	
IVA	16	30	30*	30	If one of 3 agents holds 3 assets, then $B = 60$.
	17	30*	30	30	
	18	30	30	30*	
IVB	19	30*	30	30	If one of 3 agents holds 3 assets, then $B = 10$.
	20	30	30*	30	
	21	30	30	30*	

[10]In IIA periods the HPUB or HPRB market is the same as the APUB or APRB market, and in IIB periods the LPUB or LPRB market is the same as the APUB or APRB market. Furthermore, for HPUB, LPUB, HPRB, and LPRB periods, the coordinator is marked with an asterisk in IVA and IVB periods.

Table 3. The IRPE allocations in the APUB market[11]

Periods		IRPE allocations
IA	1, 2, 3	S_{30} with $0 \le p(\{S_{30}\}) \le 60$
IB	4, 5, 6	S_{30} with $15 \le p(\{S_{30}\}) \le 35$
IIA	7, 8, 9	S_{30} with $0 \le p(\{S_{30}\}) \le 60$
IIB	10,11,12	S_{20} with $0 \le p(\{S_{20}\}) \le 50$
III	13,14,15	S_{30} with $0 \le p(\{S_{30}\}) \le 60$
IVA	16,17,18	S_{30} with $0 \le p(\{S_{30}\}) \le 60$
IVB	19,20,21	S_{30} with $15 \le p(\{S_{30}\}) \le 35$

Table 4. The IRCE allocations in the APUB market

Periods		IRCE allocations
IA	1, 2, 3	S_{20} with $0 \le p \le 50$, S_{25} with $0 \le p \le 55$, S_{30} with $0 \le p \le 60$
IB	4, 5, 6	S_{20} with $20 \le p \le 25$, S_{25} with $20 \le p \le 30$, S_{30} with $15 \le p \le 35$
IIA	7, 8, 9	S_{20} with $0 \le p \le 40$, S_{25} with $0 \le p \le 50$, S_{30} with $0 \le p \le 60$
IIB	10,11,12	S_{20} with $0 \le p \le 50$, S_{25} with $0 \le p \le 50$, S_{30} with $0 \le p \le 50$,
III	13,14,15	S_{26} with $0 \le p \le 56$, S_{28} with $0 \le p \le 58$, S_{30} with $0 \le p \le 60$
IVA	16,17,18	S_{30} with $0 \le p \le 60$
IVB	19,20,21	S_{30} with $15 \le p \le 35$

[11] At the Tables 3, 4, 5, and 6, S_α means that a coalition or an agent with the dividend value α holds 3 assets.

Table 5. The IRE allocations in the HPUB market

Periods		IRE allocations
IA	1, 2, 3	S_{30} with $0 \leq p(\{S_{30}\}) \leq 60$
IB	4, 5, 6	S_{30} with $15 \leq p(\{S_{30}\}) \leq 35$
IIA	7, 8, 9	S_{30} with $0 \leq p(\{S_{30}\}) \leq 60$
IIB	10,11,12	S_{30} with $0 \leq p(\{S_{30}\}) \leq 50$
III	13,14,15	S_{30} with $0 \leq p(\{S_{30}\}) \leq 60$
IVA	16,17,18	S_{30} with $0 \leq p(\{S_{30}\}) \leq 60$
IVB	19,20,21	S_{30} with $15 \leq p(\{S_{30}\}) \leq 35$

Table 6. The IRE allocations in the LPUB market

Periods		IRE allocations
IA	1,2,3	S_{20} with $0 \leq p(\{S_{20}\}) \leq 50$
IB	4,5,6	S_{20} with $20 \leq p(\{S_{20}\}) \leq 25$ S_{30} with $25 \leq p(\{S_{30}\}) \leq 30$
IIA	7,8,9	S_{20} with $0 \leq p(\{S_{20}\}) \leq 40$
IIB	10,11,12	S_{20} with $0 \leq p(\{S_{20}\}) \leq 50$
III	13,14,15	S_{26} with $0 \leq p(\{S_{20}\}) \leq 60$
IVA	16,17,18	S_{30} with $0 \leq p(\{S_{30}\}) \leq 60$
IVB	19,20,21	S_{30} with $15 \leq p(\{S_{30}\}) \leq 35$

Table 7. The Frequency of IRPE allocations

	Parameterizations							
	IA	IB	IIA	IIB	III	IVA	IVB	Total (%)
APUB–1	3	1	2	0	2	2	2	12/21 (57.1)
APUB–2	1	2	1	2	0	2	0	8/21 (38.1)
APUB–3	2	2	2	1	2	3	1	13/21 (61.9)
APUB–4	1	0	3	1	2	3	1	11/21 (52.4)
APUB	7	5	8	4	6	10	4	44/84 (52.4)
(%)	(58.3)	(41.7)	(66.7)	(33.3)	(50.0)	(83.3)	(33.3)	
HPUB–1	6	1	-	2	3	2	0	14/21 (66.7)
HPUB–2	6	0	-	2	3	3	1	15/21 (71.4)
HPUB–3	6	3	-	2	3	3	3	20/21 (95.2)
HPUB–4	6	3	-	2	3	3	1	18/21 (85.7)
HPUB	24	7	-	8	12	11	5	67/84 (79.0)
(%)	(100)	(58.3)	(-)	(66.7)	(100)	(91.7)	(41.7)	
LPUB–1	6	3	1	-	2	3	3	18/21 (85.7)
LPUB–2	4	1	3	-	3	3	2	16/21 (76.2)
LPUB–3	4	0	0	-	2	3	0	9/21 (42.9)
LPUB–4	3	0	2	-	2	3	2	12/21 (57.1)
LPUB	17	4	6	-	9	12	7	55/84 (65.5)
(%)	(70.8)	(33.3)	(50.0)	(-)	(75.0)	(100)	(58.3)	
PUB	48	16	14	12	27	33	16	166/252 (65.9)
(%)	(80.0)	(44.4)	(58.3)	(50.0)	(75.0)	(91.7)	(44.4)	
APRB–1	1	3	3	1	1	3	0	12/21 (57.1)
APRB–2	1	1	1	1	0	2	0	6/21 (28.6)
APRB–3	1	0	1	0	0	0	0	2/21 (9.5)
APRB–4	2	3	1	1	1	3	1	12/21 (57.1)
APRB	5	7	6	3	2	8	1	32/84 (38.1)
(%)	(41.7)	(58.3)	(50.0)	(25.0)	(16.7)	(66.7)	(8.3)	
HPRB–1	6	1	-	2	2	2	1	14/21 (66.7)
HPRB–2	4	0	-	0	2	3	0	9/21 (42.9)
HPRB–3	5	2	-	2	3	1	0	13/21 (61.9)
HPRB–4	5	1	-	3	3	2	2	16/21 (76.2)
HPRB	20	4	-	7	10	8	3	52/84 (61.9)
(%)	(83.3)	(33.3)	(-)	(58.3)	(83.3)	(66.7)	(25.0)	

Table 7. The Frequency of IRPE allocations (continued)

	Parameterizations							
	IA	IB	IIA	IIB	III	IVA	IVB	Total (%)
LPRB-1	2	1	1	-	2	2	1	9/21 (42.9)
LPRB-2	5	0	2	-	3	3	2	15/21 (71.4)
LPRB-3	3	0	1	-	2	1	1	8/21 (38.1)
LPRB-4	5	0	1	-	2	1	0	9/21 (42.9)
LPRB	15	1	5	-	9	7	4	41/84 (48.8)
(%)	(62.5)	(8.3)	(41.7)	(-)	(75.0)	(58.3)	(33.3)	
PRB	40	12	11	10	21	23	8	125/252 (49.6)
(%)	(66.7)	(33.3)	(45.8)	(41.7)	(58.3)	(63.9)	(22.2)	
TOTAL	88	28	25	22	48	56	24	291/504 (57.7)
(%)	(73.3)	(38.9)	(52.1)	(45.8)	(66.7)	(77.8)	(33.3)	

Table 8. The frequency of IRCE allocations[12]

	Parameterizations							
	IA	IB	IIA	IIB	III	IVA	IVB	Total (%)
APUB-1	3	1	3	3	2	2	2	16/21 (76.2)
APUB-2	2	3	2	3	3	2	0	15/21 (71.4)
APUB-3	3	2	2	2	3	3	1	16/21 (76.2)
APUB-4	2	0	3	2	2	3	1	13/21 (61.9)
APUB	10	6	10	10	10	10	4	60/84 (71.4)
(%)	(83.3)	(50.0)	(83.3)	(83.3)	(83.3)	(83.3)	(33.3)	
PUB	51	17	16	18	31	33	16	182/252 (72.2)
(%)	(85.0)	(47.2)	(66.7)	(75.0)	(86.1)	(91.7)	(44.4)	
APRB-1	2	3	3	2	2	3	0	15/21 (71.4)
APRB-2	3	1	2	3	2	2	0	13/21 (61.9)
APRB-3	3	0	1	1	0	0	0	5/21 (23.8)
APRB-4	2	3	2	2	3	3	1	16/21 (76.2)
APRB	10	7	8	8	7	8	1	49/84 (58.3)
(%)	(83.3)	(58.3)	(66.7)	(66.7)	(58.3)	(66.7)	(8.3)	
PRB	45	12	13	15	26	23	8	142/252 (56.3)
(%)	(75.0)	(33.3)	(54.2)	(62.5)	(72.2)	(63.9)	(22.2)	
TOTAL	96	29	29	33	57	56	24	324/504 (64.3)
(%)	(80.0)	(40.3)	(60.4)	(68.8)	(79.2)	(77.7)	(33.3)	

[12] In one assigned coordinator markets the IRCE allocations are the same as the IRPE allocations.

Table 9. Test of Hypothesis 1

(IRPE allocations)

	Number of Cases	Mean	Standard Deviation	t-value ($d.f.$)
f_{PUB}	252	0.6587	0.475	3.74
f_{PRB}	252	0.4960	0.501	(502)

(IRCE allocations)

	Number of Cases	Mean	Standard Deviation	t-value ($d.f.$)
f_{PUB}	252	0.7222	0.449	3.76
f_{PRB}	252	0.5635	0.497	(502)

Table 10. Test of Hypothesis 2

(IRPE allocations)

	Number of Cases	Mean	Standard Deviation	t-value ($d.f.$)
f_{ANY}	168	0.4524	0.499	-4.91
f_H	168	0.7083	0.456	(334)

	Number of Cases	Mean	Standard Deviation	t-value ($d.f.$)
f_{ANY}	168	0.4524	0.499	-2.19
f_L	168	0.5714	0.496	(334)

Table 10. Test of Hypothesis 2 (continued)

(IRCE allocations)

	Number of Cases	Mean	Standard Deviation	t-value ($d.f.$)
f_{ANY}	168	0.6488	0.479	1.17
f_H	168	0.7083	0.456	(334)

	Number of Cases	Mean	Standard Deviation	t-value ($d.f.$)
f_{ANY}	168	0.6488	0.479	1.45
f_L	168	0.5714	0.496	(334)

Table 11. Test for Hypotheses 3 and 4[13]

(Hypothesis 3: Analysis of Variance)

Source	d.f.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	3	6.2729	2.0910	9.2105	0.0000
Within Groups	284	64.4739	0.2270		
Total	287	70.7465			

(Hypothesis 4: Analysis of Variance)

Source	d.f.	Sum of Squares	Mean Squares	F Ratio	F Prob.
Between Groups	2	0.4525	0.2293	1.1376	0.3222
Within Groups	261	51.9111	0.1989		
Total	263	52.3636			

[13]We report the results for IRPE allocations because the results for IRCE allocations are almost identical.

Table 12. The Results of Probit Models

(IRPE allocations)

Variables	Pooled	PUB	PRB	ANY	HIGH	LOW
D	0.052	0.006	0.123	0.243	0.053	-0.159
	(0.99)	(0.09)	(1.56)	(2.54)	(0.57)	(1.71)
B	0.245	0.274	0.246	0.257	0.300	0.194
	(5.68)	(4.28)	(3.99)	(3.35)	(3.88)	(2.66)
DB	-0.016	-0.019	-0.021	-0.061	0.196	0.013
	(1.31)	(1.08)	(1.20)	(2.78)	(0.09)	(0.62)
I	-0.491			-0.027	-0.697	-0.508
	(4.05)			(1.77)	(2.93)	(2.43)
R_1	0.709	0.870	0.588			
	(4.31)	(3.57)	(2.58)			
R_2	0.274	0.361	0.201			
	(1.71)	(1.57)	(0.89)			
B_1	0.427	0.560	0.261	0.455		
	(1.44)	(1.31)	(0.63)	(1.41)		
B_2	-0.470	-0.363	-0.521	-0.529		
	(1.53)	(0.09)	(1.19)	(1.61)		
O	0.215	0.769	0.362	0.361	0.140	0.127
	(2.81)	(0.70)	(3.26)	(2.66)	(0.96)	(0.97)
Constant	-1.295	-1.006	-2.194	-1.828	-0.741	-0.336
χ^2	105.02	51.18	47.55	25.20	48.43	32.84
(d.f.)	(9)	(8)	(8)	(7)	(5)	(5)
Number of Observations	504	252	252	168	168	168

Table 12. The Results of Probit Models (continued)

(IRCE allocations)

Variables	Pooled	PUB	PRB	ANY	HIGH	LOW
D	0.462	0.014	0.108	0.240	0.053	-0.159
	(0.88)	(0.19)	(1.36)	(2.48)	(0.57)	(1.71)
B	0.265	0.300	0.269	0.320	0.300	0.194
	(6.13)	(4.57)	(4.36)	(4.04)	(3.88)	(2.66)
DB	-0.820	-0.016	-0.010	-0.032	0.196	0.013
	(0.67)	(0.90)	(0.58)	(1.37)	(0.09)	(0.62)
I	-0.513			-0.037	-0.697	-0.508
	(4.12)			(2.33)	(2.93)	(2.43)
R_1	0.247	0.453	0.074			
	(1.46)	(1.80)	(0.32)			
R_2	-0.208	-0.079	-0.338			
	(1.26)	(0.33)	(1.45)			
B_1	0.406	0.608	0.174	0.231		
	(1.27)	(1.26)	(0.40)	(0.65)		
B_2	0.290	0.584	0.400	0.010		
	(0.90)	(1.21)	(0.09)	(0.27)		
O	0.226	0.035	0.418	0.397	0.140	0.127
	(2.92)	(0.31)	(3.69)	(2.89)	(0.96)	(0.97)
Constant	-1.000	-0.643	-1.989	-1.945	-0.741	-0.336
χ^2	109.92	48.38	57.53	25.20	48.43	32.84
(d.f.)	(9)	(8)	(8)	(7)	(5)	(5)
Number of Observations	504	252	252	168	168	168

Table 13. The Results of Probit Models for Experimental Periods

Variables	IRPE allocations			IRCE allocations		
	Pooled	11–21	1–10	Pooled	11–21	1–10
D	0.052	0.095	0.031	0.046	0.071	0.025
	(0.99)	(1.28)	(0.40)	(0.88)	(0.96)	(0.31)
B	0.245	0.277	0.220	0.265	0.272	0.257
	(5.68)	(4.64)	(3.34)	(6.13)	(4.54)	(3.85)
DB	-0.016	-0.028	-0.008	-0.008	-0.013	-0.003
	(1.31)	(1.64)	(0.44)	(0.67)	(0.77)	(0.18)
I	-0.491	-0.242	-0.761	-0.513	-1.978	-0.840
	(4.05)	(1.44)	(4.23)	(4.12)	(1.14)	(4.54)
R_1	0.709	0.572	0.873	0.247	0.090	0.414
	(4.31)	(2.52)	(3.59)	(1.46)	(0.38)	(1.68)
R_2	0.274	0.148	0.399	-0.208	-0.331	-0.104
	(1.71)	(0.68)	(1.67)	(1.26)	(1.44)	(0.42)
B_1	0.427	-0.002	0.772	0.406	0.271	0.480
	(1.44)	(0.01)	(1.85)	(1.27)	(0.54)	(1.11)
B_2	-0.470	-0.625	-0.271	0.290	0.509	0.082
	(1.53)	(1.58)	(0.57)	(0.90)	(1.09)	(0.17)
O	0.215	0.026	0.360	0.226	-0.116	0.319
	(2.81)	(0.21)	(2.69)	(2.92)	(0.89)	(2.36)
Constant	-1.295	-0.917	-1.383	-1.000	-0.322	-0.990
χ^2	105.02	48.26	63.89	109.92	49.72	66.69
(d.f.)	(9)	(9)	(9)	(9)	(9)	(9)
Number of Observations	504	264	240	504	264	240

Table 14. Asset Prices at IRE allocations

(IRPE allocations)

		Parameterization						
		IA	IB	IIA	IIB	III	IVA	IVB
APUB	1st Trade	32.7	27.0	39.0	32.3	31.8	36.0	31.8
	2nd Trade	33.9	28.0	38.4	32.0	34.2	34.1	33.5
	Average	33.3	27.5	38.7	32.1	33.0	35.1	32.6
HPUB	1st Trade	35.4	25.9	-	33.5	35.5	34.9	28.6
	2nd Trade	35.3	27.9	-	33.9	38.2	35.5	28.8
	Average	35.4	26.9	-	33.7	36.8	35.2	28.7
LPUB	1st Trade	33.0	28.5	28.2	-	36.3	32.5	30.4
	2nd Trade	32.1	27.5	28.5	-	30.3	32.3	30.4
	Average	32.6	28.0	28.3	-	33.3	32.4	30.4
PUB	1st Trade	34.2	26.9	34.4	33.1	35.0	34.4	30.2
	2nd Trade	34.0	27.8	34.1	33.3	34.7	33.9	30.7
	Average	34.1	27.3	34.3	33.2	34.8	34.1	30.4
APRB	1st Trade	32.6	27.3	38.2	33.7	32.0	36.3	32.0
	2nd Trade	30.2	27.3	38.3	34.0	31.5	35.6	32.0
	Average	31.4	27.3	38.3	33.8	31.8	35.9	32.0
HPRB	1st Trade	35.7	29.5	-	29.6	40.4	35.9	28.3
	2nd Trade	35.0	28.5	-	31.1	42.1	37.8	30.3
	Average	35.3	29.0	-	30.4	41.3	36.8	29.3
LPRB	1st Trade	37.7	30.0	28.4	-	34.3	37.4	28.8
	2nd Trade	38.3	30.0	34.4	-	36.1	37.6	32.8
	Average	37.9	30.0	31.4	-	35.2	37.5	30.8
PRB	1st Trade	36.0	28.3	33.7	30.8	37.0	36.5	29.0
	2nd Trade	35.6	27.9	36.5	32.0	38.5	37.0	31.8
	Average	35.8	28.1	35.1	31.4	37.8	36.7	30.4
TOTAL	1st Trade	35.0	27.5	34.1	32.0	35.9	35.2	29.8
	2nd Trade	34.7	27.9	35.2	32.7	36.4	35.2	31.0
	Average	34.9	27.7	34.6	32.4	36.1	35.2	30.4

Table 14. Asset Prices at IRE allocations (continued)

(IRCE allocations)[14]

		Parameterization						
		IA	IB	IIA	IIB	III	IVA	IVB
APUB	1st Trade	34.8	26.7	39.7	33.7	33.6	36.0	31.8
	2nd Trade	36.0	28.3	37.0	32.6	36.7	34.1	33.5
	Average	35.4	27.5	38.4	33.2	35.2	35.1	32.6
PUB	1st Trade	34.5	26.8	35.4	33.6	35.1	34.4	30.2
	2nd Trade	34.4	27.9	33.8	33.2	35.4	33.9	30.7
	Average	34.4	27.4	34.6	33.4	35.3	34.1	30.4
APRB	1st Trade	35.0	27.3	36.9	31.8	36.7	36.3	32.0
	2nd Trade	35.9	27.3	37.3	32.1	36.9	35.6	32.0
	Average	35.5	27.3	37.1	31.9	36.8	35.9	32.0
PRB	1st Trade	36.2	28.3	33.6	30.7	37.3	36.5	29.0
	2nd Trade	36.3	27.9	36.2	31.7	38.6	37.0	31.8
	Average	36.2	28.1	34.9	31.2	38.0	36.7	30.4
TOTAL	1st Trade	35.3	27.4	34.6	32.3	36.1	35.2	29.8
	2nd Trade	35.3	27.9	34.9	32.5	36.9	35.2	31.0
	Average	35.3	27.7	34.7	32.4	36.5	35.2	30.4

[14]In one assigned coordinator markets the IRCE allocations are the same as the IRPE allocations.

Table 15. The Results of Linear Regression Models

(IRPE allocations)

Variables	Pooled	PUB	PRB	ANY	HIGH	LOW
Constant	35.467	34.848	38.952	39.547	35.982	32.760
	(15.73)	(14.34)	(8.76)	(9.72)	(9.17)	(12.00)
D	-0.850	-0.833	-0.714	-1.311	-0.462	-0.720
	(1.97)	(1.76)	(0.87)	(1.81)	(0.56)	(1.09)
B	0.935	0.761	1.151	0.363	1.470	0.726
	(3.09)	(2.28)	(1.92)	(0.65)	(2.39)	(1.83)
DB	0.105	0.124	0.048	0.133	0.042	1.084
	(1.23)	(1.31)	(0.30)	(0.91)	(0.26)	(0.84)
I	1.045			-0.109	1.634	4.045
	(2.50)			(0.97)	(1.15)	(3.50)
R_1	1.188	0.460	2.825			
	(1.09)	(0.37)	(1.45)			
R_2	-0.171	-2.036	3.052			
	(0.15)	(1.58)	(1.49)			
B_1	6.389	6.020	7.647	7.054		
	(3.17)	(2.56)	(2.20)	(3.67)		
B_2	1.921	0.759	3.505	2.516		
	(0.72)	(0.25)	(0.76)	(1.05)		
O	-2.947	-1.991	-4.677	-2.750	-3.896	-1.939
	(5.43)	(3.28)	(4.66)	(2.70)	(4.21)	(2.32)
R^2	0.242	0.243	0.282	0.347	0.252	0.247
F	9.958	6.307	5.696	5.093	7.691	5.823
(d.f.)	(9,281)	(8,157)	(8,116)	(7,67)	(5,114)	(5,90)
Number of Observations	291	166	125	75	120	96

Table 15. The Results of Linear Regression Models (continued)

(IRCE allocations)

Variables	Pooled	PUB	PRB	ANY	HIGH	LOW
Constant	36.393	36.028	39.472	40.011	35.982	-0.336
	(17.06)	(15.44)	(9.48)	(10.75)	(9.17)	(0.76)
D	-0.905	-0.865	-0.884	-1.428	-0.462	-0.159
	(2.14)	(1.84)	(1.11)	(1.98)	(0.56)	(1.71)
B	0.930	0.762	1.139	0.438	1.470	0.194
	(3.06)	(2.28)	(1.96)	(0.76)	(2.39)	(2.66)
DB	0.130	0.141	0.102	0.237	0.042	0.013
	(1.57)	(1.50)	(0.66)	(1.69)	(0.26)	(0.62)
I	1.765			-0.101	1.634	-0.508
	(2.43)			(1.13)	(1.15)	(2.43)
R_1	0.254	-0.312	1.253			
	(0.26)	(0.27)	(0.76)			
R_2	-1.092	-2.784	1.448			
	(1.08)	(2.33)	(0.83)			
B_1	4.643	4.537	5.438	4.520		
	(2.64)	(2.14)	(1.85)	(2.71)		
B_2	1.286	1.397	1.271	1.142		
	(0.73)	(0.70)	(0.43)	(0.69)		
0	-3.001	-2.267	-4.341	-2.965	-3.896	0.127
	(5.95)	(3.97)	(4.68)	(3.72)	(4.21)	(0.97)
R^2	0.243	0.258	0.264	0.312	0.252	0.247
F	11.217	7.510	5.955	6.531	7.691	5.823
(d.f.)	(9,314)	(8,173)	(8,133)	(7,101)	(5,114)	(5,90)
Number of Observations	324	182	142	109	120	96

Table 16. The Frequency of Advantageous Public Projects to Coordinators

(IRPE allocations)

	Disadvantageous	Equally Advantageous	Advantageous
APUB (%)	30 (68.2)	5 (11.4)	9 (20.4)
HPUB (%)	46 (68.7)	2 (3.0)	19 (28.3)
LPUB (%)	44 (80.0)	4 (7.3)	7 (12.7)
PUB (%)	120 (72.3)	11 (6.6)	35 (21.1)
APRB (%)	19 (59.4)	1 (3.1)	12 (37.5)
HPUB (%)	24 (46.1)	7 (13.5)	21 (40.4)
LPUB (%)	37 (90.2)	3 (7.3)	1 (2.4)
PUB (%)	80 (64.0)	11 (8.8)	34 (27.2)
TOTAL (%)	200 (68.7)	22 (7.6)	69 (11.7)

(IRCE allocations)

	Disadvantageous	Equally Advantageous	Advantageous
APUB (%)	45 (75.0)	5 (8.3)	10 (16.7)
PUB (%)	135 (74.2)	11 (6.0)	36 (19.8)
APRB (%)	36 (59.4)	1 (3.1)	12 (37.5)
PUB (%)	97 (68.3)	11 (7.7)	34 (23.9)
TOTAL (%)	232 (71.6)	22 (6.8)	70 (21.6)

Table 17. Cheap riding

APUB	HPUB	LPUB	PUB	APRB	HPRB	LPRB	PRB	TOTAL
71.0	66.9	77.5	71.8	65.8	50.8	70.8	62.5	68.4

Table 18. Free riding

Free riding	25	(16.0%)
Ran out of time	57	(37.3%)
No individually rational trades possible	60	(39.2%)
Voted to close market	9	(5.9%)
No public projects with individually rational trades	2	(1.3%)
Total	153	

Appendix 1. The Frequency of Establishing a Public Project

	Parameterizations							
	IA	IB	IIA	IIB	III	IVA	IVB	Total (%)
APUB–1	3	2	3	3	2	2	3	18/21 (85.7)
APUB–2	3	3	2	3	3	2	0	16/21 (76.2)
APUB–3	3	2	2	3	3	3	1	17/21 (81.0)
APUB–4	2	0	3	2	2	3	2	14/21 (66.7)
APUB (%)	11 (91.7)	7 (58.3)	10 (83.3)	11 (91.7)	10 (83.3)	10 (83.3)	6 (50.0)	65/84 (77.4)
HPUB–1	6	2	-	2	3	2	0	15/21 (71.4)
HPUB–2	6	2	-	2	3	3	2	18/21 (85.7)
HPUB–3	6	3	-	2	3	3	3	20/21 (95.2)
HPUB–4	6	3	-	2	3	3	1	18/21 (85.7)
HPUB (%)	24 (100)	10 (83.3)	- (-)	8 (66.7)	12 (100)	11 (91.7)	6 (50.0)	71/84 (84.5)
LPUB–1	6	0	1	-	2	3	3	15/21 (71.4)
LPUB–2	4	1	3	-	3	3	2	16/21 (76.2)
LPUB–3	4	1	2	-	2	3	0	12/21 (57.1)
LPUB–4	3	0	2	-	2	3	2	12/21 (57.1)
LPUB (%)	17 (70.8)	2 (16.7)	8 (66.7)	- (-)	9 (75.0)	12 (100)	7 (58.3)	55/84 (65.5)
PUB (%)	52 (86.7)	19 (52.7)	18 (75.0)	19 (79.2)	31 (86.1)	33 (91.7)	19 (52.7)	191/252 (75.8)

Appendix 1. The Frequency of Establishing a Public Project (continued)

	Parameterizations							
	IA	IB	IIA	IIB	III	IVA	IVB	Total (%)
APRB–1	2	3	3	2	2	3	0	15/21 (71.4)
APRB–2	3	2	3	3	2	2	2	17/21 (81.0)
APRB–3	3	0	2	3	1	1	0	10/21 (47.6)
APRB–4	2	3	2	2	3	3	1	16/21 (76.2)
APRB	10	8	10	10	8	9	3	58/84 (69.0)
(%)	(83.3)	(66.7)	(83.3)	(83.3)	(66.7)	(75.0)	(25.0)	
HPRB–1	6	3	-	3	2	2	1	17/21 (81.0)
HPRB–2	4	0	-	1	2	3	1	11/21 (52.4)
HPRB–3	5	3	-	3	3	1	0	15/21 (71.4)
HPRB–4	5	1	-	3	3	2	2	16/21 (76.2)
HPRB	20	7	-	10	10	8	4	59/84 (70.2)
(%)	(83.3)	(58.3)	(-)	(83.3)	(83.3)	(66.7)	(33.3)	
LPRB–1	2	0	1	-	2	2	1	8/21 (38.1)
LPRB–2	5	0	2	-	3	3	2	15/21 (71.4)
LPRB–3	3	0	2	-	2	1	1	9/21 (42.9)
LPRB–4	5	0	2	-	2	1	1	11/21 (52.4)
LPRB	15	0	7	-	9	7	5	43/84 (51.2)
(%)	(62.5)	(0.0)	(58.3)	(-)	(75.0)	(58.3)	(41.7)	
PRB	45	15	17	20	27	24	12	160/252 (63.4)
(%)	(75.0)	(41.7)	(70.8)	(83.3)	(75.0)	(66.7)	(33.3)	
TOTAL	97	34	35	39	58	57	31	351/504 (69.6)
(%)	(80.8)	(47.2)	(72.9)	(81.3)	(80.6)	(79.2)	(43.1)	

Appendix 2. The Frequency of IRE allocations by Experimental Periods

(IRPE allocations)

	1 – 7	8 – 14	15 – 21	TOTAL
APUB (%)	11 (39.3)	18 (64.3)	15 (53.6)	54
HPUB (%)	24 (85.7)	21 (75.0)	22 (78.6)	68
LPUB (%)	19 (67.9)	22 (78.6)	14 (50.0)	55
PUB (%)	54 (64.3)	61 (72.6)	51 (60.7)	166
APRB (%)	10 (35.7)	12 (42.9)	10 (35.7)	32
HPRB (%)	14 (50.0)	18 (64.3)	20 (71.4)	52
LPRB (%)	10 (35.7)	14 (50.0)	17 (60.7)	41
PRB (%)	34 (40.5)	44 (52.4)	47 (56.0)	125
TOTAL (%)	88 (52.4)	105 (62.5)	98 (58.3)	291

(IRCE allocations)

	1 – 7	8 – 14	15 – 21	TOTAL
APUB (%)	16 (57.1)	24 (85.7)	20 (71.4)	60
PUB (%)	59 (70.2)	67 (79.8)	56 (66.7)	182
APRB (%)	14 (50.0)	19 (67.9)	16 (57.1)	49
PRB (%)	38 (45.2)	51 (60.7)	53 (63.1)	142
TOTAL (%)	97 (57.7)	118 (70.2)	109 (64.9)	324

Appendix 3. Instructions

Thank you for your participation in this experiment.

This is an experiment in the economics of group decision-making organized through an asset market. The instructions are simple, and if you follow them carefully and make good decisions, you might earn a considerable amount of money.

At the conclusion of this experiment, you will receive **CASH** depending on your cash earnings throughout the experiment. Feel free to earn as much money as you can. It is not possible to end up owing the experimenters money at the end of the experiment.

This particular experiment involves the buying and selling (trading) of assets among the participants (or traders). No special skills are required and the instructions that follow should provide all the information you need. You should feel free to ask questions at any time.

In your folder you will find an **Information and Record Sheet** and a **Profit Sheet** which help determine the value to you of any decisions you might make. You should not reveal the information on these sheets to anyone. It is your private information.

A) General Instructions and Definitions

In the market, there will be 3 participants, called **TRADER A**, **TRADER B**, and **TRADER C**.

A **MARKET DAY** is a three-minute period during which trade may take place. There will be 21 market days in the experiment.

Each participant will be lent 100 cents at the beginning of each market day which must be repaid at the end of the market day. We will call this the **WORKING CAPITAL LOAN**. The purpose of this loan is to give you a means of exchange if you wish to trade. Remember, however, that it is not possible to end up owing money at the end of the experiment.

There are 3 **ASSETS**. On each market day each of the 3 traders will receive one of 3 assets. Each asset has value for three reasons.

(i) Any trader can buy or sell assets from any other trader for cash. An asset can be sold more than once during a market day.

(ii) Assets have value because they pay a **DIVIDEND**. The dividend is paid in cash to whoever owns the asset at the end of each market day. It is actually a liquidating dividend which means that once an asset holder is paid the dividend they no longer own the asset. Later in these instructions we will learn more about calculating your dividend earnings.

(iii) Finally, if any one of the traders[15] holds all of the assets at the end of a market day, *every trader* will receive a **BONUS**. We will learn more about this bonus later in these instructions.

[15] In one assigned coordinator markets it is the designated trader.

Any Questions?

B) **Market Organization — How trade takes place**

One of the most important features of this market is the **TRADE**. A trade occurs when one trader agrees to sell an asset to another trader at an agreed-upon price. The buyer now has one more asset and less cash. The seller now has one less asset and more cash.

If a trader wishes to buy an asset, they may make a **BID**. A bid tells **ALL** the other traders that they are willing to buy one asset from **ANY** of them at the bid price. So if Trader A bids 58 cents, he/she is showing his/her willingness to buy at that price. Either of the other traders can then decide if they want to sell Trader A an asset at that price.

If a trader wishes to sell an asset, they may make an **OFFER**. An offer tells **ALL** the other traders that they are willing to sell one asset to **ANY** of them at the offer price. So if Trader B offers 81 cents, he/she is showing his/her willingness to sell at that price. The other traders then can decide if they want to buy an asset from Trader B at that price.

REMEMBER — BID TO BUY and OFFER TO SELL!.

At any time during the market day, every trader is free to raise their hand and, when called on, to make a verbal bid to buy an asset at a price specified in the bid. Every trader also is free to raise their hand, and, when called on, to make a verbal offer to sell an asset at the price specified in the offer. For example, if Trader A wants to make a bid of 58 cents, then this person would raise their hand and, when recognized, say "Trader A bids 58 cents." I will repeat the trader number and the bid and record it on the overhead projector. Similarly, if Trader B decides to offer an asset for sale at 81 cents, this trader should raise their hand and, when recognized, say "Trader B offers 81 cents." I will repeat this information and the overhead projector will appear

Bids	Offers
A 58	B 81

Any trader may make both bids and offers during a market day.

When a trader makes a bid or offer, the other traders can then decide to accept or not accept that bid or offer. So when trader A bids 58 cents, either of the other traders can accept that bid as long as they have an asset to sell. Similarly, when trader B offers 81 cents, any of the other traders can accept that offer as long as they have at least an 81 cent cash balance. Whenever a bid or offer is accepted a trade takes place.

Once a trade is made all outstanding bids and offers are automatically canceled. If you wish, however, you may make a new bid or offer which is equal to the one which was canceled.

A market day can end in one of two ways. First, there is a 3 minute time limit for each market day. Once three minutes have passed the period will end regardless of whether the traders have completed all of the trades they wish to make. The second way that a market day can end is that the three traders may vote to close the market. At any time during a market day any trader may decide that they no longer wish to make trades during that market day. In that case they may raise their hand and say " I vote to close the market." If the other two traders agree then that market day ends and we will go on to the next market day.

We ask you to help us enforce a bid/offer improvement rule: All bids must be higher than the highest outstanding bid, should one exist, and offering prices must be lower than the lowest outstanding offer, should one exist. In the example above the next bid must be above 58 cents and the next offer must be below 81 cents.

While in the market, you will always be identified with the same trader number A, B, or C and the other trades will retain their assigned trader numbers.

To Review:

(a) You may sell an asset in one of two ways:
- (i) Make an offer that someone else accepts, or
- (ii) Accept someone else's bid to buy.

(b) You may buy an asset in one of two ways:
- (i) Make a bid that someone else accepts, or
- (ii) Accept someone else's offer to sell.

(c) You cannot sell an asset you do not have.

(d) You cannot purchase an asset with money you do not have.

(e) If any one of the traders[16] holds all of the assets at the end of a market day then *every trader* will receive a bonus.

Any Questions ?

C) Earning Money

Each trader will begin each market day with **one ASSET** and **100 cents WORKING CAPITAL LOAN**. At the end of each market day you will receive a liquidating dividend for any assets you hold and you will pay back the working capital loan.

ITEMS THAT INCREASE YOUR CASH BALANCE:
- (1) Sales prices of assets you sell,
- (2) Dividends on any assets held at market day's end,
- (3) Bonus, if any one of the traders[17] holds all assets at the end of a market day.

ITEMS THAT DECREASE YOUR CASH BALANCE:
- (1) Sales price of assets you buy.

Any Questions?

D) Recording Rules

Everyone please look at the sample **Information and Record Sheet.**

Please note the following important information:

[16] In one assigned markets it is the designated trader.

[17] In one assigned markets it is the designated trader.

(1) The market day is listed on the top of the page. Always make certain that you are using the Information and Record Sheet for the correct market day. The sample is for market day 3.

(2) Your trader number is listed on the top left hand corner. Always make certain that your trader number is correctly written on your Information and Record sheet. If it is not, please let one of the experimenters know. The sample is for Trader C.

(3) On line (0) notice that you have "Assets on Hand" of 1 and "Cash Balance" of 100. Remember that this cash is your Working Capital Loan.

(4) On line (A) you will find your Dividend Value. This is the amount that you will receive for every asset which you hold at the end of the market day. This amount may be different for the other traders in this experiment, and it may change from market day to market day – be certain to check your Dividend Value at the beginning of each market day. The sample shows a dividend value of 30.

(5) On line (G) you will find the Bonus which you will be paid if any trader holds all of the assets at the end of market day. The Bonus may be different for different market days. The Bonus may be more or less depending on which trader holds all of the assets at the end of the market day – in this case all traders will know the bonus associated with each trader holding all three assets at the end of the market day. For any market day the same size Bonus will always be paid each trader in the market. In this case the Bonus is 60 if any of the three traders holds all 3 assets at the end of the market day.[18]

After each trade:

(1) Record the Trade Price and the Trade Partner Number in the appropriate column

[18]In APRB markets, (5) On line (G) you will find the Bonus that **every** trader will receive if **you** hold all of the assets at the end of the market day. The Bonus may be different for the other traders, and it may change from market day to market day. When the trader holds all of the assets at the end of a market day, he/she will announce the Bonus. This is a realized Bonus, and for any market day the same size (realized) Bonus will always be paid each trader in the market. In this example if Trader C holds all 3 assets the Bonus will be 10, however, the Bonus if Trader A or B holds all 3 assets is unknown to Trader C.

In HPUB and LPUB markets, (5) On line G you will find the Bonus which you will be paid if the designated trader holds all of the assets at the end of market day. The Bonus may be different for different market days. The experimenter will announce the designated trader and the Bonus at the start of each market day. For any market day the same size Bonus will always be paid each trader in the market. In this case the Bonus is 60 if Trader A holds all 3 assets at the end of the market day.

In HPRB and LPRB markets, (5) On line (G) you will find the Bonus which you will be paid if the designated trader holds all of the assets at the end of the market day. The Bonus may be different for different market days. The experimenter will announce the designated trader at the start of each market day, and when the designated trader holds all of the assets at the end of a market day, he/she will announce the Bonus. This is a realized Bonus, and for any market day the same size (realized) Bonus will always be paid each trader in the market. In this example if Trader C holds all 3 assets the Bonus will be 0, however, the Bonus if Trader A, the designated trader, holds all 3 assets,is unknown to Traders B and C.

depending on the nature of the transaction. Your first trade is recorded on line (1), and succeeding trades are recorded on subsequent lines. Line (1) of the sample shows that trader C sold 1 asset to trader A for 58 cents.

(2) Calculate and record your new holdings of assets and new cash balance. If you buy an asset, your cash balance decreases by the trade price and your assets on hand increase by 1. If you sell an asset, your cash balance increase by the trade price, and your assets on hand decrease by 1. Line (1) of the sample shows that, after selling 1 asset to trader A, trader C now has 0 assets and a 158 cent cash balance.

At the end of each market day:

(1) Compute your Asset Earnings by multiplying the dividend by the number of assets you hold. The dividend value is listed on line (A) of your **Information and Record Sheet**, and is the same for every asset you hold. In other words, if your dividend value is 15, then if you hold 1 asset at the end of the market day, your Asset Earnings (shown on line (B)) are 15. If you hold 2 assets, your Asset Earnings are 30. If you hold no assets, your Asset Earnings are 0. Note that dividends may be different for different traders, and on different market days. On the sample the trader C held no assets at the end of the market day so line (B) is 0.

(2) Record your Final Cash Balance on line (C). On the sample this is 158.

(3) Calculate Total Trading Earnings as follows on line (D):

Total Trading Earnings (D) = Asset Earnings (B) + Final Cash Balance (C).

On the sample, 0 + 158 = 158 shown on line (D).

(4) You must now pay back the Working Capital Loan 100 cents listed on Line (E), and calculate Net Trading Earnings on (F) as follows:

Net Trading Earnings (F) = Total Asset Earnings (D) - 100.

On the sample, 158 - 100 = 58.

(5) If any one of the traders holds all assets at the end of each market day then everyone receives a Bonus which gets recorded on line (G). Otherwise, it is 0. On the sample trader A held all 3 assets at the end of the market day, so trader C records the 60 cent bonus. Trader A and B also receive the bonus in this sample.[19]

[19] In APRB markets, (5) If any trader holds all assets at the end of each market day then everyone receives a Bonus which gets recorded on line (G). Otherwise, it is 0. On the sample trader A held all 3 assets at the end of the market day. Trader A knew from his Information and Record Sheet that if he held all 3 assets the Bonus would be 60. When the market day ended and he did hold all 3 assets, he made an announcement that the Bonus was 60, so trader C recorded the 60 cent bonus. Traders A and B also received the Bonus.

In HPUB and LPUB markets, (5) If the designated trader holds all assets at the end of each market day then everyone receives a Bonus which gets recorded on line (G). Otherwise, it is 0. On the sample trader A held all 3 assets at the end of the market day, so trader C records the 60 cent bonus. Trader A and B also receive the bonus in this sample.

In HPRB and LPRB markets, (5) If the designated trader holds all assets at the end of each market day then everyone receives a Bonus which gets recorded on line (G). Otherwise, it is 0. On

(6) Finally, calculate and record your Total Profit as follows:

Total Profit (H) = Net Trading Earnings (F) + Bonus (G).

This is your Total Profit on this market day and is yours to keep. In the sample case 58 + 60 = 118 or 1 dollar 18 cents.

(7) Transfer Net Trading Earnings (F), Bonus (G), and Total Profit (H) on your **Profit Sheet**. Also check Yes or No depending upon whether everyone received a bonus that market day. In the final column record your Cumulative Earnings – the total amount which you have earned until that point in the experiment. Remember that at the end of the experiment you will be paid your Cumulative Earnings **in cash**.

E) **Finishing Up**

We are now ready to begin the experiment. We ask that there be no talking until after the experiment has been completed. If you have any questions from now on please raise your hand and a monitor will be around to help you.

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the sample Trader A, the designated trader, held all 3 assets at the end of the market day. Trader A knew from his Information and Record Sheet that if he held all 3 assets the Bonus would be 60. When the market day ended and he did hold all 3 assets, he made an announcement that the Bonus was 60, so Trader C recorded the 60 cent bonus. Traders A and B also received the Bonus.

Trader number (C)

Sample
Information and Record Sheet

Market day (3)

Trade number	Trade Price (Sales)	Trade Price (Purchase)	Trading Partner	Assets on Hand	Cash Balance
0	(Beginning-of-period Holdings)			1	100 cents
1	58		A	0	158
2					
3					
4					
5					
6					
7					
8					
9					
10					

A	Dividend Value	30
B	Asset Earnings: (A) × No. of Assets on Hand	0
C	Final Cash Balance	158
D	Total Trading Earnings: (B)+(C)	158
E	Working Capital Loan	100
F	Net Trading Earnings: (D)–(E)	58
G	Bonus (0)	60
H	Total Profit: (F)+(G)	118

Trader number (   )

Information and Record Sheet

Market day (   )

Trade number	Trade Price (Sales)	Trade Price (Purchase)	Trading Partner	Assets on Hand	Cash Balance
0	(Beginning-of-period Holdings)			1	100 cents
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					

A	Dividend Value	
B	Asset Earnings: (A) × No. of Assets on Hand	
C	Final Cash Balance	
D	Total Trading Earnings: (B)+(C)	
E	Working Capital Loan	100
F	Net Trading Earnings: (D)–(E)	
G	Bonus ( )	
H	Total Profit: (F)+(G)	

Trader number (   )

Profit Sheet

Market day	Net Trading Earnings (F)	Receive a Bonus?	Bonus (G)	Total Profit (F)+(G)	Cumulative Earnings
1		No Yes ( )			
2		No Yes ( )			
3		No Yes ( )			
4		No Yes ( )			
5		No Yes ( )			
6		No Yes ( )			
7		No Yes ( )			
8		No Yes ( )			
9		No Yes ( )			
10		No Yes ( )			
11		No Yes ( )			
12		No Yes ( )			
13		No Yes ( )			
14		No Yes ( )			
15		No Yes ( )			
16		No Yes ( )			
17		No Yes ( )			
18		No Yes ( )			
19		No Yes ( )			
20		No Yes ( )			
21		No Yes ( )			
Total Cash					

Chapter 4

Economies with Costly Trade Links

4.1 Introduction

We consider a trade economy with multiple (economic) sectors, each sector consisting of different industries. Trades among different sectors or different industries incur transaction costs. Between similar industries in different sectors a formal trading institution, a so-called *trade link*, can be built and thereby reduce the transaction costs. However, the establishment of trade links inevitably incurs set-up costs. For example, in an international economy there are trades among different countries and different industries. Each country bears its own costs such as transportation or contracting expenses, which depend on the quantity and quality of products, geographical locations, and tariff systems. Some industries of different countries have more productive or cheaper trade technology than others, and organize formal trade channels within those industries. Furthermore, even without a formal trade relationship among different countries the international trades can occur informally through high cost trades such as smuggling.

Our analysis of an economy with costly trade links is based on the concept of an economy with *public projects*, as introduced by Mas-Colell (1980), and developed by Diamantaras and Gilles (1994). In this approach a public project is an abstract entity without any structure. It imposes costs and widespread externalities which are to be shared by the agents. The equilibrium concept in an economy with such a public project is called a *valuation equilibrium.* In this paper we regard a trade link as a public project, and propose the notion of a *trade equilibrium*, which is an appropriate modification of the valuation equilibrium concept to an economy with costly trade links. In a trade equilibrium it is required that each trade link is separately financed and has a minimal deficit with a balanced budget separately. A trade link to be chosen is not necessarily the one which is the cheapest in costs, but the one which is the best in the overall economy.

For the traditional theory of a competitive market economy, i.e., an economy with costlessly and smoothly functioning markets, we know that the two welfare theorems hold: any Walrasian equilibrium allocation is Pareto efficient and any Pareto efficient allocation can be sustained as a Walrasian equilibrium allocation. We show that in an economy with costly trade links the two welfare theorems in the traditional sense also apply: trade equilibria are Pareto efficient and Pareto efficient allocations can be supported as trade equilibria. Furthermore, we introduce an industry-wise efficient concept and the notion of industry-wise trade equilibria based on industry-wise transaction costs which are measured by the agents who belong to a specific industry, and clarify the relationship between Pareto efficiency and industry-wise efficiency concepts: Pareto efficiency strictly implies industry-wise efficiency.

Related work has been done by many authors. Foley (1970b) modifies the traditional model in which it is possible to analyze the consequences of costs in operations of markets, and shows the existence of quasiequilibrium. In his model there are two prices in each market: a buyer's price and a lower seller's price. The difference between these yields an income which compensates the real resources used up in operations of the markets. Furthermore, Albin and Foley (1992) present the results of a simulation of exchange among geographically dispersed agents who face real costs of communication along with bounds to rationality and calculation. Exchange is accomplished by bilateral bargaining between pairs of agents. This decentralized mechanism can achieve a substantial improvement in the allocation of resources, but in contrast to the Walrasian equilibrium concept, agents who begin with endowments of equal value can end up with substantially unequal wealth. Hahn (1971) and Kurz (1974) introduce monetary transaction costs in a general equilibrium context. Transaction costs in these intertemporal or static equilibrium models are commodity related, and do not influence institutional aspects such as the market itself. The main difference of our approach with Hahn (1971) and Kurz (1974) is that we consider trade links as production units realizing certain gains from trade. In particular, Gilles, Diamantaras, and Ruys (1995) discuss the efficiency properties in an economy with a trade center which is a social institution that forms the institutional framework of a market system. In other words, the trade center is designed to provide a set of feasible allocations to the members of the economy with several types of costs.

In Section 2 we define an economy with costly trade links, and in Section 3 and 4 respectively we consider decentralization of Pareto efficient allocations and industry-wise efficient allocation with trade equilibria and industry-wise trade equilibria.

4.2 The model

We consider an economy in which exogenously there are given m different sectors, and each sector consists of n different industries. Let A be a finite set of agents, $a \in A$, and $A^i_j \subset A$ be a *locality* which belongs to the $j-$th industry of sector i for $i \in I = \{1, \ldots, m\}$ with $j \in J = \{1, \ldots, n\}$. There are two partitioning structures of A based on the sectors and the industries as follows:

(a) $A = \cup_{i \in I} A^i$ and $A^i \cap A^{i'} = \emptyset$, for $i, i' \in I$, $i \neq i'$,

where $A^i = \{A^i_j \subset A | j \in J\}$ is the collection of localities in the sector i,

(b) $A = \cup_{j \in J} A_j$ and $A_j \cap A_{j'} = \emptyset$, for $j, j' \in J$, $j \neq j'$,

where $A_j = \{A^i_j \subset A | i \in I\}$ is the collection of localities which belongs to the $j-$th industry of each sector. Note that in the economy $A^i_j = A^i \cap A_j$.

We assume that there are $\ell \in \mathbb{N}$ private goods available in the economy, and the commodity space is the nonnegative orthant of the ℓ-dimensional Euclidean space, $\mathbb{R}^\ell_+$. We denote by the function $w \colon A \to \mathbb{R}^\ell_+ \setminus \{0\}$ the *initial endowment* of private goods attributed to the agents in the economy.

It is assumed that trade of private goods is costly, even though there is no physical production in the economy. In fact, actual transactions among the localities inevitably incur costs such as transportation and contracting expenses. These are real resource costs of information gathering and processing or in the operation of "markets". Furthermore, we assume that the localities which belong to different sectors in the economy can establish an appropriate institutional trade framework that we call a *trade link*, and the possible trade links can only be built among the localities in the same industry of every sector. Note that for every sector $i \in I$, A^i is assumed to have trade links inside sector i, or every member within a given sector can trade without establishing trade links. We represent a trade link between A^i_j and $A^{i'}_j$, two localities in the sector i and i' $(i \neq i')$ of the $j-$th industry, by $y^{ii'}_j$ or $y^{i'i}_j$ with a binary indicator such that for A^i_j, $A^{i'}_j \subset A$,

$$y^{ii'}_j = y^{i'i}_j, \text{ and}$$

$$y^{ii'}_j = \begin{cases} 1 & \text{if a trade link between } A^i_j \text{ and } A^{i'}_j \text{ is established,} \\ 0 & \text{otherwise.} \end{cases}$$

We denote the set of trade link indicators between A^i_j and $A^{i'}_j$ by $\mathcal{Y}^{ii'}_j = \mathcal{Y}^{i'i}_j := \{0, 1\}$ for $j \in J$ and $i, i' \in I$. Now the trade link space can be introduced as the Cartesian product

$$\mathcal{Y} := \Pi_{\gamma \in \Gamma} \Pi_{j \in J} \mathcal{Y}^{\gamma}_j,$$

where $\Gamma := \{ii' | i, i' \in I, i > i'\}$. An element in the trade link space $\mathcal{Y}$, $y \in \mathcal{Y}$, now represents an ordered tuple of trade links between the different sectors, and it can be interpreted as a *trade structure*.

The trade links between the localities in the same industry lead to two different types of costs: One is related to the establishment of each trade link itself, and the other is related to the costs of making actual transactions which are affected by the established trade structure as well as the amount traded. The first type of costs are incurred by building trade links, and we call these the *set-up costs*. We model the set-up costs of a trade link between a pair of localities $\gamma \in \Gamma$ in the $j-$th industry as a function $c^{\gamma}_j : \mathcal{Y}^{\gamma}_j \to \mathbb{R}^{\ell}_+$: quantities of private goods $c^{\gamma}_j(y^{\gamma}_j) \in \mathbb{R}^{\ell}_+$ are used in construction and maintenance such that for all $\gamma \in \Gamma$ and all $j \in J$,

$$c^{\gamma}_j(1) \geqq c^{\gamma}_j(0) = 0,$$

where we use the vector inequalities $\gg$, $>$, and $\geqq$. These costs are assumed to be independent of the amount of trading that takes place via the trade link between the localities. The total set-up cost function can be written in the additively separable form $c(y) := \sum_{\gamma\in\Gamma}\sum_{j\in J} c_j^\gamma(y_j^\gamma)$, for $y \in \mathcal{Y}$.

The second type of costs is directly related to the actual trading among the agents without respect to localities, e.g., the costs of transportation and contracting expenses. These costs, called *transaction costs*, depend on the trade structure, and on the amount of goods traded. We assume that retrade among the agents is absent, and we can equate trade with consumption. Furthermore, we assume that the transaction costs depend linearly on the amount traded, i.e, consumed. The transaction costs are thus described by a function $t: A \times \mathcal{Y} \to \mathbb{R}_+$, called the *transaction cost function*. Here $t(a, y) \geqq 0$ is the fraction of the net consumption bundle that an agent a in the trade structure $y \in \mathcal{Y}$ should acquire additionally before consumption. If an agent $a \in A$ intends to consume $f \in \mathbb{R}_+^\ell$, then he has to acquire additionally $t(a, y)f$ to cover transaction costs, i.e., he should have the bundle $[1 + t(a, y)]f \in \mathbb{R}_+^\ell$ in order to consume f.

We do not assume that a trade link induces any other externalities than the trade opportunities provided by the trade link. Every agent $a \in A$ thus does not take into account trade links in his utility function. Hence, for each agent $a \in A$ we represent his preferences by a utility function $U_a: \mathbb{R}_+^\ell \to \mathbb{R}$, which depends only on the quantities of the private goods finally consumed by that agents. In the sequel we introduce some additional assumptions regarding the utility functions of the agents in the economy. For every $a \in A$, a utility function U_a is *monotone* if for all $f, g \in \mathbb{R}_+^\ell$, $f \gg g$ implies $U_a(f) > U_a(g)$, and *strictly monotone* if for all $f, g \in \mathbb{R}_+^\ell$, $f > g$ implies $U_a(f) > U_a(g)$.

Now we define economies with costly trade links as follows:

Definition 4.2.1 *The tuple* $\mathbb{E} := \langle A, w, \{U_a\}_{a\in A}, (\mathcal{Y}, c, t)\rangle$ *is an* **economy with costly trade links** *if for all* $y \in \mathcal{Y}$,

$$\sum_{a\in A} w(a) \gg c(y). \tag{4.1}$$

The concept of an allocation in this economy depends on the established trade structure and is derived as follows. Initial resources are spent on consumption as well as on the transaction costs mentioned above. Furthermore, we assumed that a trade link in the economy can be established with set-up costs. In this respect the initial endowment is not yet actualized, but has to be accessed through trade structure in order to be realized.

Definition 4.2.2 *An* **allocation** *for an economy* $\mathbb{E}$ *is a pair* (f, y) *where* $f: A \to \mathbb{R}^{\ell}_{+}$ *is a distribution of private goods for consumption and* $y \in \mathcal{Y}$ *is a trade structure. An allocation* (f, y) *is* **feasible** *if*

$$\sum_{a \in A} [1 + t(a, y)] f(a) + c(y) = \sum_{a \in A} w(a). \tag{4.2}$$

Note that we do not assume free disposal in consumption, although our results also hold in that case. We denote the set of feasible allocations by Φ.

4.3 Pareto efficiency and trade equilibria

In this section we investigate the relationship between efficiency and equilibrium in the economy with costly trade links. We define efficiency for the economy with costly trade links as follows:

Definition 4.3.1 *A feasible allocation* $(f, y) \in \Phi$ *is* **Pareto efficient** *in the economy* $\mathbb{E}$ *if there exists no other feasible allocation* $(g, z) \in \Phi$ *such that*

(i) *for every* $a \in A$, $U_a(g(a)) \geqq U_a(f(a))$ *and*

(ii) *there exists at least one* $b \in A$ *such that* $U_b(g(b)) > U_b(f(b))$.

The above definition of efficiency is standard, but the definition of feasibility (Definition 4.2.2) is not. We will need the usual price space for the private goods:

$$\Delta := \left\{ p \in \mathbb{R}^{\ell}_{+} \,\middle|\, \sum_{i=1}^{\ell} p_i = 1 \right\}.$$

We now introduce our trade equilibrium concept.

Definition 4.3.2 *A feasible allocation* $(f, y) \in \Phi$ *is a* **trade equilibrium** *in the economy* $\mathbb{E}$ *if there exist a price system* $p: \mathcal{Y} \to \Delta$ *and a valuation system* $V^{\gamma}_{j}: A \times \mathcal{Y} \to \mathbb{R}$ *for* $\gamma \in \Gamma$ *and* $j \in J$ *such that*

(i) *each trade link y_j^γ has a minimal deficit with a balanced budget separately, i.e., for all $z \in \mathcal{Y}$ with $\gamma \in \Gamma$ and $j \in J$:*

$$\sum_{a \in A} V_j^\gamma(a,z) - p(z) \cdot c_j^\gamma(z_j^\gamma) \leqq \sum_{a \in A} V_j^\gamma(a,y) - p(y) \cdot c_j^\gamma(y_j^\gamma) = 0, \text{ and}$$

(ii) *for every $a \in A$, the consumption bundle $f(a)$ maximizes the utility U_a on the union of the budget sets $\cup_{z \in \mathcal{Y}} B_z(a)$, where for $z \in \mathcal{Y}$*

$$B_z(a) = \left\{ g \in \mathbb{R}_+^\ell \,\middle|\, p(z) \cdot [1 + t(a,z)]g + \sum_{\gamma \in \Gamma} \sum_{j \in J} V_j^\gamma(a,z) \leqq p(z) \cdot w(a) \right\}.$$

Note that we define a price system $p: \mathcal{Y} \to \Delta$, where $p(y) \in \Delta$ is the equilibrium price vector for private goods if y is the equilibrium trade structure, and for any other trade structure $z \in \mathcal{Y}$, $z \neq y$, $p(z) \in \Delta$ is a price vector that agents *conjecture.*[1] Through the dependence of prices on the trade structure, the attractiveness of a particular trade link depends on the trade structure in which it is embedded. Furthermore, each valuation function V_j^γ for $\gamma \in \Gamma$ and $j \in J$ is defined on $A \times \mathcal{Y}$. This reflects that an agent evaluates each trade link over the economy's trade structure, and the increased attractiveness of private goods prices in one trade link can be compensated by a higher valuation for the same trade link embedded in a different trade structure. In particular, in a trade equilibrium the price system, p, as well as the valuation system are given as a price taking equilibrium. We require that each trade link is separately financed and a deficit is minimized with a balanced budget by condition (i). Condition (ii) imposes individual optimality of the assigned tuple $(f(a), y)$, given the valuation system of the trade links, i.e., the tax-subsidy system is taken as given by the agents.

The following example shows a simple economy with costly trade links.

Example 4.3.3 We consider an economy with two sectors and one industry in which each locality consists of a single agent, i.e, $A = \{a, b\}$, $a \in A^1$ and $b \in A^2$. Let $\ell = 2$

[1] An example of Diamantaras, Gilles, and Scotchmer (1994) shows that without the linear structure of Lindahl in a pure public goods economy, decentralization might require conjectural prices. In that example there is a strong complementarity between one of the private goods and the public good. There is no reason to think that problem would vanish in an economy with costly trade links, since complementarities between trade links and private goods are present there too.

and $w(a) = (3,0)$, $w(b) = (0,3)$. Furthermore, $\mathcal{Y}^{12} = \{0,1\}$ with $c^{12}(1) = (1,1)$ and $c^{12}(0) = (0,0)$. The transaction costs are

$$t(a,1) = t(b,1) = 0,\ t(a,0) = 0, \text{ and } t(b,0) = 4,$$

which represent that agent a has more productive or cheaper trade technology than agent b. For every commodity bundle $f = (f_1, f_2) \in \mathbb{R}^2_+$ the utility functions are

$$U_a(f_1, f_2) = U_b(f_1, f_2) = f_1 f_2.$$

This completes the description of the economy.

First, at the initial endowment allocation, $U_a(3,0) = U_b(0,3) = 0$. Consider a case of $y^{12} = 0$, i.e., a trade link is not established. Then two agents a and b can achieve an allocation with the given transaction costs as follows:

$$f(a) = (1.5, 1.5) \text{ and } f(b) = (0.3, 0.3),$$

with $p(0) = (0.5, 0.5)$, $V^{12}(a,0) = V^{12}(b,0) = 0$, and $p(1) = (0.5, 0.5)$, $V^{12}(a,1) + V^{12}(b,1) \leqq 1$. Furthermore, we consider a case of $y^{12} = 1$, i.e., a trade link is established and transaction costs are eliminated. Then agents a and b can improve upon their allocations such as

$$f'(a) = (1.6, 1.6) \text{ and } f'(b) = (0.4, 0.4),$$

with $p(1) = (0.5, 0.5)$, $V^{12}(a,1) = -0.1$, $V^{12}(b,1) = 1.1$, and $p(0) = (0.5, 0.5)$, $V^{12}(a,0) + V^{12}(b,0) = 0$. This allocation is indeed a trade equilibrium in this economy. ◇◇

Now the main decentralization result of our model is given by the following theorem.

Theorem 4.3.4 *Let $\mathbb{E}$ be an economy with costly trade links. Then the following statements hold:*

(a) *If for every $a \in A$ the utility function U_a is monotone, then every trade equilibrium is Pareto efficient.*

(b) *If for every $a \in A$ the utility function U_a is continuous, quasi-concave, and strictly monotone, then every Pareto efficient allocation can be supported as a trade equilibrium.*

Proof of Part (a)

Let (f, y) be a trade equilibrium with a price system p and a valuation system $V = \langle V_j^\gamma \rangle_{\gamma\in\Gamma; j\in J}$. We show that (f, y) is Pareto efficient.

Suppose to the contrary that (f, y) is not Pareto efficient. Then there exists a feasible allocation $(g, z) \in \Phi$ such that for every $a \in A$

$$U_a(g(a)) \geqq U_a(f(a)),$$

and there is $b \in A$ such that

$$U_b(g(b)) > U_b(f(b)).$$

Since $(g, z) \in \Phi$, it follows that

$$\sum_{a\in A}[1 + t(a, z)]g(a) + \sum_{\gamma\in\Gamma}\sum_{j\in J} c_j^\gamma(z_j^\gamma) = \sum_{a\in A} w(a). \tag{4.3}$$

Condition (ii) of the definition of trade equilibrium and the monotonicity of the utility functions imply that for every $a \in A$ we have that

$$p(z) \cdot [1 + t(a, z)]g(a) + \sum_{\gamma\in\Gamma}\sum_{j\in J} V_j^\gamma(a, z) \geqq p(z) \cdot w(a),$$

and for some $b \in A$ we have

$$p(z) \cdot [1 + t(a, z)]g(b) + \sum_{\gamma\in\Gamma}\sum_{j\in J} V_j^\gamma(b, z) > p(z) \cdot w(b).$$

Hence,

$$p(z) \cdot \sum_{a\in A}[1 + t(a, z)]g(a) + \sum_{a\in A}\sum_{\gamma\in\Gamma}\sum_{j\in J} V_j^\gamma(a, z) > p(z) \cdot \sum_{a\in A} w(a). \tag{4.4}$$

Condition (ii) of the definition of trade equilibrium implies that for any $\gamma \in G$ and $j \in J$,

$$\sum_{a\in A} V_j^\gamma(a, y) - p(y) \cdot c_j^\gamma(y_j^\gamma) \geqq \sum_{a\in A} V_j^\gamma(a, z) - p(z) \cdot c_j^\gamma(z_j^\gamma). \tag{4.5}$$

Since (4.3) can be written as

$$\sum_{a\in A}(w(a) - [1 + t(a, z)]g(a)) - \sum_{\gamma\in\Gamma}\sum_{j\in J} c_j^\gamma(z_j^\gamma) = 0, \tag{4.6}$$

we conclude that

$$\begin{array}{rcll} 0 & = & \sum_{\gamma\in\Gamma}\sum_{j\in J}(\sum_{a\in A} V_j^\gamma(a,y) - p(y)\cdot c_j^\gamma(y_j^\gamma)) & \text{by (4.5)} \\ & \geqq & \sum_{\gamma\in\Gamma}\sum_{j\in J}(\sum_{a\in A} V_j^\gamma(a,z) - p(z)\cdot c_j^\gamma(z_j^\gamma)) & \text{by (4.4)} \\ & > & p(z)\cdot\sum_{a\in A}(w(a) - [1+t(a,z)]g(a)) - p(z)\cdot\sum_{\gamma\in\Gamma}\sum_{j\in J} c_j^\gamma(z_j^\gamma) & \text{by (4.6)} \\ & = & p(z)\cdot 0 = 0. & \end{array}$$

This is a contradiction.

Proof of Part (b)

Let (f,y) be a Pareto efficient allocation in $\mathbb{E}$, and let $a \in A$ be arbitrary. We define

$$F(a) := \{g \in \mathbb{R}_+^\ell \mid U_a(g) > U_a(f(a))\}.$$

Note that $F(a) \neq \emptyset$ by strict monotonicity of U_a. Also it follows that $F(a)$ is open, convex, and bounded from below. Furthermore, the strict monotonicity of U_a implies that for every $h \in F(a)$, $\{h\} + \mathbb{R}_+^\ell \subset F(a)$. (This property is usually indicated as the *comprehensiveness* of $F(a)$.) Let for an arbitrary $z \in \mathcal{Y}$

$$F(z) := \sum_{a\in A}[1+t(a,z)]F(a) + \{\sum_{\gamma\in\Gamma}\sum_{j\in J} c_j^\gamma(z_j^\gamma) - \sum_{a\in A} w(a)\}.$$

The set $F(z)$ is also non-empty, open, convex, and bounded from below. An element in $F(z)$ represents the excess demand corresponding to a strictly Pareto superior allocation under the structure $z \in \mathcal{Y}$.

We now construct positive prices for private goods, $p(z) \in \Delta$ for all $z \in \mathcal{Y}$. Together with the valuation system constructed below, there will be the allocation (f,y) supported as a trade equilibrium.

Because the allocation (f,y) is efficient, we have $0 \notin F(z)$. By an application of Minkowski's separating hyperplane theorem to 0 and the convex set $F(z)$, there exists a normal vector $p(z) \neq 0$ such that $p(z)\cdot F(z) \geqq 0$. Since $F(z)$ is bounded from below as well as comprehensive, it is obvious that $p(z) > 0$. We can now scale the vector $p(z)$ without loss of generality to achieve $p(z) \in \Delta$. In this way we have defined a price system $p\colon \mathcal{Y} \to \Delta$. Next we show that p satisfies the conditions as required in Definition 4.3.2.

Let the vector $x(a,z) \in \mathbb{R}_+^\ell$ be chosen such that, in case $z \neq y$,

(a) $p(z)\cdot x(a,z) = \inf p(z)\cdot F(a) \geqq 0$;

(b) $U_a(x(a,z)) \geqq U_a(f(a))$,

and in case $z = y$, $x(a,z) = x(a,y) = f(a)$. Clearly, such vectors exist, because $F(a) \subset \mathbb{R}^\ell_+$, and $p(z) > 0$. We construct a valuation system $V_j^\gamma \colon A \times \mathcal{Y} \to \mathbb{R}$ for $\gamma \in \Gamma$ and $j \in J$. For this construction we define two disjoint sets G' and G'' as follows: for any $z \in \mathcal{Y}$,

$$\begin{aligned} G'(z) &:= \{(\gamma,j) \in \Gamma \times J \mid c_j^\gamma(z_j^\gamma) > 0\}, \\ G''(z) &:= \{(\gamma,j) \in \Gamma \times J \mid c_j^\gamma(z_j^\gamma) = 0\}. \end{aligned}$$

Note that $\sum_{\gamma \in \Gamma} \sum_{j \in J} c_j^\gamma(z_j^\gamma) = \sum_{(\gamma,j) \in G'(z)} c_j^\gamma(z_j^\gamma) = \sum_{(\gamma,j) \notin G''(z)} c_j^\gamma(z_j^\gamma)$. For $z \in \mathcal{Y}$ let $V_r^\delta(\cdot, z)$ with $(\delta, r) \in G'(z)$ be defined by

$$V_r^\delta(a,z) := \sigma_r^\delta(z)\{p(z) \cdot w(a) - p(z) \cdot [1 + t(a,z)]x(a,z)\},$$

where

$$\sigma_r^\delta(z) = \frac{p(z) \cdot c_r^\delta(z_r^\delta)}{\sum_{(\delta,r) \in G'(z)} p(z) \cdot c_r^\delta(z_r^\delta)} > 0.$$

It is obvious that such $\{\sigma_r^\delta(z)\}_{(\delta,r) \in G'(z)}$ exist for any $z \in \mathcal{Y}$.

Moreover, we define $V_s^\eta(\cdot, z)$ with $(\eta, s) \in G''(z)$ for $z \in \mathcal{Y}$ by

$$V_s^\eta(a,z) := p(z) \cdot w(a) - p(z) \cdot [1 + t(a,z)]x(a,z) - \beta_s^\eta(a,z),$$

where $\beta_s^\eta(a,z)$ is determined by

(a) $\sum_{a \in A} \beta_s^\eta(a,z) = \sum_{(\gamma,j) \notin G''(z)} p(z) \cdot c_j^\gamma(z_j^\gamma)$, and

(b) $\sum_{(\eta,s) \in G''(z)} \beta_s^\eta(a,z) = |G''(z)|\{p(z) \cdot w(a) - p(z) \cdot [1 + t(a,z)]x(a,z)\}$.

It is also obvious that for every $a \in A$ such $\{\beta_s^\eta\}_{(\eta,s) \in G''(z)}$ exist. We now check the two requirements of a trade equilibrium.

Condition (i)

For every $(\gamma, j) \in G(z)$ it holds either (A) $c_j^\gamma(z_j^\gamma) > 0$, i.e., $(\gamma, j) \in G'(z)$ or (B) $c_j^\gamma(z_j^\gamma) = 0$, i.e., $(\gamma, j) \in G''(z)$. By construction, $p(z) \cdot \inf F(a) \geqq 0$, and $z \neq y$:

$$p(z) \cdot \sum_{a \in A} [1 + t(a,z)]x(a,z) + p(z) \cdot \sum_{\gamma \in \Gamma} \sum_{j \in J} c_j^\gamma(z_j^\gamma) \geqq p(z) \cdot \sum_{a \in A} w(a).$$

(The inequality becomes weak because $x(a, z)$ is at the infimum for every $a \in A$.)

Case (A): $(\gamma, j) \in G'(z)$

$$\begin{aligned}
& \textstyle\sum_{a\in A} V_j^\gamma(a,z) - p(z)\cdot c_j^\gamma(z_j^\gamma) \\
= \; & \textstyle\sum_{a\in A} \sigma_j^\gamma(z)\{p(z)\cdot w(a) - p(z)\cdot[1+t(a,z)]x(a,z)\} - \\
& \textstyle\sigma_j^\gamma(z)\sum_{(\gamma,j)\in G'(z)} p(z)\cdot c_j^\gamma(z_j^\gamma) \\
= \; & \textstyle\sigma_j^\gamma(z)\{p(z)\cdot \sum_{a\in A} w(a) - p(z)\cdot \sum_{a\in A}[1+t(a,z)]x(a,z) - \\
& \textstyle p(z)\cdot \sum_{\gamma\in\Gamma}\sum_{j\in J} c_j^\gamma(z_j^\gamma)\} \\
\leqq \; & 0,
\end{aligned}$$

by $p(z)\cdot c_j^\gamma(z_j^\gamma) = \sigma_j^\gamma(z)\sum_{(\gamma,j)\in G'(z)} p(z)\cdot c_j^\gamma(z_j^\gamma) = \sigma_j^\gamma(z)\sum_{\gamma\in\Gamma}\sum_{j\in J} p(z)\cdot c_j^\gamma(z_j^\gamma)$. However, if $z = y$, then from the definition, $V_j^\gamma(a,y) = \sigma_j^\gamma(y)\{p(y)\cdot w(a) - p(y)\cdot[1 + t(a,y)]f(a)\}$, $(\gamma, j) \in G'(y)$, and by feasibility of (f, y),

$$\begin{aligned}
& \textstyle\sum_{a\in A} V_j^\gamma(a,y) - p(y)\cdot c_j^\gamma(y_j^\gamma) \\
= \; & \textstyle\sum_{a\in A} \sigma_j^\gamma(y)\{p(y)\cdot w(a) - p(y)\cdot[1+t(a,y)]f(a)\} - p(y)\cdot c_j^\gamma(y_j^\gamma) \\
= \; & \textstyle\sigma_j^\gamma(y)\sum_{a\in A}\{p(y)\cdot w(a) - p(y)\cdot[1+t(a,y)]f(a)\} - p(y)\cdot c_j^\gamma(y_j^\gamma) \\
= \; & \textstyle\sigma_j^\gamma(y)\sum_{(\gamma,j)\in G'(y)} p(y)\cdot c_j^\gamma(y_j^\gamma) - p(y)\cdot c_j^\gamma(y_j^\gamma) \\
= \; & 0.
\end{aligned}$$

Case (B): $(\gamma, j) \in G''(z)$

$$\begin{aligned}
& \textstyle\sum_{a\in A} V_j^\gamma(a,z) - p(z)\cdot c_j^\gamma(z_j^\gamma) = \sum_{a\in A} V_j^\gamma(a,z) \\
= \; & \textstyle\sum_{a\in A}(p(z)\cdot w(a) - p(z)\cdot[1+t(a,z)]x(a,z) - \beta_j^\gamma(a,z)) \\
= \; & \textstyle p(z)\cdot\sum_{a\in A} w(a) - p(z)\cdot\sum_{a\in A}[1+t(a,z)]x(a,z) - p(z)\cdot\sum_{\gamma\in\Gamma}\sum_{j\in J} c_j^\gamma(z_j^\gamma) \\
\leqq \; & 0,
\end{aligned}$$

by $\sum_{a\in A}\beta_j^\gamma(a,z) = \sum_{(\gamma,j)\notin G''(z)} p(z)\cdot c_j^\gamma(z_j^\gamma) = \sum_{\gamma\in G}\sum_{j\in J} p(z)\cdot c_j^\gamma(z_j^\gamma)$. However, if $z = y$, then from the definition, $V_j^\gamma(a,y) = p(y)\cdot w(a) - p(y)\cdot[1+t(a,y)]f(a) - \beta_j^\gamma(a,y)$, for $(\gamma, j) \in G''(y)$, by feasibility of (f, y), and $\sum_{a\in A} \beta_j^\gamma(a, y) = \sum_{(\eta,s)\notin G''(y)} p(y)\cdot c_s^\eta(y_s^\eta)$,

$$\begin{aligned}
& \textstyle\sum_{a\in A} V_j^\gamma(a, y) \\
= \;& \textstyle\sum_{a\in A}\{p(y)\cdot w(a) - p(y)\cdot[1 + t(a, y)]f(a) - \beta_j^\gamma(a, y)\} \\
= \;& \textstyle\sum_{a\in A}\{p(y)\cdot w(a) - p(y)\cdot[1 + t(a, y)]f(a)\} - \sum_{a\in A} \beta_j^\gamma(a, y) \\
= \;& \textstyle p(y)\cdot \sum_{\gamma\in\Gamma}\sum_{j\in J} c_j^\gamma(y_j^\gamma) - p(y)\cdot \sum_{(\eta,s)\notin G''(y)} c_s^\eta(z_s^\eta) \\
= \;& \textstyle p(y)\cdot \sum_{(\eta,s)\notin G''(y)} c_s^\eta(y_s^\eta) - p(y)\cdot \sum_{(\eta,s)\notin G''(y)} c_s^\eta(y_s^\eta) \\
= \;& 0.
\end{aligned}$$

Thus, we conclude that condition (ii) of Definition 4.3.2 is satisfied for the price system p.

CONDITION (ii)

Let $a \in A$. Now, by the continuity and the strict monotonicity of U_a, it follows that $U_a(x(a, z)) = U_a(f(a))$. First note that if $z = y$, then

$$\begin{aligned}
& \textstyle p(z)\cdot[1 + t(a, z)]f(a) + \sum_{(\delta,r)\in G'(z)} V_r^\delta(a, z) + \sum_{(\eta,s)\in G''(z)} V_s^\eta(a, z) \\
= \;& p(z)\cdot[1 + t(a, z)]f(a) + \\
& \textstyle\sum_{(\delta,r)\in G'(z)} \sigma_r^\delta(z)\{p(z)\cdot w(a) - p(z)\cdot[1 + t(a, z)]x(a, z)\} + \\
& \textstyle\sum_{(\eta,s)\in G''(z)}\{p(z)\cdot w(a) - p(z)\cdot[1 + t(a, z)]x(a, z) - \beta_s^\eta(a, z)\} \\
= \;& p(z)\cdot[1 + t(a, z)]f(a) + p(z)\cdot w(a) - p(z)\cdot[1 + t(a, z)]x(a, z) \\
= \;& p(z)\cdot w(a),
\end{aligned}$$

by $f(a) = x(a.z)$, $\sum_{(\delta,r)\in G'(z)} \sigma_r^\delta(z) = 1$, and $\sum_{(\eta,s)\in G''(z)} \beta_s^\eta(a, z) = |G''(z)|\{p(z)\cdot w(a) - p(z)\cdot[1 + t(a, z)]x(a, z)\}$.

For any $g \in \mathbb{R}_+^\ell$ with $U_a(g) > U_a(f(a)) = U_a(x(a, z))$, we have

$$\begin{aligned}
& \textstyle p(z)\cdot[1 + t(a, z)]g + \sum_{(\delta,r)\in G'(z)} V_r^\delta(a, z) + \sum_{(\eta,s)\in G''(z)} V_s^\eta(a, z) \\
= \;& p(z)\cdot[1 + t(a, z)]g + \\
& \textstyle\sum_{(\delta,r)\in G'(z)} \sigma_r^\delta(z)\{p(z)\cdot w(a) - p(z)\cdot[1 + t(a, z)]x(a, z)\} + \\
& \textstyle\sum_{(\eta,s)\in G''(z)}\{p(z)\cdot w(a) - p(z)\cdot[1 + t(a, z)]x(a, z) - \beta_s^\eta(a, z)\} \\
= \;& p(z)\cdot[1 + t(a, z)]g + p(z)\cdot w(a) - p(z)\cdot[1 + t(a, z)]x(a, z) \\
\geqq \;& p(z)\cdot w(a),
\end{aligned} \tag{4.7}$$

since $p(z) \cdot g \geqq p(z) \cdot x(a,z)$ by the definition of $x(a,z)$.

Let $z \in \mathcal{Y}$ and define the budget set of agent a in the trade structure z by

$$B_z(a) = \left\{ g \in \mathbb{R}^\ell_+ \;\middle|\; p(z) \cdot [1 + t(a,z)]g + \sum_{\gamma \in \Gamma} \sum_{j \in J} V_j^\gamma(a,z) \leqq p(z) \cdot w(a) \right\}.$$

Since by the construction of $V_j^\gamma(\cdot, z)$

$$\begin{aligned} \textstyle\sum_{\gamma \in \Gamma} \sum_{j \in J} V_j^\gamma(a,z) &= \textstyle\sum_{(\delta,r) \in G'(z)} V_r^\delta(a,z) + \sum_{(\eta,s) \in G''(z)} V_s^\eta(a,z) \\ &= p(z) \cdot w(a) - [1 + t(a,z)]x(a,z) \\ &\leqq p(z) \cdot w(a), \end{aligned}$$

we conclude that $0 \in B_z(a)$. Furthermore, by (4.1) of Definition 4.2.1 and $p(z) > 0$ we have that

$$p(z) \cdot \sum_{\gamma \in \Gamma} \sum_{j \in J} c_j^\gamma(z_j^\gamma) < p(z) \cdot \sum_{a \in A} w(a).$$

Then there is at least one agent $b \in A$ such that

$$\sum_{\gamma \in \Gamma} \sum_{j \in J} V_j^\gamma(b,z) < p(z) \cdot w(b). \tag{4.8}$$

This implies that $B_z(b)$ has a non-empty interior. For every $b \in A$ satisfying (4.8) it evidently holds that $x(b,z) \in B_z(b)$.

First, we note that $x(b,z)$ is on the boundary of $B_z(b)$. Indeed if g is in the interior of $B_z(b)$, then (4.7) implies that $U_b(g) \leqq U_b(f(b)) = U_b(x(b,z))$.

Second, we claim that $x(b,z)$ is a maximal element in $B_z(b)$. Suppose not. Then there is a $g \in B_z(b)$ such that $U_b(g) > U_b(x(b,z))$. By the continuity of U_b there is a neighborhood around g such that all bundles in that neighborhood are better than $x(b,z)$. Since $B_z(b)$ has a non-empty interior, this implies that there is a bundle $h \in \int B_z(b)$ with $U_b(h) > U_b(f(b))$. But this contradicts (4.7).

This contradiction leads to the conclusion that $x(b,z)$ is indeed a maximal element in $B_z(b)$. But then the strict monotonicity of U_b excludes the possibility that certain prices are zero, i.e., $p(z) \gg 0$. (Otherwise, the existence of a maximal element would be contradicted.)

Since $p(z) \gg 0$, it is obvious that for any agent $a \in A$ we have that $B_a(z)$ is compact and convex, and $U_a(f(a)) \geqq U_a(g)$ for any $g \in B_z(a)$.

Finally, evidently $f(a) \in B_y(a)$ for every $a \in A$. Together with the above and $U_a(f(a)) = U_a(x(a,z))$ for every $z \in \mathcal{Y}$ we conclude that for every agent $a \in A$ the bundle $f(a)$ is indeed maximal in $\cup_{z \in \mathcal{Y}} B_z(a)$. This shows condition (iii).

This completes the proof Theorem 4.3.4. □

Note that the above proof is an extension of Chapter 2 to the case of multiple public projects and many private goods.

4.4 Industry-wise efficiency and trade equilibria

In Section 3, it is required that every agent takes account of all sectors and all industries in his decision. In other words, the price system and the valuation system in a trade equilibrium are defined over the economy's trade structure. In fact, we consider a relatively perfect type of agents' behavior in the economy, and require consequently strong conditions. In this section we introduce weaker notions of efficiency and equilibrium based on a structure of industries: industry-wise efficiency and industry-wise trade equilibrium.

Definition 4.4.1 *A feasible allocation (f, y) is* **industry-wise efficient** *if there is no industry A_j, $j \in J$, and no other allocation (g, z) such that*

(i) *(g,z) is feasible for A_j, $j \in J$, i.e.,*

$$\sum_{a \in A_j} [1 + t(a,z)] g(a) + \sum_{\gamma \in \Gamma} c_j^\gamma(z_j^\gamma) = \sum_{a \in A_j} w(a),$$

(ii) *for all agents in industry j, $a \in A_j$: $U_a(g(a)) \geqq U_a(f(a))$,*
for some agents in industry j, $b \in A_j$: $U_b(g(b)) > U_b(f(b))$.

Compared with the Pareto efficiency (Definition 4.3.1), the notion of industry-wise efficiency requires that no industry can insure itself a better outcome for the industry itself by changing the industry's trade structure with industry-wise transaction costs and setup costs.

Definition 4.4.2 *A feasible allocation $(f, y) \in \Phi$ is an* **industry-wise trade equilibrium** *in the economy $\mathbb{E}$ if there exist an industry-wise price system $p_j \colon \mathcal{Y} \to \Delta$ and an industry-wise valuation system $v_j^\gamma \colon A_j \times \mathcal{Y} \to \mathbb{R}$ for $\gamma \in \Gamma$ and $j \in J$ such that*

(i) *each trade link y_j^γ minimizes the deficit with a balanced budget separately, i.e., for all $z \in \mathcal{Y}$ with $\gamma \in \Gamma$ and $j \in J$:*

$$\sum_{a \in A_j} v_j^\gamma(a,z) - p_j(z) \cdot c_j^\gamma(z_j^\gamma) \leqq \sum_{a \in A_j} v_j^\gamma(a,y) - p_j(y) \cdot c_j^\gamma(y_j^\gamma) = 0, \text{ and}$$

(ii) *for every industry $j \in J$ and every agent $a \in A_j$, the consumption bundle $f(a)$ maximizes the utility U_a on the union of the budget sets $\cup_{z \in \mathcal{Y}} B_z(a)$, where for any $z \in \mathcal{Y}$*

$$B_z(a) = \left\{ g \in \mathbb{R}_+^\ell \;\middle|\; p_j(z) \cdot [1 + t(a,z)]g + \sum_{\gamma \in \Gamma} v_j^\gamma(a,z) \leqq p_j(z) \cdot w(a) \right\}.$$

Note that each industry has its own price system p_j for $j \in J$ defined on $\mathcal{Y}$, and its own valuation system v_j^γ define on $A_j \times \mathcal{Y}$ for $\gamma \in \Gamma$ and $j \in J$. Furthermore, we require similar conditions to a trade equilibrium, but these conditions in an industry-wise trade equilibrium are satisfied *within* every industry. This implies that every industry behaves independently even though each trade link is evaluated with respect to the economy's trade structure.

Theorem 4.4.3 *Let $\mathbb{E}$ be an economy with costly trade links. Then the following statements holds:*

(a) *If for every $a \in A$ the utility function U_a is monotone, then every industry-wise trade equilibrium is industry-wise efficient.*

(b) *If for every $a \in A$ the utility function U_a is continuous, quasi-concave, and strictly monotone, then every industry-wise efficient allocation can be supported as an industry-wise trade equilibrium.*

The proofs are similar to that of Theorem 4.3.4 except that the efficiency concept is restricted to an industry, and are relegated to Appendix.

Now we clarify the relationship between the Pareto efficiency and the industry-wise efficiency.

Theorem 4.4.4 *Let $\mathbb{E}$ be an economy with costly trade links. If for every $a \in A$ the utility function is monotone, then every trade equilibrium is industry-wise efficient.*

The proof is trivial, and is omitted. Now we have an example showing that an industry-wise efficient allocation may not be Pareto efficient, and so the set of industry-wise efficient allocations is strictly larger than the Pareto efficient set.

Example 4.4.5 We extend Example 4.3.3 to the case of two sectors and two industries in which each locality also consists of a single agent. Let $\ell = 2$, and $A = \{a, a', b, b'\}$ with $a \in A_1^1$, $a' \in A_2^1$, $b \in A_1^2$, and $b' \in A_2^2$. The utility functions and initial endowments are:

$$U_a(f_1, f_2) = U_{a'}(f_1, f_2) = f_1 f_2 \quad \text{and} \quad w(a) = w(a') = (3, 0),$$
$$U_b(f_1, f_2) = U_{b'}(f_1, f_2) = f_1 f_2 \quad \text{and} \quad w(b) = w(b') = (0, 3),$$

for $a \in A_1^1$, $a' \in A_2^1$, $b \in A_1^2$, and $b' \in A_2^2$, and for every commodity bundle $f = (f_1, f_2) \in \mathbb{R}_+^2$. Furthermore, $\mathcal{Y}_1^{12} = \mathcal{Y}_2^{12} = \{0, 1\}$ with $c_1^{12}(1) = c_2^{12}(1) = (1, 1)$, and $c_1^{12}(0) = c_2^{12}(0) = (0, 0)$. For $a \in A_1^1$, $a' \in A_2^1$, $b \in A_1^2$, and $b' \in A_2^2$, the transaction costs are given as

$$t(a, 1, 1) = t(a, 1, 0) = t(a, 0, 1) = t(a, 0, 0) = 0,$$
$$t(a', 1, 1) = t(a', 1, 0) = t(a', 0, 1) = t(a', 0, 0) = 0,$$
$$t(b, 1, 1) = t(b, 1, 0) = t(b, 0, 1) = 0, \quad t(b, 0, 0) = 4,$$
$$t(b', 1, 1) = t(b', 1, 0) = t(b', 0, 1) = 0, \quad t(b', 0, 0) = 4.$$

This completes the description of the economy.

Based on the result of Example 4.3.3, we consider a case of $y_1^{12} = y_2^{12} = 1$, i.e., each industry establishes its own trade link, and for $a \in A_1^1$, $a' \in A_2^1$, $b \in A_1^2$, and $b' \in A_2^2$,

$$f(a) = (1.6, 1.6), \ f(a') = (1.6, 1.6), \ f(b) = (0.4, 0.4) \text{ and } f(b') = (0.4, 0.4).$$

This is obviously an industry-wise efficient allocation, or an industry-wise trade equilibria with an industry-wise price system $p_1(1, 1) = p_2(1, 1) = (0.5, 0.5)$ and an industry-wise valuation system v_j^{12} for $J \in \{1, 2\}$ such that $p_1(1, 1) = p_2(1, 1) = (0.5, 0.5)$,

$$v_1^{12}(a, 1, 1) = v_2^{12}(a', 1, 1) = -0.1, \ v_1^{12}(b, 1, 1) = v_2^{12}(b', 1, 1) = 1.1,$$

and for all $y \neq (1, 1)$, $p_1(y), p_2(y) \in \Delta$, $v_j^{12}(a, y) = 0$ for all $a \in A$. But, this may not be a Pareto efficient allocation. All agents can improve upon their utilities by establishing only one trade link between A^1 and A^2, i.e., $y_1^{12} = 1, y_2^{12} = 0$, or $y_1^{12} = 0, y_2^{12} = 1$, because within each sector it is not necessary to build an additional trade link since there exist no transaction costs. Therefore, we have a Pareto efficient allocation such as $(y_1^{12}, y_2^{12}) = (1, 0)$, or $(y_1^{12}, y_2^{12}) = (0, 1)$, and

$$f(a) = (1.6 + \eta_{11}, 1.6 + \nu_{11}), \quad f(a') = (1.6 + \eta_{12}, 1.6 + \nu_{12}),$$
$$f(b) = (0.4 + \eta_{21}, 0.4 + \nu_{21}), \quad f(b') = (0.4 + \eta_{22}, 0.4 + \nu_{22}),$$

where

$$\eta_{11} + \eta_{12} + \eta_{21} + \eta_{22} = 1, \quad \nu_{11} + \nu_{12} + \nu_{21} + \nu_{22} = 1, \text{ and}$$
$$\eta_{11}, \eta_{12}, \eta_{21}, \eta_{22}, \nu_{11}, \nu_{12}, \nu_{21}, \nu_{22} \geqq 0.$$

This Pareto efficient allocation depends on how to share the setup cost $c_j^{12}(1) = (1, 1)$ for $j = 1$ or $j = 2$. Furthermore, if interpreted as a replication economy, two agents from the same sector (or, two agents with the same utility function and the same endowment) may have different allocations, and are treated unequally in the Pareto efficient allocation. ◇◇

Appendix

Proof of Theorem 4.4.3 (a)

Let (f, y) be an industry-wise trade equilibrium with an industry-wise price system $p = (p_1, \ldots, p_n) \in \Delta^n$ and an industry-wise valuation system $v' = \langle v_j^{\gamma\prime} \rangle_{\gamma \in \Gamma; j \in J}$. We show that (f, y) is industry-wise efficient.

Suppose to the contrary that (f, y) is not industry-wise efficient. Then there exist an industry A_j, $j \in J$, and a feasible allocation (g, z) such that

(i) (g, z) is feasible for A_j:

$$\sum_{a \in A_j} [1 + t(a, z)] g(a) + \sum_{\gamma \in \Gamma} c_j^\gamma(z_j^\gamma) = \sum_{a \in A_j} w(a), \quad \text{and}$$

(ii) for all agents in the industry j, $a \in A_j$: $U_a(g(a)) \geqq U_a(f(a))$,
for some agents in the industry j, $b \in A_j$: $U_b(g(b)) > U_b(f(b))$.

Condition (ii) of Definition 4.4.2 and the monotonicity of the utility function imply that for all $a \in A_j$ we have that

$$p_j(z) \cdot [1 + t(a, z)] g(a) + \sum_{\gamma \in \Gamma} v_j^\gamma(a, z) \geqq p_j(z) \cdot w(a),$$

and for some $b \in A_j$ we have

$$p_j(z) \cdot [1 + t(a, z)] g(b) + \sum_{\gamma \in \Gamma} v_j^\gamma(b, z) > p_j(z) \cdot w(b).$$

Hence,

$$p_j(z) \cdot \sum_{a \in A_j} [1 + t(a, z)] g(a) + \sum_{a \in A_j} \sum_{\gamma \in \Gamma} v_j^\gamma(a, z) > p_j(z) \cdot \sum_{a \in A_j} w(a).$$

We conclude that

$$\begin{aligned} 0 &= \textstyle\sum_{\gamma \in \Gamma} (\sum_{a \in A_j} v_j^\gamma(a, y) - p_j(y) \cdot c_j^\gamma(y_j^\gamma)) \\ &\geqq \textstyle\sum_{\gamma \in \Gamma} (\sum_{a \in A_j} v_j^\gamma(a, z) - p_j(z) \cdot c_j^\gamma(z_j^\gamma)) \\ &> p_j(z) \cdot \textstyle\sum_{a \in A_j} (w(a) - [1 + t(a, z)] g(a)) - p_j(z) \cdot \sum_{\gamma \in \Gamma} c_j^\gamma(z_j^\gamma) \\ &= p(z) \cdot 0 = 0. \end{aligned}$$

This is a contradiction. □

Proof of Theorem 4.4.3 (b)

Let (f, y) be an industry-wise stable allocation, and let $a \in A_j$, $j \in J$, be arbitrary. We define

$$F_j'(a) := \{g \in \mathbb{R}_+^\ell \mid U_a(g) > U_a(f(a))\}.$$

Note that $F_j'(a) \neq \emptyset$ is defined the same as that in the proof of Theorem 4.3.4 (b), and have the same properties. Let for $j \in J$ and $z \in \mathcal{Y}$

$$F_j'(z) := \sum_{a \in A_j} [1 + t(a, z)] F_j'(a) + \{\sum_{\gamma \in \Gamma} c_j^\gamma(z_j^\gamma) - \sum_{a \in A_j} w(a)\}.$$

The set $F_j'(z)$, $j \in J$, is non-empty, open, convex, and bounded from below. An element in $F_j'(z)$ represents the excess demand corresponding to a strictly Pareto superior allocation for A_j under the arbitrary trade structure $z \in \mathcal{Y}$.

We now construct a price system for private goods in the industry $j \in J$, $p_j(z) \in \Delta$ for all $z \in \mathcal{Y}$. Because the allocation (f, y) is industry-wise efficient, we have $0 \notin F_j(z)$, $j \in J$. By an application of Minkowski's separating hyperplane theorem to 0 and the convex set $F_j'(z)$, $j \in J$, there exists a normal vector $p_j(z) \neq 0$ such that $p_j(z) \cdot F_j'(z) \geqq 0$ for $j \in J$. Since $F_j'(z)$, $j \in J$ is bounded from below as well as comprehensive, it is obvious that $p_j(z) > 0$, $j \in J$. We can now scale the vector $p_j(z)$ without loss of generality to achieve $p_j(z) \in \Delta$. In this way we have defined an industry-wise price system $p_j \colon \mathcal{Y} \to \Delta$, $j \in J$.

Before we construct an industry-wise valuation system $v_j^\gamma \colon A_j \times \mathcal{Y} \to \mathbb{R}$ for $\gamma \in \Gamma$ and $j \in J$, let for every $j \in J$ and every $a \in A_j$ the vector $x(a, z) \in \mathbb{R}_+^\ell$ be chosen such that, in case $z \neq y$,

(a) $p_j(z) \cdot x(a, z) = \inf p_j(z) \cdot F_j(a) \geqq 0$;

(b) $U_a(x(a, z)) \geqq U_a(f(a))$,

and in case $z = y$, $x(a, z) = x(a, y) = f(a)$. Clearly, such vectors exist, because $F_j(a) \subset \mathbb{R}_+^\ell$, and $p_j(z) > 0$, $j \in J$. Furthermore, for $j \in J$ we define two disjoint sets G_j' and G_j'' as follows: for any $z \in \mathcal{Y}$ and $j \in J$

$$\begin{aligned} G_j'(z) &:= \{\gamma \in \Gamma \mid c_j^\gamma(z_j^\gamma) > 0\}, \\ G_j''(z) &:= \{\gamma \in \Gamma \mid c_j^\gamma(z_j^\gamma) = 0\}. \end{aligned}$$

Note that $\sum_{\gamma\in\Gamma} c_j^\gamma(z_j^\gamma) = \sum_{\gamma\in G_j'(z)} c_j^\gamma(z_j^\gamma) = \sum_{\gamma\notin G_j''(z)} c_j^\gamma(z_j^\gamma)$.
Let $V_j^\delta(a,z)$ for $\delta \in G_j'(z)$, $z \in \mathcal{Y}$, and $a \in A_j$, $j \in J$, be defined by

$$V_j^\delta(a,z) := \sigma_j^\delta(z)\{(p_j(z)\cdot w(a) - p_j(z)\cdot[1+t(a,z)]x(a,z)\},$$

where

$$\sigma_j^\delta(z) = \frac{p_j(z)\cdot c_j^\delta(z_j^\delta)}{\sum_{\delta\in G_j'(z)} p_j(z)\cdot c_j^\delta(z_j^\delta)} > 0.$$

It is obvious that such $\{\sigma_j^\delta(z)\}_{\delta\in G_j'(z)}$ exist for any $z \in \mathcal{Y}$ and $j \in J$.

Moreover, we define $V_j^\eta(a,z)$ for $\eta \in G_j''(z)$, $z \in \mathcal{Y}$, and $a \in A_j$, $j \in J$, by

$$V_j^\eta(a,z) := p_j(z)\cdot w(a) - p_j(z)\cdot[1+t_j(a,z)]x(a,z) - \beta_j^\eta(a,z),$$

where $\beta_j^\eta(a,z)$ is determined by

(a) $\sum_{a\in A_j} \beta_j^\eta(a,z) = \sum_{\gamma\notin G_j''(z)} p_j(z)\cdot c_j^\gamma(z_j^\gamma)$, and

(b) $\sum_{\eta\in G_j''(z)} \beta_j^\eta(a,z) = |G_j''(z)|\{p_j(z)\cdot w(a) - p_j(z)\cdot[1+t(a,z)]x(a,z)\}$.

It is also obvious that for every $a \in A_j$, $j \in J$, such $\{\beta_j^\eta\}_{\eta\in G_j''(z)}$ exist. Then, in the remaining proof, i.e., in the check up of the two requirements of an industry-wise trade equilibrium, we apply the same procedure as the proof of Theorem 4.3.4 (b), and the remaining proof is omitted. □

Chapter 5

Efficiency and Egalitarian-Equivalence in Economies with a Public Project*

5.1 Introduction

Efficiency and fairness are unquestionably important issues in a resource allocation problem. Efficiency is typically formulated in the sense of Pareto efficiency: an allocation is said to be Pareto efficient if there does not exist another allocation which is unanimously preferred to it. One main concept to evaluate the fairness of an allocation is 'egalitarian-equivalence'. An allocation is said to be *egalitarian-equivalent* by Pazner and Schmeidler (1978) if there exists a fixed commodity bundle (the same for each agent) that is considered by each agent to be indifferent to the bundle that he actually gets in the allocation under consideration. Following Pazner and Schmeidler (1978), Moulin (1987) devises the concepts of 'egalitarian-equivalent cost sharing' mechanism in producing a public good and of 'egalitarian-equivalent allocation' for an economy with one private good and one public good. Moulin (1987) shows that an egalitarian-equivalent cost sharing allocation is in the core of the economy. Weber and Wiesmeth (1990) also derive a theorem that a cost share equilibrium is efficient

*This is joint work with Robert P. Gilles. A earlier version of this chapter was listed as Department Working Paper E93-26.

and egalitarian-equivalent.

This paper discusses a model of an economy with public projects. A coalition of economic agents can only attain a public project if it has sufficient resources as well as a subcoalition that is able to execute that project. Compared with a standard approach to public goods defined on an Euclidean space, our setup is a typical extension of Mas-Colell (1980) who studies a model with one private good and an abstract set of public projects without any mathematical structure such as a vector space or an ordering among public projects. Concerning the efficiency and egalitarian-equivalence in economies with a public project we prove that there exist efficient and egalitarian-equivalent allocations in an economy with arbitrary but finite number of private goods and a public project, and show that egalitarian-equivalence, however, does not guarantee efficiency. In relation to the results of recent studies such as Moulin (1987) and Weber and Wiesmeth (1990) for economies with standard public goods, every efficient and egalitarian allocation can be supported as valuation equilibrium, but valuation equilibria may not be efficient and egalitarian-equivalent. Furthermore, efficient and egalitarian-equivalent allocations also may not be in the core.

5.2 The model

Let A be a finite set of agents. We assume that the agents in A are endowed with initial bundle of ℓ private commodities as well as with abilities to cooperatively execute one and only one public project, or remain at the "status quo" situation, meaning that the unique public project is not executed. The latter situation reduces the economy to a pure exchange economy. There are various groups of agents that can execute the public project, each representing a different way to execute it. In the sequel we denote by $\mathcal{P} \subset 2^A$ the collection of all coalitions of economic agents that are able to execute the unique public project, including the "status quo" denoted by $\emptyset \in \mathcal{P}$.

Since the initial endowments of the private commodities enter the model in the usual fashion by a function $w: A \to \mathbb{R}^\ell_+$ with $\overline{w} = \sum_{a \in A} w(a) \gg 0$,[1] we may

[1]Our symbols for inequalities between vectors are $\geqq, >, \gg$.

summarize the use of private commodities in the various ways to execute the public project with a cost function $c\colon \mathcal{P} \to \mathbb{R}^\ell_+$, where we assume that for every $S \in \mathcal{P}$, $c(S) \leqq \overline{w}$, and $c(\emptyset) = 0$. In the sequel we associate a *production plan*, consisting of a coalition $S \in \mathcal{P}$ that executes the public project and a vector of optimal inputs $c(S)$, with S.

Every agent $a \in A$ evaluates bundles of private commodities alongside production plans according to the utility function $u_a\colon \mathbb{R}^\ell_+ \times \mathcal{P} \to \mathbb{R}$. We emphasize that this definition implies the existence of (widespread) externalities concerning the different production plans regarding the public project. A utility function u_a is *monotone* if for all $x, y \in \mathbb{R}^\ell_+$ and all $S \in \mathcal{P}$ with $x \gg y$, $u_a(x, S) > u_a(y, S)$. Furthermore, u_a is *strictly monotone* if for all $x, y \in \mathbb{R}^\ell_+$ and all $S \in \mathcal{P}$ with $x > y$, $u_a(x, S) > u_a(y, S)$.

An *allocation* is a pair (y, T) where $y\colon A \to \mathbb{R}^\ell_+$ is an assignment of private commodities and $T \in \mathcal{P}$ is a production plan. An allocation (y, T) is *feasible* if $\sum_{a \in A} y(a) + c(T) = \overline{w}$. We denote the collection of all feasible allocations by Φ. It is obvious that $\Phi \neq \emptyset$.

A feasible allocation $(y, T) \in \Phi$ is *Pareto efficient* if there is no feasible allocation (x, S) such that (i) for all $a \in A$, $u_a(x, S) \geqq u_a(y, T)$, and (ii) there exists $b \in A$ such that $u_b(x, S) > u_b(y, T)$.

5.3 Efficient and egalitarian-equivalent allocations

The concept of egalitarian-equivalence is proposed by Pazner and Schmeidler (1978) for allocating private goods. We adapt their definition to an economy with a public project and ℓ private goods.

Definition 5.3.1 *An allocation* (y, T) *is* **egalitarian-equivalent** *if there is an* $(x, S) \in \mathbb{R}^\ell_+ \times \mathcal{P}$ *such that* $u_a(x, S) = u_a(y(a), T)$ *for every agent* $a \in A$. *The pair* (x, S) *is called an egalitarian reference allocation.*

Before we formulate the main result about the existence of efficient and egalitarian-equivalent allocations in an economy with a public project, we have to introduce the *unanimity condition*. This condition states that in an economy with a public project there exists an allocation which every agent unanimously likes (a 'good' public project) or dislikes (a 'bad' public project).

Definition 5.3.2 *The collection of production plans $\mathcal{P}$ is said to satisfy the* **unanimity condition** *if for every bundle $x \in \mathbb{R}^\ell_+$ there exist a positive real number λ and a feasible allocation $(y, T) \in \Phi$ such that*

(i) *for every agent $a \in A$, $u_a(\lambda x, \emptyset) \leqq u_a(y(a), T)$, or*

(ii) *for every agent $a \in A$, $u_a(\lambda x, \emptyset) \geqq u_a(y(a), T)$.*

The next theorem states that there exist efficient and egalitarian-equivalent allocations in economies with a public project.

Theorem 5.3.3 *Let $\mathcal{P}$ satisfy the unanimity condition. Furthermore, assume that for every agent $a \in A$ the utility function u_a is continuous and strictly monotone. Then for every $x \gg 0$ in $\mathbb{R}^\ell_+$ there is a positive real number $\overline{\lambda}$ and an efficient egalitarian-equivalent allocation $(y, T) \in \Phi$ with $(\overline{\lambda} x, \emptyset)$ being an egalitarian reference allocation for it, i.e., for every agent $a \in A$,*

$$u_a(y(a), T) = u_a(\overline{\lambda} x, \emptyset).$$

PROOF

Let $x \in \mathbb{R}^\ell_{++}$. Suppose case (i) of unanimity condition holds.[2] For any $\lambda > 0$ define the set $M(\lambda)$ of allocation according as:

$$M(\lambda) := \left\{ (y, T) \;\middle|\; \begin{array}{l} (y, T) \in \Phi \text{ such that } u_a(\lambda x, \emptyset) \leqq u_a(y(a), T) \text{ for all } a \in A \\ \text{and } \sum_{a \in A} y(a) + c(T) = \overline{w} \end{array} \right\}.$$

By condition (i) $M(\lambda) \neq \emptyset$ for some $\lambda > 0$. Now let $\overline{\lambda} := \sup\{\lambda \mid M(\lambda) \neq \emptyset\}$. From the continuity of utility function also follows that $M(\overline{\lambda}) \neq \emptyset$.

We shall prove that $(y, T) \in M(\overline{\lambda})$ is (i) an egalitarian-equivalent and (ii) Pareto efficient allocation.

(i) Let $(y, T) \in M(\overline{\lambda})$ and suppose $u_a(y(a), T) > u_a(\overline{\lambda} x, \emptyset)$ for some $a \in A$. From the continuity of utility functions there exists $y' \colon A \to \mathbb{R}^\ell_+$ with $\sum_{b \in A} y'(b) = \sum_{b \in A} y(b)$ such that $u_a(y'(a), T) > u_a(\overline{\lambda} x, \emptyset)$, and $y'(a) < y(a)$ as well as $y'(b) > y(b)$ for $b \neq a$. In that case by the srtict monotonicity of u_b

$$u_b(y'(b), T) > u_b(y(b), T) \geqq u_b(\overline{\lambda} x, \emptyset).$$

[2]In case of (ii) of Definition 5.3.2 the proof is similar and therefore is omitted for brevity.

Thus (y', T) is Pareto superior to (y, T), since $\sum_{a\in A} y'(a)+c(T) = \sum_{a\in A} y(a)+c(T) = \overline{w}$. But then there is $\tilde{\lambda}$ such that $\tilde{\lambda} > \overline{\lambda}$ with, for all $a \in A$,

$$u_a(y'(a), T) \geqq u_a(\tilde{\lambda}x, \emptyset) > u_a(\overline{\lambda}x, \emptyset).$$

This is a contradiction to the definition of $\overline{\lambda}$. Thus, $u_a(\overline{\lambda}x, \emptyset) = u_a(y(a), T)$ for all $a \in A$ and (y, T) is an egalitarian-equivalent allocation with $(\overline{\lambda}x, \emptyset)$ being an egalitarian reference allocation.

(ii) Now suppose that (y, T) is not Pareto-efficient. Then there is a feasible allocation (y', T') such that

$$\forall a \in A : u_a(y'(a), T') \geqq u_a(y(a), T), \text{ and}$$
$$\exists b \in A : u_b(y'(b), T') > u_b(y(b), T).$$

It follows that $u_a(y'(a), T') \geqq u_a(\overline{\lambda}x, \emptyset)$ for every agent $a \in A$ and $u_b(y'(b), T') > u_b(\overline{\lambda}x, \emptyset)$ for some agent $b \in A$. The continuity and strict monotonicity of utility functions once more imply the existence of an allocation (y'', T') such that $u_a(y''(a), T') > u_a(\overline{\lambda}x, \emptyset)$ for every agent $a \in A$, which contradicts to the supremum property of $\overline{\lambda}$. So, (y, T) must be Pareto-efficient. □

From Example 5.3.4 below we conclude that in an economy with a public project egalitarian-equivalent allocations may not be Pareto efficient.

Example 5.3.4 Let there be two agents, so $A = \{a, b\}$, and let the set of production plans be given by $\mathcal{P} = \{\emptyset, \{b\}, A\}$. That is, b can build the project alone, or with the cooperation of a, but a alone cannot build the project. Let there be one private good ($\ell = 1$) and let $c(T) = 0$ for all $T \in \mathcal{P}$. Let the endowment be given by $(w(a), w(b))$, where $w(a) + w(b) = \overline{w} > 0$. Finally, let the agents' utilities be as follows, where $\alpha > \beta > \gamma > 0$:

$$u_a(y(a), \emptyset) = \gamma y(a),\ u_a(y(a), \{b\}) = \gamma y(a) \text{ and } u_a(y(a), A) = \alpha y(a);$$

$$u_b(y(b), \emptyset) = \gamma y(b),\ u_b(y(b), \{b\}) = \alpha y(b) \text{ and } u_b(y(b), A) = \beta y(b).$$

Now let w be given by $w(a) = w(b) = 2$. We shall show that (i) an allocation $(y(a), y(b), A)$ can be efficient and egalitarian-equivalent with some $(\overline{\lambda}x, \emptyset)$ being an egalitarian reference bundle, and that (ii) an allocation $(y(a), y(b), \{b\})$ can be egalitarian-equivalent with some $(\overline{\lambda}' x, \emptyset)$ being an egalitarian reference bundle, but is not efficient.

(i) From Definition 5.3.1 and Theorem 5.3.3 we get the following equations for coalition A:

$$y(a) + y(b) = \overline{w} = 4$$

$$u_a(y(a), A) = u_a(\overline{\lambda} x, \emptyset) \Rightarrow \alpha y(a) = \gamma(\overline{\lambda} x) \Rightarrow y(a) = \frac{\gamma}{\alpha}\overline{\lambda} x$$

$$u_b(y(b), A) = u_b(\overline{\lambda} x, \emptyset) \Rightarrow \beta y(b) = \gamma(\overline{\lambda} x) \Rightarrow y(b) = \frac{\gamma}{\beta}\overline{\lambda} x$$

From this it is deduced that the allocation $(\frac{4\beta}{\alpha+\beta}, \frac{4\alpha}{\alpha+\beta}, A)$ is egalitarian-equivalent with $(\frac{4\alpha\beta}{\gamma(\alpha+\beta)}, \emptyset)$ being an egalitarian reference bundle. This allocation is also Pareto efficient, and neither agent can bring upon it a unilateral improvement.

(ii) By the same way as above we get the following equations for coalition $\{b\}$:

$$y(a) + y(b) = \overline{w} = 4$$

$$u_a(y(a), \{b\}) = u_a(\overline{\lambda}' x, \emptyset) \Rightarrow \gamma y(a) = \gamma(\overline{\lambda}' x) \Rightarrow y(a) = \overline{\lambda}' x$$

$$u_b(y(b), \{b\}) = u_b(\overline{\lambda}' x, \emptyset) \Rightarrow \alpha y(b) = \gamma(\overline{\lambda}' x) \Rightarrow y(b) = \frac{\gamma}{\alpha}\overline{\lambda}' x$$

Thus, the allocation $(\frac{4\alpha}{\alpha+\gamma}, \frac{4\gamma}{\alpha+\gamma}, \{b\})$ is egalitarian-equivalent with $(\frac{4\alpha}{\alpha+\gamma}, \emptyset)$ being an egalitarian reference bundle. However, this allocation is not Pareto efficient because agent a can swith to the allocation $(4(1-\frac{\alpha\gamma}{\beta(\alpha+\gamma)}), \frac{4\alpha\gamma}{\beta(\alpha+\gamma)}, A)$ and his utility is improved such that

$$u_a(\frac{4\alpha}{\alpha+\gamma}, \{b\}) = \frac{4\alpha\gamma}{\alpha+\gamma} < u_a(4(1-\frac{\alpha\gamma}{\beta(\alpha+\gamma)}), A) = 4\alpha(1-\frac{\alpha\gamma}{\beta(\alpha+\gamma)})$$

$$u_b(\frac{4\gamma}{\alpha+\gamma}, \{b\}) = u_b(\frac{4\alpha\gamma}{\beta(\alpha+\gamma)}, A) = \frac{4\alpha\gamma}{\alpha+\gamma}.$$

Next we characterize efficient and egalitarian-equivalent allocations with the concepts of valuation equilibrium and the core. The concept of valuation equilibrium is first introduced by Mas-Colell (1980) in an economy with one private good and an abstract set of public projects. Diamantaras and Gilles (1994) extend Mas-Colell's concept to an economy with any arbitrary finite number of public goods and a set of public projects without any sturcture, and show that valuation equilibria decetralize Pareto efficient allocations.

A *valuation* is a pair (p, v) where $p: \mathcal{P} \rightarrow S^{\ell-1}$ is a function assigning to every public project $T \in \mathcal{P}$ a normalized price vector $p(T) \in S^{\ell-1} = \{p \in \mathbb{R}^{\ell}_{+} \mid \sum_{i=1}^{\ell} = 1\}$ for the private commodities given that the production plan T is selected and $v: A \times \mathcal{P} \rightarrow \mathbb{R}$ is a valuation function for the production plans. We remark that widespread externalities, the different use of resources in each production plan — as given in the cost function c — induce price changes in the private sector in case a different production plan is executed. This is reflected in the valuation of the private sector through the price function p from the collection of production plans $\mathcal{P}$ into the price simplex $S^{\ell-1}$ rather than a unique price system $p \in S^{\ell-1}$. The valuation function v for the production of the public project clearly allows agents to be taxed as well as compensated concerning the public project.

Definition 5.3.5 *A feasible allocation* $(y, T) \in \Phi$ *is a* **valuation equilibrium** *if there exists a valuation* (p, v) *such that:*

(i) $\sum_{a \in A} v_a(T) = p(T) \cdot c(T)$;

(ii) *T maximizes* $\sum_{a \in A} v_a(S) - p(S) \cdot c(S)$ *on* $\mathcal{P}$, *and*

(iii) *for every agent $a \in A$:* $(y(a), T)$ *maximizes* u_a *on*

$$\Big\{(z, S) \in \mathbb{R}^{\ell}_{+} \times \mathcal{P} \mid p(S) \cdot z + v_a(S) = p(S) \cdot w(a)\Big\} \text{ and}$$

for all $S \in \mathcal{P}$: $v_a(S) \leqq p(S) \cdot w(a)$.

We now show that efficient and egalitarian-equivalent allocations are valuation equilibria. For this result we need the following condition, which is an extension and strengthening of the indispensibility condition of Mas-Colell (1980).

Definition 5.3.6 *The collection of production plans $\mathcal{P}$ satisfies the* **essentiality condition** *if:*

(i) *For every agent $a \in A$, each bundle $y \in \mathbb{R}^\ell_+$, and all potential production plans S, $T \in \mathcal{P}$, there exists a bundle $z \in \mathbb{R}^\ell_+$ such that $u_a(z, S) > u_a(y, T)$.*

(ii) *For every agent $a \in A$, every $y \in \mathbb{R}^\ell_+ \backslash \{0\}$, and all production plans S, $T \in \mathcal{P}$, $u_a(y, S) > u_a(0, T) = 0$.*

The first condition states that all production plans in priciple can be compensated by sufficiently large amounts of private goods. In the second condition, setting $u_a(0, T) = 0$ is just a normalization, the important part of this latter statement is that $u_a(0, S) = u_a(0, T) = 0$ for all production plans S, $T \in \mathcal{P}$. This is similar to the indispensibility condition of Mas-Colell (1980).[3]

Theorem 5.3.7 *Let $\mathcal{P}$ satisfy the essentiality condition. Furthermore, assume that for every agent $a \in A$ the utility function u_a is continuous, quasi-concave, and strictly monotone. Then every efficient and egalitarian-equivalent allocation can be supported as a valuation equilibrium.*

The proof is a modification of the one given in Diamantaras and Gilles (1994).
Even though efficient and egalitarian-equivalent allocations can be supported by valuation equilibria, the following example shows that valuation equilibria do not have to be egalitarian-equivalent.

Example 5.3.8 Consider Example 5.3.4 with $w(a) = w(b) = 2$. We have additional assumptions for parameters α, β, and γ as follows: $2\beta > \alpha > \beta > \gamma > 0$ and $\frac{\beta(\alpha-\gamma)}{\alpha(\alpha-\beta)} > 1$. Since the cost of the production plan is zero, i.e., $c(T) = 0$ for all $T \in \mathcal{P}$, the valuation functions $v_a(T) + v_b(T) = 0$ for all $T \in \mathcal{P}$ from Definition 3.5 (i). Now we consider a valuation equilibrium (y, T) with valuation (p, v) such that $p(\emptyset) = p(\{b\}) = p(A)$ and $v_a(T) + v_b(T) = 0$ for all $T \in \mathcal{P}$. For example we assign for all $T \in \mathcal{P}$, $p(T) = 1$ and

$$v_a(\emptyset) = v_b(\emptyset) = 0, \ v_a(\{b\}) = v_b(\{b\}) = 0,$$

[3]About the essentiality condition, we refer to Diamantaras and Gilles (1994), and Diamantaras, Gilles, and Scotchmer (1994).

$$v_a(A) = \frac{2(\alpha-\beta)}{\beta}, v_b(A) = -\frac{2(\alpha-\beta)}{\beta}.$$

Now the allocation $(4 - \frac{2\alpha}{\beta}, \frac{2\alpha}{\beta}, A)$ is indeed a valuation equilibrium with valuation (p, v) as given above. We check whether this allocation is egalitarian-equivalent under the assumptions $2\beta > \alpha > \beta > \gamma > 0$ and $\frac{\beta(\alpha-\gamma)}{\alpha(\alpha-\beta)} > 1$. Suppose that the allocation $(y'(a), y'(b), A) = (4 - \frac{2\alpha}{\beta}, \frac{2\alpha}{\beta}, A)$ is egalitarian-equivalent with $(\overline{\lambda}x, \emptyset)$ being an egalitarian reference allocation. Then from Definition 3.1 and Theorem 3.3,

$$u_a(4 - \frac{2\alpha}{\beta}, A) = u_a(\overline{\lambda}x, \emptyset) \Rightarrow 2\alpha(4 - \frac{2\alpha}{\beta}) = \gamma(\overline{\lambda}x) \Rightarrow \overline{\lambda}x = 2\frac{\alpha}{\gamma}(4 - \frac{2\alpha}{\beta}),$$

$$u_b(\frac{2\alpha}{\beta}, A) = u_b(\overline{\lambda}x, \emptyset) \Rightarrow 2\alpha = \gamma\overline{\lambda}x \Rightarrow \overline{\lambda}x = 2\frac{\alpha}{\gamma}.$$

If $\alpha = 2\beta$, the allocation $(4 - \frac{2\alpha}{\beta}, \frac{2\alpha}{\beta}, A)$ is egalitarian-equivalent. But this contradicts the assumption $2\beta > \alpha$. Thus, the valuation equilibrium allocation $(y'(a), y'(b), A) = (4 - \frac{2\alpha}{\beta}, \frac{2\alpha}{\beta}, A)$ is not egalitarian-equivalent.

Next, we compare efficient and egalitarian-equivalent allocations with the core in an economy with a public project. Moulin (1987) shows that with one public good economy an egalitarian-equivalent allocation is in the core, and Thomson (1989) argues that even with ℓ private goods an efficient and egalitarian-equivalent allocation obtained by requiring the reference bundle to be the equal division of private goods is in the core. We define the core in our setup as follows:

Definition 5.3.9 *A feasible allocation $(y, T) \in \Phi$ is in the* **core** *if there is no coalition $S \subset A$ such that there exist $z: S \to \mathbb{R}^\ell_+$ and a subcoalition $S' \subset S$ with $S' \in \mathcal{P}$ for which it holds that $u_a(z(a), S') > u_a(y(a), T)$ for all $a \in S$, and $\sum_{a \in S} z(a) + c(S') = \sum_{a \in S} w(a)$.*

This core definition assumes free access to the production technology in the tradition of Foley (1970), but in our setting free access is natural because it inheres in the description of the technology: a coalition can execute the project if and only if a group of agents within the coalition can execute the project. This is an important difference with the formulation of the core given by Mas-Colell (1980). He assumes that *any* coalition with sufficient purchasing power is able to "buy" a production plan concerning the execution of the public project. In our case the establishment of the uniquely given public project is directly linked to a group of economic agents that is able to execute it. Hence, the public project can never be "bought", but always has to be executed by a coalition of economic agents with this abiliy.

Unfortunately, efficient and egalitarian-equivalent allocations may not be in the core in our framework. The next example illustrates this.

Example 5.3.10 Consider again Example 5.3.4 with $w(a) = w(b) = 2$. The allocation $(\frac{4\beta}{\alpha+\beta}, \frac{4\alpha}{\alpha+\beta}, A)$ is efficient and egalitarian-equivalent with $(\overline{\lambda}x, \emptyset) = (\frac{4\alpha}{\alpha+\beta}, \emptyset)$ being an egalitarian reference bundle. However, this allocation is not in the core, because there exists a feasible allocation $(2, 2, \{b\})$ such that under $\alpha > \beta > \gamma > 0$:

$$u_b(\frac{4\alpha}{\alpha+\beta}, A) = \frac{4\alpha\beta}{\alpha+\beta} < u_b(2, \{b\}) = 2\alpha.$$

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