TAAF STOPPING RULES FOR MAXIMIZING THE UTILITY OF ONE-SHOT SYSTEMS

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(ABSTRACT)

Test-analyze-and-fix (TAAF) is the most commonly recognized method of improving system reliability. The work presented here addresses the question of when to stop testing during TAAF programs involving one-shot systems when the number of systems to be produced is predetermined and the probabilities of identifying and successfully correcting each failure mode are less than one. The goal here is to determine when to cease testing to maximize utility where utility is defined as the number of systems expected to perform successfully in the field after deployment of the lot.

Two TAAF stopping rules are presented. Simulation is used to model TAAF execution under different reliability growth conditions. Four discrete reliability growth models (DRGM's) are used to generate "real world" reliability growth and to estimate reliability growth using hypothetical observed success/failure data. Ranges for the following parameters are considered: starting reliability, growth rate, maximum achievable reliability, number of systems to be produced, probability of incorrectly identifying a failure mode, and probability of an unsuccessful design modification.

Conclusions are drawn regarding stopping rule performance in terms of stopping rule signal location, utility loss, achieved reliability, and fraction tested. Both rules perform well and are implementable from a practical standpoint. Specific recommendations for stopping rule implementation are given based on the controllable factors, estimation methodology and lot size.

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CHAPTER 1: PROBLEM STATEMENT AND BACKGROUND

1.1 Problem Statement

Test-analyze-and-fix (TAAF) is the most commonly recognized method of improving system reliability. During the development of a new system, a TAAF program is often implemented to eliminate failure modes inherent in the initial system design. A TAAF program consists of testing systems individually, attempting to determine causes when failures occur, and modifying the system design in an attempt to eliminate each identified failure mode. Each successful design modification results in an increase in system reliability. The key to successful reliability growth through test-analyze-and-fix implementation is the answer to the question “How much testing is enough?”, or “When is the potential reliability gain not worth the expense of an additional test?”

The purpose of this research is to address this question for TAAF programs involving one-shot, or destructively tested, systems. More specifically, this research addresses the situation in which the number of systems to be produced is predetermined and the probabilities of identifying and successfully correcting each failure mode are less than one.

The goal here is to determine when to cease testing to maximize operational utility rather than to meet traditional TAAF stopping conditions. Operational utility is defined as the number of systems expected to perform successfully in the field after deployment of the lot. The utility is simply the product of the current reliability and the number of systems remaining and, therefore, changes after each TAAF test. The stopping rules investigated in this research are designed to be evaluated after each trial rather than to be used to calculate a predetermined stopping point. The rules aim to signal the analyst to stop testing when the utility is at its maximum value. Reliability analysts performing TAAF procedures on new one-shot systems can use the results of this research to determine if a new system undergoing TAAF testing would benefit from using one of the stopping rules examined here.

1.2 Traditional Test-Analyze-and-Fix Stopping Conditions

When the systems undergoing TAAF testing are one-shot systems and the number of systems to be produced is a predetermined fixed value, *N*, a tradeoff exists between the reliability gained from the TAAF testing and the number of systems remaining after the TAAF program is complete. In the extreme, testing all *N* systems would result in the maximum reliability gain but the minimum number of remaining systems, namely zero. As discussed in the literature review, traditional TAAF programs usually test systems successively until a specified reliability goal is realized or test a previously specified subset of systems based on economic, probabilistic, or expected value criteria.

1.3 TAAF Reliability Growth

Each test failure, and subsequent design modification, if successful, completes a test "stage." The TAAF process begins in stage one during which systems are tested individually. The result is a string of consecutive test successes (possibly zero) followed by the first test failure. The system tested after this first failure and subsequent redesign begins stage two. Stage two continues through a number of consecutive successes until the second failure is observed, at which point stage three begins, and so on. For example, in the TAAF realization in Figure 1.1, stage one consists of trials one, two, and three. Stage two consists of trials four, five, six, seven, and eight. These stages are therefore, by definition, geometric in length and theoretically increasing in mean length over time.

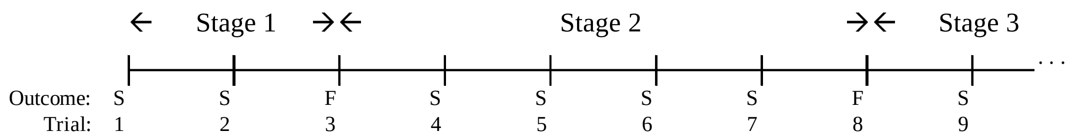

Figure 1.1 Stage Definition

The reliability growth associated with stage changes can be modeled using various discrete reliability growth models (DRGM's). Fries and Sen (1996) explain that "reliability-growth models permit the data from all of the test stages to be used to estimate reliabilities…" They describe DRGM's as models "for which the relevant data comprise sequences of dichotomous success-failure outcomes from successive system configurations of stages." Their survey paper covers three types of DRGM's: nonparametric, parametric, and miscellaneous. The genesis of DRGM's has its roots in nonparametric methods that do not directly prescribe a specific anticipated pattern for reliability growth across test stages. Parametric DRGM's can be divided into two subclasses. One parametric class can be described in terms of an initial reliability, a final reliability, and a growth rate. The other parametric class is described by the learning curve property. The third type of DRGM Fries and Sen describe includes time series approaches, smoothing models, and regression procedures.

In particular analytical applications, DRGM's may serve distinct roles. For example, in simulation, DRGM's can be used to model "real world" reliability growth. This "real world" reliability growth is the true increase in reliability associated with successful design modifications and is never known to an analyst executing a TAAF program. These DRGM's represent reliability growth based on the current stage of the TAAF program and have shapes similar to the example curves in Figure 1.2.

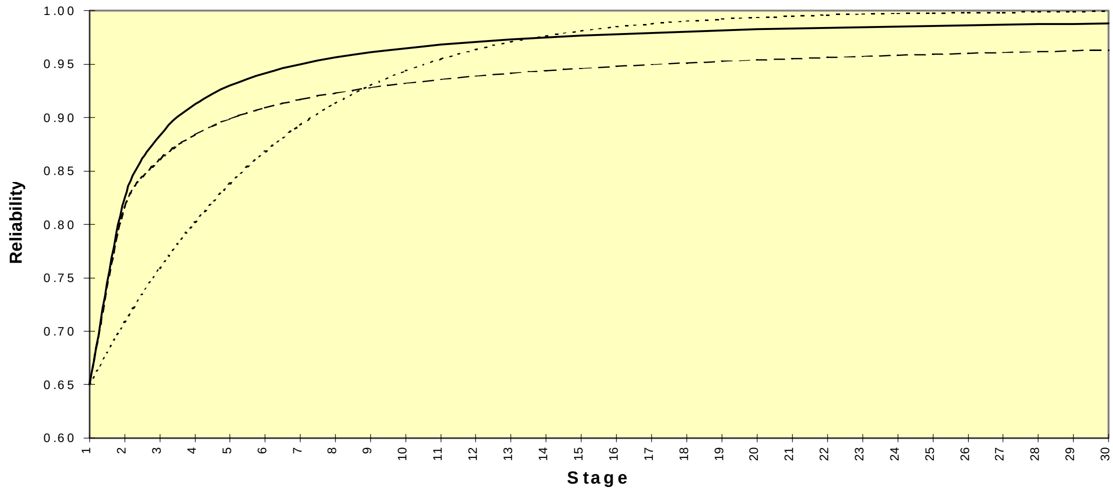

Figure 1.2 Real World Reliability Growth Model Examples

When coupled with parameter estimation techniques, DRGM's can also be used to estimate reliability growth based on observed success/failure data. Analysts performing TAAF programs use these estimation methodologies to track reliability growth and determine when to cease testing. The DRGM's selected for implementation in this research are presented in Chapter 3.

1.4 Summary of Solution Approach

The first step in this research is to define the algebraic notation required to model the TAAF process. Second, some reasonable assumptions are made. Two TAAF stopping rules for maximizing operational utility during the testing process are then developed. The rules are expressed in terms of estimated utilities and reliabilities obtained from DRGM-based computations. The rules' performances under different reliability growth conditions are analyzed.

Simulation is used to model TAAF execution under different reliability growth conditions. Four DRGM's are used to generate "real world" reliability growth scenarios and to estimate reliability growth using hypothetical observed success/failure data. Ranges for the following parameters are considered: starting reliability, growth rate, maximum achievable reliability, number of systems to be produced, probability of incorrectly identifying a failure mode, and probability of an unsuccessful design modification. Conclusions regarding stopping rule performance, in terms of stopping rule signal location, utility loss, achieved reliability, and fraction tested are drawn on the basis of performance measures calculated during the simulation.

1.5 Summary of Results

Preliminary analyses permitted the exclusion of two parameters from the simulation experimental design. Interpretation of the resulting simulation output leads to several conclusions discussed in depth in Chapter 4. In summary, the rules perform well and are implementable from a practical standpoint. All factors considered, as well as many interactions, have significant effects on the performance measures with the exception of the probability that reliability increases follow failures. Using exponential smoothing estimation rather than the Fries estimation methodology (both presented in Chapter 3) yields superior rule performances. In conjunction with exponential smoothing estimation, the U-max rule performs consistently better than the U-exp rule with the exception of cases in which the lot size is small. It is therefore recommended that if the lot size is small, the U-exp rule should be used with the exponential smoothing estimation methodology, otherwise, the U-max rule be used in conjunction with exponential smoothing estimation.

CHAPTER 2: LITERATURE REVIEW

Traditional TAAF stopping rules make use of planned reliability growth curves and signal when a predetermined reliability goal is believed to be realized.

MIL HDBK 189 (1981) treats the question of when to stop TAAF testing as a highly subjective feature of the testing program. Testing length is described as a feature built-in to the program prior to the commencement of testing and subject to mid-program decisions made by the program manager. The handbook states that a planned growth curve should be developed through a joint effort between program manager and contractor, based on several factors including: the availability of time, money, personnel, prototypes for testing, and test schedule. "This planned growth curve lays out a detailed plan of how the reliability growth will actually be achieved." The handbook also states that "the program manager should have sufficient remaining resources to take meaningful corrective action [during the TAAF program]… to achieve the reliability objectives."

Chenoweth (1986) criticizes the concepts suggested by MIL HDBK 189. He explains, as does MIL HDBK 189, that reliability testing is typically stopped when the cumulative reliability reaches a specified value. Using this type of rule parallels the construction of the planned reliability growth curve discussed in MIL HDBK 189. Chenoweth identifies the problem associated with using reliability goals to be a prior knowledge requirement. He states that implementing this type of rule requires knowledge "with some precision" of the initial reliability, growth rate, and stopping time that will satisfy the specified reliability value.

Dwyer (1987) proposes that to optimize with respect to cost and schedule, TAAF testing of one-shot systems should begin at lower levels of integration referred to as subsystems. His first step in TAAF plan development is to draw the reliability growth curve based on an instantaneous reliability of zero for the first item and a simple growth formula with an assumed growth rate value of 0.4. Using this graph, he then calculates the number of allowable failures at the system and subsystem levels that will still meet the reliability requirements imposed by the planned growth curve. He then vaguely states that "the test quantities for each of these [subsystem] levels should reflect the points on the cumulative reliability growth curve. They also need to demonstrate a high enough reliability that testing at the next higher level of integration will have a high probability of success."

More recent research on TAAF stopping rules examine the problem from the perspective of maximizing utility.

Gaver and Jacobs (1994) develop an expression for the optimal stopping point in TAAF programs for one-shot systems. This expression, based on a given lot size and Poisson seeded failure modes, maximizes the expected number of successful missions after testing is complete. Solving this equation provides the analyst with a predetermined number of trials to execute to maximize utility.

Huang, McBeth, and Vardeman (1996) investigate the question of "How much testing is enough?" for TAAF developmental testing of one-shot systems for which each test produces a binary outcome, the cost of redesign is negligible, and the procurement budget for both testing and purchase is fixed. These authors use dynamic programming to identify optimal and sub-optimal test-plans to maximize utility for a specific two-state model of system reliability. The chief weakness in their analysis is their unrealistic model under which reliability can only assume one of two fixed values.

Unlike the references cited from the 1980's in which the goal of TAAF testing is to reach a prespecified reliability, the goal of the research presented here is to maximize utility, a non-prespecified expected number of systems to function properly after deployment. Unlike Gaver and Jacobs' method in which the stopping rule is evaluated prior to beginning the TAAF process, the rules presented here are evaluated after each trial during the TAAF process. The work presented here also greatly extends that of Huang, McBeth and Vardeman in which similar rules are examined in an extremely limited scope of reliability growth scenarios. The two stopping rules presented here are evaluated after each trial and aim to signal at the peak of the true utility curve. The performance of the two rules are investigated under a wide variety of reliability growth scenarios.

CHAPTER 3: MODELS AND ANALYSES

3.1 Assumptions and Notation

The following assumptions are made:

Assume the total number of systems to be produced is predetermined but the number of systems to be dedicated to the TAAF phase is unspecified. Assume that TAAF testing proceeds in each testing stage until either a failure occurs or the stopping rule signals and a decision is made to stop testing.

Responses to test failures are not perfect. The probability, p, of incorrectly identifying a failure mode is greater than zero. In this case, the corrective action taken is ineffective because the failure mode was incorrectly identified. The probability, s, that a fix associated with a correctly identified failure mode does not eliminate its targeted failure mode is also greater than zero. In this case, the TAAF analyst may believe the fix is successful in eliminating the failure mode when it is actually unsuccessful. Combining these two probabilities into one expression yields the probability of an observed failure resulting in an increase in reliability, $(1 - p)(1 - s)$.

Stopping rule conditions are checked prior to each system test. The first check could be delayed until a number of trials or stages are completed, depending on the warm-up behavior of the DRGM being used to estimate reliability growth.

The following notation is used:

N *fixed* predetermined number of systems to be produced.

p *fixed* probability of incorrectly identifying a failure mode.

s *fixed* probability of an unsuccessful fix.

t *variable* cumulative number of completed TAAF trials.

$K(t_+)$ *variable* stage number *after* completion of trial t and any related system modifications.

$R\{K(t_+)\}$ *variable* system reliability *within* stage $K(t_+)$.

$R(t_+)$ *variable* system reliability *after* completion of trial t and any related system modifications.

$U(t_+)$
$$U(t_+) = (N - t)R(t_+) \tag{3.1}$$
variable operational utility *after* completion of trial t and all related system modifications.

$\hat{\cdot}$ statistical estimate of the quantity "·" based on a specific DRGM and the observed TAAF data.

$\cdot_+$ refers to the point in time *after* trial "·" and any related system modifications.

3.2 TAAF Stopping Rules

The utility function defined above decreases with each test success because t increases by one and the reliability does not change. The utility function can increase only if a failure occurs and the resulting increase in reliability is sufficiently large (to be demonstrated shortly). Due to the shape of the DRGM's modeling "real world" growth, the magnitude of the reliability increase associated with each fix decreases over time. Therefore, initially, the utility function is shaped as a series of peaks and descents as demonstrated in Figure

3.1. The peaks are associated with failures and the descents with series of successes. Over time the peaks in the utility function form a concave curve as demonstrated in Figure 3.2. The utility peaks initially increase until a maximum is reached, at which point the trend heads directly to zero. Stopping rules designed to maximize operational utility would ideally signal at the maximum.

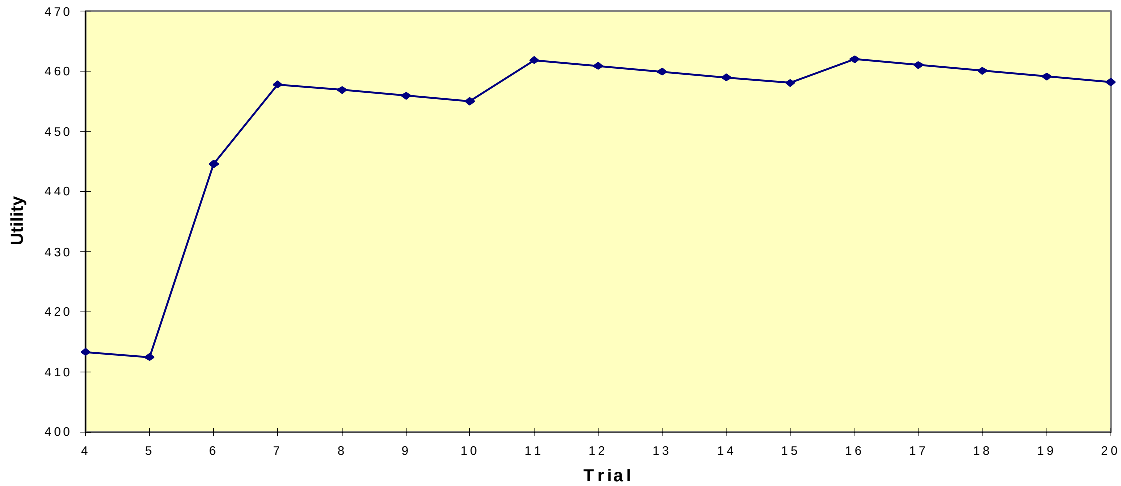

Figure 3.1 Initial Utility Function Behavior Example

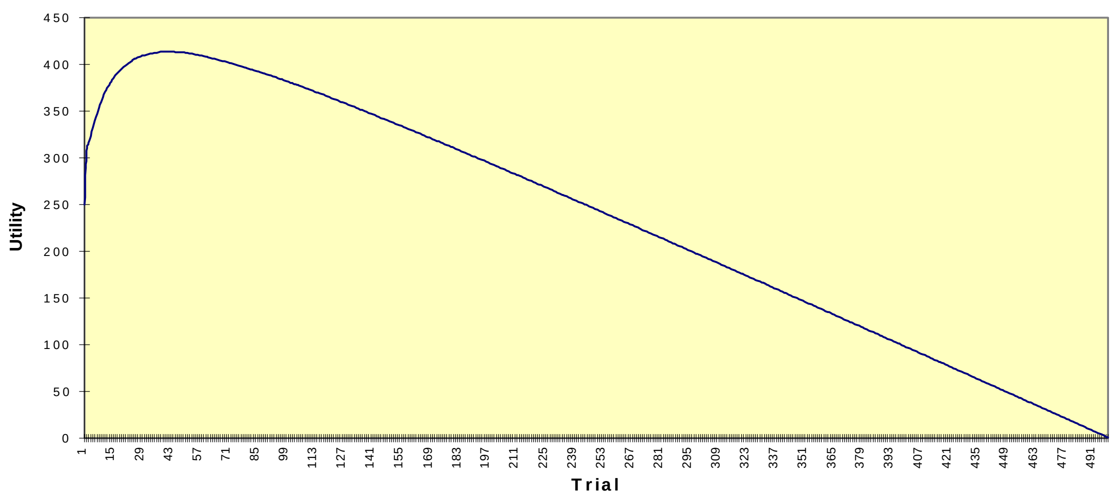

Figure 3.2 Concave Utility Function Example

The operational utility will only increase if a failure occurs and the increase in reliability associated with the failure is sufficiently large. Therefore, one logical stopping rule condition, U-max, is to stop testing if the estimated utility *given a failure on the next trial* is less than or equal to the current utility estimate, or

$$\hat{U}((t+1)_+ \,|\, \text{next trial fails}) \le \hat{U}(t_+) \,. \tag{3.2}$$

In terms of reliability,

$$\hat{U}((t+1)_+ \,|\, \text{next trial fails}) \le \hat{U}(t_+)$$

$$\hat{U}((t+1)_+ | \text{next trial fails}) - \hat{U}(t_+) + \hat{R}(t_+) \le \hat{R}(t_+) \tag{3.3}$$

$$(N-t-1)\hat{R}((t+1)_+ | \text{next trial fails}) - (N-t)\hat{R}(t_+) + \hat{R}(t_+) \le \hat{R}(t_+) \tag{3.4}$$

$$(N-t-1)\hat{R}((t+1)_+ | \text{next trial fails}) - (N-t-1)\hat{R}(t_+) \le \hat{R}(t_+) \tag{3.5}$$

$$\frac{\hat{R}((t+1)_+ | \text{next trial fails}) - \hat{R}(t_+)}{\hat{R}(t_+)} \le \frac{1}{(N-t-1)} \,. \tag{3.6}$$

Equation 3.4 follows from equation 3.3 for two reasons. The first reason is because the definition of utility holds true in the estimation world as it does in the true world. The second reason is because the definition of utility also holds true in the conditional case. This latter point is true because the condition that the next trial fails applies to a trial that is assumed to have already been executed, in this case trial (t + 1). If the current utility estimate is $\hat{U}(t_+)$ and trial $z = t + 1$ is executed and fails, the new reliability estimate given this failure is $\hat{R}(z_+)$ and the new utility estimate is

$$(N - z)\ \hat{R}(z_+) = (N - t - 1)\ \hat{R}((t+1)_+|\text{trial z failed}) .$$

The right hand side of inequality 3.6 represents the magnitude of the minimum increase in reliability necessary to produce an increase in utility. This left hand side can be viewed as the discrete equivalent of the derivative of the reliability growth curve. Because of the concavity and the nature of diminishing returns inherent in the growth curves, this derivative equivalent is always positive, yet decreases over time. Because this expression is equivalent to the U-max expression, its behavior supports the previous assertion that the utility curve is concave.

Another stopping rule condition, U-exp, is motivated by an expected value perspective. The U-exp rule signals if the estimated expected utility after the next trial (regardless of its outcome) is less than or equal to the current utility estimate, or

$$\begin{aligned}&\hat{U}((t+1)_+|\text{next trial fails})\,P(\text{next trial fails}) + \\ &\hat{U}((t+1)_+|\text{next trial succeeds})\,P(\text{next trial succeeds}) \le \hat{U}(t_+) .\end{aligned} \tag{3.7}$$

In terms of reliability,

$$\begin{aligned}&\hat{U}((t+1)_+|\text{next trial fails})\,P(\text{next trial fails}) + \\ &\hat{U}((t+1)_+|\text{next trial succeeds})\,P(\text{next trial succeeds}) \le \hat{U}(t_+)\end{aligned}$$

$$\begin{aligned}&(N - t - 1)\hat{R}((t+1)_+|\text{next trial fails})(1 - \hat{R}(t_+)) + (N - t - 1)\hat{R}(t_+)^2 \le \\ &(N - t)\hat{R}(t_+)\end{aligned} \tag{3.8}$$

$$(N-t-1)\hat{R}((t+1)_+|\text{next trial fails})(1-\hat{R}(t_+)) + (N-t-1)\hat{R}(t_+)^2 \leq (N-t-1)\hat{R}(t_+) + \hat{R}(t_+) \tag{3.9}$$

$$\hat{R}((t+1)_+|\text{next trial fails})(1-\hat{R}(t_+)) + \hat{R}(t_+)^2 - \hat{R}(t_+) \leq \frac{\hat{R}(t_+)}{(N-t-1)} \tag{3.10}$$

$$\frac{\hat{R}((t+1)_+|\text{next trial fails}) - \hat{R}(t_+)}{\hat{R}(t_+)} \leq \frac{1}{(N-t-1)(1-\hat{R}(t_+))}. \tag{3.11}$$

3.3 Discrete Reliability Growth Models Used

The DRGM's selected for implementation in this simulation study are chosen for illustrative purposes. Use of other methodologies (see Fries 1996) in conjunction with the U-max and U-exp rules should be justified with simulation prior to implementation. It is also important to note that in order to be compatible with the U-max and U-exp stopping rules, a "real world" growth generating DRGM must be able to forecast one stage ahead. Similarly, an estimation methodology must be able to forecast one trial ahead. The models selected for this study are discussed below.

Lloyd and Lipow (1984)

Two models proposed by Lloyd and Lipow are simulated: their parametric true growth model of the form

$$R\{K(t_+)\} = R_\infty - \alpha / K(t_+) \tag{3.12}$$

where $0<\alpha<1$, and their exponential smoothing estimation methodology of the form

$$\hat{R}(t_+) = \alpha\hat{X}[K(t_+)] + (1-\alpha)\hat{R}\{K(t_+) - 1\} \tag{3.13}$$

where 0<α<1 and $\hat{X}$ *[K(t_+)]* = an estimator of reliability in stage *K(t_+)* which is obtained solely from data in stage *K(t_+)*. Mann (1974) offers a Bayes point estimate of reliability for binomial sampling based on a uniform prior. This estimate is used in this research and therefore:

$$\hat{X}[K(t_=)] = \frac{c+1}{T+2} \tag{3.14}$$

where *T* = number of trials completed in stage *K(t_+)* and *c* = number of successes observed in stage *K(t_+)*.

Fries (1993)

Fries' true DRGM and corresponding estimation methodology, based on the maximum likelihood estimators for his true growth model parameters, are also simulated. His model is based on the learning curve property employed in the Crow/AMSAA model but incorporates the alternate testing strategy assumed in this research. It is of the form:

$$R\{K(t_+)\} = R_\infty - \lambda'[K(t_+)^{\beta'} - (K(t_+) - 1)^{\beta'}]^{-1} \tag{3.15}$$

where $\beta' > 1$ and $\lambda' = \lambda^{\beta'}$, $0 < \lambda' < 1$. The estimation methodology is of the same form,

$$\hat{R}(t_+) = R_\infty - \hat{\lambda}'[K(t_+)^{\hat{\beta}'} - (K(t_+) - 1)^{\hat{\beta}'}]^{-1} \tag{3.16}$$

where

$$\hat{\beta}' = \sum_{i=1}^{K(t_+)-1} \log\left(\frac{t}{T_i}\right) \Big/ (K(t_+)-1) \quad \text{and} \quad \hat{\lambda} = (K(t_+)-1) \Big/ t^{\hat{\beta}'} \tag{3.17}$$

for T_i = cumulative number of trials through the end of stage *i*.

It is important to note that the Fries estimation methodology will initially report negative reliability estimates if the first few stages do not exhibit reliability growth (i.e., stage two is shorter than stage one). Because of this behavior, when using the Fries estimation methodology, checking the stopping rule conditions is delayed until the reliability estimates are positive.

Virene (1968)

The original Gompertz DRGM, characterized by an S-shaped growth pattern and postulated by Virene, is also used to simulate true reliability growth and is of the form

$$R\{K(t_+)\} = R_\infty b^{c^{K(t_+)}} \tag{3.18}$$

where $0<b<1$ and $0<c<1$.

3.4 Performance Measures

Four performance measures are used to represent the performance of the two stopping rules. These performance measures are defined as follows:

lag = (number of trials executed) - (trial after which true utility maximum occurred).

% utility lost = ((true utility maximum) - (true utility at signal)) / (true utility maximum).

fraction tested = (number of trials executed) / *N*.

achieved reliability = true reliability at signal .

3.5 Parameter Ranges

Six factors were initially considered for inclusion in the simulation experimental design: exponential smoothing constant (α), initial reliability, growth rate, lot size (N), limiting reliability (R_∞), and (1 - p)(1 - s).

3.5.1 Preliminary Study

After conducting a preliminary study using a small subset of reliability growth scenarios, the first two factors, the exponential smoothing constant and initial reliability, were determined to have insignificant effects on rule performance.

Iterating through substitution, the exponential smoothing estimation equation (3.13) results in the following equations for end of stage reliability estimates:

$$
\begin{aligned}
&\hat{R}(1) = \hat{X}[1] \\
&\hat{R}(2) = \alpha(\hat{X}[2] - \hat{X}[1]) + \hat{X}[1] \\
&\ldots \\
&\hat{R}(5) = \alpha^4(\hat{X}[1] - \hat{X}[2]) + \alpha^3(\hat{X}[3] + 3\hat{X}[2] - 4\hat{X}[1]) + \\
&\qquad \alpha^2(6\hat{X}[1] - 3\hat{X}[2] - 2\hat{X}[3] - \hat{X}[4]) + \\
&\qquad\quad \alpha(\hat{X}[5] + \hat{X}[4] + \hat{X}[3] + \hat{X}[2] - 4\hat{X}[1]) + \hat{X}[1] \\
\\
&\hat{R}(6) = \alpha^5(\hat{X}[2] - \hat{X}[1]) + \alpha^4(5\hat{X}[1] - 4\hat{X}[2] - \hat{X}[3]) + \\
&\qquad \alpha^3(-10\hat{X}[1] + 6\hat{X}[2] + 3\hat{X}[3] - 3\hat{X}[4]) + \\
&\qquad\quad \alpha^2(10\hat{X}[1] - 4\hat{X}[2] - 3\hat{X}[3] - 2\hat{X}[4] - \hat{X}[5]) + \\
&\qquad\qquad \alpha(\hat{X}[6] + \hat{X}[5] + \hat{X}[4] + \hat{X}[3] + \hat{X}[2] - 5\hat{X}[1]) + \hat{X}[1]
\end{aligned}
\tag{3.19}
$$

The higher order alpha terms quickly become negligible, regardless of the value of α, leaving the lower order alpha terms and $\hat{X}[1]$ to determine the reliability estimate. The magnitude of the coefficients of the lower order α terms are small enough to render the choice of α insignificant. This insignificance is demonstrated in Table 3.1. For successive stage lengths increasing in length by one trial, the choice of α quickly makes little difference in the reliability estimate.

The irrelevance of the value of α is further demonstrated by a limited scope simulation study in which the initial reliability was 0.6, $R_\infty = 0.95$, $p = s = 0.0$, and $N = 500$. This reliability growth scenario was replicated 1,000 times using mid-range growth rate values for each of the three true growth models (F = Fries, LL = Lloyd and Lipow, G = Gompertz) and four values of α. As demonstrated in Table 3.2, α had little impact on the percent utility lost. The exponential smoothing constant, α, was therefore fixed at 0.7 for all other simulations.

stage		**alpha**		
lengths	**0.5**	**0.6**	**0.7**	**0.8**
4	0.67	0.67	0.67	0.67
5	0.69	0.70	0.70	0.70
6	0.72	0.73	0.74	0.74
7	0.75	0.76	0.76	0.77
8	0.77	0.78	0.79	0.79
9	0.80	0.80	0.81	0.81
10	0.81	0.82	0.83	0.83
11	0.83	0.84	0.84	0.84
12	0.84	0.85	0.85	0.85
13	0.86	0.86	0.86	0.86
14	0.87	0.87	0.87	0.87
15	0.87	0.88	0.88	0.88
16	0.88	0.88	0.89	0.89
17	0.89	0.89	0.89	0.89
18	0.89	0.90	0.90	0.90
19	0.90	0.90	0.90	0.90
20	0.90	0.91	0.91	0.91

Table 3.1 Exponential Smoothing Reliability Estimates for Various Values of α

	True Growth Model		
alpha	**F**	**LL**	**G**
0.5	0.026	0.031	0.047
0.6	0.023	0.021	0.056
0.7	0.021	0.026	0.061
0.8	0.022	0.035	0.074

Table 3.2 Average % Utility Lost Over 1,000 Replications for Various Levels of α

A second preliminary study was performed to investigate the influence of initial reliability on rule performance. In this study, R_∞ = 0.95, $p = s = 0.9$, N = 500, and growth rates were taken to be mid-range. This scenario was replicated 1,000 time for four values of initial reliability. For the true growth model/estimation methodology combinations Fries/exponential smoothing (F ESM) and Gompertz/Fries (G F), the initial reliability had a negligible effect on average percent utility lost as demonstrated in Table 3.3. The initial reliability was therefore fixed at 0.65 for all other simulations using the Fries or Gompertz true growth models. Under the Lloyd and Lipow true growth, initial reliability is dictated by the growth rate.

Initial R	F ESM	G F
0.6	0.024	0.071
0.7	0.022	0.075
0.8	0.021	0.079

Table 3.2 Average % Utility Lost Over 1,000 Replications for Various Levels of Initial Reliability

3.5.2. Experimental Design

The remaining four factors were each taken at three levels:

Growth Rate = {low, mid, high}

Fries (F): β = {0.8, 0.65, 0.5}

Lloyd and Lipow (LL): α = {0.5, 0.35, 0.2}

Gompertz (G): c = {0.9, 0.8, 0.7}

N = {75, 350, 700}

$(1 - p)(1 - s)$ = {0.85, 0.9, 0.95}

R_∞ = {0.85, 0.9, 0.95}

Using a full 3^4 factorial design for each of the twelve true-growth-model/estimation-methodology/stopping-rule combinations (F F MAX, F F EXP, F ESM MAX, F ESM EXP, LL F MAX, LL F EXP, LL ESM MAX, LL ESM EXP, G F MAX, G F EXP, G ESM MAX, G ESM EXP) would have resulted in $12(3^4) = 972$ scenarios. Due to computer time constraints, a 1/3 fraction or 3^{4-1} fractional factorial of the 3^4 design was implemented (see Montgomery, 1967).

The design space was further reduced due to an incompatibility between the Fries estimation methodology and the U-exp stopping rule. Using the Fries estimation methodology, the U-exp stopping rule is always satisfied and therefore immediately signals. The Fries estimation methodology was therefore not simulated in conjunction with the U-exp rule.

Eliminating the Fries estimation/U-exp combination resulted in $(9)3^{4-1} = 243$ scenarios. Each scenario was replicated 1,000 times bringing the total number of TAAF processes simulated to 243,000. Aggregate statistics for the four performance measures each of the scenarios are given in Appendix B.

CHAPTER 4: RESULTS AND CONCLUSIONS

4.1 ANOVA Results

The Statistical Consulting Center of Virginia Tech performed the analysis of variance calculations on the simulation output data (see Appendix A.3 for SAS code). Because the statistical consultants scaled each performance measure mean by its standard deviation prior to executing the ANOVA, the variance of each mean was transformed to one, and the mean squares reported in Table B.2 are actually χ^2 statistics. The ANOVA output is used as a screening mechanism to filter out insignificant factors. Based on the χ^2 statistics, the following factors defined in Appendix B are considered to have significant effects on the rule performance measures indicated:

Lag

Main Effects: Estimation Methodology, Growth Rate, Lot Size, Limiting Reliability, Stopping Rule, True Growth Model

Interactions: Estimation Methodology*Growth Rate, Estimation Methodology*Lot Size, Estimation Methodology*True Growth Model, Growth Rate*Lot Size, Stopping Rule*Lot Size, True Growth Model*Lot Size

% Utility Loss

Main Effects: Estimation Methodology, Growth Rate, Lot Size, Stopping Rule, True Growth Model

Interactions: Estimation Methodology*Growth Rate, Estimation Methodology*Lot Size, Estimation Methodology*True Growth Model, Stopping Rule*Lot Size, Stopping Rule*Limiting Reliability, True Growth Model*Growth Rate, True Growth Model*Limiting Reliability

Achieved Reliability

Main Effects: Estimation Methodology, Growth Rate, Lot Size, Limiting Reliability, Stopping Rule, True Growth Model

Interactions: Estimation Methodology*Lot Size, Estimation Methodology*Limiting Reliability, Estimation Methodology*True Growth Model, Lot Size*Limiting Reliability, Stopping Rule*Lot Size, Stopping Rule*True Growth Model, True Growth Model*Growth Rate, True Growth Model*Lot Size

Fraction Tested

Main Effects: Estimation Methodology, Growth Rate, Lot Size, Limiting Reliability, Stopping Rule, True Growth Model

Interactions: Estimation Methodology*N, Estimation Methodology*Growth Rate, Estimation Methodology*True Growth Model, Growth Rate*Lot Size, Stopping Rule*Lot Size, True Growth Model*Limiting Reliability

It is interesting and surprising that the $(1 - p)(1 - s)$ factor is not a significant factor for any of the performance measures. To investigate this result, a more narrowly focused simulation study was conducted. When the $(1 - p)(1 - s)$ factor was varied over the range {0.05, 0.4, 0.95}, similar results were obtained for a variety of reliability growth scenarios. Typical results over 1,000 replications from a representative reliability growth scenario are given in Figure 4.1. This figure demonstrates that smaller values of $(1 - p)(1 - s)$ result in higher utility losses. The $(1 - p)(1 - s)$ factor did not emerge significant in the large simulation study because it was varied over the realistic range {0.85, 0.9, 0.95} and does not have a significant effect in that range.

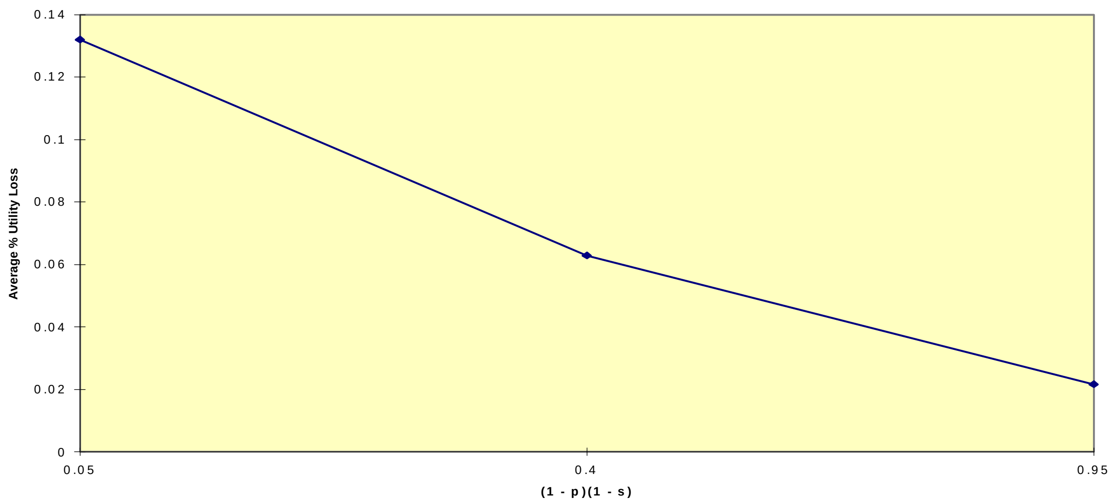

Figure 4.1 Average % Utility Loss vs. (1 - *p*)(1 - *s*)

Although several interactions between the true growth model and other factors are considered significant, these interactions are not investigated in detail. Since the underlying true reliability growth behavior of a TAAF process is never known, the nature of interactions between the growth behavior and other factors are not of interest. The true reliability growth model main effect is examined, however, to establish rule robustness with respect to true reliability growth behavior.

4.2 Simulation Results

Plots were generated for each of the significant factors to determine the nature of their effects on the performance measures. Figures 4.2, 4.3, 4.4, and 4.5 represent the average effect of each estimation methodology / stopping rule combination on each of the performance measures. As demonstrated in Figure 4.2, the Fries estimation methodology paired with the U-max rule results in a large positive lag, while the exponential smoothing estimation methodology paired with the U-max or the U-exp rule signals very close to the true utility maximum. Because of this difference in lag, the Fries estimation methodology paired with the U-max rule also results in a greater fraction tested and therefore a greater achieved reliability than the exponential smoothing method paired with either stopping rule as seen in Figures 4.3 and 4.4.

More importantly, as demonstrated in Figure 4.5, the larger lag associated with the Fries estimation methodology also results in a significantly larger loss in utility than both exponential smoothing estimation combinations. Because using the Fries estimation methodology with the U-max stopping rule results in inferior rule performance, the remainder of the analysis is restricted to rule performance under the exponential smoothing estimation methodology. It is also important to recognize the consistent superior performance of the U-max over the U-exp rule under the exponential smoothing methodology demonstrated in Figures 4.2, 4.4, and 4.5.

Figures 4.6 and 4.7 represent the effect of true reliability growth on lag and percent utility loss, respectively. Figure 4.6 illustrates that on average, under exponential smoothing estimation, the Fries true growth model yields late a signal, the Lloyd and Lipow model yields a signal very close to the true utility maximum, and the Gompertz model yields an early signal. Despite these differences in signal location, in terms of percent utility lost the rules are fairly robust with respect to different underlying reliability growth. This is depicted by the small variations in Figure 4.7.

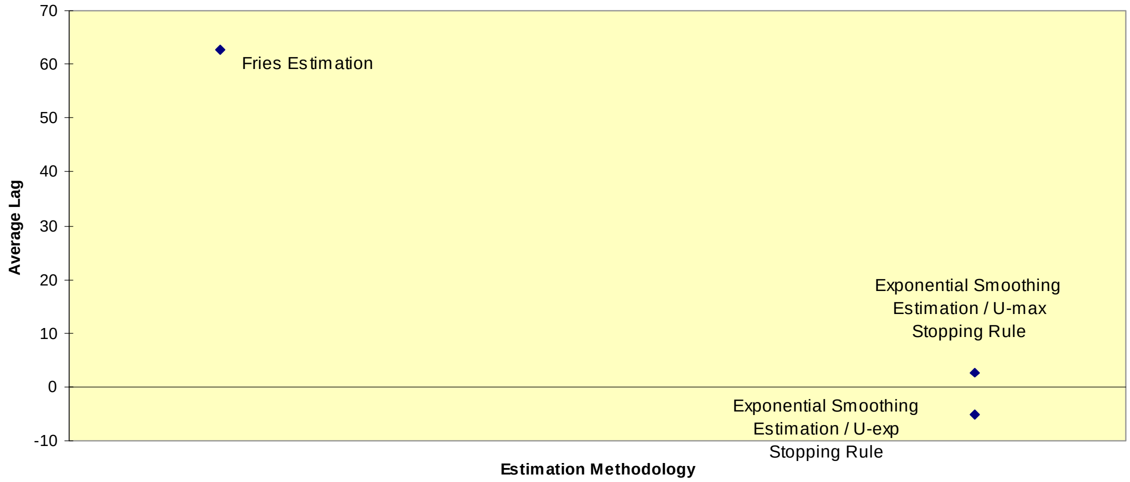

Figure 4.2 Average Lag vs. Estimation Methodology by Stopping Rule

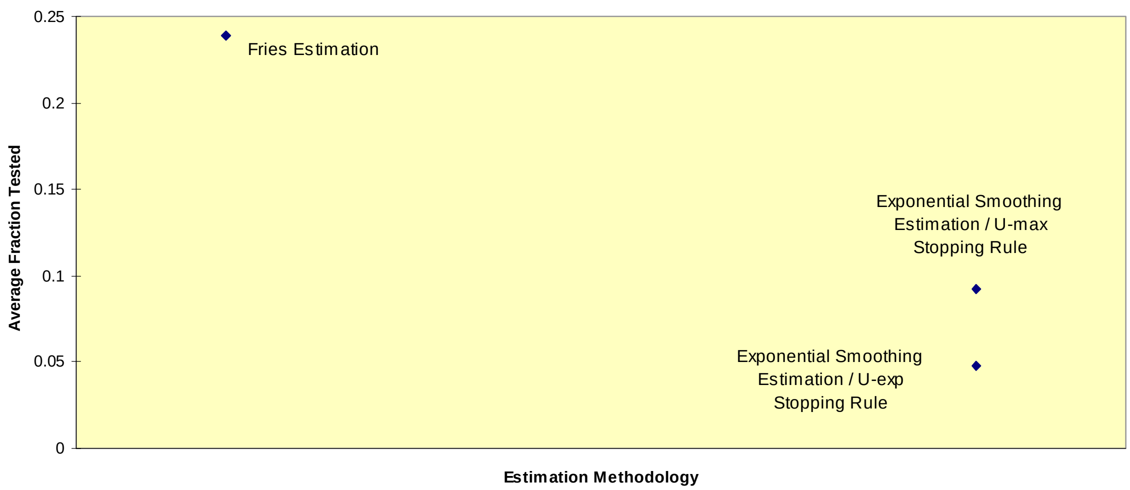

Figure 4.3 Average Fraction Tested vs. Estimation Methodology by Stopping Rule

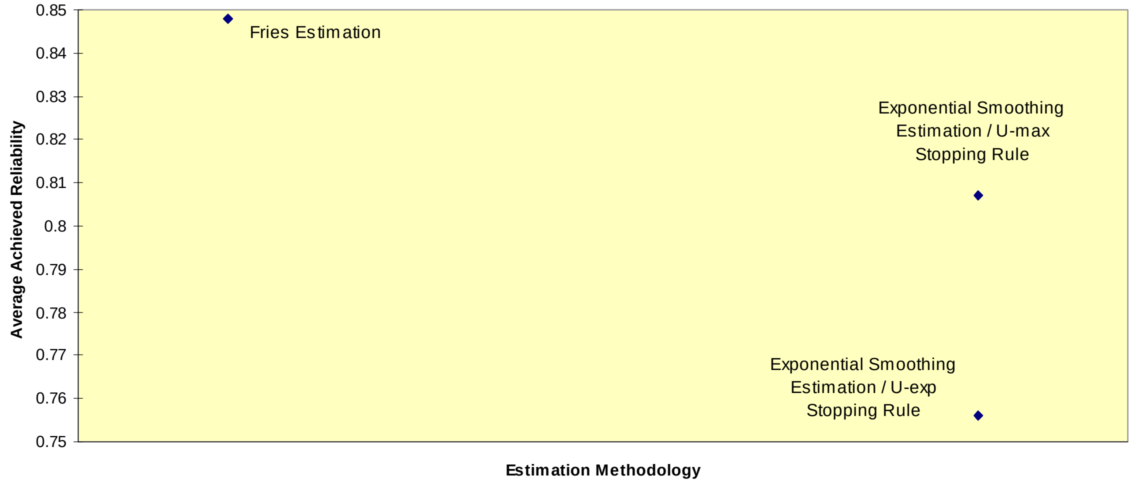

Figure 4.4 Average Achieved Reliability vs. Estimation Methodology by Stopping Rule

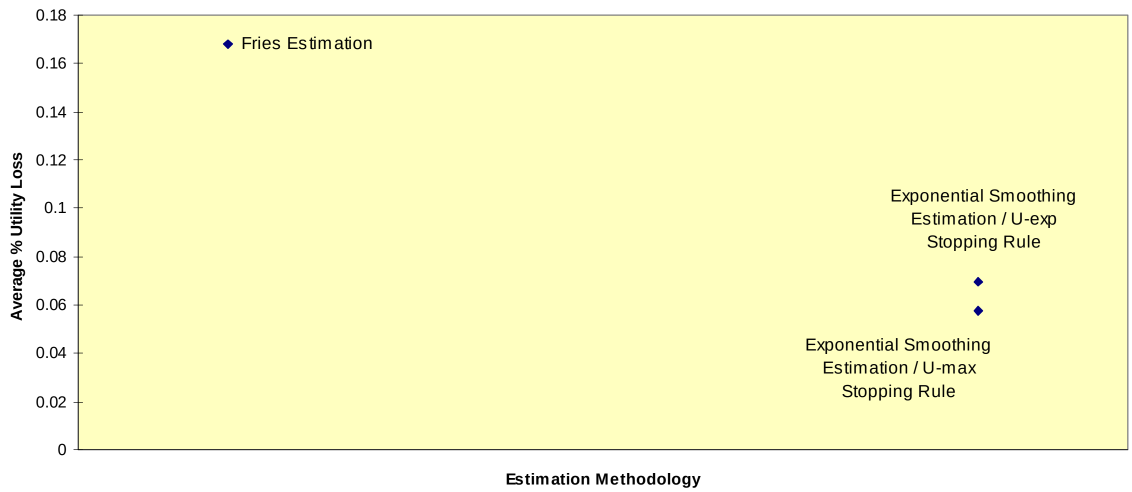

Figure 4.5 Average % Utility Loss vs. Estimation Methodology by Stopping Rule

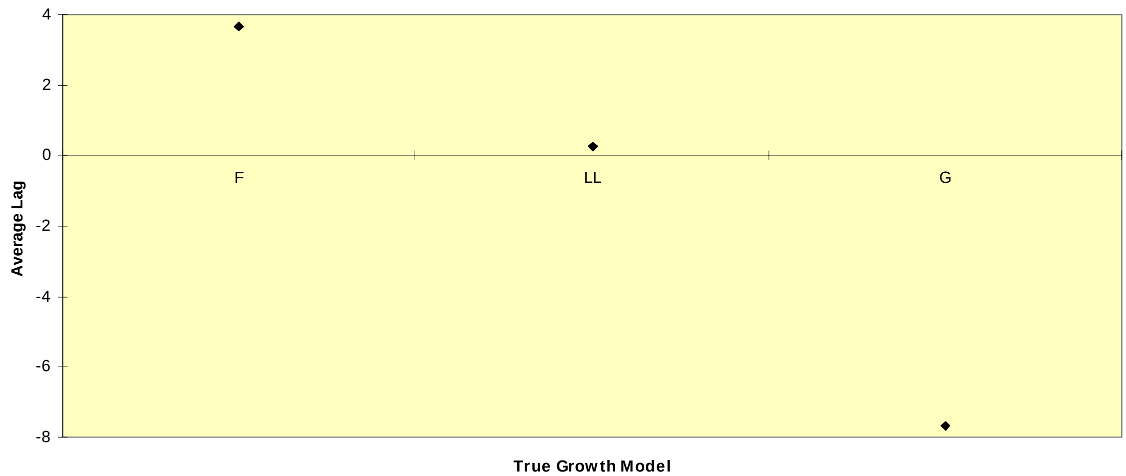

Figure 4.6 Average Lag vs. True Growth Under Exponential Smoothing Estimation

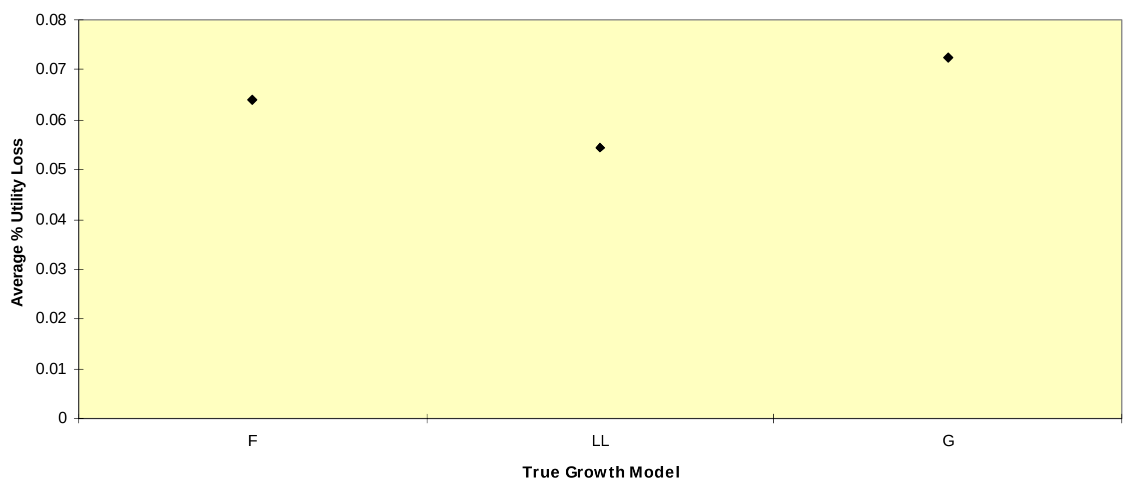

Figure 4.7 Average % Utility Loss vs. True Growth Under Exponential Smoothing Estimation

Reliability growth rate has a positive correlation with the lag performance measure. Figure 4.8 represents this relationship. The percent utility loss values in Figure 4.9 associated with the lags shown in Figure 4.8 demonstrate that signaling slightly late produces a smaller utility loss than signaling slightly early.

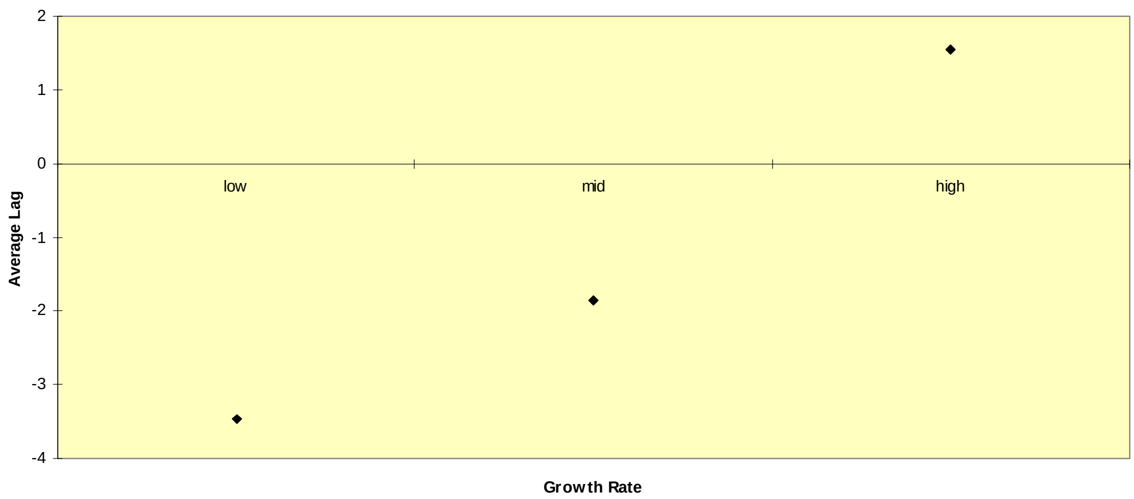

Figure 4.8 Average Lag vs. Growth Rate Under Exponential Smoothing Estimation

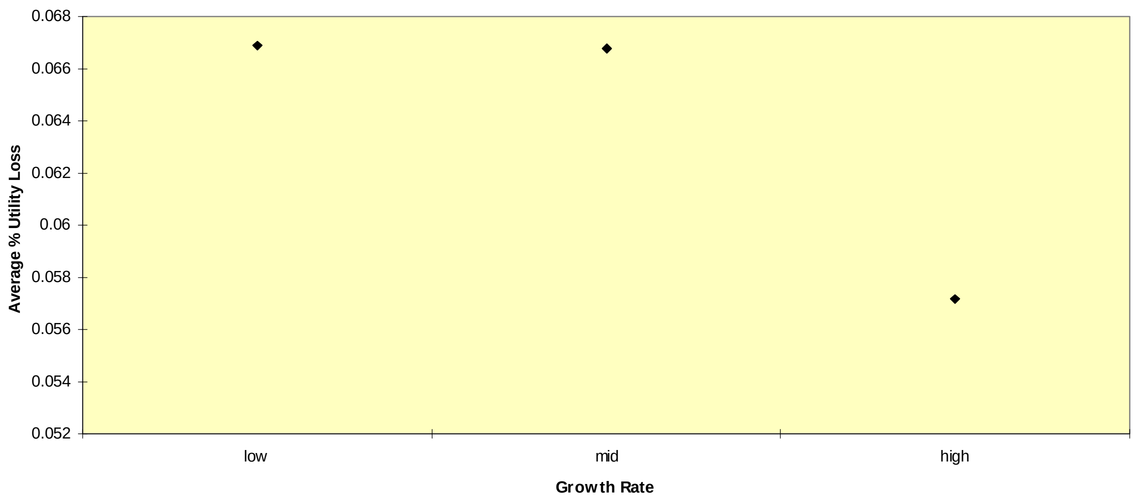

Figure 4.9 Average % Utility Loss vs. Growth Rate Under Exponential Smoothing Estimation

Figure 4.10 illustrates the inverse proportional relationship between lot size and signal location. Smaller lot sizes result in positive lag whereas larger lots result in early signals. When scaled by the lot size, the positive lag associated with the small lot size results in a much larger loss in utility than the negative lags associated with the larger lots. The average percent utility loss for lots of size 350 and 700 are nearly equivalent. The stopping rule/lot size interaction depicted in Figure 4.12 is of particular interest. Figure 4.12 shows that although the U-max rule yields smaller utility losses than the U-exp rule for the two larger values of *N*, the U-exp rule is superior for the smaller lot size of 75.

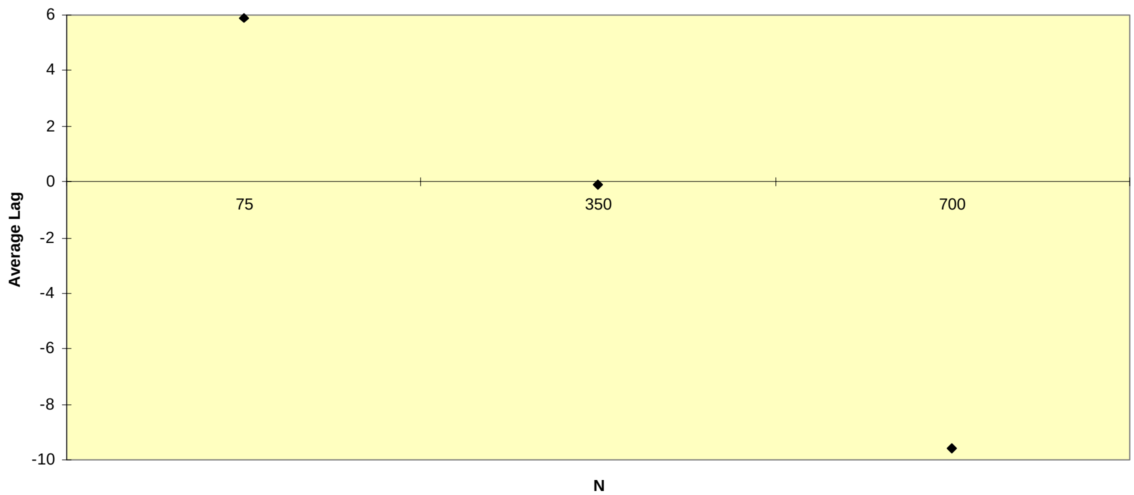

Figure 4.10 Average Lag vs. Lot Size Under Exponential Smoothing Estimation

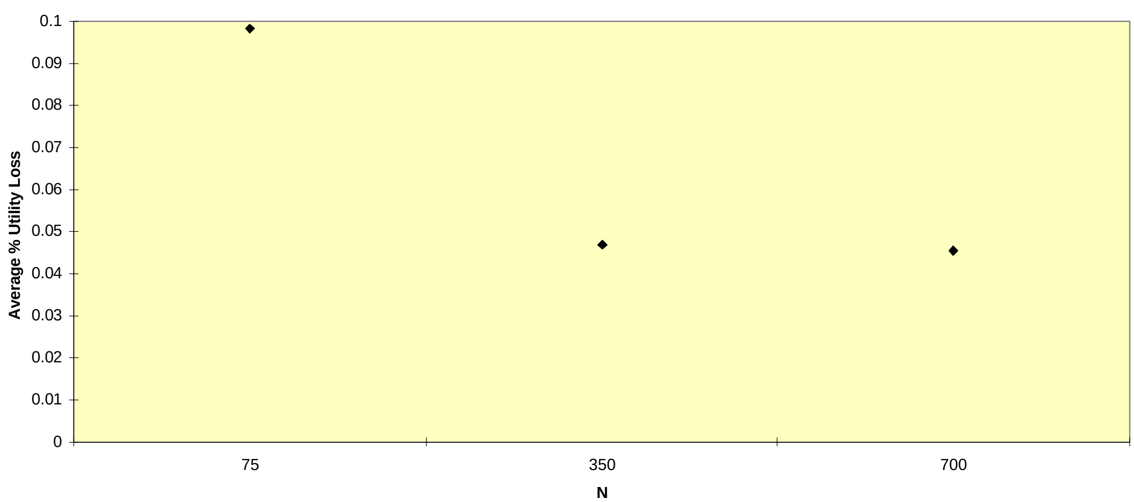

Figure 4.11 Average % Utility Loss vs. Lot Size Under Exponential Smoothing Estimation

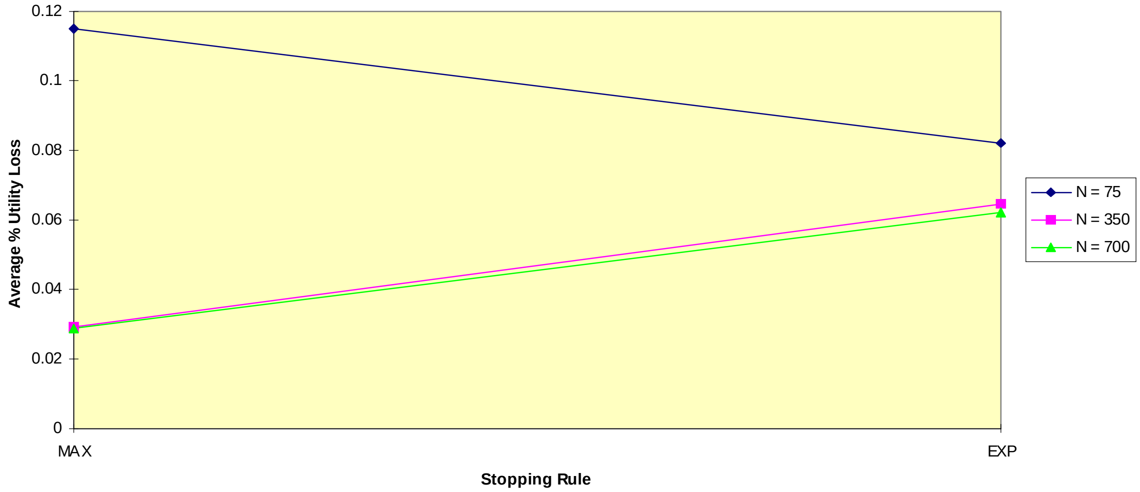

Figure 4.12 Average % Utility Loss vs. Rule*Lot Size Interaction Under Exponential Smoothing Estimation

The limiting reliability is also inversely related to lag as shown by Figure 4.13. However, the corresponding differences in utility loss are insignificant as shown in Figure 4.14. The stopping rule/limiting reliability interaction is explained by Figure 4.15. It is evident that the U-max rule is very robust relative to variations in limiting reliability whereas the U-exp rule performs better for smaller limiting reliabilities. Thus, it is the U-exp rule that is responsible for the trends in Figures 4.13 and 4.14. This last point is of little consequence however, since the U-max still outperforms the U-exp in terms of utility loss regardless of the limiting reliability value.

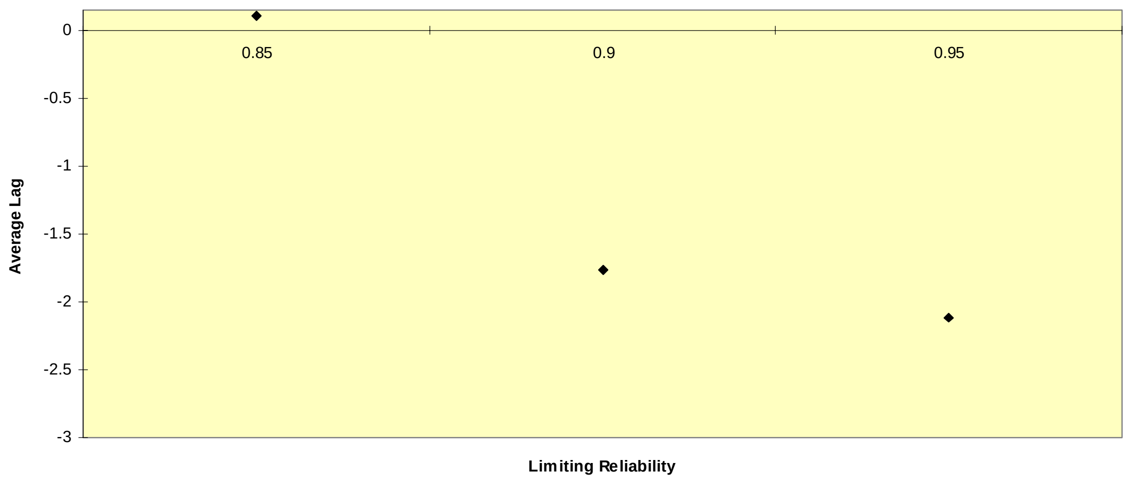

Figure 4.13 Average Lag vs. Limiting Reliability Under Exponential Smoothing Estimation

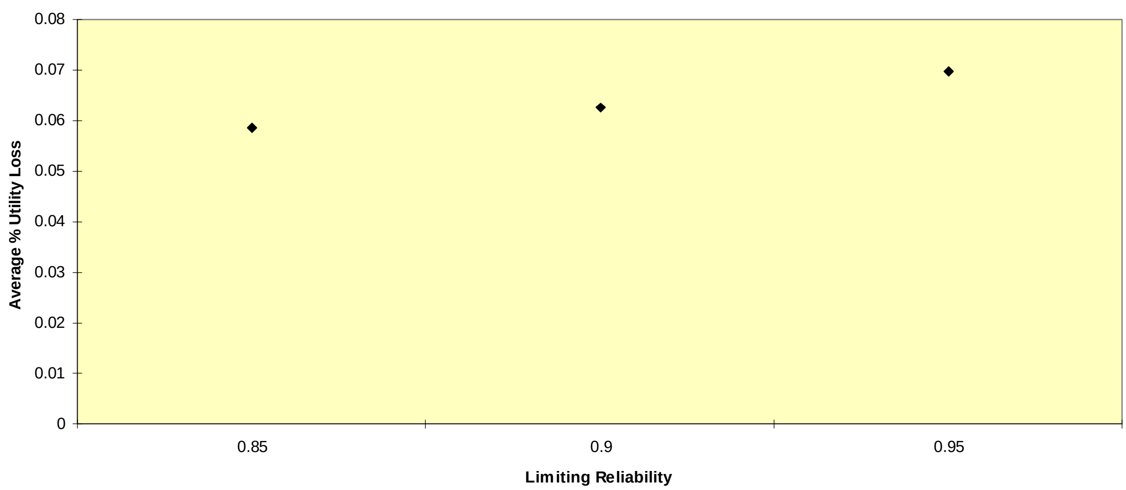

Figure 4.14 Average % Utility Loss vs. Limiting Reliability Under Exponential Smoothing Estimation

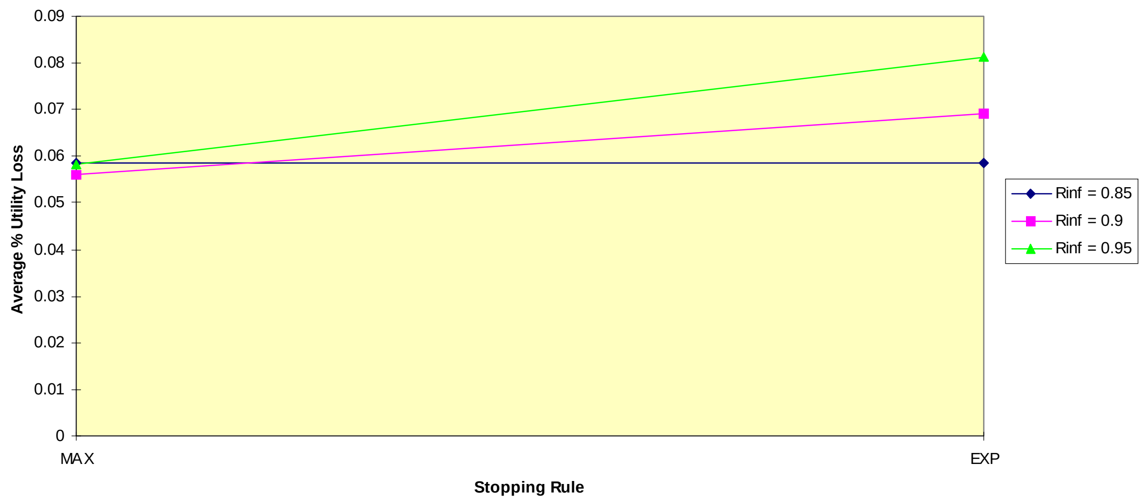

Figure 4.15 Average % Utility Loss vs. Limiting Reliability*Rule Interaction Under Exponential Smoothing Estimation

The other interactions listed previously as significant are uninformative upon further examination and are therefore omitted from this discussion.

4.3 Conclusions

Overall, the U-max and U-exp stopping rules considered here perform well. The rules perform consistently worse when paired with the Fries estimation methodology, resulting in an average lag of 62.8 trials and a corresponding average utility loss of 16.8%. When used with exponential smoothing estimation for $\alpha = 0.7$, the U-max rule results in a small average lag of 2.7 trials and an average utility loss of 5.8%. U-exp under exponential smoothing estimation with $\alpha = 0.7$ results in an average lag of -5.3 trials and an average utility loss of 7.0%. It is therefore recommended than upon implementation of either rule, the exponential smoothing estimation methodology with $\alpha = 0.7$ be used.

Under the exponential smoothing methodology with α = 0.7, the U-max rule results in lower utility losses than the U-exp for all cases except those with exceptionally small lot sizes. It is therefore recommended that the U-exp rule be used when lot sizes are small (less than 100), and the U-max rule be used otherwise.

It is also important to recognize the relationships between rule performance and other factors. In general, less utility is lost for larger lot sizes and slower growth rates. Rule performance is independent of testing errors within a realistic probabilistic range. Rule performance in terms of utility loss is also consistent across different underlying reliability growth patterns.

If these rules are implemented using exponential smoothing estimation with α = 0.7 an average of 86.9% of the limiting reliability is achieved. Separated by stopping rule, this figure is 89.7% for U-max and 84.1% for U-exp. On average, the U-max rule will allow testing to continue through 9.3% of the lot. The U-exp rule signals 4.8% of the way through the lot on average.

4.4 Future Considerations

Additional study of rule performance through more in depth simulation studies is warranted. Simulating stopping rule implementation in conjunction with other estimation methodologies and/or true growth models may prove interesting. Comparisons between the U-max and U-exp rules' performance with traditional stopping methods could also be conducted. Simulating TAAF processes for lot sizes between 75 and 350 to determine the point at which the U-max becomes more desirable than U-exp would also be worthwhile.

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APPENDIX A: SIMULATION CODE

A.1 Variable Definitions

n = lot size
k = current stage number
trial = current trial number
nk(j) = number of trials in stage j
z = signal indicator

bp = beta prime
lp = lambda prime
agrowth = alpha in equation (3.13)
rinit = initial reliability
rinf = R_∞

bpest = current estimated beta prime
bpnxtf = estimated beta prime if next trial fails
bpnxts = estimated beta prime if next trial succeeds
lpest = current estimated lambda prime
lpnxtf = estimated lambda prime if next trial fails
lpnxts = estimated lambda prime if next trial succeeds

rtrue = true current stage reliability
rest = current reliability estimate
rnxtf = estimated reliability if next trial fails
rnxts = estimated reliability if next trial succeeds

utrue = true current utility
uest = current utility estimate
unxtf = estimated utility if next trial fails
unxts = estimated utility of next trial succeeds
uexpctd = expected estimated value of the utility after the next trial

utruemax = true utility maximum
truemax = trial of true utility maximum
uestmax = estimated utility maximum
estmax = trial of estimated utility maximum
usignal = true utility at signal trial
rsignal = true reliability at signal trial
sigtrial = signal trial

d(1) = lag
d(4) = achieved reliability
d(5) = utruemax-usignal
d(6) = % utility lost
d(7) = fraction tested

A.2 FORTRAN 87 Code

The following FORTRAN code is for the specific scenario in which the true reliability growth is governed by Fries' model, equation (3.15), the estimation methodology is also governed by the Fries model, equation (3.16), and the stopping rule is the U-max rule.

```
***** DECLARE VARIABLES *****
      integer l, n, k, trial, z, t, stream, truemax, estmax, sigtrial, i, j
      real rand
      doubleprecision beta, lambda, bp, lp, utruemax, uestmax, nk(150),
     +     rtrue, u, rsignal, bpest, lpest, bpnxts, bpnxtf, lpnxts, lpnxtf,
     +     rest, rnxts, rnxtf, utrue, uest, unxtf, unxts, rinf, dmin(7),
     +     usignal, dmax(7), avg(7), sum(7), sq(7), var(7), d(7), p, s,
     +     rinit

      stream = xx
      open (unit = 1, file = 'c:\xxxxxxx')

***** SET PARAMETERS *****
      rinf = xxxx
      rinit = xxxx
      n = xxxx
      p = xxxx
      s = xxxx

***** SET GROWTH MODEL PARAMETERS *****
      beta = xxxx
      lambda = (rinf-rinit)**beta
      bp = 1/beta
      lp = lambda**bp

***** INITIALIZE STATISTIC ARRAY *****
      do 5 j = 1,7
        dmin(j) = 10000000.
        dmax(j) = 0.0
```

```
      sum(j) = 0.0
      sq(j) = 0.0
 5   continue

***** REPLICATION LOOP *****
     do 10 l = 1,1000
       rtrue = rinit
       k = 1
       trial = 0
       utruemax = float(n)*(rinf-lp/(float(k)**bp-float(k-1)**bp))          (A.1)
       uestmax = 0.0
       sigtrial = 0
       truemax = 1
       estmax = 0
       usignal = 0.0
       z = 0

***** BEGIN TAAF *****
       while (trial .lt. n) do

***** GENERATE STAGE LENGTH *****
         t = 0
         u = 0
         while (u .lt. rtrue) do
           t = t+1
           u = rand(stream)
         endwhile
         nk(k) = float(trial+t)

***** SKIP FIRST STAGE ESTIMATION *****
         if (k .eq. 1) then
           trial = t
           k = 2

***** COMPUTE RELIABILITY ESTIMATES AFTER EACH TRIAL *****
         else
           do 20 i = 1,t
             if (trial .lt. n) then
               trial = trial+1

***** ESTIMATION FOR MID-STAGE TRIALS *****
               if (i .lt. t) then
                 bpest = 0.0
                 bpnxtf = 0.0
                 bpnxts = 0.0
```

```
             do 30 j = 1,k-1
               bpest = bpest+log(trial/nk(j))/float(k-1)
               bpnxtf = bpnxtf+log((trial+1)/nk(j))/float(k)
               bpnxts =bpnxts+log((trial+1)/nk(j))/float(k-1)
 30          continue

             lpest = (float(k-1)/(trial)**(1/bpest))**bpest
             lpnxtf = (float(k)/(trial+1)**(1/bpnxtf))**bpnxtf
             lpnxts = (float(k-1)/(trial+1)**(1/bpnxts))**bpnxts

             rest = 1.-lpest/(float(k)**bpest-float(k-1)**bpest)
             rnxtf =1.-lpnxtf/(float(k+1)**bpnxtf-float(k)**bpnxtf)
             rnxts =1.-lpnxts/(float(k)**bpnxts-float(k-1)**bpnxts)

***** ESTIMATION FOR END OF STAGE TRIALS *****
            else
             bpest = 0.0
             bpnxtf = 0.0
             bpnxts = 0.0
             do 40 j = 1,k-1
               bpest = bpest+log(nk(k)/nk(j))/float(k)
 40          continue

             lpest = (float(k)/nk(k)**(1/bpest))**bpest

             do 50 j = 1,k
               bpnxtf = bpnxtf+log((nk(k)+1.)/nk(j))/float(k+1)
               bpnxts = bpnxts+log((nk(k)+1.)/nk(j))/float(k)
 50          continue

             lpnxtf = (float(k+1)/(nk(k)+1.)**(1./bpnxtf))**bpnxtf
             lpnxts = (float(k)/(nk(k)+1.)**(1./bpnxts))**bpnxts

             rest = 1.-lpest/(float(k+1)**bpest-float(k)**bpest)
             rnxtf = 1.-lpnxtf/(float(k+2)**bpnxtf-float(k+1)
   +                 **bpnxtf)
             rnxts =1.-lpnxts/(float(k+1)**bpnxts-float(k)**bpnxts)

***** GENERATE NEW TRUE REALIBILITY *****
             u = rand(stream)
             if (u .lt. (1-p)*(1-s)) then
               rtrue = rinf-lp/(float(k+1)**bp-float(k)**bp)          (A.2)
             endif

            endif
```

```
          endif

***** COMPUTE UTILITIES *****
          utrue = float(n-trial)*rtrue
          uest = float(n-trial)*rest
          unxtf = float(n-trial-1)*rnxtf
          unxts = float(n-trial-1)*rnxts

***** CHECK STOPPING RULE *****
          if ((rest .gt. 0.) .and. (unxtf .le. uest) .and.
    +         (unxts .le. uest) .and. (z .eq. 0)) then
            rsignal = rtrue
            usignal = utrue
            sigtrial = trial
            z = 1
          endif

***** TRACK TRUE UTILITY MAX *****
          if (utrue .gt. utruemax) then
            truemax = trial
            utruemax = utrue
          endif

***** TRACK ESTIMATED UTILITY MAX *****
          if (uest .gt. uestmax) then
            uestmax = uest
            estmax = trial
          endif
 20     continue

***** INCREMENT STAGE NUMBER *****
        k = k+1
      endif
    endwhile

***** COMPUTE STATISTICS *****
    d(1) = float(sigtrial-truemax)
    d(4) = rsignal
    d(5) = utruemax-usignal
    d(6) = d(5)/utruemax
    d(7) = float(sigtrial)/float(n)

    do 70 j = 1,7
      if (d(j) .gt. dmax(j)) then
        dmax(j) = d(j)
```

```
         endif
         if (d(j) .lt. dmin(j)) then
           dmin(j) = d(j)
         endif
         sum(j) = sum(j) + d(j)
         sq(j) = sq(j) + d(j)**2
70   continue

10 continue

     do 80 j = 1,7
       avg(j) = sum(j)/float(l-1)
       var(j) = (float(l-1)*sq(j)-sum(j)**2)/(float(l-1)*float(l-2))
80 continue

     do 90 j = 1,7
       if (j .eq. 1) then
         write (1,*) 'Lag to True'
       else if (j .eq. 2) then
         write (1,*) 'Lag to Estimation'
       else if (j .eq. 3) then
         write (1,*) 'Estimation/True Difference'
       else if (j .eq. 4) then
         write (1,*) 'Final Reliability'
       else if (j .eq. 5) then
         write (1,*) 'Utility Lost'
       else if (j .eq. 6) then
         write (1,*) '% Utility Lost'
       else if (j .eq. 7) then
         write (1,*) 'Fraction Tested'
       endif

       write (1,*) 'Average = ', avg(j)
       write (1,*) 'Variance = ', var(j)
       write (1,*) 'SS = ', sq(j)
       write (1,*) 'Max = ', dmax(j)
       write (1,*) 'Min = ', dmin(j)
       write (1,*)
90 continue

     endwhile

     end
```

```
      REAL FUNCTION RAND(ISTRM)

* PRIME MODULUS MULTIPLICATIVE LINEAR CONGRUENTIAL GENERATOR
* Z(I)=(630360016*Z(I-1)) (MOD(2**31 - 1), BASED ON MARSE AND ROBERTS'
* PORTABLE RANDOM-NUMBER GENERATOR UNIRAN.  MULTIPLE (100)
* STREAMS ARE SUPPORTED, WITH SEEDS SPACED 100,000 APART.
* THROUGHOUT, INPUT ARGUMENT ISTRM MUST BE AN INTEGER GIVING
* THE DESIRED STREAM NUMBER.   TO OBTAIN THE NEXT U(0,1) RANDOM
* NUMBER FROM STREAM ISTRM,  EXECUTE U = RAND(ISTRM).  THE REAL
* VARIABLE U WILL CONTAIN THE NEXT RANDOM NUMBER.  (Law and
* Kelton, 1991)

      INTEGER B2E15,B2E16,HI15,HI31,ISTRM,IZGET,IZSET,LOW15,LOWPRD,
     +        MODLUS,MULT1,MULT2,OVFLOW,ZI,ZRNG(100)
      INTEGER IRANDG,RANDST

* FORCE SAVING OF ZRNG BETWEEN CALLS.
      SAVE ZRNG

* DEFINE THE CONSTANTS.
      DATA MULT1,MULT2/24112,26143/
      DATA B2E15,B2E16,MODLUS/32768,65536,2147483647/

* SET THE DEFAULT SEEDS FOR ALL 100 STREAMS.
      DATA ZRNG/1973272912, 281629770,  20006270,1280689831,2096730329,
     +          1933576050, 913566091, 246780520,1363774876, 604901985,
     +          1511192140,1259851944, 824064364, 150493284, 242708531,
     +            75253171,1964472944,1202299975, 233217322,1911216000,
     +           726370533, 403498145, 993232223,1103205531, 762430696,
     +          1922803170,1385516923,  76271663, 413682397, 726466604,
     +           336157058,1432650381,1120463904, 595778810, 877722890,
     +          1046574445,  68911991,2088367019, 748545416, 622401386,
     +          2122378830, 640690903,1774806513,2132545692,2079249579,
     +            78130110, 852776735,1187867272,1351423507,1645973084,
     +          1997049139, 922510944,2045512870, 898585771, 243649545,
     +          1004818771, 773686062, 403188473, 372279877,1901633463,
     +           498067494,2087759558, 493157915, 597104727,1530940798,
     +          1814496276, 536444882,1663153658, 855503735,  67784357,
     +          1432404475, 619691088, 119025595, 880802310, 176192644,
     +          1116780070, 277854671,1366580350,1142483975,2026948561,
     +          1053920743, 786262391,1792203830,1494667770,1923011392,
     +          1433700034,1244184613,1147297105, 539712780,1545929719,
     +           190641742,1645390429, 264907697, 620389253,1502074852,
     +           927711160, 364849192,2049576050, 638580085, 547070247/
```

```
* GENERATE NEXT RANDOM NUMBER.
      ZI      = ZRNG(ISTRM)
      HI15    = ZI/B2E16
      LOWPRD  = (ZI-HI15*B2E16)*MULT1
      LOW15   = LOWPRD/B2E16
      HI31    = HI15*MULT1 + LOW15
      OVFLOW  = HI31/B2E15
      ZI      = (((LOWPRD-LOW15*B2E16)-MODLUS)+(HI31-OVFLOW*B2E15)
     +          *B2E16)+OVFLOW
      IF (ZI .LT. 0) ZI = ZI + MODLUS
      HI15    = ZI/B2E16
      LOWPRD  = (ZI-HI15*B2E16)*MULT2
      LOW15   = LOWPRD/B2E16
      HI31    = HI15*MULT2+LOW15
      OVFLOW  = HI31/B2E15
      ZI      = (((LOWPRD-LOW15*B2E16)-MODLUS)+(HI31-OVFLOW*B2E15)
     +          *B2E16)+OVFLOW
      IF (ZI .LT. 0) ZI=ZI+MODLUS
      ZRNG(ISTRM) = ZI
      RAND    = (2*(ZI/256)+1)/16777216.0
      RETURN

* SET THE CURRENT ZRNG FOR STREAM ISTRM TO IZSET.
      ENTRY RANDST(IZSET,ISTRM)
      ZRNG(ISTRM) = IZSET
      RETURN

* RETURN THE CURRENT ZRNG FOR STREAM ISTRM.
      ENTRY IRANDG(ISTRM)
      IRANDG = ZRNG(ISTRM)
      RETURN

      END
```

If the true reliability growth model is governed by the Lloyd and Lipow model, equation (3.12), the variable alpha is added to the doubleprecision declaration, and the “set growth model parameters” section of code reads

```
***** SET GROWTH MODEL PARAMETERS *****
      alpha = xxx                                      .
```

Equation (A.1) becomes utruemax = float(n)*(rinf-alpha/float(k)),

and equation (A.2) becomes rtrue = rinf-alpha/float(k+1).

If the true reliability growth model is governed by the Gompertz model, equation (3.18), the variable b and c are added to the doubleprecision declaration, and the "set growth model parameters" section of code reads

```
***** SET GROWTH MODEL PARAMETERS *****
   c = xxx
   b = (rinit/rinf)**(1/c)                                   .
```

Equation (A.1) becomes utruemax = float(n)*(rinf*b**(c**k)),

and equation (A.2) becomes rtrue = rinf*b**(c**(k+1)).

If the Lloyd and Lipow exponential smoothing estimation methodology is used, the variable agrowth is added to the doubleprecision declaration and the statement agrowth = xxx is added to the "set growth model parameters" section. Using this estimation methodology also changes the following sections of code to:

```
***** ESTIMATION FOR MID-STAGE TRIALS *****
        if (i .lt. t) then
         x = float(i+1)/float(i+2)
         rest = agrowth*x + (1.-agrowth)*rlast
         rnxts= agrowth*float(i+2)/float(i+3)+(1.-agrowth)*rlast
         rnxtf= agrowth*float(i+1)/float(i+3)+(1-agrowth)*rest

***** ESTIMATION FOR END OF STAGE TRIALS *****
        else
         x = float(t)/float(t+2)
         rest = agrowth*x + (1.-agrowth)*rlast
         rnxts = agrowth*2./3. + (1.-agrowth)*rest
         rnxtf = agrowth*1./2. + (1.-agrowth)*rest              .
```

Using the U-exp rule changes the following sections of code to:

```
***** COMPUTE UTILITIES *****
        utrue = float(n-trial)*rtrue
        uest = float(n-trial)*rest
        unxtf = float(n-trial-1)*rnxtf
        unxts = float(n-trial-1)*rnxts
        uexpctd = unxtf*(1-rest)+unxts*rest

***** CHECK STOPPING RULE *****
        if ((uexpctd .lt. uest) .and. (z .eq. 0)) then
          rsignal = rtrue
          usignal = utrue
          sigtrial = trial
          z = 1
        endif                    .
```

A.3 SAS Code

The following SAS code, written by Jennifer Huffman of the Statistical Consulting Center, was used to perform analysis of variance on the simulation output.

```
proc access dbms=xls;
  create work.lisa.access;
  path='a:\sas.xls';
  getnames=yes;
  list all;

  create work.lisa.view;
  select all;
run;

libname ise 'c:\users\jhuff';
run;

proc print data=lisa;
run;

data ise.lisa(drop=var15-var26);
  set lisa;
  if _n_ gt 243 then delete;
```

```
  rename var2=true;
  sclag=mlag/sqrt(vlag/1000);
  scr=mr/sqrt(vr/1000);
  sculost=mulost/sqrt(vulost/1000);
  sctest=mtest/sqrt(vtest/1000);
run;

proc means;
var sclag scr sculost sctest;
run;

Title 'Analysis for LAG';
proc glm data=ise.lisa;
class est rule true grate n rinf ps;
model sclag=est rule est|true est|grate est|n est|rinf est|ps
rule|true rule|grate rule|n rule|rinf rule|ps true|grate true|n
true|rinf true|ps grate|n grate|rinf grate|ps n|rinf n|ps rinf|ps;
run;

Title 'Analysis for R';
proc glm data=ise.lisa;
class est rule true grate n rinf ps;
model scr=est rule est|true est|grate est|n est|rinf est|ps
rule|true rule|grate rule|n rule|rinf rule|ps true|grate true|n
true|rinf true|ps grate|n grate|rinf grate|ps n|rinf n|ps rinf|ps;
run;

Title'Analysis for ULOST';
proc glm data=ise.lisa;
class est rule true grate n rinf ps;
model sculost=est rule est|true est|grate est|n est|rinf est|ps
rule|true rule|grate rule|n rule|rinf rule|ps true|grate true|n
true|rinf true|ps grate|n grate|rinf grate|ps n|rinf n|ps rinf|ps;
run;

Title'Analysis for TEST';
proc glm data=ise.lisa;
class est rule true grate n rinf ps;
model sctest=est rule est|true est|grate est|n est|rinf est|ps
rule|true rule|grate rule|n rule|rinf rule|ps true|grate true|n
true|rinf true|ps grate|n grate|rinf grate|ps n|rinf n|ps rinf|ps;
run;
```

APPENDIX B: OUTPUT

B.1 FORTRAN Simulation Output

In the following simulation output, under the column heading:

Estimation Methodology {0 = Fries, 1 = Exponential Smoothing},

Rule {0 = U-max, 1 = U-exp},

True {0 = Fries, 1 = Lloyd and Lipow, 2 = Gompertz },

Growth Rate {0 = low, 1 = mid, 2 = high},

Lot Size {0 = 75, 1 = 350, 2 = 700},

Limiting Reliability {0 = 0.85, 1 = 0.9, 2 = 0.95},

(1-p)(1-s) {0 = 0.85, 1 = 0.9, 2 = 0.95}.

Estimation Method-ology	Stopping Rule	True Growth Model	Growth Rate	Lot Size	Limiting Reliability	(1-p)(1-s)	Mean Lag	Variance Lag	Mean Achieved Reliability	Variance Achieved Reliability	Mean % Utility Lost	Variance % Utility Lost	Mean Fraction Tested	Variance Fraction Tested
0	0	0	0	0	0	0	23.7920	103.1319	0.7452	0.0008	0.3019	0.0159	0.3660	0.0181
0	0	0	1	0	0	2	18.4700	72.4956	0.7885	0.0014	0.2473	0.0105	0.3166	0.0135
0	0	0	2	0	0	1	15.7520	66.2708	0.8162	0.0019	0.2233	0.0086	0.2863	0.0117
0	0	0	0	0	1	2	20.9480	90.3637	0.7670	0.0011	0.2702	0.0130	0.3410	0.0159
0	0	0	1	0	1	1	15.9850	58.1790	0.8183	0.0020	0.2170	0.0074	0.2890	0.0109
0	0	0	2	0	1	0	13.7230	44.5228	0.8574	0.0024	0.1964	0.0052	0.2641	0.0082
0	0	0	0	0	2	1	19.4150	73.9127	0.7875	0.0015	0.2495	0.0099	0.3255	0.0133
0	0	0	1	0	2	0	14.2370	45.6004	0.8493	0.0024	0.1948	0.0049	0.2689	0.0084
0	0	0	2	0	2	2	11.3390	27.8279	0.8919	0.0033	0.1676	0.0031	0.2295	0.0052
0	0	0	0	1	0	1	106.6380	762.3173	0.7784	0.0002	0.2765	0.0056	0.3323	0.0064
0	0	0	1	1	0	0	83.2290	754.7373	0.8201	0.0002	0.2146	0.0054	0.2652	0.0063
0	0	0	2	1	0	2	71.3160	747.3595	0.8403	0.0001	0.1879	0.0059	0.2252	0.0065
0	0	0	0	1	1	0	89.9900	747.3412	0.8041	0.0003	0.2258	0.0049	0.2879	0.0063
0	0	0	1	1	1	2	61.8390	527.9410	0.8534	0.0004	0.1524	0.0033	0.2067	0.0046
0	0	0	2	1	1	1	49.5260	420.9323	0.8812	0.0002	0.1257	0.0029	0.1645	0.0039
0	0	0	0	1	2	2	76.7190	610.5005	0.8290	0.0003	0.1848	0.0038	0.2522	0.0053
0	0	0	1	1	2	1	47.9860	351.3171	0.8843	0.0005	0.1133	0.0019	0.1677	0.0033
0	0	0	2	1	2	0	34.5330	185.0339	0.9173	0.0003	0.0858	0.0011	0.1217	0.0018
0	0	0	0	2	0	2	214.1450	1450.6246	0.7901	0.0000	0.2764	0.0029	0.3304	0.0030
0	0	0	1	2	0	1	165.9630	1920.1998	0.8291	0.0001	0.2135	0.0036	0.2603	0.0040
0	0	0	2	2	0	0	143.5980	1891.4278	0.8449	0.0001	0.1909	0.0037	0.2220	0.0040
0	0	0	0	2	1	1	179.4700	1422.4936	0.8196	0.0001	0.2229	0.0026	0.2845	0.0030
0	0	0	1	2	1	0	119.5240	1299.4669	0.8665	0.0001	0.1450	0.0022	0.1971	0.0028
0	0	0	2	2	1	2	94.5530	1333.8771	0.8881	0.0001	0.1186	0.0024	0.1532	0.0030
0	0	0	0	2	2	0	145.2010	1292.2188	0.8458	0.0001	0.1725	0.0021	0.2395	0.0028
0	0	0	1	2	2	2	83.8120	951.6063	0.8978	0.0003	0.0963	0.0013	0.1476	0.0022
0	0	0	2	2	2	1	56.5080	516.0480	0.9249	0.0002	0.0671	0.0007	0.0984	0.0012
0	0	1	0	0	0	0	19.3590	51.5477	0.7753	0.0019	0.2399	0.0072	0.3419	0.0101
0	0	1	1	0	0	2	18.3330	69.4396	0.7851	0.0017	0.2363	0.0098	0.3298	0.0137
0	0	1	2	0	0	1	18.8340	90.3548	0.8104	0.0002	0.2538	0.0145	0.3173	0.0184
0	0	1	0	0	1	2	16.4770	39.7672	0.8093	0.0025	0.2025	0.0049	0.2967	0.0079
0	0	1	1	0	1	1	15.3230	52.1268	0.8221	0.0024	0.2016	0.0058	0.2896	0.0100
0	0	1	2	0	1	0	17.0750	82.1695	0.8451	0.0007	0.2070	0.0101	0.2885	0.0143
0	0	1	0	0	2	1	14.5340	29.3902	0.8470	0.0022	0.1785	0.0036	0.2657	0.0064
0	0	1	1	0	2	0	13.0840	40.6896	0.8603	0.0030	0.1794	0.0036	0.2603	0.0074
0	0	1	2	0	2	2	14.7190	67.2153	0.8808	0.0016	0.1786	0.0067	0.2588	0.0103
0	0	1	0	1	0	1	74.1690	503.6060	0.8219	0.0003	0.1843	0.0037	0.2623	0.0043
0	0	1	1	1	0	0	74.5200	608.3720	0.8265	0.0004	0.1894	0.0042	0.2573	0.0052
0	0	1	2	1	0	2	81.7820	950.5670	0.8335	0.0002	0.2097	0.0068	0.2535	0.0084
0	0	1	0	1	1	0	54.8130	354.2042	0.8583	0.0004	0.1296	0.0021	0.2042	0.0032
0	0	1	1	1	1	2	54.8230	416.1318	0.8651	0.0006	0.1341	0.0025	0.1993	0.0037
0	0	1	2	1	1	1	52.7740	576.6215	0.8725	0.0004	0.1348	0.0036	0.1884	0.0054

Table B.1 Simulation Output

0	0	1	0	1	2	2	39.9230	207.1082	0.8915	0.0006	0.0918	0.0009	0.1587	0.0019
0	0	1	1	1	2	1	39.5280	251.8270	0.8990	0.0007	0.0933	0.0013	0.1499	0.0024
0	0	1	2	1	2	0	37.1860	322.8382	0.9107	0.0007	0.0964	0.0014	0.1436	0.0028
0	0	1	0	2	0	2	143.3440	1368.9506	0.8331	0.0002	0.1785	0.0026	0.2433	0.0029
0	0	1	1	2	0	1	149.0510	1449.5760	0.8379	0.0001	0.1913	0.0029	0.2487	0.0032
0	0	1	2	2	0	0	164.3200	2406.0436	0.8415	0.0001	0.2090	0.0044	0.2519	0.0050
0	0	1	0	2	1	1	101.4380	934.2644	0.8736	0.0001	0.1191	0.0015	0.1840	0.0019
0	0	1	1	2	1	0	102.3050	1205.2873	0.8782	0.0002	0.1229	0.0019	0.1794	0.0026
0	0	1	2	2	1	2	101.9400	1706.5630	0.8831	0.0003	0.1282	0.0027	0.1738	0.0037
0	0	1	0	2	2	0	67.9610	563.2187	0.9074	0.0003	0.0755	0.0006	0.1338	0.0012
0	0	1	1	2	2	2	64.2640	658.0464	0.9124	0.0003	0.0730	0.0007	0.1229	0.0014
0	0	1	2	2	2	1	64.1930	763.7875	0.9211	0.0004	0.0775	0.0009	0.1165	0.0018
0	0	2	0	0	0	0	27.4250	96.3087	0.7508	0.0015	0.3052	0.0122	0.3891	0.0164
0	0	2	1	0	0	2	21.1910	77.1196	0.7829	0.0023	0.2407	0.0084	0.3407	0.0135
0	0	2	2	0	0	1	18.1050	65.8959	0.8009	0.0023	0.2209	0.0072	0.3172	0.0118
0	0	2	0	0	1	2	25.3510	91.5554	0.7669	0.0022	0.2725	0.0100	0.3695	0.0153
0	0	2	1	0	1	1	18.4570	61.0772	0.8062	0.0029	0.2087	0.0051	0.3180	0.0105
0	0	2	2	0	1	0	15.5000	48.7327	0.8290	0.0032	0.1911	0.0042	0.2925	0.0093
0	0	2	0	0	2	1	23.1140	83.0020	0.7814	0.0029	0.2458	0.0075	0.3536	0.0137
0	0	2	1	0	2	0	16.3180	51.5404	0.8272	0.0038	0.1880	0.0035	0.3033	0.0091
0	0	2	2	0	2	2	13.3670	36.9553	0.8542	0.0038	0.1664	0.0022	0.2711	0.0070
0	0	2	0	1	0	1	77.0540	454.0511	0.8231	0.0005	0.1755	0.0025	0.2904	0.0038
0	0	2	1	1	0	0	65.9270	485.1128	0.8380	0.0005	0.1629	0.0033	0.2489	0.0042
0	0	2	2	1	0	2	63.7310	569.6283	0.8426	0.0004	0.1656	0.0044	0.2327	0.0050
0	0	2	0	1	1	0	62.1270	294.7576	0.8517	0.0007	0.1324	0.0013	0.2564	0.0026
0	0	2	1	1	1	2	48.8270	308.0091	0.8728	0.0009	0.1135	0.0015	0.2055	0.0026
0	0	2	2	1	1	1	44.2580	294.9184	0.8824	0.0006	0.1082	0.0017	0.1818	0.0025
0	0	2	0	1	2	2	51.1110	261.0597	0.8751	0.0012	0.1046	0.0007	0.2319	0.0021
0	0	2	1	1	2	1	37.8470	200.4901	0.9015	0.0011	0.0849	0.0006	0.1790	0.0017
0	0	2	2	1	2	0	32.3930	188.1427	0.9122	0.0013	0.0794	0.0007	0.1503	0.0016
0	0	2	0	2	0	2	136.3740	957.6498	0.8422	0.0001	0.1608	0.0019	0.2594	0.0020
0	0	2	1	2	0	1	127.5190	1312.2299	0.8475	0.0001	0.1654	0.0028	0.2305	0.0028
0	0	2	2	2	0	0	126.7750	1457.8302	0.8486	0.0001	0.1710	0.0032	0.2185	0.0031
0	0	2	0	2	1	1	97.7880	559.2463	0.8767	0.0003	0.1046	0.0008	0.2115	0.0013
0	0	2	1	2	1	0	82.7020	698.9662	0.8895	0.0003	0.0973	0.0011	0.1719	0.0014
0	0	2	2	2	1	2	79.1900	832.3142	0.8935	0.0002	0.0988	0.0016	0.1547	0.0018
0	0	2	0	2	2	0	74.8780	442.3975	0.9044	0.0004	0.0743	0.0004	0.1828	0.0008
0	0	2	1	2	2	2	57.3510	401.8617	0.9226	0.0005	0.0626	0.0004	0.1389	0.0008
0	0	2	2	2	2	1	50.3300	434.9821	0.9294	0.0006	0.0592	0.0005	0.1162	0.0008
1	0	0	0	0	0	0	13.3310	52.5099	0.7351	0.0007	0.1561	0.0096	0.2270	0.0095
1	0	0	1	0	0	2	10.3350	41.1359	0.7825	0.0012	0.1317	0.0061	0.2078	0.0058
1	0	0	2	0	0	1	9.7530	45.8278	0.8126	0.0019	0.1344	0.0070	0.2037	0.0060
1	1	0	0	0	0	0	3.0360	23.0377	0.6925	0.0017	0.0636	0.0021	0.0898	0.0025
1	1	0	1	0	0	2	1.3950	23.5866	0.7217	0.0043	0.0806	0.0034	0.0886	0.0025
1	1	0	2	0	0	1	1.1170	28.2956	0.7398	0.0069	0.1007	0.0059	0.0886	0.0029

Table B.1 Simulation Output (continued)

1	0	0	0	0	1	2	11.4600	42.1966	0.7561	0.0009	0.1382	0.0068	0.2148	0.0068
1	0	0	1	0	1	1	9.2270	43.6111	0.8108	0.0018	0.1246	0.0062	0.2009	0.0054
1	0	0	2	0	1	0	8.5220	51.8514	0.8469	0.0034	0.1304	0.0080	0.1974	0.0061
1	1	0	0	0	1	2	2.0660	21.3230	0.7076	0.0026	0.0656	0.0021	0.0896	0.0022
1	1	0	1	0	1	1	1.0590	27.9895	0.7393	0.0065	0.0958	0.0051	0.0920	0.0029
1	1	0	2	0	1	0	0.7900	39.1450	0.7571	0.0109	0.1264	0.0091	0.0943	0.0043
1	0	0	0	0	2	1	10.6300	42.8640	0.7734	0.0015	0.1311	0.0063	0.2113	0.0063
1	0	0	1	0	2	0	8.7360	50.5889	0.8346	0.0037	0.1277	0.0068	0.1992	0.0060
1	0	0	2	0	2	2	8.9250	70.8843	0.8861	0.0045	0.1370	0.0121	0.2011	0.0091
1	1	0	0	0	2	1	1.6000	25.6797	0.7154	0.0036	0.0746	0.0031	0.0909	0.0027
1	1	0	1	0	2	0	0.7110	33.3148	0.7495	0.0093	0.1142	0.0072	0.0922	0.0033
1	1	0	2	0	2	2	1.3070	48.3591	0.7859	0.0153	0.1421	0.0127	0.0995	0.0055
1	0	0	0	1	0	1	11.6740	105.5313	0.7448	0.0003	0.0257	0.0004	0.0609	0.0008
1	0	0	1	1	0	0	9.9140	85.4581	0.7900	0.0006	0.0261	0.0007	0.0559	0.0005
1	0	0	2	1	0	2	10.9510	75.0016	0.8246	0.0003	0.0254	0.0006	0.0538	0.0005
1	1	0	0	1	0	1	1.9130	66.6741	0.7147	0.0017	0.0377	0.0017	0.0330	0.0003
1	1	0	1	1	0	0	1.4280	64.8437	0.7482	0.0039	0.0548	0.0044	0.0317	0.0003
1	1	0	2	1	0	2	3.1770	64.7024	0.7709	0.0063	0.0687	0.0070	0.0315	0.0003
1	0	0	0	1	1	0	9.3260	90.5763	0.7641	0.0006	0.0250	0.0005	0.0573	0.0006
1	0	0	1	1	1	2	8.5350	96.5113	0.8248	0.0004	0.0217	0.0005	0.0537	0.0005
1	0	0	2	1	1	1	10.8180	84.5935	0.8639	0.0009	0.0285	0.0013	0.0541	0.0005
1	1	0	0	1	1	0	0.5240	70.8783	0.7297	0.0025	0.0445	0.0027	0.0322	0.0003
1	1	0	1	1	1	2	1.0540	82.6337	0.7751	0.0058	0.0610	0.0064	0.0324	0.0003
1	1	0	2	1	1	1	4.0570	91.9057	0.8024	0.0096	0.0808	0.0099	0.0347	0.0005
1	0	0	0	1	2	2	7.2540	92.6822	0.7858	0.0005	0.0211	0.0004	0.0547	0.0005
1	0	0	1	1	2	1	7.8690	102.8287	0.8545	0.0011	0.0248	0.0012	0.0531	0.0005
1	0	0	2	1	2	0	12.7610	137.7997	0.9039	0.0014	0.0345	0.0021	0.0591	0.0009
1	1	0	0	1	2	2	-0.9510	76.1608	0.7417	0.0037	0.0540	0.0041	0.0312	0.0003
1	1	0	1	1	2	1	0.7300	96.3294	0.7932	0.0088	0.0767	0.0093	0.0327	0.0004
1	1	0	2	1	2	0	5.4870	129.6975	0.8347	0.0133	0.0907	0.0128	0.0384	0.0007
1	0	0	0	2	0	2	2.9710	162.6608	0.7435	0.0002	0.0123	0.0001	0.0294	0.0002
1	0	0	1	2	0	1	3.4710	129.6908	0.7911	0.0004	0.0136	0.0004	0.0279	0.0001
1	0	0	2	2	0	0	7.6810	103.3626	0.8225	0.0007	0.0154	0.0009	0.0276	0.0001
1	1	0	0	2	0	2	-5.7740	100.7116	0.7197	0.0013	0.0317	0.0019	0.0169	0.0001
1	1	0	1	2	0	1	-4.1100	98.2522	0.7619	0.0029	0.0396	0.0038	0.0170	0.0001
1	1	0	2	2	0	0	0.8220	91.4017	0.7890	0.0047	0.0464	0.0061	0.0178	0.0001
1	0	0	0	2	1	1	0.1810	160.9652	0.7643	0.0006	0.0158	0.0005	0.0286	0.0002
1	0	0	1	2	1	0	0.3890	141.3530	0.8205	0.0011	0.0183	0.0012	0.0269	0.0001
1	0	0	2	2	1	2	6.1570	134.5529	0.8649	0.0008	0.0140	0.0009	0.0271	0.0001
1	1	0	0	2	1	1	-8.0980	106.9113	0.7382	0.0020	0.0380	0.0026	0.0168	0.0001
1	1	0	1	2	1	0	-6.5330	113.4123	0.7788	0.0054	0.0591	0.0069	0.0170	0.0001
1	1	0	2	2	1	2	-0.4090	113.5993	0.8190	0.0078	0.0580	0.0092	0.0178	0.0001
1	0	0	0	2	2	0	-2.7260	174.1611	0.7852	0.0007	0.0174	0.0006	0.0276	0.0001
1	0	0	1	2	2	2	-0.0740	182.2328	0.8568	0.0007	0.0155	0.0007	0.0273	0.0001
1	0	0	2	2	2	1	8.0020	179.4374	0.9056	0.0011	0.0157	0.0013	0.0290	0.0002

Table B.1 Simulation Output (continued)

1	1	0	0	2	2	0	-10.0200	140.1477	0.7541	0.0028	0.0465	0.0037	0.0172	0.0001
1	1	0	1	2	2	2	-6.7660	148.8141	0.8145	0.0066	0.0555	0.0077	0.0178	0.0001
1	1	0	2	2	2	1	1.6580	162.2233	0.8573	0.0102	0.0602	0.0111	0.0199	0.0002
1	0	1	0	0	0	0	10.7610	42.9949	0.7548	0.0024	0.1301	0.0064	0.2309	0.0069
1	0	1	1	0	0	2	9.6620	45.7895	0.7698	0.0016	0.1173	0.0059	0.2137	0.0064
1	0	1	2	0	0	1	11.4240	54.4206	0.8011	0.0003	0.0340	0.0002	0.0537	0.0005
1	1	1	0	0	0	0	0.4530	27.4673	0.6578	0.0112	0.1096	0.0120	0.0935	0.0028
1	1	1	1	0	0	2	0.4890	25.3152	0.7091	0.0031	0.0610	0.0024	0.0914	0.0030
1	1	1	2	0	0	1	3.03	30.0142	0.7971	0.0001	0.0318	0.0033	0.1065	0.0042
1	0	1	0	0	1	2	10.2250	31.7581	0.7975	0.0018	0.1171	0.0048	0.2133	0.0049
1	0	1	1	0	1	1	8.8000	46.5806	0.8089	0.0024	0.1180	0.0058	0.2055	0.0058
1	0	1	2	0	1	0	5.6100	111.4193	0.8463	0.0003	0.0192	0.0004	0.0523	0.0006
1	1	1	0	0	1	2	1.3090	26.6201	0.7011	0.0145	0.1111	0.0154	0.0944	0.0027
1	1	1	1	0	1	1	0.1610	25.3785	0.7333	0.0060	0.0860	0.0043	0.0903	0.0026
1	1	1	2	0	1	0	3.1230	33.9879	0.8131	0.0004	0.0317	0.0027	0.1028	0.0039
1	0	1	0	0	2	1	9.7980	27.7770	0.8381	0.0021	0.1161	0.0050	0.2043	0.0045
1	0	1	1	0	2	0	8.2570	56.8458	0.8451	0.0042	0.1229	0.0079	0.1989	0.0069
1	0	1	2	0	2	2	6.6080	146.7771	0.8885	0.0008	0.0271	0.0011	0.0556	0.0008
1	1	1	0	0	2	1	1.7690	30.9065	0.7347	0.0201	0.1268	0.0207	0.0972	0.0035
1	1	1	1	0	2	0	0.6570	40.2616	0.7576	0.0102	0.1161	0.0082	0.0961	0.0042
1	1	1	2	0	2	2	3.6930	48.5973	0.8319	0.0018	0.0589	0.0041	0.1081	0.0048
1	0	1	0	1	0	1	3.8400	135.4839	0.7689	0.0012	0.0272	0.0007	0.0610	0.0006
1	0	1	1	1	0	0	4.2740	120.4213	0.7794	0.0011	0.0281	0.0008	0.0563	0.0005
1	0	1	2	1	0	2	11.4240	54.4206	0.8011	0.0003	0.0340	0.0002	0.0537	0.0005
1	1	1	0	1	0	1	-5.2590	88.2041	0.7155	0.0084	0.0706	0.0114	0.0350	0.0003
1	1	1	1	1	0	0	-4.5720	85.8246	0.7349	0.0035	0.0597	0.0037	0.0310	0.0003
1	1	1	2	1	0	2	3.4090	31.0007	0.7997	0.0001	0.0121	0.0003	0.0308	0.0003
1	0	1	0	1	1	0	3.1290	130.9853	0.8087	0.0016	0.0283	0.0014	0.0569	0.0005
1	0	1	1	1	1	2	4.2450	115.6907	0.8263	0.0009	0.0226	0.0006	0.0542	0.0005
1	0	1	2	1	1	1	5.6100	111.4193	0.8463	0.0003	0.0192	0.0004	0.0523	0.0006
1	1	1	0	1	1	0	-4.6070	95.7203	0.7555	0.0109	0.0720	0.0139	0.0348	0.0003
1	1	1	1	1	1	2	-3.6610	93.5196	0.7690	0.0062	0.0696	0.0066	0.0316	0.0003
1	1	1	2	1	1	1	-1.8170	81.7352	0.8227	0.0006	0.0253	0.0003	0.0311	0.0003
1	0	1	0	1	2	2	3.4030	130.6733	0.8523	0.0012	0.0235	0.0010	0.0556	0.0005
1	0	1	1	1	2	1	6.1320	142.7273	0.8688	0.0013	0.0261	0.0012	0.0548	0.0006
1	0	1	2	1	2	0	6.6080	146.7771	0.8885	0.0008	0.0271	0.0011	0.0556	0.0008
1	1	1	0	1	2	2	-3.7300	99.1843	0.7946	0.0138	0.0714	0.0163	0.0352	0.0003
1	1	1	1	1	2	1	-1.3810	109.6355	0.8050	0.0098	0.0786	0.0098	0.0333	0.0004
1	1	1	2	1	2	0	-1.3410	116.6434	0.8435	0.0024	0.0548	0.0015	0.0329	0.0005
1	0	1	0	2	0	2	-6.1800	204.5301	0.7702	0.0010	0.0257	0.0008	0.0306	0.0002
1	0	1	1	2	0	1	-4.5820	184.2115	0.7810	0.0010	0.0252	0.0008	0.0285	0.0001
1	0	1	2	2	0	0	7.0580	95.8405	0.8008	0.0003	0.0201	0.0002	0.0266	0.0001
1	1	1	0	2	0	2	-14.2800	135.4871	0.7326	0.0051	0.0624	0.0072	0.0191	0.0001
1	1	1	1	2	0	1	-12.3320	130.4042	0.7474	0.0028	0.0566	0.0035	0.0174	0.0001
1	1	1	2	2	0	0	0.0710	83.8278	0.7985	0.0001	0.0127	0.0002	0.0166	0.0001

Table B.1 Simulation Output (continued)

1	0	1	0	2	1	1	-6.8690	181.3452	0.8103	0.0012	0.0265	0.0011	0.0285	0.0001
1	0	1	1	2	1	0	-4.1040	170.3996	0.8226	0.0015	0.0257	0.0014	0.0271	0.0001
1	0	1	2	2	1	2	-1.1420	135.1530	0.8471	0.0003	0.0140	0.0001	0.0263	0.0001
1	1	1	0	2	1	1	-13.97	144.4825	0.7719	0.0069	0.0632	0.0092	0.0184	0.0001
1	1	1	1	2	1	0	-10.4770	137.2307	0.7869	0.0048	0.0595	0.0057	0.0179	0.0001
1	1	1	2	2	1	2	-7.5900	115.9478	0.8265	0.0006	0.0290	0.0005	0.0171	0.0001
1	0	1	0	2	2	0	-5.8920	220.8071	0.8502	0.0018	0.0279	0.0017	0.0287	0.0001
1	0	1	1	2	2	2	-1.7400	207.4599	0.8703	0.0011	0.0204	0.0009	0.0275	0.0002
1	0	1	2	2	2	1	1.1270	183.8147	0.8919	0.0005	0.0154	0.0005	0.0271	0.0002
1	1	1	0	2	2	0	-12.5550	166.1291	0.8155	0.0080	0.0587	0.0097	0.0194	0.0001
1	1	1	1	2	2	2	-8.5110	164.3162	0.8253	0.0072	0.0622	0.0082	0.0179	0.0001
1	1	1	2	2	2	1	-5.7110	151.0345	0.8523	0.0023	0.0500	0.0021	0.0173	0.0001
1	0	2	0	0	0	0	16.0250	53.6000	0.7217	0.0013	0.1618	0.0062	0.2370	0.0096
1	0	2	1	0	0	2	11.8680	29.9325	0.7600	0.0018	0.1197	0.0040	0.2160	0.0061
1	0	2	2	0	0	1	10.0470	36.2090	0.7829	0.0021	0.1140	0.0046	0.2078	0.0055
1	1	2	0	0	0	0	4.9320	11.6911	0.6762	0.0008	0.0584	0.0014	0.0891	0.0022
1	1	2	1	0	0	2	2.3590	24.1062	0.6985	0.0023	0.0589	0.0017	0.0892	0.0025
1	1	2	2	0	0	1	1.15	26.3110	0.7147	0.0039	0.0703	0.0026	0.0891	0.0024
1	0	2	0	0	1	2	14.5140	43.3031	0.7360	0.0018	0.1387	0.0045	0.2261	0.0080
1	0	2	1	0	1	1	10.3350	36.2690	0.7805	0.0025	0.1095	0.0043	0.2124	0.0060
1	0	2	2	0	1	0	8.4450	46.8759	0.8041	0.0037	0.1087	0.0057	0.2026	0.0061
1	1	2	0	0	1	2	4.3730	13.4733	0.6853	0.0012	0.0542	0.0014	0.0909	0.0023
1	1	2	1	0	1	1	1.1920	26.2194	0.7101	0.0034	0.0636	0.0020	0.0905	0.0024
1	1	2	2	0	1	0	0.0690	29.1914	0.7240	0.0057	0.0852	0.0036	0.0910	0.0024
1	0	2	0	0	2	1	13.3300	35.6267	0.7477	0.0024	0.1289	0.0036	0.2241	0.0070
1	0	2	1	0	2	0	9.0140	34.8687	0.7937	0.0039	0.1038	0.0037	0.2047	0.0052
1	0	2	2	0	2	2	7.9680	42.9619	0.8342	0.0038	0.1026	0.0057	0.2000	0.0057
1	1	2	0	0	2	1	3.4350	22.3061	0.6897	0.0017	0.0566	0.0018	0.0922	0.0028
1	1	2	1	0	2	0	0.5710	32.3293	0.7156	0.0048	0.0770	0.0030	0.0922	0.0028
1	1	2	2	0	2	2	-0.0880	34.4447	0.7438	0.0075	0.0936	0.0056	0.0926	0.0032
1	0	2	0	1	0	1	-2.4400	210.3768	0.7334	0.0014	0.0280	0.0004	0.0626	0.0008
1	0	2	1	1	0	0	-1.1200	162.4581	0.7729	0.0015	0.0311	0.0007	0.0576	0.0006
1	0	2	2	1	0	2	1.4620	136.7593	0.7984	0.0010	0.0264	0.0005	0.0547	0.0005
1	1	2	0	1	0	1	-13.5400	134.4068	0.6947	0.0014	0.0476	0.0011	0.0308	0.0003
1	1	2	1	1	0	0	-10.0130	117.9308	0.7277	0.0031	0.0633	0.0030	0.0322	0.0003
1	1	2	2	1	0	2	-6.4800	98.7884	0.7495	0.0044	0.0647	0.0045	0.0320	0.0003
1	0	2	0	1	1	0	-6.1300	206.6978	0.7503	0.0019	0.0355	0.0007	0.0615	0.0007
1	0	2	1	1	1	2	-4.0140	175.8396	0.7966	0.0018	0.0357	0.0009	0.0553	0.0005
1	0	2	2	1	1	1	0.2950	157.7297	0.8300	0.0017	0.0313	0.0011	0.0546	0.0005
1	1	2	0	1	1	0	-16.3810	145.1650	0.7072	0.0021	0.0622	0.0017	0.0322	0.0003
1	1	2	1	1	1	2	-12.2560	133.5640	0.7435	0.0044	0.0778	0.0044	0.0318	0.0003
1	1	2	2	1	1	1	-7.6170	111.8642	0.7705	0.0067	0.0801	0.0067	0.0320	0.0003
1	0	2	0	1	2	2	-9.9520	226.3921	0.7624	0.0022	0.0428	0.0010	0.0580	0.0007
1	0	2	1	1	2	1	-6.0030	184.3033	0.8173	0.0028	0.0447	0.0016	0.0544	0.0005
1	0	2	2	1	2	0	-1.1440	184.2956	0.8539	0.0028	0.0405	0.0019	0.0549	0.0006

Table B.1 Simulation Output (continued)

1	1	2	0	1	2	2	-19.4770	167.8213	0.7131	0.0026	0.0784	0.0024	0.0307	0.0003
1	1	2	1	1	2	1	-14.0780	124.5445	0.7568	0.0059	0.0941	0.0056	0.0314	0.0003
1	1	2	2	1	2	0	-8.9230	142.1893	0.7865	0.0090	0.0964	0.0086	0.0327	0.0004
1	0	2	0	2	0	2	-22.8640	325.9815	0.7337	0.0014	0.0453	0.0009	0.0310	0.0002
1	0	2	1	2	0	1	-13.7120	227.9890	0.7735	0.0015	0.0401	0.0012	0.0289	0.0001
1	0	2	2	2	0	0	-5.6610	190.9110	0.7980	0.0014	0.0318	0.0012	0.0287	0.0001
1	1	2	0	2	0	2	-32.4990	217.0951	0.7024	0.0013	0.0727	0.0014	0.0172	0.0001
1	1	2	1	2	0	1	-21.8970	163.5559	0.7368	0.0027	0.0746	0.0031	0.0172	0.0001
1	1	2	2	2	0	0	-13.6350	134.8526	0.7564	0.0039	0.0717	0.0047	0.0173	0.0001
1	0	2	0	2	1	1	-28.8840	339.7923	0.7491	0.0018	0.0594	0.0014	0.0301	0.0002
1	0	2	1	2	1	0	-17.9470	267.7680	0.7944	0.0024	0.0540	0.0019	0.0282	0.0002
1	0	2	2	2	1	2	-8.7520	216.5831	0.8325	0.0016	0.0355	0.0012	0.0282	0.0001
1	1	2	0	2	1	1	-37.85	248.4013	0.7140	0.0018	0.0913	0.0020	0.0173	0.0001
1	1	2	1	2	1	0	-25.8040	200.1517	0.7499	0.0040	0.0966	0.0045	0.0170	0.0001
1	1	2	2	2	1	2	-16.1270	154.2131	0.7900	0.0048	0.0750	0.0053	0.0176	0.0001
1	0	2	0	2	2	0	-32.6150	389.5663	0.7628	0.0025	0.0719	0.0021	0.0294	0.0002
1	0	2	1	2	2	2	-20.0580	304.4151	0.8194	0.0024	0.0594	0.0020	0.0282	0.0001
1	0	2	2	2	2	1	-11.4510	257.1988	0.8584	0.0023	0.0443	0.0019	0.0276	0.0001
1	1	2	0	2	2	0	-41.2670	306.3040	0.7219	0.0025	0.1102	0.0028	0.0171	0.0001
1	1	2	1	2	2	2	-27.6270	241.2832	0.7707	0.0050	0.1055	0.0055	0.0174	0.0001
1	1	2	2	2	2	1	-18.2780	209.5843	0.8095	0.0068	0.0901	0.0072	0.0178	0.0001

Table B.1 Simulation Output (continued)

B.2 SAS Output

In the following SAS output

EST = Estimation Methodology

RULE = Stopping Rule

TRUE = True Growth Model

GRATE = Growth Rate

N = Lot Size

RINF = Limiting Reliability

PS = (1 - *p*)(1 - *s*).

		Mean Squares			
Source	**DF**	**lag**	**achieved reliability**	**% utility lost**	**fraction tested**
EST	1	207843.566	18724171.675	120842.248	51608.437
EST*GRATE	2	6808.281	260000.354	1258.969	6043.309
EST*N	2	57878.466	16967419.460	9755.110	15739.041
EST*PS	2	51.380	27879.534	156.495	45.881
EST*RINF	2	327.062	3049738.677	390.579	228.195
EST*TRUE	2	4274.713	1212671.698	529.159	4478.056
GRATE	2	2598.245	2028686.495	4053.541	11535.240
GRATE*N	2	1407.802	272139.167	251.819	891.034
GRATE*PS	2	41.518	36831.636	3.456	1.870
GRATE*RINF	2	189.625	575570.926	47.851	58.747
N	2	450.286	21629009.500	756.778	14754.380
N*PS	2	15.779	68943.586	11.409	5.576
N*RINF	2	254.361	755266.814	205.982	180.734
PS	2	2.389	51895.537	58.878	39.234
RINF	2	2999.876	10344762.622	144.575	591.013
RINF*PS	2	41.506	127446.620	37.676	11.603
RULE	1	31539.720	3841102.749	586.678	14217.157
RULE*GRATE	2	344.779	101666.038	119.310	101.289
RULE*N	2	1789.263	606044.217	1397.230	1839.351
RULE*PS	2	72.332	104523.414	92.272	10.790
RULE*RINF	2	218.568	157044.667	564.287	50.653
RULE*TRUE	2	136.934	1315515.111	241.248	18.480
TRUE*GRATE	4	359.429	8231389.160	1858.631	438.316
TRUE*N	4	12907.347	2093172.960	363.322	574.710
TRUE*PS	4	26.048	393267.756	21.253	18.442
TRUE*RINF	4	162.730	460674.623	765.389	1016.886
TRUE	2	4440.148	2823625.761	4630.775	5734.735

Table B.2 SAS Output

VITA

Lisa Marie Maillart

Lisa Marie Maillart was born on February 1, 1974 in Fairfax, Virginia to Robert R. Maillart and Joyce L. Maillart. In May, 1995, she received her Bachelor of Science magna sum laude in Industrial and Systems Engineering with a minor in Statistics from Virginia Polytechnic Institute and State University in Blacksburg, Virginia. While completing her bachelor's degree, he was inducted into Alpha Pi Mu, Order of Omega, Gamma Sigma Alpha, and Rho Lambda. In addition, she held industrial engineering or operations research internship positions with MCI, United Parcel Service, USAir, and the Institute for Defense Analyses. In May, 1997 she will receive her Master of Science in Industrial and Systems Engineering with a concentration in operations research from Virginia Polytechnic Institute and State University. She will begin Ph.D. study in Industrial and Operations Engineering at the University of Michigan, Ann Arbor in September, 1997. She is a member of the Institute of Industrial Engineers, the Institute for Operations Research and the Management Sciences, and the American Society for Quality Control.