A STUDY OF SEMI-RIGID CONNECTIONS BETWEEN LONGSPAN JOISTS AND COLUMNS

by

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III. INTRODUCTION

This thesis is a study to provide a method of approaching an effective analysis and design of semi-rigid connections of longspan joists to columns. It is also an attempt at establishing correct testing techniques when testing full size longspan joists and column connections. Carlton E. Combs, Jr., in his thesis, An Initial Study Of Semi-Rigid Connections Between Longspan Joists and Columns, suggested certain laboratory procedures that will lead quickly to the necessary data and results. The author has adopted Combs' testing techniques in the laboratory part of this thesis.

A careful theoretical study of the semi-rigid connection has been made for four representative longspan joist and column sections. The theoretical study has been followed by design of the connections and finally followed by actual laboratory testing of the four test sections.

The purpose of a semi-rigid connection is to join the longspan joists and columns together and to make them to act as an economical organic frame. This potential economy is achieved only because the semi-rigid connection is capable of developing partial continuity in a steel building frame. This type of frame is customarily used in low, wide, one or two story buildings such as schools, warehouses and other buildings. This organic frame could possibly be a step toward lighter construction techniques in the future.

IV. THE REVIEW OF LITERATURE

There are some available references concerning semi-rigid connections and they are unanimous in their conclusion: the most productive way of examining the semi-rigid connections is by testing the actual connection, or if possible, testing a full sized mock-up of the actual building.

J. F. Baker carried on extensive testing of beam to column connections by varying the shape and thickness of the clip angles and the sizes of his beams and columns. He, in Volume I of his book, The Steel Skeleton,[1] listed testing of actual building connections. He also pointed out what he considers to be the main factors affecting the flexibility of the beam to column joint. These are as follows: magnitude of initial tension in rivets or bolts, variation in shape and thickness of the clip angles, slip between the beam and the horizontal leg of the clip angles, and position of center of rotation at low loads. He also made it clear that the behavior of a connection was complex and that it would be impractical to limit the stress in the clip angle to ordinary working stress.

Except his suggestions as to testing method, Baker's information was found of little value to this thesis since his beam connections are not appropriate for the connections of longspan joists.

L. C. Maugh, in his book, Statically Indeterminate Structures,[2] gives some interesting discussion on evaluating test results. He mentions that the strain in the connection cannot be calculated accurately as local stress concentration and deformation make the problem too complicated. The essence of his advice on testing steel connection is that it is difficult to evaluate the separate effects of such individual factors as change in length of rivets, local bending of angles or tees, shearing deformation, and local bending of the column flanges. Then he stated very clearly that the experimental results for each particular type of connection can seldom be applied to other types of connections. It is the objective of this thesis to provide for the partial testing of one type of connection.

John E. Lothers, in his book, Advanced Design in Structural Steel,[3] has a chapter concerning design of semi-rigid connections. The author found Lothers' approaching to analysis and design of semi-rigid connections to be very helpful, especially in understanding of the characteristic of semi-rigid connections. Again, those connections are of different types than the one that will be dealt with in this thesis.

Richard E. Welch, in his thesis, Semi-Rigid Connections in Trussed Beams,[4] tries to approach the design of semi-rigid connection by mathematics, but finds the solution mathematically cumbersome, and in need of laboratory verification.

Lehigh University in conjunction with the American Institute of Steel Construction has done rather extensive testing on beam to column connections and has developed tables and empirical equations expressing rigidity and economical value of certain connections.[5, 6] AISC itself has recognized certain type of semi-rigid connections and even provides examples as a guide to semi-rigid connection design.[7] At the same time, it enables the designer to take advantage of the potential economy which is achieved by partial continuity in steel building design. Here again, it is necessary to obtain data for each type of connection by testing the actual connection.

Finally, Carlton E. Combs, Jr., in his thesis, An Initial Study of Semi-Rigid Connections Between Longspan Joists and Columns,[8] developed a bolted connection of tee sections whose legs can slip in the gaps between the bottom and top chord angles of longspan joists and be welded to them. Combs' test result indicated that the connections were quite rigid and they behaved as rigid connections.

This thesis has carried on the idea of providing information for a particular type of semi-rigid connection. By using the same longspan joists and columns as Combs used, the author developed a new type of semi-rigid connection which yielded satisfactory results as obtained from testing the full size connection.

V. PROCEDURE

The objective of this investigation is to design a special semi-rigid connection suitable for joining longspan joists and columns, and to establish the behavior of this particular group of connections.

An one-story, two-bay frame was chosen with typical dimensions as representative of the use of semi-rigid connections of longspan joists to columns. This particular frame was chosen for two reasons. First, it was to make this investigation simpler since it was a symmetrical frame. Second, it was a framing pattern representative of a great many common applications of industrial longspan joist framing.

There were two conditions of interest. They were: development of strength to resist wind forces and improvement of joist efficiency through continuity provided by sufficient end connections. Wind strength is not provided by standard longspan joist construction since the longspan joists are designed to be "simply" supported on masonry bearing walls, steel beams or columns. This type of construction is adequate for most vertical loadings but lateral loadings, such as wind forces, cannot be sustained by this post and lintel construction. Therefore, lateral support or lateral stability must come from somewhere else in the building frame. In common practice for this type of building frames the lateral stability is provided by heavy masonry walls at appropriate intervals throughout the entire building and transferred to individual bents through a stiff roof deck, floor slab or from some system of trussing the building frame work.

It is possible that a more economical and organic frame can be realized when the posts and lintels are connected to make a cooperative frame. This cooperative frame will then be capable of resisting both vertical and lateral loadings simultaneously. Immediately the bearing masonry wall and trussing systems, or whatever supplied the lateral support in the post and lintel system can be greatly reduced or eliminated. It follows then, that the cost of fabricating and erecting these frames may well be less than that of providing sufficient lateral support in current longspan joist construction.

The second condition, improvement of joist efficiency through continuity provided by sufficient end connections, is merely an application of continuous construction principle to longspan joist construction. A diagram will more clearly illustrate this principle. It is as follows:

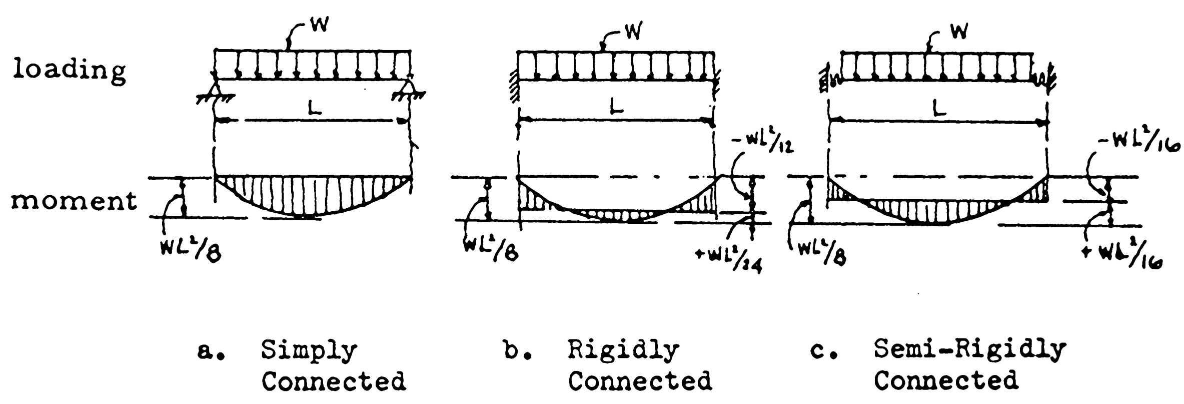


Figure 1. Comparison of Moments in Beams of Various End Restraints

The diagram graphically illustrates that, based on moment carrying capacity, the simply supported member, in which the connections at its ends provide no restraints, is least efficient. The fixed-end member, in which the end moments are two thirds that in a simply supported member, is much more efficient. Both the above extreme situations could be relieved if end restraint were provided by connections that yield somewhat. This would have the effect of increasing the mid-span moment and reducing the end-span moments. Thus the moments at the end-span and mid-span could be adjusted to equalize. A connection thus properly designed is called a "SEMI-RIGID CONNECTION". The design moment of a semi-rigidly connected member will be much less than the ones in the simply connected or rigidly connected member. With this in mind, the author chose to investigate the situation where the end moment is raised to a condition of maximum efficiency.

VI. THE INVESTIGATION

The investigation is composed of two major parts, theoretical and experimental. The theoretical part is the mathematical determination of the maximum negative moment capacity of the representative joists, the design of a semi-rigid connection to develop this negative moment, and the calculation of the rotation at the joint due to the behavior of the semi-rigid connections. The experimental part is the laboratory procedure that was followed, the data and results obtained from the four representative connections, plus a discussion and conclusion from the tests.

A. THEORETICAL

1. Selection of Longspan Joists

The four representative longspan joists were selected from a booklet which is published by Steel Joist Institute each year identifying the longspan joists that are available from the members of SJI.[9] They are listed in Table 1. There were several variables, such as, joist spans, joist depths and joist weights, in selecting the joists. In order to minimize the variables, the joists were selected so that they might all have the same net load carrying capacity. From the joist characteristic table it can be seen that they all have a net load carrying capacity of approximately 330

Table 1. Longspan Joist Characteristics

Joist	40L13	32L11	24L09	18L07
Weight	36 plf	29 plf	25 plf	21 plf
"D"	40"	32"	24"	18"
"d"	38.30"	30.29"	22.44"	16.59"
R(max)	12.02k	9.59k	8.65k	6.88k
Angles(tc)	3-1/2"x3"x3/8"	3"x3"x5/16"	3"x3"x1/4"	2-1/2"x2-1/2"x1/4"
Angles(bc)	3"x3"x5/16"	3"x3"x1/4"	2-1/2"x2-1/2"x1/4"	2-1/2"x2-1/2"x5/16"
Span	68.0'	54.0'	43.0'	34.0'
Load	340 plf	330 plf	331 plf	329 plf

pounds per linear foot.

With the span corresponding to this net load, each joist has a certain negative moment capacity. The correlation between the span and the loading are: if the load is held constant, the development of the negative moment capacity allows the span to increase or if the span is held constant, the development of the negative moment capacity allows the allowable load to increase. The author of this thesis has chosen to hold the span constant and determine the increased load carrying capacity of each joist. Then the maximum load of each joist will be determined. For this particular loading, there must exist a proper semi-rigid connection that allows both positive and negative moments to develop to optimum values simultaneously.

2. Description of Use of Longspan Joists

A typical warehouse frame of two-span, single story was chosen. The joists were analyzed in this frame and a symbolic frame is shown as follows:

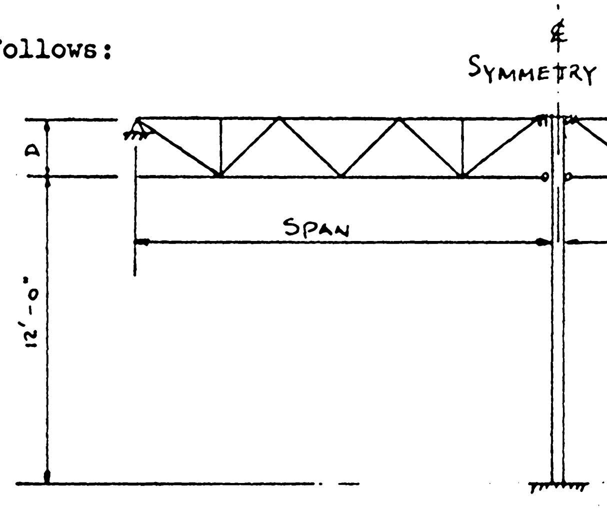


Figure 2. A Symbolic Two Span Frame

The center joint in Figure 2 is the portion of the frame that is of main interest and primary importance in this thesis. A larger scale of this section is shown in Figure 3. The connection is not shown in the sketch since this is the connection detail that is to be developed.

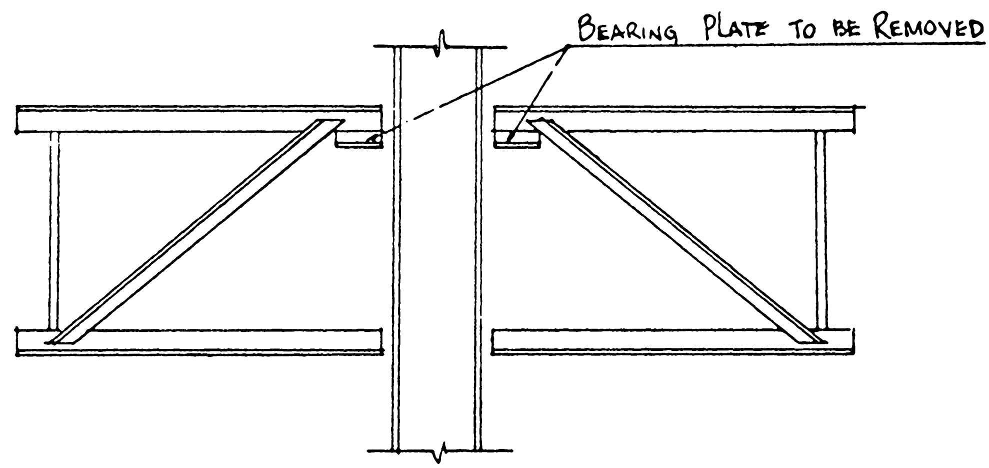


Figure 3. Column and Joist End Detail

The full size mock-up test section as shown above is made of two end panels of longspan joists, one on each side, and a section of column.

3. Characteristics of Longspan Joists

Section 103.1 of the 1959 edition of SJI Standard Specification of Open Web Steel Joists states that steel joists shall be designed as simply supported, uniformly loaded trusses. Yet the author of this thesis will try to investigate later in this thesis the negative moment capacity of certain longspan joists.

There are some limitations which the joists, by their nature, impose. First, the top chord of joists is assumed laterally supported along the entire length. This is usually done by attaching the floor slab or roof deck to the top chord to stabilize and prevent it from buckling. The bottom chords are designed as axially loaded tension members. If the connection is so designed that it is stiff enough to develop some negative moment in the longspan joist then there will be a laterally unsupported length of the bottom chord of the truss in compression. The bottom chord then will be able, to some extent, to resist buckling, but this behavior must be kept in mind.

A second limitation is that the members of a longspan joist are angles, rods and bars which are small in size compared to standard structural beams and columns used in continuous framing. This means that connections could be a problem because of their limited ability to develop bearing under bolts or rivets.

There are also qualities which are desirable in the end connections. First, it is desirable for the connection to act like a spring which yields somewhat to relieve the end restraint at joints. The spring action of a connection will cushion and reduce the degree of continuity at a rigidly framed joint or it will have the effect of modifying the moment diagram of a simply supported beam, thus showing less positive moment at mid-span and more negative moment at the ends. The joist will work more efficiently with the semi-rigid connections since it will have a negative moment while the positive moment capacity of the joist remains unchanged.

Second, the connection should be one which can be easily erected. This suggests the possibility of using construction bolts. And third, it would be desirable if present longspan joist connections could be modified simply and easily into semi-rigid connections.

4. Mathematical Analysis of the Negative Moment Capacity of the Joists

At this point the author wished to calculate the magnitude of the maximum negative moment that could be developed in the joist shown in Figure 4.

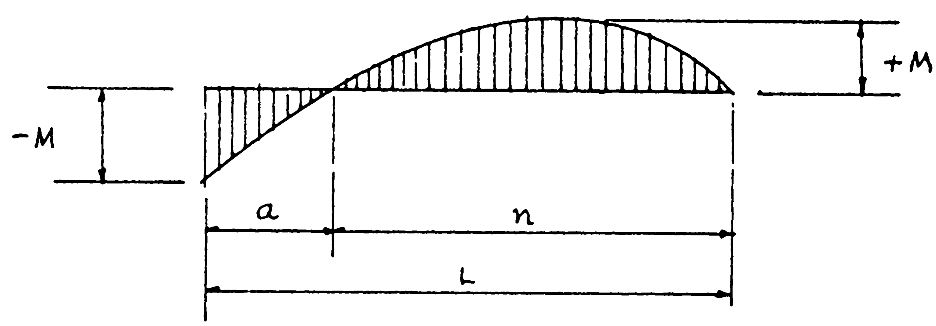


Figure 4. Moment Diagram

The tables of longspan joist characteristics list the positive moment capacity which each joist can develop. If we let the value be represented so:

$$+M = wn^2/8$$

where w = safe uniform load which each joist can carry, in pounds per linear foot

Assuming that the bottom chord will act as a column along the distance "a" we can use the allowable value given for a compression member with l/r not greater than 120 by the 1959 edition of SJI Specification, or:

$$f_c = 17{,}000 - 0.485\,(l/r)^2 \qquad \text{----------(F-1)}$$

in which "l" is the unsupported length and "r" is the corresponding least radius of gyration of a member, both in inches.

By equating equation (F-1) with the dimensional relationships of each joist, we get the maximum negative moment capacity of each joist as it is represented on the following pages.

Trial and Error Determination of Negative Moment Capacity

Longspan Joist 18L07

$L = 34'$

$A_{bc} = 1.80^{in^2}$

$d = 16.59''$

where d = distance between centroids of the top and bottom chord of Longspan Joist

$_{bc}r_{yy} = 1.33''$ (Assuming 3/4" between bottom angles)

$$-M = \frac{w}{2}(a^2 + an) \text{ ----- } (1)$$

$$-M = 1.80\left[17.0 - 0.000485\,(12a/1.33)^2\right](16.59/12) \text{ ----- } (2)$$

$$= 42.3 - 0.0983a^2$$

$a + n = 34'$

then $n = 34 - a$

substitute the above into Eq. (1), we have,

$$-M = \frac{w}{2}\left[a^2 + a\,(34 - a)\right] = 17wa \text{ ----- } (1)'$$

by equating (1)' = (2),

$$42.3 - 0.0983\,a^2 = 17\,wa$$

solving for w:

$$w = 2.49/a - 0.00578\,a \text{ ------ } (3)$$

but the positive moment capacity is,

$$+M = wn^2/8$$

for each value of "a" chosen, there are corresponding values of w, n, and +M. This can be expressed as follows:

a	$2.49/a - 0.00578\ a = w$				$n + M$
4	0.623	-	0.0231	= 0.600	30+67.5
5	0.498	-	0.0289	= 0.469	29+49.3
6	0.416	-	0.0347	= 0.381	28+37.3

By the corresponding values of "a", "w" and "+M", two curves are drawn. One is plotted on coordinates of "a" and "w" and the other on coordinates of "a" and " +M". They are designated as "w-a" curve and "+ M-a" curve on following page.

From Table 1 we have,

$W = 0.329$ klf

$L = 34.0$ ft

Consequently the maximum positive moment capacity of the Longspan Joist 18L07 can be obtained as follows,

$$+M = 0.329\ (34.0)^2/_8 = 47.5 \text{ ft-k}$$

By indicating this value on "+M-a" curve, we obtain a value of "a". Then from "w-a" curve a corresponding value of allowable load is obtained. It is:

$a = 5.14$

$n = 34.0-5.14 = 28.86$ ft

$w = 0.455$ klf

Consequently the maximum negative moment that the 18L07 can carry is calculated:

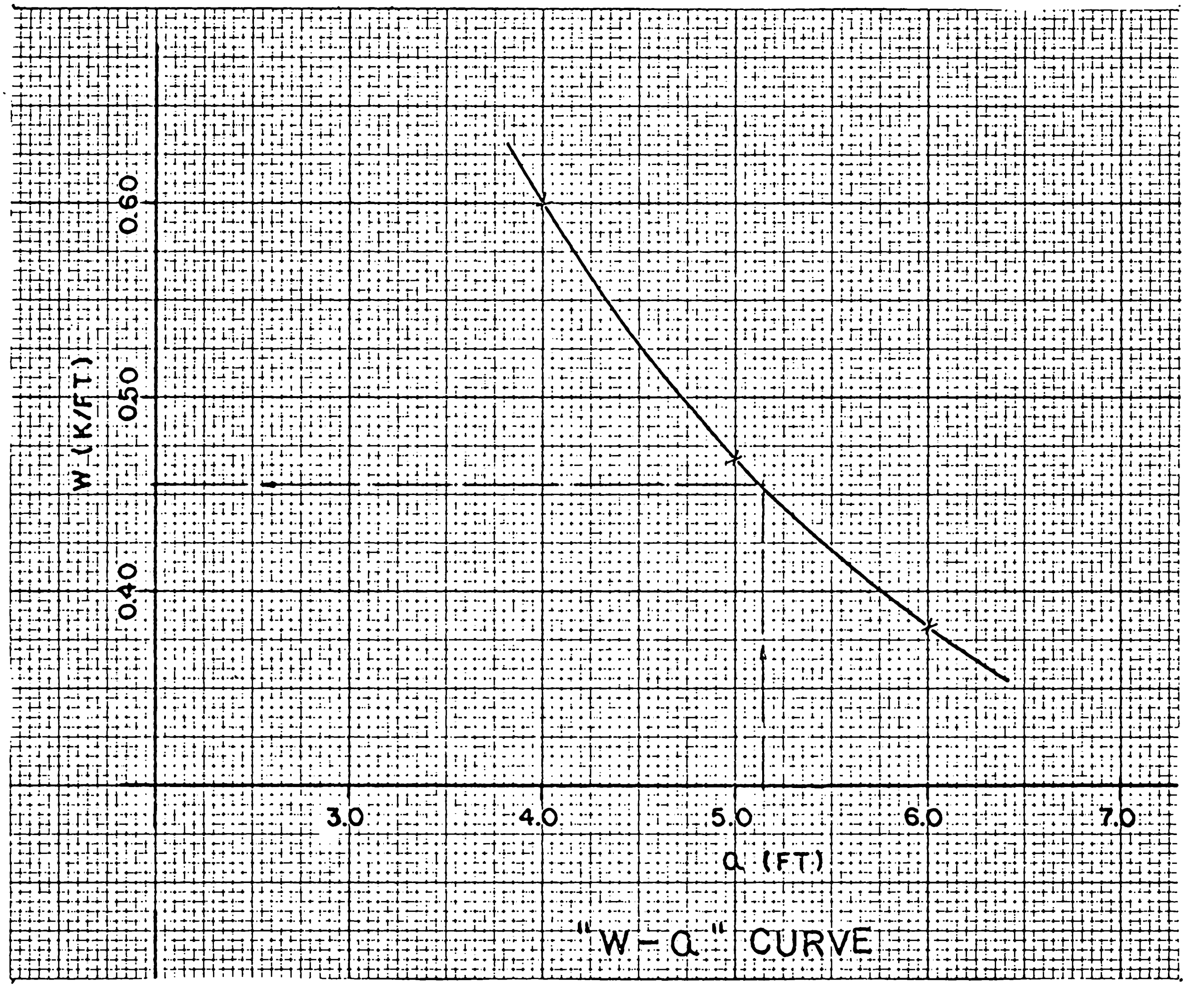


Figure 5. A.

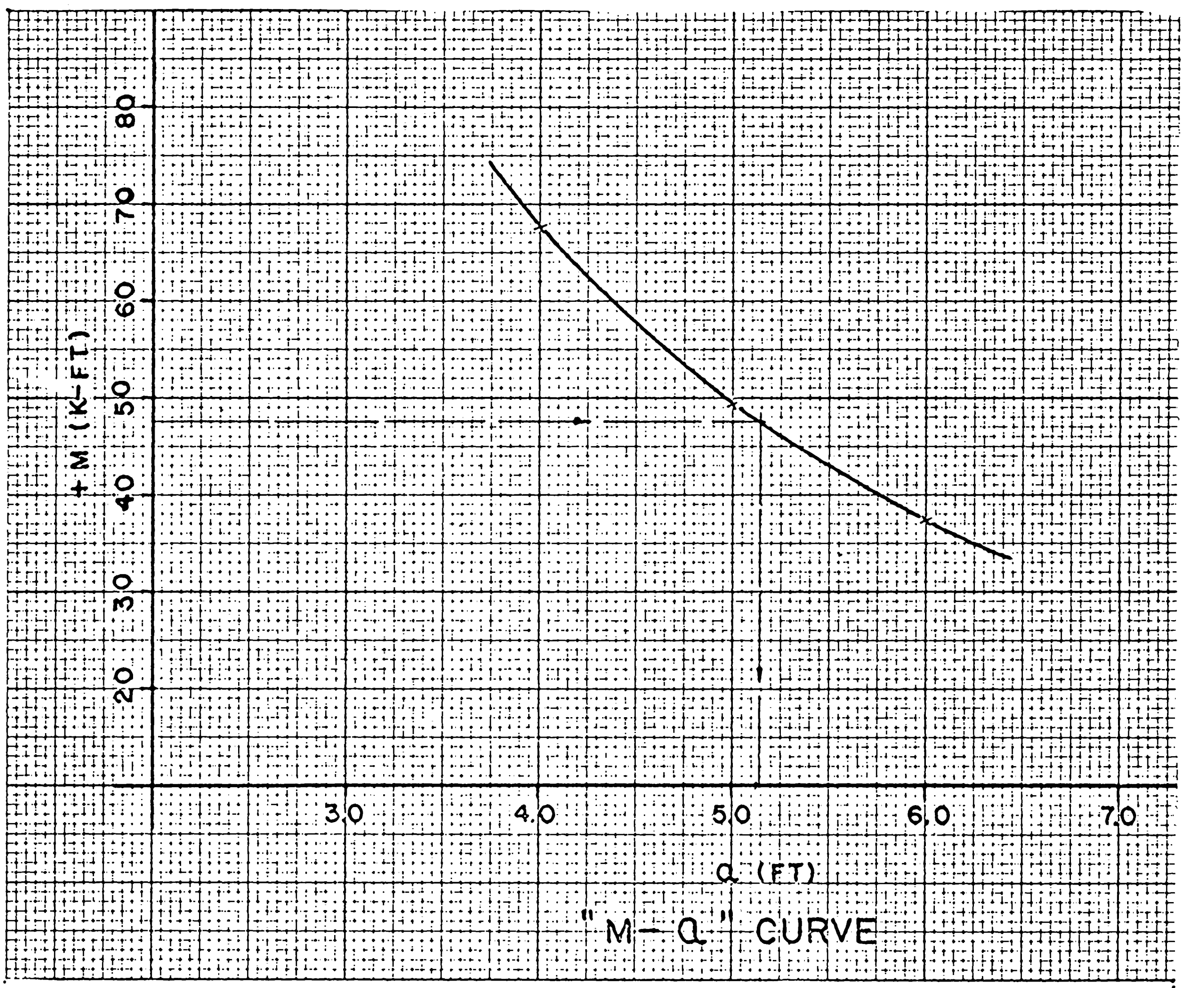


Figure 5. B.

$$-M = \frac{w}{2}(a^2 + an)$$

$$= \frac{0.455}{2}(5.14^2 + 5.14 \times 28.86) = 39.8 \text{ ft-k}$$

In a similar manner the allowable load, compression chord length, and the negative moment capacity of the remaining three joists were computed.

The data is presented as follows:

Table 2. Safe Load, Maximum Compressive Length and Maximum Moments

Section	+M (ft-k)	w (k/ft)	a (ft)	L (ft)	n (ft)	-M (ft-k)
18L07	47.5	0.455	5.14	34.0	28.86	-39.8
24L09	76.5	0.464	6.78	43.0	36.22	-67.6
32L11	120.3	0.465	8.58	54.0	45.42	-107.7
40L13	196.5	0.467	10.03	68.0	57.97	-159.5

Where: $-M = \frac{wa}{2}(a + n) = \frac{waL}{2}$

The compressive length of bottom chord "a" of each joist will be checked against the slenderness ratio l/r allowed by the 1959 Specification of the Steel Joist Institute.

Table 3. Maximum Allowable Compressive Length Governed by Vertical Buckling

18L07	2 ∠s 2-1/2 x 2-1/2 x 3/16		(Assuming 3/4" between angles)
	$r_{yy} = 1.33$		$5.14\ (12)/1.33 = 46.3 < 120$
	$r_{xx} = 0.78$		Allow $L_{xx} = 0.78\ (46.3)/12 = \underline{3.02}$ ft
24L09	2 ∠s 2-1/2 x 2-1/2 x 1/4		
	$r_{yy} = 1.34$		$6.78\ (12)/1.34 = 60.7 < 120$
	$r_{xx} = 0.77$		Allow $L_{xx} = 0.77\ (6.07)/12 = \underline{3.90}$ ft
32L11	2 ∠s 3 x 3 x 1/4		
	$r_{yy} = 1.53$		$8.58\ (12)/1.53 = 66.9 < 120$
	$r_{xx} = 0.93$		Allow $L_{xx} = 0.93\ (66.9)/12 = \underline{5.18}$ ft
40L13	2 ∠s 3 x 3 x 5/16		
	$r_{yy} = 1.54$		$10.03\ (12)/1.54 = 78.2 < 120$
	$r_{xx} = 0.92$		Allow $L_{xx} = 0.92\ (78.2)/12 = \underline{5.99}$ ft

The allowable length, which is governed by vertical buckling, is as shown above. The maximum values allowed by various manufacturers differ from one to the other. The author has designed his joists with the maximum L_{xx} above in mind.

5. Analytical Determination of Joist Dimensions and Member Stresses

Since Steel Joist Institute does not furnish the information about each member size in the 1959 edition booklet, Open Web Steel Joists, the author has to design and determine the members of all four joists.

(1) Joist Dimensions:

Since the practice of the various manufacturers varies with respect to the number of panels, the author has chosen to make the four joists alike with eleven interior panels each. They are listed as follows:

$L = 2F + 11G = 34.0$ ft

$F = 2.0$ ft

$G = (34.0 - 2 \times 2.0)/11 = 2.73$ ft $= 32.8$ in

$D = 18.0$ in

$d = 18.0 - 0.69 - 0.72 = 16.59$ in

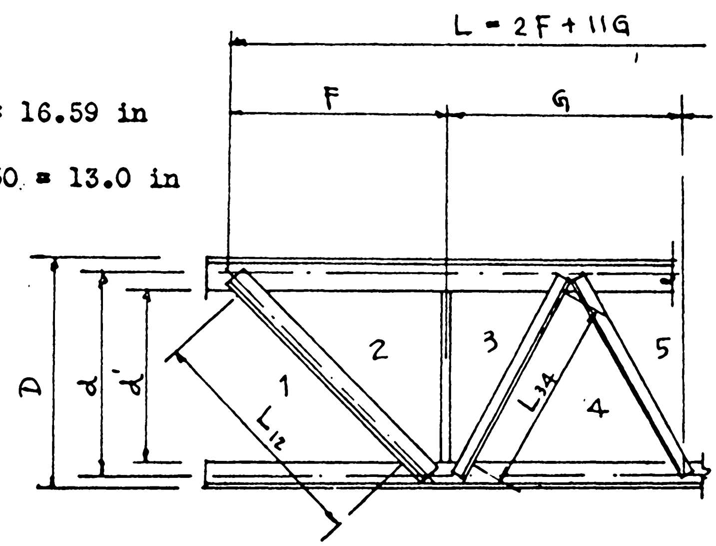
$d' = L_{2,3} = 18.0 - 2 \times 2.50 = 13.0$ in

$L^2_{1,2} = 13.0^2 + 18.8^2$

$L_{1,2} = 22.8$ in

$L^2_{3,4} = 13.0^2 + 12.82^2$

$L_{3,4} = 18.2$ in



Where $L_{1,2}$, $L_{2,3}$, $L_{3,4}$ are unsupported member lengths.

In a similar manner the joist dimensions, and unsupported member lengths of the remaining three joists were obtained. They are as follows:

Table 4. Joist Dimensions and Unsupported Member Lengths

Section	18L07	24L09	32L11	40L13
L (ft)	34.0	43.0	54.0	68.0
G (ft)*	2.73	3.45	4.36	5.45
F (ft)*	2.0	2.5	3.0	4.0
d (in)	16.59	22.44	30.29	38.30
$L_{1,2}$ (in)	22.8	30.9	40.4	53.7
$L_{2,3}$ (in)	13.0	18.5	26.0	33.5
$L_{3,4}$ (in)	18.2	25.2	34.4	44.1

*Notice these values of F and G are all less than the values of allow L_{xx} which are listed in Table 3.

The above data is useful for calculations of joint rotation.

(2) Member Stresses:

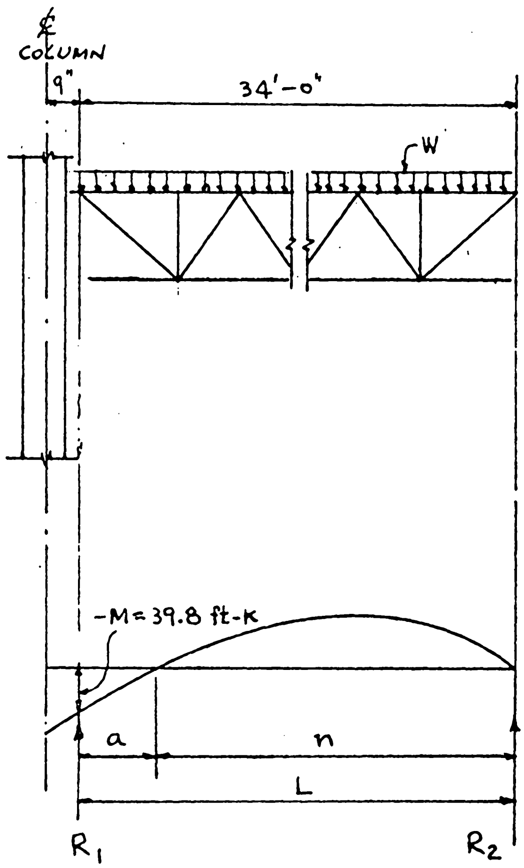
At this point it is necessary to compute the member stresses of each joist since the accurate member stress will lead to a proper and economic design of member size.

18L07

$-M = -39.8$ ft-k

$P_t = 39.8\ (12)/16.59 = 28.8$ k

The reactions at the supports are as follows: From Table 2 the given design information is obtained,



$W = 0.455$ klf

$a = 5.14$

$n = 28.86$

$-M = 39.8$ ft-k

Consequently,

$R_1 = wn/2 + wa = w\,(n/2 + a)$

$= 0.455\,(28.86/2 + 5.14) = 8.90$ k

$R_2 = Wn/2$

$= 0.455\,(28.86)/2 = 6.56$ k

Check -M from pinned end,

$-M = R_2\,L - wL^2/2$

$= 6.56\,(34.0) - 0.455\,(34.0)^2/2 = 40.0 \text{ ft-k} = 39.8 \text{ ft-k}$ O.K.

There is enough information for computing member stresses. The author used graphical method to determine the member stresses. For convenience the stress diagram of 18L07 is drawn along with the Williot-Mohr Diagram on Figure 7.

The member stresses of all four joists are listed on Table 5.

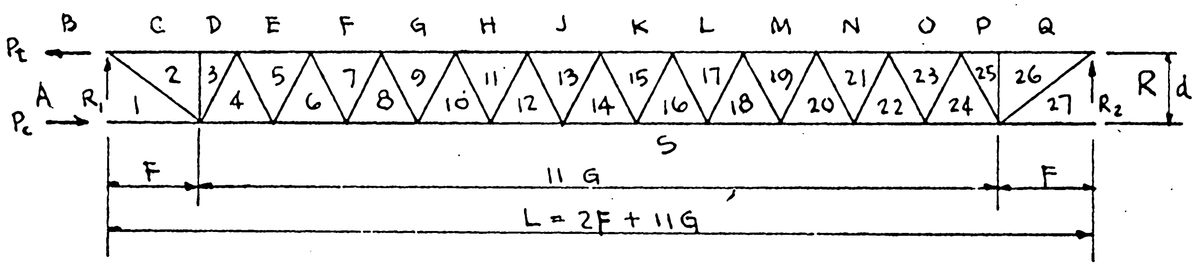


Figure 6. Typical Longspan Joist

Table 5. Member Stresses

18L07

Memb.	Stress. (kips)	Memb.	Stress. (kips)
1-2	14.8	C-2	16.7
2-3	-0.8	D-3	16.7
3-4	-10.7	E-5	2.8
4-5	9.3	F-7	-9.1
5-6	-9.3	G-9	-18.5
6-7	7.7	H-11	-25.6
7-8	-7.7	J-13	-30.1
8-9	6.0	K-15	-32.4
9-10	-6.0	L-17	-32.3
10-11	4.2	M-19	-29.9
11-12	-4.2	N-21	-25.0
12-13	2.4	O-23	-17.7
13-14	-2.4	P-25	-8.3
14-15	0.8	Q-26	-8.3
15-16	-0.8	S-1	-28.8
16-17	-0.9	S-4	-9.2
17-18	0.9	S-6	3.8
18-19	-2.6	S-8	14.4
19-20	2.6	S-10	22.7
20-21	-4.4	S-12	28.5
21-22	4.4	S-14	31.9
22-23	-6.1	S-16	32.3
23-24	6.1	S-18	31.7
24-25	-7.5	S-20	28.1
25-26	-0.8	S-22	22.0
26-27	10.4	S-24	13.5
		S-27	0

24L09

Memb.	Stress. (kips)	Memb.	Stress. (kips)
1-2	18.3	C-2	21.6
2-3	-1.0	D-3	21.6
3-4	-13.6	E-5	4.3
4-5	11.9	F-7	-10.3
5-6	-11.9	G-9	-21.9
6-7	9.7	H-11	-30.7
7-8	-9.7	J-13	-36.5
8-9	7.6	K-15	-39.4
9-10	-7.6	L-17	-39.3
10-11	5.4	M-19	-36.2
11-12	-5.4	N-21	-30.2
12-13	3.2	O-23	-21.3
13-14	-3.2	P-25	-9.7
14-15	1.0	Q-26	-9.7
15-16	-1.0	S-1	-36.1
16-17	-1.2	S-4	-12.4
17-18	1.2	S-6	3.7
18-19	-3.4	S-8	16.8
19-20	3.4	S-10	27.0
20-21	-5.5	S-12	34.3
21-22	5.5	S-14	38.7
22-23	-7.7	S-16	40.1
23-24	7.7	S-18	38.5
24-25	-9.3	S-20	33.9
25-26	-1.0	S-22	26.5
26-27	12.8	S-24	16.0
		S-27	0

Note: Positive stresses are designated without sign. They are in tension. Negative stresses are in compression.

Table 5. Member Stresses

32L11				40L13			
Memb.	Stress. (kips)	Memb.	Stress. (kips)	Memb.	Stress. (kips)	Memb.	Stress. (kips)
1-2	21.7	C-2	26.1	1-2	27.8	C-2	28.2
2-3	-1.2	D-3	26.1	2-3	-1.6	D-3	28.2
3-4	-16.8	E-5	5.5	3-4	-20.8	E-5	2.9
4-5	14.7	F-7	-11.8	4-5	18.2	F-7	-18.6
5-6	-14.7	G-9	-25.7	5-6	-18.2	G-9	-35.6
6-7	12.0	H-11	-36.1	6-7	14.8	H-11	-48.2
7-8	-12.0	J-13	-43.0	7-8	-14.8	J-13	-56.5
8-9	9.3	K-15	-46.4	8-9	11.4	K-15	-60.4
9-10	-9.3	L-17	-46.3	9-10	-11.4	L-17	-60.0
10-11	6.7	M-19	-42.8	10-11	8.1	M-19	-55.3
11-12	-6.7	N-21	-35.8	11-12	-8.1	N-21	-46.2
12-13	3.9	O-23	-25.3	12-13	4.7	O-23	-32.8
13-14	-3.9	P-25	-11.7	13-14	-4.7	P-25	-15.5
14-15	1.4	Q-26	-11.7	14-15	1.4	Q-26	-15.5
15-16	-1.4	S-1	-42.7	15-16	-1.4	S-1	-50.0
16-17	-1.4	S-4	-15.1	16-17	-1.9	S-4	-14.8
17-18	1.4	S-6	4.0	17-18	1.9	S-6	8.9
18-19	-4.0	S-8	19.6	18-19	-5.3	S-8	28.1
19-20	4.0	S-10	31.7	19-20	5.3	S-10	42.9
20-21	-6.8	S-12	40.4	20-21	-8.7	S-12	53.4
21-22	6.8	S-14	45.5	21-22	8.7	S-14	59.5
22-23	-9.5	S-16	47.3	22-23	-12.0	S-16	61.2
23-24	9.5	S-18	45.4	23-24	12.0	S-18	58.7
24-25	-11.4	S-20	40.2	24-25	-14.6	S-20	51.9
25-26	-1.2	S-22	31.4	25-26	-1.6	S-22	40.6
26-27	15.3	S-24	19.1	26-27	20.2	S-24	25.0
		S-27	0			S-27	0

Note: Positive stresses are designated without sign. They are in tension. Negative stresses are in compression.

6. Design of Joist Members

The following design is based on the 1959 Specification of the Steel Joist Institute and the 1956 Specification of the American Institute of Steel Construction. Referring to Figure 6, the joist members are designated as:

Top chords: C-2, D-3, E-5, F-7, etc.

Bottom chords: S-1, S-4, S-6, S-8, etc.

Web members: 1-2, 2-3, 3-4, 4-5, 5-6, etc.

The stresses are obtained from Table 5. They are designated by $S_{1,2}$ $S_{2,3}$ $S_{3,4}$ and so on, i.e., stresses in members 1-2, 2-3, 3-4, etc. Again, the member design which will be illustrated as follows is covered for joist 18L07 only; they are:

(1) Top Chord:

2-Angles 2-1/2" x 2-1/2" x 1/4"

$A = 2.38 \text{ in}^2$

$r_{xx} = 0.77$ in

$L = 2.727$ ft $= 32.73$ in

Max. Stress at K-15, $S = 32.4$ k

F actual $= 32.4/2.38 = 13.6$ ksi

$L/r_{xx} = 32.73/0.77 = 42.5 < 120$

$f_c = 17{,}000 - 0.485\,(L/r_{xx})^2 = 17{,}000 - 0.485\,(42.5)^2$

$= 16.13$ ksi $> f$ actual

Check the top chord for bending. (Section 103.4 (a) of the 1959 SJI Specification)

(a) At Panel Point

S_{xx} top = 2 (I/y) = 2 (0.70/0.72) = 1.95 in^3

S_{xx} bottom = 2 $\left[0.70/(2.5 - 0.72)\right]$ = 0.784 in^3

M = 0.455 $(2.727)^2$ (12)/12 = 3.4 in-k

f_b = 3.4/0.784 = 4.33 ksi

L/r_{xx} = 32.73/2(0.77) = 21.25

F_a = 16.78 ksi

$f_a/F_a + f_b/F_b$ = 13.6/16.78 + 4.33/24.0 = 0.81 + 0.18 = 0.99 <1.0 O.K.

(b) At Mid-Panel

M = 0.455 $(2.727)^2$(12)/16 = 2.54 in-k

f_b = 2.54/1.95 = 1.30 ksi

L/r_{xx} = 32.73/0.77 = 42.5

F_a = 16.13 ksi

$f_a/F_a + f_b/F_b$ = 13.6/16.13 + 1.30/20.0 = 0.84 + 0.065 = 0.905 <1.0 O.K.

(2) Bottom Chord:

2-Angles 2-1/2" x 2-1/2" x 3/16"

A = 1.80 in^2

r_{xx} = 0.78 in

Max. Tensile Stress at S-16, S = 32.3 k

F actual = 32.8/1.80 = 18.0 ksi <20.0 ksi O.K.

Max. Compressive Stress at S-1, S = - 28.8 k

f actual = 24.0/1.80 = 13.3 ksi

L/r_{xx} = 24/0.78 = 30.8 < 120

fc = 17.000 - 0.485 $(30.8)^2$ = 16.54 ksi > f actual   O.K.

(3) $S_{1,2}$ = 14.8 k

A required = $\frac{14.8}{20.0}$ = 0.74 in^2

Use 2-Angles 1" x 1" x 1/4"

A act. = 0.88 in^2

(4) $S_{26,27}$ = + 10.4 k

A required = 10.4/20.0 = 0.52 in^2

Use 2-Angles 1" x 1" x 1/4"

(5) $S_{2,3}$ = $S_{25,26}$ = -.08 k

Try 1 Bar - 3/8"

A act = 0.1406 in^2

I = 0.00165 in^4

r = $\sqrt{I/A}$ = $\sqrt{0.00165/0.1406}$ = 0.108 in

L = 13.0 in

L/r = 13.0/0.108 = 121 ≅ 120

fc = $\left(1.6 - \frac{L/r}{200}\right)\left[17.000 - 0.485\,(L/r)^2\right]$ = 9.93 ksi

f actual = 0.8/0.1406 = 5.7 ksi < fc   O.K.

Use 1 Bar - 3/8 in Ø

(6) $S_{3,4} = -10.7$ k and $S_{24,25} = -7.5$ k

Try 2-Angles 1" x 1" x 3/16"

A act = 0.68 in^2

$r_{xx} = 0.30$

L = 18.2 in

$L/r_{xx} = 18.2/0.30 = 60.7 < 120$

$f_c = 17.000 - 0.485\ (60.7)^2 = 15.2$ ksi

$f_{act} = 10.7/0.68 = 15.7$ ksi

% Over stressed = $\frac{15.7 - 15.2}{15.2} \times 100\% = 3.3\%$ O.K.

Use 2-Angles 1" x 1" x 3/16"

(7) $S_{4,5} = 9.3$ k

Try 2-Angles - 1" x 1" x 3/16"

A act = 0.68 in^2

f act = 9.3/0.68 = 13.7 ksi <20.0 ksi O.K.

(8) Members 5-6, 6-7, 8-9, 9-10

Use 2-Angles -1" x 1" x 3/16"

(9) $S_{11,12} = 4.2$ k; $S_{10,11} = 4.2$ k

Try 2-Angles - 1" x 1" x 1/8"

A act = 0.43 in^2

$L/r = 18.2/0.3 = 60.7 < 120$

$f_c = 17.000 - 0.485\ (60.7)^2 = 15.2$ ksi

$f_{actual} = 4.2/0.46 = 9.14$ ksi $< fc$ O.K.

(10) Members 12-13, 13-14, 14-15, 15-16, 16-17, 17-18

Use 2-Angles - 1" x 1" x 1/8"

(11) Members 18-19, 19-20, 20-21, 21-22, 22-23, 23-24

Use 2-Angles - 1" x 1" x 3/16"

In the same manner, the members of the remaining joists are designed and they are listed in Table 6.

7. The Axial Deformation of Members and the Rotation of Joint

By the computed stresses shown in Table 5 and the member sizes shown in Table 6, the axial deformations of members are computed. They are as follows:

The deformation of members is computed from formula, (F-2),

$$\delta = SL/AE \qquad \text{(F-2)}$$

Where

δ = Axial Deformation of Member

S = Member Stress

L = Length of member

A = Cross Sectional area of member

E = The modulus of elasticity of steel

For instance,

Table 6. Member Sizes

18L07

Memb.	Size
1-2	2 ∠s - 1 x 1 x 1/4
2-3	1 Bar 3/8" ⌀
3-4	2 ∠s 1 x 1 x 3/16
4-5	"
5-6	"
6-7	"
7-8	"
8-9	"
9-10	"
10-11	2 ∠s 1 x 1 x 1/8
11-12	"
12-13	"
13-14	"
14-15	"
15-16	"
16-17	"
17-18	"
18-19	2 ∠s 1 x 1 x 3/16
19-20	"
20-21	"
21-22	"
22-23	"
23-24	"
24-25	"
25-26	1 Bar 3/8" ⌀
26-27	2 ∠s 1 x 1 x 1/4
Top Chord	2 ∠s 2-1/2 x 2-1/2 x 1/4
Bot.Chord	2 ∠s 2-1/2 x 2-1/2 x 3/16

24L09

Memb.	Size
1-2	2 ∠s 1-1/2 x 1-1/2 x 3/16
2-3	1 Bar 1/2" ⌀
3-4	2 ∠s 1-1/2 x 1-1/2 x 3/16
4-5	"
5-6	"
6-7	"
7-8	"
8-9	"
9-10	"
10-11	2 ∠s 1 x 1 x 3/16
11-12	"
12-13	"
13-14	"
14-15	"
15-16	"
16-17	"
17-18	"
18-19	2 ∠s 1-1/2 x 1-1/2 x 3/16
19-20	"
20-21	"
21-22	"
22-23	"
23-24	"
24-25	"
25-26	1 Bar 1/2" ⌀
26-27	2 ∠s 1-1/2 x 1-1/2 x 3/16
Top Chord	2 ∠s 3 x 3 x 1/4
Bot.Chord	2 ∠s 2-1/2 x 2-1/2 x 1/4

Table 6. Member Sizes

32L11

Memb.	Size
1-2	2 ∠s 1-1/2 x 1-1/2 x 3/16
2-3	1-Bar 5/8" Ø
3-4	2 ∠s 1-3/4 x 1-3/4 x 3/16
4-5	"
5-6	"
6-7	"
7-8	"
8-9	"
9-10	"
10-11	2 ∠s 1-1/4 x 1-1/4 x 1/8
11-12	"
12-13	"
13-14	"
14-15	"
15-16	"
16-17	"
17-18	"
18-19	2 ∠s 1-3/4 x 1-3/4 x 3/16
19-20	"
20-21	"
21-22	"
22-23	"
23-24	"
24-25	"
25-26	1 Bar 5/8" Ø
26-27	2 ∠s 1-1/2 x 1-1/2 x 3/16
Top Chord	2 ∠s 3 x 3 x 5/16
Bot.Chord	2 ∠s 3 x 3 x 1/4

40L13

Memb.	Size
1-2	2 ∠s 1-1/2 x 1-1/2 x 1/4
2-3	1 Bar 3/4" Ø
3-4	2 ∠s 1-3/4 x 1-3/4 x 1/4
4-5	"
5-6	"
6-7	"
7-8	"
8-9	"
9-10	"
10-11	2 ∠s 1-1/4 x 1-1/4 x 3/16
11-12	"
12-13	"
13-14	"
14-15	"
15-16	"
16-17	"
17-18	"
18-19	2 ∠s 1-3/4 x 1-3/4 x 1/4
19-20	"
20-21	"
21-22	"
22-23	"
23-24	"
24-25	"
25-26	1 Bar 3/4" Ø
26-27	2 ∠s 1-1/2 x 1-1/2 x 1/4
Top Chord	2 ∠s 3-1/2 x 3 x 3/8
Bot.Chord	2 ∠s 3 x 3 x 5/16

18L07

(1) Member 1-2

$S_{1,2} = 14.8^K$

$L_{1,2} = 22.85''$

$A = 2 \angle s\ 1 \times 1 \times 1/4 = 0.88 \text{ in}^2$

$E = 29{,}000$ ksi

Consequently,

$\delta_{1,2} = 14.8 \times 22.85 / (0.88 \times 29.000) = 0.01325 \text{ in} = 13.25 \times 10^{-3} \text{in}$

It is an elongation because the member is under tension.

(2) Member 2-3

$S_{2,3} = 0.8$ k

$L_{2,3} = 13.0$ in

$A = 1$ Bar $3/8$ in $\varnothing = 0.1406 \text{ in}^2$

$E = 29{,}000$ ksi

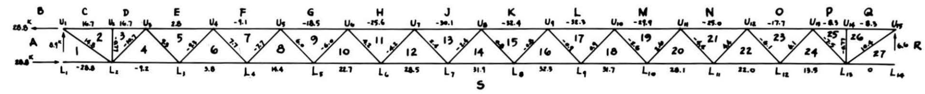
Then,

$\delta_{2,3} = -0.8 \times 13.0 / (0.1406 \times 29.000) = -0.00255 \text{ in} = -2.55 \times 10^{-3} \text{ in}$

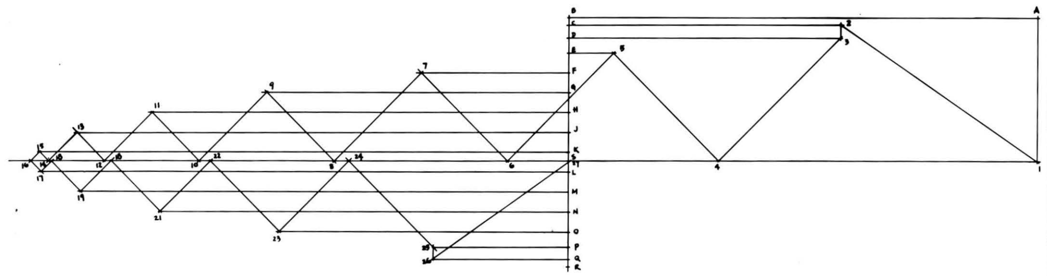
It is a shortening since this member is under compression.

In a similar manner the axial deformation of each member is computed. They are listed in Table 7.

The axial deformation of members is employed in conjunction with the Williot-Mohr diagram to determine the magnitude of rotation at the

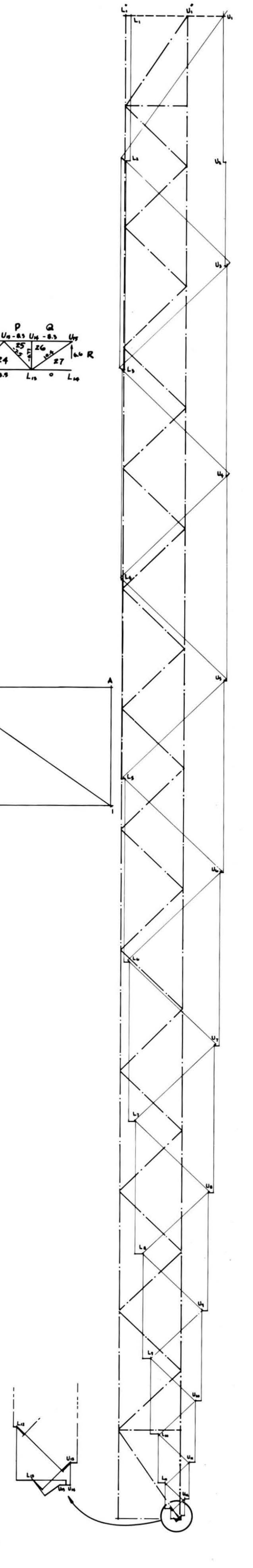


LONGSPAN JOIST 18L07



STRESS DIAGRAM

SCALE 0 1 2 3 4 5 Kips



WILLIOT—MOHR DIAGRAM

SCALE 0 20 80 100 Inches

Figure 7.

center joint. A reduced scale of the Williot-Mohr diagram of Longspan Joist 18L07 is shown in Figure 7.

From the Williot-Mohr diagram the following relationship is obtained. The relative displacement of joints U_1 and L_1, expressed as Δ, is,

$$\Delta = U_1U_1'' - L_1L_1''$$
$$= 92 - 12 = 80 \times 10^{-3} \text{ in}$$

Table 7. Axial Deformation of Members

18L07

Memb.	$\delta = SL/AE$ (10^{-3}in.)	Memb.	$\delta = SL/AE$ (10^{-3}in.)
1-2	13.25	C-2	5.81
2-3	-2.55	D-3	3.96
3-4	-9.88	E-5	1.33
4-5	8.58	F-7	-4.32
5-6	-8.58	G-9	-8.78
6-7	7.11	H-11	-12.14
7-8	-7.11	J-13	-14.27
8-9	5.53	K-15	-15.36
9-10	-5.53	L-17	-15.32
10-11	5.73	M-19	-14.18
11-12	-5.73	N-21	-11.85
12-13	3.27	O-23	-8.39
13-14	-3.27	P-25	-1.97
14-15	1.09	Q-26	-2.89
15-16	-1.09	S-1	-13.23
16-17	-1.23	S-4	-5.77
17-18	1.23	S-6	2.38
18-19	-2.40	S-8	9.03
19-20	2.40	S-10	14.23
20-21	-4.06	S-12	17.87
21-22	4.06	S-14	20.00
22-23	-5.63	S-16	20.25
23-24	5.63	S-18	19.88
24-25	-6.82	S-20	17.62
25-26	-2.55	S-22	13.80
26-27	9.32	S-24	8.46
		S-27	0

24L09

Memb.	$\delta = SL/AE$ (10^{-3}in.)	Memb.	$\delta = SL/AE$ (10^{-3}in.)
1-2	18.37	C-2	7.76
2-3	-2.55	D-3	5.36
3-4	-11.15	E-5	2.13
4-5	9.76	F-7	-5.12
5-6	-9.76	G-9	-10.87
6-7	7.95	H-11	-15.24
7-8	-7.95	J-13	-18.12
8-9	6.23	K-15	-19.56
9-10	-6.23	L-17	-19.57
10-11	6.90	M-19	-17.97
11-12	-6.90	N-21	-15.00
12-13	4.08	O-23	-10.57
13-14	-4.08	P-25	-2.41
14-15	1.28	Q-26	-3.48
15-16	-1.28	S-1	-15.70
16-17	-1.53	S-4	-7.45
17-18	1.53	S-6	2.22
18-19	-2.79	S-8	10.10
19-20	2.79	S-10	16.20
20-21	-4.51	S-12	20.60
21-22	4.51	S-14	23.20
22-23	-6.32	S-16	24.06
23-24	6.32	S-18	23.10
24-25	-7.63	S-20	20.35
25-26	-2.55	S-22	15.90
26-27	12.85	S-24	9.61
		S-27	0

Note: The Elongations are designated without sign.
The Shortenings are designated with negative sign.

Table 7. Axial Deformation of Members

32L11				40L13			
Memb.	$\delta = SL/AE$ (10^{-3}in.)	Memb.	$\delta = SL/AE$ (10^{-3}in.)	Memb.	$\delta = SL/AE$ (10^{-3}in.)	Memb.	$\delta = SL/AE$ (10^{-3}in.)
1-2	28.60	C-2	9.10	1-2	37.3	C-2	10.15
2-3	-2.76	D-3	6.62	2-3	3.29	D-3	6.92
3-4	-16.05	E-5	2.79	3-4	-19.53	E-5	1.42
4-5	14.05	F-7	-5.98	4-5	17.10	F-7	-9.13
5-6	-14.05	G-9	-13.04	5-6	-17.10	G-9	-17.50
6-7	11.47	H-11	-18.30	6-7	13.90	H-11	-23.60
7-8	-11.47	J-13	-21.80	7-8	-13.90	J-13	-27.70
8-9	8.90	K-15	-23.50	8-9	10.70	K-15	-29.65
9-10	-8.90	L-17	-23.47	9-10	-10.70	L-17	-29.45
10-11	13.25	M-19	-21.70	10-11	14.33	M-19	-27.20
11-12	-13.25	N-21	-18.15	11-12	-14.33	N-21	-22.65
12-13	7.72	O-23	-12.83	12-13	8.32	O-23	-16.10
13-14	-7.72	P-25	-2.96	13-14	-8.32	P-25	-3.80
14-15	2.77	Q-26	-4.08	14-15	2.48	Q-26	-5.58
15-16	-2.77	S-1	-18.40	15-16	-2.48	S-1	-23.25
16-17	-2.77	S-4	-9.47	16-17	-3.36	S-4	-9.38
17-18	2.77	S-6	2.51	17-18	3.36	S-6	5.64
18-19	-3.82	S-8	12.30	18-19	-4.98	S-8	17.82
19-20	3.82	S-10	19.90	19-20	4.98	S-10	27.20
20-21	-6.50	S-12	25.35	20-21	-8.16	S-12	33.85
21-22	6.50	S-14	28.52	21-22	8.16	S-14	37.70
22-23	-9.08	S-16	29.66	22-23	-11.26	S-16	38.80
23-24	9.08	S-18	28.45	23-24	11.26	S-18	37.20
24-25	-10.90	S-20	25.20	24-25	-13.70	S-20	32.90
25-26	-2.76	S-22	19.70	25-26	-3.29	S-22	25.75
26-27	20.15	S-24	11.97	26-27	27.1	S-24	15.85
		S-27	0			S-27	0

Note: The Elongations are designated without sign.
The Shortenings are designated with negative sign.

Consequently, the rotation at the restrained end of the joist is,

$$\phi = \Delta/d$$

$$= 0.080/16.59 = 0.0048 \text{ rad.}$$

In a similar manner the rotation at the restrained end of the remaining three joists are obtained. They are as follows:

Table 8. Displacement and Rotation at the Restrained Joint

Section	18L07	24L09	32L11	40L13
$U_1 U_1"$ (10^{-3} in.)	92	116	151	195
$L_1 L_1"$ (10^{-3} in.)	12	26	45	45
Δ (10^{-3} in.)	80	90	106	150
d (in.)	16.59	22.44	30.29	38.30
ϕ (radians)	0.0048	0.0040	0.0035	0.0039

8. Analysis and Design of Semi-Rigid Connection

Before discussing the design of semi-rigid connection, it is necessary to have a definition of the term, semi-rigid connection. If the beam-column connections of a building frame transmits bending moment without relative rotation between the end of the beam and the column, the connection and the structure are termed "rigid" (Fig. 1b). In such a case, the connections affords 100 per cent restraint or full continuity, and the maximum bending moments are at the ends of the beam. If the connections transmits bending moment with some relative rotation between the end of the beam and the column, the connections and the structure are termed "semi-rigid" (Fig. 1c). In such a structure, the connections resist bending moment to some degree less than in the case of full continuity, and the moment in the center of the span is always less than if the connection afforded no restraint, as in a simply supported beam (Fig. 1a).[5] Now that the negative moment capacity and joint rotation of each joist are known, an attempt can be made to design a semi-rigid connection to develop this negative moment and joint rotation in the joists.

The author of this thesis began his connection design with the connection which Combs suggested in his thesis,[8] after his experimental results failed to indicate the characteristics of semi-rigid connections (Figure 8). After some thought this form of connection was discarded because of the difficulty of determining the deformations of the bolts involved.

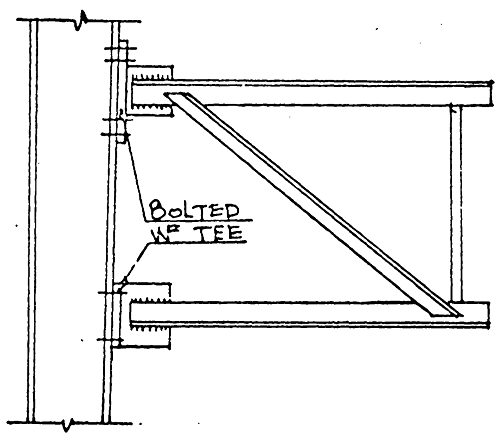


Figure 8. Preliminary Proposed Connection Detail

The final solution, based on the inadequacies of the previous one, seemed to be to eliminate the bolts and to weld the ends of flanges of the tee to the column flange (Figure 9).

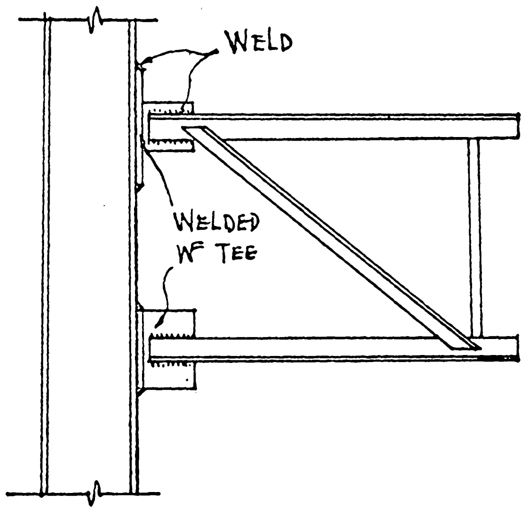


Figure 9. Final Proposed Connection Detail

Assuming this connection is a logical solution, the author now will carry out an analysis and design of this connection.

(1) Analysis of Connection

(a) Assumptions

It is necessary for the author to make several assumptions. They are as follows:

First, the connection is simply-supported at top and bottom where it is welded to column flange (Figure 10).

Secondly, the distribution of reaction at support is shaped like a section of sliced bread, formed by a parabola on top of a rectangle (Figure 11).

Thirdly, the distribution of the internal shear at the connection is like a trapezoid in the vertical direction and uniformly distributed in the horizontal direction (Figure 11).

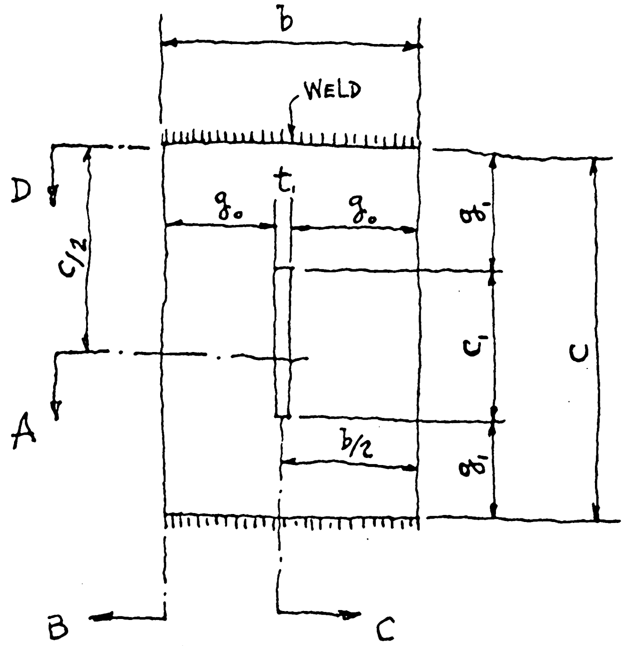


Figure 10. The Connection Detail

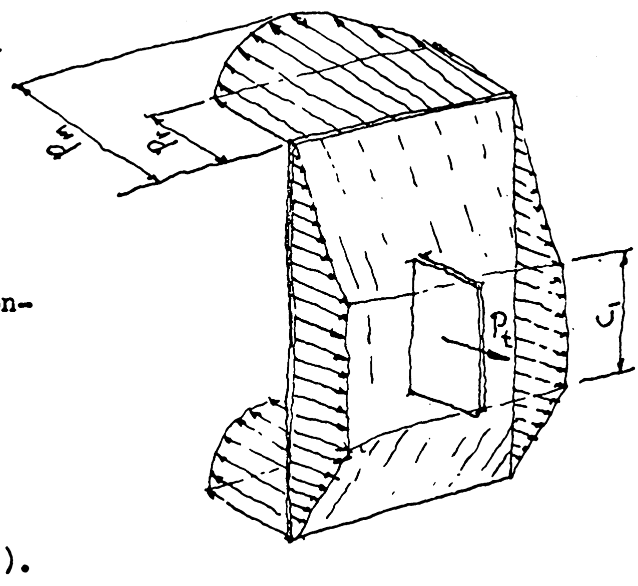
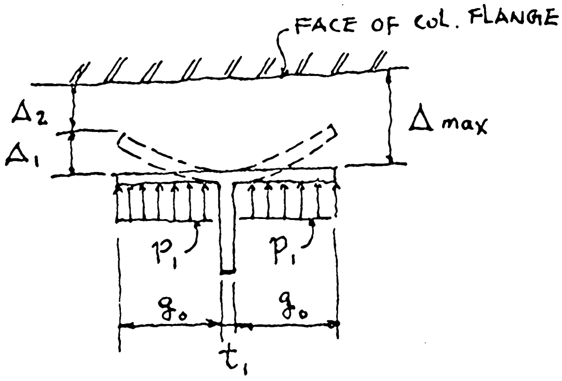
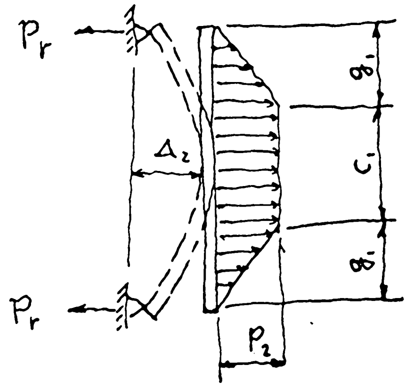


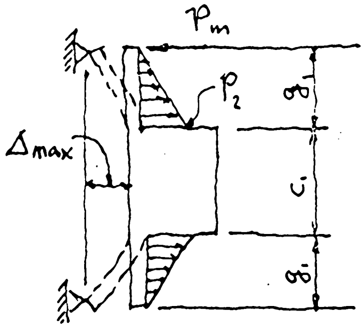
Figure 11. Distribution of Internal Shears and Reaction at Connection



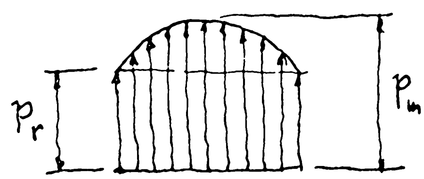
Section A.



Section B.



Section C.



Section D.

Figure 12. Sections of Connection

(b) Analysis

Referring to Figure 10, there are four sections taken at places where there are some significant relationships between the sections. The analytical procedure is shown as follows:

It is necessary to note that all the sections are taken as a unit strip. The analysis is made on the basic relationship in sections of Figure 12.

From Section A,

$$\Delta_1 = p_1 g_o^4/8EI$$

Also,

$$\Delta_{max} = \Delta_1 + \Delta_2$$

Then from Section B, the deflection Δ_2 is computed,

$$p_r = p_2(c_1 + g_1)/2$$

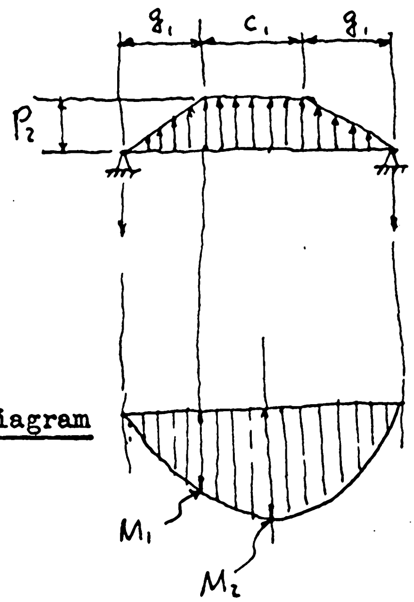
$$M_1 = p_r g_1 - (p_2 g_1/2)(g_1/3)$$

And

$$M_2 = p_r(g_1 + c_1/2) - P_2 g_1(g_1/3 + c_1/2)/2 - (P_2 c_1/2)(c_1/4)$$

Load

Mom.-Diagram

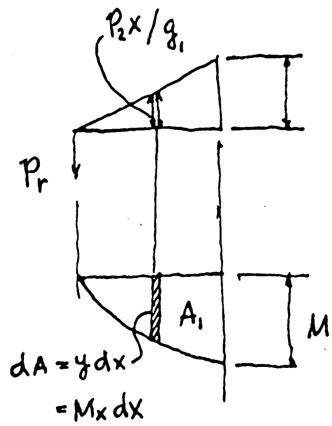


Substitute value of p_r into above equations, then,

$M_1 = p_2 g_1 (c_1 + 2g_1/3)/2$

$M_2 = p_2 (c_1\ 2/4 + c_1 g_1 + 2g_1^2/3)/2$

Since the conjugate beam method is used to compute the deflection, it is necessary to calculate all the geometric properties of the moment diagram. They are,



$$M_x = p_r x - (p_2 x/2g_1)(x)(x/3)$$

$$= p_2 x(c_1 + g_1 - x^2/3g_1)/2$$

where y = Mx in the sketch,

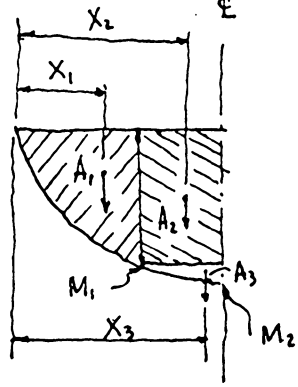
Consequently the product of A_1 and distance from the centroid to support, x_1, is,

$$x_1 A_1 = \int_0^{g_1} x\, dA = \int_0^{g_1} xy dx = \int_0^{g_1} x\, Mx\, dx$$

$$= \int_0^{g_1} p_2\, x^2/2(c_1 + g_1 - x^2/3g_1)dx = p_2(c_1 + g_1)/2\int_0^{g_1} x^2 dx -$$

$$p_2/6g_1 \int_0^{g_1} x^4\, dx$$

$$= p_2 (c_1 + g_1)/2 \left| x^3/3 \right|_0^{g_1} - p_2/6g_1 \left| x^5/5 \right|_0^{g_1} =$$



$$p_2 g_1^{3}(c_1 + 4g_1/5)/6$$

The area A_2 is expressed as,

$$A_2 = c_1 M_1/2 = p_2 g_1 c_1 (c_1 + 2g_1/3)/4$$

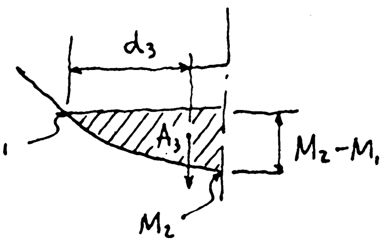
$$x_2 = g_1 + c_1/4$$

And the area A_3 is a segment of a parabola, because the load at this portion is uniformly distributed,

$$M_2 - M_1 = p_2(c_1^{2}/4 + c_1 g_1 + 2g_1^{2}/3)/2$$

$$- p_2 g_1 (c_1 + 2g_1/3)/2$$

$$= p_2 c_1^{2}/8$$



$$A_3 = (2/3)(M_2 - M_1)(c_1/2) = p_2 c_1^{3}/24$$

$$d = (5/8)(c_1/2) = 5\,c_1/16$$

$$x_3 = g_1 + d = g_1 + 5\,c_1/16$$

Then the deflection, Δ_2, is,

$$\Delta_2 = (A_1 x_1 + A_2 X_2 + A_3 x_3)/EI$$

$$= \left[p_2 g_1^{3}(c_1 + 4g_1/5)/6 + (p_2 g_1 c_1/4)(c_1 + 2g_1/3)(g_1 + c_1/4) \right.$$

$$+ (p_2 c_1^{3}/24)(g_1 + 5\ c_1/16)\Big]\ /EI$$

$$= p_2\ (4g_1^{4}/5 + 2\ g_1^{3} c_1 + 7g_1^{2} c_1^{2}/4 + 5g_1 c_1^{3}/8$$

$$+ 5c_1^{4}/64)\ /6EI$$

Let $k_o = 4g_1^{4}/5 + 2g_1^{3}c_1 + 7g_1^{2}c_1^{2}/4 + 5g_1 c_1^{3}/8 + 5\ c_1^{4}/64$

$$\Delta_2 = p_2\ k_0\ /6\ EI$$

Consequently the relationship as follows is true,

$$\Delta_{max} = \Delta_1 + \Delta_2$$

$$= p_1 g_0^{4}/8EI + p_2 k_0\ /6EI$$

But $p_1 = p_2$, Then,

$$\Delta_{max} = p_2(g_0^{4}/4 + k_0/3)\ /2EI \text{ ---------------------------------(A)}$$

From Section C, Figure 12,

$$\Delta_{max} = p_m g_1^{3}/3EI - g_1^{3}(p_2 g_1/2)\ /15EI$$

$$= g_1^{3}(p_m - p_2 g_1/10)/3EI \text{ ----------------------------------(B)}$$

Substitute (A) into (B), collect terms and simplify,

$$p_2(g_0^{4}/4 + k_0/3)/2EI = g_1^{3}(p_m - p_2 g_1/10)/EI$$

$$p_2 = 40\ p_m g_1^{3}/\ (15\ g_0^{4} + 20\ k_0 + 4\ g_1^{4})$$

$$= 40\ p_m g_1^{3}/k_1$$

where $k_1 = 15g_0^4 + 20k_0 + 4g_1^4$

substitute the value p_2 into Equation (B),

$$\Delta_{max} = g_1^3\, p_m (4g_1^4 + k_1) / 3EI\, k_1 \text{ -----------------------(B-1)}$$

from Section B, Figure 11,

$$p_r = p_2\, (c_1 + g_1) /2$$

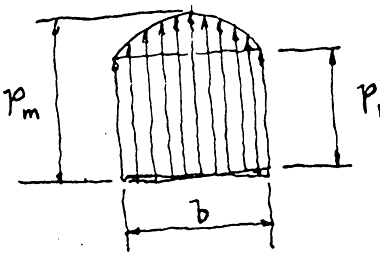
again substitute value of p_2 into above equation,

$$p_r = 20\, p_m g_1^3\, (c_1 + g_1)\, /k_1 \text{ ------------------------------(C)}$$

from Figure 11, the total reaction, V, at support is,

$$V = p_t/2 = p_r b + 2\, (p_m - p_r)\, b/3$$

$$p_m = 3\, p_t/4b - p_r/2$$



substitute Eq. (C) into above equation,

$$p_m = 3p_t/4b - 20\, p_m g_1^3 (c_1 + g_1)/2\, k_1$$

Collect terms,

$$p_m = 3\, p_t k_1/4b\, (k_1 + 10\, g_1^3\, c_1 + 10\, g_1^4) \text{ -----------------(D)}$$

substituting Eq. (D) into Eq. (B-1), the maximum deflection is obtained,

$$\Delta_{max} = g_1^3 p_m\, (4\, g_1^4 + k_1)/3EIk_1$$

$$= 3g_1^3 p_t k_1 (4g_1^4 + k_1)/4b\, (k_1 + 10g_1^3 c_1 + 10g_1^4)(3EIk_1)$$

$$\Delta_{max} = p_t g_1^3 (4g_1^4 + k_1)/4EIb\ (k_1 + 10g_1^3 c_1 + 10g_1^4)$$

or

$$\Delta_{max} = p_t g_1^3 (15g_o^4 + 20k_o + 8g_1^4)/4EIb\ (15g_o^4 + 20k_o + 14g_1^4 + 10g_1^3 c_1) \text{ -----(E)}$$

Therefore the joint rotation, ϕ is,

$$\phi = \Delta_{max/d}$$

where d = Depth of joist

The semi-rigid connection factor Z is obtained,

$$Z = \phi/M = \frac{\Delta_{max/d}}{P_t d} = \Delta_{max/p_t d^2}$$

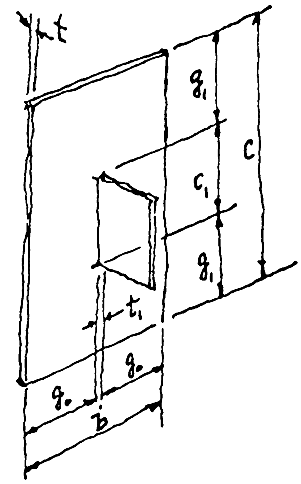
(2) Design of Connection

(a) Trial and Error Determination of Δ max and ϕ at Connection Longspan Joist 18L07

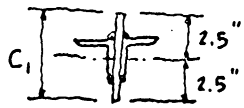
The tensile stress that is exerted at the connection, P_t, is obtained as follows:

$$P_t = M/d = 39.8(12)/16.59 = 28.8 \text{ Kip}$$

The connection is shown in the sketch. In order to let the connection deflect the exact amount so that it will give a correct rotation for this pull of 28.8 kips, the following trial and error method is used.



c = 16.5 in.



min. c_1 = 5.0 in.

max. $g_1 = \frac{1}{2}$ (16.5−5.0) = 5.75 in.

g_o = 3.717 in.

(i) if g_1 = 2.0 in.

then c_1 = 16.5 − 2 x 2.0 = 12.5 in.

and $k_o = 4g_1^4/5 + 2g_1^3c_1 + 7g_1^2c_1^2/4 + 5g_1c_1^3/8 + 5c_1^4/64$

$= 5{,}655.0 \text{ in}^4$

Eq. (E) $$\Delta_{max} = \frac{P_t g_1^3 (15g_o^4 + 20k_o + 8g_1^4)}{4EIb\,(15g_o^4 + 20k_o + 14g_1^4 + 10g_1^3c_1)} = 0.244P_t/EI$$

where b = 8.117 in.

E = 29,000 ksi

$I = bt^3/12 = 1\text{x}0.683^3/12 = 0.0265 \text{ in}^4$

consequently,

$\phi = \Delta max/d = 0.244\ P_t/EId = 0.55 \times 10^{-3}$ rad

$Z = \phi/M = \Delta max/P_td^2 = (0.244P_t/EI)(1/P_td^2)$

$= 0.244/EId^2 = 0.115 \times 10^{-5}$

(ii) if $g_1 = 3.0$ in.

$c_1 = 16.5 - 2 \times 3.0 = 10.5$ in.

then $k_o = 5,488.0 \text{ in}^4$

$\Delta_{max} = 0.808 \, P_t/EI$

$\phi = 1.81 \times 10^{-3}$ rad.

$Z = 0.378 \times 10^{-5}$

(iii) if $g_1 = 4.0$ in.

$c_1 = 8.5$ in.

then $k_o = 5,255.0$

$\Delta_{max} = 1.850 \, P_t/EI$

$\phi = 4.17 \times 10^{-3}$ rad

$Z = 0.872 \times 10^{-5}$

(iv) if $g_1 = 5.0$ in.

$c_1 = 6.5$ in.

then $k_o = 4,966.0 \text{ in}^4$

$\Delta_{max} = 3.460 \, P_t/EI$

$\phi = 7.80 \times 10^{-3}$ rad

$Z = 1.630 \times 10^{-5}$

By the use of four trials shown above there are two curves drawn. One is plotted in coordinates of ϕ and g_1 and the other in coordinates of Z and g_1. They are designated as the ϕ - g_1 curve and the Z - g_1 curve on the following pages.

By referring to Table 8 the rotation at the fixed and joint of longspan joist 18L07 is obtained. If the semi-rigid connection is capable of developing a rotation of the same amount; then the connection will satisfy the purpose of this thesis. This is achieved by plotting the value of ϕ from Table 8 on the ϕ - g_1 curve. There is a value of g_1 corresponding to this ϕ value. This means that if the stem of the tee is cut the amount g_1 at top and bottom, the connection will give the rotation of ϕ as desired. The above description is expressed as follows:

from the $\phi = 4.82 \times 10^{-3}$ rad (from Table 8)

ϕ - g_1 curve,

$g_1 = 4.2$ in.

also by the Z - g_1 curve,

$Z = 1.01 \times 10^{-5}$

The above values are indicated on the curves by arrowed lines.

(b) <u>Trial and Error Determination of Δmax and ϕ at Connection Longspan Joist 24L09</u>

$- M = 67.6$ ft-k (From Table 2)

$P_t = 67.6 \times 12/22.44 = 36.1$ kip

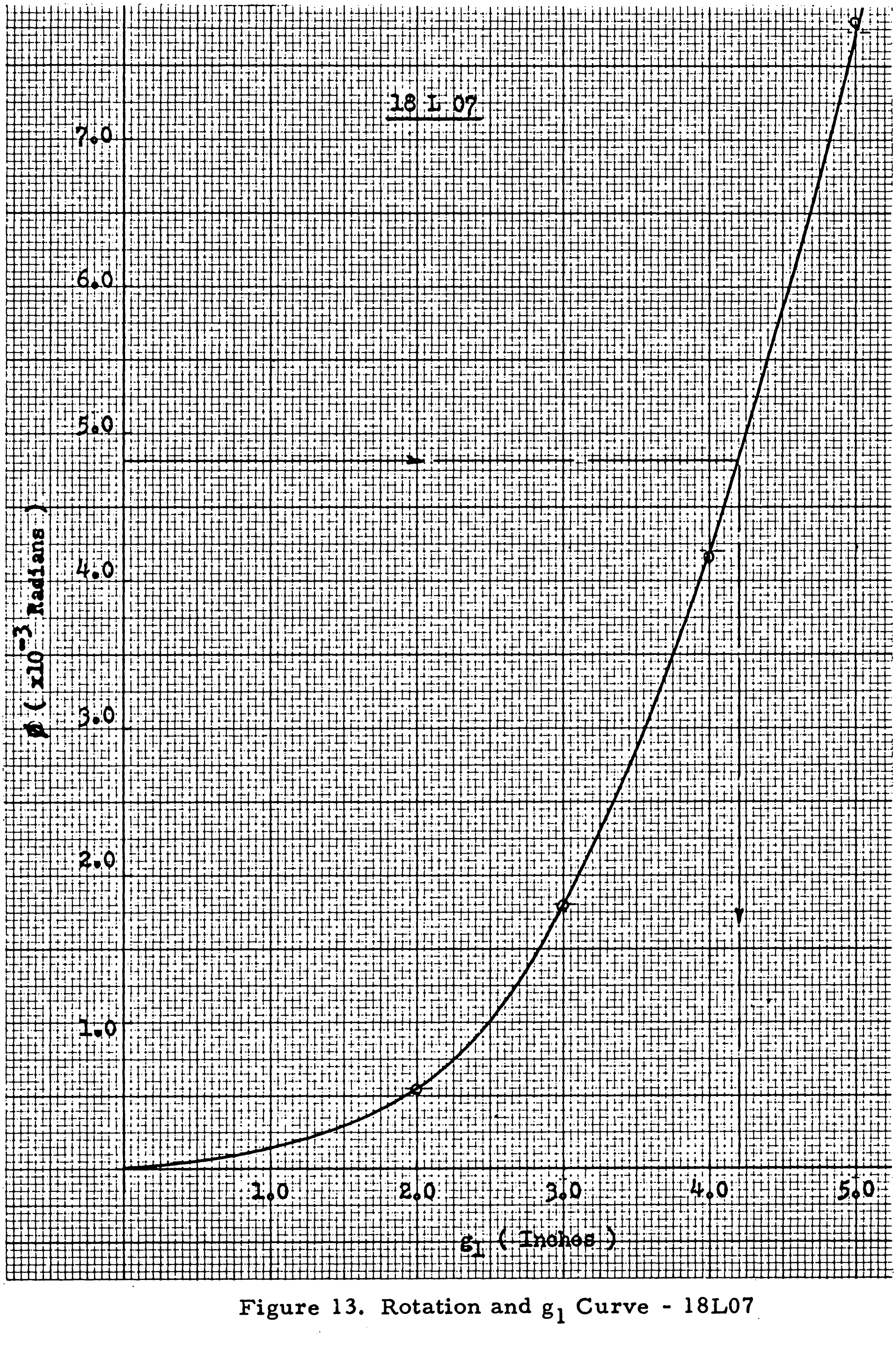


Figure 13. Rotation and g_1 Curve - 18L07

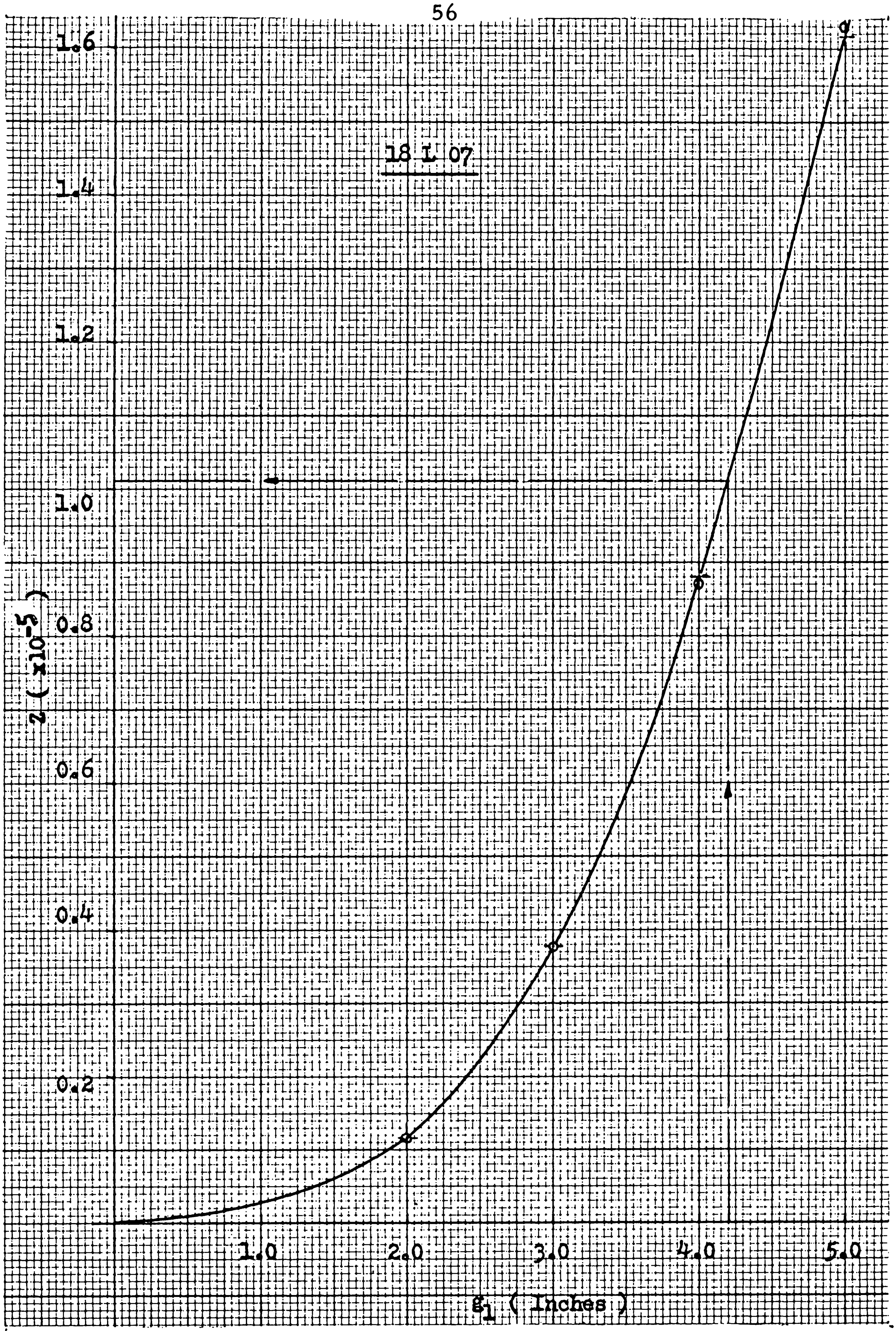


Figure 14. Z - g_1 Curve - 18L07

$d = 22.44$ in.

$g_o = 3.717$ in.

$b = 8.117$ in.

$C = 16.5$ in.

min. $C_1 = 5.0$ in.

max. $g_1 = \frac{1}{2}(16.5 - 5.0) = 5.75$ in.

(i) if $g_1 = 2.0$ in.

$c_1 = 12.5$

$k_o = 5{,}655.0 \text{ in}^4$

$\Delta_{max} = 0.244\ P_t/EI$

$\phi = \Delta_{max}/d = 0.51 \times 10^{-3}$ rad

$Z = 0.063 \times 10^{-5}$

(ii) if $g_1 = 3.0$ in.

$c_1 = 10.5$ in.

$k_o = 5{,}488.0 \text{ in}^4$

$\Delta_{max} = 0.808\ P_t/EI$

$\phi = 1.68 \times 10^{-3}$ rad

$Z = 0.207 \times 10^{-5}$

(iii) if $g_1 = 4.0$ in.

$c_1 = 8.5$ in.

$k_o = 5{,}255.0$

$\Delta_{max} = 1.850\ P_t/EI$

$\phi = 3.87 \times 10^{-3}$ rad

$Z = 0.477 \times 10^{-5}$

(iv) if $g_1 = 5.0$ in.

$c_1 = 6.5$ in.

$k_o = 4{,}966.0\ in^4$

$\Delta_{max} = 3.460\ P_t/EI$

$\phi = 7.22 \times 10^{-3}$ rad

$Z = 0.891 \times 10^{-5}$

In the same way as with 18L07 the $\phi - g_1$ and $Z - g_1$ curves are drawn. Consequently, the following data are obtained

$\phi = 4.02 \times 10^{-3}$ rad (from Table 8)

from the $\phi - g_1$ curve,

$g_1 = 4.05$ in.

from the $Z - g_1$ curve

$Z = 0.495 \times 10^{-5}$

(c) Trial and Error Determination of Δ max and ϕ at Connection Longspan Joist 32L11

$-M = 107.7$ ft-k

$P_t = 107.7 \times 12/30.29 = 42.7$ kip

$d = 30.29$ in.

$g_o = 3.717$ in.

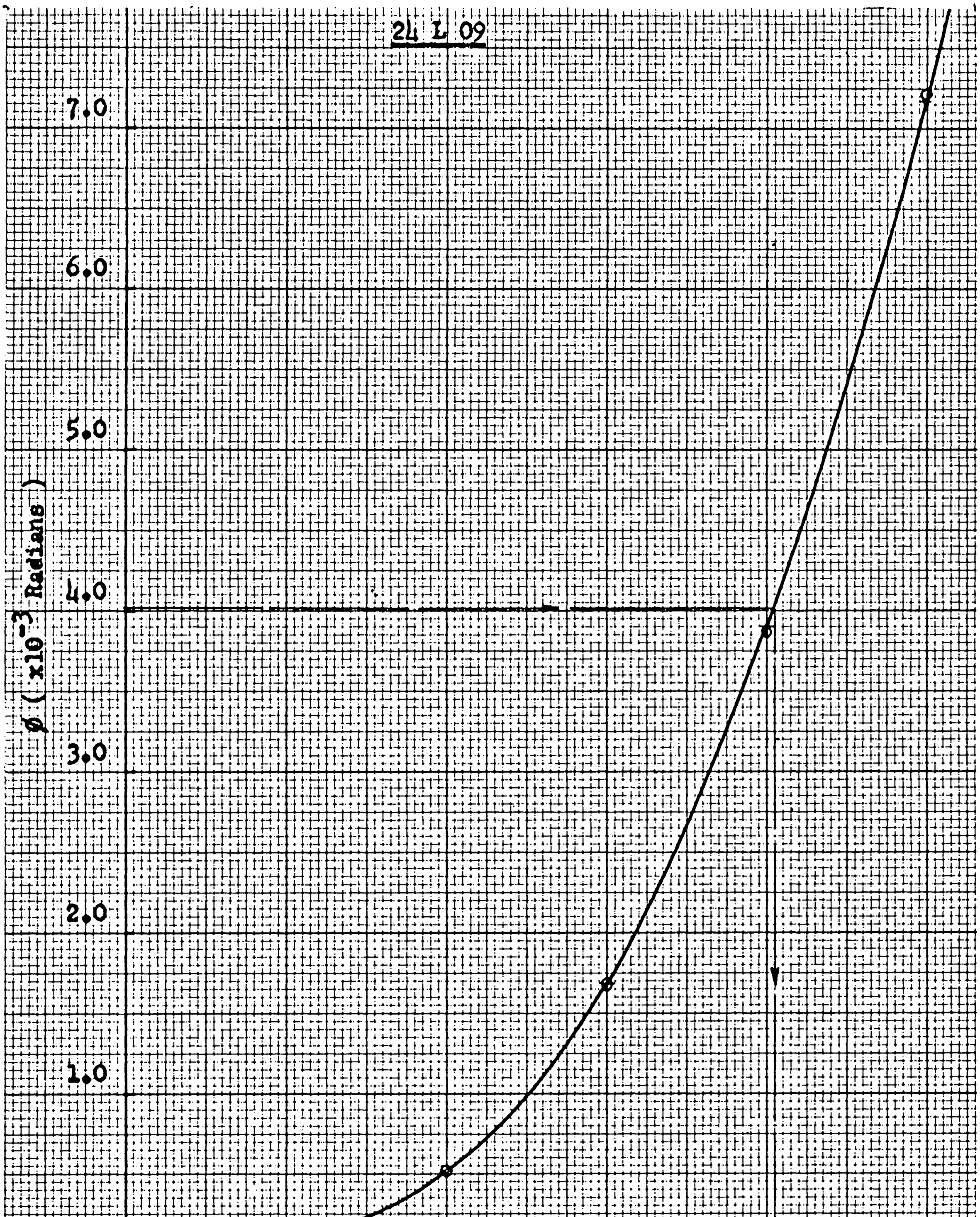


Figure 15. Rotation and g_1 Curve - 24L09

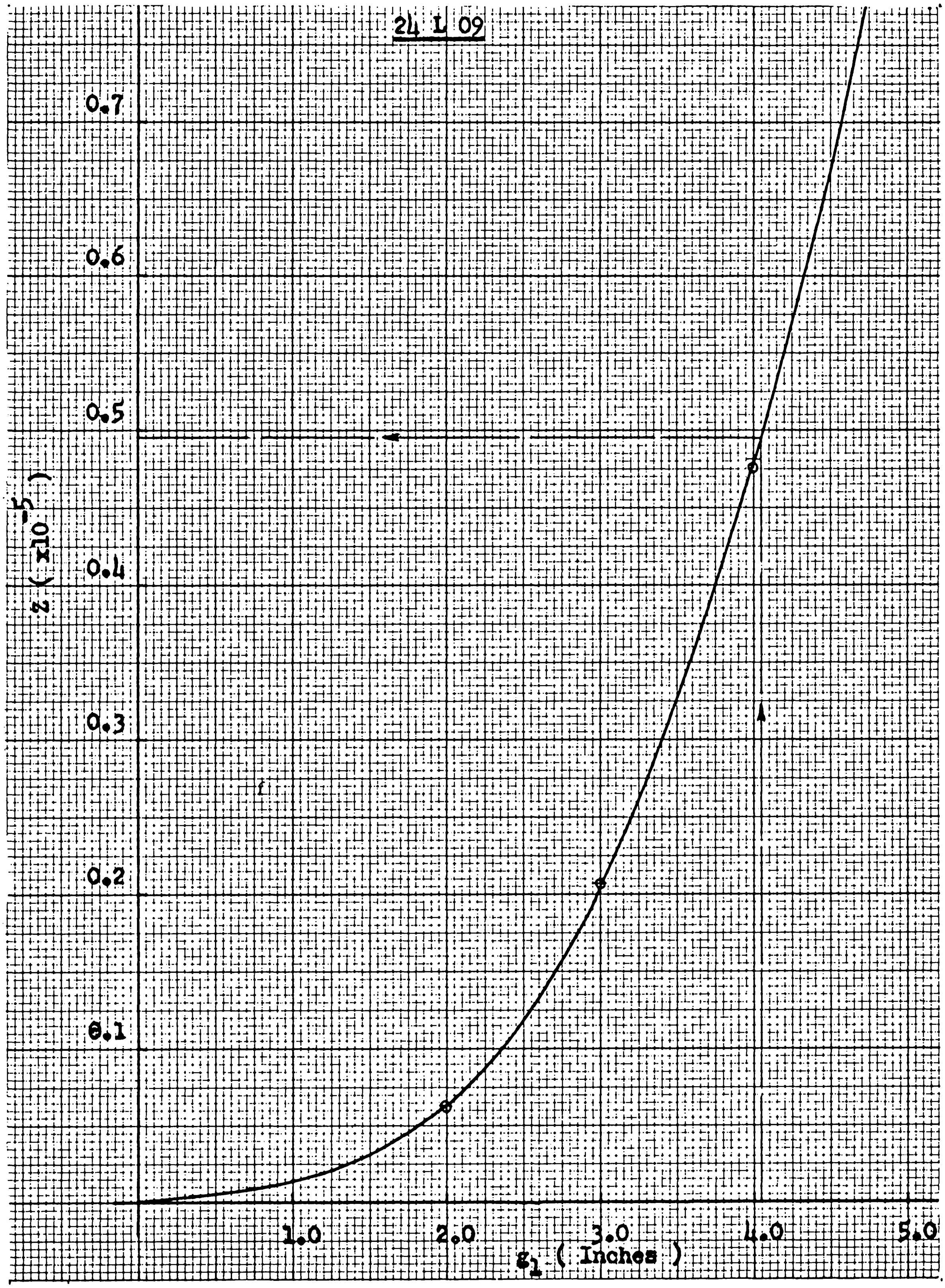


Figure 16. Z - g_1 Curve - 24L09

$b = 8.117$ in.

$c = 18.0$ in.

min $c_1 = 50$ in.

max $g_1 = \frac{1}{2}(18.0 - 5.0) = 6.5$ in.

(i) if $g_1 = 2.5$ in.

$c_1 = 18 - 2 \times 2.5 = 13.0$ in.

$k_o = 7{,}950.0$ in^4

$\Delta_{max} = 0.473\ P_t/EI$

$\phi = 0.99 \times 10^{-3}$ rad

$Z = 0.077 \times 10^{-5}$

(ii) if $g_1 = 3.5$ in.

$c_1 = 11.0$ in.

$k_o = 7{,}717.0$ in^4

$\Delta_{max} = 1.270\ P_t/EI$

$\phi = 2.33 \times 10^{-3}$ rad

$Z = 0.180 \times 10^{-5}$

(iii) if $g_1 = 5.0$ in.

$c_1 = 8.0$ in.

$k_o = 7{,}220.0$ in^4

$\Delta_{max} = 3.520\ P_t/EI$

$\phi = 6.45 \times 10^{-3}$ rad

$Z = 0.498 \times 10^{-5}$

(iv) if $g_1 = 6.5$ in.

$o_1 = 5.0$ in.

$k_o = 6{,}577.0\ in^4$

$\Delta\ max = 7.240\ P_t/EI$

$\phi = 13.25 \times 10^{-3}$ rad

$Z = 1.024 \times 10^{-5}$

From the $\phi - g_1$ and $Z - g_1$ Curves, the following results are obtained.

$\phi = 3.50 \times 10^{-3}$ rad (from Table 8)

$g_1 = 4.0$ in.

$Z = 0.270 \times 10^{-5}$

(d) <u>Trial and Error Determination of Δ max and ϕ at Connection Longspan Joist 40L13</u>

$-M = 159.5$ ft-k

$P_t = 159.5 \times 12/\ 38.30 = 60.0$ kip

$d = 38.30$ in.

$g_o = 3.717$ in.

$b = 8.117$ in.

$c = 18.0$ in.

min $o_1 = 5.0$ in.

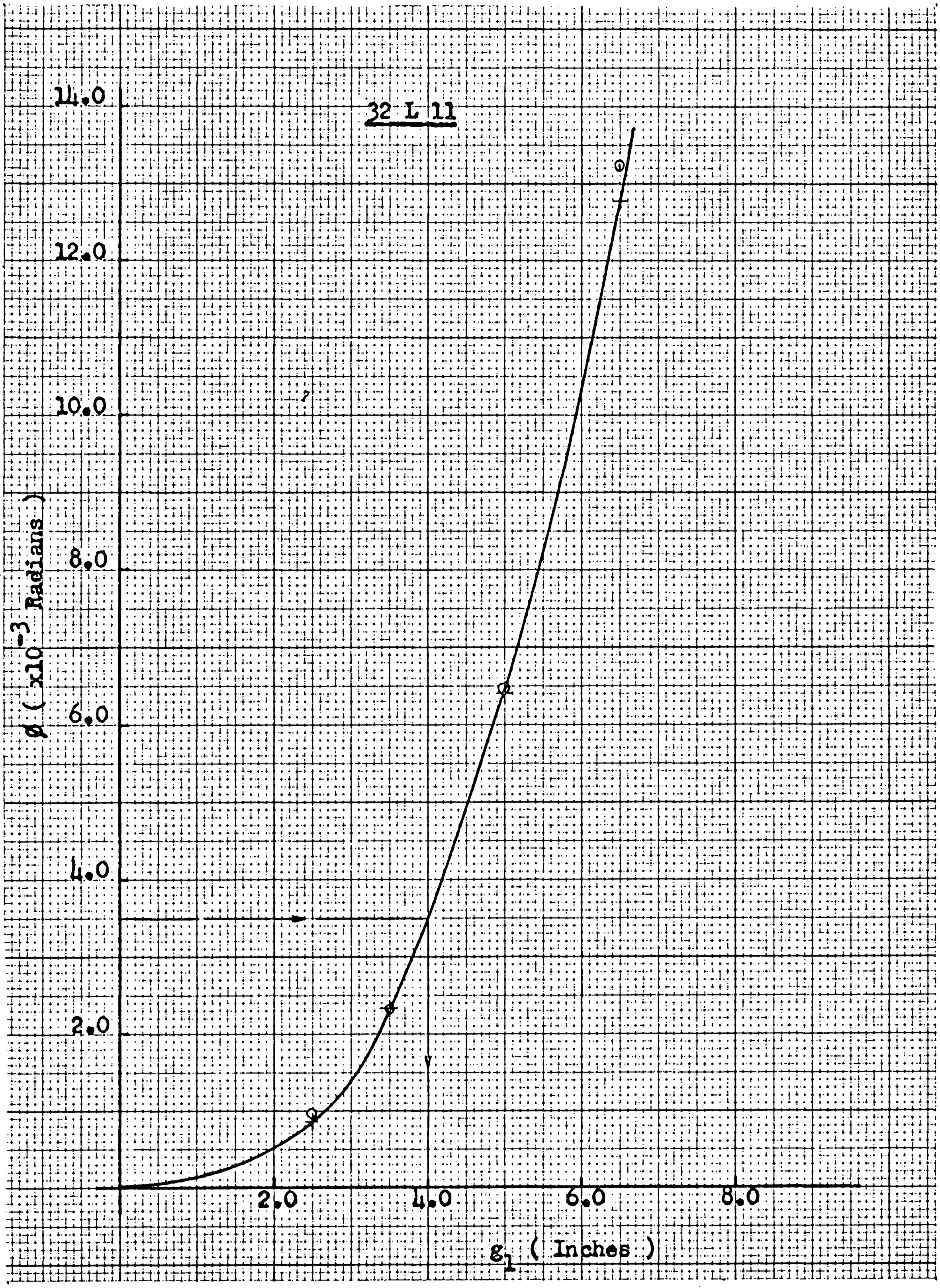


Figure 17. Rotation and g_1 Curve 32L11

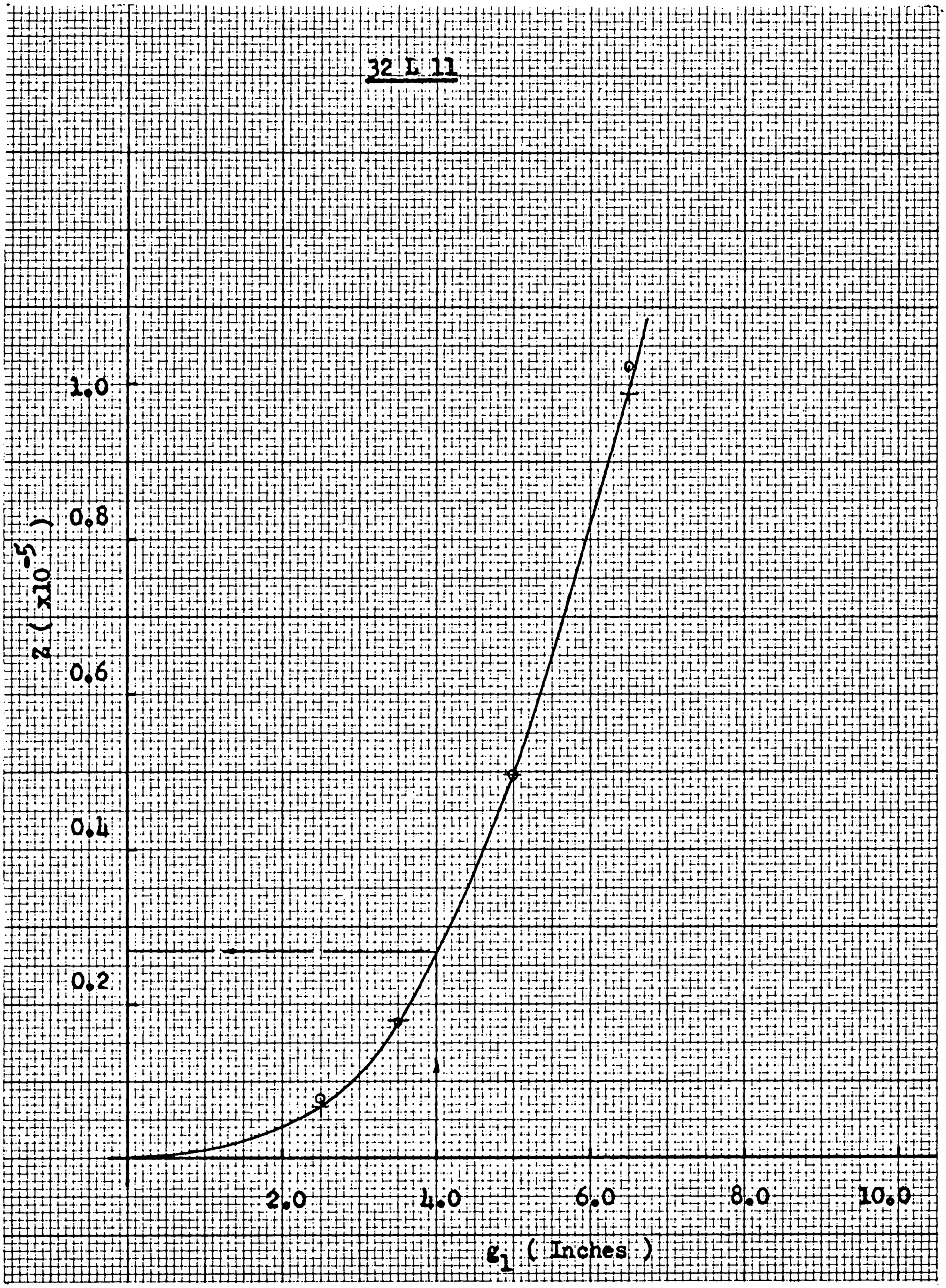


Figure 18. Z - g_1 Curve - 32 L 11

max g_1 = 6.5 in.

(i) if g_1 = 2.5 in.

c_1 = 13.0 in.

k_o = 7,950.0 in^4

Δ_{max} = 0.473 P_t/EI

ϕ = 0.92 x 10^{-3} rad

Z = 0.048 x 10^{-5}

(ii) if g_1 = 3.5 in.

c_1 = 11 in.

k_o = 7,170.0 in^4

Δ_{max} = 1.270 P_t/EI

ϕ = 2.16 x 10^{-3} rad

Z = 0.113 x 10^{-5}

(iii) if g_1 = 5.0 in.

c_1 = 8.0 in.

k_o = 7,220.0 in^4

Δ_{max} = 3.520 P_t/EI

ϕ = 5.97 x 10^{-3} rad

Z = 0.312 x 10^{-5}

(iv) if $g_1 = 6.5$ in.

$c_1 = 5.0$ in.

$k_o = 6{,}577.0\ in^4$

$\Delta_{max} = 7.240\ P_t/EI$

$\phi = 12.25 \times 10^{-3}$ rad

$Z = 0.640 \times 10^{-5}$

From the ϕ - g_1 and Z - g_1 Curves:

$\phi = 3.92 \times 10^{-3}$ rad (from Table 8)

$g_1 = 4.30$ in.

$Z = 0.210 \times 10^{-5}$

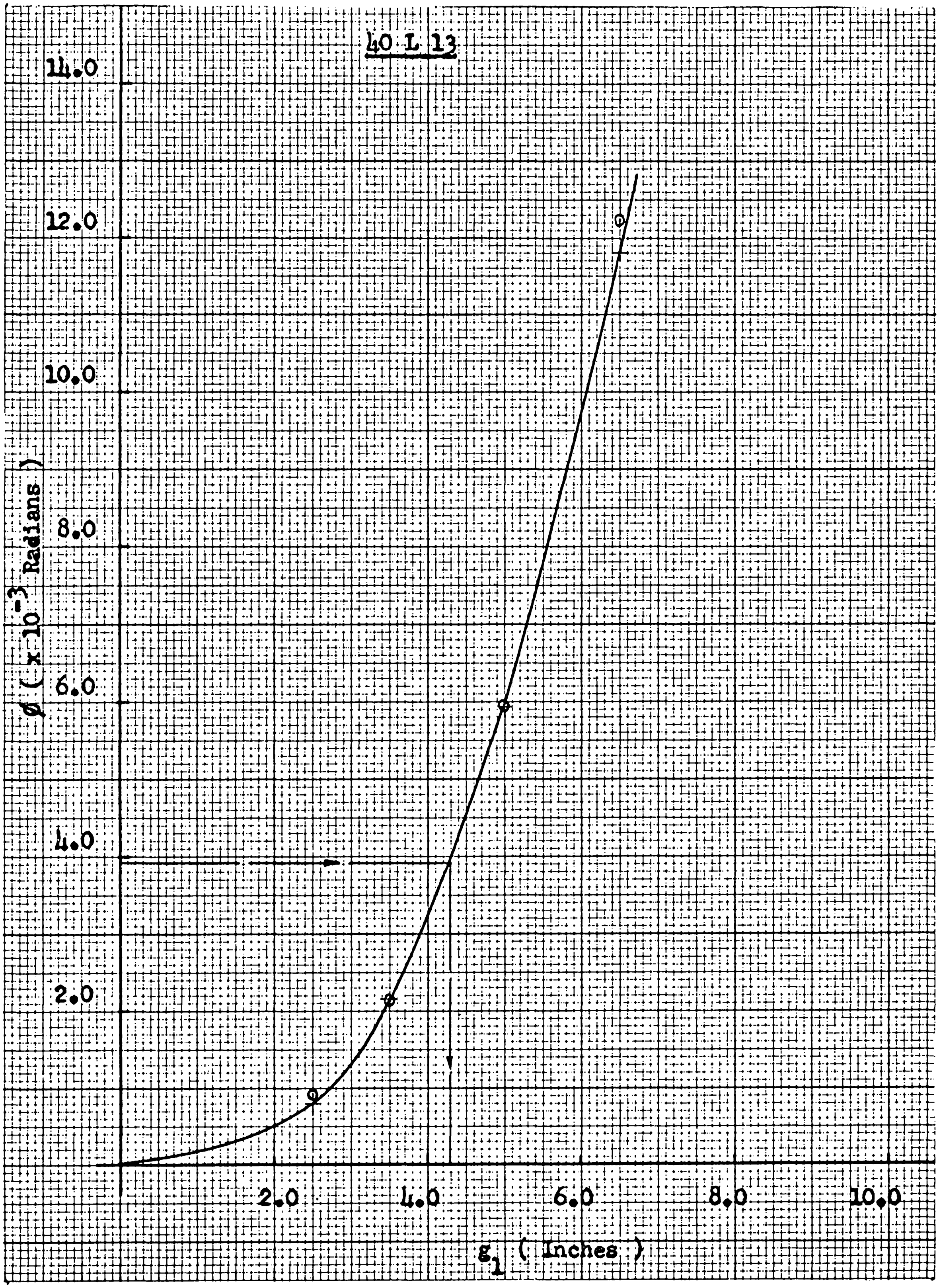


Figure 19. Rotation and g_1 Curve - 40L13

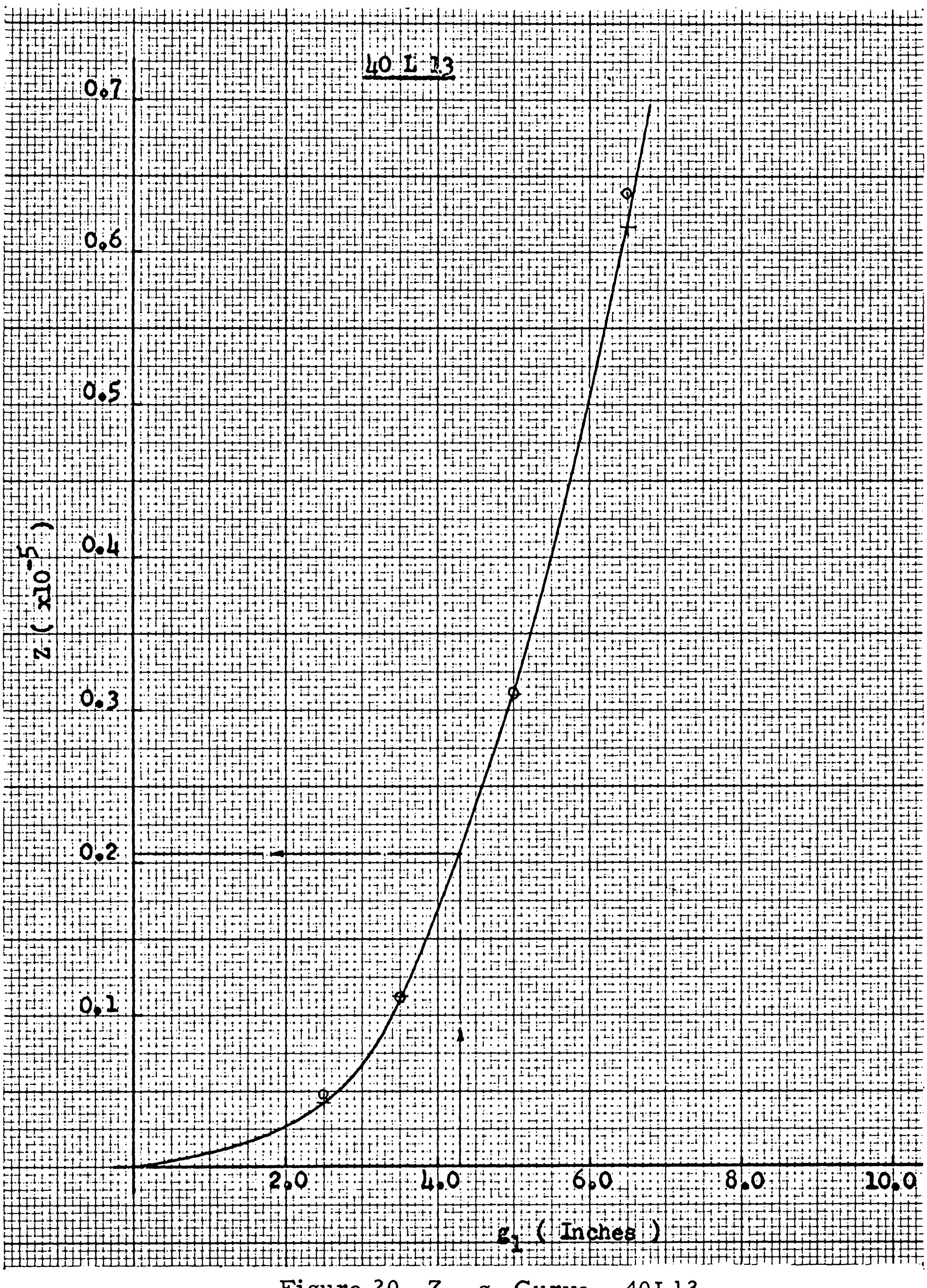


Figure 20. Z - g_1 Curve - 40L13

(3) Design of Welding at Connection

It is specified by the "Design of Welded Structural Connections"[10] on the basis of tests that a fillet weld with transverse load fails on a plane at 67-1/2 degrees to the horizontal. This is the throat section having the greatest shear stress in the conventional 45 degree equal leg fillet weld. The allowable load per inch based on the allowable unit shear stress may be calculated as follows for a leg size of "e" inches.

shear on throat = 13,600 x 1 x 0.765 x e = 10,400 e lb/in

allowable P = 10,400 e/sin 67.5° = 11,300 e lb/in

According to the formula shown above, the size of fillet weld for connection of longspan joist 18L07 is designed.

P_t = -M/d = 39.8 x 12/16.59 = 28.8 kips

if e = 5/16 in = 0.3125 in

then the length of fillet weld L, required is,

L = 28.8/(11.3 x 0.3125) = 8.2 in

the actual length of fillet weld is,

L_{act} = 2 x 8 = 16 in > L = 8.2 in   O.K.

The sizes of welding at the connections for the remaining joists are determined and will be shown on the figure showing the test sections.

9. Choice of Column Stub

The column which was chosen by Combs was a 12WF40. This column was used for all test sections as being of representative depth and possibly of representative weight.

10. The Test Sections

The test section is made of a column stub, the end panel of two joists and the connections.

(1) Stiffener Design

Since the bottom chord is under compression and top chord is under tension, it is necessary to provide stiffeners between the column flanges to resist these concentrated loads. If the stiffener is designed for the maximum load of the four test sections, it will be adequate for all sections because the column stubs are all the same size. The following design is based on the 1956 AISC Specification.

max. $P_t = 159.5 \times 12/38.30 = 50.0$ kips

length of stiffener = 10.91 in

width of stiffener = 3.125 in

try 5/8 in plate, $A = 1.95$ in^2

$I = bh^3/12 = 3.125 \times (5/8)^3/12 = 0.0636$ in^4

$r = \sqrt{I/A} = 0.18$ in

$L/r = 10.91/0.18 = 61 < 120$

$f_c = 15.08$ ksi

$f_{act} = 25.0/1.95 = 12.8$ ksi $< f_c = 15.08$ ksi O.K.

Use 10-7/8 x 3-1/8 x 5/8 Stiffener each side

(2) Vertical Web Check

The test section is designed to be supported at the vertical web members (see Figure 13). The stress is checked as follows:

Use 1 in ◻ bar for all sections

$r = 0.289$ in

(a) 18L07

$L = 13$ in, $L/r = 45$

$f_c = 16.02$ ksi

$P_v = -M/F$

$= 39.8 \times 12/24.0 = 19.9$ kips

$f_{act} = 19.9/1.0 = 19.9$ ksi $> f_c = 16.02$ ksi

The maximum test load, P_{max}, if the vertical web member is not to be over stressed, is:

$P_{max.} = 2 \times 16.02 \times 1 = 32.04$ kips

the vertical web will yield at load,

$P_{yield} = 32.04 \times 1.67 = 53.5$ kips

(b) 24L09

$L = 18.5$ in, $L/r = 64$

$f_c = 15.01$ ksi

$P_v = 67.6 \times 12/30.0 = 27.0$ kips

$f_{act} = 27.0$ ksi $> f_c = 15.01$ ksi

$P_{max.} = 2 \times 15.01 = 30.02$ kips

$P_{yield} = 30.02 \times 1.67 = 50.5$ kips

(c) <u>32L11</u>

$L = 26.0$ in, $L/r = 90$

$f_c = 13.07$ ksi

$P_v = 107.7 \times 12/36 = 35.9$ kips

$f_{act} = 35.9$ ksi $> f_c = 13.07$ ksi

$P_{max.} = 2 \times 13.07 \times 1 = 26.14$ kips

$P_{yield} = 26.14 \times 1.67 = 43.7$ kips

(d) <u>40L13</u>

$L = 33.5$ in, $L/r = 116$

$f_c = 10.47$ ksi

$P_v = 159.5 \times 12/48 = 39.9$ kips

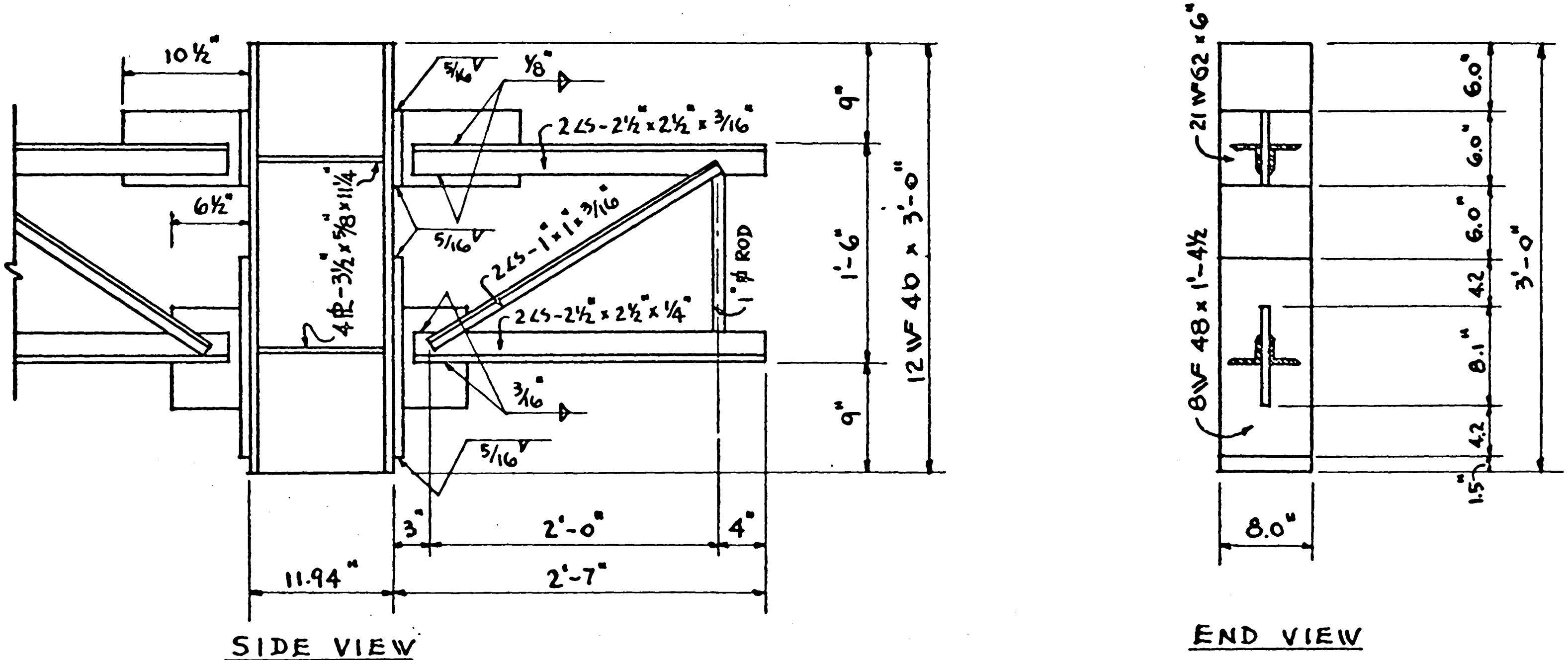
$f_{act} = 39.9$ ksi $> f_c = 10.47$ ksi

$P_{max.} = 2 \times 10.47 \times 1 = 20.94$ ksi

$P_{yield} = 20.94 \times 1.67 = 34.9$ kips

The vertical web members are all under designed. This is because the author was reusing the old test specimens which included the vertical web members which were all originally underdesigned. The yield strength of each vertical web member in comparison with the maximum test load that recorded in laboratory part of this thesis is very close.

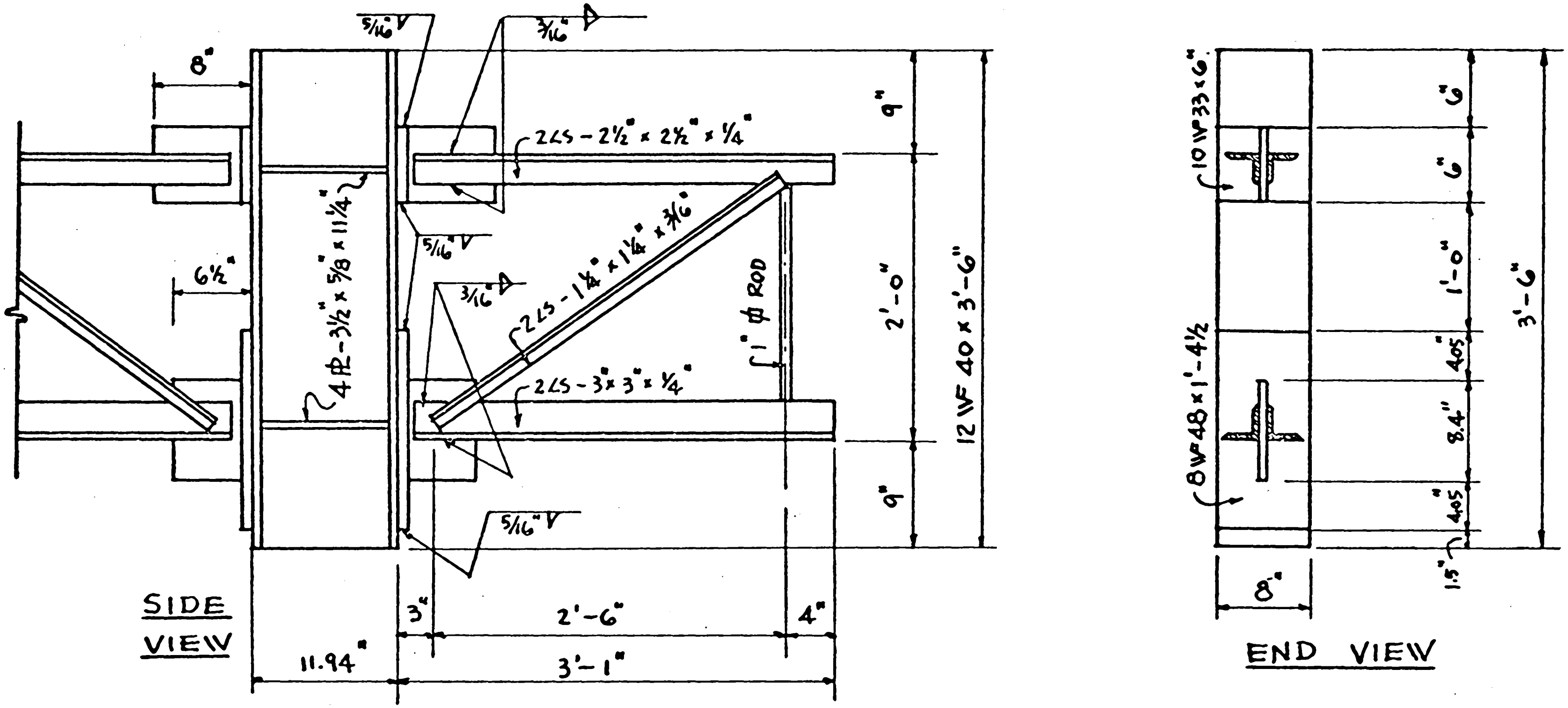
On the four following pages are scale drawings of the actual sections.



18L07 TEST SECTION

SCALE 3/4" = 1'-0"

Figure 21.



24 L09 TEST SECTION

SCALE 3/4" = 1'-0"

Figure 22.

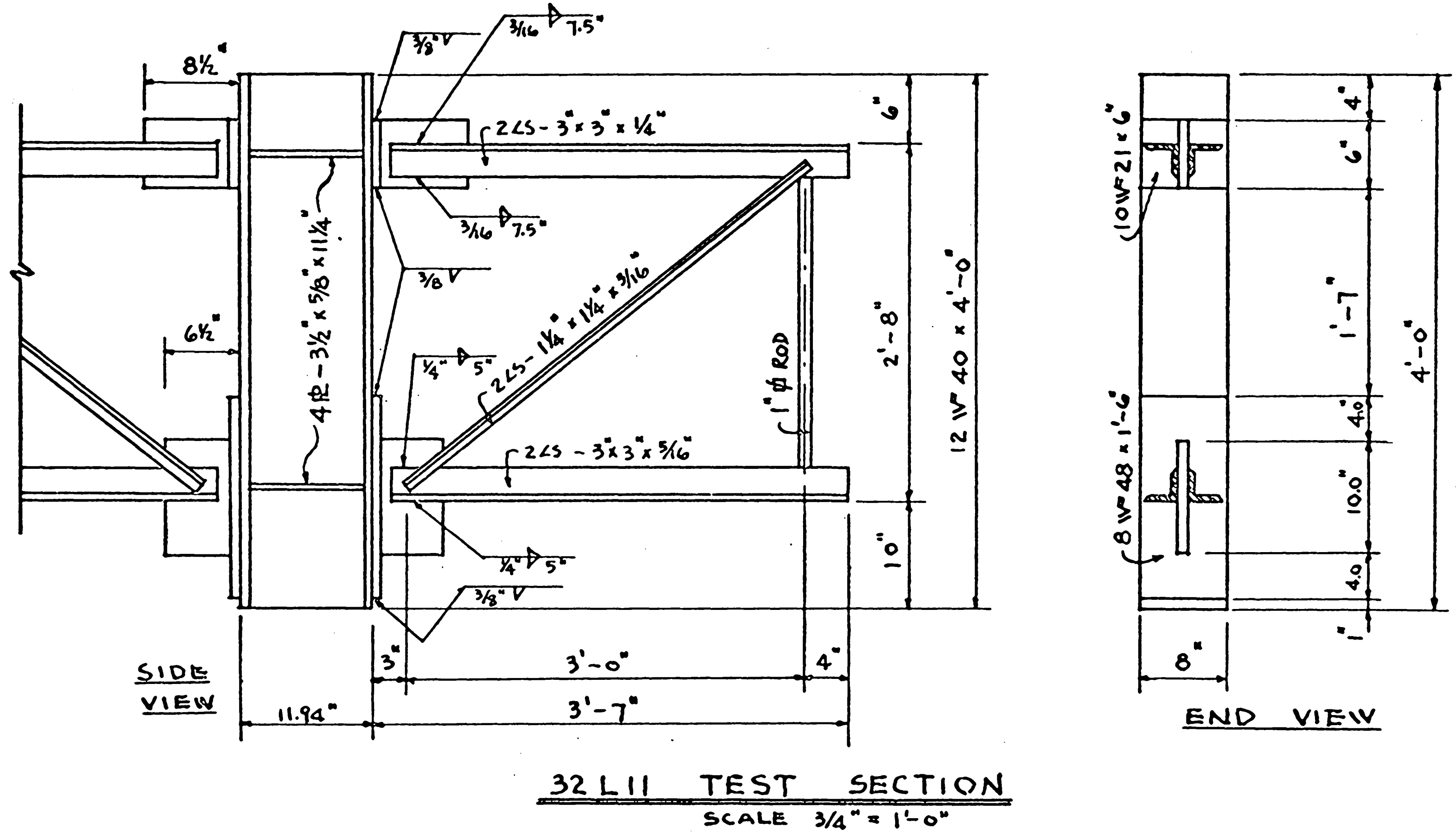
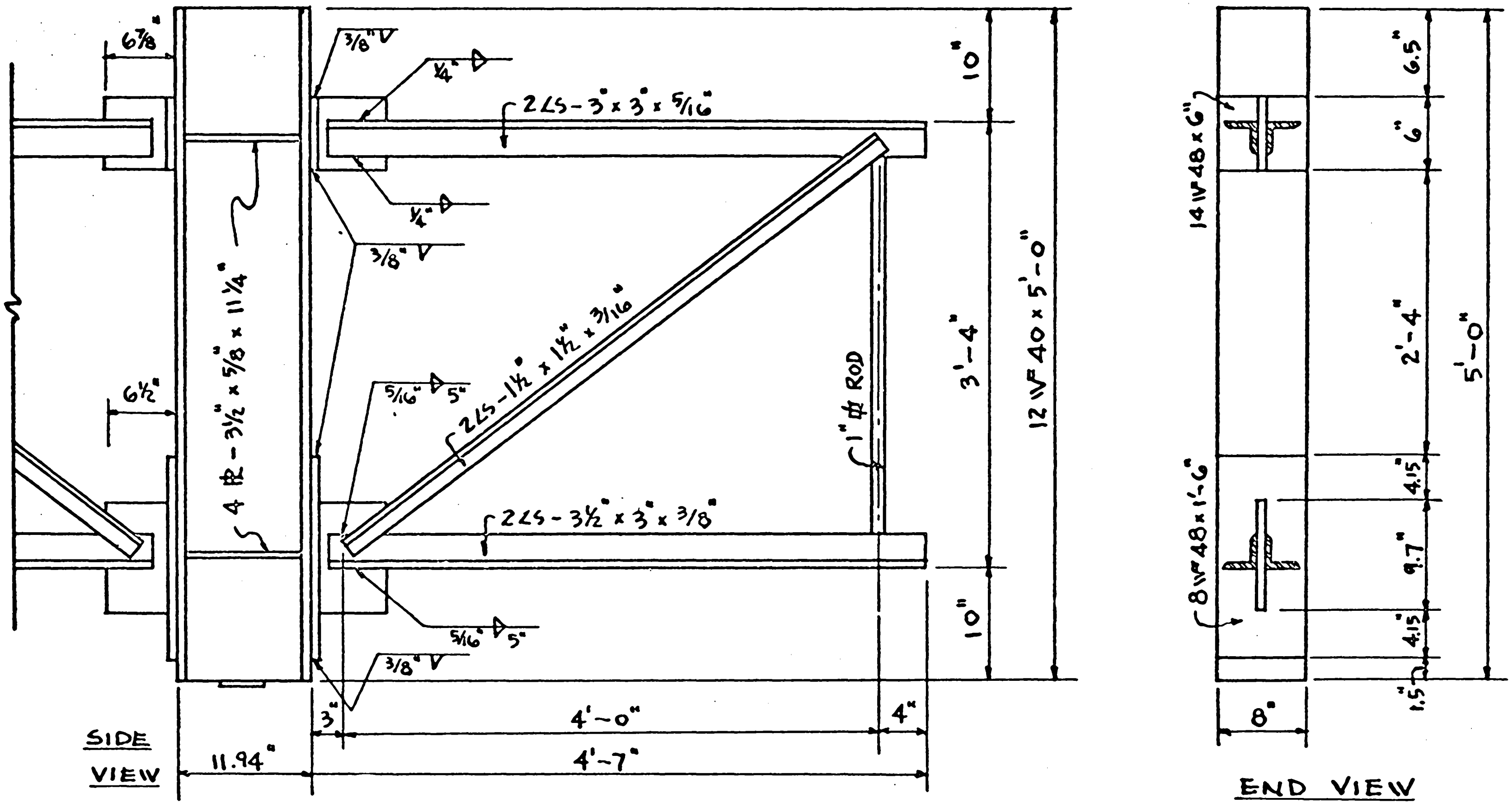

8½"
3/8" V
3/16 7.5"
2∠S - 3" x 3" x 1/4"
3/16 7.5"
4 ℞ - 3½" x 5/8" x 11¼"
3/8 V
6½"
1/4" 5"
2∠S - 1¼" x 1¼" x 3/16"
1" ϕ ROD
2∠S - 3" x 3" x 5/16"
1/4" 5"
3/8" V
6"
2'-8"
10"
12 W 40 x 4'-0"
3"
3'-0"
4"
11.94"
3'-7"
SIDE VIEW
10WF21 x 6"
8 W 48 x 1'-6"
4"
6"
1'-7"
4.0
10.0
4.0
1"
4'-0"
8"
END VIEW
32 L11 TEST SECTION
SCALE 3/4" = 1'-0"

Figure 23.



40 L 13 TEST SECTION

SCALE 3/4" = 1'-0"

Figure 24.

B. LABORATORY INVESTIGATION

1. Fabrication of Test Section

The test sections were fabricated originally in 1961 in Lynchburg, Virginia, at the Montague-Betts Steel Plant. Then the test sections were modified and refabricated by the author with the assistance of Mr. Marshall K. Smith in the Engineering Mechanics Laboratory at the Virginia Polytechnic Institute.

2. Instrumentation and Laboratory Equipment

The instrumentation that the author employed in his testing consisted of nine Ames dials which can measure in increments of thousandths of an inch. Eight of them were placed, one on each of the gravity axes of each of the eight chord angles of the longspan joists. The dials were mounted on a steel bar which was fixed to the centroid of the column stub at the stiffner. As a result, the dials measured the movement of the connections away from or toward the centroid of column stub (see Figure 25). The feeler of each of the eight Ames dials at the connections rested against a steel plate which had been welded (two inches clear from the flange of the tee) on each chord angle. The ninth dial was mounted on a steel rod which is supported from the points of support of test section and the feeler of the dial rested on a small steel plate welded to the end of the column stub (see Figure 25). It was placed so as to measure the downward deflection of the column stub.

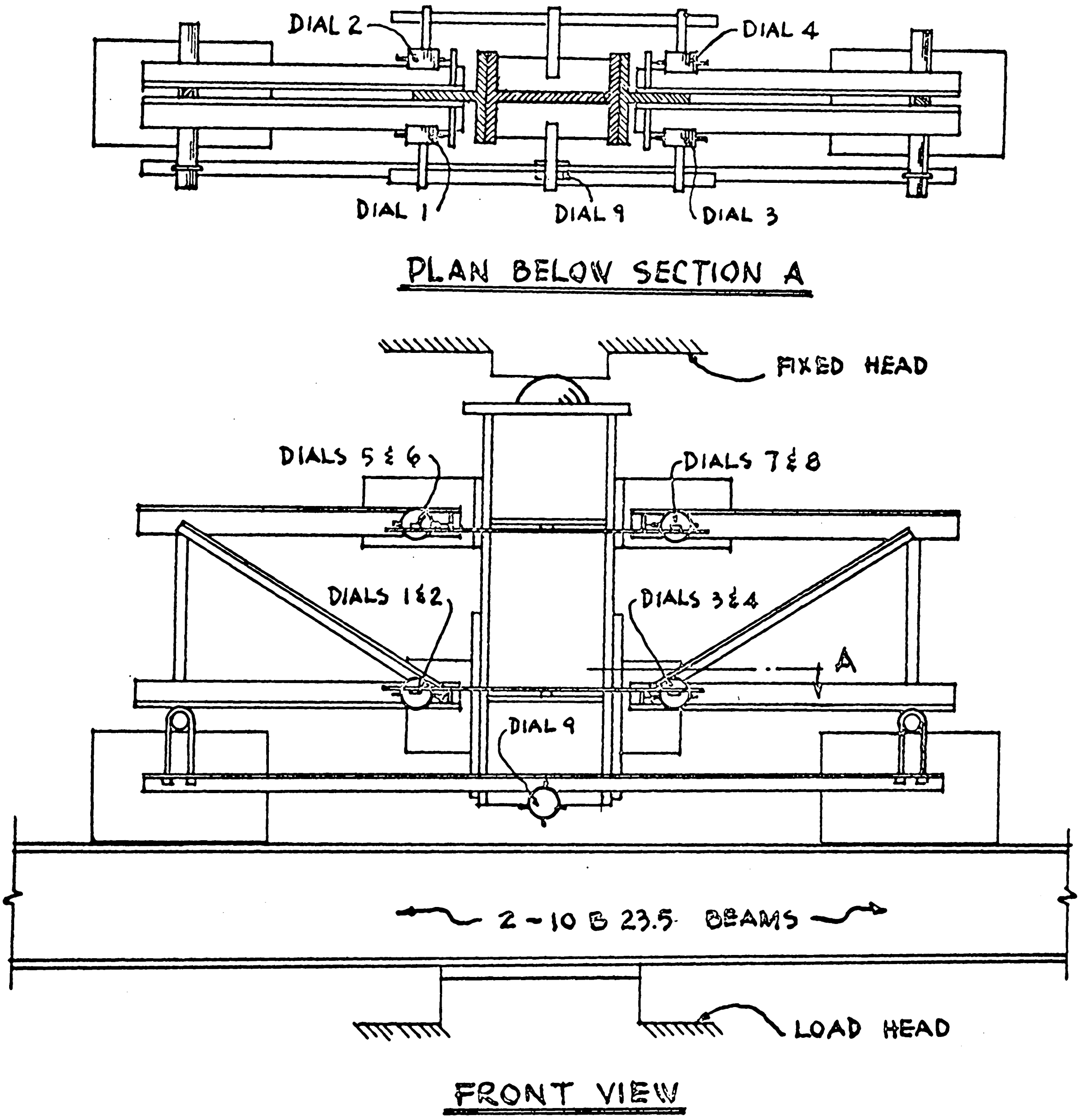


Figure 25. Set-up of Test Section

A Tinius-Olsen hydraulic load machine was used to conduct the testing. The machine has three load cells of 16,000, 80,000 and 400,000 pound capacities. The machine is operated by a hydraulic pump and controlled by two valves, a load valve and a relief valve. These valves allowed the operator to control the load within about three per cent accuracy. The test sections were lifted by an overhead two-ton capacity chain hoist and mounted in the machine.

The load machine had two eighteen inch square load heads. There was an 18-3/4 inch clearance between the upper load head and the fixed head support (see Photograph No. 1). The fixed head could be set at one-foot intervals up to ten feet in height. The test section, when in the load machine, rested on two 1-3/4 inch round rollers, which in turn rested on stiffened steel boxes. These in turn rested on a double beam composed of two 10 B 23.5 beams welded along the top and bottom flanges which rested on the movable load head of the machine (see Figure 25 and Photograph No. 1). The test sections were put into the machine upside down relative to their normal position.

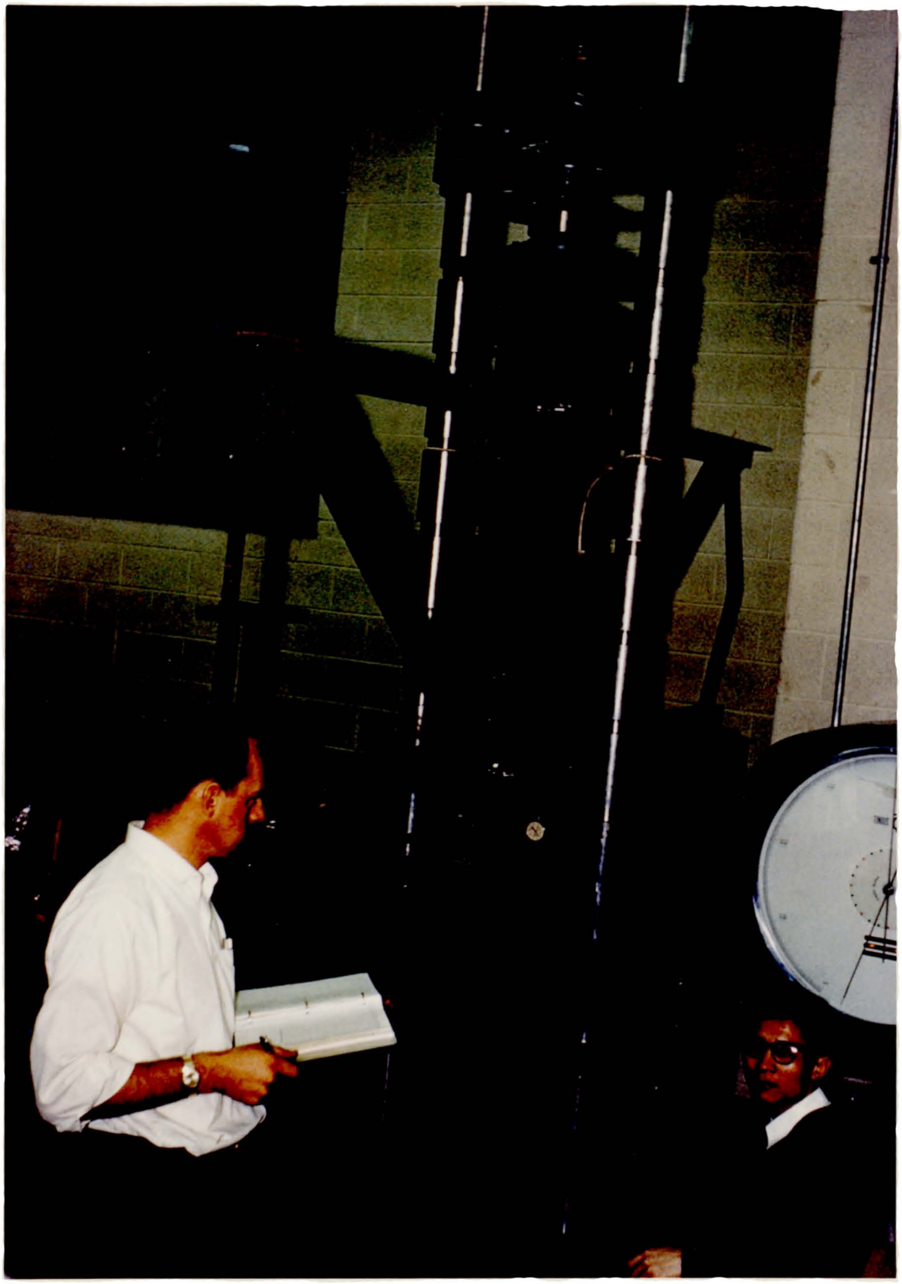
3. Laboratory Procedure

The laboratory procedure was as follows:

(1) Place test section in load machine (see Photograph No. 1).

(2) Mount instrumentation (see Figure 25).

(3) Zero instrumentation and zero load machine after a small amount of load has been applied.



Photograph No. 1

(4) Load test section to point of failure of joists.

(5) Take dial readings at appropriate intervals so that behavior curve may be plotted and drawn.

(6) Observe behavior of all the component part of the longspan joists, connection, and column stub.

(7) Note any behavior patterns or peculiar characteristics of the specimen as it is loaded.

4. Data

The arrangement of Ames dials was as follows: dials one and two were on opposite sides of top chord, dials three and four were on opposite sides of the other top chord, dials five and six were on opposite sides of bottom chord and on the top of dials one and two, and dials seven and eight were on opposite sides of bottom chord and on the top of dials three and four (see Figure 25).

The presentation of the data is as follows:

(1) A sheet of raw data for each of the four test sections.

(2) A sheet showing the average of the readings at each pair of dials at each connection.

(3) A sheet showing the total displacement at the joint, the joint rotations, and the moments corresponding to the loadings.

(4) Data curves plotted from joint rotations and moments.

Table 9. 18L07 Test Section

Load (lbs)	Dial Readings ($x10^{-3}$) Inches								
	1	2	3	4	5	6	7	8	9
400	0	0	0	0	0	0	0	0	0
800	0	1	0	1	0	0	0	0	4
1200	0	1	0	1	1	-1	1	-1.0	7
1600	0	1	1	1	1	-1	1	-1.0	12
2000	1	1	1	1	1	-1	1	-1	17
2800	1	1	1	2	2	-2	2	-2.0	32
3600	1	1	1	3	2	-2	2	-2.0	45
4400	1	1	2	3	3	-3	3	-3.0	58
5200	3	1	3	4	4	-4	3	-3.0	70
6000	5	2	3	6	4	-4	4	-4.0	77
7000	5	2	4	7	5	-5	4	-4.0	85
8000	6	3	4	8	6	-6	4	-4.0	95
9000	8	4	5	10	7	-7	4	-4.0	105
10000	9	5	6	11	8	-7	4	-4.0	112
12000	15	8	9	14	9	-8	4	-4.0	133
14000	18	11	11	18	10	-9	4	-4.0	158
16000	18	13	14	22	10	-9	4	-4.0	183
18000	28	17	20	26	10	-9	4	-3.5	205
20000	31	23	24	31	10	-0	4	-3.5	238
24000	45	35	35	42	10	-8	4	-2.0	320
28000	63	52	51	58	9	-7	4	-1.0	398
32000	73	61	61	68	9	-7	4	-1.0	478
36000	98	84	80	91	9	-6	4	-1.0	589
40000	132	120	117	128	12	-6	6	-1.0	716
45000	157	149	144	158	16	-6	11	-1.0	886
48000	Vertical Web Member Yielded at this Loading								

Table 10. 24L09 Test Section

Load (lbs)	Dial Readings ($x10^{-3}$) Inches								
	1	2	3	4	5	6	7	8	9
800	0	0	0	0	0	0	0	0	0
1600	0	1	0	1	1	0	1	-1	4
2400	0	2	-1	2	1	1	2	-2	9
3200	0	4	-1	2	2	2	2	-3	15
4000	0	6	-1	2	2	2	3	-4	18
4800	0	8	-1	4	2	2	3	-4	23
5600	0	9	-1	4	1	2	3	-4	27
6400	0	10	-1	6	1	2	3	-4	31
7200	1	12	-1	7	1	2	3	-5	35
8000	1	13	-1	10	1	-1	3.0	-5	37
10000	1	17	0	14	0	-1	4	-6	49
12000	2	22	2	19	-1	-1	5	-7	73
14000	3	28	5	23	-2	0	6	-8	90
16000	5	33	7	28	-3	1	6	-8	104
18000	7	38	12	37	-4	2	7	-10	122
20000	12	49	19	45	-6	3	8	-11	154
22000	13	52	21	47	-7	4	9	-12	160
24000	17	60	28	56	-8	5	10	-13	185
26000	19	65	31	61	-9	6	10.5	-14	198
28000	24	73	37	70	-10	6	12	-16	222
30000	30	83	45	79	-12	8	13	-17	238
32000	35	92	53	88	-13	9	14	-18	267
34000	41	101	60	99	-14	9	15	-19	286
36000	50	113	71	112	-14	10	17	-21	319
38000	59	127	81	125	-15	10	19	-24	354
40000	67	139	92	141	-15	10	23	-28	392
42000	Vertical Web Member Yielded at this Loading								

Table 11. 32L11 Test Section

Load (lbs)	Dial Readings ($x10^{-3}$) Inches								
	1	2	3	4	5	6	7	8	9
1000	0	0	0	0	0	0	0	0	0
2000	0	0	1	0	0	0	1	0	14
3000	2	0	2	0	0	0	1	0	20
4000	3	0	3	0	0	0	1	0	29
5000	3	0	3	-1	1	0	2	0	32
6000	4	0	4	-1	1	0	2	0	43
7000	5	0	6	-1	1	- 1	2	- 1	57
8000	6	1	6	-1	1	- 2	2	- 1	62
9000	7	1	7	0	1	- 3	2	- 1	68
10000	8	1	8	0	1	- 4	3	- 2	73
12000	12	1	10	0	2	- 7	3	- 3	87
14000	16	2	13	1	2	-10	3	- 3	99
16000	18	4	16	2	3	-12	3	- 4	108
18000	22	5	20	5	4	-14	3	- 4	116
20000	26	8	24	8	4	-17	3	- 4	132
22000	31	12	29	12	5	-18	3	- 5	147
24000	34	16	34	15	5	-20	3	- 5	157
26000	39	20	39	19	5	-21	4	- 6	175
28000	44	25	45	24	5	-22	4	- 6	190
30000	50	32	52	29	5	-23	4	- 7	209
32000	56	38	58	34	5	-24	5	- 8	226
34000	64	45	68	41	6	-25	6	- 9	237
36000	72	55	77	49	6	-26	6	-10	264
38000	81	63	87	56	6	-27	7	-11	280
40000	91	75	97	65	6	-27	8	-12	312
42000	103	84	108	75	4	-26	9	-14	332
44000	114	94	118	87	0	-22	11	-16	365
44000	Vertical Web Member Yielded at this Loading								

Table 12. 40L13 Test Section

Load (lbs)	Dial Readings (x10^{-3}) Inches								
	1	2	3	4	5	6	7	8	9
1000	0	0	0	0	0	0	0	0	0
2000	1	0	1	0	- 2	1	- 2	1	15
3000	1	0	1	1	- 2.5	1	- 3	2	29
4000	2	1	2	1	- 3.5	2	- 4	2	34
5000	3	1	2	2	- 5.0	2	- 5	2	44
6000	4	1	2	2	- 5.5	3	- 5	2	46
7000	4	1	3	2	- 6	3	- 6	3	52
8000	5	2	4	2	- 6	4	- 6	3	56
9000	6	2	5	3	- 8	4	- 7	4	61
10000	7	4	6	3	- 8.5	4	- 8	4	65
12000	10	8	9	5	-10	5	- 9	5	92
14000	15	11	13	8	-11	5	-10	5	100
16000	20	15	18	11	-12	6	-11	5	119
18000	25	20	22	14	-13	6	-12	5	133
20000	32	25	28	19	-14	6	-13	5	154
22000	38	30	33	22	-15.5	7	-13	5	172
24000	45	36	40	26	-17	7	-14	5	193
26000	53	44	48	30	-18	8	-16	6	206
28000	59	50	55	35	-19	9	-17	6	227
30000	70	62	66	42	-21	10	-18	7	248
32000	80	72	76	48	-22	10	-20	8	258
34000	91	82	88	56	-24	11	-21	9	280
36000	105	96	104	65	-26	12	-24	11	303
38000	120	112	121	71	-29	14	-26	13	332
38000	Vertical Web Member Yielded at this Loading								

Table 13. Average Dial Readings - 18L07 - ($x10^{-3}$) inches

Load (lbs)	(Dials 1,2)	(Dials 3,4)	(Dials 5,6)	(Dials 7,8)	Dial 9
400	0	0	0	0	0
800	0.5	0.5	0	0	4
1200	0.5	1.0	0	0	7
1600	0.5	1.0	0	0	12
2000	1.0	1.0	0	0	17
2800	1.0	1.5	0	0	32
3600	1.0	2.0	0	0	45
4400	1.0	2.5	0	0	58
5200	2.0	3.5	0	0	70
6000	3.5	4.5	0	0	77
7000	3.5	5.5	0	0	85
8000	4.5	6.0	0	0	95
9000	6.0	7.5	0	0	105
10000	7.0	8.5	0.5	0	112
12000	11.5	11.5	0.5	0	133
14000	14.5	14.5	0.5	0	158
16000	15.5	18.0	0.5	0	183
18000	22.5	23.0	0.5	0.3	205
20000	27.0	27.5	0.5	0.3	238
24000	40.0	37.5	1.0	1.0	320
28000	57.5	54.5	1.0	1.5	398
32000	67.0	64.5	1.0	1.5	478
36000	91.0	85.5	1.5	1.5	589
40000	126.0	122.5	3.0	2.5	716
45000	153.0	151.0	4.0	5.0	886

Table 14. Average Dial Readings - 24L09 - ($x10^{-3}$) inches

Load (lbs)	(Dials 1,2)	(Dials 3,4)	(Dials 5,6)	(Dials 7,8)	Dial 9
800	0	0	0	0	0
1600	0.5	0.5	0.5	0	4
2400	1.0	0.5	1.0	0	9
3200	2.0	0.5	2.0	-0.5	15
4000	3.0	0.5	2.0	-0.5	18
4800	4.0	1.5	2.0	-0.5	23
5600	4.5	1.5	1.5	-0.5	27
6400	5.0	2.5	1.5	-0.5	31
7200	6.5	3.0	1.5	-1.0	35
8000	7.0	4.5	0	-1.0	37
10000	9.0	7.0	-0.5	-1.0	49
12000	12.0	10.5	-1.0	-1.0	73
14000	15.5	14.0	-1.0	-1.0	90
16000	19.0	17.5	-1.0	-1.0	104
18000	22.5	24.5	-1.0	-1.5	122
20000	30.5	32.0	-1.5	-1.5	154
22000	32.5	34.0	-1.5	-1.5	160
24000	38.5	42.0	-1.5	-1.5	185
26000	42.0	46.0	-1.5	-1.8	198
28000	48.5	53.5	-2.0	-2.0	222
30000	56.5	62.0	-2.0	-2.0	238
32000	63.5	70.5	-2.0	-2.0	267
34000	71.0	79.5	-2.5	-2.0	286
36000	81.5	91.5	-2.0	-2.0	319
38000	93.0	103.0	-2.5	-2.5	354
40000	103.0	116.5	-2.5	-2.5	392
42000	Vertical Web Member Yielded at this loading				

Table 15. Average Dial Readings - 32L11 - ($x10^{-3}$) inches

Load (lbs)	(Dials 1,2)	(Dials 3,4)	(Dials 5,6)	(Dials 7,8)	Dial 9
1000	0	0	0	0	0
2000	0	0.5	0	0.5	14
3000	1.0	1.0	0	0.5	20
4000	1.5	1.5	0	0.5	29
5000	1.5	1.0	0.5	1.0	32
6000	2.0	1.5	0.5	1.0	43
7000	2.5	2.5	0	0.5	57
8000	3.5	2.5	- 0.5	0.5	62
9000	4.0	3.5	- 1.0	0.5	68
10000	4.5	4.0	- 1.5	0.5	73
12000	6.5	5.0	- 2.5	0	87
14000	9.0	7.0	- 4.0	0	99
16000	11.0	9.0	- 4.5	0	108
18000	13.5	12.5	- 5.0	0	116
20000	17.0	16.0	- 6.5	-0.5	132
22000	21.5	20.5	- 6.5	-1.0	147
24000	25.0	24.5	- 7.5	-1.0	157
26000	29.5	29.0	- 8.0	-1.0	175
28000	34.5	34.5	- 8.5	-1.0	190
30000	41.0	40.5	- 9.0	-1.5	209
32000	47.0	46.0	- 9.5	-1.5	226
34000	54.5	54.5	- 9.5	-1.5	237
36000	63.5	63.0	-10.0	-2.0	264
38000	72.0	71.5	-10.5	-2.0	280
40000	83.0	81.0	-10.5	-2.0	312
42000	93.5	91.5	-11.0	-2.5	332
44000	104.0	102.5	-11.0	-2.5	365

Table 16. Average Dial Readings - 40L13 - ($x10^{-3}$) inches

Load (lbs)	(Dials 1,2)	(Dials 3,4)	(Dials 5,6)	(Dials 7,8)	Dial 9
1000	0	0	0	0	0
2000	0.5	0.5	-0.5	-0.5	15
3000	0.5	1.0	-0.8	-0.5	29
4000	1.5	1.5	-0.8	-1.0	34
5000	2.0	2.0	-1.5	-1.5	44
6000	2.5	2.0	-1.3	-1.5	46
7000	2.5	2.5	-1.5	-1.5	52
8000	3.5	3.0	-1.0	-1.5	56
9000	4.0	4.0	-2.0	-1.5	61
10000	5.5	4.5	-2.5	-2.0	65
12000	9.0	7.0	-2.5	-2.0	92
14000	13.0	10.5	-3.0	-2.5	100
16000	17.5	14.5	-3.0	-3.0	119
18000	22.5	18.0	-3.5	-3.5	133
20000	28.5	23.5	-4.0	-4.0	154
22000	34.0	27.5	-4.3	-4.0	172
24000	40.5	33.0	-5.0	-4.5	193
26000	48.5	39.0	-5.0	-5.0	206
28000	54.5	45.0	-5.0	-5.5	227
30000	66.0	53.0	-5.5	-5.5	248
32000	76.0	62.0	-6.0	-6.0	258
34000	86.5	72.0	-6.5	-6.0	280
36000	100.5	84.5	-7.0	-6.5	303
38000	116.0	91.0	-7.5	-6.5	332

Table 17. Total Displacement at Joint, the Joint Rotation and Corresponding Movements of 18L07

Load (lbs)	(Dials 1,2) - (Dials 5,6) Δ ($x10^{-3}$ in.)	ϕ ($x10^{-3}$ rad.)	(Dials 3,4) - (Dials 7,8) Δ ($x10^{-3}$ in.)	ϕ ($x10^{-3}$ rad.)	Equivalent M kip - ft
400	0	0	0	0	0.45
800	0.5	0.03	0.5	0.03	0.90
1200	0.5	0.03	0.5	0.03	1.35
1600	0.5	0.03	1.0	0.06	1.80
2000	1.0	0.06	1.0	0.06	2.25
2800	1.0	0.06	1.5	0.09	3.15
3600	1.0	0.06	2.0	0.12	4.05
4400	1.0	0.06	2.5	0.15	4.95
5200	2.0	0.12	3.5	0.21	5.85
6000	3.5	0.21	4.5	0.27	6.75
7000	3.5	0.21	5.5	0.33	7.88
8000	4.5	0.27	6.0	0.36	9.00
9000	6.0	0.36	7.5	0.45	10.13
10000	6.5	0.39	8.5	0.51	11.25
12000	11.0	0.66	11.5	0.69	13.50
14000	14.0	0.85	14.5	0.87	15.75
16000	15.0	0.91	18.0	1.08	18.00
18000	22.0	1.33	22.7	1.37	20.25
20000	26.5	1.60	27.2	1.64	22.50
24000	39.0	2.35	36.5	2.20	27.00
28000	56.5	3.41	53.0	3.20	31.50
32000	66.0	3.98	63.0	3.80	36.00
36000	89.5	5.39	84.0	5.07	40.50
40000	123.0	7.42	120.0	7.23	45.00
45000	149.0	8.98	146.0	8.70	50.70

Δ = (Aver. Readings of Dials 1,2) - (Aver. Readings of Dials 5,6)
Δ = (Aver. Readings of Dials 3,4) - (Aver. Readings of Dials 7,8)
$\phi = \Delta / d = \Delta / 16.59$
$M = (\text{Load}/2)\ (27/12) = 1.125\ (\text{Load})$ ft-kip

Table 18. Total Displacement at Joint, the Joint Rotation and Corresponding Movements of 24L09

Load (lbs)	(Dials 1,2) - (Dials 5,6) Δ ($\times 10^{-3}$ in.)	ϕ ($\times 10^{-3}$ rad.)	(Dials 3,4) - (Dials 7,8) Δ ($\times 10^{-3}$ in.)	ϕ ($\times 10^{-3}$ rad.)	Equivalent M kip - ft
800	0	0	0	0	1.10
1600	0	0	0.5	0.02	2.20
2400	0	0	0.5	0.02	3.30
3200	0	0	1.0	0.04	4.40
4000	1.0	0.04	1.0	0.04	5.50
4800	2.0	0.09	2.0	0.09	6.60
5600	3.0	0.13	2.0	0.09	7.70
6400	3.5	0.16	3.0	0.13	8.80
7200	5.0	0.22	4.0	0.18	9.90
8000	7.0	0.31	5.5	0.24	11.00
10000	9.5	0.42	8.0	0.36	13.75
12000	13.0	0.58	11.5	0.51	16.50
14000	16.5	0.74	15.0	0.67	19.25
16000	20.0	0.89	18.5	0.82	22.00
18000	23.5	1.05	26.0	1.16	24.75
20000	32.0	1.42	33.5	1.49	27.50
22000	34.0	1.51	35.5	1.58	30.25
24000	40.0	1.78	43.5	1.94	33.00
26000	43.5	1.94	47.8	2.13	35.55
28000	50.5	2.25	55.5	2.47	38.50
30000	58.5	2.60	64.0	2.85	41.25
32000	65.5	2.92	72.5	3.23	44.00
34000	73.5	3.27	81.5	3.63	46.75
36000	83.5	3.72	93.5	4.17	49.50
38000	95.5	4.25	105.5	4.70	52.25
40000	105.5	4.70	119.0	5.30	55.00

$\Delta = \left(\text{Aver. Readings of Dials 1,2}\right) - \left(\text{Aver. Readings of Dials 5,6}\right)$

$\Delta = \left(\text{Aver. Readings of Dials 3,4}\right) - \left(\text{Aver. Readings of Dials 7,8}\right)$

$\phi = \Delta / d = \Delta / 22.44$

$M = (\text{Load}/2)\,(33/12) = 1.375\,(\text{Load})$ ft-kip

Table 19. Total Displacement at Joint, the Joint Rotation and Corresponding Movements of 32L11

Load (lbs)	(Dials 1,2) - (Dials 5,6) Δ ($\times 10^{-3}$ in.)	ϕ ($\times 10^{-3}$ rad.)	(Dials 3,4) - (Dials 7,8) Δ ($\times 10^{-3}$ in.)	ϕ ($\times 10^{-3}$ rad.)	Equivalent M kip - ft
1000	0	0	0	0	1.63
2000	0	0	0	0	3.25
3000	1.0	0.03	0.5	0.02	4.88
4000	1.5	0.05	1.0	0.03	6.50
5000	1.0	0.03	0	0	8.13
6000	1.5	0.05	0.5	0.02	9.75
7000	2.5	0.08	2.0	0.07	11.38
8000	4.0	0.13	2.0	0.07	13.00
9000	5.0	0.17	3.0	0.10	14.62
10000	6.0	0.20	3.5	0.12	16.25
12000	9.0	0.30	5.0	0.17	19.50
14000	13.0	0.43	7.0	0.23	22.75
16000	15.5	0.51	9.0	0.30	26.00
18000	18.5	0.61	12.5	0.41	29.25
20000	23.5	0.78	16.5	0.55	32.50
22000	28.0	0.92	21.5	0.71	35.75
24000	32.5	1.07	25.5	0.84	39.00
26000	37.5	1.24	30.0	0.99	42.25
28000	43.0	1.42	35.5	1.17	45.50
30000	50.0	1.65	42.0	1.39	48.75
32000	56.5	1.87	47.5	1.57	52.00
34000	64.0	2.11	56.0	1.85	55.25
36000	73.5	2.42	65.0	2.14	58.50
38000	82.5	2.72	73.5	2.42	61.75
40000	93.5	3.08	83.0	2.74	65.00
42000	104.5	3.45	94.0	3.10	68.25
44000	115.0	3.80	105.0	3.47	71.50

Δ = (Aver. Readings of Dials 1,2) - (Aver. Readings of Dials 5,6)
Δ = (Aver. Readings of Dials 3,4) - (Aver. Readings of Dials 7,8)
$\phi = \Delta/d = \Delta/30.29$
$M = (\text{Load}/2)\ (39/12) = 1.625\ (\text{Load})$ ft-kip

Table 20. Total Displacement at Joint, the Joint Rotation and Corresponding Movements of 40L13

Load (lbs)	(Dials 1,2) - (Dials 5,6) Δ ($x10^{-3}$ in.)	ϕ ($x10^{-3}$ rad.)	(Dials 3,4) - (Dials 7,8) Δ ($x10^{-3}$ in.)	ϕ ($x10^{-3}$ rad.)	Equivalent M kip - ft
1000	0	0	0	0	2.13
2000	1.0	0.02	1.0	0.02	4.25
3000	1.3	0.03	1.5	0.04	6.38
4000	2.3	0.06	2.5	0.07	8.50
5000	3.5	0.09	3.5	0.09	10.63
6000	3.8	0.10	3.5	0.09	12.75
7000	4.0	0.11	4.0	0.11	14.90
8000	4.5	0.12	4.5	0.12	17.00
9000	6.0	0.16	5.5	0.14	19.15
10000	7.8	0.20	6.5	0.17	21.25
12000	11.5	0.30	9.0	0.24	25.50
14000	16.0	0.42	13.0	0.34	29.75
16000	20.5	0.54	17.5	0.46	34.00
18000	26.0	0.68	21.5	0.56	38.25
20000	32.5	0.85	27.5	0.72	42.50
22000	38.3	1.00	31.5	0.82	46.75
24000	45.5	1.19	37.5	0.98	51.00
26000	53.5	1.40	44.0	1.15	55.25
28000	59.5	1.55	50.5	1.32	59.50
30000	71.5	1.87	58.5	1.53	63.75
32000	82.0	2.14	68.0	1.78	68.00
34000	92.5	2.42	78.0	2.04	72.25
36000	107.5	2.81	91.0	2.38	76.50
38000	123.5	3.22	97.5	2.54	80.75

Δ = (Aver. Readings of Dials 1,2) - (Aver. Readings of Dials 5,6)
Δ = (Aver. Readings of Dials 3,4) - (Aver. Readings of Dials 7,8)
$\phi = \Delta/d = \Delta/38.30$
M = (Load/2) (51/12) = 2.125 (Load) ft-kip

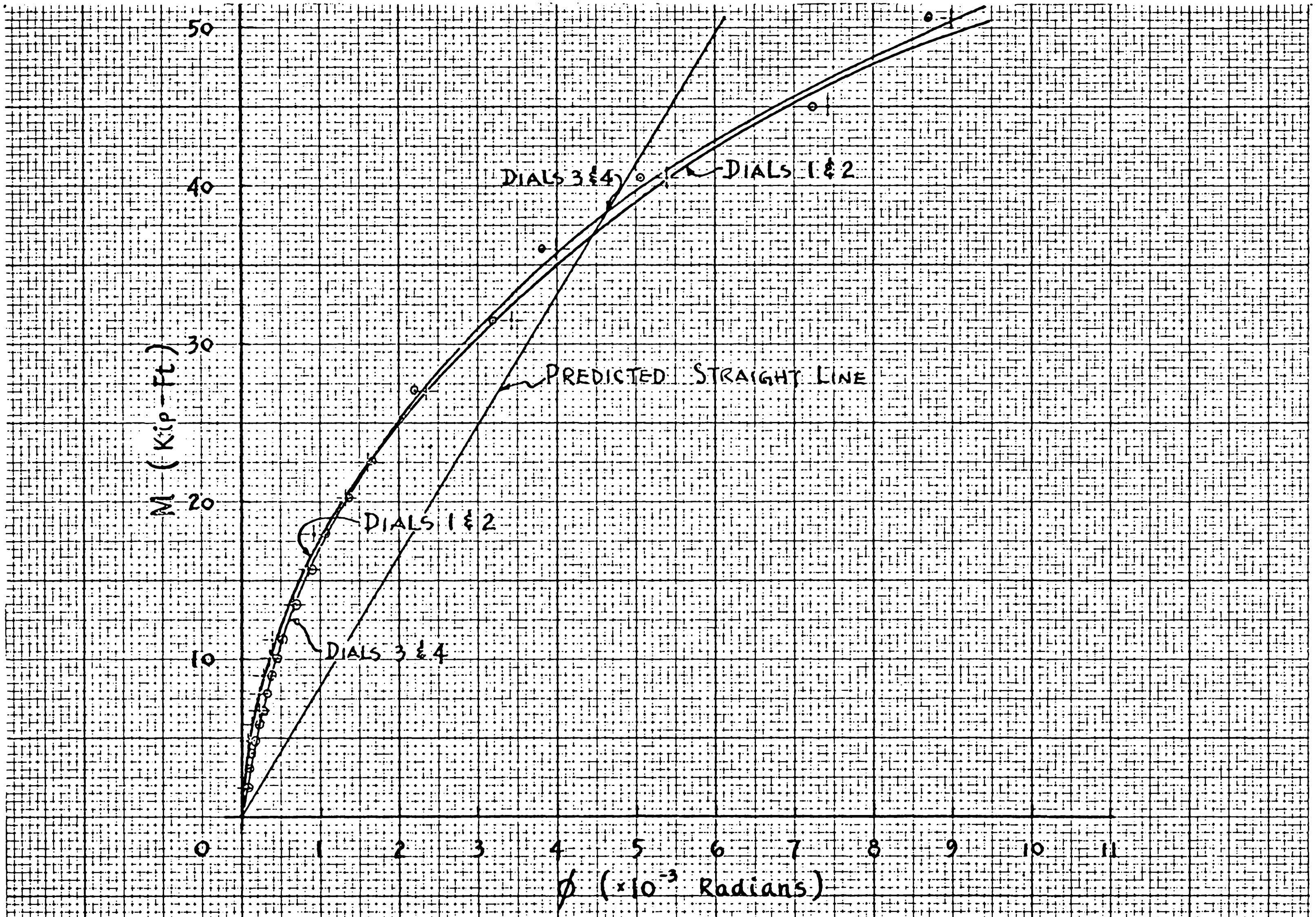


Figure 26 Moment - Rotation Curve - 18L07

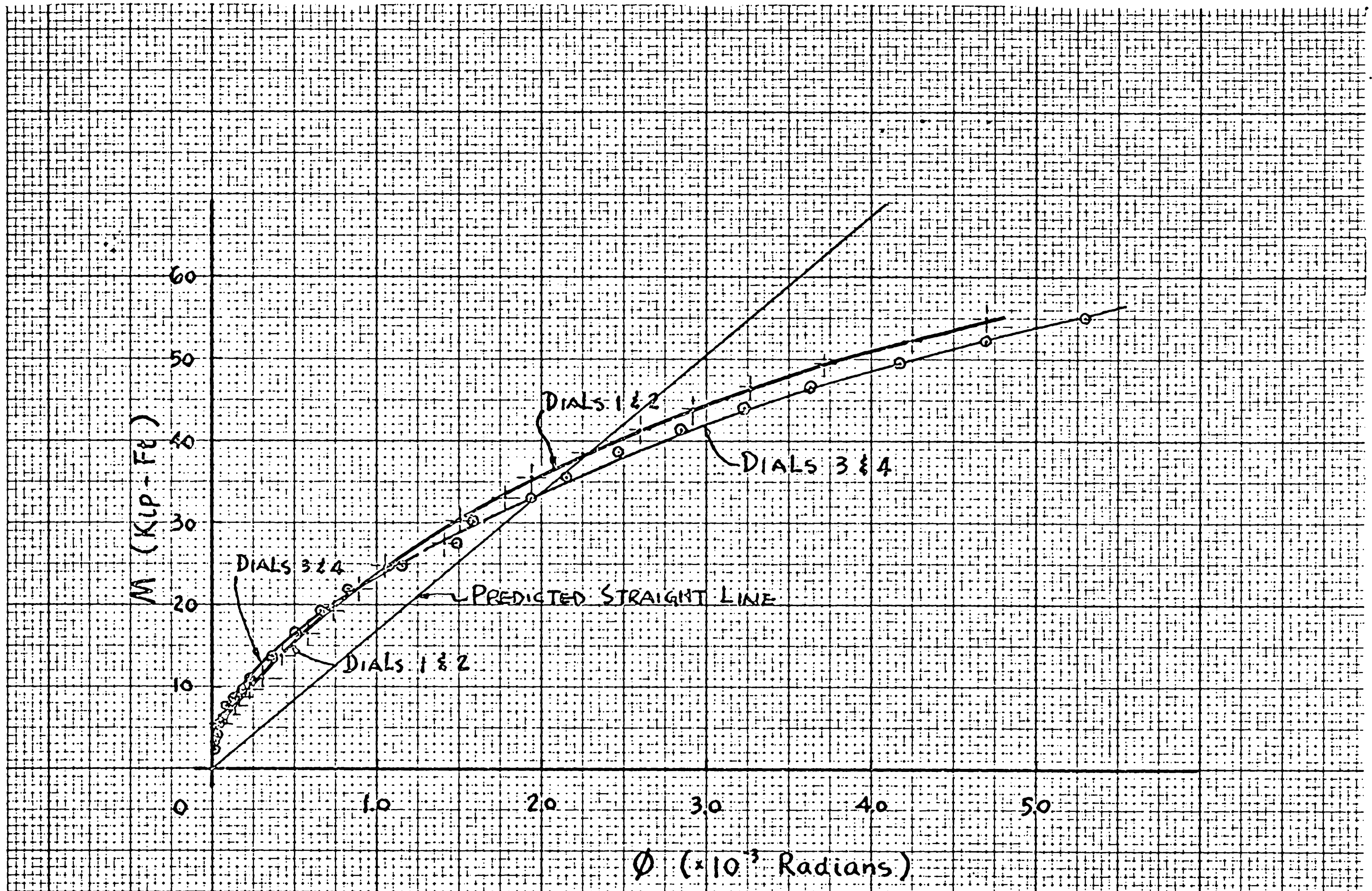


Figure 27. Moment- Rotation Curve - 24L09

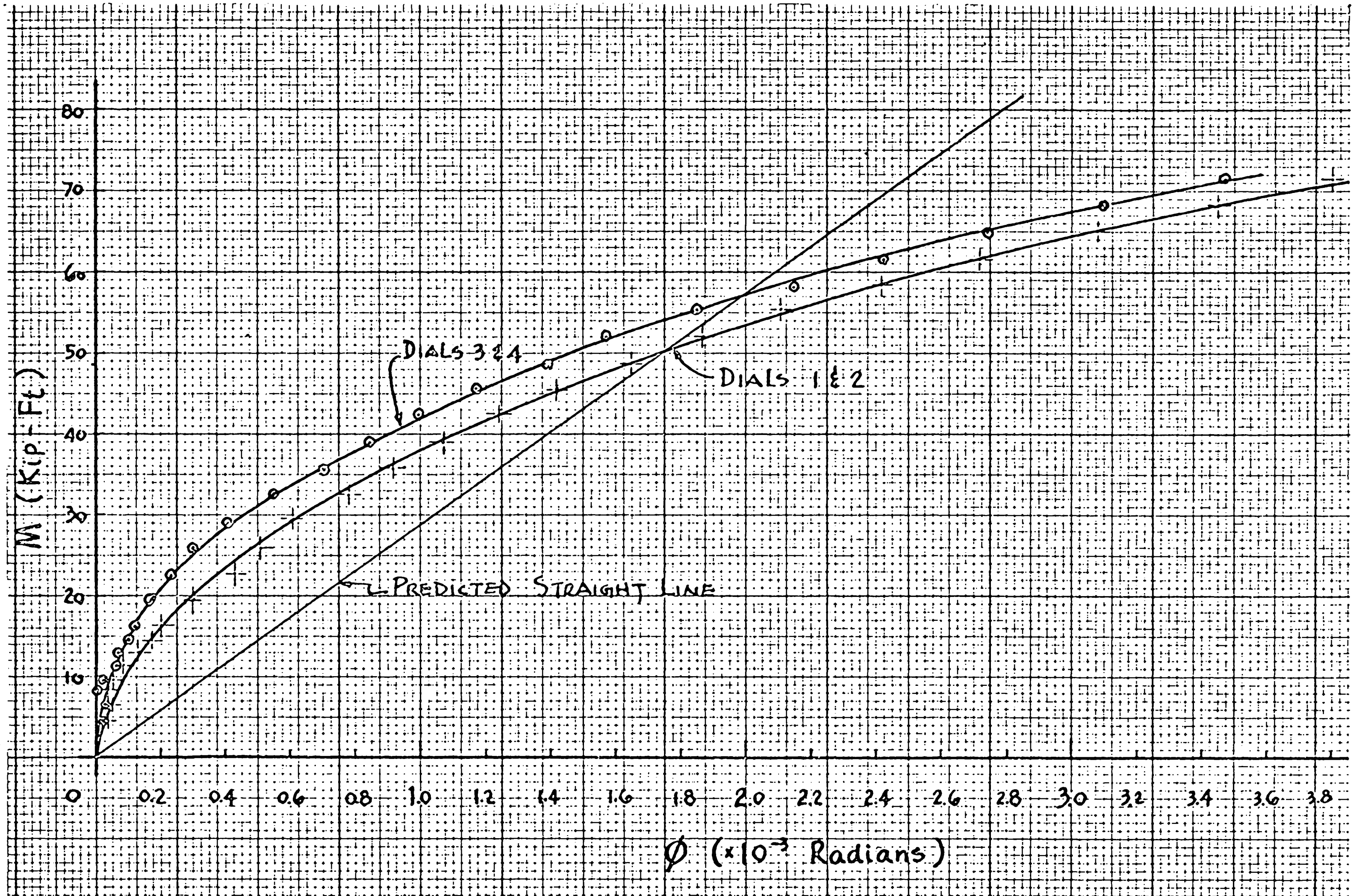


Figure 28. Moment - Rotation Curve - 32L11

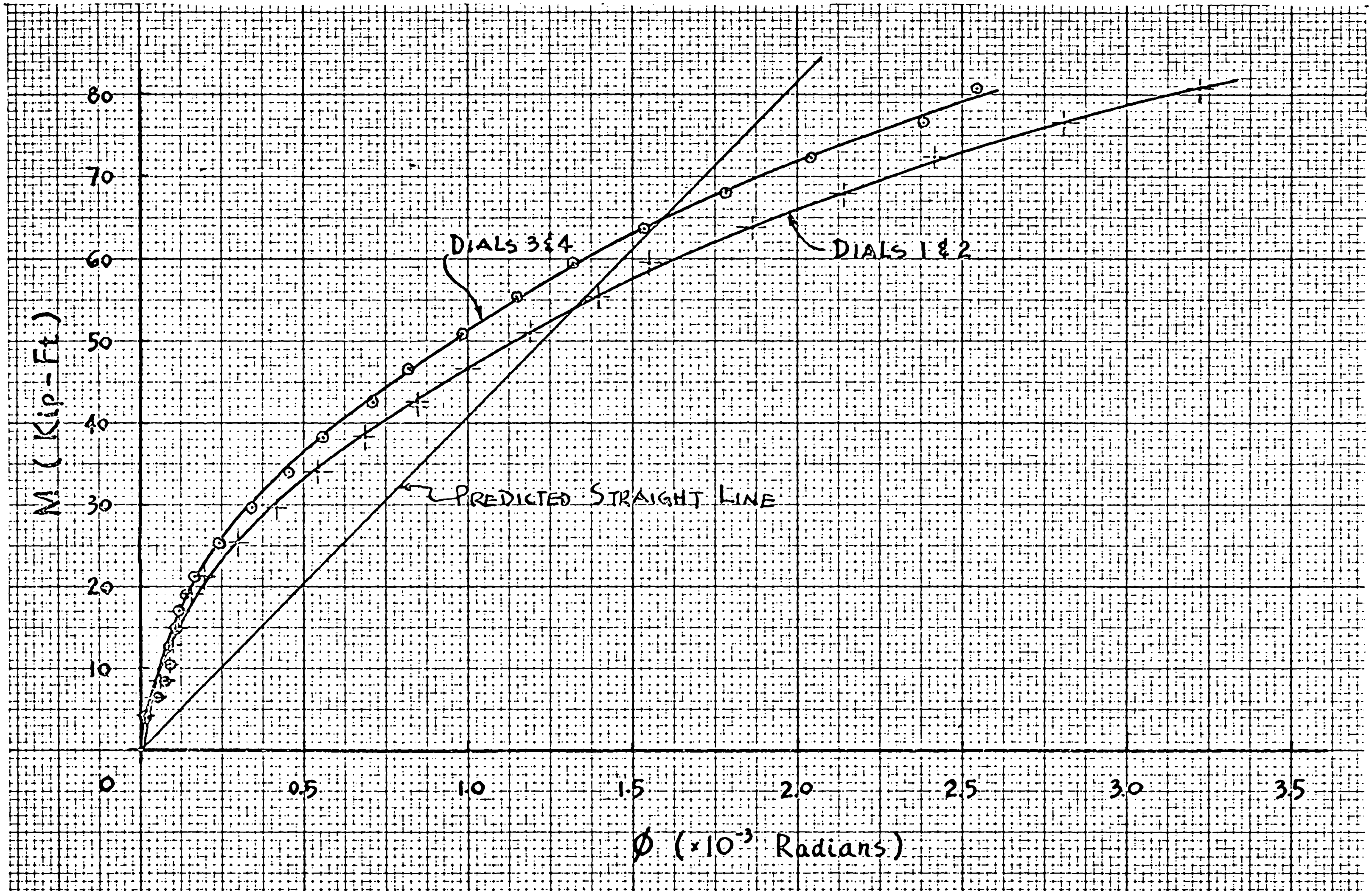


Figure 29. Moment - Rotation Curve - 40L13

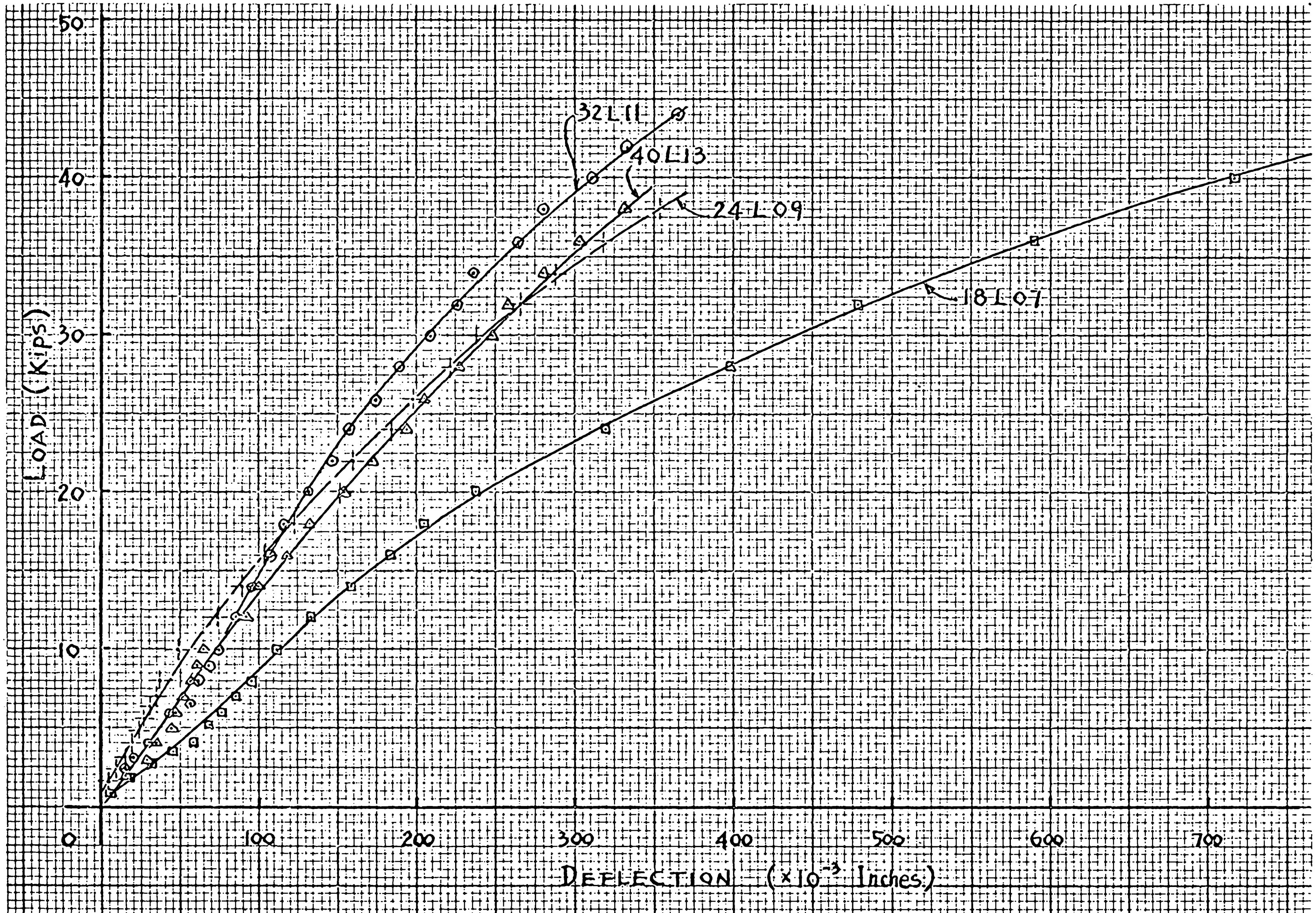


Figure 30. Deflection Curve of Dials 9 for all Test Sections

VII. DISCUSSION OF RESULTS

From the testing data it is apparent that the proposed connection behaved as a semi-rigid connection. Observing the moment-rotation curve of each test section, there is a similar character between them. That is, the connection passes through three stages: First, an initial stage with moment approximately in proportion to rotation; Second, a gradual spread of yielding in the connection; and Third, a stage of accelerated rotation, finally resulting in failure of the vertical web member.

A comparison of the predicted values with the actual values for moment-rotation relationship is given as follows. From the Z-g_1 curve on page 56, the predicted value of $Z = \phi/M$ in inch-kips of $1.05(10^{-5})$ equals $0.126(10^{-3})$ in ft-kips. This relationship is plotted on page 96 as a straight line for longspan joist 18L07. In a similar manner the predicted values of moment-rotation relationship of the remaining joists are plotted on pages 97 -99.

Observing the moment-rotation curve in relation to the predicted straight line, it is obvious that the connection behaves quite stiff at small load which is indicated by that the curve is above the straight line and gradually as the load increases the curve shifts below the straight line. This is due to the inelastic behavior of the connection. The instrumentation used was as suggested by the testing done at Lehigh University.[6]

VIII. CONCLUSION

The author's conclusion from the testing is that the proposed connection is appropriate for semi-rigid connections. This conclusion is based on the observation of the moment-rotation curves, which were plotted from the test data.

There are several points of interest that the author would like to see studied further. They are as follows:

(1) To design a vertical web member strong enough to develop the maximum negative moment capacity of each longspan joist.

(2) To observe the behavior of connection at the excessive yielding range.

(3) To establish the design range of this type of connection from testing more connections for the purpose of practical design.

(4) To establish the failure load of this type of connection.

IX. REFERENCES

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X. NOMENCLATURE

A. FIGURES

B. TABLES

C. NOTATIONS

A	area
a	length of bottom chord of longspan joist in compression
b	width of connection flange
D	overall depth of longspan joist
d	distance between centroids of the top and bottom chord of longspan joist
E	modulus of elasticity
e	length of welding
f_{act}	actual compressive stress
f_c	allowable compressive stress
g_1	length of connection stem to be cut
G	end panel length of longspan joist
H	interior panel length of longspan joist
I	moment of inertia
L	span length or laterally unsupported length
$+M$	positive moment
$-M$	negative moment
n	length of bottom chord of longspan joist in compression
P_t	axial stress at top or bottom chord
P_{max}	maximum allowable test load
P_{yield}	maximum test load that causes the vertical web member to yield

R	reaction at support
r_{xx}	radius of gyration about horizontal axis
r_{yy}	radius of gyration about vertical axis
S	longspan joist member stress
t	thickness
w	uniform load per unit of length
Z	semi-rigid connection factor
Δ	displacement, deflection
δ	axial deformation of member
ϕ	rotation
s	angles
	square

D. ABBREVIATIONS

bc	bottom chord
k	kips
k-ft	kip-foot
klf	kips per linear foot
ksi	kips per square inch
plf	pounds per linear foot
rad	radians
tc	top chord

XI. ACKNOWLEDGMENTS

The author is especially grateful and deeply indebted to Professor John F. Poulton for his guidance, patience and inspiration during two years of graduate studies at Virginia Polytechnic Institute. His counsel and enlightening suggestions for writing this thesis are sincerely appreciated.

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This investigation was made possible by using the laboratory facilities of the Department of Engineering Mechanics at the Virginia Polytechnic Institute. The author extends his appreciation to the personnel of the Department for their interest.

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ABSTRACT

Frequently in engineering design economy is sacrificed because of a lack of knowledge of the behavior of the structure. Because of the lack of information on the restraint values of beam-column connections, the beam is often over designed by failing to take advantage of partial continuity. If the degree of restraint of a beam-column connection can be determined, the optimum beam moment can be derived and then an optimum or economic beam can be designed.

This thesis is mainly concerned with the effective analysis and design of a particular type of semi-rigid connection between longspan joists and columns. It is also an attempt to establish a correct technic for testing full size longspan joists and column connections.

Although not commonly utilized in industry, in which longspan joists most frequently are used as simple-supported beams, the longspan joists have a degree of negative moment capacity. If this negative moment capacity can be utilized, a more efficient and economical building frame can be obtained. This thesis is also an attempt to analyze and determine the magnitude of the desired negative moment capacity of four representative longspan joists. A proposed semi-rigid connection based on this negative moment capacity is designed. Then the full size representative longspan joists are assembled by the semi-rigid connections to the full size representative columns.

The four test sections were fabricated and tested by the author in the Engineering Mechanic Laboratory at the Virginia Polytechnic Institute. The tests indicated that the proposed connections were quite satisfactory.