

ESTIMATING SWAP CREDIT RISK:
SIGNIFICANCE OF THE VOLATILITY INPUT
USING MONTE-CARLO SIMULATION

by

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(ABSTRACT)

Since its inception in the early 1980s, the global market for swaps has grown to over \$3 trillion in notional principal outstanding, leading some regulators and others to express concern about risks posed for the financial system. Notional principal, however, is not a measure of the risks of swaps. As a result, it is important to both businesses using swaps and regulators to develop appropriate measures of these risks. For credit risk, for example, current replacement cost measures the credit exposure in the event of default today, but does not account for the possibility of default in the future. Additional measures are required.

This thesis focuses on estimating the credit risk of swaps, accounting for both current and potential future exposure, and measuring the sensitivity of credit risk to changes in volatility. The model used is based on Monte Carlo techniques, drawing on Mark Ferron and George Handjinicolaou's article "Understanding Swap Credit Risk: The Simulation Approach". The model provides an estimate of the expected replacement cost of a swap, averaging across numerous interest rate scenarios. The

sensitivity of the model's estimate of swap credit risk to different volatility assumptions is also determined and compared to the results of Ferron and Handjinicolaou.

This analysis demonstrates that swap credit risk is highly sensitive to volatility. For example, starting with a 15% volatility level, a 100 basis point increase in volatility results in a 6.7% increase in the estimate of expected replacement cost. More generally, a given increase in volatility (e.g. from 20% to 25%) results in a proportional increase in replacement cost.

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CHAPTER ONE: INTRODUCTION

Following the increased price volatility in the financial environment of the early 1980s, a host of innovative financial instruments and strategies emerged for actively managing the risk faced by companies. The swap contract is one of the most successful of these financial tools. Since the inception of the swap contract, the market for swaps has experienced phenomenal growth. Today the swap market's size is over \$3 trillion in notional principal outstanding. Due to this tremendous growth and the considerable size of the market, many regulators have expressed concern regarding the effects of using this market. This concern has led to much continued debate among economists and financial analysts as to the appropriate measure of the credit risk firms face from engaging in swap contracts.

Determining the level of credit exposure is vitally important to businesses engaged in swap activity. Inaccurate estimates of credit exposure give rise to many problems. Since capital is a scarce resource, if financial managers misjudge the credit risk, then capital may be allocated inefficiently. In addition, because federal regulation requires businesses to meet certain capital requirements, if the credit risk is measured incorrectly by regulators, then businesses may be required to hold excessive levels of capital. For example, if a firm's credit risk is calculated to be higher than its actual exposure level, then the firm will tie up capital to meet regulatory

requirements instead of using it to invest in the firm or ultimately the economy.¹

This thesis presents a brief discussion of the historical development of the swaps market and the distinctions between different types of swaps is included, as well as the methods used in pricing these instruments. Further, the risks involved with using swaps and the issues involved with regulating the swap market are also discussed.

The analysis focuses on estimating the credit risk of swaps, accounting for both current and potential future exposure, and measuring its sensitivity to changes in the volatility of interest rates. The model used is based on Monte Carlo techniques, drawing on the work of Mark Ferron and George Handjinicolaou as presented in "Understanding Swap Credit Risk: The Simulation Approach". The model provides an estimate of the expected replacement cost of a swap, averaging across numerous interest rate scenarios. The sensitivity of the model's estimate of swap credit risk to different volatility assumptions is also determined and compared to the results of Ferron and Handjinicolaou. This analysis demonstrates that swap credit risk is highly sensitive to volatility. For example, starting with a 15% volatility level, a 100 basis point increase in volatility results in a 6.7% increase in the estimate of expected replacement cost. More generally, a 25% increase in volatility (from 20% to 25%) results in a 25% increase in replacement cost. Thus, the relationship between the volatility input and its corresponding replacement cost is proportional.

¹Jon Frye, "Underexposed and Overanxious", *Risk*, March 1992, 41.

CHAPTER TWO: OVERVIEW OF THE SWAP MARKET

A swap is simply an exchange of two things of value. A swap contract obligates two parties, referred to as counterparties, to exchange, or "swap", specified cash flows at specified intervals.² In practice, swaps have assumed three principal roles in corporate liability management: reducing the cost of current issuance, locking in the cost of future issue, and hedging the exposure to fluctuations in interest rates and foreign exchange rates faced by firms.³ In other words, swaps can reduce the cost of current issuance below that otherwise obtained through directly negotiating a loan. This type of swap often occurs between companies with different credit ratings. An example of how swaps can be used to achieve this result is presented in the later discussion of interest rate swaps.

Swaps allow firms to lock in the cost of future issuance. For example, if a firm's treasurer expects interest rates to rise in the future and knows the firm will have a need for a loan, then he may want to lock in a fixed rate on a future issuance by negotiating a forward swap. This will guarantee the firm an effective fixed-rate on its loan in the future, thus protecting it from a potential rise in future interest rates. If interest rates fall, however, then the firm loses, because the losses on the swap will

²Clifford W. Smith Jr., C. Smithson, and D. Wilford, Managing Financial Risk, 49.

³Laurie S. Goodman, "The Use of Interest Rate Swaps in Managing Corporate Liabilities", *Journal of Applied Corporate Finance*, 35, Winter 1990.

affect the benefit of the lower rates at the time of funding.⁴

The initial role of swaps was as financial instruments to hedge interest rate or foreign exchange exposure. Since financial innovation is a demand-driven phenomenon, if the financial environment is relatively stable, the market will use simple instruments. On the other hand, as exemplified in the years following the early 1970s, with a more volatile financial environment, innovative products will appear to meet the demand by companies to protect against foreign exchange, interest rate, and/or commodity price risks. Swaps were developed as a financial tool to allow firms to better manage risk and to even provide the ability to turn such risk into profit.⁵

⁴The firm does have the option to negotiate a contract with a "spread lock". This allows a party the option to set a credit spread without fixing the base rate. A firm can enter into a swap contract with a set ceiling rate and have the opportunity to set the base rate at any time during the contract life. Therefore, if interest rates fall, the firm can take advantage of this. Spread lock swaps are generally offered at a 2 to 4 basis point premium over a straight swap, due to the added hedging cost for the counterparty. (Refer to L. Goodman, "The Use of Interest Rate Swaps in Managing Corporate Liabilities", 1990.)

⁵S. Waite Rawls III and C. Smithson, "The Evolution of Risk Management Products", *Journal of Applied Corporate Finance*, 26, Winter 1989.

Evolution of the Market

Swaps developed originally as a means to hedge foreign exchange rate risk, which occurs when the value of future cash flows may change due to foreign exchange rate movements. The increased volatility of foreign exchange rates is mostly attributed to the breakdown of the Bretton Woods system of fixed exchange rates in the early 1970s.⁶ Although privately arranged, custom-tailored swaps began to appear in the 1970s, currency swaps first appeared publicly to manage foreign exchange rate risk in August 1981, marked by the World Bank - IBM swap.⁷ The prestige of these names heightened awareness of the new market for swaps and built confidence in its legitimacy.⁸

U.S. financial markets were subject to increased interest rate uncertainty after October 6, 1979. Under the leadership of Chairman Paul Volcker, the Board of Governors of the Federal Reserve System abandoned its practice of targeting interest rates and turned to targeting money supply growth instead.⁹ Banks began to offer

⁶Smith, Smithson and Wilford, *op. cit.*, 201.

⁷Rawls and Smithson, *op. cit.*, 19-21.

⁸D. K. Hargreaves, "Swaps: Versatility at Controlled Risk", *World Financial Markets*, 5, April 1991.

⁹Despite the announcement of this change, which would have based monetary policy on targeting money supply growth rates, the Federal Reserve experienced considerable difficulty in actually achieving these goals.

interest rate swaps in 1982, in response to the increased demand for interest rate risk management products.¹⁰ The commodity swap, which appeared in 1986, was created in response to unpredictable movements in commodity prices.¹¹ Commodity swaps involved the exchange of cash flows based on an underlying commodity price or index. The development of the commodity swap seems to be a logical step following the exchange of interest rates and currency .¹²

Before the innovation of the swap contract, parallel loan agreements were used to manage foreign exchange risk. For example, in the case of a U.S. parent company with a foreign subsidiary in the U.K., the exposure to foreign exchange movement could be hedged by matching them with a U.K. company that has a U.S. subsidiary. (The U.S. firm may fear that a decline in the value of the pound will result in a loss when its U.K. profits are converted into dollars for repatriation to the United States.) The U.S. company would make a dollar-denominated loan to the U.S. subsidiary of the U.K. company, and simultaneously, the U.K. company would make a pound-denominated loan of equal current value to the subsidiary of the U.S. firm. These two loans would have parallel interest and principal repayment schedules.¹³ Figure 1 on the following page illustrates the parallel loan agreement described here.

¹⁰Rawls and Smithson, *op. cit.*, 22.

¹¹Smith, Smithson and Wilford, *op. cit.*, 14.

¹²Erik Banks, Volatility and Credit Risk in the Capital Markets, 202.

¹³Smith, Smithson and Wilford, *op. cit.*, 201.

The U.S. firm could be motivated to enter into the parallel loan agreement to reduce the volatility of reported income, while the U.K. firm might enter into the agreement to counter the British government's controls on capital movements (by in effect taxing the export of capital).¹⁴ In this situation, if the value of the pound were to decrease, then the U.S. parent company would enjoy a gain on its pound-denominated loan, since it has a pound liability. Thus, the value of the pound-denominated loan would move in a direction which is opposite what would result from the firm's inherent position. In this way, the two firms could hedge their exposure to foreign exchange rate fluctuations.

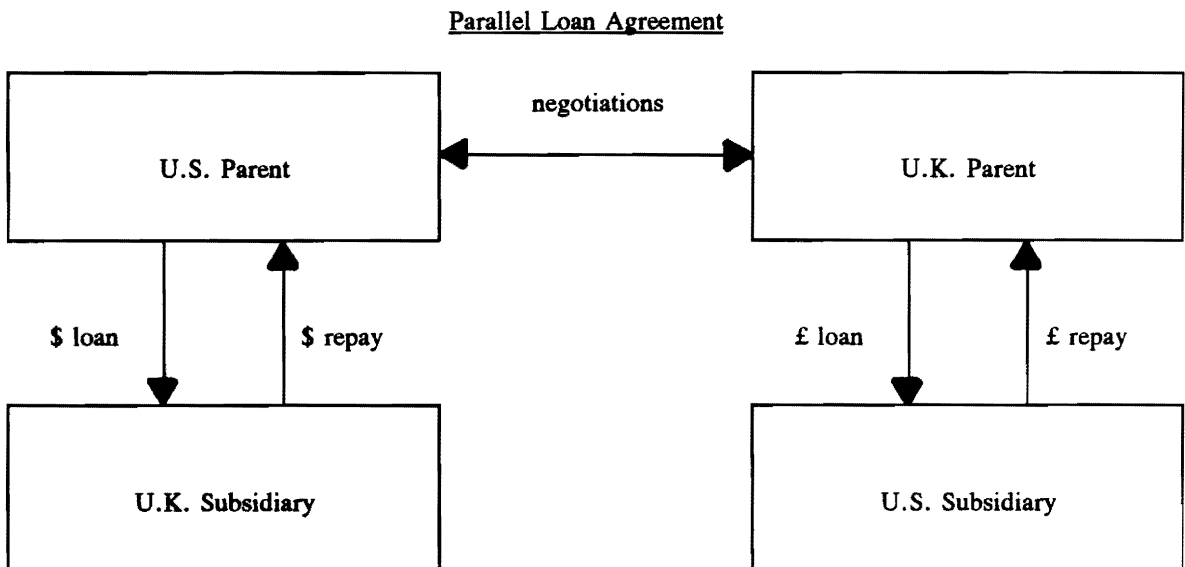


Figure 1. Diagram of Parallel Loan Agreement

¹⁴*Ibid.*, 201.

Two problems existed in the use of parallel loan agreements: default risk and the impact on the firm's balance sheet. Since the loans are independent instruments, default by one party does not release the counterparty from being contractually obligated to make payments. Moreover, if the balance sheet of the parent company and its subsidiary have to be consolidated, then the parallel loan agreement will inflate the firm's balance sheet. This may lead to potential problems when negotiating future financial contracts, because the firm carries more debt. Although the two loans effectively cancel each other out, they remain on the balance sheet for accounting and regulatory purposes.¹⁵

By changing the contract structure from two independent instruments to a single instrument, the problem of default risk can be managed, since both parties are bound by one contract. This single financial instrument is a currency swap. Following the previous example, one party agrees to pay a series of cash flows based on a fixed sterling interest rate in order to receive a series of cash flows based on a fixed dollar interest rate, while the counterparty takes the opposite position.¹⁶ Figure 2 on the following page illustrates the payment stream for this swap contract.

¹⁵*Ibid.*, 202.

¹⁶*Ibid.*, 203.

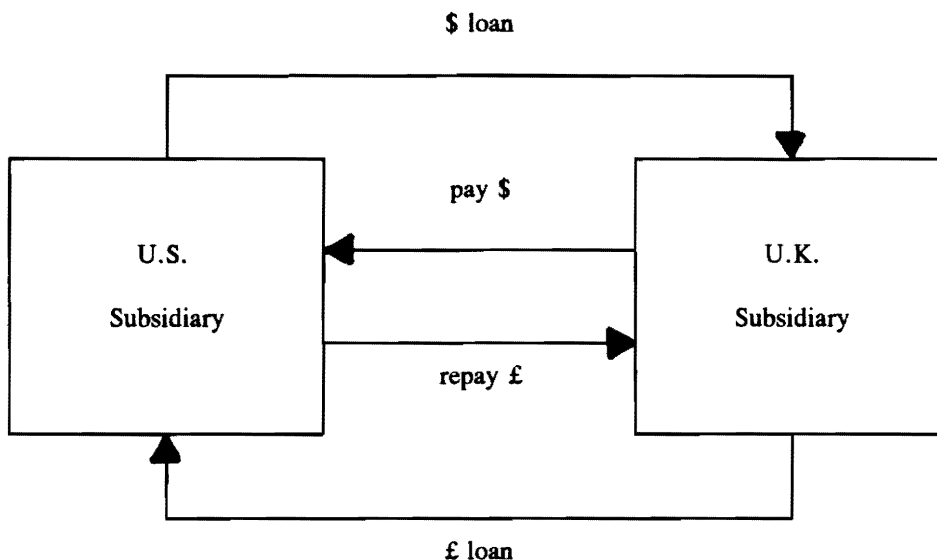
Swap Contract

Figure 2. Diagram of Swap Agreement

Another quality of swaps which provides for the further reduction of default risk or delivery exposure, is the "netting" of payments. Instead of requiring counterparties to make gross payments, the two parties can exchange a difference check. At each settlement date throughout a swap contract, the changes in the underlying's value are transferred between the counterparties, in the form of a difference check. Thus, only the difference in the value of the payments needs to be exchanged, with the higher value-paying party making a payment to the lower value-paying party equal to the difference in the values to be exchanged.¹⁷ Following the

¹⁷John F. Marshall and K. Kapner, Understanding Swap Finance, 105.

example set forth, if the value of sterling rises, the party who agreed to pay a series of cash flows based on a fixed sterling interest rate in order to receive a series of cash flows based on a fixed dollar interest rate, will pay a difference check to the counterparty. If the value of sterling falls, the party who agreed to pay sterling receives a difference check.¹⁸

The problem surrounding the impact on the balance sheet of using a parallel loan agreement is also taken care of through the use of a swap contract. Since current accounting and regulatory practices consider swaps to be off-balance sheet instruments, implementing a swap contract does not lead to explosive balance sheets.¹⁹

Since its inception in the early 1980s, the swap market has experienced phenomenal expansionary growth, so that today the market size is measured to be over \$3 trillion in notional principal outstanding.²⁰ Following the evolution of the simple currency swap from the parallel loan agreement, other types of swaps developed as increased demand for new and innovative risk management tools provided incentives for market users to design alternate forms of swaps.

¹⁸Smith, Smithson and Wilford, *op. cit.*, 204.

¹⁹*Ibid.*, 204.

²⁰IFR Self Study Workbooks, Interest Rate Swaps, v.

Varieties of Swaps

Currency swaps, involve the exchange of a fixed-rate cash flow in one currency for a fixed-rate cash flow in another currency. This simple form of swap (fixed-against-fixed currency swap) can be altered by allowing for the fixed-rate cash flow of one party to become a floating-rate cash flow, so that this type of instrument is a fixed-against-floating currency swap. In the financial market, this product is referred to as a cross currency swap.²¹

In the case of a fixed-against-fixed currency swap, there are essentially three basic stages of the life of a contract. During the first stage of the contract, an initial exchange of principal occurs. This takes place by the two counterparties exchanging principal amounts at an agreed upon rate of exchange. This initial exchange can either be on a notional basis (i.e. no physical exchange of principal amounts) or a physical exchange basis. (A standard transaction size is \$5,000,000.) In stage two of the contract, ongoing exchanges of interest occur between the two counterparties. These interest payment amounts are determined by the outstanding principal amounts and the fixed interest rates negotiated in the contract. Stage three arrives with the re-exchange of principal amounts on the maturity date. This process provides the opportunity for debt raised in one currency to be transferred into a completely hedged

²¹Barry Howcroft and Christopher Storey, Management and Control of Currency and Interest Rate Risk, 129.

fixed rate liability in another.²²

By further embellishing the currency swap, a financial instrument developed which combined a fixed rate currency swap and an interest rate swap. This type of swap, called a currency coupon swap, allows a counterparty to exchange a fixed interest rate in one currency for a floating rate of interest in another currency.²³

The typical contract period for a currency swap is between two and ten years.²⁴ By convention, the standard index for the floating interest rate in coupon swaps is six month LIBOR (London Interbank Offer Rate). Therefore, one can talk about the price or value of coupon swaps in terms of their fixed rates alone. The fixed interest rate in a coupon swap is often referred to as simply the swap rate.²⁵ This exchange rate is agreed upon in the currency swap contract before the beginning of the transaction period and remains fixed over the life of the contract. Also, currency swaps require an exchange of principal amounts upon the maturity of the contract.²⁶

Interest rate swaps are the most common type of swap, and account for \$762 billion of an estimated total swap market, measured in terms of notional principal

²²*Ibid.*, 130.

²³*Ibid.*, 132.

²⁴*Ibid.*, 129.

²⁵IFR Self Study Workbooks, Interest Rate Swaps, 10.

²⁶Howcroft and Storey, *op. cit.*, 129.

amount of new swaps written, of \$923 billion for the first half of 1991. Looking at the figures for the end of June 1991, these values are even more impressive and emphasize the dramatic growth which has occurred within the swap market to \$2,312 billion and \$2,889 billion respectively.²⁷

An interest rate swap is a type of currency coupon swap in which both the currencies are the same. More formally, an interest rate swap is a contract which commits two counterparties to exchange, over an agreed upon period, two streams of interest payments, each calculated using a different interest rate index, but applied to a common notional principal amount.²⁸ In the special case of an interest rate swap, the initial principal exchange is not necessary. Further, since all of the principal amounts are expressed in the same currency, the re-exchange at maturity is also not necessary.²⁹ By not requiring an actual exchange of physical principal, interest rate swaps can lower the default risk faced by corporations.

As mentioned previously, interest rate swaps allow companies with different credit ratings to enter into a swap so that both enjoy a reduction in the cost of current issuance. The swap mechanism which allows for this arbitrage to occur is similar to that proposed by the early 19th century English political economist David Ricardo to explain the benefits of international trade. Ricardo proposed a two country, two good

²⁷IFR Self Study Workbook, *op. cit.*, x.

²⁸*Ibid.*, 1.

²⁹Smith, Smithson and Wilford, *op. cit.*, 206.

model, in which one country was more efficient at producing both goods than the other country. If the more efficient country is not equally more efficient in producing both goods compared to the less efficient country, then the countries could each specialize in producing their most efficient good and realize a gain in overall welfare through trade. The argument being that the more efficient country would realize enough of a productivity gain to more than offset the loss of production by having the less efficient country produce the other good.

The familiar economics terminology which developed from Ricardo's model is that of the distinction between absolute advantage and comparative advantage. The more efficient country is said to have an absolute advantage in the production of both goods, but a comparative advantage in the good it produces most efficiently. In comparison, the less efficient country has an absolute disadvantage in producing both goods, but a comparative advantage in the good it produces least inefficiently.³⁰

This method can be applied to two different companies, one with an AA credit rating (called AA) and one with an A rating (called A). Assume AA can get floating rate financing for LIBOR + 100 basis points and fixed rate financing at 10.00%. Also, assume that A can get floating rate financing for LIBOR + 160 basis points and fixed rate financing for 12.50%. If one compares AA to A, then one can observe that AA has a comparative advantage in raising fixed rate funds, while A has a comparative advantage in raising floating rate funds. Further, AA has a 260 basis

³⁰IFR Self Study Workbooks, *op. cit.*, 46.

point absolute advantage in raising funds since it can borrow at 200 basis points less than A in the fixed rate market and 60 basis points less than A in the floating rate market. If AA borrows fixed rate funds by issuing a bond at 10.00% and A borrows from a bank at LIBOR + 160 basis points, then AA and A would want to engage in a swap so that AA pays a floating rate (assume LIBOR) to A in exchange for A paying a fixed rate to AA.

If AA receives fixed interest through the swap at a rate higher than 10.00%, it will be able to make more than the cost of its bond issue, and use this to cover any rise in the LIBOR it may need to pay through the swap. This will produce a net cost of borrowing for AA below LIBOR. Company A, however, will make a loss by paying LIBOR + 160 basis points on its bank loan and only receiving LIBOR through the swap. In order for A to gain from the arbitrage, the fixed interest it pays through the swap must be at least 160 basis points cheaper than its cost of borrowing directly in the bond market. Since the cost for A of directly borrowing in the bond market is 12.00%, A will require the fixed rate in the swap to be less than 10.40% (12.00% - 160 basis points). Therefore, since AA prefers to enter the swap if it receives a fixed rate above 10.00% and A will only enter the contract if it pays a fixed rate below 10.40%, the fixed rate in the swap may be set between 10.00% and 10.40%.

If the negotiated fixed rate for the swap is 10.20%, then we can calculate the gains both companies enjoy from entering the swap contract. AA has reduced the cost of its floating rate financing by 120 basis points from LIBOR + 100 basis points

to LIBOR - 20 basis points. Since AA receives 10.20% through the swap and pays only 10.00% for its bond financing, a difference of 20 basis points, it lowers the floating rate it has to pay through the swap of LIBOR to LIBOR - 20 basis points. Also, A has reduced its cost of receiving fixed rate financing by 20 basis points from 12.00% to 11.80%. This result occurs since A receives LIBOR through the swap and has to pay LIBOR + 160 basis points for its bank loan (losing 160 basis points), but makes up for this 160 basis point difference by only paying 10.20% when it would have cost 12.00% to borrow directly in the bond market. This difference of 180 basis points provides 160 basis points to cover the difference between the floating rates and an additional 20 basis points for A as an arbitrage profit.³¹

Other types of swaps include the basis swap, which is an extension of an interest rate swap. A basis swap has the same structure as an interest rate swap, with the exception that the basis swap permits floating rate cash flows calculated on one basis to be exchanged for floating rate cash flows calculated on another basis.³² For example, initially basis swaps were negotiated for the exchange of one month U.S. dollar LIBOR for six month U.S. dollar LIBOR. More recently, U.S. dollar LIBOR has been exchanged for U.S. dollar commercial paper.³³ Commercial paper is unsecured promissory notes issued by corporations in the U.S., Japan and Europe, as

³¹*Ibid.*, 41-44.

³²Smith, Smithson, and Wilford, *op. cit.*, 206.

³³Howcroft and Storey, *op. cit.*, 129.

defined by Intermarket.³⁴

These floating-against-floating swaps can use a variety of different "tenors" (maturities) of the same interest rate index. For example, three month LIBOR could be swapped against six month LIBOR. Also, an assortment of different interest rate indexes could be used. For example, three month U.S. dollar LIBOR could be swapped against the three month U.S. Treasury bill yield, or six month U.S. dollar LIBOR could be exchanged for the U.S. Prime rate. Another possibility is to enter a basis swap using the same tenor of the same interest rate index, but with one index carrying a margin. For example, three month LIBOR would be traded against three month LIBOR + 50 basis points.³⁵

Basis swaps provide a means for market users to create arbitrage spreads between different floating-rate financing sources. Also, basis swaps allow Europeans a means to simulate the U.S. commercial paper market without dealing with the need to meet the stringent U.S. requirements to enter the market.³⁶

As previously mentioned, the commodity swap was a logical development from the existing currency and interest rate swaps. Once a principal amount is agreed upon for a swap contract, and that principal is contractually converted to a flow, any set of

³⁴Intermarket, Glossary.

³⁵IFR Self Study Workbooks, *op. cit.*, 7.

³⁶Howcroft and Storey, *op. cit.*, 129.

prices can be used to calculate the cash flows.³⁷ Swaps could be priced in terms of commodities such as wheat. For example, a counterparty could agree to exchange a specified amount of dollars for a specified number of bushels of wheat, at specified periods in time. This swap is basically the same as a fixed currency swap, where the price of wheat replaces the currency price. Since the counterparties exchange a difference check paid in dollars, neither party needs to be in the wheat business. The counterparty contracting to pay in terms of the price of wheat can negotiate to receive either fixed or floating rates in any currency or commodity. Although commodity swaps for wheat have not yet appeared in the financial market, commodity swaps for oil have.³⁸

³⁷Smith, Smithson and Wilford, *op. cit.*, 207.

³⁸*Ibid.*, 207.

Regulation of Swap Activity

Given the risk of substantial losses which a firm could incur through entering a swap, and the inability to adequately hedge the credit risk portion, many economists and financial analysts have continued to debate the need for regulation within the swap market. An explanation of the risk involved with using swaps, and the inability to hedge this risk, is presented in Chapter Four of this paper.

Those who favor strict regulatory controls on swaps argue that the risk of financial institutions failing, in the event of a huge loss due to exposure to credit risk, is too great to be without supervision. The affect of a large institution or large number of institutions failing would be detrimental to the economy as a whole. Proponents of this view often support the federal enforcement of capital requirements for dealer banks which intermediate swaps.

Those who oppose the implementation of strict regulatory controls on the swap market argue that the initiation of a swap contract works to hedge the much greater and more harmful risk of exposure to fluctuations in interest rates, foreign exchange rates or commodity prices. By attempting to "encourage" prudent financial decision making, those in favor of stringent regulation could be actually "discouraging" a firm from hedging its exposure to risk, forcing it to accept more risk than is necessary.³⁹

³⁹Saul Hansell and Kevin Muehring, "Why Derivatives Rattle the Regulators", *Institutional Investor*, 1992.

Also, in the case of capital adequacy requirements, those opposed to regulation argue that in a functioning marketplace financial intermediaries will provide the optimal level of capital reserves without government intervention. Following this view, any additional reserve requirement imposed by a regulatory authority would be excessive, and lead to a reallocation in three primary dimensions of the swap market: volume, location, and credit quality.

Volume is affected because a regulatory tax (resulting from capital requirements being set at a level higher than the market optimum) will increase the cost of entering a swap contract, and thus lead to a lower volume of swap transactions. Also, the increase in the price of a swap agreement will restrict a firm's ability to manage their exposure to market volatility. Location will also be altered under the imposition of capital requirements, because swap market business will move from regulated institutions to unregulated institutions (financial intermediaries in other countries or those not covered by the regulation, such as investment banks). Further, if investment banks become the major financial intermediaries of swap contracts, instead of commercial banks, then swaps will be handled by dealers with less, not more capital. Thus, a reallocation in credit quality will occur.⁴⁰

Despite opposition, concern about credit risk led to the development of risk-based capital adequacy standards. Under the capital adequacy standards put forth by

⁴⁰Clifford W. Smith, Jr., Charles W. Smithson and Lee Macdonald Wakeman, "Credit Risk and the Scope of Regulation of Swaps", The Handbook of Financial Engineering, 623, 1990.

the Committee on Banking Regulations and Supervisory Practices of the Bank for International Settlements (the "Committee"), a bank must determine the credit risk equivalent of interest rate and currency swaps by applying one of two alternative methods.

The method preferred by the Committee recommends that a firm calculate the credit risk as equivalent to the sum of two components. The first component is the total replacement cost, which is the cost for a party to replace a payment stream in the event that the counterparty defaults today. A measure of replacement cost is obtained by valuing the swap using the settlement price on the day of default. This practice is referred to as "marking to market". Those swaps determined to have positive value to their parties (i.e., the party would have received a payment from the defaulting counterparty) are then considered in order to quantify the replacement cost. Swaps priced to have negative value are not considered, because these would not represent a loss for the party, since in the event the counterparty defaults the party is no longer required to make a settlement payment.

The second component is an amount measuring the potential future credit exposure calculated by applying a percentage to the notional principal amount of each swap. This percentage is decided by the Committee and determined by the residual maturity and the type of the swap.⁴¹ (Further discussion of replacement cost for

⁴¹Recent Amendments to the United States Bankruptcy Code Relating to Swap Agreements, 3 1990.

swaps and the pricing of these contracts is provided in Chapters Three and Four.)

The alternative approach suggested by the Committee to evaluate credit risk is to apply a percentage to the swap's notional principal amount. This percentage is similar to the one used in the preferred method because it is also dependent upon the maturity and type of swap. However, the percentage used in this method will be higher than that used in the second component of the preferred method.⁴²

Once a firm calculates the credit risk equivalent of its swap agreement using one of the above methods, the next step is to apply a weighting factor to determine the risk-weighted asset. This weighting factor will vary between 0% and 50%, depending on the type of counterparty, i.e. how risky they are. Following this risk-weighting procedure, the value of the risk-weighted asset becomes the amount the firm must maintain capital against.⁴³

To demonstrate how this rule is applied, the following example is provided. Consider a ten-year interest rate swap with a notional principal of \$100,000,000. Following the Basle committee's rule, the credit conversion factor for this swap would be 0.005.⁴⁴ Therefore, if the current exposure is estimated to equal zero, then the total credit equivalent exposure is equal to the potential credit exposure, which in this case is \$500,000. To take another example, if a swap has a current

⁴²*Ibid.*, 3.

⁴³*Ibid.*, 4.

⁴⁴Smithson, *op. cit.*, 17-22.

exposure of \$2 million, then the total credit equivalent exposure would equal \$2,500,000.⁴⁵

⁴⁵*Ibid.*, 17-22.

CHAPTER THREE: PRICING SWAPS

This chapter is provided to explain how swaps are valued. An understanding of the pricing of swap contracts is essential to comprehending the method used in the thesis model to value the swap and thus calculate a replacement cost as a measure of credit risk.

Pricing swap contracts is in practice similar to the pricing of loans. A swap contract can be viewed as economically equivalent, in terms of net cash flows, to a combined long and short position in a loan. For example, one party may hold a position of paying a fixed rate and receiving a floating rate, while the counterparty holds the alternate position of paying a floating rate and receiving a fixed rate. Therefore, the net present value of a swap contract can be set equal to zero by choice of the fixed rate.⁴⁶ Knowing this provides market users and analysts with a method for solving for the value or price of a swap. If the actual or expected floating rate payments can be determined for the origination and subsequent payment dates of the contract, and if the term of the interest rate is known, then the fixed rate that sets the net present value of the swap to zero can be solved for.⁴⁷

⁴⁶Some swap market analysts disagree with this simple assumption that a swap contract is essentially the same as a long and a short position in a loan (see Robert H. Litzenberger's article "Swaps: Plain and Fanciful", 1992).

⁴⁷Smith, Smithson and Wilford, *op. cit.*, 229.

Two-way Pricing

As previously mentioned, the value or price of coupon swaps is given in terms of their fixed rates. These fixed rates are quoted in the market in absolute terms (as the full percentage annual yield), and are referred to as "all-in-prices". The swap markets for certain major currencies, however, have discontinued the use of quoting all-in prices for the fixed rate in coupon swaps. This method has been replaced by the convention of quoting prices in two parts: a swap spread and a benchmark interest rate. The benchmark rate is usually the yield pertaining to the "on-the-run" (most liquid) government bond with a remaining maturity closest to the term of the swap. Upon the negotiation of the swap contract, both the spread and benchmark yield are fixed. This produces an "all-in" rate which is used to calculate the fixed interest payments throughout the span of the swap contract.⁴⁸

Market convention among professional swap dealers is to use a system of quoting dual prices, consisting of a buying (paying) price and a selling (receiving) price for each swap. This system is referred to in the financial market as "two-way pricing".⁴⁹ In order to interpret a two-sided pricing quote, it is important to remember that the dealer who is quoting the prices is the one aiming to make a profit

⁴⁸IFR Self Study Workbooks, *op. cit.*, 11.

⁴⁹IFR Self Study Workbooks, *op. cit.*, 10.

by negotiating a swap at the listed rates.⁵⁰ In the table of possible swap quotes below, the current price for a five year U.S. dollar coupon swap is quoted as a two-way swap spread of 45/55. The lower price of 45 means the dealer giving the quote is willing to transact swaps in which he pays a fixed rate of 45 basis points over the yield on the most liquid five year U.S. Treasury note. The higher quoted price of 55 means the dealer giving the quote is also willing to transact swaps in which he receives a fixed rate which is 55 basis points over the yield on the most liquid five year U.S. Treasury note.⁵¹

Two-sided Swap Quotes

2 year -- 40/48

3 year -- 42/50

5 year -- 45/55

By paying a lower fixed rate in one swap and receiving a higher fixed rate in another, the dealer is able to earn a dealing spread between the two rates.⁵² Two-sided prices are often referred to in the swaps market as the "bid-offer" price, and serve to distinguish between whether one wants to pay or receive the fixed rate of interest.⁵³

⁵⁰Mary S. Ludwig, Understanding Interest Rate Swaps, 42, 1993.

⁵¹IFR Self Study Workbooks, *op. cit.*, 12.

⁵²*Ibid.*, 10.

⁵³Ludwig, *op. cit.*, 42.

The market convention of quoting swap spreads was adopted from the debt capital market. Since many swap contracts are transacted to complement new bond issues (as illustrated previously, to hedge risk, or gain arbitrage profits), the fixed interest rates in swaps appear to reflect bond yields. This link between the swap and bond markets made it convenient for swap dealers to price swaps in terms comparable to bonds.⁵⁴

⁵⁴IFR Self Study Workbooks, *op. cit.*, 12.

Pricing an "At-Market" Swap

Since we can use forward rates of interest as estimates of expected floating payments, we can determine the fixed rate that sets the net present value of the contract to zero. Working through this pricing method can bring greater understanding of what is actually happening in a swap contract. The following example, which was taken from Managing Financial Risk, will show the steps involved in pricing an interest rate swap.

Consider a firm, Galactic Industries (GI), which wants to enter into a swap contract in which it will pay cash flows based on a floating rate and receive cash flows based on a fixed rate. By market convention, the rates for interest rate swaps are quoted as the floating (usually LIBOR) against the fixed rate. Suppose GI requests a quote from Dead Solid Perfect Bank (DSPB) for the following swap:

notional principal amount	=	\$100
maturity	=	1 year
floating index	=	6 month LIBOR
fixed coupon	=	? %
payment frequency	=	semiannual, day count 30/360

(The use of the 30/360 day count is for convenience in the example. The market convention for LIBOR is to use ACTUAL/360).⁵⁵

Following the pattern of a swap contract, at the six month settlement, GI will pay a "coupon" determined by the six month LIBOR rate in effect at contract

⁵⁵Smith, Smithson and Wilford, *op. cit.*, 230.

origination. At the twelve month settlement date, GI's "coupon" payment is determined by the six month LIBOR rate prevailing at month six. What remains to be determined is the amount GI will receive and how much DSPB will pay. The floating rate cash flow DSPB will receive at the first settlement date is determined by the six month rate in effect at the beginning of the transaction period, assume 8%. Therefore, at the six month settlement date, DSPB expects to receive an amount (calculated using the bond method), where:

$$\tilde{R}_1 = \$100[(180/360)0.08] = \$100 \times \frac{1}{2}(0.08) = \$4.00$$

For maturities less than one year, market practice is to quote interest rates with compounding already imbedded in the rate. Therefore, if the annualized six month rate is 8%, the amount received at the end of six months on a \$100 investment can be calculated as:

$$\$100 \times (180/360) \times 0.08 = \$4.00$$

The method uses the convention that compounding occurs annually but that the periodicity of the rate is monthly. This is in contrast to the convention used by most finance textbooks, which treat the interest rate as subject to compounding. Using the discrete compounding method to determine the amount received at the end of six months on a \$100 investment, given an annualized six month rate of 8%, the calculation is:

$$\$100 \times (1.08)^{180/360} - \$100 = \$3.92$$

where the periodicity is again monthly but the rate is compounded monthly instead of annually.⁵⁶

In order to calculate the expected floating rate inflow at the one year settlement date, one needs to estimate the six month rate in six months. This rate, the rate from $t(\text{time}) = 6 \text{ months}$ to $t = 1 \text{ year}$, can be estimated using the forward rate. The arbitrage condition guarantees that the following will hold:

$$(1 + r_{12}) = [1 + \frac{1}{2}(r_6)] \times [1 + \frac{1}{2}({}_6r_{12})]$$

where, r_{12} is the current twelve month rate and r_6 is the current six month rate.⁵⁷

Given the six month and one year rates of 8% and 10% (from the LIBOR yield curve), we can solve for the forward rate ${}_6r_{12}$, which will equal 11.5%. Plugging this value into the equation below, one can solve for the floating rate payment at the second settlement date

$$\tilde{R}_2 = \$100 \times \frac{1}{2} \times .115 = \$5.75$$

Based upon this information one can illustrate the contractual expected floating rate inflows to DSPB by the figure below:

⁵⁶*Ibid.*, 231-232.

⁵⁷*Ibid.*, 232.

$$\begin{array}{r}
 \uparrow \$4.00 (.08) \uparrow \$5.75 (.115) \\
 \hline
 \downarrow \$5.00 \quad \downarrow \$5.00 \\
 6mo. \quad 1yr.
 \end{array}$$

What still needs to be determined is the outflows, the appropriate fixed rate payments for DSPB. Since at origination of the contract for an "at-market" swap (a swap in which the fixed rate paid by one party is equal to the prevailing market fixed rate) the expected net present value is set to zero, one can use the following equation to solve for the fixed rate interest payments:

$$\frac{\$4.00 - \bar{R}_1}{1 + \frac{1}{2}(0.08)} + \frac{\$5.75 - \bar{R}_2}{1.10} = 0$$

where fixed $R_1 = \text{fixed } R_2$. Solving for the fixed rate payments we calculate an amount of \$4.85. This can be substituted into the equation below to solve for the appropriate interest rate of 9.7%:⁵⁸

$$\$100 \times (180/360) \times r = \$4.85$$

Another method of solving for this value is to consider the "par value". The par value of a swap contract is the market coupon interest rate, or the coupon rate that would put the bond trade at par. If one had followed the par value method the following equation would have given the 9.70% interest rate determined above:

⁵⁸*Ibid.*, 233.

$$100 = \frac{\frac{1}{2}r \times 100}{1 + \frac{1}{2}(0.08)} + \frac{\frac{1}{2}r \times 100}{1.10} + \frac{100}{1.10}$$

Based upon the previous calculated rates, one can expect the swap contract to be completed as follows:

notional principal amount	= \$100
maturity	= 1 year
floating index	= 6 month LIBOR
fixed coupon	= 9.70%
payment frequency	= semiannual
day count	= 30/360

The expected cash flows for DSPB can therefore be illustrated as follows:⁵⁹

↑ \$4.00 (.08)	↑ \$5.75 (.115)
↓ \$4.85 (.097)	↓ \$4.85 (.097)
6mo.	1 yr.

The type of swap used for the previous example is referred to among market users as a "plain vanilla" swap. By market convention, a plain vanilla swap is priced as LIBOR "flat" against the U.S. Treasury (par) rate "plus". For example, the following is a representation of possible market quoted prices for "at-market" interest rate swaps:

U.S. Dollar Rate Swap Quotes
[Treasury - LIBOR]

2 year	T + 70	T + 74
3 year	T + 74	T + 77
4 year	T + 74	T + 78

⁵⁹*Ibid.*, 234.

The rates listed for a three year maturity can be interpreted to mean that if a party wanted to receive a fixed rate, then the party would have to pay LIBOR and receive the three year Treasury par rate plus 77 basis points. The difference between the receive Treasuries and the pay Treasuries of 3 basis points is the "bid-ask" spread referred to in the previous section.⁶⁰

⁶⁰*Ibid.*, 235.

Pricing an "Off-Market" Swap

The described market convention of LIBOR versus Treasuries plus spread, works well as a means to price "at-market" swaps, at contract origination. This par rate convention, however, is not appropriate if one needs to determine the value of a swap after origination, or if the swap in question is an "off-market" swap. The term "off-market" swap refers to a swap contract in which the fixed rate paid by one party is higher or lower than the prevailing market rate. One must employ the zero-coupon yield curve in order to value a swap after contract origination. In determining the value of a swap after the beginning of the transaction period, one must take account of the changes in the market price of the interest rate, currency, or commodity, which the swap is based.⁶¹

In order to analyze the pricing of a swap after contract initiation, we can apply the previous example of a swap between GI and DSPB. Suppose there is a 1% upward shift in the LIBOR yield curve, which occurs on the day following the origination of the contract. Since the terms of the swap specified that DSPB will pay at an annual rate of 9.70%, DSPB's first floating rate receipt which was determined at origination to be \$4.00, will remain unchanged. For this one year swap, the only cash flow that will be altered is the expected floating rate inflow in one year. The new forward rate f_{12} resulting from the new term structure is 12.4%. The following

⁶¹*Ibid.*, 236.

equation shows the calculation for this answer:

$$(1 + 0.11) - [(1 + \frac{1}{2}(0.09)][1 + \frac{1}{2}(r_{12})]$$

Substituting this value into the equation below one can solve for the expected floating rate inflow in one year:

$$\bar{R}_2 - \$100 \times \frac{1}{2}(0.124) = \$6.22$$

Therefore, DSPB's expected cash flows can be diagrammed as follows:⁶²

↑ \$4.00 (.08)	↑ \$6.22 (.124)
↓ \$4.85 (.097)	↓ \$4.85 (.097)
6 mo.	1 yr.

Drawing upon this, one can calculate DSPB's expected net cash inflows to be -\$0.85 at the six month point and +\$1.37 at the one year point. By discounting these expected net cash inflows by the corresponding zero coupon rates from the current zero-coupon yield curve, we determine a six month rate of 9% and a one year rate of 11%. Following the equation below we can conclude that the value of the swap has risen from zero at contract origination to +\$0.42.⁶³

⁶²*Ibid.*, 237.

⁶³*Ibid.*, 237.

$$\frac{-\$0.85}{1 + \frac{1}{2}(0.09)} + \frac{\$1.37}{1.11} = \$0.42$$

The procedure used in the previous example is referred to as "marking the swap to market". In other words, we are marking the swap to market by calculating the value of the swap for different changes in the yield curve. By doing this we were able to obtain a payoff profile for the counterparties in the swap.

One also needs to consider the pricing of "off-market" swaps. As mentioned, the name "off-market" refers to the case where the fixed rate paid by one party is higher than the prevailing market fixed rate. (The fixed rate payer is paying an above the market coupon rate.) In this situation, a payment will need to be made from the floating rate payer to the fixed rate payer at contract origination. The pricing question arises due to the question of how large should the initial payment be. The amount of the initial payment is determined by the difference between the market value of a bond that carries the above-market interest rate and the notional principal of the swap.⁶⁴

In conclusion, the pricing of swaps depends on many variables, including the maturity of the swap, the structure of the swap, the availability of a counterparty, the

⁶⁴*Ibid.*, 238.

credit worthiness of the other counterparty, and the regulatory constraints on the flow of capital.⁶⁵

⁶⁵John F. Marshall and Kenneth R. Kapner, The Swaps Market, 59, 1993.

CHAPTER FOUR: CREDIT RISK

The Nature of Swap Credit Risk

With the dramatic growth in the size of the swaps market, concern has risen regarding the inherent credit risk of swaps. This credit risk exists because of the potential for loss arising from a counterparty's failure to make payments which are due. In this way, swaps act as a "bet" by one counterparty on the credit-worthiness of the other counterparty, that is the risk that one counterparty may default. Unlike a traditional variable rate loan, which requires a principal repayment at maturity, so that the amount of risk is equal to the principal amount, most swaps do not require the exchange of principal. Therefore, in the case of swaps, the credit risk can not simply be measured by the principal amount of the swap.

Credit risk is composed of two individual types of risk: default risk and market risk. Default risk depends upon the probability that default will occur during the life of the swap, and when during the life of the swap the default occurs. Default risk may lead to the event of a loss for a firm if the counterparty fails to perform according to the terms of the contract, and if interest rates (or the price of the underlying) move adversely to those negotiated in the original contract.⁶⁶ In other

⁶⁶Mark Ferron and George Handjinicolaou, "Understanding Swap Credit Risk: The Simulation Approach", The Handbook of Currency and Interest rate Risk Management. 23-5, 1990.

words, a credit loss will only occur if the counterparty defaults and the swap contract has a positive mark-to-market value to the nondefaulting party. Both conditions have to be satisfied simultaneously for a loss to be incurred.

Market risk refers to the fluctuation in the replacement cost of the swap.

Market risks arise from the possibility that market variables, such as interest rates or exchange rates, will fluctuate so that the value of the contract for the financial institution or swap dealer becomes negative.⁶⁷ Three features of market risk are that market risk varies over the life of the swap according to movements in the underlying, that market risk can be either positive or negative, and that market risk cannot be determined in advance.⁶⁸ The market risks faced by firms can be hedged by entering off-setting contracts; however, the credit risk cannot be hedged.⁶⁹

The fact that the "mark-to-market" value of a swap can be either positive or negative, implies the possibility that a counterparty might gain from the event of a default. For example, suppose in the original contract a party receives a fixed interest rate in exchange for a floating rate, and fixed rates rise during the life of the contract. In the event that the other counterparty defaults, the party would not necessarily experience a loss. In fact, the party can find another counterparty to fill

⁶⁷John Hull, Options, Futures, and Other Derivative Securities, 291, 1989.

⁶⁸Barry Howcroft and Christopher Storey, Management and Control of Currency and Interest Rate Risk, 133, 1989.

⁶⁹Hull, *op. cit.*, 291.

the existing swap contract, at the new higher rates, he can enjoy an actual gain from the default.⁷⁰

As mentioned, the value of the contract to the swap dealer could also become negative, representing a loss. Suppose a financial institution enters into off-setting contracts with two companies (A and B), and Company A defaults, leaving the financial institution obligated to honor the contract held with Company B. In this situation, the financial institution must find a third party willing to take Company A's position. In order to induce a third party to enter the swap, the financial institution must be willing to pay an amount approximately equal to the value of the contract with Company A prior to A's default.⁷¹ This supports the previous conclusion that the risk faced by counterparties is not in terms of the principal or notional amount, but rather the replacement cost of a missing interest rate stream.

The current replacement cost of a swap, which is a measure of the current exposure of the swap, is obtained from the market value of the swap contract. In the case of an interest rate swap, the value of the swap changes as the underlying interest rate changes. Panel A of Figure 3 illustrates the value profile for a fixed rate payer in an interest rate swap. If interest rates rise, the expected present value of the inflows becomes greater than that for the outflows and the value of the swap becomes positive. In contrast, if interest rates fall, the value of the swap to this fixed rate

⁷⁰Ferron and Handjinicolaou, *op. cit.*, 23-5.

⁷¹Hull, *op. cit.*, 291.

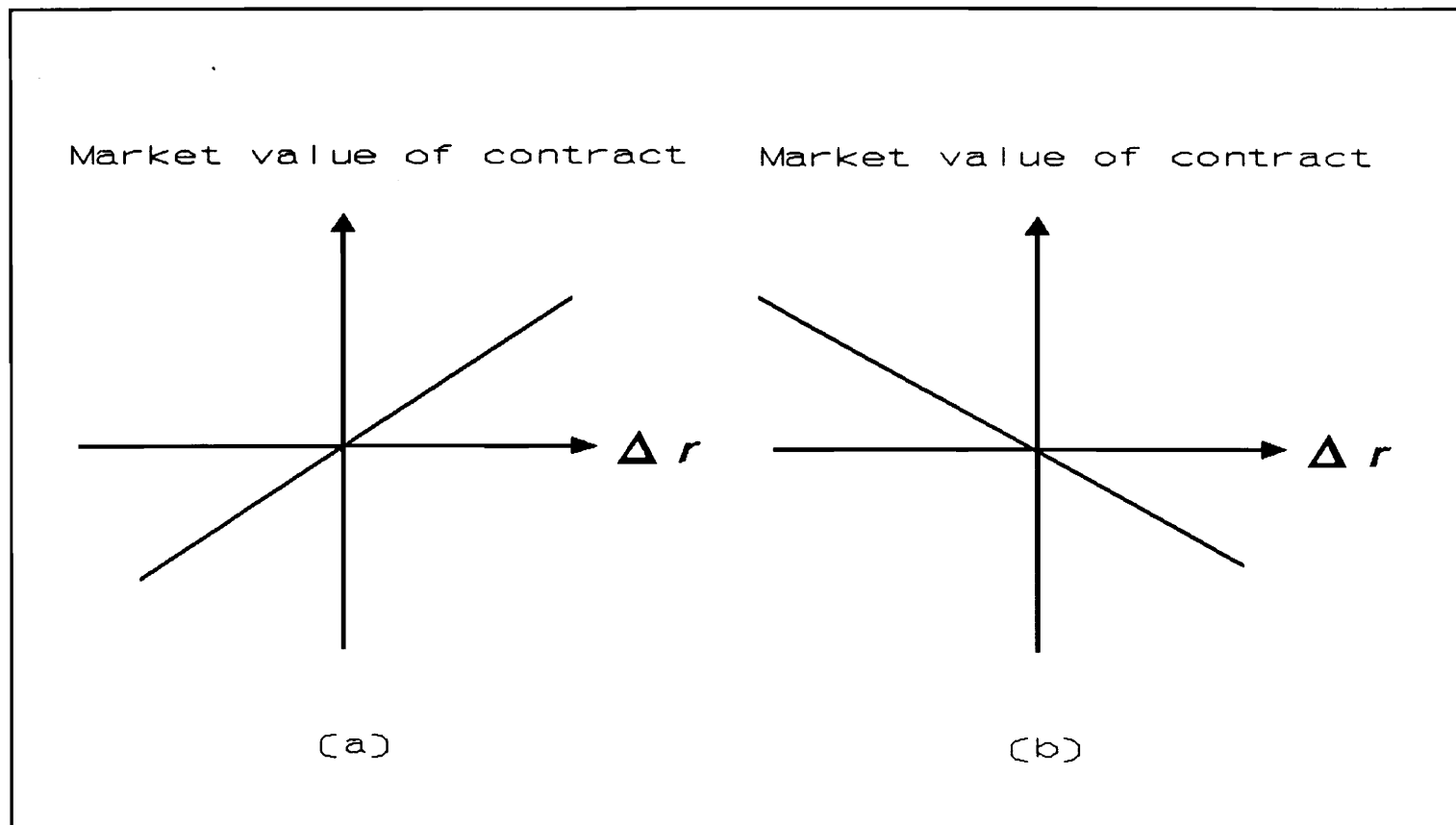


Figure 3. Value Profile

Source: Charles Smithson, "Measuring and Managing the Credit Risk", (draft), 17-6.

payer becomes negative. Panel B of Figure 3 illustrates the value profile for the floating rate payer. If interest rates rise, the expected present value of the inflows becomes less than that for the outflows and the value of the swap becomes negative. On the other hand, if interest rates fall, the value of the swap to the floating rate payer becomes positive.⁷²

In the case of the fixed rate payer, illustrated in Panel A of Figure 3, if rates decline after contract origination, he will be making payments to the floating rate counterparty. Thus, if the counterparty defaults, the fixed rate payer will not face a loss. For the fixed rate payer, a default-induced loss occurs when interest rates rise. Therefore, following this scenario, as Panel A of Figure 4 depicts, the current cost of replacing the counterparty in a swap is given by the positive segment of the market value profile for the swap contract. Panel B of Figure 4 illustrates the opposite situation faced by a floating rate payer.⁷³

Although an essential element of credit risk, current replacement cost alone does not accurately depict the credit risk faced by parties who use swaps. The cost or total exposure to a financial institution of a counterparty default is the cost of replacing the cash flow specified by the contract. This total exposure is determined by the sum of the current replacement cost and a measure of the potential future credit exposure that may arise from further movements over the remaining life of the

⁷²Charles Smithson, "Measuring and Managing the Credit Risk", (draft), 17-5.

⁷³*Ibid.*, 17-6.

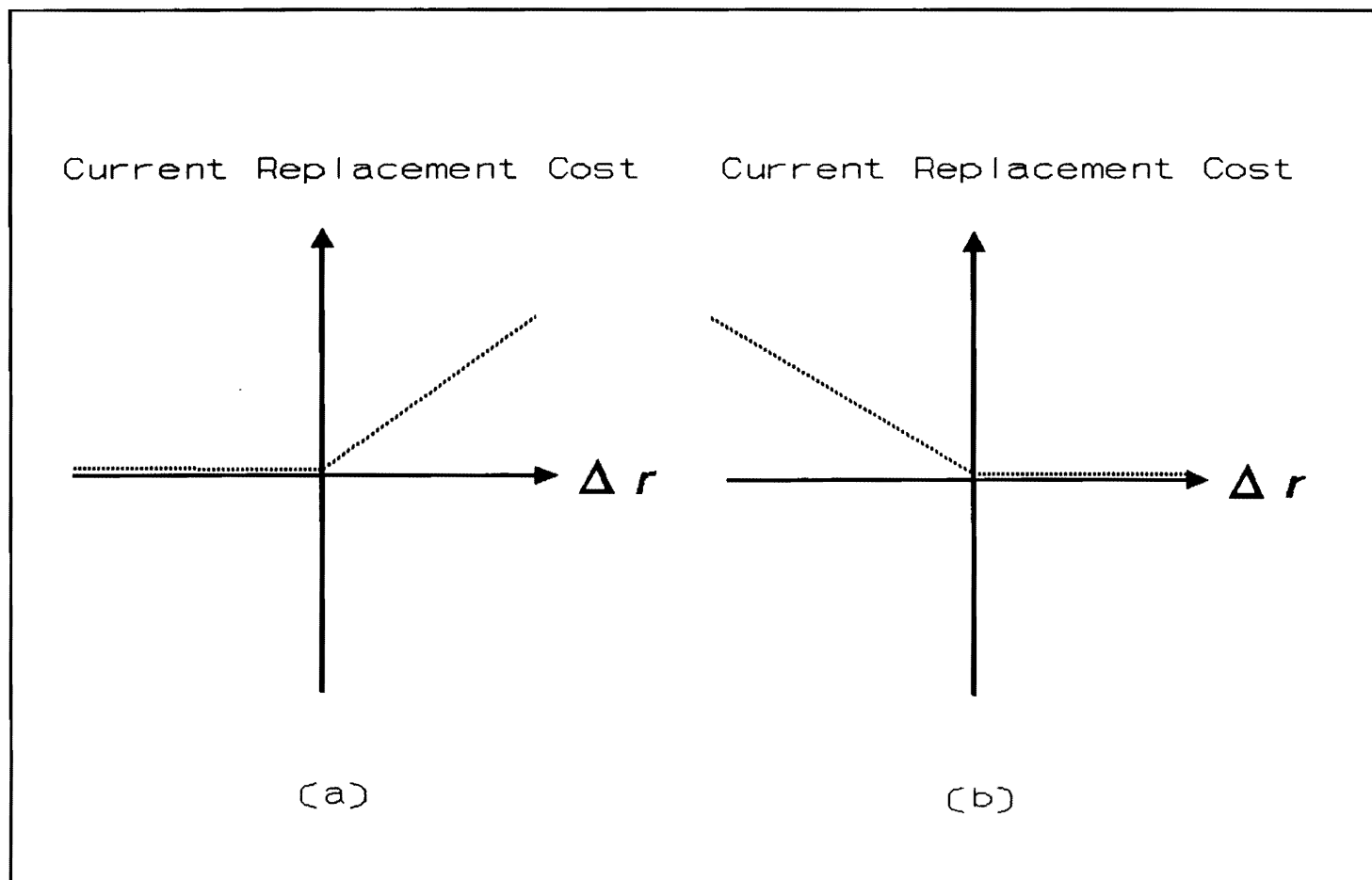


Figure 4. Current Replacement Cost

Source: Charles Smithson, "Measuring and Managing the Credit Risk", (draft), 17-7.

contract. The current replacement cost indicates only the cost of replacing the payment stream if the counterparty defaults today. The likelihood that the counterparty may default at some future date during the contract period instead of today, still needs to be accounted for.⁷⁴

In order to see the importance of accounting for the potential future credit exposure, consider the example of a fixed rate payer who's floating rate counterparty defaults. If interest rates fall following contract origination, $\Delta r_t < 0$, this situation can be illustrated by point A in Figure 5. In this case, the current replacement cost is zero. The question, however, is to determine the total exposure to loss today for default which may occur in the future. The total exposure is represented by point B in Figure 5. This is the current replacement cost plus the potential credit exposure.⁷⁵

The potential credit exposure illustrated in Figure 5 is dependent upon potential interest rate movements. In order for a default-induced loss to occur for the fixed rate payer, interest rates must rise at some future default date to exceed the rate which existed at contract origination. Further, the likelihood that this increase in rates will occur is dependent upon the volatility of interest rates, and the time remaining in the life of the contract.⁷⁶

⁷⁴*Ibid.*, 17-6.

⁷⁵*Ibid.*, 17-7.

⁷⁶*Ibid.*, 17-7.

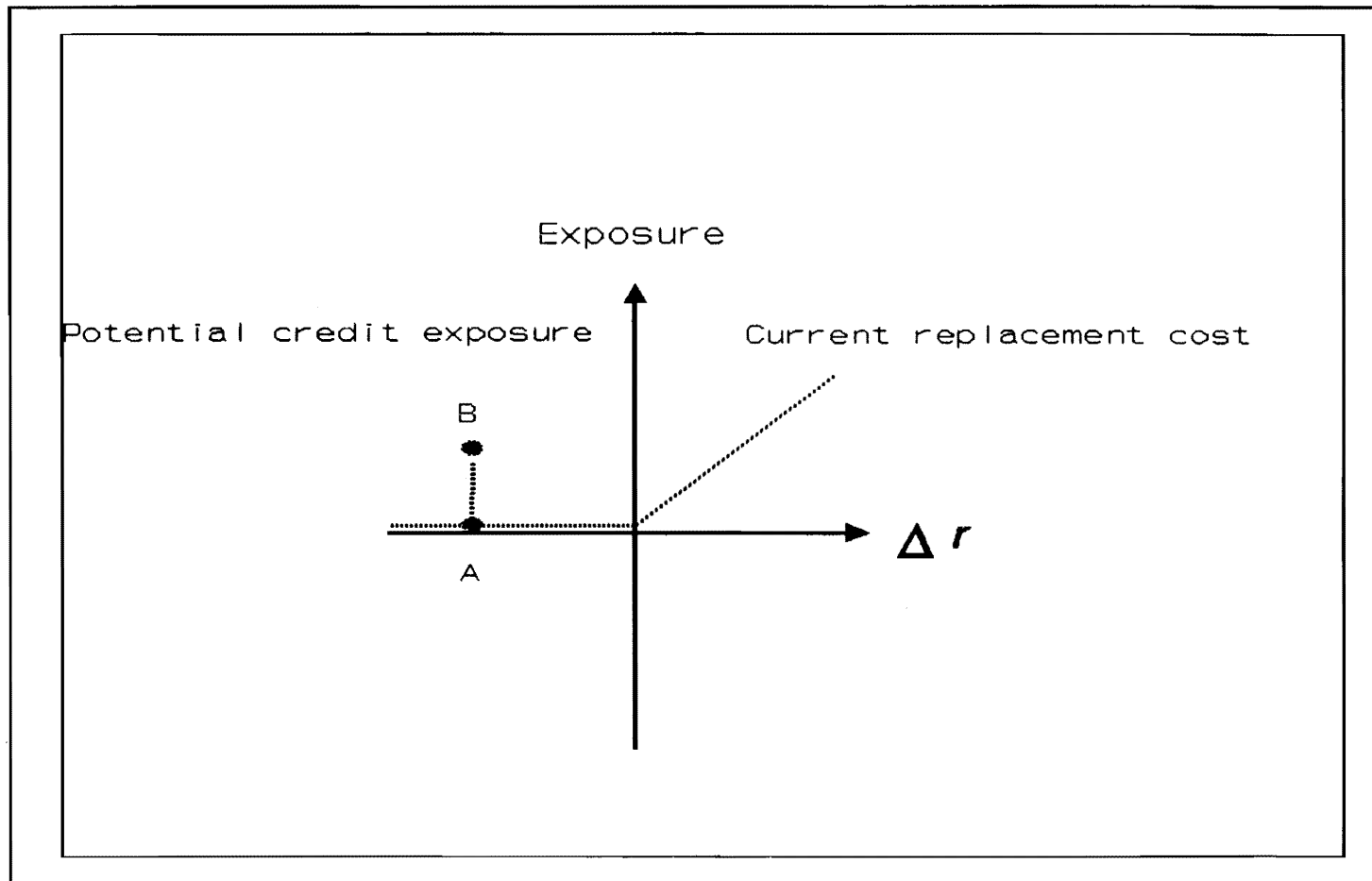


Figure 5. Potential Credit Exposure

Source: Charles Smithson, "Measuring and Managing the Credit Risk, (draft), 17-8.

The Amortization Effect

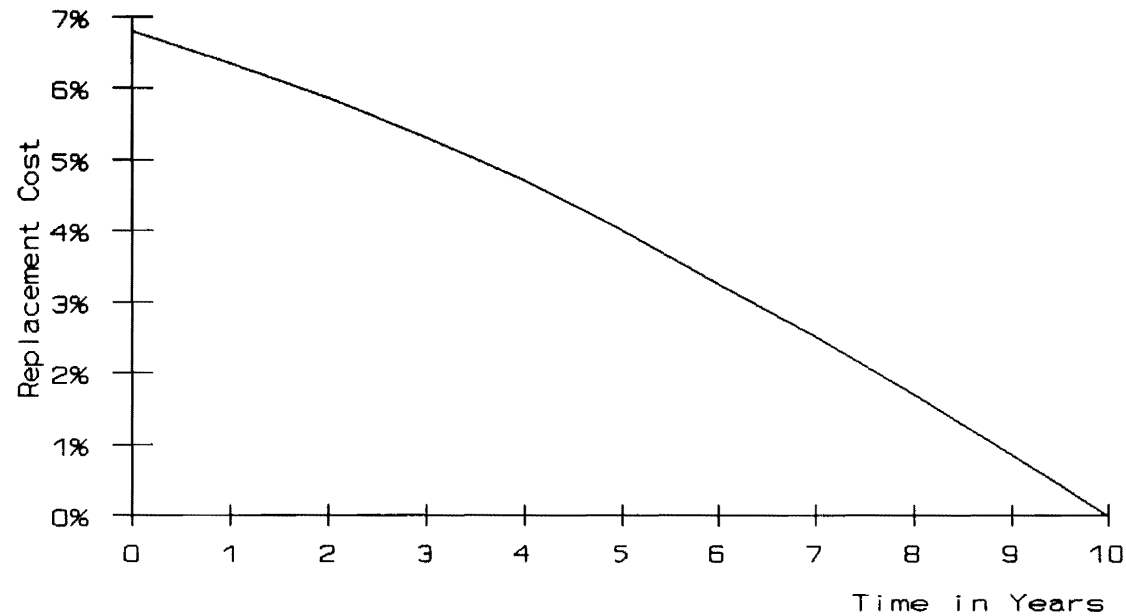
In general, the replacement cost, or credit exposure, of an interest rate swap will vary over the life of the contract as interest rates vary. In fact, even if interest rates are constant in the future, the replacement cost of the swap will adjust over time, due to the impact of the amortization effect.⁷⁷ This amortization effect reflects the relationship which exists between the credit exposure and the time remaining until maturity in the swap contract.⁷⁸ Specifically, the replacement cost declines as the time remaining to maturity declines. For example, at the initiation of the interest rate swap contract, both counterparties are exposed to potential losses on all payments. Over time, as settlement payments are made (the number of payments dates remaining falls), the potential exposure declines, until the amount at risk amortizes to zero at maturity.⁷⁹ This amortization effect is expressed graphically in Figure 6.

⁷⁷Ferron and Handjinicolaou, *op. cit.*, 23-5.

⁷⁸Smithson, *op. cit.*, 17-8.

⁷⁹Ferron and Handjinicolaou, *op. cit.*, 23-5.

The Amortization Effect



Volatility = 16.4%
Rate = 7%

Figure 6. The Amortization Effect

Source: Derived from "A Simplified Approach to Measuring Credit Risk of OTC Derivatives", DKB Financial Products, Inc., November 9, 1992.

The Diffusion Effect

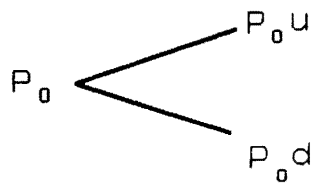
As mentioned, the credit risk exposure faced by a counterparty engaged in a swap agreement is also influenced by the degree to which the underlying price differs from the price which existed at contract origination. This effect is referred to as the diffusion effect. Diffusion deals with the amount the financial price can change over a number of periods, thus building on the concept of volatility, which deals with the amount the financial price can change in one period. Panel A of Figure 7 illustrates volatility using a lattice diagram. As the figure depicts, the financial price may move up from P_0 to P_0*u or down to P_0*d . Panel B of Figure 7 progresses to the illustration of diffusion. The figure assumes the same volatility level as Panel A, but extends the scenario to cover three periods. As the diagram reveals, the financial price resulting may be as high as P_0*u^3 or as low as P_0*d^3 .⁸⁰

This diffusion process generates a set of distributions for the underlying price, as shown in Panel C of Figure 7. The figure displays that over time, the diffusion process causes the dispersion of prices to increase.⁸¹ Further, as the number of periods increases, the dispersion increases, i.e., the probability of extreme values increases. In other words, the potential exposure becomes greater when more time

⁸⁰Smithson, *op. cit.*, 17-9.

⁸¹*Ibid.*, 17-9.

Panel A: Volatility



Panel B: Diffusion

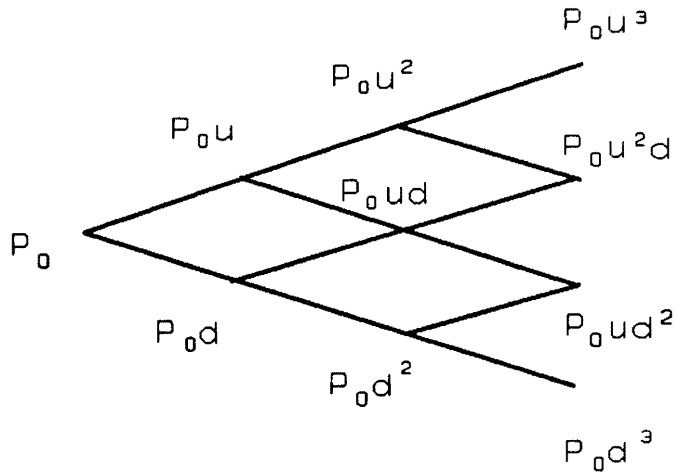


Figure 7. Volatility and Diffusion

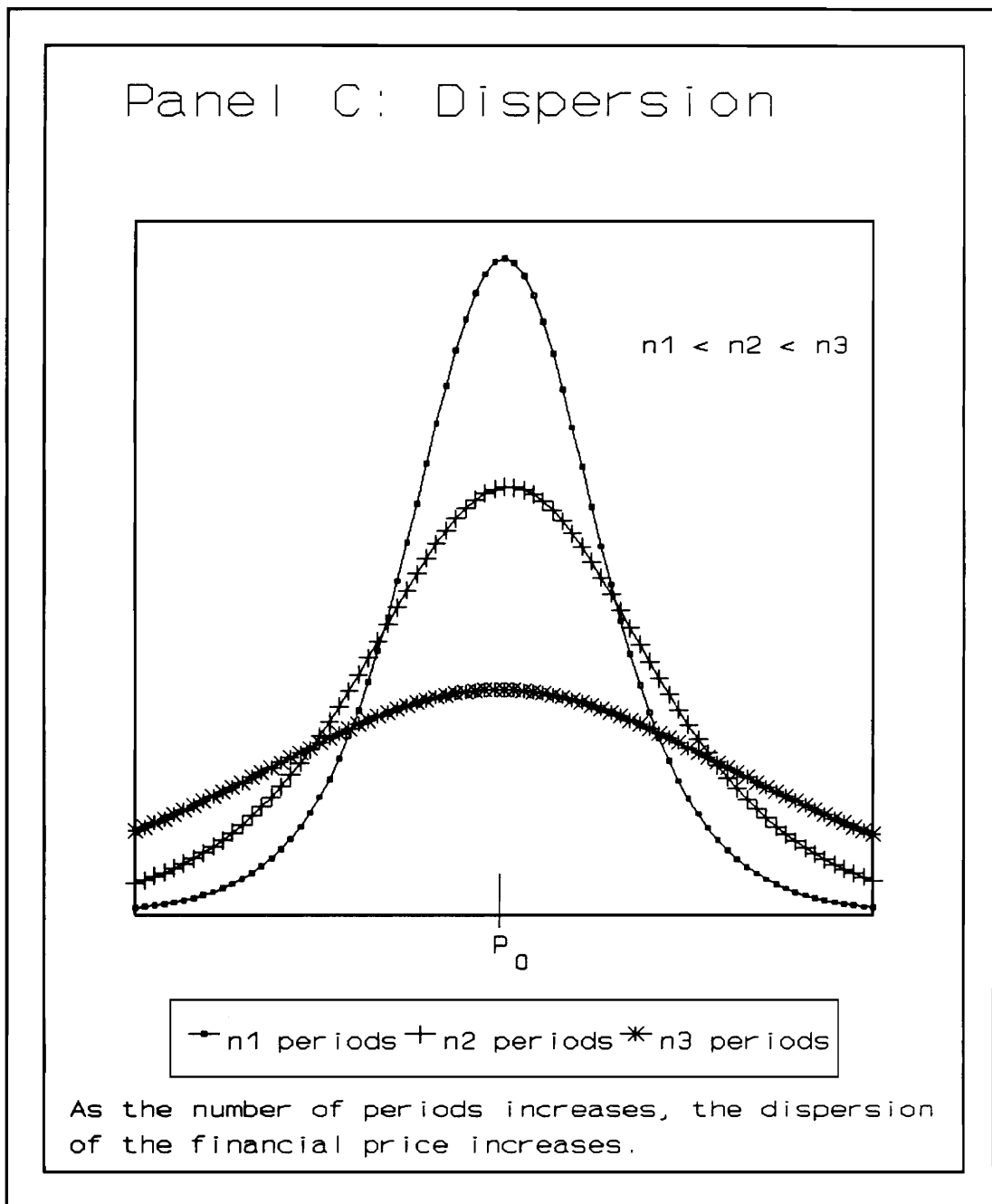


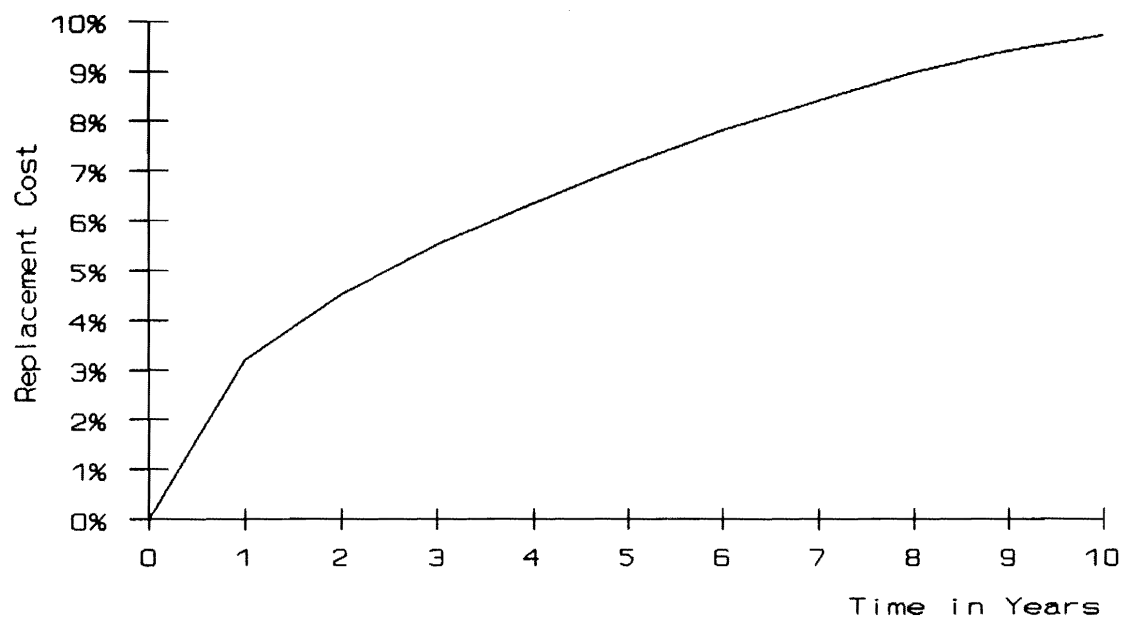
Figure 7. Volatility and Diffusion (continued)

Source: Charles Smithson, "Measuring and Managing the Credit Risk", (draft), 17-10.

remains until maturity.⁸² Figure 8 depicts the impact of the diffusion effect on a counterparty's credit exposure. The curve expresses the fact that at contract initiation the counterparty is exposed to price changes occurring over all of the settlement dates, such that the replacement cost increases. As the number of settlement dates diminishes and the contract approaches maturity, the level of the replacement cost continues to increase, but at a decreasing rate. This explains the upward sloping curve pattern depicted in the figure.

⁸²*Ibid.*, 17-10.

The Diffusion Effect



Volatility = 16.4%
Rate = 7%

Figure 8. The Diffusion Effect

Source: Derived from "A Simplified Approach to Measuring Credit Risk of OTC Derivatives", DKB Financial Products, Inc., November 9, 1992.

The Combination of the Amortization and Diffusion Effects

In order to capture the exposure profile for a swap contract, analysts need to consider the combination of both the amortization and diffusion effects, since both of these interact simultaneously to produce the credit risk profile. The interaction of these two effects creates a concave or "hump-backed" shape for the credit risk profile, which is an important and well recognized characteristic of swaps. Figure 9 provides a graphic illustration of the "hump-backed" curve which represents the behavior of swap credit risk over the life of the contract. As the figure reflects, at the commencement of the contract the expected replacement cost is zero, since the contract is written at current market rates.⁸³ The rising portion of the credit risk profile indicates that the diffusion effect is dominant early in the life of the swap contract. The falling section shows that at some point the diffusion effect is overwhelmed by the amortization effect. Eventually, upon maturity, the expected replacement cost reaches zero again.⁸⁴

This "hump-backed" profile occurs for standard interest rate swaps which have periodic payments and no final exchange of principal. Figure 10 provides an illustration of an exposure profile for a cross-currency swap. As the figure portrays, in the case of a currency swap which provides for a final exchange of principal at

⁸³Ferron and Handjinicolaou, *op. cit.*, 23-15.

⁸⁴Smithson, *op. cit.*, 17-11.

maturity, the amortization effect from the periodic settlements is swamped by the effect of diffusion on the principal reexchange.⁸⁵

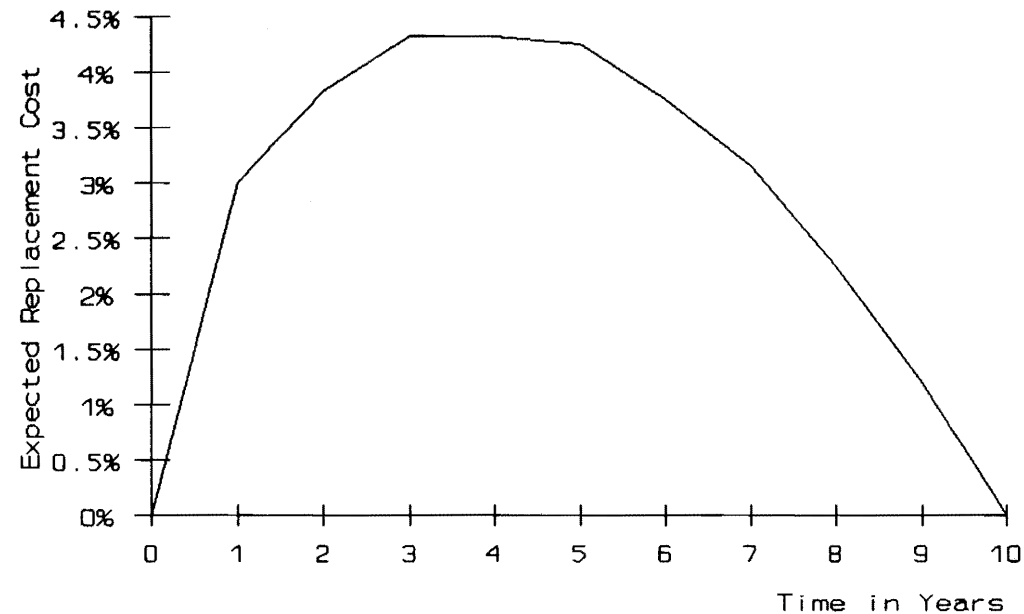
Although credit risk is discussed as "only" dealing with the replacement cost of an interest rate stream, the loss due to credit risk can be quite substantial. For example, consider a \$100,000,000 seven year swap contract which has five and a half years until maturity. Suppose a bank enters a swap at a floating rate of 11%, and the floating rate falls by 300 basis points, but the borrower continues to pay fixed rates at 11.5%. By marking the swap to market the bank can determine its exposure. Using a method like the equation below, the bank would determine its exposure as 15.3% or \$15,300,000.

$$V(t_i) = P \sum_{k=i+1}^n \Delta t (\bar{R} - \tilde{R}_i) (1 + \Delta t \tilde{R}_i)^{-(n-i)}$$

where $V(t_i)$ = the value of the swap at period i , P = the principal amount of the swap, Δt = the time period between settlements, $R \text{ BAR}$ = the fixed rate of interest, $R \text{ tilde}$ = the floating rate of interest, k = the number of remaining payments, and n = the total number of payments.

⁸⁵*Ibid.*, 17-11.

Risk Profile of Interest Rate Swaps

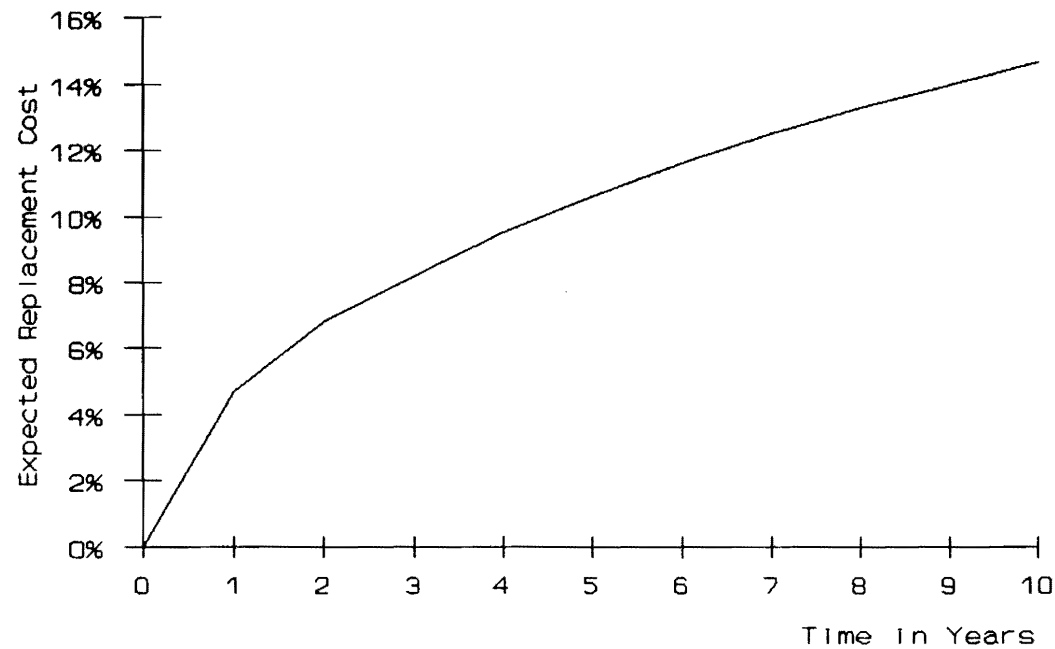


Volatility = 16.4%
Rate = 7%

Figure 9. Risk Profile of Interest Rate Swaps

Source: Derived from "A Simplified Approach to Measuring Credit Risk of OTC Derivatives", DKB Financial Products, Inc., November 9, 1992.

Risk Profile of Currency Swaps



Volatility = 12%

Figure 10. Risk Profile of Currency Swaps

Source: Derived from "A Simplified Approach to Measuring Credit Risk of OTC Derivatives", DKB Financial Products, Inc., November 9, 1992.

CHAPTER FIVE: MODELING CREDIT RISK

Alternative Approaches to Measuring Swap Credit Risk

Swap dealers and market analysts use varying methods to quantify credit risk. These methods include: the worst-case scenario approach, the historical experience approach, the options approach, and the simulated experience approach. Each of these methods has its own strengths and weaknesses.

The worst-case scenario approach developed during the early stages of credit risk analysis. This technique is based on making worst-case assumptions regarding both market and default risk. Analysts specify conservative high and low projections for the future level of replacement costs for an interest rate stream. These extreme positions are then used to calculate the maximum cost of replacing all the cash flows associated with a swap. This assumes that default occurs immediately following the initiation of the contract, which is an unrealistically conservative assumption. For instance, it is unlikely that rates will move to their worst-case level immediately following contract initiation. Further, it is unlikely that the counterparty will default immediately following contract initiation. Thus, generally the Worst-Case approach provides overly conservative estimates of swap credit risk.⁸⁶

An alternative approach which may be applied to measuring credit risk is the

⁸⁶Ferron and Handjinicolaou, *op. cit.*, 23-7.

historical experience method. This technique is based on observations of swap dealers' historical experience with default and replacement cost. Following this method, analysts attempt to capture the nature of swap replacement costs by relying on past data on interest rates to generate a series of replacement costs over time. Statistical measures are then implemented to summarize these series and provide an estimate of future market risk. Because swaps are relatively recent financial tools, however, we do not have much historical data upon which to base our model. Further, the period during which these products developed may not be representative of future environments for interest rates.⁸⁷ Therefore, this approach may not provide an accurate estimation of swap credit risk.

Another method used to determine credit risk is the options approach. This technique recognizes that a swap counterparty, in effect, holds a series of options to default on the swap contract. Implicit in this method is the assumption that the party engaging in the swap, in the instance of default, will not enjoy the benefit resulting from a negative mark-to-market value, but will be exposed to the loss from a positive mark-to-market value. Following this assumption, analysts can conclude that "the expected replacement cost of an interest rate swap is equivalent to the payouts which would be expected if the bank sold a series of call options on the level of interest rates".⁸⁸ Therefore, a default by a swap counterparty is put in terms of the

⁸⁷*Ibid.*, 23-7.

⁸⁸*Ibid.*, 23-8.

counterparty exercising the option it holds on the level of interest rates. This approach is "attractive" because there is a vast amount of past research in the area of options and options pricing. However, using a closed-form mathematical model may not capture valuable insights into the nature of replacement cost and default risk.⁸⁹

The development of the simulation approach to modeling swap credit risk corresponded with the increased understanding of this financial instrument by market participants. According to Mark Ferron and George Handjinicolaou, the simulation technique provides possibly the most powerful tool available for analyzing swap credit risk.⁹⁰ This more sophisticated method recognizes that replacement cost varies over time in response to the impact of both the amortization and diffusion effects, as well as potential changes in the market environment. By using the simulation technique to replicate reality, analysts are able to avoid the problem of insufficient historical data on swap replacement cost faced in the historical experience approach.

A variety of simulation techniques have evolved. One such technique is the application of Monte-Carlo simulation. Following this approach, analysts generate multiple economic scenarios using a statistical model. Then calculations are made to determine the replacement costs from the randomly generated interest rate scenarios. This allows the analysts to create a more complete picture of the characteristics of swap risk. After a sufficiently large number of trials are completed, then the

⁸⁹*Ibid.*, 23-8.

⁹⁰*Ibid.*, 23-3.

corresponding observations of replacement cost are analyzed statistically to provide an accurate estimate of credit risk. The Monte-Carlo simulation technique is widely accepted as a means to quantify swap credit risk. For example, the Basle committee on banking and supervision⁹¹, implemented the Monte-Carlo simulation technique in its examination of the credit exposure for interest rate swaps.⁹²

⁹¹The Basle committee is made up representatives of the central banks of Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, United Kingdom, the United States (the Group of Ten countries) plus Switzerland and Luxembourg.

⁹²Smithson, *op. cit.*, 17-18.

The Monte-Carlo Analysis of Ferron and Handjinicolaou

The Monte Carlo simulation technique applied by Mark Ferron and George Handjinicolaou in their analysis of swap credit risk provides a methodology for evaluating credit exposure. Ferron and Handjinicolaou use Monte Carlo simulation to create various interest rate scenarios. An interest rate path is generated sequentially over the life of the swap contract by starting from a given initial interest rate at contract origination and computing the interest rate level for the next settlement date. The value of next period's interest rate is determined as a function of the initial level plus or minus a change in interest rates. Following this method, the interest rate level two periods from contract origination is determined from the level of rates at the first period plus or minus a change. By repeating this process for each subsequent settlement period, a series of interest rates is generated for the entire life of the swap contract.⁹³

The nature of the interest rate scenarios generated using the above procedure is dependent upon the method used to determine the interest rate changes from period to period. The Monte Carlo model computes these changes by means of an assumed probability distribution function and a random number generator. Ferron and Handjinicolaou choose to implement a lognormal probability distribution in their analysis. The use of the lognormal distribution assumes that changes in the logarithm

⁹³Ferron and Handjinicolaou, *op. cit.*, 23-9.

of interest rates over a specified interval are normally distributed. A large body of academic research has been done which shows that the behavior of interest rates in the future is most appropriately modeled by means of a lognormal probability distribution. Therefore, the use of a lognormal distribution is common among market analysts.⁹⁴

The choice of the lognormal distribution function yields several subsequent attendant characteristics for the interest rate scenarios generated by the model. First, under the assumption of lognormality, there is no underlying trend in interest rates (i.e., there is an equal chance that interest rates will rise or fall over a period). Second, the change in interest rates between any two periods is independent of any previous change. Third, the scale of interest rate movements over a single period depends on the assumed level of the standard deviation of the lognormal distribution function. This parameter is an input in the model and is referred to as volatility. Finally, the scale of interest rate movements over the life of the swap consisting of several periods increases with the number of periods. This characteristic is the diffusion effect previously described in Chapter Four.⁹⁵

Each of the above properties has important implications. The first characteristic implies that in general there is no difference between the riskiness of a fixed-rate payer and that of a floating-rate payer. In other words, upon

⁹⁴*Ibid.*, 23-10.

⁹⁵*Ibid.*, 23-10.

commencement of the swap, both parties are equally likely to have interest rates move against them (although both cannot experience a loss simultaneously).⁹⁶

The choice of a volatility level sets a global constraint on the magnitude of possible rate movements in the future. A higher volatility level implies that there is a greater chance of interest rates reaching extreme values, where as a low volatility value implies that rates in the future will be less likely to deviate widely. Under the lognormal distribution assumption, the likelihood of a value lying within one standard deviation of the mean is roughly 84% and the likelihood of it lying within two standard deviations is about 98%. Based on these confidence intervals we can show that if the current level of interest rates is 9% and annual volatility is 20%, then we expect with approximately 84% confidence that interest rates at the end of one year will lie between 7.2% and 10.8% (one standard deviation equals $1.8\% = 20\% * 9\% = \text{volatility} * \text{mean}$). In the same manner, we can calculate with 98% certainty that rates at the end of one year will lie within approximately two standard deviations, or between 5.4% and 12.6% (two standard deviations equal $3.6\% = 2 * 20\% * 9\%$).⁹⁷

The assumption of lognormality also has significant implications for the distribution of possible interest rates prevailing in future time periods. Assuming lognormality, the degree of diffusion is a function of the square root of the time elapsed since contract initiation. This property is referred to as the "square root

⁹⁶*Ibid.*, 23-10.

⁹⁷*Ibid.*, 23-11.

rule". Applying this rule to the previous example, there is an 84% chance that interest rates lie between approximately 6.5% and 11.5% (one standard deviation for two years from contract origination is equal to $2.5\% = 20\% * 9\% * \sqrt{2}$). Also, for the 98% certainty interval, interest rates will lie between approximately 4% and 14% (two standard deviations for two years equals $2 * 2.5\% = 5\%$). Following this method, upon maturity of a ten-year swap contract, one would expect with 98% confidence that interest rates will lie between 2.5% and 30%. Thus, given an assumed volatility combined with the square root rule, a range of interest rates for each point in the future can be calculated. Table 1 provides the results from Ferron and Handjinicolaou's application of this procedure.⁹⁸

**Table 1. Range for Interest Rate Levels
Implied by Various Volatility Assumptions
Ferron and Handjinicolaou Monte Carlo Simulation Results**

RESULTS FROM FERRON AND HANDJINICOLAOU ANALYSIS		
RANGE FOR INTEREST RATE LEVELS UNDER VARYING VOLATILITY ASSUMPTIONS (10-year, 9% swap)		
Volatility Assumption	Range Within which Rates Are Expected to Lie with a Probability of 95%	
15%	4%	25%
20%	3%	30%
25%	1%	50%

⁹⁸*Ibid.*, 23-11.

Another assumption made by Ferron and Handjinicolaou in their model is that market risk and default risk can be modeled independently. This assumption implies that default occurs for reasons which are not directly related to the current value of the swap contract. Therefore, the model does not provide for the interaction between the two risks, even though this interaction seems logical. For example, one might expect there to be a higher chance of default by a party when interest rates move against the party's initial position. Ferron and Handjinicolaou justify their assumption by pointing to the examination of the joint behavior between market and default risk which has not been conclusive. In fact, studies have shown that factors which tend to increase market risk may either increase or decrease the risk of default. A benefit to imposing this assumption is that the model is applicable for counterparties of varying credit quality.⁹⁹

Also, Ferron and Handjinicolaou make a worst-case assumption that default will occur with certainty at some point over the life of the swap contract. Under this worst-case assumption, no consideration is given to the fact that a majority of swaps are negotiated between counterparties with a credit rating of "A" or better, where the probability of default is very small. Further, the model assumes that default will occur more or less at random over the life of the contract. In other words, the event of default is uniformly distributed over the life of the contract and there is an equal likelihood that default will occur at any point during the life of the swap. Under this

⁹⁹*Ibid.*, 23-6.

assumption, Ferron and Handjinicolaou quantify swap credit risk as equal to the average of the expected replacement costs for each settlement date. This assumption can be modified to allow for a probability of default of less than one over the life of the swap, by weighting the individual payment period replacement costs by the probability of default occurring during that settlement period. Thus, the process of accounting for a nonuniform probability of default simply implements a scaling factor. The above assumptions allow for the valuation of credit risk which is independent of the credit quality of the counterparty and provide a model which "can be universally applied to any swap".¹⁰⁰

In order to achieve a measure of swap credit risk, Ferron and Handjinicolaou apply the following method to compute average expected replacement cost. The Monte Carlo model is implemented to generate a series of interest rates for the life of a swap contract. Then, based on each interest rate scenario, the present discounted value of the expected loss from default is computed at each settlement date for the swap contract. This procedure is then repeated for a large number of interest rate scenarios in order to generate a probability distribution for the replacement cost at each point in time during the transaction. The expected value of the replacement cost is then calculated as the mean of the distribution.¹⁰¹

In their Monte Carlo simulation, Ferron and Handjinicolaou assume a volatility

¹⁰⁰*Ibid.*, 23-12.

¹⁰¹*Ibid.*, 23-13.

level of 20%. They arrive at this value based upon the findings of a joint study done by the Federal Reserve and the Bank of England. The study tracked the volatility of the 5-year Treasury rate over 5-year moving windows from 1975 to 1987. According to the findings, the average volatility was 15.2% and the maximum and minimum volatility values were 18.2% and 7%. Table 2 contains the average historical volatility levels for the years 1981 to 1986 for varying maturities. Since changes in swap rates seem to follow changes in Treasury yields fairly closely, Ferron and Handjinicolaou choose to use this historical data on Treasury yields as a proxy for swap rates. Ferron and Handjinicolaou state that their assumed value of 20% volatility appears to be sufficiently conservative given the range of rates shown in Table 2.¹⁰²

**Table 2. Average Annual Volatilities
U.S. Treasury Bond Yields: 1981 to 1986**

AVERAGE ANNUAL VOLATILITIES FOR U.S. TREASURY BOND YIELDS UNDER VARYING MATURITIES	
Maturity	Average Historical Volatility
1 year	19.5%
3 years	16.6%
5 years	16.0%
7 years	14.8%
10 years	14.2%

¹⁰²*Ibid.*, 23-14.

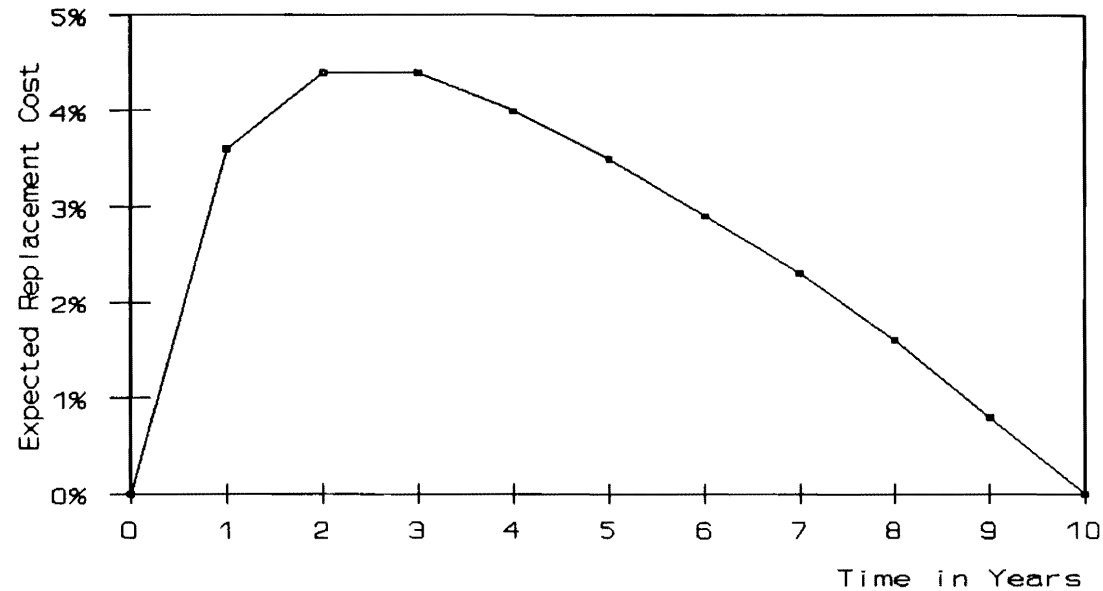
Ferron and Handjinicolaou report the results from a Monte Carlo simulation using their model to predict credit risk for a par interest rate swap (a swap in which the fixed rate coupon is equal to the current market swap rate). The swap analyzed is given the following characteristics: a term of 10 years, both a floating rate and a fixed rate of 9%, and a volatility of 20%. The results from this example are provided in Table 3 on the following page. Further, a plot of these replacement cost values is provided in Figure 11.

Table 3. Expected Replacement Cost
Ferron and Handjinicolaou Monte Carlo Simulation Results

RESULTS FROM FERRON AND HANDJINICOLAOU ANALYSIS EXPECTED REPLACEMENT COST FOR A SWAP (10-year, 9% swap, volatility = 20%)	
Year (k)	Expected Replacement Cost (Assuming Default Occurs at Time k)
0	0.0
1	3.6
2	4.4
3	4.4
4	4.0
5	3.5
6	2.9
7	2.3
8	1.6
9	0.8
10	0.0
Average Expected Replacement Cost	2.8

Source: Mark Ferron and George Handjinicolaou, "Understanding Swap Credit Risk: The Simulation Approach, 23-17.

Expected Replacement Cost



Volatility = 20%
Rate = 9%

Figure 11. Expected Replacement Cost

Source: Mark Ferron and George Handjinicolaou, "Understanding Swap Credit Risk: The Simulation Approach," 1990, 23-17.

CHAPTER SIX: METHODOLOGY

To evaluate the significance of the volatility input in calculating swap credit risk, I compared the results of applying various volatility levels using a Monte Carlo simulation model. The Monte Carlo model implemented follows the assumptions given in Ferron and Handjinicolaou's analysis, as described in the previous chapter. Ferron and Handjinicolaou, however, do not provide the equations which their model used to generate the interest rate scenarios or value the swap. Therefore, the first step in my analysis was to use the thesis model to replicate Ferron and Handjinicolaou's results. The next step was to use the derived thesis model to compute the results using various volatility input levels. Finally, I interpreted these results in order to evaluate the significance of the volatility input in determining swap credit risk. The following discussion provides an explanation of the simulation model.

Interest rate scenarios are determined by:

$$\Delta r = r \sigma \Delta z$$

In this equation, the change in the interest rate level between periods, Δr , is equal to the interest rate, r , multiplied by the volatility level, σ , times a random variable, Δz . The inclusion of the Δz term is significant for two reasons. First, the value of Δz follows a Markov process, where the Δz values for two different time periods are

statistically independent. Thus, the Δz variable captures the assumption that interest rates exhibit the property of the weak-form of market efficiency. Under this concept of weak-form market efficiency, there is no correlation between the changes in interest rates between periods because at the beginning of each period the interest rate reflects all information available. Second, Δz is related to a small increment in time, Δt , by the following equation:

$$\Delta z = \epsilon \sqrt{\Delta t} ,$$

where ϵ is a random drawing from a standardized normal distribution.¹⁰³

This allows for the first equation to be represented in a simplified form by:

$$\Delta r = r \sigma \epsilon \sqrt{\Delta t} .$$

Using Ito's lemma¹⁰⁴, the following equation may be derived for the derivative of the log of r :

$$d \ln r = -\frac{\sigma^2}{2} \Delta t + \sigma \epsilon \sqrt{\Delta t} .$$

Thus, the log of the interest rate in period $t+1$ is distributed normally with a mean represented by $\ln r_t - \sigma^2/2 \Delta t$, and standard deviation of $\sigma \sqrt{\Delta t}$. This is represented

¹⁰³Maria Tsu, "Special Study: Options and Futures Pricing Valuation of Path-Dependent Options", July 1993, 4.

¹⁰⁴Ito's lemma is the equation developed by a mathematician, Ito, in 1951, which describes a function of a stochastic variable which follows an Ito process. (For further discussion of Ito's lemma, refer to John Hull, *Options Futures and Other Derivative Securities*, 1989.)

mathematically by the following expression:

$$\ln r_{t+1} \sim \phi \left[\ln r_t - \frac{\sigma^2}{2} \Delta t, \sigma \sqrt{\Delta t} \right]$$

The equation modeling the interest rate scenarios can therefore be expressed using the following equation:

$$r_{t+1} = r_t e^{\left[-\frac{\sigma^2}{2} \Delta t + \sigma \varepsilon \sqrt{\Delta t} \right]}$$

Once the interest rate scenarios are determined, the next step is to calculate the replacement cost for the swap contracts. This replacement cost is based on the mark-to-market value of the swap for each settlement date, discounted to the present. The simulation model used to value the replacement cost assumes a flat term structure (or yield curve). In other words, the value of the future payments lost if default occurs is computed assuming that the current floating interest rate at the time of default stays constant for the remaining payment periods. The model computes the value of the swap at the time of assumed default based on the following equation:

$$V(t_i) = P \sum_{k=i+1}^n \Delta t (\bar{R} - \tilde{R}_i) (1 + \Delta t \bar{R})^{-k},$$

where $V(t_i)$ is the value of the swap at period i , P is the principal amount, Δt is the time period between settlements, $\bar{R}$ is the fixed rate of interest, $\tilde{R}$ is the floating rate of interest, and k is the number of remaining payments.

Further, the model computes credit risk from the perspective of the fixed rate

receiver (the floating rate payer). It is assumed that the credit risk will be greater than zero only when the swap has a positive value for the nondefaulting party (the floating rate payer). Therefore, the replacement cost of the swap, $RC(t_i)$, is calculated by:

$$RC(t_i) = \max[0, V(t_i)]$$

Table 4 provides the results computed for the 120th iteration, assuming a principal amount of \$100.00, a volatility of 20%, an initial floating rate of 9%, a fixed rate of 9%, a contract life of ten years, and annual payments. These results demonstrate the application of the above equation in determining the replacement cost value, given an interest rate scenario.

Table 4. Replacement Cost for Iteration 120

RESULTS FROM MONTE CARLO SIMULATION ANALYSIS		
REPLACEMENT COST FOR A SWAP		
(10-year, 9% swap, volatility = 20%)		
Iteration = 120		
Year (k)	Floating Rate	Replacement Cost
0	0.09	0.00
1	0.106000841	0.00
2	0.0883702636	0.759210408
3	0.0857082009	1.66794491
4	0.0950392485	0.00
5	0.0712894201	4.73006535
6	0.0689533949	4.06567287
7	0.0598505810	4.17482662
8	0.0845659971	0.479735434
9	0.0876395702	0.0997067094
10	0.0734929442	0.00

These results are consistent with the equation for calculating the replacement cost. For example, the floating rate rises above 9% during years one and four, and subsequently the resulting replacement costs for these swaps are given as zero. Further, by substituting the assumed parameters and the determined floating interest rate into the equation, one can compute the replacement cost of the swap, and

compare it to the computer results. For example, in order to calculate the replacement value if default occurs in year two, the equation would look like:

$$\begin{aligned}
 V(t_i) &= (100) \sum_{k=3}^{10} (1)(.09 - .0883702636)[1 + (1)(.09)]^{-k} \\
 &= (100) \sum_{k=3}^{10} (.0016298)(1.09)^{-k} \\
 &= (100)(.0075925) = (.75925)
 \end{aligned}$$

This calculated replacement cost of .75925 is very close to the computer result of .759210408. The slight difference in these values of approximately .0000396 can probably be attributed to the reduced number of digits and therefore the rounding effect.

This procedure is then repeated for 10,000 interest rate scenarios. The expected replacement cost corresponding to each settlement date is then obtained by calculating the mean of the 10,000 individual computed replacement cost values. Thus, a probability distribution is generated for the average replacement cost at each settlement date over the life of the swap contract. Next, the average expected replacement cost for the swap is computed as the mean of this distribution. The step of simply computing an average is acceptable under the model's assumption of a uniform probability of default. Therefore, the average replacement cost, $ARC(t_i)$, is calculated by taking the mean of the replacement costs after completing 10,000 runs. This is represented by the following equation:

$$ARC(t_i) = (1+j) \sum_{j=1}^{10,000} [RC(t_i)] = \bar{RC}$$

The Monte Carlo simulation model also computes the standard deviation for the estimated replacement cost values for each settlement period. The standard deviation is computed using the following equation:

$$Stdev = \left\{ \left[\sum_{j=1}^{10,000} RC_i^2 - 10,000 \bar{RC} \right] + (10,000 - 1) \right\}^{\frac{1}{2}} + 100$$

This information enhances the ability to compare and analyze the results. This analysis is provided in the following chapter.

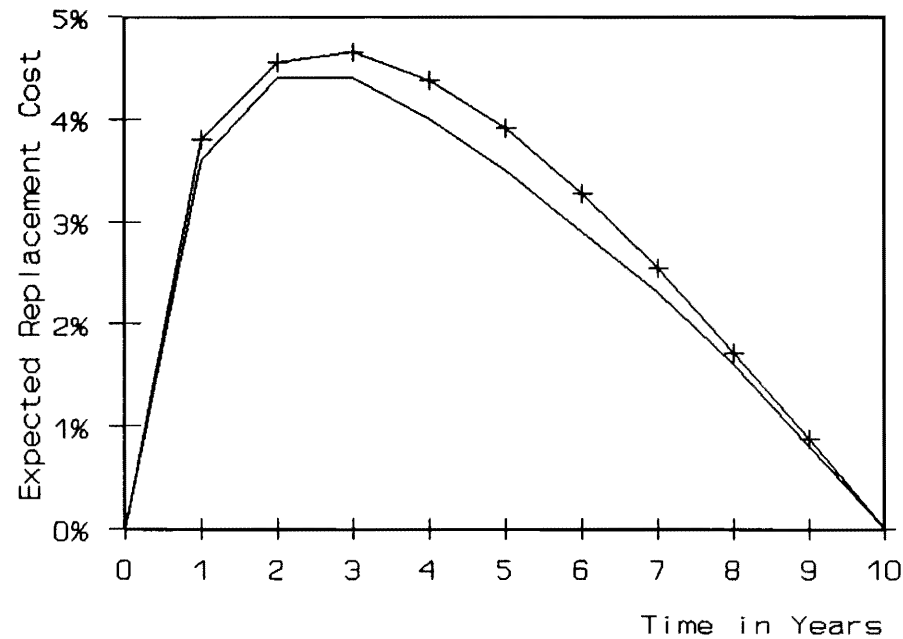
CHAPTER SEVEN: INTERPRETATION OF RESULTS

To replicate the results of Mark Ferron and George Handjinicolaou’s analysis of swap credit risk, I used the Monte Carlo simulation model described in the methodology section of this paper. I was able to obtain results very close to those of Ferron and Handjinicolaou, by computing the credit risk using semiannual time steps for the interest rate fluctuations. A comparison of my results to those of Ferron and Handjinicolaou is provided in Table 5 below and Figure 12 on the following page.

Table 5. Comparison of Results

COMPARISON OF RESULTS		
FERRON & HANDJINICOLAOU v. THESIS MODEL		
REPLACEMENT COST FOR A SWAP		
(10-year, 9% swap, volatility = 20%)		
Year (k)	Expected Replacement Cost	
	F & H	THESIS MODEL
0	0.0	0.00
1	3.6	3.80
2	4.4	4.55
3	4.4	4.65
4	4.0	4.38
5	3.5	3.91
6	2.9	3.27
7	2.3	2.54
8	1.6	1.72
9	0.8	0.87
10	0.0	0.00
Average Expected Replacement Cost	2.8	2.97

Expected Replacement Cost Ferron & Handjinicolaou v. Thesis Model



Series 1 = F & H
Series 2 = Thesis Model

Figure 12. Comparison of Results: Ferron & Handjinicolaou v. Thesis Model

The results presented in Table 5 are based on the assumption of a principal amount of \$100.00. In order to express these values in terms of a more realistic, actual swap contract, the average expected replacement cost values should be multiplied by 1,000,000. (This is based on a more practical principal amount of \$100,000,000.) Following this procedure, the credit risk for a firm entering a \$100,000,000 ten year swap contract at a rate of 9% is 2.8% or \$2,800,000, according to the results of Ferron and Handjinicolaou. This is close to the thesis model's results of 2.97% or \$2,970,000, given the properties of Monte Carlo simulation, such as the random number generator.

The similarity between these results can also be seen graphically in Figure 12. As the figure displays, the thesis model's results conform to the expected "hump-backed" shape for interest rate swaps. The peak of the thesis model's curve is slightly higher and further to the right than the curve representing the results of Ferron and Handjinicolaou's analysis.

To test the model's sensitivity to interest rate volatility, I computed results using five different levels for the volatility input. The levels chosen for volatility were 15% (the market-implied level analyzed by Ferron and Handjinicolaou in 1990), 16% (an historical level for volatility), 19% (chosen to corroborate the significance of only a 1% change in volatility), 20% (the conservative level chosen by Ferron and

Handjinicolaou), and 25% (an extremely conservative level).¹⁰⁵ The results of this comparison are presented in Table 6 on the following page.

The standard deviations, provided in the parentheses below the expected replacement cost values in Table 6, contribute to determining confidence intervals for the replacement cost of the swap contract at each settlement date. Based on the model's assumption that the log of the interest rate in period $t+1$ is normally distributed, a firm can conclude with 95% confidence that the true replacement cost will lie within the interval of the expected replacement cost plus or minus two standard deviations. Applying this procedure to the thesis model's results, assuming default occurs in year seven and volatility equals 15%, a firm can conclude with 95% confidence that its true replacement cost for the swap contract will lie between 1.864% and 1.956% [$1.91 \pm 2(0.0230)$]. This information is beneficial for firms, because they can then act to protect themselves from a loss of this size, possibly by maintaining capital reserves, or requiring the establishment of margins in the initial swap agreement.

¹⁰⁵In comparison, results were computed using current estimates of interest rates and volatility. A lower interest rate of 7% and an annual volatility of 24% (based on 6 month LIBOR) were assumed. The average replacement cost for a ten-year par interest rate swap, using these more representative values, was determined to be 3.17% of notional principal.

Table 6. Comparison of Results Using Alternate Volatility Inputs

COMPARISON OF RESULTS USING VARIOUS VOLATILITY LEVELS					
REPLACEMENT COST FOR A SWAP					
(10-year, 9% swap)					
Year (k)	Expected Replacement Cost Under Various Volatility Levels				
	15 %	16 %	19 %	20 %	25 %
0	0.00 (0.0000)	0.00 (0.0000)	0.00 (0.0000)	0.00 (0.0000)	0.00 (0.0000)
1	2.85 (0.0393)	3.04 (0.0417)	3.61 (0.0487)	3.80 (0.0510)	4.75 (0.0620)
2	3.41 (0.0451)	3.65 (0.0477)	4.33 (0.0553)	4.56 (0.0578)	5.70 (0.0694)
3	3.50 (0.0451)	3.73 (0.0476)	4.42 (0.0549)	4.66 (0.0573)	5.81 (0.0682)
4	3.29 (0.0417)	3.51 (0.0440)	4.17 (0.0505)	4.38 (0.0526)	5.47 (0.0622)
5	2.95 (0.0366)	3.14 (0.0386)	3.72 (0.0441)	3.92 (0.0459)	4.88 (0.0538)
6	2.46 (0.0301)	2.62 (0.0317)	3.11 (0.0361)	3.27 (0.0374)	4.07 (0.0437)
7	1.91 (0.0230)	2.04 (0.0241)	2.42 (0.0274)	2.54 (0.0284)	3.16 (0.0330)
8	1.30 (0.0155)	1.39 (0.0162)	1.64 (0.0184)	1.72 (0.0190)	2.14 (0.0220)
9	0.66 (0.0772)	0.71 (0.0810)	0.84 (0.0914)	0.88 (0.0947)	1.09 (0.0110)
10	0.00 (0.0000)	0.00 (0.0000)	0.00 (0.0000)	0.00 (0.0000)	0.00 (0.0000)
Average Expected Replacement Cost	2.23	2.38	2.83	2.97	3.71
Standard deviations are represented in parentheses.					

By comparing the results obtained from using different volatility inputs, one can measure the impact of a change in volatility on the average expected replacement cost. In other words, one can determine the effect of implementing a volatility level of 20%, when in fact the true market volatility level is 15%. Using the thesis model's results, one can calculate that if volatility is assumed to be 20%, then a ten year swap at a 9% par interest rate will have a credit risk equivalent to 2.97% of the notional principal amount of the swap. If volatility is instead 15%, then the credit risk for this swap decreases to 2.23% of the notional principal amount (a decrease of 24.9%). To take another example, if volatility increases from 20% to 25%, this will translate into an increase from 2.97% to 3.71% of the principal amount of the swap (an increase of 24.9%). Therefore, these results show that as volatility changes by 25% (from 20% to 15%, or from 20% to 25%), the corresponding replacement cost increases proportionally by approximately 25% (24.9%). Thus, the relationship between the volatility input and its corresponding replacement cost is approximately proportional.

Further, to see the significance of only a 1% change in volatility, one can compare the replacement costs under the volatility assumptions of 15% versus 16% and 19% versus 20%. In the case of a 1% increase in volatility from 15% to 16%, this corresponds with an increase in replacement cost from 2.23% to 2.38% of the principal amount (an increase of 6.7%). Assuming a notional principal amount of \$100,000,000, this difference in replacement cost is equivalent to \$150,000. In the

instance of an increase in volatility from 19% to 20%, this corresponds with an increase in replacement cost from 2.83% to 2.97% of the notional principal amount of the swap (an increase of 4.9%). This difference expressed in dollar terms is equal to \$140,000. Therefore, in terms of the swap contract modeled in this analysis, a 1% difference in the interest rate volatility can lead to approximately a \$150,000 difference in the expected replacement cost. Thus, these results confirm that the estimated swap credit risk is highly sensitive to the interest rate volatility input in a Monte Carlo simulation model.

Also, to determine if altering the maturity of the swap affects the sensitivity of the results to different volatilities, I compared results for three different maturities (one year, five years, and 10 years) using three various volatilities (15%, 20%, and 25%). The results from this comparison are provided in Table 7. By comparing the

Table 7. Impact of Varying Maturities on Volatility Sensitivity

COMPARISON OF RESULTS USING VARIOUS MATURITIES			
REPLACEMENT COST FOR A SWAP			
(10-year, 9% swap, semi-annual settlement)			
MATURITY	VOLATILITY		
	15%	20%	25%
1 year	.09	.12	.15
5 year	1.11	1.48	1.84
10 years	2.31	3.07	3.82

average expected replacement cost values corresponding to the various maturities and volatility levels, one can conclude that a swap's maturity does not affect the sensitivity of the results to the volatility input. For example, an increase in volatility from 20% to 25% (an increase of 25%), corresponds with an increase in replacement cost from .12% of notional principal outstanding to .15% for a 1-year maturity; 1.48% to 1.84% for a 5-year maturity; and 3.07% to 3.82% for a 10-year maturity (an increase of approximately 25% for all three examples). Therefore, the approximately proportional relationship between the volatility input and its corresponding replacement cost is not affected by a change in the maturity of the swap.

CHAPTER EIGHT: SUMMARY AND CONCLUSION

While swaps are used as tools to hedge market risk exposure, there is unavoidable credit risk inherent in the use of swaps. Questions regarding the significance and dangers of this credit risk continue to foster debate among economists and financial analysts. Further, the recognition of the credit risk associated with swaps has led to an effort by both regulators and market participants to develop an appropriate method for measuring swap credit risk.

Determining the level of credit exposure is vitally important to businesses which engage in swap contracts. Inaccurate estimates of credit exposure give rise to many problems. Since capital is a scarce resource, if financial managers misjudge the risk, then capital may be allocated inefficiently. In addition, because federal regulation requires businesses to meet certain capital requirements, if the credit risk is measured incorrectly, then businesses will hold inappropriate levels of capital in reserve. For example, if a firm's credit risk is calculated to be higher than its actual exposure level, then the firm will tie up capital to meet regulatory requirements instead of using it to invest in the firm or ultimately the economy. Therefore, an accurate measurement of credit risk is highly desirable.

As technology has progressed, so have the methods for estimating swap credit exposure. Today, the method preferred by most market analysts is the Monte Carlo simulation technique. Using a Monte Carlo simulation model, this thesis

demonstrated that swap credit risk is highly sensitive to interest rate volatility. The results of comparing average expected replacement costs from various volatility levels demonstrated that swap credit risk was approximately proportional to volatility. Further, this thesis determined that the sensitivity of the replacement cost to interest rate volatility is not affected by the maturity of the swap contract.

The results obtained in this thesis using Monte Carlo simulation to measure credit risk for interest rate swaps are highly dependent upon the volatility parameter specified in the simulation process. Therefore, since obtaining an accurate measure of credit risk is a goal for market regulators and users, significant effort should be invested by firms into determining an appropriate measure of interest rate volatility.

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APPENDIX

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C  MONTE CARLO PROGRAM FOR CALCULATING CREDIT RISK OF SWAPS
C  BY MARIA E. TSU
C  LAST EDIT AUGUST 10, 1993
C
EXTERNAL RSTART
C
C  DIMENSION ARRAYS AND ASSIGN PARAMETER VALUES
C
DIMENSION RFLOAT(0:50),RCOST(50),CRISK(50),SQRSK(50)
DIMENSION STDEV(50)
C
DATA ALPHA,A1,A2,A3/.33267,.4361836,-.1201676,.9372980/
DATA PI/3.1415927/
PRINT *, 'THIS PROGRAM CALCULATES THE CREDIT RISK FOR THE FIXED'
PRINT *, 'RATE RECEIVER. THE FIXED RATE IS USED FOR DISCOUNTING'
PRINT *, ' '
PRINT *, 'ENTER PRINCIPAL, VOLATILITY, FIXED RATE, FLOATING RATE,
$TERM OF SWAP (YRS), AND'
PRINT *, 'THE NUMBER OF PAYMENTS'
READ *, P, SIGMA, RFIX, RINIT, TIME, N
PRINT *, 'ENTER THE TIME STEPS PER YEAR AND THE ITERATION'
PRINT *, 'FOR WHICH OUTPUT IS DESIRED'
READ *, NSTPSYR, NPRINT
TPERIOD = TIME/N
NSTEPS = TPERIOD*NSTPSYR
DELT = 1.0/NSTPSYR
NITER=10000
A= -0.5*(SIGMA*SIGMA)*DELT
B=SIGMA*(DELT**.5)
PRINT *, ' '
PRINT *, 'PRINCIPAL = ',P, 'SIGMA = ',SIGMA,'RFIX = ',RFIX
PRINT *, 'INITIAL RFLOAT = ', RINIT, 'TERM (YRS) = ',TIME
C
C  INITIALIZE RANDOM NUMBER GENERATOR BY CALLING
C  SUBROUTINE RSTART(I,J,K,L)
C
CALL RSTART(12,34,56,78)
C
C  GLOBAL LOOP TO CALCULATE AVERAGE EXPOSURE PROFILE
C
RFLOAT(0)=RINIT
DO 20 KK=1,N
    CRISK(KK)=0.0
20  SQRSK(KK)=0.0
DO 1000 II=1,NITER

```

```

C
C  LOOP FOR ONE MONTE CARLO SIMULATION RUN
C
C
C  DO 800 KK = 1, N
C
C  RWEEK=RFLOAT(KK-1)
C
C  DO 650 KWEEK = 1, NSTEPS
C
C  EPSI=UNI()+UNI()+UNI()+UNI()+UNI()+UNI()
C  EPSI=EPSI+UNI()+UNI()+UNI()+UNI()+UNI()+UNI()-6.
C
C  LOOP TO TRANSFORM ETA TO EPSI, A RANDOM DRAWING FROM A
C  STANDARDIZED NORMAL DISTRIBUTION
C
C  ETA=UNI()
C  EPSI=0.
C  IF(ETA.EQ.0.5) GOTO 600
C  IF((ETA-0.5).LT.0) THEN
C    F=1.0-ETA
C  ELSE
C    F=ETA
C  ENDIF
C  EPSI=5.0*F-2.5
C  DO 500 ITER=1,5
C    DN=EXP(-0.5*EPSI*EPSI)/(2.0*PI)**0.5
C    Y=1.0/(1.0+ALPHA*EPSI)
C    VN=1.0-DN*(A1*Y+A2*Y*Y+A3*Y*Y*Y)-F
500  EPSI=EPSI-VN/DN
C  IF((ETA-0.5).LT.0) EPSI=-1.0*EPSI
600  CONTINUE
C
C  RWEEK = RWEEK*EXP(A+B*EPSI)
650  CONTINUE
C
C  RFLOAT(KK)=RWEEK
C  VSWAP=0.0
C  DO 700 LL=KK+1,N
C    VSWAP=VSWAP+TPERIOD*(RFIX-RFLOAT(KK))*
$    (1.0+RFIX*TPERIOD)**(-1.0*LL)
700  CONTINUE
C  VSWAP=VSWAP*P
C  IF(VSWAP.LT.0.0) VSWAP = 0.0
C  RCOST(KK)=VSWAP
C  CRISK(KK)=CRISK(KK)+RCOST(KK)
C  SQRSK(KK)=SQRSK(KK)+RCOST(KK)*RCOST(KK)
800  CONTINUE
C  IF(II.NE.NPRINT) GOTO 1000

```

```

PRINT *, ' '
PRINT *, 'ITERATION = ', II
PRINT *, 'TIME (YRS)', '      RFLOAT', '      RCOST'
DO 900 KK=1,N
PRINT *, KK*TPERIOD, RFLOAT(KK), RCOST(KK)
900  CONTINUE
PRINT *, ' '
1000 CONTINUE
C
C  CALCULATE AVERAGE AND STANDARD DEVIATIONS
C
DO 1500 KK=1,N
CRISK(KK)=CRISK(KK)/NITER
STDEV(KK)=(SQRSK(KK)-NITER*CRISK(KK)*CRISK(KK))/(NITER-1.0)
STDEV(KK)=(STDEV(KK)/NITER)**0.5
1500 CONTINUE
C
C  PRINT RESULTS
C
PRINT *, 'TIME (YRS)', '      EXPOSURE ($)'
AVERAGE=0.0
DO 2000 KK=1,N
PRINT *, KK*TPERIOD, CRISK(KK), '(', STDEV(KK), ')'
AVERAGE=AVERAGE+CRISK(KK)
2000 CONTINUE
AVERAGE=AVERAGE/N
PRINT *, 'AVERAGE = ', AVERAGE
C
C  END
C
C
FUNCTION UNI()
C*** FIRST CALL RSTART(I,J,K,L)
C*** WITH I,J,K,L INTEGERS
C*** FROM 1 TO 168, NOT ALL 1
C*** NOTE: RSTART CHANGES I,J,K,L
C*** SO BE CAREFUL IF YOU REUSE THEM IN THE CALLING PROGRAM.
C
REAL U(97)
COMMON /SET1/ U,C,CD,CM,IP,JP
UNI=U(IP)-U(JP)
IF(UNI.LT.0.) UNI=UNI+1.
U(IP)=UNI
IP=IP-1
IF(IP.EQ.0) IP=97
JP=JP-1
IF(JP.EQ.0) JP=97
C=C-CD
IF(C.LT.0.) C=C+CM

```

```
UNI=UNI-C  
IF(UNI.LT.0.) UNI=UNI+1.  
RETURN  
END
```

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