<u>Step 3</u>: Estimate the parameters a_{0i} , a_{1i} , a_{2i} in the least squares sense, yielding

$$\hat{\mathbf{a}} = \left(\underline{T'T}\right)^{-1} \underline{T'y} = \begin{bmatrix} \hat{a}_{0i} \\ \hat{a}_{1i} \\ \hat{a}_{2i} \end{bmatrix} \quad \text{for } \mathbf{i} = 1, \dots, \mathbf{n-1} \quad (4.3)$$

The estimated model is then given by

$$\hat{d}_{y/t} = \hat{a}_{0i} + \hat{a}_{1i}t + \hat{a}_{2i}t^2$$
 for i=1,....,n-1 (4.4)

An example of this estimation is shown in figure 4.4. For this simulation, the fault on bus 2 is assumed to self clear at t = 0.2 sec. The step integration is taken to be equal to 10 milliseconds. Ten points over a range of 100 milliseconds are saved after the fault clearance to estimate the different parameters of the model. For this example, the system is predicted to be unstable at t = 0.3 second, which leaves 300 milliseconds for control devices to operate.



(Full: real trajectory & + : polynomial model) **Figure 4.4:** Prediction of the post-fault trajectory using a second order polynomial function

It has usually been assumed [6] for real power system that it takes about 1 second for the system to lose synchronism after a fault has occurred. A power system fault is generally assumed to continue for about 100 milliseconds following a trip by the main protection. It is also assumed

in this paper that an average of 150 milliseconds of data following the clearing time are used for the prediction. In the case of first swing instability, the loss of synchronism is then predicted within 250 milliseconds after the fault occurred, which keeps 1/4 of the time for control devices to operate. In case of multi-swing instability, the prediction is started again by updating the observation matrix (4.2) using the latest machine angle and time measurements as well as the previous data recorded over 150 milliseconds. This can be thought of as a moving 150 ms data window. The addition of each new point causes the oldest point to be dropped from consideration. With a moving 150 ms data window, it could be natural to conclude that the accuracy of the prediction is then dependent with the sampling period of the machine angle measurements. As shown in Figure 4.5, a minimum of 5 points is required to get a reasonable approximation of the machine angle curves.



Figure 4.5: Machine Angle predictions using 5 points

The implementation of the proposed method needs a set of generator angle measurements. Those variables may be obtained from the phasor measurements at generator terminals [20,21]. With a minimum of 5 points, the method requires then that the machine angle measurements are sampled at least every 30 milliseconds including delay in signal transmission trip. Once the generator angles are calculated, the angular velocity and its rate of change can easily be computed. The second-order polynomial function does not include those values. Therefore in order to integrate the angular velocity and the angular acceleration, a second-order auto-regressive model is considered in the next section.