Appendix C

Derivation of the Shrinking Sphere Model

DERIVATION OF THE SHRINKING SPHERE MODEL
(Rimstidt, personal communication)

The surface area of a sphere is

$$A = 4pr^2$$

The volume of a sphere is

$$V = \frac{4}{3}pr^3$$

The general relationship between the surface are and the volume is

$$A = bV^{\frac{2}{3}}$$

$$4pr^2 = b(\frac{4}{3}pr^3)^{\frac{2}{3}}$$

b=4.84

The volume, V, of a material is defined as

$$V = nV_m$$

where V_m is the molar volume of a substance and n is the number of moles.

$$A = bn^{\frac{2}{3}}V_m^{\frac{2}{3}}$$

For a zeroth order rate law

$$\frac{dn}{dt} = -Ak$$

$$\frac{dn}{dt} = -bV_m^{\frac{2}{3}}kn^{\frac{2}{3}}$$

$$\int_{n}^{0} \frac{dn}{n^{\frac{2}{3}}} = -bV_m^{\frac{2}{3}} k \int_{0}^{t} dt$$

$$-3n^{\frac{1}{3}} = -bV_m^{\frac{2}{3}} k\Delta t$$

$$\Delta t = \frac{3n^{\frac{1}{3}}}{bV^{\frac{2}{3}}}$$

$$\Delta t = \frac{3V^{\frac{1}{3}}}{V_m bk}$$

$$\Delta t = \frac{3r(\frac{4}{3})^{\frac{1}{3}} p^{\frac{1}{3}}}{V_m 4p^{\frac{1}{3}} (\frac{4}{3})^{\frac{-2}{3}} k}$$

$$\Delta t = \frac{r}{V_m k}$$

Substituting for d=2r,

$$\Delta t = \frac{d}{2V_m k}$$