

INVESTOR PREFERENCES IN THE SECURITIES OPTIONS MARKET

by

Philip Davis Taylor

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APPROVED:

Hans Haller, Chairman

Daniel Orr

Douglas Patterson

Stephen Sheppard

William Smith

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Committee Chairman: Hans Haller
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(ABSTRACT)

Systematic mispricing by the state-of-the-art option pricing models is a paradox in financial economics as both the magnitude and direction of the mispricing is debated. The models have been found to overprice out-of-the-money and deep-in-the-money call options while underpricing in-the-money and deep-out-of-the-money calls. In addition, research has shown these biases have different signs in different time periods. We propose that when investors maximize expected utility for Friedman-Savage-Markowitz utility functions, the option mispricing observed in the market will result.

The theories and empirical tests in the literature of higher-order utility functions and risk-neutral valuation (RNV) in the options market are presented. Though investor attitudes towards risk are irrelevant in the non-arbitrage world of modern option pricing, to the extent the options market does not meet the non-arbitrage conditions, investor risk preferences will affect the pricing of options. Risk-loving traders will bid up market prices relative to risk-neutral model prices; risk-averse traders will bid down prices. And investor risk preferences can, and do, change over time as market conditions change.

New tests are run to analyze the relationship between mispricing biases and investor preferences before and after the historic stock

market crash of October 19, 1987. We find mispricing biases which imply a decreased risk aversion on the part of investors in the IBM call option markets for the period prior to the market crash and mispricing biases which imply an increased risk-averse (and decreased risk-loving) behavior in those markets following the crash. Similar analyses are also performed in the Microsoft call options markets with less conclusive results.

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Introduction

In the microeconomic theory of finance, to develop detailed and/or empirically testable hypotheses recognizing uncertainty it has become customary to follow the advice of James Tobin:

...It is very difficult to derive propositions that are simultaneously interesting and general....To get propositions with significantly more content than the prescription that the investor should maximize expected utility, it is necessary to place restrictions on his utility function or his subjective probability estimates.¹

In this effort, we believe the relationship between observed market prices for call options and model (predicted) prices, where these models assume risk neutrality, provides a unique and fruitful opportunity to observe the "representative" investor's preferences for wealth.

While empirical researchers have time and again shown the risk-neutral models tend to underprice in-the-money call options and overprice out-of-the-money calls, they have been unable to explain why these models, at the same time, overprice deep-in-the-money call options and underprice deep-out-of-the-money calls. We believe an explanation lies in the fact that when the model's assumptions are not perfectly met,² risk neutrality is no longer a valid characteristic and "true" (risk-averse and risk-loving) investor preferences lead to mispricing by the model.

Investor preferences have surfaced periodically in the literature as an indirect source of mispricing. Rubinstein (1985) writes:

...Perhaps we can correlate the bias observed in any period with the level of some macroeconomic variables....Natural candidates are the level of stock market prices, the level of stock market volatility, and the level of interest rates.³

Apart from the obvious question of volatility and interest rates already being incorporated in the option pricing equations, how will other "variables" enter the process except through investor preferences?

MacBeth and Merville (1980) find that common stock prices appear to be generated by constant elasticity of variance diffusion processes (as opposed to the constant variance assumed by many of the models, the Black and Scholes model in particular). One explanation for changes in stock return variances over time is that if in each period consumer-investors plan their consumption and investment over multiple future periods, then the variances for securities may change as new information arises and new individuals (preferences) bid for risky assets in the capital markets.⁴ Attention is then given to empirical testing of various hypotheses on variance specification, and the role of investor preferences again submerges out of sight.

We propose the existence of investor utility functions of the type proposed by Friedman and Savage (1948), as modified by Markowitz (1952), and find it is consistent with the observed mispricings in the securities options market. The presence of risk-loving behavior leads to market prices greater than model prices; the presence of risk-averse behavior leads to market prices less than model prices; and the presence of risk-neutral behavior leads to market prices equal to model prices.

In Chapter 1 we review the seminal works of Friedman and Savage and Markowitz which have established the higher order utility function as a viable alternative to the hypothesis that the investor is globally risk-averse. Machina (1982) is also discussed for his acceptance of the

higher order function. Next, we review research in experimental economics and psychology which supports the existence of higher order utility functions.

In Chapter 2 we examine the relationships which exist between a returns distribution and an investor's preference function. While we acknowledge the formal proofs in this area, in particular Müller-Machina (1987), we take a less formal and more intuitive approach. The bottom line, however, is that whenever the first m moments in a distribution are important to the investor, his utility function is an m -degree polynomial. Therefore, when the skewness (and/or kurtosis) of the returns distribution has importance for the investor, his utility function has an important characteristic, i.e, the inflection point(s), of the Friedman-Savage-Markowitz function.

Chapter 3 covers the basic concepts that are important to understanding the operation of the options market. Every security and financial market has its share of definitions and concepts. Because of its unique characteristics, the options market probably has more than its share. Call and put options are explained, the institutional framework of option trading is discussed, and the players in the market are analyzed. A glossary of common terms in options markets appears as the appendix to the chapter.

In Chapter 4 we examine the peculiar characteristics of options markets and option pricing theory which allow us to isolate the investor's preference for gambling and insurance. The primary function

of an organized options market is to provide investors with the most efficient means and most appropriate securities to hedge and speculate. We also present a simple simulation where the particular payoff structure of the call option, combined with a Friedman-Savage-Markowitz utility function and a lognormal stock return distribution, indicates risk-averse pricing of out-of-the-money calls and risk-loving pricing of in-the-money calls.

In Chapter 5 we review the development of option pricing theory in the literature. Option pricing models have fallen into two categories: (1) ad hoc models and (2) equilibrium models. Ad hoc models generally rely only upon empirical observation or curve fitting and, thus, need not reflect any of the price restrictions imposed by economic equilibrium. Equilibrium models deduce option prices as the result of maximizing behavior on the part of market participants. This latter approach to option pricing dates back to the work of Bachelier (1900), whose research pointed the way for a number of attempts to describe an equilibrium theory of option pricing, including Sprenkle (1964), Boness (1964), and Samuelson (1965). All of these models essentially equate the value of an option to the discounted expected payoff to the option which is dependent on an assumed probability distribution of future stock prices. The proper rate to discount the expected payoff must be specified as well. Thus, to complete these models it is necessary to make specific, restrictive assumptions about individual risk preferences and/or the pricing structure in equilibrium.

Black and Scholes (1973) derive an equilibrium model of option pricing that avoids restrictive assumptions on individual risk preferences and market equilibrium formation. They demonstrate that it is possible to construct a portfolio involving positions in the stock and riskless asset where the return to the portfolio over a short time interval exactly replicates the return to the option. In addition, they show precisely how the composition of the portfolio must continually change in response to movements in the stock price and passage of time such that the replication of the return to the option is maintained.

In Chapter 6 we focus on an essential feature of modern option pricing theory: the derivation of risk neutral valuation relationships (RNVRs) for the contingent claim. RNVRs are derived in the Black-Scholes arbitrage analysis by maintaining a riskless hedge which is self-financing, i.e., with a zero net investment. Failure to achieve riskless hedges may be the rule rather than the exception, however. Kreps (1980) states that phenomena such as differential information, moral hazard, individual uncertainty about future tastes, etc., represent uncertainty which is, in general, not fully reflected in the security processes.

Whether or not a contingent claim can be generated by a self-financing portfolio in the basic securities depends essentially on the distribution of the security price processes. Geometric Brownian motion is assumed in the models of Black and Scholes. Empirical studies have questioned this specification. Green and Fielietz (1977) reject the lognormality assumption. Mandelbrot (1963) and Fama (1963) favor a

logstable specification. Upton and Shannon (1979) examine the empirical validity of the logstability and lognormality assumptions. Because of the observed fat tails, Blattenberg and Gonedes (1974) model the distribution of the logarithmic price increments by a symmetric stable and by a Student distribution.

In Chapter 7 we review the literature on the empirical tests of the Black-Scholes (B-S) option pricing models. The models have been tested alone against the market data and also in comparison to other popular models against the data.⁵ There are two results that stand out from the mass of evidence. First, as a generality, the B-S pricing model works exceptionally well. On average, the pricing model is sufficiently accurate to be a useful valuation tool. Second, we wish to stress the B-S model has some biases, appearing to work best with medium maturity, at-the-money calls.

Black and Scholes (1972) test their model on price data gathered on options traded in the over-the-counter market from 1966 to 1969. Taking transaction costs into account, they demonstrate a close correspondence between model and market prices. Galai (1977) replicates and extends the Black-Scholes test using data from the first seven months of trading on the Chicago Board Options Exchange (CBOE). His results essentially reaffirm the findings of Black and Scholes.

However, other research has shown that mispricings exist with the B-S models, but what is more troublesome is there appears to be little consensus on the magnitude or even direction of the mispricing biases.

Empirical work has emphasized either the estimation of the variance rate variable (stock volatility) or the size and timing of dividends to explain the in- and out-of-the-money biases. Explanations offered for these observed mispricings are, however, not able to also explain the deep-in-the-money and deep-out-of-the-money biases.

While the magnitude of the mispricing has been reduced by several refinements of the original 1973 model, significant and systematic mispricing biases still exist. It appears that the cause(s) of this mispricing must surely lie in a direction other than dividends as options written on stocks not paying a dividend still exhibit these systematic biases.

By superimposing a linear, risk-neutral (RN) utility function onto the Friedman-Savage-Markowitz (FSM) utility function, we are able to conceptualize the potential pricing disparities which may result from risk-averse, risk-neutral, and risk-loving pricing. It is helpful to visualize the wealth axis partitioned into four regions which we give the following properties and associated pricing relationships (BS is the Black-Scholes model price, MP is market price):

Region 1:	deep-out-of-the-money	$URN > UFSM$	$BS < MP$
Region 2:	out-of-the-money	$URN < UFSM$	$BS > MP$
Region 3:	in-the-money	$URN > UFSM$	$BS < MP$
Region 4:	deep-in-the-money	$URN < UFSM$	$BS > MP$

What we propose is the presence of risk-loving market behavior in regions 1 and 3 and a risk-averse market behavior in regions 2 and 4.

This will lead to mispricing by a risk-neutral model consistent with the observations by Black (1975) and Emanuel and MacBeth (1981) in regions 1 and 4; MacBeth and Merville (1979) and Rubinstein (1985), regions 2 and 3; and Merton (1976), region 1. Only Merton's observation (1976) in region 4 is inconsistent.⁶ The presence of risk-loving traders bids up market prices; the presence of risk-averse traders bids down market prices.

Previous research, for the most part, has treated all call options alike by aggregating across various companies. Of equal importance, these studies have usually tested across various investor reference periods when investor expectations on the future can reasonably be expected to have changed.

In Chapter 8 we set out the general framework of our testing procedure and establish the magnitude and direction of the systematic biases in the Black-Scholes model for a relatively stable, current time period. Employing Black's approximation call option formula to price IBM April 1986 call options for the period 1/27/86 to 3/7/86, we find there is empirical support for our hypothesis of F-S-M preferences. In addition, we try to shed some light on the problem of the change in sign of mispricing biases observed in the previous tests by Emanuel-MacBeth (1981) and Rubinstein (1985). We hypothesize that an increase in the perceived risk in the market for the underlying stock will increase the degree of mispricing in its out-of-the-money options and decrease the degree of mispricing in its in-the-money options.

Our sample period brackets the October 19, 1987 "Market Break." Observations of September and October prices for October IBM calls serve as indicators of immediate "Pre-Break" behavior. The trend in mispricing is developed over an extended period covering July to October. October and November prices for December IBM calls indicate immediate "Post-Break" behavior. October through January prices are used to develop post-crash trends.

In Chapter 9 we perform identical tests on a quite different class of stock option, one written on Microsoft, a stock which pays no dividend and is traded over-the-counter via NASDAQ. We are looking to confirm our general conclusions on the relationship between option mispricing and investor preferences. At the same time, we wish to show that dividend-related problems are not the reasons for the option mispricing currently being observed in the market.

In Chapter 10 we present our summary and conclusions.

FOOTNOTES

¹Tobin (1969, p.13).

²Rubinstein (1976) has shown that when investors are unable to maintain a continuous hedge the combination of a lognormal market and logarithmic utility will give the risk-neutral result. We accept the lognormal market but not the logarithmic utility.

³Rubinstein (1985, p. 479).

⁴MacBeth and Merville (1980, p. 285).

⁵See Chapter 7 references for complete listing.

⁶Merton's (1980) observations were anecdotal in nature. If a case can be made for misclassifying "out" calls as "deep-out" calls, then Friedman-Savage-Markowitz preferences are consistent in all the regions. There does not appear to be a quantitative definition to distinguish the two classes of options.

Chapter 1

Review of the Literature on Higher Order Utility

1. Introduction

In this chapter we review seminal works in the literature which have established the higher order utility function as a viable alternative to the hypothesis that investors are globally risk-averse. (Here and subsequently, the notion "utility function" is meant as a shorthand for "von Neumann-Morgenstern utility function.") Next, we review the research in experimental economics and psychology which support the existence of higher order utility functions.

2. The Friedman and Savage Utility Function

The key aspect of the Friedman and Savage (1948) utility function is that it is concave, and hence locally risk averse, about low outcome levels, i.e., low levels of ultimate wealth, linear (to a second order approximation) and hence locally risk neutral at the inflection point and convex, locally risk loving, for high outcome levels. A utility function exhibiting such properties is shown in Figure 1-1.

At a later point in their 1948 article, F-S point out that an individual with a utility function as in Figure 1-1 and with initial wealth near the inflection point would always pay more for a lottery ticket offering a probability p of $\$Z$ than for a ticket with probability $2p$ of winning $\$Z/2$. On this basis, they reject a Figure 1-1 shape as inconsistent with their final observation, that lottery designers are

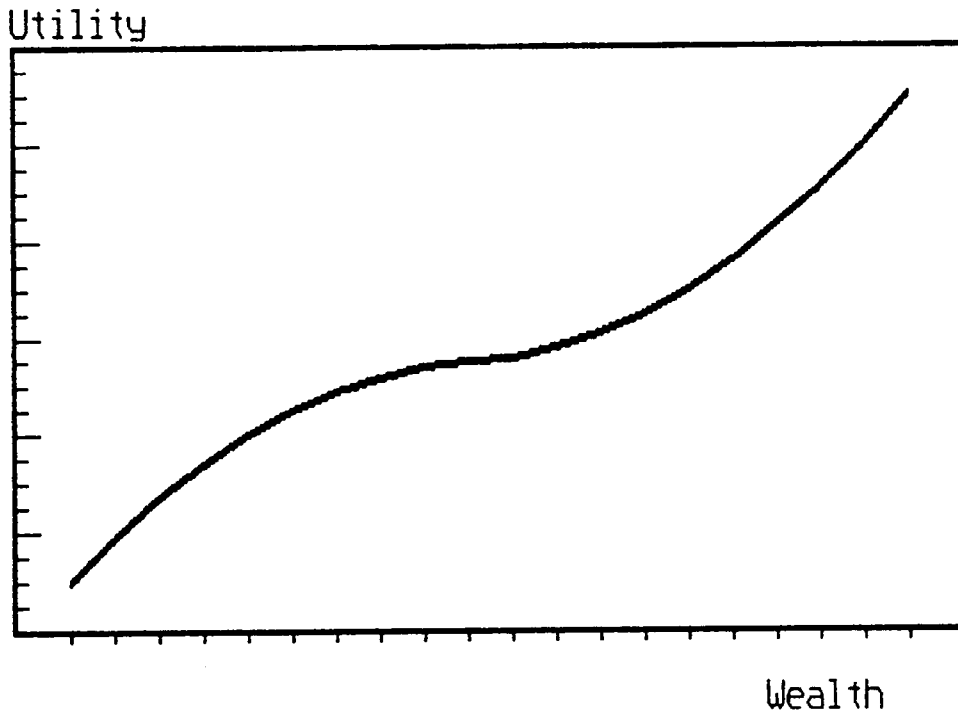


Figure 1-1. Friedman-Savage Initial Utility Curve.

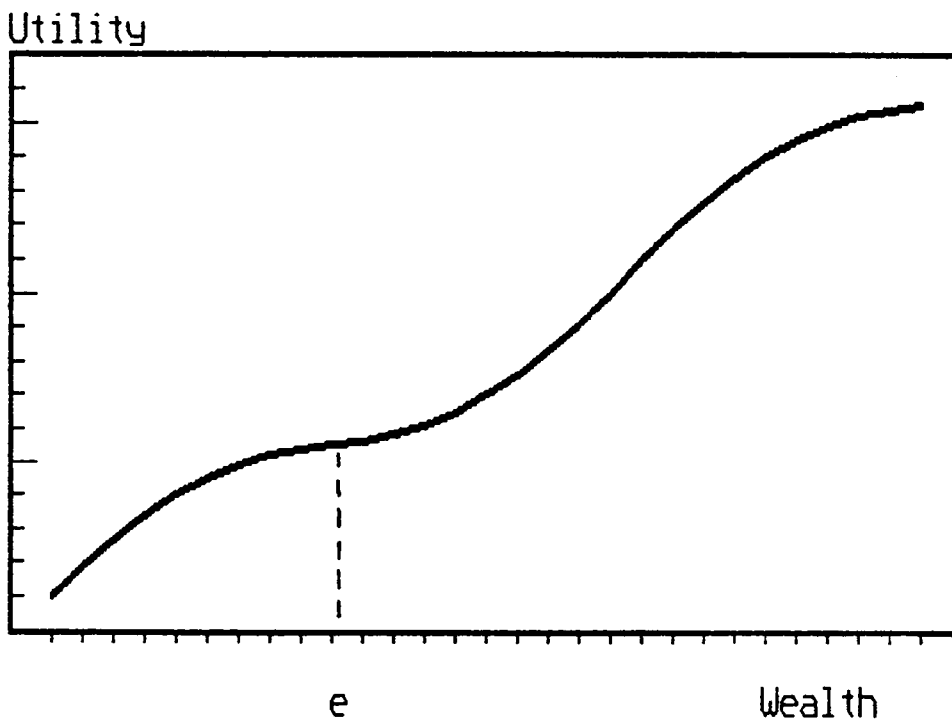


Figure 1-2. Friedman-Savage Revised Utility Curve.

presumably profit maximizers and "lotteries typically have more than one prize."¹ In light of this, F-S modified their originally proposed shape, so as to include a terminal concave section, as in Figure 1-2.

Markowitz (1952) made essentially this same point, namely that the amount the individual would pay for a $1/n$ chance of winning $\$nZ$, though possibly increasing at first, is an eventually decreasing function of n .

2.1 The Markowitz Modification

The initial objection to (and modification of) the original F-S utility function concerned not so much the typical shape of the utility function, but rather the more fundamental issue of the stability of preferences. As noted by Markowitz (1952), the assumption that the utility function of Figure 1-2 is defined over ultimate wealth levels is not consistent with the observed tendency of individuals of all wealth levels to purchase insurance and lottery tickets. Noting that individuals of all wealth levels tend to behave as if their initial wealth was near the left inflection point e in Figure 1-2, Markowitz hypothesized that changes in wealth caused the utility function to shift horizontally so as to keep this inflection point near the current level of wealth. Thus, we arrive at the Friedman-Savage-Markowitz (F-S-M) utility function in Figure 1-3.

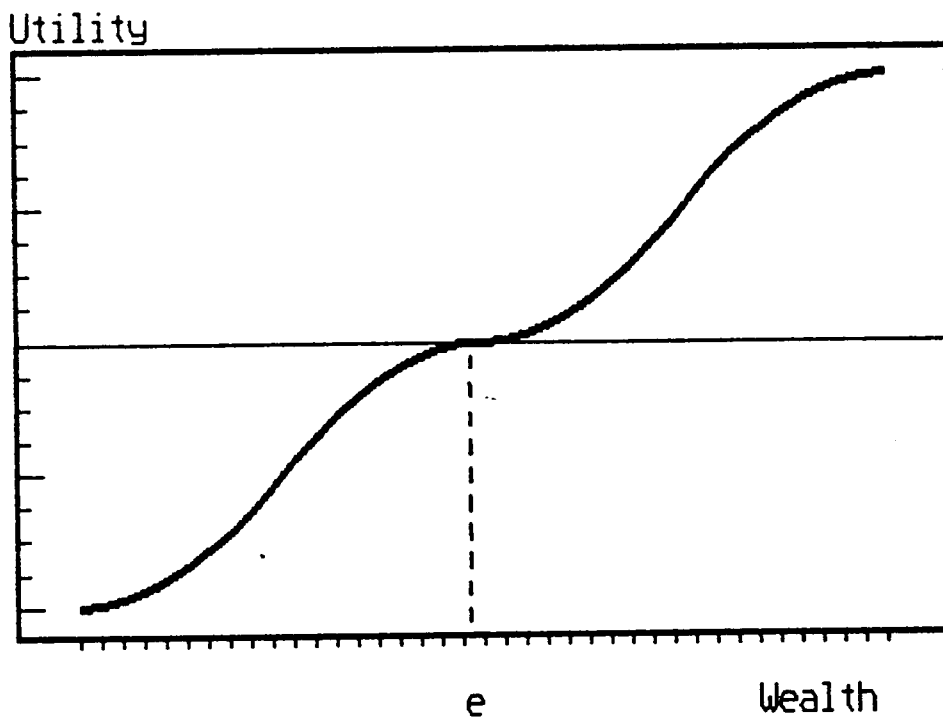


Figure 1-3. Friedman-Savage-Markowitz Utility Curve.

3. Machina: Generalized Expected Utility

Machina (1982) demonstrates that the general analytic approach, termed "generalized expected utility analysis," may be used to construct a simple, but powerful, model of individual behavior toward risk. Machina shows that two simple hypotheses² concerning the shape of a fixed nonlinear preference functional over probability distributions serve to generate predictions consistent with

...the general observations on insurance and lotteries made by Friedman and Savage in their classic article on the expected utility hypothesis...(and the) observation by Markowitz and others that preferences over alternative gambles are relatively independent of the level of current wealth (and hence that utility functions apparently shift when wealth changes),...³

Machina shows how Hypothesis I is consistent with "aversion to increases in risk involving low outcome values coupled with a preference for increases in risk involving high outcome values. A "local" utility function such as the one in Figure 1-1 would lead to the behavior described by Machina. This local utility function is later shown to be compatible with a bounded terminal concave segment such as Friedman and Savage proposed in Figure 1-2.

To the point of Markowitz's observations, Machina demonstrates that a "fixed" utility function satisfying Hypotheses I and II will rank alternative gambles which are depicted as deviations from current wealth relatively independently of the level of current wealth. That is, the preference function, such as Figure 1-1 or 1-2, will shift right (left) by the approximate increase (decrease) in current wealth.⁴

4. Experimental Measurement of Utility

Since a knowledge of $U(\cdot)$ would allow us to predict preferences (and hence behavior) in any risky situation, experimenters and applied decision analysts are frequently interested in eliciting or recovering their subjects' (or clients') von Neumann-Morgenstern utility functions. One method of doing so is termed the fractile method. This approach begins by adopting the normalization $U(0) = 0$ and $U(M) = 1$ for some large value M , then fixing a "mixture probability" p , say $p = 1/2$. The next step involves finding the individual's certainty equivalent π_1 of a $1/2:1/2$ chance of M or 0 , which implies $U(\pi_1) = (1/2) \cdot U(M) + (1/2) \cdot U(0) = 1/2$. Finding the certainty equivalents of the $1/2:1/2$ chances of π_1 or 0 and of M or π_1 yields the values π_2 and π_3 which solve $U(\pi_2) = 1/4$ and $U(\pi_3) = 3/4$. By repeating this procedure (i.e., $1/8, 3/8, 5/8, 7/8, 1/16, 3/16$, etc.), the utility function can (in the limit) be completely assessed.

Variations of this approach are found in the experimental models of Mosteller and Nogee (1951) and Teweles and Jones (1987).

4.1 The Mosteller and Nogee Model

Following the model of Friedman and Savage, Mosteller and Nogee (M-N) presented subjects with choices of the following sort:

Alternative I: risk neither loss nor gain,

Alternative II: risk the loss of 5 cents with probability $(1 - p)$ or the gain of X cents with probability p .

A subject who was offered this choice repeatedly was assumed to prefer Alternative I if he chose it more often, and to prefer II if he chose it more often. If he chose the two equally often, he was assumed to be indifferent between the them. Holding p fixed at some value, p^* , the subject was offered various values of X so as to determine the particular value, X^* , for which the subject was indifferent between I and II. This determined one point on the utility function.

Let $U(-5) = -1$,
and $U(0) = 0$.

This is equivalent to selecting the constant term and scale of the utility function. Having determined X^* such that the subject is indifferent between I and II for the given value of p^* , solve the equation

$$(1 - p^*)U(-5) + p^*U(X^*) = U(0)$$

for $U(X^*)$. This gives

$$U(X^*) = (1 - p^*)/p^*.$$

Additional points on the utility function are found by varying p and repeating the procedure.

Except for reasons of simplicity, nothing prevented M-N from using a gamble for Alternative I instead of a sure thing. They could have selected two sums of money, Y and Z and defined $U(Y) \equiv -1$ and $U(Z) \equiv 0$, where $Y < Z$. Fixing p_1 , p_2 , p_{11} , and p_{21} , they could have determined, experimentally, sums of money X_1 and X_2 such that

$$p_1 U(Y) + (1 - p_1) U(Z) = p_{11} U(X_1) + (1 - p_{11}) U(X_2)$$

and

$$p_2 U(Y) + (1 - p_2) U(Z) = p_{12} U(X_1) + (1 - p_{12}) U(X_2).$$

These equations could be solved for $U(X_1)$ and $U(X_2)$.

If the general form of the utility function is specified to be a polynomial of degree n , its coefficients can be determined provided it is possible to find, for each coefficient to be determined a pair of probability distributions, F_t and G_t with densities f_t and g_t , respectively, (where t ranges over the number of coefficients to be determined) such that the individual is indifferent between them.

Write

$$\int f_t(X) U(X) dX = \int g_t(X) U(X) dX \quad (t = 1, \dots, n-1).$$

Select zero as the constant term of U and select the scale so that the coefficient of the linear term is equal to one. Then

$$\mu_{F_t}^1 + \sum_{i=2}^n \mu_{F_t}^i a_i = \mu_{G_t}^1 + \sum_{i=2}^n \mu_{G_t}^i a_i \quad (t = 1, \dots, n-1)$$

where $\mu_{F_t}^i$ is the i^{th} moment around the origin of the distribution, F_t . These $n-1$ equations yield the coefficients a_i . Of course it is unlikely that observations obtained in such an experiment will exactly fit the postulated function, and if more than $n-1$ comparisons are made it is necessary to use some statistical technique for determining the function.

M-N, in determining seven points on the utility function, offered each subject 686 choices--168 for each point. For each value of p (seven used to obtain seven points) seven different values of X were offered. Each value of X was offered 14 times. It was found that for some value of X , X_L , the gamble was selected less than 50% of the time, and that for the next larger value of X , X_H , the gamble was selected more than 50% of the time. Linear interpolation was used to find X^* , the value of X for which the gamble would be selected exactly 50% of the time. Thus, of the 168 offers made in determining a single point on the utility function, only 28 (X_L and X_H were each offered 14 times) actually entered into the estimate of X and hence into the estimate of the point on the utility function. Figure 1-4 is the utility curve of Subject B-IV in (M-N, p. 387).

In discussing possible criticisms of their work, M-N deal with the question of the triviality of the sums involved in their experiment. The question of triviality of sums should really be regarded as two questions:

1. Are the sums so small that, over the range of possible gains and losses, changes in the level of utility cannot be detected?
2. Are the sums involved so small that during the period of the experiment uncontrolled variation in the wealth of the subjects is of an order of magnitude equal to or greater than that of the sums offered in the experiment?

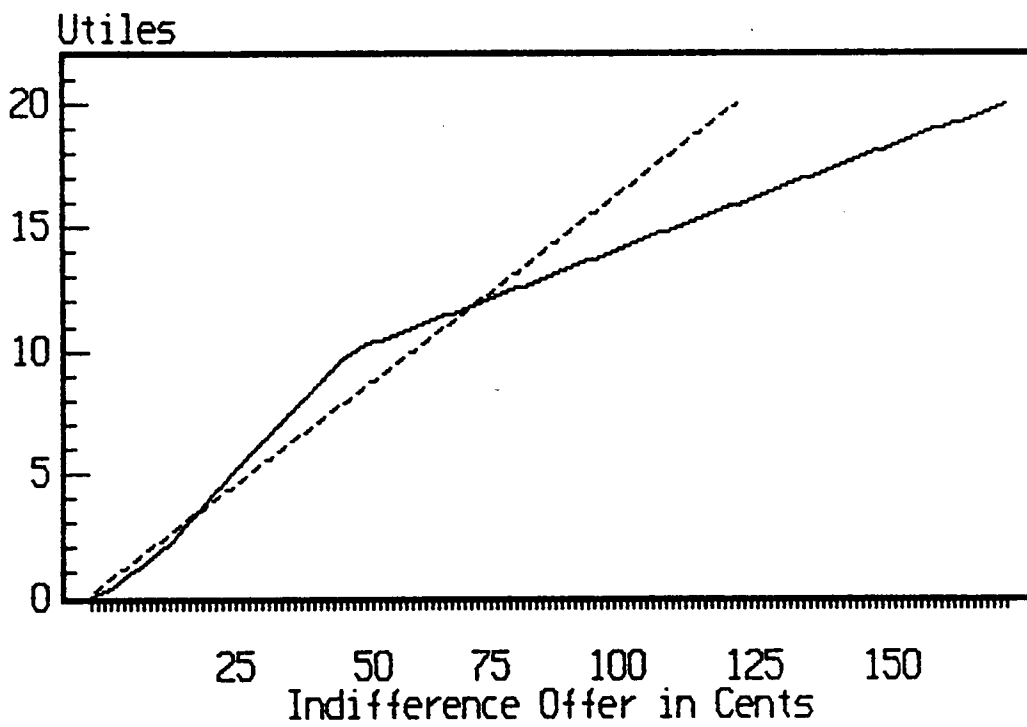


Figure 1-4. Mosteller and Nogee Cubic Utility of Wealth Curve.

Regarding the first question, M-N have provided experimental evidence to the contrary. They show considerable variation in utility over ranges as small as 50 cents. This fact unfortunately raises serious problems of the following sort: Subject B-IV was found to associate 10 utiles with the sum of 50 cents. Suppose we define his utility at the beginning of some session to be zero utiles. The first gamble offered is attractive and he accepts, winning 50 cents as a consequence. If he is now at 10 utiles, should M-N account for that in interpreting the results of the next offer? They do not; they assume that he is still at zero utiles.⁵

The question of whether M-N were measuring the utility of income or wealth might be raised here, but it would only confuse the issue. They were measuring the utility of whatever they were offering; the thing they were offering was a chance to win X cents. Starting from zero and having won X cents is the subject still at zero utiles, or is he at some higher utility level?

The 1952 paper by Markowitz offers an interpretation of the M-N results. Put most simply, Markowitz suggested that the utility function is just a device for explaining and predicting responses to choices involving risk. As wealth increases Markowitz required that the utility function satisfy certain criteria based on casually observed gambling behavior. These criteria led him to a specification of the slope of the utility function. It may be possible to raise equally plausible causal arguments against the slope specified by Markowitz, but what is

important here is that Markowitz proposed a departure from the traditional idea of utility functions enabling us to predict the choice between gambles at any level of wealth. The Markowitz utility function enables us to predict choices only at the present level of wealth since changes in wealth will change the utility function. For a small change in wealth the change in the function is small and gambling behavior remains almost unchanged.

If M-N were to argue that, over the experimental period, the changes in wealth (both those due to the experiment and those uncontrolled) were so small that changes in the Markowitz utility function were trivial, they could interpret their function as one which will predict choices in situations involving risk, but which cannot be interpreted as giving the effect of changes in income or wealth. This interpretation also disposes of the second problem connected with triviality of sums.

Except in the light of Markowitz's modification of Friedman-Savage, the Mosteller-Nogee experiment is extremely difficult to interpret. Given the modification, however, the experiment and its results make very good sense.

4.2 The Teweles and Jones Model

Teweles and Jones (1987) present a methodology to assess a futures trader's preference function through the calculation of utility values and construction of a "personal trading curve," based on the cardinal utility theory formulated by von Neumann and Morgenstern (1947). Teweles

and Jones conduct trading profile interviews (TPIs) with traders to gather the data needed to construct the individual personal trading curves.

The typical interview (with hypothetical responses) would proceed as follows:⁶

INTERVIEWER: What are the largest amounts you have had regular experience winning and losing on trades or investments?

TRADER: I've often had the experience of winning as much as \$2000 on a single venture, and I've risked as much as \$1000 on single trades.

The interviewer would now fill out the first five of the six columns in a table such as Table 1-1 below. Each line of the table represents a separate and distinct trade. He would enter \$2000 as the "best" outcome and -\$1000 as the "worst" outcome, assigning them utility values of 1 and 0, respectively. He then fills out column 3 with a fairly uniformly spaced set of probabilities to represent the respective probabilities of winning \$2000 for each of the different trades to be considered. In column 4 are listed the complementary probabilities of losing \$1000 for each of the trades. In column 5 the interviewer computes the utility of each trade by taking the average of the utilities (1 and 0 in each case); each utility weighted by its own probability.

The essence of the trading profile interview is the filling in of column 6. The trader must consider each trade separately and carefully, always remaining cognizant of the basic ground rules of the experiment.⁷

Table 1-1
Computation of Initial Set of Utilities for Trading Curve

<u>Utilities of</u>		<u>Probabilities of</u>		<u>Trader's Cash</u>	
Best outcome (col.1)	Worst outcome (col.2)	Best outcome (col.3)	Worst outcome (col.4)	Computed utility of trade (col.5)	equivalent of trade, in dollars (col.6)
1	0	1.00	0	1.00	2000
1	0	.95	.05	.95	1500
1	0	.90	.10	.90	1250
1	0	.80	.20	.80	1100
1	0	.70	.30	.70	1000
1	0	.60	.40	.60	800
1	0	.50	.50	.50	450
1	0	.40	.60	.40	0
1	0	.30	.70	.30	-400
1	0	.20	.80	.20	-600
1	0	.10	.90	.10	-800
1	0	0	1.00	0	-1000

INTERVIEWER: (describing the trade indicated by line 2 of Table 1-1) If you have a trading opportunity wherein you would either gain \$2000 or lose \$1000 with probabilities .95 and .05, respectively, would you take that trade?

TRADER: Certainly! Who wouldn't?

INTERVIEWER: Would you pay me \$800 in cash, right now, just for the opportunity to make that trade?\$1200?\$1400?\$1500?

TRADER: Maybe I would pay \$1500 and maybe not. I'm sort of indifferent at that price.

INTERVIEWER: That's fine. Then the value of that trade to you is approximately the same as the value of \$1500 to you. Therefore, your utility for the \$1500 cash is the same as your utility for that trade, already computed on line 2, column 5, as .95.⁸

Similar questioning (varying the probabilities) allows the interviewer to complete Table 1-1. For the purposes of verification and improved accuracy, and to measure the consistency of the trader's decisions, additional utility values for some of the same dollar amounts are again computed. The results are shown in Tables 1-2 and 1-3.

Figure 1-5 represents the personal trading curve of the trader based upon the data in Tables 1-1, 1-2, and 1-3. Boundaries of the trading curve could, of course, be extended beyond the original best and worst outcomes to get a graphic interpretation of personal attitudes toward the gains and losses that are larger than those with which the trader has had regular experience. Teweles and Jones, however, consider the specifics of such extensions to be less reliable.⁹

4.3 Comments on the Personal Trading Curve

The general shape of the curve is quite similar to the Friedman and Savage curve shown in Figure 1-2. From small losses of \$200 to gains as high as \$1000 the curve is slightly convex, indicating the trader's tendency to accept even an unfavorable (negative expected value) trade in

Table 1-2

Computation of Second Set of Utilities for Trading Curve

Utilities of		Probabilities of		Computed utility of trade	Trader's Cash equivalent of trade, in dollars
Best outcome (col.1)	Worst outcome (col.2)	Best outcome (col.3)	Worst outcome (col.4)		
.60	.10	1.00	0 .6	.60	800
.60	.10	.95	.05	.575	650
.60	.10	.90	.10	.55	550
.60	.10	.80	.20	.50	400
.60	.10	.70	.30	.45	250
.60	.10	.60	.40	.40	150
.60	.10	.50	.50	.35	-100
.60	.10	.40	.60	.30	-400
.60	.10	.30	.70	.25	-450
.60	.10	.20	.80	.20	-500
.60	.10	.10	.90	.15	-600
.60	.10	0	1.00	.10	-800

Table 1-3

Computation of Third set of Utilities for Trading Curve

Utilities of		Probabilities of		Computed utility of trade	Trader's Cash equivalent of trade, in dollars
Best outcome (col.1)	Worst outcome (col.2)	Best outcome (col.3)	Worst outcome (col.4)		
.95	.30	1.00	0	.95	1500
.95	.30	.95	.05	.9175	1200
.95	.30	.90	.10	.885	1150
.95	.30	.80	.20	.82	1100
.95	.30	.70	.30	.755	800
.95	.30	.60	.40	.69	600
.95	.30	.50	.50	.625	500
.95	.30	.40	.60	.56	400
.95	.30	.30	.70	.495	100
.95	.30	.20	.80	.43	-100
.95	.30	.10	.90	.365	-200
.95	.30	0	1.00	.30	-400

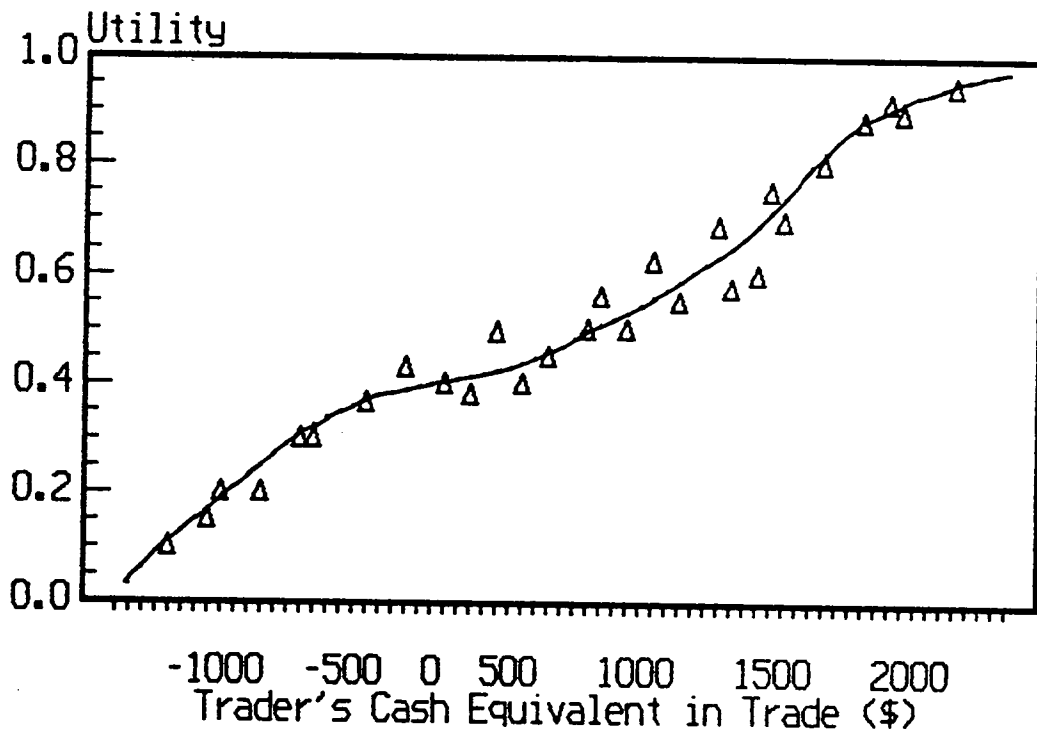


Figure 1-5. Teweles and Jones TPI Trading Curve.

those amounts. Teweles and Jones observe that "many traders act as risk lovers for small gains and losses".¹⁰

For losses greater than \$200 the trader's curve is concave, indicating an aversion to accepting fair bets, or even slightly favorable (positive expected value) bets. One of Markowitz's modifications of F-S can also be observed: the left inflection point is roughly at the trader's current (0 profit) level of wealth. The other Markowitz modification is not shown, although this risk-averse behavior would presumably end at some very large loss number with a convex segment to bound the utility function.

As our trader's gains increase beyond \$1000, the shape of the trading curve becomes concave, which indicates a decreasing utility for incremental gains. The trader's decisions in regard to larger trades will be more conservative, and he will have a tendency to take only very favorable trades. Again, Teweles and Jones have observed this specific risk-averse behavior: "Many traders act that way for large gains and losses."¹¹

5. Psychological Experiments

5.1 The Becker and Siegel Model

Becker and Siegel (1956) conducted a study where they test the hypothesis that the ordered metric utility function yields information on the individual's level of aspiration. Siegel defines level of aspiration as follows:

The level of aspiration of an individual is a point in the positive region of his utility scale of an achievement variable: it is at the least upper bound of that chord (connecting two goals) which has a maximum slope; i.e., the level of aspiration is associated with the higher of the two goals between which the rate of change of the utility function is a maximum.¹²

It may be seen from this definition that the difference in utility between achieving at the level of aspiration or achieving at the next lower level is greater than the difference in utility between achieving at the level of aspiration or achieving at the next higher level. In other words, the level of aspiration is that goal which has the largest difference in utility between it and the next lower goal. Figure 1-6 demonstrates this graphically.

Subjects were 20 students enrolled in an elementary course in statistics. These were students who volunteered to gamble with the instructor for their midterm grade, with the understanding that the grade would be entered in the course records in lieu of their score on the regularly scheduled midterm examination. An ordered metric scale of the grades A, B, C, D, and F was derived for each subject by the method described in Becker-Siegel (1956).

The subjects were required to make choices between alternatives in a number of offers. Each of the offers was numbered, and the subjects understood in advance that the offer on which the payoff (midterm grade) would be based would be selected at random from all of the offers given. Therefore each decision they made had an equal likelihood of being the crucial one on which their midterm grade depended.

After the subjects had made all the required choices, two ruses were introduced. First, the allegedly random device by which each student's

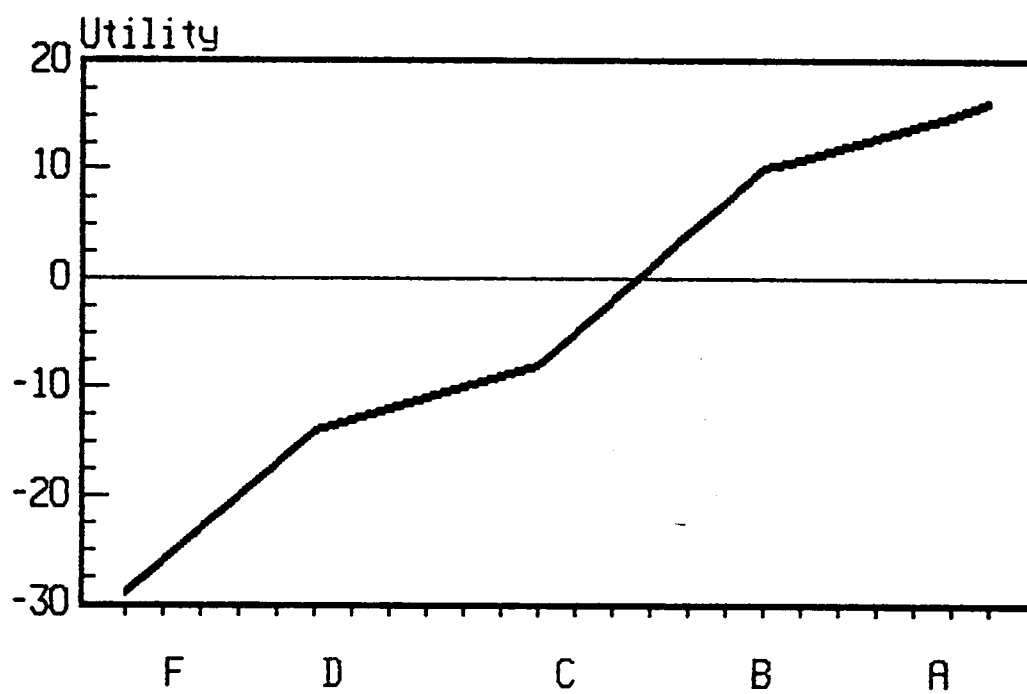


Figure 1-6. Becker and Siegel Cubic Level of Aspiration Curve.

grade would be determined was actually controlled so that each subject "won" a C for his grade. Some subjects responded to this payoff with considerable dismay and disappointment. In response to this, the experimenters perpetrated the second ruse. The group of subjects was told that if they were dissatisfied with the grade they had won, they could obtain an interview with the instructor and the other experimenter, in which perhaps some way of raising their grade could be worked out. The implication in this announcement was that performance of extra work would be the mode of raising the grade. The subjects were told that any who desired such an interview would have to wait until the instructor placed an urgent long-distance telephone call, after which he and the other experimenter would return to conduct individual interviews with those students who desired them.

The interviews were structured to obtain a careful independent measure of each subject's level of aspiration for his midterm grade. Both experimenters were present, and their independent judgments of each subject's level of aspiration were in very close agreement (.9 correlation). The interviews were conducted before either experimenter had any information concerning any subject's ordered metric scale of utility for grades. The experiment was designed to test two hypotheses:

Hypothesis I was that those subjects who would not wait for an interview with the instructor would be persons on whose ordered metric scales the largest distance was between D and F--person whose ordered metric scales would reveal that their level of aspiration was below the C which they had won. This hypothesis was supported by the data. Four

subjects left the room before the experimenters returned; all had ordered metric scales with the largest distance between D and F. Seemingly they were satisfied with the C grade they had been assured and, therefore saw no reason to seek an interview.

Hypothesis II was that subjects' levels of aspiration, as determined from their ordered metric scales of utility, would be positively correlated with their levels of aspiration as judged from the interview material. This hypothesis was confirmed: the correlation between the two independent indices of level of aspiration was .83.

In addition to the finding of utility functions with both concave and convex sections by Becker and Siegel, the research in the area of level of aspiration may also allow us to set up hypotheses on how the utility of wealth function will "change," given the outcome of a previous investment. Change, not in the general shape of the function, but in the degree of risk-averse or risk-loving behavior of the investor (i.e., change in slope along the convex (concave) portion of the utility curve given a different level of aspiration).

Child and Whiting (1954) have formulated five general statements which contain the conclusions which may reasonably be drawn from the research of level of aspiration:

1. Success generally leads to a raising of the level of aspiration, and failure to a lowering.

2. The stronger the success, the greater is the probability of a rise in level of aspiration; the stronger the failure, the greater is the probability of a lowering.

3. Shifts in level of aspiration are in part a function of changes in the subject's confidence in his ability to attain goals.

4. Failure is more likely than success to lead to withdrawal in the form of avoiding of setting a level of aspiration.

5. Effects of failure on level of aspiration are more varied than those of success.¹³

Conclusions 1 and 2 are particularly suited to empirical testing.

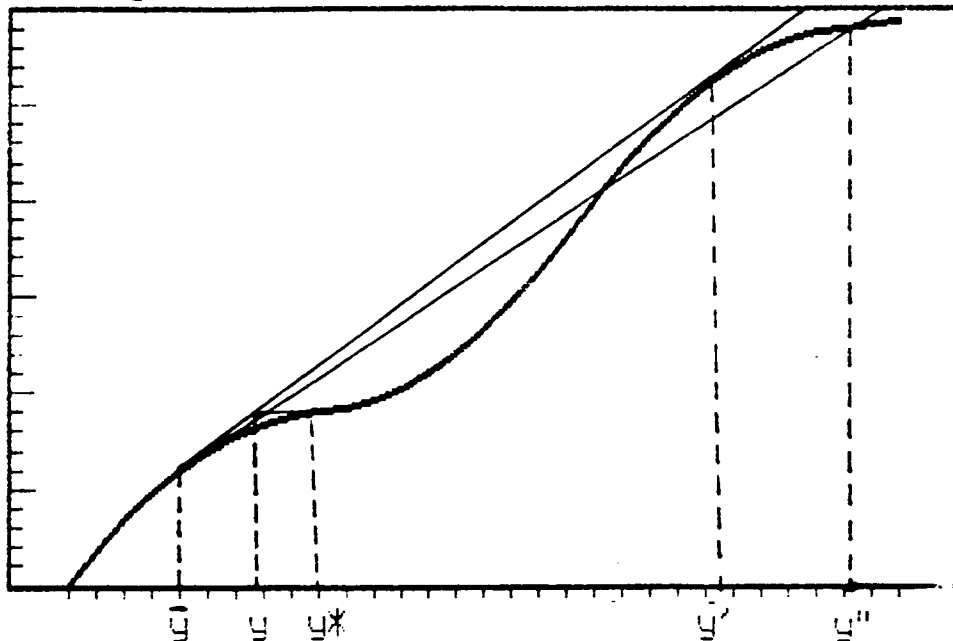
In Chapters 8 and 9, we examine the effects of a significant market "crash" on the risk preferences of traders in the options markets. If we can translate a "raising" of the level of aspiration to mean an increase in risk-loving behavior by the trader and a "lowering" to mean an increase in the risk-averting behavior, then the unparalleled rise in the market during the first eight months of 1987 should be accompanied by rising levels of aspiration (increased risk-loving behavior); the post-crash period, by lower levels (increased risk-averting behavior).

FOOTNOTES

¹Friedman and Savage (1948, p. 294). Their argument can be summarized as follows:

Consider an entrepreneur conducting a lottery--and seeking to maximize his income from it. His variables are (1) the number of tickets to sell, (2) total amount in prizes, and (3) structure of the prizes. Given (1) and (2), the optimum structure is that which maximizes the ticket price. By offering a single prize ($y'-y$) he maximizes the price (y^*-y). If he offered a larger prize ($y''-y$), the price would fall; therefore, lotteries offer multiple prizes of amount ($y'-y$) in place of larger amounts.

Utility



²Machina (1987, p. 300).

Hypothesis I: For any distribution $F(\cdot) \equiv D[0, M]$, $-U_{11}(x; F)/U_1(x; F)$ is a nonincreasing function of x over $[0, M]$.

Hypothesis II: For any $x \equiv [0, M]$ and distributions $F(\cdot) \equiv d[0, M]$, if $F^*(\cdot)$ stochastically dominates $F(\cdot)$, then

$$-U_{11}(x; F^*)/U_1(x; F^*) \geq -U_{11}(x; F)/U_1(x; F).$$

That is, with respect to the partial order on $D[0, M]$ induced by the relation of stochastic dominance, $-U_{11}(x; F)/U_1(x; F)$ is "nondecreasing in F ."

FOOTNOTES

³Machina, p. 279.

⁴Machina, p. 307. Machina notes that Hypotheses I and II are not strong enough to ensure a shift of the exact change in wealth.

⁵This is made a basic rule in subsequent experiments by other researchers. See Teweles and Jones (1987) "rules of the game" discussed in the next section.

⁶Teweles and Jones (1987, p. 127). It is not indicated in this reference whether the "hypothetical" responses are based on a "typical" trader. Given frequent references to how "many traders act," we believe the discussion, tables and figure are indeed representative of actual trader interviews.

⁷Teweles and Jones, pp.126-127.

1. The TPI is not a test. One answer is no more correct than another. Rather, the TPI measures the trader's attitudes about gains and losses...at this point in time. The same TPI taken at a later date, especially after a large gain or loss or after a significant change in the trader's capital or income tax bracket, could well reveal a significant change in attitude. Trading curves derived from the TPI can easily measure such changes.

2. The dollar amount risked, gained, or lost during the interview should be considered by the trader as real dollars with real purchasing power which, if lost, would represent some varying degree of pain. Lost dollars can be replaced by the trader only from savings or checking accounts, new earnings, or other sources. The pain of replacing \$100 is obviously less than that of replacing \$1000, and the trader's feelings should reflect that difference.

3. The rewards to the trader should also be considered to be immediate and in cash, so that there will be virtually no lag between the time dollars are risked successfully and the ensuing payoff.

FOOTNOTES

4. Many traders will be content to answer the questions in the TPI subjectively. Others will prefer to have pencil and paper handy. Either approach is satisfactory if the decisions reflect the trader's personal preferences and are not merely the result of some mechanistic computation. The trader should consider each trade and each level of risk capital separately and not relate them to preceding or subsequent questions. Each question should be considered independently and answered carefully.

*Teweles and Jones, pp. 128-129.

*Teweles and Jones, pp. 139-140.

¹⁰Teweles and Jones, p. 141.

¹¹Teweles and Jones, p. 141.

¹²Becker and Siegel (1956, no page cited).

¹³Child and Whiting (1954, p. 508).

Chapter 2

Investor Preference for the Higher Moments

1. Introduction

The shape of investors' utility of wealth functions determines which characteristics of portfolios are relevant to their decisions. If $U(w)$ is utility as a function of wealth, w , and $f(w)$ is the probability density at w , then the expected value of utility can be written as

$$E_r[U(w)] = \int U(w)f(w)dw . \quad (2.1)$$

If $U(\cdot)$ is a polynomial, $U(w) = \sum_{i=1}^n a_i w^i$, then

$$E_r[U(w)] = \sum_{i=1}^n a_i E_r[w^i] , \quad (2.2)$$

where

$$E_r[w^i] = \int w^i f(w)dw \quad (2.3)$$

is the i^{th} moment around the origin of the distribution F with density f or, for short, the i^{th} moment of f . Therefore, the first n moments summarize the properties of the probability distribution of wealth which are relevant to an investor's choice. To complement the Taylor series argument, Müller and Machina (1987) give direct non-calculus proofs of the following relationships:

$U(\cdot)$ is an m -degree polynomial implies only the first m moments matter...

and its converse

only the first m moments matter implies $U(\cdot)$ is an m -degree polynomial....¹

2. Utility Functions

For this initial look at investor preferences, we shall assume the most general form of utility function, that is, we place no restrictions whatever on its shape beyond the usual stipulation that its first derivative be positive. Such a function can of course be concave or convex, or composed of any number of concave and convex segments.

Despite the almost complete lack of specific information regarding investors' tastes we can still apply utility analysis. When making his investment decision, the investor is confronted by a probability distribution of possible returns and a market price for each alternative investment. His decision, strictly speaking, depends on all of the observations and the appropriate probabilities for each alternative distribution. Equivalently we can state that the investment decision will depend on all of the moments of the probability distribution, for if the investor knows all of the moments, he will generally know the exact features of the probability distribution.²

To illustrate this claim, let us assume a general von Neumann-Morgenstern utility function $U(W_0+r)$, where r denotes a random variable, the return on investment; W_0 is the investor's initial wealth; and U is a general function about which we know nothing except that it is analytic and its first derivative is positive.

Expanding this utility function in a Taylor series³ around the point (W_0+Er) gives

$$U(W_0+r) = U(W_0+Er) + U'(W_0+Er) \cdot [W_0 + r - (W_0+Er)] \quad (2.4)$$

$$\begin{aligned}
 &+ \frac{U''}{2!} (W_0+Er) \cdot [W_0 + r - (W_0+Er)]^2 \\
 &+ \frac{U'''}{3!} (W_0+Er) \cdot [W_0 + r - (W_0+Er)]^3 \\
 &+ \frac{U''''}{4!} (W_0+Er) \cdot [W_0 + r - (W_0+Er)]^4 + \dots
 \end{aligned}$$

where Er is the expected value of the random variable, and the expression $U'(W_0+Er)$ denotes the first derivative of the utility function at point (W_0+Er) , $U''(W_0+Er)$, the second derivative, and so on.

Since we assume that a rational investor will select the investment which maximizes his expected utility, we can write $E[U(W_0+r)]$ as follows:

$$\begin{aligned}
 E[U(W_0+r)] = & U(W_0+Er) + \frac{U''}{2!} (W_0+Er) \cdot \sigma^2 \\
 & + \frac{U'''}{3!} (W_0+Er) \cdot \mu_3 + \frac{U''''}{4!} (W_0+Er) \cdot \mu_4 + \dots
 \end{aligned} \quad (2.5)$$

where σ^2 is the variance of the distribution, μ_3 is the third moment around the mean of the distribution defined as $E(r - Er)^3$, that is, an index of the distribution's asymmetry; and μ_4 is similarly defined as the measure of the distribution's peakedness. In general, expected utility depends on all the moments of the probability distribution, and to know if a particular moment is desirable or not, one has to know the coefficient of this moment, that is, the appropriate derivative of the utility function at the point (W_0+Er) .

Assuming that an individual is rational and always prefers more money to less, an increase in the expected return E_r , other things being equal, should also increase his expected utility. With respect to the other moments, no a priori determination can be made. For example, we do not know whether raising the variance (or some other higher moment), while keeping the rest of the moments unchanged, is desirable to investors.

2.1 Implications for Utility Functions

Although decision making under uncertainty depends, in general, on all the moments of the distribution, assuming a specific utility function allows us to concentrate on a subset of the distribution moments.

Three specific utility functions (illustrated in Figure 2-1) are examined below with regard to their implications for distributions of returns from investment. Two of these functions contain inflection points and one of them is unbounded. In some instances, a limiting relationship is required between the range of r and coefficients of the utility function in order to insure that marginal utility is positive for all r .

The first of the functions to be examined is a quadratic of the form

$$U(W_0+r) = a(W_0+r) + b(W_0+r)^2 + c \quad (2.6)$$

where $a > 0$, $b < 0$, and $r < a/-2b$.

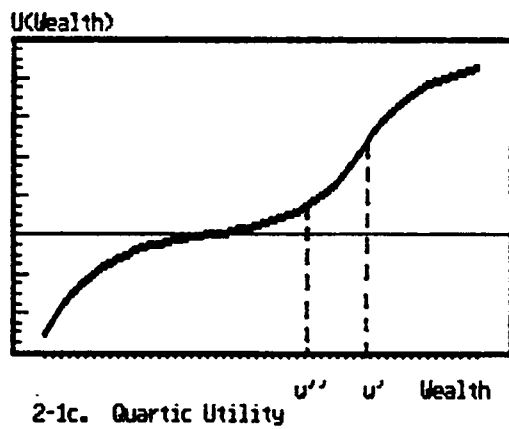
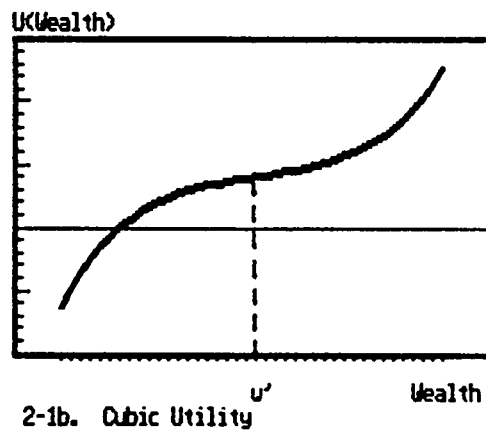
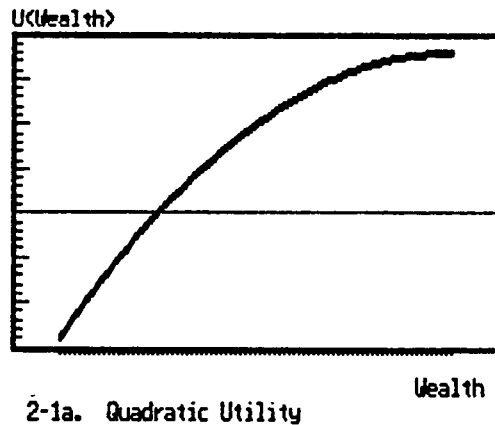


Figure 2-1. Second-, Third-, and Fourth-Degree Utility Functions.

Expanding this function about the point W_0 , we get

$$U(W_0+r) = U(W_0) + U'(W_0)r + (1/2)U''(W_0)r^2. \quad (2.7)$$

After calculating the first two derivatives of U at the point W_0 , and substituting these derivatives in the above equation, we get⁴

$$U(W_0+r) = a_1r + b_1r^2 + c_1. \quad (2.8)$$

This function is concave for $r < a/-2b$ and is illustrated in Figure 2-1a. This function is important in the financial literature because it illustrates the intuitive characteristic of decreasing marginal utility of wealth. It is important to keep in mind that the assumption of polynomial utility requires a restriction on the set of allowable outcomes if utility is to be nondecreasing. Moreover, even within its range of applicability, it involves the highly implausible implication of increasing absolute risk aversion, which both Arrow (1971) and Hicks (1962) denounced as "absurd."⁵

The second function is a cubic of the form

$$U(W_0+r) = a(W_0+r) + b(W_0+r)^2 + c(W_0+r)^3 + d. \quad (2.9)$$

where

$$b < 0, c > 0, \text{ and } c > b^2/3a > 0.$$

A single inflection point exists at $r = -b/3c$.

Expanding this function about the point W_0 , we get

$$U(W_0+r) = U(W_0) + U'(W_0)r + (1/2)U''(W_0)r^2 + (1/6)U'''(W_0)r^3. \quad (2.10)$$

After calculating the first three derivatives of U at the point W_0 , and substituting these derivative in the above equation, we get⁶

$$U(W_0+r) = a_1r + b_1r^2 + c_1r^3 + d_1. \quad (2.11)$$

This function must have both concave and convex segments.⁷ Such a function is illustrated in Figure 2-1b. From Figure 2-1b we can see that for low values of r the investor is a risk averter and therefore dislikes variance. This form of utility function is similar in many respects to the one presented by Friedman and Savage (1948); both of these utility functions have the property that they can reconcile the purchase of insurance with gambling, since in both cases investors prefer positive asymmetrical distributions (like a lottery's) but dislike negative asymmetry, and therefore buy insurance policies.

If it is assumed that the utility of percentage returns is derived from a utility of wealth function, the rate of return corresponding to the inflection point, w' ($r = -b/3c$), may be defined as a constant in terms of dollars of wealth and may, therefore, change in terms of percentage return when the initial value of wealth changes.

Alternatively, w' may be defined as constant in terms of percentage returns implying that the corresponding parameter of utility of wealth function changes as wealth changes. This is one of the Markowitz (1952) modifications to Friedman-Savage utility function.

The third function is a quartic of the form

$$U(W_0+r) = a(W_0+r) + b(W_0+r)^2 + c(W_0+r)^3 + d(W_0+r)^4 + e \quad (2.12)$$

where

$b = 0$ (sets left inflection point at W_0), $-1 < d < 0$, $0 < c < 1$,
and $[a + 3c(W_0+r)^2 + 4d(W_0+r)^3] > 0$.

This function has two inflections, at $r = 0$ and $r = -c/2d$.

Expanding this function about the point W_0 , we get

$$U(W_0+r) = U(W_0) + U'(W_0)r + (1/2)U''(W_0)r^2 \\ + (1/6)U'''(W_0)r^3 + (1/24)U''''(W_0)r^4. \quad (2.13)$$

After calculating the first four derivatives of U at the point W_0 , and substituting these derivative in the above equation, we get^a

$$U(W_0+r) = a_1r + b_1r^2 + c_1r^3 + d_1r^4 + e_1. \quad (2.14)$$

This function also has both concave and convex segments; in particular, it is bounded (is concave for large values of r). Such a function is illustrated in Figure 2-1c. From Figure 2-1c we can see that for both negative and very large values of r , the investor is a risk-averter and therefore dislikes variance. For positive, but moderate, values of r , the investor is a risk-lover. This form of utility function is similar in many respects to the Friedman and Savage (1948) "revised" utility function (Figure 1-2).

The values of the inflection points are subject to the same two interpretations as were applicable to the value w' in equation (2.6). These values may be constant in terms of initial wealth and may, consequently, vary in percentage terms as wealth varies, or they may be constant in terms of percentage returns implying that the utility of

wealth function changes as wealth changes. We view the latter as the more acceptable interpretation, though realizing the difficulties encountered when attempting to test such a function.

3. Investor Preference for Moments

$E[U(W_0+r)]$ can be found, for each of these functions, in terms of the moments of $f(r)$. The first derivative of $E[U(W_0+r)]$ can then be found with respect to each of the relevant moments in order to determine for which characteristics of the probability distribution of returns the investor has a preference (the derivative is positive) and for which an aversion (the derivative is negative).

Use of this analysis shows that, for each of the illustrated functions, the expected value of utility increases, *ceteris paribus*, as the mean return, μ_1 , increases.

For equation (2.3) expected utility decreases as $\mu_2 \equiv \sigma^2$ (the variance of return) increases. From equations (2.2) and (2.3), the expected value of utility equals

$$\begin{aligned} E[U(\cdot)] &= U(\cdot) + (1/2)U''\sigma^2 \\ &= U(\cdot) + b_1\sigma^2 \end{aligned} \tag{2.15}$$

where $b_1 = b < 0$. The higher moments (skewness, kurtosis, ...) are not present in equation (2.15) and do not affect investors with this order utility function.

For equation (2.6) expected utility decreases as σ^2 increases for values of $r < w' = -b/3c$; it increases as σ^2 increases for $r > w'$. From equations (2.2) and (2.6), the expected value of utility equals

$$\begin{aligned}
 E[U(=)] &= U(=) + (1/2)U''\sigma^2 + (1/6)U'''\mu_3 \\
 &= U(=) + (b_1 + 3c_1r)\sigma^2 + c_1\mu_3
 \end{aligned}
 \tag{2.16}$$

where $b_1 = b < 0$ and $c_1 = c > 0$. Investor utility is reduced by negative values of the third moment (distribution of r has a tail to the left) and increased by positive values (right tail). The higher moments kurtosis, ...) are not present in equation (2.16) and do not affect investors with this order utility function.

This preference pattern seems intuitively plausible, implying as it does that if expected return is below some value, w' , the possibilities of greater losses associated with larger variances are not compensated by the possibilities of greater gains. If expected return is higher, it becomes more worthwhile to run the risk of large losses, if this risk is associated with significant possibilities of large gains, i.e., if the dispersion is not predominantly in the negative direction.

For equation (2.9) expected utility decreases as σ^2 increases for values of $r < 0$ and $r > w'$; it increases as σ^2 increases for $0 < r < w'$. From equations (2.2) and (2.9), the expected value of utility equals

$$\begin{aligned}
 E[U(=)] &= U(=) + (1/2)U''\sigma^2 + (1/6)U'''\mu_3 + (1/24)U''''\mu_4 \\
 &= U(=) + (b_1 + 3c_1r + 6d_1r^2)\sigma^2 \\
 &\quad + (c_1 + 4d_1r)\mu_3 \\
 &\quad + d_1\mu_4
 \end{aligned}
 \tag{2.17}$$

Utility is increased by negative skewness if the mean is above w'' (the point of inflection in the marginal utility function) and is decreased if the mean is below w'' . Utility is decreased by increased peakedness (kurtosis) as the coefficient of μ_4 is $d < 0$.

FOOTNOTES

¹Möller and Machina (1987, p. 349).

²Requiring information regarding the distribution moments is equivalent to requiring detailed information on the distribution itself. See, for example, Mood and Graybill (1974, p. 117).

³Allen (1967, pp. 449-454).

⁴The derivatives of $U(\cdot)$ are

$$U'(\cdot) = a + 2b(\cdot) \text{ and} \\ U''(\cdot) = 2b.$$

The coefficients of the equation (2.5), a_1 , b_1 , and c_1 are

$$a_1 = a + 2bW_0 \\ b_1 = b \\ c_1 = aW_0 + bW_0^2 + c.$$

⁵The Arrow-Pratt measures of risk are

$$\text{absolute risk aversion, } R_A \equiv - \frac{U''(x)}{U'(x)} \text{ and}$$

$$\text{relative risk aversion, } R_R \equiv -x \frac{U''(x)}{U'(x)} = -xR_A.$$

$$\text{Increasing } R_A \text{ is defined by } \frac{d}{dx} \left[- \frac{U''(x)}{U'(x)} \right] > 0.$$

⁶The derivatives of $U(\cdot)$ are

$$U'(\cdot) = a + 2b(\cdot) + 3c(\cdot)^2, \\ U''(\cdot) = 2b + 6c(\cdot), \text{ and} \\ U'''(\cdot) = 6c.$$

The coefficients of the equation (2.8), a_1 , b_1 , c_1 , and d_1 are

$$a_1 = a + 2bW_0 + 3cW_0^2 \\ b_1 = b + 3cW_0 \\ c_1 = c \\ d_1 = aW_0 + bW_0^2 + cW_0^3 + d.$$

FOOTNOTES

⁷See Appendix to Chapter 2.

⁸The derivatives of $U(\cdot)$ are

$$\begin{aligned}U'(\cdot) &= a + 3c(\cdot)^2 + 4d(\cdot)^3, \\U''(\cdot) &= 6c(\cdot) + 12d(\cdot)^2, \\U'''(\cdot) &= 6c + 24d(\cdot), \text{ and} \\U''''(\cdot) &= 24d.\end{aligned}$$

The coefficients of the equation (2.13), a_1 , b_1 , c_1 , d_1 , and e_1 are

$$\begin{aligned}a_1 &= a + 3cW_0^2 + 4dW_0^3 \\b_1 &= 6cW_0 + 12dW_0^2 \\c_1 &= 6c + 24dW_0 \\d_1 &= 24d \\e_1 &= aW_0 + bW_0^2 + cW_0^3 + dW_0^4 + e.\end{aligned}$$

APPENDIX TO CHAPTER 2

Proof that a third order polynomial utility function must include a concave section followed by a convex section, as drawn in Figure 2-1b. Let us assume that $U(x)$ is a general third order polynomial of the form

$$U(x) = a_1x + a_2x^2 + a_3x^3 + e \quad (A2.1)$$

Von Neumann and Morgenstern (1953) shows that $U(x)$ is determined up to a positive linear transformation, so we may subtract e from the above equation and divide by a_1 to obtain, without loss of generality, the following function:

$$U(x) = x + bx^2 + cx^3 \quad (A2.2)$$

where $b = a_2/a_1$ and $c = a_3/a_1$.

Now making the conventional assumption that $U'(x) > 0$, we can write the first derivative as follows:

$$U'(x) = 3cx^2 + 2bx + 1 > 0 \quad (A2.3)$$

Since (A2.3) is a quadratic equation, the inequality will hold only if both of the roots of (A2.3) are imaginary, that is, if

$$(2b)^2 < 4 \cdot 3c \quad \text{or} \quad b^2 < 3c. \quad (A2.4)$$

From (A2.4) we can see that c must be positive, which means that the sign of μ_3 is positive. A second result which can be derived from (A2.4) is that b may be negative as well as positive. Taking the second derivative of $U(x)$, we get

$$U''(x) = 6cx + 2b. \quad (A2.5)$$

As c is positive, we can also determine that

$$\begin{aligned} U''(x) &\leq 0 & \text{if } x &\leq -b/3c \\ U''(x) &> 0 & \text{if } x &> -b/3c. \end{aligned}$$

If $b < 0$, the utility function is concave for some positive values of x (as well as all negative values of x). If $b > 0$, the utility function is concave for negative values of x . We shall assume that $b < 0$ to allow risk aversion in the relevant range. Thus for small values of x (or large negative values of x), marginal utility is decreasing and $U(x)$ is concave--the individual is risk-averse for low levels of wealth, but for high returns he shows a preference for risk, that is, the marginal utility increases with wealth.

Chapter 3

Securities Options Market: Overview and Terminology

1. Overview of Options Market¹

Every security and financial market has its share of definitions and concepts. Because of its unique characteristics, the options market probably has more than its share. This chapter will cover the basic concepts that are important to understanding the operation of the option market. Call and put options are explained, the institutional framework of option trading is discussed, and the players in the market are analyzed. The Appendix to the chapter contains terminology of the options market and option pricing theory.

2. Call Option

A call option is the right to buy a given number of shares of stock at a given price on or before a specific date.

As is evident by this definition, a particular call option will be characterized by four things: (1) the stock that the option refers to, (2) the number of shares of stock that the option holder has the right to buy, (3) the price at which the shares of stock can be bought, and (4) the time at which the option expires.

The stock involved in the option contract is called the *underlying stock*. The price at which the stock may be bought is called the *exercise price* or the *striking price*. The last date on which the option may be exercised is called the *expiration date* or the *maturity date*. The number of shares of stock involved in the option contract is

standardized on the various option exchanges to be 100 shares per contract.

2.1 Call Option Buying

The word *right* in the definition of the call option deserves some emphasis. It is the right rather than the obligation that makes the option market so interesting. Since the buyer has the right but not the obligation to exercise an option, the buyer will only exercise when it is profitable to do so. If the stock never rises above the striking price the buyer will obviously not exercise the option, because the stock can be purchased at a lower price in the market. The cost of the option contract will be lost. On the other hand, if the stock price rises above the striking price, then the buyer will profit from exercising the option. Thus, the option gives the buyer great potential for gain, while limiting losses to the cost of the option contract itself.

The profit profile for the option buyer is illustrated in Figure 3-1. The option giving the holder the right to buy 100 shares of stock is assumed to cost $\$[100 \cdot C]$. The profit is expressed in per-share terms. If the stock price is below the exercise price, X , on the expiration date, then the option expires worthless. The investor loses the initial investment in the option, which comes out to $\$C$ per share of stock. If the stock price is above $\$X$ per share, then the investor gains the difference between the cost of buying the stock and the higher market price at which the stock can be sold. The buyer's profit is

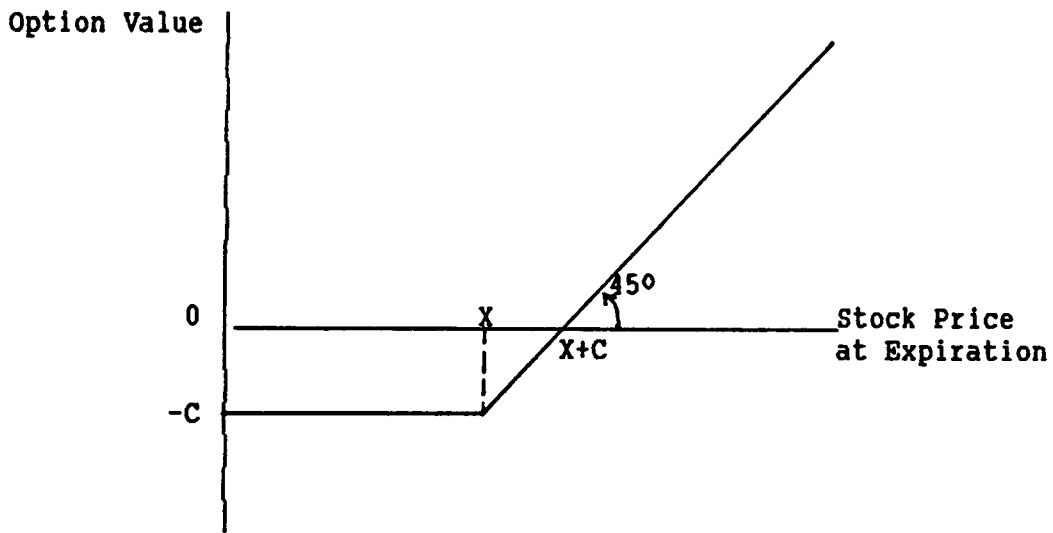


Fig. 3-1. Profit Profile for Call Option Buyer.

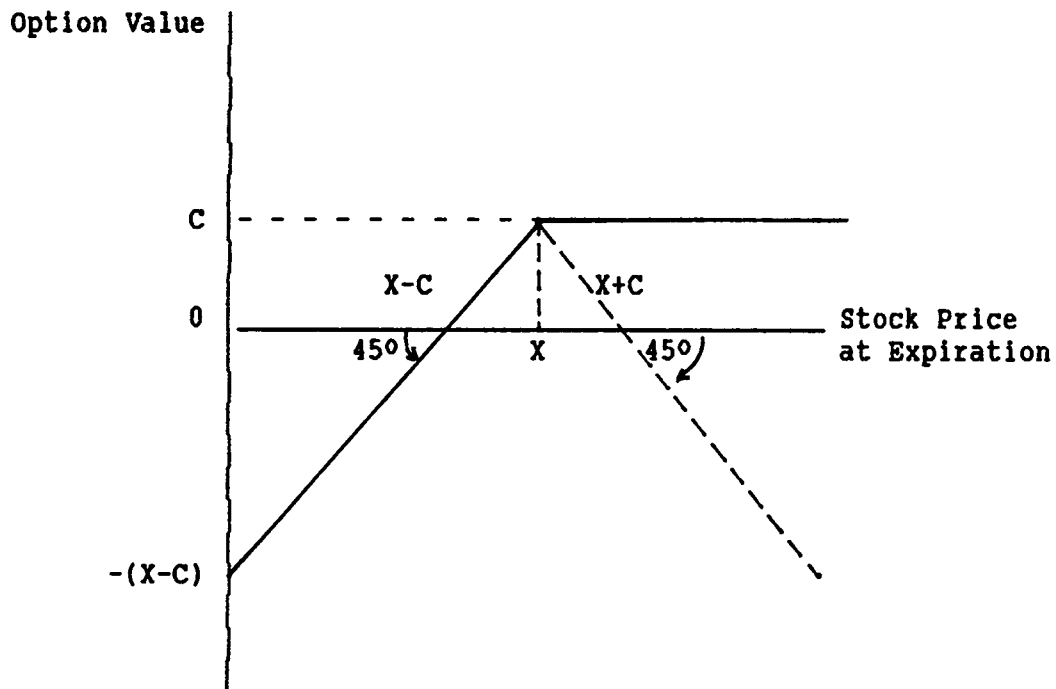


Fig. 3-2. Profit Profile for Call Option Writer for a Covered (Solid Line) and a Naked (Dotted Line) Position.

equal to the final stock price minus the \$X exercise price less the initial cost of the option, \$C.

2.2 Call Option Writing

An option contract is initiated between two parties. When an investor buys an option contract, there is someone on the other side of the contract who is agreeing to sell the buyer the stock at the exercise price. This person is called the option writer.

If the stock rises above the exercise price, the option will be exercised, and the writer will not make any gain above that price. If the writer owns the stock and writes the call, he will need to part with the stock at \$X per share, even though the stock is going for more than that in the market. On the other hand, if the stock drops in price, the option writer is left holding the bag--the option buyer will walk away without exercising, and the writer absorbs all of the loss. The writer thus has all of the potential loss from the stock, but has given all of the gain above the exercise price to the option buyer.

The writer may want to issue the option contract without holding the underlying stock. In this case the writer is said to write a *naked* option, which means the writer does not have the stock that he or she has agreed to sell to the buyer. Unlike the covered option writer who holds the stock on the option written, the writer of a naked option will not lose if the stock drops in price. However, there is a considerable risk if the stock increases in value. The profit profiles for the two types of call option writers are illustrated in Figure 3-2.

The writer receives the option premium at the time the option is issued. If the going share price for the option is $\$C$, then the writer gets $\$[100 \cdot C]$ at the time that the contract is issued.

3. Put Option

A put option is the right to sell a given number of shares of stock at a given price on or before a specific date.

The put option differs from the call option only by replacing the word "buy" with "sell." If the investor buys a put option, and the stock drops to $\$[X-5]$, the investor can buy 100 shares of the stock in the market and then turn around and sell it to the writer at $\$X$ per share, for a net gain of \$500. But if the stock is above $\$X$ per share, the investor will not exercise the option since the stock can be sold in the market for a higher price than it can through the option contract. While the call option will increase in value as the stock increases in price, the put option will increase in value as the stock decreases in price.

3.1 Put Option Buying

Once again, the use of the word "right" is important. If the contract involved the obligation rather than the right to sell the stock, then it would be the same as selling the stock short at $\$X$.² If the stock dropped in value, the short sale will yield a profit, but if the stock went above $\$X$, then the short sale will involve a loss, since the investor would have to buy the stock at the higher price and sell at the lower price. But with the put option, the investor does not face the risk from the stock rising in price. The stock does not have to be

sold at the exercise price. The investor will only do so if it is profitable. The maximum loss is the cost of buying the option.

The profit profile for the put option buyer is shown in Figure 3-3. If the stock price is above $\$X$, then the option will not be exercised, and the buyer will lose the initial cost of the put. If the stock is below the exercise price, then the investor's profit will be $\$X$ minus $\$S$, the stock price, minus $\$P$, the per-share price of the put.

3.2 Put Option Writing

Just as with the call option, for every put option bought there is an investor on the other side of the transaction who has written the contract. If the put option is exercised, the option writer will have to buy the stock at the exercise price, $\$X$, even though the going market price is lower. In return for this unfavorable possibility, the writer will receive a premium, $\$P$, from the buyer for issuing the option. Figure 3-4 illustrates the profit potential for the put writer.

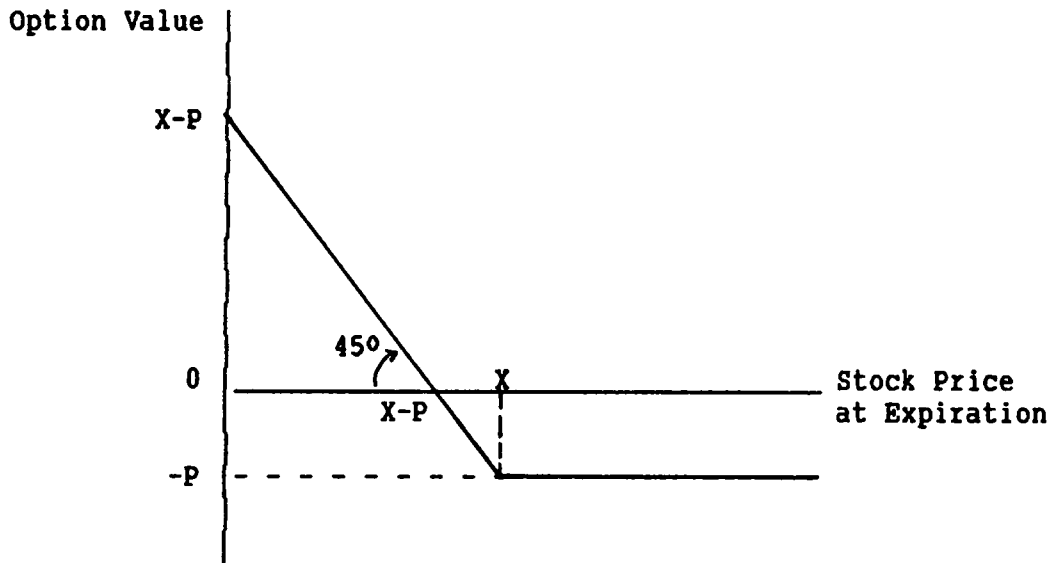


Figure 3-3. Profit Profile for Put Option Buyer.

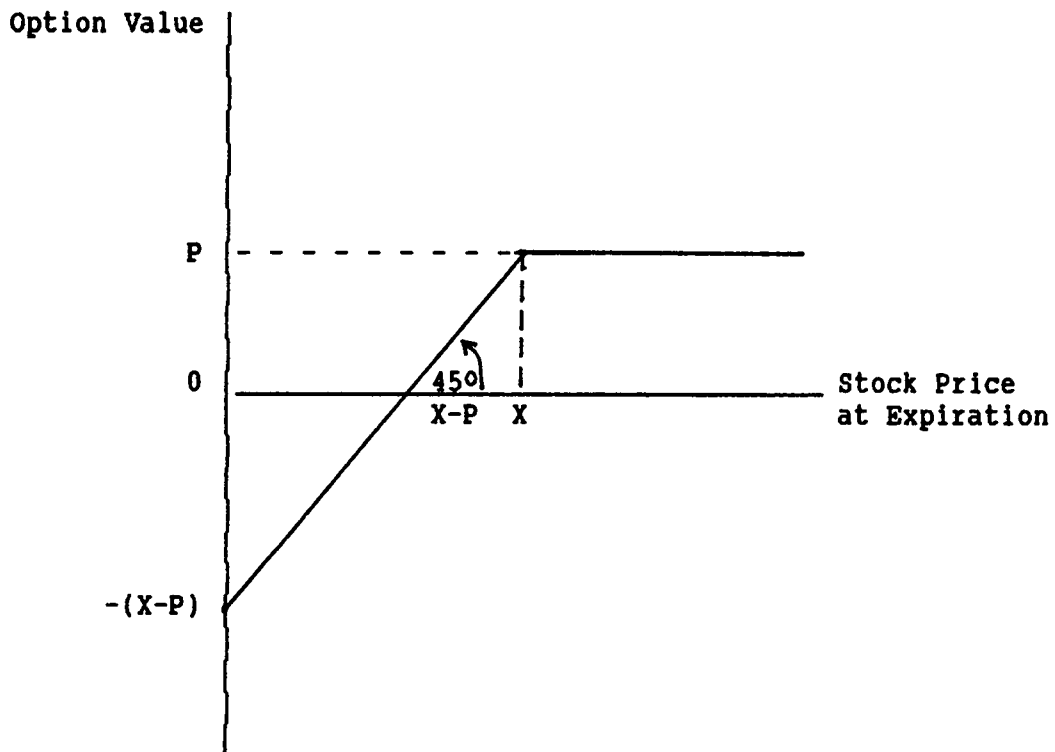


Figure 3-4. Profit Profile for Put Option Writer.

4. Investor Portfolios

Because the option buyer (writer) can combine the new option with any number of different existing positions, the question arises of whether a different existing position will lead the investor to value the new option differently. For example, suppose investor A has 100 shares of IBM and writes a (covered) call option contract for IBM. Investor B has no position in IBM and writes a (naked) call option contract for IBM. Will the price differ for these two contracts?

It is well known that if two assets with negative correlation are combined, the riskiness of the resulting portfolio will be less than the riskiness of either asset held alone. Diversification has a natural value in a market with risky assets. Since a linear combination of two negatively correlated assets is less risky than either of the two assets held alone, it might seem that the combination portfolio would be worth more than the sum of the values of the two assets. Nevertheless, the Value Additivity Theorem³ states that the price of the portfolio consisting of both assets will just be the sum of the prices of the two individual portfolios.

This is true because the equilibrium prices of assets must already reflect the value of any kind of linear portfolio manipulation. If the value of the two assets in combination exceeded the sum of the values of the assets alone, for example, then arbitrageurs could just buy the two assets and sell the "mutual fund" consisting of the combination. Similarly, if the values of the individual assets were less than the

"fund," the arbitrageurs could "unbundle" the combination and make a pure profit by doing so. Since this repackaging offers a sure profit, it cannot exist in equilibrium.

5. The Institutional Framework

Since option contracts are arranged between investors, one might anticipate considerable difficulty in coming to terms on the contract. The investors must not only negotiate the option price, but must also decide on the exercise price, the expiration date, and the number of shares of stock that the option will cover. There is the potential for an unlimited number of types of contracts. Without some restriction on the specification of the contracts, it would be impossible for an option exchange to list or trade them all. These difficulties limited interest in option trading before the opening of the Chicago Board Options Exchange (CBOE) in 1973.

The formation of the option exchanges has overcome this problem. The primary contribution of the option exchanges has been to standardize the option contracts, so only a limited number of types of contracts trade.⁴ Option contracts now have a limited number of exercise prices and expiration dates, and the number of shares traded represented by each option contract is standardized at 100 shares.

In addition to the standardization of option contracts, a second and related feature of the option exchanges is the interchangeability of option contracts. Contracts are not matched between individuals. All options are bought or written by the exchange through the Option

Clearing Corporation. No one option buyer is connected directly with another option writer.

It is this anonymity of the option participants that permits the secondary market to operate. Rather than needing to contact a particular option writer to terminate an option position, a buyer can simply sell it back to the exchange at the current market clearing price. That price is determined by the supply and demand of option contracts at the particular maturity and exercise price.

The restrictions on price increments, exercise price and expiration follow.

5.1 Price Increments

Options prices are quoted on a per-share basis, so that the cost of an option contract is given by multiplying the option price by the number of shares covered. Options trading under \$3, in terms of rights to one share, trade in sixteenths of a point (dollar), while those over \$3 trade in eighths.

5.2 Exercise Price

For prices below \$50, the exercise price is at increments of \$5 with the lowest possible exercise price being \$10. For prices between \$50 and \$200, the exercise price is at increments of \$10.⁵ For prices above \$200, the exercise price is at increments of \$20.

The exercise prices listed for a stock are determined by the stock's current price. If a stock is trading at \$41, it will have an option

with an exercise price of \$40. If the stock increases in price, a \$45 option will be added. Generally the next higher or lower exercise price is added after the stock price moves halfway between the existing exercise price and the new exercise price. Occasionally, an option will begin trade with two exercise prices listed, one on either side of the stock price. Since exercise prices are added as the stock price changes, the more volatile stocks will have a wider range of listed exercise prices.⁶

Should the stock split, say 2 for 1, during an option's lifetime, the exercise prices are adjusted to reflect the split. In this case, the new exercise prices are one-half the initial prices.⁷

5.3 Expiration Date

The expiration months for options are generally placed three months apart, with the furthest expiration month being no more than nine months away. Thus there will be three options trading at any exercise price: one with an expiration date that is three months or less away, the next three to six months away, and the next six to nine months away.⁸ The expiration date for an option is the Saturday after the third Friday of the expiration month.

5.4 Commissions, Taxes and Margins

A detailed discussion on taxes, commissions and margin requirements is omitted as it would necessarily be highly arbitrary. Taxes vary from year to year, from individual to individual. The commission charge

and margin requirement will vary across brokerage houses and market participants.

Generally, the effect of taxes and commissions on the profitability of option positions is ignored in the theoretical and empirical literature. These factors are not ignored because they are unimportant, but because their variability would make it impossible to give them a general treatment, and because they would add considerable complexity to an already complex subject with little return in additional insight.

FOOTNOTES

¹General treatments of options markets have increasingly short half-lives. An excellent current treatment can be found in Cox and Rubinstein (1985, chapter 3).

²A short sale involves selling stock the investor does not currently own, with the anticipation of covering the sale by buying the stock later at a lower price.

³Varian (1987, p. 61).

⁴"Limited" is a relative term. On any given day, for example, there are approximately 5600 different equity call option contracts traded. There are, in addition to equity options, options on the market indices, options on commodity futures, and so on.

⁵This is a general rule. Increasingly, however, we find \$5 increments in exercise prices across all ranges of stock prices. This may be due to the perceived improved accuracy of models like the Black-Scholes to price at-the-money calls. Thus, smaller increments allow traders to find a contract closer to at-the-money.

⁶While this is generally the case, high price stocks with \$5 increments in exercise prices (like IBM) will have relatively more contracts without being relatively more volatile than the average firm.

⁷See Microsoft test data in Chapter 9.

⁸This is not always the case. Quite frequently, options will be written one month apart as opposed to three. When this occurs, only the options with the three closest expiration dates are listed in The Wall Street Journal. For example, on March 1, 1988, the WSJ listed IBM calls expiring in March, April, and July; although, October options were also traded on the CBOE. Investors can access data on options with expiration dates not reported through their brokers or computer data bases.

APPENDIX TO CHAPTER 3

TERMINOLOGY

American Option--A put or call option that can be exercised any time on or before the expiration date. Options listed on the option exchanges are American options. See also European Option.

Asked Price--The quoted price at which a security is offered for sale.

At the Money--An option with an exercise price equal to the current market price of the underlying security. This is the "European" option definition; the "American" definition uses the discounted exercise price and the current market price of the underlying security. For example, if S is the current market price of the stock; X , the exercise price; r , the discount rate; and t , the time to maturity; then

$$S - Xe^{-rt} = 0$$

is an at-the-money option, where e is the base of the natural log. See European Option and American Option.

Bankers Discount--A discount calculated by subtracting the percentage discount from the face value of the bond. For example a Treasury bill with a face value of one million dollars, one year to maturity, and selling at a discount of 10.5% will have a price of $\$1,000,000(1-.105) = \$895,000$.

Bid Price--The quoted price for buying a security.

Call Option--An option that gives the buyer the right to purchase the underlying asset at a specified price (called the exercise price or the striking price), on or before a given date called the expiration date). In the case of a call option on a stock, the holder has the right to purchase 100 shares of the underlying stock.

Covered--Taking a position that assures the obligation generated from an investment strategy can be met. For example, an option writer can be covered by holding the underlying stock, since he will have the stock available to deliver if the option is exercised.

Deep in the Money--An option that has relatively large intrinsic value. For a call option, the exercise price is significantly below the current stock price. For a put option, the exercise price is significantly above the current stock price. [Note: the qualifiers "relatively large" and "significantly below (above)" have not been clearly defined in the literature.] See also Deep out of the Money.

Deep out of the Money--An option with no intrinsic value and relatively small probability of ever possessing intrinsic value. For a call option, the exercise price is significantly above the current stock price. For a put option, the exercise price is significantly below the current stock price. [Note: the qualifiers "relatively small" and "significantly above (below)" have not been clearly defined in the literature.] See also **Deep in the Money**.

Delta--The net exposure of an option position, measured by the change in the value of the position that will result from a change in the price of the underlying security. If an option will change by half a point with a one-point change in the stock, then the option has a delta of .5.

Dividend-Protected Option--All options were traded over-the-counter prior to the establishment of organized options exchanges in 1973. One of the characteristics of OTC-traded options is that their exercise price is reduced by the dividend paid on the underlying stock on the exdividend date. This automatic adjustment for dividends partially explains the omission of a dividend variable in the Black-Scholes (1973) formula.

European Option--A put or a call option that can only be exercised on the expiration date. Options listed on the option exchanges are American options, not European options. However the European option, by restricting the possible time of exercise, has provided a useful construct for developing option pricing models. "European" is not a geographic term; options traded on the European Options Exchange and the London Options Market are American options. See also **American Option**.

Exercise Price--The price at which an option can be exercised. For a call option, it is the price at which the security underlying the option can be purchased. For a put option, it is the price at which the underlying security can be sold. See also **Striking Price**.

Expiration Date--The date after which the option is void.

Hedge--Taking an investment position to reduce the risk of some other position. For example, an option position may be hedged by taking an opposing position in another option that moves closely with the first option, or by holding the underlying security according to the **Neutral Hedge ratio**.

Implied Volatility--The stock volatility that would equate the option price arising from an option pricing formula with the market price.

In the Money--An option with intrinsic value. For a call option an option with an exercise price that is below the current stock price. For a put option, an option with an exercise price that is above the current stock price. See *At the Money* for discussion of American and European distinction.

Intrinsic Value of an Option--For a call option, intrinsic value is the current price of the underlying security minus the exercise price of the option. For a put option, it is the exercise price minus the current price of the underlying security. For example, a call option with an exercise price of \$50 will have an intrinsic value of \$5 if the current stock price is \$55. The intrinsic value of an option cannot be less than zero.

Neutral Hedge--A hedge position that eliminates any profit or loss from small price changes in the underlying security. For example, if a call option moves half a point for a one-point movement in the stock (so the call option has a Delta of .5), then a hedge position consisting of buying 50 shares of stock and writing one call option will be unaffected by a small change in the price of the stock. With this position, price movement in the option position will exactly counteract the price movement of the stock. The option in this case will have a neutral hedge ratio of -.5.

Option Writing--Issuing an option contract, and thereby taking on the obligation to buy the underlying security (in the case of a put option) or sell the underlying security (in the case of a call option) at the specified exercise price at the option of the option buyer. The option writer receives the option premium for writing the contract. A covered writer holds the security underlying the option contract as a hedge for his position, while a naked writer does not hold the underlying security as a hedge.

Out of the Money--An option with no intrinsic value. For a call option with an exercise price that is above the current stock price. For a put option, an option with an exercise price that is below the current stock price. See *At the Money* for discussion of American and European distinction.

Premium--The current market price of the option. Premium is also occasionally used to refer to the difference between the market price of an option and its intrinsic value.

Risk-free Asset--A hypothetical asset that gives a known and certain payment with no risk: The riskless asset is used extensively in pricing securities and in constructing investment strategies. The best counterpart to the riskless asset in the marketplace is the U.S. Treasury Bill.

Straddle--The simultaneous purchase of (top straddle) or writing (bottom straddle) a put and a call option on the same underlying stock, with the same striking price and the same expiration date. Bottom straddles are profitable when the underlying stock has a large fluctuation (either up or down) in price; top straddles are profitable when the underlying stock does not fluctuate very much in price.

Striking Price--See Exercise Price.

U.S. Treasury Bill--A discount bond issued by the U. S. Treasury. Treasury bills are issued with maturities of three, six and twelve months. They are pure discount instruments, making no interest payments. Treasury bills are priced according to a bankers discount. See also Bankers Discount.

Warrant--A call option issued by a firm. A warrant generally has over one year to expiration at the time of issue; in some cases, there is no expiration date. In most cases, a firm will not have more than one warrant issue outstanding at the same time.

Writer--See Option Writing.

Chapter 4

The Options Market as a Source of Data for Estimating Utility of Wealth

1. Introduction

Many of the experiments reviewed in Chapter 1 were unable to establish more than a single inflection point in the utility function because measurement was constrained by a necessarily narrow range of monetary values (Mosteller-Nogee), past experience (Teweles-Jones), or grades (Becker-Siegel). The range of exercise prices (given the leveraged nature of stock options) allows us broaden the range of change (future wealth less current wealth).¹

In Section 2 we examine the payoff characteristics of stock options to emphasize the option's ability to satisfy both conservative and speculative investor needs. We illustrate the varying nature of the option through the typical choice problems of purchasing a call option versus taking a long position in the underlying stock and of purchasing a single call option from the set of options written on the same stock.

In Section 3 we discuss an important aspect of the problem of choosing among call options written on the same underlying stock. Empirical work has established significant mispricing biases when market prices are compared to the best (model) predicted prices.² We show, by way of a simple simulation experiment, that out-of-the-money options tend to be priced by risk-averse investors and in-the-money call options tend to be priced by risk-loving investors.

2. Option Payoffs

2.1 At-the-Money Call versus the Underlying Stock³

The characteristics of option payoffs are especially conducive to the gambling (or insurance) strategies of investors. In Figure 4-1, we show the expiration date profit (loss) potential of an at-the-money call option versus a long position in its underlying stock.

We can frame this analysis as the following problem. The market price for the stock (non-dividend) equals the exercise price for the call, \$175. The call option expires in six months and the market price of the call is \$10.50 on a per-share basis. Should the profit-maximizing investor choose to buy the stock or the call option?

The investor might begin by recalling what he has heard about options, that is:

(1) the call option is marketed as a very speculative investment instrument, so should the stock price increase by a large amount, the option (choice B) would be the better alternative, but

(2) the call option is also marketed as an investment with limited loss potential. That is, should the stock decrease in price, the loss to the holder of the call is limited to the price of the call. Should the stock price decrease by more than the cost of the call, the option (choice B) would be the better alternative.

(3) However, there must be some range of stock prices where holding the stock (choice A) is better. If this were not the case, no investor

interested only in the expected return would choose to hold the stock.

Figure 4-1 illustrates the profit (loss) potential of alternatives A and B. This is the diagram presented in every text and brochure⁴ to show the relative performance of call options. Without having the investor "read between the lines," Figure 4-1 presents the option from the perspective of an insurance policy against a large price drop in the stock. Dollar gains for the stock are higher for expiration prices greater than X and dollar losses for the stock are lower for expiration prices between X and X-C ($\$175 - 10.50 = \$164.50$). In effect, Figure 4-1 is a description of an investor who buys one share (one round lot) of the stock and one call (one call contract) and then rates them based on dollar gains or losses.

Financial economics teaches us that the rate of return is more important than the dollar return on an investment.⁵ Figure 4-2 is an illustration of the two alternatives from the rate of return point of view. Solving for the value of S which equates the returns on the two alternatives, we find $S = \$186.17$.⁶ That is, for stock prices greater than \$186.17 the call alternative outperforms the stock alternative. For stock prices less than \$186.17, the stock alternative is better. Again without reading between the lines, Figure 4-2 presents the option from the perspective of the speculative, leveraged, investment.

Figure 4-2 is, in effect, a comparison of equal dollar investments--one share of stock at $\$S = \$X$ and $\$X/\$C$ call options at $\$C$ each. The profit (loss) lines are still read in dollar terms, but the equal investment amounts provides a rate of return comparison as well. The

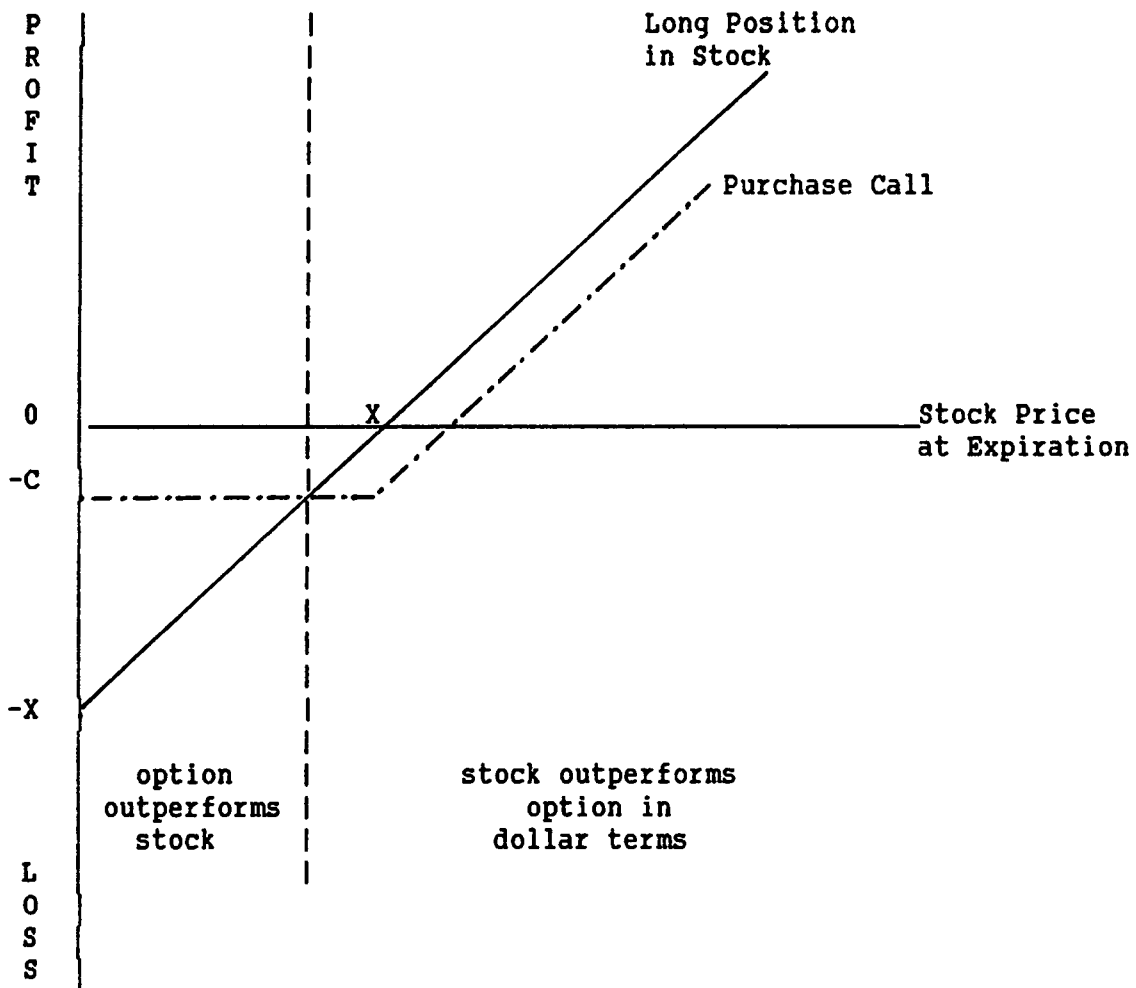


Figure 4-1. Relative Performance of At-the-Money Call and the Underlying Stock (Dollar Perspective).

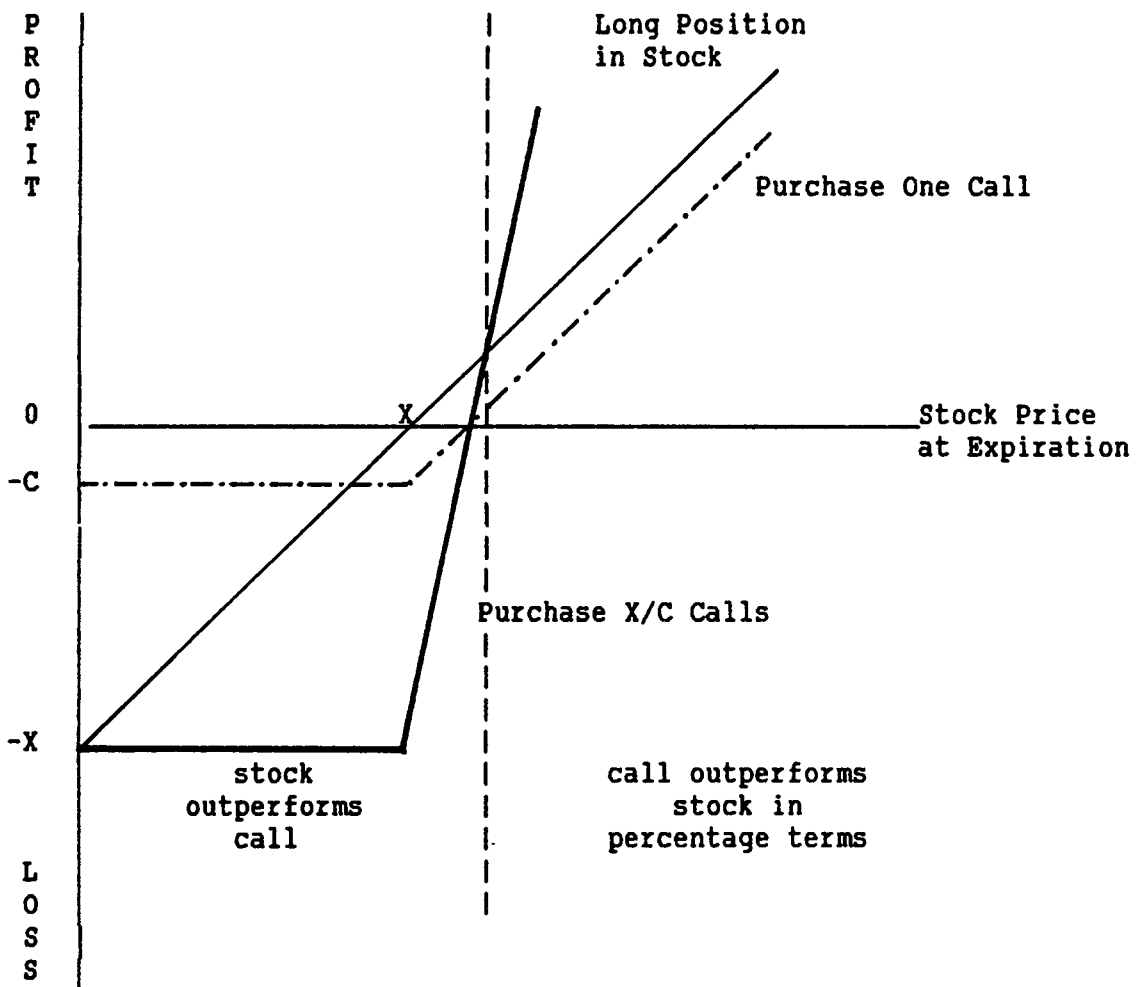


Figure 4-2. Relative Performance of At-the-Money Call and the Underlying Stock (Rate of Return Perspective).

dotted line in Figure 4-2 is to allow comparison to Figure 4-1.

Which figure is the appropriate one for the investor to employ to aid him in his choice problem? We can think of the two figures as the two extremes of the general portfolio choice problem which is beyond the scope of this exercise: Figure 4-1 represents the allocation of only the smallest part of the portfolio to the "risky" asset, while Figure 4-2 represents the allocation of the entire portfolio to the option. Given no information on investor preferences, neither of the two extremes is likely and we can reasonably posit an "intermediate" allocation. Figure 4-3 illustrates the case where alternative B is some proportion α ($0 < \alpha < 1$) of \$\$ allocated to the call option; the remainder ($1 - \alpha$) is invested in a riskless asset.

Given some intermediate level for α , the investor's choice between the stock and the option/bond mix is, therefore, a function of the expected distribution of the stock price over the lifetime of the call option. The more widespread the distribution about the current stock price, $S_0 = X$, i.e., the greater the variance, the more attractive alternative B becomes. The skewness of the returns distribution is also important to the investor; he will have a preference for positive (right-tail) skewness and an aversion to negative skewness.

In Figure 4-4 we superimpose two different return distributions to illustrate the point that were the boldprint distribution to be the relevant one, the call option alternative would be relatively more attractive to the investor.

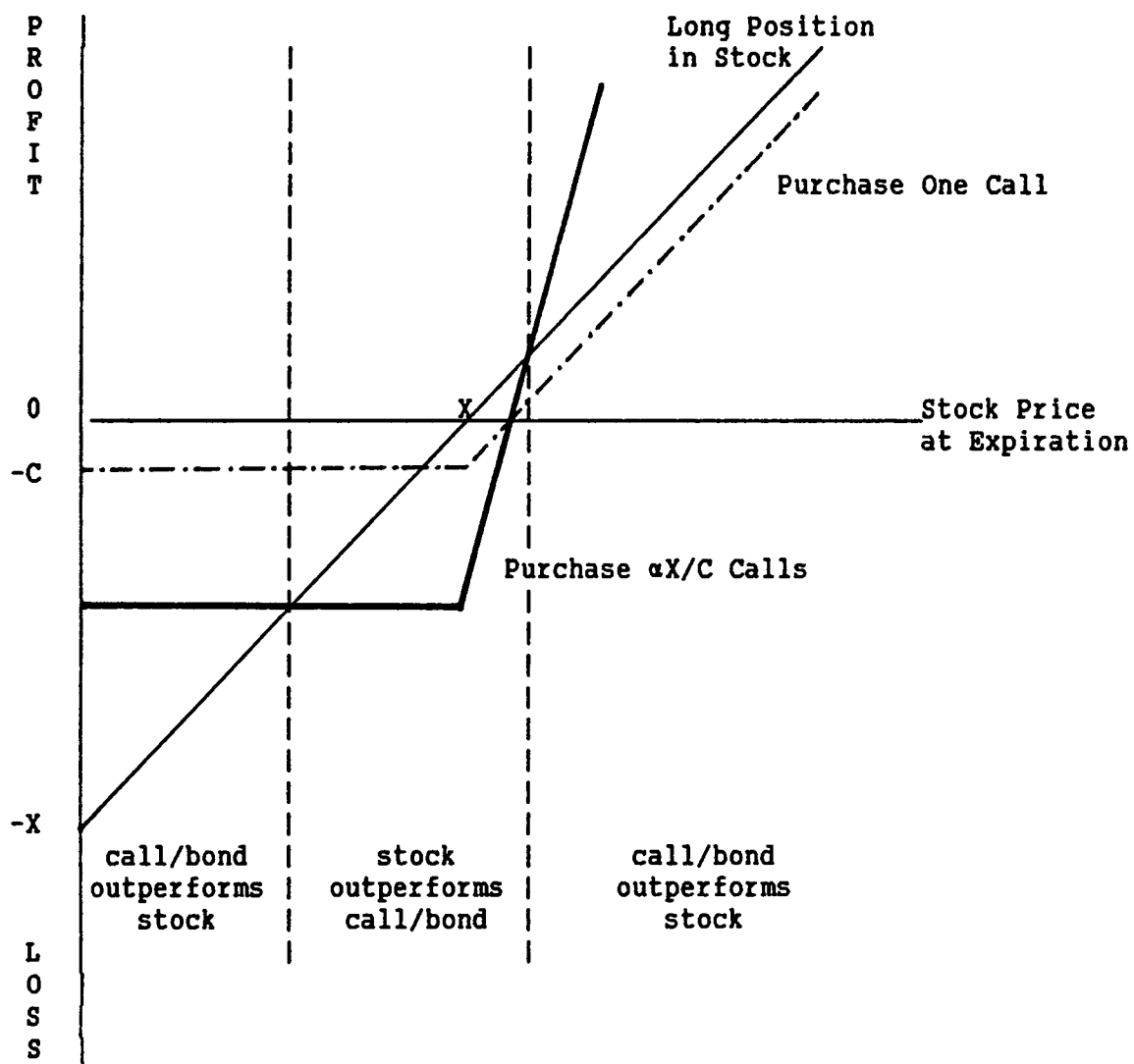


Figure 4-3. Relative Performance of At-the-Money Call and the Underlying Stock ($\alpha = .5$).

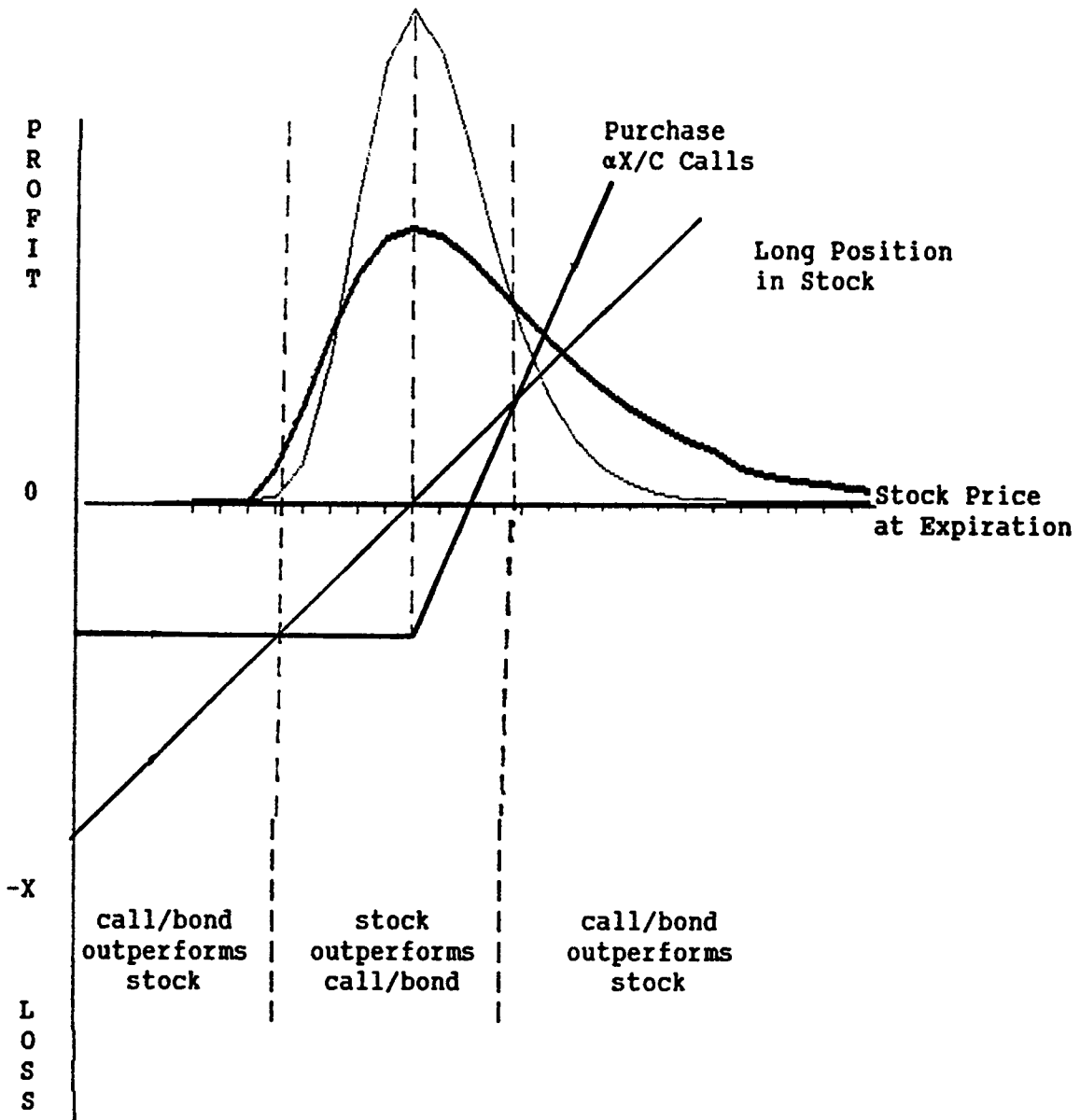


Figure 4-4. Performance of At-the-Money Call and the Underlying Stock
Investor Preference for Positive-Skewed Distribution.

2.2 At-the-Money Call versus In-the-Money and Out-of-the-Money Calls

Figure 4-5 illustrates the profit (loss) potential for the at-the-money call option ($X = \$175 = S$) of our previous example in comparison with an out-of-the-money call ($X = \$165 < S$) and an in-the-money call ($X = \$185 > S$).

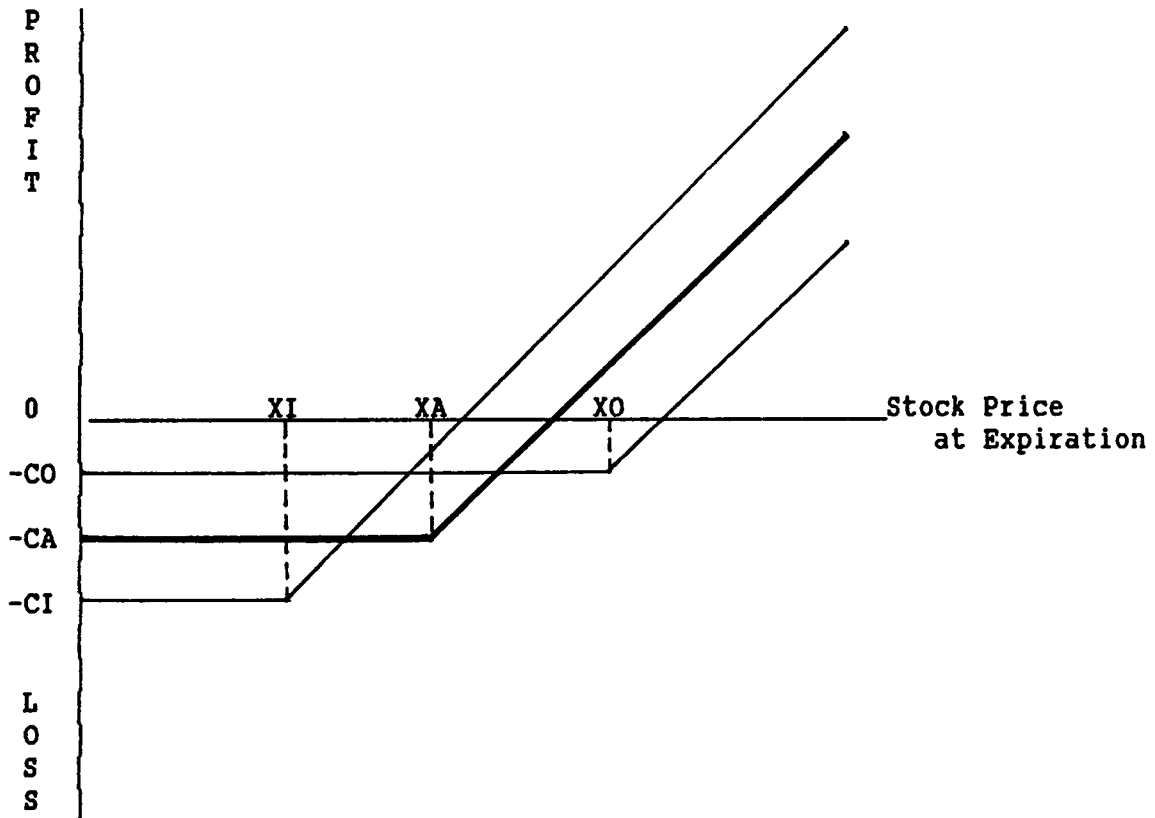


Figure 4-5. Relative Performance of At-the-Money Call and an Out-of-the-Money Call and an In-the-Money Call Written on the Same Underlying Stock.

3. Relative Attractiveness of Call Options on the Same Stock

Because we believe that the mispricing biases observed in call options come from investors exhibiting risk-averse preferences for some option contracts and risk-loving preferences for others on the same underlying stock, we now turn to the question of relative attractiveness of call options which differ only in their respective exercise prices.

3.1 Formulation of Hypotheses

Hypothesis IA:

Risk-averse traders bid down market prices relative to risk-neutral traders. Therefore, given a Friedman-Savage-Markowitz utility function and an expected distribution of final stock prices, for an out-of-the-money option, the call price conditioned on expected utility equal to zero is less than the call price conditioned on mean value equal to zero.

Hypothesis IB:

Risk-loving traders bid up market prices relative to risk-neutral traders. Therefore, given a Friedman-Savage-Markowitz utility function and an expected distribution of final stock prices, for an in-the-money option, the call price conditioned on expected utility equal to zero is greater than the call price conditioned on mean value equal to zero.

The basis of the hypotheses is that competition in a market composed of individuals who are indifferent to risk will force the price of an option up or down until its expected mean return is zero. That is, in a simplified world where the investor maximizes the expected end-of-period (expiration date of option) wealth, the call will be priced such that

$$\sum_{i=1}^N f(x_i)h(x_i) = 0 \quad (4.1)$$

where $f(x_1)$ is the probability of the price ratio (or final stock price), x_1 , and $h(x_1)$ is the payoff on the call option given the stock price.

Denote the call price which satisfies (4.1) as $C|_{mean=0}$.

Alternatively, in a market composed of individuals who are not assumed to be indifferent to risk, competition will force the price of an option up or down until its expected utility is zero. That is, again in the simplified world, the call will be priced such that

$$\sum_{i=1}^N f(x_i)U(h(x_i)) = 0 \quad (4.2)$$

where $U(h(x_i))$ is the utility of the option payoff $h(x_i)$. Denote the call price which satisfies (4.2) as $C|_{EU=0}$.

The relationship of the call option prices which satisfy these zero conditions signal the following individual risk preferences:

$$C|_{EU=0} \begin{bmatrix} > \\ = \\ < \end{bmatrix} C|_{mean=0} \begin{bmatrix} \text{Risk-Loving} \\ \text{Risk-Neutral} \\ \text{Risk-Averse} \end{bmatrix} \text{ Preferences .}$$

3.2 Return Distribution

For the purposes of the simplified analysis suggested in 3.1, it will be assumed that daily changes in the prices of stock listed on the securities exchanges are generated by a random process as follows:

$$S_t = S_{t-1}e^{X_t}, \quad (4.3)$$

where S_t is the price of a stock on day t and where X_t is a normally distributed random variable with mean μ_t and standard deviation σ_t .

Having defined the random variable as the price ratio, the next issue is to select a probability distribution function which best fits the observed data. On the basis of professionals' experiences, there are several statements which can be made with respect to the price movement of common stock. First, we know that the price cannot go below zero, but is theoretically unbounded for an upward move. Second, a 10% move in the stock price is more likely than a 50% move. Third, historically the stock market demonstrates an upward bias over the long term. Fourth, and perhaps less than obvious, the most likely future price for the average security is the current price.⁷

After a careful examination of the available statistical tools, the investor is likely to select the lognormal distribution function displayed in Figure 4-6 to model the price ratio random variable. This function possesses all the characteristics described above and has the distinct advantage of being easy to manipulate in the mathematical sense. The lognormal distribution has these mathematical characteristics because it is based on the normal distribution. When the random variable is characterized as lognormally distributed, the statement implies that the logarithm of the random variable is normally distributed.

The justification for the selection of the lognormal distribution model has been addressed by Osborne (1959) and Alexander (1961) among others. The consensus of the community appears to support the selection of the lognormal distribution function. However, several researchers, such as Fama (1963) and Mandelbrot (1963), have raised objections to the lognormal hypothesis. A point of departure in their analysis is that

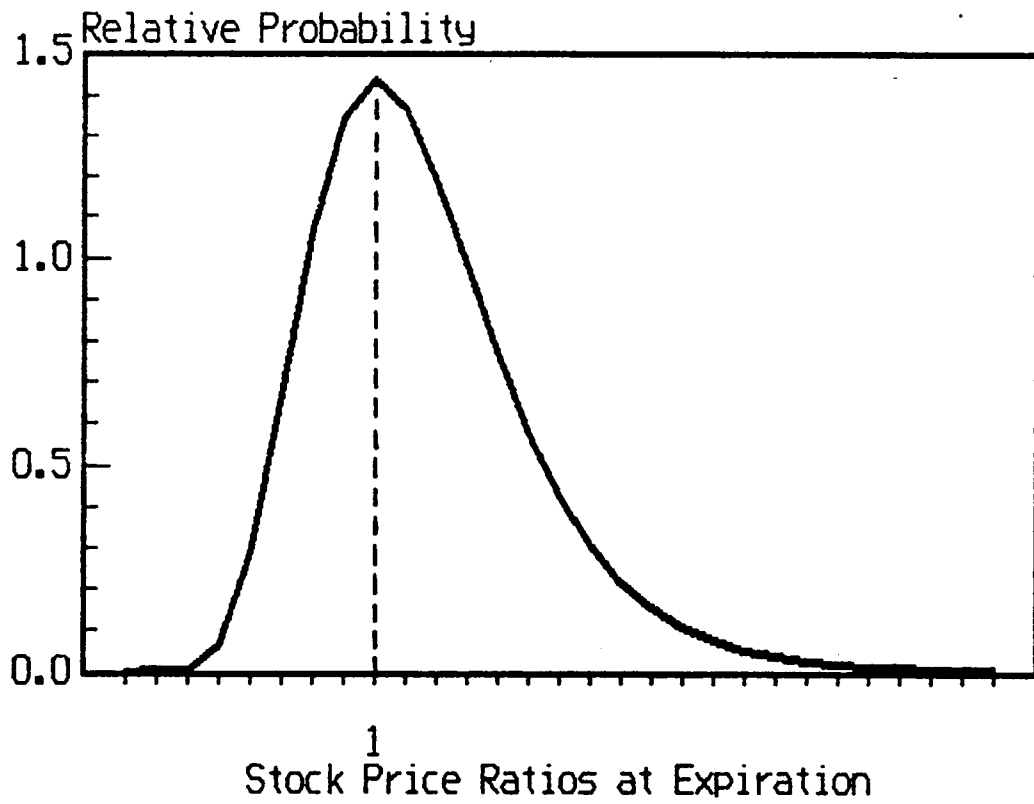


Figure 4-6. The Lognormal Return Distribution with Mode = 1.

while the price ratios are "close" to lognormal, they show a consistent tendency for a greater number of large price changes than expected.⁸

The work of Mandelbrot finds the distribution distorted from the classic lognormal form. Specifically, the distribution is higher in the peak and the tails but narrower in the medium price ratio range. From a position of practicality, however, the issue remains one of closeness of fit and utility of application. It is not clear that modifications of the lognormal assumption, proposed by Fama and Mandelbrot, will significantly improve the estimate of risk in a financial sense.⁹

Thus, it is also assumed that

$$E[(X_s - \mu_s)(X_t - \mu_t)] = 0. \quad (s \neq t) \quad (4.4)$$

In other words the ratio S_t/S_{t-1} is a lognormal random process. Dealing with a lognormal random variable involves the process of translating the random variable to a normally distributed random variable (i.e., taking the logarithm of the ratio), performing the necessary statistical transformations, and translating back into the original random variable data space. The nature of the physical process is such that the normally distributed random variable has a zero mean. This feature permits some simplification of the classical equations which describe the process.

The lognormal distribution function is described by

$$f(x) = \frac{1}{x\sqrt{2\pi V_n}} \exp \left\{ -\frac{(\ln x)^2}{2V_n} \right\} \quad (4.5)$$

where $f(x)$ is the density function for the random variable evaluated at

$x = S_t/S_{t-1}$, price ratio and V_n is the variance of the normal random variable, log of the price ratio.

In addition to this mathematical definition of the distribution function, three parameters are useful in characterizing the probability distribution and risk. These are the lognormal distribution function's mean, mode and variance:

$$\text{Mean}_{1n} = \exp (.5V_n) \quad (4.6)$$

$$\text{Mode}_{1n} = \exp (-V_n) \quad (4.7)$$

$$\text{Var}_{1n} = \text{Mean}_{1n}^2(\exp(V_n) - 1) \quad (4.8)$$

3.3 Utility Function

Since the outcome of the experiment comparing $C|_{EU=0}$ and $C|_{\text{Mean}=0}$ depends crucially on the numerical magnitudes of the payoffs, it is not sufficient to specify some general properties that our F-S-M utility function has to obey. We have to select some specific function. The following utility function was chosen for our "experiment":

$$U(x) = \begin{cases} -\frac{|x| + e^{a|x|} - 1}{e^a} & \text{for } -1 \leq x \leq 0 \\ \frac{x + e^{bx} - 1}{e^b} & \text{for } 0 < x \leq 1 \\ \frac{1 - e^{-cx} + x}{2 - e^{-c}} & \text{for } x > 1 \end{cases} \quad (4.9)$$

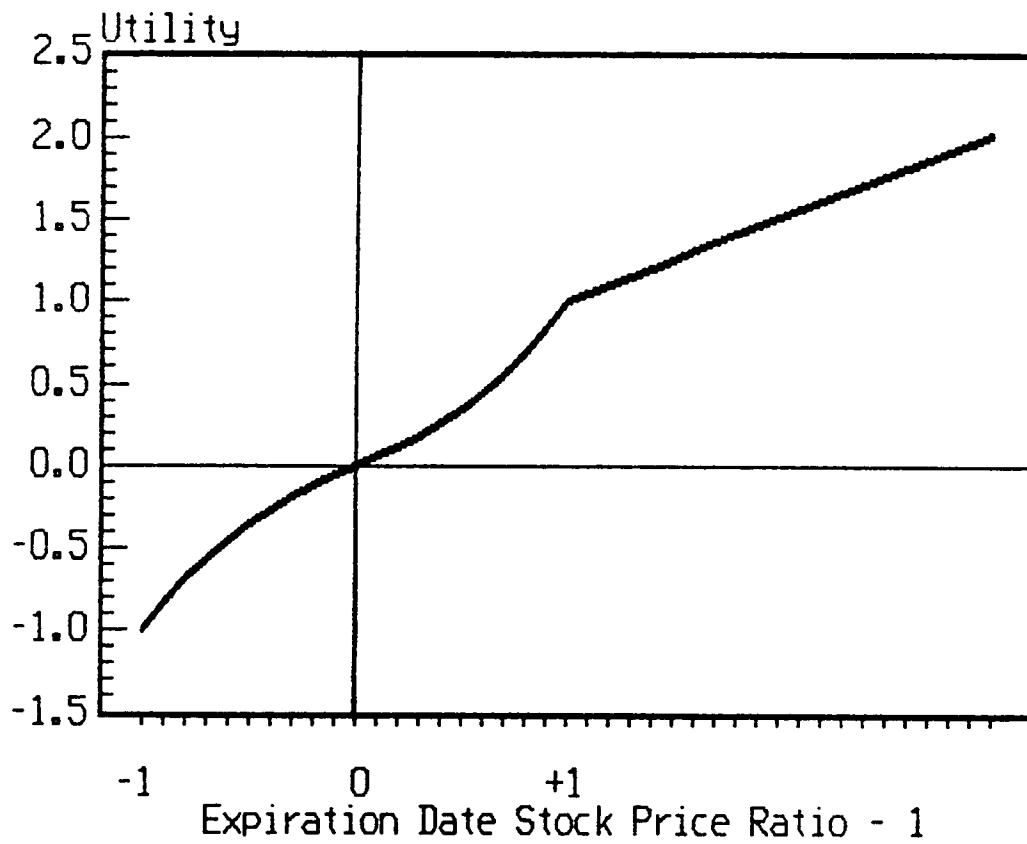


**Figure 4-7. Friedman-Savage-Markowitz Utility Function
Employed in Simulation Experiment.**

where a , b and c represent curvature parameters (the greater the value the steeper the function about the inflection point). Equation (4.9) is illustrated in Figure 4-7.

The general properties built into equation (4.9) concern the marginal utility of wealth and its rate of change as payoffs increase. The marginal utility of wealth is positive:

$$U'(x) > 0 .$$

In addition, an individual may have constant, increasing, or decreasing marginal utility of wealth. Accordingly, we employ a utility function with $U''(x) = 0$ at $x = 0$, $U''(x) > 0$ for $0 < x < 1$ and $U''(x) < 0$ for $x < 0$ to reflect the various segments of a F-S-M utility function.¹⁰

3.4 Simulation of Call Option Pricing

The results of our experiment are presented in Table 4-1 and Figure 4-8. For this particular utility function, we find that there exist generally positive differences between the call prices conditioned on zero expected utility and call prices conditioned on zero mean for in-the-money options and generally negative differences for out-of-the-money options. Note, however, some in-the-money options which also have negative differences.¹¹

4. Summary

The securities options market offers a particularly unique arena to observe individual preferences for risk. A single call option written

Table 4-1

Comparison of Call Prices Conditioned on Zero Expected Utility and Mean

Given the following data:

Stock price = \$165

Variance of Price Ratio = 6.25 %

Utility parameters $a = 2.3$, $b = c = 1.6$

Exercise Price	C EU=0	C Mean=0	Difference
120	\$63.41	\$60.82	\$ +2.59
125	58.08	56.15	+1.93
130	52.58	51.48	+1.10
135	47.77	47.12	+0.65
140	43.12	42.97	+0.15
145	38.20	38.82	-0.62
150	33.54	34.90	-1.39
155	30.16	31.49	-1.33
160	26.35	28.09	-1.74
165*	21.81	24.68	-2.87
170	19.42	22.10	-2.68
175	16.86	19.52	-2.66
180	13.85	16.94	-3.09
185	11.69	14.88	-3.19
190	10.31	13.06	-2.75

*At-the-Money Call Option

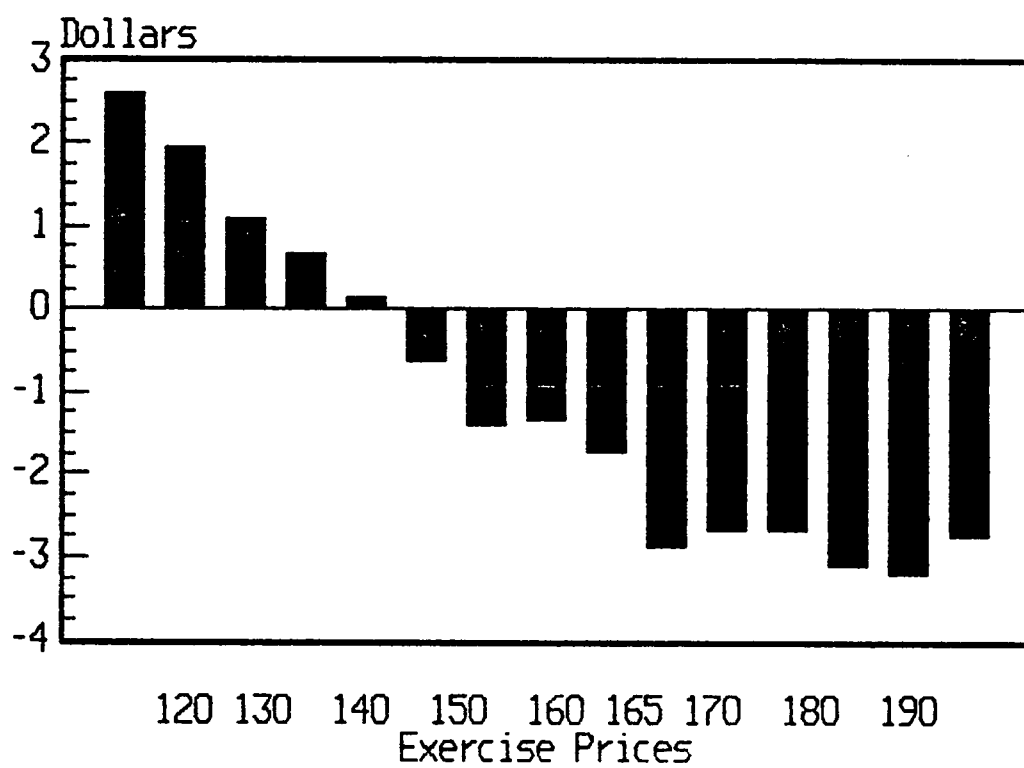


Figure 4-8. Call Price Differences: $C|EU=0$ and $C|Mean=0$.

on common stock serves as "insurance" over some final stock prices and "speculation" over other stock prices. Multiple options written on the same stock also exhibit varying relative levels of risk.

A simple simulation experiment, incorporating a Friedman-Savage-Markowitz utility function and a lognormal distribution of expected final stock prices, supports our view that risk-averse behavior generally occurs in out-of-the-money call options and risk-loving behavior generally occurs in in-the-money calls.

FOOTNOTES

¹This is especially true in the case of stock options like IBM and Microsoft which have a large number of contracts (exercise prices).

²The general direction of mispricing is that out-of-the-money call options are overpriced by the models and in-the-money calls are underpriced. See Chapter 5 for a more detailed description.

³See Chapter 3 for the definitions of In-the-Money, At-the-Money, and Out-of-the-Money options.

⁴This particular relationship is described either in the text and/or shown in a figure. Of course, there is some text I have not seen that does not have one.

⁵For example, Copeland and Weston (1986, p. 111).

$$\frac{S_T - S_0}{S_0} = \frac{S_T - (X + C)}{C}$$

For $S_0 = \$175$ and $C = \$10.50$, $S_T = \$186.17$.

⁷Fugler (1976, p. 42).

⁸See Chapter 7 for a discussion of historical and implied volatility. See Figure 12-1 for a comparison of expected returns distributions based on historical and implied variances.

⁹Reference on significance of Fama-Mandelbrot criticism of lognormal distribution.

¹⁰Furthermore, we selected this function so as to satisfy the normalization $U(0) = 0$, $U(-1) = -1$ and $U(+1) = +1$. This assumption was convenient because most of the positive payoffs are between 0 and 1 and all of the negative payoffs are between zero and -1. Should we choose to later examine other utility functions, the normalization then assures that resulting utility payoffs are of comparable magnitude, thus allowing results to reflect differences in curvature.

¹¹This may illustrate the observation by Mosteller and Nogee (1951, p. 374).

"There is not a sudden jump from no acceptances (to risk) to all acceptances at a particular offer,...the width of the discrimination band may be a characteristic that distinguishes groups of people."

Chapter 5

Review of the Literature on Option Pricing Models

1. Introduction

Historically, option pricing models have fallen into two categories: (1) ad hoc models and (2) equilibrium models. Ad hoc models generally rely only upon empirical observation or curve fitting and, therefore, need not reflect any of the price restrictions imposed by economic equilibrium. Equilibrium models deduce option prices as the result of maximizing behavior on the part of market participants.

The equilibrium approach to option pricing dates back to the work of Bachelier (1900). Although the economics and mathematics of Bachelier's work are flawed,¹ his research pointed the way for a number of attempts to describe an equilibrium theory of option pricing, including Sprenkle (1964), Boness (1964), and Samuelson (1965). All of these models essentially equate the value of an option to the discounted expected payoff to the option. The expected payoff to the option clearly depends on the assumed probability distribution of future stock prices. In addition, the proper rate to discount the expected payoff to the present must also be specified. Thus, to complete these models it is necessary to make specific and typically quite restrictive assumptions about individual risk preferences and/or the pricing structure in market equilibrium. These assumptions limit the generality (and practicality) of these early results.

Black and Scholes (1973) derive an equilibrium model of option pricing that avoids restrictive assumptions on individual risk preferences and market equilibrium formation. They demonstrate that it is possible to construct a portfolio involving positions in the stock and the risk-free asset where the return to the portfolio over a short time interval exactly replicates the return to the option. In addition, Black and Scholes show precisely how the composition of the portfolio must continually change in response to movements in the stock price and the passage of time such that the replication of the return to the option is maintained.

It is important to realize the implications of the fact that it is possible to replicate the return to options. The replication rules can be viewed as blueprints for a production technology which permits one to build synthetic options. As with any production technology, if the input markets are competitive and there is free entry into the industry, the price of the product must simply be the cost of production, i.e., there can be no excess profits.

The fact that synthetic options can be constructed from existing securities does not imply that option contracts are redundant securities with no economic purpose. In the absence of options markets, individuals or institutions could achieve the desired pattern of returns only by attempting to create options themselves using the Black-Scholes rules. The cost of this replication would necessarily exceed the price at which the options could be purchased if an option market existed

because, in a competitive market, the price will equal the cost to the least-cost producers. Thus, all but the lowest-cost producers of options gain an economic benefit from availability of options through an organized market. Moreover, all that is necessary for the Black-Scholes pricing result to obtain it that there exist enough potential producers of options who can (to a reasonable approximation) trade continuously. Such a condition appears to be met by most large security-trading firms. Hence, the price at which an option trades should be well approximated by the Black-Scholes replication cost.

2. Equilibrium Pricing Models

The major works in the development of modern option pricing theory are now reviewed in chronological order. Notation has been standardized to allow a more straightforward comparison of the models and their underlying assumptions.

2.1 The Bachelier Model

Bachelier assumes the stock price is a random variable, that prices are independent and identically distributed, and that

$$\text{Prob } \{\tilde{S}^* \leq S^* \mid \tilde{S} = S\} = F(S^* - S; T), \quad (5.1)$$

where tildes represent random variables and F is the cumulative distribution function of stock price changes. Equation (5.1) says that the probability that the stock price T periods from now, S^* , is less than or equal to a given number, S^* , given that the current stock price

S has assumed the value S , can be expressed as a function of the distance (S^*-S) and T .

This describes a Wiener process (or arithmetic Brownian motion). Bachelier's choice is unfortunate, for as T approaches infinity, then the $\text{Prob}\{S^* < S\}$ approaches $1/2$ for all S^* . Since nothing in this formulation restricts S^* to the positive numbers, there is a positive probability of negative stock prices; a violation of the property of limited liability.

Bachelier incorrectly deduces that (5.1) implies that the density function must be that of the normal,

$$F(S^*-S;T) = N \left[\frac{S^* - (S+\mu T)}{\sigma\sqrt{T}} \right], \quad (5.2)$$

where μ is the mean expected price change per time period, σ^2 the variance per time period, N the cumulative standard normal distribution.

Equation (5.1) is insufficient to deduce (5.2). Any member of the stable Paretian family of distributions satisfies equation (5.1). It must further be assumed variance is finite to deduce normality.²

Bachelier's next assumption suggests his specification of the process that generates the stock price is unsuitable as an equilibrium specification. He assumes that the mean expected price change per unit of time, μ , equals zero. Bachelier then assumes that the call is also priced to yield a mean expected return of zero. Bachelier views the stock market as a gamble; he feels that competition will reduce the

expected return to zero, which seems to deny both positive interest rates and risk aversion.

Bachelier applies the same logic to the pricing of the call option; he feels the call will be priced so that the current call price is the expected terminal call price. The terminal call price is the maximum of either the difference between the terminal stock price and the exercise price or zero,

$$C^* = \text{Max}[0, S^*-X],$$

therefore, Bachelier's model suggests that

$$C = E(C^*) = \int_X^{\infty} (S^*-X)N'(S^*)dS^*, \quad (5.3)$$

where $N'(S^*)$ is the normal density function for S^* .

Changing variables,

$$C = \int_{X-(S/\sigma\sqrt{T})}^{\infty} (Z\sigma\sqrt{T}+S-X)N'(Z)dZ, \quad (5.4)$$

where $Z = (S^*-S)/\sigma\sqrt{T}$, and

$$C = S \cdot N\left[\frac{S-X}{\sigma\sqrt{T}}\right] - X \cdot N\left[\frac{S-X}{\sigma\sqrt{T}}\right] + \sigma\sqrt{T} \cdot N'\left[\frac{S-X}{\sigma\sqrt{T}}\right]. \quad (5.5)$$

$N[\cdot]$ is the cumulative standard normal and $N'[\cdot]$ is the standard normal density function.

Note that as the time to expiration is increased the call price increases without bound. The cumulative standard normal expressions in the first two terms go to $\frac{1}{2}$ as $T \rightarrow \infty$; therefore, the first two terms go

to $\frac{1}{2}(S-X)$. In the third term, the argument of the standard normal density function goes to zero, therefore, the term goes to $\lim_{T \rightarrow \infty} \sigma\sqrt{T}$ (0.3989) which is infinity. Bachelier assumes the mean future price is positive, and because he specifies a process without drift, equal to the current stock price.

Thus, the major flaws in Bachelier's model which later theorists sought to correct are:

(1) the assumption of arithmetic brownian motion in the description of expected price movements implying both a positive probability of negative prices for the security and option prices greater than their respective security prices for large T ;

(2) the assumption that the mean expected price change is zero, suggesting both no time preference and risk neutrality; and

(3) the implicit assumption that the variance is finite, thereby ruling out other members of the stable-Paretian family except the normal.

2.2 The Sprenkle Model

Sprenkle (1964) partially removes the first two objections to Bachelier's formulation. Sprenkle assumes that stock prices are lognormally distributed, thus explicitly ruling out the possibility of negative prices for securities and removing the associated infinite prices for warrants.³ Further, he allows for drift in the random walk, thus allowing for positive interest rates and risk aversion.

The expected value of the option at the expiration date is

$$E(C^*) = \int_X^{\infty} (S^* - X)L'(S^*)dS^* , \quad (5.5)$$

where $L'(S^*)$ is a lognormal density function.

Changing variables,

$$\begin{aligned} E(C^*) = e^{rT}S \cdot N \left[\frac{\ln(S/X) + [r + (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] \\ - X \cdot N \left[\frac{\ln(S/X) + [r - (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] \end{aligned} \quad (5.6)$$

with r the expected average rate of growth in the stock price,

$e^{rT} \equiv E(S^*/S)$.

Sprenkle also assumes that "it is in general not true that the investor would be willing to pay a price for the warrant exactly equal to the expected value of it to him. In fact, he would be willing to pay exactly this price only if he were neutral to risk."⁴ Von Neumann-Morgenstern utility functions do not imply that risk-neutral individuals would be indifferent between the choice of C dollars today and a gamble with expected value $E(C^*)$ dollars at the expiration of the option. It must further be assumed that interest rates are zero. Additionally, it is selfcontradictory to assume that the random walk of stock prices has a positive bias, while assuming that investors pay the expected value for options.

The final form of Sprenkle's model containing a modification for risk is

$$E(C^*) = e^{-rT} S = N \left[\frac{\ln(S/X) + [r + (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] - (1-P_m) X = N \left[\frac{\ln(S/X) + [r - (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] \quad (5.7)$$

where P_m is an adjustment for the degree of market risk aversion.²

Sprenkle's model is flawed in that he ignores the time value of money.

2.3 The Boness Model

Boness (1964) allows for the time value of money and thus avoids Sprenkle's error. However, his assumptions are such that different levels of risk for the stock and the options are ignored. Boness assumes that:

- (A.1) The market is competitive in the sense that the equilibrium prices of all stocks of the same risk class imply the same expected rate of yield on investment. For convenience and in default of better information, all stocks on which options are traded are defined as members of the same risk class.
- (A.2) The probability distribution of expected percentage changes in the price of any stock is lognormal.
- (A.3) The variances of distributions of logarithms of expected percentage changes in the price of a given stock, where the distributions refer to changes over different time periods, are directly proportional to the respective time periods of the distributions. Thus, the variances of two distributions of expected price changes of the same stock are equal if the intervals over which the changes are anticipated are both three months. If the interval is 3 months for one distribution and six months for the other, then the variance of the second distribution will be twice as great as the variance of the first.

(A.4) Investors in puts and calls are indifferent to risk.

Boness expresses the expected terminal value of the option in terms of conditional expected values as

$$E(C^*) = [E(S^*|S^*>X) - E(X|S^*>X)] \cdot \text{Prob}(S^*>X). \quad (5.8)$$

From the definition of conditional expected values, the conditional expected value of S^* given that S^* is greater than X is

$$E(S^*|S^*>X) = \frac{\int_X^\infty S^* L'(S^*) dS^*}{\int_X^\infty L'(S^*) dS^*}. \quad (5.9)$$

Since the exercise price is nonstochastic, $E(X|S^*>X) = X$, and the probability that S^* will be greater than the exercise price is

$$\text{Prob}(S^*>X) = \int_X^\infty L'(S^*) dS^*.$$

Substituting these definitions into (5.8), Boness derives the expected terminal price of the option,

$$E(C^*) = \int_X^\infty (S^* - X) L'(S^*) dS^*. \quad (5.10)$$

To allow for the time value of money, he then discounts the expected terminal call price back to the present using the expected rate of return to the stock, r ,

$$C = e^{-rt} \int_X^\infty (S^* - X) L'(S^*) dS^*. \quad (5.11)$$

Changing the variables,

$$E(C^*) = S \cdot N \left[\frac{\ln(S/X) + [r + (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] - e^{-rT} X \cdot N \left[\frac{\ln(S/X) + [r - (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] \quad (5.12)$$

Boness' fourth assumption would suggest that he uses r as a proxy for the expected rate of return on the option, k [$e^{kT} \equiv E(C^*/C)$]. However, a different use of this assumption could make his task easier. If Boness notes that (A.4) implies that in equilibrium the returns to all assets would be equal, $r = k = r$, then he could use the appropriate risk-free rate and thus avoid the estimation of the expected average rate of growth in the stock price, r .

2.4 The Samuelson Model

Samuelson (1965) assumes stock prices follow geometric Brownian motion with positive drift, r , thus allowing for positive interest rates and risk premiums,

$$E(S^*/S) = e^{rT}. \quad (5.13)$$

If the option price grows at the rate k ,

$$E(C^*/C) = e^{kT}, \quad (5.14)$$

and the value of the option is

$$C = e^{-kT} E(C^*) \\ = e^{-kT} \int_x^\infty (S^* - X) L'(S^*) dS^*. \quad (5.15)$$

Changing variables,

$$E(C^*) = e^{(r-k)T} S N \left[\frac{\ln(S/X) + [r + (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] - e^{-kT} X N \left[\frac{\ln(S/X) + [r - (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] \quad (5.16)$$

With the additional assumption that $r = k$, then $e^{(r-k)T} = 1$ and we have the Boness model (5.12).

Samuelson examines the more difficult question—the value of an option if the return on the option is greater than the return on the stock, $k > r$. He suggests two situations in which $k > r$: (1) if the stock pays a dividend at the rate δ , it would be expected that at least $r + \delta = k$, and (2) if the market perceives the option to be more risky than the security, then the investors require that $k > r$. In the appendix to the Samuelson (1965) paper, McKean solves this problem for a perpetual option and lognormally distributed security prices. His solution is

$$\frac{C}{X} = \frac{(\theta - 1)\theta^{-1}}{\theta^{\theta}} (S/X)^{\theta}, \quad (5.17)$$

$$\text{where } \theta = \left(\frac{1}{2} - r/\sigma^2 \right) + \left[\left(\frac{1}{2} + r/\sigma^2 \right)^2 + 2(k/\sigma^2 - r/\sigma^2) \right]^{1/2} \quad (5.18)$$

It is interesting to note that if $\theta = 2$, then $C/X = \frac{1}{4}(S/X)^2$ and $C = S^2/4X$, which is the ad hoc formula Giguere (1958) assumes to define the relationship between the call and stock prices.

Samuelson's arguments as to why r and k might be expected to differ are general equilibrium in origin, but the implications of this analysis are at variance with the more general restrictions of Merton (1973). Samuelson finds with $k > r$ that there is a positive probability of premature exercise for the option. Merton shows that for a simple option on a stock which pays no dividends that it will never be advantageous to exercise before maturity.

2.5 The Black-Scholes Model

Black and Scholes (1973) demonstrate that it is possible to create a riskless hedge by forming a portfolio containing stock and European call options. The sources of change in the value of the portfolio must be the prices, since at a point in time the quantities of the assets are fixed. If the call price is a function of the stock price and the time to maturity, then changes in the call price can be expressed as a function of the changes in the stock price and changes in the time to maturity of the option. Black and Scholes then observe that at any point in time the portfolio can be made into a riskless hedge by choosing an appropriate mixture of stock and calls, e.g., if the hedge is established with a long position in the stock and a short position in the European call and if the stock price rises, then the increase in the value of the portfolio from the profit on the long position in the stock is offset by the decrease in the value of the portfolio from the loss which the increase in the stock price generates through the short position on the option and vice versa. If the quantities of the stock and option in the hedge portfolio are continuously adjusted in the appropriate manner as the asset prices change over time, then the return to the hedge portfolio becomes riskless. Therefore, the portfolio must earn the riskless rate.

The value of the hedge portfolio, V_H , can be expressed as the stock price times the number of shares of stock plus the call price times the number of calls in the hedge,

$$VH \equiv S \cdot QS + C \cdot QC, \quad (5.19)$$

where VH is the value of the hedge portfolio, QS the quantity of stock and QC the quantity of calls (for one share each).

The change in the value of the hedge, dVH, is the total derivative

$$dVH = QSdS + QCdC. \quad (5.20)$$

Black and Scholes use stochastic calculus to express dC, the change in the call price. Itô's lemma provides a technique by which certain functions of Wiener processes may be differentiated. If it is assumed that the stock price, S, follows a geometric Brownian motion, then Itô's lemma can be employed to express dC as

$$dC = \frac{\partial C}{\partial S} dS + \frac{\partial C}{\partial t} dt + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 dt. \quad (5.21)$$

Note that the only stochastic term in the expression for dC is dS. The rest are deterministic. Substituting (5.21) for dC in (5.20) yields

$$dVH = QSdS + QC \left[\frac{\partial C}{\partial S} dS + \frac{\partial C}{\partial t} dt + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 dt \right] \quad (5.22)$$

For arbitrary quantities of stock and options the change in the value of the hedge, dVH, is stochastic, but if the quantities of each asset are chosen so that

$$QSdS + QC(\partial C / \partial S)dS = 0, \quad (5.23)$$

i.e., $QS/QC = -\partial C / \partial S$, then the return to the hedge becomes riskless.

Setting $QS = 1$ and $QC = -1/(\partial C / \partial S)$ in (5.22) yields

$$dVH = -\left(\frac{1}{\partial C / \partial S}\right) \left[\frac{\partial C}{\partial t} + \frac{1}{2} \frac{\partial^2 C}{\partial S^2} \sigma^2 S^2 \right] dt. \quad (5.24)$$

In equilibrium, two perfect substitutes must earn the same return; therefore, since the hedge is riskless, its return must equal the risk-free rate,

$$\frac{dV_H}{V_H} = r \, dt. \quad (5.25)$$

Substituting (5.19) and (5.24) into (5.25) defines a differential equation for the value of the option,

$$\frac{\delta C}{\delta t} = rC - rS \left[\frac{\delta C}{\delta S} \right] - \frac{1}{2} \left[\frac{\delta^2 C}{\delta S^2} \right] \sigma^2 S^2, \quad (5.26)$$

subject to the boundary condition that at the terminal date, the option price must be equal to the maximum of either the difference between the stock price and the exercise price or zero,

$$C^* = \text{Max} [0, S^* - X]. \quad (5.27)$$

The differential equation (5.26) can be solved for the equilibrium call price. Black and Scholes transform the equation into the heat exchange equation from physics to find the solution, equation (5.29).

Call Price

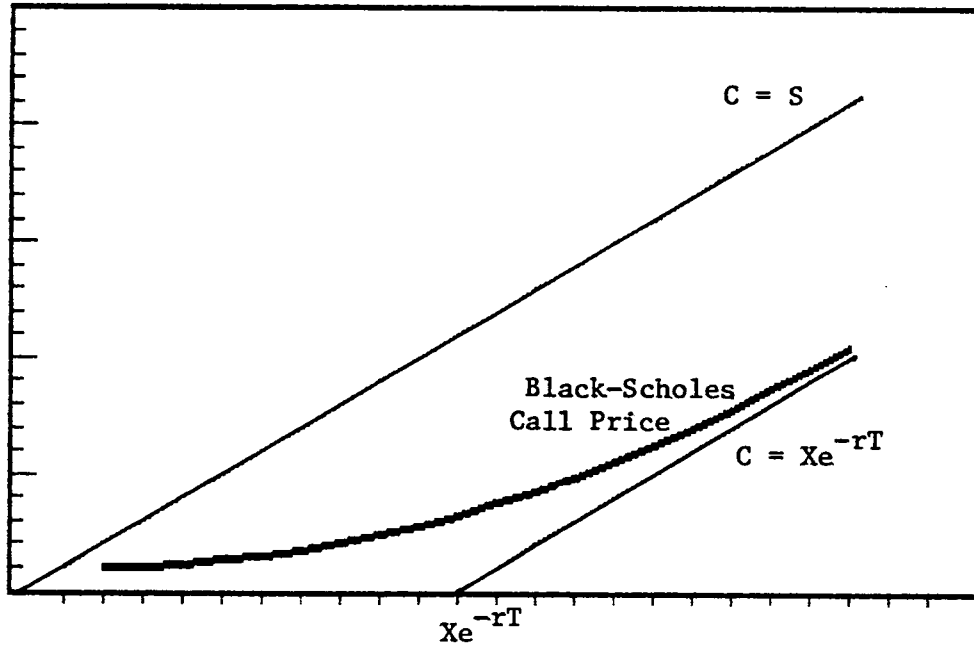


Figure 5-1. Black-Scholes European Call Price in Relation to its Boundary Conditions.

3. The Cox and Ross Technique

A more intuitive solution technique is suggested in a paper by Cox and Ross (1975). To solve equation (5.26), note two observations: First, whatever the solution to the differential equation, it is a function only of the variables in (5.26) and (5.27), i.e., r , S , T , σ^2 , and X . Second, in generating the hedge, the sole assumption involving the preferences of the individuals in the market is that two assets which are perfect substitutes must earn the same equilibrium rate of return: no assumptions involving risk are employed. This suggests that if a solution to the problem can be found which assumes one particular preference structure, it must be the solution to the differential equation for any preference structure that permits equilibrium; therefore, choose the structure which proves most tractable mathematically.

To apply this solution technique, assume the market is composed only of risk-neutral investors. In that case, the equilibrium rate of return to all assets is equal, $r = r = k$, then the option must be priced so that the current call price is the discounted expected terminal price,

$$\begin{aligned}
 C &= e^{-rT} E(C^*) \\
 &= e^{-rT} \int_X^\infty (S^* - X) L'(S^*) dS^*.
 \end{aligned}
 \tag{5.28}$$

Equation (5.28) may be solved as in previous models,

$$C = S \cdot N \left[\frac{\ln(S/X) + [r + (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] - e^{-rT} X \cdot N \left[\frac{\ln(S/X) + [r - (\sigma^2/2)]T}{\sigma\sqrt{T}} \right] \quad (5.29)$$

Equation (5.29) is the Black-Scholes pricing equation.

To derive an intuitive understanding of this equation, consider the equilibrium call price in a world of perfect certainty. Given certainty, the terminal call price would be a positive number (or no one would have purchased the call) equal to the terminal stock price minus the exercise price: $C^* = S^* - X$. In equilibrium in a world of certainty, the return to all assets must be in equilibrium, therefore, $r = \bar{r} = k$. Then since $S^* = S e^{rT}$, the current call price can be expressed as

$$C = e^{-rT}(e^{rT}S - X).$$

Substituting r for $\bar{r}$ and k yields

$$C = S - e^{-rT}X.$$

This expression differs from (5.29) only in the multiplication by the cumulative standard normal terms. These terms can be viewed as probabilities reflecting the uncertainty about the terminal stock price.

Note that the assumptions used by Black and Scholes are essentially those employed by Boness. However, Boness fails to demonstrate that a

riskless hedge can be created, a hedge which does not depend on the preference structure of the market--he does not justify his procedure in a general equilibrium framework. Regardless, a comparison with Boness' model can yield a better intuitive understanding of the terms in equation (5.29).

Boness shows that in a world of risk neutrality the equilibrium call price can be expressed in terms of conditional expected values as

$$C = e^{-kT}E(S^*|S^*>X)\text{Prob}(S^*>X) - e^{-kT}X\text{Prob}(S^*>X).$$

In a risk-neutral world the equilibrium expected rates of return on all assets would be equal, therefore $r = r = k$. Substituting r for r and k in Boness' pricing equation yields equation (5.29), the Black and Scholes solution. Hence, in a risk-neutral economy, the two terms in (5.29) have natural interpretations: the first term is the discounted expected value of the terminal stock price, given the terminal stock price exceeds the exercise price, times the probability the terminal stock price is greater than the exercise price. The second term is the discounted exercise price times the probability the terminal stock price exceeds the exercise price.

A note of caution here, for this analogy is only suggestive, as this interpretation is predicated on a world of risk neutrality. An uncritical reading of the above solution technique might suggest that this interpretation is valid for all worlds. This is not the case. The above solution technique is only a procedure to derive a solution to a

differential equation. It suggests that where a risk-free hedge can be established, the solution technique is independent of the degree of risk aversion in the economy, and therefore the mathematical solutions will be identical in any economy with any degree of risk aversion which permits a solution.

4. Adaptation of Black-Scholes Model to Pricing American Options on Stocks Which Pay Dividends

The value of a European call, denoted $c(S,T,X)$ provided by Black and Scholes (1973) is

$$c(S,T,X) = S \cdot N(d_1) - e^{-rT} X \cdot N(d_2),$$

where

(5.29)

$$d_1 = \{\ln(S/X) + (r + 0.5\sigma^2)T\} / \sigma\sqrt{T} ; d_2 = d_1 - \sigma\sqrt{T} ,$$

X is the option's exercise price, and $N(d)$ is the univariate cumulative normal density function with upper integral limit d .

The assumed absence of income distribution on the underlying security causes the B-S formula to overstate the value of an American call option on a stock with ex-dividend dates during the option's time to expiration. A dividend paid during the option's life reduces the stock price at the ex-dividend instant, and thereby reduces the probability that the stock price will exceed the exercise price at the option's expiration.

With the amount and timing of the dividend payment known, a simple approximation for the value of the American call is the value of a

European call

$$c(S', T, X), \quad (5.30)$$

where S' is the stock price net of the present value of the escrowed dividend payment, $S' = S - \alpha De^{-rt}$, and t is the time to the ex-dividend date. Note that by using the lower stock price the model's price is adjusted downward to allow for the stock price decline at the ex-dividend date.

Unfortunately, this approximation ignores a second dividend-induced effect in that it presumes that the call will not be exercised prior to expiration. Smith (1976) demonstrates that the American option holder may benefit from exercising early, just prior to the ex-dividend instant. To compensate for this possibility, Black (1975) proposed an approximate value equal to the higher of the values of a European call where the stock price net of the present value of the escrowed dividend is substituted for the stock price and a European call where the time to ex-dividend is substituted for the time to expiration, that is,

$$\max\{c(S', T, X), c(S, t, X)\}. \quad (5.31)$$

The first option within the maximum value operator assumes the probability of early exercise is zero, while the second option assumes it is one.

The American option on a stock with a known dividend, however, may be characterized by an early exercise probability between zero and one. For some time, this option pricing problem was thought to be insoluble. If the stock price follows a lognormal diffusion process, there exists

some non-zero probability that the dividend cannot be paid. Roll (1977) and Geske (1979) resolve the problem by assuming that the stock price net of the present value of the escrowed dividend follows a lognormal process. The solution to the American call option pricing problem, as provided by Whaley (1981), is

$$\begin{aligned} C(S', T, X) = & S' [N_1(b_1) + N_2(a_1, -b_1; -\rho(t/T))] \\ & - Xe^{-rT} [N_1(b_2)e^{-\rho(T-t)} + N_2(a_2, -b_2; -\rho(t/T))] \\ & + \alpha De^{-r^*t} N_1(b_2), \end{aligned}$$

where

(5.32)

$$\begin{aligned} a_1 &= \{\ln(S'/X) + (r + 0.5\sigma^2)T\} / \sigma\sqrt{T} ; a_2 = a_1 - \sigma\sqrt{T} , \\ b_1 &= \{\ln(S'/S_0^*) + (r + 0.5\sigma^2)t\} / \sigma\sqrt{t} ; b_2 = b_1 - \sigma\sqrt{t} , \end{aligned}$$

and $N_2(a, b; \rho)$ is the bivariate cumulative normal density function with upper integral limits a and b , and correlation coefficient ρ . S_0^* is the ex-dividend stock price determined by

$$c(S_0^*, T-t, X) = S_0^* + \alpha D - X ,$$

above which the option will be exercised just prior to the ex-dividend instant.

5. Summary

Modern option pricing theory began by acknowledging risk differences in stock and option portfolios. Accurate pricing of options was thus dependent upon investor attitudes towards risk and their proper formulation.

General equilibrium models advanced the applicability of the pricing process by recognizing the potential to establish a risk-free hedge and thus remove the need to consider varying risk preferences. The ability to meet this potential is discussed in the next chapter.

FOOTNOTES

¹Bachelier's model allowed for negative prices and required zero time preference and risk neutrality.

²Samuelson (1965, p. 508).

³Warrants are call options with more than one year to expiration. Some warrants have the same lifetime as the stock, i.e., no expiration. See Chapter 3 for a more detailed description.

⁴Sprenkle (1964, p. 435).

⁵Sprenkle was analyzing warrants and the risk being modeled is that associated with the leverage of the warrant versus the underlying stock. Sprenkle calls P_L "the price of leverage" and proposes $P_L < 0$ for risk-averse investors, $P_L = 0$ for risk-neutral investors, and $P_L > 0$ for risk-loving investors.

⁶Boness (1964, p. 167).

Chapter 6

Perfect Hedges, Self-financing and Risk Neutrality

1. Introduction

An essential feature of modern option pricing theory is the derivation of risk neutral valuation relationships (RNVRs) for the contingent claim. The relationship may be an equation relating the value of the contingent claim to the value of the underlying asset as, for example, the Black-Scholes (1973) formula, or it may be a differential equation relating the two asset values for which no general solution is known, for example, the Sprenkle (1964) model.¹ Because of the dominance of the general solution (B-S) model in both theory and application, our analysis will focus on the former model. If the valuation relationship is risk neutral, it is compatible with the assumption of risk-neutral preferences, under which all investors have the same expected rate of return--the riskfree rate, r .

RNVRs have been derived from two quite different general classes of model: the first class places no restrictions on investor preferences beyond the assumption of non-satiety, but assumes that asset trading takes place continuously; the second class of model places stronger restrictions on preferences, but makes the more general assumption that asset trading takes place at discrete intervals.

The first model is in the spirit of Black-Scholes (1973) and the Merton (1973) arbitrage analysis which consists of maintaining a riskless hedge which is self-financing (i.e., with a zero net

investment). From the assumption of non-satiety alone, the only results which can be obtained are those which depend upon the absence of riskless arbitrage opportunities. In this case where trading takes place continuously, both the underlying asset and the contingent claim are traded assets and at least one of them is infinitely divisible, and the price dynamics of the underlying asset can be described by an Itô process.² This no-arbitrage condition can be shown to imply a partial differential equation relating the value of the contingent claim to the value of the underlying asset, and this partial differential equation does not involve preferences. The solution of this differential equation subject to the appropriate boundary conditions is also preference free and thus provides a valuation relationship which is consistent with risk neutral preferences.

Alternatively, one can choose the simpler solution method which was originally suggested by Cox and Ross (1976) to value call options. This involves identifying the variables relevant to the pricing of the claim and then noting that once the riskless hedge can be shown to be feasible, the valuation should be independent of investor attitudes towards risk. It then follows that the solution to the problem can be obtained with arbitrary specification of investor preferences, in particular, universal risk neutrality under which the equilibrium rate of return on all assets is the riskless rate of return. The contingent claim can then be priced as the present value of its expected terminal payoffs, discounted at the riskless rate of interest.

However, it may not be possible to construct a riskless hedge in many cases, since continuous trading in the relevant assets may not be possible or, in some cases, the assets may not be traded at all. If the investor views the basic exogenous variable as the company's cash flow rather than its stock's return, then the stochastic process generating the cash flow cannot be used directly. Since cash flows are themselves not directly tradeable the construction of a riskless hedge involving them is not possible. As has been pointed out by Merton (1973), Rubinstein (1976), and Brennan (1979), this approach calls for the restriction of investor preferences so as to eliminate the need for construction and maintenance of a riskless hedge.³

2. Failure to Achieve Perfect Hedges

2.1 Differential Information

Kreps (1980) states that phenomena such as differential information, moral hazard, individual uncertainty about future tastes, etc., represent uncertainty which is, in general, not fully reflected in the security processes. That is why they are excluded. In those cases, it can hardly be expected that the model is complete, i.e., that there are valuations for all contingent claims:

At best, there are complete markets only in uncertainty which is so reflected.⁴

2.2 Redundancy

An inherent feature of the hedge approach is a kind of redundancy as pointed out by Hakansson (1979). If a contingent claim can be generated in a self-financing way by existing securities, the corresponding financial instrument does not enrich the existing financial markets; thus it is redundant. With respect to the pricing of contingent claims according to the hedge approach, Hakansson concludes:

So we find ourselves in the awkward position of being able to derive unambiguous values only for redundant assets and unable to value options which do have social value.⁵

2.3 Security Price Processes

Whether or not a contingent claim can be generated by a self-financing portfolio in the basic securities depends essentially on the distribution of the security price processes.

Probably the first treatment of the specification problem is due to Bachelier (1900). Bachelier specifies the price process of a basic security as arithmetic Brownian motion. The objection that prices cannot be negative resulted in the specification of the security price process as geometric Brownian motion by Sprenkle (1964), Boness (1964) and Samuelson (1965). This is also the specification that Black and Scholes (1973) and Merton (1973) choose in their models. Empirical studies have questioned this specification.

Feilietz (1971) and Boness, Chen and Jatusipitak (1974) examine whether the increments of the logarithmic prices are indeed stationary. Green and Fielietz (1977) reject the lognormality assumption.⁶ Mandelbrot (1963) and Fama (1963) favor a logstable specification in order to account for the large number of outliers. Upton and Shannon (1979) examine the empirical validity of the logstability and lognormality assumptions. Because of the observed fat tails, Blattenberg and Gonedes (1974) model the distribution of the logarithmic price increments by a symmetric stable and by a Student distribution. The systematic under- or over-estimation by the Black and Scholes formula in empirical studies is reduced, but not eliminated, when the geometric Brownian motion process is replaced with a diffusion process with a constant elasticity of variance. Cox and Ross (1976) extend the analysis to jump processes and obtain an option pricing formula in the case of a geometric Poisson process.

While the specification of the security price processes as geometric Brownian motion or geometric Poisson processes guarantees that every contingent claim can be generated by self-financing a portfolio in the basic securities, this is no longer true even in the case of simple mixtures of these processes. Merton (1976b) derives an option pricing formula in the case of a mixed diffusion-jump process. He assumes that the risk of infusions or withdrawals of funds associated with a generating portfolio can be diversified. An empirical study of this kind of price process was performed by Ball and Torous (1983). They

report estimation results for a stock return process which is the Bernoulli-mixture of Brownian motion. However, in those cases, Müller (1987) has provided proof of the inability to generate a perfect option hedge and "consequently classical arbitrage pricing is not applicable in those models."⁷ As Black reminds us:

If the volatility changes randomly in ways that don't depend on the stock price, then people may want to use options to bet on volatility changes and the formulas will have to be changed in ways that we don't yet understand.⁸

FOOTNOTES

¹Sprenkle's model had a "price of leverage" variable which was intended to measure the investor preference for (or aversion to) risk. See the section in Chapter 3 on Sprenkle.

²For a general description of the theory of stochastic differential equations of the Itô type, see McKean (1965). For an application to the consumption-portfolio problem, see Merton (1969). Briefly, Itô processes follow immediately from the assumption of a continuous-time stochastic process which results in continuous price changes (with finite moments) and some level of independent increments.

³Merton (1973) has stated that the B-S (1973) formula for the value of a European call option may be obtained in the discrete time model if: (i) there is a single investor whose utility function exhibits constant proportional risk aversion, (ii) returns on the underlying asset follow a lognormal distribution, and (iii) the underlying asset is aggregate national wealth.

Rubinstein (1976) has generalized Merton's conditions under by replacing the assumption of a single investor with the requirement that the conditions of aggregation are met so that securities are priced as though all investors had the same characteristics as a representative investor.

Brennan (1986) found a risk-neutral option valuation in discrete time by combining either (a) constant proportional risk-averse utility with lognormally distributed wealth (following Rubinstein), or (b) exponential utility with normally distributed wealth.

⁴Kreps (1980, p.49).

⁵Hakansson (1979, p. 723).

⁶If a security price process is specified as geometric Brownian motion, the increments of the logarithmic prices are normally distributed. This specification is referred to as the lognormality assumption. Accordingly, we speak of the logstability assumption if the increments of the logarithmic prices are assumed to be stable distributed.

⁷Möller (1987, p. 10).

⁸Black (1976, p. 177).

Chapter 7

Review of the Empirical Literature

1. Introduction

There have been a number of empirical tests of the Black-Scholes (B-S) option pricing model alone against the market data and also in comparison to other popular models against the data. There are two results that stand out from the mass of evidence. First, as a generality, the B-S pricing model works exceptionally well. There are few models in economics or finance that have such high predictive accuracy. On average, the pricing model works well; however, the result that we wish to stress is that the B-S model has some biases, appearing to work best with medium (three months and over) maturity, at-the-money calls. But what is more troublesome is there appears to be little consensus on the magnitude or even direction of the mispricing biases. For example:

1. Black (1975) reports that deep in-the-money (deep out-of-the-money) calls generally have B-S prices that are greater (less) than market prices for the 1973-1974 period.
2. Merton (1976) writes that practitioners observe B-S prices to be less than market prices for both deep in-the-money and deep out-of-the-money calls.
3. MacBeth and Merville (1979) report that B-S prices are on average less (greater) than market prices for in-the-money (out-of-the-money) calls for the 1976 calendar year. Rubinstein (1985) also found this bias in the 1976-1977 time period.
4. Emanuel and MacBeth (1981) report the striking price bias observed by Black to be the case in 1977 and 1978.

The more recent empirical work has emphasized the in-the-money and out-of-the-money biases. The explanations offered for these observed mispricings unfortunately are not able to also explain the deep in-the-money and deep out-of-the-money biases, and we do not find these extreme regions mentioned after Emanuel and MacBeth (1981).

Efforts to explain mispricing by the B-S models have emphasized either the estimation of the variance rate variable (stock volatility) or the manner in which the size and timing of dividend payments are handled in the pricing formulas.

2. Stock Volatility

Of the determinants in the call option pricing formulas, all but one are known or can be estimated with little difficulty. The exercise price and the time to expiration are terms written into the option contract; the stock price and the riskless rate are easily accessible market-determined values. The dividend information, if it is required, can be fairly easily estimated by casual inspection of the stock's historical dividend series. The problem parameter has traditionally been the expected volatility of the stock return.

An obvious candidate to proxy for the volatility expectation is an historical estimate obtained from the stock's expected return series. Black and Scholes (1972), as well as Galai (1978) and Finnerty (1978), use this estimate in valuing calls using the B-S option pricing model. Black and Scholes, however, recognize that a substantial amount of the observed deviation of the model's price from the market price may be

attributable to an "errors-in-the-variables" problem. In fact, they note that there is a tendency of the model to overprice options with high standard deviation estimates and to underprice options with low standard deviation estimates.

One approach to obtaining more accurate estimates of stock volatility has focused on using a better quality of historical data to obtain estimates of the stock volatility. To date, these studies have focused on the use of daily data. End of day prices (Jarrow and Rudd, 1983), open and closing prices (Garman and Klass, 1984), and extreme maximum and minimum daily prices (Parkinson, 1980) have also been utilized.

With the advent of intraday price data availability, the issue of estimating stock volatilities using daily closing data again comes into question. The Black-Scholes model as well as others are continuous time models which require estimates of an instantaneous variance rate. With more trading information, one would naturally expect that more accurate volatility estimates could be obtained. Kutner and Sweeney (1987) investigate this hypothesis, but find that volatility estimates using intraday data do not vary significantly as the sampling interval changes. A sensitivity to the number of observations used to estimate the volatility was observed, however, though the magnitude of the sensitivity was found to be small.

A second approach to obtaining more accurate estimates of stock volatility utilizes implied volatilities with various weighting schemes. Schmalensee and Trippi (1978) and Patell and Wolfson (1979), for

example, use an equally weighted average. Latane and Rendleman (1976), on the other hand, weight according to the partial derivative of the call price with respect to the standard deviation of stock return. In doing so, the standard deviation estimates of options which are theoretically more sensitive to the value of σ are weighted more heavily than those which are not. Chiras and Manaster (1978) follow a similar logic in using the elasticity of the call price with respect to standard deviation.

Regardless of the weighting scheme, however, there appears to be strong empirical support in favor of an implied volatility measure. Latane-Rendleman and Chiras-Manaster correlate the historical and the implied measures on the actual standard deviation of return and conclude that the implied estimate is a markedly superior predictor.

3.1 Major Empirical Tests: Volatility

Black and Scholes

Before listed option trading had commenced, Black and Scholes (1972) tested their model

$$C = S \cdot N(d_1) - e^{-rT} X \cdot N(d_2)$$

where

$$d_1 = \{\ln(S/X) + (r + \sigma^2/2)T\} / \sigma\sqrt{T}, \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

on over-the-counter (OTC) call options. From the diaries of an option broker, they recorded data on six months calls and six month straddles written on NYSE securities during a period from May, 1966 to July, 1969

(766 trading days). The sample consisted of 2,039 call contracts and 3,052 straddle contracts, written on 545 securities. To estimate the variance, they used the daily returns on the common stock for the year preceding the day on which the option was written.

Black and Scholes found that their model underpriced options on low variance stocks and near expiration options while it overpriced options on high variance stocks. Galai (1978) replicates and extends the B-S test using data from the first seven months of trading on the Chicago Board Options Exchange (CBOE). His results essentially reaffirm the findings of Black-Scholes. Geske, Roll and Shastri (1983) argue that these "biases" detected using OTC data should be expected because the OTC dividend protection (i.e., reducing the exercise price by the full amount of the dividend on the ex-dividend date) is imperfect.

Black

Black (1975) was the first to observe that actual prices on listed options tend to differ in certain systematic ways from the value by the B-S formula with regard to the relationship between the price of the underlying stock and the exercise price.

Options that are way out of the money tend to be overpriced, and options that are way in the money tend to be underpriced.²

No dates were reported, but the observations must have been in the range of mid-April, 1973, (the start of trading on the CBOE) and the end of 1974 (to allow for publication lag). No data was provided regarding the types of options investigated.

Merton

In a 1976 paper, Merton looks at the implications for option pricing should common stock return distribution be discontinuous (i.e., include a jump component). Merton noted, almost in passing, that:

...Practitioners often claim to observe in market prices for options....deep-in-the-money, deep-out-of-the-money, and shorter maturity options tend to sell for more than their Black-Scholes value, and marginally-in-the-money and longer maturity options sell for less.³

While this is one of the least substantiated observations (in terms of data and methodology) concerning mispricing biases, it is cited in almost every later study as an important paradox in option pricing theory.

MacBeth and Merville

The work by MacBeth and Merville (1979 and 1980) best illustrates the variance rate type of analysis. In their 1979 paper, based on the observation that the B-S model correctly prices at-the-money call options with at least ninety days to expiration, they assume the at-the-money implied variance rate is the "true" variance rate. They write:

Then, given $(\delta C / \delta \sigma^2) > 0$, the implication of this assumption is that the B-S model must yield call option prices which exceed observed market prices out of the money and call option prices which are less than the observed market prices for options in the money because the implied values of σ decline as the exercise price increases.⁴

The reasoning is that for out-of-the-money (relatively high exercise price), the variance is relatively (to implied variance) low, and the call price is low. As the B-S call price is calculated with the higher variance, the B-S price exceeds the market price. For in-the-money (relatively low exercise price), variance is relatively high, and the B-S price is less than the market price.

MacBeth and Merville tested the B-S model for a European call on a non-dividend-paying stock,

$$C = S' \cdot N(d_1) - e^{-rT} X \cdot N(d_2)$$

where

$$S' = S - \alpha D e^{-rT}, \text{ and}$$

$$d_1 = \{\ln(S'/X) + (r + \sigma^2/2)T\} / \sigma\sqrt{T}; \quad d_2 = d_1 - \sigma\sqrt{T}.$$

As the options used in the analysis were written on dividend-paying stocks, M-M adjusted for dividends by reducing the stock price by the present value (as measured by the riskfree rate, r) of dividends paid (assumes $\alpha = 1$)⁵ prior to the expiration date of the call option. To adjust for the possibility of early exercise, M-M eliminated those options from the sample where they were able to detect a significant probability of early exercise.

The data for M-M (1979 and 1980) consisted of daily closing prices of call options and stock prices from December 31, 1975 to December 31, 1976. The call options were those written on six "blue chip" stocks: American Telephone & Telegraph, Avon, Eastman Kodak, Exxon, IBM, and Xerox.

M-M forced the B-S model to "correctly" price at-the-money calls on the grounds that this would enable the model to better fit the data. In

order to examine the difference between B-S model prices and observed market prices, the M-M procedure must obtain an implied variance rate for an at-the-money option. Since at-the-money options are not usually available, they estimate this implied volatility using linear regression. Define M as percentage in- or out-of-the-money, i.e.,

$$M = \frac{S_{1,t} - X_{1,j}e^{-rT}}{X_{1,j}e^{-rT}} .$$

They then run the regression

$$\sigma_{1,jt} = \theta_{10t} + \theta_{11t}M_{1,jt} + \epsilon_{1,jt}$$

where $\sigma_{1,jt}$ is the implied standard deviation of option j on security i on day t .⁶ The estimated at-the-money implied volatility, $\sigma = \theta_{10t}$ is used as the true volatility of the security.

To facilitate comparison between options on different securities, M-M define the percentage difference between $C_{1,jt}$, the market price of the option, and the Black-Scholes model price of the same option, $C_{BS}(\theta_{10t})$,

$$V_{1,jt} = \frac{C_{1,jt} - C_{BS}(\theta_{10t})}{C_{BS}(\theta_{10t})} .$$

For a given stock, they provide regression statistics as well as plot the extent (V) of Black-Scholes mispricing for various options on various days in the sample, against the extent (M) that the option is in- or out-of-the-money. They summarize their findings as follows:

1. The B-S model predicted prices are on average less (greater) than market prices for in the money (out of the money) options.

2. With the lone exception of out of the money options with less than ninety days to expiration, the extent to which the B-S model underprices (overprices) an in the money (out of the money) option increases with the extent to which the option is in the money (out of the money), and decreases as the time to expiration decreases.
3. B-S model prices of out of the money options with less than ninety days to expiration are, on average, greater than market prices, but there does not appear to be any consistent relationship between the extent to which these options are overpriced by the B-S model and the degree to which these options are out of the money or the time to expiration.⁷

Emanuel and MacBeth

Building on the work by MacBeth and Merville (1980), this study added 1978 data for the same six blue chips to go with the M-M 1976 data. While the primary empirical effort was geared toward establishing values for θ , the elasticity of the instantaneous variance of the stock price with respect to the stock price, the Emanuel-MacBeth (1981) analysis also reports on the systematic biases of a constant elasticity of variance model in comparison to the B-S model.

The findings of Emanuel-MacBeth (E-M) support MacBeth-Merville results for the 1976 period. However, for the year 1978 they find the direction of the biases are reversed.

...We find periods in which it underprices in-the-money options and overprices out-of-the-money options as well as periods when it overprices in-the-money options and underprices out-of-the-money options. This changing nature of the mispricing is consistent across the stocks in our sample.⁸

In an independent work, Rubinstein (1985), using the most comprehensive data set to date, corroborates B-M that the biases are reversed in 1978 from the 1976 period.

4. Dividend Payments

Recent work suggests the bias is in part due to the limitations of the Black-Scholes formula in taking dividends and early exercise into account. But even stocks that do not pay dividends display these biases, though to a lesser degree.⁹ The work of Whaley (1982) and Sterk (1982, 1983a, 1983b) are representative of this line of inquiry.

4.1 Major Empirical Tests: Dividends

Whaley

Whaley (1982) compares the predictive ability of the three basic forms of the Black-Scholes option pricing formulas (on dividend-paying stocks) discussed in Chapter 5.

Equation (5.30) is the Black-Scholes (1973) European call, as adjusted for dividends in Merton (1973),

$$c(S', T, X) = S' \cdot N(d_1) - Xe^{-rT} \cdot N(d_2),$$

where

$$S' = S - \alpha De^{-rT},$$

$$d_1 = \{\ln(S/X) + (r + 0.5\sigma^2)T\} / \sigma\sqrt{T}, \text{ and}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

Equation (5.31) is the Black (1975) approximation for the American call,

$$\max[c(S', T, X), c(S, t, X)] ,$$

where

$$S' = S - \alpha De^{-r\tau} , \text{ and}$$

t = time to ex-dividend date.

Equation (5.32) is the Roll-Geske-Whaley (in Whaley, 1981) American call pricing formula,

$$\begin{aligned} C(S', T, X) = & S' [N_1(b_1) + N_2(a_1, -b_1; -\sqrt{t/T})] \\ & - Xe^{-rT} [N_1(b_2)e^{r(T-t)} + N_2(a_2, -b_2; -\sqrt{t/T})] \\ & + \alpha De^{-r\tau} N_1(b_2), \end{aligned}$$

where

$$a_1 = \{\ln(S'/X) + (r + 0.5\sigma^2)T\} / \sigma\sqrt{T} ,$$

$$a_2 = a_1 - \sigma\sqrt{T} ,$$

$$b_1 = \{\ln(S'/S_e^*) + (r + 0.5\sigma^2)t\} / \sigma\sqrt{t} , \text{ and}$$

$$b_2 = b_1 - \sigma\sqrt{t} .$$

The data employed in this study consisted of weekly closing prices for all CBOE call options written on 91 dividend-paying stocks during the 160 week period January 17, 1975 through February 3, 1978. Two main exclusion criteria were imposed on the data sample: First, the option's underlying stock had to have exactly one ex-dividend date during the option's remaining life. Second, options were eliminated whose premia were below fifty cents. Implied standard deviations were used as the volatility measure and α was set equal to one.

Whaley found the R-G-W American call formula "clearly dominates" the other two in terms of the grand mean and standard deviation of the relative prediction errors.¹⁰ In tests attempting to uncover systematic mispricing biases, Whaley gets results "that have been the cause of consternation." He finds the null hypothesis of a zero slope coefficient cannot be rejected at the 5 percent level for any of the three valuation models though the R-G-W model did better than the alternative models, with its slope coefficient being lower and less significant.

While we might expect the more sophisticated R-G-W model to outperform the simpler models, it is somewhat surprising to find results for the simpler models which contradict the previous research which employed them. The Whaley study is the first to find no significant mispricing biases by equations (5.30) and (5.31).

Sterk

Sterk (1982) tests the Black approximation (5.31) model against the Roll model (an earlier version of the R-G-W (5.32) model) using the testing procedure employed by MacBeth and Merville. Using a larger data set than M-M, the original biases reported by Black reappear in Sterk's analysis.

Daily call option and underlying stock price data were gathered during the month of October 1979 for CBOE options maturing in December 1979 and January and February 1980. The resulting sample consisted of data for one month on 181 options of 63 firms, in contrast to the M-M

sample of all options on six firms over a period of one year. Like Whaley, options on stocks which paid more than one dividend during the life of the option were eliminated from the sample, and α was set equal to one.

Sterk (1983a and 1983b) re-examines his data set using the R-G-W model. In response to Whaley (1982), Sterk also divides the data to separate in-the-money and out-of-the-money options. He writes:

To date, Whaley (1982) is the only author who has investigated the relationship between deviations and dividends. Upon examining both in- and out-of-the-money options simultaneously, he found no relationship between deviations and dividends.....Whaley did not attempt to uncover a relationship for in- and out-of-the-money options separately as is the case in this study.¹¹

Sterk's (1983b) results are reproduced in Tables 1 and 2 below. They show that in-the-money options on stocks with small dividends tend to be significantly overpriced and out-of-the-money options on stocks with low dividends tend to be significantly underpriced. Notably, however, this systematic bias reverses itself as the dividend increases.

5. Criticism of Previous Tests

Previous research, for the most part, has treated all call options alike. That is, they have aggregated across various companies (underlying stocks). Of equal importance, these studies have usually tested across various investor reference periods (periods when investor expectations of return distributions very likely changed).

Table 7-1
Median Difference Between Market and R-G-W Model Prices

In-the-Money Options	Number of Observations	MP-RGW	<u>MP-RGW</u> RGW
\$ 0 ≤Div< \$.25	320	-.012#	-.38%#
.25≤Div< .50	463	-.004	-.10
.50≤Div< .75	418	-.001	-.04
.75≤Div< 1.00	51	.007	.14
1.00≤Div< 1.25	85	.047*	1.26*
1.25≤Div	61	.059*	1.06*
Overall Sample	1398	-.001	-.35

*Significantly greater than zero at 5% level (one-tail test).

#Significantly less than zero at 5% level (one-tail test).

Table 7-2
Median Difference Between Market and R-G-W Model Prices

Out-of-the-Money Options	Number of Observations	MP-RGW	<u>MP-RGW</u> RGW
\$ 0 ≤Div< \$.25	280	.019*	2.12%*
.25≤Div< .50	462	.023*	1.85 *
.50≤Div< .75	479	.017*	1.06 *
.75≤Div< 1.00	105	-.049#	-2.55 #
1.00≤Div< 1.25	68	-.037#	-3.37 #
1.25≤Div	78	-.029#	-4.68 #
Overall Sample	1472	.011*	.87 *

*Significantly greater than zero at 5% level (one-tail test).

#Significantly less than zero at 5% level (one-tail test).

If the prime concern of these studies were our ability to correctly price equity options, then surely some attention should be paid to the particular return distribution of the underlying stock. But, as we noted in Chapter 6, Müller (1987) has shown that the result of combining just a few stocks into a portfolio will hide stock-specific traits. While the aggregation over several firms may not be enough to change the sign of the mispricing bias,¹¹ it may indeed affect the magnitude of the mispricing.

In the studies which looked at options written on a single stock, MacBeth and Merville (1979) and Emanuel and MacBeth (1981), one calendar year is the shortest sample period. MacBeth and Merville test over 1976; Emanuel and MacBeth test 1976 and 1978.

As we believe changing investor preferences toward risk is the primary reason for option mispricing, investor expectations should be relatively unchanged within a given sample period in order to reveal the true relationship between preferences and mispricings. The three figures below illustrate how major influences on expectations differed in a given year and differed between years (1976 and 1978, in particular).

Figure 7-1 is a plot of weekly closing prices for IBM for the years 1975-III through 1979-I. We make no assertions on the time frame of stock returns which investors use to form their expectations (calculate value of Black-Scholes volatility measure). Empirical work in this area by Latane and Rendleman (1976) and Chiras and Manaster (1978) shows that while implied volatility measures increase and decrease with historical measures, implied volatility is a better predictor of future returns than

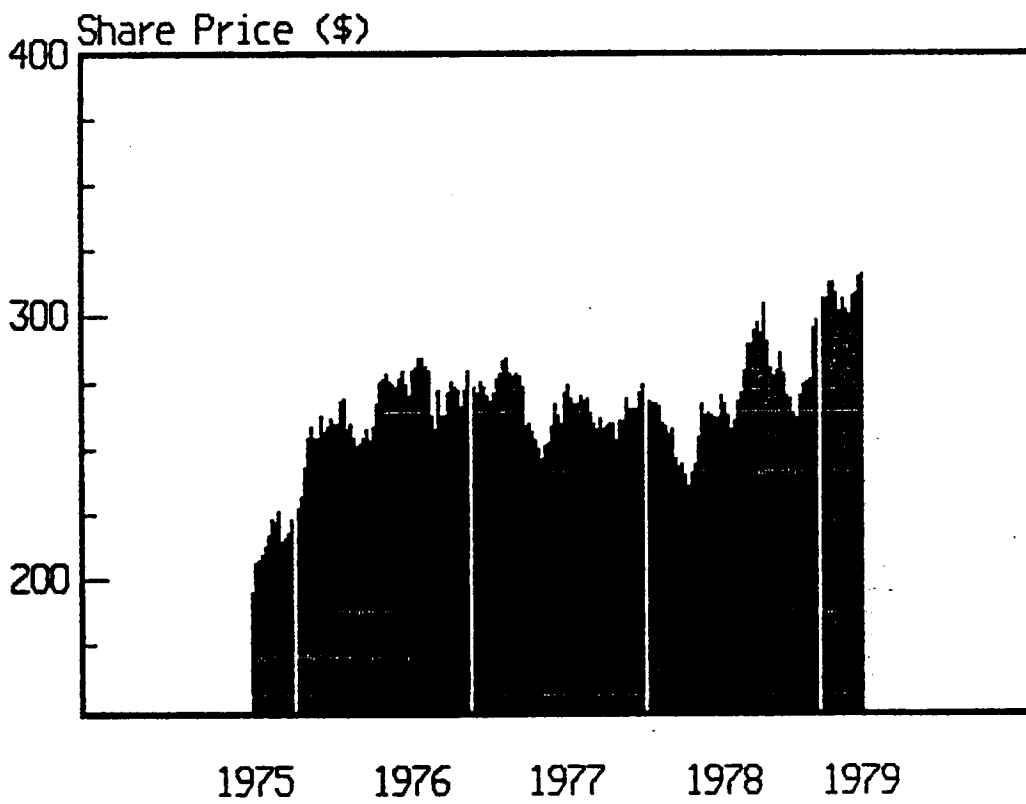


Figure 7-1. IBM Weekly Closing Prices, 1975-III to 1979-I.

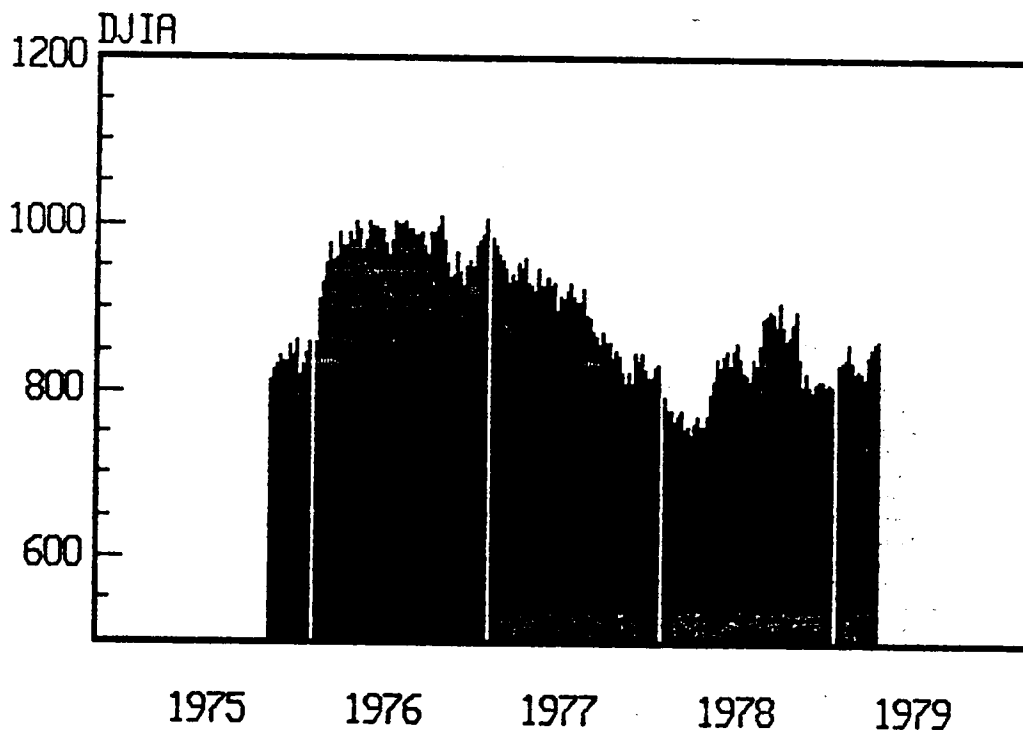


Figure 7-2. Dow Jones Industrial Average, Weekly Closing 1975-III to 1979-I.

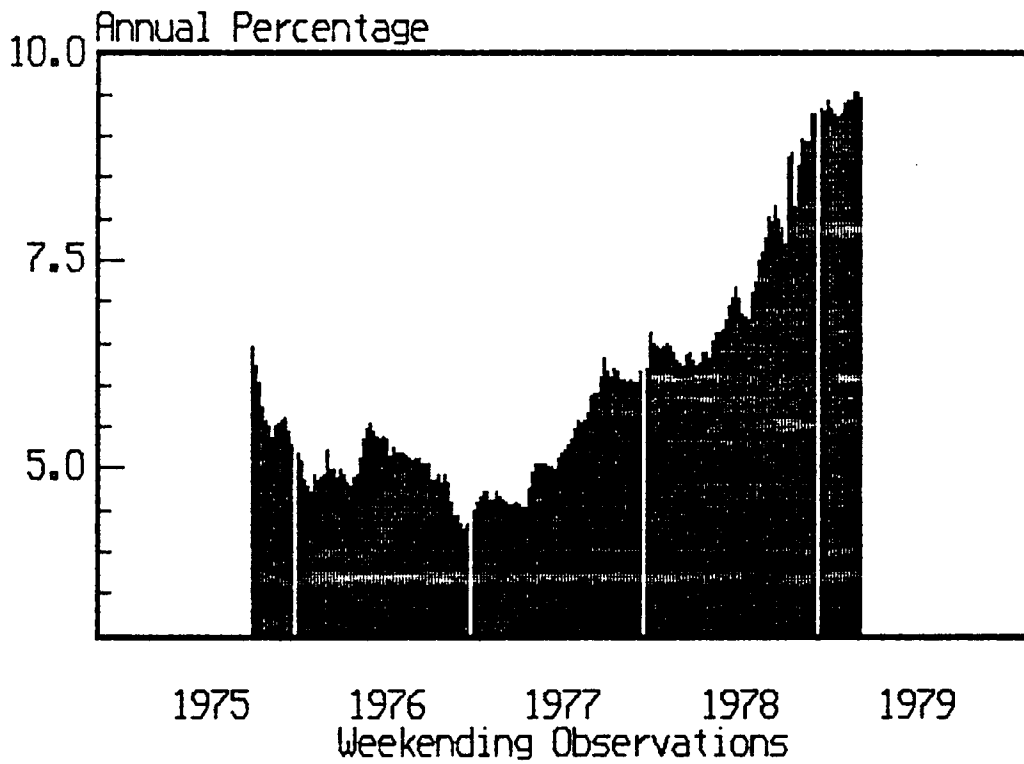


Figure 7-3. 91-Day U.S. Treasury Bill, Weekly Market Rates 1975-III to 1979-I.

historical volatility. Thus, we would wish to see 1977 as well as 1975 observations in order to fathom investor expectations in 1976. The same reasoning would require us to look at 1977 and 1979 when interpreting 1978. Figure 7-2 is a plot of weekly closing averages for the Dow Jones Industrials for the years 1975-III through 1979-I. Figure 7-3 is a plot of weekly secondary market rates on 91-day U.S. Treasury Bills for the the same period. Even casual observation shows each of these markets performed quite differently in 1976 and 1978.

6. Relationship Between Mispricing Biases and Risk Preferences

By superimposing a risk-neutral utility function onto the Friedman-Savage-Markowitz utility function (Figure 1-3), we can conceptualize the potential pricing disparities in Figure 7-4. In this diagram, we denote the risk-neutral preference set with the subscript RN, $URN(w)$, and the F-S-M preference set with FSM , $UFSM(w)$. Empirical literature showing predominant regional biases are provided for reference purposes.

It is helpful to partition the w axis into four regions which have the following properties and associated pricing relationships (BS is Black-Scholes model price, MP is market price):

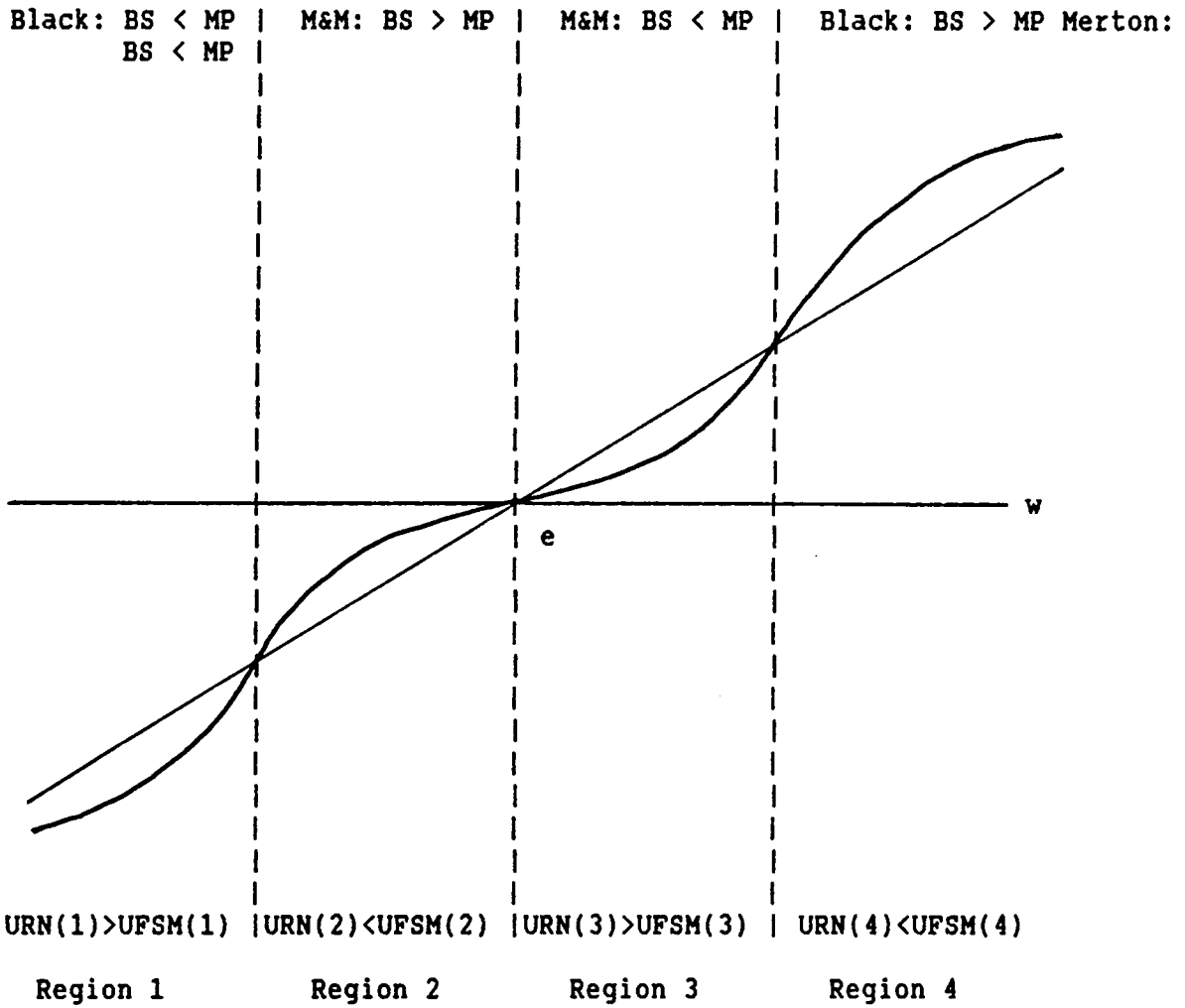


Figure 7-4. Comparison of Friedman-Savage-Markowitz Preference Set with Risk-Neutral Preference Set.

Region 1: deep-out-of-the-money $URN(1) > UFSM(1)$

Black finds $BS < MP$
 Merton finds $BS < MP$
 Emanuel & MacBeth
 finds $BS < MP$

Region 2: out-of-the-money $URN(2) < UFSM(2)$

MacBeth & Merville
 finds $BS > MP$
 Rubinstein finds $BS > MP$

Region 3: in-the-money $URN(3) > UFSM(3)$

MacBeth & Merville
 finds $BS < MP$
 Rubinstein finds $BS < MP$

Region 4: deep-in-the-money $URN(4) < UFSM(4)$

Black finds $BS > MP$
 Merton finds $BS < MP$
 Emanuel & MacBeth
 finds $BS > MP$

What we propose is the presence of risk-loving market behavior in regions 1 and 3 and a risk-averse market behavior in regions 2 and 4. This will lead to mispricing by a risk-neutral model consistent with the observations by Black and Emanuel and MacBeth (in regions 1 and 4), MacBeth and Merville and Rubinstein (in regions 2 and 3), and Merton (in region 1). Only Merton's observation in region 4 is inconsistent.¹² The presence of risk-loving traders bids up market prices; the presence of risk-averse traders bids down market prices.

The visual appeal of this relationship is complemented by the simulation of wealth maximization performed in Chapter 4, Section 3. Recall that we were able to pair negative (market - model) mispricing

biases with out-of-the-money call options and to pair positive biases with in-the-money calls.

7. Summary

Considerable attention has been paid to the Black-Scholes option pricing model since its publication fifteen years ago. It is simultaneously praised for its overall accuracy and tested for its mispricing biases. While the magnitude of the mispricing has been reduced by several refinements of the original 1973 model, particularly in the areas of measurement of volatility and dividend considerations, there still exist significant, systematic mispricing biases.

It appears that the causes of the mispricing must surely lie in directions other than dividends. The refinements have not eliminated the mispricing, and we observe (in Chapter 9) similar mispricing biases in options whose underlying stock pays no dividend.

The intuitive appeal of investors with Friedman-Savage preferences both gambling and hedging in the options market is "tested" in the next two chapters.

FOOTNOTES

¹A "straddle" is the simultaneous purchase of or writing a put and a call on the same underlying stock, with the same striking price and the same expiration date. See Terminology in Appendix to Chapter 3.

²Black (1975, p. 64).

³Merton (1976, p. 140).

⁴MacBeth and Merville (1979, p. 1177).

⁵One attempt to explain the mispricing by B-S models has been the testing the implicit assumption that $\alpha=0$ in model (6.29) or that $\alpha=1$ (in model 6.30). Barone-Adesi and Whaley (1986, p. 91) find the value of α "to be not significantly different from one."

⁶Because the implied volatility is nonlinear with respect to the amount the option is in- or out-of-the-money, this linear regression technique is somewhat inaccurate in itself. It will tend to overestimate the true variance but this is minor relative to forcing the Black-Scholes model to "correctly" price at-the-money options.

⁷MacBeth and Merville (1979, p. 1185).

⁸Emanuel and MacBeth (1981, pp. 543-544).

⁹The results were as follows:

Model	Mean	Standard deviation
$c(S', T, X)$	0.0215	0.2524
$\max[c(S', T, X), c(S, t, X)]$	0.0148	0.2396
$C(S', T, X)$	0.0108	0.2382

¹⁰Sterk (1983b, p. 49).

¹¹MacBeth and Merville (1979, pp. 1179-1181) include only the scatter diagrams for IBM as "...diagrams for corresponding options written on the other five stocks appear almost identical...."

¹²Merton's (1976) observations are anecdotal in nature. If a case is made for Merton judging "in-the-money" as "deep-in-the-money", this inconsistency disappears.

Chapter 8

Test One: Preferences for IBM Call Options Pre- and Post- Market Break, October 1987

1. Introduction

In Section 2 we set out the general framework of our testing procedure and establish the magnitude and direction of the systematic biases in the Black-Scholes model for a relatively stable, current time period. In Section 3 we examine the changing nature of the mispricing for the periods preceding and following the major "market break"¹ which occurred on October 19, 1987. In Section 4 we interpret the mispricing biases of Section 3 in terms of changing investor preferences.

2. Technique and Reference Period

In this section we discuss our technique for testing for mispricing biases in the risk-neutral Black-Scholes option pricing model adapted by Black for a dividend-paying stock. A standard of reference is also provided to better judge the level of mispricing biases in Section 3.

2.1 Model - Black's Approximation Call Option Formula

The Black-Scholes model described by Equation (5.31) is used to price IBM April 1986 call options for the period 1/27/86 to 3/7/86.

$$\max\{c(S',T,X), c(S,t,X)\} ,$$

where

$$c(S,T,X) = S \cdot N(d_1) - X e^{-rT} \cdot N(d_2),$$

$$d_1 = \{\ln(S/X) + (r + 0.5\sigma^2)T\} / \sigma\sqrt{T} ,$$

$$d_2 = d_1 - \sigma\sqrt{T} ,$$

$$c(S', T, X),$$

and

$$S' = S - \alpha D e^{-rT}.$$

The IBM exdividend date, February 7, 1986, occurred between the first nine option pricing dates and the expiration date of the April call.

2.2 Data

Empirical analysis of common stock call options should be characterized by 1) common stock inclusion on "most active list" 2) relatively large number of contracts traded, and 3) comparable option contract data used by other researchers in published works.

Options written on IBM common stock best meet the criteria set out above. With regards to 3), we feel a more current data set would be better suited to test our hypotheses.² The reference period covers 1/27/86 through 3/7/86; the testing about the October 19 crash ranges over the period July 1, 1987, through January 29, 1988.

Market Data

Option prices for IBM April Calls (all strike prices) is the Calls-Last³ price (CBOE) reported in The Wall Street Journal.

Model Data

Common stock prices are the daily NYSE closing prices for IBM common stock ("composite" closing prices are inappropriate as Western exchanges are still open after the end of trading on the CBOE; see footnote 3).

The riskfree interest rates are obtained from the current market price of a U.S. Treasury bill (using an average of bid and asked rates) maturing the day prior to the expiration date of the option contract. This data is reported daily in The Wall Street Journal.

$$\text{Current price of T-bill} = \$10,000 \left[1 - .01 \left(\frac{\text{bid} + \text{ask}}{2} \right) \left(\frac{T-1}{360} \right) \right]$$

$$1 + r^{-T} = [.]$$

$$r = [.]^{-360/(T-1)} - 1$$

Dividend information (the level of dividends and exdividend date) was taken from the Quarterly Dividend Record.

The "model" price for the call option was calculated using the Black-Scholes Option Pricing Model program in Bookstaber (1985). Estimation of stock volatility requires the market price for the at-the-money call option (see Market Data above).

2.3 Test Results

Preliminary research on IBM April 1986 call options is presented in Table 8-1 and Figure 8-1. It shows the same signs for mispricing biases for early 1986 as Macbeth and Merville (1979) found for 1976:

Out-of-the-Money - BS > MP,
At-the-Money - BS = MP (true by definition[†]), and
In-the-Money - BS < MP.

The importance of these results is that the difference between market price and model price increases then decreases for in-the-money calls. The supposition being that were additional option contracts with higher

Table 8-1

Mispricing by Black-Scholes OPM
 1/27/86 - 3/7/86 Biases for April 1986 IBM Calls

Dollar Amount In-the-Money	Sample Size	Percentage Difference	-----Mean Prices ⁵ -----		
			Market	Model	Difference
\$ -15	3	-8.59%	\$ 1.63	\$ 1.77	\$ - .14
-10	8	-7.41	2.70	2.90	- .20
- 5	21	-5.58	4.30	4.54	- .24
0	27	0	6.48	6.48	0
+ 5	27	+2.64	9.47	9.22	+ .25
+10	27	+5.13	13.26	12.58	+ .68
+15	27	+5.93	17.53	16.49	+1.04
+20	27	+7.25	22.47	20.84	+1.63
+25	26	+5.41	26.97	25.51	+1.46
+30	21	+3.64	31.36	30.22	+1.14
+35	11	+2.91	35.75	34.71	+1.04

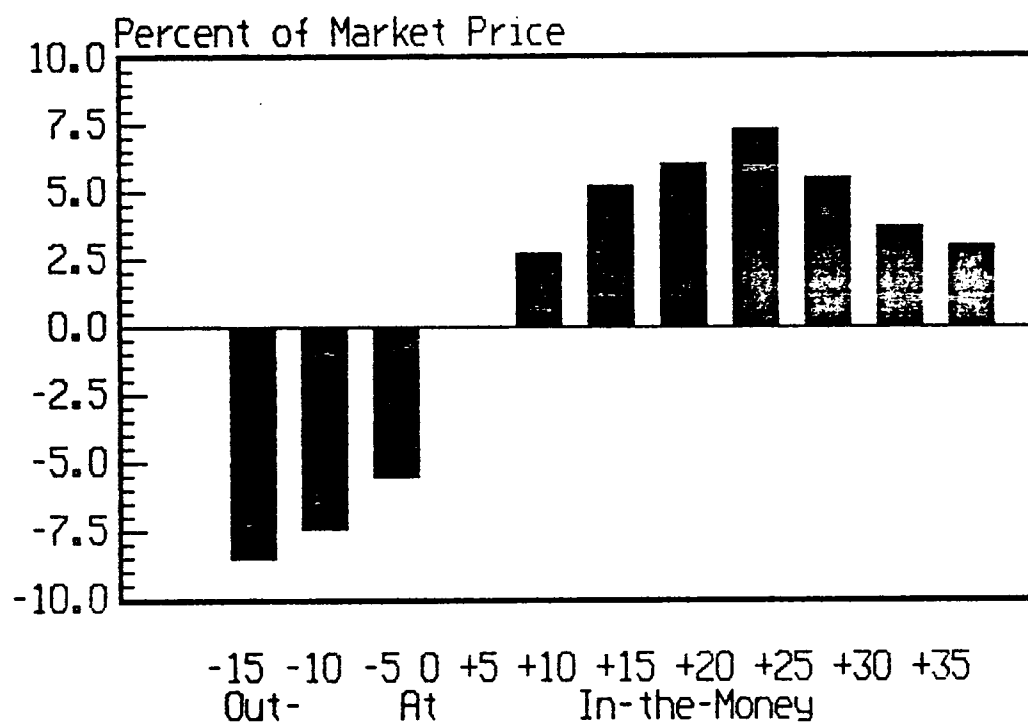


Figure 8-1. Systematic Mispricing by Black-Scholes OPM.
January-March Pricing April 1986 IBM Calls.

strike prices offered, the difference in market and model prices would continue to decrease to zero and then turn negative (for deep-in-the-money options) to illustrate Black's findings.

For this period of generally rising prices, there were not a sufficient number of out-of-the-money contracts to show the potential decrease in mispricing from that perspective.

3. Mispricing Biases in Pre- and Post- Market Break Periods

The purpose of this test is to shed some light on why a change in direction of the systematic biases was observed in the empirical work by Emanuel-MacBeth (1981) and Rubinstein (1985). Can a change in investor preference for risk result in a reversal of the biases?

The singular case in point is the October 19, 1987 "Market Break." Observations of September and October prices for October IBM calls, expiring October 16, represent investor behavior immediately preceding the break (termed the "Pre-Break" period; late October and November prices for December IBM calls represent investor behavior immediately following the break (termed the "Post-Break" period).

3.1 Model - Black's Approximation Call Option Formula

The Black-Scholes model (5.31) and described in Section 2 is used to price the call options. IBM ex-dividend date, August 6, occurred after the first twenty-five pricing dates in the "Pre-Break" sample. IBM ex-dividend date, November 5, 1987, occurred between the first six pricing dates in the "Post-Break" sample and the expiration of the December 1987 contract.

3.2 Data

The following time periods are covered:

Pre-Market Break - Monthly Average	9/ 1/87 - 10/15/87
5-Day Moving Average	7/ 1/87 - 10/15/87
Post-Market Break - Monthly Average	10/28/87 - 11/30/87
5-Day Moving Average	10/28/87 - 1/28/88.

The compilation of market and model data follows the description in Section 2.2.

3.3 Test Results

The results of testing for mispricing of IBM call options for the period prior to the October 19 "Market Break" are reported in Table 8-2 and Figure 8-2.

While the information provided by a static figure such as a mean difference is important, it is also important to observe the direction and magnitude these differences take over time. To that end, we have plotted 5-day and 20-day moving averages⁶ of the price differences for each category of in-the-money and out-of-the-money call which have a sufficient number of consecutive observations to make such a diagram meaningful.

5-day and 20-day moving averages for the extended period leading up to the market break are shown in Figures 8-3 and 8-4. The companion information for the period following the market break is given in Table 8-3 and Figures 8-5, 8-6 and 8-7.

Table 8-2

Mispricing of Black-Scholes OPM
September-October Bias for October 1987 IBM Calls
Pre-Market Break Period

Dollar Amount In-the-Money	Sample Size	Percentage Mispricing	-----Mean Prices ^S ----- Market Model Difference		
\$-20	11	7.80%	\$ 0.79	\$ 0.72	\$ + .04
-15	18	2.80	1.03	1.01	+ .02
-10	22	4.61	1.68	1.64	+ .04
- 5	26	1.85	2.80	2.75	+ .05
0	32	0	4.69	4.69	0
+ 5	32	2.07	8.15	7.93	+ .22
+10	32	0.39	11.21	11.17	+ .04
+15	31	1.62	16.20	15.91	+ .29
+20	26	2.32	20.60	20.04	+ .56
+25	19	3.02	25.34	25.48	+ .86
+30	8	0.83	29.67	29.48	+ .19

Table 8-3

Mispricing of Black-Scholes OPM
October-November Bias for December 1987 IBM Calls
Post-Market Break Period

Dollar Amount In-the-Money	Sample Size	Percentage Mispricing	-----Mean Prices ^S ----- Market Model Difference		
\$-20	13	-24.59%	\$ 1.53	\$ 1.82	\$- .24
-15	18	-22.15	2.03	2.39	- .36
-10	22	-10.83	2.81	3.05	- .24
- 5	22	- 4.15	4.48	4.57	- .09
0	23	0	6.37	6.37	0
+ 5	20	3.54	9.89	9.50	+ .39
+10	17	2.48	12.74	12.60	+ .14

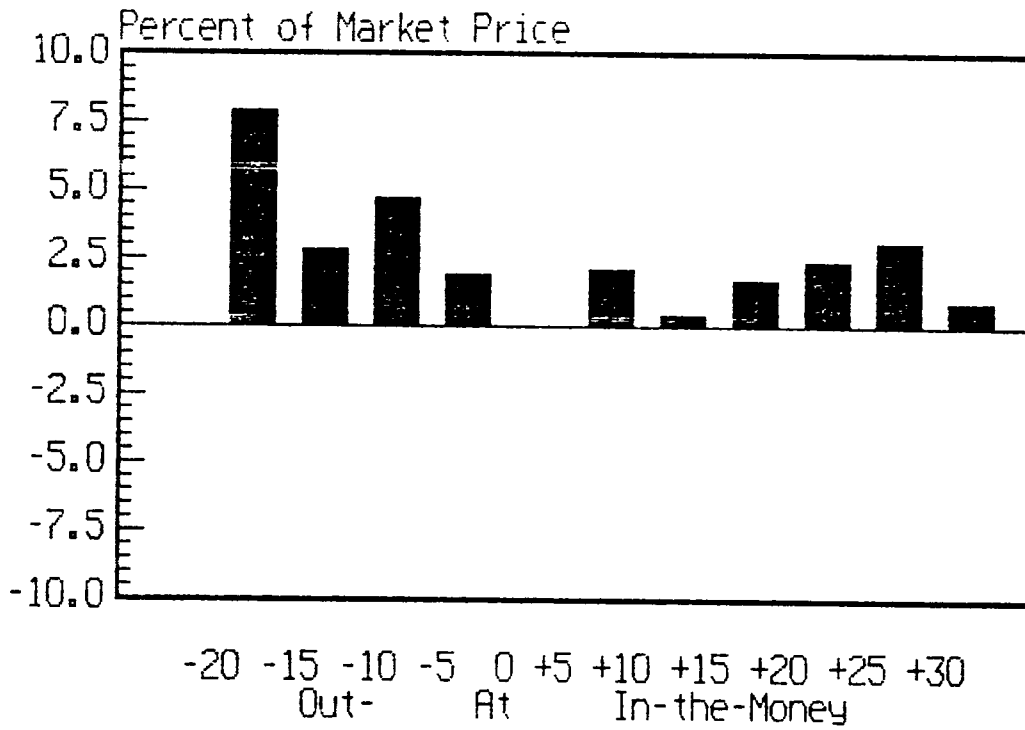


Figure 8-2. Systematic Mispricing: Pre-Market Break Period
September-October Pricing of IBM October 1987 Call.

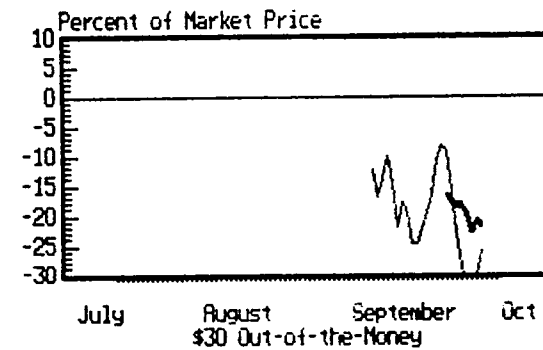
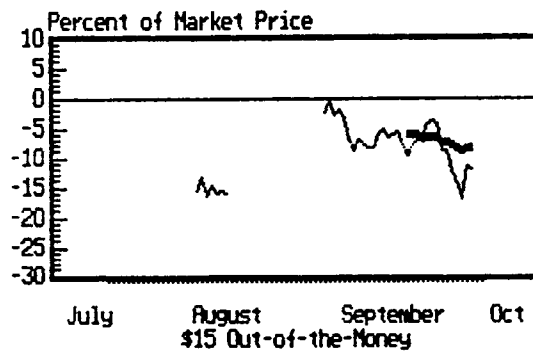
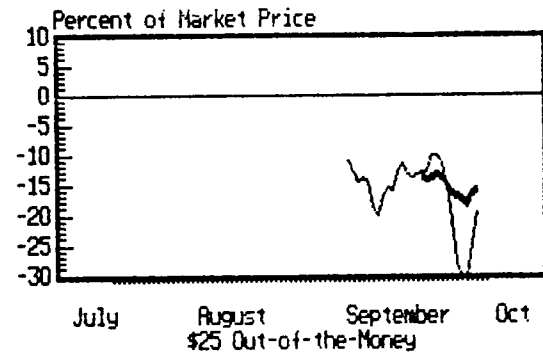
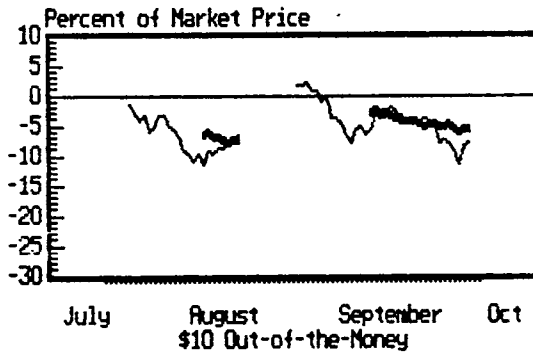
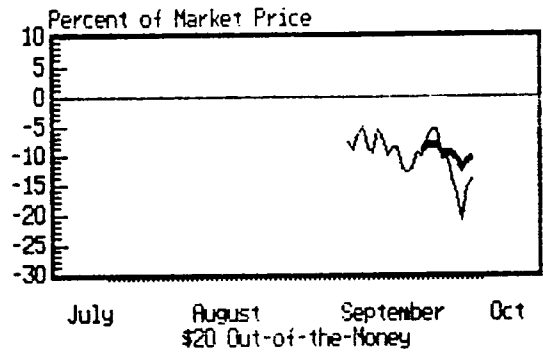
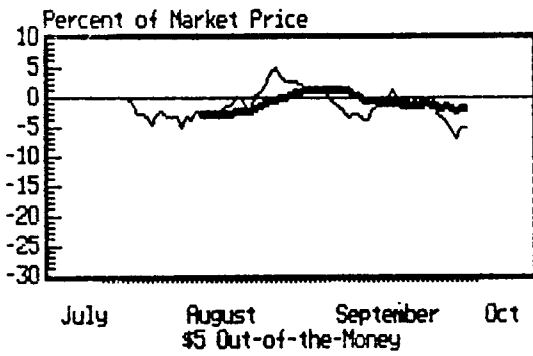


Figure 8-3. 5-Day and 20-Day Moving Averages of Mispricing Biases for the Out-of-the-Money IBM Call Options: Extended Pre-Market Break Period.

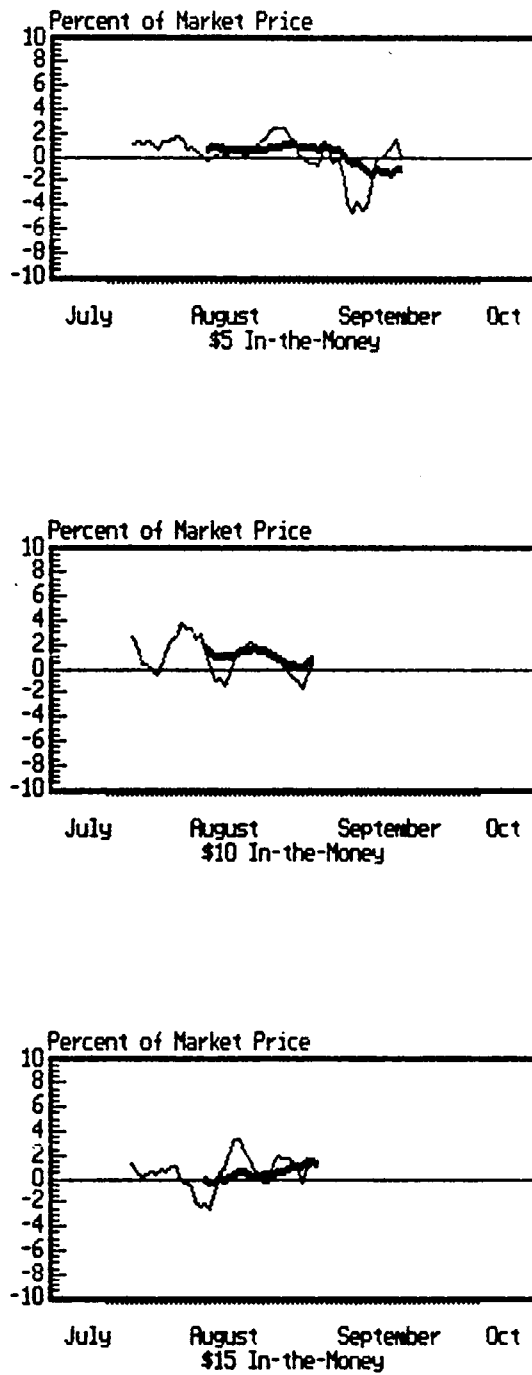


Figure 8-4. 5-Day and 20-Day Moving Averages of Mispricing Biases for the In-the-Money IBM Call Options: Extended Pre-Market Break Period.

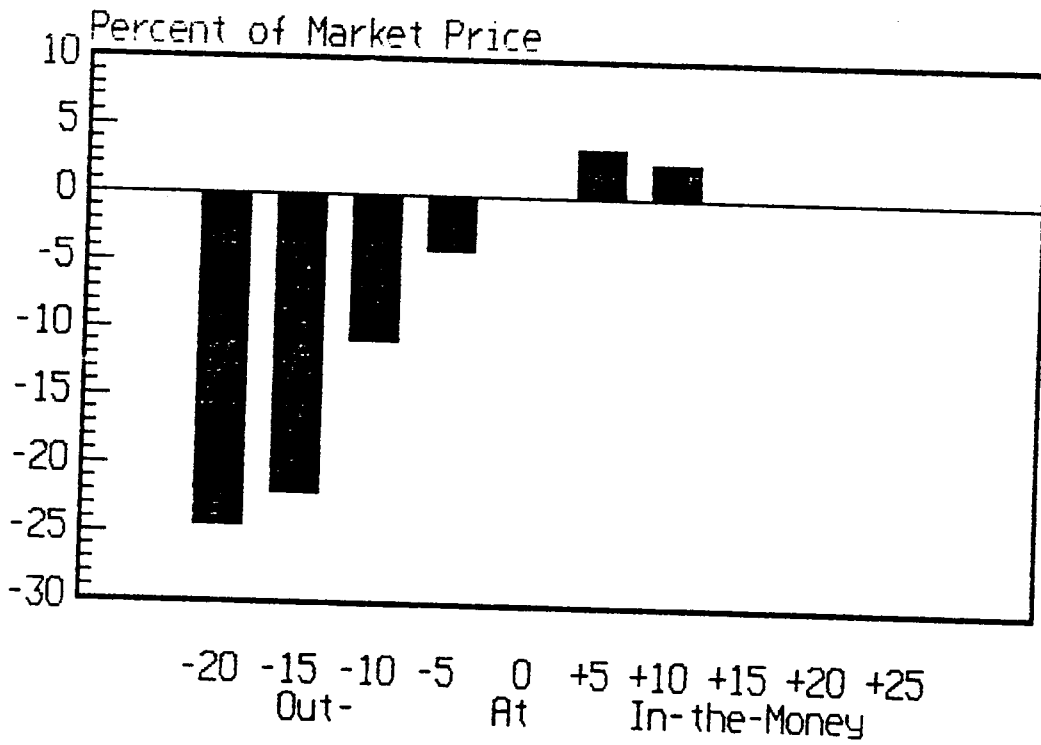


Figure 8-5. Systematic Mispricing: Post-Market Break Period
October-November Pricing of December 1987 IBM Call.

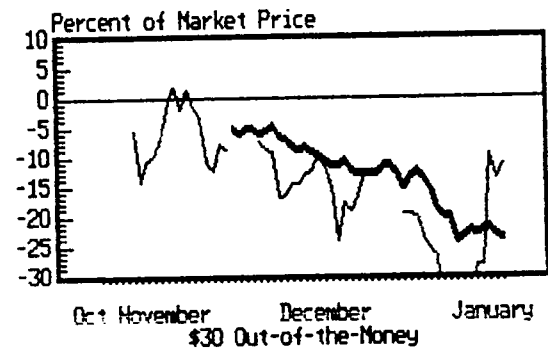
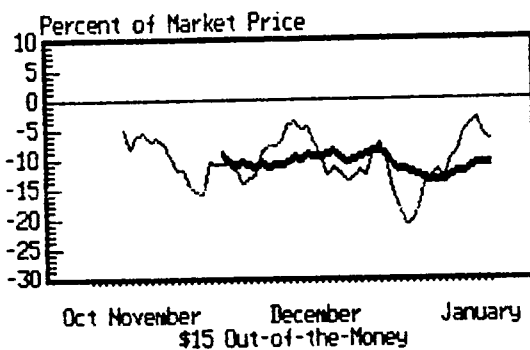
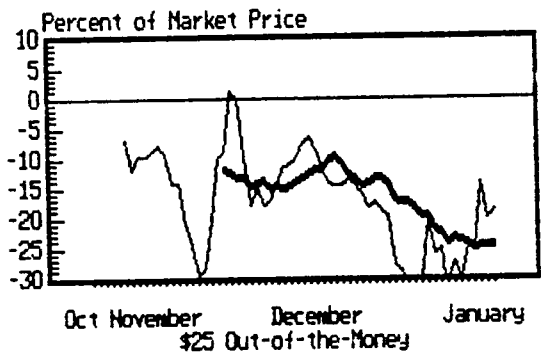
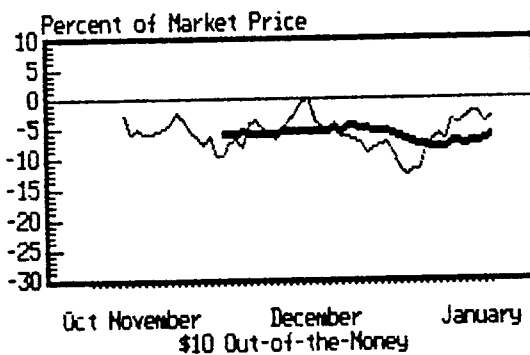
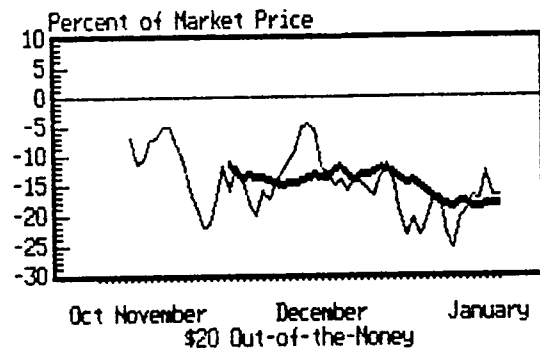
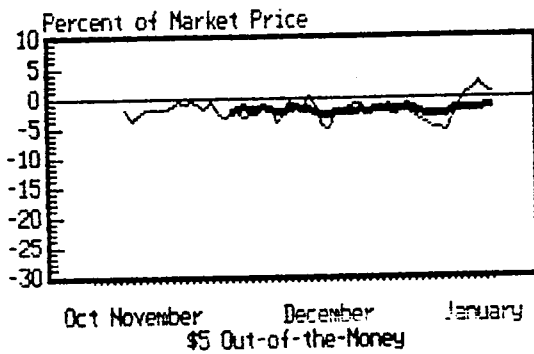


Figure 8-6. 5-Day and 20-Day Moving Averages of Mispricing Biases for the Out-of-the-Money IBM Call Options: Extended Post-Market Break Period.

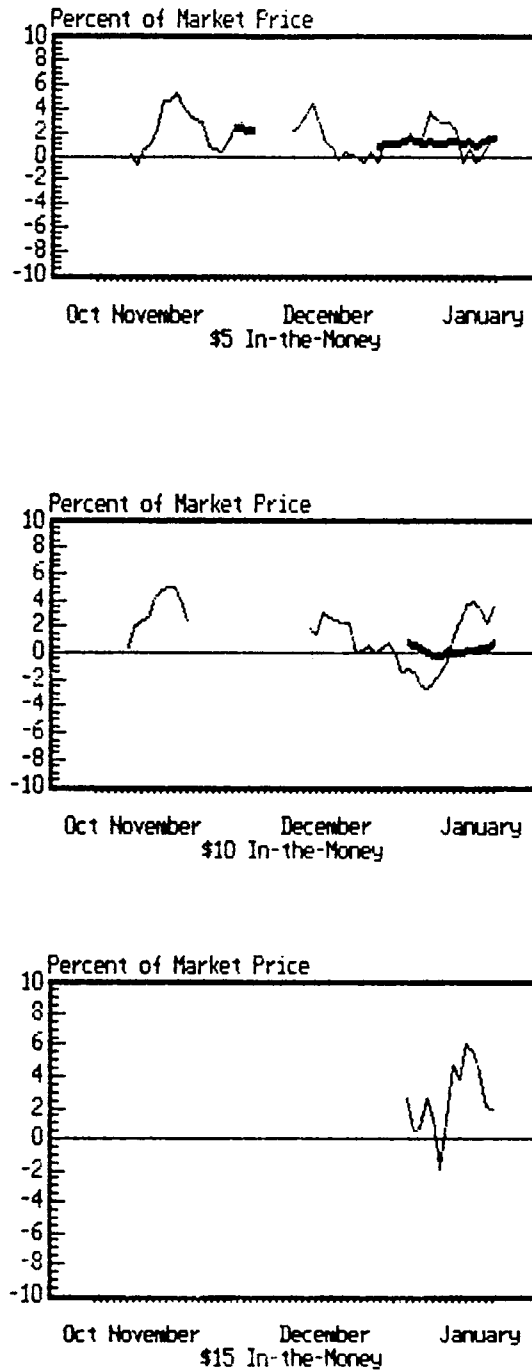


Figure 8-7. 5-Day and 20-Day Moving Averages of Mispricing Biases for the In-the-Money IBM Call Options: Extended Post-Market Break Period.

4. Mispricing Biases and Investor Preferences

In this section we attempt to explain the mispricing biases observed in Section 3 by an intuitive look at investor preferences. Market indicators (trading activity, number of traders, etc. on the CBOE and the NYSE) and institutional considerations (tighter trading limits, statements made by officials on CBOE and NYSE; advertised investment strategies; and so on) lead us to believe the degree of risk-aversion increased in the market and, conversely, the degree of risk-taking decreased following the October 19 market break.

4.1 Statement of Hypothesis

Hypothesis I: An increase in investor uncertainty in the market for the underlying stock will increase the negative magnitude of mispricing bias in its out-of-the-money call options and decrease the positive magnitude of mispricing bias in its in-the-money call options.

Corollary to I: A decrease in investor uncertainty in the market for the underlying stock will decrease the negative magnitude of mispricing in its out-of-the-money call options and increase the positive magnitude of mispricing bias in its in-the-money call options.

A decrease in investor uncertainty is characterized, in this period, by the passing of time in which no adverse events occur. The market price of the stock is our indicator of the occurrence (or lack of occurrence) of such events. The market price of IBM trended upward in the "Pre-Break" period and was, of course, significantly lower in the "Post-Break" period.

4.2 Summary: Pre-Break Monthly Average

Out-of-the-Money: $BS < MP$

1. This is inconsistent with the direction of bias in our reference period and with the findings of MacBeth and Merville (1979). The direction is the same as Emanuel and MacBeth (1982) found for 1978.

2. Positive differences (market - model) suggest the presence of risk-loving behavior in what is normally a risk-averse region.
3. It is likely that investor fears are minimal given the very long bull market.

At-the-Money: $BS = MP$

1. This is consistent with previous research, and true by definition (see implied volatility discussion in Chapter 7).

In-the-Money: $BS < MP$

1. This is consistent with our reference period and MacBeth and Merville's observations for 1976.
2. However, the magnitude of the largest difference is 3.02% (at +25) compared to 7.25% (also at +20) in our reference test.
3. It is important to note the difference between market price and model price increases then decreases, even in a relatively high risk-taking time period.

4.3 Summary of Moving Averages in Pre-Break Period

Out-of-the-Money: $d|MP-BS|/dt > 0$

1. The plots of the 20-day moving average show the general trends are downward, indicating an increasing bias leading up to October 19. According to the corollary to our hypothesis, the trend should be upward.⁸
2. The plots of the 5-day moving average show the magnitude of current mispricing are decreasing in the week prior to October 19. This week, at least, supports our argument that the degree of risk-averse behavior is decreasing prior to the crash.

In-the-Money: $d|MP-BS|/dt > 0$

1. The plots of the 20-day moving average end in late August and thus provide no information for September and October. While the Black-Scholes model performs better with options having about ninety days to expiration, when these options are not priced we must turn to the shorter maturity options which are priced in the market. Moving average plots for July-October pricing of the October call options is presented in Figure A8-2 in the appendix to this chapter. They show the general trends are upward, indicating an increase in the degree

of risk-taking by option traders. This supports the corollary to our hypothesis.

2. The (appendix) plots of the 5-day moving average show a mixed result. Options \$5, \$10 and \$15 in-the-money show decreasing price differences; those \$20 and \$25 in-the-money show increasing differences. There is no basis to give one group more weight than the other.

4.4 Summary: Post-Break Monthly Average

Out-of-the-Money: $BS > MP$

1. The direction of mispricing is consistent with our reference period and the research of Macbeth and Merville.
2. The presence of extremely risk-averse behavior bids down market price more than normal relative to the differences we would normally observe. The magnitude of the largest difference is -24.59 (at -20) compared to -8.59% (also at -15) in the reference test.

At-the-Money: $BS = MP$

1. This is consistent with previous research, and true by definition (see implied volatility discussion).

In-the-Money: $BS < MP$

1. This is consistent with our reference period and the research of MacBeth and Merville.
2. However, the magnitude of the largest difference is 3.54% (at +5) compared to 7.25% (at +20) in our reference test.
3. Again, it is important to note the difference between market price model price increases then decreases.

4.5 Summary of Moving Averages for Post-Break Period

Out-of-the-Money: $d|MP-BS|/dt > 0$

1. The plots of the 20-day moving average show the general trends are downward, indicating an increasing bias leading away from October 19. According to our hypothesis, the trend should be downward as traders become more risk-averse. The general magnitude of mispricing in out-of-the-money calls is greater than prior to the crash, as we would expect.

2. The plots of the 5-day moving average show the magnitudes of current mispricing are increasing quite rapidly in late October and again in November. It is not possible to view the mispricing which took place immediately following the crash as there were no at-the-money calls to use in calculating the implied volatility for the other calls.

In-the-Money: $d|MP-BS|/dt = ?$

1. The plots of the 20-day moving average show very little of the general trends following the crash. As one might expect, there are very few in-the-money calls after such a steep drop in stock prices. In January, we begin to pick up \$5 and \$10 in-the-money calls with (relative to the pre-break period) small positive mispricings.
2. The plots of the 5-day moving average, where they exist, are also uninformative.

FOOTNOTES

¹The general market indicators suffered their largest single-day declines (both in absolute and percentage terms) in history on October 19, 1987. Using the Dow Jones Industrial Average (DJIA) to measure the market:

October 28-29, 1929 DJIA lost 68.90 points (23.1%)
 October 16-19, 1987 DJIA lost 616.35 points (26.2%).

²Testing the mispricing biases of a portfolio of stocks is felt to be inappropriate, in that *grand* statistics may hide company-specific risk known to the traders.

Trading behavior is no doubt different today than it was a decade or more ago. Institutional changes, more sophisticated pricing models, and trader familiarity with options are valid arguments for a more contemporaneous data set than the empirical literature provides.

³Use of daily closing (last) prices for stocks and options has been criticized in the literature with the following argument: For a given firm, the simultaneous execution of the final stock trade of the day and the corresponding final option trade is unlikely because the NYSE closes earlier than the CBOE. Therefore, researchers have urged the use of transactions data to mitigate this time differential in daily closing prices.

The NYSE closes ten minutes earlier than the CBOE (3:00 p.m. Central Time versus 3:10 p.m.). With the exception that on extremely heavy trading periods when the NYSE "tape" is running behind more than ten minutes, this does not appear to pose a major problem as the option traders have the closing stock price to base their closing trades in the options. Were the CBOE the market to close prior to the NYSE closing, the problem would be more serious, and the use of transactions data might provide superior results.

Kutner and Sweeney (1987) find no significant difference in results obtained using intraday (transactions) data versus daily closing data.

⁴See discussion in Chapter 7 on the use of an implied volatility estimate versus a historical estimate.

⁵Mean prices are included for market prices, model prices and their differences to provide a "ball park" feeling for the absolute amounts involved. Individual contracts move from one category to another as the price of the stock fluctuates more than the increment between categories (in this case, \$5).

FOOTNOTES

*A plot using daily price differences versus a moving average was found to be uninformative due to the fluctuations of the daily numbers. Smoothing over five days, while having no basis in theory (i.e, there is no reason to believe traders average their prices over the previous week), maintains the relationship between current and trend data.

²To maintain continuity in the plots, missing observations (no trades occurring that day) were assigned a zero value (market equal model price). The 5-day moving average was allowed to contain no more than two zero observations; the 20-day moving average, no more than four.

*5-day and 20-day moving averages of mispricing in September and October of the October 1987 IBM call options present a dramatically different picture. For the two out-of-the-money categories with prices immediately preceding the crash, the longer term trend is up, which is what we would expect if risk-aversion is decreasing. The plots for the \$5 and \$10 out-of-the-money options are shown in Figure A8-1 in the appendix to this chapter.

APPENDIX TO CHAPTER 8

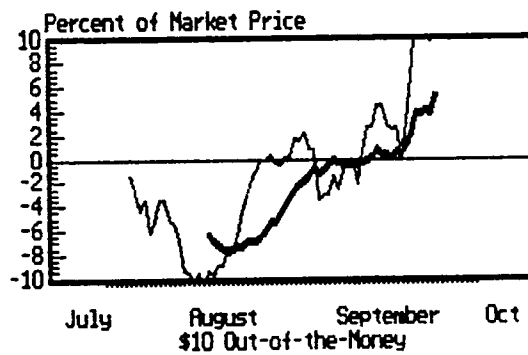
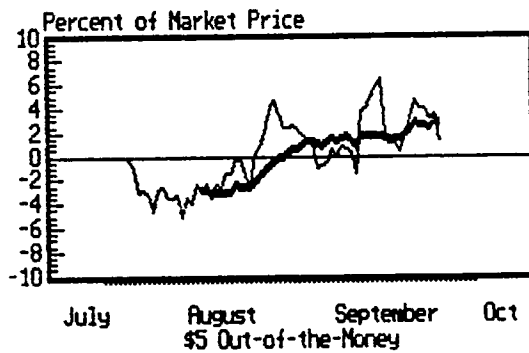


Figure A8-1. 5-Day and 20 Day Moving Averages of Mispricing Biases for the Out-of-the-Money Call Options: July-October Pricing of October 1987 IBM.

APPENDIX TO CHAPTER 8

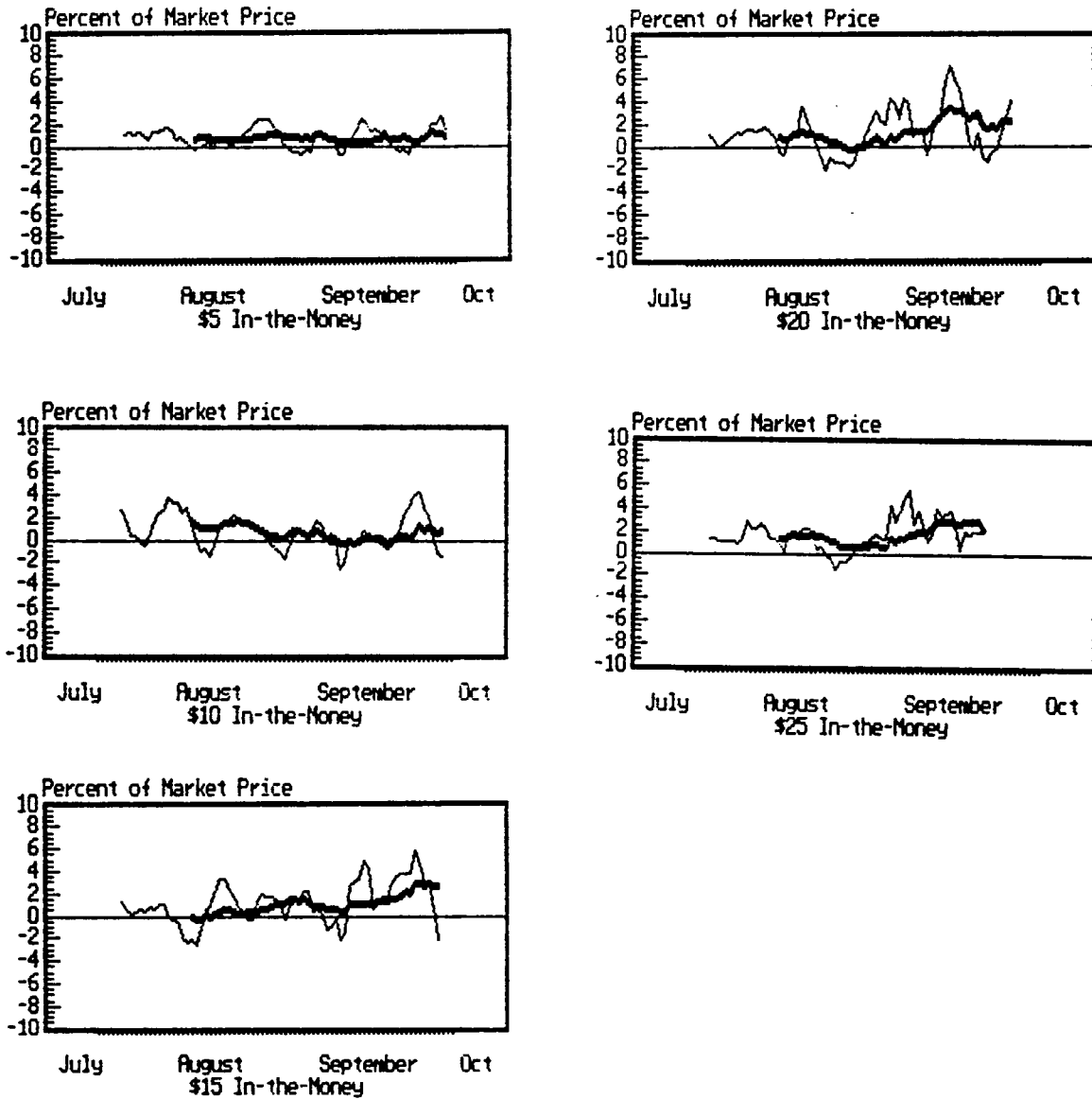


Figure A8-2. 5-Day and 20 Day Moving Averages of Mispricing Biases for the In-the-Money Call Options: July-October Pricing of October 1987 IBM.

Chapter 9

Test Two: Preferences for Microsoft Call Options Pre- and Post-Market Break, October 19, 1987

1. Introduction

The purpose of this chapter is twofold: First, we wish to examine a second class of stock option to see which, if any, of our test results in Chapter 8 are duplicated. Second, we wish to test, in general, the nature of mispricing by the Black-Scholes option pricing model of a call option whose underlying stock pays no dividend.

The presence of a non-dividend-paying stock allows us to test the claim by Geske and Roll (1984) that a major factor in our observance of mispricing biases is the improper treatment of dividends and the probability of early exercise. Obviously, these are not factors in any mispricing we might observe in a stock with no dividend.

One relationship between a stock which pays a dividend and one that does not is that the latter is generally perceived as the more risky security.¹ We wish to observe how this difference in the perceived risk of the underlying stocks translates into the mispricing biases of their respective call options.

In Section 2 we briefly review the general framework of our testing procedure and note any differences from that followed in Chapter 8. In Section 3 we report the magnitude and direction of the systematic biases in the risk-neutral pricing of Microsoft for time periods preceding and following the October 19, 1987 "Market Break." In Section 4 we compare the mispricing of Microsoft call options with that we found for IBM and interpret our findings in terms of investor preferences.

2. Test for Option Mispricing when Underlying Stock Pays No Dividend

In this section we discuss our technique of testing for mispricing biases in the risk-neutral Black-Scholes pricing of call options written on a stock which does not pay a dividend.

2.1 Model - Black-Scholes European Call Option Formula

The Black-Scholes model described by Equation (5.29) is used to price Microsoft call options for the period 7/1/87 to 1/29/88. Recall that Merton (1973) showed that an American call can be priced as a European call when the underlying stock pays no dividend. Thus, the appropriate model is Black and Scholes' original pricing model,

$$c(S, T, X) = S \cdot N(d_1) - X e^{-rT} N(d_2),$$

where

$$d_1 = \{\ln(S/X) + (r + 0.5\sigma^2)T\} / \sigma\sqrt{T} \quad \text{and}$$

$$d_2 = d_1 - \sigma\sqrt{T}.$$

2.2 Data

In the previous chapter, we set out the following criteria as desirable in the options to be examined:

- 1) underlying common stock included on "most active list,"
- 2) relatively large number of contracts traded, and
- 3) comparable option contract data used by other researchers in published works.

Microsoft options are (1) very actively traded and (2) include an exceptionally wide range of contracts (exercise prices). There is no published research that includes Microsoft data as the stock did not

begin trading over-the-counter until April 1986. The attractiveness of Microsoft from an empirical standpoint is category 2) above and the fact the common stock pays no dividend.

The following time periods are covered:

Pre-Market Break - Monthly Average	9/ 1/87 - 10/15/87
Moving Averages	7/ 1/87 - 10/15/87
Post-Market Break - Monthly Average	10/28/87 - 11/30/87
Daily Observations	10/28/87 - 1/28/88.

With two exceptions the compilation of market and model data follows the description in Chapter 8, Section 2.2. First, there is no need to reference exdividend dates and dividend payments as there are none. Second, Microsoft common stock is traded over-the-counter on the NASDAQ (National Association of Securities Dealers Automated Quotation) system, and Microsoft options are traded on the American Options Exchange. The first exception provides a simplifying element to the testing procedure. The second exception is noted for informational purposes only; it does not affect the calculations.²

3. Mispricing Biases in Pre- and Post- Market Break Periods

Observations of September and October prices for October Microsoft calls, expiring October 16, represent investor behavior immediately preceding the break (termed the "Pre-Break" period); late October and November prices for December Microsoft calls represent investor behavior immediately following the break (termed the "Post-Break" period).

To judge the behavior of the mispricing biases as we move toward and away from the periods in the neighborhood of the market break, we also examine the pricing over "extended" periods. We employ July and August pricing of October calls with September and October pricing of January calls for the period preceding the break because these contracts have the longest time to expiration of the listed options. For the same reason, October and November pricing of January calls with December and January pricing of April calls are used for the period following the break.

3.1 Pre-Market Break Mispricing

The average mispricing biases on Microsoft October 1987 call options during the period September 1 through October 15 are presented in Table 9-1 and Figure 9-1. They show the same signs for out-of-the-money and at-the-money mispricing biases as Macbeth and Merville (1979), while the in-the-money calls change sign twice:

Out-of-the-Money	-	$BS > MP$,
At-the-Money	-	$BS = MP$ (true by definition ³), and
In-the-Money	-	$BS ? MP$.

The importance of these results is that the difference between market price and model price increases then decreases for out-of-the-money calls. The supposition being that were additional option contracts with higher strike prices offered, the difference in market and model prices would continue to decrease to zero and then turn positive (for deep-out-of-the-money options) to illustrate Black's findings.

Table 9-1

**Mispricing by Black-Scholes OPM
September-October Biases for October Microsoft Calls**

Dollar Amount In-the-Money	Sample Size	Percentage Difference	-----Mean Prices ⁴ -----		
			Market	Model	Difference
\$ -20	9	- 2.52%	\$ 1.65	\$ 1.58	\$ 0.08
-15	10	- 4.96	1.80	1.83	-0.03
-10	19	- 5.28	2.66	2.67	-0.01
- 5	22	- 2.61	3.70	3.75	-0.05
0	31	0	4.84	4.84	0
+ 5	23	- 0.27	7.20	7.14	0.06
+10	25	- 1.13	7.80	8.49	-0.69
+15	22	+ 1.87	12.24	11.99	0.25
+20	18	+ 0.65	13.23	13.09	0.14
+25	16	- 1.77	15.92	16.01	-0.09
+30	9	+ 0.03	18.15	18.13	0.02

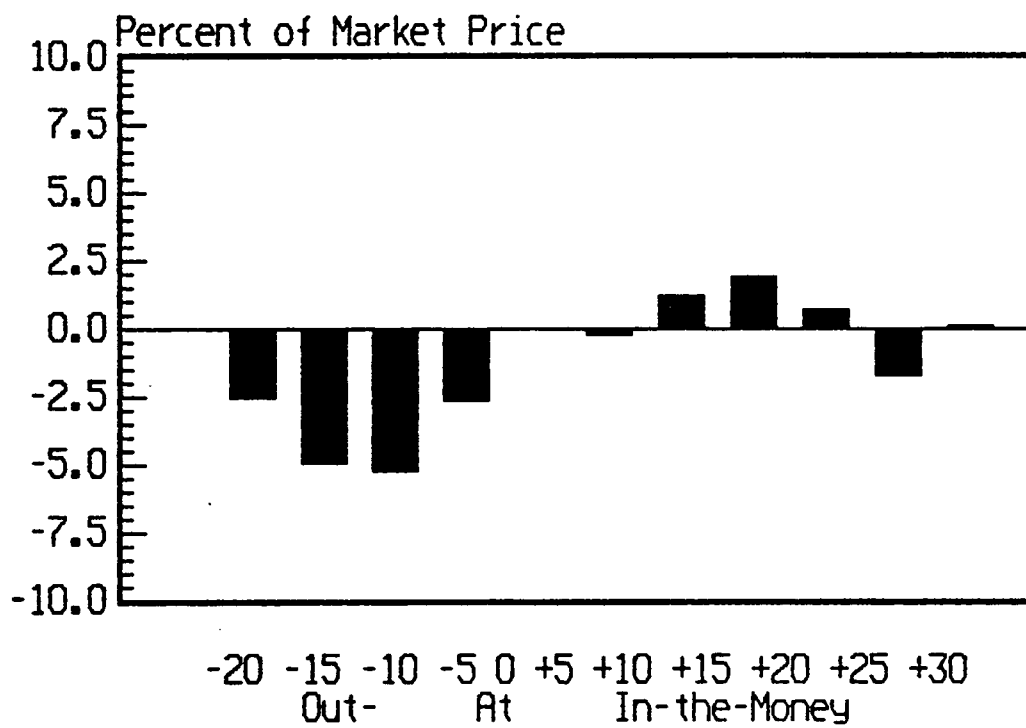


Figure 9-1. Systematic Mispricing by Black-Scholes OPM.
September-October Pricing October 1987 Microsoft Calls.

While the information provided by a static figure such as the mean difference is important, it is also important to observe the direction and magnitude these differences take over time. To that end, we have plotted 5-day and 20-day moving averages⁵ of the price differences for each category of in-the-money and out-of-the-money call which have a sufficient number of consecutive observations to make such a diagram meaningful.⁶

5-day and 20-day moving averages for the extended period leading up to the market break are shown in Figures 9-2 and 9-3. Though the number of consecutive observations in the higher out-of-the-money contracts is not sufficient to calculate the 20-day moving averages, the diagrams are worthwhile as they show the magnitude of the mispricing which is present in risk-neutral pricing of an option where no dividend problems exist.

Similar plots for the pre-break pricing of Microsoft October calls only are presented in the appendix to this chapter. While they represent (in September and October) preferences for a different distribution of returns, they are still felt to be informative.

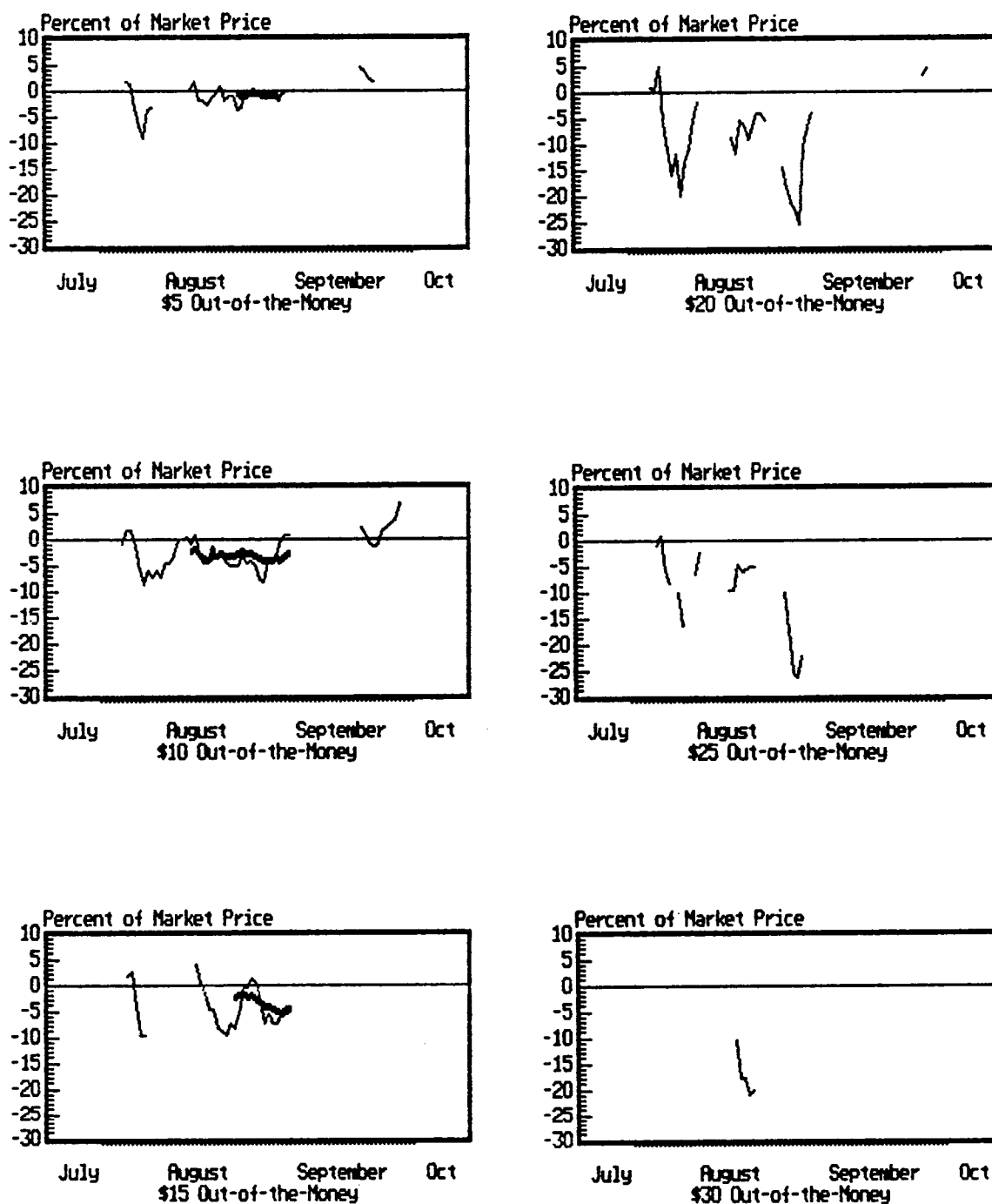


Figure 9-2. 5-Day and 20-Day Moving Averages of Mispricing Biases for the Out-of-the-Money Microsoft Call Options: Extended Pre-Market Break Period.

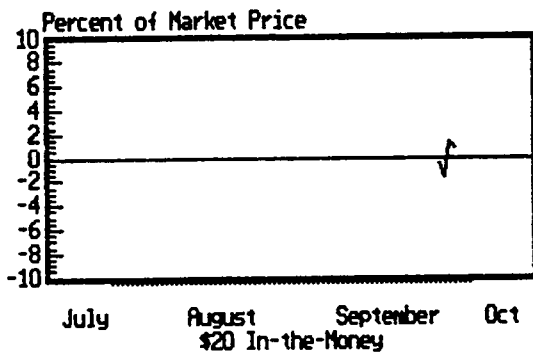
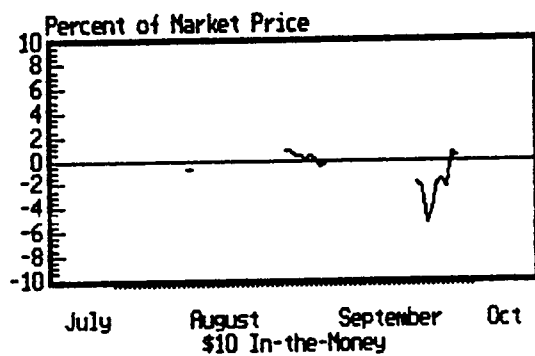
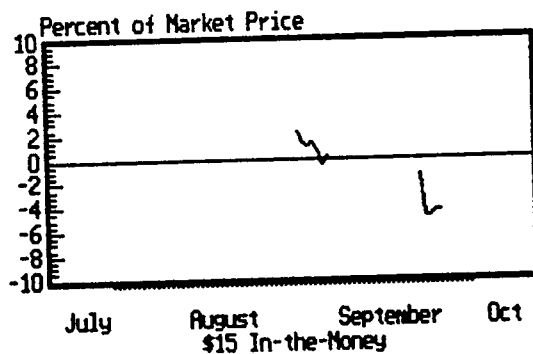
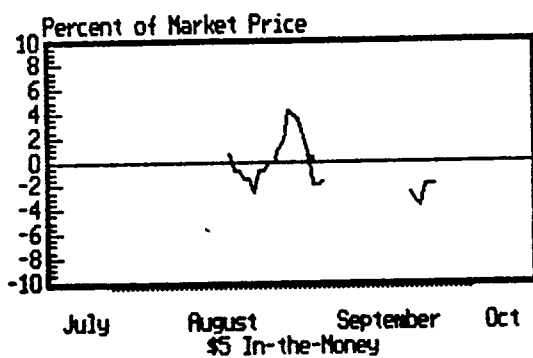


Figure 9-3. 5-Day and 20-Day Moving Averages of Mispricing Biases for the In-the-Money Microsoft Call Options: Extended Pre-Market Break Period.

3.2 Post-Market Break Mispricing

The average mispricing biases on Microsoft December 1987 call options during the period October 20 through January 29 is presented in Table 9-2 and Figure 9-4. It shows a marked lack of any systematic mispricing bias for the immediate post-break period. While the \$10 and \$30 out-of-the-money calls display sizable negative mispricing, the \$20 "out" shows a significant positive difference. The in-the-money calls show positive but insignificant mispricing.

The technique of moving averages which was used in the previous section was found to be inappropriate for Microsoft in the post-break period as the level of trading fell sharply and became concentrated in the closer expiration and \$10 (in- and out-of-the-money) calls.⁷ In order to portray the mispricing in the farther maturity calls, we have used the technique of scatter diagrams.

The diagrams for the extended period are shown in Figures 9-5 and 9-6. Again, though the number of observations is relatively small, these diagrams still show the magnitude of mispricing in options written on a stock which pays no dividend.

Table 9-2

Mispricing by Black-Scholes OPM
October-November Biases for December Microsoft Calls

Dollar Amount In-the-Money	Sample Size	Percentage Difference	-----Mean Prices ⁴ -----		
			Market	Model	Difference
\$ -30	3	-21.66%	\$ 1.14	\$ 1.27	\$-0.13
-20	11	+ 7.01	1.79	1.65	0.14
-10	13	- 6.22	2.78	2.98	-0.20
0	17	0	4.73	4.73	0
+10	6	+ 0.36	6.54	6.43	0.11
+20	3	+ 0.34	11.54	11.24	0.18

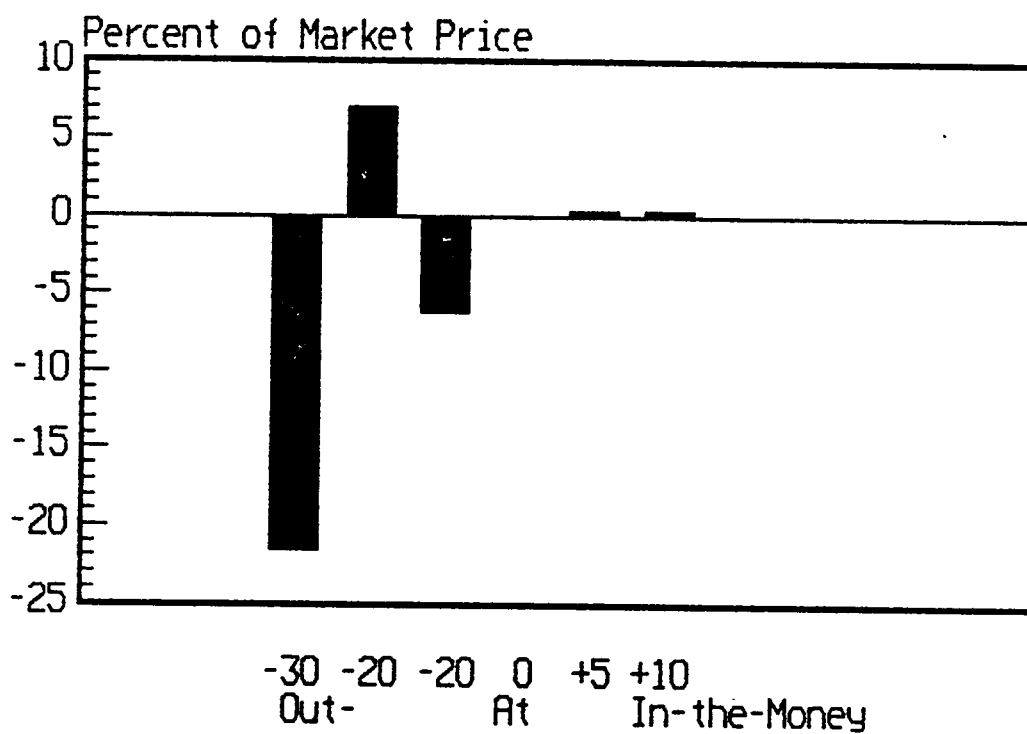


Figure 9-4. Systematic Mispricing: Post-Market Break Period
October-November Pricing of December 1987 Microsoft Call.

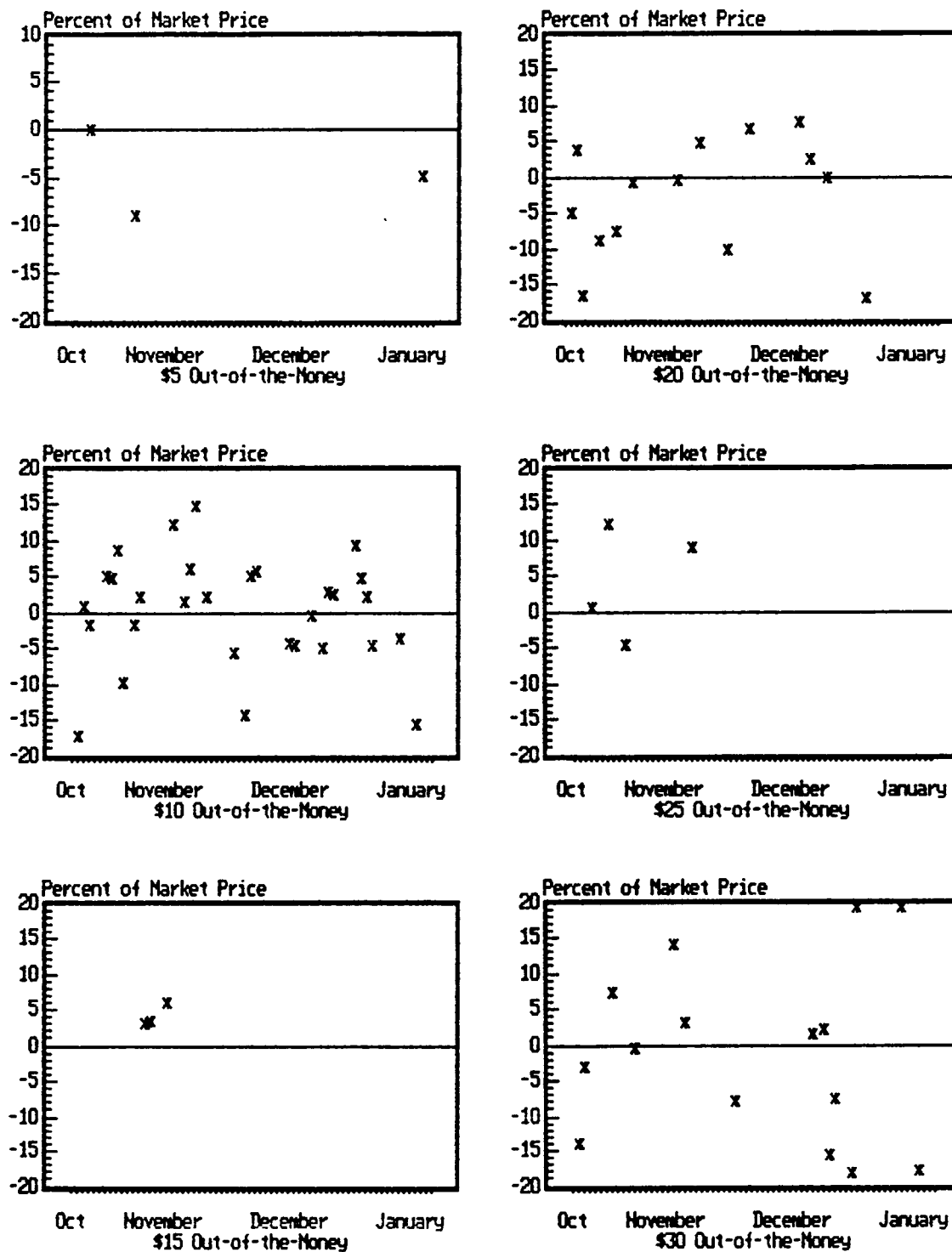


Figure 9-5. Mispricing Biases Microsoft Call Options:
Extended Post-Market Break Period.

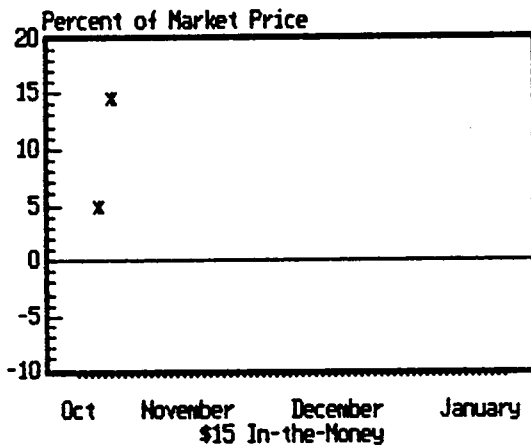
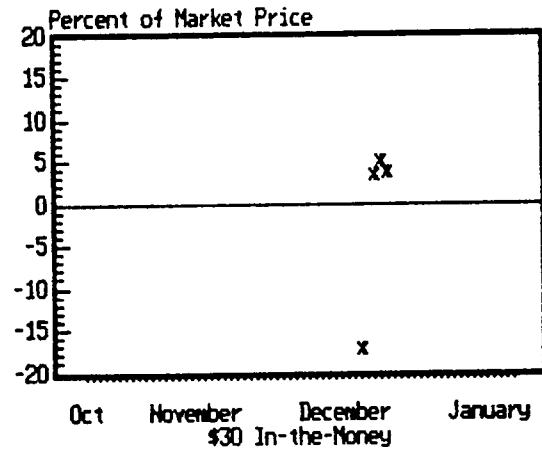
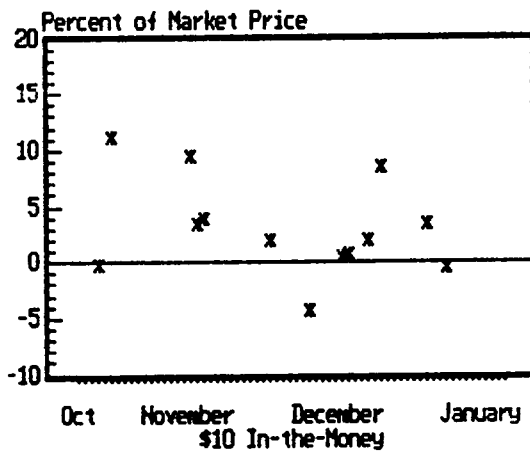
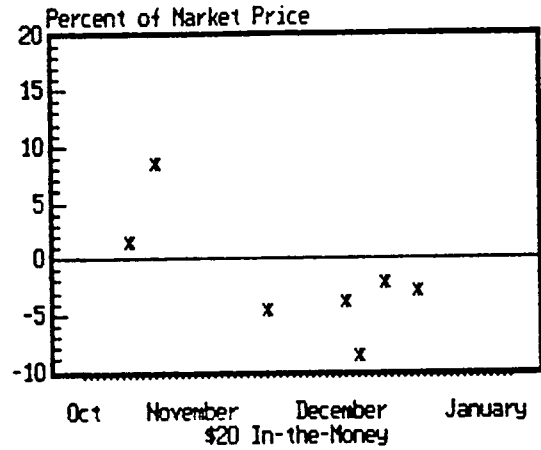
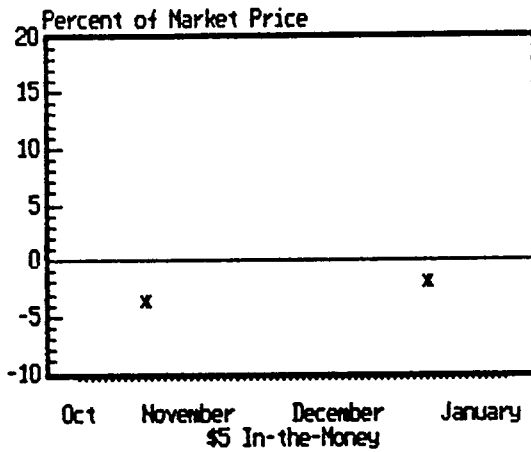


Figure 9-6. Mispricing Biases for Microsoft Call Options:
Extended Post-Market Break Period.

4. Mispricing Biases and Investor Preferences

In this section we attempt to explain the mispricing biases observed in Sections 3 by an intuitive look at investor preferences. The corresponding mispricings we observed for IBM are included for easy reference.

4.1 Summary: Pre-Break Monthly Average

The mispricing biases which we observed for Microsoft during the period immediately prior to the market break reflect the "normal" biases due to risk-averse behavior in the out-of-the-money calls and risk-loving behavior for the in-the-money calls. A more detailed discussion of the mispricing illustrated in Figure 9-1 follows.

Out-of-the-Money: Microsoft $BS > MP$

1. The investors in Microsoft were risk-averse in pricing the out-of-the-money calls immediately prior to the market break.
2. Unlike the investors in IBM ($BS < MP$), the long bull market was not sufficient to change the shape of the utility function.

At-the-Money: Microsoft $BS = MP$

1. This was consistent with previous research, and true by definition (see implied volatility discussion in Chapter 7).

In-the-Money: Microsoft $BS < MP$

1. The investors in Microsoft options were risk-loving over the in-the-money calls.
2. The negative difference for \$25 in-the-money calls may signal the deep-in-the-money region where investors turn risk-averse.
3. IBM mispricing ($BS < MP$) was consistent with Microsoft.

4.2 Summary of Mispricing in Extended Pre-Break Period

The mispricing biases which we observed for Microsoft during the period leading up to the market break were quite erratic. The generally negative bias was, we believe, due to risk-averse behavior in the out-of-the-money calls and uncertainty in the in-the-money calls. A more detailed discussion of the mispricing illustrated in Figures 9-2 and 9-3 follows.

Out-of-the-Money: Microsoft $d|MP-BS|/dt = ?$

1. The plots of the 5-day moving average show the mispricing was generally negative but extremely volatile. For the \$5, \$10 and \$20 out-of-the-money calls, the average was positive in early October. These calls lend weak support to our argument that the degree of risk-averse behavior was decreasing prior to the crash.
2. The plots of the 20-day moving average were unable to show any general trends. The averages appeared sporadically in the \$5, \$10 and \$15 out-of-the-money calls.
3. While the Black-Scholes model performs better with options having about ninety days to expiration, when these options are not priced we must turn to the shorter maturity options which are priced in the market. The plots in the appendix to this chapter, Figure A9-1, better show the trends and, although they were still quite erratic, the magnitude of the bias was clearly negative.
4. The trend for IBM was positive for this period: $d|MP-BS|/dt > 0$.

In-the-Money: Microsoft $d|MP-BS|/dt = ?$

1. The plots of the 5-day moving average show a very erratic relationship between market price and model price for both the longer time to expiration (Figure 9-3) and shorter time to expiration (Figure A9-2).
2. Given the relatively sparse trading, we were unable to calculate and plot a 20-day moving average for any of the in-the-money contracts.
3. The trend for IBM was positive for this period: $d|MP-BS|/dt > 0$.

4.3 Summary: Post-Break Monthly Average

There was a generally negative bias for this period compared to the pre-break period, implying increased risk-averse behavior in pricing. While this fits our hypothesis, the presence of a sizable positive bias for the \$10 out-of-the-money call does not and cannot be explained. A more detailed discussion of the mispricing illustrated in Figure 9-4 follows.

Out-of-the-Money: Microsoft $BS > MP$ (2 out of 3)

1. The sign and magnitude of the mispricing was supportive of our theory given the impact of the market crash on investor attitudes towards risk. We would expect an increased reluctance to buy these options to show up as an increased negative bias.
2. The positive difference for the \$20 out-of-the-money calls has no ready explanation.
3. The mispricing bias for IBM was also positive for this time period.

At-the-Money: Microsoft $BS = MP$

1. This was consistent with previous research, and true by definition (see implied volatility discussion in Chapter 7).

In-the-Money: Microsoft $BS \geq MP$

1. The mispricing was only marginally positive for Microsoft. In comparison to the pre-break period, risk-loving behavior has been substantially reduced by the crash.
2. The range of contracts was also reduced by the market crash. Prior to October 19 there were six in-the-money contracts traded; post-October 19 there were only two.
3. The mispricing bias for IBM was significantly positive for this time period.

4.4 Summary Mispricing for Extended Post-Break Period

The mispricing biases which we observed for Microsoft during the period leading away from the market break were not well defined as the level of trading dropped sharply. While we were unable to show any trends, we have tried to show the magnitudes of the mispricings through the use of scatter diagrams in Figures 9-5 and 9-6. The preference for trading in the \$10, \$20 and \$30 (actually \$5, \$10 and \$15 due to 2-for-1 split) appears to reflect the traditional \$5 increments between contracts and a market imperfection. The important information contained in these plots is the magnitude of the mispricing.

The trend in mispricing IBM out-of-the-money calls during this period was positive ($d|MP-BS|/dt > 0$), while the trend in mispricing IBM in-the-money calls was uncertain ($d|MP-BS|/dt = ?$).

4.5 General Comparison of Mispricing Biases in IBM and Microsoft

In Table 9-3, we present the correlation coefficients of the daily mispricings for the extended pre- and post-market break periods. In the pre-break period, the negative correlation we observe in the out-of-the-money contracts are indicative of risk-loving pricing of IBM at a time when Microsoft traders were risk-averse. The positive correlation in the in-the-money contracts shows a similar risk-loving behavior.

In the post-break period, we see much less agreement. The major factors here are the drop in trading activity for Microsoft options and the increased volatility of the price differences for Microsoft. While it is important to investigate different options, it is difficult to find options with the same quality of data as IBM.

Table 9-3
Correlation of Microsoft and IBM Mispricing

Dollar Amount In-the-Money	Extended Pre-Break Period	Extended Post-Break Period
\$-30	#	-0.283 (-1.02)
-25	#	#
-20	#	-0.039 (-0.15)
-15	-0.302 (-1.38)	
-10	-0.169 (-1.04)	-0.520 (-3.34)*
- 5	-0.315 (-2.27)*	#
+ 5	+0.452 (+3.32)*	#
+10	+0.457 (+3.21)*	-0.067 (-0.19)
+15	+0.373 (+2.20)*	#
+20	+0.218 (+0.92)	#
+25	+0.241 (+0.74)	#
+30	#	#

*Significant at the 95 percent level.

#Less than 10 observations.

FOOTNOTES

¹Risk is expressed in quantitative terms as the variability in total returns (current income plus capital gains). Stable dividend payments serve to reduce the volatility of this measure and, thus, the risk perceptions of investors.

²The importance of which exchanges are involved relates to the use of daily closing prices and the relative closing times of the exchanges. Though the NASDAQ is not technically an exchange but a network of dealers, the system "closes" the same time as the American Options Exchange. Therefore, the use of closing prices does not suffer the problem of one set of prices containing information the other does not.

³Refer to discussion of the use of implied volatility estimates versus historical volatility estimates in empirical testing.

⁴Mean prices are included for market prices, model prices and their differences to provide a "ball park" feeling for the absolute amounts involved. Individual contracts move from one category to another as the price of the stock fluctuates more than the increment between categories (in this case, \$5).

⁵A plot using daily price differences versus a moving average was found to be uninformative due to the fluctuations of the daily numbers. Smoothing over five days, while having no in theory (i.e., there is no reason to believe traders average their prices over the previous week), maintains the relationship between current and trend data.

⁶To maintain continuity in the plots, missing observations (no trades occurring that day) were assigned a zero value (market equal model price). The 5-day moving average was allowed to contain no more than two zero observations; the 20-day moving average, no more than four.

⁷Microsoft common stock split 2 for 1 on September 21, 1987. The effect of this in the options market is the halving of exercise prices. The incremental exercise price for the contracts in existence then changes from \$5 to \$2.50. To maintain the relationship (in percentage terms) between IBM and Microsoft, we show the new \$2.50 in-the-money/out-of-the-money as \$5, the \$5 as \$10, and so on. The tradition of trading in \$5 incremental contracts is probably the reason for such a low volume in the \$2.50-\$7.50-etc. contracts.

APPENDIX TO CHAPTER 9

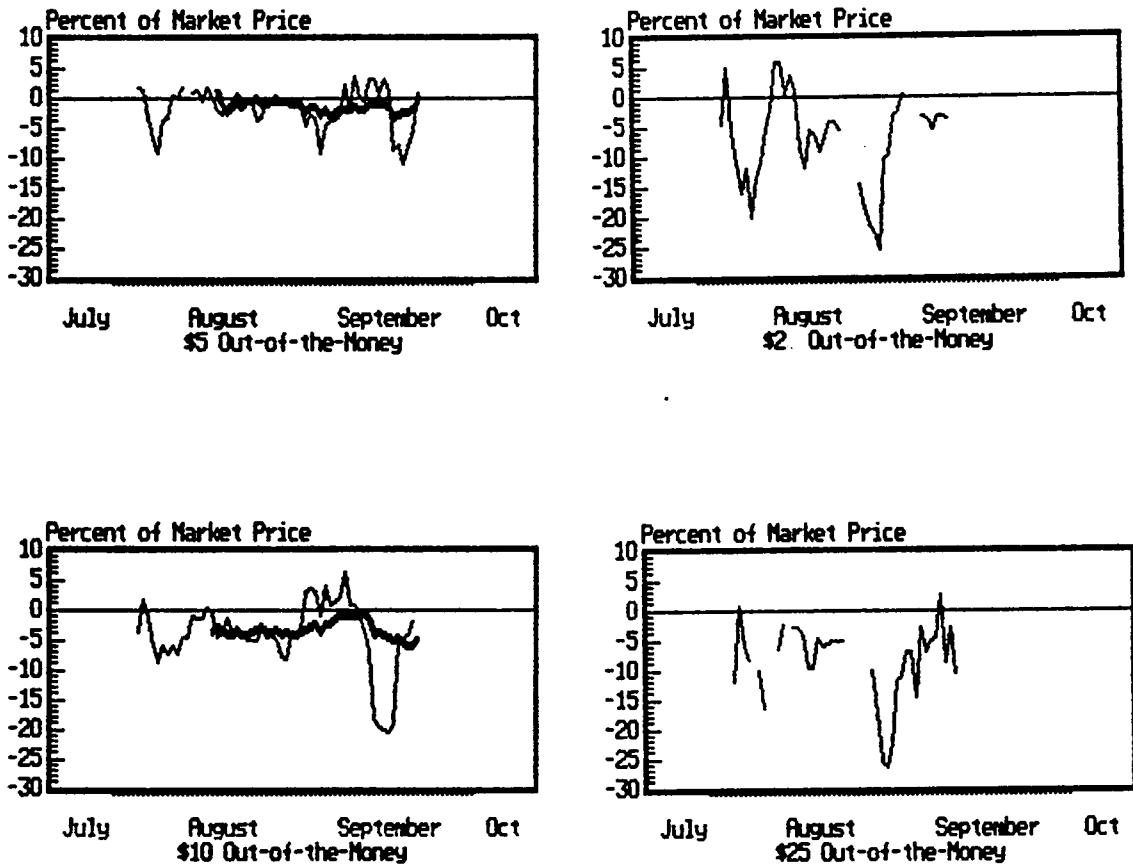


Figure A9-1. 5-Day and 20 Day Moving Averages of Mispricing Biases for the Out-of-the-Money Call Options: July-October Pricing of October 1987 Microsoft.

APPENDIX TO CHAPTER 9

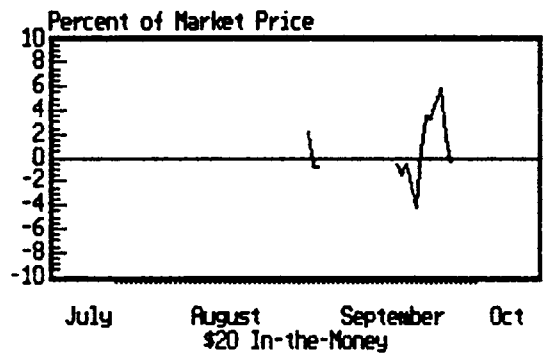
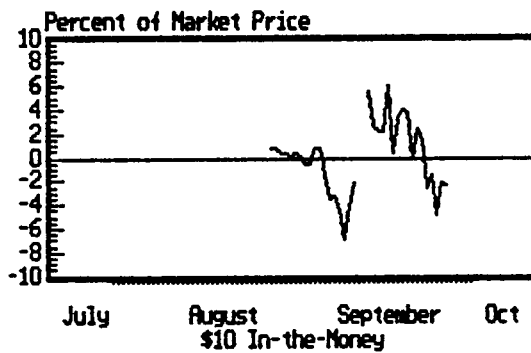
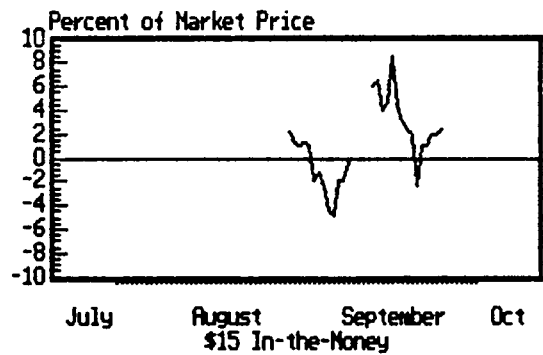
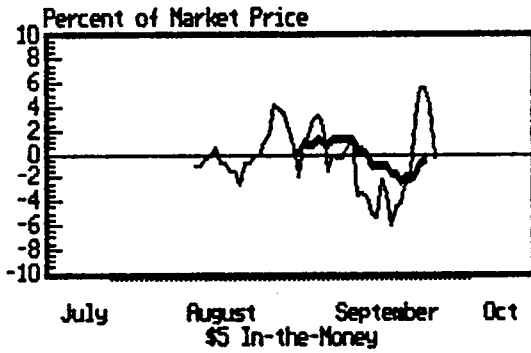


Figure A9-2. 5-Day and 20 Day Moving Averages of Mispricing Biases for the In-the-Money Call Options: July-October Pricing of October 1987 Microsoft.

Chapter 10

Summary and Conclusions

This dissertation has applied the expected utility theories of microeconomics to the problem of mispricing paradoxes in the financial theory of option pricing. In the process, we inferred risk preferences from the mispricing to support the existence of Friedman-Savage-Markowitz utility functions.

In economics, the theory that individuals who both gamble and buy insurance have utility functions which are concave over some ranges of final wealth and convex over others has a well established literature. Experiments to give substance to the theory have been criticized for their predominant use of students or in the manner in which the experiment questions were framed. Our efforts are in the real-world financial markets with participants speculating and hedging with dollar amounts infeasible in classroom experiments.

In finance, there is a very elegant model for pricing options which does very well, yet exhibits systematic mispricing biases for which no good explanation has been found. The model is based on the ability of individuals to establish riskless hedges and thus remove any influence of investor attitudes concerning risk. Our efforts are to show that the observed mispricing biases can be the result of investor risk-aversion and risk-preference. Simply stated, the presence of risk-averse behavior leads to market prices less than model prices (negative mispricing bias) and risk-loving behavior leads to market prices greater than model prices (positive mispricing bias). Of course, the presence of risk-neutral behavior leads to market prices equal to model prices (and zero bias).

The presentation in Chapter 1 of the literature on higher order utility functions led into the relationship between the shape of utility functions and preferences for the various moments of a return distribution in Chapter 2. When more than the mean and variance of a return distribution matter to an investor, his utility function must contain an inflection point.

In Chapter 4 the financial markets for options was proposed as a particularly appropriate setting in which to analyze investor attitudes for risk. Building on the general discussion and terminology of options markets provided in Chapter 3, we analyzed the characteristics of options which appeal to speculators and hedgers alike. A simple experiment showed how the combination of call option payoffs, a lognormal stock return distribution and expected utility maximization gave some empirical support to our intuition that risk-aversion played a primary role in the pricing of out-of-the-money call options; whereas risk-loving was predominant in the pricing of in-the-money call options.

In Chapter 5 we reviewed the literature on the development of modern option pricing models. The role of arbitrage and the ability to create and maintain riskless hedges was shown to be the centerpiece of the theory. When models requiring information on investor preference for risk, such as the Sprenkle equation with its individualistic parameters for expected rate of growth in stock price and measure of market risk aversion, were supplanted by the risk-neutral valuation models, such as

the Black-Scholes equation, it was heralded as a major breakthrough in asset pricing.

The problems associated with fulfilling the requirements of such risk-neutral models like the Black-Scholes were examined in Chapter 6. We believe them to be sufficient to conclude that riskless hedges cannot routinely be maintained and that the pricing models are not preference-free. If that is the case, we should be able to infer investor preferences and the general shape of investor utility functions in the neighborhood of current wealth from the mispricing of options by these models.

Chapter 7 summarized the extensive literature on mispricing of risk-neutral option pricing models. The research has, however, been at odds in the magnitude and direction of the mispricing biases. One paradox has been that while the models tend to overprice out-of-the-money calls and underprice in-the-money calls, deep-outs are underpriced and deep-ins are overpriced! Adding further confusion to the empirical results were the confirmations that the direction of mispricing seemed to reverse itself from time to time.

In our critique of the methodology employed in the empirical testing literature, we emphasized such problems as aggregating across companies and significantly different market periods. While these problems tended to reduce the magnitude of the mispricing, they were not sufficient to lead researchers to the conclusion that mispricing was not significant.

With regard to uncovering the cause of the mispricing, we believe that aggregation and testing over different markets does, in fact, mask some of the important characteristics of the mispricing which has lead to some of the confusion. In particular, it is important to follow the change in the mispricing as risk attitudes change in the market.

Addressing the first area of concern, the Friedman-Savage-Markowitz family of utility functions was shown to allow the very mispricing biases observed in the market. Individuals are risk-averse over moderate to small negative changes in wealth and over very large positive changes; they are risk-loving of large negative changes and small to medium positive changes in wealth. This translates into risk-averse pricing (negative mispricing bias) over out-of-the-money and deep-in-the-money call options; risk-loving pricing (positive mispricing bias) over in-the-money and deep-out-of-the-money call options.

The problem of a change in the sign of the mispricing bias from one period to the next is more difficult to explain. Our sense is that the changes were primarily the result of a temporary change in investor risk preferences as shown in the curvature of the utility function. The Child and Whiting observation that the level of aspiration rises with success and falls with failure might explain these changes in investor utility.

To illustrate our point, we analyzed the mispricing of a set of call options written on the same underlying stock, IBM, in Chapter 8. Being at the right place at the right time, we were able to examine the period

leading up to and following the significant market break on October 19, 1987. Prior to the market's steep drop we found relatively minor mispricing of the out-of-the-money calls. This implied relatively low risk-aversion on the part of the traders of IBM options. Immediately prior to the market crash, we found the direction of mispricing had turned positive, implying risk-loving attitudes in the pricing of out-of-the-money calls. The associated higher levels of risk-taking (i.e., larger positive mispricing bias) in the in-the-money calls were not observed, though this may be the result of spreading the risk over the out-of-the-money calls.

Following the market break, we found evidence of increased risk-aversion. In the absence of a second major market drop (which had not occurred as of May 1), the presumption was that as the effects of the market break wore off, the degree of mispricing would return to "normal." We observed the magnitude of mispricing in out-of-the-money calls remained relatively high through the end of February, implying that the market crash still had a strong influence on risk attitudes over four months later.

We attempted to confirm these results in Chapter 9 by testing a second stock, though one with different characteristics--it does not pay a dividend. The role of dividends in the mispricing literature has moved to the forefront as other problem areas have been dealt with. We sought to strengthen our case for preference-related mispricing by showing the mispricing of IBM was also present in a non-dividend case.

In the period prior to the market break, the pricing of both in-the-money and out-of-the-money Microsoft call options exhibited greater risk-aversion than their IBM counterparts. Microsoft out-of-the-money calls had a greater negative mispricing bias in the extended period than IBM and a negative bias in the immediate pre-market break period when IBM showed a positive bias. The offsetting mispricing biases for the September-October period are evidence that testing a portfolio of stocks may lead to an unfounded confidence in a risk-neutral model to correctly price individual options. The case against testing portfolios is further strengthened by negative correlation between IBM and Microsoft in three of eight categories prior to the market crash and in four of four categories afterward.

Though similar patterns were found between the two when the market was performing well, we believe there are sufficient differences to warrant the use of individualized models for each option contract, very much in the spirit of Sprenkle's model.

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