SIMULATION AND OPTIMIZATION OF A PACKAGE SEWAGE TREATMENT PLANT

by

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

Since the dawn of iniquity, Man has often tried to utilize the forces of Nature to achieve a desired product or result. And so men do research to try to tell other men how to better utilize the forces of Nature.

Purpose of Research Investigation

The purpose of this research is to investigate the interaction of the physical, biological, and chemical forces occurring in a package sewage treatment plant. The treatment plant consists essentially of a plastic medium trickling filter mounted overtop of a complete-mix activated sludge (CMAS) tank; the unit also has a primary sludge holding tank and a secondary clarifier.

The research effort consists of three stages. The first stage is a review of the literature and a careful scrutiny of the data actually obtainable from the plant in order to determine the variables of interest and the predominate theories for mathematically relating the various variables. The second stage is a computer simulation of the package sewage treatment plant based on actual plant data and a developed mathematical model. Once the behavior of the plant is accurately simulated, the final stage of the research is to optimize the various design variables with respect to cost.

LITERATURE REVIEW

For the sake of convenience, the literature is presented in four sections. First, early efforts to model trickling filters based on data from stone media are presented. The theoretical basis of certain empirical equations is discussed. Next, developments in thinking are presented including the packed bed model and biological film flow models, considering both pseudo-homogeneous and heterogeneous films. Then, a section is devoted to the simulation of the newer, plastic media trickling filters. The literature review is chronological for the most part and attempts to show every possible approach to the modelling of such filters. Lastly, the analogy between trickling filtration and activated sludge processes is examined.

Mathematical Models of Trickling Filter Performance

One of the first empirical relationships for trickling filter efficiency was developed by the National Research Council (NRC) in 1946 (1). The NRC formulation was the result of an extensive analysis of operational records from stone media trickling filter plants serving military installations in the United States. Based on data analysis, the NRC recommended the following formulation:

$$E = \frac{100}{1 + 0.0085\left(\frac{W}{VF}\right)^{\frac{1}{2}}} \qquad (1)$$

in which

E = efficiency of a single filter or the first-stage trickling filter in percent BOD_5 removal

W = applied organic loading in pounds of BOD_5 per day of settled wastewater at 20^oC

V = volume of filter medium in acre-feet

F = recirculation factor

and

$$F = \frac{1 + N}{(1 + (1 - j)\ N)^2} \qquad (2)$$

N = recirculation ratio equal to the ratio of recirculation flow to plant influent flow (N = R/I)

j = weighting factor, values range from 0.81 to 0.95 with average equal to 0.90; the j factor recognizes the change in character of the wastewater during treatment, or a decrease in treatability

The equation proposed for second-stage filters is:

$$E_2 = \frac{100}{1 + \frac{0.0085}{1 - E_1}\left(\frac{W_2}{V_2 F_2}\right)^{\frac{1}{2}}} \qquad (3)$$

where

E_1 = percent BOD_5 removal efficiency through the first-stage filter and clarifier

E_2 = percent BOD_5 removal efficiency through the second-stage filter and clarifier

V_2 = recirculation factor based on N_2, the recirculation ratio for the second-stage filter

F_2 = recirculation factor based on N_2, the recirculation ratio for second-stage filter

W_2 = BOD_5 loading (lbs./day) to the second-stage filter, not including recycle

Some of the limitations of the NRC formulas are: (1) the effect of temperature on trickling filter performance is not considered; (2) applicability is limited to concentrated domestic wastewaters because no factor is included to account for differing treatability rates; and (3) the formula for second-stage filters is based on the existence of intermediate settling tanks following the first-stage filters. The theoretical basis of the NRC formulas will be discussed in a later section.

In 1948, Velz proposed the first major formulation delineating a fundamental law as contrasted to previous attempts based on data analysis. This was the great first-order assumption. Velz postulated that: "The rate of extraction of organic matter per interval of depth of a biological bed is proportional to the remaining concentration of organic matter, measured in terms of its removability. (2) "This concept can be expressed as a first-order ordinary differential equation:

$$-\frac{dS}{dD} = k_D S \tag{4}$$

which on integration becomes:

$$\ln \frac{S_D}{S} = -k_D D \tag{5}$$

or

$$\frac{S_D}{S} = e^{-k_D D} \tag{6}$$

where

S = total removable fraction of BOD, mg/l

S_D = quantity of removable BOD_5, mg/l, at depth, D, mg/l

S_D/S = proportion of the total removable fraction remaining in the passing liquid

D = filter depth, ft.

K_D = constant

R. S. Rankin in 1953 developed empirical formulas based on the Ten-States Standards for various biofiltration schemes (3). The formulas developed by Rankin have as their theoretical basis substrate mass balances. The substrate mass balance for a first-stage trickling filter with recirculation was given as

$$(S_a - S_e)(I + R) = S_o I - S_e I \tag{7}$$

where

S_o = BOD_5 of settled raw sewage in ppm

S_e = BOD_5 of settled effluent in ppm

S_a = BOD_5 of waste applied to the filter after mixing with the recirculated flow

I = Influent flow in MGD

R = Volume of flow recirculated through the filter

S_a was also assumed to be

$$S_a = a_1 S_e \tag{8}$$

where a_1 appears to reflect the kinetic properties of the waste. Substitution of Equation (8) into Equation (7) and assuming that

$$R_1 = R/I \tag{9}$$

yielded

$$\frac{S_e}{S_o} = \frac{1}{R_1 (a_1 - 1) + a_1} \tag{10}$$

where R_1 = ratio of recirculated flow to raw sewage flow. In this analysis, the recirculated flow is filter discharge settled either in the primary clarifier or in the final clarifier and hence the value of R_1 is the ratio for the total volume recirculated through the filter and a settling tank.

The substrate utilization efficiency may be defined as

$$E = 1 - \frac{S_o}{S_e} \tag{11}$$

and substituting Equation (10) into Equation (11), Rankin found an expression for the efficiency, E_1, of a single-stage filter:

$$E_1 = \frac{R_1 + 1}{R_1 + b_1} \tag{12}$$

where

$$b_1 = \frac{a_1}{a_1 - 1}$$

When $b_2 = \frac{a_2}{a_2 - 1}$, the efficiency of a second-stage filter was given as

$$E_2 = \frac{R_2 - 1}{R_2 - b_2} \tag{13}$$

where E_2 = the efficiency of a two-stage filter system

and R_2 = ratio of recirculated flow to raw sewage flow in the second stage.

The 1951 Tentative Standards of the Upper Mississippi River Board of Public Health Engineers and the Great Lakes Board of Public Engineers state that

"For Type A (single-stage) filters, the recirculation system shall apply sufficient dilution to the settled sewage so that the BOD of the influent to the filter, recirculation included, shall not exceed three (3) times the BOD_5 of the required settled effluent...For two-stage filters, the BOD_5 load applied to the second stage filter, recirculation included, shall not exceed two times the BOD expected in the settled effluent. When the effluent of the first stage filter is applied directly to the second stage filter without intermediate settling, the assumed BOD removal by the first stage filter shall not exceed 50% (3)." Simply put, this means that a_1 = 3 or less and a_2 = 2 or less or b_1 = 1.5 and b_2 = 2.

Geber (1954) proposed a curve representing a relationship between filter efficiency and a dimensionless factor involving hydraulic loading filter depth, and BOD reaction rate (4). The filter medium consisted of trap rock varying in size from 1 to 1.5 inches in diameter. All efficiencies were based on reductions in BOD obtained solely through the filters operated without recirculation. The following relationship was computed from the curve presented by Geber:

$$\frac{S_e}{S_0} = 0.00524 \frac{Q}{D\ K_{20}} + 0.069 \qquad (14)$$

In the absence of a measured value for BOD_5 reaction rate a value of 0.2 per day at 20°C was assumed and variations with temperature were computed from the equations:

$$K_T = K_{20}\ (1.047)^{T-20} \qquad (15)$$

in which

K_{20} = BOD_5 reaction rate constant at 20°C in units of 1/day

S_e = BOD_5 at depth D in mg/l

S_o = BOD_5 applied in mg/l

Q = hydraulic dosage rate in feet per day;

and in the above Q/D K_{20} = a dimensionless product.

Fair and Geyer (1954) presented the following relationship for the rate of purification in filter (5):

$$\frac{dS}{dt} = -k_t \left(\frac{S}{S_o}\right)^u S \qquad (16)$$

The equation may be written in terms of either time of contact or bed depth, therefore alternatively

$$\frac{dS}{dD} = -k_D \left(\frac{S}{S_o}\right)^u S \qquad (17)$$

in which S_o is the amount of removable organic substances and u is a coefficient that measures the non-uniformity of the rate of purification. These equations reduce to the form of a first order reaction when u = 0. Integration of the above equations produces the following relationships:

for u = 0

$$\frac{S_t}{S_o} = \exp(-k_t t) \tag{18}$$

or alternatively

$$\frac{S_D}{S_o} = \exp(-k_D D) \tag{19}$$

Equation (19) is the same as that developed by Velz.

For u greater than zero, integration over time or distance produces

$$\frac{S_t}{S_o} = (1 + u\, k_t t) - \frac{1}{u} \tag{20}$$

or alternatively

$$\frac{S_D}{S_o} = -(1 + u\, k_D D) - \frac{1}{u} \tag{21}$$

Fairall (1956) correlated trickling filter loadings and performance data statistically and concluded that as long as the rate of absorption of oxygen was not limited, the strength of the sewage was not a factor in filter performance expressed as a fraction of BOD remaining in the settled filter effluent (6). Data were assembled from 44 plants with single-pass filters in the Upper Mississippi Valley States using 24 hour composite samples for 80 random days, and computed by the method of least squares. For filters with recirculation from the final clarifier through the primary clarifier, it was assumed that the portion of the filter feed equal to the recirculated flow had a strength equal to the settled effluent from the final clarifier. Sufficient data regarding temperature variations were not available in this study.

The empirical equations developed include:

$$\frac{S_e}{S_o} = 1.102 \left(\frac{V}{Q}\right)^{-0.322} \tag{22}$$

for filters without recirculation, and

$$\frac{S_e}{S_o} = 2.065 \left(\frac{V\ (\ 1 + N)}{Q}\right)^{-0.444} \tag{23}$$

for filters with recirculation. The equation which was developed for all data may be written as

$$\frac{S_e}{S_o} = 1.157 \left(\frac{V\ (\ 1 + N\ )}{Q}\right)^{-0.396} \tag{24}$$

where

V = volume of filter media in 1000 cubic feet

Q = volume of raw sewage in MGD

In 1953 W. E. Howland considered the effect of temperature on the biological reaction rate constant K_T (7):

$$K_T = K_{20}\ 1.035^{T-20} \tag{25}$$

The theta coefficient of 1.035 differs from the 1.047 value used by other researchers (2,4), reflecting Howland's hypothesis that trickling filtration is less sensitive to temperature changes than completely mixed systems.

In 1958 Howland proposed that the rate of BOD removal in a trickling filter is a function of the time of contact of the sewage with the slime-holding surfaces and tne form of the equation was (8):

$$\frac{S_e}{S_o} = e^{-K_T t} \tag{26}$$

The time of liquid contact with the biological mass was shown to be directly proportional to the filter depth and inversely proportional to the hydraulic loading rate; this is expressed as follows:

$$t = \frac{CD}{Q^n} \tag{27}$$

where

t = liquid contact time, minutes

C = Constant

D = Filter depth, feet

Q = Hydraulic Loading rate, gpm/sq. ft.

n = Exponent characteristic of the filter media

Based on an analysis of the time of flow over a sphere and down an inclined plate, Howland concluded that the time of travel through a stone trickling filter varies inversely with the two-thirds power of the liquid rate of application Q in MGAD. In an adaptation of the Velz theory, Howland stated the first-order equation for BOD removal in the following form:

$$\frac{S_e}{S_o} = e^{-K\,D/Q^{2/3}\,T} \tag{28}$$

Howland's relationship for the time of flow over a sphere was given as:

$$t_s = 2.6\,\left(\frac{3v}{g}\right)^{1/3}\,(2\pi)^{2/3}\,r^{5/3}\,/\,(Q')^{2/3} \tag{29}$$

in which

v = kinematic viscosity of liquid

r = radius of sphere

g = acceleration of gravity

Q' = total rate of flow on the sphere (vol. per unit time)

He assumed that all the liquid comes in at the very top and leaves the very bottom of the sphere and that the effect of forces of acceleration on the top half of the sphere and of deceleration on the bottom half of the sphere are negligible or, as seems more plausible, exactly compensating in their effect on the time of flow. The time of flow on an inclined rectangular plate was shown by Howland to be:

$$t_p = \left(\frac{3v}{\sin \beta g} \right)^{1/3} \frac{L}{\left(\frac{Q'}{w}\right)^{2/3}} \qquad (30)$$

in which

w = width of inclined plate

L = length of inclined surface

β = angular inclination

This equation is based on the assumption that laminar flow conditions prevail and the velocity of the liquid in contact with the plate surface is zero. The rate of application may be also expressed in terms of volume per unit time divided by the horizontal projected area of the sphere i.e., $Q = Q'/_{\pi} r^2$. The total time of flow through the filter becomes

$$t_p = \frac{1.3\ (2)^{2/3} \left(\frac{3v}{g} \right)^{1/3}}{r^{2/3}} D/Q^{2/3} \qquad (31)$$

If the medium of the filter consisted of n_1 plates each 1 unit wide in 1 square unit of horizontal area of the filter, inclined at an angle of B to the horizontal the liquid flowing on one side only, and if the rate of application of wastewater per unit of horizontal area of filter is equal to $Q = (\frac{Q'}{w})\ n_1$ then from Formula (30)

$$t_p = \frac{(\frac{3v}{g})^{1/3} n_1^{2/3}}{(\sin B)^{4/3}} D/Q^{2/3} \qquad (32)$$

When the plates are placed vertically and the liquid is allowed to go down both sides of each plate Formula (32) becomes

$$t_p = (\frac{3v}{g})^{1/3} (2n_1)^{2/3} D/Q^{2/3} \qquad (33)$$

For filters with recirculation, Howland modified Formula (28) to read

$$\frac{S_e}{S_o} = e^{-K D(1 + N)^{1/3}/Q^{2/3}} \qquad (34)$$

where N = recycle ratio which affects liquid film thickness. The one-thirds power appears on the recirculation factor because the thickness of a flowing film varies with the 1/3rd power of the rate of flow. For a constant influent rate to the plant, then the average time of flow through the filter would vary directly with the volume retained in the filter, i.e., with the thickness of the film, or in other words, with $(1 + N)^{1/3}$. This method of representing the data implies that recirculation effects a significant improvement of the effluent of a filter though not as significant as had been proposed by Velz whose suggestion, with qualifications, was virtually that the value 1 + N should appear

in any function to the same power as D, the depth, for he wrote "Thus R/I is 1.0 is equivalent to passing the influent through the filter twice which is tantamount to passing I once through two successive depth intervals (2)." The beneficial aspect of recirculation was seen by Howland to be due to increased contact time, comporting with Howland's general notion that time is the important factor in determining purification.

In 1960, K.L. Schulze considered hydraulic loading as the important variable affecting trickling filter efficiency (9). An experimental screen trickling filter was operated at seven different hydraulic loads ranging from 4.9 to 30 mgd/acre treating dairy waste having a BOD from 200 to 2,000 mg/l. A log plot of effiency versus hydraulic load showed that the fraction of soluble BOD remaining increased approximately linearly (on log axes) with hydraulic loading in MIAD. The curve was represented by the parabolic equation

$$\frac{L_e}{L_o} = aQ^c \tag{35}$$

where

L_e = final effluent BOD, mg/l

L_o = BOD_5 of flow to the filter, mg/l

Q = hydraulic load, mgd/acre

c = constant, and

a = intercept at Q = 1.

The constant derived from the graph was c = 0.67 and the intercept was

a = 0.079 so that

$$\frac{L_e}{L_o} = 0.079Q^{0.67} \tag{35a}$$

This means that the fraction of BOD remaining is proportional to the hydraulic load q to the 0.67 power. The constant 0.67 duplicated the two-thirds power of Q used by Howland (7) and Bloodgood, Teletzke, and Fohland (10) in computing the contact time for trickling filter models at various flow rates.

In considering the effects of depth and of temperature, Schulze used Equations (25) and (28) developed previously by Howland and reported a K_1 value of 0.3 for a hydraulic dosing rate of 5 mgd/acre and a filter depth of 6 feet. K_1 is the value of the BOD biological reaction rate constant, K, based on base ten logarithms, i.e., K = 2.303 K_1. Hence, Schulze's formula for the fraction of BOD remaining in a single-pass filter was

$$f_1 = \frac{L_e}{L_o} = 10^{-bK_{20}D/Q^{0.67}} \tag{36}$$

in which

$b = 1.035^{T-20}$

T = temperature in degrees Celsius, and

$K_{20} = 0.3$.

If recirculation is used, the hydraulic load to the filter is determined by the recirculation ratio r, so that the new hydraulic load is:

$$Q_2 = Q(1 + r) \tag{37}$$

where

Q_2 = hydraulic load under recirculation, and

$r = R/I$ = ratio of recirculation flow to incoming waste flow.

The fraction of BOD remaining under these conditions is

$$f_2 = \frac{L_{er}}{L_m} = 10^{-KD/Q_2^{2/3}} \tag{38}$$

or

$$L_m = \frac{L_{er}}{f_2} \tag{39}$$

where

L_{er} = BOD concentration of final effluent or recycle flow in mg/l,

L_m = BOD concentration of mixed flow (incoming and recycle) in mg/l, and

Q_2 = hydraulic load in mdg/acre including recirculation flow.

In this formulation, final effluent is used for recirculation and is mixed with incoming waste directly ahead of the filter. The filter is considered to operate basically the same way as a single pass filter, except that the hydraulic load is increased according to r and that the degree of BOD_5 removal is now related to the BOD_5 of the mixed flow. The BOD_5 concentration of the mixed flow is given by:

$$L_m = \frac{rL_{er} + L_o}{1 + r} \tag{40}$$

Combining Equations (39) and (40):

$$\frac{L_{er}}{f_2} = \frac{rL_{er} + L_o}{1 + r} \tag{41}$$

and rearranging:

$$L_{er} = \frac{L_o}{(\frac{1 + r}{f_2}) - r} \tag{42}$$

The fraction of BOD remaining in final effluent in relation to the actually incoming (untreated) BOD is then given by:

$$F_3 = \frac{L_{er}}{L_o} = \frac{1}{(\frac{1 + r}{f_2}) - r} \tag{43}$$

Schulze points out that according to this theoretical equation recirculation does not improve the quality of the final effluent, because what is gained by dilution of the incoming BOD_5 is lost by the increase in f_2 due to the increased hydraulic load. It should be noted that this equation applies only to filters with the mixing point ahead of the filter.

Schulze also pointed out the conformity between the adsorption process and the trickling filter process using modified Freundlich adsorption isotherms (9). The importance of this analogy is that the BOD remaining after treatment can be shown to be a constant fraction of the applied BOD_5 if the concentration of adsorbent, such as the active biological film in a trickling filter, is assumed to be constant.

In 1961 W. Wesley Eckenfelder, Jr., proposed that BOD removal is related to the surface area of active film per unit volume of filter media (11). This is analogous to the concentration of mixed liquor solids in the activated sludge process. The surface area of film frequently varies with the type, distribution and depth of filter medium and season of the year in many cases. The assumption that the amount of active surface film covering the filter medium decreases with depth was mathematically approximated as

$$C \sim \frac{1}{D^m} \tag{44}$$

in which C is related to the mean active filter film per unit volume throughout the filter depth. When the filter film is approximately uniformly distributed the exponent m is zero and when the film distribution, activity and composition vary with the filter depth the exponent m is less than 1.0.

At the low BOD_5 concentrations usually applied to a filter first-order kinetics are frequently assumed (1,2,5,8,9):

$$\frac{L_e}{L_o} = e^{-kt} \tag{26}$$

The residence time, t, in Eq. (26) has been related to the depth and hydraulic loading of the filter and to the physical characteristics of the filter media (8).

$$t = \frac{CD}{Q^n} \tag{27}$$

The constant, C, and the exponent, n, will vary with the type of filter media and the hydraulic characteristics. The exponent has been shown to approach 1/3 for turbulent flow and 2/3 for laminar flow (11). Equation (26) can be modified to include the residence and the density of film by combing with Eqs. (27) and (44):

$$\frac{L_e}{L_o} = e^{(-K)(1/D^m)(D/Q^n)} \tag{45}$$

and combining terms

$$\frac{L_e}{L_o} = e^{(-KDD^{1-m})/Q^n} \tag{45a}$$

The filter film was approximately uniformly distributed in studies conducted by McDermott (12) in a 23-ft column of 3½ in. balls and by Schulze on a screen filter(9). In these cases the exponent m in Eq. (45a) becomes zero.

Equation (45a) presumes that all the components of the organic waste are removed at the same rate. "There is considerable evidence, however, that in sewage and complex wastes the removal decreases with concentration or time, because the more readily assimilable components will be removed more rapidly." (11) Eckenfelder states that BOD removal is probably not a first-order reaction. He suggests that as the time of contact progresses the more readily treatable BOD is already removed and a so-called reduction on "treatability" of BOD_5 remaining occurs; therefore, Eq. (45a) is retarded to the form:

$$\frac{L_e}{L_o} = \frac{1}{1 + \frac{C\ D^{(1-m)}}{Q^n}} \tag{46}$$

It becomes more obvious that this is a retardant form of:

$$\exp(-C\ D^{1-m}\ /Q^n)$$

if it is expanded to:

$$\exp(-C\ D^{1-m}\ /Q^n) = \exp(-x) = \frac{1}{1 + x + (x^2/2) + x^3/3) + \ldots} \tag{47}$$

and only the first two figures of the denominator are considered.

The constant C and the exponents m and n were determined by multiple regression analysis of several sets of published performance data. For rock filters treating domestic sewage, mean values of the coefficients were determined as C = 2.5, (1-m) = 0.67, and n = 0.5. Eq. (46) then becomes

$$\frac{L_e}{L_o} = \frac{1}{1 + 2.5 \frac{D^{0.67}}{Q^{0.50}}} \tag{46a}$$

where the depth D is in feet and the hydraulic rate Q is in MGAD. The following formula was used for filters with recirculation

$$\frac{L_e}{L_o} = \frac{1}{(1 + N)\left(1 + 2.5 \frac{D^{0.67}}{Q^{0.5}}\right) - N} \tag{48}$$

where N represents the recirculation ratio. Equations (46) and (48) were considered applicable only below a limiting loading above which further removal and assimilation cannot occur.

In 1964, the last major effort to forecast the performance of stone filters was attempted by Galler and Gotaas (13) using multiple regression analysis of data from existing plants. Based on regression analysis, the following equation was developed:

$$S_e = \frac{0.464 \; a^{1.19} \; (1+N)^{0.28} \; Q^{0.12}}{(1 + D)^{0.67} \; T^{9.15}} \qquad (49)$$

where

S_o = BOD_5 of settled raw sewage in mg/l

S_e = BOD_5 of settled effluent in mg/l

S_a = BOD of domestic waste applied to the filter after mixing with the recirculated flow

D = filter depth in feet

T = temperature in degrees Centigrade

Q = hydraulic loading in MGAD.

The BOD applied to the filter in milligrams per liter was given by the mixing equation

$$S_a = \frac{I \; S_o + R \; S_e}{I + R} \qquad (7)$$

in which I is the plant in influent rate in MGD and R is the recirculation rate in MGD.

The Galler and Gotaas formula recognized the effects of recirculation, hydraulic loading, filter depth, and wastewater temperature as being important in understanding the performance of a trickling filter. They further indicated that recirculation improves the performance of a filter, but established a 4:1 ratio as a practical upper limit for recirculation. The beneficial effect of recirculation was viewed as seeding the influent sewage with organisms predominating in the filter environment, the highest concentration of which is obtained by recirculating the sludge from the secondary clarifier.

A Note on Mechanistic Versus Empirical Design Equations

Whereas empirical equations require extensive experimental data to evaluate the constants in the equation, the mechanistic equations provide the value of the constants from the knowledge of the measurable physical parameters of the trickling filter and influent. The mechanistic approach provides the understanding of the interrelationship of those parameters which affect the design and operation of trickling filters.

The empirical relationship known as the NRC formula can be written:

$$E = \frac{L_o - L_e}{L_o} = \frac{1}{1 + 0.0085 \left(\frac{W}{VF}\right)^{\frac{1}{2}}} \qquad (1)$$

Values W and V in Eq. (1) are dependent variables and can be expressed in terms of Q, D, and influent BOD, L_o. For zero recycle (F=1) then

$$\frac{L_o - L_e}{L_o} = \frac{1}{1 + 0.0085 \left(520 \frac{Q}{D} L_o\right)^{\frac{1}{2}}} \qquad (50)$$

The NRC formula was developed with a fairly narrow range of BOD values, say from 100 mg/l to 250 mg/l. Therefore over this range these values would vary:

$$\frac{L_o - L_e}{L_o} = \frac{1}{1 + 1.95 \left(\frac{Q}{D}\right)^{\frac{1}{2}}} \quad (\text{at } L_o = 100)$$

to

$$\frac{L_o - L_e}{L_o} = \frac{1}{1 + 3.04 \left(\frac{Q}{D}\right)^{\frac{1}{2}}} \qquad (\text{at } L_o = 250) \tag{51}$$

The Velz type equation proposed by Howland and reiterated by Schulze has been give as

$$\frac{L_e}{L_o} = e^{-D/Q^{2/3}} \tag{28}$$

Equation (28) can be written as

$$\frac{L_o}{L_e} = 1 + \left(\frac{k\ D}{Q^{2/3}}\right)^1 + \left(\frac{kD}{Q^{2/3}}\right)^2 \frac{1}{2!} + \left(\frac{kD}{Q^{2/3}}\right)^3 \frac{1}{2!} + \ldots + \left(\frac{kD}{Q^{2/3}}\right)^n \frac{1}{n!} \tag{52}$$

When the group, $kD/Q^{2/3}$, is sufficiently small, the empirical equation (53) can be simplified to

$$\frac{L_o}{L_e} = 1 + \frac{kD}{Q^{2/3}} \tag{53}$$

Therefore, by rearranging Eq. (53) it becomes

$$\frac{L_o - L_e}{L_e} = \frac{kD}{Q^{2/3}} \tag{53a}$$

or

$$\frac{L_o - L_e}{L_o} = \frac{1}{1 + \frac{1}{k}\left(\frac{Q}{D}\right)^{2/3}} \tag{54}$$

Because in their original forms the Velz type equation and the NRC formula appear to be quite different, they were considered to be two entirely different empirical equations. However, by the development of these equations as has been done in Eqs. (51) and (54), it is quite obvious that these equations are basically the same.

Using D = 6 ft to 12 ft, normal for stone filters, the NRC formula, Eq. (51) becomes

$$\frac{L_o - L_e}{L_o} = \frac{1}{1 + 0.564\ Q^{\frac{1}{2}}} \text{ to } \frac{1}{1 + 1.24\ Q^{\frac{1}{2}}} \tag{55}$$

Further, using K = 0.088 and D = 6 ft to 12 ft, the Velz type Eq. (54) reduces to

$$\frac{L_o - L_e}{L_o} = \frac{1}{1 + 0.95\ Q^{2/3}} \text{ to } \frac{1}{1 + 1.95\ Q^{2/3}} \tag{56}$$

Although the powers of Q and D are different in Eqs. (51) and (54), respectively, the NRC formula and Velz formulation, the values of the coefficients and the narrow range of operation provide results of the same order as shown in Eqs. (55) and (56).

These equations are inherently of the same form and it is quite probably that the differences in coefficients can be attributed to the use of different sets of experimental data (Mehta) (14).

Developments in Thinking

a. The Packed Bed Model (Mass Transfer Model)

In 1962 Ames presented a mathematical model of a trickling filter based on hypotheses that are similar in character to those that occur in chemical engineering treatment of packed beds (15). The BOD component was assumed to be transferred from the liquid phase to the solid phase by some mechanism of transport that was thought to be absorption. First order kinetics were used to describe the biochemical reactions that take place in the surface layer of the slime. A set of simultaneous partial differential equations describing the transient operation of the trickling filter were formulated and solved. Those material balance equations were also based on the assumption of a linear equilibrium relation between the mole fraction of BOD in the liquid-solid interface and the mole fraction of BOD in the solid.

Amado in 1964 further extended this work in rearranging the form of differential equation solution to incorporate by definition, an overall mass transfer coefficient (16), i.e.

$$C_e = C_r + (C_o - C_r) \exp(-K_m D/Q) \qquad (57)$$

where

K_m = overall mass transfer coefficient (moles/hr-sq ft)

C = substrate concentration, (moles/cu ft) and

Q = hydraulic loading (ft/hr).

Ames produced differential material balances of the adsorpable "food" component in the liquid and bios as follows:

In the Bulk Liquid phase

$$-vp \frac{dS}{dz} - K_L A_v (S - S^*) = h_L \frac{ds}{dt} \tag{58}$$

In the Active Biomass phase

$$-K S_s X + K_L A_v (S - S^*) = h_s \frac{dS_s}{dt} \tag{59}$$

and

$$S^* = a S_s + S_r \tag{60}$$

by analogy with gaseous adsorption as a linear equilibrium relationship for a first approximation. Utilizing equation (60), equations (58) and (59) are re-arranged into:

$$\frac{ds}{dt} = \frac{-vp}{h_L} \cdot \frac{ds}{dz} - \frac{K_L A_v}{h_L} \cdot (S - S^*) \tag{61}$$

and

$$\frac{ds^*}{dt} \quad \frac{a K_L A_v}{h_s} (S - S^*) - \frac{KX}{h_s} \cdot (S^* - S_r). \tag{62}$$

These equations (61) and (62) are transformed by using dimensionless variables into

$$\frac{ds}{du} = - (S - S^*) \tag{63}$$

and

$$\frac{ds^*}{dw} = (S - S^*) - \frac{KX}{a K_L A_v} \cdot (S^* - S_r) \tag{64}$$

where

$$u = K_L A_v Z/va \tag{65}$$

and

$$w = \frac{aK_L A_v}{h_s} \cdot \left(t - \frac{h_L z}{vp}\right). \tag{66}$$

By definition, one "transfer Unit" is $(vp/K_L A_v)$ and the kinetic reaction parameter is

$$R = \frac{KX}{a\ K_L A_v}. \tag{67}$$

Using Laplace transforms on equations (63) and (64) a unique solution can be obtained which rearranged into original variables, becomes:

$$S_e = S_r + (S_o - S_r) \exp\left(\frac{-R}{R + 1} \cdot \frac{K_L A_v D}{vp}\right) \tag{68}$$

where

s_o = influent organic "feed" concentration (moles/cu ft)

S_r = residual portion of influent organic "feed" (moles/cu ft)

S_e = effluent substrate concentration (moles/cu ft)

K_L = liquid phase film mass transfer coefficient (moles/hr-sq ft)

K = first order reaction rate constant (moles/hr-sq ft)

X = dissolved oxygen concentration (moles/cu ft)

a = specific adsorption coefficient

u,w = dimensionless distance

v = hydraulic loading (ft/hr)

p = liquid density (moles/cu ft)

D = depth of packing (ft)

h = holdup per unit volume (moles/cu ft)

* = liquid-biomass interface script

A_v = area for mass transfer per unit filter volume (sq ft/cu ft)

As the packing depth becomes infinite equation (68) predicts that S_e will approach S_r. That is, S_r may be interpreted as the non-degradable fraction of "food" presented to the biological film by the flowing liquid. The complete bracketed exponential term can be rearranged into the following form by substitution for R, above so that an overall mass transfer coefficient need be defined as

$$\frac{1}{K_m} = \frac{1}{K_L A_v} + \frac{a}{K X} \qquad (69)$$

i.e. (overall) = (mass film transfer) = (kinetic) terms contained within the exponential term $\exp(-K_m D/Q)$. Equation (68) can now be re-written as

$$S_e = S_r + (S_o - S_r) \exp(-K_m D/Q). \qquad (57)$$

Ames derived the mathematical equation (68) and Amado equation (69), but no published data have appeared as to its application in using the series term equation (69) or its interpretation. Prior information is lacking as to the effect of flow rate and temperature on liquid phase film mass transfer coefficient or on the magnitude of the individual parameters contained within the kinetic term.

b. The Vertical Wall Model (Biological Kinetics Model)

(Pseudo-homogeneous case.)

In 1963 B. Atkinson developed a vertical wall model of the trickling filter as shown in Figure 1 based on the following assumptions (17):

(a) the process is at steady state,

(b) the rate of reaction is sufficiently slow that it is the limiting factor, i.e., the concentration gradients of both "food" and oxygen are zero,

(c) the process may be described by first order irreversible reaction kinetics of the form A $\rightarrow$ B where B is an acceptable product,

(d) both components A and B are soluble in the carrier liquid,

(e) the reaction occurs at the liquid-solid interfaces throughout the liquid was flowing through a spongy mass, and

(f) the liquid film thickness is not influenced by the presence of the microbial population.

A mass balance for an element of volume dV, with a film of thickness h, width w', length dD yielded:

$$d\,(Q\,(1 + N)\,S_A) = -k_A S_A h\,W\,dD \qquad (70)$$

where

Q = influent flow rate (cfs)

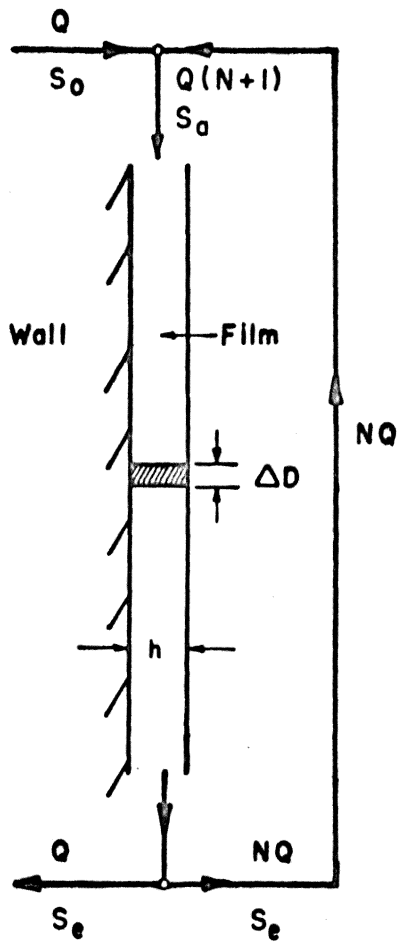

FIGURE 1. The Vertical Wall Model

N = recirculation ratio

k_A = reaction rate constant (sec^{-1})

S_A = concentration of component A (lb/cu ft).

Integration of the above expression resulted in the following equation

$$\frac{S_e}{S_o} = \frac{\exp\left(-\frac{k_A\, h\, W\, D}{Q\,(1+N)}\right)}{1+N-N\exp\left(-\frac{k_A\, h\, W\, D}{Q\,(1+N)}\right)} \tag{71}$$

and when N = 0

$$\frac{S_e}{S_o} = \exp\left(\frac{k_A\, h\, W\, D}{Q}\right) \tag{72}$$

If h W D/Q = V/Q is considered as an equivalent residence time

$$\frac{S_e}{S_o} = \exp^{-k_A t} \tag{73}$$

which is the equation proposed by Howland (1958) and reiterated by Schulze (1960).

It is interesting to note that when designing a filter for a given loading one has the freedom to select a Reynolds number for the falling film simply by choice of the peripheral length W, since

$$Re = \frac{4pQ}{\mu\, W} \quad \text{for films.} \tag{74}$$

Evaluation of equation (71) at N = 0 yields

$$\frac{S_e}{S_o} = e^{-x} \tag{75}$$

where

$$x = \frac{k_A\ h\ W\ D}{Q} \tag{76}$$

$$= k_A\ h\ D\ \frac{4p}{\mu Re} \tag{77}$$

$$= \emptyset_1\ \frac{h}{Re}\ . \tag{78}$$

This formula is of interest because it predicts higher efficiencies when an increase in recirculation causes a change from laminar to turbulent flow. Probably all filters in current operation utilize laminar films, even though Atkinson concludes: "It would appear that substantial savings in filter size could be made by the use of 'turbulent' films." Atkinson also stated, "Mechanistic analysis is the only positive technique that can be used to increase understanding of trickling filter design."

c. Inclined Plate Model (Surface Reaction)

In 1963 Swilley and Atkinson considered the inclined plate model first proposed by Howland and shown in Figure 2 from the point of view of the transport phenomena which involved the heterogeneous reaction at the interface between bacterial film and a flowing liquid film (17). Surface reaction models which included a reaction-controlled heterogeneous model and a diffusion-controlled surface reaction model with and without mixing were proposed. In these models oxygen transfer through the flowing film was not limiting and the availability of "food" at the reaction surface was the controlling factor. In the reaction-controlled model the diffusional resistance was assumed to be

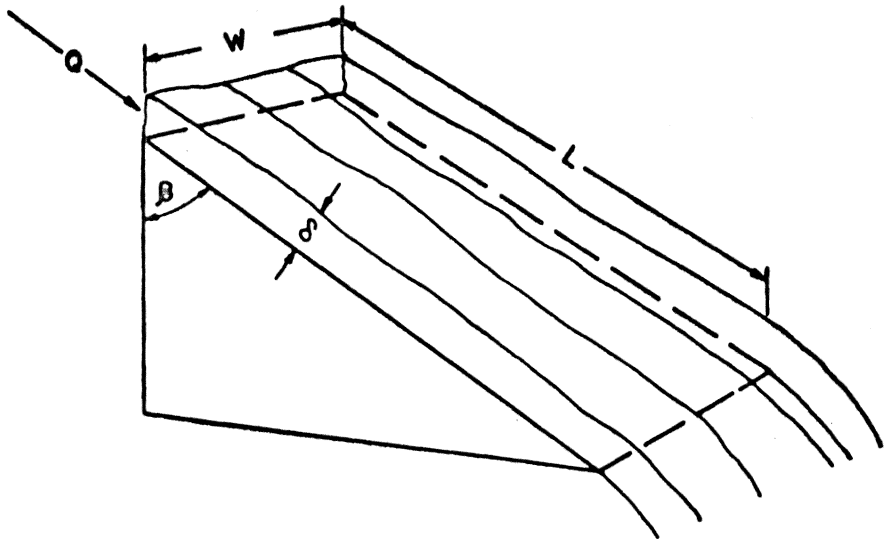

FIGURE 2. The Inclined Plate Model

negligible, so that the formula was:

$$\frac{S_e}{S_o} = \exp\left(- \frac{4\,(R_a P_a)^{1/3}}{R_e} \right) \tag{79}$$

or

$$\frac{S_e}{S_o} = \exp\left(- 1.50\ U\ K_o \right) \tag{80}$$

in which

$$K_o = \frac{D_{AB}\ L}{h^2\ v_{max}} = \frac{\text{diffusional resistance}}{\text{reaction resistance}} \tag{81}$$

and

$$U = \frac{h\ K_s}{D_{AB}} = \text{length of transfer elements} \tag{82}$$

The other terms were defined as

D_{AB} = diffusion coefficient of the substrate A in the flowing liquid B

L = length of transfer element

h = liquid film thickness

v_{max} = liquid velocity at liquid-air interface in direction parallel to flow

K_s = surface reaction coefficient

Expressing of the parameters, K_o and U as dimensionless groups yielded:

$$U = 0.9084\ S_c R_e^{1/3} Ra^{1/3} \tag{83}$$

and

$$K_o = 2.932\ Sc^{-1} Re^{4/3} Pa^{1/3} \tag{84}$$

in which

$$Sc = \text{Schmidt number}, \quad \frac{\mu}{P\, D_{AB}}$$

$$Pa = \text{packing number}, \quad \frac{L^3\, p^2 \cos B}{u^2}$$

$$Ra = \text{reaction number}, \quad \frac{K_s^2\, p}{\mu\, g \cos \beta}$$

$$Re = \text{Reynolds number}, \quad \frac{4\, J}{\mu}$$

and

μ = liquid viscosity

p = liquid density

g = gravitational acceleration

β = angle of inclination with vertical

J = peripheral mass flow rate

In the diffusion-controlled model the rate of removal and oxidation of substrate was considered to be sufficiently fast that the diffusional resistance within the liquid film becomes important. The diffusion-controlled model without mixing was expressed by the equation:

$$\frac{S_e}{S_o} = \frac{3}{2} \sum_{n=1}^{\infty} A_n e^{-b_n^2 K_o} \int_o^1 \phi_n (x,b)\, dx \qquad (85)$$

in which

$$A_n = \frac{U\, \phi_n (1,b)}{b_n^2 \qquad \phi_n^2(x,b)\, (1 - x^2)\, dx} \qquad (86)$$

$$\phi_n(x,b)=e^{\frac{-b_n x^2}{2}}\left(1-\frac{b_n(b_n-1)}{2!}x^2+\frac{b_n^2(b_n^2-5)(b_n-1)}{4!}x^4-\ldots\right) \quad (87)$$

and

b_n = Eigen values, the positive roots of the Eigen function of type:

$$\left(b-\frac{b^2(b-1)}{2!}+\frac{b^3(b-5)(b-1)}{4!}-\cdots\right)+\left(\frac{b(b-1)}{1!}-\frac{b^2(b-5)(b-1)}{3!}+\ldots\right)$$

$$-U\left(1\frac{b(b-1)}{2!}+\frac{b^2(b-5)(b-1)}{4!}+\ldots\right)=0. \quad (88)$$

Thus, Equation (85) becomes:

$$\frac{S_e}{S_o}=f(U,\ K_o) \quad (89)$$

where the parameters U and K_o as in the reaction-controlled model describe the geometry of the model, and the characteristics of the substrate and the liquid film. In a restricted sense, U represents the diffusional resistance divided by the reaction resistance and K_o represents the length of the transfer elements. The total bed depth was not considered to be as important as the total path length and special orientation of elements. All of these models assume independence of efficiency and organic loading.

It is important to note that the surface-reaction or diffusion-controlled models predict a negative effect from recirculation, in sharp contrast to the beneficial effect of recirculation due to a change in flow regime at increased Reynolds numbers discussed previously for Atkinson's vertical wall model. This is due to Atkinson's

implicit assumption of the presence of a pseudo-homogeneous biological reaction throughout the liquid film. Upon modifying this assumption to provide for a surface reaction, recirculation is shown to have a negative effect since the greater film thicknesses associated with increased Reynolds numbers will not increase the system's efficiency.

Maier, Behn and Gates in 1967 also simulated the trickling filter process by an inclined flat surface covered with a biological slime (19). A theoretical description of this physical model was developed, assuming mass transfer and the rate of microorganisms to be the main factors affecting purification. A mass balance was taken over the control surface, on the assumption that no growth occurs in the liquid film, i.e. all metabolic activity takes place in the slime layer. Using glucose as the sole food source under laminar flow conditions, studies on the effect of feed rate, glucose concentration and temperature, showed that mass transfer of biodegradable material is rate limiting.

Atkinson's experimental studies have been primarily concerned with an anaerobic environment, that is, a system in which the electron acceptor is nitrate, to eliminate the variable of interphase oxygen transport. Maier's work has been based on oxygen as electron acceptor.

d. The Heterogeneous Biological Reaction Model

In 1968 Atkinson, Daoud and Williams developed a theory for the biological film reactor which allowed prediction of performance on the basis of physical and biological variables (20). The model incorporated diffusion in both the liquid and active biomass phases, following on from the work of Atkinson (17,18). A similar differential equation to Swilley (18) was used but with modified boundary conditions, allowing for the reaction rate at the interface between the liquid and biologically active surface. This new model purports the physical situation in the biological film based on diffusion with heterogeneous chemical reaction. When the active film thickness is small, the controlling factor is that of chemical reaction. The heterogeneous model for the biological film reactor is shown schematically in Figure 3.

Atkinson and Williams in 1971 used the theoretical developments of Atkinson *et al.* to hypothesize the performance of a pilot plant trickling filter (21). It was suggested that efficiency was influenced by hold up of microbial mass within the filter. Performance of a filter was found to be independent of oxygen transfer but influence by diffusional limitations in both the microbial and liquid films. The general model also allowed for biological rate in the microbial kinetics. Inherent coefficients and functional form are not known *a priori*. The problem has to be approached by efficiency asymptotes

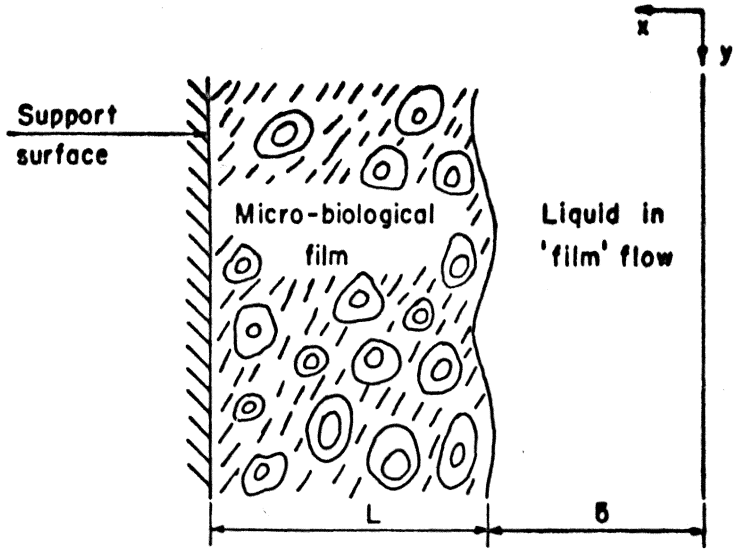

FIGURE 3. The Heterogeneous Reaction Model

at low and high substrate concentrations. A computer iteration sequence was used in the general solution since the coefficients are partially dependent upon mean interfacial substrate concentration. Prediction was reasonable at low concentrations but inaccurate at high substrate concentrations.

In 1969 Kehrberger and Busch summarized results of a theoretical analysis of the effect of recirculation on removal of total soluble organic carbon from film flow reactions (22). Three film flow models are discussed for the case where the biochemical reaction occurring in the reactor can be represented by consecutive first order reactions. The general heterogeneous model assumes no micro-organisms in the liquid phase with biochemical reaction occurring only at the liquid microbial mass interface. Organic compounds are transported by liquid film movement and diffusion into the active biomass. Partial differential equations with variable coefficients are coupled with boundary conditions and solved by computer finite difference methods. The reaction control model is a limiting case of the general heterogeneous model, above. It is assumed that the reaction rate at the active biomass surface is the controlling mechanism. First order ordinary differential equations, on integration yield

$$\frac{S_e}{S_o} = \exp\left(\frac{-k\ A\ D}{s\ v\ (1+N)Q}\right) \qquad (90)$$

where

k_s = surface reaction rate coefficient.

Both the heterogeneous and reaction control models depict a decrease in removal efficiency with increase in recirculation ratio. A film flow system for a pseudo-homogeneous model, in which liquid entering the reactor flows through or contains the microbial mass and reacts at all points in the liquid phase, is described by

$$\frac{S_e}{S_o} = \exp\left(\frac{-k_L A_V \delta D}{(1 + N)Q}\right) \tag{91}$$

where k_L = volumetric reaction rate coefficient, and

δ = liquid film thickness.

The essential difference between this homogeneous model and the two former models is the dependence of effluent concentration on liquid film thickness. Because the biochemical reaction occurs throughout the liquid phase, an increase in film thickness has a significant effect on overall efficiency. Thus an increase in recycle ratio increases the resident liquid volume and therefore film thickness, and increases removal efficiency.

Simulation of Plastic Medium Trickling Filters

As new trickling filter media were developed, researchers simply modified existing formulas to account for the new media. These media consisted of corrugated plastic sheets running parallel to each other and welded together to form a bundle or pac. It will be recalled that Howland had shown the time of contact in a trickling filter to be directly proportional to the filter depth and inversely proportional to the hydraulic loading rate, viz:

$$t = \frac{CD}{Q^n} \tag{27}$$

In 1963 Eckenfelder and Barnhart (23) showed by investigations of several types of packing media that

$$C = C'A_v^m \tag{92}$$

A general expression for the mean detention time through any type of filter medium was proposed

$$t = C'A_v^m \frac{D}{Q^n} \tag{93}$$

As the specific surface, A_v, of the media decreases, the exponent also decreases. For glass sphere, porcelain spheres and Polygrid medium without slime, the coefficients C' and m were determined as 0.7 and 0.75 respectively. The presence of filter slime was shown to increase the retention time as much as 50 percent over that without slime. The exponent n and coefficient C' were given as 0.70 and 5.6, respectively.

for Mead-Cor plastic media, and 0.50 and 4.84, respectively, for Dowpac plastic media. The specific surface area was reported as 30 sq ft/cu ft for Mead-Cor media and as 25 sq ft/cu ft for Dowpac media.

In 1965, James Germain applied the Schulze formulation to a plastic media (Dowpac) filter as follows (24):

$$\frac{S_e}{S_o} = e^{-KD/Q^n} \tag{94}$$

where

S_o = BOD of influent fed to the filter (not including recirculation), mg/l

S_e = BOD remaining, mg/l

D = Depth of filter, feet

Q = Hydraulic load, gpm/sq ft (not including recirculation)

K = Treatability constant

n = Exponent characteristic of filter media.

Germain found that K and n for Dowpac media treating domestic primary effluent were 0.088 and 0.5, respectively. The recirculation rate was not considered in the above model because studies from 10 pilot plants showed that there was no statistically significant effect on BOD removal caused by recirculation. It was the author's opinion that once a maximum effective biological film thickness is reached, no further increase in BOD removal rate will occur. Said Germain, "It has been apparent for years that a trickling filter apparatus will reduce organics to a given level and will remove additional organics with great

difficulty." He pointed out that obviously recirculation will have no effect because an increase in recirculation rate $\longrightarrow \infty$ will increase the hydraulic load Q to the filter, and for a given size the residence time t as shown by Equation (27) will approach zero; hence the treatment through the filter based on raw flow plus recycle approaches zero.

Balakrishnan (1969) studied the organic removal characteristics of 1 1/2 in. polypropylene flexiring media in an 8 foot deep pilot trickling filter and modified Eckenfelder's original equation for BOD removal as follows (25):

$$\frac{S_e}{S_o} = e^{-0.00362\ (1.035)^{T-20}\ S_o^{0.540} A_v^{0.644} D/Q^n} \qquad (95)$$

In general form equation (95) can be presented as

$$\frac{S_e}{S_o} = e^{-C'\ \theta^{T-20} S_o^{a} A_v^{b} D/Q^n} \qquad (96)$$

The mathematical manipulation of the performance data will affect the magnitude and interdependence of the various exponents.

In 1972, Gromiec, Malina and Eckenfelder used a form of Equation (96) to evaluate the performance of a steel pilot trickling filter 3 feet in diameter packed to a depth of 21.5 feet with Surfpac, a product of the Dow Chemical Company (26). The overall BOD removal rate constant, $C' = K_s$, was determined as 0.017 for the treatment of municipal wastewater by the filter operated at hydraulic loading rates ranging from 63 to 189 MGAD at the average operating temperature of $27^{o}C$.

Values for exponent n= 0.45 and n=0.50 were used for Flexirings and Surfpac, respectively, and values for specific surface area, A_v, were reported as 40 $ft^{-2}ft^{-3}$ and 27 $ft^{-2}ft^{-3}$ for Flexirings and Surfpac, respectively.

Lamb (1970) developed a simple model of a mature high rate trickle filter which incorporated the usual engineering process parameters (27):

$$\frac{S_e}{S_o} = \frac{1}{1 + \frac{V\ Av}{K\ Q}} \qquad (97)$$

where

K = quasi-rate coefficient (TL^{-1})

Q = volume flow rate (L^3T^{-1})

V = packed bed volume (L^3).

This equation excluded packed depth and irrigation rate (involving cross section area) from its development. The author extracted literature data and attempted to show agreement with this proposed equation. Actual correlation was poor, with the parameter (K) being a function of packing media, substrate type and irrigation rate.

Bruce (1970) asserted that the value of the exponent, n, in Howland's equation relating liquid retention time, t, to filter depth, D, and surface volumetric loading, Q, as follows

$$t = \frac{CD}{Q^n} \qquad (27)$$

was closer to unity than to 0.67 (28) for Flocor and Surfpac media. Bruce also stated that approximate 80 percent of the applied BOD is

removed in the upeer one-sixth of the depth of a trickling filter, the remaining five-sixths of the depth being necessary to serve the important function of removing most of the residual oxidizable organic matter and providing conditions suitable for nitrification.

Monadjemi and Behn in 1970 developed a model mechanism of substrate purification in a trickling filter, with the following assumptions (29):

(a) Liquid flow over individual surface elements is laminar, with molecular diffusion being the mode of mass transfer.

(b) Mixing occurs at point of each element.

(c) Available active surface slime film is constant on each element.

(d) The biochemical oxidation is not rate determining.

(e) The substrate is a single soluble compound.

The functional form of the developed mathematical expression was similar to that of Swilley (1963). From a variety of laboratory experiments it was found that the removal rate coefficient reflects two aspects; the first is the type and quality of microorganisms responsible for substrate transfer to their active surface, the second is the type and quality of substrate and the ease with which it can be adsorbed. Also, within experimental limitations, mass transfer was the controlling mechanism which could be enhanced by surface discontinuities.

Mehta, Davis, and Kingsbury (1972) considered biodegradation of organic wastes to be made up by three simultaneous activities (14):

(1) The transfer of oxygen from the gas phase to and through the liquid to the bacterial sites on the media surface.

(2) Transfer of organic compounds from the liquid phase to the bacterial sites.

(3) The kinetics of the biochemical reaction in the active biomass.

Complex as these three activities are, they follow the laws of chemistry and physics. Resolving this complexity into a simply stated condition, no biochemical reaction can take place until the oxygen and food are transported to the bacteria. In chemical engineering language, to achieve the most efficient biological oxidation of the wastes, the two transport factors (oxygen transport and waste transport) must be sufficiently high so that they do not impose a limit on the biochemical reaction. The next step is to locate the factors which impose rate limitations on the entire system and design around the limiting factor.

The authors postulated the controlling mechanism to be that of oxygen mass transfer. Forced convection mass transfer of oxygen into the falling liquid film was described by a model similar to Monadjemi and Behn (1970). A design expression was then developed for prediction of BOD reduction from Flocor, Surfpac, and Vinyl-Core trickling filtration systems' physical properties and process parameters. The authors note in their conclusion that in some situations, mass transfer of "food" may be the rate limiting step.

In 1973 Richard and Kingsbury noted that plastic media trickling filters were more successful in treating milk wastes than other biological treatment schemes (31). For Flocor media the authors recommended a minimum continuous irrigation rate of 0.6 gpm/sf of horizontal

tower area and a maximum irrigation rate of 2.0 gpm/sf in towers 16 to 22 feet deep treating milk wastes. Recycle is only utilized to maintain the recommended irrigation and no improvement in removal is achieved from increased recycle. The use of this irrigation as a minimum functions to flush the humus from the nearly vertical surfaces in the media to prevent plugging and insure enough opening for ventilation.

The Analogy Between Trickling Filtration and Activated Sludge

Eckenfelder (1966) proposed the equations for the removal of soluble BOD in the trickling filter to be analogous to the equations used in the activated sludge process as (31):

$$\frac{S_e}{S_o} = e^{-k' X_a t} \tag{98}$$

or

$$\frac{S_e}{S_o} = \frac{1}{1 + k' X_a t} \tag{99}$$

in which X_a is the average biological volatile solids. The equations are applicable when:

(a) the specific surface remains constant,

(b) the slime cover on the medium is uniform, and

(c) the waste is uniformly distributed to the medium.

These conditions require uniform hydraulic loading and are violated if heavy slime buildup short-circuits the filter. Putting the retention time as defined by Equation (93) into the above equations and assuming the specific surface can be used to express the available bacterial population results in the following expressions which account for film variations with filter depth:

$$\frac{S_e}{S_o} = \exp\left(-k'' A_v^{1+m} D/Q^n \right) \tag{100}$$

or

$$\frac{S_e}{S_o} = \frac{1}{1 + k''A_v^{1} + mD/Q^n} \tag{101}$$

For a specific type of medium, Equation (100) can be presented as follows:

$$\frac{S_e}{S_o} = e^{-k_s D/Q^n} \tag{102}$$

where $k_s = k'' A_v^{1\ \frac{1}{2}\ m}$

The recirculation flow was considered as a diluent to the influent waste, and for filters with recirculation the mixing equation (7) was combined with equation (102) to yield:

$$\frac{S_e}{S_o} = \frac{k^{-k_s D/Q^n}}{(1 + N) - Ne^{-k_s D/Q^n}} \tag{103}$$

in which N is the recirculation ratio and S_o is the BOD of the undiluted influent. The importance of Eckenfelder's Equation (103) is that it predicts a dramatic increase in filter efficiency with recirculation even at low hydraulic and organic loadings.

Kornegay and Andrews (1968) used a completely mixed annular reactor in order to study the reaction kinetics of a biological film and to develop a mechanistic equation (32). The authors used Monodtype growth kinetics to relate substrate removal to the active mass of attached micro-organisms. The mass of film responsible for this utilization is equal to the product of the surface area, A, the con-

centration of organisms in the biological film, X, and the depth of active organisms, d. Film accumulation continues until the actual thickness, h, equals or exceeds the active thickness, d. The active mass of organisms then becomes constant and steady-state is reached with respect to substrate utilization. For this condition a substrate mass balance combined with the Monod expression leads to the following equation:

$$Q(S_o - S_e) = \frac{\hat{\mu}}{Y} A \; X \; d \; \left(\frac{S_e}{K_s + S_e}\right) \qquad (104)$$

in which

$\hat{\mu}$ = maximum specific growth rate (T^{-1})

Y = growth yield coefficient, and

K_s = essential nutrient concentration

The utility of the above equation is that a continuous function is obtained which describes the rate of substrate removal from zero-order at high concentrations to first-order at low concentrations. Based on utilization of glucose, the limiting active film thickness, d, was found to be 70 microns and the concentration of the biological film, X, as calculated from thickness and dry weight measurements on the sludged filter slime, was 95 mg/cu cm on a dry weight basis.

While many authors have postulated that the reaction rate (or growth) of the bios is the rate limiting factor, this approach requires negligible concentration gradients. Kornegay and Andrews obtained negligible gradients by mixing. Mixing reduces the resistances to

oxygen mass transport and their conclusions reflect this condition. Equation (104) was applied to a Flocor plastic medium trickling filter by Bentley in 1973 (33).

Tucek et al. (1971) developed dimensional equations for plug flow activated sludge and by analogy for trickle filter processes. The following dimensionless numbers were introduced for the latter (34):

$$w_1 = \frac{S_o - S_e}{S_e} \; ; \quad w_2 = k\, D/Q \text{ and } w_1 = f(w_2).$$

Using early stone media results the functional form was described by:

$$w_1 = \frac{w_2}{a + b\, w_2}$$

and the coefficient, a, defined to be:

$$a = m \frac{S_o}{A_v} + n$$

with m,n being experimental coefficients.

These equations can then be rearranged to become

$$w_1 = 1/ \left(1 + \frac{mS_o}{w_2 A_v} \mp \frac{n}{w_2}\right) \text{ for } b = 1$$

which on substitution of the original variables becomes

$$\frac{S_e}{S_o} = \frac{1}{1 + (k\, D/Q(m\, S_o/A_v + n))} \qquad (105)$$

This equation is similar to Eckenfelder's retardant equation (46).

BIOLOGICAL REACTOR THEORY

Many authors have pointed out the lack of a unified theory for the design of wastewater treatment units (35). As shown in the Literature Review, this situation has arisen not from a lack of theory, but from an overabundance of conflicting theories and viewpoints. Basic understanding of wastewater treatment has progressed to be point that there is a theory for every conceivable means of mass transport, be it diffusion, adsorption, reaction, or a combination of these effects. It is now time to consider the magnitudes of these various effects and reach an understanding of what is really important.

To this author, what is really important in the design of wastewater treatment units can be seen most clearly from a biological reactor point of view. In this theory, each major biological treatment unit is considered as a biological reactor with substrate and micro-organism reactants. The theory of such reactors has been rather well-developed in the chemical process and fermentation industries, so that simple considerations of geometry, reaction kinetics, and reactor type lead at once to mechanistic design equations.

The biological reactor approach seems aptly suited to mathematically describe wastewater treatment units for two reasons. First, a macroscopic view of the world is obtained. This means that the integrated behavior of the system is described in terms of real-life design variables. And secondly, the method is applicable to any combination or arrangement of treatment units.

Reaction Kinetics (Monod-type vs. First-Order Substrate Utilization)

It is apparent that the two major biological treatment processes differ only in the geometry of the microbial population: a microbial film versus a microbial floc. The type of reaction kinetics and mixing phenomena exhibited by these two possible geometries is therefore of interest. The langmuir adsorption type of equation for a microbial floc is given by Atkinson as (39):

$$n = \frac{f\ C^*}{g + C^*} \qquad (110)$$

where

C^* = substrate concentration at the interface of the microbial mass

n = rate of removal of substrate per unit wt. of micro-organism, and

f,g = constants.

This equation is exactly analogous to Metcalf and Eddy's equation for the rate of substrate utilization in the activated sludge process (37):

$$\frac{dS}{dt} = \frac{k\ X\ S}{K_s + S} \qquad (110)$$

where

S = concentration of waste surrounding the organisms (mass/volume)

X = concentration of microorganisms (mass/volume)

K_s = waste concentration at which rate of waste utilization per unit weight of microorganism is one-half the maximum rate, i.e., when $\mu = \frac{1}{2}(\mu_{max})$, (mass/volume), and

k = maximum rate of waste utilization per unit weight of microorganisms ($time^{-1}$).

Monod-Type Growth Kinetics

The above equation (110) has been called a Monod-type substrate removal equation due to its striking similarity to Monod's equation for the growth rate of pure cultures of microorganisms on defined substrates; this equation appears graphically in Figure 8 and has been given by Jenkins and Garrison (41) as:

$$\mu = \mu_{max} \frac{S}{K_s + S} \qquad (111)$$

where

S = substrate concentration (mg/l)

K_s = substrate concentration at $\mu = 0.5\ \mu_{max}$

μ = growth rate = unit rate of increase of cell concentration

$(day^{-1}) = \frac{1}{x}\frac{dx}{dt}$, and

μ_{max} = maximum growth rate.

As shown in Figure 4, the Monod equation consists of two regions or phases. The first phase at low substrate concentration is a first-order relationship between growth rate and substrate concentration. In the second phase, at high substrate concentrations the growth rate is a maximum and independent of the substrate concentration. This zeroth-order relationship can occur because of other limiting factors, such as oxygen and nutrient transport and limiting film thickness. A zeroth-order substrate removal relationship due to limiting film thick-

ness has already been described.

First-Order Substrate Utilization Rate.

Substrate removal depends to a large extent on the stage of growth of the microorganisms. In the exponential growth phase (substrate in excess) substrate removal is zero order. In the declining growth phase (low substrate concentrations) substrate removal is dependent on the concentration of substrate remaining in solution. This latter relationship is expressed by a first-order equation (42):

$$\frac{dS}{dt} = -K_1 X_a S \qquad (112)$$

in which

S = substrate concentration at time t (milligrams per liter)

K_1 = average overall removal rate coefficient (l/mg-day)

X_a = average biological solids, usually estimated in MLVSS (mg/l)

Equation (112) has been used to preduct the substrate removal relationships in a number of biological waste treatment systems (1,2,7,8,9,43).

Reactor Type: Plug Flow vs. Complete Mixing

Reactor type is goverened by the extent of mixing. A plug flow reactor is one in which mixing occurs in two dimensions only; that is, there is a concentration gradient of substrate with respect to length. For a first-order plug flow reactor, this relationship may be expressed as follows:

$$\frac{dS}{dl} = -k X S \frac{W d}{Q}$$

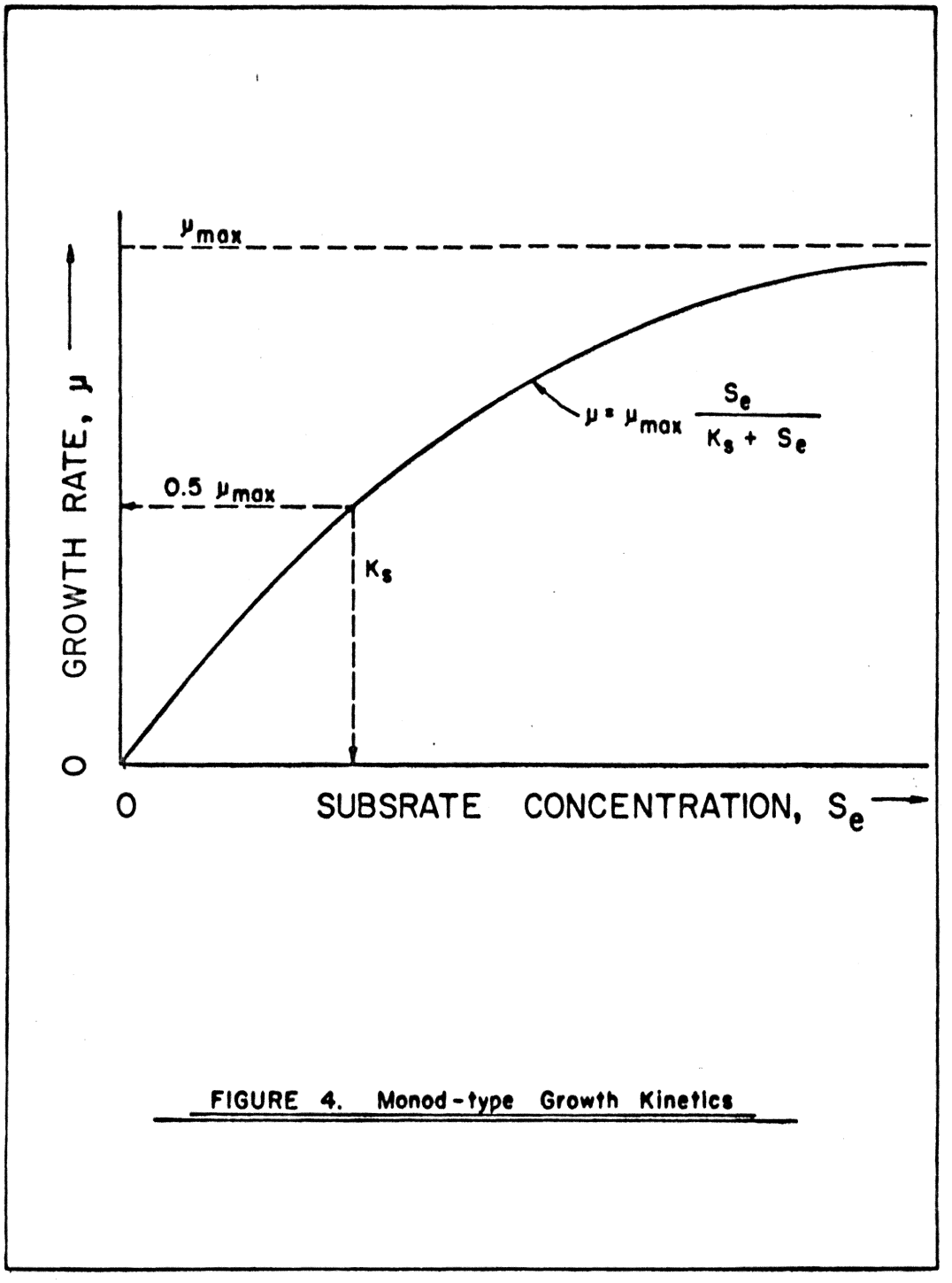

FIGURE 4. Monod-type Growth Kinetics

which on integration leads to

$$\int_{S_o}^{S_e} \frac{dS}{S} = -\int_{0}^{L} k\ X\ \frac{W\ d}{Q}\ dl$$

or

$$\ln \frac{S_e}{S_o} = -k\ X\ \frac{WdL}{Q}\ . \qquad (113)$$

This is the result obtained by Howland (1958), Shulze (1960), Eckenfelder (1961), and Atkinson (1963).

If the fraction of substrate remaining, $f = {}^{S_e}/_{S_o}$, and the area, $A = Wd$, are substituted into the above equation (113) the following formula results:

$$f_1 = e^{-K_1\ A\ X_a\ L/Q} \qquad (114)$$

where

L = length of plug flow reactor

d = depth of active biological layer

W = width of active biological layer

X_a = average microorganism concentration

Q = flow through plug flow reactor

S = substrate concentration (subscripts o and e refer to initial and effluent substrate concentrations, respectively), and

K_1 = average overall biological reaction rate constant.

Complete Mixing

In a completely mixed system, such as the conventional activated sludge, the effluent concentration is approximately equal to the concentration to the aeration tank. Therefore, a materials balance on

substrate yields:

$$Q(S_o - S_e) = - \frac{dS}{dt} V \qquad (115)$$

in which

Q = hydraulic flow rate (liters/day)

S_o = influent soluble substrate concentration (mg/l)

S_e = effluent soluble substrate concentration (mg/l), and

$t = V/Q$ = reactor detention time (days).

Substituting the first-order expression for substrate utilization rate (Equation 112) yields:

$$\frac{S_o - S_e}{X_a\, t} = K_1\, S_e \qquad (116)$$

= average substrate removal rate per unit of biological solids.

Equation (116) can be rearranged to give the fraction of soluble substrate remaining, $f = S_e/S_o$, as follows:

$$f_2 = \frac{1}{1 + K_2\, X_a (\frac{V}{Q})} \qquad (116a)$$

which is the form used by McKinney (1974) to explain the complete-mix activated sludge (CMAS) process (40).

The main difference between the plug flow and complete-mix reactor types is that plug flow considers an incremental concentration change, dS/dl, which complete-mix looks at the overall concentration change. There is no corresponding differential equation for complete-mix systems because the concentration is everywhere the same and a dS/dl expression

is useless.

For a Monod-type growth system, substitution of the Monod-type dS/dt expression (Equation 110) into the substrate mass balance (Equation 115) results in the mechanistic design equation for the complete-mix reactor with Monod-type biological reaction kinetics:

$$f_2 = \frac{K_s + S_e}{\frac{k X V}{Q} + K_s + S_e}$$

or

$$f_2 = \frac{1}{1 + \frac{k_2 X_3}{(K_s + S_e)} \frac{V}{Q}}$$

Equation (117) is the design formula used by Metcalf and Eddy (37) for complete-mix activated sludge systems and aerated lagoons without recycle.

Geometry (Physical Description of Unit)

Having discussed how each biological treatment unit may be mathematically described in terms of reaction kinetics and reactor type, I shall now describe how these units may be linked together by mass balances.

The method is perhaps best illustrated for the simple case of one trickling filtration tower with recirculation to the top of the tower; i.e., a mixing point ahead of the tower. Assuming there is some fraction of substrate remaining after the tower, f_1, equal to:

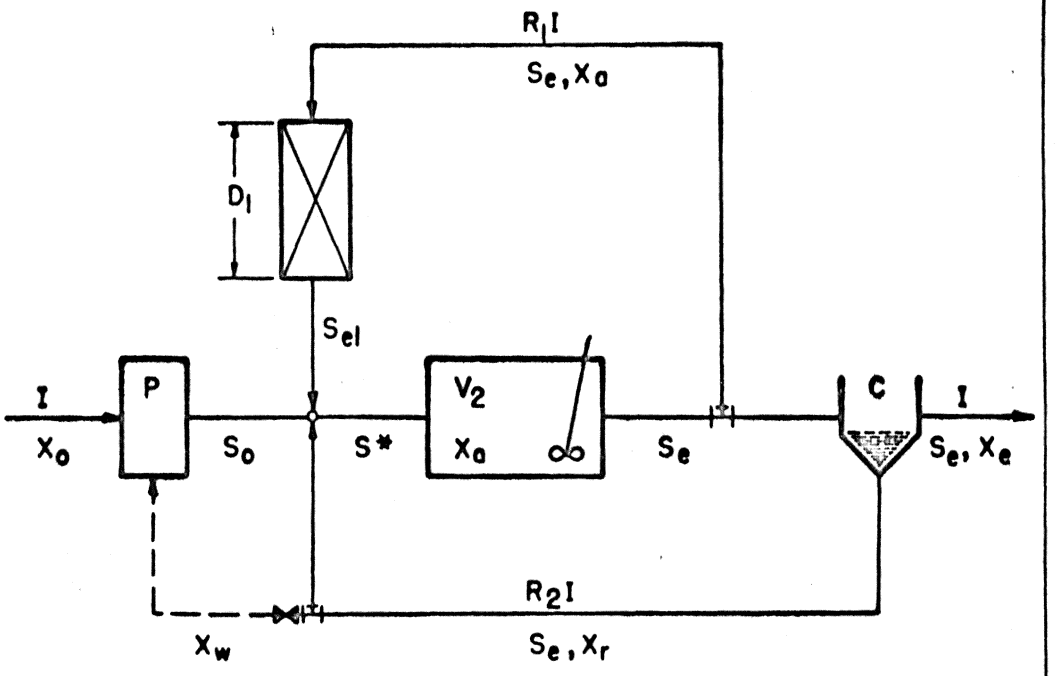

FIGURE 5. Flow Diagram of Aquatair Model P-3

$$f_1 = S_e/S' \tag{118}$$

in which S' is the mixed substrate concentration in the stream applied to the tower given by the mixing equation:

$$S' = \frac{S_o + N S_e}{1 + N} \tag{7}$$

where N = R/I = recirculation ratio. Combining Equation's (118) and (7) gives Shulze's result for the fraction of substrate remaining in relation to untreated influent (9):

$$f = \frac{S_e}{S_o} = \frac{1}{\frac{1 + N}{f_1} - N} \tag{43}$$

The method is easily extended to describe a combination of units such as that shown in Figure 9, the Aquatair Package Sewage Treatment Plant (41). The plant consists essentially of a B. F. Goodrich Vinyl-core plastic media trickling filter mounted overtop of a recirculation tank as shown diagrammatically in Figure 5. The unit also has a primary sludge holding tank, secondary clarifier and chlorinator which are of no immediate concern if S_o is defined as the BOD or COD of the settled sewage.

(Derivation of Overall f.)

Continuing from Figure 5, the derivation of the mechanistic design equation for the Aquatair combination of units is given as follows: The fraction of soluble substrate remaining after the bio-oxidation tower, f_1, and the fraction of substrate remaining in the recirculation

tank, f_2, are defined as

$$f_1 = S_{e1}/S_e, \text{ and} \tag{119}$$

$$f_2 = S_e/S^* \tag{120}$$

where

S_{e1} = effluent substrate concentration from the tower (mg/l)

S_e = final effluent substrate concentration (mg/l)

S_o = COD or BOD of settled influent (mg/l)

S^* = substrate concentration after mixing point (mg/l).

The mixed substrate concentration, S^*, is found from a substrate mass balance around the mixing point:

$$I\,S_o + R_1\,I\,S_{e1} + R_2 I S_e = (I + R_1 I + R_2 I)\,S^*$$

or

$$S^* = \frac{S_o + R_1 S_{e1} + R_2 S_e}{1 + R_1 + R_2} \tag{121}$$

Combining Equations (119) (120) and (121) results in the expression for the overall fraction of substrate remaining, f:

$$f = \frac{S_e}{S_o} = \frac{f_2}{1 + R_1(1 - f_1 f_2) + R_2(1 - f_2)} \tag{122}$$

in which R_1 = r/I = ratio of tower recycle to influent flow, and

R_2 = sludge recirculation ratio.

Additional Modifications

a. Microbial Solids Concentration

The mode of operation of a trickling filter is shown in Figure 6. "Food" and oxygen are absorbed by the microorganisms present in the liquid layer and in an attached slime growth. As the zooglea grow, an anaerobic layer develops near the media surface and the organisms loose their ability to hold on. Therefore, slime is continually sloughed from the filter. At steady-state the slime growth reaches its maximum effective thickness. The maximum effective depth of aerobic activity, d, has been given by Kornegay and Andrews (32) as 70 microns (2.297×10^{-4} ft.); the same authors reported the solid film microbial concentration, X_s, as 95 mg/cu cm. (95,000 mg/l).

Figure 6 shows that the real-life mechanism of substrate removal involves a complex assortment of transport phenomena including adsorption, diffusion, and biochemical reaction of nutrients, oxygen and oxidation products. Williamson and Chung (1975) claim that the transport of oxygen can be rate limiting because of the low diffusivity of O_2 in water. Their computer model describes the transient behavior of the trickling filter in terms of nine simultaneous ODE's which are solved by fourth-order Runga-Kutta-Gill numberical integration techniques (42).

We shall confine our model to the aerobic regions of the trickling filter and assume that there is enough mixing to keep oxygen from be-

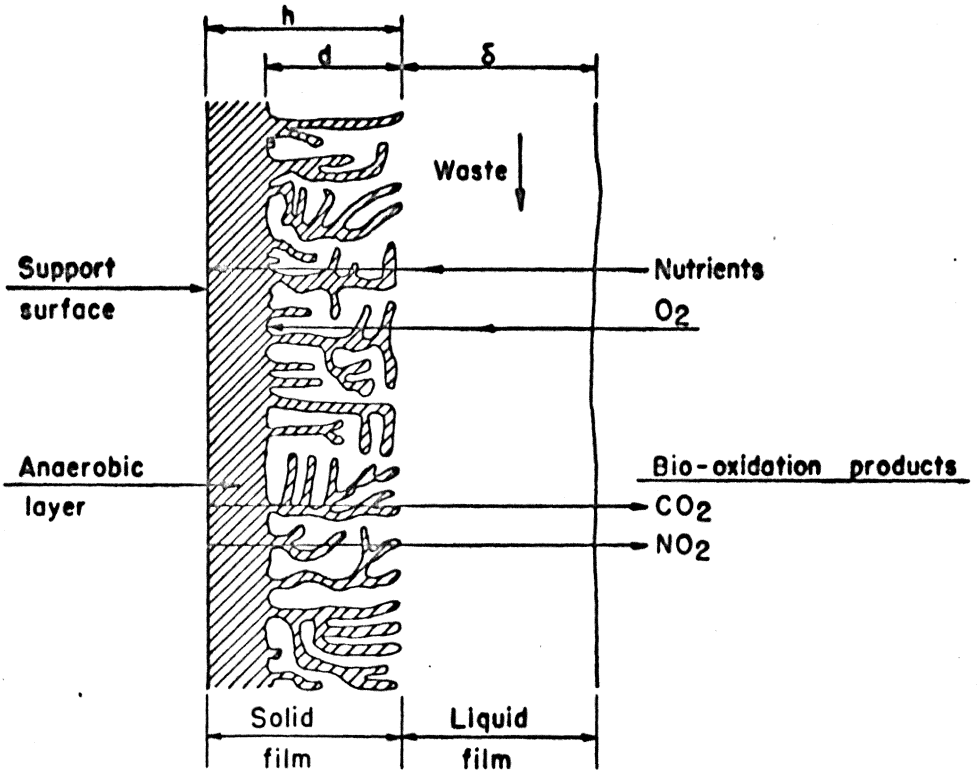

FIGURE 6. Mode of Operation of a Trickling Filter

coming rate limiting. Mixing is assumed to occur in two dimensions with a substrate concentration gradient along the third dimension, i.e. plow-flow behavior along the length of the tower. Because the feed to the tower is the effluent from the recirculation chamber, first-order reaction kinetics associated with low substrate concentrations can be expected. The endogenous nature of the tower is further supported by data which show nitrification; nitrification usually occurs in the aerobic digestion of trickling filter humus (43). This point will be elaborated on later. For the time being, it is enough to know that we shall be using the first-order assumption of Velz, Howland, Shulze, Eckenfelder, and others to describe the substrate removal kinetics of the Aquatair biological oxidation tower. The fraction of soluble substrate remaining in a plug-flow first-order biological reactor has been given as Equation (114). We shall modify this equation so that the microbial solids concentration is taken as the sum of the liquid and solid fractions. The mass of microbial solids is equal to the specific surface area of the medium, A_v, times the concentration times the appropriate film thickness. The modified formula is hence:

$$f_1 = e^{-(K_1 A_v D_1/Q_1)(X_s d + X_a \bar{d})} \qquad (123)$$

where

A_v = specific surface area of the medium, sq ft/cu ft

D_1 = depth of tower, ft

Q_1 = hydraulic loading on tower, ft/day

X_s = solid film microbial concentration, mg/l

d = solid film thickness, ft

x_a = liquid film microorganism concentration, mg/l

$\bar{d}$ = liquid film thickness, ft

K_1 = overall biological reaction rate constant, l $mg^{-1}day^{-1}$.

b. Temperature Correction

Temperature obviously affects any biological treatment process, as shown by the behavior of treatment plants after a sudden cold snap. It has been convenient for many authors to model the effect of temperature using an Arrhenius-type equation. The Arrhenius equation for pure cultures states (47):

$$k = Ae^{-\mu/RT} \tag{124}$$

where

k = biological reaction rate constant

μ = temperature characteristic

R = gas constant

T = absolute temperature

A = constant.

This equation has been applied to the design of trickling filters and activated sludge tanks as follows (7,8,9,37):

$$K_T = K_{20^oC}\theta^{T-20} \tag{125}$$

in which θ = 1.035 for trickling filters and θ = 1.047 for activated sludge tanks. It was decided to apply this temperature, even though

Gardner (45) suggests that heterogenous cultures in sewage do not strickly obey Arrhenius's equation. It will be shown later that the temperature correction does not improve the correlation between observed and predicted substrate utilization behavior, over the limited temperature range studied.

APPLICATION OF THEORY

NSF Data

Tests on an Aquatair Model P-3 unit as shown in Figure 5 were conducted by the National Sanitation Foundation Testing Laboratory, Ann Arbor, Michigan, in the summers of 1973 and 1974. The NSF protocol appears in the Appendix. All samples were taken and preserved as provided in the latest edition of Standard Methods for the Examination of Water and Wastewater (46), and all analytical methods employed were likewise those set forth in the latest edition of STANDARD METHODS. The data which comprises Table II of the Appendix was actually used in the simulation of the Model P-3 unit; it consists of measurements of flow, temperature, COD, BOD, and MLVSS at varying hydraulic and organic loadings. Essentially two drastically different dosing schemes were used at two different extremes of microbial concentration. This results in four cases of data, as follows:

(1) Case 1 involves tests conducted from June 30, 1974, to August 22, 1974, using the Subdivision flow pattern with high VSS concentrations.

(2) Case 2 uses the School (8-hr. steady-state) flow pattern with high VSS concentrations; the tests began on August 25, 1974, and ended September 24, 1974.

(3) Case 3 data start on June 17, 1973, and end July 20, 1973; Case 3 comprises the Low VSS, Subdivision flow case.

(4) Case 4 data consist of the above measurements taken from July 29, 1973 to August 23, 1973, using the Aquatair Model P-3 unit operated at low volatile suspended solids concentration and 8-hour steady-state flow.

Additional Input Parameters appear in Table 1 which follows. Amnonia

Nitrogen and Nitrate Nitrogen as given by NSF are listed in Talbe III of the Appendix.

Extraction of Biological Reaction Rate Constants.

Gromiec et al. clearly point out that the so-called rate constant (k_{20}) is dependent upon the units of flow rate, depths and whether with reference to the logarithmic base or common base (26). Note has been made in the Literature Review of particular definitions describing this parameter. It can be seen that as early as 1946, the NRC Committee considered some change in treatability of the waste material on passing through a unit. Subsequently, Schulze and most other workers who introduced a (K_{20}) parameter into their particular equation, called it a coefficient, not a constant. This coefficient was then considered to vary with temperature, active surface area of biomass per unit volume and type of waste, that is, its biodegradability or "treatability". Velz' rate constant could then be replaced by an overall mass transfer coefficient which is primarily a function of Reynolds' number (surface irrigation rate) and Schmidt's number (diffusivity of food). Biological conversion of food would be described by a kinetic rate term, for simplicity first order, because of mathematical convenience and interpretation of BOD rate-time data.

It is apparent that a given biological reactor with a set specific surface area and a given type of substrate will give rise to a constant which will characterize that particular reactor-substrate combination.

TABLE I

Input Parameters for Treatment Plant Model

Parameter	Value	Units	Reference
Q_1	192	ft/day	Aquatair
$\bar{Y}$	0.33	$\frac{\text{lb VSS}}{\text{lb COD}_r}$	(38)
K_s	22	mg/l	(37)
X_s	95000	mg/l	(32)
d	0.0002297	ft	(32)
d_f	0.0041667	ft	(47)
A_1	16.0	ft	Aquatair
V_1	128	cu ft	Aquatair
A_v	27	sq ft/cu ft	(26)
D	8.0	ft	Aquatair
k_d	0.05	day^{-1}	(40)
V_2	1875	gal	Aquatair

It is understood that diffusion of food and oxygen, adsorption and biochemical reaction all play a role in the disappearance of substrate. What is important, though, is that the rate constants characterizing these microscale processes can be combined into the one overall constant characterizing substrate utilization in the reactor on a macroscale level.

The macroscale approach enables one to quickly tie the model to experimental tests in a rather straightforward fashion. It will be recalled that a plug-flow first-order mechanistic design Equation (123) was derived to explain the behavior of the Aquatair trickling filter. Note that the only unknown parameter is K_1. It will also be recalled that a complete-mix Monod-type mechanistic design Equation (117) was derived to explain the behavior of the Aquatair reaeration tank. Again, the only unknown parameter can be thought of as the lumped constant which describes the particular reactor.

K_1 and K_2 are related by the Equation (122) derived for the overall fraction of substrate remaining, $f = S_e/S_o$. The actual fraction of substrate remaining, f_a, represents the plant's performance. The efficiency of substrate removal, E, is given by:

$$E = 1 - f_a = \frac{S_o - S_e}{S_o} \tag{11}$$

Hence the plant's performance data may be used to evaluate the biological reaction rate constants K_1 and K_2.

The evaluation of the constants K_1 and K_2 may be viewed as a

computer optimization problem. The optimization would be to minimize an objective function F which is the sum of the squares of the differences between the actual substrate removal f_a and the predicted substrate removal f, which is based ultimately on K_1 and K_2, the biological reaction rate constants for the tower and the tank. A Simplex two-dimensional optimization routine was used to solve this problem.

Simplex Optimization

A simplex is a geometric figure defined by a number of points equal to one more than the number of dimensions of the factor space. The sequential simplex method used and discussed below is essentially that given by Deming and King (48). The factor space consists of the two independent variables K_1 and K_2 shown as coordinate axes in Figure 7 and the objective function F which goes vertically through the page. Lines of constant F appear as "contours" in Figure 7. Figure 7 also shows the projection of a two-dimensional simplex (three vertices) on the plane of the independent variables (factors). Associated with each vertex is a response that could be represented as a height above the figure. Vertex W has the worst response, vertex N has the next-to-worst response, and vertex B has the best response. The strategy of the simplex algorithm is to move toward beteer response by moving away from the worst response. It does this by projecting a line from the worst point, W, through the centroid P of the triangle and expanding or contracting along this line, depending on the value of the objective function.

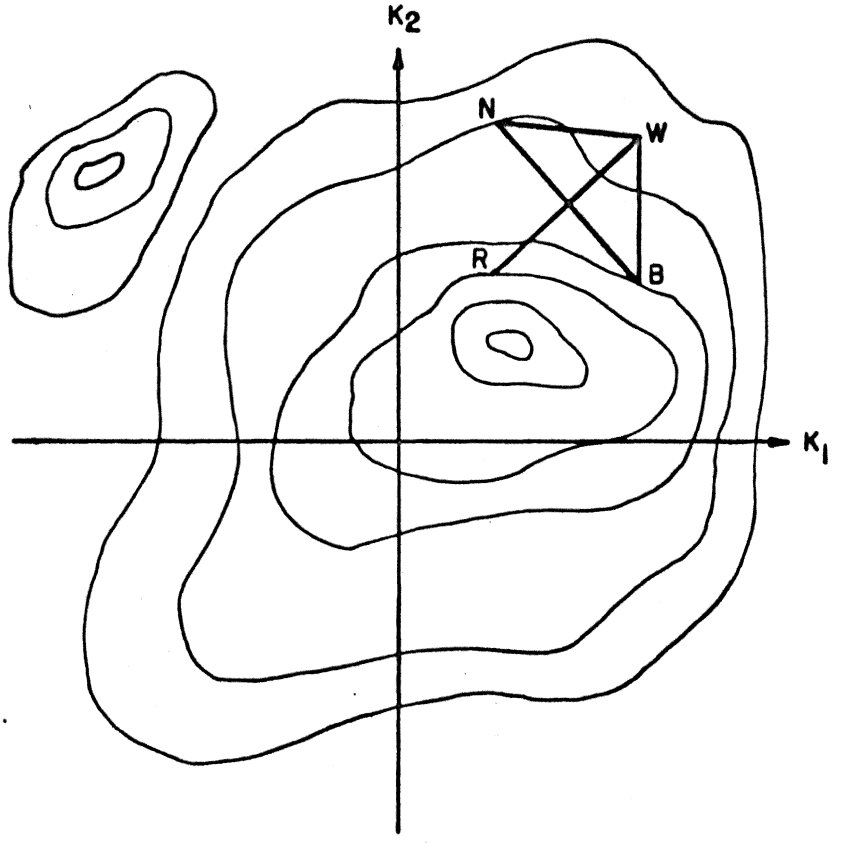

FIGURE 7. Simplex Factor Space

(Simplex moves)

Figure 8 illustrates the four possible simplex moves. The first move is always a reflection R away from the vertex of worst response W through the centroid P of the remaining hyperface and beyond a distance equal to P - W. In vector notation the coordinates of R are give by

$$R = P + (P - W)$$

The response of R is evaluated and the temporary new simplex BNR is used to determine the need for expansions or contractions. These decisions are as follows:

(1) If R is neither better than B nor worse than N, neither expansion nor contraction is indicated and BNR is accepted as the new simplex.

(2) If R is better than B, the direction P - R is favorable and an expansion is indicated. A new vertex E is calculated and the response at this vertex is evaluated

$$E = P + 2 (P - W)$$

If E is better than B, then BNE is retained as the new simplex. If E is not better than B, then we have gone "over a hill" and BNR is used as the new simplex.

(3) If R is worse than N, the direction P - R is unfavorable and a contraction is indicated. A new vertex C is calculated and the response at this vertex is evaluated. There are two possibilities for C

$$C_r = P + \tfrac{1}{2}(P - W)$$

and the new simplex is BNC_r if R is better than W.

If, on the other hand, the response at R is not better than the response at W, then

$$C_W = P - \tfrac{1}{2}(P - W)$$

and the new simplex is BNC_W. Using these rules the simplex initially expands and moves rapidly to the region of the optimum after which it contracts about the optimum and locates it with in the precision allowed by some convergence criterion.

The simplex optimization procedure was used to find the set of constants K_1 and K_2 which would give a minimum error between actual and predicted substrate removal efficiency using the mechanistic design Equation (122) to fit the 96-point data set.

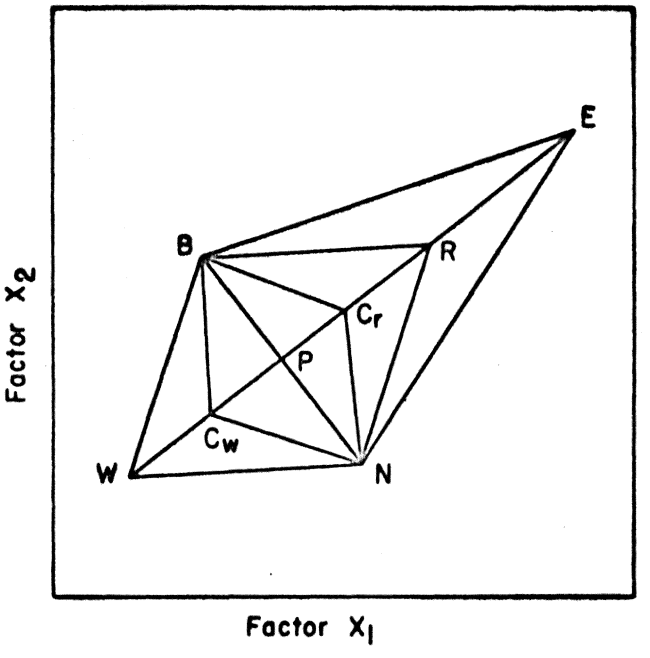

FIGURE 8. Possible Simplex Moves

Simple Linear Regression

A simple linear regression was added to the simplex simulation program in order to compare the correlation of actual versus predicted efficiency. The purpose of a linear regression given a set of data (x_i,y_i) is to determine the line of best fit $Y = a + b X$ and the correlation coefficient, r. The sample correlation coefficient r ranges between -1 and +1 and is a measure of the "scatter" or "fit" of the data to the correlation line. r close to one means that there is a strong linear behavior of Y is explained by the variable x. The defining equations for b, a, and r may be found in Walpole and Myers (49). Subroutine REGRESS is used to perform these calculations.

Perplex Optimization

The Simplex algorithm discussed previously searches for the optimum of a given objective function in two-factor space. The Perplex procedure which will now be introduced is a modification of the Box Complex Algorithm which operates in n-factor space (49). In its present form, Perplex can handle up to 15 independent variables, although only five were used in the analysis of the Aquatair Model P-3. Each combination of the n variables gives rise to an objective function value, i.e. a point in the factor space. These points are called vertices and Perplex can have as many as 30 vertices. In general, the number of vertices, m, must be greater than n + 2 and less than or equal to 2n. The combination of vertices defines the complex, which moves around the factor space by projecting a line from the worst point through the current centroid of the complex. The program takes more execution time with higher values of m, as the complex has to drag along a lot of bad points as it tries to move. An expansion factor, Alpha, must be specified along with n and m in the input data. Box recommends an alpha of 1.3, though higher values may be used if large step sizes are desired (50).

Other program input parameters include ITER, the number of projections (computer iterations) into the factor space, CONV, the desired convergence criterion, and BOUND(I), the expected lower bound of variable I. Both BOUND(I) and UPLO(I), the difference between the expected

upper and lower limits of variable I, must be specified for I = 1 to N.

Design Variables

From Figure 5, page 63, one can see that there are five primary design variables in the Aquatair Model P-3 at a given flow (Q), strength (S_o), and efficiency (E). These five variables are:

D1 = depth of plastic medium trickling filter, ft

R1 = recirculation ratio over the tower

R2 = sludge recirculation ratio

V2 = aeration tank volume, gal

XA = average microbial concentration, mg/l.

These variables are of prime importance because they are usually set by the designer or adjusted in the field. Also, they have a direct effect on both substrate removal efficiency and cost. D1, the tower depth, can be expected to reflect the effect of hydraulic residence time in the tower at constant Q1 (hydraulic load to filter); that is, D1 is chosen as the characteristic tower parameter by holding K1, AV, and Q1 constant. Similarly, V2 is chosen as the characteristic tank parameter by holding S_e, K2 and KS constant. D1 and V2 establish the size and hence cost of the two biological treatment units, as shown in Figure 5. R1 and R2 are the pumping rates for the recirculation and sludge pumps, respectively. XA, the average microbial solids concentration, is a parameter depending on XW, the concentration of microorganisms wasted, and XR, the concentration of microbial solids in the return sludge stream; therefore XA varies in the field depending upon clarifier

operation and wasting. The cost of increasing XA appears as an increase in fluid viscosity (hence higher pumping costs) as well as an increase in operator attention.

Several assumptions have been made in deriving the mechanistic simulation and constraining it to feasible space. These assumptions are:

(a) the process is at steady state,

(b) the rate of reaction is sufficiently slow that it is the limiting factor, i.e., the concentration gradients of both "food" and oxygen are zero,

(c) the process may be described at low substrate concentrations by first-order irreversible reaction kinetics of the form $A \rightarrow B$ where B is an acceptable product,

(d) both components A and B are soluble in the carrier liquid,

(e) the reaction occurs at the liquid-solid interfaces throughout the liquid film as though the liquid was flowing through a spongy mass (pseudo-homogeneous),

(f) the liquid film thickness is not influenced by the presence of the microbial population,

(g) the specific surface remains constant,

(h) the slime cover on the medium is uniform,

(i) the waste is uniformly distributed to the medium,

(j) at high substrate concentrations, the removal of substrate may be described by zeroth-order reaction kinetics, reflect-

ing the fact that the system has a maximum effective microorganism concentration,

(k) mixing in the tower may be classified as plug-flow, and

(l) mixing in the tank may be classified as complete-mix.

Cost Functions

The optimum plant design is one which yields a desired substrate removal efficiency at the lowest possible cost. Hence the Perplex optimization is constrained to a given efficiency by the subroutine CONSTRI while using cost as the external objective function, F(M). That is, only those combinations of variables which will meet the desired efficiency criterion are considered as PERPLEX searches the objective function for a minimum. The effect of the primary design variables on substrate removal efficiency has been discussed previously and comprises the biological kinetic model. The effect of the primary design variables on cost will now be considered.

The fixed capital investment of the Aquatiar Model P-3 consists of the cost of the primary sludge holding tank, biological oxidation tower, recirculation chamber, secondary clarifier, distribution nozzels, air injectors, recirculation pumps, and sludge pump. These costs are calculated in the objective function subprogram, F, for each feasible combination of N variables, M, as follows: The cost of the bio-oxidation tower is related to a charge of three dollars per cubic foot of filter medium. The volume of the tower is given by A1 x D1, where D1 is a primary design variable and A1 is allowed to vary with R1 in order to

maintain a constant Q1 (hydraulic load to the tower). The cost of the sludge tank, recirculation chamber, and secondary clarifier are related to a charge of $4.50 per square foot of ¼ in. steel plate. The sludge tank and clarifier costs are held constant for a given hydraulic load and strength, but the recirculation tank cost is allowed to change with the primary design variable V2 (volume of recirculation chamber).

The distribution nozzels at the top of the tower cost approximately ten dollars apiece; one is required for each 16 square feet of A1. The venturi air injectors cost approximately $25.00 each and can handle about 1400 gallons of V2. The cost of the recirculation pumps depends on both the capacity in gallons per minute and the total dynamic head in pounds per square inch. A capacity-head factor such as that given by Guthrie, "Capital Cost Estimating", is used to determine the capital costs of the pumps (51). The equation takes the following form:

$$\text{Cost}_2 = \text{Cost}_1 \left(\frac{\text{Size}_2}{\text{Size}_1} \right)^{\text{exponent}} = C_1 \left(\frac{Q_2 h_2}{Q_1 h_1} \right)^{0.52},$$

where the exponent of 0.52 is given by Guthrie for centrifugal pumps and drivers. With reference to Figure 5, it is noted that both D1 and R1 can be expected to increase the recirculation pump cost since an increase in D1 increases the total dynamic head. However, the depth of the recirculation chamber is held constant for a given Q -- an increase in V2 only increases the length L2. This means that R2, the sludge recirculation ratio, is the only primary design variable affecting the cost of the sludge pump.

Annual Equivalent Cost

In order to obtain a convenient basis of comparison between capital and operating expenses it is necessary to annualize the fixed capital investment of an Aquatair Model P-3 over the expected life of the plant. To do this an amortization formula by Theusen and Fabrycky is used which includes the interest charge on capital recovery (52). An interest rate of 7¼% compounded annually was assumed along with a service life of 10 years and an estimated salvage value of one thousand dollars.

Operating Costs

Two major costs arise from the operation of an Aquatiar package sewage treatment plant: these are the cost of electricity and the cost of sludge disposal. The cost of electricity is approximately 3.5¢/Kw H and is related to the horsepower required to drive the pumps by the conversion of 1.0 hp-hr for 0.745 Kw H. The motor horsepower is calculated following an equation from Linsley and Franzini (53), thusly:

$$\text{Motor Hp} = \frac{\text{Gamma (Q) (h)}}{(550)\ \text{Eta}}$$

where,

Gamma = 62.4 lbm/cu ft

Q = discharge, cfs

h = significant head, ft. of H_2O, and

Eta = pump efficiency = o.70.

The variables O1 and O3 represent the annualized electricity costs for the recirculation pumps and sludge pump, respectively, not including

the head loss through the venturi injectors. This added head loss arises from the need to supply approximately 1000 cubic feet of air for each pound of substrate removed in the recirculation chamber. The variable O2 reflects the extra electricity cost due to the oxygen requirement; the injectors also provide mixing.

Sludge Handling Costs

It is apparent that sludge handling costs are a major expense item of any sewage treatment plant. While the sludge produced from the extended aeration type of package sewage plants is small, it is nevertheless observed that most package plants of this type have to be pumped out at least once a year; the cost of sludge disposal is therefore an annual lump sum figure. The amount of sludge produced is estimated by the cell continuity equation (38). The cell continuity equation is a material balance statement for cell material or activated sludge:

$$\frac{dX}{dt} = Y\ S_r - k_d\ X_a \qquad (126)$$

where,

Y = growth yield coefficient = 0.33 lb VSS/lb COD removed (38)

k_d = decay coefficient (autoxidation rate) = 0.05 day^{-1}

S_r = substrate removed, pounds per day

X_a = average amount of microbial cells, pounds, and

$\frac{dX}{dt}$ = sludge production rate, lb/day.

The variables CELLSI and CELLS2 signify the sludge production rates of the Aquatair bio-oxidation tower and recirculation tank, respectively,

in pounds per day.

It is interesting to note that the sludge production can become negative if the microorganism mass, X_a is sufficiently large. (As the organisms become starved they enter the endogenous growth phase.) The cost associated with sludge decay would therefore be a benefit rather than a charge.

RESULTS AND DISCUSSION

Biological Reaction Rate Constants

The simplex optimization procedure was used to find the minimum of an objective function defined by the square of the difference of predicted and actual fraction of substrate remaining in an Aquatair Model P-3 package sewage treatment plant. The optimization was performed with respect to two factors, K_1 and K_2, which were thought to characterize the substrate removal behavior in the Aquatair tower and recirculation tank, respectively. As seen from a mechanistic analysis, these biological reaction constants should be true constants for a given plant, waste, and mode of operation. That is, if the assumptions upon which the mechanistic model is based, such as constant liquid film thickness, hold correct. The assumptions which comprise the Aquatair biological kinetic model are admittedly rather drastic; however, it is argued that Man has never really successfully explained Nature completely and all one can do therefore is to correlate thoughts along some level.

The biological reaction rate constants for the Aquatair Model P-3 were computed as 0.0206 l/mg-day for K_1 and 0.4510 day-1 for K_2 with a correlation coefficient of 0.7131. The correlation is based upon COD substrate data taken over 96 test days by the National Sanitation Foundation. COD values were chosen as a measure of substrate concentration because the BOD data was widely scattered, owing to the un-

reliable nature of the BOD test. Tables of the NSF data and predicted efficiency, E3, versus actual efficiency, EA, appear in the appendix.

Effect of Temperature

The Arrhenius-type temperature correction equation (125) was deleted from the Simplex simulation routine and the constants K_1 and K_2 calculated from the 96 point NSF data set. Without the temperature correction, the biological reaction rate constants for the Aquatair Model P-3 were computed as 0.0225 l/mg-day for K_1 and 0.4610 day^{-1} for K_2 with a correlation coefficient of 0.7131. These calculations indicate that the temperature correction makes no statistically significant difference in the results, confirming Gardner's notion that heterogenous cultures in sewage do not obey an Arrhenius-type equation (45). However, it is also noted that the temperature range studied was very small ($5C^{o}$), so that little or no temperature effect could be expected.

The units on K_1 include an inverse concentration because K_1 is a lumped first-order parameter. With reference to Figure 4, this means that K_1 is the dominating term for low S_e, in which small k_1 divided by a $K_s + S_e$ term appears as the single constant K_1. At higher substrate concentrations the K_s and S_e values become significant and are split out of the equation, leaving K_2 with simply units of inverse time.

Optimum Design

The optimum plant design has been defined as one which will yield a desired substrate removal efficiency at the lowest possible cost. The main optimization program finds the lowest cost feasible combination of design variables in the following manner: Figure 5 is used to select the primary design variables. These variables are D1, the depth of the trickling filter in feet, V2, the volume of the recirculation chamber in gallons, x_a, the average microbial solids concentration in mg/l, R1, the tower recirculation ration, and R2, the sludge recirculation ratio. Subroutine SIZE is used to generate an initial vertex, or combination of the five primary design variables, which is consistent with the given flow, organic load, and efficiency. The IBM utility subprogram RANDU is then used to generate an additional M-1 vertices, where M is the total number of vertices in the complex (in our case, M = 8). Each point in turn is checked for constraint satisfaction by the subroutine CONST1 and is withdrawn in 21 logarithmic steps toward the current centroid if a constraint is violated. Besides simple physical constraints, CONST1 is used to check the predicted substrate removal of a given point, based upon the biological kinetic model previously developed.

Once an initial complex has been calculated, the PERPLEX optimization procedure is used to find the minimum of the external objective cost function. It does this by projecting a line from the worst vertex of the complex through the current centroid of the n-space figure a

distance given by the expansion factor, alpha, and then testing the new point for feasibility and cost. If a constraint is violated, the point is withdrawn toward the current centroid. If a lower cost resulting from a feasible combination if found, the point is retained and the complex moves in the favorable direction until the convergence criterion or 1000 iterations are achieved. In order to make sure that a true global minimum has been found, the program then starts all over with a new initial complex consisting of random feasible points and calculates a new optimum complex.

With reference to the optimization results listed in the Appendix, it can be seen that the program converges on essentially the same optimum complex six times, though starting from different initial complexes and using different numbers of iterations and function evaluations. In every case, the desired substrate removal efficiency of 90 per cent has been achieved with an annual equivalent cost of around $483.00. The optimum design utilizes a 4-foot biological oxidation tower and a recirculation chamber of approximate 1900 gallons. The recirculation ratio over the tower is about 6.4 while the sludge recirculation ratio is held to 1.5. The optimum microbial solids concentration is approximately 2100 mg/l.

Sludge Production

The breakdown of various items associated with the optimum complex appears in the Appendix. It is interesting to note that, while the overall sludge production is positive, the variable CELLS1 shows a neg-

ative sludge production in the Aquatair biological oxidation tower. This calculation is based on the cell continuity equation and implies that aerobic digestion occurs in the Aquatair Model P-3. This implication is borne out by National Sanitation Foundation data (see Table II, Appendix) which shows the disappearance of ammonia by conversion to nitrates; nitrification usually occurs in the endogenous growth phase of microorganisms feeding upon domestic wastes.

CONCLUSIONS

The following conclusions were made based on the results of this study:

1. From consideration of reaction kinetics, reactor type, and geometry, a mechanistic model was developed to explain the phenomena of substrate removal in a package sewage treatment plant.

2. The biological reaction rate is a true constant for a given type of waste and geometrical combination of treatment units.

3. Based upon National Sanitation Foundation data, the biological reaction rate constants for the Aquatair Model P-3 were computed as 0.0206 l/mg-day for K_1 and 0.4510 day^{-1} for K_2 with a correlation coefficient of 0.7131.

4. The mechanistic approach utilized puts the two basic biological treatment processes, activated sludge and trickling filtration, on a common mathematical basis.

5. The modified COMPLEX method is shown to be a useful tool for evaluating the factor space defined by a set of mathematical assumptions which describe a wastewater treatment process.

6. Aerobic digestion occurs in the Aquatiar bio-oxidation tower as shown by nitrification data and low sludge production observations which are confirmed by calculation.

REFERENCES CITED

1. National Research Council, "Sewage Treatment at Military Installations," Sewage Works Journal, 18, 787-1028 (1946).

2. Velz, C. J., "A Basic Law for the Performance of Biological Filters," Sewage Works Journal, 20, 607-617 (1948).

3. Rankin, R. S., Performance of Biofiltration Plants By Three Methods," Proceedings Separate No. 336, ASCE, 79 (November, 1953).

4. Geber, B., "A New Concept in Trickling Filter Design," Sewage and Industrial Wastes, 26, 136-139 (February, 1954).

5. Fair, G. M., Geyer, J. C., and Okun, D. A., Water and Wastewater Engineering, Volume 2, Wiley, New York (1968).

6. Fairall, J. M., "Correlation of Trickling Filter Data," Sewage Works Journal, 28, 1069-1074 (1956).

7. Howland, W. E., "Effect of Temperature on Sewage Treatment Processes," Sewage and Industrial Wastes, 25, 161-169 (Feb., 1953).

8. Howland, W. E., "Flow Over Porous Media as in a Trickling Filter," Proceedings 12th Industrial Waste Conference, Purdue University, 94, 435-465 (1958).

9. Schulze, K.L., "Load and Efficiency of Trickling Filters," Journal WPCF, 32, 245-261 (March, 1960).

10. Bloodgood, D. E., Teletzke, G. H., and Pohlnad, F. G., "Fundamental Hydraulic Principles of Trickling Filters," Sewage and Industrial Wastes, 31, 243-251 (1959).

11. Eckenfelder, W. W., Jr., "Trickling Filtration Design and Performance," Journal Sanitary Egnineering Division, ASCE, 87, 33-45 (July, 1961).

12. Eckenfelder, W. W., Jr. and Barnhart, E. L., "Performance of a High Rate Trickling Filter Using Selected Media," Journal WPCF, 35, 1535-1551 (December, 1963).

13. Galler, W. S. and Gotaas, H. B., "Analysis of Biological Filter Variables," Journal Sanitary Engineering Division, ASCE, 90, SA6, 59-79 (December, 1964).

14. Mehta, D. S., Davis, H. H., and Kingsley, R. P., "Oxygen Theory in Biological Treatment Plant Design," Journal Sanitary Engineering Division, ASCE, 93, SA4, 91-112 (August, 1967).

15. Ames, W. F., Behn, V. C. and Collins, W. Z., "Transient Operation of the Trickling Filter," Journal Sanitary Engineering Division, ASCE, 88, 21-38 (1962).

16. Amado, M. A., "Analysis of BOD Reduction in Trickling Filters," Masters Thesis, Cornell University (February, 1974).

17. Atkinson, B., Busch, A. W. and Dawkins, G. S. "Recirculation, Reaction Kinetics, and Effluent Quality in a Trickling Filter Flow Model," Journal WPCF, 35, 1307-1317 (1963).

18. Swilley, E. L. and Atkinson, B., "A Mathematical Model for the Trickling Filter," Proceedings 18tj Industrial Waste Conference, Purdue University, 115, 706 (1963).

19. Maier, W. J., Behn, V. G. and Gates, C. D., "Simulation of the Trickling Filter Process," Journal Sanitary Engineering Division, ASCE, 93, SA4, 91-112 (August, 1967).

20. Atkinson, B., Daoud, I. S. and Williams, D. A., "A Theory for the Biological Film Reactor," Transactions of AIChE, 46 T19 (1968).

21. Atkinson, B., and Williams, D. A., "The Performance Characteristics of a Trickling Filter with Hold-up of Microbial Mass Controlled by Periodic Washing," Trans. Instn. Chem. Engrs. 49, 215 (1971).

22. Kehrberger, G. J. and Busch, A. W., "Mass Transfer Effects in Maintaining Aerobic Conditions in Film-flow Reactors," Journal WPCF, 43, 1514-1527 (July 1971).

23. Coulson, J. M. and Richardson, J. F., Chemical Engineering, Volume III, Permagon Press, Oxford (1971).

24. Germain, J. E., "Economical Treatment of Domestic Waste by Plastic Medium Trickling Filters," Journal WPCF, 38, 192-203 (1966).

25. Balakrishnan, S., Eckenfelder, W. W., Jr. and Brown, C., "Organics Removal by a Selected Trickling Filter Media, " Water and Waste Engineering, 6, A22-A25 (1969).

26. Gromiec, M. J. and Malina, J. F., "Performance of Plastic Media in Trickling Filters," Water Research, 6, 1321-1332 (1972).

27. Lamb, R., "A Suggested Formula for the Process of Biological Filtration," Water Pollution Control, 69, 209-220 (1970).

28. Bruce, A. M. and Merkens, J. C., "Recent Studies of High-rate Biological Filtration," Water Pollution Control, 69, No. 2, 113 (1970).

29. Monadjemi, P. and Behn, V. C., "Oxygen Uptake and Mechanism of Substrate Purification in a Model Trickling Filter," Proceedings 5th Int. Wat. Pollut. Res. Conf., Paper II-12-16.

30. Richard, J. G. and Kingsbury, R. P., Treating Milk Wastes by Biological Oxidation With Plastic Media," Proceedings, 24th Annual Oklahoma Industrial Waste and Advanced Wastewater Treatment Conference, Oklahoma State University, Stillwater Oklahoma (1973).

31. Eckenfelder, W. W., Jr., Industrial Water Pollution Control, McGraw-Hill Book Company, Inc., New York (1966).

32. Kornegay, B. H. and Andrews, J. F., "Kinetics of Fixed-film Biological Reactors," Journal WPCF, 40, R460-469 (Nov. 1968).

33. Bentley, T. L., "Application and Comparison of Activated Sludge Design and Operational Control Parameters to An Experimental Fixed-Bed Reactor," Masters Thesis, Oklahoma State University, (May, 1974).

34. Tucek, F., Chudoba, J., and Madera, V., "Unified Basis for Design of Biological Aerobic Treatment Processes," Water Research, 5, 647-681 (1971).

35. Roberts, J. "Towards a Better Understanding of High Rate Biological Film Flow Reactor Theory," Water Research, 7, 1561-1589 (1973).

36. Atkinson, B., Busch, A. W., Swilley, E. L., and Williams, D. H., "Kinetics, Mass Transfer, and Organism Growth in a Biological Film Reactor," Trans. Instn. Chem. Engrs. 45, T257 (1967).

37. Metcalf and Eddy, Inc., Wastewater Engineering: Collection, Treatment, Disposal, McGraw-Hill Book Company, Inc., New York (1972).

38. Jenkins, D. and Garrison, W. E., "Control of Activated Sludge by Mean Cell Residence Time," Journal WPCF, 40, 1905-1919, (Nov., 1968).

39. Adams, C. E., Jr. and Eckenfelder, W. W., Jr., "Response of Activated Sludge to Organic Transient Loadings," Journal Sanitary Engineering Division, ASCE, 96, SA2, 333-353 (April, 1970).

40. McKinney, R. E., "Design and Operational Model for Complete Mixing Activated Sludge System," Biotechnology and Bioengineering, 16, 703-722 (1974).

41. Sullins, J. K. and Self, R. W., "Process and Plant for Treating Sewage," U. S. Patent No. 3,261,779 (July 19, 1966).

42. Williamson, K. J. and Chung, T. H., "Dual Limitation of Substrate Utilization Kinetics Within Bacterial Films," Papar Presented at 49th Meeting of American Institute of Chemical Engineers, Houston, Texas (March 19, 1975).

43. Randall, C. W., Young, W. S., and King, P. H., "Aerobic Digestion of Trickling Filter Humus," Proceedings 4th Annual Environmental Engineering and Science Conference, University of Louisville (1974).

44. Topiwala, H. and Sinclair, C. G., "Temperature Relationship in Continuous Culture," Biotechnology and Bioengineering, 13, 795 (1971).

45. Gardner, R. A., "The Aerobic Digestion of Activated Sludge at Elevated Temperatures," Unpublished Masters Thesis, Virginia Polytechnic Institute and State University (1974).

46. Anon., Standard Methods for the Examination of Water and Wastewater, American Public Health Association, New York, 13th Edition (1971).

47. Clark, J. P., Personal Communication No. 1, Blacksburg, Virginia (1975).

48. Deming, S. N. and King, P. G., "Computers and Experimental Optimization," Research/Development, 22-25, (May, 1974).

49. Walpole, R. E. and Myers, R. H., Probability and Statistics for Engineers and Scientists, Macmillan Company, New York (1972).

50. Box, M. J., "A New Method of Constrained Optimization and a Comparison with Other Methods," Computer Journal, 8, 42-52 (1965).

51. Ferk, D. L., Unpublished Perplex Optimization Routine, Virginia Polytechnic Institute and State University, Blacksburg, Virginia (March, 1975).

52. Guthrie, K. M., "Capital Cost Estimating," Chemical Engineering 114-142 (March 24, 1969).

53. Thuesen, H. G., Fabrycky, W. J. and Thuesen, G. J., Engineering Economy, Prentice-Hall, Inc. Englewood Cliffs, New Jersey, 4th Edition (1971).

54. Linsley, R. K. and Franzini, J. B., Water-Resources Engineering, McGraw-Hill Book Company, New York (1972).

APPENDIX A

Table II

NSF Nitrogen Data

Data Point	Data Case	NH_3-N Influent	Effluent	NO_3-N Influent	Effluent
1	1	17.6	1.9	0.7	9.4
2	1	18.9	1.1	0.9	12.5
3	1	18.3	1.7	0.9	9.9
4	1	13.5	1.8	0.3	9.0
5	1	16.3	2.8	0.1	11.9
6	1	17.9	0.4	0.1	12.4
7	1	17.6	0.8	0.1	12.5
8	1	16.1	0.7	0.1	12.5
9	1	17.1	0.9	0.1	14.1
10	1	17.6	1.4	0.1	11.2
11	1	19.4	1.6	0.1	12.2
12	1	17.3	0.9	0.1	11.8
13	1	15.9	0.8	<0.1	12.3
14	1	16.5	0.6	0.2	13.6
15	1	17.0	0.8	<0.1	11.2
16	1	17.3	1.8	0.1	10.7
17	1	16.1	0.9	0.3	11.6
18	1	16.6	1.6	0.2	10.6
19	1	19.0	0.7	0.2	14.3
20	1	17.5	0.8	0.1	13.0
21	1	19.5	2.6	<0.1	12.7
22	1	19.4	1.0	<0.1	12.9
23	1	17.8	0.9	<0.1	12.7
24	1	18.0	1.1	<0.1	10.9
25	1	18.9	2.0	<0.1	12.1
26	1	18.5	1.7	<0.1	12.8
27	1	19.0	2.1	<0.1	13.8
28	1	21.1	1.0	<0.1	14.4
29	1	18.9	2.1	<0.1	11.1
30	2	19.8	2.4	<0.1	11.6
31	2	24.7	3.7	<0.1	15.9
32	2	25.5	5.7	1.0	14.2
33	2	23.3	5.5	0.7	14.7
34	2	21.9	6.8	<0.1	11.0
35	2	22.9	9.9	<0.1	11.8
36	2	28.1	6.7	<0.1	13.0
37	2	26.0	5.8	<0.1	12.2

Data Point	Data Case	NH_3-N, mg/l Influent	NH_3-N, mg/l Effluent	NO_3-N, mg/l Influent	NO_3-N, mg/l Effluent
38	2	25.9	6.2	0.2	13.0
39	2	24.5	6.0	0.1	16.1
40	2	22.7	5.1	< 0.1	15.3
41	2	22.7	6.6	< 0.1	14.2
42	2	25.5	6.9	< 0.1	19.7
43	2	24.1	7.0	< 0.1	16.4
44	2	27.4	7.7	0.1	15.3
45	2	24.2	10.2	0.1	13.4
46	2	23.7	10.3	0.1	15.6
47	2	25.7	9.6	0.1	7.2
48	2	24.7	7.2	0.1	7.7
49	3	18.4	2.8	0.1	6.7
50	3	18.2	1.7	0.1	8.4
51	3	19.9	3.2	0.1	8.9
52	3	18.6	2.6	0.1	7.6
53	3	16.1	0.1	0.1	8.9
54	3	11.4	0.8	0.1	7.6
55	3	10.3	1.0	0.1	8.0
56	3	14.6	1.5	0.1	7.5
57	3	15.2	2.0	0.1	7.8
58	3	14.7	1.8	0.1	7.1
59	3	12.9	2.0	0.1	8.4
60	3	15.4	2.7	0.1	8.4
61	3	17.4	2.3	0.1	1.0
62	3	16.6	2.1	0.1	8.4
63	3	15.0	1.9	0.1	3.0
64	3	16.0	2.3	0.1	3.4
65	3	17.4	1.9	0.1	7.0
66	3	14.4	0.1	0.1	7.5
67	3	16.1	2.2	0.1	7.3
68	3	19.3	0.1	0.1	9.0
69	3	14.8	1.8	0.1	10.3
70	3	16.3	2.1	0.1	8.8
71	3	15.7	2.3	0.1	17.4
72	3	16.9	2,7	0.1	9.4
73	3	18.8	9,0	0.1	7.0
74	3	16.0	6.2	0.1	6.7
75	3	14.4	5.5	0.6	1.3
76	3	16.0	2.2	0.5	1.2
77	4	17.0	3.3	0.1	10.9
78	4	12.4	2.0	0.1	10.6
79	4	15.1	3.1	0.1	11.0
80	4	14.0	3.3	0.1	10.8

Data Point	Data Case	NH_3-N, mg/l Influent	NH_3-N, mg/l Effluent	NO_3-N, mg/l Influent	NO_3-N, mg/l Effluent
81	4	15.1	3.1	0.1	9.0
82	4	17.4	2.5	0.1	11.6
83	4	17.5	3.3	0.1	9.1
84	4	18.0	1.4	0.1	0.0
85	4	17.6	3.2	0.1	8.0
86	4	12.5	3.7	0.1	9.5
86	4	20.1	0.1	0.1	9.3
88	4	18.0	1.8	0.6	11.3
89	4	18.1	1.3	0.3	10.2
90	4	19.0	2.3	0.7	11.1
91	4	18.7	5.0	0.3	10.3
92	4	18.2	2.6	0.6	2.2
93	4	14.2	2.5	0.6	11.2
94	4	17.6	1.3	0.5	9.9
95	4	20.3	3.0	0.5	11.9
96	4	18.7	2.0	0.1	11.6

Appendix B

Table III

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C       EXTRACTION OF BIOLOGICAL REACTION RATE CONSTANTS
C       MAIN PROGRAM
        REAL KD,KS,NN,LO(150),LE(150)
        INTEGER C(99)
        EXTERNAL FUN
        COMMON X(3),Y(3)
        COMMON/PARAM/  YY,KS,DELTA,CRIT,M,Q1,A1,V1,AV,D,KD,V2,XS,DS
        COMMON/NSE/SO(150),SE(150),CELLS(150),Q(150),T(150)
        COMMON/SENSE/EA(150),   E2(150),E3(150),A(150),B(150),E1(150)
        COMMON/DENOM/EA(150),E3(150),E2(150),   NN(150),R(150),Q2(150),
     6              XT(150),YT(150),LO,LE,E1(150)
    1   FORMAT (5F8.2,3X,I2,2F10.7,F8.1)
    2   FORMAT (6F10.4)
    3   FORMAT (6F10.4)
    4   FORMAT (5X,'Q1',8X,'RQ',8X,'YY',8X,'KS   ',5X,'CRIT',9X,'XS',5X,
       1        'DS',11X,'DELTA', 7X,'M')
    5   FORMAT (1X,5F10.4,F10.1,F13.7,3X,F10.7,3X,I2)
    6   FORMAT (5X,'A1',8X,'V1',8X,'AV',8X,'D',9X,'KD',11X,'V2')
    7   FORMAT (1X,5F10.4,1F10.1)
    8   FORMAT (1X,I2,I3,F5.0,2F7.1,F8.1,F10.1,F11.4,F9.4,F10.1,6F10.4)
    9   FORMAT (7F10.4,8X,I1)
        READ (5,1) Q1,RQ,YY,KS,CRIT,M,DELTA,DS,XS
        READ (5,2) A1,V1,AV,D,KD,V2
        READ (5,3) (X(J),Y(J),J=1,3)
        DO 50 J=1,M
        READ (5,9) SO(J),SE(J),CELLS(J),Q(J),T(J),LO(J),LE(J),C(J)
        NN(J)= (Q1*A1*7.5)/Q(J)
        R(J)=RQ/Q(J)
        Q2(J)= Q(J)*(1 + NN(J) + R(J))
        EA(J)=SE(J)/SO(J)
   50   CONTINUE
        CALL SIMPLE (CRIT,XB,YB,XMAX)
```

```
      WRITE (6,4)
      WRITE (6,5)  Q1,RQ,YY,KS,CRIT,XS,DS,DELTA,M
      WRITE (6,6)
      WRITE (6,7) A1,V1,AV,D,KD,V2
      PRINT,' '
      PRINT,'NSF DATA'
      PRINT,'CASE 1 = 30 JUNE 74  HIGH VSS  SUBDIVISION'
      PRINT,'CASE 2 = 25  AUG 74  HIGH VSS  SCHOOL'
      PRINT,'CASE 3 = 17  JUN 73   LOW VSS  SUBDIVISION'
      PRINT,'CASE 4 = 29  JUL 73   LOW VSS  SCHOOL'
      WRITE (6,75)
      WRITE (6,80)
   75 FORMAT (72X,'PREDICTED PERFORMANCE - COD BASIS')
   80 FORMAT (42X,'      ----------------------------------------------
     9-----------------------------------')
      PRINT,' J  C  T(J) SO(J)   SE(J) CELLS(J)  Q(J)        NN(J)    R(J
     3)     Q2(J)       XT(J)     YT(J)    EA(J)     E3(J)     E2(J)
     4E1(J)'
      DO 100 J=1,M
      WRITE (6,8) J,C(J),T(J),SO(J),SE(J),CELLS(J),Q(J),NN(J),R(J),Q2(J)
     5           ,XT(J),YT(J),EA(J),E3(J),E2(J),E1(J)
  100 CONTINUE
      CALL REGRES
      PRINT,' '
      PRINT,' '
C     BOD BASIS
      DO 150 J=1,M
      EA(J)=LE(J)/LO(J)
      SE(J)=LE(J)
  150 CONTINUE
      KS=100.0; YY=0.5
      CALL SIMPLE (CRIT,XB,YB,XMAX)
```

```
      WRITE (6,4)
      WRITE (6,5)  Q1,RQ,YY,KS,CRIT,XS,DS,DELTA,M
      WRITE (6,6)
      WRITE (6,7) A1,V1,AV,D,KD,V2
      PRINT,' '
      WRITE (6,175)
      WRITE (6,180)
  175 FORMAT (72X,'PREDICTED PERFORMANCE - BOD BASIS')
  180 FORMAT (42X,'      ----------------------------------------------
     9-------------------------------------')
      PRINT,' J  C  T(J) LO(J)   LE(J) CELLS(J)  Q(J)        NN(J)    R(J
     3)     Q2(J)       XT(J)     YT(J)    EA(J)     E3(J)     E2(J)
     4E1(J)'
      DO 200 J=1,M
      WRITE (6,8) J,C(J),T(J),LO(J),LE(J),CELLS(J),Q(J),NN(J),R(J),Q2(J)
     5           ,XT(J),YT(J),EA(J),E3(J),E2(J),E1(J)
  200 CONTINUE
      CALL REGRES
  999 STOP
      END
```

```
      SUBROUTINE SIMPLE(CRIT,XB,YB,XMAX)
      REAL NEXT,N(2)
      EXTERNAL FUN
      COMMON X(3),Y(3)
      DIMENSION W(2),B(2),E(2),P(2),R(2),F(3)
      L=0
      PRINT,'   X(1)              Y(1)              X(2)              Y(2)
     2             X(3)              Y(3)'
      PRINT,(X(J),Y(J),J=1,3)
      F(1)=FUN(X(1),Y(1));F(2)=FUN(X(2),Y(2)) ;F(3)=FUN(X(3),Y(3))
      PRINT,'   F(1)              F(2)              F(3)'
      PRINT,(F(J),J=1,3)
      BEST=AMAX1(F(1),F(2),F(3))
      WORST=AMIN1(F(1),F(2),F(3))
      DO 1 J=1,3
      IF(F(J).EQ.BEST)GOTO10
      IF(F(J).EQ.WORST)GOTO20
      IN=J
      GOTO1
   20 IW=J;GOTO1
   10 IB=J
    1 CONTINUE
      B(1)=X(IB);B(2)=Y(IB);N(1)=X(IN);N(2)=Y(IN)
      W(1)=X(IW);W(2)=Y(IW)
      PRINT,'B',B(1),B(2),'COORDINATES YIELD',F(IB),'=F(IB)'
      PRINT,'N',N(1),N(2),'COORDINATES YIELD',F(IN),'=F(IN)'
      PRINT,'W',W(1),W(2),'COORDINATES YIELD',F(IW),'=F(IW)'
  100 P(1)=(B(1)+N(1))/2
      P(2)=(B(2)+N(2))/2
      L=L+1
      PRINT,'IT NUM',L
      A=SQRT((B(1)-N(1))**2+(B(2)-N(2))**2)
```

```
      D=SQRT((B(1)-W(1))**2+(B(2)-W(2))**2)
      OR=A+D
      FB=FUN(B(1),B(2))
      FN=FUN(N(1),N(2))
      FW=FUN(W(1),W(2))
      PRINT,'B',B(1),B(2),'COORDINATES YIELD',FB,'=FB'
      PRINT,'N',N(1),N(2),'COORDINATES YIELD',FN,'=FN'
      PRINT,'W',W(1),W(2),'COORDINATES YIELD',FW,'=FW'
      IF (L.GT.100) GO TO 130
      IF(OR.LT.CRIT)GO TO 130
      R(1)=P(1)+(P(1)-W(1))
      R(2)=(P(2)-W(2))+P(2)
      FR=FUN(R(1),R(2))
      PRINT,'R',R(1),R(2),'COORDINATES YIELD',FR,'=FR'
      IF(FR.GT.FB) GO TO 80
      IF(FR.LT.FN) GO TO 90
      W(1)=N(1)
      W(2)=N(2)
      N(1)=R(1)
      N(2)=R(2)
      GO TO 100
   90 IF(FR.LT.FW) GO TO 110
      W(1)=P(1)+(P(1)-W(1))/2.
      W(2)=P(2)+(P(2)-W(2))/2.
      GO TO 100
  110 W(1)=P(1)-(P(1)-W(1))/2.
      W(2)=P(2)-(P(2)-W(2))/2.
      GO TO 100
   80 E(1)=P(1)+2.*(P(1)-W(1))
      E(2)=P(2)+2.*(P(2)-W(2))
      FE=FUN(E(1),E(2))
      PRINT,'E',E(1),E(2),'COORDINATES YIELD',FE,'=FE'
```

```
      IF(FE.GT.FUN(B(1),B(2)))GO TO 120
     W(1)=N(1)
     W(2)=N(2)
     N(1)=B(1)
     N(2)=B(2)
     B(1)=R(1)
     B(2)=R(2)
     GO TO 100
120  W(1)=N(1)
     W(2)=N(2)
     N(1)=B(1)
     N(2)=B(2)
     B(1)=E(1)
     B(2)=E(2)
     GO TO 100
130   CONTINUE
     YB=B(2)
     XB=B(1)
     PRINT,' '
     WRITE(6,11)L,CR,XB,YB
11   FORMAT( 3X, '  ITERATIONS ',I4,'   SUM OF TWO SIDES = ', F9.7,10X,
    7        'BIOLOGICAL REACTION RATE CONSTANTS ARE',F10.4,'=XB   AND',
    8         F10.4,'=YB')
     PRINT,' '
     PRINT,'PHYSICAL AND BIOLOGICAL SYSTEM PARAMETERS:'
     PRINT,' '
     RETURN
     END
```

```
      FUNCTION FUN (X,Y)
      REAL KD,KS,NN,LO(150),LE(150)
      INTEGER C(99)
      COMMON/PARAM/  YY,KS,DELTA,CRIT,M,Q1,A1,V1,AV,D,KD,V2,XS,DS
      COMMON/NSF/SO(150),SE(150),CELLS(150),Q(150),T(150)
      COMMON/SENSE/FA(150),   F2(150),F3(150),A(150),B(150),F1(150)
      COMMON/DENOM/EA(150),E3(150),E2(150),   NN(150),R(150),Q2(150),
     6             XT(150),YT(150),LO,LE,E1(150)
      SUMFUN=0.0
      DO 10 J=1,M
      XT(J)= X*1.035**(T(J)- 20)
      F1(J)=EXP((-XT(J)*AV*D/Q1)*((XS*DS) + (CELLS(J)*DELTA)))
      E1(J)= (1-F1(J)) * 100
      YT(J)= Y*1.047**(T(J)- 20)
      F2(J)= 1/(1 + (YT(J)*CELLS(J)*V2)/((KS + SE(J))*Q2(J)))
      A(J)=NN(J)*(1 - (F1(J)*F2(J)))
      B(J)=R(J)*(1 - F2(J))
      F3(J)= F2(J)/(1 + A(J) + B(J))
      EA(J)=(1-FA(J))*100;E3(J)=(1-F3(J))*100;E2(J)=(1-F2(J))*100
      SUMFUN = SUMFUN + ((F3(J) - FA(J))**2)
10    CONTINUE
      IF (X.LT.0.0) GO TO 20
      H=0; GO TO 30
20    H=1
30    IF(Y.LT.0.0) GO TO 40
      H1=0; GO TO 50
40    H1=1
50    CONTINUE
      FUN = -SUMFUN + 100*H*X + 100*H1*Y
      RETURN
      END
```

```
      SUBROUTINE REGRES
      COMMON/PARAM/  YY,KS,DELTA,CRIT,M,Q1,A1,V1,AV,D,KD,V2,XS,DS
      COMMON/DENOM/EA(150),E3(150),E2(150),   NN(150),R(150),Q2(150),
     6             XT(150),YT(150),LO,LE,E1(150)
      SUMX=0; SUMY=0; SUMX2=0; SUMY2=0; SUMXY=0
      DO 10 J=1,M
      SUMX=SUMX + EA(J)
      SUMY=SUMY + E3(J)
      SUMX2=SUMX2 + (EA(J))**2
      SUMY2=SUMY2 + (E3(J))**2
      SUMXY=SUMXY + (EA(J)*E3(J))
 10   CONTINUE
      BB=((M*SUMXY) - (SUMX*SUMY))/((M*SUMX2) - (SUMX**2))
      AA=(SUMY/M) - (BB*(SUMX/M))
      SXX=SUMX2 - (SUMX**2)/M
      SYY=SUMY2 - (SUMY**2)/M
      RHO=BB*SQRT(SXX/SYY)
      PRINT,'LINE OF BEST FIT:  Y=',AA,'+',BB,'X'
      WRITE (6,500) RHO
500   FORMAT (1X,'CORRELATION COEFFICIENT=',F10.4)
      RETURN
      END
```

PHYSICAL AND BIOLOGICAL SYSTEM PARAMETERS

Q1	RQ	YY	KS	CRIT	M	DELTA	DS	XS
0192.00	8640.00	0000.33	0022.00	0000.01	96	0.0041667	0.0002297	95000.0

A1	V1	AV	D	KD	V2
0016.0000	0128.0000	0027.0000	0008.0000	0000.0500	1875.0000

X(1)	Y(1)	X(2)	Y(2)	X(3)	Y(3)
0000.0100	0000.4270	0000.0200	0000.4400	0000.0500	0000.5500

Appendix C

NSF DATA

CASE 1 =	30	JUNE 74	HIGH VSS	SUBDIVISION
CASE 2 =	25	AUG 74	HIGH VSS	SCHOOL
CASE 3 =	17	JUN 73	LOW VSS	SUBDIVISION
CASE 4 =	29	JUL 73	LOW VSS	SCHOOL

J	C	T(J)	SO(J)	SE(J)	XA(J)	Q(J)	LO(J)	LE(J)
1	1	20.0	388.0	24.0	1595.0	4100.0	231.0	9.0
2	1	20.0	504.0	54.0	1565.0	4100.0	163.0	6.0
3	1	22.0	440.0	80.0	1810.0	3600.0	145.0	5.0
4	1	25.0	412.0	40.0	1650.0	4000.0	128.0	8.0
5	1	24.0	400.0	48.0	2515.0	4100.0	165.0	5.0
6	1	20.0	348.0	30.0	1240.0	4100.0	120.0	6.0
7	1	20.0	340.0	36.0	1230.0	4100.0	104.0	6.0
8	1	20.0	380.0	76.0	1925.0	4100.0	136.0	5.0
9	1	22.0	464.0	56.0	1875.0	4000.0	217.0	7.0
10	1	22.0	386.0	72.0	1475.0	4100.0	116.0	7.0
11	1	24.0	428.0	64.0	2010.0	4100.0	134.0	9.0
12	1	19.0	416.0	44.0	1565.0	4100.0	136.0	12.0
13	1	21.0	380.0	88.0	1615.0	4100.0	135.0	11.0
14	1	25.0	390.0	80.0	1835.0	4000.0	143.0	9.0
15	1	23.0	408.0	84.0	1715.0	4100.0	195.0	7.0
16	1	23.0	496.0	70.0	1065.0	4100.0	144.0	7.0
17	1	21.0	440.0	104.0	1225.0	4100.0	153.0	11.0
18	1	23.0	404.0	68.0	1440.0	4100.0	160.0	7.0
19	1	22.0	452.0	114.0	1610.0	4100.0	145.0	20.0
20	1	19.0	456.0	52.0	1705.0	4100.0	208.0	8.0
21	1	22.0	452.0	64.0	1760.0	4100.0	188.0	8.0
22	1	22.0	464.0	36.0	1545.0	4100.0	171.0	6.0
23	1	22.0	384.0	64.0	1540.0	4100.0	153.0	13.0
24	1	22.0	412.0	44.0	1360.0	4100.0	175.0	10.0
25	1	23.0	360.0	68.0	900.0	4100.0	138.0	10.0
26	1	23.0	320.0	34.0	1160.0	4100.0	137.0	9.0
27	1	21.0	384.0	64.0	1410.0	4100.0	187.0	9.0
28	1	23.0	364.0	36.0	1465.0	4100.0	183.0	5.0

J	C	T(J)	SO(J)	SE(J)	XA(J)	C(J)	LO(J)	LE(J)
29	2	24.0	402.0	36.0	1645.0	3200.0	186.0	8.0
30	2	22.0	432.0	30.0	1595.0	3100.0	192.0	7.0
31	2	21.0	468.0	44.0	1725.0	3100.0	240.0	9.0
32	2	19.0	536.0	36.0	1735.0	3300.0	216.0	5.0
33	2	16.0	412.0	16.0	2035.0	3100.0	189.0	5.0
34	2	18.0	452.0	36.0	1735.0	3100.0	192.0	5.0
35	2	19.0	780.0	48.0	1510.0	3200.0	309.0	5.0
36	2	20.0	868.0	42.0	1700.0	3200.0	360.0	5.0
37	2	22.0	704.0	24.0	1755.0	3100.0	304.0	5.0
38	2	23.0	772.0	56.0	1790.0	3100.0	300.0	8.0
39	2	24.0	568.0	36.0	1860.0	3100.0	224.0	8.0
40	2	23.0	436.0	36.0	1840.0	3100.0	198.0	7.0
41	2	22.0	624.0	28.0	1960.0	3100.0	247.0	8.0
42	2	20.0	536.0	28.0	1905.0	3000.0	228.0	5.0
43	2	22.0	404.0	32.0	1920.0	3300.0	197.0	6.0
44	2	14.0	876.0	20.0	1880.0	3300.0	418.0	5.0
45	2	16.0	906.0	52.0	2250.0	3100.0	322.0	10.0
46	2	16.0	1784.0	112.0	1000.0	2800.0	702.0	16.0
47	2	19.0	1072.0	56.0	1040.0	2800.0	374.0	11.0
48	2	19.0	940.0	48.0	1205.0	2800.0	338.0	12.0

J	C	T(J)	SO(J)	SE(J)	XA(J)	C(J)	LO(J)	LE(J)
49	3	21.0	392.0	64.0	30.0	2700.0	168.0	18.0
50	3	21.0	348.0	52.0	60.0	2200.0	176.0	21.0
51	3	21.0	340.0	50.0	54.0	2600.0	220.0	21.0
52	3	21.0	336.0	68.0	56.0	2600.0	220.0	32.0
53	3	20.0	412.0	76.0	28.0	2600.0	173.0	21.0
54	3	21.0	408.0	68.0	64.0	2900.0	184.0	24.0
55	3	20.0	272.0	80.0	60.0	2900.0	112.0	15.0
56	3	18.0	260.0	84.0	60.0	2900.0	132.0	21.0
57	3	20.0	272.0	52.0	24.0	2900.0	204.0	28.0
58	3	21.0	336.0	68.0	60.0	2900.0	204.0	28.0
59	3	24.0	228.0	68.0	40.0	2900.0	192.0	19.0
60	3	22.0	400.0	60.0	20.0	2900.0	126.0	17.0
61	3	20.0	352.0	52.0	80.0	2800.0	148.0	22.0
62	3	20.0	346.0	68.0	12.0	2900.0	160.0	22.0
63	3	23.0	284.0	64.0	20.0	2800.0	152.0	15.0
64	3	23.0	260.0	48.0	24.0	2800.0	124.0	15.0
65	3	23.0	368.0	64.0	36.0	2800.0	128.0	14.0
66	3	22.0	300.0	60.0	8.0	2800.0	164.0	19.0
67	3	21.0	306.0	60.0	32.0	2800.0	225.0	17.0
68	3	18.0	304.0	44.0	48.0	2800.0	146.0	20.0
69	3	23.0	304.0	84.0	52.0	2800.0	182.0	19.0
70	3	21.0	360.0	68.0	12.0	2800.0	184.0	18.0
71	3	21.0	328.0	56.0	32.0	2800.0	157.0	14.0
72	3	19.0	316.0	74.0	56.0	2600.0	148.0	18.0
73	3	19.0	292.0	92.0	44.0	1500.0	128.0	22.0
74	3	19.0	340.0	78.0	52.0	1900.0	155.0	31.0
75	3	21.0	376.0	60.0	32.0	1700.0	168.0	27.0
76	3	21.0	352.0	68.0	68.0	1000.0	172.0	22.0

J	C	T(J)	SO(J)	SE(J)	XA(J)	Q(J)	LO(J)	LE(J)
77	4	22.0	300.0	92.0	30.0	2800.0	108.0	30.0
78	4	21.0	256.0	76.0	32.0	2900.0	140.0	25.0
79	4	22.0	200.0	64.0	56.0	2900.0	140.0	23.0
80	4	21.0	204.0	50.0	44.0	2900.0	100.0	17.0
81	4	21.0	432.0	54.0	24.0	3000.0	154.0	24.0
82	4	22.0	280.0	72.0	16.0	2800.0	128.0	21.0
83	4	22.0	264.0	44.0	14.0	2900.0	151.0	19.0
84	4	24.0	292.0	88.0	24.0	2900.0	143.0	19.0
85	4	25.0	264.0	40.0	6.0	2900.0	110.0	18.0
86	4	22.0	160.0	36.0	20.0	2700.0	148.0	20.0
87	4	21.0	344.0	76.0	24.0	2900.0	190.0	31.0
88	4	21.0	256.0	52.0	30.0	2800.0	129.0	17.0
89	4	21.0	328.0	48.0	16.0	2900.0	96.0	11.0
90	4	21.0	276.0	60.0	4.0	2800.0	164.0	15.0
91	4	22.0	264.0	48.0	22.0	2900.0	166.0	24.0
92	4	22.0	280.0	56.0	22.0	2900.0	183.0	17.0
93	4	20.0	244.0	44.0	20.0	2800.0	128.0	15.0
94	4	19.0	286.0	52.0	16.0	2900.0	132.0	14.0
95	4	19.0	280.0	48.0	19.0	2800.0	131.0	15.0
96	4	20.0	208.0	68.0	36.0	2800.0	140.0	31.0

Appendix D

Table IV

PREDICTED PERFORMANCE - COD BASIS

J	C	NN(J)	R(J)	XT(J)	YT(J)	EA(J)	E3(J)	E2(J)	E1(J)
1	1	5.6195	2.1073	0.0213	0.4552	93.8144	90.9079	45.2715	49.4659
2	1	5.6195	2.1073	0.0213	0.4552	89.2857	87.5905	32.9425	49.3142
3	1	6.4000	2.4000	0.0228	0.4990	81.8182	88.8909	32.0025	52.9586
4	1	5.7600	2.1600	0.0253	0.5728	90.2913	91.1980	44.4762	55.8310
5	1	5.6195	2.1073	0.0245	0.5471	88.0000	92.4797	50.7388	58.8799
6	1	5.6195	2.1073	0.0213	0.4552	91.3793	88.4283	36.2605	47.6416
7	1	5.6195	2.1073	0.0213	0.4552	89.4118	87.6381	33.5956	47.5893
8	1	5.6195	2.1073	0.0213	0.4552	80.0000	87.4393	31.9084	51.1047
9	1	5.7600	2.1600	0.0228	0.4990	87.9310	89.6827	38.6657	53.2847
10	1	5.6195	2.1073	0.0228	0.4990	81.3472	86.5964	29.0961	51.2416
11	1	5.6195	2.1073	0.0245	0.5471	85.0467	90.0182	40.1207	56.4292
12	1	5.6195	2.1073	0.0206	0.4348	89.4231	88.1236	35.0774	48.1360
13	1	5.6195	2.1073	0.0221	0.4766	76.8421	85.8212	26.8321	50.7607
14	1	5.7600	2.1600	0.0253	0.5728	79.4872	89.0125	35.1278	56.7899
15	1	5.6195	2.1073	0.0236	0.5225	79.4118	87.3155	30.7000	53.6994
16	1	5.6195	2.1073	0.0236	0.5225	85.8871	84.8243	24.0677	50.2431
17	1	5.6195	2.1073	0.0221	0.4766	76.3636	82.9289	19.5391	48.7346
18	1	5.6195	2.1073	0.0236	0.5225	83.1683	87.1140	30.4635	52.2674
19	1	5.6195	2.1073	0.0228	0.4990	74.7788	84.8956	23.6402	51.9409
20	1	5.6195	2.1073	0.0206	0.4348	88.5965	87.9889	34.4259	48.8322
21	1	5.6195	2.1073	0.0228	0.4990	85.8407	88.4158	34.8619	52.7062
22	1	5.6195	2.1073	0.0228	0.4990	92.2414	89.9793	41.0596	51.6055
23	1	5.6195	2.1073	0.0228	0.4990	83.3333	87.4770	31.8940	51.5796
24	1	5.6195	2.1073	0.0228	0.4990	89.3204	90.4048	42.4296	53.2096
25	1	5.6195	2.1073	0.0236	0.5225	81.1111	83.7697	21.4953	49.3254
26	1	5.6195	2.1073	0.0236	0.5225	89.3750	88.6394	36.1909	50.7639
27	1	5.6195	2.1073	0.0221	0.4766	83.3333	86.4260	29.0538	49.7059
28	1	5.6195	2.1073	0.0236	0.5225	90.1099	89.9801	40.8845	52.3994

J	C	NN(J)	R(J)	XT(J)	YT(J)	EA(J)	E3(J)	E2(J)	E1(J)
29	2	7.2000	2.7000	0.0245	0.5471	91.0448	92.8674	45.4761	54.5674
30	2	7.4323	2.7871	0.0228	0.4990	93.0556	92.9144	45.2121	51.8637
31	2	7.4323	2.7871	0.0221	0.4766	90.5983	91.9021	40.1770	51.3175
32	2	6.9818	2.6182	0.0206	0.4348	93.2836	91.5326	41.0792	48.9801
33	2	7.4323	2.7871	0.0186	0.3788	96.1165	94.0324	52.2385	46.9048
34	2	7.4323	2.7871	0.0199	0.4153	92.0354	91.8709	40.7943	48.0485
35	2	7.2000	2.7000	0.0206	0.4348	93.8461	89.9414	33.5196	47.8600
36	2	7.2000	2.7000	0.0213	0.4552	95.1613	91.4420	39.3954	49.9932
37	2	7.4323	2.7871	0.0228	0.4990	96.5909	93.9140	50.6523	52.6809
38	2	7.4323	2.7871	0.0236	0.5225	92.7461	91.8514	39.2623	54.0824
39	2	7.4323	2.7871	0.0245	0.5471	93.6620	93.6503	48.6068	55.6735
40	2	7.4323	2.7871	0.0236	0.5225	91.7431	93.3617	47.1908	54.3360
41	2	7.4323	2.7871	0.0228	0.4990	95.5128	94.0561	51.3293	53.7077
42	2	7.6800	2.8800	0.0213	0.4552	94.7761	93.6540	48.3942	51.0069
43	2	6.9818	2.6182	0.0228	0.4990	92.0792	93.2513	48.7470	53.5091
44	2	6.9818	2.6182	0.0173	0.3456	97.7169	92.2215	45.3312	43.9202
45	2	7.4323	2.7871	0.0186	0.3788	94.2605	91.3242	38.3092	47.8893
46	2	8.2286	3.0857	0.0186	0.3788	93.7220	84.2293	13.3254	41.8986
47	2	8.2286	3.0857	0.0206	0.4348	94.7761	88.4008	23.9696	45.4402
48	2	8.2286	3.0857	0.0206	0.4348	94.8936	89.8190	28.9282	46.3022

J	C	NN(J)	R(J)	XT(J)	YT(J)	EA(J)	E3(J)	E2(J)	E1(J)
49	3	8.5333	3.2000	0.0221	0.4766	83.6735	78.7180	0.8986	41.9924
50	3	10.4727	3.9273	0.0221	0.4766	85.0575	82.5964	2.0939	42.1720
51	3	8.8615	3.3231	0.0221	0.4766	85.2941	79.9667	1.9177	42.1362
52	3	8.8615	3.3231	0.0221	0.4766	79.7619	79.7932	1.5962	42.1481
53	3	8.8615	3.3231	0.0213	0.4552	81.5534	78.8063	0.7064	40.9024
54	3	7.9448	2.9793	0.0221	0.4766	83.3333	78.1255	1.8047	42.1960
55	3	7.9448	2.9793	0.0213	0.4552	70.5882	77.4660	1.4312	41.0910
56	3	7.9448	2.9793	0.0199	0.4153	67.6923	76.4650	1.2585	38.9815
57	3	7.9448	2.9793	0.0213	0.4552	80.8824	76.9755	0.7942	40.8788
58	3	7.9448	2.9793	0.0221	0.4766	79.7619	78.0506	1.6937	42.1720
59	3	7.9448	2.9793	0.0245	0.5471	70.1754	79.0063	1.3011	45.3898
60	3	7.9448	2.9793	0.0228	0.4990	85.0000	77.7634	0.6556	43.0266
61	3	8.2286	3.0857	0.0213	0.4552	85.2273	78.8156	2.6065	41.2086
62	3	7.9448	2.9793	0.0213	0.4552	80.3468	76.6429	0.3280	40.8079
63	3	8.2286	3.0857	0.0236	0.5225	77.4648	78.7832	0.6564	44.1375
64	3	8.2286	3.0857	0.0236	0.5225	81.5385	78.9645	0.9647	44.1622
65	3	8.2286	3.0857	0.0236	0.5225	82.6087	79.1078	1.1753	44.2364
66	3	8.2286	3.0857	0.0228	0.4990	80.0000	78.1040	0.2640	42.9534
67	3	8.2286	3.0857	0.0221	0.4766	80.3922	78.1703	1.0013	42.0044
68	3	8.2286	3.0857	0.0199	0.4153	85.5263	77.2946	1.6159	38.9131
69	3	8.2286	3.0857	0.0236	0.5225	72.3684	79.2522	1.3746	44.3351
70	3	8.2286	3.0857	0.0221	0.4766	81.1111	77.7235	0.3443	41.8843
71	3	8.2286	3.0857	0.0221	0.4766	82.9268	78.2007	1.0522	42.0044
72	3	8.8615	3.3231	0.0206	0.4348	76.5823	78.8490	1.3683	40.0042
73	3	15.3600	5.7600	0.0206	0.4348	68.4931	86.3829	0.9394	39.9347
74	3	12.1263	4.5474	0.0206	0.4348	77.0588	83.5292	1.2467	39.9810
75	3	13.5529	5.0824	0.0221	0.4766	84.0425	85.5030	1.0339	42.0044
76	3	23.0400	8.6400	0.0221	0.4766	80.6818	91.2301	2.0244	42.2198

J	C	NN(J)	R(J)	XT(J)	YT(J)	EA(J)	E3(J)	E2(J)	E1(J)
77	4	8.2286	3.0857	0.0228	0.4990	69.3333	78.4177	0.7091	43.0876
78	4	7.9448	2.9793	0.0221	0.4766	70.3125	77.4663	0.8368	42.0044
79	4	7.9448	2.9793	0.0228	0.4990	68.0000	78.4756	1.7314	43.2457
80	4	7.9448	2.9793	0.0221	0.4766	75.4902	77.9309	1.5548	42.0763
81	4	7.6800	2.8800	0.0221	0.4766	87.5000	76.8321	0.8072	41.9564
82	4	8.2286	3.0857	0.0228	0.4990	74.2857	78.2388	0.4597	43.0023
83	4	7.9448	2.9793	0.0228	0.4990	83.3333	77.6981	0.5706	42.9900
84	4	7.9448	2.9793	0.0245	0.5471	69.8630	78.6112	0.6430	45.2895
85	4	7.9448	2.9793	0.0253	0.5728	84.8485	78.7942	0.2996	46.3178
86	4	8.5333	3.2000	0.0228	0.4990	77.5000	79.1280	0.9297	43.0266
87	4	7.9448	2.9793	0.0221	0.4766	77.9070	77.3182	0.6289	41.9564
88	4	8.2286	3.0857	0.0221	0.4766	79.6875	78.1836	1.0398	41.9924
89	4	7.9448	2.9793	0.0221	0.4766	85.3658	77.2726	0.5872	41.9084
90	4	8.2286	3.0857	0.0221	0.4766	78.2609	77.5688	0.1262	41.8362
91	4	7.9448	2.9793	0.0228	0.4990	81.8182	77.8798	0.8432	43.0388
92	4	7.9448	2.9793	0.0228	0.4990	80.0000	77.8288	0.7574	43.0388
93	4	8.2286	3.0857	0.0213	0.4552	81.9672	77.5491	0.7446	40.8552
94	4	7.9448	2.9793	0.0206	0.4348	81.8182	76.3067	0.5072	39.7721
95	4	8.2286	3.0857	0.0206	0.4348	82.8571	77.0281	0.6376	39.7896
96	4	8.2286	3.0857	0.0213	0.4552	67.3077	77.7357	0.9805	40.9496

Appendix E

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C      OPTIMIZATION MAIN PROGRAM                                                0000200
       EXTERNAL F                                                               0000300
       DIMENSION X(30,15),Z(30),SHRINK(21),UPLO(15),CENT(15),SWITX(15)          0000400
       DIMENSION  BOUND(15),XS(15),Z1(30)                                       0000500
       COMMON X,BOUND,N                                                         0000600
       COMMON/BIOL/ K1,K2,KS,XSOL,DS,DELTA                                      0000700
       COMMON/PARAM/ Q,SO,F,D1,A1,AV,Q1,V2,WIDTH,A2,L2,L3,BIGP,R1,R2,XA         0000800
       COMMON/DENOM/ SE,Q2,F1,F2,F3,E1,E2,E3                                    0000900
       COMMON/COSTS/SLUDGE,TANK,TOWER,DSTRIB,INJECT,TDH,HPSI,QGPM,R1PUMP,       0001000
      2             R2PUMP,SLANT,CLARIF,FCI,OXYGEN,O2,QCFS,GAMMA,MOTOR1,       0001100
      3             O1,QCFS2,MOTOR2,O3,MAINT,OP,LIFE,SALVAG,RATE,FACTOR,       0001200
      4             AFC(30),D2,LOAD,ETA,AYC,THETA                              0001300
       REAL DIV,COUNT,EVAL,KD                                                   0001400
       REAL K1,K2,KS,L2,L3,INJECT,LOAD,MOTOR1,MOTOR2,MAINT                      0001500
       DATA SHRINK / .978,.954,.929,.903,.875,.845,.813,.778,.740,              0001600
      1.699,.653,.602,.544,.477,.398,.301,.176,.097,.051,.027,0.0/              0001700
    1  FORMAT (1X,I2,4X,I2,F12.4,I8,F12.6)                                      0001800
    2  FORMAT (5X,'K1',8X,'K2',8X,'KS',8X,'XSOL',6X,'DS',8X,'DELTA')            0001900
    3  FORMAT (1X,3F10.4,F10.1,2F10.7)                                          0002000
    4  FORMAT (1X,I2,3F12.4)                                                    0002100
    5  FORMAT (1X,3F10.2)                                                       0002200
    6  FORMAT (5X,'A1',8X,'AV',8X,'Q1',8X,'WIDTH',5X,'A2',8X,'L2',8X,           0002300
      9         'L3',8X,'BIGP')                                                0002400
    7  FORMAT (1X,8F10.4)                                                       0002500
C RANDOM NUMBER GENERATOR STARTING VALUE                                        0002600
       IX=567653821                                                             0002700
C  READ NUMBER OF VARIABLES,N,AND NUMBER OF POINTS IN THE COMPLEX,M.            0002800
       READ,N,M                                                                 0002900
C   READ IN THE EXPANSION FACTOR                                                0003000
        READ,A                                                                  0003100
C       READ IN DESIRED NUMBER OF ITERATIONS                                    0003200
         READ,ITER                                                              0003300
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C          READ IN CONVERGENCE CRITERIA                                    0003400
         READ,CONV                                                         0003500
C                    READ DIFFERENCE BETWEEN UPPER AND LOWER BOUND AND     0003600
C    THE LOWER LIMIT FOR THE VARIABLES                                     0003700
       DO 20 I=1,N                                                         0003800
20      READ,  UPLO(I),BOUND(I)                                            0003900
C      INPUT BIOLOGICAL SYSTEM PARAMETERS                                  0004000
       READ, K1,K2,KS,XSOL,DS,DELTA                                        0004100
       PRINT,' N      M       A          ITER       CONV'                 0004200
       WRITE (6,1) N,M,A,ITER,CONV                                         0004300
       WRITE (6,2)                                                         0004400
       WRITE (6,3) K1,K2,KS,XSOL,DS,DELTA                                  0004500
C      INPUT FLOW, STRENGTH, AND DESIRED TREATMENT EFFICIENCY              0004600
       READ, Q,SO,E                                                        0004700
C      SET SPECIFIC SURFACE AREA                                           0004800
       AV = 27.0                                                           0004900
C      CALCULATE INITIAL FEASIBLE POINT AND SET CURRENT CENTROID           0005000
       CALL SIZE                                                           0005100
       PRINT,' I      UPLO(I)       BOUND(I)    X(1,I)'                  0005200
       DO 25 I=1,N                                                         0005300
       WRITE (6,4) I,UPLO(I),BOUND(I),X(1,I)                               0005400
25      CENT(I)=X(1,I)                                                     0005500
       PRINT,'      Q          SC          E'                             0005600
       WRITE (6,5) Q,SO,E                                                  0005700
       WRITE (6,6)                                                         0005800
       WRITE (6,7) A1,AV,Q1,WIDTH,A2,L2,L3,BIGP                            0005900
       DO 1500   NTRY = 1,10                                               0006000
       PRINT,'FOR NTRY =',NTRY                                             0006100
C      SET THE FUNCTION EVALUATION AND STACK COUNTERS TO ZERO              0006200
          NEVAL=0; NSTACK=0                                                0006300
       CALL SIZE                                                           0006400
C    CALCULATE THE ADDITIONAL M-1 POINTS OF THE COMPLEX                    0006500
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C      CHECK EACH POINT IN TURN FOR CONSTRAINT VIOLATIONS                0006600
C     SUBROUTINE RANDU IS USED TO GENERATE CO-ORDINATES                  0006700
        DO 100 MM=2,M                                                    0006800
        DO 30 NN=1,N                                                     0006900
         CALL RANDU(IX,IY,YFL)                                           0007000
        IX=IY                                                            0007100
30      X(MM,NN)=BOUND(NN)+YFL*UPLO(NN)                                  0007200
C      CHECK FOR CONSTRAINT SATISFACTION                                 0007300
       DO 50 J=1,21                                                      0007400
        CALL CONST1(MM,IFLAG1)                                           0007500
C     IF FLAG IS ZERO, CONSTRAINT IS SATISFIED                           0007600
C     IF FLAG IS ONE,THE POINT WILL BE LOGARITHMICALLY WITHDRAWN TO      0007700
C  THE CURRENT CENTROID                                                  0007800
       IF (IFLAG1.EQ.0) GO TO 55                                         0007900
         DO 40 NN=1,N                                                    0008000
40      X(MM,NN)=(1.-SHRINK(J))*CENT(NN)+SHRINK(J)*X(MM,NN)              0008100
 50       CONTINUE                                                       0008200
         IF(IFLAG.EQ.1)  GO TO 1050                                      0008300
C      UPDATE THE CENTROID CO-ORDINATES                                  0008400
  55   CONTINUE                                                          0008500
  60   DIV=MM                                                            0008600
         DO 80  I = 1,N                                                  0008700
         C=0.0                                                           0008800
        DO 70 J=1,MM                                                     0008900
70       C=C+X(J,I)/DIV                                                  0009000
80        CENT(I)=C                                                      0009100
100       CONTINUE                                                       0009200
C     EVALUATE THE FUNCTION AT THE COMPLEX POINTS                        0009300
        DO 150 I=1,M                                                     0009400
          Z(I)=F(I)                                                      0009500
         Z1(I)=-Z(I)                                                     0009600
150       NEVAL=NEVAL+1                                                  0009700
```

```
      PRINT,' '                                                          0009800
      PRINT,'INITIAL VERTEX'                                             0009900
      WRITE (6,1460)                                                     0010000
      DO 175 MM=1,M                                                      0010100
      WRITE (6,1470) MM,(X(MM,J),J=1,N),AEC(MM)                          0010200
 175  CONTINUE                                                           0010300
C     SET CONVERGENCE TEST COUNTER                                       0010400
        NCONV=0                                                          0010500
C     START MAIN ITERATION ROUTINE                                       0010600
      DO 1000 NCOUNT=1,ITER                                              0010700
C   ARRANGE THE VALUES IN DESCENDING ORDER,BUBBLE SORT                   0010800
       LL=M-1                                                            0010900
       DO 400 K=1,LL                                                     0011000
       DO 300 I=1,LL                                                     0011100
       IF(Z(I).GE.Z(I+1)) GO TO 300                                      0011200
       SWITZ=Z(I);Z(I)=Z(I+1);Z(I+1)=SWITZ                               0011300
       DO 200 J=1,N                                                      0011400
      SWITX(J)=X(I,J)                                                    0011500
       X(I,J)=X(I+1,J)                                                   0011600
200      X(I+1,J)=SWITX(J)                                               0011700
300      CONTINUE                                                        0011800
400      CONTINUE                                                        0011900
C   CALCULATE NEW CENTROID                                               0012000
       DIV=M-1                                                           0012100
       JJ=M-1                                                            0012200
C     THIS LOOP COMPUTES THE UNWEIGHTED CENTROID                         0012300
       DO 500 I=1,N                                                      0012400
       C=0.0                                                             0012500
       DO 450 J=1,JJ                                                     0012600
450      C=C+X(J,I)/DIV                                                  0012700
500       CENT(I)=C                                                      0012800
C    FIND VALUE OF SECOND WORST POINT                                    0012900
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```
      BAD=Z(M-1)                                                        0013000
C PROJECT WORST POINT THROUGH THE CENTROID USING EXPANSION FACTOR       0013100
      DO 600 J=1,N                                                      0013200
      XS(J)=X(M,J)                                                      0013300
600     X(M,J)=(1.+A)*CENT(J)-A*X(M,J)                                  0013400
C     CHECK FOR CONSTRAINT VIOLATIONS                                   0013500
      DO 750 J=1,21                                                     0013600
      CALL CONST1(M,IFLAG1)                                             0013700
      IF(IFLAG1.EQ.0) GO TO 800                                         0013800
      DO 700 I=1,N                                                      0013900
700   X(M,I)=(1.-SHRINK(J))*CENT(I)+SHRINK(J)*X(M,I)                    0014000
750     CONTINUE                                                        0014100
      IF (IFLAG1.EQ.1) GO TO 905                                        0014200
C       EVALUATE FUNCTION AT NEW POINT                                  0014300
 800  CONTINUE                                                          0014400
      DO 900 J=1,21                                                     0014500
      Z(M)=F(M)                                                         0014600
      NEVAL=NEVAL+1                                                     0014700
      IF(Z(M).GE.BAD)  GO TO 915                                        0014800
C     21'ST ITERATION RETURNS TO ORIGINAL WORST POINT, NO NEED TO DO    0014900
C     THE SHRINK LOOP - NO RE-EVALUATION WILL BE DONE                   0015000
      IF (J.EQ.21) GO TO 900                                            0015100
C     MOVE THE POINT BACK ALONG THE PROJECTION GRADIENT                 0015200
      DO 825 I=1,N                                                      0015300
825     X(M,I)=(1.-SHRINK(J))*XS(I)  +SHRINK(J)*X(M,I)                  0015400
900     CONTINUE                                                        0015500
 905  CONTINUE                                                          0015600
C     NO IMPROVEMENT OVER PROJECTION - PUT WORST POINT AT TOP OF STACK  0015700
      Z(M)=Z(1)                                                         0015800
      DO 910 LK=1,N                                                     0015900
910     X(M,LK)= X( 1 ,LK)                                              0016000
C     INCREMENT THE STACKING COUNTER                                    0016100
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          NSTACK=NSTACK+1                                                0016200
915       CONTINUE                                                       0016300
            NCONV=NCONV+1                                                0016400
C       CHECK FOR CONVERGENCE EVERY TENTH ITERATION                      0016500
          IF (NCONV.LT.10) GO TO 1000                                    0016600
C       FIND SUM OF DISTANCES OF VERTICES FROM THE CENTROID              0016700
920      D=0.0;BIGD=0.0                                                  0016800
         DO 950 J=1,M                                                    0016900
         DO 925   I=1,N                                                  0017000
925       D=D+(X(J,I)-CENT(I))**2                                        0017100
         BIGD=SQRT(D)+BIGD                                               0017200
950        D=0.0                                                         0017300
C       CHECK FOR CONVERGENCE                                            0017400
          IF(BIGD.LE.CONV) GO TO 1100                                    0017500
1000      CONTINUE                                                       0017600
            GO TO 1100                                                   0017700
1050        PRINT,      'BOMBED'                                         0017800
1100    CONTINUE                                                         0017900
1300       CONTINUE                                                      0018000
        NCOUNT=NCOUNT-1                                                  0018100
        IF (NCOUNT.EQ.ITER) GO TO 1400                                   0018200
        GO TO 1450                                                       0018300
1400    PRINT,'  MAX. ITERATIONS REQUIRED, MAY NOT BE STATIONARY POINT'  0018400
1450    CONTINUE                                                         0018500
        PRINT,'NUMBER OF ITERATIONS (I.E. PROJECTIONS)',NCOUNT           0018600
        PRINT,' NUMBER OF FUNCTION EVALUATIONS',NEVAL                    0018700
        COUNT=NCOUNT; EVAL=NEVAL                                         0018800
        AVG=EVAL/COUNT                                                   0018900
        PRINT,' AVERAGE NUMBER OF EVALUATIONS PER ITERATION',AVG         0019000
        PRINT,'  NUMBER OF WORST POINT STACKS=',NSTACK                   0019100
        PRINT,' '                                                        0019200
        PRINT,'OPTIMUM VERTEX'                                           0019300
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      WRITE (6,1460)                                                    0019400
1460  FORMAT (2X,'M',5X,'D1',8X,'R1',8X,'R2',8X,'V2',8X,'XA',16X,'AEC') 0019500
      DO 1475 MM=1,M                                                    0019600
      Z1(MM)= -Z(MM)                                                    0019700
      WRITE (6,1470) MM,(X(MM,J),J=1,N),Z1(MM)                          0019800
1470  FORMAT (1X,I1,3F10.4,2F10.1,8X,F10.2)                             0019900
1475  CONTINUE                                                          0020000
      PRINT,' '                                                         0020100
      WRITE (6,1480)                                                    0020200
      WRITE (6,1481) SE,Q2,F1,F2,F3,E1,E2,E3                            0020300
1480  FORMAT (5X,'SE',8X,'Q2',8X,'F1',8X,'F2',8X,'F3',8X,'E1',8X,'E2',  0020400
     5         8X,'E3')                                                 0020500
1481  FORMAT (1X,F8.2,F12.4,6F10.4)                                     0020600
      PRINT,' '                                                         0020700
      PRINT,'     FIXED CAPITAL INVESTMENT  IN DOLLARS'                 0020800
      PRINT,'     SLUDGE     TANK       TOWER      DSTRIB     INJECT    R1PUM0020900
     6P    R2PUMP     CLARIF'                                          0021000
      WRITE(6,1485) SLUDGE,TANK,TOWER,DSTRIB,INJECT,R1PUMP,R2PUMP,CLARIF0021100
1485  FORMAT (1X,8F10.4)                                                0021200
      PRINT,' '                                                         0021300
      PRINT,'     L2         D2         SLANT      TDH        HPSI       QGPM 0021400
     7      QCFS       QCFS2'                                          0021500
      WRITE (6,1485) L2,D2,SLANT,TDH,HPSI,QGPM,QCFS,QCFS2               0021600
      PRINT,' '                                                         0021700
      PRINT,'     LOAD       OXYGEN     MOTOR1     MOTOR2     ETA        A1   0021800
     8      THETA'                                                     0021900
      WRITE (6,1486) LOAD,OXYGEN,MOTOR1,MOTOR2,ETA,A1,THETA             0022000
1486  FORMAT (1X,7F10.4)                                                0022100
      PRINT,' '                                                         0022200
      PRINT,'     OPERATING COSTS IN DOLLARS PER YEAR'                  0022300
      PRINT,'     O1         O2         O3         MAINT      CP         AYC  0022400
     9      AEC'                                                       0022500
```

```
      WRITE (6,1486) O1,O2,O3,MAINT,OP,AYC, AEC(M)                          0022600
      PRINT,' '                                                             0022700
      PRINT,'    LIFE       SALVAG     RATE       FACTOR     FCI'          0022800
      WRITE (6,1490) LIFE,SALVAG,RATE,FACTOR,FCI                            0022900
1490  FORMAT (3X,I2,6X,4F10.4)                                              0023000
C     SLUDGE PRODUCTION CALCULATION                                         0023100
      Y=0.33; KD=0.05                                                       0023200
      XALB1= ((XSOL*DS) + (XA*DELTA))*AV*D1*A1*7.5*8.34E-06                 0023300
      XALB2= XA*V2*8.34E-06                                                 0023400
      SR1= SE*(E1/100)*1*A1*60*24*1.02E-06                                  0023500
      SR2= SO*(E2/100)*8.34*1.02E-06*Q                                      0023600
      CELLS1= (Y*SR1) - (KD*XALB1)                                          0023700
      CELLS2= (Y*SR2) - (KD*XALB2)                                          0023800
      PRINT, ' '                                                            0023900
      PRINT,'    SLUDGE PRODUCTION'                                         0024000
      PRINT,'    Y          KD         XALB1      XALB2      SR1        SR2  0024100
     9     CELLS1     CELLS2'                                               0024200
      WRITE (6,1485) Y,KD,XALB1,XALB2,SR1,SR2,CELLS1,CELLS2                 0024300
      WRITE (6,101)                                                         0024400
 101  FORMAT (5(1X,/))                                                     0024500
        PRINT,'  ************************************************'         0024600
        PRINT,'  ************************************************'         0024700
        PRINT,'  ************************************************'         0024800
        PRINT,'  ************************************************'         0024900
1500    CONTINUE                                                            0025000
      STOP                                                                  0025100
      END                                                                   0025200
C                                                                           0025300
C                                                                           0025400
C                                                                           0025500
C                                                                           0025550
      SUBROUTINE SIZE                                                       0025600
```

```
      DIMENSION X(30,15)                                                0025700
      COMMON X,BOUND(15),N                                              0025800
      COMMON/PARAM/ Q,SO,E,D1,A1,AV,Q1,V2,WIDTH,A2,L2,L3,BIGP,R1,R2,XA  0025900
      REAL L2,L3                                                        0026000
      IF (Q.LE.6000) GO TO 30                                           0026100
      IF (Q.LE.16000) GO TO 40                                          0026200
      IF (Q.GE.16001) GO TO 50                                          0026300
30    A1=16; WIDTH=6; BIGP=2.0*WIDTH**2; GO TO 60                       0026400
40    A1=36; WIDTH=8; BIGP=4.0*WIDTH**2; GO TO 60                       0026500
50    A1=48; WIDTH=10; BIGP=8*WIDTH**2                                  0026600
60    CONTINUE                                                          0026700
      V1=Q*SO*8.34E-05                                                  0026800
      D1=V1/A1                                                          0026900
      Q1=192                                                            0027000
      R1=(Q1*A1*7.5)/Q                                                  0027100
      R2=8640/Q                                                         0027200
      V2=Q*(1 + R1 + R2)*0.0521                                         0027300
      A2=WIDTH**2; D2=WIDTH                                             0027400
      L3=(Q*SO)/(20000*A2)                                              0027500
      L2=V2/(A2*7.48)                                                   0027600
      XA=2000                                                           0027700
      X(1,1)=D1                                                         0027800
      X(1,2)=R1                                                         0027900
      X(1,3)=R2                                                         0028000
      X(1,4)=V2                                                         0028100
      X(1,5)=XA                                                         0028200
      M=1                                                               0028300
      CALL CONST1 (M,IFLAG1)                                            0028400
      IF (IFLAG1.EQ.0) GO TO 99                                         0028500
      PRINT,'INITIAL SIZE OUTSIDE OF FEASIBLE REGION'                   0028600
99    RETURN                                                            0028700
      END                                                               0028800
```

```
C                                                                   0028900
C                                                                   0029000
```

```
      SUBROUTINE CONST1 (M,IFLAG1)                                          0029100
      DIMENSION X(30,15)                                                    0029200
      COMMON X,BOUND(15),N                                                  0029300
      COMMON/BIOL/ K1,K2,KS,XSOL,DS,DELTA                                   0029400
      COMMON/PARAM/ Q,SO,E,D1,A1,AV,Q1,V2,WIDTH,A2,L2,L3,BIGP,R1,R2,XA      0029500
      COMMON/DENOM/ SE,Q2,F1,F2,F3,E1,E2,E3                                 0029600
      REAL K1,K2,KS,L2,L3                                                   0029700
      IFLAG1=0                                                              0029800
      DO 10 I=1,N                                                           0029900
      IF (X(M,I).LE.BOUND(I)) GO TO 50                                      0030000
   10 CONTINUE                                                              0030100
      D1=X(M,1)                                                             0030200
      R1=X(M,2)                                                             0030300
      R2=X(M,3)                                                             0030400
      V2=X(M,4)                                                             0030500
      XA=X(M,5)                                                             0030600
      F1=EXP((-K1*AV*D1/Q1)*((XSOL*DS) + (XA*DELTA)))                       0030700
      SE=SO*(1 - E/100)                                                     0030800
      Q2=Q*(1 + R1 + R2)                                                    0030900
      F2= 1/(1 + (K2*XA*V2)/((KS + SE)*Q2))                                 0031000
      F3=F2/(1 + (R1*(1 - (F1*F2))) + R2*(1 - F2))                          0031100
      E1= (1 - F1)*100;  E2=(1 - F2)*100;  E3=(1 - F3)*100                  0031200
      IF (E3.GE.E) GO TO 99                                                 0031300
   50 IFLAG1=1                                                              0031400
   99 RETURN                                                                0031500
      END                                                                   0031600
C                                                                           0031700
C                                                                           0031750
C                                                                           0031800
```

```
      FUNCTION F(M)                                                          0031900
      DIMENSION X(30,15)                                                     0032000
      COMMON X,BOUND(15),N                                                   0032100
      COMMON/PARAM/ Q,SO,E,D1,A1,AV,Q1,V2,WIDTH,A2,L2,L3,BIGP,R1,R2,XA       0032200
      COMMON/COSTS/SLUDGE,TANK,TOWER,DSTRIB,INJECT,TDH,HPSI,QGPM,R1PUMP,     0032300
     2             R2PUMP,SLANT,CLARIF,FCI,OXYGEN,O2,QCFS,GAMMA,MOTOR1,      0032400
     3             Q1,QCFS2,MOTOR2,O3,MAINT,CF,LIFE,SALVAG,RATE,FACTOR,      0032500
     4             AEC(30),D2,LOAD,ETA,AYC,THETA                             0032600
      REAL L2,L3,INJECT,LOAD,MOTOR1,MOTOR2,MAINT                             0032700
      D1=X(M,1)                                                              0032800
      R1=X(M,2)                                                              0032900
      R2=X(M,3)                                                              0033000
      V2=X(M,4)                                                              0033100
      XA=X(M,5)                                                              0033200
C     CAPITAL COSTS                                                          0033300
      D2=WIDTH                                                               0033400
      THETA=0.0521                                                           0033500
      Q2=Q*(1 + R1 + R2)                                                     0033600
      V2=Q2*THETA                                                            0033700
      X(M,4)=V2                                                              0033800
      L2=V2/(A2*7.48)                                                        0033900
      SLUDGE=(A2 + (4*L3*D2))*4.50                                           0034000
      TANK=((2*A2) + (3*L2*D2))*4.50                                         0034100
      A1=(R1*Q)/(Q1*7.5)                                                     0034200
      TOWER=D1*A1*3.00                                                       0034300
      DSTRIB=(A1/16)*10.00                                                   0034400
      INJECT=(V2/1400)*25.00                                                 0034500
      TDH=D1 + 1.00 + (A1/16)*0.5 + (V2/1400)*2.00                           0034600
      HPSI = TDH*62.4/144                                                    0034700
      QGPM = (Q*R1)/(24*60)                                                  0034800
      R1PUMP=5*(1000*((HPSI*QGPM)/3000)**0.52)                               0034900
      R2PUMP=500*((Q*R2)/8640)**0.52                                         0035000
```

```
C       LATERAL SURFACE OF FRUSTUM=0.5*(SUM OF PERIMETERS OF BASES)*      0035100
C              SLANT HEIGHT                                               0035200
        SLANT=D2/1.732                                                    0035300
        CLARIF=((2*BIGP) + (SLANT*0.5)*(BIGP + 4) + 1.0)*4.50             0035400
        FCI=SLUDGE+TANK+TOWER + DSTRIB+INJECT + R1PUMP + R2PUMP + CLARIF  0035500
C       OPERATION AND MAINTENANCE COSTS                                   0035600
        LOAD=S0*(E/100)*8.34*1.02E-06                                     0035700
        OXYGEN=(LOAD*1000*6.72)/379.4                                     0035800
        O2=OXYGEN*0.745*0.035*352*24                                      0035900
        QCFS=QGPM/(60*7.48)                                               0036000
        GAMMA = 62.4; ETA = 0.70                                          0036100
        MOTOR1=(GAMMA*QCFS*TDH)/(550*ETA)                                 0036200
        C1=MOTOR1*352*24*0.745*0.035                                      0036300
        QCFS2=(Q*R2)/(1440*60*7.48)                                       0036400
        MOTOR2=(GAMMA*QCFS2*D2)/(550*ETA)                                 0036500
        C3=MOTOR2*352*24*0.745*0.035                                      0036600
        MAINT=((XA - 2000)**2)*0.0004                                     0036700
        OP= O1 + O2 + O3 + MAINT                                          0036800
C       ANNUAL EQUIVALENT COST                                            0036900
        LIFE=10; SALVAG=1000; RATE = 0.0725                               0037000
        FACTOR=RATE*((1 + RATE)**LIFE)/(((1 + RATE)**LIFE) - 1)           0037100
        AYC=(FCI - SALVAG)*FACTOR + (RATE*SALVAG)                         0037200
        AEC(M)= AYC + OP                                                  0037300
        F = -AEC(M)                                                       0037400
        RETURN                                                            0037500
        END                                                               0037600
```

```
   5      8                                                                  0037800
1                                                                           0037900
1000                                                                        0038000
1.00000                                                                     0038100
16,4                                                                        0038200
10,1                                                                        0038300
10,1.5                                                                      0038400
50000,0                                                                     0038500
10000,0                                                                     0038600
   0.0206   0.4510   22     95000     0.0002297   0.0041667                 0038700
4100   400    90.00                                                        0038800
```

Appendix F

Table V

FOR NTRY = 1

INITIAL VERTEX

M	D1	R1	R2	V2	XA	AEC
1	8.5485	5.6195	2.1073	1864.1	2000.0	531.31
2	7.4053	9.9799	6.2245	3675.0	2645.4	921.51
3	16.9983	5.9349	7.9938	3188.9	8848.5	19540.68
4	7.4440	4.2785	1.7985	1511.7	1019.5	854.29
5	12.7058	5.2592	6.3846	2700.8	345.2	1766.24
6	16.4070	6.5407	4.9547	2669.2	4079.2	2460.81
7	14.7476	4.5909	2.5896	1747.4	5514.2	5508.55
8	12.1659	1.9944	1.5330	967.1	7194.4	11205.72

NUMBER OF ITERATIONS (I.E. PROJECTIONS) = 304
NUMBER OF FUNCTION EVALUATIONS = 315
NUMBER OF WORST POINT STACKS = 2

OPTIMUM VERTEX

M	D1	R1	R2	V2	XA	AEC
1	4.6884	6.1164	1.5039	1841.4	2117.5	486.56
2	4.6884	6.1164	1.5039	1841.4	2117.5	486.56
3	4.6898	6.1162	1.5036	1841.3	2117.4	486.56
4	4.6897	6.1160	1.5035	1841.2	2117.5	486.56
5	4.6901	6.1159	1.5035	1841.2	2117.5	486.57
6	4.6916	6.1157	1.5034	1841.1	2117.5	486.57
7	4.6921	6.1156	1.5032	1841.1	2117.5	486.57
8	4.6884	6.1164	1.5039	1841.4	2117.5	486.56

SE	Q2	F1	F2	F3	E1	E2	E3
40.00	35340.43	0.6595	0.5548	0.1000	34.0547	44.5213	90.0000

FIXED CAPITAL INVESTMENT IN DOLLARS

SLUDGE	TANK	TOWER	DSTRIB	INJECT	R1PUMP	R2PUMP	CLARIF
408.00	877.84	245.01	10.88	32.88	415.15	419.50	1244.88

L2	D2	SLANT	TDH	HPSI	QGPM	QCFS	QCFS2
6.8375	6.0000	3.4642	8.8645	3.8413	17.4133	0.0388	0.0095

LOAD	OXYGEN	MOTOR1	MOTOR2	ETA	A1	THETA
0.0031	0.0542	0.0557	0.0093	0.7000	17.4133	0.0521

OPERATING COSTS IN DOLLARS PER YEAR

O1	O2	O3	MAINT	CP	AYC	AEC
12.2797	11.9486	2.0433	5.5246	31.7963	454.7700	486.5662

LIFE	SALVAG	RATE	FACTOR	FCI
10	1000.0000	0.0725	0.1440	3654.14

SLUDGE PRODUCTION IN POUNDS PER DAY

Y	KD	XALB1	XALB2	SR1	SR2	CELLS1	CELLS2
0.3300	0.0500	4.2267	32.5158	0.3484	6.2112	-0.0964	0.4239

FOR NTRY = 2

INITIAL VERTEX

M	D1	R1	R2	V2	XA	AEC
1	8.5485	5.6195	2.1073	1864.1	2000.0	531.31
2	7.0872	7.8276	5.1001	2975.1	8514.0	17631.99
3	19.5377	2.6401	3.9414	1619.5	7352.0	12003.75
4	12.1994	5.5796	2.8556	2015.5	9304.6	21933.72
5	9.7114	8.6763	5.4312	3227.1	1623.8	790.38
6	18.7779	1.8034	3.1945	1281.2	2368.6	533.62
7	16.3701	6.3619	3.7539	2374.4	197.2	1993.39
8	11.9337	9.9885	10.8042	4655.1	5831.5	6808.87

NUMBER OF ITERATIONS (I.E. PROJECTIONS) = 532
NUMBER OF FUNCTION EVALUATIONS = 545
NUMBER OF WORST POINT STACKS = 18

OPTIMUM VERTEX

M	D1	R1	R2	V2	XA	AEC
1	4.0805	6.4570	1.5000	1913.3	2076.0	483.75
2	4.0805	6.4570	1.5000	1913.3	2076.0	483.75
3	4.0805	6.4570	1.5000	1913.3	2076.0	483.75
4	4.0805	6.4570	1.5000	1913.3	2076.0	483.75
5	4.0805	6.4570	1.5000	1913.3	2076.0	483.75
6	4.0805	6.4570	1.5000	1913.3	2076.0	483.75
7	4.0801	6.4576	1.5000	1913.4	2075.8	483.75
8	4.0805	6.4570	1.5000	1913.3	2076.0	483.75

SE	Q2	F1	F2	F3	E1	E2	E3
40.00	36715.03	0.6974	0.5596	0.1000	30.2560	44.0384	90.0000

FIXED CAPITAL INVESTMENT IN DOLLARS

SLUDGE	TANK	TOWER	DSTRIB	INJECT	R1PUMP	R2PUMP	CLARIF
408.00	399.52	225.05	11.49	34.17	414.94	418.98	1244.88

L2	D2	SLANT	TDH	HPSI	QGPM	QCFS	QCFS2
7.1052	6.0000	3.4642	8.3883	3.6349	18.3844	0.0410	0.0095

LOAD	OXYGEN	MOTOR1	MOTOR2	ETA	A1	THETA
0.0031	0.0542	0.0557	0.0093	0.7000	18.3844	0.0521

OPERATING COSTS IN DOLLARS PER YEAR

O1	O2	O3	MAINT	OP	AYC	AEC
12.2680	11.9486	2.0385	2.3089	28.5640	455.1835	483.7524

LIFE	SALVAG	RATE	FACTOR	FCI
10	1000.0000	0.0725	0.1440	3657.04

SLUDGE PRODUCTION IN POUNDS PER DAY

Y	KD	XALB1	XALB2	SR1	SR2	CELLS1	CELLS2
0.3300	0.0500	3.8621	33.1259	0.3268	6.1439	-0.0853	0.3712

FOR NTRY = 3

INITIAL VERTEX

M	D1	R1	R2	V2	XA	AEC
1	8.5485	5.6195	2.1073	1864.1	2000.0	531.31
2	5.0171	2.3305	3.7622	1515.1	9229.9	21360.52
3	5.5923	3.9016	9.9531	3173.1	1545.6	720.44
4	16.5425	4.1238	9.6912	3164.6	2478.5	819.22
5	12.9156	2.1275	8.1147	2401.4	7710.2	13626.25
6	4.6365	3.9949	5.8891	2324.9	6780.6	9685.93
7	14.0153	7.5317	4.3542	2752.6	4348.9	2935.28
8	5.6622	8.0934	4.7104	2948.6	3637.7	1708.39

NUMBER OF ITERATIONS (I.E. PROJECTIONS) = 419
NUMBER OF FUNCTION EVALUATIONS = 474
NUMBER OF WORST POINT STACKS = 2

OPTIMUM VERTEX

M	D1	R1	R2	V2	XA	AEC
1	4.0040	6.3768	1.5004	1896.3	2108.7	483.28
2	4.0040	6.3776	1.5006	1896.5	2108.4	483.28
3	4.0040	6.3776	1.5006	1896.5	2108.4	483.28
4	4.0039	6.3770	1.5006	1896.4	2108.6	483.28
5	4.0039	6.3772	1.5007	1896.4	2108.5	483.28
6	4.0039	6.3770	1.5006	1896.4	2108.6	483.28
7	4.0039	6.3770	1.5006	1896.3	2108.6	483.28
8	4.0039	6.3772	1.5006	1896.4	2108.5	483.28

SE	Q2	F1	F2	F3	E1	E2	E3
40.00	36398.79	0.7012	0.5558	0.1000	29.8832	44.4169	90.0000

FIXED CAPITAL INVESTMENT IN DOLLARS

SLUDGE	TANK	TOWER	DSTRIB	INJECT	R1PUMP	R2PUMP	CLARIF
408.00	894.43	218.10	11.35	33.86	409.50	419.07	1244.88

L2	D2	SLANT	TDH	HPSI	QGPM	QCFS	QCFS2
7.0424	6.0000	3.4642	8.2804	3.5882	18.1572	0.0405	0.0095

LOAD	OXYGEN	MOTOR1	MOTOR2	ETA	A1	THETA
0.0031	0.0542	0.0543	0.0093	0.7000	18.1572	0.0521

OPERATING COSTS IN DOLLARS PER YEAR

O1	O2	O3	MAINT	OP	AYC	AEC
11.9605	11.9486	2.0393	4.7132	30.6616	452.6184	483.2800

LIFE	SALVAG	RATE	FACTOR	FCI
10	1000.0000	0.0725	0.1440	3629.20

SLUDGE PRODUCTION IN POUNDS PER DAY

Y	KD	XALB1	XALB2	SR1	SR2	CELLS1	CELLS2
0.3300	0.0500	3.7579	33.3483	0.3188	6.1967	-0.0827	0.3775

FOR NTRY = 4

INITIAL VERTEX

M	D1	R1	R2	V2	XA	AEC
1	8.5485	5.6195	2.1073	1864.1	2000.0	531.31
2	8.8473	6.4379	6.8613	3054.5	1107.4	999.80
3	16.1695	6.6691	7.0610	3146.5	4016.9	2410.29
4	8.8040	2.8631	5.6558	2023.3	1601.0	599.42
5	13.7613	3.1962	9.7699	2983.3	4690.8	3564.23
6	19.1442	5.5736	3.7558	2206.5	3929.4	2175.82
7	7.5669	9.0109	9.5016	4168.1	3452.9	1645.97
8	16.0264	5.0232	7.9907	2993.5	7995.6	15107.41

NUMBER OF ITERATIONS (I.E. PROJECTIONS) = 597
NUMBER OF FUNCTION EVALUATIONS = 641
NUMBER OF WORST POINT STACKS = 1

OPTIMUM VERTEX

M	D1	R1	R2	V2	XA	AEC
1	4.1369	6.4018	1.5162	1905.0	2083.0	484.16
2	4.1370	6.4018	1.5162	1905.0	2082.9	484.16
3	4.1370	6.4012	1.5163	1904.9	2083.1	484.16
4	4.1372	6.4013	1.5163	1904.9	2083.1	484.16
5	4.1374	6.4017	1.5162	1905.0	2082.9	484.16
6	4.1377	6.4007	1.5164	1904.8	2083.2	484.16
7	4.1375	6.4023	1.5162	1905.1	2082.8	484.16
8	4.1372	6.4016	1.5163	1904.9	2083.0	484.16

SE	Q2	F1	F2	F3	E1	E2	E3
40.00	36563.07	0.6938	0.5588	0.1000	30.6186	44.1161	90.0000

FIXED CAPITAL INVESTMENT IN DOLLARS

SLUDGE	TANK	TOWER	DSTRIB	INJECT	R1PUMP	R2PUMP	CLARIF
408.00	897.01	226.22	11.39	34.02	414.11	421.34	1244.88

L2	D2	SLANT	TDH	HPSI	QGPM	QCFS	QCFS2
7.0742	6.0000	3.4642	8.4282	3.6522	18.2266	0.0406	0.0096

LOAD	OXYGEN	MOTOR1	MOTOR2	ETA	A1	THETA
0.0031	0.0542	0.0555	0.0094	0.7000	18.2266	0.0521

OPERATING COSTS IN DOLLARS PER YEAR

O1	O2	O3	MAINT	CP	AYC	AEC
12.2205	11.9486	2.0606	2.7550	28.9846	455.1777	484.1624

LIFE	SALVAG	RATE	FACTOR	FCI
10	1000.0000	0.0725	0.1440	3656.97

SLUDGE PRODUCTION IN POUNDS PER DAY

Y	KD	XALB1	XALB2	SR1	SR2	CELLS1	CELLS2
0.3300	0.0500	3.8843	33.0928	0.3279	6.1547	-0.0860	0.3764

FOR NTRY = 5

INITIAL VERTEX

M	D1	R1	R2	V2	XA	AEC
1	8.5485	5.6195	2.1073	1864.1	2000.0	531.31
2	9.3696	9.1756	10.3495	4384.4	7456.5	12763.75
3	18.5366	8.4042	4.1566	2896.7	1901.7	829.91
4	16.3104	10.0486	6.5452	3758.2	7597.6	13455.96
5	13.7256	9.0929	5.3513	3299.0	6965.8	10674.76
6	18.9655	4.4286	7.8908	2845.2	7408.1	12422.95
7	15.2965	6.6893	2.0930	2089.6	8788.2	19080.68
8	6.4672	1.1579	8.5694	2291.5	2348.3	580.14

NUMBER OF ITERATIONS (I.E. PROJECTIONS) = 241
NUMBER OF FUNCTION EVALUATIONS = 275
NUMBER OF WORST POINT STACKS = 0

OPTIMUM VERTEX

M	D1	R1	R2	V2	XA	AEC
1	4.0001	6.4147	1.5000	1904.3	2097.7	483.20
2	4.0002	6.4139	1.5000	1904.1	2097.9	483.20
3	4.0006	6.4148	1.5000	1904.3	2097.6	483.20
4	4.0009	6.4146	1.5000	1904.2	2097.6	483.20
5	4.0000	6.4146	1.5003	1904.3	2097.6	483.21
6	4.0014	6.4153	1.5000	1904.4	2097.3	483.21
7	4.0008	6.4145	1.5001	1904.2	2097.6	483.21
8	4.0005	6.4145	1.5001	1904.2	2097.6	483.20

SE	Q2	F1	F2	F3	E1	E2	E3
40.00	36549.88	0.7018	0.5571	0.1000	29.8249	44.2889	90.0000

FIXED CAPITAL INVESTMENT IN DOLLARS

SLUDGE	TANK	TOWER	DSTRIB	INJECT	R1PUMP	R2PUMP	CLARIF
408.00	896.80	219.19	11.41	34.00	411.04	419.00	1244.88

L2	D2	SLANT	TDH	HPSI	QGPM	QCFS	QCFS2
7.0716	6.0000	3.4642	8.2916	3.5930	18.2635	0.0407	0.0095

LOAD	OXYGEN	MOTOR1	MOTOR2	ETA	A1	THETA
0.0031	0.0542	0.0547	0.0093	0.7000	18.2635	0.0521

OPERATING COSTS IN DOLLARS PER YEAR

O1	O2	O3	MAINT	OP	AYC	AEC
12.0468	11.9486	2.0386	3.8133	29.8473	453.3564	483.2036

LIFE	SALVAG	RATE	FACTOR	FCI
10	1000.0000	0.0725	0.1440	3644.32

SLUDGE PRODUCTION IN POUNDS PER DAY

Y	KD	XALB1	XALB2	SR1	SR2	CELLS1	CELLS2
0.3300	0.0500	3.7711	33.3135	0.3200	6.1788	-0.0829	0.3733

FOR NTRY = 6

INITIAL VERTEX

M	D1	R1	R2	V2	XA	AEC
1	8.5485	5.6195	2.1073	1864.1	2000.0	531.31
2	12.2088	10.6485	3.2161	3175.2	5317.6	5192.91
3	5.2184	7.7104	4.9091	2909.3	9684.1	24241.85
4	16.0887	9.1758	2.5553	2719.5	7001.9	10787.73
5	15.6194	1.5555	9.4742	2569.7	5306.2	4968.59
6	13.9617	10.6001	3.0661	3132.9	3879.8	2230.46
7	14.1058	3.9789	2.5279	1603.5	6896.0	10124.33
8	15.4480	1.8659	2.3005	1103.6	4855.7	3706.74

NUMBER OF ITERATIONS (I.E. PROJECTIONS) = 154
NUMBER OF FUNCTION EVALUATIONS = 184
NUMBER OF WORST POINT STACKS = 5

OPTIMUM VERTEX

M	D1	R1	R2	V2	XA	AEC
1	4.0278	6.3562	1.5000	1891.8	2112.5	483.42
2	4.0278	6.3562	1.5000	1891.8	2112.5	483.42
3	4.0278	6.3562	1.5000	1891.8	2112.5	483.42
4	4.0278	6.3562	1.5000	1891.8	2112.5	483.42
5	4.0278	6.3562	1.5000	1891.8	2112.5	483.42
6	4.0256	6.3557	1.5006	1891.8	2112.7	483.42
7	4.0287	6.3562	1.5002	1891.8	2112.3	483.43
8	4.0278	6.3562	1.5000	1891.8	2112.5	483.42

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SIMULATION AND OPTIMIZATION OF A PACKAGE SEWAGE TREATMENT PLANT

by

James Irvin Martin

(ABSTRACT)

Research was undertaken in order to simulate and optimize an Aquatair Model P-3 package sewage treatment plant for which operating data from the National Sanitation Foundation was available. The treatment plant consists essentially of a plastic medium trickling filter mounted overtop of a complete-mix activated sludge (CMAS) tank; the unit also has a primary sludge holding tank and a secondary clarifier. From consideration of reaction kinetics, reactor type, and geometry, a mechanistic model was developed to explain the phenomena of substrate removal in the Aquatair Model P-3. The two-factor SIMPLEX optimization procedure was used to determine the best set of biological reaction constants K_1 and K_2 which would characterize the substrate removal behavior of the Aquatair biological oxidation tower and recirculation chamber, respectively, based on actual plant data.

Once the behavior of the plant was accurately simulated, cost functions were developed in order to relate the various design variables to cost. An n-factor PERPLEX optimization routine was then used to optimize the various plant variables for a given flow, organic load, and efficiency. PERPLEX, a modification of the Box COMPLEX method, was shown to be a useful tool for evaluating the factor space defined

by a set of mathematical assumptions. The cost benefit of aerobic digestion occuring in the Aquatair bio-oxidation tower was predicted by the model and confirmed by nitrification data and sludge production calculations.