

Binary Mask Programmable Hologram

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Abstract: We report, for the first time, the concept and generation of a novel Fresnel hologram called the digital binary mask programmable hologram (BMPH). A BMPH is comprised of a static, high resolution binary grating that is overlaid with a lower resolution binary mask. The reconstructed image of the BMPH can be programmed to approximate a target image (including both intensity and depth information) by configuring the pattern of the binary mask with a simple genetic algorithm (SGA). As the low resolution binary mask can be realized with less stringent display technology, our method enables the development of simple and economical holographic video display.

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1. Concept of the proposed binary mask programmable hologram (BMPH)

The Fresnel hologram of a three-dimensional scene can be generated numerically by computing the fringe patterns emerged from each object point to the hologram plane. Recently, it has also been demonstrated that digital holograms can be generated and processed at video rates [1, 2]. These encouraging results, however, are shrouded by the lack of high resolution real-time spatial light modulators (SLMs) (5 microns or less) for displaying the digital holograms. An effective solution towards higher resolution holographic display, is realized through the integration of the optically addressed SLMs (OASLMs) and the active tiling (AT) method [3–6]. In this approach (hereafter refer as OASLM + AT), as shown in Fig. 1, a digital hologram is partitioned into tiles, and each of them is sequentially displayed

on an electrically addressed SLM (EASLM), and replicated to all the de-magnifying lens in a lens array. A shutter selects the lens that corresponds to the tile currently displayed in the EASLM, and projected the de-magnified hologram fringe patterns onto the corresponding position onto the recording material (the OASLM). Each lens in the lens array de-magnified the diffraction fringes in the tile, and increases the resolution of the hologram. As an example is shown in Fig. 1, where the fringe patterns corresponding to the top-left corner of the hologram tile is displayed on the EASLM, projected onto the OASLM via the de-magnifying lens and the shutter. Despite the success of this method, the cost and complexity of such systems are both complicated and expensive.

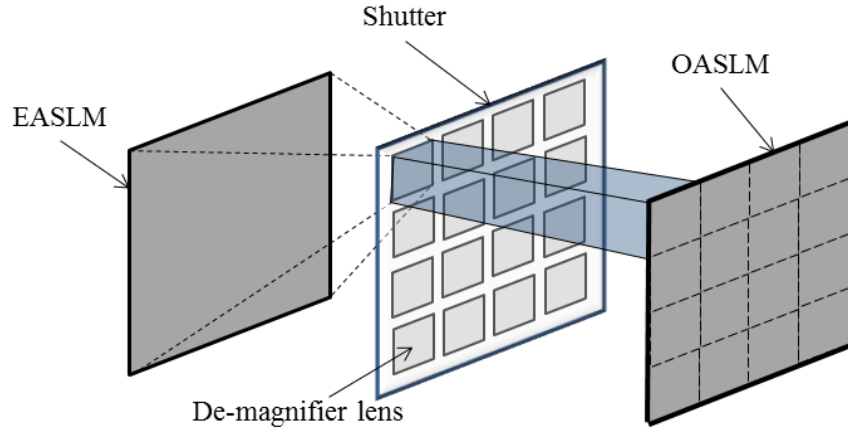


Fig. 1. High resolution holographic display based on the OASLM + AT method.

In this paper, we propose the concept and realization of a new type of digital Fresnel hologram known as the binary mask programmable hologram (BMPH). Different from the classical digital Fresnel hologram, a BMPH is a high resolution hologram that can be displayed with devices of considerably lower resolution. To our understanding, this is the first time such method is reported. A BMPH is formed by the superposition of two images as shown in Fig. 2. The first image $G(x, y)$ is a static, high resolution binary diffraction grating where each pixel is either transparent or opaque, denoted by '1' and '0', respectively. We have assigned a checkerboard for $G(x, y)$ so that the frequency of the grating pattern is maximum along both the horizontal and vertical directions, as given by

$$G(x, y) = 1 \text{ if } (x + y) = \text{odd}, \text{ and } 0 \text{ otherwise.} \quad (1)$$

The second image $M(x, y)$ is a binary mask pattern that is lower in resolution than that of the grating, and is evenly partitioned into square blocks each with a size of $k \times k$ pixels, where k is an integer that is larger than unity. Within each square block, all the pixels are identical and set to either the transparent (1) or the opaque state (0). As such, the resolution of the mask pattern is $(1/k)$ th of that of $G(x, y)$ along the horizontal and the vertical directions. Superposition of the pair of images results in the BMPH given in Eq. (2):

$$B(x, y) = G(x, y) M(x, y). \quad (2)$$

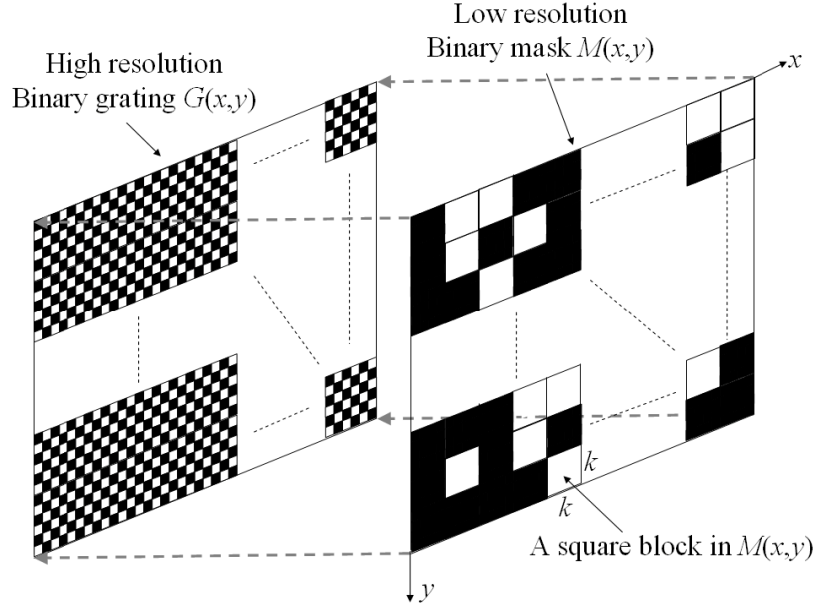


Fig. 2. Structure of the binary mask programmable hologram (BMPH): A low resolution binary mask overlaid onto a high resolution binary grating.

2. Generation of the BMPH

When the hologram is illuminated with an on-axis planar coherent beam, the magnitude of the reconstructed image at distance z_p can be expressed in Eq. (3) as [7]

$$I_d(x, y) = \left| \sum_{p=0}^X \sum_{q=0}^Y B(p, q) \exp \left(j \frac{2\pi}{\lambda} \sqrt{[(x-p)\delta d]^2 + [(y-q)\delta d]^2 + z_d^2} \right) \right|, \quad (3)$$

where $j = \sqrt{-1}$, X and Y are the horizontal and vertical extents of the hologram, respectively. λ is the wavelength of the optical beam, and δd is the width (as well as the height) of a pixel in $B(x, y)$. Without loss of generality, we assume that the hologram and the image scene have identical horizontal and vertical extents. From Eqs. (2) and (3), it can be inferred that the reconstructed image is dependent on the binary mask pattern. However, given $I_d(x, y)$ there is no explicit inverse formulation to compute $M(x, y)$. In view of this, we propose to encapsulate the inverse problem as an optimization process to determine the mask pattern that best approximates the target reconstructed image. To begin with, an objective function O_d is defined to determine the root mean square error (RMSE) between the reconstructed image $I_d(x, y)$ and a planar target image $T_d(x, y)$ that is located at a distance z_d from the hologram, as shown in Eq. (4)

$$O_d = \sqrt{\frac{1}{XY} \sum_{p=0}^X \sum_{q=0}^Y [T_d(p, q) - I_d(p, q)]^2}. \quad (4)$$

The goal is to determine $M(x, y)$ so that the value of O_d is minimized. We would like to emphasize in that this is different from determining a binary hologram $B(x, y)$ that matches

with the binary hologram of a given target image $T_d(x, y)$. If this is the case, the resolution of $B(x, y)$ will have to be identical to that of the high resolution grating. In our method, we determine a low resolution mask $M(x, y)$ so that when it is integrated with the grating, will result in a reconstructed image that is similar to the target. This is a hard problem that cannot be solved with brute force means, as there are $2^{XY/k^2}$ combinations on the mask pattern that can be represented in $M(x, y)$. For example modest square hologram with X and Y both equal to 256, and $k = 4$, the total number of patterns that can be generated is 2^{4096} . In view of this, we propose to employ a simple genetic algorithm (SGA) [8], a method that mimics the evolutionary mechanism in biological species, to determine the optimal mask pattern. Past research has demonstrated that the SGA is effective in solving hard optimization problems in many engineering applications [9]. As the principles and details of SGA have been presented in many literatures, we shall only focus on how it is applied in our proposed method. To begin with, the binary mask $M(x, y)$ is first converted into a one dimensional sequence of binary string by chaining consecutive rows of pixels. In the context of SGA, the sequence is referred as a binary chromosome and its structure is shown in Fig. 3. The chromosome is a one dimensional binary bit string of length $N = XY/k^2$, and with the n th bit ($0 \leq n < XY/k^2$) equal to $M(\text{floor}(nk/Y)k, \text{mod}(nk, Y))$ (where $\text{floor}(a)$ is the largest integer that is not greater than a , and $\text{mod}(a, b)$ is the remainder of a/b).

$M(0,0)$	$M(0,k)$	$M(0,2k)$	-	$M(0,Y-k)$	$M(k,0)$	-	$M(X-k,Y-k)$
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Fig. 3. Structure of the chromosome for representing $M(x, y)$.

A fitness measurement [8], i.e., $\text{fitness} = (1 + O_d)^{-1}$, ranging from 0 to 1, is evaluated for the chromosome. The degree of similarity between the target image $T_d(x, y)$, and the reconstructed image $I_d(x, y)$ associated with the mask $M(x, y)$, is proportional to the fitness value. A maximum fitness value of unity reflects that $T_d(x, y)$ and $I_d(x, y)$ are identical. The process of applying SGA in finding the best mask is summarized in Table 1 and outlined as follows. Initially, a population comprising of Q chromosomes are generated. Each bit in a chromosome is randomly assigned a value of either 1 or 0 with uniform probability. Next, $Q/2$ pairs of parent chromosomes are selected into a mating pool with probabilities proportional to their fitness. A new generation of population (referred to as the child population) is established by applying either one of the two genetic operations namely, crossover, or mutation operation with probabilities p_c , and p_m , to each pair of parent chromosomes, respectively. The crossover operation [10] evaluates each corresponding pair of bits in the parent strings, and swap them with a probability q_c . As for mutation, the bits in the parent chromosomes are selected randomly with a probability q_m , and complemented (i.e., 1's become 0's and 0's become 1's). In the progression from the parent to the child population the best chromosome with the highest fitness value, known as the elite, is always preserved and taken to replace the weakest candidate in the new population. The evolution process of generating a new population from the previous one repeats until the maximum allowable number of generations have elapsed.

Table 1. SGA for determining the mask for optimizing the function in Eq. (4)

Step	Operation
1	Generate an initial random population with N chromosomes.
2	Evaluate the fitness of all chromosomes in the population.
3	Select $Q/2$ pairs of parents with probabilities according to their fitness.
4	For each pair of parents, perform either reproduction, uniform crossover, or mutation operation to generate two new chromosomes, forming a child population of Q chromosomes. Evaluate the fitness of all chromosomes in the child population.
5	Apply the elite principle by finding the weakest individual and replacing it with the strongest one in the previous generation.
6	If number of generation exceeds the upper limit, end of process. Otherwise go to step 3

3. Experimental results

To start with, we shall explore what will the reconstructed image of a digital Fresnel hologram appears, if it is displayed on a low resolution SLM. We generate a complex, continuous tone digital hologram for the planar image shown in Fig. 4(a) based on the classical Fresnel diffraction equation, and with the optical setting listed in Table 2. Note that the sampling pitch of the hologram is $5\mu\text{m}$. We shall simulate the result when the hologram is directly displayed on a SLM of identical size, but with pixel sizes of $5\mu\text{m}$ and $20\mu\text{m}$. The numerical reconstructed images are shown in Figs. 4(b) and 4(c), respectively for SLM with pixel sizes of $5\mu\text{m}$ and $20\mu\text{m}$. We observe that the quality of the reconstructed image is extremely poor in the case of using a low resolution, $20\mu\text{m}$ SLM. Next, we applied our proposed method to generate a BMPH for representing the target image in Fig. 4(a) based on the setting in Table 3. Note that the pixel size of the binary mask is at $20\mu\text{m}$, which is 4 times worse than that of the Fresnel hologram. Also, from Table 3, it can be seen that the resolution of the binary grating is four times higher than that of the binary mask (i.e., $k = 4$). A BMPH is generated after applying the SGA described in Table 1 for 20000 iterations. The total time taken is around 20s when executed on a graphic processing unit (GPU). The population size Q , and the parameters p_c , p_m , q_c , and q_m are set to 16, 0.85, 0.15, 0.001, and 0.3, respectively. These values are selected empirically to provide satisfactory performance in general. The binary mask and the numerical reconstructed image of the BMPH at the focal distance are shown in Figs. 4(d) and 4(e), respectively. We observe that apart from some blurriness, the reconstructed image is similar to the target image, which is superior than the reconstructed image obtained with a low resolution SLM (Fig. 4(c)). To further demonstrate our proposed method, we have generated a sequence of holograms representing a panning scene with the target image in Fig. 4(a) as the 1st frame. The reconstructed video sequence is shown in the animation clip [Media 1](#). It can be seen that the panning scene is generated with acceptable visual quality. In addition, all the reconstructed images have attained a fitness of over 0.92, and with an overall mean value of around 0.925. Finally, we would like to draw, in Table 4, a comparison between our proposed method and the existing approach based on OASLM + AT. Briefly, our proposed method is considerably lower in hardware complexity, and faster in display rate as compared with the TDM operation in OASLM + AT. On the downside, our method requires high computation loading in determining the optimal mask image.

Table 2. Optical setting of the Fresnel hologram

Number of hologram samples	256 (horizontal) x 256 (vertical)
Sampling pitch	$5\mu\text{m}$
Wavelength of light	650nm
Distance of image from hologram	0.4m

Table 3. Optical setting of the BMPH

Size of the grating	256 x 256 pixels	Pixel size of the binary mask	20 μm square
Pixel size of the binary grating	5 μm square	Wavelength of light	650nm
Size of the binary mask	64x64 pixels	Distance of image from hologram	0.4m

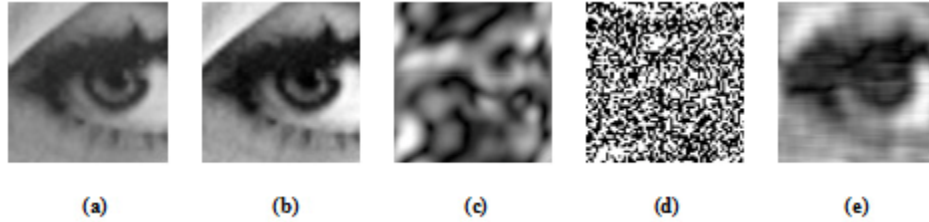


Fig. 4. (a) A planar image placed at 0.4m from the hologram. (b) Simulated reconstructed image of the hologram display by a SLM of pixel size of 5 μm at 0.4m, c) Same as (b) but with SLM of pixel size of 20 μm , (d) Binary mask corresponding to the target image in Fig. 4(a). (e) Reconstructed image of the BMPH at 0.4m (Excerpted from the first frame in Media 1)

Table 4. Comparison between OASLM + AT and our proposed method

	Hardware complexity	Display mechanism	Computation	Quality
OASLM + AT	High	Time division multiplex (TDM)	Negligible	Governed by the SLM, lens array, shutter, and the OASLM
Our proposed method	Low	Direct	High	Determined by the optimization process and the SLM

4. Conclusion

In this paper, we have proposed a novel method known as the binary mask programmable hologram (BMPH). A BMPH is comprising of a low resolution binary mask that is overlaid onto a static high resolution grating. The main feature of a BMPH is that the target image it represents can be composed by simply changing the pattern on the binary mask. In our investigation, we have employed a SGA for determining the mask pattern for a given target image. As the binary mask can be implemented with less stringent electronic devices, the proposed method can be employed as a framework for holographic display when high resolution SLMs are simply not available. The work we reported is by no means exhaustive and there are plenty of rooms for further exploration into the topic. One of the obvious directions is to increase the speed in generating the BMPH, which in the present method (about 20s per hologram) is only suitable for offline application. Another potential aspect of development is to extend the method for generating complex, continuous tone mask programmable hologram. This should lead to significant enhancement on the reconstructed image as important information could be discarded in a binary hologram. However, the complexity of the mask computation loading will also increase substantially and faster optimization methods other than SGA are required, leading to numerous research possibilities. Another aspect of potential research direction could be conducted on the generation of binary (or continuous tone) mask programmable hologram for representing 3D surface, instead of a planar image at some distance from the hologram.

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