Appendix A:

Bayesian D-Optimal Designs and Robustness Studies

Table Key	
Symbol	**Definition**
p_i	proportion of observations at design point i
p_c	proportion of observations at the control
x_i	design point i
E	efficiency

Uniform Prior

Table A1. Type I D-optimal Three Level Bayesian Designs on (EC_5, EC_{80})

Ratio	$p_1=p_2$	x_1	x_2	p_c	*E*
1	0.33	$EC_{11.1}$	$EC_{73.9}$	0.34	99.99%
1.5	0.35	$EC_{9.6}$	$EC_{75.4}$	0.30	99.73%
2	0.34	$EC_{10.6}$	$EC_{74.4}$	0.32	99.96%
2.5	0.35	$EC_{6.5}$	$EC_{78.5}$	0.30	97.33%
3	0.39	EC_5	EC_{80}	0.22	94.23%
3.5	0.38	EC_5	EC_{80}	0.24	94.85%
4	0.35	$EC_{5.2}$	$EC_{79.8}$	0.30	95.11%

Table A2. Type I D-optimal Three Level Bayesian Designs on (EC_{10}, EC_{80})

Ratio	$p_1=p_2$	x_1	x_2	p_c	*E*
1	0.36	$EC_{10.3}$	$EC_{79.7}$	0.28	99.99%
1.5	0.34	EC_{10}	EC_{80}	0.32	99.84%
2	0.36	EC_{11}	EC_{79}	0.28	99.95%
2.5	0.36	$EC_{10.7}$	$EC_{79.3}$	0.28	99.98%
3	0.35	$EC_{10.3}$	$EC_{79.7}$	0.30	99.97%
3.5	0.35	EC_{10}	EC_{80}	0.30	99.95%
4	0.35	$EC_{10.3}$	$EC_{79.7}$	0.30	99.98%

Table A3. Type I D-optimal Three Level Bayesian Designs on (EC_{20}, EC_{80})

Ratio	$p_1=p_2$	x_1	x_2	p_c	*E*
1	0.35	EC_{20}	EC_{80}	0.30	99.99%
1.5	0.34	EC_{20}	EC_{80}	0.32	99.99%
2	0.35	$EC_{20.2}$	$EC_{79.8}$	0.30	99.75%
2.5	0.31	EC_{20}	EC_{80}	0.38	99.48%
3	0.39	EC_{20}	EC_{80}	0.22	99.15%
3.5	0.38	EC_{20}	EC_{80}	0.24	99.22%
4	0.34	$EC_{20.2}$	EC_{80}	0.32	99.98%

Uniform Prior

Table A4. Type II D-optimal Three Level Bayesian Designs on (EC_5, EC_{80})

Ratio	$p_1=p_c$	x_1	x_2	p_2	***E***
1	0.44	$EC_{12.5}$	$EC_{72.5}$	0.12	111.15%
1.5	0.44	$EC_{13.0}$	$EC_{72.0}$	0.12	111.13%
2	0.44	$EC_{10.3}$	$EC_{74.7}$	0.12	110.67%
2.5	0.44	$EC_{9.1}$	$EC_{75.9}$	0.12	109.92%
3	0.44	$EC_{9.7}$	$EC_{75.3}$	0.12	110.35%
3.5	0.44	$EC_{5.5}$	$EC_{79.5}$	0.12	104.11%
4	0.44	$EC_{6.4}$	$EC_{78.6}$	0.12	106.25%

Table A5. Type II D-optimal Three Level Bayesian Designs on (EC_{10}, EC_{80})

Ratio	$p_1=p_c$	x_1	x_2	p_2	***E***
1	0.44	$EC_{12.4}$	$EC_{77.6}$	0.12	110.23%
1.5	0.44	$EC_{11.3}$	$EC_{78.7}$	0.12	110.11%
2	0.44	$EC_{11.5}$	$EC_{78.5}$	0.12	110.15%
2.5	0.44	EC_{10}	EC_{80}	0.12	109.64%
3	0.44	$EC_{10.4}$	$EC_{79.6}$	0.12	109.83%
3.5	0.44	$EC_{10.5}$	$EC_{79.5}$	0.12	109.87%
4	0.44	$EC_{10.1}$	$EC_{79.9}$	0.12	109.69%

Table A6. Type II D-optimal Three Level Bayesian Designs on (EC_{20}, EC_{80})

Ratio	$p_1=p_c$	x_1	x_2	p_2	***E***
1	0.44	EC_{20}	EC_{80}	0.12	111.01%
1.5	0.44	$EC_{20.2}$	$EC_{79.8}$	0.12	110.84%
2	0.44	EC_{20}	EC_{80}	0.12	111.01%
2.5	0.44	EC_{20}	EC_{80}	0.12	111.01%
3	0.44	$EC_{20.2}$	$EC_{79.8}$	0.12	110.83%
3.5	0.44	$EC_{20.5}$	$EC_{79.5}$	0.12	110.56%
4	0.44	$EC_{20.1}$	$EC_{79.9}$	0.12	110.92%

D-Optimal Robustness Study: Uniform Prior
Assumed β_1=-1.5, N=100

Table A7. Ratio=1, $p_1=p_{c=}$ 0.44, p_2 =0.12, $x_1=EC_{12.5}$, $x_2=EC_{72.5}$

EC Misspecification	***True β_1***	***True x_1***	***True x_2***	***E***
EC_{30}	-2.61	$EC_{2.7}$	$EC_{57.20}$	110.67%
EC_{40}	-1.98	$EC_{6.4}$	$EC_{65.37}$	112.12%
EC_{45}	-1.73	$EC_{9.11}$	$EC_{69.04}$	111.38%
EC_{55}	-1.29	$EC_{16.63}$	$EC_{75.78}$	110.38%
EC_{60}	-1.11	$EC_{21.60}$	$EC_{78.90}$	109.53%
EC_{70}	-0.77	$EC_{34.30}$	$EC_{84.75}$	107.81%

Table A8. Ratio=2.5, $p_1=p_c$= 0.44, p_2=0.12, $x_1=EC_{9.1}$, $x_2=EC_{75.9}$

EC Misspecification	***True β_1***	***True x_1***	***True x_2***	***E***
EC_{30}	-2.61	$EC_{1.56}$	$EC_{61.94}$	120.63%
EC_{40}	-1.98	$EC_{4.21}$	$EC_{69.45}$	118.12%
EC_{45}	-1.73	$EC_{6.32}$	$EC_{72.78}$	115.59%
EC_{55}	-1.29	$EC_{12.65}$	$EC_{88.83}$	110.34%
EC_{60}	-1.11	$EC_{17.09}$	$EC_{81.61}$	107.88%
EC_{70}	-0.77	$EC_{29.13}$	$EC_{86.77}$	103.46%

Table A9. Ratio=4, p_1= p_c=0.44, p_2= 0.12, $x_1=EC_{6.4}$, $x_2=EC_{80}$

EC Misspecification	***True β_1***	***True x_1***	***True x_2***	***E***
EC_{30}	-2.61	$EC_{0.84}$	$EC_{65.82}$	112.27%
EC_{40}	-1.98	$EC_{2.64}$	$EC_{72.74}$	113.32%
EC_{45}	-1.73	$EC_{4.21}$	$EC_{75.78}$	112.29%
EC_{55}	-1.29	$EC_{9.33}$	$EC_{81.25}$	109.52%
EC_{60}	-1.11	$EC_{13.19}$	$EC_{83.74}$	108.11%
EC_{70}	-0.77	$EC_{24.30}$	$EC_{88.35}$	105.45%

Normal Prior

Table A10. Type I D-optimal Three Level Bayesian Designs on (EC_5, EC_{80})

σ_{β_1}	$p_1=p_2$	x_1	x_2	p_c	E
0.00005	0.33	$EC_{11.1}$	$EC_{73.9}$	0.34	99.99%
0.15	0.34	$EC_{11.1}$	$EC_{73.9}$	0.32	99.96%
0.25	0.33	$EC_{9.4}$	$EC_{75.6}$	0.34	99.68%
0.3124	0.37	$EC_{7.9}$	$EC_{77.1}$	0.26	98.68%
0.375	0.39	$EC_{6.1}$	$EC_{78.9}$	0.22	96.35%
0.4167	0.36	$EC_{5.8}$	$EC_{79.2}$	0.28	96.20%
0.45	0.33	$EC_{5.9}$	$EC_{79.1}$	0.34	96.06%

Table A11. Type I D-optimal Three Level Bayesian Designs on (EC_{10}, EC_{80})

σ_{β_1}	$p_1=p_2$	x_1	x_2	p_c	E
0.00005	0.36	$EC_{10.3}$	$EC_{79.7}$	0.28	99.99%
0.15	0.36	$EC_{10.7}$	$EC_{79.3}$	0.28	99.98%
0.25	0.37	EC_{10}	EC_{80}	0.26	99.93%
0.3124	0.36	$EC_{11.6}$	$EC_{78.4}$	0.28	99.82%
0.375	0.34	$EC_{10.2}$	$EC_{79.8}$	0.32	99.87%
0.4167	0.36	$EC_{10.8}$	$EC_{79.2}$	0.28	99.97%
0.45	0.35	EC_{10}	EC_{80}	0.30	99.50%

Table A12. Type I D-optimal Three Level Bayesian Designs on (EC_{20}, EC_{80})

σ_{β_1}	$p_1=p_2$	x_1	x_2	p_c	E
0.00005	0.35	EC_{20}	EC_{80}	0.30	99.99%
0.15	0.35	EC_{20}	EC_{80}	0.30	99.99%
0.25	0.37	EC_{20}	EC_{80}	0.26	99.74
0.3124	0.34	$EC_{20.4}$	$EC_{79.6}$	0.32	99.54
0.375	0.34	$EC_{20.2}$	$EC_{79.8}$	0.32	99.77%
0.4167	0.36	EC_{20}	EC_{80}	0.28	99.91%
0.45	0.35	EC_{20}	EC_{80}	0.30	99.98%

Normal Prior

Table A13. Type II D-optimal Three Level Bayesian Designs on (EC_5, EC_{80})

σ_{β_1}	$p_1=p_c$	x_1	x_2	p_2	E
0.00005	0.44	$EC_{12.5}$	$EC_{72.5}$	0.12	111.15%
0.15	0.44	$EC_{12.1}$	$EC_{72.9}$	0.12	111.13%
0.25	0.44	$EC_{10.7}$	$EC_{74.3}$	0.12	110.84%
0.3124	0.44	$EC_{8.2}$	$EC_{76.8}$	0.12	109.05%
0.375	0.44	$EC_{8.9}$	$EC_{76.1}$	0.12	109.75%
0.4167	0.44	$EC_{5.6}$	$EC_{79.4}$	0.12	104.38%
0.45	0.44	$EC_{5.1}$	$EC_{79.9}$	0.12	102.95%

Table A14. Type II D-optimal Three Level Bayesian Designs on (EC_{10}, EC_{80})

σ_{β_1}	$p_1=p_c$	x_1	x_2	p_2	E
0.00005	0.44	$EC_{12.4}$	$EC_{77.6}$	0.12	110.23%
0.15	0.44	EC_{12}	$EC_{78.0}$	0.12	110.21%
0.25	0.44	$EC_{10.4}$	$EC_{79.6}$	0.12	109.83%
0.3124	0.44	$EC_{10.8}$	$EC_{79.2}$	0.12	109.98%
0.375	0.44	$EC_{10.4}$	$EC_{79.6}$	0.12	109.83%
0.4167	0.44	$EC_{10.6}$	$EC_{79.4}$	0.12	109.91%
0.45	0.44	$EC_{10.1}$	$EC_{79.9}$	0.12	109.69%

Table 15. Type II D-optimal Three Level Bayesian Designs on (EC_{20}, EC_{80})

σ_{β_1}	$p_1=p_c$	x_1	x_2	p_2	E
0.00005	0.44	EC_{20}	EC_{80}	0.12	111.01%
0.15	0.44	EC_{20}	EC_{80}	0.12	111.01%
0.25	0.44	EC_{20}	EC_{80}	0.12	111.01%
0.3124	0.44	$EC_{20.8}$	$EC_{79.2}$	0.12	110.30%
0.375	0.44	$EC_{20.4}$	$EC_{79.6}$	0.12	110.66%
0.4167	0.44	$EC_{20.6}$	$EC_{79.4}$	0.12	110.48%
0.45	0.44	$EC_{20.1}$	$EC_{79.9}$	0.12	110.92%

D-Optimal Robustness Study: Normal Prior
Assumed β_1=-1.5, N=100

Table A16. $\sigma_{\beta_1} = 0.00005$***, p_1= p_c=0.44, p_2=0.12, x_1=$EC_{12.5}$, x_2=$EC_{72.5}$***

EC Misspecification	*True β_1*	*True x_1*	*True x_2*	*E*
EC_{30}	-2.61	$EC_{2.03}$	$EC_{59.83}$	112.98%
EC_{40}	-1.98	$EC_{5.15}$	$EC_{67.64}$	113.20%
EC_{45}	-1.73	$EC_{7.54}$	$EC_{71.12}$	112.32%
EC_{55}	-1.29	$EC_{14.43}$	$EC_{77.49}$	109.99%
EC_{60}	-1.11	$EC_{19.13}$	$EC_{80.42}$	108.76%
EC_{70}	-0.77	$EC_{31.51}$	$EC_{85.89}$	106.40%

Table A17. $\sigma_{\beta_1} = 0.3214$***, p_1= p_c=0.44, p_2= 0.12, x_1=$EC_{8.2}$, x_2=$EC_{76.8}$***

EC Misspecification	*True β_1*	*True x_1*	*True x_2*	*E*
EC_{30}	-2.61	$EC_{2.0}$	$EC_{59.87}$	95.89%
EC_{40}	-1.98	$EC_{5.08}$	$EC_{67.76}$	103.99%
EC_{45}	-1.73	$EC_{7.45}$	$EC_{71.24}$	106.81%
EC_{55}	-1.29	$EC_{14.31}$	$EC_{77.58}$	111.04%
EC_{60}	-1.11	$EC_{19.00}$	$EC_{80.50}$	112.70%
EC_{70}	-0.77	$EC_{31.36}$	$EC_{85.94}$	115.46%

Table A18. $\sigma_{\beta_1} = 0.4500$***, p_1=0.44, p_2=p_c=0.12, x_1=$EC_{5.1}$, x_2=$EC_{79.9}$***

EC Misspecification	*True β_1*	*True x_1*	*True x_2*	*E*
EC_{30}	-2.61	$EC_{1.01}$	$EC_{64.80}$	90.94%
EC_{40}	-1.98	$EC_{3.03}$	$EC_{71.88}$	99.43%
EC_{45}	-1.73	$EC_{4.75}$	$EC_{75.00}$	102.48%
EC_{55}	-1.29	$EC_{10.21}$	$EC_{80.62}$	107.28%
EC_{60}	-1.11	$EC_{14.24}$	$EC_{83.19}$	109.24%
EC_{70}	-0.77	$EC_{25.64}$	$EC_{87.94}$	112.62%