A DIGITAL ADAPTIVE MODEL FOLLOWING CONTROLLER

by

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SYMBOLS

A_d	n by n desired differential transition matrix
B_d	n by m desired differential input matrix
J	scalar quadratic performance index
$Q_{1,2}$	symetric weighting matrix
u	m dimensional control vector
u_c	m dimensional control correction vector
u_p	m dimensional input control vector
u_v	m dimensional virtual control vector
x	n dimensional state vector
x_d	n dimensional desired state vector
x_v	n dimensional virtual state vector
Φ_d	n by n desired discrete transition matrix
Γ_d	n by m desired discrete input matrix
τ	digital controller cycle time

I. INTRODUCTION

The need to design control systems for non-linear systems and for systems with changing parameters has been a problem for a considerable amount of time. In the past,[1] the typical method of approach for both non-linear systems and time varying linear systems has been to linearize the plant's dynamics about one or more operating points and analize the plants about these points with several well known methods.[1,2,3] This method is very successful if the plant's parameters and/or state variables do not exceed the limits of the linearization procedure for which it is valid.

One such system for which there is considerable concern that this method will not be successful[4] is the Externally Blown Flap Short Takeoff and Landing Vehicle, hereafter referred to as the EBF-STOL, as described by Wick and Kuhn.[5] Interest in the EBF-STOL design has arisen as a possible solution to the congestion in the terminal area of our air-transportation system. This solution to the congestion problem utilizes an aircraft designed to land at slow speeds while being able to cruise at speeds that are competitive with present commercial aircraft. With this aircraft's low landing speed, it not only allows more aircraft to land in a given time period but it also requires less runway to land.

The EBF-STOL achieves this low landing speed by augmenting the lift of the aircraft with flaps that deflect the exhaust gases from the jet engines. At cruising speeds, the EBF-STOL looks and behaves similar to a conventional aircraft. However, while operating at low speeds, in addition to augmenting the lift the flaps cause a considerable change in other aerodynamic properties of the vehicle such as pitching moment, drag and the associated stability derivatives. Additional changes in the aerodynamic properties can also be caused by the large variation in the operating speed of the aircraft. These large variations of the aircraft characteristics due to changes in configuration as well as the change in speed limits the usefulness of the linearization technique described earlier.

In order for any aircraft to be acceptable it is necessary that the pilot be able to control the vehicle throughout the entire flight regime and that the aircraft be dynamically stable. It may also be desirable that the control system be designed so that the response of the aircraft to a pilot command satisfy some prescribed set of flying qualities; also, it may not be desirable for these flying qualities to remain constant, but for them to vary considerably over the flight regime. In general, it is not possible to satisfy all of the above requirements without the aid of a stability augmentation system, or SAS, in the aircraft.

The calculations that are required in order to synthesize a control system which meets the above specifications for a system with changing parameters has led to the consideration of using an on-board digital computer as an integral part of the control system. There are many advantages to be gained in using the digital computer in the SAS of the aircraft. One advantage is the ability to alter the control system's dynamics after it is installed in the aircraft by simply reprogramming the computer rather than rebuilding or rewiring the SAS. This type of control system could then be standardized in the hardware used, requiring changes only in the software for different aircraft. Another advantage of using the digital control system is that it may be incorporated into a larger digital network such as the proposed Digital Avionics Information System, or DAIS,[6] thereby increasing the reliability and decreasing the total cost of the aircraft avionics system. In fact, once the digital computer is in the control system the full transition to a digital control system becomes appealing.

Since the control system must be able to adapt to the changing environment, an adaptive control system is considered here. The concept of model-referenced adaptive control system evolved from work done by Whitaker.[7, 8] A block diagram of this configuration is shown in Figure 1. The model-referenced approach is "closed loop" with respect to the system performance, i.e., the performance is monitored and the adaptive parameters are adjusted accordingly to minimize the performance index. Here the system performance is a measure of the accuracy of the controller to perform the desired task and the adaptive parameters are any parameters that effect the system performance which may be changed. In the Whitaker system, the performance index was the integral square of the difference between the model and aircraft states and the adaptive parameters were potentiometer settings. This approach has the advantage of avoiding the system identification problem which is essential to many other methods.

The configuration used for the digital model-referenced adaptive controller proposed here is shown in Figure 2. In the study of the digital model-referenced adaptive controller, it was not considered the primary purpose of the study to choose a model. It is therefore assumed that a suitable model has already been specified for the system under consideration. The problem considered here consists of finding a control algorithm that will result in the system and model being dynamically similar, i.e., the same control input to the model and the augmented system would result in the model and augmented system state variables behaving identically. In order to achieve this objective, a mathematical model for the systems of interest will be developed for a general set of equations. The control algorithm will then be developed from optimization theory and the stability of the algorithm will be investigated. Finally the proposed control algorithm will be tested with two examples to illustrate its characteristics.

II. DERIVATION OF THE MODEL DIFFERENCE EQUATIONS

Although a specific model is not used in this study, a mathematical expression will be needed in order to develop the control algorithm. Let the model's dynamics be described by the linear vector differential equation in the form of

$$\dot{x}_d(t) = A_d x_d(t) + B_d u_p(t) \tag{2.1}$$

Here, x_d is a n-dimensional vector representing the state of the model, u_p is a m-dimensional vector representing the input to the model, A_d is an n by n matrix obtained from the desired model dynamic characteristics, and B_d is an n by m matrix determined from the desired model characteristics and the control effectiveness. In general, the solution of equation (2.1) is expressable as [9]

$$x_d(t) = \exp(A_d(t-t_o))x_d(t_o) + \int_{t_o}^{t} \exp(A_d(t-\sigma))B_d u_p(\sigma)d\sigma \tag{2.2}$$

Where t_o is a time at which the model state is known, i.e., the initial condition, and $u_p(\sigma)$ is the control function applied to the model during the time interval $[t_o,t]$.

For digital systems the state is sampled at various times and, based on these discrete samples, a control command is determined by the digital computer. This input is discrete and must be applied to the system in some sectionally continuous form. One of the most frequently used schemes to accomplish this is the zero-order hold, or clamp. The clamp receives commands from the computer and applies a constant signal at its output until a new command is received. This type of conversion will be used here in order to provide a convenient way to convert the computer commands to actual control signals.

In working with digital control systems, it is more convenient to use vector difference equations rather than the continuous equations of the form in equations (2.1) and (2.2). To obtain the difference equations, assume a time interval of τ, which represents the time interval between sampling periods. From the fundamental frequency theorem, the time interval must be at most one-half of the shortest period that may be recognized. Furthermore, if the system is to be operated by humans, the sampling period must be small enough so that the operator does not notice any significant time lag between an applied command and when the command is acknowledged by the computer. Also, it is very desirable to allow as much time as possible to the algorithm. Satisfying all of the above constraints results in the sampling period being a constant of approximately one-tenth of a second. Let $x_d^{(k)}$ be the state vector $x_d(t)$ when t equals $k\tau$. Since a zero-order hold is used, let $u_p^{(k)}$ be equal to $u_p(t)$ over the time interval $[k\tau,(k+1)\tau]$. The difference equations governing the desired system are found by solving equation (2.2) over the time interval $[k\tau,(k+1)\tau]$. The solution is given by

$$x_d^{(k+1)} = \Phi_d(\tau)x_d^{(k)} + \Gamma_d(\tau)u_p^{(k)} \qquad (2.3)$$

Where

$$\Phi_d(\tau) = \exp(A_d\tau)$$

$$\Gamma_d(\tau) = \int_0^{\tau} \exp(A_d\sigma)d\sigma B_d$$

Equation (2.3) represents the form that will be used to describe the model dynamics throughout the remainder of this paper.

III. THE OPTIMAL ADAPTIVE DIGITAL CONTROLLER

The objective of the digital adaptive controller is to minimize some performance index over a single time step. The performance index that will be used is the well known quadratic cost functional which has the form of

$$J = 1/2r'Qr \tag{3.1}$$

Here J is a scalar performance index, Q is a symetric weighting matrix and r is vector for which the minimum is desired.

Consider the state of the model and the system in two sequential time increments as shown in Figure 3. If the system is in the state $x^{(k)}$ at time $k\tau$, the application of the control $u^{(k)}$ over the interval $[k\tau,(k+1)\tau]$ will result in the system at state $x^{(k+1)}$. Similarly, if the desired model is in the state $x_d^{(k)}$, assumed here to be the same as $x^{(k)}$, at time $k\tau$, the application of the control $u_p^{(k)}$ will result in the model at state $x_d^{(k+1)}$. The algorithm is developed by considering the system at time $(k+1)\tau$ as shown in Figure 3. Now consider the optimal control which if it had been applied to the system over the time interval $[k\tau,(k+1)\tau]$ would have minimized the functional J operating on the state vector formed by the difference between the model and system state vectors at time $(k+1)\tau$. This control will be defined as the virtual control, $u_v^{(k)}$. Once the virtual control has been calculated the difference between it and the applied control, $u^{(k)}$, is added to a control correction vector, $u_c^{(k)}$. In an overall view of the controller, the control correction vector is the sum of all the previous calculated corrections to the applied control. This control correction vector is combined with the pilot input command, $u_p^{(k+1)}$, to form the input to the system, $u^{(k+1)}$, over the next sampling period. At the end of the next sampling period the procedure is repeated.

It was originally assumed by Whitaker that the model and system should be dynamically similar if the controller is working properly. This assumption is not completely correct and is the subject of the next section. For the present it will be assumed to be valid. This assumption is mathematically represented by

$$\frac{\partial x^{(k+1)}}{\partial u^{(k)}} = \frac{\partial x_d^{(k+1)}}{\partial u_p^{(k)}} \tag{3.2}$$

Taking the partial derivative of $x_d^{(k+1)}$ with respect to $u_p^{(k)}$ in equation (2.3) will result in

$$\frac{\partial x_d^{(k+1)}}{\partial u_p^{(k)}} = \Gamma_d \tag{3.3}$$

The state vector of the virtual system at the end of the sampling period can now be formulated using equations (3.2), (3.3) and the definition of the virtual control to be

$$x_v^{(k+1)} = x^{(k+1)} + \Gamma_d(u_v^{(k)} - u^{(k)}) \tag{3.4}$$

The objective is now to find $(u_v^{(k)} - u^{(k)})$ which minimizes the cost functional described by equation (3.1) with r equal to $(x_d^{(k+1)} - x_v^{(k+1)})$. The minimization of this functional can be accomplished by differention of equation (3.1) with respect to $(u_v^{(k)} - u^{(k)})$, setting the resulting derivative equal to zero, and solving the expression for $(u_v^{(k)} - u^{(k)})$. Performing the above tasks will result in

$$u_v^{(k)} - u^{(k)} = (\Gamma_d{}' Q\Gamma_d)^{-1} \Gamma_d{}' Q (x_d^{(k+1)} - x^{(k+1)}) \quad (3.5)$$

However, experience has shown that while equation (3.5) yields good results when there exists a unique solution to the minimization problem, it may yield poor results when more than one solution exists to the minimization problem. A typical result with a system with more than one control is that two or more of the controls tend to work against each other. This problem is combated by augmenting the cost functional as shown in

$$J = 1/2 \ (x_d^{(k+1)} - x_v^{(k+1)})' Q_1 (x_d^{(k+1)} - x_v^{(k+1)})$$

$$+ 1/2 \ (u_v^{(k)} - u^{(k)})' Q_2 (u_v^{(k)} - u^{(k)}) \quad (3.6)$$

Minimization of this functional gives the solution to the problem of interest with the smallest correction to the control. The minimum of this cost functional is given by

$$u_v^{(k)} - u^{(k)} = (\Gamma_d{}' Q_1 \Gamma_d + Q_2)^{-1} \Gamma_d{}' Q_1 (x_d^{(k+1)} - x^{(k+1)}) \quad (3.7)$$

Whichever equation is used, the remaining equations of the digital adaptive algorithm are given in

$$u_c^{(k+1)} = u_c^{(k)} + (u_v^{(k)} - u^{(k)}) \qquad (3.8)$$

$$u^{(k+1)} = u_p^{(k+1)} + u_c^{(k+1)} \qquad (3.9)$$

IV. STABILITY

The stability of the controller is dependent upon the accuracy of the control effectiveness, i.e., equation (3.2). In this section, an analytical expression is formulated which can be used as a stability criterion. The stability analysis will be done subject to the following restrictions:

a) The state vector of the system can be represented by the difference equation

$$x^{(k+1)} = \Phi x^{(k)} + \Gamma u^{(k)} \tag{4.1}$$

b) Φ and Γ of equation (4.1) are constant over some finite period of time.

c) The input command is constant.

From the above assumptions, it follows directly that the desired input command to the system will also be constant. Let this system command be corrupted by some error $\varepsilon^{(k)}$ as shown in

$$u^{(k)} = u^{*} + \varepsilon^{(k)} \tag{4.2}$$

In this equation, $u^{(k)}$ is the system command, u^{*} is the optimal system input which will minimize the functional J and $\varepsilon^{(k)}$ is the error in the command. Furthermore, assume the optimal control will result in the system following the model, i.e., there will be zero error between the model and system state vectors, thereby making the following equation true.

$$x_d^{(k+1)} = \phi x^{(k)} + \Gamma u^* \tag{4.3}$$

It follows from equation (4.1) that equation (4.4) may be written:

$$x^{(k+1)} = x_d^{(k+1)} + \Gamma\ \varepsilon^{(k)} \tag{4.4}$$

Let the equation defined by equation (3.5) or (3.7) be written as

$$u_v^{(k)} - u^{(k)} = G\ (x_d^{(k+1)} - x^{(k+1)}) \tag{4.5}$$

Where G is the matrix that is defined in equation (3.5) or (3.7). Combining equations (4.4) and (4.5) will result in

$$u_v^{(k)} - u^{(k)} = -\ G\ \Gamma\ \varepsilon^{(k)} \tag{4.6}$$

From equation (3.9), (4.2) and (4.6) equation (4.7) can be derived as

$$u^{(k+1)} = u^* + \varepsilon^{(k)} - G\ \Gamma\ \varepsilon^{(k)} \tag{4.7}$$

Equation (4.7) may be rewritten in the form

$$u^{(k+1)} = u^* + \varepsilon^{(k+1)}$$

$$\varepsilon^{(k+1)} = (I - G\Gamma)\varepsilon^{(k)} \tag{4.8}$$

Upon repeated application of equation (4.8) and letting the indice, k, equal zero, will result in

$$\varepsilon^{(L)} = (I - G\Gamma)^L\ \varepsilon^{(0)} \tag{4.9}$$

If the eigenvalues of the matrix $(I - G\Gamma)$ are denoted by μ_ℓ and the corresponding eigenvectors be denoted by ν_ℓ, then from matrix theory, equation (4.10) may be written as

$$(I - G\Gamma)\ \nu_\ell = \mu_\ell\ \nu_\ell \qquad (4.10)$$

Also, if the eigenvalues of the matrix $(I - G\Gamma)$ are distinct, the vector $\varepsilon^{(0)}$ can be expressed as a linear combination of the eigenvectors ν_ℓ in

$$\varepsilon^{(0)} = \sum_{\ell=1}^{m} \alpha_\ell\ \nu_\ell \qquad (4.11)$$

By combining equations (4.10) and (4.11) equation (4.9) may be rewritten as

$$\varepsilon^{(L)} = \sum_{\ell=1}^{m} \alpha_\ell\ \mu_\ell^{\ L}\ \nu_\ell \qquad (4.12)$$

As L approaches infinity, the error will not grow if the magnitude of all the eigenvalues of the matrix $(I-G\Gamma)$ are, in absolute value, less than or equal to one.

To illustrate further the causes of an unstable controller, consider the example of a system with a single state variable and a single control. With a single control there is a unique control which will minimize equation (3.1). Therefore, if the weighting matrix Q is equal to one, equation (3.5) will reduce to the following equation:

$$G = (\Gamma_d{}'\Gamma_d)^{-1}\ \Gamma_d{}' = \frac{1}{\Gamma_d}$$

From the above discussion, the eigenvalue of

$$1 - \frac{\Gamma}{\Gamma_d}$$

must be in absolute value less than one. This leads to the following inequality

$$\left|1 - \frac{\Gamma}{\Gamma_d}\right| < 1$$

There are two conditions which must be satisfied in order for the above inequality to be true. These conditions are given by the following:

$$\frac{\Gamma}{\Gamma_d} > 0 \tag{4.13}$$

$$\Gamma < 2\Gamma_d \tag{4.14}$$

In these equations, it should be noted that Γ_d is the estimate of Γ which was assumed at the beginning of the development of the digital adaptive controller in equation (3.3). Equation (4.13) shows this estimate must have the same sign as Γ in order for the controller to be stable. For a system with more than one control, the resulting matrix formed from the product of the G matrix, as defined in equation (4.5) and the Γ matrix must be positive definite. The physical interpretation of this type of instability can be seen by noting the error between the model and the system can never be made smaller by applying the correction in the opposite direction. The second form of instability is governed by equation (4.14). This instability occurs when the correction overcompensates for the error by such an amount that the resulting error is equal to or greater than, but opposite in sign, of the previous error. This type of instability is illustrated by the comparison of Figures 4 and 5 with Figures 6 and 7.

V. RESULTS AND CONCLUSIONS

Two examples are presented to illustrate the effectiveness of the controller. The first example considered is a second order system with a single control. This example was chosen in order to illustrate the controller being used to suppress an undesirable mode of vibration from the system. The system was therefore chosen to be lightly damped while the model was selected to be well damped. The values used in the model and system equations are listed in Table 1. The response of the system, model and augmented system to a step in amplitude of five units is shown in Figures 8, 9 and 10. As the figures illustrate, the system does not follow the model perfectly, but tends to lag the model. This is a typical and expected result of the behavior of the controller since the optimization was performed over the previous sampling cycle. Therefore, it is to be expected that the augmented system will lag the model. However, the controller does suppress the underdamped oscillation of the system, and although it is not shown in the figure, the augmented system will reach zero steady state error.

The second example tested was a short period approximation of a typical medium transport aircraft. The equations of motion for the system were obtained from Etkin[1] by noting that the velocity remains fairly constant during the short period mode of oscillation. The motion of the aircraft can therefore be described with the alpha, theta and pitch rate equations. The controls of the aircraft were assumed to be the elevator position and thrust coefficient. Let it be noted here that the non-linear equations of motion were linearized about a steady state operating point and the state variables used are devations from this operating point. In Table 2 the values of the equation's coefficients along with the calculated gains and pertinent parameters are listed.

In this example it was desired to test the controller against a system with changing parameters. Since the data needed to construct a simulation model for such a plant was not available, an alternate method was chosen to achieve the desired effect. To simulate a plant with changing coefficients, the original system coefficients, i.e., the A_d and B_d matries of equation (2.1), were allowed to vary randomly for a random amount of time. The amount that each coefficient was changed did not exceed in absolute value eighteen percent of the original value and it was not held constant for more than five seconds. The model used was simply the original undisturbed system. The augmented system was tested with a control input of a step in the elevator control of one-tenth of a radian. The responsesof the system, model and augmented system are shown in Figures 11 through 15. As in the previous example, the augmented system lags the model; but unlike the first example, it never has zero steady state error. Zero steady state error is never reached because the controller does not have enough time to adapt to the new system. Even with this error, the area between the desired model and the unaugmented system is greater than the area between the desired system and the augmented system.

These two examples illustrate a characteristics of the controller to require a finite amount of time to adapt to the new system. The amount of time required is related to the accuracy of the assumed value of the control effectiveness. The more accurate this estimate the faster the system will follow the model. Earlier it was shown that if this estimate exceeded the stability criteria developed previously that the controller would become unstable. For this reason it may become necessary to add to the algorithm a section dealing with system identification in order to update the assumed value of the control effectiveness used.

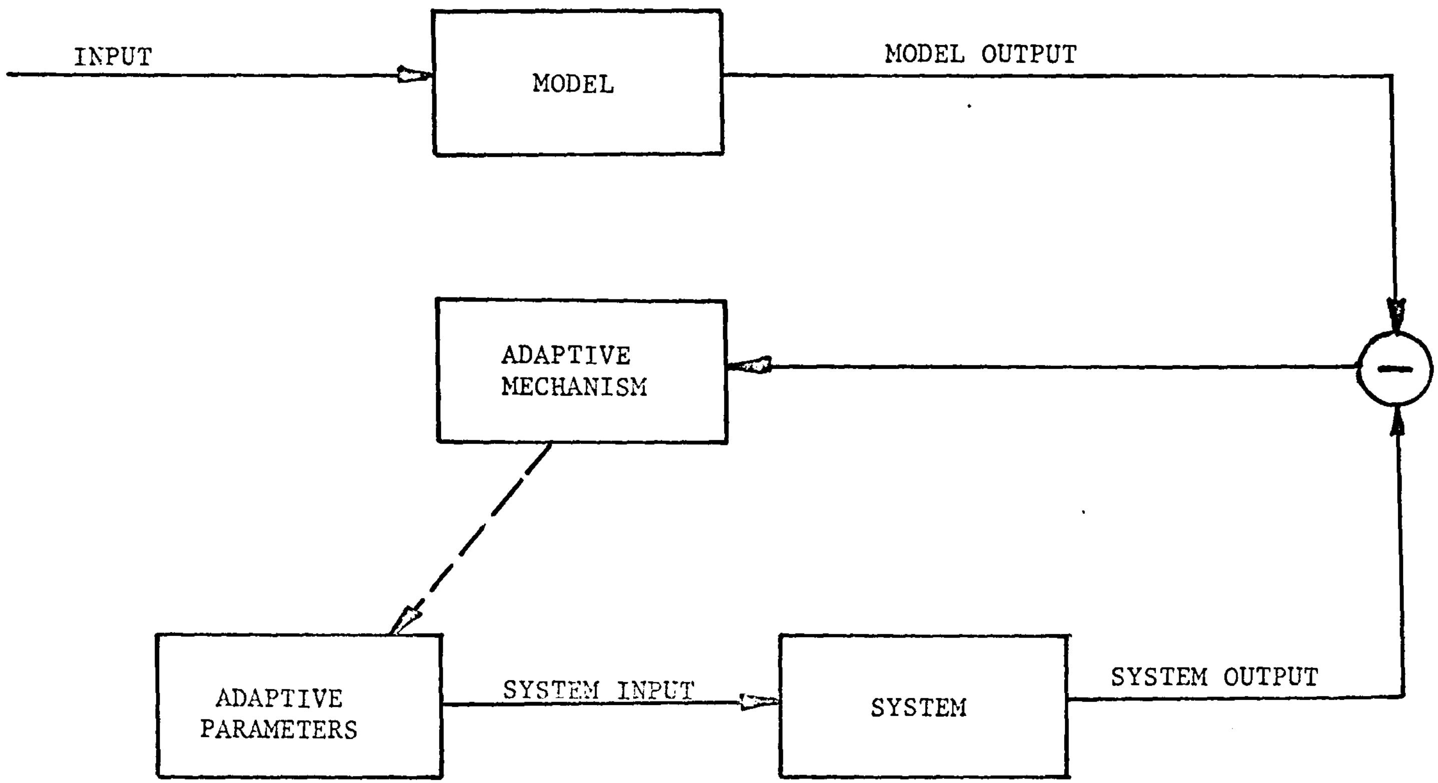

Figure 1. Model-referenced Adaptive Controller Block Diagram.

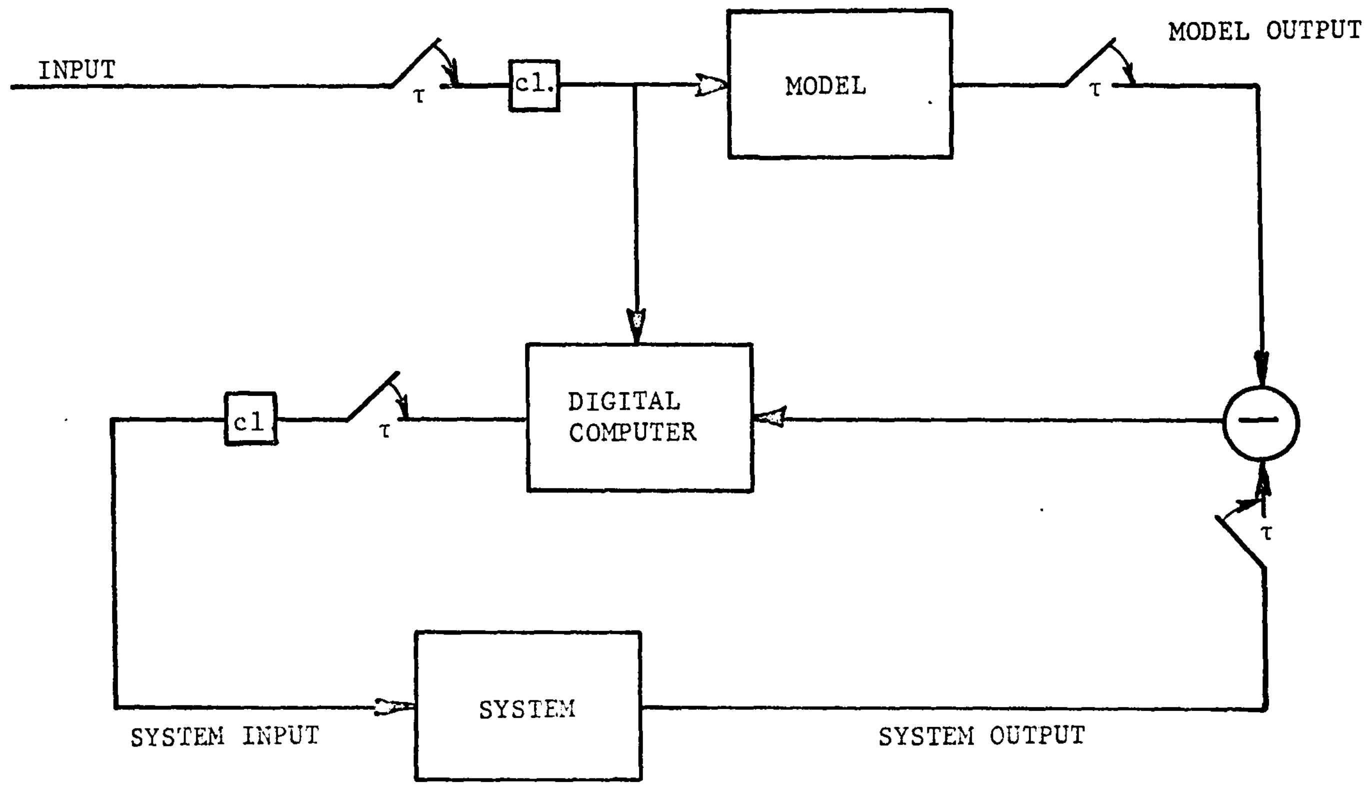

Figure 2. Digital Model-referenced Adaptive Controller Block Diagram.

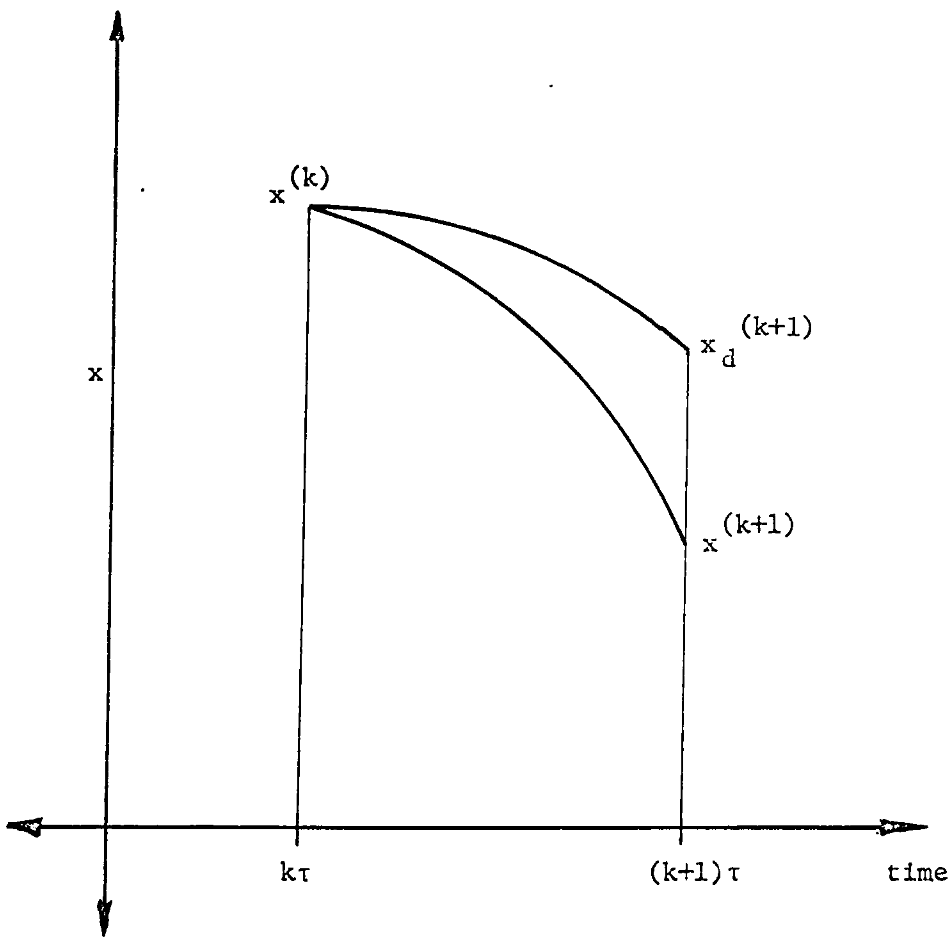

Figure 3. System and Model Space State.

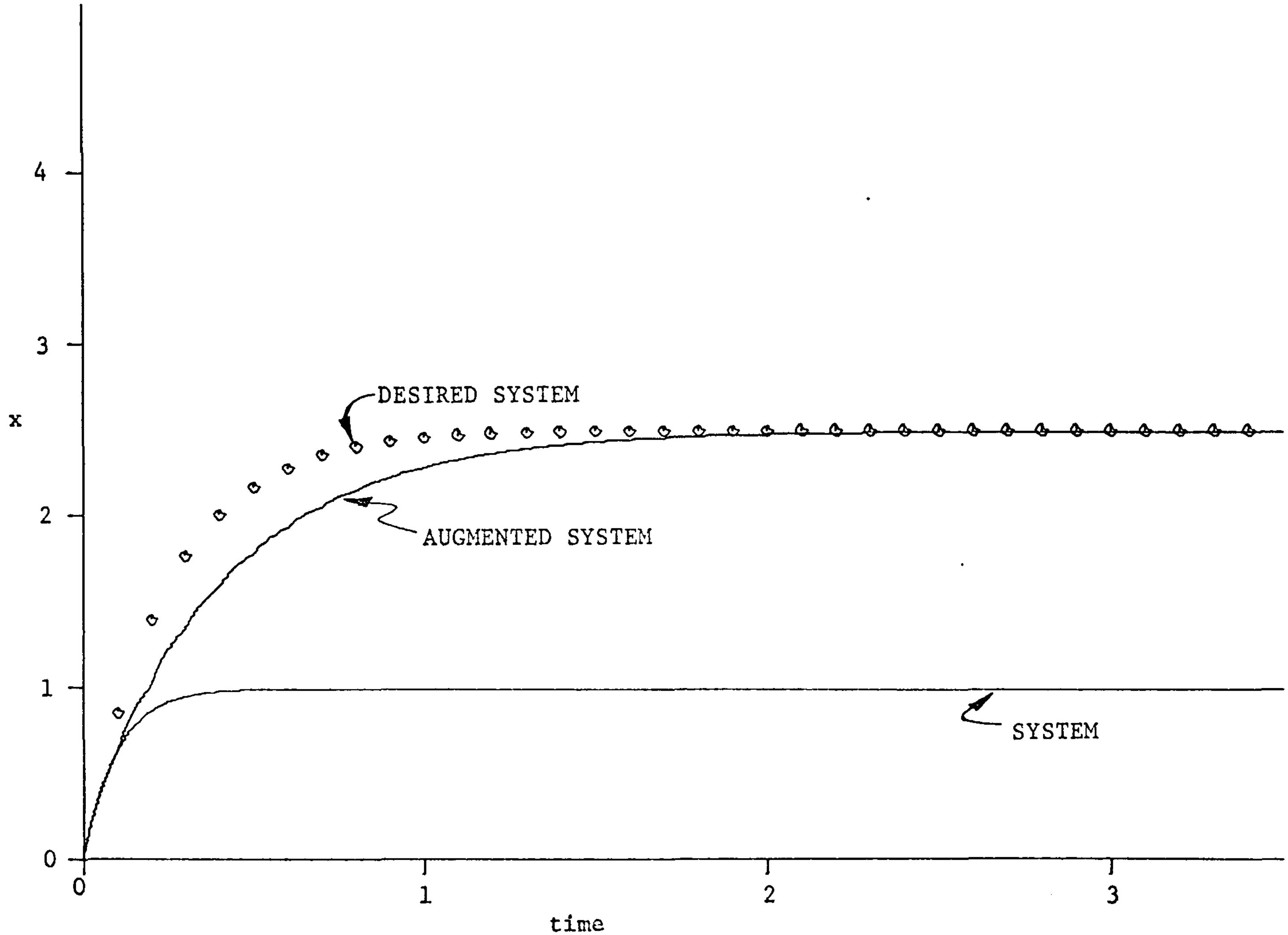

Figure 4. Stability Example, Stable System, State vs Time.

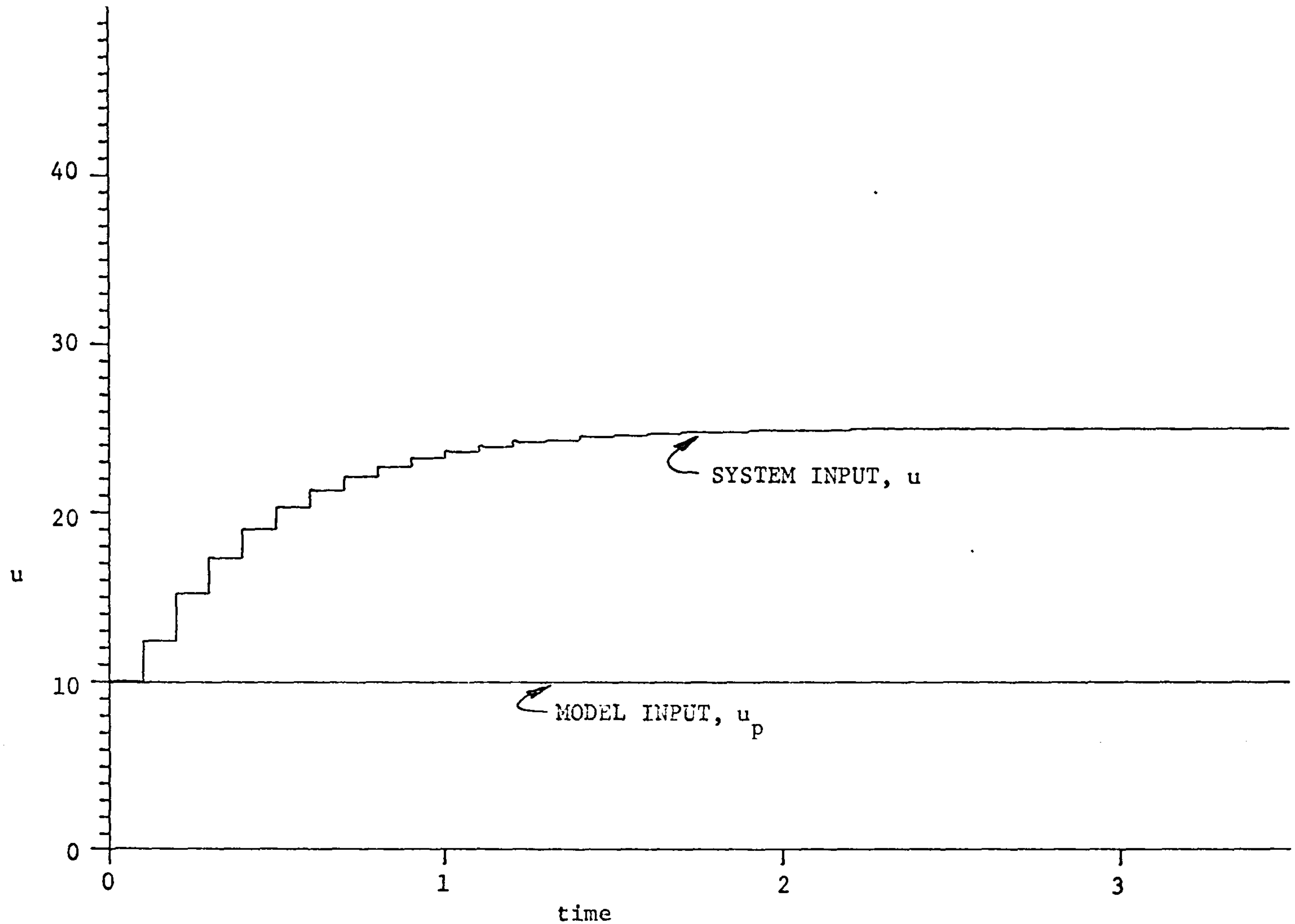

Figure 5. Stability Example, Stable System, Control vs Time.

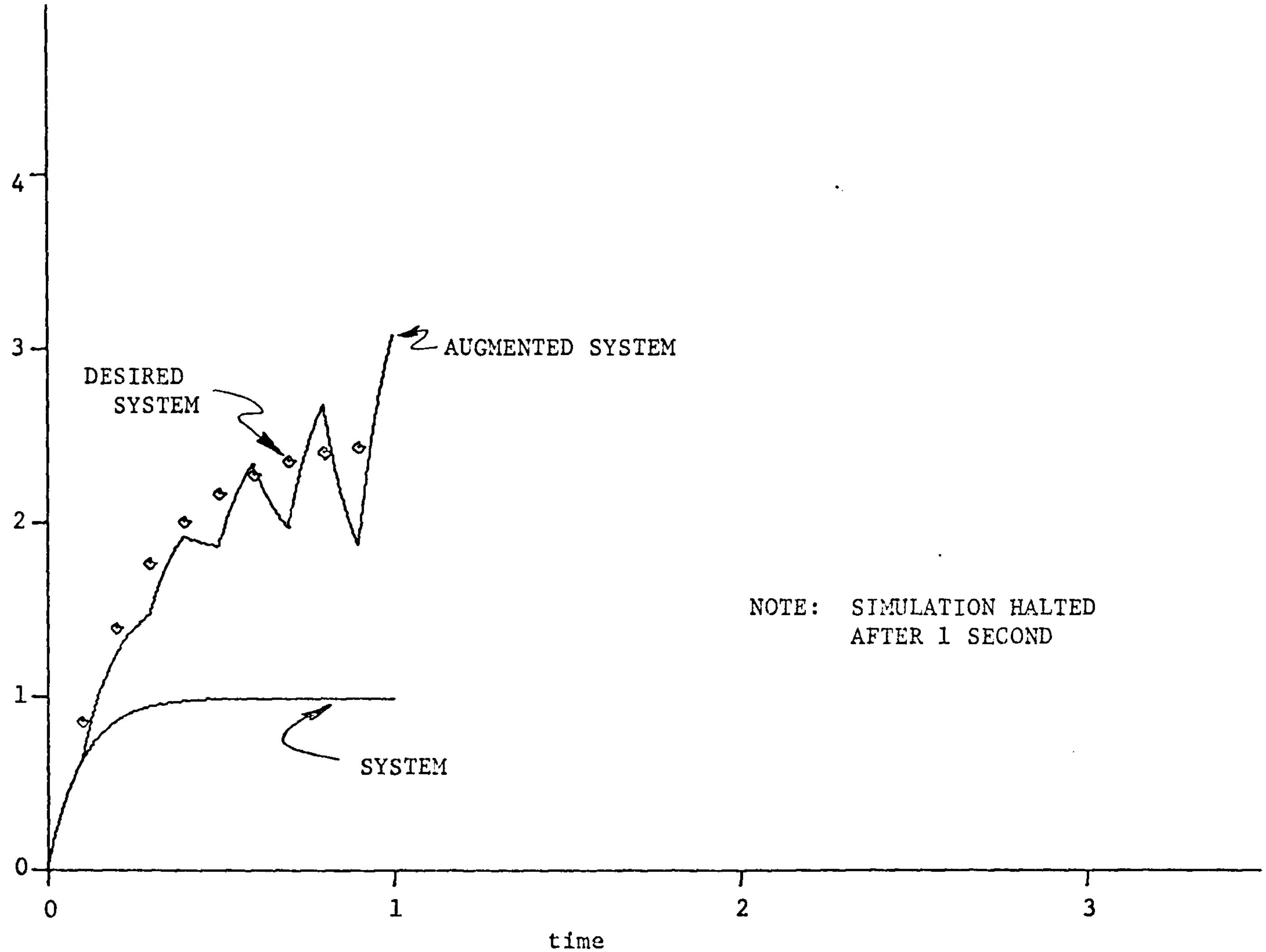

Figure 6. Stability Example, Unstable System, State vs Time.

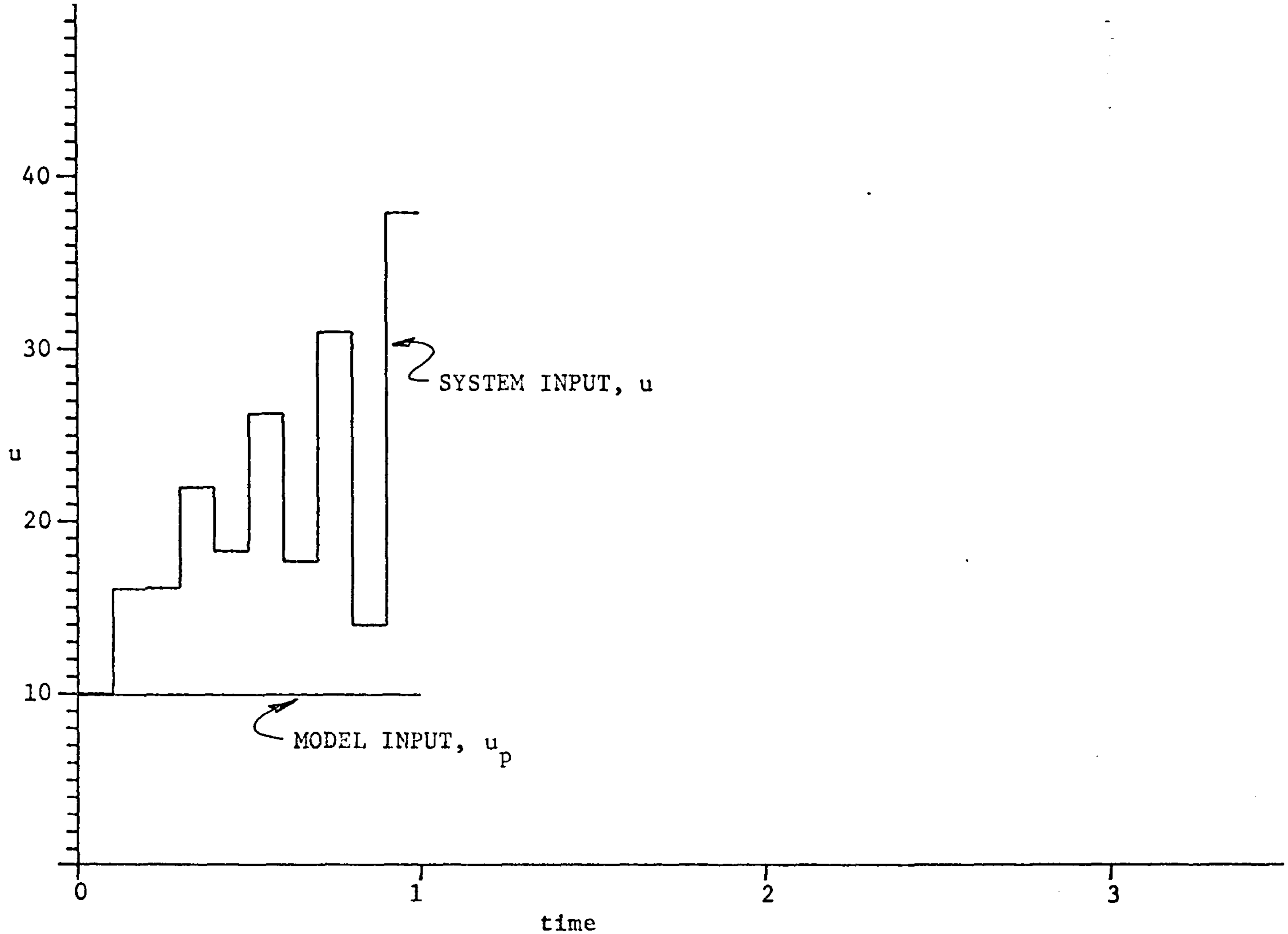

Figure 7. Stability Example, Unstable System, Control vs Time.

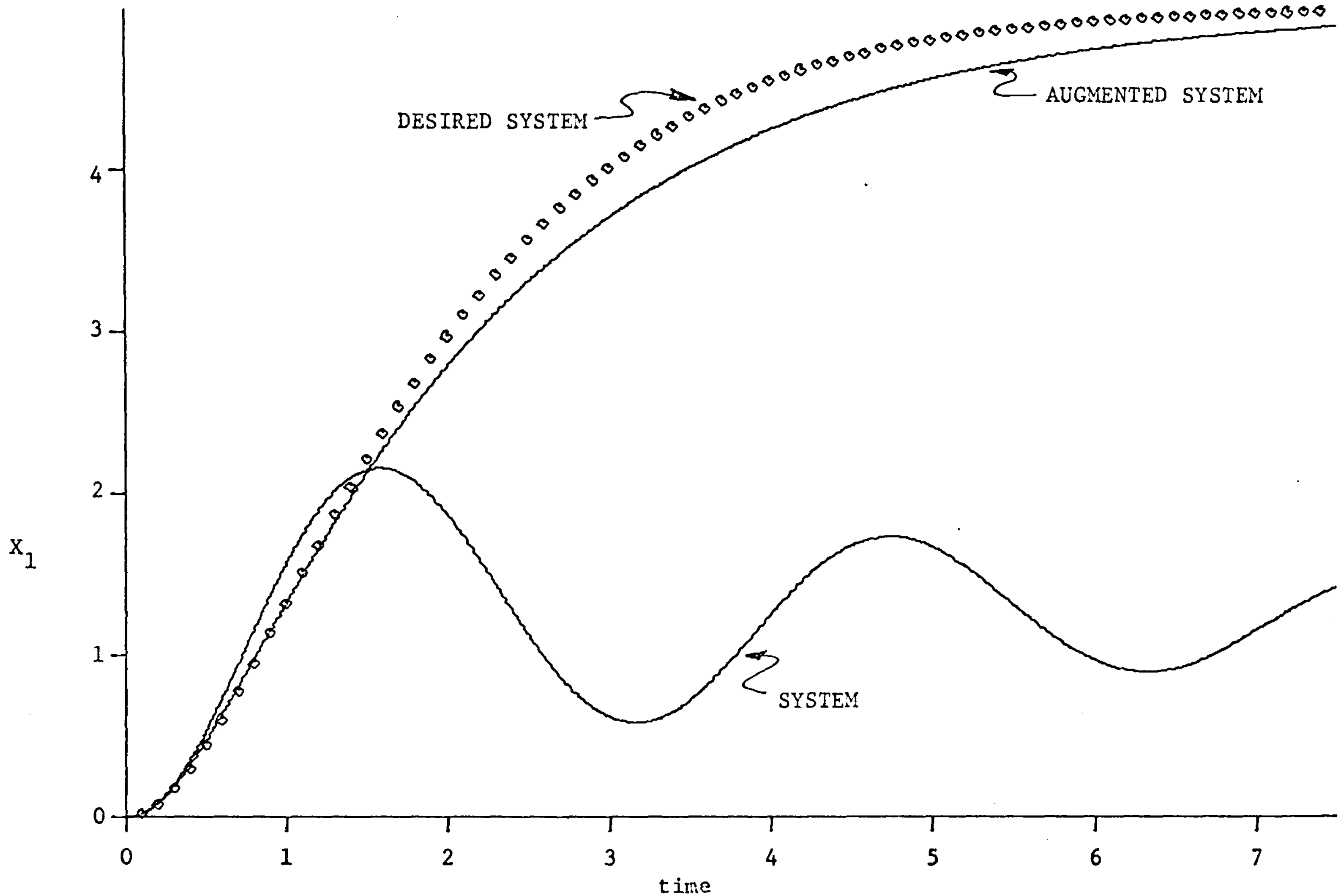

Figure 8. Second Order Example, X_1 vs Time.

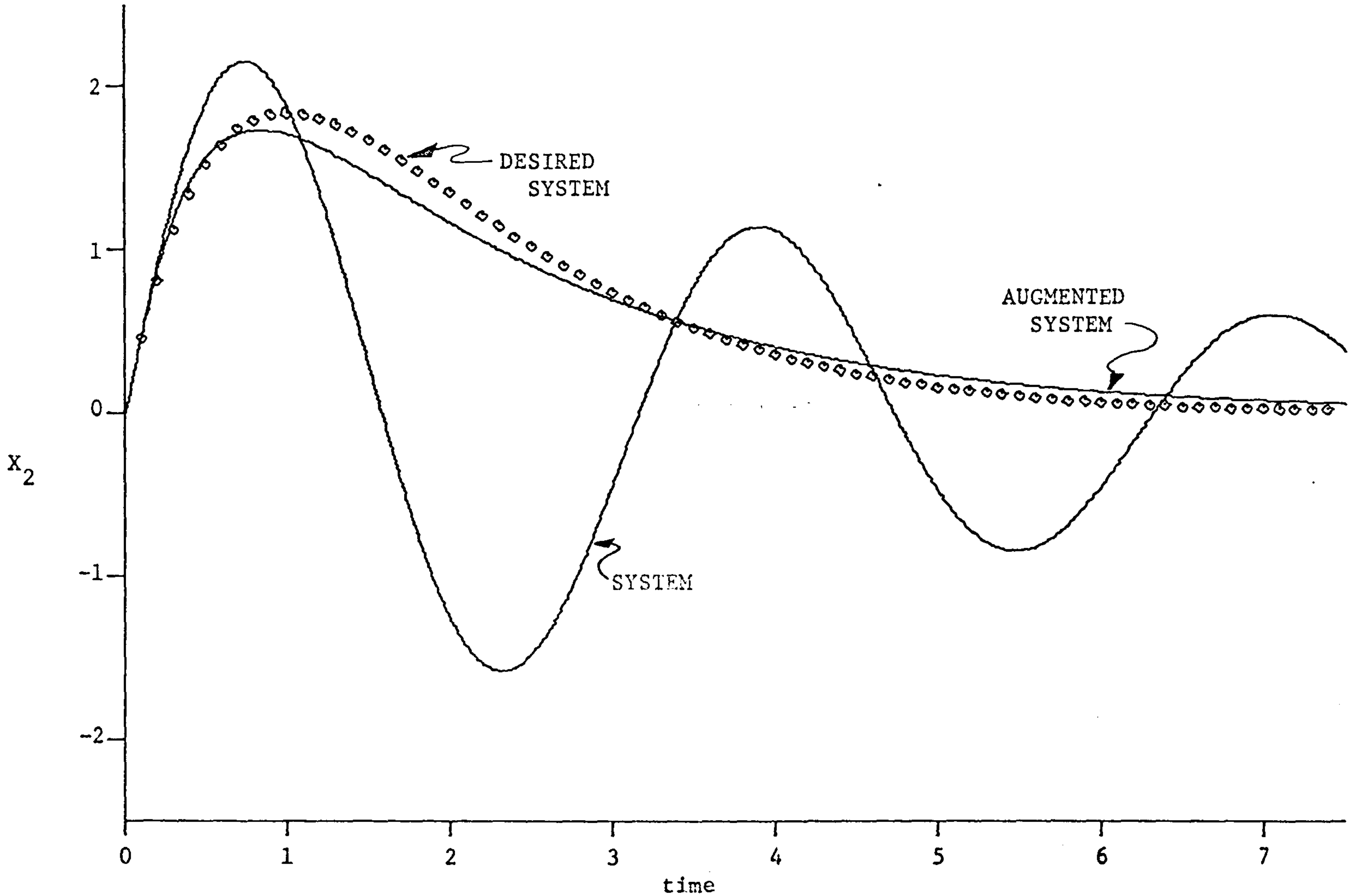

Figure 9. Second Order Example, X_2 vs Time.

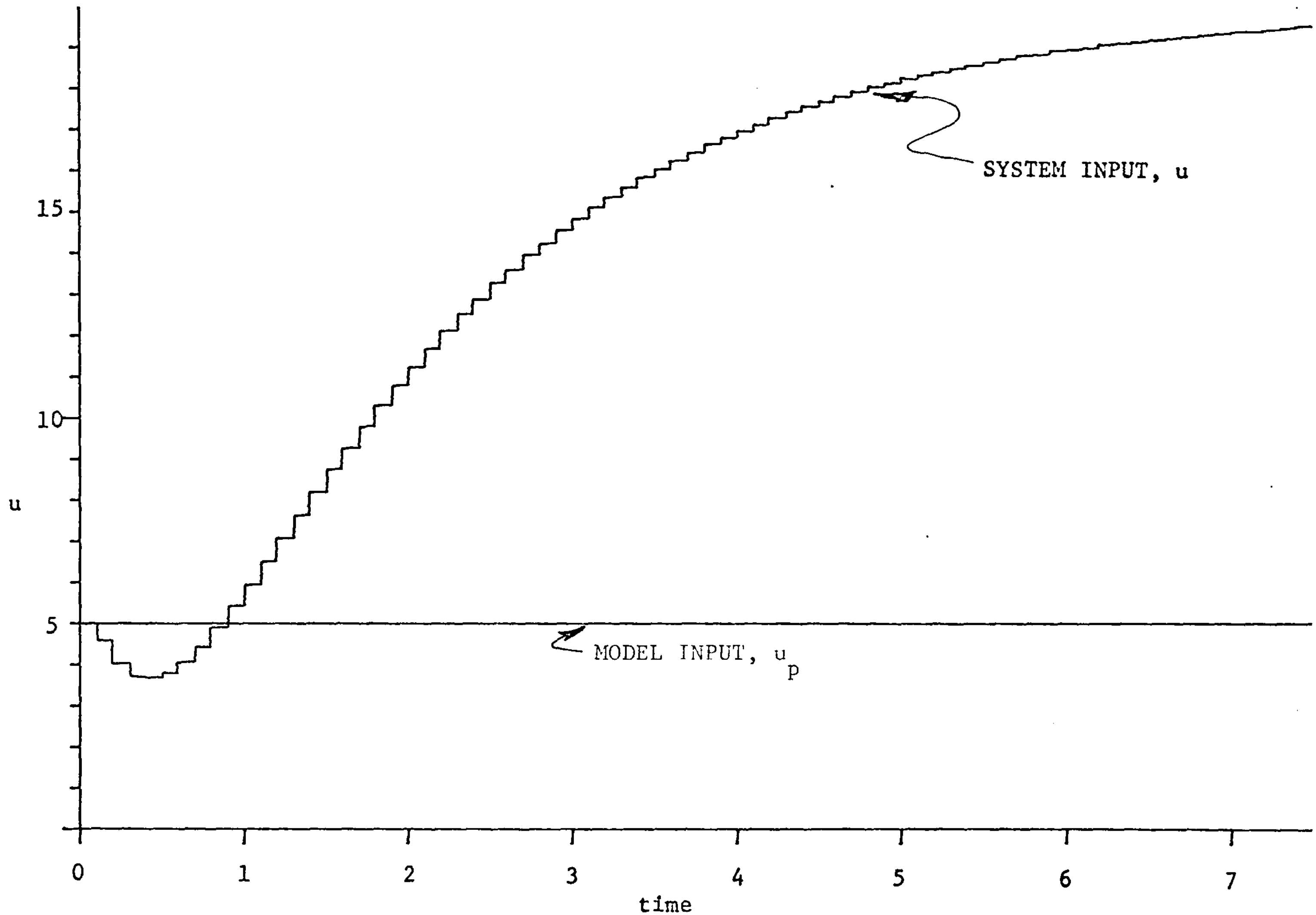

Figure 10. Second Order Example, Control vs Time.

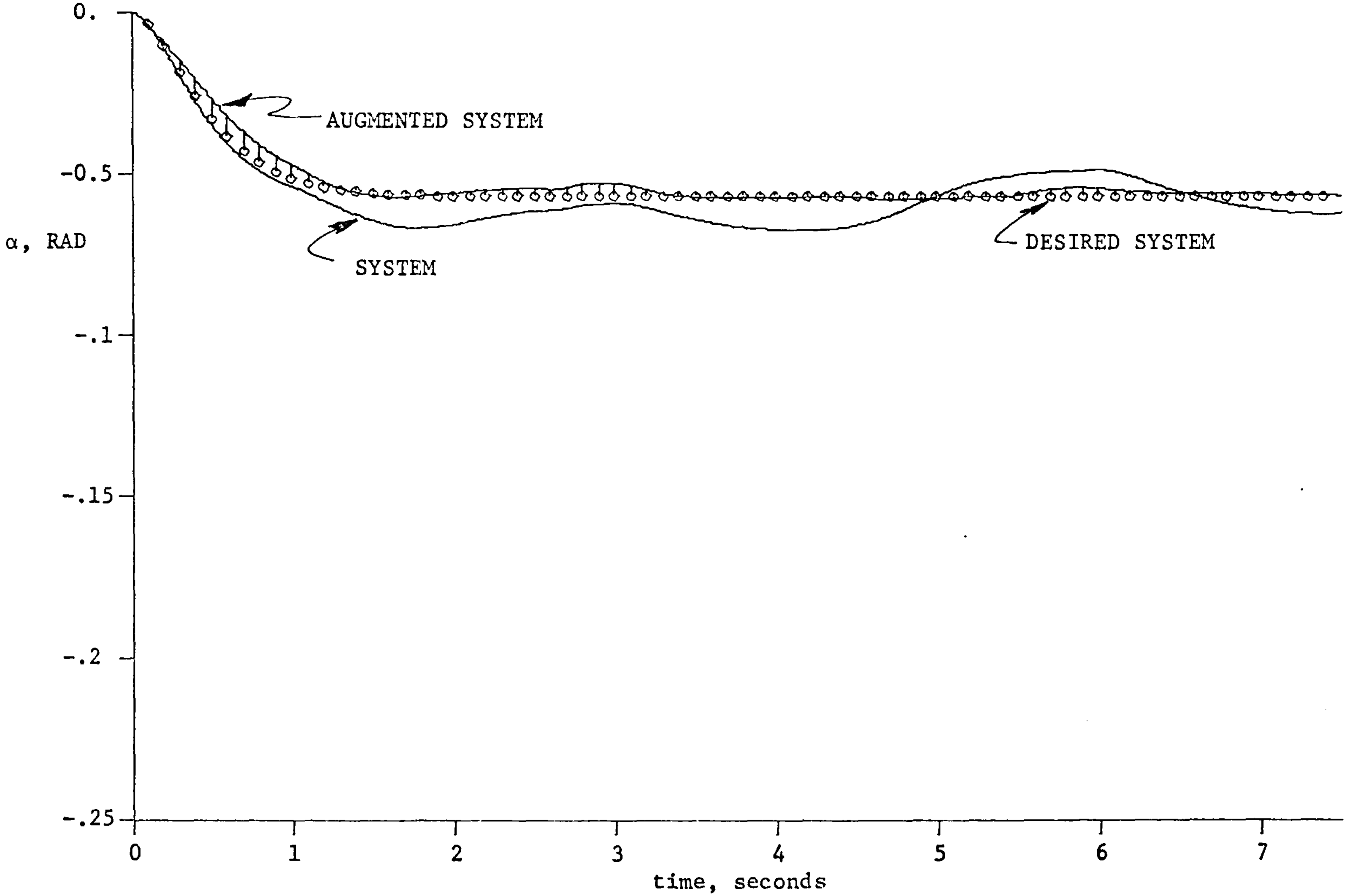

Figure 11. Third Order Example, Alpha vs Time.

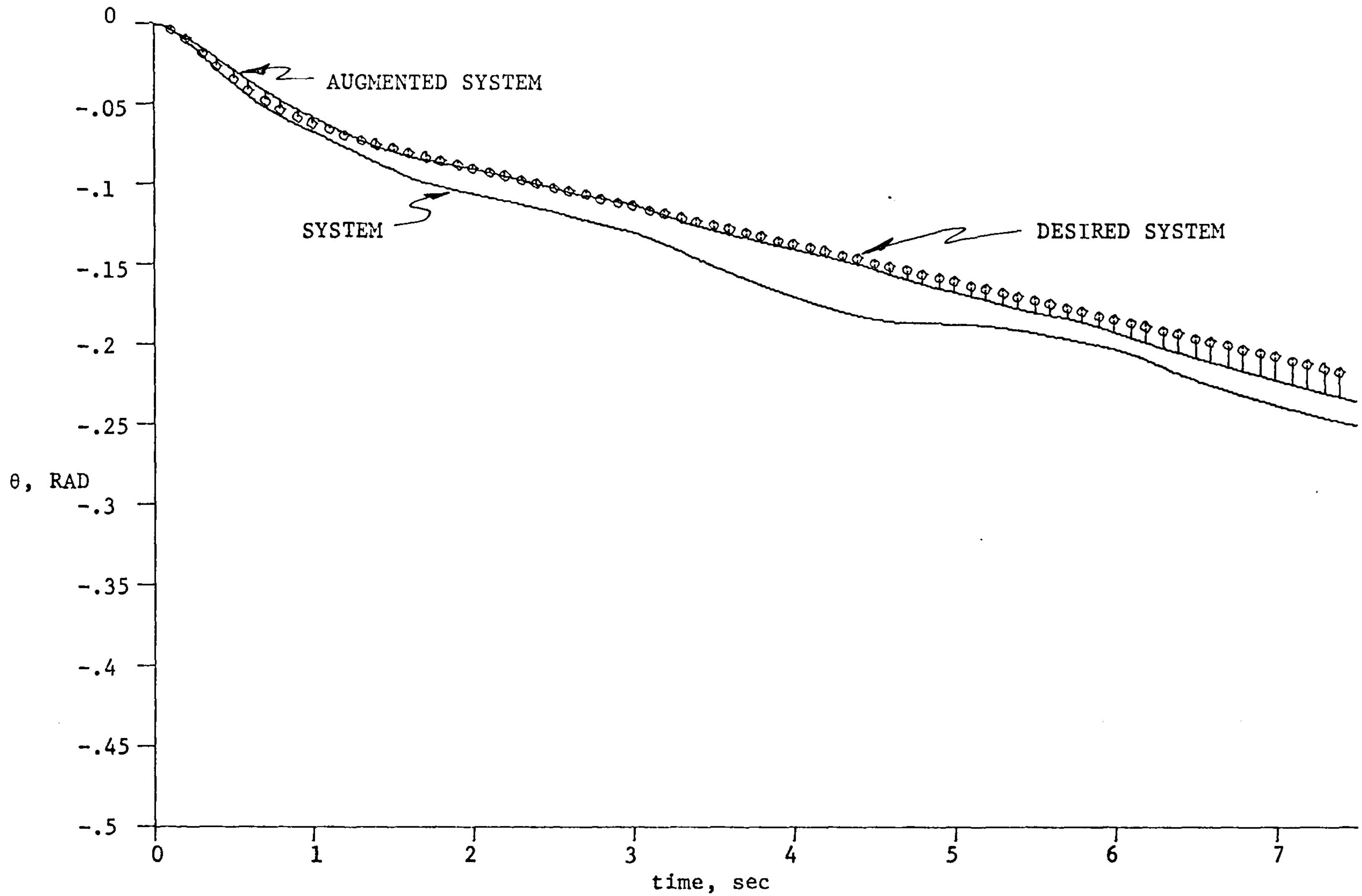

Figure 12. Third Order Example, Theta vs Time.

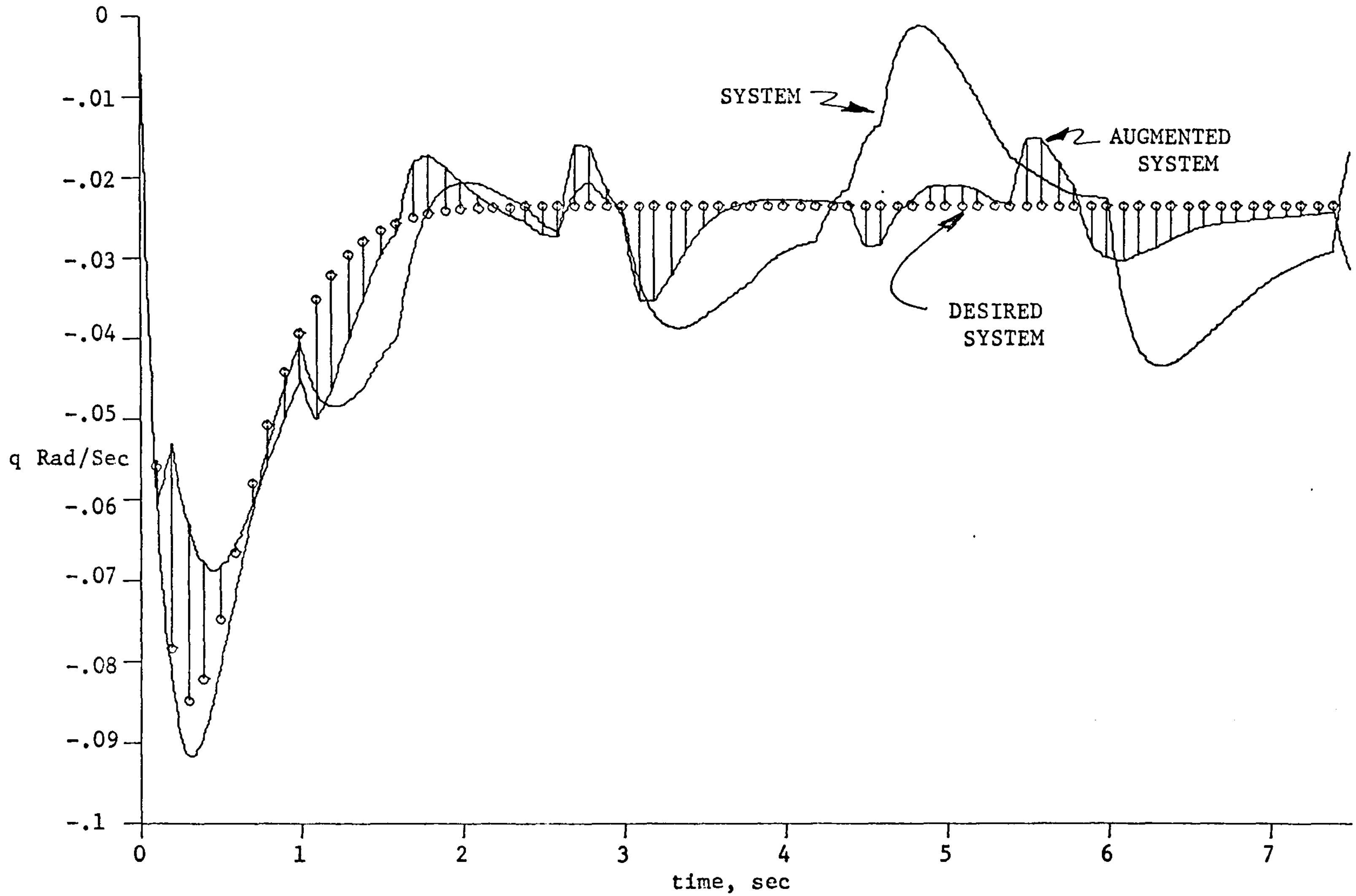

Figure 13. Third Order Example, Pitch Rate (q) vs Time.

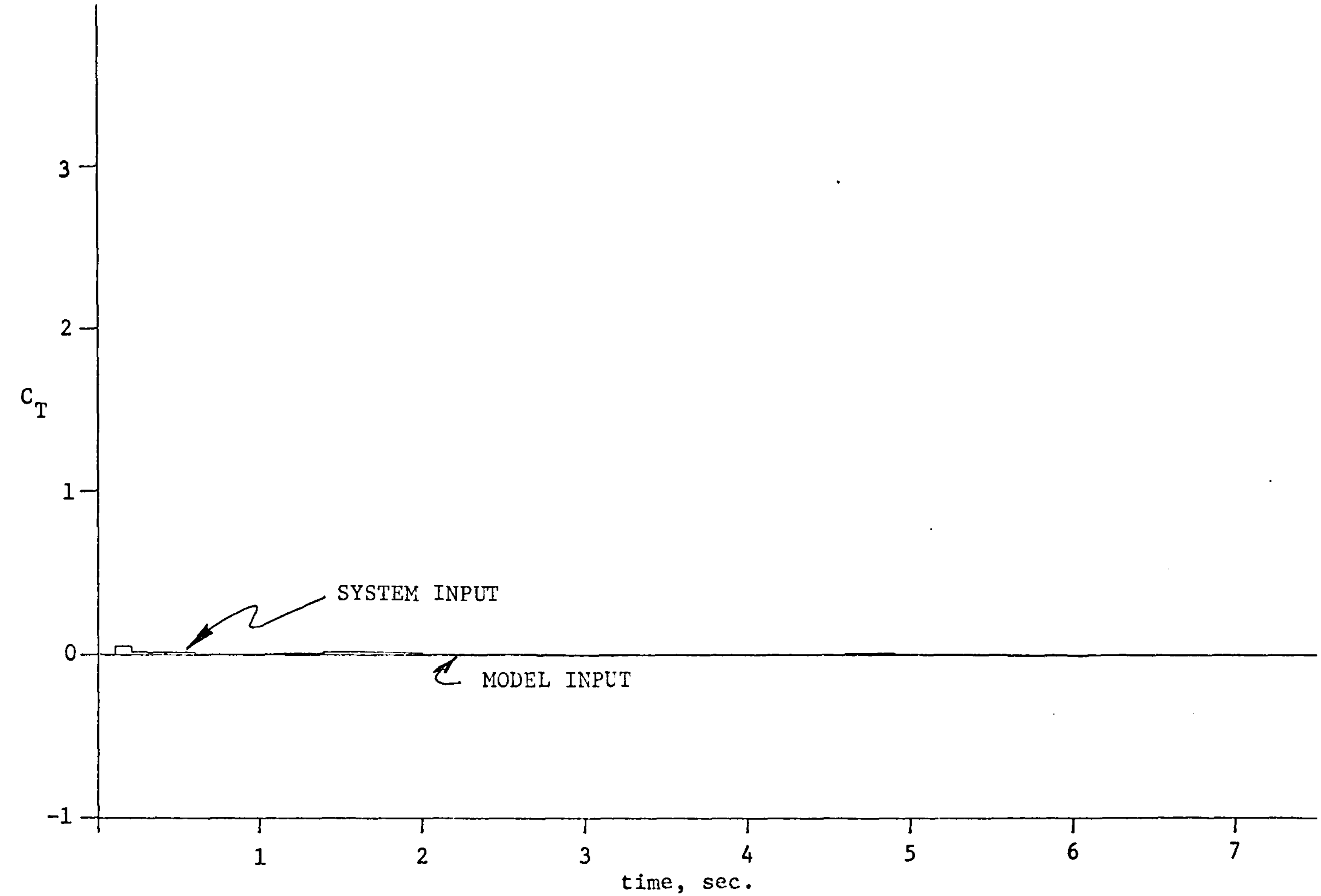

Figure 14. Third Order Example, Thrust Coefficient vs Time.

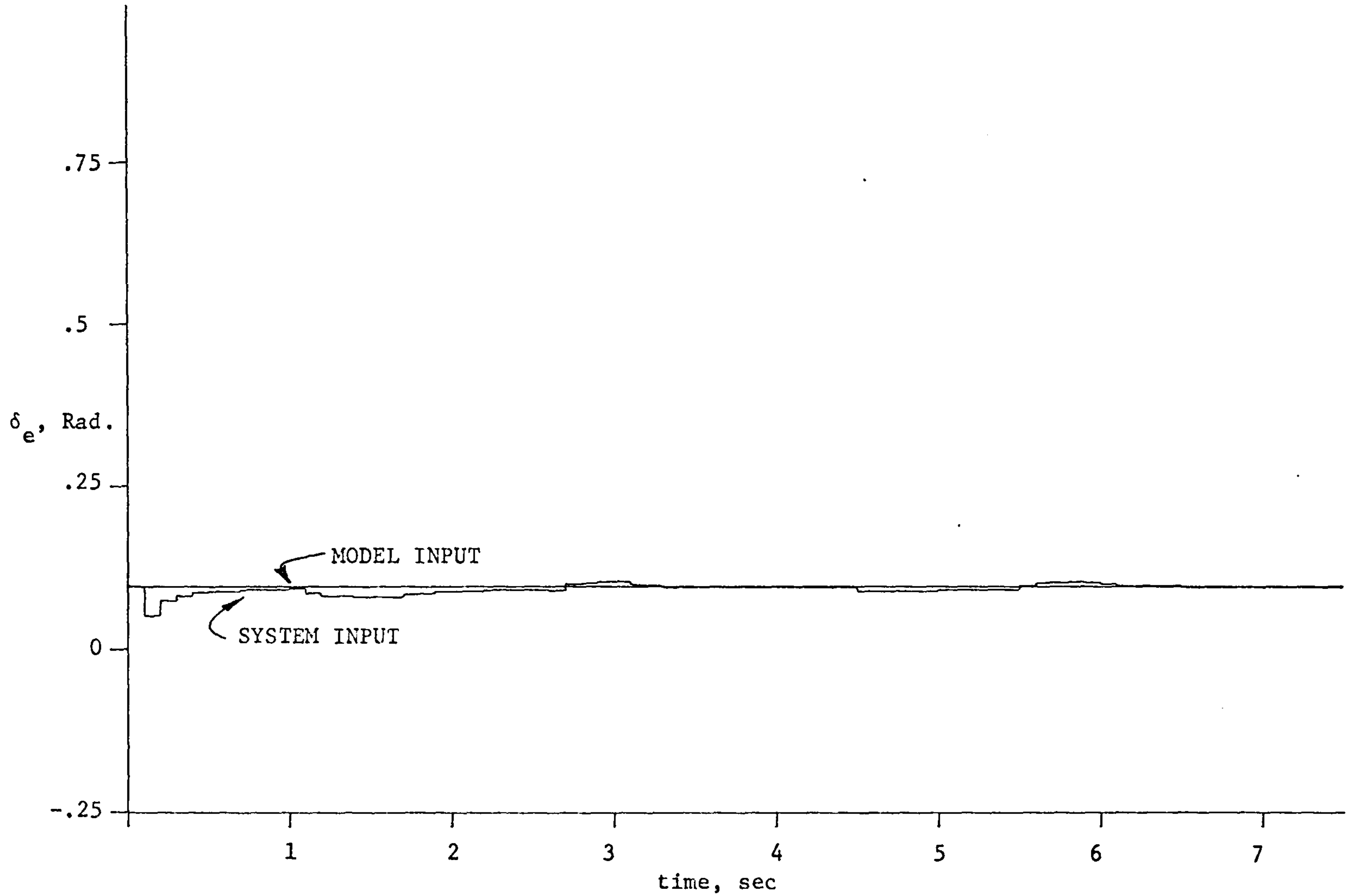

Figure 15. Third Order Example, Elevator Position (δ_e) vs Time.

TABLE 1. System Parameters Used for Second Order Example.

$$A = \begin{bmatrix} 0 & 1 \\ -4. & -.4 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 1. \end{bmatrix}$$

$$A_d = \begin{bmatrix} 0. & 1. \\ -1. & -2. \end{bmatrix} \qquad B_d = \begin{bmatrix} 0 \\ 1. \end{bmatrix}$$

$$Q = \begin{bmatrix} 1. & 0. \\ 0 & 1. \end{bmatrix}$$

$$(\Gamma_d{}' \; Q\Gamma_d)^{-1} \; \Gamma_d \; Q = [.57 \quad 11.02]$$

TABLE 2. System Parameters Used for Third Order Example.

$$\dot{x} = Ax + Bu$$

$$x' = [\alpha, \theta, q]$$

$$u' = [\delta_T, \delta_e]$$

$$D = \frac{1}{1 + \frac{\rho S \bar{c} C_{L_{\dot{\alpha}}}}{4m}} , \quad E = \frac{\rho V_e^2 S \bar{c}}{2 I_y} , \quad F = \frac{\bar{c}}{2V_e}$$

$$a_{11} = - D \left[\rho \frac{V_e}{2m} S C_{L_\alpha} + \frac{T_e \cos(\alpha_e)}{mV_e}\right]$$

$$a_{12} = 0.$$

$$a_{13} = 1 - D \left[\frac{\rho S \bar{c} C_{L_q}}{4m}\right]$$

$$a_{21} = 0$$

$$a_{22} = 0.$$

$$a_{23} = 1.$$

$$a_{31} = E(C_{m_\alpha} + F C_{m_{\dot{\alpha}}} a_{11})$$

$$a_{32} = 0$$

$$a_{33} = E F (C_{m_q} + C_{m_{\dot{\alpha}}} a_{13})$$

$$b_{11} = - \frac{D T_e \sin(\alpha_e)}{mV_e}$$

$$b_{12} = - \frac{D \rho V_e S C_{L_{\delta_e}}}{2m}$$

$$b_{21} = 0$$

$$b_{22} = 0.$$

TABLE 2. System Parameters Used for Third Order Example - continued

$b_{31} = E \quad [C_{m_{\delta_T}} + F\, C_{m_{\dot{\alpha}}} \quad b_{11}]$

$b_{32} = E \quad [C_{m_{\delta_e}} + F\, C_{m_{\dot{\alpha}}} \quad b_{12}]$

$\rho = .0015 \text{ lb sec}^2 \text{ ft}^{-4}$

$m = 6428.6 \text{ lb sec}^2 \text{ ft}^{-1}$

$I_y = 5.15 \times 10^5 \text{ ft}^4$

$S = 2892. \text{ ft}^2$

$\bar{c} = 23. \text{ ft}$

$V_e = 338 \text{ ft sec}^{-1}$

$\alpha_e = .14 \text{ rad}$

$T_e = 10^4 \text{ lb}$

$q_e = 0.$

$C_{L_\alpha} = 4.$

$C_{L_{\dot{\alpha}}} = 0.$

$C_{Lq} = 0.$

$C_{L\delta_e} = .229$

$C_{m_\alpha} = -1.03$

$C_{m_{\dot{\alpha}}} = - 4.5$

$C_{m_q} = - 13.$

$C_{m_{\delta_T}} = .1$

$C_{m\delta e} = -.688$

TABLE 2. System Parameters Used for Third Order Example - continued

$$A = \begin{bmatrix} -.46 & 0. & 1. \\ 0. & 0. & 1. \\ 1.06 & 0. & -6.59 \end{bmatrix} \qquad B = \begin{bmatrix} 0. & -.03 \\ 0. & 0. \\ 1.11 & -7.57 \end{bmatrix}$$

τ = .1 sec.

$$Q_1 = \begin{bmatrix} 1. & 0 & 0 \\ 0 & 1. & 0 \\ 0 & 0 & 10. \end{bmatrix} \qquad Q_2 = \begin{bmatrix} 1. & 0 \\ 0 & 1. \end{bmatrix}$$

$$(\Gamma_d' \, Q_1 \, \Gamma_d + Q_2)^{-1} \; \Gamma_d \, Q_1 =$$

$$\begin{bmatrix} 0. & 0. & .44 \\ -.02 & -.02 & -1.51 \end{bmatrix}$$

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A DIGITAL ADAPTIVE MODEL FOLLOWING CONTROLLER

by

Henry Jackson Dunn, III

(ABSTRACT)

A discrete algorithm for following a desired model for systems that are non-linear or time varying is developed. The controller is developed with optimal control theory by finding the optimal control that would minimize a quadratic cost functional based on the error between the model and system for the previous sampling period. With this information a correction to the system command is formulated for the next sampling period. The stability of the algorithm is investigated and a stability criterion is found. Two examples are presented to show the characteristics of the controller.