

Chapter 6: Determination of Seismologic Parameters for Proposed Liquefaction Evaluation Procedure

6.1 Introduction

The motivation for developing the liquefaction evaluation procedure presented in Chapter 5 was to establish an energy-based *Capacity* curve for use in remedial ground densification design. However, the present chapter deviates slightly from this focus and outlines procedures for determining the seismologic parameters required for performing earthquake liquefaction evaluations. Three seismologic parameters are required: peak ground acceleration at the soil surface (a_{\max}), epicentral distance from the site to the source (ED), and the magnitude of the design earthquake (M). The proposed procedures for determining these seismological parameters use the United States Geological Survey (USGS) seismic maps and are intended for use on projects where site/region specific seismologic information is not available, or for making preliminary estimates where such information is available.

The procedures outlined for determining ED and M are not new. Rather, they are commonly used by seismologists in quantifying the mean magnitude and distance of the seismic event that causes a ground-motion exceedance at a chosen return period, as determined by de-aggregating the results from probabilistic seismic hazard analyses. Accordingly, these procedures are presented in “cookbook” fashion with minimal commentary given. On the contrary, the procedure for determining a_{\max} is new and is based on the observed relationship between the characteristics of the rock outcrop motions and the peak accelerations of uniform soil profiles (i.e., soil profiles having constant shear wave velocity with depth). To validate the proposed procedure, it is used to predict a_{\max} values for soft soil sites subjected to the Loma Prieta earthquake at short and long epicentral distances. For the limited comparison conducted as part of this study, the predicted results agree remarkably well with observed values. Because the proposed approach for determining a_{\max} applies only to uniform profiles, Appendix 6a outlines a method of computing equivalent uniform profiles corresponding to more realistic non-uniform profiles.

This chapter is organized as follows:

- Proposed procedure for ED and M .
- Review of currently used procedures for determining a_{max} .
- Proposed procedure for determining a_{max} .
 - Approximate spectral shapes of USGS rock outcrop motions.
 - Response of uniform soil profiles.
 - Simplified technique for determining a_{max} of a uniform profile as a function of the spectral shape of the input motion.
- Comparison of predicted and observed results.

6.2 Procedure for determining epicentral distance (ED) and magnitude (M) of the design earthquake

The determination of the epicentral distance (ED) and magnitude (M) for use in liquefaction evaluations requires considerable judgment. The proposed procedure uses the USGS seismic hazard map and web site and is not intended for sites where detailed site/region specific seismologic information is available. As stated in the introduction, the approach outlined herein is not new, but rather has been used by seismologists for various purposes since its inception in 1981 (McGuire and Shedlock 1981). The resulting ED and M values are the averages of the mean values corresponding to the seismic event(s) causing exceedances of the pga and spectral acceleration at $1.0hz$ for approximately a 2500 year mean return period. The mean magnitudes and distances of the seismic event(s) are computed from the de-aggregation matrices of the probabilistic seismic hazard given on the USGS web site.

The de-aggregation matrices can be obtained by:

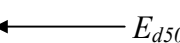
- Clicking on the link entitled “Deaggregations” listed under the heading “SEISMIC HAZARD” on the left-hand-side of the USGS web page:
<http://geohazards.cr.usgs.gov/eq/>.
- In the second paragraph, click on the link for either the CEUS map or the WUS map depending on whether the site is located in the Central-Eastern or Western United States, respectively.

- Click on the closest city to the site of interest.

The two matrices entitled “Deaggregated Seismic Hazard PE = 2% in 50 years *pga*” and “Deaggregated Seismic Hazard PE = 2% in 50 years 1.0 hz (1.0 s)” are used in determining *ED* and *M*. The basis for choosing these two matrices is that they reasonably bound the frequency range of most soil profiles. An example of a de-aggregation matrix is given in Figure 6-1.

Deaggregated Seismic Hazard PE = 2% in 50 years *pga*
 Hartford CT 41.750 deg N 72.700 deg W PGA=0.13220 g

M<=	5.0	5.5	6.0	6.5	7.0	7.5
d<= 25.	14.459	9.091	4.563	2.080	0.739	0.437
50.	7.307	8.030	6.387	4.054	1.740	1.128
75.	1.134	2.181	2.997	3.008	1.783	1.422
100.	0.255	0.727	1.481	2.051	1.538	1.482
125.	0.088	0.347	0.924	1.645	1.457	1.622
150.	0.030	0.156	0.516	1.141	1.195	1.522
175.	0.009	0.061	0.257	0.709	0.898	1.317
200.	0.003	0.023	0.118	0.410	0.619	1.078
225.	0.001	0.007	0.048	0.208	0.372	0.764
250.	0.000	0.002	0.020	0.106	0.224	0.532
275.	0.000	0.001	0.008	0.051	0.127	0.338
300.	0.000	0.000	0.003	0.025	0.073	0.224
325.	0.000	0.000	0.001	0.013	0.042	0.152
350.	0.000	0.000	0.001	0.007	0.027	0.118
375.	0.000	0.000	0.000	0.004	0.017	0.084
400.	0.000	0.000	0.000	0.002	0.011	0.062
425.	0.000	0.000	0.000	0.001	0.006	0.040
450.	0.000	0.000	0.000	0.001	0.004	0.028
475.	0.000	0.000	0.000	0.000	0.002	0.016
500.	0.000	0.000	0.000	0.000	0.001	0.007






Figure 6-1. Example de-aggregation matrix for a 2500 year return period event obtained from the USGS web site.

The numbers in the matrix represent the relative contribution of the corresponding *ED* and *M* pair to the seismic hazard at the site, having a 2500 year mean return period. For example, 8.030% of the seismic hazard at the site is attributed to a *M*5.5 earthquake having an *ED* = 50km. A sum of all the numbers in the matrix equals 100%. More information on the USGS de-aggregation matrices can be obtained from the following

web address: “http://geohazards.cr.usgs.gov/eq/html/deagg_exp.shtml.” If the numbers in the matrix are viewed as a surface, the ED and M corresponding to the mean elevation of the surface (or mean hazard) can be determined by computing the weighted average of the values given in the de-aggregation matrix, which can be determined using the following equations:

$$M_{pga} = \frac{5.0 \cdot \sum D_{M5.0} + 5.5 \cdot \sum D_{M5.5} + 6.0 \cdot \sum D_{M6.0} + 6.5 \cdot \sum D_{M6.5}}{100} + \frac{7.0 \cdot \sum D_{M7.0} + 7.5 \cdot \sum D_{M7.5}}{100} \quad (6-1)$$

$$ED_{pga} = \frac{25 \cdot \sum E_{d25} + 50 \cdot \sum E_{d50} + 75 \cdot \sum E_{d75} + 100 \cdot \sum E_{d100}}{100} + \frac{125 \cdot \sum E_{d125} + 150 \cdot \sum E_{d150} + 175 \cdot \sum E_{d175} + 200 \cdot \sum E_{d200}}{100} + \frac{225 \cdot \sum E_{d225} + 250 \cdot \sum E_{d250} + 275 \cdot \sum E_{d275} + 300 \cdot \sum E_{d300}}{100} + \frac{325 \cdot \sum E_{d325} + 350 \cdot \sum E_{d350} + 375 \cdot \sum E_{d375} + 400 \cdot \sum E_{d400}}{100} + \frac{425 \cdot \sum E_{d425} + 450 \cdot \sum E_{d450} + 475 \cdot \sum E_{d475} + 500 \cdot \sum E_{d500}}{100} \quad (6-2)$$

where: $D_{M5.0}$ are the numbers in the column under the heading “5.0” in the pga de-aggregation matrix (e.g., see Figure 6-1).

$D_{M5.5}$ are the numbers in the column under the heading “5.5”...

.

.

$D_{M7.5}$ are the numbers in the column under the heading “7.5” ...

E_{d25} are the numbers in the rows with the heading “25” in the *pga* de-aggregation matrix (e.g., see Figure 6-1).

E_{d50} are the numbers in the rows with the heading “50” ...

E_{d500} are the numbers in the rows with the heading “500” ...

$M_{1.0hz}$ and $ED_{1.0hz}$ are computed similarly from the matrix entitled “Deaggregated Seismic Hazard PE = 2% in 50 years 1.0 hz (1.0 s)”.

The averages of the mean magnitudes and epicentral distances are computed as:

$$M = \frac{M_{pga} + M_{1.0hz}}{2} \quad (6-3)$$

$$ED = \frac{ED_{pga} + ED_{1.0hz}}{2} \quad (6-4)$$

and may be used in conjunction with the proposed liquefaction evaluation procedure presented in Chapter 5.

The above procedure should be used with great care, especially if the de-aggregation matrix shows that the site is dominated by two separate events (i.e., bi-modal distribution). In such a scenario, the de-aggregation matrix will have two large numbers corresponding to distinctly different M and ED pairs and the mean values of M and ED may not correspond to a likely event.

Finally, mention needs to be given to magnitude scales. In Chapter 5, the N_{eqv} correlation was developed using the Richter magnitude (M). On the contrary, the de-aggregation

matrices given on the USGS web site is based on the Moment magnitude scale (M_W). A comparison of the various magnitude scales is shown in Figure 2-8b. Given the inherent uncertainty in determining the seismological parameters for design, the error induced by the two different magnitude scales is minimal.

6.3 Currently Used Procedures for Determining a_{\max}

In order of preference, the following approaches are currently recommended for determining a_{\max} for use in liquefaction evaluations (NCEER 1997, Youd et al. 2001):

- 1) Direct application of region specific correlations relating peak horizontal acceleration at soil surface, earthquake magnitude, and site-to-source distance (i.e., attenuation relationships).
- 2) Perform site response analysis.
- 3) Multiple rock outcrop pga by amplification ratio.

Although the author agrees with the assigned preferences of the recommended approaches, for sites in the Central and Eastern United States (CEUS), option (3) is typically the only one that can be used. The reason for this is that there are limited regional attenuation relationships for soil conditions in the CEUS. In regards to option (2), acceleration time histories are required to perform a site response analyses, and few representative time histories are available for the CEUS. Artificial time histories can be generated for this purpose, but considerable judgment and expertise is required.

Amplification ratios are typically developed from earthquake case histories and relate the peak ground accelerations recorded on rock sites to those recorded on nearby soil sites. To avoid confusion and to ease discussion, the peak acceleration on rock outcrops is designated as pga and the peak acceleration at the soil surface is designated as a_{\max} . Frequently referenced curves quantifying amplification ratios for various soil conditions are shown in Figure 6-2. Although these curves provide great insight into the qualitative behavior of various soil profiles subjected to different levels of earthquake motion, the contributions of the individual factors influencing the relationship between rock outcrop and the soil surface motion are indistinguishable. As a result, the amplification curves do

not allow the explicit incorporation of site-specific information in determining a_{\max} , other than p_{ga} and a general description of the soil profile.

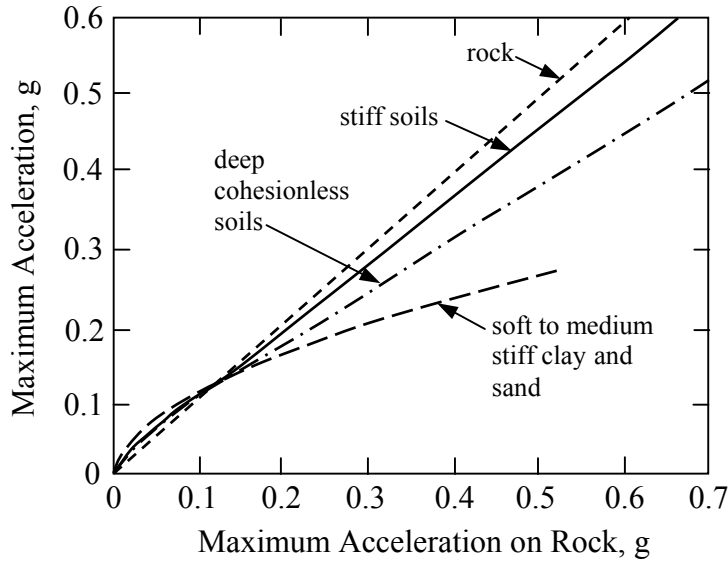


Figure 6-2a. Early curves quantifying the amplification ratios as function of site conditions and amplitude of rock acceleration. (Adapted from Seed and Idriss 1982).

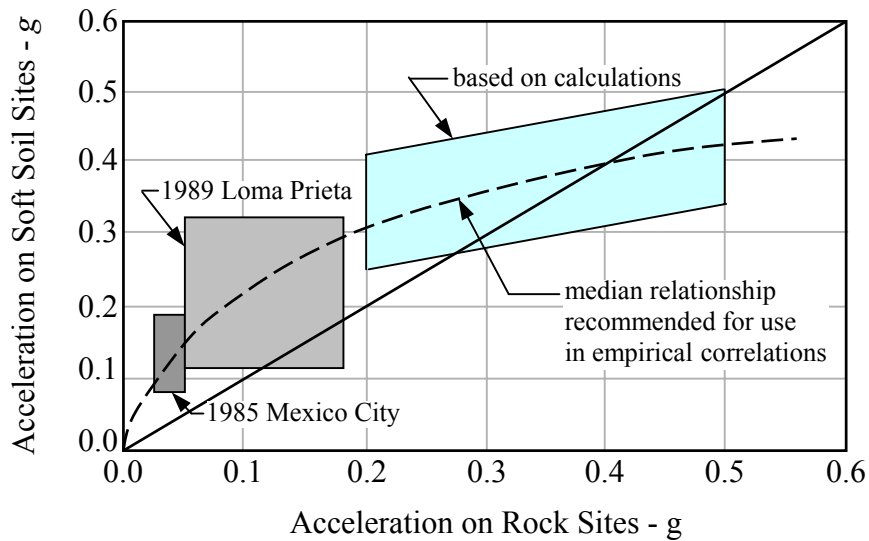


Figure 6-2b. Curve quantifying the amplification ratios for soft soil sites. (Adapted from Idriss 1990, 1991).

The amplification curves shown in Figure 6-2 were primarily developed from WUS data. Because the geology and characteristics of the earthquake motions differ in the CEUS and WUS, the validity of the amplification curves for CEUS sites is uncertain. Response spectra for CEUS and WUS events of comparable magnitudes are shown in Figure 6-3. As may be observed from this figure, the CEUS spectrum is broader banded and is richer in higher frequencies than the WUS spectrum.

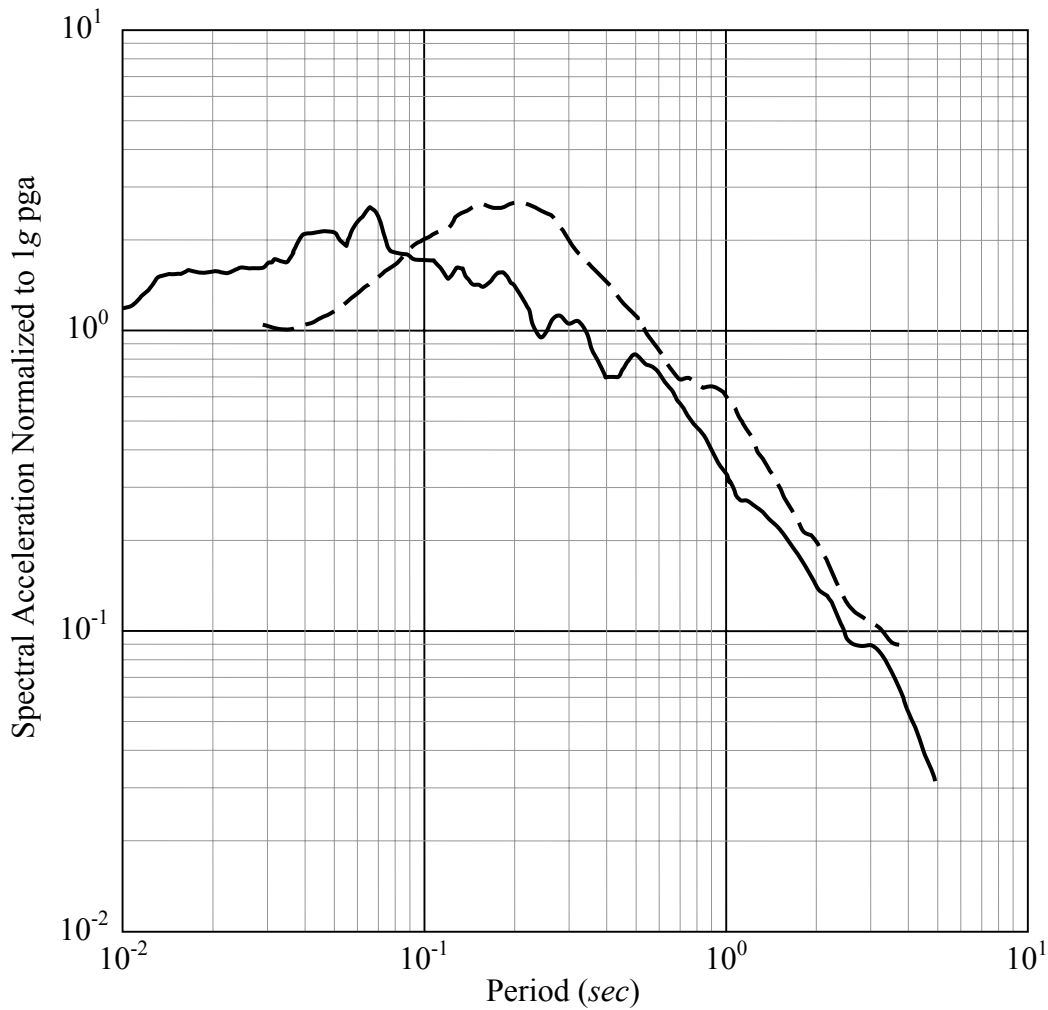


Figure 6-3. Comparison of average 5% damped response spectral shapes computed from strong-motion data recorded at rock sites in CEUS (solid line) and WUS (dashed line). CEUS average shape is from recordings of the $m_b = 6.4$ Nahanni aftershock (Canada). The WUS average shape is from recordings for the San Fernando $M_L = 6.4$ and Imperial Valley $M_L = 6.6$ events (California). (Adapted from EPRI 1988).

Variations of the curves shown in Figure 6-2 are also used in practice (Soydemir 2000). In place of the general descriptions of the site conditions (e.g., rock, stiff soil, deep cohesionless soils, soft to medium stiff clay and sand, and soft soil), the site classifications presented in the NEHRP provisions are used. The NEHRP provisions classify sites into one of five categories *A-E*, based on the average shear wave velocity of the upper 100ft (30m) of the soil profile; an additional site class *F* is used for soils requiring special consideration (NEHRP 1998; Dobry et al. 2000). The site categories are:

Site Class:	<i>A</i>	$v_{s\ ave} > 5000ft/sec$
	<i>B</i>	$2500 < v_{s\ ave} \leq 5000ft/sec$
	<i>C</i>	$1200 < v_{s\ ave} \leq 2500ft/sec$
	<i>D</i>	$600 < v_{s\ ave} \leq 1200ft/sec$
	<i>E</i>	$v_{s\ ave} < 600ft/sec$
	<i>F</i>	Soils requiring site-specific evaluation

where:

$$v_{s\ ave} = \frac{100}{\sum_{i=1}^n \frac{d_i}{v_{s,i}}} \quad (6-5)$$

$v_{s\ ave}$ = The weighted average shear wave velocity for top 100ft of profile.

d_i = The thickness of any layer between 0 and 100ft.

$v_{s,i}$ = The shear wave velocity in ft/sec.

In addition to shear wave velocity, the NEHRP provisions define the Site Class in terms of undrained shear strength and SPT *N*-values. Once the site is classified, the amplification ratio (F_a) can be determined from Table 6-1. However, the amplification ratios are specified in terms of the mapped short-period spectral acceleration (S_s), which is the spectral acceleration corresponding to 0.2sec. S_s can be determined either from the USGS seismic hazard maps that accompany the 1997 NEHRP provisions or from the USGS web site (<http://geohazards.cr.usgs.gov/eq/>).

Table 6-1. Amplification Ratios (F_a) as a Function of the Site Class and Mapped Short Period Maximum Considered Earthquake Spectral Acceleration (S_s).

Site Class	Mapped Maximum Considered Earthquake Spectral Response Acceleration at Short Periods				
	$S_s \leq 0.25$	$S_s = 0.50$	$S_s = 0.75$	$S_s = 1.00$	$S_s \geq 1.25$
<i>A</i>	0.8	0.8	0.8	0.8	0.8
<i>B</i>	1.0	1.0	1.0	1.0	1.0
<i>C</i>	1.2	1.2	1.1	1.0	1.0
<i>D</i>	1.6	1.4	1.2	1.1	1.0
<i>E</i>	2.5	1.7	1.2	0.9	<i>a</i>
<i>F</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>	<i>a</i>

NOTE: Use straight line interpolation for intermediate values of S_s .

^a Site-specific geotechnical investigation and dynamic site response analyses shall be performed.

S_s can be obtained from the USGS web site by:

- On the left-hand-side of the USGS web page (<http://geohazards.cr.usgs.gov/eq/>), click on the link entitled “Hazard by Zip Code” which is listed under the heading “SEISMIC HAZARD.” Alternatively the link entitled “Hazard by Lat/Lon” can be selected if the longitude and latitude of the site are known.
- Enter the zip code of the site and click on the button labeled “Submit Query.”
- A table will appear that provides peak ground acceleration (pga) and 3 spectral acceleration values for 3 different hazard levels (i.e., a total of 12 values are listed) for firm rock sites. An example of the information obtained by the web site is shown in Figure 6-4. S_s for 3 different hazard levels are given in the row labeled “0.2 sec SA”. Also of interest are the pga values and the spectral accelerations in the row labeled “1.0 sec SA”, which is designated as S_1 .

If the site of interest is underlain by hard rock, the pga , S_s , and S_1 values should be corrected using the following relations (NEHRP 1998):

$$\begin{aligned}
 pga_{hard\ rock} &= pga_{firm\ rock} / 1.52 \\
 S_s\ hard\ rock &= S_s\ firm\ rock / 1.76 \\
 S_1\ hard\ rock &= S_1\ firm\ rock / 1.34
 \end{aligned}
 \tag{6-6}$$

Firm and hard rocks have shear wave velocities of approximately 2500ft/sec and 8000ft/sec, respectively. NEHRP (1998) uses the term “firm rock” synonymously with geologic material commonly referred to as “soft rock” and designates hard rock sites as Site Class A and firm rock sites lying on the boundary between Site Class B and C (i.e., B-C boundary sites).

Assuming a Poisson process, the hazard levels can be expressed in terms of mean return period:

- 10 %PE in 50 yr ~ 500 year ground motion (use for typical structures)
- 5%PE in 50 yr ~ 1000 year ground motion
- 2%PE in 50 yr ~ 2500 year ground motion

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The input zip-code is 24073.
ZIP CODE                24073
LOCATION                 37.1366 Lat.  -80.4038 Long.
DISTANCE TO NEAREST GRID POINT  4.0784 kms
NEAREST GRID POINT     37.1 Lat.  -80.4 Long.
Probabilistic ground motion values, in %g, at the Nearest Grid
point
are:
      PGA      10%PE in 50 yr   5%PE in 50 yr   2%PE in 50 yr
0.2 sec SA    11.932060        19.154921        36.886810
0.3 sec SA     9.640203        15.714880        27.358530
1.0 sec SA     4.030000         6.325992        11.081900
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Figure 6-4. Example of the seismic data retrieved from the USGS web site.

Finally, the soil surface a_{max} is estimated as:

$$\begin{aligned}
 a_{max} &= F_a \cdot pga \\
 &\approx F_a \cdot \frac{S_s}{2.5}
 \end{aligned}
 \tag{6-7}$$

where $S_s / 2.5$ is approximately the rock outcrop pga (Dobry et al. 2000). A plot of a_{max} versus $S_s / 2.5$ is shown in Figure 6-5. As may be seen in this figure, the amplification curves are similar to those shown in Figure 6-2. Although this approach provides amplification factors corresponding to the specific NEHRP site categories, it is primarily based on WUS data, and therefore, its applicability to CEUS sites is uncertain. A comparison of typical CEUS and WUS response spectra was shown previously in Figure 6-3.

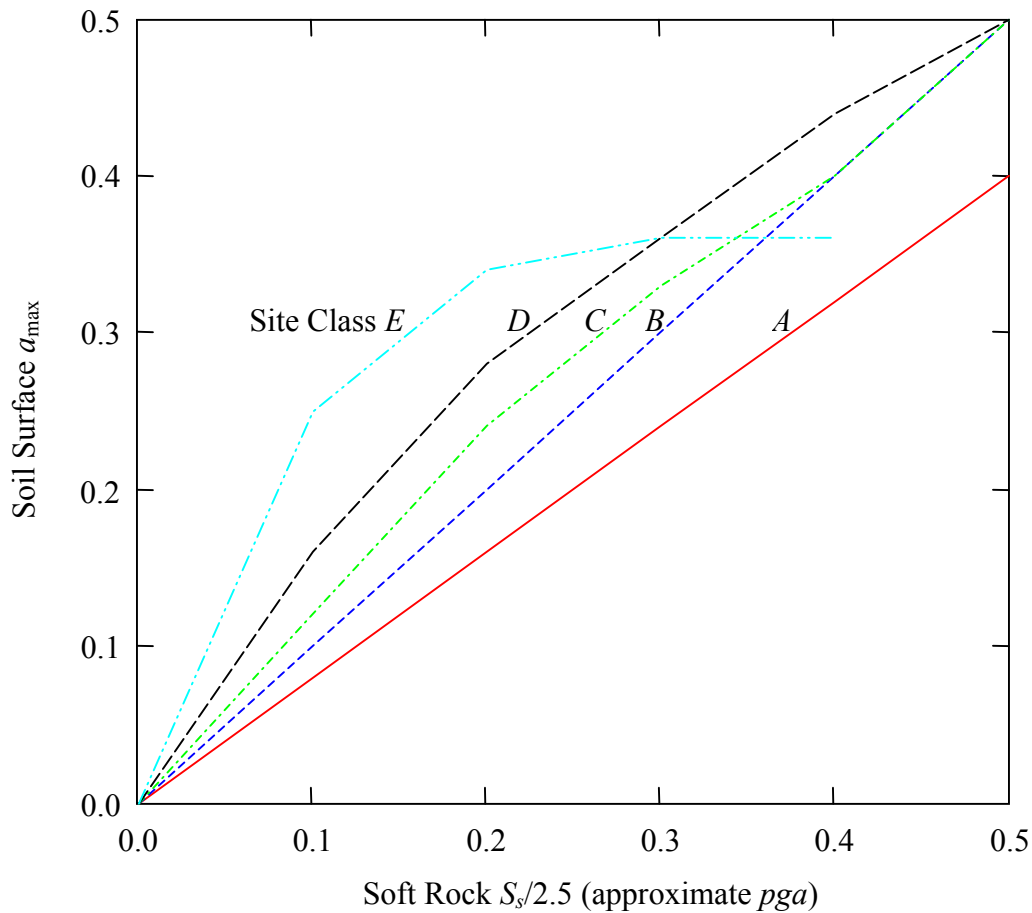


Figure 6-5. Curves quantifying the amplification ratios used in the NEHRP provisions.

6.4 Proposed Procedure for Determining a_{max}

As discussed in Hadjian and Green (2000), The relationship between rock outcrop and soil surface motions is complex and depends on numerous factors including the

fundamental period and spacing of subsequent harmonics of the soil profile, strain dependency of soil stiffness and damping, impedance contrast at bedrock and inter-lying layers, and the characteristics of the rock outcrop motion. One of the shortcomings of the amplification factors described above is that the influences of many of these factors are indistinguishable. On the contrary, the procedure proposed by the author for determining a_{\max} builds on the simplicity of the amplification factors but explicitly accounts for the frequency content of the rock outcrop motion, the impedance contrast between the soil profile and base rock, and the fundamental period of the soil profile.

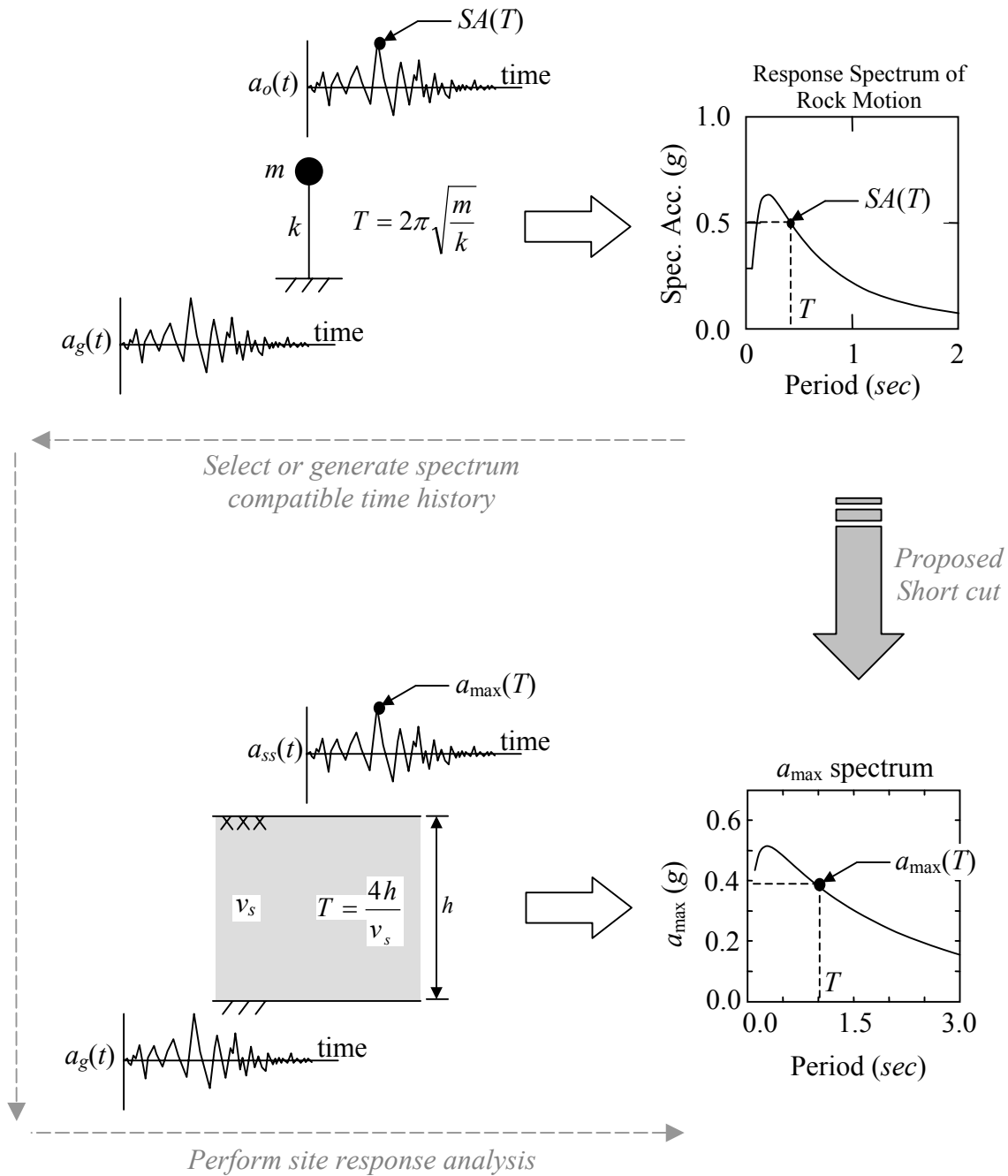
From analyzing the results of numerous site response analyses, an empirical relationship between the shape of the response spectrum of the rock outcrop motion and the corresponding a_{\max} spectrum was identified, where the a_{\max} spectrum is a plot of the maximum acceleration at the surface of a uniform soil profile as a function of the fundamental period of the profile. The proposed procedure for determining a_{\max} is just the mathematical formulation of the observed empirical relationship between the two spectra. The relationship between the proposed procedure for determining a_{\max} , the response spectrum rock outcrop motion, and the a_{\max} spectrum is conceptually illustrated in Figure 6-6.

In the top part of Figure 6-6, the computation of a response spectrum is shown, wherein a time history is used to excite similarly damped single degree of freedom oscillators having a range of periods. The response spectrum is a plot of the peak responses of the oscillators versus the corresponding oscillator periods. In the bottom part of Figure 6-6, the computation of an a_{\max} spectrum is shown. A time history excites a suite of similarly damped uniform profiles having a range of fundamental periods and the same soil-rock impedance contrast. Analogous to the response spectrum, the a_{\max} spectrum is a plot of the peak accelerations of the computed profile surface motions (a_{\max}) versus the corresponding fundamental periods of the profiles. The a_{\max} spectrum differs from acceleration response spectrum in two respects: 1) a_{\max} at the surface of a uniform profile is a function of multiple modes of vibration of the uniform layer, contrary to spectral acceleration (SA) which only represents a single mode of vibration, and 2) the a_{\max}

spectrum is a function of both material damping of the soil and radiation damping of the halfspace, whereas SA is only a function of material damping of the oscillator.

As discussed in the previous section in regards to option (2) for determining a_{\max} , an acceleration time history is selected or generated that is in general accordance with the specified design rock outcrop motions at a site, which are often specified in terms of a smoothed response spectrum. The time history is then used in the site response analysis of the soil profile, and the induced stresses and maximum soil surface acceleration are determined. The evolution of this process is shown in Figure 6-6 as the gray dotted arrows starting at the response spectrum of the design ground motions, progressing through the selection of a time history, the use of the selected time history in a site response analysis, and finally ending at the desired profile response motions.

The proposed procedure for determining a_{\max} short cuts the need for obtaining spectrum compatible time histories and performing site response analyses and relates the a_{\max} spectrum directly to the rock outcrop response spectrum. The procedure is represented in Figure 6-6 by the gray block arrow leading from the response spectrum and pointing at the a_{\max} spectrum. The proposed procedure is similar in concept to the simplified techniques for developing smooth design spectra from seismological parameters, e.g., Newmark and Hall (1982), which short cuts the need to compute and statistically analyze response spectra for representative acceleration time histories of the seismic hazard at a site.



$a_g(t)$ = rock outcrop acceleration time history.
 $a_{ss}(t)$ = soil surface acceleration time history.
 $a_o(t)$ = acceleration time history of oscillator response.

m = mass of single degree of freedom oscillator.
 k = stiffness of single degree of freedom oscillator.
 v_s = shear wave velocity of uniform soil profile.
 h = thickness of uniform soil profile.

$a_{max}(T)$ = maximum soil surface acceleration for soil profile having fundamental period T .
 $SA(T)$ = maximum acceleration of oscillator response for oscillator having fundamental period T .

Figure 6-6. Illustration of proposed approach for determining a_{max} .

Before the procedure is presented for computing the a_{\max} spectrum, the characterization of the rock outcrop motions, as represented by response spectra, is discussed. Additionally, an approach for computing equivalent uniform profiles corresponding to more complex profiles is outlined in Appendix 6a.

6.4.1 Approximate Spectral Shapes of USGS Rock Outcrop Motions

Previously, steps were presented on how to retrieve pga , S_s , and S_1 values from the USGS web site. Alternatively these values can be obtained from the seismic hazard maps that accompany the 1997 edition of the NEHRP provisions. These values represent three points on a uniform hazard spectrum (UHS) having a given mean return period, i.e., 500, 1000, or 2500 years. A UHS is a smoothed response spectrum for which each of the spectral acceleration values has an equal probability of exceedance (e.g., NRC 1988). The 500 and 2500 year UHS were examined for ten cities geographically dispersed throughout the country: San Francisco, Oakland, Los Angeles, San Diego, Salt Lake City, Memphis, St. Louis, Chicago, New York, and Charleston. These same ten cities were used in developing the current approach given in the NEHRP provisions for constructing design spectra (Algermissen and Leyendecker 1992). It was observed that when plotted on log-log scales, the UHS are characteristically parabolic in shape.

From analytical geometry, a parabola can be defined by either three points lying on it or by the location of its vertex and focus, as shown in Figure 6-7. Given that the USGS hazards maps provide the pga and spectral accelerations corresponding to 0.2 and 1.0 seconds (i.e., S_s and S_1 , respectively), these three points were used to define the parabolic UHS. However, because log-log scales are used, a non-zero period has to be assumed to correspond to the pga (i.e., $T_{pga} \neq 0$). From examination of the UHS for the ten cities and two different hazard levels, it was determined that T_{pga} equal to 0.01 and 0.044 seconds provide good approximations for the Central and Eastern United States (CEUS) and the Western United States (WUS), respectively. These values bound the $T_{pga} = 0.0303$ seconds used by Newmark and Hall (1982) for both the CEUS and WUS.

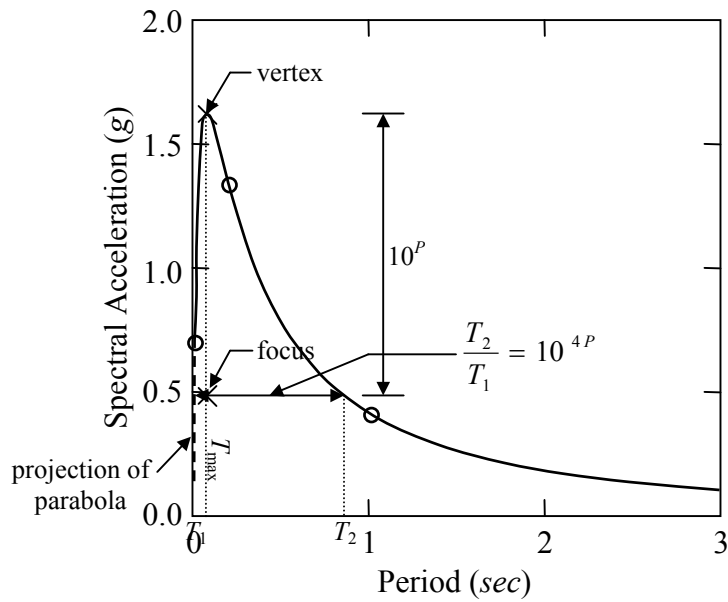
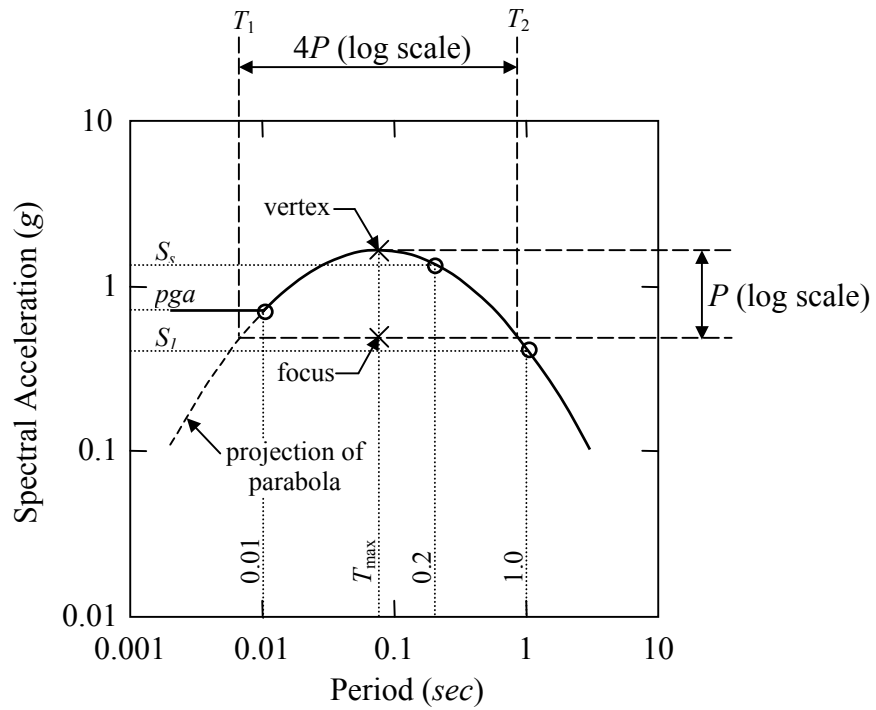


Figure 6-7. Characteristics and analytical geometry of the parabolic representation of UHS.

Using the parabolic approximation, the UHS for rock outcrop motions is given as:

$$\begin{aligned} SA(T) &= a \log\{a[\log(T)]^2 + b \log(T) + c\} & \text{for } T > T_{pga} \\ &= pga & \text{for } T \leq T_{pga} \end{aligned} \quad (6-8a)$$

where for the Central and Eastern United States (CEUS):

$$\begin{aligned} a &= 0.72 \cdot \log\left(\frac{S_1}{pga}\right) - 1.10 \cdot \log\left(\frac{S_s}{pga}\right) \\ b &= 1.93 \cdot \log\left(\frac{S_1}{pga}\right) - 2.20 \cdot \log\left(\frac{S_s}{pga}\right) \\ T_{pga} &= 0.01 \text{sec} \end{aligned} \quad (6-8b)$$

and for the Western United States (WUS):

$$\begin{aligned} a &= 1.06 \cdot \log\left(\frac{S_1}{pga}\right) - 2.19 \cdot \log\left(\frac{S_s}{pga}\right) \\ b &= 2.17 \cdot \log\left(\frac{S_1}{pga}\right) - 2.96 \cdot \log\left(\frac{S_s}{pga}\right) \\ T_{pga} &= 0.044 \text{sec} \end{aligned} \quad (6-8c)$$

and for both CEUS and WUS:

$$c = \log(S_1) \quad (6-8d)$$

These expressions were derived from the analytical geometry of a parabola. From examination of the above expressions, it may be observed that the a and b coefficients define the shape of the UHS and the c coefficient scales the spectrum. An alternate form of Equation (6-8d) for the c coefficient allows the UHS to be scaled to an arbitrary peak acceleration (pga'):

$$c = \log\left(\frac{S_1}{pga}\right) + \log(pga') \quad (6-8e)$$

Comparisons of uniform hazard spectral values and the approximated UHS computed using the above expressions are shown in Figure 6-8 for two cities, one located in CEUS and the other in WUS. The goodness-of-fit of the approximated UHS to the actual UHS values shown in this figure are typical of the twenty cases examined (i.e., ten cities and two probabilities of exceedances).

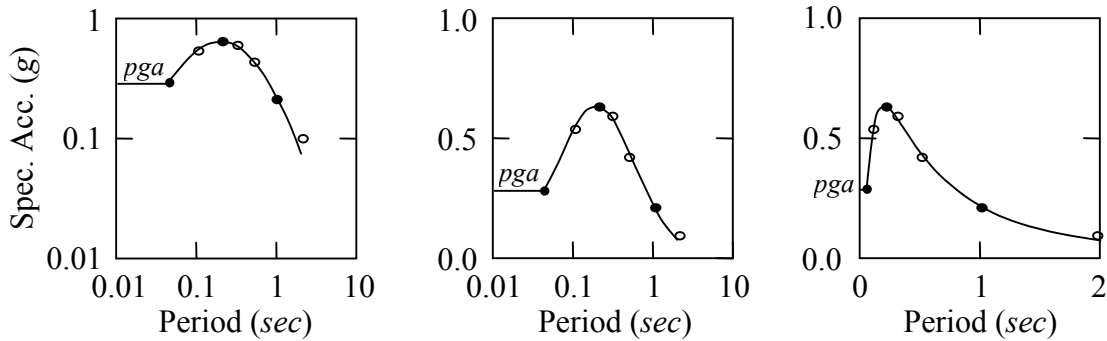


Figure 6-8a. Parabolic approximation of uniform hazard spectrum (UHS) for Salt Lake City, UT for seismic hazard having a 500 year return period (i.e., 10% probability of exceedance in 50 years). The spectrum is plotted on three scales (i.e., log-log, semi-log, and arithmetic) to facilitate the comparison of the actual spectral acceleration values and the parabolic approximation. The curves shown in the above plots were computed using Equation (6-8), and the black dots are the pga , S_s , and S_1 values obtained from the USGS web site. The open circles are additional spectral acceleration values obtained from the USGS.

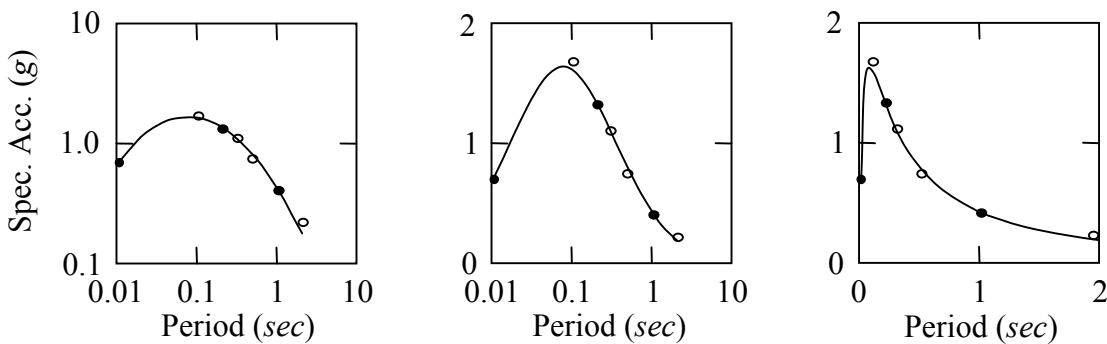


Figure 6-8b. Parabolic approximation of uniform hazard spectrum (UHS) for Memphis, TN for seismic hazard having a 2500 year return period (i.e., 2% probability of exceedance in 50 years). The spectrum is plotted on three scales (i.e., log-log, semi-log, and arithmetic) to facilitate the comparison of the actual spectral acceleration values and the parabolic approximation. The curves shown in the above plots were computed using Equation (6-8), and the black dots are the pga , S_s , and S_1 values obtained from the USGS web site. The open circles are additional spectral acceleration values obtained from the USGS.

As mentioned previously and shown in Figure 6-7, a parabola can also be defined by the location of its vertex and focus. Building on this, the spectral shape can be defined by the vertical distance between the vertex and the focus (P) and the period corresponding to the vertex (T_{max}), also shown in Figure 6-7. This allows the shape of the UHS to be characterized independent of pga . The relationships between P , T_{max} , a , and b are given in the following expressions:

$$T_{max} = 10^{-b/2a} \quad (6-9)$$

$$P = -\frac{1}{4a} \quad (6-10)$$

Both T_{max} and P have physical significance. Seed et al. (1969) used the period corresponding to the maximum spectral acceleration (which for a parabolic spectrum is T_{max}) to define the predominant period of recorded earthquakes. In a more recent study, Rathje et al. (1998) characterized the frequency content of earthquakes by the predominant period of a smoothed response spectrum, which also corresponds to T_{max} . Furthermore, P is a measure of the broadness of a spectrum. The values of P and T_{max} for the ten cities examined are listed in Table 6-2. An immediate observation from these values is that the UHS for the CEUS have considerably shorter predominant periods than those for the WUS and are considerably broader banded. These trends are consistent with the response spectra shown in Figure 6-3. Grouping the cities according to their geographic location, bounding values for the CEUS and WUS are:

$$\begin{aligned} \text{CEUS:} \quad & 0.35 < P < 0.70 \\ & 0.05 < T_{max} < 0.10 \end{aligned}$$

$$\begin{aligned} \text{WUS:} \quad & 0.15 < P < 0.45 \\ & 0.15 < T_{max} < 0.25 \end{aligned}$$

Table 6-2a. T_{\max} and P values for UHS for WUS cities.

City	Probability of Exceedance (PE)	T_{\max} (sec)	P
San Francisco	10% PE 50yrs	0.222	0.359
	2% PE 50yrs	0.245	0.391
Oakland	10% PE 50yrs	0.203	0.326
	2% PE 50yrs	0.216	0.369
Los Angeles	10% PE 50yrs	0.203	0.256
	2% PE 50yrs	0.206	0.291
San Diego	10% PE 50yrs	0.195	0.295
	2% PE 50yrs	0.197	0.310
Salt Lake City	10% PE 50yrs	0.188	0.280
	2% PE 50yrs	0.198	0.318

Table 6-2b. T_{\max} and P values for UHS for CEUS cities.

City	Probability of Exceedance (PE)	T_{\max} (sec)	P
Memphis	10% PE 50yrs	0.071	0.444
	2% PE 50yrs	0.075	0.527
St. Louis	10% PE 50yrs	0.077	0.475
	2% PE 50yrs	0.079	0.527
Chicago	10% PE 50yrs	0.089	0.540
	2% PE 50yrs	0.080	0.574
New York	10% PE 50yrs	0.071	0.437
	2% PE 50yrs	0.065	0.445
Charleston	10% PE 50yrs	0.068	0.436
	2% PE 50yrs	0.073	0.508

Using the identified ranges for P and T_{max} for the CEUS and WUS, eighteen combinations of P and T_{max} were selected to represent possible UHS for CEUS and WUS (nine UHS each for the CEUS and WUS). These combinations are listed in Table 6-3. For each combination, spectra compatible time histories were generated by combining the Fourier phase spectrum of an actual recorded earthquake time history with Fourier amplitude spectra that were iteratively modified until the resulting time-histories matched the target spectra (e.g., Silva and Lee 1987; Costantino 1995). As discussed in the next section, these time histories were used in over 2000 site response analyses of uniform soil profiles.

Table 6-3. Combinations of P and T_{max} selected to represent the UHS for the CEUS and WUS.

CEUS (P, T_{max})	WUS (P, T_{max})
(0.350, 0.050)	(0.150, 0.150)
(0.350, 0.075)	(0.150, 0.200)
(0.350, 0.100)	(0.150, 0.250)
(0.550, 0.050)	(0.300, 0.150)
(0.550, 0.075)	(0.300, 0.200)
(0.550, 0.100)	(0.300, 0.250)
(0.700, 0.050)	(0.450, 0.150)
(0.700, 0.075)	(0.450, 0.200)
(0.700, 0.100)	(0.450, 0.250)

6.4.2 Response of Uniform Soil Profiles

Employing the spectrum compatible time histories discussed above, SHAKEVT was used to perform site response analyses on 5% damped, non-degrading, uniform profiles having fundamental periods (T_1) ranging from 0.01 to 10.0 seconds (i.e., $0.01sec \leq T_1 \leq 10.0sec$) and four different soil-rock impedance ratios (IR ; defined subsequently). The purpose of the analyses was to identify trends between the shape of the UHS and the a_{max} spectrum.

Non-degrading profiles were used to lessen the importance of: the Fourier phase spectrum used in generating the time histories, the duration of the time histories, the pga of the time histories, the shear modulus and damping degradation characteristics of the soil, and the discretization of the soil profile. For strains typical during earthquakes of engineering interest, soil damping usually ranges from 2% to 15% (Whitman et al. 1972). Accordingly, using a constant 5% soil damping will result in a_{\max} values that are slightly larger than the anticipated statistical average if strain dependant damping were used.

The fundamental period and the spacing of the harmonics of a soil profile influence a_{\max} because a_{\max} is an aggregation of all the contributing modes of vibration of the profile. Single layer profiles have regularly spaced harmonic frequencies: $T_2 = T_1/3$, $T_3 = T_1/5$, $T_4 = T_1/7$, as given by the following expression:

$$T_n = \frac{4h}{v_s(2n-1)} \quad (\text{e.g., Kramer 1996}) \quad (6-11)$$

where: T_n = The period of the n^{th} mode of vibration of the profile.
 h = The thickness of the profile.
 v_s = Shear wave velocity of the profile.
 n = Mode of vibration ($n = 1$ is the fundamental mode).

Because non-uniform profiles do not have evenly spaced harmonics, the a_{\max} value determined using a uniform profile will not exactly equal that for a non-uniform profile having the same fundamental period. However, Roesset (1970), Madera (1970), and a limited study performed as part of this thesis, show that using equivalent uniform profiles for computing a_{\max} provides reasonable results. Appendix 6a outlines a procedure for computing equivalent uniform profiles.

The impedance contrasts at bedrock and inter-lying layers control the amplitudes of the incident and reflected waves at layer interfaces. A large impedance ratio (IR) at the base rock-soil interface determines the magnitude of the radiation damping that occurs (i.e., the amount of wave energy that will leave the system through the half space) and therefore influences a_{\max} . For a non-degrading uniform profile having a given

fundamental period and subjected to a given time history, the soil surface a_{\max} will increase as IR increases. Four different impedance ratios at the base rock-soil interface were used in the site response analyses: $IR = 3.5, 7.66, 11.2,$ and 17.0 . These values of IR were selected to reasonably bound possible values of actual soil profiles, i.e.:

$$\text{Soft soil overlying soft Rock: } IR = \frac{v_r \cdot \gamma_r}{v_s \cdot \gamma_s} = \frac{2500\text{ft/sec} \cdot 140\text{lb/ft}^3}{800\text{ft/sec} \cdot 125\text{lb/ft}^3} = 3.5$$

$$\text{Soft soil overlying hard Rock: } IR = \frac{v_r \cdot \gamma_r}{v_s \cdot \gamma_s} = \frac{12143\text{ft/sec} \cdot 140\text{lb/ft}^3}{800\text{ft/sec} \cdot 125\text{lb/ft}^3} = 17.0$$

- where: IR = Impedance ratio between base rock and soil.
 v_r = Shear wave velocity of half space (i.e., base rock).
 v_s = Shear wave velocity of the soil.
 γ_r = Total unit weight of base rock.
 γ_s = Total unit weight of soil.

The individual values of $v_r, \gamma_r, v_s,$ and γ_s are not important, only their combined value, as expressed by IR , influences a_{\max} .

For a given spectrum compatible time history and IR , the ratio of the a_{\max} spectrum and the UHS for the rock outcrop motion was computed from the results of the site response analyses. This ratio is referred to as the response ratio and is given by the following expression:

$$\text{Response Ratio}(T, IR) = \frac{a_{\max}(T, IR)}{SA(T)} \quad (6-12)$$

where: $a_{\max}(T, IR)$ = Spectrum of the peak acceleration at the surface of a 5% damped uniform soil profile as a function of the fundamental period of the profile.

$SA(T)$ = 5% damped response spectrum of the rock outcrop motion (i.e., UHS).

It was observed that the response ratios were parabolic in shape for the range of fundamental periods (T_1) of typical soil profiles (i.e., $0.1\text{sec} \leq T_1 \leq 3.0\text{sec}$) when plotted on log-log scales, as shown in Figure 6-9. As can be seen in this figure, the response

ratios increase as the IR of the profile increases. This is expected because the higher IR traps more seismic energy in the soil profile, resulting in higher a_{max} values.

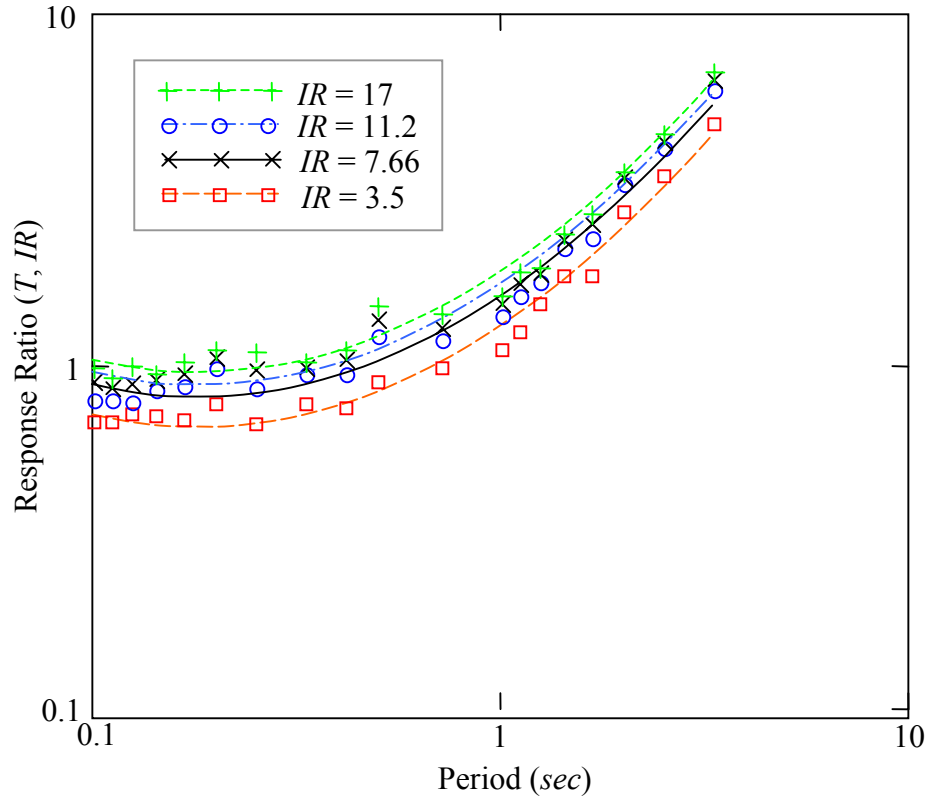


Figure 6-9. Response ratios and parabolic approximations for 4 different impedance ratios (IR). Each point in the above plot represents a separate site response analysis.

Assuming parabolic shapes, the response ratios can be approximated as:

$$\text{Response Ratio}(T, IR) = \alpha \log\{\alpha[\log(T)]^2 + \beta \log(T) + \gamma\}; \text{ for } 0.1 \leq T \leq 3.0 \text{ sec} \quad (6-13)$$

where α , β , and γ are coefficients that provided the best fit of the data and are functions of IR . As a further simplification, a relationship between the response ratio for $IR = 7.66$ to those for the other IR values was identified:

$$\text{Response Ratio}(T, IR) = \text{Response Ratio}(T, 7.66) \cdot f(IR)$$

$$\text{where: } f(IR) = \left(\frac{IR}{7.66}\right)^{m_{IR}}; \quad \begin{matrix} m_{IR} = 0.20 & \text{for } IR \geq 5 \\ m_{IR} = 0.25 & \text{for } IR < 5 \end{matrix} \quad (6-14)$$

Correlations were established between the α , β , and γ coefficients and the T_{max} and P of the UHS and are given in Table 6-4. From the parabolic approximations of the response ratio and UHS, the a_{max} spectrum is computed as:

$$\begin{aligned}
 a_{max}(T, IR) &= \text{Response Ratio}(T, IR) \cdot SA(T) \\
 &= \text{Response Ratio}(T, 7.66) \cdot f(IR) \cdot SA(T) \\
 &= \text{alog}\{a[\log(T)]^2 + b \log(T) + c\} \cdot \text{alog}\{\alpha[\log(T)]^2 + \beta \log(T) + \gamma\} \cdot \left(\frac{IR}{7.66}\right)^{m_{IR}} \\
 &= \text{alog}\{(a + \alpha)[\log(T)]^2 + (b + \beta)\log(T) + (c + \gamma)\} \cdot \left(\frac{IR}{7.66}\right)^{m_{IR}} \quad (6-15)
 \end{aligned}$$

For UHS having different T_{max} and P values than listed in Table 6-4, interpolation between computed a_{max} values for other T_{max} and P is required.

Using the above procedure, the a_{max} spectra corresponding to various IR were computed for San Francisco (i.e., $T_{max} = 0.222sec$ and $P = 0.359$) and are shown in Figure 6-10. As may be seen in this figure, a_{max} values increase for larger values of IR . The largest amplification occurs for sites having a fundamental period close to T_{max} of the rock outcrop motion (i.e., $T_1 \approx T_{max} = 0.222sec$). For an $IR = 2$, stiff profiles are predicted to amplify the rock outcrop pga , while de-amplification occurs for sites having a fundamental period greater than about $1sec$.

Table 6-4a. α , β , and γ coefficients for the Response Ratio for WUS.

		T_{\max}		
		0.15	0.20	0.25
P	0.15	$\alpha = 0.963$ $\beta = 1.797$ $\gamma = 0.696$	$\alpha = 1.156$ $\beta = 1.721$ $\gamma = 0.490$	$\alpha = 1.103$ $\beta = 1.472$ $\gamma = 0.331$
	0.30	$\alpha = 0.565$ $\beta = 0.901$ $\gamma = 0.271$	$\alpha = 0.519$ $\beta = 0.774$ $\gamma = 0.200$	$\alpha = 0.488$ $\beta = 0.651$ $\gamma = 0.095$
	0.45	$\alpha = 0.300$ $\beta = 0.494$ $\gamma = 0.151$	$\alpha = 0.265$ $\beta = 0.419$ $\gamma = 0.095$	$\alpha = 0.266$ $\beta = 0.396$ $\gamma = 0.079$

Table 6-4b. α , β , and γ coefficients for the Response Ratio for CEUS.

		T_{\max}		
		0.05	0.075	0.10
P	0.35	$\alpha = 0.281$ $\beta = 1.013$ $\gamma = 0.699$	$\alpha = 0.373$ $\beta = 0.894$ $\gamma = 0.431$	$\alpha = 0.512$ $\beta = 0.897$ $\gamma = 0.340$
	0.55	$\alpha = 0.214$ $\beta = 0.580$ $\gamma = 0.385$	$\alpha = 0.244$ $\beta = 0.548$ $\gamma = 0.246$	$\alpha = 0.200$ $\beta = 0.495$ $\gamma = 0.207$
	0.70	$\alpha = 0.158$ $\beta = 0.405$ $\gamma = 0.226$	$\alpha = 0.023$ $\beta = 0.247$ $\gamma = 0.186$	$\alpha = 0.026$ $\beta = 0.245$ $\gamma = 0.121$

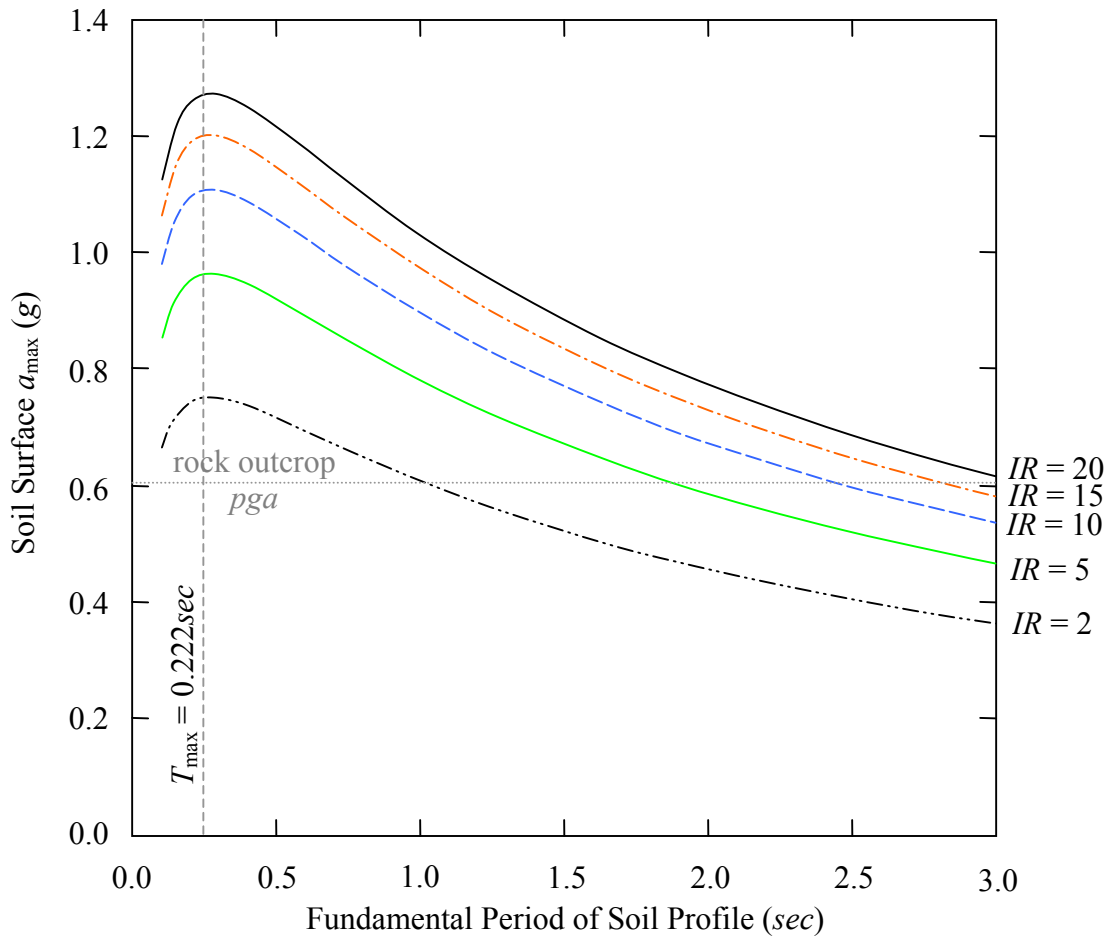


Figure 6-10. a_{\max} spectra for San Francisco, CA, corresponding to $IR = 2, 5, 10, 15,$ and 20 . The largest amplification occurs for sites having a fundamental period close to T_{\max} of the rock outcrop motion (i.e., $T_1 \approx 0.222\text{sec}$).

6.5 Comparison of proposed procedure with currently used amplification curves

As noted in Figure 6-10, the largest amplification of the pga occurs in soil profiles having a fundamental period very close to T_{\max} of the rock outcrop motion. Several proposed empirical correlations relating the characteristic period of ground motions and site-to-source distance for a $M7$ earthquake are shown in Figure 6-11. In this figure, the correlations proposed by Seed et al. (1969) and Idriss (1991) define the characteristic period of the ground motion as the period corresponding to the largest spectral acceleration, while the characteristic period for the correlation proposed by Rathje et al. (1998) is the period corresponding to the largest spectral acceleration of a smoothed

response spectrum. Both of these definitions are equivalent to T_{\max} of the parabolic UHS of the rock outcrop motions. As may be seen from examining the correlations presented in Figure 6-11, the characteristic period of the ground motion increases with site-to-source distance.

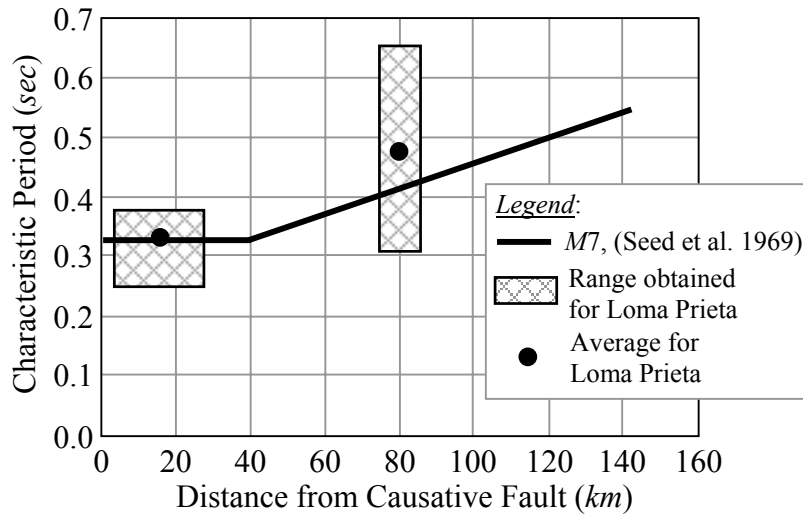


Figure 6-11a. Estimated characteristic periods for rock motions as a function of distance from the causative fault. (Adapted from Idriss 1991).

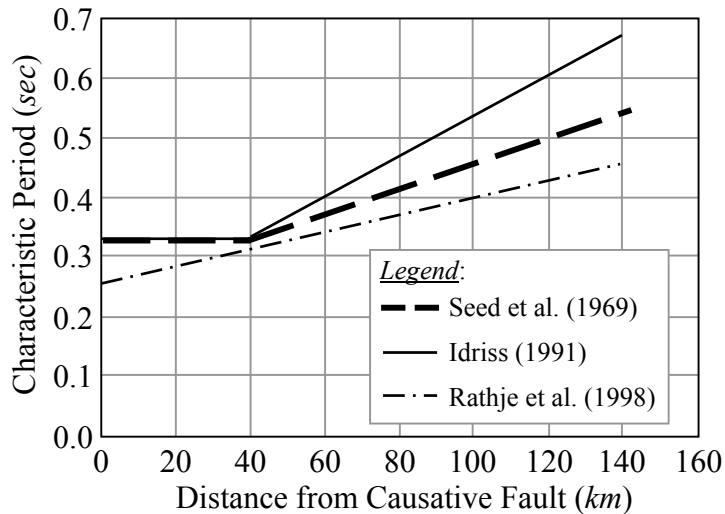


Figure 6-11b. Comparison of earthquake period – distance relations for $M7$ events. The correlations of Seed et al. (1969) and Idriss (1991) define the characteristic period of the ground motion as the period corresponding to the largest spectral acceleration value for the actual earthquake motion response spectrum. The correlation proposed by Rathje et al. (1998) defines the characteristic period as the period corresponding to the largest spectral acceleration for the smoothed response spectrum of the ground motion.

Based on the trends shown in Figures 6-10 and 6-11, it can be reasonably surmised that for a given IR , the period of the sites showing the largest amplification of the rock outcrop pga will increase with site-to-source distance (i.e., the largest amplification of the pga will occur when $T_1 \approx T_{max}$). As a corollary, soft soil sites will generally de-amplify the pga close to the source and amplify the pga at greater distances from the source. This is the same trend shown in the amplification curve for soft soil sites presented in Figure 6-2b, where the small values of the rock outcrop pga correspond to large site-to-source distances and the large values of the pga correspond to short site-to-source distances.

To examine this trend in pga amplification further, a_{max} spectra are computed for rock outcrop motions representative of those experienced at long and short epicentral distances (ED) during the 1989 Loma Prieta earthquake. The general shape of the response spectrum for the Skyline time history, recorded during the Loma Prieta earthquake on a rock outcrop just south of San Francisco at an epicentral distance of about $70km$ (Dobry 1994), was selected as being representative of long epicentral distance motions. The Skyline time history and corresponding response spectrum are shown in Figure 6-12. The general shape of the UHS having a mean return period of 500 years, computed using the USGS map spectral values and Equation (6-8) was selected as being representative of short epicentral distance rock outcrop motions experienced during the Loma Prieta earthquake. For comparison, the UHS is shown scaled to the same pga as the Skyline time history in Figure 6-12b. From this figure, it can be seen that the spectra corresponding to long and short epicentral distances differ significantly. The Skyline spectrum can be reasonably represented as a parabola having $P = 0.2$, $T_{max} = 0.6sec$, and $T_{pga} = 0.133sec$, while the UHS has $P = 0.359$, $T_{max} = 0.222sec$, and $T_{pga} = 0.044sec$. The relative values for T_{max} of these representative spectra are consistent with the correlations presented in Figure 6-11 (i.e., T_{max} for the long epicentral motion is greater than the T_{max} for the short epicentral motion).

Having established the relative shapes of the response spectra representing long and short epicentral distance motions, the spectra were scaled to representative pga 's measured on

rock outcrops during the Loma Prieta earthquake at the respective epicentral distances (i.e., short epicentral distance $pga = 0.4g$; long epicentral distance $pga = 0.1g$; Aki 1993).

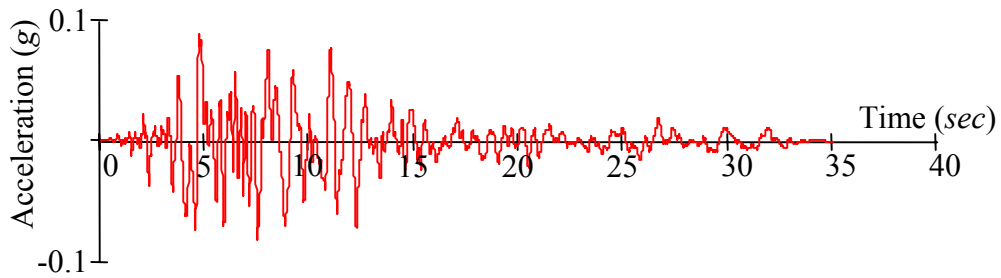


Figure 6-12a. Skyline time history recorded on a rock outcrop at approximately $70km$ from the epicenter of the Loma Prieta earthquake.

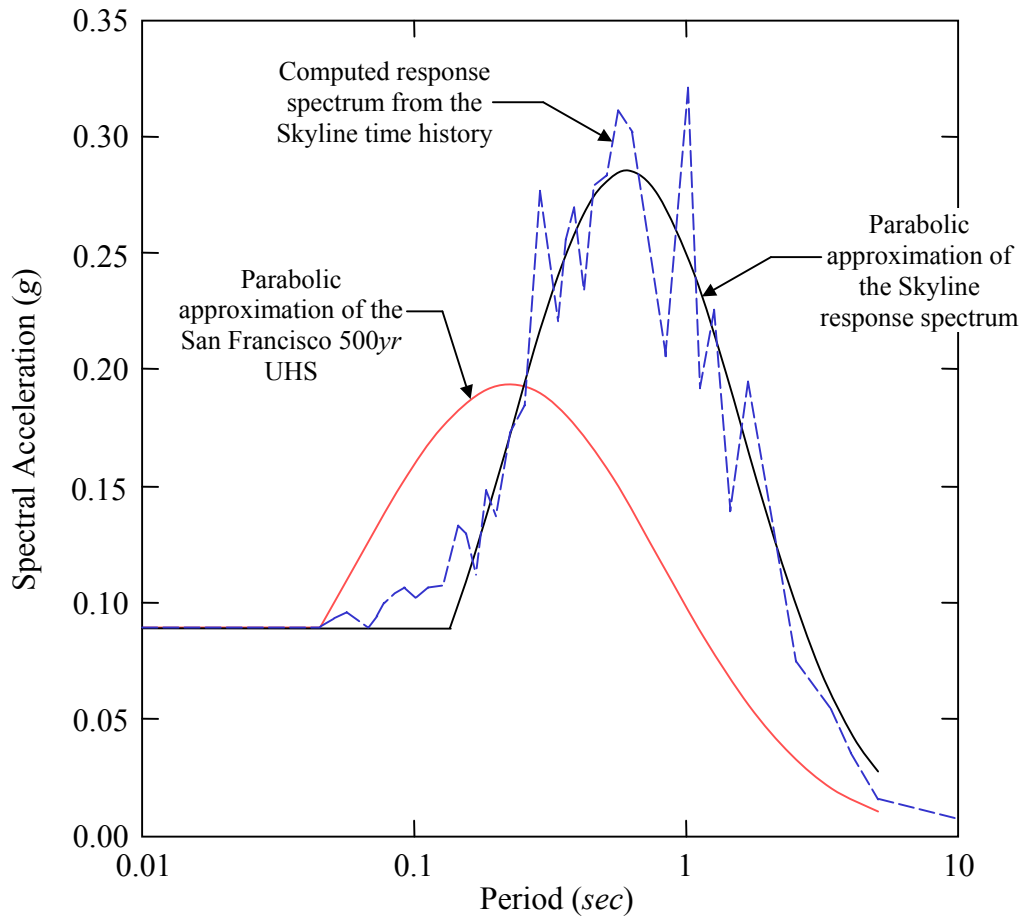


Figure 6-12b. Response spectrum of the Skyline time history and its parabolic approximation. Also shown is the 500 year UHS for San Francisco, scaled to the pga of the Skyline motion.

In computing the a_{\max} spectra corresponding to the representative response spectra shown in Figure 6-12b, IR was assumed equal to 2.28, which is a typical value for soft soil sites in the San Francisco area (e.g., the San Francisco Airport). Because the T_{pga} for the Skyline motion (i.e., 0.133sec) differs from the values assumed for the WUS and CEUS (i.e., $T_{pga} = 0.044$ and 0.01sec, respectively), the response ratio coefficients α , β , and γ given in Table 6-4 cannot be used directly to compute the corresponding a_{\max} spectrum. However, an approximate a_{\max} spectrum for the Skyline motion was determined using the following procedure, which is illustrated in Figure 6-13.

- Compute the a_{\max} spectrum for a parabolic response spectrum having a similar shape to the response spectrum of the Skyline motion, but having a $T_{pga} = 0.044\text{sec}$.
- Scale the computed a_{\max} spectrum such that it has the same relative position with the Skyline spectrum as it had with the response spectrum for which it was actually computed.

The a_{\max} spectrum corresponding to the short epicentral distance motions was computed directly using Equation (6-15) in conjunction with the response ratio coefficients α , β , and γ given in Table 6-4. The resulting a_{\max} spectra for both the long and short epicentral distance motions are shown in Figure 6-14. Similar to the observation made regarding the a_{\max} spectra in Figure 6-10, the maximum amplification of the rock outcrop pga occurs for profiles having a fundamental period approximately equal to T_{\max} .

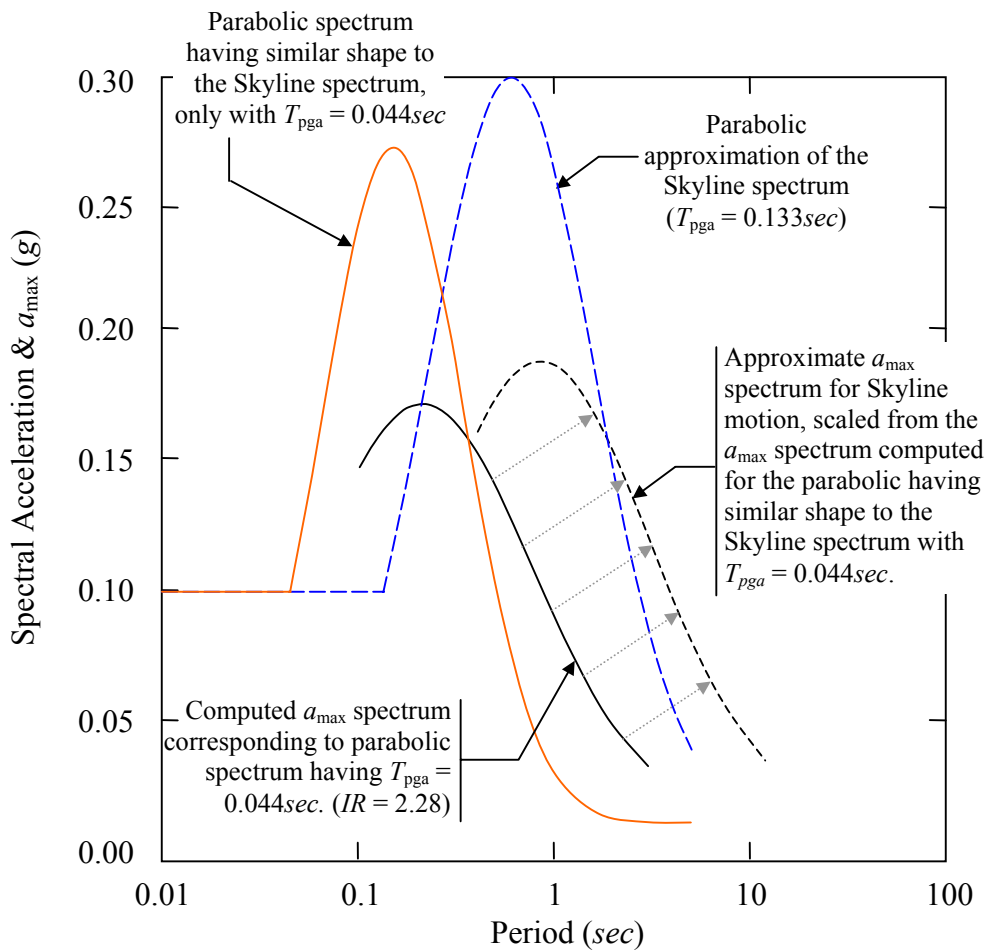


Figure 6-13. Procedure for approximation of the a_{max} spectrum for the Skyline motion ($T_{pga} = 0.133sec$) by scaling the a_{max} spectrum computed for a parabolic response spectrum having a similar shape to the Skyline spectrum with $T_{pga} = 0.044sec$.

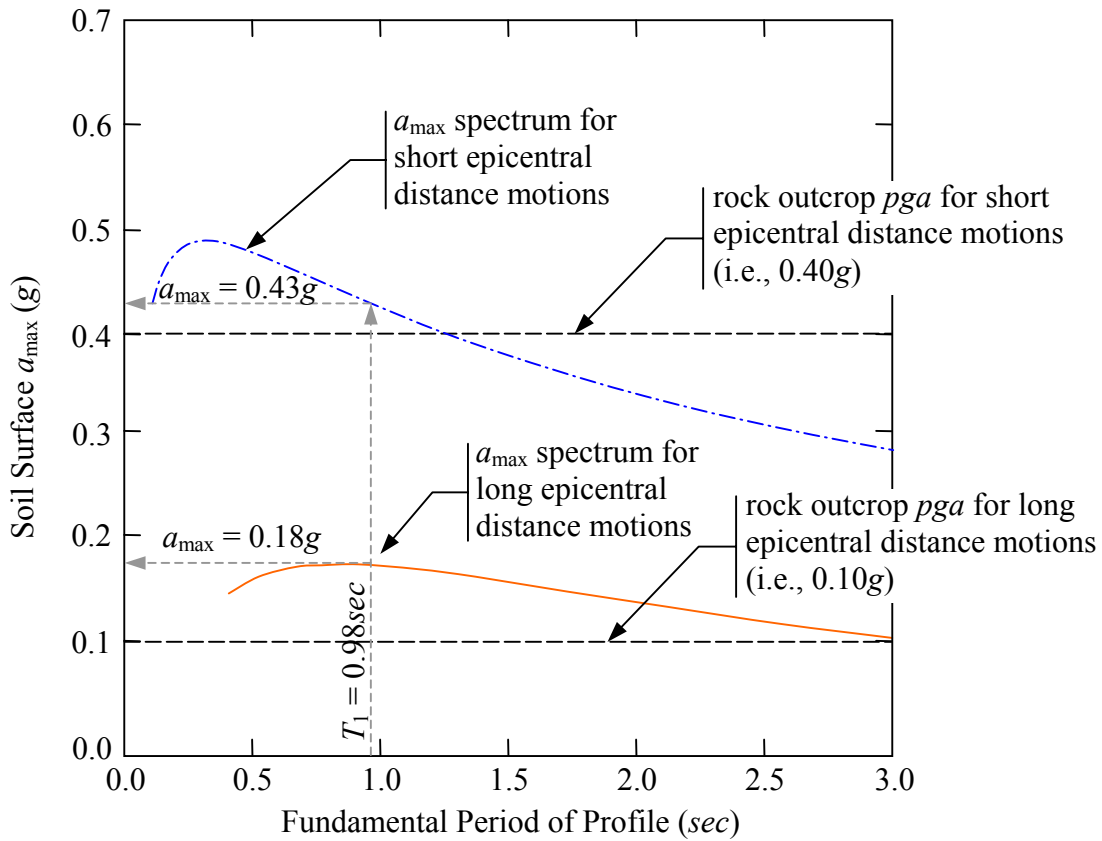


Figure 6-14. a_{max} spectra for representative long and short epicentral distance motions for the 1989 Loma Prieta earthquake for sites having $IR = 2.28$.

One of the soil profiles used by Idriss (1990, 1991) to develop the amplification curve shown in Figure 6-2b was the San Francisco Airport, which can be characterized as $T_1 = 0.98sec$ and $IR = 2.28$ (values computed from profile given in Idriss 1993). Using these site parameters and the a_{max} spectra shown in Figure 6-14, the soil surface a_{max} values at long and short epicentral distances are estimated to be 0.18g and 0.43g, respectively. When these results are superimposed on Idriss' amplification curve, as shown in Figure 6-15, the predicted and observed values agree remarkably well.

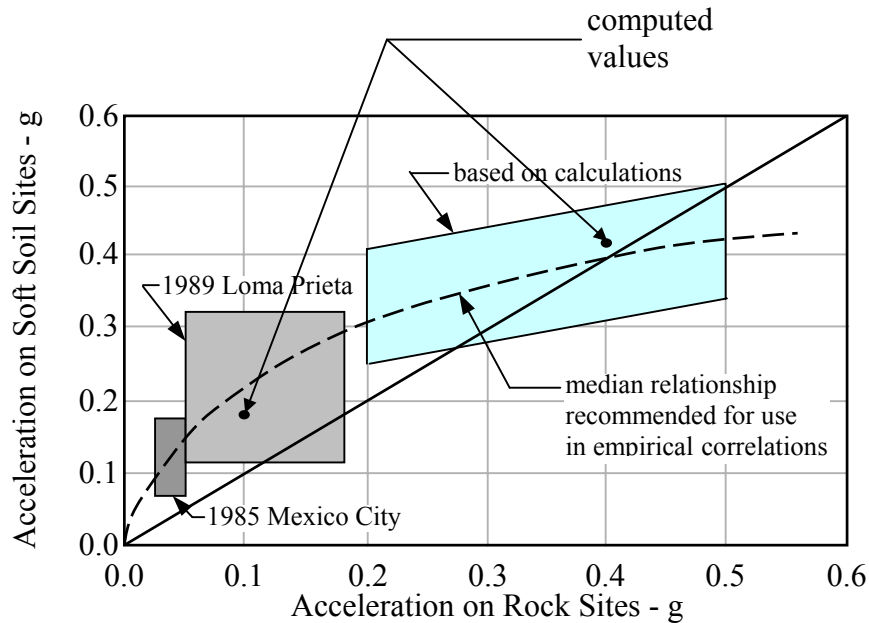


Figure 6-15. Comparison of amplification curve and a_{\max} values computed using the proposed procedure. (Adapted from Idriss 1990, 1991).

As a final commentary to the comparison shown in Figure 6-15, Idriss (1990, 1991) and several other researchers (e.g., Aki (1993)) attributed the general shape of the amplification curve in this figure to soil non-linearity. They hypothesize that for large values of rock outcrop pga , the induced shear stress in the soil approaches shear strength of the soil. As a result, there is a de-amplification of $pga > 0.4g$, while smaller pga values are amplified. However, the a_{\max} spectra shown in Figure 6-14 and the extracted a_{\max} values corresponding to $T_1 = 0.98sec$ suggest that the relative values of the characteristic period of the earthquake motion and the fundamental period of the soil profile are the dominant factors for the shape of the amplification curve, not soil non-linearity. This is not to say that soil non-linearity does not influence the site response analyses, especially when liquefaction occurs, but rather, it may not be the dominant factor.

Appendix 6a: Procedure for computing an equivalent uniform profile

As stated above, the proposed approach for determining the a_{\max} spectrum only applies to uniform profiles (i.e., profiles having constant shear wave velocity with depth). This appendix outlines a procedure for computing the equivalent uniform profile for an actual profile. To do this, there are 3 main steps: 1) determine the fundamental period of the actual soil profile, 2) estimate the unit weight and shear wave velocity of the equivalent profile, and 3) compute the impedance ratio between the underlying rock and the uniform profile. The details of each step are discussed below.

1) Determine the fundamental period of the actual profile.

There are several approaches for determining the fundamental period of an actual profile, and a comprehensive treatment of this topic is given in Dobry et al. (1976). The Successive Two Layer Approach proposed by Madera (1970) is one of the approaches reviewed in Dobry et al. (1976) and is relatively simple to implement and is accurate. This procedure is outlined below.

The Successive Two Layer Approach proposed by Madera (1970) consists of repeatedly solving the fundamental period of a two-layered profile.

- a) Starting at the soil surface and working down, compute the periods of the top two layers treating them as fixed-based shear beams: $T_1 = 4h_1/v_{s1}$ and $T_2 = 4h_2/v_{s2}$, where h_1 and h_2 are the thicknesses of layers 1 and 2, respectively, and v_{s1} and v_{s2} are the shear wave velocities of layers 1 and 2, respectively. For sand profiles, the shear wave velocity of the soil can be related to SPT N -values as follows:

$$G_{\max} = 440 \cdot (N_{1,60})^{0.333} \cdot Pa_1 \cdot \sqrt{\frac{\sigma'_{mo}}{Pa_2}} \quad (\text{Seed et al. 1986}) \quad (6a-1)$$

$$v_s = \sqrt{\frac{G_{\max} \cdot g}{\gamma_s}} \quad (\text{e.g., Kramer 1996}) \quad (6a-2)$$

- where: G_{\max} = The small strain shear modulus.
 $N_{1,60}$ = The corrected SPT N -value.
 v_s = Shear wave velocity.
 σ'_{mo} = The mean effective confining pressure.
 Pa_1 = Atmospheric pressure in the same units as G_{\max} .
 Pa_2 = Atmospheric pressure in the same units as σ'_{mo} .
 g = Acceleration due to gravity.
 γ_s = Total unit weight of the soil.

- b) Knowing the ratios of h_1/h_2 and T_2/T_1 and using Figure 6a-1, determine the period of the two-layer system, T_{1-2} (denoted as T in Figure 6a-1).
- c) Replace the top two layers with an equivalent single layer having a thickness of $h_{1-2} = h_1 + h_2$ and $v_{s, 1-2} = 4h_{1-2}/T_{1-2}$.
- d) Repeat Steps *a-c*, using the equivalent single layer computed in Step *c* as layer 1. Continue until the entire profile is modeled as an equivalent single layer having a fundamental period T_1 .

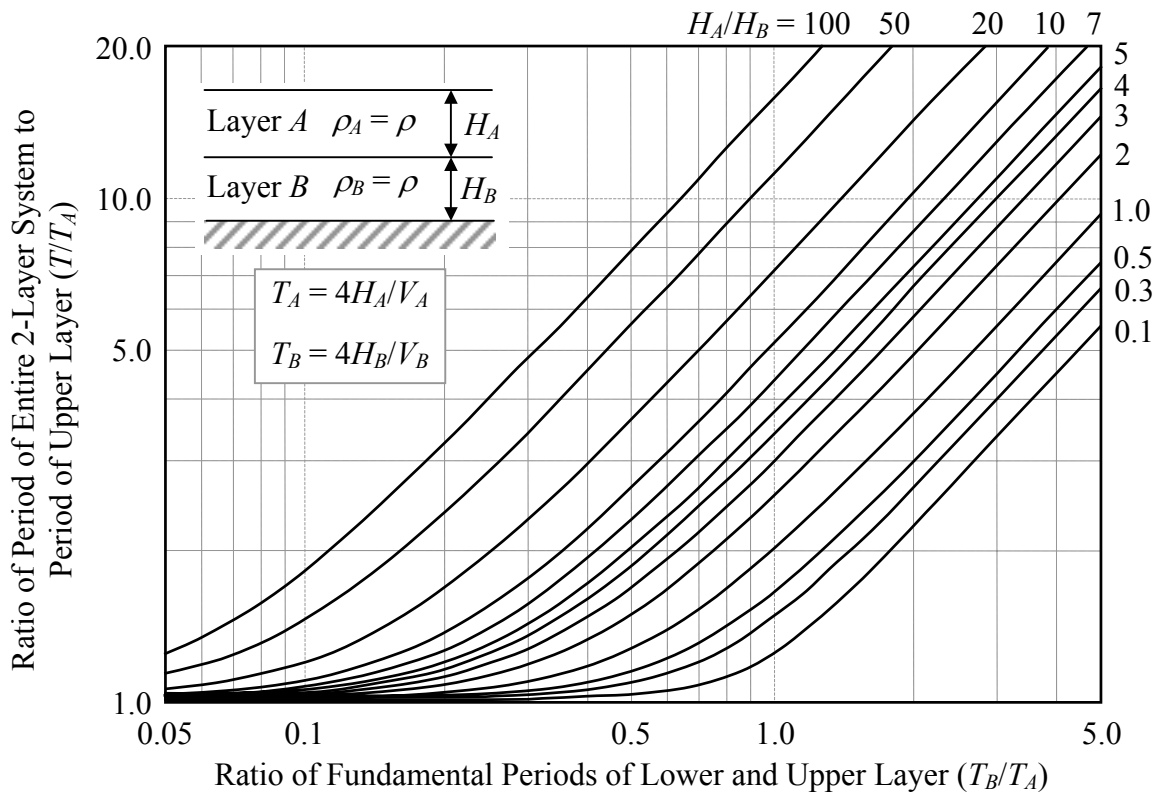


Figure 6a-1. Chart used in determining the fundamental period of soil profile. (Adapted from Oweis et al. 1975).

2) Estimate the total unit weight (γ_{equiv}) and shear wave velocity $v_{s, equiv}$ of the equivalent profile.

- a) Knowing the fundamental period of the profile (T_1), the shear wave velocity of the equivalent single layer profile ($v_{s, equiv}$) is determined as:

$$v_{s, equiv} = \frac{4H}{T_1} \quad (6a-3)$$

where H is the entire height of the actual and equivalent single layer profiles.

- b) The total unit weight of the equivalent single layer profile (γ_{equiv}) is determined as:

$$\gamma_{equiv} = \frac{h_1\gamma_1 + h_2\gamma_2 + h_3\gamma_3 + \dots + h_k\gamma_k}{H} \quad (6a-4)$$

where h_i and γ_i are the thickness and unit weight of the i^{th} layer, and k is the total number of soil layers.

3) Compute the impedance ratio (IR) between the base rock and the uniform profile.

The impedance ratio (IR) between the equivalent soil layer and the base rock is determined as:

$$IR = \frac{\gamma_r v_r}{\gamma_{equiv} v_{s,equiv}} \quad (6a-5)$$

where γ_r and v_r are the total unit weight and the shear wave velocity of the base rock, respectively.