

Stem taper functions for white birch (*Betula platyphylla*) and costata birch (*Betula costata*) in the Xiaoxing'an Mountains, northeast China

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White birch (*Betula platyphylla* Sukaczew) and costata birch (*Betula costata* Trautv.) are valuable hardwood tree species growing in northeast China. Several taper studies have analysed birch species in the countries harbouring the boreal forests. However, this study presents an initial attempt to develop stem taper models using the fixed- and mixed-effects modelling for white birch and costata birch in Xiaoxing'an Mountains, northeast China. Ten commonly used taper models were evaluated by using 228 destructively sampled trees of both tree species comprising of 4582 diameter and height measurements. The performance of these models was tested in predicting diameter at any height, total volume and merchantable volume (10 and 20 cm top diameters). We incorporated a second-order continuous-time error structure to adjust the inherent autocorrelation in the data. The segmented model of Clark best predicted the diameter and total or merchantable volume when the upper stem diameter at 5.3 m was available. When diameter measurements at 5.3 m were not available, the models of Kozak and Max and Burkhart were superior to other models for white birch and costata birch, respectively. After model comparison, the best model of Clark was refitted as the NLME model.

Introduction

Forest area in China accounts for 220 million ha with a standing volume of 17.56 billion m³, as per the latest National Forest Inventory (NFI-9). Forest cover has expanded from 12.69 to 22.96 per cent during the last 40 years. Sustainable forest management, however, remains a rigorous undertaking to meet national and international commitments. State Forestry Administration aims to maintain the forest cover at over 26 per cent by the year 2050 to bridge the supply and demand gap and achieve a sound cycle of ecological conditions (Xu *et al.*, 2019). A critical step in this direction is accurate growth and yield modelling to know the exact potential productivity of natural forests (Hou *et al.*, 2019).

Birch species naturally grow in East Asia, Scandinavia, Baltic States, Russia and North America. White birch (*Betula platyphylla* Sukaczew) and Korean/costata birch (*Betula costata* Trautv.) are major valuable tree species of China covering an area of 10.38 million ha, with a total volume of 923 million m³ (Xu *et al.*, 2019). The greatest area under these species lies in northeast China. White birch is the principal source for plywood, paper pulp and furniture industry. Recent studies indicate that it has a high medicinal value (Xu *et al.*, 2016; Wang *et al.*, 2019). Costata birch provides the raw material for pulp and fibre-based products. They

play an important role in maintaining the ecological balance of the natural broad-leaved secondary forest in Northeast China (Wang *et al.*, 2018a; Zhao *et al.*, 2019).

Stem taper models are an essential component of growth and yield modelling. Taper models can predict the stem diameter at any height (*d*), along with merchantable and total volumes (Li and Weiskittel, 2010). These models supersede the volume tables as they can estimate *d*, merchantable height to any diameter above ground, the volume of a log at any length and at any height from the ground in addition to the merchantable and total stem volume (Kozak, 2004). Additionally, taper models are useful in timber quality studies and modelling of carbon allocation in different stem sections. They are also instrumental in assessing the impact of silvicultural treatments on stem taper (Fonweban *et al.*, 2011).

Since the last century, many taper models have been developed. At present, an in-depth discussion is available about their evolution and types (Sakici *et al.*, 2008; Crecente-Campo *et al.*, 2009; Burkhart and Tomé, 2012). Of the many model forms, segmented or variable form taper models are often recommended based on taper studies (Özcelik and Brooks, 2012; Gómez-García *et al.*, 2013; Sakici and Ozdemir, 2018). Rojo *et al.* (2005) and Tang *et al.* (2017) suggested the variable form taper models

for diameter estimates of maritime pine (*Pinus pinaster* Ait.) in Spain and Himalayan birch (*Betula alnoides* Buch.-Ham. ex D.Don.) in South China, respectively. Alternatively, segmented taper models performed better than variable form models for diameter and volume estimates of Scots pine (*Pinus sylvestris* L.) in northwestern Spain (Diéguez-Aranda *et al.*, 2006) and Lebanon cedar (*Cedrus libani* A. Rich.) and Cilicica fir (*Abies cilicica* Carr.) in Bucak region, Turkey (Özcelik and Dirican, 2017). Simultaneously, the performance of segmented and variable form models was very similar for Kazdağı fir (*Abies nordmanniana* subsp. *equi-trojani* (Asc. & Sint. ex Boiss.) Coode & Cullen) and Oriental beech (*Fagus orientalis* Lipsky) in Turkey and white birch (*B. platyphylla* Sukaczew) in northeast China (Sakici and Ozdemir, 2018; Shahzad *et al.*, 2019, 2020). Therefore, it is useful to perform a thorough analysis of these taper models so that their application can be extended to other species.

The research on taper modelling has mostly focused on conifers species with an excurrent stem form. Generally, hardwood tree species exhibit a decurrent crown. This spreading crown constrains the merchantability of the main stem, in particular, for the saw timber. Hardwood trees of similar total heights have different merchantability limits (Fowler and Rennie, 1988). However, very few authors (e.g. Martin, 1981; Westfall and Scott, 2010) have evaluated taper models in estimating volume for different merchantable products (sawlog, pulpwood, etc.). This practical consideration was accounted for in the present analysis.

Over the years, mixed-effects models have gained attention in taper modelling (Trincado and Burkhart, 2006; Yang *et al.*, 2009; Cao and Wang, 2011; Özcelik *et al.*, 2011; Zhao and Kane, 2017). Compared with ordinary least squares, mixed models contain both fixed effects and random effects parameters to explain between-tree and within-tree variations in taper studies. Furthermore, this technique enables the calibration of a model for a particular tree or site, if additional data are available for that specific tree or site (Garber and Maguire, 2003; de-Miguel *et al.*, 2013; Cao and Wang, 2014).

Most of the existing studies used mixed-effects modelling to fit a single model, either variable form (Lejeune *et al.*, 2009; Arias-Rodil *et al.*, 2015; Bronisz and Zasada, 2019) or segmented type (Leites and Robinson, 2004; Trincado and Burkhart, 2006; Cao and Wang, 2011). The studies by de-Miguel *et al.* (2012) and Gómez-García *et al.* (2013) fitted different fixed-effects models and selected variable form models for developing a mixed model. However, the later study tested the fixed-effects models for diameter predictions only. Therefore, it is beneficial to further evaluate different taper models with both fixed- and mixed-effects approaches and extend their scope to other species.

There are many references to taper studies of birch species in the world (e.g. Kozak *et al.*, 1969; Martin, 1981; Laasasenaho, 1982; Gál and Bella, 1994; Westfall and Scott, 2010; Gómez-García *et al.*, 2013; Ung *et al.*, 2013). These studies accounted for paper birch (*Betula papyrifera* Marsh.), yellow birch (*Betula alleghaniensis* Britton), sweet birch (*Betula lenta* L.), grey birch (*Betula populifolia* Marsh.), river birch (*Betula nigra* L.) and downy birch (*Betula pubescens* Ehrh.) in Canada, the US and Europe. Research on white birch and costata birch is quite active in China, but it mainly focusses on biomass and genetics (Cai *et al.*, 2013; Wang *et al.*, 2018b; Wang *et al.*, 2019). Recently, Shahzad

et al. (2019, 2020) tested fixed-effects models for white birch in Daxingan Mountains, Northeast China. To our knowledge, no taper study exists for white birch and costata birch that have considered both fixed- and mixed-effects modelling.

The specific objectives of this study were to: (1) evaluate well-known segmented and variable form taper models using fixed-effects and select the best model for diameter and volume prediction of white birch and costata birch and (2) develop tree-specific mixed-effects models based on the best taper model.

Material and methods

Study area

The study area encompasses Xiaoxingan Mountains (127°42'–130°14' E, 46°28'–49°21' N) in Heilongjiang province, northeast China. The research sites fall in the cold temperate forest with continental monsoon climate. The elevation range of the area is 600–1000 m from sea level. The average annual precipitation fluctuates from 550 to 670 mm. The mean annual temperature ranges from –2 to 2°C, and the average temperatures of January and July are –25 and +22°C, respectively. The predominant species include larch (*Larix gmelinii* Rupr.), spruce (*Picea koraiensis* Nakai), Korean pine (*Pinus koraiensis* Sieb. et Zucc.), Khingan fir (*Abies nephrolepis* Maxim), white birch (*B. platyphylla* Sukaczew), costata birch (*B. costata* Trautv.) and aspen (*Populus davidiana* Dode). The typical soil of the area is dark brown forest soil (Burger and Shidong, 1988; Ma *et al.*, 2014).

Data description

A sample of 120 trees of white birch and 108 trees of costata birch was collected from uneven-aged natural stands in the study area. Data from a total of 228 trees with 4582 diameter and height measurements were utilized. The sample covered the existing range of diameter and height classes. The range of diameter and height was 4.8–37.9 and 6.9–22.2 m for white birch and 19.0–49.2 and 13.8–23.4 m for costata birch. Diameter at breast height over bark (*D*, 1.3 m) was measured to the nearest 0.1 cm for all trees. Trees were felled to measure total tree heights (*H*) and their diameters over bark (*d*) at the heights (*h*) of 0.3, 0.6, 1, 1.3 and 2 m. After the height of 2 m, *d* was measured at a fixed interval of 1 m. Measurement range fluctuated from 0.3 to 1 m along the stem except for the top section, which was considered as a cone. Two perpendicular diameters (over bark) were measured, and their average was used. Smalian's formula (Burkhart *et al.*, 2019) was used to calculate log volumes that were added to the volume of the cone to get over bark total volume. Smalian's formula is $V = A_1 + A_2/L$, where *V* is the volume of the log in m³, *A*₁ is the area of the small end of the log in m², *A*₂ is the area of the large end of the log in m² and *L* is the length of the log in m. The formula for the volume of the cone is $V_T = \pi t d_T^2/12$, where *V*_T is the volume of the terminal section, *d*_T is the diameter at the base of the last section and *t* is the distance from the base of the last section to the stem tip (West, 2015). The summary statistics of the datasets are shown in Table 1. The plots of relative diameter (*d/D*) against relative height (*h/H*) for both species are shown in Figure 1.

Table 1 Summary statistics of the datasets by species and number of trees.

Species	Variables	Minimum	Mean	Maximum	SD
White birch (n = 120)	D (cm)	4.8	18.5	37.9	5.6
	H (m)	6.9	17.2	22.2	2.8
	d (cm)	0.1	12.5	48.8	7.6
	h (m)	0.3	8.1	22.2	5.8
	V (m ³)	0.01	0.2	0.8	0.1
Costata birch (n = 108)	D (cm)	19.0	31.1	49.2	6.4
	H (m)	13.8	19.3	23.4	2.1
	d (cm)	8.0	26.0	59.4	8.8
	h (m)	0.3	7.9	23.4	5.6
	V (m ³)	0.2	0.9	2.4	0.4

D, diameter at breast height over bark (1.3 m above ground); H, total tree height; d, diameter over bark at height h; h, height above the ground to the measurement point; V, total stem volume over bark; SD, standard deviation.

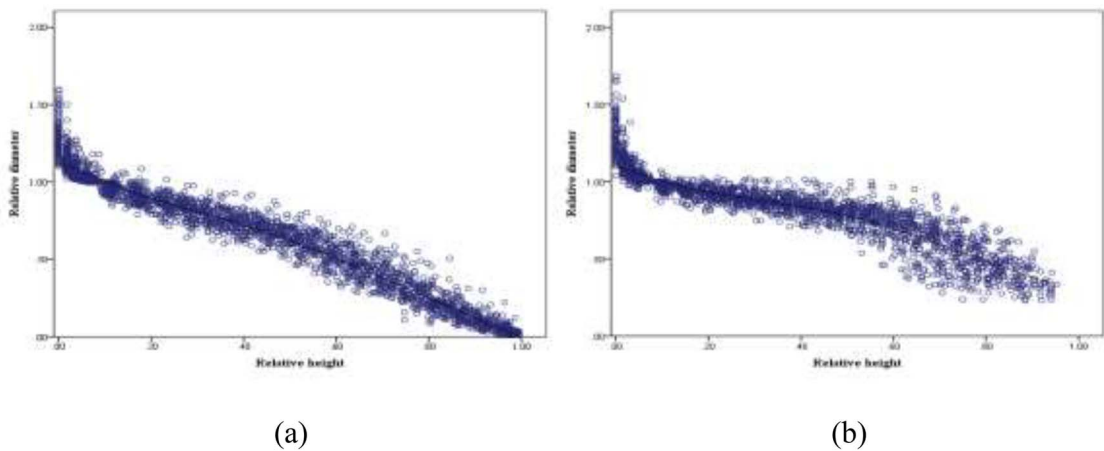


Figure 1 The plots of relative diameter against relative height for white birch (a) and costata birch (b).

Taper models

Ten taper models were selected from the literature (Sakici *et al.*, 2008; Li and Weiskittel, 2010; Özcelik and Crecente-Campo, 2016; Doyog *et al.*, 2017; Tang *et al.*, 2017). The selected models covered four segmented taper models (Max and Burkhart, 1976; Clark *et al.*, 1991; Fang *et al.*, 2000) and six variable form taper models (Muhairwe, 1999; Bi, 2000; Lee *et al.*, 2003; Kozak, 2004; Sharma and Zhang, 2004; Sharma and Parton, 2009). In the following text, these models will be written as Max and Burkhart (1976), Clark *et al.* (1991) Model 1 and Model 2, Fang *et al.* (2000), Lee *et al.* (2003), Kozak (2004), Sharma and Zhang (2004) and Sharma and Parton (2009). Table 2 shows the selected taper models along with the source of each model. In all cases, the dependent variable was the diameter along the stem (*d*). Some studies (e.g. Gregoire *et al.*, 2000; de-Miguel *et al.*, 2012) have suggested that the predictions of total or merchantable volumes are less biased when *d*² is used to fit taper models. However, we used *d* as the dependent variable because the prediction of height *h* at specific upper stem diameters is an important step

in classifying stem sections by upper diameter limits and log lengths as per different industrial requirements.

The model of Clark *et al.* (1991) requires an additional measurement of diameter at 5.3 m height. At first, these measurements were attained by linear interpolation. Afterward, they were predicted with the equation proposed by Clark *et al.* (1991) (Table 2). The Clark *et al.* (1991) Model 1 and Clark *et al.* (1991) Model 2 represented interpolation and prediction methods, respectively.

Model fitting

The model parameters were estimated with the MODEL procedure of SAS using the generalized nonlinear least-squares method, which allows direct modelling of the error variance for autocorrelation in the fitting process (SAS Institute Inc., 2008). Spatial correlation was expected within the observations due to hierarchical data of the study. We instituted a second-order continuous autoregressive error structure (CAR (2)) to adjust the

Table 2 Tested stem taper models**Segmented Taper Models**

Max and Burkhardt (1976)

$$d = D \left[b_1 (Z - 1) + b_2 (Z^2 - 1) + b_3 (a_1 - Z)^2 I_1 + b_4 (a_2 - Z)^2 I_2 \right]^{0.5}$$

where: $I_1 = 1$, if $z \leq a_1$; 0 otherwise $I_2 = 1$, if $z \leq a_2$; 0 otherwise

Clark et al. (1991)

$$d = \left\{ \begin{aligned} &I_S \left[D^2 \left(1 + \left(1 + (b_2 + b_3/D^3) \left((1 - h/H)^{b_1} - (1 - 1.3/H)^{b_1} \right) / (1 - (1 - 1.3/H)^{b_1}) \right) \right) \right] \\ &+ I_B \left[D^2 - (D^2 - F^2) \left((1 - 1.3/H)^{b_4} - (1 - h/H)^{b_4} \right) / (1 - 1.3/H)^{b_4} - (1 - 5.3/H)^{b_4} \right] \\ &+ I_T \left[F^2 (b_6 \left((h - 5.3/H - 5.3) - 1 \right)^2 + I_M (1 - b_6/b_5^2) (b_5 - (h - 5.3/H - 5.3))^2) \right] \end{aligned} \right\}^{0.5}$$

where: $I_S = \begin{cases} 1 & h < 1.3 \\ 0 & \text{otherwise} \end{cases}$, $I_B = \begin{cases} 1 & 1.3 \leq h < 5.3 \\ 0 & \text{otherwise} \end{cases}$, $I_T = \begin{cases} 1 & h > 5.3 \\ 0 & \text{otherwise} \end{cases}$, $I_M = \begin{cases} 1 & h < (5.3 + b_5(H - 5.3)) \\ 0 & \text{otherwise} \end{cases}$,

F = diameter at 5.3 m height, b_i = regression coefficients for different stem sections, i.e. b_1, b_2 and b_3 for <1.3 m, b_4 for 1.3–5.3 m and b_5, b_6 for >5.3 m. Clark et al. (1991) proposed following equation to predict F :

$$F = D (b_1 + b_2 (5.3/H)^2)$$

Fang et al. (2000)

$$d = c_1 \sqrt{H^{(k-b_1)/b_1} (1-Z)^{(k-b)/b} q_1^{I_1+I_2} q_2^{I_2}}$$

$$c_1 = \sqrt{a_0 D^{a_1} H^{(a_2-k)/b_1} / b_1 (t_0 - t_1) + b_2 (t_1 - q_1 t_2) + b_3 q_1 t_2}$$

$$\begin{cases} q_1 = (1 - p_1)^{(b_2-b_1)k/b_1 b_2} \\ q_2 = (1 - p_2)^{(b_3-b_2)k/b_2 b_3} \end{cases}, \begin{cases} t_1 = (1 - p_1)^{k/b_1} \\ t_2 = (1 - p_2)^{k/b_2} \end{cases}, t_0 = 1, \begin{cases} I_1 = 1, \text{ if } p_2 \geq Z \geq p_1; 0 \text{ otherwise} \\ I_2 = 1, \text{ if } 1 \geq Z \geq p_2; 0 \text{ otherwise} \end{cases}$$

$$b = b_1^{1-(I_1+I_2)} b_2^{I_1} b_3^{I_2}, k = 0.0000785, Z = h/H.$$

Variable Exponent Taper Models

Muhairwe (1999)

$$d = b_1 D^{b_2} \left[1 - \sqrt{Z} \right]^{(b_3 Z + b_4 Z^2 + \frac{b_5}{Z} + b_6 Z^3 + b_7 D + b_8 \left(\frac{D}{H} \right))}$$

Bi (2000)

$$d = D \left[\ln \sin \left(\frac{\pi}{2} z \right) / \ln \sin \left(\frac{\pi}{2} t \right) \right]^{b_0 + b_1 \sin \left(\frac{\pi}{2} z \right) + b_2 \cos \left(\frac{3\pi}{2} z \right) + b_3 \sin \left(\frac{\pi}{2} z \right) / z + b_4 D + b_5 z \sqrt{D} + b_6 z \sqrt{H}}$$

Lee et al. (2003)

$$d = b_1 D^{b_2} (1 - z)^{b_3 z^2 + b_4 z + b_5}$$

Kozak (2004)

$$d = b_0 D^{b_1} H^{b_2} x^{b_3 z^4 + b_4 (1/e^{D/H}) + b_5 x^{0.1} + b_6 (1/D) + b_7 H^w + b_8 x}$$

$$\text{Where: } x = w / (1 - (1.3/H)^{1/3}), w = 1 - z^{1/3}.$$

(Continued)

Table 2 (Continued)

Sharma and Zhang (2004)

$$d = D \left[b_0 \left(\frac{h}{1.3} \right)^{2 - (b_1 + b_2 z + b_3 z^2)} \left(\frac{H-h}{H-1.3} \right)^{0.5} \right]$$

Sharma and Parton (2009)

$$d = D \left[b_0 \left(\frac{H-h}{H-1.3} \right) \left(\frac{h}{1.3} \right)^{b_1 + b_2 z + b_3 z^2} \right]$$

D , diameter at breast height over bark (cm); H , total tree height (m); h , height above ground (m); d , diameter outside bark (cm) at height h (m); z , h/H ; t , $1.3/H$; a_i , b_i and p_i , parameters to be estimated.

innate autocorrelation. This specified error structure allows the practical use of a model for irregularly spaced and unbalanced data (Grégoire *et al.*, 1995). Programming for CAR (2) structure was worked out in the MODEL procedure of SAS (SAS Institute Inc., 2008).

Model comparisons

The accuracy of diameter and volume estimates was evaluated by graphical and numerical assessments of the residuals. The volumes of the following timber assortments were calculated (1) total volume (V_T), volume from stump height to tip; (2) pulpwood volume (V_P), volume from stump height to 10 cm top diameter and (3) saw log (V_S) volume from stump height to 20 cm top diameter. Stump height was 0.3 m in this analysis. The measured diameters were used to calculate sectional volumes, which were added to obtain observed V_T , V_P and V_S . Similarly, the predicted diameters were utilized to calculate the predicted volume assortments. The length of sections varied from 0.3 to 1 m along the stem. The volume of each section was calculated by Smalian's formula. Four standard goodness-of-fit statistics were tested: root mean square error (RMSE), fit index (FI), mean absolute bias (MAB) and mean percentage of bias (MPB). The notations for these statistics are as under:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{n - p}}$$

$$\text{FI} = 1 - \frac{\sum_{i=1}^n (y_i - \hat{y}_i)^2}{\sum_{i=1}^n (y_i - \bar{y})^2}$$

$$\text{MAB} = \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{n}$$

$$\text{MPB} = 100 \times \frac{\sum_{i=1}^n |y_i - \hat{y}_i|}{\sum_{i=1}^n y_i}$$

Where y_i , \hat{y}_i and \bar{y} stand for measured, predicted and average values of the response variable; n symbolizes the total number of observations and p is the number of parameters, respectively.

The models were also assessed by box plots of d residuals against position (relative heights of 5, 15, 25, up to 95 per cent).

Likewise, the total volume residuals were plotted against diameter classes. These graphs portray the domains of inadequate or acceptable predictions (Kozak and Smith, 1993).

Ranking of models

The models were also compared using the ranking method of Poudel and Cao (2013).

$$R_i = 1 + \frac{(m - 1)(S_i - S_{\min})}{S_{\max} - S_{\min}} \quad (5)$$

Where R_i indicates the relative rank of a model i ($i = 1, 2, 3 \dots m$), S_i is the goodness of fit statistics delivered by model i and S_{\max} and S_{\min} correspond to the maximum and minimum values of S_i . Rank 1 represents the best model, whereas m shows the poorest model. The ranking method was applied using RMSE, FI, MAB and MPB statistics for diameter, total and partial volumes to calculate the average rank of each model. Next, the mean of average ranks was taken to determine the overall ranks of the models for the four variables.

Nonlinear mixed-effect modelling of the selected taper model

- (1) After deciding the best model, it was refitted using nonlinear mixed-effects modelling method to account for within and between tree variations on stem profile. We used the NLMIXED procedure in SAS (SAS Institute Inc., 2008) to estimate the fixed and random parameters. Different combinations of random parameters were tested, which produced several mixed models. Mixed models were compared by Akaike's Information Criterion (AIC) (Akaike, 1974), Schwarz's Bayesian Information Criterion (BIC) (Schwarz, 1978) and twice the negative log-likelihood ($-2\ln(L)$).

$$\text{AIC} = -2 \ln(L) + 2k \quad (6)$$

$$\text{BIC} = -2 \ln(L) + k \ln(n) \quad (7)$$

Where n is the number of observations, k is the number of parameters and L is the maximum likelihood value.

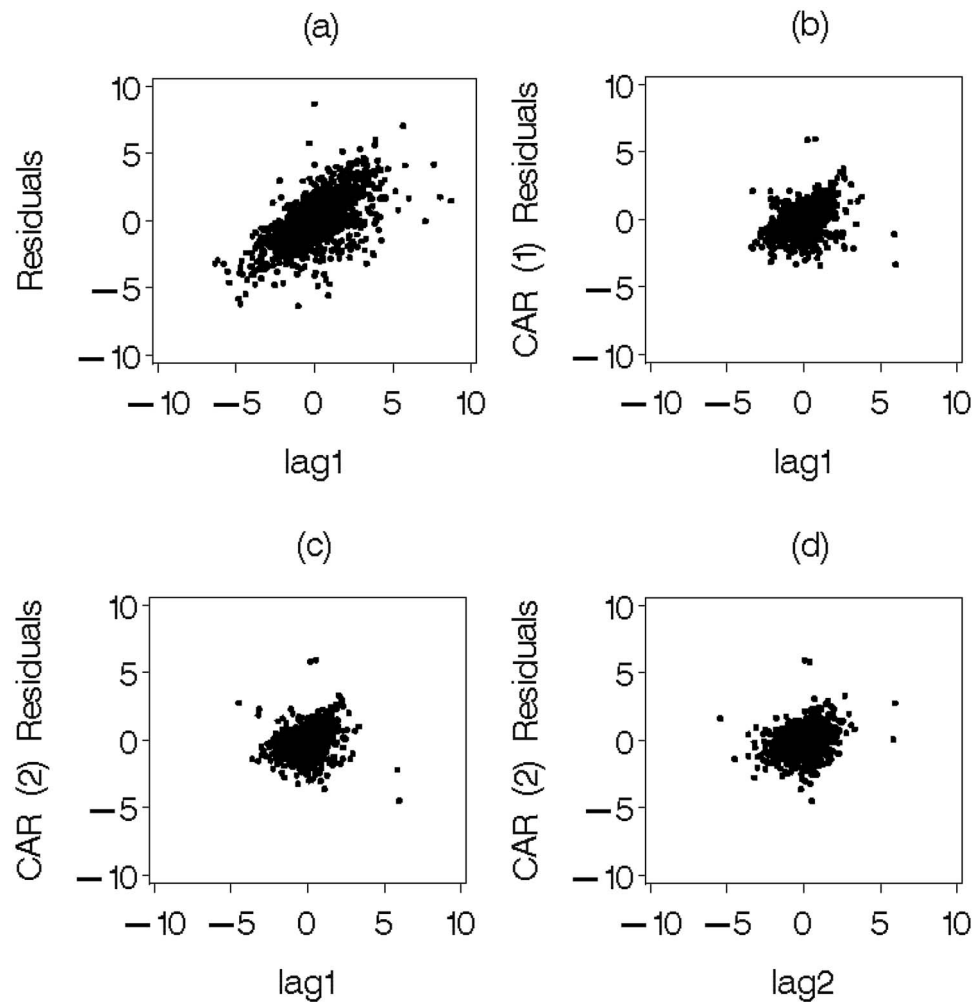


Figure 2 An example of lagged residuals of white birch for diameter plotted against: Lag1 and Lag2 residuals for Clark model fitted without considering the autocorrelation parameters (a) and using first (b) and second (c, d) order continuous autoregressive error structures.

Results

The initial fitting of the models without the error structure resulted in strong autocorrelation. As an example, the autocorrelation observed in white birch for d predictions with the Clark *et al.* (1991) Model 1 is shown in Figure 2. This correlation trend disappeared when a second-order CAR (2) was added in the model fit.

Tables 3 and 4 show the parameter estimates, and Table 5 highlights the fit-statistics of the models. Above 98 per cent of the total variance of d was explained by the models for white birch, and it was almost 95 per cent for costata birch (Table 5). The Clark *et al.* (1991) Model 1 showed the best results for both species (RMSE: 0.875–1.952 cm, MAB: 0.595–1.313 cm and MPB: 4.763–5.045 per cent). The model of Lee *et al.* (2003) indicated the largest variability and bias (MAB: 0.731 cm, MPB: 5.847 per cent for white birch; MAB: 1.574 cm, MPB: 6.048 per cent for costata birch). The models of Kozak (2004), Muhairwe (1999) and Bi (2000) provided good results in terms of error and bias for white birch, whereas the models of Max and Burkhardt (1976), Clark *et al.*

(1991) Model 2 and Kozak (2004) performed better for costata birch. Although MAB for costata birch was higher than white birch, the MPB was almost similar (5–6 per cent) for both species. The Clark *et al.* (1991) Model 1 produced better results than the Clark *et al.* (1991) Model 2. The latter increased the bias by almost 6 per cent for costata birch and 12 per cent for white birch. When diameter measurements at 5.3 m were not available, the models of Kozak (2004) and Max and Burkhardt (1976) best predicted the diameter for white birch and costata birch, respectively.

The Clark *et al.* (1991) Model 1 showed the lowest values of RMSE (0.007–0.049 m³), MAB (0.004–0.036 m³) and MPB (1.99–4.05 per cent) in predicting V_T , V_P and V_S of both species (Tables 6–8). The second best models were Kozak (2004) and Bi (2000) for all volume assortments of white birch. However, for costata birch, the next best models varied across the volume assortments. In estimating V_T and V_P , the models of Kozak (2004), Max and Burkhardt (1976), Sharma and Zhang (2004) and Lee *et al.* (2003) showed good results with marginal differences. Nevertheless, the model of Max and Burkhardt (1976) delivered the lowest RMSEs for these attributes. In predicting V_S , the models

Table 3 Parameter estimates with approximate standard errors for white birch.

	Max and Burkhardt (1976)		Clark et al. (1991) Model 1		Clark et al. (1991) Model 2		Fang et al. (2000)		Muhairwe (1999)	
	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error
a_0							5×10^{-5}	$5.3\text{E-}6$		
a_1	0.605	0.030					1.883	0.024		
a_2	0.050	0.002					0.984	0.049		
b_1	-3.124	0.081	37.129	2.126	37.623	2.322	$8.6\text{E-}6$	$2.0\text{E-}7$	1.063	0.033
b_2	1.565	0.044	0.659	0.011	0.662	0.012	3×10^{-5}	$4.4\text{E-}7$	1.031	0.011
b_3	-1.244	0.108	-21.585*	20.329	-23.861*	20.447	2×10^{-5}	$3.5\text{E-}7$	0.997	0.124
b_4	214.759	14.499	1.834	0.416	1.739	0.499			-2.225	0.306
b_5			0.312	0.027	0.322	0.031			-6×10^{-5}	$3.5\text{E-}6$
b_6			1.280	0.015	1.283	0.018			1.949	0.222
$b_7 - p_1$							0.068	0.001	6×10^{-4} *	0.001
$b_8 - p_2$							0.526	0.011	0.279	0.031
	Bi (2000)		Lee et al. (2003)		Kozak (2004)		Sharma and Zhang (2004)		Sharma and Parton (2009)	
	Estimate	Approx. std. error App. SE	Estimate	Approx. std. error App. SE	Estimate	Approx. std. error App. SE	Estimate	Approx. std. error App. SE	Estimate	Approx. std. error App. SE
b_0	0.536	0.095			0.888	0.054	1.049	0.008	1.024	0.004
b_1	0.544	0.055	1.524	0.042	0.957	0.011	2.026	6×10^{-4}	-0.011	3×10^{-4}
b_2	0.105	0.012	0.940	0.009	0.092	0.026	0.071*	0.043	0.185	0.021
b_3	-0.343	0.058	4.379	0.183	0.754	0.031	0.703	0.063	-0.140	0.031
b_4	7×10^{-4}	2×10^{-4}	-5.525	0.206	-0.727	0.083				
b_5	0.110	0.009	2.696	0.059	0.747	0.020				
b_6	-0.174	0.020			0.483*	0.428				
b_7					-0.002*	0.002				
b_8					-0.074	0.028				

*Non-significant parameters at $P < 0.05$.

Table 4 Parameter estimates with approximate standard errors for costata birch.

	Max and Burkhardt (1976)		Clark et al. (1991) Model 1		Clark et al. (1991) Model 2		Fang et al. (2000)		Muhairwe (1999)	
	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error
a_0							8×10^{-5}	1×10^{-5}		
a_1	0.708	0.081					1.926	0.031		
a_2	0.047	0.002					0.862	0.057		
b_1	-2.064	0.600	44.111	3.501	44.307	3.617	7.8E-6	2.5E-7	1.254	0.083
b_2	0.512*	0.340	0.544	0.025	0.544	0.025	4×10^{-5}	9.6E-7	0.958	0.019
b_3	-1.061	0.320	2232.48	613.0	2227.46	618.0	3×10^{-5}	6.1E-7	1.266	0.147
b_4	262.356	20.893	3.402	1.568	3.440*	1.777			-2.308	0.336
b_5			0.946	0.014	0.946	0.015			-9×10^{-5}	4.5E-6
b_6			12.682	3.114	12.702	3.295			1.516	0.227
$b_7 - p_1$							0.059	0.001	3×10^{-4}	0.001
$b_8 - p_2$							0.544	0.019	0.016	0.021
	Bi (2000)		Lee et al. (2003)		Kozak (2004)		Sharma and Zhang (2004)		Sharma and Parton (2009)	
	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error	Estimate	Approx. std. error
b_0	0.419	0.101			0.924	0.091	1.057	0.011	1.034	0.005
b_1	-0.178	0.058	1.503	0.082	0.946	0.016	2.025	8×10^{-4}	-0.010	4×10^{-4}
b_2	-0.007	0.012	0.947	0.015	0.101	0.035	-0.174	0.042	0.267	0.022
b_3	-0.156	0.062	2.899	0.164	0.397	0.028	0.154	0.056	0.169	0.029
b_4	-0.001	3×10^{-4}	-4.196	0.207	0.519	0.138				
b_5	0.016	0.008	2.081	0.066	0.206	0.023				
b_6	0.027	0.015			-2.802	1.252				
b_7					800.0-	0.002				
b_8					0.198	0.040				

* Non-significant parameters at $P < 0.05$.

Table 5 Fit statistics and average rank of taper models in estimating diameter (cm).

Models	White birch					Costata birch				
	RMSE	FI	MAB	MPB	Rank	RMSE	FI	MAB	MPB	Rank
Max and Burkhart (1976)	0.9546	0.9846	0.6815	5.4480	5.91	2.0110	0.9475	1.3940	5.3552	3.29
Clark et al. (1991) Model 1	0.8755	0.9870	0.5959	4.7638	1.00	1.9525	0.9505	1.3133	5.0454	1.00
Clark et al. (1991) Model 2	0.9428	0.9849	0.6692	5.3501	5.22	2.0106	0.9476	1.3954	5.3606	3.30
Fang et al. (2000)	0.9689	0.9841	0.6698	5.3548	5.93	2.0440	0.9459	1.4077	5.4078	4.04
Muhairwe (1999)	0.9418	0.9850	0.6545	5.2323	4.67	2.1255	0.9415	1.4910	5.7279	6.80
Bi (2000)	0.9490	0.9847	0.6517	5.2102	4.81	2.0523	0.9454	1.4272	5.4830	4.52
Lee et al. (2003)	1.0410	0.9816	0.7314	5.8472	10.0	2.2289	0.9355	1.5744	6.0482	10.0
Kozak (2004)	0.9092	0.9860	0.6351	5.0775	3.17	2.0166	0.9473	1.3974	5.3685	3.45
Sharma and Zhang (2004)	1.0201	0.9823	0.7230	5.7798	9.14	2.0753	0.9441	1.4537	5.5845	5.38
Sharma and Parton (2009)	0.9818	0.9836	0.6921	5.5332	7.05	2.0702	0.9444	1.4443	5.5486	5.13

Note: Clark et al. (1991) Model 1, diameters at 5.3 m were obtained by linear interpolation; Clark et al. (1991) Model 2, diameters at 5.3 m were obtained by prediction.

Table 6 Evaluation statistics and rank of taper models for total volume (V_T , m³) estimates.

Models	White Birch				Costata Birch			
	RMSE	MAB	MPB	Rank	RMSE	MAB	MPB	Rank
Max and Burkhart (1976)	0.0153	0.0104	4.4116	9.73	0.0575	0.0429	4.7703	8.19
Clark et al. (1991) Model 1	0.0093	0.0064	2.7064	1.00	0.0498	0.0365	4.0551	1.00
Clark et al. (1991) Model 2	0.0145	0.0104	4.4251	9.37	0.0587	0.0432	4.8001	8.75
Fang et al. (2000)	0.0132	0.0097	4.1257	7.72	0.0606	0.0433	4.8095	9.35
Muhairwe (1999)	0.0124	0.0091	3.8824	6.47	0.0598	0.0441	4.9049	9.78
Bi (2000)	0.0120	0.0090	3.8193	6.10	0.0600	0.0438	4.8705	9.59
Lee et al. (2003)	0.0128	0.0093	3.9478	6.93	0.0582	0.0429	4.7638	8.36
Kozak (2004)	0.0115	0.0085	3.6267	5.17	0.0584	0.0426	4.7307	8.18
Sharma and Zhang (2004)	0.0152	0.0104	4.4053	9.67	0.0581	0.0428	4.7535	8.26
Sharma and Parton (2009)	0.0155	0.0105	4.4691	10.0	0.0577	0.0438	4.8686	8.94

of Lee et al. (2003) and Sharma and Parton (2009) performed better by nominal values. In general, the models had a 2–5 per cent bias for the volume attributes. The models of Sharma and Zhang (2004) and Sharma and Parton (2009), and Fang et al. (2000) showed the highest RMSEs for white birch and costata birch, respectively.

According to the average rank of diameter and volume estimates, Clark et al. (1991) Model 1 showed the lowest rank for both species (Table 9). When diameter measurements at 5.3 m were not available, the best models were Kozak (2004) and Bi (2000) for white birch, whereas the Kozak (2004) and Max and Burkhart (1976) were the leading models for costata birch with a similar rank. Results also indicated that the Clark et al. (1991) Model 1 was significantly better than Clark et al. (1991) Model 2.

The box plots of d residuals reflected that the distribution of error was not similar across the relative height classes (Figures 3 and 5). The prediction errors were relatively larger for costata birch. The Clark et al. (1991) Model 1 showed the best predictions for the basal log and middle stem sections. The models of Kozak (2004), Max and Burkhart (1976), Clark et al. (1991) Model 2, Fang et al. (2000) and Bi (2000) delivered reasonable estimates for

the lower and middle stem. However, the models of Max and Burkhart (1976) and Clark et al. (1991) Model 2 were slightly biased for the middle sections (25–55 per cent), whereas Kozak (2004) and Bi (2000) models showed a minor distortion for the basal log (<10 per cent). The rest of the models had problems in their predictions as a whole.

The plots of total volume residuals indicated larger errors for bigger trees (Figures 4 and 6). The Clark et al. (1991) Model 1 predicted the total volume more accurately except for the largest diameter class, where the Kozak (2004) model performed better. The models of Max and Burkhart (1976), Clark et al. (1991) Model 2 and Fang et al. (2000) slightly underestimated smaller trees (10–20 cm) of white birch. This deviation was extended to all diameter classes of costata birch. The models of Muhairwe (1999), Bi (2000), Lee et al. (2003), Sharma and Zhang (2004) and Sharma and Parton (2009) overestimated total volume for all diameter classes. The taper simulation for a small and large tree of white birch and costata birch indicated more taper variation in the former. The species differed greatly in the middle and upper sections of the stem. The predictions of stem profiles using Clark et al. (1991) Model 1 are shown in Figure 7.

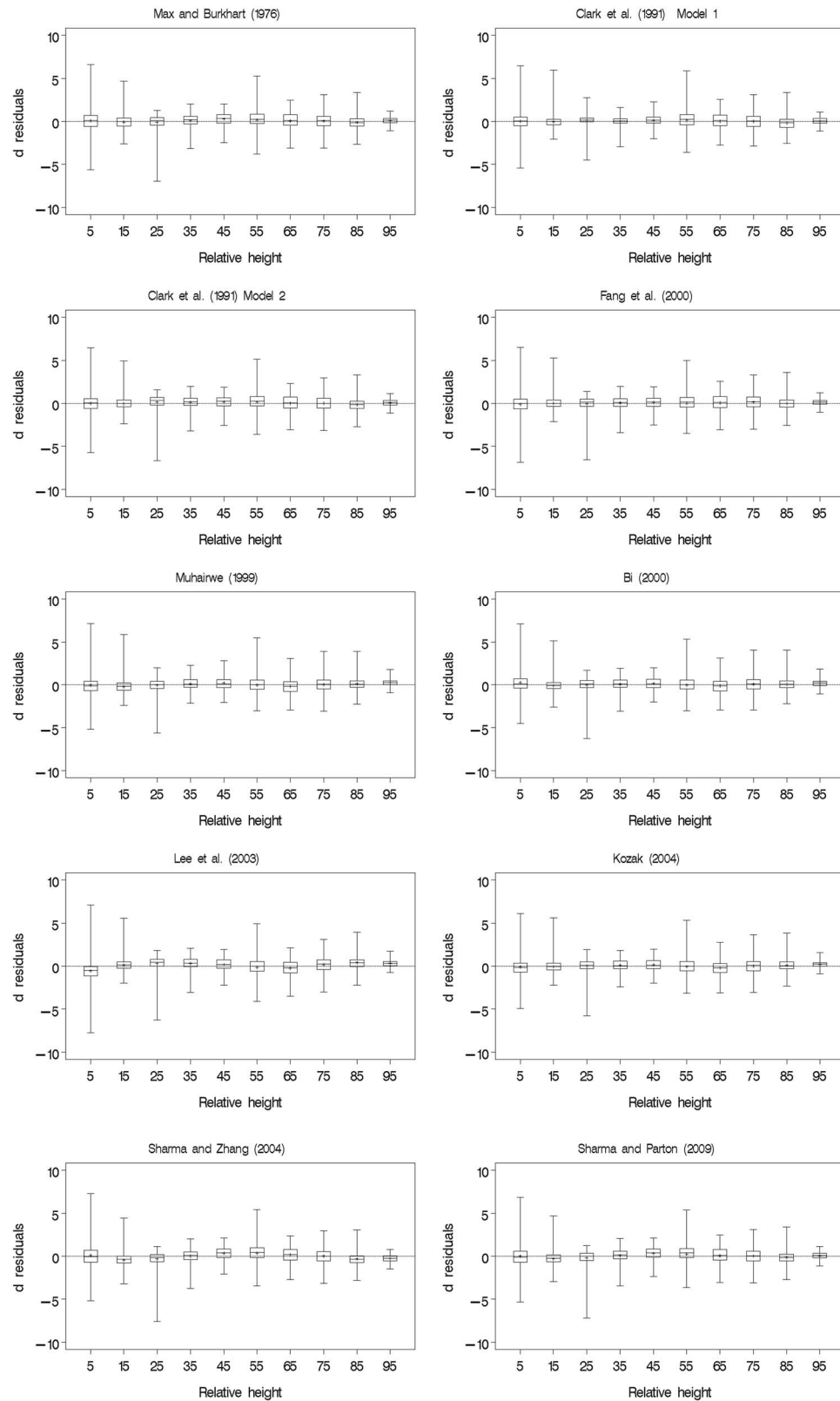


Figure 3 Box plots of d residuals (cm) against relative height classes (in percent) for white birch and costata birch.

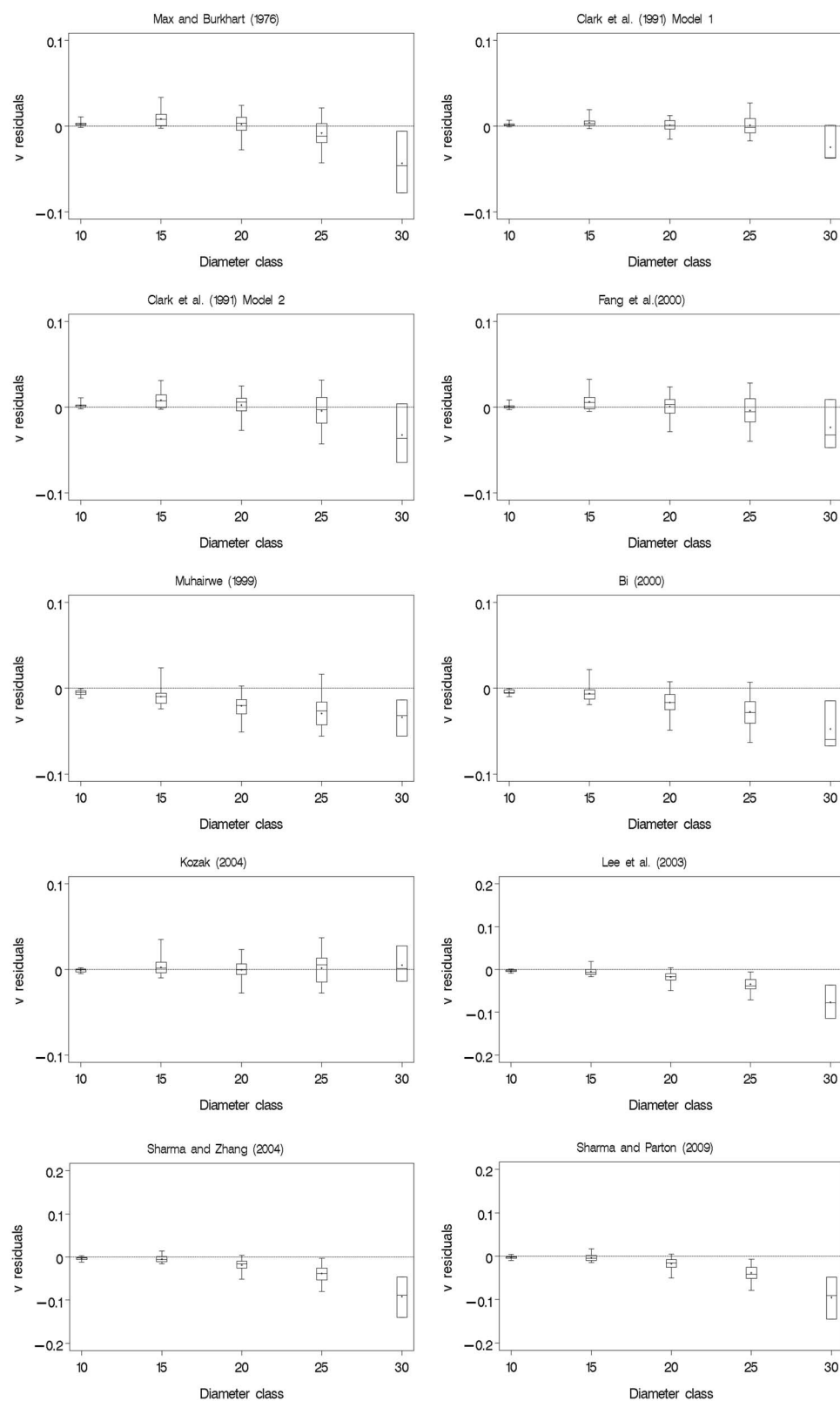


Figure 4 Box plots of total volume residuals (m^3) against diameter classes (cm) for white birch and costata birch. Note: The boxes represent interquartile ranges, with their edges being 25th and 75th percentiles. The upper and lower horizontal lines represent the maximum and minimum prediction errors. The plus signs represent mean prediction errors for the corresponding relative height and diameter classes.

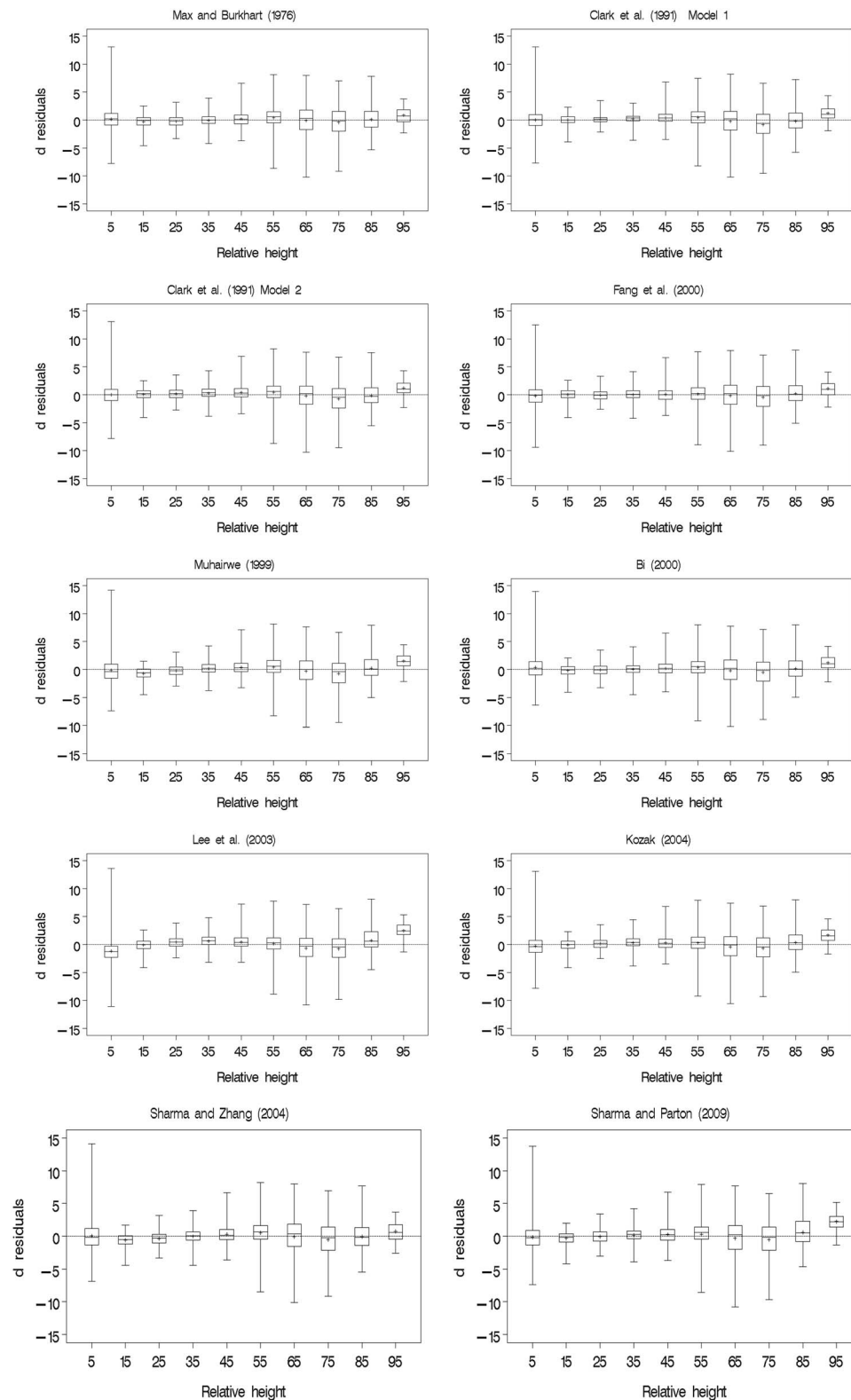


Figure 5 Box plots of d residuals (cm) against relative height classes (in percent) for white birch and costata birch.

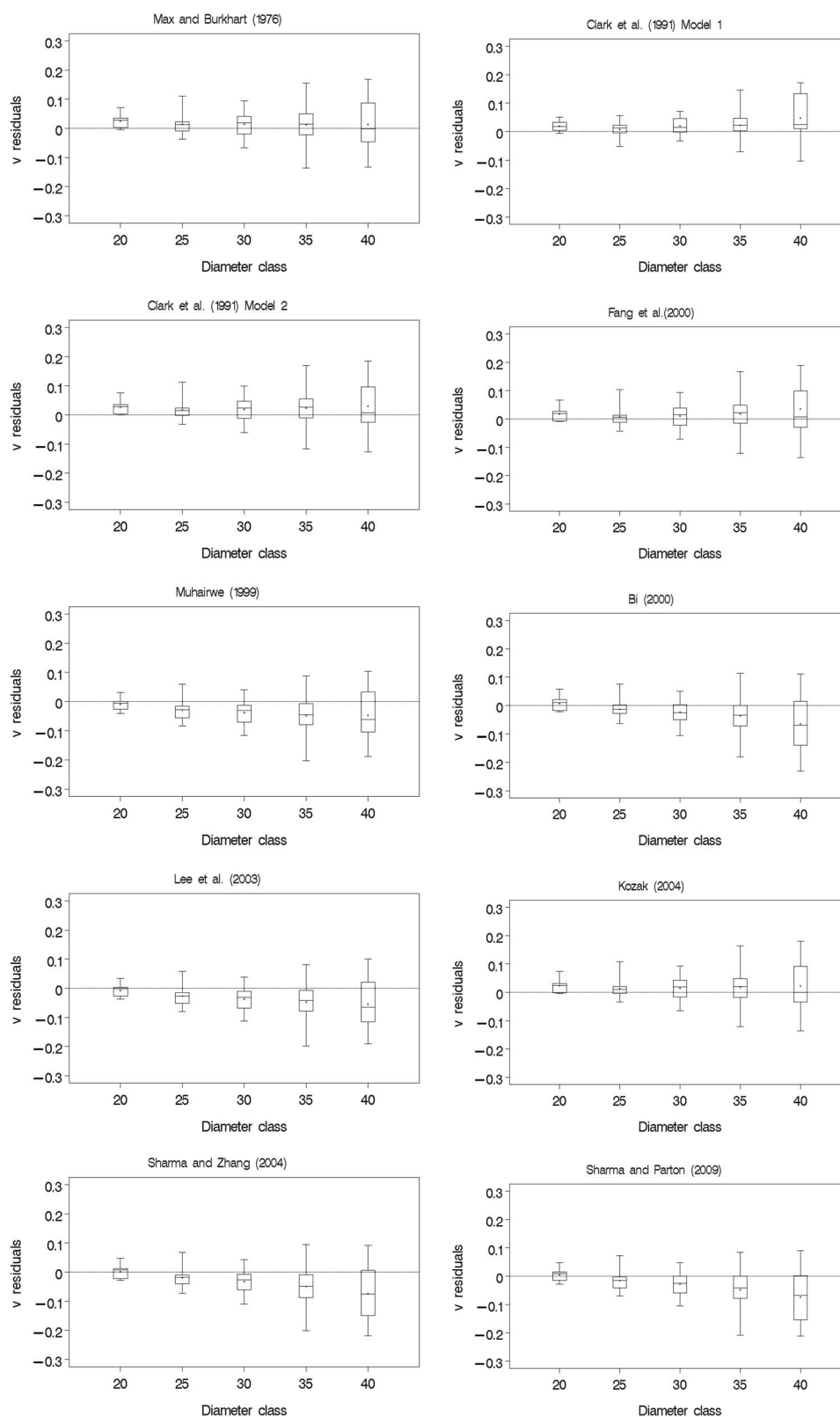


Figure 6 Box plots of total volume residuals (m^3) against diameter classes (cm) for white birch and costata birch. Note: The boxes represent interquartile ranges, with their edges being 25th and 75th percentiles. The upper and lower horizontal lines represent the maximum and minimum prediction errors. The plus signs represent mean prediction errors for the corresponding relative height and diameter classes.

Table 7 Evaluation statistics and rank of taper models for pulpwood volume (V_p , m³) estimates.

Models	White birch				Costata birch			
	RMSE	MAB	MPB	Rank	RMSE	MAB	MPB	Rank
Max and Burkhart (1976)	0.0151	0.0099	3.9639	9.61	0.0574	0.0428	4.7523	8.11
Clark <i>et al.</i> (1991) Model 1	0.0086	0.0055	2.2036	1.00	0.0498	0.0363	4.0360	1.00
Clark <i>et al.</i> (1991) Model 2	0.0142	0.0101	4.0439	9.47	0.0587	0.0430	4.7792	8.65
Fang <i>et al.</i> (2000)	0.0127	0.0093	3.7588	7.82	0.0605	0.0431	4.7930	9.24
Muhairwe (1999)	0.0120	0.0091	3.6653	7.23	0.0597	0.0441	4.9015	9.78
Bi (2000)	0.0117	0.0089	3.6143	6.88	0.0599	0.0436	4.8445	9.44
Lee <i>et al.</i> (2003)	0.0128	0.0092	3.7064	7.71	0.0577	0.0424	4.7254	7.95
Kozak (2004)	0.0111	0.0084	3.4181	5.97	0.0583	0.0422	4.6974	7.94
Sharma and Zhang (2004)	0.0150	0.0098	3.9805	9.52	0.0581	0.0427	4.7398	8.23
Sharma and Parton (2009)	0.0154	0.0100	4.0424	9.93	0.0573	0.0433	4.8176	8.50

Note: Pulpwood volume, volume up to 10 cm top diameter.

Table 8 Evaluation statistics and rank of taper models for saw log volume (V_s , m³) estimates.

Models	White birch				Costata birch			
	RMSE	MAB	MPB	Rank	RMSE	MAB	MPB	Rank
Max and Burkhart (1976)	0.0145	0.0077	3.4469	7.35	0.0511	0.0362	4.1330	7.11
Clark <i>et al.</i> (1991) Model 1	0.0074	0.0045	1.9942	1.00	0.0455	0.0298	3.3985	1.00
Clark <i>et al.</i> (1991) Model 2	0.0114	0.0063	2.8707	4.65	0.0548	0.0385	4.3813	9.76
Fang <i>et al.</i> (2000)	0.0097	0.0062	2.8397	3.94	0.0556	0.0375	4.2878	9.37
Muhairwe (1999)	0.0115	0.0076	3.5160	6.30	0.0519	0.0358	4.0831	7.06
Bi (2000)	0.0079	0.0053	2.4158	2.21	0.0549	0.0381	4.3541	9.57
Lee <i>et al.</i> (2003)	0.0096	0.0067	3.2524	4.75	0.0502	0.0350	3.9754	5.95
Kozak (2004)	0.0083	0.0053	2.4636	2.41	0.0524	0.0360	4.1007	7.33
Sharma and Zhang (2004)	0.0158	0.0096	4.2545	10.0	0.0516	0.0361	4.1281	7.21
Sharma and Parton (2009)	0.0152	0.0086	3.8506	8.66	0.0495	0.0354	4.0283	6.04

Note: Saw log volume, volume up to 20 cm top diameter.

Table 9 Ranks of the models by attribute from Tables 5–8 and average rank.

Models	White birch					Costata birch				
	d	V_T	V_p	V_s	Av. Rank	d	V_T	V_p	V_s	Av. Rank
Max and Burkhart (1976)	5.91	9.73	9.61	7.35	8.15	3.29	8.19	8.11	7.11	6.68
Clark <i>et al.</i> (1991) Model 1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Clark <i>et al.</i> (1991) Model 2	5.22	9.37	9.47	4.65	7.18	3.30	8.75	8.65	9.76	7.61
Fang <i>et al.</i> (2000)	5.93	7.72	7.82	3.94	6.35	4.04	9.35	9.24	9.37	7.99
Muhairwe (1999)	4.67	6.47	7.23	6.30	6.17	6.80	9.78	9.78	7.06	8.35
Bi (2000)	4.81	6.10	6.88	2.21	5.00	4.52	9.59	9.44	9.57	8.28
Lee <i>et al.</i> (2003)	10.0	6.93	7.71	4.75	7.35	10.0	8.36	7.95	5.95	8.06
Kozak (2004)	3.17	5.17	5.97	2.41	4.18	3.45	8.18	7.94	7.33	6.72
Sharma and Zhang (2004)	9.14	9.67	9.52	10	9.58	5.38	8.26	8.23	7.21	7.27
Sharma and Parton (2009)	7.05	10.0	9.93	8.66	8.91	5.13	8.94	8.50	6.04	7.16

Through the above model comparisons, the Clark *et al.* (1991) Model 1 was the best model owing to its accuracy in all facets of diameter and volume estimates. This model was changed to a

nonlinear mixed-effects model where random components were added to some of the fixed-effects parameters to obtain tree-specific parameter estimates for trees in the study. We fitted Clark

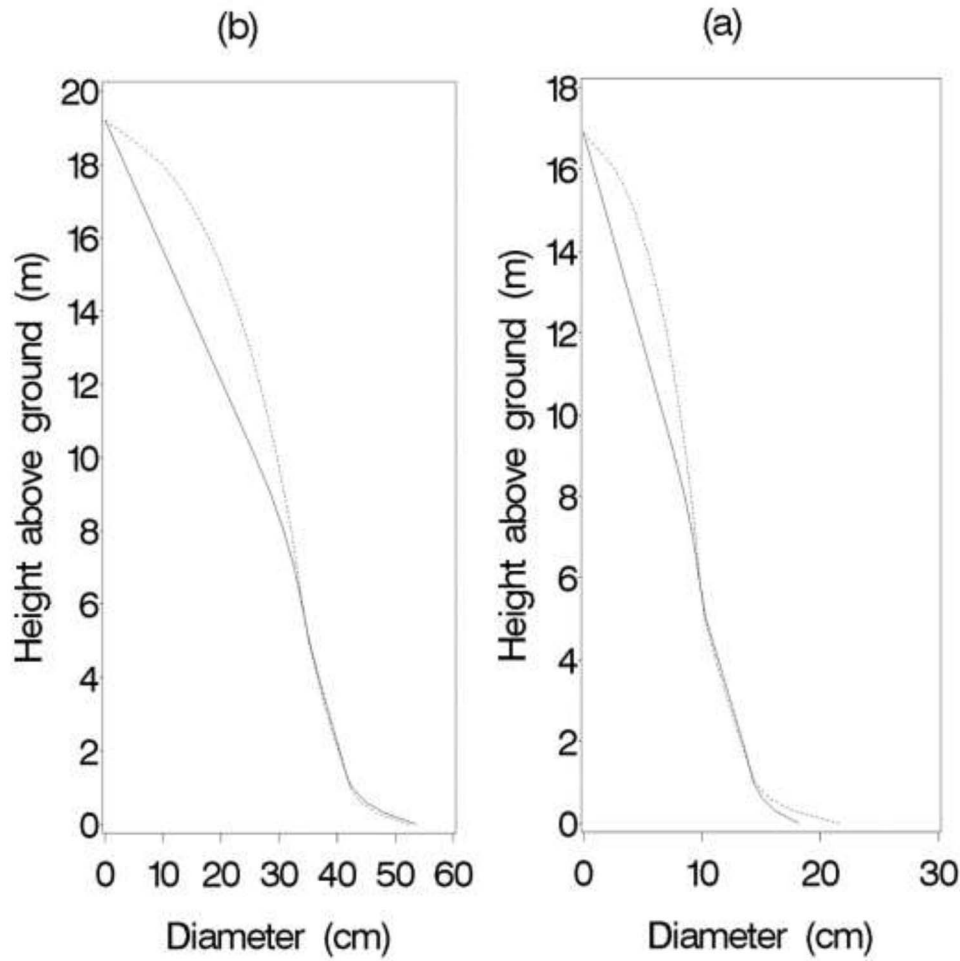


Figure 7 Stem profiles generated of white birch (solid lines) and costata birch (dot lines) using Clark *et al.* (1991) Model 1 for a small tree (a, Total height = 16.9 m and Diameter at breast height = 14.1 cm) and a large tree (b, Total height = 19.2 m and Diameter at breast height = 41.6 cm).

et al. (1991) Model 1 with possible combinations of fixed- and random-effects parameters. Fitting of the model with more than two random-effects parameters failed to achieve convergence. Therefore, we tested this model with random-effects added to one or two fixed-effects parameters only, which produced a total of 21 mixed models.

Table 10 shows the evaluation statistics of the diameter along the stem for the model with the best one and two random-effects parameters as well as fixed-effects models. The mixed models with two random parameters, u_2 and u_6 , showed the lowest values of AIC, BIC and $-2\ln(L)$. The fitting statistics of mixed models were significantly better than the fixed-effects models. Although introducing the random-effects can reduce the cor-

related errors, the existence of autocorrelation cannot be ruled out (Garber and Maguire, 2003; Trincado and Burkhardt, 2006). Therefore, CAR (1) was introduced into the mixed models, which further improved the fit statistics. Table 11 shows the evaluation statistics for the model with fixed- and mixed-effects models and fixed parameters of mixed-effects models for volume (m^3) estimates. The comparison of fixed-effects model, fixed parameters of mixed-effects model and mixed-effects model was carried out. The mixed models showed the lowest values of RMSE, MAB and MPB for both of the species. Table 12 lists the estimated fixed parameters, correlation parameter and variance components of the mixed models for white birch and costata birch.

The adaptation of Clark *et al.* (1991) Model 1 was:

$$d = \left\{ \begin{array}{l} I_5 \left[D^2 \left(1 + ((b_2 + u_2) + b_3/D^3) \left((1 - h/H)^{b_1} - (1 - 1.3/H)^{b_1} \right) / \left(1 - (1 - 1.3/H)^{b_1} \right) \right) \right] \\ + I_6 \left[D^2 - (D^2 - F^2) \left((1 - 1.3/H)^{b_4} - (1 - h/H)^{b_4} \right) / (1 - 1.3/H)^{b_4} - (1 - 5.3/H)^{b_4} \right] \\ + I_7 \left[F^2 \left((b_6 + u_6) ((h - 5.3/H - 5.3) - 1)^2 + I_M \left(1 - b_6 + u_6/b_5^2 \right) (b_5 - (h - 5.3/H - 5.3))^2 \right) \right] \end{array} \right\}^{0.5} \quad (8)$$

Table 10 Fit statistics of diameter along the stem for [Clark et al. \(1991\)](#) Model 1 with different combinations of random parameters.

Species	Random parameters	AIC (smaller is better)	BIC (smaller is better)	-2Ln (L) (smaller is better)
White birch	None*	7010	7051	6996
	u_6	6602	6625	6586
	u_2, u_6	6083	6112	6064
	$u_2, u_6 + \text{CAR (1)}$	6021	6051	5999
Costata birch	None*	9910	9949	9896
	u_6	9113	9134	9097
	u_2, u_6	8738	8765	8718
	$u_2, u_6 + \text{CAR (1)}$	8165	8195	8143

* Fixed-effects model.

Table 11 Comparison of [Clark et al. \(1991\)](#) Model 1 with fixed- and mixed-effects models for volume (m^3) estimates.

Species	Models	Volume	RMSE	MAB	MPB
White birch	FEM	V_T	0.0147	0.0099	4.2156
		V_P	0.0137	0.0082	3.2960
		V_S	0.0127	0.0083	3.7507
	FPMEM	V_T	0.0150	0.0099	4.2199
		V_P	0.0142	0.0086	3.4380
		V_S	0.0124	0.0083	3.7846
	MEM	V_T	0.0064	0.0042	1.7726
		V_P	0.0079	0.0055	2.2055
		V_S	0.0089	0.0053	2.4597
Costata birch	FEM	V_T	0.0854	0.0623	6.9255
		V_P	0.0850	0.0619	6.8819
		V_S	0.0790	0.0513	5.8231
	FPMEM	V_T	0.0878	0.0645	7.1635
		V_P	0.0865	0.0632	7.0460
		V_S	0.0790	0.0512	5.8467
	MEM	V_T	0.0219	0.0170	1.8876
		V_P	0.0217	0.0168	1.8694
		V_S	0.0293	0.0231	2.6152

FEM, fixed-effects model; FPMEM, fixed parameters of mixed-effects model; MEM, mixed-effects model.

where u_2 and u_6 are the random parameters and all other variables as defined earlier.

Discussion

Several taper studies have presented different taper models for birch species in the world. For example, [Max and Burkhart \(1976\)](#) for *B. alleghaniensis* and *B. lenta* in Virginia ([Martin, 1981](#)); [Kozak \(1988\)](#) for Saskatchewan *B. papyrifera* ([Gál and Bella, 1994](#)); [Kozak \(2004\)](#) for *B. papyrifera*, *B. alleghaniensis*, *B. lenta* and *B. populifolia* in Acadian region, USA ([Weiskittel et al., 2011](#)) and [Ung et al. \(2013\)](#) for *B. papyrifera* and *B. alleghaniensis* across Canada. Recently, [Shahzad et al. \(2019, 2020\)](#) suggested the models of [Fang et al. \(2000\)](#) and [Max and Burkhart \(1976\)](#) for *B. platyphylla* in Daxingan mountains, northeast China. However, taper models are not available for white birch and costata birch in Xiaoxingan mountains, northeast china. The Daxingan

and Xiaoxingan mountains ranges have significant variations in species composition due to different relief patterns, latitudinal domains, climate and soil formation ([Burger and Shidong, 1988](#); [Xiao et al., 2002](#)). The accuracy of a taper model depends on the particular tree species, stem-form and stand characteristics ([Muhairwe et al., 1994](#)). It is not practicable to use a single taper equation uniformly for a vast range of conditions ([Newnham, 1988](#)).

This study evaluated 10 well-known taper models for estimating diameter, total volume and merchantable volume (pulpwood and sawlog) of white birch and costata birch. Although several studies have evaluated these models, previous research has mainly focussed on conifer species. For broadleaved tree species, the model of [Clark et al. \(1991\)](#) has only been tested for yellow poplar (*Liriodendron tulipifera* L.) in West Virginia, USA ([Jiang et al., 2005](#)).

In our initial evaluation using fixed-effects models, the [Clark et al. \(1991\)](#) Model 1 provided the most accurate results across

Table 12 Parameter estimates for [Clark et al. \(1991\)](#) Model 1 with mixed effects.

Parameters	White birch		Costata birch	
	Estimate	Approx. std. error	Estimate	Approx. std. error
b_1	33.719	1.1512	39.633	1.0789
b_2	0.6389	0.0235	0.6329	0.0295
b_3	27.464	10.360	67.019	76.320
b_4	1.8141	0.2350	3.5881	0.6134
b_5	0.4106	0.0147	0.8813	0.0110
b_6	1.3506	0.0269	6.1109	0.5278
Correlation parameter				
ρ	1.0412	0.0051	1.1270	0.0053
Variance components				
σ^2	0.3632	0.0227	0.5349	0.0323
var (u_2)	0.0642	0.0094	0.0899	0.0128
var (u_6)	-0.0066	0.0072	0.1251	0.0625
cov (u_2, u_6)	0.0680	0.0116	3.9555	0.8706
Goodness-of-fit				
AIC	6021.0		8165.4	
BIC	6051.6		8194.9	
-2Ln (L)	5999.0		8143.4	

σ^2 , residual variance; ρ , correlation parameter for the CAR (1) error structure; var (u_2) and var (u_6), variances for the random effects corresponding to fixed parameters b_2 and b_6 ; cov (u_2, u_6), covariances between pairs of random effects.

the datasets, which was refitted to develop mixed-effects models. In estimating diameter, it had RMSEs that were 3.71 and 2.91 per cent lower than the next best models, [Kozak \(2004\)](#) for white birch and [Max and Burkhardt \(1976\)](#) for costata birch, respectively. In estimating V_T , V_P and V_S , the [Clark et al. \(1991\)](#) Model 1 reduced the RMSE by 6.32–22.52 per cent for white birch and 8.08–13.24 per cent for costata birch, when compared with the next models.

For the residuals' plots, the [Clark et al. \(1991\)](#) Model 1 displayed the highest accuracy in terms of mean prediction error, medians' distribution and narrowness of interquartile ranges ([Figures 3–6](#)). The models of [Kozak \(2004\)](#), [Fang et al. \(2000\)](#) and [Max and Burkhardt \(1976\)](#) have displayed a similar distribution of d residuals for pedunculate oak (*Quercus robur* L.) and Scots pine (*P. sylvestris*) in Spain and Himalayan birch (*B. alnoides*) in South China, respectively ([Barrio Anta et al., 2007](#); [Crecente-Campo et al., 2009](#); [Tang et al., 2017](#)). Contrary to the corresponding plots in [Schröder et al. \(2014\)](#), the models of [Kozak \(2004\)](#), [Max and Burkhardt \(1976\)](#) and [Bi \(2000\)](#) were more precise in this analysis. The prediction accuracy of the models gradually decreased with increasing the diameter classes. [Diéguez-Aranda et al. \(2006\)](#) and [Shahzad et al. \(2019\)](#) observed similar errors for bigger trees. [Schröder et al. \(2014\)](#) attributed this deflection to the difference in site and competition conditions that affect individual trees.

The model of [Clark et al. \(1991\)](#) is comprised of Schlaegel's form-class model and Max and Burkhardt's segmented model.

Schlaegel's model contains Girard's form class height (5.3 m above ground), which enables a single species model to predict taper formation accurately in different geographic or physiographic regions. [Clark et al. \(1991\)](#) tested their model for 58 tree species, including birch in seven different regions of southern US. The volume estimates of their model were very similar to the results of region-specific models. On the other hand, [Westfall and Scott \(2010\)](#) developed a mixed model for *B. alleghaniensis*, *B. lenta*, *B. nigra*, *B. papyrifera* and *B. populifolia* in 13 states of northeastern US. Although Westfall and Scott's model compared well with the [Clark et al. \(1991\)](#) model, the locally calibrated models were more precise in volume estimates.

[Li and Weiskittel \(2010\)](#) proposed the [Clark et al. \(1991\)](#) Model 1 for volume estimates of balsam fir (*Abies balsamea* (L.) Mill.), red spruce (*Picea rubens* Sarg.) and white pine (*Pinus strobus* L.) in North America. However, in that study, [Kozak \(2004\)](#) and [Bi \(2000\)](#) superseded [Clark et al. \(1991\)](#) Model 1 in diameter estimates. Moreover, the models of [Fang et al. \(2000\)](#) and [Kozak \(2004\)](#) were significantly superior to [Max and Burkhardt \(1976\)](#) in predicting diameter and total volume. In our analysis, the performance of the [Max and Burkhardt \(1976\)](#) model was better than [Fang et al. \(2000\)](#) and similar to [Kozak \(2004\)](#) for diameter and volume estimates of costata birch. We found that [Clark et al. \(1991\)](#) Model 1 produced lower RMSEs than [Clark et al. \(1991\)](#) Model 2 (by 2.88–7.13 per cent for diameter and 6.25–16.66 per cent for total volume). [Li and Weiskittel \(2010\)](#) recommended

other well-behaved models when diameter measurements at 5.3 m height are not available. In this analysis, the models of Kozak (2004) and Max and Burkhardt (1976) were superior to other models for white birch and costata birch, respectively. Previously, the model of Max and Burkhardt (1976) has shown excellent results for several Appalachian hardwoods and *B. alnoides* in South China (Martin, 1981; Tang et al., 2017). Similarly, the Kozak (2004) model was suggested for hardwood species in the northeastern US, and it performed very well for *Castanea sativa* Mill. in northwest Spain (Weiskittel et al., 2011; Menéndez-Miguel et al., 2014).

Similar to our results, Figueiredo-Filho et al. (1996) found that Clark et al. (1991) Model 1 best predicted the diameter and total or merchantable volume of loblolly pine (*Pinus taeda* L.) in Brazil. They also observed that the Max and Burkhardt (1976) model demonstrated quite reasonable predictions for all variables. Unlike Özcelik and Brooks (2012), the model of Max and Burkhardt (1976) performed better than Clark et al. (1991) Model 2 for costata birch in our analysis. Sakici et al. (2008) assigned a similar rank to Clark et al. (1991) Model 1 and Kozak (2004) for diameter estimates of *A. nordmanniana* in Turkey, which was not the same in this analysis. In agreement with Doyog et al. (2017), the model of Kozak (2004) was superior to the Clark et al. (1991) Model 2 in this study.

Costata birch trees indicated a different trend from white birch for diameter and volume predictions in this study (e.g. higher variance and greater bias in the models). This might be attributed to the larger size of trees available in the costata birch dataset than white birch (Figure 1, Table 1). This variation was also reflected in the taper simulations of the species (Figure 7). The species had an identical taper formation for the basal log for given tree size, but white birch showed more taper than costata birch in the middle and upper stem sections. However, the Clark et al. (1991) Model 1 best predicted the diameter, total volume and merchantable volume of both species.

We selected the Clark et al. (1991) Model 1 for developing the mixed models due to its consistent superiority across the datasets. However, the future user who likes to use the models of Kozak (2004) or Max and Burkhardt (1976), the parameters for fixed-effects are given in Tables 3 and 4. Fixed-effect modelling, even with biased parameters, can provide good predictions (de-Miguel et al., 2012, 2013).

Tree level mixed-effects models

It is well known that the taper model fitted by non-linear mixed-effects modelling approach can improve the goodness-of-fit statistics compared with ordinary least squares. However, for the prediction purpose, many authors suggest using the fixed-effects models in the absence of calibration data (de-Miguel et al., 2013; Arias-Rodil et al., 2015). The fixed-effects models are more accurate in the predictions when random parameters of the mixed models are supposed to be zero, and additional measurements are not available for model calibration (Pukkala et al., 2009; Shater et al., 2011; Guzmán et al., 2012).

de-Miguel et al. (2013) also advised reporting both fixed- and mixed-effect forms of a model since calibration may be a feasible option in some cases. Accordingly, the mixed-effects modelling of the Clark et al. (1991) Model 1 was carried out. Mixed-effects

models would be used where additional upper stem diameter measurements are available for calibration.

Table 10 shows that the addition of one random parameter improved the fitting. However, the models expanded with both u_2 and u_6 provided the best statistics. The addition of random parameters u_2 and u_6 affects the lower section and middle/upper section, respectively. The original equation is quadratic for the lower portion and linear for the upper portion (Clark et al., 1991). The adapted Clark et al. (1991) Model 1 with two random parameters allows for the variation in both segments between trees.

The expansion of the models with random effects u_2 and u_6 is tenable, considering the effects of parameters on different stem sections. The parameters b_1 – b_3 correspond to the basal log (<1.3 m), b_4 to the lower stem (1.3–5.3 m) and b_5 – b_6 to the middle and upper stem (>5.3 m) (Table 2). The random-effects affected all parameters b_1 – b_4 , but the parameter b_2 showed the best statistics amongst different combinations. The variation is usually minimum in the lower stem (see, for example, Clark et al., 1991, Figure 3/Table 4). The parameter b_2 explains the basal log and compensates for the slightly tapered lower stem. The parameter b_6 captures the variation in the middle stem and joins b_5 to control the upper stem. With a higher value, the parameter b_6 has a major impact on the model, and the parameter b_5 contributes a little. Therefore, the parameter b_6 can explain the tree-level variation in both sections. Since the Clark et al. (1991) Model uses the Max and Burkhardt (1976) type for these sections, a parallel can be drawn with the findings of Leites and Robinson (2004). While fitting the Max and Burkhardt (1976) model with mixed-effects, they used the same terms to describe the middle and upper stem sections of loblolly pine (*P. taeda*) in Uruguay.

Although improved accuracy in the mixed models is appealing, the improvement in model accuracy for individual trees depends on the calibration data that should be as accurate as modelling data. Substantial biases or inaccuracies could lead to weaker estimates than those received from the fixed-effects model (Westfall and Scott, 2010).

Conclusions

This study presents an initial attempt to develop stem taper models using fixed- and mixed-effects modelling for white birch and costata birch in northeast China. The Clark et al. (1991) Model 1 was superior to other taper models in predicting the diameter at any height and total or partial volumes across the datasets. As an additional benefit, this model is compatible, which can be integrated to estimate merchantable and total volume. The Clark et al. (1991) Model 1 was also presented as a mixed model. The addition of random effects improved the precision of the model. However, mixed-effects models would be used where additional upper stem diameter measurements are available for calibration. When diameter measurements at 5.3 m are not available, the models of Kozak (2004) and Max and Burkhardt (1976) demonstrated quite reasonable results for white birch and costata birch, respectively.

White birch and costata birch are widely distributed in northeast China, which has significant geographic differences. Future analysis with a sample of larger representation could account for

these differences and extend the scope of this study. We believe these results would contribute to modern forestry in China aimed at the conservation and commercial development of the forests and useful for other countries maintaining these species.

Conflict of interest statement

None declared.

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Data availability Statement

The data presented in this study are available on request from the corresponding author.

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