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Oblique Corrections to the W Width

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ABSTRACT

The lowest-order expression for the partial W width to $e\nu$, $\Gamma(W \to e\nu) = G_\mu M_W^3/(6\pi\sqrt{2})$, has no oblique radiative corrections from new physics if the measured W mass is used. Here $G_\mu = (1.16639 \pm 0.00002) \times 10^{-5}$ GeV/c^2 is the muon decay constant. For the present value of $M_W = (80.14 \pm 0.27)$ GeV/c^2, and with $m_t = 140$ GeV/c^2, one expects $\Gamma(W \to e\nu) = (224.4 \pm 2.3)$ MeV. The total width $\Gamma_{\rm tot}(W)$ is also expected to lack oblique corrections from new physics, so that $\Gamma_{\rm tot}(W)/\Gamma(W \to e\nu) = 3 + 6[1 + \{\alpha_s(M_W)/\pi\}]$. Present data are consistent with this prediction.

I. INTRODUCTION

Precise measurements of electroweak phenomena have reached a level of accuracy which permits the search for new phenomena, manifesting themselves through radiative corrections. A particularly interesting class of such effects occur through loops of new particles in W and Z propagators, and are known as "oblique" corrections [1].

The effects of oblique corrections have been studied in the past few years by several groups [2-6]. By expanding vacuum polarization tensors for $\gamma-\gamma$, $\gamma-Z$, $Z-Z$, and $W-W$ self-energies to order $q^2/M_{\rm new}^2$ where $M_{\rm new}$ is the mass scale associated with new physics, one can express electroweak observables as nominal values (for a specific mass of the top quark and Higgs particle) corrected by linear functions of a few phenomenological variables. These variables encapsulate the effects of new physics on the observables in a concise way. Thus, for example, in the notation of Ref. [5], one has variables S_W, S_Z, and T, where S_W and S_Z describe the effects linear in q^2 of W and Z wave function renormalization due to new particles, while T is sensitive to violations of custodial SU(2) [7] such as occur in the case of a very heavy top quark.

In the present paper we shall show that when the W partial width to $e\nu$ and total width are expressed in terms of the measured muon decay constant $G_\mu = (1.16639 \pm 0.00002) \times 10^{-5}$ GeV^{-2} and W mass $M_W = 80.14 \pm 0.27$ GeV (the average of values from Refs. [8] and [9]), the lowest-order expressions do not receive corrections proportional to S_W, S_Z, or T. The relative smallness of standard model corrections to the W partial and total widths when expressed in this manner has been noted in Refs. [10] and [11]. A recent treatment of the W width in the context of such parameters has appeared in Ref. [12], but the result mentioned here does not appear explicitly.

The predicted partial and total widths are:

$$\Gamma(W \to e\nu) = \frac{G_\mu M_W^3}{6\pi\sqrt{2}}[1+\delta^{\rm sm}] = (224.4 \pm 2.3)\ {\rm MeV} \quad , \qquad (1)$$

$$\Gamma_{\rm tot}(W) = \{3 + 6[1+\alpha_s(M_W)/\pi]\}\Gamma(W \to e\nu) = (2.07 \pm 0.02)\ {\rm GeV} \quad , \qquad (2)$$

where most of the errors come from that on M_W, and $\delta^{\rm sm}$ is a small correction in the standard model, whose value [11] is about -0.35% when evaluated for the nominal values $m_t = 140$ GeV/c^2 and $M_H = 100$ GeV/c^2.

Most standard model corrections have already been absorbed into G_μ and/or the physical value of M_W, which explains why $\delta^{\rm sm}$ is only a few parts in 10^3. Consequently, a precise measurement of $\Gamma_{\rm tot}(W)$ (to a level of 1%) would begin to check M_W itself at levels comparable to present direct measurements. Deviations from the predictions (1) and (2) would indicate physics outside the purview of the parameters S_W, S_Z, and T. We shall mention such possibilities at the end of this article.

Our discussion is organized as follows. In Section II we introduce S_W, S_Z, and T, and show that the expressions (1) and (2) do not receive corrections linear in these parameters. In Section III we discuss the full set of standard model corrections. In Section IV we present details leading to the numerical values in (1) and (2), and compare these results with recent experiments. In Section V we note the role of corrections of higher order in $q^2/M_{\rm new}^2$ which have recently been mentioned in [12] (as well as the earlier discussion of Ref. [13]). We cite possible sources of deviation from the predictions (1) and (2). A suggestion for measuring the absolute W width using continuum production of lepton pairs is noted in Section VI, while Section VII summarizes. Explicit formulae involving top quark and Higgs boson contributions to corrections to the W width are noted in an Appendix.

II. ABSENCE OF NEW-PHYSICS OBLIQUE CORRECTIONS

In this section, we will first introduce the oblique correction parameters S_W, S_Z, and T and then show that the prediction for the W width is independent of these parameters when the muon decay constant, G_μ, and the W mass, M_W, are used as inputs.

When considering oblique corrections in the $SU(2)_L \times U(1)_Y$ gauge theory of electroweak interactions, there are four types of vacuum polarizations that must be taken into account. They are the self-energies of the photon, the Z, and the W, and the Z–photon mixing, which we denote $\Pi_{AA}(q^2)$, $\Pi_{ZZ}(q^2)$, $\Pi_{WW}(q^2)$ and $\Pi_{ZA}(q^2)$, respectively [14]. We divide these vacuum polarizations into two parts:

$$\Pi_{XY}(q^2) = \Pi^{\rm sm}_{XY}(q^2) + \Pi^{\rm new}_{XY}(q^2) \tag{3}$$

for $(XY) = (AA), (ZA), (ZZ), (WW)$, where $\Pi^{\rm sm}_{XY}(q^2)$ is the contribution of the standard model, and $\Pi^{\rm new}_{XY}(q^2)$ is the contribution of new physics. If we assume the scale of new physics, $M_{\rm new}$, which contributes to the $\Pi^{\rm new}_{XY}$'s to be large compared to the W and Z masses, it is then reasonable to expand the new physics contributions around $q^2 = 0$ and neglect higher orders which will be suppressed by powers of $q^2/M_{\rm new}^2$. Keeping terms linear in q^2, we find

$$\begin{aligned}
\Pi^{\rm new}_{AA}(q^2) &= q^2 \Pi'^{\rm new}_{AA}(0) + \cdots \\
\Pi^{\rm new}_{ZA}(q^2) &= q^2 \Pi'^{\rm new}_{ZA}(0) + \cdots \\
\Pi^{\rm new}_{ZZ}(q^2) &= \Pi^{\rm new}_{ZZ}(0) + q^2 \Pi'^{\rm new}_{ZZ}(0) + \cdots \\
\Pi^{\rm new}_{WW}(q^2) &= \Pi^{\rm new}_{WW}(0) + q^2 \Pi'^{\rm new}_{WW}(0) + \cdots
\end{aligned} \tag{4}$$

Note that $\Pi^{\rm new}_{AA}(0) = \Pi^{\rm new}_{ZA}(0) = 0$ from QED gauge invariance. Thus, in this approximation, the contribution of new physics can be parametrized by just six numbers: $\Pi'^{\rm new}_{AA}(0)$, $\Pi'^{\rm new}_{ZA}(0)$, $\Pi^{\rm new}_{ZZ}(0)$, $\Pi'^{\rm new}_{ZZ}(0)$, $\Pi^{\rm new}_{WW}(0)$, and $\Pi'^{\rm new}_{WW}(0)$. Three linear combinations of these numbers will be absorbed into the renormalization of the three input parameters used to fix the theory. That will leave us with only three linear combinations that are finite and observable. A popular choice for the three combinations is [5]

$$\begin{aligned}
\alpha S_Z &= 4s^2c^2\left[\Pi'^{\rm new}_{ZZ}(0) - \frac{c^2-s^2}{sc}\,\Pi'^{\rm new}_{ZA}(0) - \Pi'^{\rm new}_{AA}(0)\right] && (5)\\
\alpha S_W &= 4s^2\left[\Pi'^{\rm new}_{WW}(0) - \frac{c}{s}\,\Pi'^{\rm new}_{ZA}(0) - \Pi'^{\rm new}_{AA}(0)\right] && (6)\\
\alpha T &= \frac{\Pi^{\rm new}_{WW}(0)}{M_W^2} - \frac{\Pi^{\rm new}_{ZZ}(0)}{M_Z^2}, && (7)
\end{aligned}$$

where

$$c = \frac{g}{\sqrt{g^2+g'^2}}, \qquad s = \frac{g'}{\sqrt{g^2+g'^2}}. \tag{8}$$

In the notation of Ref. [2], $S_Z = S$, and $S_W = S + U$.

The effect of oblique corrections from new physics to an observable, $\mathcal{O}$, can be expressed in terms of the the parameters S_Z, S_W, and T as

$$\mathcal{O}_{\rm th} = \mathcal{O}_{\rm sm}[1 + aS_W + bS_Z + cT] \tag{9}$$

where $\mathcal{O}_{\rm sm}$ is the Standard Model prediction while $\mathcal{O}_{\rm th}$ is the theoretical prediction including oblique corrections from new physics. The coefficients a, b, and c depend on the observable $\mathcal{O}$ and are easily calculable. Now, an important point which is not often mentioned explicitly is that *both the Standard Model prediction $\mathcal{O}_{\rm sm}$ and the coefficients a, b, c depend on which three observables are used as inputs to fix the theory.* To give a trivial example, consider using α, G_μ, and M_Z as inputs to predict M_W. In this case, the theoretical prediction for M_W will be given by

$$M^2_{W,{\rm th}} = M^2_{W,{\rm sm}}(\alpha, G_\mu, M_Z)\left[1 + \frac{\alpha}{c^2-s^2}\left(\frac{c^2-s^2}{4s^2}S_W - \frac{1}{4s^2}S_Z + c^2T\right)\right]. \tag{10}$$

However, if the value of M_W itself is used as one of the three inputs, then the theoretical "prediction" will be

$$M^2_{W,{\rm th}} = M^2_{W,{\rm sm}}(M_W, *, *) = M_W^2 \tag{11}$$

and there will be no extra corrections from S_W, S_Z, or T.

The observation that we would like to make in this paper is that if G_μ and M_W are used as inputs to predict the W width, Γ_W, then Γ_W doesn't receive any extra corrections from S_W, S_Z, and T. Thus

$$\Gamma_{W,\text{th}} = \Gamma_{W,\text{sm}}(G_\mu, M_W, *). \tag{12}$$

This is for the simple reason that Γ_W receives corrections from new physics through the two parameters $\Pi^{\text{new}}_{WW}(0)$ and $\Pi'^{\text{new}}_{WW}(0)$, but these happen to be the ones that are absorbed into the renormalizations of G_μ and M_W and are unobservable. We will show this more explicitly in the following.

Consider the obliquely corrected W propagator:

$$G_{WW}(q^2) = \frac{1}{q^2 - \dfrac{g^2v^2}{4} - \Pi_{WW}(q^2)} \tag{13}$$

where $g^2v^2/4$ is the bare W mass. If we rewrite this propagator in terms of the physical W mass

$$M_W^2 = \frac{g^2v^2}{4} + \Pi_{WW}(M_W^2) \tag{14}$$

and the wave function renormalization constant [15]

$$Z_W^{-1} = 1 - \Pi'_{WW}(M_W^2) \tag{15}$$

we find

$$G_{WW}(q^2) = \left[\frac{1}{1+\delta_W(q^2)}\right]\left(\frac{Z_W}{q^2 - M_W^2}\right) \tag{16}$$

where

$$\delta_W(q^2) \equiv Z_W\left[\Pi'_{WW}(M_W^2) - \frac{\Pi_{WW}(q^2) - \Pi_{WW}(M_W^2)}{q^2 - M_W^2}\right]. \tag{17}$$

Note that $\delta_W(M_W^2) = 0$. Now, since

$$-\frac{4G_\mu}{\sqrt{2}} = \frac{g^2}{2}G_{WW}(0) \tag{18}$$

(up to certain vertex and box corrections from muon decay which will be discussed in Sec. III), Eq. (16) leads to

$$g^2 Z_W = 4\sqrt{2}G_\mu M_W^2[1+\delta_W(0)] \quad . \tag{19}$$

Using this result, the partial width of the decay $W \to e\nu$ can be written as

$$\Gamma(W\to e\nu)_{\text{th}} = \frac{g^2 M_W}{48\pi}Z_W = \frac{G_\mu M_W^3}{6\pi\sqrt{2}}[1+\delta_W(0)] \quad , \tag{20}$$

where the effect of oblique corrections is summarized in $\delta_W(0)$.

Separating $\delta_W(0)$ into the standard model contribution, $\delta_W^{\rm sm}(0)$, and the contribution of new physics, $\delta_W^{\rm new}(0)$, we find

$$\begin{aligned}\Gamma(W\to e\nu)_{\rm th} &= \frac{G_\mu M_W^3}{6\pi\sqrt{2}}[1+\delta_W^{\rm sm}(0)+\delta_W^{\rm new}(0)] \\ &= \frac{G_\mu M_W^3}{6\pi\sqrt{2}}[1+\delta_W^{\rm sm}(0)][1+\delta_W^{\rm new}(0)] \\ &\equiv \Gamma(W\to e\nu)_{\rm sm}[1+\delta_W^{\rm new}(0)] \qquad (21)\end{aligned}$$

Now if we Taylor expand $\Pi_{WW}^{\rm new}(q^2)$ in the definition of $\delta_W^{\rm new}(q^2)$, we find

$$\delta_W^{\rm new}(0) = \frac{M_W^2}{2}\Pi''^{\rm new}_{WW}(0)+\cdots \qquad (22)$$

which shows explicitly that $\Pi_{WW}^{\rm new}(0)$ and $\Pi'^{\rm new}_{WW}(0)$ disappear from Eq. (21); they have been absorbed into G_μ and M_W through Eqs. (14) and (19). Therefore, in the approximation where the $\Pi_{XY}^{\rm new}(q^2)$'s are expanded only up to the linear term in q^2, $\delta_W^{\rm new}(0)$ can be safely neglected.

An exactly analogous argument can show that

$$\Gamma(Z\to\nu\bar{\nu})_{\rm th} = \frac{G_\mu M_Z^3}{12\pi\sqrt{2}}\rho[1+\delta_Z(0)] \qquad (23)$$

where

$$\delta_Z(q^2) \equiv Z_Z\left[\Pi'_{ZZ}(M_Z^2) - \frac{\Pi_{ZZ}(q^2)-\Pi_{ZZ}(M_Z^2)}{q^2-M_Z^2}\right]. \qquad (24)$$

Again, there will be no S_W, S_Z, or T dependence coming from $\delta_Z(0)$. However, $\Gamma(Z\to\nu\bar{\nu})$ will receive T dependence through the ρ–parameter:

$$\begin{aligned}\rho_{\rm th} &= 1+\delta\rho_{\rm sm}+\alpha T \\ &= (1+\delta\rho_{\rm sm})(1+\alpha T) \\ &= \rho_{\rm sm}(1+\alpha T) \qquad (25)\end{aligned}$$

Therefore, writing $\delta_Z(0)=\delta_Z^{\rm sm}(0)+\delta_Z^{\rm new}(0)$ and neglecting $\delta_Z^{\rm new}(0)$, we find

$$\Gamma(Z\to\nu\bar{\nu})_{\rm th} = \Gamma(Z\to\nu\bar{\nu})_{\rm sm}(1+\alpha T) \qquad (26)$$

where

$$\Gamma(Z\to\nu\bar{\nu})_{\rm sm} \equiv \frac{G_\mu M_Z^3}{12\pi\sqrt{2}}\rho_{\rm sm}[1+\delta_Z^{\rm sm}(0)], \qquad (27)$$

so that a measurement of the partial width (given the precise value $M_Z = (91.187 \pm 0.007)$ MeV obtained at LEP [16]) provides information on T.

The total width of the W^+ is calculated under the assumption that the open decay channels are $e^+\nu_e, \mu^+\nu_\mu, \tau^+\nu_\tau$, and three colors of $u\bar{d}$ and $c\bar{s}$. Fermion masses (treated in [10] and [11]) give negligible effects, reducing the total predicted W width by less than 1 MeV. Thus we obtain the expression (2), where the factor of $1 + \alpha_s(M_W)/\pi$ is the usual QCD correction [17] for decays into colored quarks. The expression (2), like (1), does not have any correction factors involving S_W, S_Z, or T.

In a treatment where the terms up to those that are quadratic in q^2 are kept in Eq. (4), $\delta_W^{\rm new}(0)$ and $\delta_Z^{\rm new}(0)$ cannot be neglected. In Ref. [12], Maksymyk, Burgess, and London use the notation

$$\alpha V \equiv \delta_Z^{\rm new}(0), \qquad \alpha W \equiv \delta_W^{\rm new}(0). \tag{28}$$

and discuss the possible sizes of V and W. In their notation,

$$\frac{\Gamma(W \to e\nu)_{\rm th}}{\Gamma(W \to e\nu)_{\rm sm}} = 1 + \alpha W \quad , \tag{29}$$

$$\frac{\Gamma(Z \to \nu\bar{\nu})_{\rm th}}{\Gamma(Z \to \nu\bar{\nu})_{\rm sm}} = 1 + \alpha T + \alpha V \quad . \tag{30}$$

We shall comment on possible sources of W in Sec. V.

III. FULL SET OF STANDARD MODEL CORRECTIONS

As mentioned above, the tree level expression for the partial W width,

$$\Gamma(W \to e\nu) = \frac{G_\mu M_W^3}{6\pi\sqrt{2}}, \tag{31}$$

accounts for most of the leading order standard model oblique corrections, as well as the "new" oblique corrections, parametrized by S_W, S_Z, and T. The oblique corrections not absorbed into G_μ and M_W are given by

$$\delta_W(0) = Z_W \left[\Pi'_{WW}(M_W^2) - \frac{\Pi_{WW}(M_W^2) - \Pi_{WW}(0)}{M_W^2} \right] \quad . \tag{32}$$

The complete corrected result will be given by

$$\Gamma(W \to e\nu) = \frac{G_\mu M_W^3}{6\pi\sqrt{2}} \left[1 + \delta_W^{\rm sm}(0) + \delta_V^{\rm sm} + \delta_\mu\right] \quad , \tag{33}$$

where $\delta_V^{\rm sm}$ expresses the effect of the vertex and bremsstrahlung [11,18] corrections, and

$$\delta_\mu = -\frac{G_\mu M_W^2}{2\pi^2\sqrt{2}}\left[4\left(\Delta - \ln\frac{M_W^2}{\mu^2}\right) + \left(6 + \frac{7-4s^2}{2s^2}\ln c^2\right)\right] \quad , \tag{34}$$

with

$$\Delta \equiv \frac{1}{\epsilon} - \gamma_E + \ln 4\pi \tag{35}$$

takes care of the vertex and box corrections specific to muon decay which have been omitted in Eq. (18) [19]. Note that $\delta_W^{\rm sm}(0)$ in Eq. (33) is UV finite, while the UV divergences in $\delta_V^{\rm sm}$ and δ_μ cancel against each other. However, there is an IR divergence in $\delta_W^{\rm sm}(0)$ coming from the $\gamma - W$ loop, which is cancelled by a similar divergence in $\delta_V^{\rm sm}$. The finite contributions to $\delta^{\rm sm} = \delta_W^{\rm sm}(0) + \delta_V^{\rm sm} + \delta_\mu$ are summarized in Tables I and II for $m_t = 140$ GeV/c^2 and $M_H = 100$ GeV/c^2, with $g^2 Z_W/(4\pi)^2 = G_\mu M_W^2/(2\pi^2\sqrt{2}) = 0.268\%,\ s^2 = 0.23$.

Putting all the standard model corrections together, we find that the standard model correction to Eq. (1) is $\delta^{\rm sm} = -0.35\%$. The difference between the correction for leptons and for quarks is too small to affect the ratio (2) appreciably.

IV. NUMERICAL EVALUATION

The two most precise estimates of the W mass come from the CDF and UA2 collaborations:

$$M_W(\text{measured}) = \begin{cases} 79.92 \pm 0.39 \text{ GeV}/c^2 & [8] \\ 80.35 \pm 0.37 \text{ GeV}/c^2 & [9] \\ 80.14 \pm 0.27 \text{ GeV}/c^2 & \text{(average)}, \end{cases} \tag{36}$$

where we have recalibrated the UA2 value [9] in terms of the known Z mass. For $\alpha_s(M_W)$ we use an error attributed to systematic differences among various determinations [20], and take $\alpha_s(M_W) = 0.12 \pm 0.01$.

Two recent determinations of the $W \to e\nu$ branching ratio have been performed [21,22]. The method [23] relies upon the measurement of

$$\frac{\sigma(\bar{p}p \to e^\pm\nu + \ldots)}{\sigma(\bar{p}p \to e^+e^- + \ldots)} = \frac{\sigma(\bar{p}p \to W^\pm + \ldots)}{\sigma(\bar{p}p \to Z + \ldots)}\frac{\Gamma_{\rm tot}(Z)}{\Gamma(Z \to e^+e^-)} \times \frac{\Gamma(W^+ \to e^+\nu)}{\Gamma_{\rm tot}(W)} \quad . \tag{37}$$

The measured values of the left-hand side are $10.64 \pm 0.36 \pm 0.27$ (Ref. [21]), $10.0 \pm 1.1 \pm 2.4$ (muon channels, Ref. [22]), and $10.56 \pm 0.87 \pm 1.07$ (electron channels, Ref. [22]). The first ratio on the right-hand side is taken from theory to be 3.23 ± 0.03 [24] (CDF) or 3.26 ± 0.08 [25] (D0). The ratio $\Gamma_{\rm tot}(Z)/\Gamma(Z \to e^+e^-)$ is found from LEP averages [26] to be 29.69 ± 0.13. Here we have used $\Gamma_{\rm tot}(Z) = (2.489 \pm 0.007)$ GeV, $\Gamma(Z \to e^+e^-) = (83.82 \pm 0.27)$ MeV.

The results are

$$\frac{\Gamma(W^+ \to e^+\nu)}{\Gamma_{\rm tot}(W)} = \begin{cases} 0.111 \pm 0.005 \quad [22] & , \\ \\ 0.108 \pm 0.013 \quad [23] & . \end{cases} \tag{38}$$

This is to be compared with the theoretical estimate, made assuming the open decay channels are $e\nu, \mu\nu, \tau\nu$, $u\bar{d}$, and $c\bar{s}$:

$$\frac{\Gamma(W^+ \to e^+\nu)}{\Gamma_{\rm tot}(W)} = [3 + 6(1 + \frac{\alpha_s(M_W)}{\pi})]^{-1} = 0.1084 \pm 0.0002 \quad . \tag{39}$$

The measurement of this ratio does *not* test $\Gamma(W \to e^+\nu)$ or $\Gamma_{\rm tot}(W)$ separately.

The small difference between the standard model corrections for quark and lepton final states leads to an increase of the above ratio by about 3×10^{-5}, or 0.03% of its value.

V. POSSIBLE SOURCES OF DEVIATION

The partial width $\Gamma(W \to e\nu)$ could be affected by mixing of the W with other states (e.g., new gauge bosons or vector mesons in the TeV region associated with substructure of the Higgs sector [27]. We expect, however, that constraints from other data would severely limit such mixing.

The ratio $\Gamma_{\rm tot}(W)/\Gamma(W \to e\nu)$ could be raised from its predicted value if additional exotic decay channels for the W were available. Such a channel could be $t+\bar{b}$, where the t decays to a charged Higgs boson and a b quark. The result of Ref. [21] implies $m_t > 62$ GeV under such a scenario. Another such channel would be a pair of scalar bosons H^+H^0. Comparison of the predicted and observed branching ratios places severe limits on the couplings for such decays.

As an example of the effects [12] due to higher-order oblique corrections from "new" physics, we calculate $\delta_W^{\rm new}(0)$ in the two-Higgs-doublet extension of the standard model [28]. We choose $m_1 = m_2 = m_+/4 = m_3/8$ for the scalar masses, where m_1 and m_2 are the masses of the neutral scalars, m_+ the charged scalar, and m_3 the neutral pseudoscalar. This choice is of interest since for $m_3 \geq 500$ GeV one obtains a negative contribution to the parameter ρ [29]. We plot our results as the dashed line in Fig. 1.

The authors of Ref. [12] calculate the contribution to $\delta_W^{\rm new}(0)$ (αW in their language) of a doublet of heavy degenerate leptons. We reproduce this calculation and plot the result as the dotted line in Fig. 1. Both this result and that of the previous paragraph lead to very small and probably undetectable effects on the W partial and total widths.

Very recently Lavoura and Li [30] have pointed out that one can increase some of the parameters introduced in Refs. [12] and [13] without correspondingly large increases in S_W, S_Z, and T by introducing scalar multiplets of very high weak isospin. However, it appears difficult in the cases they consider to obtain any detectable changes in the W width without appreciable effects elsewhere.

VI. MEASUREMENT OF ABSOLUTE WIDTH

The reaction $\bar{p}p \to \ell\nu_\ell = \ldots$, where $\ell = e,\ \mu,\ \tau$, is dominated by the production of real W bosons, but there is a measurable continuum of events above the W [31,32]. By comparing the signal for real and virtual W bosons, one can obtain an estimate of the total width [33].

Let us imagine that partons i and j (typically a u quark and a $\bar{d}$ antiquark) with squared center-of-mass energy $\hat{s}$ collide to form either a real or a virtual W^+, which subsequently decays to $\ell^+\nu_\ell$. The cross section for this subprocess has the form

$$\frac{d\sigma}{d\hat{s}} = {\rm Const} \times \frac{\Gamma_{ij}\Gamma_{\ell\nu_\ell}}{(\hat{s}-M_W^2)^2 + M_W^2\Gamma_{\rm tot}^2} \quad , \tag{40}$$

where Γ_{ij} is the partial width for the decay of the W into ij, while $\Gamma_{\rm tot}$ is the total W width. The integral of this cross section over $\hat{s}$ is proportional to $\Gamma_{ij}\Gamma_{\ell\nu_\ell}/\Gamma_{\rm tot}$, while far above $\hat{s} = M_W^2$ the expression is almost independent of $\Gamma_{\rm tot}$. Thus, a comparison of the real W signal with the continuum $\ell\nu_\ell$ signal above the W normalizes the production process and gives a measurement of the total W width.

The 1988-9 CDF data [31] indicate that one can count on about four or five $e^\pm\nu_e$ events above a transverse mass of 100 GeV/c^2 for each inverse pb of integrated luminosity. Thus, with one inverse femtobarn of data and detection of both $e^\pm\nu_e$ and $\mu^\pm\nu_\mu$ pairs, one can hope for a statistical accuracy of about a percent in measurement of $\Gamma_{\rm tot}$.

VII. SUMMARY

We have shown that lowest-order expression for the W^+ partial width to $e^+\nu_e$ does not receive contributions from new physics contained in the oblique correction parameters S_W, S_Z, and T when expressed in terms of the muon decay constant G_μ and the measured W mass. As a result, a measurement of Γ_W provides independent information on M_W. Any inconsistency between the value of M_W inferred from the W width and that measured directly will have to be ascribed to effects not encompassed in these three parameters.

The present method for measuring $\Gamma_{\rm tot}(W)$ at hadron colliders actually yields the *branching ratio* for $W \to e\nu$. Recent precise experiments are consistent with the prediction that this ratio should be given by approximately

$$\frac{\Gamma(W \to e\nu)}{\Gamma_{\rm tot}(W)} = \frac{1}{9}\left[1 + \frac{2}{3}\frac{\alpha_s(M_W)}{\pi}\right]^{-1} \quad . \tag{41}$$

One is still in search of an *absolute* measurement of the W partial or total width. As we have shown, there is not much room for deviations from the predictions (1) and (2) for these quantities. Comparison of production of real and virtual W bosons may begin to shed light on the total W width.

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APPENDIX: TOP QUARK AND HIGGS BOSON CONTRIBUTIONS

The standard model oblique correction due to the $t-\bar{b}$ loop is

$$\delta^t_W(0) = \frac{g^2 Z_W}{16\pi^2}\frac{3}{2}\left\{\frac{2}{3} - \frac{\xi}{2} - \xi^2 - \xi(1-\xi^2)\ln[\xi/(\xi-1)]\right\} \quad , \tag{42}$$

with $\xi \equiv m_t^2/M_W^2$. (We have neglected m_b here.) This quantity is generally small, and goes to 0 as $m_t \to \infty$. We plot $\delta^t_W(0)$ as a function of m_t as the solid line in Fig. 1. For a nominal top quark mass of 140 GeV, we get

$$\delta^t_W(0) = -\frac{0.15 g^2 Z_W}{16\pi^2} \quad . \tag{43}$$

The standard model Higgs boson's contribution is extremely small:

$$\begin{aligned}\delta_W^{\rm Higgs}(0) &= \frac{g^2 Z_W}{4(4\pi)^2}\left[\left(\frac{47}{6}-\frac{7}{2}\xi_H+\xi_H^2\right)\right.\\ &+ \frac{-4+22\xi_H-17\xi_H^2+6\xi_H^3-\xi_H^4}{2(\xi_H-1)}\ln\xi_H\\ &+ \left.(-28+20\xi_H-7\xi_H^2+\xi_H^3)\sqrt{\frac{\xi_H}{4-\xi_H}}\arctan\sqrt{\frac{4-\xi_H}{\xi_H}}\right] \qquad (44)\end{aligned}$$

where $\xi_H \equiv m_H^2/m_W^2$. With $g^2 Z_W \approx 0.4$ and for $M_H = 100$ GeV/c^2, we get $\delta_W^{\rm Higgs}(0) \approx -6\times10^{-5}$, with even smaller values for larger Higgs boson masses.

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[14] By Π_{XY} we mean the coefficient of $g_{\mu\nu}$ in the vacuum polarization tensor. The $q^\mu q^\nu$ part can be neglected. See Ref. [2] for details.

[15] The prime stands for differentiation with respect to q^2. This is not to be confused with the notation of Ref. [2], where $\Pi'_{XY}(q^2)$ stands for $[\Pi_{XY}(q^2) - \Pi_{XY}(0)]/q^2$.

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Table I. Finite parts of contributions to $\delta_W^{\rm sm}(0)$.

Contribution	Coefficient of $g^2 Z_W/(4\pi)^2$	Value (%)
Light fermions	3	0.80
$t\bar{b}$ loop	-0.15^a	-0.04^a
Photon - W	-1.00	-0.27
Z loops	0.51	0.14
Standard Higgs	-0.02^b	-0.006^b
Total	2.34	0.62

[a]For $m_t = 140$ GeV/c^2. [b]For $M_H = 100$ GeV/c^2.

Table II. Finite parts of contributions to $\delta_V^{\rm sm}$ and δ_μ.

	Leptons:		Quarks:	
Contribution	Coefficient of $g^2 Z_W/(4\pi)^2$	Value (%)	Coefficient of $g^2 Z_W/(4\pi)^2$	Value (%)
Wave function	-0.11	-0.03	0.28	0.07
Vertices	-0.91	-0.24	-2.28	-0.61
Bremsstrahlung	-0.08	-0.02	0.72	0.19
δ_μ	-2.55	-0.68	-2.55	-0.68
Subtotal	-3.65	-0.97	-3.83	-1.03
Total[a]	-1.31	-0.35	-1.49	-0.41

[a]Including contributions of Table I.

FIGURE CAPTIONS

FIG. 1. Correction term $\delta_W(0)$ affecting W partial width to $e\nu$. Solid line: contribution from top quark as function of m_t; dashed line: contribution from Higgs sector as function of charged Higgs boson mass m_+; dotted line: contribution from extra degenerate lepton doublet as function of mass m_ℓ.