

ANALYTICAL INVESTIGATION OF THE TILT

ROTOR WHIRL INSTABILITY

By

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I. INTRODUCTION

The purpose of this section is to present some background information as to the nature of the problem and to present certain assumptions or simplifications that can be introduced in order to obtain a solution. In addition, this section is intended to serve as background information for the mathematical model chosen in this dissertation, the details of which are presented in Section 2.1 entitled "Mathematical Model."

1.1 Statement of the Problem

A rigid disk mounted symmetrically on the end of a flexible shaft can vibrate on the shaft in two independent modes. These two mode shapes are shaft bending in two perpendicular planes. When the disk rotates, gyroscopic forces and moments are introduced which couple the two shaft bending modes into what are called whirl modes. In the whirl mode the disk axis rotates or precesses about the undeflected shaft axis. There are two whirl modes. One of these modes is called a forward precessional whirl mode because the disk whirls or precesses in the same direction as the shaft rotates. The other mode is termed a backward precessional whirl mode as the disk whirls in a direction opposite to the direction of the shaft rotation. The frequency of the forward whirl mode increases as the disk rotational speed is increased, whereas the frequency of the backward whirl mode decreases with increasing disk rotational speed. If one of these modes is initiated, it will persist at constant amplitude unless there is structural damping which will cause the amplitude to decay.

When the rotating rigid disk is replaced with a rotating rigid propeller blade system, and aerodynamic forces are neglected, similar constant amplitude motion will also persist. Further, this mechanical system will always be neutrally stable. When aerodynamic forces are introduced, the whirling or precessional motion of the propeller blades will produce angle-of-attack changes on the blades as they rotate and precess. These blade angle-of-attack changes will produce aerodynamic forces and moments that can introduce either negative or positive work into the system, which may make the system become stable or unstable respectively. Therefore, the importance of a good aerodynamic force representation is obvious. The general problem considered in this dissertation is: given an aircraft propeller system, determine the stability boundary of the propeller system as a function of the aircraft operating conditions and thereby define the operating restrictions for the aircraft.

1.2 Mechanical System

The mechanical system analyzed in this paper consists of members representing the propeller blades, the propeller drive shaft, and the portion of the wing between the propeller drive shaft and fuselage. The individual members of the mechanical system are considered rigid with hinges at their extremities. Each rotating blade is assumed to have both flapping and lagging hinges. (See figure 1.) The flapping hinge is a mechanical hinge with its axis perpendicular to both the axis of the rotor drive shaft and the blade radius vector, while the lagging hinge has its axis

parallel to the rotor drive shaft. Both of these hinges are offset from the rotor system center of rotation in the direction of the blade radius vector. (See figure 1.)

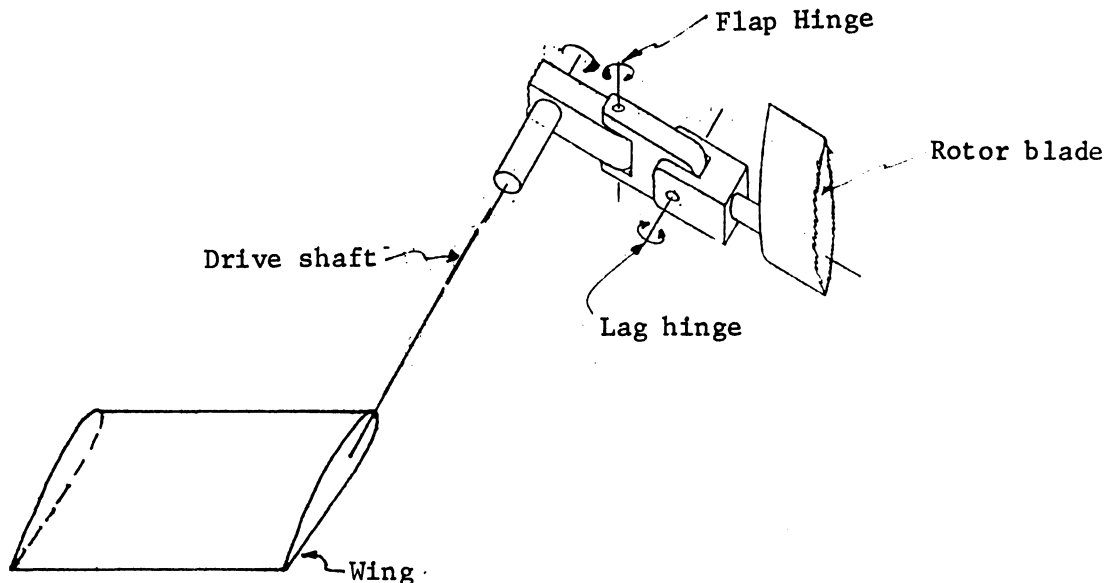


Figure 1.- Sketch of the wing-drive-shaft and rotor blade system.

The propeller blades are attached through the blade hinges to the drive shaft and the drive shaft is fastened to the wing. The wing-drive-shaft joint has three degrees of rotational freedom. The wing root is connected to the fuselage through a joint that has three additional degrees of rotational freedom. A schematic of the mechanical system is shown in figure 10. There is a linear spring and viscous damper acting across each hinge in the mechanical system. Associated with each hinge is one

degree of freedom and one differential equation of motion.

1.3 Aerodynamic System

The sophistication used in introducing the aerodynamics into the analysis can vary greatly. Attempts have been made to determine theoretically the aerodynamic force characteristics of an airfoil, for example [1,2]. In most of these attempts numerous assumptions such as neglecting viscosity, flow separation, and turbulence have been made. To eliminate such assumptions, experimentally determined aerodynamic forces and moments will be used in obtaining the solution in this dissertation.

The approach is to use a blade strip analysis technique. This technique is used to determine the force on an element of the blade and, by integration over the total blade area, determine the total forces. A knowledge of the fluid velocity at each blade strip and some force coefficient for converting these velocities to elemental forces is required. The fluid velocities are relative velocities between the blade strip and the fluid, and they consist of kinematic, aircraft flight, and induced velocities. The kinematic velocity is defined by the motions of the mechanical system. The flight velocity is defined by the aircraft flight conditions. The induced velocity is a local aerodynamic velocity caused by the difference in pressure between the upper and lower surface of the rotating blade (the velocity imparted to the air that corresponds to the blade lift and drag forces). The induced velocities can be determined from either uniform or non-uniform inflow theories. Uniform inflow theory assumes that the rotor-

induced velocity is a constant average value for the entire swept rotor blade area, whereas non-uniform inflow theory permits induced velocity variations over the swept blade area. Present analytical methods for solving the non-uniform inflow problem has not proven adequate when compared with experimental data [3,4]. Thus, the numerical solutions presented in this paper are for uniform inflow.

Another aspect in determining the elemental blade forces is to use force coefficients based on quasi-steady or unsteady aerodynamics. In the unsteady aerodynamics approach, the aerodynamic force coefficients may be time dependent. However, due to a lack of understanding for unsteady aerodynamics [5,6,7], steady aerodynamics are used in this paper. A large amount of two-dimensional force coefficient data exists for many different airfoils [8,9]. Also, Mach number effects data are available [10,11], and these effects are included in the present analysis.

1.4 Review of Literature

The literature review is presented chronologically. Published information only is referenced and reviewed.

The earliest related information about the propeller whirl problem was found in the analysis of Taylor and Browne [12]. Taylor [12] considered a rigid propeller powerplant system. The engine powerplant supports were modeled by springs and dampers. Two degrees of freedom were considered: pitch and yaw of the propeller powerplant shaft axis. The aerodynamic forces and moments on the rigid propeller were determined from the computations of Harris [13] and the experimental measurements of

Lesely [14]. Solving the equations of motion resulted in two modes of motion: (1) precession of the propeller rotational plane in the direction of the propeller rotation and (2) precession of the propeller plane in the direction opposite to the propeller rotation. These modes are presently called forward and backward whirl modes, respectively. Taylor [12] concluded from his analysis that as forward flight speed is increased, the backward whirl mode becomes unstable. Taylor [12] also concluded that a very small amount of damping in the engine mounts will prevent the backward mode from becoming unstable. Further, their actual propeller-powerplant-shaft spring stiffness was much stiffer than that required to encounter the whirl instability. It is believed that this was the primary reason for the lack of any further development or consideration of this problem at that time. However, in the 1955-1960 period, flight experience with a propeller and a rotor aircraft propulsion system revealed serious problems.

The most tragic of these problems were two fatal Lockheed Electra aircraft accidents occurring in 1960. Wind-tunnel tests conducted at Langley Research Center (NASA) indicated that if the stiffness of the propeller-engine support system for the Electra configuration was severely reduced (e.g., through structural damage), propeller whirl instability was possible. In the same period, initial flight tests of a new experimental vertical take-off and landing (VTOL) aircraft, the XV-3, revealed a wing-rotor-shaft whirl oscillation existing at high forward speed.

These aircraft accidents led to an extensive investigation at Langley Research Center [15,16,17,18,19,20]. The initial work of Reed [15] was similar to that of Taylor [12], the primary difference being that Reed [15] used modifications of the propeller aerodynamics from Ribner [21]. The investigations [15 through 20] included both analytical and experimental studies and the effect of numerous parameters.

During this same time period, numerous other flight tests wind-tunnel tests [23,24], and analytical studies [25,26,27] were conducted. Each of these studies concluded with varying degrees of success. Quigley [25] presented a physical explanation for the unstable rotor damping forces and moments. Richardson [26,27] obtained a numerical solution of whirl stability for a propeller with and without hinged blades. Richardson [26,27] could not verify the forward whirl mode with his equations. He hypothesized that the inclusion of unsteady aerodynamic considerations would be required to analytically model the forward whirl mode. In 1966-67, two additional analytical investigations were reported [28,29] along with additional full-scale wind-tunnel tests [30]. These studies were generally limited to a particular aircraft, namely, the XV-3.

An informative analytical study of the propeller whirl stability problem was published by Young [31]. However, like many of the previously published documents, the aerodynamics were simplified to expedite the numerical solution. For example, Young [31] neglected blade

stall and compressibility effects. In spite of this, Young arrives at some remarkable conclusions. Young found system stability boundaries that indicated optimum values of some physical parameters. An optimum value of drive shaft stiffness occurred when the blade-flapping natural frequency was about 1.1 to 1.2 times the rotor rotational speed. The blade-flapping natural frequency is defined as the natural frequency of the blade when subjected to a centrifugal restoring moment. Young concluded that, for flapping natural frequencies greater than this optimum value (1.1Ω to 1.2Ω), the blade motion is a backward whirl mode, whereas for lesser values the blade motion is a forward whirl mode and is self-limiting, that is, limit orbit cycle. However, both Reed [16,18] and Richardson [26,27] hypothesized that to duplicate the forward whirl mode analytically would require the inclusion of unsteady aerodynamics; they both verified the existence of the forward whirl mode experimentally. In reference [31], Young states that the windmilling case with propeller blades is more stable than the power on case which is contrary to the conclusions of Reed [18]. A few questions have arisen in trying to verify Young's [31] published equations.

In summary, the literature review indicates that the earliest propeller or rotor stability problem investigations were largely neglected until some fatal aircraft accidents occurred. Since then, the effort in trying to find engineering solutions to the problem has been expanded. An exact solution to the problem is extremely difficult and, therefore, all attempts to achieve a solution involve numerous simplifying

assumptions. The assumptions vary between investigators. To some extent, the conclusions of these investigators differ and even disagree. Certainly, some of this disagreement is due to the different simplifying assumptions used in obtaining a solution. Because of differences in conclusions between the various investigators, more work is needed. Furthermore, there is a shortage of published experimental data for correlation with some of the latest available theories.

Many of the simplifying assumptions made in previous studies will not be made herein. Further the theoretical formulation in this thesis is believed sufficiently general to include most, if not all, of the previous studies as degenerate cases. Therefore, it is hoped that some differences between existing studies can be resolved because of this study.

II. DEVELOPMENT OF THE EQUATIONS OF MOTION

This chapter presents the general procedure for the derivation of the equations of motion used in this dissertation. The complete derivation is presented in appendices A, B, and C. The general approach is to use as accurate an aerodynamic model as reasonably possible and to obtain a good mechanical system representation. The mechanical system consists of an aircraft wing, rotor drive shaft, and rotor blades. The aerodynamics include the effects of blade stall and compressibility.

2.1 Mechanical System

The mechanical model consists of rigid members connected by hinged joints. Each hinge represents one degree of freedom. Across each hinge is an ideal spring and viscous damper. The mass distribution of each rigid member is assumed to be uniformly distributed along the length of the member. The model includes pitch, flap, and lag degrees of freedom for each blade; also, three drive shaft and three wing-root hinges are included.

To obtain the inertia forces and moments, the general procedure is to use Euler angle transformations to derive equations describing the Cartesian coordinates of each elemental mass for each member of the mechanical system with respect to inertial coordinates. These equations are then differentiated twice with respect to time to find the acceleration vectors. These accelerations times the appropriate elemental masses results in the inertial force vectors and the vector cross product of the moment arm and inertial force results in the inertial moments. The

total vector forces and moments are determined by integration along the length of the members. These forces and moments are then transformed into the desired coordinate system.

In this section, the derivation contains the individual steps required to obtain the final transformation equation. (See equation A1 in appendix A.) The individual sketches provided are directed toward the final summary sketch of the complete model in figure 10. The individual transformations are provided as an equation and symbolically in each sketch. These individual transformations are combined into the final summary transformation in figure 11. The symbolic representation of the transformation written below each transformation equation will prove useful in transforming from one set of coordinates to another.

The x^0 , y^0 , z^0 coordinate axes are defined as inertial coordinates. These nonrotating coordinates are therefore restricted to constant linear velocities which can be considered the flight velocities of the aircraft. Therefore, they define the initial flight operating conditions of the aircraft or rotor system, such as aircraft angle of attack, rate of descent, inflow velocities, etc.

The aircraft wing span axis is aligned with the x^0 -axis. The first transformation consists of an angular rotation about the x^0 -axis of magnitude θ_{x0} radians. This rotation results in the new coordinate system identified by x^I , y^I , and z^I . A sketch of the procedure used to achieve this transformation is shown in figure 2.

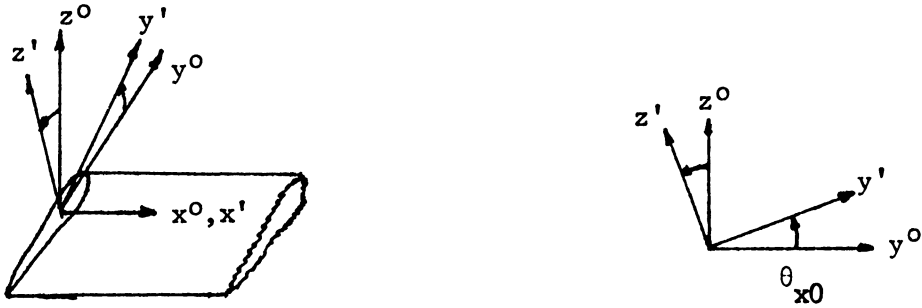


Figure 2.- Sketch of the θ_{x0} rotation.

$$\begin{Bmatrix} x^I \\ y^I \\ z^I \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta_{x0c} & \theta_{x0s} \\ 0 & -\theta_{x0s} & \theta_{x0c} \end{bmatrix} \begin{Bmatrix} x^0 \\ y^0 \\ z^0 \end{Bmatrix}$$

(1)

$$\begin{Bmatrix} x^I \\ y^I \\ z^I \end{Bmatrix} \rightarrow \begin{bmatrix} \theta_{x0} \end{bmatrix} \rightarrow \begin{Bmatrix} x^0 \\ y^0 \\ z^0 \end{Bmatrix}$$

where

$$\theta_{x0c} \equiv \cos \theta_{x0}$$

$$\theta_{x0s} \equiv \sin \theta_{x0}$$

With the use of the matrix equation (1) (fig. 2), any coordinate in the x^0, y^0, z^0 system can be converted to the corresponding coordinate in the x^I, y^I, z^I system.

The second transformation consists of an angular rotation about the y' -axis of magnitude θ_{y0} radians and results in the new coordinate system x'' , y'' , and z'' . The details of this transformation are shown in figure 3.



Figure 3.- Sketch of the θ_{y0} rotation.

$$\begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} = \begin{bmatrix} \theta_{y0C} & 0 & -\theta_{y0S} \\ 0 & 1 & 0 \\ \theta_{y0S} & 0 & \theta_{y0C} \end{bmatrix} \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix}$$

(2)

$$\begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix} \rightarrow [\theta_{y0}] \rightarrow \begin{Bmatrix} x' \\ y' \\ z' \end{Bmatrix}$$

where

$$\theta_{y0C} \equiv \cos \theta_{y0}$$

$$\theta_{y0S} \equiv \sin \theta_{y0}$$

The third transformation consists of an angular rotation about the z'' -axis of magnitude θ_{z0} radians and results in the coordinate system x''', y''', z''' . This transformation is shown in figure 4.

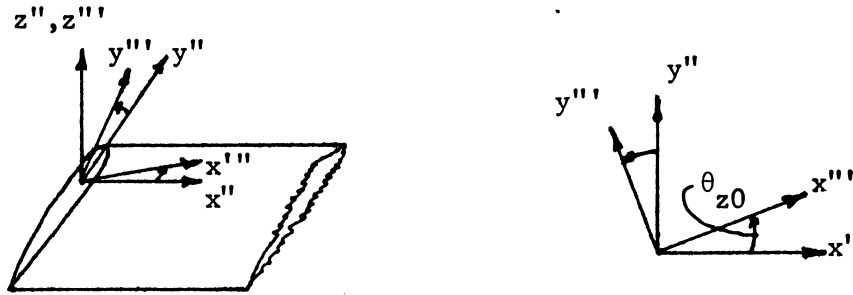


Figure 4.- Sketch of the θ_{z0} rotation.

$$\begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} = \begin{bmatrix} \theta_{z0C} & \theta_{z0S} & 0 \\ -\theta_{z0S} & \theta_{z0C} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix}$$

(3)

$$\begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} \rightarrow [\theta_{z0}] \rightarrow \begin{Bmatrix} x'' \\ y'' \\ z'' \end{Bmatrix}$$

where

$$\theta_{z0C} \equiv \cos \theta_{z0}$$

$$\theta_{z0S} \equiv \sin \theta_{z0}$$

By substitution, the three individual rotational transformations (eqs. (1), (2), and (3)) can be combined into one equation and, by renaming the matrix product, the equation can be simplified as follows:

$$\begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} = \begin{bmatrix} \theta_{zOC} & \theta_{zOS} & 0 \\ -\theta_{zOS} & \theta_{zOC} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{yOC} & 0 & -\theta_{yOS} \\ 0 & 1 & 0 \\ \theta_{yOS} & 0 & \theta_{yOC} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta_{xOC} & \theta_{xOS} \\ 0 & -\theta_{xOS} & \theta_{xOC} \end{bmatrix} \begin{Bmatrix} x^o \\ y^o \\ z^o \end{Bmatrix}$$

$$\begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} = [\theta_{iO}] \begin{Bmatrix} x^o \\ y^o \\ z^o \end{Bmatrix} \quad (4)$$

where $[\theta_{iO}]$ is defined above and by equation (A2). The matrix $[\theta_{iO}]$ is an orthogonal matrix and therefore has the property that the inverse of the matrix is equal to its transpose, thus

$$\begin{Bmatrix} x^o \\ y^o \\ z^o \end{Bmatrix} = [\theta_{iO}]^T \begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} \quad (5)$$

The aircraft wing section (to rotor shaft) is assumed to be of length L aligned along the x''' -axis with a mass distribution represented by m_5 . From equation (5) the inertial coordinates of the wing-drive-shaft joint are therefore represented by

$$\begin{Bmatrix} x^0 \\ y^0 \\ z^0 \end{Bmatrix} = [\bar{\theta}_{i0}]^T \begin{Bmatrix} L \\ 0 \\ 0 \end{Bmatrix} \quad (6)$$

Three rotations or degrees of freedom are introduced at the wing-drive-shaft joint. These three rotations follow the same order as the wing-root-fuselage joint rotations. First, an angular rotation about the x''' -axis of magnitude θ_{x1} radians is introduced which results in the x^{IV} , y^{IV} , z^{IV} coordinate system as follows:

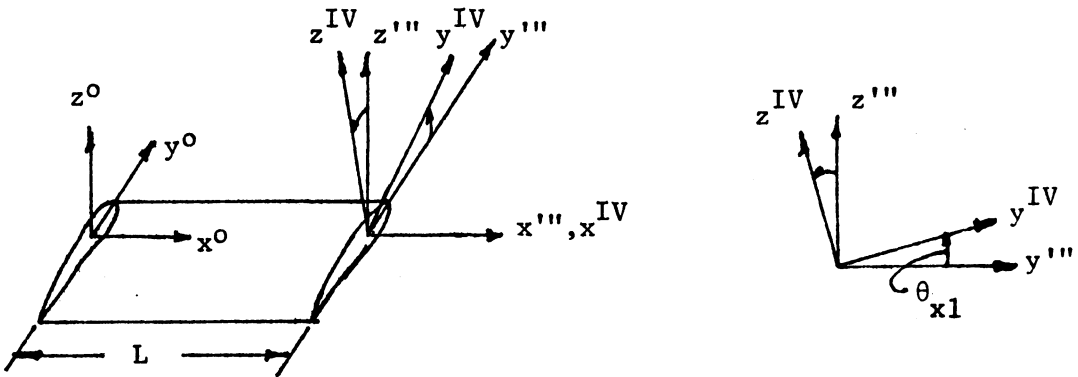


Figure 5.- Sketch of the θ_{x1} rotation.

$$\begin{Bmatrix} x^{IV} \\ y^{IV} \\ z^{IV} \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta_{x1C} & \theta_{x1S} \\ 0 & -\theta_{x1S} & \theta_{x1C} \end{bmatrix} \begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix}$$

$$\begin{Bmatrix} x^{IV} \\ y^{IV} \\ z^{IV} \end{Bmatrix} \rightarrow [\theta_{x1}] \rightarrow \begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} \quad (7)$$

where

$$\theta_{x1C} \equiv \cos \theta_{x1}$$

$$\theta_{x1S} \equiv \sin \theta_{x1}$$

Next, an angular rotation about the y^{IV} -axis of magnitude θ_{y1} radians is introduced which results in the superscript V coordinate system. An angular rotation about z^V of magnitude θ_{z1} radians is then introduced. The final summed transformation at this joint is the same as that of equation (4) except that the subscript on the transformation variables is changed to reflect the different joint. This result follows:

$$\begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix} = \begin{bmatrix} \theta_{z1C} & \theta_{z1S} & 0 \\ -\theta_{z1S} & \theta_{z1C} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{y1C} & 0 & -\theta_{y1S} \\ 0 & 1 & 0 \\ \theta_{y1S} & 0 & \theta_{y1C} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta_{x1C} & \theta_{x1S} \\ 0 & -\theta_{x1S} & \theta_{x1C} \end{bmatrix} \begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix}$$

and in a combined simplified form:

$$\begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix} = \begin{bmatrix} \theta_{i1} \end{bmatrix} \begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} \quad (8)$$

Also, as with equation (5),

$$\begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} = \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix} \quad (9)$$

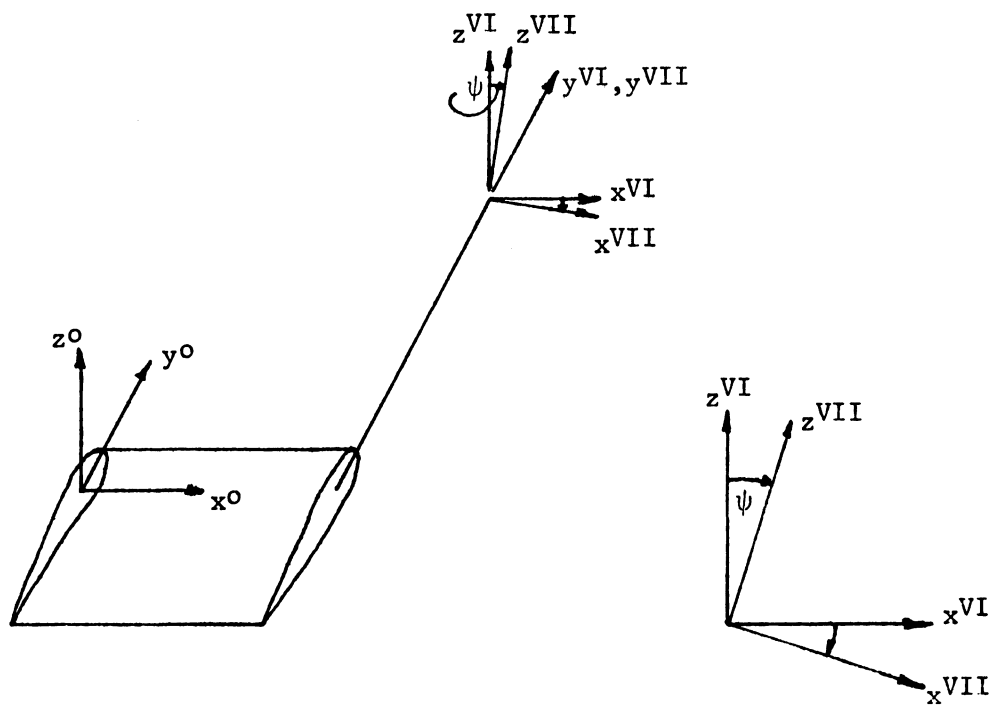
The rotor drive shaft of length ℓ with a mass distribution m_4 is assumed to be aligned along the y^{VI} -axis. The coordinates of the drive shaft end with respect to its root end (wing-drive-shaft joint) becomes

$$\begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix} = [\theta_{i1}]^T \begin{Bmatrix} 0 \\ \ell \\ 0 \end{Bmatrix} \quad (10)$$

and substituting $\begin{Bmatrix} x''' \\ y''' \\ z''' \end{Bmatrix}$ above into equation (5) will result in the inertial coordinates of the tip end of the drive shaft with respect to its root end

$$\begin{Bmatrix} x^0 \\ y^0 \\ z^0 \end{Bmatrix} = [\theta_{i0}]^T [\theta_{i1}]^T \begin{Bmatrix} 0 \\ \ell \\ 0 \end{Bmatrix} \quad (11)$$

The rotor blade system rotation is achieved by introducing an angular rotation about the y^{VI} -axis of magnitude ψ resulting in super-script VII coordinates as follows:

Figure 6.- Sketch of the ψ rotation.

$$\begin{Bmatrix} x^{VII} \\ y^{VII} \\ z^{VII} \end{Bmatrix} = \begin{bmatrix} \psi_C & 0 & -\psi_S \\ 0 & 1 & 0 \\ \psi_S & 0 & \psi_C \end{bmatrix} \begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix}$$

$$= [\psi] \begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix}$$

$$\begin{Bmatrix} x^{VII} \\ y^{VII} \\ z^{VII} \end{Bmatrix} \rightarrow [\psi] \rightarrow \begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix}$$

where

$$\psi_C \equiv \cos \psi$$

$$\psi_S \equiv \sin \psi$$

The rotor blade is chosen to lie along the z^{VII} -axis. To have the most general mechanical system, both a flapping and lagging hinge are provided herein. A common design is to have the flapping hinge closer to the center of rotation than the lagging hinge. It is not unusual to have an offset between the center of rotation and the first hinge (flapping) and also between the flapping hinge and the lagging hinge. Therefore, provisions are made herein for these offsets. The first offset of length e_1 with a mass distribution m_3 is aligned in the direction of the z^{VII} coordinate. The coordinates that are translated a distance e_1 are defined as a new set of coordinates with a superscript VIII. By using the orthogonal properties of the ψ matrix from equation (12), this offset in superscript VI coordinates becomes

$$\begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix} = [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1 \end{Bmatrix} \quad (13)$$

Transforming this element into inertial coordinates by substituting equation (13) into equation (9), (recalling that the difference between superscript VIII and VII coordinates is a translation and not a rotation), and then substituting the resulting equation (9) into equation (5)

results in

$$\begin{Bmatrix} x^0 \\ y^0 \\ z^0 \end{Bmatrix} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{bmatrix} \psi \end{bmatrix}^T \begin{Bmatrix} 0 \\ 0 \\ e_1 \end{Bmatrix} \quad (14)$$

This transformation can be followed symbolically by the use of the transformation summary in figure 11. The flapping hinge rotation is introduced by a negative rotation about axis x^{VIII} of a magnitude $+\beta$ radians as follows:

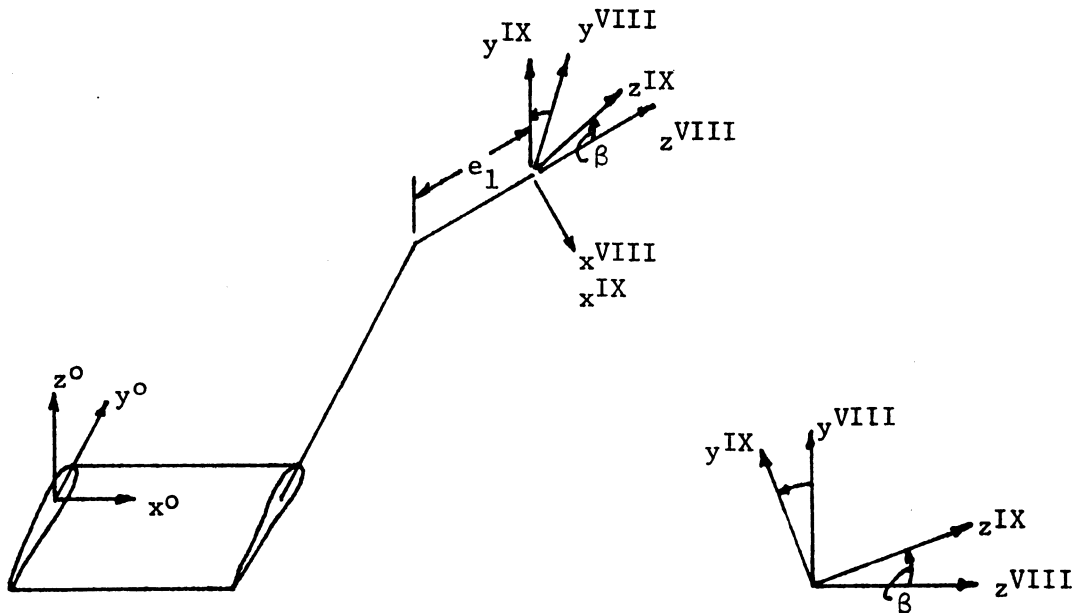


Figure 7.- Sketch of the β rotation.

$$\begin{aligned}
 \begin{Bmatrix} x^{IX} \\ y^{IX} \\ z^{IX} \end{Bmatrix} &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta_C & -\beta_S \\ 0 & \beta_S & \beta_C \end{bmatrix} \begin{Bmatrix} x^{VIII} \\ y^{VIII} \\ z^{VIII} \end{Bmatrix} \\
 &= \begin{bmatrix} \beta \end{bmatrix} \begin{Bmatrix} x^{VIII} \\ y^{VIII} \\ z^{VIII} \end{Bmatrix} ,
 \end{aligned} \tag{15}$$

where

$$\beta_C \equiv \cos \beta$$

$$\beta_S \equiv \sin \beta .$$

By using the orthogonal properties of the β matrix, equation (15) becomes

$$\begin{Bmatrix} x^{VIII} \\ y^{VIII} \\ z^{VIII} \end{Bmatrix} = \begin{bmatrix} \beta \end{bmatrix}^T \begin{Bmatrix} x^{IX} \\ y^{IX} \\ z^{IX} \end{Bmatrix} . \tag{16}$$

Introducing an additional translation of length e_2 with mass distribution m_2 along the z^{IX} -axis, redefining the translated axis with a superscript X , and rotating these coordinates into inertial coordinates by using the transformation summary in figure 11 results in

$$\begin{Bmatrix} x^0 \\ y^0 \\ z^0 \end{Bmatrix} = [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2 \end{Bmatrix} \quad (17)$$

The blade lag angle, ξ , is introduced as a negative rotation about the y^X -axis of magnitude $+\xi$ radians, resulting in the superscript XI coordinates as follows:

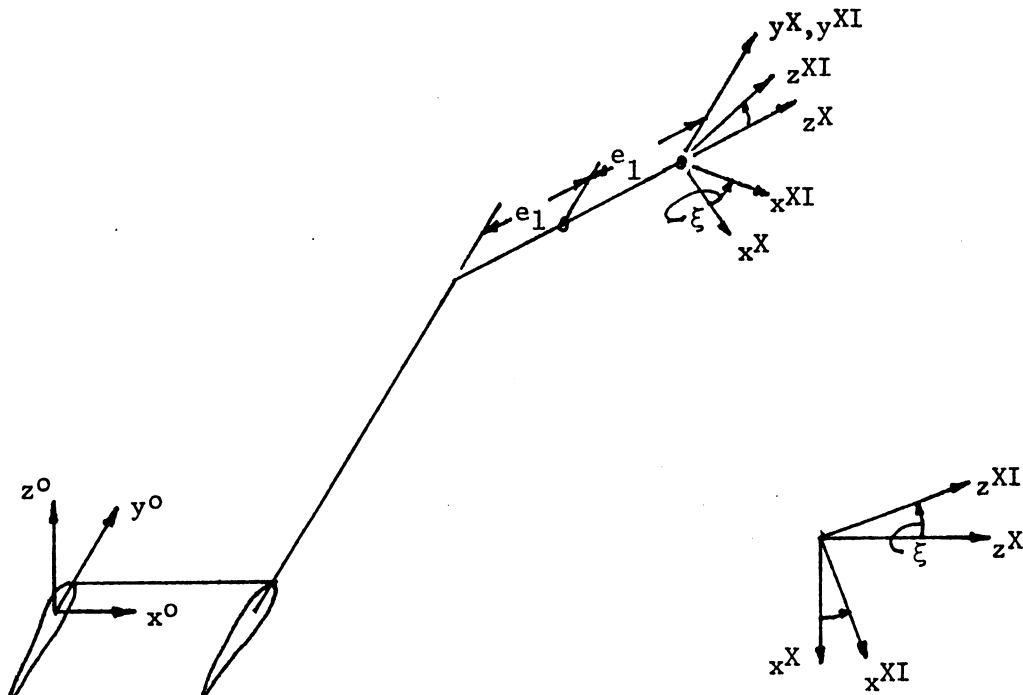


Figure 8.- Sketch of the ξ rotation.

$$\begin{aligned}
 \begin{Bmatrix} x^{XI} \\ y^{XI} \\ z^{XI} \end{Bmatrix} &= \begin{bmatrix} \xi_C & 0 & \xi_S \\ 0 & 1 & 0 \\ -\xi_S & 0 & \xi_C \end{bmatrix} \begin{Bmatrix} x^X \\ y^X \\ z^X \end{Bmatrix} \\
 &= \begin{bmatrix} \xi \end{bmatrix} \begin{Bmatrix} x^X \\ y^X \\ z^X \end{Bmatrix} \tag{18}
 \end{aligned}$$

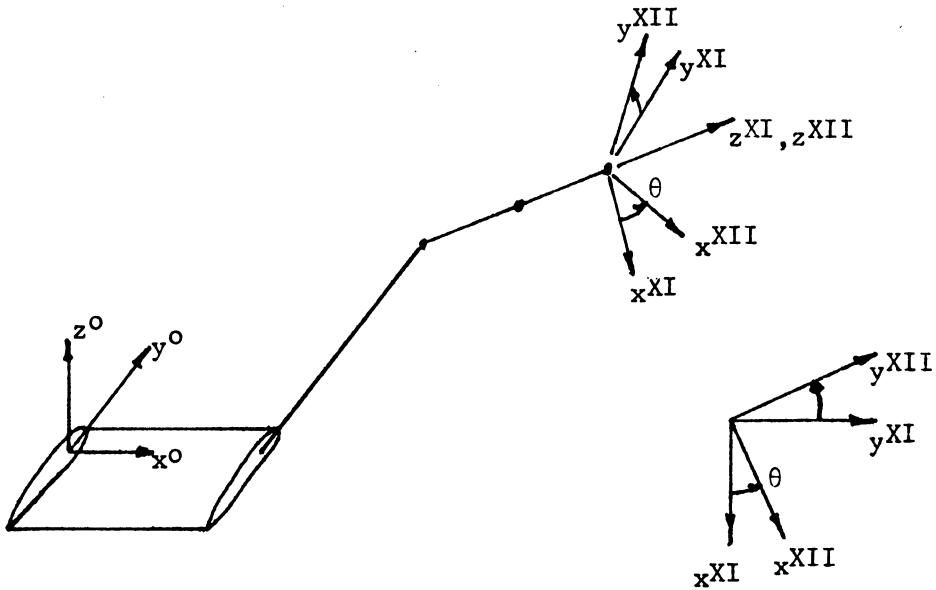
$$\begin{Bmatrix} x^{XI} \\ y^{XI} \\ z^{XI} \end{Bmatrix} \rightarrow \begin{bmatrix} \xi \end{bmatrix} \rightarrow \begin{Bmatrix} x^X \\ y^X \\ z^X \end{Bmatrix}$$

where

$$\xi_C \equiv \cos \xi$$

$$\xi_S \equiv \sin \xi .$$

The final transformation represents the blade pitch angle change. This transformation is an angular rotation about the t^{XI} -axis of an amount θ radians, resulting in the coordinate system x^{XII} , y^{XII} , and z^{XII} as follows:

Figure 9.- Sketch of the θ rotation.

$$\begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix} = \begin{bmatrix} \theta_C & \theta_S & 0 \\ -\theta_S & \theta_C & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} x^{XI} \\ y^{XI} \\ z^{XI} \end{Bmatrix}$$

$$= [\theta] \begin{Bmatrix} x^{XI} \\ y^{XI} \\ z^{XI} \end{Bmatrix}$$

(19)

$$\begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix} \rightarrow [\theta] \rightarrow \begin{Bmatrix} x^{XI} \\ y^{XI} \\ z^{XI} \end{Bmatrix}$$

where

$$\theta_C \equiv \cos \theta$$

$$\theta_S \equiv \sin \theta$$

By using the orthogonal properties of the θ matrix, equation (19) becomes

$$\begin{Bmatrix} x^{XI} \\ y^{XI} \\ z^{XI} \end{Bmatrix} = [\theta]^T \begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix} \quad (20)$$

The blade is assumed to lie along the z^{XII} -axis a distance r with a mass distribution represented by m_1 . Substituting r for the z^{XII} coordinate into equation (20) and transforming to inertial coordinates by using equations (18), (16), (12), (9), and (5), or using the transformation summary in figure 11, results in

$$\begin{Bmatrix} x^O \\ y^O \\ z^O \end{Bmatrix} = [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ r \end{Bmatrix} \quad (21)$$

The total displacement to any point on the blade is obtained by adding the displacement of the individual members. This addition must obviously be performed in the same coordinate system. Performing this addition in the inertial coordinate system (i.e., adding together equations (6),

(11), (14), (17), and (21)) results in the blade tip displacement of

$$\begin{aligned}
 \begin{Bmatrix} x^0 \\ y^0 \\ z^0 \end{Bmatrix} &= [\bar{\theta}_{i0}]^T \begin{Bmatrix} 0 \\ 0 \\ L \end{Bmatrix} + [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T \begin{Bmatrix} 0 \\ \ell \\ 0 \end{Bmatrix} + [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1 \end{Bmatrix} \\
 &+ [\bar{\theta}_{i1}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2 \end{Bmatrix} \\
 &+ [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ r \end{Bmatrix}
 \end{aligned} \tag{22}$$

where this equations corresponds to equation (A1) in the appendix. Note that the zero that signifies inertial coordinates has been omitted in the appendix. This equation (i.e. A1) is nondimensionalized by dividing all lengths by the total radius R , defined as the distance from the center of the rotor rotation to the blade tip when the blade lag and flap angles are set equal to zero.

The displacement in inertial coordinates of any member in the mechanical system can be determined from equation (22) by including the displacement of all members up to the member in question. For example, for the end point of member e_2 , omit the last term in equation (22), and for member ℓ omit the last three terms, namely, terms containing e_1 , e_2 , and r . Further, the position of any point on any member can be

determined by letting that member have a variable length. An additional subscript "p" is introduced to represent a variable length for any member; that is, e_{1p} represents member e_1 where $0 \leq e_{1p} \leq e_1$. Thus, the total displacement of any point along member e_1 is defined with the use of equation (22) as

$$\begin{Bmatrix} x^0 \\ y^0 \\ z^0 \end{Bmatrix} = [\theta_{i0}]^T \begin{Bmatrix} 0 \\ 0 \\ L \end{Bmatrix} + [\theta_{i0}]^T [\theta_{i1}]^T \begin{Bmatrix} 0 \\ \ell \\ 0 \end{Bmatrix} \\ + [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p} \end{Bmatrix}$$

From the above discussion it is seen that the displacement of any point in the mechanical system is defined w.r.t. inertial coordinates. The absolute velocities and accelerations due to the kinematics of the mechanical system can be determined by differentiating the displacement equation w.r.t. time. This differentiation is carried out in appendix A. equations (A7) and (A8), where the time variable has been nondimensionalized by multiplying the time variable by rotor rotational speed Ω in radians per second. In the following, the rotor rotational speed is assumed constant. Also, since $\frac{1}{\Omega} \frac{d}{dt} = \frac{dt}{d\psi} \frac{d}{dt} = \frac{d}{d\psi}$ nondimensionalizing time is equivalent to differentiation w.r.t. the blade azimuth angle ψ ,

this is indicated by the double bar over the variable in the acceleration equation (A9).

The elemental inertial forces are determined from the product of the acceleration and the elemental mass. The total inertial force is the integral of the elemental forces. In determining the inertial forces, the elemental mass dm_i is nondimensionalized by dividing by $\pi R^3 \rho$, where ρ is the fluid mass density. This operation results in a nondimensionalized inertial force coefficient, as follows:

$$\begin{aligned} \frac{dF_i}{\pi R^2 \rho (\Omega R)^2} &= \frac{a}{\Omega^2 R} \cdot \frac{dm_i}{\pi R^3 \rho} \\ &= \frac{a}{\Omega^2 R} \left[\frac{\mu_i}{\pi R^2 \rho} \cdot \frac{dr}{R} \right] \end{aligned}$$

where

dF_i = elemental force

a = accelerations

dm_i = elemental mass = $\frac{\mu_i}{\pi R^2 \rho} \cdot \frac{dr}{R}$

The blade inertial coefficient in inertial coordinates is determined by integrating the product of elemental mass and acceleration over the length of the blade as follows.

$$\begin{Bmatrix} C_{IFx^o} \\ C_{IFy^o} \\ C_{IFz^o} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^1 \left(1 - \frac{e_1}{R} - \frac{e_2}{R} \right) \mu_1 \left(\frac{z^{XII}}{R} \right) \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x^o/R \\ y^o/R \\ z^o/R \end{Bmatrix} \right\} d \left(\frac{z^p}{R} \right) \quad (23)$$

where the blade mass parameter μ_1 is shown as a function of the blade position z^{XIII}/R and the subscripts on the force coefficient "C" represents inertial force in inertial coordinates, respectively. For example, in the subscript for C_{IFx^o} , the IF signifies inertial force, the X signifies the X component and the o signifies the inertial coordinates. An understanding of the symbolism will aid in following the remaining derivation. Equation (23) is also shown in greater detail in the appendix A equation (A12). Of course, these forces can be transformed from the inertial coordinate system to any desired coordinate system by applying the proper transformation. (See the transformation summary figure 11.)

The blade inertial moments are determined by the integration of the vector cross product of a moment arm and elemental inertial force. The moment arm is measured from the hinge point and in the same coordinates as the inertial forces. For example, the nondimensional inertial moments of the blade about the lag hinge are

$$\begin{aligned}
\begin{Bmatrix} C_{IMx^o} \\ C_{IMy^o} \\ C_{IMz^o} \end{Bmatrix} &= \frac{1}{\pi R^2 \rho} \int_0^1 \left(1 - \frac{e_1}{R} - \frac{e_2}{R} \right) \begin{Bmatrix} x^o/R \\ y^o/R \\ z^o/R \end{Bmatrix}_{\text{BLADE}} \times \\
&\quad \mu_1 \left(\frac{z^{\text{XII}}}{R} \right) \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x^o/R \\ y^o/R \\ z^o/R \end{Bmatrix} \right\} d \left(\frac{z^{\text{XII}}}{R} \right) \quad (24)
\end{aligned}$$

where C_{IMi^o} is the moment coefficient and in the subscript IM signifies inertial moments and i^o signifies the i component in inertial coordinates. The moments in equation (24) have been nondimensionalized by dividing the moment by $\pi R^2 \rho (\Omega R)^2 R$. Equation (24) is given in greater detail in appendix A, equation (A16). The inertial forces and moments of the remaining members of the mechanical system are derived following a similar procedure as indicated above, and the details are presented in appendix A.

In this section, the mechanical system was described as made up of a number of rigid members with various degrees of freedom at the connecting joints. The system represents a wing, rotor drive shaft, and a rotating blade with flapping and lagging hinges. The inertial coordinates of the various mechanical system members were derived as a function of the various degrees of freedom. By differentiating these coordinates, the velocities and accelerations were obtained. The inertial forces were obtained by integrating the product of the elemental mass and acceleration along the member. Also, the inertial moments were

obtained by integrating the vector cross product of a moment arm and inertial force. All the equations were nondimensionalized with appropriate parameters. The definition of the aerodynamic system and the derivation of the aerodynamic force and moment equations are presented in Section 2.2. The combining of the force and moment equations to obtain the equations of motion is presented in Section 2.3.

2.2 Aerodynamic System

In this section the general procedure for including the aerodynamic forces and moments in the whirl stability problem is described; more specific details are given in appendix B. Only the aerodynamic effects on the rotating blade are considered in this dissertation; that is, aerodynamic effects on the wing are neglected.

The approach is to determine the relative fluid-blade velocity at each radial blade station and to use force coefficients and airfoil physical properties to determine the elemental blade forces. These elemental forces are integrated over the blade span to determine the total blade forces. The aerodynamic moments are determined from the integration of the vector cross product of a moment arm and the elemental aerodynamic forces. This procedure is called a strip analysis. The force coefficients, or lift and drag coefficients used in this paper, include the effects of blade stall and compressibility.

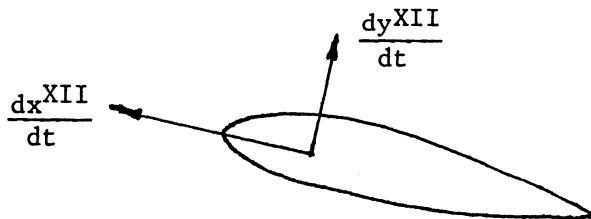
In the mechanical system, Section 2.1, the inertia coordinates of any point on the blade were presented (eq. (22)). By differentiating this equation, the velocities were determined (eq. (A7)). These

velocities originate from the kinematics of the mechanical system.

The steady-state velocities of the inertia coordinates (i.e., the flight velocities U , V , and W) must be added to the kinematic velocities in order to obtain the total velocity. (See equation (B1).) By applying the proper transformations (refer to the transformation summary figure 11), these velocities are converted to velocity components in the blade coordinates as follows:

$$\frac{1}{\Omega} \frac{d}{dt} \begin{Bmatrix} \frac{x^{XII}}{R} \\ \frac{y^{XII}}{R} \\ \frac{z^{XII}}{R} \end{Bmatrix} = [\theta][\xi][\beta][\psi] \begin{bmatrix} \theta_{i1} \\ \theta_{i0} \end{bmatrix} \begin{Bmatrix} \frac{U}{\Omega R} \\ \frac{V}{\Omega R} \\ \frac{W}{\Omega R} \end{Bmatrix} + \frac{1}{\Omega} \frac{d}{dt} \begin{Bmatrix} \frac{x^O}{R} \\ \frac{y^O}{R} \\ \frac{z^O}{R} \end{Bmatrix}$$

On a blade section, these velocity components will appear as indicated on the following sketch:



where the component of velocity along the blade span axis (z^{XII}) has been neglected. The blade section angle of attack becomes

$$\alpha = -\arctan \left\{ \frac{\frac{d}{dt} \left(\frac{y^{XII}}{R} \right)}{\frac{d}{dt} \left(\frac{x^{XII}}{R} \right)} \right\} - \theta_1 \left(\frac{z^{XII}}{R} \right) \quad (25)$$

where θ_1 is the blade twist angle from the blade root ($z^{XII}/R = 0$) to the blade tip ($z^{XII}/R = 1.0 - e_1/R - e_2/R$). The local blade total velocity is determined from the square root of the sum of the squares of $d/dt(x^{XII}/R)$ and $d/dt(y^{XII}/R)$. From this velocity, the blade section Mach number is determined as follows:

$$\text{Mach Number} = \frac{\sqrt{\left[\frac{d}{dt} \left(\frac{x^{XII}}{R} \right) \right]^2 + \left[\frac{d}{dt} \left(\frac{y^{XII}}{R} \right) \right]^2}}{(\text{speed of sound})} \quad (26)$$

From the type of geometric blade section chosen and the local angle of attack (eq. (25)) and Mach number (eq. (26)), two-dimensional force coefficients can be determined. The force coefficients used herein were determined experimentally [9,10, and 11] and are tabulated in tables 1 and 2, appendix D. The force coefficients (C_i) are used to determine the elemental blade section forces as follows:

$$dF_A = \frac{1}{2} \rho V^2 C_i c dr$$

where the definition of these various terms is presented in appendix B, equation (B7). These force coefficients are in the coordinates of the

total local blade velocity vector. Resolving these elemental forces into blade coordinates (superscript XII), integrating and nondimensionalizing the force by dividing by $\pi R^2 \rho (\Omega R)^2$ results in the following nondimensionalized aerodynamic forces:

$$C_{ATy}^{XII} = \sigma' \left\{ \int_{x_c}^B \left(1 - \frac{e_1}{R} - \frac{e_2}{R}\right) \left(\frac{V}{\Omega R}\right)^2 C_L \cos \phi \, d\left(\frac{z_p^{XII}}{R}\right) - \int_0^{\left(1 - \frac{e_1}{R} - \frac{e_2}{R}\right)} \left(\frac{V}{\Omega R}\right)^2 C_D \sin \phi \, d\left(\frac{z_p^{XII}}{R}\right) \right\} \quad (27)$$

and

$$C_{AQx}^{XII} = \sigma' \left\{ - \int_{x_c}^B \left(1 - \frac{e_1}{R} - \frac{e_2}{R}\right) \left(\frac{V}{\Omega R}\right)^2 C_L \sin \phi \, d\left(\frac{z_p^{XII}}{R}\right) - \int_0^{\left(1 - \frac{e_1}{R} - \frac{e_2}{R}\right)} \left(\frac{V}{\Omega R}\right)^2 C_D \cos \phi \, d\left(\frac{z_p^{XII}}{R}\right) \right\}$$

where

ϕ = angle between the x^{XII} blade coordinate and the total velocity vector

C_L, C_D = lift and drag force coefficients, respectively

$$\sigma' = \frac{1}{2} \frac{c}{\pi R}$$

c = blade chord

This equation corresponds to equation (B12) in appendix B. The subscript on the force coefficients AT_y^{XII} signify aerodynamic thrust in the y^{XII} coordinate and AQ_x^{XII} signifies aerodynamic drag in the x^{XII} coordinate. The moments caused by the aerodynamic forces are determined by the integration of the vector cross product of the elemental force and the moment arm (z_p^{XII} / R). This result is presented in equation (B13) where the moments have been nondimensionalized by dividing by $\pi R^2 \rho (\Omega R)^2$. With the proper transformations (see fig. 11), these aerodynamic forces and moments can be transformed into any coordinate system. Also, with the proper transfer equations, these forces and moments can be transferred to any point in the system.

In addition to the above derivation, provisions were incorporated to include a cyclic variation in the blade pitch angle, θ , and a non-uniform azimuth induced inflow (constant spanwise) effect. This deviation is presented in appendix B. Variations of these parameters were not considered in this dissertation. These variables are mentioned because they are included in the derivation and they are available for further investigation.

In this section, the method used to determine the aerodynamic forces and moments is described. The approach is to determine the relative velocity between the airflow and the blade at each radial blade station and to use two-dimensional quasi-steady airfoil force coefficients and the airfoil geometry to determine elemental blade forces. The aerodynamic moments are determined from the resultant cross product of

the moment arm and the elemental aerodynamic forces. The total forces and moments are determined by integration over an area or moment arm, respectively. The airfoil force coefficients include both stall and Mach number effects. In the next section, the aerodynamics and inertia forces and moments will be combined to obtain the differential equations of motion.

2.3 Differential Equations of Motion

This section describes the general procedures used in deriving the equations of motion, the details are presented in appendix C. The approach used is to set the sum of the external moments across each hinge equal to the inertial moments. The external moments consist of aerodynamic, spring, and damper moments. Each moment equation will result in one second-order ordinary differential equation.

The first equation of motion is obtained by setting the external moments across the blade lag hinge equal to the inertial moments. The blade inertial moments are defined in equations (24) or (A12). These moments are in inertial coordinates and must be transformed into lag hinge coordinates, namely, those with superscript XI. By using the transformation summary figure 11, this transformation follows:

$$\begin{Bmatrix} C_{IMxXI} \\ C_{IMyXI} \\ C_{IMzXI} \end{Bmatrix} = [\xi][\beta][\psi] \begin{bmatrix} \theta_{i1} \\ \theta_{i0} \end{bmatrix} \begin{Bmatrix} C_{IMx0} \\ C_{IMy0} \\ C_{IMz0} \end{Bmatrix}$$

The aerodynamic moments are defined in equation (B13); these moments are in superscript XII-coordinates. The transformation to the lag hinge coordinates (see fig. 11) follows:

$$\begin{Bmatrix} C_{AMx}^{XI} \\ C_{AMy}^{XI} \\ C_{AMz}^{XI} \end{Bmatrix} = [\theta]^T \begin{Bmatrix} C_{AMx}^{XII} \\ C_{AMy}^{XII} \\ C_{AMz}^{XII} \end{Bmatrix}$$

Since the blade lag hinge is aligned with the y^{XI} -axis, all external moments about this hinge must be set equal to the inertial moments.

Letting K_ξ and C_ξ represent the spring and damper constant, respectively, across the lag hinge and equating the sum of the moments across this hinge to zero results in

$$\begin{Bmatrix} C_{AMx}^{XI} \\ C_{AMy}^{XI} \\ C_{AMz}^{XI} \end{Bmatrix} \quad \text{y component only} \quad - \quad \begin{Bmatrix} C_{IMx}^{XI} \\ C_{IMy}^{XI} \\ C_{IMz}^{XI} \end{Bmatrix} \quad \text{y component only} \quad (28)$$

$$- K_\xi \xi - C_\xi \left(\frac{1}{\Omega} \frac{d\xi}{dt} \right) = 0$$

The spring and damper moment terms are negative because they are resisting moments. Because the aerodynamic and inertial moments in equation (28) have been nondimensionalized, the engineering units for the spring and damper constants

are divided by $\pi R^2 \rho (\Omega R)^2 R$ before putting them into equation (28). Equation (28) which is identical to equation (C2), is expanded in greater detail in equation (C3).

Equation (28) is applicable to any blade in the rotor system (e.g., see fig. 10). For other blades, the blade azimuth parameter, ψ , must be modified to reflect the proper blade position. One blade is designated the master blade and is defined by ψ_0 ; all of the other blades are defined by equally spaced azimuth angles w.r.t. the master blade. In this dissertation, provisions are made for two, three, or four blades. Since the lag motions for each blade are independent, there will be a lag degree of freedom and a lag differential equation for each blade, namely, equation (28) with the appropriate ψ value.

The second equation of motion is obtained by equating the external moments about the blade flapping hinge to the inertial moments. The moments consist of (1) the moments transferred across the lag hinge, (2) the moments associated with the transfer of the shear forces at the lag hinge to the flap hinge, and (3) the inertial moments associated with mass and motion of member e_2 .

From equation (28) it is seen that the x- and z-components of the moments are transferred across the lag hinge without being affected by the lag hinge spring or damper. The only moments transmitted across the lag hinge are:

$$\begin{Bmatrix} C_{TRMx}^{XI} \\ C_{TRMy}^{XI} \\ C_{TRMz}^{XI} \end{Bmatrix} = \begin{Bmatrix} C_{AMx}^{XI} - C_{IMx}^{XI} \\ K_{\xi}\xi + C_{\xi}\bar{\xi} \\ C_{AMz}^{XI} - C_{IMz}^{XI} \end{Bmatrix}^*$$

where

$$\bar{\xi} = \frac{d\xi}{d\psi}$$

where the subscript $TRMi^{XI}$ signifies transmitted moment in the i -direction of the XI -coordinate system. These moments are transformed into the flapping hinge coordinates (superscript IX) as follows:

$$\begin{Bmatrix} C_{TRMx}^{IX} \\ C_{TRMy}^{IX} \\ C_{TRMz}^{IX} \end{Bmatrix} = [\xi]^T \begin{Bmatrix} C_{AMx}^{XI} - C_{IMx}^{XI} \\ K_{\xi}\xi + C_{\xi}\bar{\xi} \\ C_{AMz}^{XI} - C_{IMz}^{XI} \end{Bmatrix} \quad (29)$$

*Whenever the ξ degree of freedom is omitted in the computer calculations (i.e., $\xi = \bar{\xi} = 0$), the transmitted moments in the y^{XI} direction will become zero. To avoid this problem during computations, this form of the equation is replaced with the following:

$$\begin{Bmatrix} C_{TRMx}^{XI} \\ C_{TRMy}^{XI} \\ C_{TRMz}^{XI} \end{Bmatrix} = \begin{Bmatrix} C_{AMx}^{XI} - C_{IMx}^{XI} \\ C_{AMy}^{XI} - C_{IMy}^{XI} \\ C_{AMz}^{XI} - C_{IMz}^{XI} \end{Bmatrix}$$

The second flapping moment component is the moment associated with the transfer of the blade root or lag hinge shear forces. These moment components are determined by summing the blade aerodynamic forces (eq. (27)) and the inertial forces (eq. (23)) as follows:

$$\begin{Bmatrix} C_{AIFx}^{XII} \\ C_{AIFy}^{XII} \\ C_{AIRz}^{XII} \end{Bmatrix} = \begin{Bmatrix} C_{AOx}^{XII} \\ C_{ATy}^{XII} \\ 0 \end{Bmatrix} - \begin{Bmatrix} C_{IFx}^{XII} \\ C_{IFy}^{XII} \\ C_{IFz}^{XII} \end{Bmatrix} \quad (30)$$

where the subscript on the left-hand term $AIFi^{XII}$ signifies aerodynamic and inertial forces in the appropriate direction (i.e., x, y, or z) in the XII-coordinate system. Transforming equation (30) into the flap hinge coordinates (IX) by using the summary transformation, figure 11, results in

$$\begin{Bmatrix} C_{AIFx}^{IX} \\ C_{AIFy}^{IX} \\ C_{AIFz}^{IX} \end{Bmatrix} = [\xi]^T [\theta]^T \begin{Bmatrix} C_{AIFx}^{XII} \\ C_{AIFy}^{XII} \\ C_{AIFz}^{XII} \end{Bmatrix} \quad (31)$$

This transformed shear force will produce a flapping hinge moment that is equal to the vector cross product of the moment arm and the shear force as follows:

$$\begin{Bmatrix} C_{AIMx}^{IX} \\ C_{AIMy}^{IX} \\ C_{AIMz}^{IX} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \frac{e_2}{R} \end{Bmatrix} \times \begin{Bmatrix} C_{AIFx}^{IX} \\ C_{AIFy}^{IX} \\ C_{AIFz}^{IX} \end{Bmatrix} \quad (32)$$

The third flapping moment component originates from the inertial moments of member e_2 with a mass distribution m_2 . This moment is obtained by a method similar to that used in finding the blade inertial moments (see eq. (24)). The details of the derivation are presented in appendix A, equations (A23) or (A25), for the moments in inertial coordinates and equation (A27) for the moments in flapping hinge coordinates.

The inertial moments of member e_2 in flapping hinge coordinates are designated by

$$\begin{Bmatrix} C_{IMe2x}^{IX} \\ C_{IMe2y}^{IX} \\ C_{IMe2z}^{IX} \end{Bmatrix} \quad (33)$$

Combining the three flapping hinge moments, equations (29), (32), and (33) result in the total flapping hinge moments.

$$\begin{Bmatrix} C_{TMx}^{IX} \\ C_{TM_y}^{IX} \\ C_{TMz}^{IX} \end{Bmatrix} = \begin{Bmatrix} C_{TRMx}^{IX} \\ C_{TRMy}^{IX} \\ C_{TRMz}^{IX} \end{Bmatrix} + \begin{Bmatrix} C_{AIMx}^{IX} \\ C_{AIM_y}^{IX} \\ C_{AIMz}^{IX} \end{Bmatrix} - \begin{Bmatrix} C_{IMe2x}^{IX} \\ C_{IMe2y}^{IX} \\ C_{IMe2z}^{IX} \end{Bmatrix} \quad (34)$$

Because the flapping hinge is aligned with the x^{IX} -axis, the x^{IX} component of the total moments (eq. (34)) is equated to the flap spring and damper moment to obtain the second equation of motion. This results in the following:

$$C_{TMxIX} - K_{\beta}\beta - C_{\beta}\bar{\beta} = 0 \quad (35)$$

where $\bar{\beta} = d\beta/d\psi$ and the engineering units of the spring and damper constants have again been nondimensionalized. As with the differential equation governing the blade lagging motion (eq. (28)), the flapping equation (35) is applicable to any blade in the rotor system with the appropriate blade azimuth angle ψ . Thus, there will be one blade flapping differential equation for each blade.

The third basic equation of motion is obtained by equating external moments to the inertial moments about the three hinges located at the drive-shaft-wing joint, that is, between members ℓ and L . (See figure 10.) These moments originate from numerous sources which are listed below:

(1) Moments that are transferred across the blade flapping hinge including the blade flapping spring and damper, (see equation (35)).

$$\left\{ \begin{array}{c} K_{\beta}\beta + C_{\beta}\bar{\beta} \\ C_{TMxIX} \\ C_{TMzIX} \end{array} \right\}^*$$

*As with the lag equation, whenever the β degree of freedom is omitted in the calculations, this form of the equation is replaced with the following:

$$\left\{ \begin{array}{c} C_{TMxIX} \\ C_{TMyIX} \\ C_{TMzIX} \end{array} \right\}$$

(2) Moments due to the aerodynamic and inertial shear forces acting at the blade flapping hinge. (See equation (C15).)

(3) Moments due to the inertial shears of member e_1 times the moment arm l . (See equation (C16).)

(4) Inertial moments of member e_1 due to its mass distribution. (See equation (A35).)

(5) Inertial moments due to the mass distribution of the drive shaft (member l). (See equation (A45).)

(6) Spring and damper moments across the hinge between the drive shaft and the wing joint, members l and L .

The first two of these items originate from each blade and will, in general, be different for each blade. The first four of these items must be summed over the number of blades. The detailed derivation of these equations of motion is not presented here (see appendix C).

Making use of the subscript terminology, the moments transferred across the flapping hinge for all the blades become

$$\sum_{\text{BLADES}} \begin{Bmatrix} C_{TMx}^{VI} \\ C_{TM_y}^{VI} \\ C_{TMz}^{VI} \end{Bmatrix} = \sum_{\text{BLADES}} [\psi]^T [\beta]^T \begin{Bmatrix} K_\beta \beta + C_\beta \dot{\beta} \\ C_{TM_y}^{IX} \\ C_{TMz}^{IX} \end{Bmatrix} \quad (36)$$

The aerodynamic and inertial shear forces acting at the flapping hinge are

$$\begin{Bmatrix} C_{Fx}^{VIII} \\ C_{Fy}^{VIII} \\ C_{Fz}^{VIII} \end{Bmatrix} = [\beta]^T \begin{Bmatrix} C_{AIFx}^{IX} \\ C_{AIFy}^{IX} \\ C_{AIFz}^{IX} \end{Bmatrix} - \begin{Bmatrix} C_{Ie2x}^{VIII} \\ C_{Ie2y}^{VIII} \\ C_{Ie2z}^{VIII} \end{Bmatrix}$$

and the moment is the vector cross product of the moment arm and shear force summed over all of the blades resulting in

$$\sum_{\text{BLADES}} \begin{Bmatrix} C_{FMx}^{VI} \\ C_{FMy}^{VI} \\ C_{FMz}^{VI} \end{Bmatrix} = \sum_{\text{BLADES}} \left\{ \begin{Bmatrix} 0 \\ \frac{l}{R} \\ 0 \end{Bmatrix} + [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ \frac{e_1}{R} \end{Bmatrix} \right\} \times \begin{Bmatrix} C_{Fx}^{VIII} \\ C_{Fy}^{VIII} \\ C_{Fz}^{VIII} \end{Bmatrix} \quad (37)$$

The moments due to the inertial shears of member e_1 acting at the end of the drive shaft, when summed over the number of blades, are

$$\sum_{\text{BLADES}} \begin{Bmatrix} 0 \\ \frac{l}{R} \\ 0 \end{Bmatrix} \times \begin{Bmatrix} C_{Ielx}^{VI} \\ C_{Iely}^{VI} \\ C_{Ielz}^{VI} \end{Bmatrix} \quad (38)$$

and the inertial moments from e_1 acting at the end of the drive shaft and defined by equation (A35) are

$$\sum_{\text{BLADES}} \begin{Bmatrix} C_{Melx}^{VI} \\ C_{Mely}^{VI} \\ C_{Melz}^{VI} \end{Bmatrix} \quad (39)$$

The inertial moments of member l (mass m_4) from equation (A45) are

$$\begin{Bmatrix} C_{Mlx}^{VI} \\ C_{Mly}^{VI} \\ C_{Mlz}^{VI} \end{Bmatrix} = \begin{bmatrix} \theta_{11} & \theta_{10} \end{bmatrix} \begin{Bmatrix} C_{Mlx} \\ C_{Mly} \\ C_{Mlz} \end{Bmatrix} \quad (40)$$

Combining all these moments, equations (36), (37), (38), (39), and (40), redefining the sum over all the blades as

$$\begin{Bmatrix} C_{MxVI} \\ C_{MyVI} \\ C_{MzVI} \end{Bmatrix}$$

and equating the z-component of this moment to the spring and damper moment across the z^{VI} hinge results in the equation of motion for the θ_{z1} degree of freedom

$$C_{MzVI} - K_{z1}\theta_{z1} - C_{z1}\bar{\theta}_{z1} = 0 \quad (41)$$

The moments transferred across this hinge are transformed into the IV-coordinate system. The y-component of this moment is equated to the y^{IV} spring and damper moment resulting in the equation of motion for the θ_{y1} degree of freedom

$$\theta_{z1s}C_{MxVI} + \theta_{z1c}C_{MyVI} - K_{y1}\theta_{y1} - C_{y1}\bar{\theta}_{y1} = 0 \quad (42)$$

The moment transferred across this hinge is transformed into the III-coordinate system and the X^{III} -component is equated to the corresponding spring and damper moment resulting in

$$\begin{aligned} \theta_{y1c}(\theta_{z1c}C_{MxVI} - \theta_{z1s}C_{MyVI}) + \theta_{y1s}(K_{z1}\theta_{z1} + C_{z1}\bar{\theta}_{z1}) \\ - K_{x1}\theta_{x1} - C_{x1}\bar{\theta}_{x1} = 0 \end{aligned} \quad (43)$$

The governing equation of motion for the remaining degrees of freedom, namely, θ_{z0} , θ_{y0} , and θ_{x0} , are derived in appendix C. These degrees of freedom were not utilized in determining the results in this dissertation, but they are available in the theory and the computer program for future investigation.

In this section, the governing equations of motion were derived. Each blade of the rotor system has two degrees of freedom, namely, β and ξ , and associated with each blade there are two governing equations of motion (28) and (35). In addition, there are three degrees of shaft freedom, θ_{z1} , θ_{y1} , and θ_{x1} , the governing equations of which are presented in equations (41), (42), and (43), respectively. The governing equations of motion are coupled second-order ordinary differential equations and they must be solved simultaneously.

III. METHOD OF SOLUTION AND COMPUTER PROGRAM

This chapter presents the general method of solution and a general discussion of the computer program characteristics. The basic equations to be solved are simultaneous second-order nonlinear ordinary differential equations. There are two degrees of freedom for each rotor blade plus three degrees of freedom at the drive-shaft-wing joint and three degrees of freedom at the wing root. For example, for a three-blade rotor system there are a total of 12 degrees of freedom. Provisions are available for locking out (i.e., eliminating) any of the degrees of freedom when solving the equations.

3.1 General Method of Solution

The governing equations of motion are symbolized by letting the capital Roman letters represent the time-varying coefficients of the X_i degrees of freedom ($0 \leq i \leq N$). Dots are used to represent differentiation w.r.t. time. The general form of the equations follows:

$$\begin{array}{ccccccc}
 \sum_{i=1}^N A_{1i}(t) \ddot{x}_i & + & \sum_{i=1}^N B_{1i}(t) \dot{x}_i & + & \sum_{i=1}^N C_{1i}(t) x_i & = & \sum_{i=1}^N D_{1i}(t) \\
 \sum_{i=1}^N A_{2i}(t) \ddot{x}_i & + & \sum_{i=1}^N B_{2i}(t) \dot{x}_i & + & \cdot & \cdot & \cdot & = & \sum_{i=1}^N D_{2i}(t) \\
 \cdot & & \cdot & & & & \cdot & & \cdot \\
 \cdot & & \cdot & & & & \cdot & & \cdot \\
 \cdot & & \cdot & & & & \cdot & & \cdot \\
 \sum_{i=1}^N A_{Ni}(t) \ddot{x}_i & + & \sum_{i=1}^N B_{Ni}(t) \dot{x}_i & + & \sum_{i=1}^N C_{Ni}(t) x_i & = & \sum_{i=1}^N D_{Ni}(t)
 \end{array}$$

Each of these second-order equations is reduced to two first-order equations by introducing a change in variables as follows:

$$A\ddot{x} + B\dot{x} + Cx = D$$

Let $y = \dot{x}$; therefore, $\dot{y} = \ddot{x}$, and substituting results in

$$A\dot{y} = -By - Cx + D$$

$$\dot{x} = y$$

Given the initial conditions on the right-hand side of these two equations (x and y), solve for the left-hand side. In expanded form, the complete list of first-order equations to be solved are

$$\begin{aligned} \sum_{i=1}^N A_{1i}(t)\dot{y}_i &= - \sum_{i=1}^N B_{1i}(t)y_i - \sum_{i=1}^N C_{1i}(t)x_i + \sum_{i=1}^N D_i \\ \sum_{i=1}^N A_{2i}(t)\dot{y}_i &= - \sum_{i=1}^N B_{2i}(t)y_i \cdot \cdot \cdot \cdot \\ &\vdots \\ \sum_{i=1}^N A_{Ni}(t)\dot{y}_i &= - \sum_{i=1}^N B_{Ni}(t)y_i - \sum_{i=1}^N C_{Ni}(t)x_i + \sum_{i=1}^N D_{Ni}(t) \\ \sum_{i=1}^N \dot{x}_i &= \sum_{i=1}^N y_i \end{aligned}$$

or in the matrix form

$$\begin{bmatrix} A_{11}(t) & A_{12}(t) & \cdots & A_{1N}(t) \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ & & & A_{NN}(t) \end{bmatrix} \begin{Bmatrix} \dot{y}_1 \\ \dot{y}_2 \\ \vdots \\ \dot{y}_N \end{Bmatrix} = - \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1N} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ B_{N1} & \cdots & \cdots & B_{NN} \end{bmatrix} \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{Bmatrix} \\
 - \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1N} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ & & & C_{NN} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & \cdots & D_{1N} \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ \vdots & \vdots & & \vdots \\ & & & D_{NN} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{Bmatrix} \quad (44)$$

$$\begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & & & \vdots \\ \vdots & & \vdots & & \vdots \\ \vdots & & \vdots & & \vdots \\ \vdots & & 0 & & \vdots \\ 0 & \cdots & 0 & & 1 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_N \end{Bmatrix} = [I] \begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{Bmatrix}$$

The above equations describe the general form of the equations solved in this paper. These equations are solved by the use of an electronic digital computer. A general description of the computer program is given in the next section.

3.2 Computer Program

The governing equations of motion were solved with the aid of a

digital electronic computer. This section describes the general characteristics of the computer program. A copy of the complete FORTRAN computer program is presented in appendix E.

The solution of equation (44) results in the instantaneous characteristics of the mathematical model. The characteristics at the next instant of time are determined by extrapolation. In this dissertation a fixed time interval is utilized. The integration accuracy was checked by choosing two different and constant integration time intervals and letting the computer calculate the resulting time history for approximately 10 rotor revolutions and, if the two results were within a percent or so, the constant integration time interval was judged small enough and the results accepted. Experience gained running the computer program indicated that an interval of 0.1 radian was adequate to achieve the desired accuracy.

Before making production runs on the computer, an extensive effort was expended debugging the program. The procedure consisted of taking degenerate computer input conditions (i.e., locking out various variables) and comparing the computer results with numerical hand calculations. Also, the resulting time history with these degenerate input conditions were compared with existing published data.

The computer program obtained the airfoil force coefficient characteristics from a table by a double interpolation as a function of section angle of attack and Mach number. This table was obtained from references [9,10,11] and are tabulated in appendix D, tables 1 and 2. The

airfoil characteristics have an angle-of-attack range from 0° to 360° and a Mach number range from 0 to 1.0.

The computer program requires numerous input quantities which are listed in appendix E. In addition, there are numerous default quantities or values. A default value is the value that is assumed by the computer if no input value is given. The default values greatly decrease the required number of computer input values. For example, there are default values for the airfoil characteristics (there are over 800 values tabulated in tables I and II).

Once the elemental aerodynamic and inertia forces and moments are determined, they must be integrated to find the total values. Simpson's rule for integration is used for integrating along the length of the various members in order to compute the overall forces and moments. The rotor blade is divided into 11 equal sized lengths, and the remaining members are divided into five equal sized lengths.

The computer program was written to have a very flexible and extensive output listing which is described in appendix E. In addition, there is an automatic plotting routine for plotting the results. In fact, all of the time histories in this dissertation are machine plotted.

IV. RESULTS AND DISCUSSION

Before proceeding with production runs with the computer to determine the system stability, the physical properties of the model must be chosen. In addition, the method of presentation of the results should be considered. Therefore, this chapter is presented in three sections (1) Nominal Aircraft Physical Properties, (2) Method of Presentation of the Data, and (3) Stability Characteristics From the Computer Results.

4.1 Nominal Aircraft Physical Properties

The basic rotor system used for this study is typical of current state-of-the-art design practices. In fact, a group of proposed tilt-rotor designs were tabulated and summarized to define the rotor chosen for this study. The following is a list of the assumed physical properties of the nominal aircraft.

$b = 3$ (number of blades per rotor)

$\ell = 10$ ft (drive shaft length)

$L = 20$ ft (wing span length)

$R = 20$ ft (rotor blade length)

$\gamma' = 1.4$

$\mu_1 = 0.224$ (blade mass distribution)

$\theta_1 = -18^\circ$ (blade twist from blade root to blade tip)

$\sigma = 0.09$ (rotor solidity)

In addition, the following rotor operating variables were used:

$a = 1,100$ ft per sec (speed of sound at sea level)

$V = 100$ ft per sec (forward speed of the aircraft)

$V_{\text{tip}} = 700$ ft per sec (rotor blade rotational tip speed)

$\rho = 0.002378$ lb-sec² per ft⁴ (air mass density)

$\Omega = 35.0$ radians per second (rotor rotational angular velocity)

A detailed discussion of these parameters is presented in appendix D.

4.1.1 Airfoil Characteristics

As stated in the introduction, neglecting the mechanical viscous dampers, the aerodynamic forces and moments acting on the rotor blades are the only items that can add or subtract energy from the system. Therefore, a good aerodynamic representation is vital and, since a blade strip analysis is being used, the next step is to have a good set of airfoil characteristics. Numerous airfoil characteristics are available [8]; however, [8] contains little information about Mach number effects and no data on reverse flow characteristics. Reference [9] contains reverse flow effects and [10,11] have an extensive amount of Mach number effects data for a modified NACA 0012 airfoil. In this study, a combination of data [9,10,11] were faired together and the results are presented in tabular form in tables 1 and 2, appendix D.

4.1.2 Rotor Performance

With the rotor blade physical and operating characteristics defined, the rotor system aerodynamic performance was determined. The rotor system performance is presented in figures 12 and 13 as a function of the rotor thrust and torque characteristics. These curves were obtained by varying the blade root pitch angle θ_{0s} as a computer input, and letting the computer determine the resulting net rotor system thrust and torque. The purpose of this procedure was to determine a nominal operating blade pitch angle. From these curves (figs. 12 and 13) a nominal value of 35° was chosen.

4.2 Method of Presentation of the Results

In the presentation of the results, it is desirable to have the results in a form that is easy to interpret and such that it can be correlated with other published data. In this paper a natural nominal frequency is the basic parameter. This frequency is defined as the undamped single-degree-of-freedom natural frequency. The nominal shaft natural frequency is the frequency with one degree of shaft freedom only, with nonrotating hingeless blades (no β and ξ hinges) and without damping in the system (no viscous dampers or aerodynamics). The rotor blade nominal flapping natural frequency is the flapping frequency of the rotating blade with flapping freedom only ($\xi = 0$) and without damping (i.e., $K_\beta = 0$, no aerodynamics). These nominal frequencies are used as reference values. The actual natural frequency values may be quite different than the reference values because of coupling (gyroscopics) or damping in the system. These reference frequencies were determined both algebraically and numerically with the use of the computer program and the resulting correlation was excellent. In the presentation of the results, the reference natural frequencies are non-dimensionalized by dividing by the rotor rotational speed, Ω .

The rotor shaft and blade flap damping values are presented as a fraction of their respective critical damping value. The critical damping was determined for a single-degree-of-freedom system with viscous damping only (no aerodynamics). The critical damping value was determined numerically with the use of the computer program by determining the ratio of two consecutive maximum values. The critical damping

value was also determined algebraically. The results determined by the two different methods were correlated with good results.

The results presented in this dissertation are restricted to two degrees of shaft freedom, pitch and yaw, and includes blade flapping motion only. The remaining computer program capability has been debugged but not utilized for the presentation herein.

The solution of the equation of motion, through the use of the computer, results in a time history of the system. This time history is reviewed to determine the type stability. An initial disturbance of 20° shaft pitch is incorporated into the system to encourage a more rapid convergence or divergence time history and thus conserve electronic computer time.

The time history is plotted and visually reviewed for convergence or divergence, assume the result is convergent. One variable is then changed and this case is given to the computer for computations, plotting, and finally a visual review. This process is repeated until a divergent condition is obtained. Therefore, the stability boundary lies between the convergent and divergent cases. Refinement in the stability boundary is achieved by repeating the above procedure with smaller variations in the appropriate variable.

4.3 Stability Characteristics from the Computer Results

This section of the discussion deals with the results of the numerical solution of the equations of motion. The results are presented both as stability boundaries and as motion time history plots. The stability boundary is presented for variations in rotor shaft and blade

flapping natural frequency ratios. From this, a near minimum stiffness system is defined as the basic nominal rotor. This nominal rotor is subjected to numerous operating and physical property changes. The resulting time histories are evaluated and discussed.

4.3.1 Stability Boundary for Axisymmetric Shaft Stiffness

Recall that the numerical solution of the equations of motion results in a time history of the motion. In order to obtain a stability boundary, time histories are obtained across each boundary and the degree of stability evaluated. Thus, each boundary represents numerous individual time histories.

The stability boundary for an axisymmetric shaft stiffness where the shaft natural frequency ratio is plotted versus the blade flapping nominal natural frequency ratio without viscous damping ($C_B = 0$) is shown in figure 14. The nominal natural frequency ratio is the uncoupled, undamped, single-degree-of-freedom small amplitude natural frequency divided by the rotor rotational speed.

From figure 14 it is seen that there is a strong stability relationship between the blade flapping and the shaft natural frequency ratios. Since increased natural frequency or stiffness is usually related to increased weight (or dollars), it can be seen that there is a minimum shaft stiffness available and achievable by the proper choice of the blade flapping frequency. Figure 14 shows the asymptotic value of shaft stiffness required for an infinitely large flapping natural frequency ratio; this condition corresponds to a system with propeller-

type blades. The natural flapping frequency ratio of a zero hinge offset blade with centrifugal restoring moment only is equal to 1.0. This system requires appreciably more shaft stiffness than a system with a flapping frequency ratio equal to about 1.1. (See figure 14.)

Also shown in figure 14 is the corresponding results of Young [31]. In general, the boundary where the flapping frequency ratio is about 1.0 is in good agreement. Reference [31] states that the mode of instability in this region is a limit orbit cycle and further that the mode of motion is a forward whirl mode. In the analysis herein, neither of these statements could be verified. This is in spite of the fact that initial conditions (i.e., direction of precession) were chosen such that the forward whirl and limit orbit cycle were sought. It is therefore concluded that the results of reference [31] are in error. It is believed that this error is due to assumptions made in [31] that were not made in the analysis herein. For example, [31] neglects aerodynamic Mach number and blade stall effects which is not done herein.

The remaining branch of the stability boundary, shown in figure 14, was a backward whirl mode. This branch is, in general, related to greater flapping velocities (increasing natural flapping frequencies) and blade section Mach numbers. The reason for the disagreement between the analysis herein and that of reference [31] is presently unexplainable. The next item of investigation was the effects of asymmetric drive shaft stiffness on the stability boundary.

4.3.2 Asymmetric Drive Shaft Stiffness

The effects of asymmetric drive shaft stiffness is shown in figure 15 for three values of blade nominal flapping frequency ratios. The stability boundary was determined in a manner identical to that described in obtaining figure 14, namely to obtain numerous time history plots and judge from the motion the stability characteristics and interpolate between these cases to determine the stability boundary.

For a blade nominal flapping frequency ratio of 1.0 and an axisymmetric shaft stiffness $\omega_{\theta x1} = \omega_{\theta z1}$, the shaft stiffness requirements for stability are quite high. This agrees with the results in figure 14 for $\omega_\beta/\Omega = 1.0$. Figure 15 shows that with a small asymmetry in shaft stiffness, the stiffness requirement for stability decreases quite rapidly and as the asymmetry increases further the stiffness requirements for stability again increase.

A propeller blade corresponds to blade flapping natural frequency approaching infinity. Figure 15 shows that for stability the largest shaft stiffness requirements occur for an axisymmetrical shaft. Further, the stiffness required for stability seems to decrease continuously as asymmetry is introduced. The general shape of this stability boundary agrees with that of Reed [18]; however, [18] does not provide a scale and, therefore, no direct comparison is possible.

For the case when the flapping natural frequency ratio equals 1.08, the minimum shaft stiffness requirement for stability occurs with an axisymmetrical shaft. Also, the shaft stiffness required increases as the shaft stiffness becomes asymmetrical.

In summary, for a blade flapping frequency ratio of one or infinity, asymmetry is favorable, whereas for a flapping frequency ratio of 1.08, axisymmetrical shaft stiffness is favorable. This result indicates the necessity for taking a complete and careful look at any system before going into the design or fabrication stage of development.

4.3.3 Time History Across the Stability Boundary

This section presents a few sample time histories of the motion on both sides of the stability boundary. From the discussion in this section, a nominal rotor is chosen. This nominal rotor is used to study the rotor response to various parametric changes throughout the remainder of this paper.

Figure 16 presents the stability boundary for axisymmetric shaft stiffness. Only axisymmetric shaft stiffness will be considered in the remaining sections of this report. The section marks on this figure are added to describe the regions where time histories across the stability boundaries are presented and discussed. The figure numbers by each section mark indicate the figure that depicts the time history.

All time histories are presented in two separate figures (e.g., see figure 17). The left-hand figure is a plot of the inertial coordinates of the rotor end of the drive shaft w.r.t. the wing-drive-shaft joint. The X and Z coordinates have been nondimensionalized by dividing by the rotor radius R and the superscript has been omitted. Since the wing degrees of freedom have been omitted in this analysis, this plot represents the X/R and Z/R projection of the rotor end of the drive shaft in inertial coordinates. On this plot the direction of rotor

rotation is counterclockwise and it is in the same direction throughout this report. The right-hand figure is a time history of the blade flapping angle, β , versus the blade azimuth angle, ψ . Both angles are measured in radians. In order to simplify the figure and reduce confusion, only the flapping motion of the master blade is shown.

The interval between the plotted points on the time history figures represents equal time intervals in units of radians ($\Delta\psi$). The plotting interval is not the same as the computing interval. The plotting interval, which is greater than the computing interval, depended upon the speed of the motion. Since a straight line is drawn between the plotted points, too few points would make the time histories appear discontinuous, whereas too many points would clutter up the figure. The computing intervals are always equal to or less than 0.1 radian. Also note that the scales of the ordinate or abscissa for different time history plots are not all the same.

Figure 17 is a time history plot of the motion for a nominal shaft frequency ratio of 0.55 and values of the nominal flapping frequency ratio of 1.1, 1.05, and 1.0 (figs. 17(a), 17(b), and 17(c), respectively). All three plots were chosen with the same initial conditions, namely, $\theta_{x1} = 20^\circ$ ($Z/R = 0.171$) and $\beta = 0.1$ radians. All initial velocities are equal to zero. Also, the time intervals on figures 17(a) and 17(b) are 0.5 radians, and on figure 17(c) the time interval is 0.2 radians. Since the precessional motion is clockwise, the mode of motion is a backward whirl. On figure 17 it is not possible to deter-

mine if the blade motion is going unstable and taking the shaft along or vice versa. Since there is no viscous damping in the system ($C_1 = 0$), the stability is defined entirely by the aerodynamics. From figures 16 and 17 it is seen that for $\omega_\beta/\Omega = 1.10$ the system is stable and for $\omega_\beta/\Omega = 1.0$ the system is unstable.

Figure 18 is a time history of the motion for a nominal shaft frequency ratio of 0.35 and a nominal flapping frequency ratio of 1.10, 1.06, and 1.0. All three plots indicate a backward whirl mode of motion. Figure 18(c) indicates a rapid instability; in fact, the motion was stopped at $\psi = 27$ radians because the blades reached a tip Mach number of 0.9. This Mach number indicates that the magnitude of the blade flapping velocities ($\dot{\beta}R$) are approaching the blade rotational tip speeds. Also, note on figure 18(c) that the maximum blade flapping angles are quite large. Because of the high blade flapping amplitudes and velocities, the blades would probably encounter a structural limit for this condition. The initial motion of the shaft in figure 18(c) is a cuspidal-type motion.

The time history of the motion for a nominal shaft frequency ratio of 0.20 and a blade flapping frequency ratio of 1.05, 1.08, 1.15, and 1.20 is shown in figure 19. The rotor rotated through 200 radians for the case presented in figure 19(d). Also, from figure 19(d), it can be shown that the blade flapping motion $\Delta\beta$ is about equal to the shaft angle. Since the flapping motion is measured w.r.t. the shaft axis, the position of the blade tip path plane w.r.t. inertial coordinates

cannot be determined very easily from this figure. However, from the computer printout, it can be shown that the blade tip path plane is tilted slightly forward in the direction of rotor motion and tilted laterally toward the outward side of the shaft precession. These results are in general agreement with those in [32] and [33]. Figures 19(a) and 19(c) are considered to be a near-neutral stability condition, figure 19(b) is stable, and figure 19(d) is unstable. From these plots, it can be seen that there is no clear demarcation line between stability and instability. Therefore, the boundary presented in figures 14 and 15 must be a region, and the variations in stability across this region should be studied. The mode of motion in figures 19(a) and 19(b) are clearly different than in figures 19(c) and 19(d).

Figure 20 is a time history of a model with a flapping frequency ratio of 1.5 and a shaft frequency ratio of 0.4, 0.32, and 0.30. Figure 20(a) is clearly stable, and figure 20(c) is unstable. If a system with a shaft natural frequency ratio equal to 0.4 sustained some structural damage such that the frequency ratio was reduced to 0.30, the system would become unstable. This problem has actually been experienced in commercial aviation. After a number of hard landings, compounded with some fatigue loading and eventual failures, the engine mount structure was weakened to the extent that the propeller went into a whirl instability, resulting in a fatal crash.

This section presented some insight into the effects of changes in rotor shaft and blade flapping natural frequencies. The rotor operating conditions and physical characteristics (other than stiffness) were held fixed. In order to investigate the effect of other variables on the system stability and at the same time keep this study within reasonable bounds, the shaft stiffness and blade natural frequency will now be fixed by choosing a basic rotor.

4.3.4 Basic Nominal Rotor

In order to investigate the effect of rotor operating and/or other physical variables, the spring stiffnesses for the shaft and blade flapping hinge were fixed. The values used were chosen on a basis of most practical design considerations. Increased stiffness is associated with increased weight and minimum weight is a primary aircraft design factor. Therefore, it was decided to choose the lightest system possible. In the definition of the nominal frequency, the corresponding hinge spring stiffness is a primary variable. From figure 14 it is seen that for a stable system the lowest set of stable frequencies occur when the natural flapping frequency ratio is about 1.04 and the lowest shaft frequency ratio is about 0.17. Since the stability characteristics across a section where the shaft frequency ratio is 0.20 are already available (figs. 16 or 19), it was decided to use this value as a basic parameter. Similarly, a blade flapping frequency ratio of 1.05 was chosen (see fig. 19(a)) as a basic condition. These two natural frequencies defined the basic rotor. They also defined the two corresponding hinge spring

stiffnesses. The rotor operating conditions and other rotor physical parameters are then varied from the basic rotor in order to determine their individual effect on stability.

4.3.4.1 Effect of Variations in Rotor Operating Conditions on the Stability of the Nominal Rotor

In this section, the effect of various rotor operating conditions on the stability of the system is evaluated. This section should provide some insight into rotor operating limitations and problems with the nominal rotor. Each figure in this section contains the time history of the basic rotor along with the time history for changes in some operating variable. While only one variable was changed at a time, the overall resulting effect was not always that straight forward. For example, changing the forward speed could result in blade stall with a corresponding change in the total rotor thrust, the blade aerodynamic flap damping, and the damped natural frequency.

Figures 21 and 22 are intended to indicate the effects of variation in forward speed with all other variables fixed. Increasing forward speed above the nominal value will decrease the blade local section angle-of-attack, while decreasing forward speed will increase the angle-of-attack. When some part of the rotor blade becomes stalled, all of the nonlinearities of airfoil force coefficients come into the problem. Recall that the nominal forward speed is 100 feet per second. Figures 21(a) and 21(b) for forward speeds of 150 and 125 feet per second, respectively, indicate instability increases with increasing forward speed. This is in agreement with other results (i.e., [12], [15], [22],

etc.). Figure 22(c) for a decreasing forward speed is also destabilizing. The reasons for this are explored further. For reference purposes, the blade angle-of-attack at the three-quarters blade radius is about 8.1° for a forward speed of 125 feet per second, 10.7° for 100 feet per second, and 13.4° for 75 feet per second. For the latter condition, the outer portion of the blade is definitely stalled. This stall effect has been neglected in previous analysis and this neglect has resulted in the erroneous general conclusion that only increasing forward speed is destabilizing. Thus the analysis in this dissertation indicates that decreasing forward speed can also be destabilizing. Figure 22 is a time history of the motion for these forward speeds. For increasing speed, the rotor goes unstable faster than for decreasing speeds. In fact, for increasing forward speeds, the blade flapping velocity reached the Mach number limit and the rotor did not complete the 22 revolutions allowed in the computer program. From these two figures, we conclude that the stability of the rotor system is speed sensitive. Or, in other words, flying in horizontally gusty air (e.g., at ± 25 ft/sec) could cause the nominal rotor to go unstable.

Another operating variable to investigate is the rotor rotational speed. This is measured by the ability of the pilot and/or engine to maintain constant rotational speed. The effect of a ± 10 percent change in rotor rotational speed was investigated. The resulting time history of the motion is shown in figure 23. Recall that the shaft and blade hinge springs have been fixed at a constant value as defined by the nominal rotor. The shaft nominal natural frequency was defined as

independent of rotor rotational speed. Thus, the shaft frequency will not change with changes in rotational speed; however, the nondimensionalized natural frequency (frequency ratio) will change. Also, the blade flapping natural frequency is affected by the rotational speed changes because of the change in centrifugal force and the corresponding restoring moment. Also, changes in rotor rotational speeds will affect the blade section angle-of-attack. This discussion points out that while only one variable is changed, namely, Ω , changes in many other factors must be considered in analyzing the results. Figure 23(a) indicates that increasing rotational speed increases stability, while figure 23(c) shows that decreasing rotational speed results in an unstable system. The latter instability might be explained as follows: decreasing rotational speed decreases the centrifugal restoring spring which will decrease the flapping natural frequency. Starting with the basic rotor in figure 16, decreasing the flapping natural frequency of the nominal rotor results in an unstable system, whereas increasing rotor rotational speed increases the flapping natural frequency resulting in a more stable rotor.

The effects of changes in blade pitch angle, θ_{0s} , are shown in figure 24. The section angle-of-attacks at the three-quarter blade radius for these three plots, namely, figures 24(a), 24(b), and 24(c), are 7.7° , 10.7° , and 13.7° , respectively. Since blade stall for a NACA 0012 airfoil occurs at about 12° angle-of-attack, the latter of these conditions includes some blade stall. Figure 24(a) shows that the

system becomes more stable with a decrease in blade pitch angle, whereas figure 24(c) is unstable for the increased pitch angle. This stability trend might be explained by noting that blade stall (fig 24(c)) is associated with a loss in blade aerodynamic flap damping (i.e., loss in lift-curve slope) which allows the blade flapping motion to increase with increasing time.

A similar condition occurs in figure 25 where variations in fluid density is considered. As the air density is decreased (fig. 25(a)), the blade aerodynamic flap damping is decreased and the blade flapping motion becomes unstable. When the fluid density is increased (fig. 25(c)), just the opposite occurs; the flap damping is increased and the motion is stable. In addition to the variations in rotor operating conditions in the next section variations in some physical parameters are evaluated from the stability standpoint.

4.3.4.2 Effect of Variations in Physical Parameters on the Stability of the Nominal Rotor

By physical parameters is meant parameters other than those associated with operating conditions. The shaft and blade flapping natural frequencies have already been investigated in Sections 4.3.1, 4.3.2, and 4.3.3. Before any variations were made, the system was always brought back to its original nominal conditions and/or values.

Figure 26 shows the effect of variations in number of blades on the rotor system. To arrive at this figure, everything was maintained constant except the number of blades. This means that for the rotor with two blades (fig. 26(a)), with the same blade mass per blade, the hub

inertia is lower than for that of the nominal rotor (fig. 26(b)). Also, for four blades (fig. 26(c)), the hub inertia is correspondingly increased. This variation in hub inertia with number of blades will affect the natural shaft frequency. For example, with four blades the hub mass is increased and the shaft frequency is about 78 percent of the nominal value and for two blades the hub mass is decreased and the shaft frequency is between 118 and 206 percent of the nominal value. Recall that for two blades the hub inertia will vary with the alignment of the blades. Increasing hub inertia with four blades will decrease the shaft frequency and, according to figure 26(c), the system becomes unstable. This trend agrees with the results in figure 16 for decreasing the shaft frequency from the nominal value. With two blades the shaft frequency will increase; however, figure 26(a) shows that the system is unstable. This does not agree with the results of figure 16. The reason for this is believed to be related to the different modes of vibration associated with two- and three-blade systems; recall that figure 16 was obtained for a three-blade rotor system which is axisymmetric, whereas the two-blade rotor is asymmetric.

The effects of a 10 percent variation in blade weight is shown in figure 27. Since all other variables are held constant, a change in blade weight will affect both the flapping and shaft natural nominal frequencies. The shaft frequency variations are about ± 10 percent and the flapping frequencies variations about ± 0.5 percent. From figure 27 the stability of the system does not seem to be affected by the blade

weight. The primary effect seems to be in the blade flapping motion; that is, the heavier blades have less flapping amplitude (fig. 27(c)) than the light blades (fig. 27(a)). This condition is thus different than the above case where the number of blades were changed. When the blade weight is changed, the flapping natural frequency and flapping amplitude are changed; however, when the number of blades are changed, in particular from three blades to two blades, the entire mode shape changes. Figure 16 shows that a 10 percent variation in the nominal shaft natural frequency ratio will not result in an unstable system.

The effect of variations in shaft length is shown in figure 28. Decreasing the shaft length will decrease the shaft inertia or increase the shaft natural frequency. In fact, for a shaft length of 7-1/2 feet (fig. 28(b)) and 5 feet (fig. 28(c)), the natural shaft frequency is increased about 16 and 80 percent, respectively. Recall that the nominal shaft length is 10 feet. From figures 28(b) and 28(c) it can be seen that both systems become unstable for the decreased shaft lengths. This result agrees with that of figure 16 for a corresponding increase in the shaft natural frequency ratio. And again, the advantages for the definition of the nominal natural frequency is seen and the value of figure 16 in predicting results is appreciated.

4.3.4.3 Effect of Blade Flap Damping on the Stability of the Nominal Rotor

To determine the effect of damping on the system, the critical damping ratio was determined. Various percentages of the critical damping were then introduced across the blade flapping hinge. For each

damping value the shaft spring stiffness was increased or decreased and the time histories computed. This process was continued until the new stability boundary was determined for each flap damping value chosen. The object was to determine if the introduction of blade flap damping would reduce the shaft stiffness stability requirements still further. If the additional damping reduced the stiffness requirements, then the additional damper weight must be traded against the savings in structural stiffness weight. Damping values as high as 20 percent of critical were introduced and no noticeable change in stability was noticed. This result implies that for the basic nominal rotor the blade flapping motion does not have a significant effect on the overall rotor aerodynamics.

4.3.4.4 Effect of Drive Shaft Damping on the Stability of the Nominal Rotor

To evaluate shaft damping effects, the critical damping ratio was determined. Then, as with flap damping, various amounts of damping were introduced and their effect on the stability criteria evaluated. The conditions that were evaluated were restricted to axisymmetric shaft stiffness. The resulting stability boundary is shown in figure 29. Again, the boundary shown should be considered a region rather than an absolute boundary. Of course, the energy absorbed by the viscous damper will have to be dissipated, usually in the form of heat, and the damper weight will have to be traded against the structural stiffness weight saved. From figure 29 it is seen that a small amount of damping will result in an appreciable decrease in the required stiffness. However, this advantage decreases beyond a critical damping ratio of about 10 percent.

V. CONCLUDING REMARKS AND RECOMMENDATIONS

5.1 Concluding Remarks

The literature has been reviewed and background information obtained. The literature review indicates a shortage of experimental data available for correlating with theory. The results of the analytical solutions indicate some conflicting differences. These differences in analytical results are believed related to differences in assumptions in formulating the mathematical model. The model in this dissertation contains more flexibility, is more sophisticated, and has fewer assumptions than any model published to date. Of course, this sophistication results in a highly complex and difficult solution.

Some of the numerical results herein do not agree with those of other researchers. For example, the forward whirl mode obtained by Young [31] over part of the stability boundary could not be obtained in the analysis herein. Reed [15] indicates that he observed the model in a forward precessional mode; however, neither he nor Richardson [26,27] could verify this mode analytically. Reed [15] and Richardson [26,27] speculate that this mode must arise because of unsteady aerodynamic effects, in particular the phase lag in the aerodynamics. As a result of this analysis, the author tends to agree with this conclusion.

In this study it was found that decreasing forward speed can result in a whirl instability. As forward speed is decreased, the rotor begins to stall and the blades begin to lose their flap damping until the system eventually becomes unstable. This has not been confirmed by

other researchers. This result is believed due to the fact that other researchers have not included stall effects in their analyses. Also, for practical reasons, experimental testing is usually not conducted in the deep stall region.

Another finding in this study is that there is a difference in stability when the rotating inertia is decreased by different methods. For example, the rotating inertia can be decreased by decreasing individual blade weight or by decreasing the number of blades on the rotor system. The differences in stability for these two examples are due to mode shape changes when going from three blades to two blades; these mode shape changes do not occur when changing blade weights.

Many rotor physical and operating variables have been investigated in this dissertation. For example, changes have been made in the rotor rotational speed, the blade pitch angle, atmospheric density (flight altitude effects), drive shaft length, and blade flap damping. Many of these variables have not been investigated previously. Blade Mach number effects evaluated herein have not been evaluated before. In this analysis, a number of cases were encountered in which the Mach number was a dominant factor. It is concluded in this dissertation that blade flap damping has very little effect on the whirl stability boundary.

It is difficult to generalize from a complex mathematical model because there are so many interacting variables. This is one of the disadvantages of complexity. A number of specific numerical solutions have been obtained. The trends indicated might be typical of what to

expect for a different physical model. However, it cannot be stated categorically the same trend will always occur. For example, changing blade weight while operating at a much higher operating speed than considered herein could indicate different trends. However, the computer program is operating and available, and is capable of handling a very large number of variables.

5.2 Recommendations

Based on the experience obtained in working on the study of this dissertation, the following recommendations are offered:

(1) Obtain sufficient experimental information to verify the theoretical results. There is only one company known to have experimental model data. The data that they have published have not been complete enough to use for correlation with theory. The completion of this item might require additional wind-tunnel tests.

(2) It is recommended that the numerical value of the variables that have been investigated in this dissertation be expanded in value. For example, investigate the flight speed over the whole aircraft operating range and investigate the stability for various rotor shaft tilt angles such as the complete aircraft transition region. Provisions are written into the computer program to investigate cyclic pitch control and non-uniform inflow effects. It is recommended that these items be investigated.

(3) It has been suggested and one company has investigated (unpublished) the possibility of obtaining whirl stability by a feed-

back control system. This approach is attractive in that an automatic feedback control system could provide a much wider range of stability boundaries by using the existing rotor control system to keep the rotor from going into a whirl instability. The present computer program does not have provisions for a feedback system. However, by using the existing cyclic control system, incorporating a feedback system does not appear to be too difficult a task.

(4) The last recommendation is to modify the computer program so that it included blade elasticity. The reason for this feature is to have a computer program available for determining the blade bending moments. It is not believed that blade elasticity will affect the whirl stability boundaries very much; this is because the first mode blade motion is the blade rigid-body flapping mode which is included in this analysis. Providing for blade elasticity would not be an easy task, either normal modes or another iterative computer loop could be incorporated.

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APPENDIX A - DERIVATION OF THE INERTIA FORCES AND MOMENTS

A1 - Absolute Accelerations of a Blade Element

By making consecutive x, y, z right-hand axis rotations about each joint (figure 10), and introducing linear translations for each member and starting from the inertial axes and transferring to the blade axes, using standard matrix terminology, we arrive at:

$$\begin{aligned}
 \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} &= [\theta_{i0}]^T \begin{Bmatrix} L \\ 0 \\ 0 \end{Bmatrix} + [\theta_{i0}]^T [\theta_{i1}]^T \begin{Bmatrix} 0 \\ l \\ 0 \end{Bmatrix} \\
 &+ [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix} \quad (A1) \\
 &+ [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2 \end{Bmatrix} + [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1 \end{Bmatrix}
 \end{aligned}$$

Where the following matrix notation has been introduced:

$$\begin{aligned}
 [\theta_{i0}] &= [\theta_{z0}] [\theta_{y0}] [\theta_{x0}] \\
 &= \begin{bmatrix} \theta_{z0c} & \theta_{z0s} & 0 \\ -\theta_{z0s} & \theta_{z0c} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{y0c} & 0 & -\theta_{y0s} \\ 0 & 1 & 0 \\ \theta_{y0s} & 0 & \theta_{y0c} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta_{x0c} & \theta_{x0s} \\ 0 & -\theta_{x0s} & \theta_{x0c} \end{bmatrix} \quad (A2)
 \end{aligned}$$

herein the last subscript symbol signifies the appropriate trigonometric function, that is:

$$\theta_{z0C} \Rightarrow \cos \theta_{z0}; \theta_{y0C} \Rightarrow \cos \theta_{y0}$$

$$\theta_{z0S} \Rightarrow \sin \theta_{z0}; \theta_{y0S} \Rightarrow \sin \theta_{y0}$$

Since these matrices are orthogonal and the determinant equals unity; $[\theta_{i0}]^{-1} = [\theta_{i0}]^T$
 Multiplying out the $[\theta_{i0}]$ matrix and taking the transpose results in:

$$[\theta_{i0}]^T = \begin{bmatrix} \theta_{z0C}\theta_{y0C} & -\theta_{z0S}\theta_{y0C} & \theta_{y0S} \\ \theta_{z0S}\theta_{x0C} + \theta_{z0C}\theta_{y0S}\theta_{x0S} & \theta_{z0C}\theta_{x0C} - \theta_{z0S}\theta_{y0S}\theta_{x0S} & -\theta_{y0C}\theta_{x0S} \\ \theta_{z0S}\theta_{x0S} - \theta_{z0C}\theta_{y0S}\theta_{x0S} & \theta_{z0C}\theta_{x0S} + \theta_{z0S}\theta_{y0S}\theta_{x0C} & \theta_{y0C}\theta_{x0S} \end{bmatrix} \quad (A3)$$

Similarly for the second joint rotation (the wing-drive-shaft joint, see figure 10)

$$[\theta_{i1}]^T = \begin{bmatrix} \theta_{z1C}\theta_{y1C} & -\theta_{z1S}\theta_{y1C} & \theta_{y1S} \\ \theta_{z1S}\theta_{x1S} + \theta_{z1C}\theta_{y1S}\theta_{x1S} & \theta_{z1C}\theta_{x1C} - \theta_{z1S}\theta_{y1S}\theta_{x1S} & -\theta_{y1C}\theta_{x1S} \\ \theta_{z1S}\theta_{x1S} - \theta_{z1C}\theta_{y1S}\theta_{x1S} & \theta_{z1C}\theta_{x1S} + \theta_{z1S}\theta_{y1S}\theta_{x1C} & \theta_{y1C}\theta_{x1S} \end{bmatrix} \quad (A4)$$

where as with $[\theta_{i0}]$

$$\begin{aligned}
[\theta_{i1}] &= [\theta_{z1}] [\theta_{v1}] [\theta_{x1}] \\
&= \begin{bmatrix} \theta_{z1C} & \theta_{z1S} & 0 \\ -\theta_{z1S} & \theta_{z1C} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \theta_{y1C} & 0 & -\theta_{y1S} \\ 0 & 1 & 0 \\ \theta_{y1S} & 0 & \theta_{y1C} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \theta_{x1C} & \theta_{x1S} \\ 0 & -\theta_{x1S} & \theta_{x1C} \end{bmatrix} \quad (A5)
\end{aligned}$$

The remaining matrix used in equation (A1) follow, recall that the last subscript symbol signifies the appropriate trigonometric function.

$$[\psi]^T = [\psi]^{-1} = \begin{bmatrix} \psi_C & 0 & \psi_S \\ 0 & 1 & 0 \\ -\psi_S & 0 & \psi_C \end{bmatrix}$$

$$[\beta]^T = [\beta]^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \beta_C & \beta_S \\ 0 & -\beta_S & \beta_C \end{bmatrix}$$

(A6)

$$[\xi]^T = [\xi]^{-1} = \begin{bmatrix} \xi_C & 0 & -\xi_S \\ 0 & 1 & 0 \\ \xi_S & 0 & \xi_C \end{bmatrix}$$

$$[\theta]^T = [\theta]^{-1} = \begin{bmatrix} \theta_C & -\theta_S & 0 \\ \theta_S & \theta_C & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The remaining column matrix in equation (A1) are linear dimensions (translations) and are defined on figure 10.

Equation (A1) represents the coordinates of any point on the rotor blade $\begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix}$ with respect to (w.r.t.) a point at the root end of the wing, center of gravity, or reference point, on the aircraft, see figure 10. Since this reference point is not accelerating or rotating, w.r.t. inertial space, i.e. translating at a constant velocity, then the $\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$ axes can be considered inertial axes and equation (A1) defines the coordinates of any point on the blade w.r.t. inertial coordinates. Therefore differentiation of equation (A1) w.r.t. time will result in the absolute inertial accelerations. The absolute velocities result from the sum of the absolute velocities of the reference point (i.e. flight velocities) and the first derivative w.r.t. time of equation (A1). Before proceeding with the differentiation of equation (A1), we will nondimensionalize the equation by dividing all linear variables by the rotor radius "R" and multiplying the time variable by the rotor rotational speed Ω rad/sec.

Differentiating equation (A1) once w.r.t. time and nondimensionalizing results in

$$\begin{aligned}
\frac{1}{\Omega} \frac{d}{dt} \begin{Bmatrix} \mathbf{x}/R \\ \mathbf{y}/R \\ \mathbf{z}/R \end{Bmatrix}_{\text{INERTIA}} &= \frac{d}{dt} \left(\frac{[\bar{\theta}_{i0}]^T}{\Omega} \right) \begin{Bmatrix} \mathbf{L}/R \\ 0 \\ 0 \end{Bmatrix} + \frac{d}{dt} \left(\frac{[\bar{\theta}_{i0}]^T}{\Omega} \right) [\bar{\theta}_{i1}]^T \begin{Bmatrix} 0 \\ \mathbf{L}/R \\ 0 \end{Bmatrix} + \left(\frac{[\bar{\theta}_{i0}]^T}{\Omega} \right) \frac{d}{dt} [\bar{\theta}_{i1}]^T \begin{Bmatrix} 0 \\ \mathbf{L}/R \\ 0 \end{Bmatrix} \\
&+ \frac{d}{dt} \left(\frac{[\bar{\theta}_{i0}]^T}{\Omega} \right) [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} \mathbf{x}^{\text{XII}}/R \\ \mathbf{y}^{\text{XII}}/R \\ \mathbf{z}^{\text{XII}}/R \end{Bmatrix} + \left(\frac{[\bar{\theta}_{i0}]^T}{\Omega} \right) \frac{d}{dt} [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} \mathbf{x}^{\text{XII}}/R \\ \mathbf{y}^{\text{XII}}/R \\ \mathbf{z}^{\text{XII}}/R \end{Bmatrix} \\
&+ \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T}{\Omega} \frac{d}{dt} [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} \mathbf{x}^{\text{XII}}/R \\ \mathbf{y}^{\text{XII}}/R \\ \mathbf{z}^{\text{XII}}/R \end{Bmatrix} + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T}{\Omega} \frac{d[\beta]^T}{dt} [\xi]^T [\theta]^T \begin{Bmatrix} \mathbf{x}^{\text{XII}}/R \\ \mathbf{y}^{\text{XII}}/R \\ \mathbf{z}^{\text{XII}}/R \end{Bmatrix} \\
&+ \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T}{\Omega} \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} \mathbf{x}^{\text{XII}}/R \\ \mathbf{y}^{\text{XII}}/R \\ \mathbf{z}^{\text{XII}}/R \end{Bmatrix} + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T [\xi]^T}{\Omega} \frac{d[\theta]^T}{dt} \begin{Bmatrix} \mathbf{x}^{\text{XII}}/R \\ \mathbf{y}^{\text{XII}}/R \\ \mathbf{z}^{\text{XII}}/R \end{Bmatrix} \\
&+ \frac{1}{\Omega} \frac{d[\bar{\theta}_{i0}]^T}{dt} [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ \mathbf{e}_2/R \end{Bmatrix} + \frac{[\bar{\theta}_{i0}]^T}{\Omega} \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ \mathbf{e}_2/R \end{Bmatrix}
\end{aligned}$$

(A7)

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$$\begin{aligned}
& + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T}{\Omega} \frac{d[\psi]^T}{dt} [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T}{\Omega} \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} + \frac{1}{\Omega} \frac{d[\bar{\theta}_{i0}]^T}{dt} [\bar{\theta}_{i1}]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \\
& + \frac{[\bar{\theta}_{i0}]^T}{\Omega} \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T}{\Omega} \frac{d[\psi]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix}
\end{aligned} \tag{A7}$$

Differentiating equation (A1) twice w.r.t. time results in

$$\begin{aligned}
\frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \text{INERTIA} &= \frac{1}{\Omega^2} \frac{d^2 [\bar{\theta}_{i0}]^T}{dt^2} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \frac{1}{\Omega^2} \frac{d^2 [\bar{\theta}_{i0}]^T}{dt^2} [\bar{\theta}_{i1}]^T \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d [\bar{\theta}_{i0}]^T}{dt} \frac{d [\bar{\theta}_{i1}]^T}{dt} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} \\
& + \frac{[\bar{\theta}_{i0}]^T}{\Omega^2} \frac{d^2 [\bar{\theta}_{i1}]^T}{dt^2} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \frac{1}{\Omega^2} \frac{d^2 [\bar{\theta}_{i0}]^T}{dt^2} [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \\
& + \frac{2}{\Omega^2} \frac{d [\bar{\theta}_{i0}]^T}{dt} \frac{d [\bar{\theta}_{i1}]^T}{dt} [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d [\bar{\theta}_{i0}]^T}{dt} [\bar{\theta}_{i1}]^T \frac{d [\psi]^T}{dt} [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix}
\end{aligned} \tag{A8}$$

(continued on next page)

$$\begin{aligned}
& + \frac{2}{\Omega^2} \frac{d[\bar{\theta}_{i0}]^T}{dt} [\bar{\theta}_{i1}]^T [\psi]^T \frac{d[\beta]^T}{dt} [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\bar{\theta}_{i0}]^T}{dt} [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \\
& + \frac{2}{\Omega^2} \frac{d[\bar{\theta}_{i0}]^T}{dt} [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\xi]^T \frac{d[\theta]^T}{dt} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{[\bar{\theta}_{i0}]^T}{\Omega^2} \frac{d^2[\bar{\theta}_{i1}]^T}{dt^2} [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \\
& + \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} \frac{d[\psi]^T}{dt} [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{2[\bar{\theta}_{i0}]^T}{\Omega^2} \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T \frac{d[\beta]^T}{dt} [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \quad (A8) \quad \infty \\
& + \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T [\beta]^T \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{2[\bar{\theta}_{i0}]^T}{\Omega^2} \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T [\beta]^T [\xi]^T \frac{d[\theta]^T}{dt} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \\
& + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T}{\Omega^2} \frac{d^2[\psi]^T}{dt^2} [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{2[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T}{\Omega^2} \frac{d[\psi]^T}{dt} \frac{d[\beta]^T}{dt} [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix}
\end{aligned}$$

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$$\begin{aligned}
& + \frac{2 [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T}{\Omega^2} \frac{d[\psi]^T}{dt} [\beta]^T \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{2 [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T}{\Omega^2} \frac{d[\psi]^T}{dt} [\beta]^T [\xi]^T \frac{d[\theta]^T}{dt} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \\
& + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T}{\Omega^2} \frac{d^2[\beta]^T}{dt^2} [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{2 [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T}{\Omega^2} \frac{d[\beta]^T}{dt} \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \\
& + \frac{2 [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T}{\Omega^2} \frac{d[\beta]^T}{dt} [\xi]^T \frac{d[\theta]^T}{dt} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T}{\Omega^2} \frac{d^2[\xi]^T}{dt^2} [\theta]^T \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \quad (A8) \\
& + \frac{2 [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T}{\Omega^2} \frac{d[\xi]^T}{dt} \frac{d[\theta]^T}{dt} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \frac{[\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T [\xi]^T}{\Omega^2} \frac{d^2[\theta]^T}{dt^2} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \\
& + \frac{1}{\Omega^2} \frac{d^2 [\bar{\theta}_{i0}]^T}{dt^2} [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d [\bar{\theta}_{i0}]^T}{dt} \frac{d [\bar{\theta}_{i1}]^T}{dt} [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix}
\end{aligned}$$

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$$+ \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} [\theta_{i1}]^T \frac{d[\psi]^T}{dt} [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} [\theta_{i1}]^T [\psi]^T \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix}$$

$$+ \frac{1}{\Omega^2} [\theta_{i0}]^T \frac{d^2[\theta_{i1}]^T}{dt^2} [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \frac{2}{\Omega^2} [\theta_{i0}]^T \frac{d[\theta_{i1}]^T}{dt} \frac{d[\psi]^T}{dt} [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\theta_{i0}]^T \frac{d[\theta_{i1}]^T}{dt} [\psi]^T \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \frac{1}{\Omega^2} [\theta_{i0}]^T [\theta_{i1}]^T \frac{d^2[\psi]^T}{dt^2} [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\theta_{i0}]^T [\theta_{i1}]^T \frac{d[\psi]^T}{dt} \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \frac{1}{\Omega^2} [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T \frac{d^2[\beta]^T}{dt^2} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix}$$

$$+ \frac{1}{\Omega^2} \frac{d^2[\theta_{i0}]^T}{dt^2} [\theta_{i1}]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} \frac{d[\theta_{i1}]^T}{dt} [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} [\theta_{i1}]^T \frac{d[\psi]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix}$$

(A8)

(continued on next page)

$$\begin{aligned}
& + \frac{1}{\Omega^2} [\theta_{i0}]^T \frac{d^2 [\theta_{i1}]^T}{dt^2} [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} + \frac{2}{\Omega^2} [\theta_{i0}]^T \frac{d [\theta_{i1}]^T}{dt} \frac{d [\psi]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} + \frac{[\theta_{i0}]^T [\theta_{i1}]^T}{\Omega^2} \frac{d^2 [\psi]^T}{dt^2} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \\
& \hspace{25em} (A8)
\end{aligned}$$

For simplicity, the inertial accelerations have been redefined as follows:

$$\begin{aligned}
\frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} &= \{D_1\} + \bar{\bar{\theta}}_{x0} \{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \bar{\bar{\theta}}_{y0} \{D_3\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \bar{\bar{\theta}}_{z0} \{D_4\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \bar{\bar{\theta}}_{x0} \{D_5\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} \\
& + \bar{\bar{\theta}}_{y0} \{D_6\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \bar{\bar{\theta}}_{z0} \{D_7\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \bar{\bar{\theta}}_{x1} \{D_8\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \bar{\bar{\theta}}_{y1} \{D_9\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \bar{\bar{\theta}}_{z1} \{D_{10}\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} \\
& + \bar{\bar{\theta}}_{x0} \{D_{11}\} \begin{Bmatrix} x^{XII}/R \\ y^{XII}/R \\ z^{XII}/R \end{Bmatrix} + \bar{\bar{\theta}}_{y0} \{D_{12}\} \begin{Bmatrix} x^{XII}/R \\ y^{XII}/R \\ z^{XII}/R \end{Bmatrix} + \bar{\bar{\theta}}_{z0} \{D_{13}\} \begin{Bmatrix} x^{XII}/R \\ y^{XII}/R \\ z^{XII}/R \end{Bmatrix} + \bar{\bar{\theta}}_{x1} \{D_{14}\} \begin{Bmatrix} x^{XII}/R \\ y^{XII}/R \\ z^{XII}/R \end{Bmatrix} \\
& \hspace{25em} (A9)
\end{aligned}$$

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$$\begin{aligned}
& + \bar{\theta}_{y1} \{D_{15}\} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \bar{\theta}_{z1} \{D_{16}\} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \bar{\psi} \{D_{17}\} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \bar{\beta} \{D_{18}\} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} \\
& + \bar{\xi} \{D_{19}\} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \bar{\theta} \{D_{20}\} \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} + \bar{\theta}_{x0} \{D_{21}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \bar{\theta}_{y0} \{D_{22}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \\
& + \bar{\theta}_{z0} \{D_{23}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \bar{\theta}_{x1} \{D_{24}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \bar{\theta}_{y1} \{D_{25}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \bar{\theta}_{z1} \{D_{26}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \bar{\psi} \{D_{27}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \\
& + \bar{\beta} \{D_{28}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \bar{\theta}_{x0} \{D_{29}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \bar{\theta}_{y0} \{D_{30}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \bar{\theta}_{z0} \{D_{31}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \bar{\theta}_{x1} \{D_{32}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \\
& + \bar{\theta}_{y1} \{D_{33}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \bar{\theta}_{z1} \{D_{34}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \bar{\psi} \{D_{35}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix}
\end{aligned}$$

(A9)

The $\{D_i\}$ matrix coefficients are defined at the end of this appendix.

Also, since $\frac{1}{\Omega} \frac{dA}{dt} = \frac{dA}{d\psi}$, the bars over the variable are defined by

$$\bar{A} = \frac{dA}{d\psi} \quad \text{and} \quad \overline{\overline{A}} = \frac{d^2 A}{d\psi^2} \quad \text{where } A \text{ is a dummy variable.}$$

Equation (A8) represents the absolute acceleration of any point on

the blade $\begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix}$. The absolute acceleration of any other member in

the system can be obtained from this equation (A8) by including all terms up to the member in question. For example, the absolute accelerations of member "e2" are obtained from equation (A8) by setting

$$\begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} = 0 \quad \text{and letting "e2" be the variable "e2p" where}$$

$0 \leq e2p \leq e2$, see figure 10, and the absolute accelerations of member

$$\text{"l" are obtained by setting } \begin{Bmatrix} x^{XII/R} \\ y^{XII/R} \\ z^{XII/R} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ e2/R \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ e1/R \end{Bmatrix} = 0 \quad \text{and}$$

letting "l" be the variable ℓ_p where $0 \leq \ell_p \leq \ell$.

The inertial forces are determined from the product of mass and acceleration or more precisely the integral taken along the member of the product of the inertial acceleration and the elemental mass. The inertial moments are determined by the vector cross product of the moment arm and inertial force. For a distributed mass the inertial

moment becomes the integral of the vector cross product. The governing equations of motion are obtained by equating the external aerodynamic, spring and damper moments to the inertial moments at the hinge. In this appendix the inertial shear forces and the inertial moments at each hinge are derived.

A2 - Blade Inertial Forces and Moments About the Lag Hinge

The blade inertial forces are determined by the integral of the product of the inertial acceleration and the elemental blade mass as follows:

$$\begin{Bmatrix} C_{IFx} \\ C_{IFy} \\ C_{IFz} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{1.0} \left(1.0 - \frac{e_1}{R} - \frac{e_2}{R} \right) \mu_1 \left(\frac{z^{XII}}{R} \right) \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{INERTIA}} \right\} d \left(\frac{z^{XII}}{R} \right) \quad (A10)$$

It has been assumed that the blade mass is concentrated along the z^{XII} axis only. This mass is defined by μ_1 , a function of (z^{XII}/R) , mass per unit length. The inertial force has been nondimensionalized by dividing by $\pi R^2 \rho (\Omega R)^2$ where ρ is the air density. This force vector is resolved into blade coordinates as follows:

$$\begin{Bmatrix} C_{IFx}^{XII} \\ C_{IFy}^{XII} \\ C_{IFz}^{XII} \end{Bmatrix} = [\theta][\xi][\beta][\psi] \begin{bmatrix} \bar{\theta}_{11} \\ \bar{\theta}_{10} \end{bmatrix} \begin{Bmatrix} C_{IFx} \\ C_{IFy} \\ C_{IFz} \end{Bmatrix} \quad (A11)$$

The inertial force matrix is expanded in terms of "D_i" variables as follows:

$$\begin{aligned}
 \begin{Bmatrix} C_{IFx} \\ C_{IFy} \\ C_{IFz} \end{Bmatrix} &= \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \{D_1\} d\left(\frac{z^{XII}}{R}\right) \\
 &+ \frac{\bar{\theta}_{x0}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_5\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \{D_{11}\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\
 &+ \{D_{21}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \{D_{29}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left. \right] d\left(\frac{z^{XII}}{R}\right) \\
 &+ \frac{\bar{\theta}_{y0}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\{D_3\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_6\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \{D_{12}\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\
 &+ \{D_{22}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \{D_{30}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left. \right] d\left(\frac{z^{XII}}{R}\right) \\
 &+ \frac{\bar{\theta}_{z0}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\{D_4\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_7\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \{D_{13}\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right.
 \end{aligned} \tag{A12}$$

(continued on next page)

$$\begin{aligned}
& + \left\{ D_{23} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{ D_{31} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left[d \left(\frac{z^{XII}}{R} \right) \right. \\
& + \frac{\bar{\theta}_{x1}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_8 \right\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \left\{ D_{14} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\{ D_{24} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \right. \\
& + \left. \left\{ D_{32} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d \left(\frac{z^{XII}}{R} \right) + \frac{\bar{\theta}_{y1}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_9 \right\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right. \\
& + \left. \left\{ D_{15} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\{ D_{25} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{ D_{33} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d \left(\frac{z^{XII}}{R} \right) \quad (A12) \\
& + \frac{\bar{\theta}_{z1}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_{10} \right\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \left\{ D_{16} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\{ D_{26} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \right. \\
& + \left. \left\{ D_{34} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d \left(\frac{z^{XII}}{R} \right) + \frac{\bar{\beta}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_{18} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\
& + \left. \left\{ D_{28} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \right] d \left(\frac{z^{XII}}{R} \right) + \frac{\bar{\xi}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_{19} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right] d \left(\frac{z^{XII}}{R} \right)
\end{aligned}$$

This equation can be simplified by redefining the coefficients as follows:

$$\begin{Bmatrix} C_{IFx} \\ C_{IFy} \\ C_{IFz} \end{Bmatrix} = \{E_{40}\} + \bar{\theta}_{x0} \{E_{41}\} + \bar{\theta}_{y0} \{E_{42}\} + \bar{\theta}_{z0} \{E_{43}\} \\ + \bar{\theta}_{x1} \{E_{44}\} + \bar{\theta}_{y1} \{E_{45}\} + \bar{\theta}_{z1} \{E_{46}\} + \bar{\beta} \{E_{47}\} + \bar{\xi} \{E_{48}\} \quad (A13)$$

where the "E_i" coefficients are defined at the end of this appendix.

As stated previously the inertial moments are determined from the integral of the vector cross product of the moment arm and the inertial force. The blade moment arm vector in inertial coordinates can be obtained from equation (A1) and is

$$\begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} = [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII}/R \\ y^{XII}/R \\ z^{XII}/R \end{Bmatrix} \quad (A14)$$

With this moment arm the inertial moments about the blade lag hinge in inertial coordinates become:

$$\begin{Bmatrix} C_{IMx} \\ C_{IMy} \\ C_{IMz} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{1.0} \left(1.0 - \frac{e_1}{R} - \frac{e_2}{R} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \right) \times \mu_1 \left(\frac{z^{XII}}{R} \right) \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \right\} d \left(\frac{z^{XII}}{R} \right) \quad (A15)$$

Here the blade distributed mass is defined by μ_1 a function of $\left(\frac{z^{XII}}{R}\right)$ and the moment is nondimensionalized by dividing by $\pi R^2 \rho (\Omega R)^2 R$. The cross inside the integral indicates vector cross products. Inciden-

tally, the blade radius vector $\begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}}$ can be rewritten as $\begin{Bmatrix} R_x \\ R_y \\ R_z \end{Bmatrix} \left(\frac{z^{XII}}{R}\right)$

for purposes of integration. This is not done here for simplicity.

Rewritting this equation in terms of "D" variables results in:

$$\begin{aligned} \begin{Bmatrix} C_{IMx} \\ C_{IMy} \\ C_{IMz} \end{Bmatrix} &= \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \{D_1\} \mu_1 d\left(\frac{z^{XII}}{R}\right) \\ &+ \frac{\bar{\theta}_{x0}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \left[\{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_5\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} \right. \\ &+ \left. \{D_{11}\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \{D_{21}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \{D_{29}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] \mu_1 d\left(\frac{z^{XII}}{R}\right) \\ &+ \frac{\bar{\theta}_{y0}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \left[\{D_3\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_6\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} \right] \mu_1 d\left(\frac{z^{XII}}{R}\right) \end{aligned} \quad (A16)$$

(Continued on next page)

$$\begin{aligned}
& + \left\{ \langle D_{12} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \langle D_{22} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{30} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right\} \mu_1 d\left(\frac{z^{XII}}{R}\right) \\
& + \frac{\bar{\bar{\theta}}_{z0}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \left[\langle D_4 \rangle \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \langle D_7 \rangle \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} \right. \\
& + \left. \langle D_{13} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \langle D_{23} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{31} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] \mu_1 d\left(\frac{z^{XII}}{R}\right) \\
& + \frac{\bar{\bar{\theta}}_{x0}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \left[\langle D_8 \rangle \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \langle D_{14} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\
& + \left. \langle D_{24} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{32} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] \mu_1 d\left(\frac{z^{XII}}{R}\right) \\
& + \frac{\bar{\bar{\theta}}_{y0}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \left[\langle D_9 \rangle \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \langle D_{15} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\
& + \left. \langle D_{25} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{33} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] \mu_1 d\left(\frac{z^{XII}}{R}\right)
\end{aligned} \tag{A16}$$

(continued on next page)

$$\begin{aligned}
& + \frac{\bar{\theta}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \left\{ \begin{matrix} x/R \\ y/R \\ z/R \end{matrix} \right\}_{\text{BLADE}} \times \left[\{D_{10}\} \begin{Bmatrix} 0 \\ z/R \\ 0 \end{Bmatrix} + \{D_{16}\} \begin{Bmatrix} 0 \\ 0 \\ z^{\text{XII}}/R \end{Bmatrix} \right. \\
& + \left. \{D_{26}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \{D_{34}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] \mu_1 d\left(\frac{z^{\text{XII}}}{R}\right) \\
& + \frac{\bar{\beta}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \left\{ \begin{matrix} x/R \\ y/R \\ z/R \end{matrix} \right\}_{\text{BLADE}} \times \left[\{D_{18}\} \begin{Bmatrix} 0 \\ 0 \\ z^{\text{XII}}/R \end{Bmatrix} \right. \\
& + \left. \{D_{28}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \right] \mu_1 d\left(\frac{z^{\text{XII}}}{R}\right) + \frac{\bar{\xi}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \left\{ \begin{matrix} x/R \\ y/R \\ z/R \end{matrix} \right\}_{\text{BLADE}} \times \\
& \times \left[\{D_{19}\} \begin{Bmatrix} 0 \\ 0 \\ z^{\text{XII}}/R \end{Bmatrix} \right] \mu_1 d\left(\frac{z^{\text{XII}}}{R}\right)
\end{aligned}$$

These moments are in inertial coordinates and, to be utilized, they must be transformed into XI coordinates by the following transformation:

$$\begin{Bmatrix} C_{\text{IMx}^{\text{XI}}} \\ C_{\text{IMy}^{\text{XI}}} \\ C_{\text{IMz}^{\text{XI}}} \end{Bmatrix} = [\xi][\beta][\psi] \begin{bmatrix} \theta_{i1} \\ \theta_{i0} \end{bmatrix} \begin{Bmatrix} C_{\text{IMx}} \\ C_{\text{IMy}} \\ C_{\text{IMz}} \end{Bmatrix} \quad (\text{A17})$$

Rewriting this equation in terms of "E" variables results in

$$\begin{aligned}
 \begin{Bmatrix} C_{IMxXI} \\ C_{IMyXI} \\ C_{IMzXI} \end{Bmatrix} &= \{E_{10}\} + \bar{\bar{\theta}}_{x0} \{E_{11}\} + \bar{\bar{\theta}}_{y0} \{E_{12}\} + \bar{\bar{\theta}}_{z0} \{E_{13}\} + \bar{\bar{\theta}}_{x1} \{E_{14}\} \\
 &+ \bar{\bar{\theta}}_{y1} \{E_{15}\} + \bar{\bar{\theta}}_{z1} \{E_{16}\} + \bar{\bar{\beta}} \{E_{17}\} + \bar{\bar{\xi}} \{E_{18}\}
 \end{aligned}
 \tag{A18}$$

A3 - Member "e₂" Inertial Forces and Moments About the Flap

$$\text{Hinge Between Members } \begin{Bmatrix} 0 \\ 0 \\ e_2 \end{Bmatrix} \text{ and } \begin{Bmatrix} 0 \\ 0 \\ e_1 \end{Bmatrix}$$

Using the above procedures, the inertial forces from member "e₂" in inertial coordinates are:

$$\begin{Bmatrix} C_{Ie2x} \\ C_{Ie2y} \\ C_{Ie2z} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left(\frac{e_{2p}}{R} \right) \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \right\}_{e_2} d \left(\frac{e_{2p}}{R} \right) \quad (A19)$$

where e_{2p} represents the variable length for member "e₂"

($0 \leq e_{2p} \leq e_2$), μ_2 is the member mass per unit length and a function of $\left(\frac{e_{2p}}{R} \right)$. The inertial accelerations for member "e₂", namely

$$\left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \right\}_{e_2}, \text{ are obtained from equation (A8) by letting}$$

$$\begin{Bmatrix} x^{XII}/R \\ y^{XII}/R \\ z^{XII}/R \end{Bmatrix} = 0. \text{ Rewriting this equation in terms of "D}_1\text{" variables,}$$

where "e₂/R" must be changed to "e_{2p}/R" for purposes of integration, results in:

$$\begin{aligned}
\begin{Bmatrix} c_{Ie2x} \\ c_{Ie2y} \\ c_{Ie2z} \end{Bmatrix} &= \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \{D_{50}\} d\left(\frac{e_{2p}}{R}\right) \\
&+ \frac{\bar{\theta}_{x0}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_5\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{21}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right. \\
&+ \left. \{D_{29}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \right] d\left(\frac{e_{2p}}{R}\right) + \frac{\bar{\theta}_{y0}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\{D_3\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_6\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} \right. \\
&+ \left. \{D_{22}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} + \{D_{30}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \right] d\left(\frac{e_{2p}}{R}\right) \tag{A20} \\
&+ \frac{\bar{\theta}_{z0}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\{D_4\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_7\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{23}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right. \\
&+ \left. \{D_{31}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \right] d\left(\frac{e_{2p}}{R}\right) + \frac{\bar{\theta}_{x1}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\{D_8\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{24}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right.
\end{aligned}$$

(Continued on next page)

$$\begin{aligned}
& + \{D_{32}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left[d\left(\frac{e_{2p}}{R}\right) + \frac{\bar{\theta}_{y1}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} D_9 \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \{D_{25}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right. \\
& + \{D_{33}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left[d\left(\frac{e_{2p}}{R}\right) + \frac{\bar{\theta}_{z1}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} D_{10} \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right. \\
& + \{D_{26}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} + \{D_{34}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left. d\left(\frac{e_{2p}}{R}\right) \right. \\
& + \frac{\bar{\beta}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} D_{28} \end{Bmatrix} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \left. d\left(\frac{e_{2p}}{R}\right) \right]
\end{aligned}$$

Redefining this equation in terms of "E_i" results in

$$\begin{aligned}
\begin{Bmatrix} C_{Ie2x} \\ C_{Ie2y} \\ C_{Ie2z} \end{Bmatrix} &= \{E_{30}\} + \bar{\theta}_{x0} \{E_{31}\} + \bar{\theta}_{y0} \{E_{32}\} + \bar{\theta}_{z0} \{E_{33}\} + \bar{\theta}_{x1} \{E_{34}\} \\
&+ \bar{\theta}_{y1} \{E_{35}\} + \bar{\theta}_{z1} \{E_{36}\} + \bar{\beta} \{E_{37}\}
\end{aligned} \tag{A21}$$

Here again the $\{D_i\}$ and $\{E_i\}$ coefficients are defined at the end of this appendix. These inertial forces (equation A21) are transformed into "VIII" coordinates by the following transformation:

$$\begin{Bmatrix} C_{Ie2x}^{VIII} \\ C_{Ie2y}^{VIII} \\ C_{Ie2z}^{VIII} \end{Bmatrix} = [\psi] [\theta_{i1}] [\theta_{i0}] \begin{Bmatrix} C_{Ie2x} \\ C_{Ie2y} \\ C_{Ie2z} \end{Bmatrix} \quad (A22)$$

The inertial moments of member "e₂" about the flap hinge (between members "e₁" and "e₂") becomes:

$$\begin{Bmatrix} C_{Ie2x} \\ C_{Ie2y} \\ C_{Ie2z} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \left\{ [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right\} \times \left\{ \mu_2 \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \right\}_{e_2} d\left(\frac{e_{2p}}{R}\right) \quad (A23)$$

The cross inside the integral represents vector cross product. The moment arm (in inertial coordinates) is obtained from equation (A1) by letting all member lengths be zero save "e₂". Letting the radius vector be written as:

$$[\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} = \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \quad (A24)$$

and rewriting moment in terms of "D" variables results in:

$$\begin{aligned}
 \begin{Bmatrix} C_{IMe2x} \\ C_{IMe2y} \\ C_{IMe2z} \end{Bmatrix} &= \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \times \{D_{50}\} d\left(\frac{e_{2p}}{R}\right) \\
 &+ \frac{\bar{\theta}_{x0}}{\pi R^2 \rho} \int_0^{e_2/R} \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \times \left\{ \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_2\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \{D_{21}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right. \\
 &+ \left. \{D_{29}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} \mu_2 d\left(\frac{e_{2p}}{R}\right) + \frac{\bar{\theta}_{y0}}{\pi R^2 \rho} \int_0^{e_2/R} \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \times \left\{ \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_3\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} \right. \\
 &+ \left. \{D_6\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \{D_{22}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} + \{D_{30}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} \mu_2 d\left(\frac{e_{2p}}{R}\right) \\
 &+ \frac{\bar{\theta}_{z0}}{\pi R^2 \rho} \int_0^{e_2/R} \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \times \left\{ \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_4\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \{D_6\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \{D_{23}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right. \\
 &+ \left. \{D_{31}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} \mu_2 d\left(\frac{e_{2p}}{R}\right) + \frac{\bar{\theta}_{x1}}{\pi R^2 \rho} \int_0^{e_2/R} \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \times \left\{ \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \{D_8\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} \right.
 \end{aligned} \tag{A25}$$

(continued on next page)

$$\begin{aligned}
& + \left\{ D_{24} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} + D_{32} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right\} \mu_2 d\left(\frac{e_{2p}}{R}\right) + \frac{\bar{\bar{\theta}}_{y1}}{\pi R^2 \rho} \int_0^{e_2/R} \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \\
& \times \left\{ D_9 \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + D_{25} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} + D_{33} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right\} \mu_2 d\left(\frac{e_{2p}}{R}\right) \\
& + \frac{\bar{\bar{\theta}}_{z1}}{\pi R^2 \rho} \int_0^{e_2/R} \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \times \left\{ D_{10} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + D_{26} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right. \\
& + \left. D_{34} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right\} \mu_2 d\left(\frac{e_{2p}}{R}\right) + \frac{\bar{\bar{\beta}}}{\pi R^2 \rho} \int_0^{e_2/R} \begin{Bmatrix} R_{xe2} \\ R_{ye2} \\ R_{ze2} \end{Bmatrix} \times \\
& \left\{ D_{28} \begin{Bmatrix} 0 \\ 0 \\ e_{2p}/R \end{Bmatrix} \right\} \mu_2 d\left(\frac{e_{2p}}{R}\right)
\end{aligned}$$

In terms of "E" variables, the inertial moment for member "e₂" in terms of inertial coordinates becomes:

$$\begin{aligned}
\begin{Bmatrix} C_{IME2x} \\ C_{IME2y} \\ C_{IME2z} \end{Bmatrix} &= \{E_{20}\} + \bar{\bar{\theta}}_{x0} \{E_{21}\} + \bar{\bar{\theta}}_{y0} \{E_{22}\} + \bar{\bar{\theta}}_{z0} \{E_{23}\} + \bar{\bar{\theta}}_{x1} \{E_{24}\} \\
&+ \bar{\bar{\theta}}_{y1} \{E_{25}\} + \bar{\bar{\theta}}_{z1} \{E_{26}\} + \bar{\bar{\beta}} \{E_{27}\}
\end{aligned} \tag{A26}$$

These inertial moments are transformed into IX coordinates by the following transformation:

$$\begin{Bmatrix} C_{\text{IMe2x}}^{\text{IX}} \\ C_{\text{IMe2y}}^{\text{IX}} \\ C_{\text{IMe2z}}^{\text{IX}} \end{Bmatrix} = [\beta][\psi] \begin{bmatrix} \theta_{i1} \\ \theta_{i0} \end{bmatrix} \begin{Bmatrix} C_{\text{IMe2x}} \\ C_{\text{IMe2y}} \\ C_{\text{IMe2z}} \end{Bmatrix} \quad (\text{A27})$$

The $\{D_i\}$ and $\{E_i\}$ coefficients appear at the end of this appendix.

A4 - Member "e₁" Inertia Forces and Moments About Point

$$\text{Between Member } \begin{Bmatrix} 0 \\ 0 \\ e_1 \end{Bmatrix} \text{ and } \begin{Bmatrix} 0 \\ l \\ 0 \end{Bmatrix}$$

Member "e₁" is rigidly fixed to and rotates with the drive shaft (member "l"), in fact the axis of "e₁" is perpendicular to the axis of "l". Member "e₁" is part of the blade retension system and there will be as many "e₁" members in the system as there are blades (see figure 10). Therefore some of the inertial forces and moments of the total number of "e₁" members will cancel each other. However, there may be some of these forces and moments that do not cancel, such as gyroscopic moments, and therefore "e₁" will be treated as an independent member. The inertial forces originating from the mass of member "e₁" are:

$$\begin{Bmatrix} C_{Ie1x} \\ C_{Ie1y} \\ C_{Ie1z} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left(\frac{e_{1p}}{R} \right) \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} \bar{x}/R \\ y/R \\ z/R \end{Bmatrix}_{e_1} \right\} d \left(\frac{e_{1p}}{R} \right) \quad (A28)$$

The same comments as with the previous derivation of member "e₂" are in order here. Namely, μ_3 is the mass distribution of member "e₁", "e_{1p}" is the variable for e_1 ($0 \leq e_{1p} \leq e_1$) and the inertial accelerations of "e₁" are determined from equation (A8). Rewriting this equation in terms of "D" variables as before results in:

$$\begin{aligned}
\begin{Bmatrix} C_{Ie1x} \\ C_{Ie1y} \\ C_{Ie1z} \end{Bmatrix} &= \frac{1}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \{D_{75}\} d\left(\frac{e_{1p}}{R}\right) \\
&+ \frac{\bar{\theta}_{x0}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_5\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{29}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} d\left(\frac{e_{1p}}{R}\right) \\
&+ \frac{\bar{\theta}_{y0}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \{D_3\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_6\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{30}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} d\left(\frac{e_{1p}}{R}\right) \\
&+ \frac{\bar{\theta}_{z0}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \{D_4\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_7\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} \right\} \tag{A29} \\
&+ \{D_{31}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} d\left(\frac{e_{1p}}{R}\right) + \frac{\bar{\theta}_{x1}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \{D_8\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} \right. \\
&+ \{D_{32}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} d\left(\frac{e_{1p}}{R}\right) + \frac{\bar{\theta}_{y1}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \{D_9\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} \right. \\
&+ \{D_{33}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} d\left(\frac{e_{1p}}{R}\right) + \frac{\bar{\theta}_{z1}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \{D_{10}\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} \right.
\end{aligned}$$

(continued on next page)

$$+ \{D_{34}\} \left\{ \begin{array}{c} 0 \\ 0 \\ e_{1p}/R \end{array} \right\} d\left(\frac{e_{1p}}{R}\right) \quad (A29)$$

Rewriting this in terms of "E" variables results in

$$\begin{Bmatrix} C_{Ielx} \\ C_{Iely} \\ C_{Ielz} \end{Bmatrix} = \{E_{50}\} + \bar{\bar{\theta}}_{x0} \{E_{51}\} + \bar{\bar{\theta}}_{y0} \{E_{52}\} + \bar{\bar{\theta}}_{z0} \{E_{53}\} + \bar{\bar{\theta}}_{x1} \{E_{54}\} \\ + \bar{\bar{\theta}}_{y1} \{E_{55}\} + \bar{\bar{\theta}}_{z1} \{E_{56}\} \quad (A30)$$

Transforming these inertial forces into VI coordinates results in

$$\begin{Bmatrix} C_{IelxVI} \\ C_{IelyVI} \\ C_{IelzVI} \end{Bmatrix} = \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix} \begin{Bmatrix} C_{Ielx} \\ C_{Iely} \\ C_{Ielz} \end{Bmatrix} \quad (A31)$$

The inertial moments for member "e₁" are the integral of the vector cross product of the moment arm and inertial force.

$$\begin{Bmatrix} C_{Melx} \\ C_{Mely} \\ C_{Melz} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{e_{1p}/R} \mu_3 \left\{ \left\{ \begin{bmatrix} \bar{\theta}_{i0} \end{bmatrix}^T \begin{bmatrix} \bar{\theta}_{i1} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} \times \right. \\ \left. \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} e_1 \right\} \right\} d\left(\frac{e_{1p}}{R}\right) \quad (A32)$$

Rewriting this in terms first of "D" and then of "F" variables results

in

$$\begin{aligned}
 \begin{Bmatrix} C_{Me1x} \\ C_{Me1y} \\ C_{Me1z} \end{Bmatrix} &= \frac{1}{\pi R^2 \rho} \int_0^{e_{1/R}} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p/R} \end{Bmatrix} \right\} \times \right. \\
 \left. \begin{Bmatrix} D_{75} \end{Bmatrix} \right\} & d\left(\frac{e_{1p}}{R}\right) + \frac{\bar{\theta}_{x0}}{\pi R^2 \rho} \int_0^{e_{1/R}} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p/R} \end{Bmatrix} \right\} \times \right. \\
 \left. \left\{ \begin{Bmatrix} D_2 \end{Bmatrix} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \begin{Bmatrix} D_5 \end{Bmatrix} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \begin{Bmatrix} D_{29} \end{Bmatrix} \begin{Bmatrix} 0 \\ 0 \\ e_{1p/R} \end{Bmatrix} \right\} \right\} & d\left(\frac{e_{1p}}{R}\right) \\
 + \frac{\bar{\theta}_{y0}}{\pi R^2 \rho} \int_0^{e_{1/R}} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p/R} \end{Bmatrix} \right\} \times \left\{ \begin{Bmatrix} D_3 \end{Bmatrix} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \right. \right. & \\
 \left. \left. + \begin{Bmatrix} D_6 \end{Bmatrix} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \begin{Bmatrix} D_{30} \end{Bmatrix} \begin{Bmatrix} 0 \\ 0 \\ e_{1p/R} \end{Bmatrix} \right\} \right\} & d\left(\frac{e_{1p}}{R}\right) \\
 + \frac{\bar{\theta}_{z0}}{\pi R^2 \rho} \int_0^{e_{1/R}} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p/R} \end{Bmatrix} \right\} \times \left\{ \begin{Bmatrix} D_4 \end{Bmatrix} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \right. \right. &
 \end{aligned} \tag{A33}$$

(continued on next page)

$$\begin{aligned}
& + \left\langle D_7 \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \left\langle D_{31} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \left. \vphantom{\begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix}} \right\} d\left(\frac{e_{1p}}{R}\right) \\
& + \frac{\bar{\bar{\theta}}_{x1}}{\pi R^2 \rho} \int_0^{e_{1p}/R} \mu_3 \left\{ \left[\bar{\theta}_{i0} \right]^T \left[\bar{\theta}_{i1} \right]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} \times \left\langle D_8 \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \\
& + \left\langle D_{32} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \left. \vphantom{\begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix}} \right\} d\left(\frac{e_{1p}}{R}\right) + \frac{\bar{\bar{\theta}}_{y1}}{\pi R^2 \rho} \int_0^{e_{1p}/R} \mu_3 \left\{ \left[\bar{\theta}_{i0} \right]^T \left[\bar{\theta}_{i1} \right]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} \\
& \times \left\langle D_9 \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \left\langle D_{33} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \left. \vphantom{\begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix}} \right\} d\left(\frac{e_{1p}}{R}\right) \\
& + \frac{\bar{\bar{\theta}}_{z1}}{\pi R^2 \rho} \int_0^{e_{1p}/R} \mu_3 \left\{ \left[\bar{\theta}_{i0} \right]^T \left[\bar{\theta}_{i1} \right]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \right\} \times \left\langle D_{10} \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \\
& + \left\langle D_{34} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix} \left. \vphantom{\begin{Bmatrix} 0 \\ 0 \\ e_{1p}/R \end{Bmatrix}} \right\} d\left(\frac{e_{1p}}{R}\right)
\end{aligned}$$

$$\begin{Bmatrix} C_{Me1x} \\ C_{Me1y} \\ C_{Me1z} \end{Bmatrix} = \{F_{30}\} + \bar{\theta}_{x0} \{F_{31}\} + \bar{\theta}_{y0} \{F_{32}\} + \bar{\theta}_{z0} \{F_{33}\} + \bar{\theta}_{x1} \{F_{34}\} \\ + \bar{\theta}_{y1} \{F_{35}\} + \bar{\theta}_{z1} \{F_{36}\} \quad (A34)$$

Transforming these inertial moments into VI coordinates results in

$$\begin{Bmatrix} C_{Me1xVI} \\ C_{Me1yVI} \\ C_{Me1zVI} \end{Bmatrix} = \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix} \begin{Bmatrix} C_{Me1x} \\ C_{Me1y} \\ C_{Me1z} \end{Bmatrix} \quad (A35)$$

Each of the above derivations for inertial forces and moments apply to any one rotor blade and one blade retention system (members "e₁" and "e₂"). Each blade is attached to a common drive shaft through its "e₁" member. The "β" and "ξ" degrees of freedom are independent for each blade. Therefore each of the above derivations for inertial forces and moments will have to be applied to each of "N" blades independently. This will be accomplished as follows: choose one blade, say the master blade, to be located at $\psi = \psi_1$, then the next blade will be at $\psi = \psi_1 + \frac{2\pi}{N}$, and so on, for each of the remaining blades. In other words, the above derivations will be applicable to each of the blades with the appropriate ψ . Also, ψ_1 and ψ are also functions of time (the blades rotate), therefore a more precise equation for ψ is:

$$\psi(t, N) = \psi_1 + \frac{d\psi}{dt} dt + \frac{2\pi}{N} \quad (A36)$$

It is assumed herein that the powerplant has sufficient torque or the pilot sufficient control to maintain a constant rotor rotational speed Ω . Therefore $\frac{d\psi_1}{dt} = \Omega = \text{constant}$ or

$$\psi(t, N) = \psi_1 + \Omega t + \frac{2\pi}{N} = \psi_N(t) \quad (\text{A37})$$

A5 - Member "ℓ" Inertial Forces and Moments

About the Wing Drive-Shaft Joint

The inertial force for member "ℓ" is equal to the integral of the product of the elemental mass and absolute accelerations.

$$\begin{Bmatrix} C_{F\ell x} \\ C_{F\ell y} \\ C_{F\ell z} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left(\frac{\ell_p}{R} \right) \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} \bar{x}/R \\ y/R \\ z/R \end{Bmatrix} \right\} d \left(\frac{\ell_p}{R} \right) \quad (A38)$$

Again, this force is nondimensionalized by dividing by $\pi R^2 \rho (\Omega R)^2$, μ_4 is the variable mass per unit length, the acceleration term comes from equation (A8) and " ℓ_p " is the integration variable ($0 \leq \ell_p \leq \ell$).

Rewriting this in " D_i " variables, where the " ℓ " in the $\{D_i\}$ coefficients must be changed to " ℓ_p 's" for purposes of integration, results in:

$$\begin{Bmatrix} C_{F\ell x} \\ C_{F\ell y} \\ C_{F\ell z} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \{D_{76}\} d \left(\frac{\ell_p}{R} \right) + \frac{\bar{\theta}_{x0}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{Bmatrix} D_2 \end{Bmatrix} \begin{Bmatrix} \bar{L}/R \\ 0 \\ 0 \end{Bmatrix} \right\} \\ + \left\{ \begin{Bmatrix} D_5 \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell_p/R \\ 0 \end{Bmatrix} \right\} d \left(\frac{\ell_p}{R} \right) + \frac{\bar{\theta}_{y0}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{Bmatrix} D_3 \end{Bmatrix} \begin{Bmatrix} \bar{L}/R \\ 0 \\ 0 \end{Bmatrix} \right\}$$

(Continued on next page)

$$\begin{aligned}
& + \left\{ D_6 \right\} \begin{Bmatrix} 0 \\ \ell_p/R \\ 0 \end{Bmatrix} d\left(\frac{\ell_p}{R}\right) + \frac{\bar{\bar{\theta}}_{z0}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ D_4 \right\} \begin{Bmatrix} \ell/R \\ 0 \\ 0 \end{Bmatrix} \\
& + \left\{ D_7 \right\} \begin{Bmatrix} 0 \\ \ell_p/R \\ 0 \end{Bmatrix} d\left(\frac{\ell_p}{R}\right) + \frac{\bar{\bar{\theta}}_{x1}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ D_8 \right\} \begin{Bmatrix} 0 \\ \ell_p/R \\ 0 \end{Bmatrix} d\left(\frac{\ell_p}{R}\right) \quad (A39) \\
& + \frac{\bar{\bar{\theta}}_{y1}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ D_9 \right\} \begin{Bmatrix} 0 \\ \ell_p/R \\ 0 \end{Bmatrix} d\left(\frac{\ell_p}{R}\right) + \frac{\bar{\bar{\theta}}_{z1}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ D_{10} \right\} \begin{Bmatrix} 0 \\ \ell_p/R \\ 0 \end{Bmatrix} d\left(\frac{\ell_p}{R}\right)
\end{aligned}$$

Resolving these forces into "III" coordinates and introducing new variables results in

$$\begin{Bmatrix} C_{F\ell x}^{III} \\ C_{F\ell y}^{III} \\ C_{F\ell z}^{III} \end{Bmatrix} = [\bar{\bar{\theta}}_{i0}] \begin{Bmatrix} C_{F\ell x} \\ C_{F\ell y} \\ C_{F\ell z} \end{Bmatrix} \quad (A40)$$

$$\begin{aligned}
& = \{F_{10}\} + \bar{\bar{\theta}}_{x0} \{F_{11}\} + \bar{\bar{\theta}}_{y0} \{F_{12}\} + \bar{\bar{\theta}}_{z0} \{F_{13}\} \\
& + \bar{\bar{\theta}}_{x1} \{F_{14}\} + \bar{\bar{\theta}}_{y1} \{F_{15}\} + \bar{\bar{\theta}}_{z1} \{F_{16}\} \quad (A41)
\end{aligned}$$

The inertial moments for member "l" are obtained from the integral of the vector cross product of the moment arm and the elemental inertial force as follows:

$$\begin{Bmatrix} C_{M\ell x} \\ C_{M\ell y} \\ C_{M\ell z} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left(\frac{\ell_p}{R} \right) \left\{ \begin{Bmatrix} \theta_{i0} \\ \theta_{i1} \end{Bmatrix}^T \begin{Bmatrix} \ell_p/R \\ 0 \end{Bmatrix} \right\} \times \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \right\} d \left(\frac{\ell_p}{R} \right) \quad (A42)$$

Introducing "D" variables, this equation becomes:

$$\begin{aligned}
\begin{Bmatrix} C_{M\ell x} \\ C_{M\ell y} \\ C_{M\ell z} \end{Bmatrix} &= \frac{1}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{Bmatrix} \theta_{i0} \\ \theta_{i1} \end{Bmatrix}^T \begin{Bmatrix} \ell_p/R \\ 0 \end{Bmatrix} \right\} \times \{D_{76}\} d \left(\frac{\ell_p}{R} \right) \\
&+ \frac{\bar{\theta}_{x0}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{Bmatrix} \theta_{i0} \\ \theta_{i1} \end{Bmatrix}^T \begin{Bmatrix} \ell_p/R \\ 0 \end{Bmatrix} \right\} \times \left\{ \{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \right\} \\
&+ \{D_5\} \begin{Bmatrix} 0 \\ \ell_p/R \\ 0 \end{Bmatrix} \left\{ d \left(\frac{\ell_p}{R} \right) + \frac{\bar{\theta}_{y0}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{Bmatrix} \theta_{i0} \\ \theta_{i1} \end{Bmatrix}^T \begin{Bmatrix} \ell_p/R \\ 0 \end{Bmatrix} \right\} \times \right. \\
&\left. \left\{ \{D_3\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_6\} \begin{Bmatrix} 0 \\ \ell_p/R \\ 0 \end{Bmatrix} \right\} d \left(\frac{\ell_p}{R} \right) \right\} \quad (A43)
\end{aligned}$$

(continued on next page)

$$\begin{aligned}
& + \frac{\bar{\bar{\theta}}_{z0}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_{p/R} \\ 0 \end{Bmatrix} \right\} \times \begin{Bmatrix} \ell/R \\ 0 \\ 0 \end{Bmatrix} \\
& + \left\{ \begin{Bmatrix} D_7 \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell_{p/R} \\ 0 \end{Bmatrix} \right\} \left\{ d\left(\frac{\ell_p}{R}\right) + \frac{\bar{\bar{\theta}}_{x1}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_{p/R} \\ 0 \end{Bmatrix} \right\} \right. \\
& \times \left\{ \begin{Bmatrix} D_8 \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell_{p/R} \\ 0 \end{Bmatrix} \right\} \left\{ d\left(\frac{\ell_p}{R}\right) + \frac{\bar{\bar{\theta}}_{y1}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_{p/R} \\ 0 \end{Bmatrix} \right\} \right. \\
& \times \left\{ \begin{Bmatrix} D_9 \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell_{p/R} \\ 0 \end{Bmatrix} \right\} \left\{ d\left(\frac{\ell_p}{R}\right) + \frac{\bar{\bar{\theta}}_{z1}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_{p/R} \\ 0 \end{Bmatrix} \right\} \right. \\
& \times \left\{ \begin{Bmatrix} D_{10} \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell_{p/R} \\ 0 \end{Bmatrix} \right\} \left\{ d\left(\frac{\ell_p}{R}\right) \right.
\end{aligned} \tag{A43}$$

Transforming equation A43 into VI coordinates by using the transformation summary on figure 11 results in:

$$\begin{Bmatrix} C_{M\ell_x}^{VI} \\ C_{M\ell_y}^{VI} \\ C_{M\ell_z}^{VI} \end{Bmatrix} = \begin{bmatrix} \theta_{i1} & \theta_{i0} \end{bmatrix} \begin{Bmatrix} C_{M\ell_x} \\ C_{M\ell_y} \\ C_{M\ell_z} \end{Bmatrix} \tag{A44}$$

and introducing "F" variables results in:

$$\begin{aligned} \begin{Bmatrix} C_{M\ell x}^{VI} \\ C_{M\ell y}^{VI} \\ C_{M\ell z}^{VI} \end{Bmatrix} &= \{F_{20}\} + \bar{\theta}_{x0} \{F_{21}\} + \bar{\theta}_{y0} \{F_{22}\} + \bar{\theta}_{z0} \{F_{23}\} \\ &+ \bar{\theta}_{x1} \{F_{24}\} + \bar{\theta}_{y1} \{F_{25}\} + \bar{\theta}_{z1} \{F_{26}\} \end{aligned} \quad (A45)$$

A6 - Member "L" Inertial Forces and Moments

About the Wing-Root-Fuselage Joint

The inertial force of member "L" is determined from the integral of the product of the inertial acceleration and elemental mass

$$\begin{Bmatrix} C_{FLx} \\ C_{FLy} \\ C_{FLz} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left(\frac{L_p}{R} \right) \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_L \right\} d \left(\frac{L_p}{R} \right) \quad (A46)$$

In "D" variables this equation becomes:

$$\begin{Bmatrix} C_{FLx} \\ C_{FLy} \\ C_{FLz} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \{D_{76}\} d \left(\frac{L_p}{R} \right) + \frac{\bar{\bar{\theta}}_{x0}}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \{D_2\} d \left(\frac{L_p}{R} \right) \\ + \frac{\bar{\bar{\theta}}_{y0}}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \{D_3\} d \left(\frac{L_p}{R} \right) + \frac{\bar{\bar{\theta}}_{z0}}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \{D_4\} d \left(\frac{L_p}{R} \right) \quad (A47)$$

The inertial forces from member "L" will produce root shears at the origin. There is no translational degree of freedom at the origin in this derivation. However, for completeness and to provide some insight into the system forces and moments acting on the aircraft, the shear forces and moments are determined.

The inertial moments from member "L" are determined from the integral of the vector cross product of the moment arm and inertial force

$$\begin{Bmatrix} C_{MLx} \\ C_{MLy} \\ C_{MLz} \end{Bmatrix} = \frac{1}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\theta_{i0} \right]^T \begin{Bmatrix} L_p/R \\ 0 \\ 0 \end{Bmatrix} \left\{ \frac{1}{\Omega^2} \frac{d^2}{dt^2} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_L \right\} \right\} d\left(\frac{L_p}{R}\right) \quad (A48)$$

Transforming into the III coordinate system and using "D" variables results in:

$$\begin{aligned} \begin{Bmatrix} C_{MLxIII} \\ C_{MLyIII} \\ C_{MLzIII} \end{Bmatrix} &= \frac{[\theta_{i0}]}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\theta_{i0} \right]^T \begin{Bmatrix} L_p/R \\ 0 \\ 0 \end{Bmatrix} \times \left\{ \{D_{40}\} \begin{Bmatrix} L_p/R \\ 0 \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{L_p}{R}\right) \\ &+ \frac{\bar{\theta}_{x0} [\theta_{i0}]}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\theta_{i0} \right]^T \begin{Bmatrix} L_R/R \\ 0 \\ 0 \end{Bmatrix} \times \left\{ \{D_2\} \begin{Bmatrix} L_p/R \\ 0 \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{L_p}{R}\right) \\ &+ \frac{\bar{\theta}_{y0} [\theta_{i0}]}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\theta_{i0} \right]^T \begin{Bmatrix} L_p/R \\ 0 \\ 0 \end{Bmatrix} \times \left\{ \{D_3\} \begin{Bmatrix} L_p/R \\ 0 \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{L_p}{R}\right) \\ &+ \frac{\bar{\theta}_{z0} [\theta_{i0}]}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\theta_{i0} \right]^T \begin{Bmatrix} L_p/R \\ 0 \\ 0 \end{Bmatrix} \times \left\{ \{D_4\} \begin{Bmatrix} L_p/R \\ 0 \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{L_p}{R}\right) \end{aligned} \quad (A49)$$

Finally, changing to "F" variables results in:

$$\begin{Bmatrix} c_{MLx}^{III} \\ c_{MLy}^{III} \\ c_{MLz}^{III} \end{Bmatrix} = \{F_{50}\} + \bar{\bar{\theta}}_{x0} \{F_{51}\} + \bar{\bar{\theta}}_{y0} \{F_{52}\} + \bar{\bar{\theta}}_{z0} \{F_{53}\} \quad (A50)$$

$$\begin{aligned}
\{D_1\} &= \{D_{40}\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_{40}\} [\theta_{11}]^T \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \\
&+ \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} \frac{d[\theta_{i1}]^T}{dt} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + [\theta_{i0}]^T \{D_{41}\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \\
&+ \{D_{40}\} [\theta_{i1}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix} \\
&+ \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} \frac{d[\theta_{i1}]^T}{dt} [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix} \\
&+ \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} [\theta_{i1}]^T \frac{d[\psi]^T}{dt} [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix} \\
&+ \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} [\theta_{i1}]^T [\psi]^T \frac{d[\beta]^T}{dt} [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix} \\
&+ \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} [\theta_{i1}]^T [\psi]^T [\beta]^T \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix} \\
&+ \frac{2}{\Omega^2} \frac{d[\theta_{i0}]^T}{dt} [\theta_{i1}]^T [\psi]^T [\beta]^T [\xi]^T \frac{d[\theta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}
\end{aligned}$$

(Continued on next page)

$$+ [\bar{\theta}_{i0}]^T \{D_{41}\} [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} \frac{d[\psi]^T}{dt} [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T \frac{d[\beta]^T}{dt} [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T [\beta]^T \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T [\beta]^T [\xi]^T \frac{d[\theta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi_1] [\beta]^T [\xi]^T [\theta]^T \psi^{-2} \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

(Continued on next page)

$$+ \frac{2}{\Omega^2} [\theta_{i0}]^T [\theta_{i1}]^T \frac{d[\psi]^T}{dt} \frac{d[\beta]^T}{dt} [\xi]^T [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ \frac{2 [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T}{\Omega^2} \frac{d[\psi]^T}{dt} [\beta]^T \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T \frac{d[\psi]^T}{dt} [\beta]^T [\xi]^T \frac{d[\theta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ [\theta_{i0}]^T [\theta_{i1}]^T [\psi]^T [\beta_1] [\xi]^T [\theta]^T \bar{\beta}^2 \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T \frac{d[\beta]^T}{dt} \frac{d[\xi]^T}{dt} [\theta]^T \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T \frac{d[\beta]^T}{dt} [\xi]^T \frac{d[\theta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

$$+ [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T [\xi_1]^T [\theta]^T \bar{\xi}^2 \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix}$$

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$$\begin{aligned}
& + 2 [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T \frac{d[\xi]^T}{dt} \frac{d[\theta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ z^{XII/R} \end{Bmatrix} \\
& + \{D_{40}\} [\bar{\theta}_{i1}]^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\bar{\theta}_{i0}]^T}{dt} \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \\
& + \frac{2}{\Omega^2} \frac{d[\bar{\theta}_{i0}]^T}{dt} [\bar{\theta}_{i1}]^T \frac{d[\psi]^T}{dt} [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\bar{\theta}_{i1}]^T}{dt} [\bar{\theta}_{i0}]^T [\psi]^T \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \\
& + [\bar{\theta}_{i0}]^T \{D_{41}\} [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} \frac{d[\psi]^T}{dt} [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \\
& + [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi_1] [\beta]^T \bar{\psi}^2 \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \\
& + \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T \frac{d[\psi]^T}{dt} \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi]^T [\beta_1] \bar{\beta}^2 \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \\
& + \{D_{40}\} [\bar{\theta}_{i1}]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\bar{\theta}_{i0}]^T}{dt} \frac{d[\bar{\theta}_{i1}]^T}{dt} [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix}
\end{aligned}$$

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$$\begin{aligned}
& + \frac{2}{\Omega^2} \frac{d[\bar{\theta}_{i0}]^T}{dt} [\bar{\theta}_{i1}]^T \frac{d[\psi]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} + [\bar{\theta}_{i0}]^T \{D_{41}\} [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \\
& + \frac{2}{\Omega^2} [\bar{\theta}_{i0}]^T \frac{d[\bar{\theta}_{i1}]^T}{dt} \frac{d[\psi]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} + [\bar{\theta}_{i0}]^T [\bar{\theta}_{i1}]^T [\psi_1]^T \bar{\psi}^2 \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix}
\end{aligned}$$

$$\{D_2\} = [\bar{\theta}_{x02}] [\bar{\theta}_{y0}]^T [\bar{\theta}_{z0}]^T$$

$$\{D_3\} = [\bar{\theta}_{x0}]^T [\bar{\theta}_{y02}] [\bar{\theta}_{z0}]^T$$

$$\{D_4\} = [\bar{\theta}_{x0}]^T [\bar{\theta}_{y0}]^T [\bar{\theta}_{z02}]$$

$$\{D_5\} = [\bar{\theta}_{x02}] [\bar{\theta}_{y0}]^T [\bar{\theta}_{z0}]^T [\bar{\theta}_{i1}]^T$$

$$\{D_6\} = [\bar{\theta}_{x0}]^T [\bar{\theta}_{y02}] [\bar{\theta}_{z0}]^T [\bar{\theta}_{i1}]^T$$

$$\{D_7\} = [\bar{\theta}_{x0}]^T [\bar{\theta}_{y0}]^T [\bar{\theta}_{z02}] [\bar{\theta}_{i1}]^T$$

$$\{D_8\} = [\bar{\theta}_{i0}]^T [\bar{\theta}_{x12}] [\bar{\theta}_{y1}]^T [\bar{\theta}_{z1}]^T$$

$$\{D_9\} = [\bar{\theta}_{i0}]^T [\bar{\theta}_{x1}]^T [\bar{\theta}_{y12}] [\bar{\theta}_{z1}]^T$$

$$\{D_{10}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z12} \end{bmatrix}$$

$$\{D_{11}\} = \begin{bmatrix} \theta_{x02} \end{bmatrix} \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T$$

$$\{D_{12}\} = \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \begin{bmatrix} \theta_{y02} \end{bmatrix} \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T$$

$$\{D_{13}\} = \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \begin{bmatrix} \theta_{z02} \end{bmatrix} \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T$$

$$\{D_{14}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x12} \end{bmatrix} \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z1} \end{bmatrix}^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T$$

$$\{D_{15}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y12} \end{bmatrix} \begin{bmatrix} \theta_{z1} \end{bmatrix}^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T$$

$$\{D_{16}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z12} \end{bmatrix} [\psi]^T [\beta]^T [\xi]^T [\theta]^T$$

$$\{D_{17}\} = 0 \quad \text{since } \bar{\psi} = 0$$

$$\{D_{18}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta_2] [\xi]^T [\theta]^T$$

$$\{D_{19}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta]^T [\xi_2] [\theta]^T$$

$$\{D_{20}\} = 0 \quad \text{since } \bar{\theta} = 0$$

$$\{D_{21}\} = \begin{bmatrix} \theta_{x02} \end{bmatrix} \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta]^T$$

$$\{D_{22}\} = \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \begin{bmatrix} \theta_{y02} \end{bmatrix} \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta]^T$$

$$\{D_{23}\} = \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \begin{bmatrix} \theta_{z02} \end{bmatrix} \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta]^T$$

$$\{D_{24}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x12} \end{bmatrix} \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z1} \end{bmatrix}^T [\psi]^T [\beta]^T$$

$$\{D_{25}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y12} \end{bmatrix} \begin{bmatrix} \theta_{z1} \end{bmatrix}^T [\psi]^T [\beta]^T$$

$$\{D_{26}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z12} \end{bmatrix} [\psi]^T [\beta]^T$$

$$\{D_{27}\} = 0 \quad \text{since} \quad \bar{\bar{\psi}} = 0$$

$$\{D_{28}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T [\beta_2]$$

$$\{D_{29}\} = \begin{bmatrix} \theta_{x02} \end{bmatrix} \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T$$

$$\{D_{30}\} = \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \begin{bmatrix} \theta_{y02} \end{bmatrix} \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T$$

$$\{D_{31}\} = \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \begin{bmatrix} \theta_{z02} \end{bmatrix} \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi]^T$$

$$\{D_{32}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x12} \end{bmatrix} \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z1} \end{bmatrix}^T [\psi]^T$$

$$\{D_{33}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y12} \end{bmatrix} \begin{bmatrix} \theta_{z1} \end{bmatrix}^T [\psi]^T$$

$$\{D_{34}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z12} \end{bmatrix} [\psi]^T$$

$$\{D_{35}\} = \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T [\psi_2] \quad \bar{\bar{\psi}} = 0$$

RECALL: $\frac{1}{\Omega^2} \frac{d^2[A]^T}{dt^2} = [A_1] \left(\frac{d(A/\Omega)}{dt} \right)^2 + [A_2] \frac{d^2(A/\Omega)}{dt^2} = [A_1] \bar{A}^2 + [A_2] \bar{\Lambda}$

ONE ADDITIONAL SUBSCRIPT IS ADDED TO THE BASIC SUBSCRIPT WITHIN

THE MATRIXES

$$\begin{aligned} \left\{ D_{40} \right\} &= \begin{bmatrix} \theta_{x0} \end{bmatrix} \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \bar{\theta}_{x0}^2 + \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \begin{bmatrix} \theta_{y0} \end{bmatrix} \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \bar{\theta}_{y0}^2 \\ &+ \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \begin{bmatrix} \theta_{z0} \end{bmatrix} \bar{\theta}_{z0}^2 + \frac{2d \begin{bmatrix} \theta_{x0} \end{bmatrix}^T}{\Omega^2 dt} \frac{d \begin{bmatrix} \theta_{y0} \end{bmatrix}^T}{dt} \begin{bmatrix} \theta_{z0} \end{bmatrix}^T \\ &+ \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{x0} \end{bmatrix}^T}{dt} \begin{bmatrix} \theta_{y0} \end{bmatrix}^T \frac{d \begin{bmatrix} \theta_{z0} \end{bmatrix}^T}{dt} + \frac{2}{\Omega^2} \begin{bmatrix} \theta_{x0} \end{bmatrix}^T \frac{d \begin{bmatrix} \theta_{y0} \end{bmatrix}^T}{dt} \frac{d \begin{bmatrix} \theta_{z0} \end{bmatrix}^T}{dt} \end{aligned}$$

$$\begin{aligned} \left\{ D_{41} \right\} &= \begin{bmatrix} \theta_{x1} \end{bmatrix} \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z1} \end{bmatrix}^T \bar{\theta}_{x1}^2 + \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y1} \end{bmatrix} \begin{bmatrix} \theta_{z1} \end{bmatrix}^T \bar{\theta}_{y1}^2 \\ &+ \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \begin{bmatrix} \theta_{z1} \end{bmatrix} \bar{\theta}_{z1}^2 + \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{x1} \end{bmatrix}^T}{dt} \frac{d \begin{bmatrix} \theta_{y1} \end{bmatrix}^T}{dt} \begin{bmatrix} \theta_{z1} \end{bmatrix}^T \\ &+ \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{x1} \end{bmatrix}^T}{dt} \begin{bmatrix} \theta_{y1} \end{bmatrix}^T \frac{d \begin{bmatrix} \theta_{z1} \end{bmatrix}^T}{dt} + \frac{2}{\Omega^2} \begin{bmatrix} \theta_{x1} \end{bmatrix}^T \frac{d \begin{bmatrix} \theta_{y1} \end{bmatrix}^T}{dt} \frac{d \begin{bmatrix} \theta_{z1} \end{bmatrix}^T}{dt} \end{aligned}$$

= $\left\{ D_{40} \right\}$ WITH ALL SUBSCRIPT ZERO'S CHANGED TO SUBSCRIPT ONE

$$\left\{ D_{50} \right\} = \left\{ D_{40} \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \left\{ D_{40} \right\} \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{i0} \end{bmatrix}^T}{dt} \frac{d \begin{bmatrix} \theta_{i1} \end{bmatrix}^T}{dt} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix}$$

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$$\begin{aligned}
& + \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{Bmatrix} D_{41} \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \begin{Bmatrix} D_{40} \end{Bmatrix} \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} \\
& + \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{Bmatrix} D_{41} \end{Bmatrix} [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} + \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T [\beta_1] \bar{\beta}^2 \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} \\
& + \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{10} \end{bmatrix}^T}{dt} \frac{d \begin{bmatrix} \theta_{11} \end{bmatrix}^T}{dt} [\psi]^T [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{10} \end{bmatrix}^T}{dt} \begin{bmatrix} \theta_{10} \end{bmatrix} \frac{d[\psi]^T}{dt} [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} \\
& + \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi_1] [\beta]^T \bar{\psi}^2 \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{10} \end{bmatrix}^T}{dt} \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} \\
& + \frac{2}{\Omega^2} \begin{bmatrix} \theta_{10} \end{bmatrix}^T \frac{d \begin{bmatrix} \theta_{11} \end{bmatrix}^T}{dt} [\psi]^T \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} + \frac{2}{\Omega^2} \begin{bmatrix} \theta_{10} \end{bmatrix}^T \frac{d \begin{bmatrix} \theta_{11} \end{bmatrix}^T}{dt} \frac{d[\psi]^T}{dt} [\beta]^T \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} \\
& + \frac{2}{\Omega^2} \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T \frac{d[\psi]^T}{dt} \frac{d[\beta]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{2/R} \end{Bmatrix} + \begin{Bmatrix} D_{40} \end{Bmatrix} \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \\
& + \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{Bmatrix} D_{41} \end{Bmatrix} [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{10} \end{bmatrix}^T}{dt} \frac{d \begin{bmatrix} \theta_{11} \end{bmatrix}^T}{dt} [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \\
& + \frac{2}{\Omega^2} \frac{d \begin{bmatrix} \theta_{10} \end{bmatrix}^T}{dt} \begin{bmatrix} \theta_{11} \end{bmatrix}^T \frac{d[\psi]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} + \frac{2}{\Omega^2} \begin{bmatrix} \theta_{10} \end{bmatrix}^T \frac{d \begin{bmatrix} \theta_{11} \end{bmatrix}^T}{dt} \frac{d[\psi]^T}{dt} \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix} \\
& + \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi_1] \bar{\psi}^2 \begin{Bmatrix} 0 \\ 0 \\ e_{1/R} \end{Bmatrix}
\end{aligned}$$

$$\begin{aligned}
\{D_{75}\} &= \{D_{40}\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_{40}\} [\theta_{i1}]^T \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\theta_{i0}]}{dt}^T \frac{d[\theta_{i1}]}{dt}^T \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \\
&+ [\theta_{i0}]^T \{D_{41}\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \{D_{40}\} [\theta_{i1}]^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \\
&+ \frac{2}{\Omega^2} \frac{d[\theta_{i0}]}{dt}^T \frac{d[\theta_{i1}]}{dt}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\theta_{i0}]}{dt}^T [\theta_{i1}]^T \frac{d[\psi]}{dt}^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \\
&+ [\theta_{i0}]^T \{D_{41}\} [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} + \frac{2}{\Omega^2} [\theta_{i0}]^T \frac{d[\theta_{i1}]}{dt}^T \frac{d[\psi]}{dt}^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \\
&+ [\theta_{i0}]^T [\theta_{i1}]^T [\psi_1] \bar{\psi}^2 \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} = \{D_1\} \text{ with } e_2 = z^{XII} = 0 \\
\{D_{76}\} &= \{D_{40}\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_{40}\} [\theta_{i1}]^T \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \frac{2}{\Omega^2} \frac{d[\theta_{i0}]}{dt}^T \frac{d[\theta_{i1}]}{dt}^T \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \\
&+ [\theta_{i0}]^T \{D_{41}\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \\
&= \{D_1\} \text{ with } e_1 = e_2 = z^{XII} = 0
\end{aligned}$$

$$\begin{aligned}
\{E_1\} &= \frac{[\beta][\psi][\theta_{i1}][\theta_{i0}]}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \{D_1\} d\left(\frac{z^{XIII}}{R}\right) \\
\{E_2\} &= \frac{[\beta][\psi][\theta_{i1}][\theta_{i0}]}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_5\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right]
\end{aligned}$$

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$$+ \left\{ D_{11} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\{ D_{21} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{ D_{29} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left] d\left(\frac{z^{XII}}{R}\right)\right.$$

$$\{E_3\} = \frac{[\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_3 \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \left\{ D_6 \right\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right. \\ \left. + \left\{ D_{12} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\{ D_{22} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{ D_{30} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_4\} = \frac{[\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_4 \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \left\{ D_7 \right\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right.$$

$$\left. + \left\{ D_{13} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\{ D_{23} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{ D_{31} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_5\} = \frac{[\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_8 \right\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \left\{ D_{14} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right.$$

$$\left. + \left\{ D_{24} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{ D_{32} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_6\} = \frac{[\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\left\{ D_9 \right\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \left\{ D_{15} \right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right.$$

$$\left. + \left\{ D_{25} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{ D_{33} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_7\} = \frac{[\beta][\psi][\underline{0}_{i1}][\underline{0}_{i0}]}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\begin{aligned} &\left\{D_{10}\right\} \begin{Bmatrix} 0 \\ y/R \\ 0 \end{Bmatrix} + \left\{D_{16}\right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \\ &+ \left\{D_{26}\right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{D_{34}\right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \end{aligned} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_8\} = \frac{[\beta][\psi][\underline{0}_{i1}][\underline{0}_{i0}]}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\begin{aligned} &\left\{D_{18}\right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \\ &+ \left\{D_{28}\right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \end{aligned} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_9\} = \frac{[\beta][\psi][\underline{0}_{i1}][\underline{0}_{i0}]}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left\{D_{19}\right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{10}\} = \frac{[\xi][\beta][\psi][\underline{0}_{i1}][\underline{0}_{i0}]}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\begin{aligned} &\begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \left\{D_1\right\} \end{aligned} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{11}\} = \frac{[\xi][\beta][\psi][\underline{0}_{i1}][\underline{0}_{i0}]}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left\{ \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \left[\begin{aligned} &\left\{D_2\right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \\ &+ \left\{D_5\right\} \begin{Bmatrix} 0 \\ y/R \\ 0 \end{Bmatrix} + \left\{D_{11}\right\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\{D_2\right\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\{D_{29}\right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \end{aligned} \right] \right\} d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{12}\} = \frac{[\xi][\beta][\psi][\underline{0}_{i1}][\underline{0}_{i0}]}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left\{ \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{\text{BLADE}} \times \left[\begin{aligned} &\left\{D_3\right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \end{aligned} \right] \right\}$$

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$$\begin{aligned}
& + \left\langle D_6 \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \left\langle D_{12} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\langle D_{22} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\langle D_{30} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left] d\left(\frac{z^{XII}}{R}\right) \right. \\
\{E_{13}\} &= \frac{[\xi][\beta][\psi] \begin{bmatrix} \theta_{i1} \\ \theta_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left\{ \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \times \left[\left\langle D_4 \right\rangle \begin{Bmatrix} \ell/R \\ 0 \\ 0 \end{Bmatrix} \right. \right. \\
& + \left\langle D_7 \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \left\langle D_{13} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\langle D_{23} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\langle D_{31} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left] d\left(\frac{z^{XII}}{R}\right) \right. \\
\{E_{14}\} &= \frac{[\xi][\beta][\psi] \begin{bmatrix} \theta_{i1} \\ \theta_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left\{ \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \times \left[\left\langle D_8 \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right. \right. \\
& + \left\langle D_{14} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\langle D_{24} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\langle D_{32} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left] d\left(\frac{z^{XII}}{R}\right) \right. \\
\{E_{15}\} &= \frac{[\xi][\beta][\psi] \begin{bmatrix} \theta_{i1} \\ \theta_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left\{ \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \times \left[\left\langle D_9 \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right. \right. \\
& + \left\langle D_{15} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\langle D_{25} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\langle D_{33} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left] d\left(\frac{z^{XII}}{R}\right) \right. \\
\{E_{16}\} &= \frac{[\xi][\beta][\psi] \begin{bmatrix} \theta_{i1} \\ \theta_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left\{ \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \times \left[\left\langle D_{10} \right\rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right. \right. \\
& + \left\langle D_{16} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \left\langle D_{26} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \left\langle D_{34} \right\rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left] d\left(\frac{z^{XII}}{R}\right) \right.
\end{aligned}$$

$$\{E_{17}\} = \frac{[\xi][\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \times \text{BLADE}$$

$$\left[\{D_{18}\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \{D_{28}\} \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{18}\} = \frac{[\xi][\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \times \text{BLADE}$$

$$\left[\{D_{19}\} \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{20}\} = \frac{[\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{e_2} \times \{D_{50}\} d\left(\frac{e_{2P}}{R}\right)$$

$$\begin{aligned} \{E_{21}\} = & \frac{[\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{e_2} \times \left[\{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_5\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} \right. \\ & \left. + \{D_{21}\} \begin{Bmatrix} 0 \\ 0 \\ e_{2P}/R \end{Bmatrix} + \{D_{29}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{e_{2P}}{R}\right) \end{aligned}$$

$$\{E_{22}\} = \frac{[\beta][\psi] \begin{bmatrix} \bar{\theta}_{i1} \\ \bar{\theta}_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{e_2} \times \left[\{D_3\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_6\} \begin{Bmatrix} 0 \\ L/R \\ 0 \end{Bmatrix} \right]$$

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$$+ \left\{ D_{22} \right\} \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} + \left\{ D_{30} \right\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \left. \right\} d\left(\frac{e_{2P}}{R}\right)$$

$$\{E_{23}\} = \frac{[\beta][\psi] \overline{[\theta_{i1}]} \overline{[\theta_{i0}]}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{e_2} \times \left[\{D_4\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_7\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right.$$

$$\left. + \{D_{23}\} \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} + \{D_{31}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{e_{2P}}{R}\right)$$

$$\{E_{24}\} = \frac{[\beta][\psi] \overline{[\theta_{i1}]} \overline{[\theta_{i0}]}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{e_2} \times \left[\{D_8\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right.$$

$$\left. + \{D_{24}\} \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} + \{D_{32}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{e_{2P}}{R}\right)$$

$$\{E_{25}\} = \frac{[\beta][\psi] \overline{[\theta_{i1}]} \overline{[\theta_{i0}]}}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix}_{e_2} \times \left[\{D_9\} \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right.$$

$$\left. + \{D_{25}\} \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} + \{D_{33}\} \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{e_{2P}}{R}\right)$$

$$\begin{aligned}
\{E_{26}\} &= \frac{[\beta][\psi][\bar{\theta}_{11}][\bar{\theta}_{10}]}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left\{ \begin{pmatrix} x/R \\ y/R \\ z/R \end{pmatrix}_{e_2} \times \left[\{D_{10}\} \begin{pmatrix} 0 \\ \ell/R \\ 0 \end{pmatrix} \right. \right. \\
&\quad \left. \left. + \{D_{26}\} \begin{pmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{pmatrix} + \{D_{34}\} \begin{pmatrix} 0 \\ 0 \\ e_1/R \end{pmatrix} \right] \right\} d\left(\frac{e_{2P}}{R}\right) \\
\{E_{27}\} &= \frac{[\beta][\psi][\bar{\theta}_{11}][\bar{\theta}_{10}]}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left\{ \begin{pmatrix} x/R \\ y/R \\ z/R \end{pmatrix}_{e_2} \times \left\{ \{D_{28}\} \begin{pmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{pmatrix} \right\} \right\} d\left(\frac{e_{2P}}{R}\right) \\
\{E_{30}\} &= \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \{D_{50}\} d\left(\frac{e_{2P}}{R}\right) \\
\{E_{31}\} &= \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\{D_2\} \begin{pmatrix} L/R \\ 0 \\ 0 \end{pmatrix} + \{D_5\} \begin{pmatrix} 0 \\ \ell/R \\ 0 \end{pmatrix} + \{D_{21}\} \begin{pmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{pmatrix} \right. \\
&\quad \left. + \{D_{29}\} \begin{pmatrix} 0 \\ 0 \\ e_1/R \end{pmatrix} \right] d\left(\frac{e_{2P}}{R}\right) \\
\{E_{32}\} &= \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\{D_3\} \begin{pmatrix} L/R \\ 0 \\ 0 \end{pmatrix} + \{D_6\} \begin{pmatrix} 0 \\ \ell/R \\ 0 \end{pmatrix} + \{D_{22}\} \begin{pmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{pmatrix} \right. \\
&\quad \left. + \{D_{30}\} \begin{pmatrix} 0 \\ 0 \\ e_1/R \end{pmatrix} \right] d\left(\frac{e_{2P}}{R}\right)
\end{aligned}$$

$$\{E_{33}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\langle D_4 \rangle \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \langle D_7 \rangle \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \langle D_{23} \rangle \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} + \langle D_{31} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{e_{2P}}{R}\right)$$

$$\{E_{34}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\langle D_8 \rangle \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \langle D_{24} \rangle \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} + \langle D_{32} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{e_{2P}}{R}\right)$$

$$\{E_{35}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\langle D_9 \rangle \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \langle D_{25} \rangle \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} + \langle D_{33} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{e_{2P}}{R}\right)$$

$$\{E_{36}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \left[\langle D_{10} \rangle \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \langle D_{26} \rangle \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} + \langle D_{34} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{e_{2P}}{R}\right)$$

$$\{E_{37}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_2/R} \mu_2 \langle D_{28} \rangle \begin{Bmatrix} 0 \\ 0 \\ \frac{e_{2P}}{R} \end{Bmatrix} d\left(\frac{e_{2P}}{R}\right)$$

$$\{E_{40}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \langle D_1 \rangle d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{41}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\langle D_2 \rangle \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \langle D_5 \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right. \\ \left. + \langle D_{11} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \langle D_{21} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{29} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{42}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\langle D_3 \rangle \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \langle D_6 \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \right. \\ \left. + \langle D_{12} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \langle D_{22} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{30} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{43}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\langle D_4 \rangle \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \langle D_7 \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \langle D_{13} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\ \left. + \langle D_{23} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{31} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{44}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\langle D_8 \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \langle D_{14} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\ \left. + \langle D_{24} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{32} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{45}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\langle D_9 \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \langle D_{15} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\ \left. + \langle D_{25} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{33} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{46}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\langle D_{10} \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \langle D_{16} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} \right. \\ \left. + \langle D_{26} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} + \langle D_{34} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{47}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \left[\langle D_{18} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} + \langle D_{28} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \right] d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{48}\} = \frac{1}{\pi R^2 \rho} \int_0^{1.0 - \frac{e_1}{R} - \frac{e_2}{R}} \mu_1 \langle D_{19} \rangle \begin{Bmatrix} 0 \\ 0 \\ z^{XII}/R \end{Bmatrix} d\left(\frac{z^{XII}}{R}\right)$$

$$\{E_{50}\} = \frac{1}{\pi R^2 \rho} \int_0^{\frac{e_1}{R}} \mu_3 \langle D_{75} \rangle d\left(\frac{e_{1P}}{R}\right)$$

$$\{E_{51}\} = \frac{1}{\pi R^2 \rho} \int_0^{\frac{e_1}{R}} \mu_3 \left[\{D_2\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_5\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{29}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right] d\left(\frac{e_{1P}}{R}\right)$$

$$\{E_{52}\} = \frac{1}{\pi R^2 \rho} \int_0^{\frac{e_1}{R}} \mu_3 \left[\{D_3\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_6\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{30}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right] d\left(\frac{e_{1P}}{R}\right)$$

$$\{E_{53}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left[\{D_4\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \{D_7\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{31}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right] d\left(\frac{e_{1P}}{R}\right)$$

$$\{E_{54}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left[\{D_8\} \begin{Bmatrix} 0 \\ l/R \\ 0 \end{Bmatrix} + \{D_{32}\} \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right] d\left(\frac{e_{1P}}{R}\right)$$

$$\{E_{55}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left[\langle D_9 \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \langle D_{33} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right] d\left(\frac{e_{1P}}{R}\right)$$

$$\{E_{56}\} = \frac{1}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left[\langle D_{10} \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \langle D_{34} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right] d\left(\frac{e_{1P}}{R}\right)$$

$$\{F_1\} = [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} C_{AQX} \\ C_{ATY} \\ 0 \end{Bmatrix} + [\bar{\theta}_{i1}]^T [\bar{\theta}_{i0}]^T \left\{ \langle E_{30} \rangle + \langle E_{40} \rangle + \langle E_{50} \rangle \right\}$$

$$\{F_2\} = [\bar{\theta}_{i1}] [\bar{\theta}_{i0}] \left\{ \langle E_{31} \rangle + \langle E_{41} \rangle + \langle E_{51} \rangle \right\}$$

$$\{F_3\} = [\bar{\theta}_{i1}] [\bar{\theta}_{i0}] \left\{ \langle E_{32} \rangle + \langle E_{42} \rangle + \langle E_{52} \rangle \right\}$$

$$\{F_4\} = [\bar{\theta}_{i1}] [\bar{\theta}_{i0}] \left\{ \langle E_{33} \rangle + \langle E_{43} \rangle + \langle E_{53} \rangle \right\}$$

$$\{F_5\} = [\bar{\theta}_{i1}] [\bar{\theta}_{i0}] \left\{ \langle E_{34} \rangle + \langle E_{44} \rangle + \langle E_{54} \rangle \right\}$$

$$\{F_6\} = [\bar{\theta}_{i1}] [\bar{\theta}_{i0}] \left\{ \langle E_{35} \rangle + \langle E_{45} \rangle + \langle E_{55} \rangle \right\}$$

$$\{F_7\} = [\bar{\theta}_{i1}] [\bar{\theta}_{i0}] \left\{ \langle E_{36} \rangle + \langle E_{46} \rangle + \langle E_{56} \rangle \right\}$$

$$\{F_8\} = [\bar{\theta}_{i1}] [\bar{\theta}_{i0}] \left\{ \langle E_{37} \rangle + \langle E_{47} \rangle \right\}$$

$$\{F_9\} = [\bar{\theta}_{i1}] [\bar{\theta}_{i0}] \langle E_{48} \rangle$$

$$\left\{ F_{10} \right\} = \frac{\left[\theta_{10} \right]}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ D_{76} \right\} d\left(\frac{\ell_P}{R} \right)$$

$$\left\{ F_{11} \right\} = \frac{\left[\theta_{10} \right]}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left[\left\{ D_2 \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \left\{ D_5 \right\} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \right] d\left(\frac{\ell_P}{R} \right)$$

$$\left\{ F_{12} \right\} = \frac{\left[\theta_{10} \right]}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left[\left\{ D_3 \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \left\{ D_6 \right\} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \right] d\left(\frac{\ell_P}{R} \right)$$

$$\left\{ F_{13} \right\} = \frac{\left[\theta_{10} \right]}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left[\left\{ D_4 \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \left\{ D_7 \right\} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \right] d\left(\frac{\ell_P}{R} \right)$$

$$\left\{ F_{14} \right\} = \frac{\left[\theta_{10} \right]}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ D_8 \right\} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix}$$

$$\left\{ F_{15} \right\} = \frac{\left[\theta_{10} \right]}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ D_9 \right\} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} d\left(\frac{\ell_P}{R} \right)$$

$$\left\{ F_{16} \right\} = \frac{\left[\theta_{10} \right]}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ D_{10} \right\} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} d\left(\frac{\ell_P}{R} \right)$$

$$\left\{ F_{20} \right\} = \frac{\begin{bmatrix} \theta_{i1} \end{bmatrix} \begin{bmatrix} \theta_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_{P/R} \\ 0 \end{Bmatrix} \times \left\{ D_{76} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\left\{ F_{21} \right\} = \frac{\begin{bmatrix} \theta_{i1} \end{bmatrix} \begin{bmatrix} \theta_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_{P/R} \\ 0 \end{Bmatrix} \times \left\{ D_2 \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \right.$$

$$\left. + \left\{ D_5 \right\} \begin{Bmatrix} 0 \\ \ell_{P/R} \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\left\{ F_{22} \right\} = \frac{\begin{bmatrix} \theta_{i1} \end{bmatrix} \begin{bmatrix} \theta_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_{P/R} \\ 0 \end{Bmatrix} \times \left\{ D_3 \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \right.$$

$$\left. + \left\{ D_6 \right\} \begin{Bmatrix} 0 \\ \ell_{P/R} \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\left\{ F_{23} \right\} = \frac{\begin{bmatrix} \theta_{i1} \end{bmatrix} \begin{bmatrix} \theta_{i0} \end{bmatrix}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{i0} \end{bmatrix}^T \begin{bmatrix} \theta_{i1} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_{P/R} \\ 0 \end{Bmatrix} \times \left\{ D_4 \right\} \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \right.$$

$$\left. + \left\{ D_7 \right\} \begin{Bmatrix} 0 \\ \ell_{P/R} \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\left\{ F_{24} \right\} = \frac{\begin{bmatrix} \theta_{11} \end{bmatrix} \begin{bmatrix} \theta_{10} \end{bmatrix}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \times \right. \\ \left. \left\{ \begin{Bmatrix} D_8 \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\left\{ F_{25} \right\} = \frac{\begin{bmatrix} \theta_{11} \end{bmatrix} \begin{bmatrix} \theta_{10} \end{bmatrix}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \times \right. \\ \left. \left\{ \begin{Bmatrix} D_9 \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\left\{ F_{26} \right\} = \frac{\begin{bmatrix} \theta_{11} \end{bmatrix} \begin{bmatrix} \theta_{10} \end{bmatrix}}{\pi R^2 \rho} \int_0^{\ell/R} \mu_4 \left\{ \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \times \right. \\ \left. \left\{ \begin{Bmatrix} D_{10} \end{Bmatrix} \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\left\{ F_{30} \right\} = \frac{\begin{bmatrix} \theta_{11} \end{bmatrix} \begin{bmatrix} \theta_{10} \end{bmatrix}}{\pi R^2 \rho} \int_0^{\frac{e_1}{R}} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \times \right. \\ \left. \left\{ \begin{Bmatrix} D_{75} \end{Bmatrix} \right\} \right\} d\left(\frac{e_{1P}}{R}\right)$$

$$\begin{aligned}
\{F_{31}\} &= \frac{\begin{bmatrix} \theta_{11} \\ \theta_{10} \end{bmatrix}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \right. \\
&\quad \times \left. \left\{ \langle D_2 \rangle \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \langle D_5 \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \langle D_{29} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \right\} d\left(\frac{e_{1P}}{R}\right) \\
\{F_{32}\} &= \frac{\begin{bmatrix} \theta_{11} \\ \theta_{10} \end{bmatrix}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \right. \\
&\quad \times \left. \left\{ \langle D_3 \rangle \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \langle D_6 \rangle \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} + \langle D_{30} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \right\} d\left(\frac{e_{1P}}{R}\right)
\end{aligned}$$

$$\begin{aligned}
\{F_{33}\} &= \frac{\begin{bmatrix} \theta_{11} \\ \theta_{10} \end{bmatrix}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \right. \\
&\quad \times \left. \left\{ \langle D_4 \rangle \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} + \langle D_7 \rangle \begin{Bmatrix} 0 \\ \ell_P/R \\ 0 \end{Bmatrix} + \langle D_{31} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \right\} d\left(\frac{e_{1P}}{R}\right) \\
\{F_{34}\} &= \frac{\begin{bmatrix} \theta_{11} \\ \theta_{10} \end{bmatrix}}{\pi R^2 \rho} \int_0^{e_1/R} \mu_3 \left\{ \left\{ \begin{bmatrix} \theta_{10} \end{bmatrix}^T \begin{bmatrix} \theta_{11} \end{bmatrix}^T [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \right. \\
&\quad \times \left. \left\{ \langle D_8 \rangle \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + \langle D_{32} \rangle \begin{Bmatrix} 0 \\ 0 \\ e_{1P}/R \end{Bmatrix} \right\} \right\} d\left(\frac{e_{1P}}{R}\right)
\end{aligned}$$

$$\{F_{35}\} = \frac{[\theta_{11}][\theta_{10}]}{\pi R^2 \rho} \int_0^{e_{1/R}} \mu_3 \left\{ \left[\begin{array}{c} \theta_{10} \end{array} \right]^T \left[\begin{array}{c} \theta_{11} \end{array} \right]^T [\psi]^T \left\{ \begin{array}{c} 0 \\ 0 \\ e_{1P/R} \end{array} \right\} \right\} \\ \times \left\{ \left\{ D_9 \right\} \left\{ \begin{array}{c} 0 \\ \ell/R \\ 0 \end{array} \right\} + \left\{ D_{33} \right\} \left\{ \begin{array}{c} 0 \\ 0 \\ e_{1P/R} \end{array} \right\} \right\} d\left(\frac{e_{1P}}{R}\right)$$

$$\{F_{36}\} = \frac{[\theta_{11}][\theta_{10}]}{\pi R^2 \rho} \int_0^{e_{1/R}} \mu_3 \left\{ \left[\begin{array}{c} \theta_{10} \end{array} \right]^T \left[\begin{array}{c} \theta_{11} \end{array} \right]^T [\psi]^T \left\{ \begin{array}{c} 0 \\ 0 \\ e_{1P/R} \end{array} \right\} \right\} \\ \times \left\{ \left\{ D_{10} \right\} \left\{ \begin{array}{c} 0 \\ \ell/R \\ 0 \end{array} \right\} + \left\{ D_{34} \right\} \left\{ \begin{array}{c} 0 \\ 0 \\ e_{1P/R} \end{array} \right\} \right\} d\left(\frac{e_{1P}}{R}\right)$$

$$\{F_{50}\} = \frac{[\theta_{10}]}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\begin{array}{c} \theta_{10} \end{array} \right]^T \left\{ \begin{array}{c} L_P/R \\ 0 \\ 0 \end{array} \right\} \times \left\{ D_{40} \right\} \left\{ \begin{array}{c} L_P/R \\ 0 \\ 0 \end{array} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\{F_{51}\} = \frac{[\theta_{10}]}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\begin{array}{c} \theta_{10} \end{array} \right]^T \left\{ \begin{array}{c} L_P/R \\ 0 \\ 0 \end{array} \right\} + \left\{ D_2 \right\} \left\{ \begin{array}{c} L_P/R \\ 0 \\ 0 \end{array} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\{F_{52}\} = \frac{[\theta_{10}]}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\begin{array}{c} \theta_{10} \end{array} \right]^T \left\{ \begin{array}{c} L_P/R \\ 0 \\ 0 \end{array} \right\} \times \left\{ D_3 \right\} \left\{ \begin{array}{c} L_P/R \\ 0 \\ 0 \end{array} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

$$\{F_{53}\} = \frac{[\theta_{10}]}{\pi R^2 \rho} \int_0^{L/R} \mu_5 \left\{ \left[\begin{array}{c} \theta_{10} \end{array} \right]^T \left\{ \begin{array}{c} L_P/R \\ 0 \\ 0 \end{array} \right\} \times \left\{ D_4 \right\} \left\{ \begin{array}{c} L_P/R \\ 0 \\ 0 \end{array} \right\} \right\} d\left(\frac{\ell_P}{R}\right)$$

APPENDIX B

DERIVATION OF THE AERODYNAMIC FORCES AND MOMENTS

The forces and moments originating from aerodynamic effects are determined by the blade strip analysis technique. In this technique the airfoil is divided into elemental lengths. Each element is first assumed to be independent of the adjacent elements. The total forces and moments are determined by integration of each elemental force or moment over the total member length. The elemental moments are determined by the vector cross product of the moment arm and the appropriate elemental force. Making the second assumption namely that there is no flow along the direction of the blade length leads to the applicability of two-dimensional airfoil theory. Admittedly, these are two important but generally accepted assumptions; however, they admit access to an enormous amount of available experimental force data. This force data is presented in nondimensional coefficients as a function of Mach number, Reynolds number, airfoil dimensions, and angle-of-attack. The angle-of-attack is the angle between the airfoil cord line and the relative fluid velocity vector component in a plane perpendicular to elemental length direction. The relative velocity of an element is made up of the kinematic velocity of the system and the velocity of the origin. In this dissertation, aerodynamic forces are assumed to act on the rotating blade members only (i.e. neglect wing aerodynamics). The rotating blade elements will be subjected to the highest relative fluid

velocities and, therefore, they will produce the largest aerodynamic forces on the system. As will be seen later, it is a relatively simple matter to introduce additional aerodynamic forces into the system (e.g. to account for wing aerodynamics).

The kinematic velocity of a blade element has already been determined in Appendix A. Equation A7, Appendix A, is the first derivative with respect to (w.r.t) time of the absolute position vector of an ele-

ment of the blade $\begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix}$. This velocity must be added to the constant velocity of the origin $\begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$. The velocity of the origin is defined by

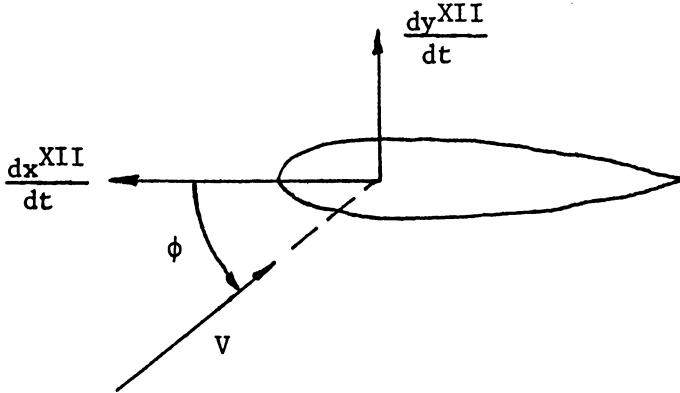
the initially assumed flight velocity of the vehicle. The flight velocity $\begin{Bmatrix} U \\ V \\ W \end{Bmatrix}$ is nondimensionalized by dividing by ΩR , the rotor blade rotational tip speed. Summing these velocities results in

$$\frac{1}{\Omega} \frac{d}{dt} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \underset{\text{RELATIVE BLADE}}{=} = \begin{Bmatrix} U/\Omega R \\ V/\Omega R \\ W/\Omega R \end{Bmatrix} + \frac{1}{\Omega} \frac{d}{dt} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \underset{\text{KINEMATIC}}{\quad} \quad (B1)$$

This velocity can be resolved into blade coordinates by the following transformation (see figure 11):

$$\frac{1}{\Omega} \frac{d}{dt} \begin{Bmatrix} x^{XII}/R \\ y^{XII}/R \\ z^{XII}/R \end{Bmatrix} = \frac{[\theta][\xi][\beta][\psi][\theta_{11}][\theta_{10}]}{\Omega} \frac{d}{dt} \begin{Bmatrix} x/R \\ y/R \\ z/R \end{Bmatrix} \underset{\text{RELATIVE BLADE}}{\quad} \quad (B2)$$

The following sketch shows the blade coordinates and blade angle ϕ .



$$\tan \phi = - \frac{\frac{d}{dt} \left(\frac{y^{XII}}{R} \right)}{\frac{d}{dt} \left(\frac{x^{XII}}{R} \right)} \quad (B3)$$

The elemental airfoil section angle-of-attack α becomes

$$\alpha = - \arctan \left\{ \frac{\frac{d}{dt} \left(\frac{y^{XII}}{R} \right)}{\frac{d}{dt} \left(\frac{x^{XII}}{R} \right)} \right\} - \theta_1 \left(\frac{z^{XII}}{R} \right) \quad (B4)$$

when θ_1 is the blade twist distribution and is a constant, and z^{XII}/R is the nondimensionalized spanwise blade coordinate (in the direction of the elemental blade length). The elemental airfoil velocity in a plane perpendicular to the elemental length direction is

$$\left(\frac{v}{\Omega R}\right)_{\text{ELEMENT}}^2 = \left[\frac{1}{\Omega} \frac{d}{dt} \left(\frac{x^{\text{XII}}}{R}\right)\right]^2 + \left[\frac{1}{\Omega} \frac{d}{dt} \left(\frac{y^{\text{XII}}}{R}\right)\right]^2 \quad (\text{B5})$$

and the Mach number then becomes

$$M = \frac{\left(\frac{v}{\Omega R}\right)_{\text{ELEMENT}} (\Omega R)}{a} \quad (\text{B6})$$

where "a" is the speed of sound. The force coefficients can now be determined from numerous charts and/or curves (e.g., refs. [9, 10 and 11]), from the angle-of-attack α and Mach number M.

The blade pilot control parameter θ_s is developed in this appendix on page 153. This parameter can be coupled with numerous parameters to give various feedback systems.

From two-dimensional airfoil theory, force is defined by

$$dL = \frac{1}{2} \rho V^2 C_i c dr \quad (\text{B7})$$

where

ρ = fluid mass density, lb-sec² per ft⁴

V = relative fluid velocity, ft/sec

c = chord of airfoil, ft

dr = elemental length of blade, ft

C_i = force coefficient determined from experimental data
($i = L$ for lift coefficient, $i = D$ for drag coefficient)

dL = elemental force, lb

The force equation (B7) is nondimensionalized by dividing by $\pi R^2 \rho (\Omega R)^2$ resulting in

$$dC_L = \frac{dL}{\pi R^2 \rho (\Omega R)^2} = \frac{1}{2} \frac{cR}{\pi R^2} \left(\frac{V}{\Omega R} \right)^2 C_L d \left(\frac{z^{XII}}{R} \right) \quad (B8)$$

Letting $\sigma' = \frac{1}{2} \frac{cR}{\pi R^2}$ results in

$$dC_L = \sigma' \left(\frac{V}{\Omega R} \right)^2 C_L d \left(\frac{z^{XII}}{R} \right) \quad (B9)$$

and similarly for the drag force

$$dC_D = \sigma' \left(\frac{V}{\Omega R} \right)^2 C_D d \left(\frac{z^{XII}}{R} \right) \quad (B10)$$

Since these elemental forces are parallel and perpendicular to the relative velocity vector, they are resolved into blade coordinates as follows:

$$dC_{ATy}^{XII} = dC_L \cos \phi - dC_D \sin \phi \quad (B11)$$

$$dC_{AQx}^{XII} = -dC_L \sin \phi - dC_D \cos \phi$$

Integrating along the blade to determine the total aerodynamic forces results in

$$C_{ATy}^{XII} = \sigma' \left\{ \int_{x_c}^{B(1 - e_1/R - e_2/R)} \left(\frac{V}{\Omega R} \right)^2 C_L \cos \phi d \left(\frac{z^{XII}}{R} \right) \right.$$

(Continued on next page)

$$\begin{aligned}
& - \int_0^{1.0 - e_1/R - e_2/R} \left(\frac{V}{\Omega R} \right)^2 C_D \sin \phi \, d \left(\frac{z^{XII}}{R} \right) \Bigg\} \\
C_{AQx}^{XII} = \sigma' & \left\{ - \int_{x_o}^{B(1 - e_1/R - e_2/R)} \left(\frac{V}{\Omega R} \right)^2 C_L \sin \phi \, d \left(\frac{z^{XII}}{R} \right) \right. \\
& \left. - \int_0^{1.0 - e_1/R - e_2/R} \left(\frac{V}{\Omega R} \right)^2 C_D \cos \phi \, d \left(\frac{z^{XII}}{R} \right) \right\} \quad (B12)
\end{aligned}$$

The airfoil will have an effective drag force along its complete length, namely, $x_c \leq \frac{z^{XII}}{R} \leq (1 - e_1/R - e_2/R)$, whereas the lift force will not exist unless the airfoil extends along the complete span. Because of structural reasons, the airfoil does not extend to zero, and because of three-dimensional effects, the lift force does not extend to the blade tip but to about $BR \approx 0.98R$. Therefore the limits of the lift integration are $x_c R \leq z^{XII} \leq B(R - e_1 - e_2)$.

The aerodynamic moments are obtained by multiplying the force by a moment arm, z^{XII}/R , resulting in

$$C_{AMx}^{XII} = \sigma' \left\{ - \int_{x_c}^{B(1 - e_1/R - e_2/R)} \left(\frac{V}{\Omega R} \right)^2 C_L \cos \phi \left(\frac{z^{XII}}{R} \right) d \left(\frac{z^{XII}}{R} \right) \right.$$

(Continued on next page)

$$\begin{aligned}
& + \int_0^{1.0 - e_1/R - e_2/R} \left(\frac{V}{\Omega R} \right)^2 C_D \sin \phi \left(\frac{z^{XII}}{R} \right) d \left(\frac{z^{XII}}{R} \right) \Bigg\} \\
C_{AQy}^{XII} = \sigma' & \left\{ - \int_{x_0}^{B(1 - e_1/R - e_2/R)} \left(\frac{V}{\Omega R} \right)^2 C_L \sin \phi \left(\frac{z^{XII}}{R} \right) d \left(\frac{z^{XII}}{R} \right) \right. \\
& \left. - \int_0^{1.0 - e_1/R - e_2/R} \left(\frac{V}{\Omega R} \right)^2 C_D \cos \phi \left(\frac{z^{XII}}{R} \right) d \left(\frac{z^{XII}}{R} \right) \right\}
\end{aligned} \tag{B13}$$

The total aerodynamic force matrix becomes

$$\begin{Bmatrix} C_{ADx}^{XII} \\ C_{ATy}^{XII} \\ 0 \end{Bmatrix} \tag{B14}$$

and the moment matrix becomes

$$\begin{Bmatrix} C_{AMx}^{XII} \\ C_{AQy}^{XII} \\ 0 \end{Bmatrix} \tag{B15}$$

To incorporate induced flow effects and a time dependent pitch angle, the following assumptions are made: (1) assume uniform inflow, and (2) assume the inflow is perpendicular to the control (swash plate) plane. The blade pitch angle, θ_s , defined w. r. t. the shaft is

$$\theta_s = \theta_{os} + \theta_{lsc} \cos \psi + \theta_{lss} \sin \psi$$

the maximum value of θ_s occurs when $\psi = P_o$ determined from

$$\tan P_o = \frac{\theta_{lss}}{\theta_{lsc}} \quad (B16)$$

The maximum angle between the shaft axis and a perpendicular to the control axis, $\Delta\theta$, becomes

$$\Delta\theta = \sqrt{\theta_{lsc}^2 + \theta_{lss}^2} \quad (B17)$$

and the induced velocity component along the shaft axes (coordinates x^{VI} , y^{VI} , z^{VI}) become

$$v_{SHAFT} = -v \cos \Delta\theta \quad (B18)$$

$$v_{\perp SHAFT} = v \sin \Delta\theta$$

Further, resolving the induced velocity component perpendicular to the shaft into the shaft axis results in the following induced velocity components:

$$\{\text{induced velocity}\}^{\text{VI}} = \begin{Bmatrix} v \sin \Delta\theta \sin P_o \\ -v \cos \Delta\theta \\ v \sin \Delta\theta \cos P_o \end{Bmatrix} \quad (\text{B19})$$

This is then rotated into blade axis coordinates and added to the relative blade velocities as follows:

$$\begin{Bmatrix} \frac{dx_v^{\text{XII}}}{dt} \\ \frac{dy_v^{\text{XII}}}{dt} \\ \frac{dz_v^{\text{XII}}}{dt} \end{Bmatrix} = [\theta][\xi][\beta][\psi] \{\text{induced velocity}\}^{\text{VI}} \quad (\text{B20})$$

$$\begin{Bmatrix} \frac{dx^{\text{XII}}}{dt} \\ \frac{dy^{\text{XII}}}{dt} \\ \frac{dz^{\text{XII}}}{dt} \end{Bmatrix} = \begin{Bmatrix} \frac{dx^{\text{XII}}}{dt} \\ \frac{dy^{\text{XII}}}{dt} \\ \frac{dz^{\text{XII}}}{dt} \end{Bmatrix}_{\text{OLD}} + \begin{Bmatrix} \frac{dx_v^{\text{XII}}}{dt} \\ \frac{dy_v^{\text{XII}}}{dt} \\ \frac{dz_v^{\text{XII}}}{dt} \end{Bmatrix} \quad (\text{B21})$$

The above modification is incorporated into the computer program in a loop that operates as follows.

Assume an initial guess for v , called v_0 and go through the above equations until C_{ATyXII} (equation B12) is computed. Compute a new value for v from

$$v = \frac{1/2(C_{ATyXII})}{\left\{ \left(\frac{dx^{VI}}{dt} \right)^2 + \left(\frac{dy^{VI}}{dt} \right)^2 + \left(\frac{dz^{VI}}{dt} \right)^2 \right\}^{1/2}} \quad (B22)$$

where C_{ATyXII} is the last computed value. Introduce this new value for v into equation (B18) and repeat until a new value of C_{ATyXII} (equation B-12) is obtained. This value is compared with the previous value (C_{ATyXII}) and if it passes a test criteria, the computer goes on to the next blade. If it does not pass, it continues in the above loop. Once the blade has moved to a new azimuth position ($\Delta\psi$), the computer uses the previously computed value of C_{ATyXII} (at ψ) to compare with (at $\psi + \Delta\psi$).

APPENDIX C

DERIVATION OF THE GOVERNING EQUATIONS OF MOTION

In this appendix the results of the two previous appendices are used to obtain the governing equation of motion. The governing equations are obtained by assuming that the only moments transmitted across the mechanical hinge joints can be defined by an equivalent spring and viscous damper. These springs and dampers are assumed to have linear characteristics. Although in the electronic computer programming, it would be a relatively simple matter to introduce nonlinear characteristics. The hinge location and spring constant can be used to define an equivalent elastic system by matching the first bending mode shape and fundamental natural frequency. For example take an elastic system, determine its fundamental natural frequency and mode shape, now find a hinge location and spring constant of a rigid system such that the actual natural frequency and mode shape are closely matched.

C1 - Satisfying The Boundary Conditions At The Lag Hinge

The first governing equation is determined by equating the external moments to the inertial moments across the first hinge, namely the lag hinge. The inertial moment about the lag hinge have already been derived in appendix A. Also the aerodynamic moments about the lag hinge have been derived in appendix B. The inertial moments in terms of XI- coordinates are given in equation (A18). The aerodynamic moments are given in XII- coordinates in equation (B15), this moment is transformed to XI- coordinates by

$$\begin{Bmatrix} C_{AMx}^{XI} \\ C_{AMy}^{XI} \\ C_{AMz}^{XI} \end{Bmatrix} = [\theta]^T \begin{Bmatrix} C_{AMx}^{XII} \\ C_{AMy}^{XII} \\ 0 \end{Bmatrix} \quad (C1)$$

Since the lag hinge is aligned with the y^{XI} axis, all moments about this hinge must be summed and equated to the inertial moments. Letting K_ξ be the spring constant and C_ξ be the viscous damper constant across the lag hinge results in the following governing equation for the ξ variable.

$$\begin{Bmatrix} C_{AMx}^{XI} \\ C_{AMy}^{XI} \\ C_{AMz}^{XI} \end{Bmatrix}_{y \text{ component}} - \begin{Bmatrix} C_{IMx}^{XI} \\ C_{IMy}^{XI} \\ C_{IMz}^{XI} \end{Bmatrix}_{y \text{ component}} - K_\xi \xi - C_\xi \left(\frac{1}{\Omega} \frac{d\xi}{dt} \right) = 0 \quad (C2)$$

Expanding this equation results in

$$\begin{aligned} & C_{AMx}^{XII} \theta_s + C_{AOy}^{XII} \theta_c - \{E_{10}\}_{y \text{ only}} - \bar{\theta}_{xo} \{E_{11}\}_{y \text{ only}} - \bar{\theta}_{yo} \{E_{12}\}_{y \text{ only}} \\ & - \bar{\theta}_{zo} \{E_{13}\}_{y \text{ only}} - \bar{\theta}_{x1} \{E_{14}\}_{y \text{ only}} - \bar{\theta}_{y1} \{E_{15}\}_{y \text{ only}} - \bar{\theta}_{z1} \{E_{16}\}_{y \text{ only}} \\ & - \bar{\beta} \{E_{17}\}_{y \text{ only}} - \bar{\xi} \{E_{18}\}_{y \text{ only}} - K_\xi \xi - C_\xi \bar{\xi} = 0 \end{aligned} \quad (C3)$$

where $\bar{\xi} = \frac{d\xi}{d\psi}$. At this point no mention was made of a particular blade, therefore equation (C3) is applicable to any blade and a similar equation will exist for each blade in accordance with the discussion on page 110 of appendix A.

C2 - Satisfying The Boundary Conditions At The Flap Hinge

The second governing equation is obtained by summing external moments to the inertial moments about the flapping hinge. Here, we must include the moments transferred across the lag hinge, the inertial moments from member "e₂", and the moments due to the shear forces at the lag hinge. Combining the blade root aerodynamic and inertial shears acting at the lag hinge results in:

$$\begin{Bmatrix} C_{AIFx}^{XII} \\ C_{AIFy}^{XII} \\ C_{AIFz}^{XII} \end{Bmatrix} = - \begin{Bmatrix} C_{IFx}^{XII} \\ C_{IFy}^{XII} \\ C_{IFz}^{XII} \end{Bmatrix} + \begin{Bmatrix} C_{AQx}^{XII} \\ C_{ATy}^{XII} \\ 0 \end{Bmatrix} \quad (C4)$$

resolving this into IX-coordinates results in

$$\begin{Bmatrix} C_{AIFx}^{IX} \\ C_{AIFy}^{IX} \\ C_{AIFz}^{IX} \end{Bmatrix} = [\xi]^T [\theta]^T \begin{Bmatrix} C_{AIFx}^{XII} \\ C_{AIFy}^{XII} \\ C_{AIFz}^{XII} \end{Bmatrix} \quad (C5)$$

and to obtain the moments we take the vector cross product of the moment arm and shear force

$$\begin{Bmatrix} C_{AIMxIX} \\ C_{AIMyIX} \\ C_{AIMzIX} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ e_2/R \end{Bmatrix} \times \begin{Bmatrix} C_{AIFxIX} \\ C_{AIFyIX} \\ C_{AIFzIX} \end{Bmatrix} \quad (C6)$$

The moment transferred across the lag hinge becomes

$$\begin{Bmatrix} C_{TRMxXI} \\ C_{TRMyXI} \\ C_{TRMzXI} \end{Bmatrix} = \begin{Bmatrix} C_{AMxXI} \\ 0 \\ C_{AMzXI} \end{Bmatrix} - \begin{Bmatrix} C_{IMxXI} \\ 0 \\ C_{IMzXI} \end{Bmatrix} + \begin{Bmatrix} 0 \\ K_{\xi}\xi + C_{\xi}\bar{\xi} \\ 0 \end{Bmatrix}$$

$$= \begin{Bmatrix} C_{AMxXI} - C_{IMxXI} \\ K_{\xi}\xi + C_{\xi}\bar{\xi} \\ C_{AMzXI} - C_{IMzXI} \end{Bmatrix} \quad (C7)$$

and transferring to IX-coordinates

$$\begin{Bmatrix} C_{TRMxIX} \\ C_{TRMyIX} \\ C_{TRMzIX} \end{Bmatrix} = [\xi]^T \begin{Bmatrix} C_{AMxXI} - C_{IMxXI} \\ K_{\xi}\xi + C_{\xi}\bar{\xi} \\ C_{AMzXI} - C_{IMzXI} \end{Bmatrix} \quad (C8)$$

The inertial moments originating from the distributed mass of member "e₂" was derived in appendix A equation (A27) and is given below.

$$\begin{Bmatrix} C_{IMe2xIX} \\ C_{IMe2yIX} \\ C_{IMe2zIX} \end{Bmatrix} = [\beta][\psi][\theta_{11}][\theta_{10}] \begin{Bmatrix} C_{IMe2x} \\ C_{IMe2y} \\ C_{IMe2z} \end{Bmatrix} \quad (C9)$$

Combining these three moments together, equations (C6), (C8), and (C9), we arrive at

$$\begin{Bmatrix} C_{TMxIX} \\ C_{TMyIX} \\ C_{TMzIX} \end{Bmatrix} = \begin{Bmatrix} C_{AIMxIX} \\ C_{AIMyIX} \\ C_{AIMzIX} \end{Bmatrix} + \begin{Bmatrix} C_{TRMxIX} \\ C_{TRMyIX} \\ C_{TRMzIX} \end{Bmatrix} - \begin{Bmatrix} C_{IMe2xIX} \\ C_{IMe2yIX} \\ C_{IMe2zIX} \end{Bmatrix} \quad (C10)$$

The blade flapping hinge is aligned with the x^{IX} axis. Combining moments about the flapping hinge and equating to the flapping spring and damper moments results in:

$$C_{AIMx}^{IX} + C_{TRMx}^{IX} - C_{IMe2x}^{IX} \quad (C11)$$

$$- K_{\beta} \beta - C_{\beta} \bar{\beta} = 0$$

where

$$\bar{\beta} = \frac{d\beta}{d\psi}$$

$$K_{\beta} = \frac{\text{flapping spring constant}}{\pi R^2 \rho (\Omega R)^2 R}$$

$$C_{\beta} = \frac{\Omega (\text{flapping damping constant})}{\pi R^2 \rho (\Omega R)^2 R}$$

This equation is the second set of governing differential equations.

It is a set of equations because it applies to each blade of the rotor system. In other words, there will be one differential equation for each and every rotor blade in the system. The remaining moments will be transferred across this hinge, namely

$$\left\{ \begin{array}{c} C_{TMx}^{IX} \\ C_{TMy}^{IX} \\ C_{TMz}^{IX} \end{array} \right\} \quad (C12)$$

C3 - Satisfying The Boundary Conditions At The Wing-drive-shaft Joint

The third basic governing equation is obtained by satisfying the hinge boundary conditions at the wing-drive-shaft joint. As before the boundary conditions at the hinge are that the sum of the external moments must be equated to the inertial moments. These hinge moments consist of the following:

- (1) moments transferred across the flapping hinge of each blade, equation (C12)
- (2) moments due to the aerodynamic and inertial shear forces acting at the flapping hinge
- (3) moments due to the inertial shears of member " e_1 " acting on the rotor end of the shaft axis
- (4) inertial moments of member " e_1 " due to its mass distribution equation (A35)
- (5) inertial moment due to the mass distribution of member " ℓ ," equation (A45)
- (6) spring and damper moments across the wing-drive-shaft joint.

The moments transferred across the flapping hinge, equation (C12) transferred into the VI-coordinate system becomes:

$$\begin{Bmatrix} C_{TMx}^{VI} \\ C_{TMy}^{VI} \\ C_{TMz}^{VI} \end{Bmatrix} = [\psi]^T [\beta]^T \begin{Bmatrix} C_{TMx}^{IX} \\ C_{TMy}^{IX} \\ C_{TMz}^{IX} \end{Bmatrix} \quad (C13)$$

This moment must be summed over all the blades.

The moment due to the aerodynamic and inertial shear forces acting at the flapping hinge is equal to the vector cross product of the moment arm, members " e_1 " plus " ℓ " and the total shear force acting at the flap hinge. The total shear force per blade at the flapping hinge is the vector sum of the total shear force at the lag hinge, equation (C5), and the inertial shear from member " e_2 ", derived in appendix A equation (A22). Transforming these shears into similar coordinates and adding results in the total flap hinge shear forces:

$$\begin{Bmatrix} C_{Fx}^{VIII} \\ C_{Fy}^{VIII} \\ C_{Fz}^{VIII} \end{Bmatrix} = [\beta]^T \begin{Bmatrix} C_{AIFx}^{IX} \\ C_{AIFy}^{IX} \\ C_{AIFz}^{IX} \end{Bmatrix} - \begin{Bmatrix} C_{Ie2x}^{VIII} \\ C_{Ie2y}^{VIII} \\ C_{Ie2z}^{VIII} \end{Bmatrix} \quad (C14)$$

The shaft moment per blade for the total flap hinge shear force becomes:

$$\begin{Bmatrix} C_{FMx}^{VI} \\ C_{FMy}^{VI} \\ C_{FMz}^{VI} \end{Bmatrix} = \left\{ \begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} + [\psi]^T \begin{Bmatrix} 0 \\ 0 \\ e_1/R \end{Bmatrix} \right\} \times \left\{ [\psi]^T \begin{Bmatrix} C_{Fx}^{VIII} \\ C_{Fy}^{VIII} \\ C_{Fz}^{VIII} \end{Bmatrix} \right\} \quad (C15)$$

Where the moment arm is the vector sum of members " ℓ " and " e_1 ". This moment must be summed over the total number of blades.

The moment due to the inertial shears of member " e_1 " is equal to the vector cross product of the moment arm, member " ℓ ", and the inertial shears from member e_1 , equation (A31).

Performing these operations results in:

$$\begin{Bmatrix} 0 \\ \ell/R \\ 0 \end{Bmatrix} \times \begin{Bmatrix} C_{Ielx}^{VI} \\ C_{Iely}^{VI} \\ C_{Ielz}^{VI} \end{Bmatrix} \quad (C16)$$

Summing all the moments at the wing-drive-shaft joint between numbers " ℓ " and " L " and redefining this sum results in:

$$\begin{Bmatrix} C_{Mx}^{VI} \\ C_{My}^{VI} \\ C_{Mz}^{VI} \end{Bmatrix} = \sum_{\text{BLADES}} \begin{Bmatrix} C_{TMx}^{VI} \\ C_{TM_y}^{VI} \\ C_{TMz}^{VI} \end{Bmatrix}$$

(Continued on next page)

$$+ \left[\begin{array}{c} 0 \\ \ell/R \\ 0 \end{array} \right] + [\psi]^T \left[\begin{array}{c} 0 \\ 0 \\ e_1/R \end{array} \right] \times \left[[\psi]^T \left[\begin{array}{c} C_{Fx}^{VIII} \\ C_{Fy}^{VIII} \\ C_{Fz}^{VIII} \end{array} \right] \right] \quad (C17)$$

$$- \left[\begin{array}{c} 0 \\ \ell/R \\ 0 \end{array} \right] \times \left[\begin{array}{c} C_{Ielx}^{VI} \\ C_{Iely}^{VI} \\ C_{Ielz}^{VI} \end{array} \right] - \left[\begin{array}{c} C_{Melx}^{VI} \\ C_{Mely}^{VI} \\ C_{Melz}^{VI} \end{array} \right] - \left[\begin{array}{c} C_{Mlx}^{VI} \\ C_{Mly}^{VI} \\ C_{Mlz}^{VI} \end{array} \right]$$

Equating the z component of this moment to the spring and damper moment results in the third governing equation:

$$C_{Mz}^{VI} - K_{z1} \theta_{z1} - C_{z1} \bar{\theta}_{z1} = 0 \quad (C18)$$

where K_{z1} and C_{z1} have again been nondimensionalized. The moments transferred across this joint to the V -coordinates are

$$[\theta_{z1}]^T \left[\begin{array}{c} C_{Mx}^{VI} \\ C_{My}^{VI} \\ K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1} \end{array} \right]$$

Transforming into IV-coordinates and setting the y component equal to the spring and damper moment results in:

$$[\theta_{y1}]^T [\theta_{z1}]^T \left\{ \begin{array}{c} C_{Mx}^{VI} \\ C_{My}^{VI} \\ K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_z \end{array} \right\}_{y \text{ only}} - K_{y1} \theta_{y1} - C_{y1} \bar{\theta}_{y1} = 0$$

or

$$\theta_{z1s} C_{Mx}^{VI} + \theta_{z1c} C_{My}^{VI} - K_{y1} \theta_{y1} - C_{y1} \bar{\theta}_{y1} = 0 \quad (C19)$$

This is the fourth general governing equation of motion. The moment transferred across this joint to the IV-coordinates become:

$$[\theta_{y1}]^T \left\{ \begin{array}{c} \theta_{z1c} C_{Mx}^{VI} - \theta_{z1s} C_{My}^{VI} \\ K_{y1} \theta_{y1} + C_{y1} \bar{\theta}_{y1} \\ K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1} \end{array} \right\}$$

or

$$\left\{ \begin{array}{c} \theta_{y1c} (\theta_{z1c} C_{Mx}^{VI} - \theta_{z1s} C_{My}^{VI}) + \theta_{y1s} (K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1}) \\ K_{y1} \theta_{y1} + C_{y1} \bar{\theta}_{y1} \\ -\theta_{y1s} (\theta_{z1c} C_{Mx}^{VI} - \theta_{z1s} C_{My}^{VI}) + \theta_{y1c} (K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1}) \end{array} \right\}$$

and transforming to the III-coordinate system results in:

$$[\theta_{x1}]^T \left\{ \begin{array}{l} \theta_{y1c} (\theta_{z1c} C_{Mx}^{VI} - \theta_{z1s} C_{My}^{VI}) + \theta_{y1s} (K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1}) \\ K_{y1} \theta_{y1} + C_{y1} \bar{\theta}_{y1} \\ -\theta_{y1s} (\theta_{z1c} C_{Mx}^{VI} - \theta_{z1s} C_{My}^{VI}) + \theta_{y1c} (K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1}) \end{array} \right\}$$

Summing moments about the x^{III} axis to zero results in the equation of motion for θ_{x1} namely:

$$\begin{aligned} & \theta_{y1c} (\theta_{z1c} C_{Mx}^{VI} - \theta_{z1s} C_{My}^{VI}) + \theta_{y1s} (K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1}) \\ & - K_{x1} \theta_{x1} - C_{x1} \bar{\theta}_{x1} = 0 \end{aligned} \quad (C20)$$

This equation is the fifth general governing equation of motion and it is the primary governing equation for the θ_{x1} variable. Each moment coefficient contains acceleration terms of all the variables (except the blade variables β and ξ) and therefore each governing equation will have an influence on each of the variables. In fact the governing equations must be solved simultaneously. The governing equations for each blade include only the blade variables for that blade (β , ξ).

C4 - Satisfying The Boundary Conditions At The Wing-root-fuselage Joint

The last set of governing equations are obtained by satisfying the zero hinge moment requirement at the origin of coordinates.

The moment at this hinge consists of the following:

- (1) moment transferred across the wing-drive-shaft joint
- (2) moment due to shear forces acting at the wing-drive-shaft joint
- (3) inertial moments due to the mass distribution of member "L", equation (A48)
- (4) the moments associated with the springs and damper at the wing-root-fuselage joint.

The moments transferred across the hinge in III-coordinates are:

$$\left\{ \begin{array}{l} K_{x1} \theta_{x1} + C_{x1} \bar{\theta}_{x1} \\ \theta_{x1c} (K_{y1} \theta_{y1} + C_{y1} \bar{\theta}_{y1}) - \theta_{x1s} \left[-\theta_{y1s} (\theta_{z1c} C_{Mx}^{VI} - \theta_{z1s} C_{My}^{VI}) + \theta_{y1c} (K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1}) \right] \\ \theta_{x1s} (K_{y1} \theta_{y1} + C_{y1} \bar{\theta}_{y1}) + \theta_{x1c} \left[-\theta_{y1s} (\theta_{z1c} C_{Mx}^{VI} - \theta_{z1s} C_{My}^{VI}) + \theta_{y1c} (K_{z1} \theta_{z1} + C_{z1} \bar{\theta}_{z1}) \right] \end{array} \right\}$$

redefining this matrix as follows:

$$\begin{Bmatrix} C_{TRMx}^{III} \\ C_{TRMy}^{III} \\ C_{TRMz}^{III} \end{Bmatrix}$$

The moment due to the shear forces is equal to the vector cross product of the moment arm, member "L", and the shear force at the wing-drive-shaft joint. The total shear force acting at this joint is equal to the sum of:

- (1) the total shear force at the flap hinge, equation (C14)
- (2) inertial shear of member "e₁", equation (A31)
- (3) inertial shear of member "ℓ", equation (A40).

The moment due to the above shear forces is:

$$\begin{Bmatrix} C_{IMLx}^{III} \\ C_{IMLy}^{III} \\ C_{IMLz}^{III} \end{Bmatrix} = \begin{Bmatrix} L/R \\ 0 \\ 0 \end{Bmatrix} \times \left\{ - \begin{Bmatrix} C_{Flx}^{III} \\ C_{Fly}^{III} \\ C_{Flz}^{III} \end{Bmatrix} + [\theta_{11}]^T \sum_{\text{BLADES}} \left[[\psi]^T \begin{Bmatrix} C_{Fx}^{VIII} \\ C_{Fy}^{VIII} \\ C_{Fz}^{VIII} \end{Bmatrix} - \begin{Bmatrix} C_{Ielx}^{VI} \\ C_{Iely}^{VI} \\ C_{Ielz}^{VI} \end{Bmatrix} \right] \right\}$$

Combining all of these moments, and setting the z component equal to the spring and damper moment results in:

$$[\theta_{zo}]^T \left\{ \begin{matrix} C_{TRMxIII} \\ C_{TRMyIII} \\ C_{TRMzIII} \end{matrix} \right\} + \left\{ \begin{matrix} C_{IMLxIII} \\ C_{IMLyIII} \\ C_{IMLzIII} \end{matrix} \right\}_{z \text{ only}} - [\theta_{yo}][\theta_{xo}] \left\{ \begin{matrix} C_{MLx} \\ C_{MLy} \\ C_{MLz} \end{matrix} \right\}_{z \text{ only}}$$

$$- K_{zo} \theta_{zo} - C_{zo} \bar{\theta}_{zo} = 0$$

or

$$C_{TRMzIII} + C_{IMLzIII} - \theta_{yos} C_{MLx} - \theta_{yoc} \left(-\theta_{xos} C_{MLy} + \theta_{xoc} C_{MLz} \right)$$

$$- K_{zo} \theta_{zo} - C_{zo} \bar{\theta}_{zo} = 0 \quad (C21)$$

The moment transferred across the θ_{zo} hinge in II-coordinates become:

$$\left\{ \begin{matrix} \theta_{zoc} (C_{TRMxIII} + C_{IMLxIII}) - \theta_{zos} (C_{TRMyIII} + C_{IMLyIII}) \\ \theta_{zos} (C_{TRMxIII} + C_{IMLxIII}) + \theta_{zoc} (C_{TRMyIII} + C_{IMLyIII}) \\ 0 \end{matrix} \right\} - \left\{ \begin{matrix} \theta_{yoc} C_{MLx} - \theta_{yos} (-\theta_{xos} C_{MLy} + \theta_{xoc} C_{MLz}) \\ \theta_{xoc} C_{MLy} + \theta_{xos} C_{MLz} \\ 0 \end{matrix} \right\} + \left\{ \begin{matrix} 0 \\ 0 \\ K_{zo} \theta_{zo} + C_{zo} \bar{\theta}_{zo} \end{matrix} \right\}$$

Equating the y component of this moment to the θ_{yo} spring and damper moment results in one more governing equation.

$$\theta_{zos} (C_{TRMx}^{III} + C_{IMLx}^{III}) + \theta_{zoc} (C_{TRMy}^{III} + C_{TMyl}^{III}) \quad (C22)$$

$$- \theta_{xoc} C_{MLy} - \theta_{xos} C_{MLz} - K_{yo} \theta_{yo} - C_{yo} \bar{\theta}_{yo} = 0$$

The transferred moments in II-coordinates are:

$$\left\{ \begin{array}{l} \theta_{zoc} (C_{TRMx}^{III} + C_{IMLx}^{III}) - \theta_{zos} (C_{TRMy}^{III} + C_{IMLy}^{III}) \\ K_{yo} \theta_{yo} + C_{yo} \bar{\theta}_{yo} \\ K_{zo} \theta_{zo} + C_{zo} \bar{\theta}_{zo} \end{array} \right\}$$

and transforming into I-coordinate system results in:

$$[\theta_{yo}]^T \left\{ \begin{array}{l} \theta_{zoc} (C_{TRMx}^{III} + C_{IMLx}^{III}) - C_{zos} (C_{TRMy}^{III} + C_{IMLy}^{III}) \\ K_{yo} \theta_{yo} + C_{yo} \bar{\theta}_{yo} \\ K_{zo} \theta_{zo} + C_{zo} \bar{\theta}_{zo} \end{array} \right\}$$

or

$$\left\{ \begin{array}{l} \left\{ \begin{array}{l} \theta_{yoc} \left[\theta_{zoc} (C_{TRMxIII} + C_{IMLxIII}) - \theta_{zos} (C_{TRMyIII} + C_{IMLyIII}) \right] \\ + \theta_{yos} (K_{zo} \theta_{zo} + C_{zo} \bar{\theta}_{zo}) \end{array} \right\} \\ K_{yo} \theta_{yo} + C_{yo} \bar{\theta}_{yo} \\ \left\{ \begin{array}{l} -\theta_{yos} \left[\theta_{zoc} (C_{TRMxIII} + C_{IMLxIII}) - \theta_{zos} (C_{TRMyIII} + C_{IMLyIII}) \right] \\ + \theta_{yoc} (K_{zo} \theta_{zo} + C_{zo} \bar{\theta}_{zo}) \end{array} \right\} \end{array} \right\}$$

Combining the x component of these moments to the θ_{xo} spring and damper moments results in the final governing equation of motion.

$$\begin{aligned} & \theta_{yoc} \left[\theta_{zoc} (C_{TRMxIII} + C_{IMLxIII}) - \theta_{zos} (C_{TRMyIII} + C_{IMLyIII}) \right] \\ & + \theta_{yos} (K_{zo} \theta_{zo} + C_{zo} \bar{\theta}_{zo}) - K_{xo} \theta_{xo} - C_{xo} \bar{\theta}_{xo} = 0 \end{aligned} \quad (C23)$$

C5 - Governing Equations

Each blade of the rotor system is associated with two degrees of freedom, namely β and ξ , and associated with each blade there are two governing equations, namely (C3) and (C11). Each blade motion is independent of the other blades. Therefore, there will be two additional degrees of freedom and two additional differential equations (C3 and C11) for each additional blade.

In addition to the blade degrees of freedom there are six other degrees of freedom, namely θ_{xo} , θ_{yo} , θ_{zo} , θ_{x1} , θ_{y1} , and θ_{z1} . The governing six differential equations to be satisfied for these variables are presented in equations (C18) through (C23). The governing equations are second order ordinary coupled differential equations and they must be solved simultaneously.

APPENDIX D

BASIC PHYSICAL CHARACTERISTICS

The basic rotor system used for this study is typical of existing present-day state-of-the-art practices. In fact, a group of proposed tilt-rotor designs were tabulated and summarized. This result is reflected in the basic rotor chosen herein. The assumed and computed physical properties are derived and summarized at the end of this section.

Assume a rotor radius, R , of 20 feet and a rotor system with three blades ($b = 3$). A typical rotor solidity, σ , equals 0.09. By definition:

$$\sigma = \frac{bcR}{\pi R^2}$$

where c is the blade chord dimension. Therefore,

$$c = \frac{\sigma \pi R}{b}$$

$$= \frac{.09 \pi 20}{3}$$

$$= .6\pi$$

$$= 1.884 \text{ feet}$$

Let the rotational tip speed equal 700 feet per second, then since

$$V_{TIP} = \Omega R$$

where Ω is the angular velocity. Therefore

$$\Omega = \frac{V_{TIP}}{R}$$

$$= \frac{700}{20}$$

$$= 35.0 \text{ rad/sec}$$

Let the blade twist, θ_1 , from the rotor center of rotation to the blade tip, equal -18° . For a mathematical definition of θ_1 , see Appendix B. The blade Lock number, γ , (nondimensional ratio of the blade flapping aerodynamic forces to the flapping mass forces) is assumed equal to 8.0. By definition

$$\gamma = \frac{\rho c a R^4}{I_H}$$

where

ρ = fluid mass density ($\rho = .002378 \text{ lbs-sec}^2 \text{ per ft}^4$)

a = an equivalent aerodynamic section lift curve slope

$$\left(\text{i.e., } a = \frac{C_L}{\alpha} \right) \quad (a = 5.73)$$

I_H = the blade mass moment of inertia about the flapping hinge.

Rearranging,

$$\begin{aligned}
 I_H &= \frac{\rho c a R^4}{\gamma} \\
 &= \frac{(.002378)(1.884)(5.73)(20)^4}{8.0} \\
 &= 512.0 \text{ lb-sec}^2\text{-ft}
 \end{aligned}$$

Assuming that the blade mass is uniformly distributed over the length of the blade

$$I_H = \frac{\mu_1 R^3}{3}$$

assume further that the blade mass does not start at the center of rotation but at the one-foot radius. Referring to Appendix A, one can see that μ_2 is the uniform mass distribution of member e_2 (i.e., between the flap hinge and the lag hinge) and further μ_3 is the uniform mass distribution of member e_1 (i.e., between the center of rotation and the flap hinge). With this in mind the uniform blade mass becomes

$$\begin{aligned}
 \mu_1 &= \frac{3I_H}{R^3} \\
 &= \frac{3(512.0)}{(19)^3}
 \end{aligned}$$

$$= 0.224 \frac{\text{lb-sec}^2}{\text{ft}^2}$$

Let "ℓ" the rotor drive shaft length equal 10 feet. The speed of sound was assumed equal to 1100 feet per second and the wing root to drive shaft length, L, was assumed equal to 20 feet. The forward speed of the aircraft was assumed equal to 100 ft/sec. The aerodynamic characteristics of the blade airfoil section, which is a modified NACA 0012 section, is shown in the following tables (page 178 and 179).

Given the rotor operating characteristics and the airfoil characteristics the performance curves were determined (see figures 12 and 13), where

$$C_T = \text{rotor thrust coefficient}$$

$$= \frac{\text{thrust}}{\pi R^2 \rho (\Omega R)^2}$$

$$C_Q = \text{rotor torque coefficient}$$

$$= \frac{\text{torque}}{\pi R^2 \rho (\Omega R)^2 R}$$

From these curves the rotor collective pitch angle, θ_{0S} , was chosen equal to 35° . For a mathematical definition of θ_{0S} see appendix B.

Table 1.- Airfoil lift coefficients for the nominal rotor blade.

[illegible]

Table 2.- Airfoil drag coefficients for the nominal rotor blade.

α , Deg	Mach Number									
	.1	.2	.3	.4	.5	.6	.7	.75	.8	.9
0	.008	.008	.008	.008	.008	.008	.008	.01	.0185	.07
1	.00805	.00805	.00805	.00805	.00805	.00825	.0081	.013	.023	.075
2	.0081	.0081	.0081	.0081	.0081	.0085	.01	.02	.04	.0915
3	.00815	.00815	.00815	.00815	.00815	.00875	.02	.033	.057	.108
4	.0082	.0082	.0082	.0082	.0082	.009	.03	.055	.08	.126
5	.0085	.0085	.0085	.0091	.009	.014	.0455	.0775	.105	.144
6	.0088	.0088	.0088	.01	.0115	.019	.061	.1	.12	.162
7	.0091	.0091	.0096	.012	.014	.0345	.0805	.123	.14	.18
8	.01012	.01012	.01073	.014	.019	.05	.1	.1466	.15	.19
9	.01115	.01115	.01186	.02	.04175	.075	.125	.17	.16	.2
10	.01218	.01218	.013	.026	.0645	.1	.15	.1791	.17	.22
11	.0132	.0132	.019	.032	.08725	.16	.175	.1882	.18	.24
12	.0181	.0181	.025	.066	.11	.18	.2	.1973	.19	.2433
13	.023	.023	.0625	.1	.145	.2	.20875	.2063	.2	.2467
14	.0615	.0615	.1	.14	.18	.22	.2175	.2154	.21	.25
15	.1	.1	.14	.18	.195	.24	.22625	.2245	.22	.2533
16	.155	.155	.18	.198	.21	.26	.235	.2336	.23	.2567
17	.18	.18	.21	.25	.27	.29	.12	.12	.12	.12
18	.21	.21	.25	.27	.29	.31	.20	.20	.20	.20
19	.24	.24	.2475	.252	.255	.2675	.2612	.2609	.26	.2667
20	.274	.274	.27	.27	.27	.27	.27	.27	.27	.27
21	.302	.302	.337	.348	.378	.409	.31	.31	.31	.31
22	.33	.33	.375	.378	.411	.448	.33	.33	.33	.33
23	.364	.364	.4	.409	.444	.478	.36	.36	.36	.36
24	.394	.394	.428	.447	.478	.519	.38	.38	.38	.38
25	.421	.421	.489	.486	.514	.553	.40	.40	.40	.40
26	.45	.45	.489	.509	.553	.6	.42	.42	.42	.42
27	.478	.478	.519	.563	.60	.645	.46	.46	.46	.46
28	.513	.513	.549	.602	.648	.695	.50	.50	.50	.50
29	.53	.53	.53	.53	.53	.53	.53	.53	.53	.53
30	.57	.57	.57	.57	.57	.57	.57	.57	.57	.57
40	.96	.96	.96	.96	.96	.96	.96	.96	.96	.96
50	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32	1.32
60	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
70	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84	1.84
80	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
90	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7	2.7
100	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0	2.0
110	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86	1.86
120	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62	1.62
130	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33	1.33
140	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
150	.66	.66	.66	.66	.66	.66	.66	.66	.66	.66
160	.3	.3	.3	.3	.3	.3	.3	.3	.3	.3
170	.08	.08	.08	.08	.08	.08	.08	.08	.08	.08
180	.02	.02	.02	.02	.02	.02	.02	.02	.02	.02

A summary of the basic rotor characteristics follow:

$R = 20$ feet (rotor radius)

$b = 3$ blades

$\sigma = 0.09$ (solidity)

$c = 1.884$ feet (blade chord)

$V_{TIP} = 700$ ft/sec

$\Omega = 35$ radians per second (rotor rotation speed)

$\theta_1 = -18^\circ$ (blade twist)

$\gamma = 8.0$ (Lock number)

$a = 5.73$ (Lift curve slope)

$\mu_1 = 0.224$ lbs-sec²/ft² (blade uniform mass distribution)

$\ell = 10$ feet (drive shaft length)

$L = 20$ feet (wing semispan length)

airfoil modified NACA 0012

$\theta_{OS} = 35^\circ$ (collective pitch angle)

APPENDIX E

COMPUTER PROGRAM

This section of the report deals with the details of the computer program. A description of the input variables, a list of a sample input deck and a list of the complete computer program are included in this appendix. The contents are extensive and will be of interest to a specific few. However, this study would not be considered complete without a listing of the program. Thus, the information is put in the appendix.

The following listing is of input variables. On the left-hand column is the computer language (FORTRAN) symbol and on the right-hand side, just opposite, is the equivalent variable used in the derivation of the equations (Appendix A, B, or C). Also on the right-hand side is a general description of the variable and the corresponding engineering units.

A	a, the speed of sound (ft/sec)
*B	b, the number of blades
BETA(I)	an array of the initial values of the blade flapping motion, the index I refers to blade I, in units of degrees
BETADOT(I)	$\dot{\beta}$, an array of the initial values of the blade flapping velocities, the index I refers to blade I, in units of degrees per second. If the value is greater than Ω , then the β angle is held constant.

BETAMAX	$\beta_{\max}$, the maximum value that any of the blade flapping angles may compute, in degrees. If any blade flapping angle exceeds BETAMAX, the execution of that case is terminated.
C	c, the blade chord, feet
CAPL	L, the wing semispan distance, feet
CBETA	c_{β} , the viscous damping constant across the flapping hinge, ft - lbs - sec. The same value for each blade.
CNO	C_{n_0} , the initial guess for the normal force coefficient, used in variable in-flow routine.
CXI,CXO,CXI	C_{ξ} , C_{x_0} , C_{x_1} , C_{y_0} , C_{y_1} , C_{z_0} , and C_{z_1} , the viscous damping
CYO,CY1	constant across ξ , θ_{x_0} , θ_{x_1} , θ_{y_0} , θ_{y_1} , θ_{z_0} , and
CZO,CZ1	θ_{z_1} , respectively, ft-lb-sec.
DPSIC	$\Delta\psi_c$, the interval of ψ_0 which is used for the integration, degrees
DPSIP	$\Delta\psi_p$, the interval of ψ_0 at which the user desires output listing and/or plotting, degrees. If DPSIP = 0, then the program treats it as if it equals DPSIC.
E1	e_1 , flapping hinge offset, ft
E2	e_2 , distance between the flapping hinge and the lag hinge, feet
*ISTOP	= 0 indicates the present case is the last case. ≠ 0 indicates there is at least one case following the present case.

KBETA, KXI	$K_\beta, K_\xi, K_{x_0}, K_{x_1}, K_{y_0}, K_{y_1}, K_{z_0}$, and K_{z_1} , the spring constants
KX0, KX1	across $\beta, \xi, \theta_{x_0}, \theta_{x_1}, \theta_{y_0}, \theta_{y_1}, \theta_{z_0}$, and θ_{z_1} , hinge
KY0, KY1	respectively,
KZ0, KZ1	ft-lbs
L	ℓ , shaft length from wing to rotor plane, ft
MU1	μ_1 , the blade mass distribution, slugs per ft. An array of eleven values which represent linear density at 11 equally spaced points on the blade.
MU2	μ_2 , the mass distribution between the flapping hinge and the lag hinge, slugs per ft. An array of five values which represent the linear density of five equally spaced points on the segment e_2 .
MU3	μ_3 , the mass distribution between the center of rotation and the flapping hinge, slugs per ft. An array of five values which represent the linear density at five equally spaced points on the segment e_1 .
MU4	μ_4 , the mass distribution of the rotor shaft, slugs per ft. An array of five values which represent the linear density at five equally spaced points on the segment ℓ .

- MU5 μ_5 , the mass distribution of the wing, slugs per ft. An array of five values which represent the linear density at five equally spaced points on the segment L.
- *NIT Maximum number of iteration allowed for convergence on the blade inflow; used in the nonuniform inflow theory only.
- *NN1 = 0, do not print any of the data in this group.
 > 0, print ψ_0 , C_{AQx}^{XI} , C_{ATy}^{XI} , C_{AMx}^{XI} , and C_{AMy}^{XI}
 > 1, print $\frac{z^{XII}}{R}$, α , $\left(\frac{x}{R}\right)_{TOTAL\ BLADE}$, $\left(\frac{y}{R}\right)_{TOTAL\ BLADE}$ and $\left(\frac{z}{R}\right)_{TOTAL\ BLADE}$
 > 2, print ϕ , M, C_L and C_D
- *NN2 ≤ 0 , do not print any of the data in this group.
 > 0, print ψ_0 , C_{IMx}^{XI} , C_{IMy}^{XI} and C_{IMz}^{XI}
 > 1, print C_{IFx}^{XI} , C_{IFy}^{XI} and C_{IFz}^{XI}
 > 2, print C_{IMx} , C_{IMy} and C_{IMz}
 > 3, print dC_{IMx} , dC_{IMy} , dC_{IMz} and $\left(\frac{z}{R}\right)^{XII}$
- *NN3 ≤ 0 , do not print any of the data in this group.
 > 0, print ψ_0 , β_0 , $\dot{\beta}_0$ and $\ddot{\beta}_0$.
 > 1, print ψ_1 , β_1 , $\dot{\beta}_1$ and $\ddot{\beta}_1$.
 > 2, print ψ_2 , β_2 , $\dot{\beta}_2$, and $\ddot{\beta}_2$ if $B > 2$.
 > 3, print ψ_3 , β_3 , $\dot{\beta}_3$, and $\ddot{\beta}_3$ if $B = 4$.
- *NN4 ≤ 0 , do not print any of the data in this group.
 > 0, print ψ_0 , ξ_0 , $\dot{\xi}_0$, and $\ddot{\xi}_0$.
 > 1, print ψ_1 , ξ_1 , $\dot{\xi}_1$, and $\ddot{\xi}_1$.
 > 2, print ψ_2 , ξ_2 , $\dot{\xi}_2$, and $\ddot{\xi}_2$ if $B > 2$.
 > 3, print ψ_3 , ξ_3 , $\dot{\xi}_3$, and $\ddot{\xi}_3$ if $B = 4$.

- *NN5 ≤ 0 , do not print any data in this group.
 > 0 , print ψ_0 , C_{TMx}^{IX} , C_{TMy}^{IX} and C_{TMz}^{IX}
 > 1 , print C_{AIMx}^{IX} , C_{AIMy}^{IX} and C_{AIMz}^{IX}
 > 2 , print C_{AIFx}^{IX} , C_{AIFy}^{IX} and C_{AIFz}^{IX}
 > 3 , print C_{IMe2x}^{IX} , C_{IMe2y}^{IX} and C_{IMe2z}^{IX}
- *NN6 ≤ 0 , do not print any data in this group.
 > 0 , print ψ_0 , C_{FMx}^{VIII} , C_{FMy}^{VIII} and C_{FMz}^{VIII}
 > 1 , print C_{Fx}^{VIII} , C_{Fy}^{VIII} and C_{Fz}^{VIII}
 > 2 , print C_{AIFx}^{VIII} , C_{AIFy}^{VIII} and C_{AIFz}^{VIII}
 > 3 , print C_{Melx}^{VI} , C_{Mely}^{VI} and C_{Melz}^{VI}
- *NN8 ≤ 0 , do not print any data in this group.
 > 0 , print ψ_0 , C_{Mx}^{VI} , C_{My}^{VI} and C_{Mz}^{VI}
 > 1 , print C_{Flx}^{III} , C_{Fly}^{III} and C_{Flz}^{III}
- *NN9 ≤ 0 , do not print any data in this group.
 > 0 , print ψ_0 , C_{MLx}^{III} , C_{MLy}^{III} and C_{MLz}^{III}
 > 1 , print C_{Fx}^{III} , C_{Fy}^{III} and C_{Fz}^{III}
- *NN10 ≤ 0 , do not print any data in this group.
 > 0 , print θ_{x_0} , θ_{y_0} , θ_{z_0} , θ_{x_1} , θ_{y_1} , and θ_{z_1} .
 > 1 , print θ_{x_0} , θ_{y_0} , θ_{z_0} , θ_{x_1} , θ_{y_1} and θ_{z_1} .
- NUO v_0 , initial guess for the induced velocity, ft/sec. Used only for the case of variable inflow.
- OMEGA Ω , rotor rotational speed, radians per second.
- PSIMAX ψ_{max} , the maximum value for ψ_0 , when ψ_0 exceeds ψ_{max} , execution of the present case is terminated, degrees
- PSIO ψ_0 , the initial position of the master blade, degrees

R R, the length of the blade, from the center of rotation, ft

PHO ρ , the density of air, lbs-sec² per ft⁴

THMAX $\theta_{\max}$, the maximum value allowed for θ_{mn} , degrees. If any of the θ variables exceeds $\theta_{\max}$, the execution of the present case is terminated.

THXO, THX1 θ_{x_0} , θ_{x_1} , θ_{y_0} , θ_{y_1} , θ_{z_0} , and θ_{z_1} the initial values
 THYO, THY1 x_0 , x_1 , y_0 , y_1 , z_0 , z_1
 THZO, THZ1 of the θ_{mn} variable, degrees

THXODOT, THX1DOT $\dot{\theta}_{x_0}$, $\dot{\theta}_{x_1}$, $\dot{\theta}_{y_0}$, $\dot{\theta}_{y_1}$, $\dot{\theta}_{z_0}$ and $\dot{\theta}_{z_1}$, the initial angular
 THYODOT, THY1DOT velocity of the θ_{mn} variable, degrees per second.

THZODOT, THZ1DOT If the initial value is greater than Ω , then the corresponding angle is held constant.

THOS θ_{0s} , blade root collective pitch angle, degrees

TH1 θ_1 , blade twist, degrees

THISC θ_{sc} , longitudinal cyclic pitch angle, degrees

THISS θ_{ss} , lateral cyclic pitch angle, degrees

U,V,W u,v,w linear velocity of the aircraft center of gravity
 in the x,y, and z directions, respectively, feet per second

XC x_c , distance from the lag hinge to the start of the airfoil section, feet

XI(I) ξ_1 , an array indicating the initial values for the blade lag angle, degrees. The index I refers to the Ith blade.

XIDOT(I) $\dot{\xi}_1$, an array indicating the initial values of the blade lagging velocity, degrees per second. The index I refers to the Ith blade. If the value is greater than Ω , then the corresponding angle will be held constant.

XIMAX $\xi_{\max}$, the maximum value that any of the blade lag angles may compute, in degrees. If any lag angle exceeds XIMAX, the execution of that case is terminated.

* Represent fixed point variables, all other variables are floating point.

The following is a list of the variables which contain default values. These values will be used if no value is specified in the input on the initial case. For subsequent cases, the values of the previous case become the default values.

Variable or Array Name	Default value(s)
A	1100
B	3
BETA	0,0,0,0
BETADOT	0,0,0,0
CC	1000
CNO	0
DELTA3	0
DPSIP	0
E1	0
E2	0
ISTOP	0
NIT	0
NN1	0
NN10	0
NN2	0

NN3	0
NN4	0
NN5	0
NN6	0
NN7	0
NN8	0
NN9	0
PHO	0.0023769
THXO	0
THXODOT	0
THXI	0
THXIDOT	0
THYO	0
THYODOT	0
THYI	0
THYIDOT	0
THZO	0
THZODOT	0
THZ1	0
THZ1DOT	0
U	0
V	0
W	0
XC	0
XI	0,0,0,0
XIDOT	0,0,0,0

In addition to the basic computer program inputs there is a plotting subroutine. This routine has an additional set of inputs. A sample of the total input deck is listed in this appendix. The definition of the terms for the plotting routine follows:

XMIN	minimum scale value for θ_{x_1}
XMAX	maximum scale value for θ_{x_1}
ZMIN	minimum scale value for θ_{z_1}
ZMAX	maximum scale value for θ_{z_2}
PSMIN	minimum scale value for ψ_0
PSMAX	maximum scale value for ψ_0
BMIN	minimum scale value for β
BMAX	maximum scale value for β
XIMIN	minimum scale value for ξ
XIMAX	maximum scale value for ξ

If both the minimum and maximum values for any of the above variables are set equal to zero then the computer will choose a suitable set of scales

LOR	value of ℓ/R
PC	percentage of the data points that the computer can drop before rescaling occurs (between 0 and 1); default value equals zero
IPEN	= 0 for ball point plotted figures ≠ 0 for Leroy plotted figures
INERTIA	= 0 then X/R versus Z/R is not plotted ≠ 0 then X/R versus Z/R is plotted

IBETA(I) = 0 the flapping angle for the Ith blade is not plotted

≠ 0 the flapping angle for the Ith blade is plotted

IXI(I) = 0 the lag angle for the Ith blade is not plotted

≠ 0 the lag angle for the Ith blade is plotted

As stated in the discussion, the airfoil characteristics used in this study are stored in the memory of the computer. Also, it was stated that these airfoil characteristics could be changed with the proper input data. In order to make an airfoil characteristics change the following procedure should be followed. Divide the lift and drag airfoil characteristics into three separate sections as a function of angle of attack from 0 to 180°. Submit the following input cards.

AL1 first value of α in the first section of C_L table

AL2 first value of α in the second section of C_L table

AL3 first value of α in the third section of C_L table

AD1 first value of α in the first section of C_D table

AD2 first value of α in the second section of C_D table

AD3 first value of α in the third section of C_D table

DAL1 difference in α in the first section of the C_L table

DAL2 difference in α in the second section of the C_L table

DAL3 difference in α in the third section of the C_L table

DAD1 difference in α in the first section of the C_D table

DAD2 difference in α in the second section of the C_D table

DAD3 difference in α in the third section of the C_D table

NAL1 number of α 's in the first section of the C_L table

NAL2 number of α 's in the second section of the C_L table

NAL3 number of α 's in the third section of the C_L table
NAD1 number of α 's in the first section of the C_D table
NAD2 number of α 's in the second section of the C_D table
NAD3 number of α 's in the third section of the C_D table

Replace the letter A with M to incorporate the Mach number part of the table.

CL list the lift coefficients starting with the lowest α and Mach number. List in order of increasing M to the largest value of M; then to the next α and repeat until the entire table is tabulated.

CD list the drag coefficients consecutively in the same order as the lift coefficients.

The sample listing of the input cards is self explanatory (along with the above information). A portion of the input listing consists of binary cards associated with the plotting routine. The second listing is the FORTRAN language of the computer program.

```

JOB:1,1500,63000,      D3101      RPC436      BLOC.643
USER:SCHEINMAN, JAMES      000778600  35200
LINECNT(10000)
NOMAP.
NORFL.
FETCH(D3101,SPRA06,BINARY)
BNFILE.
REWIND(TAPE1)
INPUT.
REWIND(CALTP)
REQUEST TAPE2,H1,X,CALTP,R,M,J-S
REWIND(TAPE2)
COPYBF(CALTP,TAPE2)
UNLOAD(TAPE2)
EXIT.
REWIND(TAPE1)
INPUT.
REWIND(CALTP)
REQUEST TAPE2,H1,X,CALTP,R,M,J-S
REWIND(TAPE2)
COPYBF(CALTP,TAPE2)
UNLOAD(TAPE2)
-

```

STABILITY WITH BLADE DAMPING OMBOOM=1.08 CBOCBC=.5 OMX1=OMZ1=.1 DPSIP=.5

```

SINPUT
OBETA   = 0.572957795131E+01, 0.572957795131E+01, 0.572957795131E+01, 0.0.
BETADOT=3*0.0.
OBETAMAX = 0.90000095867618E+02.
OC       = 0.18849555921539E+01.
OCAPL    = 0.2E+02.
OCC       = 0.1E+07.
OCNO     = 0.0.
CBETA=0.0,CX1=0.0,CX0=0.0,CX1=0.0,CY0=0.0,CY1=0.0,CZ0=0.0,CZ1=0.0.
CBETA=.32322415E-07.
ODPSIC   = 0.572957795131E+01.
ODPSIP    = 0.286478897565E+02.
ICAPLXP=.1E+02,ILXP=.1+02.
KBETA=0.,KX1=0.,KX1=0.,KX0=0.,KY0=0.,KY1=0.,KZ0=0.,KZ1=0..
KBETA=.1422222E-09.
KX1=.3196937E-10.
KZ1=.3196937E-10.
OL        = 0.1E+02, MU1=.224,.224,.224,.224,.224,.224,.224,.224,.224.
          .224,.224, MU2=0.,0.,0.,0.,0.,MU3=0.,0.,0.,0.,0..
MU4=0.,0.,0.,0.,0.,0.,0.,0.,0.,0..
NIT=10..

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NMJ=1,NN9=0,NM10=1,
NUO=0,0,OMEGA=334,225,PS10=0,0,R=20,PS1MAX=8021,409,TMAX=180,0,
THX0=0,0,THX0DOT=.1E+99,
THY0=0,0,THY0DOT=.1E+99,
THZ0=0,0,THZ0DOT=.1E+99,
THZ1=0,0,THZ1DOT=0,0,
THX1=20,0,THX1DOT=0,0,
THOS=35,0,THI=18,0,THISC=0,0,THISS=0,0,U=0,0,V=100,0,W=0,0,XC=.0025,
X1=30,0,0,X1DOT=3,0,1E+99,X1MAX=90,0,
S
--00
((X- PX( PL0TXZ 0001 9
(- /B PX(8X((U9V8 PX( 39+8 3 PX(-+5 39,480YPL0TXZ 0002 8
((0 ---3 P7=-P*-7-3 - X -7-X-XX*-((K3 W-(L9((R0-0IZ X9((-PL0TXZ 0003 9
((XP-7-3 - +5 35YV697X9(-XP-7-3 - P7((-P*-7-3 - X -7-X-XP*-((KPL0TXZ 0004 7
((3 W-(L9((R0-0IZ P7((-P*-7+5 3-WT6(.3 - 0IZ P--*L3 - N=-X-PL0TXZ 0005 -
((G3 0IZ X -7-X-PP-(-Z6-*)R(U-*ZX- X(-X-*7-W-*RX+5 3=-3-S(-7-PL0TXZ 0006 -
((7 )-9X-*--P*-7-3 - X-*)*-9 N=-X-3 N=-X-3 X -7-X-((XP-WX-PL0TXZ 0007 -
((XT=(8(+5 3-0SW9ZG,(0-*(P-3 X-*S*X-((PM6 W-(RX(-7-7 (((3 PL0TXZ 0008 6
((7X-7P-3 - -9ZV-0-*7-3 -3 +5 37((,7XX-+Z(3 -3 X-(Z(3 -3 X-(ZXZ--IXPL0TXZ 0009 =
((X-D*)X7ZX-U-(Z(G(PP-P-((-P*-7-3 - X-+Z)*)-9 +5 3-,6X-(X-*W9*-9 PL0TXZ 0010 =
((U-*/(+*-9 0IZ W-PL9((10TX=(RX*-+09((7-0-)RX*-+*-3 -1 0-(R(P7*IXPL0TXZ 0011 =
((X+5 3-((X+3 -1 0-(RX*-+1X3 -1 3 - U-Z1-3 - X-,4-0-+*- P-((-P----PL0TXZ 0012 7
((X -((*(R(((-X(-R(+5 3-((Y7(-ZX(-R((-0-(-RX(-0-(-R((-0-PL0TXZ 0013 -
(((-7-3 --0-(-P-((-P----X-**(R(((-X+5 36((UW(-R((-ZX(-R(PL0TXZ 0014 -
((P(-0-(-RX(-0-(-R((-0-(-7-3 --X -7-X-PX-(-Z1-X-*P--8(-+5 3-PL0TXZ 0015 -
((1866FFX-*7-P-((XP--((0-(-P-+*-P-(-(((X-X)-P-X-XV9X-XP9X-XV9X-GP9PL0TXZ 0016 5
((X-XV9X-PP9X-XV9+5 3-1YX,(X-PP9X-XV93 --P-(ZXT=(8(G(IZ(P -ZGM-((-PL0TXZ 0017 -
((X -((*(R(((-X(-R((-Z((-R((-0+5 3-(YW,F(-7(-0-(----(-ZXPL0TXZ 0018 -
(((-7-3 --W-*RX(-7-78*P-X-+9X-PX-(-Z1-X-*P--9--9X-*7+5 37FSFFFPL0TXZ 0019
((P7) XP--((0-)-P-+*-P-(-(((X-X)-P-X-XV9X-XP9X-XV9X-GP9X-XV9X-GP9PL0TXZ 0020 7
((X-XV9X-GP9+5 3-)-Y*(X-GV93 --P7(ZXT=(8(G(IZ(P -ZGM-((-X-((-*(R(PL0TXZ 0021 -
((6((-X(-R((-ZX(-R((-0-(----+5 3-((-XW(-0-(----(-ZX(-7-3 --PL0TXZ 0022 -
((EW-*RX(-7-78(0-P-(0-3 -6 P---P*-7-3 - 0IZ X-XX+5 3-ZY9D23 - PL0TXZ 0023 -
((PU-XX-3 - P-=-P*-7-3 - 0IZ X-WX-3 - W-WX-3 - =VP 7,v 3-F 7PL0TXZ 0024 7
((9 9+5 3122222 7,0 3-A 7 9 -+V 3-D 7 9 -+0 3-E 7 9 5*( PL0TXZ 0025 -
((3-G 7 9 5*7 3-0 7 9+5 3-22222 8,v 3-T 7 9 8,0 3-S 7 9PL0TXZ 0026 -
((X 7((3-W 7 9 717 3-/ 7 9 7 3-C 7 9+5 3-22222 56 3-B 7PL0TXZ 0027 -
(( 9 =(P,3-U 8 9 -R-3-T 83 7 +, 3-U 83 7 +*V 3-V 8 9PL0TXZ 0028 7
((X+8 3- +5 3- 9( 9 (*J -(NP (R*+ 9R*C BR*/ *PL0TXZ 0029 -
((((*J ( (7(-R*+ R*C (NP ( +5 3- X((7X(-(-XXIX(PL0TXZ 0030 -
(((X(X(X1,(P,)((X)*(-S-S _R*/ F (X(X*----X*R*-)-+5 37 PL0TXZ 0031 -
((VU X((-)-) - 9 - 7 *((( (-R((( PL0TXZ 0032 4
(( 9 7+5 3- *((( (-R((( *((( ,((( PL0TXZ 0033 -
((8 *((( ,(((+5 3- ( PL0TXZ 0034

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(X/ + (((Z(X-7(((7.1,8*((-XX-X-*(G)) -U +9 3- PLOTXZ 0035 -
(-(X-7(B7 73719P3918X3719737A7 *X* R =VP6 73-R U371** R (PPLOTXZ 0036 7
(7V 73-(NR R P3-1 X3-1 G3719X3719X3-R9P3-(#Q R G371 G3-1 P3-PLOTXZ 0037 -
((R X3-Z973-(-- R9(( 73=R9.( 73-.0 R9G3-R9.7-X 73-(*T R9*(X PLOTXZ 0038 -
(XX 73-R873-RBM37R8X3-R8X36R9P3-R917PJ 73-PX R9P3-R9(X 73-+8 =X PLOTXZ 0039 -
(G (((((+8 =( X((P+8 =X +(((+8 =( (((+8 =( PLOTXZ 0040 7
(((P+8 =( +8 =( +8 =( +8 =( PLOTXZ 0041 -
-P+8 =( E9 PX( PLOTXZ 0042 -
-00
(IX- .0 MAX 0001 9
(L /8 .0 -(((U9V8 .0 39+8 3 .0 7+5 39 6WFFMAX 0002 8
(( )*(P.9*(P(9.X-X )X-X )PX(=5=P4 6 W F*PX F*NX -7-X (-9X OX-MAX 0003 9
(*((X-X ( X+5 356W6XFP--((XW*(- OP-(((07*(X-X ( XP--((X0*(- WP-MAX 0004 7
(X((07X -L9.* 7-W (P-( -7-7 (X-+7 3-1 1 3 X GX-3 - +- 3- MAX 0005 -
(((7-(.*(( B9 73-A8 -1 R C MAX 0006 -
-
-

```

STABILITY WITH BLADE DAMPING OMBOOM=1.08 CBOCBC=.5 OMX1=OMZ1=.1 DPSIP=.5

SINPUT

IPEN=0.

XMJN=0.0,XMAX=.0,ZMJN=0.0,ZMAX=.0,PSMJN=0.0,PSMAX=0.0,BMJN=0.0,BMAX=.0.

XIMAX=0.0,XIMIN=0.0,LOR=.5,PC=25.,INERTIA=1.

INERTIA=1.,IBETA=3*1,IXI=3*0.

```

PROGRAM SCHEIM N (INPUT, OUTPUT, TAPES=INPUT,TAPE6=OUTPUT, TAPE1=
A 1001)
REAL BETA(4), TH(6)
DIMENSION BBAR(4), THBAR(6)
$FROM 12
COMMON/BLANK /CAIF939(3,9), CAIM939(3,9),CAM11(3,1), CAPLOR,
$ CA12(4), CB, CX, CXC, CX1, CY0, CY1, CZ0, CZ1,
A CFL337(3,7), CF39(3,9), CF639(3,9), CF839(3,9), CIME2(48),
$ C1FM(54),
B C1F1239(3,9), CIME1(42), CIM139(3,9), CME1637(3,7),
$ CIME29(3,8),
C CML334(3,4), CML6(3), CML637(3,7), CM635(3,9),CONST, CTM939(3,9),BLAN0040
D CUVAR(29), DER(29),
$ ILXP,
E DPSR, EIMAT(3), KB, KX, KX0, KX1, KY0, KY1, KZ0, KZ1,LILLOR,M,N, BLAN0060
F PSI(4), SPR, THMAT(3,3), THT(3,3), UOOR, VOOR, WUOR, XBAR(4), XC,BLAN0070
G XMAT(3,3),
$ XI(4)
H , EZMAT(3)
I , CILXP
J , CD3, SD3, SSQD3, CSCC3
K , NOOR(4), CATY12P(4), COTH, COTHSP0, SOTHCP0, M10CDTH, NIT
M , UPI, UP97
N , NPB, NMI
$FROM 13
COMMON /ALL/ NB, J, K, RET
COMMON /ANGLES/ AUI, AD2, AD3, AL1, AL2, AL3,DAD1,DAD2,DAD3, DAL1,
A DAL2, DAL3, OMEGA, SPEC, TH1, VAR(29), THOS, THISC, THISS,DELTA3,
B R MAX, CI , P S MAX, X MAX
C , THMAX
COMMON /CHK/ SKIP(87), MC1,MC2, MD3, ML1, ML2, ML3
COMMON /IN/ NN10, NN3, NN4, NN5, NN6, NN7, NN8, NN9, OR, PRSQRH,
$ SOUND,
A ILILLXP, NAD1, NAD2, NAD3, NAL1, NAL2, NAL3, NMD1, NMD2,
B NMD3, NML1, NML2, NPL3
COMMON /SCDAF/ A1, BIASA1, BIASA2, BIASA3, BIASA11, BIASA12,
A BIASA13, COTAB(480), CLTAB(480),
B C81(3,3), G6SAVE(3,9,11)
C , NAD, NAL, NMI, NA2
D PTBB839(3,9,11), T31(3), NMD, DMD1, DMD2,
E DMD3, BIASMD1, BIASMD2, BIASMD3, NML, DML1, DML2, DML3,
F BIASML1, BIASML2, BIASML3
G , PMAX
H , II
$FROM J1
INTEGER H, RET
$FROM I1
REAL ANG(14), CAIF8(3,1), CAIF9(3,1), CAIM9(3,1),
A CFL3(3), CFM8(3), CF3(3), CF6(3), CF8(3), CIE1(3), CIE137(3,7), REAL0002
B CIE238(3,8), CIE28(3), CIE11(3,1),CIM(3),CIME2IX(3), CIME238(3,8)REAL0003
C , CIM11(3), CIM39(3,9), CME16(3), CML3(3), CM6(3), CTM9(3), REAL0004
D DCIM(3), ELFI(28), ELE2(28), ERRVAL(28), E1OR, E2OR, MARE2, REAL0005
$ ILXP,
$ KB, KX, KX0, KX1, KY0, KY1, KZ0, KZ1, MCD, MCL,
$ LILLOR,
$ M1, M2, M3, M4,
E M5, NAU, NAL, PTBB8(3), THOOT(6)
F , ILILLXP
G , NOOR, NOORO, M10CDTH
H , NAD1, NAD2, NAU3, NAL1, NAL2, NAL3, MD1, MD2, MD3, ML1, ML2,
I ML3, NMD1, NMD2, NMD3, NML1, NML2, NML3, MDBIAS1, MDBIAS2,
J MDBIAS3, MLBIAS1, MLBIAS2, MLBIAS3, NML, NMD
K , NML3M1, NMD3M1, PMAX
$FROM L2
EQUIVALENCE (CIME2, CIE238), (CIME2(25), CIME238), (CIME1,
A CIE137), (CIME1(2), CME137), (ELMAT(3), E1OR), (DER(8), BBAR),
B (DER(2), THXOBAR), (DER(3), THYOBAR), (DER(4), THZOBAR), (DER(5),
C THX1BAR), (DER(6), THY1BAR), (DER(7), THZ1BAR)
$FROM L3
EQUIVALENCE(IVAR,PS0), (IVAR(8), BETA), (IVAR(2), THX0),
A (IVAR(3), THY0), (IVAR(4), THZ0), (IVAR(5), THX1), (IVAR(6), THY1),
B (IVAR(7), THZ1), (THMAT, THC), (THMAT(1,2), THS), (DER(2), THBAR),
C (IVAR(2), TH)
F , (CA12, CATY12), (CA12(3), CAQX12)
G , (C1FM(28), CIM39)
H , (EZMAT(3), EZUR)
I , (NOOR, NOORO), (CATY12P, CATY120)
NAMELIST /SCHE/ AUI, AC2, AD3, AL1, AL2, AL3, A1, ROOT,
A BUOTOJ1, BETA, BIASA1, BIASA2, BIASA3, BIASA11, BIASA12,
B BIASA13, BIASMD1, BIASMD2, BIASMD3, BIASML1, BIASML2, BIASML3,
C BMAX, CAIF8, CAIF9, CAIF939, CAIM9, CAIM939, CAM11,
D CAQX11, CAQX12, CATY11, CATY12, CATY120, CC, COTH,
E COTHSP0, CD3, CFL3, CFL337, CFM8, CF3, CF339, CF6, CF639, CF8,
F CF639, CI, CIE1, CIE137, CIE238, CIE28, CIE11, CIE1239, CILXP,
G C1F, CIME2IX, CIME29, CIM11, CIM139, CIM39, CME16, CME1637,
H CML3, CML6, CML637, CF6, CF639, CONST, CSQD3, CTM9, CTM939,
I CUVAR, COT, DAD2, DAD3, DAL2, DAL3, DCIM, DELTA3, DER, DMD2,
J DMD3, DML2, DML3, DPSR, DT, DZ12OR, EIMAT, E1OR, E2OR, H, ID,
K II, ILILLXP, ISTUP, J, K, L, M, MD1, MD2, MD3, MLL, ML2, ML3,
L MMAX, MML3, M10CDTH, N, NAD, NAD1, NAD2, NAD3, NAL, NAL1, NAL2,
M NAL3, NB, NMD, NMD1, NMD2, NMD3, NMD3M1, NML, NML1, NML2, NML3,
N NML3M1, NN, NN1, NN10, NN2, NN3, NN4, NN5, NN6, NN7, NN8, NN9,
O NOORO, OMEGA, OR, PRSQRH, P S MAX, PS0, PTBB8, PO, RAD, SOTHCP0,
P SD3, SOUND, SPEC, SSQD3, THOOT, THMAT, THT, THX1, THZ1, TH1,
Q THISC, THISS, T31, UPI, UP97, VAR, XMAT, XMAX, ZUOT, ZOOTDOT,
R ZETA, Z12OR
$FROM M/A
COMMON /ICKY/ IC
DIMENSION IC(14)
ASSIGN 10 TO RET
10 CALL IN

```

```

UP1 = 1. - E10R - E2CR
UP47 = .47 * UP1
DZ12GR = .1 * UP1
NML3M1 = NML3 - 1.
IF (NML3M1 .LT. 0) NML3M1 = 0
NMD3M1 = NMD3 - 1.
IF (NMD3M1 .LT. 0) NMD3M1 = 0
MMAX = AMIN1 (MC3 + NMD3M1 * DMD3, ML3 + NML3M1 * DML3)
CONST = -1. / PRSQRT
12 WRITE (1) N3
BIAS
AAD 1 = -AD1
BIAS
AAL 1 = -AL1
BIAS
AMD 1 = -MD1
BIAS
AML 1 = -ML1
BIAS
AAD 2 = DA02*NAU1 - AC2
BIAS
AAL 2 = DAL2*NAL1 - AL2
BIAS
AMD 2 = DMD2*NMU1 - MD2
BIAS
AML 2 = DML2*NML1 - ML2
NAD = NAD1 + NAD2
NAL = NAL1 + NAL2
NMD = NMD1 + NMD2
NML = NML1 + NML2
BIAS
AAD 3 = DA03*NAU - AC3
BIAS
AAL 3 = DAL3*NAL - AL3
BIAS
AMD 3 = DMD3*NMU - MD3
BIAS
AML 3 = DML3*NML - ML3
NAL = NAL + NAL3
NAD = NAD + NAD3
NMD = NMD + NMD3
NML = NML + NML3
CILXP = CONST * ILILLXP
DPSB = 0.26318530717958 / FLOAT (NB)
IF (NN3 .GT. NB) NN3 = NB
IF (NN4 .GT. NB) NN4 = NB
A1 = OR / SOUND
CD3 = CUS(DELTA3)
SU3 = SIM(DELTA3)
SSQD3 = SD3**2
CSQD3 = CD3**2
$FROM M3A
DO 16 J=2,NB
NOCR(J) = NOCR0
16 CATY12P(J) = CATY12C
DTH = SQRT (THISS**2 + THISS**2)
CDTH = CUS (DTH)
IF (THISS .NE. 0) GC TC 37
IF (THISSC .NE. 0) GC TC 37
PD = 0
GO TO 39
37 PD = ATAN2 (THISS, THISSC)
39 CONTINUE
COTHSPJ = COTH*SIN (PD)
SOTHGPD = SIN (DTH) * COS (PC)
MLOCOTH = -1./COTH
C THX0BAR, THY0BAR, TZ0BAR, TFX1BAR, THY1BAR, THZ1BAR, AND
C BBAK(1) ARE EQUIVALENCED TO THE 2ND THRU THE 8TH ELEMENTS OF
C DER
NN = 2 * N - 2
$FROM M3
II = 0
C PSJ, THX0, THY0, THZ0, THX1, THY1, THZ1, AND BETA(1) ARE EQUI-
C VALENCED TO THE 1ST 8 ELEMENTS OF VAR IN SCHEIMV ONLY
50 CALL INI2A (II, NN, CI, SPFC, VAR, CUVAR, DER, ELE1, ELE2)
$FROM M3A
DO 40 J = 1, NB
XI(J) = VAR(M+J)
XBAR(J) = DER(M+J)
40 CONTINUE
WRITE (1) PSO, TFX1, THZ1, (BETA(J), XI(J), J=1,NB)
L = 2 * NB + 8
IF (NN1 .LE. 0) GO TO 90
CAQX11 = THT(1,1) * CAQX12 + THT(1,2) * CATY12
CATY11 = THT(2,1) * CAQX12 + THT(2,2) * CATY12
$FROM M4
PRINT 80, PSO, CAQX11, CATY11, CAM11(1), CAM11(2)
80 FORMAT ('OAEODYNAMIC DATA/ *OPSIO* 9X *CAQX11* 7X *CATY11* 7X
A *CAMX11* 7X *CAMY11* / E14.5, 4E13.5)
90 IF (NN2 .LE. 0) GO TO 190
CALL NINE (CIM11, CIM139, DER11)
PRINT 100, PSO, CIM11
100 FORMAT ('OBLADE INERTIA DATA/ *OPSIO* 9X *CIMX11* 7X *CIMY11* 7X
A *CIMZ11* / E14.5, 3E13.5)
IF (NN2 .LE. 1) GO TO 190
CALL NINE (CIF11, CIF1239, DER11)
* CIF11 = THT * CIF11
CALL MATRIX (203 ,3 ,1 ,THT ,3 ,CIF11
X ,3 ,CIF11 ,3
$FROM M5
PRINT 110, CIF11
110 FORMAT (110 13X *CIF11* 7X *CIFY11* 7X *CIFZ11* / 14X 3E13.5)

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10200000
10300000
10400000
10500000
10600000
10700000
10800000
10900000
11000000
11100000
11200000
11300000
11400000
11500000
11600000
11700000
11800000
11900000
12000000
12100000
12200000
12300000
12400000
12500000
12600000
12700000
12800000
12900000
13000000
13100000
13200000
13300000
13400000
13500000
13600000
13700000
13800000
13900000
14000000
14100000
14200000
14300000
14400000
SCHE0420 14500000
SCHE0430 14600000
SCHE0440 14700000
SCHE0560 14800000
14900000
15000000
15100000
15200000
SCHE0570 15300000
15400000
15500000
15600000
15700000
15800000
15900000
16000000
16100000
16200000
16300000
16400000
16500000
16600000
16700000
SCHE620 16800000
SCHE630 16900000
SCHE640 17000000
SCHE660 17100000
SCHE670 17200000
17300000
SCHE690 17400000
SCHE700 17500000
17600000
SCHE730 17700000
SCHE740 17800000
SCHE750 17900000
SCHE760 18000000
SCHE770 18100000
18200000
18300000
18400000
SCHE0840 18500000
SCHE0850 18600000
SCHE0860 18700000
SCHE0870 18800000
SCHE0880 18900000
19000000
SCHE0900 19100000
SCHE0920 19200000
SCHE0930 19300000
SCHE0940 19400000
19500000
SCHE0960 19600000
SCHE0970 19700000
SCHE0980 19800000
PREPROCS 19900000
PREPROCS 20000000
SCHE0990 20100000
SCHE1000 20200000
SCHE1010 20300000

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```

      IF (NN2 .LE. 2) GO TO 190
      CALL NINE (CIM, CIM39, DER(L))
      PRINT 120, CIM
120  FORMAT (1H0, 13X, *CIM* 9X *CIMY* 9X *CIMZ* / 14X 3E13.5)
      IF (NN2 .LE. 3) GO TO 150
      Z12OR = 0
      DO 180 M = 1, 11
      CALL NINE (DCIM, G6SAVE(1,1,H), DER(L))
$FROM M6
      DO 130 K = 1, 3
      DCIM(K) = DCIM(K) + G6SAVE(K,1,H)
130  CONTINUE
      PRINT 140, DCIM, Z12OR
140  FORMAT (1H0 13X *DCIP* 8X *DCIMY* 8X *DCIMZ* 8X *Z12/R* / 14X
      A 4E13.5)
      IF (NN2 .LE. 4) GO TO 170
      CALL NINE (PTB39, PTB39(1,1,H), DER(L))
      PRINT 160, PTB39
$FROM M7
160  FORMAT (14X 1H= 2(18X 1H=) / 14X *X/R(TOTAL BLADE) Y/R(TOTAL BLAS
      ADE) 2/R(TOTAL BLADE)* / 8X 3(6X E13.5))
170  Z12OR = Z12OR + DZ12OR
180  CONTINUE
190  IF (NN3 .LE. 0) GO TO 220
      PRINT 195
195  FORMAT (*OBETA MOTICN*)
      J = NB
200  BOOT = OMEGA * EBAR(J)
      BOOTDOT = OMEGA**2 * DER(N+J+6)
      PRINT 210, PSI(J), ZETA(J), BOOT, BOOTDOT
210  FORMAT (28X 1H. 12X 2H.. / 5H PSI 3(9X, 4HBETA)
      A / 4E13.5)
$FROM M8
      IF (NN4 .EQ. 1) GO TO 220
      IF (J .EQ. NB) J = 0
      J = J + 1
      IF (J .LT. NN3) GO TO 200
220  IF (NN4 .LE. 0) GO TO 250
      PRINT 225
225  FORMAT (*OXI MOTICN*)
      J = NB
230  ZETA = - XI(J)
      ZDOT = -XBAK(J) * OMEGA
      ZDOTDOT = -DER(N+NB+J+6) * OMEGA**2
      PRINT 240, PSI(J), ZETA, ZDOT, ZDOTDOT
240  FORMAT (28X 1H. 12X 2H.. / 5H PSI 3(9X, 4HZETA) /
      A 4E13.5)
$FROM M9
      IF (NN4 .EQ. 1) GO TO 250
      IF (J .EQ. NB) J = 0
      J = J + 1
      IF (J .LT. NN4) GO TO 230
      IF (NN5 .LE. 0) GO TO 280
      CALL NINE (CTM9, CTM9(1,1,H), DER(L))
      PRINT 260, PSO, CTM5
260  FORMAT (*OPSIO* 9X *CTMX9* 8X *CTMY9* 8X *CTMZ9* / X 4E13.5)
      IF (NN5 .LE. 1) GO TO 280
$FROM M10
      CALL NINE (CAIM9, CAIM9(1,1,H), DER(L))
      PRINT 270, CAIM9
270  FORMAT (*O* 13X *CAIPX9* 7X *CAIMY9* 7X *CAIMZ9* / 14X 3E13.5)
280  IF (NN5 .GT. 2) GO TO 285
      IF (NN5 .LE. 2) GO TO 310
285  CALL NINE (CAIF9, CAIF9(1,1,H), DER(L))
      PRINT 290, CAIF9
290  FORMAT (*O* 13X *CAIFX9* 7X *CAIFY9* 7X *CAIFZ9* / 14X 3E13.5)
      IF (NN5 .LE. 3) GO TO 310
      CALL EIGHT (CIME21X, CIME29, DER(L))
$FROM M11
      PRINT 300, CIME21X
300  FORMAT (*O* 13X *CIME2X9* 6X *CIME2Y9* 6X *CIME2Z9* / 14X 3E13.5)
310  IF (NN5 .LE. 0) GO TO 360
      CALL NINE (CFB9, CFB9(1,1,H), DER(L))
      DATA EIMAT(1), EIMAT(2)/2*0/
      CALL CRUSS (CFM8, EIPAT, CFB, 1)
      PRINT 320, PSO, CFB8
320  FORMAT (*OPSIO* 9X *CFMX8* 8X *CFMY8* 8X *CFMZ8* / X 4E13.5)
      IF (NN6 .LE. 1) GO TO 360
      PRINT 330, CFB8
$FROM M12
330  FORMAT (1H0 13X *CFX8* 9X *CFY8* 9X *CFZ8* / 14X 3E13.5)
      IF (NN6 .LE. 2) GO TO 310
      *CAIF8 = XMAT * CAIF9
      CALL MATRIX (20,3 ,3 ,1 ,XMAT ,3 ,CAIF9
      X 3 ,CAIF8 ,3
      PRINT 340, CAIF8
340  FORMAT (1H0 13X *CAIFX8* 7X *CAIFY8* 7X *CAIFZ8* / 14X 3E13.5)
      IF (NN6 .LE. 3) GO TO 310
      CALL EIGHT (T31, CIE28, DER(L))
      CALL MATRIX (23,3,3,1,CAT,3,CIE28,3,T31,3)
      CALL MATRIX (23,3,3,1,CBT,3,T31,3,CIE28,3)
      PRINT 350, CIE28
350  FORMAT (1H0 13X *CIE2X8* 7X *CIE2Y8* 7X *CIE2Z8* / 14X 3E13.5)
$FROM M13
360  IF (NN7 .LE. 0) GO TO 410
      CALL NINE (CM6, CM6(1,1,H), DER(L))
      PRINT 370, PSO, CM6
370  FORMAT (*OPSIO* 9X *CFX6* 13X *CFY6* 9X *CMZ6* / X 4E13.5)
      IF (NN7 .LE. 1) GO TO 410
      CALL NINE (CF6, CF6(1,1,H), DER(L))
      PRINT 380, CF6
380  FORMAT (1H0 13X *CFX6* 13X *CFY6* 13X *CFZ6* / 14X 3E13.5)

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SCHE1020 20400000
SCHE1030 20500000
SCHE1040 20600000
SCHE1050 20700000
SCHE1060 20800000
SCHE1070 20900000
SCHE1080 21000000
SCHE1100 21100000
SCHE1110 21200000
SCHE1120 21300000
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SCHE1160 21700000
SCHE1170 21800000
SCHE1180 21900000
SCHE1230 22000000
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SCHE1830 26800000
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SCHE1860 27000000
SCHE1870 27100000
SCHE1890 27200000
SCHE1900 27300000
PREPROCS 27400000
PREPROCS 27500000
SCHE1910 27600000
SCHE1920 27700000
SCHE1930 27800000
SCHE1940 27900000
SCHE1955 28000000
SCHE1960 28100000
SCHE1970 28200000
SCHE1980 28300000
SCHE1990 28400000
SCHE2000 28500000
SCHE2010 28600000
SCHE2030 28700000
SCHE2040 28800000
SCHE2050 28900000
SCHE2060 29000000

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IF (NN7 .LE. 2) GO TO 410
CALL SEVEN (CIE1, CIE137, DERIL1)
$FROM M14
PRINT 390, CIE1
390 FORMAT (1H0 13X *CIE1* 8X *CIE1* 8X *CIE12* / 14X 3E13.5)
IF (NN7 .LE. 3) GO TO 410
CALL SEVEN (CIE16, CMF1637, DERIL1)
PRINT 400, CME16
400 FORMAT (1H0 13X *CME16* 7X *CME16* 7X *CME126* / 14X 3E13.5)
410 IF (NN8 .LE. 0) GO TO 440
CALL SEVEN (CML6, CML637, DERIL1)
PRINT 420, PS0, CML6
$FROM M15
420 FORMAT (5H0PS10, 9X, *CM16* 8X *CM16* 8X *CM126* / X 4E13.5)
$FROM M16
IF (NN9 .LE. 1) GO TO 440
CALL SEVEN (CFL3, CFL337, DERIL1)
PRINT 430, CFL3
430 FORMAT (1H0 13X *CFL3* 8X *CFL3* 8X *CFL123* / 14X 3E13.5)
440 IF (NN9 .LE. 0) GO TO 480
DO 460 K = 1, 3
DO 450 J = 1, 3
CML3(K) = CML334(K,J+1) * DERIL+J-1)
450 CONTINUE
CML3(K) = CML3(K) + CML334(K,J)
$FROM M17
460 CONTINUE
PRINT 465, PS0, CML3
465 FORMAT (*0PS10* 9X *CML3* 8X *CML3* 8X *CML23* / X 4E13.5)
IF (NN9 .LE. 1) GO TO 480
CALL NINE (CF3, CF335, DERIL1)
PRINT 470, CF3
470 FORMAT (1H0 13X *CF3* 9X *CF3* 9X *CF23* / 14X 3E13.5)
480 IF (NN10 .LE. 0) GO TO 510
PRINT 485, (ANG(J),J=1,6), (VAR(J),J=1,7)
485 FORMAT (*0PS10* 3X 6R13 / X 7E13.5)
IF (NN10 .LE. 1) GO TO 510
C DER12) IS EQUIVALENCED TO THEAR(1)
DO 490 K = 1, 6
THOOT(K) = OMFGA * THEAR(K)
490 CONTINUE
$FROM M18
PRINT 500, THOOT
500 FORMAT (16X1H, 5(12X 1H, ) / 14X *THETAX0* 6X *THETAY0* 6X
A *THETA20* 6X *THETA1* 6X *THETA1* 6X *THETA21* / 14X 6E13.5)
DATA ANG /7RTHETAXC, 7RTHETAY0, 7RTHETA20, 7RTHETA1, 7RTHETA1,
A 7RTHETA21, 5RRETA1, 5RRETA2, 5RRETA3, 5RRETA0, 3RX11, 3RX12,
$ 3RX13,
B 3RX10/
510 K = 0
C VAR(2) IS EQUIVALENCED TO TH(1)
DO 520 J = 1, 6
IF (TH(J) .GE. THMAX) K = J
520 CONTINUE
DO 530 J = 1, NB
IF (BETA(J) .GE. BMAX) K = J + 6
$FROM M19
IF (X1(J) .GE. XMAX) K = J + 10
530 CONTINUE
IF (K .EQ. 0) GO TO 550
PRINT 540, ANG(K)
540 FORMAT (1H0 #7, * IS TOO LARGE. IT IS TIME TO RAIL OUT. *)
GO TO 10
550 IF (PS0 .LT. PSMAX) GO TO 50
GO TO 10
END
SUBROUTINE AFLAP (EIOR)
$FROM V1
COMMON /ALL/ NB, J, K
COMMON /DERAF/ B10(3,3), B2(3,3), B3(3,3), B4(3,3), B4T(3,3),
A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
B B10(3,3), B11(3,3), B12(3,3), B13(3,3), B14(3,3),
C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
E D29(3,3), D3(3,3), D30(3,3), D31(3,3), D32(3,3), D33(3,3),
F D34(3,3), D4(3,3), D4C(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
G D9(3,3), JJ,
H PE2333(3,3), PLNB34(3,4), PLNB37(3,7), TH10T(3,3)
I , B30(3)
J , 1
COMMON /SCJAF/ A1, BIASA01, BIASA02, BIASA03, BIASA11, BIASA12,
A BIASA13, CDAB(460), CLTAB(460),
B CBT(3,3), G6SAVE(3,5,11)
C , F, NAD, NAL, NN1, NN2,
D PT89839(3,9,11), T31(3), AMO, DMD1, DMD2,
E DMD3, BIASA01, BIASA02, BIASA03, NML, DML1, DML2, DML3,
F BIASML1, BIASML2, BIASML3
REAL D75(3), D76(3)
$FROM L2
EQUIVALENCED (PLNB34, C77), (PLNB37, C76), (PE1837, D75),
A (PE2838, D50), (PK8835, D1)
NAMLIST /AFLA/ B5, C29, D30, D31, D32, D33, D34, D75, D76, E10R,
A PE1837, PLNB37
$FROM V2
DO 10 K = 1, 3
D75(K) = D76(K) + B5(K,3) * E10R
PE1837(1,1) IS EQUIVALENCED TO D75(1)
PE1837(K,2) = PLNB37(K,2) + D29(K,3) * E10R
PE1837(K,3) = PLNB37(K,3) + D30(K,3) * E10R
PE1837(K,4) = PLNB37(K,4) + D31(K,3) * E10R
PE1837(K,5) = PLNB37(K,5) + D32(K,3) * E10R
PE1837(K,6) = PLNB37(K,6) + D33(K,3) * E10R

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SCHE2070 30600000
SCHE2080 30700000
SCHE2090 30800000
SCHE2100 30900000
SCHE2110 31000000
SCHE2120 31100000
SCHE2130 31200000
SCHE2140 31300000
SCHE2150 31400000
SCHE2160 31500000
SCHE2170 31600000
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SCHE2190 31800000
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SCHE2660 37100000
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AFLA0010 37300000
COMMO260 37400000
37500000
37600000
DERA0020 37700000
DERA0040 37800000
DERA0050 37900000
DERA0060 38000000
DERA0070 38100000
DERA0080 38200000
DERA0090 38300000
38400000
38500000
38600000
38700000
38800000
38900000
39000000
39100000
39200000
AFLA0030 39300000
EQUI0010 39400000
EQUI0060 39500000
EQUI0070 39600000
39700000
39800000
AFLA0040 39900000
AFLA0050 40000000
AFLA0060 40100000
AFLA0070 40200000
AFLA0080 40300000
AFLA0090 40400000
AFLA0100 40500000
AFLA0110 40600000
AFLA0120 40700000

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PE18837(K,7) = PL8837(K,7) + D34(K,3) * E10R
$FROM V3
10 CONTINUE
RETURN
END
SUBROUTINE ALAG (E2CR)
$FROM W1
COMMON /ALL/ NB, J, K
COMMON /DEKAF/ B10(3,3), B2(3,3), B3(3), B4(3,3), B4T(3,3),
A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
B D10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
E D29(3,3), D3(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
F D34(3,3), D4(3,3), D4C(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
G D9(3,3), JJ, PE18837(3,7),
H PE28838(3,8), PL8834(3,4), PL8837(3,7), TH10T(3,3)
I , B30(3)
J , I
COMMON /SCDAF/ A1, BIASA01, BIASA02, BIASA03, BIASA11, BIASA12,
A BIASA13, CDTAB(48C), CLTAB(480),
B C0T(3,3), G6SAVE(3,9,11)
C , H, NAD, NAL, NAL, NN2,
D PTHB839(3,9,11), T31(3), NMD, DMD1, DMD2,
E DMD3, BIASMD1, BIASPD2, BIASMD3, NML, DML1, DML2, DML3,
F BIASML1, BIASML2, BIASML3
REAL D50(3)
A , D75(3)
EQUIVALENCE (PL8834, D77), (PL8837, D76), (PE18837, D75),
A (PE28838, D50), (PKB839, D1)
NAMELIST /ALAGNAM/ B8, D21, D22, U23, D24, D25, D26, D28, D50,
A D75, E2OR, PE18837, PE28838
$FROM W2
DO 10 K = 1, 3
D50(K) = D75(K) + B8(K,3) * E2OR
C PE28838(1,1) IS EQUIVALENCED TO D50(1)
PE28838(K,2) = PE18837(K,2) + D21(K,3) * E2OR
PE28838(K,3) = PE18837(K,3) + D22(K,3) * E2OR
PE28838(K,4) = PE18837(K,4) + D23(K,3) * E2OR
PE28838(K,5) = PE18837(K,5) + D24(K,3) * E2OR
PE28838(K,6) = PE18837(K,6) + D25(K,3) * E2OR
$FROM W3
PE28838(K,7) = PE18837(K,7) + D26(K,3) * E2OR
PE28838(K,8) = D28(K,3) * E2OR
10 CONTINUE
RETURN
END
SUBROUTINE ASHAFT (LILLOR)
$FROM P1
COMMON /ALL/ NB, J, K
COMMON /DEKAF/ B10(3,3), B2(3,3), B3(3), B4(3,3), B4T(3,3),
A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
B D10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
E D29(3,3), D3(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
F D34(3,3), D4(3,3), D4C(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
G D9(3,3), JJ, PE18837(3,7),
H PE28838(3,8), PL8834(3,4), PL8837(3,7), TH10T(3,3)
I , B30(3)
J , I
COMMON /SCDAF/ A1, BIASA01, BIASA02, BIASA03, BIASA11, BIASA12,
A BIASA13, CDTAB(480), CLTAB(480),
B C0T(3,3), G6SAVE(3,9,11)
C , H, NAD, NAL, NAL, NN2,
D PTHB839(3,9,11), T31(3), NMD, DMD1, DMD2,
E DMD3, BIASMD1, BIASPD2, BIASMD3, NML, DML1, DML2, DML3,
F BIASML1, BIASML2, BIASML3
REAL D76(3), D77(3), LILLOR
EQUIVALENCE (PL8834, D77), (PL8837, D76), (PE18837, D75),
A (PE28838, D50), (PKB839, D1)
NAMELIST /ASHA/ H10, C5, C6, D7, D76, D77, LILLOR, PL8834, PL8837
$FROM P2
DO 10 K = 1, 3
D76(K) = D77(K) + B10(K,2) * LILLOR
C PL8837(1,1) IS EQUIVALENCED TO D76(1)
PL8837(K,2) = PL8834(K,2) + D5(K,2) * LILLOR
$FROM P3
PL8837(K,3) = PL8834(K,3) + D6(K,2) * LILLOR
PL8837(K,4) = PL8834(K,4) + D7(K,2) * LILLOR
PL8837(K,5) = D8(K,2) * LILLOR
PL8837(K,6) = D9(K,2) * LILLOR
PL8837(K,7) = D10(K,2) * LILLOR
10 CONTINUE
RETURN
END
SUBROUTINE AWING (CAPLER)
$FROM D1
COMMON /ALL/ NB, J, K
COMMON /DEKAF/ B10(3,3), B2(3,3), B3(3), B4(3,3), B4T(3,3),
A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
B C10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
E D29(3,3), D3(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
F D34(3,3), D4(3,3), D4C(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
G D9(3,3), JJ, PE18837(3,7),
H PE28838(3,8), PL8834(3,4), PL8837(3,7), TH10T(3,3)
I , B30(3)
J , I
COMMON /SCDAF/ A1, BIASA01, BIASA02, BIASA03, BIASA11, BIASA12,
A BIASA13, CDTAB(48C), CLTAB(480),

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AFLA0130 408C0000
AFLA0140 40900000
AFLA0150 410C0000
AFLA0160 41100000
AFLA0170 41200000
ALAG0020 41300000
ALAG0010 41400000
COMM0260 415C0000
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41700000
41800000
DERA0040 41900000
DERA0050 42000000
DERA0060 42100000
DEKAC070 42200000
42300000
42400000
DERA0090 42500000
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ALAG0030 43400000
ALAG0035 43500000
EQUIT060 43600000
EQUIT070 43700000
43800000
43900000
ALAG0040 44000000
ALAG0050 44100000
ALAG0060 44200000
ALAG0070 44300000
ALAG0080 44400000
ALAG0090 44500000
ALAG0110 44600000
ALAG0120 447C0000
ALAG0140 44800000
ALAG0130 44900000
ALAG0150 45000000
ALAG0150 45100000
ALAG0160 452C0000
ALAG0170 45300000
ALAG0180 45400000
ASHA0020 455C0000
ASHA0010 45600000
COMM0260 457C0000
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45900000
46000000
DERA0040 46100000
DERA0050 462C0000
DERA0060 46300000
DERA0070 464C0000
46500000
DERA0090 46600000
467C0000
46800000
469C0000
47000000
47100000
47200000
47300000
47400000
47500000
ASHA0030 476C0000
EQUIT060 47700000
EQUIT070 47800000
479C0000
ASHA0040 48000000
ASHA0050 48100000
ASHA0060 48200000
ASHA0070 48300000
ASHA0080 484C0000
ASHA0090 48500000
ASHA0100 48600000
ASHA0110 48700000
ASHA0120 48800000
ASHA0130 48900000
ASHA0140 49000000
ASHA0150 49100000
ASHA0160 49200000
ASHA0170 49300000
AWINC020 494C0000
AWINC010 49500000
COMM0260 496C0000
49700000
49800000
499C0000
50000000
DERA0040 50100000
DERA0050 50200000
DERA0060 50300000
DERA0070 50400000
50500000
DERA0090 50600000
50700000
50800000
50900000

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      B CRT(3,3), G6SAVE(3,9,11)
      C , F, NAU, NAL, AN1, AN2,
      D PTBB33(3,9,11), T1(3), NMD, DMD1, DMD2,
      E DMD3, BIASMD1, BIASMD2, BIASMD3, NML, DML1, DML2, DML3,
      F BIASML1, BIASML2, BIASML3
      REAL D77(3)
      EQUIVALENCE (PLB834, C77), (PLB837, D76), (PE18837, D75),
      A (PE28338, D50), (PKB839, D1)
      NAMELIST /AWIN/ CAPLOR, C2, C3, C4, D40, D77, PLB834
SFRJH 02
      DO 10 K = 1, 3
      D77(K) = D40(K) * CAPLOR
      C      PLB834(1,1) IS EQUIVALENCED TO D77(1)
      PLB834(K,2) = D2(K) * CAPLOR
      PLB834(K,3) = D3(K) * CAPLOR
      PLB834(K,4) = D4(K) * CAPLOR
      10 CONTINUE
      RETURN
      END
      SUBROUTINE BLADE (Z1ZUR, G)
SFRJH 01
      COMMON /BLANK/ SKIP(506), PSO
      COMMON /ALL/ NB, J, K
      A ,RET
      COMMON /ANGLES/ AD1, AD2, AD3, AL1, AL2, AL3, DAD1, DAD2, DAD3, DAL1,
      A DAL2, DAL3, OMEGA, SPEC, TH1, VAR(29), THOS, THISC, THISS, DELTA3,
      B GETAMA, UPSIC, PSIMAX, XIMAX
      COMMON /C-K/ SKIP1(8), MD2, MD3, ML1, ML2, ML3
      COMMON /IN/ NN10, NN3, NN4, NN5, NN6, NN7, NN8, NN9, OR, PRSQRH,
      A SOUND, ILLXP, NAD1, NAD2, NAD3, NAL1, NAL2, NAL3, NMD1, NMD2,
      B NMD3, NML1, NML2, NML3
      COMMON /DERAF/ B10(3,3), B21(3,3), B31(3,3), B41(3,3),
      A B57(3,3), B67(3,3), B71(3,3), B81(3,3), B91(3,3),
      C D10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
      D D15(3,3), D16(3,3), D17(3,3), D18(3,3), D19(3,3), D21(3,3),
      E D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
      F D29(3,3), D31(3,3), D30(3,3), D31(3,3), D32(3,3), D33(3,3),
      G D34(3,3), D41(3,3), D42(3,3), D51(3,3), D61(3,3), D71(3,3), D81(3,3),
      H D91(3,3), JJ, PE18837(3,7),
      I , B30(3)
      J , I
      COMMON /INT/ IFLAG
      COMMON /SCDAF/ A1, BIASAD1, BIASAD2, BIASAD3, BIASAL1, BIASAL2,
      A BIASAL3, CUTAB(480), CLTAB(480),
      B CRT(3,3), G6SAVE(3,9,11)
      C , F, NAU, NAL, AN1, AN2,
      D PTBB33(3,9,11), T1(2), NMD, DMD1, DMD2,
      E DMD3, BIASMD1, BIASMD2, BIASMD3, NML, DML1, DML2, DML3,
      F BIASML1, BIASML2, BIASML3
      G , MMAX
      H , II
      DIMENSION G(8), PTBBAR(3,1), P12BAR(3,1)
      INTEGER RET
      EQUIVALENCE (PLB834, D77), (PLB837, D76), (PE18837, D75),
      A (PE28338, D50), (PKB839, D1)
      C      G(1), G(2), G(3), AND G(4) CORRESPOND TO G1, G3, G2, AND G4,
      C      RESPECTIVELY
      REAL MACH
      A , NAD, NMD, MD2, MD3, NAL, NML, ML2, ML3
      B , MMAX
      NAMELIST /BLAD/ ARSA, ACC, AC2, AD3, ALPHA, AL2, AL3, A1, BIASAD1,
      A BIASAD2, BIASAD3, BIASAL1, BIASAL2, BIASAL3, BIASMD1, BIASMD2,
      B BIASMD3, BIASML1, BIASML2, BIASML3, B2, B3, B30, B4, CD, CL, CPH,
      C DAD1, DAU2, DAU3, DAL1, DAL2, DAL3, DMD1, DMD2, DMD3, DML1, DML2,
      D DML3, I, IFLAG, II, J, K, MACH, MD2, MD3, ML2, ML3, MMAX, NMD,
      E NAL, JND, NML, NN1, PFI, PSC, PTBBAR, P12BAR, SPH, TEMP, TEMP1,
      F TH1, VOORSEC, VOORSEC
SFRJH 02
      DO 10 K = 1, 3
      PTBBAR(K) = B31(K) + P2(K,3) * Z1ZUR
      10 CONTINUE
      CALL MATRX(20,3,3,1,4,3,PTBBAR,3,P12BAR,3)
      DO 15 J=1,3
      15 P12BAR(J) = P12BAR(J) + B30(J)
      PHI = ATAN2 (P12BAR(2), P12BAR(1))
      ALPHA = -PHI - TH1 * Z1ZUR
      VOORSEC = P12BAR**2 + P12BAR(2)**2
      VOORSEC = SQRT (VOORSEC)
      MACH = VOORSEC * A1
      IF (MACH .GT. 4MAX) GO TO 50
      IF (ALPHA .LT. -3.14159265358979) ALPHA = ALPHA +
      A 6.28318530717958
      ARSA = ARS(ALPHA)
      CALL FUNG2 (COTAB, CD, NMD, NAU, MACH, MD2, MD3, DMD1, DMD2,
      A DMD3, BIASMD1, BIASMD2, BIASMD3, ARSA, AD2, AD3, DAD1, DAD2,
      B DAD3, BIASAD1, BIASAD2, BIASAD3)
      CALL FUNG2 (CLTAB, CL, NML, NAL, MACH, ML2, ML3, DML1, DML2,
      A DML3, BIASML1, BIASML2, BIASML3, ARSA, AL2, AL3, DAL1, DAL2,
      B DAL3, BIASAL1, BIASAL2, BIASAL3)
      IF (ALPHA .LT. 0) CL = -CL
      IF (NN1 .LE. 1) GO TO 40
      IF (IFLAG .NE. 0) GO TO 40
      PRINT 20, PSO, ALPHA, PTBBAR, Z1ZUR, I
      20 FORMAT (27X 1H- Z1ZUR 1H-) / * PSIO * 9X *ALPHA* 8X *X/(TOTAL BLAD
      A) Y/(TOTAL BLADE) Z/(TOTAL BLADE) Z1ZUR * 10X *BLADE * /
      B X 3E13.5, 3E19.5, 1R)
      IF (NN1 .GT. 2) PRINT 30, PHI, MACH, CL, CD
      30 FORMAT (*PHI*, 10X, 4FPACH, 9X, 2HCL, 11X, 2HCD / 4E13.5)
      40 CPH = COS (PHI)
SFRJH 04
      SPH = SIN (PHI)

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TEMP = VOORSQ * CL
TEMP1 = VOORSQ * CC
G(5) = TEMP * CPH
G(1) = TEMP1 * SPH
G(6) = TEMP * SPH
G(2) = TEMP1 * CPH
G(7) = G(5) * Z1ZUR
G(3) = G(1) * Z1ZUR
G(8) = G(6) * Z1ZUR
G(4) = G(2) * Z1ZUR
RETURN
50 PRINT 6J, MMAX
60 FORMAT (* ERROR: THE MACH NUMBER IS GREATER THAN* F5.3)
GO TO RET
END
SUBROUTINE CROSS (A, B, C, N)
$FROM S1
DIMENSION A(3,9), B(3), C(3,9)
DO 10 I = 1, N
  A(1,I) = B(2) * C(3,I) - B(3) * C(2,I)
  A(2,I) = B(3) * C(1,I) - B(1) * C(3,I)
  A(3,I) = B * C(2,I) - B(2) * C(1,I)
10 CONTINUE
RETURN
END
SUBROUTINE DERSUB
REAL BETA(4)
DIMENSION BBAR(4)
$FROM N1
COMMON/BLANK /CAIF939(3,9), CAIM939(3,9), CAM1(3,1), CAPLOK,
$ CA12(4), CR, CX, CXC, CX1, CY0, CY1, CZ0, CZ1,
A CFL337(3,7), CF339(3,9), CF639(3,9), CF839(3,9), CIME2(48),
$ C1FM(54),
B C1F1239(3,9), C1FM1(42), C1M139(3,9), CME1637(3,7),
$ CIME29(3,8),
C CML334(3,4), CML6(3), CML637(3,7), CM639(3,9), CONST, CTM939(3,9), BLAN0040
D CUVAR(29), DER(29),
$ ILXP,
E DPS0, EIMAT(3), KB, KY, KX0, KX1, KY0, KY1, KZ0, KZ1, LILLUR, M, N, BLANCC60
F PS1(4), SPR, THMAT(3,3), THT(3,3), UOOR, VOOK, WOOD, XHAR(4), XC, BLAN0070
G XMAT(3,3),
$ XI(4)
H, EZMAT
I, CILXP
J, C03, SD3, SSQD3, CSQD3
K, NOUR(4), CATY12P(4), COTH, COTHSP0, SDTHCP0, M10CDTH, NIT
M, UP1, UP97
N, NPB, NMI
COMMON /ALL/ NB, J, F
A, RET
COMMON /ANGLES/ AD1, AC2, AD3, AL1, AL2, AL3, DAD1, DAD2, DAD3, DAL1,
A DAL2, DAL3, OMEGA, SPEC, TH1, VAR(29), TH03, TH1SC, TH1SS, DELTA,
B BETAMAX, OPSIC, PSIPAX, XIMAX
COMMON /CHK/ SKIP(53), CC
COMMON /DEKAF/ B10(3,3), B2(3,3), B3(3), B4(3,3), B4T(3,3),
A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
C C10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
D D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
E D29(3,3), D3(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
F D34(3,3), D4(3,3), D40(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
G D9(3,3), JJ, PE18B37(3,7),
H PE28B36(3,8), PLB834(3,4), PLB837(3,7), TH10T(3,3)
I, B30(3)
J, I
COMMON /ICKY/ IC(14)
COMMON /SCDAF/ A1, BIASAD1, BIASAD2, BIASAD3, BIASAL1, BIASAL2,
A BIASAL3, CUTAB(48C), CLTAB(480),
B CBT(3,3), G6SAVE(3,9,11)
C, P, NAD, NAL, NM1, NM2,
D PT88B39(3,9,11), T31(3), NMD, DMD1, DMD2,
E DMD3, BIASMD1, BIASMD2, BIASMD3, NML, DML1, DML2, DML3,
F BIASML1, BIASML2, BIASML3
INTEGER H
A, RET
REAL BET1(3,3), BET2(3,3), BMAT(3,3), BT(3,3),
A B1(3), B5(3,3), B6(3,3), CAIM9(3,9), CAIMH1(3,9), CAIF12(3,9)
B, CANU12(3,1), CFL37(3,7), CF939(3,9),
C C1E137(3,7), C1E238(3,8), C1F39(3,9), CIME238(3,8),
D C1M39(3,9), CME137(3,7), CML34(3,4), CML37(3,7),
E CM3(3), CTFM939(3,9), C1(3,3), C10(3,3), C11(3,3),
F C12(3,3), C13(3,3), C14(3,3), C15(3,3), C16(3,3), C17(3,3),
G C18(3,3), C19(3,3), C2(3,3), C20(3,3), C21(3,3), C22(3,3),
H C23(3,3), C24(3,3), C25(3,3), C26(3,3), C27(3,3), C3(3,3),
I C30(3,3), C32(3,3), C33(3,3), C4(3,3), C5(3,3), C5T(3,3),
J C6T(3,3), C7(3,3), C71(3,3), C8(3,3), J1(3), D41(3,3), D50(3),
K D75(3), D76(3), D77(3), E2MAT(3), KB, KX, KX0, KX1, KY0, KY1,
L KZ0, KZ1, LILLUR, PSBART(3,3), PSBRT(3,3), PSMAT(3,3), PST(3,3),
M SCF337(3,7), SCF637(3,7), SCN637(3,7), TEMP1(3), TEMP2(3,3),
N TEMP3(3,3), TH10(3,3), TH10BT(3,3), TH11(3,3), TH11T(3,3),
O TH11T(3,3), THXOUT(3,3), THXOMAT(3,3), THXOT(3,3), THXO1(3,3),
P THXO2(3,3), THX1BT(3,3), THX1MAT(3,3), THX1T(3,3), THX11(3,3),
Q THX12(3,3), THYOBT(3,3), THYOMAT(3,3), THYOT(3,3), THY01(3,3),
R THY02(3,3), THY1BT(3,3), THY1MAT(3,3), THY1T(3,3), THY11(3,3),
S THY12(3,3), THZOBT(3,3), THZOMAT(3,3), THZOT(3,3), THZ01(3,3)
REAL THZ02(3,3), THZ1BT(3,3), THZ1MAT(3,3), THZ1T(3,3),
A THZ11(3,3), THZ12(3,3), T33(3,3), T37(3,7), T39(3,9), U33(3,3),
B V33(3,3), X1(3,3), X2(3,3)
C, A1(4,15), RBART(3,3), CFML1(42)
D, XBART(3,3), XT(3,3), C6(3,3)
$, ILXP
E, NOUR, M10CDTH, PCHAR(3), NSOOR

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BLAD0310 61200000
BLAD0329 61300000
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DERS0342 61700000
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DRRL0025 71200000
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EQUIVALENCE (CIFM2, CIE238), (CIFM2(25), CIME238), (CIME1,
A CIE137), (CIME1(22), CME137), (EIMAT(3), EIOR), (UER(8), BBAR),
B (DER12), THXOBAR, (DER13), THYOBAR), (DER14), THZOBAR), (DER15),
C THX1BAR), (DER16), THY1BAR), (DER17), THZ1BAR)
EQUIVALENCE (PLB834, D77), (PLB837, D76), (PE1B837, D75),
A (PE2B838, D50), (PKB839, D1)
EQUIVALENCE (CUVAR(1), PSO), (CUVAR(2), THXO), (CUVAR(3), THYO),
A (CUVAR(4), THZO), (CUVAR(5), THX1), (CUVAR(6), THY1), (CUVAR(7),
B THZ1), (CUVAR(8), BETA(1)), (E2MAT(3), E2OR), (BMAT(2,2), BC),
C (BMAT(3,2), BS), (THXOMAT(2,2), THXOC), (THXOMAT(2,3), THXOS),
D (THYOMAT, THYOC), (THYOMAT(3), THYOS), (THZOMAT, THZOC),
E (THZOMAT(1,2), THZCS), (THXIMAT(2,2), THXIC), (THXIMAT(2,3),
F THXIS), (THYIMAT, THYIC), (THYIMAT(3), THYIS), (THZIMAT, THZIC),
G (THZIMAT(1,2), THZIS), (XPAT, XIC), (XMAT(1,3), XS), (PSMAT, PSC),
H (PSMAT(1,1), PSS)
$FROM N1
EQUIVALENCE (CUVAR(1), PSO), (CUVAR(2), THXO), (CUVAR(3), THYO),
A (CUVAR(4), THZO), (CUVAR(5), THX1), (CUVAR(6), THY1), (CUVAR(7),
B THZ1), (CUVAR(8), BETA(1)), (E2MAT(3), E2OR), (BMAT(2,2), BC),
C (BMAT(3,2), BS), (THXOMAT(2,2), THXOC), (THXOMAT(2,3), THXOS),
D (THYOMAT, THYOC), (THYOMAT(3), THYOS), (THZOMAT, THZOC),
E (THZOMAT(1,2), THZCS), (THXIMAT(2,2), THXIC), (THXIMAT(2,3),
F THXIS), (THYIMAT, THYIC), (THYIMAT(3), THYIS), (THZIMAT, THZIC),
G (THZIMAT(1,2), THZIS), (XPAT, XIC), (XMAT(1,3), XS), (PSMAT, PSC),
H (PSMAT(1,1), PSS)
$FROM N1A
EXTERNAL BLADE, FBLADE, FFLAP, FLAG, FSHAFT, FWING
EQUIVALENCE (CFML, CFL37), (CFML(22), CML37), (CIFM, CIF39),
A (CIFM(28), CIM39), (CA12, CATY12), (CA12(2), CAMX12), (CA12(3),
B CAQX12), (CA12(4), CAQY12)
$FROM N2
C (CTM939, CTFM939)
D (THMAT, THC), (THMAT(1,2), THS)
C PSO, THXO, THYO, THZO, THX1, THY1, THZ1, AND BETA(1), ARE
EQUIVALENCED TO THE 1ST 8 ELEMENTS OF CUVAR, RESPECTIVELY, IN
C DERSUB ONLY
NAMELIST /DERS/ A, A10, A12, A13, A14, A15, A16, A17, A18, A19,
A A2, A20, A21, A22, A23, A24, A25, A26, A27, A28, A29, A3, A30,
B A31, A32, A34, A35, A36, A37, A4, A6, A8, A9, BBAR, BBART, BC,
C BCSQ, BETA, BET1, BET2, BMAT, BS, BSSQ, BT, B1, B10, B2, B3, B30,
D B4, B4T, B5, B5T, B6, B6T, B7, B8, B9, CA1B11, CA1B9, CA1F12,
E CA1F39, CA1F939, CAMC12, CAMX12, CAM11, CAPLOR, CAQX12, CAQY12,
F CA1Y12, LATY12P, C12, CR, CC, COTH, COTHSPD, C03, CFL337, CFL37,
G CFPL, CF339, CF639, CF839, CF939, CIE137, CIE238, CIFM, CIFM2,
H CIF123, CIF39, CILXP, CIME1, CIME238, CIME29, CIM1139, CIM39,
I CML37, CME1637, CML334, CML34, CML37, CML6, CML637, CM3, CM639,
J CUNST, CSQD3, CTFM939, CTF939, CUVAR, CX, CX0, CX1, CY0, CY1,
K C20, C21, C1, C10, C11, C12, C13, C14, C15, C16, C17, C18, C19,
L C2, C20, C21, C22, C23, C24, C25, C26, C27, C3, C30, C32, C33,
M C4, C5, C5T, C6, C6T, C7, C7T, C8, C8T, DER, DPS0, D1, D10, D11,
N D12, D13, D14, D15, D16, D18, D19, D2, D21, D22, D23, D24, D25,
O D26, D28, D29, D3, D3C, D31, D32, D33, D34, D4, D40, D41, D5,
P D6, D7, D8, D9, EIMAT, EIOR, E2MAT, E2OR,
Q G6SAVE, H, I, IC, IFLAG, II, ILXP, J, JJ, K, KB, KK, KX, KX0,
R KX1, KY0, KY1, KZ0, KZ1, LILLOR, LRD, M, MIOCPTH, N, NB, NIT,
S N1, N11, N10R, ASCR, PSBART, PSBHT, PSC, PSI, PSMAT, PSS, PST
NAMELIST /DERS2/ PSC, P6BAR, SCF317, SCF637, SCM637, SOTHCPD, SD3,
A SPH, SSQD3, TEMP, TEMP2, TEMP3, THC, THETA, THIO, THIOBT, THIOU,
B TH11, TH11T, TH11T, THMAT, THS, TMT, THXO, THXOBAR, THXOBT,
C THXOC, THXOMAT, THXCS, THXOT, THXO1, THXO2, THX1, THX1BAR,
D THX1BT, THXIC, THXIPAT, THXIS, THXIT, THX11, THX12, THYO,
E THYOBAR, THYOBT, THYOC, THYOCAT, THYOS, THYOT, THY01, THY02,
F THY1, THY1BAR, THY1BT, THYIC, THYIMAT, THYIS, THYIT, THY11,
G THY12, THZO, THZOBAR, THZBT, THZOC, THZOMAT, THZOS, THZOT,
H THZ01, THZ02, THZ1, THZ1BAR, THZ1BT, THZIC, THZIMAT, THZIS,
I THZ1T, THZ11, THZ12, THOS, THISC, THISS, T11, T33, T37, T39,
J UDOR, U1, UP97, U33, VAR, V0OR, V33, W0OR, X, XBAR, XBART, XC,
K X1, X1C, XMAT, XPF, XS, XT, X1, X2, Y, YPR, Z, ZPR
DO 10 J = 1, NB
XI(J) = CUVAR(M+J)
XBAR(J) = CUVAR(NPB + J+6)
BBAR(J) = CUVAR(N+J+6)
10 CONTINUE
THXOBAR = CUVAR(N+1)
THYOBAR = CUVAR(N+2)
THZOBAR = CUVAR(N+3)
THX1BAR = CUVAR(N+4)
THY1BAR = CUVAR(N+5)
THZ1BAR = CUVAR(N+6)
THXOC = COS (THXO)
THXOS = SIN (THXO)
THYOC = COS (THYO)
THYOS = SIN (THYO)
$FROM N3
DO 20 J = 1, 21
SCF637(J) = 0
SCM637(J) = 0
20 CONTINUE
$FROM N4
I = 0
THZOC = COS (THZO)
THZCS = SIN (THZO)
THXIC = COS (THX1)
THYIS = SIN (THY1)
THYIC = COS (THY1)
THYIS = SIN (THY1)
THZIC = COS (THZ1)
THZIS = SIN (THZ1)
C THXOMAT(2,2) AND THXOMAT(2,3) ARE EQUIVALENCED TO THXOC AND
THXOS, RESPECTIVELY
$FROM N5
DATA (THXOMAT(J), J=1,4) /1., 3*0/, THXOMAT(7)/0/
THXOMAT(3,2) = -THXCS
THXOMAT(3,3) = THXCC
C THYOMAT(1,1) AND THYOMAT(3,1) ARE EQUIVALENCED TO THYOC AND
THYOS, RESPECTIVELY
C DATA THYOMAT(2,1), THYOMAT(1,2), THYOMAT(2,2), THYOMAT(3,2),
A THYOMAT(2,3) /0, 0, 1., 0, 0/
THYOMAT(1,3) = -THYCS
THYOMAT(3,3) = THYOC
C THZOMAT(1,1) AND THZOMAT(1,2) ARE EQUIVALENCED TO THZOC AND

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EQUI0020 71400000
EQUI0030 71500000
EQUI0040 71600000
EQUI0050 71700000
EQUI0060 71800000
EQUI0070 71900000
DERS0370 72000000
DERS0380 72100000
DERS0390 72200000
DERS0400 72300000
DERS0410 72400000
DERS0420 72500000
DERS0430 72600000
DERS0440 72700000
DERS0450 72800000
DERS0460 72900000
DERS0470 73000000
DERS0480 73100000
SCHE0130 73200000
SCHE0140 73300000
SCHE0150 73400000
DERS0490 73500000
73600000
73700000
73800000
73900000
DERS0520 74000000
74100000
74200000
74300000
74400000
74500000
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75300000
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DERS0530 77300000
DERS0540 77400000
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77600000
DERS0560 77700000
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78300000
DERS0570 78400000
DERS0580 78500000
DERS0590 78600000
DERS0600 78700000
DERS0610 78800000
DERS0620 78900000
DERS0630 79000000
DERS0640 79100000
DERS0650 79200000
DERS0660 79300000
DERS0670 79400000
DERS0680 79500000
DERS0690 79600000
DERS0700 79700000
DERS0710 79800000
DERS0720 79900000
DERS0730 80000000
DERS0740 80100000
DERS0750 80200000
DERS0760 80300000
DERS0770 80400000
DERS0780 80500000
80600000
80700000
DERS0800 80800000
DERS0810 80900000
DERS0820 81000000
DERS0830 81100000
DERS0840 81200000
DERS0850 81300000
DERS0860 81400000
DERS0870 81500000
DERS0880 81600000

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C      THZOS, RESPECTIVELY
$FROM N6
  DATA THZOMAT(3), THZOMAT(3,2), THZOMAT(1,3), THZOMAT(2,3),
  A THZOMAT(3,3) /4*0, 1./
  THZOMAT(2) = - THZOC
  THZOMAT(2,2) = THZOC
C      THXIMAT(2,2) AND THXIMAT(2,3) ARE EQUIVALENCED TO THXIC AND
C      THXIS, RESPECTIVELY
  DATA THXIMAT(1,1), THXIMAT(2,1), THXIMAT(3,1), THXIMAT(1,2),
  A THXIMAT(1,3) /1., 4*0/
  THXIMAT(3,2) = - THXIS
  THXIMAT(3,3) = THXIC
C      THXIMAT(1,1) AND THYIMAT(3,1) ARE EQUIVALENCED TO THYIC AND
C      THYIS, RESPECTIVELY
$FROM N7
  DATA THYIMAT(2), THYIMAT(1,2), THYIMAT(2,2), THYIMAT(3,2),
  A THYIMAT(2,3) /0, 0, 1., 0, 0/
  THYIMAT(1,3) = -THYIS
  THYIMAT(3,3) = THYIC
C      THZIMAT(1,1) AND THZIMAT(1,2) ARE EQUIVALENCED TO THZIC AND
C      THZIS, RESPECTIVELY
  DATA THZIMAT(3), THZIMAT(3,2), THZIMAT(1,3), THZIMAT(2,3),
  A THZIMAT(3,3) /4*0, 1./
  THZIMAT(2) = -THZIS
  THZIMAT(2,2) = THZIC
  DATA THXOI(1), THXOI(2), THXOI(3), THXOI(1,2), THXOI(1,3) /5*0/
  THXOI(2,2) = -THXOC
  THXOI(4,2) = -THXOS
  THXOI(2,3) = THXCS
$FROM N8
  THXOI(3,3) = -THXOC
  DATA (THXOI(J),J=1,4), THXOI(7) /5*0/
  THXOI(2,2) = -THXOS
  THXOI(3,2) = THXOC
  THXOI(2,3) = -THXOC
  THXOI(3,3) = -THXOS
  DATA THYOI(2), (THYCI(J),J=4,6), THYOI(8) /5*0/
  THYOI = THYOI(3,3) = -THYOC
  THYOI(3) = THYOS
  THYOI(1,3) = -THYOS
$FROM N9
  DATA THYOI(2), (THYOI(J),J=4,6), THYOI(8) /5*0/
  THYOI = THYOI(3,3) = -THYOS
  THYOI(3) = -THYOC
  THYOI(1,3) = THYOC
  DATA THZOI(3), (THZCI(J),J=6,9) /5*0/
  THZOI = THZOI(2,2) = -THZOC
  THZOI(2) = -THZOS
  THZOI(1,2) = THZCS
  DATA THZOI(3), (THZOI(J),J=6,9) /5*0/
  THZOI = THZOI(2,2) = -THZOS
$FROM N10
  THZOI(2) = THZOC
  THZOI(1,2) = -THZOC
  DATA (THXII(J),J=1,4), THXII(7) /5*0/
  THXII(2,2) = THXII(3,3) = -THXIC
  THXII(3,2) = -THXIS
  THXII(2,3) = THXIS
  DATA (THXII(J),J=1,4), THXII(7) /5*0/
  THXII(2,2) = THXII(3,3) = -THXIS
  THXII(3,2) = THXIC
  THXII(2,3) = -THXIC
$FROM N11
  DATA THYII(2), (THYII(J),J=4,6), THYII(8) /5*0/
  THYII = THYII(3,3) = -THYIC
  THYII(3,1) = THYIS
  THYII(1,3) = -THYIS
  DATA THYII(2), (THYII(J),J=4,6), THYII(8) /5*0/
  THYII = THYII(3,3) = -THYIS
  THYII(3) = -THYIC
  THYII(1,3) = THYIC
  DATA THZII(3), (THZII(J),J=6,9) /5*0/
  THZII = THZII(2,2) = -THZIC
$FROM N12
  THZII(2) = -THZIS
  THZII(1,2) = THZIS
  DATA THZII(3), (THZII(J),J=6,9) /5*0/
  THZII = THZII(2,2) = -THZIS
  THZII(2) = THZIC
  THZII(1,2) = -THZIC
  DO 30 K = 1, 9
    THXOBT(K) = THXOBT * THXOBT(K)
    THYOBT(K) = THYOB * THYOBT(K)
    THZOBT(K) = THZOBT * THZOBT(K)
    THXIBT(K) = THXIBT * THXIBT(K)
$FROM N13
  THYIBT(K) = THYIBT * THYIBT(K)
  THZIBT(K) = THZIBT * THZIBT(K)
30 CONTINUE
* THXOT = .TR. THXOMAT
CALL MATRIX ( 0,3 ,3 ,C,THXOMAT,3 ,THXOT ,
X 3
* THYOT = .TR. THYOPAT
CALL MATRIX ( 0,3 ,3 ,C,THYOMAT,3 ,THYOT ,
X 3
* THZOT = .TR. THZOPAT
CALL MATRIX ( 0,3 ,3 ,C,THZOMAT,3 ,THZOT ,
X 3
* THXIT = .TR. THXIMAT
CALL MATRIX ( 0,3 ,3 ,C,THXIMAT,3 ,THXIT ,
X 3
* THYIT = .TR. THYIMAT

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DERS0893 81600000
DERS0900 81700000
DERS0910 81800000
DERS0920 81900000
DERS0930 82000000
DERS0940 82100000
DERS0950 82200000
DERS0960 82300000
DERS0970 82400000
DERS0980 82500000
DERS0990 82600000
DERS1000 82700000
DERS1010 82800000
DERS1020 82900000
DERS1030 83000000
DERS1040 83100000
DERS1050 83200000
DERS1060 83300000
DERS1070 83400000
DERS1080 83500000
DERS1090 83600000
DERS1100 83700000
DERS1110 83800000
DERS1120 83900000
DERS1130 84000000
DERS1140 84100000
DERS1150 84200000
DERS1160 84300000
DERS1170 84400000
DERS1180 84500000
DERS1190 84600000
DERS1210 84800000
DERS1220 84900000
DERS1230 85000000
DERS1240 85100000
DERS1260 85300000
DERS1270 85400000
DERS1280 85500000
DERS1290 85600000
DERS1310 85700000
DERS1320 85800000
DERS1330 85900000
DERS1350 86200000
DERS1360 86300000
DERS1370 86400000
DERS1390 86600000
DERS1400 86700000
DERS1410 86800000
DERS1420 86900000
DERS1440 87100000
DERS1450 87200000
DERS1460 87300000
DERS1480 87500000
DERS1490 87600000
DERS1500 87700000
DERS1510 87800000
DERS1530 87900000
DERS1540 88000000
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DERS1570 88400000
DERS1580 88500000
DERS1590 88600000
DERS1610 88800000
DERS1620 88900000
DERS1630 89000000
DERS1640 89100000
DERS1660 89300000
DERS1670 89400000
DERS1680 89500000
DERS1690 89600000
DERS1700 89700000
DERS1710 89800000
DERS1720 89900000
DERS1730 90000000
DERS1740 90100000
DERS1750 90200000
DERS1760 90300000
DERS1770 90400000
DERS1780 90500000
PREPROCS 90600000
PREPROCS 90700000
DERS1790 90800000
PREPROCS 90900000
PREPROCS 91000000
DERS1800 91100000
PREPROCS 91200000
PREPROCS 91300000
DERS1810 91400000
PREPROCS 91500000
PREPROCS 91600000
DERS1820 91700000

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CALL MATRIX ( 0,3      ,3      ,0,THYIMAT,3      ,THYIT ,
X 3
*THZIT = .TR. THZIPAT
CALL MATRIX ( 0,3      ,3      ,0,THZIMAT,3      ,THZIT ,
X 3
* CALL MATRIX (20, 3, 3, 3, THZOMAT, 3, THZOMAT, 3, THIO , 3)
CALL MATRIX (20,3,3,3, TPZOMAT, 3, THYOMAT, 3, THIO, 3)
*THIO = THIO * THXOPAT
CALL MATRIX (20, 3, 3, 3, THIO , 3, THXOMAT, 3, THIO , 3)
*THI1 = THZIMAT * THYIMAT
CALL MATRIX (20, 3, 3, 3, THZIMAT, 3, THYIMAT, 3, THI1 , 3)
*THI1 = THI1 * THXIMAT
CALL MATRIX (20, 3, 3, 3, THI1 , 3, THXIMAT, 3, THI1 , 3)
*THIOT = .TR. THIO
CALL MATRIX ( 0,3      ,3      ,0,THIO ,3      ,THIOT ,
X 3
*THIIT = .TR. THI1
CALL MATRIX ( 0,3      ,3      ,0,THI1 ,3      ,THIIT ,
X 3
2 CONTINUE
* C1 = THYOT * THZOT
CALL MATRIX (20, 3, 3, 3, THYOT , 3, THZOT , 3, C1 , 3)
* C2 = THXOT * THYOT
CALL MATRIX (20, 3, 3, 3, THXOT , 3, THYOT , 3, C2 , 3)
*THIOBT = THXOBT * C1
CALL MATRIX (20, 3, 3, 3, THXOBT , 3, C1 , 3, THIOBT , 3)
* T33 = THXOT * THYOBT
CALL MATRIX (20, 3, 3, 3, THXOT , 3, THYOBT , 3, T33 , 3)
* T33 = T33 * THZCT
CALL MATRIX (20, 3, 3, 3, T33 , 3, THZOT , 3, T33 , 3)
DO 40 K = 1, 9
THIOBT(K) = THIOBT(K) + T33(K)
40 CONTINUE
$FROM N15
* T33 = C2 * THZOBT
CALL MATRIX (20, 3, 3, 3, C2 , 3, THZOBT , 3, T33 , 3)
DO 50 K = 1, 9
THIOBT(K) = THIOBT(K) + T33(K)
50 CONTINUE
* C3 = THYIT * THZIT
CALL MATRIX (20, 3, 3, 3, THYIT , 3, THZIT , 3, C3 , 3)
* C4 = THXIT * THYIT
CALL MATRIX (20, 3, 3, 3, THXIT , 3, THYIT , 3, C4 , 3)
*THI1BT = THXIBT * C3
CALL MATRIX (20, 3, 3, 3, THXIBT , 3, C3 , 3, THI1BT , 3)
* T33 = THXIT * THYIBT
CALL MATRIX (20, 3, 3, 3, THXIT , 3, THYIBT , 3, T33 , 3)
* T33 = T33 * THZIT
CALL MATRIX (20, 3, 3, 3, T33 , 3, THZIT , 3, T33 , 3)
DO 60 K = 1, 9
THI1BT(K) = THI1BT(K) + T33(K)
60 CONTINUE
* T33 = C4 * THZIBT
CALL MATRIX (20, 3, 3, 3, C4 , 3, THZIBT , 3, T33 , 3)
DO 70 K = 1, 9
THI1BT(K) = THI1BT(K) + T33(K)
70 CONTINUE
$FROM N16
* C13 = THIOBT * THIIT
CALL MATRIX (20, 3, 3, 3, THIOBT , 3, THIIT , 3, C13 , 3)
* C15 = THIOBT * THI1BT
CALL MATRIX (20, 3, 3, 3, THIOBT , 3, THI1BT , 3, C15 , 3)
* B6T = THIOBT * THIIT
CALL MATRIX (20, 3, 3, 3, THIOBT , 3, THIIT , 3, B6T , 3)
* D4 = C2 * THZOT
CALL MATRIX (20, 3, 3, 3, C2 , 3, THZOT , 3, D4 , 3)
* D2 = THXO2 * C1
CALL MATRIX (20, 3, 3, 3, THXO2 , 3, C1 , 3, D2 , 3)
* D7 = D4 * THIIT
CALL MATRIX (20, 3, 3, 3, D4 , 3, THIIT , 3, D7 , 3)
* D3 = THXOT * THYO2
CALL MATRIX (20, 3, 3, 3, THXOT , 3, THYO2 , 3, D3 , 3)
* D3 = D3 * THZOT
CALL MATRIX (20, 3, 3, 3, D3 , 3, THZOT , 3, D3 , 3)
* D6 = D3 * THIIT
CALL MATRIX (20, 3, 3, 3, D3 , 3, THIIT , 3, D6 , 3)
* D5 = D2 * THIIT
CALL MATRIX (20, 3, 3, 3, D2 , 3, THIIT , 3, D5 , 3)
$FROM N17
* D10 = THIOBT * C4
CALL MATRIX (20, 3, 3, 3, THIOBT , 3, C4 , 3, D10 , 3)
* D10 = D10 * THZIT
CALL MATRIX (20, 3, 3, 3, D10 , 3, THZIT , 3, D10 , 3)
* D8 = THIOBT * THX12
CALL MATRIX (20, 3, 3, 3, THIOBT , 3, THX12, 3, D8 , 3)
* D8 = D8 * C3
CALL MATRIX (20, 3, 3, 3, D8 , 3, C3 , 3, D8 , 3)
* D9 = THIOBT * THXIT
CALL MATRIX (20, 3, 3, 3, THIOBT , 3, THXIT , 3, D9 , 3)
* D9 = D9 * THY12
CALL MATRIX (20, 3, 3, 3, D9 , 3, THY12 , 3, D9 , 3)
* D9 = D9 * THZIT
CALL MATRIX (20, 3, 3, 3, D9 , 3, THZIT , 3, D9 , 3)
* B6 = .TR. B6T
CALL MATRIX ( 0,3      ,3      ,0,B6T ,3      ,B6 ,
X 3
* D40 = THXO1 * C1
CALL MATRIX (20, 3, 3, 3, THXO1 , 3, C1 , 3, D40 , 3)
T11 = THXOAR**2
$FROM N18
DO 75 K = 1, 9
75 D40(K) = T11 * D40(K)

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PREPROCS 91800000
IPREPROCS 91900000
DERS1830 92000000
PREPROCS 92100000
IPREPROCS 92200000
DENS1845 92300000
92400000
DERS1850 92500000
DERS1855 92600000
DERS1870 92700000
DERS1875 92800000
DERS1880 92900000
DERS1885 93000000
DERS1890 93100000
PREPROCS 93200000
IPREPROCS 93300000
DERS1900 93400000
PREPROCS 93500000
IPREPROCS 93600000
93700000
DERS1910 93800000
DERS1915 93900000
DERS1920 94000000
DERS1925 94100000
DERS1930 94200000
DERS1935 94300000
DERS1940 94400000
DERS1945 94500000
DERS1950 94600000
DERS1955 94700000
DERS1960 94800000
DERS1970 94900000
DERS4000 95000000
DERS4010 95100000
DERS4020 95200000
DERS4025 95300000
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DERS4070 95900000
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DERS4205 97900000
DERS4210 98000000
DERS4210 98100000
DERS4220 98200000
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DERS4250 98800000
DERS4255 98900000
DERS4260 99000000
DERS4265 99100000
DERS4270 99200000
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DERS4285 99500000
DERS4290 99600000
DERS4300 99700000
DERS4305 99800000
DERS4310 99900000
DERS4315 10000000
DERS4320 100100000
100200000
DERS4330 100300000
DERS4335 100400000
DERS4340 100500000
DERS4345 100600000
DERS4350 100700000
DERS4355 100800000
DERS4360 100900000
DERS4365 101000000
DERS4370 101100000
PREPROCS 101200000
IPREPROCS 101300000
DERS4380 101400000
DERS4385 101500000
DERS4390 101600000
DERS4400 101700000
DERS4410 101800000
DERS4420 101900000

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* *T33 = THXOT * THY01
CALL MATRIX (20, 3, 3, 3, THXOT, 3, THY01, 3, T33, 3)
* *T33 = T33 * THZCT
CALL MATRIX (20, 3, 3, 3, T33, 3, THZOT, 3, T33, 3)
T11 = THYOBAR**2
DO 80 K = 1, 9
D40(K) = D40(K) + T33(K) * T11
80 CONTINUE
* *T33 = C2 * THZC1
CALL MATRIX (20, 3, 3, 3, C2, 3, THZ01, 3, T33, 3)
T11 = THZOBAR**2
DO 90 K = 1, 9
T33(K) = T11 * T33(K)
D40(K) = D40(K) + T33(K)
90 CONTINUE
$FROM N19
* *T33 = THXOBT * THYCBT
CALL MATRIX (20, 3, 3, 3, THXOBT, 3, THYCBT, 3, T33, 3)
* *T33 = T33 * THZCT
CALL MATRIX (20, 3, 3, 3, T33, 3, THZOT, 3, T33, 3)
* *U33 = THXOBT * THYOT
CALL MATRIX (20, 3, 3, 3, THXOBT, 3, THYOT, 3, U33, 3)
* *V33 = THXOT * THYCBT
CALL MATRIX (20, 3, 3, 3, THXOT, 3, THYCBT, 3, V33, 3)
DO 100 K = 1, 9
U33(K) = U33(K) + V33(K)
100 CONTINUE
* *U33 = U33 * THZCBT
CALL MATRIX (20, 3, 3, 3, U33, 3, THZCBT, 3, U33, 3)
DO 110 K = 1, 9
D40(K) = 2. * (T33(K) + U33(K)) + D40(K)
110 CONTINUE
* *D41 = THX11 * C3
CALL MATRIX (20, 3, 3, 3, THX11, 3, C3, 3, D41, 3)
$FROM N20
T11 = THX1BAR**2
DO 120 K = 1, 9
D41(K) = T11 * D41(K)
120 CONTINUE
CALL MATRIX (20, 3, 3, 3, THX1T, 3, THY11, 3, T33, 3)
T11 = THY1BAR**2
CALL MATRIX (20, 3, 3, 3, T33, 3, THZ1T, 3, T33, 3)
DO 130 K = 1, 9
D41(K) = U41(K) + T11 * T33(K)
130 CONTINUE
CALL MATRIX (20, 3, 3, 3, C4, 3, THZ11, 3, T33, 3)
T11 = THZ1BAR**2
DO 140 K = 1, 9
D41(K) = D41(K) + T11 * T33(K)
140 CONTINUE
* *T33 = THX1BT * THY1BT
CALL MATRIX (20, 3, 3, 3, THX1BT, 3, THY1BT, 3, T33, 3)
* *T33 = T33 * THZ1T
CALL MATRIX (20, 3, 3, 3, T33, 3, THZ1T, 3, T33, 3)
* *U33 = THX1BT * THY1T
CALL MATRIX (20, 3, 3, 3, THX1BT, 3, THY1T, 3, U33, 3)
* *V33 = THX1T * THY1BT
CALL MATRIX (20, 3, 3, 3, THX1T, 3, THY1BT, 3, V33, 3)
DO 150 K = 1, 9
U33(K) = U33(K) + V33(K)
150 CONTINUE
CALL MATRIX (20, 3, 3, 3, U33, 3, THZ1BT, 3, U33, 3)
DO 160 K = 1, 9
D41(K) = D41(K) + 2. * (T33(K) + U33(K))
160 CONTINUE
$FROM N22
* *C11 = D40 * TH11T
CALL MATRIX (20, 3, 3, 3, D40, 3, TH11T, 3, C11, 3)
DO 170 K = 1, 9
C10(K) = TH10BT(K) + TH10RT(K)
170 CONTINUE
* *C10 = C10 * TH11BT
CALL MATRIX (20, 3, 3, 3, C10, 3, TH11BT, 3, C10, 3)
* *C21 = TH10T * D41
CALL MATRIX (20, 3, 3, 3, TH10T, 3, D41, 3, C21, 3)
DO 180 K = 1, 9
B10(K) = C11(K) + C10(K) + C21(K)
C14(K) = C13(K) + C13(K)
C16(K) = C15(K) + C15(K)
180 CONTINUE
CALL AWTING (CAPLOC)
JJ = 0
$FROM N23
C *CFL(1) AND CFML(22) ARE EQUIVALENCED TO CFL37(1,1) AND
C *CFL37(1,1), RESPECTIVELY
CALL SIMP (0, CFML, LILLER, FSHAFT, 42, 5)
DO 190 J = 1, 42
CFML(J) = CFML(J) * COAST
190 CONTINUE
$FROM N23A
A2 = THZ1C*THX1S
A4 = THZ1S*THY1S
A3 = A4*THX1C
A6 = THZ1C*THX1C
A8 = THZ1S*THY1S
A9 = THZ1C*THY1C
Y = A6 - A8*THY1C
Z = A2 + A3
A10 = ASIN (Z/SQRT(Y**2+Z**2))
A12 = THZ1C*THY1S*THX1C
X = -THZ1S*THY1C

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DERS4430 10200000
DERS4435 102100000
DERS4440 102200000
DERS4445 102300000
DERS4450 102400000
DERS4460 102500000
DERS4470 102600000
DERS4480 102700000
DERS4490 102800000
DERS4495 102900000
DERS4500 103000000
DERS4510 103100000
DERS4520 103200000
DERS4530 103300000
DERS4540 103400000
DERS4550 103500000
DERS4560 103600000
DERS4565 103700000
DERS4570 103800000
DERS4575 103900000
DERS4580 104000000
DERS4585 104100000
DERS4590 104200000
DERS4595 104300000
DERS4600 104400000
DERS4610 104500000
DERS4620 104600000
DERS4630 104700000
DERS4635 104800000
DERS4640 104900000
DERS N19 105000000
DERS4660 105100000
DERS4670 105200000
DERS4675 105300000
DERS4680 105400000
DERS4690 105500000
DERS4700 105600000
DERS4710 105700000
DERS4720 105800000
DERS4735 105900000
DERS4750 106000000
DERS4745 106100000
DERS4760 106200000
DERS4770 106300000
DERS4780 106400000
DERS4795 106500000
DERS4800 106600000
DERS4810 106700000
DERS4820 106800000
DERS4830 106900000
DERS4840 107000000
DERS4850 107100000
DERS4855 107200000
DERS4860 107300000
DERS4865 107400000
DERS4870 107500000
DERS4875 107600000
DERS4880 107700000
DERS4885 107800000
DERS4890 107900000
DERS4900 108000000
DERS4910 109100000
DERS4915 108200000
DERS4930 108300000
DERS4940 108400000
DERS4950 108500000
DERS4960 108600000
DERS4970 108700000
DERS4975 108800000
DERS4980 108900000
DERS4990 109000000
DERS5000 109100000
DERS5010 109200000
DERS5015 109300000
DERS5020 109400000
DERS5020 109500000
DERS5030 109600000
DERS N22 109700000
DERS5050 109800000
DERS5060 109900000
DERS5070 110000000
DERS5080 110100000
DERS5090 110200000
DERS5100 110300000
DERS5110 110400000
DERS5120 110500000
DERS5130 110600000
DERS5140 110700000
DERS5150 110800000
DERS5160 110900000
111000000
111100000
111200000
111300000
111400000
111500000
111600000
111700000
111800000
111900000
112000000
112100000

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A13 = ASIN (X/SQRT(X**2+Y**2))
A14 = THXIC**2
A15 = THXIS*A12
A16 = THXIS**2
A17 = THYIS*A9
A19 = THXIS*THXIC
A18 = A19*X
A20 = THYIS*A9
$FROM N23B
A21 = THXIS*A3
A22 = THYIS*X
A23 = A19*THYIC
A24 = THZOS*THXOC
A25 = THZOC*THYOS*THXOS
YPR = A24 + A25
A26 = THZUC*THXOC
ZPR = THZOS*THXOS - A26*THYOS
A28 = THXOC**2
A27 = A28*THZOS
A29 = THXOS**2
A30 = A29*THXOC
XPR = THZOC*THYOC
A31 = THYOS*XPR
A32 = THXOS*THYOC
A34 = THXOS * THYOS
A35 = THZOS*THYOC
A36 = ASIN (ZPR/SQRT(XPR**2+ZPR**2))
A37 = ASIN (YPR/SQRT(YPR**2+XPR**2))
$FROM N23
CML6 = KX1 * THX1 + CX1 * THX18AR
CML6(2) = KY1 * THY1 + CY1 * THY18AR
CML6(3) = KZ1 * THZ1 + CZ1 * THZ18AR
*CM3 = TH1IT * CML6
CALL MATRIX (20, 3, 3, 1, TH1IT, 3, CML6, 3, CM3, 3)
JJ = 0
CALL SIMP (0, CML34, CAPLOR, FWING, 12, 5)
CML34(1,2) = ILXP
DO 200 K = 1, 12
CML34(K) = CML34(K) * CONST
200 CONTINUE
* CML334 = TH10 * CML34
CALL MATRIX (20, 3, 3, 4, TH10, 3, CML34, 3, CML334, 3)
DO 210 K = 1, 3
$FROM N24
A(1,K) = CML334(1,K+1)
210 CONTINUE
NM1 = N - 1
DO 220 K = 4, NM1
A(1,K) = 0
220 CONTINUE
A(1,N) = THXOAR*XPR*CXO - CML334(1,1) - CM3(1) + KXO*THXO +
A THYOBAR*(CYO*(A27+A3C)+CZC*(A29*THZOS-A3O))+ THZOBAR*(CXO*A31-
BCYO*(A32*A24+A25*A31))+CZC*(A24 *A32-A28*A31))
230 IF (I.EQ. 0) PSII(1) = PSO + DPSB
IF (I.EQ. 0) PSII(1+1) = PSII(1) + DPSB
I = I + 1
IF (I.EQ. NB) PSII(1) = PSO
BC = CDS (BETA(1))
$FROM N25
RS = SIN (BETA(1))
XIC = COS (XI(1))
XS = SIN (XI(1))
PSC = COS (PSII(1))
PSS = SIN (PSII(1))
C BMAT(2,2) AND BMAT(3,2) ARE EQUIVALENCED TO BC AND RS, RESPEC-
C TIVELY
DATA (BMAT(J),J=1,4), BMAT(7) /1., 4*0/
BMAT(2,3) = -BS
BMAT(3,3) = BC
THETA = THOS + THISC*PSC + THISS*PSS
THC = COS (THETA)
THS = SIN (THETA)
C THC AND THMAT(1,1) ARE EQUIVALENCED
C THS AND THMAT(1,2) ARE EQUIVALENCED
DATA THMAT(3), (THMAT(J),J=6,9) /4*0, 1./
THMAT(2) = -THS
THMAT(2,2) = THC
CALL MATRIX(0,3,3,0,THPAT,3,THT,3)
C XMAT(1,1) AND XPAT(1,3) ARE EQUIVALENCED TO XIC AND XS,
C RESPECTIVELY
$FROM N26
DATA XMAT(2), (XMAT(J),J=4,6), XMAT(8) /2*0, 1., 2*0/
XMAT(3) = -XS
XMAT(3,3) = XIC
DATA (BET1(J),J=1,4), BET1(7) /5*0/
BET1(2,2) = BET1(3,3) = -BC
BET1(3,2) = RS
BET1(2,3) = -BS
DATA BET2 /9*0/
BET2(2,2) = BET2(3,3) = -BS
$FROM N27
BET2(3,2) = -BC
BET2(2,3) = BC
DATA X2(2), (X2(J),J=4,6), X2(8) /5*0/
X2 = X2(3,3) = -XS
X2(3) = XIC
X2(1,3) = -XIC
DATA X1(2), (X1(J),J=4,6), X1(8) /5*0/
X1 = X1(3,3) = -XIC
X1(3) = -XS
X1(1,3) = XS
$FROM N28

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112200000
112300000
112400000
112500000
112600000
112700000
112800000
112900000
113000000
113100000
113200000
113300000
113400000
113500000
113600000
113700000
113800000
113900000
114000000
114100000
114200000
114300000
114400000
114500000
114600000
114700000
114800000
114900000
115000000
DERS5170 115100000
DERS5180 115200000
DERS5190 115300000
DERS5200 115400000
DERS5205 115500000
DERS5210 115600000
115700000
115800000
DERS5230 115900000
DERS5240 116000000
DERS5250 116100000
DERS5260 116200000
DERS5265 116300000
DERS5270 116400000
DERS5280 116500000
116600000
DERS5300 116700000
DERS5310 116800000
DERS5320 116900000
117000000
DERS5340 117100000
117200000
117300000
117400000
DERS5360 117500000
DERS5370 117600000
DERS5380 117700000
117800000
DERS5390 117900000
DERS5400 118000000
DERS5410 118100000
DERS5420 118200000
DERS5430 118300000
DERS5440 118400000
DERS5450 118500000
DERS5460 118600000
DERS5470 118700000
118800000
DERS5490 118900000
DERS5500 119000000
119100000
119200000
119300000
119400000
119500000
119600000
119700000
119800000
119900000
DERS5510 120000000
DERS5520 120100000
DERS5530 120200000
120300000
DERS5550 120400000
DERS5560 120500000
120600000
DERS5580 120700000
DERS5590 120800000
DERS5600 120900000
DERS5610 121000000
DERS5620 121100000
DERS5630 121200000
DERS5640 121300000
DERS5650 121400000
121500000
DERS5670 121600000
DERS5680 121700000
DERS5690 121800000
121900000
DERS5710 122000000
DERS5720 122100000
DERS5730 122200000
DERS5740 122300000

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DATA PSBBI(2), (PSBBI(J), J=4,6), PSBBI(8) /5*0/
PSBBI = PSBBI(3,3) = -PSC
PSBBI(3) = PSS
PSBBI(1,3) = -PSS
DO 235 K = 1, 9
235  *BART(K) = BBART(1) * BET2(K)
DATA PSBART /9*0/
PSBART = PSBART(3,3) = -PSS
PSBART(3) = -PSC
PSBART(1,3) = PSC
DO 240 K = 1, 9
XBART(K) = XBART(1) * X2(K)
240 CONTINUE
$FROM N29
DATA PSMAT /4*0, 1., 4*0/
C PSMAT(1,1) AND PSMAT(3,1) ARE EQUIVALENCED TO PSC AND PSS,
C RESPECTIVELY
PSMAT(1,3) = -PSS
PSMAT(3,3) = PSC
* PST = .TR. PSMAT
CALL MATRIX ( 0,3 ,3 ,0,PSMAT ,3 ,PST ,
X 3
* BT = .TR. BMAT
CALL MATRIX ( 0,3 ,3 ,0,BMAT ,3 ,BT ,
X 3
* XT = .TR. XMAT
CALL MATRIX ( 0,3 ,3 ,0,XMAT ,3 ,XT ,
X 3
* C18 = C13 * PST
CALL MATRIX (20,3,3,3,C13 ,3,PST ,3,C18 ,3)
* C23 = C15 * PST
CALL MATRIX (20,3,3,3,C15 ,3,PST ,3,C23 ,3)
* C25 = B6T * PSBART
CALL MATRIX (20,3,3,3,B6T ,3,PSBART ,3,C25 ,3)
* C8T = B6T * PST
CALL MATRIX (20,3,3,3,B6T ,3,PST ,3,C8T ,3)
$FROM N29A
* C5 = THMAT * XMAT
CALL MATRIX ( 20,3 ,3 ,3 ,THMAT ,3 ,XMAT ,
X 3 ,C5 ,3
* C7 = C5 * BMAT
CALL MATRIX (20,3,3,3,C5 ,3,BMAT ,3,C7 ,3)
* C7T = .TR. C7
CALL MATRIX ( 0,3 ,3 ,0,C7 ,3 ,C7T ,
X 3
* CST = XT * THT
CALL MATRIX ( 20,3 ,3 ,3 ,XT ,3 ,THT ,
X 3 ,CST ,3
* C32 = BBART * CST
CALL MATRIX ( 20,3 ,3 ,3 ,BBART ,3 ,CST ,
X 3 ,C32 ,3
* C30 = BT * XBART
CALL MATRIX ( 20,3 ,3 ,3 ,BT ,3 ,XBART ,
X 3 ,C30 ,3
$FROM N29
DO 250 K = 1, 9
$FROM N30
TEMP(K) = C1(K) + C23(K) + C25(K)
TEMP3(K) = C32(K) + C30(K)
250 CONTINUE
DO 260 K = 1, 3
B1(K) = THIOB(K) * CAPLOR + (C13(K,2) + C15(K,2)) * LILLOR +
A (TEMP(K) * RT(1,3) + TEMP(K,2) * RT(2,3) + TEMP(K,3) * RT(3,3) +
B CBT(K) * BBART(1,3) + CBT(K,2) * BBART(2,3) + CBT(K,3) * BBART(3,3) +
C 3)) * EZOR + TEMP(K,3) * E1CR
260 CONTINUE
$FROM N31
* B2 = TEMP * C7T
CALL MATRIX ( 20,3 ,3 ,3 ,TEMP ,3 ,C7T ,
X 3 ,B2 ,3
* T33 = CBT * TEMP3
CALL MATRIX (20,3,3,3,CBT ,3,TEMP3 ,3,T33 ,3)
DO 270 K = 1, 9
B2(K) = B2(K) + T33(K)
270 CONTINUE
B3 = UOUR + B1
B3(2) = VOOR + B1(2)
B3(3) = WOUR + B1(3)
$FROM N32
* C8 = .TR. CBT
CALL MATRIX ( 0,3 ,3 ,0,CBT ,3 ,C8 ,
X 3
* B4 = C7 * C8
CALL MATRIX (20,3,3,3,C7 ,3,C8 ,3,B4 ,3)
* B4T = .TR. B4
CALL MATRIX ( 0,3 ,3 ,0,B4 ,3 ,B4T ,
X 3
$FROM N32A
IF (NWL .LE. 1) GO TO 2805
IF (IFLAG .EQ. 0) PRINT 280
280 FORMAT ('OAEERUDYNAMIC DATA')
2805 CONTINUE
DO 281 J=1,NIT
DO 281 J=1,3
281 P6BAR(J) = B6(J) * UCOR + B6(J,2) * VOOR + B6(J,3) * WOUR
P6BAR(2) = P6BAR(2) + NGOR(1)
NSOOR = .5 * CATY12P(1) / SQRT (P6BAR(1)**2 + P6BAR(2)**2 + P6BAR(3)**2)
A 2)
NDOR(1) = MLOCOT + NSOOR
B30 = NJOK(1) + COT + SPC
B30(2) = -NDOR(1) * COT
B30(3) = NJOK(1) * SOT + CPO

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122400000
DERS5760 122500000
DERS5770 122600000
DERS5780 122700000
DERS5800 122800000
DERS5810 122900000
DERS5820 123000000
DERS5830 123100000
DERS5840 123200000
DERS5850 123300000
DERS5860 123400000
DERS5870 123500000
DERS5880 123600000
DERS5890 123700000
DERS5900 123800000
DERS5910 123900000
DERS5920 124000000
DERS5927 124100000
124200000
DERS5930 124300000
PREPROCS 124400000
IPREPROCS 124500000
DERS5940 124600000
PREPROCS 124700000
IPREPROCS 124800000
DERS5950 124900000
PREPROCS 125000000
IPREPROCS 125100000
DERS5960 125200000
DERS5965 125300000
DERS5970 125400000
DERS5975 125500000
DERS5980 125600000
DERS5985 125700000
DERS5990 125800000
DERS5995 125900000
DERS6000 126000000
DERS6010 126100000
PREPROCS 126200000
IPREPROCS 126300000
DERS6020 126400000
DERS6025 126500000
DERS6030 126600000
PREPROCS 126700000
IPREPROCS 126800000
DERS6040 126900000
PREPROCS 127000000
IPREPROCS 127100000
DERS6050 127200000
PREPROCS 127300000
IPREPROCS 127400000
DERS6060 127500000
PREPROCS 127600000
IPREPROCS 127700000
DERS6080 127800000
DERS6090 127900000
DERS6100 128000000
DERS6110 128100000
DERS6120 128200000
DERS6130 128300000
DERS6140 128400000
DERS6150 128500000
DERS6160 128600000
DERS6170 128700000
DERS6180 128800000
DERS6190 128900000
DERS6200 129000000
DERS6210 129100000
PREPROCS 129200000
IPREPROCS 129300000
DERS6220 129400000
DERS6225 129500000
DERS6230 129600000
DERS6240 129700000
DERS6250 129800000
DERS6260 129900000
DERS6270 130000000
DERS6280 130100000
DERS6290 130200000
DERS6300 130300000
PREPROCS 130400000
IPREPROCS 130500000
DERS6310 130600000
DERS6315 130700000
DERS6320 130800000
PREPROCS 130900000
IPREPROCS 131000000
DERS6330 131100000
131200000
131300000
DERS6350 131400000
131500000
131600000
131700000
131800000
131900000
132000000
132100000
132200000
132300000
132400000
132500000

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DER\$6360	132600000
DER\$6380	132700000
DER\$6390	132800000
	132900000
	133000000
DER\$6410	133100000
	133200000
	133300000
	133400000
	133500000
	133600000
	133700000
	133800000
	133900000
	134000000
DER\$6420	134100000
	134200000
DER\$6440	134300000
DER\$6450	134400000
DER\$6460	134500000
DER\$6465	134600000
DER\$6470	134700000
DER\$6475	134800000
DER\$6480	134900000
DER\$6485	135000000
DER\$6490	135100000
DER\$6495	135200000
DER\$6500	135300000
DER\$6505	135400000
DER\$6510	135500000
DER\$6515	135600000
DER\$6520	135700000
DER\$6525	135800000
DER\$6530	135900000
DER\$6535	136000000
DER\$6540	136100000
DER\$6545	136200000
DER\$6550	136300000
DER\$6555	136400000
DER\$6560	136500000
DER\$6565	136600000
DER\$6570	136700000
DER\$6575	136800000
DER\$6580	136900000
DER\$6585	137000000
DER\$6590	137100000
DER\$6595	137200000
PREPROC\$	137300000
PREPROC\$	137400000
DER\$6600	137500000
DER\$6605	137600000
DER\$6610	137700000
PREPROC\$	137800000
PREPROC\$	137900000
DER\$6620	138000000
DER\$6625	138100000
DER\$6630	138200000
DER\$6635	138300000
DER\$6640	138400000
DER\$6645	138500000
DER\$6650	138600000
DER\$6660	138700000
DER\$6665	138800000
DER\$6670	138900000
DER\$6675	139000000
DER\$6680	139100000
DER\$6685	139200000
DER\$6690	139300000
DER\$6695	139400000
DER\$6700	139500000
DER\$6705	139600000
DER\$6710	139700000
DER\$6715	139800000
DER\$6720	139900000
DER\$6725	140000000
DER\$6730	140100000
DER\$6735	140200000
DER\$6740	140300000
DER\$6745	140400000
DER\$6750	140500000
DER\$6755	140600000
DER\$6760	140700000
DER\$6770	140800000
DER\$6775	140900000
DER\$6780	141000000
DER\$6785	141100000
DER\$6790	141200000
DER\$6800	141300000
DER\$6810	141400000
DER\$6820	141500000
DER\$6830	141600000
DER\$6840	141700000
DER\$6850	141800000
DER\$6860	141900000
DER\$6870	142000000
DER\$6880	142100000
DER\$6890	142200000
DER\$6900	142300000
DER\$6910	142400000
DER\$6920	142500000
DER\$6925	142600000
DER\$6930	142700000

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CALL MATRIX (20, 3, 3, 3, TEMP2, 3, BBART, 3, T33, 3)
DO 310 K = 1, 9
  B8(K) = B8(K) + T33(K)
310 CONTINUE
* *T33 = C8T * BET1
CALL MATRIX (20, 3, 3, 3, C8T, 3, BET1, 3, T33, 3)
T11 = BBART(11)*2
DO 315 K = 1, 9
  B8(K) = B8(K) + T11 * T33(K)
315 CONTINUE
CALL ALAG (E2OR)
H = 0
* *B7 = B9 * C7T
CALL MATRIX (20, 3, 3, 3, B9, 3, C7T, 3, B7, 3)
* *T33 = TEMP2 * TEMP3
CALL MATRIX (20, 3, 3, 3, TEMP2, 3, TEMP3, 3, T33, 3)
$FROM N38
DO 320 K = 1, 9
  B7(K) = B7(K) + T33(K)
320 CONTINUE
* *T33 = BET1 * C5T
CALL MATRIX (20, 3, 3, 3, BET1, 3, C5T, 3, T33, 3)
DO 330 K = 1, 9
  T33(K) = T11 * T33(K)
  U33(K) = BBART(K) + BBART(K)
330 CONTINUE
* *U33 = U33 * XBART
CALL MATRIX (20, 3, 3, 3, U33, 3, XBART, 3, U33, 3)
* *U33 = U33 * THT
CALL MATRIX (20, 3, 3, 3, U33, 3, THT, 3, U33, 3)
DO 340 K = 1, 9
  T33(K) = T33(K) + U33(K)
340 CONTINUE
* *T33 = C8T * T33
CALL MATRIX (20, 3, 3, 3, C8T, 3, T33, 3, T33, 3)
DO 350 K = 1, 9
  B7(K) = B7(K) + T33(K)
350 CONTINUE
* *T33 = B5T * X1
CALL MATRIX (20, 3, 3, 3, B5T, 3, X1, 3, T33, 3)
$FROM N39
* *T33 = T33 * THT
CALL MATRIX (20, 3, 3, 3, T33, 3, THT, 3, T33, 3)
T11 = XBART(11)*2
DO 360 K = 1, 9
  B7(K) = B7(K) + T11 * T33(K)
360 CONTINUE
CALL SIMP (3, C1FM, UP1, FLADE, 54, 11)
C C1FM(1) AND C1FM(28) ARE EQUIVALENCED TO C1F39(1,1) AND
C C1M39(1,1), RESPECTIVELY
DO 370 K = 1, 54
  C1FM(K) = CONST * C1FM(K)
370 CONTINUE
* *C1F1239 = B4 * C1F39
CALL MATRIX (20, 3, 3, 9, B4, 3, C1F39, 3, C1F1239, 3)
* *C1M1139 = XMAT * B5
CALL MATRIX (20, 3, 3, 3, XMAT, 3, B5, 3, C1M1139, 3)
* *C1M1139 = C1M1139 * C1M39
CALL MATRIX (20, 3, 3, 9, C1M1139, 3, C1M39, 3, C1M1139, 3)
$FROM N40
CAMQ12 = CAMX12
CAMQ12(2) = CAQY12
DATA CAMQ12(3) /C/
* *CAM11 = THT * CAMQ12
CALL MATRIX (20, 3, 3, 3, THT, 3, CAMQ12, 3, CAM11, 3)
DO 375 K = 1, 6
  A(1+NB+6,K) = C1M1139(2,K+1)
375 CONTINUE
DO 390 J = 1, NB
$FROM N41
  IPNB = 1+NB
  IF (I.EQ. J) GO TO 380
  A(IPNB+6,J+6) = A(1+NB+6,J+NB+6) = 0
  GO TO 390
380 A(IPNB+6,J+6) = C1M1139(2,8)
  A(IPNB+6,J+NB+6) = C1M1139(2,9)
390 CONTINUE
  A(1+NB+6,N) = -C1M1139(2,1) - CAM11(2) - KX * X(1) - CX * XHAR(1)
  JJ = 0
  CALL SIMP (0, C1FM2, E2CR, FLAG, 48, 5)
C C1FM2(1) AND C1FM2(25) ARE EQUIVALENCED TO C1E238(1,1) AND
C C1ME238(1,1), RESPECTIVELY
DO 400 K = 1, 48
  C1FM2(K) = CONST * C1FM2(K)
400 CONTINUE
* *C1ME2913,8) = B5 * C1ME238
CALL MATRIX (20, 3, 3, 8, B5, 3, C1ME238, 3, C1ME29,3)
$FROM N42
CAIF12 = C1F1239 + CACX12
CAIF12(2) = C1F1239(2) + CATY12
DO 410 J = 3, 27
  CAIF12(J) = C1F1239(J)
410 CONTINUE
* *CAIF939 = C5T * CAIF12
CALL MATRIX (20, 3, 3, 9, C5T, 3, CAIF12, 3, CAIF939, 3)
DATA E2MAT(1), E2MAT(2) /0,0/
C E2MAT(3) IS EQUIVALENCED TO E2UR
CALL CROSS (CAIM939, E2MAT, CAIF939, 9)
$FROM N43
DO 430 K = 1, 3
  DERS6939 142800000
  DERS6940 142900000
  DERS6950 143000000
  DERS6960 143100000
  DERS6970 143200000
  DERS6975 143300000
  DERS6980 143400000
  DERS6990 143500000
  DERS N37 143600000
  DERS7010 143700000
  DERS7C20 143800000
  DERS7030 143900000
  DERS7040 144000000
  DERS7045 144100000
  DERS7050 144200000
  DERS7C55 144300000
  DERS7060 144400000
  DERS7070 144500000
  DERS7080 144600000
  DERS7090 144700000
  DERS N38 144800000
  DERS7105 144900000
  DERS7110 145000000
  DERS7120 145100000
  DERS7130 145200000
  DERS7140 145300000
  DERS7150 145400000
  DERS7155 145500000
  DERS7160 145600000
  DERS7165 145700000
  DERS7170 145800000
  DERS7180 145900000
  DERS7190 146000000
  DERS7200 146100000
  DERS7205 146200000
  DERS7210 146300000
  DERS7220 146400000
  DERS7230 146500000
  DERS7240 146600000
  DERS7245 146700000
  DERS7245 146800000
  DERS7250 146900000
  DERS7255 147000000
  DERS7260 147100000
  DERS7270 147200000
  DERS7280 147300000
  DERS7290 147400000
  DERS7310 147500000
  DERS7320 147600000
  DERS7330 147700000
  DERS7340 147800000
  DERS7350 147900000
  DERS7360 148000000
  DERS7365 148100000
  DERS7370 148200000
  DERS7370 148300000
  PREPROCS 148400000
  PREPROCS 148500000
  DERS7380 148600000
  DERS7385 148700000
  DERS7390 148800000
  DERS7400 148900000
  DERS7410 149000000
  DERS7420 149100000
  DERS7430 149200000
  PREPROCS 149300000
  PREPROCS 149400000
  DERS7440 149500000
  DERS7440 149600000
  DERS7460 149700000
  DERS7470 149800000
  DERS7480 149900000
  DERS7490 150000000
  DERS7490 150100000
  DERS7510 150200000
  DERS7510 150300000
  DERS7510 150400000
  DERS7510 150500000
  DERS7540 150600000
  DERS7540 150700000
  DERS7560 150800000
  DERS7570 150900000
  DERS7580 151000000
  DERS7590 151100000
  DERS7590 151200000
  DERS7610 151300000
  DERS7620 151400000
  DERS7630 151500000
  DERS7640 151600000
  DERS7640 151700000
  DERS7650 151800000
  DERS7660 151900000
  DERS7670 152000000
  DERS7680 152100000
  DERS7690 152200000
  DERS7700 152300000
  DERS7700 152400000
  DERS7700 152500000
  DERS7720 152600000
  DERS7730 152700000
  DERS7740 152800000
  DERS7750 152900000

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CA18M11(K) = CIM139(K,1) + CAM11(K)
DO 420 J = 2, 9
CA18M11(K,J) = CIM139(K,J)
420 CONTINUE
430 CONTINUE
*CA18M9 = XT * CA18M11
CALL MATRIX ( 20,3 ,3 ,9 ,XT ,3 ,CA18M11
X ,3 ,CA18M9 ,3
DO 440 K = 1, 27
CTM939(K) = CA18M9(K) + CA1M939(K)
440 CONTINUE
DO 450 K = 1, 24
CTM939(K) = CTM939(K) + C1ME29(K)
450 CONTINUE
DO 460 K = 1, 6
$FROM N44
A(I+6,K) = CTM939(1,K+1)
A * CD3 - CTM939(3,K+1)*SD3
460 CONTINUE
DO 480 J = 1, NB
IF (I .EQ. J) GO TO 47C
A(I+6,J+6) = A(I+6,J+NB+6) = 0
GO TO 480
470 A(I+6,J+6) = CTM939(1,8)
A * CD3 - CTM939(3,8)*SD3
A(I+6,J+NB+6) = CTM939(1,9)
A * CD3 - CTM939(3,9)*SD3
480 CONTINUE
HCSQ = BC**2 $ BSSC = RS**2
A(I+6,N) = CTM939(3,1) * SD3 - KB * SIGN (ACOS (BC * CD3 / SQRT
A (BSSQ + BCSQ*CSQD3)), BETA(I)) - CR * (1. + BCSQ * SSQD3 * SQRT
B (BSSQ + BCSQ * CSQD3)) * CD3 * BBAR(I) - CTM939(1,1) * CD3
$FROM N45
CF939(1,9) = CF939(2,9) = CF939(3,9) = 0
*CF939(3,8) = R5 * C1E238
CALL MATRIX (20, 3, 3, 8, B5 , 3, C1E238 , 3, CF939 , 3)
DO 500 K = 1, 27
CF939(K) = CAIF939(K) + CF939(K)
500 CONTINUE
*CF839 = BT * CF939
CALL MATRIX ( 20,3 ,3 ,9 ,BT ,3 ,CF939
X ,3 ,CF839 ,3
JJ = 0
CALL SIMP (0, C1ME1, E1CR, FFLAP, 42, 5)
C C1ME1(1) AND C1ME1(2) ARE EQUIVALENCED TO C1E137(1,1) AND
C C1E137(1,1), RESPECTIVELY
DO 510 K = 1, 42
C1ME1(K) = CONST * C1ME1(K)
510 CONTINUE
*CF639 = PST * CF839
CALL MATRIX ( 20,3 ,3 ,9 ,PST ,3 ,CF839
X ,3 ,CF639 ,3
* T37 = R6 * C1E137
CALL MATRIX (20, 3, 3, 7, R6, 3, C1E137, 3, T37, 3)
$FROM N46
DO 520 K = 1, 21
CF639(K) = CF639(K) + T37(K)
520 CONTINUE
* C6T = .TR. C6
CALL MATRIX ( 0,3 ,3 ,0,C6 ,3 ,C6T ,
X ,3
* CME1637 = R6 * CME137
CALL MATRIX (20, 3, 3, 7, R6 , 3, CME137 , 3, CME1637, 3)
* CM639 = C6T * CTFH939
CALL MATRIX (20, 3, 3, 9, C6T , 3, CTFH939, 3, CM639 , 3)
CALL CROSS (T39, E1MAT, CF839, 9)
* T39 = PST * T39
CALL MATRIX (20, 3, 3, 9, PST, 3, T39, 3, T39, 3)
DO 523 K = 1, 21
CM639(K) = CM639(K) + CME1637(K) + T39(K)
523 CONTINUE
DO 525 K = 22, 27
CM639(K) = CM639(K) + T39(K)
525 CONTINUE
* CFL337 = TH10 * CFL37
CALL MATRIX (20, 3, 3, 7, TH10 , 3, CFL37 , 3, CFL337, 3)
DO 530 K = 1, 21
SCF637(K) = SCF637(K) + CF639(K)
SCM637(K) = SCM637(K) + CM639(K)
530 CONTINUE
* CML637 = R6 * CML37
CALL MATRIX (20, 3, 3, 7, R6 , 3, CML37 , 3, CML637, 3)
CML637(2,6) = CILAP
A(4,I+6) = CF639(3,8)*LILLOR + CM639(1,8)
$FROM N47
A(4,I+NB+6) = CF639(3,9)*LILLOR + CM639(1,9)
A(5,I+6) = CM639(2,8)
A(5,IPNB+6) = CM639(2,9)
A(6,I+6) = CM639(3,8) - CF639(1,8)*LILLOR
A(6,IPNB+6) = CM639(3,9) - CF639(1,9) * LILLOR
* SCF337 = TH11 * SCF637
CALL MATRIX (20, 3, 3, 7, TH11 , 3, SCF637 , 3, SCF337, 3)
DO 540 K = 1, 21
SCF337(K) = SCF337(K) + CFL337(K)
540 CONTINUE
* CF339 = TH11 * CF639
CALL MATRIX (20, 3, 3, 9, TH11 , 3, CF639 , 3, CF339 , 3)
DO 550 K = 1, 21
CF339(K) = CF339(K) + CFL337(K)
550 CONTINUE
$FROM N48
A(2,I+6) = -CF339(3,9)*CAPLOR

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DERS7760 153000000
DERS7770 153100000
DERS7780 153200000
DERS7790 153300000
DERS7800 153400000
DERS7810 153500000
PREPROCS 153600000
IPREPROCS 153700000
DERS7820 153800000
DERS7830 153900000
DERS7840 154000000
DERS7850 154100000
DERS7860 154200000
DERS7870 154300000
DERS7880 154400000
DERS7890 154500000
154600000
154700000
DERS7910 154800000
DERS7920 154900000
DERS7930 155000000
155100000
DERS7950 155200000
155300000
155400000
155500000
155600000
DERS7980 155700000
155800000
155900000
156000000
156100000
DERS8000 156200000
DERS N45 156300000
DERS8070 156400000
DERS8075 156500000
DERS8080 156600000
DERS8090 156700000
DERS8100 156800000
DERS8110 156900000
PREPROCS 157000000
IPREPROCS 157100000
DERS8120 157200000
DERS8130 157300000
DERS8140 157400000
DERS8150 157500000
DERS8160 157600000
DERS8170 157700000
DERS8180 157800000
DERS8190 157900000
PREPROCS 158000000
IPREPROCS 158100000
DERS8200 158200000
158300000
DERS8210 158400000
DERS8220 158500000
DERS8230 158600000
DERS8240 158700000
DERS8500 158800000
PREPROCS 158900000
IPREPROCS 159000000
DERS8510 159100000
DERS8515 159200000
DERS8520 159300000
DERS8525 159400000
DERS 159500000
159600000
159700000
DERS N46 159800000
DERS 159900000
160000000
DERS 160100000
DERS 160200000
DERS8550 160300000
DERS8560 160400000
DERS8565 160500000
DERS8570 160600000
DERS8580 160700000
DERS8590 160800000
DERS8600 160900000
DERS8610 161000000
DERS8615 161100000
161200000
161300000
DERS8630 161400000
161500000
161600000
161700000
161800000
161900000
DERS8690 162000000
DERS8695 162100000
DERS8700 162200000
DERS8710 162300000
DERS8720 162400000
DERS8730 162500000
DERS8735 162600000
DERS8740 162700000
DERS8750 162800000
DERS8760 162900000
DERS8770 163000000
163100000

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A(2,IPNB*6) = -CF335(3,9) * CAPLOR
A(3,I*6) = CF339(2,8)
A(3,IPNB*6) = CF335(2,9)
IF(1.EQ.NBIGOT0553
GO TO 700
553 CONTINUE
IF (1,LT, NB) GE TC 230
DO 555 K = 1,6
A(4,K) = SCF637(3,K+1)*LILLOR + SCM637(1,K+1) + CML637(1,K+1)
$FROM N48A
A(5,K) = SCM637(2,K+1) + CML637(2,K+1)
A(6,K) = SCM637(3,K+1) + CML637(3,K+1) - SCF637(1,K+1)*LILLOR
$FROM N48
555 CONTINUE
$FROM N48A
A(4,N) = KX1*A10*A9 - SCM637(1,1) - CML637(1,1) - SCF637(3,1)*LILLOR
A - KZ1*(THZ15*THX15-A12)*A13 + THX1BAR*A9*CX1 + THY1BAR*(CY1*(A14*
B THZ15+A15) + CZ1*(A16*THZ15-A15)) + THZ1BAR*(CX1*A17+CY1*(A18-A16
C *A20)-CZ1*(A18+A14*A20))
A(5,N) = THX1BAR*CX1*X - CML637(2,1) + KY1*THY1 - SCM637(2,1) +
A THY1BAR*(CY1*(A14*THZ15-A21) + CZ1*(A16*THZ15+A21)) + THZ1BAR*
B (CX1*A22-CY1*(A19*A5+A16*A22)+CZ1*(A19*A9-A14*A22))
$FROM N49A
A(6,N) = SCF637(1,1)*LILLOR-SCM637(3,1)-CML37(3,1)+ KX1*THY15*A10-
A KZ1*THY1C*THX1C*A13 + THX1BAR*CX1*THY15 - THY1BAR*(CY1-CZ1)*
B A23 + THZ1BAR*(CX1*THY15**2+(CY1*A16+ CZ1*A14)*THY1C**2
C)
A(2,N) = SCF337(3,1)*CAPLOR - CML334(2,1) - CM3(2) + KY0*(A26 -
A THZ05*A34)*A36 + KZ0*(THZ0C*THX05+A24*THY05)*A37 - THX0BAR*CX0*
B A35+THY0BAR*(CY0*(A28*THZ0C - A34*A24)+ CZ0*(A29*THZ0C + A34*
C A24)) - THZ0BAR * (CX0 * THY05 *A35 + CY0 * (THX05 * THX0C * XPR
D - A29 * THY05 * A35) - CZ0 * (THX05 * THX0C * XPR + A27 *
E THY05 * THY0C))
A(3,N) = KZ0*THY0C*THX0C*A37 - CML334(3,1) - CM3(3) - SCF337(2,1)*
A CAPLOR - KY0*A32*A36 + THX0BAR*CX0*THY05 + THY0BAR*A32*THX0C*(CZ0
B - CY0) + THZ0BAR*(CX0*THY05**2 + (CY0*THX05**2 + CZ0*A28)*
C THY0C**2)
$FROM N49
DO 560 K = 1, 3
A(3,K) = CML334(3,K+1) + SCF337(2,K+1)*CAPLOR
A(2,K) = CML334(2,K+1) - SCF337(3,K+1)*CAPLOR
560 CONTINUE
DO 570 K = 4, 6
A(2,K) = -SCF337(3,K+1)*CAPLOR
$FROM N50
A(3,K) = SCF337(2,K+1)*CAPLOR
570 CONTINUE
K = 2 * NB + 6
GO TO 700
571 CONTINUE
* CALL MATRIX (10,K ,N ,1,A ,15 ,T11
CALL MATRIX (10, K, A, 0, A, 14, T11)
DO 580 J = 2, N
DER(J) = CUVAR(K+J)
DER(K+J) = A(J-1,N)
580 CONTINUE
DO 3 J=1,NM1
IF (IC(J).EQ. 1) GO TO 3
DER (J+1) = 0
DER(J+NI)=0
3 CONTINUE
RETURN
700 CONTINUE
DO 3000 KK=1,NM1
DO 3000 JJ=1,N
IF (KK.NE. JJ .A. IC(KK).FC. 0) A(KK,JJ) = 0
3000 CONTINUE
IF (1.EQ. NR) 571, 230
END
SUBROUTINE EIGHT (S, T, X)
COMMON /ALL/ NB, J, K
DIMENSION S(3), T(3,8), X(14)
DO 20 K = 1, 3
DO 10 J = 1, 6
S(K) = T(K,J+1) * X(J)
10 CONTINUE
20 CONTINUE
S(K) = S(K) + T(K,8) * X(NB+6) + T(K)
RETURN
END
SUBROUTINE FBLACE (Z1ZCR, C)
$FROM X1
COMMON /ALL/ NB, J, K
COMMON /CHK/ SKIP(55), M(11)
COMMON /JERAF/ B10(3,3), D2(3,3), B3(3), B4(3,3), B4T(3,3),
A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
B D10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D21(3,3), D21(3,3),
D D21(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
E D29(3,3), D31(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
F D34(3,3), D4(3,3), D40(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3), DERACC70
G D9(3,3), JJ, PE18837(3,7),
H PE28838(3,8), PLB834(3,4), PLR837(3,7), TH10T(3,3)
I , H30(3)
J , I
COMMON /SCUAF/ A1, BIASA1, BIASA2, BIASA3, BIASA11, BIASA12,
A BIASA13, CDAR(480), CLTAR(480),
B CRT(3,3), GSAVE(3,6,11)
C , F, NAD, NAL, NN1, NN2,
D PTBB39(3,9,11), T31(3), NMC, DMD1, DMD2,
E DMD3, BIASMD1, BIASMD2, BIASMD3, NML, DML1, DML2, DML3,
F BIASML1, BIASML2, BIASML3

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DIMENSION G(3,18), C50(3), PBLADE(3,11), PKB839(3,9)
C      G(1,1) AND G(1,1C) CORRESPOND TO G5(1,1) AND G6(1,1), RESPEC-
C      TIVELY
      INTEGER M
      REAL M1, D1(3)
      EQUIVALENCE (PLB834, D77), (PLB837, D76), (PE18837, D75),
      A (PE28838, D50), (PKB839, D1)
      NAMELIST /FBLA/ B4T, B7, C1, D11, D12, D13, D14, D15, D16, D18,
      A D19, D50, G, M1, PELADE, PE28838, PKB839, Z120R,
      B H
$FRM X2
      H = H + 1
      DO 10 K = 1, 3
      PBLADE(K,H) = B4T(K,3) * Z120R
      D1(K) = D50(K) * D7(K,3) * Z120R
C      PKB839(1,1) IS EQUIVALENT TO D1(1)
      PKB839(K,2) = PE28838(K,2) * D11(K,3) * Z120R
      PKB839(K,3) = PE28838(K,3) * D12(K,3) * Z120R
      PKB839(K,4) = PE28838(K,4) * D13(K,3) * Z120R
      PKB839(K,5) = PE28838(K,5) * D14(K,3) * Z120R
$FRM X3
      PKB839(K,6) = PE28838(K,6) * D15(K,3) * Z120R
      PKB839(K,7) = PE28838(K,7) * D16(K,3) * Z120R
      PKB839(K,8) = PE28838(K,8) * D18(K,3) * Z120R
      PKB839(K,9) = D19(K,3) * Z120R
      DO 20 J = 1, 9
      G(K,J) = M1(H) * PKB839(K,J)
20 CONTINUE
10 CONTINUE
      CALL CROSS (G(1,10), PBLADE(1,H), G, 9)
      IF (NN2 .LE. 3) GO TO 40
$FRM X4
      DO 30 K = 1, 27
      G6SAVE(K,1,H) = G(K,1C)
      PTRB839(K,1,H) = PKP839(K)
30 CONTINUE
40 RETURN
      END
      SUBROUTINE FFLAP (E1CR, G)
$FRM Z1
      COMMON /ALL/ NB, J, K
      COMMON /CHK/ SKIP(71), M3(5)
      COMMON /DERAF/ B10(3,3), B2(3,3), B3(3,3), B4(3,3), B4T(3,3),
      A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
      B      D10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
      C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
      D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
      E D29(3,3), D3(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
      F D34(3,3), D4(3,3), D4C(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
      G D9(3,3), JJ,
      H PE28838(3,8), PLB834(3,4), PLB837(3,7), TH10T(3,3)
      I , B30(3)
      J , I
      COMMON /SCDAF/ A1, RIASAD1, RIASAD2, RIASAD3, RIASAL1, RIASAL2,
      A RIASAL3, CUTAB(460), CLTAB(430),
      B C8T(3,3), G6SAVE(3,9,11),
      C , F, NAD, NAL, NAL1, NN2,
      D PTRB839(3,9,11), T31(3), NMD, DMD1, DMD2,
      E DMD3, RIASMD1, RIASMD2, RIASMD3, NML, DML1, DML2, DML3,
      F RIASML1, RIASML2, RIASML3
      DIMENSION G(3,14)
C      G(1,1) AND G(1,8) CORRESPOND TO G9(1,1) AND G10(1,1), RESPEC-
C      TIVELY
      REAL M3
      EQUIVALENCE (PLB834, D77), (PLB837, D76), (PE18837, D75),
      A (PE28838, D50), (PKB839, D1)
      NAMELIST /FBLA/ B4T, G, E1CR, M3, PE18837, T31
      JJ = JJ + 1
$FRM Z2
      CALL AFLAP (E1CR)
      DO 10 K = 1, 21
      G(K) = M3(JJ) * PE18837(K)
10 CONTINUE
      DO 20 K = 1, 3
      T31(K) = C8T(K,3) * E1CR
20 CONTINUE
      CALL CROSS (G(1,8), T31, G, 7)
      RETURN
      END
      SUBROUTINE FLAG (E2CR, G)
$FRM Y1
      COMMON /ALL/ NB, J, K
      COMMON /CHK/ SKIP(66), M2(5)
      COMMON /DERAF/ B10(3,3), B2(3,3), B3(3,3), B4(3,3), B4T(3,3),
      A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
      B      D10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
      C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
      D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
      E D29(3,3), D3(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
      F D34(3,3), D4(3,3), D4C(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
      G D9(3,3), JJ,
      H PE28838(3,8), PLB834(3,4), PLB837(3,7), TH10T(3,3)
      I , B30(3)
      J , I
      COMMON /SCDAF/ A1, RIASAD1, RIASAD2, RIASAD3, RIASAL1, RIASAL2,
      A RIASAL3, CUTAB(460), CLTAB(430),
      B C8T(3,3), G6SAVE(3,9,11),
      C , F, NAD, NAL, NAL1, NN2,
      D PTRB839(3,9,11), T31(3), NMD, DMD1, DMD2,
      E DMD3, RIASMD1, RIASMD2, RIASMD3, NML, DML1, DML2, DML3,
      F RIASML1, RIASML2, RIASML3
      DIMENSION G(3,16)
      FLAG0030 173400000
      FLAG0040 173500000
      FLAG0050 173600000
      FLAG0060 173700000
      FLAG0070 173800000
      EQUI0060 173900000
      EQUI0070 174000000
      174100000
      174200000
      174300000
      FBLA0080 174400000
      FBLA0090 174500000
      FBLA0100 174600000
      FBLA0110 174700000
      FBLA0120 174800000
      FBLA0130 174900000
      FBLA0140 175000000
      FBLA0150 175100000
      FBLA0160 175200000
      FBLA0170 175300000
      FBLA0180 175400000
      FBLA0190 175500000
      FBLA0200 175600000
      FBLA0210 175700000
      FBLA0220 175800000
      PBLA X3 175900000
      FBLA0240 176000000
      FBLA0250 176100000
      FBLA0260 176200000
      FBLA0270 176300000
      FBLA0280 176400000
      FBLA0290 176500000
      FBLA0300 176600000
      FBLA0310 176700000
      FBLA0320 176800000
      FBLA0330 176900000
      FBLA0340 177000000
      FBLA0350 177100000
      FLAG0200 177200000
      FLAG0190 177300000
      COMM0260 177400000
      177500000
      177600000
      177700000
      177800000
      DERA0040 177900000
      DERA0050 178000000
      DERA0060 178100000
      DERA0070 178200000
      178300000
      DERA0090 178400000
      178500000
      178600000
      178700000
      178800000
      178900000
      179000000
      179100000
      179200000
      179300000
      FLAG0210 179400000
      FLAG0220 179500000
      FLAG0230 179600000
      FLAG0240 179700000
      EQUI0060 179800000
      EQUI0070 179900000
      180000000
      FLAG0250 180100000
      FLAG0260 180200000
      FLAG0270 180300000
      FLAG0280 180400000
      180500000
      RAF 100 180600000
      180700000
      FLAG0290 180800000
      FLAG0300 180900000
      FLAG0310 181000000
      FLAG0320 181100000
      FLAG0330 181200000
      FLAG0020 181300000
      FLAG0010 181400000
      COMM0260 181500000
      181600000
      181700000
      181800000
      181900000
      DERA0040 182000000
      DERA0050 182100000
      DERA0060 182200000
      DERA0070 182300000
      182400000
      DERA0090 182500000
      182600000
      182700000
      182800000
      182900000
      183000000
      183100000
      183200000
      183300000
      183400000
      FLAG0030 183500000

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C      G(1,1) AND G(1,9) CORRESPOND TO G7(1,1) AND G8(1,1), RESPEC-
C      TIVELY
      REAL M2
      EQUIVALENCE (PLB834, D77), (PLB837, D76), (PE18B37, D75),
      A (PE28B38, D50), (PKR839, D1)
      NAMELIST /LAGNAM/ B5T, E2CR, C, M2, PE28B38, T31
      JJ = JJ + 1
$FROM Y2
      CALL ALAG (E2CR)
      DO 10 K = 1, 24
      G(K) = M2(JJ) * PE28B38
      A(K)
10 CONTINUE
      DO 20 K = 1, 3
      T31(K) = B5T(K,3) * E2CR
20 CONTINUE
      CALL CROSS (G(1,9), T31, G, 8)
      RETURN
      END
      SUBROUTINE FSHAFT (LILLOR, G)
$FROM R1
      COMMON /ALL/ N8, J, K
      COMMON /CHK/ SKIP(176), M4(5)
      COMMON /DERAF/ B10(3,3), B2(3,3), B3(3), B4(3,3), B4T(3,3),
      A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
      B      D10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
      C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
      D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
      E D29(3,3), D3(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
      F D34(3,3), D4(3,3), D4C(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
      G D9(3,3), JJ, PE18B37(3,7),
      H PE28B38(3,8), PLB834(3,4), PLB837(3,7), TH10T(3,3)
      I , B30(3)
      J,1
      COMMON /SCDAF/ A1, BIASA01, BIASA02, BIASA03, BIASA11, BIASA12,
      A BIASA13, CDTAB(480), CLTAB(480),
      B CST(3,3), G6SAVE(3,9,11)
      C , H, NAD, NAL, NNL, NN2,
      D PT8B839(3,9,11), T31(3), NMD, DMD1, DMD2,
      E DMD3, BIASMD1, BIASMD2, BIASMD3, NML, DML1, DML2, DML3,
      F BIASML1, BIASML2, BIASML3
      DIMENSION G(3, 14)
      REAL LILLOR, M4
      EQUIVALENCE (PLB834, D77), (PLB837, D76), (PE18B37, D75),
      A (PE28B38, D50), (PKR839, D1)
      NAMELIST /FSHA/ B6T, G, LILLOR, M4, PLB837, T31
      JJ = JJ + 1
$FROM R2
      CALL ASHAFT (LILLOR)
      DO 10 K = 1, 21
      G(K) = M4(JJ) * PLB837(K)
10 CONTINUE
      DO 20 J = 1, 3
      T31(J) = B6T(J,2) * LILLOR
20 CONTINUE
      CALL CROSS (G(1,9), T31, G, 7)
      RETURN
      END
      SUBROUTINE FWING (CAPLER, G13)
$FROM T1
      COMMON /ALL/ N8, J, K
      COMMON /CHK/ SKIP(181), M5(5)
      COMMON /DERAF/ B10(3,3), B7(3,3), B3(3), B4(3,3), B4T(3,3),
      A B5T(3,3), B6T(3,3), B7(3,3), B8(3,3), B9(3,3),
      B      D10(3,3), D11(3,3), D12(3,3), D13(3,3), D14(3,3),
      C D15(3,3), D16(3,3), D18(3,3), D19(3,3), D2(3,3), D21(3,3),
      D D22(3,3), D23(3,3), D24(3,3), D25(3,3), D26(3,3), D28(3,3),
      E D29(3,3), D3(3,3), D3C(3,3), D31(3,3), D32(3,3), D33(3,3),
      F D34(3,3), D4(3,3), D4C(3,3), D5(3,3), D6(3,3), D7(3,3), D8(3,3),
      G D9(3,3), JJ, PE18B37(3,7),
      H PE28B38(3,8), PLB834(3,4), PLB837(3,7), TH10T(3,3)
      I , B30(3)
      J,1
      COMMON /SCDAF/ A1, BIASA01, BIASA02, BIASA03, BIASA11, BIASA12,
      A BIASA13, CDTAB(480), CLTAB(480),
      B CST(3,3), G6SAVE(3,9,11)
      C , H, NAD, NAL, NNL, NN2,
      D PT8B839(3,9,11), T31(3), NMD, DMD1, DMD2,
      E DMD3, BIASMD1, BIASMD2, BIASMD3, NML, DML1, DML2, DML3,
      F BIASML1, BIASML2, BIASML3
      DIMENSION G13(3,4)
      REAL M5
      EQUIVALENCE (PLB834, D77), (PLB837, D76), (PE18B37, D75),
      A (PE28B38, D50), (PKR839, D1)
      NAMELIST /FWIN/ CAPLER, G13, M5, PLB834, TH10T, T31
      JJ = JJ + 1
$FROM T2
      CALL AWING (CAPLER)
      DO 10 J = 1, 3
      T31(J) = TH10T(J) * CAPLER
10 CONTINUE
      CALL CROSS (G13, T31, PLB834, 4)
      DO 20 J = 1, 12
      G13(J) = M5(JJ) * G13(J)
20 CONTINUE
      RETURN
      END
      SUBROUTINE IN
      DIMENSION MU1(11), MU2(5), MU3(5), MU4(5), MU5(5)
      COMMON /BLANK/ SKIP1(57), CAPLER, CA12(4), CHG1(8), SKIP2(494),
      A ILXPS, SKIP3(3), FICR, CHG2(8), LILLOR, M, N, PSI(4), SPK,
      S SKIP4(18), CUOR, VCCR, WCCR,

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FLAGCC40 1836C0000
FLAGC050 183700000
FLAGC060 183800000
EQUI0060 183900000
EQUI0070 1840C0000
18410C000
FLAGC0070 184200000
FLAGC0080 184300000
FLAGC0090 184400000
FLAGC100 184500000
FLAGC110 1846C0000
184700000
FLAGC120 1848C0000
FLAGC130 184900000
FLAGC140 185000000
FLAGC150 1851C0000
FLAGC160 185200000
FLAGC170 1853C0000
FLAGC180 1854C0000
FSHA0020 185500000
FSHA0030 185600000
FSHA0040 185700000
FSHA0050 1858C0000
FSHA0060 185900000
FSHA0070 1860C0000
FSHA0080 1861C0000
FSHA0090 186200000
FSHA0100 1863C0000
FSHA0110 1864C0000
FSHA0120 1865C0000
FSHA0130 1866C0000
FSHA0140 1867C0000
FSHA0150 1868C0000
FSHA0160 186900000
FSHA0170 187000000
FSHA0180 1871C0000
FSHA0190 1872C0000
FSHA0200 1873C0000
FSHA0210 1874C0000
FSHA0220 1875C0000
FSHA0230 1876C0000
FSHA0240 1877C0000
FSHA0250 1878C0000
FSHA0260 1879C0000
FSHA0270 1880C0000
FSHA0280 1881C0000
FSHA0290 1882C0000
FSHA0300 1883C0000
FSHA0310 1884C0000
FSHA0320 1885C0000
FSHA0330 1886C0000
FSHA0340 1887C0000
FSHA0350 1888C0000
FSHA0360 1889C0000
FSHA0370 1890C0000
FSHA0380 1891C0000
FSHA0390 1892C0000
FSHA0400 1893C0000
FSHA0410 1894C0000
FSHA0420 1895C0000
FSHA0430 1896C0000
FSHA0440 1897C0000
FSHA0450 1898C0000
FSHA0460 1899C0000
FSHA0470 1900C0000
FSHA0480 1901C0000
FSHA0490 1902C0000
FSHA0500 1903C0000
FSHA0510 1904C0000
FSHA0520 1905C0000
FSHA0530 1906C0000
FSHA0540 1907C0000
FSHA0550 1908C0000
FSHA0560 1909C0000
FSHA0570 1910C0000
FSHA0580 1911C0000
FSHA0590 1912C0000
FSHA0600 1913C0000
FSHA0610 1914C0000
FSHA0620 1915C0000
FSHA0630 1916C0000
FSHA0640 1917C0000
FSHA0650 1918C0000
FSHA0660 1919C0000
FSHA0670 1920C0000
FSHA0680 1921C0000
FSHA0690 1922C0000
FSHA0700 1923C0000
FSHA0710 1924C0000
FSHA0720 1925C0000
FSHA0730 1926C0000
FSHA0740 1927C0000
FSHA0750 1928C0000
FSHA0760 1929C0000
FSHA0770 1930C0000
FSHA0780 1931C0000
FSHA0790 1932C0000
FSHA0800 1933C0000
FSHA0810 1934C0000
FSHA0820 1935C0000
FSHA0830 1936C0000
FSHA0840 1937C0000

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B          XBARI(4), XCS, SKIP5(15), E2OR, SKIP6(5), NOORO.
$ SKIP11(3),
C CNO, SKIP10(7), NIT
D , SKIP12(2), NPB
COMMON /ALL/ B, J, K, RET
COMMON /ANGLES/ ANG(53)
COMMON /CHK/ CHECK(92)
COMMON /SCDA/ SKIP7(7), CD(480), CL(480), SKIP8(307), NAD, NAL,
A NNL, NM2, PTB8839(3,9,11), T31(3), NMD, DMD1, DMD2, DMD3, SKIP9(4
B), DML1, DML2, DML3
COMMON /IN/ NN10, NA3, NN4, NN5, NN6, NN7, NN8, NN9, OR, PRSQRH,
AA,I LXP,NAD1, NAD2, NAD3, NAL1, NAL2, NAL3, NMD1, NMD2, NMD3,
B NML1, NML2, NML3
COMMON /ICKY/ IC(14)
INTEGER RET, B, IO(16), RP6
A , TMOB
REAL ANGDI(53), CHK1(14), CHK2(14), CHK3(39), CHK4(2), CHK5(3),
A CHK6(6), CHK7(3), CHK8(3), ICAPLXP, ILXP, KBETA, KXI,
B KX0, KX1, KY0, KY1, KZ0, KZ1, L, MD1, MD2, MD3, ML1, ML2, ML3,
C MU1, MU2, MU3, MU4, MU5, NAD1, NAD2, NAD3, NAL1, NAL2, NAL3,
D NMD1, NMD2, NMD3, NML1, NML2, NML3, NUO, CHG3(8), CHG4(8),
E LILLOR, VAR(29)
F , BETA(4), XI(4), BETADCT(4), XIDOT(4)
REAL NUJRO
A , ILXPS
EQUIVALENCE (CHECK, ANGDI), (CHECK1(53),CHK3(5)), (CHECK1(88),CHK6)
EQUIVALENCE (CHK1,THX0), (CHK1(2),THY0), (CHK1(3),THZ0), (CHK1(4),
A THX1), (CHK1(5),THY1), (CHK1(6),THZ1), (CHK1(7),BETA), (CHK1(11),
B X1), (CHK2,THXODOT), (CHK2(2),THYODOT), (CHK2(3),THZODOT),
C (CHK2(4),THXIDOT), (CHK2(5),THYIDOT), (CHK2(6),THZIDOT),
D (CHK2(7),BETADOT), (CHK2(11),XIDOT), (CHK3,BETAMAX), (CHK3(2),DPS
E IC), (CHK3(3),PSIMAX), (CHK3(4),XIMAX), (CHK3(5),THMAX), (CHK3(6),CC),
F (CHK3(7),L), (CHK3(8),MU1), (CHK3(19),MU2), (CHK3(24),MU3),
G (CHK3(29),MU4), (CHK3(34),MU5), (CHK3(39),RHO), (CHK4,C),
HICHK4(2),XC), (CHK5,CAPL), (CHK5(2),F1), (CHK5(3),E2), (CHK6,MD1),
I (CHK6(2),MD2), (CHK6(3),MD3), (CHK6(4),ML1), (CHK6(5),ML2),
J (CHK6(6),ML3), (CHK7,THISS), (CHK7(2),THISS), (CHK7(3),DELTA3),
K (CHK8,U), (CHK8(2),V), (CHK8(3),W), (ANGDI,AD1), (ANGDI(2),AD2),
L (ANGDI(3),AD3), (ANCE(4),AL1), (ANGDI(5),AL2), (ANGDI(6),AL3),
M (ANGDI(7),DAD1), (ANGDI(8),DAD2), (ANGDI(9),DAD3), (ANGDI(10),DAL1),
N (ANGDI(11),DAL2), (ANGDI(12),DAL3), (ANGDI(13),UMEGA), (ANGDI(14),
O DPSIP), (ANGDI(15),TH1), (ANGDI(16),PSI0), (ANGDI(17),CHK1), (ANGDI(3
P),CHK2), (ANGDI(45),THOS), (ANGDI(46),CHK7), (CHG3,CBETA),
Q (CHG3(2),CX1), (CHG3(3),CX0), (CHG3(4),CX1), (CHG3(5),CY0),
R (CHG3(6),CY1), (CHG3(7),CZ0), (CHG3(8),CZ1), (CHG4,KBETA),
S (CHG4(2),KX1), (CHG4(3),KX0), (CHG4(4),KX1), (CHG4(5),KY0)
EQUIVALENCE (CHG4(6),KY1), (CHG4(7),KZ0), (CHG4(8),KZ1),
A (ANGDI(16),VAR)
B , (UMEGA,GMD)
C , (B,NB)
NAMELIST /INPUT/ A, AD1, AD2, AD3, AL1, AL2, AL3, B, BETA,
A BETADOT, BETAMAX, C, CAPL, CBETA, CC, CD, CL, CNO, CX1, CX0, CX1,
B CY0, CY1, CZ0, CZ1, DAD1, DAD2, DAD3, DAL1, DAL2, DAL3, DELTA3,
C DMD1, DMD2, DMD3, DML1, DML2, DML3, DPSIC, DPSIP, E1, E2, ICAPLXP
D , ILXP, ISTOP, KBETA, KXI, KX0, KX1, KY0, KY1, KZ0, KZ1, L, MD1,
E MD2, MD3, ML1, ML2, ML3, MU1, MU2, MU3, MU4, MU5, NAD1, NAD2,
F NAD3, NAL1, NAL2, NAL3, NIT, NMD1, NMD2, NMD3, NML1, NML2, NML3,
G NNL1, NNL2, NNL3, NN4, NN5, NN6, NN7, NN8, NN9, NUO, OMEGA,
H PSIMAX, PSI0, R, RPE, THMAX, THX0, THXODOT, THX1, THXIDOT, THY0,
I THYODOT, THY1, THYIDOT, THZ0, THZODOT, THZ1, THZIDOT, THOS, TH1,
J THISC, THISS, U, V, W, XC, XI, XIDOT, XIMAX
K DATA A, AD1, AD2, AD3, AL1, AL2, AL3, /1100., 0, 30., 180., 0,
A 30., 60., B, BETA, BETADCT(3), B*O/, CC, CNO/1000., 0/, DAD1,
B DAC2, DAD3, DAL1, DAL2, DAL3, DELTA3, DMD1, DMD2, DMD3, DML1,
C DML2, DML3, DPSIP /1., 10., 0, 1., 30., 70., 0, .1., .05., .1., .1.
D .05., .1., 0/, E1, E2 /20/, ISTOP /0/, MD1, MD2, MD3, ML1, ML2,
E ML3, NAD1, NAD2, NAD3, NAL1, NAL2, NAL3, NIT, NMD1, NMD2, NMD3,
F NML1, NML2, NML3, AN1, NN10, NN2, NN3, NN4, NN5, NN6, NN7, NN8,
G NN9 /1., .7., .8., .1., .7., .8., 30., 15., 1., 30., 1., 3., 0,
H $ 6., 2*2., 6., 2., 3., 10*0/,
I RHOD /0.023769/, THX0, THXODOT, THX1, THXIDOT, THY0, THYODOT,
J THY1, THYIDOT, THZ0, THZODOT, THZ1, THZIDOT, TH1, THISC, THISS/
K 15*0/, U, V, W /3*C/, XC, XI, XIDOT /9*0/
L DATA CU /7*.008, .01, .0185, .07, 5*.00805, .00825, .0081, .013, .023,
A .075, 5*.0081, .0085, .01, .02, .04, .0915, 5*.00815, .00875,
B .02, .033, .057, .108, 5*.0082, .009, .03, .055, .08, .126,
C 3*.0085, .0091, .009, .014, .0455, .0775, .0105, .144, 3*.0088,
D .01, .0115, .019, .061, .11, .12, .162, .0091, .0091, .0096, .012,
E .014, .0345, .0805, .1233, .14, .18, .010125, .010125, .01073,
F .014, .019, .05, .1, .1466, .15, .19, .01115, .01115, .01186,
G .02, .04175, .075, .125, .17, .18, .2, 2*.012175, .013, .026,
H .0645, .1, .15, .1791, .17, .22, 2*.0132, .019, .032, .08725,
I .16, .175, .1982, .18, .24, 2*.0181, .025, .066, .11, .18, .2,
J .1973, .19, .2433, 2*.023, .0625, .1, .145, .2, .20875, .2063,
K .2, .2467, .0615, .0615, .1, .14, .18, .22, .2175, .2154, .21,
L .25, 2*.1, .14, .18, .195, .24, .22625, .2245, .22, .2533, 2*
M .155, .18, .198, .21, .26, .235, .2336, .23, .2567, 2*.18, .21,
N .25, .27, .29, 4*.12, .21, .21, .25, .27, .29, .31, 4*.2, 2*.24,
O .2475, .252, .255, .2675, .26125, .2609, .26, .2667, 2*.274,
P 8*.27, 2*.302, .337, .348, .378, .409, .31, 3*.31, 2*.33, .375,
Q 3.78, .411, .448, 4*.33, 2*.364, .4, .409, .444, .478, 4*.36, 2*
R .394, .428, .447, .478, .519, 4*.38, .421, .421, .489, .486/
L DATA (CU,K),K=25,400) / .516, .553, 4*.4, 2*.45, .489, .509,
A .553, .6, 4*.62, 2*.478, .519, .563, .6, .645, .66, 3*.46, 2*
B .513, .549, .602, .646, .695, .5, 3*.5, 10*.53, 10*.57, 10*.96,
C 10*1.32, 10*1.62, 10*1.84, 10*2., 10*2.7, 10*2., 10*1.86,
D 10*1.62, 10*1.33, 10*1., 10*.66, 10*.3, 10*.08, 10*.02/
L DATA CL/1100,2*.11,2*.12,13.,14.,16.,18.,21,2*.09,2*.22,.23,.24,
A .25,.27,.32,.36,.38,2*.19,2*.33,.34,.35,.38,.41,.46,2*.52,2*.3,2*
B .43,.45,.46,.51,.54,.57,.63,.61,2*.39,2*.54,.56,.57,.62, 2*.66,.72,
C .665,2*.52*.65,.67,.68,3*.74,.76,.72,2*.57,2*.75,.77,.8.,84,2*.79
D .794,.7488,2*.64,2*.66,.88,.91,.97,.66,.83,.828,.7775-.7*.68,2*.96

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E..99,1.01,.96,.89,.86,.862,.8025,2*.72,2*1.06,1.1,1.08,.98,.92,
F..89,.89,.835,2*.76,2*1.17,1.18,1.12,.98,.93,.92,.93,.86375,2*.77,
G1.27,1.27,1.25,1.12,.98,.94,.93,.92,.8925,2*.78,2*1.33,1.27,1.11,
H..98,.95,.91875,.91,.92125,2*.79,1.33,1.33,1.22,1.08,.98,.96,.9075,
I..95,2*.8,2*1.2,1.08,1.05,2*.97,.89625,.89,.9317,2*.8067,2*1.04,
J..98,1.2*.97,.885,.88,.9133,.8133,2*.96,.9,.93,2*.96,.87375,
K..87,.8945,2*.82,2*.85,.85,.87,.93,.94,.8625,.86,.8767,.8267,.8267,
L2*.85,.82,.845,2*.85,.85125,.85,.85835,2*.8335,2*.82,.81,.82,2*
M..86,5*.84,2*.8,.805,.824,.84,.85,5*.866,2*.8,.82,2*.84,.86,5*.892,
N2*.81,.83,.853,.863,.91,5*.918,2*.825,.853,.88,.89,.95,5*.944,.85,
O..85,.88,.902,.92,.99,.97,4*.97,2*.873,.901,.93,.96,1.03,.996,4*
P..996,2*.9,.925,.96,.99,1.08,1.022,4*1.022,2*.92,.953,.99,1.02,1.11
Q,5*1.048,2*.95,.98,1.01,1.05,1.15,1.074,1.074,3*1.074,2*.98,
R 1.01,1.03,1.07,1.17,5*1.1,11*.95,22*-1./
DATA III /O/
NAMELIST /NAME/ VAR, XI, XI00T, ANG, ANGD, M, N, B, CHK2, OMD,
A TMOB, DEG, NMI
IF (III.EQ. 0) GO TO 111
IF (ISTUP.EQ. 0) STOP
111 III = 1
J = 0
READ 1, ID
1 FORMAT (8A10)
IF (EOF,5) 2,3
2 STOP
3 PRINT 4, IC
4 FORMAT (47(/),(X, 8A10))

*****
*BOUNDS HAVE BEEN PLACED ON THE FOLLOWING INPUT VARIABLES: *
*VARIABLE NAME LOWER BOUND UPPER BOUND *
* A 0 2000 *
* B 2 4 *
* BETA -190 190 *
* BETADJT -OMEGA(IN DEG/SEC) OMEGA(IN DEG/SEC)*
* BETAMAX 0 NONE *
* C R *
* CAPL NONE R *
* CC 0 NONE *
* CNO -1 1 *
* DELTA3 -90 90 *
* DPSIC 0 NONE *
* E1 NONE R *
* E2 ACNE R *
* L C NONE *
* MD1 0 2 *
* MD2 0 2 *
* MD3 C 2 *
* ML1 C 2 *
* ML2 0 2 *
* ML3 0 2 *
* MU1 C NONE *
* MU2 C NONE *
* MU3 C NONE *
* MU4 C NONE *
* MU5 0 NONE *
* PSMAX C NONE *
* PSIO -360 360 *
* RHO C NONE *
* THMAX C NONE *
* THXC -190 190 *
* THXDDJT -OMEGA(IN DEG/SEC) OMEGA(IN DEG/SEC)*
* THX1 -190 190 *
* THXIDDJT -OMEGA(IN DEG/SEC) OMEGA(IN DEG/SEC)*
* THYO -190 190 *
* THYDDJT -OMEGA(IN DEG/SEC) OMEGA(IN DEG/SEC)*
* THY1 -190 190 *
* THYIDDJT -OMEGA(IN DEG/SEC) OMEGA(IN DEG/SEC)*
* THZO -190 190 *
* THZDDJT -OMEGA(IN DEG/SEC) OMEGA(IN DEG/SEC)*
* THZ1 -190 190 *
* THZIDDJT -OMEGA(IN DEG/SEC) OMEGA(IN DEG/SEC)*
* THOS -180 180 *
* THISC -90 90 *
* THISS -90 90 *
* U ACNE A *
* V ACNE A *
* W ACNE A *
* XI -190 190 *
* XI00T -OMEGA(IN DEG/SEC) OMEGA(IN DEG/SEC)*
* XIMAX C NONE *
*****
READ (5, INPUT)
IF (EOF, 5) 6, 7
6 PRINT 8
8 FORMAT (* INPUT ERROR: NO DATA FOUND.*)
STOP 1
7 WRITE (6, INPUT)
CALL SCREEN (A, 1, C, 2000.)
CALL SCREEN (FLOAT(P), 1, 2., 4.)
CALL SCREEN (CHK1, 14, -190., 190.)
OMD = 6. * OMEGA
CALL SCREEN (CHK2, 14, -CMD, OMD)
CALL LJ (CHK3, 39, C)
CALL SCREEN (CHK4, 2, C, P)
CALL HI (CHK5, 3, R)
CALL SCREEN (CNO, 1, -1., 1.)
CALL SCREEN (CHK6, 6, 0, 2.)
CALL SCREEN (PSIO, 1, -360., 360.)
CALL SCREEN (THOS, 1, -180, 180.)
CALL SCREEN (CHK7, 3, -90., 90.)
CALL HI (CHK8, 3, A)

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204000000
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213800000
213900000
214000000
214100000

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M = B + 7
N = M + 8
NPB = N + 8
NM1 = N - 1
TWOB = 2*B
DO 9 J = 1, 6
  VAR(M+J) = XI(J)
9 VAR(B+J+21) = XICCT(J)
DO 10 J = 2, N
10 VAR(N+J-1) = CHK2(J-1)
DO 11 J = 1, NM1
  IF (VAR(J+N).LT.1E3C) GO TO 12
  IC(J) = 0 $ VAR(J+N) = 0
  GO TO 11
12 IC(J) = 1
11 CONTINUE
  DATA DEG /.017453252515594321/
  DO 13 J = 1, 53
13 ANG(J) = DEG * ANG(J)
  DO 15 J = 01, NM1
15 ANG(J+15+N) = ANG(J+15+N) / ANG(13)
  CAPLOR = CAPL / R
  DATA PI /3.14159265358979/
  PRSQ = PI * R**2
  PRSQRM = PRSQ * RHC
  CK = ANG(13) * R
  TEMP = PRSQRM * CK**2 * R
  DO 16 J = 1, 8
16 CHG2(J) = CHG4(J) * TEMP
  TEMP = ANG(13) / TEMP
  DO 17 J = 1, 8
17 CHG1(J) = CHG3(J) / TEMP
  EIOR = E1 / R
  EZOR = E2 / R
  LILLOR = L / R
  SPR = C * R / (2. * PRSQ)
  UOOR = U / OR
  VOOR = V / UR
  WOOR = W / OR
  XCS = XC * R
  NOORO = NUO / OR
  ILXPS = -ICAPLXP
  RETURN
  END
  SUBROUTINE NINE (S, T, X)
$FROM BETA1
  COMMON /ALL/ NB, J, K
  DIMENSION S(3), T(3,9), X(14)
$FROM BETA2
  L = 2 * NB + 6
  CALL MATRIX (20, 3, 6, 1, T(1,2), 3, X, 9, S, 3)
  DO 10 K = 1, 3
  S(K) = S(K) + T(K,2) * X(NB+6) + T(K,9) * X(L) + T(K,1)
10 CONTINUE
  RETURN
  END
  SUBROUTINE SCREEN (X, N, A, B)
  COMMON /ALL/ NB, J, K, RET
  INTEGER RET, L(90)
  REAL X(38)
  DATA L /1RA, 1RB, 4RTHXC, 4RTHYO, 4RTHZO, 4RTHX1, 4RTHY1, 4RTHZ1,
  A 7RBETA(1), 7RBETA(2), 7RBETA(3), 7RBETA(4), 5RXI(1), 5RXI(2),
  B 5RXI(3), 5RXI(4), 7RTHXDOT, 7RTHYDOT, 7RTHZDOT,
  C 7RTHXIDOT, 7RTHYIDOT, 7RTHZIDOT, 10RBETADOT(1), 10RBETADOT(2),
  D 10RBETADOT(3), 10RBETADOT(4), 8RXIDOT(1), 8RXIDOT(2), 8RXIDOT(3),
  E 8RXIDOT(4), 7RBETAPAX, 5RDPSC, 6RPSIMAX, 5RXIMAX, 5RTHMAX,
  F 5ZCC, 1RL,
  G 6RMU1(1), 6RMU1(2), 6PMU1(3), 6RMU1(4), 6RMU1(5), 6RMU1(6),
  H 6RMU1(7), 6RMU1(8), 6RMU1(9), 7RMU1(10), 7RMU1(11), 6RMU2(1),
  I 6RMU2(2), 6RMU2(3), 6RMU2(4), 6RMU2(5), 6RMU3(1), 6RMU3(2),
  J 6RMU3(3), 6RMU3(4), 6RMU3(5), 6RMU4(1), 6RMU4(2), 6RMU4(3),
  K 6RMU4(4), 6RMU4(5), 5RMU5(1), 6RMU5(2), 6RMU5(3), 6RMU5(4),
  L 6RMU5(5), 3RRHC, 1RC, 2RXC, 4RCAPL, 2RE1, 2RE2, 3RCNU, 3RMD1,
  M 3RMD2, 3RMD3, 3RML1, 3RML2, 3RML3, 4RPSIO, 4RTHOS, 5RTHISC,
  N 5RTHISS, 6RDELTA3, 1RL, 1RV, 1RW/
  DO 1 K = 1, N
  J = J + 1
  IF (X(K).LT. A) GO TO 2
  IF (X(K).GT. B) GO TO 2
1 CONTINUE
  RETURN
  ENTRY LG
  DO 3 K = 1, N
  J = J + 1
  IF (X(K).LT. A) GO TO 2
3 CONTINUE
  RETURN
  ENTRY HI
  DO 4 K = 1, N
  J = J + 1
  IF (X(K).GT. A) GO TO 2
4 CONTINUE
  RETURN
2 IF (J.LT. 17) GO TO 6
  IF (J.GT. 30) GO TO 6
  IF (X(K).GE. 1E30) RETURN
6 CONTINUE
  PRINT 5, L(J)
5 FORMAT (1X INPUT ERROR: * R11, * IS OUT OF RANGE.*)
  GO TO RET
  END
  SUBROUTINE SEVEN (S, T, X)

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214200000
214300000
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217800000
217900000
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218100000
218200000
218300000
218400000
218500000
NINEC020 218600000
NINEC010 218700000
COMM0260 218800000
NINE0030 218900000
NINE0040 219000000
NINE0050 219100000
NINE0060 219200000
NINE0070 219300000
NINEC080 219400000
NINE0090 219500000
NINE0100 219600000
NINE0110 219700000
219800000
219900000
220000000
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220200000
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224000000
224100000
224200000
SEVFC010 224300000

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$FROM GAMMA1
COMMON /ALL/ NB, J, K
DIMENSION S(3), T(3,7), X(14)
$FROM GAMMA2
DO 20 K = 1, 3
DO 10 J = 1, 6
S(K) = T(K,J+1) * X(J)
10 CONTINUE
S(K) = S(K) + T(K)
20 CONTINUE
RETURN
END
SUBROUTINE SIMP (A, AREA,B, GRAND, M, N)
COMMON /ALL/ NB, J, K
DIMENSION AREA(54), EVEN(54), ODD(54), Y(54), YA(54), YB(54),
A YXPH(54)
$FROM Q2
CALL GRAND (A, YA)
X = A
H = (B - A) / FLOAT (N - 1)
DO 10 I = 1, M
EVEN(I) = 0
ODD(I) = 0
10 CONTINUE
DO 40 I = 4, N, 2
X = X + H
CALL GRAND (X, Y)
DO 20 J = 1, M
EVEN(J) = EVEN(J) + Y(J)
20 CONTINUE
X = X + H
$FROM Q3
CALL GRAND (X, Y)
DO 30 J = 1, M
ODD(J) = ODD(J) + Y(J)
30 CONTINUE
40 CONTINUE
CALL GRAND (X+H, YXPH)
CALL GRAND (B, YB)
DO 50 I = 1, M
AREA(I) = (YA(I) + 4. * (EVEN(I) + YXPH(I)) + ODD(I) + ODD(I) +
A YB(I)) * H * .333333333333333
50 CONTINUE
RETURN
END
SUBROUTINE INT2A (II, A, CI, SPEC, VAR, CUVAR, DER, ELE1, ELE2)
DIMENSION DER(29), ELE1(28), ELE2(28), ERRVAL(28)
DIMENSION TEMP(28), IERR(28), DER2(28), DER3(28)
DIMENSION SIVAR(29), VAR(29), CUVAR(29)
COMMON /INT/ IFLAG
COMMON /ALL/ SKIP(3), RET
INTEGER RET
NAMELIST /INT2/ A, CUVAR, DEL, DELP, DELT, DER, DER1, DER2, DER3,
A OM, OJUB, ELE1, ELE2, ERRVAL, H, IERR, IFLAG, J, MODE, NI, SIVAR,
B TEMP, TO, VAR
IFLAG = 1
A=0 & IF(II)3,3,2
INITIALIZATION SECTION
SAVE CI
3 H=CI
18 CONTINUE
TO = VAR(1)
MODE=1
II= 1
NI=NI
DO 5 J=1,NI
CUVAR(J)=VAR(J)
5 CONTINUE
C EVALUATION SECTION HERE
8 IFLAG = 0
CALL DEKSUB
IF (MODE.LE.1) GO TO 7
36 CONTINUE
37 DO 38 J=1,NI
38 VAR(J)=CUVAR(J)
IF(II-3)6,7,7
7 RETLRN
6 IF (SPEC) 9,7,9
9 DEL= VAR(1) -TO
DELP=DEL*(1.+1.CE-11)
IF (ABS(DELP)-ABS(SPEC)) 2,10,10
10 TO = VAR(1)
GO TO 7
2 II=1
IF (MODE-4) 11,12,12
C RUNGE-KUTTA
11 DO 20 J=2,NI
DER3(J-1)=DER2(J-1)
DER2(J-1)=DER1(J-1)
DER1(J-1)=DER(J)
ELE1(J-1)=DER(J)
CUVAR(J)=A
DELT=0.4*ELE1(J-1)*H
SIVAR(J)=VAR(J)
CUVAR(J)=SIVAR(J)+DELT
20 CONTINUE
SIVAR(1)=VAR(1)
CUVAR(1)=SIVAR(1)+0.4*H
CALL DEKSUB
23 CUVAR(1)=SIVAR(1)+C.45573725421879*H
DO 24 J=2,NI
ELE2(J-1)=DER(J)

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EIGH0120 224400000
COMMON0260 224500000
SEVE0020 224600000
SEVE0030 224700000
SEVE0040 224800000
SEVE0050 224900000
SEVE0060 225000000
SEVE0070 225100000
SEVE0080 225200000
SEVE0090 225300000
SEVE0100 225400000
SEVE0110 225500000
SIMPC020 225600000
COMMON0260 225700000
SIMPC030 225800000
SIMPC040 225900000
SIMPC050 226000000
SIMPC055 226100000
SIMPC060 226200000
SIMPC070 226300000
SIMPC080 226400000
SIMPC090 226500000
SIMPC100 226600000
SIMPC110 226700000
SIMPC120 226800000
SIMPC130 226900000
SIMPC140 227000000
SIMPC150 227100000
SIMPC160 227200000
SIMPC170 227300000
SIMPC180 227400000
GRAN0010 227500000
GRAN0020 227600000
GRAN0030 227700000
GRAN0040 227800000
GRAN0050 227900000
GRAN0060 228000000
GRAN0080 228100000
GRAN0090 228200000
GRAN0100 228300000
GRAN0110 228400000
GRAN0120 228500000
GRAN0130 228600000
GRAN0140 228700000
GRAN0150 228800000
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D2.4 6 229100000
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D2.4 9 230100000
D2.4 14 230200000
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D2.4 18 230600000
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D2.4 22 231000000
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D2.4 24 231200000
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D2.4 26 231500000
231600000
D2.4 30 231700000
D2.4 31 231800000
D2.4 32 231900000
D2.4 33 232000000
D2.4 34 232100000
D2.4 35 232200000
D2.4 36 232300000
D2.4 37 232400000
D2.4 38 232500000
D2.4 39 232600000
D2.4 40 232700000
D2.4 41 232800000
D2.4 42 232900000
D2.4 43 233000000
D2.4 44 233100000
D2.4 45 233200000
D2.4 46 233300000
D2.4 47 233400000
D2.4 48 233500000
D2.4 49 233600000
D2.4 50 233700000
D2.4 51 233800000
D2.4 52 233900000
D2.4 53 234000000
D2.4 54 234100000
D2.4 55 234200000
D2.4 57 234300000
D2.4 58 234400000
D2.4 59 234500000

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      DELT=(J.29697760524775*ELE1(J-1)+0.15875964497104*ELE2(J-1))*H
      CUVAR(J)=SIVAR(J)+DELT
24  CONTINUE
      CALL DEKSUB
25  CUVAR(1)=SIVAR(1)+h
      DO 26 J=2,N1
      TEMP(J)=DER(J)
      DELT=(J.21810038822552*ELE1(J-1)-3.05096514464293*ELE2(J-1)
1+3.83236476046701*TEMP(J-1))*H
      CUVAR(J)=SIVAR(J)+DELT
26  CONTINUE
      CALL DEKSUB
27  DH=H
      CUVAR(1)=VAR(1)+DH
      DO 28 J=2,N1
      DOUB= (0.17476028226269*ELE1(J-1)-0.55148066287873*ELE2(J-1)
1+1.20553559939652*TEMP(J-1)+0.17118478121952*DER(J))
      CUVAR(J)=VAR(J)+DH*DOUB
28  CONTINUE
      MODE=MODE+1
      GO TO 8
C      ADAMS-MULTUN
C      ADAMS-WASHFORTH PREDICTOR
12 CUVAR(1)=VAR(1)+h
      DH=H/24.0
      DO 13 J=2,N1
      DOUB= (55.0*DER(J)-59.0*DER1(J-1)+37.0*DER2(J-1)-9.0*DER3(J-1))
      CUVAR(J)=VAR(J)+DH*DOUB
13 CONTINUE
      DO 14 J=1,N
      DER3(J)=DER2(J)
      DER2(J)=DER1(J)
14 DER1(J)=DER(J+1)
      CALL DEKSUB
C      ADAMS-MULTUN CORRECTOR
15 DO 16 J=2,N1
      TEMP=CUVAR(J)
      DOUB= (9.0*DER(J)+19.0*DER1(J-1)-5.0*DER2(J-1)+DER3(J-1))
      CUVAR(J)=VAR(J)+DH*DOUB
16 CONTINUE
19 GO TO 8
      END

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D2.4 60 234600000
D2.4 61 234700000
D2.4 62 234800000
D2.4 63 234900000
D2.4 65 235000000
D2.4 66 235100000
D2.4 67 235200000
D2.4 68 235300000
D2.4 69 235400000
D2.4 70 235500000
D2.4 71 235600000
D2.4 72 235700000
D2.4 74 235800000
D2.4 75 235900000
D2.4 76 236000000
D2.4 77 236100000
D2.4 78 236200000
D2.4 79 236300000
D2.4 80 236400000
D2.4 81 236500000
D2.4 82 236600000
D2.4 83 236700000
D2.4 84 236800000
D2.4 85 236900000
D2.4 86 237000000
D2.4 87 237100000
D2.4 88 237200000
D2.4 89 237300000
D2.4 90 237400000
D2.4 91 237500000
D2.4 92 237600000
D2.4 93 237700000
D2.4 94 237800000
D2.4 95 237900000
D2.4 97 238000000
D2.4 98 238100000
D2.4 99 238200000
D2.4 100 238300000
D2.4 101 238400000
      238500000
D2.4 103 238600000
D2.4 104 238700000

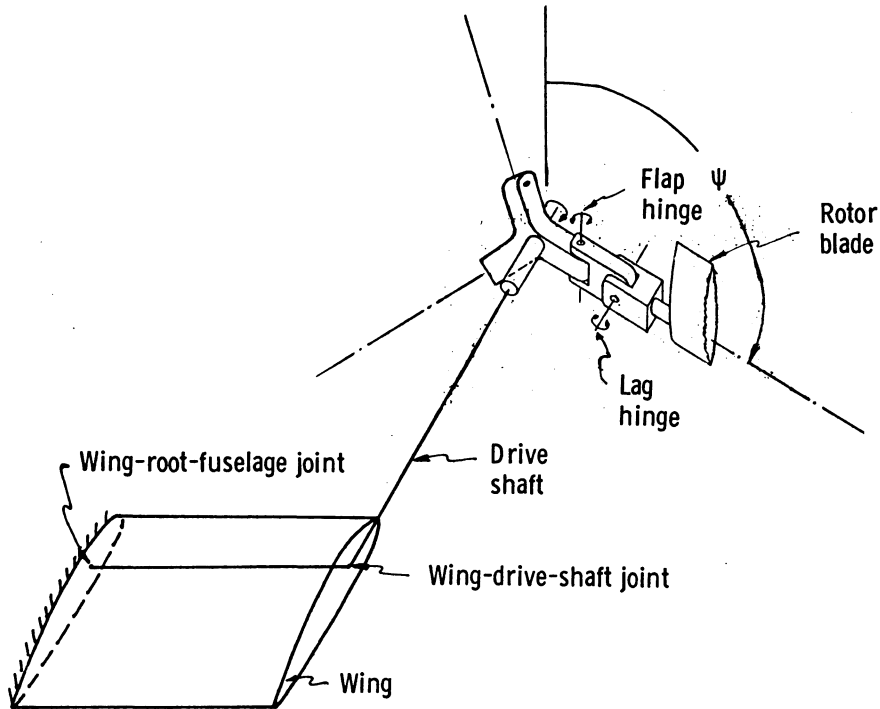
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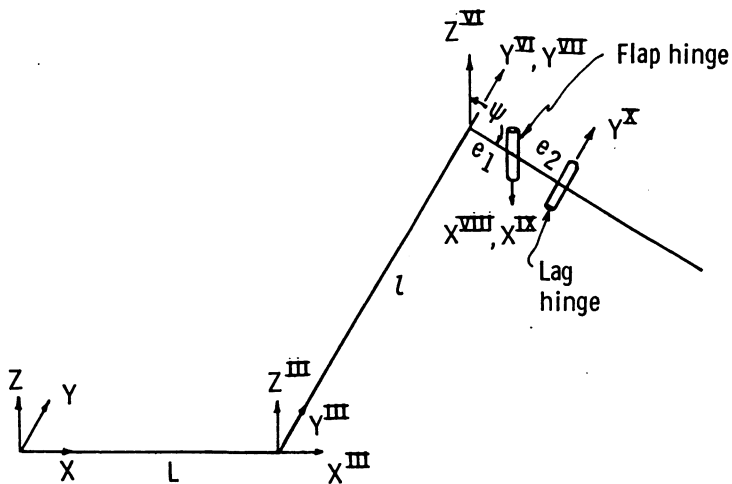
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(a) Physical model (three blades).



(b) Mathematical model.

Figure 10. - Sketch of the physical and mathematical model used in this analysis. Various coordinate systems and member lengths are identified.

Transformation Summary

$$\begin{aligned}
 \begin{Bmatrix} x \\ y \\ z \end{Bmatrix} &\rightarrow [\theta_{10}] \rightarrow \begin{Bmatrix} x^{III} \\ y^{III} \\ z^{III} \end{Bmatrix} \rightarrow \begin{Bmatrix} L \\ 0 \\ 0 \end{Bmatrix} \rightarrow \begin{Bmatrix} x^{III} \\ y^{III} \\ z^{III} \end{Bmatrix} \rightarrow [\theta_{11}] \rightarrow \begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix} \rightarrow \begin{Bmatrix} 0 \\ \ell \\ 0 \end{Bmatrix} \rightarrow \\
 &\rightarrow \begin{Bmatrix} x^{VI} \\ y^{VI} \\ z^{VI} \end{Bmatrix} \rightarrow [\psi] \rightarrow \begin{Bmatrix} x^{VII} \\ y^{VII} \\ z^{VII} \end{Bmatrix} \rightarrow \begin{Bmatrix} 0 \\ 0 \\ e_1 \end{Bmatrix} \rightarrow \begin{Bmatrix} x^{VIII} \\ y^{VIII} \\ z^{VIII} \end{Bmatrix} \rightarrow [\beta] \rightarrow \\
 &\rightarrow \begin{Bmatrix} x^{IX} \\ y^{IX} \\ z^{IX} \end{Bmatrix} \rightarrow \begin{Bmatrix} 0 \\ 0 \\ e_2 \end{Bmatrix} \rightarrow \begin{Bmatrix} x^X \\ y^X \\ z^X \end{Bmatrix} \rightarrow [\xi] \rightarrow \begin{Bmatrix} x^{XI} \\ y^{XI} \\ z^{XI} \end{Bmatrix} \rightarrow [\theta] \rightarrow \begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix}
 \end{aligned}$$

To Transform from Blade Coordinates to Inertial Coordinates

$$\begin{Bmatrix} x \\ y \\ z \end{Bmatrix} = [\theta_{10}]^T [\theta_{11}]^T [\psi]^T [\beta]^T [\xi]^T [\theta]^T \begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix}$$

To Transform from Inertial Coordinates to Blade Coordinates

$$\begin{Bmatrix} x^{XII} \\ y^{XII} \\ z^{XII} \end{Bmatrix} = [\theta] [\xi] [\beta] [\psi] [\theta_{11}] [\theta_{10}] \begin{Bmatrix} x \\ y \\ z \end{Bmatrix}$$

Figure 11.- A summary of the numerous transformations, coordinate systems and the general transformation equation are presented.

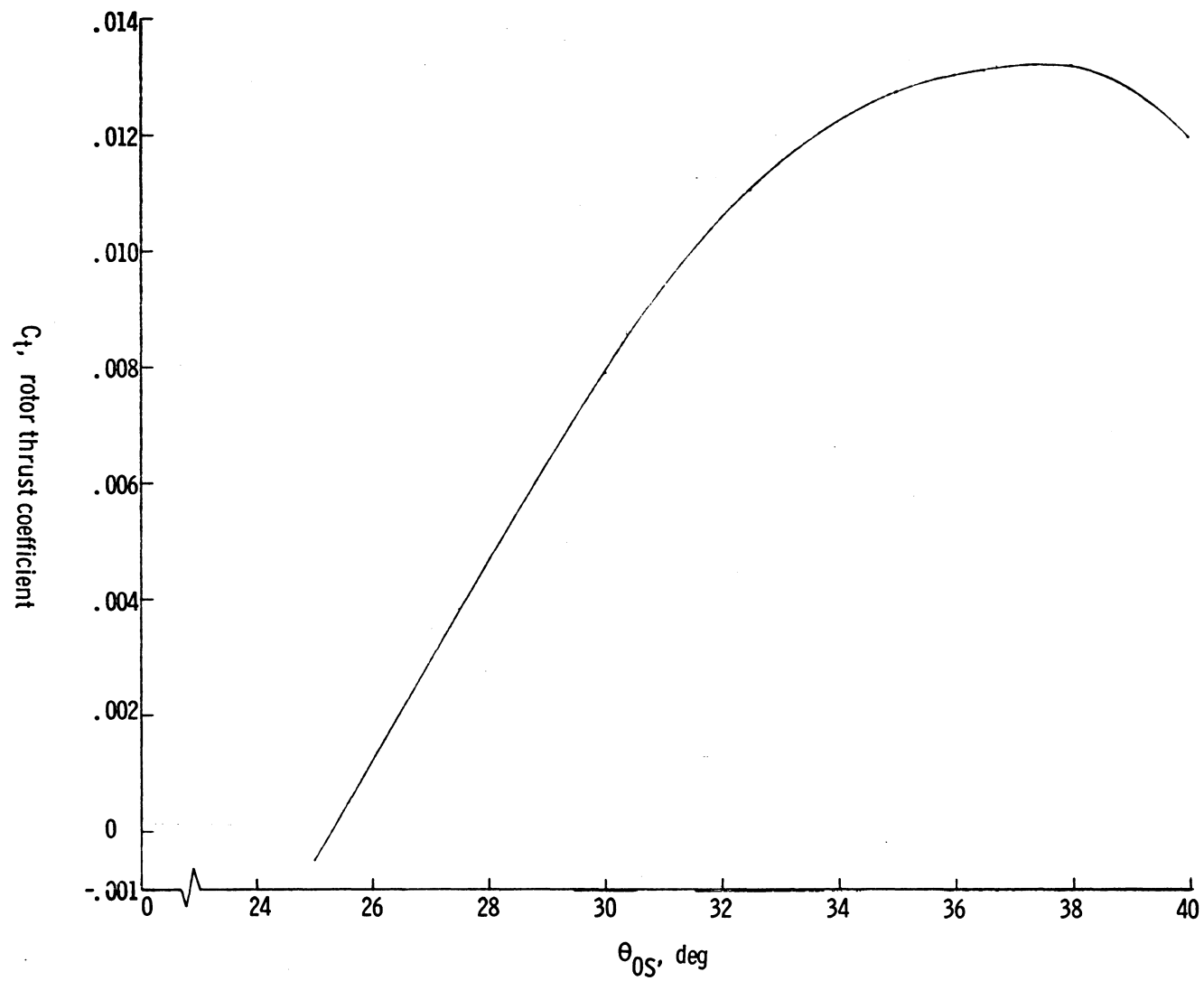


Figure 12.- Thrust coefficient versus blade pitch angle for the nominal rotor.

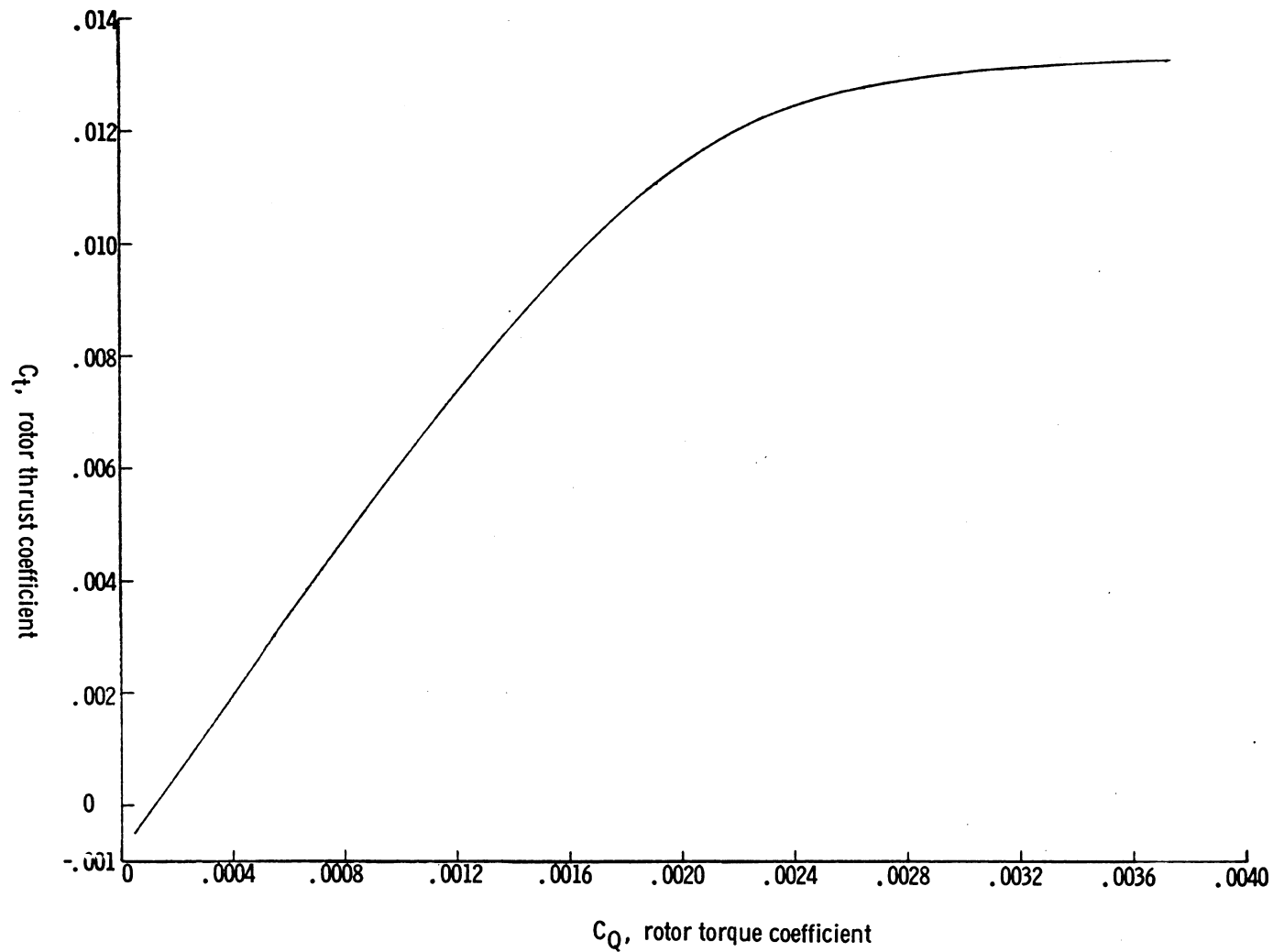


Figure 13.- Thrust and torque coefficients for the nominal rotor with variations in the blade pitch angle.

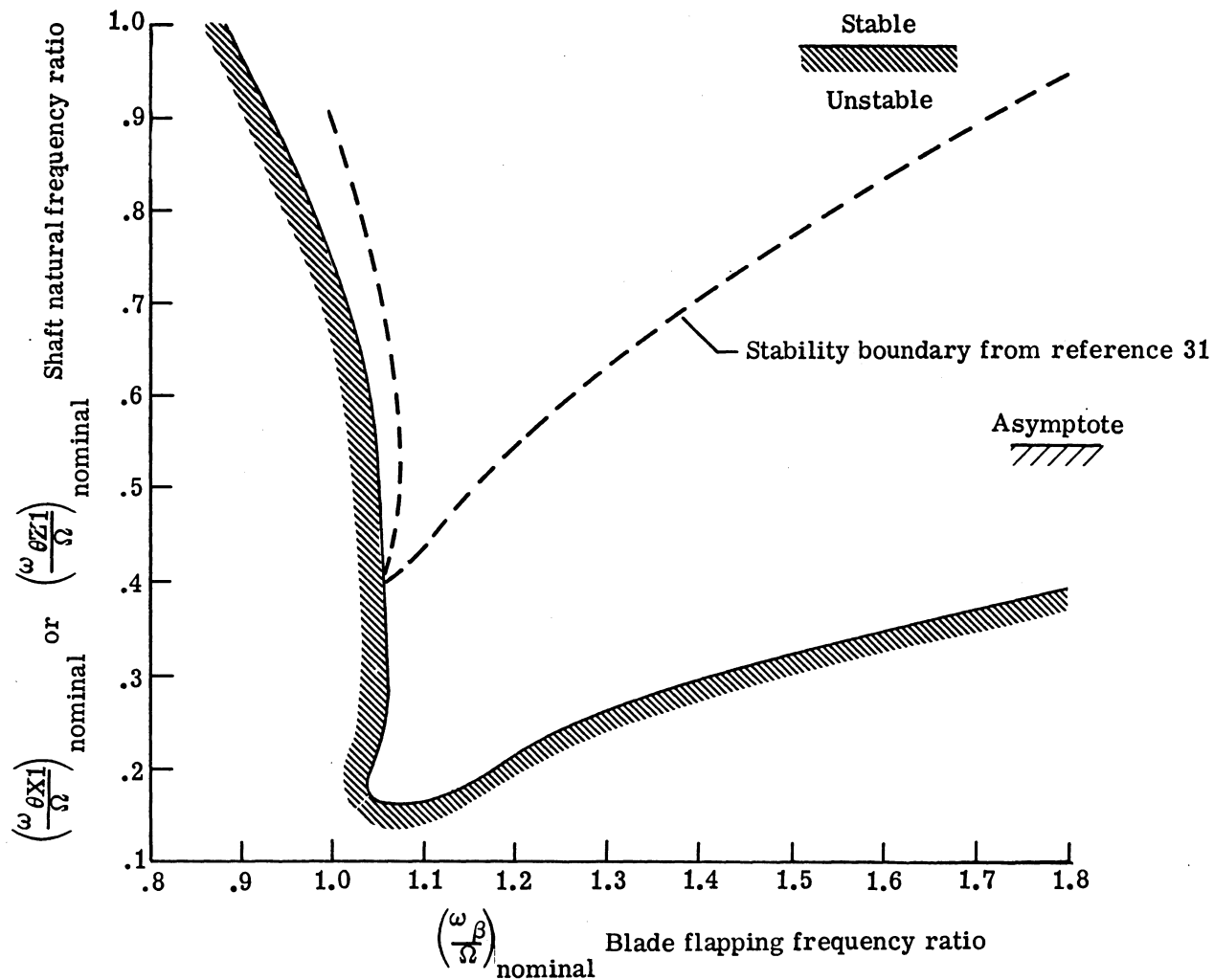


Figure 14.- Rotor stability boundary for the nominal rotor with variations in the rotor shaft and blade flapping natural frequency; $\left(\frac{\omega_{\theta x1}}{\Omega}\right) = \left(\frac{\omega_{\theta z1}}{\Omega}\right)$.

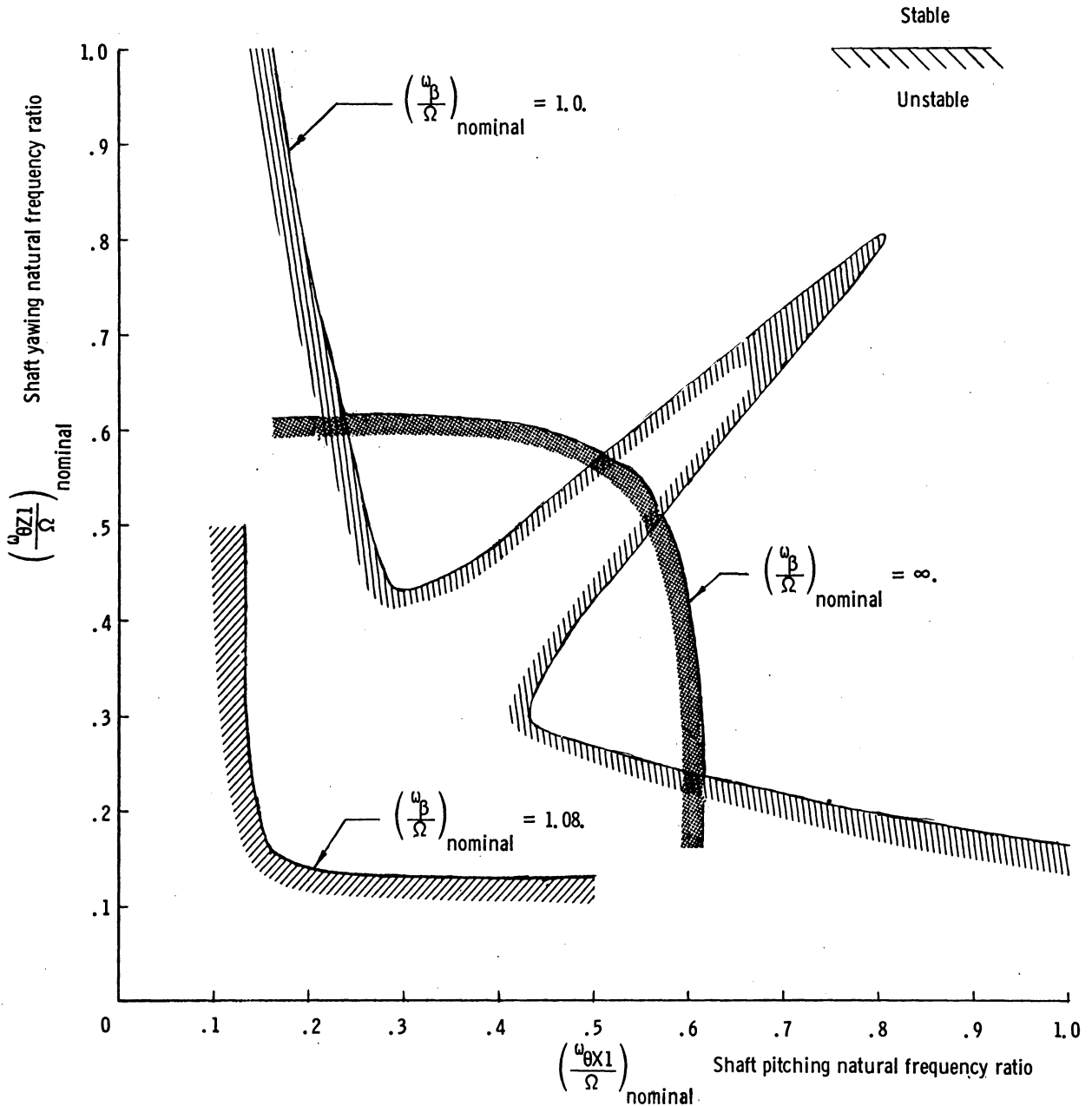


Figure 15.- Rotor stability boundary for the nominal rotor with variations of $\left(\frac{\omega_{\theta X1}}{\Omega}\right)_{\text{nominal}}$ and $\left(\frac{\omega_{\theta Z1}}{\Omega}\right)_{\text{nominal}}$ for fixed values of the blade flapping natural frequency, $\left(\frac{\omega_{\beta}}{\Omega}\right)_{\text{nominal}}$.

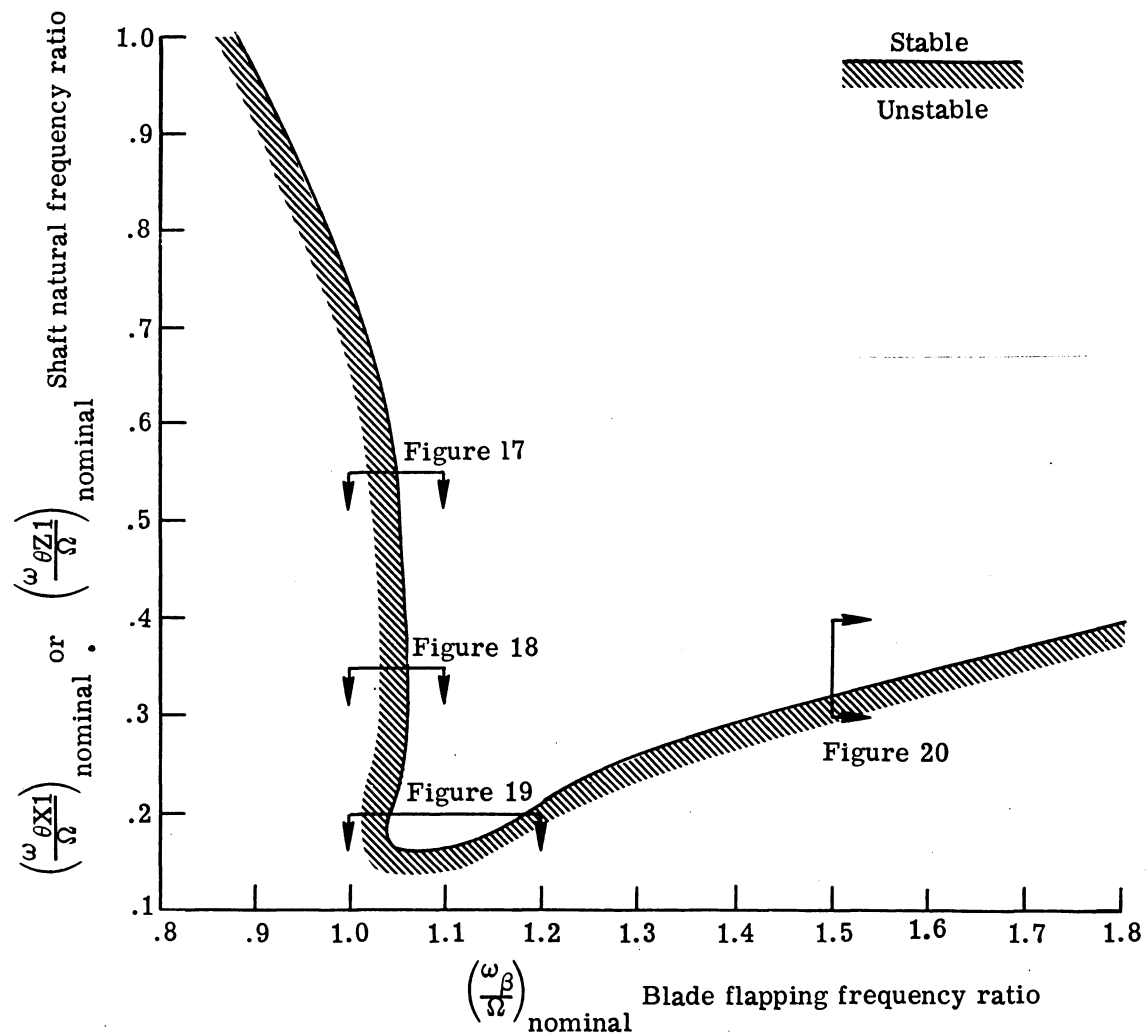


Figure 16.- Rotor stability boundary for the nominal rotor with variations in the rotor shaft and blade flapping natural frequency; $\left(\frac{\omega_{\theta X1}}{\Omega}\right) = \left(\frac{\omega_{\theta Z1}}{\Omega}\right)$.

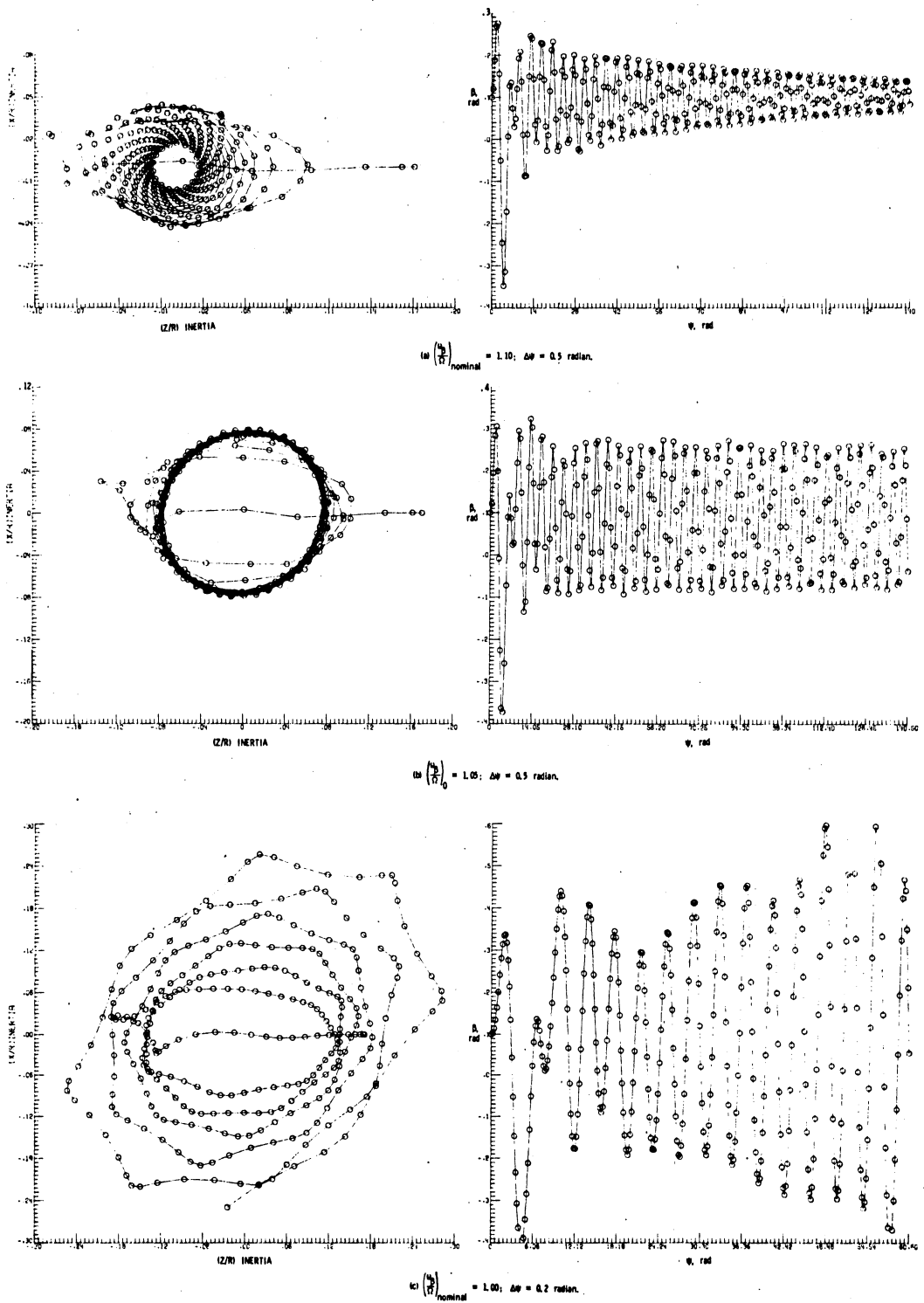


Figure 17.- Drive shaft and rotor blade motion across a stability boundary.

$$\left(\frac{\omega_{\theta x1}}{\Omega}\right)_{\text{nominal}} = \left(\frac{\omega_{\theta z1}}{\Omega}\right)_{\text{nominal}} = 0.55$$

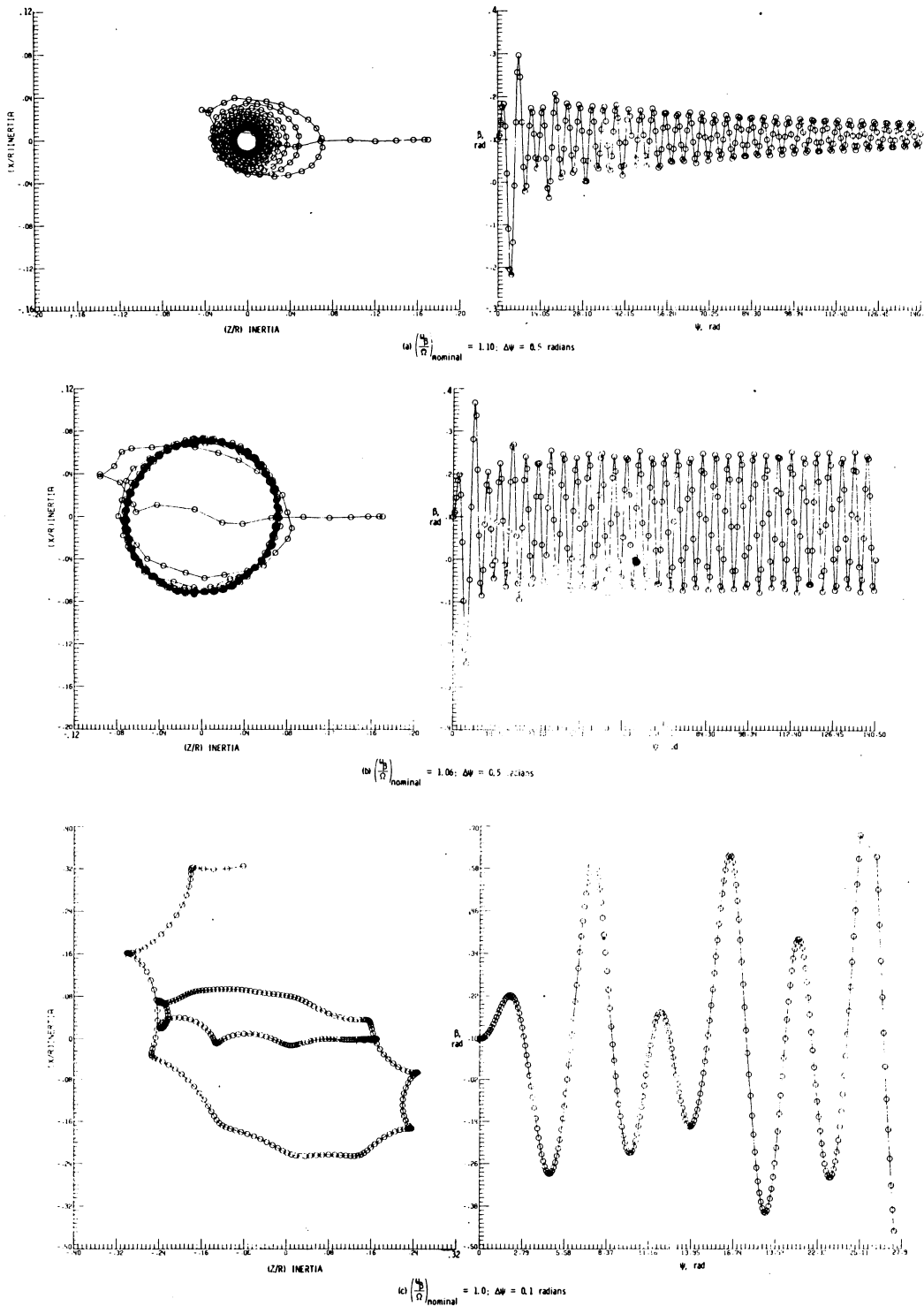


Figure 18.- Drive shaft and rotor blade motion across a stability boundary.

$$\left(\frac{\omega_{\theta X1}}{\Omega}\right)_{\text{nominal}} = \left(\frac{\omega_{\theta Z1}}{\Omega}\right)_{\text{nominal}} = 0.35$$

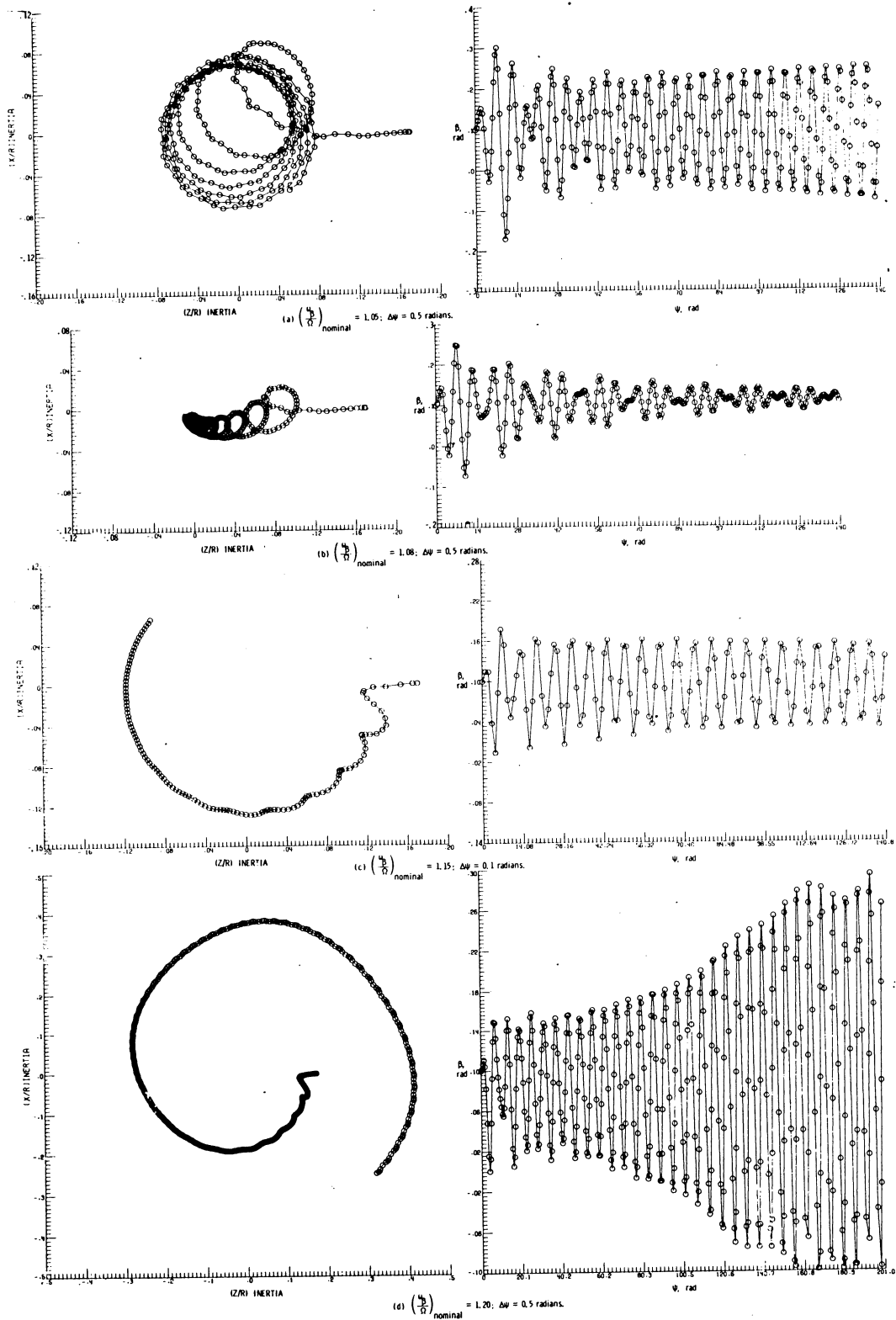


Figure 19.- Drive shaft and rotor blade motion across a stability boundary.

$$\left(\frac{\omega_{\theta x1}}{\Omega}\right)_{\text{nominal}} = \left(\frac{\omega_{\theta z1}}{\Omega}\right)_{\text{nominal}} = 0.20$$

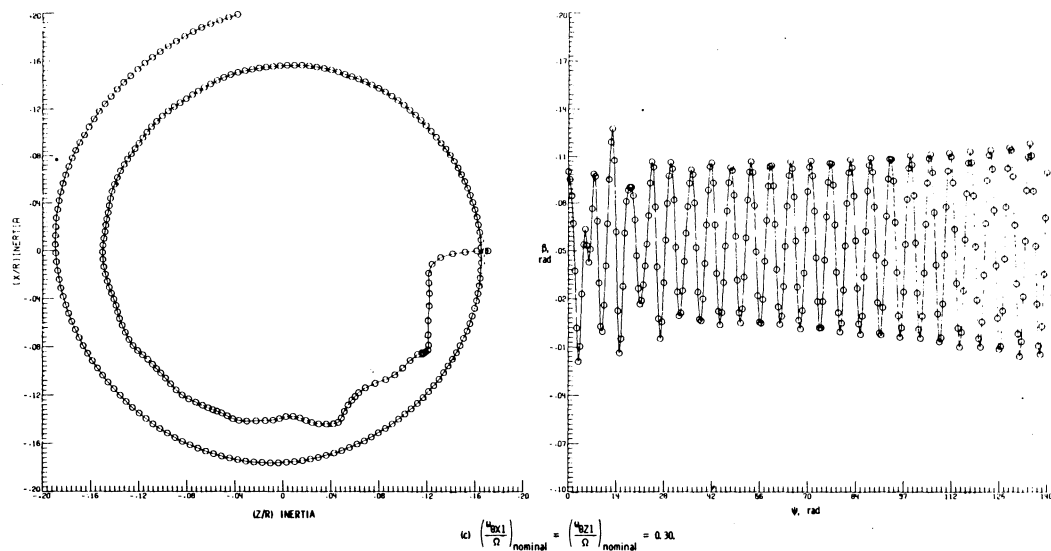
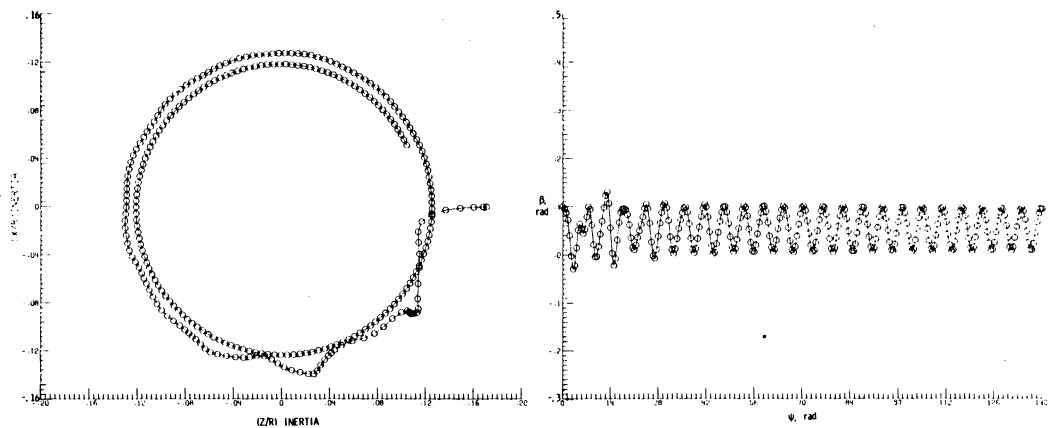
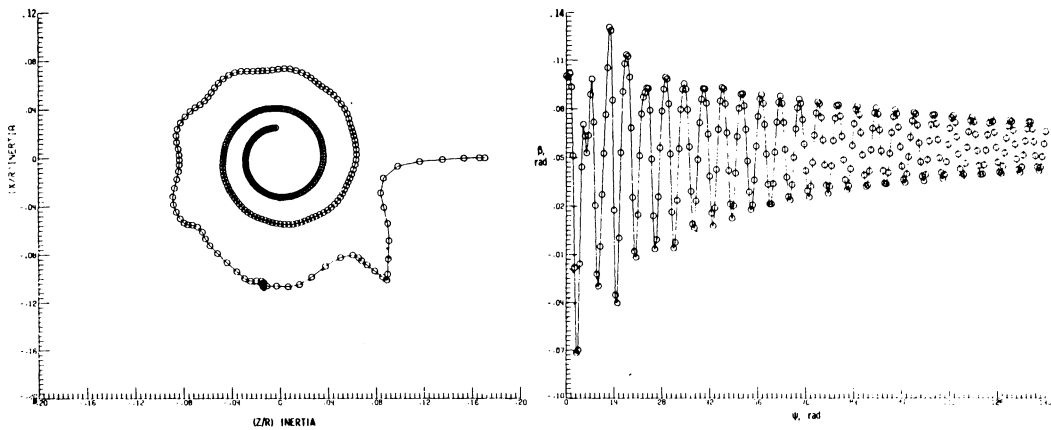


Figure 20.- Drive shaft and rotor blade motion across a stability boundary.

$$\left(\left(\frac{\omega_B}{\Omega}\right)_{\text{nominal}} = 1.5\right) \Delta\psi = 0.5 \text{ radian.}$$

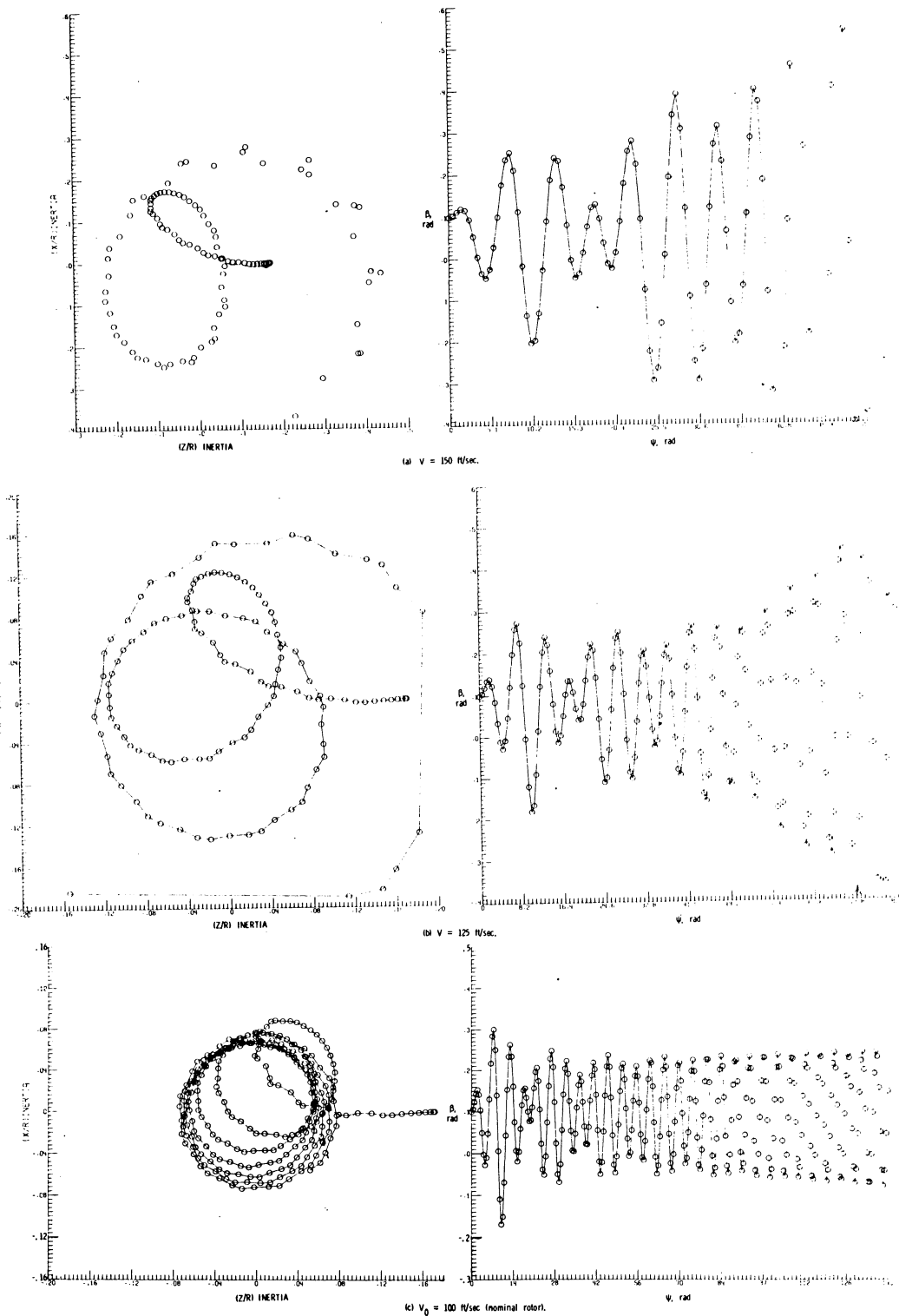


Figure 21.- The effect of the forward speed on the stability of the nominal rotor $\left(\left(\frac{\omega_{\theta X1}}{\Omega}\right)_0 = \left(\frac{\omega_{\theta Z1}}{\Omega}\right)_0 = 0.2; \left(\frac{\omega_{\beta}}{\Omega}\right)_0 = 1.05\right)$; $\Delta\psi = 0.5$ radian.

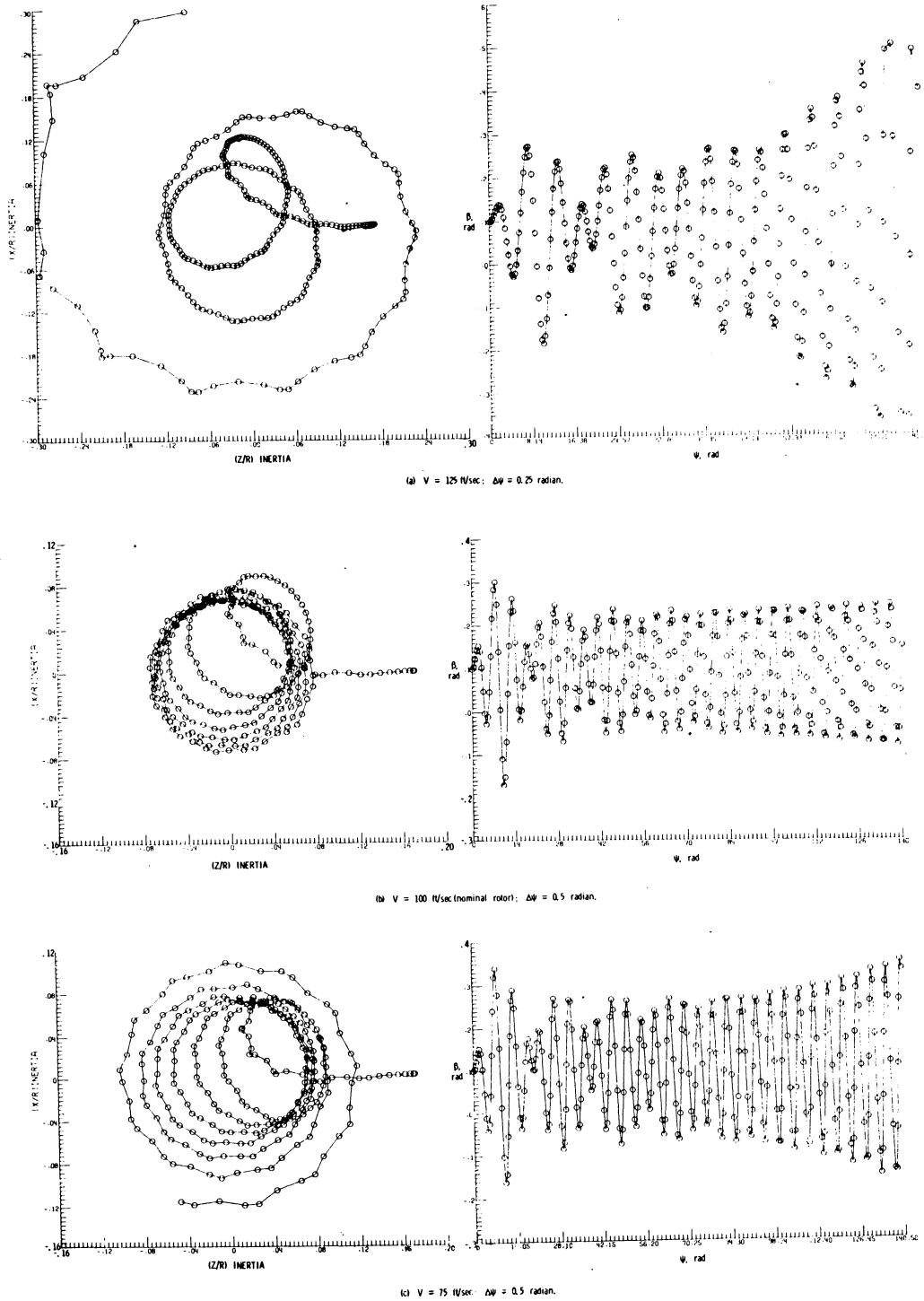


Figure 22.- The effect of the forward speed on the stability of the nominal rotor $\left(\left(\frac{\omega_{\theta x1}}{\Omega}\right)_0 = \left(\frac{\omega_{\theta z1}}{\Omega}\right)_0 = 0.2; \left(\frac{\omega_{\beta}}{\Omega}\right)_0 = 1.05\right)$.

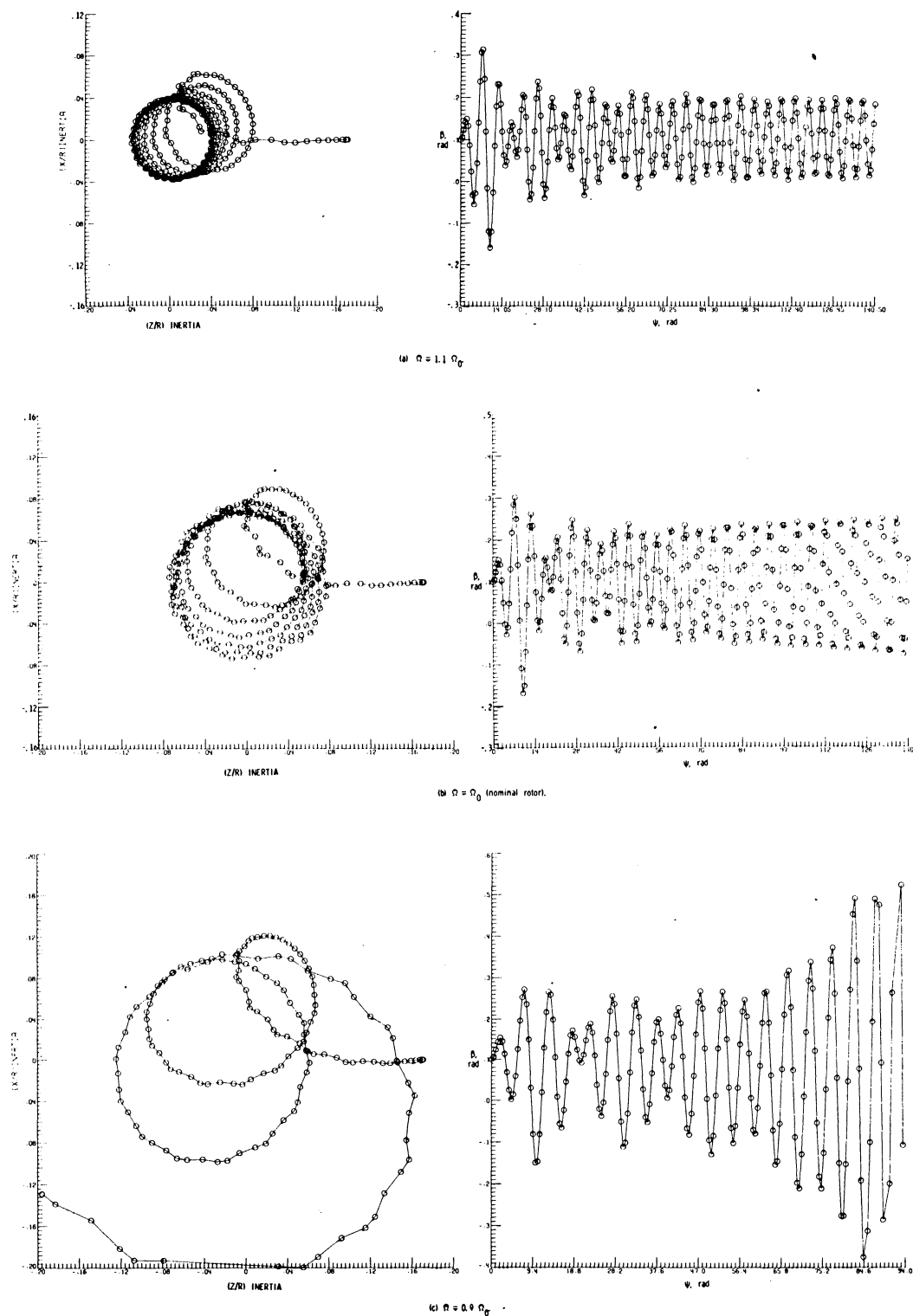


Figure 23.- The effect of the rotor rational speed on the stability of the nominal rotor $\left(\frac{\omega_3}{\Omega}\right)_0 = 1.05$. $\Delta\psi = 0.5$ radian.

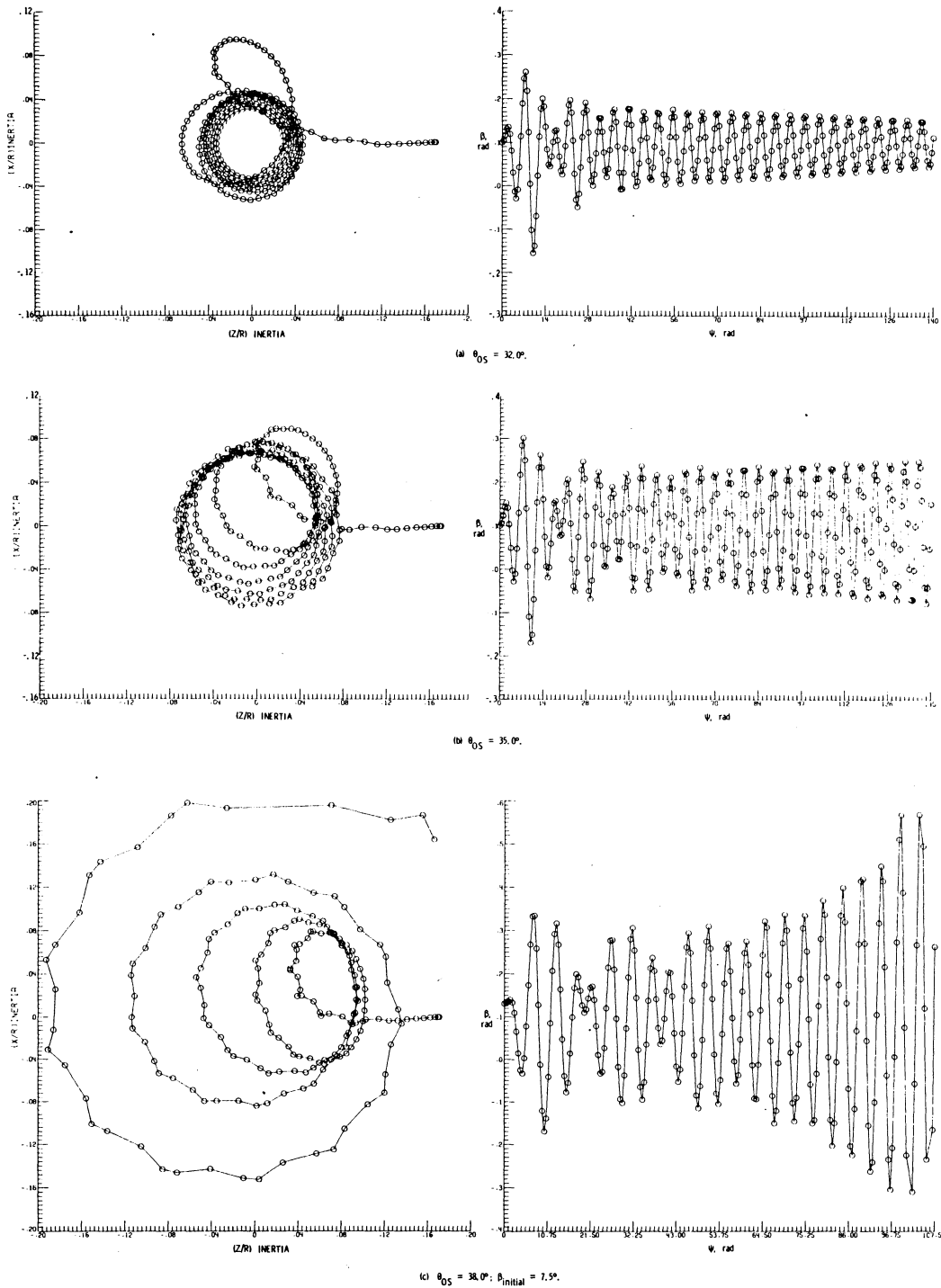


Figure 24.- The effect of the blade root pitch angle on the stability of the nominal rotor $\left(\left(\frac{\omega_{\beta}}{\Omega}\right)_0 = 1.05; \left(\frac{\omega_{\theta X1}}{\Omega}\right)_0 = \left(\frac{\omega_{\theta Z1}}{\Omega}\right)_0 = 0.2\right)$;
 $\Delta\psi = 0.5$ radian.

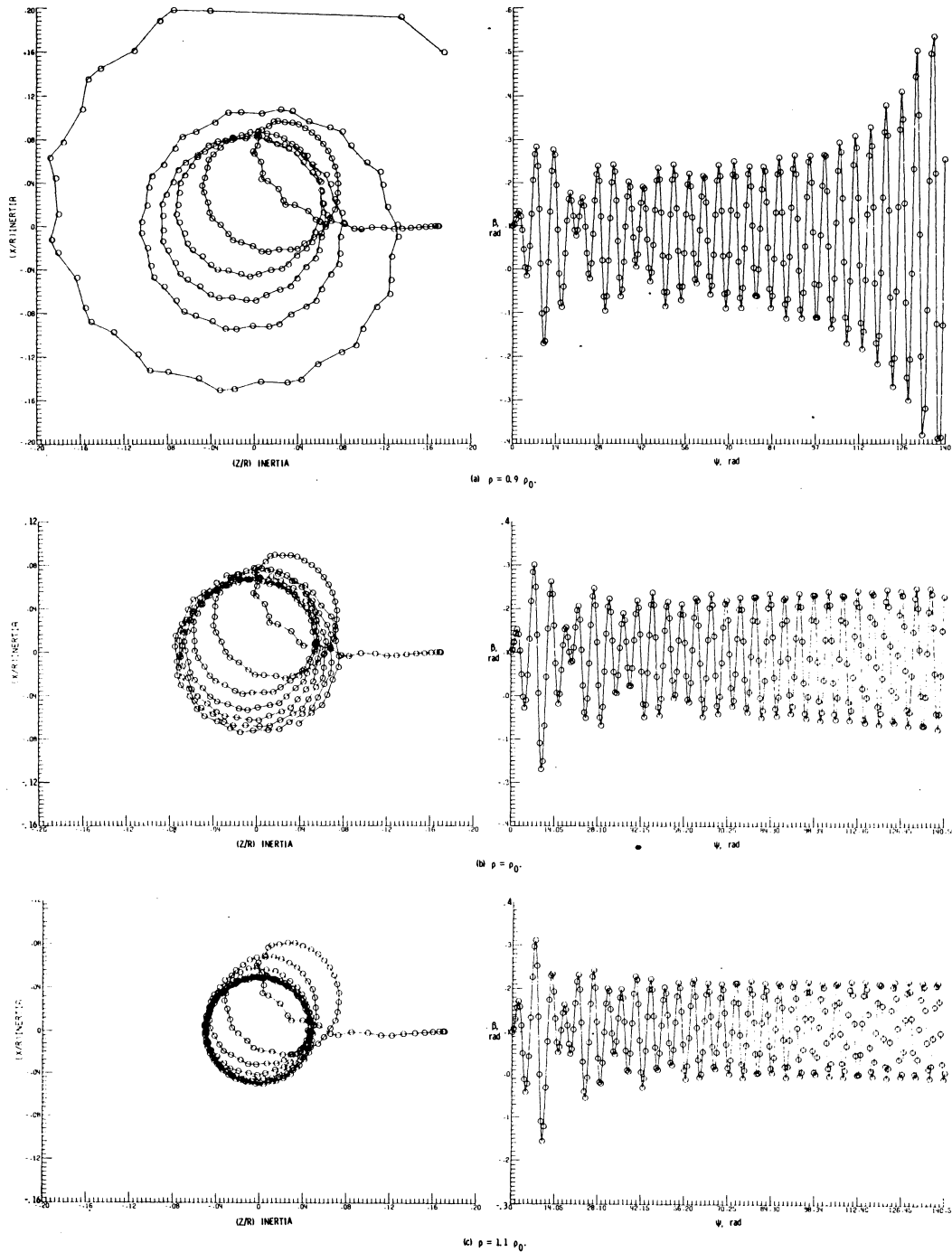


Figure 25.- The effect of the air density on the stability of the nominal rotor $\left(\left(\frac{\omega_\beta}{\Omega}\right)_0 = 1.05; \left(\frac{\omega_{\theta X1}}{\Omega}\right)_0 = \left(\frac{\omega_{\theta Z1}}{\Omega}\right)_0 = 0.2\right)$; $\Delta\psi = 0.5$ radian.

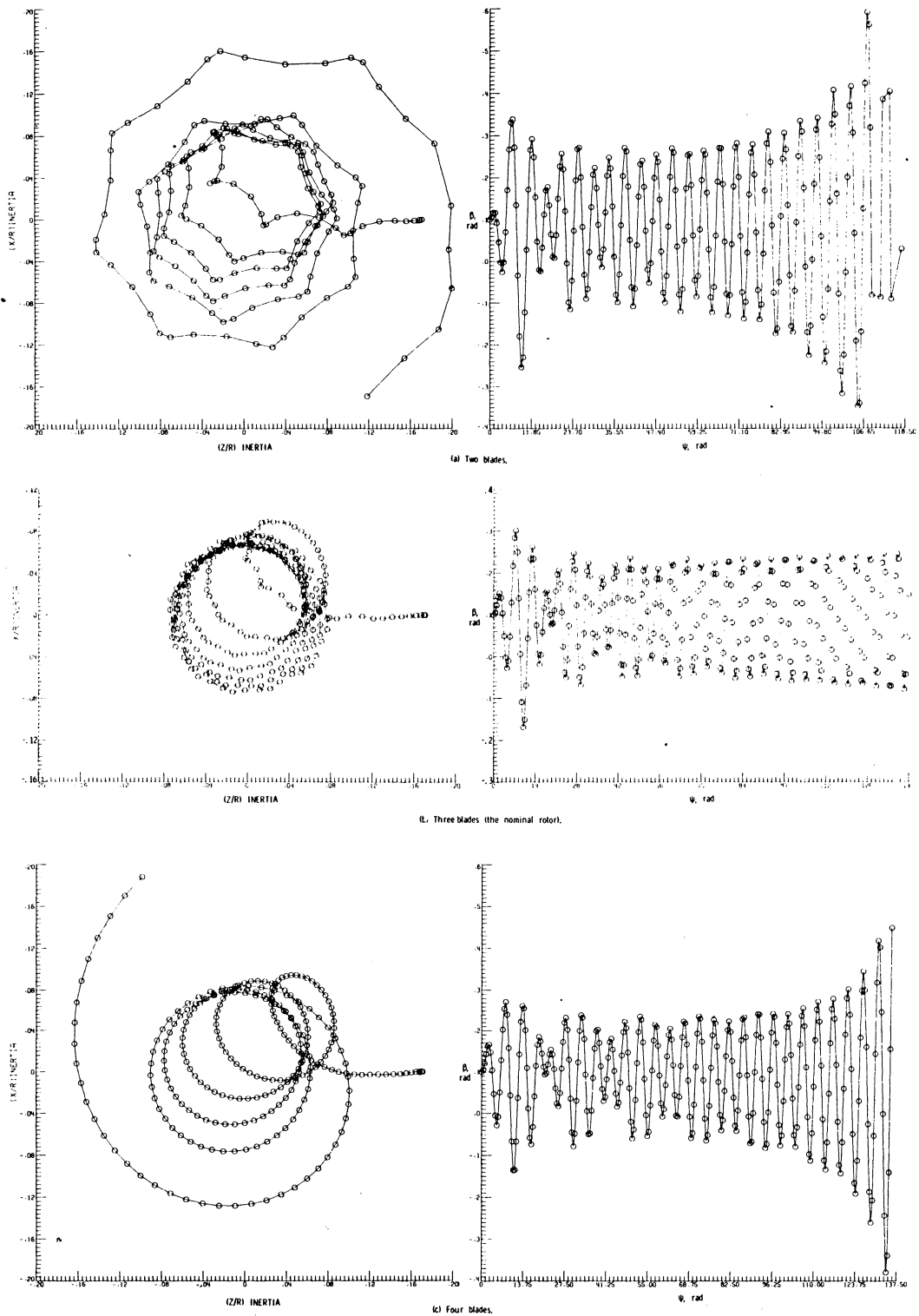


Figure 26.- The effect of the number of blades on the rotor system of the nominal rotor $\left(\frac{\omega_B}{\Omega}\right)_0 = 1.05$. $\Delta\psi = 0.5$ radian.

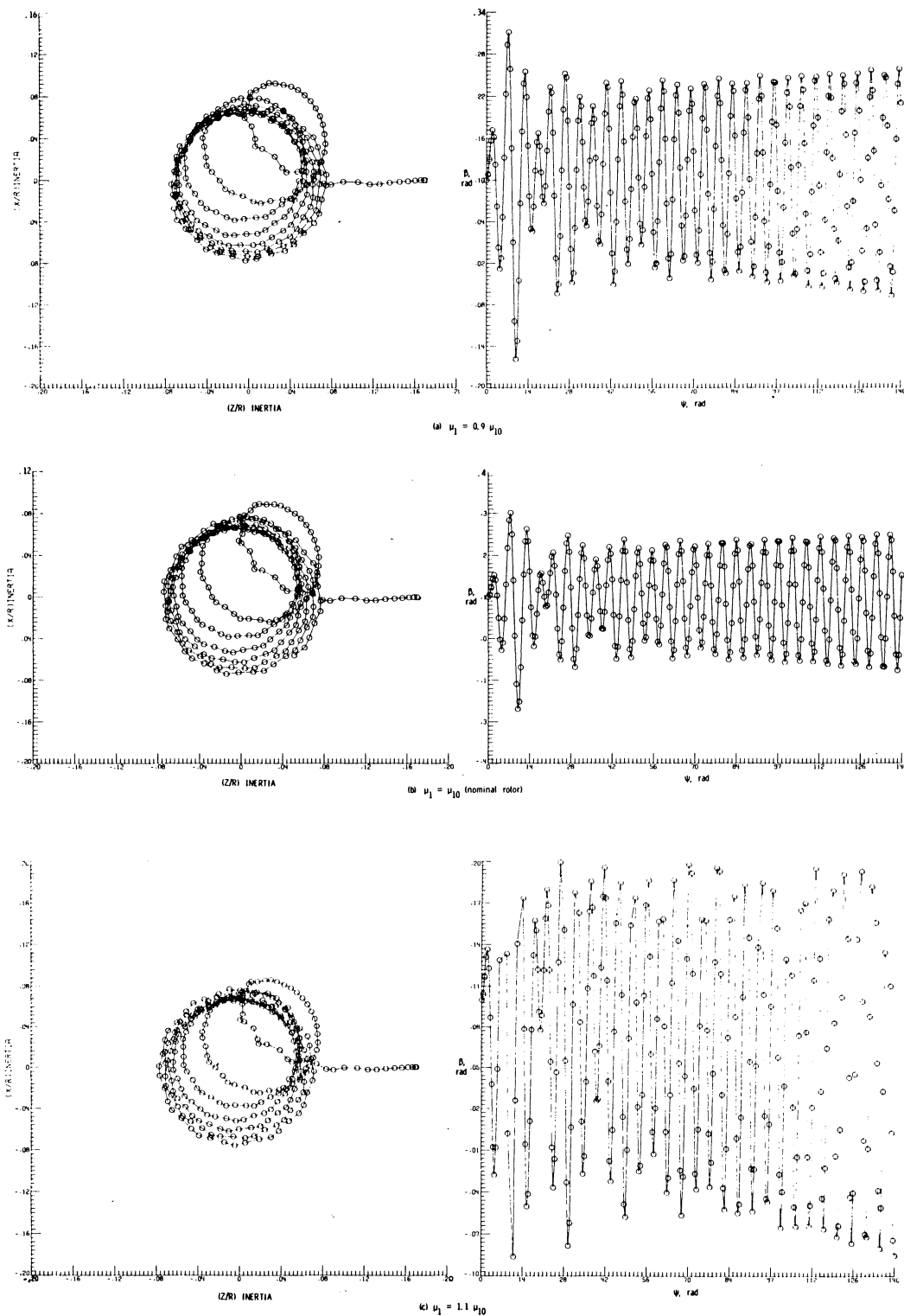


Figure 27.- The effect of rotor blade weight on the stability of the nominal rotor $\Delta\psi = 0.5$ radian.

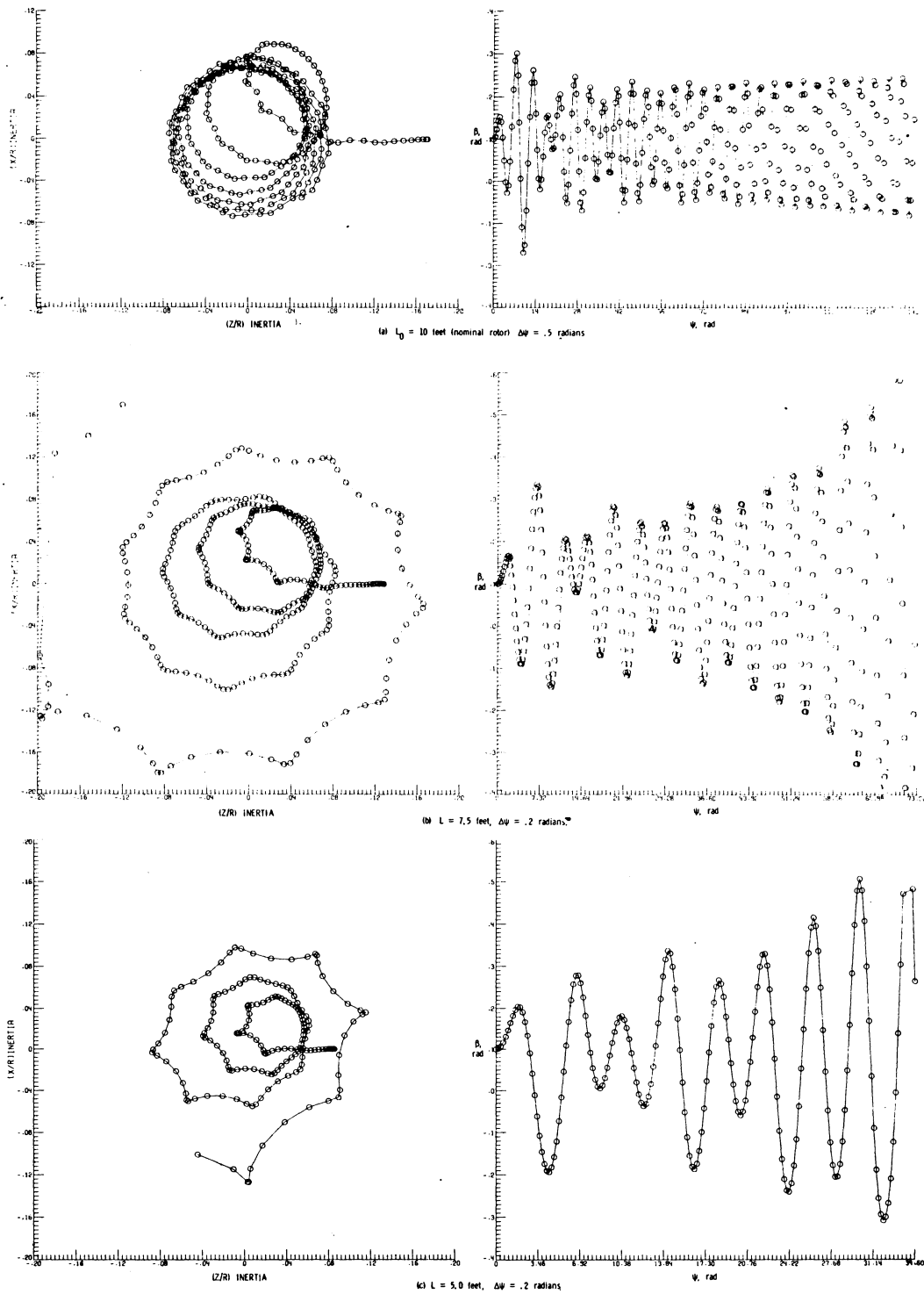


Figure 28.- The effect of the drive shaft length on stability of the nominal rotor $\left(\frac{\omega_B}{\Omega}\right)_0 = 1.05$.

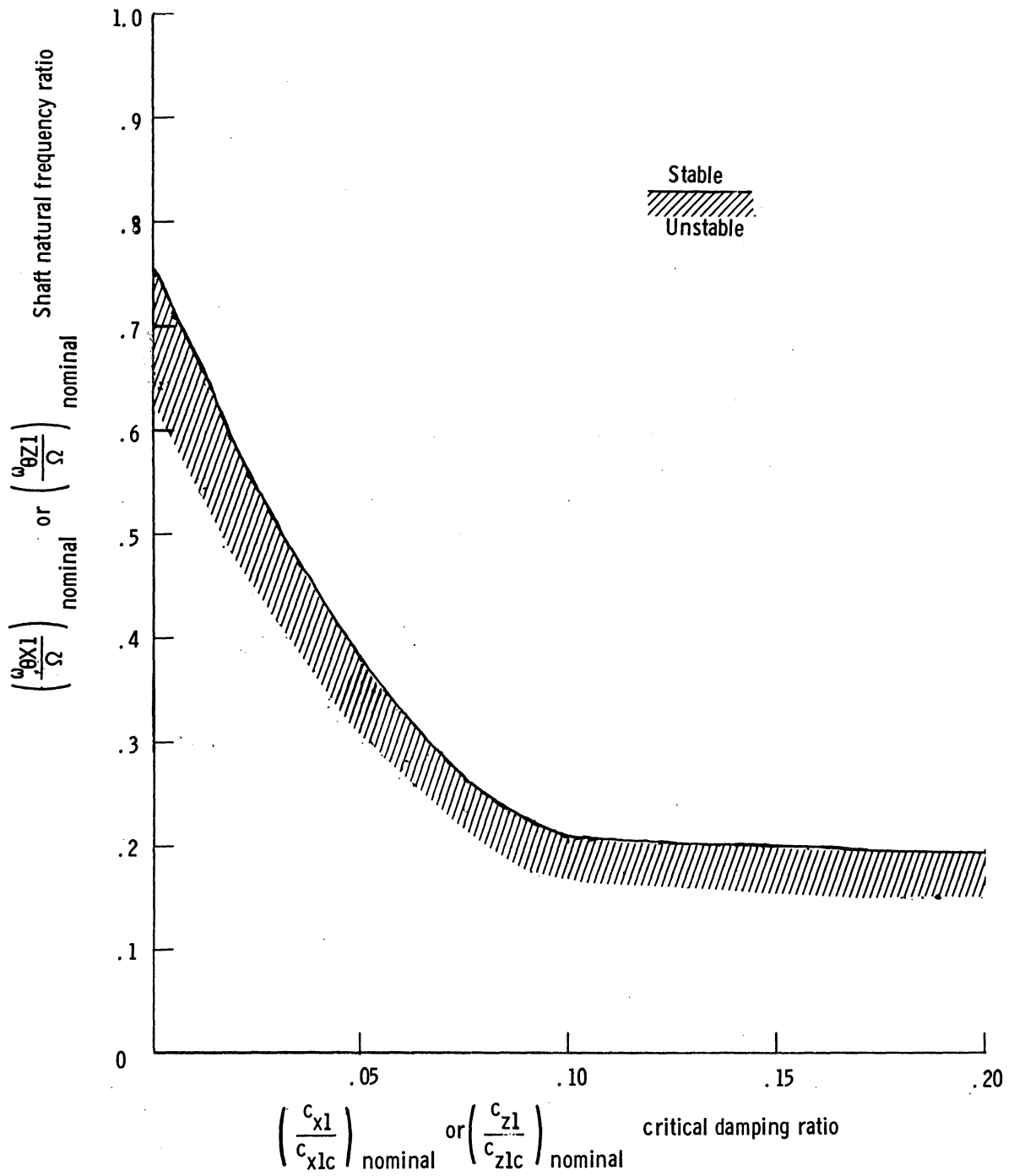


Figure 29.- Effect of shaft damping on nominal shaft stiffness requirements.

ANALYTICAL INVESTIGATION OF THE TILT
ROTOR WHIRL INSTABILITY

By

By James Scheiman

ABSTRACT

This dissertation presents a generalized analytical study of the whirl instability of a rotating propeller or rotor system. The mathematical model consists of rigid members with three degrees-of-freedom at the wing root and the wing-drive-shaft joints. In addition, each blade has lagging and flapping freedom. Two dimensional aerodynamics, including compressibility and stall effects, are included.

Whirl stability boundaries and characteristics are presented. The parameters investigated include both mechanical and operating variations such as shaft stiffness, number of blades and air density. It was found that decreasing forward speed can result in a whirl instability when blade stall is taken into account. No condition was found in which the rotor went into a forward precessional mode of motion.