

Chapter 1

Introduction and Background

1.1 Motivation

Fluid shear layers play a very important role in the performance of the combustion flow train components of air breathing propulsion devices. Shear layers are responsible for mixing oxidizer with fuel, reactants with hot combustion products and hot combustion products with cooling and dilution flows.

The functions performed by the shear layers are in large part made possible by their dynamics which produce coherent structures of various scales and lifetimes. Depending on the role of a particular shear layer both large and small scale mixing may be desired. Currently, shear layer behaviour in the primary combustor is not optimized for the desired effects, and generally, the dynamics will inevitably also affect performance detrimentally. In order to minimize detrimental effects and maximize performance enhancing dynamics, the shear layer dynamics must be first predicted and then controlled using passive geometrical design changes, semi-active operating point based control or fully active control.

The research described herein deals with the application of linear stability analysis to isothermal turbulent free shear layer flows and represents a stepping stone to the even more complex scenario of reacting turbulent free shear layer flows. Still, it is expected that the results from the present study will provide useful knowledge for applications of flow control downstream of the flame in the primary combustor to improve mixing and decrease the pattern factor at the exit of the combustor.

1.2 Background

1.2.1 Applied combustion systems research

Few studies performed using full-scale combustors have been published. Several full-size combustor configurations have been studied in the last decade, examining the dependence of the pattern factor on secondary flows and other parameters. The Allied Signal F109 combustor was the subject of a study by Crocker et al. (1994). They studied the optimal strength and direction for secondary mixing jets installed

between the combustor exit and the first turbine stage to lower the pattern factor at the turbine inlet. They found that the secondary mixing jets needed to be oriented against the direction of residual swirl and that most of the momentum of the jets was converted to turbulent kinetic energy. Mixing enhancement was thus achieved at the expense of increased turbulent kinetic energy and therefore enhanced heat-transfer to turbine blades, as well as increased pressure drop through the combustor. The powerful dilution jets were oriented so as to maximize the shear with the primary flow-field of the combustor. The shear layer structures thus generated provide good large scale mixing at the expense of a larger residual turbulence energy.

Another study dealing with full-size combustor flows and their exit characteristics is the study by Goebel et al. (1993). The study used various configurations of bypass, cooling, dilution and nozzle flows to examine the influence of each on the turbulence characteristics downstream of the combustor exit. The study concluded that the dilution jets served to remove residual swirl and thus removed a major source of shear-induced turbulence production. The dilution jets were observed to actually lower the amount of turbulence downstream of the combustor exit.

Hassa et al. (2002) studied the isothermal flow field in a combustor sector model of a MTU AeroEngines (Engine3E program) gas turbine. The results showed that for the particular arrangement of dilution jets used in this prototype combustor, the primary zone flow and dilution jet flow were very closely coupled. The dilution jets clearly affected the recirculation zone of the isothermal flow and the recirculation zone itself caused the jets to deflect upstream from the expected downstream deflection. Interaction or lack thereof between primary zone and dilution flows may explain the large differences in the results obtained in the studies above.

The contrasting results of Goebel et al. (1993) and Crocker et al. (1994) illustrate the wide variety of results that can be obtained depending on the specific configuration used. At the same time, combustor technology has not changed significantly in gas turbines for propulsion in the last 15 to 20 years, and even the flow configuration for modern ultra-lean premixed systems in gas turbines for power generation are closely related to their predecessor designs. Yet still, relatively small differences in combustor

geometry and flow arrangement yield entirely different behaviour.

The key to achieving future combustor performance gains and decreasing design cycle time is therefore a fundamental understanding leading to control of the underlying flow processes that cause the seemingly unrelated behaviour observed in different full-scale and research configurations.

1.2.2 Flow characteristics in gas turbine combustors

Most combustor configurations utilize the excellent mixing and flame stabilization characteristics of swirling flow in the design of the primary combustor flow. The mixing characteristics of swirling flow stem in part from the inherent turbulence producing shear present in this type of flow. The presence of swirl assists flame stabilization through the enhancement of recirculation zones present behind bluff bodies (Biswas et al., 1997) or otherwise in the generation of recirculation zones through vortex breakdown (Sattelmayer et al., 1990).

Flame stabilization with similar characteristics can also be achieved using bluff bodies in the absence of swirl. Bluff body flame stabilization is achieved through the recirculation zone that is formed on the downstream side of the inserted bluff body. Bluff body flame stabilization also involves complex fluid dynamics which promote rapid mixing and reaction of the incoming fuel and oxidizer.

Whether continuous combustion is stabilized by swirl, swirl assisted by a bluff body or a bluff body alone, shear layers play an essential role. Compared to the relatively low amplification rates found in wall-bounded shear flow modes (pipe-flow for example) free shear layer modes exhibit very large growth rates, giving rise to the large coherent structures that promote large scale and small scale mixing necessary for continuous high rate combustion.

For combustion stabilized by swirl alone, the unstable swirling jet breaks down through an instability referred to as vortex breakdown and forms the recirculation zone, shown schematically in Figure 1.1a. The recirculation zone brings hot combustion products in contact with fresh mixture providing a source of active chemical radicals and thermal energy to continuously initiate the combustion of the incoming

reactants mixture. The location of the combustion zone is not fixed for this case but may be more stationary by subjecting the flow to a sudden expansion, triggering the vortex breakdown. However, the presence of the sudden expansion adds a free shear layer that affects combustor performance and generates its own recirculation zone. An example of the interaction of the dynamic behaviors of the swirling jet with the sudden expansion is described in Paschereit et al. (1999) and Paschereit and Gutmark (1998). Those papers show that depending on the flow conditions either an axisymmetric instability related to the sudden expansion or an asymmetric instability related to vortex breakdown was observed. In the case where the natural swirl recirculation zone is enhanced by the presence of a bluff body the central swirling vortex core is replaced by yet another shear layer on the inside of the flow, shown in Figure 1.1b. Even without swirl, and using only a bluff body along with a sudden expansion (Figure 1.1c) there are still two unstable free shear layers that interact and make continuous combustion possible. The importance of the interaction of the shear layers mentioned in each of the three configurations goes beyond providing an environment suitable for rapid continuous combustion. The combined shear layer interaction and associated dynamics also determine the structure of the downstream flow field and turbulence levels, flame stability characteristics and hence also the cooling and dilution flow requirements.

Shear layer dynamics also play an important role in the performance of one of the most recently developed combustor concepts, the trapped vortex combustor as presented by for example Hsu et al. (1998). In order to trap the vortex in the annular groove in isothermal flow, the shear layer between the bypass flow and the cavity flow must extend from the upstream edge and impinge on the downstream end. To enable trapped vortex combustion, fuel and air are injected into the cavity in such a way as to strengthen the vortex. The shear layer between the annular flow and the cavity flow no longer impinges on the downstream end of the cavity but due to the strengthening of the vortex through injection, vortex stability is maintained. The dynamics of the shear layer between the bypass flow and the cavity flow still play a crucial role in the performance of the combustor (Katta and Roquemore, 1998).

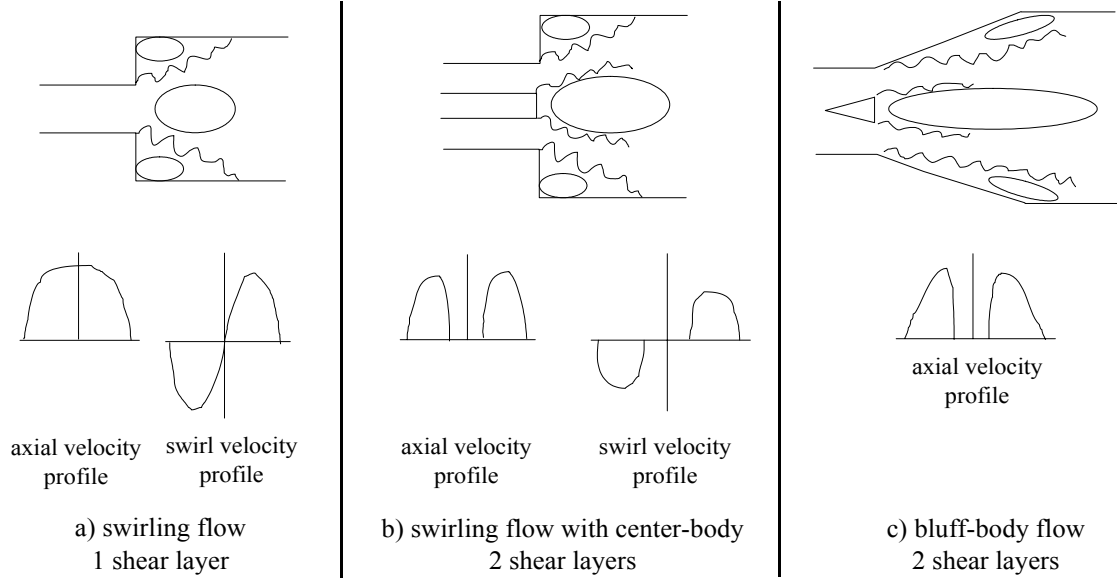


Figure 1.1: Schematic of shear layers found in combustors

1.2.3 Flow Control Opportunities

In order to take full advantage of all the optimization and control opportunities such complex flows offer for the further improvement in combustor performance, shear layer dynamics must be quantified and able to be predicted reliably with and without combustion. Although combustion modifies the flow-field significantly, at the base of the shear layer, where the dynamics and mode structure of the shear layer instabilities are largely determined, the flow does not fundamentally change under the influence of combustion. The results shown by Paschereit et al. (1999) bear this out as the instabilities found during combustion were reproduced in a water tunnel experiment. The instability frequency scaled with Strouhal number and as in the combustion experiment, two different shear layer instability modes were observed to dominate depending on the flow-rate. Similar behavior was observed by Stone and Menon (2001) who studied the dynamic behaviour of a swirl combustor using a LES computational model. Increasing swirl was found to discourage axisymmetric vortex shedding modes. Flow instabilities are not confined to single nozzle combustor flow

studies. Hassa et al. (2002) reported instabilities in their sector model of a MTU Aeroengines annular combustor.

The complexity of a multiple free shear layer flow does not merely present a challenging research subject, it offers possibilities for a variety of control strategies with reasonable control cost. The existence of a number of modes in free shear layers offers the ability for a minimal geometry alteration or amount of forcing to effect large changes in the flow field characteristics through the sensitivity of the shear layer dynamics to the flow profile characteristics and the natural amplification the shear layer modes provide.

There have been many flow control studies that have tried to take advantage of the above mentioned amplification mechanisms inherent in shear layer dynamics. Temporal excitation using acoustic drivers and speakers is most common (Panda and McLaughlin, 1994; Collier, 2000) but piezo-electric actuators (Vandsburger and Ding, 1995; Parekh et al., 1996) as well as other mechanical excitation techniques (Vandsburger and Ding, 1993; Lee and Taghavi, 1996) have been studied. Synthetic jets comprise a large sub-class of actuators in shear layer control. These actuators are small oscillatory sources of momentum that have no net fluid mass contribution. The capabilities of these actuators have been studied extensively by Smith and Glezer (1998) and Rediniotis et al. (1999) among others. In most experiments the goal of flow excitation is the increase of mixing between fluid streams separated by the shear layer. Vandsburger and Ding (1993) report a doubling of the shear layer spreading rate for excitation with a wire tuned to a natural frequency of 200 Hz using its tension. Shear layer spreading and the associated mixing of fluid is also used by Collier (2000) in experiments and modeling attempting to increase the pressure recovery in a separating diffuser flow.

Spatial mode excitation has been used by a number of researchers and is an interesting extension of simple temporal excitation of shear layers. Spatial mode excitation uses spatially distributed excitation of the shear layer to increase the effect of excitation. Cohen and Wygnanski (1987a) used circumferentially arranged acoustic actuators around a circular jet to promote the development of targeted azimuthal

modes of instability. The superposition of several azimuthal modes of excitation was also studied. The study showed that under judicious choice of the azimuthal modes used for excitation, the jet could be distorted into non-circular shapes in the near field. Cohen and Wygnanski (1987a) were able to interpret the results using linear stability analysis along with some elementary concepts in non-linear interaction. The results of Cohen and Wygnanski (1987a) were extended by Vandsburger and Ding (1995) for triangular jets. Vandsburger and Ding (1995) used a piezo-ceramic amplified brass shim to excite the shear layers at the exit of the triangular jet. The results showed that far-field flow modulation could be achieved using counter-propagating azimuthal waves. Far-field flow modulation could not be achieved using single azimuthal mode excitation.

Swirling flow excitation has been less successful. Panda and McLaughlin (1994) observed no coherent structures and predominant frequencies in the velocity spectra of the natural (unexcited) swirling flow. Several types of coherent structures could be identified by exciting different azimuthal modes. However, the excitation did not yield significant changes in the flow-field as may have been expected given the large excitation amplitudes used. The results of Panda and McLaughlin (1994) cannot be considered typical because significant coherent structures have been observed in combustion environments (Paschereit and Gutmark, 1998; Froud et al., 1995; Gouldin et al., 1984) and isothermal environments (Paschereit et al., 1999; Garg and Leibovich, 1979; Gouldin et al., 1984) including the basic vortex breakdown studies of for example Cassidy and Falvey (1970)(see Section 1.3). Still, it is clear that excitation in swirling shear layers must be implemented differently from excitation in non-swirling flows and that swirling flow characteristics are geometry dependent. The variety of results found for swirling flow excitation are revisited in Section 1.4.1 and interpreted in terms of flow stability characteristics.

All of the flow control studies discussed above employ open-loop control, where some predetermined control signal is applied to the flow and the resulting influence on the flow is measured. Feedback control complexity ranges from simple phase shift type controllers to model based and adaptive controllers. For phase shift controllers,

the control signal is proportional to the measured output, but shifted by a certain phase which is determined by trial and error to achieve the optimum result. Model based controllers allow all the tools of optimal control to be applied to help guarantee that the optimal result can be achieved. Feedback control has been applied to the problems of external flows with some success by a number of researchers, for example Caruana et al. (2001). Feedback control in the wakes of cylinders has been studied experimentally using simple phase-shift control algorithms by Tao et al. (1996) among others. Model based feedback control is much more attractive in terms of optimization but is often difficult in complex systems. Gillies (1998) analytically designed a low order model based on proper orthogonal decomposition for the cylinder wake problem and implemented nonlinear neural network control of the wake. Gillies (1998) also showed that erroneous conclusions can be drawn from single sensor observations in a feedback control system designed to eliminate the wake behind a cylinder. Collier (2000) demonstrated modeling of the dynamic system underlying the flow in a diffuser and was able to reproduce some of the global effects of open-loop control experiments using synthetic jets. The model however still required 700 computational elements, making the model too large to implement in real-time model based feedback control. Noack et al. (2000) were able to prove controllability for the simple configuration of a vortex in a corner subject to potential flow. Noack et al. (2000) demonstrated the power of achieving controllability by formulating a measure of mixing and optimizing control to achieve maximum mixing.

Another difficulty in flow control is the problem of quantifying the control input beyond the basic actuation amplitude. It is very difficult to compare acoustic excitation to piezo-mechanical or synthetic jet actuation except on qualitative terms because the actual control input is not measured. The essential principal of all flow control actuators seems to be the modulation of some amount of vorticity at the boundary of the flow. In piezo-mechanical actuation the vorticity generation happens at the end of a moving flap. For synthetic jets, the vorticity is generated at the lip of the orifice. In plain acoustic actuation, the main flow control input also occurs by a conversion of acoustic energy to vorticity (Crighton, 1981). The problem with

quantifying these vorticity inputs is that the location of conversion from mechanical or acoustic energy to vorticity is very small and difficult to measure. A possibility for the quantification of forcing in the case of acoustic excitation is offered by the interaction between flow and acoustics at sharp boundaries. In studies aimed at quantifying and understanding jet noise several researchers found significant dissipation of acoustic power at the exit of a jet Bechert et al. (1977); Cargill (1982). They found acoustic energy dissipation greater than 15 dB for low Mach number, low frequency excitation, whereas for higher frequencies, acoustic energy was conserved. In effect, the jet acted as a low frequency muffler.

Multiple models have been derived that to varying degrees of accuracy and complexity, all quantitatively capture the attenuation effects (Munt, 1990; Moore, 1977; Cargill, 1982; Bechert, 1980). Among these, the model by Bechert (1980) is the simplest, specifically aimed at capturing the low frequency attenuation observed in his experiments (Bechert et al., 1977). The common denominator for all of these models is the reliance on the explicit or implicit application of a Kutta condition (Crighton, 1985) at the jet exit. The Kutta condition is used to remove the singularity in the flow field that would be encountered in most shear layer type expansions of the flow near the nozzle lip. The Kutta condition is indirectly responsible for the conversion of acoustic energy to vorticity. The vorticity generated then couples into the base of the shear layer and excites instabilities. These instabilities however are not generally effective sound sources Crighton (1981), causing an attenuation of the transmitted sound. In terms of flow actuation however, the above results indicate that measuring acoustic power transmission properties allows the quantification of forcing in the particular flow under study. Furthermore, the results show that the forcing is very localized and limited to the very base of the shear layer where there is an effective boundary layer discontinuity. It should be noted that the sound radiating properties of flows can be expected to change significantly under the addition of swirl, as demonstrated in a simple model by Howe and Liu (1977).

Although the flow control studies described above have demonstrated the ability to affect shear layer development significantly using various excitation techniques,

there has not been to date a significant introduction of flow control in actual devices. The demonstrated ability to affect shear layer development will enable the type of flow control necessary to improve real device performance. The technology developed in these basic flow control studies must be transitioned to flow fields found in real applications. The research performed for this dissertation represents a first step in such a transition by examining linear stability analysis methods for the identification of flow control opportunities in complex turbulent free shear flows.

1.2.4 Predictive tools for flow design

The reliable prediction of primary combustor flows is still difficult even without considering combustion or secondary flows. Chen and Lin (1999) found that strongly swirling flows, as used in gas turbine combustors, require non-linear pressure-strain modeling to obtain good accuracy in prediction in addition to requiring a full Reynolds stress turbulence closure. The often used $k - \epsilon$ turbulence closure seems to be inadequate for swirling flows even when used with Richardson number type corrections (Leschziner and Rodi, 1984; Armfield and Fletcher, 1989). Added to these difficulties in the prediction of swirling flows is the fact that the flow, although often modeled as axisymmetric, actually has significant three dimensional and unsteady features such as the precessing vortex core (Gupta et al., 1984; Froud et al., 1995). These three dimensional and unsteady features have been found to play a prominent role in combustion instabilities (Paschereit et al., 1999). Few studies have been performed solving the full three dimensional Navier-Stokes equations. The calculations by Biswas et al. (1997) used a swirl modified $k - \epsilon$ turbulence model and showed reasonable agreement with selected experimental results even with combustion. The experimental data used for comparison however seems too coarse to allow a conclusive evaluation of the model used.

Computational prediction abilities in non-swirling bluff body flows are superior to those in swirling flows, although modeling in these flows still cannot be considered fully reliable, especially when turbulent combustion is considered. Finally, the computational expense in calculating even bluff-body flows is still considerable.

The prediction of complex three dimensional turbulent flows will benefit from further advancements in large eddy simulation (LES). LES is attractive because modeling of turbulence is essentially restricted to the dissipative scales whose interaction with other scales is more fully understood and therefore more accurately modeled. The required grid resolution for high Reynolds number LES calculations is however still prohibitive in the computational effort required. Stone and Menon (2001) reported requiring two weeks of computational time on a PC cluster calculating one case of a premixed swirl combustor operating at a Reynolds number of 500,000. The study did not incorporate finite chemical rate effects which would have further increased computational expense. In view of the difficulties involved in the prediction of complex separated shear flows there is a need for simple computational tools that allow the prediction of selected important aspects of these separated flows. Among these the most promising tools appear to be linear stability analysis and discrete vortex methods.

Discrete vortex methods offer an attractive alternative to complete modeling of the complex flows encountered in propulsion devices. Discrete vortex methods are based on the principle that in inviscid two-dimensional flows, vorticity is neither produced nor destroyed. Saffman and Schatzman (1982) studied the von Karman vortex street using a discrete finite size inviscid vortex model. Vortex strength and spacing was determined by global energy and momentum conservation arguments. The convergence of vortex methods for two dimensional inviscid flows was proven by Hald (1979) and for three dimensional inviscid flows by Beale and Majda (1982). The application of vortex methods in three dimensional flows is complicated by the fact that vorticity altering processes such as vortex stretching exist and must be modeled. A discrete vortex method was used by Coller (2000) to model the ability of synthetic jet control to increase pressure recovery in a diffuser. The main problems encountered in the development of the model was the correct description of the vorticity source at the base of the separating shear layer and the modeling of the dissipation of vortices near the diffuser wall. The simulation tracked about 700 vortices to describe the flow-field dynamics adequately. Although discrete vortex methods are most suited to

model inviscid flows, adaptations of the method to account for vorticity generation near walls has been accomplished by algorithms such as the random vortex method used by Gharakhani and Ghoniem (1996) in the calculation of the flow field inside a combustion chamber with a moving piston.

The computational expense for these methods is however once again higher and there are significant difficulties in handling circular wall geometries along with the previously mentioned difficulties of vorticity generation in a free shear layer. The importance of discrete vortex methods should not be underestimated, but underlying the predictive abilities of these methods is a knowledge of the vorticity sources which are very complex in nature for separated turbulent flows. A description of these sources and further information on the complete modal structure of the shear layer dynamics can be derived from linear stability analysis as described by for example Wygnanski and Petersen (1987) and Michalke (1965) (See Section 1.4.

1.3 Experimental Studies in Swirling Flows

Experimental studies in swirling flows have been quite numerous. The swirling flow studies range from the basic vortex breakdown studies in laminar flow of Harvey (1962) to the full combustor model experiments of Paschereit et al. (1999). With only very few exceptions these studies do not report spectral data and these types of measurements are of central importance for the research effort reported on in this thesis. The following paragraphs represent an overview of the various studies and their results. More detailed aspects of the studies are discussed with respect to the experimental results of the present study in Chapter 4 and Chapter 5.

One of the landmark studies in turbulent vortex breakdown is the study by Cassidy and Falvey (1970). Starting from the axial momentum equation in cylindrical coordinates the authors derive the non-dimensional parameters important for swirling flows. Among the parameters found are the Reynolds number, a form of the Strouhal number (fD^3/Q), a non-dimensional pressure drop and the swirl number. The swirl number as derived by Cassidy and Falvey (1970) is given in Equation 1.1.

$$S_{falvey} = \frac{D \int \rho u w dS}{\rho Q^2} \quad (1.1)$$

The experimental facility used incorporated a radial inflow swirler without center-body and various lengths and diameters of pipe attached to the exit of the swirler. Frequency was extracted from both wall pressure measurements and wall hot film anemometry measurements. The results showed that for each swirl number, a Reynolds number exists above which the frequency parameter is found to remain constant for further changes in Reynolds number. Below this Reynolds number, the frequency parameter decreases. This shows that above this Reynolds number, frequency will vary linearly with flow rate. Plotting the Strouhal number against the swirl number a linear relationship is found over the wide range of swirl number studied.

Another important experimental study in turbulent vortex breakdown is that by Garg and Leibovich (1979). The authors study vortex breakdown in a water tunnel where the experimental flow section is a slowly diverging pipe. Similar experimental setups were used by Harvey (1962) and Sarpkaya (1971) in their studies of laminar vortex breakdown. The study of Garg and Leibovich (1979) is noteworthy because it is one of the first studies to use the non-intrusive LDV to measure the flow profiles before and after breakdown. Additionally, Garg and Leibovich (1979) report on the power spectra of the measured velocities. Mean profiles were measured for two Reynolds numbers and at a wide range of swirl numbers. All the profiles contained an axial momentum surplus in the center of the pipe before breakdown and a deficit of axial momentum in the center of the pipe after breakdown. In some instances, the flow profiles measured did not line up exactly with the pipe centerline. The radial width of the axial momentum surplus can be associated with the size of the vortex core. The vortex core is defined as that region of the flow field that exhibits solid body rotation, i.e. where the rotational speed of the flow is constant and the swirl velocity is linear. The size of the vortex core was hypothesized to be associated with the boundary layer thickness shed from the center-body. Vortex breakdown increases the size of the core noticeably but the swirl profile shape does not change appreciably.

Garg and Leibovich (1979) report that the type of vortex breakdown observed is

a sensitive function of the maximum swirl angle of the flow. The swirl angle is defined to be the inverse tangent of the maximum swirl to axial velocity ratio upstream of breakdown. Angles of 49-52 degrees resulted in a bubble type vortex breakdown and angles of 44-46 degrees resulted in a spiral vortex breakdown.

The power spectra reported by Garg and Leibovich (1979) contain large peaks at around 13 Hz and 23.4 Hz in the wake of a bubble breakdown for a Reynolds number of 11,480. The axial velocity fluctuations are most energetic at an off-axis location approximately coinciding with the maximum swirl level. The swirl velocity fluctuations are most energetic in the very center of the pipe. It must be mentioned that the velocity fluctuations measured at the center of the pipe using the 2-D LDV are actually radial velocity fluctuations. Nevertheless, the absence of axial velocity fluctuations in the center of the pipe represents strong evidence for the observed coherent motion to be associated with a circumferentially periodic flow structure and not an axisymmetric flow structure. The equations of motion do not allow axial velocity fluctuations for circumferentially periodic fluctuations. The frequency peaks observed in the near vortex breakdown wake dissipate downstream. Power spectra taken in the wake of a spiral vortex breakdown exhibit a single peak at approximately the same frequency as the lower of the two bubble vortex breakdown frequencies. The energy contained in these fluctuations is significantly lower than those observed in the bubble breakdown, supporting the hypothesis that the bubble form of vortex breakdown is the more "violent" form of breakdown.

Garg and Leibovich (1979) also used the analytical results of Lessen et al. (1974) to compare the frequencies of maximum growth predicted by linear stability theory with the frequencies observed in the experiments in the wake of the vortex breakdown. Calculations show that the flow upstream of breakdown is stable, even to non-axisymmetric disturbances, whereas the downstream flow, while still stable with respect to axisymmetric disturbances is unstable to non-axisymmetric, circumferentially periodic disturbances. The frequencies calculated from theory and the experimentally observed frequencies compare well, and on average are well within 20% of each other. Garg and Leibovich (1979) do not compare the disturbance eigenfunctions

with the experimentally determined distribution of the fluctuations.

At the highest Reynolds number studied (22660), upstream low frequency coherent fluctuations were measured that were absent in all other cases. The authors were not able to explain this phenomenon except to speculate that it is perhaps related to the existence of neutral waves excited downstream traveling upstream.

Gouldin et al. (1985) and Gouldin et al. (1984) report comparisons between the flow structure and dynamics observed with and without combustion. The design of the experimental facility includes the ability to have a core jet flow with one swirl direction and an outer annular flow with the opposite swirl direction. Both works include results from flows employing co-swirl and counter swirl. Of particular interest to the research performed for this dissertation is the case of co-swirl. Periodic oscillations are observed in the density fluctuation data for the combustion case (co-swirl and counter swirl) and the velocity fluctuation data for the isothermal cases. The density fluctuations are observed at 100 Hz while the velocity fluctuations are observed at 356 Hz even though the inlet conditions and geometrical dimensions of the facilities were very similar. Some concentrated low frequency energy around 100 Hz is observed for the counter swirl case but is reported absent for the co-swirl case. Additionally, while the co-swirl combustion flow field exhibited flow reversal, the isothermal case did not. For this flow geometry, the addition of combustion completely modified the dynamics of the downstream flow field. This stands in contrast to the already mentioned study of Paschereit et al. (1999) to be discussed further below. The observed oscillations in the isothermal case were not distributed evenly over the cross section. The peak energy of oscillation was observed on the half-radius of the inner flow. Gouldin et al. (1985) suspect circumferentially symmetric but not axisymmetric instabilities (azimuthal wavenumber = 1) are to blame for the observed coherent isothermal fluctuations.

Ahmed (1998a) and Ahmed (1998b) report on the results from a confined swirling flow. The experimental setup is of particular note. Instead of moving the measurement location axially along the experiment, the location of the expansion relative to the probe was changed using a system sliding sleeves. In addition to the ingenious

experimental design, radial velocities are reported by performing not only horizontal scans (giving azimuthal velocity) but also vertical scans. To this end a special window was machined into the test section and the window was rotated as the probe was translated vertically. The results of the study were used to examine the turbulent kinetic energy budget under the assumption of circumferential symmetry. The production term found most responsible for increases in turbulent fluctuations was the term: $\overline{u'v'}\partial U/\partial r$. The conclusion relies on the assumption that the $\overline{v'w'}$ stress can be modeled by $\overline{v'v'}$ which may be a valid assumption for conditions without swirl. As soon as swirl is introduced however, the anisotropy associated with such a flow does not allow this assumption. At any rate, the underlying reason for the prominence of the term determined dominant is the large radial gradient in axial velocity. The term that may have contributed significantly but was dismissed involves the radial gradient of the azimuthal velocity.

Dellenback et al. (1988) studied swirling flow in an axisymmetric expansion without center-body, similar to the present experimental setup. To produce swirl, a variable amount of water was directed through tangential inlets in the upstream pipe. Measurements are not only reported downstream of the expansion but also in the nozzle. Dellenback et al. (1988) represents the only confined swirling flow study found reporting velocity profiles upstream of the expansion. Dellenback et al. (1988) reports inlet and downstream dump velocity profiles for swirl numbers ranging from 0.6 to 1.16. For these conditions, the inlet axial velocity profile has the form of a redeveloping wake which in the sudden expansion develops negative axial velocities. The study also included lower swirl number results but the mean and RMS velocities for these cases were not reported. Like other studies, the reported RMS velocities are very high (20% when normalized with the mean inlet axial velocity). In addition to these measurements, Dellenback et al. (1988) report on the measurement of the frequency of velocity fluctuations in the flow field. Unfortunately, the strengths of these oscillations are not reported. Similar to Cassidy and Falvey (1970), the study finds that the non-dimensional oscillation frequency (Strouhal number) is close to linear with swirl strength and that the non-dimensional frequency is in general insensitive

to changes in Reynolds number.

Paschereit et al. (1999) report on a study comparing the combustion dynamics and isothermal flow dynamics of a novel swirl inducing burner nozzle. Combustion measurements showed that depending on operating condition either a helical mode or axisymmetric mode of dynamic burning could be observed. A model of the burner was then studied in a water tunnel where the same types of instabilities at the same non-dimensional frequencies were observed. Unfortunately, the normalized data presented cannot be compared directly because not enough information is given on the characteristic quantities used in the normalization of both velocity and frequency data. Paschereit et al. (1999) also report on the radial distribution of the magnitudes of the coherent fluctuations. These distributions reveal that the helical instability is associated with the inner shear layer (vortex core) and that the axisymmetric instability is associated with the outer shear layer at the nozzle exit sudden expansion. In the case with combustion, phase-locked images of OH^* chemiluminescence were taken showing the azimuthal periodicity of the fluctuations. Additionally, relative phase measurements of OH^* chemiluminescence at two locations are reported. As the second of the two measurement points moves diametrically away from the first, the phase relative to the first location changes for helical modes of instability whereas it remains constant for axisymmetric modes. A 180 degree phase change was measured for the helical mode indicating an azimuthal mode with wavenumber 1.

Experimental studies involving excitation of swirling flows are very rare. One of the few studies is that by Panda and McLaughlin (1994) who studied the response of a free swirling jet to external excitations. Acoustic actuators were placed just outside the lip of a swirling flow nozzle. The actuators were controlled so that azimuthal modes could be specifically targeted in excitation. Flow visualization studies showed how a spiral vortex breakdown becomes more and more compact as the Reynolds number is increased. The breakdown was no longer identifiable at a Reynolds number of 40,000. The frequencies associated with the breakdown movement were reported to be in the single Hertz range. Velocity measurements use hot-wire anemometry, even though the authors report interference for some measurement locations and it

has been shown that the insertion of hot-wire probes potentially causes significant changes in the flow-field characteristics (e.g. Döbbeling, 1990). The power spectra of velocity taken without forcing do not exhibit any traces of coherent motion, i.e. frequency peaks. The power spectra do show an offset indicating that perhaps noise contaminated the dynamic velocity measurement.

The excitation experiments of Panda and McLaughlin (1994) show that only for very high excitation levels, is it possible to produce spectral peaks related to the excitation in the power spectrum. The excitation level was measured a priori using microphones to measure the acoustic pressure and then deducing the acoustic velocity from a plane wave assumption. Based on these measurements, the excitation level was determined to be only several percent of the mean velocity. No hot-wire measurements were performed to verify that the excitation generated was actually at the level calculated. Excitation was most successful at Strouhal numbers of 1.5 for a Reynolds number of 57,000, corresponding to 56 Hz. Panda and McLaughlin (1994) report that the percentage of total fluctuation energy contained in the coherent excited velocity fluctuations is about 25% for axisymmetric excitation and 40% for helical excitations. The downstream growth of fluctuations is however only modest and coherence is lost relatively quickly downstream. The radial distribution of fluctuations is nearly identical for axisymmetric and helical excitation. The modest growth of fluctuation energy downstream and the nearly identical radial distribution of fluctuations points to the possibility that the applied excitation was too high in amplitude to allow linear growth. Furthermore, it is possible that the point of excitation is chosen in a region that is not optimal because it is located too far downstream to couple into the thin areas of the shear layer. (The swirler hub in this experiment is set back into the nozzle exit so that separation occurs inside the nozzle). Another reason for the poor response may be that the location of excitation is at an axial station that experiences large flow divergence so that the stability characteristics of the flow are changing extremely quickly downstream. The flow divergence in this free swirling jet is larger than that for a confined swirling jet.

1.4 Linear Stability Analysis

1.4.1 Overview of results

Linear stability analysis consists of the study of the evolution of normal mode perturbations to the underlying mean flow. The analysis has been instrumental in obtaining insight into the instabilities of jets (Michalke, 1971; Wygnanski and Petersen, 1987; Gaster et al., 1985). Cohen and Wygnanski (1987b) showed that linear stability analysis could predict the radial distribution of velocity oscillations in a turbulent jet. Cohen and Wygnanski (1987b) were also able to show that the initial perturbations of the flow at the exit of a jet were amplified by the shear layer according to the amplification rate as a function of frequency predicted by linear stability analysis. Linear stability analysis has also been used to explain the evolution of trailing line vortices generated from wings (Spall, 1993; Khorrami, 1995, 1991; Mayer and Powell, 1992). Application of linear stability analysis to jets of various shapes allows a good characterization of the mixing enhancement achieved through the action of fluid dynamic instability (Huang et al., 1994; Shozo et al., 1989). Linear stability analysis was used successfully to predict the occurrence of the von Karman vortex street behind bluff bodies (Jackson, 1987; Kim and Pearlstein, 1990; Kelkar and Patankar, 1992).

Linear stability analysis on swirling jets emerging from nozzles has been performed to predict the occurrence and some characteristics of vortex breakdown. Garg and Leibovich (1979) showed that linear stability analysis could predict the main frequency peaks observed in the wake of their turbulent vortex breakdown study with reasonable accuracy. Loiseleux et al. (1998) and Loiseleux and Delbende (2000) studied analytical representations of swirling jets and wakes and were able to derive a criterion for the occurrence of vortex breakdown in high Reynolds number flows which compared reasonably well with experimental data. Michalke (1999) was able to predict the main frequency component of swirling jet noise with reasonable accuracy by calculating the vortex breakdown instability frequency using linear stability analysis. Vortex breakdown was also studied in detail by Wang and Rusak (1997) (also see Section 1.4.4) who analyzed the inviscid steady states of swirling flows. Lin-

ear stability analysis was used in that study to show how the different steady states found changed stability depending on the inlet swirl ratio. In both the studies by Loiseleux et al. (1998) and Michalke (1999) the effects of viscosity and turbulence were neglected. Turbulence and turbulent viscosity can play a very important role in shear layer instability development as can be seen from the analysis done by Marasli et al. (1989). Marasli et al. (1989) found that stability predictions for the varicose mode of a turbulent wake were accurate only when the analysis accounted for the presence of random turbulent fluctuations through a mean turbulent viscosity. The influence of turbulent fluctuations may be more complicated for swirling flows due to the anisotropic distribution of turbulence in these flows (Shtern et al., 2000). Furthermore, the more complex case of multiple free shear-layer flows such as bluff body flows with sudden expansion geometries has not been studied. The effect of asymmetry in the flow-field on stability has also not been studied.

1.4.2 Basic stability problem setup

Linear stability analysis examines the stability of a given steady state. To analyze stability, perturbations are added to the basic flow quantities and the evolution of these perturbations in time and space is studied. In general, the perturbations are of normal mode form. For the study of 2-D parallel flow (i.e. flow does not evolve in the axial direction and is uniform in the z direction) stability, the normal mode expansion takes the form given in Equation 1.2. An example of such a flow is Poiseuille flow between two infinite parallel plates.

$$f'(x, y, t) = F(y)e^{i\alpha x - i\omega t} \quad (1.2)$$

The governing equation (in stream function form) for the evolution of disturbances in a 2-D parallel incompressible viscous flow can be obtained from the incompressible Navier–Stokes equations and is known as the Orr–Sommerfeld equation, shown in Equation 1.3. In Equation 1.3 F represents a perturbation to the mean stream function. The use of the stream function allows the evolution of the pertur-

bations to be analyzed using just one fourth order differential equation as shown in Equation 1.3.

$$(U - c)(F'' - \alpha^2 F) - FU'' = \frac{1}{i\alpha Re} (F'''' - 2\alpha^2 F'' + \alpha^4 F) \quad (1.3)$$

Note that setting the right-hand side of Equation 1.3 equal to zero neglects the effects of viscosity and Rayleigh's equation is recovered from which the well known inflection point theorem can be derived. With boundary conditions, the Orr-Sommerfeld equation becomes an eigenvalue problem where $F(y)$ is the eigenfunction and either c or α in general is the eigenvalue. Early solutions fixed the value of α to some real value and calculated the corresponding complex phase speed c . An analysis of this type is often called a temporal stability analysis because disturbance growth is in time rather than strictly in space. To compare results from the temporal stability analysis with results from experiments, the temporal growth has to be transformed to a spatial growth using the group velocity, the velocity with which wave energy is convected. Gaster (1962) noted that such a transformation is only accurate for small temporal growth rates and that in the case of larger growth rates, spatial growth must be calculated directly by considering a complex wavenumber α . Such an analysis is more intuitive since it uses a real frequency ω which can then be interpreted as a forcing frequency and easily related to the physical parameters of a given experiment.

Researchers attempted to explain the transition from laminar to turbulent flow using solutions to the Orr-Sommerfeld equations, reasoning that instabilities cause the transition. However, for Poiseuille flow between infinite parallel plates (as well as for Couette flow) the attempts failed. In the case of Poiseuille flow, the predicted Reynolds number for instability was calculated to be 5772 by Orszag (1971) whereas experiments exhibited transition at much lower Reynolds numbers. The discrepancy between linear stability analysis and experiments observed for these flows will be addressed in more detail below in Section 1.4.7.

1.4.3 Flow stability in cylindrical coordinates

The cylindrical coordinate versions of the inviscid stability equations were studied with much more success, starting with Batchelor and Gill (1962), who studied the stability of axisymmetric jets. Batchelor and Gill (1962) also derived the equations governing viscous stability in a parallel axisymmetric flow, along with the non-trivial boundary conditions for this case. Batchelor and Gill (1962) also generated a stability criterion for axisymmetric inviscid flows and proved the validity of Howard’s semi-circle theorem (Sherman, 1990) in axisymmetric inviscid parallel flows. The inviscid form of the axisymmetric disturbance equation is given in Equation 1.4 (Plaschko, 1979).

$$P'' + \left(\frac{1}{r} - \frac{2U'}{U - c} \right) P' - \left(\frac{m^2}{r^2} + \alpha^2 \right) P = 0 \quad (1.4)$$

As noted by Cohen and Wygnanski (1987b), the restriction of the azimuthal wavelength of the disturbances implied by Equation 1.4 introduces another important scale to the stability problem, namely the relative size of the shear layer thickness compared to the diameter of the jet. When the jet shear layer is very thin compared to the diameter of the jet, the influence of curvature is negligible and results are a very weak function of azimuthal wave number. As the shear layer increases in thickness, the curvature becomes more important (Cohen and Wygnanski, 1987b). Cohen and Wygnanski (1987b) were also able to demonstrate the physical relevance of the disturbance eigenfunctions by selectively exciting a certain azimuthal mode and comparing the radial variation in induced velocity oscillations with the radial variation of the magnitude of the corresponding eigenfunction. Linear stability analysis results matched the distribution of velocity fluctuations well at eight locations downstream of the nozzle. For each downstream location, the instability properties were calculated from fits to the local velocity profile, thus accounting for the jet spreading slowly upon exiting from the nozzle. The analysis was able to predict the change in magnitude in the power spectrum over a wide range of frequencies when a short axial distance is considered ($x/D = 0.125$ to $x/D = 0.25$). It is important to underline that the cor-

responsiveness between linear stability analysis and experiment was achieved assuming locally parallel flow and solving the parallel flow stability equations.

The linear stability analysis of Cohen and Wygnanski (1987b) did not match all the experimental results. The agreement with calculated eigenfunctions was demonstrated only to $x/D = 0.70$ with the agreement becoming less and less complete with increasing downstream distance. Truly significant discrepancies are encountered when integral effects are compared such as total amplification of a wave starting from the exit of the jet. As pointed out by Cohen and Wygnanski (1987b), an extremely important factor in this type of comparison is the magnitude of the initial disturbance. A larger initial disturbance causes quicker linear saturation and consequently a more significant exaggeration in the predicted wave amplitude when linear analysis is compared with experiment. The idea of wave saturation is also important in explaining the process of "vortex pairing". Wygnanski and Petersen (1987) use purely linear ideas to explain the appearance of vortex merging. The vorticity distribution associated with the linear amplifying wave exhibits two maxima. Wygnanski and Petersen (1987) observed similar to Michalke (1965) that these two maxima approach each other due to the spreading of the jet. The two maxima are only vertically displaced when the wave reaches neutral stability. Wygnanski and Petersen (1987) postulate that the appearance of subharmonic frequencies is not related to vortex merging but should be associated with lower frequency flow disturbances (approximately subharmonic) that are amplified strongly in the region of neutral stability of the original instability wave. Further evidence that vortex merging can be understood in terms of lower frequency wave amplification can be derived from the work of Smith and Glezer (1998) who shows that the synthetic jet produced in the experiments does not exhibit vortex merging. The jet is produced by discrete vortices appearing at the given forcing frequency. The vortices move under the self-induced velocity and eventually disintegrate into turbulence but never merge. If vortex merging were a phenomenon strictly related to vortex dynamics, then these vortices would merge. These qualitatively valuable insights from linear stability still do not improve important integral estimates such as final wave amplitude.

The effects of flow divergence on jet stability were first explicitly calculated by Crighton and Gaster (1976) using a multiple scale expansion. The analysis took advantage of the relatively slow mean flow divergence relative to the axially varying exponential growth of the instability wave. The results of Crighton and Gaster (1976) showed that flow divergence alters the growth rates and phase speed of the waves at first order compared to the strictly parallel analysis. Furthermore, the divergence causes the growth rate and phase speed to be different for different flow variables (pressure, velocity, energy density, etc.), a characteristic also observed in experiments. However, the radial eigenfunction of the disturbance is unaltered from its strictly parallel flow approximation, explaining the good agreement obtained by Cohen and Wygnanski (1987b) in their flow excitation study. The analysis of Crighton and Gaster (1976) was extended to include the first two azimuthal modes by Plaschko (1979). Both studies found reasonable agreement with experiments limited in general by nonlinear amplitude effects. Parallel linear theory, slowly diverging linear theory and experiment were compared in Wygnanski and Petersen (1987) for the axial development of the wave amplitude. The results show that the amplitude predicted by linear parallel analysis is four times as high as the amplitude measured in the experiment. Non-parallel linear analysis improves the estimate significantly to reduce the error to 40%. The remaining error can be attributed to nonlinear wave amplitude effects.

In all the research on jets mentioned so far, the effects of viscosity or background turbulence have been neglected. The influence of turbulence on the development of unstable shear layers can be expected to be important due to the spatially non-uniform viscosity-like effects introduced by turbulence. Many turbulence closure models are based on the idea that turbulence acts similar to an enhanced, spatially varying viscosity. High Reynolds number flows are associated with the insignificance of laminar viscous forces. However, high Reynolds number flows are also turbulent and therefore the viscous-like effect of turbulence becomes important. Consequently an inviscid approach to calculating the stability of high Reynolds number flows does not consider the potentially important effects introduced by the presence of turbulence. Legner

and Finson (1980) made the observation that ship wakes, which are dominated by the small scale boundary layer turbulence shed from the ship's hull, seem to damp out the large scale free surface waves incident upon the wake. Legner and Finson (1980) performed a linear stability analysis for a 2-D turbulent wake, assuming uniform, fine-scaled, background turbulence. The governing equations were derived similarly to the works of Hussain and Reynolds (Hussain and Reynolds, 1970; Reynolds and Hussain, 1972; Hussain and Reynolds, 1972) with the exception that turbulence closure in the work by Legner and Finson (1980) is effected at second order compared to the first order closure invoked by Hussain and Reynolds (1970). The equations for the turbulent stresses considering an infinitesimal wave disturbance in velocity were derived by Reynolds and Hussain (1972). Chen and Crighton (1999) discusses the influence of various second order closure models on 2-D jet stability, especially in regards to the importance of the viscoelastic modeling of the turbulence. The results of the study of Legner and Finson (1980) confirm the important viscous effect that the presence of fine-scaled turbulence has on the growth or damping of incident velocity waves. Lower order closure models have also been used in accounting for turbulence effects.

Ragab and Wu (1989) used a Prandtl mixing length model to account for turbulence effects in the development of a compressible mixing layer. Marasli et al. (1989) simply found an effective Reynolds number based on the eddy viscosity concept and measured turbulence intensities. Marasli et al. (1989) were able to show improvement in the agreement of the distribution of the velocity perturbations in a two dimensional turbulent wake. However, the relative phase of the velocity perturbation was not predicted correctly, a fact that may be attributed to the non-uniform distribution of turbulence in the wake. To account for turbulence or solve lower Reynolds number stability problems it becomes necessary to solve the viscous disturbance equations. In Equation 1.3, the 2-D cartesian version of the disturbance equations is given. Equations 1.5 through 1.8 show the disturbance equations for axisymmetric parallel flow with swirl and radial velocity similar to those given by Lessen and Singh (1973) and Mayer and Powell (1992). It is clear that a non-zero radial component of velocity

violates the parallel flow assumption, but the term will nevertheless not be dropped for now. The results of Lessen and Singh (1973) interestingly pointed to the possibility that the viscous axisymmetric jet has a range of Reynolds numbers where growth rates exceed that observed in the inviscid case. The findings were confirmed by Morris (1976) who was able to use an energy argument to explain why the flow exhibits this unexpected behaviour.

Continuity:

$$F' + \frac{F}{r} + \frac{mG}{r} + \alpha H = 0 \quad (1.5)$$

Radial Momentum:

$$\begin{aligned} -\frac{iF''}{Re} + i\left(V_r - \frac{1}{Rer}\right)F' + \left[\omega + iV_r' - \frac{mW}{r} - \alpha U + \frac{i}{Re}\left(\frac{m^2 + 1}{r^2} + \alpha^2\right)\right]F \\ + \frac{2}{r}\left(\frac{im}{Rer} - W\right)G + P' = 0 \end{aligned} \quad (1.6)$$

Azimuthal Momentum:

$$\begin{aligned} -\frac{G''}{Re} + \left(V_r - \frac{1}{Rer}\right)G' + \left[-i\omega + \frac{V_r}{r} + \frac{imW}{r} + i\alpha U + \frac{1}{Re}\left(\frac{m^2 + 1}{r^2} + \alpha^2\right)\right]G \\ + \left[iW' + \frac{1}{r}\left(\frac{2m}{Rer} + iW\right)\right]F + \frac{im}{r}P = 0 \end{aligned} \quad (1.7)$$

Axial Momentum:

$$\begin{aligned} -\frac{H''}{Re} + \left(V_r - \frac{1}{Rer}\right)H' + \left[-i\omega + \frac{imW}{r} + i\alpha U + \frac{1}{Re}\left(\frac{m^2 + 1}{r^2} + \alpha^2\right)\right]H \\ + iU'F + i\alpha P = 0 \end{aligned} \quad (1.8)$$

It should be mentioned that for mathematical convenience, the introduction of the disturbances for the velocity components is slightly different. The radial perturbation is introduced as i times the usual normal mode form as discussed above. The set of equations 1.5 to 1.8 was solved by Lessen and Singh (1973) for the viscous stability of jets and wakes in the self-similar region.

1.4.4 Stability of swirling flows

Viscosity is generally thought to have a stabilizing effect on the growth of instabilities and a negligible effect on their frequency and eigenvectors. Since inviscid

analysis is much more straightforward, it is thus generally thought to provide a worst case scenario able to provide reasonably accurate frequency estimates. However, as was seen above in the case of the 2-D wake (Marasli et al., 1989) and axisymmetric jet (Lessen and Singh, 1973), the general thought may be misleading. Khorrami (1991) as well as Mayer and Powell (1992) found purely viscous modes of instability in the analysis of the trailing line (Batchelor) vortex. The swirling flow exhibited very long wave instabilities, one axisymmetric and one asymmetric ($m=1$) that become stable as the Reynolds number is increased. Even though the growth rates of these modes were much lower than that of their inviscid counterparts Khorrami (1991) was able to provide supporting evidence for their existence in the form of photographs of aircraft contrails. In view of the results obtained by Marasli et al. (1989) the existence of these instabilities in high Reynolds number flows may point to the viscous influence of turbulence on the development of instabilities.

Swirling flow stability is not only important in the behavior of trailing line vortices. Another major field of research involving swirling flows is that of the vortex breakdown. Vortex breakdown is the abrupt transition a swirling flow undergoes under certain conditions. Explanations for the occurrence of vortex breakdown vary and still today no consensus has been reached. Many elegant studies have been performed, starting with the analysis of Benjamin (1962) who compared vortex breakdown in swirling flows to the hydraulic jump in free surface flows. According to Benjamin (1962), vortex breakdown thus represents a transition from a supercritical flow state to a corresponding subcritical flow state brought about by certain changes in the flow environment such as an adverse pressure gradient. Many accompanying fascinating flow visualizations were published of the phenomenon. Among these are the studies by Harvey (1962) and Sarpkaya (1971). Figure 1.2 shows a photograph taken from Sarpkaya (1971). To obtain the picture the vortex core as well as an off-axis location was seeded with dye. The dye from the core is seen to accumulate in a recirculation bubble before being ejected in a spiraling jet. The conditions for these amazing pictures however are laminar and in turbulent high Reynolds number flows, the detail structure of vortex breakdown is more difficult to visualize. In several studies how-

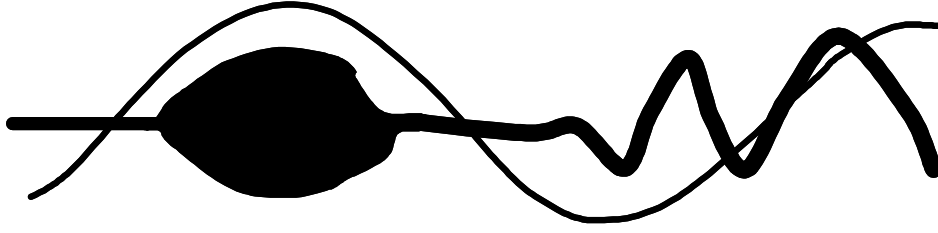


Figure 1.2: Sketch of vortex breakdown structure based on (Sarpkaya, 1971) and (Harvey, 1962)

ever, it has been possible to discover one particular structure. The precessing vortex core (PVC) has been identified in combustion and isothermal flows (Froud et al., 1995; Kihm et al., 1989) as a jet that whips around the central recirculation zone with a speed approximately equal but not identical to the speed of rotation of the swirling flow.

Not surprisingly, alternate explanations of vortex breakdown are based on the stability of the flow. Leibovich and Stewartson (1983) derived a sufficient condition for the instability of a columnar vortex such as a trailing line vortex. The criterion is based on the assumption of inviscid flow and is given in Equation 1.9.

$$\text{Instability if: } W\Omega' [\Omega'\Gamma' + (U')^2] < 0 \quad \text{for any } r. \quad (1.9)$$

Garg and Leibovich (1979) had some success comparing their spectral velocity measurements to the theory advanced by Lessen and Paillet (1974). More recently, vortex breakdown has been associated with absolute instability. The matter of absolute instability will be addressed in greater detail in Section 1.4.5. Loiseleux et al. (1998) and Loiseleux and Delbende (2000) solved the inviscid parallel stability equations for various idealizations of swirling flow profiles and used the results in terms of absolute stability to predict the occurrence of vortex breakdown as a function of the swirl strength and the axial flow form (wake or jet). The analysis proved reasonably successful for a variety of quoted studies (see Loiseleux et al., 1998). A similar analysis was performed by Michalke (1999) for the velocity profile measured by Lehmann et al. (1997). The study was able to confirm the fact that the observed oscillations

were largely radial in nature. The quantitative estimate for the instability frequency however was 40% lower than the measured frequency.

The understanding of vortex breakdown has continued to evolve and though no consensus on the mechanics of vortex breakdown has been reached, recent studies have been able to unify stability and transition concepts to some extent. The analysis by Benjamin (1962) was extended by Keller et al. (1985, 1988) who studied vortex breakdown as a three stage process, extending the analogy with the hydraulic jump. The first transition is hypothesized to occur without dissipation, whereas the second transition can be dissipative. The next major step in the evolution of vortex breakdown analysis was the analytical work performed by Leibovich and Kribus (1990) who studied large amplitude waves on swirling flows. Linear stability analysis in Kribus and Leibovich (1994) showed that these large amplitude waves are stable to axisymmetric perturbations but unstable to azimuthal mode perturbations beginning with $m = 1$. The stability characteristics observed are thus consistent with the spiraling downstream character of the recirculation bubble often observed in vortex breakdown.

The most comprehensive unifying analysis in the last decade however was performed by a research group lead by Z. Rusak at RPI. In a series of papers, the existence of multiple steady solutions and the physical selection process of which steady state is observed in experiments was connected to the stability characteristics of these solutions. Wang and Rusak (1997) summarize and extend two papers (Wang and Rusak, 1996a,b) on swirling flows in straight pipes. The work is able to show that columnar flows (flows without breakdown) become unstable (in a global sense) as swirl is increased and transition to a corresponding non-columnar state (flow with recirculation, i.e. vortex breakdown). The non-columnar state satisfying the governing equations becomes stable near the point where the columnar state loses stability and vice-versa. For swirling flows in a pipe, the transition is a particular point of swirl strength. For swirling flow in a diverging pipe, the transition occurs over a range of swirl ratios with strong unsteady motion bridging the two flow regimes (Rusak and Judd, 2001; Rusak et al., 1997). Additionally, the analysis in Wang and Rusak (1997) was able to relate the loss of stability of the columnar flow state to the ideas of sub-

critical and supercritical flow put forth originally by Benjamin (1962) and continued by Keller et al. (1985, 1988). Finally, Rusak et al. (1998) shows that near the critical swirl level, transition can be initiated by subjecting the flow to finite but still small perturbations.

The stability analysis employed in this work is not unlike that described here with the exception that the entire flow field was analyzed and the parallel flow assumption was not required. It should also be mentioned that all the work reported has dealt with entirely axisymmetric flows and stability to circumferentially non-uniform perturbations has not been studied.

The problem with applying linear parallel stability analysis to these flow fields is that they are in no reasonable approximation parallel. Still, an impressive amount of information can be obtained by the study of these equations (see Section 1.4.5), even in flows where the assumptions underlying the equations are categorically violated. It should be noted that the growth rates seen in swirling flows are still larger than the growth rates observed in jets. On a scale relative to the axial amplification of the disturbance, therefore, swirling flows are not as non-parallel as one may initially be inclined to think.

1.4.5 Absolute and convective instability

The distinction between absolute and convective instability was first introduced in the field of plasma stability. A phenomenological description of the difference is given by Huerre and Monkewitz (1990). A flow that is convectively unstable behaves as a spatial amplifier to specific ranges of frequencies whereas an absolutely unstable flow behaves like a self-excited oscillator. As pointed out by Monkewitz and Sohn (1986) in their study of the instabilities of hot jets, convectively unstable flows lend themselves very well to control due to their intrinsic amplifying capabilities. Absolutely unstable flows such as hot jets, under certain conditions (Monkewitz and Sohn, 1986), behave as self excited oscillators and are therefore much more difficult to control. From an observability standpoint however, self-excited flows are much more easily monitored because the global instability has a certain form in space. Therefore

the instability state can be identified with a relatively low amount of monitoring. Still as reported later by Monkewitz (1989) the control of absolutely unstable flow is challenging because control of one global mode will likely lead to the development of a secondary global mode. Convectively unstable flows on the other hand have axially continuously changing amplifying properties that would require, ideally, continuous axial monitoring to obtain a full description of the flow characteristics. All cases discussed above in Section 1.4.3 are not absolutely unstable anywhere in the flow.

Absolute and convective instability is generally distinguished using the impulse response of a flow. If in response to the impulse the flow over all space returns to its original state as time approaches infinity, the flow is at most convectively unstable. If there is a period of time and space over which the impulse has excited growing waves which are then however convected downstream, the flow is at least convectively unstable. It follows that a necessary condition for absolute instability is convective instability. To obtain a mathematical form for the condition of absolute instability, it is sufficient to consider what happens to the initial impulse at the location the impulse was given. For disturbances to grow in place it is necessary for the group velocity (the velocity with which energy is transported) to be equal to zero. The condition is expressed mathematically in Equation 1.10. Expanding $\omega(\alpha)$ in a Taylor series around α_o immediately leads to the conclusion that α_o is a saddle point of the complex function $\omega(\alpha)$. The mathematical condition for absolute stability then depends on the sign of the imaginary part of ω . If the imaginary part is positive the flow is absolutely unstable, whereas if it is negative the flow is convectively unstable (Monkewitz, 1990). Furthermore, the branch point described by Equation 1.10 must be the result of the coalescence of an upstream and a downstream mode (Huerre and Monkewitz, 1985). This so called 'pinching condition' has the obvious physical interpretation that absolute instability is the result of wave amplitude not only growing in the downstream direction but also in the upstream direction. The pinching requirement is illustrated in Figure 1.3. Figure 1.3 shows level curves, keeping the imaginary part of the frequency ω constant. Each opposed pair of curves corresponds to the variation of two eigenvalues as the real part of the frequency is varied at a given level of the

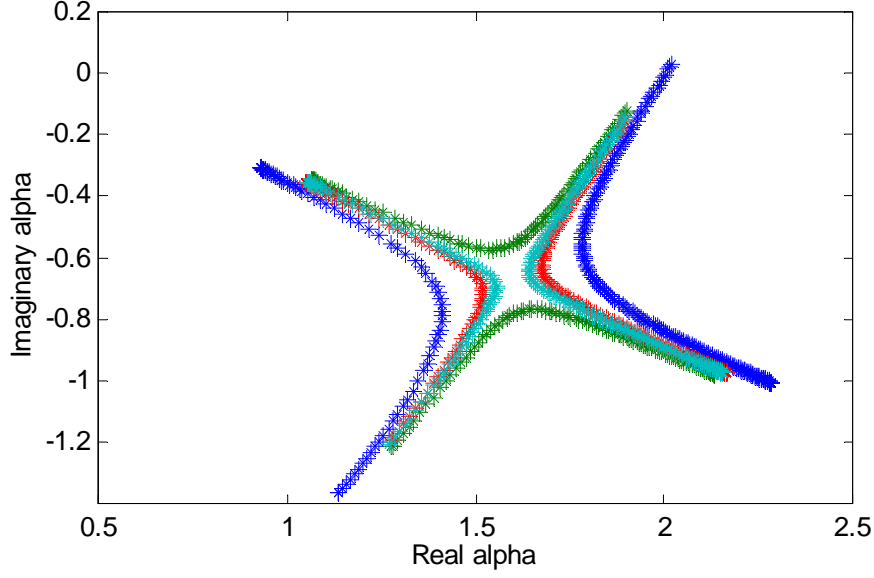


Figure 1.3: Pinching condition for determination of absolute stability

imaginary part of the frequency. The two eigenvalues correspond to the upstream and downstream traveling modes respectively. As the the imaginary part of the frequency is increased past the location of the saddle point, the two opposed curves switch from left right to top bottom.

$$\frac{d\omega}{d\alpha}(\alpha_o) = 0 \quad \text{with} \quad \omega_o = \omega(\alpha_o) \quad (1.10)$$

A locally absolutely unstable flow is globally unstable if in fact the flow is parallel. However, even in the case of non-parallel flows a connection between local and global stability exists. Chomaz et al. (1988) explicitly found this connection under the assumption of a slowly varying mean flow. Specifically, they found that a necessary condition for global instability is a finite area of local absolute instability. Even before the connection between local absolute stability and global stability was positively established, three global mode frequency selection criteria were advanced to attempt to calculate the frequency of the oscillations observed. Pierrehumbert (1984) argued that the observed oscillations should be dominated by the frequency of the maximum

absolute growth rate found in the area of absolute instability. In other words, the absolute growth rates are calculated for all axial locations, say, and the frequency of the observed oscillations is the real part of the branch point value where the imaginary part of the branch point frequency (ω_o) has a maximum. Koch (1985) argued that any point in flow where the absolute frequency is real acts as an effective narrow band reflector for instability waves of that frequency. The presence of another reflector would thus enable waves to be amplified between the two reflectors. The second reflector can take the form of a solid boundary, which is an excellent broad band reflector of instability waves or possibly a second point in the flow where the absolute frequency is real. Finally, Monkewitz and Nguyen (1987) suggest that the frequency observed is dominated by the first local resonance with a non-negative absolute growth rate that is encountered by the flow. Analytically none of these criteria were able to be verified except that the criterion due to Pierrehumbert (1984) gives a leading order estimate for frequency in the case of the simple model used by Chomaz et al. (1988). In fact, Huerre and Monkewitz (1990) show, based on the analysis of the Ginzburg-Landau equation with variable coefficients (discussed further in Section 1.4.6), that the absolute frequency of instability is given by another saddle point criterion, this time between the local absolute complex frequency and a complex axial location. It is unclear how this selection criterion should be implemented for real flows and we are left with the first three selection criteria as summarized by Monkewitz and Nguyen (1987) and described above.

Monkewitz and Nguyen (1987) noted that each of the three criteria appear to work best with three types of absolutely unstable flows. The criterion of Pierrehumbert (1984) appears best suited to describe the evolution of a global instability in flows that are largely unbounded such as the flows investigated by Pierrehumbert (1984). The mechanism for global instability advanced by Koch (1985) appears best suited to flows in which there is a pocket of absolute instability where the absolute growth rates are never large and a solid boundary exists for efficient instability wave reflection. Accordingly, the criterion performs best for bluff body flows just beyond global instability. The frequency selection mechanism proposed by Monkewitz and

Nguyen (1987) appears to perform best in flows with a solid boundary that contain significant absolute instability.

The three criteria were evaluated by Hannemann and Oertel (1989) in a numerical study of the wake behind a flat plate at the supercritical Reynolds number of 200. Hannemann and Oertel (1989) were able to obtain a pseudo steady state in their numerical calculation from which they performed a time-accurate simulation of the developing instability. They also solved the local Orr-Sommerfeld equation (Equation 1.3) to determine local stability properties. Hannemann and Oertel (1989) found that none of the three selection criteria outlined above were able to predict both frequency and growth rate. Both the criteria of Monkewitz and Nguyen (1987) and Pierrehumbert (1984) predict the same frequency and growth rate because the maximum absolute growth rate occurs immediately at the downstream edge of the plate. The growth rate predicted by these criteria was more than double that observed in the numerical simulation. The predicted frequency was close to the saturation frequency even though strictly speaking the criteria should only hold for the frequency during the onset of instability. It should be noted that as expected, during the exponential growth phase of the disturbance there was no change in frequency. As the saturation state is approached, the frequency increases by approximately 8%. The criterion due to Koch (1985) was also able to predict the saturation frequency. The criterion does not provide an estimate for the initial growth rate. All these results are questionable however since the saturation frequency prediction is based on the flow profiles obtained in the pseudo steady state. The time mean flow in the saturation state differs significantly from the pseudo steady state, and so the stability properties of the pseudo steady state may only be coincidentally related to the dynamic properties of the saturation state. A study of how the stability properties change from the pseudo steady state to the time mean saturation state was not reported. It should be noted that the absolute growth rate exhibited a local maximum in the middle of the pocket of absolute instability. The corresponding frequency matched the onset frequency of instability very well. The success of the criterion of Pierrehumbert (1984) when the region immediately behind the bluff body is ignored is not surprising considering the

fact that the criterion was designed for unbounded flows.

Hannemann and Oertel (1989) then used base bleed in their numerical simulation to suppress the instability and studied the stability properties of the generated mean velocity profiles to see if base bleed control was mirrored in the stability properties. As base bleed is increased, the absolute instability pocket predicted by analysis shrinks. However, as predicted by Chomaz et al. (1988), global stability is attained before local absolute instability is lost. Hannemann and Oertel (1989) conclude that linear stability analysis can only provide an upper bound on the amount of bleed necessary to eliminate the oscillations. However, the trend for a linear variation of onset frequency with increasing base bleed was predicted by the criteria due to Monkewitz and Nguyen (1987) and Pierrehumbert (1984). If the local maximum is used for the application of the criterion due to Pierrehumbert (1984) as described above, linear stability also predicts the quantitative variation of the onset frequency. Again however, no results were reported to answer the question whether or not the saturation frequency is at all related to the local stability properties of the time mean flow state in saturation.

The ideas of Monkewitz and Nguyen (1987) were extended to axisymmetric bluff-body wakes by Monkewitz (1988) with limited success. Whereas 2-D bluff body flows seem insensitive to the actual geometry of the bluff body, the occurrence of vortex shedding and its characteristics are very geometry sensitive in axisymmetric bluff body flows. Still, Monkewitz (1988) was able to show that vortex shedding, when it occurs, is likely due to global instability similar to the 2-D bluff body flows.

The extension of local linear stability analysis to flows that have a stronger non-parallel character is difficult. The analysis of Chomaz et al. (1988) indicates that as the flow becomes more non-parallel, boundary conditions for the flow become more important. Boundary effects have indeed been shown to be important in a study by Chao et al. (1991) who examined the spectral characteristics of swirling flows under changes of the downstream boundary condition. The study also found the flow to be very sensitive to downstream actuation, a view that is consistent with that of Benjamin (1962) who predicted that the flow downstream of vortex breakdown should be subcritical and thus admit upstream traveling waves.

1.4.6 Global stability

Global stability was connected to local stability by Chomaz et al. (1988) using a simplified model for the global evolution of the instability based on the Ginzburg-Landau relation shown in Equation 1.11. The equation was formally derived by Stewartson and Stuart (1971) in the study of how a wave packet is amplified in plane Poiseuille flow. Equation 1.11 can be solved not only for the strictly parallel case where the coefficients are all constant but also for the case where coefficients vary. Most interesting here is the case where $\omega_{\alpha\alpha}$ remains constant but both ω_o and α_o are allowed to vary slowly with the axial coordinate (Hunt and Crighton, 1991). Using a quadratic variation in coefficients, Chomaz et al. (1988) proved the necessity of a finite expanse of local absolute instability for global instability to become possible.

$$\frac{\partial \Psi}{\partial t} - \omega_{\alpha\alpha} \alpha_o \frac{\partial \Psi}{\partial x} - \frac{i}{2} \omega_{\alpha\alpha} \frac{\partial^2 \Psi}{\partial x^2} + i \left[\omega_o + \frac{\omega_{\alpha\alpha}}{2} \alpha_o^2 \right] \Psi = 0 \quad (1.11)$$

Monkewitz (1989) used Equation 1.11 (including a forcing term) to study the prospect for feedback control in locally absolutely unstable but still globally stable flows as well as globally unstable flows. The study showed that it was relatively straightforward to bring the system to oscillate by applying forcing at the damped global frequency. The control of an existing self-excited global instability however proved far more difficult. In fact control was only achievable for very small global mode growth rates, for more supercritical flows, controlling the unstable global mode caused the excitation of a higher global mode. This observation is consistent with several experimental observations quoted in Monkewitz (1989) such as the difficulties in obtaining control over the von Karman vortex street at larger supercritical Reynolds numbers (Tao et al., 1996).

Linear stability analysis does not have to be performed on the local level. It is possible to perform a 2-D linear stability analysis where the form of the perturbation is such that the normal mode is no longer periodic in the axial direction. The axial dependence is then absorbed into the eigenfunction, leaving only the temporal frequency as periodic. Global linear stability analysis has been performed by Jack-

son (1987) for variously shaped bluff bodies. Zebib (1987) performed global linear stability analysis on flow over a long cylinder. Kim and Pearlstein (1990) performed the analysis on flow past a sphere. Global stability analysis of course is not able to predict the saturation state but the onset frequencies are routinely well predicted, at the cost of significant additional computational expense and complexity of course.

Finally, it is important to mention a different type of global instability where the feedback is not strictly internal to the flow but rather occurs through the acoustics of the system. It is possible for a perturbation to be amplified by the shear layer and then transported to the system boundary via acoustics and fed back to the base of the shear layer as a coherent excitation. The feedback loop can cause the entire system to become unstable. Interestingly, some of the same characteristics observed in attempting to model control of self-excited globally unstable flows were seen in acoustic feedback control experiments with jets. As reported in Monkewitz (1989), the jet became self-excited above a certain feedback control gain. The nature of the self-excited mode was found to be greatly dependent on speaker and sensor locations, a feature which has also been observed in the globally unstable bluff-body flows discussed in detail above. The acoustic feedback also played an important role in the study of Gutmark et al. (1991) who examined isothermal and reacting flows in sudden expansions. Gutmark et al. (1991) found that fluid dynamic instabilities played an important role in determining which acoustic mode becomes unstable in the presence of combustion.

1.4.7 Amplification without unstable eigenvalues

The global stability of a flow does not guarantee that all disturbances are immediately damped, because the flow is indeed locally unstable and some transient growth can be expected. Cossu and Chomaz (1997) were able to analytically show how such amplification is possible. Once again the Ginzburg-Landau model given in Equation 1.11 was analyzed for a quadratic axial variation of the coefficients. However, in this analysis the interest lay in the maximum amount of transient amplification that can be obtained from a perturbation whether coherent or random. The analysis made

use of the pseudo-spectrum which holds the key to defining such transient growth. The pseudo-spectrum is a term due at least in part to Trefethen (1992).

The pseudo-spectrum of an operator or in a discrete sense, a matrix is the spectrum of eigenvalues of a randomly perturbed matrix. If the operator is orthogonal then perturbations of order ϵ will result in changes in the spectrum of eigenvalues of order ϵ . However if the operator is non-orthogonal, the pseudo spectrum will be significantly different from the original operator spectrum, allowing the existence of unstable perturbed eigenvalues even if all eigenvalues of the original operator are stable. The Ginzburg-Landau operator is a non-orthogonal operator. Interestingly, the more parallel the flow is the more non-orthogonal the operator becomes (Cossu and Chomaz, 1997). Cossu and Chomaz (1997) showed explicitly that excitation of the flow near the globally damped resonance can result in very large amplification of the input signal. In general, the amount of transient gain achieved depends on the form of the excitation. Furthermore, the eventual decay in the amplified disturbance is governed by the actual least damped eigenvalue, underlining the fact that the original spectrum is only able to determine asymptotic time behaviour. Cooper and Crighton (2000) used global mode analysis for the case of a slowly diverging isothermal jet to show that the preferred Strouhal number, as given by the damped global frequency, is close to those observed in most experiments, i.e. 0.44. The analysis was able to show remarkable similarity between experiment and model in the near and far field directivity pattern of the acoustic field generated by the instability. The calculation of the damped global frequency involves the search for yet another saddle point, this time in the complex temporal frequency and spatial coordinate planes. The saddle point search in complex spatial coordinates is performed using rational function extrapolation.

In Section 1.4.2 it was pointed out that instability theory failed to predict turbulent transition for 2-D Poiseuille flow. Since the early results of Orszag (1971) much work has been done to explain the apparent premature transition to turbulence in Poiseuille flow. Gustavsson (1991) showed that 3-D disturbances can be amplified significantly over a finite period of time before eventually decaying. The 3-D dis-

turbances considered by Gustavsson (1991) represent a favorable phase relationship between independent vorticity and velocity perturbations. The form of the perturbation in the velocity resembles cross-stream streaks which are also observed in experiments. Butler and Farrell (1992) used variational methods and principles based on the pseudo-spectrum to find the form of the perturbation that would achieve maximal gain in a given time frame. The Orr-Sommerfeld operator also is a non-orthogonal operator. The results were very similar to those of Gustavsson (1991) in terms of the optimal form of the perturbation and the maximum amplification achieved. An excellent introduction to the subject of pseudo spectra and calculation of transient amplification is given by Trefethen et al. (1993). The issue of transient amplification may also be important in swirling and convectively stable flows if the governing operator is non-orthogonal.

1.5 Summary

1.5.1 Experiments

Although many studies report distributions of mean and RMS velocities, only very few report power spectra or other dynamic details of the velocity fluctuations. Garg and Leibovich (1979) studied turbulent vortex breakdown in a slowly expanding pipe and found significant concentrations of spectra energy for the bubble form of vortex breakdown. Panda and McLaughlin (1994) studied the dynamics of a free swirling jet and found no concentration of spectral energy and reported difficulty in exciting the flow field. The present research will exhaustively report on the dynamics observed in the swirling flows measured in Chapter 5.

1.5.2 Analysis

Many forms of analysis have been performed in order to obtain a more complete understanding of the behavior of swirling flows. Wang and Rusak (1997) analyzes the global stability of a developing swirling flow and finds that vortex breakdown is

associated with a loss of stability of the columnar solution branch as the amount of swirl in the flow field is increased. Keller et al. (1985, 1988) take the approach of Benjamin (1962) and exploit the analogy between vortex breakdown and a hydraulic jump to explain the occurrence and structure of vortex breakdown. These global approaches to the analysis of swirling flows are analytically involved and in general are difficult to apply to real non-idealized flow fields. The approach taken here is similar to that of Michalke (1999) who analyzed a local velocity profile typical of swirling flows with vortex breakdown. The analysis determined the local stability characteristics of the flow field. However, absolute instability was found so that the entire flow field can be expected to be affected by the instability. The locally parallel flow assumption makes it possible to analyze a very large variety of flow fields.

Other dynamic analysis approaches exist in the field of fluid dynamics but have so far not been applied to swirling flows. The most interesting of these is the pseudo-spectral analysis as presented by Trefethen et al. (1993). Pseudo-spectral analysis is not reported in this study, though the rapid broadband production of turbulence observed in swirling flows may be closely related to mode-to-mode distribution of energy facilitated by a non-orthogonal dynamic flow operator (similar to the Orr-Sommerfeld operator).