THE SUSTAINABILITY OF DOMESTIC BUDGET DEFICITS IN OPEN ECONOMIES

by

Farrokh K. Langdana

Dissertation submitted to the Faculty of the

Virginia Polytechnic Institute and State University

in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

in

ECONOMICS

APPROVED:

Allan Mandelstamm　　　　Douglas McTaggart

Chairmen

Barbara Craig　　　　Richard Cothren

Daniel Orr

May 1987

Blacksburg, Virginia

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Committee Chairmen: Allan Mandelstamm, Douglas McTaggart
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(ABSTRACT)

This paper presents a framework for exploring the sustainability of U.S. domestic budget deficits in the presence of the currently experienced capital inflows. A 'sustainable' deficit-financing policy is defined as one in which the combination of debt-financing and seigniorage precludes the creation of a large unanticipated inflation to wipe out the debt in real terms. The model implemented is a rational-expectations model of the open economy and two separate cases are analyzed.

In Case I, domestic money creation is held 'fixed' and any increases in the deficit are financed by the sale of one-year discounted government bonds to domestic and foreign residents. In Case II domestic money and bonds are both endogenously determined. The asset market, in both the cases, is characterized by perfect capital mobility as defined by uncovered nominal interest parity. Real interest parity, however, does not exist as domestic and foreign goods are not perfect substitutes.

In Case I, the solution of the domestic price level exhibits price-neutrality with repect to the deficits. The nominal and real

exchange rates, however, are found to appreciate with increases in deficits and the situation is aggravated further by an exodus of domestic real wealth.

In Case II, on the other hand, deficits are found to be inflationary and both nominal and real exchange rates depreciate with increases in the deficit. Furthermore, increases in the amount of debt being rolled over cause even greater upward pressures on domestic inflation and result in the further weakening of the dollar. The solutions also provide us with an expression for the maximum amount of debt that can be rolled over without causing the domestic price level to explode or the currency to collapse. This 'critical value' of debt is found to bear an inverse relationship to the rate of growth of the domestic deficit.

Bond-financed deficits are therefore non-sustainable in both the cases discussed, and the arithmetic, it seems is unpleasant indeed.

Any bonds today?
Bonds of Freedom, that's what I'm selling,
Any bonds today?
Scrape up the most you can, here comes
the freedom man,
Asking you to buy a share of freedom
today.

Treasury Secretary Henry J. Morgenthau copywrited this ballad on behalf of the Treasury of 1941 for use as the Offical Theme Song of the National Defense Savings Program. The words and music were a gift from Irving Berlin.

ACKNOWLEDGMENTS

I wish to thank the members of my committee, Professors Barbara Craig, Richard Cothren, Allan Mandelstamm, Douglas McTaggart and Daniel Orr for the time, patience and comments required to finish this dissertation. Their confidence in me was, at times, greater than my confidence in myself. For that I thank them.

I would like to thank my wife, Mary, for her love, patience and tolerance as I turned the house into a repository of scores of equation-bearing sheets of paper.

Last, but not the least, I wish to thank my parents without whose sacrifices all this would not be possible.

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CHAPTER I

INTRODUCTION AND LITERATURE SURVEY

This thesis presents a framework for exploring the sustainability of U.S. domestic budget deficits by implementing a rational-expectations model under two separate monetary policy regimes. Sustainability is viewed in the context of the ability of the fiscal authority (Treasury/ Congress) to incur a continually increasing stream of deficits by issuing new debt to retire the principal plus interest on that of the previous period. A 'sustainable' deficit-financing policy is defined as one in which an upper limit of debt financing, characterized by adverse effects on the price level, nominal exchange rates and real wealth, is not attained, and the possibility of future unanticipated inflation to wipe out the debt in real terms, is non-existent. The present state of the U.S. economy can be characterized by burgeoning domestic and current account deficits, increasing amounts of foreign capital inflows and an exchange-rate that has only recently come off its 1985 record-high, and this scenario makes the sustainability issue timely and pertinent. The time-plots of domestic and current account deficits, nominal exchange rates and interest rates, and the price level for the period 1974-85, are presented in figures 1 to 4.

The rapidly growing net inflow of capital from abroad, mirroring the extraordinary deterioration of the U.S. current-account balance, has played a significant role in equilibrating overall saving and investment in the United States in the face of unprecedentedly large and persistent domestic budget deficits during the 1980s. The size of the current domestic budget deficit is of the order of $200 billion, or 4 to 5 percent of the gross national product. This was exceeded only in World War II when the deficit was as large as 28 percent of the GNP. This situation has raised a host of issues, some of which are:

1. Can the government continue on a path of large fiscal deficits without accommodation from the monetary authority, or, how long can the government 'roll over' its debt by issuing more bonds and without any help from the monetary authority?
2. How does the inflow of capital affect the sustainability of domestic budget deficits, or, how is the 'rolling over' policy affected in a milieu of large capital inflows?
3. How should fiscal and monetary policy be conducted in the above situation, and what are the effects of these policies on key macroeconomic variables such as the price level, nominal and real interest rate and exchange rates, and individual's wealth holdings?
4. And, most importantly, is there an upper limit on the amount of debt the government can 'roll over' by issuing new debt? Will monetary accommodation be inevitable, and if so, when? In other words, how sustainable is the U.S. domestic budget deficit in the presence of capital inflows and under the present policies of rolling over a substantial portion of the existing debt by issuing new government bonds?

The following simple example of a closed economy, with no domestic money creation illustrates the meaning of a 'sustainable' debt-financing policy as used in the context of this dissertation.

Let P = the price level

y = real output or GNP

D = the real primary deficit (the non-interest part of the actual deficit)

i = the after-tax gross nominal interest paid by the government on its debt.

B = the nominal debt outstanding

The change in the real value of the debt, B/P, over time can be shown to be:

$$\frac{d(B/P)}{dt} = D + r\,B/P \qquad \text{(a)}$$

Here r is the real interest rate, and the right-hand side is the primary deficit D plus the real interest on public debt, or the inflation-corrected deficit, while the left-hand side is the increase in real debt.

Defining the ratios, d = D/y, as the primary deficit as a fraction of GNP, and b = B/Py as the debt to income ratio, equation (a) can be written as:

The inflation-corrected deficit as a fraction of GNP = d + rb

The important question is that given d and r, what is the behavior of b? The examination of the deficit-GNP ratio over time has been one of the central issues in determining the sustainability of domestic deficits.

Using the definition of b, and $g = \dot{y}/y$, the growth rate of output, we have the following expression:[1]

$$\dot{b} = d + (r-g)b \qquad \text{(b)}$$

Models incorporating various forms of this equation have been fairly common in the deficit dynamics literature. In (b) the debt-income ratio rises if the primary deficit, d, exceeds the debt-income ratio multiplied by the excess of the real interest rate over the growth rate of output. Equation (b) has been used to analyze the existence, and the behavior, of steady states in which b reaches a constant level.

From (b) it can be seen that if the primary deficit, d, is positive, and if the real interest rate, r, exceeds the economy's growth rate, g, then the debt-income ratio keeps on increasing. Given a positive primary deficit, there can be no steady state debt-income ratio unless the growth rate exceeds the real interest rate. The intuition behind this is that with a primary deficit, the government is always issuing new debt and the only way the debt-income ratio can be kept from increasing is if income is growing fast. But if the interest rate on existing debt exceeds the growth rate of GNP, the rate at which interest payments on the debt mount up outweighs the effects of the growth of GNP in permitting debt issues that do not increase the debt-income ratio. With debt rising relative to income, one or more of the following three things must eventually happen:

1. The government raises taxes to increase revenue with which to service the debt and slow down the growth of the debt to below the rate of income growth.
2. The government creates a big unanticipated inflation to wipe out the debt in real terms.
3. The public debt is repudiated or written down.

In this thesis, a 'weakly non-sustainable' deficit-financing policy is defined as one in which the primary debt, d, is positive, the real interest rate, r, exceeds g, the economy's growth rate, and the fiscal/monetary authorities are <u>forced</u> to resort to one of the three extreme measures listed above. It is assumed here that the second option, the large-scale monetization of the deficit resulting in a large unanticipated inflation, is the one most likely to occur and therefore a weakly non-sustainable debt-financing policy is taken to be one in which monetary accommodation is expected in the future.

We define a 'strongly non-sustainable' debt-financing policy as one in which the monetary authority has had to purchase a large amount of government debt due to the reluctance of the public to accept any more government bonds in their portfolios. This economy is characterized by an exploding price level and a collapse of the domestic currency. Both the weak and the strong form of non-sustainability will be made clearer with the discussion of the results of cases I and II, respectively.

In the period post-World War II till 1982, the growth rate of the output, g, has been in the 3-4% range while the real after-tax rate on U.S. Treasury securities has been of the order of 2-3%. Consequently, debt dynamics have never been explosive. Post 1982, however, the real rate has either exceeded the growth rate, or come dangerously close to it. This economic backdrop makes the issues that this dissertation tackles timely and pertinent.

Sargent and Wallace (1981) show that in a closed economy where the monetary base is closely connected to the price level and the monetary authority can raise seigniorage, the monetary authority's control over inflation is very limited. By assuming that the real rate of return exceeds the growth rate of the economy, they establish inherently unstable debt dynamics. They give particular attention to the inflation option, especially when fiscal policy 'dominates' monetary policy. In this case they have the fiscal authority independently setting its budgets and announcing all current and future deficits and surpluses, thereby determining the amounts of revenue that must be raised through the sale of government bonds and seigniorage. Sargent and Wallace demonstrate that if the demand for government bonds implies an interest rate on bonds greater than the economy's growth rate, then, in the presence of deficits, the monetary authority is unable to contain the growth rate of the money supply forever, i.e., monetary accommodation is inevitable.

The objective of this dissertation is to extend the Sargent and Wallace analysis by incorporating the foreign sector and to determine if debt-financed deficits are indeed non-sustainable even when domestic savings are supplemented by foreign savings in the form of capital inflows. These inflows are a direct consequence of the large current account deficits experienced by the U.S. since 1980, and their present persistently large rate of growth makes this a relevant topic. Furthermore, the model constructed in this thesis is amenable to an analysis of price and exchange rate volatility, particularly in the context of changes in domestic fiscal and monetary policies in a regime of forward-looking individuals with rational expectations. Exchange rate volatility, in particular, must emerge as fluctuations either in the prices of tradeable goods or in the profits of the firms producing them. This is especially important in a situation such as the present where studies indicate that Japanese firms are choosing to sacrifice profits by absorbing any systematic downward movement in the dollar resulting from the G-5 agreement in September, 1985, thereby leaving the trade in real terms, largely unchanged. Therefore, in addition to determining the sustainability of bond-financed deficits, this thesis analyzes the effects of these debt-financing policies on the volatility of domestic prices and exchange rates.

Before proceeding with the model description, a brief overview of the recent open-economy macroeconomic literature is in order. This undertaking follows next.

Literature Survey

Currently there are two broad classes of macroeconomic models of the open economy. The first class is one in which the models represent a short-run partial equilibrium portfolio balance perspective. In this class (Mundell-Fleming 1962, Driskill and McCafferty, 1980), the recent strengthening of the U.S. dollar from 1982-1985, for example, could be explained in terms of the effect the increased government spending has on the domestic and foreign interest rate differential. High and rising U.S. real interest rates associated with domestic budget deficits, in this view, have created an interest rate differential that has attracted foreign capital inflow. The inflow has,in turn, caused a temporary appreciation of the dollar exchange rate above long-run equilibrium value associated with purchasing power parity. In this class of models, for the example under consideration, the real value of the dollar would gradually fall back to its former level either because interest rates would eventually fall, or because investors might become reluctant to invest an increasingly large share of their portfolios in dollar-denominated securities.

The second class of models does not only consider the short-run portfolio adjustments but also incorporates the long-run effects on goods markets and interest rates in a world of perfect capital mobility.

This broader asset market approach (Dornbusch 1976, Branson 1977, 1985, McTaggart 1985) assumes two-way causation. This exchange rate, in this view, is determined proximately by financial market equilibrium conditions. It, in turn, influences the current account balance. The latter, in its turn, is the foreign rate of accumulation of domestic national debt, and this feeds back into the financial market equilibrium. Thus, the general equilibrium asset market approach contains a dynamic feedback mechanism in assets and exchange rates.

This second class of models has various sub-classes. One of these is the equilibrium models based on first principles of explicit utility maximization subject to budget constraints--primarily overlapping generations and two-period models. Frenkel and Razin [1984(a)(b)] are good examples of these. These general equilibrium models have been used quite frequently in the recent literature to assess the effects of permanent versus transitory fiscal policies as well as changes in countries' net debtor positions on world rates of interest, domestic and foreign wealth, and spending. However, even modest complications render these models quite intractable. For instance, most of these models are simplified 'real' models and hence they ignore the effects stemming from the interaction of fiscal and monetary policy. These effects, on the contrary, are deemed to be of central importance in this thesis, as (i) the sustainability of domestic and current account deficits is viewed in the context of expected future monetization, and (ii) the behavior of prices and particularly, exchange rates, is explained in terms of present and anticipated monetary and fiscal (primary tax) policies. For the reasons mentioned above, the model implemented in this dissertation is not of the general equilibrium class of models.

McTaggart (1985, 1986) constructs a model that lies within another sub-class of the second class of models. The two-sector open-economy, log-linear rational-expectations model is rich enough to capture the important feedback effects from asset markets to the product markets, and vice-versa. It is specified so that solution techniques developed in conjunction with linear rational expectation models can be applied, and the evolution of the economy be simulated under different policy experiments. He explicitly models a medium to large economy producing a single tradeable good in an economy where the markets are those of domestic output, domestic money, domestic bonds and foreign exchange. The agents are domestic households, the treasury, the Federal Reserve, foreign residents and later when the model is extended to two sectors, agricultural and manufacturing households. Omission of any simplifying assumptions and the explicit inclusion of all the various different goods and agents, renders this model suitable for stochastic simulation methods.

Another important paper in the above sub-class is Turnovsky (1976) where he provides a good brief survey of the important contributions up to and including 1976. He found that much of the closed economy research was essentially static and that the models did not address the intrinsic dynamics of the economy. Some examples are Heliwell (1969) and Takayama (1969). Others such as Oates (1966) and McKinnon (1969) have recognized the role of the government budget constraint in an open economy but emphasis here has been only on the equilibrium steady state of the system with the dynamics being ignored by the analysis. Yet others such as Towers (1972) and Floyd (1969) explicitly considered the dynamics of the capital flows but ignored the manner in which the deficit was financed.

Turnovsky extends the work of Blinder and Solow (1973) to analyze the dynamics of fiscal policy in a small fixed-exchange-rate open economy.[2] The research done by Blinder and Solow shows how the government's deficit-financing mechanism affects the stability of fiscal policy and the long-run real effects of government expenditure.[3] They find that if the deficit were financed entirely by money creation, then the system will be stable, but if it were purely bond financed, then fiscal policy will not be stable. Furthermore, they demonstrate that if it were stable, then the long-run efforts of increased government expenditure would be more expansionary than in the pure money case.

Turnovsky emphasizes three aspects which are lacking in the studies prior to 1976.[4] He introduces two policy parameters, (i) the mix of money and debt financing, and (ii) the extent to which the monetary authority indulges in the sterilization of changes in foreign reserves through open market operations. By introducing these parameters which have impacts on the supply of domestic financial assets, he, in effect, endogenizes the government's fiscal and monetary policies. The model constructed in this thesis incorporates certain features of the Turnovsky flexible rate model. The similarities and differences will become apparent in Case II.

The analysis in this thesis is purely theoretical and it subscribes to a trade-off between greater computational ease and theoretical flexibility, and a loss of richness of structure, relative to McTaggart/Turnovsky.

The outline of this thesis is as follows. The model is described in Chapter II, and the solution-technique and the solutions are summarized in Chapter III. The solutions and the results of the various policy experiments are then interpreted in Chapter IV and the conclusions are presented in Chapter V.

CHAPTER II

THE MODEL DESCRIPTION

The log-linear rational expectations model constructed in this dissertation incorporates the feedback effects between the goods and the asset markets. This two-way causation (Dornbusch, 1976; Turnovksy, 1976, 1980; Branson, 1985; McTaggart, 1985) has the exchange rate determined proximately by financial market equilibrium conditions. It, in turn, influences the current account balance. The latter, in its turn, is the rate of accumulation of national claims on foreigners, and this feeds back into the financial market equilibrium. In this manner, the asset market approach incorporated in the following model, contains a dynamic feedback mechanism in foreign assets and exchange rates.

The model lies within the Turnovksy (1976) and McTaggart (1985) class of open-economy macro-models and it provides a sufficiently rich theoretical framework within which the issues of sustainability can be studied.

The outline of this section is as follows: Section II(i) presents the conceptual framework within which the model is constructed, Section II(ii) and II(iii) describe the model under the two monetary regimes, and Section II(iv) presents the model summary for Cases I and II, respectively.

2.1 Conceptual Framework

The mechanism by which budget deficits influence the current account, and the channels through which this influence is transmitted under a floating exchange rate regime, are described here. The basic reason the budget and current account deficits are related is because budget deficits represent a 'use' of saving, and current account deficits a 'source' of saving. This may be seen from the national saving identity:[5]

(G - T)	=	(S - I)	+	(M + R - X)
Budget Deficit or Surplus		Private Domestic Saving Surplus or Deficit		Current Account Deficit or Surplus

The government budget deficit (expenditures less taxes, G - T) must equal, or be financed by, the excess of private domestic saving (S) over private investment (I) plus the current account deficit.

In flow of funds technology, the budget deficit and private investment constitute competing 'uses' of savings. The 'sources' of this saving are private domestic saving (S) and the funds from the foreign sector represented by the current account deficit. Not only does a current account deficit require a net inflow of foreign funds to finance it, but a nation can sustain a net financial inflow from abroad <u>only</u> by incurring an equal current account deficit, in a regime of floating exchange rate.

One possible mechanism linking the budget deficit to the current account deficit could be the following: in an open economy, large and growing domestic budget deficits, in the absence of accommodative monetary policy, might cause domestic real interest rates to exceed those of the rest of the world. This will cause investors to attempt to shift out of foreign assets and into domestic assets in order to take advantage of higher domestic real yields.

The rise in demand for domestic assets, in turn, will put upward pressure on the domestic currency in the foreign exchange market. As investors move to sell foreign currency for domestic currency and use the receipts to purchase higher yielding domestic bonds, they will bid up the exchange rate.

Real domestic currency appreciation associated with higher real interest rates also represents a rise in the price of domestically produced goods relative to those produced abroad. This weakens export demand and spurs imports, causing the current account balance to deteriorate gradually. Current account deterioration, in turn, is the mechanism that allows foreign savings to begin to supplement domestic savings in financing domestic government budget deficits and private domestic investments, as in the post-1980 period in the United States.

In an economy characterized at present by ballooning levels of government spending, it is important to identify the various mechanisms by which the government can obtain a large share of the current output.[6]

Firstly, it is reasonable to suppose that the increased deficit has resulted from the government attempting to increase its consumption share of current output while not directly reducing the share going to private consumption (via increased taxation). As the demand for current output rises, short of an increase in output stemming from a Keynesian response, some prices have to rise to ration national current domestic output and induce voluntary transfers from private consumption to government consumption. There are several ways in which this price rationing can occur. The aggregate price level might be pushed up by the excess demand, resulting in inflation even in the absence of monetary influences. Individuals will then be induced to save more to restore their eroding real cash balances, and in this way the inflation tax releases resources from private consumption to current government consumption.

The second, third and fourth mechanisms by which the government can achieve a larger share of current output are related to the manner of deficit financing. Influences through the asset markets will be of importance when deficits are financed by borrowing from the private sector. The increased supply of government debt drives down bond prices. Yields on these bonds increase, with arbitrage pushing up interest rates generally. If prices are stable, this rise in both the nominal and real interest rates will reduce current private consumption and increase current savings (or expected future consumption). This is because the real interest rate is the opportunity cost of current consumption in terms of future consumption, and the higher nominal rates cause some crowding out by increasing the cost of real investment. The reduction in both private consumption and investment release current output directly to government consumption. The higher domestic interest rates attract a capital inflow from overseas which in turn enables the government (Treasury/Congress) to increase its current consumption, and this has been described earlier.

Finally, an 'inflation tax' can be imposed upon the public by simply having the Federal Reserve purchase government debt directly--or monetize the debt. The prices are generally presumed to increase in proportion to the increase in the money supply, and this releases real resources to government consumption.

This conceptual framework forms the underlying superstructure on which the model is constructed. Case I incorporates the first three mechanisms, while Case II incorporates all of them.

2.2 Model Description: Case I

Consider a small country that produces a good that is imperfectly substitutable with goods in the rest of the world.[7] This small country exports part of its output to the rest of the world, and the quantity exported depends on the real exchange rate (relative price of foreign output in terms of domestic output) and the level of foreign income. Similarly, the country imports output from the rest of the world and the level of imports is a function of domestic income and real exchange rates. The system is a 3x3 matrix system with the endogenous variables being the domestic price level, p_t, the nominal exchange rate s_t, and the domestic real wealth balances, w_t. Domestic private consumption is a function of real disposable income and wealth holdings, and government consumption is a function of domestic income.

These considerations imply the real private domestic demand of domestic output in levels (a ^ signifies a level):

$$\hat{Y} = \hat{C}(\hat{Y}^d,\hat{W}) + \hat{X}[\hat{Y}^*,\frac{\hat{S}\hat{P}^*}{\hat{P}}] - \frac{\hat{S}\hat{P}^*}{\hat{P}} \cdot \hat{M}[\hat{Y}^d,\frac{\hat{S}\hat{P}^*}{\hat{P}}] + \hat{G}(\hat{Y}) \qquad (0)$$

where $\hat{C}$ is the consumption demand or domestic absorbtion, $\hat{X}$ and $\hat{M}$ are the levels of exports and imports, respectively. $\hat{S}$ is the nominal exchange rate which is in units of domestic currency per unit of foreign currency, $\hat{P}$ is the domestic currency price of domestic output, and $\hat{G}$ is government demand or the level of government spending. Because of the small country assumption, the domestic residents view $\hat{P}^*$ and $\hat{Y}^*$ as some exogenous stochastic processes. The real disposable income, $\hat{Y}_d$, is given by

$$\hat{Y}_d = \hat{Y}(1 - \tau)$$

where $\hat{Y}$ is the level of gross real income and τ is the marginal tax rate

Assuming that the level of trade is initially balanced, equation (1) is derived as a log-linear approximation of (0). All coefficients are non-negative and all lower case variables, other than interest rates, are in natural logarithms.[8]

$$y_t = k^d - \gamma_0\tau_t + \gamma_1\omega_t + \gamma_2 g_t + \gamma_3\rho_t$$

$$+ \gamma_4 y_t^* + \varepsilon_t^d \qquad (1)$$

where the following variables, not yet defined, are:

K^d = exogenously determined trend component of domestic output

g_t = real government consumption of domestic output

ρ_t = real exchange rate, which is the relative price of foreign goods in the domestic currency with respect to the price of domestic goods in the domestic currency. In levels

$\hat{\rho}_t = \frac{\hat{p}_t^{*} \hat{s}_t}{\hat{p}_t}$ where the variables are as defined earlier.

The parameter γ_3 captures the substitution of both domestic and foreign residents towards domestic goods as their relative price falls. It is thus a measure of the responsiveness of the trade account to relative price changes. The ε_t is a series of iid shocks, with zero means and a constant variance. It is assumed that the government (Treasury/Fiscal authority) can fix the fraction of output that it wants to consume and to obtain as tax revenues (T_t). In levels, this is:

$$\hat{G}_t = K_1 \hat{Y}_t \; , \qquad 0 < K_1 < 1.$$

$$\hat{T}_t = \tau \hat{Y}_t \; , \qquad 0 < \tau < 1.$$

$$\text{Domestic Deficit} = \hat{G}_t - \hat{T}_t = (\kappa_1 - \tau) . \hat{Y}_t$$

These functional forms are chosen for analytical convenience and conceptually they could be thought of as being derived from some time-variant government utility maximization problem.

Domestic and foreign bonds are assumed to be perfectly substitutable in this economy. Capital is therefore 'perfectly mobile' internationally with 'perfect capital mobility' being defined as uncovered nominal interest parity. The operative criterion of this interest rate parity is:

$$i_t = i_t^* + (E_t s_{t+1} - s_t) \tag{c}$$

where $(E_t s_{t+1} - s_t)$ is the expected percentage depreciation of the dollar over the coming period. In other words, investors respond to any differentials in expected returns so as to arbitrage them away. It is assumed here that investors are risk neutral, or exchange risk is completely diversifiable.

While capital is thus defined to be perfectly mobile in the sense of uncovered nominal interest parity, stemming from the perfect substitutability of domestic and foreign bonds, it should be noted that real interest parity, $r_t = r_t^*$ need not exist in this economy. This is because real interest parity exists <u>only</u> when <u>both</u> the following conditions are met: (i) nominal interest arbitrage exists, (ii) purchasing power parity holds, or there is perfect substitutability between domestic and foreign goods. Since the goods are <u>not</u> perfectly substitutable here, condition (ii) is not met, and real interest parity therefore need not exist, i.e., $r_t \neq r_t^*$.

This inequality of the real interest rates is a crucial mechanism for ensuring a flow of capital across national boundaries. Hence the presence of the real exchange rate, ρ_t, implying the absence of perfect substitutability, cannot be overemphasized.

But would not the real interest differences be arbitraged away just as the nominal interest rates are?

The answer lies in the fact that international portfolio investors have reason to arbitrage away gaps in countries' nominal rates of return when expressed in a common numeraire, but they have no reason to arbitrage away a gap between the domestic rate of return expressed in terms of domestic goods and the foreign rate of return expressed in terms of foreign goods, when these goods are not perfect substitutes.

The output supply for the small domestic country is given by:[9]

$$y_t = \bar{y} + \gamma(p_t - E_t p_{t+1}) + \varepsilon_t^s \tag{2}$$

where y_t is the logarithm of domestic output, p_t is the logarithm of the domestic price level, ε_t^s is an iid supply side shock. $\bar{y}$ is a systematic supply term that is intended to capture systematic changes in technology, population, etc. γ is the absolute value of the relative price elasticity of current output supply. E_t is the mathematical expectation operator conditional on information possessed at date t. More formally, the relevant price expectation could be written as

$E_t p_{t+1}/\Omega_t$ where Ω_t is the information set which includes knowledge of all variables through time t as well as knowledge of the structure of the model.

This equation embodies the 'natural rate' hypothesis and any deviations of output from this 'natural rate' respond positively to the term $p_t - E_t p_{t+1}$. As in Barro (1976) this can be viewed as an effect of speculation over time associated with the intertemporal substitutability of leisure.

The money supply is given by:

$$m_t^s = \psi_d \; d_{t-1} + \bar{f} + \varepsilon_t^m \tag{3}$$

where m_t^s is the nominal supply of domestic money, d_t is the domestic component of the domestic money supply, ψ_d is a feedback policy parameter of the money growth process, ε_t^m is an iid shock and $\bar{f}$ is the volume of foreign reserves which is a constant amount in this economy of floating exchange rates. The d_t is supplied exogenously by the Federal Reserve, in this model, by means of open market operations, and is of the form:

$$d_t = \psi_d \; d_{t-1} + \varepsilon_t^m$$

In this economy, the fiscal authority independently chooses K_1 and announces it, while the monetary authority (Fed) maintains a money supply given above.

The domestic demand for real money balances depends on the nominal interest rate and the level of real output. This is given by:

$$m_t^d = \gamma_5 y_t + p_t - \gamma_6 i_t \qquad (4)$$

the variables are as defined earlier. The domestic trade sector is assumed to be small and hence the price level in question is not a weighted average of domestically consumed and imported goods.

The exports for the small domestic country, which were described in levels earlier in (0) are

$$X_t = K_x + \gamma_7 y_t^* + \gamma_8 \rho_t + \varepsilon_t^x$$

where the variables are as defined earlier.

The imports are

$$I_t = K_n + \gamma_9 y_t - \gamma_{10} \rho_t + \varepsilon_t^n$$

The net exports, therefore, can be written as:

$$X_t - \rho I_t = K_N + \gamma_7 y_t^* + (\gamma_8 + \gamma_{10})\, \rho_t - \gamma_9 y_t - \varepsilon_t^N$$

where $K_N = K_x - K_n$. Therefore, the balance of payments equation, which is the sum of the current and capital amounts is (for a flexible exchange rate regime):

$$0 = [K_N + \gamma_7 y_t^* + (\gamma_8 + \gamma_{10}) \rho_t - \gamma_9 y_t] + \gamma_{11} (r_t - r^*_t) \qquad (5)$$

The first term (in brackets) on the right-hand side is the net exports expression (or the current account). The second term is the short-term, capital flow (capital account) which is assumed to be a function of $r_t - r^*_t$, the domestic and foreign real interest differential, given by $\gamma_{11} (r_t - r^*_t)$. This specification is similar to that of Turnovsky (1976) and Krugman (1985).

Using the expression for uncovered nominal interest parity (c) and the Fisher equations:

$$r_t = i_t - (E_t p_{t+1} - p_t) \text{ and } r_t^* = i_t^* - (E_t p_{t+1}^* - p_t^*)$$

where the real rate is the difference between the nominal interest rate and expected inflation for the domestic and foreign economy, respectively, and the definition $\rho_t = p_t^* + s_t - p_t$, the following expression for the real interest rate differential is obtained:

$$r_t - r_t^* = E_t \rho_{t+1} - \rho_t = (E_t p_{t+1}^* - p_t^*)$$

$$+ (E_t s_{t+1} - s_t) - (E_t p_{t+1} - p_t) \qquad (6)$$

The real interest differential is thus equal to the expected depreciation of the real exchange rate.

The domestic budget deficit, in any particular time period, is financed by domestic bond holdings and foreign capital inflow, and this is achieved by the Treasury's sale of one-period discounted government bonds to domestic and foreign residents. The domestic money creation, seigniorage, is obtained by the Federal Reserve's indulging in open market operations. There are only two assets held in the portfolios of the domestic private sector: domestic money and bonds. Private citizens at home and abroad do not hold foreign currencies as assets and the Treasury does not hold cash balances. The bonds are viewed by domestic and foreign residents as perfect substitutes and therefore (c) holds, as discussed. For the sake of simplicity, the foreign country is assumed to possess a balanced budget.

The consolidated budget constraint is given by (in levels):

$$\frac{\hat{B}_t^d}{1+i_t} + \gamma_{11}\hat{p}_t\,(r_t - r_t^*) + \hat{D}_t = \hat{p}_t(\hat{G}_t - \hat{T}_t) + \hat{B}_{t-1}^p$$

$\hat{B}_t^d$ is the domestic demand in period t for one-period discounted government bonds with \$1 face value and $\frac{\hat{B}_t^d}{1+i_t}$ represents the current nominal value of bond sales to the domestic residents, $\gamma_{11}\hat{p}_t\,(r_t - r_t^*)$

is the bond sales to foreign residents, and D_t is the nominal money creation (seigniorage) or bond sales to the Federal Reserve. The first term on the right-hand side is the budget deficit and $\hat{B}^p_{t-1}$ is the amount required to retire the last period's debt, both domestic and foreign. It is assumed that the domestic budget was balanced at and prior to time (t-2), and this initial condition is necessary to pin down a unique solution in Section III.

The amount needed to pay the principal on the last periods debt, $\hat{B}^p_{t-1}$, is simply the total demand of domestic and foreign residents in period t-1 for the one-period discounted government bonds. The balanced budget assumption for, and prior to, period t-2 precludes the presence of principal repayments in time t-1.

The consolidated government budget constraint in logarithmic form is given by:

$$\gamma_{12} b^d_t - \gamma_{13} i_t + \gamma_{11}(r_t - r^*_t) + \gamma_{14} d_t$$

$$= (p_t + y_t)\ (\gamma_{15} - \gamma_{16}) + \gamma_{15} K_1 -$$

$$\gamma_{16} \tau + \gamma_{16} b^p_{t-1} \qquad (7)$$

The definition for domestic real wealth is:[10]

$$w_t = \gamma_{20} b_{t-1} + \gamma_{21} d_{t-1} - p_t \qquad (8)$$

where w_t is the real wealth with which individuals enter period t, or, in levels, $\hat{B}_{t-1} + \hat{D}_{t-1}$ represents the current dollar value of household wealth carried over from the last period. Individuals in this economy view deficits as permanent because government bonds are not backed by taxes, and they incorporate these bonds into their wealth holdings. Consequently, Ricardian Equivalence does not hold in Case I.

2.3 Model Description: Case II

The model with an accommodative monetary policy regime is described in this section. Domestic money creation is endogenized here as opposed to the fixed money rule in Case I. The system is a 4 x 4 matrix system with the endogenous variables being the domestic price level, p_t, the nominal exchange rate, s_t, domestic demand for one-period discounted government bonds, b_t, and domestic real wealth balances, w_t.

The equations for the real private demand for domestic output, and the supply of domestic output, are identical to Equations (1) and (2) in Case I. However, domestic real wealth holdings are not defined as in Case I, where they (the real wealth holdings) were comprised of domestic bonds and money. In Case I when the possibility of debt monetization was non-existent, and the tax rule was clearly defined, bonds were incorporated into the definition of wealth. But now, with the endogenized domestic money supply, there is always some accommodation, the extent of which depends on the 'mix' of money and bond-financing and the exact amount of which is fixed by the fiscal and monetary authority, as explained later in this section. Hence, real wealth, w_t, is an endogenous variable, without any imposed a priori definition of its components, and the extent to which w_t is affected by money and bonds is determined endogenously within the model.

The real government spending, g_t, and tax revenues, t_t are given by the following announced, and adhered to, rules:

$$g_t = \psi_g g_{t-1} \qquad \text{II(a)}$$

$$t_t = \psi_t t_{t-1} \qquad \text{II(b)}$$

These rules imply that once again, the fiscal policy dominates the monetary policy because by fixing these rules the Treasury has announced the string of deficits that it is going to incur into the future.

The domestic money supply is again given by:

$$m_t^s = d_t + \bar{f} + \varepsilon_t^m \qquad (3)$$

Here d_t, the components of domestic money creation, is an endogenous variable and the $\bar{f}$ and ε_t^m are as defined earlier.

The domestic demand for real money balances depends on the nominal interest rate and the level of real output. This is identical to that of Case I and is given by:

$$m_t^d = \gamma_5 y_t + p_t - \gamma_6 i_t \qquad (4)$$

The balance of payments is again identical to equation (5) in Case I and is given by:

$$0 = [K_N + \gamma_7^* y_t + (\gamma_8 + \gamma_{10}) \rho_t - \gamma_9 y_t] + \gamma_{11}(r_t - r_t^*) \qquad (5)$$

where the domestic and foreign real interest rate differential is once again:

$$r_t - r_t^* = E_t\rho_{t+1} - \rho_t = (E_t p_{t+1}^* - p_t^*) + (E_t s_{t+1} - s_t) - (E_t p_{t+1} - p_t) \qquad (6)$$

The real interest rate differential is thus equal to the expected depreciation of the real exchange rate.

The financing of the domestic budget deficit is radically different from the financing mechanism in Model I. In this economy a sequence of deficits is announced and it extends into future periods, by virtue of the rules II(a) and II(b). These deficits, in any particular time period, can be financed by sales of government one-period discounted bonds to domestic and foreign residents, with the discrepancy between total amounts needed to finance the deficit and total bond sales, made up by domestic money creation. The amount of debt that is 'rolled over," or the proportion of the principal plus interest on debt issued in the last period that is retired by selling yet more bonds, is 'fixed' in this economy. This value θ_1, is fixed by some assumed coordination between the Treasury and the Federal Reserve, and a higher value of θ_1 implies a larger portion of debt being 'rolled over'.

Similarly, as an example, let the amount of the primary deficit that is financed by issuing one-period government bonds be fixed by the parameter α_1. If α_1 and θ_1 which are percentages of the primary deficit and the principal plus interest payments, have values of $\alpha_1 = 1$ and $\theta_1 = 1$, then this would imply an entirely bond-financed economy with no reason for domestic money creation.

It should be noted that even though α_1 and θ_1 are fixed by the Treasury and the Fed, the quantities of government bonds issued and the amount of domestic money creation are still endogenously determined by the model because the nominal interest rates that prevailed between time t-1 and t, and the domestic price level, are both endogenous. It is only the extent of the rolling over of domestic debt that is exogenously determined in this economy.

The bonds sold to domestic and foreign residents can thus be expressed by, in levels:

Government bonds sold in period t to domestic and

$$\text{foreign residents} = \alpha_1 \hat{P}_t(\hat{g}_t - \hat{t}_t) + \theta_1(\hat{B}^p_{t-1}) \qquad \text{II(c)}$$

where the first term on the right-hand side is the fraction of the current period nominal deficit and the second term is the fraction of the principal plus interest payments ($\hat{B}^p_{t-1}$) that are bond-financed. Here $\hat{B}^p_{t-1}$ is the total demand by domestic and foreign residents in period t-1 for government bonds. The domestic budget is once again assumed to be balanced for, and prior to, period t-1, and this precludes principal repayments in time t-1.

In this model, for the sake of computational simplicity, it is assumed that the entire primary deficit and a portion θ_1 of the principal and interest payments are bond-financed i.e. $\theta_1 < 1$ and $\alpha_1 = 1$. This means that domestic money is created to finance the remainder of the principal plus interest payments--it does not finance the primary deficit in the current period, at all.

Here we have log [government bonds sold to domestic and foreign residents] = [log (nominal time t deficit)] + θ_1 [log (principal plus interest payments to retire last period's debt)].

Denoting bonds sold to domestic residents as b_t and those to foreign residents by the expression for net capital inflow, $_{11}$ (r_t - we have the following expression for bond financing:

$$\gamma_{12}b^d_t - \gamma_{13}i_t + \gamma_{11}(r_t - r^*_t) = (\gamma_{15} - \gamma_{16})p_t$$

$$+ \gamma_{15}g_t - \gamma_{16}t_t + \theta_1 (\gamma_{15} - \gamma_{16}) p_{t-1}$$

$$+ \theta_1\gamma_{15}g_{t-1} - \theta_1 \gamma_{16} \frac{t_t}{\psi_t} - \theta_1(s_{t-1} - \frac{i^*_t}{\psi_t} - \emptyset)$$

Substituting in for i_t from the expression for uncovered nominal interest arbitrage, and for $(r_t - r_t^*)$ from the expression (6) we obtain:

$$\gamma_{12}b_t + (\gamma_{13} - \gamma_{11})s_t + [\gamma_{11} + (\gamma_{15} - \gamma_{16})]p_t$$

$$= \gamma_{11}E_t p_{t+1} + (\gamma_{13} - \gamma_{11})E_t s_{t+1} - \theta_1 s_{t-1}$$

$$+ \theta_1(\gamma_{15} - \gamma_{16})p_{t-1} + \gamma_{15}g_{t-1}(\theta_1 + \psi_g)$$

$$- \gamma_{16}t_{t-1}(\theta_1 + \psi_t) - \gamma_{11}(1-\overset{*}{\psi})\overset{*}{p}_t$$

$$+ i_t^*\left(\gamma_{13} + \frac{\theta_1}{\overset{*}{\psi}_i}\right) + \theta_1 \emptyset \qquad (22)$$

where the variables are as defined earlier, $\emptyset$ is $E_t(E_{t-1}s_t)$. The domestic money creation is given by, in levels: d_t = [total amount required for principal plus interest payments on last period's debt net of amount of debt rolled over by issuing new debt].

The logarithmic expression for domestic money creation is then:

$$d_t = \theta_2 p_{t-1}(\gamma_{15} - \gamma_{16}) + \theta_2 \gamma_{15} g_{t-1}$$

$$- \theta_2 \gamma_{10} t_{t-1} + \theta_2 i_{t-1}^* + {}_2\emptyset - \theta_2 s_{t-1} \qquad (23)$$

where $\theta_2 = - K_1\theta_1$, i.e., a greater amount of debt rolled over necessitates less domestic money creation.

It should be noted here that the rule (choice of θ_1 and θ_2) for determining the amount of debt-financing need not necessarily be a conscious deficit-financing strategy hammered out by the Treasury and the Federal Reserve. It could be the result of the independent behaviors of the above two decision-making bodies. W. Michael Cox (1985) has determined that a 'rule' similar to the one used in this thesis does, in fact, exist. Over the 1950-81 period, according to Cox, each $1 of interest paid on the Federal Government's debt was, on average, financed with only 41 cents in taxes. The 59 cent remainder was deficit-financed. Furthermore, a statistical analysis of federal deficits has revealed a shift in this 'rule' in the early 1970's--interest payments post January 1971 have been totally deficit-financed. In the economy of this thesis, tax rates are held exogenously fixed and any change in the expected time-paths of these revenues is fungible between principal and primary deficit financing.

2.4 Model Summary: Case I

1. $$y_t^d = k_d - \gamma_0\tau + \gamma_1 w_t + \gamma_2 g_t + \gamma_3 \rho_t + \gamma_4 y_t^* + \varepsilon_t^d$$

Note: $\rho_t = p_t^* + s_t - p_t$

2. $$y_t^s = \bar{y} + \gamma(p_t - E_t p_{t+1}) + \varepsilon_t^s$$

3. $m_t = \psi_d d_{t-1} + \overline{f} + \varepsilon_t^m$

4. $m_t^d = \gamma_5 y_t + p_t - \gamma_6 i_t$

5. $0 = [K_n + \gamma_7 y_t^* + (\gamma_8 + \gamma_{10})\rho_t - \gamma_9 y_t + \varepsilon_t^n]$

$+ \gamma_{11} (r_t - r_t^*)$

6. $r_t - r_t^* = E_t\rho_{t+1} - \rho_t = (E_t p_{t+1}^* - p_t^*)$

$+ (E_t s_{t+1} - s_t) - (E_t p_{t+1} - p_t)$

7. $\gamma_{12} b_t^d - \gamma_{13} i_t + \gamma_{11}(r_t - r_t^*) + \gamma_{14} d_t$

$= (\gamma_{15} - \gamma_{16}) (p_t + y_t)$

$+ (\gamma_{15} K_1 - \gamma_{16}\tau) + \gamma_{17} b_{t-1}^p$

8. $w_t = \gamma_{20} b_{t-1} + \gamma_{21} d_{t-1} - p_t$

(a) $g_t = K_1 + y_t$

(b) $t_t = \tau + y_t$

(c) $i_t = i_t^* + [E_t s_{t+1} - s_t]$

(d) $r_t = i_t - [E_t p_{t+1} - p_t]$

(e) $r_t^* = i_t^* - [E_t p_{t+1}^* - p_t^*]$

where

y_t^d = demand for domestic output

y_t^s = real domestic output supply

p_t = price of domestic output in units of domestic currency (dollars)

s_t = nominal exchange rate (the number of units of domestic currency per unit of foreign currency).

i_t = domestic nominal interest rate

w_t = real domestic wealth

τ = tax collection (marginal rate)

K_1 = government's consumption share of current output

g_t = real government consumption of domestic output

ρ_t = real exchange rate (the relative price of foreign goods in the domestic currency with respect to the price of domestic goods in the domestic currency)

p_t^* = price of foreign output in units of foreign currency

y_t^* = foreign real output

$\bar{y}$ = trend rate of growth of the domestic output

m_t = total money supply

d_t = domestic component of the money supply

f_t = level of reserves

b_t^d = domestic nominal demand for one-period discounted government bonds

b_t^p = principal repayment for government bonds purchased in period t by domestic and foreign residents

ε_t = iid shock, zero mean finite variance.

Equations (1) - (4) are the goods and money market supplies and demands, respectively. Equation (5) is the balance of payments equation, (6) is the expression for the difference in domestic and foreign real interest rates, (7) is the consolidated budget constraint, (8) is the definition for real wealth and (a) - (h) are definitions of fiscal policy, uncovered nominal interest parity, and the Fisher equations.

2.5 Model Summary: Case II

The equations for supply and demand in the goods and money market, as well as the expressions for the balance of payments and the real interest rate difference, are identical to those of Case I, i.e. Equations (1), (2), (5), (6). The Case II-specific equations are:

22. $$\gamma_{12}b_t^d + (\gamma_{13} - \gamma_{11})s_t$$

$$+ [\gamma_{11} + (\gamma_{15} - \gamma_{16})]p_t$$

$$= \gamma_{11}E_t p_{t+1} + (\gamma_{13} - \gamma_{11})E_t s_{t+1}$$

$$- \theta_1 s_{t-1} + \theta_1 (\gamma_{15} - \gamma_{16})p_{t-1}$$

$$+ \gamma_{15} g_{t-1} (\theta_1 + \psi_g) - \gamma_{16} t_{t-1} (\theta_1 + \psi_t)$$

$$- \gamma_{11}(1 - \psi_p^*)p_t^* + i_t^* (\gamma_{13} + \frac{\theta_1}{\psi_i^*}) + \theta_1 \emptyset$$

$$d_t = \theta_2 p_{t-1} (\gamma_{15} - \gamma_{16}) + \theta_2 \gamma_{15} g_{t-1}$$

$$- \theta_2 \gamma_{15} t_{t-1} + \theta_2 i^*_{t-1} + \theta_2 \emptyset - \theta_2 s_{t-1}$$

These are the expressions for the 'rules' of bond and money financing incorporated into the government budget constraint. Here, θ_1 is the fixed portion of principal payment that is rolled over, and $\theta_2 = -K_1.\theta_1$ where θ_2 is the portion of principal payment that is paid off by domestic money creation.

CHAPTER III

THE SOLUTION TECHNIQUE

The solution-technique and the solutions for Case I and Case II are presented here. This technique is an adaptation of the method of undetermined coefficients to a matrix system, and is similar to that of Aoki and Canzoneri (1979).

3.1 Solutions for Case I

The systems of equations for Case I is reduced to a system of three equations in quasi-reduced form, with p_t, s_t and w_t as the endogenous variables. This 3 x 3 matrix system is presented in Appendix A.

Defining the following vectors:

$$\underline{p}_t = [p_t \; s_t \; w_t]$$

$$\underline{p}_t^* = [p_t^* \;\; y_t^* \;\; i_t^*]$$

$$\underline{d}_{t-1} = [d_{t-1}{}^{\tau} \;\; K_1]$$

$$\underline{\bar{y}} = [\bar{y} \;\; \bar{f} \;\; \emptyset]$$

$$\underline{\varepsilon}_t = [\varepsilon_t^s \;\; \varepsilon_t^d \;\; \varepsilon_t^m]$$

Using this vector notation, it is possible to reduce the model to the quasi-reduced form given below:

$$A\underline{p}_t = BE_t\underline{p}_{t+1} + \lambda\underline{p}_{t-1} + C\underline{\bar{y}} + D\underline{d}_{t-1}$$

$$+ E\underline{p}^*_{t-1} + F\underline{\varepsilon}_t \qquad (12)$$

$\underline{p}_t$ is the vector of endogenous variables. $\underline{p}^*_t$ is the vector of exogeneous disturbances emanating from the rest of the world, $\underline{d}_{t-1}$ is the vector of domestic policy instruments, $\underline{\varepsilon}_t$ is the vectors of domestic and foreign unobservable taste shift disturbances. ε_t represents the underlying uncertainty in the model and it is assumed to be iid, zero mean, finite variance. A, B, λ , C, D, E, and F are all 3 x 3 matrices with model parameters as the elements.

Suppose that:

$$\underline{p}^*_t = \underline{\psi}^* p^*_{t-1} \qquad (13)$$

That is, domestic residents perceive the foreign price level, interest rate and output to follow an exogeneously determined process given by (13) above--the United States is considered 'small' relative to the rest of the world, and hence it takes foreign prices, output and interest rates as exogenously given. Again, ψ is a 3 x 3 matrix. Assuming the above process is not really restrictive, as any general autoregressive process or a relatively large class of general ARMA processes could have been chosen.

In (12) the current and expected future values of the vector of foreign prices, output, and interest rates have been expressed in terms of the values of the above three variables lagged one period. This has been accomplished by applying the Weiner-Kolmogorov procedure to equation (13). This procedure is described in Appendix B.

Using the method of Undetermined Coefficients for a matrix system, we 'guess' the following solution, and then verify that it is in fact correct. A reasonable guess is:

$$\underline{p_t} = \underline{\pi_o p_t^*} + \underline{\pi_1 d_t} + \pi_2 \underline{\bar{y}} + \pi_3 \underline{\varepsilon}_t + \pi_4 \underline{v}_t^* \tag{14}$$

The next step is to solve for the unknown coefficients π_o to π_3, where each coefficient is a 3 x 3 matrix of unknowns. To substitute (14) into (12), the following manipulations have been made:

Leading (14) one period, taking expectations at time t, and applying Weiner-Kolmogorov:

$$E_t \underline{p}_{t+1} = \pi_o \psi^{*2} \underline{p}^*_{t-1} + \pi_1 \psi_d^2 \underline{d}_{t-1} + \pi_1 \psi_d E_t \underline{\varepsilon}_t$$

$$+ \pi_2 \underline{\bar{y}} + \pi_3 E_t \underline{\varepsilon}_{t+1} \tag{15}$$

The vector $E_t \varepsilon_{t+1}$ is zero by definition, but $E_t \underline{\varepsilon}_t$ is not. This is because the conditional expectation of the vector $\underline{\varepsilon}_t$, $E_t[\varepsilon_t/\Omega_t]$, where

Ω_t is the set of all available information, namely the current observed price level, the nominal exchange rate, domestic wealth, and the exogeneously given policy and trend variables, is non-zero. From equation (14) it can be seen that if agents observe the vector of endogeneous variables, $\underline{p}_t$ (which they do) and if $\underline{P}_t^*$, $\underline{d}_t$ and $\bar{y}$ are exogeneously given and known to all, then the expectation of ε_t, conditional on the observed pieces of information can be explictly determined by agents so that $E_t[\varepsilon_t/\Omega_t]$ is simply $\underline{\varepsilon}_t$. It should be noted that since $E_t\underline{\varepsilon}_t$ can be explicitly determined in this economy, agents are never in doubt regarding the source of the fluctuations (i.e., stemming from real or nominal shocks) in the observed endogeneous variables, P_t, s_t, and ω_t, and consequently they are not forced to indulge in the signal extractions characterized by a Lucasian economy. Three combinations of the three shocks, ε_t^s, ε_t^d and ε_t^m are observed here, and this reduces the signal extraction process to a simple conditional forecast.

Lagging the guess (14) by one period we obtain:

$$\underline{p_{t-1}} = \pi_0 \underline{p^*_{t-1}} + \pi_1 \underline{d_{t-1}} + \pi_2 \underline{\bar{y}} + \pi_3 \underline{\varepsilon}_{t-1} \qquad (16)$$

The next step is to solve for the undetermined coefficient 3 x 3 matrices, the π's. The solutions are divided into two parts with respect to the coefficients of the policy variables and the shocks. One advantage of using the solution-technique of undetermined coefficients is that, in this case, all the π's do not have to be solved simultaneously. This piece-wise solution technique enables us to solve for the coefficients of the policy instruments without simultaneously solving for the coefficients of the stochastic elements.

The former, π_1, presented below, followed by the solution of the coefficient of the vector of the shocks, π_3.

Substituting (14), (15) and (16) minus the stochastic element terms into (12) we obtain:

$$A\,[\pi_0 \underline{p}_t^* + \pi_1 \underline{d}_t + \pi_2 \bar{\underline{y}}]$$

$$= B[\pi_0\, \psi^2 \underline{p}_{t-1} + \pi_1 \psi_d \underline{d}_{t-1} + \pi_2 \bar{\underline{y}}]$$

$$+ \lambda[\pi_0 p^*_{\underline{t-1}} + \pi_1 d_{\underline{t-1}} + \pi_2 \underline{\bar{y}}\,]$$

$$+ \underline{c}\bar{y} + dd_{\underline{t-1}} + ep^*_{\underline{t-1}} \qquad (17)$$

Rational expectations have been imposed here implicitly in the solution technique because Equation (14), which is the 'guessed' solution is in fact in the form of the guessed stochastic process governing p_t. The right-hand side of (14), $\pi_0 \underline{p}_t^* + \pi_1 \underline{d}_t + \pi_2 \bar{\underline{y}} + \pi_3 \underline{\varepsilon}_t$, is a probability distribution whose specific nature is yet to be determined. Expectations are based on this distribution, conditional on all available information. By solving for the parameters of the stochastic process in terms of the underlying structural parameters of the model, conditional expectations formed by agents will, on average, be correct.

Using (13) and the rule for domestic money creation, $d_t = \psi_d \, d_{t-1} + \varepsilon_t^m$, and equating coefficients of the vector of foreign variables, $\underline{p}_{t-1}^*$:

$$A\pi_o = B\pi_o + \lambda \, \pi_o(\psi^*)^{-1} + E$$

Simplifying:

$$\pi_o = [A - B + \lambda(\psi^*)^{-1}]^{-1} \; . \; E$$

Equating coefficients of the vector of policy instruments, d_{t-1}:

$$A\pi_1 \, \psi_d = B\pi_1 \psi^2{}_d + \lambda\pi_1 + D$$

Simplifying:

$$\pi_o = [A \, \psi_d - B \, \psi_d - \lambda]^{-1} \; . \; D$$

and equating coefficients of the vector of constants, $\underline{\bar{y}}$,

$$A \, \pi_2 = B \, \pi_2 + \pi_2 + C$$

Simplifying:

$$\pi_2 = [A - B - \lambda]^{-1} \; . \; C$$

The next step is to solve for each of the elements of the 3 x 3 matrix, π_1, where π_k^{ij} will be the element in row i and column j of the matrix k. The matric system is represented in the following truncated form for computational simplicity:

$$\begin{bmatrix} a_0 & -a_1 & -a_2 \\ a_3 & a_4 & 0 \\ a_5 & a_6 & 0 \end{bmatrix} \begin{bmatrix} p_t \\ s_t \\ w_t \end{bmatrix} = \begin{bmatrix} b_0 & 0 & 0 \\ b_1 & b_2 & 0 \\ b_3 & b_4 & -b_5 \end{bmatrix} \begin{bmatrix} E_t p_{t+1} \\ E_t s_{t+1} \\ E_t w_{t+1} \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda_0 & -\lambda_0 & \lambda_1 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ s_{t-1} \\ w_{t-1} \end{bmatrix} +$$

$$+ \begin{bmatrix} 0 & -d_0 & d_1 \\ d_2 & 0 & 0 \\ d_3 & -d_4 & d_5 \end{bmatrix} \begin{bmatrix} d_{t-1} \\ \tau \\ k_1 \end{bmatrix} + \begin{bmatrix} e_0 & e_1 & 0 \\ 0 & 0 & e_2 \\ e_3 & e_4 & e_5 \end{bmatrix} \begin{bmatrix} p^*_{t-1} \\ y^*_{t-1} \\ i^*_{t-1} \end{bmatrix} + \ldots \tag{18}$$

The elements a_i, b_i, λ_i, d_i and e_i $(0 \leq i \leq 6)$ directly coincide with their corresponding elements in the detailed matrix system, 12(a), presented in Appendix A.

The system (19) then simplifies to:

$$\begin{bmatrix} p_t \\ s_t \\ w_t \end{bmatrix} = \frac{1}{|D|} \begin{bmatrix} 0 & a_i(b_5-\lambda_1)-a_2\lambda_0 & 0 \\ -(a_3\psi_d-b_1\psi_d^2)(b_5-\lambda_1) & i_0 i_6+i_2 i_4 & -a_2(a_3\psi_d)-b_1\psi_d^2) \\ (a_3\psi_d - b_1\psi_d^2)\ \lambda_0) & (-1)(i_0 i_5+i_1 i_4) & i_1 i_3 \end{bmatrix} \cdot$$

$$\cdot \begin{bmatrix} 0 & -d_0 & d_1 \\ d_2 & 0 & 0 \\ d_3 & -d_4 & d_5 \end{bmatrix} \begin{bmatrix} d_{t-1} \\ \tau \\ K_1 \end{bmatrix} + \ldots$$

where $|D|$ is the determinant of the inverted matrix:

$$|D| = (a_3\psi_d - b_1 \psi_d^2)\, [a_1(b_5 - \psi_1) - a_2 \lambda_o]$$

The final solutions of the three domestic endogenous variables with respect to the policy instruments are:

$$p_t = \left[\frac{1}{a_3 - b_1\psi_d}\right] d_{t-1} + \dots \tag{19}$$

$$s_t = \left[\left[\frac{(a_o-b_o\psi_d)(b_5-\lambda_1)+a_2(a_5-b_3\psi_d)}{(a_3-b_1\psi_d)[a_1(b_5-\lambda_1)-a_2\lambda_o]}\right]\psi_d - \frac{a_2d_3}{a_1(b_5-\lambda_1)-a_2\lambda_o}\right] d_{t-1}$$

$$+ \left[\frac{\beta_5(b_5-\lambda_1) + a_2d_4}{a_1(b_5-\lambda_1) - a_2d_o}\right] \tau \;\dot{-}\; \left[\frac{\beta_7(b_5-\lambda_1) + a_2d_5}{a_1(b_5-\lambda_1) - a_2\lambda_o}\right] K_1 + \dots \tag{20}$$

$$w_t = \left[\frac{-\psi_d[a_o\psi_d-b_o\psi_d^2)\lambda_o + a_1i_4]}{\psi_d(a_3-b_1\psi_d)[a_1(b_5-\lambda_1)-a_2\lambda_o]} + \frac{a_1d_3}{a_1(b_5-\lambda_1)-a_2\lambda_o}\right] d_{t-1}$$

$$+ \left[\frac{-1(\lambda_o\beta_5+a_1d_4}{[a_1(b_5-\lambda_1]-a_2\lambda_o]}\right] \tau + \left[\frac{\lambda_o\beta_7 + a_1d_5}{[a_1(b_5-\lambda_1)-a_2\lambda_o]}\right] K_1 + \dots \tag{21}$$

The solution of π_3, the coefficient of the 3 x 1 vector of the stochastic elements, is presented below. This vector ε_t in (14) is comprised of three identical and independently distributed shocks with zero means and finite variances. The supply and demand shocks, ε_t^s and ε_t^d, are real, while the nominal shock stemming from the domestic money supply process is ε_t^m.

Substituting the guess, (14), into the vector representation of the matrix system, (12) and following the same procedure used to obtain the π_1's above, the coefficient of the vector of the stochastic elements is obtained. This coefficient π_3, obtained after equating the coeffcients of $\underline{\varepsilon}_t$, is:

$$\pi_3 = (a)^{-1} [\ \pi_1 \ (b\ \psi_d - a) + f]$$

The next step then is to solve for each of the elements of the 3 x 3 matrix π_3, where π_3^{ij} is the coefficient in row i and column j of the matrix π_3.

The matrix $(b\psi_d - a)$ is:

$$\begin{bmatrix} \gamma - a_0 & a_1 & a_2 \\ \gamma_5 - a_3 & 0 & 0 \\ \gamma_{15} - \gamma_{16} & 0 & \frac{-\gamma_{12}}{\gamma_{13}} \psi_d \end{bmatrix}$$

To premultiply this by π_1:

$$\begin{bmatrix} \pi_1^{11} & 0 & 0 \\ \pi_1^{21} & \pi_1^{22} & -\pi_1^{23} \\ -\pi_1^{31} & \pi_1^{32} & -\pi_1^{33} \end{bmatrix} \begin{bmatrix} \gamma - a_0 & a_1 & a_2 \\ \gamma_5 - a_3 & 0 & 0 \\ \gamma_{15}-\gamma_{16} & 0 & \frac{-\gamma_{12}}{\gamma_{13}}\psi_d \end{bmatrix}$$

Simplifying, we obtain the following matrix for $\pi_1(b\psi_d - a) + \frac{f}{J}$:

$$\begin{bmatrix} \pi_1^{11}(\gamma-a_o)-1 & \pi_1^{11}a_1 + \frac{1}{1-a_2} & \pi_1^{11}a_2 \\ \pi_1^{21}(\gamma-a_o)+\pi_1^{22}(\gamma-a_3)-\pi_1^{23}(\gamma_{15}-\gamma_{16})-\gamma_5 & \pi_1^{21}a_1 & (\pi_1^{21}a_2)(\pi_1^{23}\frac{\gamma_{12}}{\gamma_{13}}\psi_d) + 1 \\ -\pi_1^{31}(\gamma-a_o)+\pi_1^{32}(\gamma_5-a_3)-\pi_1^{33}(\gamma_{15}-\gamma_{16})+(\gamma_{15}-\gamma_{16}) & \pi_1^{31}a_1+f_3 & \pi_1^{31}a_2+\frac{\gamma_{12}}{\gamma_{13}}\psi_2\,\pi_1^{33} \end{bmatrix}$$

The next step is to obtain $(a)^{-1}(\pi_1(b\psi_d - a) + f)$. The determinant of (a), $|A|$, is $-a_2a_3a_6$ or $-\beta_2(1+\gamma\gamma_5)(\gamma_{13}-\gamma_{11})$ and it is < 0. The matrix $\pi_1(b\psi_d - a) + f$ is then pre-multiplied by the inverse of (a), $(a)^{-1}$, which is presented below.

$$\frac{1}{|A|}\begin{bmatrix} 0 & -a_2a_5 & a_2a_4 \\ 0 & a_2a_5 & -a_2a_3 \\ a_oa_6-a_4a_5 & -(a_oa_6+a_1a_5) & a_oa_4+a_1a_3 \end{bmatrix}$$

This finally simplifies to the following solution of the domestic price level, p_t, with respect to the real and nominal shocks;

$$p_t = \frac{1}{|A|}[\, \pi_3^{11}\ \varepsilon_t^s - \pi_3^{12}\ \varepsilon_t^d - \pi_3^{13}\ \varepsilon_t^m] + \ldots \qquad (22)$$

where $|A| < 0$ and the coefficients π_3^{ij}, $i,j \leq 3$ are presented in detail in Appendix C.

Similarly the solutions of the nominal exchange rate and the domestic wealth balances with respect to the shocks, are:

$$s_t = \frac{1}{|A|}[\, -\pi_3^{21}\ \varepsilon_t^s + \pi_3^{22}\ \varepsilon_t^d - \pi_3^{23}\ \varepsilon_t^m] + \ldots \qquad (23)$$

and

$$\omega_t = \frac{1}{|A|} [-\pi_3^{31} \varepsilon_t^s + \pi_3^{32} \varepsilon_t^d + \pi_3^{33} \varepsilon_t^m] + \ldots \quad (24)$$

The explicit solutions of each of the coefficients π_3^{ij} are presented in Appendix C.

3.2 Solutions for Case II

The system of equations for Case II is reduced to a system of four equation sin quasi-reduced form, with p_t, s_t, b_t^d and ω_t as the endogeneous variables. This 4 x 4 matrix system is presented in Appendix D.

Defining the following vectors:

$$\underline{p}_t = [p_t \ \ s_t \ \ b_t^d \ \ \omega_t]$$

$$\underline{p}_t^* = [p_t^* \ \ y_t^* \ \ i_t^*]$$

$$\underline{g}_{t-1} = [g_{t-1} \ \ t_{t-1}]$$

$$\underline{\varepsilon}_t = [\varepsilon_t^s \ \ \varepsilon_t^d \ \ \varepsilon_t^m]$$

The matrix system can then be represented in the quasi-reduced form given below:

$$A_2 \underline{p}_t = B_2 E_t \underline{p}_{t+1} + C_2 \underline{p}_{t-1} + D_2 \underline{g}_{t-1} + E_1 \underline{p}^*_{t-1} + F_2 \underline{\varepsilon}_t + G_2 \bar{\underline{y}}$$

The vectors $\underline{p}_t$, $\underline{p}^*_t$ and $\underline{g}_{t-1}$ are the vectors of the domestic endogeneous variables, foreign prices, and domestic policy instruments, respectively. The matrices A_2-G_2 are 4 x 4 matrices of structural parameters. The vector of foreign prices follows an exeogeneous process identical to (13) in Case I. The vector $\bar{\underline{y}}$ is a vector of the trend components.

Using a solution technique identical to that used earlier, we 'guess' a solution and then verify that the guess is in fact correct. This is, once again, the method of undetermined coefficients adapted to a matrix system.

A reasonable guess is;

$$\underline{p}_t = \pi_0 \underline{p}^*_t + \pi_1 \underline{g}_{t-1} + \pi_2 \underline{\varepsilon}_t + \pi_3 \bar{\underline{y}}$$

obtaining expressions for p_{t-1} and $E_t p_{t+1}$ by manipulating the above guess and using the Weiner-Kolmogrov procedure (Appendix B) we get the solution for the 4 x 4 matrix π_1. This is the coefficient on the vector of policy instruments, $\underline{g}_{t-1}$, in the solution of the endogeneous vector $\underline{p}_t$.

The solution of π_1 is:

$$\pi_1 = [\ a - b\psi - c/\psi\ ]^{-1}\ [d]$$

The 4 x 4 matrix is to be inverted, $a - b\psi - c/\psi$ is:

$$\begin{bmatrix} a_o - b_o\psi_g & -a_1 & 0 & -a_2 \\ a_3 - b_1\psi_g + c_o/\psi_g & a_4(1 - \psi_T) - \theta/\psi_T & 0 & 0 \\ -(a_5 + b_2\psi_g) & a_6 - b_3\psi_T & 0 & 0 \\ a_7 + b_3\psi_g - c_o/\psi_g & a_8(1-\psi_T) + \theta/\psi_T & a_9 & 0 \end{bmatrix}^{-1}$$

For the sake of computational simplicity, the above matrix is re-written as:

$$\begin{bmatrix} k_o & -k_1 & 0 & -k_2 \\ k_3 & k_4 & 0 & 0 \\ -k_5 & k_6 & 0 & 0 \\ k_7 & k_8 & k_9 & 0 \end{bmatrix}^{-1}$$

where each of the k's correspond to the symmetrically located element in the previous matrix, $a-b\psi-c/\psi$. This inverse is multiplied to the 4 x 4 matrix d:

$$\frac{1}{|J|}\begin{bmatrix} 0 & k_2k_6k_9 & -k_2k_4k_6 & 0 \\ 0 & k_2k_5k_9 & k_2k_3k_9 & 0 \\ 0 & -k_2(k_5k_8+k_6k_7) & -k_2(k_3k_8-k_4k_7) & k_2(k_3k_6+k_4k_5) \\ -k_a(k_3k_6-k_4k_5) & k_9(k_ok_6-k_1k_5) & -k_9(k_ok_4+k_1k_3) & 0 \end{bmatrix}\begin{bmatrix} d_o & -d_1 & 0 & 0 \\ -d_2 & d_3 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ d_2 & -d_3 & 0 & 0 \end{bmatrix}$$

where $|J| = k_2K_9(k_3k_6 - k_4k_5) < 0$

The solution of the domestic price level with respect to the policy intruments, obtained after performing the above computation, is:

$$p_t = \left[\frac{-d_2k_6}{k_3k_6 + k_4k_5}\right] g_{t-1} + \left[\frac{d_3k_6}{k_3k_6 + k_4k_5}\right] t_{t-1} + \ldots \qquad (28)$$

and the solution of the domestic nominal exchange rate with respect to the policy instruments is:

$$s_t = \left[\frac{-d_2 k_5}{k_3 k_6 + k_4 k_5} \right] g_{t-1} + \left[\frac{d_3 k_5}{k_3 k_6 + k_4 k_5} \right] t_{1-1} + \ldots \qquad (29)$$

CHAPTER IV

RESULTS

This chapter is divided into three sections. In IV(i), the solutions of Case I for the three endogenous variables p_t, s_t, w_t with respect to the policy instruments are analyzed and interpreted, respectively. The solutions of the vector of the stochastic elements, ε_t, and their effect on the volatility of prices and exchange rates for Case I is presented in IV(ii). This is followed by the presentation and analysis of the solutions of the coefficients of the policy variables for Case II, in IV(iii), and a historical analog of Case II in IV(iii)(a).

4.1 Solutions with Respect to the Policy Instruments: Case I

Examining the solution of p(t), the domestic price level, we obtain our first result--budget deficits are non-inflationary in this economy. The solution for the domestic price level as a function of policy instruments resembles a standard quantity theory result. The price level is affected by the stock and growth rate of domestic money only, and given by:

$$p_t = [\frac{1}{a_3 - b_1 \psi_d}] \quad d_{t-1} + \quad \ldots \tag{19}$$

where d_{t-1} is last period's domestic money creation, given by

$$d_t = \psi_d d_{t-1} \text{ with } \psi_d > 1,$$

$$a_3 = 1 + \gamma\gamma_5$$

$$b_1 = \gamma_5$$

Result (19) states that the domestic price level p_t, is completely unaffected, or neutral to, change in fiscal policy, i.e. neither K_1 nor τ influence the domestic price level. Furthermore, this implies that domestic output, y_t is also neutral to changes in the government's share of current output or the tax rate. The intuition and interpretation of these results is provided after first examining the solutions of the nominal exchange rate and the interest rate, because these following solutions shed further light on the neutrality results obtained above.

The solution of the nominal exchange rate as a function of the policy instruments is:

$$s_t = \left[\left[\frac{(a_0-b_0\psi_d)(b_5-\lambda_1)+a_2(a_5b_3\psi_d)}{(a_3-b_1\psi_d)[a_1(b_5-\lambda_1)-a_2\psi_0]}\right]\psi_d - \frac{a_2d_3}{a_1(b_5-\lambda_1)-a_2\lambda_0}\right]d_{t-1}$$

$$+ \left[\frac{\beta_5(b_5-\lambda_1) + a_2d_4}{a_1(b_5-\lambda_1) - a_2d_0}\right]\tau - \left[\frac{\beta_7(b_5-\lambda_1) + a_2d_5}{a_1(b_5-\lambda_1) - a_2\lambda_0}\right]K_1 + \ldots \qquad (20)$$

The parameters $a_i - e_i$, $0 \leq i \leq 6$, have been defined earlier in Section III and Appendix A.

Examining the solution of the nominal exchange rate, the following results are obtained.

(i) Increases in either domestic money creation or the tax rate cause the domestic currency to depreciate--the dollar gets 'weaker'. An increase in either d_t or τ would mean that fewer discounted government bonds would have to be sold to finance the deficit. This decreased supply of government debt drives up bond prices. This in turn causes bond yields to fall, resulting in a lowering of domestic nominal interest rates. With stable prices, this means that the domestic and foreign real interest rate differential, $r_t - r^*_t$, would decrease, resulting in a lower foreign demand for U.S. government bonds. In a regime of floating exchange rates, this decrease in overseas demand for dollar-denominated assets and the consequent decrease in the demand for the U.S. dollar in the foreign exchange markets, causes the domestic currency to depreciate, or, the dollar gets weaker.

(ii) As the government increases its consumption share of current output, K_1, ceteris paribus, it causes the domestic currency to appreciate--the dollar gets 'stronger.' In this case, with domestic money held fixed, any increases in K_1, with the tax rate, τ, held constant, have to be financed with the issuance of larger numbers of bonds to domestic and foreign residents. This increased supply of government debt drives down bond prices, causes domestic and real interest rates to rise, resulting in a greater demand for dollar-denominated assets and hence an appreciation of the domestic currency. In this economy, both the nominal and real exchange rates, s_t and ρ_t, appreciate.

We are now in a position to interpret the neutrality of the domestic price level, p_t, and the domestic output, y_t, with respect to increases in domestic budget deficits. More specifically, we examine why prices and output are neutral to changes in fiscal policy, even when individuals treat government deficits as permanent and incorporate the bonds which finance these deficits into their current wealth holdings.

The intuition behind this result is as follows. In this economy, the government can increase its consumption share of current output, K_1, only by issuing more bonds. As discussed earlier, this increased supply of bonds causes the domestic nominal and real interest rates to rise. Individuals do not discount future tax liabilities here, and they incorporate these bonds into their wealth holdings. But the reason that there are no aggregate demand or price effects stemming from changes in K_1, is that with the incipient nominal and real appreciation of the domestic currency (explained earlier), domestic imports are 'cheaper' for the U.S. residents, and the increase in domestic aggregate demand caused by the increased wealth holdings literally overflows into the foreign sector.[11] That is, the domestic country incurs a current account deficit due to the appreciation of the real exchange rate, ρ_t, and this is why the domestic aggregate demand effects manifest themselves in the form of stronger real exchange rates and a subsequent current account deficit without affecting domestic prices or output.

The solution for domestic real wealth is:

$$w_t = \left[\frac{[a_0\psi_d - b_0\psi_d^2]\lambda_o + a_1 i_4}{\psi_d(a_3-b_1\psi_d)]a_1(b_5-\lambda_1)-a_2\lambda_o} + \frac{a_1 d_3}{a_1(b_5-\lambda_1)-a_2\lambda_o)}\right] d_{t-1}$$

$$+ \left[\frac{-1(\lambda_o\beta_5+a_1d_4)}{a_1(b_5-\lambda_1)-a_2\lambda_o}\right]\tau + \left[\frac{\lambda_o\beta_7+a_1d_5}{a_1(b_5-\lambda_1)-a_2\lambda_o}\right] K_1 + \ldots \qquad (21)$$

where, $i_4 = \psi_d(a_5 - b_3\,\psi_d) - \lambda_o$, and the rest of the parameters are as defined in Chapter III.

$$[a_1(b_5 - \lambda_1) - a_2\lambda_o] < 0, \quad i_4 < 0, \quad \lambda_o < 0.$$

From the solution (21), domestic real wealth balances decrease with increases in the component of domestic money, d_t, or its rate of growth ψ_d. This is because increases in the domestic price level caused by increases in domestic money, erode real wealth balances. From (21) it can also be seen thatiIncreases in the fraction of GNP the Treasury receives as tax revenues, τ , cause domestic real wealth balances to increase. These increases in τ cause the outstanding component of government debt to decrease, thus freeing resources for the private sector. Furthermore, they cause the domestic currency to depreciate, as depicted in solution (20) by causing real and nominal rates to fall, and by leading to a decrease in the real interest differential. Therefore, there is a decreased demand for domestic government bonds from the foreign sector, and a larger percentage of the bonds outstanding are held by domestic residents. The 'mix' of government debt outstanding (held by domestics and foreigners) has shifted, so that a greater debt load is borne by domestic residents as government bonds are now less attractive to foreigners.

The conclusion of the presentation of interpretation of the results of the three endogenous variables brings us to the issue of sustainability. Are debt-financed deficits sustainable in an economy when debt is continuously rolled over and domestic money creation is held fixed? The answer is no; a debt-financing policy such as the one in this economy, is weakly non-sustainable as explained below.

The domestic output is neutral with respect to systematic changes in the fiscal policy and deviations from trend are obtained only by unanticipated disturbances. Therefore, we have a situation identical to the simple example discussed earlier in chapter I; increases in the deficit (increases in K_1 ceteris paribus) cause the real interest rate to increase without having any effect on the trend rate of growth of domestic output. As larger deficits are incurred each period due to the mounting outstanding principal payments on previous debt issuance, it is inevitable that the real rate of interest will equal and eventually exceed the rate of growth of domestic output, i.e., finally $r_t - \bar{y} > 0$. This situation cannot continue indefinitely, and as discussed in chapter I, one or more of three drastic measures (a large future tax increase, an unanticipated inflation to wipe out real debt, or a repudiation of the outstanding debt) will eventually have to be taken. Therefore, we conclude that a debt-financing policy such as the one described in Case I is weakly non-sustainable because in the long-run, the world economy will refuse to absorb additional government bonds and the Federal Reserve will be forced to step in and monetize a substantial part of the government debt.

In addition to this non-sustainability result, there is the adverse effect of domestic real wealth holdings. Increases in the domestic deficit lead to a deterioration of domestic real wealth balances. Increases in the Treasury's current consumption as a function of GNP, with taxes held fixed, cause an outflow of domestic real wealth. The incurrence of greater domestic deficits leads to higher domestic nominal and real interest rates. These, in turn, cause the domestic and foreign real interest rate differential to increase, resulting in an excess demand for U.S. government bonds from the foreign sector which then accumulates ever-increasing shares of domestic debt. The percentage of U.S. government bonds outstanding held by foreigners thus increases and that held by domestic residents decreases, thus leading to a deterioration of domestic real wealth balances.

The results obtained in this section, closely resemble the behavior of the U.S. economy from 1980-84. This period was characterized by record-high nominal and real exchange rates, unsurpassed domestic and current account deficits, persistently high nominal interest rates, a real rate that has come dangerously close to the rate of growth of output post 1982, and a low and stable inflation rate. These are presented in figures 1 to 4. The monetary and fiscal policy prescription implemented in this model, closely resembles the actual policies exercised during this period in the U.S. The Federal Reserve adopted a 'tight money' policy aimed at curbing the inflation rate with which the U.S. entered this decade, and the Treasury incurred larger deficits as a result of the across-the-board tax cuts. Therefore, the results obtained from the 'tight' monetary and debt-financed policies of Case I, namely, a real and nominal appreciation of the domestic currency, a deterioration of current account and real wealth balances, high nominal and real interest rates relative to the rate of growth of domestic output, and a price level neutral to changes in fiscal policies, replicate and explain the behavior of the U.S. economy from 1980-1984 in a satisfactory and consistent manner.

4.2 Solutions with Respect to the Stochastic Elements: Case I

Modern open-economy macroeconomic models emphasize the idea that the exchange rate is proximately determined in the financial markets, and consequently should be expected to fluctuate like a stock price. Frenkel and Mussa (1976) have described the exchange rate as the relative price of national monies, and Frenkel (1981) shows that exchange rates fluctuate more like stock prices rather than goods prices.

All in all, our experience with flexible nominal exchange rates has been rather sobering. We are constantly reminded that short-term fluctuations in nominal exchange rates are unpredictable. In view of these inherent difficulties, market analysts have adopted one of the following two alternative strategies; (i) to make long-term exchange-rate forecasts, some of which currently include various 'hard-landing' and 'soft- landing' scenarios, and (ii) to predict often by making frequent, and at times quite meaningless, forecasts.

The solution of Case I, in addition to providing information on the sustainability of bond-financed deficits, offers an explanation of the causes of the volatility of prices and nominal exchange rates. This rational-expectations open-economy model is particularly amenable to the analysis of price and exchange rate movements because the forward-looking expectations structure modeled here brings the future conequences of policy into the present, and this is essential for a meaningful analysis of the fluctuations in prices and exchange rates.

The solutions of p_t and s_t with respect to the real and nominal shocks, ε_t^s, ε_t^d and ε_t^m are reproduced here from Chapter III;

$$p_t = \frac{1}{|A|} [\pi_3^{11} \varepsilon_t^s - \pi_3^{12} \varepsilon_t^d - \pi_3^{13} \varepsilon_t^m] + \ldots \quad (22)$$

$$s_t = \frac{1}{|A|} [-\pi_3^{21} \varepsilon_t^s + \pi_3^{22} \varepsilon_t^d - \pi_3^{23} \varepsilon_t^m] + \ldots \quad (23)$$

where $|A|$ is < 0 and presented explicitly in Appendix C along with the π_3^{ij}'s.

The unconditional variance of the price level is obtained from (22) and the fact that ε_t is a vector of i.i.d shocks with zero mean and a finite variance. This is

$$\sigma_p^2 = \frac{1}{|A|^2}\left[(\pi_3^{11})^2\,\sigma_{\varepsilon}^2 s + (\pi_3^{12})^2\,\sigma_{\varepsilon}^2 d + (\pi_3^{13})^2\,\sigma_{\varepsilon}^2 m\right]$$

And the unconditional variance of the nominal exchange rate is:

$$\sigma_s^2 = \frac{1}{|A|^2}\left[(\pi_3^{21})^2\,\sigma_{\varepsilon}^2 s + (\pi_3^{22})^2\,\sigma_{\varepsilon}^2 d + (\pi_3^{23})^2\,\sigma_{\varepsilon}^2 m\right]$$

The first result that is obtained from an analysis of the above variances is that the volatility of the domestic price level and the nominal exchange rate increases with an increase in the growth rate of domestic money, ψ_d. Both the coefficients π_3^{13} and π_3^{23} are functions of ψ_d, and as ψ_d increases representing a 'looser' domestic monetary policy, both these coefficients increase thereby resulting in an increase in the unconditional variance of prices and exchange rates.

The intuition behind this result is as follows. The Fed increases the rate of growth of domestic money and holds it fixed at this new and higher rate of growth, with domestic money creation now being given by:

$$d_t = \psi_d' \, d_{t-1} + \varepsilon_t^m$$

where ψ_d' is the new rate of growth of domestic money, and $\psi_d' - \psi_d > 0$. Since current prices and exchange rates in this rational-expectations economy are functions of future prices and exchange rates, it is imperative that expectations of future monetary policy, say K periods ahead, be formed in the present. Using the Weiner-Kolmogorow procedure outlined in Appendix B, it can be shown that this expectation of future monetary policy, $E_t d_{t+k}$ can be represented by a moving average in terms of observed current and past nominal shocks. This turns out to be

$$E_t d_{t+k} = (\psi_d)^k d_t$$

And since $d_t = \psi_d d_{t-1} + \varepsilon_t^m$ and given that $|\psi_d| < 1$, we implement the backward solution technique using lag operators to obtain an expression for d_t in terms of current and past shocks:

$$d_t(1 - \psi_d L) = \varepsilon_t^m$$

$$\therefore \quad d_t = \sum_{i=0}^{\infty} (\psi_d)^i \varepsilon_{t-i}^m \quad \text{or} \quad d_t = \varepsilon_t^m + \sum_{i=1}^{\infty} (\psi_d)^i \varepsilon_{t-i}^m$$

Substituting this into the expression obtained for $E_t d_{t+k}$ we get:

$$E_t d_{t+k} = (\psi_d)^k \left[\varepsilon_t^m + \sum_{i=1}^{\infty} (\psi_d)^i \varepsilon_{t-i}^m \right]$$

Therefore current prices and exchange rates, which are functions of future prices and exchange rates and consequently of future paths of monetary policy, can be represented as functions of current and past nominal shocks weighted by the rate of growth of domestic money, $(\psi_d)^k$. Thus, it stands to reason that as a 'looser' domestic money policy is adopted and as ψ_d therefore increases, the coefficients of the money shocks (or the 'weights' attributed to these shocks) increase in magnitude thereby resulting in greater unconditional variances of domestic prices and exchange rates.

This result has disturbing implications for domestic monetary policy. As demonstrated earlier in this chapter, the bond-financed deficits of Case I are weakly non-sustainable, and the main ingredient of this non-sustainability is the fact that the real rate of return equals and exceeds the growth rate of the economy. Any attempts by the monetary authority to rectify the situation by adopting a 'looser' money stance to diminish this disparity between the real interest rate and the growth rate would only result in the greater volatility of domestic prices and nominal exchange rates. This result indicates that if the Fed adopts, say, two different Friedman-like x % money growth rates, one with a higher rate of growth of domestic money ('looser') than the other, then these rules will not be equivalent. The rule with the 'looser' domestic money growth will result in a greater volatility of prices and nominal exchange rates, than the rule with 'tighter' domestic money and hence the latter rule is clearly preferable. Therefore, in an economy characterized by Case I, any attempts to shift from a 'tighter' money rule to a 'looser' one, will prove detrimental.

4.3 Solutions with Respect to the Policy Instruments: Case II

The reduced-form solutions of the domestic price level, p_t, the nominal exchange rate, s_t, the domestic demand for government bonds, b_t and the domestic real wealth holdings, w_t, with respect to the policy instrument, g_t and t_t, are presented and interpreted in this section.

The solution of the price level is given by:

$$p_t = \left[\frac{-d_2 k_6}{k_3 k_6 + k_4 k_5} \right] g_{t-1} + \left[\frac{d_3 k_6}{k_3 k_6 + k_4 k_5} \right] t_{t-1} + \ldots \quad (28)$$

The first important result is that in this case where the amount of debt that is rolled over is fixed, the price level is not neutral wtih respect to changes in domestic fiscal policies. In contrast to the results in Case I, increased domestic government spending causes domestic inflation. In this economy, deficits are inflationary.

As taxes remain 'fixed' into the indefinite future, any increase in government expenditures over revenues is financed by the issuance of government bonds, and the remainder is made up by the creation of domestic money. The traditional assumption that government debt is temporary, therefore cannot be made here. In this case, the issuance of debt in lieu of current taxation does stimulate aggregate demand and is, therefore, inflationary. Here, debt issued today is not matched by additional savings because there are no foreseeable future taxes which might be imposed to retire the debt.

This tends to corroborate the findings of Cox that in the period 1950-1984, Federal government debt has not been temporary; i.e., backed by taxes. Government bonds have not been matched by future taxes and bond-financing has contributed to the increase in the price level. He finds that over the 1950-84 period, inflation in the United States was as closely related to the growth in outstanding government debt as to the growth in the monetary base.

The change in the domestic price level, in this model, is influenced by two channels of influence. First, increased sales of bonds drive up the aggregate demand as discussed above. Second, increases in the deficit mean that the absolute amount of money creation for the retirement of last period's debt, is larger, and this in turn exerts an upward force on the price level. Conversely, decreases in the deficit stemming from decreases in governing spending, g_t, or increases in taxes, t_t, cause p_t to fall.

Now that it has been determined that the level of government spending does affect the price level, the next task is to determine how the composition of the government deficits affects p_t--or how θ_1, which is the fraction of principal payment that is bond financed, affects the domestic price level. This, after all, lies at the heart of the issue of the sustainability of domestic budget deficits.

From the non-singularity requirement for matrix $[A\psi_d - B\psi_d - \lambda]$, we get the following restriction which provides us with an expression for the maximum amount of debt that can be rolled over, before domestic money ceases to have any value.

$$\theta_1 < \frac{(a_5-b_3\psi_t)(a_3-b_1\psi_g) + [\beta_0 + (\gamma_{11} - \gamma\gamma_9)\psi_g]\,[a_4(1-\psi_t)]}{\psi_g[\beta_0 + (\gamma_{11} - \gamma\gamma_9)\psi_g] - \psi_t\,(\gamma_{15} - \gamma_{16})(a_6-b_3\psi_t)} = \theta_c \quad (32)$$

According to this restriction, the amount of principal plus interest payments that are bond-financed has to be less than the right-hand side, θ_c, or the 'critical value' of θ.

If $\theta_1 = \theta_c$, $p_t = \infty$ i.e., the sequence of domestic prices explodes. In this case, the rate of return on domestic bonds has exceeded the rate of growth of the domestic output leading to an explosive debt/income ratio. An upper bound on the number of bonds that can be absorbed is reached when the fraction of debt rolled over, θ_1 equals the amount denoted as θ_c. At this point the monetary authority is forced to purchase government debt directly--or monetize the debt. The θ_c expression involves parameters that correspond to coefficients of current and expected future price and nominal exchange rates. The latter are 'weighted' by the rates of growth of domestic government spending and tax revenues. The intuition behind the inequality (32) is that as increasingly larger amounts of government bonds are rolled over, or as θ_1 increases, p_t increases too ($\frac{dp_t}{d\theta_1} > 0$) <u>but</u> when $\theta_1 = \theta_c$, an <u>upper limit</u> on the amount of bond-financing is reached as individuals are loath to hold everything denominated in worthless U.S. dollars in their portfolios. Any bond financing at θ_1 =

θ_c produces an exploding price level and this is the 'critical amount' of debt that can be 'rolled over' in this economy. This result is similar to an aggregated version of the upper bound of debt that the 'rich' individuals are willing to hold in Sargent and Wallace's 'Some Unpleasant Monetarist Arithmetic' (1981).

Furthermore, from (32), we obtain two interesting results regarding the effect the rates of growth of government spending and tax revenues have on this 'critical value' of θ_1. First, $\frac{d\theta_c}{d\psi_g} < 0$, and second, $\frac{d\theta_c}{d\Psi_t} > 0$. These are important results. In this economy, as the fiscal authority incurs bigger deficits by either increasing Ψ_g or decreasing ψ_t, the 'critical' value or the upper limit of the amount of total debt that can be rolled over without causing the domestic price level to explode, is <u>lowered</u>. The domestic price level would now explode at a lower value of θ_c, and hence the increase in the domestic deficit causes the upper limit on the amount of debt that can be financed, θ_c, to decrease. The intuition behind this result is that there is some absolute maximum amount of government debt that the world economy is willing to absorb, and therefore in an economy characterized by large and rapidly growing deficits, this absolute amount is reached <u>much sooner</u> than it would in one with a marginally increasing deficit.

From result (24),

$$s_t = \left[\frac{-d_2k_5}{k_3k_6 + k_4k_5}\right] g_{t-1} + \left[\frac{d_3k_5}{k_3k_6 + k_4k_5}\right] t_{1-1} + \cdots \qquad (29)$$

The behavior of domestic nominal exchange rates with respect to changes in g_t, t_t and θ_1 can be determined. It is found that in this model, the domestic nominal exchange rate, s_t, depreciates with either an increase in government spending or a decrease in tax revenues. The reason is that deficits are inflationary in this economy, and this causes the domestic and foreign real interest differential to diminish. This, in turn, leads to a decline in foreign residents' excess demand for U.S. bonds, thereby causing the nominal exchange rate to depreciate. This result is in stark contrast to that obtained for s_t in Case I where the exchange rate appreciated and the dollar grew 'stronger' as a result of the neutrality of the price level with respect to domestic deficits.

In (29), $\frac{ds_t}{d\theta_1}$ is positive and the dollar gets weaker as the amount of debt being rolled over as a proportion of total principal payments increases. When θ_1 equals θ_c, the domestic currency <u>collapses</u> ($s_t \rightarrow 0$) and the price level explodes, and this corresonds to the 'hard fall' of the dollar which has gained some attention lately.[12] Domestic currency rapidly ceases to be of any value and at this point bond financed deficits are clearly 'strongly non-sustainable' as defined in Section I.

The results $\frac{dp_t}{d\theta_1}$, $\frac{ds_t}{d\theta_1} > 0$ are disconcerting for policy-makers in that they make the arithmetic very unpleasant indeed. Any coordinated attempt by the fiscal and monetary authorities to 'tighten money today' by simply rolling over the last period's debt and thereby indulging in lesser amounts of monetary accommodation would <u>only serve</u> to exacerbate the domestic inflation further and cause the dollar to depreciate even more. In a sense these results corroborate the Sargent and Wallace result that 'tighter money today would mean more inflation today' in a rational-expectations economy. Therefore, in this economy where the permanency of the deficit is ensured by the Reagan administrations' inflexibility on the tax issue, any attempt to prevent deficits from being inflationary, short of decreasing g_t, would prove futile. And furthermore, as larger amounts of bonds are retired by selling still more bonds, the domestic economy runs a greater risk of having its currency collapse and its price level explode, as θ_1 approaches θ_c--especially in an environment where run-away government spending causes this upper limit of bond-financing, θ_c, to be lower due to the $\frac{d\theta_c}{d\psi_g} < 0$ result.

Let us now examine the mechanism causing this upper limit of debt-financing to decrease with increases in the growth of domestic deficits. In an economy characterized by persistent inflation due to increases in the deficit and in the amount of debt being rolled over, y_t decreases from trend. This is because, (i) individual demanders substitute foreign goods for the higher priced domestic goods, and (ii) suppliers of output, due to the intertemporal substitution of leisure incorporated into the aggregate supply equation, postpone supply indefinitely as the prices are always higher in the next period. Therefore, the perpetual postponement of output supply due to the continuous rise in the domestic price level caused by larger amounts of debt rolled over, and the substitution of cheaper foreign goods for domestic goods cause a decline in national income, and this in turn causes the upper limit on debt holdings to be lowered.

4.3.1 Historical Analog to Case II

The results obtained here resemble the Austrian, Hungarian, Polish and German hyperinflations of the period 1919-24, quite closely. Immediately after World War I, these countries, unlike the United States, were not on a gold standard. Under the gold standard, a government was obligated to convert into gold under certain specified conditions the demand notes and long-term debt that it issued. This imposed a certain discipline on the government because large bond-financed deficits could not be incurred as the government debt was 'backed' to some degree by its gold reserves and by its commitment to levy taxes in the future to service its outstanding debt. A country on the gold standard, therefore, had to honor its debts and could not engage in inflationary finance.

Since the four European countries in question were not on the gold standard, they had no such budgetary discipline imposed on them, and their currencies were unbacked or 'fiat'. This led the governments of those countries to resort to the printing of new unbacked money to finance their domestic budget deficits. The government debt (notes and treasury bills) could not be expected to be paid off by levying taxes in the future and tremendous amounts of debt were simply 'rolled over' by issuing yet more treasury bills, as in Case II.

Consequently, these countries experienced what we have defined as a 'strongly non-sustainable' deficit-financing policy characterized by domestic hyper-inflation, a collapse of the currency, and a declining national output--all of which are results obtained in Case II of this model.

Germany experienced the most severe case of debt non-sustainability. The German domestic inflation rate assumed immense proportions towards the end of 1923, as shown in figure 5. This was aggravated by an event that took place earlier that year--the military occupation of the Ruhr by the French in January 1923 as a result of the inability of the Germans to make the staggering repartions payments. The German government, determined to fight this French occupation by a policy of "passive resistance", made direct payments to striking workers which were financed by the issuance of discounted treasury bills to the public or the Reichsbank, i.e., by 'rolling over' the debt. Relevant plots pertaining to the German hyperinflation are in figures 5-8, and an examination of the plots of the price level, the exchange rate, government debt outstanding and domestic money creation, reveal a close similarity to the behaviors of these variables for the economy in Case II.

CHAPTER V

CONCLUSIONS

Domestic bond-financed deficits, in an open-economy regime in which money creation is held exogenously fixed, as in Case I, are weakly non-sustainable. This economy is characterized by output and price levels that are neutral with respect to changes in the domestic budget deficit. Larger issuances of government bonds cause exchange rate appreciation, an exodus of domestic real wealth, and an increase in the real rate of returns on bonds that increases with the amount of debt that is 'rolled over'. As the output is neutral to changes in the deficit, the real interest rate equals and exceeds the rate of growth of the economy, thus making domestic monetary accommodation inevitable. This prospect of future inflation makes a debt-financed policy of holding money creation fixed and financing further increases in the deficit by simply 'rolling over' the debt, weakly non-sustainable. Furthermore, any attempts to reduce the disparity between the real interest rate and the growth rate by adopting a 'looser' path of money creation, only serves to aggravate the volatility of the price level and the nominal exchange rate.

The behavior of prices, exchange rates, domestic current account balances, and the amounts of government debt outstanding observed in the United States economy for the period 1980-1984, bears close resemblance to the behavior of these variables as obtained from the results of Case I.

When domestic money creation and government bond issuance are both endogenously determined, as in Case II, we find that domestic deficits are inflationary. Here, a portion of government debt outstanding is monetized in every period and the mix of government bond issuance and domestic money creation is held exogenously fixed. As more bonds are rolled over, or as the mix of money and bonds changes so that a larger portion of the principal payment is now bond financed, we obtain an increase in the domestic inflation rate and a depreciation of the domestic currency, i.e., any attempt to put a restraint on domestic money creation only results in an exacerbation of the inflation rate and a weakening of the dollar.

Additionally, Case II provides us with an upper limit on the amount of government debt that can be absorbed by the world economy. As larger fractions of principal repayments, θ_1, are financed by issuing yet more government bonds, an upper limit on the amount of debt that can be issued is reached when $\theta_1 = \theta_c$. Domestic and foreign residents are reluctant to absorb more government bonds in their portfolios and the monetary authority is therefore forced to monetize the deficit by directly purchasing these bonds from the Treasury. The economy at this stage is characterized by a rapidly escalating domestic price level and a collapsing national currency. Furthermore, this upper limit on the amount of debt that can be issued is found to bear an inverse relationship to the rate of growth of the domestic deficit, and this leads us to the conclusion that there is some finite, absolute, amount of debt that can be absorbed by the world economy. The German hyperinflation experience of the period 1919-1924, closely resembles that experienced by the economy in Case II.

In conclusion, large inflows of capital from abroad, stemming from the burgeoning current account deficits such as those currently experienced by the United States, do not overturn the Sargent and Wallace proposition--bond-financed deficits are still non-sustainable in both cases of this open-economy rational-expectations model. This has some rather disconcerting implications for policy-makers. Short of reducing the domestic deficit, tighter money policies and greater amounts of debt being rolled over would only serve to deplete domestic real wealth balances and to make monetary accommodation inevitable in Case I, and to exacerbate the domestic inflation rate and drastically weaken the dollar in Case II. The arithmetic, it seems, is very unpleasant indeed.

DATA DESCRIPTION FOR THE PLOTS OF THE UNITED STATES, FIGURES 1-4

Effective Exchange Rates (EFFEX)

The effective exchange rate is an index combining the exchange rate between the U.S. currency and the currencies of its ten largest trading partners, with weights derived from the International Monetary Fund's Multilateral Exchange Rate Model. Each weight represents the models' estimate of the effect of the trade balance of the country in question caused by a change of one percent in the domestic currency price of one of the other countries.

Current Account Balance (CURAC)

The current account balance, in units of billions of U.S. dollars, equals the trade balance plus the balance on other goods, services, income, private unrequited transfers, and official unrequited transfers.

Debt

The debt outstanding relates to the direct debt of the central government and excludes loans guaranteed by the government. It is expressed in units of billions of U.S. dollars.

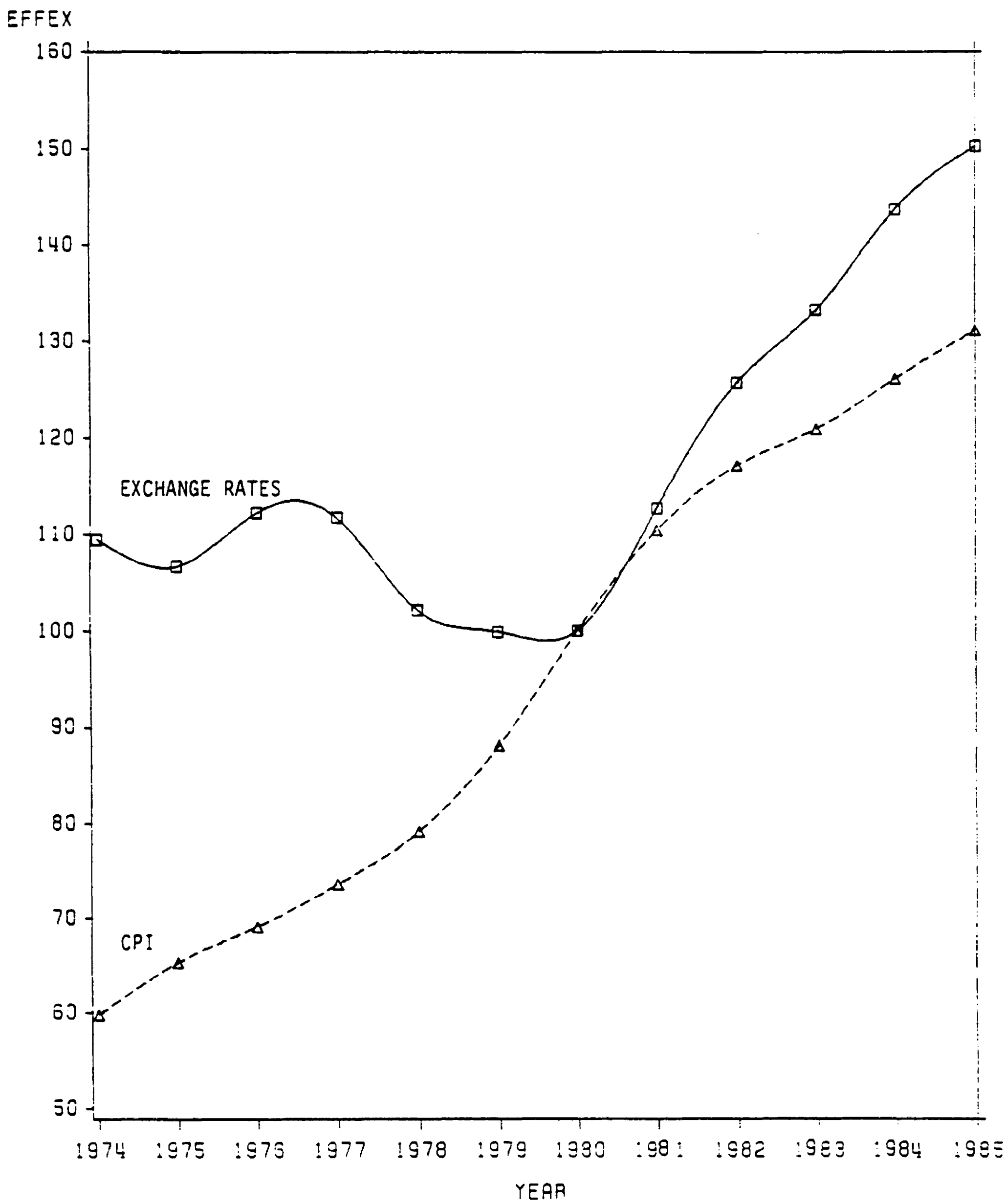
Nominal Interest Rates (NOMIN)

This group of nominal interest rates consists of one or more representative short-term money market rates, i.e., the rate at which short-term borrowings are effected between financial institutions or the rate at which short-term government paper is issued or traded in the market. The U.S. Treasury Bill rate for the period 1974-1985 has been plotted here.

Deficit

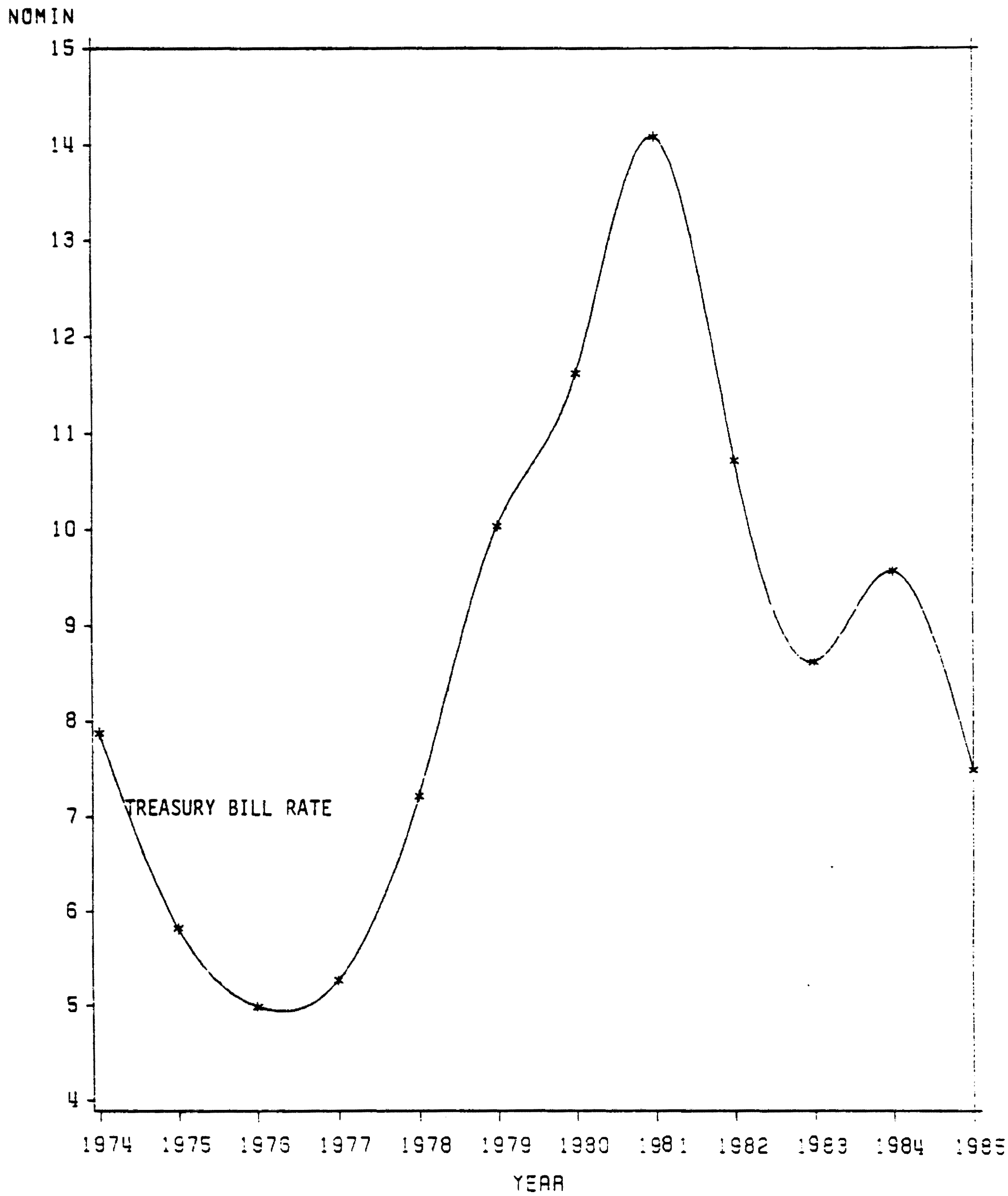
The deficit or surplus, in units of billions of U.S. dollars, is calculated as the difference between revenue, and, if applicable, grants received on the one hand, and expenditures and lending minus repayments on the other. Lending minus repayments includes purchases of equities and is net of repayments of lending and sales of equities previously purchased. In determining the deficit or surplus, lending minus repayments is grouped with expenditures, since it is presumed to represent a means of pursuing government policy objectives and not to be an action undertaken to acquire a financial asset. Lending minus repayments includes foreign lending which meets this criterion.

Source: International Financial Statistics Yearbook, 1985, IMF



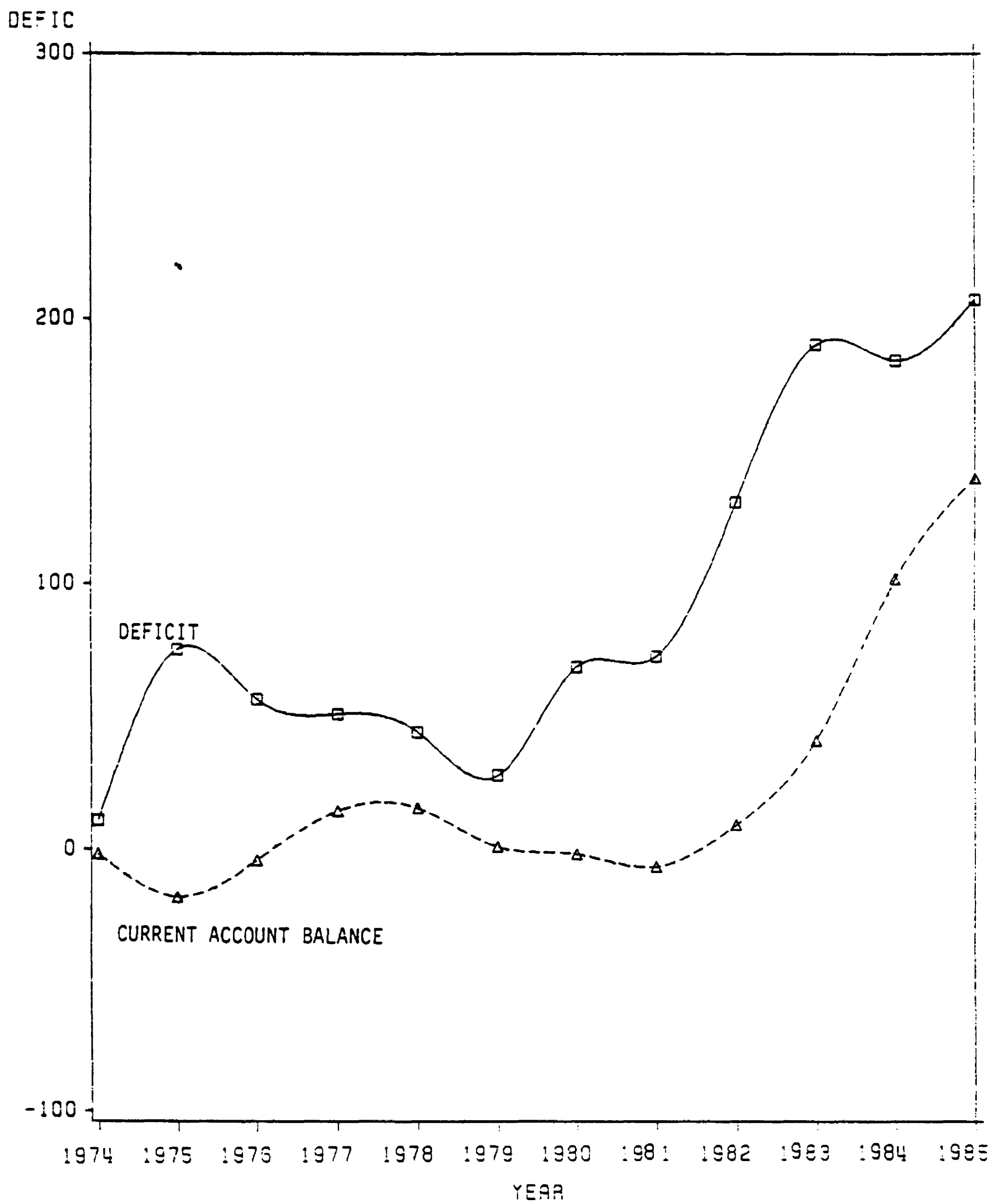
CPI AND EXCHANGE RATE VS. TIME

Figure 1



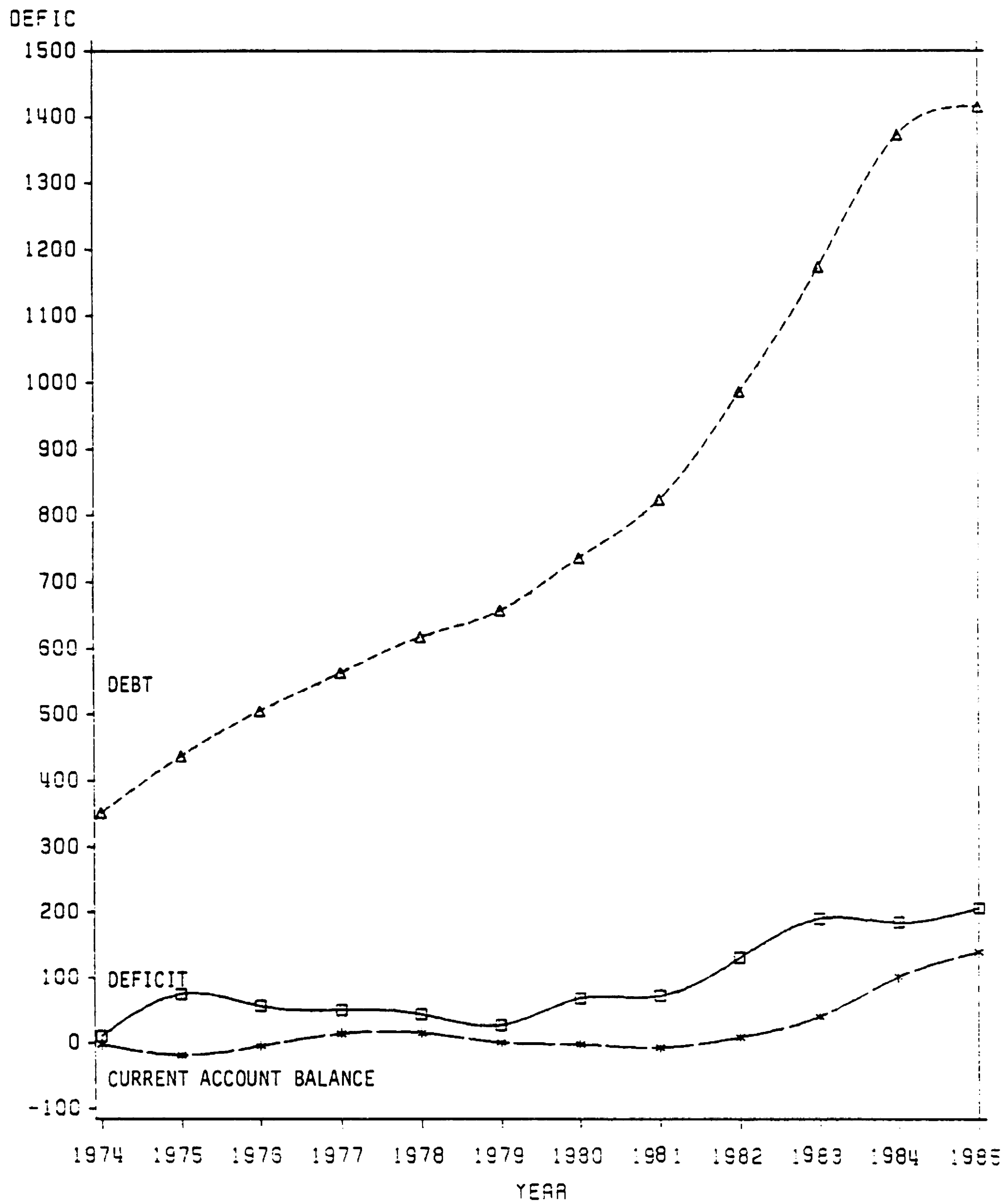
NOMINAL INTEREST RATES VS. TIME

Figure 2



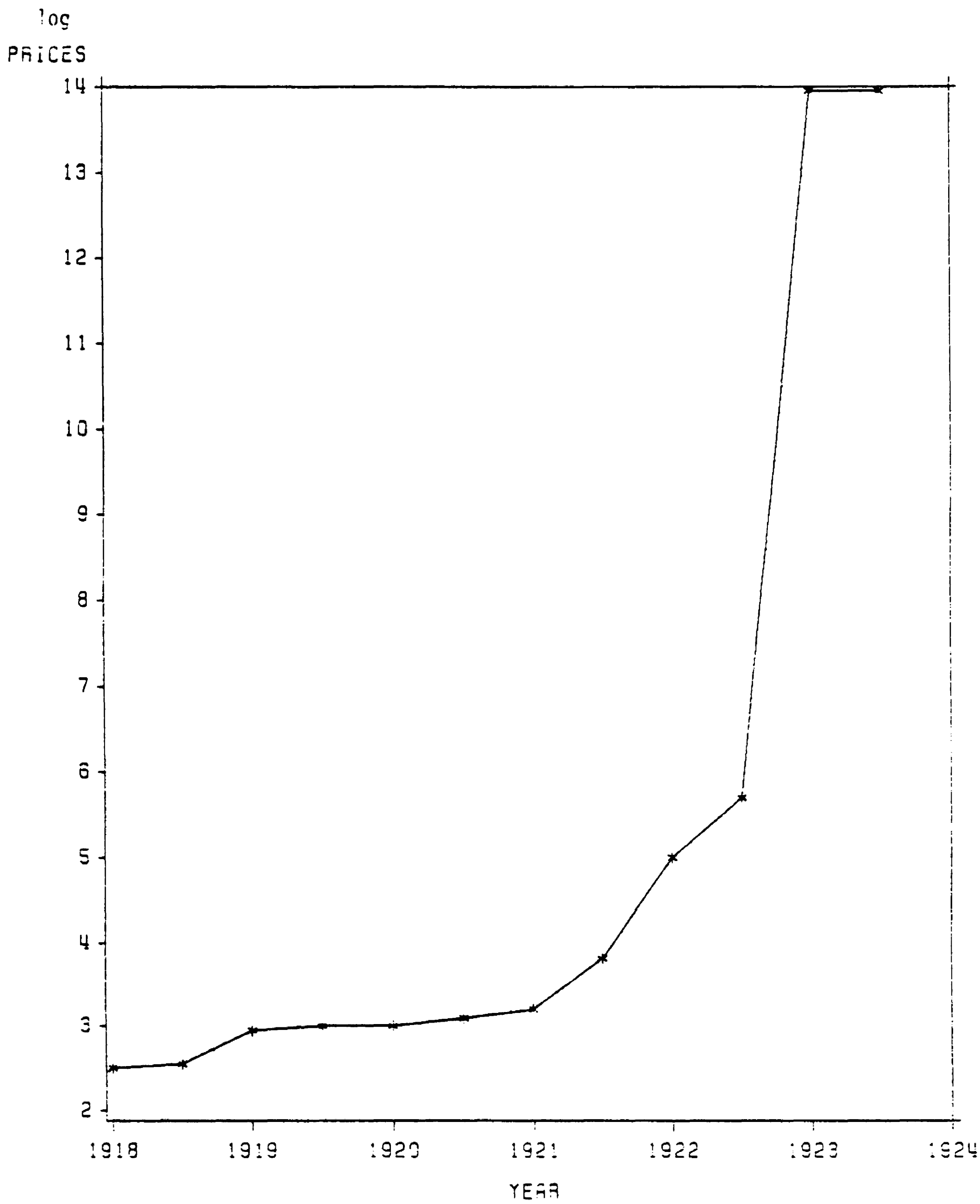
DEFICIT, CURRENT ACCOUNT VS. TIME

Figure 3



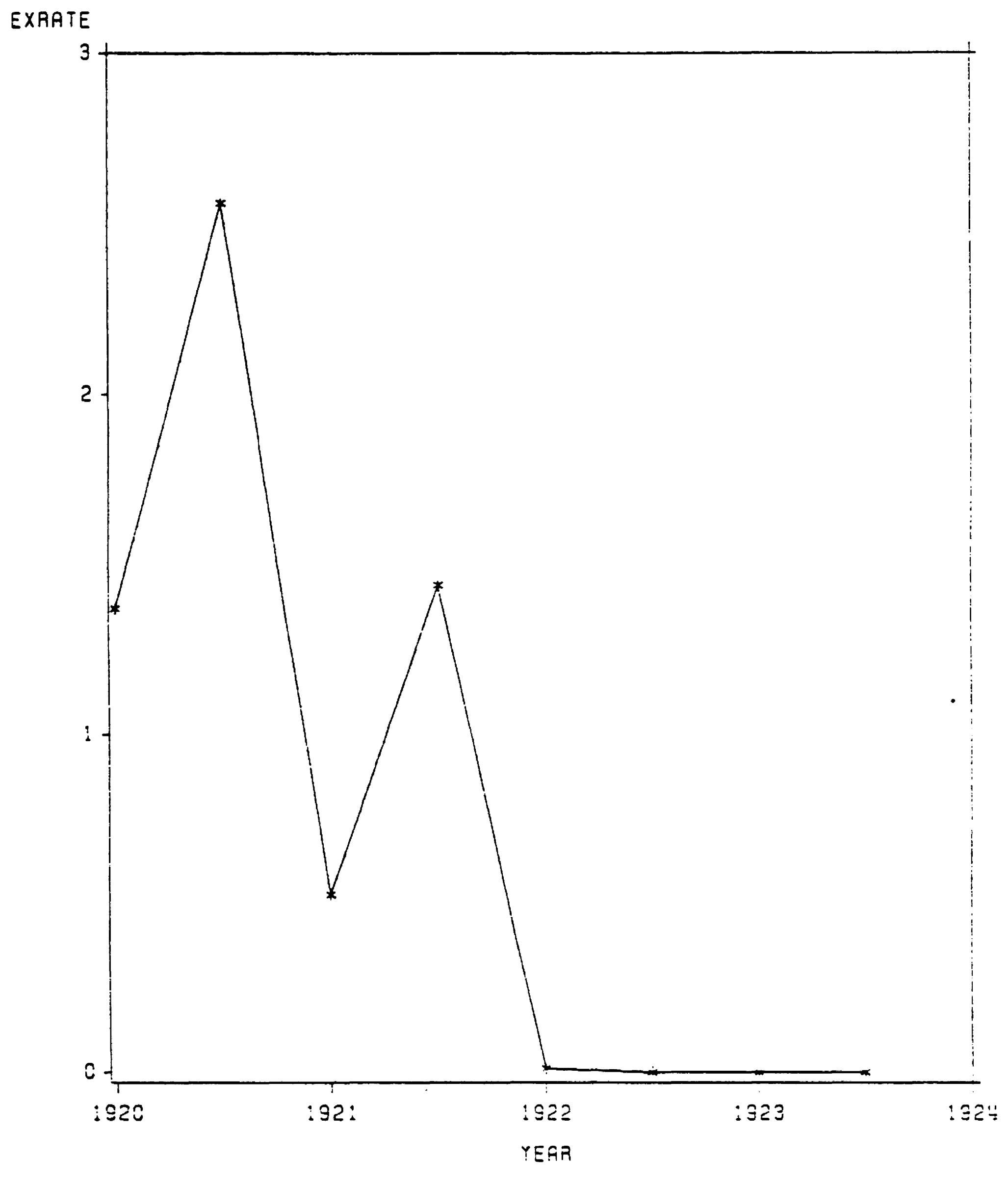
DEFICIT DEBT CURRENT ACCOUNT VS. TIME

Figure 4



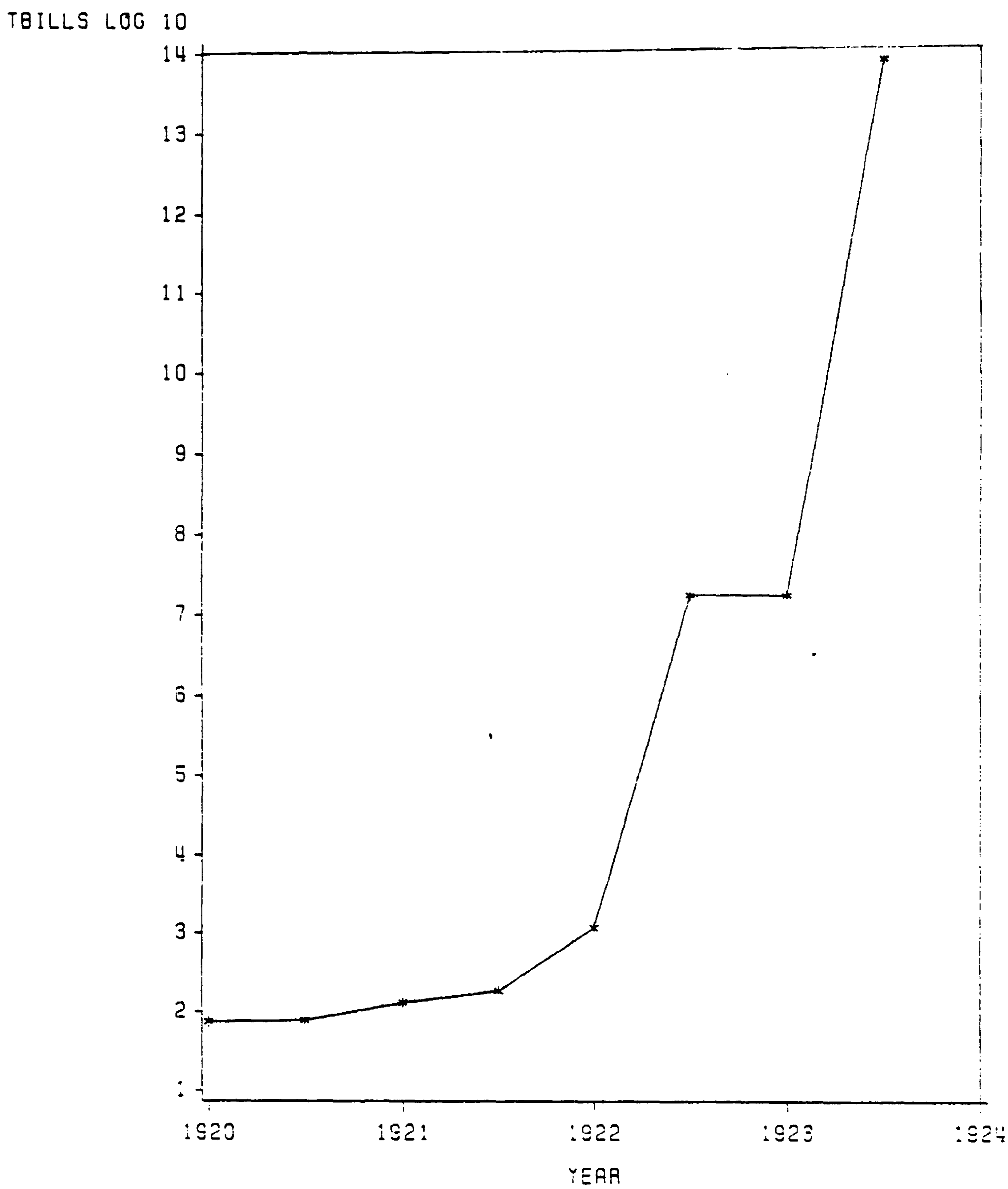
GERMAN HYPERINFLATION: 1918–1923.
WHOLESALE PRICES VS. TIME

Figure 5



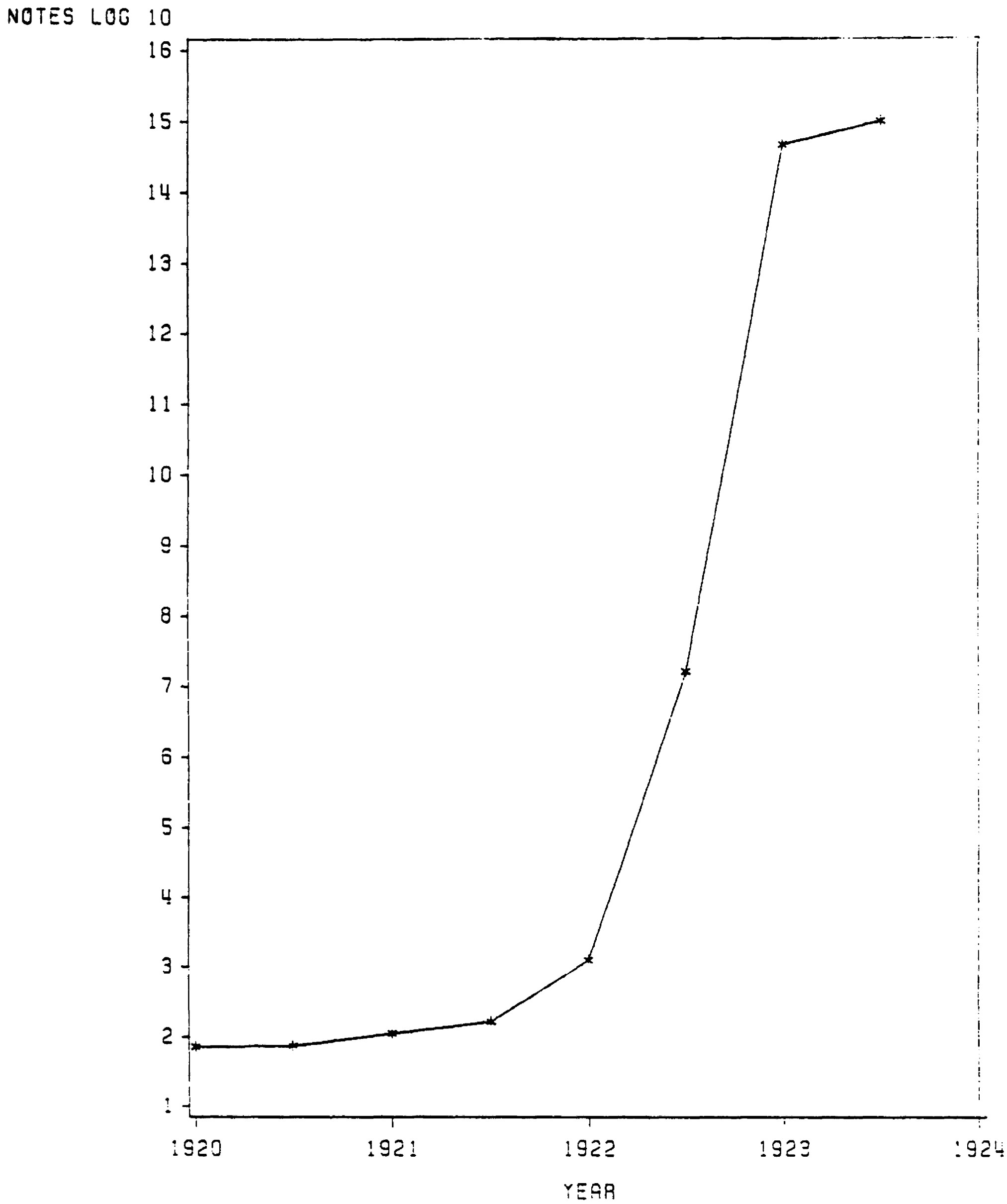
GERMAN HYPERINFLATION: 1918–1923.
EXCHANGE RATES IN CENTS PER MARK VS. TIME

Figure 6



GERMAN HYPERINFLATION: 1918–1923.
DISCOUNTED TREASURY BILLS VS. TIME

Figure 7



GERMAN HYPERINFLATION: 1918–1923.
NOTES IN CIRCULATION VS. TIME

Figure 8

FOOTNOTES

1 Expressing b in terms of growth rates, we have:

$\dot{b}/b = \dot{B}/B - \dot{p}/p - g$

where the variables are as defined.. This says that the debt/income ratio rises when the nominal debt (B/B) grows faster then the nominal GNP ($\dot{p}/p + g$).

Using the following expression;

$\dot{B} = PD + iB$

which states that the increase in debt is equal to the primary deficit plus the nomial interest payments on existing debt (i is the nominal interest paid by the government on its debt) and substituting it into the expression for $\dot{b}/b$, we obtain equation (b) in the text. (Dornbusch 1984)

2 It should be noted that the model in this dissertation is one with a regime of flexible exchange rates.

3 Stability is viewed in the context of the increase in the size of the budget deficit over time, along the lines of the simple Dornbusch example discussed earlier.

4 He discusses this in greater detail in his text-book, 'Macroeconomics Analysis and Stabilization Policy' (1977).

5 This expression is obtained from the standard National Income Accounting Identity.

6 See McTaggart (December 1985) for a nice discussion of these mechanisms.

7 The significance of the imperfect substitutability of domestic and foreign goods will be made clear during the discussion of real interest rate parity, later in this section.

8 We assume that the log of an expected value is equal to the expected value of the log of that variable i.e. $\ln E_t X_{t+1} = E_t \ln X_{t+1} = E_t X_{t+1}$.

9 This is a fairly standard short-run rational expectations aggregate supply function. The systematic supply term, $\bar{y}$, is assumed to be independent of changes in fiscal policy i.e. the marginal product of labor is not a function of changes in fiscal policy and the aggregate supply is therefore not directly affected by increases in goverment spending. Furthermore, equation (2) abstracts from the

effect of capital accumulation on the trend rate of growth of the domestic output. A stripped-down version of this model explicitly incorporating capital growth is planned for future work. The author is grateful to participants of the macroeconomic workshop at Rutgers University for the above comments and suggestions.

10 Turnovsky (1977) has emphasized the importance of incorporating beginning period wealth in the analysis of government policies.

11 The reason that the increase in domestic aggregate demand manifests itself in a 100% complete overflow is the following. There are two goods (or two composite goods) in this economy, and these are domestic and foreign goods. They are similar but not identical to each other. Individuals therefore behave in a rational manner and choose the good with the lower price i.e. they choose either the domestic or the foreign good and not a combination of both. This is why the overflow is 'complete'.

12 Money ceases to have any value and individuals are therefore loathe to hold dollar-denominated U.S. bonds.

APPENDIX A

The system of equations for Case I is reduced to a system of three equations in quasi-reduced form, with p_t, s_t, and w_t as the endogenous variables. In matrix form, these equations can be represented as:

$$
\begin{bmatrix} \beta_1 & -\beta_3 & -\beta_2 \\ 1+\gamma\gamma_5 & \gamma_6 & 0 \\ a_5 & (\gamma_{13}-\gamma_{11}) & 0 \end{bmatrix}
\begin{bmatrix} p_t \\ s_t \\ \omega_t \end{bmatrix}
=
\begin{bmatrix} \gamma & 0 & 0 \\ \gamma_5 & \gamma_6 & 0 \\ b_3 & (\gamma_{13}-\gamma_{11}) & -\frac{\gamma_{12}}{\gamma_{20}} \end{bmatrix}
\begin{bmatrix} E_t p_{t+1} \\ E_t s_{t+1} \\ E_t \omega_{t+1} \end{bmatrix}
+
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \lambda_o & -\lambda_o & \lambda_1 \end{bmatrix}
\begin{bmatrix} p_{t-1} \\ s_{t-1} \\ \omega_{t-1} \end{bmatrix}
$$

$$
+
\begin{bmatrix} \beta_3 \psi_p^* & \beta_6 \psi_y^* & 0 \\ 0 & 0 & \gamma_6 \psi_i^* \\ e_3 & \gamma_{17}(\gamma_{15}-\gamma_{16})\gamma_4 & \gamma_{17}+\gamma_{13}\psi_i^* \end{bmatrix}
\begin{bmatrix} p_{t-1}^* \\ y_{t-1}^* \\ i_{t-1}^* \end{bmatrix}
+
\begin{bmatrix} 0 & -\beta_5 & \varepsilon_7 \\ \psi_d & 0 & 0 \\ d_3 & -q_o & q_1 \end{bmatrix}
\begin{bmatrix} d_{t-1} \\ \tau \\ K_1 \end{bmatrix}
+
\begin{bmatrix} \frac{1}{1-\gamma_2}-1 & 0 & 0 \\ -\gamma_5 & 1 & 0 \\ C_2 & 0 & \gamma_{17} \end{bmatrix}
\begin{bmatrix} \bar{y} \\ \bar{f} \\ \emptyset \end{bmatrix}
$$

$$
+
\begin{bmatrix} -1 & \frac{1}{1-\gamma_2} & 0 \\ -\gamma_5 & 0 & 1 \\ (\gamma_{15}-\gamma_{16}) & f_3 & 0 \end{bmatrix}
\begin{bmatrix} \varepsilon_t^s \\ \varepsilon_t^d \\ \varepsilon_t^m \end{bmatrix}
\qquad 12(a)
$$

In this system the following substitutions have been made for notational simplicity:

$$a_5 = [\gamma_{11} - (1 + \gamma)\ (\gamma_{15} - \gamma_{16})]$$

$$b_3 = [\gamma_{11} - \gamma(\gamma_{15} - \gamma_{16})]$$

$$\lambda_o = \gamma_{17}\ (\gamma_{15} - \gamma_{16})(1 - \frac{\gamma_3}{1-\gamma_2})$$

$$\gamma_1 = \gamma_{17}\ (\gamma_{15} - \gamma_{16}) + \frac{\gamma_1}{1-\gamma_2}$$

$$d_3 = \psi_d\ [\gamma_{15}\ (\frac{\gamma_{12}}{\gamma_{13}}) - 2)] - \gamma_{17}$$

$$e_3 = \gamma_{17}\ (\gamma_{15} - \gamma_{16})\ \frac{\gamma_3}{1-\gamma_2} + \gamma_{11}\ (1 - \psi_p^*)\ \psi_p^*$$

$$q_o = [\gamma_{16} + \gamma_{17}\ (\gamma_{15} - \gamma_{16})\ \frac{o}{1-\gamma_2} + \gamma_{15}\ \gamma_{17})$$

$$q_1 = [\gamma_{15} + \gamma_{17}\ (\gamma_{15} - \gamma_{16})\ \frac{\gamma_2}{1-\gamma_2} + \gamma_{15}\ \gamma_{17}]$$

$$c_2 = (\gamma_{15} - \gamma_{16})(\frac{\gamma_{17}}{1-\gamma_2} - 1)$$

$$f_3 = \gamma_{17}\ \frac{(\gamma_{15} - \gamma_{16})}{1-\gamma_2}$$

APPENDIX B

The Wiener-Kolmogorov (W-K) k-step ahead, linear least squares forecast is presented here.

Consider the first-order autoregressive process $(1 - \rho L)\, y_t = \varepsilon_t$ where ε_t is white noise, $|\varepsilon|<1$.

To obtain the expectation of y_t, k periods from now, based on all available information at time t, i.e., to obtain $E_t y_{t+k}$, we use the W-K prediction formula.

$$E_t y_{t+k} = \left[\frac{\rho(L)}{L^k} \right]_+ \varepsilon_t$$

Here $\rho(L)$ is $(1 - \rho L)^{-1}$, and $[\quad]_+$ is the annihilator operator that ignores negative powers of L.

Substituting in for the white noise from the given process;

$$E_t y_{t+k} = \left[\frac{\rho(L)}{L^k} \right]_+ \rho(L)^{-1} y_t$$

$$= \frac{\rho^k}{(1-\rho L)} \cdot (1 - \rho L)\, y_t$$

Therefore, $E_t y_{t+k} = \rho^k y_t$, is the expectation of y_t in period t+k based on all available information at time t.

APPENDIX C

The explicit solutions of the elements π_3^{ij} are presented here.

$$\pi_3^{11} = -a_2 a_6\left[\pi_1^{21}(\gamma - a_0) + \pi_1^{21}(\gamma_5 - a_3) - \pi_1^{23}(\gamma_{15} - \gamma_{16}) - \gamma_5\right] + a_2 a_4\left[-\pi_1^{31}(\gamma - a_0) + \pi_1^{31}(\gamma_5 - a_3) + (\gamma_{15} - \gamma_{16})(1 - \pi_1^{33})\right]$$

where the $_1^{ij}$ elements are the coefficients of the policy variables d_{t-1}, the domestic money supply, , the marginal tax rate, and k_1, the governments consumption of current output and they presented earlier in Chapter III.

$$\pi_3^{12} = -a_2 a_6 \pi_1^{21} a_1 + a_2 a_4\left(f_3 - \pi_1^{31} a_1\right)$$

$$\pi_3^{13} = -a_2 a_6\left[\pi_1^{21} a_2\left(\pi_1^{23} \frac{\gamma_{12}}{\gamma_{13}} \phi_d\right) + 1\right] + a_2 a_4\left(\frac{\gamma_{12}}{\gamma_{13}} \phi_d \pi_1^{31} - \pi_1^{31} a_2\right)$$

$$\pi_3^{21} = a_2 a_5\left[\pi_1^{21}(\gamma - a_0) + \pi_1^{22}(\gamma_5 - a_3) - \pi_1^{23}(\gamma_{15} - \gamma_{16} - \gamma_5\right] - a_2 a_3\left[-\pi_1^{31}(\gamma - a_0) + \pi_1^{32}(\gamma_5 - a_3) - \pi_1^{23}(\gamma_{15} - \gamma_{16}) + (\gamma_{15} - \gamma_{16})\right]$$

$$\pi_3^{22} = a_2 a_5 \left(\pi_1^{21} a_1\right) - a_2 a_3 \left(f_5 - \pi_1^{31} a_1\right)$$

$$\pi_3^{23} = a_2 a_5 \left[\pi_1^{21} a_2 \left(\pi_1^{23} \frac{\gamma_{12}}{\gamma_{13}} \psi_d\right) + 1\right] + a_2 a_3 \left(\pi_1^{31} a_2 + \frac{\gamma_{12}}{\gamma_{13}} \psi_d \pi_1^{33}\right)$$

$$\pi_3^{31} = \left(a_2 a_6 - a_4 a_3\right)\left[\pi_1^{11}\left(\gamma - a_0 - 1\right)\right] - \left(a_0 a_6 + a_1 a_5\right)\left[\pi_1^{21}\left(\gamma - a_0\right) + \pi_1^{22}\left(\gamma_5 - a_3\right)\right.$$

$$\left. - \pi_1^{23}\left(\gamma_{15} - \gamma_{16}\right) - \gamma_5\right] + \left(a_0 a_4 + a_1 a_3\right)\left[-\pi_1^{31}\left(\gamma - a_0\right) + \pi_1^{32}\left(\gamma_5 - a_3\right)\right.$$

$$\left. - \pi_1^{33}\left(\gamma_{15} - \gamma_{16}\right) + \left(\gamma_{15} - \gamma_{16}\right)\right]$$

$$\pi_3^{32} = \left(a_0 a_6 - a_4 a_3\right)\left(\pi_1^{11} a_1 + \frac{1}{1 - a_2}\right) - \left(a_0 a_6 + a_1 a_5\right)\left(\pi_1^{21} a_1\right) + \left(a_0 a_4 + a_1 a_3\right) \cdot$$

$$\left(f_3 + \pi_1^{31} a_1\right)$$

$$\pi_3^{33} = \left(a_0 a_6 - a_4 a_3\right)\left(\pi_1^{11} a_2\right) - \left(a_0 a_6 + a_1 a_5\right)\left[\pi_1^{21} a_2 \left(\pi_1^{23} \frac{\gamma_{12}}{\gamma_{13}} \psi_d\right) + 1\right] + \left(a_0 a_4\right.$$

$$\left. + a_1 a_3\right)\left(-\pi_1^{31} a_2 + \frac{\gamma_{12}}{\gamma_{13}} \psi_d \pi_1^{33}\right)$$

APPENDIX D

The system of equations for Case II can be reduced to the following four equations in quasi-reduced form:

$$\begin{bmatrix} (\gamma+\gamma_3) & -\gamma_3 & 0 & 0 \\ \beta_2 & \gamma_6 & 0 & 0 \\ -\beta_0 & (\gamma_8+\gamma_{10}-\gamma_{11}) & 0 & 0 \\ \beta_1 & (\gamma_{15}-\gamma_{11}) & \gamma_{12} & 0 \end{bmatrix} \begin{bmatrix} p_t \\ s_t \\ b_t^d \\ \omega_t \end{bmatrix} \quad \begin{bmatrix} \gamma & 0 & 0 & 0 \\ \gamma_5 & \gamma_6 & 0 & 0 \\ (\gamma_{11}-\gamma\gamma_9) & -\gamma_{11} & 0 & 0 \\ \gamma_{11} & (\gamma_{13}-\gamma_{11}) & 0 & 0 \end{bmatrix} .$$

$$\cdot \begin{bmatrix} E_t\, p_{t+1} \\ E_t\, s_{t+1} \\ E_t\, b_{t+1}^d \\ E_t\, \omega_{t+1} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ -\theta(\gamma_{15}-\gamma_{16}) & \theta & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \theta(\gamma_{15}-\gamma_{16}) & -\theta & 0 & 0 \end{bmatrix} \begin{bmatrix} p_{t-1} \\ s_{t-1} \\ b_{t-1}^d \\ \omega_{t-1} \end{bmatrix} +$$

$$\begin{bmatrix} \gamma_2\psi_g & -\gamma_0\psi_t & 0 & 0 \\ -\gamma_{15}(\theta+\alpha_1\psi_g) & \gamma_{16}(\theta+\alpha_1\psi_t) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \gamma_{15}(\theta+\alpha_1\psi_g) & -\gamma_{16}(\theta+\alpha_1\psi_t) & 0 & 0 \end{bmatrix} \begin{bmatrix} g_{t-1} \\ t_{t-1} \\ \\ \\ \end{bmatrix} + \quad \rightarrow$$

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ -\gamma_s & 1 & -\theta & 0 \\ \gamma_q & 0 & 0 & -1 \\ 0 & 0 & \theta & 0 \end{bmatrix} \begin{bmatrix} \bar{y} \\ \bar{f} \\ \phi \\ K_N \end{bmatrix} + \begin{bmatrix} -1 & 1 & 0 & 0 \\ -\gamma_s & 0 & 1 & 0 \\ \gamma_q & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_t^s \\ \varepsilon_t^d \\ \varepsilon_t^m \\ \end{bmatrix}$$

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